# FUZZY DYNAMIC PROGRAMMING FOR ECONOMIC DISPATCH INCORPORATING FACT DEVICES

#### A DISSERTATION

Submitted in partial fulfilment of the requirements for the award of the degree

of

#### MASTER OF ENGINEERING

in

# ELECTRICAL ENGINEERING (With Specialization in Power System Engineering)

By

#### PRAVEEN KUMAR TRIVEDI



DEPARTMENT OF ELECTRICAL ENGINEERING UNIVERSITY OF ROORKEE ROORKEE-247 667 (INDIA)

MARCH, 2000

ID NO. MT-173 | BD-NPP/2000

# <u>CANDIDATE'S DECLARATION</u>

I hereby declare that the work presented in this dissertation entitled "FUZZY DYNAMIC PROGRAMMING FOR ECONOMIC DISPATCH INCORPORATING FACT DEVICES", submitted in partial fulfillment of the requirement for the award of the degree of Master of Engineering, in Electrical Engineering, with specialization in Power System Engineering, in the Department of Electrical Engineering, University of Roorkee, Roorkee, is an authentic record of my own work, carried out with effect from July 1999 to March 2000, under the guidance of Dr. B. Das and Dr. N. P. Padhy, Department of Electrical Engineering, University of Roorkee, Roorkee.

The matter embodied in this thesis has not been submitted for the award of any other degree.

DATE : 25<sup>th</sup>March, 2000.

(PRAVEEN KUMAR TRIVEDI)

It is certified that the above statement made by the candidate is correct to the best of our knowledge and belief.

Biswamp Dass (Dr. B. Das) 25.3.2000

(Dr. N. P. Padhy)

Department of Electrical Engineering University of Roorkee Roorkee – 247667 (INDIA)

# <u>ACKNOWLEDGEMENT</u>

I am greatly indebted to my guides Dr. B. Das, and Dr. N. P. Padhy, Department of Electrical Engineering, University of Roorkee, Roorkee, for their kind support and guidance during my work. Their co-operation and in-depth knowledge have made my work possible.

I am also thankful to Sri Bharat Gupta, O.C. Power System Simulation Lab, for providing me the best of facilities, which enabled my work to roll faster.

Last but not least, I am also grateful to Power System Engg. Faculty and my classmates whose presence have made a workable atmosphere in the lab, conducive for my work.

in coli (PRAVEEN KUMAR TRIVEDI)

## ABSTRACT

In this thesis, economic dispatch problem with transmission capacity constraints has been solved accounting for the uncertainty in system load demand. The uncertainty in the load-demand has been incorporated by fuzzy membership functions. To solve this problem, fuzzy dynamic programming technique has been used. Effectiveness of using thyristor controlled series capacitor (TCSC) for achieving dispatch pattern with stringent transmission capacity constraints has been demonstrated.

# CONTENTS

CANDIDATE'S D	ECLAI	RATION	(i)
ACKNOWLEDGE	EMENT		( ii )
ABSTRACT			( iii )
CHAPTER -1	INTI	RODUCTION	1
CHAPTER -2	ECO	NOMIC DISPATCH USING DYNAMIC	
	PRO	GRAMMING	· 5
	2.1	Economic dispatch	5
	2.2	Existing methods for solving ED	·
		problem	6
	2.3	Solution of ED problem through	
		DP approach	8
· .	2.4	Conclusion	10
CHAPTER –3	ECO	NOMIC DISPATCH USING FUZZY	
	DYN	IAMIC PROGRAMMING	13
	3.1	Fundamental of fuzzy set theory	13
	3.2	Solution of ED problem under	
		uncertainty	- 15
	3.3	Conclusion	19
CHAPTER – 4		ULTS AND DISCUSSION	22
	4.1	Classical dynamic programming approach	23
	4.2	Fuzzy dynamic programming approach	28
	4.3	Comparison of results	34
CHAPTER - 5	CON	ICLUSION	35
REFERENCES			36
APPENDIX A	DC p	power flow	38
APPENDIX B	Syste	em data	39

## **INTRODUCTION**

One of the most important operational functions of any modern day energy management system is economic dispatch.

The economic dispatch (ED) aims to minimize the total cost of real power generation from thermal power plants at various stations while supplying the loads and losses in a power transmission system. The objective is to distribute the total load demand and total loss among the units connected on-line while simultaneously minimizing the generation costs and satisfying the power balance equations and other constraints [1].

The economic dispatch problem assumes that there are N thermal generation units already connected to the system. The purpose of economic dispatch problem is to find the optimum generation policy for these N units, such that the total generation cost is minimized while simultaneously satisfying the power balance equations and various other constraints in the system.

The most common constraints in the operation of a power transmission system are constraints on the voltage magnitudes of the buses and constraints on the reactive power generated by the generators. For reliable operation of a power system, the voltage magnitudes of the buses and the reactive power generated by the generators are constrained to stay within certain specified minimum and maximum limits. However, due to the increasing load demand, more and more power is required to be pushed over the existing transmission lines. But as it is a very common knowledge, increasing the power flow over the transmission corridors above a certain operating level decreases the overall stability of the system. Hence it is very necessary to limit the power flows over the transmission lines to their respective operating limits and consequently this constraint should also be considered as the system constraint for the economic dispatch problem. Hence, economic dispatch is an involved constrained optimization problem.

To solve the economic dispatch problem subjected to commonly used constraints along with the transmission capacity constraints, many researchers have proposed different solution methodologies in the literature. Different techniques such as artificial

neural network approach [2], dynamic queuing method [3], lagrangian relaxation method [4], quadratic programming approach [5], fast Newton-Raphson method [6] etc. have been proposed in the literature. In all the above methods, the system losses have been adequately considered. Pallanichamy and Srikrishna [7] has solved the dynamic economic dispatch problem using B-coefficients to account for the system losses. This method has been found to be very efficient and capable of solving a large size problem in a time span short enough to be compatible for on line applications.

Among the above approaches, lagrangian relaxation method, quadratic programming approach, fast Newton Raphson method etc. require the existence of continuously differentiable cost functions. However, in real life scenario, the cost functions of the generators are not always continuously differentiable. In these cases, the optimization problem is solved using discretization techniques.

Among the various discretization techniques, the most commonly used methods to solve ED problem are neural network approach, dynamic programming approach, linear programming approach [8], genetic algorithm approach [9] etc. The linear programming approach, neural network approach and genetic algorithm approach have been found to be quite heavy in terms of computational burden and execution time. On the other hand, the dynamic programming approach has been found to be quite efficient compared to the other discretization methods in terms of computational burden and execution time.

However, in all the above studies, the system load demands have been assumed to be precisely known. But in real-life scenario, this is not so. In a practical power system, at any instant of time, some of the loads are being cut "OFF", while some of other loads are being switched "ON" and almost always, the amount of load being switched "OFF" or switched "ON" is not known precisely. Due to this, at any instant of time, the system load demand is not known precisely, but is known approximately. For example, at any instant of time, it would be more appropriate to say that the load demand in the system is approximately 1000 MW (say), rather than exactly 1000 MW. Hence, there is an amount of 'uncertainty' regarding the system load demand at any instant of time. This uncertainty or variation in load demand can be taken into account by classifying the load demand into some categories (e.g. small, medium and high). To represent more

realistically the generating patterns of the generators when the load is in either of the classification categories, the total generating capacity of an individual generator is also divided into the same categories. For example, if the total load demand is classified into three categories (low, medium and high), the total generating capacity of an individual generator is also classified into the same three categories, e.g. low, medium and high. The rationale behind this same classification is the fact that for a low load demand the generation from an individual generator is most likely to the low. Similarly, for a high load demand in the system, the individual generators are also most likely to produce high output power. However, the extent to which an individual generator would participate to meet the system load demand is somewhat uncertain or 'fuzzy'. This 'uncertainty' or 'fuzziness' can best be described by 'fuzzy logic theory' [10]. To represent this uncertainty, the generation of an individual generator in each category is represented as a fuzzy generation pattern.

As it has been already noted, increasing the power flow over a transmission line decreases the stability margin of the system. To achieve better power flow control over the transmission lines without risking the stability margin of the system, application of Flexible ac transmission system (FACTS) technology [11 - 12] is currently being pursued very intensively. FACTS technology is essentially the art and science of achieving better controllability over various electrical quantities in a power transmission system by the suitable application of various power electronic device and controller to power transmission system.

Different FACTS devices, such as static var compensator (SVC), solid state synchronous compensator (STATCOM), thyristor controlled series capacitor (TCSC), unified power flow controller (UPFC) etc, are among the most potential candidates for application to power system to achieve better controllability. SVC and STATCOM essentially control the voltage of a bus in a system. TCSC essentially controls power flow over a line and UPFC controls both the bus voltage and power flow over a line.

Now if the ED problem is solved without any transmission capacity constraints, the system is most likely to be operated at the optimum point. However if the transmission capacity constraints are included, there is a possibility that the original optimum point (i.e. without any transmission capacity constraints) obtained from ED

algorithm would not be able to satisfy the transmission line capacity constraints for all lines. Hence if it is desired that any acceptable solution for ED problem must also satisfy the transmission constraints, then there may be a possibility that some sub-set of the original sub-optimal points (i.e. without any network constraint) may satisfy this criterion. But obviously in that case the cost of the generation will be higher than that at the optimum point.

In this scenario, the judicious application of FACTS devices (for example a TCSC) may be very helpful in achieving the objective of satisfying the transmission line constraints at the optimum solution point of the ED problem. The logic goes as follows. At any operating point of a transmission system there may be some transmission lines through which the actual power flow is quite small compared to their operational limits. Hence, at the optimum solution point of ED problem, there are also some lines which are under-utilized (i.e. the actual flow through them is below their operational limits). Now, if the power flow through these lines can be increased by reducing their reactances (i.e. by putting a TCSC in the lines), then some power from the other over-loaded lines may be diverted to these lines and consequently, the power flow in the overloaded lines may reduce and ultimately come down to a level which is below their operating limits.

In this thesis, the ED problem under uncertain load demand has been solved by fuzzy dynamic programming approach. To satisfy the transmission capacity constraints, options of putting TCSCs at different lines have been investigated in detail and their effects on the final solutions of ED problem have also been studied thoroughly. This thesis report is organized as follows. Chapter 2 discusses the classical dynamic programming approach for the solution of ED problem, in which constant load demands have been assumed. In Chapter 3, solution of ED problem through fuzzy dynamic programming is described, where the uncertainties in load demands are taken into account. Chapter 4 presents the main results of this work. Finally, Chapter 5 gives the main conclusions of this work.

# **CHAPTER 2**

# ECONOMIC DISPATCH USING DYNAMIC PROGRAMMING 2.1 ECONOMIC DISPATCH

The basic purpose of the Economic Dispatch (ED) problem is to schedule the outputs of the power generators which are connected on-line and serving a particular area, so as to meet the net load of that particular area with minimum cost subjected to various operating constraints. Mathematically, the economic dispatch problem can be formulated as follows :

The objective is to minimize the total cost of generation, i.e. minimize

$$F_{T} = \sum_{i=1}^{NG} F_{i}(P_{Gi})$$

where,

F <sub>T</sub>	=	total cost of generation
$F_i(P_{Gi})$	=	The cost of $P_{Gi}$ generation by $i^{th}$ generator
NG		Number of generators
$F_i(P_{Gi}) = a_i P_G^2$	? ;; +	$b_i P_{Gi} + c_i$

Where,  $a_i$ ,  $b_i$ ,  $c_i$  are the cost coefficients for  $i^{th}$  generator.

The above minimization problem is subjected to certain system constraints. The most common constraints are :

### (a) Active power balance equation for the System

The total generation in the system must be equal to total load plus the total loss in the system, i.e.

$$\sum_{i=1}^{NG} P_{Gi} = P_{load} + P_{loss}$$

where,  $P_{load}$  = Total active load in the system

 $P_{loss}$  = Total active power loss in the system

#### (b) Limits on the outputs of the generation units

The output of each generating unit must be within some specified minimum and maximum limits, i.e.

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max}, \qquad \text{for } i = 1, 2, \dots NG$$

where,

 $P_{Gi}$  = The power generated by i<sup>th</sup> generator in MW.

 $P_{Gi}^{min}$  = The specified minimum MW generation by i<sup>th</sup> generator

 $P_{Gi}^{max}$  = The specified maximum MW generation by i<sup>th</sup> generator

#### (c) Operating line constraints

The power flow over a transmission line should not exceed the specified maximum limit because of stability considerations, i.e.

$$p_{ij} \le p_{ij}^{\max}$$
 i=1, 2,.....n  
j=1, 2,....n

where,

 $p_{ii}$  = active power flowing in line joining i<sup>th</sup> & j<sup>th</sup> bus

 $p_{ij}^{max}$  = maximum allowable active power flow in line joining i<sup>th</sup> & j<sup>th</sup> bus

n = number of buses in the system

Several methods have been reported in the literature to solve the above constrained optimization problem. Some of these methods are discussed briefly here.

#### 2.2 EXISTING METHODS FOR SOLVING ED PROBLEM

#### (a) Lagrangian Relaxation Method (LR)

Lagrangian relaxation method involves decomposition of the problem into sequence of master problem and easy sub-problems, whose solution converges to an optimal solution to the original problem [4]. In this method, a set of lagrangian multipliers are determined which subsequently generate a solution that meets all the system constraints. However, the main problem associated with LR method is to get a good starting value for lagrangian multipliers to speed up the iteration process.

#### (b) Newton's Method

This method [6] solves the ED problem in presence of non monotonous incremental cost (NMIC) function of generating units. The traditional equal incremental cost method, which requires monotonously increasing incremental cost curve, fails in the presence of NMIC. This method exhibits fast convergence to the optimum value.

#### (c) Linear Programming Method

It is difficult to handle inequality constraints in the Newton's method. On the other hand, linear programming method is very adaptable at handling inequality constraints. In this method [8], the cost function is linearized. The various constraints are dealt in very efficient way. However, this method is very cumbersome in terms of computational burden and execution time.

#### (d) Artificial Intelligence Method

The Artificial intelligence method consists of artificial neural network [2], genetic algorithm [9], expert system and fuzzy logic [10] techniques. By using energy function, the ANN can very easily handle inequality and equality constraints. The genetic algorithm approach aims at searching for optimum value with random search through strategic techniques. Both of the above techniques, although can advantageously deal with various equality and inequality constraints, are quite ineffective in reducing the computational burden for finding the optimum solution.

#### (e) Dynamic Programming (DP) Method

In many cases, the cost functions of the generators are not continuous in nature. Hence the traditional analytical methods, such as LR, Newton's method etc. can not be applied in these cases as they depend upon the existence of the continuously differentiable functions. Under such circumstances dynamic programming approach has been found to be very efficient. The dynamic programming approach to solve ED problem is done as an allocation problem [1]. In this method the non-convex cost functions can be handled very easily. The DP approach for ED problem can reduce the computational efforts in finding the optimum solution to a very large extent.

In this thesis work, the ED problem has been solved by using the dynamic programming approach. Hence, in the next section, method of solving the ED problem by using the DP approach is discussed in detail.

#### 2.3 SOLUTION OF ED PROBLEM THROUGH DP APPROACH

Dynamic programming method is a mathematical technique dealing with the optimization of multistage decision problems. This technique was originated by Richard Bellman and G. B. Dantzig in the year 1952.

In this technique, the decision regarding a certain optimization problem is typically solved in stages, rather than simultaneously. The original problem is broken into sub-problems (stages) which can then be handled more efficiently. The final solution of the original problem is achieved through a series of decisions reached in the stages where the decision at each stage depends upon the results of the previous stages. In this process, individually each decision may not be optimal. A sacrifice at one stage may result in greater gains in some other subsequent stages. Fig. 2.1 shows the dynamic programming forward propagation through different stages to reach the final optimum solution.

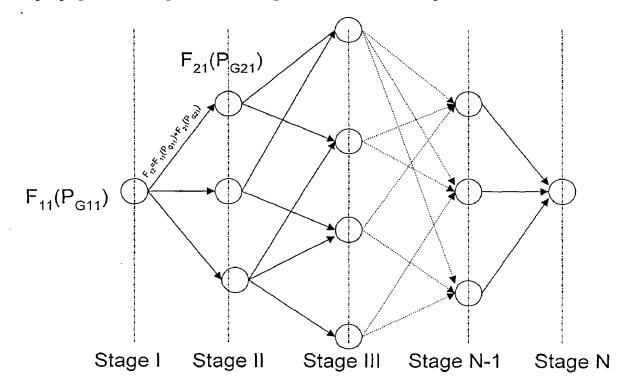


Figure 2.1 : Dynamic Programming Forward Propagation Diagram

Where, in the above figure,

 $F_{11}(P_{G11}) =$  The cost of generation for  $I^{st}$  discrete generation  $(P_{G11})$  of  $I^{st}$  stage .  $F_{21}(P_{G21}) =$  The cost of generation for  $II^{nd}$  discrete generation  $(P_{G21})$  of  $II^{nd}$  stage .

 $F_{12}$  = The effective cost of generation for Stage I<sup>st</sup> and stage II<sup>nd</sup> taken together.

In this work, a new method for solving ED problem with DP technique considering the system power loss has been developed. The step-by-step algorithm of the developed method is given below.

#### Step 1 : Read input data

- (a) Generation output limits ( $P_{Gi}^{min}$ ,  $P_{Gi}^{max}$ )
- (b) Transmission line limits  $(p_{ij}^{max})$
- (c) Generator cost coefficients  $(a_i, b_i, c_i)$
- (d) Line parameters (resistance, reactance and half line charging susceptance), for each existing line
- (e) Loads at different buses,  $p_{busload}$ .
- Step 2 : Set  $P_{loss} = 0$

Step 3:  $P_{load}^{new} = P_{load} + P_{loss}$ , where  $P_{load} = \sum P_{busload}$ .

- Step 4: Run DP (Dynamic programming). Store all optimum and sub-optimal solutions.
- Step 5:Compute the bus load angles using DC power flow technique<br/>corresponding to the generation pattern of the most optimum solution.Then calculate the new total system loss ( $P_{loss}^{new}$ ) by using the simple loss<br/>formula suggested in [13]. The DC power flow technique is described in<br/>detail in Appendix A.
- **Step 6 :** Calculate error =  $|P_{loss} P_{loss}^{new}|$
- Step 7: If error >  $\varepsilon$  (specified tolerance), set  $P_{loss} = P_{loss}^{new}$  and go to step 3. If error <  $\varepsilon$ , total loss in the system is obtained and go to step 8.

Step 8: Run DP. Store the optimum and sub-optimal sets (each set consists of :  $P_{G1}, P_{G2}, \dots P_{GNG}$ ).

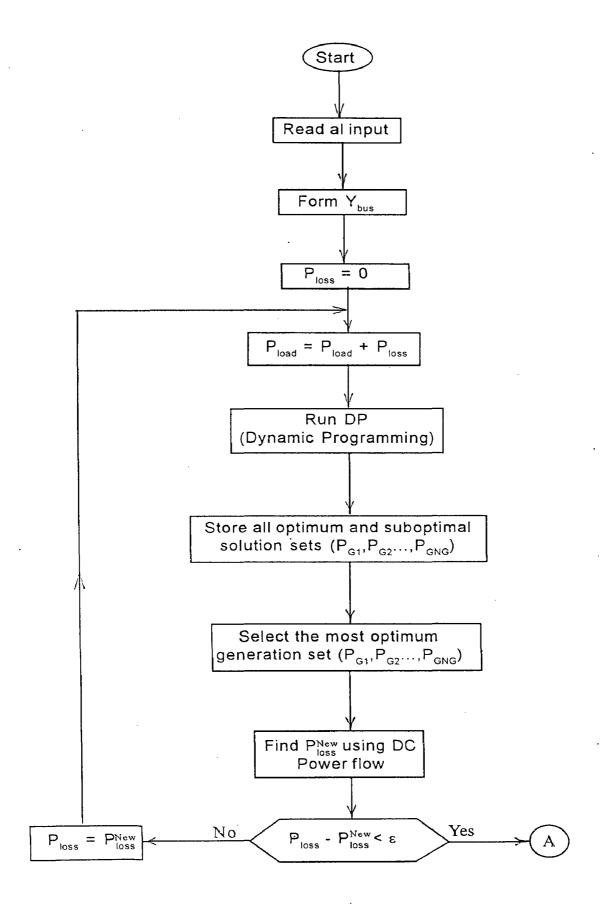
Step 9 : Select the most optimum solution set.

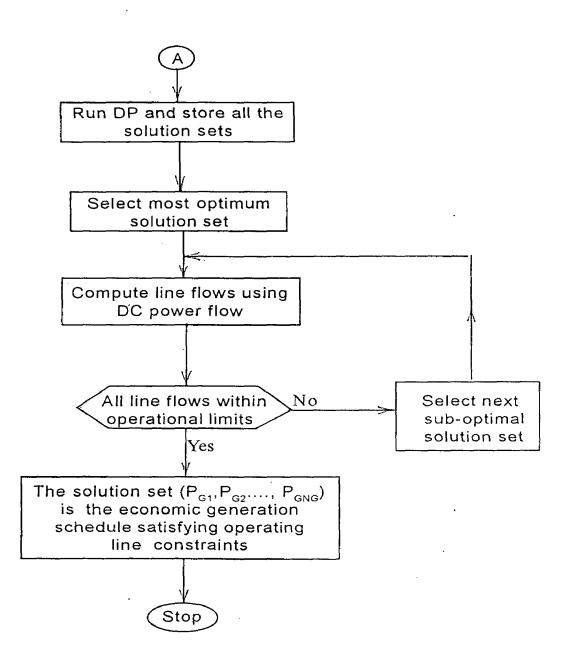
- Step 10: Compute line flows corresponding to the generation pattern using DC power flow.
- Step 11: Check whether on any transmission line the line flow limit is violated or not.
- Step 12: If on all the transmission lines the power flow constraints are satisfied then select the generation pattern for this solution set and compute the total cost.
- Step 13: If there is any power flow violation, then select the next sub-optimal solution set and go to step 10.

The detail flowchart of the above algorithm is shown in the next two pages (page 11 and page 12).

#### 2.4 CONCLUSION

In this chapter, solution methodology for ED problem through DP technique has been described. A new method to solve the ED problem accounting for the system loss by the use of DP technique has been proposed.





# **CHAPTER 3**

# ECONOMIC DISPATCH USING FUZZY DYNAMIC PROGRAMMING

In the previous chapter, solution of economic dispatch problem by using DP approach has been described where the system load demand has been assumed to be known precisely. However, in practical scenario, the load demand is not always known precisely, rather an approximate value of the load demand is more likely to be known. In other words, there is an uncertainty or vagueness associated with the system load demand. This uncertainty can not be adequately expressed by a crisp variable. On the other hand, this vagueness can be adequately handled in fuzzy set theory. This theory provides a strict mathematical frame work in which vague conceptual phenomenon can be studied rigorously. In this theory, the variables, functions, etc. connected with the imprecise phenomenon to be studied are expressed as fuzzy variables and fuzzy functions. Consequently, to solve economic dispatch problem under vague load demand. the objective function and all or some of the constraints need to be expressed as fuzzy objective function and fuzzy constraints. The final solution point is reached by applying various fuzzy operations on the fuzzy objective functions and the constraints. In the next section, the basic mathematical operations in fuzzy set theory which are useful to solve ED problem under fuzzy environment are described.

#### 3.1 FUNDAMENTAL OF FUZZY SET THEORY

#### (a) Fuzzy set

Let X be a collection of objects (X is the universal set). Then a fuzzy set  $\widetilde{A}$  in X is defined to be a set of ordered pairs:

$$\widetilde{A} = \{ (x, \mu_A(x)), | x \in X \}$$
 (3.1)

where  $\mu_A(x)$  is called the membership function of x in  $\widetilde{A}$ .

Note that the membership function  $\mu_A(x)$  denotes the degree that x belongs to  $\widetilde{A}$  and is normally limited to values between 0 and 1. A high value of  $\mu_A(x)$  implies that it is very likely for x to be in  $\widetilde{A}$ . Elements with a zero degree of membership are normally not listed. If we limit the values of the membership function to be either 0 or 1, then  $\widetilde{A}$  becomes a crisp (non-fuzzy) set.

#### (b) The union of two fuzzy sets

Let  $\widetilde{A}$  and  $\widetilde{B}$  be two fuzzy sets with membership functions  $\mu_A(x)$  and  $\mu_B(x)$  respectively. The membership function of the union  $\widetilde{C} = \widetilde{A}U\widetilde{B}$  is pointwise defined by,

$$\mu_{c}(x) = \max(\mu_{A}(x), \mu_{B}(x)), x \in X$$
(3.2)

#### (c) The Intersection of two fuzzy sets

Let  $\widetilde{A}$  and  $\widetilde{B}$  be two fuzzy sets with membership functions  $\mu_A(x)$  and  $\mu_B(x)$ respectively. The membership function of the intersection  $\widetilde{D} = \widetilde{A} \cap \widetilde{B}$ , is defined by,

$$\mu_{D}(x) = \min(\mu_{A}(x), \mu_{B}(x)), x \in X)$$
(3.3)

#### (d) The complement of a fuzzy set

Let A be a fuzzy set with membership function  $\mu_A(x)$ . The membership function of complement of set A is defined by

$$\mu_{\overline{A}}(\mathbf{x}) = 1 - \mu_{A}(\mathbf{x}) \tag{3.4}$$

By using the basic fuzzy operations described above, a methodology to solve ED problem under fuzzy environment has been developed. In the next section, the developed solution methodology is described in detail.

### 3.2 SOLUTION OF ED PROBLEM UNDER UNCERTAINTY

As has been discussed earlier, the objective is to solve ED under uncertainties in the system load demand. This uncertainty in load demand can be handled in two ways. First, the load demand can be treated as a fuzzy variable. In this approach, a suitable membership function in chosen to represent the ambiguous load demand (e.g. approximately 100 MW). Alternatively, in this work, another approach is suggested. In this second approach the load demand is treated as a crisp variable instead of as a fuzzy variable. However, the variation in load demand in taken into account by classifying the load demand into some categories (e.g. small, medium and high). To represent more realistically the generating patterns of the generators when the load is in either of the classification categories, the total generating capacity of an individual generator is also divided into the same categories. For example, if the total load demand is classified into three categories (low, medium and high), the total generating capacity of an individual generator is also classified into the same three categories, e.g. low, medium and high. The rationale behind this same classification is the fact that for a low load demand the generation from an individual generator is most likely to the low. Similarly, for a high load demand in the system, the individual generators are also most likely to produce high output power. However, the extent to which an individual generator would participate to meet the system load demand is somewhat uncertain. To represent this uncertainty, the generation of an individual generator in each category is represented as a fuzzy generation pattern.

In this work, the system load demand has been classified into three categories ; low, medium and high. Consequently, the total generation capacity of an individual generator is also classified into these three categories. For each category of loads the three categories of generation are represented by appropriate membership functions to take into account the uncertainty in generator participation. Thus, in this work, total 9 generation membership functions have been considered. These membership functions are shown in Fig. 3.1.

As the generations of individual generators have been considered as fuzzy variables, obviously the cost functions corresponding to these fuzzy generations also need to be considered as fuzzy functions.

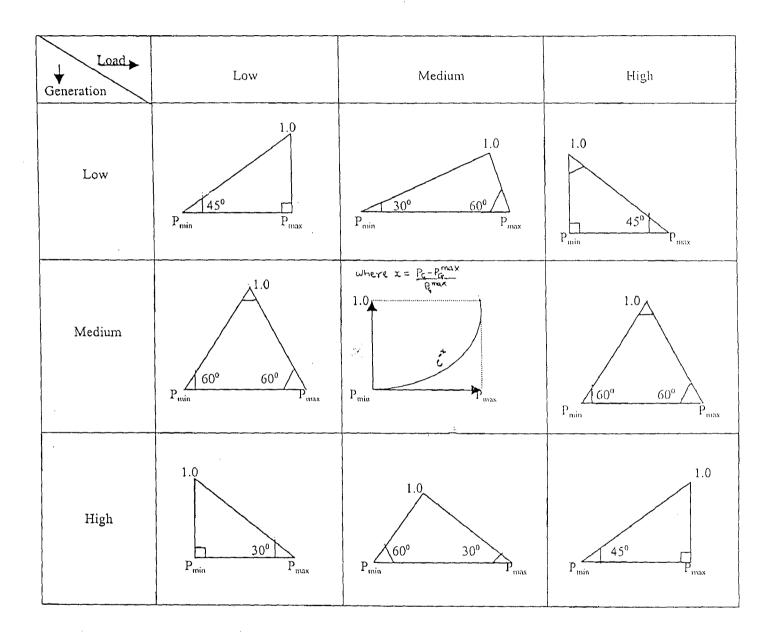


Figure 3.1 : Membership functions for generations under various loads

The membership function chosen for the cost is written as,

$$\mu_{c}(i,j) = \exp(-\Delta c(i,j)) \tag{3.5}$$

where,

$$\Delta c(i,j) = \frac{C(i,j) - C_{\min}(i,j)}{C_{\min}(i,j)}$$

where,

 $\mu_{c}(i, j) =$  The cost membership value for i<sup>th</sup> bus generator with j<sup>th</sup> discrete generation (P<sub>Gij</sub>).

C(i,j) = cost per MW for i<sup>th</sup> bus generator with j<sup>th</sup> discrete generation given by the expression.

$$C(i,j) = a_i P_{Gij} + b_i + c_j / P_{Gij}$$

 $a_i$ ,  $b_i$ ,  $c_i$  are the cost coefficients for  $i^{th}$  bus generator.

 $C_{min}(i,j)$  = minimum cost per MW for i<sup>th</sup> generator bus with j<sup>th</sup> discrete generation, given by the expression,

$$C_{\min}(i,j) = b_i + 2\sqrt{a_j}c_j$$

Fig 3.2 depicts the cost membership function ( $\mu_c$ ) for the i<sup>th</sup> generator.

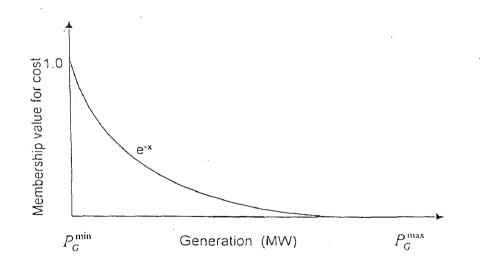


Figure 3.2 : Cost membership function

Where, in the above figure,  $x = \Delta c$  (i,j) defined above.

With the consideration of the membership functions as described above, the objective function of the ED problem, becomes a fuzzy objective function. However, the various constraints as described in chapter 2, are still considered to be crisp. To solve this optimization problem with fuzzy objective function and crisp constraints, fuzzy dynamic programming (FDP) technique been used. The step-by-step algorithm of FDP is given below :

Step 1 : Read all inputs data.

- (a) Minimum and maximum Generation limits,  $P_{Gi}^{min} \& P_{Gi}^{max}$ , for  $i = 1, 2, \dots, NG$
- (b) Transmission line limits  $p_{ij}^{max}$  for i = 1, 2, ..., n. j = 1, 2, ..., n
- (c) Cost coefficients  $a_i$ ,  $b_i$ ,  $c_i$  for i = 1, 2, ..., NG
- (d) Line parameters (resistance, reactance and line charging susceptance, for each existing line between i<sup>th</sup> and j<sup>th</sup> bus.
- (e) System load demand,  $P_{load}$ .

Step 2 : Set  $P_{loss} = 0$ .

Step 3 :  $P_{load}^{new} = P_{load} + P_{loss.}$ 

Step 4: Find all combination of generations for each set  $(P_{G1}, P_{G2}, P_{G3}, \dots, P_{NG})$  for

$$\sum_{i=1}^{NG} P_{Gi} = P_{\text{load}}^{\text{new}}$$

Step 5: For each  $P_{Gi}$  in each generation set ( i  $\epsilon$  NG),  $\mu_C(i, j)$  and  $\mu_G(i, j)$  are found. The  $\mu_R(i, j)$  for each  $P_{Gi}$ , i  $\epsilon$ NG is calculated by

$$\mu_{R_k}(i, j) = \min \left( \mu_c(i, j), \mu_G(i, j) \right)$$
 for k = 1, 2, 3.....NG

Step 6 : The set membership value (SMV) for each set is found as  $\mu_{\text{smv}_{W}} = \max(\mu_1(i, j), \mu_2(i, j), \dots, \mu_{NG}(i, j))$  for  $w = 1, 2, 3, \dots$  number of generation sets

Step 7:  $\mu_{smv}$  for all sets are arranged in descending order.

Step 8 : Solution corresponding to the highest membership value is selected.

Step 9 : Run the DC power flow and find  $P_{loss}^{new}$ .

Step 10 : Compute Error =  $|P_{loss} - P_{loss}^{new}|$ .

Step 11 : If error >  $\varepsilon$  (tolerance), set  $P_{loss} = P_{loss}^{new}$  and go to step 3. Else, go to step 12.

Step 12 : Total loss is obtained and all sets of  $\mu_{smv}$  are arranged in descending order.

Step 13 : Select largest of all  $\mu_{smv}$ . Select the corresponding generation set.

Step 14 : The power flow in all lines are found for the set corresponding to the selected  $\mu_{smv}$  using DC power flow.

Step 15 : All line constraints are checked.

Step 16 : If all line constraints are within their individual limits, then this set  $(P_{G1}, P_{G2}, \dots, P_{NG})$  is the solution set and the pattern  $P_{G1}, P_{G2}, \dots, P_{NG}$  is the most economic solution satisfying the transmission capacity constraints.

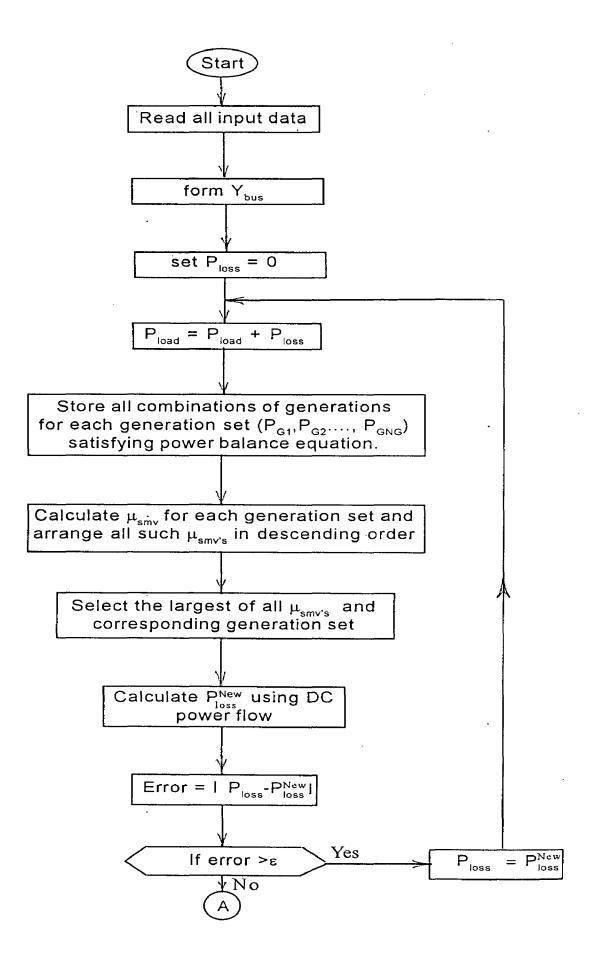
Step 17 : If any line constraint is violated, select the next lower  $\mu_{smv}$  and go to step 14.

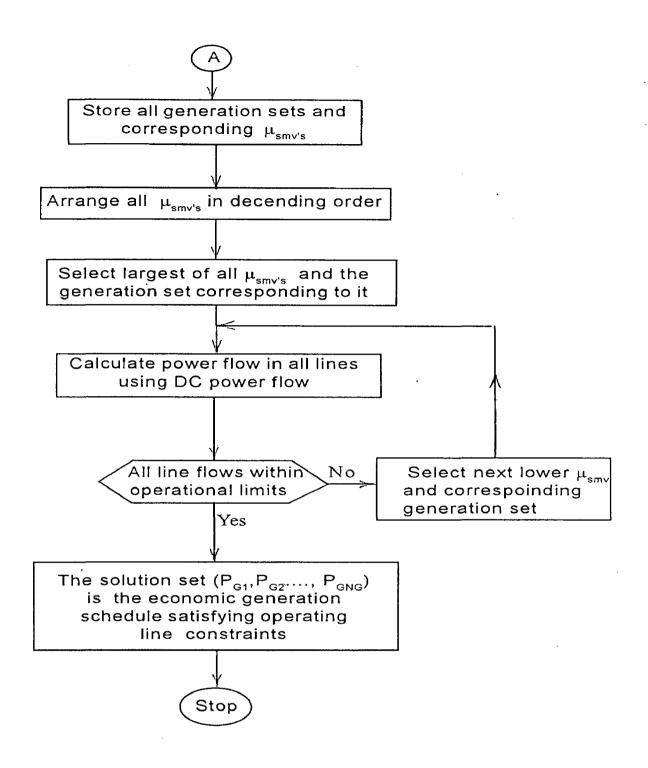
The flowchart of the FDP algorithm is shown in the next two pages (page 20 and 21).

#### 3.3 CONCLUSION

In this chapter, a fuzzy dynamic programming methodology is proposed to consider uncertainty in system load demand for the solution of ED problem. The uncertainty in load demands has been taken into account by classifying the load demand into various categories. The uncertainty of participation of each generator to meet the load demand in various categories is incorporated in this study through the use of fuzzy generation patterns.

#### **FLOW CHART**





# **CHAPTER 4**

# **RESULTS AND DISCUSSION**

To illustrate the application of DP techniques to solve ED problem, a 5 generator, 10-bus system taken from [5] is considered. The one line diagram of the system is shown in Fig. 4.1. The load and line data of this system are tabulated in Appendix B.

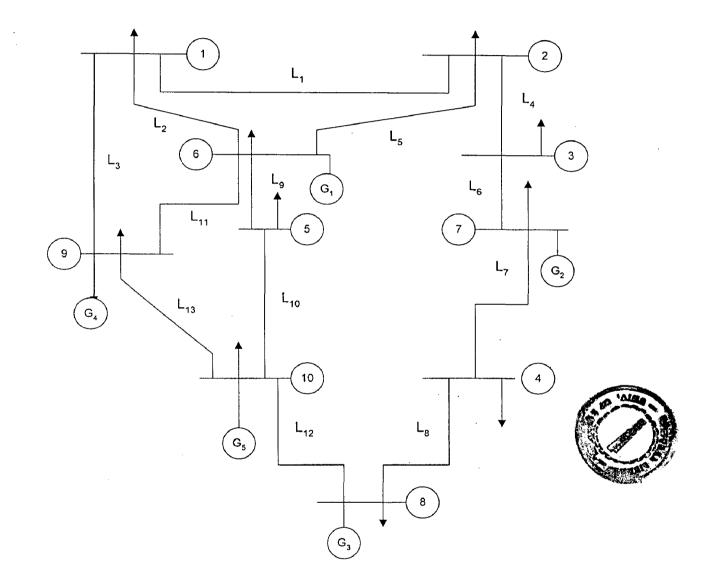


Figure 4.1 : One line diagram of the test system

To compare between the classical dynamic programming technique and fuzzy dynamic programming method, two different cases have been considered. In the first case, the load demands as given in Appendix B have been assumed to be crisp in nature. Subsequently in the second case, on this same loading pattern, fuzzy generation membership functions have been applied. In the next two sections, these results are discussed in detail.

#### 4.1 CLASSICAL DYNAMIC PROGRAMMING APPROACH

To find the optimum generation patterns when the load demands are assumed to be crisp in nature, classical dynamic programming technique has been applied following the algorithm described in chapter 2. The most optimum generation pattern is tabulated in table 4.1.

Unit No.	Generation (MW)	Generating cost (units)
1	115.00	29.637441
2	25.00	29.31875
3	40.00	24.71200
4	40.00	26.76000
5	103.286774	28.68650

Table 4.1 : Optimum generation pattern

Total generation cost = 138.544144 units.

In this table, the cost of individual generation is also given. From this table, the total cost of generation is found to be 138.544144 units of money. The complex power flows over all the lines at this most optimum generating condition are tabulated in Table 4.2. From this table, it is found that on four lines, 1-9, 2-6, 4-8 and 8-10, the power flows are more than the specified limits. Hence, from practical point of view, the optimum generation pattern may not be feasible for implementation. To find out a feasible generation pattern which satisfies all the operating constraints, the sub-optimal solution patterns, it is found that at 129<sup>th</sup> sub-optimal solution pattern, (the most optimum solution is ranked as 1, the immediate next sub-optimal solution is ranked as 2 and so on), all the system constraints are satisfied. Table 4.3 and Table 4.4 tabulate the generation pattern and the line flows at the 129<sup>th</sup> sub-optimal solution point respectively.

Line joining		Line flow (p.u)
Bus	Bus	
1	2	0.260567
1	6	0.223816
1	9	0.357582
2	3	0.140030
2	6	0.307331
3	7	0.075030
4	7	0.025355
4	8	0.345846
5	6	0.064826
5	10	0.254226
6	. 9	0.015720
8	_ 10	0.348445
9	10	0.162226

Table 4.2 : Line flows for economic dispatch

Table 4.3 : Economic dispatch at 129<sup>th</sup> sub-optimal solution

Unit No.	Generation (MW)	Generating cost (units)
1.	115.00	29.637006
2.	55.00	32.282749
3.	40.00	24.712000
4.	25.00	25.312500
5.	88.286774	26.286506

Total generation cost = 138.630768 units.

÷

Line joining		Line flow (p.u)
Bus	Bus	
1	2	0.160426
2	6	0.199196
1	9	0.281843
2	3	0.015870
2	6	0.250592
3	7	0.229787
4	7	0.140035
4	8	0.179506
5	6	0.072074
5	10	0.246963
6	9	0.058751
8	10	0.180926
9	10	0.171564

Table 4.4 : Line flows at 129th sub-optimal solution

It is observed that the total cost of generation at that sub-optimal solution point is 138.630768 units of money, which is more than the cost at optimum solution point. Now, the cost at any sub-optimal point should be more than the cost at the optimum point and the results also corroborate this.

It has been already discussed in Chapter 1, that it is possible to relieve the over loaded lines by applying thyristor controlled series capacitor (TCSC) at some strategic lines. To investigate this possibility, TCSCs have been applied at different lines with various compensation levels and the solutions of the economic dispatch problem are obtained with the TCSCs in the lines. The obtained solutions are again ranked in the ascending order starting from the most optimum solution. The compensation levels of the TCSC have been varied from 10% to 80% of the corresponding line reactances. It has been found that when the line 3-7 is 34% compensated, at the 26<sup>th</sup> solution pattern (corresponding to this compensation level) all the constraints in the system are satisfied. The generation pattern and line flows at this solution point are tabulated in Tables 4.5 &

4.6 respectively. It has been found that at this solution point, the total cost of generation is 138.625458 units of money.

Unit No.	Generation (MW)	Generating cost (units)
1.	115.00	29.637001
2.	55.00	32.282749
3.	25.00	23.293751
4.	25.00	25.31250
5.	103.111298	28.099459

Table 4.5 : Economic dispatch with TCSC installed on line 3-7

Total generation cost = 138.625458 units.

Line j	Line joining	
Bus	Bus	
1	2	0.144234
1 .	6	0.193614
1	9	0.271073
2 .	6	0.239819
4	7	0.189094
5	6	0.084577
6	9	0.062849
2	3	0.042538
3	7	0.274192
4	8	0.136776
5	10	0.234435
8	10	0.291810
9	10	0.156548

Table 4.6 : Line flows with TCSC installed on line 3-7

Comparison of the costs in Tables 4.1, 4.3 & 4.5 reveals that it is possible to satisfy all the system constraints with the help of TCSC at a higher cost than that obtained at the most optimum point. In addition to the higher generation cost, the cost of TCSC also contributes to the higher cost which is required to satisfy all the system constraints. However, the cost of generation with TCSC is less than the cost without TCSC when all the operating constraints are required to be satisfied. Hence in the long term, the savings in cost obtained from the difference in costs of Tables 4.3 & 4.5 would offset the cost of TCSC.

To explore other possibilities of TSCS application, economic dispatch problem has been solved when other lines have been compensated with varying compensation levels. It has been found that when the line 4-7 is 50% compensated, all the system operating constraints are satisfied. The results at this solution point are tabulated in Tables 4.7 & 4.8 respectively.

Unit No.	Generation (MW)	Generating cost (units)
1	115.00	29.637001
2	55.00	32.282749
3	25.00	23.293751
4	25.00	25.312500
5	103.38992	28.126358

Table 4.7 : Economic dispatch with TCSC installed on line 4-7

Total generation cost = 138.652359 units.

It is found that the total generation cost at this solution level in 138.652359 units of money. As this total cost is higher than the total cost in Table 4.3, there is no savings in the generation cost and hence in the long run, the operating cost at the solution point of Table 4.7 would be more than that obtained at the operating point of Table 4.3. As this higher operating cost would also add to the cost of TCSC implementation, clearly this operating point is not economically feasible. Hence the operating point of Table 4.7 is not recommended.

Line joining		Line flow (p.u)
Bus	Bus	
1	2	0.154968
1	6	0.197314
1	9	0.278213
2	3	0.024859
2	6	0.246961
3	7	0.238684
4	7	0.262576
4	8	0.096571
5	6	0.076288
5	10	0.242740
6	9	0.060132
8	10	0.257316
9	10	0.166502

Table 4.8: Line flows with TCSC installed on line 4-7

Based on the above results, it is found that when the line 3-7 is 34% compensated, the solution of the ED problem is most economic at which all the system constraints are also satisfied.

#### 4.2 FUZZY DYNAMIC PROGRAMMING APPROACH

To consider the uncertainties in loads, the ED problem has been solved using the generation membership functions and the cost membership functions as discussed in Chapter 3. To generate the generation membership functions, load demands have been classified into three categories. These are as follows :

i) Load in the range :  $(L_{min}) - (L_{min} + 0.25L_{dif}) \rightarrow low range$ 

ii) Loads in the range :  $(L_{min} + 0.25L_{dif}) - (L_{max} - 0.25L_{dif}) \rightarrow$  medium range

iii) Loads in the range : 
$$(L_{max} - 0.25L_{dif}) - (L_{max}) \rightarrow high range$$

Where,  $L_{min} \rightarrow minimum$  of the the bus load demands in the system

 $L_{max} \rightarrow$  maximum of the bus load demands in the system

 $L_{dif} = (L_{max} - L_{min})$ 

To classify the generators into the above same three categories, following criteria have been adopted.

- i) Low range :  $P_{Gmax} < 50 \text{ MW}$
- ii) Medium range :  $50 \text{ MW} \le P_{\text{Gmax}} \le 150 \text{ MW}$
- iii) High range :  $P_{Gmax} > 150 \text{ MW}$

Where,  $P_{Gmax} \rightarrow maximum$  generating capacity of a generator.

Thus there are total 9 generation membership functions.

With these 9 generation membership functions and the associated cost functions, fuzzy dynamic programming method as described in chapter 3 has been used to determine the most optimum generating schedule. Tables 4.9 and 4.10 tabulate the results at this most optimum solution.

Table 4.9 : Optimum economic dispatch with uncertain loads

Unit No.	Generation (MW)	Generating cost (units)
1.	85.00	29.816999
2.	70.00	33.839001
3.	55.00	26.161751
4.	55.00	28.252501
5.	59.110260	24.008343

Total generation cost = 139.078595 units.

It is observed that at this most optimum solution, the total generation cost is 139.07895 units of money and in this condition, two lines, namely lines 1-9 and 3-7 are violating the maximum power flow constraints.

Line joining		Line flow (p.u)
Bus	Bus	SP2
1	2	0.134694
2	6	0.146318
1	9	0.308809
2	3	0.093831
2	6	0.189410
. 3	7	0.312321
4	7	0.187601
4	8	0.131157
5	6	0.063340
5	10	0.255540
6	9	0.160411
8	10	0.020003
9	10	0.127892

Table 4.10 : Line flows for optimum economic dispatch

However, in this solution set, it is observed that at the 36<sup>th</sup> sub-optimal solution set, all the system constraints are satisfied. The results at this condition are shown in Tables 4.11 and 4.12.



Table 4.11 : Economic dispatch at 36<sup>th</sup> sub-optimal solution

Unit No.	Generation (MW)	Generating cost (units)
1	115.00	29.637001
2	70.00	33.839001
3	55.00	26.161751
4	40.00	26.76000
5	44.110260	22.684479

Total generation cost = 139.082245 units.

It is also observed that the total generation cost at this condition is 139.082245 units of money, which is naturally higher than the total generation cost at the most optimum case.

Line joining		Line flow (p.u)	
Bus	Bus		
1	2	0.1285940	
1	6	0.1768700	
1	9	0.2713670	
2	3	0.0709960	
2	6	0.2180210	
3	7 ·	0.2878110	
4	7	0.2124870	
4	8	0.1061760	
5	6	0.145000	
5	10	0.1738680	
6	9	0.0099870	
8	10	0.046722	
9	10	0.0658310	

Table 4.12 : Line flows at 36<sup>th</sup> sub-optimal solution

To explore the possibility of using TCSC to relieve the overloaded lines, ED problem has been solved using TCSC at various lines with varying compensation levels. It has been found that for two TCSC installations, all the system constraints are satisfied. These two cases are :

- a) TCSC installed on the line 1-6 with 50% compensation level and
- b) TCSC installed on the line 2-6 with 30% compensation level.

The results for case (a) are tabulated in Tables 4.13 and 4.14 and those corresponding to case (b) are tabulated in Tables 4.15 and 4.16 respectively.

Unit No.	Generation (MW)	Generating cost (units)
1	85.00	26.816999
2	70.00	33.839000
3	55.00	26.161751
4	55.00	28.282501
5	57.972691	23.287486

Table 4.13 : Economic dispatch with TCSC installed on line 1-6

Total generation cost = 138.887741 units.

Line joining		Line flow (p.u)
Bus	Bus	
1	2	0.162617
1	6	0.246011
1	9	0.270788
2	3	0.078211
2	6	0.175043
3	7	0.295580
4	7 0.20	
4	8	0.114070
5	6	0.098512
5	10	0.220388
6	9	0.157183
8	10	0.038306
9	10	0.083626

Table 4.14 : Line flows with TCSC installed on line 1-6

,

Unit No.	Generation (MW)	Generating cost (units)
1	85.00	26.816999
2	70.00	33.839001
3	55.00	26.161751
4	55.00	28.252501
5	57.001785	23.820080

Table 4.15 : Economic dispatch with TCSC installed on line 2-6

Total generation cost = 138.890320 units.

Line j	Line joining	
Bus	Bus	
1	2	0.106635
1	6	0.136582
1	9	0.290260
2	3	0.081786
2	6	0.243865
3	7	0.299429
4	7	0.200732
4	8	0.117982
5	6	0.085812
5	10	0.233096
6	9 0.15370	
8	8 10 0.034	
9	10	0.100775

Table 4.16 : Line flows with TCSC installed on line 2-6

From these tables it is observed that for both these two cases, the total generation costs are less than that obtained at the operating condition given in Table 4.9. Hence both these cases are economically viable as in both these cases, the cost of TCSC installation

would be offset by the savings accrued in the generation cost. However, in the case (a) the generation cost is less than the cost in case (b). Hence, solution set (a) is preferable as the savings in this case is higher and consequently, the cost of the TCSC installation would be offset more quickly.

Hence, from the above discussion, TCSC with 50% compensation on the line 1-6 is recommended.

#### 4.3 COMPARISON OF RESULTS

Comparison of the results of the classical dynamic programming and fuzzy dynamic programming methods reveals the following facts :

- The generation cost at the recommended operating point with fuzzy dynamic programming is higher than the generation cost at the recommended operating point with classical dynamic programming. In other words, if the uncertainty of loads is taken into consideration, the total generation cost at the most feasible operating point increases than in the case where the uncertainty is not considered. Intuitively, this conclusion seems to be logical, as the cost is supposed to increase when more flexibility is imparted in the solution.
- ii) The generation pattern obtained when the load uncertainty is considered is more reliable than the generation pattern obtained without the consideration of the load uncertainty. This is so because the generation pattern in the former case is more uniformly spread among the generators than in the latter case. For example, in the latter case, unit no. 1 in delivering 115 MW whereas in the former case, no unit is supplying more than 85 MW. Hence, in the latter case, the stress on unit 1 is more than the stress on any generator in the former case. Hence, there is more probability that the unit 1 in the latter case may develop some problem. If this generation unit develops any fault, the deficiency in power in the system would be much more than the deficiency in the system if any unit in the former case develops any fault.

## **CHAPTER 5**

#### CONCLUSION

In this thesis work, economic dispatch problem has been solved considering the uncertainty in the load demand. The uncertainty in the load demand has been incorporated in the study by the use of fuzzy logic. To gain insights regarding the advantage or demerits of using fuzzy logic for ED problem, the results have also been compared with the results where the uncertainty in the load demand has been neglected. Also, possibility of using TCSCs to satisfy the transmission capacity constraints during ED has also been explored. The main conclusions of this work are :

- (a) Consideration of uncertainty in the load demand results into higher generation cost at the most feasible operating point compared to the cost when the uncertainty in the load demand is not considered.
- (b) The optimum generation pattern when the load uncertainty is considered is more reliable than the optimum generation pattern obtained without the consideration of the uncertainty in load demand.
- (c) If transmission capacity constraints are not satisfied at the solution of the ED problem, it is possible to install TCSC with suitable compensation level at some strategic line so that all the transmission capacity constraints are satisfied.
- (d) With the use of TCSC, the total generating cost comes out to be less than that obtained without TCSC. Hence the savings obtained because of the difference in generation cost would offset the cost of TCSC implementation.

#### FUTURE SCOPE OF WORK

In the present work, system loss has been computed based on DC power flow technique. However, to get more accurate results, an AC power flow technique needs to be used. Also, different fuzzy functions can be used and a comparison of the results would reveal the best fuzzy membership function for practical implementation.

# REFERENCES

- Wood, A.J. Wollenberg, B.F., "Power Generation and Control", John Wiley and Sons, 1984.
- T. Yalcinoz and M.J. Short, "Neural Networks Approach for Solving Economic Dispatch Problem with Transmission Capacity Constraints", IEEE Transactions on Power Systems, Vol. 13, No. 2, pp. 307-313, May 1998.
- Qing Xia, Y.H. Song, Boming Zhang, Changqing Kang and Niande Xiang, "Dynamic Queuing Approach to Power System Short Term Economic and Security Dispatch", IEEE Transactions on Power Systems, Vol. 13, No. 2, pp. 280-285, May 1998.
- Yung Chung Chong, Wei Tzen Yang and Chun Chang Liu, "On-Line Economic Dispatch Using a New Loss Coefficient Formula", Electric Power System Research, Vol. 25, pp. 131-136, May 1992.
- Ji. Yuan Fan, Lan Zhang, "Real Time Economic Dispatch with Line Flow and Emission Constraints Using Quadratic Programming", IEEE Transactions on Power Systems, Vol. 13, No. 2, pp. 320-324, May 1998.
- R. Ramanathan, "Fast Economic Dispatch Based on Penalty Factors from Newton's Method", IEEE Transactions on Power Systems, Vol. 104, pp. 1624-1629, 1985.
- 7. C. Palanichamy and K. Srikrishna, "Simple Algorithm for Economic Power Dispatch", Electric Power System Research, Vol. 21, pp. 147-153, 1991.
- Dragan Vlaisovlijevic, Miodrag B. Djukanovic, Dejan, J. Sobajic, Barvoje S. Babic,", Fuzzy Linear Programming Based Optimal Power System Rescheduling Including Preventive Dispatch", IEEE Transactions on Power Systems, Vol. 14, No.2, May 1999.
- P.H. Chen, H.C. Chang, "Large Scale Economic Dispatch Approach by Genetic Algorithm", IEEE Transactions on Power Systems, Vol. 10, No.4, pp. 1919-1926, Nov. 1995.
- H.J. Zimermann, "Fuzzy Sets, Decision Making and Expert Systems", Kluwer Academic Publishers, 1987.

- N. Hingorani, "Flexible AC Transmission", IEEE Spectrum, Vol. 30, No. 4, pp. 40-45, April, 1993.
- Douglas, J. Gotham and G.T. Haydt, "Power Flow Control in Systems with FACTS devices", Electrical Machines and Power Systems, Vol. 26, pp. 951-962, 1998.
- Preecha Preedavichit and S.C. Srivastava, "Optimal Reactive Power Dispatch Considering FACTS Devices", Electrical Power Systems Research, Vol. 46, pp. 251-257, 1998.

#### **APPENDIX A : DC POWER FLOW**

The DC power flow method assumes that the magnitudes of all the bus voltages are known and equal to 1.0 p.u. This method is least accurate but simple and fast.

#### (a) DC Power Flow and Active Power Losses

The active power losses can be found from DC power flow using equation :

 $[P^*] = [B'] [\delta]$ 

where,

 $[\delta] =$  Vector denoting load angles at buses

[B'] = -[B]

[B] = Bus admittance matrix with zero conductance element

$$[P^*] = [P_{Gi}] - [P_{bus load}]$$

 $[P_{Gi}]$  = Vector denoting bus generations.

 $[P_{bus load}] =$  Vector denoting bus loads.

$$\sum_{i=1}^{n} P_{Bus \, load} = P_{load}$$

Loss expressions is given by [13],

 $P_{Lij} = 2 G_{ij} (1 - \cos (\delta_i - \delta_j))$ 

where,

(b)

 $P_{Lij}$  = active power loss on line joining  $i^{th}$  and  $j^{th}$  bus

 $G_{ij} = i^{th}$  row and  $j^{th}$  column element of  $Y_{bus}$  (Y = G+j B)

Total active power loss  $P_{loss} = \sum_{i=1}^{n} \sum_{i=1}^{n} = P_{Lij}$ , i = 1, 2, ..., ni = 1, 2, ..., n

DC Power Flow and MVA Flows

The MVA flows in line can be found by using the DC power flow. The  $[\delta]$  can be found as described above.

The active power flow in line joining  $i^{th}$  and  $j^{th}$  bus is given by,

$$p_{ij} = \frac{(\delta_i - \delta_j)}{X_{ij}}$$

The reactive power flow in line joining  $i^{th}$  and  $i^{th}$  bus is given by,

$$q_{ij} = \frac{(\cos(\delta_i - \delta_j) - 1)}{X_{ij}}$$

The MVA power flow in line joining  $i^{th} & j^{th}$  Bus is given by

$$s_{ij} = \sqrt{p_{ij}^2 + q_{ij}^2}$$
 where  $X_{ij}$  = reactance of line joining i<sup>th</sup> & j<sup>th</sup> bus.

# **APPENDIX B**

# SYSTEM DATA

Unit#	Bus#	Cost function	$P_{max}(p.u)$	$P_{\min}(p.u)$
1	6	$20 + 7p + 1.2 P^2$	1.2	0.1
2	7	$20 + 9p + 1.1 P^2$	1.2	0.1
3	8	$21 + 9p + 0.7 P^2$	1.2	0.1
4	9	$23 + 9p + 1.0 P^2$	1.2	0.1
5	10	$19 + 8p + 0.8 P^2$	1.2	0.1

# Table 1 : Generator Unit Characteristics

 Table 2 : Load Data

Bus #	Load (p.u)	Bus #	Load (p.u)
1	0.3 + j0.1	6	0.6 + j0.15
2	0.4 + j0.15	7	0.2 + j0.1
3	0.2 + j0.1	8	0.4 + j0.1
4	0.3 + j0.15	9	0.2 + j0.1
5	0.3 + j0.1	10	0.6 + j0.1

From Bus	To Bus	Impedance (p.u)	Sus. (p.u)	Limit (p.u)
1	2	0.02 + j0.08	0.01	0.6
1	6	0.06 + j0.25	0.02	0.3
1	9	0.04 + j0.16	0.02	0.3
2	3	0.06 + j0.25	0.02	0.3
2	6	0.06 + j0.25	0.02	0.3
3	7	0.06 + j0.25	0.02	0.3
4	7	0.04 + j0.16	0.02	0.3
4	8	0.06 + j0.25	0.02	0.3
5	6	0.04 + j0.16	0.02	0.3
5	10	0.06 + j0.25	0.02	0.3
6	9	0.02 + j0.08	0.01	0.6
8	10	0.04 + j0.16	0.02	0.3
9	10	0.08 + j0.32	0.025	0.2

Table 3 : Line data

.