

# PARAMETRIC STUDIES OF SEISMIC GROUND RESPONSE USING NONLINEAR ANALYSIS

**A DISSERTATION**

*Submitted in partial fulfilment of the  
requirements for the award of the degree*

*of*

**MASTER OF ENGINEERING**

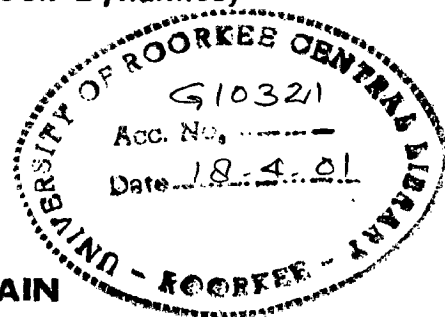
*in*

**EARTHQUAKE ENGINEERING**

**(With Specialization in Soil Dynamics)**

*By*

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**FEBRUARY, 2001**

# CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in this dissertation entitled, **“Parametric Studies of Seismic Ground Response using Nonlinear Analysis”** in partial fulfillment of the requirements for the award of the degree of MASTER OF ENGINEERING in EARTHQUAKE ENGINEERING with specialization in SOIL DYNAMICS, submitted in the Department of Earthquake Engineering, University of Roorkee, Roorkee, is an authentic record of my own work carried out for a period of about six months from August, 2000 to January 2001 under the guidance of Dr. V.H. Joshi, Professor, Department of Earthquake Engineering, University of Roorkee, Roorkee, India.

The matter embodied in this dissertation has not been submitted by me for award of any other degree or diploma.

Dated: February 28, 2001

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## CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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# ABSTRACT

One part of the dynamic analysis is determination of spatial/temporal distribution of seismic ground response along the boundaries chosen for the system that include the structure and the surrounding medium. Seismic ground motion records indicate that spatial variations in ground motions can be significant, especially when the structure has a considerable length. Some structure like dam, bridge, nuclear power plant and under ground structures are critical from an overall viewpoint and the provision of relief services after earthquakes. Thus it is very important to determine as precise as possible the spatial/temporal distribution of the seismic ground response.

The work described herein may be divided into two major parts : the determination of spatial/temporal distribution of seismic ground response; and the parametric studies. For the first part, a computer program was developed for linear as well as nonlinear analysis, using principles of wave propagation. Effect of inclined propagation of seismic wave and for the nonlinear analysis, dynamic properties of the soil as a function of time was taken into account. In case of absence of experimental data, a stress-strain relationship has also been proposed.

For the presentation of capability of software the effect of magnitude and period of input sinusoidal excitation (at base layer level) on a particular five layer system has been studied.

## LIST OF SYMBOLS

- $D$  : Downward propagation shear wave
- $Db_i(t)$  : Component response at the bottom of  $i^{\text{th}}$  layer due to downward propagating wave at time  $t$
- $Dt_i(t)$  : Component response at the top of  $i^{\text{th}}$  layer due to downward propagating wave at time  $t$
- $G$  : Modulus of rigidity of the surface layer
- $G_i$  : Modulus of rigidity for  $i^{\text{th}}$  layer
- $H$  : Thickness of surface layer
- $H_i$  : Thickness of  $i^{\text{th}}$  sub-surface layer
- $H_{\text{eq}}$  : Equivalent thickness of surface layer
- $K_i$  : Impedance ratio of  $i^{\text{th}}$  layer
- $Rd_i$  : Coefficient of downward reflection at  $i^{\text{th}}$  interface
- $Ru_i$  : Coefficient of upward reflection at  $i^{\text{th}}$  interface
- $t$  : Time station
- $T$  : Period of component vibration of shear wave
- $Td_i$  : Coefficient of downward transmission
- $Tg$  : Predominant period of surface layer
- $Tg_{\text{eq}}$  : Equivalent predominant period of surface layer
- $Tg_{\text{equi}}$  : Period of vibration of equivalent layer system
- $Tu_i$  : Coefficient of upward transmission
- $U$  : Upward propagating shear wave
- $Ub_i(t)$  : Component response at the bottom of  $i^{\text{th}}$  layer due to upward propagating wave at time  $t$

$U_i(t)$  : Component response at the top of  $i^{\text{th}}$  layer due to upward propagating wave at time  $t$

$u$  : Response at any point

$V_{s_s}$  : Shear wave velocity in surface layer

$V_{s_{si}}$  : Shear wave velocity in  $i^{\text{th}}$  sub-surface layer

$V_{s_b}$  : Shear wave velocity in base layer

$V_s$  : Shear wave velocity

$V_{s_{eq}}$  : Equivalent shear wave velocity in surface layer

$\Delta t$  : The time difference between two consecutive time station

$\theta$  : Angle of incidence

$[DVHJ1]\mu$  : Amplification factor

$\phi$  : Angle of shearing resistance

$\gamma$  : Shear Strain

$\gamma_o$  : Shear Strain at failure of soil

$\gamma_p$  : Shear Strain at junction of 3 deg. curve and ellipse

$\gamma_{i(t)}$  : Strain induced in  $i^{\text{th}}$  sub-surface layer at  $nt$

$\rho$  : Mass density

$\rho_s$  : Mass density of base layer

$\rho_s$  : Mass density of surface layer

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## INTRODUCTION

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### 1.1 Preamble

Earthquakes are one of the many natural disasters against which it is necessary to design engineering structures to minimize the damage caused by earthquakes. The study of earthquake-induced damages indicates that the local geology as well as surface formations have strong influence on the seismic ground response and affect significantly the damage pattern.

When earthquake occur, P-waves and S-waves are set into motion of which S-waves are much important and cause damage to structure. These wave when incidence at the interface of layered media and especially at ground level, generate surface waves. R-waves are the most important in this category. When shear waves are incident at interface of two media, it is partially reflected back and partially transmitted. When seismic wave propagate from the source, it originate from a considerable depth (many km) below the ground level. In this process, it passes through base rock layer and progressively encounters after formation like soil as especially near the ground level. A incidence wave is transmitted from a denser medium to rare medium is more close to the normal to the interface of incidence and when it arrives at the ground level it propagate nearly in the vertical direction making very small angle with the vertical. Hence, vertical propagation of shear wave is convenient and useful assumption in computation of seismic ground response.

However, it is important to consider propagation of waves making an angle with the vertical for dynamic analysis of structure such as water, sewer, oil and gas pipelines, transmission cable conduits, irrigation, power and transportation tunnels, mining shafts and

power plants which involves horizontal directions. Many of these important facilities are termed as lifeline systems. Some of these facilities such as nuclear power plant components are critical to overall safety. For these reasons, reliable procedures for the analysis and design of such structures are very important.

One part of the dynamic analysis is determination of spatial/temporal distribution of seismic ground response along the boundaries chosen for the system that include the structure and the surrounding medium. Seismic ground motion records indicate that spatial variations in ground motions can be significant, especially when the structure has a considerable length (Nair,1974). In general seismic waves propagate through the ground with direction of propagation at some angle with the vertical. As such, response should be treated as a three dimensional quantity (Suzuki, 1932). Soil is a nonlinear material and it has a very low proportionality limit. Therefore, earthquake induced strains may easily exceed proportional limit of such materials. In such cases, dynamic properties do not remain constant. So, assumption of constant material properties always leads, to less precise response and extra safe design of structures. Thus for precise determination of ground response, nonlinear analysis should be carried out.

The method of analysis using shear wave propagation is well known. When linear material properties are used, the method of analysis is very simple. Many investigators have carried out research investigations in this field.

When the response is known at the ground level, it is possible to obtain the component response of upward and downward propagating shear wave. However such a determination of these component responses is not possible when response is not known at base rock level. Joshi(1980) proposed a method for obtaining such component responses at

the top of base rock level. He assumed the base rock formation extended upto infinite depth below so that the reflected wave propagating in the downward direction has no chance of returning back to the base rock level considering within the time duration of the proposed analysis.

Using this proposition Srivastava(1995), proposed method of analysis considering non-linear stress strain characteristics of the soil. However, he did not consider the presence of ground water table and the initial range of elastic behavior of soil over range of strain level. Besides the nonlinear property of any soil layer of the layer system considered to be the same for the entire thin sublayer within that soil layer. This is incorrect, because, the shear modulus increases with increasing octahedral stress due to increasing depth below the ground level within the layer. Hence, it is desirable to consider a separate nonlinear stress-strain relationship for each sub layer.

From the above discussion it is clear that there is need for developing a new method of analysis, which is free from above sighted shortcomings.

## **2.2 The Objective and Scope of the Proposed Investigation**

The objective of this investigation is to obtain spatial/temporal distribution of seismic ground motion by using shear wave propagation in vertical/inclined direction.

The linear/nonlinear behaviour of soil is also considered. The analysis is performed in time domain. A suitable computer program has been developed for this purpose. To account for the nonlinear behaviour of soil, at every time station of the analysis the displacement are computed at every interface of the layered system by double integration of

acceleration using Willson  $\theta$  method. Wilson  $\theta$  method gives unconditionally stable results. The layered system is sub divided into many thin layers so that the strain with in each sublayer may be considered reasonably uniform. The nonlinear stress strain relationship for the soil may be obtain from suitable laboratory test data or may be assumed to be represented by a suitable mathematical function. The analysis in the time domain is carried out at very small interval of time. So that properties of the soil may be considered to remain the same with in this small time interval. Using this strain dependent shear modulus, velocity of shear wave propagation is computed at each time station. Based on these velocities, various reflection and transmission coefficient at each interface are revised at every time station. The seismic excitation is assumed to be known at the base rock/firm ground level. Even though it is possible to consider it at any other place.

In addition to the development of computer program, it is aimed to compare the results of linear and nonlinear analysis and to study the effect of excitation and period of excitation on induced strain and acceleration response. It is proposed to carry out parametric studies using this program to study the influence of various parameters on the seismic ground response.

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# LITERATURE REVIEW

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### 2.1 Preamble

Although quite a number of studies have been reported on linear analysis of seismic ground response, the nonlinear analysis has not yet received much attention until recently. This is largely due to huge computational effort required in considering time dependent material properties. The advents of digital computers have enabled to use the more precise technique in the analysis of seismic ground response.

A brief review of the state of the art for linear and nonlinear analysis is presented here. A more comprehensive review can be found elsewhere (Joshi, 1980).

### 2.2 Period of Ground and Amplification of Ground Response

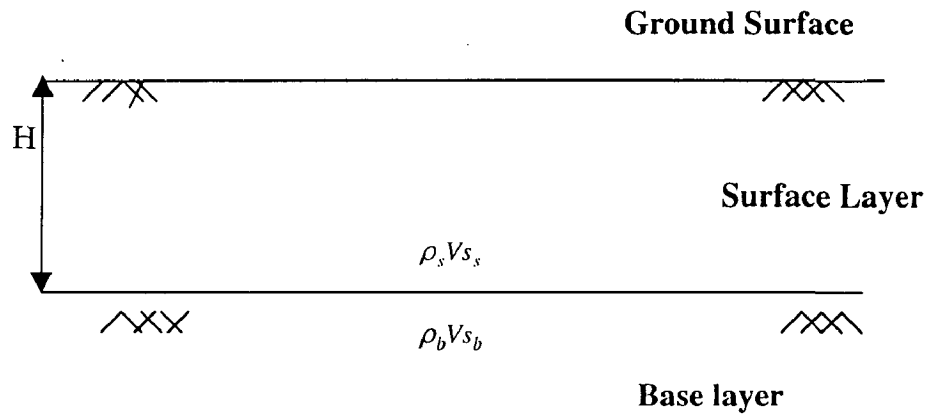
#### Ground Amplification Factor

Ground amplification factor is defined as the ratio of maximum response at any point to the maximum response of base excitation for a given layered system. The amplification factor, computed for ground level is generally of great interest.

For alluvial soil deposits, velocities of seismic wave for surface layer are lower compared to those for layer below. Shear wave travel nearly vertical near ground level and a multi-reflection phenomenon will occur in the surface layers. As a result, ground vibrates appreciably, with the appearance of certain dominant period, which is called as predominant



period of the ground. This is depending on the structure of the surface layer and it's material properties.



**Figure 2.2.1 Single Layer System over Base Layer System**

Let  $T$  be the period of component vibration of shear wave,  $\rho_s$  &  $\rho_b$  the velocity  $V_{s_s}$  &  $V_{s_b}$  the velocity of shear wave in surface layer and base layer. For a single surface layer system (Fig. 2.2.1) of predominant period,  $T_g$ , and thickness,  $H$ , Kanai (1951) recommended an empirical formula cited below for amplification factor,  $\mu$ , by combining the results of actual measurements with theoretical calculation :

$$\mu = 1 + \frac{1}{\sqrt{\left[ \frac{0.3 T}{\sqrt{T_g} T_g} \right]^2 + \left[ \frac{1+k}{1-k} \left\{ 1 - \left( \frac{T}{T_g} \right)^2 \right\} \right]^2}} \quad \dots(2.2.1)$$

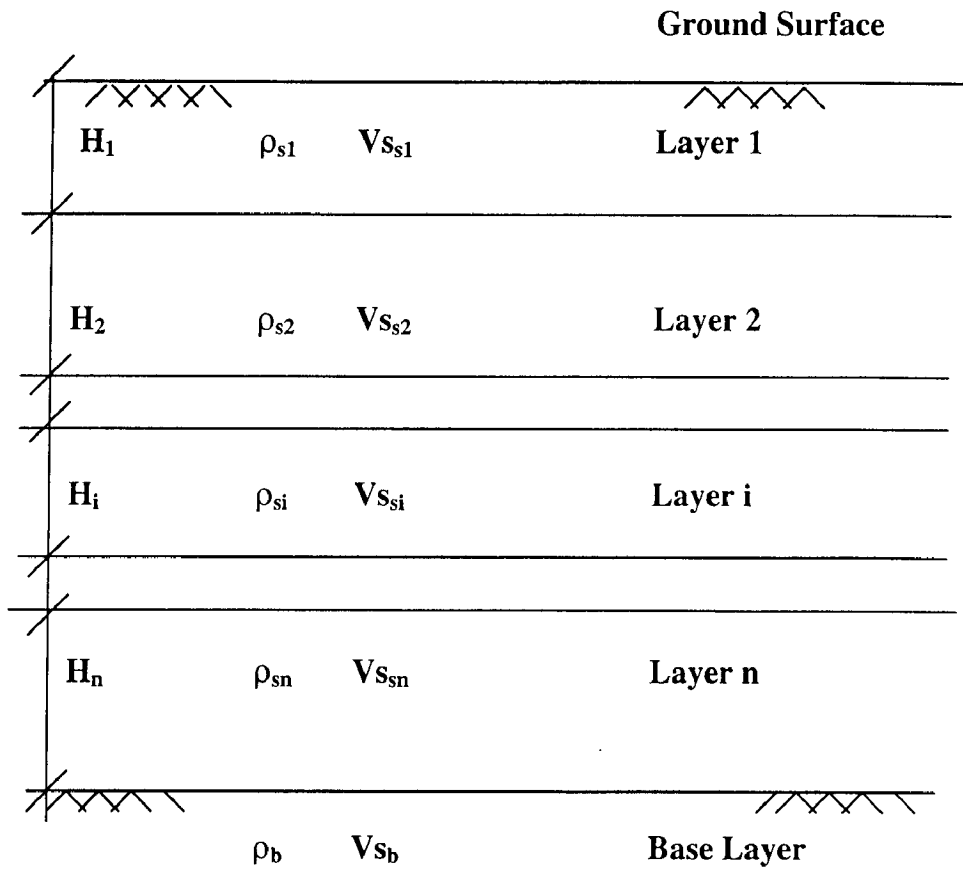
where,  $k = \frac{\rho_s V_{s_s}}{\rho_b V_{s_b}} = \text{impedance ratio}$  and  $\dots(2.2.2)$

$$T_g = \frac{4H}{v_{s_s}} = \text{fundamental period of vibration of surface layer.} \quad \dots(2.2.3)$$

After observing about fifty cases, Suzuki (1932) concluded that the angle of incidence of shear waves at Hongo was about  $4^\circ$ , Ocamoto (1973) has given a table in which the value of amplification are given for various  $T/T_g$  ratios.

Equation 2.2.1 is valid for single layer resting on base layer. However in reality a system of layers rest on base layer for which above equation is not directly applicable. For such a case it is necessary to workout a single layer which is equal in depth to the system of layers under consideration and having equivalent characteristics. Kanai proposed that the depth of equivalent single layer,  $H_{eq}$  is given by:

$$H_{eq} = \sum_{i=1}^n H_i \quad \dots(2.2.4)$$



**Figure 2.2.2 Multi Layer System over Base Layer System**

Similarly, he proposed the expression for equivalent shear wave velocity,  $V_{s_{eq}}$ , and equivalent unit weight,  $\rho_{eq}$ , for a equivalent single layer are given as:

$$V_{s_{eq}} = \frac{\left[ \sum_{i=1}^n V_{s_{si}} H_i \right]}{\sum_{i=1}^n H_i} \quad \dots(2.2.5)$$

$$\rho_{eq} = \frac{\left[ \sum_{i=1}^n \rho_i H_i \right]}{\sum_{i=1}^n H_i} \quad \dots(2.2.6)$$

Fig. 2.2.2 shows various symbols used in above equation.

It is also know the theory of vibration that if a layer vibrates in the fundamental mode, the wavelength of shear wave is equal to  $4H$  and the fundamental period  $T_g$  is given by

$$T_g = \frac{4H}{V_s}$$

It may be noted here that  $H_{eq}$  is real quantity where as  $V_{seq}$  is mathematical quantity which represents average property of layer system using thickness of layer as weighting function. The degree of error between the  $V_s$  of the layer (in particular strongest and weakest layer) is not readily understandable. The larger the difference between  $V_{s_{max}}$  and  $V_{s_{min}}$  the greater will be discrepancy between those two quantities and  $V_{seq}$ . Therefor, if the error has to be kept low, the difference between  $V_{s_{max}}$  and  $V_{s_{min}}$  should be small. Similarly the layer with smallest thickness become irrelevant. Therefor, if there is a relatively very deep layer with few other layer which are much thinner the error is relatively smaller.

Kanai in his expression for  $T_g$  does not propose to obtain

$$T_g = 4 \frac{H_{eq}}{V_{seq}} \quad \dots(2.2.7)$$

Instead of it, he proposed to use equation

$$T_g = \sum_{i=1}^n T_{gi} = \sum \frac{4H_i}{V_{s_i}} \quad \dots(2.2.8)$$

However, if each layer vibrate in it's fundamental mode simultaneously, which can be assumed together in a continuous mode.

It may be noted that Equation 2.2.8 is a real quantity. Hence, Equation 2.2.8 is not a fictitious quantity like Equation 2.2.7. Hence, this way of computing  $T_g$  proposed by Kanai appear to be more logical than obtain as equation 2.2.7

It has been reported (Okamoto 1973) that for Mexico city, the fundamental period of vibration of layer system at point for the same earthquake actually obtained. The  $T_g$  obtained was 2.63 sec by Okamoto method and 2.12 sec by using equivalent depth and shear wave velocity at one of the two site the fundamental period obtained from record was exactly 2.63 sec. at the other site it was 2.15 sec which was closer to ground vibration obtain by using equivalent depth and shear wave velocity. From the above discussion, it may be noted that both this method was computing  $T_g$  are reasonable. However the Okamoto equation is likely to be more reasonable specially when the discrepancy between shear wave velocity of weaker layer is appreciable different from the strongest layer.

The period of component vibration as seismic wave is best obtained by the study of seismic ground vibration record at or near firm ground. It depends upon material properties and the depth of firm ground from ground level as well as magnitude, focal depth and epicentral distance of earthquake. If focal distance is large, most of the frequency vibration will be absent as they tend to die down very rapidly. If the magnitude of earthquake is large, it has enough energy to excite deeper layer below the ground level which result into predominance of long period vibration. Therefore, for same site, the period of component seismic wave vibration may vary depending upon the magnitude epicentral distance, and

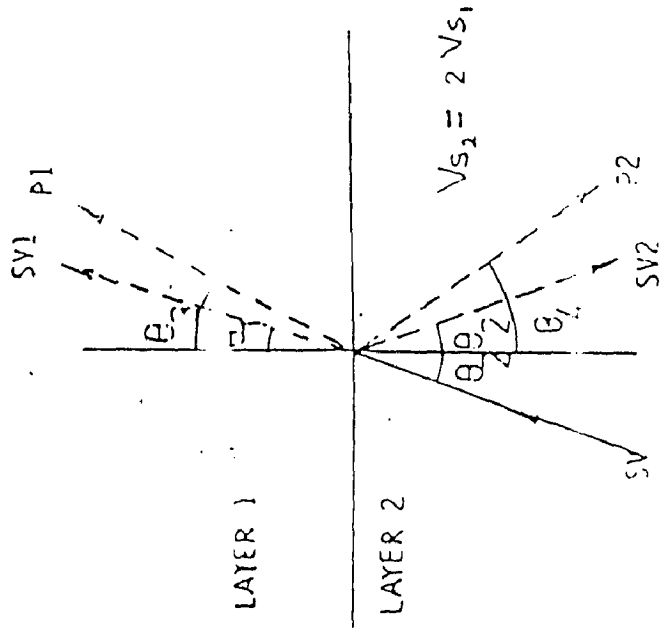
focal depth of the earthquake. All these factors need be considered in estimating the predominant period of shear wave motion at the top of base layer. (Joshi, 1980)

### 2.3 DIRECTION OF PROPAGATION

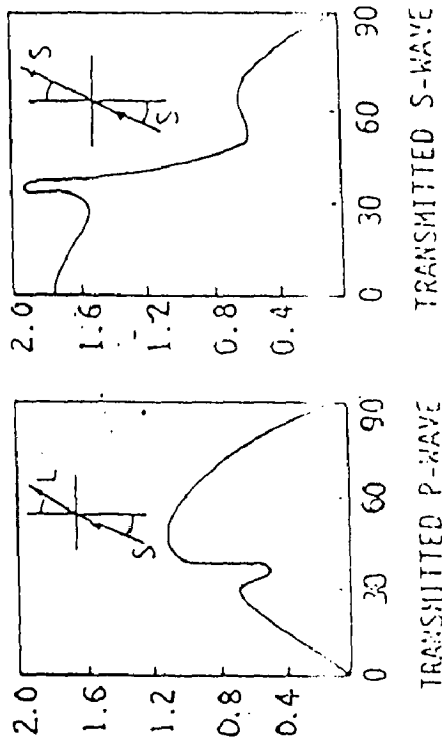
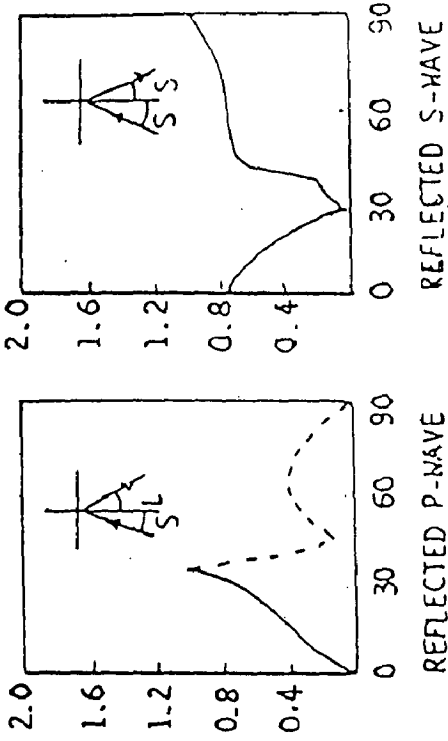
The amount of energy, which reaches the ground surface depends upon the angle of incidence of wave at different interfaces of layers. Zoeppritz (1919) determined the nature of reflected and transmitted waves and the distribution of energy between these layers. He concluded that for two stratified media the amplitude of resultant waves is a function of the incident angle only. Using Snell's law, the equations for incident SV-waves are solved for ratio of resultant wave amplitude in terms of incident angle and a graph is plotted between amplitude ratio and incident angle for  $\rho_1 > \rho_2$  and  $V_{s1} > V_{s2}$  (Fig. 2.3.1) where  $\rho_1$  &  $\rho_2$  are mass density and  $V_{s1}$  &  $V_{s2}$  are shear wave velocity for first layer and second layer respectively. From Fig. (2.3.1), it can be concluded that when angle of incidence is zero, i.e., when shear wave falls normally over the interface, P-waves are not generated. Only reflected and refracted S-waves are generated. For SV-waves, the reflected and refracted amplitudes of P-waves are not significant for small angles of incidence less than  $30^\circ$ . For the same range, the SV-waves are almost completely refracted. For angles greater than  $30^\circ$ , most of the incident wave is reflected downwards. As such, at distant sites, most of the energy incident as SV-wave is directed away from the site. This tends to reduce the amplitudes of seismic ground motion in the formation above.

When the velocity of a reflected or refracted wave is greater than that of the incident wave, there will be critical angle of incidence for which the angle of reflection or refraction will be  $90^\circ$ . For angle of incidence greater than the critical angle a disturbance which decay rapidly with distance from the interface is created, which does not transmit energy away

$$\frac{\sin \theta_1}{V_{s_1}} = \frac{\sin \theta_2}{V_{s_2}} = \frac{\sin \theta_3}{V_{p_1}} = \frac{\sin \theta_4}{V_{p_2}}$$



a. DECOMPOSITION OF SHEAR WAVE AT INTERFACE BETWEEN TWO ELASTIC MEDIA



b. RELATION BETWEEN AMPLITUDE RATIO OF EMITTED WAVES AND INCIDENT ANGLE

Fig.2.3.1 : Decomposition of Shear Waves with Incident Angle

## **2.4 SPATIAL/TEMPORAL DISTRIBUTION OF RESPONSE**

### **Methods of Analysis**

Nair (1974), Kobayashi (1972) and Joshi (1980) gave methods for determination of component response in multi layer system if response at base rock level is known.

#### **Nair's Method (1974)**

Nair assumed that the total response at the base rock level is equal to component response due to the upward propagating wave in base rock layer and in layer immediately above base rock and that there is no downward reflection, downward transmission and upward reflection at the base rock interface.

#### **Kobayashi's Method (1972):**

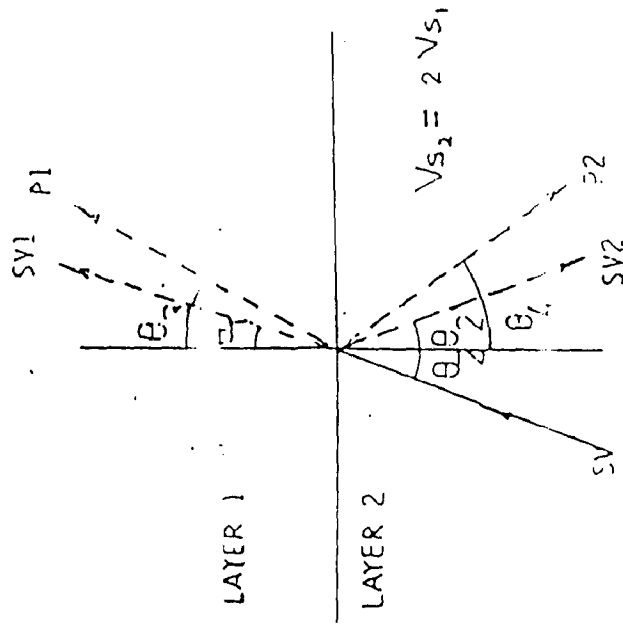
Kobayashi made the same assumption but considered upward reflection and downward transmission at the base rock level, which makes it better than Nair's method.

However, the assumptions made in both of these methods violate principle of shear wave propagating when applied to the interface at base rock level. These two methods also result in over-estimation of response of surface layer, which may be of the order of about 30 to 50 percent in some cases, which is not desirable.

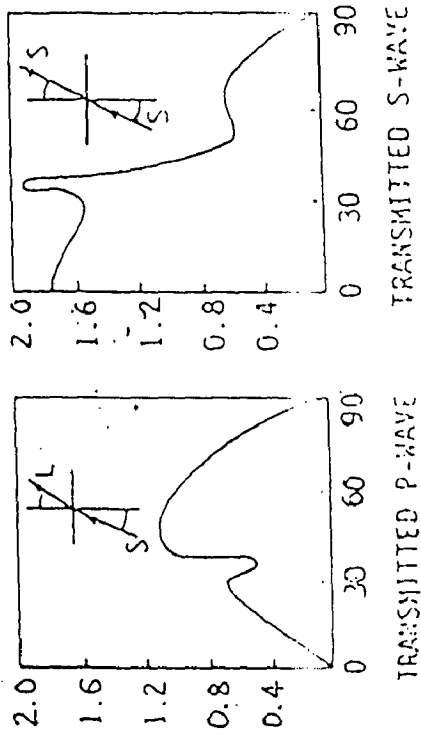
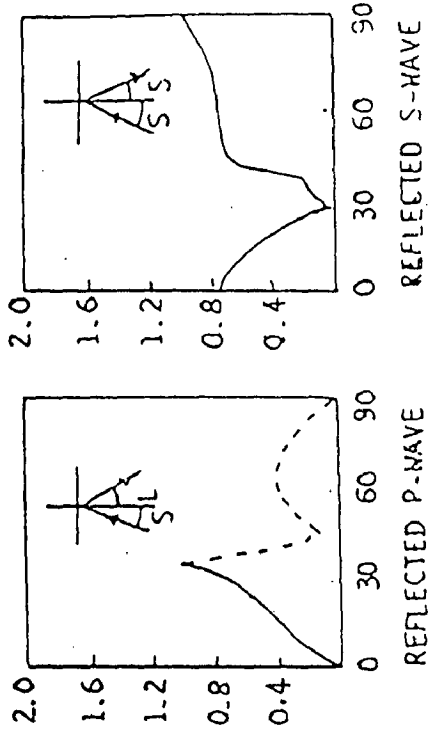
#### **Joshi's Method (1980):**

Joshi proposed a method in which he considered the reflection and refraction in the upward as well as in the downward directions at the base rock level in accordance with the principles of shear wave propagation. He assumed the base rock formation extend to infinite depth below the base rock level, which is reasonable for all practical purposes. As such, a downward propagating wave has no chance of being reflected upwards within the total duration of the earthquake under consideration.

$$\frac{\sin \theta_j}{V_{S_1}} = \frac{\sin \theta_2}{V_{S_2}} = \frac{\sin \theta_3}{V_{P_1}} = \frac{\sin \theta_4}{V_{P_2}}$$



a. DECOMPOSITION OF SHEAR WAVE AT INTERFACE BETWEEN TWO ELASTIC MEDIA



b. RELATION BETWEEN AMPLITUDE RATIO OF EMITTED WAVES AND INCIDENT ANGLE

Fig.2.3.1 : Decomposition of Shear Waves with Incident Angle



from interface, and in this case, complex function must be introduced in equations. This doubles the number of equations that must be solved. However, the imaginary amplitude ratios for resultant waves found from these equations have no physical significance. The dashed portion of the curve in Fig. (2.3.1) represents this condition.

The angle of incidence of seismic shear wave at the ground level depends upon the epicentral distance of site, shear wave velocity in the surface layer and the focal depth. The value of angle of incidence for a surface layer with shear wave velocity of 3.4 km/sec have been computed and are available as a set of tables for epicentral distances greater than 2172km (Chandra, 1972). These tables are based on theoretical model of earth and are not suitable for sites in the epicentral region. At 2172 km the angles of incidence are given as 36.21°, 35.67° and 34.67° for the focal depths of 0, 40 and 104 respectively. This indicates that the focal depth has relatively little influence on the angle of incidence.

Nair (1974) suggested that angle of incidence in any layer is proportional to the shear wave velocity of the layer. By allowing a model adjustment factor of 1.5, the angle of incidence at depth of 450km may be considered for epicentral distance 2172km to be 54 degrees. Based on this finding, he has suggested following empirical formula for angle of incidence in the surface layer.

For nearby site:

$$\theta \text{ (in degree)} = \frac{\text{Velocity of shear wave in the layer (m/s)}}{73.15} \quad \dots(2.3.1)$$

For distant site:

$$\theta = 0$$

For most engineering problems, what seismologists refer to as the angle of incidence at the ground level is actually the angle of incidence at the base rock level. In many cases, it may much below the formations generally referred to as bed rocks in engineering. The shear wave velocity of surface rock formations in general is of the order of 2km/sec. Snell's law holds good for the incidence of shear wave on interface of two layers, i.e.,

$$\sin(r) = \sin(i) V_{s_r} / V_{s_i} \quad \dots(2.3.2)$$

where,  $V_{s_i}$  = Shear wave velocity in the layer through which it is incident and

$V_{s_r}$  = Shear wave velocity in the layer into which the wave is refracted.

Joshi (1980) proposed that Snell's law holds good for incidence of shear waves on the interface of two layers. The largest possible angle of incidence is 90 degree at base layer level. Even for this case, the angle of refraction works out to be only 35°, if shear wave velocity in surface and base layer 2.0 km/sec and 3.4 km/sec, respectively. This for engineering problems, the largest angle of incidence at base layer level would be of the order of 30° to 35° and much less in many cases.

After observing about fifty cases, Suzuki (1932) concluded that the angle of incidence of the shear wave at Hongo about 4°, it's fluctuations being small. The velocity of shear waves for the surface layer was 1140 m/s. A more desirable presentation would be to consider the curves for the shallow, intermediate and deep focus earthquakes separately. Nevertheless, it may be noted that angles as high as 12° occur at epicentral distances comparable to the focal depth. At large epicentral distances the angle of incidence decreases as expected.

## **2.4 SPATIAL/TEMPORAL DISTRIBUTION OF RESPONSE**

### **Methods of Analysis**

Nair (1974), Kobayashi (1972) and Joshi (1980) gave methods for determination of component response in multi layer system if response at base rock level is known.

#### **Nair's Method (1974)**

Nair assumed that the total response at the base rock level is equal to component response due to the upward propagating wave in base rock layer and in layer immediately above base rock and that there is no downward reflection, downward transmission and upward reflection at the base rock interface.

#### **Kobayashi's Method (1972):**

Kobayashi made the same assumption but considered upward reflection and downward transmission at the base rock level, which makes it better than Nair's method.

However, the assumptions made in both of these methods violate principle of shear wave propagating when applied to the interface at base rock level. These two methods also result in over-estimation of response of surface layer, which may be of the order of about 30 to 50 percent in some cases, which is not desirable.

#### **Joshi's Method (1980):**

Joshi proposed a method in which he considered the reflection and refraction in the upward as well as in the downward directions at the base rock level in accordance with the principles of shear wave propagation. He assumed the base rock formation extend to infinite depth below the base rock level, which is reasonable for all practical purposes. As such, a downward propagating wave has no chance of being reflected upwards within the total duration of the earthquake under consideration.

Okamoto (1973) observed from field records that the seismic ground response decreases with increasing depth below the ground level. This is clear from the recorded ground motions at various elevations along a deep shaft at Kinugawa Power Station in Japan and excited by Niigata earthquake in 1964. He concluded that decrease in the intensity of ground motion with depth below is expected for two reasons. Firstly, the layers at greater depth below usually have larger elastic modulus by virtue of higher confining pressures and overburden pressure. A higher modulus results into smaller response for a given level of force. Secondly, the material at greater depth has to carry a substantial overburden pressure when it vibrates under earthquake-induced forces. These two factors lead to smaller seismic response. He recommended that these factors should be considered while obtaining seismic ground motion distribution in a layered system, which is done by assigning an appropriate value of shear modulus and other material properties for different layers. He also observed that buried structures like tunnels, underground power houses, basements of tall buildings etc experience much smaller seismic ground vibrations compared to those near the ground level.

## **2.5 STUDIES OF NONLINEAR ANALYSIS**

Kanai (1953) studied the relationship between nature of the surface layer and amplitude of displacements considering the problem of the oscillations of doubly stratified visco-elastic layer excited by seismic waves. He found that amplitude at ground surface in general becomes maximum when the period of exciting wave synchronizes with the fundamental period of first layer. But actually damping in the first layer is not zero, because, damping increases with strain levels. If the first layer is rather thin, the amplitude,

influenced by damping, cannot become very large even if the period of seismic excitation synchronizes with that of the first layer.

Okamoto (1973) considered an elasto-plastic surface layer resting over an elastic base layer and took three different levels of strain at the elastic limit of the surface layer. He observed that when the elastic limit is lower, the vibration amplification is no more prominent and the period of component vibrations having largest amplitude becomes longer.

## **2.6 MISCELLANEOUS**

Biot (1956) concluded that presence of water table in the soil mass changes the wave propagation characteristics of the soil medium (Sherman, 1945). The soils above and below the water table tend to behave as if they are separate layers. The upper layer transmits energy through soil structure while the lower layer, which is saturated, transmits energy through both soil and fluid. When shear waves are incident at saturated layer, they are divided into S-waves and P-waves. S-waves propagate through soil structure and P-wave through the fluid. In saturated layer, since fluid has no shearing stiffness, there is no structural coupling between the elastic structure and the fluid. He proposes a formula for computation of shear wave velocity in saturated soil layer. It may be noted that due to presence of pore fluid, the unit weight of the soil medium changes due to buoyancy. Besides, its inertia forces also change due to presence of pore fluid.

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### RESPONSE OF LAYERED MEDIA

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#### 3.1 PREAMBLE

When earthquake occur, two types of body waves are set into motion. The primary waves or P-waves have the particle movements in the direction of propagation of wave. The secondary waves or S-waves have the direction of particle movement transverse to the direction of wave propagation. P-wave are do not travel for a long distance from the source, because, they spend more energy while propagating. Shear waves travel relatively a longer distance with lesser attenuation of amplitude of vibration. Hence, for most sites where seismic ground motion are strong enough to cause structural damage, the potential cause to damage is predominantly due to shear waves. Body waves when incident at the interface of two layers give rise to surface waves. Raleigh waves are the most important of such waves and are also known to cause significant damage to engineering structures.

Because of the non uniform nature of the materials making up the earth's crust and the waves themselves, simplifying assumptions are necessary in choosing a representative system. Figure 3.1.1 gives a highly idealized system. At nearby sites with epicentral distance less than  $2.25 d$  ( $d$  being the focal depth), the seismic ground response is due to shear waves as well as surface waves (Okamoto, 1973). At distant sites, the difference between arrival times of shear and surface waves is so large and hence, there is no possibility to their combined effect. For intermediate sites, surface waves are often masked by shear waves. As such, a portion of the seismic ground response is actually due to surface waves. Love waves are possible only when the shear wave velocity for the top most sub-surface layer is less

than that for any underlying layer. They are not very much important to most of the engineering structures (Joshi, 1980).

In this investigation, only response due to shear waves has been considered. This chapter deals with the method of linear and nonlinear analysis of layered systems.

## **3.2 LINEAR ANALYSIS**

### **Plane Shear Wave**

After generation of shear waves, it moves in outward direction from the source in form of circular wave front. When seismic wave travel larger distance from the source, the curvature of wavefront becomes smaller. If the site under consideration is sufficiently small (which is often the case for most engineering problem), then curvature of the wave may be neglected and assumption of plane wave propagation may be justified. This assumption have advantage, i.e., response of the wave in x-z vertical plane is independent of the response in the y-z vertical plane (at right angle to the x-z plane).

### **Linear Wave Propagation**

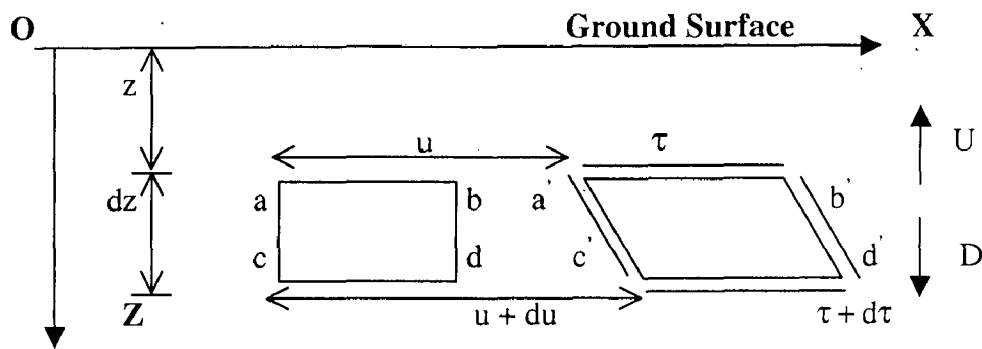
The strain induced by the seismic wave in the medium of propagation depends upon the amplitude of response and the material properties of the medium. The moduli of the medium depend upon the strain level. The material may be considered to be in the elastic domain for low strain level. With increasing strain level, the nonlinear behaviour of the material assumes greater prominence. Nevertheless, it is commonly observed that the material may be considered to be in elastic domain upto the strain level of the order of  $10^{-5}$  to  $10^{-4}$  or smaller. In this chapter, the material is assumed to be in the elastic domain only. Such a analysis is called linear analysis. This chapter deals with the derivation of differential equation of motion of vertical propagation of shear wave and it's solution.

### 3.2.1 Equation of Motion for Shear Wave Propagation

The following assumptions are made in developing the equation of motion for shear wave propagation:

1. Linear elastic medium.
2. Layer and interfaces are horizontal, and extend to infinity.
3. Vertical propagation of plane shear waves.

Due to shear wave propagation, the particles of the medium are subjected to movements within the plane of propagation in the direction perpendicular to the direction of propagation. Let X and Z is a horizontal and vertical coordinate axes, and let  $\rho$ ,  $V_s$  and G be the mass density, shear wave velocity and the modulus of rigidity of the medium respectively (Fig. 3.2.1.1).



**Figure 3.2.1.1 Vertical Propagation of Plane Shear Waves**

If  $u$  is the response at any general point at a distance of  $z$  below the ground level and at any time station,  $t$  the shear stress differential  $d\tau$ , is obtained as:

$$d\tau = \frac{\partial \tau}{\partial z} . dz = \frac{\partial^2 u}{\partial t^2} . \rho . dz \quad \dots(3.2.1.1)$$



From the definition of shear modulus 'G'

Where,  $\tau$  and  $\frac{\partial u}{\partial z}$  are shear stress and shear strain respectively.

$$\tau = G \frac{\partial u}{\partial z} \quad \dots(3.2.1.2)$$

From these relationships, we have

$$\rho \frac{\partial^2 u}{\partial t^2} - G \frac{\partial^2 u}{\partial z^2} = 0 \quad \dots(3.2.1.3)$$

i.e., 
$$\frac{\partial^2 u}{\partial t^2} - V_s^2 \frac{\partial^2 u}{\partial z^2} = 0 \quad \dots (3.2.1.4)$$

where, 
$$V_s^2 = \frac{G}{\rho}$$

Equation 3.2.4 is differential equation of shear vibrations for propagation in vertical directions. This equation has two solution  $U(z, t)$  and  $D(z, t)$ . In any layer, there are two waves; one is rising upward and second one going downward.  $U(z, t)$  represents the response due to upward propagating wave and  $D(z, t)$  represents the response due to downward propagating wave. Both these propagation may access simultaneously in the layer.

Net response,  $u(z, t)$ , at any time  $t$  and at depth  $z$  is given by:

$$u(z, t) = U(z, t) + D(z, t) \quad \dots (3.2.1.5)$$

If shear wave velocity in the medium is  $V_s$ , then  $U(z, t)$  &  $D(z, t)$  at any depth,  $z$ , and any time,  $t$  are give by  $U\left(t + \frac{z}{V_s}\right)$  and  $D\left(t - \frac{z}{V_s}\right)$  respectively. From this, we get:

$$u(z, t) = U\left(t + \frac{z}{V_s}\right) + D\left(t - \frac{z}{V_s}\right) \quad \dots(3.2.1.6)$$

which is the solution of equation 3.2.1.4

### 3.2.2 REFLECTION AND TRANSMISSION OF SHEAR WAVES

#### Reflection and Transmission at Ground Surface

When shear waves propagate upward, they are ultimately incident at the ground level. It is not possible for them to get transmitted into the medium of air above the ground level, because, the velocity of shear wave in the air is zero. The shear modulus for air is also zero for all geotechnical engineering purposes. As such, all the energy associated with the upward travelling shear waves is totally reflected downward. In other words, the component response of upward travelling incident shear wave is equal to that of the downward travelling reflected shear wave at the ground level i.e.,

$$\begin{aligned} -U(t) + D(t) &= 0 \\ \Rightarrow D(t) &= U(t) \\ \Rightarrow u &= 2U(t) \end{aligned} \quad \dots(3.2.2.1)$$

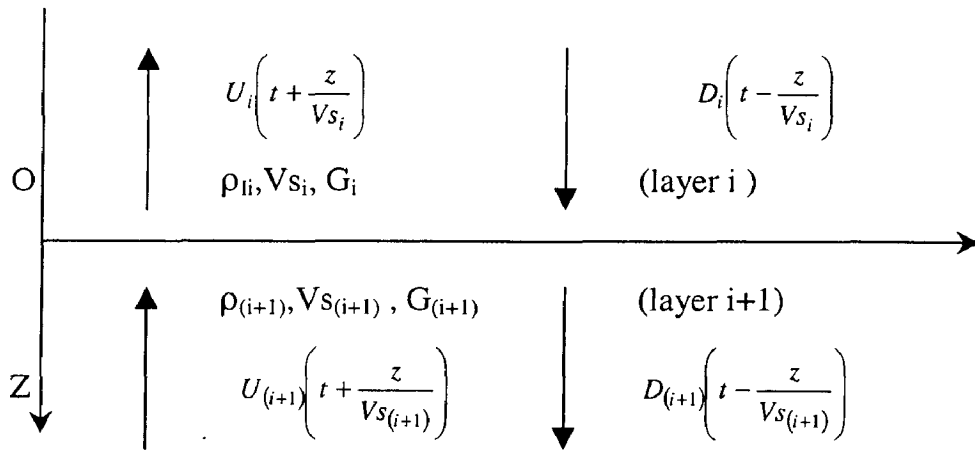
When the incident SV-wave is not exactly vertical, it gives rise to reflected P-waves in addition to reflected SV-waves. However, for small angle of incidence, the reflected P-wave may be neglected for all practical purposes. When SH-waves are incident, they generate no P-waves. This has already been discussed in Article 2.3 earlier.

The explicit knowledge of component responses of upward and downward travelling waves at the ground level is useful in obtaining the component responses of upward and downward travelling waves in all the layer of the system. From this, it is possible to obtain the spatial/temporal variations of seismic ground motions for a known response at the ground level for the entire time period, using principles of wave propagation.

## Reflection and Transmission at Interface

As shown in previous chapter the layer system (Fig. 2.2.2) of soil medium of different properties, consider the  $i^{\text{th}}$  interface shown in Fig. 3.2.2.1 for the  $i^{\text{th}}$  layer with impedance  $(\rho_i V_{s_i})$  and  $(i+1)^{\text{th}}$  layer with impedance  $(\rho_{(i+1)} V_{s_{(i+1)}})$ . The response in  $i^{\text{th}}$  layer at a distance  $z$  from the interface in terms of the component responses at the bottom of  $i^{\text{th}}$  layer at any time,  $t$ , is give below:

$$u_i = Ub_i \left( t + \frac{z}{V_{s_i}} \right) + Db_i \left( t - \frac{z}{V_{s_i}} \right) \quad \dots(3.2.2.2)$$



**Figure 3.2.2.1 Reflection and Transmission at Interface**

Similarly, the response at any point in layer  $i+1$ , distant  $z$  from the interface may also be expressed in terms of component responses at the top of same layer as:

$$u_{(i+1)} = Ut_{i+1} \left( t + \frac{z}{V_{s_{(i+1)}}} \right) + Dt_{(i+1)} \left( t - \frac{z}{V_{s_{(i+1)}}} \right) \quad \dots(3.2.2.3)$$

These two responses should be identical to satisfy compatibility at the interface, i.e.,

$$(u_i)_{z=0} = (u_{(i+1)})_{z=0} \quad \dots(3.2.2.4)$$

$$\left( G_i \frac{\partial u_i}{\partial z} \right)_{z=0} = \left( G^{(i+1)} \frac{\partial u^{(i+1)}}{\partial z} \right)_{z=0} \quad \dots(3.2.2.5)$$

The component response at any time,  $t$ , due to upward travelling wave at the top end of  $(i+1)^{th}$  layer may be denoted by  $ut_{(i+1)}(t)$  and that at any bottom of the  $i^{th}$  layer may be denoted by  $ub_i(t)$ . Similarly, the corresponding component responses due to downward travelling wave at the interface may be denoted by  $Dt_{(i+1)}(t)$  and  $Db_i(t)$ . Therefore,

$$Ub_i(t) + Db_i(t) = Ut_{(i+1)}(t) + Dt_{(i+1)}(t) \quad \dots(3.2.2.6)$$

$$\frac{G_i}{V_{s_i}} [Ub_i(t) - Db_i(t)] = \frac{G^{(i+1)}}{V_{s^{(i+1)}}} [Ut_{(i+1)}(t) + Dt_{(i+1)}(t)] \quad \dots(3.2.2.7)$$

Integrating above equation, we get :

$$Ub_i(t) - Db_i(t) = \frac{1}{k_i} [Ut_{(i+1)}(t) + Dt_{(i+1)}(t)] \quad \dots(3.2.2.8)$$

$$\text{where, } k_i = \frac{G_i V_{s^{(i+1)}}}{G^{(i+1)} V_s} = \frac{\rho_i V_{s_i}}{\rho^{(i+1)} V_{s^{(i+1)}}}$$

Solving equation (3.2.2.6) and (3.2.2.8), simultaneously, we get :

$$Ub_i(t) = Ut_{(i+1)}(t) \cdot Tu_i + Db_i(t) \cdot Ru_i \quad \dots(3.2.2.9)$$

$$Dt_{(i+1)}(t) = Ut_{(i+1)}(t) \cdot Rd_i + Db_i(t) \cdot Td_i \quad \dots(3.2.2.10)$$

where:

$$Rd_i = \frac{1 - k_i}{1 + k_i} \rightarrow \text{coefficient of downward reflection}$$

$$Tu_i = 1 + Rd_i \rightarrow \text{coefficient of upward transmission}$$

$$Ru_i = -Rd_i \rightarrow \text{coefficient of upward reflection}$$

$$Td_i = 1 - Rd_i \rightarrow \text{coefficient of downward transmission}$$

From the above discussion it is clear that if we know the component responses incident at any interface, the component responses generate by them in the upward and downward direction can be evaluated using the coefficients of reflection and transmission in upward and downward directions.

### 3.2.3 COMPONENT RESPONSE OF LAYERED MEDIUM

Soil properties are not uniform both in vertical and horizontal extents. It consists of different layers of different material properties. Dynamic response depends upon material properties. Therefore seismic response computation of layered system is necessary. Natural process of soil formation normally favours formation of horizontal strata. Hence, it is usually assumed that layers have horizontal orientation.

In a soil layer, waves propagate simultaneous in upward and downward directions. Response of ground due to these waves is vector. Hence, total response at any point is a vector sum of response due to upward and downward propagation of waves. Using wave propagation theory, it may be possible to compute the upward and downward responses separately and which are called as component responses.

Consider a multi-layered system shown in Fig. (3.2.3.1). Surface layer is divide into  $n$  layers and base rock layer is  $(n+1)^{th}$  layer. For  $i^{th}$  layer, the nterface at its bottom!level is bottom is  $i^{th}$  interface and that it's top level is  $(i-1)^{th}$  interface. For layer no.1, the interface at it's bottom is 1<sup>st</sup> interface and for base rock layer, the interface at it's top is  $n^{th}$  interface. The responses  $Ub_i(t)$  and  $Ut_i(t)$  represent the response at the bottom and top of the  $i^{th}$  layer due to upward propagating waves respectively and  $Db_{i1}(t)$  and  $Dt_i(t)$  are those due to the downward propagating waves at time station,t. Symbol use as U & D denotes the direction

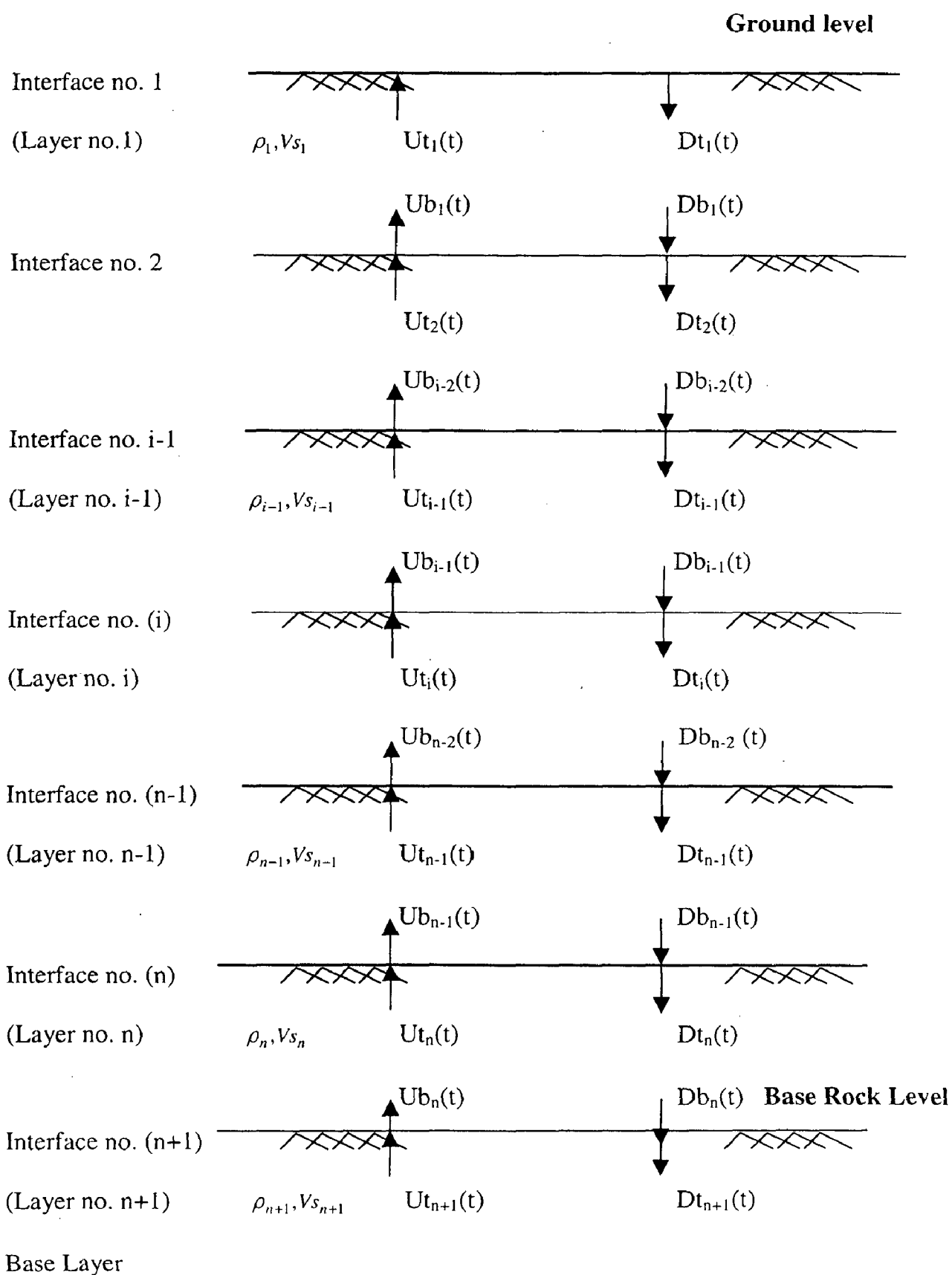


Figure 3.2.3.1 Multi Layer System

of propagating waves in upward and downward direction respectively and t & b are denoting the responses at top and bottom level of a that layer. For  $i^{\text{th}}$  layer,  $V_{s_i}$  and  $\rho_i$  are the shear wave velocity and mass density respectively. The coefficient of upward reflection  $R_{u_i}$ , downward reflection  $R_{d_i}$ , upward transmission  $T_{u_i}$ , and downward transmission  $T_{d_i}$  for the  $i^{\text{th}}$  layer already defined in article 3.2.2 The expression for component responses are:

$$U_{b_i}(t) = U_{t_{(i+1)}}(t) \cdot T_{u_i} + D_{b_i}(t) \cdot R_{u_i} \quad \dots(3.2.3.1)$$

$$D_{t_i}(t) = D_{b_{(i-1)}}(t) \cdot T_{d_{(i-1)}} + U_{t_i}(t) \cdot R_{d_{(i-1)}} \quad \dots(3.2.3.2)$$

$$U_{t_i}(t) = U_{b_i}(t - t_i) \quad \dots(3.2.3.3)$$

$$D_{b_i}(t) = D_{t_i}(t - t_i) \quad \dots(3.2.3.4)$$

where,  $t_i$  is the time of travel for the seismic wave to cover the depth of the  $i^{\text{th}}$  layer. Similar expressions may be obtained for other layers. The component responses of various layers at top and bottom interfaces are shown in Fig. 3.2.3.1.

### Component Response for Topmost Layer:

As discussed earlier in Article 3.2.2, the component responses at ground level are obtained as:

$$U_{t_1}(t) = D_{t_1}(t) = X_1(t)/2 \quad \dots(3.2.3.5)$$

where,  $X_1(t)$  is response of ground level at time t.

Thus if the surface response is known for a given system it is possible to compute component responses in the top layer and hence by using Equations 3.2.3.1 to 3.2.3.5, the net response at any interface for any time station may be obtained. If response at base rock is known for all time stations, then equation is not useful to obtain component responses at base rock level. For this purpose additional expressions are needed. In this case, the method proposed by Joshi (1980) may be used for computation of component responses.

## Component Response at Base Rock Level

Joshi (1980), assumed that the base rock formation extend to infinite depth below the base rock level, which is reasonable for all practical purposes. As such, a downward propagating wave has no chance of being reflected upward within the total duration of the earthquake under consideration.

Denoting the base rock as  $(n+1)^{\text{th}}$  layer, the total response of the base rock,  $Xt_n(t)$ , at any time,  $t$ , is :

$$Xt_n(t) = Ut_{(n+1)}(t) + Dt_{(n-1)}(t)$$

$$\text{i.e.,} \quad Dt_{(n+1)}(t) = Xt_n(t) - Ut_{(n+1)}(t) \quad \dots(3.2.3.6)$$

$$= Ut_{(n+1)}(t) \cdot Rd_n + Db_n(t) \cdot Td_n$$

$$\text{i.e.,} \quad Ut_{(n+1)}(t) \cdot Rd_n + Db_n(t) \cdot Td_n = Xt_n(t) - Ut_{(n+1)}(t)$$

$$\text{i.e.,} \quad Ut_{(n+1)}(t) = \frac{Xt_n(t) - Db_n(t) \cdot Td_n}{1 + Rd_n} \quad \dots(3.2.3.7)$$

The quantity  $Db_n(t)$  does not exist till the downward reflected wave inside the  $n^{\text{th}}$  layer reaches the base rock level which needs a time interval equal to  $2t_n$ . For any other time stations,  $Db_n(t)$  is also a known quantity. Using this information and equations 3.2.3.1 to 3.2.3.7 the component responses of the level at which response is known may be obtained from the known response. Using these component responses and net response at any other interface below can also be compared.

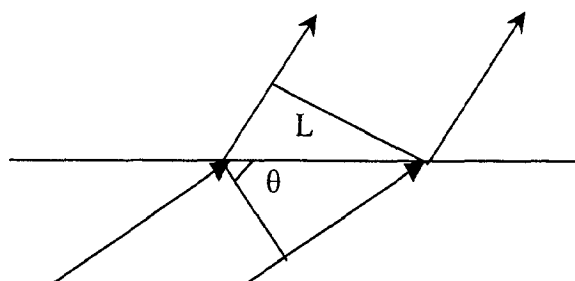


### 3.2.4 INCLINED PROPAGATION OF SHEAR WAVE

The seismic ground motion due to vertical propagation of shear wave through layered media has been discussed earlier in this presentation. However, the shear wave may often propagate at angle to the vertical also, particularly in the epicentral region and for nearby sites. The consideration inclined propagation of seismic waves in a layer medium becomes very important for structures with larger horizontal dimensions, because, this includes a relatively large time lake between response of two points in the same horizontal plane but separated by a large distance in the plane of propagation of wave. In such case, it is necessary to obtain seismic ground motions with due consideration to angle of incidence of shear waves. In view of the angle of incidence being not equal to zero, there will be a vertical component of response also generated, which may be obtained as the product of horizontal response and angle of incidence in radians.

When the wave front AB of shear wave propagating at an angle,  $\theta$ , with the vertical is incident, as shown in Fig. (3.2.4.1), at the interface AB' at point A, the response due to this wave along the wave front AB is same at all points. The response at B will travel with a velocity equal to  $V_{s2}$  in the lower layer to reach the interface at B' after a time interval equal to  $\Delta t$ ; which is given by:

$$\Delta t = \frac{BB'}{V_{s2}} = \frac{L \sin \theta}{V_{s2}} \quad \dots(3.2.4.1)$$



**Fig. 3.2.4.1 Non vertical Propagation of Shear waves**

In other words response at B' at time station,  $t$ , is the same as the response at time station,  $(t - \Delta t)$  at B which in turn is same as response at A at time  $(t - \Delta t)$ .

Using this methodology, it is possible to obtain spatial/temporal variation of ground response for any given layered system for a given history of seismic ground motion at any point within the system.

The consideration of inclined propagation of shear waves through the layered media becomes necessary when the lateral extent of the structure under consideration is significant with respect to the wavelength of the shear waves. Structures like very long buildings, sea front structures, retaining wall, dam, embankment, tunnels and canals etc have considerable lateral length and as such, they may be expected to experience different seismic ground motions at point separated by appreciable lateral distance for a given time station. This may cause stress differences at these points. As such, use of spatial/temporal distribution of seismic ground motion with due consideration to angle of incidence of seismic would be essential. However, for structures with small lateral extent, the inclined propagation of seismic wave is important only for computation for vertical response.

### **3.3 Nonlinear Analysis**

Soil is nonlinear material and it has very low proportionality limit. Therefore, earthquake induced strains may easily exceed proportional limit of such materials. In such cases, dynamic properties do not remain constant. So, assumption of constant material properties always leads, to less precise response and extra safe design of structures. Thus for precise determination of ground response, nonlinear analysis should be carried out.

All the assumptions, criteria and procedures adopted for linear analysis has been used for nonlinear analysis except consideration of strain level independent material property.

### 3.3.1 Outline of Procedure for Non Linear Analysis

For nonlinear analysis, following procedure is adopted:

- i. Each layer of the layered system under consideration is divided into adequate number of thin sub-layers,
- ii. Assume some initial material properties of each sub-layer ( $V_{s_i}, G_i, \text{etc.}$ ),
- iii. For each time station:
  - a. Compute response (in particular displacement) at interfaces of sub-layers considering linear material properties by double integration of acceleration using Wilson  $\theta$  method. Wilson  $\theta$  method gives unconditionally stable results.
  - b. The shear strain in any sub-layer is obtained as the ratio of the difference between the displacement at the top and bottom of that sub-layer divided by the depth of that sub-layer.
  - c. If the difference of strain induced at current time station and at that previous time station is less than a predefined value of small strain tolerance, then, don't revise the shear modulus and the step (f) may be followed. Otherwise follow step (d) , (e) and (f) before going to step (g),
  - d. Using stress strain relationship for the sublayer, compute shear modulus of material of sub-layer, corresponding to strain induced in that sub-layer.

- e. Using the modified shear modulus, the velocity of propagation of shear wave in the sub-layer may be computed which may be used for computation of response
- f. Based on these velocities, various reflection and transmission coefficient at each interface are revised at every time station.
- g. Compute the response of the sub-layers at the next time station (with revised sublayer properties if applicable).
- h. The above procedure is repeated for the entire history of earthquake to obtain the response of the system using nonlinear material properties.

### **Elastic Limit**

Elastic limit is defined as that limit upto which material behaviour is linear. After elastic limit nonlinear behaviour of material starts. Elastic limit is obtained by the experimental investigation. If this experimental investigation is not possible, elastic behaviour of the material is considered upto a strain level of  $10^{-4}$ , beyond which nonlinear behaviour of the material starts.

### **3.3.2 Division of Layers into Sub-layers**

Layer should be divided into number of sublayer for nonlinear/linear analysis otherwise computed strain level may be incorrect. For example, if thickness of layer is equal to the wavelength of propagating wave, then difference between extreme point at top and bottom of one wave cycle may be zero, which is less, then the actual response. For getting precise results, thin layer should be taken. For this each layer of the assumed layer system is

subdivided into smaller sub layers such that the time of travel of shear wave in that sub layer is not more than a specified small time interval. The thickness of sub layer is also not more than  $1/10^{\text{th}}$  of the wavelength of the shear wave in that material.

Time of travel through each of the sub-layer should be greater than small time interval ( $t_i$ ). The distance traveled in  $t_i$  time is  $t_i * V_s$ . The corresponding thickness of layer is  $(t_i * V_s * \cos(\theta))$ . It is proposed to normally form sub-layer of thickness of  $(n t_i * t_i * V_s * \cos(\theta))$ . Any layer thinner than  $(2 * n t_i * t_i * V_s * \cos(\theta))$  shall not be subdivided. Thickness of sublayer are not more than  $1/10^{\text{th}}$  of the wave length of the shear wave in that material, because, minimum eight points are required for getting a sinusoidal response but beyond 16 points accuracy does not increase so much, hence, it is only wastage of time. For reliable solution 10, 12, 14 points are enough. In this discussion for making a sinusoidal response 10 points are considered. Where  $n t_i$ ,  $t_i$  and  $\theta$  are multiple of minimum thickness of sublayer, time interval between two consecutive time station and angle of propagation of shear wave in the sublayer respectively.

### 3.3.3 Computation of Strain in Sublayers

Let at time station  $n t$  computed acceleration response at top and bottom of sublayer  $i$  are  $\ddot{Z}_{ti}(n t)$  and  $\ddot{Z}_{bti}(n t)$  respectively. By applying Wilson  $\theta$  method corresponding displacements  $Z_{ti}(n t)$  and  $Z_{bi}(n t)$ , can be computed. Therefore, strain level induced in layer  $i$  at time station  $n t$  is given by:

$$\gamma_{i(n t)} = \frac{Z_{ti}(n t) - Z_{bi}(n t)}{\text{thickness.of.layer.i}} \quad \dots(3.3.3.1)$$

### Wilson $\theta$ Method:

The basic assumption in the Wilson  $\theta$  Method is that the acceleration varies linearly over the time interval from  $t$  to  $t + \Delta t$  and is determined to obtain optimum stability and accuracy characteristics. Let  $Z_t, \dot{Z}_t$  and  $\ddot{Z}_t$  are displacement, velocity, and acceleration respectively, at current time station and  $Z_{t+\Delta t}, \dot{Z}_{t+\Delta t}$  and  $\ddot{Z}_{t+\Delta t}$  are displacement, velocity and acceleration respectively, at next time station. Thus,

$$\dot{Z}_{t+\Delta t} = \dot{Z}_t + (\ddot{Z}_{t+\Delta t} + \ddot{Z}_t) \frac{\Delta t}{2} \quad \dots(3.3.3.2)$$

$$Z_{t+\Delta t} = Z_t + \dot{Z}_t \Delta t + (\ddot{Z}_{t+\Delta t} + 2\ddot{Z}_t) \frac{\Delta t^2}{2} \quad \dots(3.3.3.3)$$

where,  $\Delta t$  is time interval.

### **3.3.4 Computation of Shear Strain and Shear Stress Relationship**

The shear modulus at any strain level is obtained as secant modulus define as shear stress at that instant divided by shear strain at that instant. Stress-Strain properties of the soil to obtained from experimental investigation. If this is not available, artificial stress-strain relationship may be considered in the form of elliptical, parabola or composite curve with flat slope at the failure strain.

Proposed stress-strain relationship is developed as follows:

1. Define the failure strain ( $\gamma_f$ ), (it is assumed that material of layer fails when induced strain in layer exceeds defined failure strain).
2. Compute shear strength of material by,

$$\tau_f = c + \bar{\sigma} * \tan \phi \quad \dots(3.3.4.1)$$

where,  $c$  : cohesion,

$\phi$  : angle of shearing resistance,

$\bar{\sigma}$  : effective normal stress at middle of layer.

The non-linear stress strain characteristics for any sub layer is represented by unit stress strain characteristics for which the strength and shear strains at failure are considered to be unity. The actual stress and strain are obtained by multiplying the stresses and strain read from this stress strain relationship by shear stress and strain at failures for that sub layer obtained from experimental investigations. The stress strain relationship is digitized with 1000<sup>th</sup> equal intervals of strain along the strain axis.

The relationship is assumed to be linear for an initial range of shear strain ( $\gamma_0$ ) of the order of  $10^{-4}$ . Following condition is imposed for linear behaviour of material:

- a. Straight line passes through (0,0) and ( $\gamma_0, \tau_0$ ),

Fit a three-degree curve from ( $\gamma_0, \tau_0$ ) to a strain level ( $\gamma_p$ ) which is specified fraction of the failure strain. Equation of three-degree curve is:

$$\gamma = a + b * \tau + c * \tau^2 + d * \tau^3 \quad \dots(3.3.4.2)$$

constants a, b, c and d are computed by following condition:

- a. Curve passes through ( $\gamma_0, \tau_0$ ) and ( $\gamma_p, \tau_p$ ),
- b. At point ( $\gamma_p, \tau_p$ ), slope of three degree curve and circle is same,
- c. At point ( $\gamma_0, \tau_0$ ), slope of three degree curve and straight is same,

Beyond strain level ( $\gamma_p$ ), the variation is assumed to be defined by a circle. For fitting circle in remaining part, ( $\gamma = \gamma_p$ , to  $\gamma = \gamma_f$ ), following condition is imposed:

a. Curve passes through  $(\gamma_p, \tau_p)$  and  $(\gamma_f, \tau_f)$ ,

b. Slope at  $(\gamma_f, \tau_f)$  is zero.

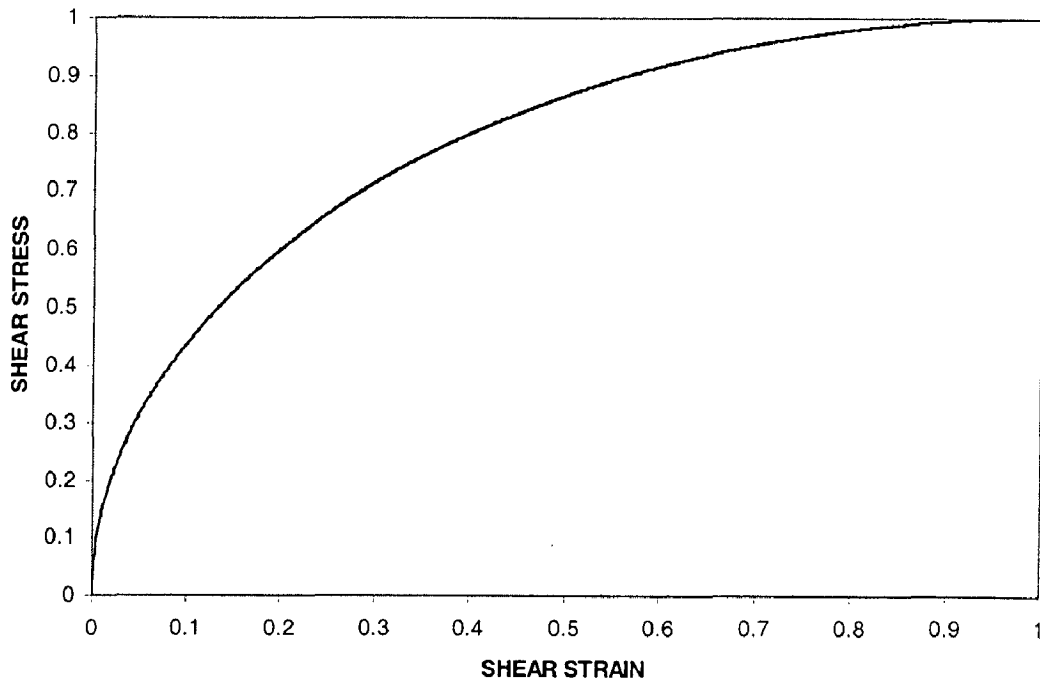


FIGURE 3.3.4.1 NORMALIZE STRESS-STRAIN CURVE



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# PRESENTATION OF RESULT AND DISCUSSION

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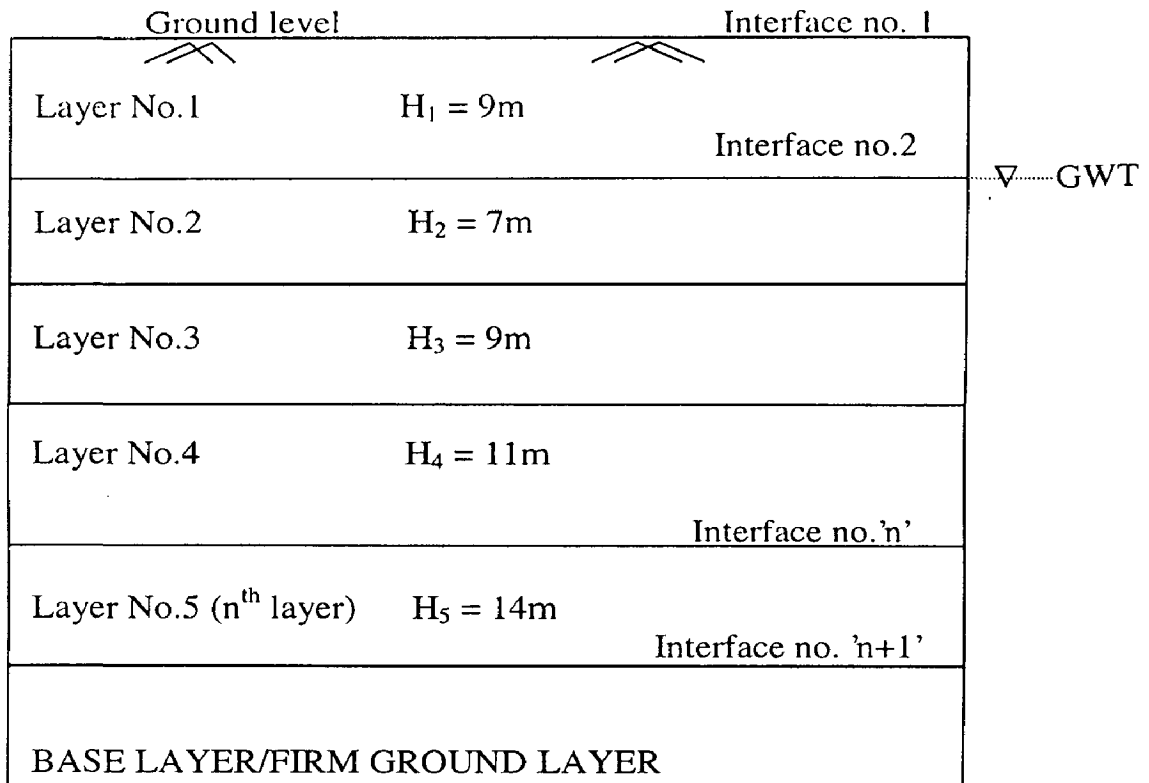
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### 4.1 Preamble

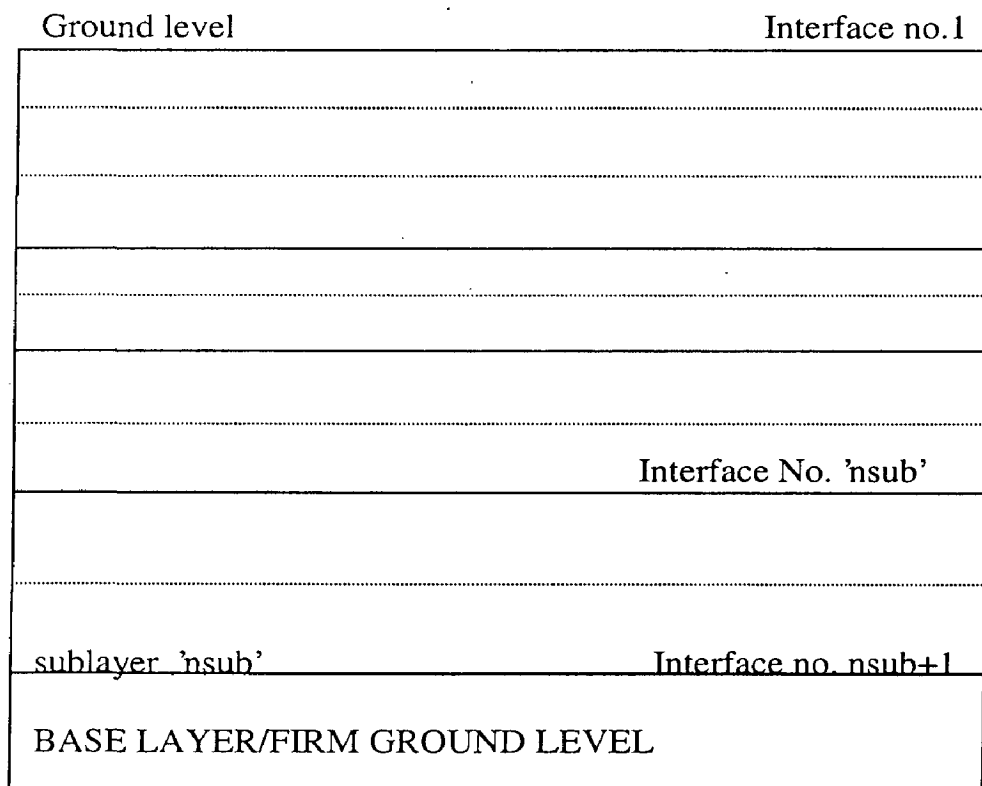
Results of investigations obtained by using the computer program “SENORE” for different layered system for a variety of excitation at the firm ground/base rock level are presented in this chapter. The parametric studies vary various parameter to study their influence on seismic ground response and other factors based on the ground response. The analysis is carried out in linear as well as nonlinear domains and the results obtained are compared. The results obtained are also used for the demonstrating the capability of the program “SENORE” for the study of ground response for a layered system as well as for preparation of data for developing seismic microzonation maps for areas/regions under consideration.

### 4.2 Input data for investigation

For this investigation, a layered system consisting of ‘n’ main layers resting on base rock/firm ground is employed. Fig 4.2.1 shows details of the system with five main layers (n=5) such as thickness, cohesion, failure strain, angle of shearing resistance, specific gravity, void ration and water content of each layer as well as properties of base layer which is  $n^{\text{th}}$  layer(n+1). Table no.4.2.1 and Table 4.2.2 gives various details of the layer which include input data computed data respectively. The layers are numbered serially with top most layer named as layer no.1. The interface are also numbered serially with ground level denoted as interface no.1.



**Figure 4.2.1 Five layer system**



**Figure 4.2.2 Division of layers**

**Table No.4.2.1 Details of soil properties as input data used for parametric studies**

Layer No.	Thickness (H)m	Cohesion (c)kN	Failure strain(%)	Angle of shearing resistance (degree)	Void ratio (e)	Specific gravity	Water content
1	9	0.981	12.5	10	0.9	2.47	0.2
2	7	1.4715	10.0	15	0.8	2.57	1.0
3	9	0.0	9.0	20	0.7	2.67	1.0
4	11	0.0	8.0	25	0.6	2.67	1.0
5	14	0.0	7.0	30	0.5	2.67	1.0

**Table 4.2.2 Details of computed data**

Layer No	Bulk ut.wt (kN/m <sup>3</sup> )	Saturated ut.wt (kN/m <sup>3</sup> )	Shear wave velocity (m/sec)	Limiting shear stress(kN/m <sup>2</sup> )
1	15.30	17.30	184.56	22.056
2	18.37	18.37	230.9591	30.207
3	19.45	19.45	259.780	60.77
4	20.05	20.05	294.4882	85.556
5	20.73	20.73	232.6374	100.393

**Table 4.2.3 Details of input data for parametric studies**

Base Excitation Amplitude	Period of Excitation (sec)	Time interval (ti sec)	Factor of minimum thickness
0.05g	0.25	0.005	1
0.1g	0.5	0.01	2
0.2g	1.0	0.02	3
0.4g	2.0	0.03	4

The ground water table is always considered as an interface. In case the input data dose not complies with this requirement, the program automatically rennumbers the layer and interfaces show that ground water table becomes an interface.

As explained in the earlier chapters the main layers of the system are subdivided into thinner layers or sublayers so that each sublayer is reasonably thin to the desire extend. Sublayers of each main layer have the same thickness. For obvious region, the ground water table becomes an interface between two sublayers. The interfaces of the sublayer are also numbered serially from one to nsubp where nsubp tends for largest rank of interface number. Interface no 1 remains to be at the ground level. Sublayer no 1 is the top most sublayer, nsub<sup>th</sup> layer is the lower most sublayer and baserock or firm ground layer is denoted by the number nsubp<sup>th</sup> (nsub+1). Figure 4.2.2 shows details of the system with sublayer. Other input data employed for parametric studies are the base excitation (amplitude of acceleration  $a_{eq}$ ,  $f_{eq}$  and duration of excitation  $t_{eq}$ ), depth of ground water table below the ground level, duration of time interval  $t_1$  and minimum thickness of the sublayer are as cited in the table 4.2.3.

Even though the computer program is capable of reading digitized acceleration time history of actual earthquakes, in this investigation only sinusoidal base excitation is employed. Sinusoidal excitation is useful in carrying out parametric studies to study the influence of amplitude and frequency of base excitation. It is also useful in the study of resonance and quais-resonance condition on the response of layered system.

In this investigation, unless otherwise stated, the value of input data will be the first value of the parameter under consideration. The second and subsequent values of any parameters are considered only when the influence of that particular parameter is being considered.

### 4.3 Computation of Properties of Main-layer and Sublayer

#### Properties of Main-layer

The program computes bulk unit weight and saturated unit weight of each main layer using the input data. The ultimate shear strength of each main layer is also computed by using coulomb equation:

$$\tau_u = c + \bar{\sigma} \tan \phi \quad \dots(4.3.1)$$

where  $c$  : cohesion

$\bar{\sigma}$  : effective stress which is effective vertical stress ( $\sigma_1$ )

$\phi$  : angle of shearing resistance of the main layer under consideration

The properties of main layer are worked out on the bases of stresses computed at the mid depth of that layer. The effective stress at the mid depth of each main layer is obtained and the lateral stresses  $\sigma_2$  and  $\sigma_3$  are computed by assuming at rest earth pressure condition by assuming  $K_o$  given by:

$$K_o = (1 - \sin \phi) \quad \dots(4.3.2)$$

The octahedral stress ( $\sigma_{oct}$ ) is obtained as:

$$\sigma_{oct} = (\sigma_1 + \sigma_2 + \sigma_3)/3.0 \quad \dots(4.3.3)$$

The shear modulus ( $G$ ) corresponding to this stress condition is obtained by using the expression (Seed & Idriss):

$$G = K_2(\sigma_{oct})^{0.25} \quad \dots(4.3.4)$$

where  $K_2$  is constant which depends on void ratio and is given by:

$$K_2 = 14760 * (2.973 - e)^2 / (1 + e) \quad \dots(4.3.5)$$

The velocity of shear wave  $V_s$  for the layer is obtained as:

$$V_s = \sqrt{G / \rho} \quad \dots(4.3.6)$$

where  $\rho$  is the mass density of the layer.

This velocity and corresponding shear modulus are considered to be initial properties of the layer under consideration for shear strain ( $\gamma$ ) less than or equal to  $10^{-4}$ , which is considered the acceptable by many.

### **Material Properties of the Sublayer**

The material properties like bulk unit weight, saturated unit weight, void ratio, water content, degree of saturation, cohesion, angle of shearing resistance and failure strain of any sublayers are the same as the corresponding properties of main layer in which the sublayer under consideration is situated. However, the shear modulus and shear wave velocity are dependent on octahedral shear stress in the sublayer. Hence, their values are different for different sublayers each when the sublayer may be in the same main layer. Obviously, computing stress dependent velocity of shear wave for each sublayer is helpful in considering variation of material properties of the same layer as a function of depth, which is desirable and which include accuracy of the analysis. The computation of  $V_s$ ,  $G$  and ultimate shear strength for each sublayer is done on the lines explained earlier for the main layer.

#### **4.4 Influence of the Minimum Thickness of Sublayers**

The thickness of the sublayer causes time lag in the propagation of shear wave from bottom to top interface of the sublayer under consideration. The influence of this time lag is more important than the physical dimension of the layer thickness. Therefore, the minimum thickness of sublayer increases if the velocity of shear wave propagation of the sublayer is higher. To study the influence of this parameter on computed response, the minimum time lag specified for each sublayer was set to 2, 3 and 4 times the period of time interval  $t_i$  considered for the dynamic analysis in time domain. Figure 4.4.1 shows variation of period of ground vibration with time for the three cases. It may be observed from the figure that at the time station where discrepancy between periods obtained for the three cases is the largest. The discrepancy is of the order of 1 to 3% only, which is negligible. Therefore, in this investigation the thickness of the sublayers is computed by using  $2nt_i$  as criteria which is adequate.

#### **4.5 Influence of Time Interval between two Consecutive Time Stations**

The analysis is done in time domain. The time interval between two consecutive stations is an important parameter, which determines, the accuracy of the analysis, the required computational effort and cost of analysis. The computational effort increases significantly with decreasing value of this time interval the storage requirement of computers also increases substantially with reducing value of this interval. Figure 4.5.1 shows variation of period of ground vibration with time. When duration of earthquake excitation is 3 sec, the discrepancy in computed values of  $T_g$  with respect to that obtained for the values of  $t_i$  equal to 0.01 sec work out to be 3.9% and 9.8% respectively. Therefore,

the interval of 0.01sec between two consecutive time stations may be considered reasonably adequate.

#### **4.6 Influence of Amplitude of Base Excitation**

Amplitude of base excitation is the most important feature of the seismic excitation. It indicates the sensitivity of excitation. Generally it ranges from 5% to 20% of 'g' even though it could be as high as 0.364g for some sites in the country.

Figure 4.6.1 shows response at ground level for base excitation amplitude ranging from 0.05g to 0.3g and frequency of base excitation 2Hz. It may be observed that as level of base excitation increases the degree of nonlinearity also increases and the difference between linear and non linear response becomes more and more evident. This is expected.

Figure 4.6.2 shows strain in the sublayer three as a function of time for the above excitation. It may be observed that strain level predicted by nonlinear analysis could be quite significant when compared with those obtained by linear analysis. This highlights the need for carrying out nonlinear analysis for computing seismic response.

Figure 4.6.3 shows the variation of predominant period of ground vibration  $T_g$  computed by using the relationship

$$T_g = 4H_{eq}/V_{s,eq} \quad \dots(4.6.1)$$

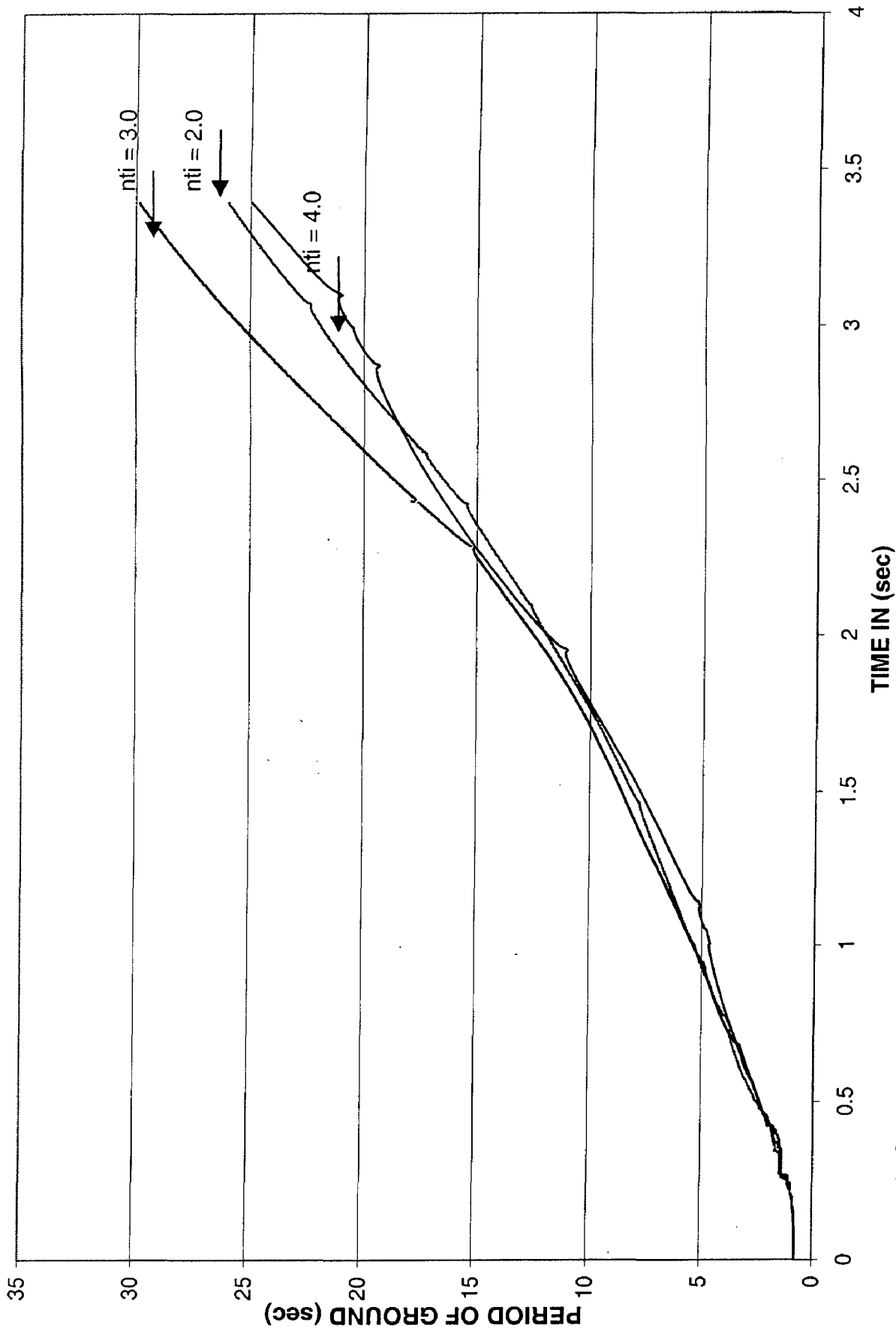


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**FIGURE 4.4.1 VARIATION OF PERIOD OF GROUND WITH TIME FOR DIFFERENT MINIMUM THICKNESS OF SUBLAYER**

PERIOD OF GROUND VIBRATION FOR LINEAR ANALYSIS = 0.765324 sec

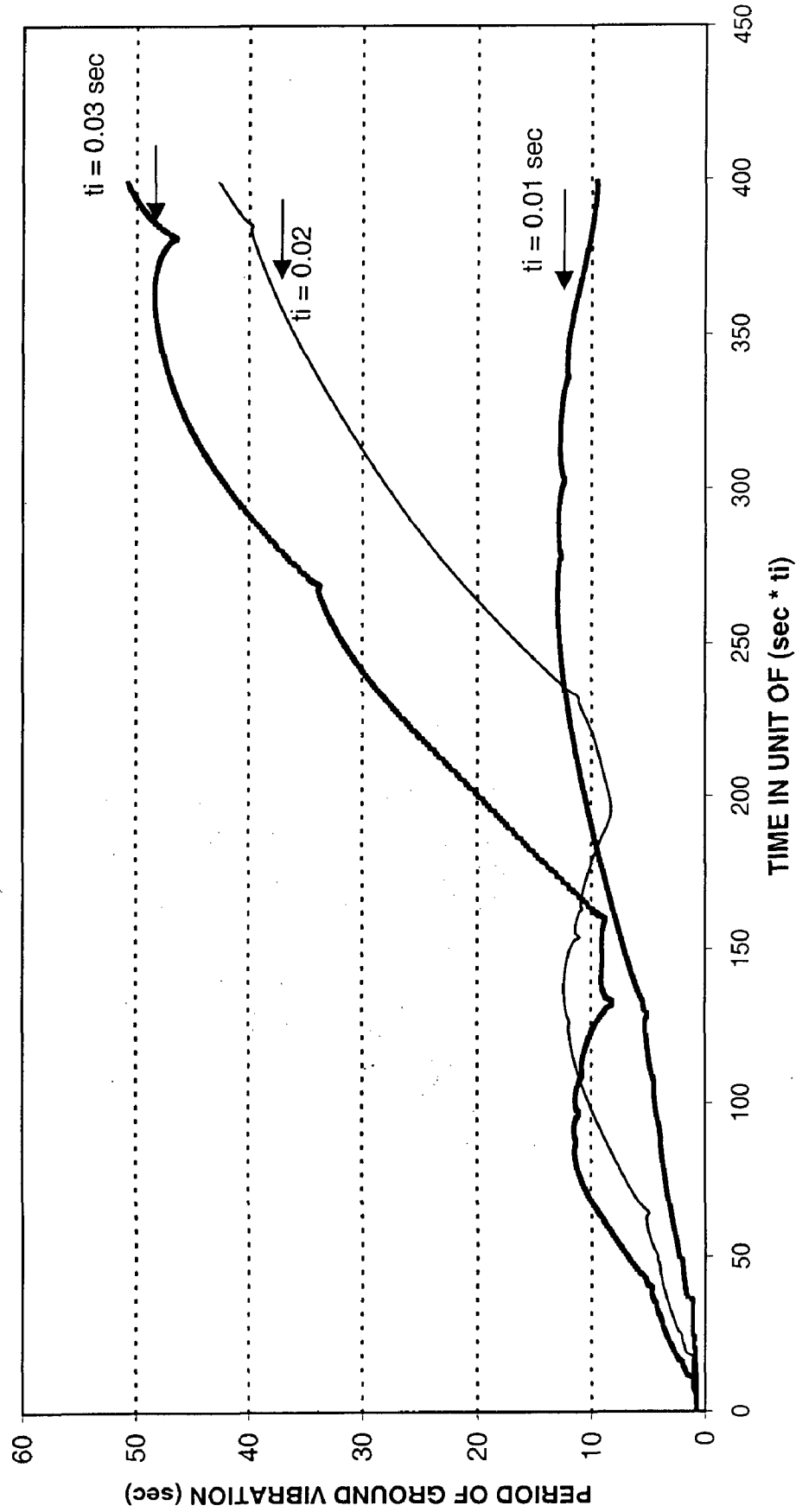


FIGURE 4.5.1 VARIATION OF PERIOD OF GROUND VIBRATION WITH TIME FOR DIFFERENT TIME INTERVAL ( $t_i$ )

the interval of 0.01sec between two consecutive time stations may be considered reasonably adequate.

#### **4.6 Influence of Amplitude of Base Excitation**

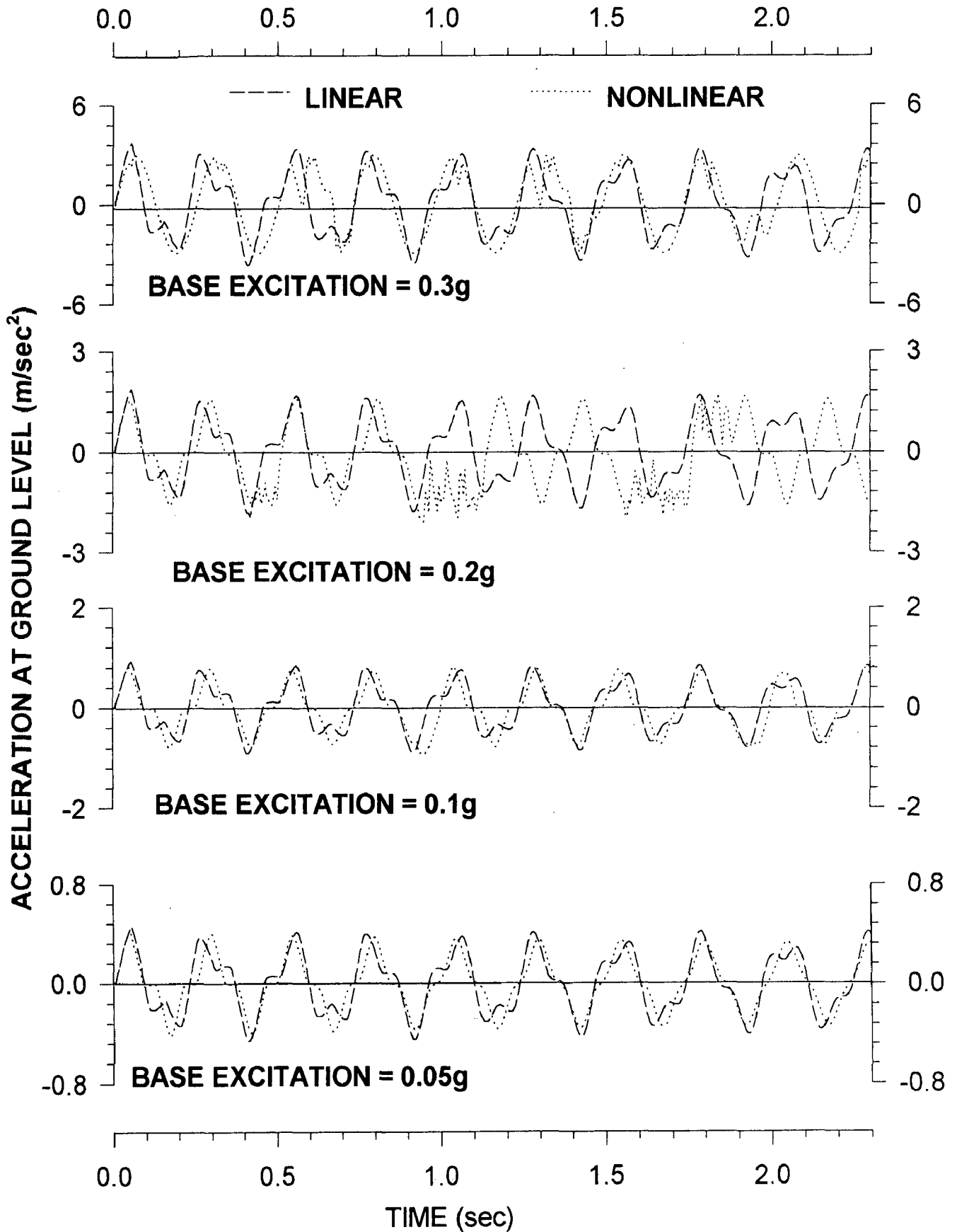
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Figure 4.6.3 shows the variation of predominant period of ground vibration  $T_g$  computed by using the relationship

$$T_g = 4H_{eq}/V_{s,eq} \quad \dots(4.6.1)$$



**FIGURE 4.6.1: VARIATION OF ACCELERATION AT GROUND LEVEL  
TIME FOR DIFFERENT BASE EXCITATION AMPLITUDE**

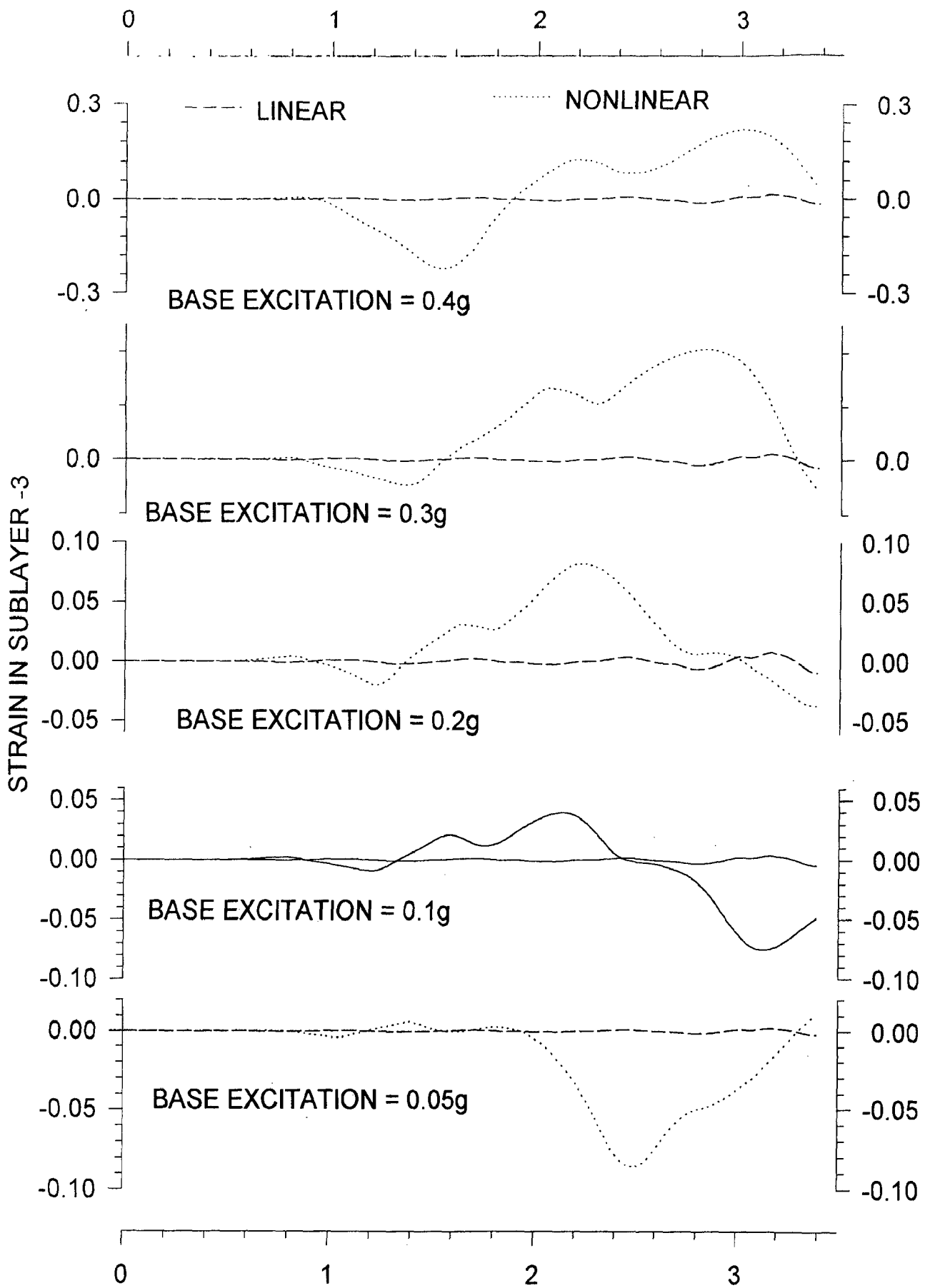
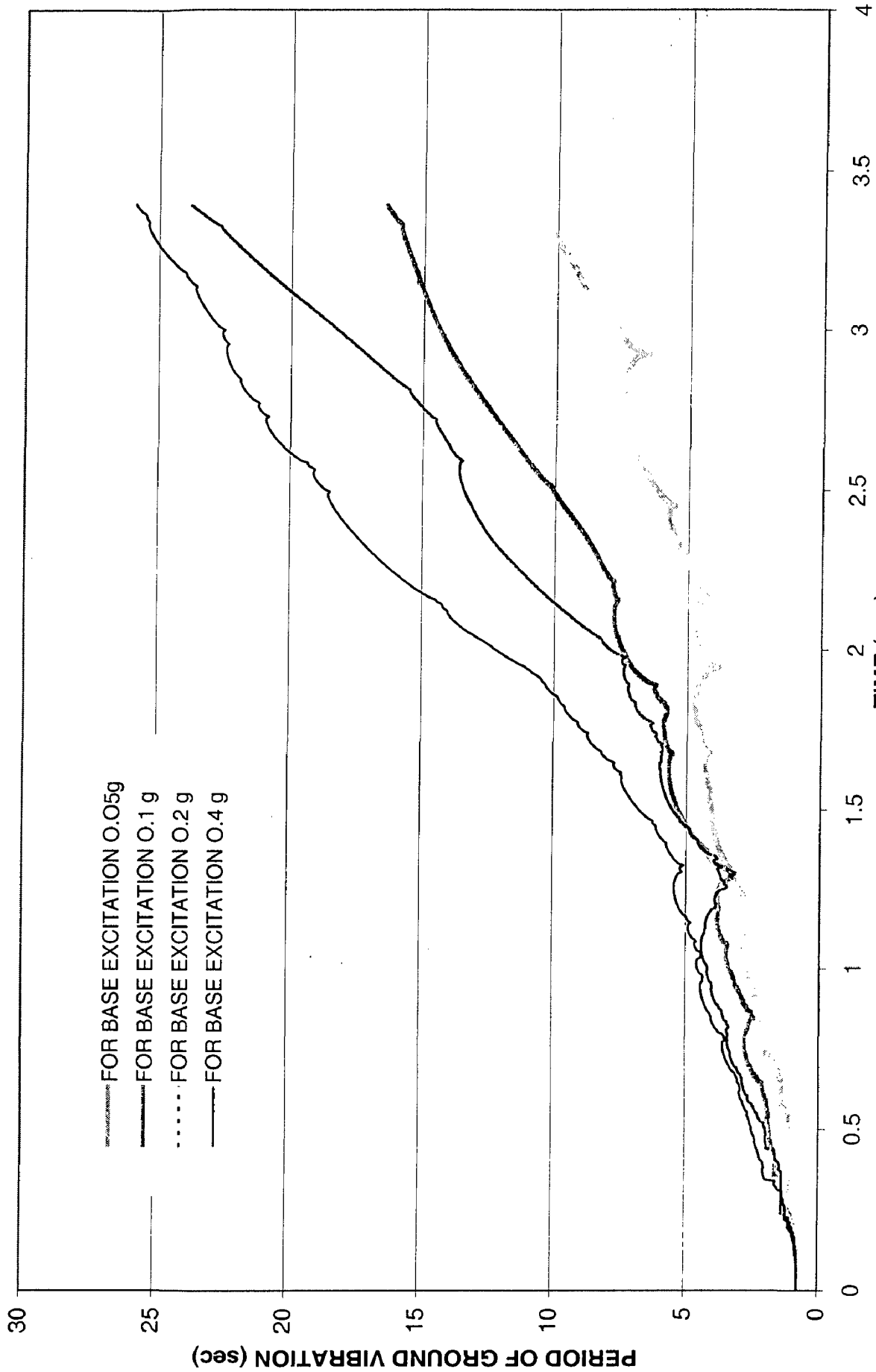


FIGURE 4.3.2 : VARIATION OF STRAIN FOR SUBLAYER -3 WITH TIME FOR DIFFERENT BASE EXCITATION AMPLITUDE



**FIGURE 4.6.3: VARIATION OF PERIOD OF GROUND VIBRATION WITH TIME FOR DIFFERENT BASE EXCITATION AMPLITUDES**

As the nonlinear behavior of the soil becomes more and more prominent, the shear modulus reduces and hence, the velocity of shear wave reduces. This results into increase in the period of ground vibration. This is clearly brought out by the increase in the period of ground vibration for all the base excitation level considered as in the above analysis. The degree of nonlinearity is higher for higher level of base excitation which is manifested in the form of largest period of ground vibration for the larger excitation which is reasonable.

Figure 4.6.4 shows the variation of mobilized shear stress as a function of time for different base excitation level. It may be observed that the shape of curve changes appreciably and the time instant of occurrence of peak stress changes considerably with change in base excitation. The level of mobilized shear stress also increases with increasing level of base excitation which is expected.

Figure 4.6.5 shows variation of peak strain amplitude computed at the mid depth of each sublayer as a function of depth below the ground level. It may be observed that with increasing depth below the ground level the value of peak strain usually reduces. This is again on the expected lines. It is also supported by actual record of seismic ground vibration from the field.

#### **4.7 Influence of Period of Ground Excitation**

Period of excitation is yet another parameter of excitation which is important. This controls the occurrence of resonance/quake resonance as associated high level of strain and stresses.



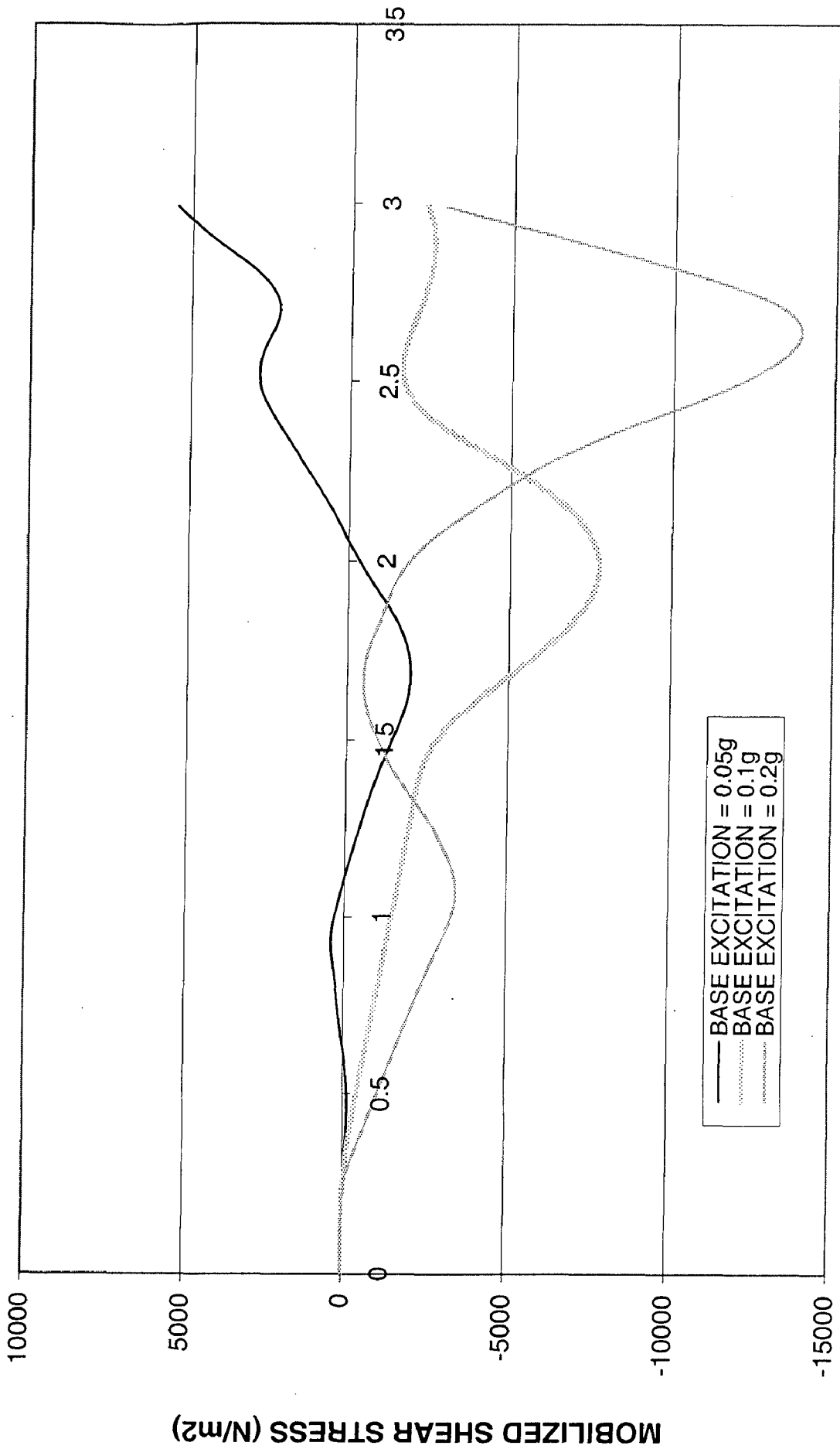
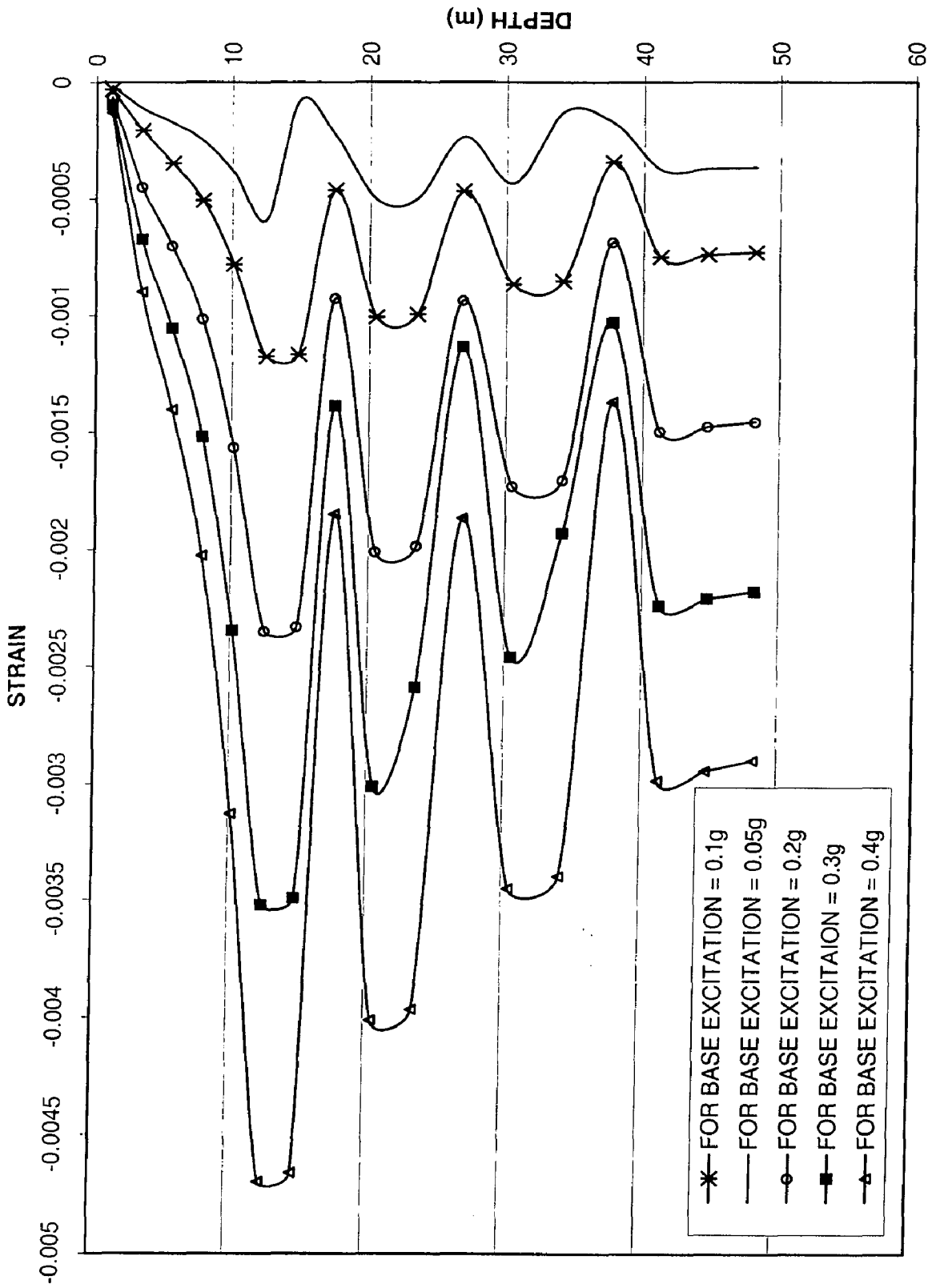


FIGURE 4.6.4 VARIATION OF MOBILIZED SHEAR STRESS WITH TIME FOR DIFFERENT BASE EXCITATION AMPLITUDE (NONLINEAR)





**FIGURE 4.3.4: VARIATION OF STRAIN WITH DEPTH WHEN -ve STRAIN IS MAXIMUM FOR LINEAR CASE**

Figure 4.7.1 shows variation of strain in sublayer three when period of base excitation reduces from 2 sec to 0.5sec. It may be observed from figure that the pattern of variation of strain alter radically with changing period of excitation. The level of strain and rate of change of strain with time vary sharply when period of ground vibration is 0.5sec, because, this excitation is very close to resonance. The strain level fall appreciably when base excitation is 0.5sec or 2.0sec which are reasonably different from resonance period of 0.75sec.

Figure 4.7.2 shows period of ground vibration as a function of time when period of base excitation varies. For all these cases the amplitude of base excitation is 0.1g. The figure shows that the period of ground vibration is highest for the case of excitation period of 0.5sec. This represents occurrence of resonance. The period of ground vibration reduces when the period of excitation reduces below the fundamental period of ground vibration. This behavior of the system is on the desired lines and hence reasonable.

Figure 4.7.3 shows variation of shear modulus with time obtained for different base excitation with amplitude of 0.1g and periods varying from 0.5sec to 4sec. It may be observed that the initial modulus of the soil is the highest value of modulus for the entire duration of excitation. As nonlinearity of soil behavior increases, the value of shear modulus keeps on reducing. This behavior is on the expected lines and hence considered reasonable.

Figure 4.7.4 shows variation of mobilized shear stress as a function of time for various base excitations. It may be observed that the largest value of mobilized shear stress changes appreciable with period of base excitation. The instant of occurrence of peak value of mobilized shear stress also changes with period of base excitation.

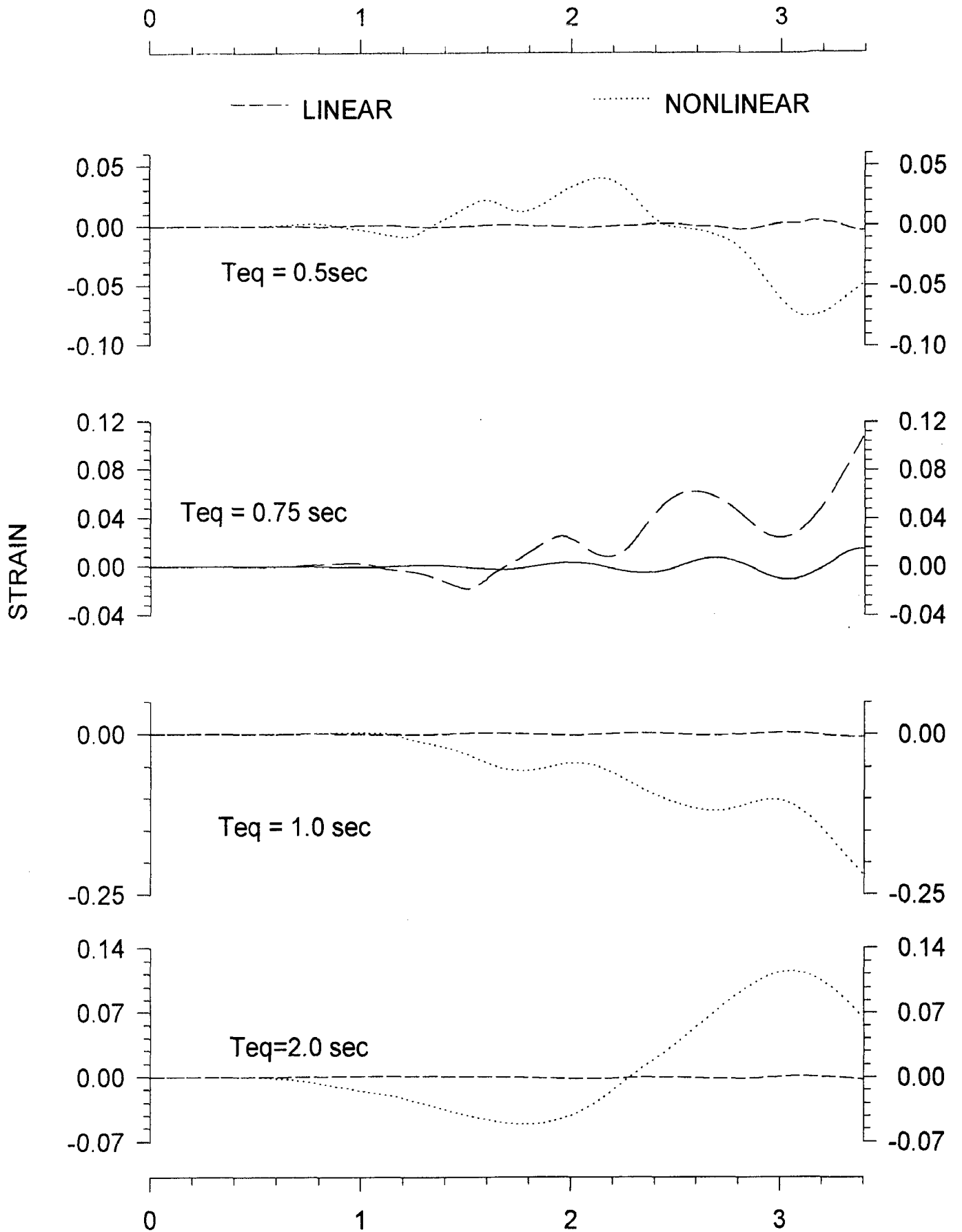
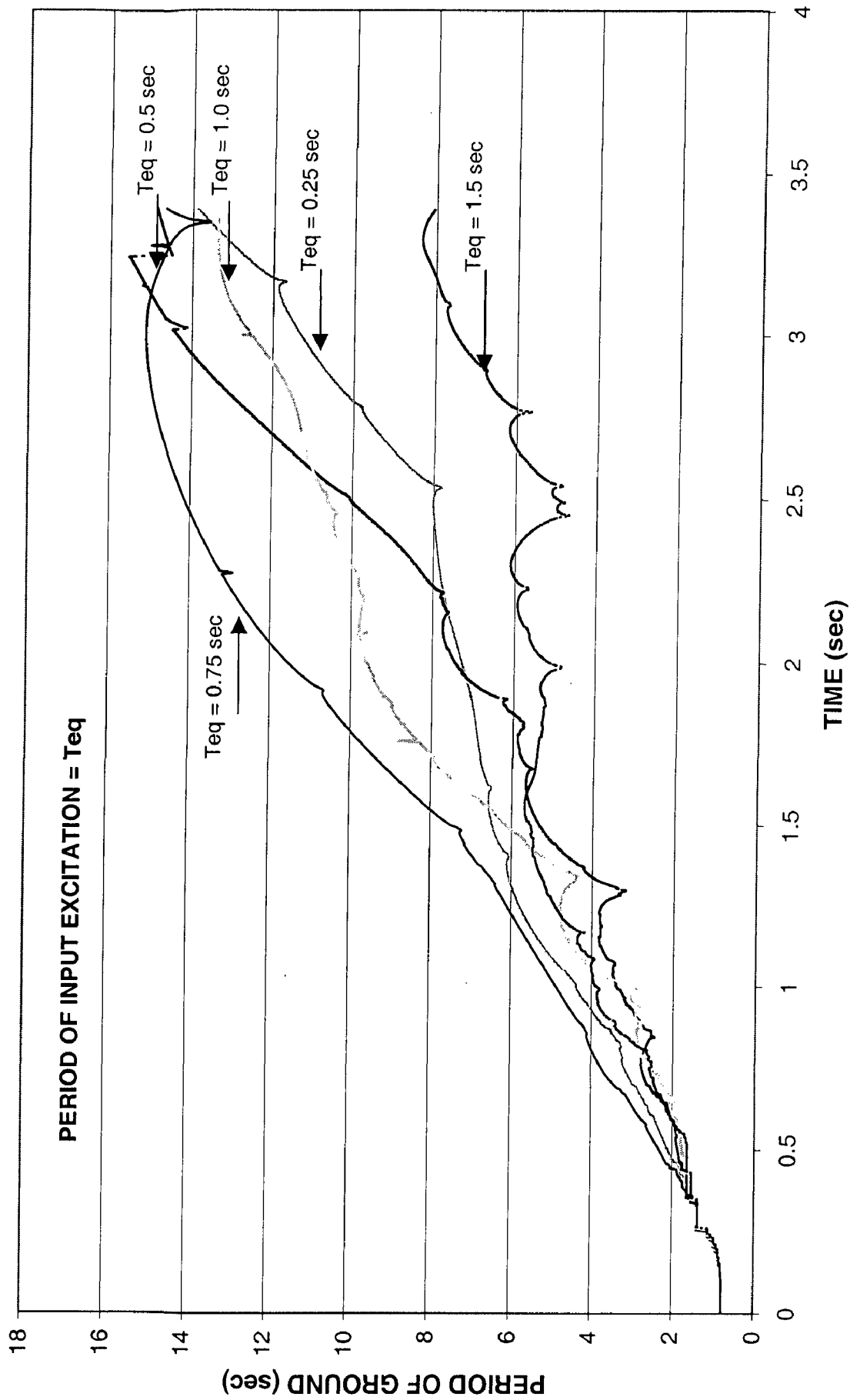
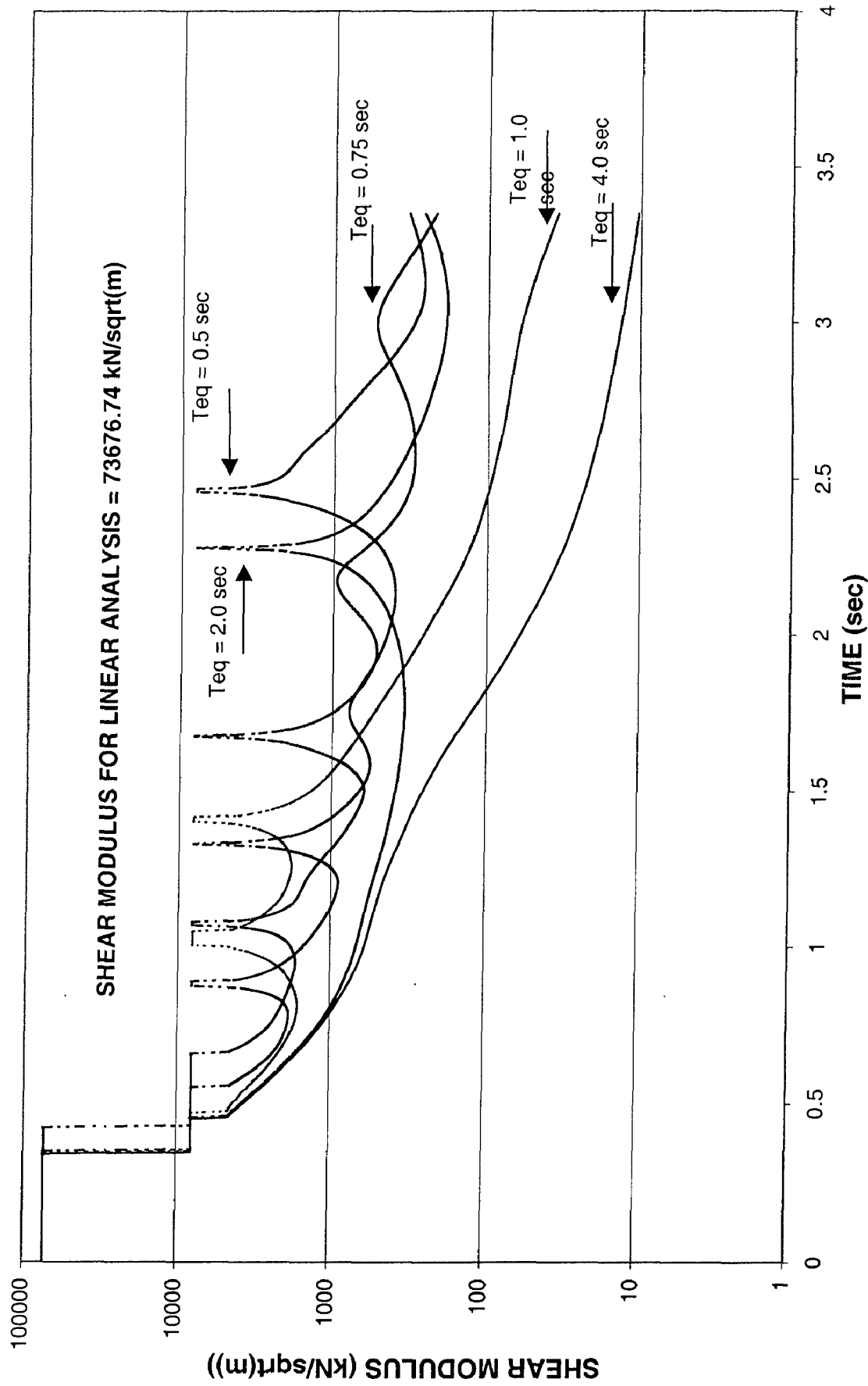


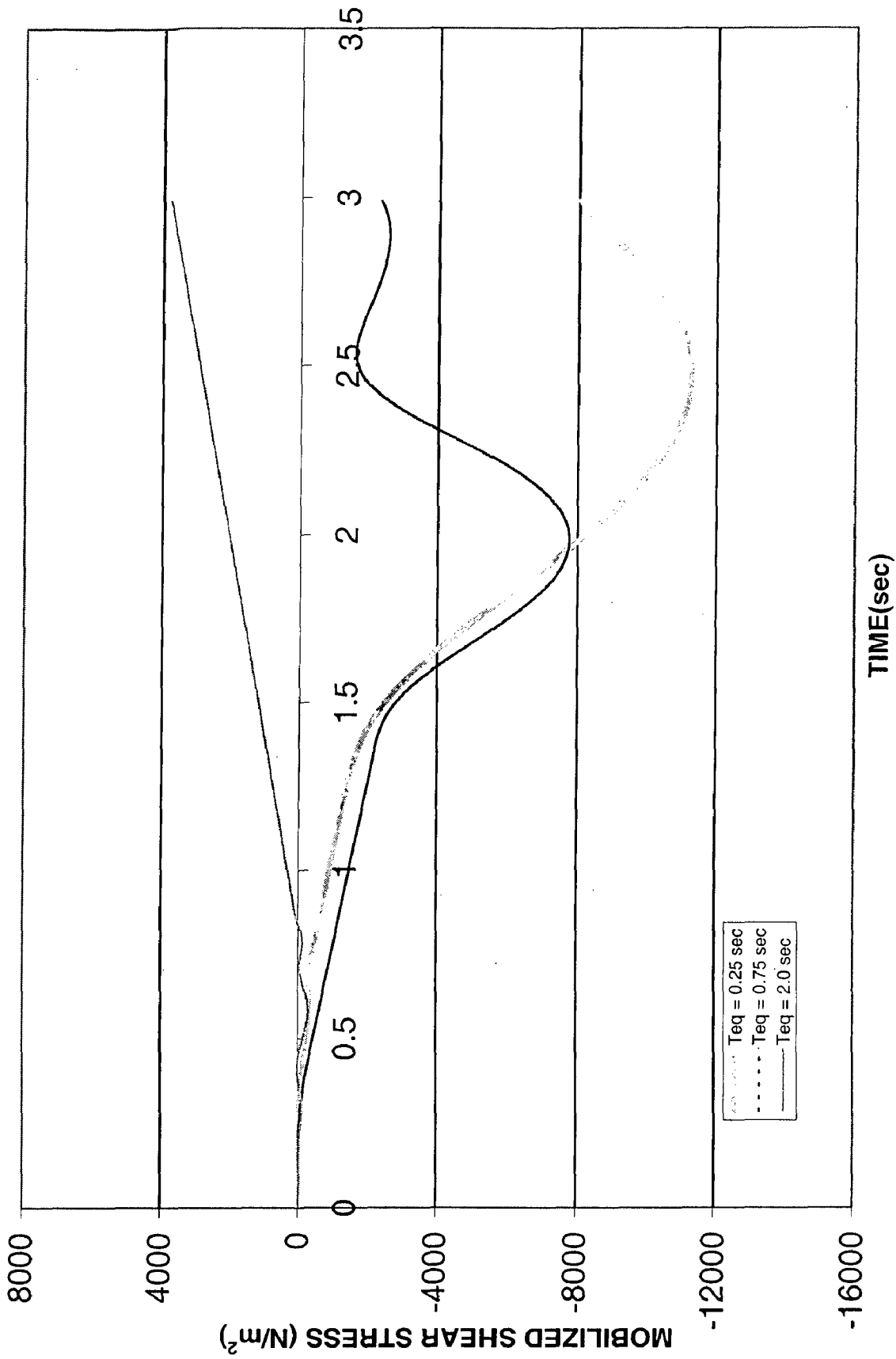
FIGURE 4.7.1: VARIATION OF STRAIN IN SUBLAYER-3 WITH TIME FOR DIFFERENT BASE EXCITATION PERIOD



**FIGURE 4.7.2 : VARIATION OF PERIOD OF GROUND VIBRATION WITH TIME FOR DIFFERENT BASE EXCITATION PERIOD (NONLINEAR)**



**FIGURE 4.7.3: VARIATION OF SHEAR MODULUS OF SUBLAYER 3 WITH TIME FOR DIFFERENT INPUT BASE EXCITATION PERIOD (NONLINEAR)**



**FIGURE 4.7.4 VARIATION OF MOBILIZED SHEAR STRESS WITH TIME FOR DIFFERENT BASE EXCITATION PERIOD (NONLINEAR)**

#### 4.8 Variation of Amplification Factor with Depth

Amplification factor for any depth below the ground level is defined as the ratio of response at the depth to the amplitude of seismic base excitation considered for the analysis. Fig.4.8.1 shows variation of amplification factor with depth obtained by linear and nonlinear seismic ground response obtained for base excitation with amplitude of 0.1g and period of 2.0sec.

It may be observed that both these curves for ground amplification factor vary linearly with depth. However, the  $\mu$  predicted by linear analysis is larger than nonlinear analysis which is on the expected lines. The figure also show variation of seismic ground response with depth as recommended by IS code. As per the code of practice, the response at a depth of 30m below ground level is 50% of the response at ground level. Below 30m depth the response is assumed to be constant and equal to response at the 30m below ground level.

It may be observed from the figure that the variation of ground response with depth predicted by the IS code is not in reasonable agreement with the computed response. Infact it under estimates for the depth of vicinity of 30m below the ground level. It is possible that the response specified by the code arbitrarily may be higher than or lower than the computed response depending on the nature of the base excitation as well as the nature of the layered system under consideration. Therefore, it is advisable to compute the variation of seismic ground response with depth below the ground level by making use of wave propagation method explained in this investigation.



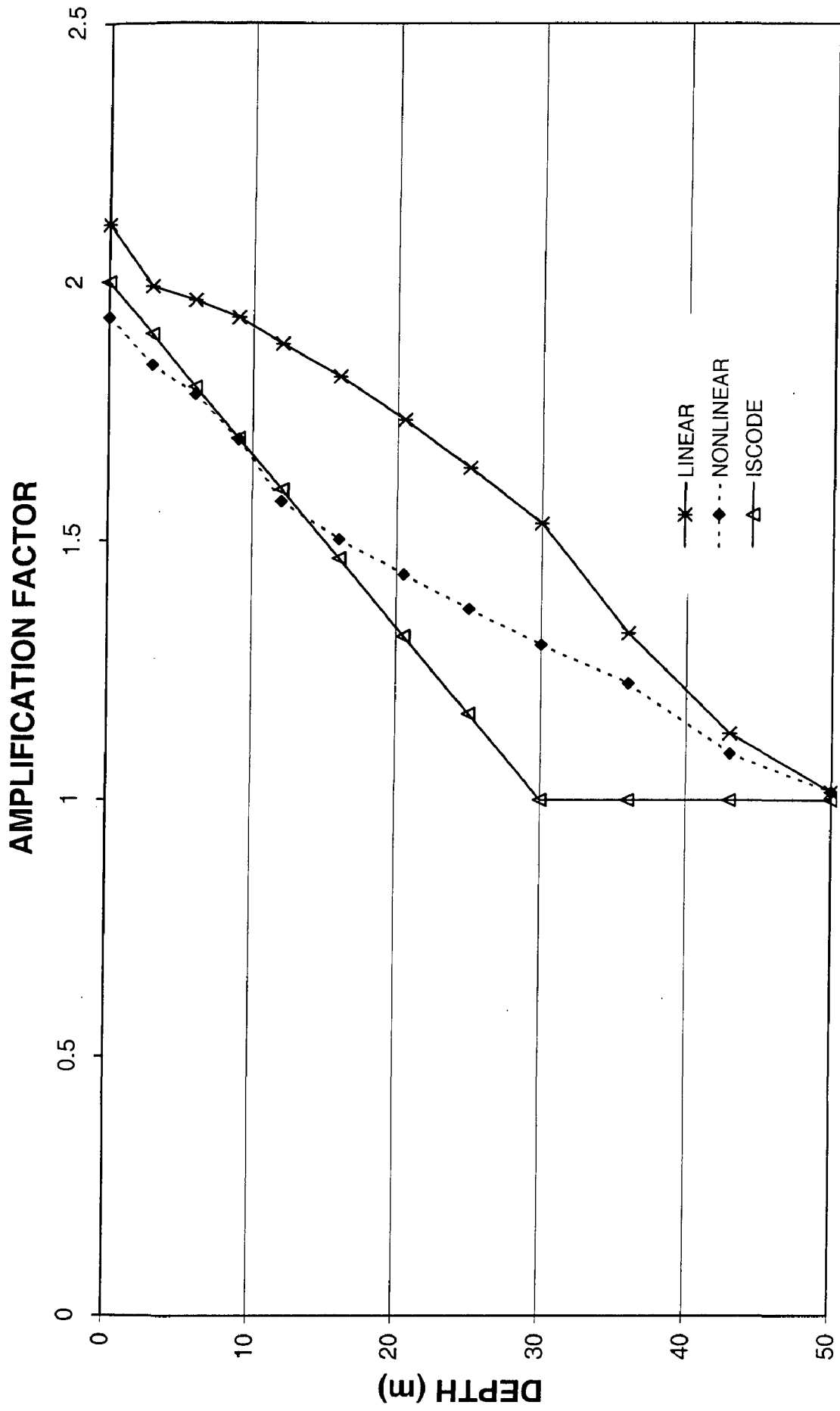


FIGURE 4.8.1 VARIATION OF AMPLIFICATION FACTOR WITH DEPTH

#### **4.9 Merits of Nonlinear Analysis for Computing Seismic Response**

Soil is a relatively soft material. Its properties vary considerably with depth below the ground level even though its relative density may be the same. Moreover, even for the same soil and the same depth below the ground level the shear modulus of the soil is strain dependent and decreases with increasing strain level. The influence of the ground water table on the material properties of the soil is also significant, because the presence of ground water significantly reduces the effective pressure at any depth below the water table. The reduction in effective stress results into reduced shear modulus.

When seismic base excitation varies with time it sets into motion the seismic wave which induced different levels of accelerations, velocity, displacement, stresses and strain various depth below the ground level at various instant of time. Therefore, the continually changing shear modulus and hence shear wave velocity have to be accounted for accurately in the computation of seismic ground response. It is difficult to choose equivalent properties of the soil which will remain the same for the entire duration of the earthquake. Computed ground response using linear analysis may lead to significant error particularly when the level of base excitation is high and the period of base excitation is comparable to the natural period of ground vibration. Under such conditions the linear analysis would give rise to very large inaccuracies in the computed response. Hence the nonlinear analysis of seismic ground response assumed great importance particularly for important structures like nuclear power plants etc. This is also important for the structure with large structures like dams, bridges, tunnels etc.

With passing time the quality of computation facilities available is improved significantly in terms of speed of computation and storage space available.

### CONCLUSIONS

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The following are the conclusions drawn from the study:

- a. Nonlinear analysis is required for soft layer seismic response.
- b. Even through base response may be small the ground response may be high resulting into nonlinearity due to amplification.
- c. Plane wave propagation method is ideal for seismic response analysis as it accounts for radiational damping completely.
- d. The method of computation of component response due to upward and downward propagating shear wave for a given seismic response history at the base rock level recommended by the Joshi (1980) with due consideration to compatibility conditions at the base rock level is useful in analysis of seismic ground response of layered system.
- e. Stress-Strain properties of the soil to obtained from experimental investigation. If this is not available, artificial stress-strain relationship may be considered in the form of elliptical, parabola or composite curve with flat slope at the failure strain.
- f. The computer program SENERO developed for obtaining spatial/temporal distribution of seismic ground response. This program is easy to use and efficient to handle the response of any system with horizontal layers with due regard to nonlinear behaviour. The program may be run on commonly available personal computers. As such it can be used for obtaining the design seismic data for analysis and earthquake resistant design seismic design of structure by various design office/research organizations.

- g. The computer program the following quantities for linear as well as nonlinear analysis.
- 1) History of acceleration, velocity & displacement at selected or overall points within the system at the discretion of the user.
  - 2) The history of strain, mobilized shear stress, mobilized shear modulus of each sublayer of the system.
  - 3) History of the fundamental period of equivalent single layer system.
  - 4) History of average strain in any main layer.
  - 5) +ve and -ve maximum value of strain of sublayer anywhere with in the system and the instant of time of their occurrence as well as the variation of strain in the entire layered system at that instant of time.
- h. For a given sinusoidal base excitation, the acceleration increases with increasing base excitation amplitude as long as nonlinearity does not come into play. Once the nonlinearity begins to appear, the rate of increase of the amplitude of seismic vibration will increase at a much slower rate with the increase of base excitation. In contrast, the amplitude of ground acceleration predicted by linear analysis increases considerably with increasing base excitation amplitude to unrealistically high values.
- i. Th strain level obtained by nonlinear analysis is much larger than those obtained by linear analysis. This is reasonable, because, with increasing nonlinearity the material becomes softer and gets easily deformed resulting into large deformation and strain.
- j. The period of vibration of the equivalent layer remains unchanged as long as the system vibrates in the elastic domain. When nonlinear behaviour is manifested, the material becomes softer and the value of shear modulus decreases with increasing strains. This results into the fluctuation in the value of the shear modulus with time. The larger the degree of nonlinearity, the greater the period of vibration of the system.

- k. The maximum strain amplitude occurs in the softer layers near the ground level which is expected. With increasing depth below the ground level where velocity of shear wave is larger, the strain levels decreases with depth. Besides, the strain levels generally increases with increasing amplitude of acceleration of base excitation . this effect is much more pronounced in case of nonlinear analysis.
- l. With decrease in the time interval between two consecutive time stations, the accuracy of the determination of the response reasonably increased.
- m. When period of sinusoidal base excitation is varied from 2.05 sec to 0.5 sec the period of single layer system changes considerably due to non linear behaviour. Importantly the computed ground period appreciably increased, when the excitation period is very close to fundamental period of system. This is basically due to occurrence of resonance/quasi-resonance condition prevailing. For such a case the resultant strains are very large and the mobilized value of the shear modulus are very low.
- n. The amplification factor near the ground level is generally larger than that at the lower depth below where the layers are generally stronger. The decreasing value of amplification factor obtained by the analysis clearly indicates this phenomenon. The amplification factor obtained by the linear analysis are in general larger than obtained by the non linear analysis, which is also expected. Moreover, most of the build up of  $\mu$  is mostly with in the top to soft layers of the system which is reasonable.

### SUGGESTION FOR FUTURE RESEARCH

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The objective of the investigation have been to make a computer program for nonlinear analysis for obtaining spatial/temporal distribution of seismic ground response. Only limited parametric studies have been carried out. Therefore, an extensive parametric study may be considered for future research using this program. Only sinusoidal base excitation considered in the results presented. It would be desirable to use the actual seismic ground response obtained for a variety of earthquakes in carrying out the future studies. Besides, the propagation of plane waves in XZ as well as YZ plane may also be incorporated for obtaining the three-dimensional seismic ground response.

In the proposed analysis only shear waves are accounted. The primary waves are neglected which is justifiable in most of the cases. However, the Rayleigh waves are quite significant and may exist simultaneously with the shear wave. Therefor, suitable method has to be formulated to segregate the contribution of response due to shear waves and Rayleigh waves to account for them separately.

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**COMPUTER PROGRAMME**  
**“SENORE”****INTRODUCTION**

Shri A. K. Srivastava originally prepared this program for his M.E. Thesis (1995) entitled “Nonlinear Response of Seismic Ground Response”. It was written for running it in Unix system. However, in view of the prevailing DOS system being used in the office now, the programme has been re-written to run it in DOS. Besides, the programme of Shri Srivastava was not written to consider certain design considerations and input data. This has been rectified in the revised programme. He carried out only limited parametric studies. The present investigation aims to make up for this shortcoming to complete the parametric studies.

**THEORETICAL CONSIDERATIONS**

1. Considers numerical analysis in time domain using plane shear wave propagation.
2. Nonlinear properties of layers considered by using strain dependent shear modulus (secant modulus) at the end of each time station of the analysis.
3. Digitized stress-strain curve to obtain strain dependent  $G$  is obtained either from tests or generated artificially using appropriate stresses and strains at failure. A dimensionless form of digitized strains at uniform strain intervals of 0.001 considering unit failure shear stress and unit failure shear strain is employed. The actual shear stress is obtained by multiplying the corresponding strain coordinate by the actual failure strain of the material and the corresponding shear stress by multiplying the corresponding digitized shear stress.
4. The digitized acceleration considered at base rock level only. Time interval used is 0.005, 0.01, 0.02 and 0.03 second. Displacements at end of each time station are obtained by double integration of the acceleration curve using standard numerical methods (willson  $\theta$  method). Using the displacements, the average shear strain is computed and based on this

the strain dependent secant modulus of shear as well as the velocity of shear wave propagation are obtained. The reflection and transmission coefficients are worked out again to carry out the wave propagation analysis till the complete time history of the input acceleration is completed.

5. Kanai magnification factor and period of fundamental ground vibrations are computed.
6. Each layer is subdivided into smaller sub-layers such that time of travel in any layer is not less than the time interval considered in wave propagation analysis and also its thickness is not more than one tenth of the wave length computed for equivalent single layer system with equivalent velocity of shear wave propagation.

## MAIN PROGRAMME

The programme is written in Fortran language. Most of the variables are placed in commons. Only accelerations are declared as dimensioned arrays. To reduce chances of running programme with wrong data, various **traps** are devised which stop programme with suitable error messages in main as well as in various subroutines. While making computations, programme always uses angles in the form of radians only. The dimensions are adequate to accommodate 20 layers and 100 sub-layers. The digitized stress strain curve has 1000 divisions of unit failure strains and corresponding unit failure stresses. All formats are bunched together at the end of the main. Statement numbers smaller than 100 are reserved for formats. Statement number 100 and larger than 100 are used for the rest of the main programme. Some important do loops are given special statement numbers like 100, 1000, 2000 etc. for easy identification. While computations are in progress, quantities are in Newtons only.

Various subroutine use in this program name as Divide, Gdigit, Kanai, Tausig, reftra, timer, bndrev, inixil, tsreve, timrev, vcdca4, base, baupin and badoin.

## **Input Data**

Total no. of main layer (n), time interval (ti), depth of water table (wtb), no of boundary point, maximum horizontal dimension (aln) read as input data. Various soil properties such as angle of shearing resistance (p(i)), failure strain (ga(i)), cohesion(c(i)), specific gravity (sg(i)), void ratio(e(i)) and water content (wc(i)) read in Main.

There are some redundant input data. For base rock, depth, angle of shear resistance, cohesion and failure strain is all cited as zeros in input data file. These are never used in computations. This is only for satisfying computer data reading only.

## **Output Data**

Computer performs computations with angles in Radians. It converts angles from radians to degrees just before routing the angles to printer. All input data is printed with necessary hollarith captions for easy understanding. Pressures, moduli, forces etc involving Newtons are converted into kilo Newtons before routing the data to printer. Suitable hollarith caption shall appear before any data is printed out regarding the force being in kilo Newtons etc.

## **SUBROUTINE TAUSIG**

This routine is called from the Main Program, before any other routine is called. It computes static effective stress (signe), total normal stress (sigt), pore water pressure (pwt) and limiting shear stress (ta) at mid depth of main layer. It also computes initial dynamic shear modulus for main layer as a function of octahedral normal stress.

## **SUBROUTINE KANAI**

Kanai computes equivalent single layer properties like equivalent depth, heq, equivalent shear wave velocity, beq, and equivalent unit weight, req. Thicknesses of main layers are used as

weighting function in these calculations. The fundamental period of vibration of the equivalent single layer,  $t_{nka}$ , is obtained as sum of  $(4 * dp(i)/bt(i))$ . Kanai magnification factor,  $amka$ , is also computed.

Fundamental period,  $t_{nj}$ , proposed by Dr. Joshi, is computed as  $[4 * heq/beq]$  where  $heq$  and  $beq$  are computed as cited above. Amplification factor proposed by Dr. Joshi is computed by using the expression proposed by Kanai, but by using  $t_{nj}$  in place of  $t_{nka}$ .

Kanai also computes other significant frequencies of layered system using  $(4h/Vs)$ . For this, cases of first layer only, then first and second layers, then first, second and third layers, then first, second, third and fourth layers and so on and so forth. The approach is similar to that of Kanai and Joshi approaches cited above.

## **SUBROUTINE DIVIDE**

It divides each main layer into smaller sub-layers for nonlinear/linear analysis. Besides, using equivalent shear wave velocity and fundamental period of vibration of equivalent single layer system computed in the SR Kanai, wave length,  $\lambda$ , given by  $\{bt(k) * t_{nka}\}$  may be computed. Thickness of any sub-layer,  $k$ , should not be more than  $(\lambda/10)$ .

It computes static effective stress ( $sge$ ), total normal stress ( $sgt$ ), pore water pressure ( $pw$ ) and limiting shear stress ( $\tau_{ul}$ ) at mid depth of sublayer. It also computes initial dynamic shear modulus for sublayer as a function of octahedral normal stress. Initial strain level considered as 0.0001. Based of this strain level, computes the slope( $sn(k)$ ) in elastic range, strain station no. ( $in(k)$ ) upto which soil behave like elastic material and corresponding shear wave velocity ( $b(i)$ ).  $dsg(k)$  is computed as  $\{h(k) * \text{unit wt. of soil}\}$  and is supposed to be total vertical stress caused by sub-layer  $h(k)$ .

## **SUBROUTINE GDIGIT**

This routine digitizes the stress-strain curve common to all main layers and with unit ultimate shear stress and unit failure shear strain. The range of unit stress from 0 to 1 is divided into

1000 equal divisions of  $d_{\text{gam}}=0.001$  and corresponding shear stresses ranging from 0 to ultimate shear stress of unity are also worked out.

The composite curve using normalized shear stress and normalized shear strain is supposed to be a composite curve with straight line upto strain level  $1 \times 10^{-4}$ , three degree function passing through the GACO and elastic limit and circle with unit normalized shear stress and unit normalized shear strain. The three degree function and circle are to have a common tangent at GACO, the common point shear strain where circle and parabola meet each other. The present program is written this way.

It may be noted that the failure shear stress for loose soils is of the order of 15-20%, for medium dense soils it is from 5-10% and for dense soils it is from 3 to 5 %. Even for 3% (i.e. 0.03) failure strain, the strain at GACO is  $(0.14) \times (0.03) = 42 \times 10^{-4}$  which is much more than  $1 \times 10^{-4}$ . Therefore, there is practically no chance that for common dense soils, GACO will be any where close to  $10^{-4}$ . It is much less likely to occur for medium dense and loose soils.

### **SUBROUTINE REFTRA**

This should be called only after SR GDIGIT. It computes reflection & transmission coefficient for interface of each sublayer. Assume base rock level is infinitely deep. Coefficient of downward reflection & downward transmission are unity at ground level.

### **SUBROUTINE TIMER**

Timer is called repeatedly in nonlinear analysis. In this subroutine compute the travel time of wave in term of  $t_z(i)$ . It gives the travel time as wave to reach wave to sublayer. After it convert  $t_z(i)$  in  $t_i$  unit as a  $m_z(i)$ . It computes  $ndats$ ,  $nsum$ ,  $nsuml$  and arrays  $nd$ ,  $rd$  and  $rt$ . At  $m_z(i)$  response at interface is zero because wave don't reach at the interface in time  $m_z(i)$ . So for computation of response at interface interpolation should be done. We considered both vertical as well as inclined

propagation of wave.  $n_{sum}$  is a cumulative travel time of wave,  $n_{temp}$  is a time to travel extra distance due to inclination of wave in base rock level.  $n_{dats}$  give the total travel time of wave from base rock level to ground level & ground level to base rock level.

#### **SUBROUTINE BNDREV**

In the SR `bndrev` compute the travel time of wave to reach upto boundary point. We deal with three-dimensional case. Boundary point may be in between the sublayer. So computation of response at boundary point is determine by the interpolation. The coordinates are choosen in such a way that all the coordinates are positive. i.e. the entire size of problem lies in the first quadrant of coordinate axis. Consider origin at GL and positive direction of waves traveling in downward direction. None of the  $x(i)$  &  $y(i)$  coordinate is larger than maximum horizontal dimension ( $aln$ ) and none of the  $z(i)$  coordinate is larger than total depth ( $tdp$ ).

#### **SUBROUTINE INIXI1**

This sr consider when  $ld$  equal to 0. i.e.  $xz$  plane response only. Set initial old and current value of acceleration, velocity and displacement equal to zero for time station  $n_{dats}$ . And set initial value of vertical acceleration  $a1(i), a2(i), a3(i)$  and  $a4(i)$  equal to zero and set current shear strain  $gamc(i)$  and old shear strain  $gamo(i)$  arrays to zero for all sub-layers.

Initialization the above value equal to zero is done for safety purpose if any value become nonzero and we considered it as zero initial value, than, they give incorrect result.

#### **SUBROUTINE VCDCA4**

Vertical accn  $a4(i)$  computed in this sr. First determine the  $a1(i), a2(i), a3(i), a0(i)$  and  $ac(i)$  for each time station. By old and current value of acceleration compute the current value of velocity and displacement by using the WILSON  $\theta$  METHOD. The basic assumption of this method is that

the acceleration varies linearly over the small time interval and is determined to obtain optimum stability and accuracy characteristics. At time  $t$ , velocity is  $v_o$  and displacement is  $d_o$  and for  $t+t_i$  time velocity is  $v_c$  and displacement is  $d_c$ .  $a_c$ ,  $v_c$  and  $d_c$  are current horizontal acceleration, velocity and displacement respectively and  $a_o$ ,  $v_o$  and  $d_o$  are old horizontal acceleration, velocity and displacement respectively.  $a_1, a_2$  and  $a_3$  are vertical acceleration at  $(fnt-2)*t_i$ ,  $(fnt-1)*t_i$  and  $fnt*t_i$  time station. Values of  $a_1, a_2, a_3$  are known at time  $fnt$  already. Hence,  $a_4$  at time  $(fnt+1)$  can be computed by extrapolation.

### **SUBROUTINE TSREVI**

This subroutine revises shear stress  $\tau_{ul}(i)$  at mid-depth of sub-layer with due consideration to vertical acceleration in nonlinear analysis.  $d_{sg}(i)$  is mass of column of  $i$ 'th sublayer of  $1m*1m$  size and thickness of column equal to thickness of sub-layer. In this we determine the inertia force. Effect of vert accn  $a_4(i)$  at mid depth computed in  $vcdca4$  to revise is cumulative inertia forces. Shear stress is revised with effect of inertia forces. For this cohesion  $c(i)$  read in main is added to  $(sge+inertia)*\tan(p(i))$  to get  $\tau_{ul}(i)$  at mean depth of respective sublayer. Inertia force computed for each sublayer separately after it computed cumulative inertia force.

### **SUBROUTINE TIMREV**

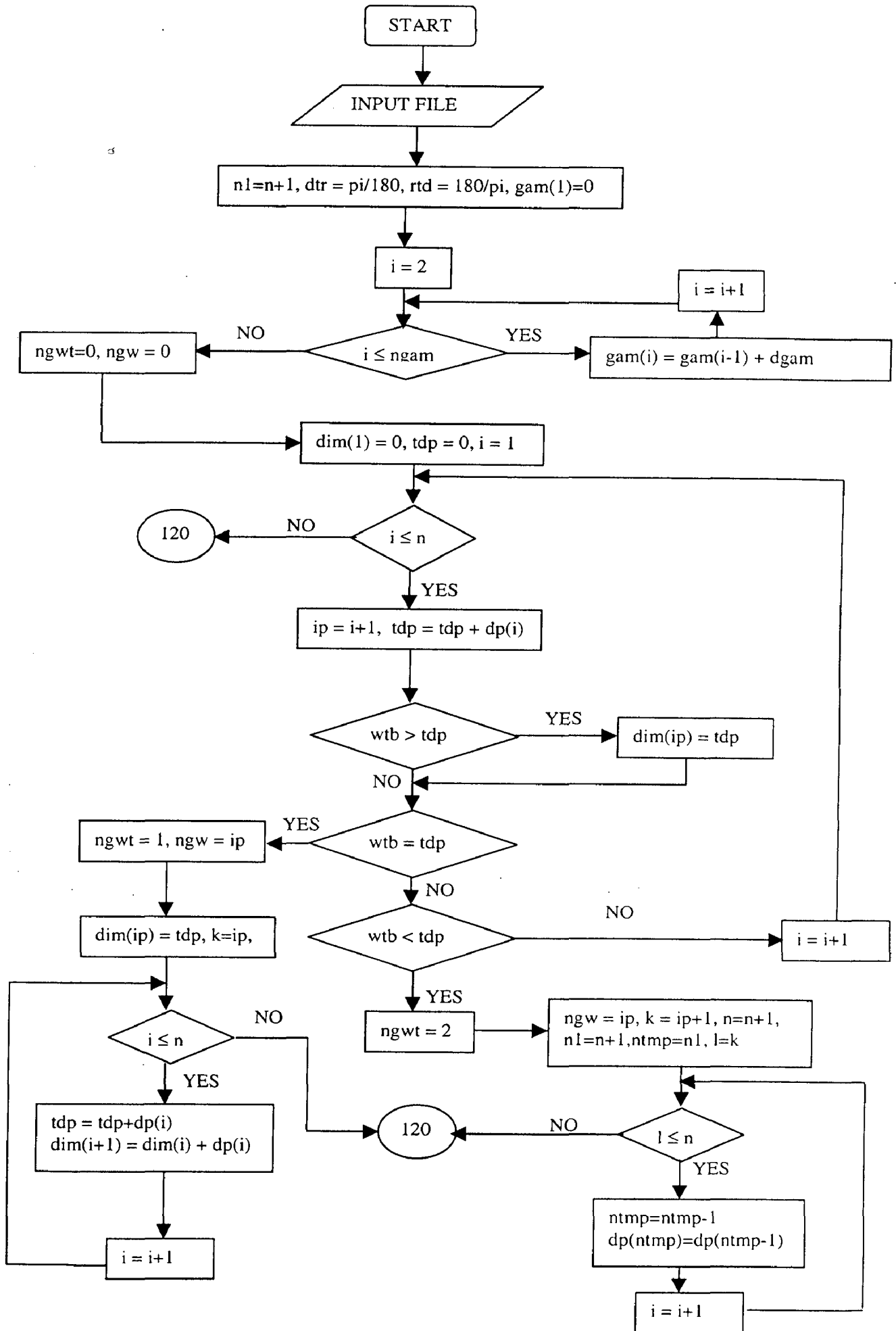
In this  $sr, nd, rd, rt, th, tz$  and  $mz$  computed again with revised shear wave velocity,  $b(i)$  when changes in  $b(i)$  with nonlinearity. This  $sr$  is similar to  $Timer$  except that  $b(i)$  is revised velocity in this  $sr$  &  $b(I)$  is initial velocity in  $Timer$ .

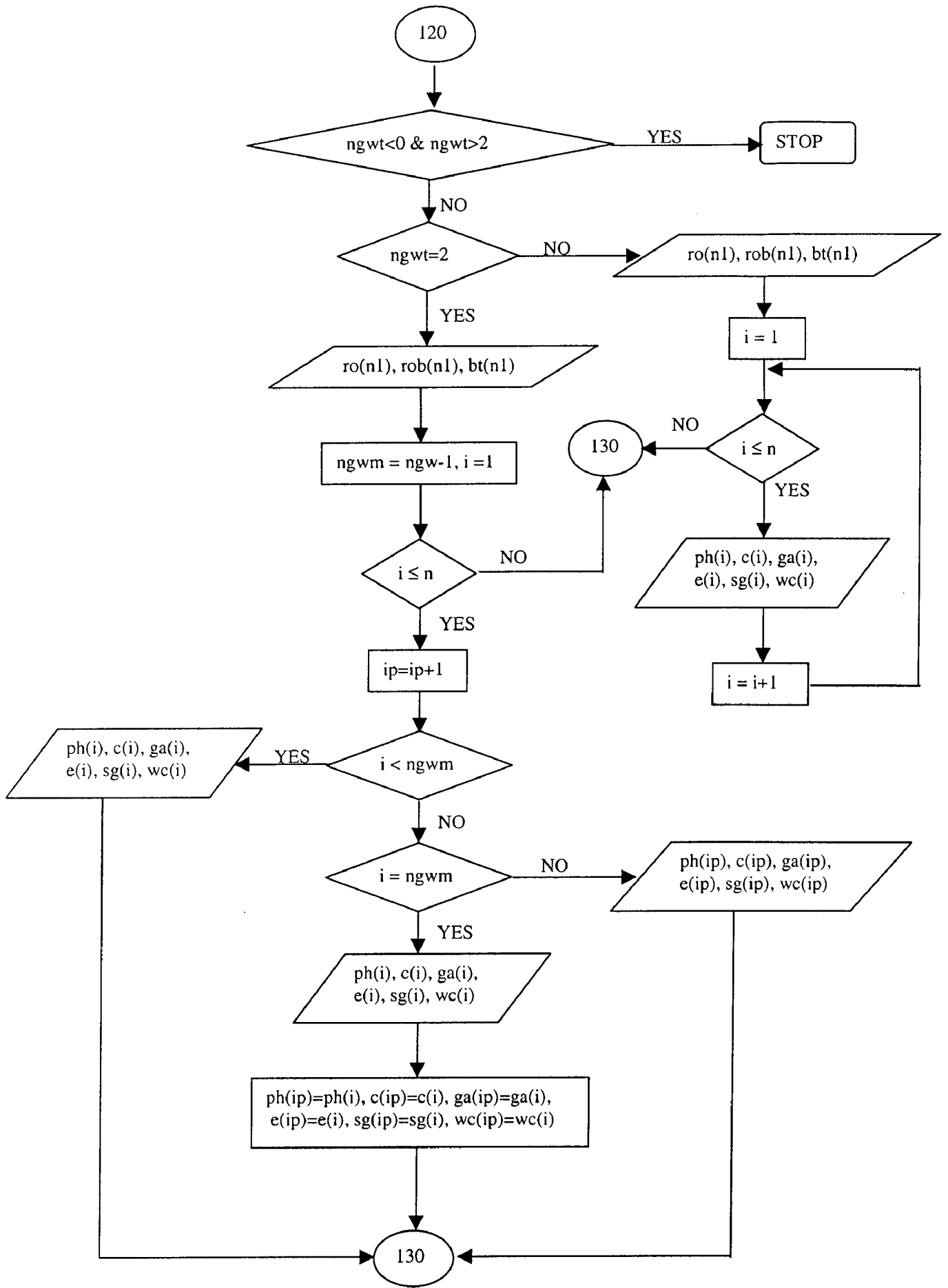
### **SUBROUTINE BASE**

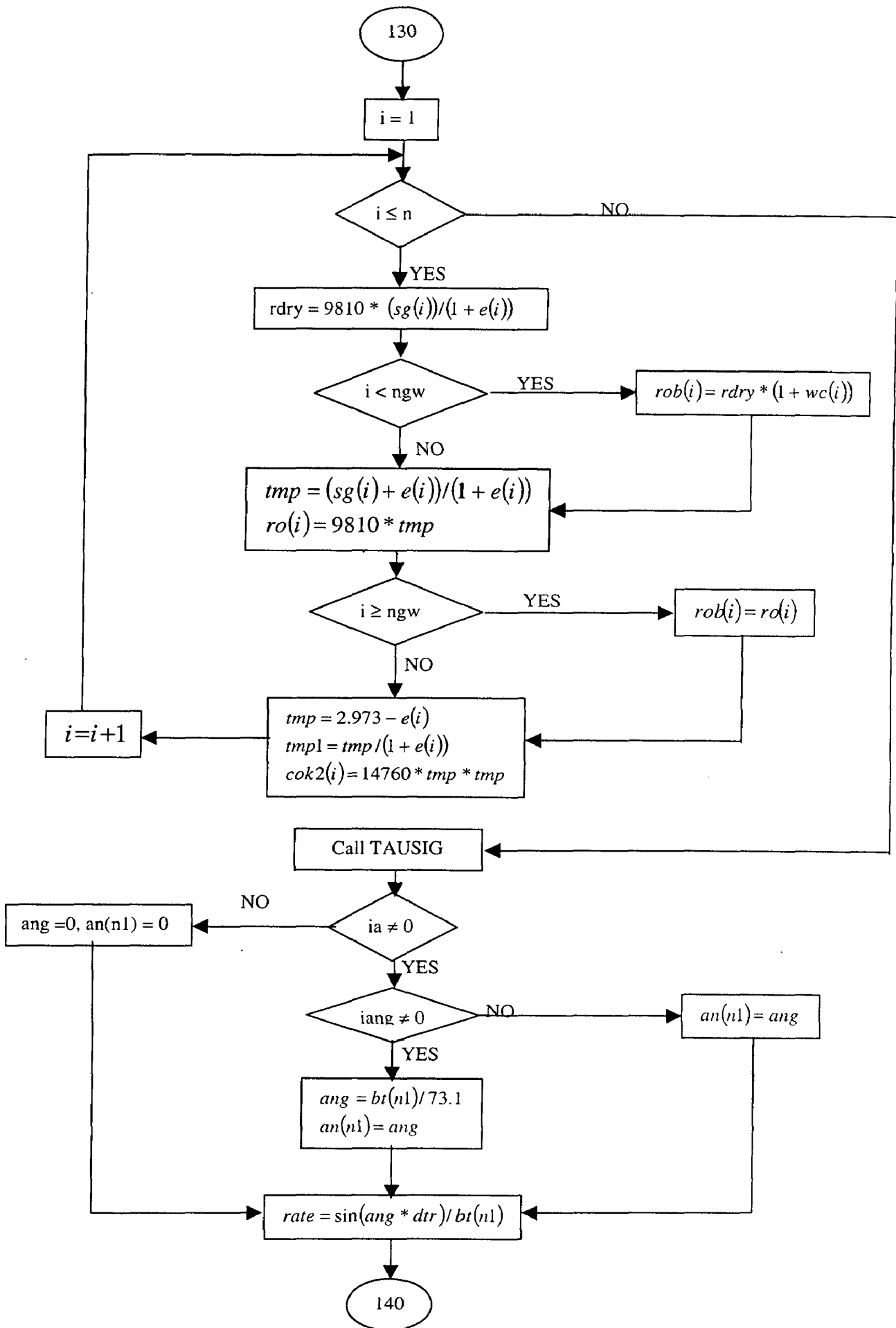
This subroutine computes the acceleration at interface of sublayer due to upward and downward traveling wave.

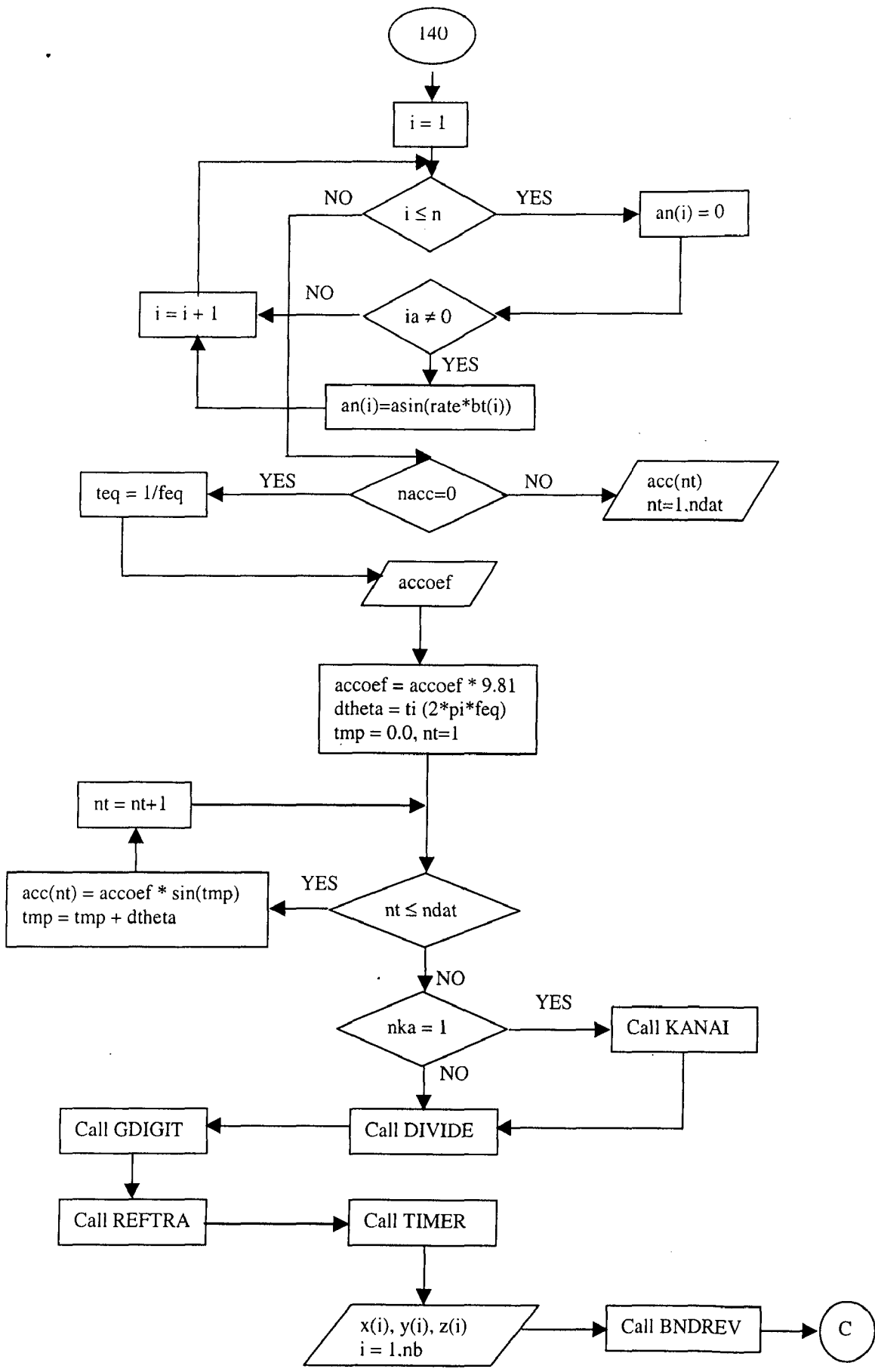


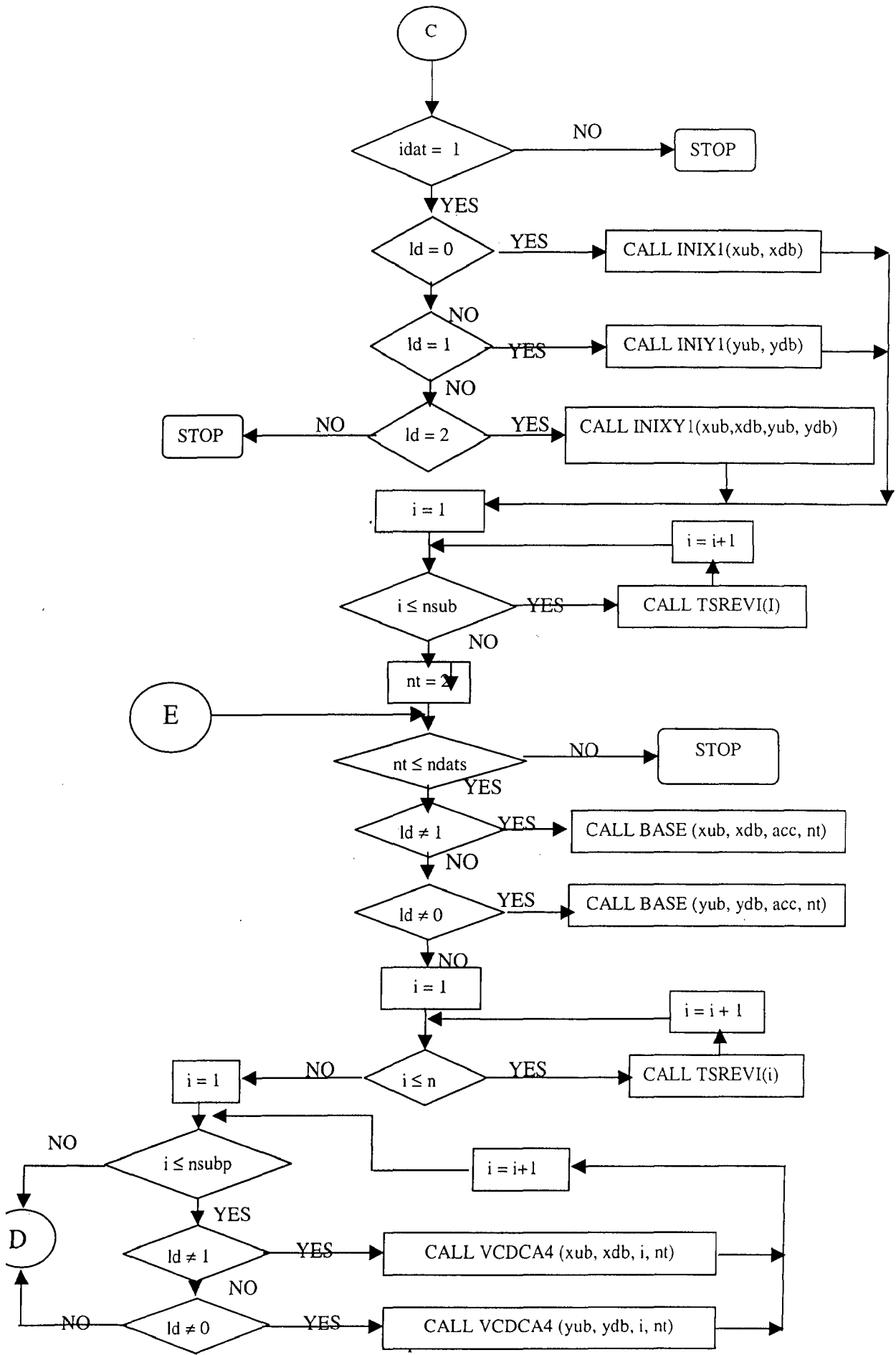
# FLOWCHART

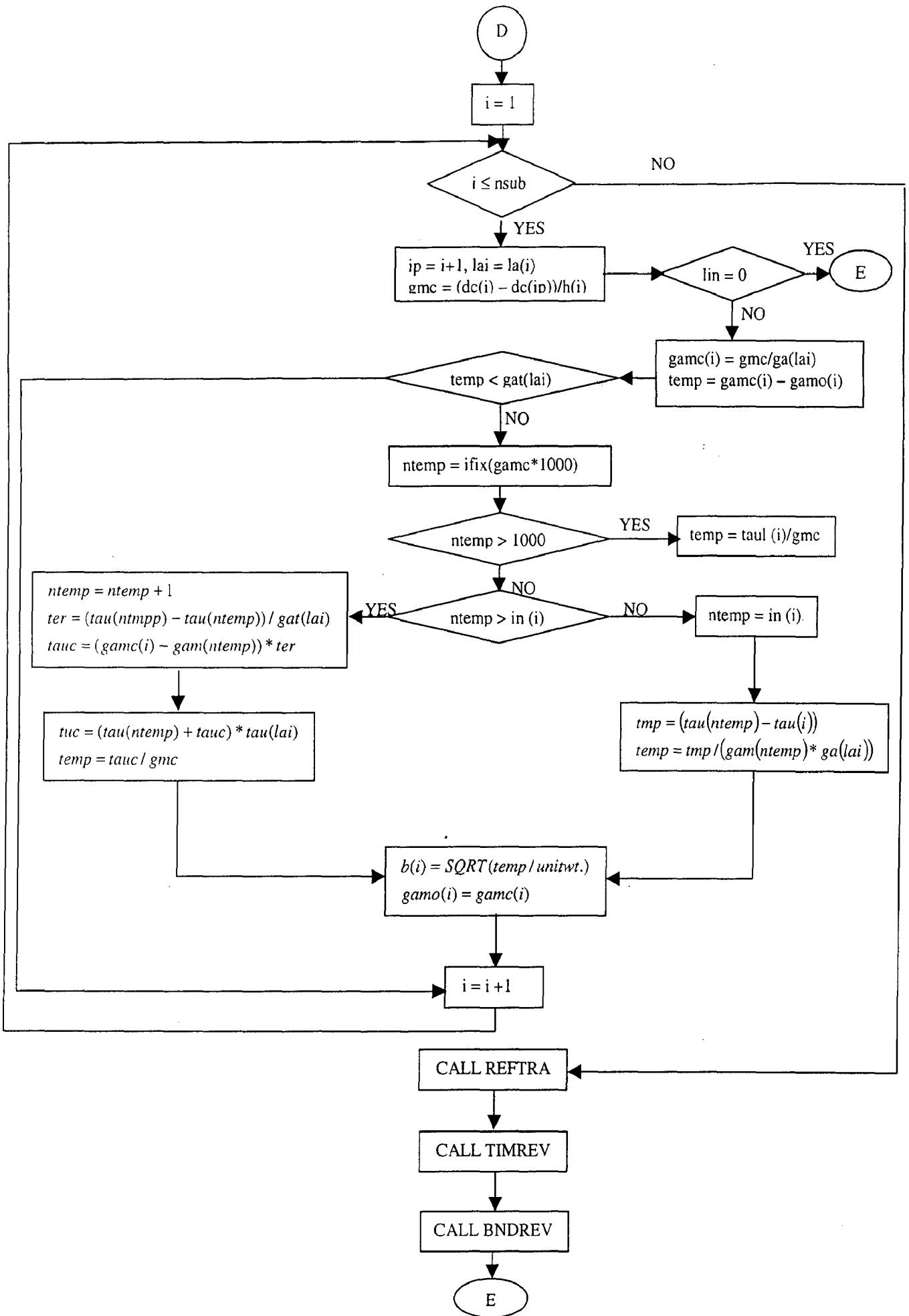












## SYMBOLS USED AND THEIR MEANING

acc(nt)	Base rock acceleration–time history at time intervals $t_i$ from 1 to ndat (duration in $t_i$ ).
accoef	Acceleration coefficient read in main when nacc=1
ang	Angle of propagation (in deg.) of shear wave in base rock layer. Computed as (velocity of shear wave/73.1) in degrees.
an(i)	Angle of propagation of shear wave in i'th main layer, defined first for base rock layer in Main, assuming $\{\tan[an(i)]\}/bt(i)=\text{constant}$ .
bt(i)	Shear wave velocity in m/s in i'th main layer.
c(i)	Shear parameter cohesion ( $N/m^2$ ) for i'th main layer.
cok2(i)	It is a constant which give the value of shear modulus when effective stress is unity
dgam	Interval of normalized unit shear strain used in assembling the strain array gam(i) in subroutine Gdigit
dtr	An multiplication converts angles in degrees to radians, $dtr = \pi/180.0$
e(i)	Void ratio
feq	Predominant freq. of earthquake or component vibration of eq. Considered; Kanai.
ga(i)	Failure strain in percentage for i'th main layer.
gaco	Strain (as a fraction of failure strain) at junction of two curves representing stress- strain curves for all layers. This common strain is taken as 0.14 times failure strain of material
gam(i)	Normalized digitized 1001 long strain array at equal intervals of 0.001
gamc(i)	Current value of gam(i)
gamo(i)	Old value of gam(i)
gat(i)	$gat(i)=ga(i)/1000.0$ normalized tolerance strain, where ga(i) is in actual.
ia	If ia = 0 vertical propagation of wave; ia.ne.0 – inclined propagation of wave.
iang	If iang=0 angle of prop. in base rock given as input data. If iang=1, angle of prop. of wave in base rock is worked out by program as $(Vs/73.1)$
idat	Index to define at what level seismic response is known; idat=0 at surf.; idat=1 at base rock; idat=2 at inter-mediate level
ld	ld=0 xz plane only ld=1 yz plane only ld=2 xz and yz planes
lin	lin=0 linear analysis, lin=1 nonlinear analysis, if li.lt.0.and.lin.gt.1 program Stops



n	Number of main layers excluding base rock.
nacc	Index; nacc=0 Computer generates sinusoidal acceleration. nacc=1 Excitation such as El Centro record is supplied in input data file.
nb	Number of boundary point
n1	Number of main layers, including base rock layer. Base layer number
ndat	No. of time stations for which input base rock accel. is read at time intervals of $t_i$ .
ndats	Time duration = ndata + twice the time to travel from base rock to ground level.
ngt	It is a no. strain station in norm stress-strain curve.
ngw	Main layer number in which GWT is situated at top interface of that main layer.
ngwt	ngwt=0, GWT is below base rock level. ngwt=1, GWT is at interface. ngwt=2, GWT is within the main layer.
nka	Index to activate call to Kanai. nka=1 call Kanai. If nka.ne.1 do not call Kanai.
nsub	Number of sub-layers within the n main layers.
nt	Number of time stations in acceleration history of input base rock response, at intervals of $t_i$
nw1	Index to activate some write statements in programme.
nw2	Index to activate some write statements in programme.
nw3	Index to activate some write statements in programme.
nws	Index to activate only short out put in programme.
nwl	Index to activate long detailed output in programme.
pi	Angle $P_i$ in radians, $p_i = 4.0 * \text{atan}(1.0)$
p(i)	Angle of shear resistance (in deg.) of main layer, i,
rate	Constant used to compute angle of propagation of wave in each layer.
ro(i)	Saturated unit weight in $\text{kN/m}^3$ for i'th main layer.
rob(i)	Bulk unit weight in $\text{kN/m}^3$ for i'th main layer.
sc	Multiplying factor to increase or reduce intensity of input acceleration history.
sg(i)	Specific gravity
tdp	Total depth of n-layer system, computed in Do 100 in Main.
teq	Period of component vibration of earthquake considered. Not in any common.
th(k)	Angle of propagation of shear wave with vertical in kth sub-layer.
ti	Time interval, in sec, at which accelerations are read.
wc(i)	water content
wtb	Depth of water table below Ground Level, in meters.