POWER SYSTEM DAMPING USING STATCOM

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

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MASTER OF TECHNOLOGY

in

ELECTRICAL ENGINEERING

(With Specialization in Power System Engineering)

By

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CANDIDATE'S DECLARATION

I hereby declare that the work which is being presented in the dissertation entitled **"POWER SYSTEM DAMPING USING STATCOM"** in partial fulfillment of the requirements for the award of degree of **Master of Technology** in **Electrical Engineering** with specialization in **Power System Engineering,** submitted in the Department of Electrical Engineering, Indian Institute of Technology Roorkee, Roorkee is an authentic record of my own work carried out from July, 2008 to June, 2009 under the guidance of **Dr. Vinay Pant**, Assistant Professor and **Dr. Biswarup Das**, Associate Professor, Department of Electrical Engineering, Indian Institute of Technology Roorkee, Roorkee, India.

The matter embodied in this dissertation has not been submitted by me for award of any other degree or diploma.

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CERTIFICATE

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ABSTRACT

Owing to various reasons like ever-increasing demand of electric power, deregulation, restrictions on construction of new transmission corridors etc., the modern power system now a days are forced to carry quite a large amount of power, thereby operating in a much stressed condition. Under this stressed condition, power transfer between two areas over a tie-line often exhibits low frequency inter-area oscillations. Low frequency oscillations were first observed in Northern America Power Network in early 1960's during a trial interconnection of the Northwest Power pool and Southwest Power pool. Later they have been also reported in many other countries. As the low frequency oscillations constrain the power carrying capacity of the line, it threatens power system security as well as limits the most efficient utilization of the transmission system. Therefore, this problem has caused wide concern and attracted the attention of the researchers since 1960's.

In the early days, the excitation control of the synchronous machines was remedy available for controlling these oscillations. Later, power system stabilizers (PSS) were developed and added to the excitation system loop to enhance the damping of these low frequency oscillations. However, a PSS is generally more effective in damping local modes, but in certain cases it may not be sufficient to provide the necessary damping for inter-area oscillations. Therefore, for enhancing the power system security, development of most effective method of damping the inter-area mode of low-frequency oscillation is still a major area of research today.

Now, in the last one and half decades power electronic device based equipments, commonly known as Flexible AC Transmission (FACTS) controllers, have been implemented in various parts of the world to achieve better utilization of the existing power grid. Because of their capability of fast power control, these FACTS controllers can be used effectively to damp out the inter-area oscillations. Among the various FACTS controllers, Static Synchronous Compensator (STATCOM) is one of the most prominent equipments. It is a shunt connected equipment whose main objective is to control the bus voltage. However, by using the voltage controller only, the STATCOM may not be able to damp out the inter-area oscillations effectively and in some cases may even aggravate the problem. Therefore, an auxiliary damping controller is often needed to be designed, which would provide a superimposed damping signal to its voltage loop.

The static synchronous compensator (STATCOM) is one of the recently developed converter-based flexible AC transmission systems (FACTS) controllers. Usually the reactive output of a STATCOM is regulated to maintain the desired AC voltage at the bus, to which a STATCOM is connected. This report describes a voltage control functional model of a STATCOM for power system steady-state operations. Voltage control mode of a STATCOM is presented. A numerical example on the 3-machine 9-bus system and 10-machine 39-bus system is used to illustrate the voltage control functional model of a STATCOM.

The first step for design of the damping controller is calculation of the initial operating conditions of the system, which can be obtained through load flow analysis. After the initial operating conditions were computed, LQR technique has been applied to design the STATCOM damping controller in multi-machine system under different loading conditions. Towards this goal, the detailed linearized state space model of a power system with a STATCOM has been developed. From initial condition response simulation (unforced response of a state-space model) studies carried out in multi-machine system under different loading conditions, the efficacy of the developed local signal based controller has been found to be satisfactory for improving the system damping.

Apart from LQR technique, GA (genetic algorithm) technique has also been used for calculating the suitable value of the gain matrix K, for which damping is improved. From initial condition response simulation studies carried out in multi-machine system under different loading conditions, it has been found that the GA based controller are better capable of damping out the oscillations effectively in the power system than LQR technique.

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LIST OF SYMBOLS

A : State matrix of the system

B : Control input matrix

C : Output matrix

D : Output control matrix

E'_{di} : Direct axis internal voltage of i^{th} machine behind transient reactance in p.u.

E'_{q} : Quadrature axis voltage of machine behind transient reactance in p.u.

E'_{qi} : Quadrature axis voltage of i^{th} machine behind transient reactance in p.u.

E'_{q0} : Initial value of quadrature axis voltage of machine behind transient reactance in p.u.

E_{fd} : Generator field winding voltage in p.u.

E_{fdi} : Field winding voltage of i^{th} machine in p.u.

$S_{Ei}\left(E_{fdi}\right)$: Field saturation function

I_{dc} : STATCOM d.c. current

I_{di} : Direct axis current of i^{th} generator in p.u.

I_{dst} : Direct axis component of STATCOM current in multi-machine system in p.u.

I_{qi} : Quadrature axis current of i^{th} generator in p.u.

I_{qst} : Quadrature axis component of STATCOM current in multi-machine system in p.u.

j : Complex number operator

K_{A} : Exciter gain in SMIB system

K_{Ai} : Amplifier gain of i^{th} machine

K_{Ei} : Exciter gain of i^{th} machine

K_{Fi} : Stabilizing transformer gain for i^{th} machine m : Number of synchronous machines in multi-machine system

n : Number of buses in multi-machine system

P_g : Generator real power at bus in p.u.

P_i : Injected active power at i^{th} bus in p.u.

P_{Line} : Active power flow

P_{Loss} : Active power loss

P_m : Mechanical power input of generator in p.u.

Q_g : Generator reactive power at bus in p.u.

Q_i : Injected reactive power at i^{th} bus in p.u.

Q_{Line} : Reactive power flow

Q_{Loss} : Reactive power loss

R_{fi} : Rate feedback

R_{si} : Leakage resistance of STATCOM step down transformer to represent the ohmic loss

t : Time

T : Transpose

T_A : Exciter time constant in SMIB system

T_{Ai} : Amplifier time constant of i^{th} machine

T'_{do} : Direct axis open circuit field time constant of machine

T'_{doi} : Direct axis open circuit field time constant of i^{th} machine

T_{Ei} : Exciter time constant of i^{th} machine

T_{Fi} : Stabilizing transformer time constant of i^{th} machine

T'_{qoi} : Quadrature axis time constant of i^{th} machine

V_{dc} : STATCOM capacitor dc voltage in p.u. in multi-machine system

V_i : Voltage at i^{th} bus

V_{Ri} : Output voltage of regulator of i^{th} machine in p.u.

V_{st} : Magnitude of STATCOM output voltage in p.u. in multi-machine system

V_t : Magnitude of generator terminal voltage in p.u.

x_d : Direct axis steady state synchronous reactance of generator in p.u.

x_{dc} : STATCOM dc capacitance in p.u. in multi-machine

x_{di} : Direct axis steady state synchronous reactance of i^{th} machine in p.u.

x_d' : Direct axis transient synchronous reactance of generator in p.u.

x_{di}' : Direct axis transient synchronous reactance of i^{th} machine in p.u.

x_q : Quadrature axis steady state synchronous reactance of machine in p.u.

x_{qi}' : Quadrature axis transient synchronous reactance of i^{th} machine in p.u.

x_{si} : Leakage reactance of STATCOM step down transformer in p.u.

z_{si} : Impedance of the step down transformer

Ψ : Angle of STATCOM output voltage in degrees

δ : Generator rotor angle in degrees

δ_0 : Initial value of the generator rotor angle

δ_i : Rotor angle of the i^{th} machine

ω : Generator speed in p.u.

ω_i : Angular speed for i^{th} machine in p.u.

ω_s : Synchronous speed for i^{th} machine in p.u.

θ_i : Angle of V_i in degree

θ_t : Angle of STATCOM bus voltage in p.u. in multi-machine system in degrees

LIST OF ABBREVIATIONS

FACTS	Flexible AC Transmission System
GA	Genetic Algorithm
LQR	Linear Quadratic Regulator
LTI	Linear Time Invariant
MATLAB	Matrix Laboratory
MIMO	Multi Input Multi Output
MVA	Mega Volt Ampere
MVAR	Mega Volt Ampere Reactive
MW	Mega Watt
NR	Newton Raphson
p.u.	Per Unit
PQ	Load Buses
PSS	Power System Stabilizer
PV	Voltage Control or Generator Bus
SISO	Single Input Single Output
SMIB	Single Machine Infinite Bus
SSSC	Static Synchronous Series Compensator
STATCOM	Static Synchronous Compensator
SVC	Static Var Compensator
VSC	Voltage Source Converter

CHAPTER 1

INTRODUCTION

Owing to various reasons like ever-increasing demand of electric power, deregulation, restriction on construction of new transmission corridors etc., the modern power system now-a-days is forced to carry quite a large amount of power, thereby operating in much stressed condition. Under this stressed condition, power transfer between two areas over a tie line is often restricted by low frequency inter area oscillations in the power system[1-3], which in turn, threaten the power system security. Traditionally, Power System Stabilizers (PSS) have been applied [4] to damp out low frequency oscillations. However, a PSS is generally most effective in damping local modes [5], but it is not very much effective in damping the inter-area modes. Therefore, for enhancing the power system security, development of most effective method of damping the inter-area mode of low-frequency oscillation is still a major area of research today.

It is nowadays well recognized that the stability limits can be further improved by real time, dynamic control of various network parameters by employing Flexible AC Transmission System (FACTS) controllers [6-9]. FACTS controllers are solid state devices that provide fast and reliable control over the transmission line parameters, such as voltage, phase angle and line impedance. Increased interest in use of these devices (FACTS devices) is due to two reasons: development in high power electronic devices and increased loading of power systems combined with deregulation of power industry. FACTS controllers are becoming an integral component of modern power transmission systems. The development of FACTS controllers has followed two distinctly different technical approaches both resulting in a comprehensive group of controllers which are able to address targeted transmission problems. The first group employs reactive impedances or a tap changing transformer with thyristor switches as controlled elements. The second group uses self-commutated static converters as controlled voltage sources [6, 8]. Use of these controllers, which are essentially high speed power electronic controllers, results in enhanced utilization of the existing transmission system, allowing

- flexible control of power, so that flow is on established transmission routes.

-secure loading (but not overloading) of transmission lines to levels nearer their thermal limits

-greater ability to transfer power between controlled areas, with consequent generation reserve margin

-prevention of cascading outages by limiting the effects of faults and equipment failures

-damping of power system oscillations

Among the various FACTS controllers, STATCOM(Static Synchronous Compensator) is one of the most prominent equipments. It is basically a voltage source converter (VSC) based shunt-connected equipment whose main objective is to control the bus voltage. As compared to other shunt connected FACTS equipment, namely SVC (Static Var Compensator), there are several technical advantages of a STATCOM over SVC as follows [10]:

- Faster response
- Require less space
- Modular and relocatable design
- Can be interfaced with real power sources such as battery, fuel cell or SMES(Superconducting Magnetic Energy Storage)
- Even during low voltage conditions, STATCOM is capable of maintaining constant reactive current
- If devices (such as GTO, IGBT etc.) are rated for transient overload, under transient conditions it is capable of supplying increased reactive current.

As mentioned above, a STATCOM is mainly used for controlling the voltages of the grid. As a result, several publications in the literature have addressed this issue. Different techniques such as PI controller [11], loop shaping technique [12], linear optimal control based on LQR strategy [13] etc., have been proposed in the literature.

Application of STATCOM has also been extensively studied for improving the Power System Stability. Apart from the schemes for improving the Sub Synchronous Resonance (SSR) [14] and Torsional damping [15], different other STATCOM control techniques for enhancing small signal and transient stability have also been suggested in the literature. For transient stability enhancement with STATCOM, various techniques such as trajectory sensitivity analysis [16], Lyapunov stability theory [17], nonlinear H_∞ based control [18] etc. have been suggested in the literature.

Samir and Baiyat [18] presented robust control design for a SMIB with a STATCOM. They designed the controller for the nonlinear system using the recently developed nonlinear H_∞ theory. The approach combines state feedback exact linearization with linear H_∞ principle, which avoids the difficulty of solving Hamilton–Jacoby–Issacs inequality. Simulation results with a number of disturbances like torque pulses and three-phase faults on the generator show that the proposed robust controller can ensure transient stability of the power system over a wide range of operating points.

In [19] it has been shown that the shunt connected FACTS devices (SVC or STATCOM), though primarily used for voltage support, can also be used for effectively damping the electromechanical oscillations if properly placed and controlled. As a result, they can be quite attractive alternatives of the series connected FACTS devices, as their overall cost is lower.

Haque [20] determined additional damping provided by STATCOM and a SSSC by using the concept that the rate of dissipation of transient energy as a measure of system damping. They derived Analytical expressions for additional damping provided by these devices and compared for classical model of a simple power system. They tested the proposed technique of evaluating system damping is on a SMIB system and for some special faults in the 10-machine New England system.

In [21] it is reported that under certain circumstances a STATCOM can provide better damping as compared to a SSSC. Thus, quite significant amount of effort has been made in the literature to study the effectiveness of STATCOM for improving the damping of oscillations in a power system.

For carrying out different studies or for designing suitable control schemes, suitable models of STATCOMs for dynamic studies as well as for computing the initial conditions (from load flow analysis) are required. As a result, various models have been suggested in the literature in this regard. Average circuit model and linear frequency domain model of a STATCOM have been discussed in [22] and [23] respectively.

Wang [24] designed a STATCOM stabiliser to improve power system oscillation stability. First he establishes the linearized Phillips-Heffron model of a power system installed with a STATCOM in the case of SMIB system as well as multi machine systems and demonstrates the application of this model in analysing the damping effect of the STATCOM and then he designed a STATCOM stabiliser to improve power system oscillation stability.

To compute the initial conditions, different steady state models of STATCOM suitable for load flow analysis have been suggested in [25-28]. In [25], detailed steady state equations for STATCOM, which can also appropriately handle different operating limits, has been suggested. An extension of Newton-Raphson (NR) power flow technique for incorporating STATCOM has been presented in [26], A power injection model of a STATCOM suitable for NR formulation has been described in [27]. Strategies for incorporating various control functions of STATCOM in load flow analysis have been elaborated in [28].

Now, for designing the damping controller, the initial steady state operating point must be found out through load flow analysis of a power system incorporating STATCOM. Traditionally, a STATCOM has been represented in a power flow study as a PV bus [29], In this approach, generally the internal losses in the STATCOM (both switching losses as well as the ohmic losses of the step down transformer) are neglected and as a result, the specified real powers at these PV buses are set to zero. As the internal losses are neglected, this method, although very popular, is not very accurate. In [30] an improved model of the STATCOM has been proposed to consider the ohmic losses of the step down transformer in the power flow solution method. This method however neglects the switching losses in the STATCOM. In [25] detailed equations for steady state and transient analysis for the STATCOM have been presented but no numerical results have been given to illustrate the application of these equations in load flow. In [26, 28] only ohmic losses have been considered whereas in [27] both switching loss as well as the ohmic loss have also been taken into account. However, for implementation of the algorithms proposed in [26-28], modification of the main load flow Jacobian matrix is necessary.

In this dissertation, a simple and indirect approach for load flow with STATCOM has been developed to compute the initial conditions. In the proposed technique, the main load flow equations and the STATCOM equations are solved separately and therefore there is no need to make any changes in the main load flow Jacobian. The developed technique is also capable of incorporating STATCOM and switching losses in the power flow calculation. The effectiveness of this proposed technique has been tested on 3-machine 9-bus and 10-machine 39-bus system with STATCOM. The developed algorithm has also been found to be equally effective for different loading conditions in all test systems and the results obtained are in complete agreement with the expected behavior of the STATCOM.

Acha et.al. [31] discussed about the power flow model of the STATCOM in the fifth chapter. They also discussed the operational characteristics of STATCOM in the second chapter. They also took the numerical example of voltage magnitude control using on STATCOM on 5 Bus system. They showed that use of the STATCOM results in an improved voltage profile.

Ramirez and Perez [32] showed that in an actual power system, each phase of critical buses may be subjected to a different voltage stability margin, depending on the load level, and that the STATCOM inclusion can help to improve such margins. The STATCOM improves the voltage stability margin and the voltage magnitude at its surrounding nodes; it also diminishes the total losses in the system.

Haque [33] explained about how to determine the additional damping provided by a STATCOM and evaluated by examining the rate of dissipation of transient energy in post fault period. First he evaluated the additional damping provided by a STATCOM in the case of single machine system, in which he also defined the control law of STATCOM current, then he explained the same for the multi machine system, then he tested his technique in a SMIB system and 10-machine new England. It was also observed that the rate of dissipation of transient energy increases as the STATCOM rating is increased.

Ghafouri et.al. [34] explained the application of fuzzy logic controlled STATCOM to improve the transient stability of the power system. First they expressed the STATCOM model for transient stability analysis and its effect on transient stability. Their control strategy is based on Mamdani's Fuzzy logic controller and then they tested their technique on a SMIB system and 9-bus IEEE system and they found that the controller can improve the stability margin of system in case of transient stability and provides considerable damping of power oscillation.

Puleston et.al. [35] designed different algorithms for STATCOM voltage control to inject extra friction in order to eliminate under damped power oscillation in transmission system. First they described power transmission system dynamic model and their fault behaviour, then different control algorithms are analysed and the variable structure controller design control law is synthesised. Finally, they observed that better results were obtained combining the discontinuous friction with continuous linear friction instead of high order polynomial friction.

Son and Park [36] discussed the procedure to design of the robust TCSC controller for power system oscillation damping enhancement using LQG (Linear Quadratic Gaussian) technique. First they discussed about LQG technique, then they designed TCSC controller for 3-machine 9-bus system. They discussed about the controller design procedure, in which they considered the input signal selection, comments on the classical control and the LQG control, model order reduction, input signal filtering, weighting matrices selection, robustness against modelling error, time domain simulations.

Ferreira [37] used self-tuning Linear Quadratic Gaussian control with Loop Transfer Recovery (LQG/LTR), applied in TCSC installed in interconnected power systems. First they presented robust adaptive control. The proposed robust adaptive controller can improve power system stability margins, damping properly the system's dominant electromechanical modes and its properties can be verified using non-linear simulations for a four-machine power system, in different operation conditions.

Noroozian [38] developed control laws for damping electromechanical oscillations using SVCs and TCSCs by energy function method. Their paper examines the enhancement of power system stability properties by use of TCSCs and SVCs. Each device (TCSC and SVC) will contribute to the damping of power swings without deteriorating the effect of the other power oscillation damping (POD) devices using his control strategy. The damping effect is robust with respect to loading condition, fault location and network structure. Furthermore, the control inputs are based on local signals.

Cai et.al. [39] proposed a novel robust adaptive modulation controller (RAMC) for TCSC in interconnected power systems to damp low frequency oscillation. First they derived centre of inertia (COI) co-ordinate based model, then RAMC for TCSC is derived using back stepping method with COI dynamic signals from WAMS. Then they tested the new controller performance is tested through a three-area five machine system and they found that the new controller using COI signals has superior performance as compared with the conventional controller.

Rahim et.al. [40] designed the robust controller for providing damping to power system transients through STATCOM devices. They employed method of multiplicative uncertainty to model the variations of the operating points in the system. They employed loop shaping method to select a suitable open-loop transfer function from which the robust controller is constructed. Then they tested the proposed controller through a number of disturbances including three-phase faults. They found that the robust design is much superior to conventional PI and other similar controllers.

Panda and Padhy [41] discussed about optimal location of a STATCOM and its coordinated design with power system stabilizers (PSSs) for power system stability improvement. First they formulated the location of STATCOM to improve transient stability as an optimization problem and then they employed particle swarm optimization (PSO) method for its optimal location. They used four-machine two-area system to show the effectiveness of the proposed approach for determining the optimal location of STATCOM and controller parameters for power system stability improvement. They found that the inter-area and local modes of oscillations are well damped with the proposed PSO-optimized controllers.

Morris et.al. [42] presented two new variable structure fuzzy control algorithms for controlling the reactive component of the STATCOM current in a power system. The control signal is obtained from a combination of generator speed deviation and STATCOM bus voltage deviation fed to the variable structure fuzzy controller. These new fuzzy controllers for STATCOM provide a wide range of gain variations for controlling the electromechanical oscillations of a SMIB and multi machine power systems.

Haque [43] presented a simple method to evaluate the first swing stability of a large power system in the presence of various FACTS devices. First a unified power flow controller (UPFC) and the associated transmission line are considered and represented by an equivalent circuit model. Then the above model is interfaced to the power network to obtain the system reduced admittance matrix which is needed to generate the machine swing curves. He tested the effectiveness of the proposed method of generating dynamic response and hence evaluating first swing stability of a power system in the presence of various FACTS devices on the 10-machine New England system and the 20-machine IEEE test system.

The organisation of this dissertation is as follows:

Chapter 2 describes the load flow method for power system with STATCOM. Chapter 3 describes the LQR design technique for damping controller. Chapter 4 describes the Genetic algorithm based controller design technique. In Chapter 5, damping controller design for STATCOM in multi-machine system using LQR and GA technique are presented. Chapter 6 describes the conclusions and scope for future work.

CHAPTER 2

LOAD FLOW ANALYSIS WITH STATCOM

2.1 Introduction [7]

The STATCOM is a shunt connected reactive power compensation device, which is capable of generating or absorbing reactive power and in which the output can be varied to control the specific parameters of an electric power system.

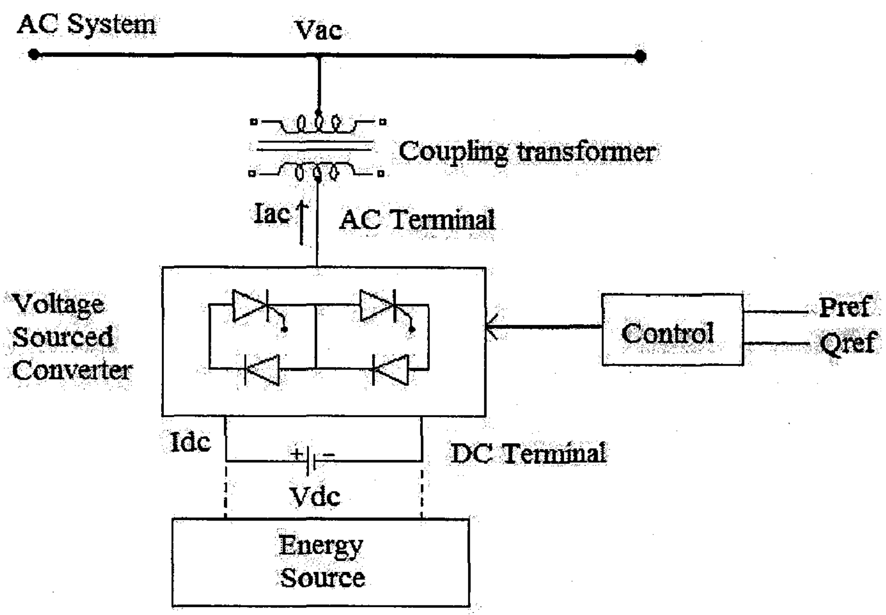

Figure 2.1: Functional block diagram model of STATCOM

The basic electronic block of STATCOM is the voltage sourced converter, which in general converts an input dc voltage into three phase output voltage at fundamental frequency with rapidly controllable amplitude and phase angle. In addition to this the controller has a coupling transformer and a dc capacitor. References Q_{ref} and P_{ref} define the magnitude and phase angle of the converter voltage V_{cnv} necessary to exchange the desired reactive and active power between the solid state voltage source converter and ac system.

2.2 Principle of Operation [7]

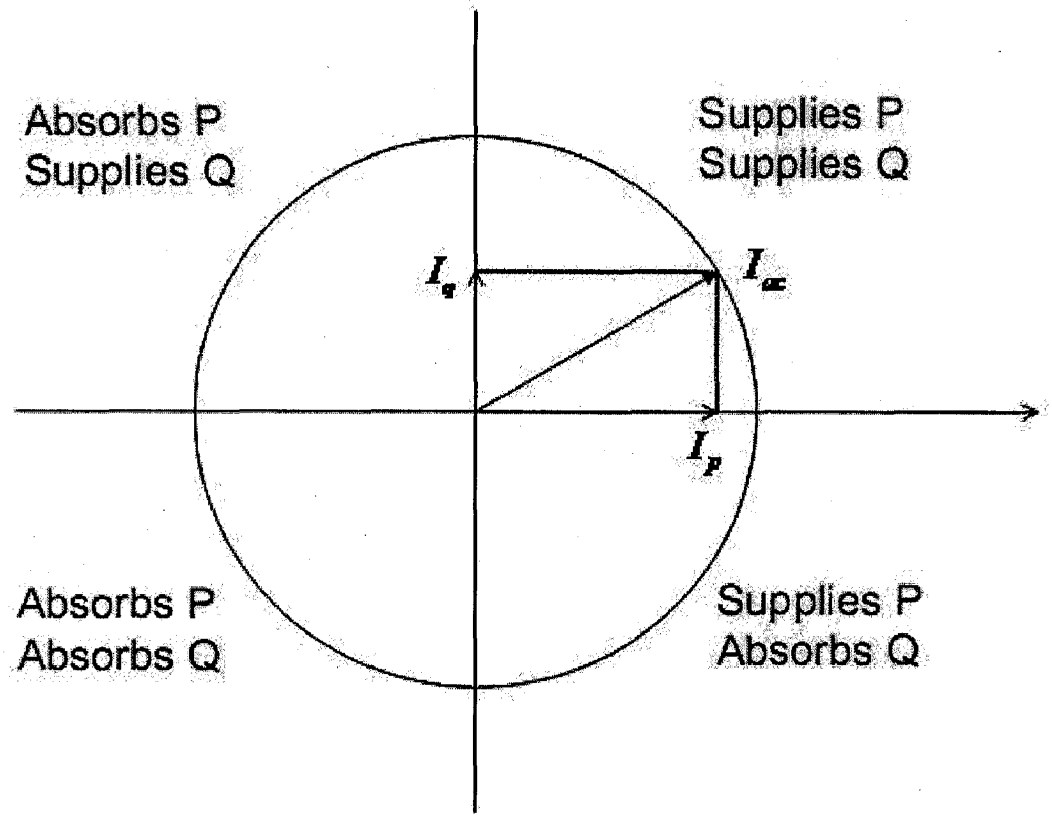

Figure 2.2: Power exchange with energy storage device

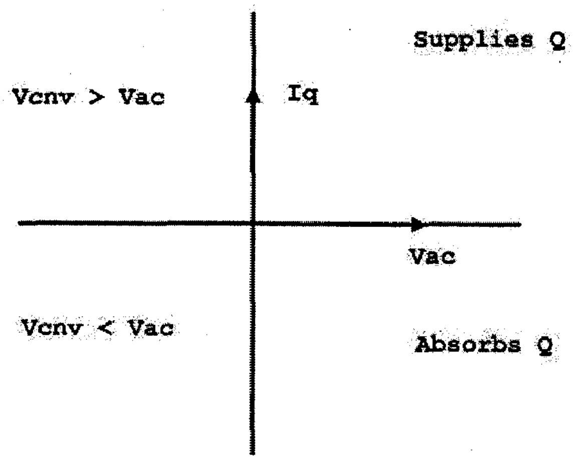

Figure 2.3: Power exchange without energy storage device

The STATCOM supplies reactive power to the ac system, if converter voltage V_{cnv} is greater than ac terminal voltage V_{ac} and consumes reactive power, if V_{cnv} is lower than V_{ac}. The device can be designed to maintain the magnitude of the bus voltage constant by controlling the magnitude and/or phase shift of the VSC output voltage with energy storage of suitable rating. The STATCOM can exchange both reactive and real power with the ac system. The power exchange between the STATCOM and the ac system with and without energy storage are shown in Figure 2.2 and 2.3 respectively.

The STATCOM can provide both capacitive and inductive compensation and it is able to control its output current over the rated maximum capacitive or inductive range independently of the ac system voltage. That is, the STATCOM can provide full capacitive output current at any system voltage practically down to zero.

2.3 Power Flow Constraints of the STATCOM [28]

Based on operating principle of the STATCOM, the equivalent circuit will be derived, which is shown in figure 2.4. In the derivation, it is assumed that

(a) Harmonics generated by the STATCOM are neglected,

(b) The system as well as the STATCOM is three phase balanced.

Then, the STATCOM can be equivalently represented by a controllable fundamental frequency positive sequence voltage source V_{sh}. In principle, the STATCOM output voltage can be regulated such that the reactive power of the STATCOM can be changed.

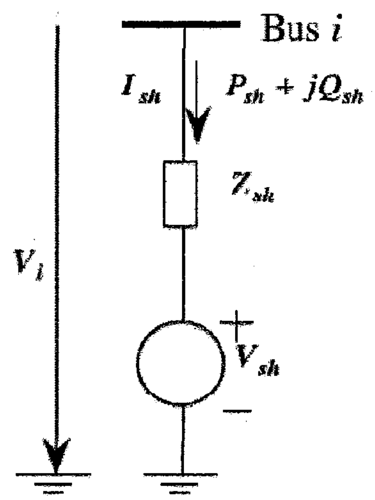

Figure 2.4: STATCOM equivalent circuit

According to the equivalent circuit of STATCOM shown in Figure 2.3, suppose $V_{sh} = V_{sh}\angle\theta_{sh}$, $V_i = V_i\angle\theta_i$, then the power flow equations of the STATCOM are:

$$P_{sh} = V_i^2 g_{sh} - V_i V_{sh}(g_{sh}\cos(\theta_i - \theta_{sh}) + b_{sh}\sin(\theta_i - \theta_{sh})) \tag{2.1}$$

$$Q_{sh} = -V_i^2 b_{sh} - V_i V_{sh}(g_{sh}\sin(\theta_i - \theta_{sh}) - b_{sh}\cos(\theta_i - \theta_{sh})) \tag{2.2}$$

Where, $g_{sh} + jb_{sh} = 1/Z_{sh}$

The operating constraints of the STATCOM are the active power exchange via the DC-link as described by:

$$\text{PE} = \text{Re}(V_{sh} I_{sh}^*) = 0 \tag{2.3}$$

Where $\text{Re}(V_{sh} I_{sh}^*) = V_{sh}^2 g_{sh} - V_i V_{sh}(g_{sh}\cos(\theta_i - \theta_{sh}) - b_{sh}\sin(\theta_i - \theta_{sh}))$

2.4 Control Functions of the STATCOM [28]

In the practical applications of a STATCOM, it may be used for controlling one of the following parameters:

1. Voltage magnitude of the local bus, to which the STATCOM is connected;
2. Reactive power injection to the local bus, to which the STATCOM is connected;
3. Impedance of the STATCOM;
4. Current magnitude of the STATCOM while the current I_{sh} leads the voltage injection V_{sh} by 90°.
5. Current magnitude of the STATCOM, while the current I_{sh} lags the voltage injection V_{sh} by 90°.
6. Voltage injection;
7. Voltage magnitude at a remote bus;
8. Reactive power flow;
9. Apparent power or current control of a local or remote transmission line.

Among these control options, control of the voltage of the local bus, which the STATCOM is connected to, is the most-recognized control function. The other control possibilities have not fully been investigated in power flow analysis. The mathematical description of the Bus voltage control function is presented as follows:

Control mode: Bus voltage control

The bus control constraint is as follows:

$$V_i - V_i^{spec} = 0 \tag{2.4}$$

Where $V_i^{spec} = 1$ is the bus voltage control reference.

2.5 Voltage Control Functional Model of STATCOM in Power Flow

A STATCOM has only one degree of freedom for control since the active power exchange with the DC link should be zero at any time. The STATCOM may be used to control one of the nine parameters.

Here, n bus system has taken out of which m buses are generator bus including one slack bus and there is p number of STATCOMs are connected in the n bus system to improve the voltage profile of desired bus. Hence, the model equation can be written as following:

$$J \bullet X = M \tag{2.5}$$

$$\text{So, } X = J^{-1}M$$

$$
\begin{bmatrix}
\frac{\partial P_2}{\partial \theta_2} & \cdots & \frac{\partial P_2}{\partial \theta_n} & \frac{\partial P_2}{\partial V_{m+1}} & \cdots & \frac{\partial P_2}{\partial V_n} & \frac{\partial P_2}{\partial V_{sh1}} & \cdots & \frac{\partial P_2}{\partial V_{shp}} & \frac{\partial P_2}{\partial \theta_{sh1}} & \cdots & \frac{\partial P_2}{\partial \theta_{shp}} \\
\vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\frac{\partial P_n}{\partial \theta_2} & \cdots & \frac{\partial P_n}{\partial \theta_n} & \frac{\partial P_n}{\partial V_{m+1}} & \cdots & \frac{\partial P_n}{\partial V_n} & \frac{\partial P_n}{\partial V_{sh1}} & \cdots & \frac{\partial P_n}{\partial V_{shp}} & \frac{\partial P_n}{\partial \theta_{sh1}} & \cdots & \frac{\partial P_n}{\partial \theta_{shp}} \\
\frac{\partial Q_{m+1}}{\partial \theta_2} & \cdots & \frac{\partial Q_{m+1}}{\partial \theta_n} & \frac{\partial Q_{m+1}}{\partial V_{m+1}} & \cdots & \frac{\partial Q_{m+1}}{\partial V_n} & \frac{\partial Q_{m+1}}{\partial V_{sh1}} & \cdots & \frac{\partial Q_{m+1}}{\partial V_{shp}} & \frac{\partial Q_{m+1}}{\partial \theta_{sh1}} & \cdots & \frac{\partial Q_{m+1}}{\partial \theta_{shp}} \\
\vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\frac{\partial Q_n}{\partial \theta_2} & \cdots & \frac{\partial Q_n}{\partial \theta_n} & \frac{\partial Q_n}{\partial V_{m+1}} & \cdots & \frac{\partial Q_n}{\partial V_n} & \frac{\partial Q_n}{\partial V_{sh1}} & \cdots & \frac{\partial Q_n}{\partial V_{shp}} & \frac{\partial Q_n}{\partial \theta_{sh1}} & \cdots & \frac{\partial Q_n}{\partial \theta_{shp}} \\
\frac{\partial PE_1}{\partial \theta_2} & \cdots & \frac{\partial PE_1}{\partial \theta_n} & \frac{\partial PE_1}{\partial V_{m+1}} & \cdots & \frac{\partial PE_1}{\partial V_n} & \frac{\partial PE_1}{\partial V_{sh1}} & \cdots & \frac{\partial PE_1}{\partial V_{shp}} & \frac{\partial PE_1}{\partial \theta_{sh1}} & \cdots & \frac{\partial PE_1}{\partial \theta_{shp}} \\
\vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\frac{\partial PE_p}{\partial \theta_2} & \cdots & \frac{\partial PE_p}{\partial \theta_n} & \frac{\partial PE_p}{\partial V_{m+1}} & \cdots & \frac{\partial PE_p}{\partial V_n} & \frac{\partial PE_p}{\partial V_{sh1}} & \cdots & \frac{\partial PE_p}{\partial V_{shp}} & \frac{\partial PE_p}{\partial \theta_{sh1}} & \cdots & \frac{\partial PE_p}{\partial \theta_{shp}} \\
\frac{\partial F_1}{\partial \theta_2} & \cdots & \frac{\partial F_1}{\partial \theta_n} & \frac{\partial F_1}{\partial V_{m+1}} & \cdots & \frac{\partial F_1}{\partial V_n} & \frac{\partial F_1}{\partial V_{sh1}} & \cdots & \frac{\partial F_1}{\partial V_{shp}} & \frac{\partial F_1}{\partial \theta_{sh1}} & \cdots & \frac{\partial F_1}{\partial \theta_{shp}} \\
\vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\frac{\partial F_p}{\partial \theta_2} & \cdots & \frac{\partial F_p}{\partial \theta_n} & \frac{\partial F_p}{\partial V_{m+1}} & \cdots & \frac{\partial F_p}{\partial V_n} & \frac{\partial F_p}{\partial V_{sh1}} & \cdots & \frac{\partial F_p}{\partial V_{shp}} & \frac{\partial F_p}{\partial \theta_{sh1}} & \cdots & \frac{\partial F_p}{\partial \theta_{shp}}
\end{bmatrix}
\begin{bmatrix} \Delta\theta_2 \\ \vdots \\ \Delta\theta_n \\ \Delta V_{m+1} \\ \vdots \\ \Delta V_n \\ \Delta V_{sh1} \\ \vdots \\ \Delta V_{shp} \\ \Delta\theta_{sh1} \\ \vdots \\ \Delta\theta_{shp} \end{bmatrix}
=
\begin{bmatrix} -\Delta P_2 \\ \vdots \\ -\Delta P_n \\ -\Delta Q_{m+1} \\ \vdots \\ -\Delta Q_n \\ -\Delta PE_1 \\ \vdots \\ -\Delta PE_p \\ -\Delta F_1 \\ \vdots \\ -\Delta F_p \end{bmatrix}
$$

$$
J = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} \\ J_{21} & J_{22} & J_{23} & J_{24} \\ J_{31} & J_{32} & J_{33} & J_{34} \\ J_{41} & J_{42} & J_{43} & J_{44} \end{bmatrix}
$$

$$
X = \left[\Delta\theta_2 \quad \dots\dots\dots \quad \Delta\theta_n \quad \Delta V_{m+1} \quad \dots\dots\dots \quad \Delta V_n \quad \Delta V_{sh1} \quad \dots\dots\dots \quad \Delta V_{shp} \quad \Delta\theta_{sh1} \quad \dots\dots\dots \quad \Delta\theta_{shp}\right]^t
$$

$$
M = \left[\Delta P_2 \quad \dots\dots\dots \quad \Delta P_n \quad \Delta Q_{m+1} \quad \dots\dots\dots \quad \Delta Q_n \quad \Delta PE_1 \quad \dots\dots\dots \quad \Delta PE_p \quad \Delta F_1 \quad \dots\dots\dots\dots \quad \Delta F_p\right]^t
$$

Where,

J = Jacobian matrix,

X =The incremental vector of state variables and

M = Power and control mismatch vector

ΔP = Real power mismatch

ΔQ = Reactive power mismatch

ΔPE =Active power exchange via DC link

$\Delta F = V_i - V_i^{sp}$ = Control variable

$\Delta\theta$ =Bus angle

ΔV = Bus voltage magnitude

ΔV_{sh} = STATCOM voltage magnitude

$\Delta\theta_{sh}$ = STATCOM angle magnitude

Derivatives of Jacobian Elements

$J_{11}:((n-1)\times(n-1)):\frac{\partial P}{\partial\theta}$:

$$J_{11}(i,i) = -\sum_{k=1}^{n} V_i V_k Y_{ik}\sin(\theta_i - \theta_k - \alpha_{ij}) + V_i V_{shl}(g_{sh}\sin(\theta_i - \theta_{shl}) - b_{sh}\cos(\theta_i - \theta_{shl})) \quad (2.6)$$

$$J_{11}(i,j) = V_i V_j (G_{ij}\sin(\theta_i - \theta_j) - B_{ij}\cos(\theta_i - \theta_j)) \quad (2.7)$$

$J_{12}:((n-1)\times(n-m)):\frac{\partial P}{\partial V}$:

$$J_{12}(i,i) = \sum_{k=1}^{n} V_k Y_{ik}\cos(\theta_i - \theta_k - \alpha_{ij}) + 2V_i g_{sh} - V_{shl}(g_{sh}\cos(\theta_i - \theta_{shl}) + b_{sh}\sin(\theta_i - \theta_{shl})) \quad (2.8)$$

$$J_{12}(i,j) = V_i (G_{ij}\cos(\theta_i - \theta_j) + B_{ij}\sin(\theta_i - \theta_j)) \quad (2.9)$$

$J_{13}:((n-1)\times p):\frac{\partial P}{\partial V_{shl}}$:

$$J_{13}(i,i) = -V_i(g_{sh}\cos(\theta_i - \theta_{shl}) + b_{sh}\sin(\theta_i - \theta_{shl})) \quad (2.10)$$

$$J_{13}(i,j) = 0 \quad (2.11)$$

$J_{14}:((n-1)\times p):\frac{\partial P}{\partial\theta_{shl}}$:

$$J_{14}(i,i) = -V_i V_{shl}(g_{sh}\sin(\theta_i - \theta_{shl}) - b_{sh}\cos(\theta_i - \theta_{shl})) \quad (2.12)$$

$$J_{14}(i,j) = 0 \quad (2.13)$$

$J_{21}:((n-m)\times(n-1)):\dfrac{\partial Q}{\partial \theta}$:

$$J_{21}(i,i)=\sum_{k=1}^{n}V_iV_kY_{ik}\sin(\theta_i-\theta_k-\alpha_{ij})-V_i^2V_{shl}+V_iV_{shl}(-g_{sh}\sin(\theta_i-\theta_{shl})+b_{sh}\cos(\theta_i-\theta_{shl})) \quad (2.14)$$

$$J_{21}(i,j)=-V_iV_j(G_{ij}\cos(\theta_i-\theta_j)+B_{ij}\sin(\theta_i-\theta_j)) \quad (2.15)$$

$J_{22}:((n-m)\times(n-m)):\dfrac{\partial Q}{\partial V}$:

$$J_{22}(i,i)=\sum_{k=1}^{n}V_kY_{ik}\sin(\theta_i-\theta_k-\alpha_{ij})-2V_ib_{sh}+V_{shl}(-g_{sh}\sin(\theta_i-\theta_{shl})+b_{sh}\cos(\theta_i-\theta_{shl})) \quad (2.16)$$

$$J_{22}(i,j)=V_i(G_{ij}\sin(\theta_i-\theta_j)-B_{ij}\cos(\theta_i-\theta_j)) \quad (2.17)$$

$J_{23}:((n-m)\times p):\dfrac{\partial Q}{\partial V_{shl}}$:

$$J_{23}(i,i)=V_i(b_{sh}\cos(\theta_i-\theta_{shl})-g_{sh}\sin(\theta_i-\theta_{shl})) \quad (2.18)$$

$$J_{23}(i,j)=0 \quad (2.19)$$

$J_{24}:((n-m)\times p):\dfrac{\partial Q}{\partial \theta_{shl}}$:

$$J_{24}(i,i)=V_iV_{shl}(b_{sh}\sin(\theta_i-\theta_{shl})+g_{sh}\cos(\theta_i-\theta_{shl})) \quad (2.20)$$

$$J_{24}(i,j)=0 \quad (2.21)$$

$J_{31}:(p\times(n-1)):\dfrac{\partial \mathrm{PE}}{\partial \theta}$:

$$J_{31}(i,i)=V_iV_{shl}(g_{sh}\sin(\theta_i-\theta_{shl})+b_{sh}\cos(\theta_i-\theta_{shl}) \quad (2.22)$$

$$J_{31}(i,j)=0 \quad (2.23)$$

$J_{32}:(p\times(n-m)):\dfrac{\partial \mathrm{PE}}{\partial V_i}$:

$$J_{32}(i,i)=-V_{shl}(g_{sh}\cos(\theta_i-\theta_{shl})-b_{sh}\sin(\theta_i-\theta_{shl}) \quad (2.24)$$

$$J_{32}(i,j)=0 \quad (2.25)$$

$J_{33}:(p\times p):\dfrac{\partial \mathrm{PE}}{\partial V_{shl}}$:

$$J_{33}(i,i)=2V_{shl}g_{sh}-V_i(g_{sh}\cos(\theta_i-\theta_{shl})-b_{sh}\sin(\theta_i-\theta_{shl})\qquad(2.26)$$

$$J_{33}(i,j)=0\qquad(2.27)$$

$J_{34}:(p\times p):\dfrac{\partial \mathrm{PE}}{\partial \theta_{shl}}$:

$$J_{34}(i,i)=-V_iV_{shl}(g_{sh}\sin(\theta_i-\theta_{shl})+b_{sh}\cos(\theta_i-\theta_{shl}))\qquad(2.28)$$

$$J_{34}(i,j)=0\qquad(2.29)$$

$J_{41}:(p\times(n-1)):\dfrac{\partial F_l}{\partial \theta_i}$:

$$J_{41}(i,i)=0\qquad(2.30)$$

$$J_{41}(i,j)=0\qquad(2.31)$$

$J_{42}:(p\times(n-m)):\dfrac{\partial F_l}{\partial V_i}$:

$$J_{42}(i,i)=1\qquad(2.32)$$

$$J_{42}(i,j)=0\qquad(2.33)$$

$J_{43}:(p\times p):\dfrac{\partial F_l}{\partial V_{shl}}$:

$$J_{43}(i,i)=0\qquad(2.34)$$

$$J_{43}(i,j)=0\qquad(2.35)$$

$J_{44}:(p\times p):\dfrac{\partial F_l}{\partial \theta_{shl}}$:

$$J_{44}(i,i)=0\qquad(2.36)$$

$$J_{44}(i,j)=0\qquad(2.37)$$

2.6 RESULT AND DISCUSSIONS

To verify the STATCOM model and explore the Voltage Control capability of the STATCOM, numerical studies have been carried out on the 3-machine 9-bus system and 10-machine 39-bus system. In these tests, a convergence tolerance of 1.0e-12 p.u. is used for maximal absolute bus power mismatches and power flow control mismatches. STATCOM has injected at bus no. 5 to improve the voltage profile of that mentioned bus. STATCOM has also injected at bus no. 35 to improve the voltage profile of that mentioned bus.

2.6.1 Results of 3-machine 9-bus system and 10-machine 39-bus system

Table 2.1

Load flow solutions of 3-machine 9-bus System with and without STATCOM

Bus no.	Without STATCOM				With STATCOM			
	Bus voltage p.u.	Bus voltage Angle degrees	P_g	Q_g	Bus voltage p.u.	Bus voltage Angle degrees	P_g	Q_g
1	1.04	0	0.7164	0.2705	1.04	0	0.7161	0.2408
2	1.025	9.28	1.63	0.0665	1.025	9.2719	1.63	0.0486
3	1.025	4.6648	0.85	-0.1086	1.025	4.6721	0.85	-0.1173
4	1.0258	-2.2168	-0	0	1.0274	-2.2122	0	0
5	0.9956	-3.9888	-0	0	1	-3.9888	0	-0
6	1.0127	-3.6874	-0	0	1.014	-3.6762	-0	-0
7	1.0258	3.7197	0	0	1.0269	3.7174	-0	-0
8	1.0159	0.7275	-0	0	1.0167	0.7338	0	0
9	1.0324	1.9667	0	0	1.0328	1.9753	-0	0

Table 2.2

Result of STATCOM Data

Bus no.	STATCOM Bus	V_{sh}	Thst	Q_{sh}
1.	5	1.0048	-4.0165	-0.0484

Table 2.3

Load flow solutions of 10-machine 39-bus System with and without STATCOM

Bus no.	Without STATCOM				With STATCOM			
	Bus voltage p.u.	**Bus voltage Angle degrees**	P_g	Q_g	**Bus voltage p.u.**	**Bus voltage Angle degrees**	P_g	Q_g
1	0.982	0	14.8161	6.3369	0.982	0	14.8552	4.7409
2	0.97	-11.2875	1102.96	248.2204	1.03	-10.8306	1102.96	248.687
3	0.9831	1.9984	6.5	1.7794	0.9831	1.9231	6.5	0.3409
4	1.0123	2.5439	5.08	1.569	1.0123	2.7065	5.08	1.2854
5	0.9972	3.5695	6.32	0.8771	0.9972	3.6973	6.32	0.2598
6	1.0493	4.4118	6.5	2.838	1.0493	4.4621	6.5	2.1238
7	1.0635	6.8561	5.4	2.2844	1.0635	6.8924	5.4	1.8859
8	1.0278	1.3792	5.4	0.4915	1.0278	1.7371	5.4	-0.0208
9	1.0265	7.0264	8.3	0.663	1.0265	7.3299	8.3	0.3536
10	1.0475	-4.382	2.5	2.252	1.0475	-4.1169	2.5	1.3548
11	0.9946	-9.5781	-0	-0	1.0383	-9.2538	-0	0
12	1.0095	-6.8345	0	0	1.025	-6.5324	0	-0
13	0.9807	-9.9103	318.78	2.376	1.0049	-9.504	318.78	2.376
14	0.9461	-10.7912	495	182.16	0.9827	-10.3399	495	182.16
15	0.9469	-9.4619	-0	-0	0.9899	-9.1147	-0	-0
16	0.9486	-8.6687	-0	0	0.9889	-8.3709	0	0
17	0.9397	-11.1586	231.462	83.16	0.9934	-10.6799	231.462	83.16
18	0.94	-11.7331	516.78	174.9342	1	-11.2231	516.78	177.6912
19	0.9691	-11.5094	0	0	1.0299	-11.0308	-0	0
20	0.9561	-5.9515	0	-0	0.9851	-5.7914	0	0
21	0.984	-5.0503	271.26	113.85	1.0021	-4.8585	271.26	113.85
22	1.0145	-0.5975	0	0	1.0242	-0.4996	0	0
23	1.0119	-0.9274	271.755	83.8134	1.022	-0.8241	271.755	83.8134
24	0.9721	-7.4644	305.514	91.278	0.996	-7.1892	305.514	91.278
25	1.0208	-5.4624	221.76	46.728	1.0323	-5.0444	221.76	46.728
26	1.0095	-6.745	137.61	16.83	1.0231	-6.3544	137.61	16.83

Bus no.	Without STATCOM				With STATCOM			
	Bus voltage p.u.	**Bus voltage Angle degrees**	P_g	Q_g	**Bus voltage p.u.**	**Bus voltage Angle degrees**	P_g	Q_g
27	0.989	-8.8919	278.19	74.745	1.0075	-8.481	278.19	74.745
28	1.0149	-2.9944	203.94	27.324	1.0218	-2.6489	203.94	27.324
29	1.0177	-0.0637	280.665	26.631	1.0224	0.2589	280.665	26.631
30	0.9839	-2.6421	621.72	101.97	0.989	-2.4675	621.72	101.97
31	0.9522	-6.8784	0	-0	0.9851	-6.6791	-0	0
32	0.9322	-6.8789	7.425	87.12	0.965	-6.6781	7.425	87.12
33	0.9534	-6.76	-0	-0	0.9847	-6.5653	0	0
34	0.9518	-8.6505	-0	-0	0.9879	-8.3596	-0	0
35	0.9547	-9.1106	316.8	151.47	1	-8.8505	316.8	151.47
36	0.972	-7.5211	326.106	31.977	0.9981	-7.2499	326.106	31.977
37	0.9788	-8.6788	0	0	1.0025	-8.3267	0	-0
38	0.978	-9.6251	156.42	29.7	1.002	-9.2302	156.42	29.7
39	0.9843	-1.6403	0	0	0.9931	-1.4913	0	0

Table 2.4

Result of STATCOM Data

Bus no.	STATCOM Bus	V_{sh}	Thst	Q_{sh}
1.	35	1.2374	-10.2126	-2.3765

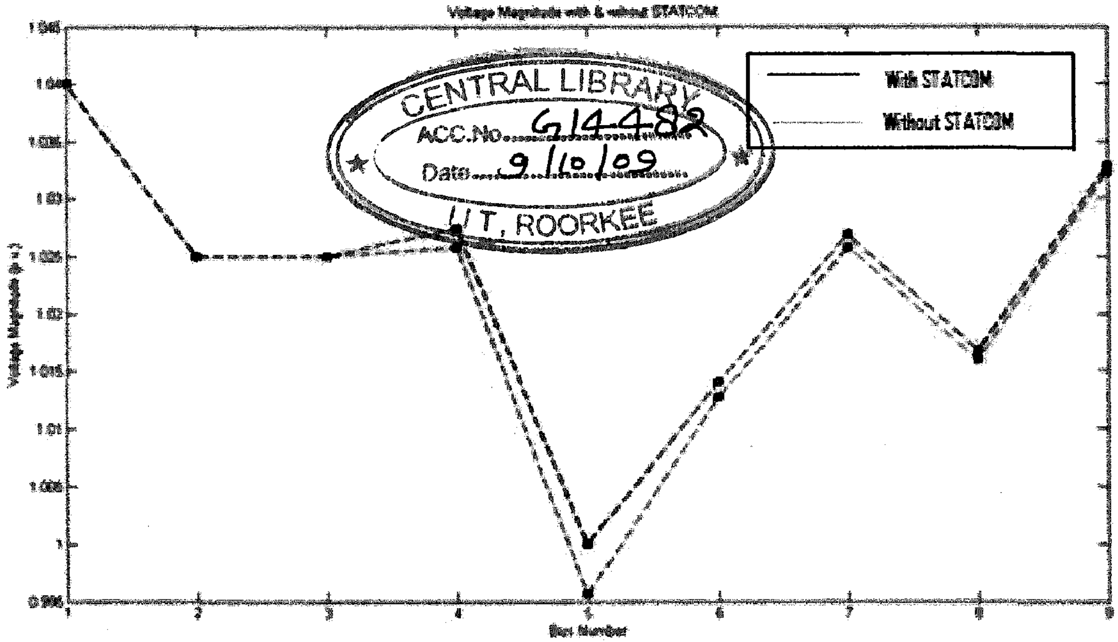

Figure 2.5: Voltage magnitude versus Bus Number in 3 machine 9 bus system

The upper curve shows the voltage magnitude of the each bus, when STATCOM is injected at bus no. 5 and the lower curve shows the voltage magnitude of the each bus when there is no STATCOM. From the above result, it is found that the bus voltage at bus no.5 is improved and it becomes equal to 1 p.u., when STATCOM is injected at bus no. 5.

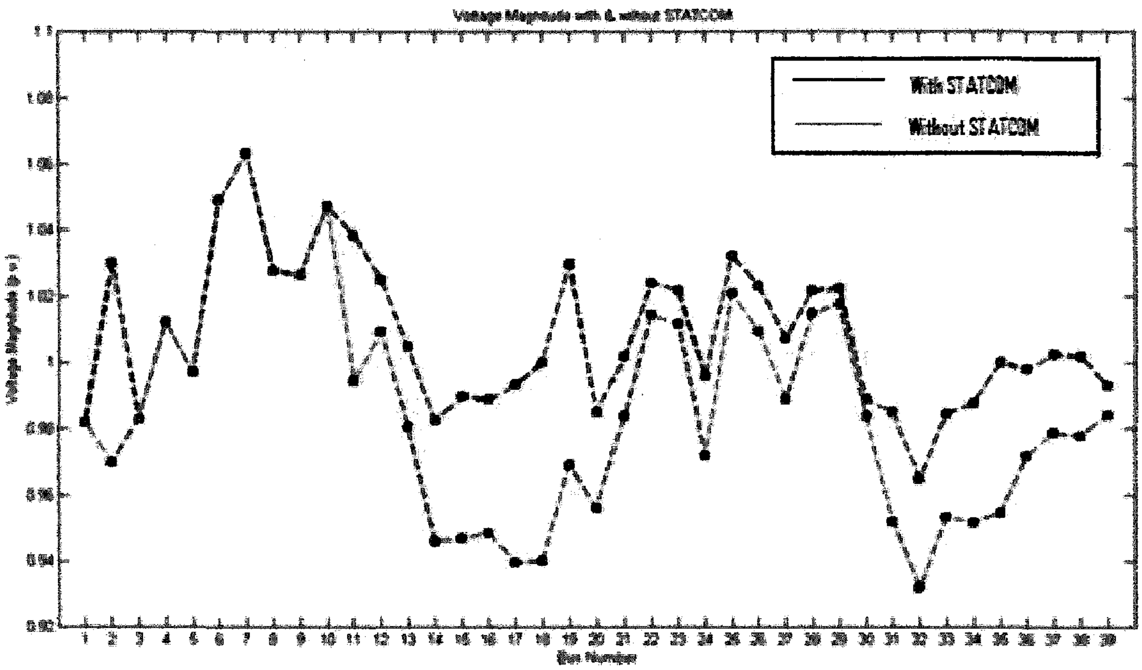

Figure 2.6: Voltage magnitude versus Bus Number in 10 machine 39 bus system

The upper curve shows the voltage magnitude of the each bus, when STATCOM is injected at at bus no. 35 and the lower curve shows the voltage magnitude of the each bus when there is no STATCOM. From the above result, it is found that the bus voltage at bus no.35 is improved and it becomes equal to 1 p.u., when STATCOM is injected at bus no. 35.

CHAPTER 3

LQR DESIGN

3.1 The General Optimal Control Formulation for Regulators [45]

A linear plant described by the following state-equation is considered as:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{3.1}$$

A time-varying plant is chosen in eqn. (3.1), because the optimal control problem is generally formulated for time-varying systems. It is supposed to design a full-state feedback regulator for the plant described by eqn. (3.1) so as the control input vector is given by

$$u(t) = -K(t)x(t) \tag{3.2}$$

The control law given by eqn. (3.2) is linear. Since the plant is also linear, the closed-loop control system would be linear. The control energy can be expressed as $u^T(t)R(t)u(t)$, where *R(t)* is a square, symmetric matrix called the control cost matrix. The expression for control energy is called a quadratic form, because the scalar function $u^T(t)R(t)u(t)$, contains quadratic functions of the elements of *u(t)*. Similarly, the transient energy can also be expressed in a quadratic form as $x^T(t)Q(t)x(t)$, where *Q(t)* is a square, symmetric matrix called the state weighting matrix. The objective function can then be written as follows:

$$J(t,t_f) = \int_t^{t_f} \left[x^T(\tau)Q(\tau)x(\tau) + u^T(\tau)R(\tau)u(\tau)\right]d\tau \tag{3.3}$$

Where, t and t_f are the initial and final times respectively, for the control to be exercised, i.e., the control begins at $\tau = t$ and ends at $\tau = t_f$, where τ is the variable of integration. The optimal control problem consists of solving for the feedback gain matrix *K(t)*, such that the scalar objective function $J(t,t_f)$ given by eqn. (3.3) is minimized. However, the minimization must be carried out in such a manner that the state-vector *x(t)* is the solution of the plant's state-equation eqn. (3.1). Equation (3.1) is called a constraint (because in its absence, *x(t)* would be free to assume any value), and the resulting minimization is said to be a constrained minimization. Now a regulator gain matrix *K(t)*, which minimizes $J(t,t_f)$ subject to the constraint given by eqn. (3.1) is required to be designed. The transient term $x^T(t)Q(t)x(t)$ in the objective function implies that a departure of the system's state $x(\tau)$ from the final desired state $x(t_f)=0$ is to be minimized. In other words, the design objective is to bring $x(\tau)$ to a constant value of zero at final time $\tau = t_f$.

By substituting eqn. (3.2) into eqn. (3.1), the closed-loop state-equation can be written as follows:

$$\dot{x}(t) = [A(t) - B(t)K(t)]x(t) = A_{CL}(t)x(t) \tag{3.4}$$

Where, $A_{CL}(t) = [A(t) - B(t)K(t)]$ is the closed loop state-dynamics matrix. The solution to eqn. (3.4) can be written as follows:

$$x(t) = \Phi_{CL}(t,t_0)x(t_0) \tag{3.5}$$

Where, $\Phi_{CL}(t,t_0)$ is the state-transition matrix of the time-varying closed-loop system represented by eqn. (3.4). Equation (3.5) indicates that the state at any time *x(t)* can be obtained by post-multiplying the state at some initial time $x(t_0)$ with $\Phi_{CL}(t,t_0)$. On substituting eqn. (3.5) into eqn. (3.3), the following expression for the objective function is obtained as:

$$J(t,t_f) = \int_t^{t_f} x^T(\tau)\Phi_{CL}^T(\tau,t)[Q(\tau) + K^T(\tau)R(\tau)K(\tau)]\Phi_{CL}(\tau,t)x(\tau)d\tau \tag{3.6}$$

or, taking the initial state vector *x(t)* outside the integral sign, we can write

$$J(t,t_f) = x^T(t)M(t,t_f)x(t) \tag{3.7}$$

$$\text{Where, } M(t,t_f) = \int_t^{t_f} \Phi_{CL}^T(\tau,t)[Q(\tau) + K^T(\tau)R(\tau)K(\tau)]\Phi_{CL}(\tau,t)d\tau \tag{3.8}$$

Equation (3.7) shows that the objective function is a quadratic function of initial state *x(t)*. Hence the linear optimal regulator problem posed by eqns. (3.1)-(3.3) is also called the linear quadratic regulator (LQR) problem. From eqn. (3.8), it is observed that $M(t,t_f)$ is a symmetric matrix, i.e. $M^T(t,t_f) = M(t,t_f)$, as both *Q(t)* and *R(t)* are symmetric. On substituting eqn. (3.5) into eqn. (3.6), the objective function can be written as follows:

$$J(t,t_f)=\int_t^{t_f} x^T(\tau)\left[Q(\tau)+K^T(\tau)R(\tau)K(\tau)\right]x(\tau)d\tau \tag{3.9}$$

On differentiating eqn. (3.9) partially with respect to the lower limit of integration t according to the Leibniz rule we get the following:

$$\frac{\partial J(t,t_f)}{\partial t}=-x^T(t)\left[Q(t)+K^T(t)R(t)K(t)\right]x(t) \tag{3.10}$$

Where, ∂ denotes partial differentiation. Also, partial differentiation of eqn.(3.7) with respect to t results in the following:

$$\frac{\partial J(t,t_f)}{\partial t}=[\dot{x}(t)]^T M(t,t_f)x(t)+x^T(t)\left[\frac{\partial M(t,t_f)}{\partial t}\right]x(t)+x^T(t)M(t,t_f)\dot{x}(t) \tag{3.11}$$

On substituting $\dot{x}(t)=A_{CL}(t)x(t)$ from eqn. (3.4) into eqn. (3.11), we can write

$$\frac{\partial J(t,t_f)}{\partial t}=[\dot{x}(t)]^T M(t,t_f)x(t)+x^T(t)\left[\frac{\partial M(t,t_f)}{\partial t}\right]x(t)+x^T(t)M(t,t_f)\dot{x}(t) \tag{3.12}$$

Equations (3.10) and (3.12) are quadratic forms for same scalar function $\frac{\partial J(t,t_f)}{\partial t}$ in terms of the initial state *x(t)*. Equating eqns. (3.10) and (3.12), the following matrix differential equation is obtained, which is to be satisfied by $M(t,t_f)$:

$$-\left[Q(t)+K^T(t)R(t)K(t)\right]=A_{CL}^T(t)M(t,t_f)+\frac{\partial M(t,t_f)}{\partial t}+M(t,t_f)A_{CL}(t) \tag{3.13}$$

or

$$-\frac{\partial M(t,t_f)}{\partial t}=A_{CL}^T(t)M(t,t_f)+M(t,t_f)A_{CL}(t)+\left[Q(t)+K^T(t)R(t)K(t)\right] \tag{3.14}$$

Equation (3.14) is a first order matrix partial differential equation in terms of the initial time t solution $M(t,t_f)$ is given by eqn. (3.8). As the state transition matrix $\Phi_{CL}(\tau,t)$ of the general time-varying, closed-loop system is not known, eqn. (3.8) is useless for determining $M(t,t_f)$. Hence, the only way to find the unknown matrix $M(t,t_f)$ is by solving the matrix differential equation (3.14). Only one initial condition is needed to solve the first order matrix differential equation eqn. (3.14). The simplest initial condition can be obtained by putting $t = t_f$ in eqn. (3.8), resulting in

$$M(t_f, t_f) = 0 \qquad (3.15)$$

The linear optimal control problem is thus posed as finding the optimal regulator gain matrix *K(t)* such that the solution $M(t,t_f)$ to eqn.(3.14) and hence the objective function $J(t,t_f)$ is minimized, subject to the initial condition eqn.(3.15). The choice of the matrices *Q(t)* and *R(t)* is left to the designer. These two matrices specifying performance objectives and control effort, cannot be arbitrary, but must obey certain conditions are described below.

3.2 Optimal Regulator Gain Matrix and the Riccati Equation [45]

The optimal feedback gain matrix that minimizes $M(t,t_f)$ is denoted by $K_0(t)$. The minimum value of $M(t,t_f)$ which results from the optimal gain matrix $K_0(t)$ is denoted by $M_0(t,t_f)$ and the minimum value of the objective function is denoted by $J_0(t,t_f)$. For simplicity of notation, let us drop the functional arguments for the time being, and denote $M(t,t_f)$ by *M*, $J(t,t_f)$ by *J* etc. Then, according to eqn.(3.7), the minimum value of the objective function is the following:

$$J_0 = x^T(t) M_0 x(t) \qquad (3.16)$$

Since J_0 is the minimum value of *J* for any initial state *x(t)*, we can write $J_0 \le J$, or

$$x^T(t) M_0 x(t) \le x^T(t) M x(t) \qquad (3.17)$$

Expressing M as follows:

$$M = M_0 + m \qquad (3.18)$$

and substitute eqn. (3.18) into eqn. (3.17), the following condition must be satisfied:

$$x^T(t) M_0 x(t) \le x^T(t) M_0 x(t) + x^T(t) m x(t) \qquad (3.19)$$

or

$$x^T(t) m x(t) \ge 0 \qquad (3.20)$$

A matrix *m* which satisfies eqn. (3.20) is called a positive semi-definite matrix. Since *x(t)* is an arbitrary initial state-vector, hence all eigenvalues of *m* must be greater than or equal to zero.

It now remains to derive an expression for the optimum regulator gain matrix $K_0(t)$ such that M is minimized. If M_0 is the minimum value of M, then M_0 must satisfy eqn. (3.14) when $K(t) = K_0(t)$, i.e.

$$-\frac{\partial M_0}{\partial t} = A_{CL}^T(t)M_0 + M_0A_{CL}(t) + \left[Q(t) + K_0^T(t)R(t)K_0(t)\right] \tag{3.21}$$

The gain matrix *K(t)* can be expressed in terms of the optimal gain matrix $K_0(t)$ as follows:

$$K(t) = K_0(t) + k(t) \tag{3.22}$$

On substituting eqn. (3.18) and (3.21) into eqn. (3.14), we can write

$$-\frac{\partial (M_0 + M)}{\partial t} = A_{CL}^T(t)(M_0 + M) + (M_0 + M)A_{CL}(t) + \left[Q(t) + \{K_0(t) + k(t)\}^T R(t)\{K_0(t) + k(t)\}\right] \tag{3.23}$$

On subtracting eqn. (3.21) from eqn. (3.23), we get

$$-\frac{\partial m}{\partial t} = A_{CL}^T(t)m + mA_{CL}(t) + S \tag{3.24}$$

Where,

$$S = \left[K_0^T(t)R(t) - M_0B(t)\right]k(t) + k^T(t)\left[R(t)K_0(t) - B^T(t)M_0\right] + k^T(t)R(t)k(t) \tag{3.25}$$

Comparing eqn. (3.24) with eqn. (3.14), we find that the two equations are of the same form, with the term $\left[Q(t) + K^T(t)R(t)K(t)\right]$ in eqn. (3.14) replaced by S in eqn. (3.24). Since the non-optimal matrix M in eqn. (3.14) satisfies eqn. (3.8), it must be true that *m* satisfies the following equation:

$$m(t,t_f) = \int_{t_f}^{t} \Phi_{CL}^T(\tau,t)S(\tau,t_f)\Phi_{CL}(\tau,t)d\tau \tag{3.26}$$

Recall from eqn. (3.20) that *m* must be positive semi-definite. However, eqn. (3.26) requires that for *m* to be positive semi-definite, the matrix S given by eqn. (3.25) must be positive semi-definite. S can be positive semi-definite if and only if the linear terms in eqn. (3.25) are zero, i.e. which implies

$$K_0^T(t)R(t) - M_0B(t) = 0 \tag{3.27}$$

or the optimal feedback gain matrix is given by

$$K_0(t) = R^{-1}(t)B^T(t)M_0 \tag{3.28}$$

Substituting eqn. (3.28) into eqn. (3.21), we get the following differential equation to be satisfied by the optimal matrix M_0:

$$-\frac{\partial M_0}{\partial t} = A^T(t)M_0 + M_0A(t) + M_0B(t)R^{-1}(t)B^T(t)M_0 + Q(t) \tag{3.29}$$

Equation (3.29) has a special name the matrix Riccati equation. The matrix Riccati equation is special because it's solution M_0 substituted into eqn. (3.28) gives us the optimal feedback gain matrix $K_0(t)$. Exact solutions to the Riccati equation are rare, and in most cases a numerical solution procedure is required. Note that Riccati equation is a first order, nonlinear differential equation, and can be solved by numerical methods for solving the nonlinear state-equations such as the Runge-Kutta method. However, in contrast to the state-equation, the solution in a matrix rather than a vector, and the solution procedure has to march backwards in time, since the initial condition for Riccati equation is specified eqn. (3.15) at the final time $t = t_f$ as follows:

$$M_0(t_f, t_f) = 0 \tag{3.30}$$

For this reason, the condition given by eqn. (3.30) is called the terminal condition rather than initial condition. Note that the solution to eqn. (3.29) is $M_0(t_f, t_f)$ where $t \langle t_f$.

In summary, the optimal control procedure using full-state feedback consists of specifying an objective function by suitably selecting the performance and control cost weighting matrices, *Q(t)* and *R(t)* and solving the Riccati equation subject to terminal condition in order to determine the full-state feedback matrix $K_0(t)$. In most cases, rather than solving the general time-varying optimal control problem, certain simplifications can be made which result in an easier problem as seen in the next section.

3.3 Infinite-Time Linear Quadratic Regulator Design [45]

A large number of control problems are such that the control interval $(t_f - t)$ is infinite. In a specific steady-state behavior of the control system (i.e. the response *x(t)* when $t_f \to \infty$), the control interval is infinite. The approximation of an infinite control interval results in a simplification in the optimal control problem, as shown below. For infinite final time, the quadratic objective function can be expressed as follows:

$$J_{\infty}(t) = \int_{t}^{\infty}\left[x^{T}(\tau)Q(\tau)x(\tau) + u^{T}(\tau)R(\tau)u(\tau)\right] \qquad (3.31)$$

Where, $J_{\infty}(t)$ indicates the objective function of the infinite final time (or steady-state) optimal control problem. For the infinite final time, the backward time integration of the matrix Riccati equation eqn. (3.29), beginning from $M_0(\infty,\infty) = 0$ would result in a solution $M_0(t,\infty)$ which is either a constant or does not converge to any limit. If the numerical solution to the Riccati equation converges to a constant value, then $\frac{\partial M_0}{\partial t} = 0$ and the Riccati equation becomes

$$A^{T}(t)M_0 + M_0A(t) - M_0B(t)R^{-1}(t)B^{T}(t)M_0 + Q(t) = 0 \qquad (3.32)$$

Eqn. (3.32) is no longer a differential equation, but an algebraic equation. Hence, eqn. (3.32) is called the algebraic Riccati equation. The feedback gain matrix is given by eqn. (3.28), in which M_0 is the solution to the algebraic Riccati equation. It is relatively much easier to solve eqn. (3.32) rather than eqn. (3.29). However, a solution to the algebraic Riccati equation may not always exist.

The conditions for the existence of the positive semi-definite solution to the algebraic Riccati equation are explained as, "*If either the plant is asymptotically stable, or the plant is controllable and observable with the output,* $y(t) = C(t)x(t)$, *where R(t) is a symmetric, positive definite matrix and* $Q(t) = C^{T}(t)C(t)$ *must be a symmetric and positive semi-definite matrix, then there is a unique, positive definite solution* M_0 *to the algebraic Riccati equation.*" These all sufficient (but not necessary) conditions for the existence of a unique solution to the algebraic Riccati equation, i.e. there may be plants that do not satisfy these conditions, and yet there may exist a unique, positive definite solution for such plants. A less restrictive set of sufficient conditions for the existence of a unique, positive definite solution to the algebraic Riccati equation is that the plant must be stabilizable and detectable with the output $y(t) = C(t)x(t)$, where $C^{T}(t)C(t) = Q(t)$ and *R(t)* is a symmetric, positive definite matrix.

While eqn. (3.32) has been derived for linear optimal control of time-varying plants, its usual application is to time-invariant plants, for which the algebraic Riccati equation is written as follows:

$$A^{T}M_0 + M_0A - M_0BR^{-1}B^{T}M_0 + Q = 0 \qquad (3.33)$$

In eqn. (3.33), all the matrices are constant matrices. MATLAB contains a solver for the algebraic Riccati equation for time-invariant plants in the M-file named are.m. The command care is used as follows:

$$\rangle\rangle x = are(a,b,c) \ \langle enter \rangle$$

Where, a=A, $b = BR^{-1}B^T$, c=Q in eqn. (3.33) and the returned solution is x = M_0. For the existence of a unique, positive definite solution to eqn. (3.33), the sufficient conditions remains the same, i.e. the plant with coefficient matrices A, B must be controllable, Q must be symmetric and positive semi-definite and R must be symmetric and positive definite.

MATLAB's Control System Toolbox (CST) provides the functions *lqr* and *lqr2* for the solution of the linear optimal control problem with a quadratic objective function, using two different numerical schemes. The command *lqr* or *(lqr2)* is used as follows:

$$\rangle\rangle [K_0, M_0, E] = lqr(A,B,Q,R) \ \langle enter \rangle$$

Where, A, B, Q, R are the same as in eqn. (3.33), Mo=M_0 the returned solution of eqn. (3.33), $K_0 = R^{-1}B^T M_0$ the returned optimal regulator gain matrix, and E is the vector containing the closed loop eigenvalues (i.e. the eigenvalues of $A_{CL} = A - BK_0$). The command *lqr* (or *lqr2*) is more convenient to use, since it directly works with the plant's coefficient matrices and the weighting matrices.

Results of LQR based damping controller in multi-machine power system are presented in the Chapter 5.

CHAPTER 4

GENETIC ALGORITHM

4.1 Background

Genetic Algorithms (GAs) were invented by John Holland and developed by him and his students and colleagues. This lead to Holland's book "Adaption in Natural and Artificial Systems" published in 1975.

In 1992 John Koza has used genetic algorithm to evolve programs to perform certain tasks. He called his method "genetic programming" (GP). LISP programs were used, because programs in this language can be expressed in the form of a "parse tree", which is the object the GA works on.

Genetic Algorithms were invented to mimic some of processes observed in natural evolution. Many people, biologists included, are astonished that life at the level of complexity that we observe could have evolved in the relatively short time suggested by fossil records. There is constant debate regarding the truth of Darwinian evolutionary theory as one scientist stated [46]:

"The chance that a functioning cell could evolve in that time can be linked to the probability that a tornado sweeping through a junkyard might assemble a Boeing 747."

--- Sir Fredrick Hoyle

Regardless of its validity, the idea with GA is to use this power of evolution to follow the principles first laid down by Charles Darwin in his "survival of the fittest" theory. In nature, competition among individuals for scarce resources results in the fittest individuals dominating over weaker ones. GAs attempt to find the optimal solution from the search space. Genetic Algorithms (GA) are search algorithms based on the mechanics of natural selection and natural genetics [48, 46]. GAs are adaptive search techniques which simulate both natural inheritance by genetics and a Darwinian struggle for survival.

A GA starts with a population of candidate solutions that evolve through generations of competition and reproduction until convergence to one solution. As such they represent an intelligent exploitation of random search used to solve problems. Although randomized, GAs are by no means random, instead they exploit historical information to direct the search into the region of better performance within the search space.

4.2 Introduction [46, 47]

Genetic algorithms are one of the best ways to solve a problem for which little is known. They are a very general algorithm and so will work well in any search space. Genetic algorithms use the principles of selection and evolution to produce several solutions to a given problem.

A Genetic algorithm (GA) is a search technique used in computing to find exact or approximate solutions to optimization and search problems. Genetic algorithms are categorized as global search heuristics. Genetic algorithms are a particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover (also called recombination).

Genetic algorithms tend to thrive in an environment in which there is a very large set of candidate solutions and in which the search space is uneven and has many hills and valleys. True, genetic algorithms will do well in any environment, but they will be greatly outclassed by more situation specific algorithms in the simpler search spaces. Therefore you must keep in mind that genetic algorithms are not always the best choice. Sometimes they can take quite a while to run and are therefore not always feasible for real time use. They are, however, one of the most powerful methods with which to (relatively) quickly create high quality solutions to a problem.

Genetic algorithms are implemented in a computer simulation in which a population of abstract representations (called chromosomes or the genotype of the genome) of candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem evolves toward better solutions. Traditionally, solutions are represented in binary as strings of 0s and 1s, but other encodings are also possible. The evolution usually starts from a population of randomly generated individuals and happens in generations. In each generation, the fitness of every individual in the population is evaluated, multiple individuals are stochastically selected from the current population (based on their fitness), and modified (recombined and possibly randomly mutated) to form a new population. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. If the algorithm has terminated due to a maximum number of generations, a satisfactory solution may or may not have been reached.

A typical genetic algorithm requires:

1. A genetic representation of the solution domain,
2. A fitness function to evaluate the solution domain.

A standard representation of the solution is as an array of bits. Arrays of other types and structures can be used in essentially the same way. The main property that makes these genetic representations convenient is that their parts are easily aligned due to their fixed size, which facilitates simple crossover operations. Variable length representations may also be used, but crossover implementation is more complex in this case. Tree-like representations are explored in genetic programming and graph-form representations are explored in evolutionary programming.

The fitness function is defined over the genetic representation and measures the quality of the represented solution. The fitness function is always problem dependent. A representation of a solution might be an array of bits, where each bit represents a different object, and the value of the bit (0 or 1) represents whether or not the object is in the knapsack. Not every such representation is valid, as the size of objects may exceed the capacity of the knapsack. The fitness of the solution is the sum of values of all objects in the knapsack if the representation is valid or 0 otherwise. In some problems, it is hard or even impossible to define the fitness expression; in these cases, interactive genetic algorithms are used.

4.3 Procedure of Genetic Algorithms [47]

A genetic algorithm for a particular problem must have the following five components.

1. A genetic representation for the potential solution to the problem.
2. A way to an initial population of potential solutions.
3. An evaluation function that plays the role of environment, rating solution in terms of their 'fitness'.
4. Genetic operators that alter the composition of the offspring.
5. Values for the various parameters that the genetic algorithm uses(population size, probabilities of applying genetic operators. etc).

Principles of genetic algorithms are as follows:

1. Encoding of the problem in a binary string.
2. Random generation of a population. This one includes a genetic pool representing a group of possible solutions.

3. Reckoning of a fitness value for each subject. It will directly depend on the distance to the optimum.
4. Selection of the subjects that will mate according to their share in the population global fitness.
5. Genomes crossover and mutations.
6. And then start again from point 3.

This continues until a suitable solution has been found or a certain number of generations have passed, depending on the needs of the programmer. Flow chart for the GA and for the GA based controller design is shown below.

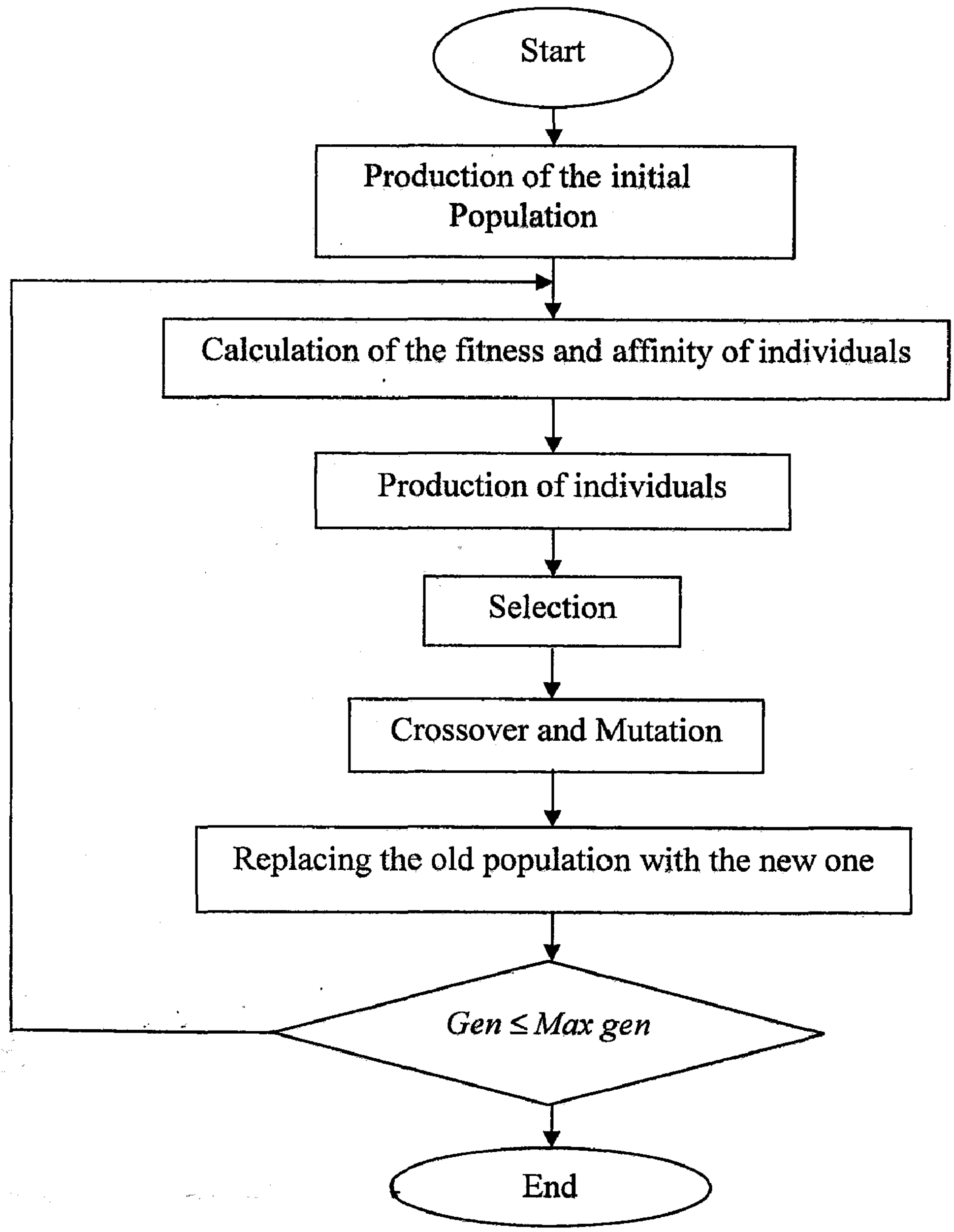

Figure 4.1: Flow chart for the GA [47]

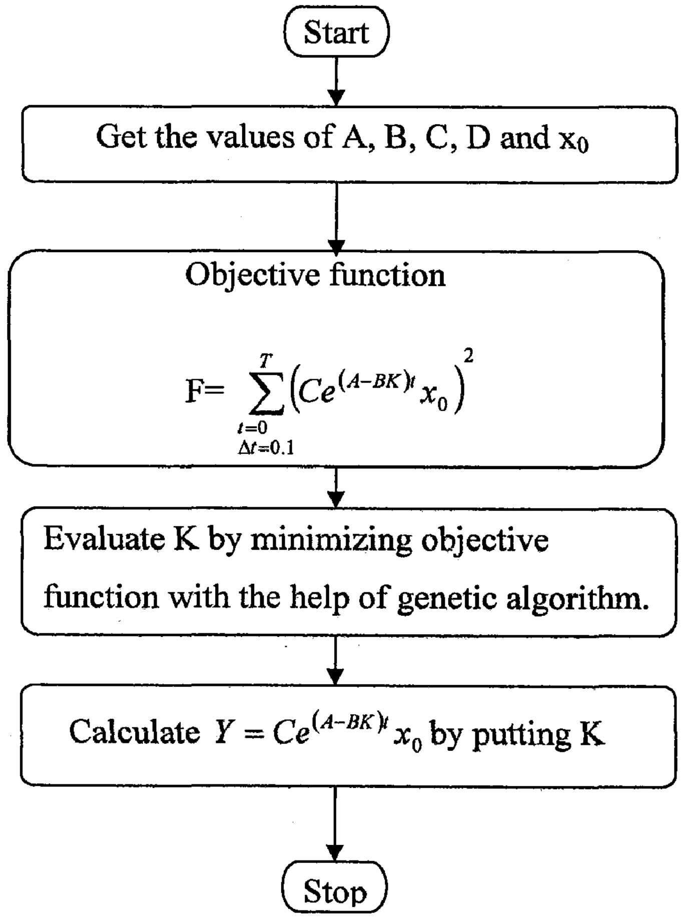

Figure 4.2: Flow chart for the design of GA based controller

Results of GA based damping controller in multi-machine power system are presented in the next Chapter 5.

CHAPTER 5

DAMPING CONTROLLER DESIGN FOR STATCOM IN A MULTIMACHINE SYSTEM

5.1 MATHEMATICAL MODEL OF A MULTI-MACHINE SYSTEM WITH STATCOM

Consider a 'm' machine, 'n' bus power system in which a STATCOM is assumed to be installed at bus number 's'. Without any loss of generality, it is also assumed that the 'm' generators are connected at bus numbers 1, 2,...m. the detailed equations, in per unit, for the machines, STATCOM and network are as follows:

Machine differential equations [44]

$$\frac{d\delta}{dt} = \omega_i - \omega_s \tag{5.1}$$

$$\frac{d\omega_i}{dt} = \frac{\omega_i}{2H_i}\left[T_{mi} - E'_{qi}I_{qi} - \left(x'_{qi} - x'_{di}\right)I_{di}I_{qi} - E'_{di}I_{di} - D_i\left(\omega_i - \omega_s\right)\right] \tag{5.2}$$

$$\frac{dE'_{qi}}{dt} = \frac{1}{T'_{doi}}\left[E_{fdi} - E'_{qi} - \left(x_{di} - x'_{di}\right)I_{di}\right] \tag{5.3}$$

$$\frac{dE'_{di}}{dt} = \frac{1}{T'_{qoi}}\left[-E'_{di} - \left(x_{qi} - x'_{qi}\right)I_{qi}\right] \tag{5.4}$$

$$\frac{dE_{fdi}}{dt} = \frac{1}{T_{Ei}}\left[-\left(S_{Ei}\left(E_{fdi}\right) + K_{fdi}\right)E_{fdi}\right] + V_{Ri} \tag{5.5}$$

$$\frac{dR_{fi}}{dt} = \frac{1}{T_{Fi}}\left[\frac{K_{Fi}}{T_{Fi}}E_{fdi} - R_{fi}\right] \tag{5.6}$$

$$\frac{dV_{Ri}}{dt} = \frac{1}{T_{Ai}}\left[-V_{Ri} + K_{Ai}R_{fi} - \frac{K_{Ai}K_{Fi}}{T_{Fi}}E_{fdi} + K_{Ai}\left(V_{refi} - V_i\right)\right] \tag{5.7}$$

for $i = 1........m$

Stator Algebraic Equations [44]

$$E'_{di} - V_i \sin\left(\delta_i - \theta_i\right) - R_{si}I_{di} + x'_{qi}I_{qi} = 0 \tag{5.8}$$

$$E'_{qi} - V_i \cos\left(\delta_i - \theta_i\right) - R_{si}I_{qi} + x'_{di}I_{di} = 0 \tag{5.9}$$

for $i = 1........m$

Network Equations at Generator Buses [44]

$$I_{di}V_i \sin(\delta_i - \theta_i) + I_{qi}V_i \cos(\delta_i - \theta_i) + P_{Li} - \sum_{k=1}^{m} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) = 0 \quad (5.10)$$

$$I_{di}V_i \cos(\delta_i - \theta_i) - I_{qi}V_i \sin(\delta_i - \theta_i) + Q_{Li} - \sum_{k=1}^{m} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) = 0 \quad (5.11)$$

for $i = 1........m$

Network Equations at Load Buses(except the STATCOM bus) [44]

$$P_{Li} - \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) = 0 \quad (5.12)$$

$$Q_{Li} - \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) = 0 \quad (5.13)$$

$for\, i = m+1,......n$

$i \neq s$

The STATCOM equations

$$\frac{dI_{dst}}{dt} = -\frac{\omega_s R_{st}}{X_{st}} I_{dst} + \omega_s I_{qst} - \frac{\omega_s \sin(\alpha + \theta_i)}{X_{st}} V_{dc} + \frac{\omega_s}{X_{st}} V_t \sin\theta_t \quad (5.14)$$

$$\frac{dI_{qst}}{dt} = -\frac{\omega_s R_{st}}{X_{st}} I_{qst} - \omega_s I_{dst} - \frac{\omega_s \cos(\alpha + \theta_i)}{X_{st}} V_{dc} + \frac{\omega_s}{X_{st}} V_t \sin\theta_t \quad (5.15)$$

$$\frac{dV_{dc}}{dt} = -\sqrt{3}\omega_s X_{dc} \sin(\alpha + \theta_i) I_{dst} - \sqrt{3}\omega_s X_{dc} \cos(\alpha + \theta_i) I_{qst} + \frac{\omega_s X_{dc}}{R_{dc}} V_{dc} \quad (5.16)$$

In the above equations angle α denotes the angle difference between the voltage of bus at which the STATCOM is connected and the voltage of the STATCOM.

Network Equations at the STATCOM bus

$$P_{STATCOM} + P_{Li} - \sum_{k=1}^{n} V_s V_k Y_{sk} \cos(\theta_s - \theta_k - \alpha_{sk}) = 0 \quad (5.17)$$

$$Q_{STATCOM} + Q_{Li} - \sum_{k=1}^{n} V_s V_k Y_{sk} \sin(\theta_s - \theta_k - \alpha_{sk}) = 0 \quad (5.18)$$

The STATCOM real power $P_{STATCOM}$ and reactive power $Q_{STATCOM}$ equations

$$P_{STATCOM} = \frac{V_i V_{dc} R_{st} \cos\alpha + V_i V_{dc} X_{st} \sin\alpha - R_{st} V_t^2}{R_{st}^2 + X_{st}^2} \quad (5.19)$$

$$Q_{STATCOM} = \frac{V_i V_{dc} X_{st} \cos\alpha - V_i V_{dc} R_{st} \sin\alpha - X_{st} V_t^2}{R_{st}^2 + X_{st}^2} \tag{5.20}$$

The details of equations (5.19) and (5.20) are given in the section B.5 of Appendix B.

As the primary job of a STATCOM is to control the bus voltage, it is always equipped with a bus voltage regulator. The schematic diagram of the STATCOM bus voltage regulator is shown in Figure 5.1. From this diagram, following dynamic equation can be written for the regulator.

$$\frac{d\alpha}{dt} = -\frac{\alpha}{T_s} + \frac{K_s}{T_s}\left(V_{ref} + V_{aux} - V_t\right) \tag{5.21}$$

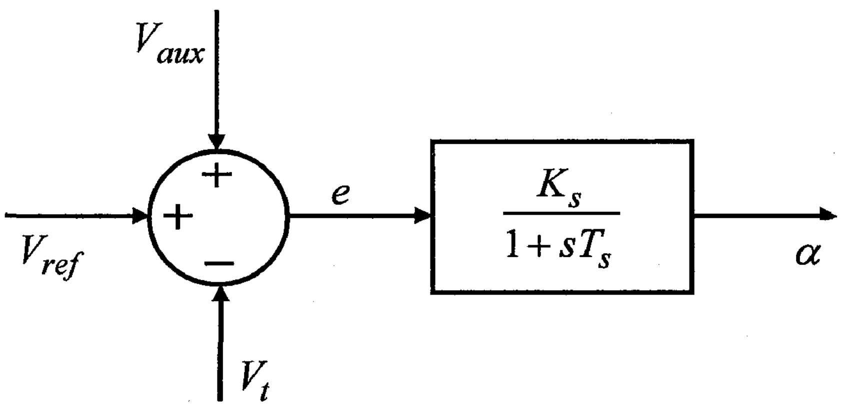

Figure 5.1: STATCOM bus voltage regulator

Now, the equations (5.8) - (5.13) and (5.17) - (5.20) are linearized. The matrix representation of these linearized equations is as follows.

$$\begin{bmatrix} D_1 & D_4 & D_{32} & 0 & 0 & D_3 \\ D_5 & D_{34} & D_8 & 0 & 0 & D_7 \\ D_{10} & 0 & 0 & D_{11} & D_{13} & D_{12} \\ D_{14} & 0 & 0 & D_{15} & D_{17} & D_{16} \\ D_{18} & D_{24} & D_{23} & D_{19} & D_{21} & D_{20} \\ D_{25} & D_{31} & D_{30} & D_{26} & D_{28} & D_{27} \end{bmatrix} \begin{bmatrix} \Delta V_g \\ \Delta I_q \\ \Delta I_d \\ \Delta V_l \\ \Delta\theta_l \\ \Delta\theta_g \end{bmatrix} = \begin{bmatrix} -D_2 & 0 & -D_{32} & 0 & 0 \\ -D_6 & -D_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & -D_{35} & -D_{36} \\ 0 & 0 & 0 & -D_{37} & -D_{38} \\ -D_{22} & 0 & 0 & 0 & 0 \\ -D_{29} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta E_q^{'} \\ \Delta E_d^{'} \\ \Delta V_{dc} \\ \Delta\alpha \end{bmatrix} \tag{5.22}$$

Where,

$\left[\Delta V_g\right]=\left[\Delta V_1 \;\; \Delta V_2 ....\Delta V_m\right]^T$ is the vector of perturbed generator voltage magnitudes

$\left[\Delta V_l\right]=\left[\Delta V_{m+1} \;\; \Delta V_{m+2} ....\Delta V_n\right]^T$ is the vector of perturbed load voltage magnitudes

$\left[\Delta I_d\right]=\left[\Delta I_{d1} \;\; \Delta I_{d2} ....\Delta I_{dm}\right]^T$ is the vector of perturbed generator d-axis currents

$\left[\Delta I_q\right]=\left[\Delta I_{q1} \;\; \Delta I_{q2} ....\Delta I_{qm}\right]^T$ is the vector of perturbed generator q-axis currents

$\left[\Delta \theta_g\right]=\left[\Delta \theta_1 \;\; \Delta \theta_2 ....\Delta \theta_m\right]^T$ is the vector of perturbed generator bus voltage angles

$\left[\Delta \theta_l\right]=\left[\Delta \theta_{m+1} \;\; \Delta \theta_{m+2} ....\Delta \theta_n\right]^T$ is the vector of perturbed load bus voltage angles

$\left[\Delta \delta_g\right]=\left[\Delta \delta_1 \;\; \Delta \delta_2 ....\Delta \delta_m\right]^T$ is the vector of perturbed generator rotor angles

$\left[\Delta E_q^{'}\right]=\left[\Delta E_{q1}^{'} \;\; \Delta E_{q2}^{'} ....\Delta E_{qm}^{'}\right]^T$ is the vector of perturbed voltage behind q-axis transient reactance

$\left[\Delta E_d^{'}\right]=\left[\Delta E_{d1}^{'} \;\; \Delta E_{d2}^{'} ....\Delta E_{dm}^{'}\right]^T$ is the vector of perturbed voltage behind d-axis transient reactance

The expression for various D coefficient sub-matrices are given in Appendix B. Equation (5.22) can be expressed in compact form as,

$$[\Delta D_L][\Delta XX]=\left[\Delta D_g\right][\Delta S] \qquad (5.23)$$

Where,

$$[\Delta XX]=\left[\Delta V_g \; \Delta I_q \; \Delta I_d \; \Delta V_l \; \Delta \theta_l \; \Delta \theta_g\right]^T$$

$$[\Delta S]=\left[\Delta \delta_g \; \Delta E_q^{'} \; \Delta E_d^{'} \; \Delta V_{dc} \; \Delta \alpha\right]^T$$

From eqn. (5.23) one can get,

$$[\Delta XX]=[D_L]^{-1}\left[\Delta D_g\right][\Delta S] \qquad (5.24)$$

Expanding the above equation, one can write,

$$\begin{bmatrix} \Delta V_g \\ \Delta I_q \\ \Delta I_d \\ \Delta V_l \\ \Delta \theta_l \\ \Delta \theta_g \end{bmatrix} = \begin{bmatrix} Y_1 & Y_2 & Y_2 & Y_2 & Y_2 \\ Y_2 & Y_2 & Y_2 & Y_2 & Y_2 \\ Y_2 & Y_2 & Y_2 & Y_2 & Y_2 \\ Y_2 & Y_2 & Y_2 & Y_2 & Y_2 \\ Y_2 & Y_2 & Y_2 & Y_2 & Y_2 \\ Y_2 & Y_2 & Y_2 & Y_2 & Y_2 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta E_q^{'} \\ \Delta E_d^{'} \\ \Delta V_{dc} \\ \Delta \alpha \end{bmatrix} \tag{5.25}$$

The linerized dynamic equations for machines (eqns. (5.1)-(5.7)), STATCOM (eqns. (5.14) - (5.16)) and the STATCOM bus voltage regulator eqn. (5.21) can be written as,

$$\dot{\Delta\delta} = C_1 \Delta\omega \tag{5.26}$$

$$\dot{\Delta\omega} = C_2 \Delta T_m + C_3 \Delta E_q^{'} + C_4 \Delta I_q + C_5 \Delta I_d + C_6 \Delta\omega + C_{19} \Delta E_d^{'} \tag{5.27}$$

$$\dot{\Delta E_q^{'}} = C_7 \Delta E_{fd} + C_8 \Delta E_q^{'} + C_9 \Delta I_d \tag{5.28}$$

$$\dot{\Delta E_{fd}} = C_{10} \Delta E_{fd} + C_{11} \Delta V_R \tag{5.29}$$

$$\dot{\Delta E_d^{'}} = C_{20} \Delta I_q + C_{21} \Delta E_d^{'} \tag{5.30}$$

$$\dot{\Delta R_f} = C_{22} \Delta E_{fd} + C_{23} \Delta R_f \tag{5.31}$$

$$\dot{\Delta V_R} = C_{26} \Delta E_{fd} + C_{25} \Delta R_f + C_{24} \Delta V_R + C_{27} \Delta V_{ref} + C_{28} \Delta V_g \tag{5.32}$$

$$\dot{\Delta I_{dst}} = C_{35} \Delta I_{dst} + C_{36} \Delta I_{qst} + C_{37} \Delta V_{dc} + C_{38} \Delta\alpha + C_{39} \Delta\theta_L + C_{40} \Delta V_L \tag{5.33}$$

$$\dot{\Delta I_{qst}} = C_{41} \Delta I_{dst} + C_{42} \Delta I_{qst} + C_{43} \Delta V_{dc} + C_{44} \Delta\alpha + C_{45} \Delta\theta_L + C_{46} \Delta V_L \tag{5.34}$$

$$\dot{\Delta V_{dc}} = C_{47} \Delta I_{dst} + C_{48} \Delta I_{qst} + C_{51} \Delta V_{dc} + C_{49} \Delta\alpha + C_{50} \Delta\theta_L \tag{5.35}$$

$$\dot{\Delta\alpha} = C_{75} \Delta\delta + C_{76} \Delta E_q^{'} + C_{77} \Delta E_d^{'} + C_{78} \Delta V_{dc} + C_{79} \Delta\alpha + C_{80} \Delta V_{aux} \tag{5.36}$$

The expression for various C coefficient sub-matrices are given in Appendix B. Using eqn. (5.25), the variables ΔI_d , ΔI_q , ΔV_g , ΔV_l , $\Delta\theta_l$ are eliminated from eqns. (5.26) - (5.36) and the resulting equations can be expressed in compact form as,

$$\left[\dot{\Delta X}\right] = [A][X] + [B][U] \tag{5.37}$$

Where,

$[\Delta X] = \left[\Delta\delta \ \ \Delta\omega \ \ \Delta E_q^{'} \ \ \Delta E_{fd} \ \ \Delta E_d^{'} \ \ \Delta R_f \ \ \Delta V_R \ \ \Delta i_{Dst} \ \ \Delta i_{Qst} \ \ \Delta v_{dc} \ \ \Delta\alpha\right]^T$ is the state variable matrix.

$[U] = \left[\Delta T_m \ \ \Delta V_{ref} \ \ \Delta V_{aux}\right]^T$ is the vector of input quantities.

The matrices A and B are given in Appendix B.

Equation (5.37) represents the state space representation of power system with STATCOM. In order to apply LQR technique and GA as described in Chapters 3 and 4, one need to select the proper STATCOM location and suitable stabilizing signal for designing an effective controller.

5.2 SELECTION OF PROPER STATCOM LOCATION AND STABILIZING SIGNAL

***Step 1:** Screening of possible* STATCOM *location buses*

Generally in power system, no STATCOM is placed either at any of the generator buses or at the secondary bus of the generator transformer. Therefore, in 'm'-machine, 'n'-bus system, only (n-2m) buses remain to be considered as possible candidate buses for STATCOM placement. In this work, two different multi-machine power systems, namely 3 machine 9 bus system [44] and 10 machine 39 bus system [49] have been used to demonstrate the design procedure for STATCOM damping controller. The single line diagrams of these two systems are shown in figure 5.2 and 5.3 respectively. The system data of these two test systems are given in APPENDIX B.

Now, with reference to the diagrams of these two systems, as shown in Figs. 5.2 and 5.3 respectively, for the 3 machine system the possible candidate buses for locating STATCOM are $\{5, 6\, and\, 8\}$ as per the above discussion. Similarly for the 10 machine system, the possible candidate buses are $\{11,13,14,15,17,18,19,21,24,26,27,28,31,32,33,34,35,36,37\, and\, 38\}$.

Step 2: Choice of stabilizing signal

The line real power (P_{Line}) and the line reactive power flow (Q_{Line}) on the lines incident to the STATCOM bus could be considered as possible choices of stabilizing signals. The detailed derivatives of the output matrix 'C' for these line real and reactive power flow signals are given in Appendix B.

P_{Line} from bus 5 to 7 is selected as the final choice of stabilizing signal for STATCOM damping controller design in the 3 machine system with the STATCOM assumed to be located at bus 5. Similarly, for the 10 machine system, the signal P_{Line} from bus 35 to 36 has been chosen as the final stabilizing signal for STATCOM damping controller with the STATCOM assumed to be located at bus no. 35.

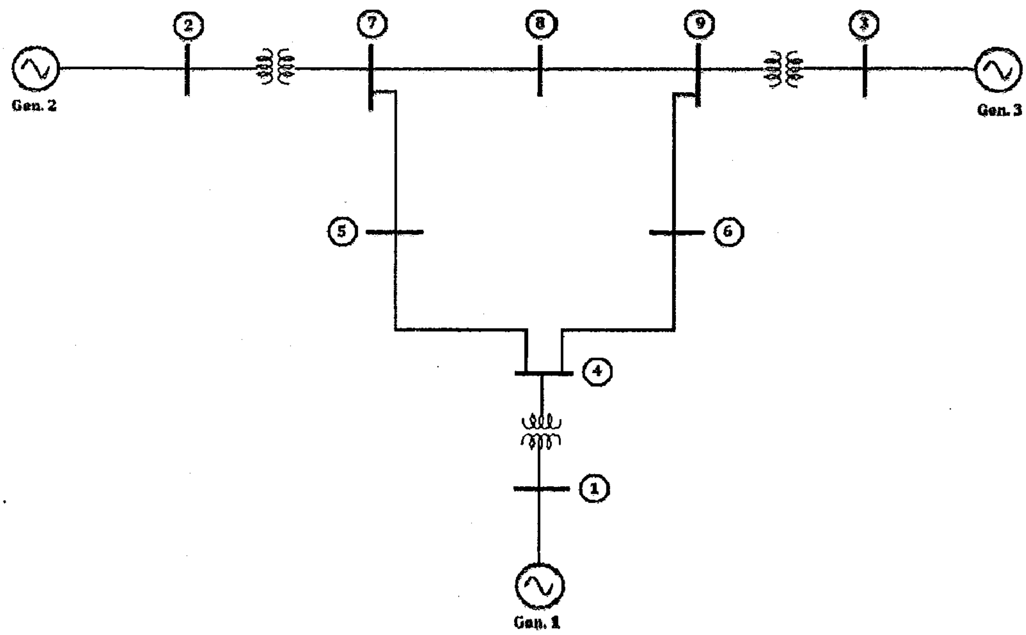

Figure 5.2: Single Line Diagram of 3 -machine 9-bus Power System

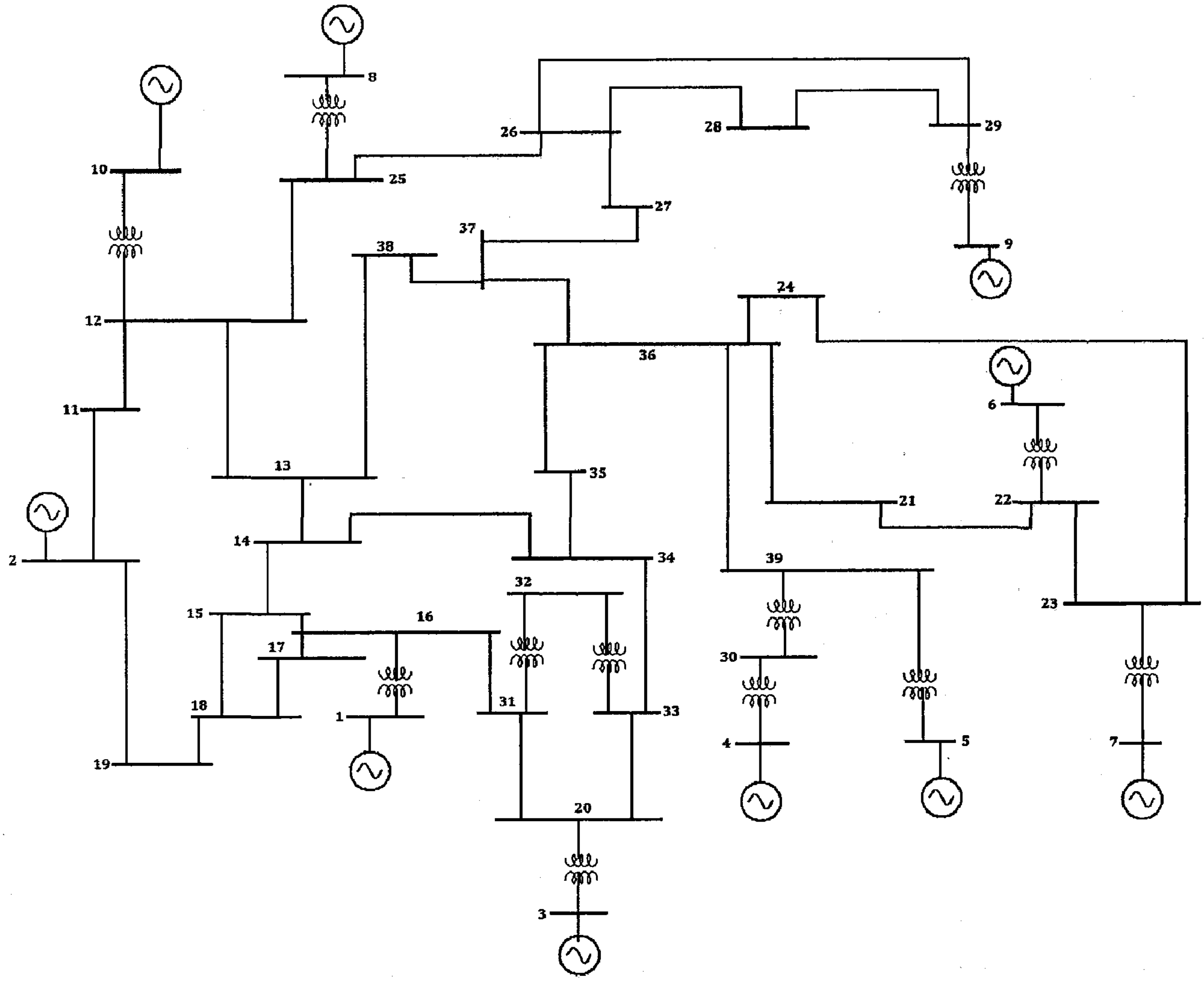

Figure 5.3: Single Line Diagram of 10-machine 39-bus Power System

5.3 RESULTS AND DISCUSSION

After having selected the location and the stabilizing signal for the STATCOM, the damping controller as shown in Figure 5.4 has been designed by the using two techniques LQR and GA (described in Chapters 3 and 4 respectively). Essentially, as shown in Figure 5.4, based on the stabilizing signal presented at its input the damping controller produces an output signal V_{aux} which in turn modulates the reference voltage of the voltage controller of the STATCOM. The initial condition response simulation (unforced response of a state-space model) results for demonstrating the effectiveness of the designed damping controllers are presented below.

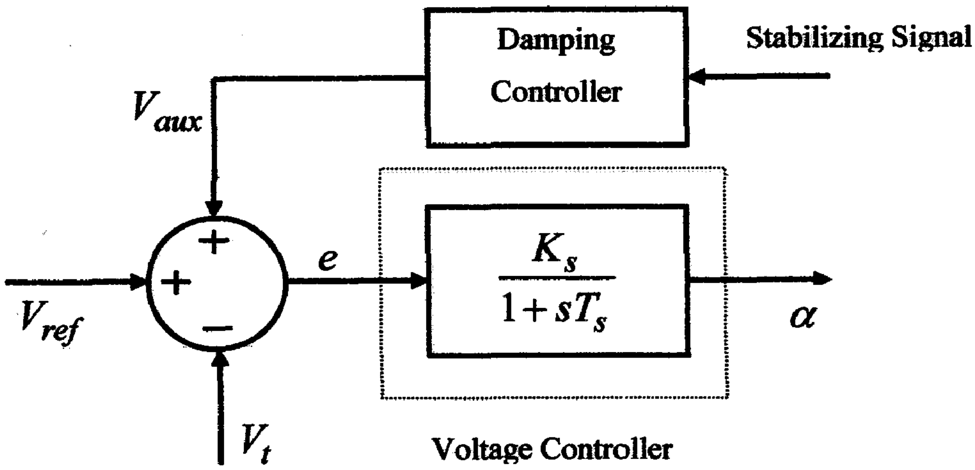

Figure 5.4: STATCOM damping controller

5.3.1 Results of 3 machine system

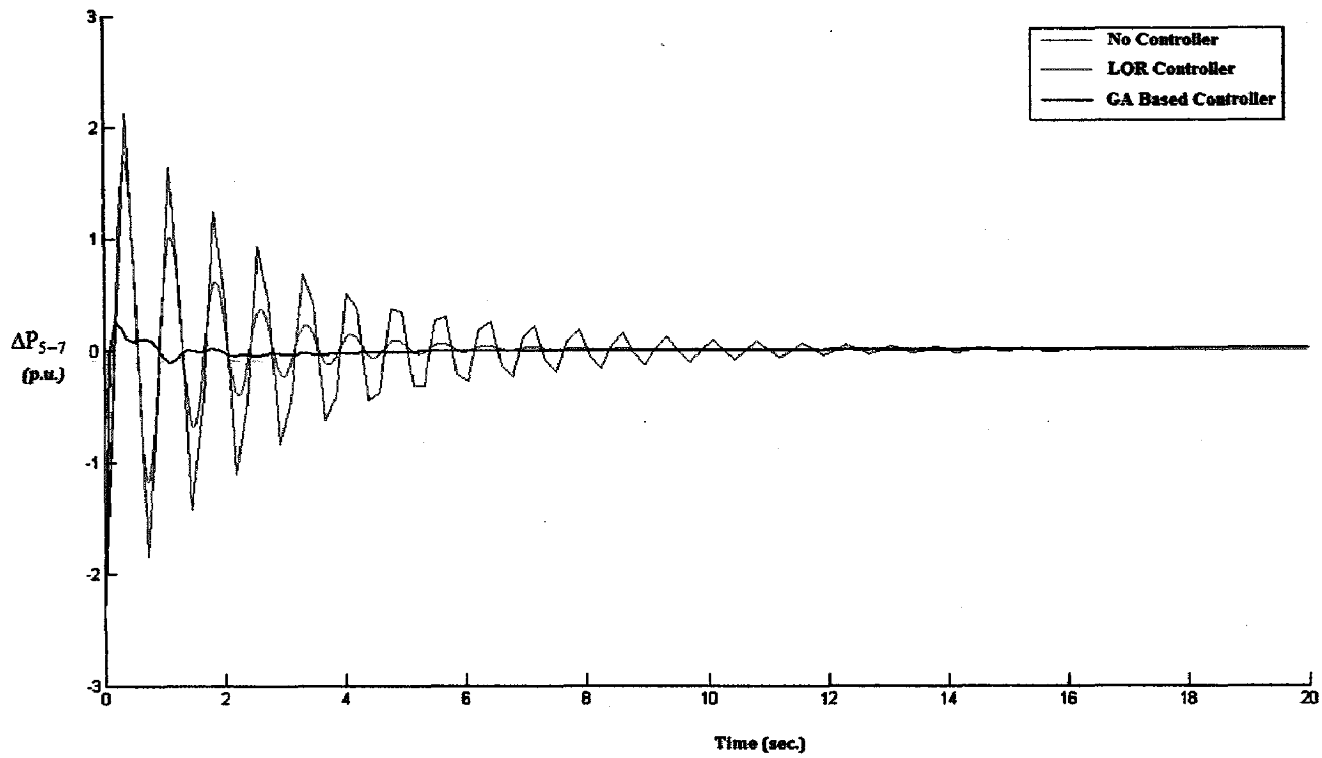

Figure 5.5: ΔP_{5-7} versus time at 80% loading

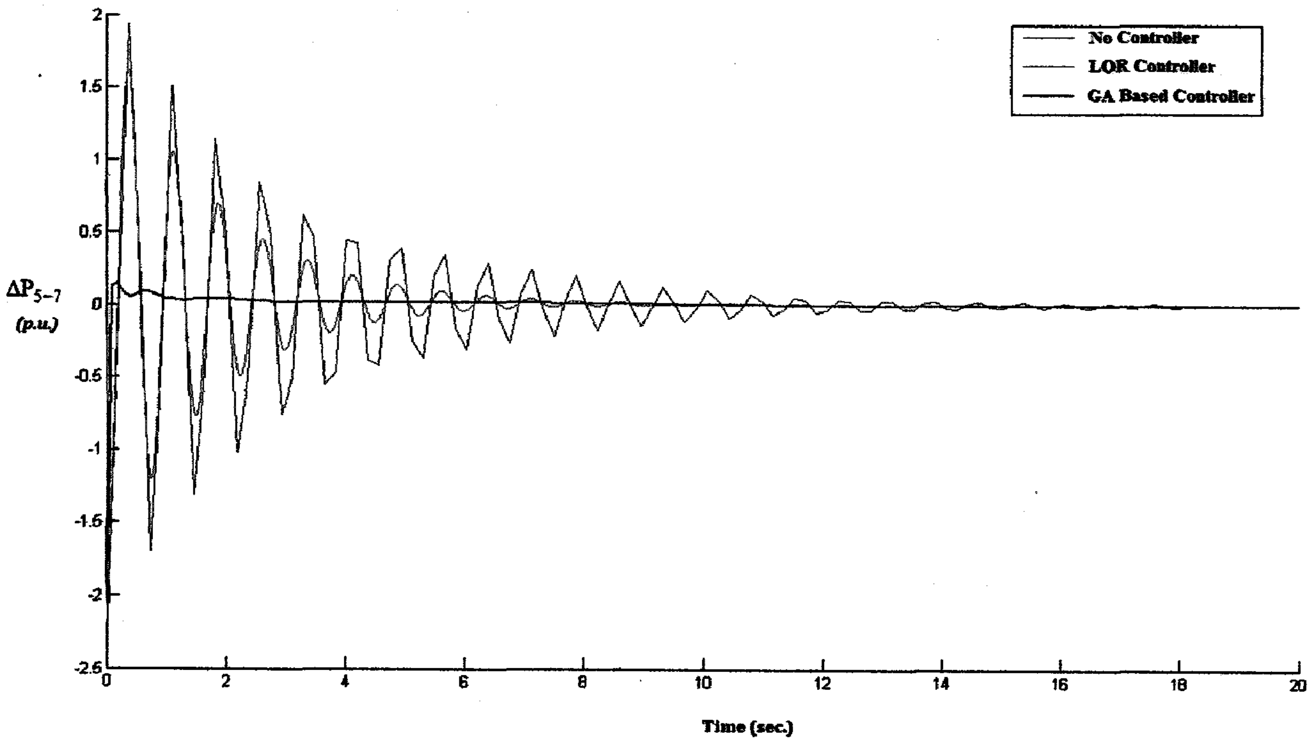

Figure 5.6: ΔP_{5-7} versus time at 100% loading

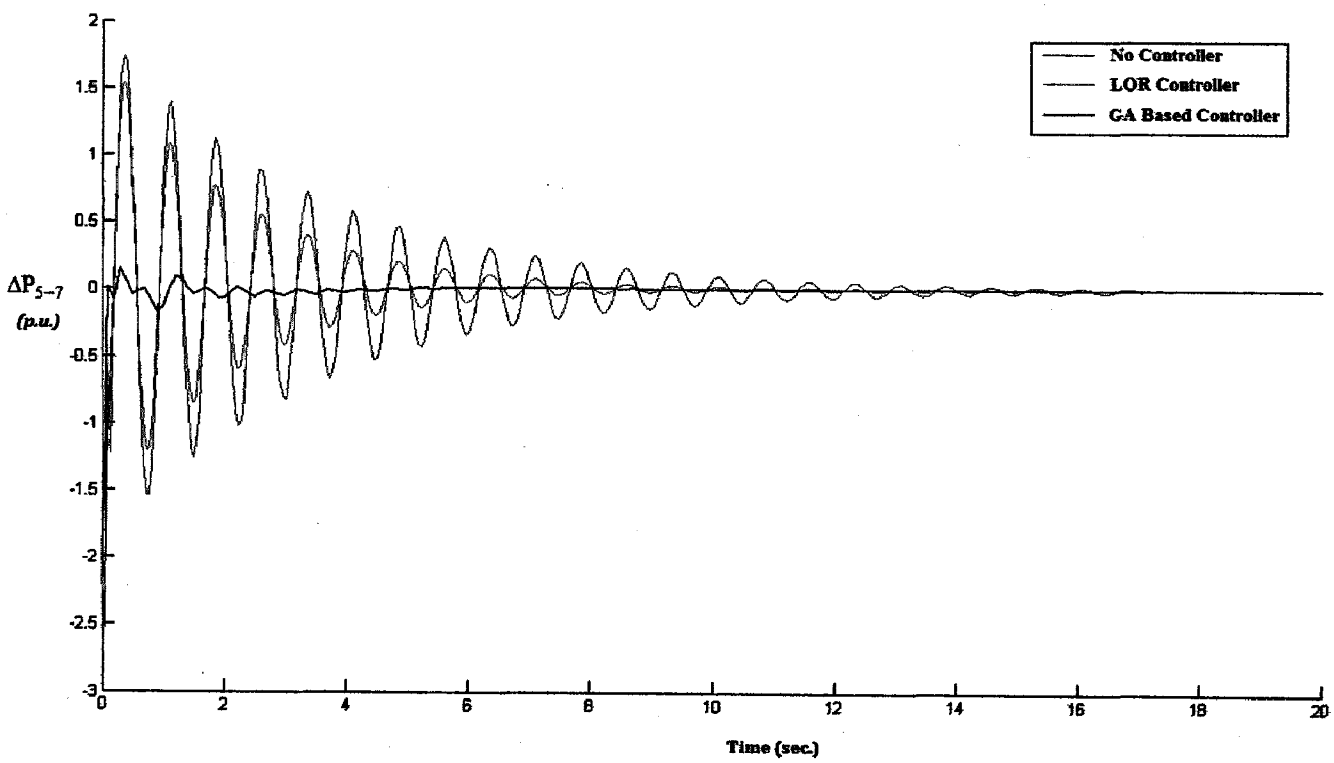

Figure 5.7: ΔP_{5-7} versus time at 120% loading

5.3.2 Results of 10 machine system

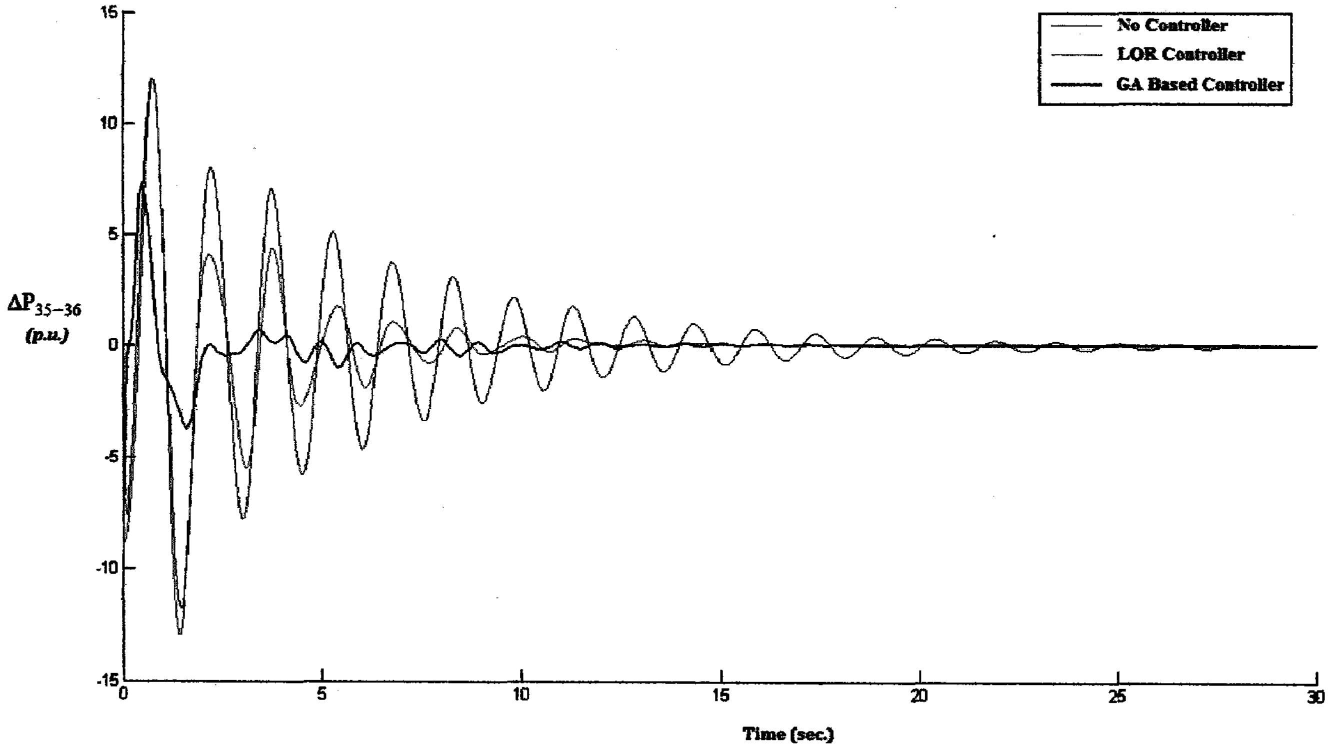

Figure 5.8: ΔP_{35-36} versus time at 80% loading

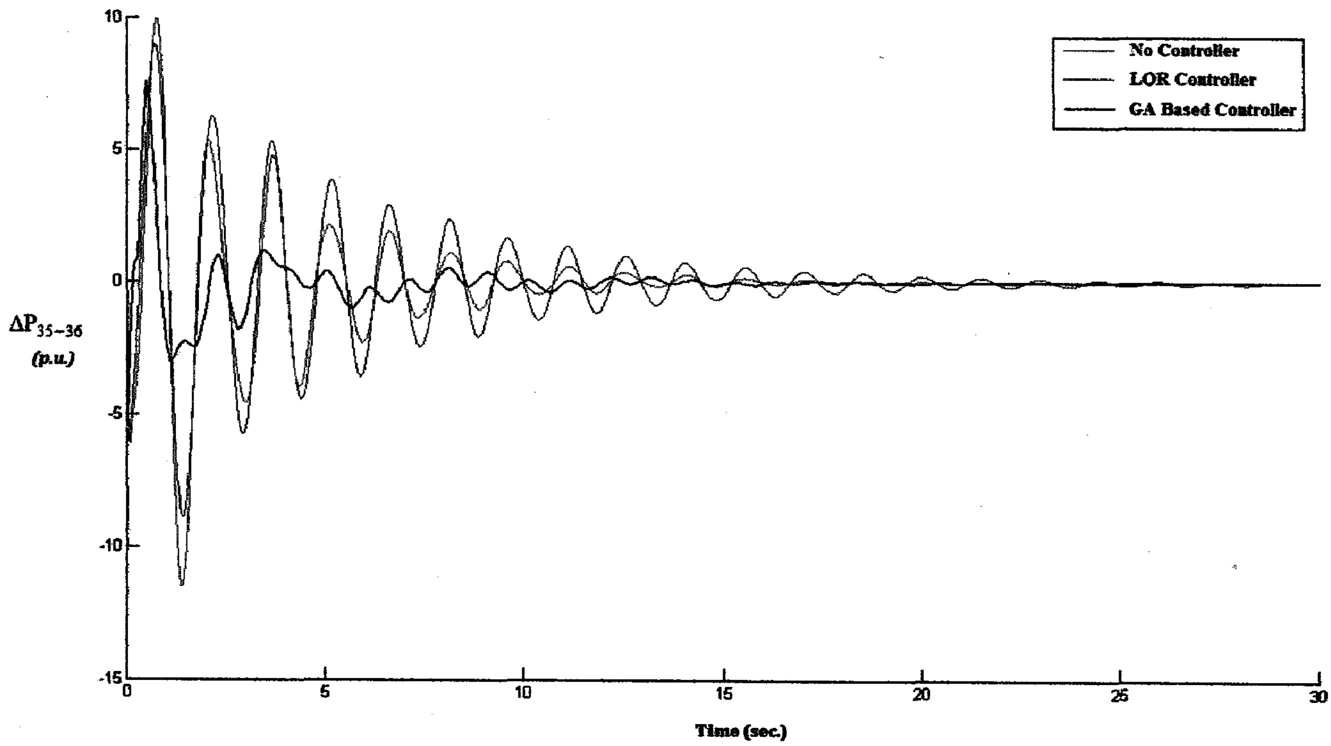

Figure 5.9: ΔP_{35-36} versus time at 100% loading

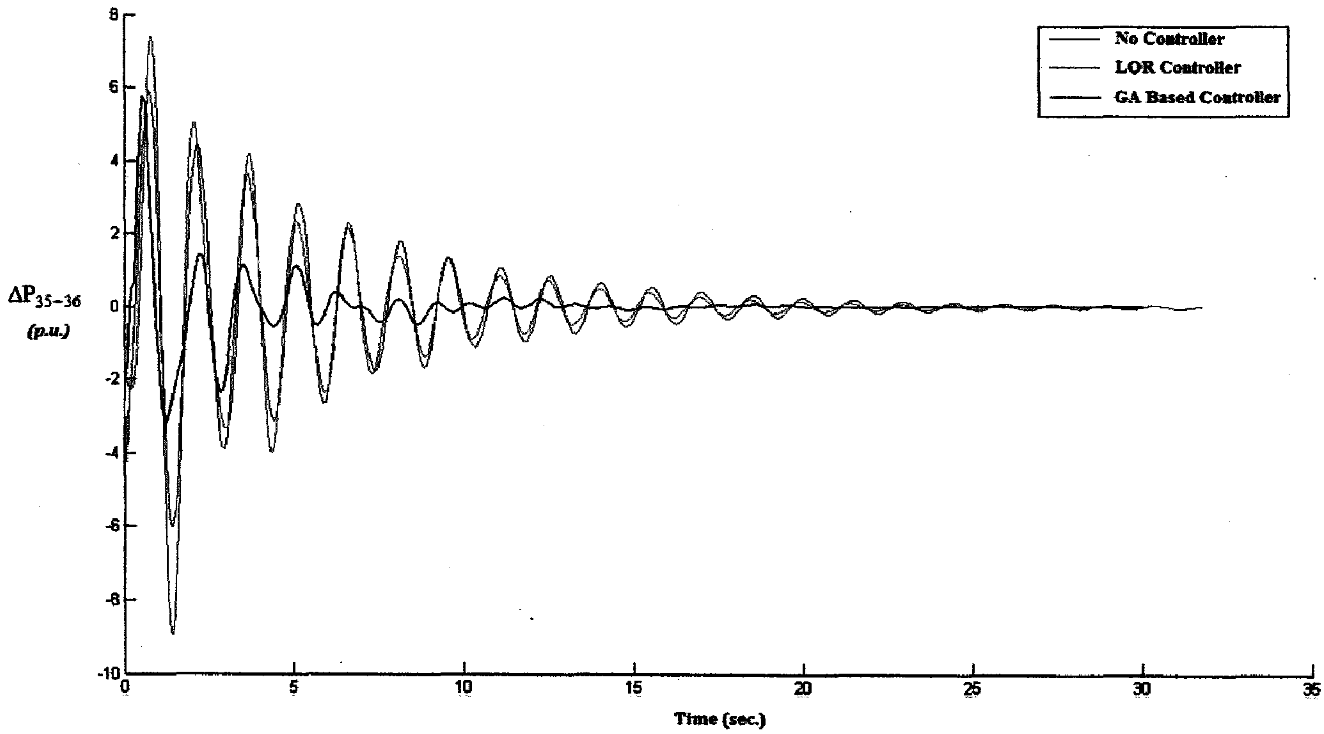

Figure 5.10: ΔP_{35-36} versus time at 120% loading

5.4 CONCLUSIONS

In this chapter, performance of LQR and GA based damping controller in multi-machine power system is presented. For this purpose, initially a linearized model of a multi-machine power system with a STATCOM has been developed. Further, the suitable location of the STATCOM and the stabilizing signal selection has been carried out. The designed controllers have been rigorously tested using initial condition response simulation (unforced response of a state-space model) studies for two study systems under different loading conditions. In Figure 5.5-5.10, the upper curve shows the result, when there is no controller, the middle curve shows the result, when there is LQR based controller and the lower curve shows the result, when there is GA based controller with STATCOM. It has been observed that the damping controllers are able to substantially enhance the damping in both the multi-machine systems considered in this work.

The main conclusions of the dissertation are given in the next chapter.

CONCLUSIONS AND SCOPE FOR FUTURE WORK

6.1 CONCLUSIONS

A voltage control functional model for STATCOM suitable for power system steady-state operational studies is presented. Voltage control mode has been incorporated into the STATCOM model. Numerical results based on the 3-machine 9-bus and 10-machine 39-bus system with the voltage control functional STATCOM have demonstrated the feasibility and effectiveness of the presented voltage control functional STATCOM model for power system steady-state operation and control. Numerical results show that voltage magnitude profile of the system is improved with a STATCOM on both the multi-machine systems. More control functions of a STATCOM may be explored.

LQR technique has been applied to design the STATCOM damping controller in multi-machine system in different loading condition. Towards this goal, the detailed linearized state space model of a power system with a STATCOM has been developed. From initial condition response simulation (unforced response of a state-space model) studies carried out in two study systems under different loading conditions, the efficacy of the developed local signal based controller has been found to be satisfactory for improving the system damping.

Apart from LQR technique, GA (genetic algorithm) technique has also been used for calculating the suitable value of the gain matrix K, for which damping is improved. From initial condition response simulation studies carried out in 3-machine 9-bus and 10-machine 39-bus system, it has been found that the GA based controller are better capable of damping out the oscillations effectively in the power system than LQR technique.

6.2 SCOPE FOR FUTURE WORK

- For improving the system damping still further, some machines may be provided with PSS and a coordinated design of PSS, STATCOM voltage controller and damping controller can be carried out.
- Use of multiple stabilizing signals available from wide area measurement system for damping controller design can be investigated.
- Coordinated design of damping controllers for multiple STATCOMs can also be explored.

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APPENDIX A

10-machine 39-bus system BUS DATA

Bus No.	Type	Voltage	Angle	P_l	Q_l	P_g	Q_g	Q_{min}	Q_{max}
1	1	0.9820	0	9.200	4.60	0	0	0	0
2	2	1.0300	0	1104	250	1000	0	-400	500
3	2	0.9831	0	0	0	650	0	-400	500
4	2	1.0123	0	0	0	508	0	-400	500
5	2	0.9972	0	0	0	632	0	-400	500
6	2	1.0493	0	0	0	650	0	-400	500
7	2	1.0635	0	0	0	540	0	-400	500
8	2	1.0278	0	0	0	540	0	-400	500
9	2	1.0265	0	0	0	830	0	-400	500
10	2	1.0475	0	0	0	250	0	-500	500
11	3	1	0	0	0	0	0	0	0
12	3	1	0	0	0	0	0	0	0
13	3	1	0	322	2.40	0	0	0	0
14	3	1	0	500	184	0	0	0	0
15	3	1	0	0	0	0	0	0	0
16	3	1	0	0	0	0	0	0	0
17	3	1	0	233.80	84	0	0	0	0
18	2	1	0	522	176	0	0	-500	500
19	3	1	0	0	0	0	0	0	0
20	3	1	0	0	0	0	0	0	0
21	3	1	0	274	115	0	0	0	0
22	3	1	0	0	0	0	0	0	0
23	3	1	0	274.50	84.66	0	0	0	0
24	3	1	0	308.60	92.20	0	0	0	0
25	3	1	0	224	47.20	0	0	0	0
26	3	1	0	139	17	0	0	0	0
27	3	1	0	281	75.50	0	0	0	0
28	3	1	0	206	27.60	0	0	0	0
29	3	1	0	283.50	26.90	0	0	0	0
30	3	1	0	628	103	0	0	0	0
31	3	1	0	0	0	0	0	0	0
32	3	1	0	7.50	88	0	0	0	0
33	3	1	0	0	0	0	0	0	0
34	3	1	0	0	0	0	0	0	0
35	3	1	0	320	153	0	0	0	0
36	3	1	0	329.40	32.30	0	0	0	0
37	3	1	0	0	0	0	0	0	0
38	3	1	0	158	30	0	0	0	0
39	3	1	0	0	0	0	0	0	0

10-machine 39-bus system LINE DATA

Line no.	From bus no.	To bus no.	Series impedance p.u.		Half line charging susceptance p.u.
			Resistance	Reactance	
1	22	6	0	0.01430	0
2	16	1	0	0.02500	0
3	20	3	0	0.02000	0
4	39	30	0.00070	0.01380	0
5	39	5	0.00070	0.01420	0
6	32	33	0.00160	0.04350	0
7	32	31	0.00160	0.04350	0
8	30	4	0.00090	0.01800	0
9	29	9	0.00080	0.01560	0
10	25	8	0.00060	0.02320	0
11	23	7	0.00050	0.02720	0
12	12	10	0	0.01810	0
13	37	27	0.00130	0.01730	0.16080
14	37	38	0.00070	0.00820	0.06595
15	36	24	0.00030	0.00590	0.03400
16	36	21	0.00080	0.01350	0.12740
17	36	39	0.00160	0.01950	0.15200
18	36	37	0.00070	0.00890	0.06710
19	35	36	0.00090	0.00940	0.08550
20	34	35	0.00180	0.02170	0.18300
21	33	34	0.00090	0.01010	0.08615
22	28	29	0.00140	0.01510	0.12450
23	26	29	0.00570	0.06250	0.51450
24	26	28	0.00430	0.04740	0.39010
25	26	27	0.00140	0.01470	0.11980
26	25	26	0.00320	0.03230	0.25650
27	23	24	0.00220	0.03500	0.18050
28	22	23	0.00060	0.00960	0.09230
29	21	22	0.00080	0.01350	0.12740
30	20	33	0.00040	0.00430	0.03645
31	20	31	0.00040	0.00430	0.03645
32	19	2	0.00100	0.02500	0.60000
33	18	19	0.00230	0.03630	0.19020
34	17	18	0.00040	0.00460	0.03900
35	16	31	0.00070	0.00820	0.06945
36	16	17	0.00060	0.00920	0.05650
37	15	18	0.00080	0.01120	0.07380
38	15	16	0.00020	0.00260	0.02170
39	14	34	0.00080	0.01290	0.06910
40	14	15	0.00080	0.01280	0.06710
41	13	38	0.00110	0.01330	0.10690
42	13	14	0.00130	0.02130	0.11070
43	12	25	0.00700	0.00860	0.07300
44	12	13	0.00130	0.01510	0.12860
45	11	12	0.00350	0.04110	0.34935
46	11	2	0.00100	0.02500	0.37500

3-machine 9-bus system BUS DATA

Bus No.	Type	Voltage	Angle	P_l	Q_l	P_g	Q_g	Q_{min}	Q_{max}
1	1	1.040	0	0	0	0	0	-400	400
2	2	1.025	0	1.63	0	0	0	-400	400
3	2	1.025	0	0.85	0	0	0	-400	400
4	3	1	0	0	0	0	0	0	0
5	3	1	0	0	0	1.25	0.50	0	0
6	3	1	0	0	0	0.90	0.30	0	0
7	3	1	0	0	0	0	0	0	0
8	3	1	0	0	0	1	0.35	0	0
9	3	1	0	0	0	0	0	0	0

3-machine 9-bus system LINE DATA

Line no.	From bus no.	To bus no.	Series impedance p.u.		Half line charging susceptance p.u.
			Resistance	Reactance	
1	1	4	0	0.0576	0
2	2	7	0	0.0625	0
3	3	9	0	0.0586	0
4	4	5	0.0100	0.0850	0.0880
5	4	6	0.0170	0.0920	0.0790
6	5	7	0.0320	0.1610	0.1530
7	6	9	0.0390	0.1700	0.1790
8	7	8	0.0085	0.0720	0.0745
9	8	9	0.0119	0.1008	0.1045

APPENDIX B

Expression for Various Sub-Matrices in Multi-Machine Modelling

B.1 The expressions for Various 'D' sub-matrices are given below:

D sub matrices	Indices	
$D_1(i,i) = -\sin(\delta_i - \theta_i)$ $D_1(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.1)
$D_2(i,i) = -V_i \cos(\delta_i - \theta_i)$ $D_2(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.2)
$D_3(i,i) = V_i \cos(\delta_i - \theta_i)$ $D_3(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.3)
$D_4(i,i) = x_{qi}^{'}$ $D_4(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.4)
$D_5(i,i) = -\cos(\delta_i - \theta_i)$ $D_5(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.5)
$D_6(i,i) = V_i \sin(\delta_i - \theta_i)$ $D_6(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.6)
$D_7(i,i) = -V_i \sin(\delta_i - \theta_i)$ $D_7(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.7)
$D_8(i,i) = -x_{di}^{'}$ $D_8(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.8)
$D_9(i,i) = 1$ $D_9(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.9)
$D_{10}(i,k) = -V_i Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$	$i = m+1,.....n$ $k = 1,.....m$	(B.10)

D sub matrices	Indices	
$D_{11}(i,i) = -2V_iG_{ii} - \sum_{k=1,k\neq i}^{n} V_iY_{ik}\cos(\theta_i-\theta_k-\alpha_{ik})$ $D_{11}(i,k) = -V_iY_{ik}\cos(\theta_i-\theta_k-\alpha_{ik})$ $D'_{11}(s,s) = -2V_sG_{ss} - \sum_{k=1,k\neq s}^{n} V_iY_{sk}\cos(\theta_s-\theta_k-\alpha_{sk})$ $D_{11}(s,s) = D'_{11}(s,s) + \frac{1}{R_{st}^2+X_{st}^2}\left(V_{dc}R_{st}\cos\alpha - 2V_sR_{st} + V_{dc}X_{st}\sin\alpha\right)$	$i=m+1,.....n$ $k=m+1,.....n$ $i,k\neq s$ $k\neq i$	(B.11)
$D_{12}(i,k) = -V_iV_kY_{ik}\sin(\theta_i-\theta_k-\alpha_{ik})$	$i=m+1,.....n$ $k=1,.....m$	(B.12)
$D_{13}(i,i) = \sum_{k=1,k\neq i}^{n} V_iV_kY_{ik}\sin(\theta_i-\theta_k-\alpha_{ik})$ $D_{13}(i,k) = -V_iV_kY_{ik}\sin(\theta_i-\theta_k-\alpha_{ik})$	$i=m+1,.....n$ $k=m+1,.....n$ $k\neq i$	(B.13)
$D_{14}(i,k) = -V_iY_{ik}\sin(\theta_i-\theta_k-\alpha_{ik})$	$i=m+1,.....n$ $k=1,.....m$	(B.14)
$D_{15}(i,i) = 2V_iB_{ii} - \sum_{k=1,k\neq i}^{n} V_iY_{ik}\sin(\theta_i-\theta_k-\alpha_{ik})$ $D_{15}(i,k) = -V_iY_{ik}\sin(\theta_i-\theta_k-\alpha_{ik})$ $D'_{15}(s,s) = 2V_sB_{ss} - \sum_{k=1,k\neq s}^{n} V_kY_{sk}\sin(\theta_s-\theta_k-\alpha_{sk})$ $D_{15}(s,s) = D'_{15}(s,s) + \frac{1}{R_{st}^2+X_{st}^2}\left(V_{dc}X_{st}\cos\alpha - 2V_sX_{st} - V_{dc}R_{st}\sin\alpha\right)$	$i=m+1,.....n$ $k=m+1,.....n$ $i,k\neq s$ $k\neq i$	(B.15)
$D_{16}(i,k) = V_iV_kY_{ik}\cos(\theta_i-\theta_k-\alpha_{ik})$	$i=m+1,.....n$ $k=1,.....m$	(B.16)
$D_{17}(i,i) = -\sum_{k=1,k\neq i}^{n} V_iV_kY_{ik}\cos(\theta_i-\theta_k-\alpha_{ik})$ $D_{17}(i,k) = V_iV_kY_{ik}\cos(\theta_i-\theta_k-\alpha_{ik})$	$i=m+1,.....n$ $k=m+1,.....n$ $k\neq i$	(B.17)
$D'_{18}(i,i) = -2V_iG_{ii} - \sum_{k=1,k\neq i}^{n} V_kY_{ik}\cos(\theta_i-\theta_k-\alpha_{ik})$ $D_{18}(i,i) = D'_{18}(i,i) + I_{di}\sin(\delta_i-\theta_i) + I_{qi}\cos(\delta_i-\theta_i)$ $D_{18}(i,k) = -V_iY_{ik}\cos(\theta_i-\theta_k-\alpha_{ik})$	$i,k=1,.....m$ $k\neq i$	(B.18)
$D_{19}(i,k) = -V_iY_{ik}\cos(\theta_i-\theta_k-\alpha_{ik})$	$i=1,.....m$ $k=m+1,.....n$	(B.19)

$D_{20}(i,k) = -V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$	$i = 1,.....m$ $k = 1,.....n$	(B.20)

D sub matrices	Indices	
$D_{21}(i,k) = -V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$	$i = 1,.....m$ $k = m+1,.....n$	(B.21)
$D_{22}(i,i) = I_{di} V_i \cos(\delta_i - \theta_i) - I_{qi} V_i \sin(\delta_i - \theta_i)$ $D_{22}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.22)
$D_{23}(i,i) = V_i \sin(\delta_i - \theta_i)$ $D_{23}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.23)
$D_{24}(i,i) = V_i \cos(\delta_i - \theta_i)$ $D_{24}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.24)
$D_{25}(i,i) = 2V_i B_{ii} - \sum_{k=1,k\neq i}^{n} V_i Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$ $D_{25}(i,i) = D'_{25}(i,i) + I_{di} \cos(\delta_i - \theta_i) - I_{qi} \sin(\delta_i - \theta_i)$ $D_{25}(i,k) = -V_i Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$	$i,k = 1,.....m$ $k \neq i$	(B.25)
$D_{26}(i,k) = -V_i Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$	$i = 1,.....m$ $k = m+1,.....n$	(B.26)
$D'_{27}(i,i) = -\sum_{k=1,k\neq i}^{n} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$ $D_{27}(i,i) = D'_{27}(i,i) + I_{di} V_i \sin(\delta_i - \theta_i) + I_{qi} V_i \cos(\delta_i - \theta_i)$ $D_{27}(i,k) = V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$	$i,k = 1,.....m$ $k \neq i$	(B.27)
$D_{28}(i,k) = V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$	$i = 1,.....m$ $k = m+1,.....n$	(B.28)
$D_{29}(i,i) = -I_{di} V_i \sin(\delta_i - \theta_i) - I_{qi} V_i \cos(\delta_i - \theta_i)$ $D_{29}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.29)
$D_{30}(i,i) = V_i \cos(\delta_i - \theta_i)$ $D_{30}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.30)
$D_{31}(i,i) = -V_i \sin(\delta_i - \theta_i)$ $D_{31}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.31)
$D_{32}(i,i) = -R_{si}$ $D_{32}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.32)
$D_{33}(i,i) = 1$ $D_{33}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.33)
$D_{34}(i,i) = -R_{si}$ $D_{34}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.34)

D sub matrices	Indices	
$D_{35}(s,1)=\frac{1}{R_{st}^2+X_{st}^2}(V_s R_{st}\cos\alpha+V_s X_{st}\sin\alpha)$ $D_{35}(i,1)=0$	$i=1,\ldots..m$ $i\neq s$	(B.35)
$D_{36}(s,1)=\frac{1}{R_{st}^2+X_{st}^2}(-V_s V_{dc} R_{st}\sin\alpha+V_s V_{dc} X_{st}\cos\alpha)$ $D_{36}(i,1)=0$	$i=1,\ldots..m$ $i\neq s$	(B.36)
$D_{37}(s,1)=\frac{1}{R_{st}^2+X_{st}^2}(V_s X_{st}\cos\alpha-V_s R_{st}\sin\alpha)$ $D_{37}(i,1)=0$	$i=1,\ldots..m$ $i\neq s$	(B.37)
$D_{38}(s,1)=\frac{1}{R_{st}^2+X_{st}^2}(-V_s V_{dc} X_{st}\sin\alpha-V_s V_{dc} R_{st}\cos\alpha)$ $D_{38}(i,1)=0$	$i=1,\ldots..m$ $i\neq s$	(B.38)

B.2 The expressions for Various 'C' sub-matrices are given below:

C sub matrices	Indices	
$C_1(i,i)=1$ $C_1(i,k)=0$	$i,k=1,\ldots..m$ $k\neq i$	(B.39)
$C_2(i,i)=\frac{\omega_s}{2H_i}$ $C_2(i,k)=0$	$i,k=1,\ldots..m$ $k\neq i$	(B.40)
$C_3(i,i)=-\frac{\omega_s}{2H_i}I_{qi}$ $C_3(i,k)=0$	$i,k=1,\ldots..m$ $k\neq i$	(B.41)
$C_4(i,i)=-\frac{\omega_s}{2H_i}\left\{E'_{qi}+\left(x'_{qi}-x'_{di}\right)I_{di}\right\}$ $C_4(i,k)=0$	$i,k=1,\ldots..m$ $k\neq i$	(B.42)
$C_5(i,i)=-\frac{\omega_s}{2H_i}\left\{E'_{di}+\left(x'_{qi}-x'_{di}\right)I_{qi}\right\}$ $C_5(i,k)=0$	$i,k=1,\ldots..m$ $k\neq i$	(B.43)
$C_6(i,i)=-\frac{\omega_s}{2H_i}D_i$ $C_6(i,k)=0$	$i,k=1,\ldots..m$ $k\neq i$	(B.44)

C sub matrices	Indices	
$C_7(i,i)=\frac{1}{T'_{doi}}$ $C_7(i,k)=0$	$i,k=1,.....m$ $k\neq i$	(B.45)
$C_8(i,i)=-\frac{1}{T'_{doi}}$ $C_8(i,k)=0$	$i,k=1,.....m$ $k\neq i$	(B.46)
$C_9(i,i)=-\frac{(x_{di}-x'_{di})}{T'_{doi}}$ $C_9(i,k)=0$	$i,k=1,.....m$ $k\neq i$	(B.47)
$C_{10}(i,i)=-\frac{1}{T_{Ei}}\{K_{Ei}+S_{Ei}(\Delta E_{fdi})\}$ $C_{10}(i,k)=0$	$i,k=1,.....m$ $k\neq i$	(B.48)
$C_{11}(i,i)=\frac{1}{T_{Ei}}$ $C_{11}(i,k)=0$	$i,k=1,.....m$ $k\neq i$	(B.49)
$[C_{13}]=[C_5][Y_5]+[C_4][Y_3]$		(B.50)
$[C_{14}]=[C_3]+[C_4][Y_4]+[C_5][Y_6]$		(B.51)
$[C_{15}]=[C_9][Y_5]$		(B.52)
$[C_{16}]=[C_8]+[C_9][Y_6]$		(B.53)
$[C_{17}]=[C_{20}][Y_3]$		(B.54)
$[C_{18}]=[C_{20}][Y_4]$		(B.55)
$C_{19}(i,i)=-\frac{\omega_s}{2H_i}I_{di}$ $C_{19}(i,k)=0$	$i,k=1,.....m$ $k\neq i$	(B.56)
$C_{20}(i,i)=-\frac{(x_{qi}-x'_{qi})}{T'_{qoi}}$ $C_{20}(i,k)=0$	$i,k=1,.....m$ $k\neq i$	(B.57)
$C_{21}(i,i)=-\frac{1}{T'_{qoi}}$ $C_{21}(i,k)=0$	$i,k=1,.....m$ $k\neq i$	(B.58)
$C_{22}(i,i)=\frac{K_{Fi}}{T^2_{Fi}}$ $C_{22}(i,k)=0$	$i,k=1,.....m$ $k\neq i$	(B.59)

C sub matrices	Indices	
$C_{23}(i,i) = -\frac{1}{T_{Fi}}$ $C_{23}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.60)
$C_{24}(i,i) = -\frac{1}{T_{Ai}}$ $C_{24}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.61)
$C_{25}(i,i) = \frac{K_{Ai}}{T_{Ai}}$ $C_{25}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.62)
$C_{26}(i,i) = -\frac{K_{Ai}K_{Fi}}{T_{Ai}T_{Fi}}$ $C_{26}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.63)
$C_{27}(i,i) = \frac{K_{Ai}}{T_{Ai}}$ $C_{27}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.64)
$C_{28}(i,i) = -\frac{K_{Ai}}{T_{Ai}}$ $C_{28}(i,k) = 0$	$i,k = 1,.....m$ $k \neq i$	(B.65)
$[C_{29}] = [C_{19}] + [C_4][Y_{14}] + [C_5][Y_{15}]$		(B.66)
$[C_{30}] = [C_9][Y_{15}]$		(B.67)
$[C_{31}] = [C_{21}] + [C_{20}][Y_{14}]$		(B.68)
$[C_{32}] = [C_{28}][Y_1]$		(B.69)
$[C_{33}] = [C_{28}][Y_2]$		(B.70)
$[C_{34}] = [C_{28}][Y_{13}]$		(B.71)
$C_{35}(1,1) = -\frac{\omega_s R_{st}}{X_{st}}$		(B.72)
$C_{36}(1,1) = \omega_s$		(B.73)
$C_{37}(1,1) = -\frac{\omega_s \cos(\alpha + \theta_t)}{X_{st}}$		(B.74)
$C_{38}(1,1) = -\frac{\omega_s V_{dc} \sin(\alpha + \theta_t)}{X_{st}}$		(B.75)

C sub matrices	Indices	
$C_{39}(1,s) = \frac{\omega_s V_t \sin\theta_t}{X_{st}} - \frac{\omega_s V_{dc} \sin(\alpha+\theta_t)}{X_{st}}$ $C_{39}(1,i) = 0$	$i = 1,.....m$ $i \neq s$	(B.76)
$C_{40}(1,s) = -\frac{\omega_s \cos\theta_t}{X_{st}}$ $C_{40}(1,i) = 0$	$i = 1,.....m$ $i \neq s$	(B.77)
$C_{41}(1,1) = -\omega_s$		(B.78)
$C_{42}(1,1) = -\frac{\omega_s R_{st}}{X_{st}}$		(B.79)
$C_{43}(1, 1) = \frac{\omega_s \sin(\alpha+\theta_t)}{X_{st}}$		(B.80)
$C_{44}(1, 1) = \frac{\omega_s V_{dc} \cos(\alpha+\theta_t)}{X_{st}}$		(B.81)
$C_{45}(1,s) = \frac{\omega_s V_{dc} \cos(\alpha+\theta_t)}{X_{st}} - \frac{\omega_s V_t \cos\theta_t}{X_{st}}$ $C_{45}(1,i) = 0$	$i = 1,.....m$ $i \neq s$	(B.82)
$C_{46}(1,s) = -\frac{\omega_s \sin\theta_t}{X_{st}}$ $C_{46}(1,i) = 0$	$i = 1,.....m$ $i \neq s$	(B.83)
$C_{47}(1, 1) = \sqrt{3}\omega_s X_{dc} \cos(\alpha+\theta_t)$		(B.84)
$C_{48}(1, 1) = \sqrt{3}\omega_s X_{dc} \sin(\alpha+\theta_t)$		(B.85)
$C_{49}(1, 1) = \sqrt{3}\omega_s X_{dc} I_{qst} \cos(\alpha+\theta_t) - \sqrt{3}\omega_s X_{dc} I_{dst} \sin(\alpha+\theta_t)$		(B.86)
$C_{50}(1, 1) = \sqrt{3}\omega_s X_{dc} I_{qst} \cos(\alpha+\theta_t) - \sqrt{3}\omega_s X_{dc} I_{dst} \sin(\alpha+\theta_t)$ $C_{50}(1,i) = 0$	$i = 1,.....m$ $i \neq s$	(B.87)
$C_{51} = -\frac{\omega_s X_{dc}}{R_{dc}}$		(B.88)
$[C_{52}] = [C_4][Y_{21}] + [C_5][Y_{23}]$		(B.89)
$[C_{53}] = [C_4][Y_{22}] + [C_5][Y_{24}]$		(B.90)
$[C_{54}] = [C_9][Y_{23}]$		(B.91)
$[C_{55}] = [C_9][Y_{24}]$		(B.92)
$[C_{56}] = [C_{20}][Y_{21}]$		(B.93)
$[C_{57}] = [C_{20}][Y_{22}]$		(B.94)

C sub matrices	Indices	
$[C_{58}]=[C_{28}][Y_{19}]$		(B.95)
$[C_{59}]=[C_{28}][Y_{20}]$		(B.96)
$[C_{60}]=[C_{37}]+[C_{39}][Y_{27}]+[C_{40}][Y_{25}]$		(B.97)
$[C_{61}]=[C_{38}]+[C_{39}][Y_{28}]+[C_{40}][Y_{26}]$		(B.98)
$[C_{62}]=[C_{39}][Y_{9}]+[C_{40}][Y_{7}]$		(B.99)
$[C_{63}]=[C_{39}][Y_{10}]+[C_{40}][Y_{8}]$		(B.100)
$[C_{64}]=[C_{39}][Y_{17}]+[C_{40}][Y_{16}]$		(B.101)
$[C_{65}]=[C_{43}]+[C_{45}][Y_{27}]+[C_{46}][Y_{25}]$		(B.102)
$[C_{66}]=[C_{44}]+[C_{45}][Y_{28}]+[C_{46}][Y_{26}]$		(B.103)
$[C_{67}]=[C_{45}][Y_{9}]+[C_{46}][Y_{7}]$		(B.104)
$[C_{68}]=[C_{45}][Y_{10}]+[C_{46}][Y_{8}]$		(B.105)
$[C_{69}]=[C_{45}][Y_{17}]+[C_{46}][Y_{16}]$		(B.106)
$[C_{70}]=[C_{51}]+[C_{50}][Y_{27}]$		(B.107)
$[C_{71}]=[C_{49}]+[C_{50}][Y_{28}]$		(B.108)
$[C_{72}]=[C_{50}][Y_{9}]$		(B.109)
$[C_{73}]=[C_{50}][Y_{10}]$		(B.110)
$[C_{74}]=[C_{50}][Y_{17}]$		(B.111)
$[C_{75}]=-\frac{K_s}{T_s}[Y_{7s}]$		(B.112)
$[C_{76}]=-\frac{K_s}{T_s}[Y_{8s}]$		(B.113)
$[C_{77}]=-\frac{K_s}{T_s}[Y_{16s}]$		(B.114)
$[C_{78}]=-\frac{K_s}{T_s}[Y_{25s}]$		(B.115)
$[C_{79}]=-\left(\frac{1}{T_s}+\frac{K_s}{T_s}[Y_{26s}]\right)$		(B.116)
$[C_{80}]=-\frac{K_s}{T_s}$		(B.117)

Where $[Y_{7s}],[Y_{8s}],[Y_{16s}],[Y_{25s}]$ and $[Y_{26s}]$ are the rows corresponding to the STATCOM bus 's' of the matrices $[Y_7],[Y_8],[Y_{16}],[Y_{25}]$ and $[Y_{26}]$ as described in eqn.(5.25).

B.3 The matrices A and B of eqn. (5.36) are given as

$$[A]=\begin{bmatrix} 0 & C_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{13} & C_6 & C_{14} & 0 & C_{29} & 0 & 0 & 0 & 0 & C_{52} & C_{53} \\ C_{15} & 0 & C_{16} & C_7 & C_{30} & 0 & 0 & 0 & 0 & C_{54} & C_{55} \\ 0 & 0 & 0 & C_{10} & 0 & 0 & C_{11} & 0 & 0 & 0 & 0 \\ C_{17} & 0 & C_{18} & 0 & C_{31} & 0 & 0 & 0 & 0 & C_{56} & C_{57} \\ 0 & 0 & 0 & C_{22} & 0 & C_{23} & 0 & 0 & 0 & 0 & 0 \\ C_{32} & 0 & C_{33} & C_{26} & C_{34} & C_{25} & C_{24} & 0 & 0 & C_{58} & C_{59} \\ C_{62} & 0 & C_{63} & 0 & C_{64} & 0 & 0 & C_{35} & C_{36} & C_{60} & C_{61} \\ C_{67} & 0 & C_{68} & 0 & C_{69} & 0 & 0 & C_{41} & C_{42} & C_{65} & C_{66} \\ C_{72} & 0 & C_{73} & 0 & C_{74} & 0 & 0 & C_{47} & C_{48} & C_{70} & C_{71} \\ C_{75} & 0 & C_{76} & 0 & C_{77} & 0 & 0 & 0 & 0 & C_{78} & C_{79} \end{bmatrix} \quad \text{(B.118)}$$

$$[B]=\begin{bmatrix} 0 & 0 & 0 \\ C_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & C_{27} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & C_{80} \end{bmatrix} \quad \text{(B.119)}$$

B.4 Stabilizing signals coefficient are calculated as

From the FigureB.1 the power over line connected between i^{th} and j^{th} bus can be written as

$$P_{ij} = V_i^2 y_{ij} \cos\alpha_{ij} - V_i V_j y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij}) \quad \text{(B.120)}$$

$$Q_{ij} = -V_i^2 y_{ij} \sin\alpha_{ij} - V_i V_j y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij}) - V_i^2\left(\frac{B_c}{2}\right) \quad \text{(B.121)}$$

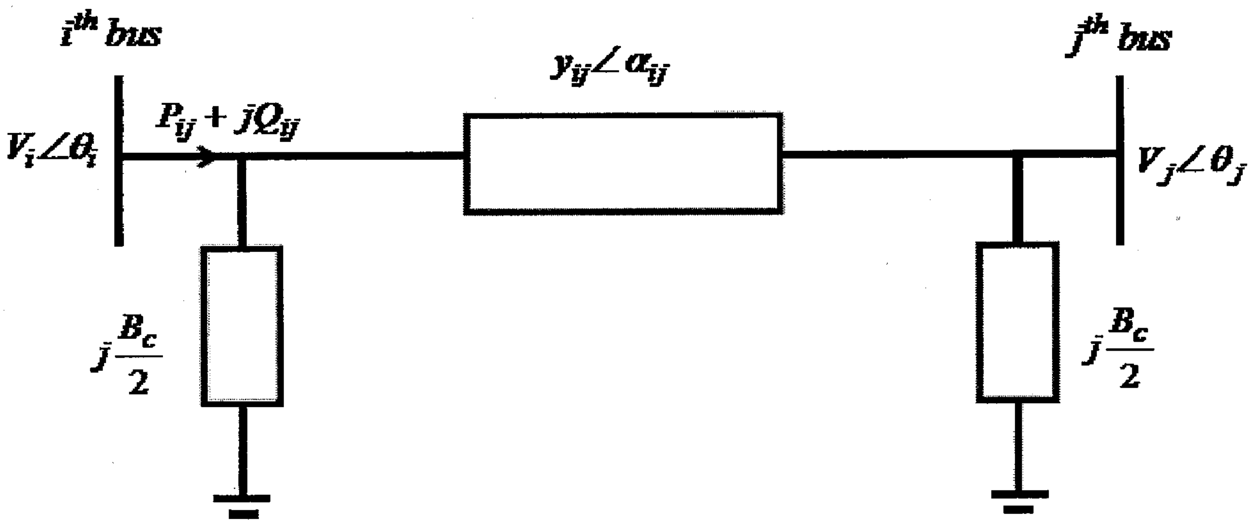

Figure B.1 Typical line element connected between i^{th} and j^{th} bus

For Line power P_{ij} as stabilizing signal

Linearizing eqn. (B.120)

$$\Delta P_{ij} = L_1\Delta V_i + L_2\Delta V_j + L_3\Delta\theta_i + L_4\Delta\theta_j \tag{B.122}$$

The coefficients $L_1 - L_4$ are given by

$$L_1 = 2V_i g_{ij} - V_j y_{ij}\cos(\theta_i - \theta_j - \alpha_{ij}) \tag{B.123}$$

$$L_2 = -V_j y_{ij}\cos(\theta_i - \theta_j - \alpha_{ij}) \tag{B.124}$$

$$L_3 = V_i V_j y_{ij}\sin(\theta_i - \theta_j - \alpha_{ij}) \tag{B.125}$$

$$L_4 = -V_i V_j y_{ij}\sin(\theta_i - \theta_j - \alpha_{ij}) \tag{B.126}$$

Expressing the variables $\Delta V_i, \Delta V_j, \Delta\theta_i, \Delta\theta_j$ in terms of the state variable using eqn. (5.25) of Chapter 5, ΔP_{ij} can be written as

$$\Delta P_{ij} = L_5\Delta\delta + L_6\Delta E_q^{'} + L_7\Delta E_d^{'} + L_8\Delta v_{dc} + L_9\Delta\alpha \tag{B.127}$$

The coefficients $L_5 - L_9$ are given by

$$L_5 = L_1 Y_{7i} + L_2 Y_{7j} + L_3 Y_{9i} + L_4 Y_{9j} \tag{B.128}$$

$$L_6 = L_1 Y_{8i} + L_2 Y_{8j} + L_3 Y_{10i} + L_4 Y_{10j} \tag{B.129}$$

$$L_7 = L_1 Y_{16i} + L_2 Y_{16j} + L_3 Y_{17i} + L_4 Y_{17j} \tag{B.130}$$

$$L_8 = L_1 Y_{25i} + L_2 Y_{25j} + L_3 Y_{27i} + L_4 Y_{27j} \tag{B.131}$$

$$L_9 = L_1 Y_{26i} + L_2 Y_{26j} + L_3 Y_{28i} + L_4 Y_{28j} \tag{B.132}$$

For Reactive power flow Q_{ij} over the line as the stabilizing signal

Linearizing the Reactive power eqn. (B.121)

$$\Delta Q_{ij} = L_{10}\Delta V_i + L_{11}\Delta V_j + L_{12}\Delta\theta_i + L_{13}\Delta\theta_j \tag{B.133}$$

The coefficients $L_{10} - L_{13}$ are given by

$$L_{10} = -2V_i b_{ij} - V_j y_{ij}\sin(\theta_i - \theta_j - \alpha_{ij}) - 2V_i\left(\frac{B_c}{2}\right) \tag{B.134}$$

$$L_{11} = -V_j y_{ij}\sin(\theta_i - \theta_j - \alpha_{ij}) \tag{B.135}$$

$$L_{12} = -V_i V_j y_{ij}\cos(\theta_i - \theta_j - \alpha_{ij}) \tag{B.136}$$

$$L_{13} = V_i V_j y_{ij}\cos(\theta_i - \theta_j - \alpha_{ij}) \tag{B.137}$$

Expressing the variables $\Delta V_i, \Delta V_j, \Delta\theta_i, \Delta\theta_j$ in terms of the state variable using eqn. (5.25) of Chapter 5, ΔQ_{ij} can be written as

$$\Delta Q_{ij} = L_{14}\Delta\delta + L_{15}\Delta E_q' + L_{16}\Delta E_d' + L_{17}\Delta v_{dc} + L_{18}\Delta\alpha \tag{B.138}$$

The coefficients $L_{14} - L_{18}$ are given by

$$L_{14} = L_{10} Y_{7i} + L_{11} Y_{7j} + L_{12} Y_{9i} + L_{13} Y_{9j} \tag{B.139}$$

$$L_{15} = L_{10} Y_{8i} + L_{11} Y_{8j} + L_{12} Y_{10i} + L_{13} Y_{10j} \tag{B.140}$$

$$L_{16} = L_{10} Y_{16i} + L_{11} Y_{16j} + L_{12} Y_{17i} + L_{13} Y_{17j} \tag{B.141}$$

$$L_{17} = L_{10} Y_{25i} + L_{11} Y_{25j} + L_{12} Y_{27i} + L_{13} Y_{27j} \tag{B.142}$$

$$L_{18} = L_{10} Y_{26i} + L_{11} Y_{26j} + L_{12} Y_{28i} + L_{13} Y_{28j} \tag{B.143}$$

Where $[Y_{7i}],[Y_{8i}],[Y_{16i}],[Y_{25i}]$ and $[Y_{26i}]$ and $[Y_{7j}],[Y_{8j}],[Y_{16j}],[Y_{25j}]$ and $[Y_{26j}]$ are the rows corresponding to the i^{th} and j^{th} buses of the matrices $[Y_7],[Y_8],[Y_{16}],[Y_{25}]$ and $[Y_{26}]$ as described in eqn. (5.25).

B.5 STATCOM Power equations

The connection of the STATCOM to the system is shown in Figure B.2. STATCOM is connected to the system at bus 's'.

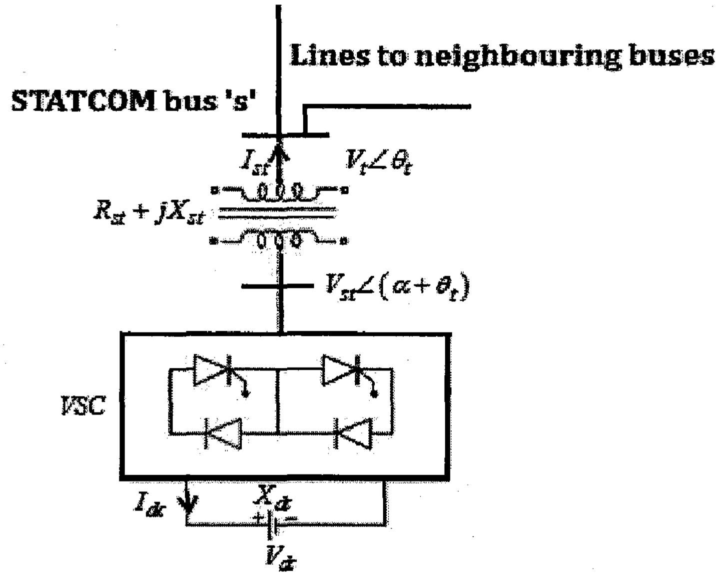

Figure B.2 STATCOM connection to the system at bus 's'.

The STATCOM power is given by

$$P_{STATCOM} + j\,Q_{STATCOM} = V_t I_{st}^* \tag{B.144}$$

The STATCOM current can be written as

$$I_{st} = \frac{V_{st}e^{j(\alpha+\theta_t)} - V_t e^{j\theta_t}}{R_{st} + jX_{st}} \tag{B.145}$$

Substituting I_{st} in eqn. (B.144) and simplifying the resulting expression, the STATCOM power can be written as

$$P_{STATCOM} + j\,Q_{STATCOM} = \frac{V_t V_{st} e^{-j\alpha} - V_t^2}{R_{st} - jX_{st}} \tag{B.146}$$

Since $V_{st}(pu) = V_{dc}(pu)$, the real and imaginary parts of eqn. (B.146) can be written as

$$P_{STATCOM} = \frac{V_t V_{dc} R_{st}\cos\alpha + V_t V_{dc} X_{st}\sin\alpha - R_{st}V_t^2}{R_{st}^2 + X_{st}^2} \tag{B.147}$$

$$Q_{STATCOM} = \frac{V_t V_{dc} X_{st}\cos\alpha - V_t V_{dc} R_{st}\sin\alpha - X_{st}V_t^2}{R_{st}^2 + X_{st}^2} \tag{B.148}$$

APPENDIX C

System Data

C.1 System Data for 3-Machine 9-Bus System [44]

Base MVA 100 MVA

The system data for 3-Machine 9-Bus System is as follows:

MACHINE DATA

Parameter	Gen. 1	Gen. 2	Gen. 3
H(sec)	23.6400	6.400	3.0100
$x_d\,(p.u.)$	0.14600	0.8958	1.3125
$x_d'\,(p.u.)$	0.06080	0.1198	0.1813
$x_q\,(p.u.)$	0.09690	0.8645	1.2578
$x_q'\,(p.u.)$	0.09690	0.1969	0.2500
T_{do}' (sec)	8.9600	6.0000	5.8900
T_{qo}' (sec)	0.3100	0.5350	0.6000

EXCITER DATA

Parameter	Gen. 1	Gen. 2	Gen. 3
K_A	20.0	20.0	20.0
T_A (sec)	0.20	0.20	0.20
K_E	1.0	1.0	1.0
T_E (sec)	0.314	0.314	0.314
K_F	0.063	0.063	0.063
T_F (sec)	0.35	0.35	0.35
R_S	0	0	0
A_{ex}	0.0039	0.0039	0.0039
B_{ex}	1.555	1.555	1.555

LINE DATA

Line Number	Bus		Line Parameters		
	From	To	$R(p.u.)$	$X(p.u.)$	Half line charging admittance (p.u.)
1	2	7	0	0.0625	0
2	1	4	0	0.0576	0
3	3	9	0	0.0586	0
4	4	6	0.1700	0.0920	0.0790
5	4	5	0.0100	0.0850	0.0880
6	5	7	0.0320	0.1610	0.1530
7	6	9	0.0390	0.1700	0.1790
8	9	8	0.0119	0.1008	0.1045
9	8	7	0.0085	0.0720	0.0745

BUS DATA

BUS	TYPE	$P_L(p.u.)$	$Q_L(p.u.)$	$P_G(p.u.)$	$Q_G(p.u.)$
1	*swing**	0	0	0.716	0.27
2	PV	0	0	1.63	0.067
3	PV	0	0	0.85	-0.109
4	PQ	0	0	0	0
5	PQ	1.25	0.5	0	0
6	PQ	0.9	0.3	0	0
7	PQ	0	0	0	0
8	PQ	1.00	0.35	0	0
9	PQ	0	0	0	0

* The swing bus voltage is 1.04 p.u.

C.2 System Data for 10-Machine 39-Bus System [49]

Base MVA 100 MVA

MACHINE DATA

Parameter	x_d p.u.	$x_d^{'}$ p.u.	x_q p.u.	$x_q^{'}$ p.u.	$T_{do}^{'}$ sec	$T_{qo}^{'}$ sec	H sec	D p.u.	x_l p.u.
Gen 1	0.295	0.0647	6.56	0.282	0.0647	1.5	30.3	0	0.0518
Gen 2	0.02	0.006	6	0.019	0.006	0.7	500	0	0.0048
Gen 3	0.2495	0.0531	5.7	0.237	0.0531	1.5	35.8	0	0.0425
Gen 4	0.33	0.066	5.4	0.31	0.066	0.44	26	0	0.0528
Gen 5	0.262	0.0436	5.69	0.258	0.0436	1.5	28.6	0	0.0349
Gen 6	0.254	0.05	7.3	0.241	0.05	0.4	34.8	0	0.04
Gen 7	0.295	0.049	5.66	0.292	0.049	1.5	26.4	0	0.0392
Gen 8	0.29	0.057	6.7	0.28	0.057	0.41	24.4	0	0.0456
Gen 9	0.2106	0.057	4.79	0.205	0.057	1.96	34.5	0	0.0456
Gen 10	0.2	0.004	5.7	0.196	0.004	0.5	42	0	0.0032

EXCITER DATA

	K_A p.u.	T_A sec.	K_E p.u.	T_E sec.	$E_{fd\,min}$ p.u.	$E_{fd\,max}$ p.u.	K_F p.u.	T_F (sec) sec.	A_{ex} p.u.	B_{ex} p.u.
Ex1	6	0.05	-0.63	0.41	-6	6	0.25	0.5	0.705	0.288
Ex2	20	0.2	1.0	0.314	-6	6	0.063	0.35	0	0
Ex3	5	0.06	-0.02	0.5	-6	6	0.08	1.0	0.0184	0.625
Ex4	40	0.02	1.0	0.73	-6	6	0.03	1.0	0	0
Ex5	5	0.06	-0.05	0.5	-6	6	0.08	1.0	0.0035	0.82
Ex6	5	0.02	-0.04	0.47	-6	6	0.075	1.25	0.0021	0.857
Ex7	40	0.02	1.0	0.73	-6	6	0.03	1.0	0.493	0.311
Ex8	5	0.02	-0.05	0.53	-6	6	0.085	1.26	0.0028	0.837
Ex9	40	0.02	1.0	1.4	-6	6	0.03	1.0	0.61	0.3
Ex10	25	0.06	-0.02	0.50	-6	6	0.08	1.0	0	0

LINE DATA

Line number	BUS		LINE PARAMETERS		
	From	To	*R* *p.u.*	*X* *p.u.*	Half Line Charging Admittance
1	22	6	0	0.0143	0
2	16	1	0	0.0250	0
3	20	3	0	0.0200	0
4	39	30	0.0007	0.0138	0
5	39	5	0.0007	0.0142	0
6	32	33	0.0016	0.0435	0
7	32	31	0.0016	0.0435	0
8	30	4	0.0009	0.0180	0
9	29	9	0.0008	0.0156	0
10	25	8	0.0006	0.0232	0
11	23	7	0.0005	0.0272	0
12	12	10	0	0.0181	0
13	37	27	0.0013	0.0173	0.1608
14	37	38	0.0007	0.0082	0.06595
15	36	24	0.0003	0.0059	0.0340
16	36	21	0.0008	0.0135	0.1274
17	36	39	0.0016	0.0195	0.1520
18	36	37	0.0007	0.0089	0.0671
19	35	36	0.0009	0.0094	0.0855
20	34	35	0.0018	0.0217	0.1830
21	33	34	0.0009	0.0101	0.08615
22	28	29	0.0014	0.0151	0.1245
23	26	29	0.0057	0.0625	0.5145
24	26	28	0.0043	0.0474	0.3901
25	26	27	0.0014	0.0147	0.1198
26	25	26	0.0032	0.0323	0.2565

Line number	BUS		LINE PARAMETERS		
	From	To	*R* *p.u.*	*X* *p.u.*	Half Line Charging Admittance
27	23	24	0.0022	0.0350	0.1805
28	22	23	0.0006	0.0096	0.0923
29	21	22	0.0008	0.0135	0.1274
30	20	33	0.0004	0.0043	0.03645
31	20	31	0.0004	0.0043	0.03645
32	19	2	0.0010	0.0250	0.6000
33	18	19	0.0023	0.0363	0.1902
34	17	18	0.0004	0.0046	0.0390
35	16	31	0.0007	0.0082	0.06945
36	16	17	0.0006	0.0092	0.0565
37	15	18	0.0008	0.0112	0.0738
38	15	16	0.0002	0.0026	0.0217
39	14	34	0.0008	0.0129	0.0691
40	14	15	0.0008	0.0128	0.0671
41	13	38	0.0011	0.0133	0.1069
42	13	14	0.0013	0.0213	0.1107
43	12	25	0.0070	0.0086	0.0730
44	12	13	0.0013	0.0151	0.1286
45	11	12	0.0035	0.0411	0.34935
46	11	2	0.0010	0.0250	0.3750

BUS DATA

BUS	TYPE	P_L *p.u.*	Q_L *p.u.*	P_G *p.u.*	Q_G *p.u.*
1	*swing***	0.0920	0.0460	5.4282	1.5724
2	PV	11.0400	2.5000	10.000	2.2624

BUS	TYPE	P_L p.u.	Q_L p.u.	P_G p.u.	Q_G p.u.
3	PV	0	0	6.5000	1.6606
4	PV	0	0	5.0800	1.5510
5	PV	0	0	6.3200	0.8381
6	PV	0	0	6.5000	2.8105
7	PV	0	0	5.6000	2.2967
8	PV	0	0	5.4000	0.2757
9	PV	0	0	8.3000	0.5970
10	PV	0	0	2.5000	1.8388
11	PQ	0	0	0	0
12	PQ	0	0	0	0
13	PQ	3.2200	0.0240	0	0
14	PQ	5.0000	1.8400	0	0
15	PQ	0	0	0	0
16	PQ	0	0	0	0
17	PQ	2.3380	0.8400	0	0
18	PQ	5.2200	1.7600	0	0
19	PQ	0	0	0	0
20	PQ	0	0	0	0
21	PQ	2.7400	1.1500	0	0
22	PQ	0	0	0	0
23	PQ	2.7450	0.8466	0	0
24	PQ	3.0860	0.9220	0	0
25	PQ	2.2400	0.4720	0	0
26	PQ	1.3900	0.1700	0	0
27	PQ	2.8100	0.7550	0	0
28	PQ	2.0600	0.2760	0	0
29	PQ	2.8350	0.2690	0	0
30	PQ	6.2800	1.0300	0	0
31	PQ	0	0	0	0
32	PQ	0.0750	0.8800	0	0

BUS	TYPE	P_L p.u.	Q_L p.u.	P_G p.u.	Q_G p.u.
33	PQ	0	0	0	0
34	PQ	0	0	0	0
35	PQ	3.2000	1.5300	0	0
36	PQ	3.2940	0.3230	0	0
37	PQ	0	0	0	0
38	PQ	1.5800	0.3000	0	0
39	PQ	0	0	0	0

** The swing bus voltage is 0.982 p.u.

C.3 STATCOM Parameters used for Two Test Systems

Base MVA 100 MVA

$R_{st} = 0.01\ p.u.\quad X_{st} = 0.1\ p.u.\quad R_{dc} = 1\times10^5\ p.u.\quad X_{dc} = 0.0165\ p.u.$

STATCOM Voltage Controller Parameters

$K_s = -2,\ T_s = 0.05\,\text{sec}$

Size of STATCOM

- **3 Machine 9 bus system**
  - ±200 MVAR
- **10 Machine 39 bus system**
  - ±350 MVAR