# ERROR IDENTIFICATION AND STATE ESTIMATION IN POWER SYSTEM

**A DISSERTATION**

*Submitted in partial fulfilment of the requirements for the award of the degree*

*of*

MASTER OF ENGINEERING

*in*

ELECTRICAL ENGINEERING

(With Specialization in Power System Engineering)



By

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# CANDIDATE'S DECLARATION

I hereby declare that the work presented in this dissertation titled **"ERROR IDENTIFICATION AND STATE ESTIMATION IN POWER SYSTEM"** in partial fulfilment of the requirements for the award of the degree of **MASTER OF ENGINEERING** in **POWER SYSTEM ENGINERING**, submitted in the Department of Electrical Engineering, University of Roorkee, Roorkee is an authentic record of my own work carried out during the period Sept. 1998 to January 1999 under the guidance of **Dr. H.O. Gupta**, Professor, Department of Electrical Engg., and **Dr. S.N. Singh**, Assistant Professor, Department of Electrical Engg., University of Roorkee, Roorkee. The matter embodied in this dissertation has not been submitted by me for the award of any other degree or diploma.

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## CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of our knowledge and belief.

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# ABSTRACT

This dissertation work describe WLS state estimation algorithm, decomposition approach of state estimation, Bad data detection and topological error identification. The algorithm is tested on IEEE-14 bus system. The power Network is decomposed into number of blocks by identifying boundary buses and branches. The boundary buses are modelled as slack, bus for decomposition approach.

# CONTENTS

# CHAPTER 1

# INTRODUCTION

Attempts of precise control of Engineering system proved the validity of paradox "What it appears is not as it is". The crude information obtained by various measurements is insufficient to explain the state of operation of the system due to its inherent errors. This has led to the evolution of statistical estimation theory as a concept of approximate the state variables of a system from its erroneous measurement.

Estimation theory has been extensively used for navigation of air craft and space craft as well as post experimental analysis. In the field of power system, the objective is to provide a reliable and consistent data base for security monitoring. Contingency analysis and system control . To meet the objective SE is required to

* provide a "Best estimate of the bus voltage and angles
* Detect identify and suppress gross measurement errors.
* produced on estimate of non-metered or lost data points.

State estimation was first applied to power system by scheweppe et al. [1,2,3] in 1970 followed by a series of papers [4,11] in the same year. In the early seventies foundation for the computer solution of state estimation were firmly established [1-4]. Short coming of the classical weighted least square formation have been practically overcome during these two decades with the introduction of fast decoupled estimator [5-6] as well as more robust techniques which are less sensitive to numerical ill conditioning [2-9] or bad data. Attempts have been made to make state estimation solution algorithm fast, numerically stable in early eighties. Several algorithm were reported such as normal equation, orthogonal transformation, hybrid method, normal education with constrains (NE/C) and Hachtel's augumented matrix method (HACHTEL) to increase the computational speed of the state estimation [15,20].

Van custem et al [44] and Tripathy et al [45] have suggested two level HSE in which a network if divided into K sub network K+1 solution are obtained. One solution for each area and K+1th solution for interconnecting area formed by boundary nodes and the lines. The first level state estimation provides estimate of local area utilizing its own measurement. The second level state estimator uses the states of boundary buses as pseudo measurements and the measurement of the line flow for state estimation.

Seidu et al [46] had stretched the logic further, to develop coupling equations in respect of the interconnection so that overall effect of the system is reflected on boundary parameters.

In 1993 Iwamato et al [47] had developed HSE mainly based on second order load flow method. Recently a decomposition approach for load flow solution of large system has been reported.

H. SINGH and Liu had developed another method called constrained LAV state estimation using penalty functions. He introduces a simple technique that allows enforcement of inequality constrains in $l_1$ norm problem without any modification in existing program [41].

K.A. elements and P.W. Davis developed an accelerated interior point methods for the least absolute value state estimation in power system [42]. C-N. Lu, R.C. Leoa, K.C. L.U. developed a fuzzy based approach to solve state estimation problem [43].

In 1997 Ali Abur proposed a method to Detect multiple solutions in state estimation in the presence of current magnitude measurements. He analyzed the method by the use of branch current magnitude measurements which leads to extend the observability of a given network [13].

The method of normalized lagrange multipliers to detect topology error was developed by Kevin A. clements. The method is an extension of the normalized residual method. Calculation of normalized lagrange multipliers enables detection of errors in constrains as well as in measurement errors [20].

Load dispatcher in power system control centres is required to know at all times the value of voltages, currents and power throughout the network. Some of the values such as bus voltage magnitude and power line flows can be measured within a certain degree of variance. Difficulties are further encountered when some of data is missing either due to meter being out of order or missing transmission. Moreover, the size of the present day power system is prohibitive to manual calculations or even on a small computer to generate on line missing information state estimation utilizes the available redundancy for systematic cross checking of the measurements, to approximate the states as well as generate information in respect of missing observation or gross measurement errors, called. Bad data. The prerequisite for state estimation is that the system must be observable with the available measurements. The states of power system can also be computed with the load flow calculation based on equal number of measurements, assuming them to be accurate. However, the implicit error will lead to imperfect data base and prejudice the security monitoring whereas, the state estimator is a data processing algorithm for use on a digital computer to transform meter reading (measurement vector) an estimate of the system's state (state vector which is not accurate but the best reliable estimate. A comparison between load flow calculation and state estimation has been shown in fig. 1.1.

The state estimator, apart from security monitoring, bad data and topological error detection and identification has wider applications in central control of power system as shown in fig. 1.2. The state estimate is an essential tool of load dispatchers. the state estimators are classified into three categories.

(i) Static state estimator: It converts observation vector into state vector without regard to past information [15]. Here system changes are considered enough to be assumed static.

(ii) Tracking state estimator : It is a discrete feedback loop which uses real time measurements to track the static state as it varies during the daily load cycle [20]. The comparison of static and tracking state estimator is shown in fig. 1.3. In real sense tracking state estimator extends techniques developed for static state estimation to the time varying case without explict definition of the dynamic models.

(iii) Dynamic state estimator. It is based on time behaviour of the state vector and requires knowledge of past states alongwith the present measurement vector [18]. Power system under normal operating conditions since behave in quasi static manner the state trajectory is discretised in small time intervals. the dynamics state estimation approach is based on Kalman filtering technique, using simplified model of the dynamic behaviour of the power system [19] . This dynamic state estimator in real sense is a tracking estimator with memory, because model is not sufficiently accurate under rapidly changing conditions [13]. A true dynamic models, using magnetic flux linkages in all the synchronous generators in the Network as state vector.

The use of static state estimator in real time operation, security and monitoring has received such a wide acceptance that, unless dynamic or tracking state estimation is specified, state estimation is synonym to static state estimation. The estimator with its functional constituents is illustrated in fig. 1.4.

The state estimator, has since to cater the needs of on line application, computational speed plays a vital role specially when systems are large Newer methods of state estimation are being reported to optimize on (i) Numerical stability, (ii) computation efficiency and (iii) implementation complexity. [15]. Due to large number of interconnections and ever growing demand, the size and complexity of the present day power system have increased tremendously. Therefore it is becoming difficult and time consuming to solve the large and complex power networks.

To solve the large size, interconnection power network. There is need for an efficient decomposition technique.

The aim of the present thesis is to develop an efficient decomposition method for state estimation and to compare the result with weighted least square method. Further Bad data has been analyzed and topological error identification algorithm has been also proposed. The decomposition method [] has been used.

The contents of thesis in remaining chapter are briefly as under.

**CHAPTER II** The state of art of state estimation has been brought out. The weighted least square technique for solving state estimation problem has been used Jacobian has been formulated. the result for IEEE-14 bus is also featured.

**CHAPTER III** It deals with Network decomposition approach [57] for the solution of state estimation problem. The result is tabulated for IEEE 14 bus system using decomposition approach [57].

**CHAPTER IV** The chapter briefly summarizes bad data problem, its detection and observability analysis. For IEEE-14 bus bad data is detected from available measurement data.

**CHAPTER V** Topological error concept has been discussed. Algorithm for detection of topological error is explained. The result is tested on IEEE-14 bus system for two types of topological errors.

**CHAPTER VI** It deals with conclusions.

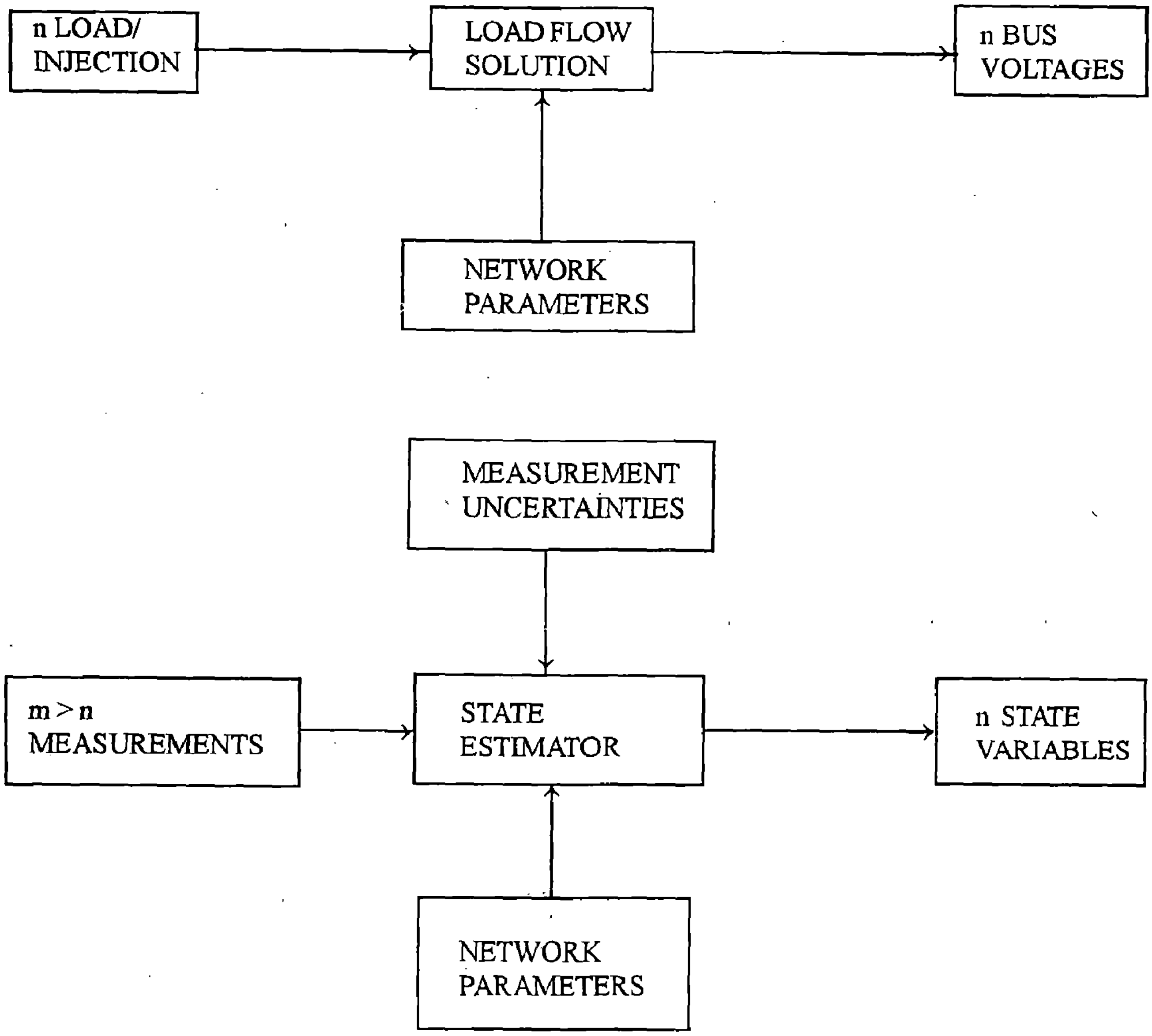



**FIG. 1.1. COMPARISON BETWEEN LOAD FLOW CALCULATIONS AND STATE ESTIMATOR**

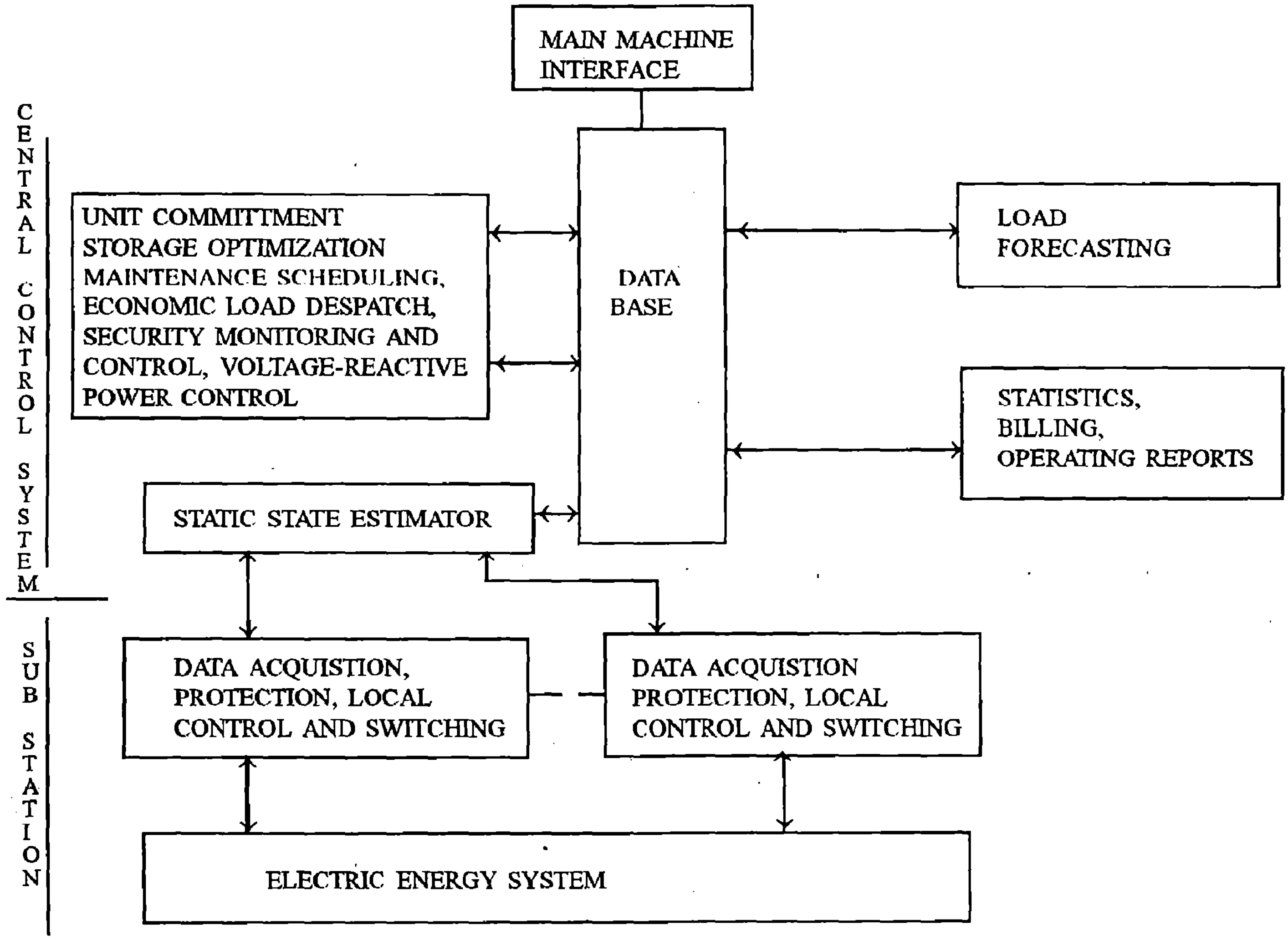



**FIG.1.2 : SCHEMATIC OF LOAD CONTROL CENTRE**

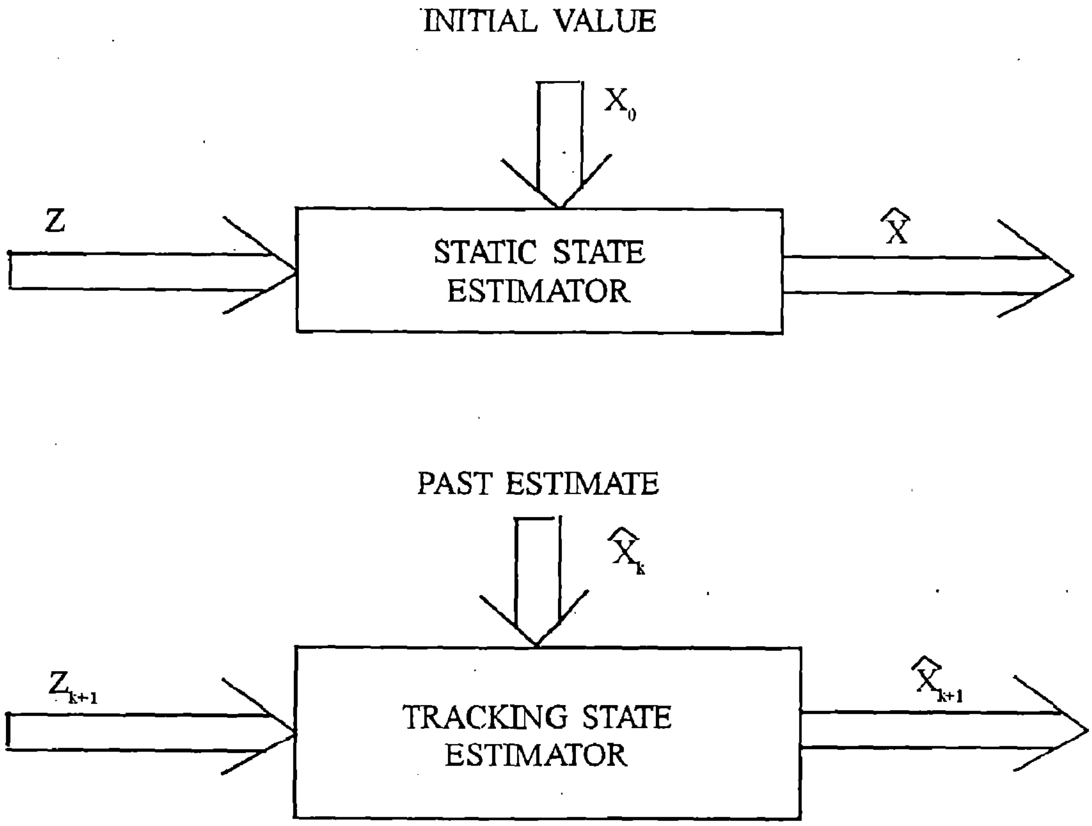



**FIG. 1.3 COMPARISON BETWEEN STATIC AND TRACKING STATE ESTIMATOR**

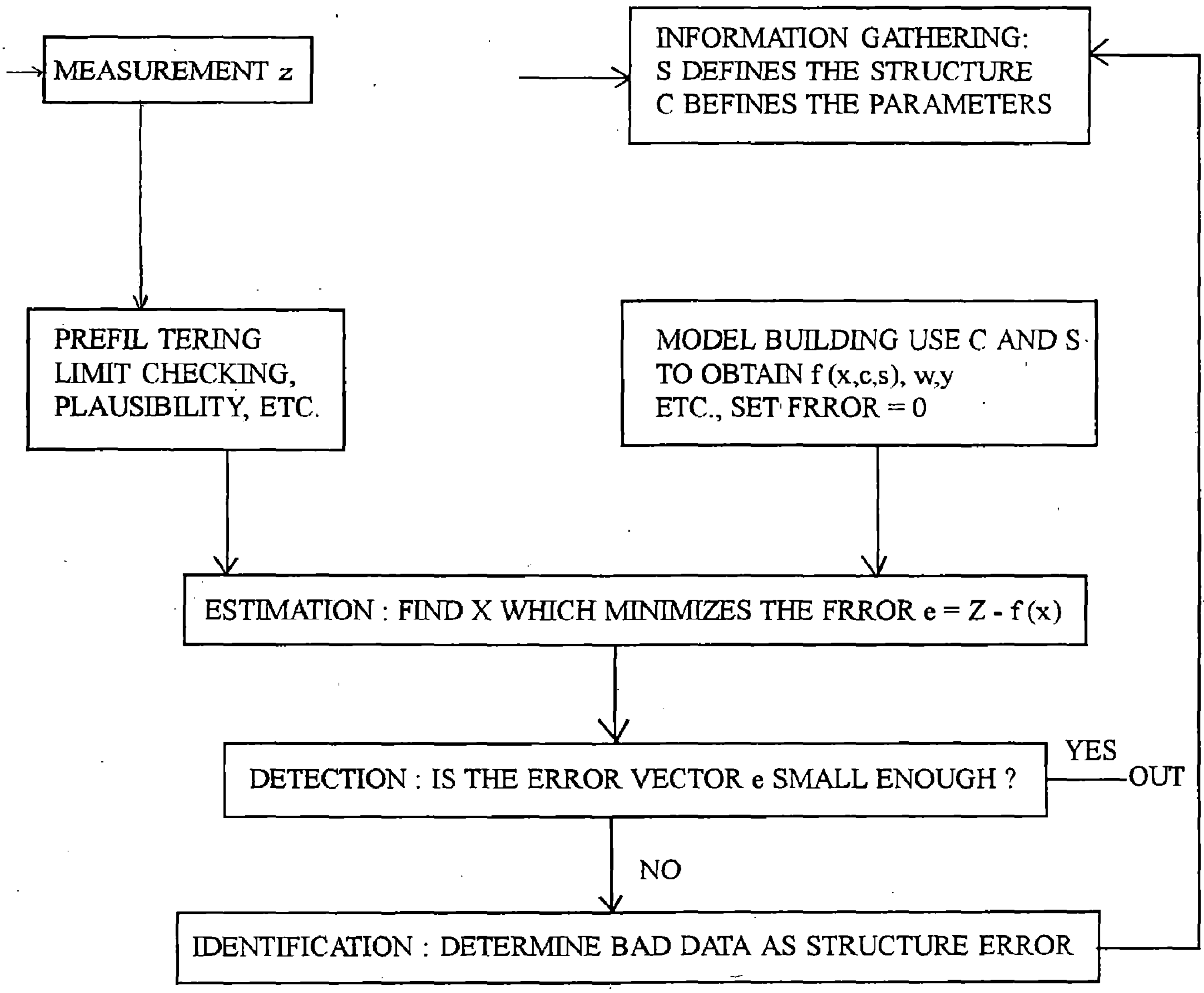



**FIG. 1.4 BASIC STATE ESTIMATOR**

# CHAPTER 2

# STATE ESTIMATION PROBLEM FORMULATION AND SOLUTION ALGORITHM

## 2.1 INTRODUCTION

Power system state estimation is closely related to the regression methods used by statisticians. The non linear equation relating the measurement vector z(m x 1) and state vector x (nx1) are

$$Z = h(x) + \varepsilon$$

where $\varepsilon$ is an (mx1) random noise vector with zero mean Gausion distribution. Traditionally the state estimation is formulated as a WLS Problem and solved by iterative scheme [5]. In this scheme objective is to minimize the sum of the square of the weighted deviation of the estimated measurement Z from the actual measurement Z.

Thus if we are estimating a single parameter x using Nm Measurements, we would write the expression

$$\underset{x}{\mathrm{Min}}\ J(x) = \sum_{i=1}^{Nm} \frac{[Z_i^{Meas} - f_i(x)]^2}{\sigma_i^2} \qquad (2.1)$$

where

$f_i$ = function that is used to calculate the value being measured by the ith measurement

$\sigma^2$ = Variance of ith measurement

J(x) = Measurement residuals

$N_m$ = Number of independent Measurements

$Z_i^{mes}$ = ith measured quantity.

If we were to try to estimate Ns unknown Parameters using Nm Measurements, we would write.

$$\min_{x_1, x_2 \ldots x_{Ns}} J(x_1, x_2, x_3 \ldots x_{Ns}) = \sum_{i=1}^{Nm} \frac{[Z_i - f_i (x_1, x_2 \ldots x_{Ns})]^2}{\sigma_i^2} \quad (2.2)$$

**MATRIX FORMULATION:**

If the functions fi $(x_1, x_2, \quad X_{Ns})$ are linear functions, eq$^n$. (2.2) has a closed form solution. Let us write the function fi $(x_1, x_2 \quad X_{Ns})$ as

fi $(x_1, x_2, \quad XNS)$ = fi (X) = $hi_1 x_1 + hi_2 x_2 + \quad hi_{Ns} X_{Ns}$ Then if we place all the fi functions in a vector, we may write.

$$f(x) = \begin{bmatrix} f_1 (x) \\ f_2 (x) \\ ! \\ f_{Nm}(x) \end{bmatrix} = [H] \, x$$

where

[H] = an Nm by NS Matrix Containing Coefficients of linear functions $f_i(x)$

$N_m$ = No of Measurements

$N_s$ = No of state variables

placing the measurements in vector.

$$Z^m = \begin{bmatrix} Z_1^{\ m} \\ Z_2^{\ m} \\ ! \\ Z_{Nm}^{\ m} \end{bmatrix}$$

Now Eq$^n$ (2.2) Can be written in a compact form as

$$\underset{x}{Min}\ J(x) = [Z^m - f(x)]^T . [R]^{-1} . [Z^m - f(x)]$$

$$\text{where. } R = \begin{bmatrix} \sigma_1^2 & & \\ & \sigma_2^2 & \\ & & \sigma_{NS}^2 \end{bmatrix}$$

[R] is called the Co-variance matrix of Measurement errors For minimum of J(x). It is essential that.

$$\frac{\partial J(x)}{\partial (x_1)} = 0 \text{ for } i = 1.. Ns$$

$$\text{Now } J(x) = [Z-[H]x]^T R^{-1} [Z-[H]x]$$

$$= Z^T Z - x^T [H]^T Z - Z^T [H] x + x^T [H]^T [H] x.$$

$$= Z^T Z - 2z^T [H] x + x^T [H]^T [H] x.$$

Now $\nabla J(x) = 0$ gives

$$X^{est} = [H^T R^{-1} H]^{-1} . [H^T].[R^{-1}] [Z^m] \quad (2.3)$$

Eq$^n$ (2.3) holds true for NS <Nm

when NS = Nm then

$$x^{est} = [H]^{-1} . Z^m \quad (2.4)$$

When NS > Nm, then still there exist a closed form sol$^n$ but in this case one is not estimating x to maximize a likelihood function since NS > Nm usually implies that many different values for $x^{est}$ can be found that cause fi ($x^{est}$) to equal $z^m$ for all i = 1 Nm, exactly. Rather the objective is to find $x^{est}$ such that the sum of squares of $x_1^{est}$ can be minimized i.e.

$$\underset{x}{\text{Min}} \sum_{i=1}^{Ns} x_i^2 = x^T.x.$$

subject to condition that $Z^m = [H]\, x$ the closed form sol$^n$ for this case is

$$x^{est} = [H]^T.\ [H.H^T]^{-1}.\ Z^m \qquad (2.5)$$

In power system state estimation the under determined problem (i.e. Ns > Nm) are not solved by eq$^n$ (2.5). Rather "pseudo measurements" are added to the measurement set to give a completely determined or overdetermined problem.

| Case | Description | Closed form solution |
|---|---|---|
| Ns < Nm | overdetermined | $X^{est} = [H^T.\ R^{-1}.\ H]^{-1}.[H^T][R^{-1}].Z^m$ |
| Ns = Nm | Completely determined | $X^{est} = [H^{-1}]\ Z^m$ |
| Ns > Nm | Under determined | $X^{est} = [H^T]\ [H.H^T]^{-1}.Z^m$ |

If the relationship between the states and power flow is not linear then we have to resort to an iterative technique to minimize J(x). A commonly used technique is to calculate gradient of J(x) and force it to zero using newton's method.

$$\text{if} \quad \underset{x}{\text{Min}}\ J(x) = \sum_{i=1}^{Nm} \frac{[Zi - fi\ (x)]^2}{\sigma i^2}$$

Then gradient of J(x) is given by

$$\nabla_X J(x) = \begin{bmatrix} \frac{\partial J(x)}{\partial x_1} \\ ! \\ \frac{\partial J(x)}{\partial x_1} \end{bmatrix}$$

$$= -2 \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \cdots \\ \frac{\partial f_1}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma_1^2} & \\ & \frac{1}{\sigma_2^2} \end{bmatrix} \begin{bmatrix} Z_1 - f_1(x) \\ Z_2 - f_2(x) \end{bmatrix}$$

$$\text{or } \nabla J(x) = \left[ -2[H]^T [R]^{-1} \begin{bmatrix} (Z_1 - f_1(x) \\ (Z_2 - f_2 (x) \end{bmatrix} \right] \tag{2.6}$$

$$\text{where } H = \frac{\partial f(x)}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \\ \frac{\partial f_1}{\partial x_2} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_3}{\partial x_2} \end{bmatrix} = \text{JaCobian of } f(x).$$

$$\text{and } H^T = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_2} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \\ \vdots & \vdots & \vdots \end{bmatrix}$$

According to newton's method $eq^n$ (2.6) becomes

$$\Delta x = \left[ \frac{\partial \nabla x.j(x)}{\partial x} \right] \left[ -\nabla x.J(x) \right]$$

$$\frac{\partial}{\partial x}\left[\nabla x J(x)\right] = \frac{\partial}{\partial x}\left[-2H^T . R^{-1} . \begin{bmatrix} Z_1 - f_1 (x) \\ Z_2 - f_2 (x) \end{bmatrix}\right]$$

$$= - 2H^T . R^{-1} [-H]$$

$$\Delta x = [2H^T R^{-1} . H]^{-1} \left[2H^T R^{-1} \begin{bmatrix} Z_1 - f_1 (x) \\ Z_2 - f_2 (x) \end{bmatrix}\right]$$

$$\text{or } \Delta x = [A]^{-1} . [b] \qquad (2.7)$$

and $[b] = H^T . R^{-1} . [Z_1^m - fi (x)]$ for $i = J$ Nm

$[A] = H^T R^{-1} . H$

Eq$^n$ (2.7) is a set of linear equations, if higher order terms of the Taylor series were really negligible. The solution would yield correct x . A Jacobian H is itself a function of x. We must view eq$^n$ (2.7) as a prescription for an iterative procedure which in a finite number of steps will compute x to a certain degree of accuracy vector x should therefore be changed accordingly [49].

$$x^{i+1} = x^1 + (H_I^T W H_I)^{-1} * H_I^T * (Z - f_I)$$

$$= x^{(I)} + \Delta x^{(I)} . \text{ (I} \Rightarrow \text{iteration count)} \qquad (2.8)$$

Until convergence is reached. This is weighted least squares minimization.

## 2.2 JACOBIAN FORMULATION

No of state variables = 2 * total no of bus - 1

= 2 * n-1

total No of Measurements = m

these measurement may include quantities as

$[p_1, q_i, P_{iJ}, q_{iJ}$. etc]

Pi, qi - Real and imaginary part of injected power respectively

PiJ, qiJ- Real and imaginary part of line respectively

Dimension of Jacobian matrix = m * (2n-1)

m - rows

2n-1 column

$\Delta Z = Z^{measured} - Z^{calculated}$

So in general

$$\begin{bmatrix} \Delta Pi \\ \Delta Qi \\ \Delta PiJ \\ \Delta QiJ \end{bmatrix} = \begin{bmatrix} H_1 & H_2 \\ H_3 & H_4 \\ H5 & H6 \\ H_7 & H_8 \end{bmatrix} \begin{bmatrix} \Delta \delta i \\ \Delta Vi \end{bmatrix} \quad (2.9)$$

injected real power $P_i = PG_i - P_{Di}$

injected imaginary power $Q_i = QG_i - Q_{Di}$

where subscripts G and D denotes Generation and demand respectively.

$$S_i = P_i + J\,Q_i = V_i.\ I_i^*$$

$$= Pi - JQi = V_1^*.I_i \quad (2.10)$$

(Si, Vi & Ii are complex quantities)

XiJ is the element from admittance matrix

$$P_i = \sum_{J=1}^{N} |V_i| V_j| \, |Y_{ij}J| \cos(\delta_i - \delta_j - \theta_{ij}) \quad (2.11)$$

$$Q_i = \sum_{j=1}^{N} |V_i||V_j| \; Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij}) \tag{2.12}$$

$$H_1 = \begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \frac{\partial P_1}{\partial \delta n-1} \\ \frac{\partial P_{n-1}}{\partial \delta_i} & \frac{\partial P_{n-1}}{\partial \delta n-1} \end{bmatrix} = \frac{\partial P_i}{\partial \delta_j}$$

$H_{1(ii)}$ diagonal terms is by differentiating eq[n] (2.11) with respect to delta ($\delta$) we get.

$$H_{1(ii)} = - \sum_{J=1}^{N} |V_i||V_j|.|Y_{ij}|. \text{ Sin } (\delta_i - \delta_j - \theta_{ij}) \tag{1.12}$$

$$= - |V_i|^2. \; |Y_{ii}| \sin \theta_{ii} - \sum_{J=1}^{N} |V_i| \; |V_i| \; |Y_{ij}| \sin (\delta_i - \delta_j - \theta_{ij})$$

$$H_{1(ii)} = - |V_i|^2. \; B_{ii} - Q_i^{cal} \tag{2.13}$$

$$H_{1(iJ)} = \frac{-\partial P_i}{\partial \delta_j} = |V_i||V_j||Y_{ij}| \sin (\delta_i - \delta_j - \theta_{ij}). \tag{2.14}$$

Similarly $H_3 = \frac{\partial Q_j}{\partial \delta_j}$

differentiating Eq. (1.12) and solving as above.

$$H_{3(ii)} \text{ (diagonal elements)} = P_i^{cal} - G_{ii} \; |V_i|^2 = \frac{\partial Q_i}{\partial \delta_i} \tag{2.15}$$

$Bii = Y_{ii} \sin \theta_{ii}$, $G_{ii} = Y_{ii} \cos \theta_{ii}$

$$H_{3(ij)} = - |V_i| \; |V_j|. \; |Y_{ij}| \cos (\delta_i - \delta_j - \theta_{ij}) \tag{2.16}$$

$$H_2 = \frac{\partial P_i}{\partial |V_i|};$$ by differentiating Eq. (2.11) with respect to $v_i$ and simplifying will yield.

$$\text{diagonal, } H_{2_{(ii)}} = (P_I^{cal} + G_{ii}\ |V_i|^2)/|V_i| = \frac{\partial P_i}{\partial |V_i|} \quad (2.17)$$

$$H_{2_{(ij)}} = |V_i||Y_{iJ}| \cos(\delta_i - \delta_j - \theta_{ij}) \quad (2.18)$$

Similarly for $H_{4_1}$ differentiating Eq. (2.12).

$$\text{diagonal } H_{4(ii)} = (Q_i^{cal} - B_{ii}.\ |V_i|^2)/|V_i| = \frac{\partial Q_i}{\partial |V_i|}$$

$$H_{4(ij)} = |V_i||Y_{ij}| \sin(\delta_i - \delta_J - \theta_{iJ}) - \quad (2.19)$$

**JACOBIAN FOR LINE FLOW**

$Y_{iJ}$ Primitive element admittance between ith and Jth bus.

$Y_{J0}$ and $Y_{i0}$ $\Rightarrow$ Shunt admittance from corresponding bus to ground respectively

Complex line flow from ith to Jth bus ($S_{iJ}$) is,

$$S_{iJ} = V_i\ I_i^*$$

$$= V_i\ [(V_i^* - V_J^*).\ Y_{iJ} + V_i^*.\ Y_{i0}^*]$$

$$V_i = |V_i|\angle \delta_i\ ,\ V_J = |V_J|.\angle\delta_J,\ Y_{iJ} = |Y_{iJ}|\angle \theta_{iJ},\ Y_{i0} = |Y_{i0}|\angle \alpha_{i0}$$

$$\therefore \quad S_{iJ} = |V_i|\angle \delta_i\ [(|V_i|\angle{-\delta_i} - |V_J|\angle{-\delta_J})\ |Y_{iJ}|\angle{-\theta_{iJ}} + |V_i|\angle{-\delta_i}.|Y_{i0}|\angle{-\delta_{i0}}]$$

$$= |V_i^2|\ |Y_{iJ}|\ \angle{-\theta_{iJ}} - |V_i||V_J||Y_{iJ}|\ \angle\delta_i - \delta_J - \theta_{iJ} + |V_i^2||Y_{i0}|\ \angle{-\alpha_{i0}}$$

$$\therefore P_{iJ} = |V_i|^2\ |Y_{iJ}| \cos(\theta_{iJ}) - |V_i||V_J||Y_{iJ}| \cos(\delta_i - \delta_J - \theta_{iJ}) \quad (2.20)$$

$$Q_{iJ} = |V_i|^2|Y_{iJ}| \sin(-\theta_{iJ}) - |V_i||V_J||Y_{iJ}| \sin(\delta_i - \delta_J - \theta_{iJ}) \quad (2.21)$$

Jacobian $H_5$ (Real power flow)

$$H_5 = \frac{\partial P_{iJ}}{\partial \delta_i} = |V_i|V_J| \ |V_{iJ}| \ \sin(\delta_i - \delta_J - \theta_{iJ}) \text{ in column (i)} \quad (2.22)$$

$$= \frac{\partial P_{iJ}}{\partial \delta_J} = -|V_i|V_J| \ |V_{iJ}| \ \sin(\delta_i - \delta_J - \theta_{iJ}) \text{ in column (J)} \quad (2.23)$$

Jacobian $H_6$ (Real power flow)

Differentiating eq$^n$ (2.20) with respect $|V_i|$ and $|V_J|$

$$H_6 = 2 \ |V_i||Y_{iJ}| \cos\theta_{iJ} - |V_J| \ |Y_{iJ}| \cos(\delta_i - \delta_J - \theta_{iJ}) \text{ in column (i)} \quad (2.24)$$

$$= - |V_i||V_J| \cos(\delta_i - \delta_J - \theta_{iJ}) \text{ in column (J)} \quad (2.25)$$

Reactive power flow Jacobian

$$H_7 = - |V_i||V_J| \ |Y_{iJ}| \cos(\delta_i - \delta_j - \theta_{iJ}) \text{ in column (i)} \quad (2.26)$$

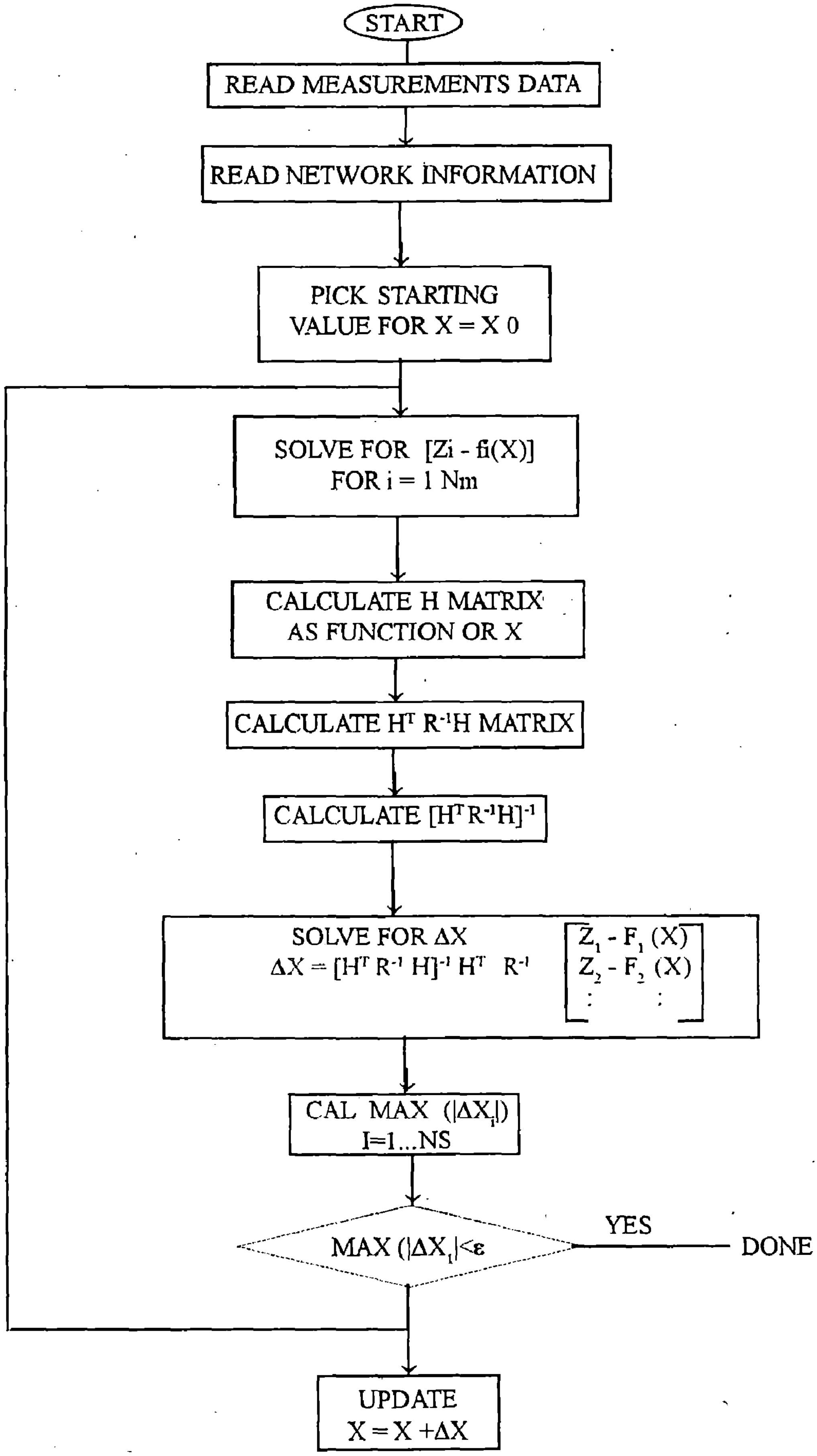
$$= |V_i||V_J| \ |Y_{iJ}| \cos(\delta_i - \delta_j - \theta_{iJ}) \text{ in column (J)} \quad (2.27)$$

$$H_8 = 2 \ |V_i||Y_{iJ}| \sin(-\theta_{iJ}) - |V_i| \ |Y_{iJ}| \sin(\delta_i - \delta_J - \theta_{iJ}) \text{ in column (i)} \quad (2.28)$$

$$= - |V_i||Y_{iJ}| \sin(\delta_i - \delta_J - \theta_{iJ}) \text{ in column (J)} \quad (2.29)$$

For line flow Jacobian, each Jacobian ($J_5$- $J_8$) has only two entries, one is at ith column and other is at Jth column.

**STATE ESTIMATION SOLUTION ALGORITHM**



## 2.4 RESULT

TABLE 2.1

| BUS NUM | ESTIMATED CASE VOLTAGE (P.U) | ESTIMATED CASE ANGLE (degree) | BASECASE VOLTAGE (P.U) | BASECASE ANGLE (degree) |
|---|---|---|---|---|
| 1 | 1.057 | 0.000 | 1.0600 | 0.000 |
| 2 | 1.043 | -4.495 | 1.0450 | -4.5158 |
| 3 | 1.075 | -11.535 | 1.0700 | -11.6712 |
| 4 | 1.007 | -11.839 | 1.0100 | -11.9482 |
| 5 | 1.103 | -4.272 | 1.0900 | -12.0492 |
| 6 | 1.077 | -11.836 | 1.0638 | -12.0491 |
| 7 | 1.067 | -13.168 | 1.0568 | -13.4228 |
| 8 | 1.029 | -7.812 | 1.0296 | -7.9038 |
| 9 | 1.024 | -9.265 | 1.0248 | -9.4149 |
| 10 | 1.061 | -13.152 | 1.0514 | -13.4012 |
| 11 | 1.065 | -12.464 | 1.0569 | -12.6759 |
| 12 | 1.061 | -12.423 | 1.0554 | -12.5973 |
| 13 | 1.057 | -12.574 | 1.0503 | -12.7529 |
| 14 | 1.038 | -13.969 | 1.0359 | -14.1332 |

ACTIVE POWER INJECTION ESTIMATION

| BUS NUM. | MEASURED VALUE ( P.U.) | ESTIMATED VALUE (P.U.) |
|---|---|---|
| 1 | 2.115 | 2.111 |
| 2 | 0.183 | 0.173 |
| 3 | -0.942 | -0.938 |
| 4 | -0.478 | -0.468 |
| 5 | -0.076 | -0.083 |
| 6 | -0.091 | 0.078 |
| 7 | -0.295 | -0.289 |
| 8 | -0.090 | -0.096 |
| 9 | -0.035 | -0.031 |
| 10 | -0.060 | -0.062 |
| 11 | -0.135 | -0.125 |
| 12 | -0.149 | -0.153 |

REACTIVE POWER INJECTION ESTIMATION

| | | |
|---|---|---|
| 1 | -0.171 | -0.173 |
| 2 | 0.180 | 0.181 |
| 3 | 0.000 | 0.007 |
| 4 | 0.039 | 0.036 |
| 5 | 0.162 | 0.162 |
| 6 | -0.016 | -0.012 |
| 7 | 0.114 | 0.119 |
| 8 | -0.166 | -0.167 |
| 9 | -0.058 | -0.057 |
| 10 | -0.017 | -0.020 |
| 11 | -0.016 | -0.014 |
| 12 | -0.058 | -0.064 |

ACTIVE POWER FLOW ESTIMATION

| | | | |
|---|---|---|---|
| 1 | 2 | 1.427 | 1.431 |
| 2 | 1 | -1.393 | -1.395 |
| 1 | 8 | 0.680 | 0.680 |
| 8 | 1 | -0.657 | -0.657 |
| 2 | 4 | 0.705 | 0.701 |
| 4 | 2 | -0.684 | -0.679 |
| 2 | 9 | 0.510 | 0.507 |
| 9 | 2 | -0.497 | -0.494 |
| 2 | 8 | 0.361 | 0.360 |
| 8 | 2 | -0.355 | -0.353 |
| 4 | 9 | -0.259 | -0.258 |
| 9 | 4 | 0.265 | 0.263 |
| 9 | 8 | -0.610 | -0.623 |
| 8 | 9 | 0.615 | 0.628 |

| | | | |
|---|---|---|---|
| 9 | 6 | 0.245 | 0.245 |
| 6 | 9 | -0.245 | -0.245 |
| 9 | 7 | 0.145 | 0.141 |
| 7 | 9 | -0.145 | -0.141 |
| 8 | 3 | 0.289 | 0.300 |
| 3 | 8 | -0.289 | -0.300 |
| 3 | 11 | 0.109 | 0.106 |
| 11 | 3 | -0.108 | -0.105 |
| 3 | 12 | 0.078 | 0.081 |
| 12 | 3 | -0.078 | -0.081 |
| 3 | 13 | 0.195 | 0.189 |
| 13 | 3 | -0.194 | -0.187 |
| 6 | 7 | 0.245 | 0.245 |
| 7 | 6 | -0.245 | -0.245 |
| 7 | 10 | 0.018 | 0.022 |
| 10 | 7 | -0.019 | -0.022 |
| 7 | 14 | 0.073 | 0.075 |
| 14 | 7 | -0.073 | -0.074 |
| 10 | 11 | -0.073 | -0.074 |
| 11 | 10 | 0.072 | 0.075 |
| 12 | 13 | 0.020 | 0.010 |
| 13 | 12 | -0.020 | -0.018 |
| 13 | 14 | 0.078 | 0.081 |
| 14 | 13 | -0.078 | -0.003 |

REACTIVE POWER FLOW ESTIMATION

| | | | |
|---|---|---|---|
| 1 | 2 | -0.171 | -0.171 |
| 2 | 1 | 0.220 | 0.222 |
| 1 | 8 | -0.001 | -0.001 |
| 8 | 1 | 0.039 | 0.039 |
| 2 | 4 | 0.004 | 0.039 |
| 4 | 2 | 0.001 | 0.005 |
| 2 | 9 | -0.046 | -0.046 |
| 9 | 2 | 0.048 | 0.048 |
| 2 | 8 | -0.033 | -0.033 |
| 8 | 2 | 0.017 | 0.017 |
| 4 | 9 | 0.003 | 0.002 |
| 9 | 4 | -0.003 | -0.027 |
| 9 | 8 | 0.086 | 0.082 |
| 8 | 9 | -0.083 | -0.080 |
| 9 | 6 | -0.074 | 0.074 |
| 6 | 9 | 0.087 | -0.087 |
| 9 | 7 | -0.006 | -0.006 |
| 7 | 9 | 0.003 | -0.003 |
| 8 | 3 | 0.011 | -0.011 |
| 3 | 8 | 0.010 | -0.009 |
| 3 | 11 | 0.019 | 0.020 |
| 11 | 3 | -0.017 | -0.018 |
| 3 | 12 | 0.022 | 0.023 |
| 12 | 3 | -0.021 | -0.021 |
| 3 | 13 | 0.064 | 0.067 |

| | | | |
|---|---|---|---|
| 13 | 3 | -0.059 | -0.062 |
| 6 | 5 | -0.158 | -0.158 |
| 5 | 6 | 0.162 | 0.162 |
| 6 | 7 | 0.071 | 0.071 |
| 7 | 6 | -0.065 | -0.064 |
| 7 | 10 | 0.060 | 0.059 |
| 10 | 7 | -0.060 | -0.059 |
| 7 | 14 | 0.048 | 0.047 |
| 14 | 7 | -0.046 | -0.045 |
| 10 | 11 | 0.002 | 0.002 |
| 11 | 10 | -0.001 | -0.001 |
| 12 | 13 | 0.006 | 0.007 |
| 13 | 12 | -0.006 | 0.006 |
| 13 | 14 | 0.060 | 0.050 |

Maximum error = 0.008

# CHAPTER - 3

## DECOMPOSITION APPROACH FOR STATE ESTIMATION

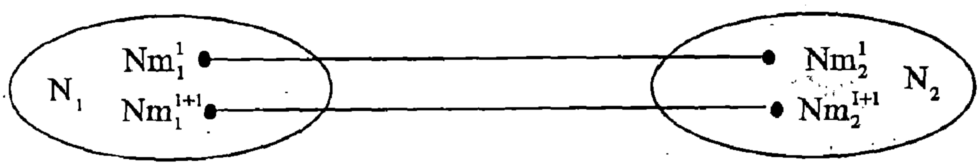
Due to large number of interconnection and ever growing demand, the size and complexity of the present day power system, have increased Tremendously. Therefore it becomes difficult and time consuming to solve the large and complex power networks. To solve the large size inter-connected power network, there is need for an efficient deomposition technique [48],[51].

The existing methods decompose the large network into small sub network and these sub networks solved independently. The solution of different sub networks are coordinated to get the solution of complete network. the decompostion method used by Subhas Joshi [51] is implemented for the method descirbed in chapter No. 3.

### 3.1 DECOMPOSITION METHOD FOR POWER NETWORK [51]

Assume that a large power network $N^o$ is decomposed into sub networks $N^1$, $N^2$ as shown in Fig. 3.1. These sub networks are called block.

A boundary bus of the block under consideration, through its boundary lines is known as external boundary bus, to the block under consideration. As shown in Fig. 3.1 block $N^1$ is connected to buses $Nm_{2'}^2, Nm_{2-1}^2$ of Block $N^2$. These buses are the external boundary buses of block $N^1$.



In this method with the pre-estimated states of external boundary buses state estimation is carried out for an area. This provides sub optimal states of internal boundary buses of the area under interaction . These sub optimal estimates serve as pre estimated states of external boundary buses of the connected area. One complete cycle of inter area changes from one co-ordinating cycle or global solution.

The set of buses used in local state estimation of Ith area comprise of the internal buses of the ith area and external boundary buses of the connected area and is expressed as

$$B^I = B^I_I \cup B^I_{bII} \; . \; . \; . \; . \; \cup B^{I-1}_{bI-1,I} \cup Bb^{I+1}_{I+1} \; . \; . \; . \; . \cup B^n_{b,I} \qquad (3.1)$$

Similarly set of lines used in local state estimation of ith area are the internal ines of the Ith area and lies lines connected to the Ith area -

$$L^I = L^I_I \cup L^I_{t_{1,I}} \; . \; . \; . \; . \cup L^{I-1}_{t_{I-1,I}} \cup L^{I+1}_{t_{I+1,I}} \; .... \cup L^n_{t_{n,I}} \qquad (3.2)$$

Likewise measurements are the internal measurements of the area, measurements on the lines connected to the Kth area and prestimate states of the external boundary buses as pseudo measurements.

$$M^I = M^I_I \cup \left\{ M^I_{t_{1,I}} \; . \; . \; . \; . \cup M^{I-1}_{t_{I-2,II,I}} \cup M^{I+1}_{t_{I+1,I}} \; . \; . \; . \cup M^n_{t_{n,I}} \right\}$$

$$\cup \left\{ \hat{xb}^I_{II} \cup \hat{xb}^{I-1}_{I-1}, \cup \hat{xb}^{I+1}_{I+1} \; . \; . \; . \cup xb^n_{n,I} \right\}$$

The algorithm has been shown in fig. 3.1

Step wise solution procedure of this method is [5]

(i) Read system data and decomposed the network into N blocks prepare block data and boundary data.

(2) Read boundary data, internal and external boundary buses of each area and the tie lines.

(3) Set Max$^{m}$ iteration count for global solution $K_{max}$.

(4) Set K $\longleftarrow$ 1

(5) Initialize convergence tolerance $\in$ for global solution.

(6) Read measurement data and sort them blockwise

(7) Read data of ith block

(8) Update V and $\delta$ of external boundary buses of ith block from boundary data table.

(9) Perform state estimation of ith block using solution step given in (2.2)

(10) Uptdate boundary bus data V and $\delta$ corresponding to the internal boundary bus of ith block

(11) IF $I \neq N$ then $I \leftarrow I + 1$ go to Step 1.

(12) Increment global iteration count $K \leftarrow K + 1$

(13) If Maxi $|\Delta xb^{i}| < \in$ thin transfer states to database. initialize global iteration count K $\leftarrow$ goto step 6.

(19) IF $K > K_{max}$ modify convergence tolerance. Goto step 7.

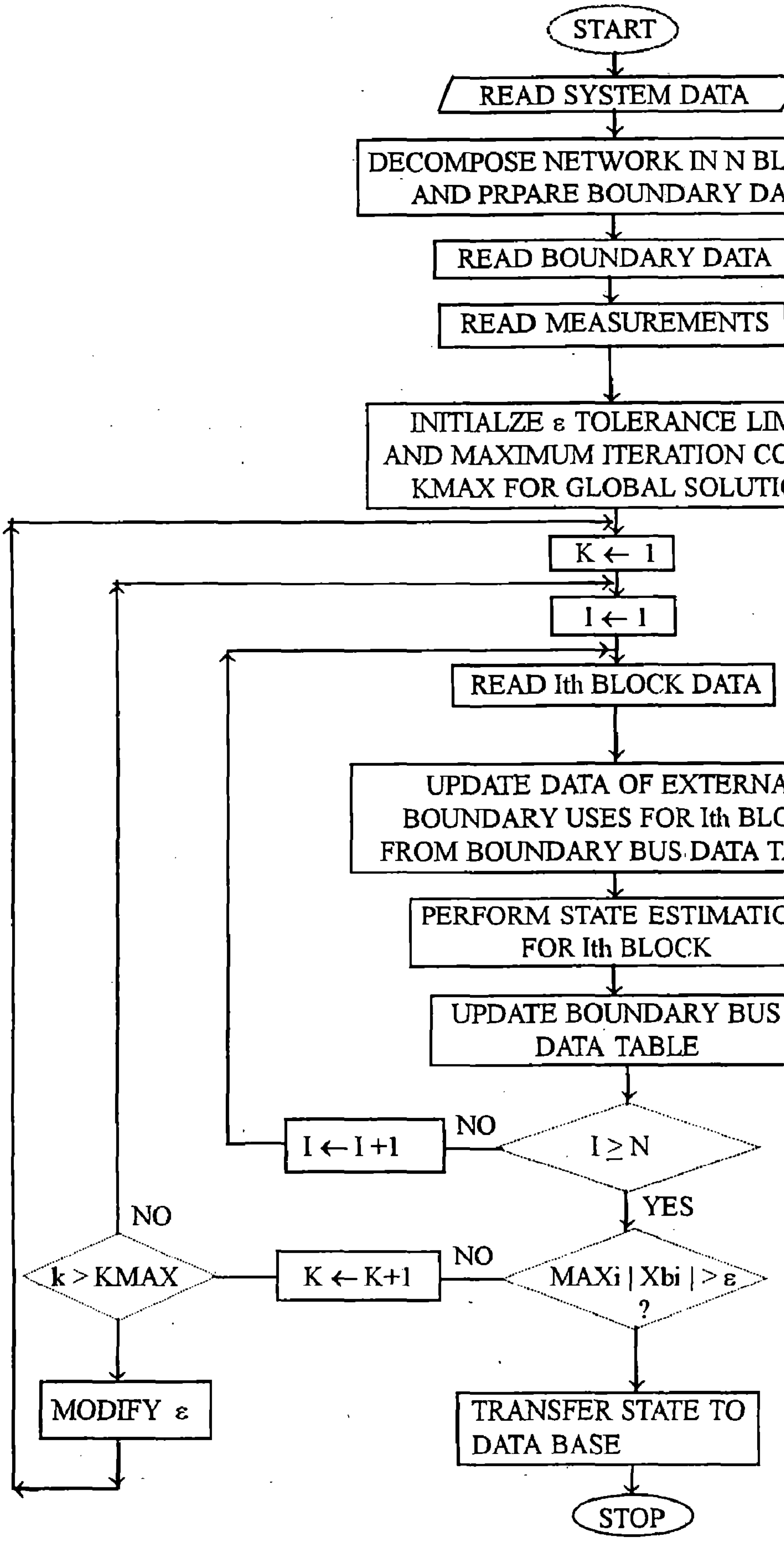



**FIG. 3.1 ALGORITHM OF STATE ESTIMATION BY DECOMPOSITION [51]**

### 3.2 DISCUSSION OF RESULT

The decomposition method [51] is tested on IEEE 14 bus system. The Fig. 3.2 shows IEEE 14 bus systrem. Line x-xx shows the decomposition of Network into two blocks. Block 1 contains 5 buses and Block 2 contains 9 buses. Bus numbers 5 and 10 are external boundary buses for the Block 1 and Bus number 6, 11 and 14 are the external boundary buses for the block 2. The external boundary buses are medelled as slack buses in solving the blocks.

Table 3.1 and 3.2 the estimated result fo the IEEE 14 bus. Table 3.3 gives the comparison of CPU time when solved as decomposed Network and intact system for the state estimation solution. As Block 1 contains 5 buses in which 3 buses are modelled as swing bus so size of Jacobian will be (2x5-3) x $m_1$ where $m_1$ = no of measurements vectors. Similarly for Block 2 out of 9 buses two are modelled as swing bus so size of Jacobian will be (2x 9-2) x $m_2$ where $m_2$ = no of measurements vector.

### 3.3 CONCLUSION

The decomposition method [51] leads to large saving in computational time and memory. The method enables to solve the large size power Network with a smaller computer. The method described in chapter 3 is used to demostrate the suitability of the state estimation method with the decomposition method [51].

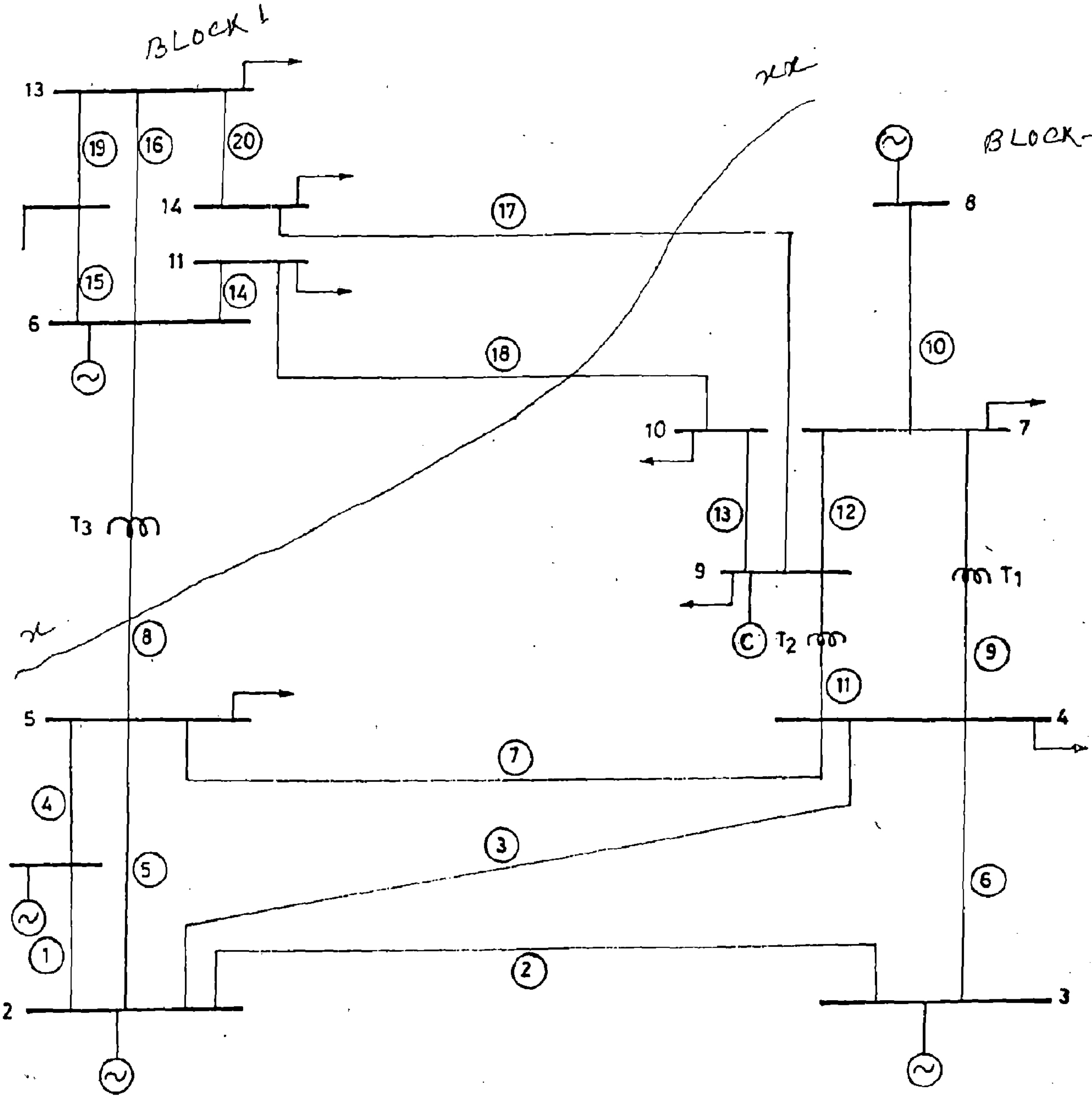



FIG. C.3 14 - BUS (IEEE) SYSTEM

TABLE 3.1

| BUS NUM | VOLTAGE (P.U.) | ANGLE (degree) |
|---|---|---|
| 1 | 1.067 | 4.345 |
| 2 | 1.053 | -4.785 |
| 3 | 1.095 | -11.545 |
| 4 | 1.017 | -12.959 |
| 5 | 1.113 | -5.272 |
| 6 | 1.066 | 0.000 |
| 7 | 1.067 | -13.168 |
| 8 | 1.029 | -9.912 |
| 9 | 1.024 | -9.265 |
| 10 | 1.063 | -16.152 |
| 11 | 1.005 | 0.000 |
| 12 | 1.061 | -15.673 |
| 13 | 1.098 | -15.674 |
| 14 | 1.068 | 0.000 |

TABLE 3.2

| | | |
|---|---|---|
| 1 | 1.076 | -4.543 |
| 2 | 1.078 | -5.678 |
| 3 | 1.089 | -12.897 |
| 4 | 1.043 | -13.987 |
| 5 | 1.053 | 0.000 |
| 6 | 1.078 | -8.986 |
| 7 | 1.098 | -12.345 |
| 8 | 1.076 | -13.435 |
| 9 | 1.067 | -8.543 |
| 10 | 1.065 | 0.000 |
| 11 | 1.096 | -15.543 |
| 12 | 1.098 | -14.897 |
| 13 | 1.078 | -13.789 |
| 14 | 1.069 | -15.678 |

Maximum Error = 0.007

TABLE 3.3

COMPARISION OF C.P.U TIME

| TEST SYSTEM | NUMBER OF ITERATION | C.P.U TIME IN SECOND | METHOD OF SOLUTION |
|---|---|---|---|
| IEEE_14 | 6 | 48.1 | WLS |
| IEEE_14 | 3 | 4.5 | DECOMPESED (For Block 1) |

# CHAPTER - 4

# BAD DATA PROCESSING AND OBSERVABILITY

## 4.1 BAD DATA PROCESSING

A data which is more inaccurate than assumed by mathematical model is called Bad data (BD). The presence of B.D can be due to number of reasons. viz. failure of communication link, intermittent fault in meters, change of system states far off from that assumed for pseudo measurements. The presence of BD causes very poor estimates. The latter effort on development of state estimation for practical application has deserving share on bad data processing. The bad data processing is a three tier exercise.

(i) Defection (ii) identification (iii) Estimate correction.

### 4.1.1 Defection

The state estimation equation can be written as

$$Z = f(x) + r$$

$$\text{or } r = Z - f(\hat{x}) \qquad (4.1)$$

Where r is the estimation Residual. The deviation in r can be expressed as

$$\Delta r = \frac{\partial r}{\partial z}\Delta z - \frac{\partial r}{\partial x}\Delta x$$

$$= I\Delta z - H\Delta x$$

$$\text{Since } \frac{\partial r}{\partial z} = I \text{ and } \frac{\partial r}{\partial x} = F'(x) = H$$

The sensitivity of the residual to the measurements is called the residual sensitivity matrix and is expressed as $\partial r/\partial z$, expression for which can be developed from (4.1) as under [27]

$$\frac{\partial r}{\partial z} = I - H\frac{\partial x}{\partial z}$$
$$= I - H\,(H^T WH)^{-1} H^T W = R \qquad (4.3)$$

The residual sensitivity matrix is of vital importance in bad data processing. the properties of this matrix are shown in Appendix A [28].

In absence of BD the measurement Residual vector is distributed N(O, $RW^{-1}R^T$) or N(O,WR). The presence of BD is currently detected through one of the variables below [28-29].

(i) Weightage residual vector $rW = \sqrt{W}.\ r$

(ii) Normalized residual vector $rN = \sqrt{D^{-1}}.\ r$

Where $D = \text{diag}\ (RW^{-1})$

The detection of BD is based on a hypothesis testing with two hypothesis. $H_0$ and $H_1$.

where

$H_0$ No bad data are present

$H_1$ $H_0$ is not true i.e. there are bad data.

Denoting by Pe the probability of rejecting Ho when Ho is actually true and Pd the probability of accepting $H_1$ when $H_1$ is true. The hypothesis consists of testing J(x), $|r_{N1}|$ or $|r_w|$ with a detection Threshold $\gamma$ which depends on Pe. For example, considering the normalized residuals one is led to :

- accept $H_0$ if $|r_{N+1}| < \gamma$ , $i = 1,2...m$

- reject $H_0$ otherwise

The $r_N$ has some interesting properties for acceptance as detection test i.e. the $R_N$ test [29].

(i) For a same detection threshold the $\gamma_N$ test is more sensitive since $|r_{N1}| > |r_{W1}|$

(ii) $r_N$ provides a more powerful test than $r_W$

(iii) Within linearlized approximation and provided e=0, the largest normalized residual $|r_{N1}|$ max corresponds to the erroreous measurements in the presence of single bad data.

(iv) in the presence of multiple BD the property (iii) above does not hold true.

### 4.1.2 Identification

A set of BD being known, it is interesting to determine whether measurement configuration is rich enough to allow their proper identification. A set of BD is said to be topologically identifiable if their suppression does not cause

- Systems unobservability
- Creation of basic or critical measurements.

i.e. those measurements whose errors are undetectable. It is desired that if f is BD then f< m-n. It is necessary condition but not sufficient, as it must satisfy the observability criteria discussed in 4.2.

#### 4.1.2.1 Identification by Estimation

Conceptually it is the continuation of BD detection step implying residual vector $r_N$ or ($r_W$). In the event of positive detection test, a first list of BD is drawn up on the basis of on $r_N$ test. Then successive cycles of elimination, reestimation redetection are performed until the test become positive.

The Bad data is detected for IEEE-14 bus data and successive elimination reestimation- redetection are performed. The result for BD detection is given in Table 4.1.

### 4.2 OBSERVABILITY

A system is said to be observable if with available set of measurements it is possible to determine the states of the system. It requires the measurements to be well distributed geographically. Sufficient redundancy in measurements will allow processing of BD as discussed in section 4.1. Thus at the stage of design of a state estimator following questions must be positively replied.

(i) Are there sufficient measurements to make state estimation possible.

(ii) If not, where additional meters should be placed so that state estimation is possible.

Temporary unobservability may still occur due to unanticipated network topology changes or failure of communication link. However, a system is designed to be observable for most operating conditions. Therefore the observability test algorithm must satisfy following requirements. -

(i) Test whether there are enough real time measurements to make state estimation possible.

(ii) If requirement (i) is not met, it should provide information in respect to the part of the network whose states can still be estimated with available measurements i.e. ovservable islands.

(iii) It should assist in estimation of the states of observalbe islands.

(iv) Selection of pseudo-measurements to be included in the measurement set to make the state estimation possible.

(v) It should guarantee that inclusion of additional pseudo measurements will not contaminate the results of the state estimation.

These considerations lead to redefinition as under [33].

A network is said to be observable if for all $\phi$ such that $H\phi = 0$, $A^T\phi=0$. Any state $\phi^*$ for which $H\phi^* = 0$, $A^T\phi^* = 0$ is called unobservable state. For an unobservable $\phi^*$, let $\delta^* = A^T \phi^*$ if $\delta^*_1 = 0$ then the corresponding branch is an unobservable branch.

Here H is the B′ matrix of the fast decoupled load flow. A is the incidence matrix and $\phi$ is the angle vector.

Mathematically network observability is related to the rank of the Jacobian matrix. The rank of matrix is very sensitive to the numerical values of its elements, whereas the oibservability should not. Therefore most of the methods proposed on network observability are combinational in nature and use no flatting point calculation. Clements and Wollenberg [34] proposed a beuristic procedure to process measurements for observability. Allemong et al. [35] proposed a modified version of the Clement's method as it was conservative in the sense that it may label an observable systems as unobservable. Handschin et al. [36] proposed a method which tests connectivity of the Jacobian matrix. Krumpholz et al. [37, 38, 39] utilized concept of graph theory to develop theoretic topological basis of a algorithm for network observability. These combinatoric methods were since very complex and computationally expensive Monticelli et al. [33, 40] developed.

## 4.3 DISCUSSION OF RESULT

The bad data is detected for IEEE-14 bus system from given measurement data by $r_N$ test. Table 4.1 shows the BD detection result from Table the highlighted lines highest residual. So their exist bad data in the measurement data for line 9-6.

TABLE 4.1

BAD DATA IDENTIFICATION TABLE

| BUS | NUM | MEASURED (REAL & REACTIVE POWER) | ESTIMATED | RESIDUAL |
|---|---|---|---|---|
| 1 | 0 | 2.115 | 2.109 | 0.0060 |
| 2 | 0 | 0.183 | 0.169 | 0.0140 |
| 3 | 0 | -0.942 | -0.940 | -0.0017 |
| 4 | 0 | -0.478 | -0.464 | -0.0136 |
| 5 | 0 | -0.076 | -0.094 | 0.0180 |
| 6 | 0 | 0.091 | 0.084 | 0.0066 |
| 7 | 0 | 0.000 | -0.003 | 0.0026 |
| 8 | 0 | 0.000 | -0.002 | 0.0017 |
| 9 | 0 | -0.295 | -0.301 | 0.0060 |
| 10 | 0 | -0.090 | -0.078 | -0.0123 |
| 11 | 0 | -0.035 | -0.016 | -0.0186 |
| 12 | 0 | -0.060 | -0.060 | -0.0003 |
| 13 | 0 | -0.135 | -0.122 | -0.0134 |
| 14 | 0 | -0.149 | -0.167 | 0.0177 |
| 1 | 0 | -0.171 | -0.174 | 0.0026 |
| 2 | 0 | 0.180 | 0.179 | 0.0005 |
| 3 | 0 | 0.000 | 0.005 | -0.0050 |
| 4 | 0 | 0.039 | 0.038 | 0.0006 |
| 5 | 0 | 0.162 | 0.162 | -0.0002 |
| 6 | 0 | -0.016 | -0.024 | 0.0084 |
| 7 | 0 | 0.114 | 0.121 | -0.0069 |
| 8 | 0 | -0.166 | -0.200 | 0.0339 |
| 9 | 0 | -0.058 | -0.009 | -0.0493 |
| 10 | 0 | -0.017 | -0.013 | -0.0041 |
| 11 | 0 | -0.016 | -0.016 | 0.0004 |
| 12 | 0 | -0.058 | -0.045 | -0.0127 |
| 13 | 0 | -0.050 | -0.063 | 0.0126 |

REAL AND REACTIVE POWER FLOW

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 1.427 | 1.431 | -0.0036 |
| 2 | 1 | -1.393 | -1.395 | 0.0024 |
| 1 | 8 | 0.680 | 0.678 | 0.0016 |
| 8 | 1 | -0.657 | -0.656 | -0.0012 |
| 2 | 4 | 0.705 | 0.701 | 0.0038 |
| 4 | 2 | -0.684 | -0.679 | -0.0043 |
| 2 | 9 | 0.510 | 0.505 | 0.0048 |
| 9 | 2 | -0.497 | -0.492 | -0.0052 |
| 2 | 8 | 0.361 | 0.358 | 0.0030 |
| 8 | 2 | -0.355 | -0.351 | -0.0033 |
| 4 | 9 | -0.259 | -0.261 | 0.0016 |
| 9 | 4 | 0.265 | 0.265 | -0.0008 |
| 9 | 8 | -0.610 | -0.623 | 0.0133 |
| 8 | 9 | 0.615 | 0.628 | -0.0133 |
| 9 | 6 | 0.245 | 0.245 | -0.0001 |

| | | | | |
|---|---|---|---|---|
| 6 | 9 | -0.245 | -0.245 | -0.0001 |
| 9 | 7 | 0.145 | 0.140 | 0.0054 |
| 7 | 9 | -0.145 | -0.140 | -0.0054 |
| 3 | 11 | 0.109 | 0.095 | 0.0137 |
| 11 | 3 | -0.108 | -0.095 | -0.0135 |
| 3 | 12 | 0.078 | 0.081 | -0.0023 |
| 12 | 3 | -0.078 | -0.080 | 0.0024 |
| 3 | 13 | 0.195 | 0.194 | 0.0015 |
| 13 | 3 | -0.194 | -0.191 | -0.0022 |
| 6 | 7 | 0.245 | 0.241 | 0.0046 |
| 7 | 6 | -0.245 | -0.241 | -0.0047 |
| 7 | 10 | 0.018 | 0.000 | 0.0180 |
| 10 | 7 | -0.019 | 0.000 | -0.0190 |
| 7 | 14 | 0.073 | 0.079 | -0.0061 |
| 14 | 7 | -0.073 | -0.078 | 0.0052 |
| 10 | 11 | -0.073 | -0.078 | 0.0045 |
| 11 | 10 | 0.072 | 0.078 | -0.0058 |
| 12 | 13 | 0.020 | 0.020 | -0.0002 |
| 13 | 12 | -0.020 | -0.020 | 0.0001 |
| 13 | 14 | 0.078 | 0.090 | -0.0119 |
| 14 | 13 | -0.078 | -0.089 | 0.0106 |
| 1 | 2 | -0.171 | -0.171 | 0.0006 |
| 2 | 1 | 0.220 | 0.222 | -0.0023 |
| 1 | 8 | -0.001 | -0.002 | 0.0012 |
| 8 | 1 | 0.039 | 0.040 | -0.0014 |
| 2 | 4 | 0.004 | 0.039 | -0.0354 |
| 4 | 2 | 0.001 | 0.005 | -0.0041 |
| 2 | 9 | -0.046 | -0.048 | 0.0019 |
| 9 | 2 | 0.048 | 0.050 | -0.0015 |
| 2 | 8 | -0.033 | -0.034 | 0.0008 |
| 8 | 2 | 0.017 | 0.018 | -0.0010 |
| 4 | 9 | 0.003 | 0.000 | 0.0025 |
| 9 | 4 | -0.003 | -0.024 | 0.0217 |
| 9 | 8 | 0.086 | 0.087 | -0.0006 |
| 8 | 9 | -0.083 | -0.084 | 0.0011 |
| 9 | 7 | -0.006 | -0.003 | -0.0032 |
| 7 | 9 | 0.003 | -0.007 | 0.0099 |
| **9** | **6** | **-0.074** | **0.074** | **-0.1483** |
| **6** | **9** | **0.087** | **-0.087** | **0.1783** |
| 3 | 11 | 0.019 | 0.024 | -0.0052 |
| 11 | 3 | -0.017 | -0.023 | 0.0056 |
| 3 | 12 | 0.022 | 0.021 | 0.0010 |
| 12 | 3 | -0.021 | -0.019 | -0.0016 |
| 3 | 13 | 0.064 | 0.059 | 0.0054 |
| 13 | 3 | -0.059 | -0.054 | -0.0050 |
| 6 | 5 | -0.158 | -0.158 | 0.0003 |
| 5 | 6 | 0.162 | 0.162 | -0.0001 |
| 6 | 7 | 0.071 | 0.056 | 0.0147 |
| 7 | 6 | -0.065 | -0.050 | -0.0147 |
| 7 | 10 | 0.060 | 0.000 | 0.0600 |
| 10 | 7 | -0.060 | 0.000 | -0.0600 |
| 7 | 14 | 0.048 | 0.056 | -0.0078 |
| 14 | 7 | -0.046 | -0.054 | 0.0075 |
| 10 | 11 | 0.002 | -0.009 | 0.0109 |

| | | | | |
|---|---|---|---|---|
| 11 | 10 | -0.001 | 0.010 | -0.0108 |
| 12 | 13 | 0.006 | 0.003 | 0.0028 |
| 13 | 12 | -0.006 | -0.003 | -0.0027 |
| 13 | 14 | 0.060 | 0.000 | 0.0484 |
| 14 | 13 | -0.004 | 0.000 | 0.0051 |

ALL VALUES ARE IN P.U.

# CHAPTER - 5

## TOPOLOGICAL ERRORS : IDENTIFICATION

This section considers the effect of topological errors on the estimation and bad data detection procedures. The following model errors are considered :

* A line is not it included in the model when it is in service in the actual system.
* A line is included in the model when it is actually out of service in the system.

The above inconsistencies can be detected if redundant information on the status of the line is available. That is, inconsistency of the status of the line can be detected by comparisons with the analog measurements of the line. This can be performed as a front end pre-processing of the estimation process.

Extensive analysis of the effect of topological errors on the WLS estimator bring out the following observation which are pertinent to the topological error detection method [50].

1. The estimated state vector (Bus voltage and angles) is not affected by topological errors if the line in question and the injections at the end buses are either not measured on their measurements have been suppressed. This follows from the fact that the model of the line in question is not included in the WLS algorithm equations.
2. The primarily effect of a wrong line status is to produce a wrong estimate of the injection of the end buses of the line. If sufficient

redundancy exists, the voltage magnitude and angles and the other line flows are affected at a much lesser degree than the injections. If the injections at the end buses in question are metered they are most likely to be identified and suppressed as "bad" data. Once suppressed, the state vector produced is independent of the topological error.

3. An important corollary to the above is that the practice of not allowing suppression of injection measurements at certain buses (zero injection or equality constraint buses) is not advisable from the point of view of topological errors. If the model inconsistencies exist in the direct vicinity of such a bus, the estimator is forced to fit the error into the estimate instead of suppressing the appropriate measurements to uncouple the questionable line from the estimator equations.

## 5.1 TOPOLOGICAL ERROR DETECTION :

Based on observation, the following condition are set as requirements for the topological error detection procedures.

* The end buses of a line in question must have injection measurements :
* A good state vector is produced ; ie the injection measurements at the end buses of the questionable line have been suppressed and are observable.

Given the estimation results and a list of the suppressed measurements, post processing is performed to defect topological errors. The following hypothesis testing problem is based :

HO : A line is configured different from the one modeled.

HI : No conclusions can be obtained as to the line configuration.

Accept HO if the tested line status results in the suppressed injection measurements at the end buses passing the residual test. Otherwise, either the line status is bad or injection measurements are bad.

Specifically the method reduces to the post-process retesting of the suppressed bus injection taking into account the following.

* Outage a line which is not metered or whose measurements have been suppressed.
* Reconnect a line which is not included in the model given the state vector, the blows of the line being investigated are readily calculated and the bus injection estimates are modified correspondingly.

The procedure is shown in the block diagram.

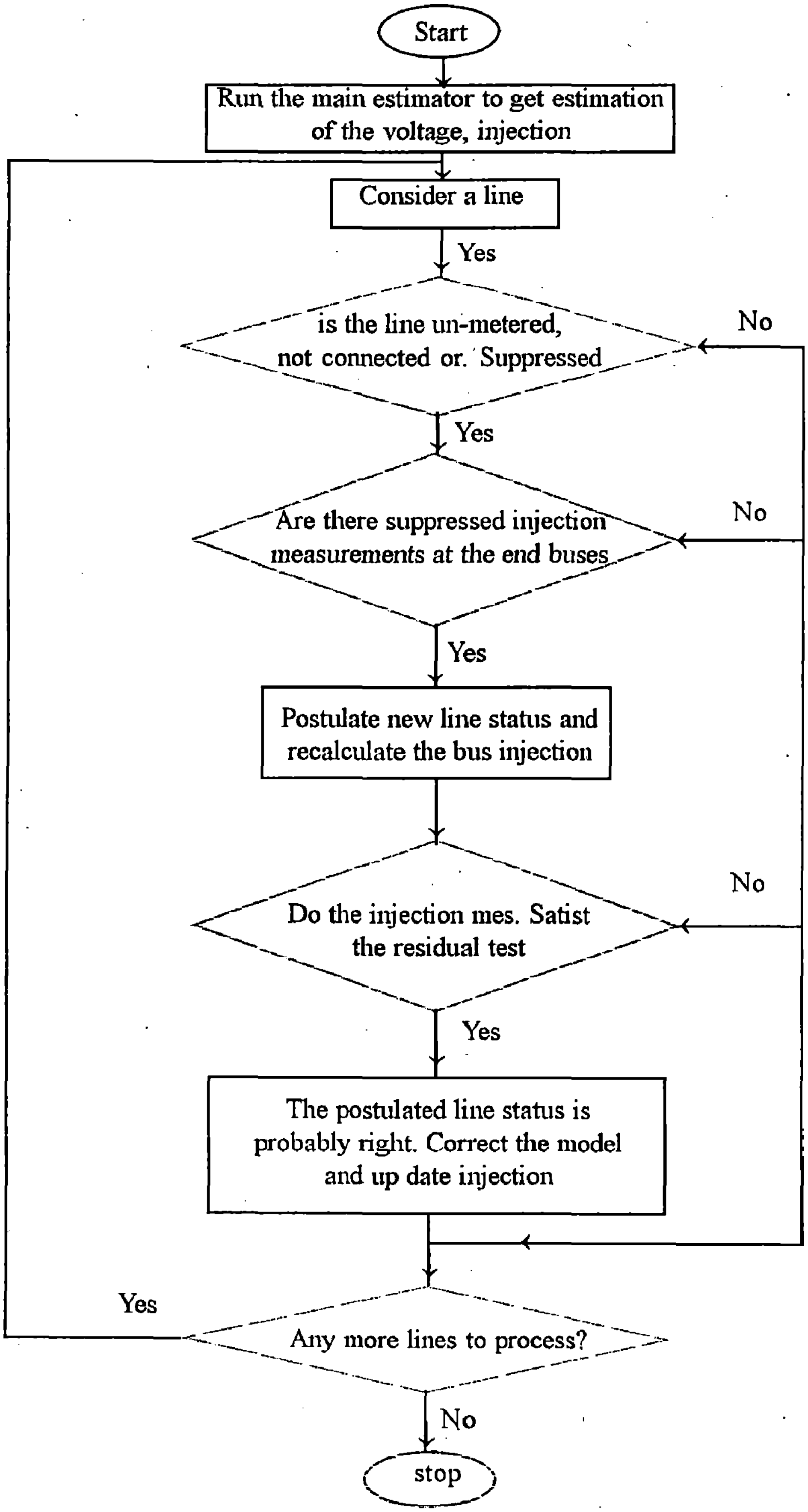



**Fig. 5.1 FUNCTIONAL FLOW CHART OF TOPOLOGICAL ERROR DETECTION PROCEDURE**

## 5.2 DISCUSSION OF RESULTS

Two types of the topological error are created on IEEE 14 bus systems and they are defected by available algorithm :

1. One of the parallel lines between buses 2 and 4 simulates an unmetered line included in the model that is actually out of service in real system.
2. The line between buses 1 and 8 is disconnected from the model whereas it is actually in service in the real system.

All two types of topological error were detected by the program. The result are shown in tables 5.1 and Table 5.2. The values under the true column represent the system state as obtained by a load flow solution. The column under measured value consists of input measurements to the estimator. The remaining columns represent the estimated values for the following case :

**Case I :** These are estimated result without topological errors. These are used to compare the estimates produced with topological errors.

**Case II :** These are estimator results with the described topological errors. But no error detection or correction is included.

The estimates for real power injection of the buses having topological errors are found to be far from correct estimates. These are indication of topological errors. Table5.2 shows the voltage and angle estimation of the buses. It is found that estimates are correct with suppressed incorrect injections.

TABLE 5.1
TOPOLOGICAL ERROR DETECTION

| BUS NUM | TRUE (P.U.) MW | TRUE (P.U.) MVAR | MEASURED (P.U.) MW | MEASURED (P.U.) MVAR | CASE1 (P.U.) MW | CASE1 (P.U.) MVAR | CASE2 (P.U.) MW | CASE2 (P.U.) MVAR |
|---|---|---|---|---|---|---|---|---|
| 1 | 2.1061 | -.1713 | 2.115 | -.1710 | 2.111 | -.1810 | **12.113** | -.1809 |
| 2 | 0.1830 | .1791 | .184 | .1891 | .1730 | .1792 | **5.175** | .1812 |
| 3 | 0.0800 | .1148 | .090 | .1149 | .090 | .1242 | 0.086 | .1397 |
| 4 | -.9420 | .0085 | -.953 | .0100 | -.938 | .0200 | **9.938** | .0199 |
| 5 | 0.0000 | .1620 | .008 | .1720 | .009 | .1730 | 0.010 | .1830 |
| 6 | 0.0000 | 0.000 | .000 | .0000 | .006 | .002 | 0.009 | .0060 |
| 7 | -.2950 | -.1660 | -.296 | -.1770 | -.296 | -.1780 | -.305 | -.1797 |
| 8 | -.0760 | -.0160 | -.067 | -.0170 | -.076 | -.0170 | **-9.017** | -.0163 |
| 9 | -.4780 | .0390 | -.488 | .0400 | -.588 | .0430 | -.555 | .0450 |
| 10 | -.0900 | -.0580 | -.095 | -.0590 | -.096 | -.0580 | -.069 | -.0575 |
| 11 | -.0350 | -.0180 | -.036 | -.0190 | -.046 | -.0187 | -.045 | -.0187 |
| 12 | -.0610 | -.0160 | -.063 | -.0170 | -.064 | -.0168 | -.067 | -.0167 |
| 13 | -.1350 | -.0580 | -.145 | -.0590 | -.155 | -.0587 | -.156 | -.0578 |
| 14 | -.1490 | -.0500 | -.1510 | -.060 | -.1615 | -.0670 | -.1616 | -.0650 |

TABLE 5.2

| BUS NUM | VOLTAGE (P.U.) | ANGLE (degree) |
|---|---|---|
| 1 | 1.0567 | 0.0000 |
| 2 | 1.044 | -4.495 |
| 3 | 1.067 | -11.545 |
| 4 | 1.007 | -11.839 |
| 5 | 1.103 | -4.272 |
| 6 | 1.076 | -11.837 |
| 7 | 1.068 | -12.689 |
| 8 | 1.028 | -7.812 |
| 9 | 1.023 | -9.265 |
| 10 | 1.062 | -13.215 |
| 11 | 1.063 | -12.464 |
| 12 | 1.061 | -12.423 |
| 13 | 1.056 | -12.564 |
| 14 | 1.037 | -13.959 |

# CHAPTER 6

## CONCLUSION AND SCOPE FOR FUTURE WORK

An approach based on decomposition of Network to solve state estimation problem developed. Further a method has been presented for the detection of bad data and erroneous status of lines and Transformers in the Network model used by state estimator. The result obtained on test system reveals the followings :

* Large saving in computation time has been achieved using decomposition method.[51] .
* Decomposition method enables to solve the large size power network with smaller computers due to its reduce memory requirement. Therefore, this method can be used with mini/micro computers for state estimation.
* WLS method is highly sensitive to Bad data.
* Proposed $R_N$ test for bad data identification works satisfactory for single as well as non interacting bad data.
* The method for topological error detection is a simple and fast post processing of the results of currently available estimation techniques. This makes it very suitable for real time application.

Detection and identification of bad data in measurements can be done by decomposition technique. Further method of topological error detection can be easily extended to consider various combinations of questionable line status within a given neighborhood.

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# APPENDIX - A

## PROPERTIES OF RESIDUAL SENSITIVITY MATRIX - R

1. R is an idempotent matrix i.e.

   $R^2 = R$ ... (A.1)

2. The eigen values of R matrix must be either 1 or 0, i.e. it is semi-positive definite.

3. R is a matrix with eigen values of K set of ones an n set of zeros. Where K is the degree of freedom (m-n) and n is the number of state variables.

4. R is a singular matrix of rank K.

5. The weighted residual sensitivity matrix $R_W$ is symmetrical.

   $R_W^T = R_W$

6. If there is no redundancy i.e. number of measurements m=n, then

   $R = 0$

7. If it is assumed that measuring points are evenly distributed in a network and $m \rightarrow \infty$ then

   $R_{\lim_{m \rightarrow \infty}} = I$

8. Utilizing above properties

   $r = Rr$

   and when $m \rightarrow \infty$, then $r = e$

9. The value of diagonal elements $R_{ii}$ may have the range of

   $0 < R_{ii} < 1$

   It has been reported that performance of identification of bad data are better at measurement points where $R_{ii} \geq 0.5$.

# APPENDIX - B

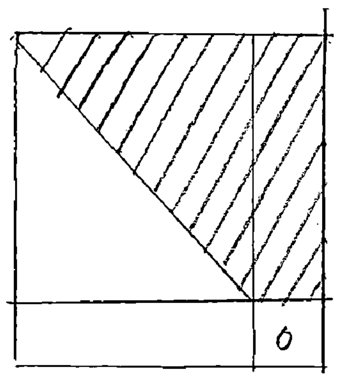
## NETWORK OBSERVABILITY THEOREMS

Theorem-1 Assume that there is no voltage measurement, then the following statements are equivalent.

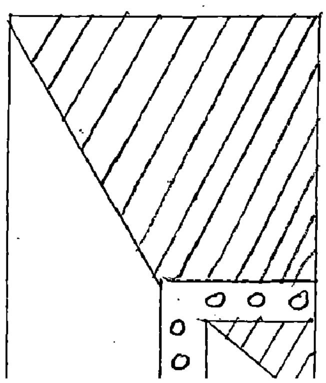
(i) The network is observable.

(ii) Let H be obtained from H by deleting any column, then H is of full rank.

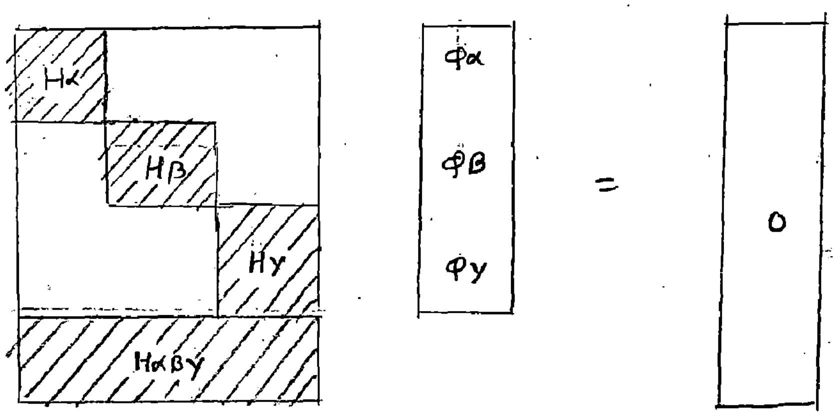
(iii) The triangular factorisation reduces the gain matrix $G = H^T H$ in the following form.




Theorem-2 In the triangular factorisation of the gain matrix G, if a zero pivot is encountered, then the remaining elements of row and column are all zeros, i.e., G is reduced to the form.



Theorem-3 $\phi\alpha$ is not an unobservable state for the subnetowrk $\alpha$ with measurement $H\alpha$, SIMILARLY $\Phi B$ AND $\Phi R$




Theorem-4 Consider state Estimation model

$\tau = H\phi + r$

Suppose that the measurement set consists of the $\phi s$ pseudo measurements and all other measurements equal to zero, then the residual $r = 0$.

Theorem-5 If minimal set of additional non-redundant (pseudo) measurements is so selected that they make the network barely observable, then the estimated states of the already observable islands will not be affected by these pseudo measurements.