STRENGTH AND DUCTILITY OF JACKETTED R.C. MEMBERS

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

EARTHQUAKE ENGINEERING

(With Specialization in Structural Dynamics)

By

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CANDIDATE'S DECLARATION

I hereby declare that the work presented in this dissertation **"STRENGTH AND DUCTILITY OF JACKETTED R.C. MEMBERS"** in partial fulfillment of the requirements for the award of the degree of **"MASTER OF TECHNOLOGY"** in Earthquake Engineering with specialization in ***Structural Dynamics*** of the Indian Institute of Technology Roorkee, Roorkee, is an authentic record of my own work carried out during the period from July 2003 to June 2004 under the guidance of **Dr. Yogendra Singh,** ***Asst. Professor,*** and **Dr. D.K. Paul,** ***Professor,*** Department of Earthquake Engineering, Indian Institute of Technology Roorkee, Roorkee.

The matter embodied in this dissertation has not been submitted by me for the award of any other degree or diploma.

Dated: 28 June, 2004

Place: Roorkee

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CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

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ABSTRACT

A large number of RC buildings, in India, have been constructed without giving adequate attention to earthquake forces. These buildings require strengthening. Different strengthening strategies can be used for different structures, depending on the particular conditions and performance levels desired. Strengthening of individual beams and columns using RC jacketing is a commonly used strategy for seismic retrofitting.

Seismic safety of a structure demands sufficient strength and ductility. RC members such as beams and columns must possess sufficient strength to resist internal forces generated due to earthquake. At the same time, these members should also have adequate ductility which imparts capacity to undergo large inelastic deformation without significant loss of strength.

Generally, the earthquake forces encountered by a structure are much higher than those for which it is designed. This deficiency can be partially recuperated, by provision of ductility which induces capacity to absorb energy by hysteretic behavior of member. The ductility in a member can be obtained with ductile constituent materials. Since concrete is brittle, it requires confinement, which imparts ductility to it. Quantification of effect of passive confinement on ductility is very important parameter, which helps the designer to predict the behavior of the member under earthquake load.

Generally the strength available in a member is more than that for which it is designed. Exact quantification of this over-strength in member is required for capacity design method, which involves choosing a failure mechanism prior to design.

In this dissertation strength and ductility of unjacketed and jacketted beams and columns have been studied. Effect of confinement on ductility has been studied using the Kent and Park model of confined concrete. Effect of axial load on ductility of columns has also been studied. Contribution of original section in the strength of a jacketed column has been studied and it has been found that ignoring the steel in original column does not make much error, in the normal range of reinforcement. Strength of beams and columns for different material strength i.e., Limit State Strength, Characteristic Strength and Most Probable Strength has been studied and overstrength available has been estimated.

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INTRODUCTION

Voltaire, one of the greatest writer of France, gives a realistic description of 1775 Lisbon earthquake in "Candid" as **"The boiling sea heaves in the harbor, smashing the anchored ships to smithereens. Whirlwinds of fire and ashes rage in the streets and squares. Houses collapse, roof fall to the ground, wall crumble... Thirty thousand young and old of both sexes perished in the ruins..."** [1].

Every year, hundreds of earthquake occur all over the world, but most of them are either insignificant or take place in uninhabited regions. Some earthquakes occur dangerously close to populated localities. These earthquakes cause damage to structures, which are not adequately strong to resist it.

Past experience regarding behaviour of buildings and increased knowledge enabled engineers to develop some theories and methods that can more or less cushion the disastrous effect of earthquake. .

Occurrence of earthquake is known to be irregular in space, sizes and time. Along with irregularity in occurrence, each earthquake has a "temper" of its own. Therefore, it may so happen that anti-seismic measures, which are helpful to one earthquake, will be useless in another. These differences put restrains to use a "common to all" anti-seismic measure. Still, some rules seasoned by many years of previous experience may be regarded as useful in any situation.

Three properties of structure namely Stiffness, Strength and Ductility have been regarded as most important for its seismic performance. Effect of these three parameters can be easily incorporated in design and construction of new structures. Since a large number of R.C buildings, in India have been constructed either without any consideration to resist earthquake forces or without using current codal practice on earthquake resistant design. Such deficient buildings require enhancement in the above mentioned three structural properties in order to improve their seismic performance. There are various methods to improve strength, stiffness and ductility of structural elements; one of them is jacketing of members.

This dissertation focuses on study of seismic behaviour of R.C jacketed members. Behaviour of Beams and Columns for different confinement and R.C jacketing is studied here. Specially increase in moment capacity and ductility is studied. Effect of confinement, grade of concrete, grade of steel, depth of jacket, placement of bars etc are dealt, and their effect are plotted and tabulated.

1.1 ORIGIN AND CAUSE OF EARTHQUAKES

"Earthquakes have always been terror to mankind, which is why man has always been prone to regard them as a heavenly punishment for his sin, or a kind of devil's witchcraft, or a whim of an underground monster"[1]

Yet even in ancient times, progressive-minded people tried to explain the nature of earthquake by linking them to other surrounding phenomena. Although the way they put it was far from being true, the fact that heaven's (or hell's) administration was relieved of the responsibility was an outstanding achievement. [1]

1.1.1 Internal Structure of the Earth

Earth is roughly spherical, with its equatorial radius of 6370 km and a polar radius of 6350 km. Based on geophysical methods researchers have been able to establish that earth is a multi-layered body. In fact researchers have found that earth consist of series of shells differentiating from one another in mineralogy, density, elastic properties, temperature and pressure. Fig 1 shows a section through part of globe and characteristics of constituent layers.

Crust: It is the outermost shell of earth. In turn it is composed of several layers. Thickness of crust varies from 5km to 70km with an average thickness of 35km.

Different layers of crust are shown in Fig 2

Mohorovicic discontinuity: The outer crust is separated from underlying stratified material (i.e. mantle) by surface called Mohorovicic discontinuity. A distinct change in propagation velocity of seismic waves has been observed due to this layer.

Mantle: Mantle is about 2850 km thick. It is divided into upper and lower Mantle. Researchers have found that average temperature of mantle is about 4000°F, which makes its material in a viscous or semi-molten state. No earthquake has been found to occur in lower mantle.

Gutenberg discontinuity: Discontinuity between Mantle and Outer Core is called as Gutenberg discontinuity.

Core: Core has been divided into outer core and inner core. Outer core is in liquid state with thickness of 2260km, consists of molten iron. Since it is in liquid state shear wave cannot pass through it. Inner core is solid due to high pressure and temperature (about 5000°F). Core consists of solid nickel-iron material.

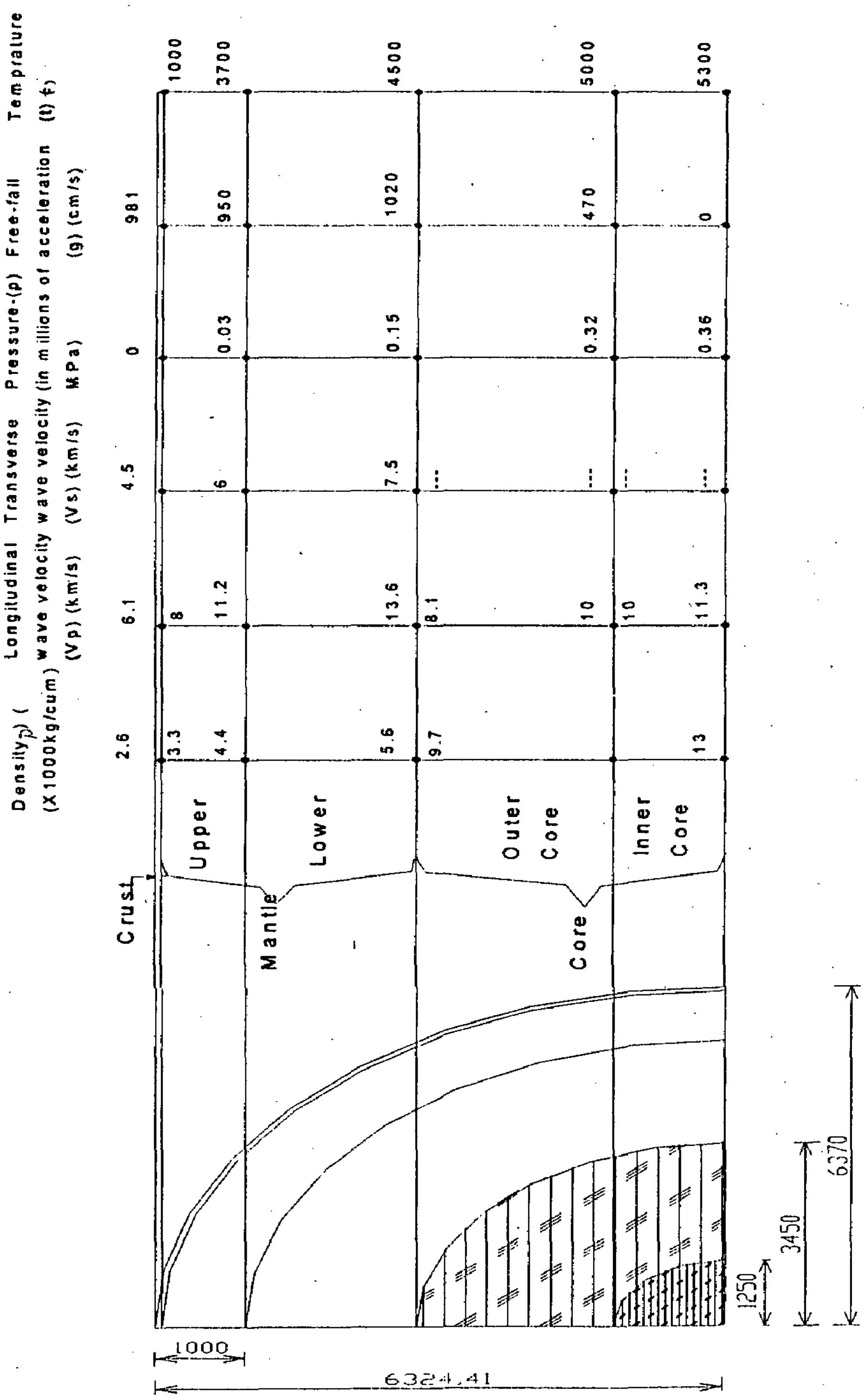


Fig. 1.1 Section through part of the globe

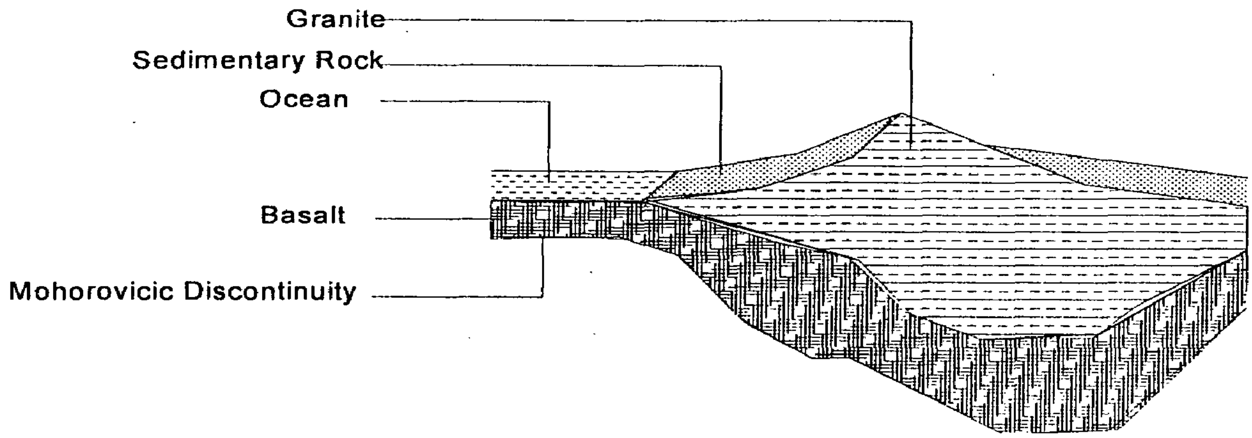


Fig. 1.2 Structure of earth's crust

1.1.2 Plate Tectonics

It has been established that earth's surface consists of plates which move with respect to each other. Plates have been classified based on its size as, six continental plates (African, American, Antarctic, Australia-Indian, Eurasian and Pacific), fourteen sub-continental plates and many micro-plates. Due to movement of these plates, inter-plate boundary zones are prone to suffer deformation.

Movement of plates has been explained based on thermo-mechanical equilibrium of earth's material. Due to temperature variation between crust-mantle boundary and mantle-core boundary, there exists temperature gradient. Variation of material density of mantle with temperature creates a unstable situation of denser material resting on less dense material. Thus denser material sinks down from above. During sinking its temperature increases and becomes less dense and eventually starts moving laterally and begins to rise again as subsequent cooled material begins to sink. This lateral movement of material, termed as convection current, imposes shear stress on bottom of plates dragging them in various directions. This dragging initiates building of strain between plate boundaries due to friction between them, which is suddenly released causing earthquakes.

1.1.3 Propagation of Seismic Waves

The region from which an earthquake originates below earth surface has been termed as hypocenter. Energy from this region propagates in all direction through seismic waves. Seismic waves have been broadly classified as surface waves and body waves. P-waves and S-waves constitute body waves where as Rayleigh waves and Love waves constitute surface waves. These waves when reach surface of earth, causes shaking of earth surface and buildings resting on it.

1.1.4 Effect of Seismic Waves on Structures

Seismic excitation imparts kinetic energy to foundation of structures causing its deformation, which results in vibration of the structure. This energy, during successive phase of vibration of structures, alternates continuously from kinetic to potential energy and vice-versa, until it is dissipated. This dissipation of energy occurs through the process of damping.

1.2 BEHAVIOR OF STRUCTURES UNDER EARTHQUAKE

An earthquake causes vibration of structure, which imparts deformation to it. A structure vibrates in one of the following four ways of deformation or their combination

a. Extensional
b. Bending
c. Shearing
d. Torsional

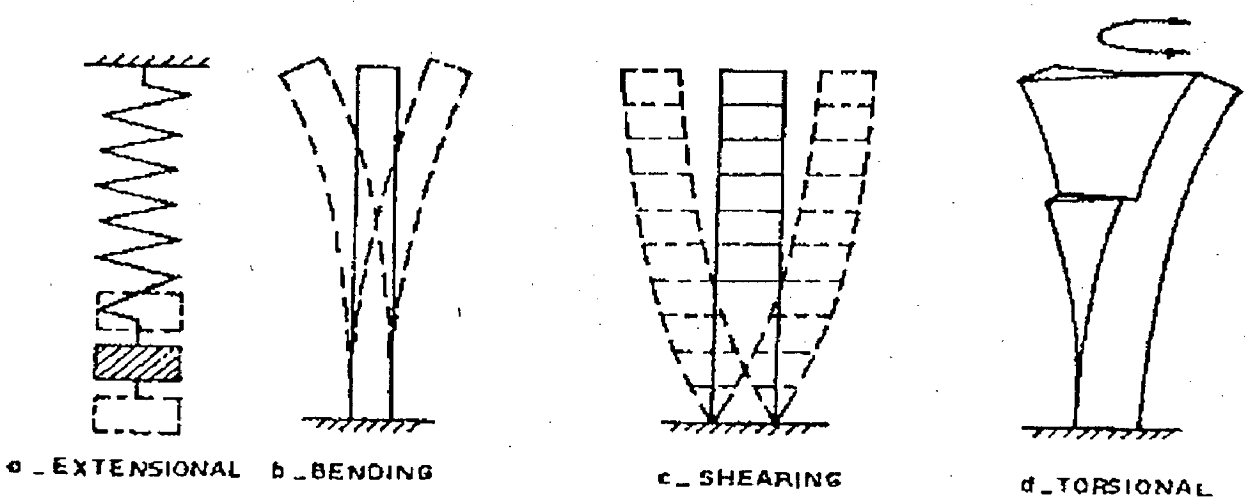


Fig 1.3. Types of deformation during vibration [2]

Extensional vibration of a structure would be in vertical plane. Bending is generally encountered by chimney-stack. Multistoried buildings, stiffened by rigid floor beams at different levels mostly encounter shear in horizontal direction. Torsion will be encountered by a building when its supporting parts have different stiffness or irregularity.

1.2.1 Structural Properties

Structural properties which is considered to provide seismic protection are

- Stiffness
- Strength
- Ductility

1.2.1.1 Stiffness

Stiffness relates load or forces to ensuing structural deformations [3]. Response of a reinforced concrete member indicates a non-linear relationship of force and displacement. During the analysis of a structure, inter-storey drifts are calculated based on stiffness, to satisfy requirement for serviceability.

1.2.1.2 Strength

Strength is used to express the resistance of a structure, or a member, or a particular section. A structure should have sufficient strength to resist internal force generated due to earthquake. Strength of structures depends on material property, section dimensions and various other factors. Since these parameters are not constants but vary between probable limits, it forces a designer to define various types of strength terms.

(a) Required Strength (S_u): It is the strength demand arising from the application of prescribed load and forces.

(b) Ideal Strength (S_i): It is nominal strength of section, based on prescribed limit state with respect to failure of that section. In India ideal strength is based on characteristic strength of the material. Characteristic strength is defined as the strength of material below which not more than 5 % of test results are expected to fall. Table 2 of IS 456:2000 gives specified characteristic compressive strength of 150 mm cube at 28 days in MPa. In case of reinforcing steel specified yield strength may be treated as characteristic strength. Characteristic strength of steel is defined by proof strain of 0.002.

$$\varphi Si > Su \tag{1.1}$$

Where ϕ is a factor less than 1

(c) Probable strength (S_p): It takes into account the fact that the material strength, which can be utilized in a member, is generally greater than the nominal strength specified by codes. Mean strength and standard deviation of concrete can be obtained by testing samples. According to CL: 9:2:2 (IS 456:2000). The target mean strength of concrete mix should be equal to the characteristic strength plus 1.65 times the standard deviation. Characteristic strength and standard deviation can be obtained from table 2 and 8 respectively of IS 456:2000.

$$Sp = \varphi_p Si \tag{1.2}$$

Where ϕ_P is greater than 1

(d) Over strength (S_o): It takes into account all possible factors that may contribute to strength exceeding the nominal or ideal value. It includes strength of steel due to strain hardening, concrete and masonry strength at a given age of the structure being higher than specified, strength increment due to confinement, strain rate effect etc.

$$So = \lambda_o Si \tag{1.3}$$

Where λ_o is greater than 1

1.2.1.3 Ductility

Ductility of a structure or its member is the capacity to undergo large inelastic deformation without significant loss of strength or stiffness. This is most important parameter while considering earthquake forces. Generally force arising due to earthquake will be too high. Designing structures for that level of force, will be too uneconomical, so it is common to design for a strength which are a fraction, perhaps 10% to 25% of elastic response, and rest of the forces are expected to be taken care by large inelastic deformations and energy dissipation corresponding to material distress. Ductility induces the ability to sustain large deformation, and a capacity to absorb energy by hysteritic behavior of a member.

There are various ways of quantifying ductility.

(a) Strain ductility: It is the ability of the constituent materials to sustain plastic strains without significant reduction of stress. Strain ductility is considered as fundamental source of ductility. It is denoted by the symbol μ_{ε}.

$$\mu\varepsilon = \frac{\varepsilon}{\varepsilon_y} \tag{1.4}$$

Where ε is total imposed strain and ε_y is the yield strain

Unconfined concrete exhibits very limited strain i.e., 0.002 in axial compression and 0.0035 in flexure. By effectively confining concrete, strain ductility can be significantly increased. In a member it is desired that inelastic strain should develop over a reasonable length, rather than being concentrated in a small area.

(b) Curvature ductility:

Curvature ductility is defined as the ratio of maximum curvature expected to be attained or relied on to yield curvature.

$$\mu_\theta = \frac{\varphi_m}{\varphi_y} \tag{1.5}$$

Where ϕ_m is maximum curvature expected to be attained or relied on and ϕ_y is yield curvature.

Curvature ductility of section depends on how yield curvature is defined. Pauley and Priestely [3] have defined yield curvature as following.

First yield curvature ϕ'_y is defined on basis of yielding of steel or concrete and it is related to yield curvature.

If steel is yielding, first yield curvature is defined as

$$\varphi'_y = \frac{\varepsilon_y}{(d - C_y)} \tag{1.6}$$

If steel is not yielding, first yield curvature is defined as

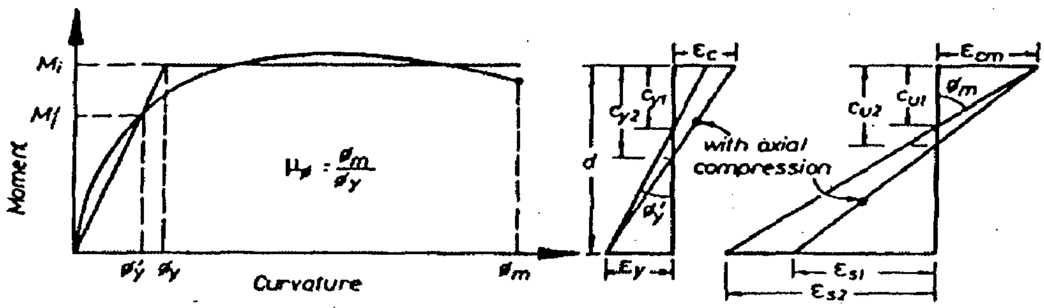
$$\varphi'_y = \frac{\varepsilon_y}{C_y} \tag{1.7}$$

Where ε_y is yield strain of steel given as $\varepsilon_y = \dfrac{f_y}{E_s}$, C_y is corresponding depth of neutral axis, ε_c is taken as 0.0015.

Yield curvature is given as

$$\varphi_y = \frac{M_i}{M'_i}\varphi'_y \tag{1.8}$$

Where M_i is ideal moment and M'_i is moment corresponding to first yield curvature.



(a) Moment curvature relationship (b) First-yield curvature (c) 'Ultimate' curvature

Fig. 1.4. Definition of curvature ductility [3]

Parameters affecting curvature ductility are ultimate compression strain, axial force, compressive strength of concrete and reinforcement yield strength.

(c) Rotational ductility: It is defined as ratio of maximum rotation to yield rotation.

$$\mu_\theta = \frac{\theta_u}{\theta_y} \tag{1.9}$$

Where θ_u is maximum rotation and θ_y is yield rotation.

(d) Displacement ductility: It is defined as total imposed displacement at any instant to that at onset of yield.

$$\mu_\Delta = \frac{\Delta}{\Delta_y} \tag{1.10}$$

Where Δ is the total imposed displacement at any instant and Δ_y is displacement onset of yield.

$$\Delta = \Delta_y + \Delta_p \qquad (1.11)$$

Δ_p is fully plastic component of total lateral deflection.

It is considered to be most convenient quantity to evaluate the ductility demand imposed on structure by an earthquake. Displacement ductility factor μ_Δ relates inter-storey deflection. For most of reinforced concrete and masonry structures, yield deflection of cantilever Δ_y is assumed to occur simultaneously with yield curvature ϕ_y at the base. Displacement ductility can be obtained by integrating rotational ductility.

1.2.2 Plastic Hinge

When a section reaches its plastic moment capacity, plastic hinge is formed. Plastic hinge allows large rotation to occur at constant plastic moment. In seismic analysis, plastic hinge dissipates energy; therefore it is preferable to call such mechanism as energy dissipating mechanism. Generally plastic hinge is considered to occur at a point, but in reality plastic hinge extends along short length of member and depends on loading condition, characteristics of steel and concrete, geometry of section and on shear which can increase the length of inelastic region substantially. This length over which plastic hinge is formed is called equivalent plastic hinge length (l_p). Several relations has been suggested by researchers to determine equivalent plastic hinge length, one suggested by Pauley & Priestley [3] is in following form of equation

$$l_p = 0.08 l_0 + 0.022 f_y d_b \qquad (1.12)$$

Where l_0 is length of member considered, d_b is diameter of tensile reinforcement bar and f_y is yield strength of steel reinforcement used.

ESTIMATION OF STRENGTH

Primary aim of Structural design is to ensure sufficient strength and serviceability so that it can fulfill its intended purpose during its expected lifetime. Safety of a structure implies that there is acceptably low chance of partial or total collapse under normal loads and probable overloads. Serviceability implies satisfactory performance of structure under normal loading. To achieve these two goals with minimum cost is a challenge to structural designer.

Over the years, various design philosophies have evolved in different parts of world. During these years, the understanding about material behaviour and structural behaviour of reinforced concrete has increased tremendously. This understanding forced to sideline some philosophies and has given way to others.

There are basically three philosophies for design of Reinforced concrete:

- Working Stress Method
- Ultimate Strength Method
- Limit State Method

Early studies of reinforced concrete member were based on ultimate strength theories (Thullie 1897, Kitter1893)[4]. But due to its mathematical complexities, and little knowledge about behaviour of reinforced concrete at ultimate load, working stress theory (Elastic) became generally accepted. After more than half century of development of R.C.C. again ultimate strength theory gained a wider acceptance. It is due to practical experience and laboratory tests, knowledge of the behaviour of structural concrete has vastly increased and deficiencies of elastic theory become evident. Recently a new philosophy called Limit State theory has been developed. This theory combines the best features of ultimate strength and working stress method.

2.1 WORKING STRESS METHOD

This has been the traditional method used for reinforced concrete design. According to this method structures are analysed by elastic – linear structural theory and stress resultants acting on members are computed, then members are proportioned such that actual stress caused in the member should be limited to an allowable stress.

Assumptions of working stress method is given as below: [5]

(1) A section which is plane before bending remains plane after bending (Euler – Bernoulli assumption)

(2) Bond between steel and concrete is perfect within the elastic limit of steel.

(3) The tensile strength of concrete is ignored

(4) Concrete is elastic, that is, the stresses in concrete varies from zero at the neutral axis to a maximum at the extreme fiber

(5) The modular ratio m has the value ($280/3\sigma_{cbc}$) where σ_{cbc} is the permissible compressive stress in bending in MPa.

Factor of Safety is given as

$$Factor\ of\ Safety = \frac{Yield\ Stress}{Allowable\ Stress} \tag{2.1}$$

Main drawbacks of the working stress design are as follows:

Concrete is not elastic. The inelastic behaviour of concrete starts right from very low stresses. So triangular stress diagram cannot describe the actual stress distribution in a concrete section.

(1) Since factor of safety is on the stresses under working loads, there is no way to account for different degree of uncertainty associated with different types of loads. With elastic theory it is impossible to determine the actual factor of safety with respect to loads.

(2) It is difficult to account for shrinkage and creep effects by using working stress method.

2.2 ULTIMATE STRENGTH METHOD

According to this philosophy, the section of the structural member is designed taking inelastic strains into account to reach ultimate strength, when an ultimate load, equal to sum of each service load multiplied by its respective load factor, is applied to structure. In this method, the nonlinear stress-stain curves of concrete and steel are used.

$$Load\ Factor = \frac{Ultimate\ Load}{Working\ Load} \qquad (2.2)$$

Assumption in ultimate strength method are given below:

(1) A section which is plane before bending remains plane after bending.

(2) At ultimate strength stress and strain are not proportional and the distribution of compressive stress diagram may be assumed as a rectangle, trapezoid, parabola or any other shape which gives ultimate strength in reasonable agreement with test.

(3) Maximum fiber strength in concrete does not exceed $0.68\sigma_{cu}$. As in Whitney's theory, the actual stress diagram can be replaced by a rectangular stress block whose height 'a' is taken $0.43d$ and average stress is assumed to be $0.55\sigma_{cu}$. Here σ_{cu} is ultimate compressive strength of concrete cubes at 28-days and d is effective depth.

(4) Tensile strength of concrete is ignored in section subjected to bending.

Main drawbacks of ultimate load design are given below:

(1) Since Load Factor is used on the working loads, there is no way to account for different degree of uncertainty associated with variation in material stresses.

(2) There is complete disregard for control against excessive deflections.

2.3 LIMIT STATE METHOD

Limit state method aims for a comprehensive and rational solution to the design problem, by considering safety at ultimate loads and serviceability at working loads. The Limit State concept of design of reinforced concrete structures takes into account the probabilistic and structural variation in the material properties, loads and safety factors.

There are two types of Limit States. [6]

(a) Ultimate Limit States: It deals with strength, overturning, sliding, buckling, fatigue fracture etc.

(b) Serviceability Limit State: It deals with discomfort to occupancy and (or) malfunction, caused by excessive deflection, crack width, vibration, and also loss of durability.

These two Limit State can be achieved by using multiple safety factor formats.

Limit State of collapse can be expressed by

$$\mu R > \sum_{i=1}^{n} \lambda_i L_i \tag{2.3}$$

Where R is resistance of a structural element arises due to variation in material properties, workmanship etc., μ is safety factor which is always less than unity reflects uncertainty associated with R, L_i different loads and λ_i is factor normally greater than unity evaluated on basis of randomness in the evaluation of respective load L_i. Summation sign denotes the combination of load effect from different load sources. μ and λ together called partial safety factors.

Limit State of Serviceability can be expressed by

$$\frac{\delta}{L} \leq \frac{1}{\alpha} \tag{2.4}$$

Where δ is deflection, L is length or height or span of the structural element and α is a non-dimensional number.

2.4 ANALYSIS OF REINFORCED CONCRETE SECTION

Analysis of R.C. section deals with determination of

- The Stresses in the materials under given load.
- The allowable or ultimate bending moments that the member can resist.

2.4.1 Analysis of Beam Section

As described above there are three philosophies, based on which stresses in material under given load and allowable or ultimate bending moment of member can be determined.

2.4.1.1 Analysis of Beam Section Based on Working Stress Method:

Reinforced Concrete section is changed to a transformed section using modular ratio. The neutral axis is located by line passing through the centroid of transformed section and perpendicular to plane of bending. The concrete on the tension side of neutral axis is neglected. Linear variation of stress is assumed. Compressive and tensile force is determined on basis of allowable stress in corresponding materials as given in table 2.1 and table 2.2 and further moment of resistance of section can be calculated.

Table 2.1 Permissible stress in steel

Steel Grade	Permissible Stress (Tension)	Permissible Stress (Comp.)
Fe 250	140 MPa	130 MPa
Fe 415	230 MPa	180 MPa
Fe 500	275 MPa	190 MPa

Table 2.2 Permissible compressive stress of concrete in bending

Concrete Grade	σ_{cbc} (MPa)
M 15	5
M 20	7
M 25	8.5
M 30	10
M 35	11.5
M 40	13

2.4.1.2 Analysis of Beam Section Based on Limit State Method:

This method assumes plane section normal to the beam axis remain plane after bending. In this method maximum compressive stain in concrete at outermost fiber shall be taken as 0.0035 and a partial safety factor of 1.5 is conceded. Tensile stain in steel is taken as

$$\varepsilon_{st} = \left(\frac{0.87 f_y}{E_s}\right) + 0.002. \qquad (2.5)$$

Where ε_{st} is tensile strain in steel, f_y is yield stress, E_s is Modulus of Elasticity

It is based on 0.2% of Proof Strain and a partial safety factor of 1.15 is considered. Determination of Limiting depth of neutral axis is based on grade of steel

$$\frac{x_u}{d} = \frac{0.0035}{0.0035 + \varepsilon_{st}} \qquad (2.6)$$

Where X_u is depth of neutral axis, d is effective depth of section and ε_{st} is strain in steel at yield.

Compressive force in concrete is calculated based on concrete stress block parameter. Concrete stress block assumes a parabolic variation of stress from zero strain upto strain equal to 0.002, and then it is constant upto 0.0035 strain. Maximum stress at extreme fiber from neutral axis is equal to 0.447 of characteristic strength of concrete, obtained by using a partial safety factor 1.5 applied to characteristic strength of concrete i.e., $\left(\frac{0.67 f_{ck}}{1.5} = 0.447 f_{ck}\right)$. And yield stress of steel it is obtained using partial safety factor of 1.15 on Yield strength of steel i.e., $\left(\frac{f_y}{1.15} = 0.87 f_y\right)$. Based on above stress compressive and tensile forces are calculated, and corresponding depth of neutral axis is determined. Moment of resistance can be obtained by multiplying these forces with lever arm between them

Design stress strain curve for concrete:

Relation between design stresses and corresponding strain is given as

$$f_c = \begin{cases} 0.447 f_{ck}\left[2\left(\frac{\varepsilon}{0.002}\right) - \left(\frac{\varepsilon}{0.002}\right)^2\right] & for\ \varepsilon < 0.002 \\ 0.447 f_{ck} & for\ 0.002 \le \varepsilon \le 0.0035 \end{cases} \qquad (2.7)$$

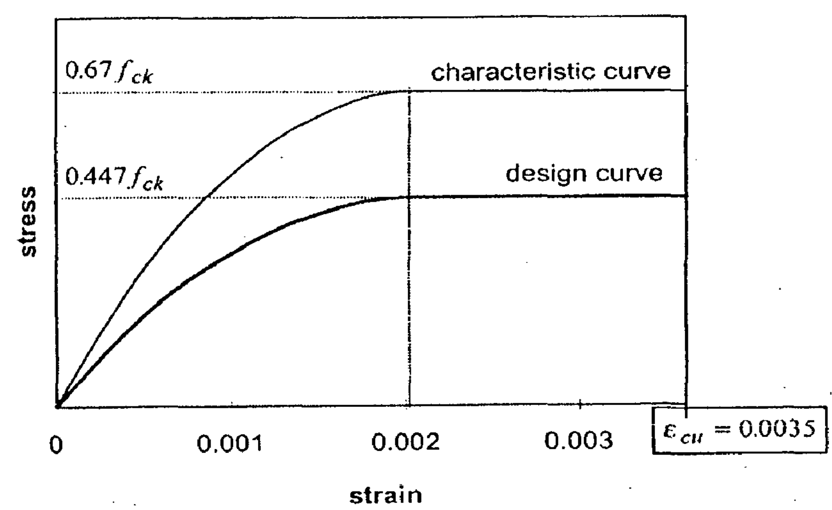


Fig. 2.1 Design and characteristic stress strain curve for concrete

Design stress-strain curve for reinforcing steel:

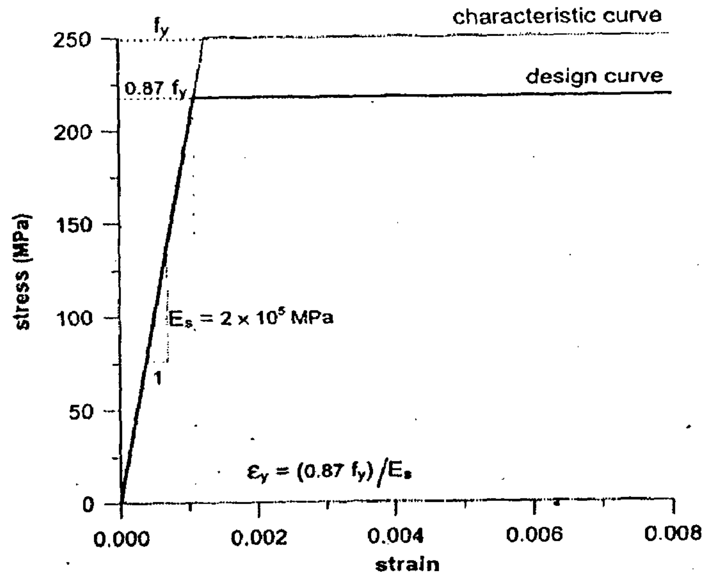


Fig.2.2 Design and characteristic curve for Fe250 grade steel

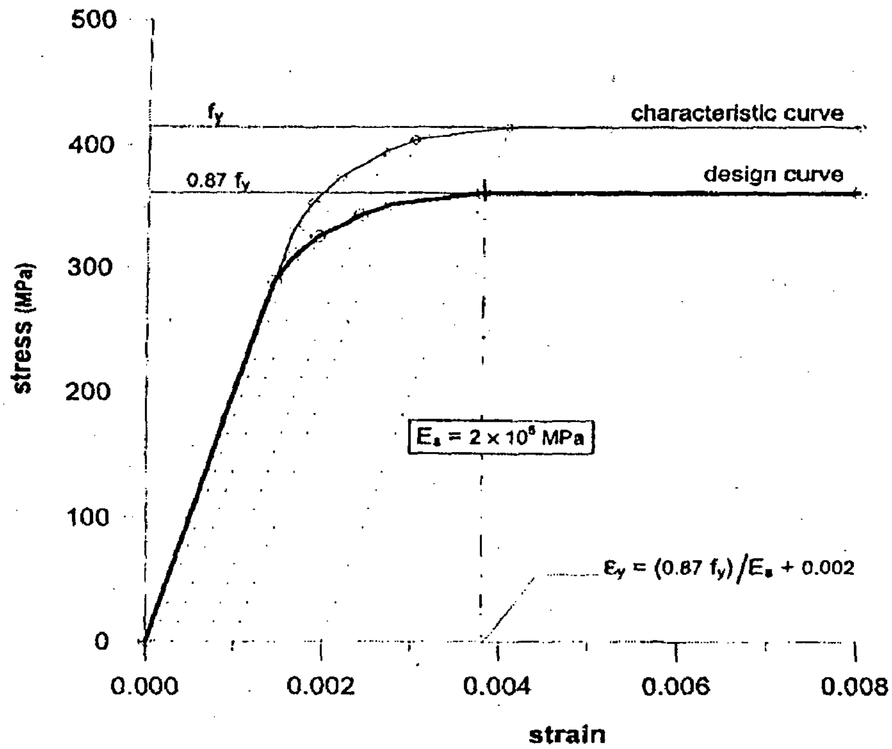


Fig.2.3 Design and characteristic curve for Fe415 grade steel

Characteristic curve for cold worked bars can be obtained by following table.

Table 2.3 Salient points of characteristic stress-strain curve for cold work bars

Sl. No	Elastic Strain	Inelastic Strain	Total Strain	Total Stress
1	0.000	0.000	0.0000	0.0000
2	f_y/E_s	0.0000	f_y/E_s	$0.8f_y$
3	f_y/E_s	0.0001	f_y/E_s+0.0001	$0.85f_y$
4	f_y/E_s	0.0003	f_y/E_s+0.0003	$0.9f_y$
5	f_y/E_s	0.0007	f_y/E_s+0.0007	$0.95f_y$
6	f_y/E_s	0.0010	f_y/E_s+0.0010	$0.975f_y$
7	f_y/E_s	0.0020	f_y/E_s+0.0020	$1.0f_y$

To obtain design curve same table can be use with replacing f_y from $0.87f_y$.

2.4.1.3 Analysis of Beam Section Based on Ultimate State Method:

In this method no safety factor is applied on material properties. Maximum compressive strain in unconfined concrete is 0.0035 and corresponding stress is $0.67f_{ck}$. Maximum strain in steel is $\varepsilon_{st} = \left(\frac{f_y}{E_s}\right) + 0.002$. Euler-Bernoulli assumption is valid also tensile strength of concrete is ignored. Therefore characteristic stress-strain curve of concrete can be used as in table 4.

Relation between stress and strain of concrete in this case given as

$$f_c = \begin{cases} 0.67 f_{ck}\left[2\left(\frac{\varepsilon}{0.002}\right) - \left(\frac{\varepsilon}{0.002}\right)^2\right] & for\ \varepsilon < 0.002 \\ 0.67 f_{ck} & for\ 0.002 \le \varepsilon \le 0.0035 \end{cases} \qquad (2.8)$$

Factor of 0.67 is due to end zone effect, h/d ratio of specimen, size effect etc. The characteristic stress-strain curve for steel can be obtained by using table-2.3. This curve can be extended upto ultimate strain (generally ranges from 14.5% to 25% for reinforcing beams in R.C.C.)

Depth of balanced neutral axis can be calculated as

$$\frac{x_u}{d} = \frac{0.0035}{0.0035 + \frac{f_y}{E_s}} \qquad (2.9)$$

Where X_u is depth of balance neutral axis, d is effective depth of section and E_s is modulus of elasticity of steel.

Based on above stress-strain values forces in section and thereby corresponding moments can be determined.

2.4.2 Analysis of Column Section

2.4.2.1 Analysis of column section based on working stress method

This theory involves the concept of permissible stresses, modular ratio and transformed section. Codal recommendations [7] for allowable stresses in concrete and steel are as given in the tables below.

Table 2.4 Allowable stress in concrete under compression.

Sl.No.	Grade of Concrete	Allowable Stress (σ_{cc})
1	M 15	4 MPa
2	M 20	5 MPa
3	M 25	6 MPa
4	M 30	8 MPa
5	M 35	9 MPa

Table 2.5 Allowable stresses in steel

Sl. No.	Grade of Steel	Allowable Stress(σ_{sc})
1	Fe 250	130
2	Fe 415	190
3	Fe 500	190

Design equation based on W.S.M is given as

$$P_o = \sigma_{cc} A_g + (\sigma_{sc} - \sigma_{cc}) A_{sc} \tag{2.10}$$

Where P_o is load carrying capacity of section, σ_{cc} is permissible stress in concrete, σ_{sc} is permissible stress in steel, A_g is gross area of cross-section, A_{sc} is area of steel.

2.4.2.2 Analysis of Column Section Based on Limit State Method

This method is based on assumption that plane section normal to axis of the member remains plane after bending. Tensile strength of concrete is ignored. The strain in tension reinforcement is to be not less than $\left(\frac{0.87 f_y}{E_s}\right) + 0.002$, corresponding stresses can be determined from table 2.3 based of $f_y = 0.87 f_y$.

For purely axial compression, he strain is assumed to be uniformly equal to 0.002 across the section. The maximum strain in concrete at the outermost compression fiber is 0.0035 is applicable when the neutral axis lies within the section and in the limiting case when the neutral axis lies along one edge of the section. When the neutral axis lies outside of the section, strain at the highly compressed edge is given by

$$\varepsilon_{c(Hc)} = 0.0035 - (0.75 \varepsilon_{c(Lc)} \varepsilon_{c(hc)}) \qquad (2.11)$$

Where $\varepsilon_{c(Hc)}$ is strain is highly compressed edge and $\varepsilon_{c(Lc)}$ is strain in least compressed edge.

Stress – Strain curve for concrete and steel in same as described in section (Analysis of R.C. beam section based on W.S.M)

For short column subjected to Axial load with minimum eccentricity, design strength can be obtained by using simplified formula as below [8]

$$\overline{P}_{uo} = 0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) A_{sc} \qquad (2.12)$$

Where $\overline{P}_{uo}$ denotes the design strength in uniaxial compression permitted by the code, f_{ck} is characteristic strength, A_g is gross area of cross section, f_y is yield strength of steel and A_{sc} is total area of longitudinal reinforcement.

Design strength: Axial Load – Moment Interaction

When a column is subjected to load along with a moment, or load is placed at some eccentricity, design strength has two components

- Axial Compression component (P_{uR})
- Corresponding uniaxial moment component ($M_{uR} = P_{uR}$ x e)

These two components can be obtained by following equations.

$$P_{uR} = C_c + C_s \qquad (2.13)$$

$$M_{uR} = M_c + M_s \qquad (2.14)$$

Where C_c and C_s resultant forces in concrete and steel and M_c and M_s are corresponding moments.

Interactive Curve: It is a complete graphical representation of the design strength of a uniaxially eccentrically loaded column. The interaction curve defines the different (M_{uR}, P_{uR}) combination for all possible eccentricity.

Salient Points on Interaction Curve (shown in figure 2.4):

- Point 1 – Pure axial loading (e = 0), $M_{uR} = 0$ and $P_{uR} = P_{uo}$
- Point 1' – Axial loading with minimum eccentricity ($e = e_{min}$), $P_{uR} = \overline{P}_{uo}$
- Point 2 – Neutral axis is outside section ($e<e_D$) where e_D is eccentricity of load at which depth of neutral axis is equal to depth of section.
- Point 3 – Corresponds to X_u (depth of neutral axis) equal to D, ($e=e_D$)
- Point 4 – Balance failure ($e = e_b$) and ($X_u = X_{ub}$) corresponding design strength in P_{ub} and M_{ub}.
- Point 5 – Pure bending condition ($e = \propto$, $P_{uR} = 0$), resulting ultimate moment M_{u0}.

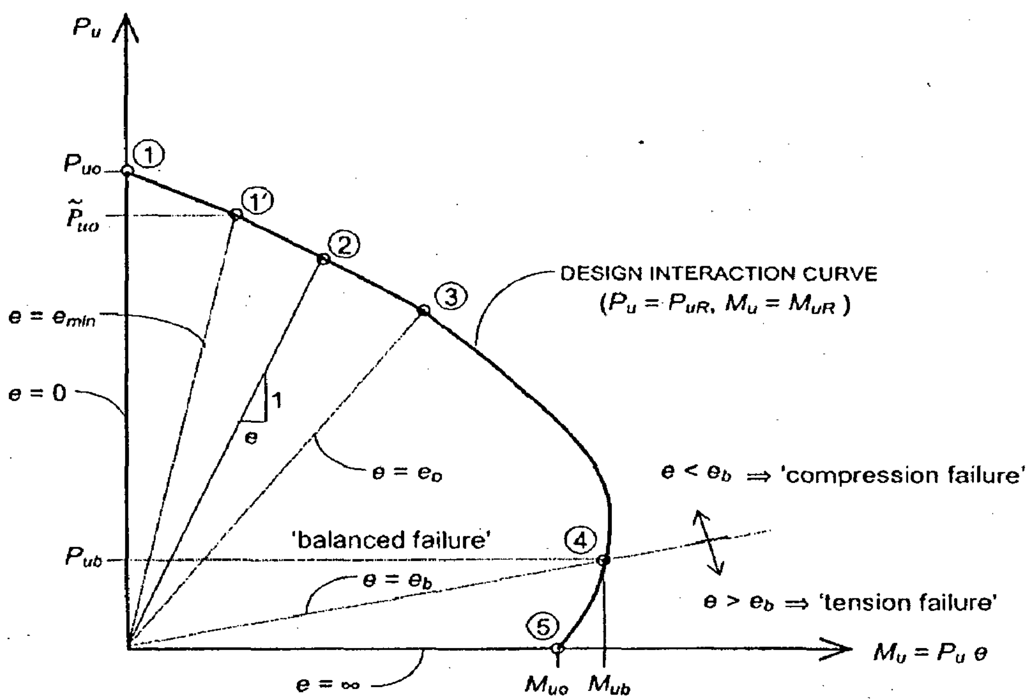


Fig. 2.4 Typical P_u - M_u interaction diagram [6]

If load has eccentricity along both the axis, interaction surface rather than interaction curve is generated. Interaction surface is a three dimensional lot of $P_{uR} - M_{ux} - M_{uy}$. It envelop a large number of design interaction curves for different axes of bending. However interaction curve for biaxially loaded column is out of scope of present thesis.

2.4.2.3 Analysis of Column Section Based on Ultimate Load Method

In this method all analysis procedure remains same as in Limit State Method except the partial factor on material is removed. Maximum stress in concrete is taken as $0.67f_{ck}$ and in steel as f_y. Other assumption is same as given in section 2.2.

2.5 NUMERICAL STUDIES

2.5.1 Overstrength Factor

Capacity design method for RC frames is considered to be very useful for earthquake resistant design. It involves that the dissipation of seismic energy should take place mainly in element which posses' adequate ductility and which does not makes structure unstable. For RC buildings this principal leads to the requirement that beam failure be precede column failure. This means the strength of column should be higher than that of beam. Since the design of members is based on characteristic strength of material but actual strength comes to be much larger than the design strength. This puts an emphasis to determine over strength of members, which can be effectively incorporated in capacity design.

Here study is made to determine over strength of section based on material strength variation. Stress-strain curve for concrete and reinforcing steel are used based on limit state value, 95% confidence value and 50% confidence value. Mean and standard deviation of concrete has been taken from IS456:2000 [8], and for steel it has been taken from research work of S.A .Mirza and J.G.MacGregor [9]. Limit state value of stress-strain of concrete has been based on equation 2.6, and for steel it has been taken from table 1 using $0.87f_y$. 95% confidence value of concrete stress-strain is based on equation 2.7, and for steel it is from table 1 using f_y. 50% confidence value of concrete stress-strain is based on equation 2.7 using f_c in place of f_{ck}. Where f_c is given as

$$f_c = f_{ck} + 1.65\sigma_c \tag{2.15}$$

Where, f_{ck} is characteristics strength of concrete and σ_c is standard deviation of concrete.

Stress –strain curve of steel has been obtained from table 2.1 using mean strength proposed by S.A .Mirza and J.G.MacGregor [9].

2.5.1.1 Overstrength of Beam Section

Moment-curvature relation for different stress-strain curve for concrete and steel based on limit state value, 95% confidence value and 50% confidence value has been determined. Concrete is assumed to be confined by transverse hoops, according to model proposed by Kent and Park [10] (this model is explained in section 3.2.1 as model-1). Figure 2.5 shows moment curvature relation.

Data of the section analysed is given below.

Beam section: 300mm×500mm and 40mm of clear cover

Top reinforcement: 4 bars of 20mm diameter placed at equal spacing

Bottom reinforcement: 4 bars of 25mm diameter at equal spacing

Stirrup details: 10mm diameter stirrup at 100mm spacing

Grade of materials: M20 concrete and Fe415 steel

From this study increase in moment capacity has been calculated. Maximum moment from Limit State is calculated to be 294.94 kNm, for 95% confidence value it calculated as 341.86 kNm (an increase of 15.9% from Limit State value) and for 50% confidence value it is calculated as 416.46 kNm (an increase of 41.2% from Limit State value).

From study change in curvature has also been observed. Maximum curvature for Limit State has been calculated as 3.01×10^{-4} rad./mm. For 95% confidence value it has been calculated as 2.37×10^{-4} rad./mm (a decrease of 21.26% from Limit State), this decrease may be due to use of different partial safety factor for concrete and steel. For 50% confidence value it has been calculated as 3.13×10^{-4} rad./mm (a increase of 4% from Limit State).

This study shows that there is significant reserve strength or over strength is available in section which requires a precise estimation for capacity design of frames.

2.5.1.2 Overstrength of Column Section

Interaction curve for different stress-strain curve for concrete and steel based on limit state value, 95% confidence value and 50% confidence value has been determined. Figure 2.6 showing interaction curve.

Data of the section analysed is given below.

Column size: 400mm×400mm with an effective cover of 40mm

Reinforcement: 8 numbers of 20mm diameter bars placed uniformly all around perimeter of column section

Grade of materials: M20 concrete and Fe415 steel

From this study increase in moment capacity is calculated. Maximum moment for Limit State value has been calculated to 172.52 kNm, for 95% confidence value has been calculated to 219.4 kNm (an increase of 27% from Limit State value) and for 50% confidence value it has been calculated to 336.97 kNm (an increase of 95.3% from Limit State value).

Also increase in axial capacity is calculated. Maximum axial load for Limit State value has been calculated to 2222.21 kN, for 95% confidence value it has been calculated to 2999.98 kN (an increase of 35% from Limit State value), and for 50% confidence value it has been calculated be 5138.02 kNm (an increase of 131% from Limit State value).

This shows there is significant reserve strength or over strength (both, moment carrying capacity and axial load carrying capacity) is available to section. A precise determination is required to apply it in capacity design method.

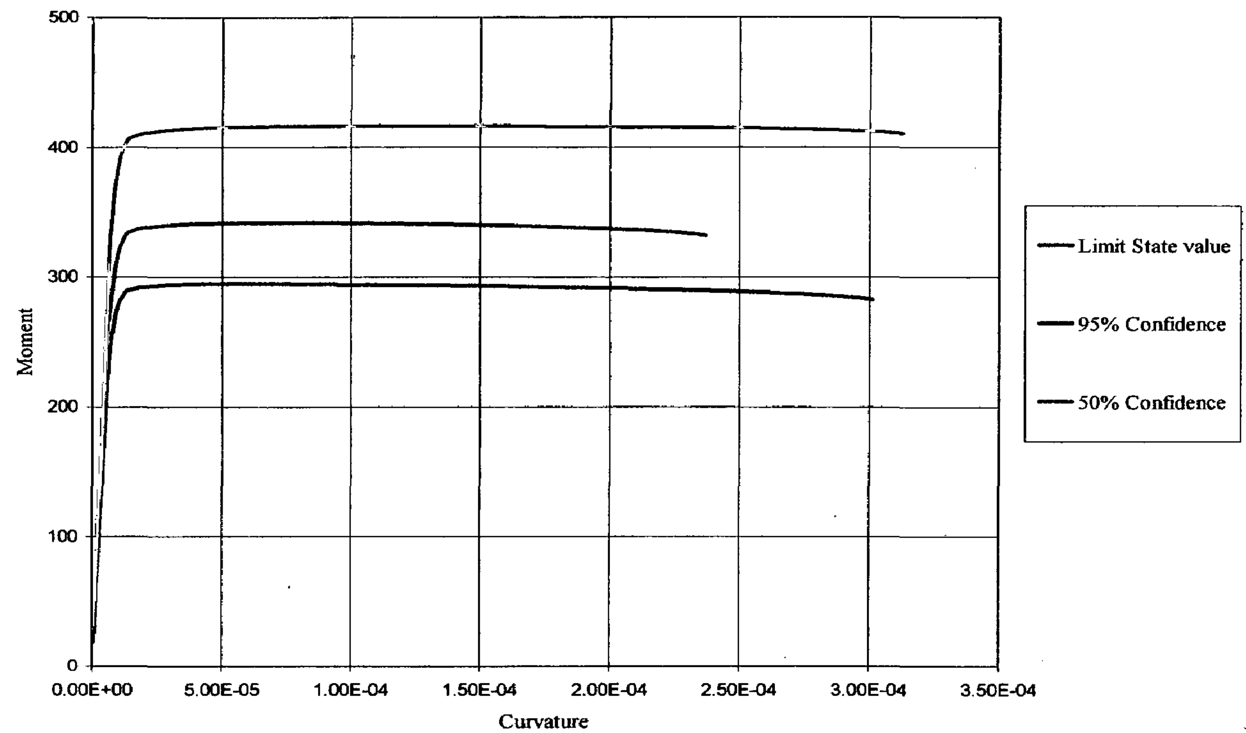


Fig.2.5 Moment-Curvature diagram for limit state value, 95% and 50% confidence value

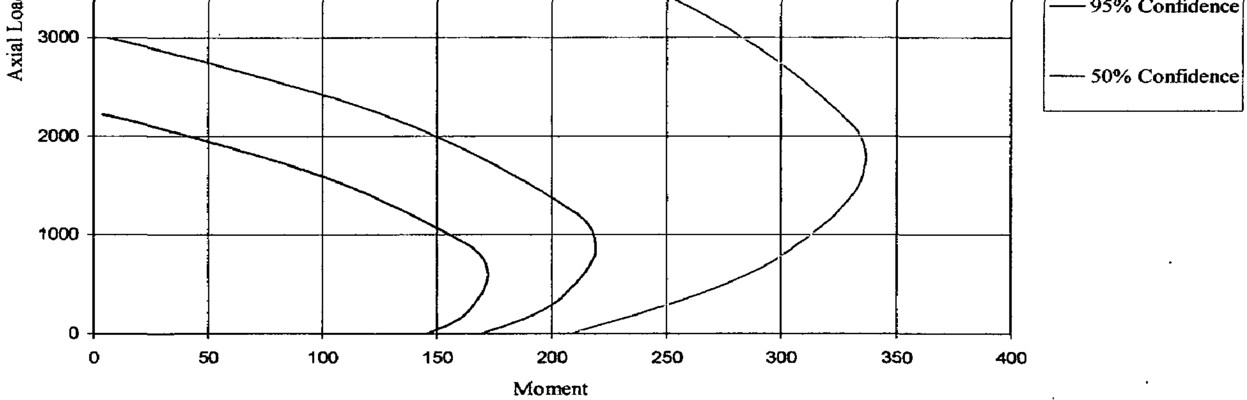


Fig2.6 Interaction Curve for Limit State, 95% and 50% confidence value

ESTIMATION OF DUCTILITY

Ductility in a building imparts it the capacity to deform in-elastically without significant decrease in strength. It is very important parameter for earthquake resistant design, since buildings are designed for only a fraction of load expected to be encountered during earthquakes. A discussion on ductility and various ways to quantify it has undertaken in *Chapter-1*.

To achieve ductility in a building, it is required that its component members should be sufficiently ductile. Ductility of members can be achieved only with ductile constituent materials. Generally in the case of reinforced concrete members, reinforcing steel is ductile material whereas concrete is brittle in nature. To obtain a ductile behaviour, it is desired primarily that failure of member should occur due to yielding in steel rather than in concrete. Since failure of member occurs finally due to crushing of concrete, it is necessary to increase its ductility. Concrete can be made ductile by confining it.

3.1 CONCRETE CONFINEMENT BY REINFORCEMENT

Concrete, which is restrained in the direction at right angles to the applied stress, will be referred to as confined concrete. Active confinement is, when the transverse stress is induced from some externally applied action. In practice concrete may be confined by transverse reinforcement in the form of closely spaced steel spirals or hoops. Such confinement is called passive.

Transverse reinforcement is generally provided in form of closely spaced steel spirals or hoops. Confining property of transverse reinforcement comes into picture after the concrete reaches a certain level of stress. Generally at stress level equal to uniaxial strength of concrete, transverse reinforcement starts applying confining pressure. This is attributed to high transverse strain due to progressive internal cracking which makes concrete to push the transverse reinforcement which then applies confining reaction to concrete.

Many researchers [4, 10, 12, 13, 14] have found that at high strains, stress-strain characteristic of concrete is greatly improved due to confinement by transverse reinforcement. At high strain, shape of stress-strain curve of concrete is a function of many variables such as, [4]

(1) Ratio of volume of transverse steel to volume of concrete core → If transverse steel is more, more will be transverse confining pressure.

(2) Yield strength of transverse steel → it gives upper limit of confining pressure.

(3) Spacing of transverse steel → smaller spacing leads to more effective confining pressure.

(4) The ratio of diameter of the transverse bar to unsupported length of transverse bars (for rectangular stirrups or hoops) → small diameter hoops or stirrups will act merely as ties between the corners because the flexural stiffness of hoop bar is small and it bends outward rather than effectively confining the concrete in regions between the corners. Whereas large diameter transverse bars (compared to unsupported length) will have greater flexural stiffness and it will have lesser tendency to bend outward thus area of concrete will be more effectively confined. In case of spirals this variable has no significance. This is because spirals will be in axial tension and will apply a uniform radial pressure to the concrete.

(3) Amount and size of longitudinal reinforcement → Usually longitudinal reinforcement are of large diameter and ratio of diameter to unsupported length is such that it can effectively confine concrete. However longitudinal steel should be tightly placed and tied with transverse reinforcement such that there should not be any allowance for its moment.

(4) Strength of concrete → Low strength concrete is generally more ductile than high strength concrete.

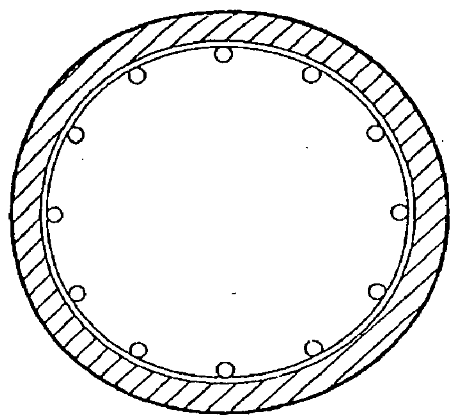
(5) Type of transverse steel i.e., rectangular or spiral → it has been found that circular spirals confine concrete more effectively than rectangular or square hoops.

3.1.1 Types of Transverse Reinforcement and their Effect on Ductility

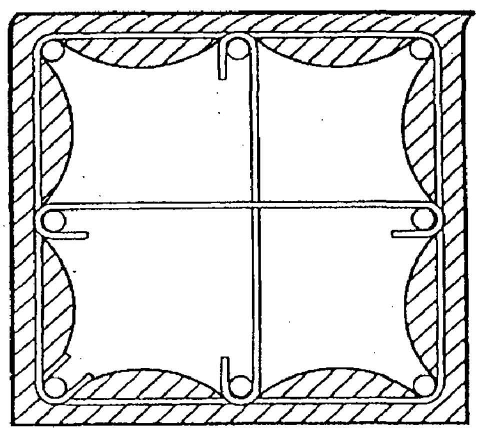
Generally two types of transverse reinforcement are used.

- Circular Spirals
- Rectangular or Square hoops.

Generally use of a particular type of transverse reinforcement is governed by shape of members. Types of transverse reinforcement is shown in Fig. 3.1



(a) Circular hoops or spiral



(b) Rectangular hoops with cross ties

Fig. 3.1 Types of transverse reinforcement.[11]

Various researches have been done to determine confinement property (such as increase in strength and ductility) of different types of transverse reinforcement. It has been found that there is a significant increase in strength and ductility of members due to transverse steel. Stress-strain curves for concrete with different types of confinement are shown in Fig. 3.2.

Tests performed by many researchers have demonstrated that circular spirals confine concrete much more effectively than rectangular and square hoops as shown in Fig. 3.2. The reason for above is due to the fact that circular spirals are in axial hoop tension and provide a continuous confining pressure around the circumference. Where as square hoops can only apply confining reactions near the corners of hoops since the pressure of concrete against the sides of he hoops tends to bend it outward as shown in Fig. 3.3.

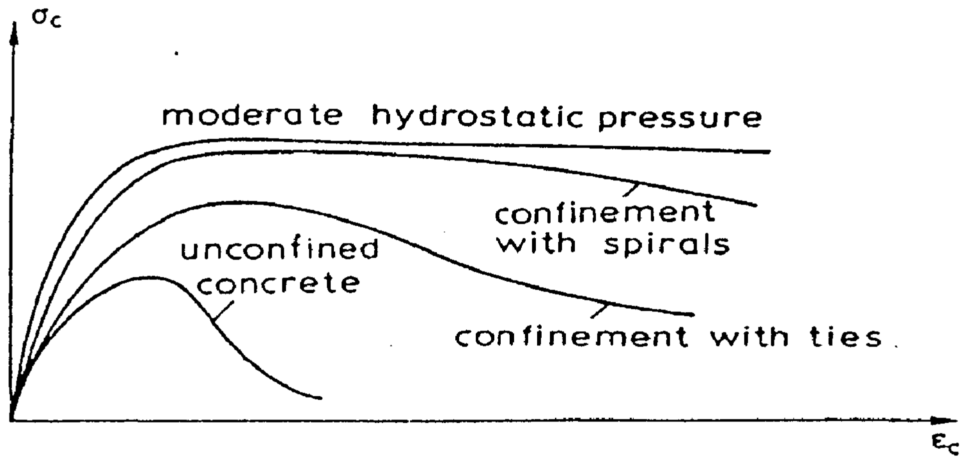


Fig. 3.2 Stress strain diagrams for concrete subjected to various types of confinement.[12]

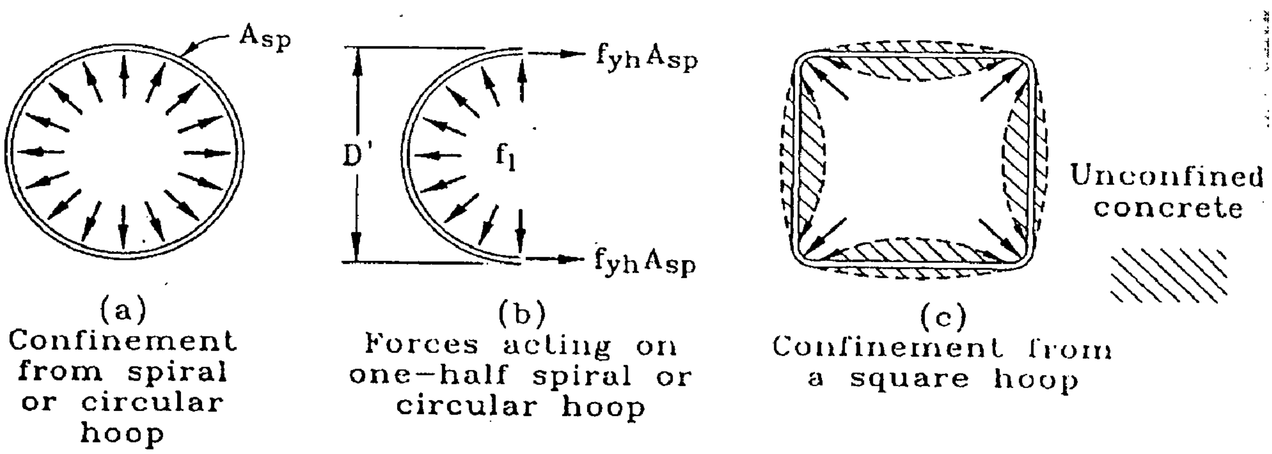


Fig. 3.3 Confinement of concrete by circular and square hoops.[11]

3.1.2 Concrete Confined by Circular Spirals

Assuming that the spirals are sufficiently close to apply a near uniform pressure, the confining pressure applied is given as

$$f_l = \frac{2f_y A_{sp}}{d_s S} \tag{3.1}$$

Where d_s is diameter of spiral, A_{sp} is area of spiral bars, S is pitch of the spiral, f_l is lateral pressure in concrete, f_y is yield strength of spiral reinforcement.

Richart, Brandtzaeg and Brown [10] have found relationship for strength of concrete cylinders loaded axially to failure while subjected to confining fluid pressure as

$$f'_{cc} = f'_{c} + 4.1 f_{l} \tag{3.2}$$

Where f'_{cc} is the axial compressive strength of confined specimen, f'_c is the uniaxial compressive strength of unconfined specimen and f_l is the lateral confining pressure.

For spirals the axial compressive strength of confined specimen is given as

$$f'_{cc} = f'_{c} + 8.2 \frac{f_y A_{sp}}{d_s S} \tag{3.3}$$

3.1.3 Concrete Confined by Rectangular Hoops

Commonly in building frames, the members are rectangular. To confine these rectangular sections rectangular hoops are required. Along with many factors as described in section 3.1, configuration of transverse steel also effects the stress-strain characteristics in this case [13]. The effect of configuration is shown in Fig. 3.4.

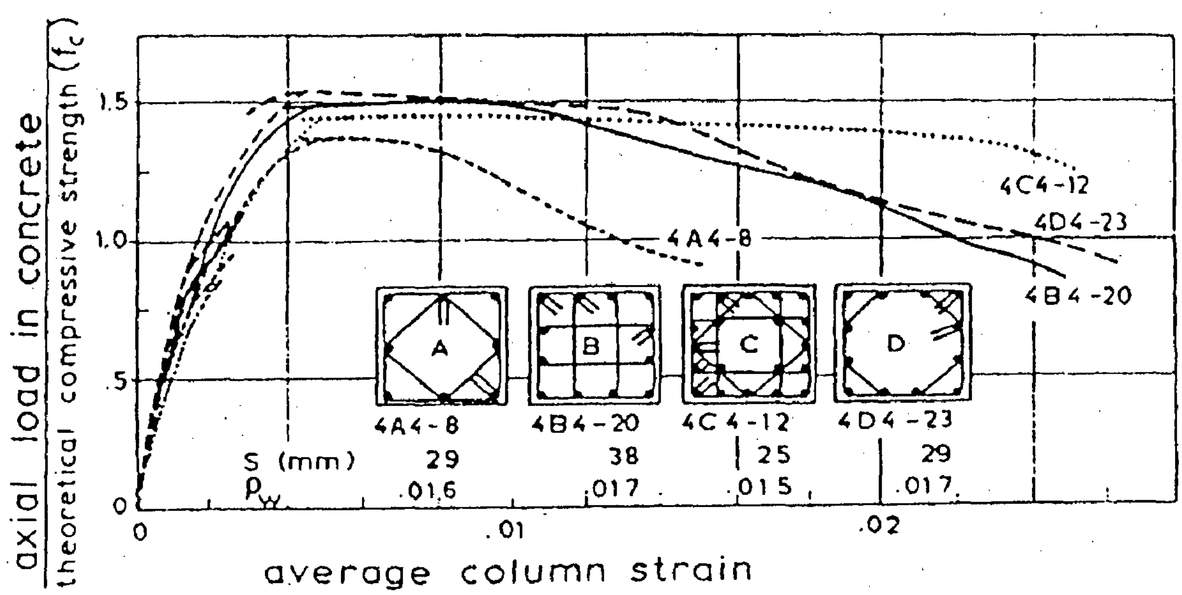


Fig. 3.4 Effect of steel configuration on confinement. [13]

Section 4b4-20 shows a higher stress value even at large strain. It is also reported [13] that the gain in strength of concrete is not directly proportional to amount of lateral reinforcement.

3.2 STRESS-STRAIN MODELS FOR CONFINED CONCRETE

Various researchers have proposed stress-strain relationship for concrete confined by rectangular hoops. Two models of confined concrete are discussed here.

3.2.1 Model –1: Kent and Park Model [10]

On basis of existing experimental evidences Kent and Park proposed stress-stain curve for concrete. This proposed stress-strain relationship is shown in Fig. 3.5.

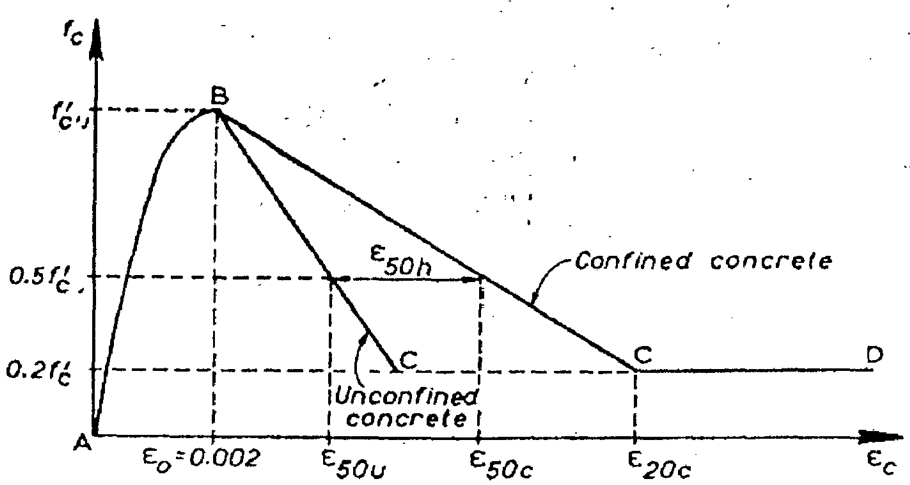


Fig 3.5 Stress strain relationship for unconfined and confined concrete. [10]

Characteristic curves are suggested as

For stain $\varepsilon_c \leq 0.002$

$$f_c = f'_c\left[\left(\frac{2\varepsilon_c}{0.002}\right)-\left(\frac{\varepsilon_c}{0.002}\right)^2\right] \tag{3.5}$$

Where ε_c is compressive strain in concrete, f'_c is uniaxial unconfined compressive stress in concrete, f_c is compressive stress in confined concrete.

Confining steel has no effect on shape of the ascending part of the curve which is represented by second order parabola, maximum stress is reached equal to uniaxial strength of unconfined specimen, at strain 0.002.

For strain $0.002 \leq \varepsilon_c \leq \varepsilon_{20c}$

$$f_c = f'_c \left[1 - Z(\varepsilon_c - 0.002)\right] \tag{3.6}$$

Where f_c is stress in confined concrete, ε_c is strain in concrete and f'_c is uniaxial strength of unconfined concrete, and

$$Z = \frac{0.5}{\varepsilon_{50u} + \varepsilon_{50h} - 0.002} \tag{3.7}$$

The parameter Z defines slope of linear falling branch.

$$\varepsilon_{50u} = \frac{3 + 0.002 f'_c}{f'_c - 1000} \tag{3.8}$$

$$\varepsilon_{50h} = \frac{3}{4}\rho_s \sqrt{\frac{b''}{S_h}} \tag{3.9}$$

Where, ρ_s is the ratio of volume of transverse reinforcement to volume of concrete core, measured to outside of hoops, b'' is the width of confined core measured to outside of hoops, S_h is spacing of hoops, f'_c is uniaxial strength of unconfined concrete

For strain $\varepsilon_c \geq \varepsilon_{20c}$

$$f_c = 0.2 f'_c \tag{3.10}$$

Where f_c is stress in confined concrete and f'_c is uniaxial strength of unconfined concrete.

3.2.2 Model-2: Mander, Priestley and Park Model [14]

This model is applicable to different types of cross-section with different levels of confinement. This model also incorporates effect of number of hoop legs. Proposed stress strain model is shown in Fig. 3.6.

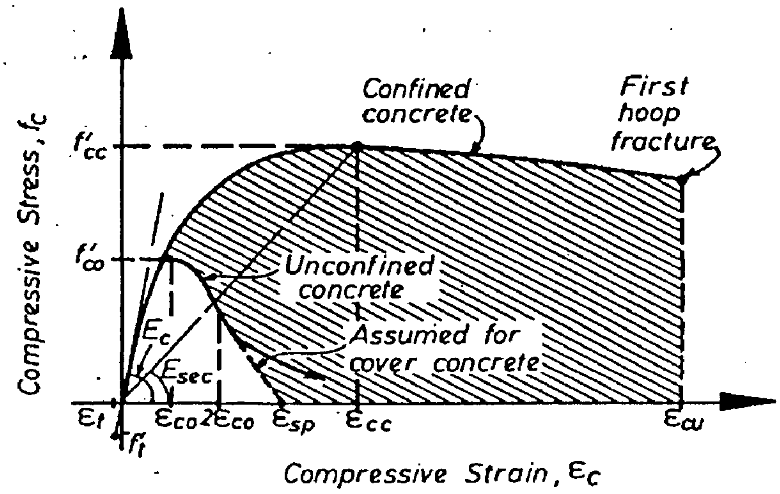


Fig. 3.6 Stress strain proposed for confined and unconfined concrete. [14]

It is defined by following equations:

$$f_c = \frac{f'_{cc}\, xr}{r - 1 + x^r} \tag{3.11}$$

Where f_c is compressive stress of confined concrete, f'_{cc} is concrete stress at peak, r is given as

$$r = \frac{E_c}{E_c - E_{scc}} \tag{3.12}$$

Where $E_c = 5000\sqrt{f'_{co}}$ and $E_{sec} = \dfrac{f'_{cc}}{\varepsilon_{cc}}$, f'_{co} is unconfined concrete strength.

$$x = \frac{\varepsilon_c}{\varepsilon_{cc}} \tag{3.13}$$

Where ε_c is longitudinal compressive concrete strain, ε_{cc} is given as

$$\varepsilon_{cc} = \left[1 + 5\left(\frac{f'_{cc}}{f'_c} - 1\right)\right]\varepsilon_{co} \tag{3.14}$$

Where ε_{co} is strain corresponding to unconfined concrete strength (0.002).

$$f'_{cc} = f'_{co}\left(2.254\sqrt{1 + \frac{7.94f'_l}{f'_c}} - 2\frac{f'_l}{f'_{co}} - 1.254\right) \tag{3.15}$$

Where f'_l is effective lateral confining stress, it can be computed in both x and y directions as

$$f'_{lx} = K_e \rho_x f_{yh} \tag{3.16}$$

$$f'_{ly} = K_e \rho_y f_{yh} \tag{3.17}$$

Where K_e is confinement effectiveness coefficient given as ratio of area of effectively confined concrete to area of the concrete within the centerline of particular spiral or hoop: ρ_x is ratio of total area of transverse bars running in x direction to product of pitch and depth of confined concrete, ρ_y is ratio of total area of transverse bars running in y direction to product of pitch and width of confined concrete and A_{sh} is yield strength of transverse reinforcement.

In this study Model-1 for stress-strain curve of confined concrete has been used.

3.3 ESTIMATION OF DUCTILITY

Curvature, defined as rotation per unit length, is associated with bending moment of section. Curvature, *R* is given as

$$\frac{1}{R} = \frac{d\theta}{dx} \tag{3.18}$$

Where, $d\theta$ is rotation over small length dx

For a elastic beam moment, *M* is related to curvature, *R* as

$$\frac{1}{R} = \frac{M}{EI} \quad (3.19)$$

Where, E is modulus of elasticity and I is moment of inertia of section

Curvature at a RC section can be obtained as

$$\varphi = \frac{\varepsilon_C + \varepsilon_S}{d} \quad (3.20)$$

Where, ε_C is strain in concrete, ε_S is strain in steel and d is effective depth of section.

Expression for curvature ductility is given by equation 1.5.

Relationship curvature between and rotational ductility is given by following expression [12]

$$\mu_\varphi = 1 + \frac{l}{\lambda * l_p}(\mu_\theta - 1) \quad (3.21)$$

Where, l is length of member, l_p is plastic hinge length (defined in 1.2.2), and

$$\lambda * = \lambda(\frac{EI_{ef}}{EI_r}) \quad (3.26)$$

Where, EI_{ef} and EI_r are effective stiffness and cracked section stiffness respectively, and λ is determined on the basis of ratio, β of end moments.

$$\lambda = \frac{6\beta}{(2\beta - 1)} \quad (3.27)$$

3.4 NUMERICAL STUDIES

3.4.1 Effect of Confinement on Ductility

Confinement by transverse reinforcement can effectively increase strain capacity of concrete. The effect of confinement on moment curvature relation and increase in ductility capacity has been studied using the following numerical examples.

3.4.1.1 Effect of Confinement by Transverse Hoops and Steel Ratio on Ductility of Beam Section

In this section, effect of confinement on moment-curvature relation of beam has been studied. In this regard effect of variation in spacing of transverse hoops, diameter of transverse hoops and grade of concrete on ductility of section has been studied. Effect of quantity of longitudinal reinforcement at tension and compression face of beam has also been studied.

Details of section analysed:

Beam section: 300mmX500mm with effective cover of 40mm

Grade of steel: Fe415

Grade of concrete: M20 and M40 denoted as $f_{ck} = 20$ and $fck = 40$ respectively.

Diameter of hoops: 8mm and 10mm denoted as $sd = 8$ and $sd = 10$ respectively.

Spacing of hoops: ∞, 300mm and 100mm denoted as *uncon, con-300* and *con-100 respectively.*

Steel Ratio, *Rr*: 0, 0.5, 1 and 2 It is defined as

$$Rr = \frac{\rho - \rho'}{\rho_{Bal}} \tag{3.29}$$

Where ρ is bottom steel, ρ' is top steel and ρ_{Bal} is balance steel for section.

Details of analysis are given below in Table 3.1 to 3.4, and Fig. 3.7 to 3.10 show the moment curvature relations.

Table 3.1 Effect of confinement on ductility for Fe415, *fck* 20 and 8mm dia. hoops

Spacing	∞	300		100	
Rr	μ_ϕ	μ_ϕ	% increase	μ_ϕ	%increase
0	6.78	37	445.7	47.9	600.6
0.5	3.03	9.71	220.5	16.5	444.5
1	1.62	4.48	176.5	7.73	377.2
2	1.36	3.92	188.2	6.20	355.8

Table 3.2 Effect of confinement on ductility for Fe415, fck 20 and 10mm dia. hoops

Spacing	∞	300		100	
Rr	μ_ϕ	μ_ϕ	% increase	μ_ϕ	%increase
0	6.78	41	504.7	47.8	605
0.5	3.03	10.5	246.5	21.2	599.7
1	1.62	4.90	202.5	9.93	513
2	1.36	4.17	206.6	7.60	458.8

Table 3.3 Effect of confinement on ductility for Fe415, fck 40 and 8mm dia. hoops

Spacing	∞	300		100	
Rr	μ_ϕ	μ_ϕ	% increase	μ_ϕ	%increase
0	6.61	17.3	161.72	47.4	617.1
0.5	3	5.26	75.3	11.9	296.7
1	1.6	2.54	58.7	5.59	249.4
2	1.36	2.54	86.8	4.81	253.6

Table 3.4 Effect of confinement on ductility for Fe415, fck 40 and 10mm dia. hoops

Spacing	∞	300		100	
Rr	μ_ϕ	μ_ϕ	% increase	μ_ϕ	%increase
0	6.73	21.5	219.5	48.3	617.7
0.5	3	6.14	104.7	16.2	440
1	1.6	2.91	81	7.75	384.4
2	1.36	2.85	109.5	5.24	285.3

The above study shows that spacing of stirrups has significant effect on ductility. Effect of change of stirrup diameter from 8 mm to 10 mm, on ductility ranges from 8% to 30%. There is slight decrease in ductility for change of grade of concrete from M20 to M40. Steel ratio has most pronounced effect on ductility. Ductility is maximum for $Rr = 0$ and the increase in ductility with confinement is also maximum for $Rr = 0$. It can also be observed that for $Rr = 0$, the maximum ductility is almost independent of concrete grade.

3.4.1.2 Effect of Confinement by Transverse Hoops and Axial Load on Ductility of Column Sections

Details of section analysed:

Column size: 400mmX400mm with effective cover of 40mm.

Grade of materials: M20 concrete and Fe415 steel.

Longitudinal steel percentage: 1%, 2% and 4% of cross sectional area placed equal spacing all round the section.

Axial load level (*Pu*): Three levels of axial load (*Pu*) is considered. Denoted as $Pu = 0$, *Pu_bal* and *Pu_50%* for Pu=0 i.e. section in pure flexure, $Pu = Pu\_bal$, i.e. balance failure condition and $Pu = Pu\_50\%$ i.e. at 50% of maximum axial load.

Diameter of transverse reinforcement: 10mm

Spacing of transverse reinforcement: ∞, 300mm and 100mm denoted as unconfined, confined-300 and confined-100 respectively.

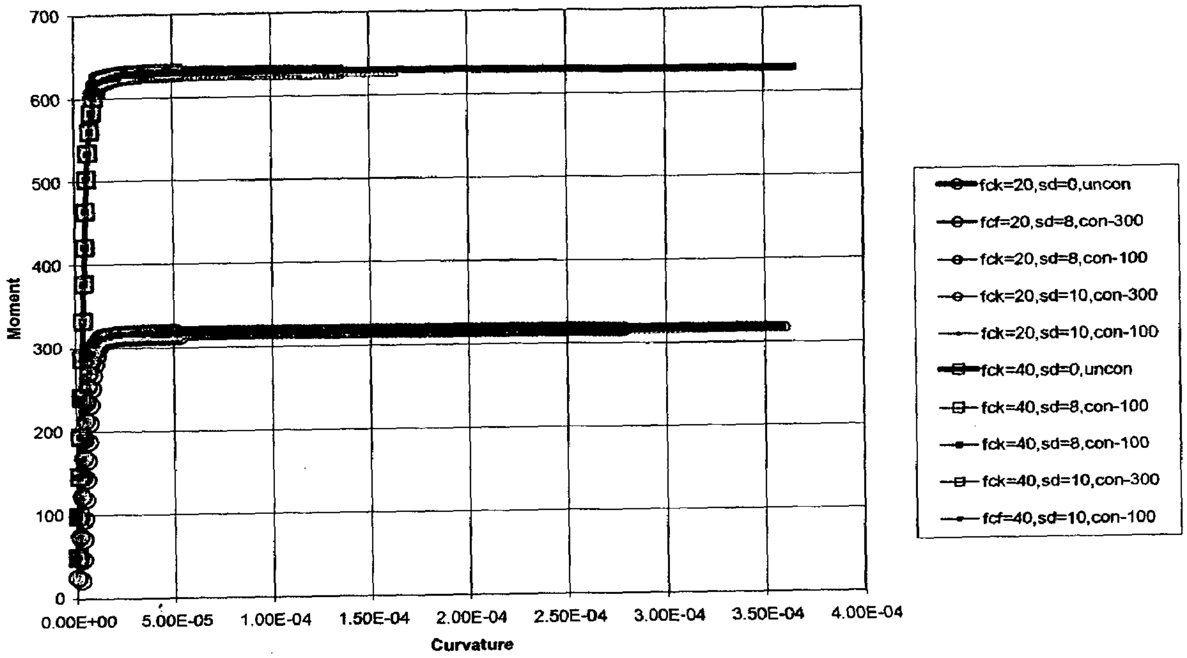


Fig. 3.7 Moment Curvature relation for $Rr = 0, fy=415$

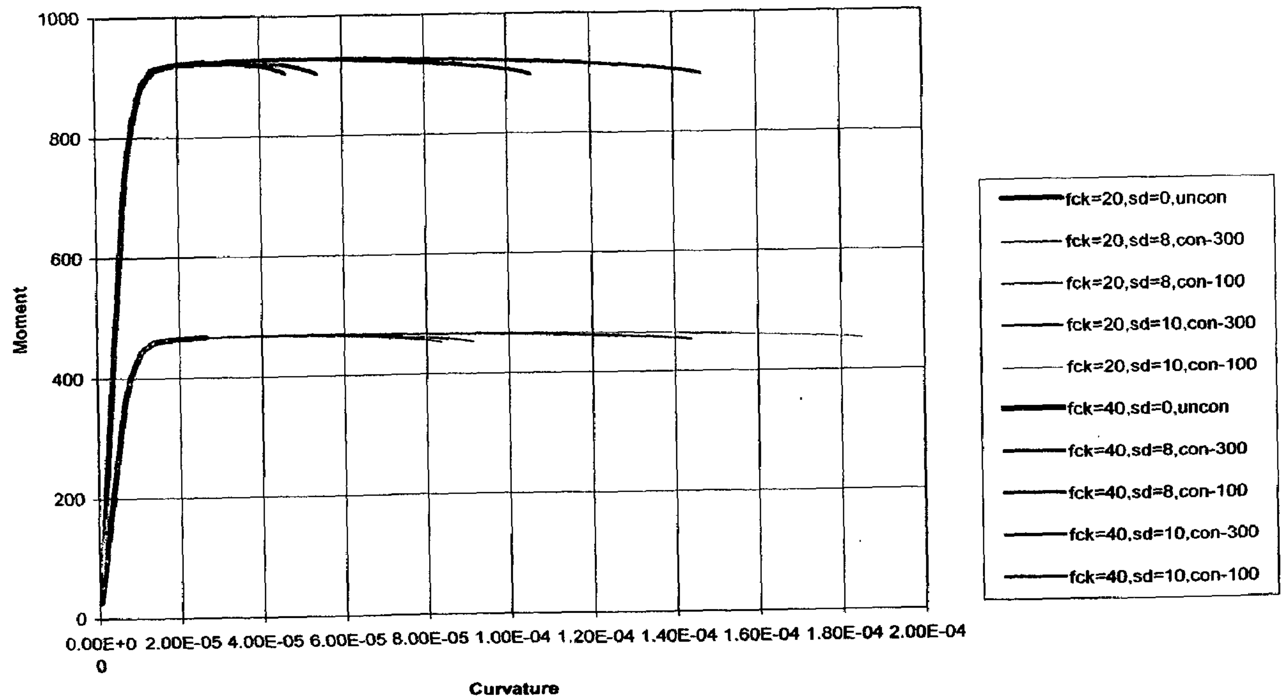


Fig. 3.8 Moment Curvature relation for $Rr = 0.5, fy=415$

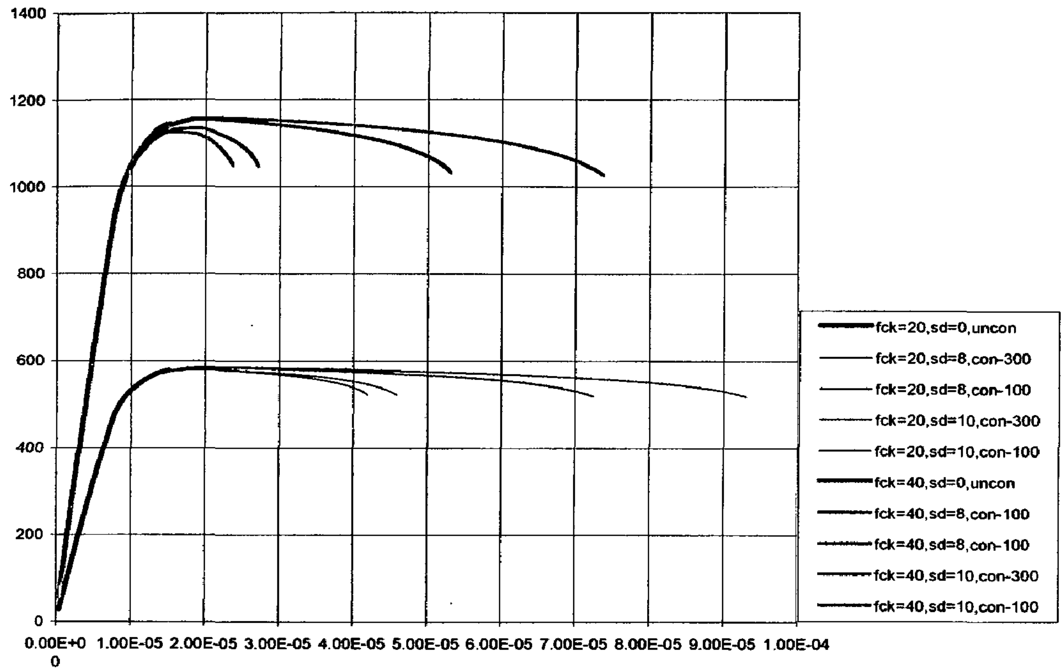


Fig. 3.9 Moment Curvature relation for *Rr* =1, *fy=415*

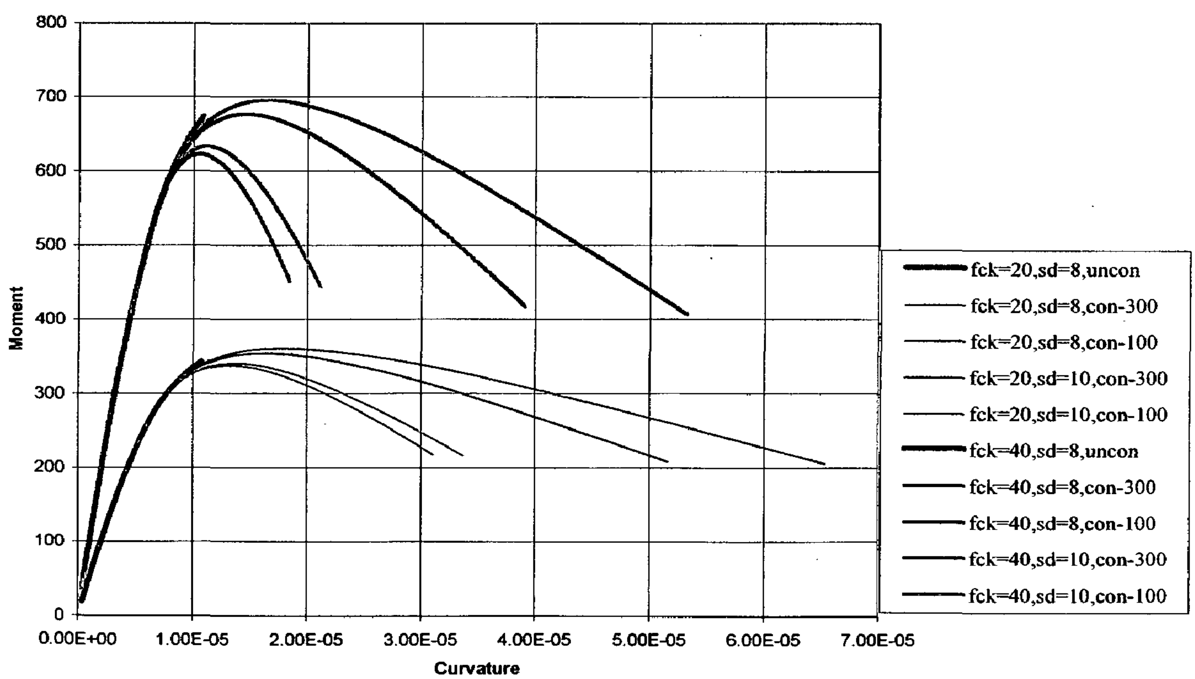


Fig. 3.10 Moment Curvature relation for *Rr* =2, *fy=415*

Tables 3.5-3.7 show details of analysis and figure 3.11 to 3.13 show moment curvature curves with different axial load for different percentage of steel.

Table 3.5 Effect of confinement and axial load on ductility for column section with 1% steel.

Spacing	∞	*300*		*100*	
Axial Load	μ_ϕ	μ_ϕ	*% increase*	μ_ϕ	*% increase*
Pu = 0	4.11	16.7	306.3	35.4	761.3
Pu_bal	1.76	6.02*	242	11.6*	559.1
Pu_50%	1.71	6.60*	286	12.9*	654.4

* *These values of ductility have been obtained, ignoring the reduction in moment capacity. Therefore these values do not represent the true ductility.*

Table 3.6 Effect of confinement and axial load on ductility for column section with 2% steel.

Spacing	∞	*300*		*100*	
Axial Load	μ_ϕ	μ_ϕ	*% increase*	μ_ϕ	*% increase*
Pu = 0	2.23	7.3	227.4	15.5	595.1
Pu_bal	1.52	6.25	311.2	13.1	761.8
Pu_50%	1.58	5.76	264.6	9.86	524.1

Table 3.7 Effect of confinement and axial load on ductility for column section with 4% of steel.

Spacing	∞	300		100	
Axial Load	μ_ϕ	μ_ϕ	% increase	μ_ϕ	% increase
Pu = 0	1.82	5.75	216	11.7	542.9
Pu_bal	1.46	5.93	306.2	11.6	694.5
Pu_50%	1.45	4.53	212.4	7.64	427

It can be seen from the tables that, there is significant increase in ductility by confinement. It can also be seen that, ductility is maximum at zero axial load. It can be observed from the moment-curvature plots that Ductility reduces drastically with increase in axial loads. For low percentage of steel (1%), there is hardly any ductility available in columns subjected to axial loads. However the ductility improves with increased percentage of steel (4%). On the other hand, the ductility at zero axial load (at zero axial load the column behaves essentially as a beam) reduces with increase in percentage of steel. Further, as percentage of steel is increased ductility for zero and balance failure load approaches to be equal.

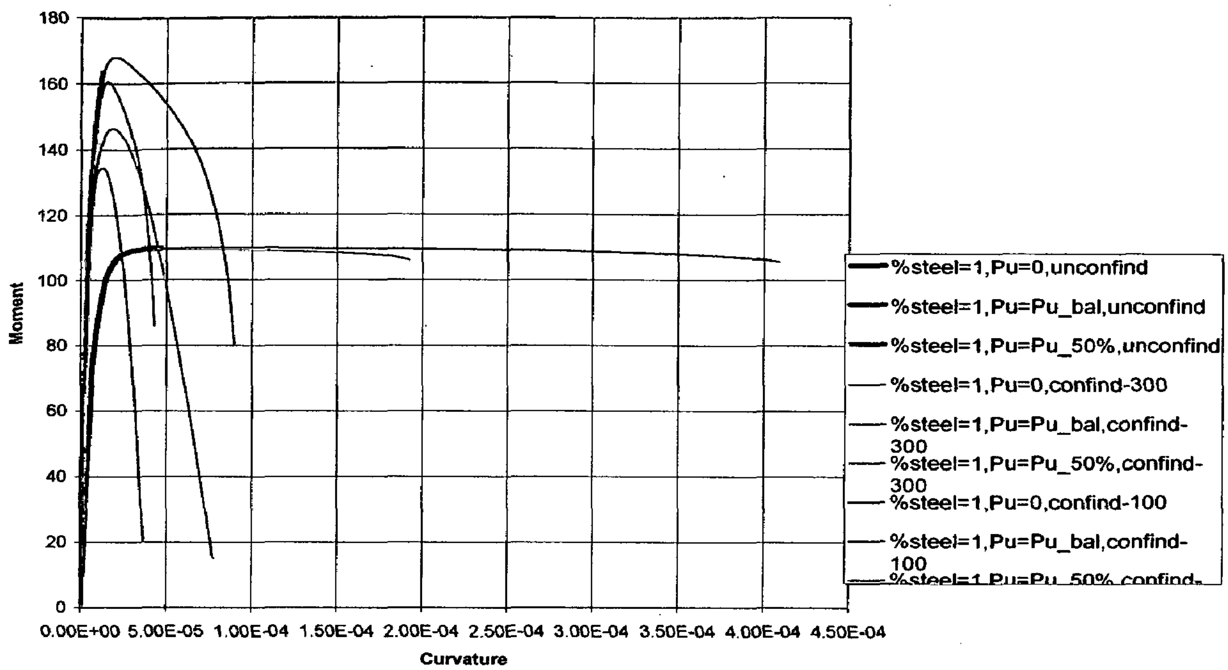


Fig. 3.11 Effect confinement and axial load on moment-curvature curvè, with 1% steel, for unconfined, confind-300 and confined-100.

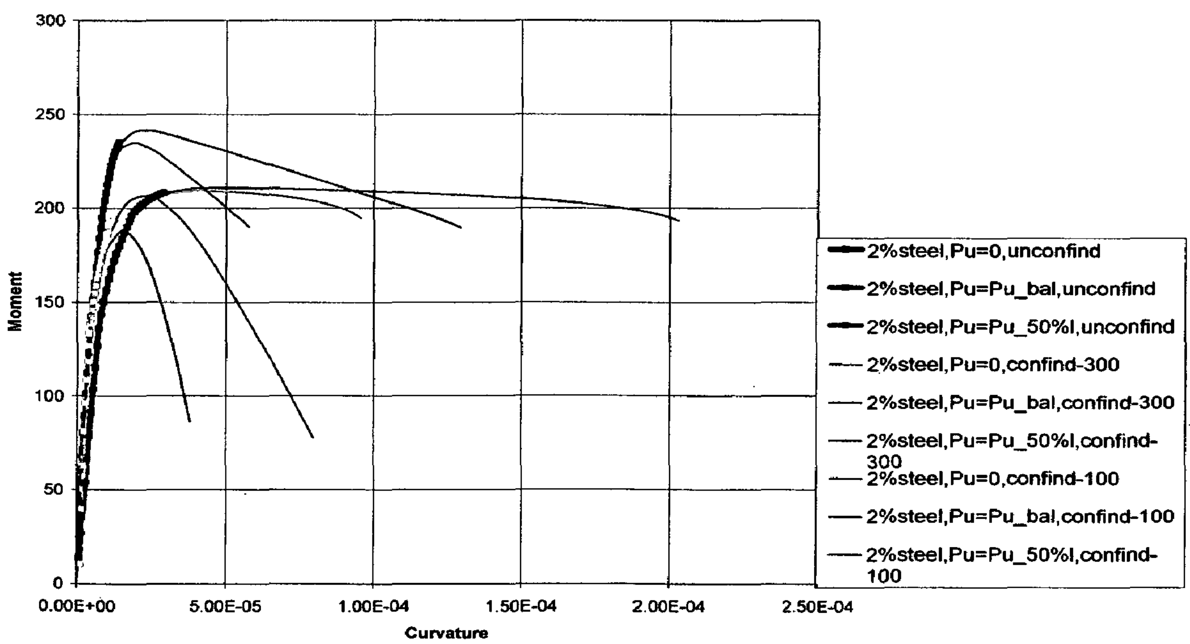


Fig. 3.12 Effect of confinement and axial load on moment curvature curve, with 2% steel, for unconfined, confind-300 and confined-100.

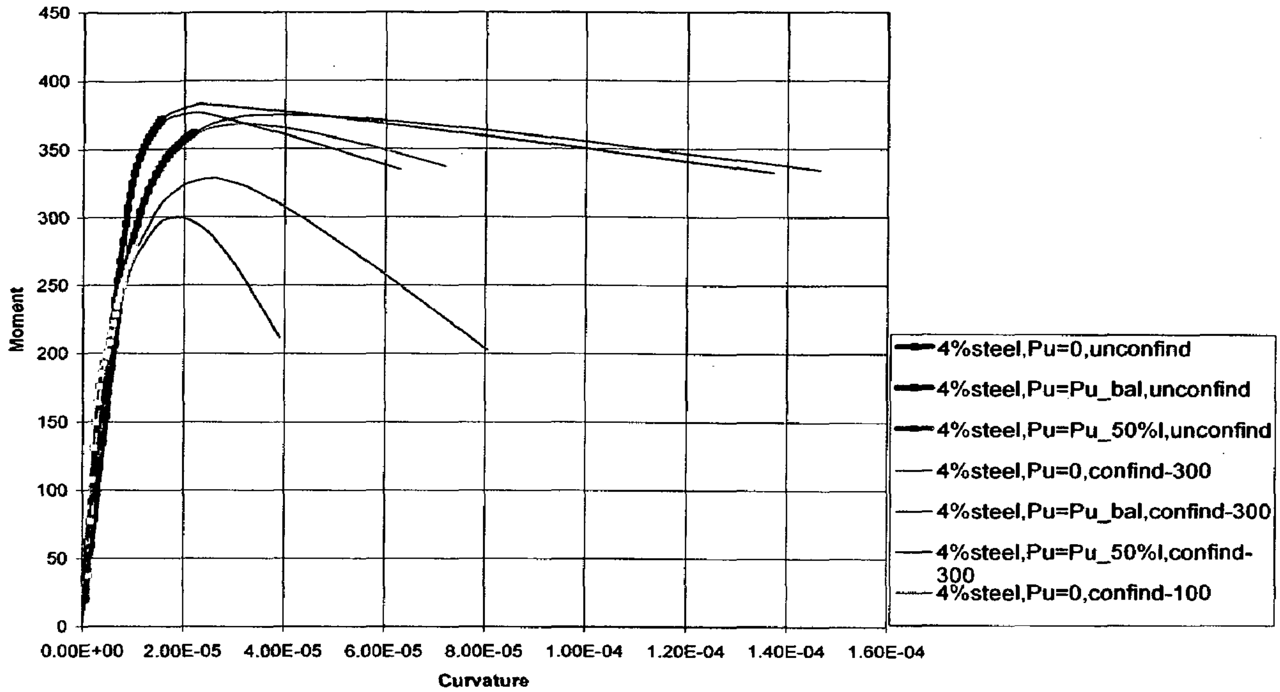


Fig. 3.13 Effect of confinement and axial load on moment curvature curve, with 4% steel, for unconfined, confined-300 and confined-100.

STRENGTH AND DUCTILITY OF RC JACKETTED MEMBERS

A large number of RC buildings, in India, have been constructed either without any consideration to resist earthquake forces or without using current codal practice on earthquake resistant design. Such deficient buildings require strengthening of their members. Strengthening of structural elements depends on the desirable seismic resistance and acceptable damage level. It can be done in several ways as described below.

- Addition of new structural members such as shear walls, braced frames, buttresses and moment resisting frames.
- Addition of dampers.
- Base Isolation
- Strengthening of existing members, by strengthening slabs, beams, columns, walls, joints and foundation.

Following ways can be used to strengthen beams and columns of existing deficient building.

- Local interventions, by injecting resign mortar.
- Glued metal or FRP sheets.
- RC jacketting.

This study focuses on strengthening of beams and column by RC jacketting.

4.1 RC JACKETTING

Reinforced concrete jackets can be applied by adding new concrete and reinforcement to any or all of the four sides of members.

4.1.1 RC Jacketting of Beams

Jacketting a beam from four sides, three sides or by providing RC underlay depends on availability of space and location of member in existing structure. Generally there has been restriction imposed by structural configuration and member location, for jacketting it from all four sides. Fig. 4.1 shows jacketting of beam on three sides and on four sides.

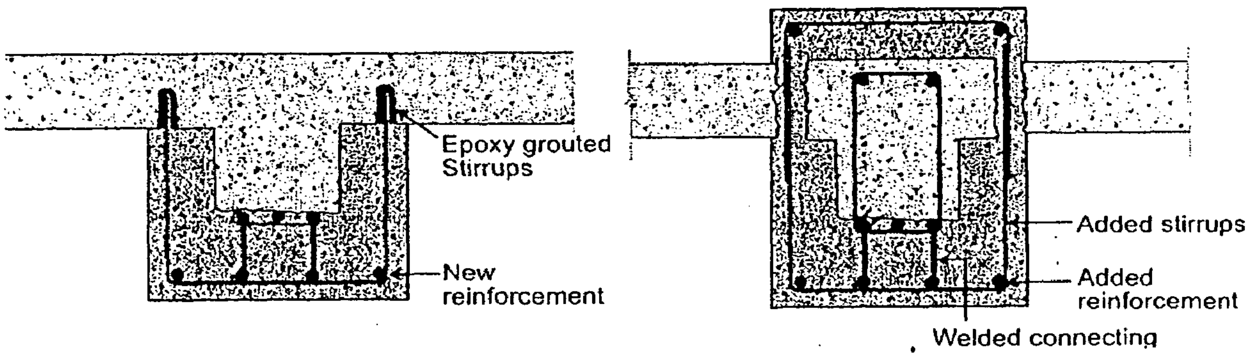


Fig. 4.1 RC jacketting of beam.

All Around Jacketting

Jacketting on all four sides of the beam is most effective solution. In this case thickness of concrete which is added to the upper face should be such that it can be accommodated within floor thickness. Placement of ties can be achieved through holes which are opened in the slab at close distances, which can also be used for pouring of concrete. Longitudinal reinforcing bars of jacket should be welded to the existing bars as shown in Fig. 4.1.

Three Side Jacketting

In case of jacketting on three sides, the stirrups are either to be grout-anchored into slab or nailed into beam web using strands, as shown in Fig. 4.1. In this case pouring of concrete is not possible and shotcreting is to be used.

RC Underlays

Beams can also be strengthened by providing RC underlays on their bottom faces. However it can only increase beam's flexural capacity. It is not very effective in resisting earthquake loads due to inadequate ductility.

4.1.2 RC Jackctting of Columns

Depending on the local existing conditions, jacket can be provided either all around the perimeter or can be applied on one or more sides.

All Around Jacketting

When jacket is provided all around the column, it's axial and shear capacities increases. Also by providing special confining reinforcement, its ductility can be appreciably increased.

Jacketting of Column in One or More Sides

Some times, due to existing local conditions it is forced to jacket column by one or more sides but not all around. In this case special care is required to maintain unified action between old and jacketted part.

4.2 STRENGTH AND DUCTILITY OF RC JACKETTED MEMBERS

Since jacketting of beams and columns is widely used as an effective way of seismic strengthening, it is important to estimate strength and ductility of jacketted members. The following assumptions are made in the present study:

- Bond between existing and jacket material is perfect. Transfer of forces will be same as in case of a member with monolithically casted section.
- Plane section before bending remains plane even after bending.
- Concrete strength in tension has been neglected.
- Maximum strain in unconfined concrete is *0.0035* and stress corresponding to this is equal to $0.67f_{ck}$.
- Stress-strain relation for confined concrete is based on model-1 (*Chapter-1*)
- Maximum strain in steel at failure is *0.15*. Stresses in steel corresponding to any strain can be obtained from Table 1.3 (*Chapter-1*).

4.2.1 Bond between Existing Material and Jacket Material

Bond refers to the adhesion between two materials of a member. The key to success of strengthening depends on attaining high degree of bonding between old and new concrete. Bond in case of jacketting transfers force to jacket concrete through interface of existing member and jacket. Bond between two concrete surfaces can be accomplished by following ways:

- By roughening the surface of the old concrete.
- By coating the surface with epoxy or other type of resign before concreting.
- By welding reinforcement bars.
- By using steel dowels.

4.3 NUMERICAL STUDIES

4.3.1 Effect of Jacketting on Strength and Ductility of Beams

Details of section analysed

Original beam section: Beam of size 300mmX500mm with effective cover of 60mm.

Size of Jacketted Beam: Width of jacketted beam has been taken as 300mm and depth has been taken as *d2* + effective cover in jacket. Effective cover in jacket has been taken as 40mm.

Steel ratio, *Rr*: is defined as:

$$Rr = \frac{\rho + \rho'' - \rho'}{\rho_{bal}} \tag{4.1}$$

Where ρ is area of bottom steel in original section,ρ' is area or top reinforcement, ρ'' is area of reinforcement provided in jacket and ρ_{bal} is area of steel for balanced section.

Rr =0, 0.5 and 1 has been considered in the present study.

Transverse reinforcement: 8mm diameter Hoops at spacing of ∞, 300mm and 100mm have been considered, denoted as unconfined, confined-300 and confined-100, respectively.

Ratio of effective depths of jacketted and original beams: It is denoted as d2/d1, as shown in Fig. 4.2, depth ratio equal to 1.5, 1.4, 1.25, and 1.15 has been considered.

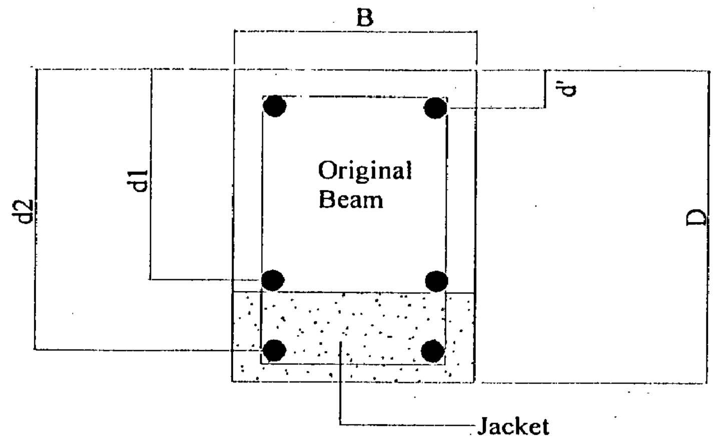


Fig.4.2 Details of jacketted section.

Tables 4.1 to 4.3 summarize the moment and ductility of jacketted beams with varying depth of jacket, confinement, and *Rr*.

Table 4.1 Maximum moment and ductility for $Rr = 0$

Spacing	∞		*300*	*100*
d2/d1	*Max. Moment*	μ_ϕ	μ_ϕ	μ_ϕ
1.5	385.3	8.12	47.6	59.7
1.4	368.9	7.9	46.9	57.9
1.25	344	6.95	37.3	51.1
1.15	327.5	6.84	37	50.2

Table 4.2 Maximum moment and ductility for $Rr = 0.5$

Spacing	∞		*300*	*100*
d2/d1	*Max. Moment*	μ_ϕ	μ_ϕ	μ_ϕ
1.5	609	4	12.5	20.7
1.4	570	3.8	11.7	19.6
1.25	526.5	3.47	10.8	18.3
1.15	493.5	3.11	9.75	16.6

Table 4.3 Maximum moment and $Rr = 1$

Spacing	∞		*300*	*100*
d2/d1	*Max. Moment*	μ_ϕ	μ_ϕ	μ_ϕ
1.5	715.4	1.96	5.39	8.94
1.4	682.4	1.86	5.11	8.5
1.25	632.9	1.77	4.89	8.26
1.15	599.9	1.65	4.6	7.84

The study shows that moment capacity increases as *d2/d1* ratio increases. The effect of *d2/d1* ratio is almost independent of *Rr*. Spacing of hoops has significant effect on ductility. Effect of *d2/d1* ratio on ductility is not significant. However, the ductility decreases significantly with increase in *Rr*.

Figs. 4.3-4.5 show the moment-curvature curves for jacketted beams with different values of *Rr*. It can be seen from the Fig. 4.3 that for $Rr = 0$ the outer layer of tension reinforcement yields before yielding of the inner layer. This gives rise to a peculiar shape of Moment-Curvature curve.

4.3.2 Effect of Jacketting on Strength of Columns

4.3.2.1 Interaction Curve for Design of Jacketted Column

Figures 4.7 to 4.9 show the design interaction curves for jacketted columns in terms of usual parameters defined in SP-16. In addition to the parameters defined in SP-16, few other parameters are also defined here. Percentage of steel in exterior layer and interior layer is denoted by *p1* and *p2* respectively. Location of exterior and interior reinforcement layer is given as *d'* and *d"*, respectively. Details of sectional parameters are shown in Fig. 4.6.

These interaction curves can be used for design of RC jacketted columns.

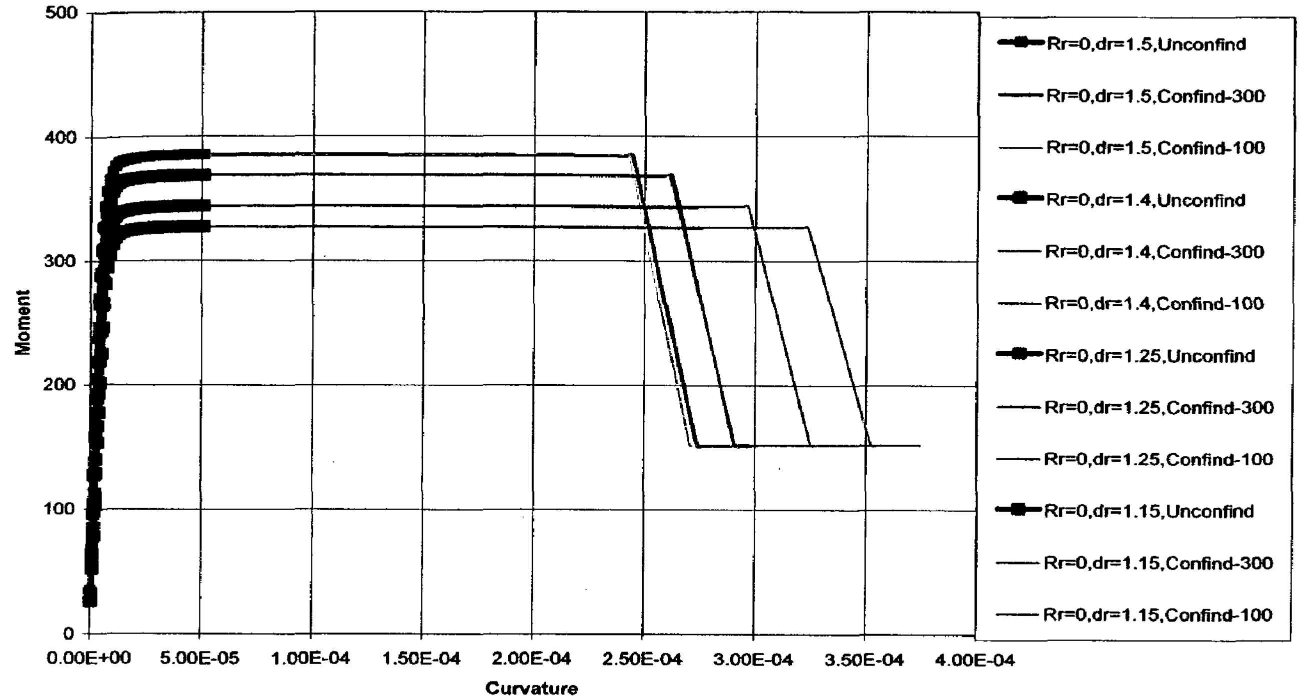


Fig. 4.3. Moment-curvature curve, *Rr*=0.

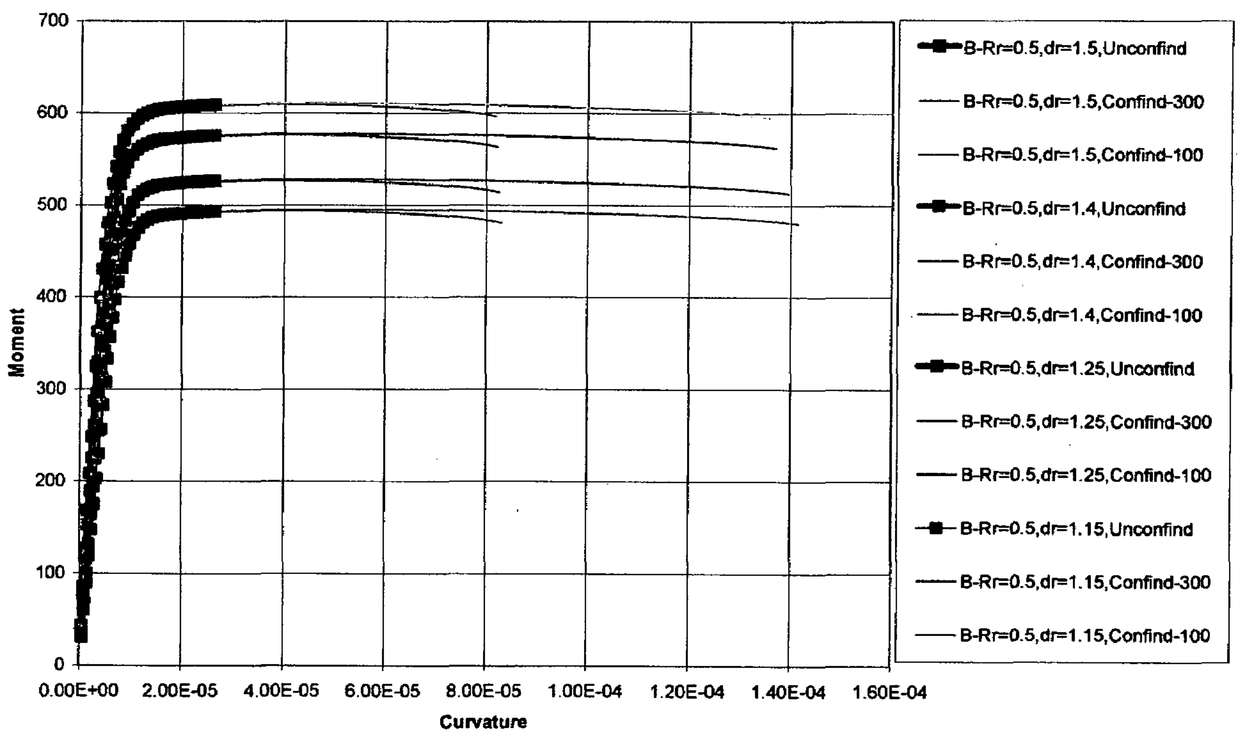


Fig. 4.4. Moment-curvature curve, *Rr*=0.5.

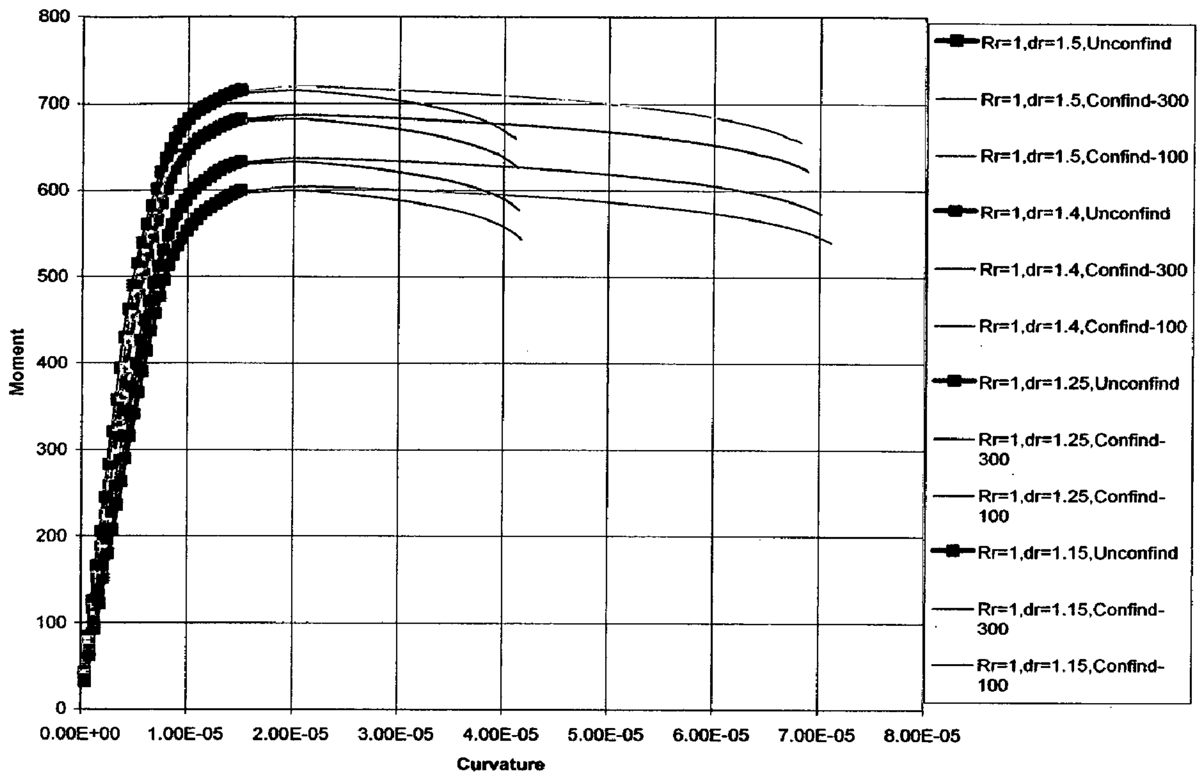


Fig. 4.5. Moment-curvature curve, Rr=1.

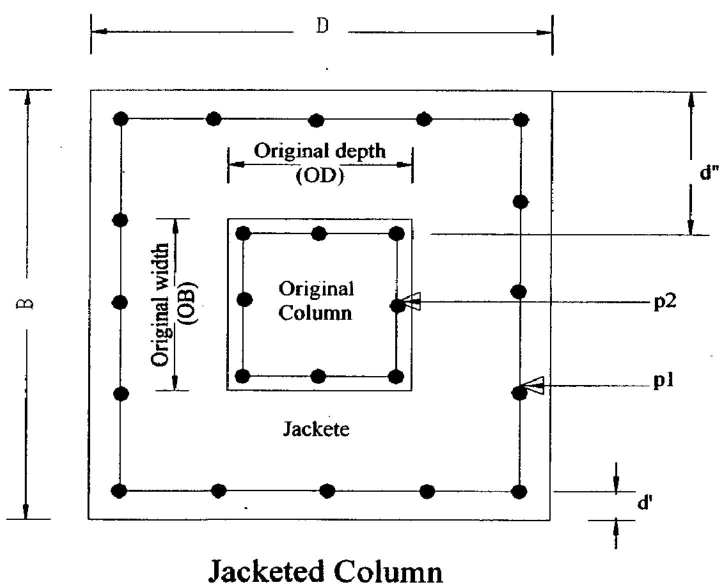


Fig. 4.6 Parameters of design interaction curve

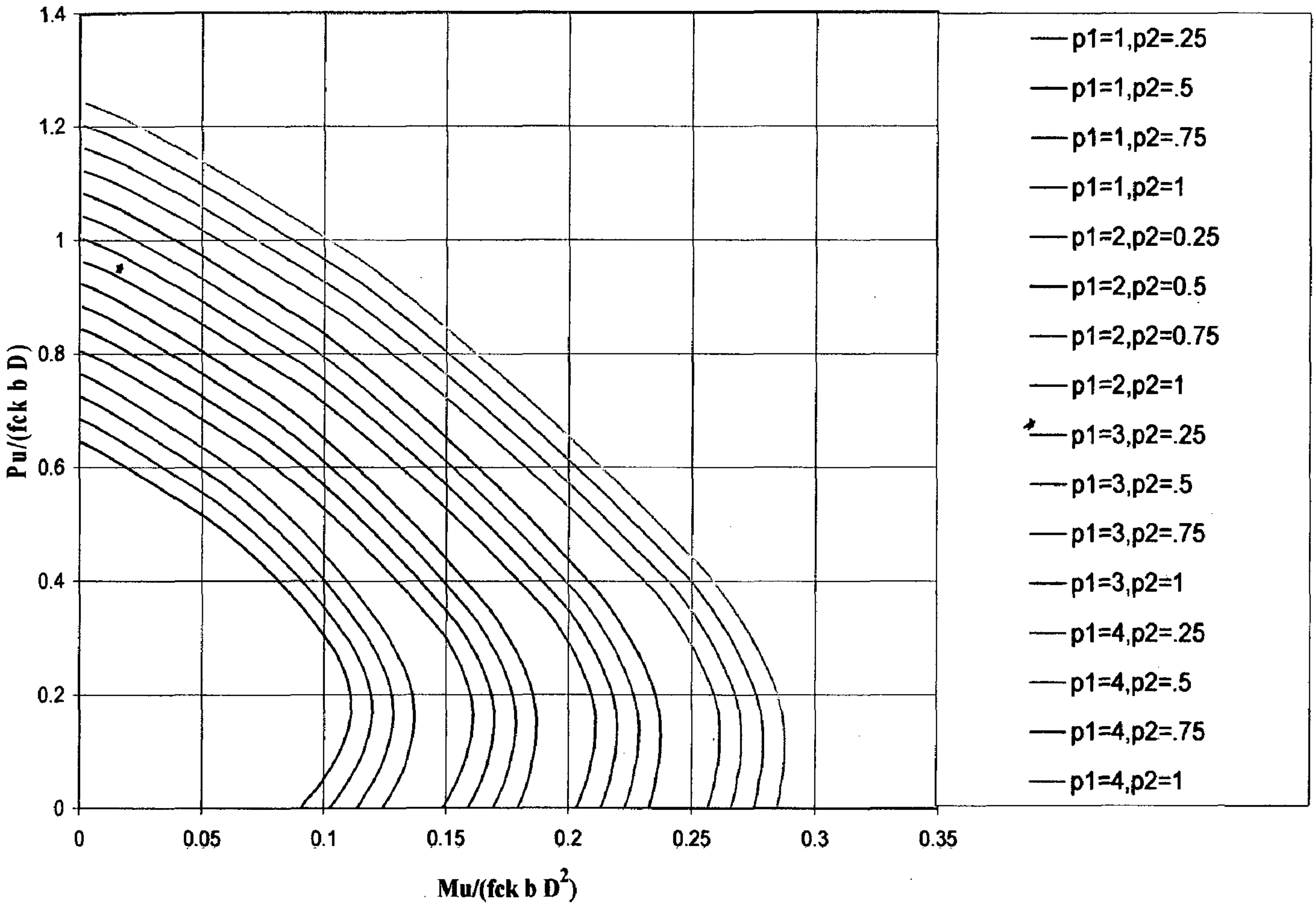


Fig. 4.7 Interaction curve for jacketted column with $\frac{d'}{D} = 0.1$ and $\frac{d''}{D} = 0.2$

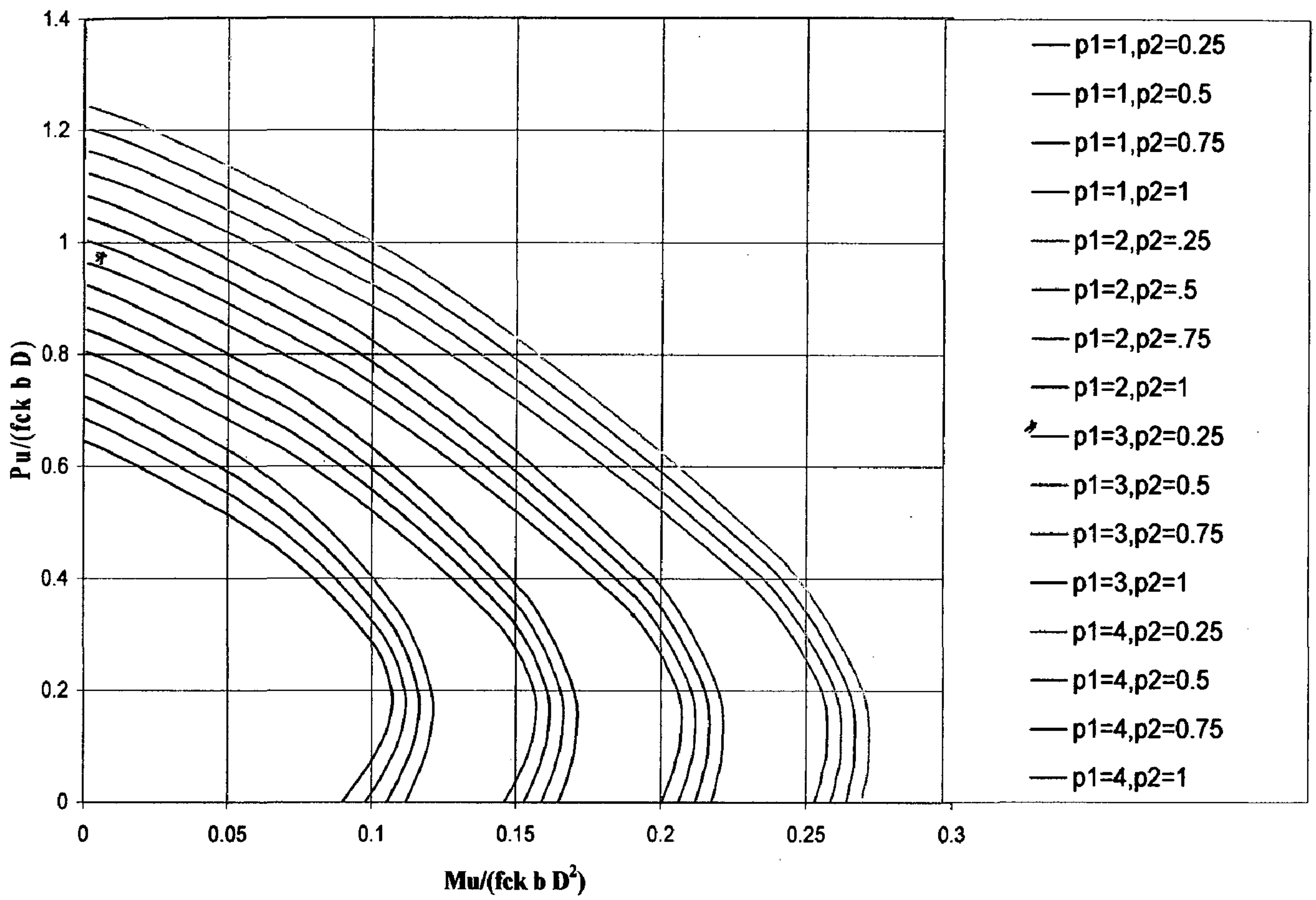


Fig. 4.8 Interaction curve for jacketted column with $\frac{d'}{D} = 0.1$ and $\frac{d''}{D} = 0.3$

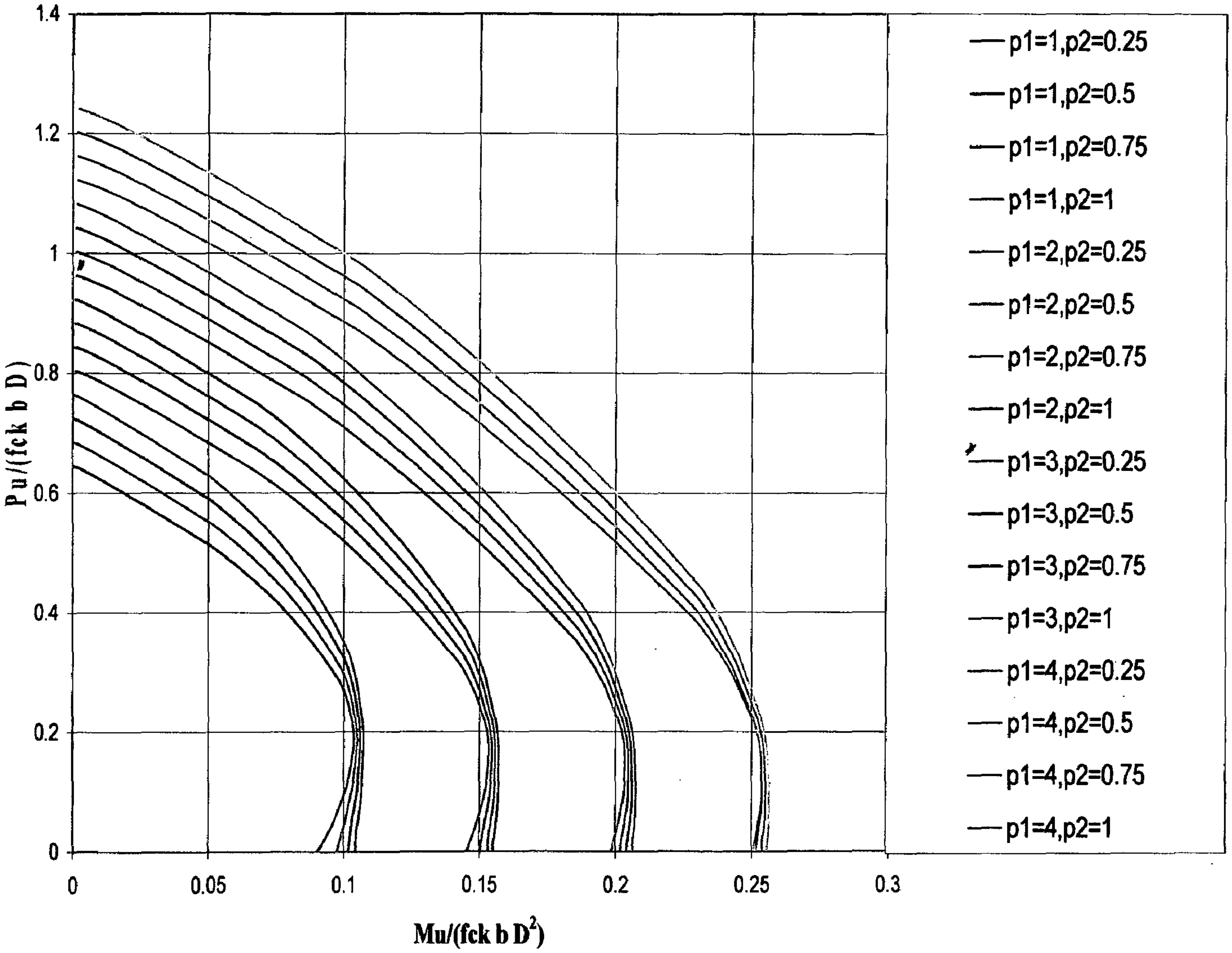


Fig. 4.9 Interaction curve for jacketted column with $\frac{d'}{D} = 0.1$ and $\frac{d''}{D} = 0.4$

4.3.2.1 Estimation of contribution in strength by reinforcement layer p1 and p2.

Many times the drawings of existing buildings are not available Member sizes can be measured at site but getting details of reinforcement inside the concrete members is very-very difficult. A study has been performed to estimate the contribution of reinforcement in original column for normal ranges of reinforcement.

Details of section considered

Original column: Size 400mm×400mm with effective cover of 40mm.

Grade of materials: *M20* concrete and *Fe415* steel

Longitudinal steel: 1% steel in original column placed at equal spacing denoted as *p2*. Steel in jacket has been provided as 1%, 2%, 3% and 4% of original column area denoted as *p1*.

Size of jacketted column: Uniform jacket on all the four sides has been considered. Size of jacketted column is, *original size + 2 x (d" + effective cover in jacket (d') – effective cover in original column)*.

Sectional details are shown in Fig. 4.6. Interaction curves for this study are given in Figs. 4.10 to 4.12. It is observed from figures that an error is made in the estimation of strength if the reinforcement in original column is ignored. This error varies from 5% to 8% for different percentage of steel and different values of $\frac{d''}{d}$. It can be concluded that this error is marginal in the normal range of reinforcement. Therefore, reinforcement in the original column can be ignored if the details are not available. If the designers do not have access to interaction curves / software for jacketted columns, he may think of designing columns by assuming total reinforcement at the outer layer. Figs. 4.10 to 4.13 shows that this assumption doesn't cause any error in estimation of maximum axial load capacity but the error in bending moment at a given axial load increases near the balance failure point. This error ranges from 10% to 12% for the range of parameters selected in this study. It can be concluded that, although the error in this case is more that that in previous case, still it can be ignored.

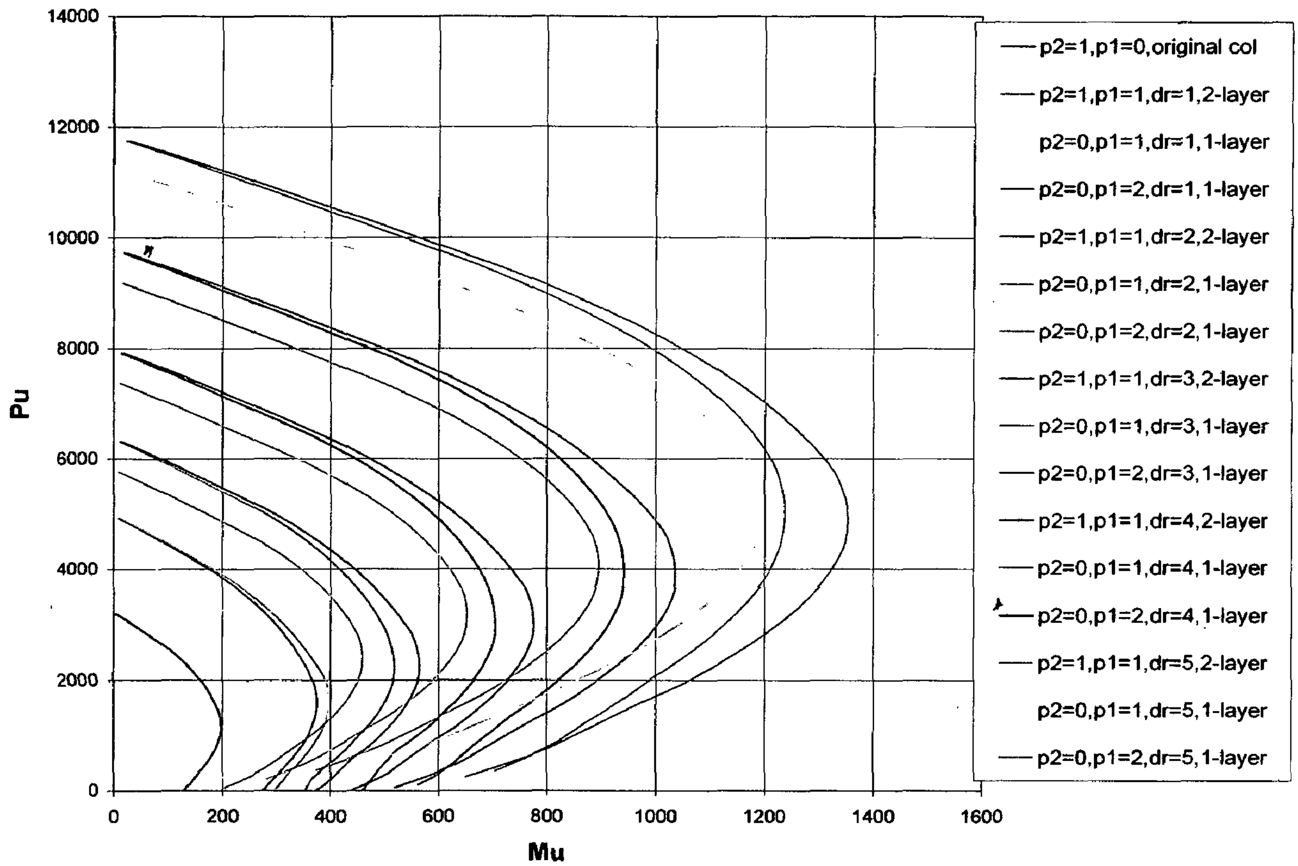


Fig. 4.10 Effect of jacketting, for p2=1% (1600 mm^2), p1=1% (1600 mm^2) of original column area, for varying thickness (*dr*=1, 2, 3, 4, 5), for steel placed in two layers, only jacket steel has been placed (1600 mm^2) and total steel placed in outer layer (3200 mm^2). p2= % of bars considered in original column considered for a particular analysis. p1= % of bars considered in jacket of column considered for a particular analysis.

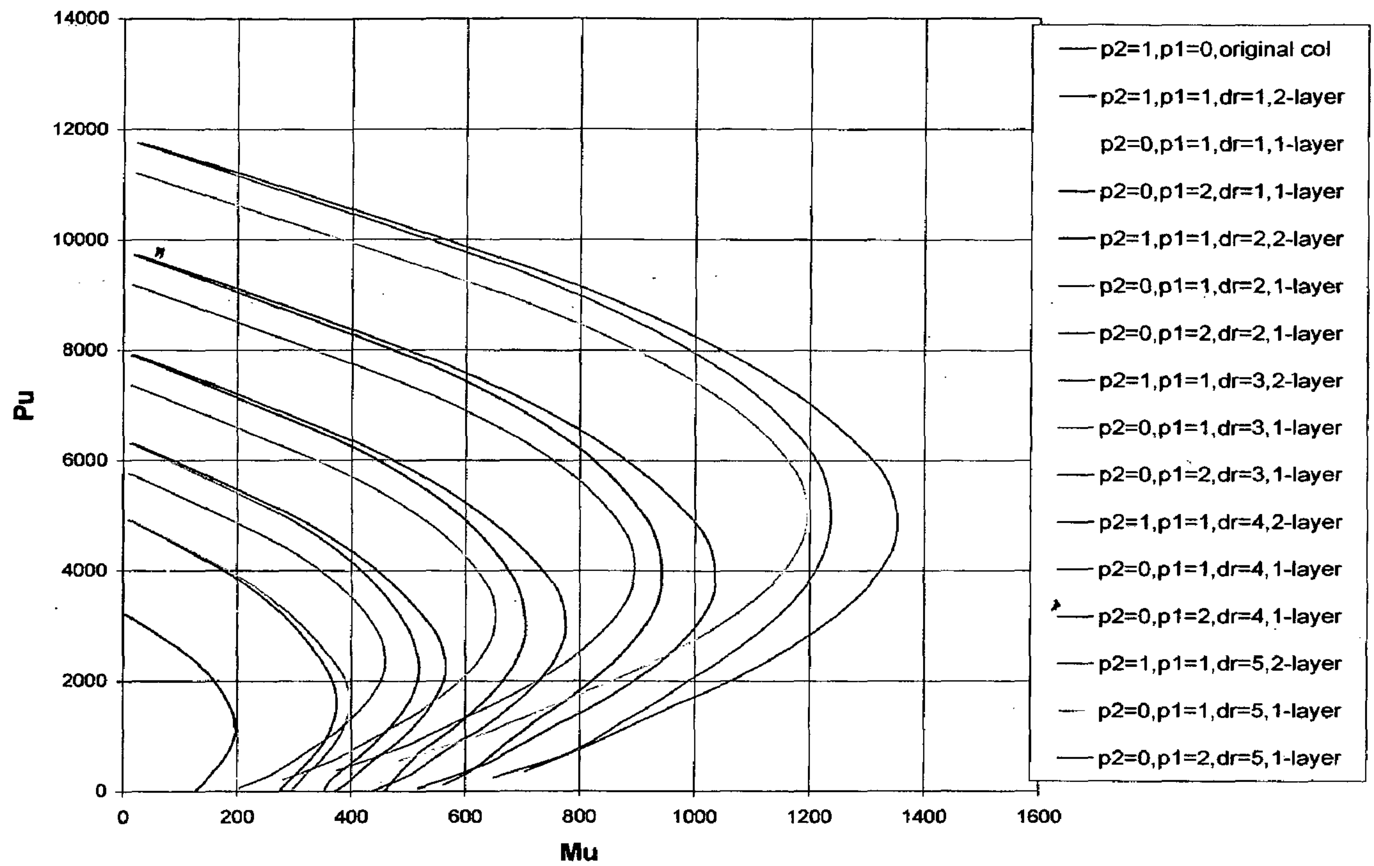


Fig. 4.10 Effect of jacketting, for p2=1% (1600 mm^2), p1=1% (1600 mm^2) of original column area, for varying thickness (*dr*=1, 2, 3, 4, 5), for steel placed in two layers, only jacket steel has been placed (1600 mm^2) and total steel placed in outer layer (3200 mm^2). p2= % of bars considered in original column considered for a particular analysis. p1= % of bars considered in jacket of column considered for a particular analysis.

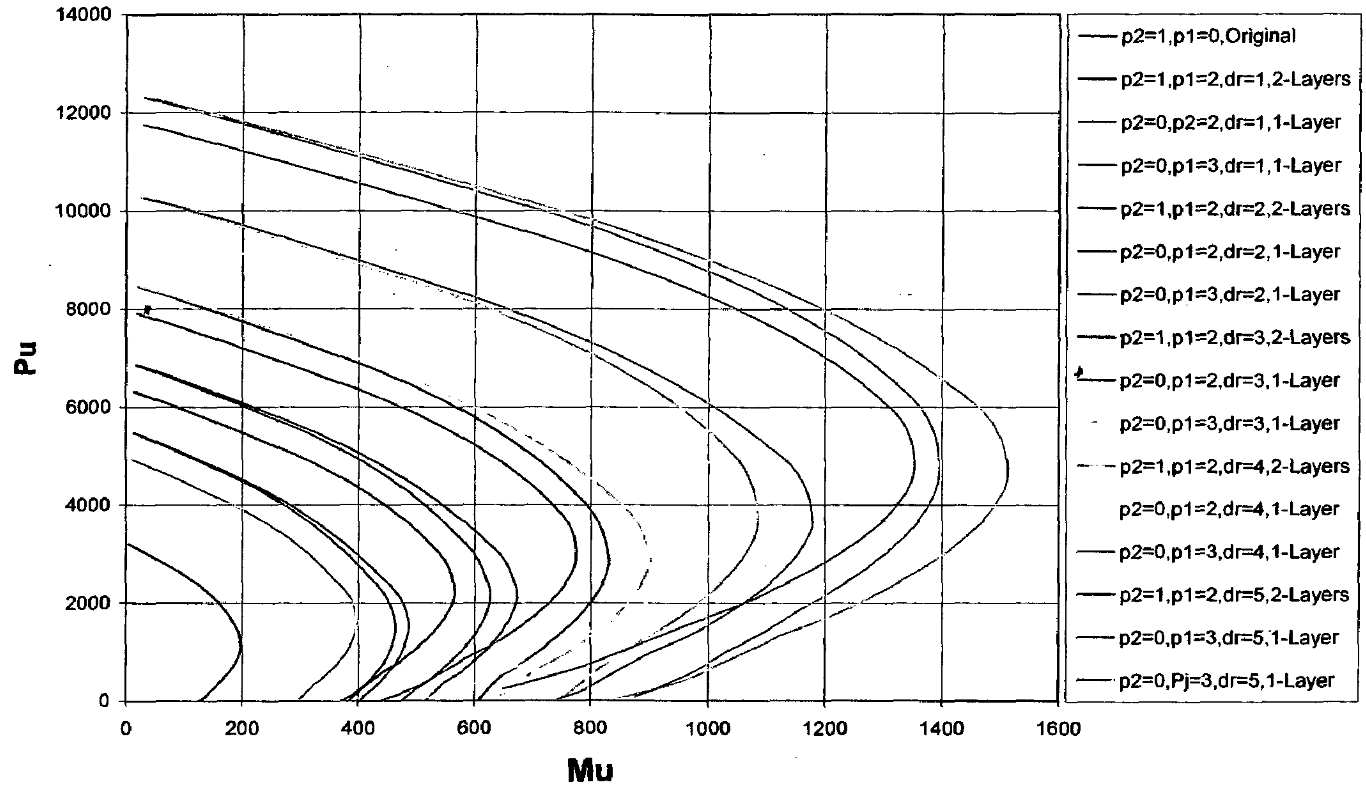


Fig.4.11 Effect of jacketting, for p2=1% (1600 mm^2), p1=2% (3200 mm^2) of original column area, for varying thickness (*dr*=1, 2, 3, 4, 5), for steel placed in two layers, only jacket steel has been placed (3200 mm^2) and total steel placed in outer layer (4800 mm^2). p2= % of bars considered in original column considered for a particular analysis. p1 = % of bars considered in jacket of column considered for a particular analysis.

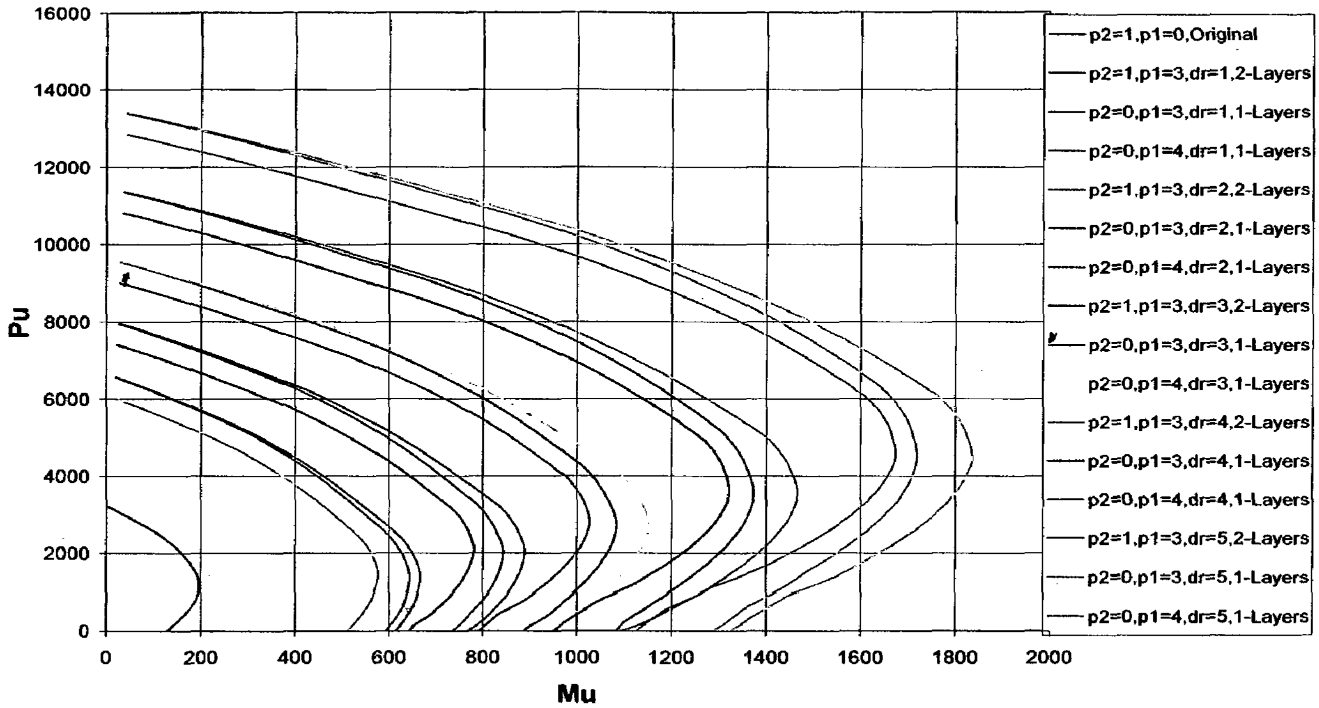


Fig.4.12 Effect of jacketting, for p2=1% (1600 mm^2), p1=3% (4800 mm^2) of original column area, for varying thickness (*dr*=1, 2, 3, 4, 5), for steel placed in two layers, only jacket steel is placed (4800 mm^2) and total steel placed in outer layer (6400 mm^2). p2= % of bars considered in original column considered for a particular analysis. p1 = % of bars considered in jacket of column considered for a particular analysis.

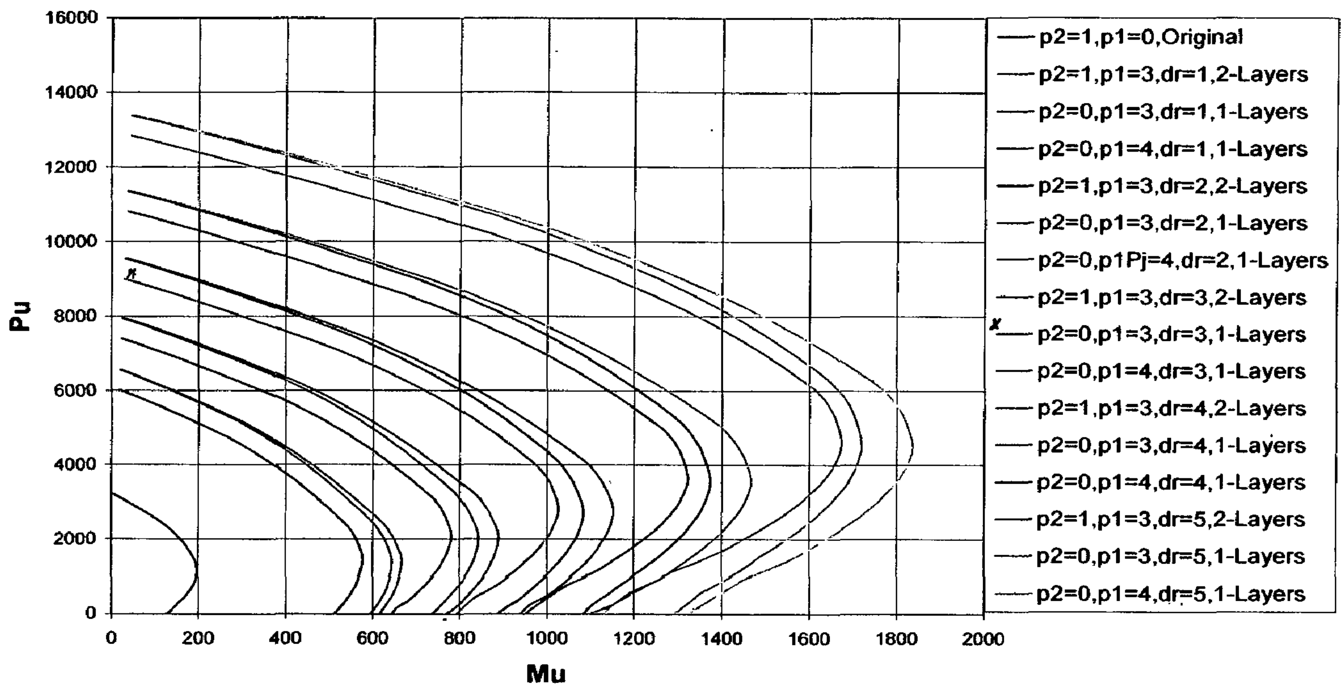


Fig.4.13 Effect of jacketting, for p2=1% (1600 mm^2), p1=4% (6400 mm^2) of original column area, for varying thickness (*dr*=1, 2, 3, 4, 5), for steel placed in two layers, only jacket steel is placed (6400 mm^2) and total steel placed in outer layer (8000 mm^2). p2= % of bars considered in original column considered for a particular analysis. p1 = % of bars considered in jacket of column considered for a particular analysis.

CONCLUSIONS

Strength and ductility of non jacketted and jacketted RC beams and columns have been studied. Following are the main conclusions of the study:

(1) In case of beams, Steel ratio has most pronounced effect on ductility. Ductility is maximum for $Rr = 0$ and the increase in ductility with confinement is also maximum for $Rr = 0$. It has been also observed that for $Rr = 0$, the maximum ductility is almost independent of concrete grade.

(2) In case of beams, confining reinforcement has significant effect on ductility. Change of spacing of confining reinforcement from 300mm to 100mm results in 50% to 100% increase in ductility. Effect of change of stirrup diameter from 8 mm to 10 mm, on ductility ranges from 8% to 30%.

(3) There is slight decrease in ductility for change of grade of concrete from M20 to M40 in case of beam section.

(4) Confinement of column results in significant increase in ductility. It has been observed that, ductility is maximum at zero axial load and reduces drastically with increase in axial loads. For low percentage of steel (1%), there is hardly any ductility available in columns. However, the ductility improves with increased percentage of steel (4%). On the other hand, the ductility at zero axial load (at zero axial load the column behaves essentially as a beam) reduces with increase in percentage of steel. Further, as percentage of steel is increased, ductility for zero and balance failure load approaches to be equal.

(5) The study on jacketted beams has shown that moment capacity increases as $d2/d1$ ratio increases. The effect of $d2/d1$ ratio is almost independent of Rr.

(6) In study of jacketted beams, effect of $d2/d1$ ratio on ductility is not significant.

(7) For jacketted beams with $Rr = 0$, the outer layer of tension reinforcement yields before yielding of the inner layer. This gives rise to a peculiar shape of Moment-Curvature curve.

(8) In case of jacketted columns, it has been observed that if the reinforcement in original column is ignored, error in moment capacity varies from 5% to 8% for different percentage of steel and different value of $\frac{d''}{d}$. It can be concluded that the error is marginal in the normal range of reinforcement. Therefore, the reinforcement in the original column can be ignored if the details are not available.

(9) Assuming total reinforcement in jacketted columns at the outer layer has shown that, this assumption doesn't cause any error in estimation of maximum axial load capacity but the error in bending moment at a given axial load increases near the balance failure point. This error ranges from 10% to 12% for the range of parameters selected in this study.

(10) The overstrength, in beams and columns, available due to variation in strength of materials has been studied for Limit State Strength, Characteristic Strength (95% confidence strength) and Most Probable Strength (Mean Strength). The Most Probable Strength of the considered beam in flexure is about 41% higher than Limit State Strength. For the considered columns, the Most Probable Strength in axial load is about 135% higher, and in flexure it is about 95% higher than the corresponding Limit State Strengths.

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