# SEISMIC CODE ANALYSIS OF ASYMMETRIC BUILDING WITHOUT LOCATING CENTRES OF RIGIDITY

**A DISSERTATION**

*Submitted in partial fulfilment of the requirements for the award of the degree*
*of*
MASTER OF ENGINEERING
*in*
EARTHQUAKE ENGINEERING
(With Specialization in Structural Dynamics)



*By*

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## CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in this dissertation entitled " **Seismic Code Analysis of Asymmetric Building Without Locating Centers of Rigidity**" in partial fulfillment of the requirements for the award of **Master of Engineering** Degree in Earthquake Engineering with Specialization in **Structural Dynamics**, submitted to the Department of Earthquake Engineering, University of Roorkee, Roorkee,is an authentic record of my own work carried out for a period of five to six months from September, 1999 to February, 2000 under the supervision of **Dr.Vipul Prakash**, Assistant Professor and **Dr.G.I.Prajapati**, Professor, Department of Earthquake Engineering, University of Roorkee, Roorkee.

The matter embodied in this dissertation has not been submitted by me for the award of any other degree or diploma.

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This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

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Place: Roorkee

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# ABSTRACT

The most building codes state that the lateral earthquake force at each story level of an asymmetric-plan building should be applied eccentrically from their centres of rigidity [6]. So to implement such procedures, it is necessary to determine the locations of centres of rigidity of each floor level in case of asymmetric-plan building. The location of centres of rigidity of an asymmetric-plan building in itself is a very difficult and time consuming process. In this dissertation, an evaluation test of the approach given by Chopra and Goel for lateral force analysis of an asymmetric-plan building without locating the centre of rigidity of its floors is given. The shear forces in the asymmetric-plan building are determined with the help of Staad-III package.

In one approach,the lateral forces and torques are applied on the centre of rigidity of the floor and in the second approach,the lateral forces and torques are applied at the centre of mass of the floor. The evaluation test of approach given by Chopra and Goel presented in this thesis dispels the long held view that locations of centres of rigidity must be determined for the implementation of the code procedure, and therefore, it removes a major difficulty in the analysis of buildings.

# CONTENTS

# LIST OF NOTATIONS

| | | |
|---|---|---|
| $e_{dj}$ | = | Design eccentricity at $j^{th}$ floor level |
| $e_{sj}$ | = | Eccentricity for the $j^{th}$ floor |
| $b_j$ | = | Floor plan dimension of the building perpendicular to the direction of ground motion |
| $\alpha$, $\beta$, $\delta$, $\gamma$ | = | Coefficients used by the 1987 Mexico Federal District Code |
| $h_j$ | = | Height of the building at the $j^{th}$ floor level |
| CRs | = | Centres of rigidity |
| CMs | = | Centre of masses |
| M.I. | = | Moment of inertia |
| $K_{yy}$, $K_{y\theta}$, $K_{\theta\theta}$ | = | Elements of stiffness matrix |
| $u_y$ | = | Lateral displacement of y-direction at floor centre of mass |
| $u_{\theta j}$ | = | Torsional displacement at a selected reference point on the $j^{th}$ floor at floor centre of mass |
| $\tilde{u}_y$ and $\tilde{u}_\theta$ | = | Vectors of lateral and torsional displacements, respectively, at the CRs. |
| $F_y$ | = | Vector of floor forces at floor centre of mass |
| $F_{yj}$ | = | Lateral force in y-direction |
| $F_{\theta j}$ | = | Floor torque applied at a selected reference point at floor centre of mass |
| $k_{xi}$ | = | Lateral stiffness matrix of element i in x-direction |
| $k_{yi}$ | = | Lateral stiffness matrix of element i in y-direction |

| | | |
|---|---|---|
| $x_i$ | = | Distance from the reference point to the $i^{th}$ frame oriented in the y-direction with lateral stiffness matrix $k_{yi}$ |
| $y_i$ | = | Distance from the reference point to the $i^{th}$ frame oriented in the x-direction with lateral stiffness matrix $k_{xi}$ |
| $X_R$ | = | Diagonal matrix with diagonal element $x_{Rj}$ |
| $x_{Rj}$ | = | x-co-ordinate of the CR at the $j^{th}$ floor |
| $\tilde{F}_y$ and $\tilde{F}_\theta$ | = | Vectors of lateral forces and floor torques respectively, applied at the CRs |
| $x_R$ | = | Vector of $x_{Rj}$ |
| $[F_y]$ | = | Diagonal matrix of $F_{yj}$ |
| $V_{jA}, V_{jB}, V_{jC}$ | = | $j^{th}$ story shears in frames A, B, C |
| $x_A, x_B, x_C$ | = | x-distances of frames A, B and C from the reference point |
| $r^{(1)}, r^{(2)}, r^{(3)}$ | = | Desired response obtained by steps 1, 2 and 3 |
| $r^{(a)}$ | = | Response obtained by first analysis of new approach |
| $r^{(b)}$ | = | Response obtained by second analysis of new approach |
| $V_j^{(a)}$ | = | Shear obtained by first analysis of new approach |
| $V_j^{(b)}$ | = | Shear obtained by second analysis of new approach |
| $W_j$ | = | Floor weight at $j^{th}$ floor level |
| $C$ | = | Seismic coefficient of 1987 Mexico Federal District Code |
| $Q$ | = | Yield reduction factor |
| $V_B$ | = | Base shear |
| $a_{xi}, a_{yi}$ | = | Transformation matrices |

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# CHAPTER - 1

# INTRODUCTION

Buildings subjected to earthquake force simultaneously undergo lateral as well as torsional motions if their structural plans do not have two axes of mass and stiffness symmetry. So the lateral forces experienced by the various resisting elements such as frames, shear walls, etc., of an unsymmetrical building would differ from those experienced by the same elements if the building had symmetric plan [3]. The lateral force analysis of most building codes requires that earthquake force at each floor level of an asymmetric-plan building should be applied eccentrically from the centre of rigidity at a distance equal to the design eccentricity. The design eccentricity $e_{dj}$ to be used at the $j^{th}$ floor level is specified in most seismic codes as

$$e_{dj} = \alpha\, e_{sj} + \beta b_j \qquad (1.1)$$

and $$e_{dj} = \delta e_{sj} - \beta b_j \qquad (1.2)$$

where,

$e_{sj}$ = eccentricity for the $j^{th}$ floor

$b_j$ = floor -plan dimension of the building perpendicular to the direction of ground motion.

$\alpha$, $\beta$ and $\delta$ = specified coefficients of 1987 Mexico Federal District code

For each structural frame or wall of a building, $e_{dj}$ value leading to the larger design force is to be used. The term $e_{sj}$ is intended to account for the coupled lateral torsional response of the building arising from the lack of symmetry in plan. The other term, often called the accidental eccentricity, is specified as a fraction of the plan dimension $b_j$, and is included to consider torsional effects due to other factors, such as rotational component of ground motion about a vertical axis, yield strengths and unfavourable distribution of live-load masses [2].

Several codes, such as Uniform Building Code (1991) and New Zealand Standard NZS 4230 (1984), specify that the lateral force be applied at a distance equal to $\pm \beta b_j$ from the centre of mass, which is equivalent to $\alpha = \delta = 1$ in eq. (1.1). In implementing the lateral force analysis procedures of such building codes, it is not necessary to determine the locations of the CRs at the various floor levels. However, in building codes where the design eccentricity formula uses $\alpha$ or $\delta$ different than 1, e.g., National Building Code of Canada (1990) and the 1987 Mexico Federal District Code, it seems necessary to determine the locations of the CRs.

Unlike one-story buildings, there are several difficulties in establishing locations of the centre of rigidity at various floor levels of a multistory building unless it belongs to a special class known as proportional buildings. For a proportional building, the lateral stiffness matrix of all its resisting elements along one direction are proportional to each other. For non-proportional buildings, it is cumbersome to determine the centres of rigidity because of the tedious calculations involved.

In 1993, Chopra and Goel [2] presented an approach that did not require the determination of the centres of rigidity. This motivated this study. In their study, Chopra and Goel [2] enumerated the different definitions of the centres of rigidity as given by earlier researchers, as well as restated their own definition [3].

Using their definition of centres of rigidity [3] as a starting point, Chopra and Goel [2] presented an approach to apply the building code provisions for lateral load analysis of asymmetric-plan multistory buildings. This approach did not require the explicit determination of the centres of rigidity.

This study shows that by using the definition of the centres of rigidity as given by Chopra and Goel [2], the following difficulties arise :

1. The location of the centres of rigidity for a asymmetric-plan multistory building becomes dependent on the height-wise distribution of the lateral loads. This implies the following :

(i) The centres of rigidity of Chopra and Goel [2] are not the intrinsic property of the building alone.

(ii) The centres of rigidity of Chopra and Goel [2] are not unique, as they are a function of the height-wise distribution of lateral loads, i.e., the lateral load pattern.

(iii) For certain lateral load patterns, it can be shown that the location of these centres of rigidity cannot be determined, i.e., they do not exist.

(iv) If the location of the centres of rigidity cannot be determined, then the eccentricities ($e_{sj}$) required for implementing the code procedures can also not be determined.

2. The lateral load pattern specified by building codes is largely based on an assumed first mode shape. If higher mode effects were also desired to be considered, then the centres of rigidity determined for the fundamental mode shape cannot be used for other modes.

3. During an earthquake, the displaced shape of the structure at any instant of time contains the contributions from all mode shapes, and the lateral load pattern varies from instant to instant. This would imply that the location of centres of rigidity of Chopra and Goel [2] would also vary from instant to instant, even for assumed linear behaviour of the structure.

the inelastic domain of response is known as ductility. It includes the ability to sustain large deformations and a capacity to absorb energy by hysteretic behaviour. Therefore it is the most important property sought by the designer of buildings located in regions of significant seismicity. In general, the term ductility defines the ability of a structure and selected structural components to deform beyond elastic limits without excessive strength or stiffness degradation.

Ductility is also defined as the ratio of the total imposed displacements $\Delta$ at any instant to that at the onset of yield $\Delta_y$.

Ductility, $\mu = \Delta/\Delta_y > 1$

The displacements $\Delta_y$ and $\Delta$ may represent strain, curvature, rotation or deflection.

Ductility in structural members can be developed easily if the constituent material itself is ductile. Thus, it is relatively easy to achieve the desired ductility if resistance is to be provided by steel in tension. However, precautions need to be taken when steel is subjected to compression. To ensure that premature buckling does not interfere with the development of the desired large inelastic strains in compression [5].

## 2.2 Definitions

**1. Centre of Mass :** During an earthquake, the acceleration-induced inertia forces will be generated at each floor level where the mass of an entire story may be assumed to be concentrated. Hence, the location of a force at a particular level will be determined by the centre of the accelerated mass at that level. This point where the mass of an entire story is assumed to be concentrated is known as centre of mass [5].

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# CHAPTER - 2

# STRUCTURAL PROPERTIES OF BUILDINGS

## 2.1 Structural Properties

There are basically three structural properties of a seismic building.

1. **Stiffness :** If deformations under the action of lateral forces are to be reliably quantified and subsequently controlled, then the designers must make a realistic estimate of the relevant property called stiffness. This quantity relates loads or forces to the ensuing structural deformations [5].

2. **Strength :** If a concrete or masonry structure is to be protected against damage or the damage which is repairable during a selected or specified seismic event, an inelastic excursions during its dynamic response should be prevented. This means that the structure must have adequate strength to resist internal actions generated during the elastic dynamic response of the structure. Therefore, the appropriate technique for the evaluation of earthquake-induced actions is an elastic analysis based on the stiffness properties of the structure. The term strength in fact is the resistance of a structure or a member or a particular section to the internal forces.

3. **Ductility :** To minimise major damage and to ensure the survival of buildings with moderate resistance with respect to lateral forces, the structures must be capable of sustaining a high proportion of their initial strength when a major earthquake imposes large deformations. These deformations may be well beyond the elastic limit. This ability of the structure or its components, or of the materials used to offer resistance in

the inelastic domain of response is known as ductility. It includes the ability to sustain large deformations and a capacity to absorb energy by hysteretic behaviour. Therefore it is the most important property sought by the designer of buildings located in regions of significant seismicity. In general, the term ductility defines the ability of a structure and selected structural components to deform beyond elastic limits without excessive strength or stiffness degradation.

Ductility is also defined as the ratio of the total imposed displacements $\Delta$ at any instant to that at the onset of yield $\Delta_y$.

Ductility, $\mu = \Delta/\Delta_y > 1$

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2. **Centre of Rigidity :** There are several definitions of the centre of rigidity. The various definitions given by various scientists are as follows :

(i) Poole in 1977 defined the centre of rigidity of a story as the location of the resultant of shear forces of various resisting elements in that story when the building is subjected to a static lateral loading that causes no rotation in any of the stories.

(ii) Humar in 1984 defined the centre of rigidity at any floor as the point such that application of a lateral load through this point would not cause rotation of that floor, other floors may rotate.

(iii) Cheung and Tso in 1986 defined the centres of rigidity as the set of points located on the building floors through which application of lateral forces would cause no rotation of any of the floors.

(iv) Hejal and Chopra in 1987 defined the centres of rigidity of the building as the points in the plane of the floors through which any set of static horizontal forces must be applied in order that it may cause all floors to translate without torsion [2].

3. **Floor Eccentricity :** It is defined as the distance between the floor centre of mass and the floor centre of rigidity. When the eccentricities of all floors of building are zero, lateral motions of the building are independent of its torsional motions and the building is said to be uncoupled [3].

## 2.3 Influence of Building Configuration on Seismic Response

The building configuration greatly effects the seismic response of a building. By adopting the following fundamental principles relevant to seismic response, more suitable structural system can be adopted.

1. Simple and regular plans are preferable. Buildings with articulated plans such as T and L shapes should be avoided or they should be subdivided into simpler forms.
2. Symmetry in plan should be provided where possible. Lack of symmetry may lead to significant torsional response. Much greater damage due to earthquakes has been observed in buildings situated at street corners, where structural symmetry is more difficult to achieve than in those along streets, where a more simple rectangular and often symmetrical structural plan could be utilised.
3. An integrated foundation system should tie together all vertical structural elements in both principal directions. Foundations resting partly on rock and on soils should preferably be avoided.
4. Lateral force resisting systems within one building, with significantly different stiffnesses such as structural walls and frames should be arranged in such a way that at every level symmetry in lateral stiffness is not grossly violated. Thereby undesirable torsional effects will be minimized.
5. Regularity should prevail in elevation, in both the geometry and the variation of story stiffnesses.
6. Avoid concentration of mass especially near the top of high rise buildings because it helps in exciting the higher modes of vibration [5].

# CHAPTER - 3

# METHOD OF SEISMIC CODE ANALYSIS

## 3.1 Analysis Using Centres of Rigidity

The first step in this approach is to determine the locations of the centres of rigidity for all floor levels of the asymmetric-plan building by the matrix approach or by the plane frame analysis approach.

## 3.2 Computation of Centres of Rigidity

### 3.2.1 Matrix approach

Consider an n-story building with orthogonal arrangement of lateral-load resisting elements connected by rigid floor diaphragms as shown in Fig. 3.1. For lateral force analysis in the y-direction, building plan is considered as symmetric about the x-axis. The equilibrium equations for such systems are :

$$\begin{bmatrix} K_{yy} & K_{y\theta} \\ K_{\theta y} & K_{\theta\theta} \end{bmatrix} \begin{Bmatrix} u_y \\ u_\theta \end{Bmatrix} = \begin{Bmatrix} F_y \\ F_\theta \end{Bmatrix} \tag{3.1}$$

where,

$$u_y^T = < u_{y1},\ u_{y2},\ u_{y3},\ \ldots\ldots\ldots\ u_{yj},\ \ldots\ldots\ldots\ u_{yn} >$$

and $u_\theta^T = < u_{\theta 1},\ u_{\theta 2},\ u_{\theta 3},\ \ldots\ldots\ldots\ u_{\theta j},\ \ldots\ldots\ldots\ u_{\theta n} >$

are displacement vectors.

and $u_{yj}$ = lateral displacement in y-direction at a selected reference point on the j[th] floor, where $j = 1, 2, \ldots, n$

and $u_{\theta j}$ = torsional displacement on the j[th] floor

$$F_y^T = < F_{y1}, F_{y2}, \ldots\ldots\ldots F_{yj}, \ldots\ldots\ldots F_{yn} >$$

and $$F_\theta^T = < F_{\theta 1}, F_{\theta 2}, \ldots\ldots\ldots F_{\theta j}, \ldots\ldots\ldots F_{\theta n} >$$

are vectors of floor forces.

$F_{yj}$ = lateral force in y-direction at the selected point on the j[th] floor

and $F_{\theta j}$ = floor torque applied on the j[th] floor

Various submatrices of stiffness matrix given by eq. (3.1) can be expressed in terms of lateral stiffness matrices of individual resisting elements as follows [2].

$$K_{yy} = \sum_i^n k_{yi} \tag{3.2a}$$

$$K_{y\theta} = K_{\theta y} = \sum_i^n x_i\, k_{yi} \tag{3.2b}$$

$$K_{\theta\theta} = \sum_i^n x_i^2\, k_{yi} + \sum_i^n y_i^2\, k_{xi} \tag{3.2c}$$

where

$x_i$ = distance from the reference point to the i[th] frame oriented in the y-direction with lateral stiffness matrix $k_{yi}$

$y_i$ = distance from the reference point to the i[th] frame oriented in the x-direction with lateral stiffness matrix $k_{xi}$

Now equilibrium equations w.r.t. the degrees of freedom

$\tilde{u}^T = < \tilde{u}_y^T \; \tilde{u}_\theta^T >$ at the CRs is

$$\begin{bmatrix} K_{yy} & K_{y\theta} - K_{yy} X_R \\ K_{\theta y} - X_R K_{yy} & K_{\theta\theta} + X_R K_{yy} X_R - X_R K_{y\theta} - K_{\theta y} X_R \end{bmatrix} \begin{Bmatrix} \tilde{u}_y \\ \tilde{u}_\theta \end{Bmatrix} = \begin{Bmatrix} \tilde{F}_y \\ \tilde{F}_\theta \end{Bmatrix} \quad (3.3)$$

where,

$X_R$ = diagonal matrix with its diagonal elements = $x_{Rj}$

$x_{Rj}$ = x-coordinate of the CR of the j[th] floor

and

$\tilde{F}_y = F_y$ and $\tilde{F}_\theta = F_\theta - X_R F_y$

where,

$\tilde{F}_\theta$ = vector of floor torques at the CR

If the lateral forces are applied at the CRs then $\tilde{F}_\theta = 0$ and the system would undergo pure translation, i.e. $\tilde{u}_\theta = 0$

Thus eq. (3.3) becomes

$$K_{yy} \tilde{u}_y = \tilde{F}_y = F_y$$

$$\tilde{u}_y = K_{yy}^{-1} F_y \quad (3.4)$$

and $$(K_{\theta y} - X_R K_{yy}) \tilde{u}_y = 0 \quad (3.5)$$

substituting value of $\tilde{u}_y$ from eq. (3.4) in eq. (3.5) we get

$$(K_{\theta y} - X_R K_{yy}) K_{yy}^{-1} F_y = 0 \tag{3.6}$$

For the special class of buildings, the locations of CRs are independent of the distribution of lateral force. Hence, eq. (3.6) becomes

$$(K_{\theta y} - X_R K_{yy}) K_{yy}^{-1} = 0$$

$$X_R = K_{\theta y} K_{yy}^{-1} \tag{3.7}$$

where,

$X_R$ = Vector of $X_{Rj}$

For buildings not belonging to the special class, location of CRs depends upon the distribution of lateral force. Hence, eq. (3.6) becomes

$$K_{\theta y} K_{yy}^{-1} F_y - x_R F_y = 0$$

$$x_R = [F_y]^{-1} K_{\theta y} K_{yy}^{-1} F_y \tag{3.8}$$

where

$x_R$ = Vector of $x_{Rj}$

$[F_y]$ = Diagonal matrix of $F_{yj}$

$F_y$ = Vector of $F_{yj}$

where $j = 1, 2, \ldots, n$

## 3.3 Plane Frame Analysis Approach

The location of the CRs at each floor level can be determined from equilibrium analysis of the free body diagram of that floor when the lateral forces $F_{yj}$ are applied at those floor levels.

Free body diagrams of each story are shown in Fig. 3.2.

Let $V_{jA}$, $V_{jB}$ and $V_{jC}$ be the j[th] story shears in frames A, B and C, respectively.

Equilibrium of the forces in the y-direction at each floor level gives :

$$(V_{jA} - V_{j+1,A}) + (V_{jB} - V_{j+1,B}) + (V_{jC} - V_{j+1,C}) = F_{yj} \tag{3.9}$$

where,

$F_{yj}$ = lateral force at j[th] floor

Equilibrium of the moments about the vertical axis gives the location of the CR at the floor level j as

$$X_{Rj} = \frac{\left(V_{JA} - V_{J+1,A}\right) x_A + \left(V_{JB} - V_{J+1,B}\right) x_B + \left(V_{JC} - V_{J+1,C}\right) x_C}{F_{yj}} \tag{3.10}$$

where $x_A$, $x_B$ and $x_C$ = x-distances of frames A, B and C from the reference point, respectively.

### 3.4 Implementation of Code Procedure

After determining the locations of the CRs at various floor levels, the code procedure can be implemented by applying the lateral forces $F_{yj}$ at a distance equal to $e_{dj}$ from the centre of rigidity at each floor as shown in Fig. 3.3. This implies the need for two analysis.

For first analysis.

$$e_{dj} = \alpha\, e_{sj} + \beta\, b_j \tag{3.11}$$

and for second analysis

$$e_{dj} = \delta e_{sj} - \beta b_j \tag{3.12}$$

These two conditions can be combined together as

$$e_{dj} = \gamma e_{sj} \pm \beta b_j \tag{3.13}$$

where, $\gamma = \alpha$ and $+ \beta b_j$ to arrive at (3.11).

and $\gamma = \delta$ and $- \beta b_j$ to arrive at (3.12).

For buildings with rigid diaphragms, the load condition of Fig. 3.3 is equivalent to the superposition of three load cases.

1. Lateral force $F_{yj}$ at the floor CRs.
2. Floor torques = $\gamma e_{sj} F_{yj}$ at the floor CRs
3. Floor torques = $\beta b_j F_{yj}$ at the floor CRs

The floor torques of second and third load cases can be combined and need not be considered separately and is equal to $e_{dj}F_{yj}$.

where,

$$e_{dj} = \gamma e_{sj} + \beta b_j \tag{3.14}$$

For the building considered in chapter 4, the floor torques are computed in accordance with the 1987 Mexico Federal District Code which specifies the value of coefficients, as

$\alpha = 1.5$, $\delta = 1.0$ and $\beta = 0.1$,

substituting these values in eq. (3.11) and (3.12) we get,

For first analysis

$$e_{dj} = 1.5\, e_{sj} + 0.1\, b_j \tag{3.15}$$

For second analysis

$$e_{dj} = e_{sj} - 0.1\, b_j \tag{3.16}$$

### 3.5 Analysis Without Using Centres of Rigidity

This new approach to implement the code procedure for asymmetric-plan buildings combines the results of three sets of analysis. In these sets of analysis, the forces are applied at the floor CMs, implying that the locations of CRs are not needed. The steps involved in this approach are given below:

1. The first step in the analysis is to apply the lateral forces $F_{yj}$ at the floor CMs and building was restricted to deform only in the y-direction. For the example building, this analysis was done by Staad-III package. The forces applied at the CMs were assumed to be distributed equally at the nodes or in proportion to its distance from the nodes. The resulting value of the force or deformation obtained is $r^{(1)}$ [2].
2. In the second step, the lateral forces were applied at the floor CMs and the asymmetric-plan building was analysed as a three dimensional system to obtain the value $r^{(2)}$ of the desired response,i.e.,force or deformation. For this analysis by Staad-III package, the forces were assumed to be distributed equally or in proportion to their distances from the nodes.
3. In the third step, the asymmetric-plan building was analysed for floor torques = $\beta b_j F_{yj}$ to obtain the value $r^{(3)}$ of the desired response, i.e., force or deformation. In this analysis, the moments were assumed to be distributed in proportion to their distances from the nodes. This analysis was also done by Staad-III package.
4. In the fourth step the responses $r^{(a)}$ and $r^{(b)}$ associated with design eccentricities of eq. (3.11) and eq. (3.12) are obtained by combining $r^{(1)}$, $r^{(2)}$, and $r^{(3)}$ as follows :

$$r^{(a)} = r^{(1)} + \alpha'(r^{(2)} - r^{(1)}) + r^{(3)}$$

$$r^{(b)} = r^{(1)} + \delta\,(r^{(2)} - r^{(1)}) - r^{(3)}$$

$$r^{(a)} = r^{(1)}\,(1 - \alpha) + \alpha r^{(2)} + r^{(3)} \tag{3.17}$$

and $$r^{(b)} = r^{(1)}\,(1 - \delta) + \delta r^{(2)} - r^{(3)} \tag{3.18}$$

Here, $\alpha = 1.5$ and $\delta = 1.0$

$$r^{(a)} = -0.5\,r^{(1)} + 1.5r^{(2)} + r^{(3)} \tag{3.19}$$

and $$r^{(b)} = r^{(2)} - r^{(3)} \tag{3.20}$$

5. The design value of the desired response is the larger of the two values $r^{(a)}$ and $r^{(b)}$.

   For the example building the design value of shear is the larger of $V_j^{(a)}$ and $V_j^{(b)}$ [2].

# CHAPTER - 4

# ANALYSIS OF ASYMMETRIC PLAN BUILDING

## 4.1 Dimensions of Building

The building considered in the analysis of code procedure is a four story asymmetric plan building having three frames A, B and C as shown in Fig. 4.1.

The building is symmetric in the x-direction. The building possesses stiffness only in the y-direction. The building has following properties.

| | | |
|---|---|---|
| Height of building | = | 16 m (4 m each story) |
| No. of Bays | = | 2 of 10 m each |
| Moment of inertia for all beams, M.I. | = | 0.3 $m^4$ |
| Moment of inertia of columns of frames A and C | = | 0.05 $m^4$ |
| Moment of inertia of columns of frame B | = | 0.1 $m^4$ |
| Floor weights, W | = | 20 KN for each of two bottom floors and 10 KN for each of two top floors |

The four story asymmetric plan building as shown in Fig. 4.1 has been designed as per the seismic provisions of the 1987 Mexico Federal District Code in which :

| | | |
|---|---|---|
| Seismic coefficient, C | = | 0.6 |
| Yield reduction factor, Q | = | 2.0 |
| Total floor weight, W | = | 20 x 2 + 10 x 2 |

W = 60 KN

Base shear, $V_B$ = C.W/Q

= (0.6 x 60) / 2

= 18 KN

## 4.2 Determination of Centres of Rigidity

### 4.2.1 Matrix Approach

The building possesses stiffness only in the y-direction. Most buildings, however, possesses stiffness in the x-direction also. For such buildings the stiffness is given by eq. (3.2c).

The lateral stiffness matrices of frames A, B and C as determined from matrix structural analysis program are given as :

$$k_{yA} = \frac{E}{100}\begin{bmatrix} 3.57 & -1.87 & 0.18 & -0.01 \\ -1.87 & 3.40 & -1.86 & 0.16 \\ 0.18 & -1.86 & 3.37 & -1.67 \\ -0.01 & 0.16 & -1.67 & 1.52 \end{bmatrix}$$

$$k_{yB} = \frac{E}{100}\begin{bmatrix} 6.93 & -3.75 & 0.57 & -0.05 \\ -3.75 & 6.37 & -3.68 & 0.50 \\ 0.57 & -3.68 & 6.23 & -3.06 \\ -0.05 & 0.50 & -3.06 & 2.61 \end{bmatrix}$$

$$k_{yC} = \frac{E}{100}\begin{bmatrix} 3.55 & -1.69 & 0 & 0 \\ -1.69 & 1.53 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Putting these values in eq. (3.2a) and (3.2b) the values of $K_{yy}$ and $K_{y\theta}$ can be determined as :

$$K_{yy} = \sum k_{yi} = \sum_{A}^{C} k_{yi} = k_{yA} + k_{yB} + k_{yC}$$

$$K_{yy} = \frac{E}{100}\begin{bmatrix} 14.05 & -7.31 & 0.75 & -0.06 \\ -7.31 & 11.30 & -5.54 & 0.66 \\ 0.75 & -5.54 & 9.60 & -4.73 \\ -0.06 & 0.66 & -4.73 & 4.13 \end{bmatrix} \tag{4.1}$$

Now $K_{y\theta} = K_{\theta y} = \sum_{i} x_i\, k_{yi}$

Taking frame A as the reference frame, we get $x_A = 0$, $x_B = 10$ m and $x_C = 20$ m

$$K_{y\theta} = \sum_{A}^{C} x_i\, k_{yi} = 0 + x_B\, k_{yB} + x_C\, k_{yC}$$

$$K_{y\theta} = \frac{E}{100}\begin{bmatrix} 140.30 & -71.30 & 5.70 & -.05 \\ -71.30 & 94.30 & -36.80 & 5.00 \\ 5.70 & -36.80 & 62.30 & -30.60 \\ -0.50 & 5.00 & -30.60 & 26.10 \end{bmatrix} \tag{4.2}$$

### 4.2.2 Storywise Distribution of Lateral Force

Lateral force at each floor level is calculated as shown in Table 4.1.

**Table 4.1 : Storywise Distribution of Lateral Force**

| Story j<br><br>(1) | Height, $h_j$ (m)<br><br>(2) | Weight, $W_j$ (KN)<br><br>(3) | $Wjhj$<br><br>(4) | Lateral force (KN) $F_{yj} = \frac{Wjhj}{\sum Wjhj} V_B$<br><br>(5) |
|---|---|---|---|---|
| 1 | 4.0 | 20.0 | 80 | 2.77 |
| 2 | 8.0 | 20.0 | 160 | 5.54 |
| 3 | 12.0 | 10.0 | 120 | 4.15 |
| 4 | 16.0 | 10.0 | 160 | 5.54 |
| | | | $\sum = 520$ | |

Lateral force vector is given as

$$F_y^T = < 2.77 \quad 5.54 \quad 4.15 \quad 5.54 >$$

### 4.2.3 Calculation of Centres of Rigidity

Hence from eq. (3.8) location of centre of rigidity is

$$x_R = [F_y]^{-1} K_{\theta y} K_{yy}^{-1} F_y \qquad (4.3)$$

A computer program was made for calculating the inverse with the help of book "A numerical library in C for Scientists and Engineers [4].

$$K_{yy}^{-1} = \frac{1}{E}\begin{bmatrix} 15 & 18 & 18 & 17 \\ 18 & 37 & 40 & 40 \\ 18 & 40 & 67 & 70 \\ 17 & 40 & 70 & 99 \end{bmatrix}$$

$$\left[F_y\right]^{-1} = \begin{bmatrix} 0.3610 & 0 & 0 & 0 \\ 0 & 0.1805 & 0 & 0 \\ 0 & 0 & 0.2410 & 0 \\ 0 & 0 & 0 & 0.1805 \end{bmatrix}$$

Hence, after substituting these values in eq. (4.3) we get

$$x_R = \frac{1}{E} \times \frac{E}{100}\begin{bmatrix} 0.3610 & 0 & 0 & 0 \\ 0 & 0.1805 & 0 & 0 \\ 0 & 0 & 0.2410 & 0 \\ 0 & 0 & 0 & 0.1805 \end{bmatrix}\begin{bmatrix} 140.3 & -71.3 & 5.7 & -0.5 \\ -71.3 & 94.3 & -36.8 & 5 \\ 5.7 & -36.8 & 62.3 & -30.6 \\ -0.5 & 5 & -30.6 & 26.10 \end{bmatrix}$$

$$\text{X} \begin{bmatrix} 15 & 18 & 18 & 17 \\ 18 & 37 & 40 & 40 \\ 18 & 40 & 67 & 70 \\ 17 & 40 & 70 & 99 \end{bmatrix}\begin{Bmatrix} 2.77 \\ 5.54 \\ 4.15 \\ 5.54 \end{Bmatrix}$$

$$x_R = \begin{Bmatrix} 8.55 \\ 17.57 \\ 6.14 \\ 5.99 \end{Bmatrix}$$

### 4.3 Plane Frame Analysis Approach

From eq. (3.10) the location of centre of rigidity of each floor level j is given as :

$$X_{Rj} = \frac{\left(V_{jA} - V_{j+1,A}\right) x_A + \left(V_{jB} - V_{j+1,B}\right) x_B + \left(V_{jC} - V_{j+1,C}\right) x_C}{F_{yj}} \qquad (4.4)$$

Free body diagram of each floor is shown in Fig. 3.2.

**4.3.1** The story shears in various frames at each floor level are given in Table 4.2. These are obtained by making the two dimensional model of frames A, B and C as shown in Fig. 4.2.

**Table 4.2 : Story Shears in Various Frames of Building**

| Floor j | Story Shear in Frame A (KN) | Story Shear in Frame B (KN) | Story Shear in Frame C (KN) |
|---|---|---|---|
| 1 | 4.80 | 8.42 | 4.78 |
| 2 | 4.26 | 6.32 | 4.64 |
| 3 | 3.82 | 5.88 | - |
| 4 | 2.22 | 3.32 | - |

#### 4.3.2 Determination of Centre of Rigidity

To determine the location of centres of rigidity at each floor level, take frame A as the reference frame and making use of eq. (4.4), we get

$x_A = 0$, $x_B = 10$ m and $x_C = 20$ m

Location of centre of rigidity at first floor

$$X_{R1} = \frac{(4.80 - 4.26) \times 0 \; + \; (8.42 - 6.32) \times 10 + (4.78 - 4.64) \times 20}{2.77}$$

$X_{R1} = 8.59$ m

Similarly

$$X_{R2} = \frac{(4.26 - 3.82) \times 0 \; + \; (6.32 - 5.88) \times 10 + (4.64 - 0) \times 20}{5.54}$$

$X_{R2} = 17.55$ m

$$X_{R3} = \frac{(3.82 - 2.22) \times 0 \; + \; (5.88 - 3.32) \times 10 + 0}{4.15}$$

$X_{R3} = 6.17$ m

$$X_{R4} = \frac{(2.22 - 0) \times 0 + (3.32 - 0) \times 10 + 0}{5.54}$$

$X_{R4} = 5.99$ m

Hence, the eccentricity of the various CRs of the floors from the CMs are given as

$CR_1$ = 8.59 - 10 = - 1.41 m

$CR_2$ = 17.55 - 10 = 7.55 m

$CR_3$ = 6.17 - 5 = 1.17 m

$CR_4$ = 5.99 - 5 = 0.99 m

Table 4.3 shows the location of the CRs and the floor eccentricity of the 4-story asymmetric-plan building.

**Table 4.3 : Location of Centres of Rigidity and Floor eccentricities of Building**

| Floor (j) | Location of CRs (m) | Floor eccentricity $e_{sj}$ (m) |
|---|---|---|
| 1 | 8.59 | -1.41 |
| 2 | 17.55 | 7.55 |
| 3 | 6.17 | 1.17 |
| 4 | 5.99 | 0.99 |

## 4.4 Implementation of Code Procedure

In this section, the values of floor torques are calculated in accordance with the 1987 Mexico Federal District Code Provisions for two values of $e_{dj}$ given by eq. (3.15) and eq. (3.16) as

For first analysis $e_{dj} = 1.5\, e_{sj} + 0.1\, b_j$ (4.5)

For second analysis $e_{dj} = e_{sj} - 0.1\, b_j$ (4.6)

The values of floor torques calculated for both the analysis are given in Tables 4.4 and 4.5.

Clockwise torque is considered positive and anticlockwise torque is negative.

For first Analysis :

**Table 4.4 : Calculation of Floor Torques for Building by first analysis**

| Floor j (1) | Lateral force $F_{yj}$ (KN) (2) | Floor eccentricity $e_{sj}$ (m) (3) | Floor-plan Dimension of Building $b_j$ (m) (4) | Design eccentricity $edj = 1.5\ esj + 0.1\ b_j$ (m) (5) | Floor Torques $F_{\theta j} = e_{dj} F_{yj}$ (KN-m) (6) |
|---|---|---|---|---|---|
| 1 | 2.77 | -1.41 | 20 | -0.115 | -0.32 |
| 2 | 5.54 | 7.55 | 20 | 13.325 | 73.82 |
| 3 | 4.15 | 1.17 | 10 | 2.755 | 11.43 |
| 4 | 5.54 | 0.99 | 10 | 2.485 | 13.77 |

For second analysis :

**Table 4.5 : Calculation of floor Torques for building**

| Floor j (1) | Lateral force $F_{yj}$ (KN) (2) | Floor eccentricity $e_{sj}$ (m) (3) | Floor-plan Dimension of building $b_j$ (m) (4) | Design eccentricity $e_{dj} = e_{sj} - 0.1\ b_j$ (m) (5) | Floor Torques $F_{\theta j} = e_{dj} F_{yj}$ (KN-m) (6) |
|---|---|---|---|---|---|
| 1 | 2.77 | -1.41 | 20 | -3.41 | -9.45 |
| 2 | 5.54 | 7.55 | 20 | 5.55 | 30.75 |
| 3 | 4.15 | 1.17 | 10 | 0.17 | 0.705 |
| 4 | 5.54 | 0.99 | 10 | -0.010 | -0.055 |

Now the shears in columns of Frame A due to lateral force $F_{yj}$ and floor torques $\tilde{F}_{\theta j}$ are calculated from both the analyses. This analysis was carried out by using Staad-III package. The forces acting at the centres of rigidity are assumed to be distributed in proportion to their distances at the nodes of that floor. Similarly, for applying the floor torques at the floor CRs similar assumption is made that the floor torques are distributed in proportion to their distances at the nodes of that floor.

### 4.4.1 First analysis

Apply the lateral forces at the floor CRs. as shown in column (2) of Table 4.4 and in Fig. 4.3. The loads applied at each floor are distributed in proportion to its distance at the nodes.

The joint loads are shown in Table 4.6.

**Table 4.6 : Distribution of Lateral Force at Floor Centres of Rigidity**

| Joint | Force-X (KN) | Force-Y (KN) | Force-Z (KN) |
|---|---|---|---|
| 13 | .00 | .00 | 1.110 |
| 14 | .00 | .00 | 1.660 |
| 28 | .00 | .00 | 1.110 |
| 29 | .00 | .00 | 1.660 |
| 10 | .00 | .00 | 0.790 |
| 11 | .00 | .00 | 1.285 |
| 25 | .00 | .00 | 0.790 |
| 26 | .00 | .00 | 1.285 |
| 7 | .00 | .00 | 0.340 |
| 9 | .00 | .00 | 2.430 |
| 22 | .00 | .00 | 0.340 |
| 24 | .00 | .00 | 2.430 |
| 4 | .00 | .00 | 0.790 |
| 6 | .00 | .00 | 0.595 |
| 19 | .00 | .00 | 0.790 |
| 21 | .00 | .00 | 0.595 |

The forces obtained for this loading are shown in Table 4.7

**Table 4.7 : Shears in Columns of Frame A for Lateral Forces**

| Member | Load | JT Joint | Axial (KN) | Shear-Y (KN) | Shear-Z (KN) | Torsion (KN-M) | Mom-Y (KN-M) | Mom-Z (KN-M) |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 1 | 13.49 | 0.411 | -4.369 | -0.133 | 9.191 | 0.833 |
| | | 4 | -13.49 | -0.411 | 4.369 | 0.133 | 8.285 | 0.813 |
| 6 | 3 | 4 | 7.61 | 0.351 | -3.883 | -0.002 | 7.783 | 0.655 |
| | | 7 | -7.61 | -0.351 | 3.883 | 0.002 | 7.752 | 0.752 |
| 11 | 3 | 7 | 2.849 | 0.182 | -2.349 | -0.009 | 4.590 | 0.349 |
| | | 10 | -2.849 | -0.182 | 2.349 | 0.009 | 4.806 | 0.378 |
| 16 | 3 | 10 | 0.662 | 0.166 | -1.262 | -0.041 | 2.449 | 0.315 |
| | | 13 | -0.662 | -0.166 | 1.262 | 0.041 | 2.598 | 0.351 |

2. Apply the floor torques at the floor CRs obtained from first analysis as shown in column (6) of Table 4.4 and Fig. 4.4.

The forces obtained for these floor torques are shown in Table 4.8.

**Table 4.8 : Shears in Columns of Frame A for Floor Torques**

| Member | Load | JT Joint | Axial (KN) | Shear-Y (KN) | Shear-Z (KN) | Torsion (KN-M) | Mom-Y (KN-M) | Mom-Z (KN-M) |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | -3.392 | 0.034 | -0.045 | 0.010 | 0.131 | 0.086 |
| | | 4 | 3.392 | -0.034 | 0.045 | -0.010 | 0.052 | 0.053 |
| 6 | 2 | 4 | -3.303 | -0.023 | -0.047 | 0.012 | 0.205 | -0.033 |
| | | 7 | 3.303 | 0.023 | 0.047 | -0.012 | -0.017 | -0.061 |
| 11 | 2 | 7 | -1.720 | 0.020 | -0.049 | 0.021 | 0.072 | 0.028 |
| | | 10 | 1.720 | -0.020 | 0.049 | -0.021 | 0.125 | 0.052 |
| 16 | 2 | 10 | -0.920 | 0.021 | -0.012 | 0.007 | 0.041 | 0.042 |
| | | 13 | 0.920 | -0.021 | 0.012 | -0.007 | 0.009 | 0.043 |

Add the two forces to obtain the shear in columns of frame A by first analysis. It is shown in Table 4.9.

**Table 4.9 : Total Shear in Columns of frame A by first analysis**

| Floor (j) | Shear in columns of frame A due to lateral force $F_{yj}$ applied at CRs (KN) | Shear in columns of frame A due to floor torques applied at CRs (KN) | Total Shear in columns of frame A from first analysis (KN) |
|---|---|---|---|
| 1 | 4.369 | 0.045 | 4.41 |
| 2 | 3.883 | 0.047 | 3.93 |
| 3 | 2.349 | 0.049 | 2.398 |
| 4 | 1.262 | 0.012 | 1.27 |

### 4.4.2 Second analysis

1. Apply the lateral forces at the floor CRs as shown in column (2) of Table 4.5.

   The values obtained of shear forces in columns of frame A are same as obtained in first step of first analysis and are shown in Table 4.7.

2. Apply the floor toques obtained from second analysis as shown in column (6) of Table 4.5 and in Fig. 4.5.

   The forces obtained for these applied floor torques at the CRs are shown in Table 4.10. Add both the forces to obtain the shear in columns of frame A by second analysis. The shears in columns of frame A are given in Table 4.11. The greater of the forces obtained from the two analysis will be the design shear for the frame A of the building. It is shown in Table 4.12.

**Table 4.10 : Shears in Columns of Frame A for Floor Torques of Second Analysis**

| Member | Load | JT Joint | Axial (KN) | Shear-Y (KN) | Shear-Z (KN) | Torsion (KN-M) | Mom-Y (KN-M) | Mom-Z (KN-M) |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | 0.089 | 0.023 | -0.027 | 0.005 | 0.014 | 0.050 |
| | | 4 | -0.089 | -0.023 | 0.027 | -0.005 | 0.096 | 0.043 |
| 6 | 2 | 4 | -0.693 | -0.002 | -0.025 | 0.005 | 0.133 | 0.001 |
| | | 7 | 0.693 | 0.002 | 0.025 | -0.005 | -0.029 | 0.011 |
| 11 | 2 | 7 | -0.044 | 0.012 | -0.023 | 0.007 | 0.015 | 0.019 |
| | | 10 | 0.044 | -0.012 | 0.023 | -0.007 | 0.079 | 0.030 |
| 16 | 2 | 10 | -0.0009 | 0.011 | -0.006 | 0.001 | 0.011 | 0.023 |
| | | 13 | 0.0009 | -0.011 | 0.006 | -0.001 | 0.014 | 0.023 |

Second analysis

**Table 4.11 : Total Shears in Columns of frame A by Second Analysis**

| Floor (j) | Shear in columns of frame A due to lateral force $F_{yj}$ applied at CRs (KN) | Shear in columns of frame A due to floor torques applied at CRs (KN) | Total Shear in columns of frame A from second analysis (KN) |
|---|---|---|---|
| 1 | 4.369 | 0.027 | 4.39 |
| 2 | 3.883 | 0.025 | 3.90 |
| 3 | 2.349 | 0.023 | 2.37 |
| 4 | 1.262 | 0.0065 | 1.26 |

**Table 4.12 : Value of Design Shear in Columns of frame A of Building**

| Floor (j) | Total Shear in columns of frame A from first analysis (KN) | Total Shear in columns of frame A from second analysis (KN) | Design Shear (KN) |
|---|---|---|---|
| 1 | 4.41 | 4.39 | 4.41 |
| 2 | 3.93 | 3.90 | 3.93 |
| 3 | 2.398 | 2.37 | 2.398 |
| 4 | 1.27 | 1.26 | 1.27 |

## 4.5 Analysis Without using Centre of Rigidity

In the new approach to implement the code procedure the forces are applied at the floor CMs, implying that the locations of CRs are not needed. This approach consists of following steps :

(1) In the first step, apply the lateral forces $F_{yj}$ given by column (2) of Table 4.4 at the floor CMs. The building was restricted to deform in the y-direction. It is shown in Fig. 4.6. The joint loads are shown in Table 4.13. The forces obtained for this loading are shown in Table 4.14

(2) In the second step, the asymmetric-plan building was analysed as a three-dimensional system for the lateral forces $F_{yj}$ at floor CMs as shown in Fig. 4.7. The joint loads are shown in Table 4.13. The forces obtained for this loading are shown in Table 4.15.

(3) In the third step, the building was analysed for floor torques = $\beta b_j F_{yj}$. The value of floor torques is shown in Table 4.16 and Fig. 4.8. The value of shear forces obtained are shown in Table 4.17.

(4) The responses are calculated in accordance with eq. (3.19) and eq. (3.20). All these calculations are shown in Table 4.17. The values of shear forces obtained in the first three steps are shown in columns (2), (3) and (4) of Table 4.18 as $V_j^{(1)}$, $V_j^{(2)}$ and $V_j^{(3)}$.

(5) The design value of the desired shear is the greater of the two values $V_j^{(a)}$ and $V_j^{(b)}$ [2].

**Table 4.13 : Distribution of Lateral Force at Floor Centres of Mass**

| Joint | Force-X (KN) | Force-Y (KN) | Force-Z (KN) |
|---|---|---|---|
| 13 | .00 | .00 | 1.3850 |
| 14 | .00 | .00 | 1.3850 |
| 28 | .00 | .00 | 1.3850 |
| 29 | .00 | .00 | 1.3850 |
| 10 | .00 | .00 | 1.0375 |
| 11 | .00 | .00 | 1.0375 |
| 25 | .00 | .00 | 1.0375 |
| 26 | .00 | .00 | 1.0375 |
| 7 | .00 | .00 | 1.3850 |
| 9 | .00 | .00 | 1.3850 |
| 22 | .00 | .00 | 1.3850 |
| 24 | .00 | .00 | 1.3850 |
| 4 | .00 | .00 | 0.6925 |
| 6 | .00 | .00 | 0.6925 |
| 19 | .00 | .00 | 0.6925 |
| 21 | .00 | .00 | 0.6925 |

**Table 4.14 : Shears in Columns of Frame A for Lateral Forces**

| Member | Load | JT Joint | Axial (KN) | Shear-Y (KN) | Shear-Z (KN) | Torsion (KN-M) | Mom-Y (KN-M) | Mom-Z (KN-M) |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 1 | -12.94 | -0.40 | -4.120 | -0.123 | 8.931 | -0.743 |
| | | 4 | 12.94 | 0.40 | 4.120 | 0.123 | 7.880 | -0.713 |
| 6 | 3 | 4 | -7.468 | -0.332 | -3.644 | -0.0016 | 6.970 | -0.4455 |
| | | 7 | 7.468 | 0.332 | 3.644 | -0.0016 | 6.738 | -0.55752 |
| 11 | 3 | 7 | -2.684 | -0.162 | -1.868 | -0.0089 | 4.400 | -0.329 |
| | | 10 | 2.684 | 0.162 | 1.868 | 0.0089 | 4.725 | -0.356 |
| 16 | 3 | 10 | -0.550 | -0.156 | -1.08 | -0.034 | 2.382 | -0.275 |
| | | 13 | 0.550 | 0.156 | 1.08 | -0.034 | 2.437 | -0.291 |

**Table 4.15 : Shears in Columns of Frame A for Lateral Forces**

| Member | Load | JT Joint | Axial (KN) | Shear-Y (KN) | Shear-Z (KN) | Torsion (KN-M) | Mom-Y (KN-M) | Mom-Z (KN-M) |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 1 | -13.494 | -0.411 | -4.370 | -0.134 | 9.193 | -0.833 |
| | | 4 | 13.494 | 0.411 | 4.370 | 0.134 | 8.288 | -0.813 |
| 6 | 3 | 4 | -7.608 | -0.351 | -3.877 | -0.0018 | 7.770 | -0.655 |
| | | 7 | 7.608 | 0.351 | 3.877 | 0.0018 | 7.738 | -0.752 |
| 11 | 3 | 7 | -2.849 | -0.182 | -2.354 | -0.00965 | 4.600 | -0.349 |
| | | 10 | 2.849 | 0.182 | 2.354 | 0.00965 | 4.815 | -0.378 |
| 16 | 3 | 10 | -0.662 | -0.166 | -1.261 | -0.041 | 2.448 | -0.315 |
| | | 13 | 0.662 | 0.166 | 1.261 | 0.041 | 2.597 | -0.351 |

**Table 4.16 : Calculation of Floor Torque = $\beta\, b_j\, F_{yj}$ for Building**

| Floor j | Lateral Force, $F_{yj}$ (KN) | Floor-Plan Dimension of Building $b_j$ (m) | Constant $\beta$ | Floor Torques = $\beta\, b_j\, F_{yj}$ (KN-m) |
|---|---|---|---|---|
| 1 | 2.77 | 20 | 0.1 | 5.54 |
| 2 | 5.54 | 20 | 0.1 | 11.08 |
| 3 | 4.15 | 10 | 0.1 | 4.15 |
| 4 | 5.54 | 10 | 0.1 | 5.54 |

**Table 4.17 : Value of Shear Forces in Columns of Frame for Floor Torques**

| Member | Load | JT Joint | Axial (KN) | Shear-Y (KN) | Shear-Z (KN) | Torsion (KN-M) | Mom-Y (KN-M) | Mom-Z (KN-M) |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | -1.373 | -0.009 | 0.0054 | -0.0005 | 0.014 | -0.016 |
| | | 4 | 1.373 | 0.009 | -0.0054 | 0.0005 | -0.035 | -0.021 |
| 6 | 2 | 4 | -1.165 | -0.013 | 0.0072 | -0.0004 | 0.008 | -0.024 |
| | | 7 | 1.165 | 0.013 | -0.0072 | 0.0004 | -0.037 | -0.030 |
| 11 | 2 | 7 | -0. 743 | -0.014 | 0.0096 | -0.0004 | -0.017 | -0.027 |
| | | 10 | 0. 743 | 0.014 | -0.0096 | 0.0004 | -0.021 | -0.029 |
| 16 | 2 | 10 | -0.431 | -0.012 | 0.0084 | -0.0002 | -0.006 | -0.022 |
| | | 13 | 0.431 | 0.012 | -0.0084 | 0.0002 | -0.027 | -0.026 |

**Table 4.18 : Shears in Columns of frame A of Building by New Approach**

| Floor (j) (1) | Shears calculated in step 1 $V_j^{(1)}$ (KN) (2) | Shears calculated in step 2 $V_j^{(2)}$ (KN) (3) | Shears calculated in step 3 $V_j^{(3)}$ (KN) (4) | $V_j^{(a)} = -0.5\ V_j^{(1)} + 1.5\ V_j^{(2)} + V_j^{(3)}$ (KN) (5) | $V_j^{(b)} = V_j^{(2)} - V_j^{(3)}$ (KN) (6) | Design Shear (KN) (7) |
|---|---|---|---|---|---|---|
| 1 | 4.120 | 4.370 | 0.0054 | 4.50 | 4.36 | 4.50 |
| 2 | 3.644 | 3.877 | 0.0072 | 3.99 | 3.87 | 3.99 |
| 3 | 1.868 | 2.354 | 0.0096 | 2.60 | 2.34 | 2.60 |
| 4 | 1.08 | 1.261 | 0.0084 | 1.36 | 1.25 | 1.36 |

# CHAPTER - 5

# EQUIVALENCE OF TWO APPROACHES

## 5.1 Mathematical Proof

The equivalence of two approaches, which uses centres of rigidity and the new approach without using the centres of rigidity for the seismic code analysis of asymmetric-plan building with orthogonal lateral resisting elements and rigid diaphragm, is established mathematically in this chapter. It is achieved here by demonstrating that the deformations at the floor centre of rigidity of the building obtained by the two approaches are identical. The mathematical proof for equivalence of two approaches is as follows.

## 5.2 The equation of equilibrium with respect to the CMs are

$$\begin{bmatrix} K_{yy} & K_{y\theta} \\ K_{\theta y} & K_{\theta\theta} \end{bmatrix} \begin{Bmatrix} u_y \\ u_\theta \end{Bmatrix} = \begin{Bmatrix} F_y \\ F_\theta \end{Bmatrix} \tag{5.1}$$

where,

$u_y$ and $u_\theta$ = vectors of lateral and torsional displacements, respectively at the CMs.

$F_y$ and $F_\theta$ = vectors of lateral forces and floor torques respectively, applied at the CMs.

Similarly, equation of equilibrium with respect to CRs is

$$\begin{bmatrix} \tilde{K}_{yy} & \tilde{K}_{y\theta} \\ \tilde{K}_{\theta y} & \tilde{K}_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \tilde{u}_y \\ \tilde{u}_\theta \end{Bmatrix} = \begin{Bmatrix} \tilde{F}_y \\ \tilde{F}_\theta \end{Bmatrix} \tag{5.2}$$

where

$\tilde{u}_y$ and $\tilde{u}_\theta$ = vectors of lateral and torsional displacements, respectively, at the CRs.

$\tilde{F}_y$ and $\tilde{F}_\theta$ = vectors of lateral forces and floor torques, respectively, applied at the CRs

For buildings with rigid diaphragms and orthogonal arrangement of the lateral-load resisting elements various submatrices in the stiffness matrix of eq. (5.2) are related to those of eq. (5.1) as

$$\tilde{K}_{yy} = K_{yy} \tag{5.3a}$$

$$\tilde{K}_{y\theta} = K_{y\theta} - K_{yy}\, e_s \tag{5.3b}$$

$$\tilde{K}_{\theta y} = K_{\theta y} - e_s\, K_{yy} \tag{5.3c}$$

$$\tilde{K}_{\theta\theta} = K_{\theta\theta} + e_s\, K_{yy}\, e_s - e_s\, K_{y\theta} - K_{\theta y}\, e_s \tag{5.3d}$$

where,

$e_s$ = diagonal matrix with diagonal terms $e_{sj}$

If only lateral force $\tilde{F}_y\,(=F_y)$ is applied at the CRs, i.e. $\tilde{F}_\theta = 0$, then system would undergo pure translation with $\tilde{u}_\theta = 0$.

Thus eq. (5.2) becomes

$$\tilde{K}_{yy}\;\tilde{u}_y = \tilde{F}_y = F_y$$

$$\tilde{u}_y = \tilde{K}_{yy}^{-1} F_y \quad (5.4)$$

and $$\tilde{K}_{\theta y} \tilde{u}_y = 0 \quad (5.5)$$

Substituting value of $\tilde{u}_y$ from eq. (5.4) in eq. (5.5), we get

$$\tilde{K}_{\theta y} \tilde{K}_{yy}^{-1} F_y = 0 \quad (5.6)$$

Now using eq. (5.3a) and eq. (5.3c),we get

$$(K_{\theta y} - e_s K_{yy}) K_{yy}^{-1} F_y = 0 \quad (5.7)$$

$$K_{\theta y} K_{yy}^{-1} F_y = e_s F_y \quad (5.8)$$

**5.3 Deformation at CRs by the Procedure Using CRs**

The load condition in this approach is to apply the lateral forces $F_{yj}$ at the floor CRs and floor torques, $F_{\theta j} = -(\gamma e_{sj} F_{yj} \pm \beta b_j F_{yj})$ ;

-ve sign is used for the floor torques to be compatible with the direction of rotational degrees of freedom.

Hence, eq. (5.2) becomes

$$\begin{bmatrix} \tilde{K}_{yy} & \tilde{K}_{y\theta} \\ \tilde{K}_{\theta y} & \tilde{K}_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \tilde{u}_y \\ \tilde{u}_\theta \end{Bmatrix} = \begin{Bmatrix} F_y \\ -(\gamma e_s F_y + \beta b F_y) \end{Bmatrix} \quad (5.9)$$

where,

$b$ = diagonal matrix with diagonal elements equal to $b_j$

Now solving the first eq. of (5.9), we get

$$\tilde{K}_{yy}\ \tilde{u}_y + \tilde{K}_{y\theta}\ \tilde{u}_\theta = F_y$$

$$\tilde{u}_y = \tilde{K}_{yy}^{-1} F_y - \tilde{K}_{yy}^{-1}\ \tilde{K}_{y\theta}\ \tilde{u}_\theta \tag{5.10}$$

Now solving the second eq. of (5.9), we get

$$\tilde{K}_{\theta y}\ \tilde{u}_y + \tilde{K}_{\theta\theta}\ \tilde{u}_\theta = -(\gamma e_s F_y \pm \beta b F_y) \tag{5.11}$$

Substituting value of $\tilde{u}_y$ from eq. (5.10) in above equation and solving for $\tilde{u}_\theta$ gives

$$\tilde{u}_\theta = -(\tilde{K}_{\theta\theta} - \tilde{K}_{\theta y}\ \tilde{K}_{yy}^{-1}\ \tilde{K}_{y\theta})^{-1}\ (\gamma e_s F_y + \beta b F_y + \tilde{K}_{\theta y}\ \tilde{K}_{yy}^{-1} F_y) \tag{5.12}$$

For the equivalence of the two approaches, it is useful to write the submatrices $\tilde{K}_{yy}$, $\tilde{K}_{y\theta}$, $\tilde{K}_{\theta y}$ and $\tilde{K}_{\theta\theta}$ defined at the CR in terms of submatrices $K_{yy}$, $K_{y\theta}$, $K_{\theta y}$ and $K_{\theta\theta}$ defined at CM.

This is achieved by utilising eq. (5.3). Now

$$
\begin{aligned}
(\tilde{K}_{\theta\theta} - \tilde{K}_{\theta y}\ \tilde{K}_{yy}^{-1}\ \tilde{K}_{y\theta}) &= K_{\theta\theta} + e_s K_{yy} e_s - e_s K_{y\theta} - K_{\theta y} e_s \\
&\quad - (K_{\theta y} - e_s K_{yy})\ K_{yy}^{-1}\ (K_{y\theta} - K_{yy} e_s) \\
&= K_{\theta\theta} + e_s K_{yy} e_s - e_s K_{y\theta} - K_{\theta y} e_s \\
&\quad - K_{\theta y} K_{yy}^{-1} K_{y\theta} + K_{\theta y} K_{yy}^{-1} K_{yy} e_s \\
&\quad + e_s K_{y\theta} - e_s K_{yy} e_s
\end{aligned}
$$

After simplifying, we get

$$\tilde{K}_{\theta\theta} - \tilde{K}_{\theta y}\ \tilde{K}_{yy}^{-1}\ \tilde{K}_{y\theta} = K_{\theta\theta} - K_{\theta y}\ K_{yy}^{-1} K_{y\theta} \tag{5.13}$$

and $\tilde{K}_{yy}^{-1} \tilde{K}_{y\theta} = K_{yy}^{-1} (K_{y\theta} - K_{yy}\, e_s)$

$$= K_{yy}^{-1} K_{y\theta} - K_{yy}^{-1} K_{yy}\, e_s$$

$$\tilde{K}_{yy}^{-1} \tilde{K}_{y\theta} = K_{yy}^{-1} K_{y\theta} - e_s \tag{5.14}$$

Using eq. (5.6), eq. (5.13) and eq. (5.14) in eq. (5.12) gives

$$\tilde{u}_\theta = - (K_{\theta\theta} - K_{\theta y}\, K_{yy}^{-1} K_{y\theta})^{-1} \gamma\, e_s\, F_y - (K_{\theta\theta} - K_{\theta y}\, K_{yy}^{-1} K_{y\theta})^{-1} \beta\, b\, F_y \tag{5.15}$$

Substituting value of $\tilde{u}_\theta$ from above in eq. (5.10), we get

$$\tilde{u}_y = K_{yy}^{-1} F_y - K_{yy}^{-1} (K_{y\theta} - K_{yy}\, e_s) \{ - (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \beta\, b\, F_y$$

$$- (K_{\theta\theta} - K_{\theta y}\, K_{yy}^{-1} K_{y\theta})^{-1} \gamma\, e_s\, F_y\}$$

After solving, we get

$$\tilde{u}_y = K_{yy}^{-1} F_y + K_{yy}^{-1} K_{y\theta} (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \gamma\, e_s\, F_y$$

$$+ K_{yy}^{-1} K_{y\theta} (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \beta\, b\, F_y$$

$$- e_s (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \gamma\, e_s\, F_y$$

$$- e_s (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \beta\, b\, F_y \tag{5.16}$$

## 5.4 Deformation at CRs by New Approach

The deformations at the CMs and CRs from step 1 would be equal to

$$\tilde{u}_y^{(1)} = \tilde{K}_{yy}^{-1} \tilde{F}_y = K_{yy}^{-1} F_y \tag{5.17}$$

Since the torsional deformations are zero,

$$\tilde{u}_\theta^{(1)} = 0$$

Deformations at the CMs in step 2 are computed from

$$\begin{bmatrix} K_{yy} & K_{y\theta} \\ K_{\theta y} & K_{\theta\theta} \end{bmatrix} \begin{Bmatrix} u_y^{(2)} \\ u_\theta^{(2)} \end{Bmatrix} = \begin{Bmatrix} F_y \\ 0 \end{Bmatrix} \tag{5.18}$$

On solving

$$u_y^{(2)} = K_{yy}^{-1} F_y - K_{yy}^{-1} K_{y\theta} u_\theta^{(2)} \tag{5.19}$$

and

$$u_\theta^{(2)} = -(K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} K_{\theta y} K_{yy}^{-1} F_y \tag{5.20}$$

Substituting eq. (5.8) in eq. (5.20), we get

$$u_\theta^{(2)} = -(K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} e_s F_y \tag{5.21}$$

Substituting value of $u_\theta^{(2)}$ in eq. (5.19), we get

$$u_y^{(2)} = K_{yy}^{-1} F_y - K_{yy}^{-1} K_{y\theta} \{-(K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} e_s F_y\}$$

$$u_y^{(2)} = K_{yy}^{-1} F_y + K_{yy}^{-1} K_{y\theta} \{(K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} e_s F_y\} \tag{5.22}$$

Having determined the deformations at the CMs, the deformations at the CRs are obtained as

$$\tilde{u}_y^{(2)} = u_y^{(2)} + e_s u_\theta^{(2)} \tag{5.23a}$$

and $$\tilde{u}_\theta^{(2)} = u_\theta^{(2)} \tag{5.23b}$$

Substituting value of $u_\theta^{(2)}$ from eq. (5.21) and $u_y^{(2)}$ from eq. (5.22) in eq. (5.23) we get

$$\tilde{u}_y^{(2)} = K_{yy}^{-1} F_y + K_{yy}^{-1} K_{y\theta} (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} e_s F_y$$

$$- e_s (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} e_s F_y \tag{5.24}$$

and $\tilde{u}_\theta^{(2)} = -(K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \; e_s F_y$ (5.25)

Similarly, the deformations in step 3 due to floor torques, $F_{\theta j} = -\beta b \; F_y$ at CMs are calculated from

$$\begin{bmatrix} K_{yy} & K_{y\theta} \\ K_{\theta y} & K_{\theta\theta} \end{bmatrix} \begin{Bmatrix} u_y^{(3)} \\ u_\theta^{(3)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -\beta b F_y \end{Bmatrix}$$

On solving the above equation, we get

$$u_y^{(3)} = -K_{yy}^{-1} K_{y\theta} u_\theta^{(3)} \quad (5.26)$$

and $u_\theta^{(3)} = -(K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \beta b \, F_y$ (5.27)

Substituting value of $u_\theta^{(3)}$ from eq. (5.27) in eq. (5.26), we get

$$u_y^{(3)} = K_{yy}^{-1} K_{y\theta} (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \beta b \, F_y \quad (5.28)$$

and the deformations at the CRs are given by using eq. (5.23)

$$\tilde{u}_y^{(3)} = K_{yy}^{-1} K_{y\theta} (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \beta b \, F_y$$
$$- e_s (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \; \beta b \, F_y \quad (5.29)$$

and $\tilde{u}_\theta^{(3)} = -(K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \; \beta b \, F_y$ (5.30)

Combining the deformation obtained in the steps 1, 2 and 3 according to the combination rule

$r = r^{(1)} + \gamma (r^{(2)} - r^{(1)}) + r^{(3)}$, we get

$$\tilde{u}_y = \tilde{u}_y^{(1)} + \gamma (\tilde{u}_y^{(2)} - \tilde{u}_y^{(1)}) + \tilde{u}_y^{(3)}$$

$$\tilde{u}_y = K_{yy}^{-1}\ F_y + K_{yy}^{-1} F_y + K_{yy}^{-1} K_{y\theta}(K_{\theta\theta} - K_{\theta y} k_{yy}^{-1} K_{y\theta})^{-1}\ x\ \gamma\ e_s F_y -$$

$$-\ e_s (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \gamma\ e_s F_y - K_{yy}^{-1} F_y +\ K_{yy}^{-1} K_{y0}$$

$$x\ (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \beta\ b\ F_y - e_s (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \beta\ b\ F_y$$

$$\tilde{u}_y = K_{yy}^{-1}\ F_y + K_{yy}^{-1} K_{y\theta}(K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \gamma\ e_s F_y +$$

$$K_{yy}^{-1} K_{y\theta} (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \beta\ b\ F_y$$

$$-\ e_s (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \gamma\ e_s F_y -$$

$$e_s (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \beta\ b\ F_y \qquad (5.31)$$

and $\tilde{u}_\theta = \tilde{u}_\theta^{(1)} + \gamma\,(\tilde{u}_\theta^{(2)} - \tilde{u}_\theta^{(1)}) + \tilde{u}_\theta^{(3)}$

$$\tilde{u}_\theta = 0 + \gamma\,(-(K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} e_s F_y) - (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \beta\ b\ F_y$$

$$\tilde{u}_\theta = -\ (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \gamma e_s F_y - (K_{\theta\theta} - K_{\theta y} K_{yy}^{-1} K_{y\theta})^{-1} \beta\ b\ F_y \qquad (5.32)$$

Eq. (5.31) and eq. (5.32) are identical to eq. (5.16) and eq. (5.16), indicating that the response obtained by the new approach is identical to that obtained by the procedure using centres of Rigidity.

# CHAPTER - 6

## DISCUSSION AND CONCLUSIONS

In 1993, Chopra and Goel [2] presented an approach that did not require the determination of the centres of rigidity. This motivated this study. In their study, Chopra and Goel [2] enumerated the different definitions of the centres of rigidity as given by earlier researchers, as well as restated their own definition [3].

Using their definition of centres of rigidity [3] as a starting point, Chopra and Goel [2] presented an approach to apply the building code provisions for lateral load analysis of asymmetric-plan multistory buildings. This approach did not require the explicit determination of the centres of rigidity.

This study shows that by using the definition of the centres of rigidity as given by Chopra and Goel [2], the following difficulties arise :

1. The location of the centres of rigidity for a asymmetric-plan multistory building becomes dependent on the height-wise distribution of the lateral loads. This implies the following :

(i) The centres of rigidity of Chopra and Goel [2] are not the intrinsic property of the building alone.

(ii) The centres of rigidity of Chopra and Goel [2] are not unique, as they are a function of the height-wise distribution of lateral loads, i.e., the lateral load pattern as given by eq. (3.8).

(iii) For certain lateral load patterns, as if $[F_y]$ is singular in eq. (3.8), it can be shown that the location of these centres of rigidity cannot be determined, i.e., they do not exist.

(iv) If the location of the centres of rigidity cannot be determined, then the eccentricities ($e_{sj}$) required for implementing the code procedures can also not be determined.

2. The lateral load pattern specified by building codes is largely based on an assumed first mode shape. If higher mode effects were also desired to be considered, then the centres of rigidity determined for the fundamental mode shape cannot be used for other modes.

3. During an earthquake, the displaced shape of the structure at any instant of time contains the contributions from all mode shapes, and the lateral load pattern varies from instant to instant. This would imply that the location of centres of rigidity of Chopra and Goel [2] would also vary from instant to instant, even for assumed linear behaviour of the structure.

4. In the paper presented by Goel and Chopra equations (33), (34), (39), (43) were written incorrectly and they have been corrected in this dissertation.

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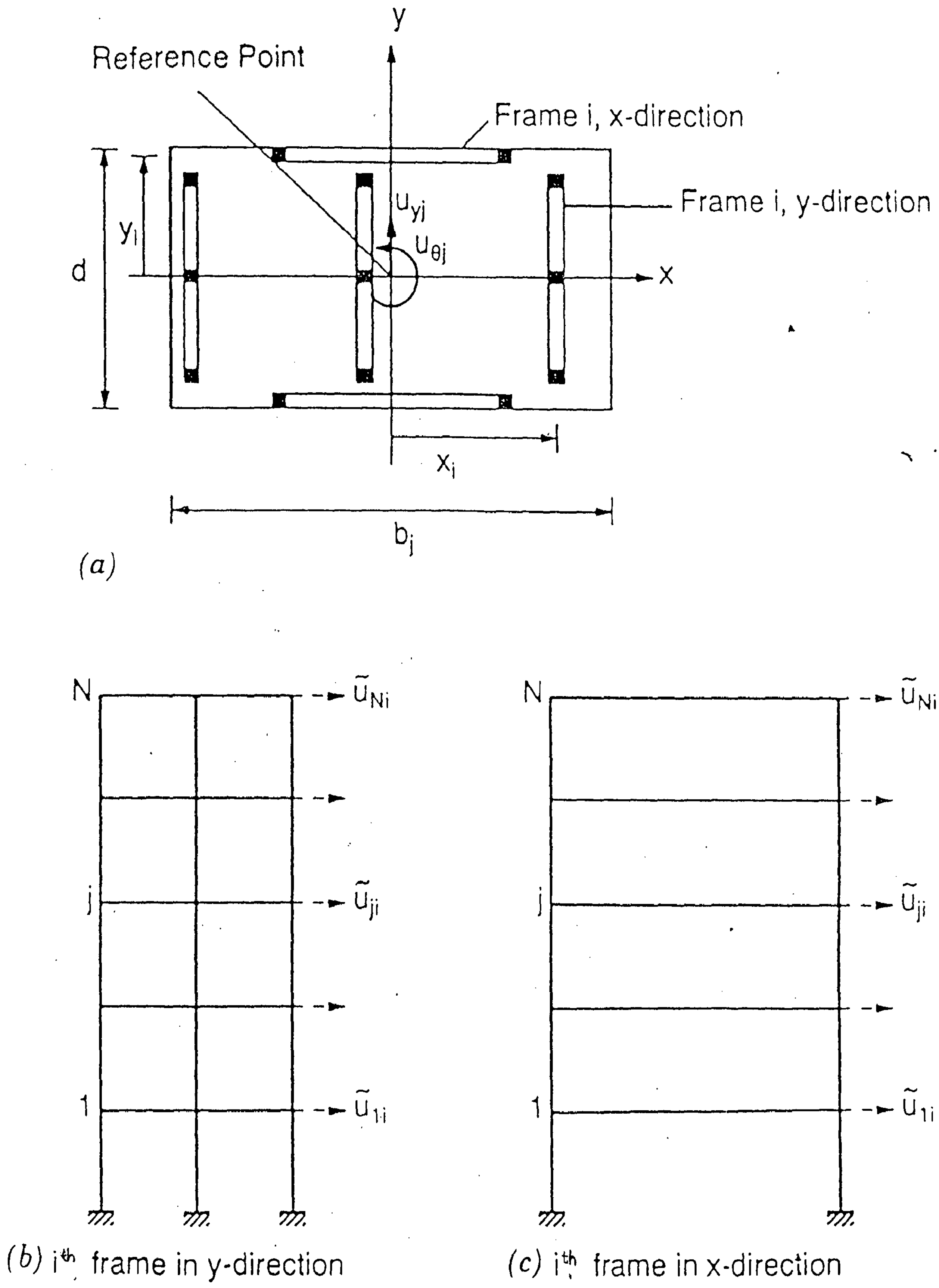



Fig. 3.1 Multistory System Considered

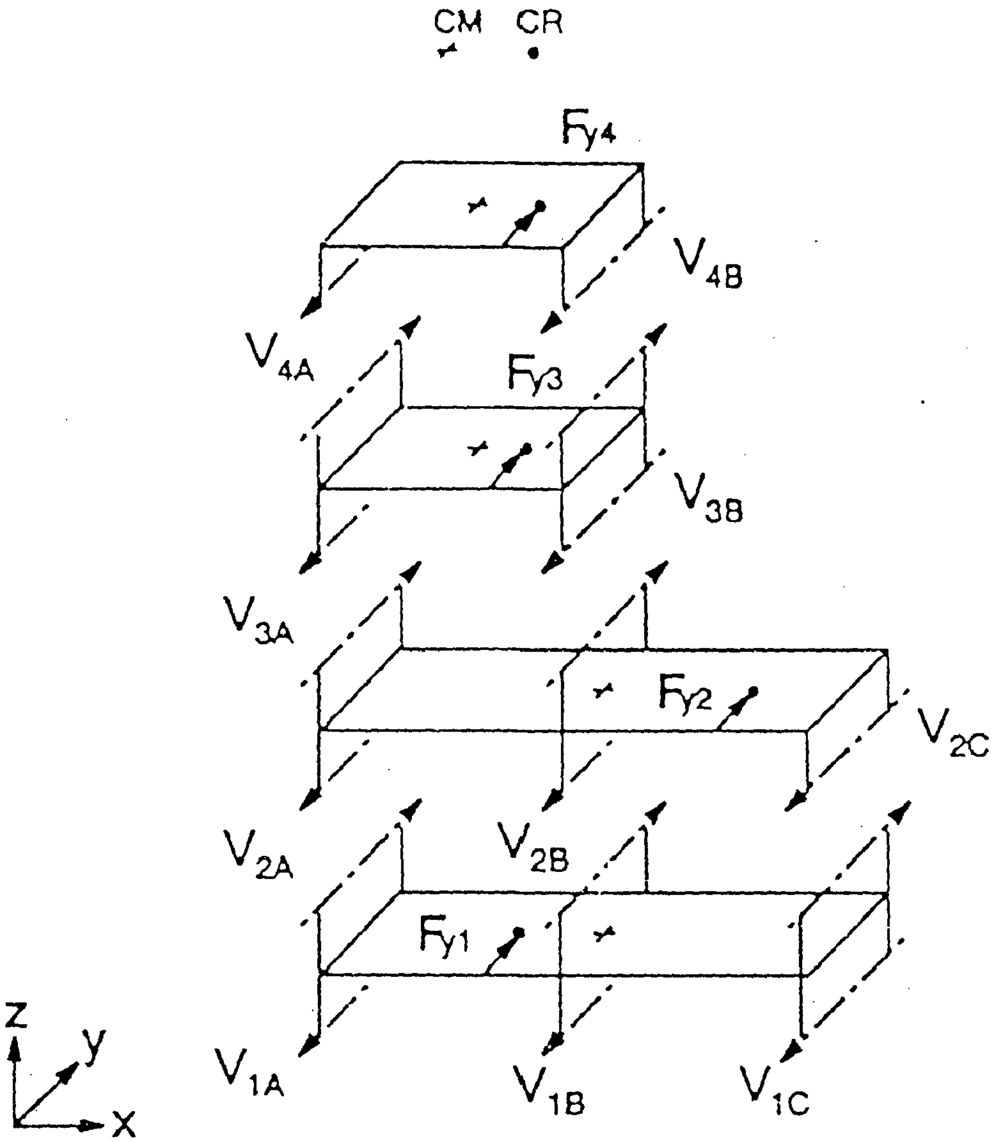



Fig. 3.2 Free Body Diagram of Each Story

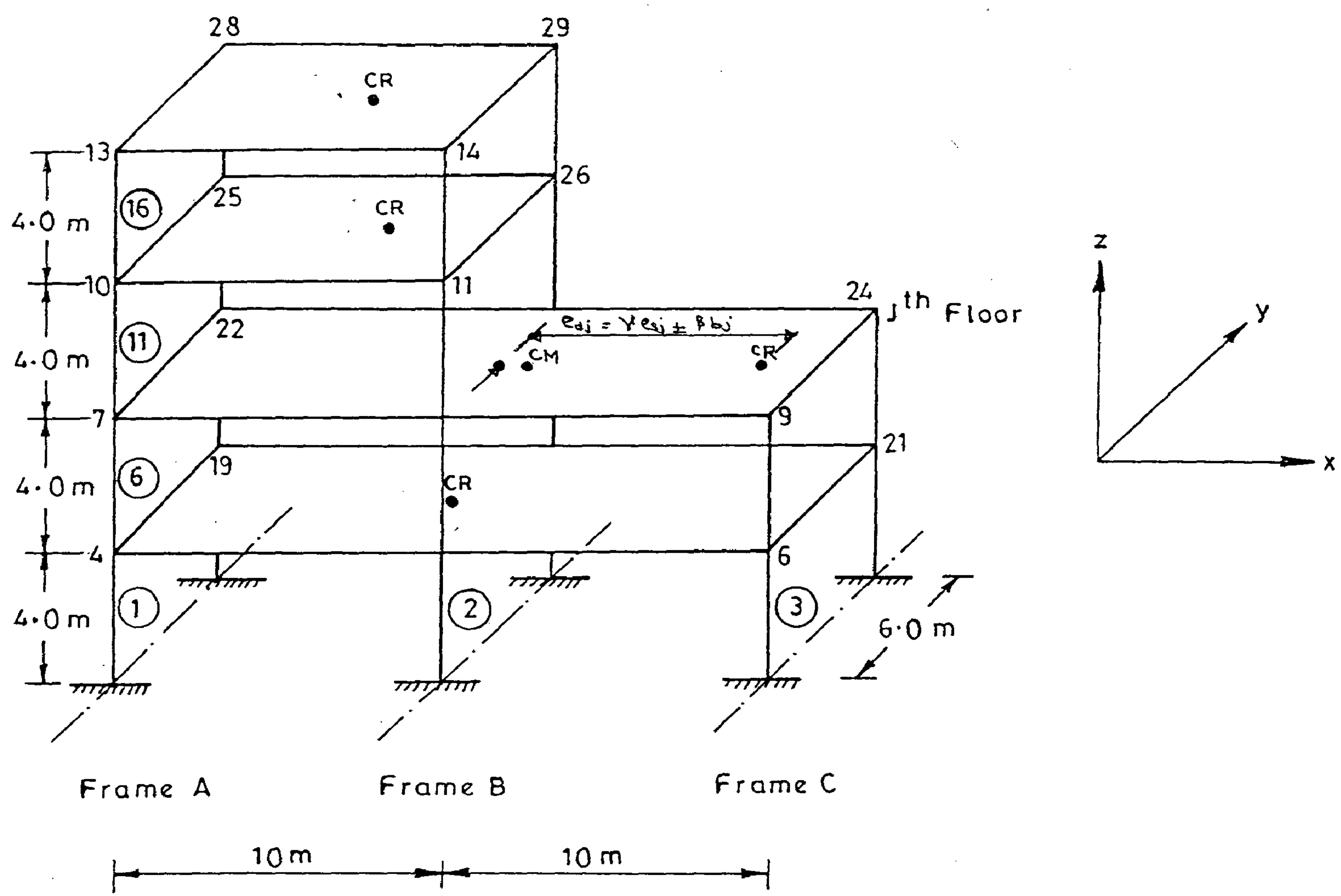



Fig. 3.3 Lateral Forces Applied at a Distance Equal to $e_{dj}$ from the CRs at Each Floor

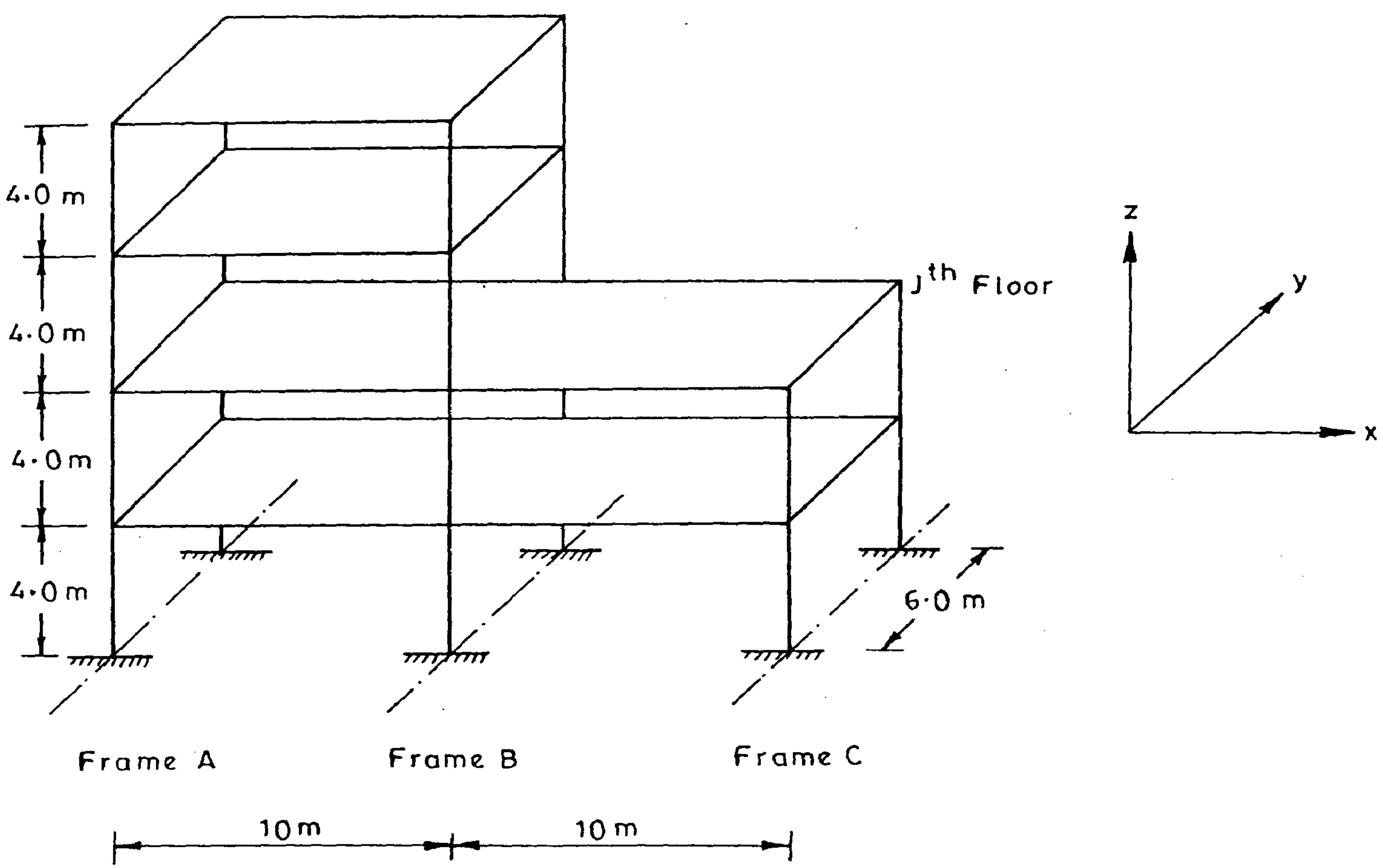
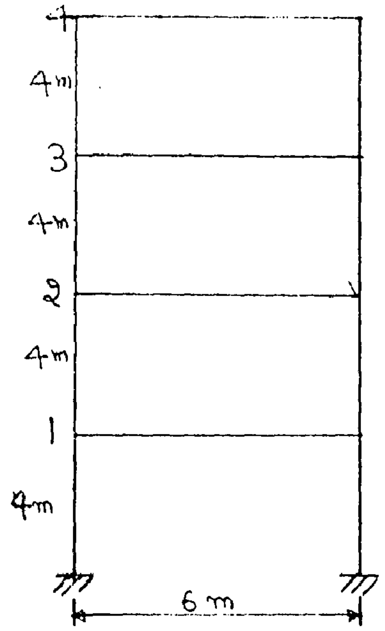
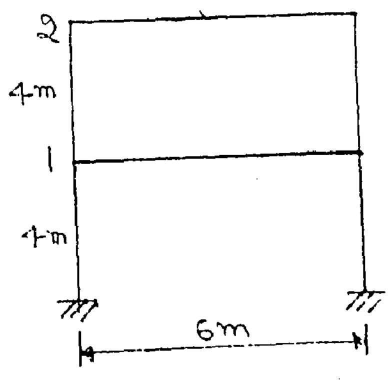



Fig. 4.1 Four Story Asymmetric-Plan Building having Three Frames A, B and C




(a) Frame A and B in Y-Z direction




(b) Frame C in Y-Z direction

Fig. 4.2 Two Dimensional Model of Frames A, B and C

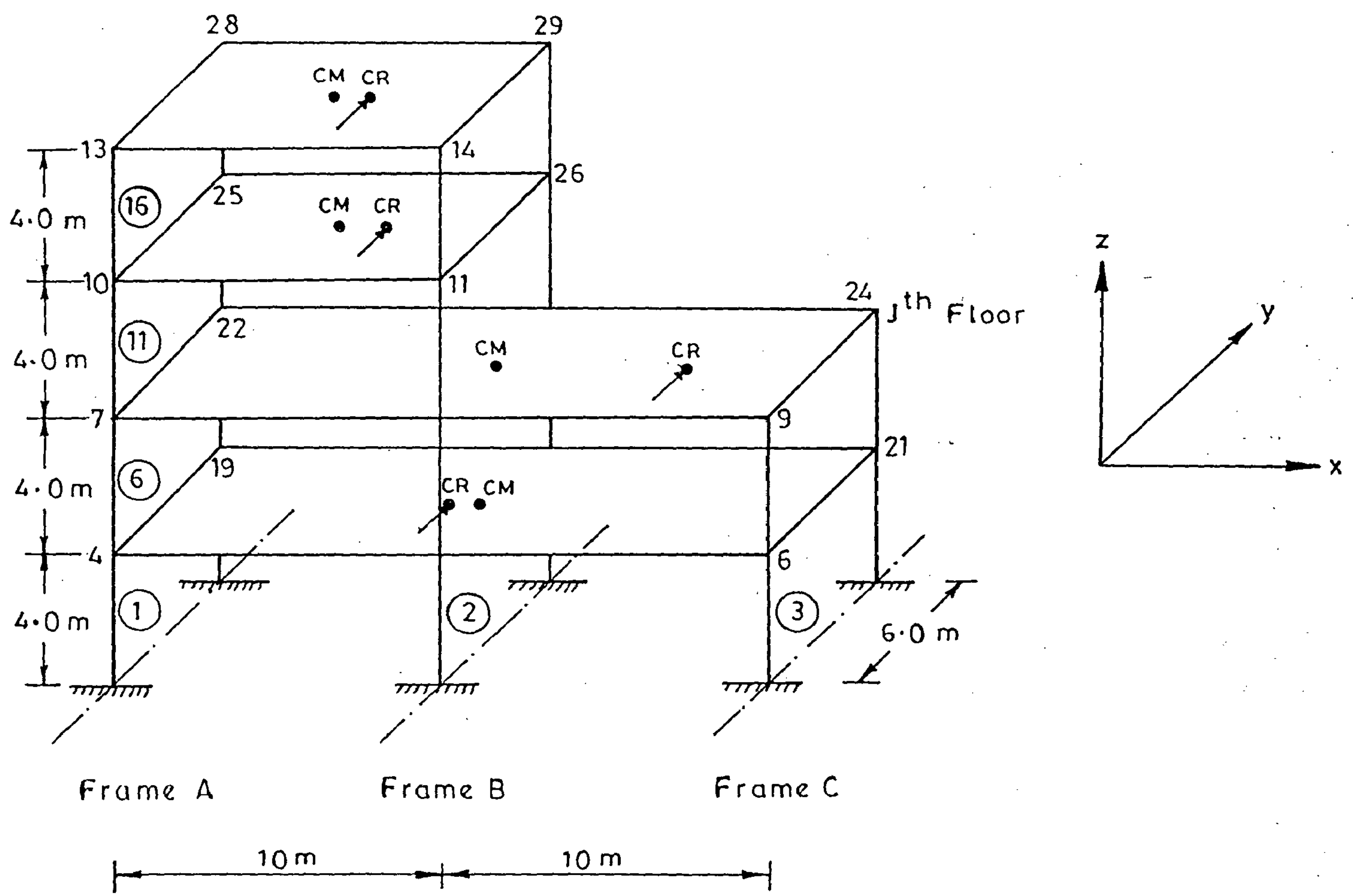



Fig. 4.3 Lateral Force Applied at the Floor CRs

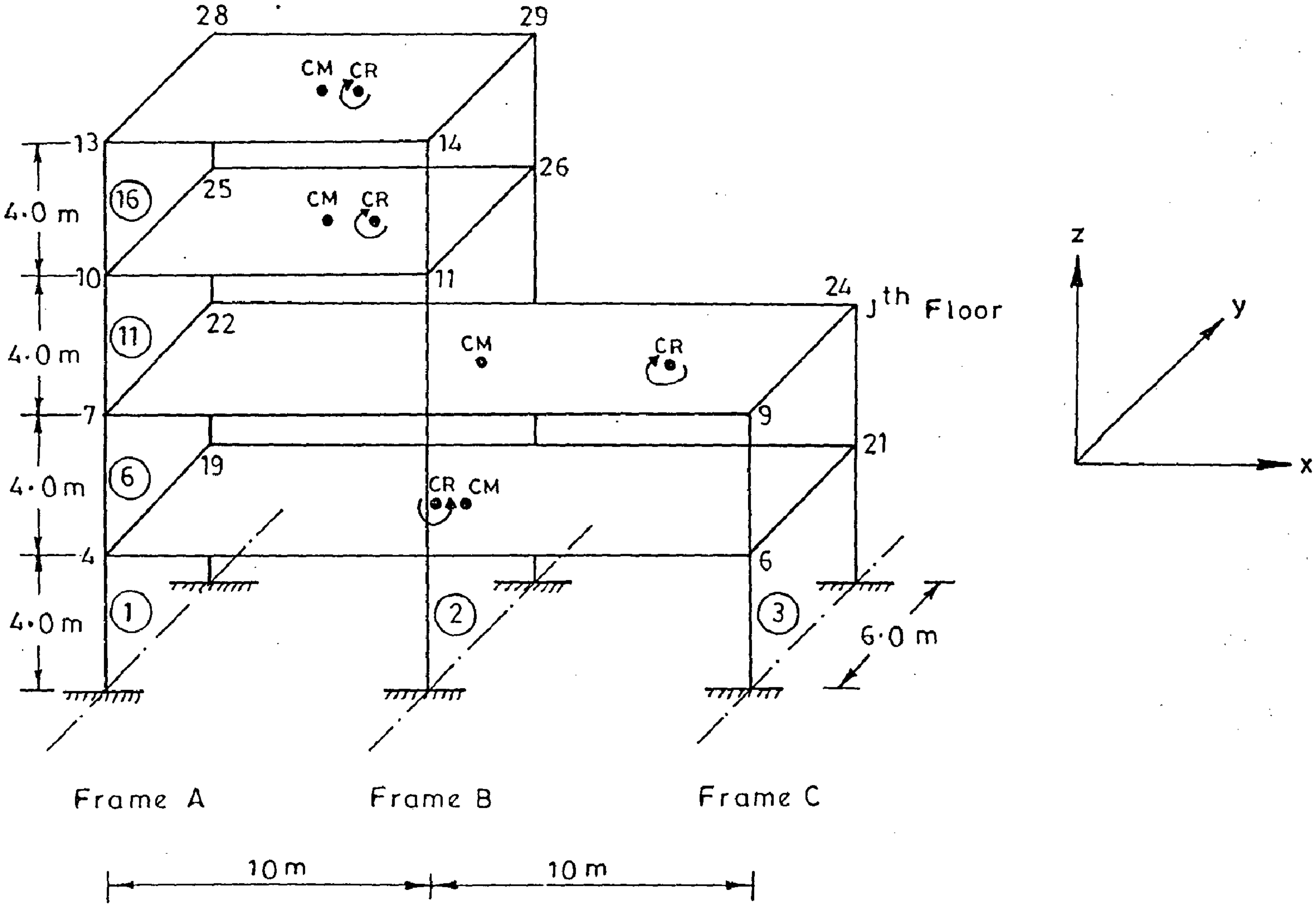



Fig. 4.4 Floor Torque of First Analysis Applied at the Floor CRs

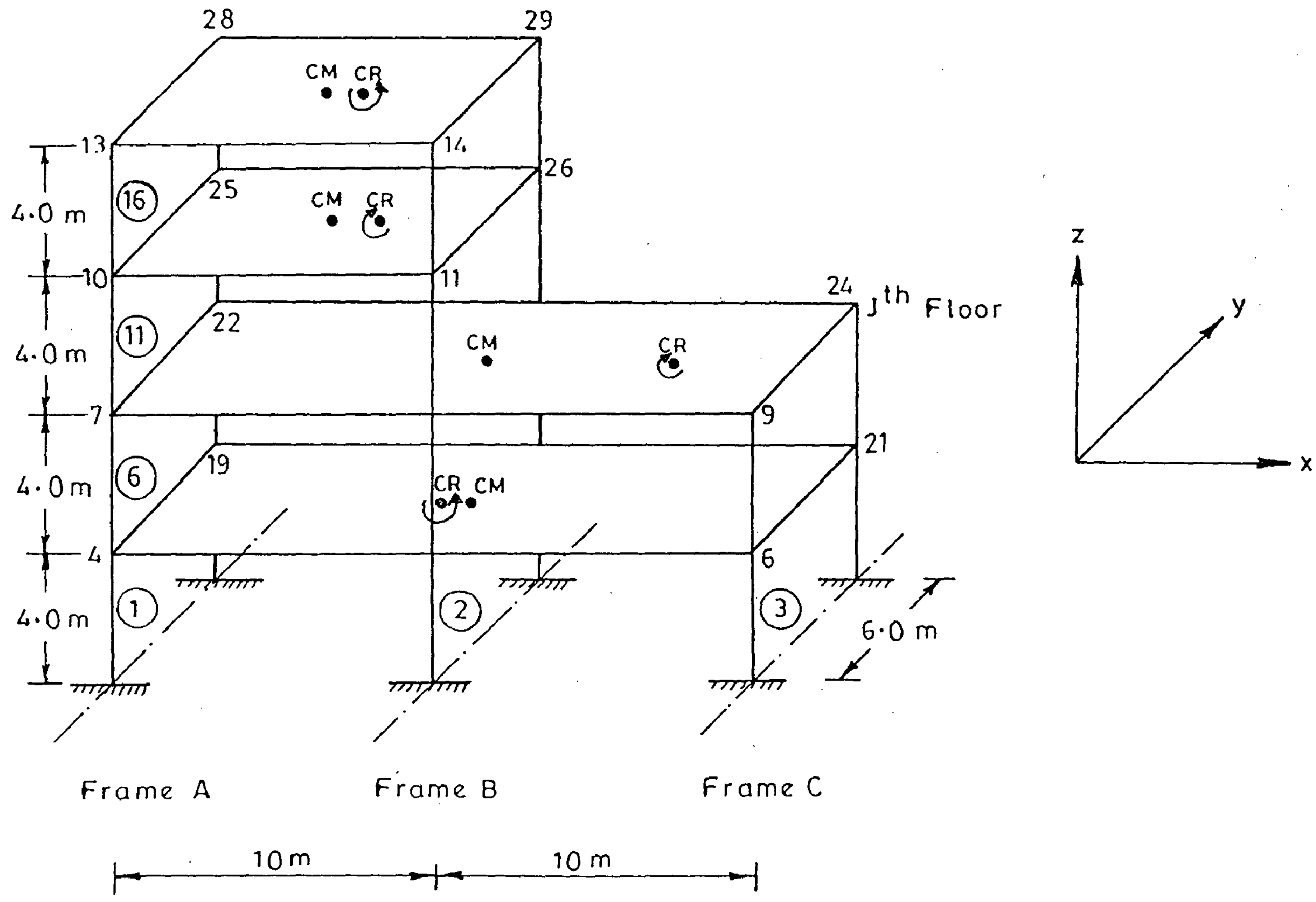



Fig. 4.5 Floor Torque of Second Analysis Applied at the Floor CRs

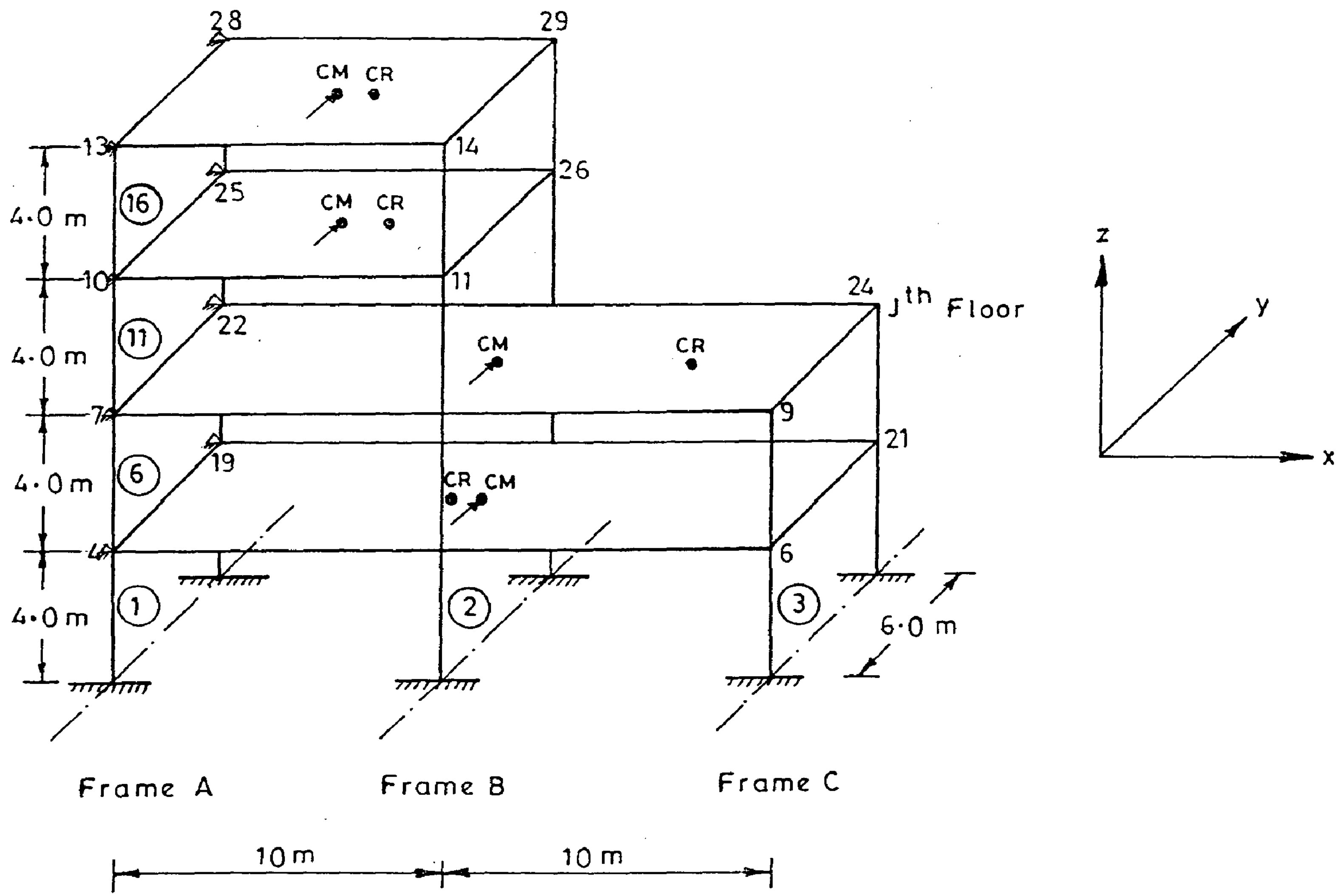



Fig. 4.6 Lateral force Applied at the Floor CMs for First Step of New Approach

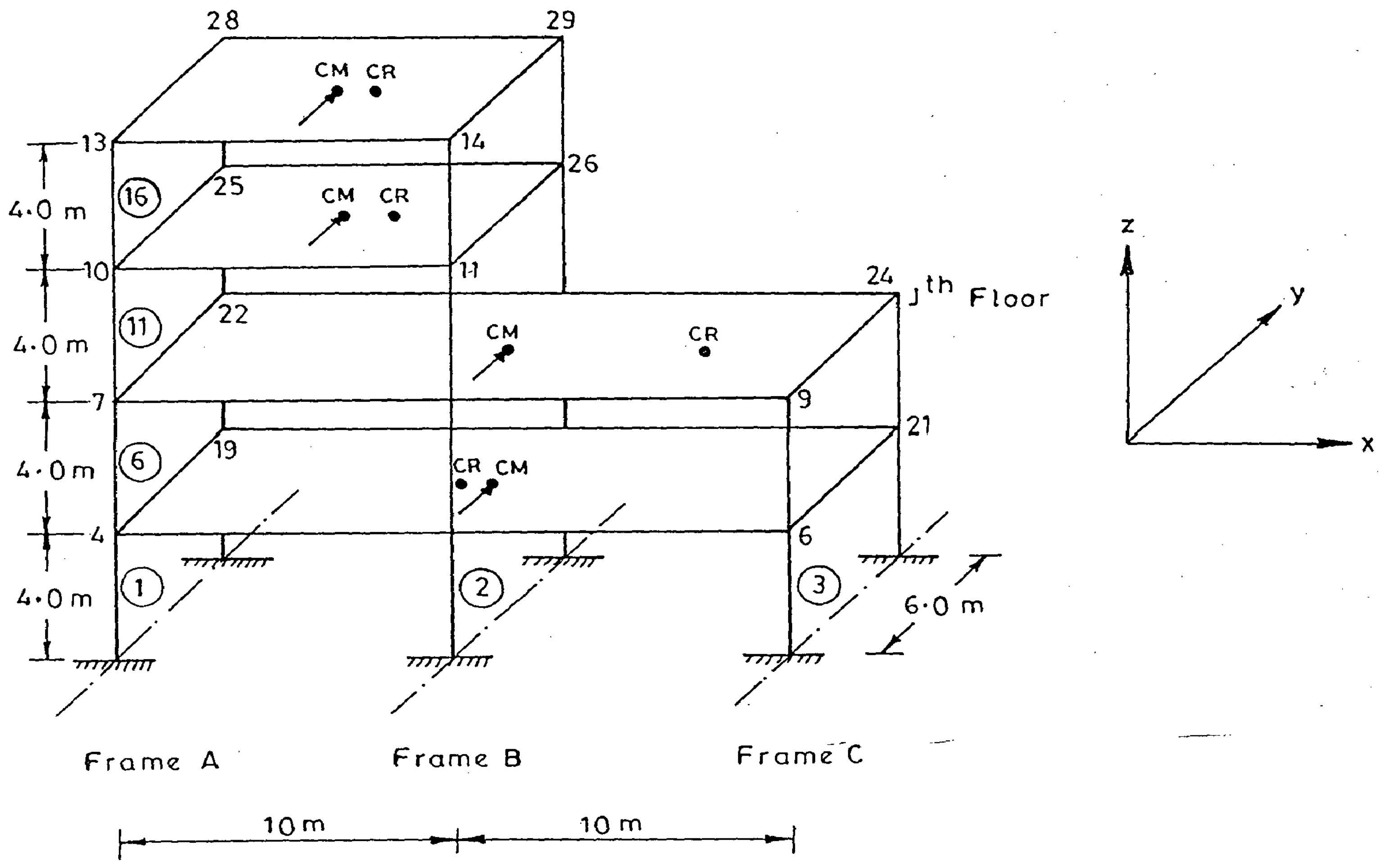



Fig. 4.7 Lateral Force Applied at the Floor CMs for Second Step of New Approach

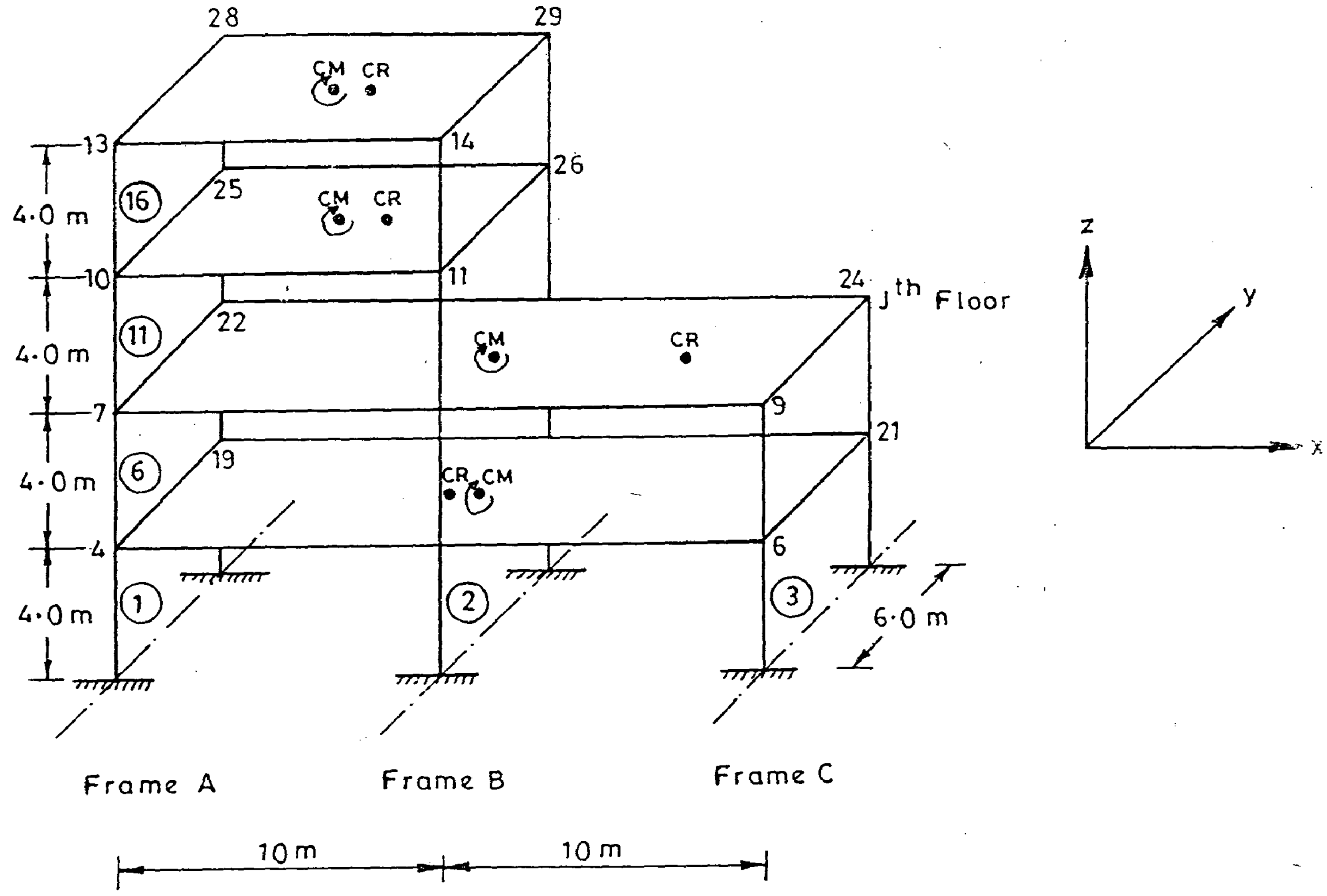



Fig. 4.8 Floor Torque Applied at the Floor CMs for Third Step of New Approach

# APPENDIX - A

## FORMATION OF STIFFNESS MATRIX

It consists of following steps :

(1) Determine the lateral stiffness matrix for each frame. For the $i^{th}$ frame it is determined by defining the degree of freedom of the frame as lateral displacements at floor levels, $u_i^T = \langle u_{i1}, u_{i2}, \ldots\ldots, u_{ij}, \ldots\ldots u_{iN} \rangle$ and torsional displacement at nodes. Then obtain the complete stiffness matrix for the $i^{th}$ frame with reference to the frame degree of freedom. Statically condense all the rotational and vertical degrees of freedom to obtain the $N$ x $N$ lateral stiffness matrix of the $i^{th}$ frame, denoted by $k_{xi}$ if the frame is oriented in the x-direction, or by $k_{yi}$ if the frame is oriented in the y-direction.

(2) Determine the displacement transformation matrix relating the lateral degrees of freedom $u_i$ for the $i^{th}$ frame to the global degree of freedom u for the building. This $N$ x $2N$ matrix is denoted by $a_{xi}$ if the frame is oriented in x-direction or $a_{yi}$ if frame is oriented in y-direction[1].

Thus,

$$u_i = ax_i u \qquad or \qquad u_i = ay_i u$$

These transformation matrices are

$$a_{xi} = [O \quad -y_i] \qquad \text{or} \qquad a_{yi} = [1 \quad x_i] \qquad (A.1)$$

where,

$O$ = square matrix of order N with all elements equal to zero

3. Transform the lateral stiffness matrix for the i$^{th}$ frame to the building degree of freedom u to obtain

$$k_i = a_{xi}^T \, k_{xi} \, a_{xi} \qquad \text{or} \qquad k_i = a_{yi}^T \, k_{yi} \, a_{yi} \tag{A.2}$$

The $2N$ x $2N$ matrix $k_i$ is the contribution of the i$^{th}$ frame to the building stiffness matrix.

4. Add the stiffness matrices for all frames to obtain the stiffness matrix for the building :

$$K = \sum_i k_i$$

Substituting the value of $a_{xi}$ and $a_{yi}$ from eq. (A.1) in eq. (A.2) to obtain $k_i$

$$k_i = \begin{bmatrix} 0 \\ -y_i \end{bmatrix} \begin{bmatrix} k_{xi} \end{bmatrix} \begin{bmatrix} 0 & -y_i \end{bmatrix}$$

$$k_i = \begin{bmatrix} 0 \\ -k_{xi} y_i \end{bmatrix} \begin{bmatrix} 0 & -y_i \end{bmatrix}$$

$$k_i = \begin{bmatrix} 0 & 0 \\ 0 & y_i^2 k_{xi} \end{bmatrix} \tag{A.3}$$

and

$$k_i = \begin{bmatrix} 1 \\ x_i \end{bmatrix} \begin{bmatrix} k_{yi} \end{bmatrix} \begin{bmatrix} 1 & +x_i \end{bmatrix}$$

$$k_i = \begin{bmatrix} k_{yi} \\ x_i k_{yi} \end{bmatrix} \begin{bmatrix} 1 & +x_i \end{bmatrix}$$