

**FLOOD DISCHARGE ESTIMATION ONLY USING  
AT-SITE STAGE INFORMATION**

**A DISSERTATION**

*Submitted in partial fulfillment of the  
requirements for the award of the degree*

of

**Master of Technology  
in  
Hydrology**

By

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## CANDIDATE'S DECLARATION

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This is to certify that I have personally worked on the seminar report entitled, “*Flood Discharge estimation only using at-site stage Information*”, a prerequisite towards fulfillment for the award of the Degree of Master of Technology in Hydrology submitted to the Department of Hydrology of the Indian Institute of Technology Roorkee, India, is an authentic record of my genuine work. This has been carried under the supervision of Dr. M. Perumal, Professor, Dr. Sumit Sen, Asst. Professor, Department of Hydrology, Indian Institute of Technology, Roorkee, India.

I have not submitted the matter embodied in this dissertation for award of any degree of this or any other institutes.

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## LIST OF ABBREVIATION/SYNONYM/NOTATION

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%	percentage
<	less than
=	equal to
>	greater than
≤	less than or equal to
≥	greater than or equal to
A	cross sectional area
ACD	Approximate convection diffusion
B	channel width
C	wave celerity
Cusec	cubic feet per second
Cusec	cubic metre per second
Eq.	equation
et al.	and others
etc.	et cetera
F	Froude's number
FDM	finite difference method
Fig	figure
ft	feet
GH	Georgia highway
HEC-RAS	Hydraulic engineering center river analysis system
hh:mm	hour:minute
hr	hour
m	metre
n_Fread	Manning's n on Fread method
n_Fread*1.05	1.05 times of Manning's n on Fread method
n_Q	Manning's n based on discharge hydrograph
n_y	Manning's n based on stage hydrograph
NSE	Nash Sutcliff efficiency
P	wetted perimeter
Q	Discharge

$Q_o$	Normal Discharge
$R$	ratio of bed slope to average wave slope
$R$	hydraulic radius
RMSE	root mean square error
$S$	second
$S_f$	frictional slope
$S_o$	channel bed slope
$V_e$	variance explained by model
$V_r$	remaining variance
$V_t$	total variance
$Y$	Stage

## ABSTRACT

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Flood discharge estimation at a river site is generally made by converting observed flood stage hydrograph at the site using the established stage-discharge relationship developed for that site based on either few direct observations of flood discharges pertaining to a flood event or their indirect estimation based on average velocity measurements corresponding to the observed stages or flow depths of that flood event. However estimation of discharge using rating curve characterizing a steady flow relationship is not theoretically correct as the actual rating curve developed for a flood event exhibits a loop-rating curve. It would be more desirable in field practices to estimate discharge using the measured flood stage hydrograph and the rating curve pertaining to that site, but duly accounting for the loop-rating nature of the observed flood hydrograph at that site. A number of discharge estimation methods using observed stage hydrograph taking into account the hysteresis in rating curve are available. In order to compare various discharge estimation methods, study based on conversion of hypothetical stage hydrograph into discharge is first used for the assessment of suitability of these methods by reproducing the benchmark discharge hydrograph corresponding to the hypothetical discharge hydrograph used. In most of the cases, the refined Jones method produced higher efficiency than the Jones, the modified Jones and the iterative Jones methods, except for channels characterized by the Manning's roughness  $n < 0.02$  and channels with bed slopes  $S_o > 0.0008$ . Moreover, the refined method is not suitable for applications in channel having slope  $< 0.0002$  with  $n > 0.02$ . Also the refined Jones method performs better than the Fread's method in channels characterized by roughness  $n > 0.03$  and bed slope,  $S_o < 0.0006$ . In the remaining cases, the Fread method gives better result as compared to other methods. The applicability of the methods are assessed by estimating discharge hydrographs at few sites of Chattahoochee River, USA and Bhadrachalam station of Godavari River, India and comparing the estimate discharge hydrograph with the corresponding benchmark discharge hydrograph. In most cases, the practical applicability gives an estimate of Nash Sutcliffe Efficiency (NSE)  $> 90\%$



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# Chapter 1 INTRODUCTION

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## 1.1 General

River discharge is required for hydrological analysis studies such as rainfall-runoff modelling and flood routing, and for water balance studies. These studies are required for design flood estimation, catchment erosion estimation and flow forecasting. Monitoring river discharge at a river station continuously is usually very expensive, time consuming and dangerous. But stage of the river at a station can be monitored continuously with comparative ease and economy. Generally the routine measurement of stage at a river section is relative to an arbitrary datum level. Apart from using stage in marking the danger level of flood of a river at the section of interest in a river, the measured stage can be related to discharge by establishing a stage-discharge relationship.

Generally discharge estimation using stage measurement involves a two stage process:

- 1) Establishing the relationship, known as the rating curve/stage-discharge relationship between the measured stage and the corresponding measured discharge based on sufficient number measurements of both variables, and
- 2) Using the rating curve, the conversion of measured stage into discharge.

The general form of the rating curve is given as

$$Q = C_r(G - a)^\beta \quad (1.1)$$

where  $Q$ =stream discharge,  $G$ =stage,  $a$ = constant corresponding to zero discharge,  $C_r$ , and  $\beta$  are constants.

Under steady flow condition of the river, the stage-discharge relationship gives a one-to-one relationship as given by Eq. (1.1). Thus the measured stage can be converted into discharge with much accuracy during steady flow condition.

But during unsteady condition, the approaching flood wave has more velocity than the steady flow corresponding to a given stage. So the discharge at the same stage is more in unsteady condition than at the steady condition. But in falling stage, the flood wave has lesser velocity in unsteady flow condition than in steady flow condition for a given stage. Thus the discharge at the falling stage has lesser discharge in at unsteady condition than at steady condition for the same stage. This leads to the formation of hysteresis in rating curve known as the loop rating curve during unsteady flow. Figure 1.1 shows the loop rating curve in unsteady flow condition.

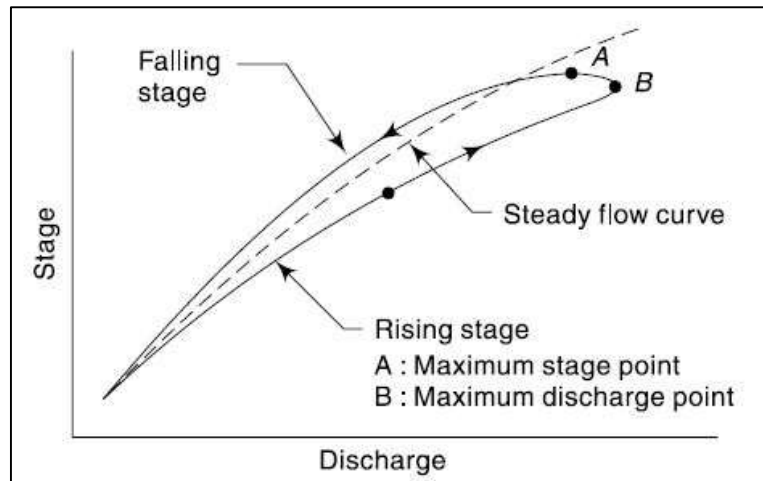


Figure 1.1 Loop rating curve (Adapted from Subramanya, (1994))

The loop rating curve is due to variable energy slope that arises from inertial and pressure forces. There is no one-to-one relationship between the stage and discharge as the relationship is affected by hysteresis and most of the natural rivers are unsteady in nature. So the accurate estimation of discharge using the measured stage becomes a huge challenge for then hydrologists and hydraulic engineers. The preciseness of the estimated discharge is sensitive to the channel roughness, channel slope, hydraulic geometry etc. Discharge is generally not measured all the time during unsteady flow and the estimation of river discharge is usually done using rating curve.

## 1.2 Scope of the Study

The stage-discharge relationship or rating curve in steady flow condition represents a one-to-one relationship. In such condition, with much accuracy we can estimate discharge hydrograph using stage information. In case of unsteady flow condition, stage-discharge relationship is no longer follows a one-to-one relationship. The scope of this study is to analyse various flood discharge estimation methods available in literature only using at-site stage information and comparison of their performances in reproducing the hypothetical/observed discharge hydrographs.

## 1.3 Objectives of the Study

The following are the objectives of the proposed study:

- a) Flood discharge estimation only using at-site stage information, and
- b) Comparison with other similar discharge estimation methods.

#### **1.4 Limitations of the Study**

The discharge estimation formulae considered in this study are applicable only to steep and the moderate slope channels/rivers. In order to compare various discharge estimation methods, the study also uses hypothetical data as the field data is influenced by uncertainties of Manning's roughness coefficient used and errors of stage measurement.

#### **1.5 Conclusions**

Discharge estimation using only at-site stage data under unsteady river flow condition is discussed. The objectives and limitations of the study also has been discussed.

## Chapter 2 REVIEW OF LITERATURE

---

### 2.1 General

A rating curve at a river section is established with a number of stage and discharge data recorded at a channel section. During unsteady flow discharge is not a one-to-one function of stage variable which changes with time. So while estimating unsteady using rating curve this fact should be kept in mind.

Analytically, discharge,  $Q$  and stage,  $y$  can be related as

$$Q = f(y) \quad (2.1)$$

Algebraic relationship and graphical rating can be used for conversion of stage to discharge and in visualizing the relationship respectively.

Because of the difficulty in measuring flow frequently and flow at high stages, *a priori* relationship between measured stage and the corresponding discharge can be established for a flood event based on which a one-to-one relationship between the measured stages and estimated discharge based on velocity measurement can be established. This relationship known as rating curve can be suitably used for enabling the estimation of unsteady discharge using stage measured during unsteady flow condition. This chapter discusses such methods of estimating unsteady discharge.

### 2.2 Literature Review

#### 2.2.1 Jones formula

Jones (1916) estimated unsteady discharge,  $Q$  only using stage data and his method is known as the Jones formula. Unsteady flow is estimated by the Jones formula without considering inertial forces in the frictional slope expression and this formula is expressed as

$$Q = Q_o \left[ 1 - \frac{1}{S_o} \frac{\partial y}{\partial x} \right]^{1/2} \quad (2.2)$$

where  $Q_o$ =normal discharge for a given stage,  $y$ =stage and  $S_o$ =channel bed slope.

Further, Jones (1916) deduced  $\partial y/\partial x$  term into temporal derivative term of  $y$  as

$$\frac{\partial y}{\partial x} = -\frac{1}{c} \frac{\partial y}{\partial t} \quad (2.3)$$

where  $c$ = wave celerity.

Using Eq. (2.3) in Eq. (2.2), Jones formula can be written as

$$Q = Q_o \left[ 1 + \frac{1}{s_{oc}} \frac{\partial y}{\partial t} \right]^{1/2} \quad (2.4)$$

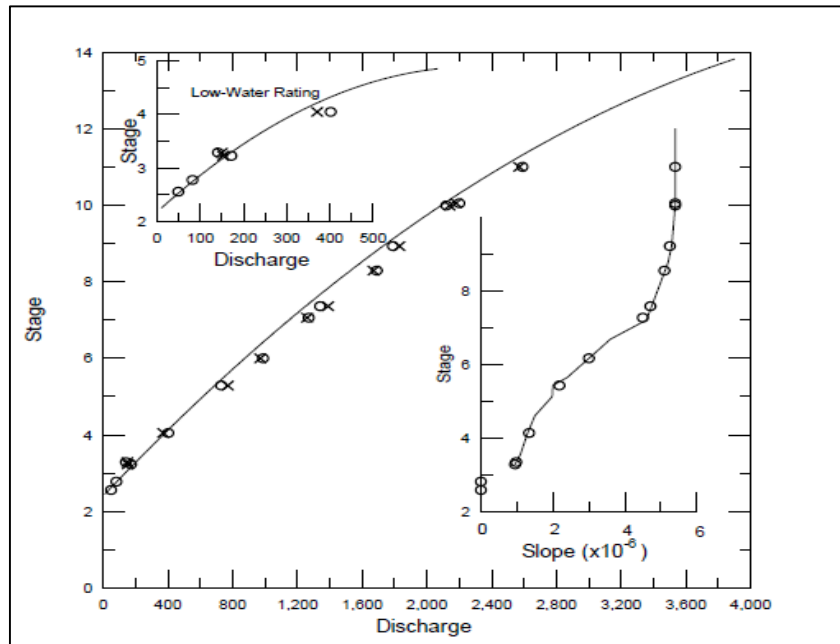


Figure 2.1 Jones method's application for correcting discharge in changing stage (Jones, 1916)

### 2.2.2 Henderson Method

Henderson formula (1966) for estimating unsteady discharge is based on the parabolic approximation for flood wave moving in wide rectangular channel accounting for wave subsidence. The unsteady discharge estimation by Henderson (1966) formula is given by

$$Q = Q_o \left[ 1 + \frac{1}{s_{oc}} \frac{\partial y}{\partial t} + \frac{2}{3r^2} \right] \quad (2.5)$$

### 2.2.3 Method of Perumal et al. (2004)

Perumal et al. (2004) gave credence to the use of Jones formula and verified the logic of the ACD equation in producing unsteady discharge flow. Moreover, two formulae were developed which were found to yield better results than that of the Jones formula i.e. the Modified and refined Jones formula.

(a) Modified Jones formula:

Inertial terms were considered in frictional slope of Jones formula, which was given as

$$S_f = S_o - \frac{\partial y}{\partial x} \left[ 1 - m^2 F^2 \left( P \left( \frac{\frac{\partial R}{\partial y}}{\frac{\partial A}{\partial y}} \right) \right)^2 \right] \quad (2.6)$$

where,  $m=2/3$  for the Manning's friction law,  $F$ =Froude's number.  $R$ =hydraulic radius (m),  $A$ =area of cross section ( $m^2$ ). Thus the modified Jones formula was obtained as

$$Q = Q_o \left[ 1 + \frac{1}{S_o} \frac{\partial y}{\partial t} \left[ 1 - \frac{4}{9} F^2 \left( P \left( \frac{\frac{\partial R}{\partial y}}{\frac{\partial A}{\partial y}} \right) \right)^2 \right] \right]^{1/2} \quad (2.7)$$

(b) Refined Jones formula: The convection-diffusion equation in stage formulation is given as

$$\frac{\partial y}{\partial t} + c \frac{\partial y}{\partial x} = \frac{Q}{2B \left( S_o - \frac{\partial y}{\partial x} \right)} \frac{\partial^2 y}{\partial x^2}$$

Using the above equation, Perumal et al. (2004) proposed the following expression for unsteady flow estimation using stage data as

$$Q = \frac{Q_o}{\sqrt{2}} \left[ 1 + \frac{1}{S_o c} \frac{\partial y}{\partial t} + \sqrt{\left( 1 + \frac{1}{S_o c} \frac{\partial y}{\partial t} \right)^2 - \frac{2Q}{BS_o^2 c^3} \frac{\partial^2 y}{\partial t^2}} \right]^{1/2} \quad (2.8)$$

where all notations represent the same variable as discussed earlier.

It was found that the refined estimate of longitudinal water gradient estimated from the convection-diffusion equation performs better than the other two methods. The refined Jones formula is capable of producing discharge hydrograph when  $|(1/S_o) \partial y / \partial x| < 0.5$ .

**Perumal and Moramarco (2005)** made a reassessment of different formulae such as Jones, Fenton and Marchi available for the estimation of discharge hydrographs using stage hydrographs.

Marchi formula (Marchi, 1976) for computing discharge using stage data is given as

$$Q = \alpha A^{(m+1)} + \frac{A}{2(m+1)S_o B} \left[ 1 - m^2 \frac{Q^2 B}{g A^3} \right] \frac{\partial A}{\partial t} \quad (2.9)$$



where, A=area, B=width of channel, S<sub>o</sub>=channel bed slope,  $\alpha$  and m are constants.

The Fenton formula (Fenton, 1999) for discharge estimation using stage data as

$$Q = Q_o \sqrt{1 + \frac{1}{cS_o} \frac{\partial y}{\partial t} - \frac{D}{S_o c^3} \frac{\partial^2 y}{\partial t^2}} \quad (2.10)$$

where, notations described as above, Q<sub>o</sub>=normal discharge, and D is diffusion parameter given as

$$D = \frac{Q_o}{2S_o B} \quad (2.11)$$

when D → 0 Fenton formula reduces to Jones formula.

The assessment was based on estimating discharge hydrograph using hypothetical stage hydrograph. The hypothetical discharge hydrograph corresponding to a given stage hydrograph was obtained by routing the given stage hydrograph in a channel reach using the Saint Venant equations which govern one-dimensional unsteady flow movement in channels. The modified Nash-Sutcliffe (Perumal et al. 2004) was used for assessing the performance which is expressed as

$$V_e = \frac{V_t - V_r}{V_t} \quad (2.11)$$

where, V<sub>e</sub> = the variance explained by the model, V<sub>t</sub> = the total variance of simulated discharge hydrograph and V<sub>r</sub> = the remaining variance.

It was found that in mild slope channels the Refined Jones formula is able to reproduce the hypothetical benchmark discharge hydrograph better than the Jones, Fenton's and Marchi's formulae. It was also found that the Jones, Fenton and Marchi formulae are special cases of the Refined Jones formula.

#### 2.2.4 Fread Method

Fread (1975) presented a general mathematical model to convert stage hydrograph data into discharge hydrograph data and vice-versa. In both cases, to compute either discharge or stage hydrograph, temporal data of the other is required. The method is based on one dimensional unsteady flow equation and Manning's equation. It was applied at several stations of Lower

Mississippi river, Red River and Atchafalaya River in USA. Root mean square error (RMSE) of the computed and observed discharges were found to be 3-7 %. It was significant when  $S_o \leq 0.001$  ft/ft.

$$Q - \frac{AD^{2/3}}{n} \left\{ S_o + \left[ \frac{A}{KQ} + \left( 1 - \frac{1}{K} \right) \frac{BQ}{gA^2} \right] \delta h_s + \frac{Q'/A' - Q/A}{g\Delta t} + \frac{2S_o}{3r^2} \left( 1 - \frac{BQ^2}{gA^3} \right) \right\}^{1/2} = 0 \quad (2.12)$$

where,  $D \approx R = A/B$  which is applicable for wide channels,

A=area of cross section, B=width of channel, n= Manning's channel roughness coefficient

$$K = \frac{5}{3} - \frac{2A}{3B^2} \frac{dB}{dh'} \quad (2.12a)$$

$$\delta h_s = \left( \frac{y-y'}{\Delta t} \right),$$

y=stage,  $y'$ =stage at time  $t-\Delta t$ ,  $Q'$  and  $A'$  are the discharge and area at time  $t-\Delta t$ .

### 2.2.5 Other studies

**Faye and Cherry (1980)** developed a mathematical model based on continuity and momentum equations to compute discharge hydrograph from stage hydrograph in highly sensitive stations. Model can estimate discharge in open channels for one-dimensional unsteady flow. Model was applied to Chattahoochee River with a reach length of 17 miles. Further, model was also used to check the sensitivity of the parameters. It was found that the Manning's roughness coefficient as well as slope of the channel were most sensitive to estimated discharge while velocity coefficient as less sensitive.

**Asgeir Petersen-Øverleir (2006)** developed a method for estimating discharge hydrograph using stage hydrograph which was based on non-linear regression and Jones formula. The regression model was developed based on monoclinal rising wave and Manning's friction law. Moreover, the method assumes simple hydraulic and geometry properties of the channel gauging stations.

The method was applied to the Chattahoochee River having high dynamic flow and Ohio River as well as Tennessee River which are large river affected by hysteresis. The average error in Chattahoochee River was found to be 3.2% or less in most of the gauging stations. In one of the gauging station, errors was found upto 50%. It showed that the method can be applied to medium large rivers only.

The study concluded that the model is suitable for post-modelling hydraulic and statistical validation and assessment.

### **2.3 Conclusions**

In this chapter, review of literature on discharge estimation using only stage information is discussed. Moreover, the performance comparison of Jones formula and its variants also discussed.

## Chapter 3 METHODOLOGY

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### 3.1 General

There are various methods available to estimate discharge hydrographs by using only at-site stage data. Some well-known methods like Jones formula and Fread are used to compare with other methods like the Iterative Jones formula, the modified Jones formula and the Refined Jones formula. Perumal et al. (2004) already compared the performance of the Jones formula with Modified as well as Refined Jones formulae. In this study, the discharge is estimated by representing the derivatives is charge using stage data of the Chattahoochee River,USA and at Bhadrachalam site of Godavari River.

### 3.2 Methods Studied

The methods for reproducing discharge hydrograph using stage hydrograph are evaluated in this study. The following approach is used in the study:

I) Various forms of Jones formula such as the original formula (Jones,1916), the modified and refined Jones formulae were discussed in Chapter-2. Herein, temporal changes of stage are estimated using the finite difference approximation as follows

The first order derivative of stage with reference to time is obtained using backward and centred difference schemes as

Case-I: Using backward finite difference scheme:

$$\frac{\partial y}{\partial t} = \frac{y(t) - y(t - \Delta t)}{\Delta t} \quad (3.1)$$

where,  $\Delta t$ =time interval,  $y(t)$  and  $y(t - \Delta t)$  are stage at time 't' and '(t - Δt)' respectively.

Case-II: Using centred finite difference scheme.

$$\frac{\partial y}{\partial t} = \frac{y(t + \Delta t) - y(t - \Delta t)}{2 * \Delta t} \quad (3.2)$$

where,  $y(t + \Delta t)$  =stage at time '(t + Δt)' and

the second order derivative of stage is obtained as

$$\frac{\partial^2 y}{\partial t^2} = \frac{y(t + \Delta t) - 2 * y(t) + y(t - \Delta t)}{\Delta t^2} \quad (3.3)$$

In addition to the above cases, Jones formula was also modified by replacing velocity as  $Q/A$

Accordingly the wave celerity  $c$  is expressed as

In the Jones formula, using wave celerity ' $c$ ' as

$$c = \left[ 1 + mP \left( \frac{\frac{\partial R}{\partial y}}{\frac{\partial A}{\partial y}} \right) \right] \frac{Q}{A} \quad (3.4a)$$

where  $m=2/3$  for Manning's,  $A$ =area of cross section,  $P$ =wetted perimeter of the river cross section,  $R$ =hydraulic radius of cross section,  $Q$ =discharge and  $y$ =stage.

And the iterative form of Jones formula is expressed as

$$Q = Q_0 \left[ 1 + \frac{A}{S_0 \left[ 1 + mP \left( \frac{\frac{\partial R}{\partial y}}{\frac{\partial A}{\partial y}} \right) \right] Q} \frac{\partial y}{\partial t} \right]^{1/2} \quad (3.4b)$$

II) Fread Formula:

To compute discharge hydrograph using stage information in unsteady flow condition, Fread formula used the one dimensional equation and Manning equation.

According to Fread (1975), the variable energy slope,  $S$  due to channel boundary resistance is given by

$$S = S_0 + S_o + \left[ \frac{A}{KQ} + \left( 1 - \frac{1}{K} \right) \frac{BQ}{gA^2} \right] \frac{\partial y}{\partial t} + \frac{\left( \frac{Q'}{A'} - \frac{Q}{A} \right)}{g\Delta t} + \frac{2S_0}{3r^2} \left( 1 - \frac{BQ^2}{gA^3} \right) \quad (3.5)$$

Using Eq. (3.5) in Manning equation gives the Fread formula as

$$Q - \frac{AD^{2/3}}{n} \left\{ S_0 + \left[ \frac{A}{KQ} + \left( 1 - \frac{1}{K} \right) \frac{BQ}{gA^2} \right] \frac{\partial y}{\partial t} + \frac{\left( \frac{Q'}{A'} - \frac{Q}{A} \right)}{g\Delta t} + \frac{2S_0}{3r^2} \left( 1 - \frac{BQ^2}{gA^3} \right) \right\}^{1/2} = 0 \quad (3.6)$$

where, all the notations are already discussed above.

### 3.3 Comparison of various methods

I) In order to compare various discharge estimation methods, hypothetical stage data is used. This stage hydrograph is routed in rectangular channel reaches each characterized by a set of channel width, bed slope and Manning's roughness. The channel configurations used in the study are given in Table 3.1.

The hypothetical stage hydrograph for converting discharge hydrograph is obtained using four parameter Pearson type III distribution which is given by the following equation

$$y_t = y_o + (y_p - y_o) \left[ \frac{t}{t_p} \right]^{\frac{1}{(\gamma-1)}} \exp \left[ - \frac{1 - \frac{t}{t_p}}{(\gamma-1)} \right] \quad (3.7)$$

where,  $y_o$ = initial stage,  $y_p$ = peak stage,  $t_p$ = time to peak and  $\gamma$ =shape factor.

Figure 3.1 shows the hypothetical stage hydrograph used for producing discharge hydrograph at the channel having  $Q_o=100 \text{ m}^3/\text{s}$ ,  $y_o=1.53 \text{ m}$ ,  $y_p=12\text{m}$ .  $t_p=12 \text{ hr}$ ,  $\gamma=1.15$ ,  $n=0.04$ ,  $S_o=0.0004$  and  $B=100\text{m}$ .

The discharge hydrograph required corresponding to the given stage hydrograph is obtained by routing it in the considered rectangular channel for a specified reach length using the HEC-RAS model (Hydrologic Engineering Centre, 2008). The downstream boundary condition considered for routing using HEC-RAS in rectangular channel was a one-to-one stage-discharge relationship. It was assumed that this downstream condition is located at a faraway downstream of the section at which the routed discharge is estimated. The discharge hydrograph estimated by the HEC-RAS at the inlet of the channel of the reach was considered as the benchmark discharge hydrograph to be reproduce by the considered discharge estimation formula

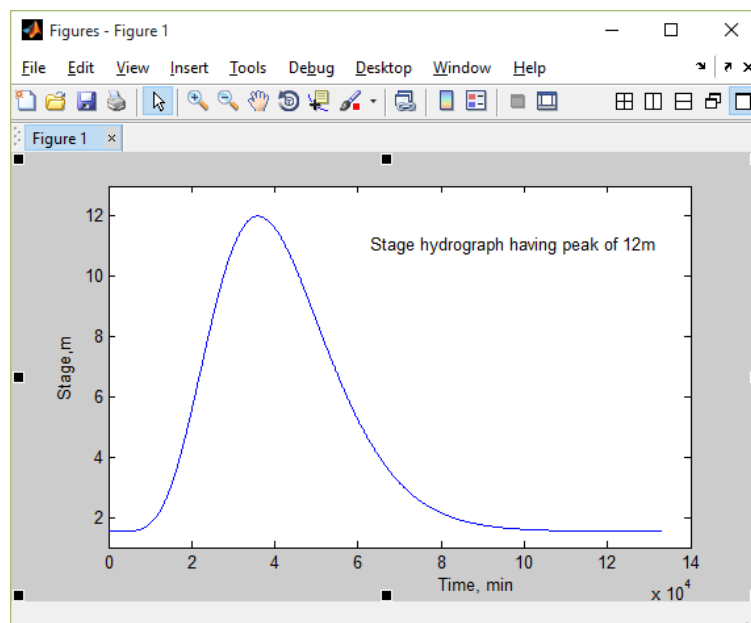


Figure 3.1 Stage hydrograph at Channel 17.

II) Hydrologic Engineering Center River Analysis System (HEC-RAS) Model discharge hydrograph is used a benchmark solution to compare the discharge hydrograph reproduces by various methods.

Table 3.1 shows the various channel configurations used for comparing the various discharge estimation methods. The Manning's roughness coefficient ranges from 0.02-0.04. The bed slope ranges from 0.0002 -0.001. Moreover, the width of the rectangular channel is differs as 50, 100, 150 and 200m at different conditions.

Table 3.1 Channel configurations used for comparison of various discharge estimation methods

Channel No.	Manning's n	Bed slope, So	Channel width, B (m)
1	0.02	0.001	100
2	0.02	0.0008	100
3	0.02	0.0006	100
4	0.02	0.0004	100
5	0.02	0.0002	100
6	0.03	0.001	100
7	0.03	0.0008	100
8	0.03	0.0006	100
9	0.03	0.0004	50
10	0.03	0.0004	100
11	0.03	0.0004	150
12	0.03	0.0004	200
13	0.03	0.0002	100
14	0.04	0.001	100
15	0.04	0.0008	100
16	0.04	0.0006	100
14	0.04	0.0004	100
18	0.04	0.0002	100

(Source: Perumal et al. 2004)

III) To compare the performance of various methods for producing discharge hydrograph, modified Nash-Sutcliffe efficiency (Perumal et al. 2004) is used for evaluating the efficacy of the method. In the modified Nash-Sutcliffe efficiency criterion, normal discharge is used instead of the average discharge while estimating the total variance of the discharge hydrograph,  $V_{tm}$ . The modified Nash-Sutcliffe efficiency is expressed as

$$V_{em} = \frac{V_{tm} - V_r}{V_{tm}} \quad (3.8)$$

where  $V_{em}$ =total variance of the model.

$V_{tm}$ = total variance of discharge hydrograph given as

$$V_{tm} = \frac{1}{(n-1)} \sum_{i=1}^n (Q_i - Q_{oi})^2 \quad (3.8a)$$

And  $V_r$ =reaming variance given as remaining variance,

$$V_r = \frac{1}{(n-1)} \sum_{i=1}^n (Q_i - Q_{mi})^2 \quad (3.8b)$$

where,  $Q_i$ =observed discharge,  $Q_o$ = normal discharge,  $Q_m$ = model discharge and  $n$ = no. of observations.

### 3.4 Application to field data

The various methods for producing discharge hydrograph using stage hydrograph are applied to the following stations. The analysis and estimation of discharge at Chattahoochee River, USA and Bhadrachalam has done using US unit system and Metric system respectively.

#### 3.4.1 Chattahoochee River, USA

The upper Chattahoochee river basin has an area of 3550 square miles in which the entire Chattahoochee River lies within it. The entire reach length extends for 250 miles. The reach length for research interest extends for about 17 miles which is bound at upstream by Buford Dam and downstream by Georgia Highway-141. The discharge from the Buford dam of Chattahoochee River were collected during 23 March 1976 at different stations. The total reach length is 17 miles downstream of Buford dam. Georgia Highway 141 is located at 17 miles downstream of Buford Dam. Little ferry which is located 8 miles downstream of Buford dam (Source: Faye and Cherry, 1980).

Figure 3.2 and Figure 3.3 below gives the observed stage and discharge hydrograph at GH-141 and Little Ferry respectively. The peak stage for GH-141 and Little Ferry, were 12.54 ft and 19.59 ft respectively. Moreover, the peak discharge at GH-41 and Little Ferry, were 6550 Cusec and 7670 Cusec respectively.



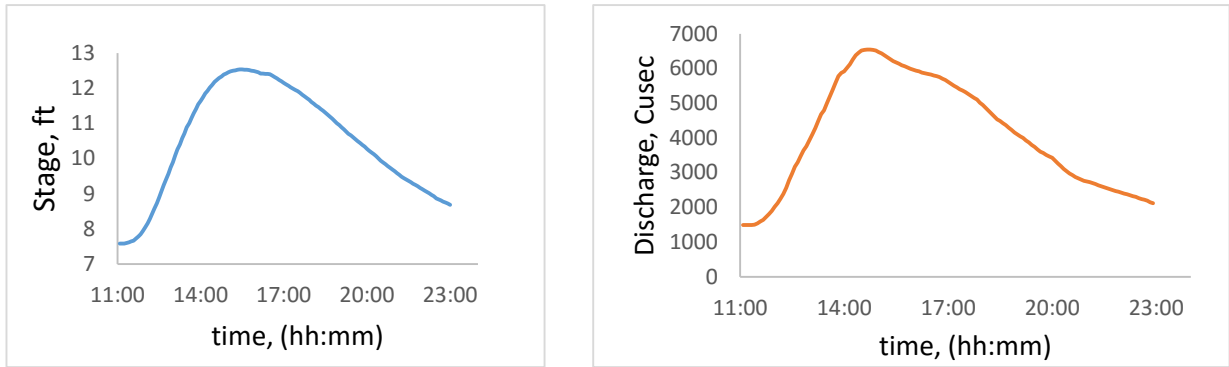


Figure 3.2 Observed stage and discharge hydrograph at GH-141, 23 March 1976 (11:10-23:00).

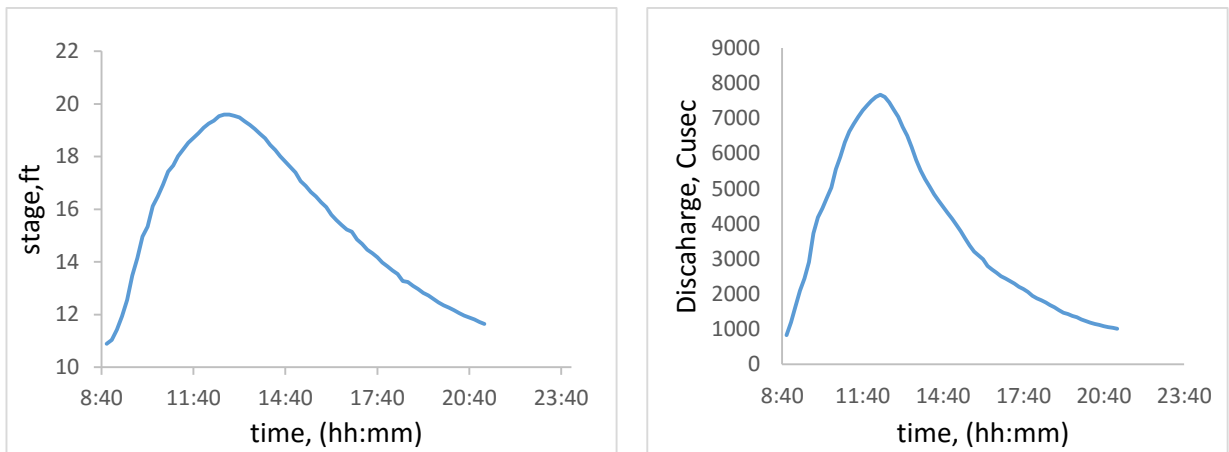


Figure 3.3 Observed Stage and discharge hydrograph at Little Ferry, 23 March 1976 (08:50-21:10).

Faye and Cherry (1980) obtained the channel cross section data at Chattahoochee River by measuring the water surface from a known datum by leveling. The water depth is then measured with fathometer or sounding technique. Again, the channel section above water surface is obtained using stadia and leveling technique. Appendix I and Tables 3.2 show the channel cross sectional data as well as roughness data for both GH-141 and Little Ferry respectively. The channel cross sectional width at GH-141 and Little Ferry were 290 ft and 201 ft respectively.

The channel bed slope at GH-141 and Little Ferry were 0.00031 and 0.00036 respectively. Moreover, the channel roughness coefficient at GH-141 ranges from 0.033 to 0.049. The channel roughness coefficient at Little Ferry was very sensitive to stages which ranges from 0.048 to 0.088 (Faye and Cherry,1980).

Table 3.2 The channel roughness data at GH-141 and Little Ferry

Station name	Little Ferry			Georgia Highway 141		
	March 20	March 22	March 23	March 22	March 22	March 23
Date in 1976	March 20	March 22	March 23	March 22	March 22	March 23
Time	1505	1700	1240	1080	3880	6550
Discharge (ft <sup>3</sup> /s)	774	4180	7450	1080	3880	6550
Hydraulic Radius (ft)	5.13	9.66	12.2	3.92	6.8	8.62
Effective channel slope (ft/ft)	0.00036	0.00036	0.00036	0.00031	0.00031	0.00031
Mean flow velocity (ft/s)	0.96	2.31	3.1	1.34	2.62	3.39
Manning's n (s/ft <sup>1/3</sup> )	0.088	0.056	0.048	0.049	0.036	0.033
Maximum flow depth (ft)	10.61	16.48	19.59	6.92	10.18	12.29

(Source: Faye and Cherry, 1980)

#### 3.4.1.1 Channel roughness coefficient

The channel roughness coefficient value is estimated using three approach as

- 1)  $n_{\text{Fread}}$  : It is defined by the Fread method of estimating roughness as

$$n = n_{L_0} + \frac{(n_{L_1} - n_{L_0})}{(h_{L_1} - h_{L_0})} (h - h_{L_0}) \quad (3.9)$$

(Source: Fread, 1975)

where  $n_{L_1}$  and  $n_{L_0}$  are roughness value at stages  $h_{L_1}$  and  $h_{L_0}$  respectively and

$h_{L_0} < h < h_{L_1}$ .

- 2)  $n_Q$ : Interpolated roughness value with respect to discharge hydrograph.
- 3)  $n_y$ : Interpolated roughness value with respect to stage hydrograph.

#### 3.4.2 Bhadrachalam Site at Godavari River, India

Bhadrachalam lies at Godavari River, India. The total length of Godavari River is around 1,450 km. It is the second longest river in India which rises from Trimbakeshwar, Nasik about 380 km from the Arabian Sea. It flows down to many states and finally empties to Bay of Bengal. The Bhadrachalam station is located at 80°52'54"E, 17°40'33.44"N. The channel roughness coefficient and bed slope for estimating discharge hydrograph at Bhadrachalam are 0.032 and 0.00034 respectively.

Appendix-II shows the channel cross sectional data at Bhadrachalam. The depth-area relationship is given by

$$A = 0.0275y^5 - 1.4292y^4 + 24.423y^3 - 109.6y^2 + 208.56y \quad (3.10)$$

where, A=area of cross section and y=river stage.

Figure 3.4 and Figure 3.5 shows the stage, discharge hydrograph and normal rating curve for event 2005\_ev1 at Bhadrachalam respectively. The observed stage hydrograph is used for conversion to discharge hydrograph. The estimated discharge hydrograph is used as reference hydrograph. The peak stage and estimated discharge during 2005\_ev1 was 17.04 m and 29712.18 Cumec respectively. It can be seen from the Figure 3.5 below that, for discharge less than 10,000 Cumec there is one-one relationship between the stage and discharge relationship.

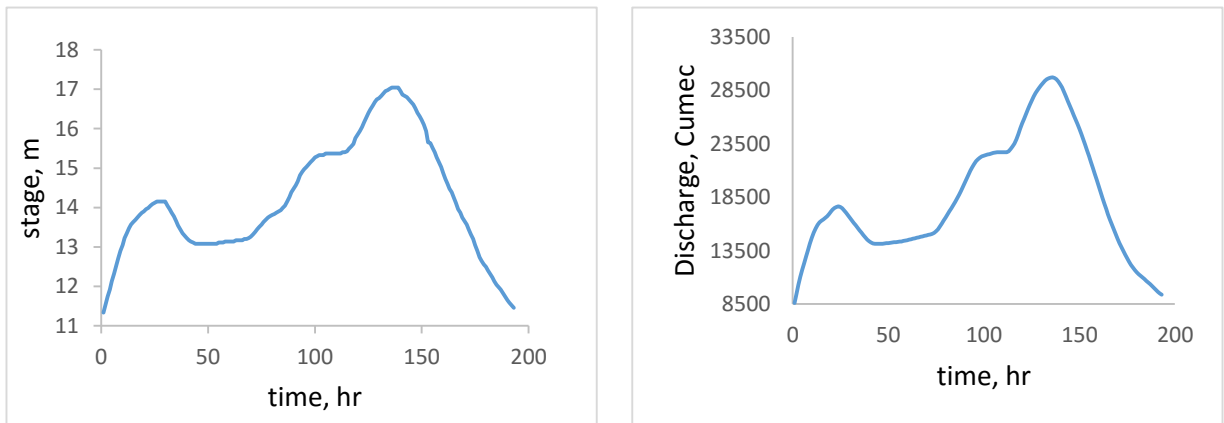


Figure 3.4 Observed Stage and estimated hydrograph curve at Bhadrachalam during 2005\_ev1

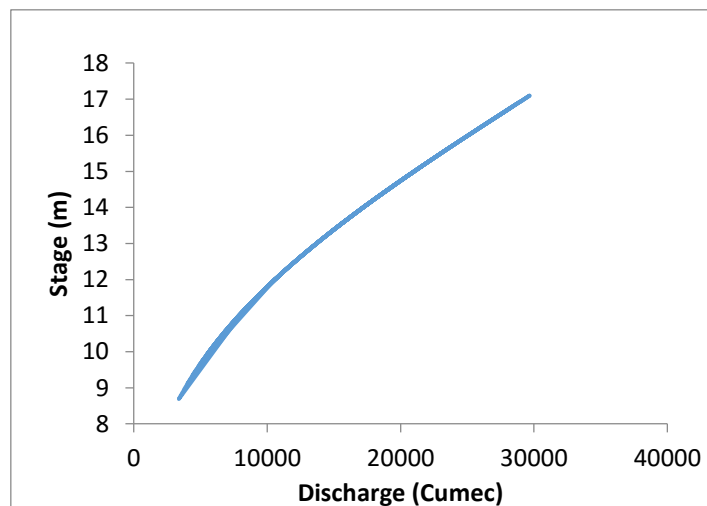


Figure 3.5 Normal rating curve at Bhadrachalam.

### 3.5 Evaluation criteria

In order to evaluate the performance of producing discharge hydrograph from stage hydrograph, Nash Sutcliff efficiency (NSE) is used. NSE is defined as

$$\text{NSE}(\%) = \frac{V_{\text{tm}} - V_r}{V_r} * 100 \quad (3.11)$$

where,  $V_r$ =remaining variance given as

$$V_r = \frac{1}{n} \sum_{i=1}^n (Q_i - Q_{mi})^2 \quad (3.11a)$$

where,  $Q_i$ =observed discharge,  $\bar{Q}$ =average discharge,  $Q_m$ = model discharge and  $n$ = no. of observations.

### 3.6 Software Used for analysis

HEC-RAS solution is considered as benchmark solution for comparing the various discharge estimation methods. Moreover, computer codes were developed using MATLAB for estimation of discharge hydrograph using stage information. Computer codes for various discharge estimation methods is enclosed in Appendix. Further, for digitisation of stage, discharge hydrograph *WebPlotDigitizer* was used.

### 3.7 Conclusions

The various discharge estimation methods using stage information has compared. Modified Nash-Sutcliffe efficiency has been used as evaluation criteria for comparing methods. The methods has again applied to Chattahoochee River, USA and Bhdrachalam, Godavari River, India. In both cases, the approach has done using finite difference scheme for estimation of temporal derivative of stage.

### 4.1 General

Various discharge estimation methods using stage information discussed in Chapter 3 are evaluated to compare their performances. In all the methods, finite difference schemes are used for estimating the derivatives of stage with either central or backward finite difference scheme. The methods are then applied for estimating discharge of Chattahoochee River, USA and at Bhadrachalam site of Godavari River.

### 4.2 Hypothetical data

To compare various methods used for estimating discharge from stage information, HEC-RAS solution is taken as benchmark solution. Figure 4.1 shows the stage and discharge hydrograph obtained using HEC-RAS for channel having  $n=0.04$ ,  $S_o=0.0004$  and  $B=100$  m (Channel no.17). The dotted line shows the stage hydrograph and continuous line shows the discharge hydrograph respectively. The peak discharge is found to be 2888.25 Cumec with a peak stage of 12m. Figure 4.2 shows the loop rating curve obtained using HEC-RAS at Channel 17. In steady flow condition, there is one-one stage-discharge relationship but due to unsteady condition there is a loop in the rating curve. The loop is due to the inertial and pressure force which leads to more velocity in rising portion as compare to steady condition and vice versa. It leads to the formation of hysteresis in the stage-discharge relationship.

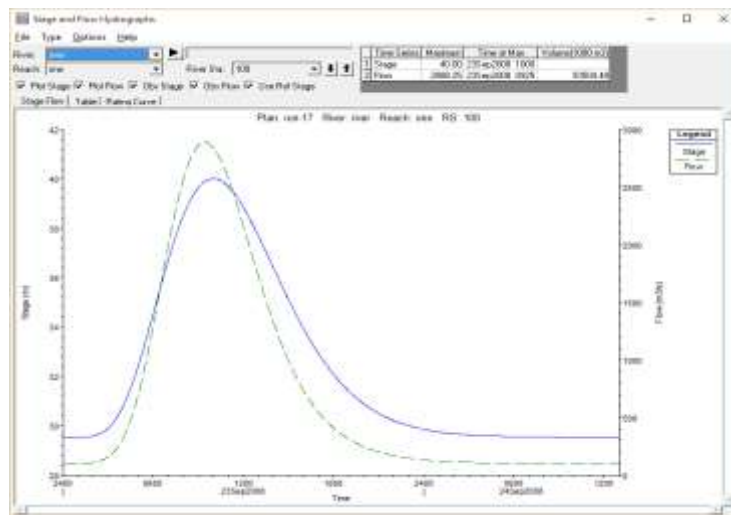


Figure 4.1 Stage and discharge hydrograph for Channel no.17 using HEC-RAS

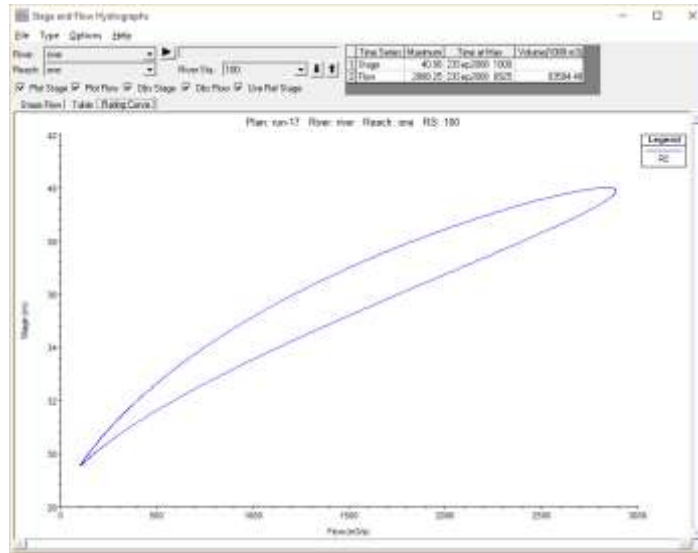


Figure 4.2 Rating curve for Channel no.17 using HEC-RAS

The comparison of various discharge hydrograph and rating curve using Jones, Modified Jones, Refined Jones, Fread, Iterative Jones and HEC-RAS methods with normal discharge for Channel no.17 is shown below in Figures 4.3 and 4.4 respectively. From the figure it can be seen that the Refined Jones formula is producing more closely to the HEC-RAS solution. In Refined Jones formula, longitudinal gradient of depth is closer than that used in Jones, Modified Jones formulas (Perumal et al. (2004)). Moreover, it may be due to lesser variable energy slope in case of Fread method.

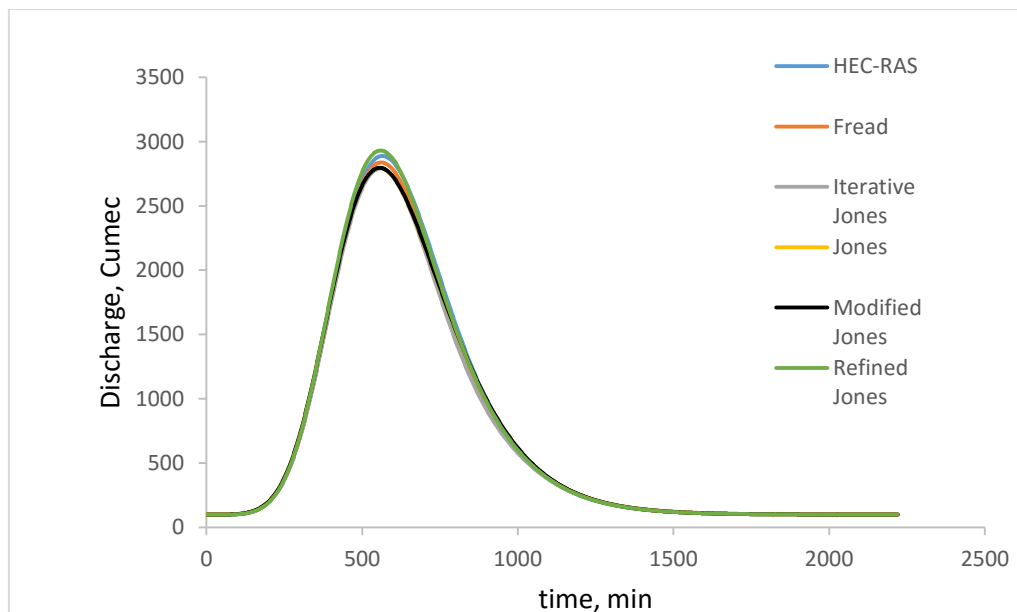


Figure 4.3 Comparison of discharge hydrograph of various methods for channel no.17 using central FDM.

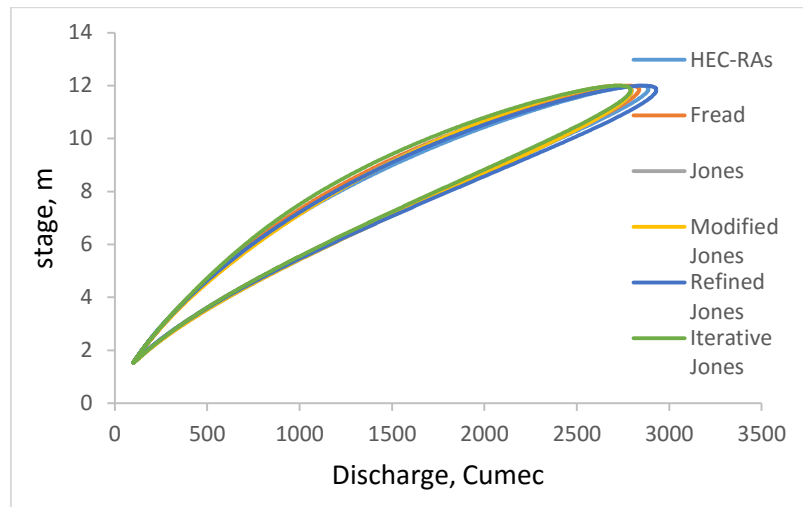


Figure 4.4 Comparison of rating curve for Channel no.17 using central FDM.

Likewise, it is observed from Table 4.1 that Fread formula and Refined Jones formula are giving higher efficiency than other methods except in the case of Channel having bed slope of 0.001 and Manning's roughness coefficient of 0.02. Fread formula is giving better efficiency than the Refined Jones formula in case of channel having Manning's roughness coefficient  $< 0.02$  as well as slope  $> 0.0008$  with  $n=0.03$ . In remaining cases, Refined Jones method is superior to other methods. Lastly, Jones and Modified Jones methods are able to reproduce in case of flatter slope of  $S_o = 0.0002$ . From Table 4.2, it is seen that central finite difference method is giving better result in case of Refined Jones method.

Table 4.1 Comparative Modified NSE of various methods for estimation of discharge using backward FDM.

Channel	n	S <sub>o</sub>	Modified Nash-Sutcliffe efficiency (%)				
			Jones	Modified Jones	Refined Jones	Fread	Iterative Jones
1		0.001	97.332	99.544	97.422	99.924	97.464
2		0.0008	97.974	99.3	98.297	99.882	97.993
3	0.02	0.0006	97.937	98.704	98.816	99.752	97.684
4		0.0004	96.104	96.69	98.757	99.144	94.755
5		0.0002	82.514	83.77	92.986	error	Error
6		0.001	99.063	99.426	99.432	99.829	98.972
7		0.0008	98.777	99.032	99.513	99.698	98.511
8		0.0006	97.795	98.022	99.417	99.309	97.048
9	0.03	0.0004	92.83	93.111	97.695	96.679	88.622
10		0.0004	94.284	94.631	98.530	97.64	90.845
11		0.0004	94.723	95.098	98.711	97.877	91.459
12		0.0004	94.927	95.318	98.782	97.972	91.725
13		0.0002	70.615	71.73	error	error	Error
14		0.001	99.082	99.204	99.711	99.687	98.836
15		0.0008	98.482	98.596	99.651	99.427	97.962
16	0.04	0.0006	96.919	97.073	99.343	98.708	95.555
17		0.0004	91.734	92.032	97.465	95.065	83.457
18		0.0002	51.827	53.102	error	error	Error

Table 4.2 Comparative Modified NSE of various methods for estimation of discharge using central FDM.

Channel	n	S <sub>o</sub>	Modified Nash-Sutcliffe efficiency (%)				
			Jones	Modified Jones	Refined Jones	Fread	Iterative Jones
1		0.001	97.305	99.495	97.435	99.901	97.429
2		0.0008	97.929	99.24	98.303	99.853	97.938
3	0.02	0.0006	97.864	98.624	98.818	99.709	97.598
4		0.0004	95.981	96.566	98.761	99.072	94.61
5		0.0002	82.255	83.515	93.075	-	-
6		0.001	99.017	99.372	99.435	99.796	98.917
7		0.0008	98.712	98.962	99.515	99.654	98.435
8		0.0006	97.701	97.925	99.422	99.246	96.937
9	0.03	0.0004	92.664	92.945	97.743	96.565	88.419
10		0.0004	94.131	94.478	98.555	97.539	90.66
11		0.0004	94.574	94.951	98.729	97.779	91.281
12		0.0004	94.781	95.173	98.796	97.877	91.55
13		0.0002	70.292	71.41	-	-	-
14		0.001	99.023	99.141	99.714	99.645	98.767
15		0.0008	98.402	98.514	99.655	99.371	97.87
16	0.04	0.0006	96.805	96.959	99.357	98.629	95.421
17		0.0004	91.553	91.852	97.515	94.94	83.233
18		0.0002	51.439	52.717	-	-	-



### 4.3 Chattahoochee River

(a) Georgia Highway-141(GH-141):

Figures 4.5 and 4.6 show the comparison of discharge hydrograph estimated by different methods considered in the study with the observed hydrograph using Manning's roughness  $n_Q$  with backward finite difference method. The peak observed discharge is 6550 Cusec. The peak discharge using Jones, Modified Jones, Refined Jones, Fread and Iterative Jones are 6904.64, 6894.75, 7474.42, 6873.27 and 6890.31 Cusec respectively. The methods are able to reproduce closely to the observed discharge except in the case of Refined Jones method. In case of hypothetical data, Refined Jones methods gives a better result than other methods. But, in application to field data it's not giving better result rather scattered discontinuously. It may be due to the randomness of the data.

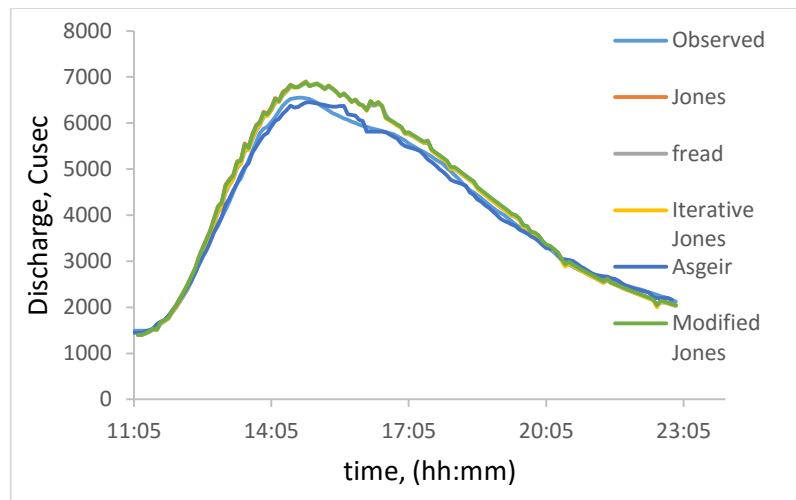


Figure 4.5 Comparison of discharge hydrograph using backward FDM at GH141, 23 March 1976 (08:50-21:10).

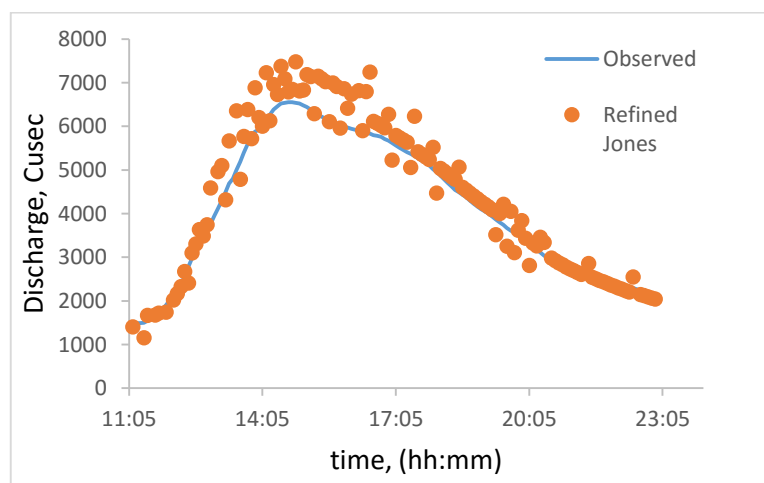


Figure 4.6 Observed discharge hydrograph with Refined Jones using backward FDM at GH-141, 23 March 1976 (08:50-21:10).

Figures 4.7 and 4.8 show the comparison of rating curve using Manning's roughness of  $n_Q$  with backward finite difference method. Likewise, the methods are producing discharge hydrograph closely to the observed hydrograph.

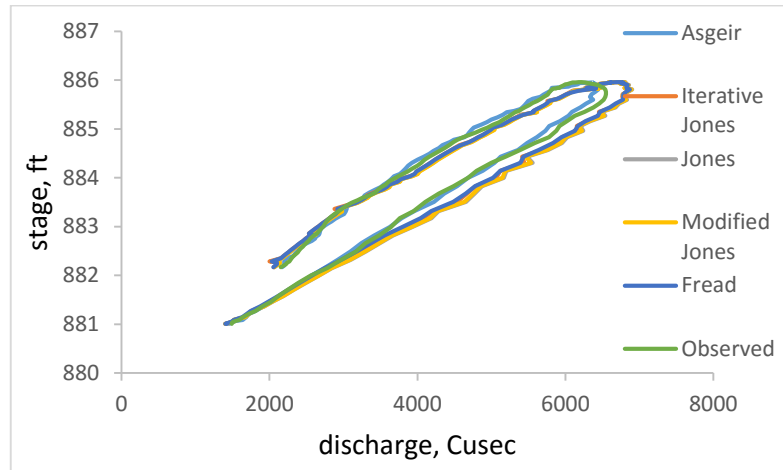


Figure 4.7 Comparison rating curve using backward FDM at GH-141, 23 March 1976 (08:50-21:10).

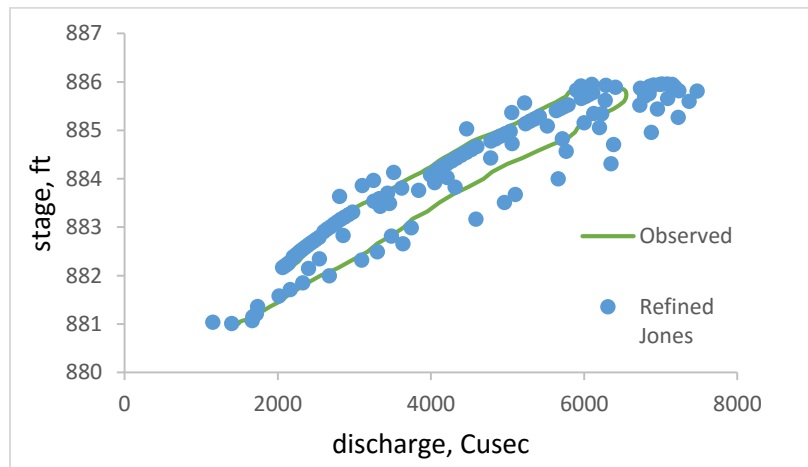


Figure 4.8 Observed rating curve with Refined Jones using backward FDM at GH-141, 23 March 1976 (08:50-21:10).

Figures 4.9 and 4.10 show the comparison of discharge hydrograph with Observed hydrograph using Manning's roughness of  $n_Q$  with central finite difference method. In the case of central FDM also, all methods are closely producing to observed discharge hydrograph except in case of Refined Jones method.

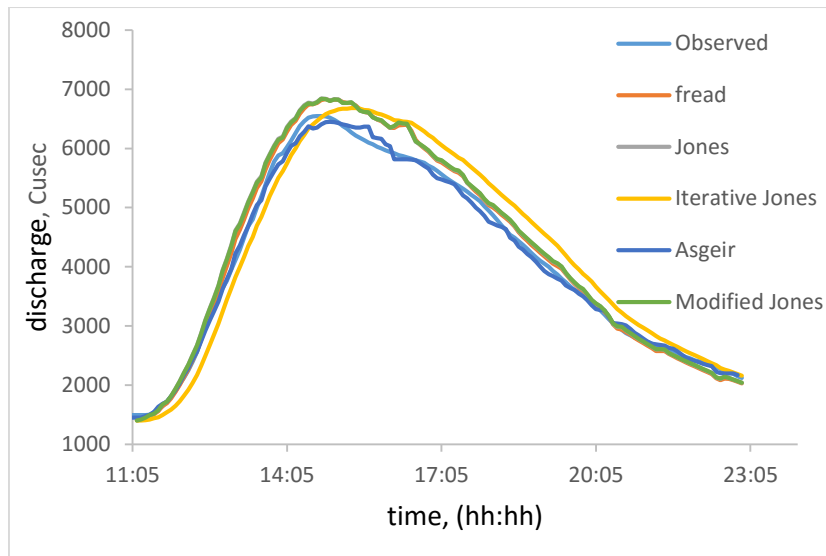


Figure 4.9 Comparison of discharge hydrograph using central FDM at Gh-141, 23 March 1976 (08:50-21:10).

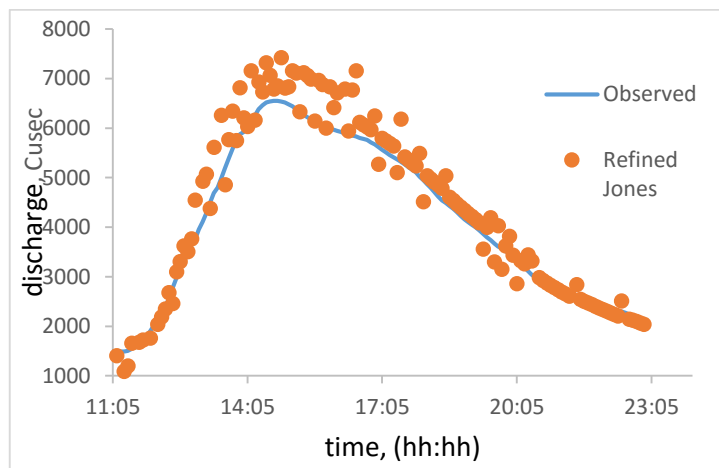


Figure 4.10 Observed discharge hydrograph with Refined Jones using central FDM at GH-141, 23 March 1976 (08:50-21:10).

Figures 4.11 and 4.12 show the comparison of rating curve with Observed hydrograph using Manning's roughness of  $n_Q$  with central fine difference method at Georgia Highway 141. The observed peak discharge is 6550 Cusec. The peak discharge 6682.024 Cusec using Iterative Jones method gives closer to the observed peak than other methods.

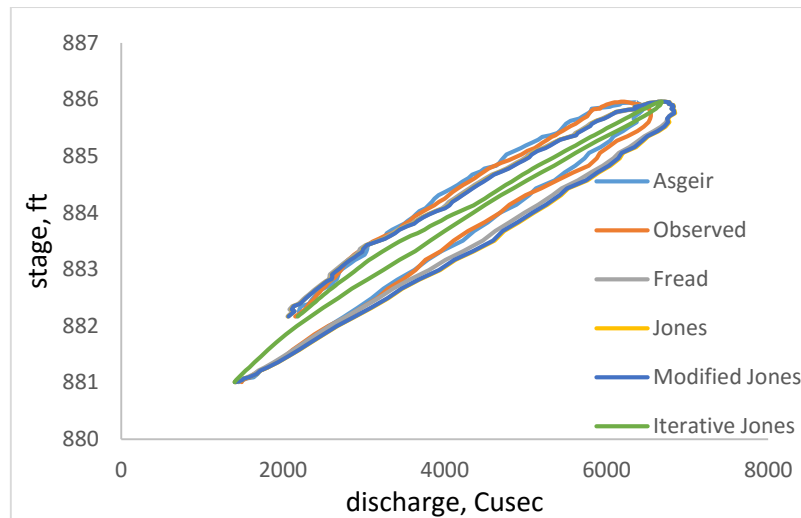


Figure 4.11 Comparison of rating curve using backward FDM at GH-141, 23 March 1976 (08:50-21:10).

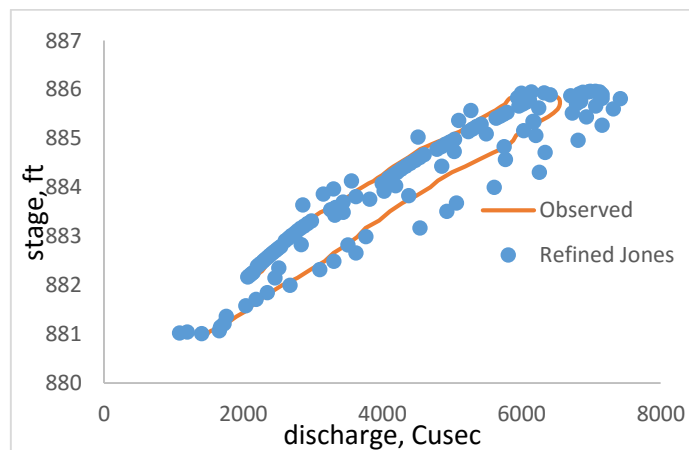


Figure 4.12 Observed rating curve with Refined Jones using central FDM at GH-141, 23 March 1976 (08:50-21:10).

Tables 4.3 and 4.4 show the Nash Sutcliffe efficiency for the Chattahoochee River at Georgia Highway-141 using backward and central method respectively. Different Manning's roughness coefficient viz.  $n_{\text{Fread}}$ ,  $n_y$  and  $n_Q$  used to compare the efficiency.

The Manning's roughness coefficient  $n_Q$  which is interpolated with respect to discharge hydrograph gives higher efficiency than other Manning's roughness estimation method. But increasing 1.05 times to  $n_{\text{Fread}}$  gives superior to others. Among the various methods, Fread method gives better result in case of  $n_Q$  and Modified Jones method in case of  $1.05 \cdot n_{\text{Fread}}$ .

Table 4.3 Comparison of various discharge estimation methods at GH-141 using backward FDM, 23 March 1976 (08:50-21:10).

Nash Sutcliff Efficiency (fraction)				
Methods	n_Fread	1.05*n_Fread	n_Q	n_y
Jones	0.9646	0.984	0.972	0.956
Modified Jones	0.9645	0.9841	0.9727	0.9561
Refined Jones	0.901	0.919	0.913	0.897
Fread	0.966	0.980	0.978	0.963
Asgeir	0.997	0.997	0.997	0.997
Iterative jones	0.965	0.980	0.976	0.961

In case of central FDM also, Manning's roughness  $n_Q$  gives better result than others. If roughness  $n_{Fread}$  is increased by 1.05, the result become superior to others. Among the various methods, Fread method gives better result in case of  $n_Q$  and Modified Jones method in case of  $1.05*n_{Fread}$ .

Table 4.4 Comparison of various discharge estimation methods at GH-141 using central FDM, 23 March 1976 (08:50-21:10)

Nash Sutcliff Efficiency (fraction)				
Methods	n_Fread	1.05*n_Fread	n_Q	n_y
Jones	0.966	0.985	0.974	0.959
Modified Jones	0.966	0.985	0.975	0.958
Refined Jones	0.911	0.934	0.922	0.907
Fread	0.967	0.980	0.979	0.965
Asgeir	0.997	0.997	0.997	0.997
Iterative Jones	0.967	0.981	0.978	0.964

(a) Little Ferry:

Figures 4.13 and 4.14 show the comparison of discharge hydrograph and rating curve respectively with observed hydrograph using Manning's roughness of  $n_y$  with backward finite difference method. The observed peak discharge is 7670 Cusec. The peak discharge using Jones, Modified Jones, Refined Jones, Fread and Iterative Jones are 7790.96, 7784.82, 8726.05, 7740.17 and 7748.25 Cusec respectively. The methods are able to reproduce closely to the observed discharge except Refined Jones method. Likewise, may be due to randomness of the data Refined Jones doesn't produce smoothly hydrograph rather a scattered data.

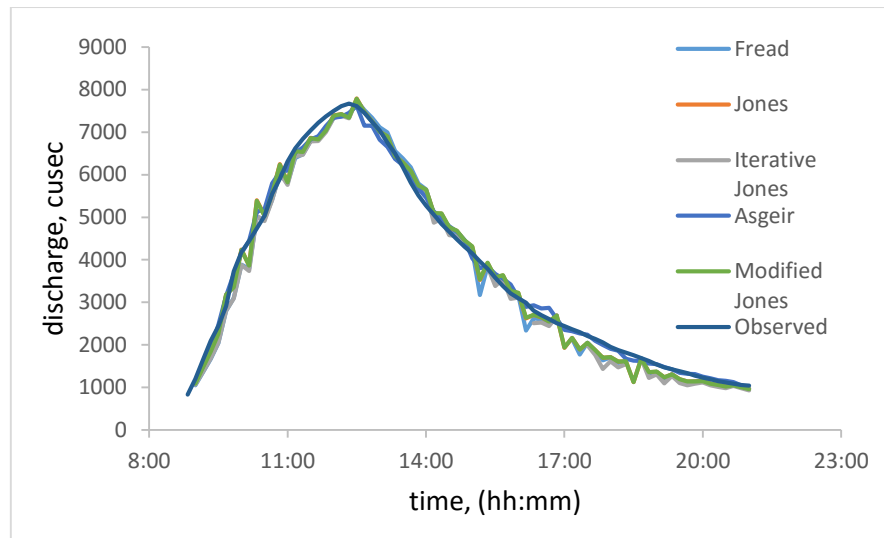


Figure 4.13 Comparison of discharge hydrograph at Little Ferry using backward FDM, 23 March 1976 (11:10-23:00).

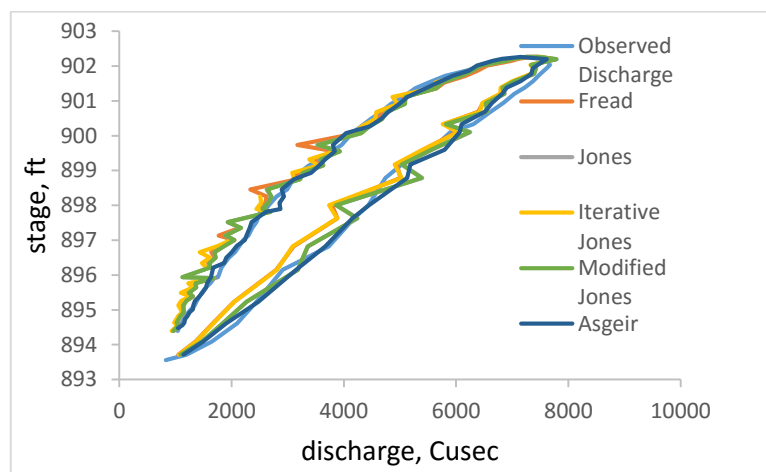


Figure 4.14 Comparison of rating curve at Little Ferry using backward FDM, 23 March 1976 (11:10-23:00).

Figures 4.15 and 4.16 show the comparison of discharge hydrograph with observed hydrograph using Manning's roughness of  $n_y$  with central finite difference method. The methods are able to reproduce closely to the observed discharge except Refined Jones method. In case of central finite difference scheme, the results are giving better as the methods are producing more smoothly than backward FDM.

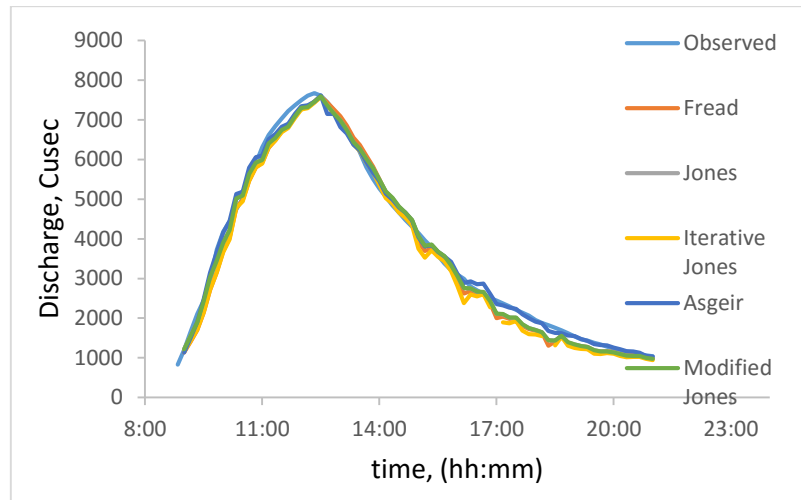


Figure 4.15 Comparison of discharge hydrograph at Little Ferry using central FDM, 23 March 1976 (11:10-23:00).

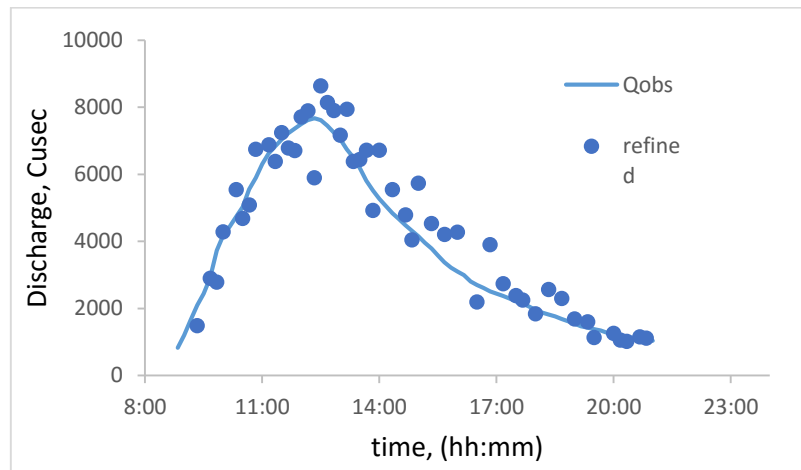


Figure 4.16 Observed discharge and Refined Jones hydrograph at Little Ferry using central FDM, 23 March 1976 (11:10-23:00).

Figures 4.17 and 4.18 show the comparison of rating curve using Manning's roughness of  $n_y$  with central finite difference method. Likewise, rating curve shows that the methods also produce discharge hydrograph closely to the observed hydrograph.

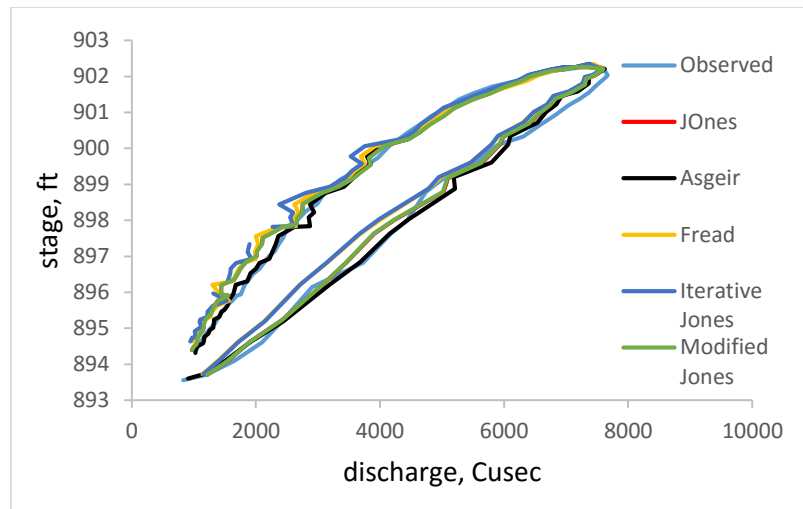


Figure 4.17 Comparison of rating curve at Little Ferry using central FDM, 23 March 1976 (11:10-23:00).

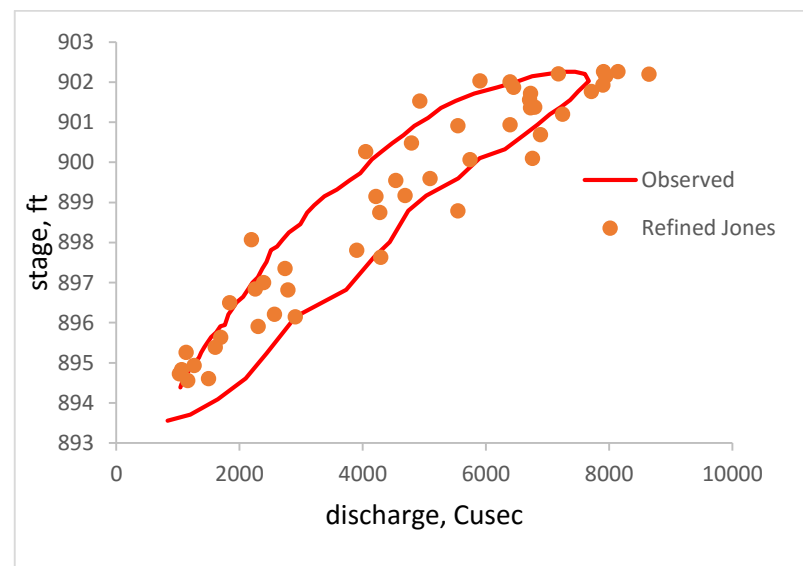


Figure 4.18 Observed and Refined Jones rating curve at Little Ferry using central FDM, 23 March 1976 (11:10-23:00).



Tables 4.5 and 4.6 below show the NSE for the Chattahoochee River at Little Ferry using backward and central method respectively. Different Manning's roughness coefficient viz.  $n_{Fread}$ ,  $n_y$  and  $n_Q$  used to compare the efficiency. The Manning's roughness coefficient  $n_y$  which is interpolated with respect to stage hydrograph gives higher efficiency than other Manning's roughness estimation method. Among the various methods, Jones and Modified Jones method gives better result than other methods. The central FDM gives better result than the backward FDM. In case of Jones and Modified Jones method central FDM gives a NSE (fraction) of 0.992 against backward FDM of 0.987.

Table 4.5 Comparison of various discharge estimation methods at Little Ferry using backward FDM, 23 March 1976 (11:10-23:00).

Nash Sutcliff Efficiency (fraction)			
Methods	$n_{Fread}$	$n_Q$	$n_y$
Jones	0.976	0.975	0.987
Modified Jones	0.976	0.976	0.987
Refined Jones	0.928	0.910	0.911
Fread	0.973	0.980	0.984
Asgeir	0.996	0.996	0.996
Iterative Jones	0.971	0.976	0.986

In case of central FDM also, the Jones and Modified Jones method give better result. It is observed that central FDM is superior to backward FDM method.

Table 4.6 Comparison of various discharge estimation methods at Little Ferry using central FDM, 23 March 1976 (11:10-23:00).

Nash Sutcliff Efficiency (fraction)			
Methods	$n_{Fread}$	$n_Q$	$n_y$
Jones	0.980	0.982	0.992
Modified Jones	0.980	0.982	0.992
Refined Jones	0.920	0.930	0.913
Fread	0.972	0.981	0.988
Asgeir	0.996	0.996	0.996
Iterative Jones	0.962	0.969	0.985

### 4.3 Bhadrachalam

Jones, Modified Jones, Refined Jones, Fread and Iterative Jones method were used to estimate discharge from stage hydrograph at Bhadrachalam. The methods approaches using backward as well as central finite difference method from 2002 to 2010 having 16 events.

Figures 4.19 and 4.20 show the comparison of discharge hydrograph and rating curve respectively for event 2005\_ev1 using backward finite difference method. The peak estimated

discharge is 29712.18 Cumec. The peak discharge using Jones, Modified Jones, Refined Jones, Fread and Iterative Jones are 29522.56, 29521.4436, 29625.87, 29520.58 and 29522.28 Cumec respectively. The methods are able to reproduce closely to the observed discharge hydrograph.

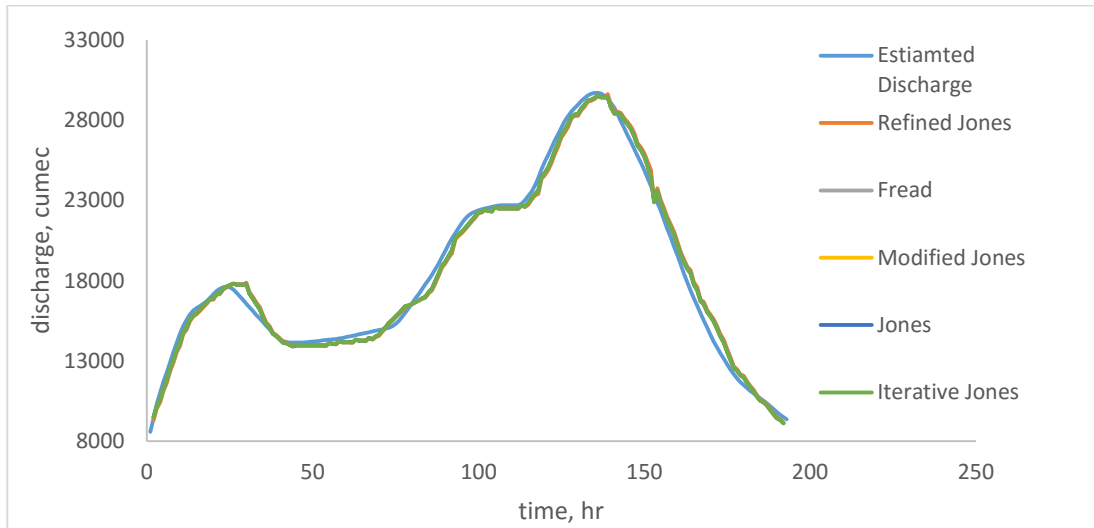


Figure 4.19 Comparison of discharge hydrograph during 2005\_ev1 using backward FDM.

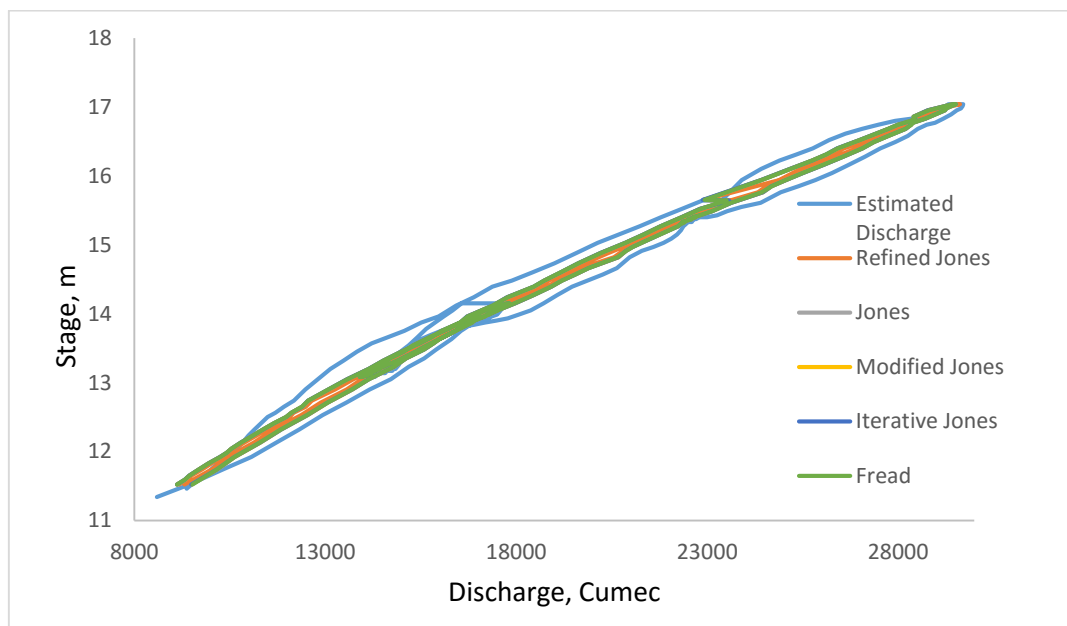


Figure 4.20 Comparison of Rating curve during 2005\_ev1 using backward FDM

Figures 4.21 and 4.22 show the comparison of discharge hydrograph and rating curve respectively for event 2005\_ev1 using central FDM. The peak estimated discharge in observed hydrograph is 29712.18 Cumec. Refined Jones formula with peak discharge of 29625.87 Cumec gives a closer to the observed peak than any other methods. The peak discharge using

Jones, Modified Jones, Fread and Iterative formula are 29522.56, 29521.44, 29520.58 and 29522.28 Cumec respectively.

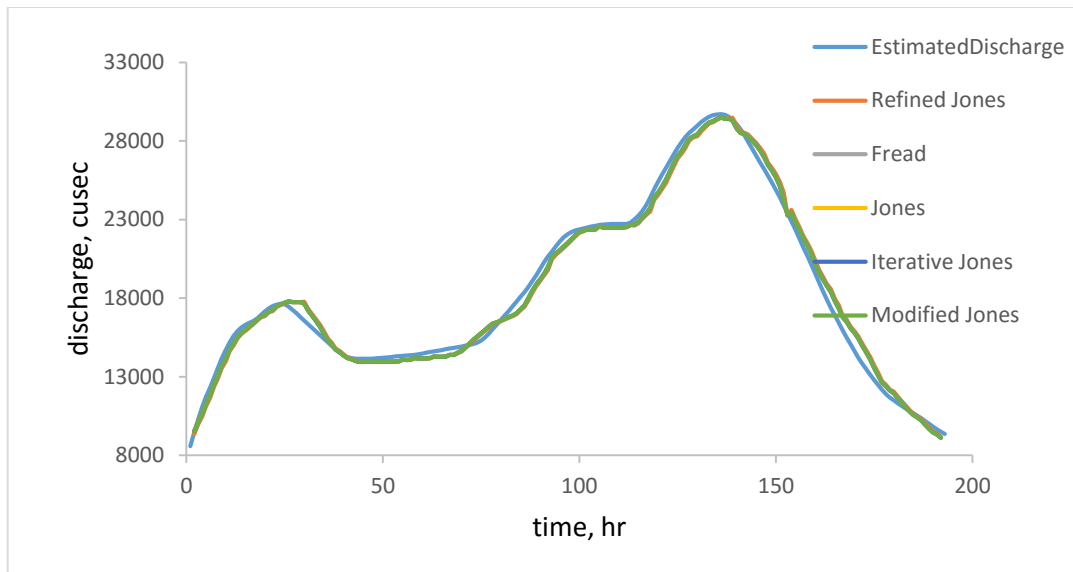


Figure 4.21 Comparison of discharge hydrograph during 2005\_ev1 using central FDM

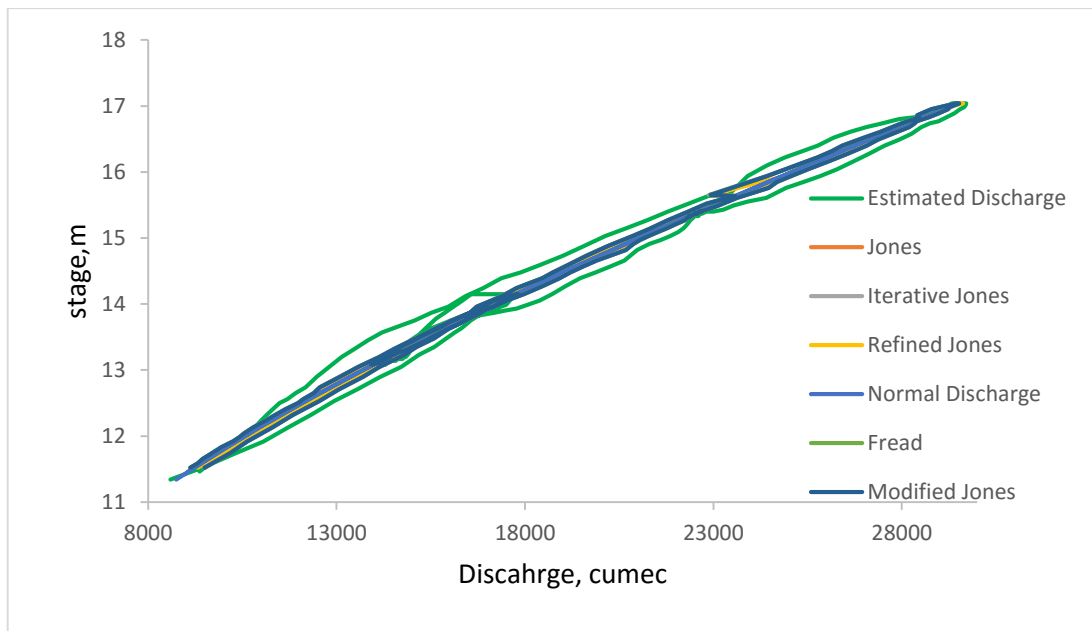


Figure 4.22 Comparison of rating curve during 2005\_ev1 using central finite difference method.

Tables 4.7 and 4.8 show the comparison of Nash Sutcliff efficiency of various methods for estimation of discharge using stage hydrograph data using backward and central method respectively. In most of the cases, Jones formula is giving better result than others with  $NSE > 90\%$  except in the cases of events having normal discharge less than 10000 Cumec.

Table 4.7 Comparison of NSE for various methods for estimation of discharge backward FDM at Bhadrachalam.

Nash Sutcliff efficiency (%)					
Event	Jones	Modified Jones	Refined	Fread	Iterative Jones
2002_ev1	97.807	97.802	98.145	97.776	97.759
2002_ev2	98.026	98.016	97.324	97.982	97.997
2003_ev1	98.598	98.596	98.348	98.59	98.593
2003_ev2	99.429	99.427	99.291	99.422	99.424
2004_ev1	97.076	97.073	96.869	97.061	97.065
2004_ev2	98.409	98.397	97.6	98.38	98.402
2005_ev1	99.153	99.147	98.82	99.145	99.153
2005_ev2	99.357	99.353	99.111	99.347	99.354
2006	98.262	98.245	-	98.228	98.224
2007_ev1	97.835	97.833	97.716	97.826	97.825
2007_ev2	98.942	98.936	98.644	98.928	98.937
2008_ev1	97.828	97.827	97.636	97.83	97.83
2008_ev2	82.79	82.799	82.875	82.824	82.797
2009_ev1	76.74	76.743	76.925	76.754	76.703
2009_ev2	58.113	58.11	57.648	58.063	58.025
2010	98.956	98.954	98.779	98.945	98.951

Table 4.8 Comparison of NSE for various methods for estimation of discharge central FDM at Bhadrachalam.

Nash Sutcliff efficiency (%)					
Event	Jones	Modified Jones	refined	Fread	Iterative Jones
2002_ev1	97.758	97.754	98.116	97.731	97.716
2002_ev2	98.033	98.022	97.334	97.989	98.003
2003_ev1	98.612	98.615	98.374	98.607	98.618
2003_ev2	99.436	99.434	99.3	99.428	99.431
2004_ev1	97.084	97.081	96.88	97.069	97.074
2004_ev2	98.447	98.434	97.646	98.416	98.44
2005_ev1	99.179	99.173	98.853	99.169	99.18
2005_ev2	99.361	99.356	99.117	99.349	99.357
2006	98.247	98.231	-	98.212	98.211
2007_ev1	97.835	97.833	97.717	97.827	97.826
2007_ev2	98.97	98.964	98.677	98.954	98.964
2008_ev1	97.837	97.836	97.653	97.837	97.839
2008_ev2	83.084	83.089	83.221	83.083	83.076
2009_ev1	76.706	76.709	76.894	76.719	76.674
2009_ev2	58.18	58.177	57.781	58.124	58.106
2010	98.98	98.978	98.805	98.968	98.974

#### **4.4 Conclusions**

Jones formula and its variants formula has been compared with finite difference scheme approaches. In addition to this, Fread formula along with Iterative Jones formula has included for comparing the performance of discharge estimation using stage information. The various methods has again applied to field data. The methods are able to reproduce the discharge hydrograph using stage hydrograph.

## Chapter 5 CONCLUSIONS

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### Conclusions

This study was undertaken to assess the suitability of various discharge estimation methods available in literature for converting the observed stage/flow depth hydrographs at a river/channel gauging site to discharge hydrograph duly accounting for hysteresis behaviour of stage-discharge relationship exhibited during unsteady flow. To achieve this evaluation both hypothetical and field applications approaches were studied in reproducing the respective cases benchmark discharge hydrographs.

The following conclusion can be arrived at from the analysis of the results of the study.

1. In most of the cases, the refined Jones method produced higher efficiency of reproducing the benchmark hydrographs than the other methods except for channels characterized by steep slopes. None of the methods studied is found to be suitable for channels characterized by slope  $< 0.0002$  when Manning's roughness  $> 0.02$ .
2. The Fread method gives better results of reproducing the benchmark discharge hydrograph for channels having slope  $> 0.0008$  with roughness  $n > 0.02$ .

While applying the methods to field data of Chattahoochee River due to fluctuating flow depths, the discharge estimated using the methods under study could not perform well, but for the Bhadrachalam site of the Godavari River these methods can be applied effectively with an efficiency of reproduction  $NSE > 90\%$ .

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## APPENDIX-I

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Channel cross sectional data at			
GH-141		Little Ferry	
Horizontal Distance (ft)	Altitude (ft)	Horizontal Distance (ft)	Altitude (ft)
90	893.2	8	918
130	891.4	12	901.55
150	885.92	15	891.57
156	880.22	25	887.4
166	877.42	35	885.4
176	878.62	40	883.85
191	876.62	45	882.71
216	877.02	50	882.74
236	877.02	55	882.67
256	876.52	60	883.31
276	876.92	65	883.85
296	875.12	70	884.35
316	875.42	80	885.98
326	873.92	90	887.51
331	873.42	100	888.03
346	874.72	110	888.86
355	877.62	120	889.29
364	881.62	130	891.62
368	885.92	145	892.15
380	889.4	160	892.67
		170	892.7
		180	894.9
		190	897.7
		193	902.21
		209	914

(Source: Faye and Cherry, 1980)

## APPENDIX-II

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### Channel cross section at Bhadrachalam site of Godavari River

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Depth, m	Area, m <sup>2</sup>
0	0.00
1	121.92
2	152.00
3	189.44
4	305.76
5	548.06
6	942.36
7	1496.87
8	2205.29
9	3050.13
10	4006.00
11	5042.90
12	6129.53
13	7236.61
14	8340.12
15	9424.69
16	10486.80
17	11538.15

## APPENDIX-III

```

% Mat Lab: Jones METHOD for reproducing discharge hydrograph using stage
data for channel no.17
% Units are in SI-Unit
clear all
clc
yo= 1.534168; % initial stage w.r.t initial flow of 100 Cumec.
n=0.04;      % Manning's roughness coefficient
B=100;
So=0.0004;  % Channel bed slope
dT=60;
yp=12;      % Peak stage of the stage hydrograph
tp=36000;   % time to peak
Q1=100.0018485; % approximate Q at beginning
Q0=100.0018485; % initial fzero
t=60:60:133200; % time interval
sum1=0;
sum2=0;
maatt =
xlsread('D:\Dissertaio_VIP\Dissertation\hypothetical_data\HEC_RAS.xlsx','HEC
','B3:S2222');
HEC= maatt(:,17);
for i=1:length(t)
    z=yo+(yp-yo)*((t(i)-60)/tp).^(1/0.15)*exp(-(t(i)-60)/tp)/(0.15));
    y=yo+(yp-yo)*(t(i)/tp).^(1/0.15)*exp(-(t(i)/tp)/(0.15));
    y1=yo+(yp-yo)*((t(i)+60)/tp).^(1/0.15)*exp(-(t(i)+60)/tp)/(0.15));
    D1=(y-z)/60; % Backward FDM
    D1=(y1-z)/(2*dT); % Central FDM
    A=B*y;
    P=(B+2*y);
    V=1/n*(A/P).^(2/3)*So.^(0.5);
    c=(1+(2/3)*(B/(B+2*y)))*V;
    Qo=A*V;
    Q(i)=Qo*sqrt(1+D1/(So*c));
    xx=(HEC(i)-Qo)^2;
    sum1=sum1+xx;
    yy=(HEC(i)-Q(i))^2;
    sum2=sum2+yy;
end
i
Jones=Q';
Vr=1/2220*sum2;
Vtm=1/2220*sum1;
NSE=100*(Vtm-Vr)/Vtm; %Nash Sutcliffe efficiency
fprintf('Nash sutcliffe efficiency for channel No. 17 using Jones formula is
%.3f\n', NSE)

```

## APPENDIX-IV

```
% Mat Lab: Modified Jones for reproducing discharge hydrograph using stage
data for channel no.17
% Units are in SI-Unit
clear all
clc
yo= 1.534168;
n=0.04;
B=100;
So=0.0004;
dT=60;
yp=12;
tp=36000;
t=60:60:133200;
sum1=0;
sum2=0;
maatt =
xlsread('D:\Dissertaio_VIP\Dissertation\hypothetical_data\HEC_RAS.xlsx','HEC
','B3:S2222');
HEC= maatt(:,17);
for i=1:length(t)
    z=yo+(yp-yo)*((t(i)-60)/tp).^(1/0.15)*exp((1-(t(i)-60)/tp)/(0.15));
    y=yo+(yp-yo)*(t(i)/tp).^(1/0.15)*exp((1-t(i)/tp)/(0.15));
    y1=yo+(yp-yo)*((t(i)+60)/tp).^(1/0.15)*exp((1-(t(i)+60)/tp)/(0.15));
    % D1=(y-z)/60; %backward FDM
    D1=(y1-z)/(2*dT); % Central FDM
    A=B*y;
    P=(B+2*y);
    V=1/n*(A/P).^(2/3)*So.^(0.5);
    c=(1+(2/3)*(B/(B+2*y)))*V;
    F=sqrt(V^2/(9.81*y));
    Qo=A*V;
    Q(i)=Qo*sqrt(1+D1/(So*c)*(1-4/9*F^2*(B/P)^2));
    xx=(HEC(i)-Qo)^2;
    sum1=sum1+xx;
    yy=(HEC(i)-Q(i))^2;
    sum2=sum2+yy;
end
modified_jones=Q';
Vr=1/2220*sum2;
Vtm=1/2220*sum1;
NSE=100*(Vtm-Vr)/Vtm; %Nash Sutcliff efficiency
fprintf('Nash sutcliff efficiency for channel 17 using Modified JOnes
formula is %.3f\n', NSE)
```

## APPENDIX-V

```

% Mat Lab: Refined Jones for reproducing discharge hydrograph using stage
data for channel no.17
% Units are in SI-unit
clear all
clc
yo= 1.534168;
n=0.04;
B=100;
So=0.0004;
dT=60;
yp=12;
tp=36000;
t=60:60:133200;
Q0=100;
sum1=0;
sum2=0;
maatt =
xlsread('D:\Dissertaio_VIP\Dissertation\hypothetical_data\HEC_RAS.xlsx','HEC
','B3:S2222');
HEC= maatt(:,17);
for i=1:length(t)
    z=yo+(yp-yo)*((t(i)-60)/tp).^(1/0.15)*exp((1-(t(i)-60)/tp)/(0.15));
    y=yo+(yp-yo)*(t(i)/tp).^(1/0.15)*exp((1-t(i)/tp)/(0.15));
    za=yo+(yp-yo)*((t(i)+60)/tp).^(1/0.15)*exp((1-(t(i)+60)/tp)/(0.15));
%    D1=(y-z)/(2*dT); % backward FDM
    D1=(za-z)/(2*dT); %central FDM
    D2=(za-2*y+z)/3600;
    A=B*y;
    P=(B+2*y);
    V=1/n*(A/P).^(2/3)*So.^(0.5);
    c=(1+(2/3)*(B/(B+2*y)))*V;
    R=A/P;
    Qo=A*V;
    f=@(Q) Q-(Qo/sqrt(2))*(1+D1/(So*c)+sqrt((1+D1/(So*c)).^2-
(2*Q*D2/(B*So.^2*c.^3))))).^0.5;
    Q(i)=fzero(f,Q0);
    Q0=Q(i);
    xx=(HEC(i)-Qo)^2;
    sum1=sum1+xx;
    yy=(HEC(i)-Q(i))^2;
    sum2=sum2+yy;
    [i, t(i)]
end
refined_jones=Q';
Vr=1/2220*sum2;
Vtm=1/2220*sum1;
NSE=100*(Vtm-Vr)/Vtm; %Nash Sutcliffe efficiency
fprintf('Nash sutcliffe efficiency forchannel no 17 using Refined Jones
formula is %.3f\n', NSE)

```

## APPENDIX-VI

```

% Mat Lab: Iterative Jones for reproducing discharge hydrograph using stage
data for channel no.17
% Units are in SI-Unit.
clear all
clc
yo= 1.534168; % initial stage w.r.t initial flow of 100 Cumec.
n=0.04;      % Manning's roughness coefficient
B=100;
So=0.0004;  % Channel bed slope
dT=60;
yp=12;      % Peak stage of the stage hydrograph
tp=36000;   % time to peak
Q1=100.0018485; % approximaate Q at beginning
Q0=100.0018485; % initial fzero
t=60:60:133200; % time interval
sum1=0;
sum2=0;
maatt =
xlsread('D:\Dissertaio_VIP\Dissertation\hypothetical_data\HEC_RAS.xlsx','HEC
','B3:S2222'); % data source
HEC= maatt(:,17); % HEC-RAS solution
for i=1:length(t)
    z=yo+(yp-yo)*((t(i)-60)/tp).^(1/0.15)*exp(-(t(i)-60)/tp)/(0.15)); %
stage at time t-dt
    y=yo+(yp-yo)*(t(i)/tp).^(1/0.15)*exp(-(t(i)/tp)/(0.15)); %stage at
time t
    y1=yo+(yp-yo)*((t(i)+60)/tp).^(1/0.15)*exp(-(t(i)+60)/tp)/(0.15));
%stage at time t+dt
    D1=(y-z)/60; % backward finite difference method
    % D1=(y1-z)/(2*dT);% central finite difference method
    A=B*y;
    P=(B+2*y);
    dpdy=((B+2*y)-(B+2*z))/(y-z);
    V=1/n*(A/P).^(2/3)*So.^(0.5);
    c=(1+(2/3)*(B/(B+2*y)))*V;
    R=A/P;
    Qo=A*V;
    f=@(Q) Q-Qo*sqrt(1+A*D1/(So*Q*(5/3-(2/3)*dpdy*R/B));
    Q(i)=fzero(f,Q0);
    Q1 = Q(i);
    Q0=Q(i);
    xx=(HEC(i)-Qo)^2;
    sum1=sum1+xx;
    yy=(HEC(i)-Q(i))^2;
    sum2=sum2+yy;
[i, t(i)]
end
Iterative_Jones=Q';
Vr=1/2220*sum2;
Vtm=1/2220*sum1;
NSE=100*(Vtm-Vr)/Vtm; %Nash Sutcliffe efficiency
fprintf('Nash sutcliffe efficiency for the channel no. 17 using Iterative
Jones formula is %.3f\n', NSE)

```

## APPENDIX-VII

```

% Mat Lab: Fread method for reproducing discharge hydrograph using stage
% data for channel no.17. Units are inn SI-unit
clear all
clc
yo= 1.534168;
n=0.04;
B=100;
So=0.0004;
tp=565*60;
Qp=2888.25; %peak discharge
Q_o=100.25; % initial discharge
dT=60;
yp=12;
tp=36000;
Q1=100.0018485;
Q0=100.0018485;
t=60:60:133200;
maatt =
xlsread('D:\Dissertaio_VIP\Dissertation\hypothetical_data\HEC_RAS.xlsx','HEC
','B3:S2222');
HEC= maatt(:,17);
sum1=0;
sum2=0;
for i=1:length(t)
    y=yo+(yp-yo)*(t(i)/tp)^(1/0.15)*exp((1-t(i)/tp)/(0.15));
    z=yo+(yp-yo)*((t(i)-60)/tp)^(1/0.15)*exp((1-(t(i)-60)/tp)/(0.15));
    y1=yo+(yp-yo)*((t(i)+60)/tp)^(1/0.15)*exp((1-(t(i)+60)/tp)/(0.15));
    dHs=(y-z)/60 ; %Backward FDM
    dHs=(y1-z)/(2*60) ; %Central FDM
    A=B*y;
    K=(5/3)-(4/3)*(y/(B+2*y));
    r=So*(Qp+Q_o)*(tp)*K/((12^2-yo^2)*B); %ratio of channel slope to avg.
    wave slope
    P=(B+2*y);
    R=A/P;
    A1=B*z;
    Qo=A*1/n*R^(2/3)*So^0.5;
    f=@(Q) Q-A*R^(2/3)/n*sqrt(So+dHs*(A/(K*Q)+...
        (1-1/K)*(B*Q/(9.81*A.^2)))+(Q1/A1-Q/A)/(9.81*dT)+...
        (2*So)/(3*(r.^2))*(1-B*(Q.^2)/(9.81*A.^3.0)));
    Q(i)=fzero(f,Q0); %discharge w.r.t stage hydrograph using FREAD(1975)
    Q1 = Q(i);
    Q0=Q(i);
    xx=(HEC(i)-Qo)^2;
    sum1=sum1+xx;
    yy=(HEC(i)-Q(i))^2;
    sum2=sum2+yy;
[i, t(i)]
end
Fread=Q';
Vr=1/2220*sum2;
Vtm=1/2220*sum1;
NSE=100*(Vtm-Vr)/Vtm; %Nash Sutcliff efficiency
fprintf('Nash sutcliff efficiency for channel NO.17 using Fread formula is
%.3f\n',NSE)

```

## APPENDIX-VIII

---

```
% Mat Lab code: Jones formula for producing discharge hydrograph at Georgia
Highway-141
%Units are in US-Unit
clear all
clc;
So=0.00031;
mat = xlsread('D:\Dissertaio_VIP\Dissertation\buford-image\GH-
141.xlsx','Vip','B2:O145');
y = mat(:,1); % observed stages
A = mat(:,2); % observed area w.r.t stage
B = mat(:,3); % width of the river w.r.t stage
R = mat(:,4); % hydraulic radius w.r.t stage
n = mat(:,11); % roughness n=1.05*n_Fread
% n = mat(:,12); % n=n_y
% n = mat(:,13); % n=n_Q
% n = mat(:,14); % n=n_Fread
dp_dy_b = mat(:,5);
dT=5*60;
for i=2:length(mat)-1
    D1=(y(i)-y(i-1))/(dT); % backward FDM
%    D1=(y(i+1)-y(i-1))/(2*dT); %% central FDM
    V=(1.49/n(i))*R(i)^(2/3)*So^(0.5);
    c=V*(1+(2/3)*(1-(R(i)/B(i))*dp_dy_b(i)));
    Qo=A(i)*V;
    Q(i)=Qo*sqrt(1+D1/(So*c));

    i
end
Jones_GH141=Q';
```



## APPENDIX-IX

```

% Mat Lab code: Iterative Jones formula for producing discharge hydrograph
at Georgia Highway-141
% Units are in US-unit
clear all
clc;
So=0.00031;
mat = xlsread('D:\Dissertaio_VIP\Dissertation\buford-image\GH-
141.xlsx','Vip','B2:P144');
y = mat(:,1);
A = mat(:,2);
B = mat(:,3);
R = mat(:,4);
n = mat(:,11);          % n=1.05*n_Fread
% n = mat(:,12);        % n=n_y
% n = mat(:,13);        % n=n_Q
% n = mat(:,14);        % n=n_Fread
dp_dy_b = mat(:,5);
dT=5*60;
Q0=1500;
Qobs= mat(:,15);
sum1=0;
sum2=0;
for i=2:length(mat)-1
    D1=(y(i)-y(i-1))/(dT); % backward FDM
%    D1=(y(i+1)-y(i-1))/(2*dT); %% central FDM
    V=(1.49/n(i))*R(i)^(2/3)*So^(0.5);
    c=V*(1+(2/3)*(1-(R(i)/B(i))*dp_dy_b(i)));
    Qo=A(i)*V;
    f=@(Q) Q-Qo*sqrt(1+A(i)*D1/(So*Q*(5/3-(2/3)*dp_dy_b(i)*R(i)/B(i))));
    Q(i)=fzero(f,Q0);
    Q0=Q(i);
    Q_mean=mean(Qobs);
    xx=(Qobs(i)-Q_mean)^2;
    sum1=sum1+xx;

    yy=(Qobs(i)-Q(i))^2;
    sum2=sum2+yy;
    i
end
computed_Jone=Q';
Vr=1/142*sum2;
Vtm=1/142*sum1;
NSE=100*(Vtm-Vr)/Vtm; %%Vem=(Vtm-Vr)/Vtm;
fprintf('Nash sutcliff efficiency for the channel at GH-141 using Iterative
Jones is %.3f\n', NSE)

```

## APPENDIX-X

---

```
% Mat Lab code: Fread formula for producing discharge hydrograph at Georgia
Highway-141
% Units are in US-unit
clear all
clc;
So=0.00031;
mat = xlsread('D:\Dissertaio_VIP\Dissertation\buford-image\GH-
141.xlsx', 'Vip', 'B2:O145');
y = mat(:,1);
A = mat(:,2);
B = mat(:,3);
R = mat(:,4);
n = mat(:,11);           % n=1.05*n_Fread
% n = mat(:,12);         % n=n_y
% n = mat(:,13);         % n=n_Q
% n = mat(:,14);         % n=n_Fread
dp_dy_b = mat(:,5);
dT=5*60;
Q0=1490;
Q1=1490;
dT=5*60;
for i=2:length(mat)-1
    D1=(y(i)-y(i-1))/(dT); % backward FDM
    % D1=(y(i+1)-y(i-1))/(2*dT); %% central FDM
    A1=A(i-1);
    K=5/3-(2/3)*(R(i)/B(i))*dp_dy_b(i);

    f=@(Q) Q-1.486*A(i)*R(i)^(2/3)/n(i)*sqrt(So+D1*(A(i)/(K*Q)+(1-
1/K)*(B(i)*Q/(32.19*A(i).^2)))+(Q1/A1-Q/A(i))/(32.19*dT));
    Q(i)=fzero(f,Q0);
    Q1 = Q(i)
    Q0=Q(i)
    i
end
Fread=Q';
```

## APPENDIX-XI

```
% refined Jones with Finite difference method at Georgia Highway-141
% Units are in US-unit
clear all
clc;
So=0.00031;
mat = xlsread('D:\Dissertaio_VIP\Dissertation\buford-image\GH-
141.xlsx','Vip','B2:O145');
y = mat(:,1);
A = mat(:,2);
B = mat(:,3);
R = mat(:,4);
n = mat(:,11);           % n=1.05*n_Fread
% n = mat(:,12);         % n=n_y
% n = mat(:,13);         % n=n_Q
% n = mat(:,14);         % n=n_Fread
dp_dy_b = mat(:,5);
dT=5*60;
Q0=1490;
for i=2:length(mat)-1
    D1=(y(i)-y(i-1))/(dT); % backward FDM
    % D1=(y(i+1)-y(i-1))/(2*dT); %% central FDM
    D2=(y(i+1)-2*y(i)+y(i-1))/(dT*dT);
    V_J=(1.49/n(i))*R(i)^(2/3)*So^(0.5);
    c_J=V_J*(1+(2/3)*(1-(R(i)/B(i))*dp_dy_b(i)));

    Qo=A(i)*V_J;
    Q_J=Qo*sqrt(1+D1/(So*c_J));
    V=Q_J/A(i);

    aa=1+D1/(So*c_J);
    bb=(2*D2/(B(i)*So^2*c_J.^3));

    Q(i)=(Qo/sqrt(2))*sqrt(aa+sqrt(aa.^2-(Q_J*bb)));
    Q(imag(Q) ~= 0) = NaN;
    i
end

Jones03=Q';
```

## APPENDIX-XII

---

```
% Matlab code: Modified Jones formula for producing discharge hydrograph at
Georgia Highway-141
clear all
clc;
So=0.00031;
mat = xlsread('D:\Dissertaio_VIP\Dissertation\buford-image\GH-
141.xlsx','Vip','B2:O145');
y = mat(:,1);
A = mat(:,2);
B = mat(:,3);
R = mat(:,4);
% n = mat(:,11);           % n=1.05*n_Fread
n = mat(:,12);           % n=n_y
% n = mat(:,13);           % n=n_Q
% n = mat(:,14);           % n=n_Fread
dp_dy_b = mat(:,5);
dT=5*60;
Q0=1500;
for i=2:length(mat)-1
%   D1=(y(i+1)-y(i-1))/(2*dT);
   D1=(y(i)-y(i-1))/(dT);
   V=(1.49/n(i))*R(i)^(2/3)*So^(0.5);
   c=V*(1+(2/3)*(1-(R(i)/B(i))*dp_dy_b(i)));
   K=1-(R(i)/B(i))*dp_dy_b(i);
   Qo=A(i)*V;
   F=sqrt(V^2/(9.81*y(i)));
   Q(i)=Qo*sqrt(1+D1/(So*c)*(1-4/9*F^2*(K.^2)));

   i
end
Modified_J=Q';
```

## APPENDIX-XIII

```
% Mat Lab Code: Jones formula for reproducing discharge hydrograph at
Bhadrachalam for event 2005_ev01.
% Units are in SI-unit
clear all
clc;
So=0.00034; % channel bed slope
mat = xlsread('D:\Dissertaio_VIP\Dissertation\bhadrachalam\New
folder\Book1.xlsx','2005_ev1','A2:H194');
y = mat(:,2); % stage
A = mat(:,3); % area w.r.t stage
B = mat(:,4); % width w.r.t to stage
P=mat(:,5); % wetted perimeter w.r.t stage
dp_dy_b = mat(:,6); % dp/dy
R = mat(:,7); % hydraulic radius
n=0.032 % Manning'n roughness coefficient
dT=60*60; % time interval
Qobs= mat(:,8); % observed discharge
sum1=0;
sum2=0;
for i=2:length(mat)-1
% D1=(y(i)-y(i-1))/(dT); % backward FDM
D1=(y(i+1)-y(i-1))/(2*dT); % central FDM
V=(1/n)*R(i)^(2/3)*So^(0.5);
c=V*(1+(2/3)*(1-(R(i)/B(i))*dp_dy_b(i)));
Qo=A(i)*V;
Q(i)=Qo*sqrt(1+D1/(So*c));
Q_mean=mean(Qobs);
xx=(Qobs(i)-Q_mean)^2;
sum1=sum1+xx;
yy=(Qobs(i)-Q(i))^2;
sum2=sum2+yy;
i
end
Jones=Q';
Vr=1/192*sum2;
Vtm=1/192*sum1;
NSE=100*(Vtm-Vr)/Vtm; %Nash Sutcliff efficiency
fprintf('Nash sutcliff efficiency for Bhadrachalam during 2005_ev1 using
Jones formula is %.3f\n', NSE)
```

## APPENDIX-XIV

```
% Mat Lab Code: Iterative Jones formula for reproducing discharge hydrograph
at Bhadrachalam for event 2005_ev01.
% Units are in SI-unit
clear all
clc;
So=0.00034;
mat = xlsread('D:\Dissertaio_VIP\Dissertation\bhadrachalam\New
folder\Book1.xlsx','2005_ev1','A2:H194');
y = mat(:,2);
A = mat(:,3);
B = mat(:,4);
P=mat(:,5);
dp_dy_b = mat(:,6);
R = mat(:,7);
n=0.032
dT=60*60;
Qobs= mat(:,8);
sum1=0;
sum2=0;
Q0=9400;
for i=2:length(mat)-1
    %D1=(y(i)-y(i-1))/(dT); % backward FDM
    D1=(y(i+1)-y(i-1))/(2*dT); % central FDM
    V=(1/n)*R(i)^(2/3)*So^(0.5);
    c=V*(1+(2/3)*(1-(R(i)/B(i))*dp_dy_b(i)));
    Qo=A(i)*V;
    f=@(Q) Q-Qo*sqrt(1+A(i)*D1/(So*Q*(5/3-(2/3)*dp_dy_b(i)*R(i)/B(i)));
    Q(i)=fzero(f,Q0);
    Q0=Q(i);
    Q_mean=mean(Qobs);
    xx=(Qobs(i)-Q_mean)^2;
    sum1=sum1+xx;
    yy=(Qobs(i)-Q(i))^2;
    sum2=sum2+yy;
i
end
Q=Q';
Vr=1/192*sum2;
Vtm=1/192*sum1;
NSE=100*(Vtm-Vr)/Vtm; %Nash Sutcliff efficiency
fprintf('Nash sutcliff efficiency for Bhadrachalam during 2005_ev1 using
Iterative Jones is %.3f\n', NSE)
```

## APPENDIX-XV

```

% Mat Lab Code: Refined Jones formula for reproducing discharge hydrograph
at Bhadrachalam for event 2005_ev01.
% Units are in SI-unit
clear all
clc;
So=0.00034;
mat = xlsread('D:\Dissertaio_VIP\Dissertation\bhadrachalam\New
folder\Book1.xlsx','2005_ev1','A2:H194');
y = mat(:,2);
A = mat(:,3);
B = mat(:,4);
P=mat(:,5);
dp_dy_b = mat(:,6);
R = mat(:,7);
n=0.032
dT=60*60;
Qobs= mat(:,8);
sum1=0;
sum2=0;
Q0=9400;
for i=2:length(mat)-1
    %D1=(y(i)-y(i-1))/(dT); % backward FDM
    D1=(y(i+1)-y(i-1))/(2*dT); % central FDM
    V=(1/n)*R(i)^(2/3)*So^(0.5);
    c=V*(1+(2/3)*(1-(R(i)/B(i))*dp_dy_b(i)));
    Qo=A(i)*V;
    D2=(y(i+1)-2*y(i)+y(i))/(dT*dT);
    f=@(Q) Q-(Qo/sqrt(2))*(1+D1/(So*c)+sqrt((1+D1/(So*c)).^2-
(2*Q*D2/(B(i)*So.^2*c.^3))))).^0.5;
    Q(i)=fzero(f,Q0);
    Q0=Q(i);
    Q_mean=mean(Qobs);
    xx=(Qobs(i)-Q_mean)^2;
    sum1=sum1+xx;
    yy=(Qobs(i)-Q(i))^2;
    sum2=sum2+yy;
    i
end
Q=Q';
Vr=1/192*sum2;
Vtm=1/192*sum1;
NSE=100*(Vtm-Vr)/Vtm; %Nash Sutcliff efficiency
fprintf('Nash sutcliff efficiency for Bhadrachalam during 2005_ev1 using
Refined Jones formula is %.3f\n', NSE)

```

## APPENDIX-XVI

```
% Mat Lab Code: Modified Jones formula for reproducing discharge hydrograph
at Bhadrachalam for event 2005_ev01.
% Units are in SI-unit
clear all
clc;
So=0.00034; % channel bed slope
mat = xlsread('D:\Dissertaio_VIP\Dissertation\bhadrachalam\New
folder\Book1.xlsx','2005_ev1','A2:H194');
y = mat(:,2); % stage
A = mat(:,3); % area w.r.t stage
B = mat(:,4); % width w.r.t to stage
P=mat(:,5); % wetted perimeter w.r.t stage
dp_dy_b = mat(:,6); % dp/dy
R = mat(:,7); % hydraulic radius
n=0.032 % Manning'n roughness coefficient
dT=60*60; % time interval
Qobs= mat(:,8); % observed discharge
sum1=0;
sum2=0;
for i=2:length(mat)-1
% D1=(y(i)-y(i-1))/(dT); % backward FDM
D1=(y(i+1)-y(i-1))/(2*dT); % central FDM
V=(1/n)*R(i)^(2/3)*So^(0.5);
c=V*(1+(2/3)*(1-(R(i)/B(i))*dp_dy_b(i)));
K=1-(R(i)/B(i))*dp_dy_b(i);
Qo=A(i)*V;
F=sqrt(V^2/(9.81*y(i)));
Q(i)=Qo*sqrt(1+D1/(So*c)*(1-4/9*F^2*(K.^2)));
Q_mean=mean(Qobs);
xx=(Qobs(i)-Q_mean)^2;
sum1=sum1+xx;
yy=(Qobs(i)-Q(i))^2;
sum2=sum2+yy;
i
end
Jones=Q';
Vr=1/192*sum2;
Vtm=1/192*sum1;
NSE=100*(Vtm-Vr)/Vtm; %Nash Sutcliffe efficiency
fprintf('Nash sutcliffe efficiency for Bhadrachalam during 2005_ev1 using
Modified Jones formula is %.3f\n', NSE)
```



## APPENDIX-XVII

```

% Mat Lab Code: Fread formula for reproducing discharge hydrograph at
Bhadrachalam for event 2005_ev01.
% Units are in SI-unit
clear all
clc;
So=0.00034;
mat = xlsread('D:\Dissertaio_VIP\Dissertation\bhadrachalam\New
folder\Book1.xlsx', '2005_ev1', 'A2:H194');
y = mat(:,2);
A = mat(:,3);
B = mat(:,4);
P=mat(:,5);
dp_dy_b = mat(:,6);
R = mat(:,7);
n=0.032
dT=60*60;
Qobs= mat(:,8);
sum1=0;
sum2=0;
Q0=9546.49716021511;
Q1=9546.49716021511;
for i=2:length(mat)-1
%     D1=(y(i)-y(i-1))/(dT); % backward FDM
    D1=(y(i+1)-y(i-1))/(2*dT); % central FDM
    V=(1/n)*R(i)^(2/3)*So^(0.5);
    c=V*(1+(2/3)*(1-(R(i)/B(i))*dp_dy_b(i)));
    Qo=A(i)*V;
    A1=A(i-1);

    K=5/3-(2/3)*(R(i)/B(i))*dp_dy_b(i);

    f=@(Q) Q-A(i)*R(i)^(2/3)/n*sqrt(So+D1*(A(i)/(K*Q)+(1-
1/K)*(B(i)*Q/(9.81*A(i).^2)))+(Q1/A1-Q/A(i))/(9.81*dT));
    Q(i)=fzero(f,Q0);
    Q1 =Q(i);
    Q0=Q(i);
    Q_mean=mean(Qobs);
    xx=(Qobs(i)-Q_mean)^2;
    sum1=sum1+xx;
    yy=(Qobs(i)-Q(i))^2;
    sum2=sum2+yy;
    i
end
Q=Q';
Vr=1/192*sum2;
Vtm=1/192*sum1;
NSE=100*(Vtm-Vr)/Vtm; %Nash Sutcliff efficiency
fprintf('Nash sutcliff efficiency for Bhadrachalam during 2005_ev1 using
Fread is %.3f\n', NSE)

```