

# SYSTEM RELIABILITY STUDIES USING FUZZY LOGIC CONCEPTS

## A DISSERTATION

*Submitted in partial fulfilment of the requirements for the award of the degree*

*of*

MASTER OF TECHNOLOGY

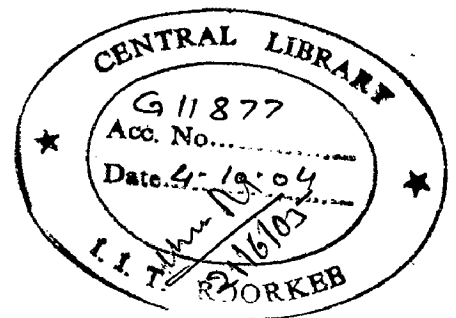
*in*

ELECTRICAL ENGINEERING

(With Specialization in System Engineering and Operations Research)

*By*

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**CANDIDATE'S DECLARATION**

I hereby declare that the work which is being presented in this dissertation entitled, "System Reliability Studies Using Fuzzy Logic Concepts", submitted in the partial fulfillment of the requirements for the award of degree of Master of Technology in the specialization System Engineering and Operation Research, submitted in Department of Electrical Engineering, Indian Institute of Technology, Roorkee, is an authentic record of my own work carried out during the period July 2003 to June 2004 under the supervision and guidance of Dr. Rajendra Prasad Associate Professor and Dr. Surendra Kumar, Assistant Professor, Department of Electrical Engineering, Indian Institute of Technology, Roorkee.

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**CERTIFICATE**

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## ABSTRACT

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It is well known that the conventional reliability analysis using probabilities has been found to be inadequate to handle uncertainty of failure data and modeling. To overcome this problem the concept of fuzzy probability has been used in the evaluation of reliability of systems. It has been amply demonstrated that the fuzzy set theory can be conveniently used for system reliability evaluation, particularly when there exists uncertainty in the failure data. In reality, even though one may use the best data collection procedures, failure data uncertainty always exists. In the presented work a concept of possibility of failure i.e. fuzzy set defined on probability space is used to evaluate system reliability. The notion of the possibility of failure is more predictive than that of probability of failure, the latter is the limiting case of the former.

In the presented approach a fuzzy-set analysis is made for various system structures viz. fault tree, event trees etc. such analysis can not be made by hand calculations due to complexity of trees. Hence a computer algorithm is developed for each case. Also some approach of neural network is described for the reliability analysis. A feed-forward recursive neural network is used to perform the reliability analysis.

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## INTRODUCTION

In recent years, network reliability theory has been applied extensively in areas other than electronics, such as chemical engineering and energy production etc.

Reliability assessment as such not new, engineers have always strived to operate systems that are relatively free from failures. In the past however, this reliability generally has been achieved from the subjective and qualitative experience of design and operating engineers. Once it is decided that quantitative reliability evaluation is needed it becomes necessary to decided on the method to use the indices required. In essence all technique are concerned with failure behavior of system cannot be defined in deterministic but as stochastic in nature i.e. it varies randomly with time complete assessment of a stochastic process can only be achieved using probability techniques. However, probability theory alone cannot predict either the reliability of system. The assessment requires a through understanding of system, its design, the way it operates the way it fails, its environment and the stresses to which it is subjected.

The analysis of system reliability often requires the use of subjective judgment, uncertain data, the approximate system models. By allowing imprecision and approximate analysis fuzzy logic provide an effective tool for characterizing system reliability in this circumstances, it does not force precision, where it is not possible. In the present work the main concepts of fuzzy logic, fuzzy arithmetic and linguistic variables are applied to the analysis of system structure, fault tree, event trees, the reliability of degradable system, and the assessment of system criticality based on the severity of a failure and its probability of occurrence.

Fault tree analysis is a logical and diagrammatic method to evaluate the probability of an accident resulting from sequence and combinations of fault and failure events. In conventional fault tree analysis, the failure probabilities of a component of a system are treated as exact values in estimating the failure probability of the top event. For many systems it is often difficult to evaluate the failure probability of component

from the past occurrences because the environment of the system changes. In these cases a concept of “possibility of failure” i.e. a fuzzy set defined in probability space is used. The notion of possibility of failure is more predictive than that of the probability of failure, the latter is limiting case of former. By resorting the concept, we can allocate a degree of uncertainty to each value of the probability, in this manner, different aspects of uncertainty probability and possibility can be simultaneously treated. In the present work, we will consider the application of notions and techniques from fuzzy logic, fault trees, and Markov modeling to robot fault tolerance.

The Markov model is a method of determining system behaviors by using information about certain probabilities of event within the system. Markov Models treat a system as a series of states with specific, constant rate transition between them. At all the time  $s$ , the system is in exactly one state. The only information available is the current state, the allowed transition, and the probability of this transition. Such a system is referred to as memoryless, and is said to possess the Markov property. As an illustration, fuzzy Markov modeling of a power generator with a derated state has been developed in this present work.

Event trees are useful for system reliability analysis and risk quantification since they illustrate the logic of combination of probabilities and consequences of event sequences. For many systems, estimation of single number for the probabilities and consequences has to be used in the analysis. Fuzzy logic is used to account for imprecision and uncertainty in data while employing event tree analysis. The application of fuzzy event trees is further demonstrated by using set of event trees for an electric power system protection system to assess the viability of the method in computing the risk associated with a failure in an electric power system.



Another approach to the reliability analysis, based on neural networks, is introduced in this work. The reliability analysis of a simple non-redundant digital system, Simplex system, with repair is used to illustrate the neural network approach. The discrete time Markov model of simplex systems and the TMR system is realized using feed-forward recursive neural network. The energy function and update equations for the weight of neural network are established such that the network converges to the desired reliability of the simplex system under design. The failure rate and repair rate, satisfying the desired reliability, are extracted from the neural weights at convergence.

## FUZZY LOGIC: AN OVERVIEW

Among the various paradigmatic changes in science and mathematics in this century one such change concerns the concept of uncertainty. Uncertainty is concerned essential to science, it is not only an unavoidable plague, but it has in fact a great utility.

It is generally agreed that an important point in the evaluation of the modern concept of uncertainty was the publication of a paper by Lotif A. Zadeh (1965). In this paper, Zadeh introduced a theory whose objects – Fuzzy sets – are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation but rather a matter of a degree.

The significant of Zadeh's paper was that it challenged not only probability theory as the sole agent for uncertainty but the very foundations upon which probability theory is based: two value logic. The capability of fuzzy sets to express gradual transition from membership to non-membership and vice-versa has a broad utility. It provides us not only with a meaningful and powerful representation of measurement of uncertainty, but also with a meaningful representation of vague concepts expressed in natural language. For example instead of describing the weather today in terms of the exact percentage of cloud cover, we can just say it is sunny. While the latter description is vague and less specific it is often more useful. Research on the theory of fuzzy sets has been growing steadily since the inception of the theory in the mid 1960. The body of concept and result pertaining to the theory is now quit impressive.

The four features that make the fuzzy logic superior to classical theory are:

1. The fuzzy logic allow us to express irreducible observation and measurement uncertainties in their various manifestations and make these uncertainties intrinsic to empirical data. Such data, which are based on graded distinction among states of relevant variables are usually called fuzzy data, when fuzzy data are processed, their intrinsic uncertainties are processed as well, and the result obtained are more meaningful than their counterparts obtained by processing the usual crisp data.

2. Fuzzy logic offers far greater recourses for managing complexity and controlling computational cost.
3. Fuzzy logic has considerably greater expressive power, consequently it can effectively deal with a broader class of problem. In particular, it has the capability to capture and deal with the meanings of sentence expressed in natural language.
4. The fuzzy logic has greater capability to capture human common sense reasoning, decision making and other aspects of human cognition.

## 2.1 FUZZY ARITHMATIC

A set, A of points or objects in some relevant universe, X is defined as those elements of X that satisfy the membership property defined for A. In traditional or crisp set theory each element of X either is or is not an element of A. Element in fuzzy set can have a continuum of degrees of membership ranging from complete membership to complete non-membership.

The membership function  $\mu(x)$  gives the degree of membership for each elements  $x \in X$ .  $\mu(x)$  is defined on  $[0, 1]$  where 1 represents elements that are completely in A, and 0 represents that are completely not in A and values between 0 and 1 represents partial inclusion in A.

Formally, A is represented as the order pair  $(X, \mu(x))$ .

$$A = \{(X, \mu(x)) \mid x \in X, \text{ and } 0 \leq \mu(x) \leq 1\}$$

The use of numerical scale for the degree of membership provides a convenient way to represent graduation in the degree of membership. Precise degrees of membership generally do not exist; instead they tend to reflect a sometime subjective ordering of the elements in the universe.

## 2.2 FUZZY NUMBERS

Fuzzy numbers are numerical approximation such as "about 5". Formally a fuzzy number is defined as a close interval on R (the real line), N(the integers) or any totally ordered set whose membership function is normal and convex and reaches its maximum values (1.0) at the number (e.g. 5). For simplicity fuzzy numbers are often represented with triangular membership functions as illustrated in fig.2.1

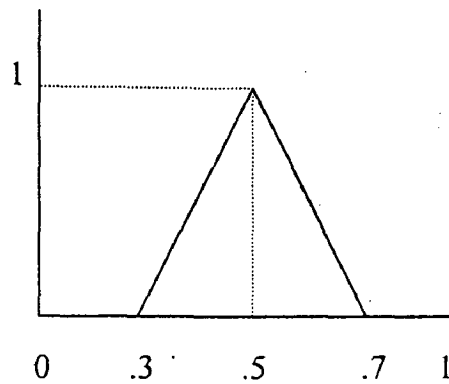


Fig 2.1

The width of membership function shows range of possible values.

### 2.3 BASIC OPERATIONS ON FUZZY SETS:

1. A fuzzy set A is contained in fuzzy set B,  $A \subset B$  if  $\mu_A(x) \leq \mu_B(x)$  for all  $x \in X$ .
2. The basic set operations for two fuzzy sets A and B

: Intersections

$$A \cap B = \min [\mu_A(x), \mu_B(x)]$$

: Union

$$A \cup B = \max [\mu_A(x), \mu_B(x)]$$

: Complement

$$\bar{A} = 1 - \mu_A(x)$$

These operations satisfy the associativity and distributivity properties of ordinary sets.

3. A  $\alpha$ -cut is the set of element in fuzzy sets that have a degree of membership greater than  $\alpha$  :

$$A_\alpha = \{x \in X, \text{ and } \mu(x) > \alpha\}$$

4. Arithmetic operations on fuzzy numbers

Fuzzy numbers are represented by

$$A = [a_1 \ a_2]$$

$$B = [b_1 \ b_2]$$

$$C = [c_1 \ c_2]$$

Fuzzy arithmetic operations as defined above are equivalent to the corresponding interval arithmetic operation for each  $\alpha$  cut. For the basic arithmetic operations we have:

$$A_\alpha + B_\alpha = [a_1^\alpha, a_2^\alpha] + [b_1^\alpha, b_2^\alpha]$$

$$= [a_1^\alpha + b_1^\alpha, a_2^\alpha + b_2^\alpha] \quad \text{----- (1)}$$

$$A_\alpha - B_\alpha = [a_1^\alpha, a_2^\alpha] - [b_1^\alpha, b_2^\alpha]$$

$$= [a_1^\alpha - b_2^\alpha, a_2^\alpha - b_1^\alpha] \quad \text{----- (2)}$$

$$A_\alpha \times B_\alpha = [a_1^\alpha, a_2^\alpha] \times [b_1^\alpha, b_2^\alpha]$$

$$= [a_1^\alpha \times b_1^\alpha, a_2^\alpha \times b_2^\alpha] \quad \text{----- (3)}$$

$$A_\alpha \div B_\alpha = [a_1^\alpha, a_2^\alpha] \div [b_1^\alpha, b_2^\alpha]$$

$$= [a_1^\alpha \div b_2^\alpha, a_2^\alpha \div b_1^\alpha] \quad \text{----- (4)}$$

$$1 \div B_\alpha = 1 \div [b_1^\alpha, b_2^\alpha]$$

$$= [1 \div b_2^\alpha, 1 \div b_1^\alpha] \quad \text{----- (5)}$$

$$A_\alpha \times K = [a_1^\alpha, a_2^\alpha] \times K$$

$$= [Ka_1^\alpha, Ka_2^\alpha] \quad \text{----- (6)}$$

Operation (4) and (5) are undefined if the interval contains 0 as resulting interval goes to infinity.

It is easy to show from the proportionality properties of triangles that addition and subtraction of triangular fuzzy numbers and a multiplication by a constant result in a triangular number. Multiplication, division and inversion of fuzzy number generally do not give a triangular result. Fig. 2.2 illustrates these properties of fuzzy number. This approximation simplifies the fuzzy arithmetic.

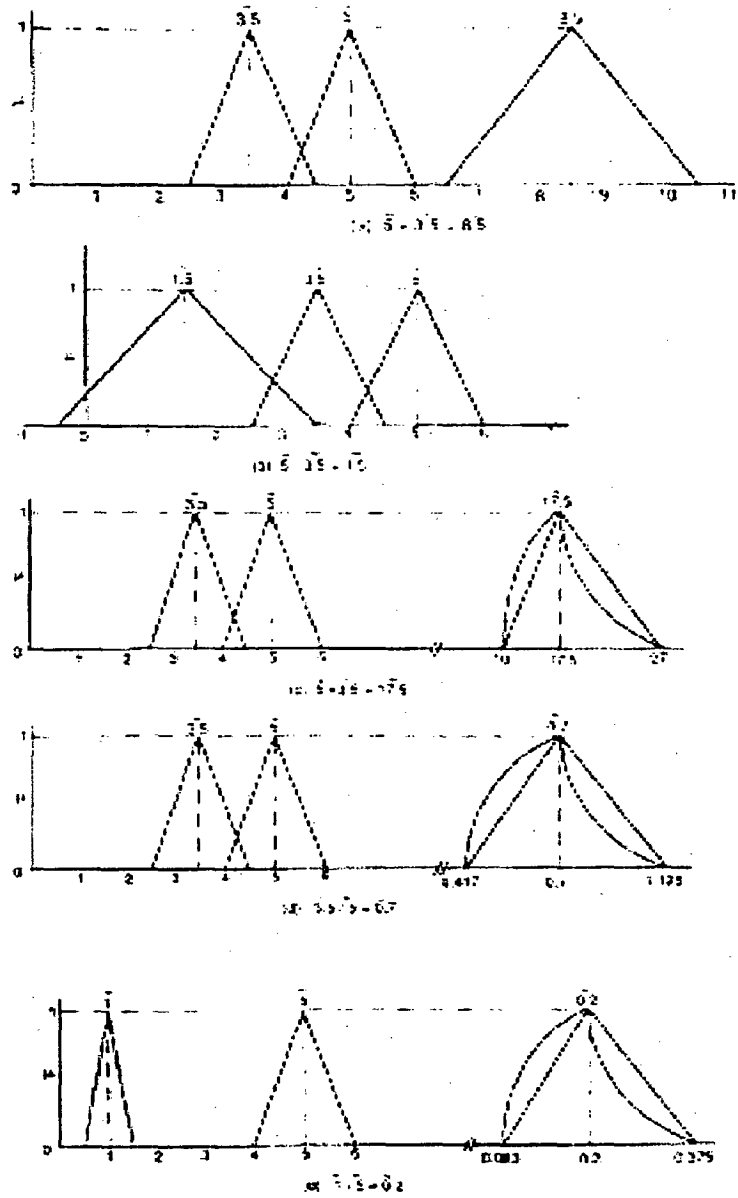


Fig.2.2 Fuzzy arithmetic operations.

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## RELIABILITY EVALUATION OF ENGINEERING SYSTEMS

In our modern society, professional engineers are responsible for the planning, design, manufacture and operation of products and systems ranging from the simple product to the complex system. The failure of these can often cause effects which range from inconvenience and irritation to a severe impact on society and on its environment. Users expect that the products and the systems they purchase should be reliable and safe. A question arises is how reliable or how safe will the system be during its future operating life. This question can be answered in parts by the use of quantitative reliability evaluation. There are many variations on the definitions of reliability but a widely expected form is as follows:

“Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered”.

The definition breaks down into four basic parts: probability – adequate performance – time – operating conditions. The first part, probability provides the numerical input for the assessment of reliability and also the first index of system adequacy. In many instances it is most significant index but there are many more parameters calculated and used, the most appropriate being dependent on the system and its requirement. These parameters are generally all termed reliability indices. Typical examples of additional indices are:

- The expected number of failure that will occur in a specified period of time.
- The average time between failure
- The expected loss of output due to failure.

The appropriate reliability index or indices are determined using probability theory.

### 3.1 BASIC PROBABILITY THEORY

The word probability is used frequently in a loose sense implying that a certain event has a good chance of occurring

#### 3.1.1 General Concepts

A. Permutations:

The number of permutation of  $n$  different items is the number of different ways these items can be arranged. If all the items are used in the arrangement, the number of permutations is designated as  ${}^n P_n$ . If only some are used say  $r$  ( $r < n$ ) the number of permutations as  ${}^n P_r$ .

B. Combinations:

The number of combination of  $n$  different items is the number of different selections of  $r$  items, each without regard to the order or arrangement of the items in the group. It is this regard for order which distinguishes combination from permutations. The number of combinations of  $r$  items from  $n$  items is the designated as  ${}^n C_r$ . In practical reliability evaluation the concept of combinations is usually, but not universally of more use and importance than permutations.

C. Independent events:

Two events are said to be independent if the occurrence of one event does not affect the probability of occurrence of the other event.

D. Mutually exclusive events:

Two events are said to be mutually exclusive (or disjoint) if they cannot occur at the same time.

E. Complimentary events:

Two outcomes of an event are said to be complimentary if, when one outcome does not occur the other must.

F. Probability distributions:

In practice the knowledge of design, geometry or specification of events, experiments or systems is not readily available and a series of experiments must be



performed or a data collection scheme instituted to deduce sufficient knowledge about the system behavior for the application of probability theory to reliability evaluation.

### 3.1.2 Probability Distribution in Reliability Evaluation:

In practice the parameters that are normally associated with reliability evaluation are describe by probability distributions. This can easily be appreciated by considering that all components of a given type construction, manufacture and operating condition will not all fail after the same operating time but will fail at different time in future. Consequently, these time to failure obeys a probability distributions.

The most useful continuous distributions are the normal and exponential and the most important discrete distributions are binomial and poisson distributions.

The cumulative distribution function increases from zero to unity as the random variable increases from its smallest to its largest value. In reliability evaluation, the random variable is time. If at  $t=0$ , the component or system is known to be operating then its probability of failure at  $t=0$  is zero. As  $t \rightarrow \infty$  however, the probability of failure tends to unity, as it is certainty that the component or system will fail given that the exposure time to failure is long enough. This characteristic is therefore equivalent to cumulative function distribution function and is measure of the probability of failure as a function of time. In reliability technology, this cumulative distribution function is known as the cumulative failure distribution function  $Q(t)$ . From this probability of surviving

$$R(t) = 1 - Q(t)$$

The derivative of cumulative distribution function of a continuous random variable gives the probability density function

$$f(t) = \frac{dQ(t)}{dt} = \frac{dR(t)}{dt}$$

$$Q(t) = \int_0^t f(t) dt$$

$$R(t) = 1 - \int_0^t f(t) dt$$

General reliability function

$$R(t) = e^{-\lambda t}$$

$\lambda$  = failure rate

## 3.2 NETWORK MODELLING AND RELIABILITY EVALUATION OF SIMPLE SYSTEM

The previous topics consider the application of basic probability techniques to reliability assessment. However, the system is frequently represented as a network in which the system components are connected together either in series or parallel, meshed or combination of these.

### 3.2.1 Series-Systems

The component are said to be in series from reliability point of view if they must all work for system success or only one need to fail for system failure consider a system consisting of two independent components A and B. Both components must work for system success.

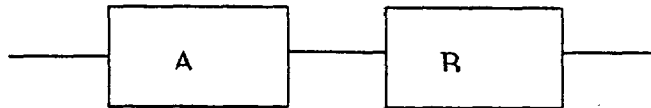


Fig. 3.1

The system reliability is given by

$$R_S = R_A R_B$$

$R_A$  = Probability of successful operation of A.

$R_B$  = Probability of successful operation of B.

For n component system

$$R_s = \prod_{i=1}^n R_i$$

### 3.2.2 Parallel Systems:

The component are said to be in parallel from reliability point of view if only one need to be working for the system success or all must fail for system failure. Consider a system consisting of two independent components A and B connected in parallel.

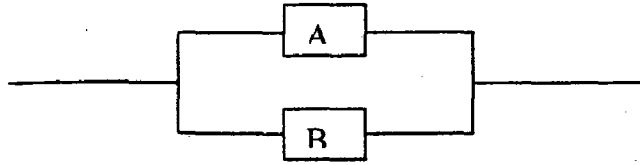


Fig. 3.2

System reliability is given by

$$R_S = 1 - Q_A Q_B$$

$Q_A, Q_B$ : probability of failure of A and B respectively

For n component system

$$R_S = 1 - \prod_{i=1}^n Q_i$$

### 3.2.3 Series-Parallel Systems

In these types of systems, the general principle used is to reduce sequentially the complicated configuration by combining appropriate series and parallel branches of the reliability model until a single equivalent block lefts. This equivalent element then represents the reliability of the original configuration.

### 3.3 NETWORK MODELLING AND RELIABILITY EVALUATION OF COMPLEX SYSTEMS

Many systems either do not have this simple type of structure or have complex operational logic. Additional modeling and evaluation techniques are necessary in order to determine the reliability of such systems. A typical system not having a series/parallel structure is the bridge type network shown in fig.3.3.

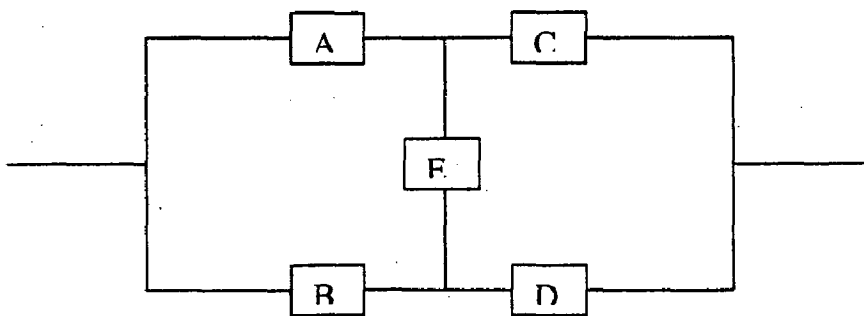


Fig. 3.3 /stem often occurs in many engineering problems. There are number of technique available for solving this type of network including the:

- conditional probability approach

- cut set – tie set analysis
- tree diagram, logic diagram
- connection matrix method

Most of these more advance techniques are formalized method for transforming the logical operation of the system or the topology of the system into structure that consist of only series and parallel components and path or branches. The main difference between them is in the formal presentation or logic of the method and not the essential underlying concept.

### 3.3.1 Cut Set Method

The cut-set method a powerful one for evaluating the reliability of a system for two main reasons:

- (i) It can be easily programmed on a digital computer for the fast and efficient solution of any general network.
- (ii) The cut-set are directly related to the modes of failure and therefore identify the distinct and discrete ways in which a system may fail.

“A cut set is a set of system components which when failed, causes failure of the system”

In terms of reliability network or block diagram, the above definition can be interpreted as asset of components, which must fail in order to disrupt all paths between input and output of the reliability network.

A minimum subset of any given set of components that cause system failure is known as a minimal cut-set. It can be define as follows: A minimal cut set is a set of system components, which, when failed, cause failure of the system but when any one component of the set has not failed does not cause system failure. This definition means that all components of a minimal cut-set must be failed to cause system failure. Using this definition, the minimal cut set of the system shown in fig.3.3 are listed in table 3.1.

Table 3.1

| number of minimal cut-set | component of the cut-set |
|---------------------------|--------------------------|
| 1                         | AB                       |
| 2                         | CD                       |
| 3                         | AED                      |
| 4                         | BEC                      |

In order to evaluate the system reliability the minimal cut sets identified from the reliability network must be combined. From the definition of minimal cut set it is evident that all components of each cut must fail in order for the system to fail. Consequently, the components of the cut set are effectively connected in parallel and the failure probabilities of the components cut set may combined using the principle of parallel systems. In addition, the system fails if any one of the cut sets, occurs and consequently each cut is effectively in series with all other cuts. The use of this principle gives the reliability diagram in fig.3.4.

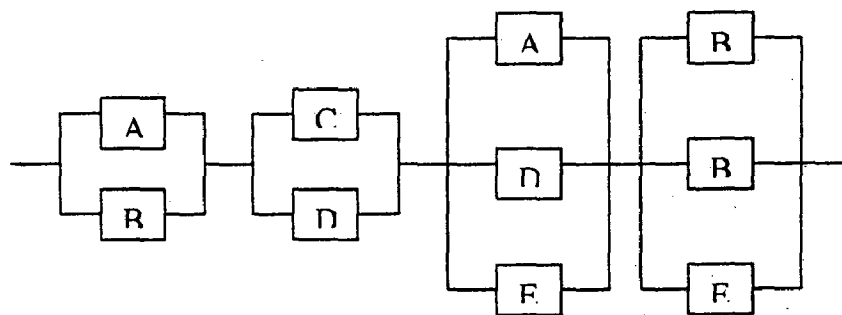


Fig. 3.4

Although, these cut-set are in series, the concept of series systems cannot be used because the same component can appear in two or more of cut sets. The concept of union

does apply however and if the  $i^{\text{th}}$  cut is designated as  $C_i$  and its probability of occurrence is designated as  $P(C_i)$ , then the unreliability of the system is given by

$$Q_s = P(C_1 \cup C_2 \cup C_3 \cup \dots \cup C_i \dots \cup C_n)$$

This method calculate the unreliability and hence the reliability of the system precisely, but it can be an exhaustive and time consuming exercise which can become prohibitive with large systems. To overcome this problem, approximation can be made in the evaluation, which although they reduce precision, permit much faster evaluation. The degree of imprecision introduced is negligible and often within the tolerance associated with the data of the component reliabilities for systems, which have high value of component reliability.

$$Q_s = P(C_1) + P(C_2) + \dots + P(C_i) + \dots + P(C_n)$$

$$= \prod_{i=1}^n P(C_i)$$

For the system shown in fig.3.4

$$Q_s = Q_A Q_B + Q_C Q_D + Q_A Q_D Q_E + Q_B Q_C Q_E$$

### 3.3.2 Tie Set Method

The tie set method is complement of cut-set method. It is used less frequently, in practice, as it does not directly identify the failure modes of the system. It has certain special applications and therefore is discussed briefly.

A tie set is a minimal path of the system and in therefore a set of system components connected in series. Consequently a tie set fails if any one of the components in it fails and this probability can be evaluated using the principle of series system. For the system to fail however all of the tie sets must fail therefore all tie sets are effectively connected in parallel. The reliability diagram for fig 3.3 is shown in fig. 3.5.

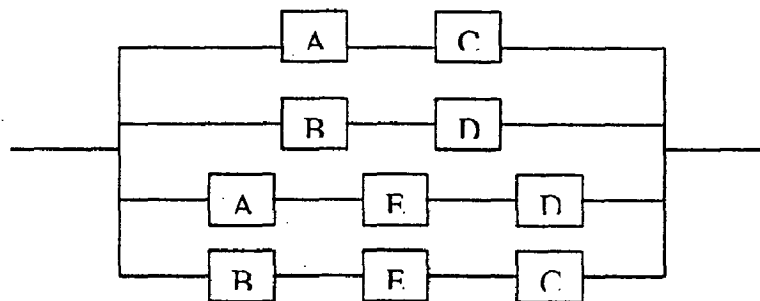


Fig. 3.5

### **3.4 EVENTS TREES**

An event tree is a pictorial representation of all the events which can occur in system. It is defined as a tree because the pictorial representation gradually fans out like branches of a tree as an increasing number of events are considered. Event trees are useful for system reliability analysis and quantification since they illustrate the logic of combination of probabilities and consequences of event sequences.

### **3.5 FAULT TREES**

Fault trees use a logic that is essentially the reverse of that used in event trees. In this method a particular failure condition is considered and a tree is constructed that identifies the various combinations and sequence of other failures that leads to the failure being considered.

## APPLICATION OF FUZZY LOGIC TO RELIABILITY ENGINEERING

The analysis of system reliability often requires subjective judgment, uncertain data, and approximates system models. By allowing imprecision and approximation analysis fuzzy logic provides an effective tool for characterizing system reliability in these circumstances, it does not force precision where it is not possible.

In the case of reliability, uncertainty is due to the fact since failures are relatively rare events. Collecting enough data on which to base a statistical "probability of failure" is a costly and difficult undertaking and the relevance of the data to any particular system as well its validity is often questionable. Furthermore, especially early in the design the item whose probability of failure is needed often does not exist and it must be estimated based on engineering judgment or experience from similar items. Extrapolating these failure probabilities through statistical method to calculate a system level reliability only increases the uncertainty.

### 4.1 PARALLEL, SERIES AND NON-SERIES-PARALLEL SYSTEM ANALYSIS

The organizational structure of most system can be described as a series, parallel or a non-series parallel system. The reliability of such systems is easily analyzed probabilistically.

let.

$P_i$  = probability of failure of component  $i$

$R_i$  = reliability of component  $i$

$P_{sys}$  = System probability of failure

$R_{sys}$  = system reliability =  $1 - P_{sys}$

#### 4.1.1 Parallel Systems

For parallel system the system probability of failure  $P_{sys}$  is the product of individual component probabilities.

$$P_{sys} = P_1 \times P_2 \times P_3 \times \dots \times P_n$$



Although the equation is precise, as observed earlier, the numbers used in its computation seldom are. Fuzzy logic provides an intuitively appealing way of handling uncertainty by treating the probability of failure as a fuzzy number. This allows the analyst to specify a range of value with an associated probability distribution for the failure probabilities. If associated a triangular membership function with the interval, we assume that the analyst has more confidence that the actual parameter lies near the center of the interval than at the edges. Here the analysis of a simple 2 component parallel system is considered. Let  $P_A$  and  $P_B$  be the component failure probabilities of a simple two component parallel system then:

$$P_{sys} = P_A P_B \approx (P_A P_B) \Delta$$

If  $P_A$  and  $P_B$  are fuzzy probabilities on the interval  $[P_{A1}, P_{A2}]$  and  $[P_{B1}, P_{B2}]$  respectively, the result  $P_{sys}$  will be a fuzzy probability  $P_A P_B$  with range  $[P_{A1} \times P_{B1}, P_{A2} \times P_{B2}]$ . This is illustrated in figure 4.1.

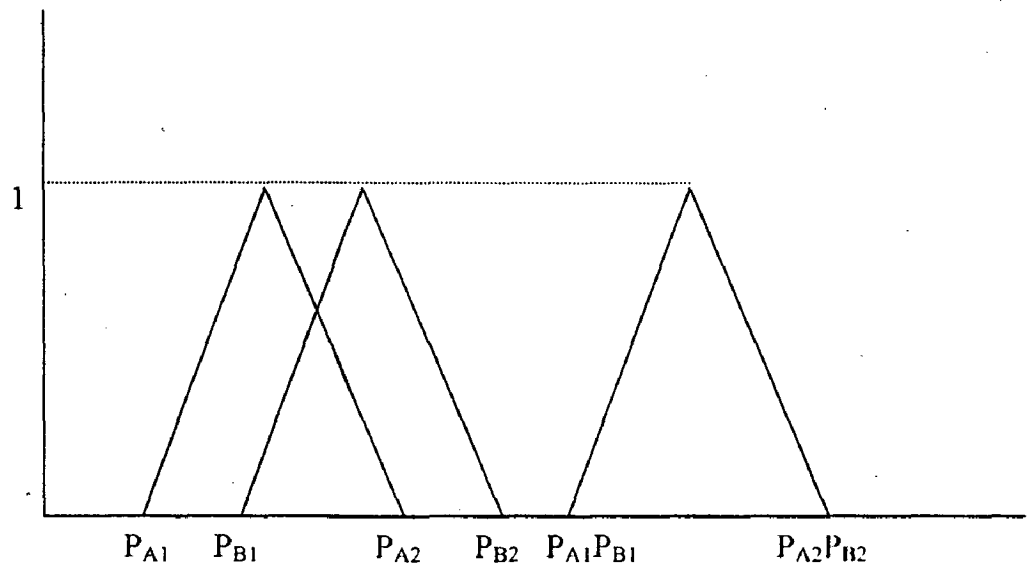


Fig.4.1

The system probability of failure can also be evaluated using linguistic description of the component failure probabilities. Such descriptions are often used as guidelines for estimating numerical reliabilities. Although the descriptions are less precise than numerical failure probabilities, they may in many cases be more accurate in the sense that they give a truer assessment of the knowledge of the component reliability than a numerical value. Each term identifies a range of reliability than a numerical value.

Each term identifies a range of reliabilities specified as a fuzzy set. A parallel system is evaluated using the linguistic rule:

$$\text{If Fail}(A) = P_A \text{ and Fail}(B) = P_B \text{ then Fail}(\text{sys}) = P_A P_B$$

And corresponding fuzzy arithmetic operations to yield the (fuzzy) system probability of failure and its associated degree of possibility of possibility. If desire the fuzzy result can be defuzzified to give the system probabilities of failure.

To illustrate the procedure, consider a 2 component parallel system, the failure probability of two components can be expressed using following term;

|           |    |                    |
|-----------|----|--------------------|
| Very high | if | $P_1 \geq .8$      |
| High      | if | $.6 \leq P_1 < .8$ |
| Moderate  | if | $.4 \leq P_1 < .6$ |
| Low       | if | $.2 \leq P_1 < .4$ |
| Very low  | if | $0 \leq P_1 < .2$  |

These five fuzzy sets are shown in Fig. 4.2.

To calculate  $1-P$  where  $P$  is a fuzzy probability is defined as moving to the corresponding fuzzy set at the opposite end of the scale. Thus

- 1 – low = high
- 1 – very high = very low
- 1 – moderate = moderate
- etc.

The analyses of parallel system using above concepts are shown in fig. 4.3

#### 4.1.2 Series System

In series system all system components must be operational for the system to work. Series systems are most easily analysed in terms of their component reliabilities

$$R_{\text{sys}} = R_1 \times R_2 \times \dots \times R_n$$

The analysis of series system is identical to that of parallel system. One example of series system is shown in fig. 4.4.

### **4.1.3 Combination System**

Series parallel systems are arranged as combinations of series and parallel elements. Series parallel systems can be analyzed by successively analyzing their series and parallel subsystems. Non-series parallel systems can be analyzed using either pathset or cutset method. The analysis is carried out by treating each of the probabilities of failure as a fuzzy probability or as a linguistic value.

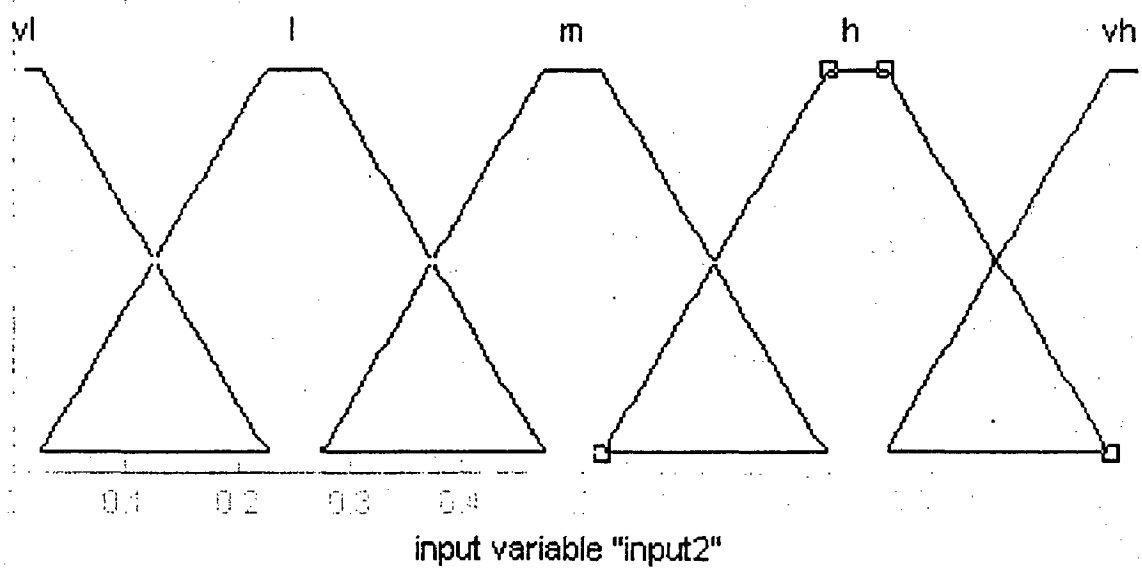
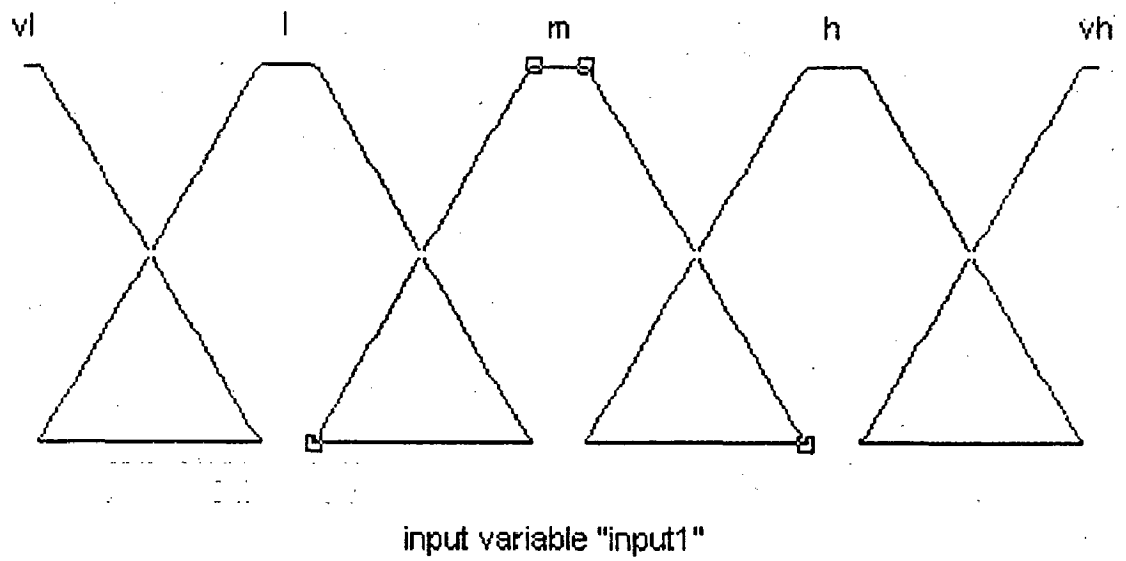


Fig. 4.2 fuzzy sets used for fuzzy calculations of two component systems

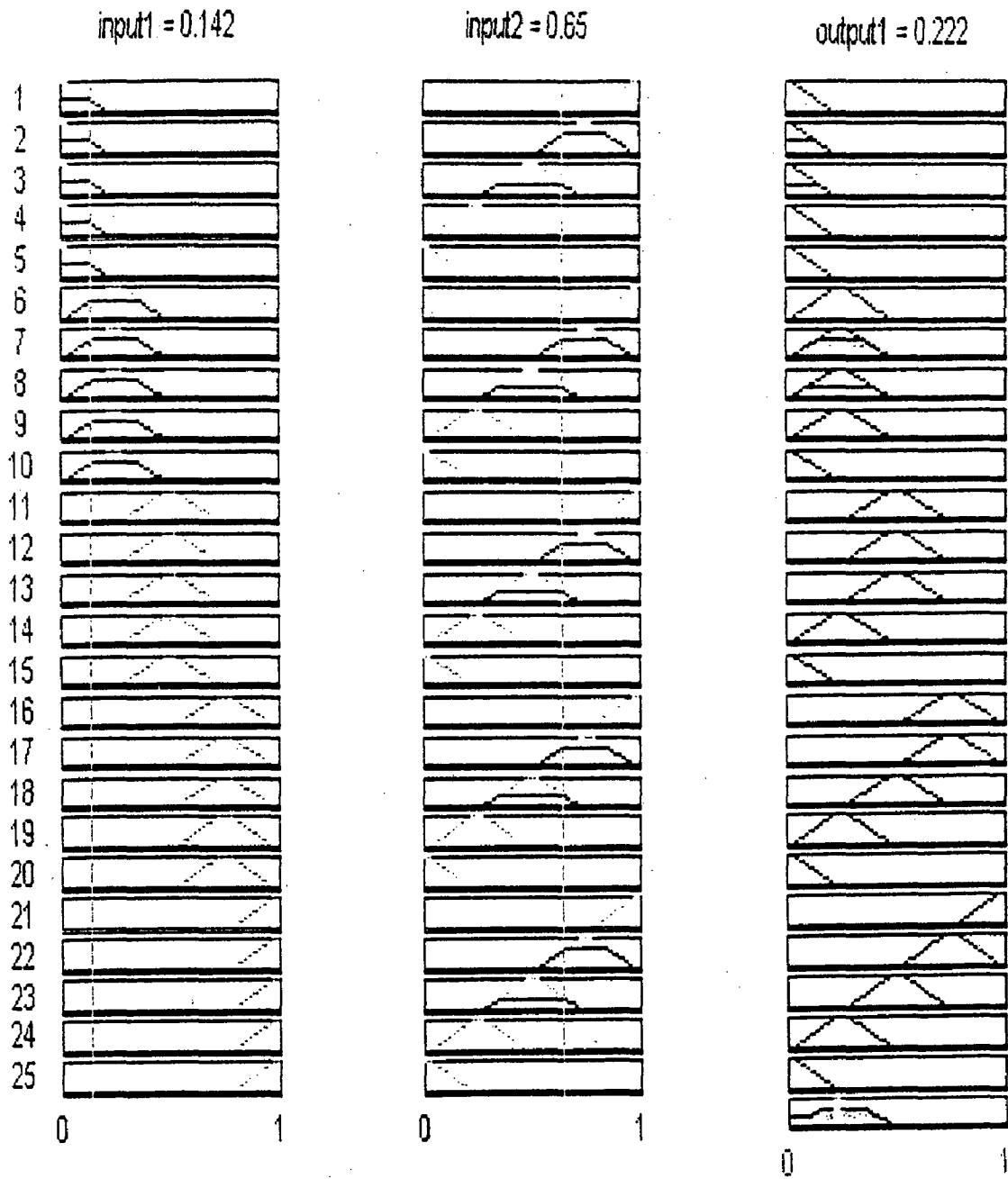


Fig.4.3 fuzzy reliability for parallel system

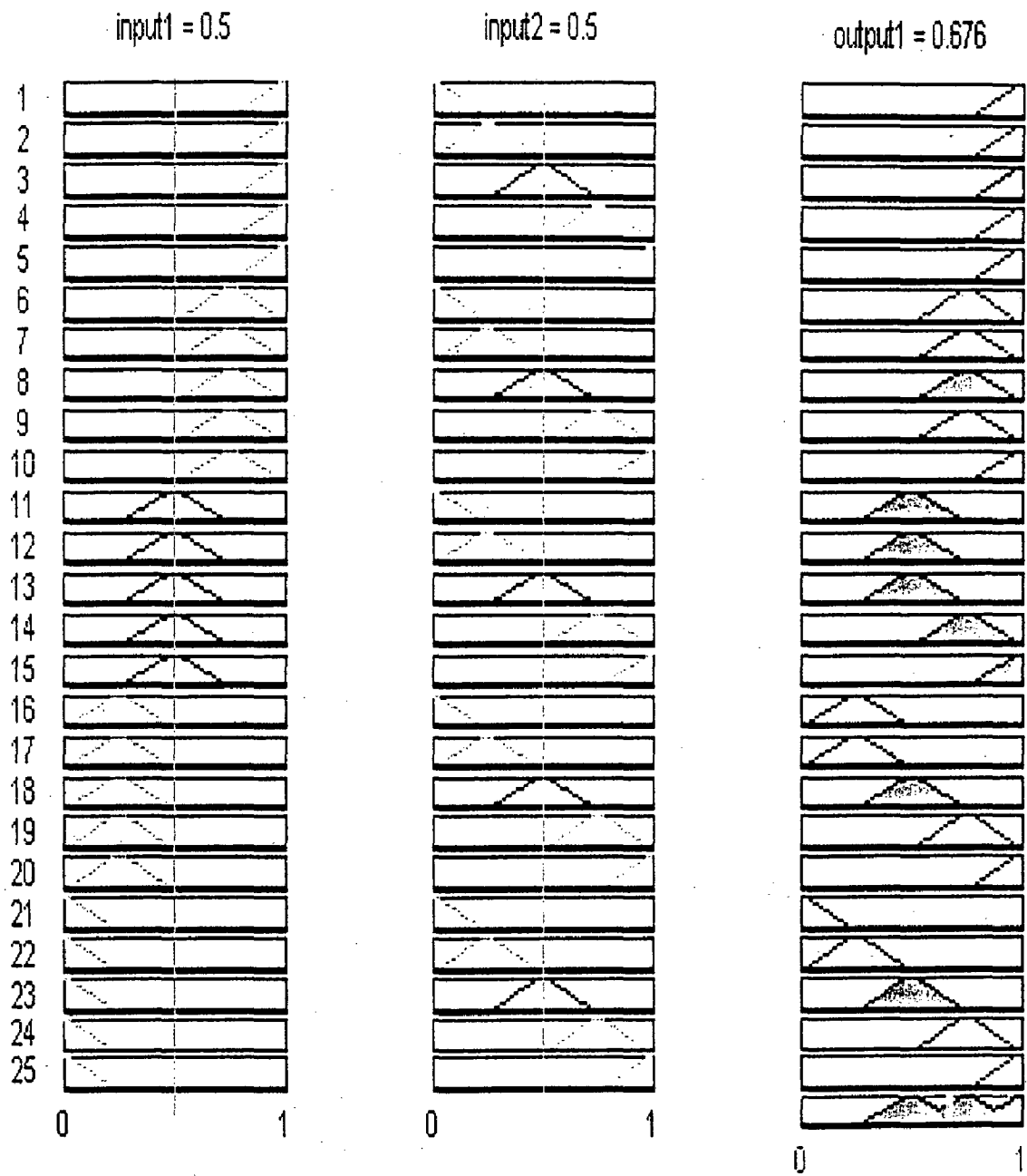


Fig.4.4 fuzzy reliability for series system

## EVALUATION OF FUZZY RELIABILITY OF PRACTICAL SYSTEMS

### 5.1 EVALUATION OF FUZZY RELIABILITY OF NON-SERIES PARALLEL NETWORK

#### 5.1.1 Concept of Fuzzy Probability

Fuzzy probability represents a fuzzy number, between zero and one assigned to the probability of event. One can choose different types of membership functions for fuzzy probabilities. For example a fuzzy probability may have trapezoidal membership function. The fuzzy probability of an event  $i$  can then be denoted by a four parameter function i.e.

$$P_i = (\alpha_{i1}, \alpha_{i2}, \beta_{i2}, \beta_{i1})$$

as shown in Fig.5.1.

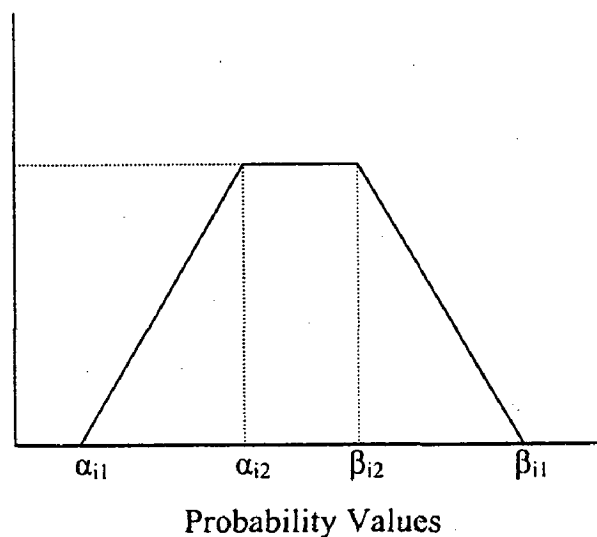


Fig. 5.1:

The membership function is given by

$$\mu_{pi}(p) = \begin{cases} 0 & \text{for } 0 \leq p \leq \alpha_{i1} \\ 1 - \left( \frac{\alpha_{i2} - p}{\alpha_{id}} \right) & \text{for } \alpha_{i1} \leq p \leq \alpha_{i2} \\ 1 & \text{for } \alpha_{i2} \leq p \leq \beta_{i2} \\ 1 - \left( \frac{p - \beta_{i2}}{\beta_{id}} \right) & \text{for } \beta_{i2} \leq p \leq \beta_{i1} \\ 1 & \beta_{i1} \leq p \leq 1 \end{cases}$$

where  $\alpha_{id} = (\alpha_{i2} - \alpha_{i1})$ ,  $\beta_{id} = \beta_{i1} - \beta_{i2}$  operation used in computing fuzzy reliability:

Basically only two operations i.e. multiplication and compliment, need to be performed in assessing system reliability of non-series parallel system. A procedure is developed to carry out these operations efficiently. Let  $P_i$  and  $P_j$  be the two fuzzy sets that have membership function given by  $\mu_{pi}(p)$  and  $\mu_{pj}(p)$  respectively. The operations used in fuzzy reliability can be defined as follows

### Multiplication

The multiplication of two fuzzy sets  $P_i$  and  $P_j$  that have trapezoidal membership functions represented by

$$P_{ij} = P_i \cdot P_j$$

would have membership function given by

$$\begin{cases} 0 & \text{for } 0 \leq p \leq \alpha_{i2} \cdot \alpha_{j2} \\ 1 - \alpha_{ij} + \left[ (p - \alpha_{i2} \cdot \alpha_{j2}) / (\alpha_{id} \cdot \alpha_{jd}) + \alpha_{ij}^2 \right]^2 & \text{for } \alpha_{i1} \cdot \alpha_{j1} \leq p \leq \alpha_{i2} \cdot \alpha_{j2} \\ 1 & \text{for } \alpha_{i2} \cdot \alpha_{j2} \leq p \leq \beta_{i2} \cdot \beta_{j2} \\ 1 + \beta_{ij} - \left[ (p - \beta_{i2} \cdot \beta_{j2}) / (\beta_{id} \cdot \beta_{jd}) + \beta_{ij}^2 \right]^2 & \text{for } \beta_{i2} \cdot \beta_{j2} \leq p \leq \beta_{i1} \cdot \beta_{j1} \\ 0 & \text{for } \beta_{i1} \cdot \beta_{j1} \leq p \leq 1 \end{cases}$$

where

$$\alpha_{ij} = \frac{\alpha_{id} \alpha_{j2} + \alpha_{jd} \alpha_{i2}}{2 \alpha_{id} \alpha_{jd}}$$

$$\beta_{ij} = \frac{\beta_{id} \beta_{j2} + \beta_{jd} \beta_{i2}}{2 \beta_{id} \beta_{jd}}$$



However, an approximate formula can be used

$$P_{ij} = P_i \cdot P_j = (\alpha_{i1} \cdot \alpha_{j1} \cdot \alpha_{j2} \cdot \alpha_{i2}, \beta_{i2} \cdot \beta_{j2} \cdot \beta_{i1} \cdot \beta_{j1})$$

### Complementation

The complementation of a fuzzy set  $P_i$

$$P_i = 1 - \mu_{pi}$$

For example in case of a trapezoidal membership function as shown in fig.

$$P_i = (1 - \beta_{i1}, 1 - \beta_{i2}, 1 - \alpha_{i2}, 1 - \alpha_{i1})$$

### 5.1.2 Bridge Network

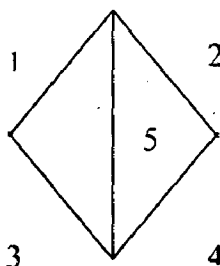


Fig.5.2 A Bridge Network (Non-Series Parallel)

System reliability of a non-series parallel network can be assessed through the use of two operations i.e. multiplication and complementation. Consider the bridge network shown in fig 5.2. There are four minimum pathsets in a bridge network.

$$\{1,2\}, \{3,4\}, \{1,4,5\}, \{2,3,5\}$$

The system reliability can be expressed as

$$R_s = P_1 \cdot P_2 + P_{12} \cdot P_3 \cdot P_4 + P_{12} \cdot P_{34} \cdot P_1 \cdot P_4 \cdot P_5 + P_{12} \cdot P_{34} \cdot P_{145} \cdot P_2 \cdot P_3 \cdot P_5$$

Where;

$$P_{12} = P_1 \cdot P_2$$

$$P_{34} = P_3 \cdot P_4$$

$$P_{145} = P_1 \cdot P_4 \cdot P_5$$

$$P_{235} = P_2 \cdot P_3 \cdot P_5$$

Where  $P_i$  denotes the fuzzy reliability of  $i^{\text{th}}$  element. The fuzzy system reliability can be computed using following procedures.

Let

$$P_{A1} = P_1.P_2$$

$$P_{A2} = P_{12}.P_3.P_4$$

$$P_{A3} = P_{12}.P_{34}.P_1.P_4.P_5$$

$$P_{A4} = P_{12}.P_{34}.P_{145}.P_2.P_3.P_5$$

So system reliability can be expressed as

$$R_s = 1 - (P_{A1}.P_{A2}.P_{A3}.P_{A4})$$

Now let us assume that the trapezoidal membership functions and the fuzzy probabilities of elements 1,2,3,4,5 are given as follows:

$$P_1 = (0.18, 0.9, 0.94, 0.98)$$

$$P_2 = (0.26, 0.7, 0.80, 0.94)$$

$$P_3 = (0.38, 0.55, 0.72, 0.92)$$

$$P_4 = (0.17, 0.32, 0.45, 0.55)$$

$$P_5 = (0.16, 0.18, 0.26, 0.55)$$

respectively. These fuzzy probabilities are shown in fig.5.3. Now carry out multiplications and complementation, the system fuzzy reliability is found to be,

$$R_s = (0.18, 0.70, 0.80, 0.94) \text{ as shown in fig. 5.4.}$$

The inference that can easily be drawn is that the system reliability lies between limits 0.7- 0.80 with a 100% possibility.

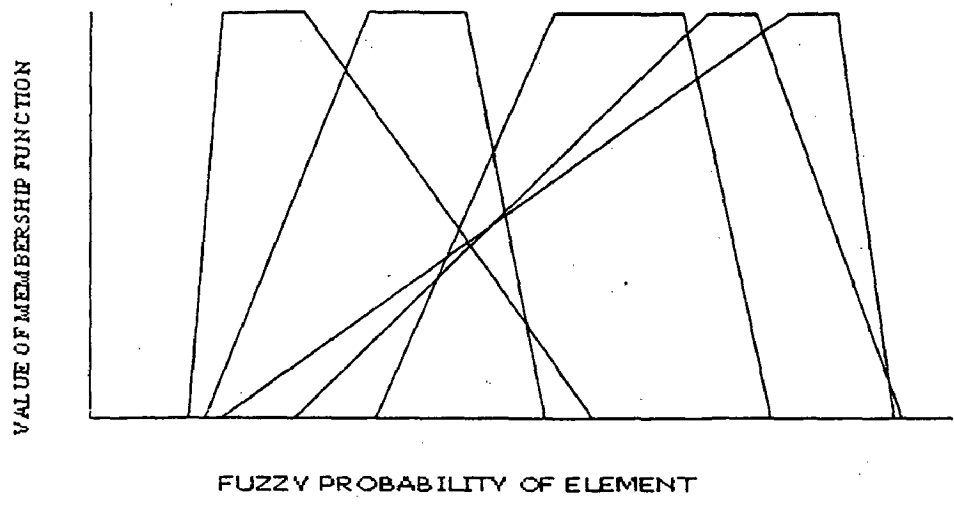


Fig.5.3

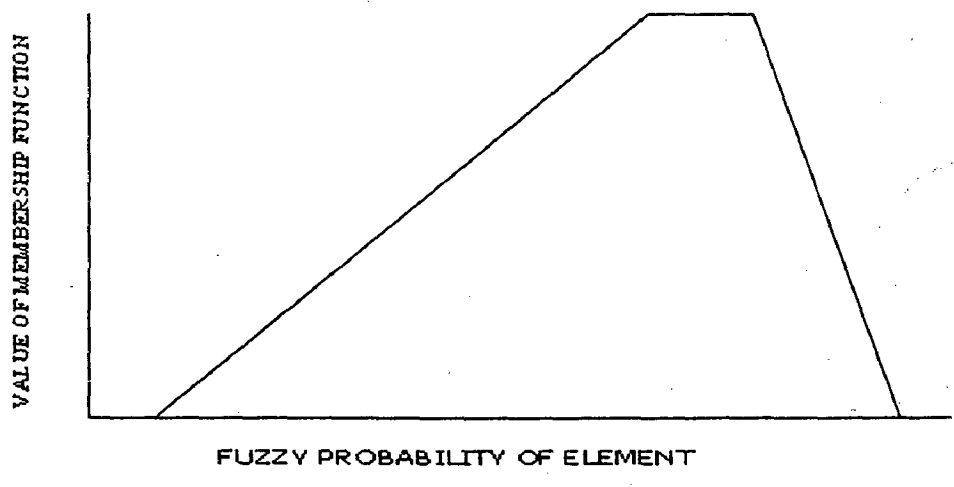


Fig 5.4

## 5.2 FUZZY RELIABILITY MODELING – LINGUISTIC APPROACH

A fuzzy process, similar to stochastic process is carried out for reliability network modeling. As an illustration, fuzzy Markov modeling for a power generator with a derated state has been developed. The model indicates the possibility of the power generator being in different states after some transition period by considering the experts subjective opinions about the state transition probabilities and it is seen that this model serve better.

Reliability of elements in a network depends on numerous factors. To evaluate the probability associated with the failure in an analytic manner considering all such factors would become very difficult. As an alternative, fuzzy modeling has been applied for subjective evaluation, which replaces the analytical approach.

The basic operations on fuzzy sets can be represented by simple networks as in the theory of reliability networks. The intersection operation on fuzzy sets can be represented by a parallel network and so on. A fuzzy process is carried out for reliability modeling of a power generator with a derated state. Linguistic values are assigned to the state transition probabilities.

### 5.2.1 Comparison between Reliability Networks and Basic Operation on Fuzzy Sets

Let P and Q be two fuzzy sets and  $\mu_P(x)$  and  $\mu_Q(x)$  are respective grades of membership of x in P and Q. The union of fuzzy sets P and Q is denoted by  $P \cup Q$  and is defined by

$$P \cup Q = \mu_P(x) \vee \mu_Q(x) \quad \forall x$$

Where  $\vee$  is the symbol for maximum. The intersection of fuzzy set P and Q is denoted by  $P \cap Q$  and is defined by

$$P \cap Q = \mu_P(x) \wedge \mu_Q(x) \quad \forall x$$

Where  $\wedge$  is the symbol for minimum. The result obtained from these operations can be compared with the reliability values obtained from the networks. For two elements  $R_1$  and  $R_2$  connected in series, the system reliability is given by

$$R_s = R_1 * R_2 \text{ and } R_s \text{ must be less than or equal to the minimum of } R_1 \text{ and } R_2$$

These results resemble the intersection operation on fuzzy sets. If these elements are connected in parallel, the reliability value will be

$$R_p = 1 - (1 - R_1)(1 - R_2)$$

Which resemble the grade of membership obtained from the probabilistic algebraic sum of two fuzzy subset.  $R_p$  must be greater than or equal to the maximum of  $R_1$  and  $R_2$ . This resembles the union operation on fuzzy sets. If the operation has to be done with three fuzzy subsets, it is as given below:

$$g(A,B,C) = (A \wedge B \wedge C) \vee (A \wedge B) \vee (A \wedge B \wedge C)$$

where  $g(A,B,C)$  is the grade of membership obtained from the above fuzzy operation can be shown in fig. 5.5

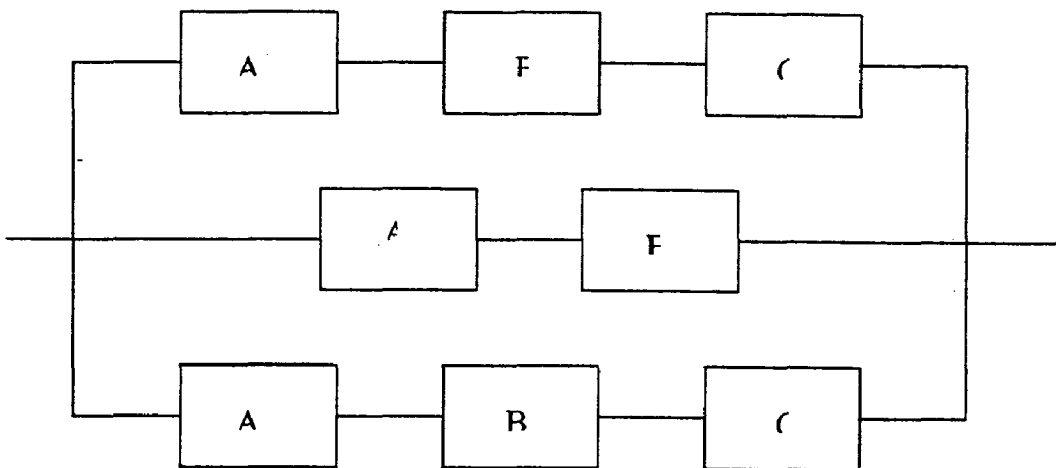


fig. 5.5 Sample reliability network representing  $g(A,B,C)$ .

The reliability value obtained from the above network must be greater than or equal to  $g(A,B,C)$

### 5.2.2 FUZZY RELIABILITY MODELING

If we have the fuzzy elements in a reliability network, that is, the reliability values of those elements are not known precisely, we can assign linguistic variables such as Low, Medium, High etc., as reliability values and then we can evaluate the total network reliability. The final result obtained is also fuzzy in nature yet it convey a meaningful solution if we quantify the linguistic variables. The fuzzy grades of membership of linguistic variables Low, Medium and High defined by the following fuzzy sets

Low:  $[(0.5, 0.2), (0.7, 0.3), (1.0, 0.4), (0.7, 0.5), (0.5, 0.6)]$

Medium:  $[(0.5, 0.4), (0.7, 0.5), (1.0, 0.6), (0.7, 0.7), (0.5, 0.8)]$

High:  $[(0.5, 0.7), (0.7, 0.8), (0.9, 0.9), (1.0, 1.0)]$

Therefore, the fuzzy sets for Very low, Very medium, and Very low are as follows:

Very Low = Low<sup>2</sup>

Very Low:  $[(0.25, 0.2), (0.49, 0.3), (1.0, 0.4), (0.49, 0.5), (0.25, 0.6)]$

Very Medium:  $[(0.25, 0.4), (0.49, 0.5), (1.0, 0.6), (0.49, 0.7), (0.25, 0.8)]$

Very High:  $[(0.25, 0.7), (0.49, 0.8), (0.81, 0.9), (1.0, 1.0)]$

Similar to stochastic process, a fuzzy process can be defined as follows:

A finite process in discrete time with a discrete state space  $Q = \{q_1, q_2 \dots q_n\}$  is called a finite fuzzy process if it satisfies the following conditions.

i) The matrix F which describe the state transition has the following for

$$F = \begin{matrix} & \begin{matrix} q_1 & q_2 & & q_r \end{matrix} \\ \begin{matrix} q_1 \\ q_2 \\ \dots \\ q_r \end{matrix} & \begin{bmatrix} f_{11} & f_{12} & & f_{1r} \\ f_{21} & f_{22} & \dots & f_{2r} \\ \dots & \dots & \dots & \dots \\ f_{r1} & f_{r2} & & f_{rr} \end{bmatrix} \end{matrix}$$

where  $0 \leq f_{ij} \leq 1$  denotes the grade of membership of state transition from state  $q_i$  to  $q_j$ . This matrix will be called the fuzzy state transition matrix of the fuzzy process.

ii) Let A be a fuzzy set defined on S, and

$$w_A^{(0)} = [\phi_{q_1}^{(0)} \phi_{q_2}^{(0)} \dots \phi_{q_r}^{(0)}]$$

be a row vector, called initial state designator of A, where  $\phi$  is the grade of membership of states with respect to A. Then the state designator of A at  $t = n$  is obtained by,

$$w_A^{(n)} = w_A^{(0)} \cdot F^n = w_A^{(0)} [f_{ij}^{(n)}]$$

where

$$f_{ij}^{(n)} = \begin{matrix} \max & \min \\ \text{over all} & \text{over all} \\ \text{parallel paths} & \text{series paths} \\ \text{from } q_i \text{ to } q_j & \text{from } q_i \text{ to } q_j \end{matrix} \{ \mu_{ij} \}$$

$\mu_{ij}$  = the set of grade of membership of state transition of a path from  $q_i$  to  $q_j$

An example is shown for the application of a fuzzy process in the reliability modeling of power generator systems.

### CASE STUDY

#### **Markov Modelling of a Power Generator with a Derated States – Linguistic Approach**

In this model a power generator is said to have three states:

- i) Fully operational ( full power generation )
- ii) Partial operation (power generation at a derated level )
- iii) Failed ( no power output )

For example, at a coal-fired power station, the generator derated state may occur due to failure of some of the unit pulverizers.

The state space diagram is given in fig. 5.6

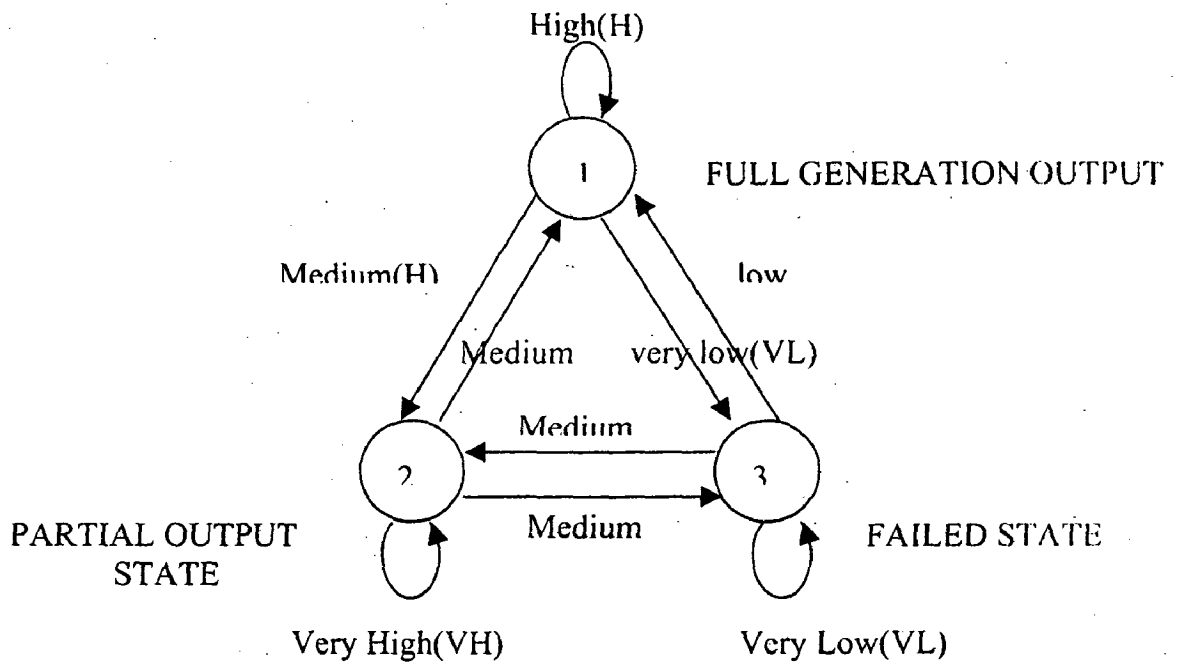


Fig.5.6 State Space Diagram

The generating system can either be derated or fail completely and can be repaired. Here it is assumed the state transition probability are known imprecisely and defined in terms of linguistic variables.

$$F = \begin{bmatrix} H & M & VL \\ M & VH & M \\ L & M & VL \end{bmatrix}$$

The tree diagram for this state transition matrix is shown in fig. 5.7. This tree diagram explains the status of the power generator after two steps given that the fuzzy process started in state 1, state 2, and state 3 respectively. That is,

$$F^2 = \begin{bmatrix} H & M & M \\ M & VH & M \\ M & M & M \end{bmatrix}$$

If the initial state designator  $w_A^{(0)}$  is defined as,

$$w_A^{(0)} = [M \ H \ L]$$

then the state designator at  $t = 2$  can be obtain as follows:



$$w_A^{(2)} = w_A^{(0)} F^2$$

$$= [M \quad H \quad L] \begin{bmatrix} H & M & M \\ M & VH & M \\ M & M & M \end{bmatrix}$$

By using the fuzzy basic operation

$$w_A^{(2)} = [M \quad H \quad M]$$

The first element in the above vector is obtained as follows:

$$(M \cap VH) \cup (H \cap M) \cup (L \cap H) = [M]$$

Using the quantitative measure of linguistic variables, the following  $w_A^{(2)}$  can be obtained

$$w_A^{(2)} = \{ [(0.5, 0.4), (0.7, 0.5), (1.0, 0.6), (0.7, 0.7), (0.5, 0.8)], [(0.5, 0.7), (0.7, 0.8), (0.9, 0.9), (1.0, 1.0)], [(0.5, 0.4), (0.7, 0.5), (1.0, 0.6), (0.7, 0.7), (0.5, 0.8)] \}$$

$$w_A^{(2)} \approx [0.6 \quad 1.0 \quad 0.6]$$

The approximation  $w_A^{(2)}$  vector contains the values those have highest grade of membership in the quantitative definitions of linguistic terms and expresses that there is a high probability of the system being in state 2 (partial operation state) after two transitions.

The fuzzy steady-state designator vector is determine by the following equation

$$\bigcup_j \pi_i \wedge f_{ij} = \pi_i \quad i, j = 1, 2, \dots, k$$

where k is the total number of state

that is for the above considered 3-state problem,

$$(\pi_1 \wedge f_{11}) \vee (\pi_2 \wedge f_{21}) \vee (\pi_3 \wedge f_{31}) = \pi_1 \quad (1)$$

$$(\pi_1 \wedge f_{12}) \vee (\pi_2 \wedge f_{22}) \vee (\pi_3 \wedge f_{32}) = \pi_2 \quad (2)$$

$$(\pi_1 \wedge f_{13}) \vee (\pi_2 \wedge f_{23}) \vee (\pi_3 \wedge f_{33}) = \pi_3 \quad (3)$$

Since the operation on the left hand side (LHS) of the above equations, are fuzzy operations, to solve them, the following procedure is used.

**STEP 1:**

Start with an initial approximation for  $\pi_1, \pi_2, \pi_3$ .

**STEP 2:**

Compute left hand side (LHS) of equation (1), (2), and (3). Let  $\epsilon$  be the tolerance limit.

If  $(LHS_i - RHS_i) \leq \epsilon$  for  $i = 1, 2, 3$

then

the obtained  $\pi_i$  vector is the fuzzy steady state designation vector.

Stop.

else

perturb the values of  $\pi_1, \pi_2, \pi_3$

go to step 2.

**Example**

if  $\pi_i$  vector is [ 0.6 1.0 0.4 ],

then the fuzzy steady state designation vector is given as,

[ 0.6 1.0 0.6 ]

| t=0            | t=1            | t=2            | F <sup>2</sup>       |                                    |                                    |
|----------------|----------------|----------------|----------------------|------------------------------------|------------------------------------|
|                |                |                | min{μ <sub>ij</sub>  | max.min{ μ <sub>ij</sub>           |                                    |
| q <sub>1</sub> | q <sub>1</sub> | q <sub>1</sub> | f <sub>11</sub> = H  | f <sub>11</sub> <sup>(2)</sup> = H |                                    |
|                |                | q <sub>2</sub> | f <sub>12</sub> = M  |                                    |                                    |
|                |                | q <sub>3</sub> | f <sub>13</sub> = VL |                                    |                                    |
|                | q <sub>3</sub> | q <sub>3</sub> | q <sub>1</sub>       | f <sub>11</sub> = M                | f <sub>12</sub> <sup>(2)</sup> = H |
|                |                |                | q <sub>2</sub>       | f <sub>12</sub> = M                | f <sub>13</sub> <sup>(2)</sup> = H |
|                |                |                | q <sub>3</sub>       | f <sub>13</sub> = M                | f <sub>21</sub> <sup>(2)</sup> = H |
|                | q <sub>3</sub> | q <sub>3</sub> | q <sub>1</sub>       | f <sub>11</sub> = VL               | f <sub>22</sub> <sup>(2)</sup> = H |
|                |                |                | q <sub>2</sub>       | f <sub>12</sub> = VL               | f <sub>23</sub> <sup>(2)</sup> = H |
|                |                |                | q <sub>3</sub>       | f <sub>13</sub> = VL               | f <sub>31</sub> <sup>(2)</sup> = H |
| q <sub>2</sub> | q <sub>1</sub> | q <sub>1</sub> | f <sub>21</sub> = M  | f <sub>32</sub> <sup>(2)</sup> = H |                                    |
|                |                | q <sub>2</sub> | f <sub>22</sub> = M  | f <sub>33</sub> <sup>(2)</sup> = H |                                    |
|                |                | q <sub>3</sub> | f <sub>23</sub> = VL |                                    |                                    |
|                | q <sub>2</sub> | q <sub>2</sub> | q <sub>1</sub>       | f <sub>21</sub> = M                |                                    |
|                |                |                | q <sub>2</sub>       | f <sub>22</sub> = VH               |                                    |
|                |                |                | q <sub>3</sub>       | f <sub>23</sub> = M                |                                    |
|                | q <sub>3</sub> | q <sub>3</sub> | q <sub>1</sub>       | f <sub>21</sub> = L                |                                    |
|                |                |                | q <sub>2</sub>       | f <sub>22</sub> = M                |                                    |
|                |                |                | q <sub>3</sub>       | f <sub>23</sub> = VL               |                                    |
| q <sub>3</sub> | q <sub>1</sub> | q <sub>1</sub> | f <sub>31</sub> = L  |                                    |                                    |
|                |                | q <sub>2</sub> | f <sub>32</sub> = L  |                                    |                                    |
|                |                | q <sub>3</sub> | f <sub>33</sub> = VL |                                    |                                    |
|                | q <sub>2</sub> | q <sub>2</sub> | q <sub>1</sub>       | f <sub>31</sub> = M                |                                    |
|                |                |                | q <sub>2</sub>       | f <sub>32</sub> = M                |                                    |
|                |                |                | q <sub>3</sub>       | f <sub>33</sub> = M                |                                    |
|                | q <sub>3</sub> | q <sub>3</sub> | q <sub>1</sub>       | f <sub>31</sub> = VL               |                                    |
|                |                |                | q <sub>2</sub>       | f <sub>32</sub> = VL               |                                    |
|                |                |                | q <sub>3</sub>       | f <sub>33</sub> = VL               |                                    |

Fig. 5.7 TREE DIAGRAM

### **5.3 EVENT TREE ANALYSIS BY FUZZY PROBABILITY**

#### **(Calculation of Risk associated with an Electric Power System )**

Event trees are useful for system – reliability analysis and risk quantification since they illustrate the logic of combination of probabilities and consequence of event sequences. For many systems, estimation of the single number for the probability and consequences is difficult due to uncertainty and imprecision of data. Here I uses fuzzy-set logic to account for imprecision and uncertainty in data while event tree analysis

#### **CASE STUDY**

Consider the electric power system as shown in fig. 5.8. Each of the transformer is protected by differential scheme i.e. both circuit breaker protecting each of the transformer are operated by the sane fault detector, FD, and a combine relay / trip signal device, RTS. Given that fault occurs on the transformer TR1 it is desired to evaluate the probability of successful operation of the protection system.

Generally electric power protection system involves the sequential operation of set of components and devices. Event trees are particularly useful because they recognize the sequential operational logic of system. Fig.5.9 shows the fuzzy event tree for the network shown in fig. 5.8.

#### **FUZZY PROCEDURE FOR EVALUATING RISK**

There is an important consideration is the severity of the effect of the failure. The ‘risk’ associated with a failure increases as either the severity of the effect of the failure or the failure probabilities increases.

Fuzzy logic provides a more flexible and meaningful way of assessing risk. The analysis uses linguistic variables to describe the severity and frequency of occurrence of the failure. These parameters are “fuzzified” to determine their degree of membership in each input class using membership functions. The resulting “fuzzy inputs” are evaluated using a linguistic rule base and fuzzy logic operations to yield a classification of the “riskiness” of the failure and an associated degree of membership in each risk class. This “fuzzy conclusion” is then “defuzzified” to give a single risk priority for the failure.

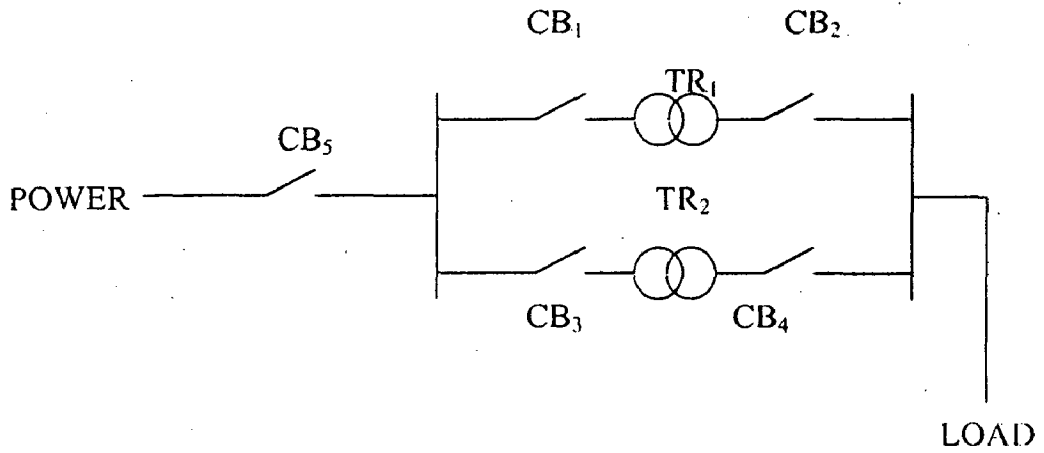


Fig. 5.8 POWER SYSTEM NETWORK

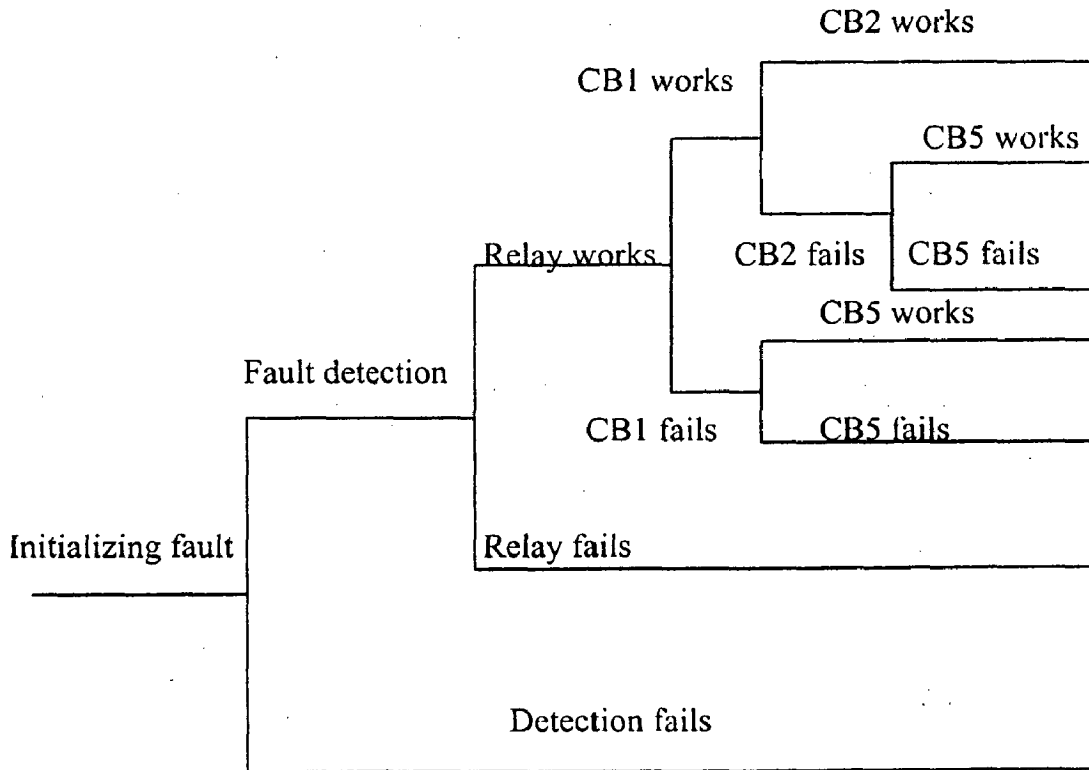


Fig.5.9 EVENT TREE FOR ALL POSSIBLE CASES

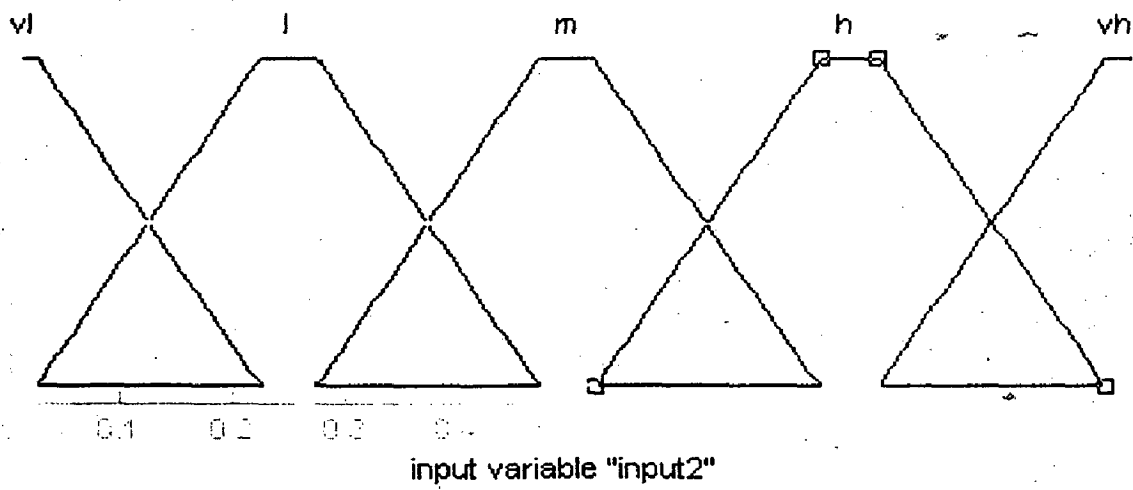
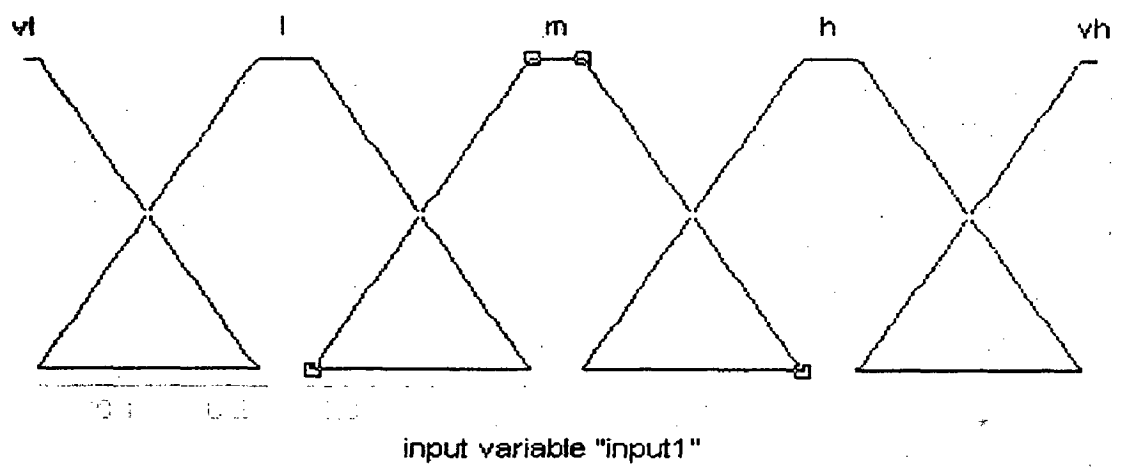


Fig. 5.10 fuzzy sets used for fuzzy calculations of two component systems

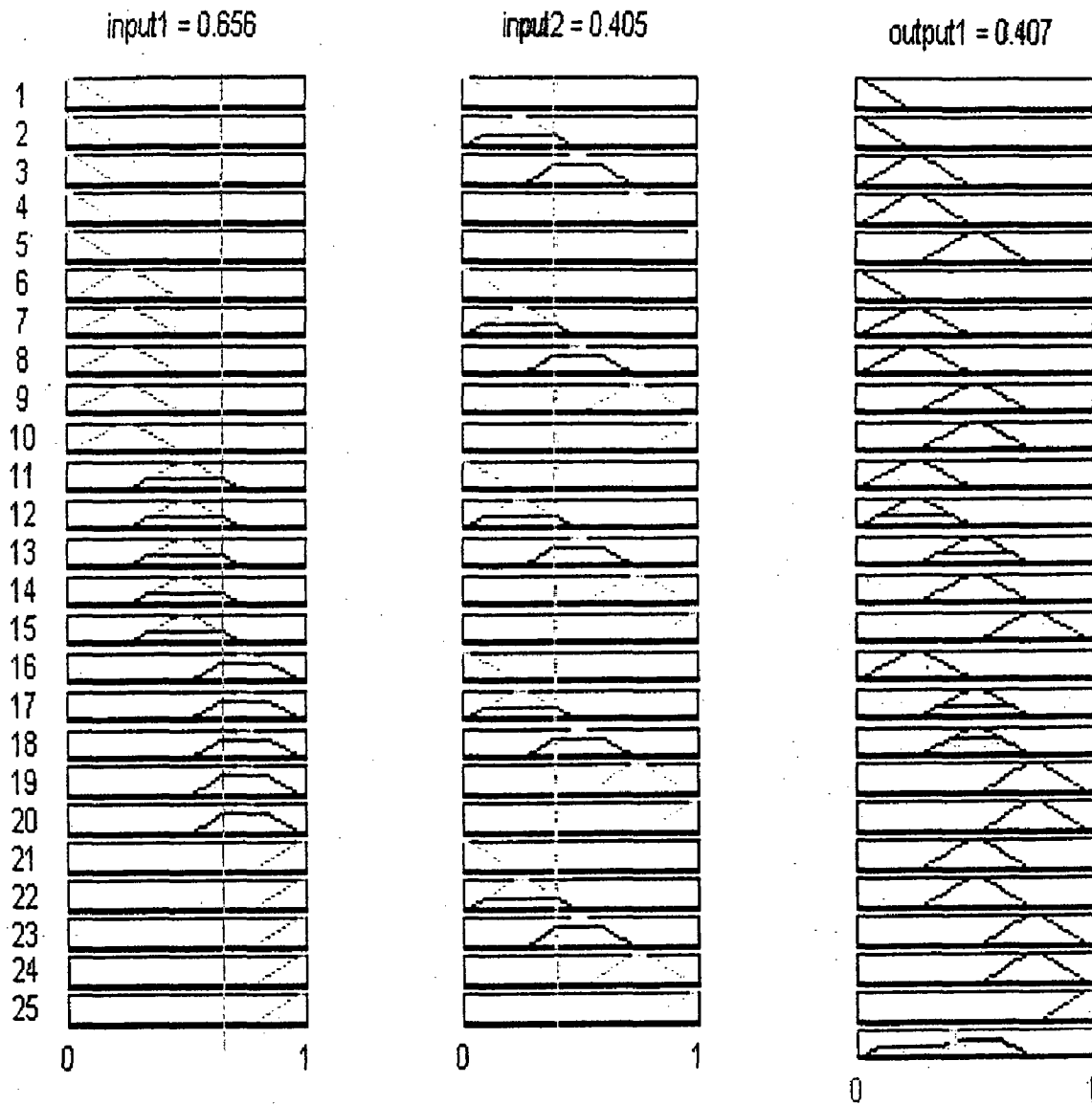


Fig.5.11 “fuzzy conclusion” and then “defuzzified” to give a single risk priority for the failure.

## **5.4 ROBOT RELIABILITY USING FUZZY FAULT TREE AND MARKOV MODELS**

Robot reliability has become an increasingly important issue in the last few years, in part due to the increased application of robots in hazardous and unsaturated environments. We will consider the application of notion and techniques from fuzzy logic, fuzzy fault tree, and Markov modeling to robot fault tolerance. Fuzzy logic lends itself to quantitative reliability calculations in robotics. Fault trees are standard reliability tool that can easily assimilate fuzzy logic. Markov modeling allows evaluation of multiple failure modes simultaneously, and is thus an appropriate method of modeling failures in redundant robotic system.

The increasing desire to produce more reliable robots has created interest in several tools used in fault tolerance design. Such tools seek to evaluate the effectiveness of new designs. The extra components needed for fault tolerance robot design obviously add extra cost and extra possibilities of failure. Reliability analysis tool such as fault trees and Markov models are needed to give hard numbers showing that the benefits of the fault tolerant design are tangible and worth the effort.

### **TEST PROBLEM**

The classic test problem is the two degree of freedom, planar manipulator. From a reliability engineering point of view, it is interesting to investigate the effect of redundant systems on this robot. Kinematics redundancy arises when more degree of freedom are available than are needed to perform the task. For the planner robot interested only in end- effector position, the required number of degree of freedom is two. If a robot in this situation possesses three degree of freedom, it can still reach a significant fraction of its workspace if one of its joint is frozen. Sensor redundancy occurs when there is more than one sensor at each joint, allowing sensor failure without joint failure.

Four distinct robots are examining her as shown in fig.5.12:

The non redundant robot with two joints and one sensor per joint

The partial redundant robot with just sensor or kinematic redundancy, and

The fully redundant robot with both sensor and kinematic redundancy.

The aim is to determine how much more reliable the redundant robots are.



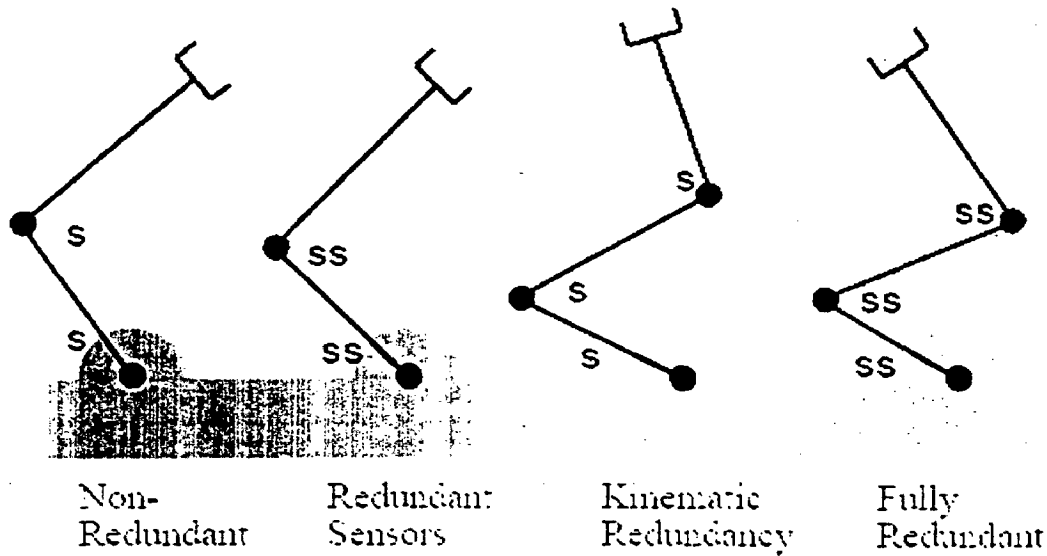


Fig.5.12 Four distinct robots

### 5.3.1 FAULT TREE

#### Basics

Fault tree is a common tool in reliability analysis. Basic events are connected through a series of logic gates to a terminal event that usually represents the failure of the system. The classic And and Or gates are the basic gates needed to represent most systems. Additionally, the N/M (N out of M) gate is useful in the redundant system. An And gate represent a so called parallel system. All of the components must fail for the system to fail. An Or gate corresponds to a series system. The system fails if any of the components fail. An N/M system is a type of redundant system. N out of the M elements in the system must fail before the system itself fails. Fault trees for the four robots are shown in fig.5.13.

If the probability of failure of all the parts on the 'leaves' of the tree is known, these probabilities can be propagated through the tree using the following rules:

Or gate:  $P_c = 1 - (1 - P_a)(1 - P_b)$

And gate:  $P_c = P_a \cdot P_b$

N/M gates are best decomposed into an equivalent set of And and Or gates. One Or gate is used, with its input being the  $\binom{n}{m}$  possible N member combination of the M inputs. For example, the failure probability for a 2/3 could be calculated as:

$$P_d = 1 - (1 - P_a P_b) (1 - P_a P_c) (1 - P_b P_c)$$

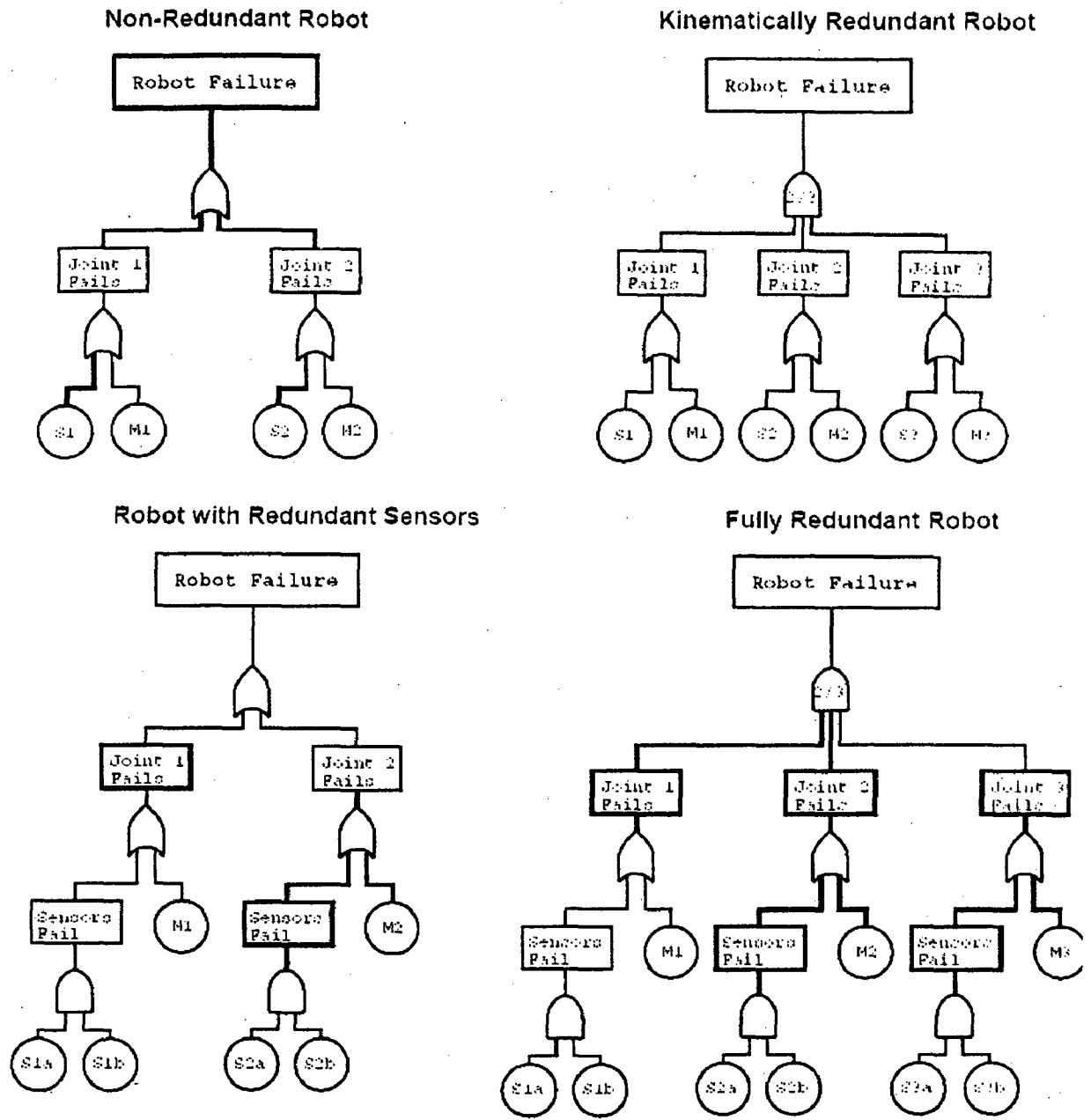


fig.5.13 Fault Tree for example Robots

### 5.3.2 Fuzzy Fault Tree

The probabilities for the basic events in a fault tree are often not known with great accuracy. Fuzzy member are a natural way to represent uncertainties such as these. The fuzzy representation of a failure probability can be propagate through a fault tree using fuzzy arithmetic. The resulting fuzzy number will cover a rage of possible results, giving an accurate view of what is actually known about the system.

### 5.3.3 Markov Modeling

The Markov model is a method of determining system behaviors by using information about certain probabilities of event within the system. Markov Models treat a system as a series of states with specific, constant rate transition between them. At all the time  $s$ , the system is in exactly one state. The only information available is the current state, the allowed transition, and the probability of this transition. Such a system is referred to as memoryless, and is said to possess the Markov property.

A useful way of looking at Markov models is to consider a large population of such systems. The probability of being in each state will be rough equivalent to the relative number of systems in each state in a large population. Thus 'the probability of being in state X at time T' is interchangeable with 'the population of state X at time T'

A simple Markov model for a repairable one component system is shown in fig.5.14.

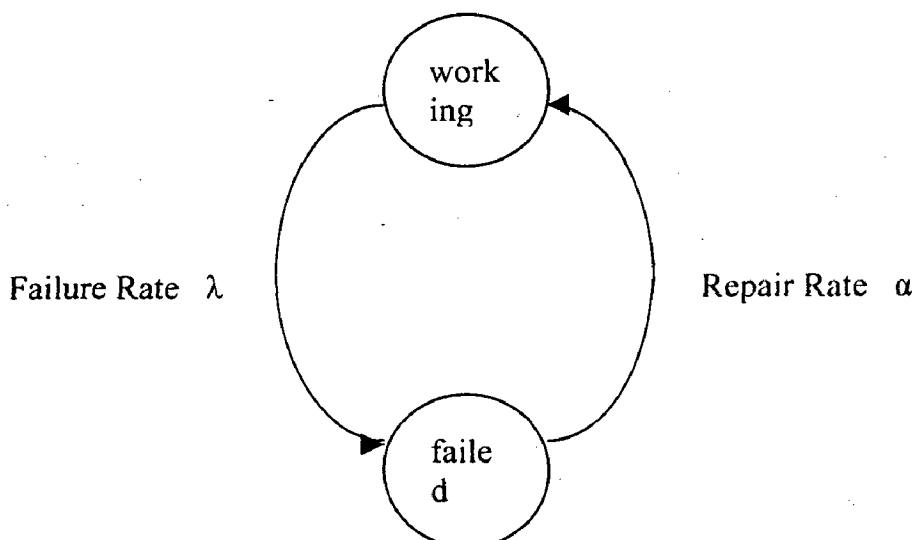


Fig. 5.14 Markov model for a Repairable One component system

The system failed with a constant rate  $\lambda$  while it is in the working state. Once failed, repairs proceed at rate  $\alpha$ . This system exponentially approaches the steady state where it has probability  $\alpha/(\lambda + \alpha)$  of being in the working state and the probability  $\lambda/(\lambda + \alpha)$  of being in the field state.

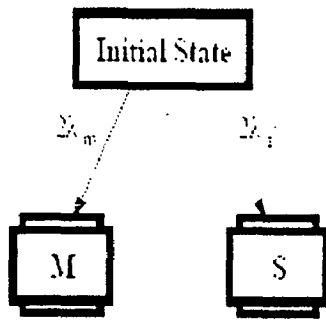
#### 5.3.4 Markov Model for the Example Robot

For the robots we are considering, the Markov models become complicated quickly. Even the simple non-redundant robot has four separate part components of interest (one sensor and one motor per joint), each of one can either working or failed. This leads to  $2^4$ , or 16 possible states of the system. The fully redundant robot has 9 components, leading to 512 possible states. This is too many states to deal with effectively. It is necessary to group and cut states until a reasonable number is reached. We do this in part by grouping the components into three categories:

- i) Motor
- ii) Single sensor, and
- iii) Pairs of sensors on the same joint

State with one or more of this parts failed are represented by states with labels M, S, and P respectively. States are characterized only by how many motors, sensors, and matching pairs of sensors are failed. Another reduction is accomplished by lumping all system and joint failures which have the same cause together, regardless of extraneous subsystem failures. For example, if the fully redundant robot fails from a working state into the state where two motor have failed, additional failures are ignored, and the state is referred to as 'MM+', where the '+' indicates that there may be other failed components. Our final reduction of the model was to make the standard Markov simplification and assume the sensor and motor failure rates were constant across the robot and time. The Markov models of the example robots are shown in fig.5.15 (a), (b), (c)

Non-Redundant Robot



Robot with Rdundant Sensors

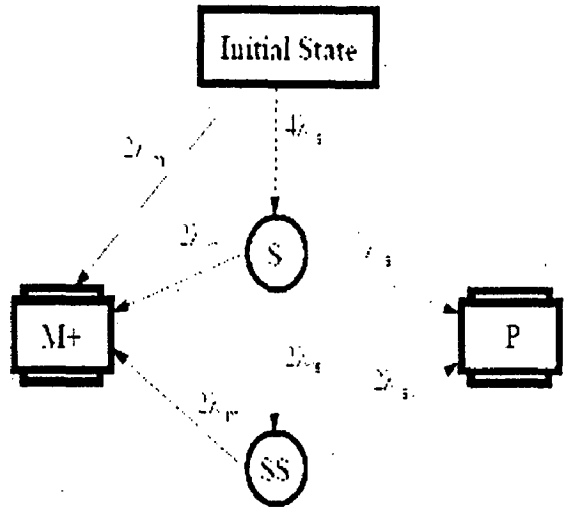
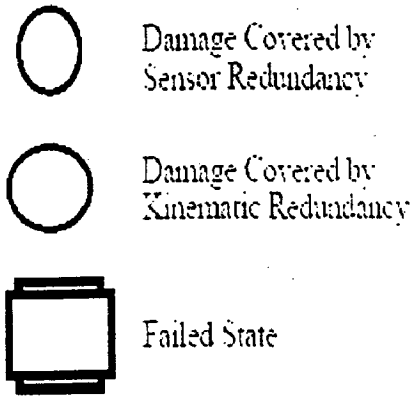


Fig. 5.15(a) Markov models for first two robots.

Kinematically Redundant Robot



M Failed Motor

S Failed Sensor

p Failure of Both Sensors on a Single Joint

+ Possibility of More Than Listed Failures

Motor Failure →

Sensor Failure - - - - ->

Sensor Failure Rate  $\lambda_s$

Motor Failure Rate  $\lambda_m$

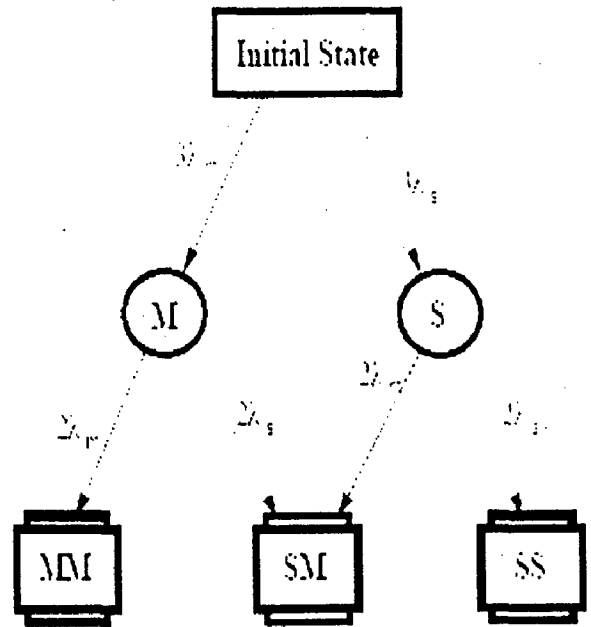


Fig. 5.15 (b) Markov model for Kinematically Redundant Robot

Fully Redundant Robot

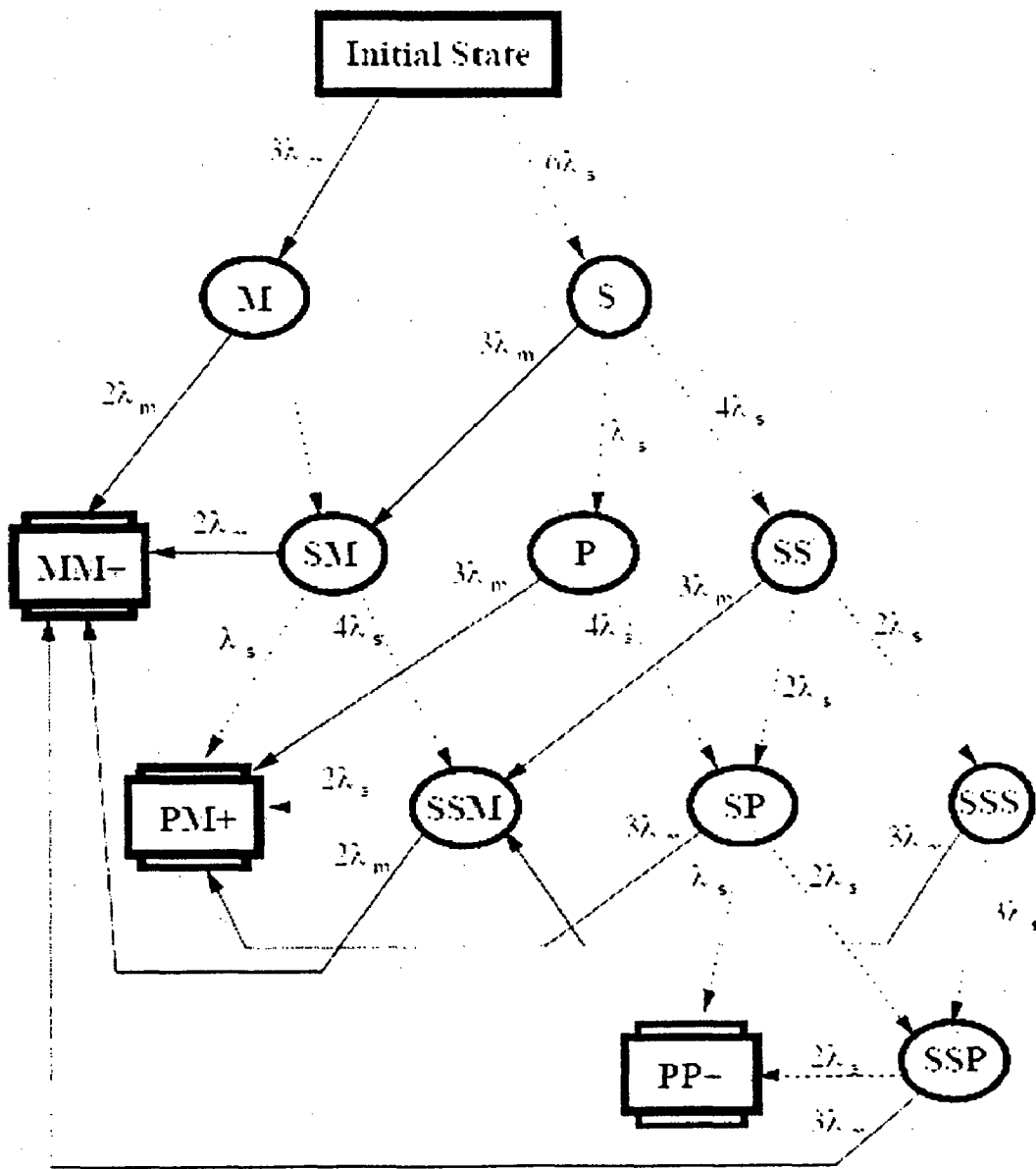
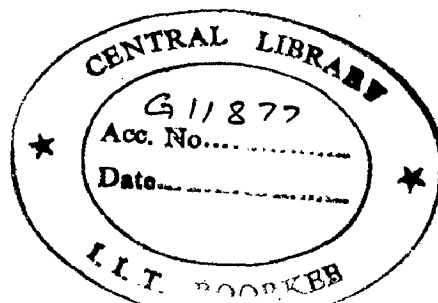


Fig. 5.15 (c) Markov model for Fully Redundant Robot





In addition to the state shown, the utility of the robot is calculated. The utility in a Markov model does not in general corresponds to any one state of the model, but instead is a weighted sum of several states where all the robot joints are working is weighted by one, and a state where two out of three joints are working is weighted by 0.5. The utility of the robot is the topic of great interest in regard to reliability, as it shows a measure of how useful the robot is expected to be over time. It results in a numerical value of usefulness to compare different robot configuration.

### 5.3.5 Fuzzy Markov Modeling of the Robots

Component failure rates can be very difficult to calculate accurately during the design process, as environmental factor and component interactions cannot be easily determined before several prototypes are built. This can leads to crisp values being given for a order of magnitude estimate. Even if the result is only viewed as a rough guess, exactly 'how fuzzy' the guess is not known. In addition unlike fuzzy fault tree, one cannot simply take the extreme values and propagate them through using fuzzy arithmetic. This method results in nonsensical results for the transient state.

The method used here for generating Markov models were using a straightforward method, the explicitly used the extension principle and computing power to get the results. Only the zero and one  $\alpha$ -cuts were generated, to keep the computing reasonable. For each of these cuts we needed to generate a set of points that roughly cover the available intervals for that  $\alpha$ -cut.

For our examples, each cut was divided into twenty geometrically-spaced intervals, and all possible combinations of these intervals were run through a Markov model of the system.

All of the resulting curves were compared, and the highest probability and the lowest probability outputs for each time step made bounds of the new  $\alpha$ -cut

The fuzzy Markov model generated has a 3-D membership function with axes of time, probability of being in state, and membership. If the trapezoidal representation is used, this can be represented by 2-D plots of the four edge of function. The highest and the lowest lines on the probability axis represent the zero  $\alpha$ -cut, while the inner two lines represent the one  $\alpha$ -cut.

The output of the fuzzy Markov model for four different robots is as shown in fig.

## A) NON REDUNDANT ROBOT:

The first and simplest model is the non-redundant robot. The output of the fuzzy Markov model is as shown in fig.5.16.

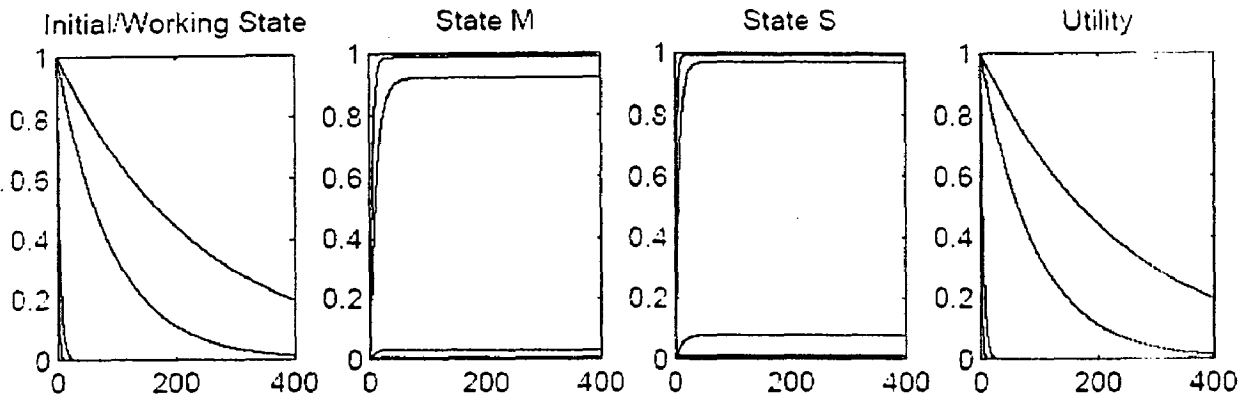


Fig.5.16: Non Redundant Robot

- \* The initial/working state in this robot has the highest population of all the robots. This is not surprising, as this robot has the fewest components (two motors, two sensors) and thus there are fewer parts to fail. However, all part failures lead instantly to system failure, so the robot has the lowest utility of all the robots as well.
- \* The lowest bounds of the fuzzy sets are much lower than the upper bounds. For both the **M** and **S** states, the one  $\alpha$ -cut alone covers most of the range of possibilities. This indicates that it is not possible to isolate one or the other failure mode as being predominant with the given data.
- \* The fuzzy sets give a somewhat misleading impression of the error possibilities when considered together. Both states have high memberships in high probabilities at the same time. This does not allow both states to have very high probabilities or very low probabilities at the same time, as the axioms of probability would not allow this. Instead, these membership functions indicate that it is highly possible that either type of failure could be that probable, and that with the fuzzy probabilities we have, this range of probabilities we have.

## B) ROBOT WITH REDUNDANT SENSOR:

The model of the redundant sensor as shown in the fig.5.15(a) and 5.17 tells us the following:

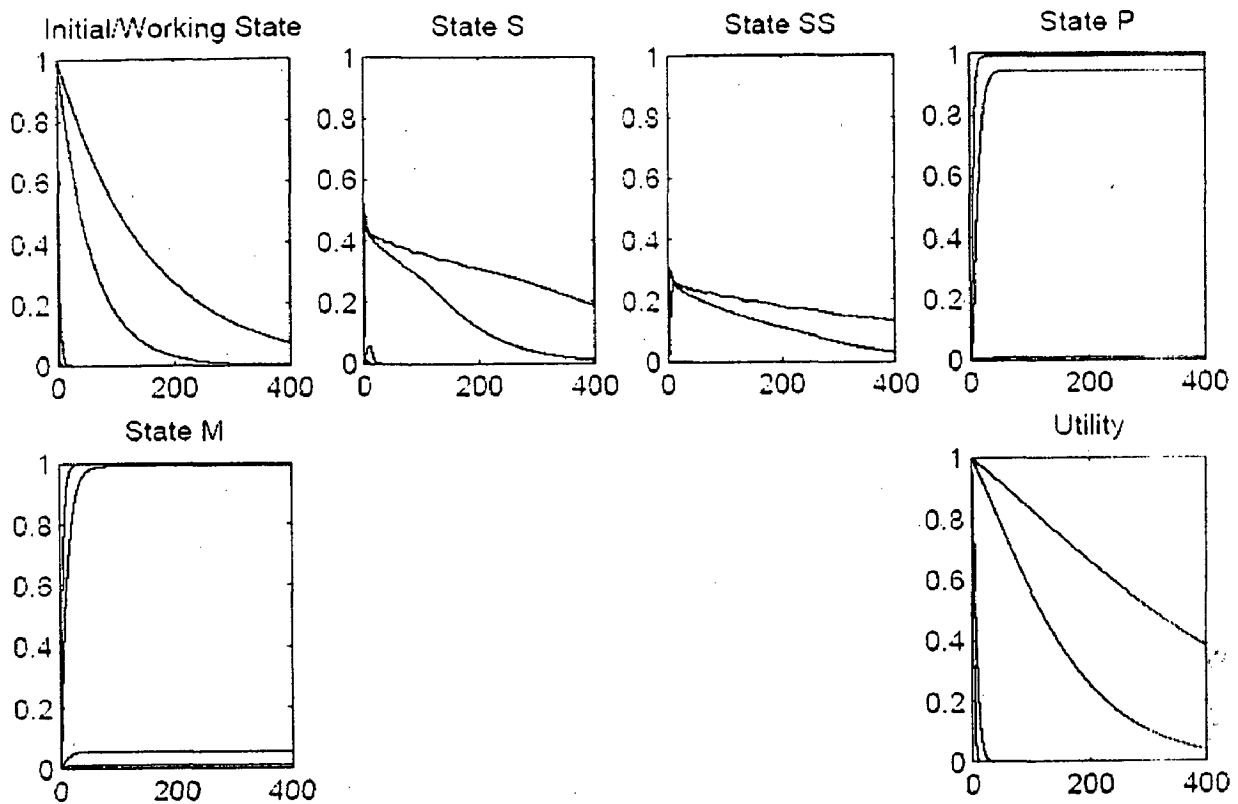


fig.5.17 Robot with Redundant Sensor.

- Although the robot fails out of the initial state more quickly than the previous robot, it has a higher utility. There are six components in the failure models, so failures are more common. However many of this are not fatal to the system.
- Note that the lower bound of utility are not significantly better than those for the non-redundant robot. This robot has a 'weakness' – high motor failure rates bypass its redundancy.
- This model, unlike the previous one, has transitory states, or state that have no population in both the initial (new) and final (failed) states of the robot. The states S and SS are very possible for a large set of probabilities and times. This is because the time at which these functions hit a maximum is highly dependent on the failure rates. For high rates this peak is very early, and for low rates, it

happens very late. The positions in between are filled by various intermediate failure rates. Similarly, the lower bounds for these states are very low, as the low failure rates grow very slowly, and before they get too large, the high failure rates grow very slowly, and before they get too large, the high failure rates have already peaked and soon drop below them. Thus the wide range of possibilities for these states.

- Several anomalies can be observed on the transitory states. Notably, a small spike at the beginning of the state and a series of bumps along the top edge of the state.

### C) KINEMATICALLY REDUNDANT ROBOT:

The model of the kinematically redundant robot shown in fig.5.15(b) and 5.18. shows us some serious flaws

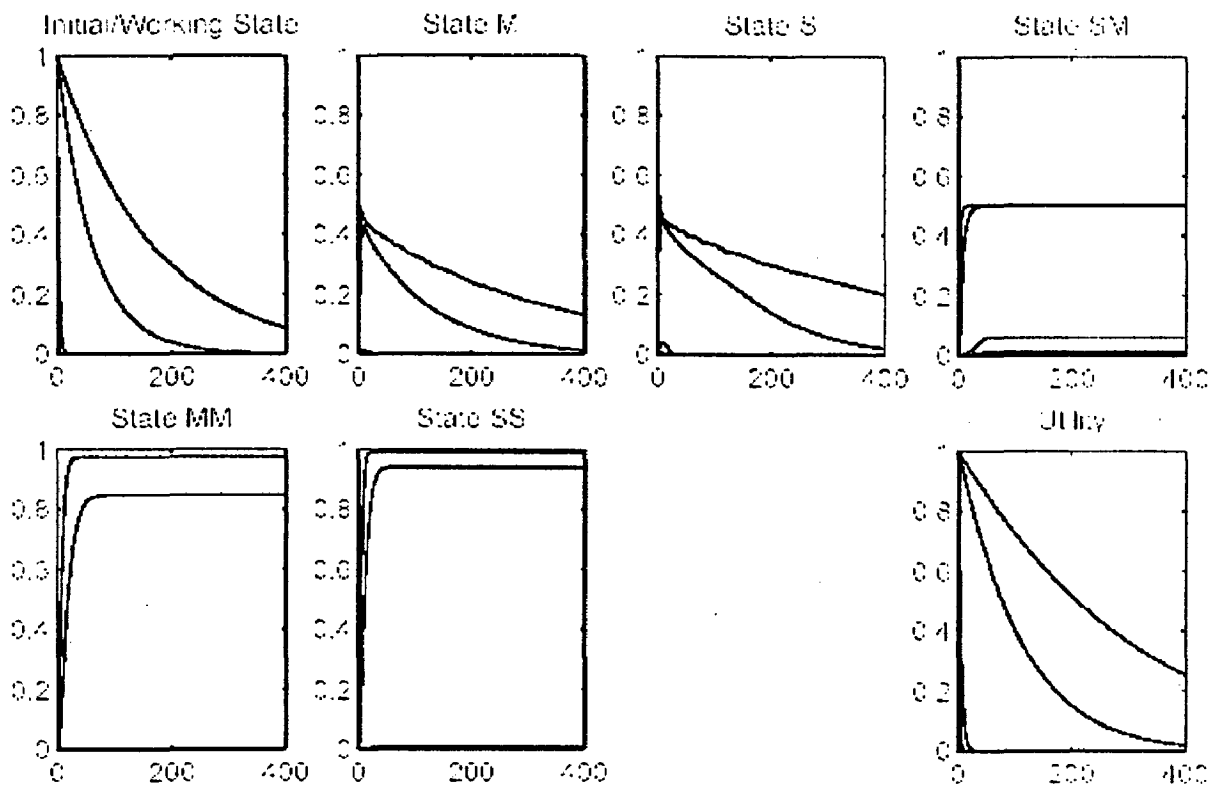
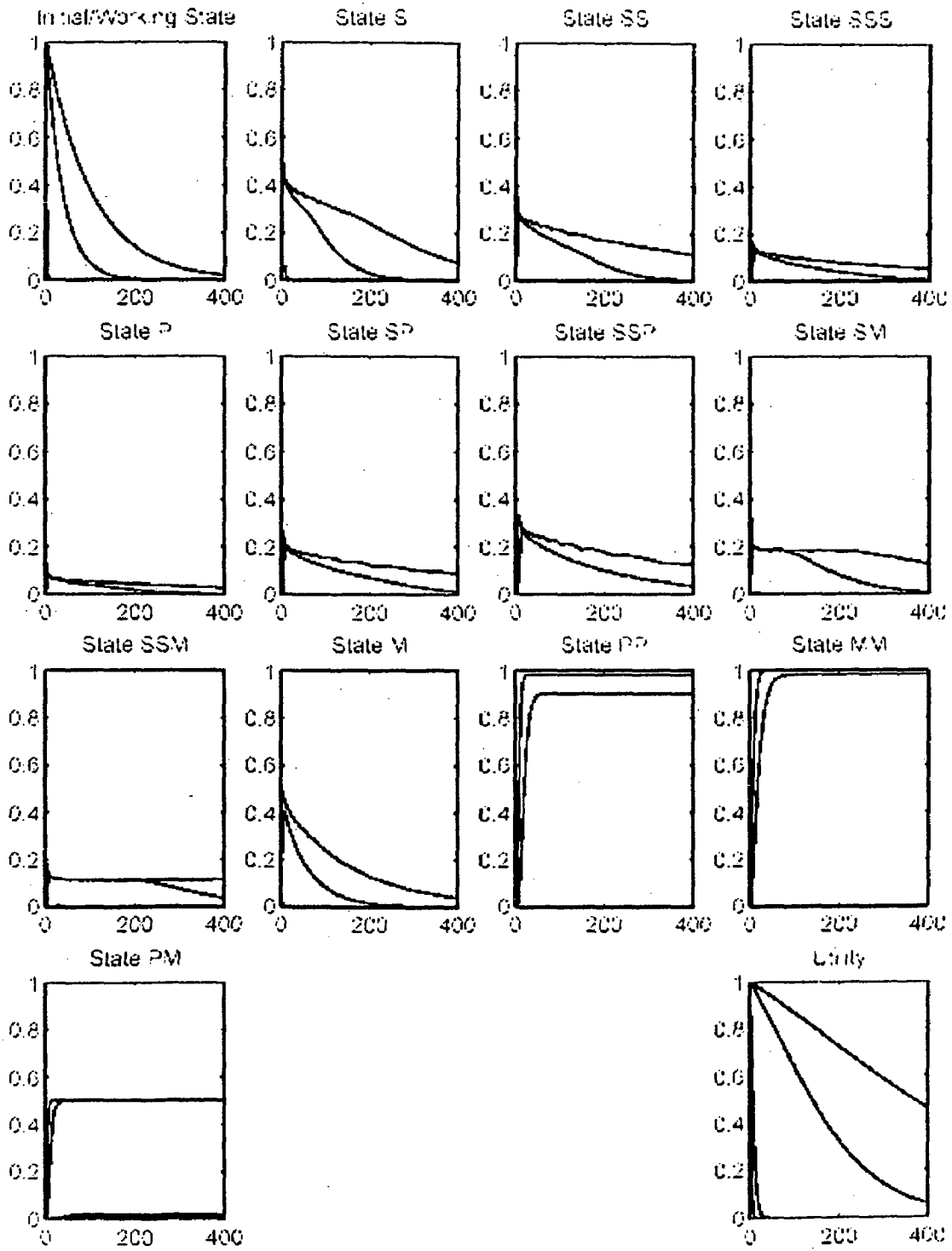


fig.5.18 Kinematically Redundant Robot

- Although the initial state decays approximately as fast as for the previous robot (both have six component), the utility of these robot is considerably lower. This is because of the effect of the lower utility of the partially degraded states
- This robot does not have the weakness in the way the previous robot did. The kinematic redundancy applies equally to sensor and motor failures immobilizing a joint. However the higher number of components meant that very high failure rates had a strong effect than on the non redundant robot, so the utility lower bound is still low
- This robot, like those previous, also has very uncertain failure states. It is interesting to note the limited range of the **SM** failure state. This is the first state we encounter that requires both a sensor and a motor failure. This causes this state to be limited in its maximum population, as it is most populous when both failure rates are high, and thus the **MM** and **SS** failure rates are also high. This state also has the same upper bound on the one and the zero cuts after a certain amount of time has passed. This is because the population of this state is influenced strongly by the ratio of the two failure rates

### D) FULLY REDUNDANT ROBOT:

The model of a fully redundant robot as shown in the fig.5.15(c) and 5.19 tell us the following:



- With nine components, this robot fails out of the initial state faster than any other robots. However, its utility is the highest by a wide margin, as it has protection from both kinds of component failure. Even the lower bounds show noticeable improvement.
- The **PM** state is similar to the **SM** state in the previous robot in that it requires both types of failure to happen, and is thus limited to 0.5 in its upper bound for both the zero and one  $\alpha$ -cuts. However, its lower bound is less prominent, as it requires a pair of sensor failures, which is less likely to happen than a single sensor failure.
- The transient state **SM** and **SSM** exhibit similar behavior. Although these states are transient, they also contain both the motor and sensor faults, and thus they exhibit the same ratio based convergence of  $\alpha$ -cuts.
- The large size of the states **M** and **S** is expected, as they result from the initial transition out of the working state. However, the large possible population in the **SSP** state has fewer working sensors than any other states, so failure out of this state would be at an unusually low rate.

Several themes can be found in the fuzzy Markov models above. The increasing in the rate of component failure as reliability schemes are implemented is made clear. Higher reliability will paradoxically require us to deal with more component failures. However those failures will be mitigated by the reduced rate of system failure that is evidenced by the higher utilities displayed by the fault-tolerant robots. Sensor redundancy provides a lot of this reliability for little effort, assuming that the motors are somewhat reliable. Kinematic redundancy adds a little more margin, but it not as useful, as the damaged robot is not as useful as the initial one. On the other hand, kinematic redundancy guarantees us that the first failure will not be fatal, and give us improvement no matter which components has the higher fault rate, so it should be considered.

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## APPLICATION OF NEURAL NETWORK FOR THE RELIABILITY ANALYSIS

### 6.1 Neural Network for the Reliability Analysis of Simplex System

Another approach to the reliability analysis, based on neural networks, is introduced in this work. The reliability analysis of a simple non-redundant digital system, Simplex system, with repair is used to illustrate the neural network approach. The discrete time Markov model of simplex systems is realized using feed-forward recursive neural network. The energy function and update equations for the weight of neural network are established such that the network converges to the desired reliability of the simplex system under design. The failure rate and repair rate, satisfying the desired reliability, are extracted from the neural weights at convergence.

The reliability of hardware under design is usually arrived at by assuming suitable values for certain parameters such as the failure rate, coverage factor, and the repair rate, whichever is applicable to the design. The reliability of the system is, then, computed using discrete or continuous time analysis. If the resultant reliability does not meet the design requirements, then the whole process is repeated to obtain another set of values. This technique is lengthy and complicated when dealing with complex fault-tolerance systems.

Neural networks have demonstrated advantages in speed and development effort over conventional computers in performing some basic operations. The collective computation capabilities of neural networks will be employed in the reliability analysis. A neural network will be used to determine the design feature features of a system, given its reliability after a specified period of operation. The initial conditions and the desired reliability are feed into the neural network. When the neural network converges, its different weight will indicate the appropriate parameter, and hence the features of the system under investigation. A feed-forward recursive neural network is employed to represent the Markov model of a simplex non-redundant system with repair (Simplex



system). The energy function and the update equations of the weights are derived using the least mean square, gradient learning procedure.

### 6.1.1 Simplex System

Consider the Markov model of a simple non-redundant system with a constant failure rate  $\lambda$  and a constant repair rate  $\mu$ , as shown in fig. 6.1. During the time interval  $\Delta t$ , the system will have probability of failure given by  $\lambda\Delta t$  and a probability of repair  $\mu\Delta t$ . State o represents the condition of the system being completely operational, and state f represents the failed condition. If the system is in state o, the probability of the system transitioning to state f during the time period  $\Delta t$  is  $\lambda\Delta t$ . Also if the system is in state f the probability of transitioning to state o is  $\mu\Delta t$ . The discrete time-equation for the Markov model are given by

$$P_o(t + \Delta t) = P_o(t)(1 - \lambda\Delta t) + P_f(t)(\mu\Delta t) \quad (6.1.1)$$

$$P_f(t + \Delta t) = P_o(t)(\lambda\Delta t) + P_f(t)(1 - \mu\Delta t) \quad (6.1.2)$$

Initially the system is assumed operational; that is  $P_o(0) = 1$  and  $P_f(0) = 0$

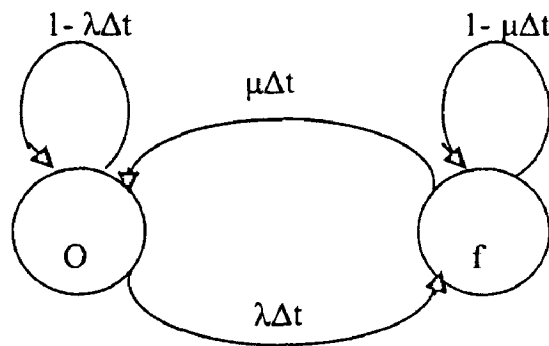


Fig 6.1 Markov Model of a Simple Non-Redundant System With Repair

### 6.1.2 The Neural Network

A feed forward recursive network is set to represent the simplex system. As shown in fig. 6.2, the network consists of two layers of neurons: one forms the input and the other forms the output. The number of neurons in each layer equals to two, which is the number of states in the Markov model. The weight connecting the input and output neurons represent the entries of the transition matrix of the discrete-time equations. In

other words, the weights of the neural network are related as follows to the Markov model:

$$\begin{aligned}
 W_{21} &= \lambda \Delta t \\
 W_{12} &= \mu \Delta t \\
 W_{11} &= 1 - W_{21} \\
 W_{22} &= 1 - W_{12}
 \end{aligned}
 \tag{6.1.3}$$

At any time  $t$  during operation of the system

$$\begin{aligned}
 X_1 &= P_o(t) \\
 X_2 &= P_i(t) \\
 Y_1 &= P_o(t + \Delta t) \\
 Y_2 &= P_i(t + \Delta t)
 \end{aligned}
 \tag{6.1.4}$$

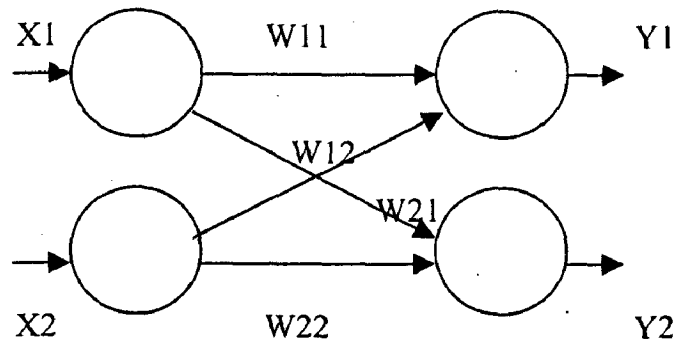


Fig. 6.2- Neural Network for Simplex system

The initial conditions are given by  $X_1 = 1$ , and  $X_2 = 0$ . the basic equation of the neural network are:

$$\begin{aligned}
 Y_1 &= W_{11} X_1 + W_{12} X_2 \\
 Y_2 &= W_{21} X_1 + W_{22} X_2
 \end{aligned}
 \tag{6.1.5}$$

The energy function  $E$  for the neural network and the update equations are obtained using the least mean square, gradient descent learning procedure as follows:

$$E = (Y_1 - D_1)^2 + (Y_2 - D_2)^2
 \tag{6.1.6}$$

where  $D_1$  and  $D_2$  are the desired outputs of the network which are, respectively, equivalent to the required reliability and unreliability of the system; while  $Y_1$  and  $Y_2$  are

the actual outputs of the network. The change in the weight  $W_{ij}$ , denoted by  $\Delta W_{ij}$ , is related to the energy function by the following update relation:

$$\Delta W_{ij} = -K \frac{\partial E}{\partial W_{ij}} \quad (6.1.7)$$

where  $K$  is the constant of proportionality. Using the chain rule

$$\frac{\partial E}{\partial W_{12}} = \frac{\partial E}{\partial Y_1} \frac{\partial Y_1}{\partial W_{12}} + \frac{\partial E}{\partial Y_2} \frac{\partial Y_2}{\partial W_{12}} \quad (6.1.8)$$

After substituting from equation (6.1.5) and (6.1.6), we get

$$\begin{aligned} \frac{\partial E}{\partial W_{12}} &= 2X_2((D_2 - Y_2) - (D_1 - Y_1)) \\ &= 2X_2(\text{error}_2 - \text{error}_1) \end{aligned} \quad (6.1.9)$$

where  $\text{error}_1 = D_1 - Y_1$  and  $\text{error}_2 = D_2 - Y_2$ . Similarly

$$\begin{aligned} \frac{\partial E}{\partial W_{21}} &= 2X_1((D_1 - Y_1) - (D_2 - Y_2)) \\ &= 2X_1(\text{error}_1 - \text{error}_2) \end{aligned} \quad (6.1.10)$$

where  $\text{error}_1 = D_1 - Y_1$  and  $\text{error}_2 = D_2 - Y_2$ . Finally the update equations for the two weights are:

$$\Delta W_{12} = 2KX_2(\text{error}_1 - \text{error}_2) \quad (6.1.11)$$

$$\Delta W_{21} = 2KX_1(\text{error}_2 - \text{error}_1) \quad (6.1.12)$$

### 6.1.3 SIMULATION RESULTS

Computer simulation of a neural network representing the Simplex System is performed. The time of operation of the system under design is take as  $t = 10$  hours and  $\Delta t = 0.1$  sec. The initial failure and repair rates are chosen within an attainable practical range. A sample of the result obtained from the simulation is shown in table 6.1.

For example consider the case when the software is run with initial repair rate  $\mu = 0.2$  repair/hour ( $W_{12} = 5.5 * 10^{-6}$  repair/0.1 sec) and initial failure rate  $\lambda = 0.3$  failure/hour ( $W_{21} = 8.3 * 10^{-6}$  failure/0.1 sec), the desired probability being  $P_o = 0.92$  and  $P_f = 0.08$  after 10 hour of operation ( $t = 10$  hour). The network coverage in only five iteration to  $P_o = 0.9200089$  and  $P_f = 0.079991$ . These results are within the 0.001 accuracy limits from the desired values. The different weights at convergence are  $W_{12} = 1.07462 * 10^{-5}$  repair/0.1 sec and  $W_{21} = 9.4965 * 10^{-7}$  failure/0.1 sec. Form these values the required failure and repair rates are calculated as:

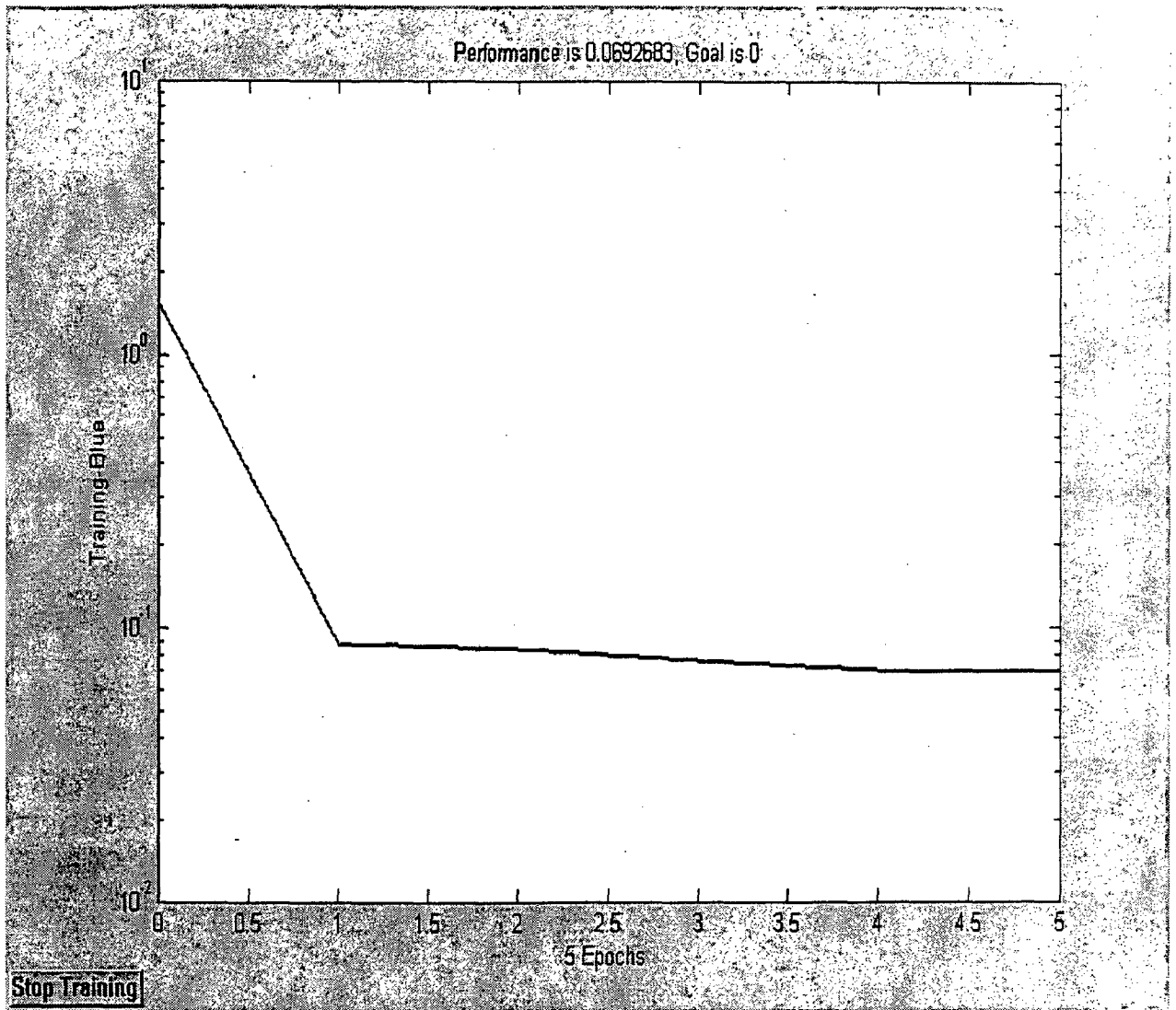
$$\lambda = W_{21} / \Delta t = 0.034187 \text{ failure/hour} \quad (6.1.13)$$

$$\mu = W_{12} / \Delta t = 0.386864 \text{ repair/hour} \quad (6.1.14)$$

To verify these results,  $\lambda$ ,  $\mu$  and the time ( $t = 10$  hour) values are substituted in the continuous time solution of equation (6.1) and (6.2). The same magnitude for  $P_o$  and  $P_f$  are obtained.

| Initial Values |       | Desired Values |          | Values at Convergence<br>N = Number of Iteration |           |       |    |
|----------------|-------|----------------|----------|--|-----------|-------|----|
| $\lambda$      | $\mu$ | $P_o(t)$       | $P_f(t)$ | $P_o(t)$   | $\lambda$ | $\mu$ | N  |
| 0.03           | 0.20  | 0.39           | 0.61     | 0.4001   | 0.2510    | 0.150 | 10 |
| 0.03           | 0.20  | 0.52           | 0.48     | 0.5119   | 0.2010    | 0.198 | 8  |
| 0.03           | 0.20  | 0.65           | 0.35     | 0.6499   | 0.1764    | 0.324 | 13 |
| 0.03           | 0.20  | 0.80           | 0.20     | 0.7996   | 0.0923    | 0.365 | 8  |
| 0.03           | 0.20  | 0.87           | 0.13     | 0.8699   | 0.0574    | 0.378 | 6  |
| 0.03           | 0.20  | 0.92           | 0.08     | 0.9200   | 0.0342    | 0.387 | 5  |

Table 6.1 -- Simulation results of the Simplex System



## 6.2 RELIABILITY ANALYSIS OF THE TMR SYSTEMS ON NEURAL NETWORK

A six-neuron feed-forward recursive neural network is used to perform the reliability analysis of a Triple Modular Redundancy (TMR) digital system. The network represents the discrete-time Markov model of the TMR system with a minimal number of states. The initial conditions and the desired reliability of the TMR system after a specified period of operation are fed into the neural network. At convergence, the different weights of the neural network indicate the appropriate failure rate for the modules to be used.

### 6.2.1 TMR SYSTEM

The TMR is the most common form of passive hardware redundancy. Its basic concept, as shown in the fig.6.3, is to triplicate the hardware and perform a majority vote to determine the output of the system. If one module becomes faulty, the two remaining fault-free modules mask the result of the faulty module when the majority vote is performed.

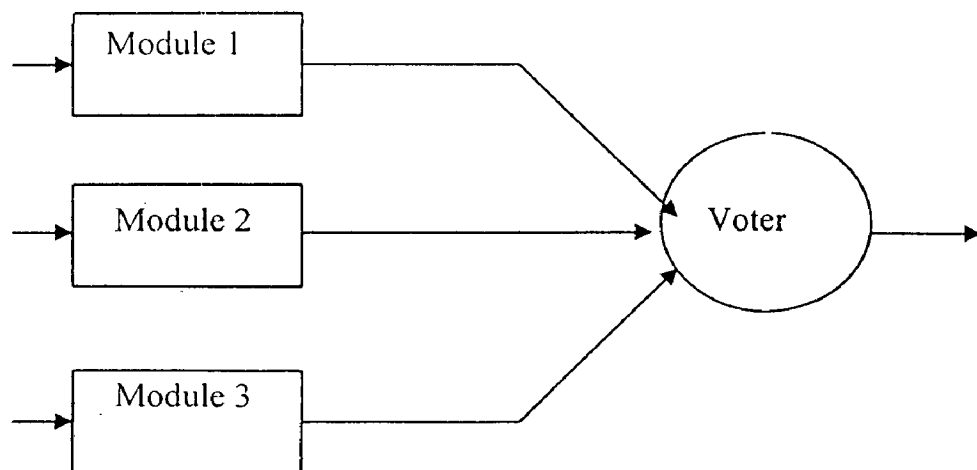


Fig.6.3. The TMR System

Fig.6.4 illustrates the reduced Markov model of the TME system with the minimal no of state. State three indicates all three modules in the TMR system are working correctly. In state 2, only two modules are functioning correctly. Sate f is the failed state in which two or more modules have failed. The probability that the system being in any given state S at some time  $t+\Delta t$  depends on the probability that the system was in a state from which it could transition to state S and the probability of that transition occurring. Denoting the failure rate by  $\lambda$ , the equations of the discrete Markkov model of the TMR system are:

$$P_3(t + \Delta t) = P_3(t)(1 - 3\lambda\Delta t) \quad (6.2.1)$$

$$P_2(t + \Delta t) = P_3(t)(3\lambda\Delta t) + P_2(t)(1 - 2\lambda\Delta t) \quad (6.2.2)$$

$$P_f(t + \Delta t) = P_2(t)(2\lambda\Delta t) + P_f(t) \quad (6.2.3)$$

Initially, all three modules are assumed be fault-free; that is  $P_3(0) = 1$ ,  $P_2(0) = 0$  and  $P_f(0) = 0$ .

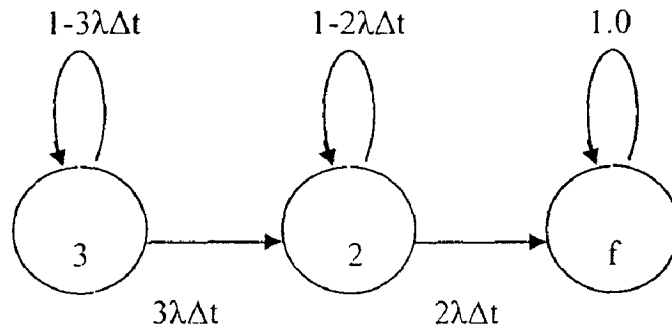


fig.6.4. Reduced Markov Model of a TMR System with a Minimal Number of State

### 6.2.2 The Neural Network

A feed-forward cascade recursive network is set to represent the TMR system. As illustrated in fig.6.5, the network consists of six neurons; three in the output layer. In fact the number of neurons in each layer is equal to the number of states in the Markov model. The weight connecting the neurons of the input and output layers represent the entries of the transition matrix

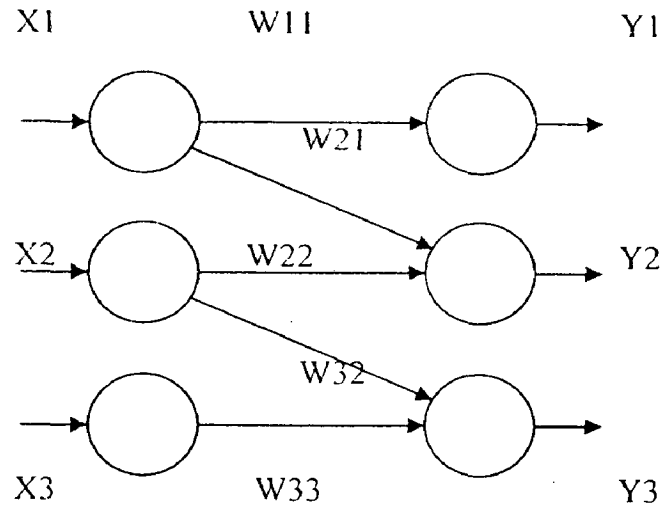


Fig.6.5. Neural Network for TMR System

Now let

$$\begin{aligned}
 W_{11} &= 1 - 3W \\
 W_{21} &= 3W \\
 W_{22} &= 1 - 2W \\
 W_{32} &= 2W \\
 W_{33} &= 1
 \end{aligned} \tag{6.2.4}$$

where  $W$  is equal to  $\lambda\Delta t$ . The inputs of the networks are

$$\begin{aligned}
 X_1 &= P_3(t) \\
 X_2 &= P_2(t) \\
 X_3 &= P_1(t)
 \end{aligned}$$

The outputs of the networks are

$$\begin{aligned}
 Y_1 &= P_3(t + \Delta t) \\
 Y_2 &= P_2(t + \Delta t) \\
 Y_3 &= P_1(t + \Delta t)
 \end{aligned}$$

The initial conditions for the neural network are given by  $X_2 = X_3 = 0$  and  $X_1 = 1$ . The input and the output of the neural network is related as follows

$$\begin{aligned}
 Y_1 &= W_{11} X_1 \\
 Y_2 &= W_{21} X_1 + W_{22} X_2 \\
 Y_3 &= W_{32} X_2 + W_{33} X_3
 \end{aligned}$$



### 6.2.3 SIMULATIONS RESULTS

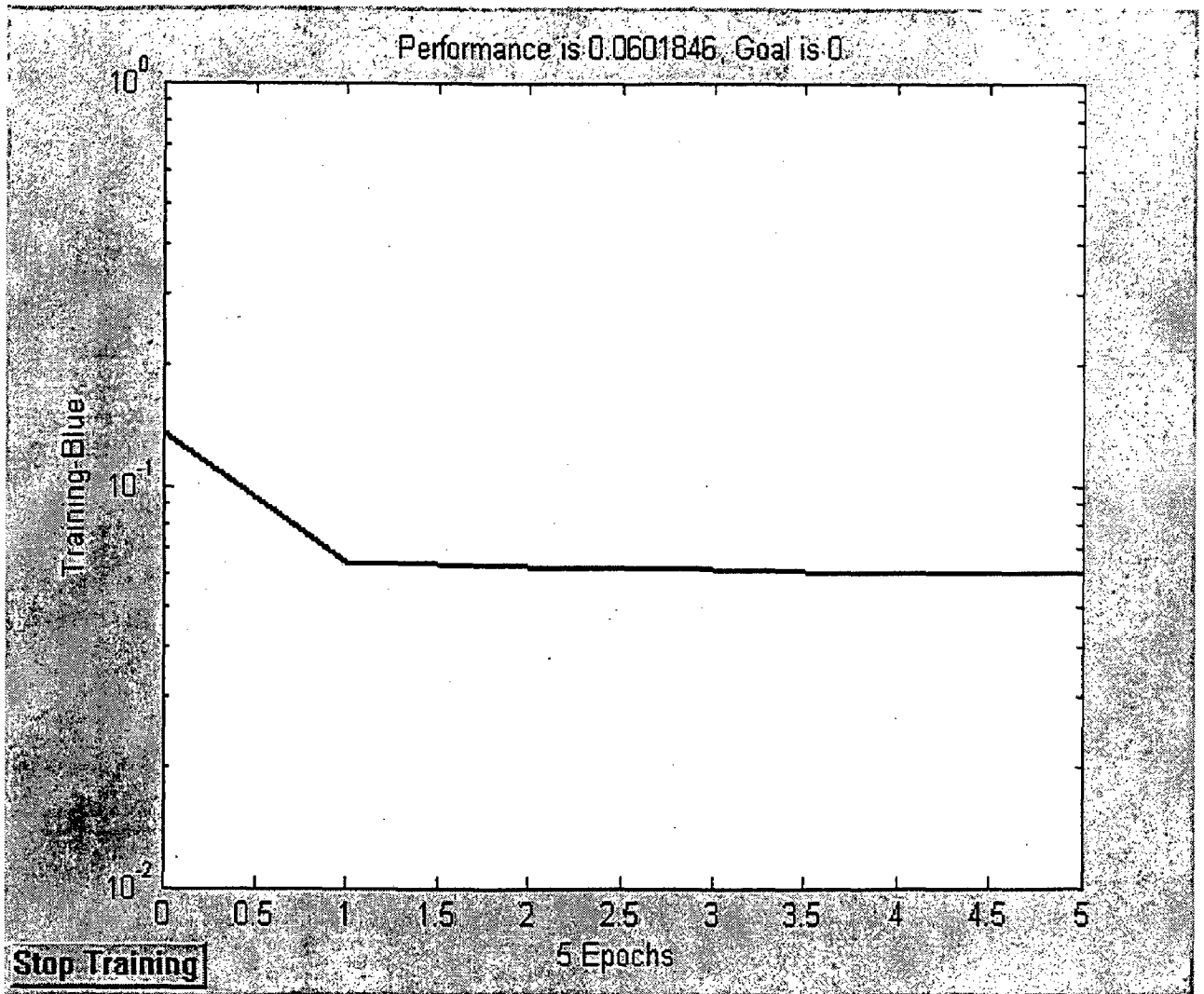
Computer simulation of a six-neuron network realizing the TMR system is performed. The time of operation of the system under design is taken as  $t = 10$  hours and  $\Delta t = 0.1$  sec. Table 6.2 shows sample results of the simulation. Considered the case when the desired probability value  $P_3(t) = 0.75$  and the initial failure rate  $\lambda = 40 \times 10^{-5}$  failure/hour (weight  $W = 1.111 \times 10^{-8}$  failure per 0.1 sec) are fed in the simulation program. The network coverage after 29 iteration to  $P_3(t) = 0.750875$ ,  $P_2(t) = 0.225746$  and  $P_1(t) = 0.023379$  within a 0.01 implied accuracy. At convergence  $W = 2.652915 \times 10^{-7}$  failure per 0.1 sec, from which the value of the failure rate is calculated:

$$\lambda = W / \Delta t = 0.0096 \text{ failure/hour}$$

These results are verified using the continuous time solution of the Markov model equations.

| Initial $\lambda$ | Desire<br>$P_3(t)$ | Value at Convergence |          |          |                          |    |
|-------------------|--------------------|----------------------|----------|----------|--------------------------|----|
|                   |                    | $P_3(t)$             | $P_2(t)$ | $P_1(t)$ | $\lambda \times 10^{-3}$ | N  |
| 40                | 0.40               | 0.3998               | 0.4285   | 0.1709   | 30.55                    | 37 |
| 40                | 0.50               | 0.5003               | 0.3899   | 0.1097   | 23.00                    | 35 |
| 40                | 0.60               | 0.5994               | 0.3342   | 0.0663   | 17.00                    | 33 |
| 40                | 0.75               | 0.7508               | 0.2257   | 0.0237   | 09.60                    | 29 |
| 40                | 0.90               | 0.9001               | 0.0963   | 0.0037   | 03.50                    | 20 |

Table6.2 Simulation Results of the TMR System



## CONCLUSION AND FUTURE SCOPE

### 7.1 GENERAL

In reliability evaluation, uncertainty is due to the fact that since failure are relatively rare events (typically only a few per million hours of operation) collecting enough data on which to base a statistical "probability of failure" is a costly and difficult undertaking and the relevance of the data to any particular system as well its validity is often questionable. Furthermore especially early in the design, the item whose probability of failure is needed often does not exist and it must be estimated based on "engineering judgment" or "experience" from "similar" items. Extrapolating these failure probabilities through statistical methods to calculate a system level reliability only increase the uncertainty.

The analysis of system reliability often requires the use of subjective judgment, uncertain data and approximate system models. Fuzzy set theory provides an effective tool for characterizing system reliability in these circumstances. Probability theory alone is not sufficient to deal with the problem of subjectivity. Reliability information can best be expressed using fuzzy set, because it can seldom be defined crisply and the use of natural language expressions about reliability offers a powerful approach to bundle the uncertainties more effectively.

### 7.2 FUZZY FAULT TREE ANALYSIS

Fault tree analysis (FTA) is a logical and diagrammatic method to evaluate the probability of an accident resulting from sequences and combinations of fault and failure events. Thus a fault tree is useful for understanding logically the mode of occurrence of an accident. In conventional fault tree analysis, the failure probabilities of system components are treated as exact values. For many systems, however it is often difficult to evaluate the failure probabilities of components from past occurrence, because the environment of the system is changes. Fuzzy fault tree provides a powerful and computationally efficient technique for developing fuzzy probability based on independent events. The probability of any event that can be described in terms of

sequence of independent unions, intersections and complements may be calculated by fuzzy fault tree. The application of fuzzy fault tree is further demonstrated by using the application to robot fault tolerance. In this work, we discussed the existing fault tree methods of dealing with the fuzzy data that is common in reliability situations, and expanded them into the Markov domain. The new method of fuzzy Markov modeling showed much promise for increasing the flexibility of fuzzy reliability analysis, just as crisp Markov expands the possibilities of crisp analysis.

### **7.3 FUZZY EVENT TREE ANALYSIS**

Event trees are useful for system reliability analysis and risk quantification since they illustrate the logic of combination of probabilities and consequences of probabilities and consequences of event sequences. The application of fuzzy event trees is further demonstrated by using set of event trees for an electric power system protection system to assess the viability of the method in computing the risk associated with a failure in an electric power system. The "risk" associated with a failure increases as either the severity of the effect of the failure or the failure probability increases. Fuzzy event tree provides a more flexible and meaningful way of assessing risk.

### **7.4 MARKOV MODEL**

The Markov model is a method of determining system behaviors by using information about certain probabilities of event within the system. Markov Models treat a system as a series of states with specific, constant rate transition between them. At all the time  $s$ , the system is in exactly one state. The only information available is the current state, the allowed transition, and the probability of this transition. Such a system is referred to as memoryless, and is said to possess the Markov property. As an illustration, fuzzy Markov modeling of a power generator with a derated state has been developed in this present work.

### **7.5 NEURAL NETWORK ANALYSIS**

Reliability analysis based on neural networks, is introduced in this work. The reliability analysis of a simple non-redundant digital system, Simplex system, with repair is used to illustrate the neural network approach. The discrete time Markov model of

simplex systems and the TMR system is realized using feed-forward recursive neural network. The main interesting feature of the introduced method is the utilization of the collective computational abilities of neural networks in the analysis in the various aspects of the reliability analysis.

## **7.6 FUTURE SCOPE**

Reliability information can best be expressed using fuzzy sets, because it can seldom be defined crisply, and the use of natural language expressions about reliability offer a powerful approach to handle the uncertainties more effectively

Future work in the area of fuzzy Markov modeling is like to focus on four areas. The first and most obvious of these is reduction of computational complexity of the model. Similarly, further method of simplification of the model should be considered. Additionally, Markov modeling is a very broad area, and expanding this technique to some of the modified Markov model show promise. Finally, application of this technique to other systems is an interesting research issue.

Future work in the area of neural network work may include the repair rate and coverage factors in the reliability analysis of TMR system.

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