OPTIMAL PLACEMENT OF SUPPLEMENTAL DAMPERS IN A STRUCTURAL SYSTEM

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

EARTHQUAKE ENGINEERING

(With Specialization in Structural Dynamics)

By

AJEET SHANKAR KOKIL

DEPARTMENT OF EARTHQUAKE ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE-247 667 (INDIA)

JUNE, 2004

CANDIDATE'S DECLARATION

I hereby declare that the work which is being presented in this dissertation titled OPTIMAL PLACEMENT OF SUPPLEMENTAL DAMPERS IN A STRUCTURAL SYSTEM as a partial fulfillment of the requirements for the award of the Degree of MASTER OF TECHNOLOGY in Earthquake Engineering, with specialization in Structural Dynamics, submitted to the Department of Earthquake Engineering, Indian Institute of Technology Roorkee, Roorkee, is the record of my own work carried out during the period from August 2003 to June 2004 under the supervision of Dr. Manish Shrikhande, Asst. Professor, Department of Earthquake Engineering, Indian Institute of Technology Roorkee, Roorkee.

This matter embodied in this dissertation has not been submitted for the award of any other degree.

Dated: 30 June 2004
Place: Roorkee

(AJEET SHANKAR KOKIL)

CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

Shrikhande
Manish Shrinkhande
Asstt. Professor

ACKNOWLEDGEMENT

I would like to take this opportunity to express my deep regards and sincere gratitude to Dr. Manish Shrikhande, *Asstt. Professor*, Department of Earthquake Engineering, Indian Institute of Technology Roorkee, Roorkee, for his expert guidance, valuable suggestion and inspiration throughout the course of my dissertation. I would like to thank him for his painstaking efforts to complete the dissertation report.

I am extremely grateful to all my well wishers for their love and support. I would like to thank my those friends who used good floppies and Cds and saved my computer from getting infected.

I would like to thank our Jawahar Bhawan Canteen for providing a great refreshment during those sleepless nights. Finally, I would like to thank our computer lab for giving an opportunity to chat with my beloveds.

AJEET KOKIL

ABSTRACT

With the increase in the seismicity of the region, as indicated by the upward increment in the zone factor by the current code of practice, alarming number of structures fall into the category of seismically deficient structures. Seismic retrofitting, if economical, for such structures provides the plausible solution to their seismic safety problem. One of the seismic retrofitting scheme which is gaining popularity now a days is provision of energy dissipation mechanism in a structure. This energy dissipation is achieved by providing external energy dissipation devices known as supplemental dampers. These dampers dissipates the input energy and reduces the demand on the structural members. However, noting that excessive damping does not reduce the response beyond certain value and effectiveness of dampers depends upon the location in which they are installed in the structure, an attempt has been made to place available number of dampers in the optimal locations to get response reduction. Moreover, effects of soil-structure interaction on the response of the system and hence on the damper locations are studied.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Prelude

Rapid urbanisation and experience gained from the performance of structures during recent earthquakes worldwide has led to an increased awareness about the seismic safety of existing structures. The problem of seismic safety becomes more acute taking into account the enhanced understanding of the earthquake process and the seismicity of a region. In view of this, the structural response control presents itself as a means to enhance the seismic withstand capacities of existing structures. Aseismic design via response control is radically different from the conventional paradigm of earthquake resistant design, which is primarily based on the considerations of strength and ductility. In the conventional design, structures are proportioned for a fraction of the estimated seismic forces and structures are especially detailed to dissipate a part of imposed loads during an earthquake by means of inelastic deformations. These inelastic deformations in structural members reduce the effective stiffness of a member, and thereby, of the entire structural system.

Every structure is designed and detailed to satisfy some performance criteria depending upon its usage and importance. Though it is possible to design a structure to remain elastic even during a severe earthquake, such a design is not economically viable for ordinary structures considering the low probability of earthquake occurrence. However, critical structures like dams, hazardous plants, etc. are always designed for elastic behaviour. As the epistemic uncertainty regarding the seismic hazard at a site reduces with increasing database of ground motions and mapping of geological features, seismic zoning maps – representing average seismic hazard in a region – are subsequently revised. These revisions are, more often than not, always in the direction of increased perception of seismic hazard in the region. A direct fall-out of such a revision of perceived seismic threat is the increased seismic vulnerability of existing structures (designed for lower hazard levels). It is, generally, more economical and expedient to retrofit the existing structure instead of constructing it anew. Several different strategies for seismic retrofit of structures exist, such as, increasing member capacities, reducing demands on structural members, providing alternate load paths, etc. A typical seismic retrofit solution would primarily depend on the functional requirements of the structure and structural deficiencies. A popular seismic retrofit solution consists of providing energy dissipation devices, called dampers, in the structure with little, or no down-time. In such cases, the demand on structural members is reduced as a part of the energy input due to ground vibrations is taken up by these dampers. The mechanism of energy dissipation in a damper can be very diverse, such as, yielding of metal, sliding friction, dashpot with viscous fluid, or viscoelastic action of polymeric materials. Apart from dissipating input energy, the dampers may also contribute to the structural stiffness.

1.2 Optimal Placement of Dampers

More and more of seismic retrofit solutions now employ the supplemental damping devices. However, the increase in system damping can be beneficial only upto a certain limit and thereafter, the additional dampers become ineffective. As shown in Fig. 1.1, the increase in damping beyond, say 20%, does not lead to any significant reduction in the spectrum ordinates. Moreover, the damping devices are generally very expensive , and therefore, it is imperative that the best possible use is made of the available number of damping devices. Several investigators have considered the problem of optimal location of dampers in structures in the past. A brief account of their findings is given in the following.

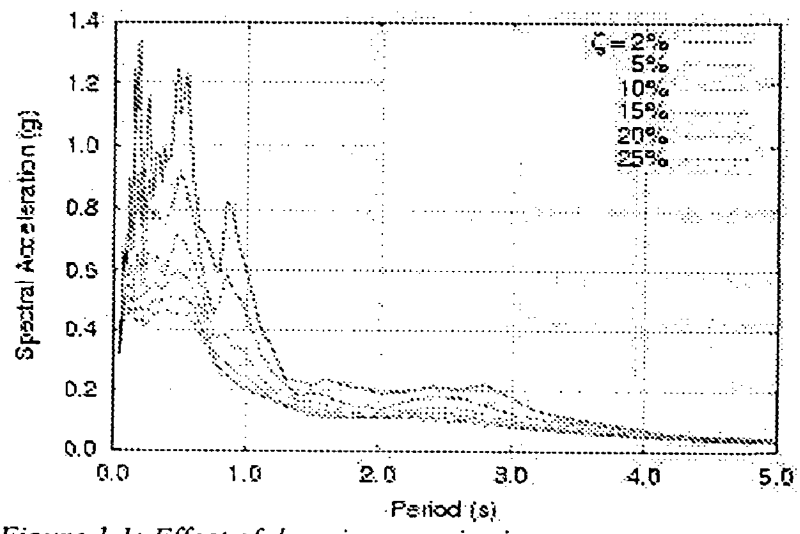

Figure 1.1: Effect of damping on seismic response

Li and He (1992)[1] studied the torsional response of a tall building. The parameters of fluid viscous dampers were optimized to control the seismic response of the tall buildings. The additional dampers were assumed to be installed on each storey along the longitudinal (X) and transverse (Y) axes. Root mean square (RMS) of lateral displacements along X and Y directions and the rotational angle about vertical (Z) axis was taken as the objective function. This function was minimized subject to constraints on lateral displacements and rotations. For El-Centro (1940) earthquake excitation they found that not only the maximum lateral displacement but also torsional rotation reduced by 30-35% by the optimal placement of dampers.

Gluck and Reinhorn (1996)[2] suggested a method for design of supplemental dampers (VED) for use in multistory buildings. Optimization was based on the principle of minimizing a performance cost function that produces the most suitable minimal configuration of devices with the aim that they will maximize their (dampers') effect. The optimal linear control approach was used to determine the constant coefficients for the damping devices. They found that the structure without dampers has largest response and the response using design based on considering first mode only is almost identical to the optimal solution. They concluded that the single mode approach is suitable for tall structures subjected to earthquake load, for which the first mode is governing. They investigated the response of clustered supplemental dampers considering the possibility that it may not be feasible to provide damper at a particular floor/floors.

Wu and Ping Ou (1997)[3] studied the effects of torsional and translational responses which are not accounted in plane frame models and interstory drift. In the first case, translational response due to torsion was taken as the performance index for evaluating optimum number and location of supplemental dampers to control structural response. They employed transfer function matrix to construct the objective function. An iterative procedure was applied to evaluate optimum parameters. It was found that with the decrease in torsional response the maximum inter storey drift is also reduced. They also concluded that addition of excess dampers may not always results in better structural performance. Interstory drift caused by interstory translational displacements of the mass centre and inter storey torsional response of an unsymmetrical structure has been taken as performance index. Optimal locations for damping devices correspond to the position where the relative displacements of the structure are largest. A sequential procedure was proposed to seek optimum device locations for 3-D shear building. They found that symmetric structure with asymmetrically located supplemental dampers is no longer symmetric and hence its response must be calculated using 3-D model instead of plane model. They concluded that to have a symmetric structure with asymmetric placement of dampers, it is necessary that they should be placed at the locations those are closest to the geometric centre of the structure to reduce the torsional effect to minimum degree.

Takewaki (1997)[4] proposed an effective and systematic procedure for finding the optimal damper placement to minimize the sum of the transfer function evaluated at undamped fundamental natural frequency of a structural system subjected to a constraint on the sum of the damping coefficients of added dampers. A systematic algorithm for optimal damper placement was proposed for structural system with an arbitrary damping (proportional/non-proportional). The amplitude of the transfer function of an inter-storey drift evaluated at the undamped fundamental natural circular frequency was treated as the controlling quantity. It was found that the optimal dampers locations in a building, with uniform distribution of mass and stiffness properties, correspond to those storeys where the largest inter-storey drifts were attained in the initial design.

Shukla and Datta (1999)[5] studied the seismic response of multi-storey shear building with optimally placed VEDs. Optimal locations of dampers were determined with respect to a controllability index related to root mean square value of inter-storey drift of a multi-storey building. They argued that a passive controller is optimally located if it is placed at a position where the displacement response of the uncontrolled structure is largest i.e. best position for first damper is found from uncontrolled response to be the point with the maximum inter storey drift. They found that for optimally placed VEDs response reduction is not significant beyond certain number of VEDs. They also concluded that the choice of using more dampers of small capacity instead of a small number of large capacity to achieve the economy is inferior.

Takewaki *et. al.* (1999)[6] proposed a procedure for finding the optimal locations of dampers in 3-D shear building. The inter-storey drift in an undamped fundamental mode of vibration was taken as the performance index, which was then minimized with a constraint on sum of the damping coefficients of supplemental dampers. Optimum positioning of supplemental dampers was determined using steepest direction search algorithm, to obtain damper position sequentially for gradual increase in the dampers capacity levels (sum of damping coefficients). They found that for a 3-D shear building, increase in number of additional dampers need not always reduce structural response.

Zhang and Soong (1990)[7] studied the seismic performance enhancement of a symmetric building associated with the application of visco-elastic dampers (VEDs). A sequential procedure for optimization of VED location based on the concept of minimizing the performance index was proposed. The optimal location for placing a damper was supposed to be one where displacement response of the uncontrolled structure was maximum. It was found that addition of each damper modified the response of the structure and optimum locations were a function of the excitation.

Singh and Moreschi (2002)[8] studied the optimum placement of viscoelastic dampers to achieve desired performance of structure under the earthquake load. They used genetic algorithm to obtain the optimal size and location of supplemental dampers. The objective function for optimization was taken as 60% reduction in base shear, 50% reduction in floor acceleration and 65% reduction in storey drift and it was found that 72, 65, and 65 numbers of dampers are necessary to accomplish respective performance indices with totally different distribution throughout the structure. They found that, same numbers of dampers are obtained for nearly same percentage of reduction of acceleration and storey drift, indicating a strong correlation.

Moreschi and Singh (2002)[9] studied the utilization of the viscous or visco-elastic dampers in an optimal manner to achieve the best response reduction in structures. A gradient-based approach was used for optimal design of VEDs. The RMS value of displacement response of a system was taken as the performance index with the constraint to minimize the difference between the summation of coefficients of added dampers and total amount of damping distributed throughout the system. A 40% reduction in objective function was achieved with a total of 37 dampers in a 24-storeyed building.

Singh and Moreschi (2003)[10] carried out a study on optimal design of yielding metallic dampers (YMDs) and friction dampers. A methodology was presented to determine the optimal design parameters for the devices installed at different locations in a building for the desired performance objective. It was found that the optimal damping parameters were different for different storeys of a 10-storey example building. Moreover, it was found that the use of RMS value of floor acceleration as objective function to be minimized was more effective than a weighted sum of the RMS floor acceleration and inter-storey drifts.

1.3 Scope and Organisation of Dissertation

All of the above mentioned studies on optimal placement of dampers are aimed at determining the optimum number of dampers and their placement in a given structural system. However, it is often the case that only a limited number of dampers are available for use due to budgetary constraints and it is worthwhile to determine the best possible locations for a certain specified number of dampers. Moreover, the effects of soil-structure interaction on the optimal damper placements need to be investigated for a more rational assessment of response reduction due to addition of dampers. These two issues form the thrust of the present dissertation. First, a problem for optimal location of a specified number of dampers is formulated for the simple test case of a multi-storey building. The problem formulation is described in Chapter 2 on Modeling and Analysis. The effects of dynamic soil-structure interaction are also considered in the models developed in this chapter. The formulation of the optimal damper placement problem and the solution is discussed in Chapter 3 on Optimization. The results of the optimization procedure and their discussions are presented in Chapter 4 on Results and Discussions. Chapter 5 concludes the dissertation with the conclusions of this study.

Modeling and Analysis

2.1 Introduction

The primary objective of this dissertation is to develop an optimization procedure for deriving maximum possible benefit from the placement of a given number of dampers in a structure for reduction in vibration response. In this regard, a 10-storey building has been considered as a typical structure for testing out the proposed optimization procedure. The choice of a multi-storey building as an example structure also helps in deriving some useful conclusions regarding enhancement in seismic withstand capacity of existing buildings by provision of supplemental dampers.

2.2 Model of Example Building

A 10-storey shear building as shown in Fig. 2.1 has been considered for the parametric study. It is assumed that the center of stiffness (CS) and center of mass (CM) for all floors individually align in the same vertical line, *i.e.*, the eccentricities at all floor levels are identical. There are 3 degrees of freedom at each floor, namely, the translations in X and Y directions and a rotation in the XY plane about the vertical axis passing through the center of mass. It is further assumed that the dampers can only be placed as diagonal

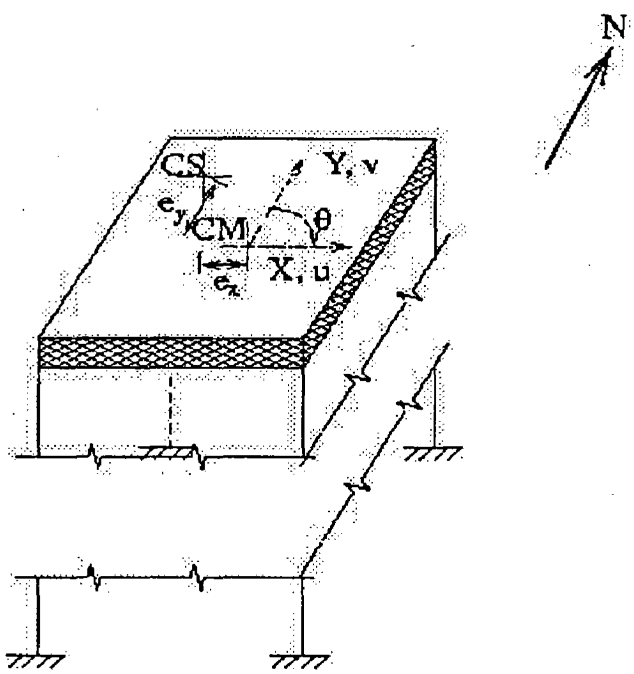

Figure 2.1: Schmatic diagram of example building

braces along any of the building face at any floor. Thus the damper location can be uniquely specified by a combination of the face (North, South, East, or West) and storey level identifiers. This notation has some practical advantage in the formulation of optimization problem as discussed in the next chapter. The physical parameters of the example building are as follows:

a) column sizes: 0.3m x 0.3m
b) column height: 3.0m
c) depth of floor slab: 0.25m
d) plan dimensions: 3m x 3m
e) 5% damping in all modes before the installation of dampers

A bi-directional excitation by ground acceleration, characterized by the 5% damped response spectrum for rocky site as specifed in IS-1893(Part 1):2002 has been considered. The inertia and stiffness matrices for a storey of the building can be formulated by using the equilibrium equations. The global matrices for inertia and stiffness may then be derived by a suitable assembly of storey-level matrices. The modal damping of 5% in each mode of vibration has been assumed and for this assumption an appropriate damping matrix can be formulated by using the undamped mode shapes, natural frequencies and the desired modal damping ratio. The equation of motion can then be written as

$$M\ddot{U} + C\dot{U} + KU = -M\,r\,\ddot{u}_g$$

where, M, C, and K represent the global inertia, damping and stiffness matrices, U represents the vector of degrees of freedom, r is the matrix of rigid body influence coefficients and $\ddot{u}_g$ denotes the vector of instantaneous ground acceleration values in each of the two orthogonal directions in horizontal plane. Assuming a linear behaviour, the probable maximum value of any response quantity of interest may be estimated by the response spectrum method using mode-superposition.

2.3 Soil-Structure Interaction

The soil-structure interaction can significantly influence the dynamic response of strucutres. These effects can be very pronounced in cases where the superstructure is much rigid in comparison to the strata supporting the foundations, or when the structural foundations are very massive and rigid in comparison to the neighbouring soil. In both of these situations the vibrations of structure and soil present a complex interacting system, wherein the response of one sub-system affects the response of the other sub-system. This phenomenon, in which the vibrations of soil influence the dynamic response of structure and vice-versa, is referred to as *soil structure interaction* (SSI). The SSI effects can be broadly classified into two categories:

a) *Kinematic interaction*: If the foundations are very massive and very rigid in comparison to the neighbouring soil deposits, deformations at the soil-foundation interface are constrained as the foundation cannot deform by the same amount as the soil. Therefore, rigid foundations act as a low pass filter by averaging out the high frequency components in seismic motions due to kinematic constraints imposed by the rigid foundation. This modification of the motion at the soil-foundation interface is only due to kinematic constraints on the propagation of elastic waves in a elastic medium and dynamic response of structure has no role to play. The actual seismic input motion to the structural foundation is the result of kinematic interaction analysis considering only the geometry and stiffness properties of structural foundation and soil. This effect is significant only in the case of massive, rigid foundations embedded in soils. For surface footings these effects are generally negligible in comparison to the inertial effects.
b) *Inertial interaction*: The other form of SSI effects involves the deformations and stresses in the supporting soil induced due to the vibrations of superstructure. The ensuing deformation of soil further leads to a modification of the dynamic response of structural system and thereby creating a dynamically interactive system.

Only inertial interaction effects have been considered in this study. The parameters of the soil-foundation system considered in the analysis are as follows:

a) Soil has been modeled as a visco-elastic halfspace medium and is represented by equivalent frequency-independent translational and rotational springs and dashpots attached to the foundation basemat. The coefficients of soil springs and dashpots have been estimated from approximate analytical expressions for frequency-dependent impedance functions for circular footings resting on homogeneous halfspace.

b) Poisson ratio for the soil medium is 0.3.

c) The flexibility of soil, as characterized by the shear wave velocity (V_S), is varied from 300 m/s (soft) to 4000 m/s (very stiff) for parametric study.

d) The basemat is assumed to be circular with radius 3.0 m and 0.45 m depth.

e) The embedment ratio (E/R) is assumed to be 2.0.

The analytical model for building including SSI effects is similar to the one developed for rigid base structure except that additional 3 degrees of freedom (related to translation and twist of the basemat) enter the formulation. The inertia properties of the basemat contribute to the inertia matrix for the foundation level while the stiffness contribution comes from the building columns and soil springs. These stiffness and damping parameters for soil have been derived from the values of the frequency dependent impedance functions (Pais and Kausel (1988)[12]) evaluated at the first mode frequency of the structure as a first order approximation. The real part of the impedance function contributes to the stiffness of soil spring, while the imaginary provides the damping coefficient. It may be noted that by augmenting the structure damping matrix by the damping terms contributed by soil dashpots, the system damping matrix can no longer be treated as of the classical form, *i.e.*, the damping matrix can not be diagonalized by using the undamped mode shapes in mode-superposition analysis. Therefore, the response spectrum method for non-classically damped system, as proposed by Singh (1980)[11] has been used for dynamic analysis.

2.4 Dynamic Analysis of Non-classically Damped Systems

If the system damping matrix is non-classical, the state-space formulation is used by transforming a system of N-coupled second order ordinary differential equations into a system of $2N$-coupled first order ordinary differential equations. For this, we define a vector of generalized coordinates $y = \begin{bmatrix} U & \dot{U} \end{bmatrix}^T$ and the equation of motion can be transformed as

$$A \dot{y} + B y = f$$

where, $A = \begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}$, $B = \begin{bmatrix} 0 & -I \\ C & K \end{bmatrix}$, and $f = \begin{Bmatrix} 0 \\ -M r \ddot{u}_g \end{Bmatrix}$. The associated free vibration problem can be given by for the ith mode. Here, p_i $p_i A y^{(i)} + B y^{(i)} = 0$ and $y^{(i)}$ respectively denote the complex eigenvalue and corresponding complex eigevector for the ith mode. The pair of complex conjugate eigenvalues of the first order system are related to the undamped natural frequency and modal damping ratio of the associated second order system by the following relations:

$$\omega_i = (a_i^2 + b_i^2)^{0.5} \quad \text{and} \quad \zeta_i = a_i / \omega_i$$

where, a_i and b_i respectively denote the real and imaginary parts of the complex eigenvalue p_i of the ith mode of first order system, while ω_i and ζ_i represent the undamped natural frequency and modal damping ratio for the ith mode of vibration of the second order system. The response spectrum method of analysis for non-classically damped systems as proposed by Singh (1980)[11] has been used to estimate the maximum probable base shear and the maximum probable inter-storey drifts.

Chapter 3

Optimization

3.1 Introduction

Optimization is the process of getting the best result under given circumstances. Ultimate goal of all managerial or technological decisions taken at several stages is to either minimize the effort required or maximize the desired benefit. In short, optimization is best possible allocation of available resources to achieve the feasible solution. Mathematically, optimization is defined as the process of finding the condition that gives the maximum or minimum value of the function. No single method of optimization is suitable for solving all optimization problems. Hence, a number of optimization techniques have evolved for solving different types of problems over the years.

Optimization methods are broadly categorized in two types viz. constrained and unconstrained methods of optimization. In constrained method, objective function is subjected to design and functional constraints. First type of constraints are the restrictions those must be satisfied in order to produce an acceptable design and latter constraints represents the limitation on the behaviour or performance of the system. When minimization or maximization of objective function is carried out without any restrictions or constraints then applied optimization method is referred to as unconstrained.

3.2 Objective function

The conventional design procedures are aimed at finding an acceptable or adequate design. There could be more than one acceptable design. The purpose of optimization is to choose the best out of the many acceptable designs available. Thus, some criteria has to be chosen to select the best design amongst the several alternate designs. The criterion with respect to which the design is to be optimized when expressed as a function of the design variables is known as objective function. The choice of objective function for minimization/maximization depends upon nature of problem. It could be strength/serviceability or cost based.

In some situations, there may be more than one criterion to be satisfied simultaneously. An optimization problem involving multiple objective functions is known as a multi-objective programming problem. Thus if $f_1(X)$ and $f_2(X)$ denote two different design criteria depending on the same set of design variables X, a composite objective function can be constructed as weighted sum of the two as:

$$f(X)=c_1 f_1(X)+c_2 f_2(X)$$

where, c_1 and c_2 are suitable scaling parameters which can be appropriately selected to introduce a bias in the composite objective function, if so required. These parameters need to be selected carefully to avoid accidental dominance of one term over the other.

Since the design of civil engineering structural systems is governed by the dual criteria of strength and seviceability conditions, a composite index accounting for both of these factors is considered to be a good choice for deciding the optimal locations of dampers. Therefore the objective function is taken to be a weighted sum of maximum base shear and maximum inter-storey drift in the building.

$$f=\frac{V}{Vu}+\frac{\mathrm{SD}}{\mathrm{SDu}}$$

where, V = maximum base shear in the structure after the placement of current damper in position, Vu = maximum base shear in the structure without any supplemental damper, SD = maximum inter-storey drift in the structure after the placement of current damper in position, and SDu = maximum inter-storey drift in the structure without any supplemental damper.

3.3 Optimization technique

The optimization procedure invloves the search for the best location for a damper in the structure. For automating the search process, an automated system of modification of the system damping matrix following the placement of a damper in a trial position is necessary. This is achieved by the use of a 2-dimensional damper location matrix with 4 rows and 10 columns. The row index corresponds to the face (North, South, East, or West) of the building and the column index corresponds to the building storey level. A value of '1' at any location of this matrix indicates the presence of a damper in the corresponding storey and building face, whereas a '0' value indicates the absence of a supplemental damper in the designated location. A damping matrix with dampers installed in all possible locations is derived from the equilibrium considerations. An appropriate damping matrix for some arrangement of supplemental dampers can then be obtained by weighing the contributions of all individual dampers with the respective entry in the damper location matrix. This damping matrix due to supplemental dampers is then added to the original damping matrix of the structure (derived for 5% modal damping). Thus the seismic response of structure with any arbitrary number of damper installations can be readily evaluated automatically as required for any optimization procedure.

Since the optimization procedure now involves a discrete search amongst patterns of '0's and '1's in the damper location matrix, none of the gradient-based optimization methods can be used. Moreover, to facilitate a sequential search of optimal locations for a given number of dampers, a sequential search algorithm like that of Hooke and Jeeves' method is well suited. The Hooke and Jeeves' method essentially consists of a sequence of two steps (i) to explore the objective function variation in a specific search direction in the local neighbourhood of the current configuration, and (ii) a search for the maximum step that can be taken in a favourable direction as identified by the exploratory move in the first step (Rao (1996)[13]) . For the problem of optimal damper locations, the exploratory move involves repeated placement of a damper in various available feasible locations and investigating the variation in the objective function. The optimal damper location is one for which the objective function is the minimum of all pattern searches for the placement of current damper. There is, therefore, no scope for any pattern move and hence has been dispensed with in this study. Once a best possible location has been identified for a damper, the pattern of damper location matrix is preserved. In the next cycle, for the search of optimal location of the next damper, only those location are considered to be feasible which have a '0' entry in the previously preserved damper location matrix. This cycle is repeated till either the given number of dampers, or the available feasible locations have been exhausted.

Results and Discussions

A model 10-storeyed shear building has been considered to investigate the optimal location of a specified number (say, 5) of (fluid viscous) dampers for seismic response reduction. A detailed parametric study to investigate the effects of plan irregularities, as charaterized by eccentricities, and compliance of the soil-foundation system has been performed. The optimal placement of dampers in a symmetric building on rigid foundation is shown in Table 4.1. In this table a '1' in a cell represents the presence of an external damper in the corresponding storey and face of the building, on the other hand a '0' entry represents the absence of an external damper. A 37% reduction in the objective function was achieved in this case for the optimal placement of 5 dampers. Simlar results for the case of un-symmetrical building with eccentricies of (i) e_x=0 and e_y=0.05L, (ii) e_x=0.075L and e_y=0, and (iii) e_x=0.075L and e_y=0.05L are shown in Tables 4.2–4.4. Here, L refers to the maximum plan dimension of the building. The maximum reduction in the objective funtions for 5 optimal damper placements is found to be (i) 32.9%, (ii) 31.6%, and (iii) 29.2% for each of the three example cases of plan irregularity.

Table 4.1: Damper location matrix for a 10-storeyed symmetric building

Storey	1	2	3	4	5	6	7	8	9	10
N	0	0	0	0	0	0	0	0	0	1
S	0	1	0	0	0	0	0	0	0	0
E	0	0	0	1	0	0	0	0	0	0
W	0	1	1	0	0	0	0	0	0	0

Table 4.2: Damper location matrix for e_y=5%L

Storey	1	2	3	4	5	6	7	8	9	10
N	0	0	0	1	0	0	0	0	0	1
S	0	0	0	0	0	0	0	0	0	0
E	0	1	0	0	0	0	0	0	0	0
W	0	0	1	0	0	0	0	0	0	1

Table 4.3: Damper location matrix for e_x=7.5%L

Storey	1	2	3	4	5	6	7	8	9	10
N	0	1	0	0	0	0	0	0	0	1
S	0	0	1	0	0	0	0	0	0	0
E	0	0	0	1	0	0	0	0	0	0
W	0	0	0	0	0	0	0	0	0	1

Table 4.4: Damper location matrix for e_x=7.5%L and e_y=5%L

Storey	1	2	3	4	5	6	7	8	9	10
N	0	1	0	0	0	0	0	0	0	0
S	0	0	0	1	0	0	0	0	0	1
E	0	0	0	0	0	0	0	1	0	0
W	0	0	1	0	0	0	0	0	0	0

The dampers appear to be more effective in the lower storeys in the case of symmetric building, on the other hand, as the plan irregularities increase the effectiveness of dampers increases in the intermediate storeys. This shift in the optimal damper placement pattern is caused due to the increased contribution of higher (torsional) modes in the seismic response of unsymmetric building. Figures 4.1 and 4.2 show the variation of modal damping for the first three modes of the 4 building types on rigid foundation and on soft soil (V_S=300 m/s). It can be seen that with placement of supplemental dampers in the building, the modal damping ratio increases and tends to saturate quickly as evident from the *flattening* of the damping ratio curve as the number of dampers increase in Fig. 4.1. Moreover, with saturation of modal damping in lower modes, further addition of dampers contributes to the increase in the modal damping in higher modes.

The effect of flexible foundations on the optimal damper layout has been investigated for different foundation conditions as characterized by the shear wave velocity (V_s) of the soil strata, which in turn is idealized as a linear visco-elastic halfspace. A variation from 150 m/s to 4000 m/s of V_s is considered to study the effect of different foundation compliance conditions on the optimal damper layout in all four cases, namely, (i) symmetric building, (ii) building with eccentricities e_x=0 and e_y=0.05L, (iii) e_x=0.075L and e_y=0, and (iv) e_x=0.075L and e_y=0.05L. The results of this parametric study have been summarized in Tables 4.5–4.8, respectively for each of the above mentioned cases. In all cases, it has been observed that the effectiveness of the supplemental dampers in reducing the seismic response diminishes with increasing foundation compliance. Moreover, the maximum number of dampers required to reach the saturation point (beyond which there is no significant reduction in objective function) also increases with increasing flexibility of the soil-foundation system. Further, for a given number of dampers, the total response reduction in an unsymmetric building is always less than that in the case of symmetric building for all soil conditions. The effectiveness of dampers seems to reduce with the increase in plan irregularity as well as with increase in foundation compliance. It has also been found that the shift in the

Table 4.5:Summary of results for a symmetric building

Shear wave velocity (m/s)	Number of available dampers	% reduction in objective function	Max. reduction in objective function till saturation (%)	Max. number of dampers placed
300	5	15.00	26.02	11
600	5	17.95	32.72	10
900	5	19.52	36.79	10
1200	5	23.70	38.01	10
1500	5	24.65	39.79	9
2000	5	26.09	42.69	9
2500	5	29.15	44.01	8
3000	5	30.05	45.14	8
3500	5	31.40	45.72	8
4000	5	36.75	46.14	7
∞ Fixed Base	5	37.15	46.89	7

optimal location of dampers from lower storeys in the case of symmetric building to the intermediate storeys storeys in the case of unsymmetric buildings is more pronounced in the case of flexible foundations (as infered from the optimal damper location patterns given in Table 4.9.

Table 4.6:Summary of results for a building with e_y=5%L

Shear wave velocity in m/s	Number of available dampers	% reduction in objective function	Max. reduction in objective function till saturation (%)	Max. number of dampers placed
300	5	13.95	18.42	10
600	5	15.45	24.79	10
900	5	16.30	27.02	10
1200	5	18.45	31.45	10
1500	5	22.85	34.89	9
2000	5	25.15	36.02	9
2500	5	26.60	39.78	9
3000	5	29.20	41.82	8
3500	5	31.25	43.79	8
4000	5	32.30	44.27	8
∞ Fixed Base	5	32.90	44.79	8

Table 4.7:Summary of results for a building with e_x=7.5%L

Shear wave velocity in m/s	Number of available dampers	% reduction in objective function	Max. reduction in objective function till saturation (%)	Max. number of dampers placed
300	5	10.40	17.01	10
600	5	16.15	22.72	10
900	5	17.30	26.17	10
1200	5	20.90	28.65	10
1500	5	24.40	32.72	9
2000	5	26.15	34.12	9
2500	5	29.20	37.17	8
3000	5	30.85	39.41	8
3500	5	31.25	42.01	8
4000	5	31.45	41.99	8
∞ Fixed Base	5	31.60	42.29	8

Table 4.8: Summary of results for a building with e_x=7.5%L, e_y=5%L

Shear wave velocity in m/s	Number of available dampers	% reduction in objective function	Max. reduction in objective function till saturation (%)	Max. number of dampers placed
300	5	9.87	14.56	11
600	5	14.25	19.47	10
900	5	16.20	22.97	10
1200	5	18.87	26.65	10
1500	5	21.26	29.12	10
2000	5	23.47	31.82	9
2500	5	26.20	36.17	8
3000	5	29.52	37.89	8
3500	5	30.04	39.69	8
4000	5	30.88	40.72	8
∞ Fixed Base	5	31.02	41.02	8

Table 4.9: Damper location matrix for e_x=7.5%L, V_S=300 m/s

Storey	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
N	0	0	1	0	0	0	0	0	1	0	0
S	0	0	0	0	0	0	0	0	0	0	1
E	0	1	0	0	0	0	1	0	0	0	0
W	0	0	0	0	1	0	0	0	0	0	0

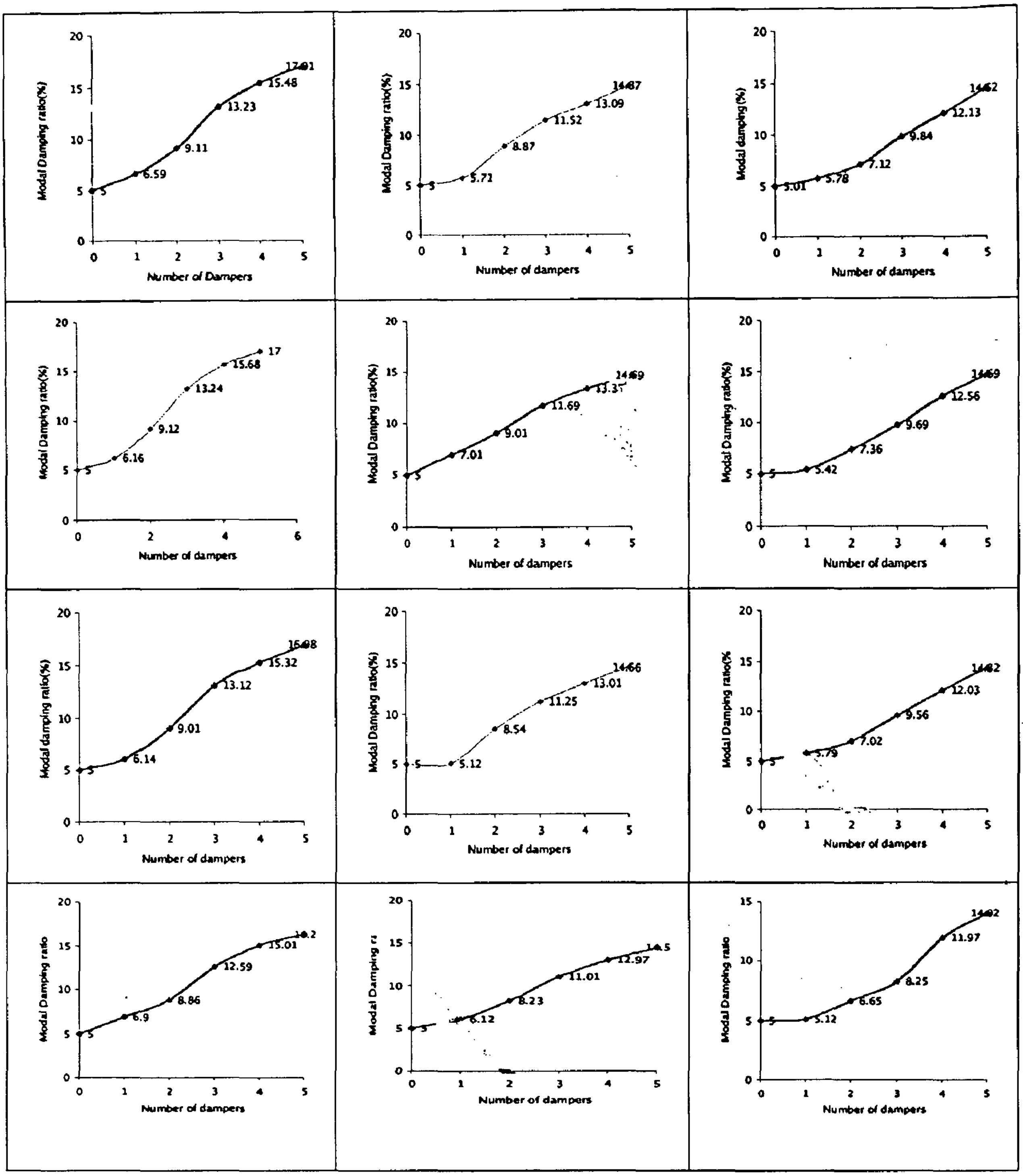

Figure 4.1: Modal damping variation in first three modes for a fixed base symmetric and unsymmetric buildings (First row: Symmetric, Second row: e_x=7.5%L, Third row: e_y=5%L, Fourth row: e_x=7.5%L and e_y=5%L, First column: First mode, Second column: Second mode, Third column: Third mode)

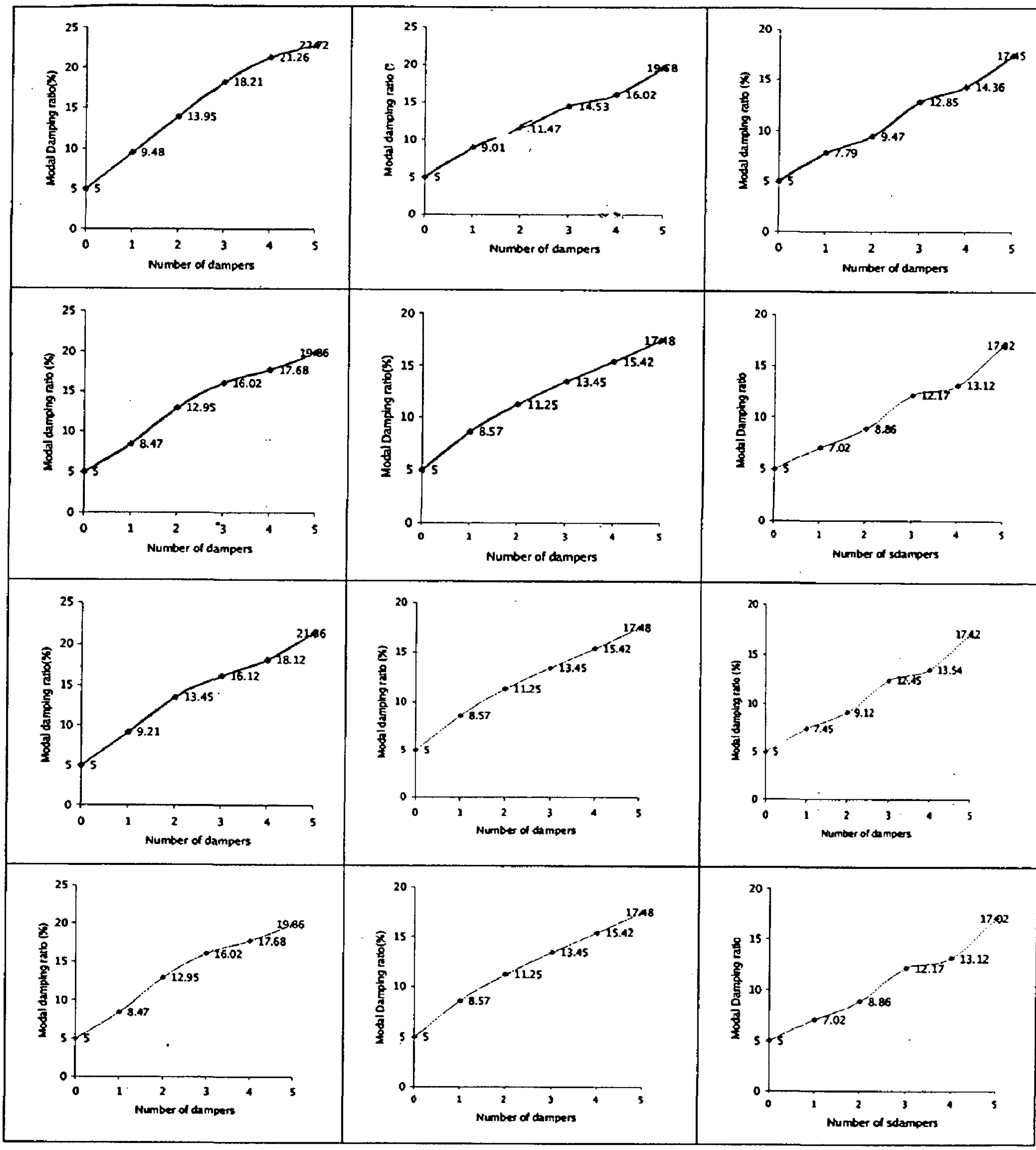

Figure 4.2: Modal damping variation in first three modes for symmetric and unsymmetric buildings on soft soil V_s=300 m/s (First row: Symmetric, Second row: e_x=7.5%L, Third row: e_y=5%L, Fourth row: e_x=7.5%L and e_y=5%L, First column: First mode, Second column: Second mode, Third column: Third mode)

Chapter 5
Conclusions

Based on the study conducted on a 10-storey model building with different plan irregularities and founded on different soil types to determine the optimal damper locations, the following conclusions can be drawn:

1. Dampers are effective in reducing the seismic response of a symmetric building. Its effectiveness reduces as the plan irregularity increases.
2. In a symmetric building, effectiveness of dampers is found to be maximum in the lower storeys and in intermediate storeys as unsymmetry increases.
3. Reduction in seismic response with addition of supplemental dampers is not very significant for buildings resting on compliant soils. The effectiveness of supplemental dampers increases as the soil stiffness increases.

Increase in number of dampers in a building beyond a certain limit does not lead to any further reduction in the seismic response of a building. This limiting number of dampers, however, depends on structural configuration and soil flexibility.

References

1. Li, Z.X. and He, Y.A. "Optimal damper control for 3-dimensional tall buildings under earthquake excitations", *Tenth World Conference on Earthquake Engineering, Barcelona, Spain,* 4159–4164, 1992.
2. Gluck, A. and Reinhorn, A. "Design supplemental dampers for control of structure", *Journal of Structural Engineering, ASCE,* 122 (12), 1394–1399, 1996.
3. Wu, B. and Ou, J.P. "Optimal placement of energy dissipation devices for three dimensional structures", *Engineering Structures,* 30 (2), 113–125, 1997.
4. Takewaki I. "Optimal damper placement for minimum transfer function", *Earthquake Engineering and Structural Dynamics,* 26, 1113–1124, 1997.
5. Shukla, A. K. and Datta, T.K. "Optimal use of visco-elastic dampers in building frame for seismic force", *Journal of Engineering, ASCE,* 125(4), 401–409, 1999.
6. Takewaki I, Yoshitomi S, Uetani K. and Tsuji M. "Non-monotonic optimal damper placement via steepest direction search", *Earthquake Engineering and Structural Dynamics, 28,* 665–670, 1999.
7. Zhang R. and Soong T. " Seismic Design of visco-elastic dampers for structural applications", *Journal of Structural Engineering, ASCE,* 118(5), 1375–1392
8. Singh M.P. and Moreschi L. M. "Optimal design of structures with added linear visco-elastic devices", *Earthquake Engineering and Structural Dynamics,* 28, 1113–1124, 2002.
9. Singh M.P. and Moreschi L.M. "Optimal placement of dampers for passive response control", *Earthquake Engineering and Structural Dynamics,* 31:955–976, 2002.
10. Singh, M.P. and Moreschi, L. "Design of yielding metallic and friction dampers for optimal seismic performance", *Earthquake Engineering and Structural Dynamics, 32,* 1291–1311, 2003.
11. Singh, M.P. "Seismic Response by SRSS for non-proportional damping" *Journal of Engieering Mechanics Division, Proceedings of the American Society of Civil Engineers",* 106, (EM6), December 1980.
12. Pais, A. and Kausel, E. "Approximate formulas for dynamic stiffnesses of rigid foundation", *Soil Dynamics and Earthquake Engieering,* 7(4), 213 –226, 1988.
13. Rao, S.S. *Engineering Optimization,* New Age International (P) Ltd., Publishers, New Delhi, 1996.

Appendix A

System Matrix Formulation

Degrees of freedom are assumed to be located at the center of mass. These degrees of freedom are as shown in Figure 2.1. The DOFs u and v are translational degrees of freedom and θ is the torsional degree of freedom.

Floor mass matrix :

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_o \end{bmatrix} \text{ where}$$

m is the mass of the floor and I_o is the mass moment of inertia about the Z axis.

Floor Stiffness Matrix:

$$K = \begin{bmatrix} \sum_{i=1}^{2}(K_{i1}+K_{i3}) & 0 & \sum_{i=1}^{2}(K_{i1}-K_{i3})(\frac{L}{2}) \\ 0 & \sum_{i=1}^{2}(K_{i2}+K_{i4}) & \sum_{i=1}^{2}(K_{i2}-K_{i4})(\frac{L}{2}) \\ \sum_{i=1}^{2}(K_{i1}-K_{i3})(\frac{L}{2}) & \sum_{i=1}^{2}(K_{i2}-K_{i4})(\frac{L}{2}) & \sum_{i=1}^{2}\sum_{j=1}^{4}(K_{ij})(\frac{L^2}{2}) \end{bmatrix}$$

Eccentricities are implicitly accounted in this formulation in terms of stiffness.
In order to produce eccentricity e in the building along an axis, the required total stiffness of columns on stiffer side can be derived in terms of the stiffness of columns on the side as,

$$K_s = \frac{(50+e)}{(50-e)} K_f$$

where, K_s is the total stiffness of columns on stiff side, and K_f is the stiffness of columns on flexible side, and e is the eccentricity expressed in percent.

Damping Matrix:

$$C = \begin{bmatrix} \sum_{i=1}^{2}(C_{i1}+C_{i3}) & 0 & \sum_{i=1}^{2}(C_{i1}-C_{i3})(\frac{L}{2}) \\ 0 & \sum_{i=1}^{2}(C_{i2}+C_{i4}) & \sum_{i=1}^{2}(C_{i2}-C_{i4})(\frac{L}{2}) \\ \sum_{i=1}^{2}(C_{i1}-C_{i3})(\frac{L}{2}) & \sum_{i=1}^{2}(C_{i2}-C_{i4})(\frac{L}{2}) & 0 \end{bmatrix}$$

System Matrices for Soil-Structure Interaction:

The spring stiffnesses are evaluated at fundamental natural frequency of undamped system. Expressions for soil spring stiffnesses as given by Pais and Kausel(1988) [12] for a footing resting on the surface.

Translational Springs:

$$K^s_H = K^o_H(1+\frac{E}{R})$$

$$K^o_H = \frac{8GR}{(2-\nu)}$$

$$K^d_H = K^s_H(k+i\,a_o c)$$

$$k = 1.0$$

$$c = \pi \frac{[1.0+(1.0+\alpha)\frac{E}{R}]}{(\frac{K^s_H}{GR})}$$

$$\alpha = \frac{V_p}{V_s}$$

where, K^d_H is the dynamic stiffness of horizontal spring, K^s_H is the static stiffness of horizontal spring, V_p and V_s are P and S wave velocities in the soil, R is the radius of the foundation. G Shear modulus of the soil, ν Poisson's ratio of the homogeneous half space, E/R is the embedment ratio, E being the depth of foundation

Torsional spring:

$$K_T^s = K_T^o(1 + 2.67\frac{E}{R})$$

$$K_T^o = \frac{16GR^3}{3}$$

$$K_T^d = K_T^s(K + i\,a_o c)$$

$$k = 1.0 - \frac{(0.35\,a_o^2)}{(1.0 + a_o^2)}$$

$$c = \frac{\pi}{2}(1 + 4.0\frac{E}{R})a_o^2/[(b + a_o^2)\frac{(K_T^s)}{GR^3}]$$

$$b = \frac{1}{(0.37 + 0.87(\frac{E}{R})^{0.67})}$$

where,

K_t^d and K_t^s are respectively the dynamic and static stiffness of torsional spring, E and R are the embedment depth and radius of the foundation base mat.

Stiffness matrix:

Knowing the translational and torsional spring stiffnesses, a stiffness matrix corresponding to the soil-springs can be constructed as,

$$K = \begin{bmatrix} K_d^H & 0 & 0 \\ 0 & K_d^H & 0 \\ 0 & 0 & K_T^d \end{bmatrix}$$

Mass matrix:

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_o \end{bmatrix}$$

where m and I_o are the appropriate mass and mass moment of inertia about vertical axis passing through the center of mass of base mat.

Response Spectrum Analysis for Non-classically Damped Systems

Response Spectrum Analysis for a System with Classical Damping Matrix:

Governing equation of motion for a multi degree freedom system subjected to the ground motion is given by,

$$M \ddot{U}(t)+C \dot{U}(t)+K U(t)=-M r \ddot{U}g(t) \tag{B1}$$

where,

M, C and K are system mass, damping and stiffness matrices respectively and r is the static coefficient vector.

Eigenvalue problem:

Expressing design response U as,

$$U=\Phi V \tag{B2}$$

where,

Φ is the function of space and V is the function of time

Eigenvalue problem for undamped MDOF system is formulated as,

$$\lambda_i M \Phi = K \Phi \tag{B3}$$

λ is the spectral matrix and Φ is the modal matrix

Knowing the time period of vibration of a structure and modal damping ratio in each mode, pseudo spectral acceleration can be obtained from the given design response spectrum.

Peak displacement of a structure in *i*th mode can be calculated as,

$$\{U\}_i = A_i \phi_i \frac{(Sa(T_i, \zeta_i))}{\omega_i^2} \tag{B4}$$

where,

$X_i = \dfrac{(\phi_i^T M r)}{(\phi_i^T M \phi_i)}$ is the modal participation factor in *i*th mode of vibration.

Similarly, knowing the mode shapes, peak displacement can be computed and combined appropriately by SRSS method to get the design response.

Response Spectrum Analysis of a Non-Classically Damped System:

A modal damping ratio prior to installation supplemental damper is assumed to be constant in each mode of vibration.

Formulation of a structural damping matrix:

Assuming that the damping matrix C is a classical damping matrix, we have from modal analysis,

$$C = \Phi^T c \Phi = \begin{bmatrix} 2M_1\omega_1\zeta_1 & 0 & 0 \\ 0 & 2M_i\omega_i\zeta_i & 0 \\ 0 & 0 & 2M_n\omega_n\zeta_n \end{bmatrix} \tag{B5}$$

where, Φ is the modal matrix.

knowing the natural frequency of vibration in each mode(ω_i) and taking constant modal damping ratio (ζ_i) as 5% in each mode C matrix thus obtained is easily used to compute the structural damping matrix c as,

$$c = (\Phi^T)^{-1} C \Phi^{-1}$$

this structural damping matrix when added to damping matrix formulated for added supplemental dampers gives total damping matrix. This damping matrix is often found to be non-classical i.e it cannot be diagonalized by undamped mode shapes. Hence, Response Spectrum Method of analysis for a non-classical damping matrix given by Singh[11] is used. Various steps involved in the analysis are summarized below:

State Space Formulation:

Governing equation of motion for a system with a non-classical damping subjected to the ground motion is given by equation A.1

A state-space formulation is used to transform a system of *N*-coupled second order ordinary differential equation into *2N*- coupled first order ordinary differential equation.

Defining generalized co-ordinates $y = \begin{Bmatrix} U \\ \dot{U} \end{Bmatrix}$.

$$A\dot{y} + By = f \tag{B6}$$

where,

$$A = \begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -I \\ C & K \end{bmatrix} \text{ and } f = \begin{Bmatrix} 0 \\ -Mr\ddot{U}g \end{Bmatrix} \tag{B7}$$

in which I is the $N \times N$ size identity matrix and A, B are the matrices of size $2N \times 2N$ (where N is the generalized degrees of freedom of the system) and f is the a vector of size $N \times 1$.

Free vibration solution of Eq. A.6 yields eigenvalues and eigenvectors of second order system. This requires the solution of following $2N \times 2N$ dimension eigenvalue problem,

$$p_i A\, y^{(i)} + B\, y^{(i)} = 0 \tag{B8}$$

Here, p_i and $y^{(i)}$ denote the complex eigenvalue and complex eigenvector respectively in ith mode of vibration.

The pair of complex conjugate eigenvalues of the first order system are related to the undamped natural frequency and modal damping ratio of second order system by following relation,

$\omega_i = (a_i^2 + b_i^2)^{0.5}$ and $\zeta_i = \dfrac{a_i}{\omega_i}$ where a_i and b_i are respectively the real and imaginary parts of complex eigenvalue p_i of first order system while ω_i and ζ_i respectively denotes the ith undamped natural frequency and modal damping ratio.

Using following transformation,

$y = \Phi z$ where Φ is the modal matrix. Using the orthogonal property of the eigenvectors with respect to matrices A and B, equation A.1 can be transformed into a decoupled set of equations as follows:

$A^x \dot{z} + B^x z = -\Phi^T f$ where,

$A^x = \Phi^T A \Phi$ and $B^x = \Phi^T B \Phi$ are the diagonal matrices.

SRSS Response spectrum method of analysis:

The formulation of SRSS method of analysis suggested by Singh(1980)[11] is given in the following.

$$R_x^2(u) = \sum_{j=1}^{j=n} 4\left[a_j F(\omega_j) + A_j\right] \frac{(I_1(\omega_j))}{\omega_j^2}$$

$$+ 2 \sum_{j=1}^{j=n} \sum_{k=j+1}^{n} \left[\frac{Q}{r^2} + F(\omega_j)\right] \frac{(I_1(\omega_j))}{\omega_j^2} + \left[S + F(\omega_k) R\right] \frac{(I_1(\omega_k))}{\omega_k^2} ¿ \tag{B9}$$

in which

$R_x^2(u)$ is the design response; $r = \dfrac{\omega_j}{\omega_k}$ and a_j and b_j are the real and imaginary parts of f vector.

$$I_1(\omega_j) = \frac{(R^2(\omega_j))}{(1+4\beta_j^2)} \;\; ; \;\; I_2(\omega_j) = I_1(\omega_j) - \frac{(\ln\omega_j - \ln\omega_l)}{(\ln\omega_u - \ln\omega_l)} A_g^2 \;\; ;$$

$$F(\omega_j) = \frac{(I_2(\omega_j))}{(I_1(\omega_j))}$$ where A_g is the maximum ground acceleration

$_l$ is the control frequency at which the ground spectra starts to drop

$_u$ is the frequency beyond which there is no spectral amplification of the ground motion

Constants P, Q, R and S are obtained from the solution of the following simultaneous equations:

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ u & 1 & s & 1 \\ v & u & t & s \\ 0 & v & 0 & t \end{bmatrix} \begin{Bmatrix} P \\ Q \\ R \\ S \end{Bmatrix} = \begin{Bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{Bmatrix}$$

where, the terms u, v,s,t, *and* W_1, W_2, W_3, and W_4 are defined as follows:

$$s = -2r^2(1-2\beta_j^2); \qquad t = r^4; \qquad u = -2r^2(1-2\beta_k^2); \quad v = 1;$$

$$C_1 = -(1+r^2-4\beta_j\beta_k r) \;\; ; \qquad C_2 = r^2 \;\; ; \qquad D_1 = 4a_j a_k \;\; ;$$

$$D_2 = 4r(a_j a_k \beta_j \beta_k + b_j b_k \sqrt{1-\beta_j^2}\sqrt{1-\beta_k^2} - a_j b_k \beta_k \sqrt{1-\beta_j^2} - b_j a_k \beta_j \sqrt{1-\beta_k^2}) \;\; ;$$

$$E_2 = -8(\beta_j r - \beta_k)[a_j a_k(\beta_k - \beta_j r) - (a_j b_k \sqrt{1-\beta_k^2} - b_j a_k r\sqrt{1-\beta_j^2})] \;\; ;$$

$$E_3 = -8r(\beta_k r - \beta_j)[a_j a_k(\beta_k - \beta_j r) - (a_j b_k \sqrt{1-\beta_k^2} - b_j a_k r\sqrt{1-\beta_j^2})] \;\; ;$$

$$W_1 = D_1 \;\; ; \;\; W_2 = C_1 D_1 + D_2 + E_2 \;\; ; \;\; W_3 = C_1 D_2 + C_2 D_1 + E_3 \;\; ; \;\; W_4 = C_2 D_2$$

A Parametric Study

The physical parameters of example symmetric building are as follows:

1. Column size 0.3m x 0.3m
2. Column height 3.0m
3. Floor thickness 0.15m
4. Plan dimensions 3m x 3m
5. Damping coefficient C = 1.55e5Ns/m for fluid viscous dampers
6. 5% damping in all modes before the installation of dampers

A bi-directional excitation by ground acceleration, characterized by the 5% damped response spectrum for rocky site as specified in IS-1893(Part I):2002 has been considered.

To study the effect of compliant soils on the response of a system and hence on optimal location of dampers, in addition to above parameters mentioned for a fixed based symmetric building following parameters are assumed.

1. Poisson ratio for the soil medium is 0.3
2. Flexibility of the soil, as reflected by the shear wave velocity (V_s) is varied from 300 m/s to 4000 m/s.
3. Average safe bearing capacity of the soil is assumed to be 450 KN/m^2.
4. Base mat assumed to be a circular with radius 1.2m. and depth 0.25m
5. Embedment ratio (E/R)=0

Results:

Table 1:Damping location matrix for a fixed base symmetric building

Face	Storey									
	1	2	3	4	5	6	7	8	9	10
N	0	0	1	0	0	0	0	0	0	1
S	0	1	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	1	0
W	0	1	0	0	0	0	0	0	0	0

Table 2:Damping location matrix for a fixed base unsymmetric building with e_x =5%L

Face	Stoery									
	1	2	3	4	5	6	7	8	9	10
N	0	1	0	0	0	0	0	0	0	1
S	0	0	0	0	0	0	0	0	0	0
E	0	0	1	1	0	0	0	0	0	0
W	0	0	0	0	0	0	0	0	0	1

Table 3: *Damping location matrix for a fixed base unsymmetric building with e_y= 7.5%L*

Face	Storey									
	1	2	3	4	5	6	7	8	9	10
N	0	0	0	0	0	0	0	0	0	1
S	0	0	0	1	0	0	0	0	0	0
E	0	1	0	0	0	0	0	0	0	0
W	0	0	1	0	0	0	0	0	0	1

Table 4: *Damping location matrix for a fixed base unsymmetric building with e_x =5%L and e_y=7.5%L*

Face	Storey									
	1	2	3	4	5	6	7	8	9	10
N	0	1	0	0	0	0	0	0	0	1
S	0	0	1	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	1	0	0
W	0	0	0	1	0	0	0	0	0	0

Table 5: *Summary of results for a symmetric building*

Shear wave velocity m/s	Number of available dampers	% reduction in objective function	Maximum response reduction till saturation(%)	Saturation limit
300	5	16.72	29.14	11
600	5	20.05	35.1	11
900	5	22.29	38.01	10
1500	5	27.69	42.69	10
2000	5	34.98	44.01	9
4000	5	40.06	48.35	8
Infinity	5	40.24	48.59	8

Table 6: *Summary of results for a unsymmetric building with e_x =5% L*

Shear wave velocity m/s	Number of available dampers	% reduction in objective function	Maximum response reduction till saturation (%)	Saturation limit
300	5	14.89	18.42	11
600	5	18.72	24.79	10
900	5	21.58	27.02	10
1500	5	25.68	34.89	9
2000	5	31.89	36.02	9
4000	5	36.45	44.98	8
Infinity	5	36.72	45:02	8

Table 7: *Summary of results for an unsymmetric building with e_y = 7.5%L*

Shear wave velocity m/s	Number of available dampers	% reduction in objective function	Maximum response reduction till saturation (%)	Saturation limit
300	5	12.03	16.58	11
600	5	16.15	22.72	11
900	5	19.84	24.2	10
1500	5	24.40	31.52	10
2000	5	28.96	35.01	9
4000	5	35.59	42.58	8
Infinity	5	35.79	42.58	8

Table 8: *Summary of results for an unsymmetric building with e_x = 5%L and e_y = 7.5%L*

Shear wave velocity m/s	Number of available dampers	% reduction in objective function	Maximum response reduction till saturation (%)	Saturation limit
300	5	10.87	16.08	12
600	5	13.99	21.35	11
900	5	17.89	22.98	10
1500	5	22.98	30.01	10
2000	5	26.12	33.9	10
4000	5	32.59	41.68	9
Infinity	5	32.79	41.72	9

Table 9:Damping location matrix for a unsymmetric building resting on a soil with e_x=5%L with V_s = 300 m/s

Face	Storey										
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
N	0	0	1	0	0	0	0	0	0	0	0
S	0	0	0	0	0	0	0	0	0	0	1
E	0	1	0	0	1	0	1	0	0	0	0
W	0	0	0	0	0	0	0	0	0	0	0

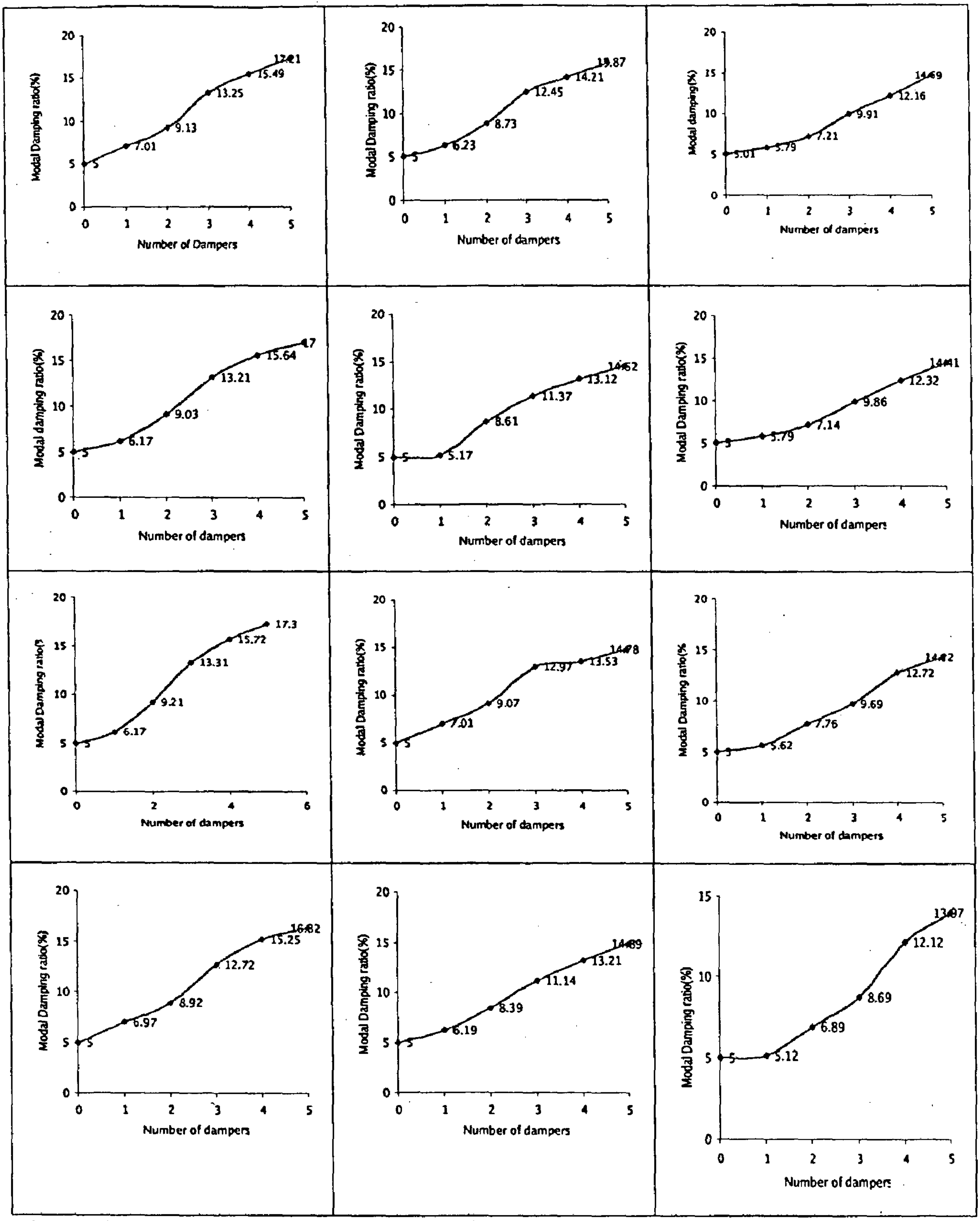

Figure 2: *Modal damping variation in first three modes for a fixed base symmetric and unsymmetric buildings (First row: Symmetric, Second row:* $e_x = 5\%L$, *Third row:* $e_y=7.5\%L$ *and Fourth row:* $e_x=5\%L$ *and* $e_y=7.5\%L$. *First column: First mode, Second column: Second mode and Third column: Third mode)*

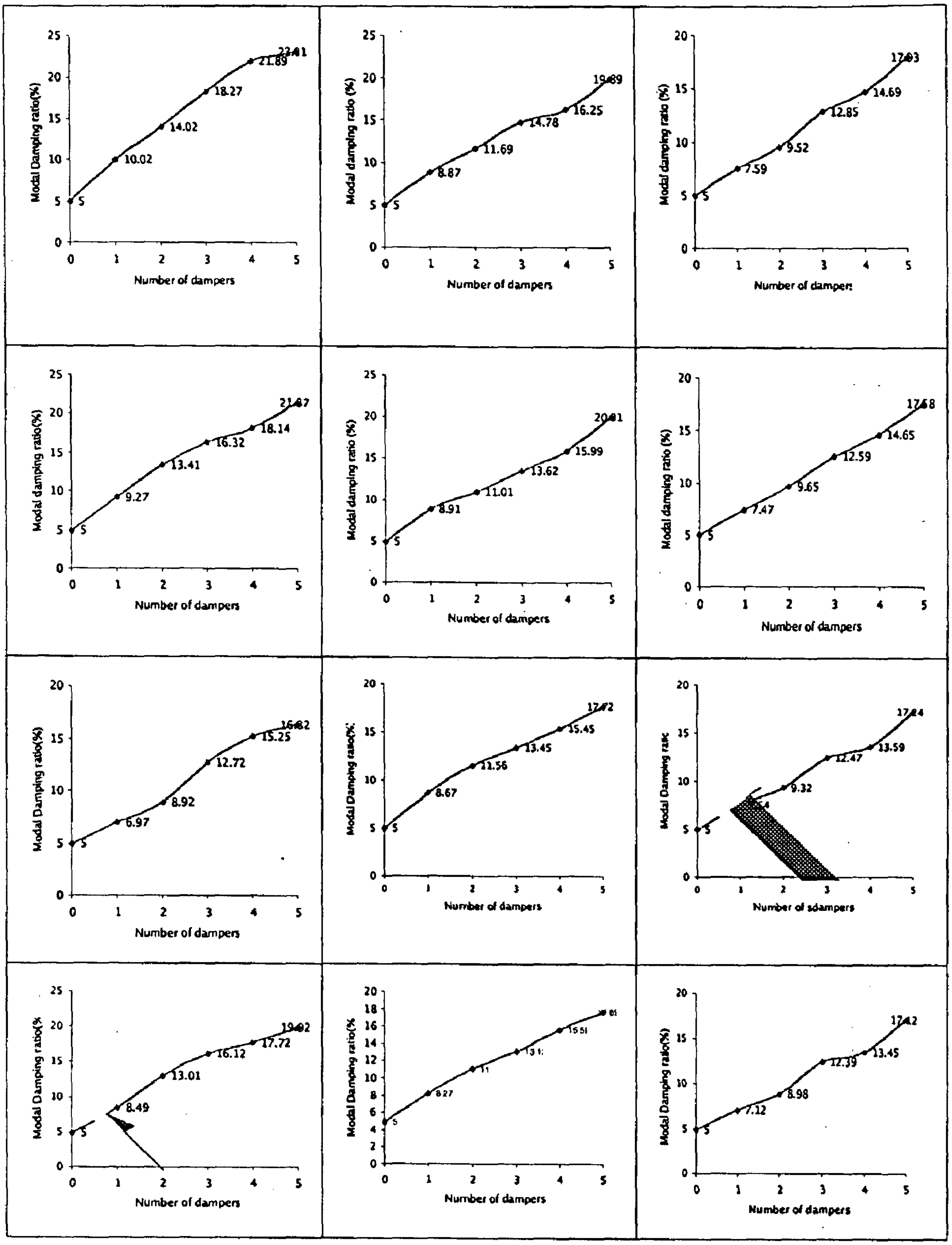

Figure 3: *Modal damping variation in first three modes for a a symmetric and unsymmetric building resting on a soil with V_s =300 m/s (First row: Symmetric, Second row: e_x = 5%L, Third row: e_y=7.5%L and Fourth row: e_x=5%L and e_y=7.5%L. First column: First mode, Second column: Second mode and Third column: Third mode)*