"THE EFFECT OF SATURATION ON CONSTANTS AND STABILITY OF SYNCHRONOUS MACHINES"

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of the requirements for the degree of
Master of Engineering (Electrical Machine Design)

By

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I N D E X

LIST OF SYMBOLS

X_d = reactance of synchronous machine in direct axis.

X_q = reactance of synchronous machine in quadrature axis.

X_{al} = armature leakage reactance

X_{ad} = armature reaction reactance in direct axis.

X_{aq} = armature reaction reactance in quadrature axis

X_{fl} = exciter leakage reactance

X_{Ddl} = Damper winding leakage reactance in direct axis

X_{Qql} = Damper winding leakage reactance in quadrature axis.

X''_d = Subtransient reactance in direct axis.

E_i = Internally induced e.m.f

K_s = Saturation factor = $1 + \frac{\text{m.m.f required for iron path for a certain flux}}{\text{m.m.f required for air gap for the same flux.}}$

K_c = Carter's coefficient

b_s = Slot width

b_c = Conductor width in the slot

b_o = Slot opening

l_s = length of the slot

N_c = No. of conductors per slot

ϕ = Flux

ψ = Flux linkages

p = No. of poles

q	=	Slots per pole per phase
τ_p	=	Pole pitch
δ	=	length of air gap
μ	=	harmonic number
k_{dp}	=	kd x kp = winding factor
m	=	No. of phases
X_e	=	external reactance connected in between synchronous machine and infinite bus.
b_p	=	Pole width
A_d	=	Armature Ampere turns in direct axis.
A_q	=	Armature Ampere turns in quadrature axis.
c_d	=	X_{ad}/X_a
c_q	=	X_{aq}/X_a
B_d	=	Maximum flux density due to A_d
B_{d1}	=	Fundamental component of B_d
A_a	=	Armature ampere turns per unit periphery at full load.
N	=	No. of turns per phase
θ	=	load angle
u	=	Terminal voltage
I_{fd}	=	field current required for ideal short circuit
I_{fnl}	=	field current required for no load rated voltage.

X_p = Potier's reactance

X_{du} = unsaturated value of X_d

X_{qu} = unsaturated value of X_q

X_{d1} = Saturated value of X_d at the saturation defined by $K_s = K_{s1}$

r = $\dfrac{\text{field ampere conductors per unit periphery for armature reaction compensation}}{\text{armature reaction ampere conductors per unit periphery at full load.}}$

E_b = infinite bus voltage

T'_{do} = direct axis armature open circuit time constant.

$$= \frac{L_e}{r_e} = \frac{\text{Equivalent field circuit inductance}}{\text{field resistance}}$$

T_D' = Load time constant

I = Moment of Inertia of rotor

T_f = Frictional Power lost per unit angular speed

K = Mechanical stiffness constant.

HISTORICAL DEVELOPMENT OF THE SUBJECT

The history of the subject relating to the effect of saturation on the constants and stability of synchronous machines traces back as early as 1934 when a paper was published by Crary, Shildneck and March giving an idea of equivalent reactance of a new equivalent machine replaced for an original one for any small steady state changes on the operating point of O.C.C. The equivalent reactance was discussed to be a function of not only the particular load on the machine, voltage and power factor at which the machine would be operating but also upon the system it was connected to. The paper also gave formulae to get at these equivalent reactances.

In 1935 a paper published by C. Kingsley in Trans. A.I.E.E. criticized the paper given by Crary as regards the load angle of a cylindrical rotor synchronous machine not being the same in the actual machine as would be in case of an equivalent machine replaced by Crary in his paper. Robertson, Rogers and Dalziel collectively published another paper in 1937 giving some empirical relations to get at the saturated values of the machine constants and finally calculated the effect on maximum power available. In 1950, Saad L. Mikhail gave his paper in Trans. A.I.E.E. dealing with saturation considerations from Potier's reactance.

The present work deals in a systematic manner the effect of saturation on all the constants of an equivalent circuit of a salient pole synchronous machine from the design data considerations and an attempt is made to find the effect due to all leakages occurring in the machine with saturation. Certain assumptions have, of course, been made but these are quite valid. Lateron experimental test results are compared with the calculated ones and a close resemblence is found with the results obtained from Kapp's diagram method which is discussed to be the most logical one as it takes into consideration the non-linearity of O.C.C. for all additions unlike A.S.A. method of dealing with saturation by calculations on air gap line and correcting these for saturation by a linear addition.

INTRODUCTION

Stability of a power system is the ability of the system to remain in synchronous equilibrium under steady operating conditions, and to regain the state of equilibrium after a disturbance has taken place.

The criterion of stability of a power system may be studied under two broad headings, - the steady state stability and the transient state stability. The former deals with the stability of the system under steady load conditions and with strictly constant armature and field currents in all synchronous machines. The latter criterion investigates the stability of the system under transient disturbance due to sudden load increases, circuit isolation and due to switching operations.

Inspite of the above two broad classifications as regards to the steady state and the transient state stability, it is important to mention that such a classification is necessarily for the purpose of analysis and should not conceal the basic oneness of the phenomena or a complete physical conception of the problem.

The present dissertation deals with the problem of analysing the stability limits of salient pole synchronous machine supplying load to an infinite bus taking saturation effects in the determination of the stability limits so as to have full utilization of the stability factor determined from the above condition.

Various methods are suggested for the saturation consideration particularly in steady state stability limit calculations and the methods of calculation are illustrated in Chapter -5 from the experimental work carried out on a synchronous machine in the laboratory A comparative study of the various methods is also made.

Chapter - 3 deals with the dynamic stability of synchronous machines and the equations are deduced taking saturation into consideration.

As far as the stability question of a salient pole synchronous machine connected to an infinite bus due to the transient disturbances is concerned, Chapter -4 deals with the effect of saturation, although it is concluded that the effects are ~~quite~~ comparatively small.

Differential equations of motion for the rotor during transient disturbances have been set up and their method of solution is indicated in the Appendix at the end.

In short the subject deals with the changes of stability limits as affected by taking saturation into account to give a better idea of the stability limits to a generation engineer working over a somewhat saturated portion of the O.C.C. of a synchronous machine.

CHAPTER - I

EFFECT OF SATURATION ON STABILITY OF SYNCHRONOUS MACHINE

1.1. Equivalent circuit of synchronous machine.

The direct axis and quadrature axis equivalent circuit of a synchronous machine can be put down as in Fig. 1.1.

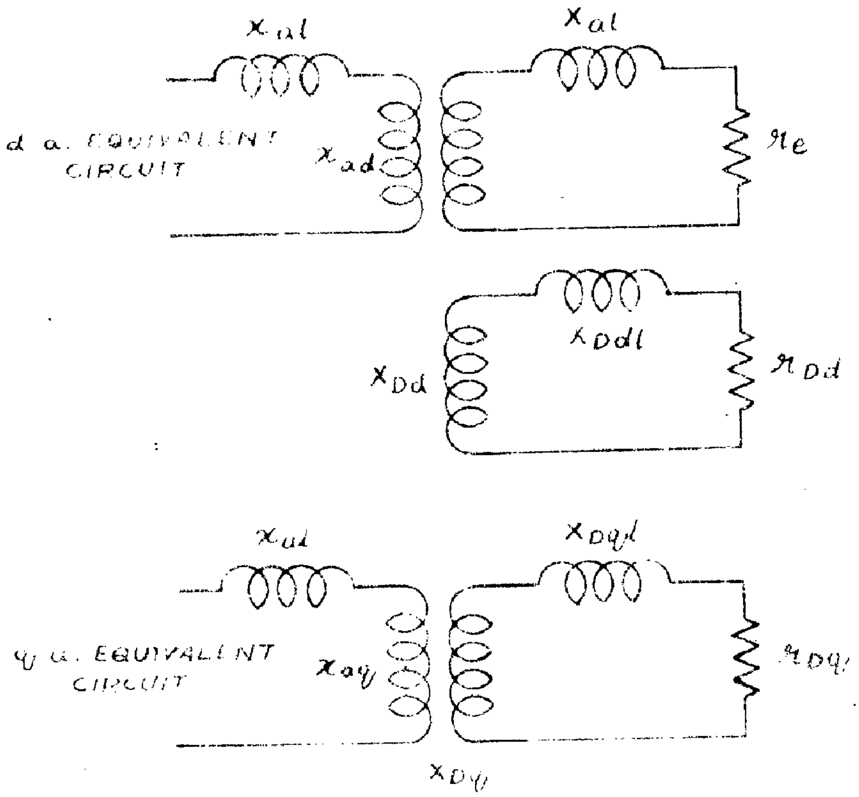

Fig. 1.1

Equivalent circuit of a synchronous machine in direct and quadrature axes.

The values of X_d, X_q and X_d' can be put as

$$X_d = X_{al} + X_{ad} \quad \ldots \quad 1.1.1$$

$$X_q = X_{al} + X_{aq} \quad \ldots \quad 1.1.2$$

$$X_d' = X_{al} + \frac{X_{ad} \cdot X_{el}}{X_{ad} + X_{el}} \quad \ldots \quad 1.1.3$$

We shall proceed to investigate the effect of saturation on the constants indicated in Fig. 1.1 and shall find the effect on X_d, X_q and X_d' wherever required in this study.

1.2. Armature Leakage reactance:

This is the representation of all the leakage fluxes linking with the armature and includes the effect of

(i) Slot leakage flux

(ii) End winding leakage flux

(iii) Tooth top leakage flux

(iv) Harmonic leakage or differential leakages.

1.3. Slot leakage flux:

It is assumed that the m.m.f spent up in forcing the leakage flux in the iron portion of the slot is negligible as compared to the m.m.f required to force it in the slot width. The assumption is valid inspite of the saturation in the main magnetic circuit.

CHAPTER - I

EFFECT OF SATURATION ON CONSTANTS OF SYNCHRONOUS MACHINE.

1.1. Equivalent circuit of Synchronous machine.

The direct axis and quadrature axis equivalent circuit of a synchronous machine can be put down as in fig. 1.1.

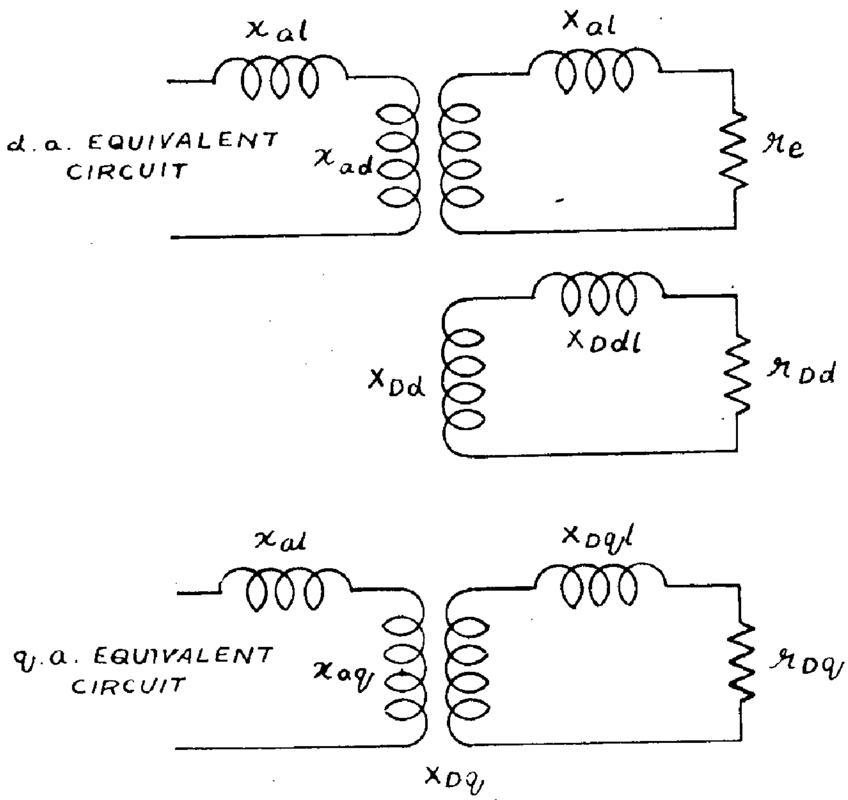

Fig. 1.1

Equivalent circuit of a synchronous machine in direct and quadrature axes.

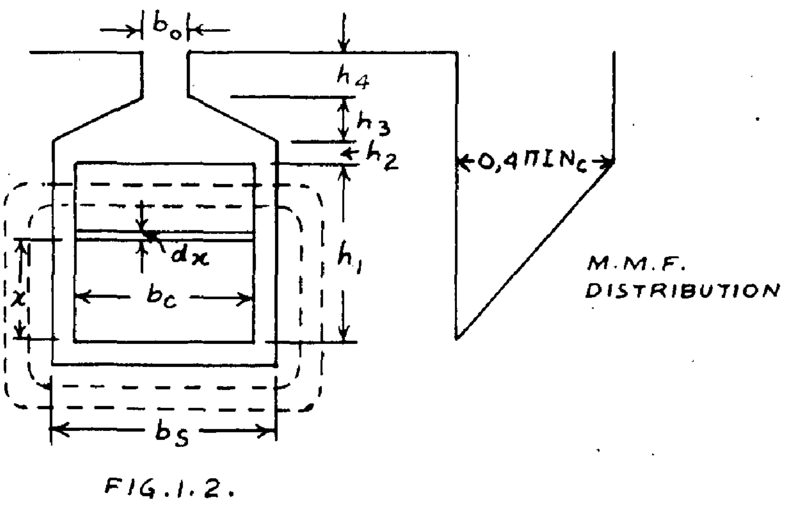

Fig. 1.2.

Path of leakage flux in the slot and the m.m.f. distribution along the height of the slot.

Let the current carried by the conductor be I amps.

The m.m.f. varies from zero at bottom to $\frac{4\pi IN_c}{10}$ at the top of the conductor and remains constant. The path of the leakage flux is shown in fig. 1.2.

Leakage flux linkages in portion h_1

$(M.M.F.)_x$

$$= \frac{4\pi}{10} \cdot IN_c \cdot \frac{x}{h_1} \qquad \ldots \quad 1.3.1$$

Reluctance of element d_x

$$= \frac{b_s}{l_s . dx} \qquad \ldots \quad 1.3.2$$

ϕx

$$= \frac{4\pi}{10} \cdot IN_c \frac{x}{h_1} \cdot \frac{l_s . dx}{b_s} \qquad \ldots \quad 1.3.3.$$

Flux linkages

$$= \frac{4\pi}{10} . I N_c^2 . \frac{x^2}{h_1^2} . \frac{l_s}{b_s} . dx \qquad .... \quad 1.3.4$$

Integrating from zero to h_1

$$\psi = \frac{4\pi}{10} . I N_c^2 . \frac{l_s}{b_s} . \int_0^{h_1} \frac{x^2}{h_1^2} . dx \qquad .... \quad 1.3.5$$

Total flux linkages =

$$\text{per ampere} = \frac{4\pi}{10} N_c^2 . \frac{l_s}{b_s} . \frac{h_1}{3} \qquad ... \quad 1.3.6$$

Leakage flux in h_2: =

$$\text{M.M.F.} = \frac{4\pi}{10} . I N_c \qquad ... \quad 1.3.7$$

$$\text{Reluctance} = \frac{b_s}{h_2 . l_s} \qquad .... \quad 1.3.8$$

Leakage flux, linkage per ampere

$$= \frac{4\pi}{10} . N_c^2 . \frac{h_2 . l_s}{b_s} \qquad .... \quad 1.3.9$$

Leakage flux linkages in h_3 :-

$$\text{Mean reluctance} = \frac{\frac{b_s + b_o}{2}}{h_3 . l_s} \qquad ....$$

Leakage flux linkages per ampere

$$= \frac{4\pi}{10} N_c^2 . l_s . \frac{2 h_3}{b_s + b_o} \qquad .... \quad 1.3.10$$

Leakage flux linkages in h_4 :-

Reluctance $= \frac{b_o}{h_4 . l_s}$ 1.3.11

Leakage flux $= \frac{4\pi}{10} . I N_c \frac{h_4}{b_o} . l_s$ 1.3.12

Linkage per ampere

$$= \frac{4\pi}{10} . N_c^2 . l_s . \frac{h_4}{b_o}$$

Total leakage flux linkages :-

$$= \frac{4\pi}{10} . N_c^2 . l_s \left\{ \frac{h_1}{3b_s} + \frac{h_2}{b_s} + \frac{2h_3}{b_s + b_o} + \frac{h_4}{b_o} \right\} \quad \text{.... 1.3.13}$$

Leakage inductance due to slot
leakage flux

$$= 0.4 \pi \times 10^{-8} N_c^2 . l_s \left\{ \frac{h_1}{3b_s} + \frac{h_2}{b_s} + \frac{2h_3}{b_s + b_o} + \frac{h_4}{b_o} \right\} \quad \text{.... 1.3.14}$$

As such we see that the slot leakage flux is indipendent of saturation in the main magnetic circuit.

1.4. End Winding Leakage flux:

Saturation in the main magnetic circuit cannot affect this leakage flux as the path of this leakage flux lies in air. Hence the armature leakage reactance due to end winding leakage flux remains constant.

1.5. Tooth top leakage flux:

The path of leakage flux is from the top of one tooth to the top of another tooth in the air gap and it can be assumed that the saturation of the teeth does not affect the reluctance of the path as the path of the leakage flux is mostly in the air. Hence these leakages are independent of saturation in the main magnetic circuit of the machine.

1.6. Harmonic Leakages or Differential leakages:

With the flow of current in the armature, different harmonic m.m.fs are set up and are given by

$$M_{a\mu} = 1{,}13\ \frac{m}{2} . I\ N_c .\ q .\ \frac{k_{dp\mu}}{\mu}$$

These harmonic m.m.fs will cause a flux of the corresponding harmonic number and a voltage will be induced which may be taken as drop in the armature leakage inductance.

Armature m.m.f. due to
μ th harmonic $M_{a\mu} = M_{a_1} \cdot \frac{1}{\mu} \cdot \frac{k_{dp\mu}}{k_{dp1}}$ 1.6.1

$$B_{a\mu} = \frac{k_{dp\mu}}{k_{dp1}} \cdot \frac{1}{\mu} \cdot \frac{M_{a_1}}{\delta k_c k_s} \quad \text{.... 1.6.2}$$

Where δ = air gap

K_c = carter's coefficient

K_s = Saturation factor

= Total effective reluctance magnetic path / Reluctance of the air gap

= 1 + m.m.f. required for magnetic path for a certain flux / m.m.f. required for air gap for the same flux.

$$B_{a\mu} = 1{,}13\ \frac{m}{2} \cdot I N_c \cdot q \cdot \frac{k_{dp\mu}}{\mu} \cdot \frac{1}{\delta k_c k_s} \quad \text{.... 1.6.3}$$

$$= 1{,}13\ m \cdot I \frac{N}{p} \cdot \frac{k_{dp\mu}}{\mu} \cdot \frac{1}{\delta k_c k_s} \quad \text{.... 1.6.4}$$

$$\phi_{a\mu} = \frac{2}{\pi} \cdot B_{a\mu} \left(\frac{\tau_p}{\mu}\right) \cdot \ell \quad \text{.... 1.6.5}$$

$$= 1{,}13\ \frac{2}{\pi} \cdot m \cdot I \cdot \frac{N}{p} \cdot \frac{\tau_p}{\delta k_c k_s} \cdot \ell \cdot \frac{k_{dp\mu}}{\mu^2} \quad \text{.... 1.6.6}$$

Voltage drop in the harmonic leakage reactance has to be the same as the voltage induced due to flux given in equation 1.6.6.

Voltage induced = E_μ

$$E_\mu = 4{,}44\, f \cdot N \cdot k_{dp\mu} \cdot \Phi_{a\mu} \times 10^{-8} \text{ volts} \quad \text{---- 1.6.7}$$

$$\text{Total voltage} = \sum_{\mu=5}^{\infty} E_\mu$$

$$= \sum 4{,}44 \cdot 1{\cdot}13 \times \frac{2}{\pi} \cdot m \cdot f \; \frac{I N^2}{p} \cdot \frac{\tau_p \cdot l}{\delta k_c k_s} \left(\frac{k_{dp\mu}}{\mu}\right)^2 \times 10^{-8} \quad \text{... 1.6.8}$$

Harmonic leakage

reactance = $X_h = \frac{\sum E_\mu}{I}$

$$\therefore X_h = 3{,}99 \frac{N^2}{p} \cdot \frac{\tau_p \cdot l}{\delta k_c k_s} \cdot \sum_{\mu \neq 1} \left(\frac{k_{dp\mu}}{\mu}\right)^2 \times 10^{-8} \quad \text{... 1.6.9}$$

$$L_h = \frac{1{,}6}{\pi} m \cdot \frac{N^2}{p} \cdot \frac{\tau_p \cdot l}{\delta k_c k_s} \cdot \sum_{\mu \neq 1} \left(\frac{k_{dp\mu}}{\mu}\right)^2 \times 10^{-8} \quad \text{... 1.6.10}$$

$$= 1{,}6\,\pi \frac{N^2}{pq} \cdot l \cdot \left\{ \frac{m}{\pi^2} \frac{q \cdot \tau_p}{\delta k_c k_s} \cdot \sum_{\mu \neq 1} \left(\frac{k_{dp\mu}}{\mu}\right)^2 \times 10^{-8} \right\} \quad \text{.. 1.6.11}$$

Thus we find that the harmonic or differential leakage reactance depends upon the saturation factor Ks. Thus the armature leakage reactance gets partially affected because a part of it is formed of diffrential leakage reactance.

Hence we may write

$$X_{al} = a + \frac{b}{K_s} \quad \text{.... .... 1.6.12}$$

1.7. Armature Reaction in direct and quadrature axes

Resolving the space component of m.m.f along the two axes of symmetry – the direct axis and the quadrature axis, we have the component in direct axis as Ha Sin ψ and the component of armature m.m.f in quadrature axis as Ha Cos ψ.

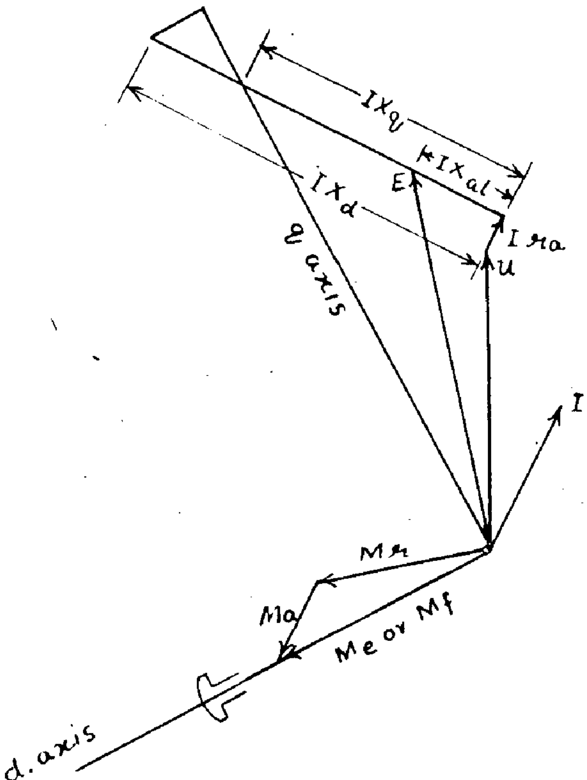

Fig. 1.3.
Vector diagram of a Salient pole synchronous machine.

The direct axis component of armature reaction m.m.f would result in a flux density distribution as shown in fig. 1.4.

The equation of the flux density can be written as

$$f(x) = 0 \qquad 0 < x \leq \frac{\tau_p - l_p}{2}$$

$$= A_d \sin \frac{\pi x}{\tau_p} \qquad \frac{\tau_p - l_p}{2} < x \leq \frac{\tau_p + l_p}{2}$$

$$= 0 \qquad \frac{\tau_p + l_p}{2} < x \leq \tau_p \qquad \ldots 1.7.1$$

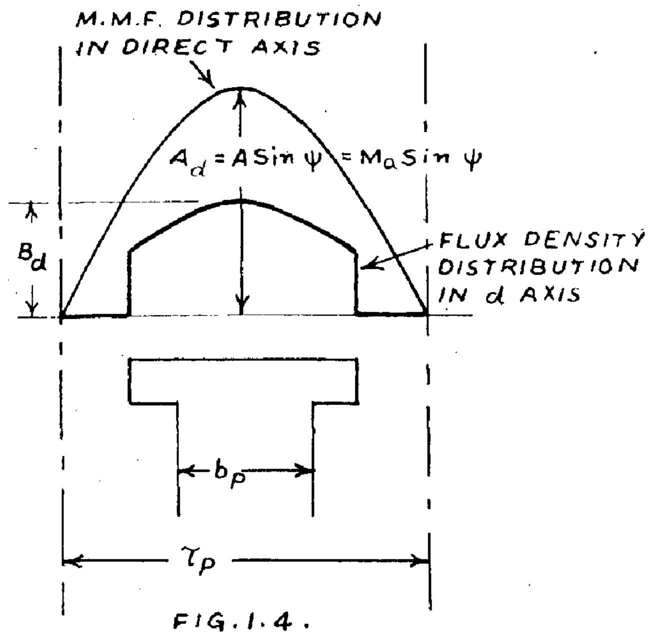

FIG. 1.4.

Fig. 1.4.
Armature reaction m.m.f and flux density distribution in direct axis.

Analysing the above equation in fourier series

$$A_{d_1} = \frac{4}{\tau_p} \cdot \int_0^{\tau_p/2} f(x) \cdot \sin \frac{\pi x}{\tau_p} \cdot dx \qquad \ldots 1.7.2$$

$$= \frac{4}{\tau_p} \cdot \int_{\frac{\tau_p - b_p}{2}}^{\frac{\tau_p + b_p}{2}} A_d \cdot \sin \frac{\pi x}{\tau_p} \cdot \sin \frac{\pi x}{\tau_p} \cdot dx \qquad \ldots 1.7.3$$

$$= A_d \left[\frac{b_p}{\tau_p} + \frac{1}{\pi} \cdot \sin \pi \frac{b_p}{\tau_p} \right] \qquad \ldots 1.7.4$$

Since $\frac{A_d}{k_c k_s \delta} = B_d$ and $\frac{A_{d_1}}{k_c k_s \delta} = B_{d_1}$

$$B_{d_1} = B_d \left[\frac{b_p}{\tau_p} + \frac{1}{\pi} \cdot \sin \pi \frac{b_p}{\tau_p} \right] \qquad \text{.... 1.7.5}$$

$$= B_d \cdot C_d \qquad \text{....}$$

$$\text{where } C_d = \frac{b_p}{\tau_p} + \frac{1}{\pi} \cdot \sin \pi \frac{b_p}{\tau_p} \qquad \text{---- 1.7.6}$$

C_d is the ratio of the amplitudes of the flux density distribution due to fundamental and the flux density corresponding to $\frac{A_d}{\delta}$ where A_d is amplitude of the m.m.f in direct axis.

The quadrature axis component can similarly be analysed and the m.m.f as well as the flux density distribution curves are as shown in fig. 1.5

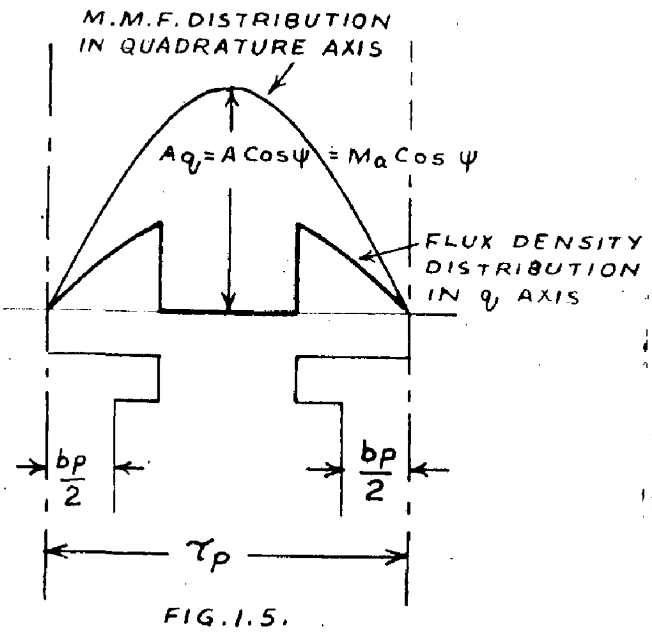

FIG. 1.5.

Fig. 1.5
Armature m.m.f and flux density distribution in quadrature axis.

The equation for the flux density curves are

$$f(x) = A_q \sin \frac{\pi x}{\tau_p} \qquad 0 < x \leq \frac{\tau_p - l_p}{2}$$

$$= 0 \qquad \frac{\tau_p - l_p}{2} < x \leq \frac{\tau_p + l_p}{2}$$

$$= A_q . \sin \frac{\pi x}{\tau_p} \qquad \frac{\tau_p + l_p}{2} < x \leq \tau_p . \qquad \text{---- 1.7.7}$$

Analysing the above equation in fourier series

$$A_{q_1} = \frac{4}{\tau_p} . \int_0^{\tau_p/2} f(x) \sin \frac{\pi x}{\tau_p} . dx \qquad \text{.... 1.7.8}$$

$$= \frac{4}{\tau_p} \int_0^{\frac{l_p}{2}} A_q . \sin^2 \frac{\pi x}{\tau_p} . dx \qquad \text{---- 1.7.9}$$

$$= A_q . \left[\frac{l_p}{\tau_p} - \frac{1}{\pi} \sin \pi \frac{l_p}{\tau_p} \right] \qquad \text{.... 1.7.10}$$

Again since $\frac{A_{q_1}}{\delta k_c k_s} = B_{q_1}$

and $\frac{A_q}{\delta k_c k_s} = B_q$

$$B_{q_1} = B_q . \left\{ \frac{l_p}{\tau_p} - \frac{1}{\pi} . \sin \pi \frac{l_p}{\tau_p} \right\} \qquad \text{.... 1.7.11}$$

$$= B_q . C_q$$

where $C_q = \frac{l_p}{\tau_p} - \frac{1}{\pi} . \sin \pi \frac{l_p}{\tau_p} .$ 1.7.12

C_q is the ratio of the amplitude of the flux density distribution due to fundamental and the flux density corresponding to A_q/δ where A_q is the amplitude of m.m.f in quadrature axis.

The fundamental component of armature reaction flux density in the direct axis B_{ad_1} can be written as follows

$$B_{ad_1} = B_a \sin\psi . C_d \qquad \text{.... 1.7.13}$$

Where B_a = amplitude of the flux density due to armature reaction m.m.f in the core of a cylindrical rotor.

or $$E_{ad_1} = E_a \sin\psi \, C_d \qquad \text{.... 1.7.14}$$

E_{ad} and E_a refer to the corresponding Voltages induced due to B_{ad} and B_a

or $$\frac{E_{ad_1}}{I_d} = \frac{E_a}{I_d} . \sin\psi . C_d \qquad \text{.... 1.7.15}$$

or $$X_{ad} = X_a \, C_d \qquad \text{.... 1.7.16}$$

Since $I_d = I \sin\psi$

Similarly $$X_{aq} = X_a . C_q \qquad \text{.... 1.7.17}$$

Now we proceed to calculate the value of X_a i.e. the armature reaction reactance in the case of a cylindrical rotor and then to determine X_{ad} and X_{aq} from the two above equations 1.7.16 and 1.7.17, the

The m.m.f distribution of an m phase armature (expressed in A.T.) by fourier analysis is known to be (ref. 1, 13.)

$$Ma_{(x)} = 0{,}9 . \frac{m}{2} . I N_c . q . \left\{ k_{dp_1} \sin\left(\omega t - \frac{\pi x}{\tau_p}\right) - \frac{k_{dp5}}{5} \sin\left(\omega t + \frac{5\pi x}{\tau_p}\right) + \frac{k_{dp7}}{7} \sin\left(\omega t - \frac{7\pi x}{\tau_p}\right) - \cdots \right\} \quad \ldots\ 1.7.18$$

and the amplitude of the fundamental is equal to

$$0{,}9 \frac{m}{2} I N_c . q . k_{dp_1} \quad \ldots\ 1.7.19$$

For a cylindrical rotor 3-phase machine

$$B_{a_1} = \frac{A.}{\delta k_c k_s} = 0{,}9 . \frac{3}{2} . \frac{I_m . N_c . q . k_{dp_1}}{k_c k_s . \delta} \quad \ldots\ 1.7.20$$

$$= 0{,}45\ A_a\ \tau_p . \frac{k_{dp_1}}{k_c k_s . \delta} \quad \ldots\ 1.7.21$$

$$\phi_{a_1} = \frac{2}{\pi} \tau_p . l_e\ B_{a_1}$$

$$= \frac{2}{\pi} . \tau_p . l_e . 0{,}45 . \frac{A_a . \tau_p . k_{dp_1}}{\delta k_c k_s .} \quad \ldots\ 1.7.22$$

$$E_{a_1} = 4{,}44\ f.N.\ K_{dp_1} \times 10^{-8}.\ \phi_{a_1} \qquad \ldots\ldots \qquad 1.7.23$$

$$= 4{,}44\ f.N.\ K_{dp_1} \times 10^{-8}.\left\{\frac{2}{\pi}\ \tau_p.l_e.\ 0{,}45\ A_a \frac{\tau_p . k_{dp_1}}{\delta\ K_c k_s}\right\} \qquad \ldots\ldots \qquad 1.7.24$$

X_a per phase

$$= \frac{E_{a_1}}{I_m}$$

$$= 4{,}44\ f.N.\ K_{dp_1}.\ \frac{10^{-8}}{I_m}.\left\{\frac{2}{\pi}\ \tau_p.l_e\ 0{,}9.\frac{3}{2}\ I_m.\ \frac{N_c\ q.\ k_{dp_1}}{K_c k_s.\delta}\right\}$$

$$= \frac{3{,}88}{120}.\ n_s\ N^2.\ \frac{k_{dp_1}^2}{K_c k_s.\delta}.\ \tau_p.l_e \qquad \ldots\ldots \qquad 1.7.25$$

$$\alpha.\ \frac{1}{k_s} \qquad \ldots\ldots \qquad 1.7.26$$

Also since

$$X_{ad} = X_a C_d$$

$$X_{aq} = X_a C_q$$

$$X_{ad} = \frac{3{,}88}{120}.\ n_s.\ N^2.\ \frac{k_{dp_1}^2}{K_c k_s.\delta}\ \tau_p.l_e.\ C_d = \frac{c}{k_s} \qquad \ldots\ldots\ 1.7.27$$

$$X_{aq} = \frac{3{,}88}{120}\ n_s.\ N^2.\ \frac{k_{dp_1}^2}{K_c k_s.\delta}.\ \tau_p.l_e.\ C_q = \frac{d}{k_s} \qquad \ldots\ldots\ 1.7.28$$

Hence

$$X_d = X_{al} + X_{ad} = a + \frac{b}{k_s} + \frac{c}{k_s} \qquad \ldots\ 1.7.29$$

$$X_q = X_{al} + X_{aq} = a + \frac{b}{k_s} + \frac{d}{k_s} \qquad \ldots\ 1.7.30$$

a, b, c and d being constants depending upon the dimensions and other constants of the machine.

1.8.8 Leakage reactance in the rotor circuit.

The leakage in the rotor circuit takes place at the following places.

Pole shoe leakages

Let ϕ_i = Leakages line of force from one pole shoe to another pole shoe throughout the length of the pole

ϕ_{ii} = Leakage lines of force from one pole shoe to adjecent pole shoe at their flat ends.

Pole shaft leakages

ϕ_{iii} = leakage lines of force from one pole shaft to another pole shaft along the length of the pole.

ϕ_{iv} = Leakage lines of force from one pole shaft to another pole shaft at the flat ends.

(1) Calculations of Ψ_i i.e. the flux linkages due to ϕ_i (refer fig. 1.6)

Let i_e = excitation current in the field winding.

N_p = No. of turns on one pole shaft.

M.M.F available for the leakage flux $\varphi_i = 2 i_e . N_p$ A.T. 1.8.1

$$\varphi_i = 2 i_e N_p . \frac{l_i . l_p}{a_{ps}} \quad \text{.... 1.8.2}$$

$$\text{flux linkages per amp.} = 4 N_p^2 . \frac{l_i . l_p}{a_{ps}} \quad \text{.... 1.8.3}$$

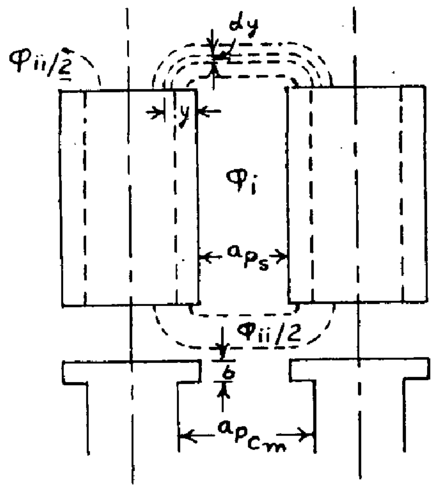

FIG.1.6.

FIG. 1.6

Leakage flux at the pole shoe ends and along the length of the pole shoe.

(ii) Calculations of

Considering an element dy as shown in fig. 1.6 length of flux path

$$= 2.\frac{\lambda}{2}.y + a_{ps}$$

Area of cross section = $b \times dy$

$$d\varphi_{ii} = 2\left\{ i_e . N_p . \frac{b.dy}{\lambda y + a_{ps}} \right\} \times 2 \qquad \text{.... 1.8.4}$$

$$d\psi_{ii} = 8\, i_e\, N_p^2 . \frac{b \times dy}{\lambda y + a_{ps}} \qquad \text{.... 1.8.5}$$

Integrating $d\psi_{ii}$ between the limits o and $b_{p/2}$ we have flux linkages per ampere as

$$\frac{\psi_{ii}}{i_e} = 8\, N_p^2 . \frac{b}{\lambda} . \left\{ \log_e \left(1 + \frac{\lambda}{2} . \frac{b_p}{a_{ps}}\right) \right\} \qquad \text{.... 1.8.6}$$

(iii) Calculation of

$$\varphi_{iii} = 2\frac{i_e}{2} N_p . \frac{h_p . l_p}{a_{pcm}} \qquad \text{.... 1.8.7}$$

$$\psi_{iii} = 2\frac{i_e}{2} N_p^2 . \frac{h_p . l_p.}{a_{pcm}} \qquad \text{.... 1.8.8}$$

$$\psi_{iii}/i_e = N_p^2 . \frac{h_p . l_p.}{a_{pcm}.} \qquad \text{.... 1.8.9}$$

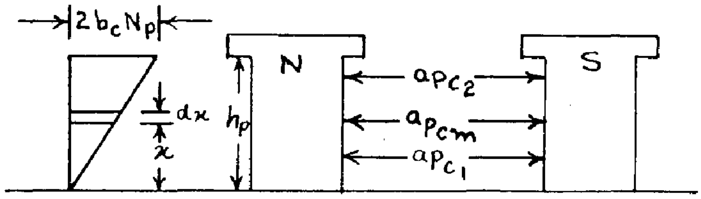

FIG. 1.7.

M.M.F. distribution along the height of the pole.

(iv) Calculation of φ_{iv}

Length of the leakage path

$$= 2.\frac{\hat{\lambda}}{2}.y + a_{pcm} \qquad \text{---- 1.8.10}$$

$$\varphi_{iv} = \int_0^{\frac{l_c}{2}} \frac{2 i_e N_p}{2} . \frac{h_p . dy}{2.\frac{\hat{\lambda}}{2}.y + a_{pcm}} \qquad \text{---- 1.8.11}$$

From which

$$\frac{\varphi_{iv}}{i_e} = 2 N_p^2 . \frac{h_p}{\lambda} . \log_e\left\{1 + \frac{\hat{\lambda}}{2} . \frac{b_c}{a_{pcm}}\right\} \qquad \text{---- 1.8.12}$$

Total leakage inductance of the excitor winding taking leakage from all sides of the pole shaft would be

$$L_{el} = 2\left\{ 4.N_p^2 . \frac{l_c \, l_p}{a_{ps}} + N_p^2 . \frac{h_p . l_p}{a_{pcm}} + 8 N_p^2 . \frac{b}{\lambda} \log_e . \left(1 + \frac{\hat{\lambda}}{2} . \frac{l_p}{a_{ps}}\right) + 2 N_p^2 . \frac{h_p}{\lambda} . \log_e . \left(1 + \frac{\hat{\lambda}}{2} . \frac{l_c}{a_{pcm}}\right) \right\} \times 10^{-8}$$

---- 1.8.13

Hence it xx is seen the expression for the leakage inductance of the rotor winding and as such the leakage reactance X_{el} is independent of saturation factor K_s. There are, however, certain assumptions involved in the calculations above.

(i) M.M.F spent up in the pole body is negligible as compared to the m.m.f required to force the total value of flux.

(ii) The leakage flux ϕ_{li} takes place at the middle of the pole height and a mean length of magnetic path a_{pm} is taken, although the length of the magnetic path does not remain constant throughout the height of the pole.

CHAPTER - II

EFFECT OF SATURATION ON STEADY STATE STABILITY OF SYNCHRONOUS MACHINES.

2.1. Steady state stability criterian:

Steady state stability is the stability of a system under conditions of gradual or slow changes. The load is assumed to be applied at a rate which is slow when compared with either the natural frequency of oscillation of the major parts of the system or with the rate of change of field flux in the synchronous machine in response to the change in loading.

Steady state stability may be considered as the system in static equilibrium under which pull out power for constant field current is to be determined or under dynamic equilibrium. The latter one is discussed in detail in Chapter -III

2.2. Pull out power with constant field current:-

From the vector diagram of a salient pole Synchronous machine, as shown in figure. 2.1, we get

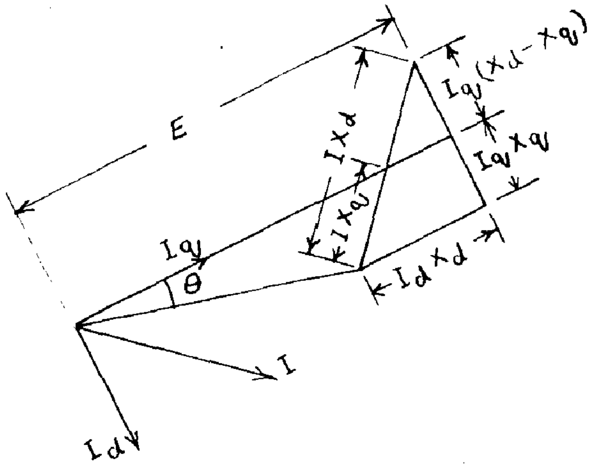

FIG. 2.1.

Vector diagram of a salient pole synchronous machine.

$$I_q = \frac{u \sin\theta}{X_q} \qquad \ldots\; 2.2.1$$

$$I_d = \frac{E - u\cos\theta}{X_d} \qquad \ldots\; 2.2.2$$

Also ... 2.2.3

$$P = u . I_q . \cos\theta + u\, I_d . \sin\theta$$

$$= u . \frac{u . \sin\theta}{X_q} . \cos\theta + u . \frac{E - u . \cos\theta}{X_d} . \sin\theta$$

$$= \frac{E . u}{X_d} . \sin\theta + \frac{u^2}{2} . \left\{ \frac{1}{X_q} - \frac{1}{X_d} \right\} \sin 2\theta$$

$$= \frac{E . u}{X_d} . \sin\theta + \frac{u^2}{2} \left\{ \frac{X_d - X_q}{X_d . X_q} \right\} . \sin 2\theta \qquad \ldots\; 2.2.4$$

The second term in the expression 2.2.4 represents the reluctance power due to the saliency of the synchronous machine. This power is available even at zero field current when internally induced e.m.f is also zero.

The variation of power available with a certain load angle is shown in fig. 2.2. from which it can be seen that the effect of saliency is to bring the maximum of the function at a load angle less than 90°

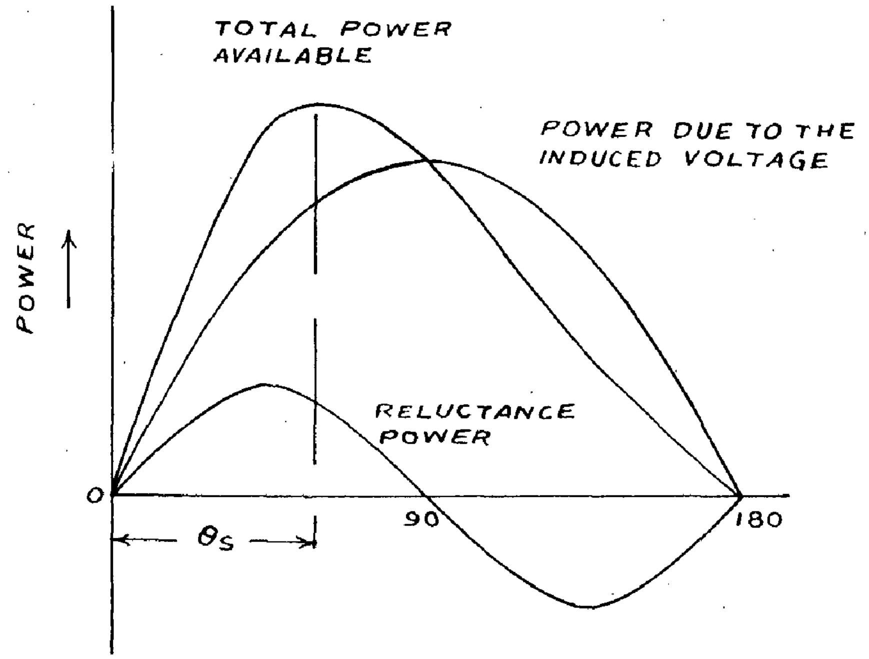

Fig. 2.2.
Variation of reluctance and total power available for a salient pole machine.

The maximum power available at any load angle can be obtained by differentiating the expression 2.2.4 so that:

$$\frac{\partial P}{\partial \theta}_{(E,u,X_d\ \text{Const})} = \frac{E\,u}{X_d}\cdot \cos\theta + \frac{u^2}{2}\cdot\frac{X_d - X_q}{X_d \cdot X_q}\cdot \cos 2\theta \times 2 \quad \text{.... 2.2.5}$$

from which

$$\frac{\cos 2\theta_s}{\cos\theta_s} = -\frac{u}{E}\cdot\frac{X_d - X_q}{X_q}. \quad \text{.... 2.2.6}$$

Putting $\theta = \theta_s$, the load angle for maximum power in eq. 2.24

$$P_{max} = \frac{E.u}{X_d} \cdot \sin\theta_s + \frac{u^2}{2} \cdot \left\{ \frac{X_d - X_q}{X_d \cdot X_q} \right\} \cdot \sin 2\theta_s \qquad \ldots 2.2.7$$

For a load angle less than θ_s the value of

D $\frac{dp}{d\theta}$ would be positive and hence with a gradual increase in load, the load angle would increase. The machine would remain in stability as long as θ reaches the value θ_s. For a working load angle $\theta = \theta_s$, any addition load cannot be supplied as the curve (iii) in fig. 2.2 droopes down and $\frac{dp}{d\theta}$ is negative beyond that. Hence θ_s represents the load angle for maximum power that can be delivered at constant excitation.

The figure represents the power load characteristics of a synchronous machine at one particular excitation or at one particular value of induced e.m.f. With different values of field excitation currents, a family of such curves is obtained according to the variation of E with the field current. The locus of the maximas of the different curves (refer Fig. 2.3) would give the maximum power available for any field current.

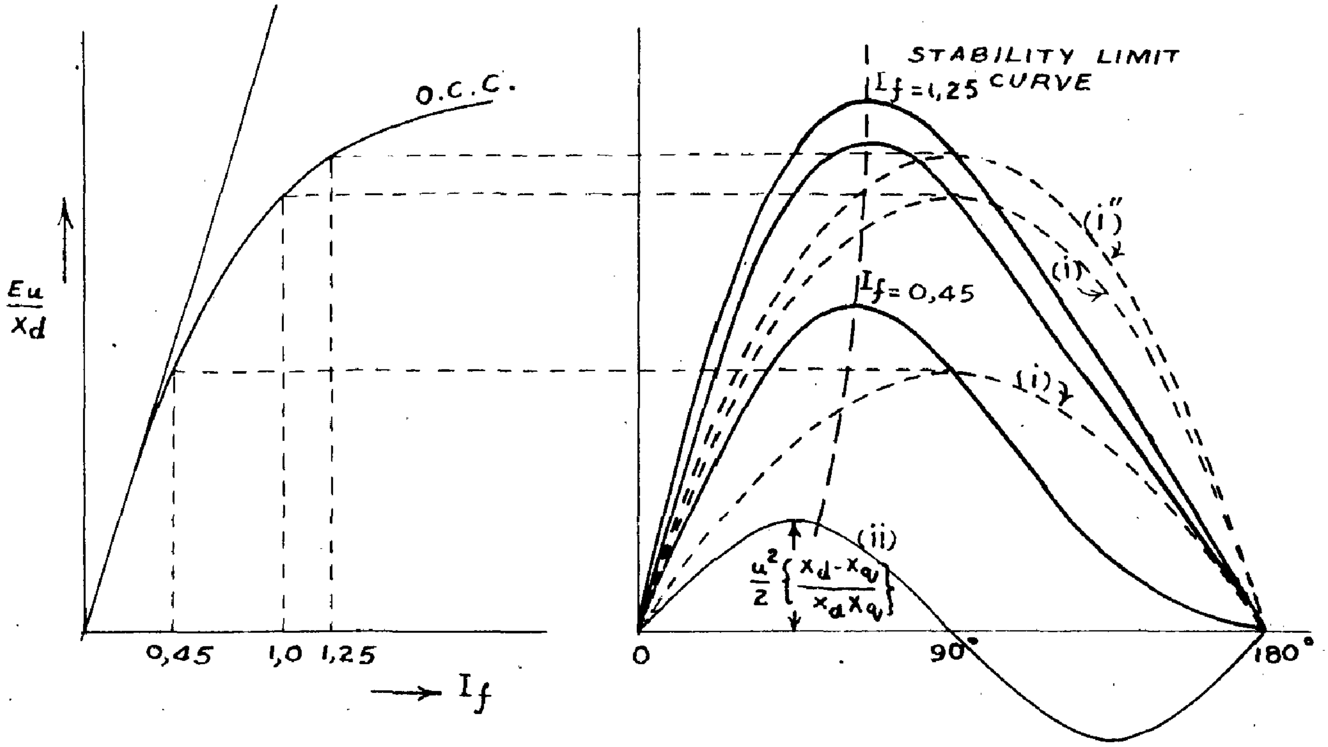

FIG. 2.3.

Variation of stability limit
with saturation of the machine.

In the above construction the variation of X_d and X_q with field currents is not taken into account.

2.2(a) Power equation of a salient pole connected to infinite bus through external reactance.

The external reactance may represent a transformer when unit system has been used. The power equation can be written as follows for this case.

$$P = \frac{E.u}{X_d + X_e} \cdot \sin\theta + \frac{u^2}{2} \left\{ \frac{X_d - X_q}{(X_e + X_d)(X_q + X_e)} \right\} \sin 2\theta$$

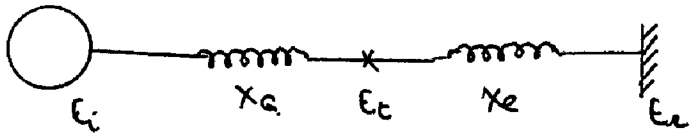

Fig. 2.3.(a)

Generator connected through an external reactance to infinite bus.

2.3. Saturation considerations on steady state Stability:

We have only discussed the effect of saliency on the pull out power of a synchronous generator and have not considered the effect of saturation which will be brought in. There are various methods by which we can determine the pull out power knowing the initial loading conditions. These methods are discussed below and take into the effect of saturation from different considerations.

2.4. Short Circuit Ratio Methods

Short circuit ratio of a synchronous machine can be obtained from open circuit characteristics and the short circuit characteristic. Refer Fig2.8

$$S.C.R. = \frac{I_{fn}}{I_{fsi}}$$

where I_{fnl} = field current required to produce rated voltage on no load saturation curve.

I_{fsi} = field current required to produce rated armature current with a 3-phase symmetrical short circuit at the generator terminals.

$$X_d = \frac{1}{S.C.R}$$

The quantity $\frac{1}{S.C.R.}$ is roughly equivalent to the generator unsaturated synchronous reactance differing only in the fact that it takes into account a certain amount of saturation. The saturation included is that at rated voltage on the no load saturation curve (fig. 2.5) and if this value of saturation is designated as S_{nl}, it can be shown that

$$\frac{1}{S.C.R.} = X_d \left\{ \frac{1}{1 + S_{nl}} \right\} \quad \ldots\ldots \quad \ldots\ldots 2.4.3$$

In this method, therefore, a certain amount of correction for saturation at pull out is obtained but it is a constant approximation where as the true saturation is variable depending upon the operating conditions. No correction for saturation is made for X_q.

The max^m. power output can be obtained as follows

$$E = E_t + I_d\left(\frac{1}{S.C.R.}\right) + I_q X_q \quad \ldots\ 2.4.4$$

$$P = \frac{E_i u}{\frac{1}{S.C.R} + X_e} \sin\theta + \frac{u^2}{2}\left\{\frac{\frac{1}{S.C.R} - X_q}{\left(\frac{1}{S.C.R} + X_e\right)(X_q + X_e)}\right\} \sin 2\theta \quad 2.4.5$$

The calculations are illustrated in Chapter V based on the test data obtained from the tests on a small synchronous machine in the laboratory.

1.5. Potier voltage Method:

X_G = Generator reactance

= X_d in direct axis and X_q in quadrature axis.

X_e = external reactance

The following equations can be written from the vector diagram in fig. 2.4.

$$\vec{E}_r = \vec{E}_t - \vec{I}X_e \quad \ldots\ 2.5.1$$

Voltage behind Potier's reactance $\vec{E}_p = \vec{E}_t + \vec{I}X_p$ 2.5.2

Internal voltage of the machine $\vec{E}_i = \vec{E}_t + \vec{I}_d X_d + \vec{I}_q X_q$ 2.5.3

The saturation S is taken from O.C.C. and is the difference between the excitation required to produce $\vec{E}_p$ on no load saturation curve and the excitation required to produce $\vec{E}_p$ on air gap line refer Fig. 2.5. The excitation voltage $\vec{E}_X$ which is equivalent to the total field current under initial

Load conditions is the internal voltage plus the saturation S.

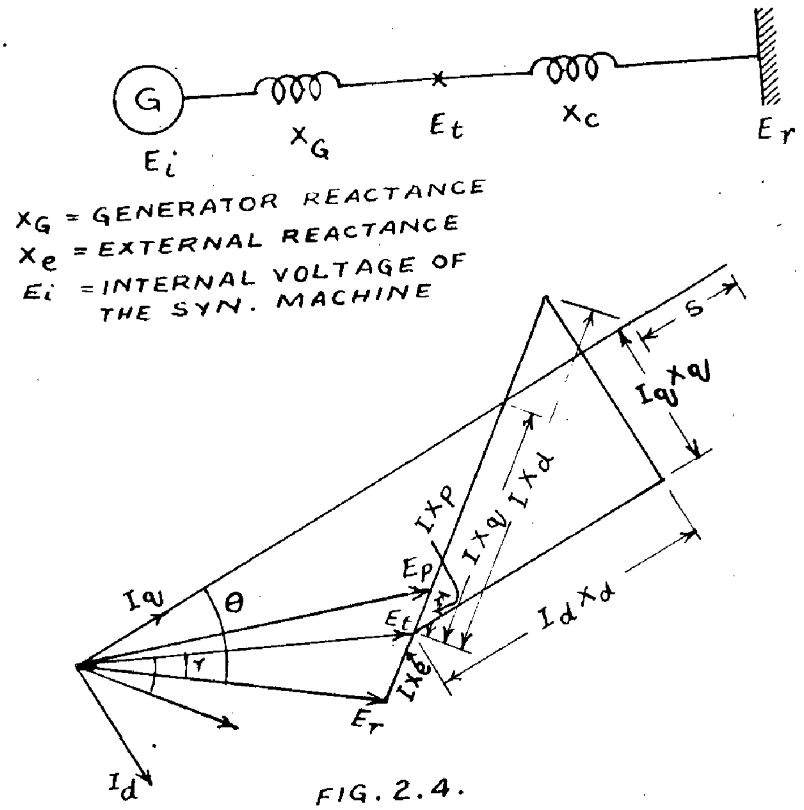

Fig. 2.4

Vector diagram during initial operating conditions prior to pull out.

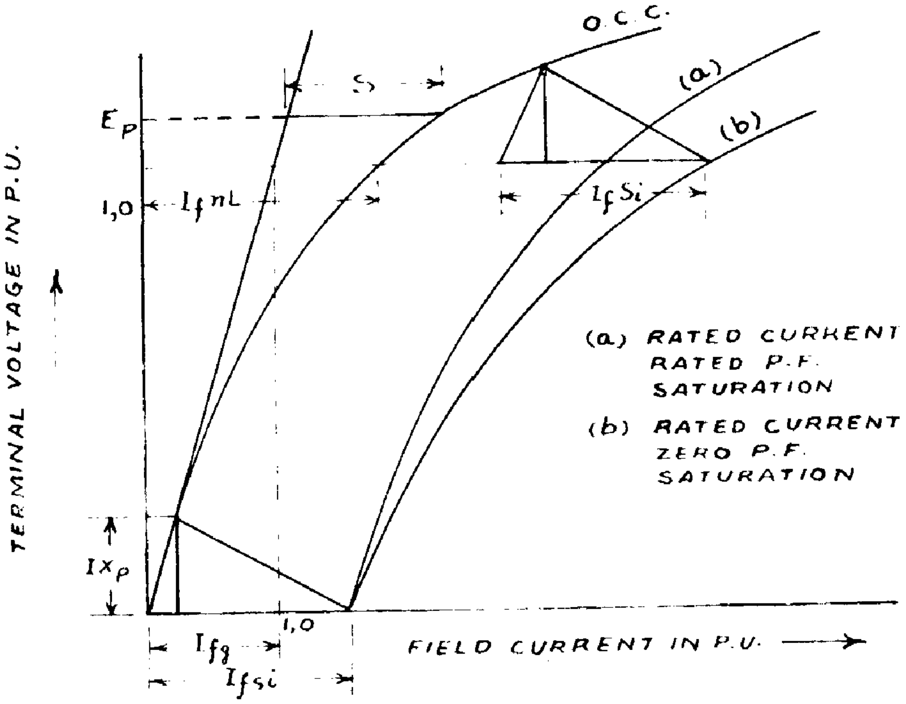

FIG.2.5.

O.C.C. and Zero p.f. characteristics for estimating Potier's reactance.

Power transferred from Generator to infinite bus is

$$\frac{E_1 E_2}{X_d+X_e}\cdot \sin 2\theta + \frac{E_2^2}{2}\left\{\frac{X_d-X_q}{(X_d+X_e)(X_q+X_e)}\right\}\sin 2\theta \quad \ldots \quad 2.5.4.$$

Under the hypothesis of constant excitation in the transition from the initial load condition to pull out conditions, the excitation voltage E_1 remains constant. The infinite bus voltage E_r would also remain constant so that these two voltages and the reactances X_g and X_p and X_e and the angle θ are only quantities known at the time of pull out and the internal voltage at pull out E must be determined.

Fig. 2.6 gives the vector position for maximum power transfer. The angle θ would under these conditions be θ_s corresponding to the load angle of maximum power transfer.

The quantities $\vec{E}_x$, $\vec{E}_r$, X_g, X_p, X_e and θ are known and it is required to calculate E_i' under pull out conditions.

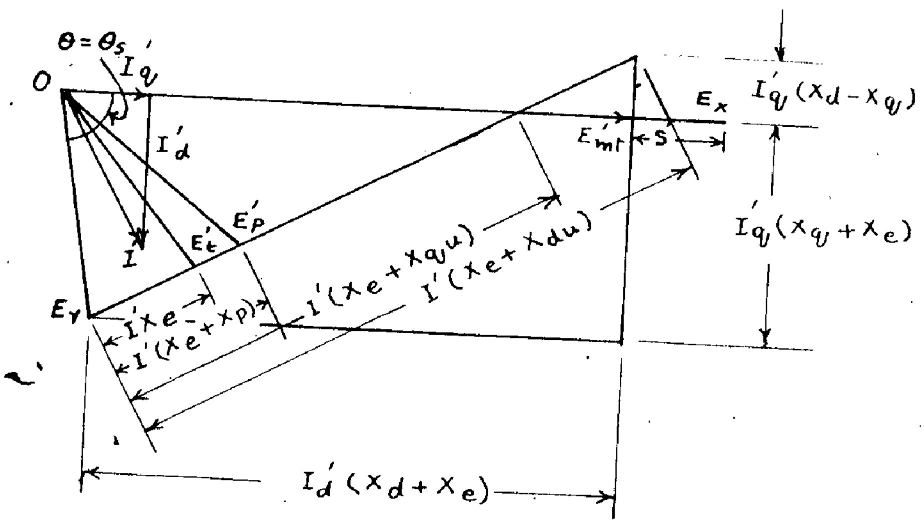

FIG. 2.6.

Vector diagram of salient pole synchronous machine under pull out conditions.

The following expressions can be written:

$$\vec{E}_i' = \vec{E}_x - \vec{S}' \qquad \text{.... 2.5.5}$$

$$E_r . \sin\theta_s = I_q'(X_q + X_e)$$

$$\text{or } I_q' = \frac{E_r . \sin\theta_s}{X_q + X_e} \qquad \text{.... 2.5.6}$$

$$E_i' - E_r \cos\theta_s = I_d'(X_d' + X_e)$$

$$\text{or } I_d' = \frac{E_i' - E_r . \cos\theta_s}{X_d + X_e} \qquad \text{.... 2.5.7}$$

$$I' = \left[I_d'^2 + I_q'^2\right]^{1/2} \qquad \text{.... 2.5.8}$$

$$\vec{E}_t' = \vec{E}_r + \vec{I}' X_e \qquad \text{.... 2.5.9}$$

$$\vec{E}_p' = \vec{E}_r + \vec{I}'(X_e + X_p) \qquad \text{.... 2.5.10}$$

A method of successive approximation must be used to determine S' which leads to the correct solution of the vector diagram. The method is as follows:

A value of S' is assumed. The above equations 2.5.5. to 2.5.10 are used to calculate E_p'. The actual value of S' is found from the saturation curve and is compared with the assumed value. The assumed value is then adjusted until the actual value found by repeating calculations of E_p' is equal to the assumed value.

The value of E_p is the vector sum of terminal voltage and potier's reactance drop IX_p where X_p is the Potier reactance at terminal voltage. Strictly saturation has to be considered at voltage behind the Potier reactance i.e. E_p and the value of

E_p at E_p should be taken. However the value of E_p at terminal voltage may be taken without any appreciable error.

When the correct value of S' and E_1' are found the pull out power can be calculated by

$$P_{max} = \frac{E_i' \cdot E_r'}{X_d + X_e} \cdot \sin\theta_s + \frac{E_i'^2}{2} \cdot \left\{ \frac{X_d - X_q}{(X_d+X_e)(X_q+X_e)} \right\} \sin 2\theta_s \qquad \text{.... 2.5.11}$$

The method is illustrated in Chapter V and is based on the data obtained from tests on a small salient pole synchronous machine in the laboratory.

2.6. Synchronous Reactance method:

The synchronous machine is represented by a reactance equal to the unsaturated synchronous reactance and an internal voltage equal to the voltage behind unsaturated synchronous reactance as determined by the initial load conditions. This voltage E_i' is the same as determined in section 2.5. The saturation effect is taken in the reactances of the synchronous machine by selecting the value of X_{ds} and X_{qs} at the voltage corresponding to the voltage behind the Potier's reactance. This is true only under the assumption that the change in X_p from terminal voltage to the value of the voltage behind X_p is very little.

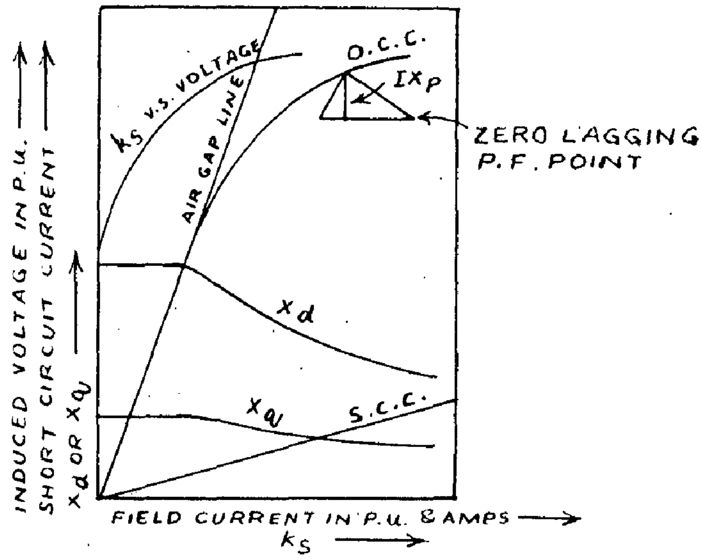

FIG. 2.7.

Variation of K_s, X_d and X_q with field current.

The variation of X_{ds} and X_{qs} with voltage can be determined from the following equations and finally plotted as shown in figure 2.7 and 5.1

From equations 1.7.29 and 1.7.30 we have

$$X_d = a + \frac{b}{k_s} + \frac{c}{k_s} \qquad \ldots\ 1.7.29$$

$$X_q = a + \frac{b}{k_s} + \frac{d}{k_s} \qquad \ldots\ 1.7.30$$

At $k_s = k_{s_u} = 1$

$$X_{du} = a + b + c \qquad \ldots\ 2.6.2$$

$$X_{qu} = a + b + d \qquad \ldots\ 2.6.3$$

At $k_s = k_{s_1}$

$$X_{d_1} = a + \frac{b+c}{k_{s_1}} \qquad \ldots\ 2.6.4$$

$$X_{du} - X_{d_1} = (b+c)\left\{1 - \frac{1}{K_{s_1}}\right\} \qquad \ldots\ 2.6.5$$

$$\text{or} \quad b+c = \frac{X_{du} - X_{d_1}}{1 - \frac{1}{K_{s_1}}} \qquad \ldots\ 2.6.6$$

$$a = X_{du} - \frac{X_{du} - X_{d_1}}{1 - \frac{1}{K_{s_1}}} \qquad \ldots\ 2.6.7$$

$$c = X_{du} - X_{al_u}$$

$$b+d = X_{qu} - \left\{ X_{du} - \frac{X_{du} - X_{d_1}}{1 - \frac{1}{K_{s_1}}} \right\} \qquad \ldots\ 2.6.8$$

$$X_{q_1} = X_{du} - \left\{ \frac{X_{du} - X_{d_1}}{1 - \frac{1}{K_{s_1}}} \right\} + \frac{1}{K_{s_1}} \left\{ X_{qu} - \left(X_{du} - \frac{X_{du} - X_{d_1}}{1 - \frac{1}{K_{s_1}}} \right) \right\} \qquad \ldots\ 2.6.9$$

Having determined the values of K_s corresponding to the voltage behind the Potier's reactance, X_{ds} and X_{qs} can be calculated or observed from fig 2.7 and 5.1. The power equation can therefore be written as

$$P = \frac{E_i . E_r}{X_{d_1} + X_e} \sin\theta + \frac{E_r^2}{2} . \left\{ \frac{X_{d_1} - X_{q_1}}{(X_{d_1} + X_e)(X_{q_1} + X_e)} \right\} \sin 2\theta \qquad \ldots\ 2.6.10$$

The calculations based on the test data from a small synchronous machine in the laboratory are illustrated in Chapter - V

2.7. Saturation effect taken into consideration from Kapp's diagram.

The method differs from that described in section 2.5 in determining the value of E_1 and taking saturation into consideration. As a matter of fact we do not have any justification for the addition of saturation S along the vector E_1 and strictly speaking we must operate on O.C.C. for all such additions in a non-linear fashion. It is not the voltages but the field currents which have to be added linearly and the net effect is observed on O.C.C. for the total voltage which would result due to the added field currents. The values of the reactances are however taken at the voltage E_p.

Aa = armature reaction ampere conductors per unit periphery at full load.

r = Field ampere conductors per cm. periphery for armature reaction compensation at full load.
―――――――――――――――――
armature reaction ampere conductors per unit periphery at full load.

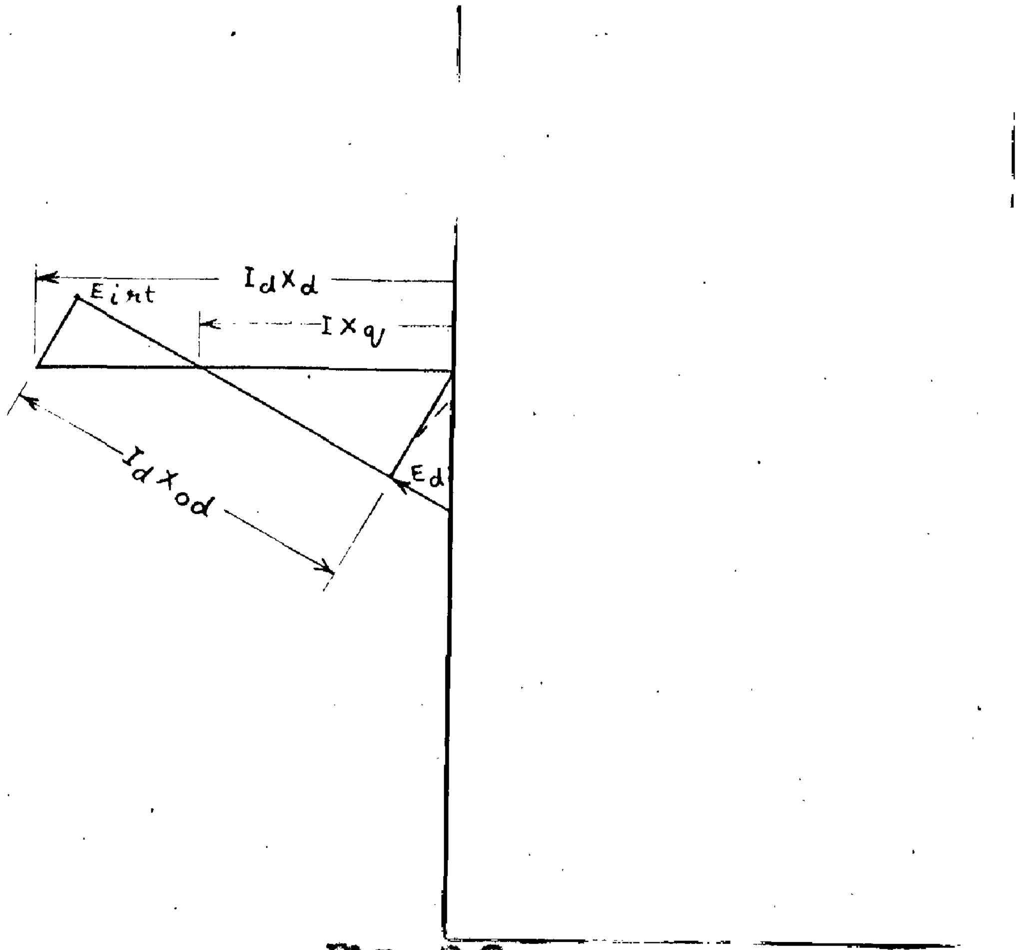

Fig. 2.8.
Calculations of field current and internally induced e.m.f for a salient pole synchronous machine.

Therefore γA_a = field ampere conductors per cm periphery for armature reaction compensation at full load required to overcome the drop IX_{ad}

$\gamma A_a \dfrac{C_q}{C_d}$ = field ampere conductors per cm periphery required in direct axis for q. axis armature reaction compensation.

Field ampere conductors per cm periphery to overcome a drop of $I(X_{al} + X_{aq})$ would be the sum of the field a.c. required for IX_{al} drop and $\gamma A_a \dfrac{C_q}{C_d}$.

Having determined IX_q , the quadrature axis can be found out and the angle ψ determined. The vector diagram can be completed as in figure 2.8.

Now E_{d1} is the resultant voltage on O.C.C. due to the sum of a.c. per cm. required for E_{d1} and I_dX_{ad}. Ampere conductors per cm periphery required for E_{d1} is Ad_1. In addition to this certain amount of ampere conductors per cm. periphery are required for I_dX_{ad} or IX_{ad} Sin ψ and there would be γAa Sin.ψ

Hence the total field current or field ampere conductors per unit periphery would be $Ad_1 + \gamma Aa \sin\psi$ the total internally induced e.m.f being E_1 shown as LM' in fig. 2.8.

The voltage which gives the flux conditions in the machine is Ed_1, the rest of flux due to $\gamma Aa \sin\psi$ getting neutralised by armature reaction. The reactance can be considered at the voltage Ed_1 or at the voltage E_p as in section 2.6

Hence we can write down the power equation as

$$P = \frac{E_i . E_r}{X_{d_1} + X_e} . \sin\theta + \frac{E_r^2}{2} . \left\{ \frac{X_{d_1} - X_{q_1}}{(X_{d_1}+X_e)(X_{q_1}+X_e)} \right\} \sin 2\theta \quad \ldots 2.21$$

$k_{s_1} = k_s$ at a voltage E_{d_i}

The method is illustrated by calculations based on test results of a small salient pole synchronous machine in the laboratory and is detailed in Chapter - V

CHAPTER - III

DYNAMIC STABILITY OF SYNCHRONOUS MACHINES UNDER SATURATED CONDITIONS.

Up till now the effect of saturation only under steady state stability conditions was considered having assumed that the operation is only under a fixed excitation on the power - load angle characteristics. It was also studied that the steady state conditions could not exist after a load angle $\theta = \theta_s$ and determined the value of θ_s for a salient pole synchronous machine while for cylindrical rotor machines it is found to be exactly 90° .

There are however certain conditions under which the operation is possible at a load angle $\theta > \theta_s$. Although at this point the system is statically unstable, high speed regulation of the field current with the help of high speed regulators can make the operation possible.

The operation of the machine under these conditions is said to be working under dynamic stability.

Considering once again a system of a salient pole synchronous generator supplying load to an infinite bus through an external reactance, the power equation can be written as

$$P = \frac{E_1 . E_2}{X_d + X_e} \sin\theta + \frac{E_2^2}{2} . \left\{ \frac{X_d - X_q}{(X_d + X_e)(X_q + X_e)} \right\} . \sin 2\theta \qquad \ldots 3.1.1$$

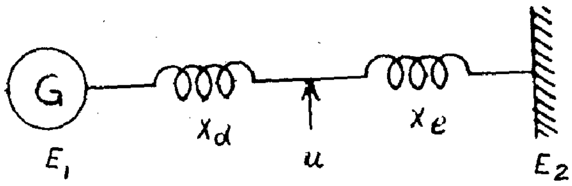

FIG. 3.1.

Arrangement showing the Generator connected to an infinite bus through an external reactance representing a transformer in a case of unit system.

The value of the terminal voltage u can be written as follows from figure 3.2

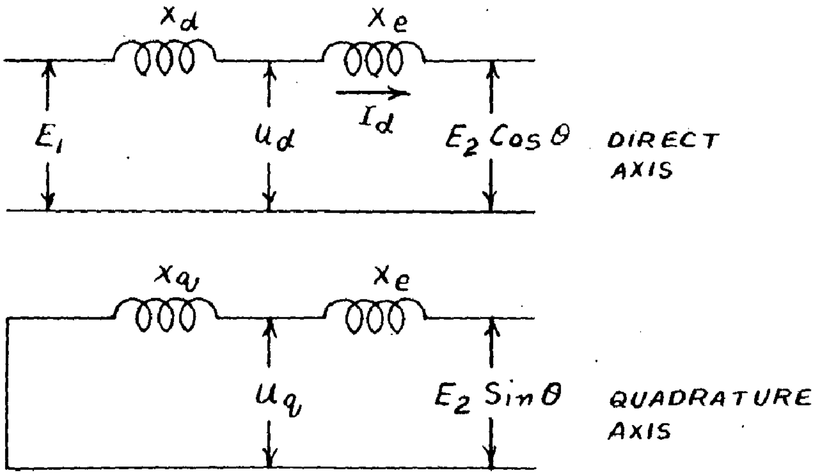

FIG. 3.2.

Fig. 3.2.
Equivalent circuit in direct and q axis for system shown in fig. 3.1

$$u_d = E_2 . \cos\theta \left\{ 1 - \frac{X_e}{X_d + X_e} \right\} + E_1 . \frac{X_e}{X_d + X_e} \qquad \dots\ 3.1.2$$

$$u_q = E_2 . \frac{X_q}{X_q + X_e} . \sin\theta \qquad \dots\ 3.1.3$$

$$u = (u_d^2 + u_q^2)^{1/2}$$

$$= \sqrt{\left[E_2 \cos\theta \left\{1 - \frac{X_e}{X_d + X_e}\right\} + E_1 \cdot \frac{X_e}{X_d + X_e}\right]^2 + \left[E_2 \cdot \frac{X_q}{X_q + X_e} \cdot \sin\theta\right]^2} \quad \ldots\ 3.1.4$$

Thus knowing the value of E_1, E_2, X_e, X_d the variation of the u with θ can be plotted as shown in figure 3.3.

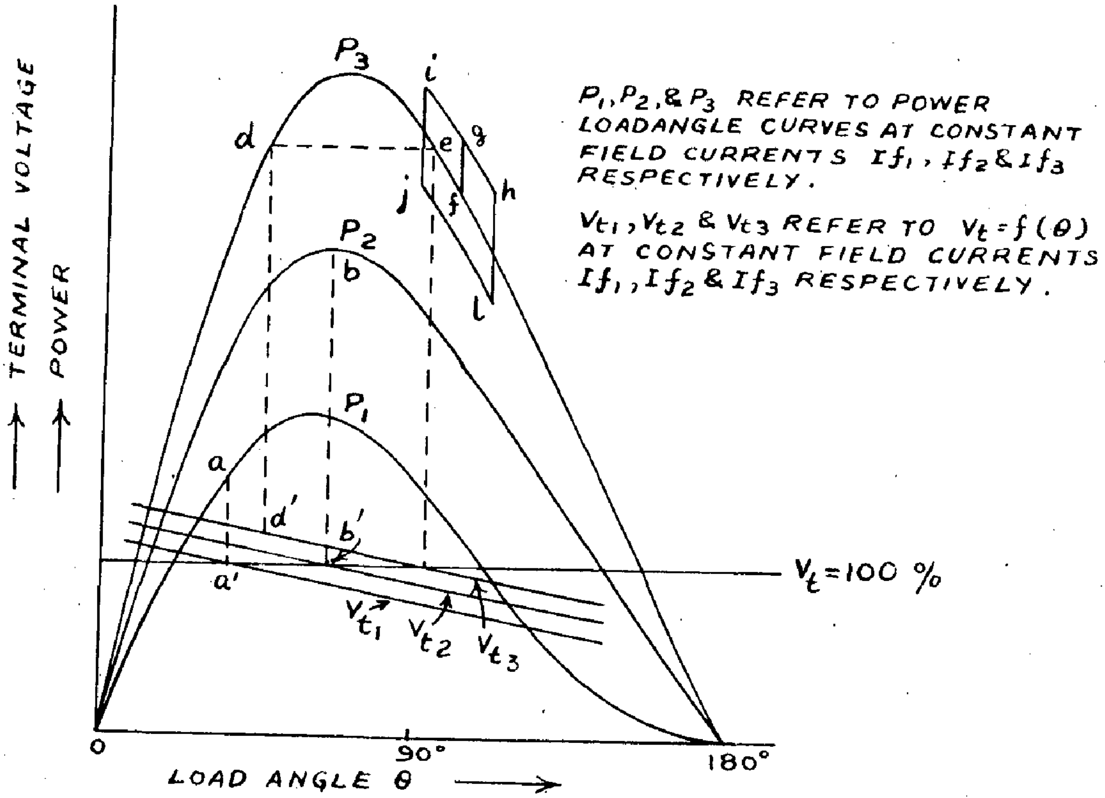

FIG. 3.3.

Fig. 3.3.
Showing the power+terminal voltage variations with load angle.

It is assumed first of all that the working point is a on P_1 with field current of I_{f_1} and

V_{t1} corresponding to point a reads 100%. The system is working with an automatic voltage regulator such that any variation in the terminal voltage goes to adjust the voltage to 100%. The maximum power under steady state conditions that can be supplied is the peak of P_1, I_{g_1} being kept constant.

Now if the field current is increased to I_{g_3} as well as the load, the working point lies at point d, the corresponding T. V. would be higher than 100% as read on V_{t3} line at point d'. If an automatic voltage regulator, were fitted, the excess voltage would operate the regulator and decrease the field current so as to bring the working point on the curve in between P_1 and P_2 and at the same time to adjust V_t at 100%. As such there is a definite limit of power that can be delivered represented by point b, beyond which the steady state conditions do not exist and the voltage is just 100% on the curve V_{t2} - the field current being I_{g_2}

Assume the generation on the third power angle at a point d. At this point steady state stability is present but the terminal voltage is more than 100%

corresponding to the point G' on curve Vg on the field current is I_{g3}. The cusp power can however be supplied in the inherently unstable part of this power [illegible] curve at [illegible] until such that the point is indicated by c. At this point the system is unstable and a high speed voltage regulator is necessary in order to make the operation at all possible.

Owing to the incidental load increases tending to increase the load angle and consecutively slow the [illegible] the influence of inertia in rotating parts. If, once the machine has changed its load angle, it were possible to increase the generator field current sufficiently so as to make the power at that larger than the input, the rising speed might be stopped or reduced. This is just what the high speed regulator in conjunction with a suitably designed excitation system accomplishes.

As the machine increases in its load angle, the terminal voltage drops actuating the regulator which in turn causes the excitation to begin building up. When the machine has swung to the point f, for instance, the field current may be assumed to be increased to that corresponding to the fourth power angle curve just above the first one drawn; the operating point is g. As to steady-state [illegible] however the [illegible] it continues to swing [illegible] past to point h at which the kinetic energy have been given up and will then swing back along this power angle curve till the point 'i' is reached. The terminal voltage has once again increased so that the regulator will operate and reduce the field current probably corresponding to a fifth power angle curve just below the 3rd one. The generation point has changed to 'j' and the swing might continue up till 'k' where stored and kinetic energy has become equal.

The cycle keeps on regulating and instead of perfectly steady state situation there seems that has been called as quasi-steady state stability.

There are three important things required for the possibility of maintaining dynamic equilibrium.

(i) the ability of the regulator to act quickly

(ii) the ability of the excitation system to perform without too much of lag.

(iii) presure of inertia in the rotating parts.

3.2. Dynamic Stability Limits-

Equation 3.1.4 can be written once again as

$$v = \sqrt{\left[E_2 \cos\theta \left\{1 - \frac{X_e}{X_d + X_e}\right\} + E_1 \cdot \frac{X_e}{X_d + X_e}\right]^2 + \left[E_2 \frac{X_q}{X_q + X_e} \cdot \sin\theta\right]^2}$$

Since the value of u has been kept constant at a value equal to 1 p.u through out the stability operation, we can write

$$\left[E_1 \cos\theta\left\{1-\frac{X_e}{X_d+X_e}\right\} + E_1\left\{\frac{X_e}{X_d+X_e}\right\}\right]^2 + \left[E_2\,\frac{X_q}{X_q+X_e}\cdot\sin\theta\right]^2 = 1 \qquad \text{..3.2.1}$$

E_2, being the infinite bus voltage, is constant. Hence the value of E_1 can be determined in terms of E_2 as follows

$$E_1 = \frac{1}{\frac{X_e}{X_d+X_e}}\cdot\left[\sqrt{1-\left\{E_2\cdot\frac{X_q}{X_q+X_e}\cdot\sin\theta\right\}^2} - E_2\cos\theta\left(\frac{X_d}{X_d+X_e}\right)\right] \qquad \text{....3.2.2}$$

$$= \frac{X_d+X_e}{X_e}\left[\sqrt{1-\frac{E_2^2\cdot X_q^2\cdot\sin^2\theta}{(X_q+X_e)^2}} - E_2\cdot\cos\theta\cdot\frac{X_d}{X_d+X_e}\right] \qquad \text{....3.2.3}$$

The power equation can be written as

$$P = \frac{E_1E_2}{X_d+X_e}\cdot\sin\theta + \frac{E_2^2}{2}\cdot\left\{\frac{X_d-X_q}{(X_d+X_e)(X_q+X_e)}\right\}\sin 2\theta$$

Substituting the value of E_1 from 3.2.3, we get

$$P = \frac{E_2}{X_e}\left[1-\frac{E_2^2\cdot X_q^2\cdot\sin^2\theta}{(X_q+X_e)^2} - E_2\cdot\cos\theta\cdot\frac{X_d}{X_d+X_e}\right]\sin\theta$$

$$+ \frac{E_2^2}{2}\left[\frac{X_d-X_q}{(X_d+X_e)(X_q+X_e)}\right]\sin 2\theta \qquad \text{....3.2.4}$$

In order to calculate E_1 from equation 3.2.3 we require the values of X_d, and X_q, at the different portion of O.C.C. for which E_1 it self is required. Hence only a step by step method can be adopted.

The values of X_{d_s} and X_{q_s} are to be found at a voltage behind the leakage reactance and not at the total internally induced e.m.f since it is the former one which defines the state of saturation. The latter also takes into account the flux which got neutrilised due to armature reaction and actually is not causing any saturation. The voltage behind the leakage reactance can be obtained by vectorial summation of terminal voltage and the armature leakage reactance drop. (refer eq. 1.6.12). The latter itself depends upon the amount of saturation or for that matter the voltage behind the leakage reactance ultimately required to find K_s, X_{d_s} and X_{q_s}

The following method is adopted to solve the equation:

A certain value of K_s only slightly higher than that at terminal voltage is assumed. Leakage reactance is calculated at that value of K_s with the use of equation 1.6.12. The voltage behind the leakage reactance can now be determined and the new value of K_s observed. The solution has to be obtained by method of successive approximation.

The values of X_{d_s} and X_{q_s} are observed over the calculated value of K_s (refer Chapter-V for an illustration) and the value of E_1 is calculated from Equation 3.2.3. This when substituted in power equation would yield $P = f(\theta)$ under dynamic loading conditions.

3.3. Equation of Motion of Synchronous machine under Dynamic state - Equilibrium and the analysis of stability from those conditions.

In order to analyse the dynamic response it is necessary to know the exciter response with its regulator and the constants involved in the equation of motion of the rotor.

(a) Variations of field current of a separately excited exciter with voltage Regulator.

For an open circuited armature, if a voltage V_e' is suddenly applied to the field terminals, the exciter current at any instant is given by

$$L_e \frac{di_e}{dt} + r_e i_e = V_e' \quad \ldots 3.3.1$$

$$\text{or } T_{do}' \frac{di_e}{dt} + i_e = V_e'/r \quad \ldots 3.3.2$$

where L_e = equivalent field Ckt inductance

r_e = field resistance.

i_F = E in p.u system, unit field current is such as to produce unit voltage as air gap line

$\frac{V_d}{r}$ = Field current = U_0 in p.u if unit of field current is such as to force unit current through resistance.

The equation can now be rewritten as

$$T_{d0}' \frac{dE}{dt} + E = U_e \quad \ldots\ldots\ldots\ldots 3.3.3$$

Under loaded armature conditions, the variations of field terminal voltage with time is given by

$$U_e(t) = T_E' \cdot \frac{dE}{dt} + E \quad \ldots\ldots\ldots\ldots 3.3.4$$

where T_E' = load line constant.

The above equations do not take saturation into account and all these relations are true for air gap line. Under dynamic equilibrium, as explained before (section 3.1) the working takes place in the neighbourhood of a particular power angle characteristics and as such voltage behind the leakage reactance is constant with the result that a correction 'S' can be taken and supposed to be constant through out the study of dynamic stability at that point on power angle characteristics.

Hence the equation 3.3.4 can be modified to take saturation effect into consideration as

$$T_D'. \frac{dE}{dt} + E = U_e(t) - U_{e0} \quad ....3.3.5$$

where $U_e(t)$ refers to the total excitation on actual O.C.C. and not for air gap line only.

The regulator system is assumed as follows:

At time $t = 0$, a disturbance is there on the operationpoint as regards, say a small increase in load causing the terminal voltage to go down.

After a time h depending upon the regulator time lag, a direct voltage is suddenly impressed in the exciter field circuit and consequently a voltage across the exciter circuit starts building up.

An assumption of exponential build up of exciter voltage between its initial and final values, gives a reasonably accurate representation of actual build up voltage line curve (ref.(9) and (10). This can be put as

$$U_e(t) = (U_{ec} - U_{e1}) \varepsilon^{t/Te} \quad ---- 3.3.6$$

U_{ec} = ceiling voltage

U_{e1} = initial exciter voltage

T_e = exciter voltage build up time constant depending upon exciter response.

If this voltage given by above equation is applied after a time h, using Heaviside delayed unit function H (t - h) the exciter voltate becomes

$$U_e(t) = U_{e1} - U_{e1}\, H(t-h) + U_{e_e} H(t-h) - (U_{ec} - U_{e1})$$

$$= U_{e_1} + (U_{ec} - U_{e_1})\left\{1 - \epsilon^{-\frac{t-h}{T_e}}\right\} H(t-h) \qquad \ldots 3.3.7$$

Substituting eq. 3.3.7 in eq. 3.3.5

$$\frac{dE}{dt} + \frac{E}{T_B'} = \frac{1}{T_B'}\left[U_{e_1} + (U_{ec} - U_{e_1})\left(1 - \epsilon^{-\frac{t-h}{T_e}}\right) H(t-h) - U_{es}\right]$$

Taking Laplace Transform and putting $(E)_{p=0} = E_1$

$$\left(p + \frac{1}{T_B'}\right)\bar{E} = \frac{1}{T_B'}\left[\frac{U_{e_1} - U_{es}}{p} + (U_{ec} - U_{e_1})\left\{\frac{\epsilon^{-ph}}{p} - \frac{\epsilon^{-ph}}{p + \frac{1}{T_e}}\right\}\right] + E_1 \qquad \ldots 3.3.8$$

$$\text{or}\quad \bar{E} = \frac{1}{T_B'}\left[\frac{U_{e_1} - U_{es}}{\left(p + \frac{1}{T_B'}\right)} + (U_{ec} - U_{e_1})\left\{\frac{\epsilon^{-ph}}{p\left(p + \frac{1}{T_B'}\right)} - \frac{\epsilon^{-ph}}{\left(p + \frac{1}{T_e}\right)\left(p + \frac{1}{T_B'}\right)}\right\}\right] + \frac{E_1}{p + \frac{1}{T_B'}} \qquad \ldots 3.3.9$$

$$\text{or}\quad \bar{E} = \frac{1}{T_B'}\left[(U_{e_1} - U_{es}).T_B'\left\{\frac{1}{p} - \frac{1}{p + \frac{1}{T_B'}}\right\} + (U_{ec} - U_{e_1})\epsilon^{-ph}\left\{\frac{T_B'}{p} - \frac{T_B'}{p + \frac{1}{T_B'}} - \left(\frac{T_e T_B'}{T_e - T_B'}\cdot\overline{\frac{1}{p + \frac{1}{T_e}} - \frac{1}{p + \frac{1}{T_B'}}}\right)\right\}\right] + \frac{E_1}{p + \frac{1}{T_B'}} \qquad \ldots 3.3.10$$

Taking Laplace inverse, we get

$$E(t) = (U_{e_1} - U_{e_s})\left(1 - \epsilon^{-t/T_B'}\right) + (U_{e_c} - U_{e_1})\left\{1 - \epsilon^{-\frac{t+h}{T_B'}} \frac{T_e}{T_e - T_B'}\left(\epsilon^{-\frac{t+h}{T_e}} - \epsilon^{-\frac{t+h}{T_B'}}\right)\right\} + E_1 \cdot \epsilon^{-t/T_B'} \quad \text{.... 3.3.11}$$

$$= (U_{e_c} - U_{e_s}) + \left[E_1 - (U_{e_1} - U_{e_s}) + \frac{U_{e_c} - U_{e_1}}{T_e - T_B'}\, \epsilon^{h/T_B'}\right] \epsilon^{-t/T_B'} - \frac{U_{e_c} - U_{e_1}}{T_e - T_B'}\, \epsilon^{-\frac{t+h}{T_e}} \quad \text{.... 3.3.12}$$

So this equation gives us the variation of internal induced e.m.f and if substituted in the power equation curves with the values of X_d as X_{du} and X_q as X_{qu} since the saturation effect is already considered in E(t) we have

$$P_{(t)} = \frac{E_{(t)} \cdot E_2}{(X_{du} + X_e)} \cdot \sin\theta + \frac{E_2^2}{2}\left\{\frac{X_{du} - X_{qu}}{(X_{du} + X_e)(X_{qu} + X_e)}\right\} \cdot \sin 2\theta \quad \text{--- 3.3.13}$$

$$P_{(t=0)} = \frac{E_{(t=0)} \cdot E_2}{X_{du} + X_e} \cdot \sin\theta_1 + \frac{E_2^2}{2}\left\{\frac{X_{du} - X_{qu}}{(X_{du} + X_e)(X_{qu} + X_e)}\right\} \sin 2\theta_1 \quad \text{---- 3.3.14}$$

where E(t) = E (t = 0) at t = 0

and $\theta = \theta_1$ at t = 0

Power or which in other words is nothing else but the torque (since the synchronous speed is constant) tending to swing the motor would be

$$P_{(t)} - P_{(t=0)}$$

$$= \frac{E_2}{X_{du}+X_e} \cdot \left\{ E_{(t)} . \sin\theta - E_{(t=0)} \sin\theta_1 \right\}$$

$$+ \frac{E_2^2}{2} \cdot \left\{ \frac{X_{du} - X_{qu}}{(X_{du}+X_e)(X_{qu}+X_e)} \cdot \right\} (\sin 2\theta - \sin 2\theta_1)$$

.... 3.3.15

where E(t) is given by equation 3.3.11.

Now the equation of motion of rotor of a salient pole machine can be written as

$$I . \frac{d^2\theta}{dt^2} + T_f . \frac{d\theta}{dt} + K\theta = P_{(t)} - P_{(t=0)}$$

$$= f(\theta, t)$$

all other constants being known from the machine and from the excitor reference.

The values of I_1, T_f and K can be determined from empirical relations available and experimental data and if the above differential equation could be solved to get $\theta = f(t)$ and the plot obtained, the dynamic of the dynamical stability would be analysed and the system would be stable in dynamic equilibrium if the value of θ does not exceed beyond reasonable limits.

CHAPTER – 4

TRANSIENT STABILITY AND THE EFFECT OF SATURATION.

4.1. Transient stability refers to the amount of power that can be transmitted with stability when the system is subjected to an "aperiodic disturbance". The three principle type of disturbances that receive consideration in stability studies are :

(1) Sudden load changes

(2) Switching operations

(3) Faults with subsequent circuit isolations.

The stability of the system under the above heads is discussed as follows :

4.2. (1) Sudden Load Changes :

Load increases can result in transient disturbances that are important from stability point of view if

((i) the total load exceeds the steady state stability limit for specific voltage and circuit reactance conditions.

(ii) The load increase sets up a condition for oscillation that the rotor swings beyond the critical angle from where the recovery would be impossible.

Assuming loss less conditions, during the swing of the rotor due to the transient disturbances from sudden load, changes, equal area criteria can be used to find out if the swing is within the limits of critical angle. Beyond this angle the rotor would not come back and the limit of transient stability would be exceeded.

Fig. 4.1. gives the power load angle characteristic for a synchronous machine with the operation point A delivering load of P_a at a load angle θ_a.

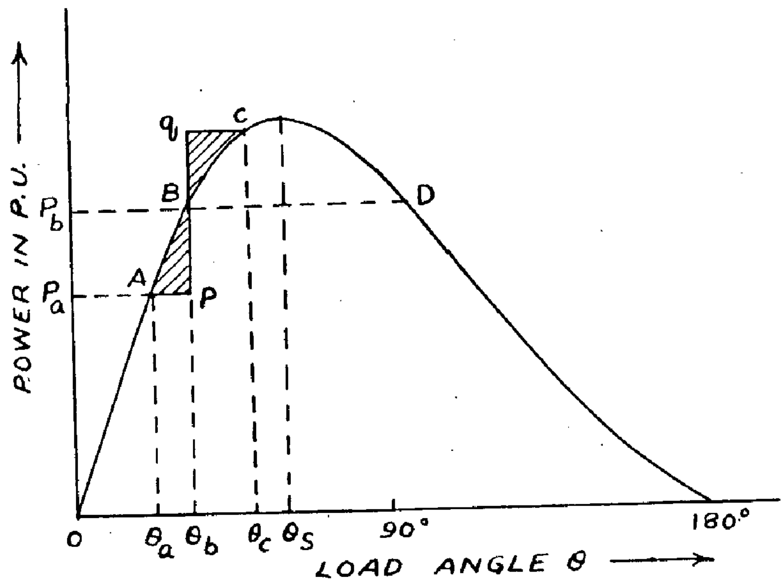

FIG. 4.1.

Steady state Power - Vs Load angle characteristics of a salient pole rotor synchronous machine.

We assume that the input from the prime-mover is suddenly increased from P_a to P_b. The output of the machine is the same as P_a and there is a net difference of power which goes to accelerate the rotor. Depending upon the inertia of the rotating system the rotor takes some time to reach the point B, and because of the kinetic energy stored due to the swing from A to B, the rotor overshoots the point B and reaches the point C such that area A_bB is equal to area B_aC (assuming no losses). The rotor would oscillate between the limits A & C and would ultimately stabilise at the point B because of the losses which would always decay the oscillations.

If the point C remains to the left of the critical point Q, the system is under the limits of transient stability. The critical point is fixed from the new load conditions, the internal voltage of the machine and the reactances.

Mathematically the conditions can be represented as follows

$$I \frac{d^2\theta}{dt^2} + K_d \frac{d\theta}{dt} + K\theta = P_a + (P_b - P_a).\, H(t) - P_\theta \quad ..4.2.1$$

Where P_θ = Load under any angle θ during the swing = $f(\theta)$

P_b = Load under Transient disturbance conditions

$$P_\theta = \frac{E.i.u}{X_e+X_d}\sin\theta + \frac{u^2}{2}\left\{\frac{X_{ds}-X_{qs}}{(X_{ds}+X_e)(X_{qs}+X_e)}\right\}\sin 2\theta \qquad \ldots 4.2.2$$

The equation when solved would result in a plot of θ as a function of time. If the maximum value of θ is more than critical angle θ_c given by the following eq. 4.2.3.

the transient stability limit would be exceeded. The maximum value of P_b can thus be determined

$$P_c = \frac{E_i.u}{X_d}\sin\theta_c + \frac{u^2}{2}\left\{\frac{X_{ds}-X_{qs}}{(X_{ds}+X_e)(X_{qs}+X_e)}\right\}\sin 2\theta_c \qquad \ldots 4.2.3$$

where value of $\theta_c > \theta_0$ is to be adopted

the values of X_{d_s} and X_{q_s} (i.e. corrected) for saturation for a value of Es corresponding to E_i has to be taken as given in equation 2.6.2 to equation 2.6.9

The solution of the differential equation 4.2.1 is given in Appendix I and II for the cases of an undamped and damped rotors. The stability can be determined by a plot of $\frac{d\theta}{dt} - \theta$ which would be a closed curve if $F < F_c$ showing that the system is stable and periodic. If on the other hand $F > F_c$ the curve is open which entails instability. For a value $F = F_c$ the curve is a separatrix, showing the limit of the stability range.

4.3. Switching Operation

Equal area criteria is used to analyse the stability limit due to the switching operation. Two power angle characteristics would be required - one for the conditions before the switching has taken place and another for the conditions after the switching is done. Those curves are marked I and II in Fig.

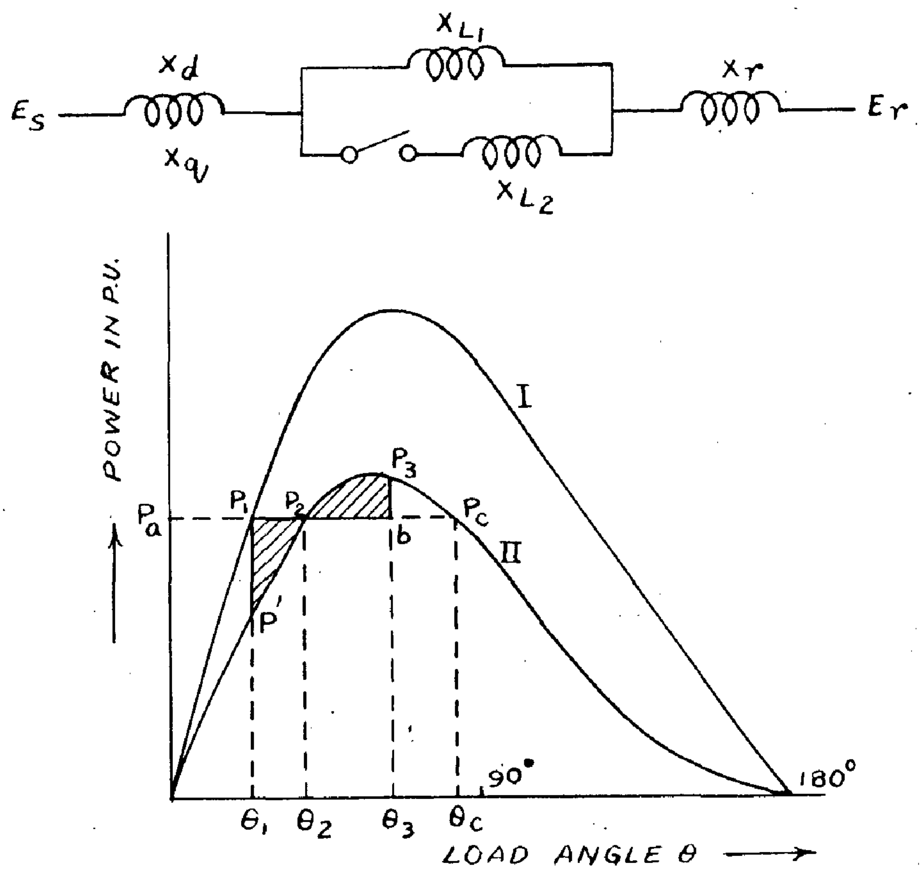

Fig. 4.2.
Stability criteria due to sudden switching operation.

A load P_1 is to be supplied so that the operating point before switching is shown at P_1 on curve-I giving a corresponding load angle of θ_1. After the switching has been done the load is supplied with an operation point P_2 on curve II at a load angle of θ_2.

The change produces an increment power of $P_a - p'$ which will accelerate the rotor to increase its load angle from θ_1 to θ_2. At a load angle of θ_2. At a load angle of θ_2, the rotor will overshoot and reach a load angle of θ_3 such that area P_2P_3b is equal to $P_1P'P_2$. The limit of transient stability is reached when P_3 coincides with P_c, the critical point beyond which the recovery is not possible. Once again it can be expressed mathematically as

$$I \frac{d^2\theta}{dt^2} + T_g \cdot \frac{d\theta}{dt} + k\theta = (P_a - P) \quad \ldots 4.3.1$$

where P_a = Power delivered under initial conditions.

$$P = \frac{E_g \cdot E_a}{X_{d_g} + X_T + \frac{X_{L_1} \cdot X_{L_2}}{X_{L_1} + X_{L_2}}} \sin\theta$$

neglecting the effect of saliency as the external reactance is high as compared to X_d and X_q

We can write down

$$P = k_2 \sin\theta$$

$$\text{where} \quad k_2 = \frac{E_s \cdot E_2}{X_{d_s} + X_2 + \frac{X_{L_1} \cdot X_{L_2}}{X_{L_1} + X_{L_2}}}$$

The value of K_2 will not be much affected by saturation but even then the saturation can be taken into account by calculating X_{d_s} from X_{d_u} knowing the values of K_s at the value of E_g.

The above equation 4.3.1 can be written as

$$I \frac{d^2\theta}{dt^2} + T_f \frac{d\theta}{dt} + E\theta + K_2 \sin\theta = P_a$$

which when solved would result in a plot of $\theta = f(t)$. The maximum value of θ when put equal to θ_c would result in a condition of limit of transient stability. θ_c can be determined from the following equation.

$$P_a = \frac{E_g E_r}{X_{d_g} + X_r + \frac{X_{L1} X_{L2}}{X_{L1} \quad X_{L2}}} \cdot \sin\theta$$

$$\theta_c > 90^0$$

4.4. Transient disturbances under fault conditions and subsequent circuit isolations:

For such disturbances, three or more circuit conditions are to be considered. Let us say that the power - load angle characteristics under the three conditions namely before the fault, during the fault and after the removal of the fault are given by curves I, II and II respectively. The power delivered under condition I is P such that the operation is at point a, the load angle being θ_1

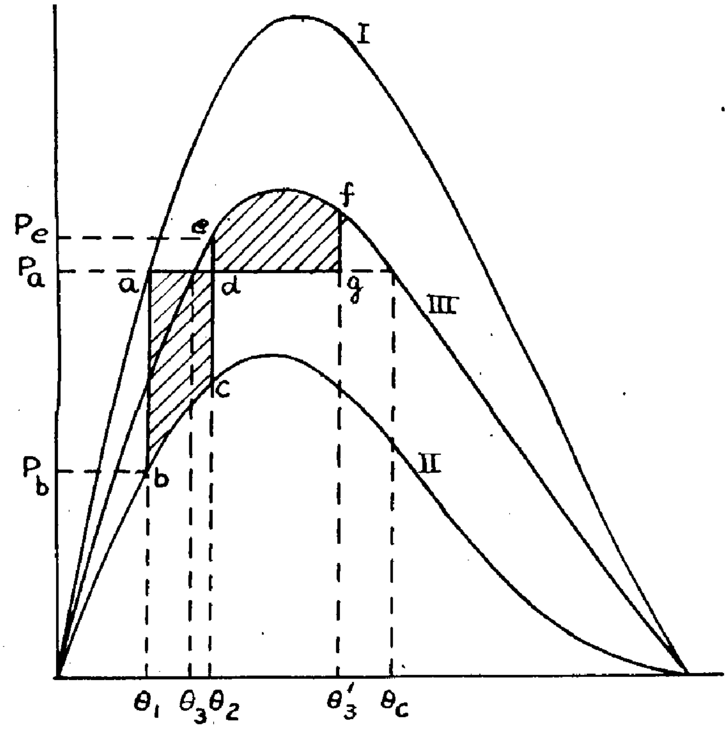

FIG. 4.3. STABILITY CRETERIAN UNDER TRANSIENT DISTURBANCE ON FAULTS

When the fault occurs, the load that can be supplied from conditions under II is only P_b, with the result that generator accelerates and the load angle increases, till θ_2 when the fault gets isolated and power supplied corresponds to P_e. The rotor swings to the point f (as explained earlier) such that area defg is equal to the area abcd assuming no losses in the swing. The limit of transient stability is represented by the conditions when point f coincides with point m and the highest value of θ reaches θ_c

If the severity of the fault increases as is indicated by the reduction in amount of power that can be transmitted during the fault conditions, or if the duration of the fault is increased as is indicated by a larger θ_2, or if the power angle diagram for the final condition has a lower maximum, the largest angle during the system oscillation is increased beyond θ'_3 and under some conditions would reach the critical angle θ_c, for the transmitted power under the final circuit conditions. Mathematically the limit of transient stability can be determined as follows:

$$I.\frac{d^2\theta}{dt^2} + T_f\frac{d\theta}{dt} + k\theta = (P_a - P) \quad \ldots\ldots\ldots 4.4.1$$

where $P = P_2 \sin\theta$ for $\theta_1 < \theta \leq \theta_2$

$P = P_3 \sin\theta$ for $\theta > \theta_2$

P_2 and P_3 refer to the maximum power that can be delivered under conditions II and III as specified earlier.

$$I.\frac{d^2\theta}{dt^2} + T_f\frac{d\theta}{dt} + k\theta = P_a - \left\{P_2 \sin\theta + (P_3 \sin\theta - P_2 \sin\theta).H(t-T)\right\}$$

where T = time required to clear up the fault.

The equation 4.4.2 can be solved knowing the numer-

[illegible]

numerical values of I, and T_f as given in appendix II. The plot of θ as a function of time or $\frac{d\theta}{dt}$ as a function of θ can indicate the stability of the system.

If the maximum value of θ is less than θ_c the latter given by

$P_a = P_3 \sin \theta_c$

and θ_c being greater than $\pi/2$

the system shall be in a position to stand the transient disturbance.

4.5. Torque Angle characteristics of a salient pole synchronous machine during transient period.

The vector diagram for the transient state is well known to be as follows

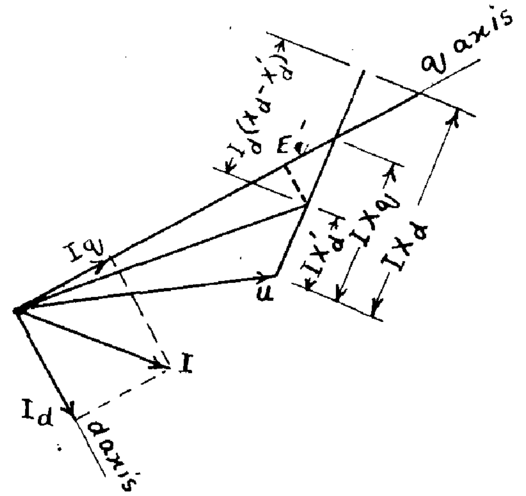

FIG. 4.4. VECTOR DIAGRAM UNDER TRANSIENT STATE.

Proceeding the same way as in equation 2.2.1 to equation 2.2.4

$$P = u\, I_q \cdot \cos\theta + u \cdot I_d \cdot \sin\theta \quad \ldots\ldots\ldots\ldots \quad 4.5.1$$

$$= u \cdot \frac{u \sin\theta}{X_q} \cdot \cos\theta + u \cdot \frac{E_q' - u\cos\theta}{X_d'} \cdot \sin\theta \quad \ldots\ 4.5.1$$

$$= \frac{E_q' \cdot u}{X_d'} \cdot \sin\theta + \frac{u^2}{2} \cdot \left\{ \frac{X_d' - X_q}{X_d' \cdot X_q} \right\} \cdot \sin 2\theta \quad \ldots\ 4.5.2$$

Since the speed is constant at r_s.

Torque in p.u $= \frac{E_q' \cdot u}{X_d'} \cdot \sin\theta + \frac{u^2}{2} \cdot \left\{ \frac{X_d' - X_q}{X_d' \cdot X_q} \right\} \sin 2\theta \quad \ldots\ 4.5.3$

with an external reactance X_e before the infinite bus the torque equation would change to

Torque in p.u =

$$\frac{E_q' \cdot u}{X_d' + X_e} \cdot \sin\theta + \frac{u^2}{2} \left\{ \frac{X_d' - X_q}{(X_d' + X_e)(X_q' + X_e)} \right\} \sin 2\theta \quad \ldots\ 4.5.4$$

Torque in p.u under saturated condition =

$$\frac{E_q' \cdot u}{(X_{ds}' + X_e)} \cdot \sin\theta + \frac{u^2}{2} \cdot \left\{ \frac{X_{ds}' - X_{qs}}{(X_{ds}' + X_e)(X_{qs} + X_e)} \right\} \cdot \sin 2\theta \quad \ldots\ 4.5.5$$

The value of X_d' can be written from the equivalent circuit as shown in fig. 4.5.

$$X_d' = X_{al} + \frac{X_{el} \cdot X_{ad}}{X_{el} + X_{ad}} \quad \ldots\ 4.5.6$$

$$= a + \frac{b}{k_s} + \frac{X_{el} \cdot c/k_s}{X_{el} + c/k_s} \quad \ldots\ 4.5.7$$

Knowing the values of constants a, b and c from the equation 2.6.2 to equation 2.6.9, the value of X_d' can be found for any particular saturation factor or for any particular internally induced voltage. The saturation effect in transient stability can be accounted by substituting proper values of X_{d_s} and X_{q_s} in 4.55. Actual calculations based on test data of a small salient pole synchronous machine in laboratory is given in Section 5.9.

The plot of the equation 4.5.5 would lie above steady state load angle characteristic as $X_q' < X_q$ and the limit of transient state stability can be obtained for any particular load angle.

4.6. Effect of saturation on Transient stability.

Machine saturation affects the transient stability by reducing the value of effective reactance X_q'. The variations of X_q' with the saturation factor are shown figure 5.10, Calculated from the constants of a small laboratory machine and the effect of saturation on transient stability is estimated in Chapter - V.

CHAPTER - V

EXPERIMENTAL TEST RESULTS

5.1. The chapter deals with the experimental determination of the machine constants for a small salient pole synchronous machine and the calculationss of its stability limits by the methods discussed in chapter - II

The tests were carried out on a 3-phase Δ/Y type, 230/400 volts 7.5/4.35 Amps, 3 KVA, 0.8 Pf 1500 r.p.m. 50 cycles rated salient pole synchronous machine manufactured by Elcktromotorn Werke Kaiser, Berlin. The machine used an excitation voltage of 230 volts and was coupled to a 230 volts 19.3 Amps 5 H.P. 1500 r.p.m. motor.

The open circuit and short circuit characteristics are shown in Fig. 5.1. Zero p.f, full load rated voltage point is also obtained to determine the Potier's reactance and is found to be 15.625%

The value of X_{du} found from air gap line and S.C.C. is 1.084 p.u and the value of X_{qu} from slip test was determined to be 0.415 p.u. The value of X_{qu} is quite low but the result was confirmed by negative excitation test and the reluctance power available.

The values of Ks at any particular voltage is the ratio of field current required to induce this voltage on O.C.C. to that required in air gap line. (refer defination of Ks in section 1.6). The variations of Ks with voltages induced are plotted as shown in Fig. 5.1.

Equations 2.6.2 to 2.6.9 give us the method of calculating X_{ds} and X_{qs} at any value of Ks or for that matter at any value of voltage or field current. The variations of X_{ds} and X_{qs} are shown in fig.5.1 and the values at the rated voltage are found to be 0.909 p.u and 0.36 p.u respectively. The values of constants in equations 1.7.29 and 1.7.30 when solved is found to be

$$a = 0.036$$

$$b + c = 0.944$$

$$b + d = 0.327$$

5.2. Calculations of Pull out Powers

The calculations of pull out power are based on rated initial conditions i.e. the machine is assumed to supply unit current at unit voltage at a P.f of 0.8 lagging. The pull out power is calculated by all the methods described in section 2.4 to 2.7. The maximum power is also calculated without taking saturation into account in section 2.7 and results are compared in section 2.8 to show the effect of saturation on steady state stability.

5.3. Short circuit Ratio Method:

As per the equation 2.4.4

$$\vec{E}_1 = \vec{E}_t + \vec{I}_d \left(\frac{1}{S.C.R}\right) + \vec{I}_q.(X_q) \quad \ldots.$$

$$\frac{1}{S.C.R.} = \frac{1}{1.09} = 0.918$$

The vectorial summation required has been made in figure 5.2. and is given by 04', the value being 1.762 p.u. Substituting these values in equation 2.4.5, we have

$$P = \frac{1.762 \times 1.0}{\left(\frac{1}{1.09}\right)} \sin \theta + \frac{(1.0)^2}{2} \; \frac{\frac{1}{1.09} - 0.36}{\frac{1}{1.09} \times 0.36} \sin 2\theta$$

and the maximum power

$$P_{max} = \frac{1.762 \times 1.0}{\frac{1}{1.09}} \sin \theta_s + \frac{(1.0)^2}{2} \; \frac{\frac{1}{1.09} - 0.36}{\frac{1}{1.09} \times 0.36} \sin 2\theta_s$$

where $\frac{\cos 2\theta_s}{\cos \theta_s} = -\left\{\frac{0.918 - 0.36}{0.36}\right\} \cdot \frac{1}{1.762}$

from equation 2.2.7

S = 0,313 p.u.
Eint = 1,825 p.u.
E_x = 2,138 p.u.
LOAD ANGLE : 14,4°
I_q = 0,629 p.u.
I_d = 0,777 p.u.
IX_{qu} = 0,415 p.u.
IX_{du} = 1,08 p.u.

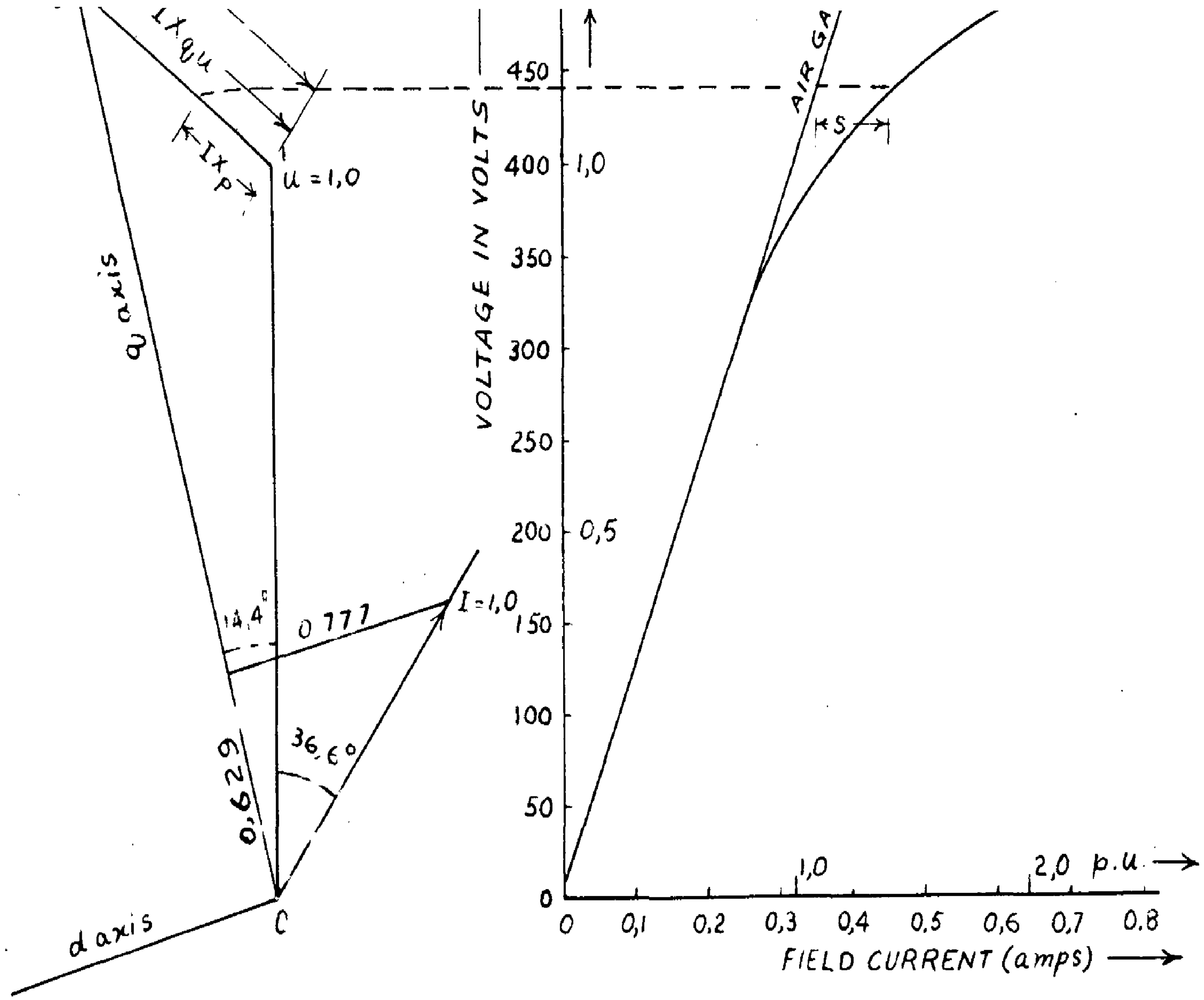

FIG. 5,3.

or $\theta_s = 58.62^0$

$$P_{max} = \frac{1.762 \times 1.0}{\frac{1}{1.09}} \sin 58.62^0 + \frac{(1.0)^2}{2} \left\{ \frac{\frac{1}{1.09} - 0.36}{\frac{1}{1.09} \times 0.36} \right\} \sin 117.24^0$$

$= 2.382$ p.u

5.4. Potier Reactance Method:

The desciption of the method is given in section 2.5. The following are the constants known to us under initial operating conditions.

$E_t = 1.0$ p.u $= 1.0 + 0j$

Load current $= 1.0$ p.u at p.f. 0.8 lag.

$= -36.6^0$

$X_{du} = 1.084$

$X_{qu} = 0.415$

Referring to the vector diagram given in fig. 5.2. we got :

$E_X = 2.318$

$E_1 = 1.325$

$S = 0.313$

Load angle $= 14.4^0$

$I_q = 0.629$

$I_d = 0.777$

Under pull out conditions, the load angle would be θ_s as defined by equation 2.2.7 Hence

VECTOR DIAGRAM UNDER INITIAL OPERATING CONDITIONS OF FULL LOAD & RATED P.F. POTIER TRIANGLE METHOD

S = 0,313 P.U.
Eint = 1,825 p.u.
Ex = 2,138 p.u.
LOAD ANGLE = 14,4°
Iq = 0,629 p.u.
Id = 0,777 p.u.
IXqu = 0,415 p u
IXdu = 1,08 p.u.

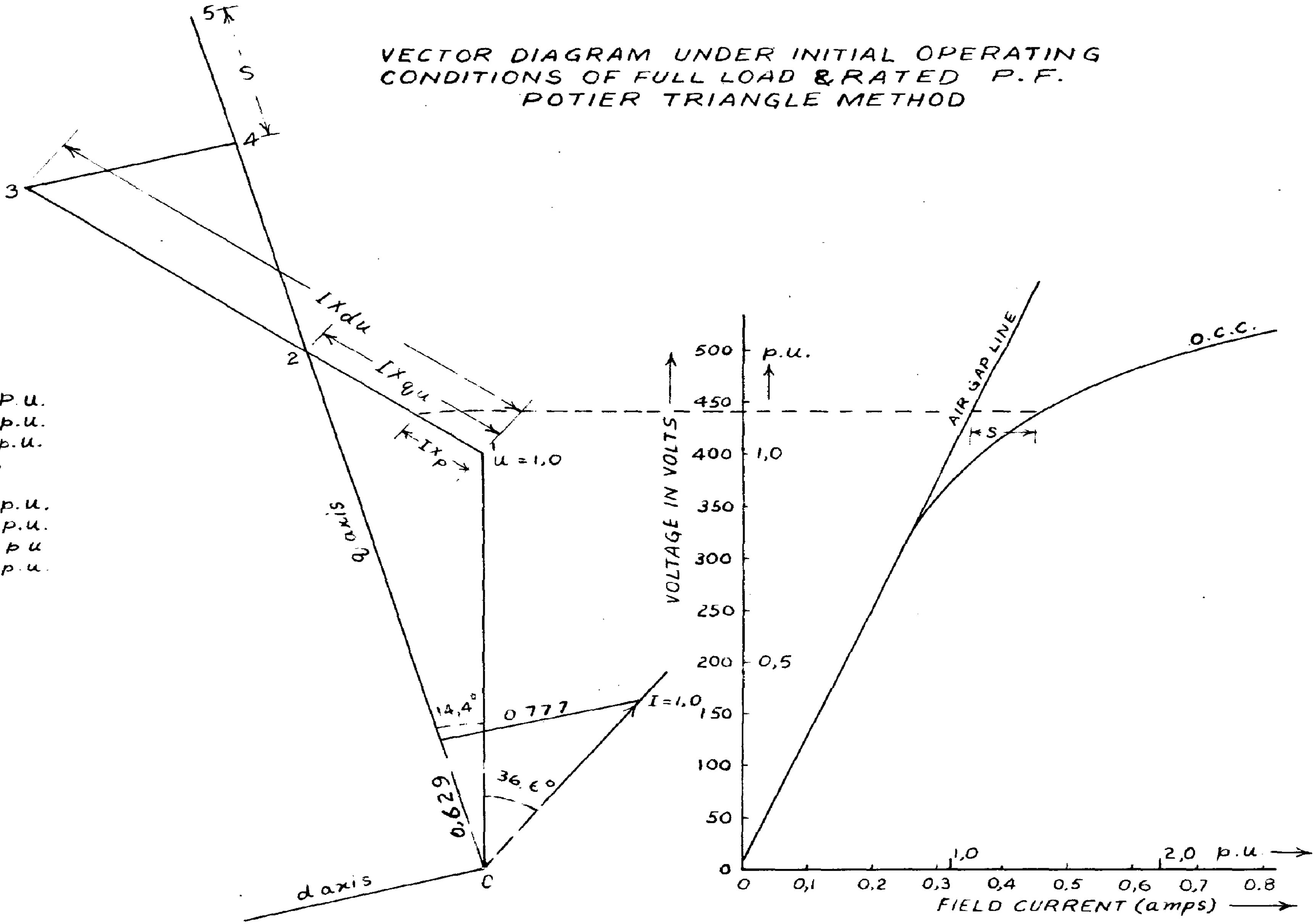

FIG. 5.3.

$$\frac{\cos 2\theta_s}{\cos \theta_s} = -\frac{1.084 - 0.415}{0.415} \cdot \frac{1}{E_1} = -\frac{1.61}{E_1}$$

For Ist trial let us assume S' = 0.1 so that

$E_1' = 2.318 - 0.1 = 2.218$

$$\frac{\cos 2\theta_s}{\cos \theta_s} = -\frac{1.61}{2.218} = 0.726$$

This gives $\theta_s = 56.78^0$

$E_T' = 1.0 \angle -56.78^0$

$$I_q' = \frac{\sin 56.78}{X_q} = \frac{0.8366}{0.413} = 2.015$$

$$I_d' = \frac{2.218 - 0.503}{1.084} = \frac{1.715}{1.084} = 1.584$$

$$I' = \left\{(I_d')^2 \quad (I_q')^2\right\}^{1/2}$$

$$= 2.55 \angle -38.15^0$$

$$E_T' = 1.0 \angle -56.78 + 2.55 \angle -38.15 \times 0.1625 \angle 90^0$$

$$= 0.954$$

This value of E_T' exactly gives S' = 0.1, the value we started with. The procedure will have to be repeated if the value of S' originally assumed and ultimately found are not equal.

$$P = \frac{2.218 \times 1.0}{1.084} \cdot \sin 56.78^0 + \frac{(1.0)^2}{2} \left\{ \frac{0.669}{1.084 \times 0.415} \right\} \sin 113.56^0$$

$$= 2.392 \text{ p.u}$$

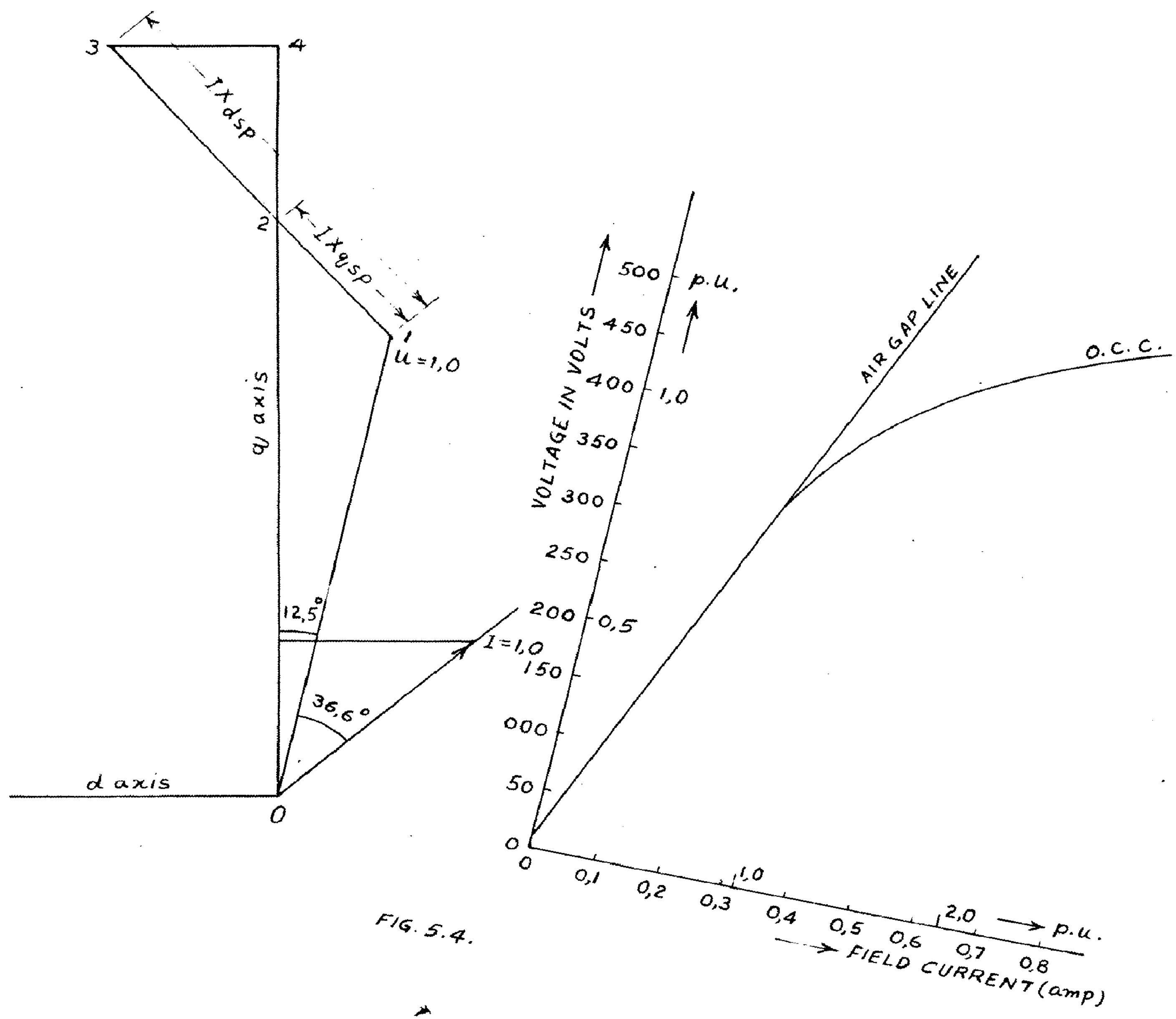
3
4
IXdsp
2
IXqsp
1
U=1,0
q axis
12,5°
I=1,0
36,6°
d axis
0
VOLTAGE IN VOLTS
500
450
400
350
300
250
200
150
000
50
0
p.u.
1,0
0,5
AIR GAP LINE
O.C.C.
0
0,1
0,2
0,3
1,0
0,4
0,5
0,6
2,0
0,7
0,8
p.u.
FIELD CURRENT (amp)

FIG. 5.4.

5.5 Synchronous Reactance Method

The internally induced e.m.f is determined by using the saturated values of X_{ds} and X_{qs} at the voltage behind the Potier's reactance as shown in figure 5.4

The value of internally induced e.m.f is observed to be 1.62 p.u. Using equation 2.6.10 we have

$$P = \frac{1.62 \times 1}{0.828} \cdot \sin\theta + \frac{(1.0)^2}{2} \cdot \left\{ \frac{0.828 - 0.334}{0.828 \times 0.334} \right\} \cdot \sin 2\theta$$

Maximum power is obtained at angle θ_m given by :

$$\frac{\cos 2\theta_m}{\cos \theta_m} = -\frac{0.494}{0.334} \times \frac{1}{1.62} \quad \text{refer equation 2.2.6}$$

$$= 0.914$$

$$\theta_m = 59^{\circ}$$

$$\therefore P = \frac{1.62 \times 1}{0.828} \cdot 0.8572 + \frac{(1.0)^2}{2} \cdot \left\{ \frac{0.828 - 0.334}{0.828 \times 0.334} \right\} \times 0.8829$$

$$= 1.675 + 0.788$$

$$= 2.463 \text{ p.u}$$

5.6 Kapp's diagram method

The method of calculation has been discussed in section 2.7 and details out the procedure. The following calculations are required before the graphical solution for E_1 given in figure 5.5 can be started.

$$\frac{C_q}{C_d} = \frac{X_{aq}}{X_{ad}} \qquad \text{refer equation 1.7.16 and 1.7.17}$$

$$= \frac{X_q - X_{al}}{X_d - X_{al}} \qquad \text{--------} \qquad 5.6.1$$

$$= \frac{c/K_s}{d/K_s} \qquad \text{refer equations 1.7.27 and 1.7.28}$$

$$= c/d$$

Hence the ratio C_q/C_d is independent of saturation factor Ks.

X_q at unit voltage = 0.36

X_d at unit voltage = 0.91

X_{al} at unit voltage = 0.156

Putting the above values of X_{al}, X_d and X_q in equation 5.6.1, we have

$$\frac{C_q}{C_d} = \frac{0.36 - 0.156}{0.91 - 0.156} = 0.271$$

FA_a = (0.37 - 0.065) on amps scale

$$FA_a \frac{C_q}{C_d} = (0.37 - 0.065) \cdot 0.271$$

$$= 0.0832$$

Angle ψ as observed in vector diagram of fig. 5.5 is 49.5°

FIG. 5.5. CALCULATIONS OF INTERNALLY INDUCED E.M.F. FOR A SALIENT POLE SYN. MACHINE BY KAPP'S METHOD

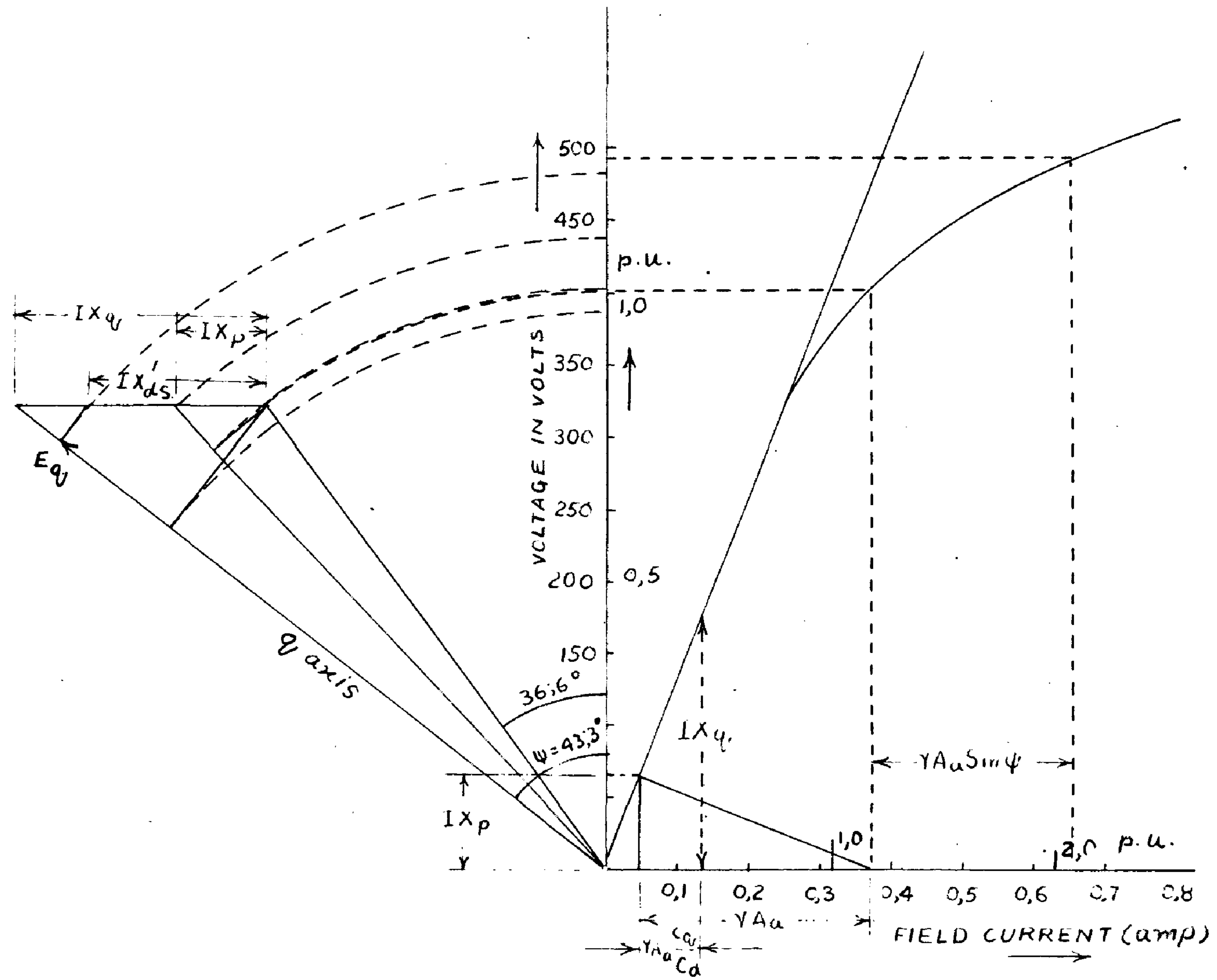

FIG. 5.5.

$VA_a \sin\varphi = 0.325 \times 0.6884 = 0.223$

E_1 as determined from O.C.C. is 492.5 volts or 1.228 p.u voltage behind the transient reactance is 438 volts or 1.095 p.u the values of X_{d_s} and X_{q_s} being 0.765 and 0.31 p.u respectively as observed from fig. 5.1

Using this data in equation 2.7.1 we have

$$P_{max} = \frac{1.228 \times 1.0}{0.765} \sin\theta_s + \frac{(1.0)^2}{2} \cdot \left\{ \frac{0.765 - 0.31}{0.765 \times 0.31} \right\} \cdot \sin 2\theta_s$$

$$\text{Where } \frac{\cos 2\theta_s}{\cos\theta_s} = - \frac{0.765 - 0.31}{0.31} \times \frac{1}{1.228}$$

$$\text{or } \theta_s = 61.88^\circ$$

$$\text{Hence } P_{max} = \frac{1.228 \times 1.0}{0.765} \sin 61.88^\circ + \frac{1}{2} \cdot \left\{ \frac{0.765 - 0.31}{0.765 \times 0.31} \right\} \cdot \sin 123.76^\circ$$

$$= 2.212 \text{ p.u}$$

5.7 Maximum Power without taking saturation into consideration.

It is interesting to determine the pull out power without taking saturation into consideration in order to give an estimate of the effect of saturation on the steady state stability of the synchronous machine.

Internal voltage using X_{du} and X_{qu} is found from figure 5.3. as is estimated as 1.825 p.u

$$P_{max} = \frac{1.825 \times 1.0}{1.084} . \sin\theta_s + \frac{(1.0)^2}{2}\left\{\frac{1.084 - 0.415}{1.084 \times 0.415}\right\} \sin 2\theta_s$$

Where

$$\frac{\cos 2\theta_s}{\cos\theta_s} = \frac{1.084 - 0.415}{0.415} \times \frac{1}{1.825}$$

$$\text{or } \theta_s = 68.1^0$$

Therefore

$$P_{max} = \frac{1.825 \times 1.0}{1.084} \sin 68.1^0 + \frac{(1.0)^2}{2} . \left\{\frac{1.084 - 0.415}{1.084 \times 0.415}\right\} . \sin 136.2^0$$

$$= 2.096 \text{ p.u}$$

Hence the pull out power the synchronous machine without considering the saturation effect turns out to be only 2.098 p.u

5.8. Critical study of the various methods used to take saturation into consideration.

The results of the pull out power obtained is shown in Table 5.1 and shows an increase in stability which gets affected due to saturation.

TABLE -5.1. showing the pull-out power obtained by various methods:

Sl.No.	Method	Pull out power (p.u.)
(1)	Short circuit Method ...	2.332
(2)	Potier's Method ...	2.392
(3)	Synchronous reactance Method	2.463
(4)	Kapp's diagram Method ...	2.212
(5)	Results without considering Saturation ...	2.098
(6)	Pull out powers of the test machine by an actual load test from initial rated conditions ...	2.28

The pull out power for the machine under test was actually determined by an experiment by loading the machine as a motor from the initial conditions of unit voltage, unit current at 0.8 P.F. overexcited for which the calculations have been made by various method. The pull out power is found to be 2.28 p.u.

Although the result in method 4 listed in Table 5.1 compares some what different from that given in method (1), (2) and (3), yet the method is most logical and gives the result nearest to the actual stability limit.

Short circuit method takes saturation effect only to a limited extent as given by equation 2.4.3.

The Potier's method estimates a too high value of internal voltage as the saturation effect S is added linearly for which we do not have any justification. This objection is also valid for the synchronous reactance method.

Kapp's diagram method determines the internally induced e.m.f. on the open circuit characteristics and takes saturation into account from point to point. As a matter of fact we do not have any justification for adding up saturation linearly to induced e.m.f. as is done in Potier's method. Strictly speaking we must operate on O.C.C. for all such additions in a non-linear way. The closest approximation would be to add the field currents linearly and to observe the voltage on O.C.C. This would automatically result in voltages getting added up in a non-linear way as required.

The effect of saturation on steady state stability is very well revealed from table 5.1 and shows a definite increase in the steady state stability.

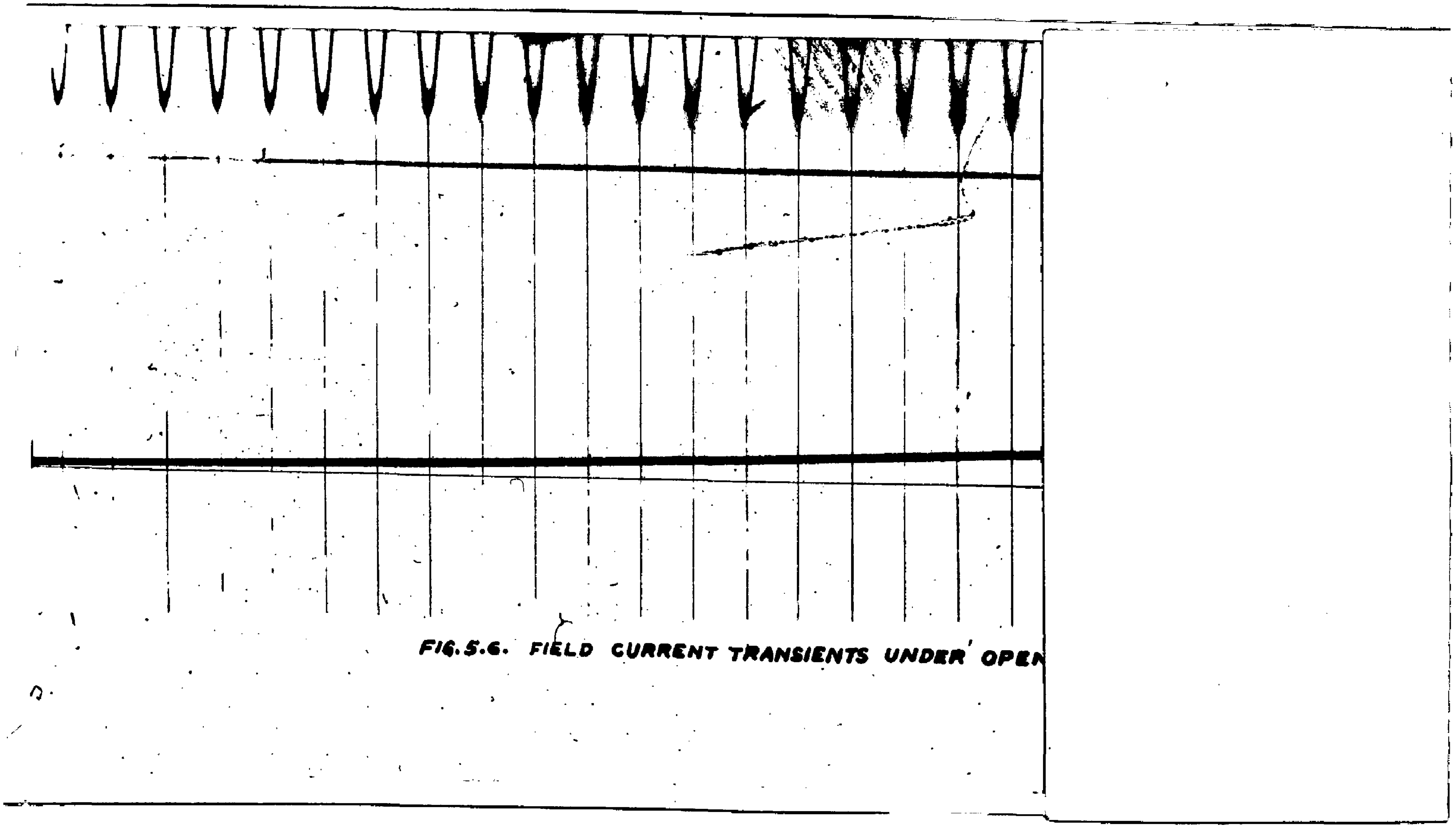

FIG. 5.6. FIELD CURRENT TRANSIENTS UNDER OPEN

FIELD CURRENT UNDER OPEN CIRCUITED ARMATURE

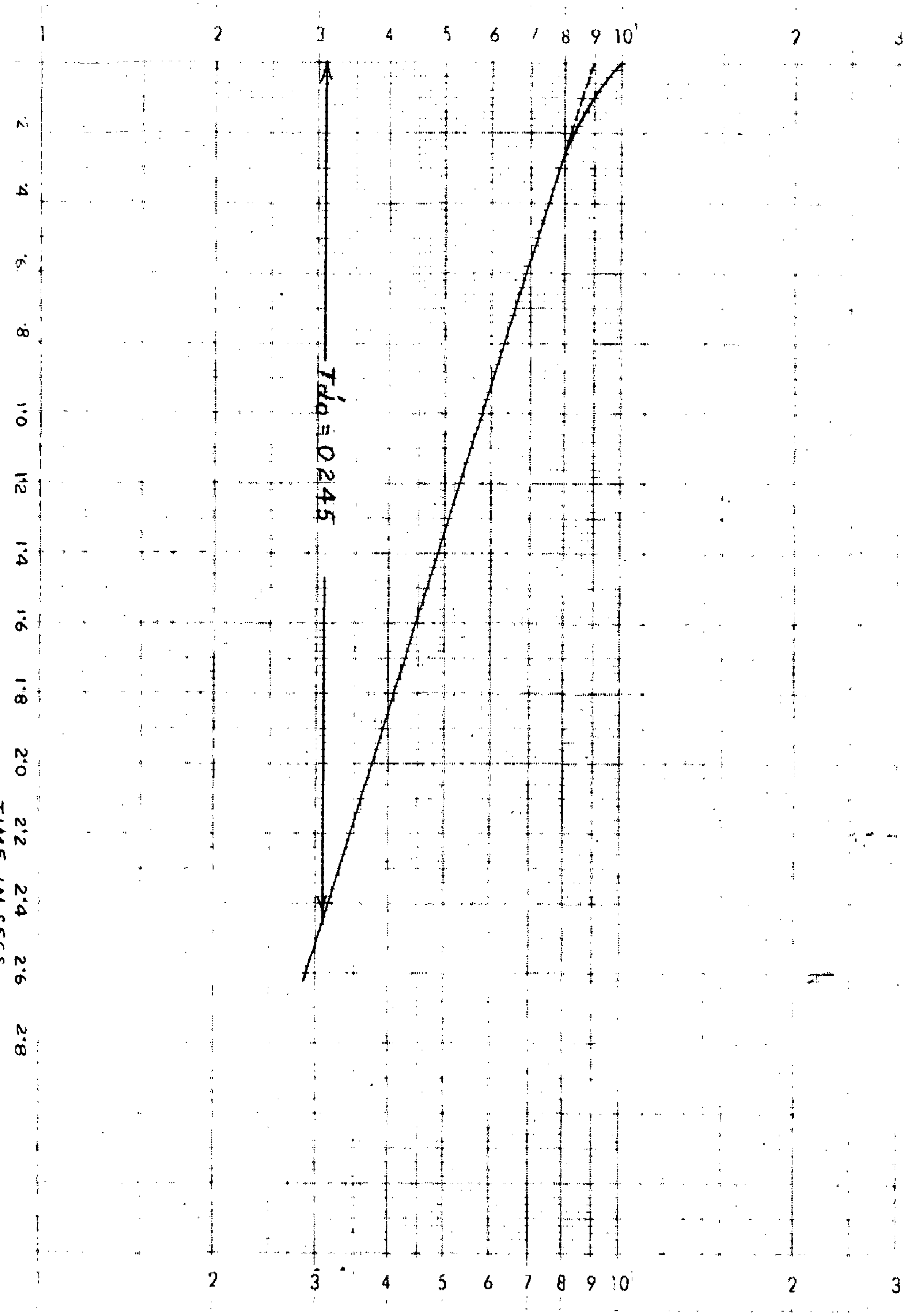

FIG. 5·8.

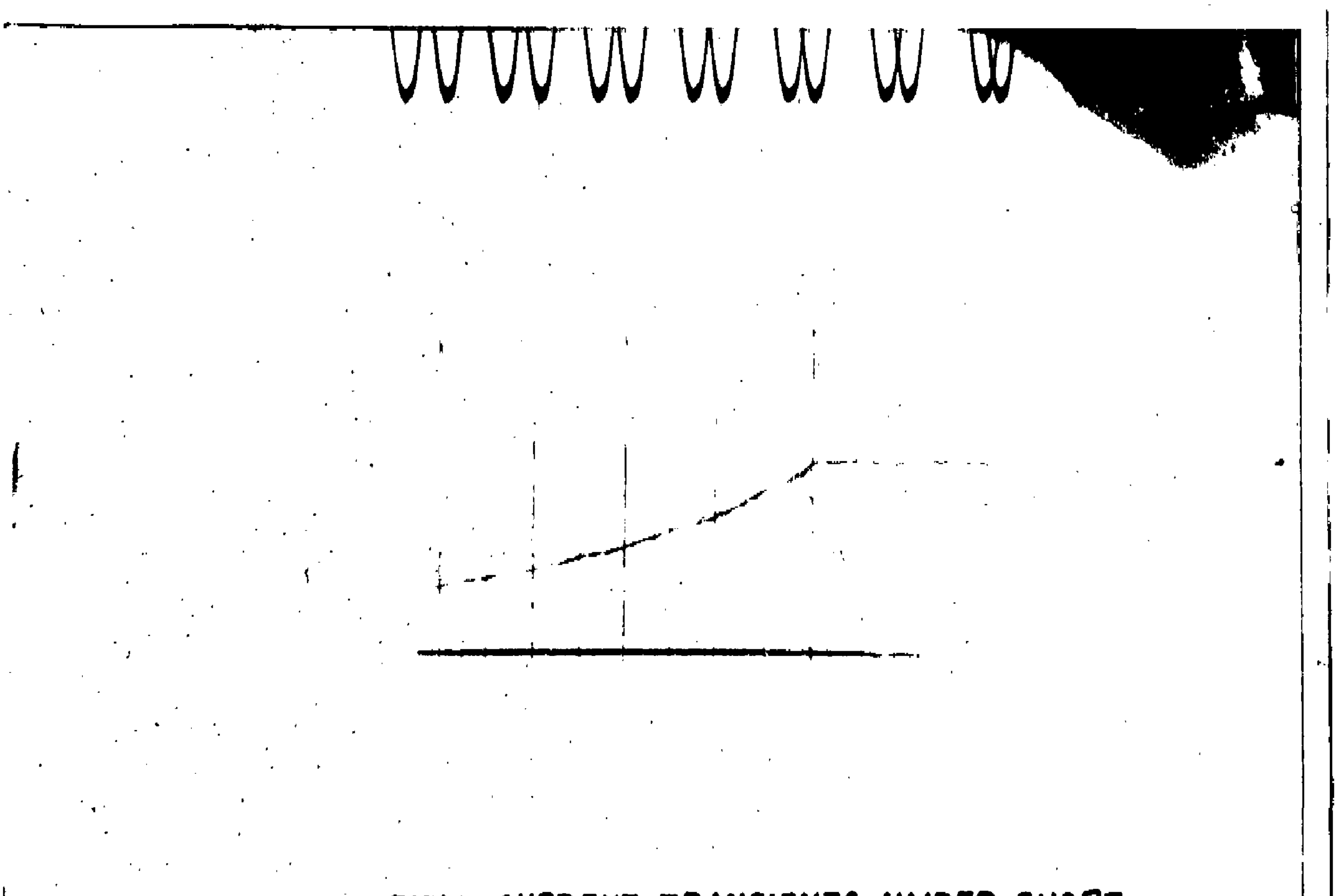

FIG.5.7. FIELD CURRENT TRANSIENTS UNDER SHORT CIRCUITED ARMATURE.

FIELD CURRENT UNDER OPEN CIRCUITED ARMATURE

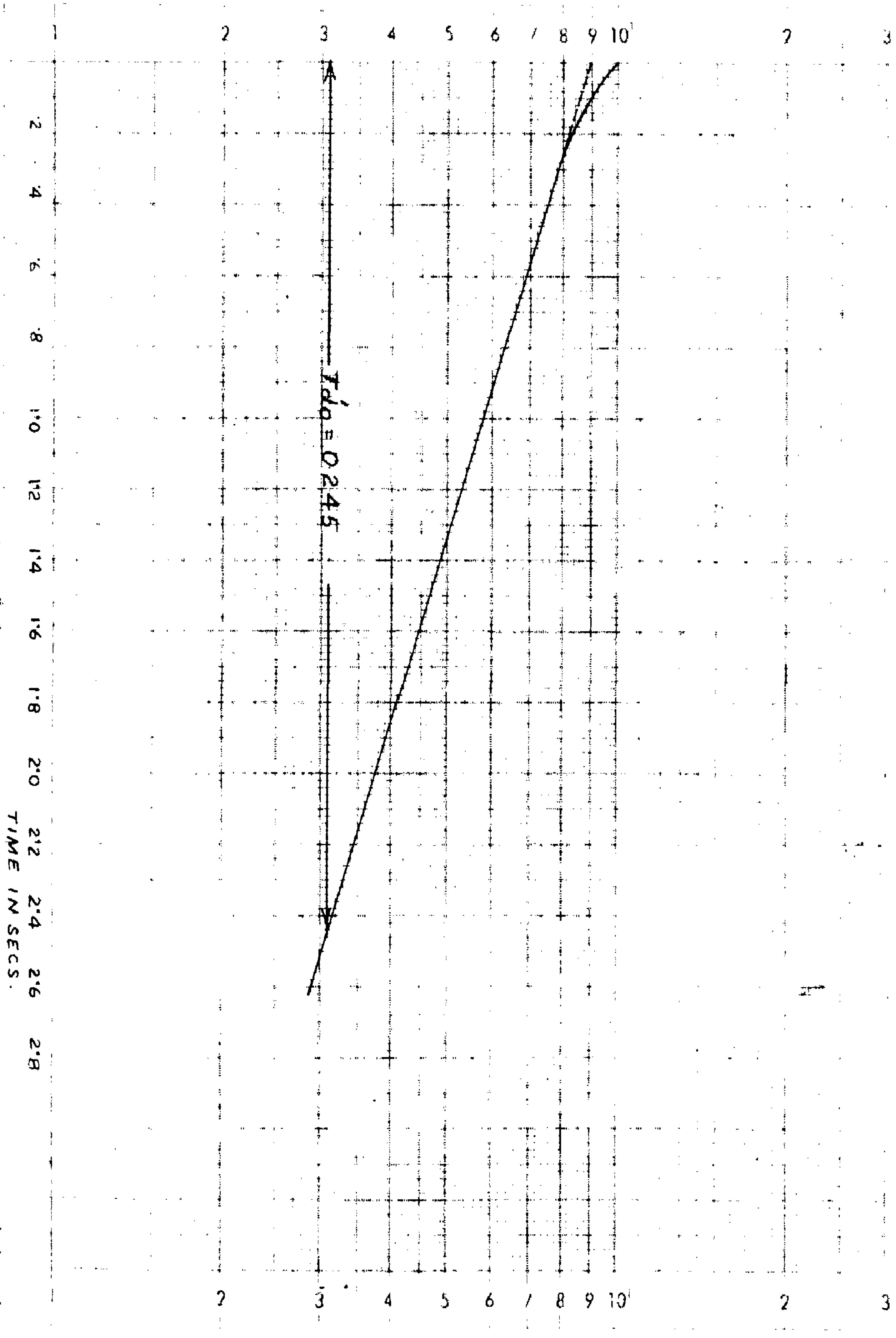

FIG. 5·8.

FIELD CURRENT UNDER SHORT CIRCUITED ARMATURE

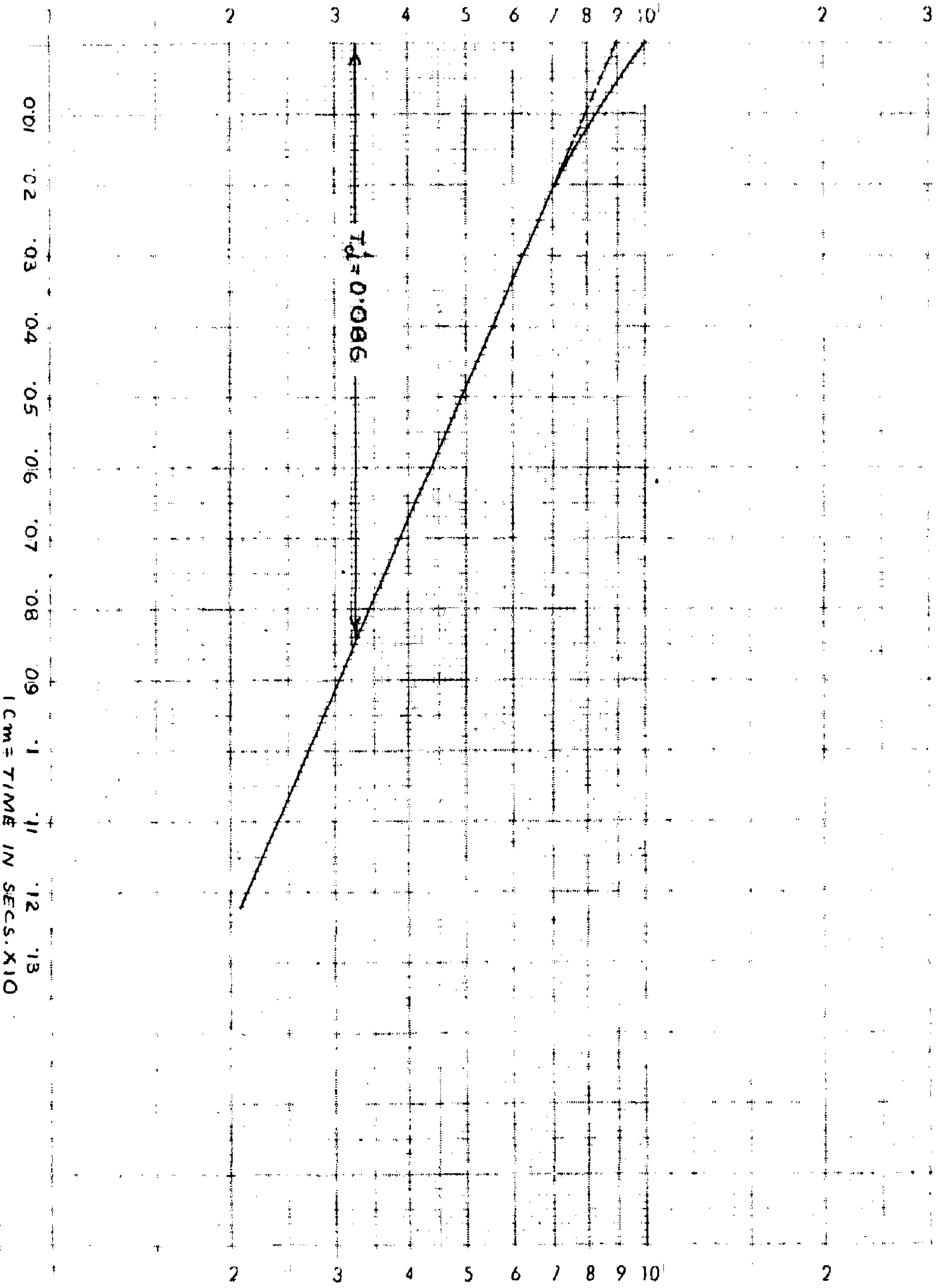

FIG. 5.9.

VARIATION OF X_d' WITH SATURATION FACTOR FOR SYN. M/C UNDER TEST

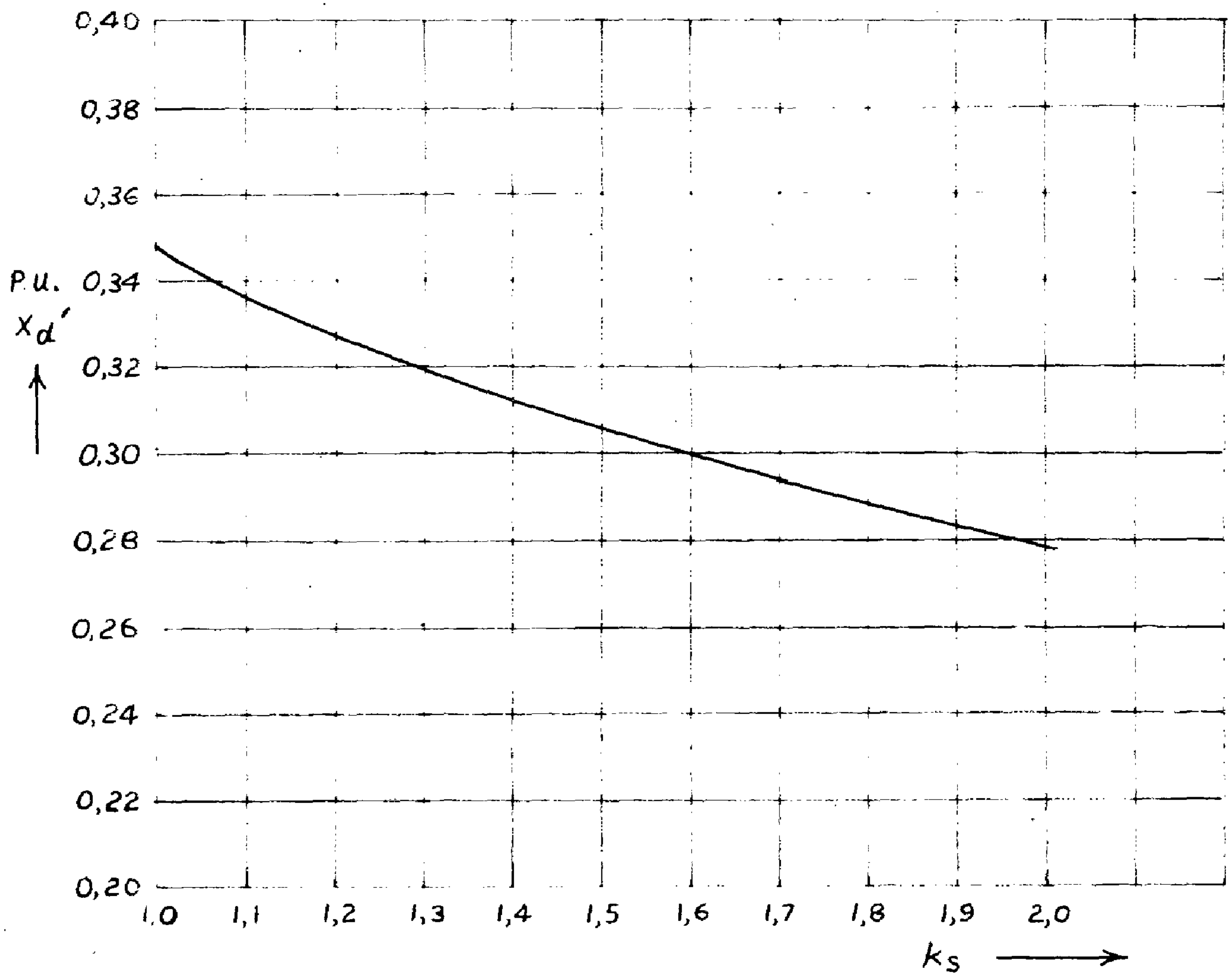

FIG. 5.10.

5.9. Effect of saturation on transient stability:

The value of Xd' can be calculated from the measurements of armature open circuit field time constant and armature short circuit transient time constant as shown in figures 5.6 to 5.9 as follows:

$$Xd'_u = \frac{T_d'}{T_{do}'} \; xd_u$$

$$= \frac{0.085}{0.245} \times 1.804 = 0.347$$

This is the unsaturated value of X_d' as the times constants were measured at a voltage in the St. line portion of O.C.C.

From equation 1.1.3

$$Xd' = X_{al} + \frac{X_{al} \cdot X_{ad}}{X_{al} + X_{ad}}$$

Knowing the values of constants a, b, c and d from equations 2.6.1 to 2.6.9 and found out in section the value of Xd'_s at any saturation factor can be determined.

The max^m. power output can be determined by substituting the proper values of Xd'_s, X_{qs} and voltage behind the transient reactance in equation 4.5.5 illustrated as under:

Voltage behind the potier's reactance (Fig.5.5) = 1.1 p.u

Saturation factor for 1.1 p.u (Fig.5.1) = 1.20

X_d' for K_s = 1.39 (Fig. 5.10) = 0.314

Voltage behind the transient reactance =(Fig.5.5)

= ~~0.964~~ 1.2 p.u.

X_q for voltage = 1.1 (Fig 5.1) is 0.325 p.u.

$$P = \frac{1.2 \times 1.0}{0.314} \sin\theta + \frac{(1.0)^2}{2} \cdot \left\{ \frac{0.314 - 0.325}{0.314 \times 0.325} \right\} \cdot \sin 2\theta$$

$$= 3.82 \sin\theta - 0.01078 \sin\theta$$

$$P_{max} \approx 3.82 \text{ p.u.}$$

5.10 Pull out power under transient conditions without saturation considerations

Using the unsaturated values of X_d' and X_q in equation

Voltage behind transient reactance = 1.231 pu

$$P = \frac{1.231 \times 1.0}{0.347} \cdot \sin\theta + \frac{(1.0)^2}{2} \left\{ \frac{0.347 - 0.41}{0.347 \times 0.41} \right\} \sin 2\theta$$

$$P_{max} \approx 3.56 \text{ p.u.}$$

Sections 5.9 and 5.10 show that the effect of saturation is quite important and cannot be neglected to get an accurate figure for the stability, limit.

C O N C L U S I O N S

The pull out power of a synchronous machine is considerably affected by the saturation occurring in the machine. The fact is revealed by the comparison of the figures in Table 5.1 in which the value of stability limit is found by an actual test as 2.28 p.u. as against the value obtained without taking saturation into account is only 2.098 p.u. The closest approximation of saturation consideration is obtained by Kapp's diagram method because of the calculations made adhere most closely to the O.C.C. as explained in section 5.8. Hence taking saturation into consideration in a proper manner is important to get at the nearest value to the actual pull out figure for any particular machine.

The saturation effect on the constants of the machine are very important as detailed out in theoretical considerations of Chapter I. Except for a certain portion of armature leakages, (slot leakages, tooth top leakages and end winding leakages) all the other armature circuit constants are inversely proportional to the saturation factor. This in turn affects the value of X_d and X_q so as to vary these machine constants as shown in figure 5.1 for the machine put under test.

As regards the effect on transient stability is concerned, the change of pull out power with saturation considerations cannot be expected so high since the value of X_{al} is independent of saturation factor. This results in X_q' getting affected comparatively to a lower degree (fig 5.10) as the two reactances X_{ad} (inversely proportional to Ks) and X_{al} are appearing in parallel (fig 4.7). However, to get at a more accurate value of X_q' and finally the stability limit under transient conditions, the effect of saturation must be considered as shown in section 2.9.

The steady state stability under dynamic condition and the effect of saturation there upon has been discussed in Chapter IV. The net effect could not be determined on the machine under test because of the non-availability of the proper fast acting voltage regulator required to load the machine in order to keep it stable under dynamic conditions. However, the effect can be taken into consideration by knowing the excitation response of the regulator fitted to the machine (references 8, 9 & 10)

APPENDIX-I

Solution of the 2nd order differential equation involved in transient state stability due to sudden unloading.

Considering the equation 4.2.1

$$I \frac{d^2\theta}{dt^2} + K_d \frac{d\theta}{dt} + K\theta = P_o + (P_1 - P_o)\, H(t) - P_\theta \quad \ldots 4.2.1$$

Where P_θ according to equation 4.2.2 can be written as

$$P_\theta = a' \sin\theta \;\; b' \sin 2\theta \quad \ldots\ldots\ldots\ldots 4.2.4$$

a' and b' being constants.

Substituting equation 4.2.4 in equation 4.2.1 we have

$$I \frac{d^2\theta}{dt^2} + K_d \frac{d\theta}{dt} + K\theta + a' \sin\theta + b' \sin 2\theta$$

$$= P_o \; (P_1 - P_o) \; H(t)$$

$$= P_1 \quad \sin t > 0 \quad \ldots\ldots\ldots 4.2.5$$

The mechanical stiffness K can be neglected in comparison to stiffness of the system given by $\frac{d}{d\theta}(a \sin\theta + b \sin 2\theta)$ (Ref.14). We can write

$$\frac{d^2\theta}{dt^2} + \frac{K_d}{I} \cdot \frac{d\theta}{dt} + \frac{1}{I} \left(a' \sin\theta + b' \sin 2\theta \right)$$

$$= \frac{P_1}{I}, \quad t > 0 \quad \ldots\ldots\ldots 4.2.6$$

or $=$

$\frac{d^2\theta}{dt^2}$ $=$

$$\frac{d^2\theta}{dt^2} + 2\gamma \frac{d\theta}{dt} + a \sin\theta + b \sin 2\theta = F, \quad t > 0 \qquad \ldots\ldots 4.27$$

Neglecting the damping to obtain an approximate solution

$$\frac{d^2\theta}{dt^2} + a \sin\theta + b \sin 2\theta = F \qquad \ldots\ldots\ldots\ldots\ldots\ldots 4.28$$

Put $v = \frac{d\theta}{dt}$ in 4.28

$$v \frac{dv}{d\theta} + a \sin\theta + b \sin 2\theta = F \qquad \ldots\ldots\ldots 4.29$$

Integrating we get

$$\frac{v^2}{2} = F\theta + a \cos\theta + \frac{b \cos 2\theta}{2}$$

$$v^2 = 2(F\theta + a\cos\theta) + b \cos 2\theta + A \qquad \ldots\ldots\ldots 4.30$$

At $\theta = \theta_0$, $v = \dot{\theta} = 0$ and the [illegible] [illegible] [illegible] a [illegible] at θ_0 such that $0 < \theta_0 < 2\pi$

Also $\frac{d^2\theta}{dt^2} = 0$ at [illegible]

$$F = a \sin\theta_0 + b \sin 2\theta_0$$

Also $v = 0$ for $\theta = \theta_a$

$$A = -2(F_b\theta_a + a\cos\theta_a) - b\cos 2\theta_a \qquad \ldots\ 4.2.11$$

For $t > 0$

$$v^2 = 2(F_b\theta_a + a\cos\theta) + b\cos 2\theta$$
$$\qquad - 2(F_b\theta_a + a\cos\theta_a) - b\cos 2\theta_a$$

or
$$v = \pm\left[2(F_e\overline{\theta_a} - \theta_a) + 2a(\cos\theta - \cos\theta_a) + b.(\cos 2\theta - \cos 2\theta_a)\right]^{\frac{1}{2}} \qquad \ldots\ 4.2.12$$

When $v = 0$ excluding the case when $\theta = \theta_a$ we have

$$F_e = -\frac{a(\cos\theta - \cos\theta_a) + b.(\cos 2\theta - \cos 2\theta_a)}{\theta - \theta_a}$$

$$= \frac{a(\cos\theta - \cos\theta_a) + b.(\cos 2\theta - \cos 2\theta_a)}{\theta_a - \theta} \qquad \ldots\ 4.2.13$$

The critical conditions corresponding to the maximum value of F_b which satisfies the above equation. Accordingly we must have

$$\frac{\partial}{\partial\theta}\left\{\frac{a(\cos\theta - \cos\theta_a) + b(\cos 2\theta - \cos 2\theta_a)}{\theta_a - \theta}\right\} = 0$$

This would result in

$$(\theta - \theta_a)\left(a\sin\theta + b.\frac{\sin 2\theta}{2}\right) - a(\cos\theta_a - \cos\theta) - b(\cos 2\theta_a - \cos 2\theta) = 0. \qquad \ldots\ 4.2.14$$

A value of θ in [illegible] $\kappa < \theta < \kappa$ [illegible] which satisfies [illegible] (4.3.11) is the critical angle θ_c. [illegible] in equation (4.3.10) we have

$$V_{crit} = V_0 = [illegible](\cos\theta_0 - \cos\theta) + [illegible](\sin\theta_0 - \sin\theta) \qquad \theta = \theta_0 \quad \ldots\ldots(4.3.13)$$

The [illegible] [illegible] which [illegible] [illegible] [illegible] corresponding to $V_0 = V_3$

The $V-\theta$ curve may be plotted using equation (4.3.12). If $V < V_{c}$, it is [illegible] portion of the [illegible] parabola and [illegible]. If $V > V_0$, it is open [illegible] [illegible]. If $V = V_0$ the $V-\theta$ curve is a [illegible] i.e. a line between stability and instability.

APPENDIX A.

[illegible]

[illegible] equation [illegible] can be written [illegible]

[illegible]

$$[illegible] > 0 \quad \ldots\ldots \quad 4.2.18$$

[illegible]

$$v^2 = 2\left[F_e\theta + a\cos\theta\right] + b\cos 2\theta - 4k\int_{\theta_0}^{\theta} v\,d\theta + A \quad \ldots \quad 4.2.17$$

[illegible] $\theta = \theta_0$, $V = \dot{\theta} = 0$ [illegible]

[illegible] $\theta_0 + a\cos\theta_0$) [illegible] θ_0 ...... [illegible]

[illegible] the value of A in equation [illegible] we have

$$v^2 = 2\left\{(\theta-\theta_a)F_b + a(\cos\theta - \cos\theta_a)\right\} + b(\cos 2\theta - \cos 2\theta_a) - 4k\int_{\theta_0}^{\theta} v\,d\theta \quad \ldots \quad 4.2.19$$

[illegible] $v = 0$ [illegible]

[illegible] θ [illegible]

$$F_e(\theta-\theta_a) = a(\cos\theta_a - \cos\theta) + \frac{e_1}{2}(\cos 2\theta_a - \cos 2\theta) - 2k\int_{\theta_0}^{\theta} v\,d\theta \quad \ldots \quad 4.2.20.$$

We can [illegible] a [illegible] quadrature to evaluate w for a suitable θ range using the [illegible] $= 0$ as the upper limit, equation 4.2.11. Then the integral [illegible] $w\,d\theta$ can be evaluated numerically. This numerical value can be substituted in equation 4.2.17 and a new value of V obtained.

This procedure can be repeated to get an accurate value of the integral $\int_{\theta_0}^{\theta} w\,d\theta$.

The curve $a(\cos\theta_a - \cos\theta) + \frac{[illegible]}{[illegible]}(\cos\theta_c - \cos\theta)$ as a function of θ can be plotted. From the straight line $V_b(\theta - \theta_c)$ tangential to the above curve, a value of V_c and θ_c will be obtained.

Knowing the value of V_c, the value of critical load which can be suddenly applied [illegible] can be determined.

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