S O M E   A S P E C T S   O F
A   R E L U C T A N C E   M O T O R

BY

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"A C K N O W L E D G E M E N T"

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"I N D E X"

Pages

SUMMARY

Salient pole synchronous motors, without excitation are capable of delivering certain amount of power, known as the reluctance power and the synchronous motors working under such conditions are called Reluctance Motors.

The following aspects about a Reluctance Motor are presented in this dissertation.

A brief description of the working principle and the scope of the Reluctance Motors is given as introduction. This is followed by a discussion on the m.m.f. of a poly-phase winding.

After this, the vector diagram of a Reluctance Motor is developed and from the vector diagram, the general power equation is deduced. The general power equation determines the performance characteristics of the Reluctance Motor.

Due to saturation, the performance of a Reluctance motor is affected. The effects of saturation are also discussed in this dissertation.

The last part of this dissertation consists of the following experimental work on a 2.8 K.W., 12.8A, 1000 r.p.m. 220 V., three phase synchronous motor working as a Reluctance Motor.

The direct axis synchronous reactance of the machine, X_d, is determined from the open circuit and short circuit characteristics and also under purely inductive loading of the machine.

To determine the values of the quadrature and direct axis synchronous reactances, X_q and X_d, the slip test is performed on the synchronous machine.

Using the experimentally determined values of X_q and X_d, a theoretical current locus and the theoretical power curves are plotted.

The current locus and the power curves of the reluctance motor are also determined experimentally and compared with the theoretical results mentioned above. The discrepancy in the two results is explained.

"INTRODUCTION"

If a salient pole motor, without winding on the poles, is brought up near synchronous speed, it will lock into synchronism. The motor will then continue to run at its synchronous speed, provided the load is not more than only a fraction of the full load. The reason for this action is that the magnetic lines of force due to armature m.m.f. tend to become as short as possible. Thus there is a tendency of the rotor to align itself in the minimum reluctance position with respect to the synchronously revolving field set up due to armature currents. (Figs. 1 and 2).

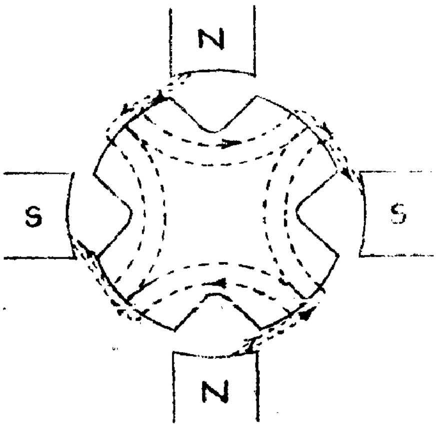


FIG. 1.

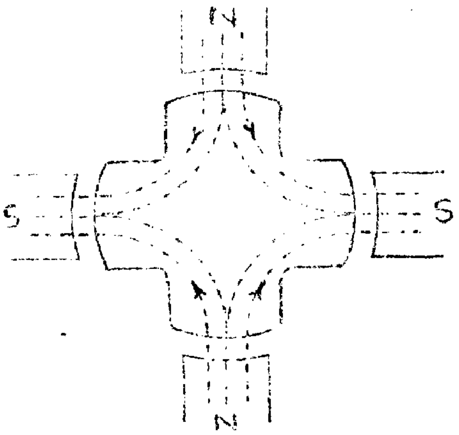


FIG. 2.

Fig. 1 illustrates the flux path as the rotor slips backwards and the magnetic lines are expanded.

Fig. 2 shows flux paths with lines fully contracted.

Had it been a cylinderical rotor, there could have been no maximum or minimum reluctance position for the rotor and the motion at synchronous speed, without excitation, could have been impossible. The torque in the reluctance motors is produced due to the saliency in poles or due to the variation in the reluctance of the air gap, the reluctance being less in the direct axis and more in the quadrature axis. The higher the ratio of these reluctances the better are the properties of these motors in respect of the over-load capacity and the power factor.

A salient pole synchronous motor will carry load at synchronous speed without excitation, though with poor power factor and efficiency. In the case of small synchronous motors, under discussion, these characteristics are not very important and a very great saving in cost is effected by eliminating the field windings, rings and brushes and the necessity for providing a separate D.C. supply and controls.

Compared with an excited type, the unexcited synchronous motor has only about one quarter of the output for the same dimensions. Because of this, these motors are used for light duties only. Examples are timing mechanism of a stroboscope, driving the contact maker of a transient visualiser or the rotating mirror in an Oscillograph, electric clocks and the tachometer checking etc.

M.M.F. OF A POLYPHASE WINDING

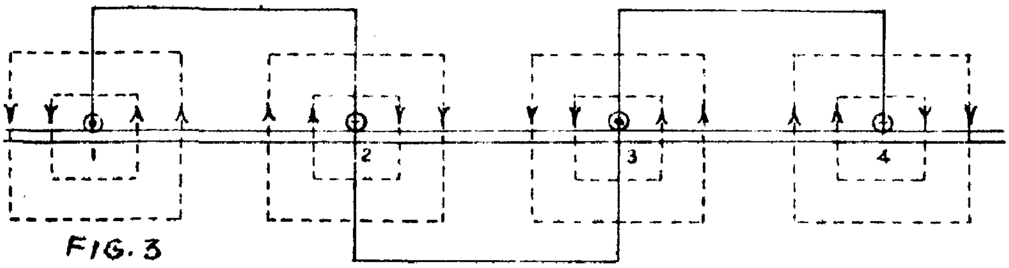


FIG. 3

Consider one phase of a full pitch polyphase winding with one slot/pole/phase. The directions of the currents and fluxes are shown for a particular instant of time.

Total m.m.f. required to set up the flux around one slot, say slot No.1 = $\sqrt{2}\ I\ N_c$ where N_c = the total no. of conductors/slot and I is the effective value of the current/conductor.

This m.m.f. sends flux around the slot in two series paths, one on the right and the other on the left of the slot.

Therefore, m.m.f. required for slot No.1 to send an upward flux through the air gap on the right side of it only is $\frac{\sqrt{2}\ N_c\ I}{2}$.

Similarly for slot No.2, the m.m.f required to send an upward flux through the air gap on the left of the slot is $\frac{\sqrt{2}\ I\ N_c}{2}$. Thus inbetween slots 1 & 2, the m.m.f. for an upward flux through the air gap is $\frac{\sqrt{2}\ I\ N_c}{2}$

Similarly between slots 2 and 3, the m.m.f. for a downward flux is $\frac{\sqrt{2}\ I\ N_c}{2}$, but acts in the reverse direction.

This m.m.f. is of rectangular wave form and hence of constant amplitude over a pole pitch because we have assumed the winding to be concentrated with one slot per pole.

We get an m.m.f. curve as shown. This is a space distribution of the m.m.f. round the armature periphery but the instantaneous amplitude of this space distribution varies sinusoidally with respect to time.

Now to consider a single phase winding with several slots per pole per phase and a polyphase winding with several slots per pole per phase, the relevant m.m.f. curve can be analysed into its fundamental component and higher harmonics.

If the ordinate is taken through the centre of the pole, the space distribution function is an even one and therefore $f(-x) = f(x)$ and hence only cosine terms will be present in the Fourier components.

Further since the function is periodic with a period 2π, only odd harmonics will be present.

Therefore, $f(x) = \sum_{n=1}^{n=\infty} a_n \cos \frac{n\pi x}{T}$

where $a_n = \frac{1}{T}\int_0^{2T} f(x) \cos \frac{n\pi x}{T}\, dx$ & T, the pole pitch

From the m.m.f. distribution curve:

$f(x) = \frac{\sqrt{2}}{2} I N_c \sin \omega t$ from $x = 0$ to $x = \frac{T}{3}$

$f'(x) = -\frac{\sqrt{2}}{2} I N_c \sin \omega t$ from $x = \frac{T}{3}$ to $x = \frac{2T}{3}$

$f(x) = \frac{\sqrt{2}}{2} I N_c \sin \omega t$ from $x = \frac{2T}{3}$ to $x = T$.

Now in order to find the resultant m.m.f curve of a 3 phase winding, we add the m.m.fs of the three windings.

Taking the simple case of full pitch coils, the m.m.fs of three windings are given by

$$f_1(x) = 0.9 N_c I \sin \omega t \left[\cos \frac{\pi x}{T} - \frac{1}{3}\cos \frac{3\pi x}{T} + \frac{1}{5}\cos \frac{5\pi x}{T} - \dots\dots\right]$$

$$f_2(x) = 0.9 N_c I \sin(\omega t - 120)\left[\cos\left(\frac{\pi x}{T} - 120\right) - \frac{1}{3}\cos 3\left(\frac{\pi x}{T} - 120\right) + \frac{1}{5}\cos 5\left(\frac{\pi x}{T} - 120\right) - \dots\dots\right]$$

$$f_3(x) = 0.9 N_c I \sin(\omega t + 120)\left[\cos\left(\frac{\pi x}{T} + 120\right) - \frac{1}{3}\cos 3\left(\frac{\pi x}{T} + 120\right) + \frac{1}{5}\cos 5\left(\frac{\pi x}{T} + 120\right) - \dots\dots\right]$$

Sum of the three fundamentals gives

$$f(x) = 0.9 N_c I \left[\sin \omega t \cos \frac{\pi x}{T} + \sin(\omega t - 120)\cos\left(\frac{\pi x}{T} - 120\right) + \sin(\omega t + 120)\cos\left(\frac{\pi x}{T} + 120\right)\right]$$

$$= \frac{0.9 I N_c}{2}\left[\sin\left(\omega t + \frac{\pi x}{T}\right) + \sin\left(\omega t - \frac{\pi x}{T}\right) + \sin\left(\frac{\pi x}{T} + \omega t - 240\right) + \sin\left(\omega t - \frac{\pi x}{T}\right) + \sin\left(\omega t + \frac{\pi x}{T} + 240\right) + \sin(\omega t - \dots\right.$$

But $\sin\left(\omega t + \frac{\pi x}{T}\right) + \sin\left(\omega t + \frac{\pi x}{T} - 240\right) + \sin\left(\omega t + \frac{\pi x}{T} + 240\right)$

$$= \sin\theta + \sin(\theta - 240) + \sin(\theta + 240)$$

where $\theta = \left(\omega t + \frac{\pi x}{T}\right)$

$$= \sin\theta + \sin\theta\cos 240 - \cos\theta\sin 240 + \sin\theta\cos 240 + \cos\theta\sin 240$$

$$= \sin\theta + 2\sin\theta\cos 240 = \sin\theta + 2\sin\theta\left(-\frac{1}{2}\right) = 0$$

$$\therefore f(x) = \frac{0.9}{2} N_c I \; 3\sin\left(\omega t - \frac{\pi x}{T}\right) = \frac{3}{2}(0.9) N_c I \sin\left(\omega t - \frac{\pi x}{T}\right)$$

Thus the resultant m.m.f. of the three fundamental m.m.fs of three phases, is a rotating m.m.f., rotating at synchronous speed.

DEVELOPMENT OF THE VECTOR DIAGRAM OF A RELUCTANCE MOTOR

In a synchronous machine, if the armature current I_a lags behind the induced voltage in the armature winding by 90°, the armature reaction m.m.f. axis coincides with the field axis and the two m.m.fs act against each other, as shown in figs. 4 and 5.

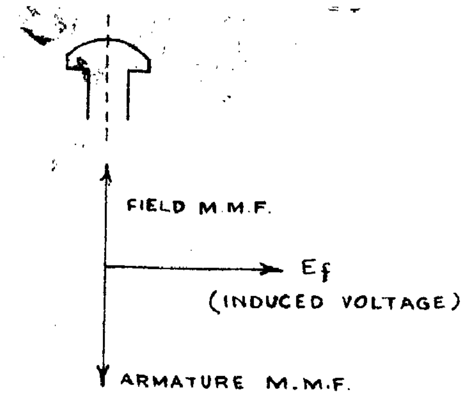


FIG.4.

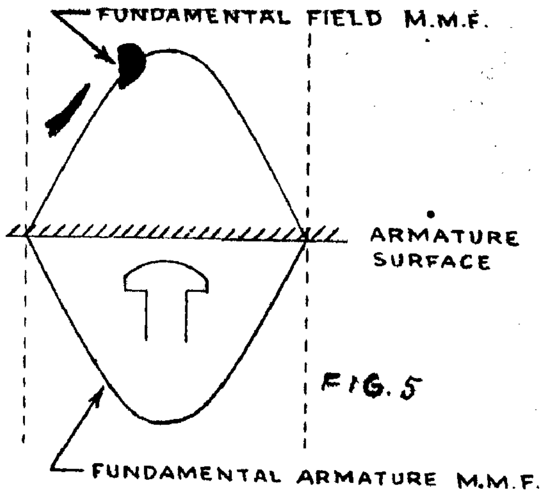


FIG.5

Conditions are quite different, when the armature current is in phase with the induced voltage (figs. 6 & 7).

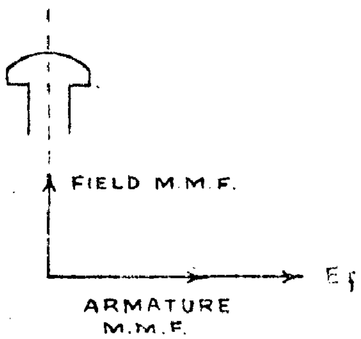


FIG.6.

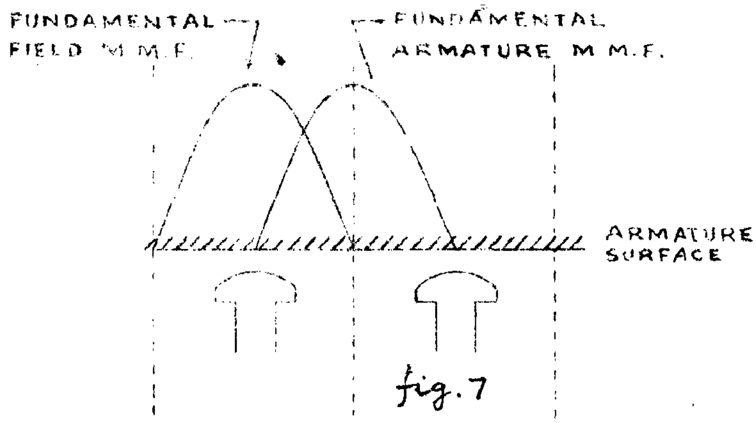


fig.7

The axis of the armature reaction m.m.f., in this case, coincides with the interpolar axis.

Thus the spatial position of the armature reaction m.m.f. wave with respect to the field poles depends on the phase angle of the armature current with respect to the induced voltage.

In a cylindrical Rotor machine, the air gap is uniform. Therefore the reluctance of the magnetic circuit is independent of the spatial position of the armature m.m.f. Therefore in such a machine, the inductive effects of balanced polyphase armature currents can be accounted for by a synchronous reactance, which is independent of power factor.

In a salient pole machine, however, quite a different state of affairs exists. Due to the non-uniformity in the air gap, the reluctance of the magnetic circuit is also non uniform and it is the spatial position of the armature m.m.f. which determines the flux density curve due to armature m.m.f.

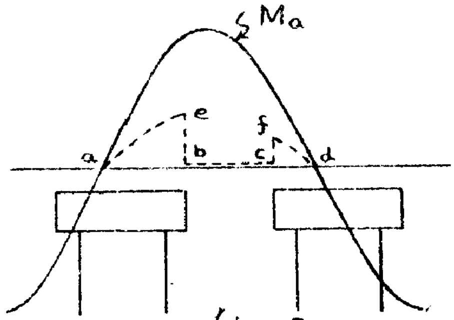


fig. 8

In Fig. 8 is shown, a position of the armature m.m.f. wave in space. From a to b & from c to d, the reluctance is uniform and therefore the flux density wave is sinusoidal. From b to c, the reluctance is infinite, therefore, the flux density is Zero.

Thus armature m.m.f. M_a gives a flux distribution curve aebcfd. It is difficult to take into account the effect of such a flux distribution curve due to armature currents, on the main field system, since with various positions of the armature m.m.f. the shape of the field distribution curve would be different.

Therefore, the synchronously rotating armature m.m.f. M_a is resolved into two components. The peak value of one component is always in the direct axis of the synchronously rotating poles and the peak value of the other component is always in the quadrature axis of the poles as shown in Fig. 9

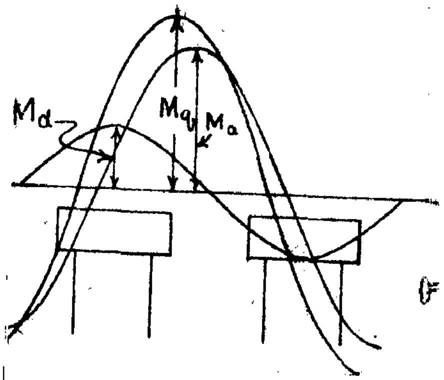


FIG. 9

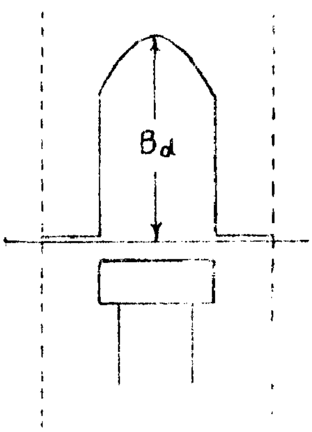


FIG. 10.

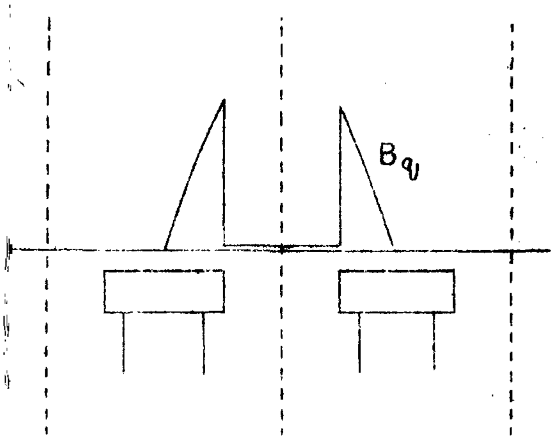


FIG. 11.

Due to M_d and M_q, the flux density curves in the direct and quadrature axes are shown in figs. 10 and 11. These simple flux density curves can easily be made use of to determine the inductive effects of armature reaction fluxes in the direct and quadrature axis separately. In each of the direct and quadrature axis, the inductive effect of armature reaction flux is combined with the inductive effect of armature leakage flux.

That is, corresponding to both direct and quadrature axes, direct and quadrature axes synchronous reactances are taken into account for the inductive drops in the two axes separately.

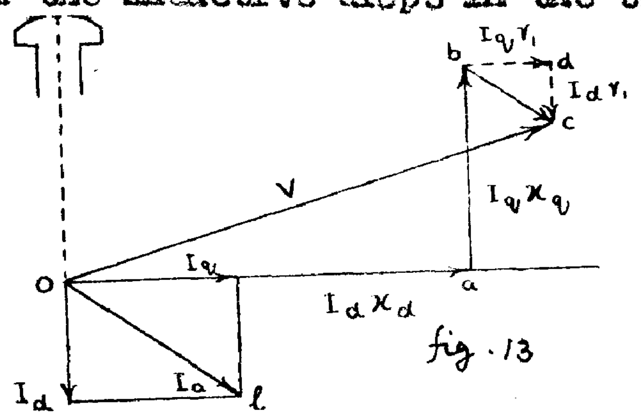


fig. 13

Thus the effect of salient poles can be taken into account by resolving armature m.m.f. or the armature current I_a into two components, one in time quadrature with and the other in time phase with the induced voltage E_f (Fig. 12).

In the vector diagram of a reluctance motor (fig. 13) we know that the induced voltage is Zero, there being no

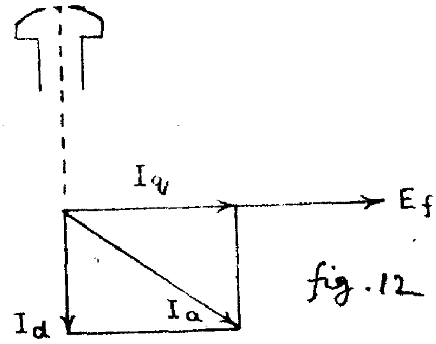


fig. 12

field excitation and the armature current I_a, as explained above, is resolved into two components I_d and I_q. The synchronous reactance drop $I_d X_d$ represents the inductive drop due to leakage flux in the direct axis plus an inductive drop equivalent to armature reaction in the direct axis. This $I_d X_d$ drop must lead I_d by 90°. Similarly the synchronous reactance drop in the quadrature axis $I_q X_q$ must lead I_q by 90°.

Thus in the vector diagram shown

$oa = I_d X_d$

$ab = I_q X_q$

bc is a vector giving Ir drop in the armature winding and therefore is parallel to the current vector I.

bc can further be resolved into two components,

$bd = I_q r$

and $dc = I_d r$

Thus the total voltage V, applied to the stator of the reluctance Motor is given by

$V = \overline{oa} + \overline{ab} + \overline{bc}$

$= \overline{oa} + \overline{ab} + \overline{bd} + \overline{dc}$

Therefore, the complete vector diagram of the reluctance Motor is as shown in Fig. 13.

[GENE]RAL POWER EQUATION OF A RELUCTANCE MOTOR

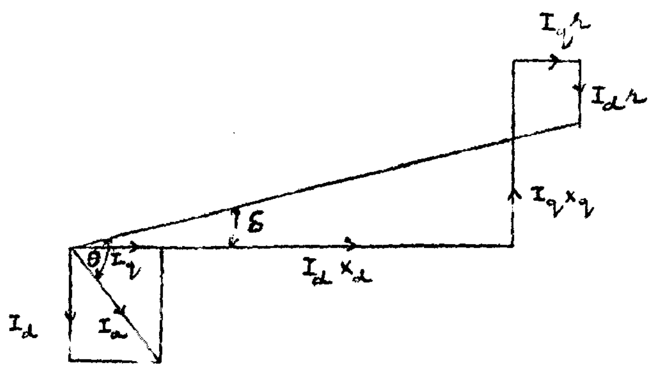


fig. 14

From the vector diagram

Power input $= VI\cos\theta$

Resolving horizontally and vertically

$$V\cos\delta = I_d x_d + I_q r \quad \text{——— (1)}$$

$$V\sin\delta = I_q x_q - I_d r \quad \text{——— (2)}$$

But $I_d = I\sin(\theta-\delta)$ ——— (3)

and $I_q = I\cos(\theta-\delta)$ ——— (4)

$$\therefore V\cos\delta = I\sin(\theta-\delta)\,x_d + I\cos(\theta-\delta)\,r \quad \text{——— (5)}$$

$$V\sin\delta = I\cos(\theta-\delta)\,x_q - I\sin(\theta-\delta)\,r \quad \text{——— (6)}$$

From equation (5)

$$V\cos\delta = I x_d\left[\sin\theta\cos\delta - \cos\theta\sin\delta\right] + I r\left[\cos\theta\cos\delta + \sin\theta\sin\delta\right]$$

$$\therefore \left[V\cos\delta + I x_d\cos\theta\sin\delta - I r\cos\theta\cos\delta\right]$$

$$= I\sin\theta\,(x_d\cos\delta + r\sin\delta)$$

$$\therefore I\sin\theta = \frac{V\cos\delta + I x_d\cos\theta\sin\delta - I r\cos\theta\cos\delta}{x_d\cos\delta + r\sin\delta} \quad \text{——— (7)}$$

From equation (6)

$$V \sin\delta = I x_q \left[\cos\theta \cos\delta + \sin\theta \sin\delta\right] - I r \left[\sin\theta \cos\delta - \cos\theta \sin\delta\right]$$

$$I \sin\theta \left[x_q \sin\delta - r\cos\delta\right] = V\sin\delta - I x_q \cos\theta \cos\delta - I r \cos\theta \sin\delta$$

$$\therefore\ I \sin\theta = \frac{V\sin\delta - I x_q \cos\theta\cos\delta - I r\cos\theta\sin\delta}{x_q \sin\delta - r\cos\delta} \quad \text{——— (8)}$$

From equations (7) and (8)

$$\frac{V\cos\delta + I x_d \cos\theta \sin\delta - I r \cos\theta\cos\delta}{x_d \cos\delta + r\sin\delta} = \frac{V\sin\delta - I x_q \cos\theta\cos\delta - I r\cos\theta\sin\delta}{x_q \sin\delta - r\cos\delta}$$

Cross multiplying and simplifying

$$V x_q \sin\delta\cos\delta + I x_d \cos\theta\, x_q \sin^2\delta - V r\cos^2\delta + I r^2 \cos\theta\cos^2\delta$$
$$= V x_d \sin\delta\cos\delta - I x_q x_d \cos\theta\cos^2\delta + V r\sin^2\delta - I r^2 \cos\theta \sin^2\delta$$

$$\therefore\ I\cos\theta\left[x_d x_q + r^2\right] = V\sin\delta\cos\delta\,(x_d - x_q) + Vr$$

$$\therefore\ I\cos\theta = \frac{\frac{V}{2}\sin 2\delta\,(x_d - x_q) + Vr}{x_d x_q + r^2}$$

$\therefore$ Power input $= V I \cos\theta$

$$= \frac{\frac{V^2}{2}\sin 2\delta\,(x_d - x_q) + V^2 r}{x_q x_d + r^2}$$

If $r = 0$

Power input $= \frac{V^2}{2}\left(\frac{1}{x_q} - \frac{1}{x_d}\right)\sin 2\delta$ and the power input curve is as shown in fig. 15

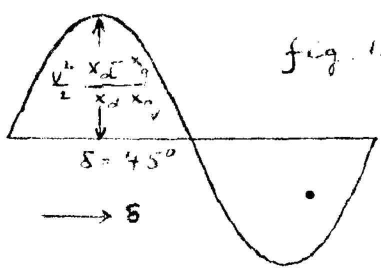


fig. 15

EFFECTS OF SATURATION ON THE PERFORMANCE OF RELUCTANCE MOTORS

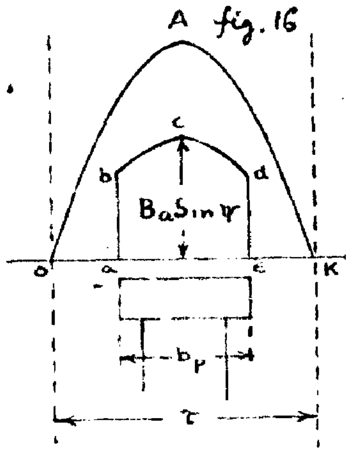
We have already seen that in the case of motors with salient poles, the curves of flux density due to armature m.m.f. will be quite different from those of armature m.m.f. curves. It is so because the reluctance of the gaps under the poles is much lower than at points between the poles. We shall now study such curves in details. The flux density curve can be represented in a generalised form by Fourier Analysis.

$y = a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + \text{----------}$

The amplitude of the fundamental component a_1 is given by the equation

$$a_1 = \frac{2}{\pi}\int_0^{\pi} y \sin x \, dx$$

Fig. 16 shows the armature m.m.f. curve OAK in the direct axis.



From O to a and e to K, the reluctance being infinite, the flux or flux density is Zero.

From a to e, because of the constant air gap reluctance, the flux density curve is of the same shape as the armature m.m.f. wave i.e. sinusoidal.

Since this flux density wave is due to the direct axis component, $I_a \sin\psi$, of armature current, its maximum value is represented as $B_a \sin\psi$. Fig. 17 shows the two components of I_a

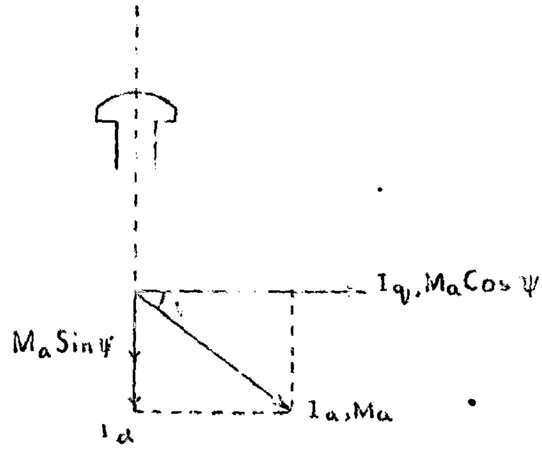


FIG. 17.

Now taking O as origin

$$ox = \frac{T}{2} - \frac{b_p}{2}$$

$$= \frac{T}{2}\left(1 - \frac{b_p}{T}\right)$$

$$= \frac{\pi}{2}(1-\alpha) \quad \text{where } \alpha = \frac{b_p}{T}$$

$$oc = oa + xc$$

$$= \frac{T}{2} - \frac{b_p}{2} + b_p = \frac{T}{2} + \frac{b_p}{2}$$

$$= \frac{T}{2}(1+\alpha)$$

$$= \frac{\pi}{2}(1+\alpha)$$

$$\therefore a_1 = \frac{2}{\pi}\int_0^{\pi} f(x)\sin x\, dx$$

$$f(x) = 0 \quad \text{for } 0 \leq x < \frac{\pi}{2}(1-\alpha)$$

$$f(x) = B_a \sin\psi \sin x \quad \text{for } \frac{\pi}{2}(1-\alpha) \leq x \leq \frac{\pi}{2}(1+\alpha)$$

$$f(x) = 0 \quad \text{for } \frac{\pi}{2}(1+\alpha) < x \leq \pi$$

$$\therefore a_1 = \frac{2}{\pi}\int_{(1-\alpha)\frac{\pi}{2}}^{(1+\alpha)\pi/2} B_a \sin\psi \sin x \sin x\, dx$$

$$= \frac{2}{\pi}\,\frac{B_a \sin\psi}{2}\int_{(1-\alpha)\frac{\pi}{2}}^{(1+\alpha)\frac{\pi}{2}} 2\sin^2 x\, dx$$

$$= \frac{B_a \sin\psi}{\pi}\int_{(1-\alpha)\frac{\pi}{2}}^{(1+\alpha)\frac{\pi}{2}} (1-\cos 2x)\, dx$$

$$= \frac{B_a \sin\psi}{\pi} \left[x - \frac{\sin 2x}{2} \right]_{(1-\alpha)\frac{\pi}{2}}^{(1+\alpha)\frac{\pi}{2}}$$

$$= \frac{B_a \sin\psi}{\pi} \left[\frac{\pi}{2} + \frac{\alpha\pi}{2} - \frac{\pi}{2} + \frac{\alpha\pi}{2} - \frac{1}{2} \sin(\pi + \alpha\pi) + \frac{1}{2} \sin(\pi - \alpha\pi) \right]$$

$$= \frac{B_a \sin\psi}{\pi} \left[\alpha\pi + \sin\alpha\pi \right]$$

[illegible] ψ.

$$B_a \sin\psi \quad \frac{\alpha\pi + \sin\alpha\pi}{\pi}$$

[illegible] $= \frac{\alpha\pi + \sin\alpha\pi}{\pi}$

$$a_1 = \frac{2}{\pi} \int_0^{\pi} f(x) \sin x \, dx$$

$$= \frac{2}{\pi} \left[\int_{27}^{65} B_a \sin\psi \sin x \sin x \, dx + \int_{65}^{115} K \sin x \, dx + \int_{115}^{153} B_a \sin\psi \sin x \sin x \, dx \right]$$

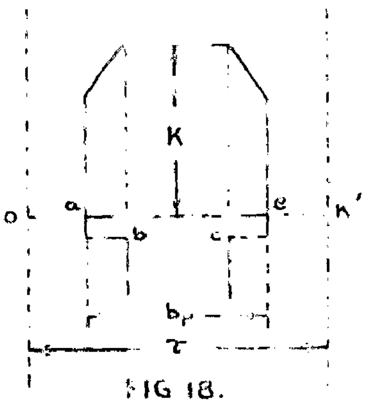


FIG 18.

$$1\text{st term} = B_a \sin\psi \cdot \frac{1}{2}\int_{27^\circ}^{65^\circ} (1-\cos 2x)\,dx$$

$$= B_a \sin\psi \ \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right)_{27^\circ}^{65^\circ}$$

$65^\circ = .362\pi$

$27^\circ = .150\pi$

$$= B_a \sin\psi \ \frac{1}{2}\left[.212\pi + .0215\right]$$

$$= 0.343\, B_a \sin\psi$$

$$2\text{nd term} = K(-\cos x)_{65^\circ}^{115^\circ}$$

$$= 0.84\, K$$

$$3\text{rd term} = \frac{B_a \sin\psi}{2}\left[x - \frac{\sin 2x}{2}\right]$$

$115^\circ = .638\pi$

$$= B_a \sin\psi\ (0.775)$$

$$\therefore\ a_1 = \frac{2}{\pi}\left[0.343 + 0.775\right] B_a \sin\psi + \frac{2}{\pi}(0.84)K$$

$$= 0.71\, B_a \sin\psi + 0.535\, K \quad \text{———} (A)$$

Under unsaturated Condition

$$a_1 = B_a \sin\psi \ \frac{\alpha\pi + \sin\alpha\pi}{\pi}$$

if $\alpha = \frac{b_p}{T} = 0.7$

$$\therefore a_1 = B_a \sin\psi\left[0.7 + \frac{\sin 126^\circ}{\pi}\right]$$

$$= 0.958\, B_a \sin\psi \quad \text{———} (B)$$

From equations (A) and (B)

a_1 at Saturation is $<$ a_1 at undersaturation

if $0.71\, B_a \sin\psi + 0.535\, K < B_a \sin\psi\ (0.958)$

if $2.15\, K < B_a \sin\psi$ or $B_a \sin\psi > 2.15\, K$

But $B_a \sin\psi$ is obviously $> K$ ...

Therefore in the direct axis, the flux density at undersaturation is greater than at saturation.

The value of the direct axis synchronous reactance depends upon the flux density in the direct axis. This will be clear from the following.

In fig. 19, let OA and OB be the short circuit and open circuit characteristics respectively.

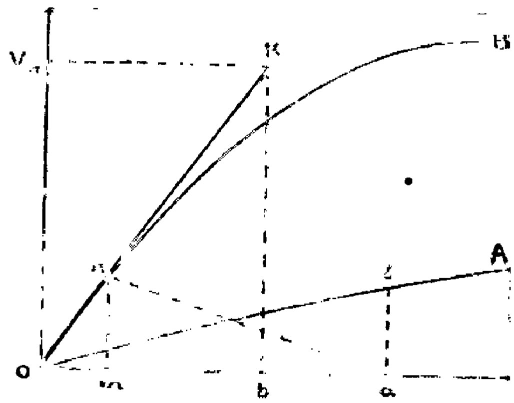


FIG. 19

Also let V_n = the rated voltage

and I_n = the rated current

oa = excitation required to circulate the full load current = aI, through armature winding on short circuit.

ob = the excitation required to get the rated voltage V_n at unsaturated condition.

= I_f

then P.U. unsaturated direct axis synchronous reactance

X_d = oa/ob

If we neglect armature resistance, out of total excitation oa, om is required to compensate for $I_a X_l$ drop and ma is the excitation required to counteract the armature reaction in the direct axis. Let it be = X_a.

Therefore X_d = om + ma/ob

But om/mn = ob/bk

$om/X_a \; X_l = X_g/V_a$

Therefore $om = X_g/V_a \cdot X_a \; X_l$

$= X_g X_l$ p.u.

Therefore $X_d = X_l + X_{ad}$

$= om + mc/ob$

$= (X_g X_l + X_d)/X_g = X_l + X_d / X_g$

Therefore $X_{ad} \propto X_g$.

$\propto$ excitation required to compensate armature reaction in the direct axis.

$\propto$ flux density due to armature m.m.f. in the direct axis.

We have already seen that flux density in the direct axis is reduced at saturation. Therefore X_{ad}, which is directly proportional to flux density due to armature m.m.f. in the direct axis, is also reduced at saturation.

Therefore X_d is also reduced at saturation.

Because $X_d = X_{ad} + X_l$.

The m.m.f. and flux density distribution in the quadrature axis is shown in Fig. 20. From o to b, the flux density is a sine wave. Between b and c, the reluctance being infinite, the flux density is zero and from c to o, it is again sinusoidal.

Taking o as the origin

$ob = b_p/2$

$oc = ob + bk + kc$

$= b_p/2 + 2bk = b_p/2 + 2(\tau/2 - b_p/2)$

$= \pi\left(1 - \frac{b_p}{2\ell}\right) = \pi\left(1 - \frac{\alpha}{2}\right)$

$\omega = \pi - \frac{b_p}{2} + \frac{b_p}{2} = \pi$

Again by Fourier series, the flux density wave can be represented as

$y = a' \sin x + a'' \sin 2x + a''' \sin 3x + \cdots$

where $a' = \int_0^{\pi} f(x) \sin x \, dx$

$f(x) = B_a \cos\psi \sin x$ for $0 \leq x < \frac{b_p}{2}$

$f(x) = 0$ for $\frac{b_p}{2} \leq x \leq \pi\left(1 - \frac{\alpha}{2}\right)$

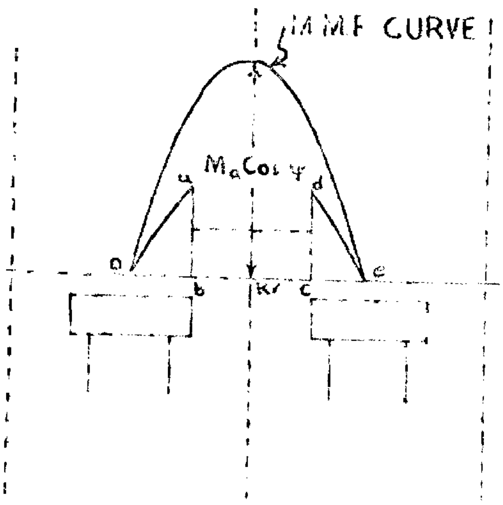


FIG 20.

$f(x) = B_a \cos\psi \sin x$ for $\pi\left(1 - \frac{\alpha}{2}\right) < x \leq \pi$

$$\therefore a' = 2/\pi \left[\int_0^{b_p/2} B_a \cos\psi \sin^2 x \, dx + \int_{\pi(1-\alpha/2)}^{\pi} B_a \cos\psi \sin^2 x \, dx \right]$$

$$= \frac{2 B_a \cos\psi}{\pi} \cdot \frac{1}{2} \left[\left(x - \frac{\sin 2x}{2}\right)_0^{b_p/2} + \left(x - \frac{\sin 2x}{2}\right)_{\pi(1-\alpha/2)}^{\pi} \right]$$

$$= \frac{B_a \cos\psi}{\pi} \left[\frac{b_p}{2} - \sin \frac{b_p}{2} + \frac{\alpha \pi}{2} - \frac{1}{2} \sin \alpha \pi \right]$$

$$= \frac{B_a \cos\psi}{\pi} \left[\alpha \pi - \sin \alpha \pi \right] \text{ ---- } (C'), \; b_p = \alpha \pi$$

Because of the shape of poles the flux density in the pole-arc area has been reduced from $B_a \cos\psi$ is $B_a \cos\psi$ to $B_a \cos\psi \frac{\alpha\pi - \sin\alpha\pi}{\pi}$ or $C_v B_a \cos\psi$ where $C_v = \frac{\alpha\pi - \sin\alpha\pi}{\pi}$

Because of saturation, the flux density in the interpolar gap increases. We assume a uniform air gap flux density $= K'$ from b to c (ref. fig 10).

Then $f(x) = B_a \cos\psi \sin x$ for $0 \leq x < \frac{\alpha\pi}{2}$

$f(x) = K$ for $\frac{\alpha\pi}{2} \leq x \leq \pi(1 - \frac{\alpha}{2})$

$f(x) = B_a \cos\psi \sin x$ for $\pi(1 - \alpha/2) < x \leq \pi$

$$\therefore a' = \frac{2}{\pi}\left[\int_0^{\frac{\alpha\pi}{2}} B_a \cos\psi \sin^2 x \, dx + \int_{\frac{\alpha\pi}{2}}^{\pi(1-\alpha/2)} K \sin x \, dx + \int_{\pi(1-\alpha/2)}^{\pi} B_a \cos\psi \sin x \sin x \, dx \right]$$

$$= \frac{B_a \cos\psi}{\pi}\left[\alpha\pi - \sin\alpha\pi\right] - \frac{2K}{\pi}\left[\cos x\right]_{\frac{\alpha\pi}{2}}^{\pi(1-\alpha/2)}$$

$$= \frac{B_a \cos\psi}{\pi}(\alpha\pi - \sin\alpha\pi) + \frac{2K}{\pi} \cdot 2\cos\frac{\alpha\pi}{2}$$

———— (D′)

From equations (C′) and (D′)

the fundamental component of the flux density at saturation is greater than at undersaturation.

Therefore in the quadrature axis, the flux density increases because of saturation..

Therefore, the quadrature axis synchronous reactance, which is proportional to the flux in the quadrature axis also increases with saturation.

i.e. due to saturation X_q increases and X_d decreases.

From the power equation of the reluctance motor

$P = V^2/2\,(1/X_q - 1/X_d)\sin 2\delta$, neglecting the armature resistance.

$(1/X_q - 1/X_d)$ at saturation decreases, therefore the Power output of the Reluctance motor decreases at saturation.

EXPERIMENTAL WORK

ON

A 3.2 K.W. 10.5 A 1000 r.p.m.

220 V THREE PHASE SYNCHRONOUS MOTOR

WORKING AS A RELUCTANCE MOTOR

DETERMINATION OF X_d

In order to determine the performance characteristics of a Reluctance Motor, it is evident from the general power equation that we require to determine the value of X_d and X_q of the machine under test.

To determine X_d, the open and short circuit test on the machine is performed. Then from the open and short circuit characteristics (Ref. Graph No.1).

Excitation required to get the rated voltage on open circuit = 1.8 amps.

Excitation required to circulate the full load current through armature winding on short circuit = 1.2 amps.

Therefore the short circuit ratio = 1.8/1.2 = 3/2.

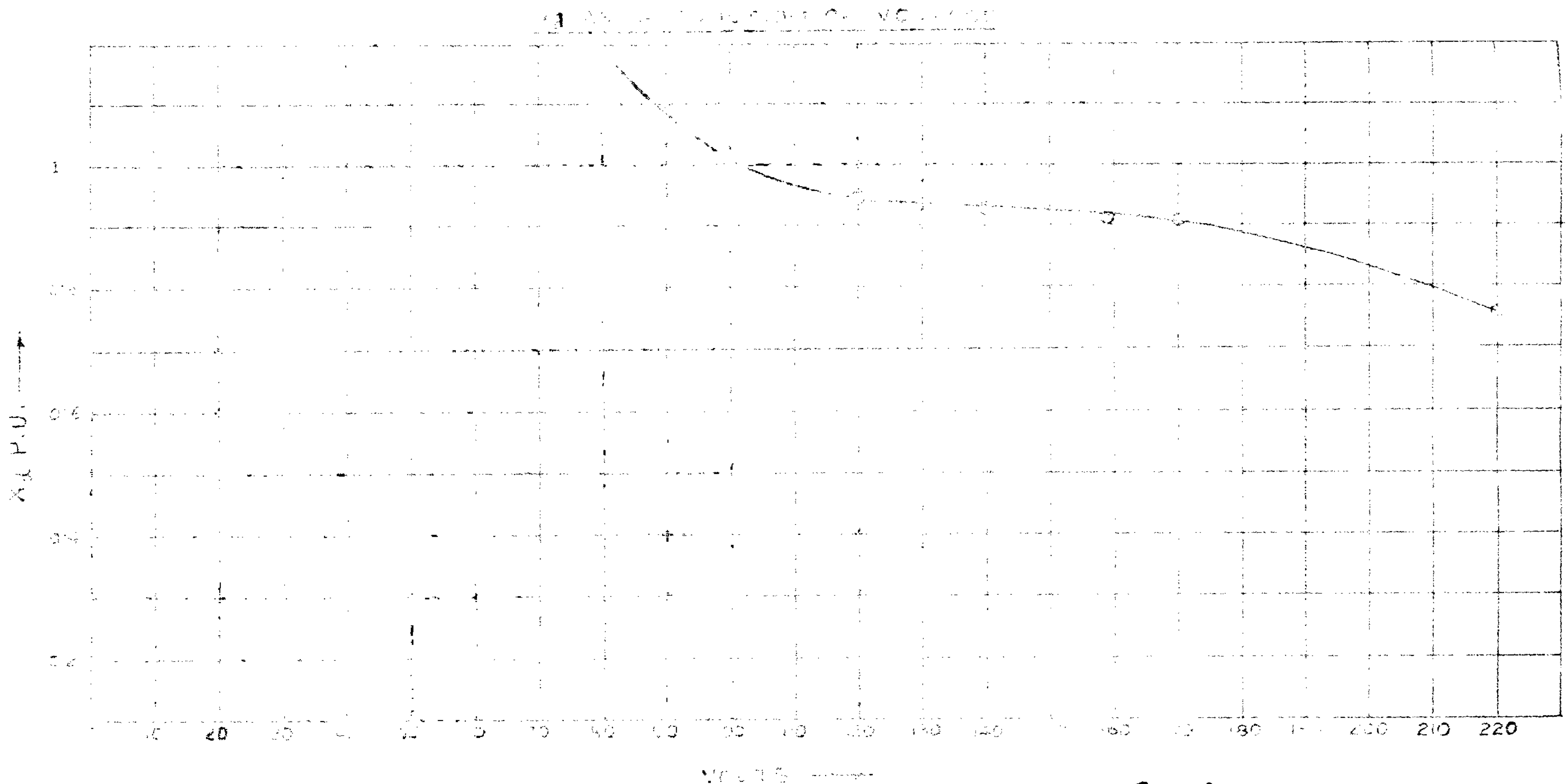
Therefore P.U. X_d = 1/short circuit ratio = 0.66.

This is the P.U. value of X_d at 220 volts.

It has been already pointed out that X_d decreases due to saturation. The higher the voltage across the terminals of the synchronous machine, the higher the degree of saturation. Therefore X_d decreases as the voltage increases.

In order to find out the value of X_d at different voltages, purely inductive loading test, is carried out on the machine.

The synchronous machine is run with a separately excited D.C. Motor. Both the armature and field of the D.C. motor are fed from a constant D.C. voltage source.



Graph 2

When the synchronous motor, Fig. 1, has come up to synchronous speed, the armature of the synchronous machine is fed from an induction Regulator. If the

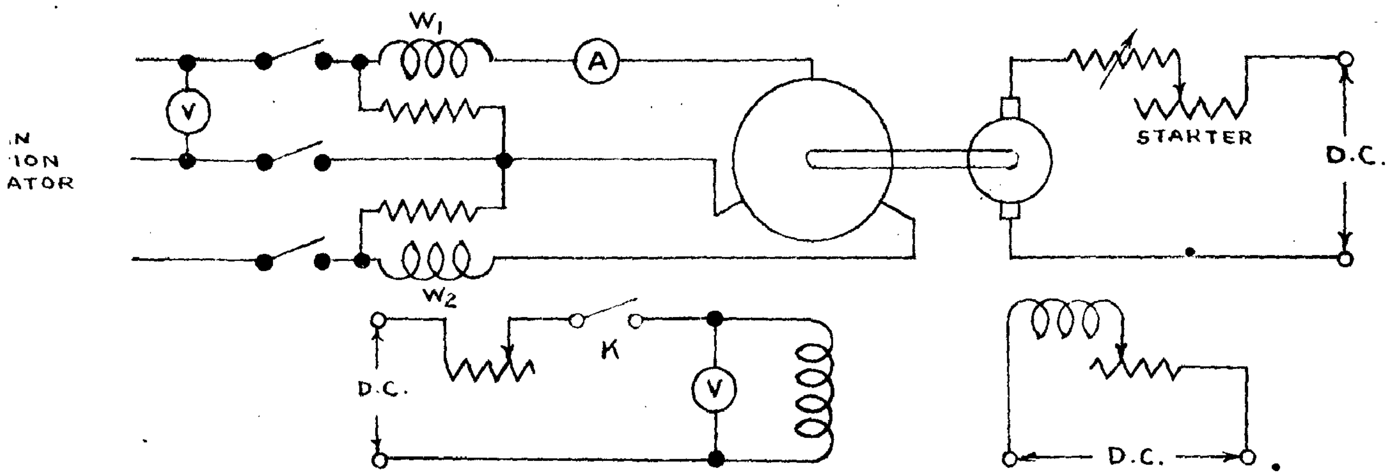


FIG. 1.

synchronous machine is not running at exact synchronous speed, which can be observed in the oscillations of the measuring instruments on A.C. side, the field of the synchronous machine is excited for a few seconds, so that the machine falls into step and, then its field excitation is removed and, then, this field circuit is kept open throughout the test.

Now the input of the D.C. machine is so adjusted with the rheostats in its armature and field circuits, that the active power drawn from the A.C. side is minimum i.e. the wattmeters on the A.C. side record almost Zero power. Under such conditions, there is only reactive load on the A.C. side. All the active power, losses etc. are supplied by the D.C. Motor.

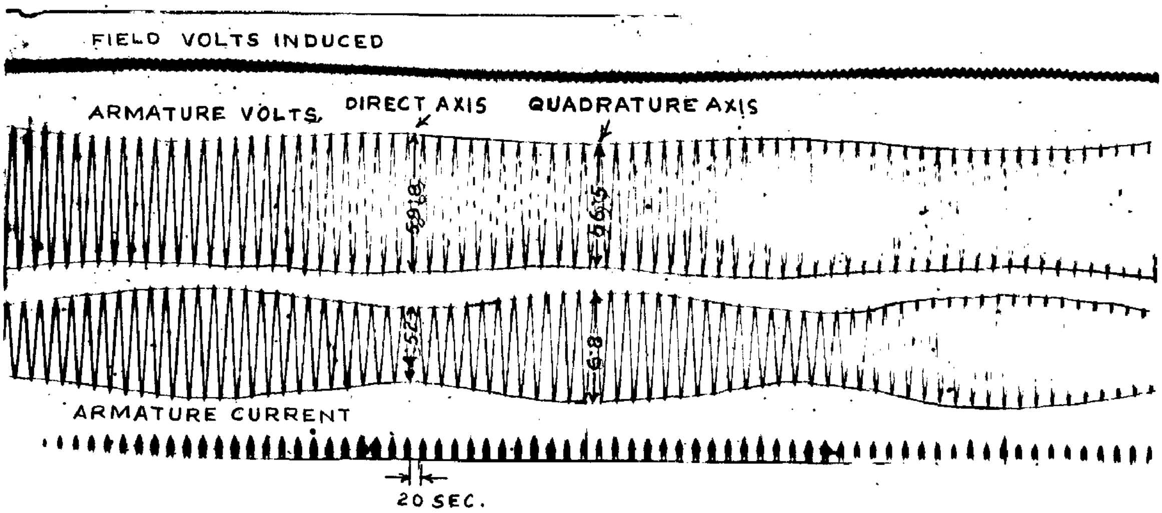
Different voltages are impressed on the stator of the synchronous machine. For every voltage impressed, the wattmeter readings are adjusted to nearly Zero value and the corresponding reactive current drawn is noted.

Then $X_d = \frac{\text{Volts impressed on Armature of synchronous machine}}{\text{Reactive current drawn from the mains}}$

Graph No. 2 shows the variation of X_d with voltage, as obtained from the above test.

In this test, with 170 volts across the armature terminals, a current of 10.1 amps. (very nearly the rated value of the current of the machine) started flowing in the armature. Therefore, the test was not carried out at voltage higher than 170 volts.

But from the open and short circuit tests, the value of X_d at 220 V, the rated voltage has already been determined Therefore on Graph 2, the value of X_d at 220 V is taken from the open circuit and short circuit characteristics. Thus the graph shows completely the variation of X_d with voltage, righ upto the rated voltage.



SLIP TEST AT 2·08 % SLIP

DETERMINATION OF X_d AND X_q FROM THE SLIP TEST

The synchronous motor is run with a separately excited D.C. Motor at a speed slightly less than the synchronous speed. The field of the synchronous machine is kept open except for putting a high resistance voltmeter accross it. To the stator of the synchronous machine, 25% of the rated voltage is applied.

Before putting in the switch on the A.C. side, the sequence of the voltages induced in the stator windings due to the residual magnetism present in the poles of the synchronous machine, is checked.

Thus, at a reduced voltage applied to the stator, an oscillographic record is obtained of the current variations in the stator and the voltage variations across it. Also an oscillographic record of the voltage variations, induced in the field of the synchronous machine, are obtained then

$$X_d = \frac{\text{Maximum volts}}{\text{Minimum current}}$$

$$X_q = \frac{\text{Minimum volts}}{\text{Maximum current}}$$

Minimum current or Maximum voltage point occurs when the voltage induced in the field of the synchronous motor is Zero. Also maximum current or minimum voltage point occurs when the voltage induced in the field of the synchronous motor is maximum.

Thus from the slip test:- $X_d = 0.63$ P.U.

$X_q = 0.396$ P.U.

$\frac{X_q}{X_d} = 0.63$ P.U.

THEORETICAL DETERMINATION OF THE CURRENT LOCUS OF A RELUCTANCE MOTOR

From the general power equation of a reluctance Motor

$$P = \frac{\frac{V^2}{2}\sin 2\delta\,(x_d - x_q) + V^2 r}{x_d x_q + r^2}$$

if r, the armature resistance, is taken negligibly small.

$$P = \frac{V^2}{2}\sin 2\delta\left(\frac{x_d - x_q}{x_d x_q}\right)$$

$$= V\,\frac{V}{2}\sin 2\delta\left(\frac{1}{x_q} - \frac{1}{x_d}\right)$$

Therefore the wattful component of the current

$$= \frac{V}{2}\sin 2\delta\left(\frac{1}{x_q} - \frac{1}{x_d}\right)$$

$$= R\sin 2\delta \quad \text{where } R = \frac{1}{2}\left(\frac{V}{x_q} - \frac{V}{x_d}\right)$$

Therefore the locus of the current is a circle with radius $= \frac{1}{2}(V/X_q - V/X_d)$ and the wattful component of the current corresponding to any current such as OP is given by PH which is $= \frac{1}{2}(V/X_q - V/X_d)\,\sin 2\delta$, Ref. Graph No. 3.

Next stage is to determine this current locus for a definite voltage across the stator terminals.

The value of voltage chosenin this case is 120 V. Later on, the theoretical current locus at this particular voltage is compared with the experimental current locus obtained at the same voltage.

From Graph No. 2, X_d at 120 V = 0.83 P.U.

rated current of the synchronous motor = 10.5 amps.

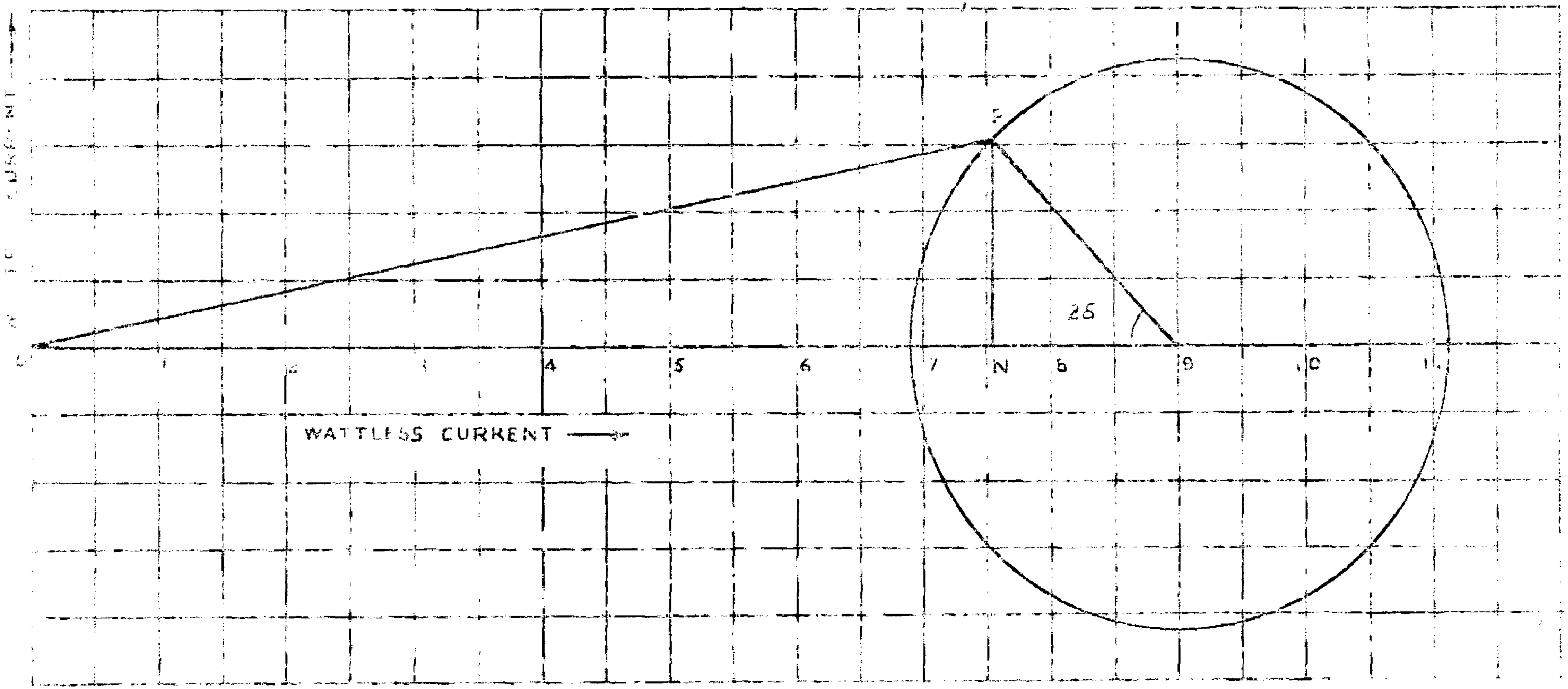
rated voltage of the synchronous motor = 220 volts.

Therefore X_d in ohms per phase $= \frac{220 \times 0.83}{\sqrt{3} \times 10.5} = 10$

$X_q = 0.63 \times 10 = 6.3$ ohms per phase.

Assuming the ratio X_q/X_d, determined by the slip test, to have approximately the same value at 120 volts.

Therefore, if the armature resistance is neglected



Graph 3

the theoretical current locus of a reluctance motor at 120V (line voltage) or $\frac{120}{\sqrt{3}}$ V (Phase value) is a circle with

$$\text{radius} = \frac{1}{2}\left[\frac{120/\sqrt{3}}{x_q} - \frac{120/\sqrt{3}}{x_d}\right]$$

$$= \frac{1}{2}\left[\frac{120/\sqrt{3}}{6.3} - \frac{120/\sqrt{3}}{10}\right]$$

$$= 2.05$$

This locus is shown as curve D in Graph No.4.

The theoretical current locus diagram is modified when the armature resistance also is considered. Armature resistance of the machine under test, as determined by Kelvin's Double Bridge = 0.356 ohms per phase at 30°C and the modified expression for the wattful current

$$= \frac{V/_2 \sin 2\delta\ (x_d - x_q) + Vr}{x_d x_q + r^2}$$

For different values of δ and

r = 0.356 ohms per phase.

X_d = 10 ohms per phase.

X_q = 6.3 ohms per phase.

V = $120/\sqrt{3}$ volts per phase.

r^2 - negligible

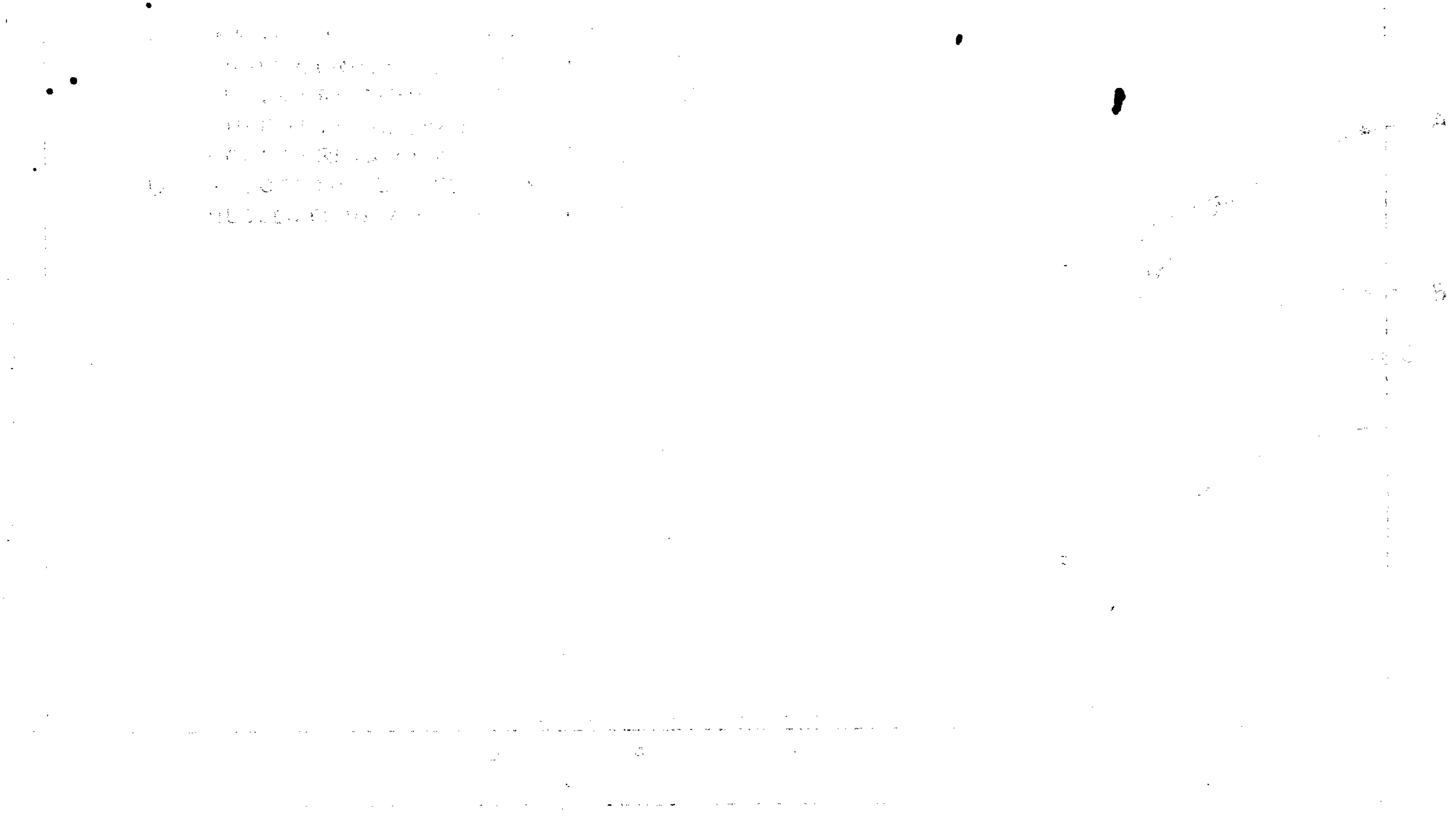
the modified expression for the wattful current

$$= \frac{V}{2} \sin 2\delta\ \left(\frac{x_d - x_q}{x_d x_q}\right) + \frac{Vr}{x_d x_q}$$

$$= \frac{V}{2} \sin 2\delta\ \left(\frac{x_d - x_q}{x_d x_q}\right) + \frac{120}{\sqrt{3}} \times \frac{0.356}{10 \times 6.3}$$

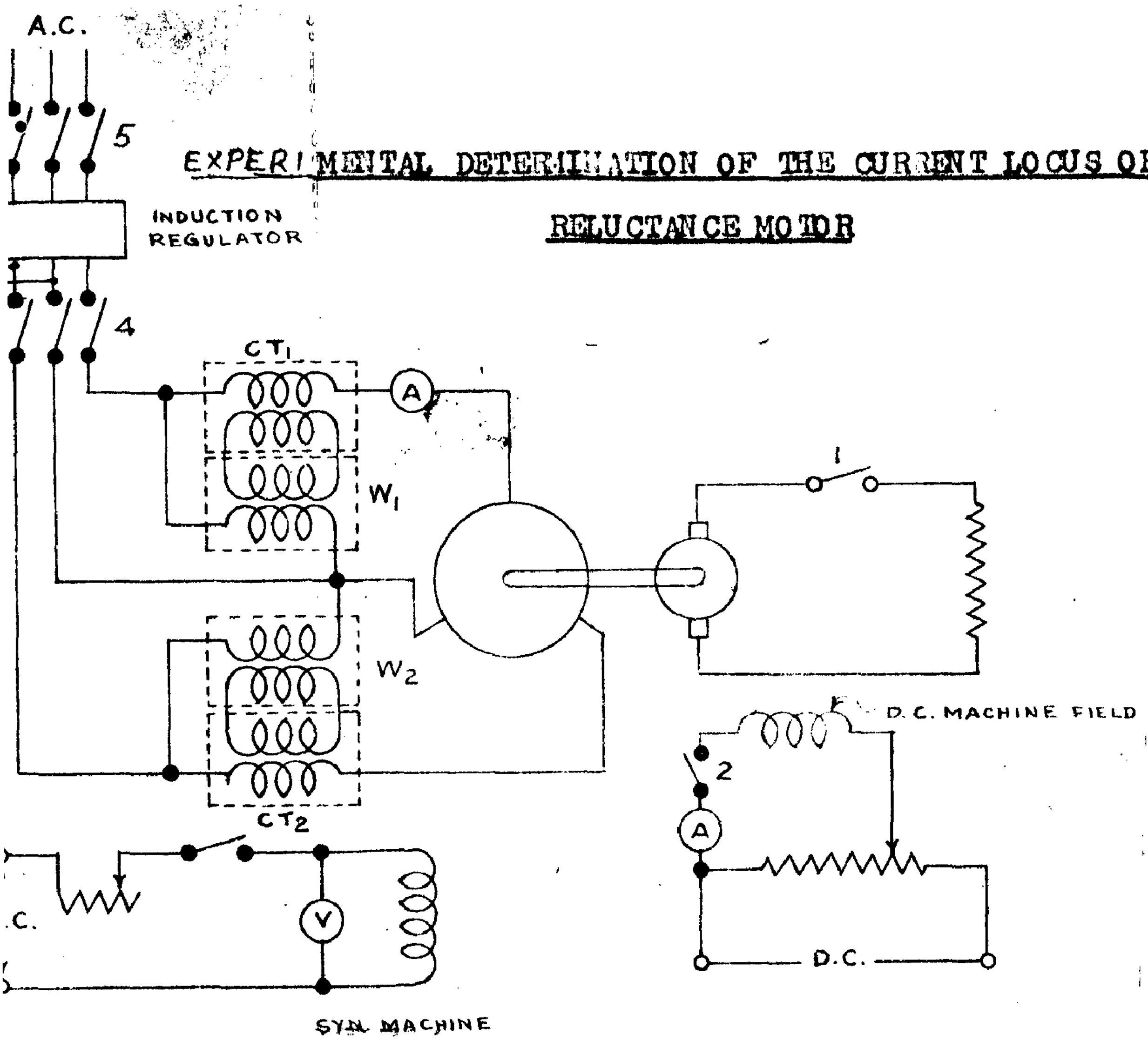
Therefore ordinate of every point on curve D is increased by an amount = $120/\sqrt{3} \times 0.356/ 63$.

By adding these ordinates to all the points on curve D, the curve C on graph No.4 is obtained.



Graph. 4

EXPERIMENTAL DETERMINATION OF THE CURRENT LOCUS OF THE RELUCTANCE MOTOR



The synchronous machine is coupled to a D.C. machine and the D.C. machine is loaded as shown. The resistance load connected across the armature terminals on D.C. side is fixed and the excitation of the D.C. machine is varied.

The synchronous motor is run from the A.C. side by keeping its field open. All the switches on D.C. side are also kept open.

The machine starts as an induction motor due to the damper windings in the field poles and runs as a reluctance motor. In case, some difficulty is experienced to bring the machine into synchronism, it is given the small excitati[on] for a few seconds but removed as soon as the motor falls int[o] step.

Now the load switch 1 and field switch 2 on the D.C. side are closed. Gradually the D.C. field excitation is increased thereby increasing the voltage generated at the D.C. end. Greater voltage across the armature on D.C. side circulates greater current through the fixed resistance load across it, which in turn demands a greater load from the A.C. end.

Wattmeters and ammeter readings are noted, for different D.C. excitations and the following results are obtained, which show the values of currents at different power factors.

The voltage at A.C. end is kept constant at 120V.

Ammeter Reading on A.C. side	$W_1 \times 10$	$W_2 \times 10$	$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$	ϕ
7.2	- 16	52	3.23	73°
7.6	-13.5	56	2.84	70° 36'
8	- 11	62	2.42	67° 36'
8.6	- 9	67	2.27	66° 18'
9.7	- 9	76	2.2	65° 36'
10	-10.5	79	2.27	66° 18'

With the above readings, an experimental current locus of the reluctance motor is obtained. Ref. curve A on Graph No. 4.

In order to get a point on this curve at a still lower power factor, which could not be obtained with the above setting, we proceed as follows.

The A.C. machine is run from a separately excited D.C. motor and brought nearly to synchronous speed. The synchronous machine field is kept open and the voltage is gradually raised on the A.C. side upto 120 volts and the D.C. machine input is so adjusted that the wattmeters indicate the minimum power drawn from the A.C. side.

Under these conditions, a reading recorded showed armature current = 7 amps.

Reading on one wattmeter = -330 watts = W_1.

Reading on the other wattmeter = 350 watts = W_2.

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = 59$$

$$\phi = 89^\circ$$

This point corresponds to the point 'a' on curve A on Graph No.4.

Still another point 'b' on this curve is obtained by running the A.C. machine light at 120 V. The field of the machine is kept open i.e. it runs as a reluctance motor without any load on it. The input under these conditions is only to meet the iron, friction and armature copper losses. The current and power drawn, as observed are as follows:

arematuro current = 7.1 amps.

W_1 = -250 watts

W_2 = 400 watts.

$$\phi = \tan^{-1} \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = 81^\circ 24'$$

Thus the curve A, which is an experimental input current locus, is determined.

EFFECTS OF LOSSES ON EXPERIMENTAL CURRENT LOCUS OF THE RELUCTANCE MOTOR

Had there been no internal losses such as copper, friction and iron losses, in the synchronous machine, the current locus would have been different. We proceed to determine this current locus, when all the losses in the machine are absent. To do this, we must estimate the amount of each one of these losses.

First of all iron and friction losses are determined. For this purpose the synchronous machine is run without any load on it, as a reluctance motor at 120 volts. The motor, as pointed out earlier starts as an induction motor due to dampers, and then pulls into synchronism due to saliency in the poles. So far the field of the machine was kept open but it is now excited. Then the excitation is adjusted until the ammeter shows the minimum current drawn from the mains. Whatever the wattmeter readings, now, will be approximately a measure of iron and friction losses.

Iron and friction losses measured in this way, in this case = 130 watts.

The copper losses can easily be known for different values of the current drawn. The table below gives a complete information to get a current locus without various losses. We know that armature resistance R_a = 0.356 ohms per phase and Iron plus friction losses = 130 watts.

Armature current I amps.	cu Losses $3I^2R_a$ watts	cu Losses plus Iron & friction losses watts	wattful current corresponding to the losses in amps= $\frac{Watts}{\sqrt{3} \times 120}$
7.1	54	184	0.882
7.2	55.5	185.5	0.882
7.6	61.8	191.8	0.922
8	68.4	198.4	0.955
8.6	79.2	209.2	1.0
9.7	90.5	220.5	1.06
10	106.8	236.8	1.14

The wattful components of the currents supplying the losses, tabulated, in the last column above are subtracted from corresponding points on curve A to get the curve B, which is the current locus of the reluctance motor without cu, friction and Iron Losses, as determined experimentally Ref. Graph No.4.

THEORETICAL AND EXPERIMENTAL POWER CURVES OF THE RELUCTANCE MOTORS

Curves A', B', C' and D' on Graph No. 5 are the Power curves of the reluctance motor under test. These are derived from the curves A, B, C & D of Graph No.4.

Curves A' and D' are plotted by noting the wattful current corresponding to an instantaneous value of 2δ, which is found for any point on curves A or D, by joining that point with the centre O'. 2δ is then an angle which the line joining that point with O' makes with OO'.

Curve C' is obtained by adding to curve D' a constant ordinate $\propto \frac{V^2}{X_d X_q}$ (Ref. Page 32)

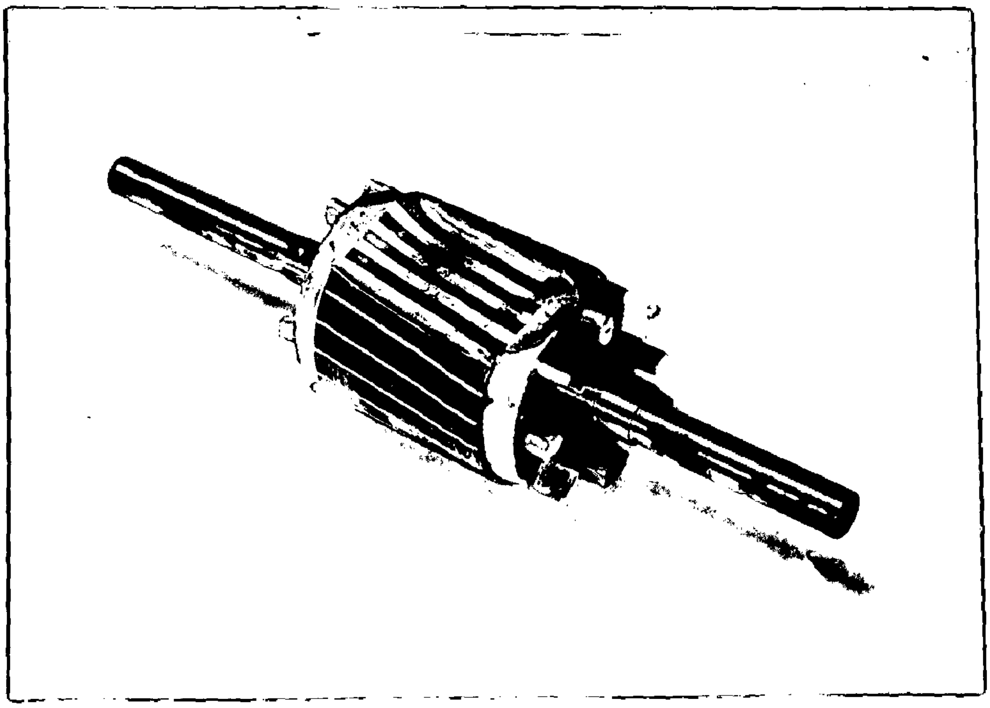
Curve B' is obtained by subtracting from A', the ordinates corresponding to iron, friction and cu losses.

CONCLUSION OF EXPERIMENTAL WORK

The discrepancy in the experimental and theoretical results is firstly due to the effects of residual magnetism, secondly due to the use of ordinary and not the precision type measuring instruments and lastly the supply net work was not stable and the mean values of the instrument readings were taken.

"MANUFACTURING OF RELUCTANCE MOTORS."

The manufacturing of reluctance motors can be easily and economically introduced in small scale industries manufacturing small induction motors. In the squirrel cage rotor of a small induction motor, salient poles are created by removing some iron from diametrically opposite sides of the rotor. The rotor in its new shape can be employed in small reluctance motors, which can be very usefully put in service, where small power output and constant speed is demanded.



Reluctance Motor Rotor

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