PROBABILISTIC LOAD FLOW COMPUTATION USING GRAM-CHARLIER EXPANSION

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

ELECTRICAL ENGINEERING

(With Specialization in Power System Engineering)

By

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JUNE, 2006

IDNO: MT-338/2006-33/ NM-VP-BD

CANDIDATE'S DECLARATION

As partial fulfillment of the requirements for the Master of Engineering (Electrical) Degree (Honours), I hereby submit for your considerations this thesis entitled:

"Probabilistic load flow computation using Gram-Charlier expansion"

I declare that the work submitted in this thesis is to the best of my knowledge and ability, and is an authentic record of my own work carried out during the period from July 2005 to June 2006 under the supervision of **Dr.Vinay Pant** Assistant professor and **Dr.B.Das,** Associate professor, Electrical Engineering Department, Indian Institute of Technology, Roorkee.

This work has not been previously submitted for a degree at the Indian Institute of Technology Roorkee or any other institutes.

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CERTIFICATE

This is to certify that the above statements made by the student are correct to the best of my knowledge.

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Acknowledgements

First and foremost, I would like to express my sincere appreciation to my thesis supervisors, Dr.Vinay Pant and Dr.B.Das, for their patience and guidance throughout the entire duration of my thesis. Without their supervision, this thesis would never be a success.

I would like to take this opportunity to express my deep sense of gratitude to my family for their support and encouragement they have provided me over the years. Last but not least, I would also like to thank all my friends who have offered me their unrelenting assistance throughout the course.

Abstract

This thesis report deals with the application of probabilistic load flow computation using Gram-Charlier expansion and Probabilistic load flow using Complex Random Variable analysis to the radial distribution system. Two types of load distributions have been considered namely, normally distributed loads and discrete random loads. A typical system is analyzed for nodal powers when they are (a) independent and when they are (b) dependent. The mean and standard deviations of bus voltages, active and reactive powers have been calculated. The results obtained have been compared with Deterministic load flow, basic probabilistic distribution load flow and Monte Carlo simulation and are found to be in good agreement. The probability density functions for these variables also have been plotted. In this thesis, for probabilistic load flow with method of moments, the input parameters viz. loads and line data are assumed as complex random variables. The probability distribution functions for bus voltages have been calculated. The results can be used for adequacy analysis of the distribution system.

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List of Acronyms

CDF	Cumulative Distribution Function
CRV	Complex Random Variable
CV	Coefficient of Variation
DISTFLOW	Distribution Load Flow
DLF	Distribution Load Flow
FDLF	Fast Decoupled Load Flow
FFT	Fast Fourier Transform
FOR	Forced Outage Rate
IEEE-RTS	IEEE- Reliability Test System
L.T	Laplace Transform
MCS	Monte Carlo Simulation
PDF	Probabilistic Distribution Function
PLF	Probabilistic Load Flow
PMF	Probability Mass Function
RDS	Radial Distribution System
RV	Random Variable
SLF	Stochastic Load Flow
WSCC	Western System Coordinating Council

Chapter1

Introduction

1.1 General

Electrical power system is an interconnected structured system composed of generating stations and distribution substations together. In terms of operations and characteristics, the transmission system is distinctly different from the distribution system. The principal difference between these two is in their associated voltage levels and network structure. The former usually featured as a loop structure, while the latter generally is of radial structure. It has been realized that the precise solution state of distribution system can be acquired with a robust and efficient power (load) flow solution method [1-2]. These power flow solution methods must be able to model the specific features of radial distribution system in sufficient detail. Some of the more prominent features of radial distribution systems are as follows.

- Multiphase, unbalanced, grounded or ungrounded operation.
- Imperfection and uncertainties of network parameters.
- Unbalanced distributed loads.
- Extremely large number of nodes and branches

Since then power flow analysis has been one of the most fundamental and widely used tool by power engineers. The power flow analysis yields the system's solution state on solving a set of precisely known non-linear algebraic equations simultaneously. Due to peculiar features of radial structure and wide-ranging resistance, reactance values, the distribution system got status as ill-conditioned power system.

The popularly used Newton–Raphson and fast-decoupled load flow (FDLF) solution techniques are unsuitable for solving load flow for radial distribution systems. Consequently many other load flow analysis methods have been developed that suits to distribution system characteristics on assumption that input parameters (line resistance, reactance and load at different buses) as fixed quantities [3-5]. However, in realistic condition, the situation is quite different and input parameters for the load flow study are relatively uncertain [6]. These uncertainties arises by virtue of

1. Error in calculation or measurement of the feeder parameters (resistance and reactance)
2. Error in the magnitude of assumed load demand at system buses

Even if parameter uncertainties were not an issue, the power flow problem would be nothing more than a "snapshot" of the system at a given instant. Solutions obtained would be valid only for a single specific system configuration and operation condition. However, the system evolves through time. It appears that it would be reasonable to ask not what the system looks like at a given instant, but rather ask for the ranges of all plausible system conditions that might be encountered as result of expected uncertainties in power injection and other parameters.

1.2 Types of uncertainties: Broadly, speaking uncertainties can be classified in to two types.

1. Quantitative uncertainty: The uncertainty is quantifiable in numerical terms by a mathematical function with deterministic parameters.

Examples are:

a). Probabilistic variables. The uncertainty is defined by a probability density function: uniform, normal, Poisson, etc. [7] or by means of moments of a distribution and the method of cumulants.

b). Interval variables. An interval variable is a closed set of real numbers [x1, x2] such that any x in the interval x1<x<x2 is in the set [6, 8].

2. Qualitative uncertainty: This uncertainty is initially expressed in vague, non-numeric (usually verbal) terms such as "approximately equal to" and "a small percentage." By using

the concept of degree of membership of a value to a set, it is possible to establish the notion of fuzzy sets and fuzzy arithmetic. Qualitative uncertainty is quantified using fuzzy sets [9].

The purpose of a formal characterization of uncertainty is to gain a greater understanding of a system or process. Single-solution answers, although pleasing in traditional engineering terms, often give an incomplete picture of the behavior of a system. A characterization that explicitly considers uncertainty allows us to create models and answer questions that are either impossible or difficult to answer with deterministic methods. Several roles in the characterization of uncertainty are:

a). Uncertainty as an aid in the decision making process. Decision makers often consider the risk associated with a particular decision. The nature of the uncertainty also has an influence on the decision: overestimating a number may result in slightly higher costs of operation, but underestimating the same number could result in severe effects on a system, which will translate into considerably higher costs.

b). Deterministic solutions in the presence of uncertainty give deterministic answers that are guaranteed to almost never take place. Of greater value would be to bracket the solution and either give intervals guaranteed to contain the solution, or probabilistic measures guaranteed to contain the solution at a given level of confidence.

c). Uncertainty is essential when reconciling mathematical models with measurements on physical systems. The classic example of this use of uncertainty is the state estimation problem in power systems, where more measurements than strictly needed are made on a system, and the state of the system is determined under the assumption that measurements are subject to error.

Methods for handling uncertainty can be applied to determine both engineering and economic parameters, such as current flows, voltages, cost and reliability (or security). Of increasing interest are methods capable of characterizing important externalities of a power system, such as environmental effects. These externalities are often associated with greater degrees of uncertainty than is customary within traditional engineering models. The fact that uncertainty exists is no reason, however, for simply ignoring an important concern. Rather, methods for capturing the inherent uncertainty must be used and incorporated into the more traditional ways of assessing the system.

1.3 Representation of Uncertainty:

Many papers have been published to deal with uncertain power flow analysis problem [6-10]. According, applied mathematical techniques these works can be classifieds in to the three categories.

a. Interval analysis load flow methods
b. Fuzzy load flow methods
c. Probability power flow methods

Each method uses the notion of an "uncertain variable." An uncertain variable is a variable that can take more than one numeric value according to the point of view of the method. For probabilistic methods and Monte Carlo Simulation (MCS), uncertain variables are better known **as random** ***variables,*** for interval methods they are known **as** ***interval variables*** and for fuzzy arithmetic methods are known **as** ***fuzzy*** or ***possibility variables.***

This thesis mainly focuses on Probabilistic Load Flow (PLF) Methods.

1.4 Probabilistic methods:

Probabilistic methods are based on the interpretation of belief in the possibility of an event outcome as a numeric probability. The determination of the probabilities of basic event outcomes is done either by heuristic reasoning or based on historical information [7]. The fundamental characterization of probability is the probability density function (PDF). Areas under this curve denote probabilities. Fig. 1.1 illustrates the most commonly used probability density functions for representation of loads. These are.

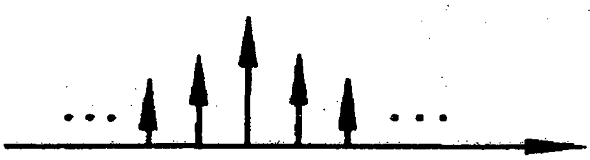
(a) Uniform.

(b) Gaussian.

(c) Binomial.

(a) Uniform.

(b) Gaussian.



(c) Binomial.

Fig. 1-1: Probability Density Functions.

Probabilistic Load Flow (PLF) uses linear or quadratic approximations of deterministic load flow equations. With these approximations, load flow equations are solved in a direct fashion and the probabilistic density function (PDF) of state variables (voltages and currents) are obtained from the given probabilistic description of measured variables (generation outputs and loads). PLF calculates both possible power flows and their possibilities of occurrence. There are two methods adopted in past research to obtain the probabilistic distribution functions of the state vector and line flows: Monte Carlo simulation and convolution method.

Monte Carlo Simulation is one of the methods to compute the Probabilistic Distribution Function (PDF) of line flows and voltages. This method consists of running and probabilistically processing several cases of individual load flows, where the data are generated by pseudo-random numbers. Monte Carlo solutions have been the backbone of probabilistic computation. This also serves as benchmark for comparison with other methods.

The Convolution technique is another method to compute Probabilistic Distribution Function (PDF) of line flows and voltages based on probabilistic distribution of data [11-15]. The probabilistic load flow transforms these input random variables, defined in terms of probability density functions, into output random variables also defined in terms of density functions using statistic theory.

1.5 Problem Statement:

The purpose of this thesis is avoids complex convolution and replaces them with simple arithmetic process due to unique properties of cumulants. This method combines the cumulants and Gram-Charlier Expansion theory to compute the power flows and voltages in balanced radial distribution system. This method has significantly reduced the computational time while maintaining a high degree of accuracy.

The deregulated and competitive power markets are widely spread in the world and bring about new aspects to system planning [16]. Probabilistic Load Flow solution based on the method of moments is one of the method used for security assessment of bus voltages in power systems. In this method, bus loads and network parameters are treated as complex random variables. Probabilistic Load Flow solution using method of moments is fast, because the process of convolution of various complex random variables is performed in moment and cumulant domain.

1.6 Overview of the thesis: This thesis is organized as follows

Chapter 1 as stated above serves as a general introduction to Probabilistic Load Flow.

Chapter 2 provides the research contribution from the past to present in the area of probabilistic load flow in distribution as well as transmission system.

Chapter 3 provides the brief discussion on moments, cumulants, relationship between moments and cumulants, properties of cumulants, complex random variables, Gram-Charlier expansion and correlation between the input nodal powers.

Chapter 4 describes the Probabilistic Load Flow (PLF) using Gram-Charlier detailed algorithm and also describes the PLF with Method of moments and its algorithm used in this thesis.

Chapter 5 provides the details on the simulation program results.

Chapter 6 concludes the thesis with a final regard to the improvements for future development.

Chapter 2

Literature review

Literature survey shows that the most commonly used techniques for radial distribution load flow (DISTFLOW) are, one based on a Newton like method involving formation of jacobians and computation of power mismatches at the end of the feeder and laterals [1] and the other based on the backward and forward sweeps involving computation of branch flows [2-3]. This method is an efficient solution for weakly meshed distribution and transmission networks. Nanda and Srinivas [4] proposed a method similar to the previous method, but differing in the formulation of its algorithm and in the convergence criterion. Chiag and Zimmerman et al [5] presented a load flow methodology based on current computations, was applied for multi phase radial distribution networks.

Load flow analysis of a distribution system, when the load demand is varying over an interval, can be performed by either repeated application of normal load flow or by the use of interval arithmetic load flow. Wang and Alvarado [6-7] first proposed the application of interval arithmetic method for power flow analysis of transmission networks. This paper, discusses uncertainty in power flow computations by coming up with simple bounds on the solutions that are, in some sense, as small as possible. These results were compared with Monte Carlo Simulation (MCS) results. Later Das [8] extended this technique for power flow analysis in balanced radial distribution systems. This paper, uncertainties only in the input load parameters are considered and the results are compared with the results obtained from repeated load flow simulation.

Satpathy and Das [9] proposed the application of the fuzzy set theory and possibility theory for power flow analysis of transmission networks. The uncertain power injections are usually given in fuzzy numbers with known possibility distributions. The modeling part, the system loads and available generations are modeled with the help of trapezoidal membership functions. This paper also discusses a case study on IEEE 14- bus test system. The most important results of these models are membership functions, for instance of branch flows or voltages, that reflected, in aggregated way, the uncertainty of a set of specified power scenarios. Later Ghosh [10] applied fuzzy techniques for radial distribution systems. This paper also discusses real, reactive power losses and voltage magnitude at every node with respect to membership function.

The first notion of probabilistic power flow appeared in the early 1970s. Borkowska, Allen et al. [11, 12] have proposed a simplified probabilistic load flow. In this paper two assumptions were introduced: (1) the electric power system is represented with dc network model (2) The real part of the bus electric loads are independent random variables with these assumptions, a conventional deterministic power flow is solved. Later, this basic method has been extended to AC network model [13-14]. This paper presents two possible formulations of the problem that permit the probability density curves of angles, voltages, injected active and reactive power flows to be computed.

The papers [11-14] assumed that the nodal powers are independent. The assumption of independence of the nodal electric loads is unrealistic. However, there are various reasons for correlations to exist between nodal powers. These reasons depend on whether load/load, generation/load or generation/generation behavior is being considered. For example, a group of loads existing in the same area will tend to increase and decrease in a like manner due to environmental or social factors. Therefore there will be certain degree of dependence between them. Al-Shakarchi et al. [15, 16] proposed a method in which he has taken all correlations in to account as explained above. Da Silva et al.[17] proposed a linear dependence model of electric loads. Using a linearized power flow model, they proposed a method, which combines Monte Carlo Simulations and convolutions. DopaZo et al. [18] proposed a method, which models the correlation between the loads at any two buses. Their proposed method assumes that circuit flows and bus voltage magnitude are Gaussian distributed and, thus, only the variance must be computed With Monte Carlo Simulation Technique.

Burchett [19] has proposed a method for obtaining a probabilistic load flow solution using a discrete frequency domain convolution technique which is based on the Fast Fourier Transform (FFT). Patra [20] proposed a method, probabilistic load flow using method of moments to consider the network outages. In this method, the load and generated power was considered as complex random variables. The probability density functions of bus voltage and line currents are evaluated using method of moments and cumulants. Tae-KyunKim [21] proposed a method of probabilistic load flow analysis using method of moments for the security assessment of bus voltages. The PDF of bus voltage readily provide probabilities of threshold violations for the entire planning period, reflecting the random variation of loads, generation uncertainties, dispatching effects and outages. Zhang [22] used a dc load-flow model combining the concept of Cumulants and Gram-Charlier expansion theory to consider the bus injection uncertainties and to compute Probabilistic and cumulative distributions of network branch flows with less computation effort. Chun-Lien [23] proposed a method for probabilistic load flow based on an efficient point estimate method and the uncertainty of bus injections and line parameters can be estimated or measured efficiently.

When considering distribution networks the problem is simplified since there are no generation/generation relations [24-25]. These papers, discuss derivation of much simpler relations between input, output and state random variables based on the following assumptions. At every node voltage is considered as rated voltage and imaginary part of voltage drop is neglected. Karakatsanis and Hatziargyriou [26-27] presented a load flow in distribution network with dispersed wind power. This paper discusses probabilistic model for the Active power produced and reactive power absorbed by the wind turbines equipped with induction generator, which takes in to account the probabilistic nature of short-term wind velocity forecasts.

Tande [28, 30] discusses Probabilistic load flow calculation using Monte Carlo Simulation (MCS) for distribution network with wind generation. This paper, the total number of hours with over voltage per year was estimated for a distribution network with several wind turbines.

From literature review it is evident that the application of Gram-Charlier expansion in Distribution system probabilistic load flow has not been explored in-depth. This thesis application of Gram-Charlier expansion in distribution probabilistic load flow has been carried out.

Chapter 3

moments and Cumulants

Suppose X is a random variable and that all of the moments $E(X^k)$ exist. The probability distribution of X is completely determined by its moments, i.e., there is no other probability distribution with the same sequence of moments. If $\lim_{n\to\infty} E\,(X_n^k)^n = E(X^k)$ for all values of k, then the sequence $\{X_n\}$ converges to X in distribution.

3.1 Significance of the moments [33]:

The First moment about zero, if it exists, is the expectation of X, i.e. the mean of the probability distribution of X, designated α. In higher orders, the central moments are more interesting than the moments about zero. The first central moment is thus 0; the second central moment is the variance, the square root of which is the standard deviation. The normalized nth central moment is the nth central moment divided by σ^n; the nth moment of t=(x-α)/σ. These normalized central moments are dimensionless quantities, which represent the distribution independently of any linear change of scale.

3.1.1 Skewness: the third central moment represents the lopsidedness of the distributional any symmetric distribution will have a third central moment of zero. The normalized third central moment is called the skewness. Fig 3.1 represents the skewness of the probability distribution functions have the same mean and standard deviation. The one on the left is positive skewness. The one on the right is negative skewness.

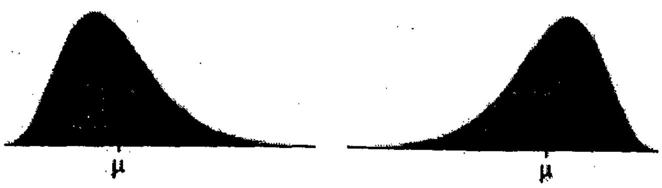


Figure 3.1: skewness

3.1.2 Kurtosis: the fourth central moment determines whether the distribution is tall and skinny or short and squat, compared to the normal distribution of the same variance. Since it is the expectation of fourth power, the fourth central moment is always positive. Fig.3.2 represents the kurtosis. The PDF on the right has higher kurtosis than the PDF on the left. It is more peaked at the center, and it has fatter tails.

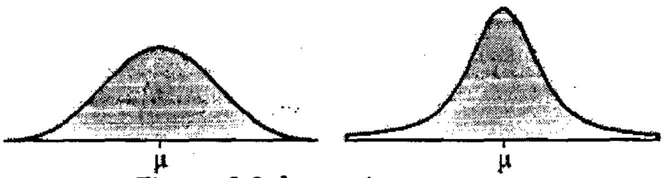


Figure 3.2: kurtosis

3.2 Moments about origin

If, for a positive integer v, the function X^v is integrable with respect to F(x) over interval (-∞, +∞),

$$\alpha_v = E(\xi^v) = \int_{-\infty}^{+\infty} x^v dF(x) \qquad \text{Where v =1, 2....n} \qquad .... (3.1)$$

The above equation is called the moment of order v or the vth moment of the distribution [22].

3.3 Moments about mean

The most important set of moments in statistical theory is obtained by shifting the origin to the arithmetic mean. These moments, m, are often called central moments.

$$\beta_v = E[(\xi - m)^v] = \int_{-\infty}^{+\infty} (x-m)^v dF(x) \qquad \text{Where v=1, 2...n} \qquad ..... (3.2)$$

3.4 Cumulants

The mean value of the particular function $e^{it\xi}$ will be written

$$\varphi(t) = E(e^{it\xi}) = \int_{-\infty}^{+\infty} e^{itx} dF(x) \quad \text{Where t=1, 2...n} \qquad ..... (3.3)$$

This is a function of the real variable t, and will be called the characteristic function of the variable ξ. If the k th moment of the distribution exists, the characteristic function can be developed in MacLaurin's series for small values of t:

$$\varphi(t) = 1 + \sum_{1}^{k} \frac{\alpha_\nu}{\nu!}(it)^\nu + o(t^k) \qquad \text{..... (3.4)}$$

$$\log \varphi(t) = \sum_{1}^{k} \frac{\gamma_\nu}{\nu!}(it)^\nu + o(t^k) \qquad \text{..... (3.5)}$$

The coefficient γ_ν is called the semi-invariants or cumulants of the distribution.

3.5 Relation ship between Moments and cumulants

The relationship between the moments and the cumulants can be deduced by substituting $\varphi(t)$ in (3.4) to (3.5).

$$\log(1 + \sum_{1}^{k} \frac{\alpha_\nu}{\nu!}(it)^\nu) = \sum_{1}^{k} \frac{\gamma_\nu}{\nu!}(it)^\nu + o(t^k) \qquad \text{..... (3.6)}$$

It is seen that γ_n is polynomial in $\alpha_1, \alpha_2, \ldots\ldots\alpha_n$ and conversely α_n is a polynomial in $\gamma_1, \gamma_2, \ldots\ldots\gamma_n$.

$$\begin{aligned} \gamma_1 &= \alpha_1 = m \\ \gamma_2 &= \alpha_2 - \alpha_1^2 \\ \gamma_3 &= \alpha_3 - 3\alpha_1\alpha_2 + 2\alpha_1^3 \end{aligned} \qquad \text{..... (3.7)}$$

And conversely

$$\begin{aligned} \alpha_1 &= m = \gamma_1 \\ \alpha_2 &= \gamma_2 + \gamma_1^2 \\ \alpha_3 &= \gamma_3 + 3\gamma_2\gamma_1 + \gamma_1^3 \end{aligned} \qquad \text{..... (3.8)}$$

Where

m denotes the mean value.

In order to facilitate calculation of higher order cumulants, a recursive relationship between the moments and cumulants of any order of a Probability Distribution Function (PDF) has been developed [31]. The relationship is as follows.

$$\gamma_1 = \alpha_1 = m \qquad \text{...... (3.9)}$$

$$\gamma_{k+1} = \alpha_{k+1} - \sum_{j=1}^{k} \binom{k}{j} \alpha_j \gamma_{k-j+1} \qquad \text{..... (3.10)}$$

And conversely

$$\alpha_1 = m = \gamma_1 \qquad \text{..... (3.11)}$$

$$\alpha_{k+1} = \gamma_{k+1} + \sum_{j=1}^{k} \binom{k}{j} \alpha_j \gamma_{k-j+1} \qquad \text{..... (3.12)}$$

Where $\binom{k}{j}$ are the binomial coefficients and γ_1 and α_1 are the k[th] order cumulants and moments respectively.

The binomial coefficient $\binom{k}{j} = \dfrac{k!}{j!(k-j)}$

In terms of the central moments β_v, the expression of the γ_v become

$$\begin{aligned} \gamma_1 &= m \\ \gamma_2 &= \beta_2 = \sigma^2 \\ \gamma_3 &= \beta_3 \\ \gamma_4 &= \beta_4 - 3\beta_2^2 \end{aligned} \qquad \text{..... (3.13)}$$

Where σ denotes standard deviation and conversely

$$\begin{aligned} \beta_1 &= 0 \\ \beta_2 &= \gamma_2 = \sigma^2 \\ \beta_3 &= \gamma_3 \\ \beta_4 &= \gamma_4 + 3\gamma_2^2 \end{aligned} \qquad \text{..... (3.14)}$$

A new recursive relationship between the central moments and cumulants of any order of a Probability Distribution Function (PDF) has been developed. The relationship is as follows.

$$\gamma_1 = m \qquad \text{..... (3.15)}$$

$$\gamma_{r+1} = \beta_{r+1} + \sum_{j=2}^{r-1} \binom{r}{j} \gamma_j \beta_{r-j+1} \qquad \text{...... (3.16)}$$

And conversely

$$\beta_1 = 0 \qquad \text{...... (3.17)}$$

$$\beta_{r+1} = \gamma_{r+1} + \sum_{j=2}^{r-1} \binom{r}{j} \beta_j \gamma_{r-j+1} \qquad \text{...... (3.18)}$$

Where $\binom{r}{j}$ are the binomial coefficients and γ_1 and β_1 are the k[th] order cumulants and central moments respectively.

The binomial coefficient $\binom{r}{j} = \dfrac{r!}{j!(r-j)}$

3.6 Properties of cumulants [22]

Let ξ and η be independent random variables with known cumulative function F_1 and F_2. The cumulative function F(x) of the sum of two independent variables is given by

$$F(x) = \int_{-\infty}^{+\infty} F_1(x-z)dF_2(z) = \int_{-\infty}^{+\infty} F_2(x-z)dF_1(z) \qquad \text{..... (3.19)}$$

$$F(x) = F_1(x) * F_2(x) \qquad \text{..... (3.20)}$$

For the sum $\xi_1 + \xi_2 + \ldots\ldots + \xi_n$ of n independent variables, the cumulative function

$$F = F_1 * F_2 * \ldots\ldots. * F_n \qquad \text{..... (3.21)}$$

Let $\varphi_1(t), \varphi_2(t)$, and $\varphi(t)$ denote the characteristic function of ξ, η, and $\xi+\eta$ respectively.

$$\varphi(t) = E[e^{it(\xi+\eta)}] = E[e^{it\xi}] * E[e^{it\eta}] = \varphi_1(t) * \varphi_2(t) \qquad \text{...... (3.22)}$$

If $\xi_1, \xi_2, \ldots\ldots \xi_n$ are independent variables with the characteristic function $\varphi_1(t), \varphi_2(t)$ $,\ldots\ldots \varphi_n(t)$, the characteristic function $\varphi(t)$ of the sum $\xi_1 + \xi_2 + \ldots\ldots + \xi_n$ is thus given by

$$\varphi(t) = \varphi_1(t) * \varphi_2(t) * \ldots\ldots * \varphi_n(t) \qquad \text{...... (3.23)}$$

The multiplication theorem for characteristic function gives

$$\log\varphi(t) = \log\varphi_1(t) + \log\varphi_2(t) + \ldots\ldots \log\varphi_n(t) \qquad \text{..... (3.24)}$$

Therefore

$$\gamma_v = \gamma_v^1 + \gamma_v^2 + \ldots\ldots\ldots + \gamma_v^n \qquad \text{..... (3.25)}$$

According to (3.13) to (3.18) it can be observed that

$$m = m_1 + m_2 + \ldots\ldots\ldots + m_n \qquad \text{..... (3.26)}$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \ldots\ldots + \sigma_n^2 \qquad \text{..... (3.27)}$$

3.7 Complex Random Variable (CRV)

Let $Z = X + jY$ be a complex random variable (CRV) with a Probability density Function $f(Z)$. Clearly X and Y are real random variables (RRV), defined in the same probability space with a joint Probability Density Function f(x, y). Similarly in terms of magnitude and phase angle, let Z= $e^{j\theta}$ then joint Probability Density Function (PDF) is defined as shown in Fig. 3.3. This Probability Density Function (PDF) consists of three discrete functions at Z_1, Z_2 and Z_3 these are the values which Complex Random Variable Z may assume) with corresponding probabilities p_1, p_2 and p_3 respectively.

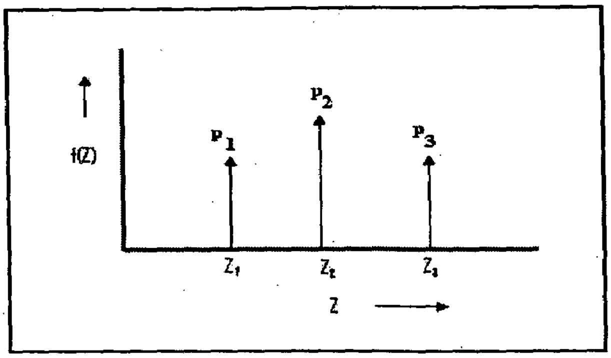


Figure: 3.3 Discrete PDF of Complex Random Variable

Where Z_1, Z_2 and Z_3 are clearly complex random numbers and the sum of each probability p_1, p_2, p_3 is one. The interpretation to be the given above Probability Density Function (PDF) is as follows [21].

The complex random variable Z assumes three complex values: Z= z_1 with probability p_1, Z= z_2 with probability p_2 and Z= z_3 with probability p_3. Therefore, for the case Z= $z_1 = x_1 + jy_1$, with probability of existence p_1, is equivalent to X= x_1 and Y= y_1, both with the same probability existence, i.e. p_1. Another way of interpreting this is to think of occurrence of x_1 and y_1 together as one event.

3.8 Moments and Cumulants of Complex Random Variable

Referring to the Probability Density Function of Complex Random Variable as shown in Fig 3.3, the moments about origin of order t, of the Complex Random Variable Z are defined as [20]

$$\alpha_t = E[Z^t] = \sum_{i=1}^{3} Z_i^t p_i \qquad \text{Where t=1, 2...n} \qquad \text{..... (3.28)}$$

Where,

E [.]: the expected value of random variable

Z_i^t : i- th value of t-th order Complex Random Variable Z

p_i : Probability of i-th value of Complex Random Variable Z

For t= 1, 2, 3, 4 the above expression is

$$\begin{aligned}\alpha_1 &= z_1p_1 + z_2p_2 + z_3p_3 \\ \alpha_2 &= z_1^2p_1 + z_2^2p_2 + z_3^2p_3 \\ \alpha_3 &= z_1^3p_1 + z_2^{32}p_2 + z_3^3p_3 \\ \alpha_4 &= z_1^4p_1 + z_2^4p_2 + z_3^4p_3\end{aligned} \quad \text{..... (3.29)}$$

Clearly these moments are Complex Random Variables. The corresponding cumulants are obtained using the relationship between moments and cumulants.

3.9 Relation ship between CRV moments and cumulants [20]: the recursive relation between cumulants and moments is

$$\begin{aligned}\gamma_1 &= \alpha_1 \\ \gamma_t &= \alpha_t - \sum_{i=1}^{j-1}\binom{j-1}{i}\gamma_{j-1}\alpha_i \quad (j \geq 2)\end{aligned} \quad \text{Where t= 1, 2...n}^{\text{th}} \text{ order} \quad \text{..... (3.30)}$$

For t=2, 3, 4 the above expression is

$$\begin{aligned}\gamma_2 &= \alpha_2 - \alpha_1^2 \\ \gamma_3 &= \alpha_3 - 3\alpha_2\alpha_1 + 2\alpha_1^3 \\ \gamma_4 &= \alpha_4 - 4\alpha_3\alpha_1 + 6\alpha_2\alpha_1^2 - 3\alpha_1^4\end{aligned} \quad \text{...... (3.31)}$$

Similarly the recursive relation between moments and cumulants is

$$\begin{aligned}\alpha_1 &= \gamma_1 \\ \alpha_t &= \gamma_t + \sum_{i=1}^{j-1}\binom{j-1}{i}\gamma_{j-i}\alpha_i \quad (j \geq 2)\end{aligned} \quad \text{Where t= 1, 2...n}^{\text{th}} \text{ order} \quad \text{..... (3.32)}$$

For t=2, 3, 4 the above expression is

$$\begin{aligned}\alpha_2 &= \gamma_2 + \gamma_1^2 \\ \alpha_3 &= \gamma_3 + 3\gamma_2\gamma_1 + \gamma_1^3 \\ \alpha_4 &= \gamma_4 + 4\gamma_3\gamma_1 + 3\gamma_2^2 + 6\alpha_2\alpha_1^2 + \alpha_1^4\end{aligned} \quad \text{..... (3.33)}$$

The above moments and cumulants in Probabilistic Load Flow calculation are used each in multiplication and addition.

3.10 Random Variables used in Load Flow Analysis

To model the load of the system discrete distribution is considered in this thesis. However, continuous distribution like normal distribution or any general distribution can also be used. The load is assumed to be expressible in terms of active and reactive powers. The simplest way to stimulate this is by means of the Probability density Function shown in Fig. 3.4.

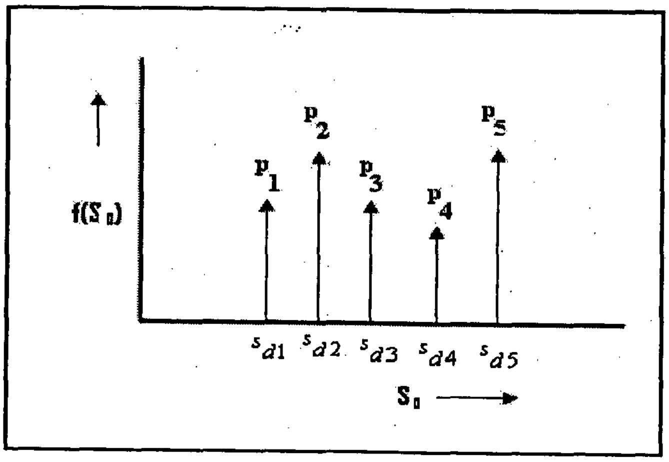


Figure: 3.4 Probability Distribution Function of load with uncertainty

The Probability Distribution Function (PDF) as shown in Fig. 3.4 assumes that the uncertainty applies to both active and reactive powers. If only the active power is in with an assumed uncertainty; while reactive power is constant. In this case there is no difficulty in obtaining the moments of complex load. The moments can be obtained from the knowledge of the moments of active and reactive components [appendix A]. From the Fig. 3.4 various moments can be obtained as follows.

$$m_t(S_D) = \sum_{i=1}^{5} s_{di}^t p_i \qquad \text{Where t=1, 2...nth order} \qquad \text{..... (3.34)}$$

where $s_d = p + jq$

p_i: Probability of i-th value of Complex Random Variable s_d

The transmission lines are represented by their series impedance or admittance. The impedance of the line is assumed to be a random variable Z_L. In its simplest form this random variable may take two values.

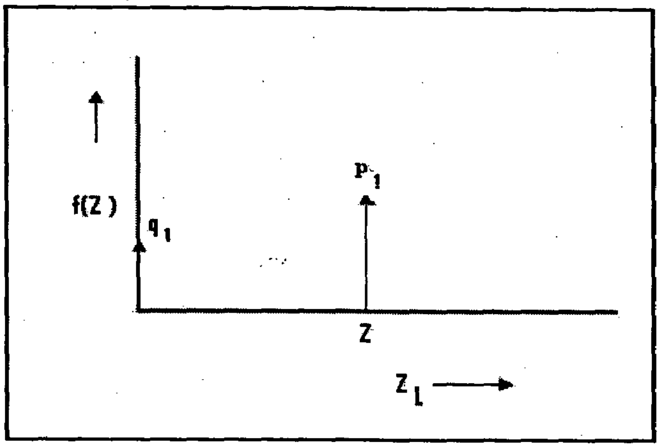


Figure: 3.5 Probability Distribution Function of Available impedance

Z_L =z when the line is in operation with probability p_1 and Z_L =0 when the line is down, with probability q_1, clearly $p_1+q_1=1$.The probability Density Function (PDF) of available line impedance is shown in Fig. 3.3.

3.11 Gram-Charlier Type-A Series [32]

Consider a random variable ξ with a distribution of a continuous type and denote the mean value as μ and the standard deviation as σ. For the standardized variable $(\xi-\alpha)/(\sigma)$, its cumulative and Probability density function are denoted as F(x) and f(x) respectively. According to Gram-Charlier expansion, the cumulative and the Probability density functions can be written as

$$F(x) = -\sum_{j=0}^{\infty} c_j H_{j-1}(x)\phi(x) \qquad \text{..... (3.35)}$$

$$f(x) = \sum_{j=0}^{\infty} c_j H_j(x)\phi(x) \qquad \text{..... (3.36)}$$

Where

$$c_j = \frac{1}{j!} \sum_{k=0}^{j/2} \left(\frac{-1}{2}\right)^k \frac{j!\beta_{j-2k}}{k!(j-2k)!} \left(\frac{1}{\sigma^{j-2k}}\right)$$

$$H_j(x) = \sum_{k=0}^{j/2} \left(\frac{-1}{2}\right)^k \frac{j!x^{j-2k}}{k!(j-2k)!}$$

$$\phi(x) = \frac{1}{\sqrt{2\Pi}} e^{\frac{-x^2}{2}}$$

[n/2] denotes the largest integer<=n/2

The expression $H_j(x)$ are known as Hermite Polynomials.

Some of the expressions for probability distribution function are given in Appendix A

3.12 Edgeworth form of the Gram-Charlier Type-A series

Any Probability Density Function f(x), with finite moments, may be expressed in terms of orthogonal polynomials. Consider the Probability Density Function f(x) of a random variable ξ, expanded in terms of a standardized random variable x and its corresponding normal Probability Density Function $\phi(x)$, as follows

$$f(x) = \phi(x) - G_1\phi^3(x)/3! + G_2\phi^4(x)/4! + G_1\phi^6(x)/6! + \ldots \qquad \text{..... (3.37)}$$

Where

$x = (\xi - \alpha)/(\sigma)$

$$\phi(x) = \frac{1}{\sqrt{2\Pi}} e^{\frac{-x^2}{2}}$$

The parameters G_1 and G_2 determines the skewness and peakedness of the distribution and defined in terms of the cumulants of the random variable x, as follows.

$$G_1 = \gamma_3(X)/[\gamma_2(X)]^{\frac{3}{2}}$$

$$G_2 = \gamma_4(X)/[\gamma_2(X)]^2$$

The equation (3.37) is referred to as the Edgeworth form Gram-Charlier expansion.

The equation (3.36) is formally identical to equation (3.37), even though it has some difference between them. The probability distribution function is obtained by Edge worth form Gram-Charlier expansion in terms of the cumulants of the desired random variable whose probability distribution is to be evaluated where as Gram-Charlier type-A series in terms of central moments of the desired random variable whose probability distribution is to be evaluated. For practical purposes it is necessary to take only a finite number of terms in the series and to neglect the remainder [31].

3.13 Correlation between nodal powers

In long-term planning studies, possible variations in load demands are due to forecast uncertainties. The demands can only be predicted within certain statistical uncertainties and are described by the normal distribution. In these cases, demands are completely random and can be independent. When the behavior of the system for relatively short term, say a few months or less, is being considered, assumption of independence between the load demands is less valued. The demands may be characteristically independent, e.g. for different types of consumer, but may be correlated owing to common effects such as weather conditions and human-behavior patterns [15].

There are various reasons for correlation between nodal powers to exist, and these reasons tend to depend on whether load/load, generation/generation, or generation/load correlation behavior is being considered. The extreme case would be the total correlation. A group of loads existing in the same area, will for example, for which all demands would rise and fall "in step" because of environmental or social factors. Some of the most relevant reasons are discussed below [16-17].

3.13.1 Load/load correlation

In long term-planning problems, the probabilistic variation of loads is, generally, not one involving time, but, instead, is associated with load forecasting at a specific time in the future. In such cases, total independence between loads is a reasonable assumption. In operational planning problems, however, the probabilistic variation of loads is associated with time and a group of loads existing in the same area will tend to increase and decrease in a like manner; i.e. a certain degree of correlation exists between them. The most important reasons for this correlation are due mainly to common environmental factors such as temperature, sunset, rainfall etc., and to social factors such as sporting events, television programmes, meal times, working habits etc. As these factors are likely to affect all loads of a similar nature in a like manner, a degree of correlation will exist. When the loads rise and fall together, the correlation is positive. Similarly, in the event of a load failing while another rises, the correlation is negative.

3.13.2 Generation/generation correlation

In practice, generation output into the system may sometimes be controlled so that the output of a specific group of generation is kept constant. Consequently, if the output of one source of generation in that group is decreased for one reason or another, the output of the other sources of generation is increased by the same amount within the output limits of each source. In this case, the correlation is such that, as one nodal power increases, another decreases and therefore generation/generation correlation is negative.

3.13.3 Generation/load correlation

Frequently, in the operation of a power system, a group of generators is controlled to meet the load within a certain load area, this being known as area control. In such cases, there must be correlation between those generation assigned to the area load and the load itself; i.e. as the load rises and falls, the output from the relevant group of generators is increased and decreased likewise. In this case, the correlation is again positive.

All of the above types of correlations are utilized in operational planning of power system.

3.14 Representation of correlation of nodal powers

As discussed above that load/load, generation/generation, and generation/load correlations are approximately linear. Therefore, for many practical applications a linear representation of such correlations is all that is required, and the approximations introduced because of this assumption can be neglected [17].

If two random variables, X and Y, are linearly dependent, they can be related simply using the equation of a straight line; i.e.

$$Y = aX + b \qquad \text{..... (3.38)}$$

Where

a and b are constants

If the correlation is positive, then 'a' is positive value. In other words, if the correlation is negative, then 'a' has negative value. These will be referred to as positive and negative linear dependence, respectively. If the assumption of linear dependence is considered inappropriate, then the above representation can be modified quite readily to give a relationship between X and Y when they are not exactly linear. In this case the variable Y can be divided in to two random variables Y' and Y''. Where Y' is linearly dependent on Z and obeys equation (3.38). Y'' Is relatively small, has an expected value of zero and is independent of X. this is shown in Fig. 3.6 for two arbitrary variables related by positive linear dependency.

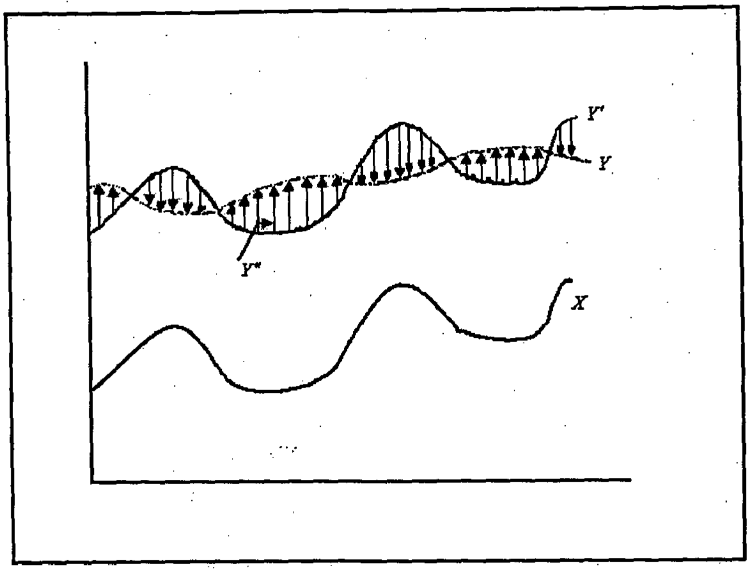


Figure: 3.6 Two independent random variables that are not exactly linear

To include linear dependence in probabilistic load flow, each group of linearly Dependent random variables is considered to be independent of all other groups and all other independent random variables including the variables Y'' introduced to account for dependence that is not exactly linear. If it is considered that all the random variables are linearly dependent (positive or negative), then only one such group exists, and there are no independent groups or random variables i.e. Y" does not exist.

3.15 Linear dependence between random variables

Consider the case of two random variables X and Y having expected values μ_x and μ_y, standard deviation σ_x and σ_y, respectively. The covariance and correlation functions [33] are convenient parameters for indicating the measure of linear dependence between them.

The covariance of η_{xy} of X and Y is

$$\eta_{xy} = E\{(X-\mu_x)(Y-\mu_y)\} \quad ......(3.39)$$

Where $E\{\ \}$ represents the expected value.

Equation (3.4) can be expressed as

$$\begin{aligned}\eta_{xy} &= E\{XY - X\mu_y - \mu_x Y + \mu_x\mu_y\} \\ &= E\{XY\} - \mu_y E\{X\} - \mu_x E\{Y\} + \mu_x\mu_y \\ &= E\{XY\} - \mu_y\mu_x - \mu_x\mu_y + \mu_x\mu_y \\ &= E\{XY\} - \mu_y\mu_x\end{aligned} \qquad \text{..... (3.40)}$$

The correlation coefficient γ_{xy} is defined as

$$\gamma_{xy} = \frac{\eta_{xy}}{\sigma_x\sigma_y} \qquad \text{...... (3.41)}$$

Where the value γ_{xy} is between -1 and 1

γ_{xy} =0 if X and Y are linearly independent and $|\gamma_{xy}|$ =1 if X and Y are linearly dependent [17].

Consider now the case of two linearly dependent random variables X and Y represented by the discrete distribution and consider that they are to be combined to give a third random variable Z, such that

$$Z = cX + eY + d \qquad \text{..... (3.42)}$$

Where

c, d, e are constants.

Since X and Y are linearly dependent. They are related by equation (3.38), and since they are linearly combined to give Z. For each value of X, there are corresponding values of Y and Z all of which have the same value of probability. Therefore if X takes a value x_i with probability f_i, then Y takes the value y_i with probability f_i and Z takes the value z_i with probability f_i this is shown in Fig.3.7

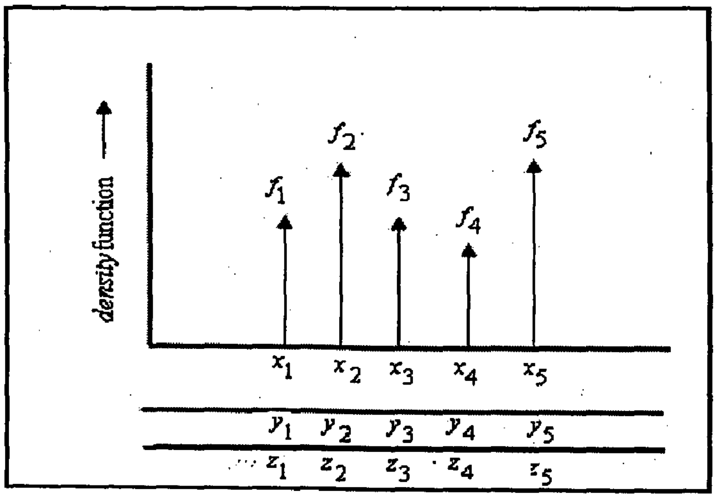


Figure: 3.7 discrete distribution functions of dependent random variables

$$z_i = cx_i + ey_i + d \qquad \text{..... (3.43)}$$

The resultant derivation function is [32]

$$\mu_z = c\mu_x + e\mu_y + d \qquad \text{..... (3.44)}$$

$$\sigma_z = c\sigma_x + e\sigma_y \quad \text{(For positive correlation)} \qquad \text{..... (3.45)}$$

$$\sigma_z = c\sigma_x - e\sigma_y \quad \text{(For negative correlation)} \qquad \text{..... (3.46)}$$

This concept can be extended to any number of random variables therefore, if

$$Z = c_1X_1 + c_2X_2 + \text{.......} + c_iX_i + c_nX_n + c_{n+1} \qquad \text{..... (3.47)}$$

Then

$$\mu_z = c_1\mu_1 + c_2\mu_2 + \text{.......} + c_i\mu_i + c_n\mu_n + c_{n+1} \qquad \text{...... (3.48)}$$

$$\sigma_z = c_1\sigma_1 \pm c_2\sigma_2 \pm \text{......} \pm c_i\sigma_i \pm c_n\sigma_n \qquad \text{..... (3.49)}$$

In equation (3.49), the positive sign is used if the relationship between X_i and X_1 is a positive linear dependence, i.e. $X_1 = a_i X_i + b_i$ and the negative sign is used if the relationship is a negative linear dependence.

The above equation has been derived by assuming the random variables were represented by the discrete distributions. It is, also applicable for normally distributed random variables. This is evident since a normal distribution can be approximated to a very large number of discrete impulses. Therefore, if all X_i in equation (3.45) are normal distributions, Z will also be a normally distributed with an expected value of μ_z and a standard deviation of σ_z.

Chapter4

Probabilistic Load Flow

In this chapter the following probabilistic load flow methods for distribution system have been discussed.

1. Monte Carlo simulation method
2. Probabilistic load flow using Laplace transform Method
3. Probabilistic load flow computation using Gram-Charlier expansion
4. Probabilistic load flow using Complex Random Variable analysis.

Methods 3 and 4 have been modified in this work and applied to distribution system and results have been compared those of 1 and 2.

4.1 Monte Carlo Simulation [34]

4.1.1 Introduction:

In fact, simulation methods can often be the only means of obtaining the solution to the system model, especially when the system studied is large and complex or when the probability distributions rather than only the means and variances, are required. A numerical simulation is a process of selecting a set of values of system parameters and obtaining a solution of the system model for a selected set. Repeating the simulation process for different sets of system parameters, obtain different sample solutions. The key activity in the simulation process is the selection of system parameters to obtain sample solutions.

Monte Carlo simulation is repeating the simulation process. In each simulation process a particular set of values of the random variables generated in accordance with the corresponding probability distribution function.

4.1.2 Algorithm:

Step1. Generate random numbers from the given distribution function (normal or discrete or binomial distribution).

Step2. This step is deterministic, in which the mathematical model is solved to obtain the parameters voltage, angle and power flows.

Step3. Above two steps are repeated a sufficient number of times, a statistical analysis of simulation results is then performed.

4.1.3 Generation of random numbers [35]:

Generation of appropriate values of random numbers in accordance with respective given probability distribution. For each variable is first generate a uniformly distributed random number between 0 and 1.0 and then, through appropriate transformations, obtain the corresponding random numbers with the specified probability distribution.

Generating uniformly distributed random numbers is based on recursive calculations of residues of modulus m from a linear transformation. The multiplicative congruential method (or power residue method) is frequently used at present. In this method one takes residues of successive powers of a number 'x' to be the successive numbers in the random sequence: that is,

$$X_i = X^i (\text{mod m}) \qquad \text{...... (4.1)}$$

Equation (4.1) an equivalent expression is

$$X_i = \rho X_{i-1} (\text{mod m}) \qquad \text{..... (4.2)}$$

Where ρ is constant, m=modulus=2^{31}-1. The uniform variates are obtained from

$$u_i = x_i / m \qquad \text{.....(4.3)}$$

Equation (4.2) and (4.3) are used for generation of uniform random numbers.

The value of variable 'x' therefore is obtained by evaluating an inverse of cumulative distribution of respective distribution function.

$$X_i = F_X^{-1}(u_i) \quad \text{where i=1, 2 ... n} \qquad \text{.....(4.4)}$$

4.1.4: Discrete random variable generation: The probability mass function $p(X_1)$, $p(X_2)$....$p(X_i)$, on the integers $S(x_1, x_2.....x_i)$, is shown in Fig.4.1. The Cumulative Probability distribution function is shown in Fig.4.2. In Fig.4.2 represents $P(X_1)= p(X_1)$, $P(X2)=p(X_1)+p(X_2)$ $P(X_i)=p(x_1)+p(X_2)+............+p(X_i) =1.0$

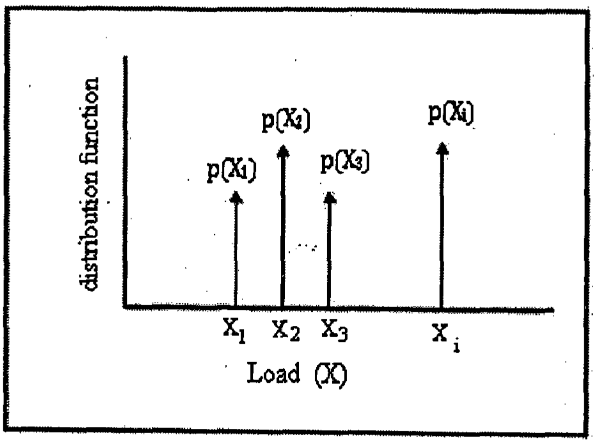


Figure 4.1: Probability Mass Function

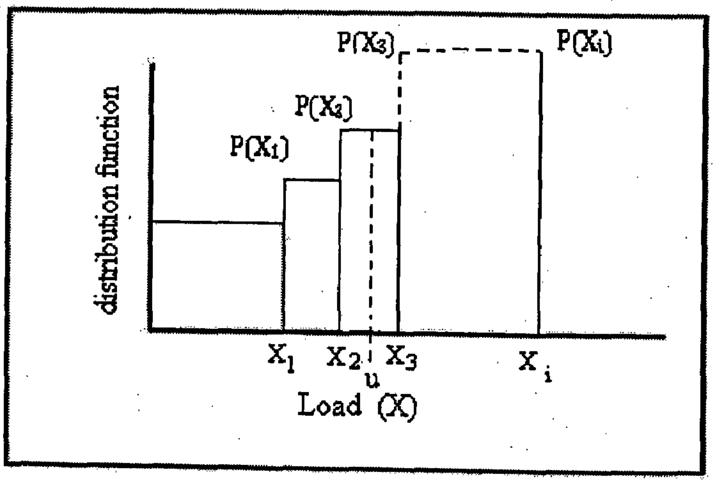


Figure 4.2: Cumulative density function

The generation of discrete random variable for the probability mass function is as follows.

Step1: Generate **u** random value between 0 and 1: **u (0, 1)**

Step2: the integer X =X_I if it satisfies the following equation

$$\sum_{j=0}^{I-1} P(j) \le u < \sum_{j=1}^{I} P(j)$$

Step 3: return

4.1.5 Flow chart:

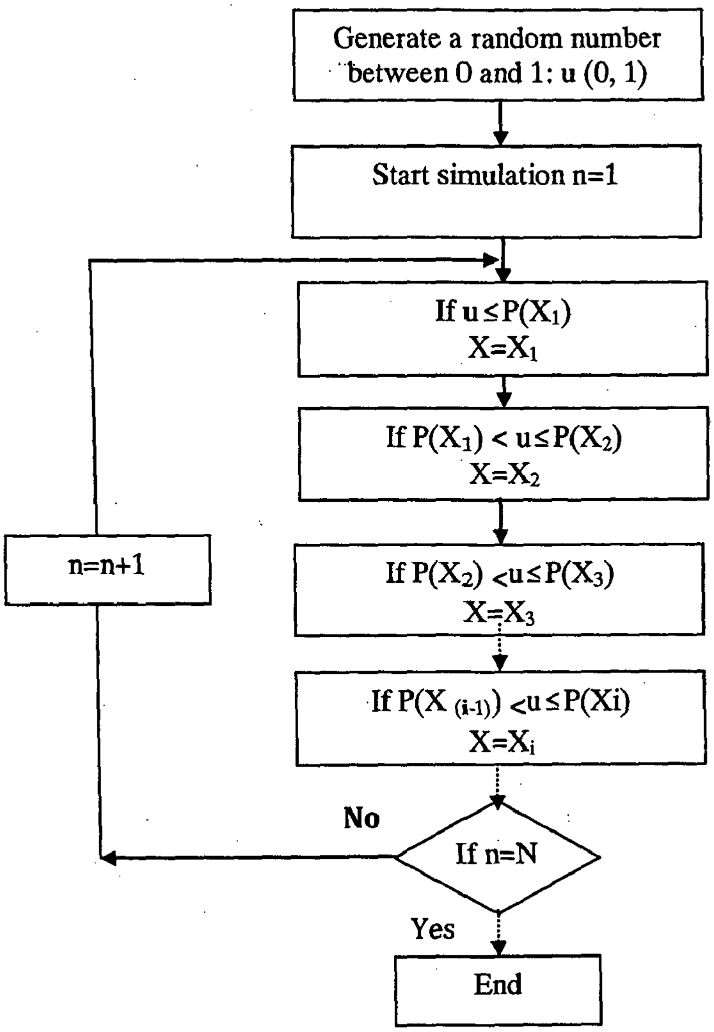


Figure 4.3: discrete random generation

4.1.6 Solution of Deterministic equations:

1. Nodal current calculation:

The nodal current injection $I_i^{(k)}$, at network node i is calculated as

$$I_i^{(k)} = (S_i / V_i^{(k-1)})^* - Y_i V_i^{(K-1)} \quad i= 1, 2... n \qquad .... (4.5)$$

Where

$V_i^{(k-1)}$ Is the voltage at node i calculate during (k-1) th iteration

S_i is the specified power injection at node i

Y_i is the sum of all shunt elements at node i

2. Branch current calculation:

Starting from the branches in the last layer and moving towards the branches connected to the root node. The current in branch L, is calculated as

$J_{L2}{}^{(k)} = -I_{L2}{}^{(k)} + \sum$(currents in branches emanating from node L2) ...... (4.6)

L=b, b-1 1

Where $I_{l2}^{(k)}$ is the current injection at node L2.

3. Voltage calculation:

Nodal voltages are updated in a forward sweep starting from branches in the first layer toward those in the last. For each branch, L, the voltage at node L2 is calculated using the up dated voltage at node L1 and the branch current calculated in the preceding backward sweep

$$V_{l2}^k = V_{L1}^k - Z_L J_L^k \qquad L= 1, 2........ b \qquad ..... (4. 7)$$

Where

Z_L is the series impedance of branch L

Flow chart:

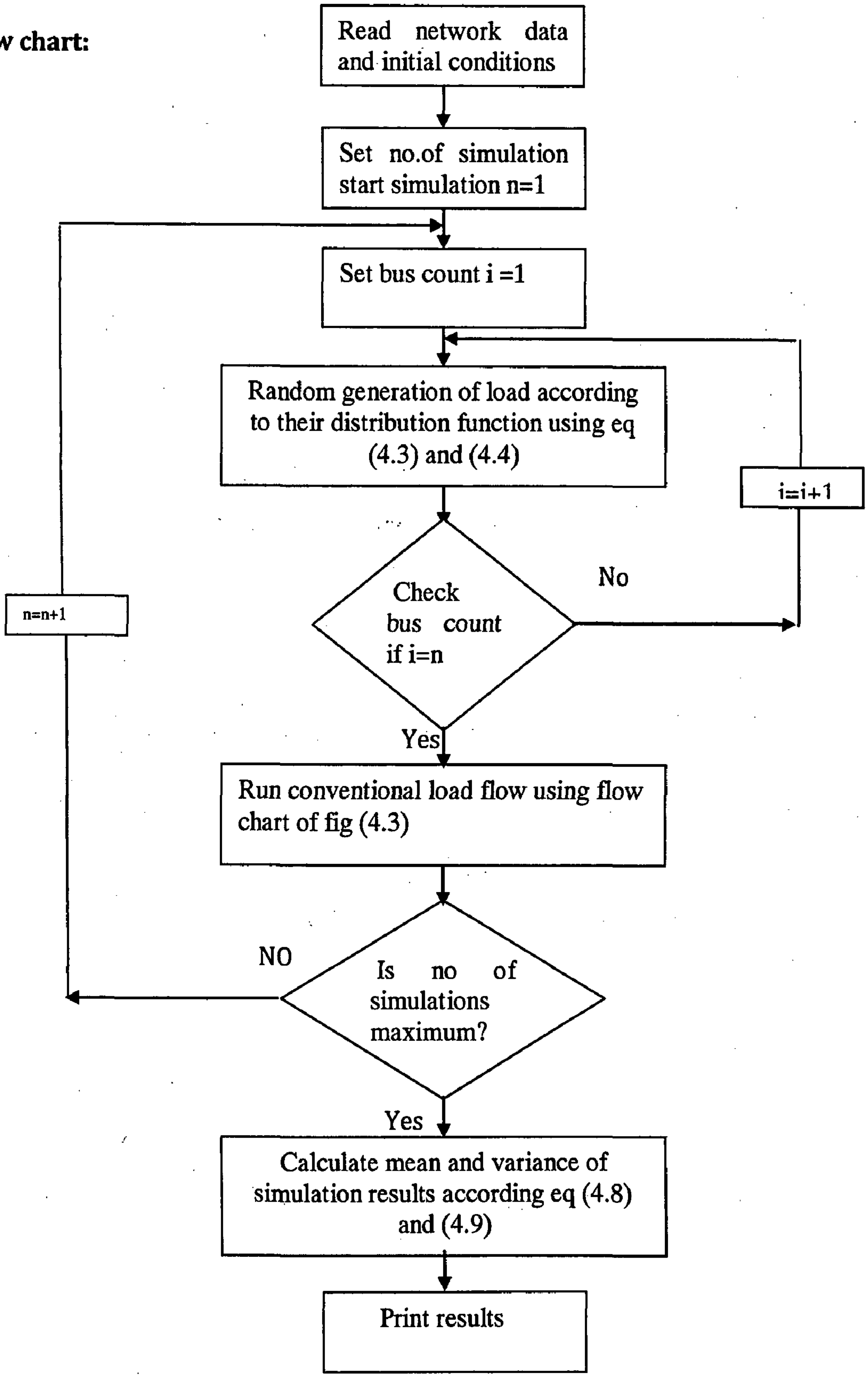


Figure 4.4: Flow chart of Monte Carlo Simulation

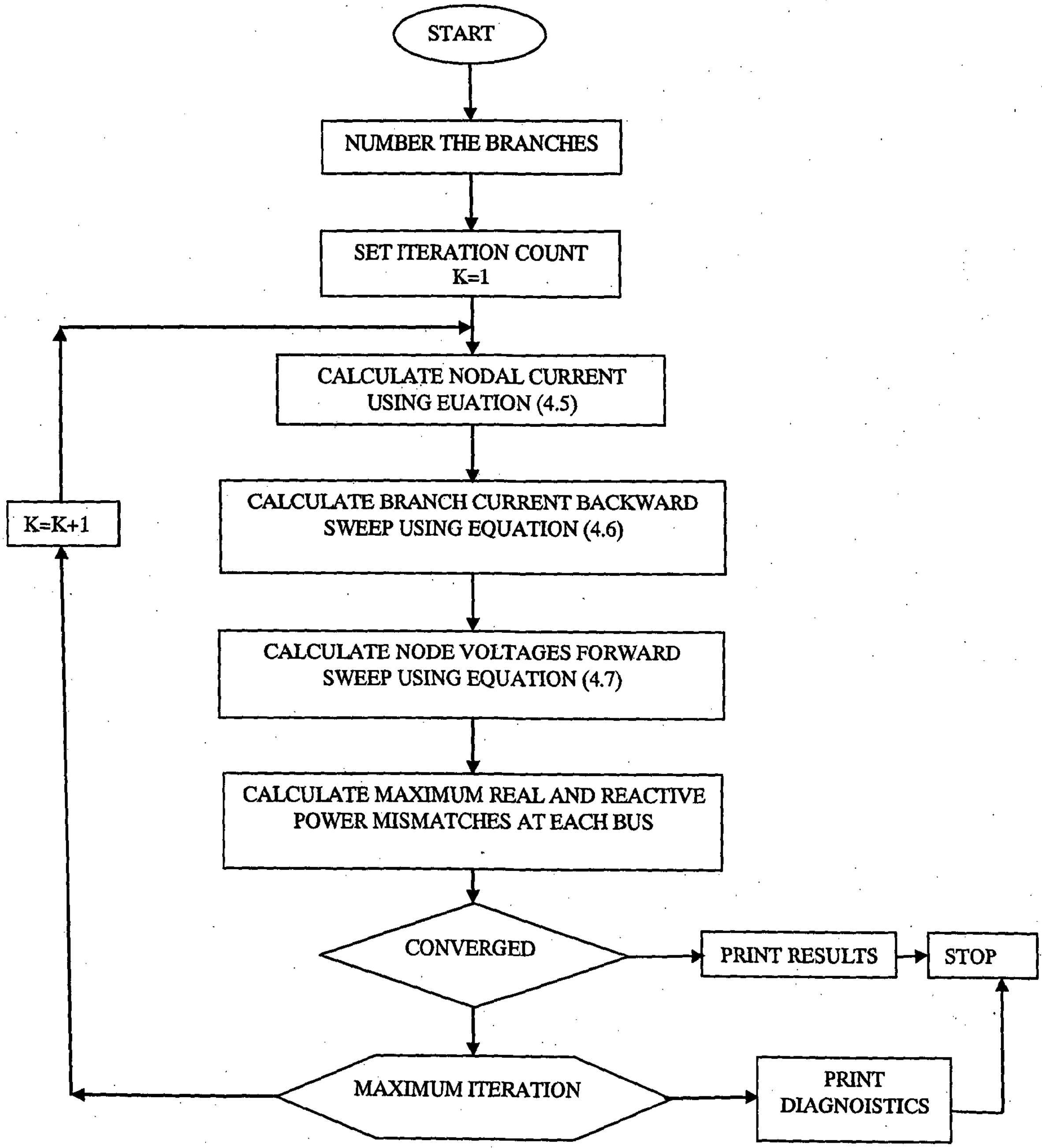


Figure 4.5: Flow chart of load flow solution

4.1.7 General evaluation:

The Monte Carlo Simulation is a synthetic sampling process. Generation of n values y1, y2..........y_n of Y and computed sample mean and its variance is

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad \text{..... (4.8)}$$

$$Var(Y)=E[(Y-E(Y))^2] \qquad \text{.....(4.9)}$$

Where E(Y) is the expected or average value of Y

In any sampling experiment, the accuracy of results increases with the number of samples; therefore the accuracy of Monte Carlos analysis will depend on the number of simulations. Flow chart of Monte Carlo Simulation method is given in Fig.4.4.

4.2 Probabilistic Distribution Load Flow Technique Using Laplace transforms:

4.2.1General formulation:

The load flow problem can be mathematically described by two sets of nonlinear equations [6-7] as:

$$Y=g[x] \qquad \text{..... (4.10)}$$

$$Z=h[x] \qquad \text{..... (4.11)}$$

Where, in the case of probabilistic load flow (PLF)

Y- Input random vector (real and reactive injections)

X-state random vector (voltage magnitudes and angles)

Z-output random vector (power flows)

g, h-load flow functions.

Once the input vector Y is specified, the state vector X must be evaluated in order to determine the output vector Z. As it is a well-known fact, the main problem is solving (4.10) since it is not possible to explicitly express X in terms of Y. Therefore, (4.10) is linearized around the specified values Y_0. In the case of probabilistic load flow (PLF) where input variables are given in terms of their respective probability distribution function (PDF) s, the most appropriate values to linearize around are the expected values. So, let Y_0 denotes the expected value of Y.

$$Y_0=g[X_0] \qquad \text{..... (4.12)}$$

$$Z_0=h[X_0] \qquad \text{..... (4.13)}$$

Z_0 and X_0 are only approximations for the expected values of Z and X due to the nonlinear load flow functions.

Linearizing (4.10) and (4.11) around the points (Y_0, X_0) and (Z_0, X_0), respectively, gives the following:

Where

$$X \approx X_0 + A.\Delta Y = X_0' + A.Y \qquad \text{..... (4.14)}$$

$$Z \approx Z_0 + A.\Delta Y = Z_0' + B.Y \qquad \text{..... (4.15)}$$

$$A=(\frac{\partial g}{\partial x} \mid x = x_0)^{-1}$$

$$B = (\frac{\partial h}{\partial x} \mid x = x_0).A$$

$$\Delta Y = Y - Y_0$$

$$X_0' = X_0 - A.Y_0,\ Z_0' = Z_0 - B.Y_0$$

Equations (4.14) and (4.15) express each element of the random vectors X and Z as a linear combination of random elements of the input vector Y. The random elements of vectors X and Z can be computed from a "weighted" sum of the random elements of vector ΔY. The weighting coefficients are defined sensitivity coefficients. The sum of independent (or in some cases linearly dependent) random variables can be made [7] using mathematical convolution techniques. The convolution implied by equations (4.14) and (4.15) can be written as

$$f(X_i) = f(Y_1') * f(Y_2') * \dots\dots\dots f(Y_n') \qquad \text{..... (4.16)}$$

Where

Y_k' *represents* $(Y_k - Y_{0k})a_{ik}$

* Denotes convolution.

a_{ik} *is an element of A*

Equation (4.16) can be evaluated with different ways. One is to use numerical methods based on Laplace transforms, which is referred to as the conventional method. Another method, transforms the equation in to frequency domain using fast Fourier (FFT) techniques. The remaining deterministic part of (4.14) and (4.15), x_{oj} and z_{oj}, which is related to the point of linearization, affects only the position of the resultant PDF.

4.2.2 Algorithm: Step1:

When the system being considered has radial operating structure, the linearization of (4.10) and (4.11) can be significantly simplified. Firstly, if we neglect power losses in the elements of the system, i.e. $\Delta P = \Delta Q = 0$, real and reactive power flow in the element "i-j" would be.

$$P_{i-j} = \sum_{k \in A_j} P_k \quad \text{..... (4.17)}$$

$$Q_{i-j} = \sum_{k \in A_j} Q_k \quad \text{..... (4.18)}$$

Where A_j denotes the set of all nodes supplied via node "j", including "j" itself.

Because of the radial operating structure of the system, each element of the output vector (whether they are random or not) can be determined directly as a sum of some or all the elements of the input vector (injected real and reactive powers).

Step2:

When calculating the voltages in distribution networks, having in mind that those are operating on medium and low voltage level, the following two approximations can be made

- The imaginary part of voltage drop in any element of the network compared to the real one is much smaller; hence, can be neglected.
- Since voltages in every node of the network do not differ much from the rated Voltage, it can be used rated voltage can be used instead of the actual voltage, when calculating the voltage drop.

Consequently, for the voltage drop in the element "i-j", ΔV_{i-j} we can write the following:

$$\Delta V_{i-j} = \frac{P_{i-j}R_{i-j}+Q_{i-j}X_{i-j}}{V_j} + \frac{P_{i-j}X_{i-j}-Q_{i-j}R_{i-j}}{V_j} \qquad \text{..... (4.19)}$$

Where R_{i-j}, and X_{i-j} denote resistance and reactance of the element "i-j". One consequence of the first approximation is that the voltages in all the nodes of the network have the same phase angle. Therefore, the voltage drop in any sequence of the consecutive elements in the network is the algebraic sum of the respective voltage drops calculated in accordance with (4.19). Thus, the total voltage drop from the feeding node to the node "k" will be sum of voltage drops in all the elements found on the path of supply, starting from the feeding point up to the node "k", i.e:

$$\Delta V_k = \sum_{(i-j)\in\pi_k} \Delta V_{i-j} \qquad \text{..... (4.20)}$$

Where π_k denotes the set of elements found on the supply path for the node k.

Step3:

For the voltage in node "k", we can write:

$$V_k = V_0 - \Delta V_k \qquad \text{..... (4.21)}$$

Where V_0 denotes the voltage of the feeding point.

Expression (4.19) from which we calculate (4.20) and (4.21) is not quite suitable, however, for evaluation of the resultant PDF of the voltage in node "k", because it contains output random variables (P_{ij}, Q_{ij}) Although these variables are not just numbers, but random variables defined with their respective PDFs which need not be statistically independent. Therefore, it is much more suitable to express the voltage drops in terms of input random variables. Real and reactive power in (4.19) should be substituted with (4.18) and (4.19). When this new expression for the element's voltage drop is entered in (4.20) and the summations are interchanged, the total voltage drop from the feeding node to the node "k" is given as:

4.2.3 Flow chart:

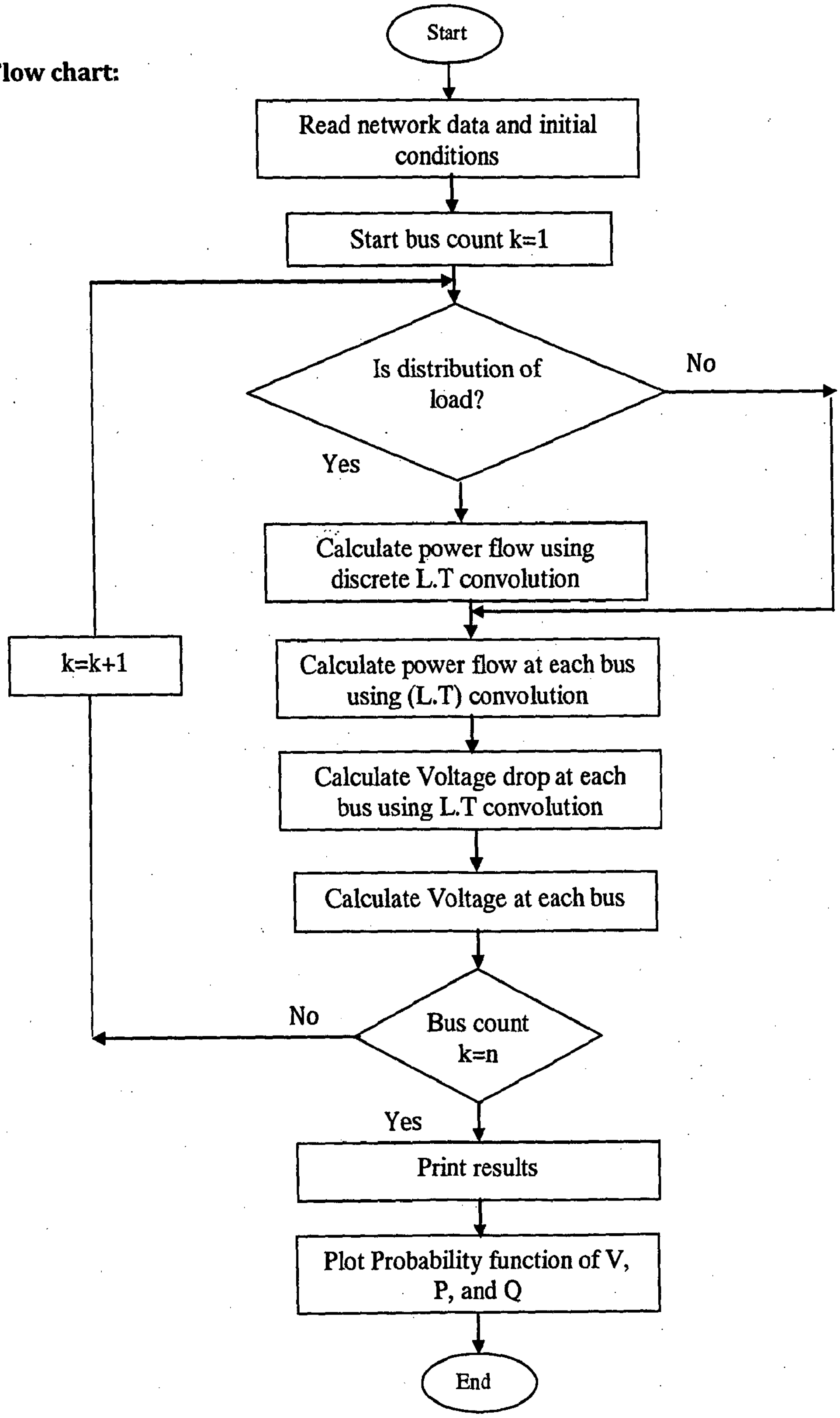


Figure 4.6: Flow chart

$$\Delta V_k = \frac{1}{V_{nom}} \sum_{i=1}^{n} (P_i R_{ik} + Q_i X_{ik}) \qquad \text{..... (4.22)}$$

Where R_{ik} and X_{ik} denote total resistance and reactance of the common elements found on the supplying paths of nodes "i" and "k" (intersection). The flow chart for this method is given in Fig.4.6.

4.3 Probabilistic Load flow Computation using Gram-Charlier expansion

P. Zhang and S. T. Lee [22] first proposed the application of probabilistic load flow computation using combined cumulants and Gram-Charlier expansion for power flow analysis of transmission networks. They also compared their results with the results obtained from Monte-Carlo Simulation method. In their study they have taken WSCC test system for illustration.

In this thesis the probabilistic load flow computation using Gram-Charlier expansion for power flow solution of a balanced radial distribution system has been developed. In this study, the load demands in the system at different buses are uncertain. The load flow algorithm chosen is essentially a probabilistic load flow algorithm [24]. This proposed method has been tested with 30 bus test system and the results have been compared with Monte Carlo and Probabilistic Load flow using L.T convolution method.

The major problem in the conventional convolution method is to compute the equivalent discrete function since a function represented by r impulses convolved with another represented by s impulses will have r times s impulses. Reference [11] clearly stated that, even to obtain the PDF of a single line flow, the final number of discontinuous points could be extremely large when the number of discontinuous curve to be convoluted are large or each curve is represented by a large number of points. This process requires a large amount of storage and time. Compared with other methods used by previous researchers [7]-[12], the method incorporate in this thesis avoids complex convolution calculation and replaces them with simple arithmetic process due to unique properties of Cumulants. Moreover, this method is able to obtain the PDF of line flows with one load flow. This method significantly reduces the storage since low order Gram-Charlier expansion is able to achieve enough accuracy to approximate PDF of line flows. Study results have shown that the proposed method can calculate the probability distribution accurately with much less computation effort.

4.3.1 Algorithm:

Step1: Read the given Probabilistic load data and network data

Step2: Set the order of moments and cumulants

Step 3: check the type of distribution of load data

Step 4: if the distribution of load is normal go to step 6 else step5.

Step 5: Calculate the moments of injected active power as well as reactive power according to the equation (3.1).

Step 6: Compute the cumulants of injected power according to the relationship between moments and cumulants of equation (3.7) or (3.9 & 3.10).

Step 7: Compute the moments of resistance and reactance from the given network probabilistic data based on the equation (3.1)

Step 8: Calculate the cumulants of line flow from the following equation

For the i th line flow.

$$P_{linei} = h_{i1}p_1 + h_{i2}p_2 + \dots\dots\dots\dots + h_{in}p_n \qquad \dots\dots (4.23)$$

$$Q_{linei} = h_{i1}Q_1 + h_{i2}Q_2 + \dots\dots\dots\dots + h_{in}Q_n \qquad \dots\dots (4.24)$$

Where

h_{ij} represents the sensitivity coefficient

h_{ij} =1 if j^{th} load is in the path to the i^{th} line else 0

For the cumulants (γ) related with i^{th} line flow

$$\gamma_v = h_{i1}^v \gamma_v^1 + h_{i2}^v \gamma_v^2 + \dots\dots\dots\dots + h_{in}^v \gamma_v^n \qquad \text{..... (4.25)}$$

Where V= 1, 2………

Step 9: Compute the central moments of each line according to the relation ship between central moments and cumulants expressed using equation (3.14) or (3.17 & 3.18).

Step 10: Calculate the Gram-Charlier expansion coefficients using equation (3.38)

Step 11: the Probability Distribution Function of line flow can be obtained using equati on (3.38)

Step 12: Compute resultant moments of active and reactive voltage drops using the following equations.

$$\alpha_v(\Delta V_{ik})_{active} = \alpha_v(P_{ik})\alpha_v(R_{ik}) \qquad \text{..... (4.26)}$$

$$\alpha_v(\Delta V_{ik})_{reactive} = \alpha_v(Q_{ik})\alpha_v(X_{ik}) \qquad \text{..... (4.27)}$$

Where

α denotes moments about origin

Step 13: Calculate cumulants of voltage drop from their moments using equation (3.6)

Step 14: Compute the voltage drop cumulants using the following equation

$$\gamma_v(\Delta V_k) = \gamma_v(P_{ik}R_{ik}) + \gamma_v(Q_{ik}R_{ik}) \qquad \text{..... (4.28)}$$

Step 15: Compute the voltage cumulants at each node

$$\gamma_v(V_k) = \gamma_v(V_1) - \gamma_v(\Delta V_k) \qquad \text{..... (4.29)}$$

Where

$\gamma_v(V_1)$ = Voltage cumulants at substation node

$\gamma_v(\Delta V_k)$ = Voltage drop cumulants

Step 16: Calculate central moments of voltage at each node based on equation (3.14) or (3.17 & 3.18).

Step 17: Compute the Gram-Charlier coefficients using equation (3.36).

Step 18: Compute the Probability Distribution Function of voltage using equation (3.38).

4.4Probabilistic Load Flow using Complex Random Variable Analysis:

S.Patra and R.B.Misra [20] first proposed the probabilistic load flow using Method of moments for power flow analysis of transmission networks. Later Tae-Kyunkim [21] applied this method for security assessment of bus voltages in transmission network.

In this thesis the probabilistic load flow using Complex random Variable analysis for power flow solution of distribution system has been carried out. In this study, the load outages and network outages has also been considered. The load flow algorithm chosen is essentially a Z Loop load flow algorithm [24]. This method has been tested with 13 and 30 bus test system.

4.4.1 Slack Bus Representation: At the slack bus the voltage remains constant. If the slack bus is usually assumed to be bus number one, this constancy of bus voltage may be probabilistically simulated by a PDF as depicted in Fig. 4.7.

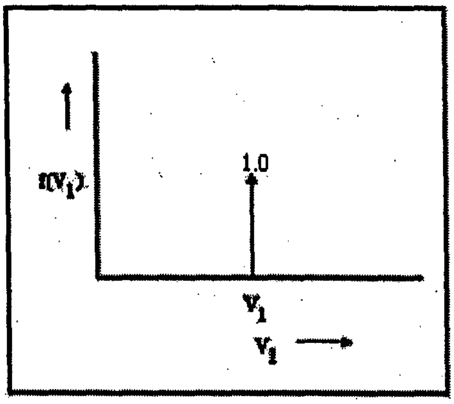


Figure 4.7: PDF of Slack bus voltage

The moments of this CRV V_1 are simply given by

$$\alpha_t(V_1) = V_1, \quad t = 1,2 \ldots\ldots \qquad \ldots\ldots (4.30)$$

The cumulants are given by

$$\gamma_t(V_1) = \alpha_1(V_1) = V_1 \qquad \ldots\ldots (4.31)$$

$$\gamma_t(V_1) = 0 \qquad t \geq 2 \qquad \ldots\ldots (4.32)$$

The slack bus is, therefore, assumed capable, by means of the generating unit connected to it, to maintain a constant voltage for all possible contingencies of the generating units, distribution lines as well as load variation and uncertainty. This clearly implies that a 100% reliable generating unit is appropriately fixed so as to provide the required active and reactive powers for all contingencies.

4.4.2 PQ-Bus Representation: At a PQ- bus the active and reactive powers are known since the bus powers are given by the difference between the generated powers and the demand as

$$S = S_G - S_D \qquad \text{..... (4.33)}$$

This assumes that the PDF of generated power as well as the PDF of demand are known. There are clearly three cases to consider

(i) PQ-bus with no generation such that $S = S_D$, (ii) PQ-bus with no demand such that $S = S_G$ and (iii) PQ-bus with both generation and demand such that $S = S_G - S_D$

In the first case, the PDF of demand is assumed to be expressible in terms of active and reactive powers. In this thesis two types of loads have been considered.

a. Deterministic load:

The load is represented by $S_D = P_D + jQ_D$ with known probability $p_d = 1.0$ and may be depicted as in Fig.4.8.

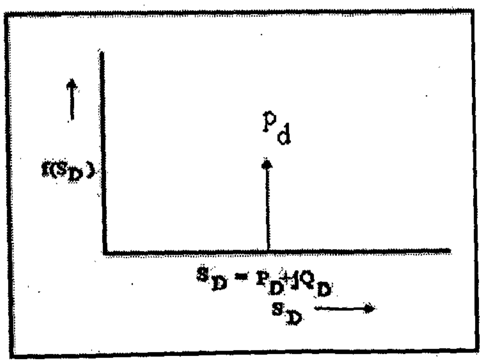


Figure: 4.8 PDF of available Load or demand

b. Load with Uncertainty:

The simplest way to simulate this is by means of The PDF as shown in Fig. 4.9.

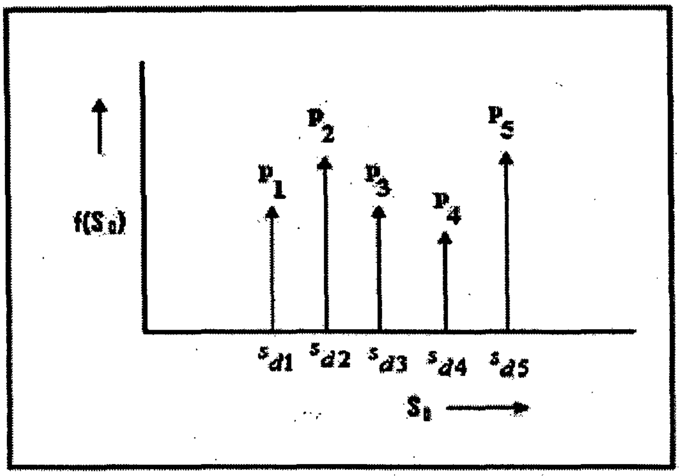


Figure 4.9: PDF of load with Uncertainty

The moments and cumulants may be evaluated, therefore, from these PDFs in a straight forward manner.

In the second case, the PDF of available capacity is assumed known. A typical PDF for a unit may be depicted as shown in Fig. 4.10. The total PDF of available capacity at a bus may be obtained convolving the individual PDFs of the units connected at that bus. The implicit assumption that is made is that outages of these units occur independently.

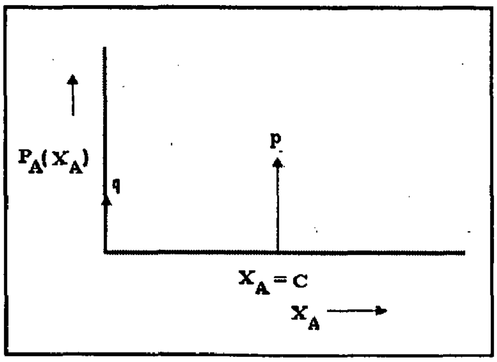


Figure: 4.10 PDF of available capacity of a generating unit

In the third case, the PDF of generation must be convolved with PDF of demand (negative demand S_D) to obtain the PDF of bus power. Since these CRVs are assumed independent. That cumulants can be added. The procedure can be put into the following algorithm.

4.4.2.1 Algorithm:

1. Obtain the moments of S_G *and* $-S_D$

2. Obtain the cumulants using the relation between moments and cumulants using equation (3.30)

3. Add the cumulants of S_G *and* $-S_D$

4. Obtain the moments of the sum using the relation between cumulants and moments using equation (3.32)

4.4.3 Formulation of Stochastic equations:

Assume that voltage (V) and power (S) are independent complex random variables with known probability density functions. The moments of arbitrary i-th bus currents are calculated by the complex power equation

$$I.V^* = S^* \qquad \text{..... (4.34)}$$

$$\alpha_t(I_i) = \alpha_t(S_i^*) / \alpha_t(V_i^*) \qquad \text{..... (4.35)}$$

Where

t: the t-th order moments,

i: the i-th bus

$\alpha = moments\ about\ origin$

Branch current stochastic equation

$$\gamma_t(J_i) = -\gamma_t(I_{i-1}) + \gamma_t(\Sigma\, currents \text{ eminating from (i-1) th node}) \qquad \text{..... (4.36)}$$

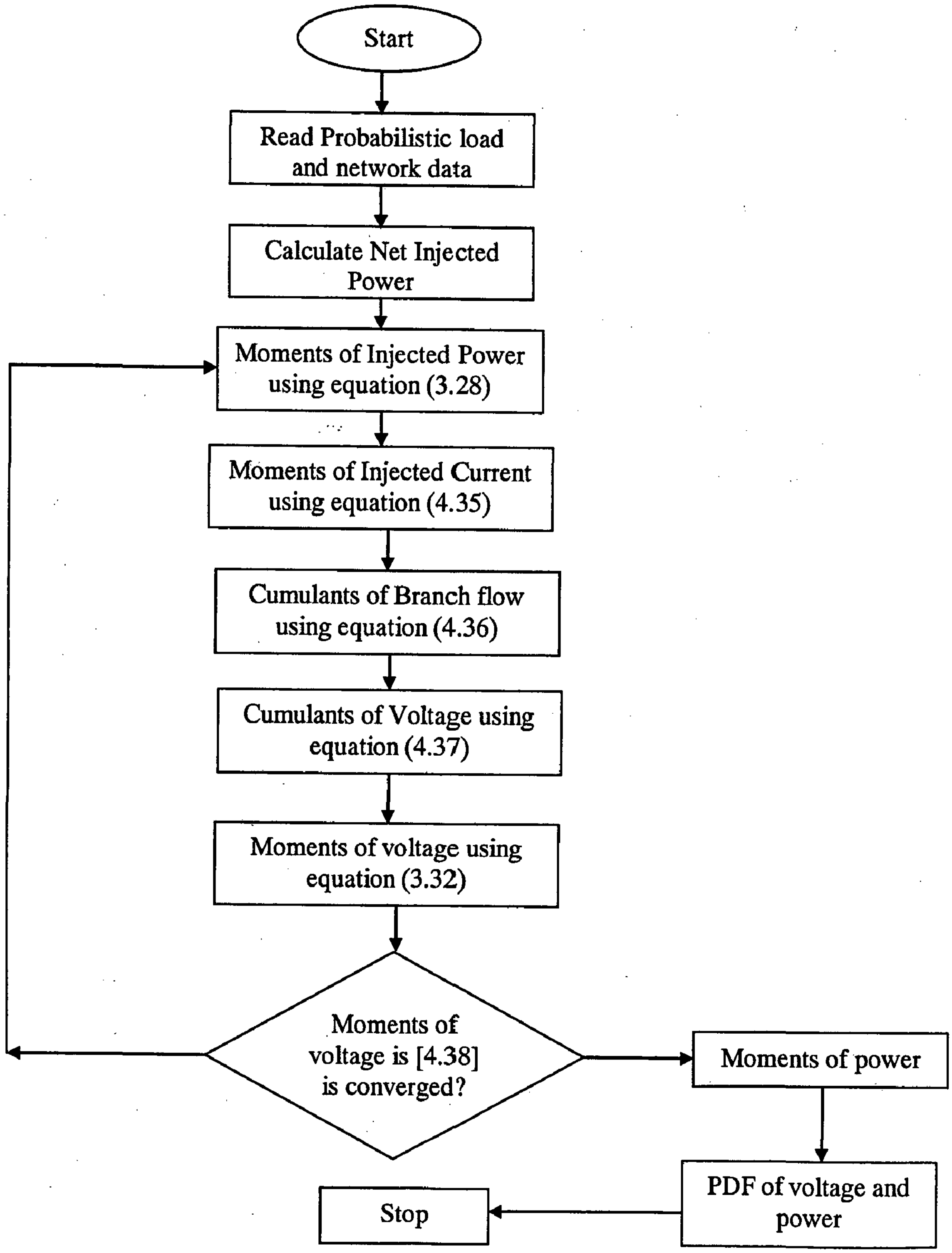


Figure 4.11: Flow chart

Voltage equation in the stochastic form

$$\gamma_v(V_i) = \gamma_v(V_{i-1}) - \gamma_v(Z_i J_i) \quad \text{..... (4.37)}$$

The cumulants of $\gamma_v(Z_i J_i)$ can be calculated using transformation between the moments and cumulants.

$$\alpha_v(Z_i J_i) = \alpha_v(Z_i)\alpha_v(J_i) \quad \text{..... (4.38)}$$

Moments of Voltage Converge if

$$|\alpha(V^{(k-1)}) - \alpha(V^k)| \leq \epsilon \quad \text{..... (4.39)}$$

Where

$\alpha\,(V^k) =$ *moment of* k^{th} *iteration of voltage*

$\alpha\left(V^{(k-1)}\right) =$ *moment of* $(k-1)th$ *iteration of voltage*

$\epsilon = 1 \times 10^{-3}$

The flow chart of Probabilistic Load Flow using Complex Random Variable analysis is given in Fig 4.11.

In the next chapter the simulation results are presented

Chapter5

Simulation Results and Discussions

Probabilistic Load Flow using Laplace Transform, Probabilistic load flow computation using Gram-Charlier have been applied on a test system as shown in Fig 3.1, and the data are given in table B1 an B2 in appendix B. Out of the total number of 30 buses, 20 buses are load buses. The root node has a specified voltage of 1.05p.u. Two types of load probability distribution function (normal and discrete) with four different cases have been considered. The results have been compared with Monte Carlo Simulation and deterministic load flow methods.

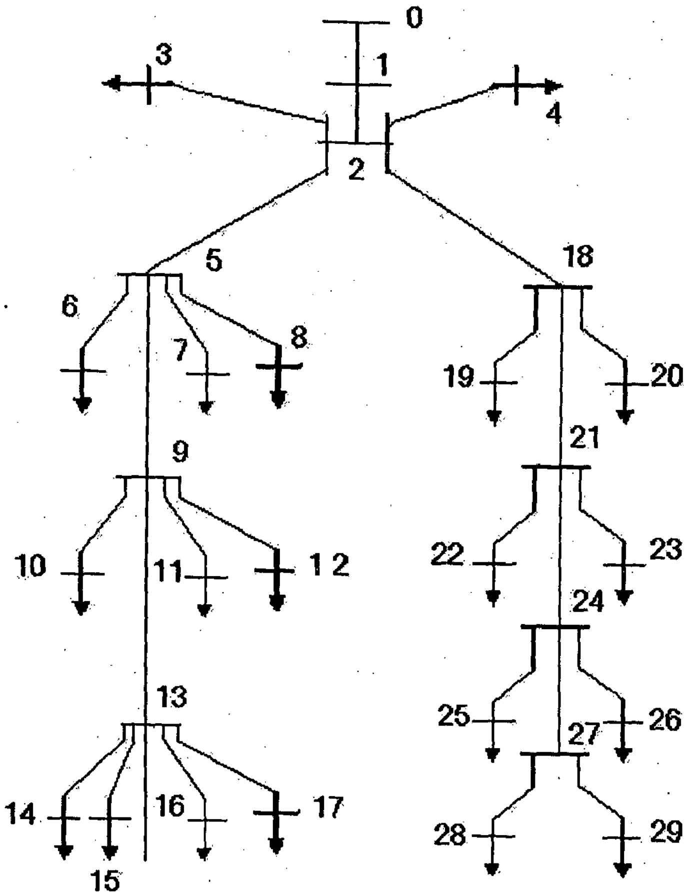


Figure 5. 1 Radial distribution network

In deterministic load flow the loads are deterministic i.e. loads are with out uncertainty. The power losses and voltage drops have been considered. In Monte Carlo Simulation method the loads are probabilistic instead of deterministic.

To select appropriate number of simulations for Monte Carlo Simulation method the bus voltages, the bus active power and the bus reactive power variations with number of simulations were plotted (FIG. 5.2) and compared with base case load flow results. The Monte Carlo Method converges after 800 simulations for voltages, 1400 simulations for bus active power and 1400 simulations for bus reactive power. Hence 1400 simulations are used in Monte Carlo Simulations.

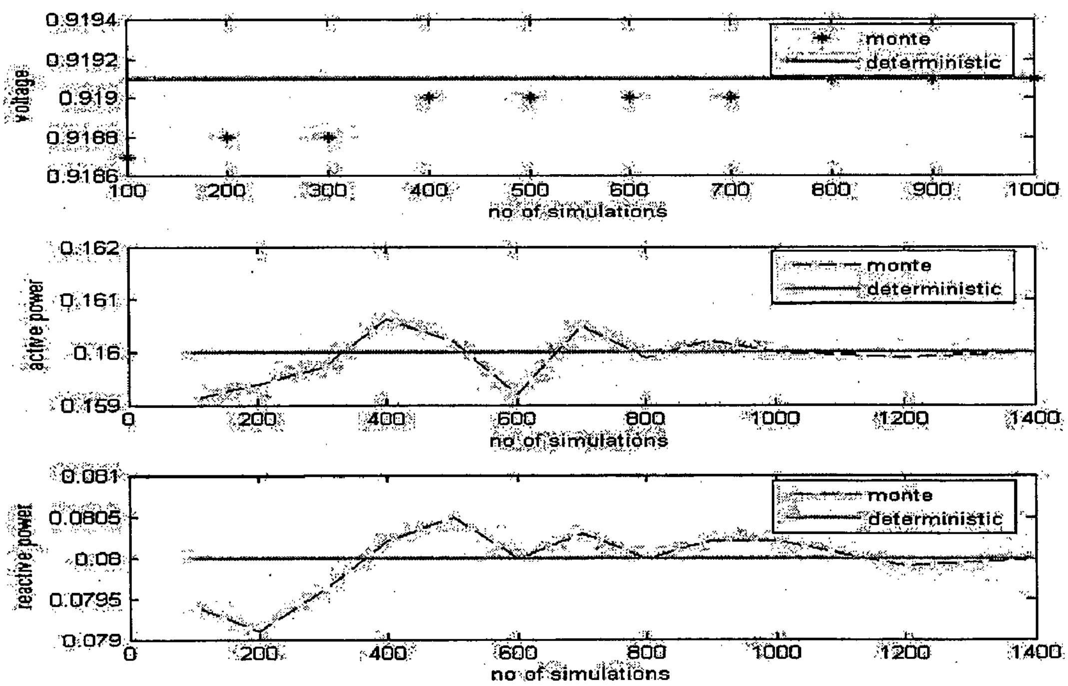


Figure5.2: Variation of bus voltage, bus active power, bus reactive power with no of simulations

5.1.1 Case (1): When all Bus loads are independent and normal:

Load flow using Monte Carlo Technique, probabilistic using L.T technique and PLF using Gram-Charlier have been carried out for the case when all loads are independent and have normal distribution. The results for bus voltage, bus active powers and bus reactive powers are given below.

Table 5.1: Comparison of bus voltage PDF

DETERMINISTIC		MONTE-CARLO		PROBABILISTIC USING (L.T)		GRAM-CHARLIER	
Bus no	base(V)	mean(V)	std(V)	mean(V)	Std(V)	mean(V)	Std(V)
1	1.05	1.05	0	1.05	0	1.05	0
2	0.9265	0.9264	0.0025	0.9372	0.0019	0.9372	0.0019
3	0.9252	0.9251	0.0026	0.936	0.0019	0.936	0.0019
4	0.9243	0.9242	0.0026	0.9352	0.0019	0.9352	0.0019
5	0.9241	0.924	0.0026	0.935	0.0019	0.935	0.0019
6	0.9187	0.9186	0.0027	0.93	0.002	0.93	0.002
7	0.9181	0.9181	0.0027	0.9295	0.002	0.9295	0.002
8	0.9181	0.9181	0.0027	0.9295	0.002	0.9295	0.002
9	0.9172	0.9172	0.0027	0.9287	0.002	0.9287	0.002
10	0.9166	0.9165	0.0027	0.9282	0.0021	0.9282	0.0021
11	0.916	0.9159	0.0027	0.9276	0.0021	0.9276	0.0021
12	0.9157	0.9156	0.0027	0.9273	0.0021	0.9273	0.0021
13	0.9161	0.916	0.0027	0.9277	0.0021	0.9277	0.0021
14	0.9159	0.9158	0.0027	0.9275	0.0021	0.9275	0.0021
15	0.9154	0.9154	0.0028	0.9271	0.0021	0.9271	0.0021
16	0.9154	0.9153	0.0028	0.9271	0.0021	0.9271	0.0021
17	0.9147	0.9146	0.0028	0.9264	0.0021	0.9264	0.0021
18	0.9151	0.9151	0.0028	0.9268	0.0021	0.9268	0.0021
19	0.923	0.923	0.0026	0.934	0.0019	0.934	0.0019
20	0.9228	0.9228	0.0026	0.9339	0.0019	0.9339	0.0019
21	0.9219	0.9219	0.0026	0.933	0.002	0.933	0.002
22	0.9214	0.9213	0.0026	0.9325	0.002	0.9325	0.002
23	0.9213	0.9212	0.0027	0.9324	0.002	0.9324	0.002
24	0.9208	0.9207	0.0027	0.932	0.002	0.932	0.002

25	0.9206	0.9205	0.0027	0.9318	0.002	0.9318	0.002
26	0.9201	0.92	0.0027	0.9313	0.002	0.9313	0.002
27	0.92	0.9199	0.0027	0.9313	0.002	0.9313	0.002
28	0.9202	0.9201	0.0027	0.9314	0.002	0.9314	0.002
29	0.9191	0.919	0.0027	0.9304	0.002	0.9304	0.002
30	0.9199	0.9198	0.0027	0.9311	0.002	0.9311	0.002

From the table (5.1) it can be observed that Monte Carlo Simulation results are closer to the results obtained from deterministic load flow method. The probabilistic using L.T method and Gram-Charlier results are exactly matching. The probabilistic load flow using L.T method, Gram-Charlier results are also in good agreement with Monte Carlo simulation. The maximum error in both cases is 1.2737%.

This difference in bus voltages in both cases due to

(a) Assuming 1.0p.u voltage at all busses.

(b) Using approximate expression for calculating the line voltage drop.

Table 5.2: Comparison of bus active power PDF

DETERMINISTIC		MONTE-CARLO		PROBABILISTIC USING (L.T)		GRAM-CHARLIER	
Bus no	P	mean(P)	std(P)	mean(P)	std(P)	mean(P)	std(P)
1	3.5849	3.5866	0.0871	3.2	0.0716	3.2	0.0716
2	3.2271	3.2282	0.0723	3.2	0.0716	3.2	0.0716
3	3.223	3.2241	0.0721	3.2	0.0716	3.2	0.0716
4	0.16	0.1601	0.0157	0.16	0.016	0.16	0.016
5	0.16	0.16	0.0163	0.16	0.016	0.16	0.016
6	1.6044	1.6055	0.05	1.6	0.0506	1.6	0.0506
7	0.16	0.1604	0.016	0.16	0.016	0.16	0.016
8	0.16	0.1597	0.0161	0.16	0.016	0.16	0.016
9	0.16	0.1605	0.016	0.16	0.016	0.16	0.016
10	1.1215	1.1221	0.042	1.12	0.0423	1.12	0.0423
11	0.16	0.1601	0.0161	0.16	0.016	0.16	0.016
12	0.16	0.1602	0.0159	0.16	0.016	0.16	0.016
13	0.16	0.1601	0.0161	0.16	0.016	0.16	0.016

14	0.6406	0.6408	0.0318	0.64	0.032	0.64	0.032
15	0.16	0.1597	0.0163	0.16	0.016	0.16	0.016
16	0.16	0.1601	0.0159	0.16	0.016	0.16	0.016
17	0.16	0.1601	0.0159	0.16	0.016	0.16	0.016
18	0.16	0.1603	0.016	0.16	0.016	0.16	0.016
19	1.2833	1.2831	0.046	1.28	0.0453	1.28	0.0453
20	0.16	0.1603	0.0159	0.16	0.016	0.16	0.016
21	0.16	0.16	0.0162	0.16	0.016	0.16	0.016
22	0.9614	0.9609	0.0396	0.96	0.0392	0.96	0.0392
23	0.16	0.1599	0.0159	0.16	0.016	0.16	0.016
24	0.16	0.1598	0.0159	0.16	0.016	0.16	0.016
25	0.6407	0.6405	0.0321	0.64	0.032	0.64	0.032
26	0.16	0.1597	0.0161	0.16	0.016	0.16	0.016
27	0.16	0.16	0.0163	0.16	0.016	0.16	0.016
28	0.3203	0.3204	0.0228	0.32	0.0226	0.32	0.0226
29	0.16	0.1601	0.0161	0.16	0.016	0.16	0.016
30	0.16	0.16	0.016	0.16	0.016	0.16	0.016

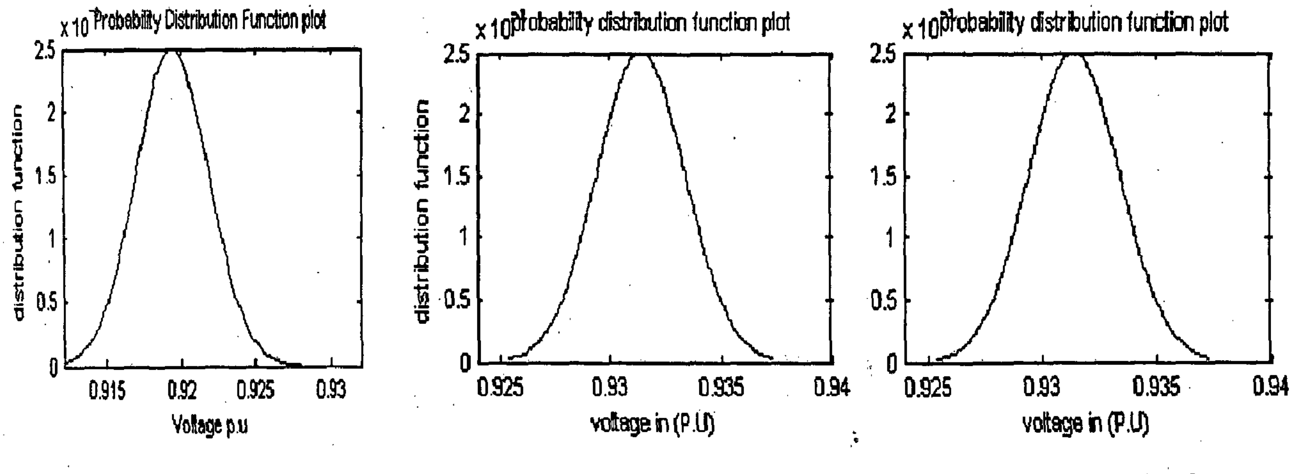
Table 5:3: Comparison of bus reactive power PDF

DETERMINISTIC		MONTE-CARLO		PROBABILISTIC USING (L.T)		GRAM-CHARLIER	
Bus no	Q	mean(Q)	std(Q)	mean(Q)	std(Q)	mean(Q)	std(Q)
1	1.966	1.9672	0.0431	1.6	0.0358	1.6	0.0358
2	1.6127	1.6133	0.0362	1.6	0.0358	1.6	0.0358
3	1.6097	1.6103	0.0362	1.6	0.0358	1.6	0.0358
4	0.08	0.0801	0.008	0.08	0.008	0.08	0.008
5	0.08	0.08	0.0078	0.08	0.008	0.08	0.008
6	0.802	0.8024	0.0256	0.8	0.0253	0.8	0.0253
7	0.08	0.0799	0.0079	0.08	0.008	0.08	0.008
8	0.08	0.0799	0.008	0.08	0.008	0.08	0.008
9	0.08	0.08	0.0081	0.08	0.008	0.08	0.008
10	0.5604	0.5611	0.0215	0.56	0.0212	0.56	0.0212
11	0.08	0.0801	0.008	0.08	0.008	0.08	0.008
12	0.08	0.0801	0.0081	0.08	0.008	0.08	0.008
13	0.08	0.0801	0.0081	0.08	0.008	0.08	0.008
14	0.3201	0.3203	0.0161	0.32	0.016	0.32	0.016
15	0.08	0.0801	0.0079	0.08	0.008	0.08	0.008
16	0.08	0.0799	0.008	0.08	0.008	0.08	0.008

17	0.08	0.0801	0.0081	0.08	0.008	0.08	0.008
18	0.08	0.0802	0.0082	0.08	0.008	0.08	0.008
19	0.6415	0.6416	0.0226	0.64	0.0226	0.64	0.0226
20	0.08	0.0801	0.0081	0.08	0.008	0.08	0.008
21	0.08	0.0799	0.0079	0.08	0.008	0.08	0.008
22	0.4805	0.4806	0.0194	0.48	0.0196	0.48	0.0196
23	0.08	0.0799	0.008	0.08	0.008	0.08	0.008
24	0.08	0.0802	0.0081	0.08	0.008	0.08	0.008
25	0.3201	0.3202	0.016	0.32	0.016	0.32	0.016
26	0.08	0.08	0.0081	0.08	0.008	0.08	0.008
27	0.08	0.08	0.0082	0.08	0.008	0.08	0.008
28	0.16	0.1601	0.0114	0.16	0.0113	0.16	0.0113
29	0.08	0.08	0.008	0.08	0.008	0.08	0.008
30	0.08	0.0801	0.008	0.08	0.008	0.08	0.008

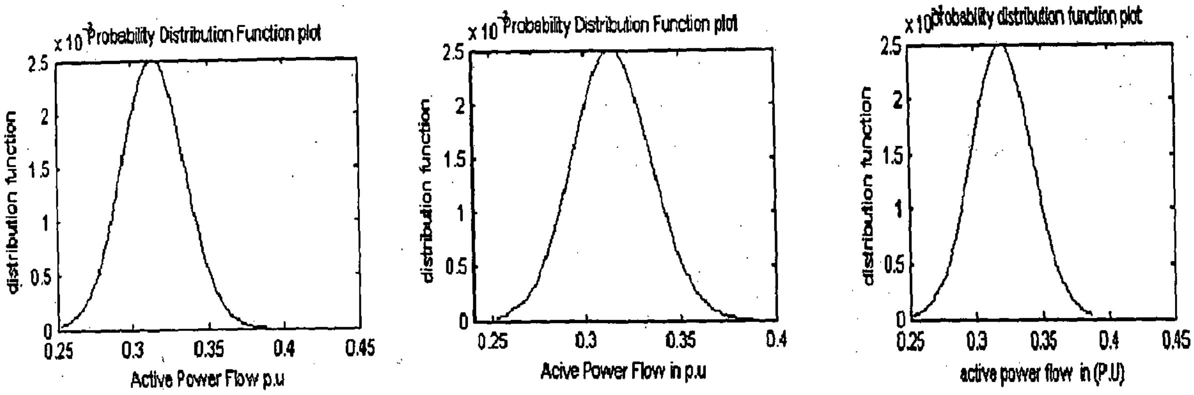
From the tables (5.2) and (5.3) comparing the bus active and reactive powers A good agreement in results between the four methods can be seen. Differences can be observed in the bus powers at the slack bus. The difference is of the order of 10.77%.

The difference in the slack bus power is due to the assumption of neglecting power losses in the lines, in this case active power loss is 0.3849pu and reactive power loss is 0.366pu. These losses are nearly equal to the difference in the value of P and Q at slack bus.



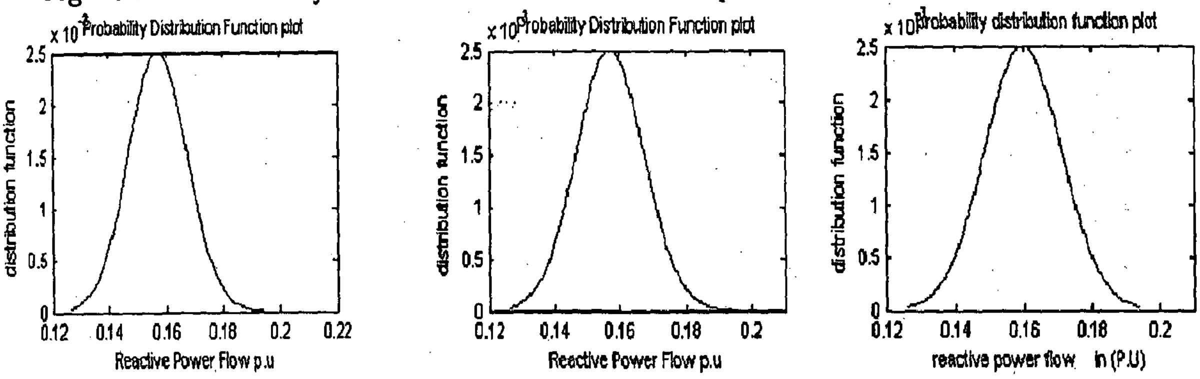
Monte Carlo Probabilistic [L.T] Gram-Charlier

Figure 5.3: Probability distribution function of the Voltage at node 28



Monte Carlo Technique | Probabilistic Technique [L.T] | Gram-Charlier

Figure 5.4: Probability distribution function of Active power flow in element 25-28



Monte Carlo Technique | Probabilistic Technique [L.T] | Gram-Charlier

Figure 5.5: Probability distribution function of Reactive power flow in element 25-28

In figures 5.3, 5.4 and 5.5 are shown Probability distribution function(PDF) of the voltage at node 28, the real and reactive power flow in element 25-28, for the case when all input variables independent and normal with Monte Carlo technique, Probabilistic technique using L.T and Gram-Charlier respectively. As it can be seen, these functions have the similar "bell" shape of the normal probability distribution functions. This is what should be expected for all the resultant variables in this case, since all inputs are normal. The PDF of above techniques compare well with each other.

The time taken by the probabilistic load flow computation using Gram-Charlier method is 1.9375sec, while the Monte Carlo Simulation requires 15sec (1400simulations). The results obtained by Monte Carlo Simulation are very accurate while the probabilistic load flow Computation using Gram-Charlier results are slightly less accurate. But the maximum error is 1- 2% and is with in acceptable limits. The results are of acceptable accuracy. The slight inaccuracy is attributed to the simplifying assumptions. How ever, these assumptions result in substantial simplification in the modeling.

5.1.2 Case (ii): At least one load has a discrete distribution function:

In order to demonstrate the impact on the resultant probability distribution function of the shape of the input probability distribution functions, for the load in 30 has been assumed to be a discrete distribution as shown in Fig 5.6, in percentage of the mean value. All the other inputs are independent and normal.

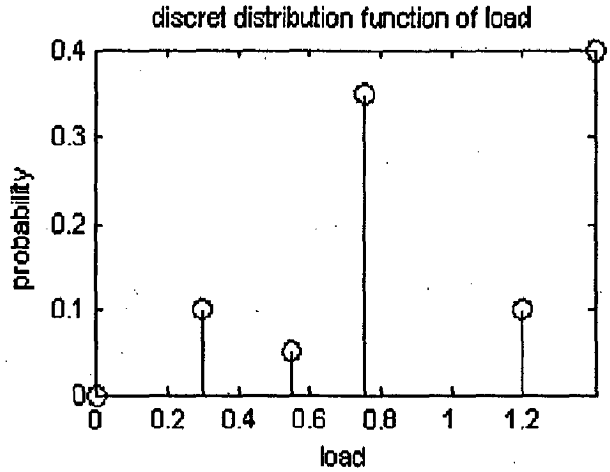


Figure 5.6: Probability mass function for the load at 30th bus

Load flow using probabilistic method using L.T and Gram-Charlier have been carried out and the results have been compared with Monte Carlo Simulation, Probabilistic load flow using Laplace transform and deterministic load flow, for the case when at least one of the load has been considered as a discrete distribution function and remain loads are independent and normal. The results for bus voltage, bus active powers are given below.

Table 5.4: Comparison of Bus Voltages

DETERMINISTIC		MONTE-CARLO		PROBABILISTIC USING L.T		GRAM-CHARLIER	
Bus no	base(V)	mean(V)	std(V)	mean(V)	std(V)	mean(V)	std(V)
1	1.05	1.05	0	1.05	0	1.05	0
2	0.9265	0.9263	0.0038	0.9372	0.0024	0.9372	0.0024
3	0.9252	0.9251	0.0038	0.936	0.0025	0.936	0.0025
4	0.9243	0.9241	0.0038	0.9352	0.0025	0.9352	0.0025
5	0.9241	0.9239	0.0038	0.935	0.0025	0.935	0.0025
6	0.9187	0.9185	0.0039	0.93	0.0026	0.93	0.0026
7	0.9181	0.918	0.0039	0.9295	0.0026	0.9295	0.0026
8	0.9181	0.918	0.0039	0.9295	0.0026	0.9295	0.0026
9	0.9172	0.9171	0.0039	0.9287	0.0026	0.9287	0.0026
10	0.9166	0.9165	0.0039	0.9282	0.0026	0.9282	0.0026
11	0.916	0.9159	0.004	0.9276	0.0026	0.9276	0.0026
12	0.9157	0.9156	0.004	0.9273	0.0026	0.9273	0.0026
13	0.9161	0.916	0.004	0.9277	0.0026	0.9277	0.0026
14	0.9159	0.9158	0.004	0.9275	0.0026	0.9275	0.0026
15	0.9154	0.9153	0.004	0.9271	0.0026	0.9271	0.0026
16	0.9154	0.9153	0.004	0.9271	0.0026	0.9271	0.0026
17	0.9147	0.9146	0.004	0.9264	0.0026	0.9264	0.0026
18	0.9151	0.915	0.004	0.9268	0.0026	0.9268	0.0026
19	0.923	0.9229	0.0039	0.934	0.0026	0.934	0.0026
20	0.9228	0.9227	0.0039	0.9338	0.0026	0.9338	0.0026
21	0.9219	0.9218	0.0039	0.933	0.0026	0.933	0.0026
22	0.9214	0.9213	0.004	0.9325	0.0026	0.9325	0.0026
23	0.9213	0.9211	0.004	0.9324	0.0026	0.9324	0.0026
24	0.9208	0.9206	0.004	0.932	0.0026	0.932	0.0026
25	0.9206	0.9205	0.0041	0.9318	0.0027	0.9318	0.0027
26	0.9201	0.9199	0.0041	0.9313	0.0027	0.9313	0.0027
27	0.92	0.9199	0.0041	0.9313	0.0027	0.9313	0.0027
28	0.9202	0.92	0.0041	0.9314	0.0027	0.9314	0.0027
29	0.9191	0.9189	0.0042	0.9304	0.0027	0.9304	0.0027
30	0.9199	0.9197	0.0042	0.9311	0.0028	0.9311	0.0028

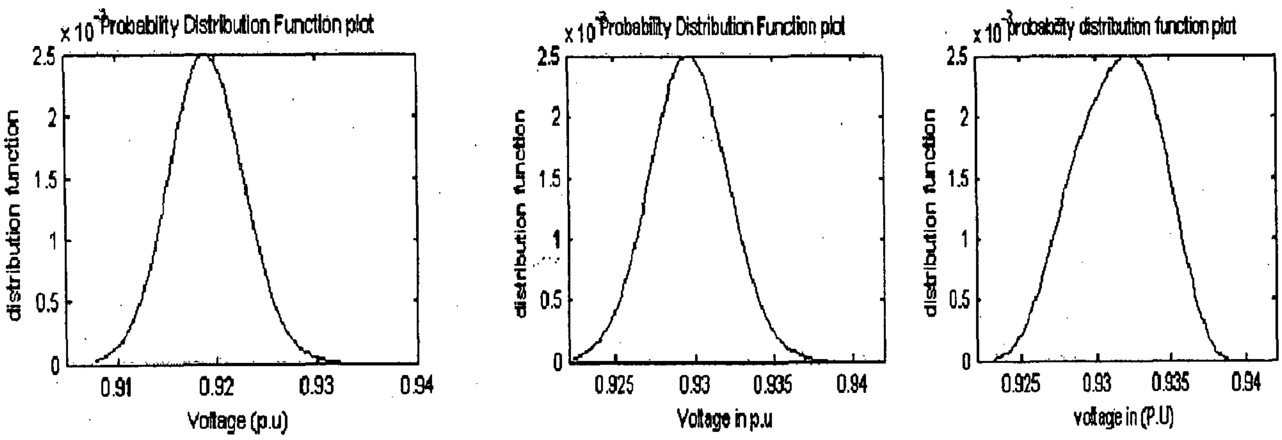
Table 5.5: Comparison of Bus active power flow

DETERMINISTIC		MONTE CARLO		PROBABILISTIC Using (L.T)		GRAM-CHARLIER	
bus no	P	mean(P)	std(P)	mean(P)	std(P)	mean(P)	std(P)
1	3.5849	3.5895	0.1167	3.2002	0.0931	3.2002	0.0931
2	3.2271	3.2305	0.0951	3.2002	0.0931	3.2002	0.0931
3	3.223	3.2264	0.0948	3.2002	0.0931	3.2002	0.0931
4	0.16	0.1597	0.0159	0.16	0.016	0.16	0.016
5	0.16	0.1599	0.016	0.16	0.016	0.16	0.016
6	1.6044	1.6057	0.051	1.6	0.0506	1.6	0.0506
7	0.16	0.1602	0.016	0.16	0.016	0.16	0.016
8	0.16	0.1604	0.0161	0.16	0.016	0.16	0.016
9	0.16	0.1602	0.0161	0.16	0.016	0.16	0.016
10	1.1215	1.122	0.0426	1.12	0.0423	1.12	0.0423
11	0.16	0.1599	0.0161	0.16	0.016	0.16	0.016
12	0.16	0.1603	0.0159	0.16	0.016	0.16	0.016
13	0.16	0.1601	0.016	0.16	0.016	0.16	0.016
14	0.6406	0.6408	0.0322	0.64	0.032	0.64	0.032
15	0.16	0.1603	0.0158	0.16	0.016	0.16	0.016
16	0.16	0.1599	0.016	0.16	0.016	0.16	0.016
17	0.16	0.1602	0.016	0.16	0.016	0.16	0.016
18	0.16	0.1598	0.0159	0.16	0.016	0.16	0.016
19	1.2833	1.2857	0.0749	1.2802	0.0748	1.2802	0.0748
20	0.16	0.16	0.0157	0.16	0.016	0.16	0.016
21	0.16	0.1602	0.0159	0.16	0.016	0.16	0.016
22	0.9614	0.9636	0.0714	0.9602	0.0713	0.9602	0.0713
23	0.16	0.1602	0.0161	0.16	0.016	0.16	0.016
24	0.16	0.16	0.0159	0.16	0.016	0.16	0.016
25	0.6407	0.6427	0.0677	0.6402	0.0676	0.6402	0.0676
26	0.16	0.1598	0.0159	0.16	0.016	0.16	0.016
27	0.16	0.1602	0.0159	0.16	0.016	0.16	0.016
28	0.3203	0.3223	0.0635	0.3202	0.0637	0.3202	0.0637
29	0.16	0.1604	0.0156	0.16	0.016	0.16	0.016
30	0.16	0.1616	0.0617	0.1602	0.0617	0.1602	0.0617

Table 5.6: Comparison of bus Reactive power Flow

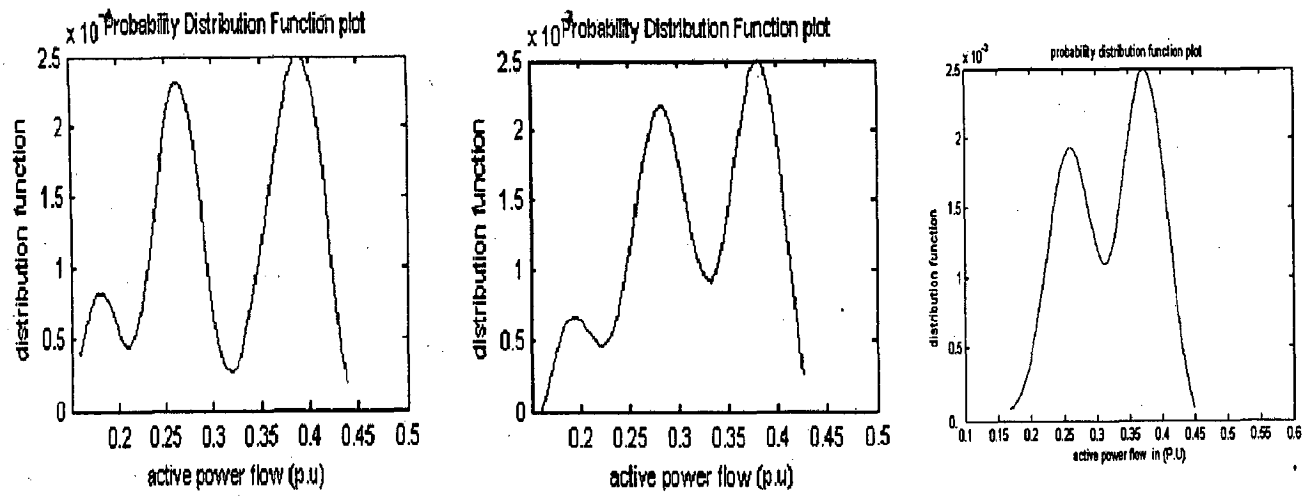
DETERMINISTIC		MONTE CARLO		PROBABILISTIC Using (L.T)		GRAM-CHARLIER	
bus no	Q	mean(Q)	std(Q)	mean(Q)	std(Q)	mean(Q)	std(Q)
1	1.966	1.9674	0.0631	1.6001	0.0465	1.6001	0.0465
2	1.6127	1.6129	0.0469	1.6001	0.0465	1.6001	0.0465
3	1.6097	1.6099	0.0468	1.6001	0.0465	1.6001	0.0465
4	0.08	0.0801	0.0081	0.08	0.008	0.08	0.008
5	0.08	0.08	0.0079	0.08	0.008	0.08	0.008
6	0.802	0.8015	0.0251	0.8	0.0253	0.8	0.0253
7	0.08	0.08	0.008	0.08	0.008	0.08	0.008
8	0.08	0.0799	0.008	0.08	0.008	0.08	0.008
9	0.08	0.0799	0.0081	0.08	0.008	0.08	0.008
10	0.5604	0.5602	0.021	0.56	0.0212	0.56	0.0212
11	0.08	0.0799	0.008	0.08	0.008	0.08	0.008
12	0.08	0.0799	0.008	0.08	0.008	0.08	0.008
13	0.08	0.08	0.0079	0.08	0.008	0.08	0.008
14	0.3201	0.32	0.0158	0.32	0.016	0.32	0.016
15	0.08	0.0803	0.008	0.08	0.008	0.08	0.008
16	0.08	0.0799	0.0079	0.08	0.008	0.08	0.008
17	0.08	0.0799	0.0079	0.08	0.008	0.08	0.008
18	0.08	0.0798	0.0081	0.08	0.008	0.08	0.008
19	0.6415	0.6421	0.0376	0.6401	0.0374	0.6401	0.0374
20	0.08	0.08	0.008	0.08	0.008	0.08	0.008
21	0.08	0.0798	0.0081	0.08	0.008	0.08	0.008
22	0.4805	0.4812	0.0358	0.4801	0.0356	0.4801	0.0356
23	0.08	0.08	0.008	0.08	0.008	0.08	0.008
24	0.08	0.0799	0.0081	0.08	0.008	0.08	0.008
25	0.3201	0.3208	0.034	0.3201	0.0338	0.3201	0.0338
26	0.08	0.0799	0.008	0.08	0.008	0.08	0.008
27	0.08	0.08	0.0081	0.08	0.008	0.08	0.008
28	0.16	0.1608	0.032	0.1601	0.0318	0.1601	0.0318
29	0.08	0.08	0.008	0.08	0.008	0.08	0.008
30	0.08	0.0808	0.0309	0.0801	0.0308	0.0801	0.0308

The shape of the voltage probability distribution function remains the bell shaped and is shown in Fig 5.7, but the impact on the active and reactive power flow probability distribution function is as shown in Fig.5.8 and 5.9. The probability distribution functions (PDF) becomes non-normal. The difference in the shape of PDF in Gram-Charlier method can be attributed to the fact that Gram-Charlier series is based on normal probability function, and the resultant function in this case is multi model.



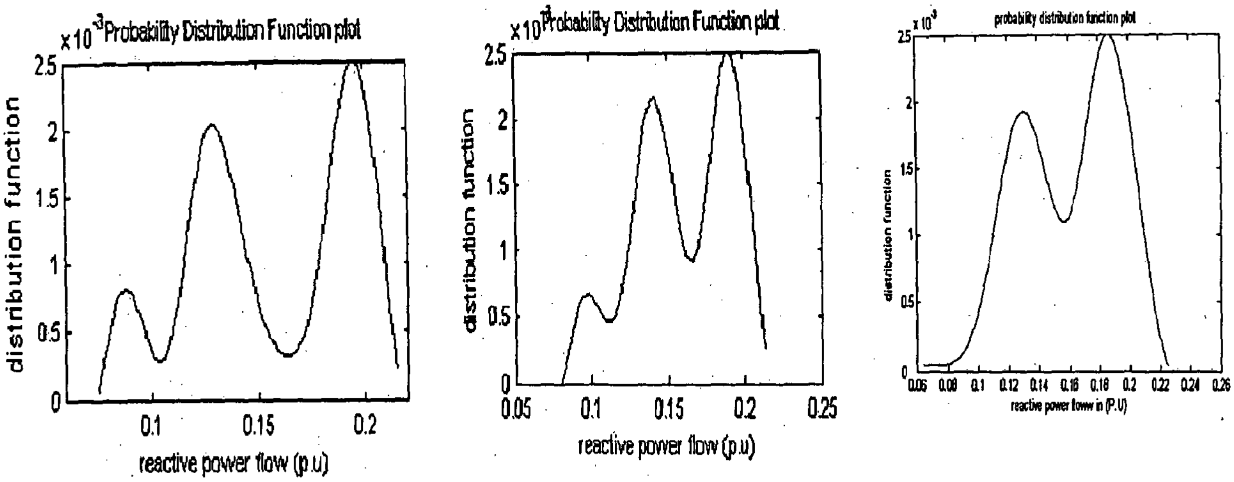
Monte Carlo Probabilistic [L.T] Gram-Charlier

Figure 5.7: Voltage PDF at 28 th bus



Monte Carlo Probabilistic [L.T] Gram-Charlier

Figure 5.8: Active power flow PDF 25-28



Monte-Carlo　　Probabilistic [L.T]　　Gram-Charlier

Figure 5.9: Active power flow PDF 25-28

But the expected values and the range of randòm variables are predicted accurately by Gram-Charlier.

5.1.3 Case (iii): When All Bus loads have a discrete distribution function:

To stimulate a practical system the IEEE Discrete hourly load data has been considered. From these data, the probabilistic distribution of load at each bus has been calculated. The Probabilistic method using Laplace transform and Probabilistic using Gram-Charlier has been carried out when the case of all bus loads have discrete distribution function as shown in Fig 5.10, in percentage of the mean value.

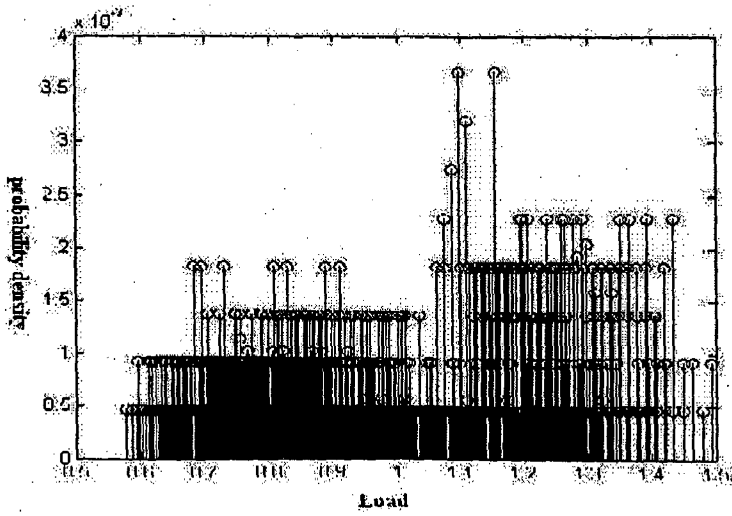


Figure 5.10: IEEE Probabilistic discrete distribution of load

The bus powers and voltages are given below.

Table 5.7: Comparison of Bus voltages

DETERMINISTIC		MONTE-CARLO		PROBABILISTIC USING (L.T)		GRAM-CHARLIER	
Bus no	base(V)	mean(V)	std(V)	mean(V)	std(V)	mean(V)	std(V)
1	1.05	1.05	0	1.05	0	1.05	0
2	0.9265	0.9264	0.0073	0.9372	0.0041	0.9372	0.0041
3	0.9252	0.9251	0.0073	0.936	0.0041	0.936	0.0041
4	0.9243	0.9242	0.0074	0.9352	0.0041	0.9352	0.0041
5	0.9241	0.924	0.0074	0.935	0.0042	0.935	0.0042
6	0.9187	0.9186	0.0077	0.93	0.0044	0.93	0.0044
7	0.9181	0.9181	0.0077	0.9295	0.0044	0.9295	0.0044
8	0.9181	0.9181	0.0077	0.9295	0.0044	0.9295	0.0044
9	0.9172	0.9172	0.0078	0.9287	0.0044	0.9287	0.0044
10	0.9166	0.9166	0.0078	0.9282	0.0044	0.9282	0.0044
11	0.916	0.916	0.0078	0.9276	0.0045	0.9276	0.0045
12	0.9157	0.9156	0.0079	0.9273	0.0045	0.9273	0.0045
13	0.9161	0.9161	0.0078	0.9277	0.0045	0.9277	0.0045
14	0.9159	0.9158	0.0079	0.9275	0.0045	0.9275	0.0045
15	0.9154	0.9154	0.0079	0.9271	0.0045	0.9271	0.0045
16	0.9154	0.9154	0.0079	0.9271	0.0045	0.9271	0.0045
17	0.9147	0.9147	0.0079	0.9264	0.0045	0.9264	0.0045
18	0.9151	0.9151	0.0079	0.9268	0.0045	0.9268	0.0045
19	0.923	0.923	0.0075	0.934	0.0042	0.934	0.0042
20	0.9228	0.9228	0.0075	0.9339	0.0042	0.9339	0.0042
21	0.9219	0.9219	0.0075	0.933	0.0042	0.933	0.0042
22	0.9214	0.9214	0.0076	0.9325	0.0042	0.9325	0.0042
23	0.9213	0.9212	0.0076	0.9324	0.0042	0.9324	0.0042
24	0.9208	0.9208	0.0076	0.932	0.0043	0.932	0.0043
25	0.9206	0.9206	0.0076	0.9318	0.0043	0.9318	0.0043
26	0.9201	0.92	0.0076	0.9313	0.0043	0.9313	0.0043
27	0.92	0.92	0.0076	0.9313	0.0043	0.9313	0.0043
28	0.9202	0.9201	0.0076	0.9314	0.0043	0.9314	0.0043
29	0.9191	0.919	0.0077	0.9304	0.0043	0.9304	0.0043
30	0.9199	0.9198	0.0076	0.9311	0.0043	0.9311	0.0043

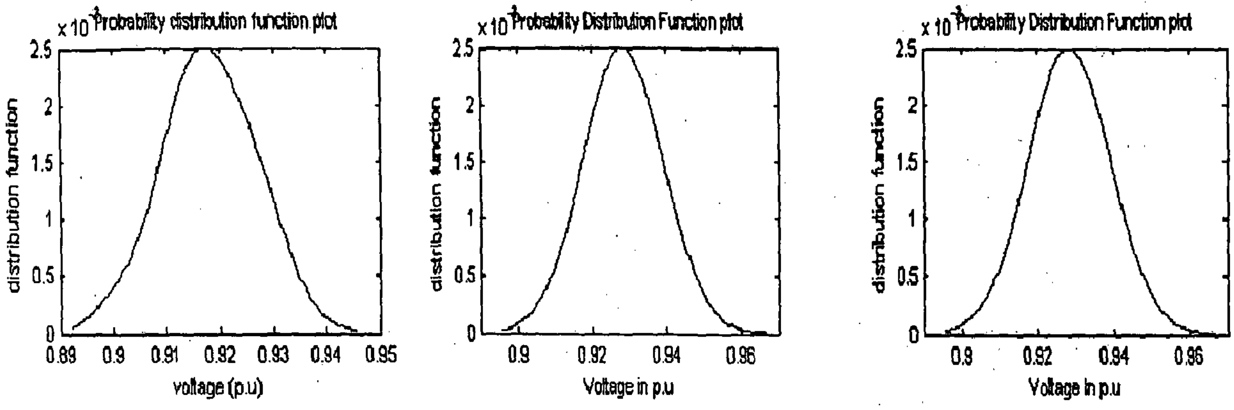
Table 5.8: Comparison of Bus active power

DETERMINISTIC		MONTE-CARLO		PROBABILISTIC USING (L.T)		GRAM-CHARLIER	
Bus no	P	mean(P)	std(P)	mean(P)	std(P)	mean(P)	std(P)
1	3.5849	3.5855	0.1975	3.1998	0.1543	3.1998	0.1543
2	3.2271	3.2264	0.1571	3.1998	0.1543	3.1998	0.1543
3	3.223	3.2223	0.1567	3.1998	0.1543	3.1998	0.1543
4	0.16	0.1598	0.0345	0.16	0.0345	0.16	0.0345
5	0.16	0.1593	0.0348	0.16	0.0345	0.16	0.0345
6	1.6044	1.6055	0.1097	1.5999	0.1091	1.5999	0.1091
7	0.16	0.1595	0.0347	0.16	0.0345	0.16	0.0345
8	0.16	0.1599	0.0342	0.16	0.0345	0.16	0.0345
9	0.16	0.1602	0.0348	0.16	0.0345	0.16	0.0345
10	1.1215	1.1229	0.0916	1.1199	0.0913	1.1199	0.0913
11	0.16	0.1603	0.0346	0.16	0.0345	0.16	0.0345
12	0.16	0.1601	0.0344	0.16	0.0345	0.16	0.0345
13	0.16	0.1603	0.035	0.16	0.0345	0.16	0.0345
14	0.6406	0.6412	0.0693	0.64	0.069	0.64	0.069
15	0.16	0.1615	0.0347	0.16	0.0345	0.16	0.0345
16	0.16	0.1596	0.0344	0.16	0.0345	0.16	0.0345
17	0.16	0.1597	0.0346	0.16	0.0345	0.16	0.0345
18	0.16	0.1597	0.0348	0.16	0.0345	0.16	0.0345
19	1.2833	1.2822	0.0989	1.2799	0.0976	1.2799	0.0976
20	0.16	0.1603	0.0343	0.16	0.0345	0.16	0.0345
21	0.16	0.1601	0.0345	0.16	0.0345	0.16	0.0345
22	0.9614	0.9598	0.0855	0.9599	0.0845	0.9599	0.0845
23	0.16	0.1598	0.035	0.16	0.0345	0.16	0.0345
24	0.16	0.1591	0.0348	0.16	0.0345	0.16	0.0345
25	0.6407	0.6402	0.0685	0.64	0.069	0.64	0.069
26	0.16	0.1601	0.0347	0.16	0.0345	0.16	0.0345
27	0.16	0.1601	0.0343	0.16	0.0345	0.16	0.0345
28	0.3203	0.3197	0.049	0.32	0.0488	0.32	0.0488
29	0.16	0.1595	0.0346	0.16	0.0345	0.16	0.0345
30	0.16	0.1599	0.0345	0.16	0.0345	0.16	0.0345

Table 5.9: Comparison of bus reactive power

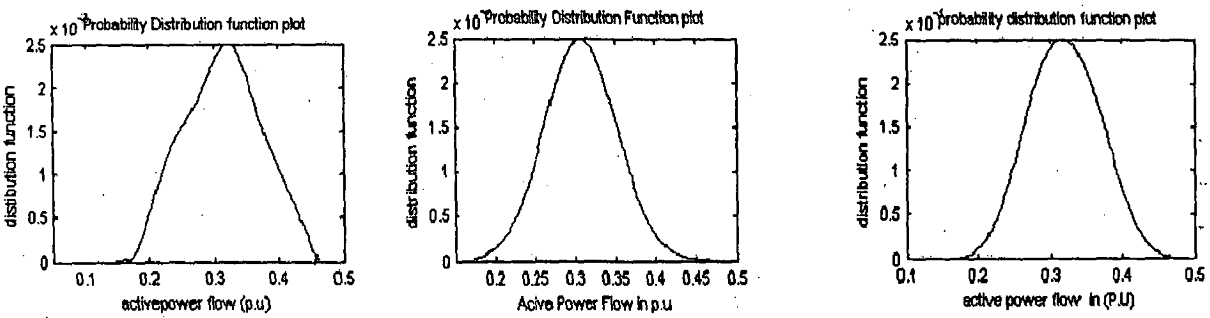
DETERMINISTIC		MONTE-CARLO		PROBABILISTIC USING (L.T)		GRAM-CHARLIER	
Bus no	P	mean(P)	std(P)	mean(P)	std(P)	mean(P)	std(P)
1	1.966	1.9671	0.1183	1.5999	0.0772	1.5999	0.0772
2	1.6127	1.6125	0.0785	1.5999	0.0772	1.5999	0.0772
3	1.6097	1.6094	0.0781	1.5999	0.0772	1.5999	0.0772
4	0.08	0.0799	0.0173	0.08	0.0173	0.08	0.0173
5	0.08	0.0797	0.0174	0.08	0.0173	0.08	0.0173
6	0.802	0.8026	0.0548	0.7999	0.0546	0.7999	0.0546
7	0.08	0.0798	0.0173	0.08	0.0173	0.08	0.0173
8	0.08	0.08	0.0171	0.08	0.0173	0.08	0.0173
9	0.08	0.0801	0.0174	0.08	0.0173	0.08	0.0173
10	0.5604	0.5611	0.0458	0.56	0.0457	0.56	0.0457
11	0.08	0.0802	0.0173	0.08	0.0173	0.08	0.0173
12	0.08	0.0801	0.0172	0.08	0.0173	0.08	0.0173
13	0.08	0.0802	0.0175	0.08	0.0173	0.08	0.0173
14	0.3201	0.3204	0.0346	0.32	0.0345	0.32	0.0345
15	0.08	0.0808	0.0173	0.08	0.0173	0.08	0.0173
16	0.08	0.0798	0.0172	0.08	0.0173	0.08	0.0173
17	0.08	0.0799	0.0173	0.08	0.0173	0.08	0.0173
18	0.08	0.0799	0.0174	0.08	0.0173	0.08	0.0173
19	0.6415	0.6411	0.0494	0.64	0.0488	0.64	0.0488
20	0.08	0.0802	0.0172	0.08	0.0173	0.08	0.0173
21	0.08	0.0801	0.0172	0.08	0.0173	0.08	0.0173
22	0.4805	0.4798	0.0427	0.48	0.0423	0.48	0.0423
23	0.08	0.0799	0.0175	0.08	0.0173	0.08	0.0173
24	0.08	0.0796	0.0174	0.08	0.0173	0.08	0.0173
25	0.3201	0.3199	0.0342	0.32	0.0345	0.32	0.0345
26	0.08	0.08	0.0174	0.08	0.0173	0.08	0.0173
27	0.08	0.08	0.0172	0.08	0.0173	0.08	0.0173
28	0.16	0.1597	0.0245	0.16	0.0244	0.16	0.0244
29	0.08	0.0797	0.0173	0.08	0.0173	0.08	0.0173
30	0.08	0.08	0.0173	0.08	0.0173	0.08	0.0173

The shapes of voltage, active and reactive power flow probability distribution function remain the bell shaped and is shown in Fig 5.11, 5.12 and 5.13



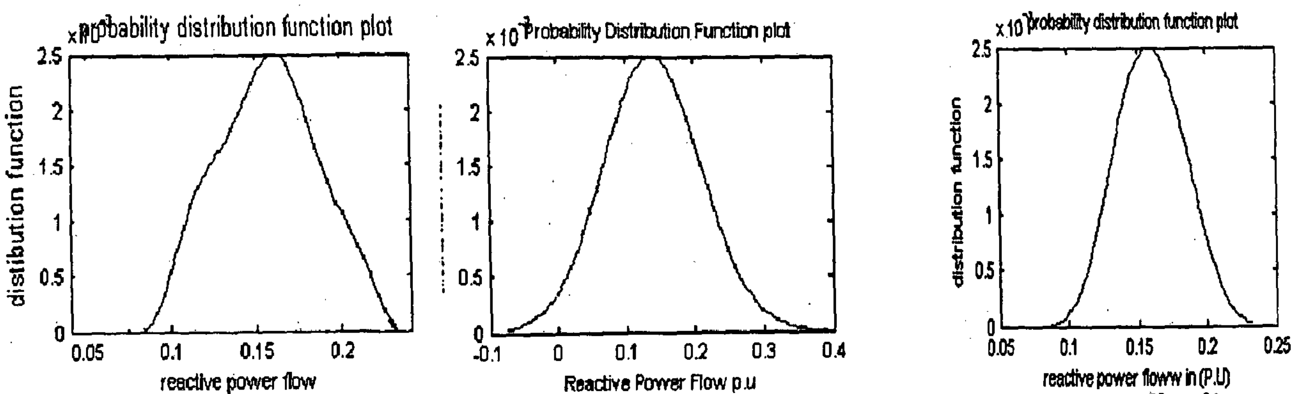
Monte-Carlo Probabilistic [L.T] Gram-Charlier

Figure5.11: Probability distribution function of the voltage at node 28



Monte-Carlo Probabilistic [L.T] Gram-Charlier

Figure 5.12: Probability distribution function of Active power flow in element 25-28



Monte-Carlo Probabilistic [L.T] Gram-Charlier

Figure 5.13: Probability distribution function of reactive power flow

While the voltage PDFs match well slight difference in, the shape of active and reactive power PDFs are slightly difference from Monte Carlo Simulation. The difference is due to omission of line losses and approximations in voltage drops.

5.1.4 Case (IV): When all the loads are dependent and normal with positive correlation:

Probabilistic method using Laplace transform and Gram-Charlier has been carried out when the case of all loads are dependent and normal with positive correlation. The bus voltages, active and reactive power flow are given below.

Table 5.10: Bus Voltages:

Bus no	base(V)	mean(V)	std(V)	mean(V)	std(V)
1	1.05	1	0	1.05	0
2	0.9265	0.9372	0.0113	0.9372	0.0113
3	0.9252	0.936	0.0114	0.936	0.0114
4	0.9243	0.9352	0.0115	0.9352	0.0115
5	0.9241	0.935	0.0115	0.935	0.0115
6	0.9187	0.93	0.012	0.93	0.012
7	0.9181	0.9295	0.012	0.9295	0.012
8	0.9181	0.9295	0.012	0.9295	0.012
9	0.9172	0.9287	0.0121	0.9287	0.0121
10	0.9166	0.9282	0.0122	0.9282	0.0122
11	0.916	0.9276	0.0122	0.9276	0.0122
12	0.9157	0.9273	0.0123	0.9273	0.0123
13	0.9161	0.9277	0.0122	0.9277	0.0122
14	0.9159	0.9275	0.0122	0.9275	0.0122
15	0.9154	0.9271	0.0123	0.9271	0.0123
16	0.9154	0.9271	0.0123	0.9271	0.0123
17	0.9147	0.9264	0.0124	0.9264	0.0124
18	0.9151	0.9268	0.0123	0.9268	0.0123
19	0.923	0.934	0.0116	0.934	0.0116
20	0.9228	0.9339	0.0116	0.9339	0.0116
21	0.9219	0.933	0.0117	0.933	0.0117
22	0.9214	0.9325	0.0117	0.9325	0.0117
23	0.9213	0.9324	0.0118	0.9324	0.0118
24	0.9208	0.932	0.0118	0.932	0.0118

25	0.9206	0.9318	0.0118	0.9318	0.0118
26	0.9201	0.9313	0.0119	0.9313	0.0119
27	0.92	0.9313	0.0119	0.9313	0.0119
28	0.9202	0.9314	0.0119	0.9314	0.0119
29	0.9191	0.9304	0.012	0.9304	0.012
30	0.9199	0.9311	0.0119	0.9311	0.0119

Table 5.11: Bus active power Flow

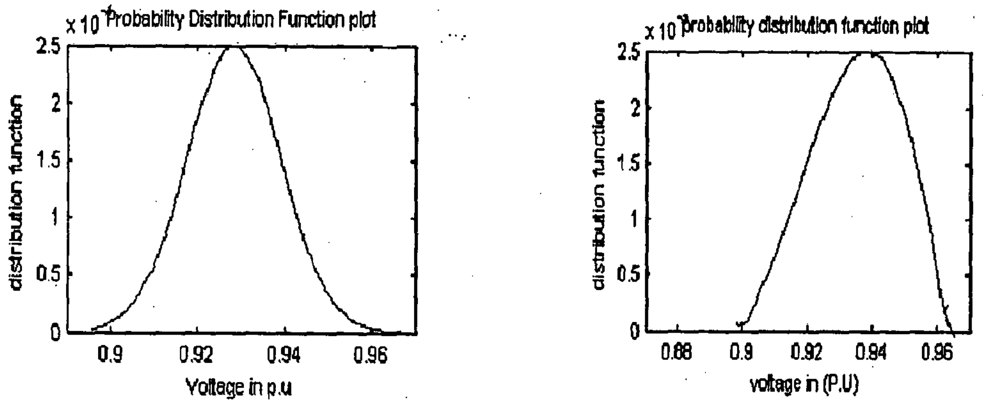
Bus no	P	mean(P)	std(P)	mean(P)	std(P)
1	3.5849	3.2	0.32	3.2	0.32
2	3.2271	3.2	0.32	3.2	0.32
3	3.223	3.2	0.32	3.2	0.32
4	0.16	0.16	0.016	0.16	0.016
5	0.16	0.16	0.016	0.16	0.016
6	1.6044	1.6	0.16	1.6	0.16
7	0.16	0.16	0.016	0.16	0.016
8	0.16	0.16	0.016	0.16	0.016
9	0.16	0.16	0.016	0.16	0.016
10	1.1215	1.12	0.112	1.12	0.112
11	0.16	0.16	0.016	0.16	0.016
12	0.16	0.16	0.016	0.16	0.016
13	0.16	0.16	0.016	0.16	0.016
14	0.6406	0.64	0.064	0.64	0.064
15	0.16	0.16	0.016	0.16	0.016
16	0.16	0.16	0.016	0.16	0.016
17	0.16	0.16	0.016	0.16	0.016
18	0.16	0.16	0.016	0.16	0.016
19	1.2833	1.28	0.128	1.28	0.128
20	0.16	0.16	0.016	0.16	0.016
21	0.16	0.16	0.016	0.16	0.016
22	0.9614	0.96	0.096	0.96	0.096
23	0.16	0.16	0.016	0.16	0.016
24	0.16	0.16	0.016	0.16	0.016
25	0.6407	0.64	0.064	0.64	0.064
26	0.16	0.16	0.016	0.16	0.016
27	0.16	0.16	0.016	0.16	0.016
28	0.3203	0.32	0.032	0.32	0.032
29	0.16	0.16	0.016	0.16	0.016
30	0.16	0.16	0.016	0.16	0.016

Table 5.12: Bus reactive power flow

Bus no	Q	mean(Q)	std(Q)	mean(Q)	std(Q)
1	1.966	1.6	0.16	1.6	0.16
2	1.6127	1.6	0.16	1.6	0.16
3	1.6097	1.6	0.16	1.6	0.16
4	0.08	0.08	0.008	0.08	0.008
5	0.08	0.08	0.008	0.08	0.008
6	0.802	0.8	0.08	0.8	0.08
7	0.08	0.08	0.008	0.08	0.008
8	0.08	0.08	0.008	0.08	0.008
9	0.08	0.08	0.008	0.08	0.008
10	0.5604	0.56	0.056	0.56	0.056
11	0.08	0.08	0.008	0.08	0.008
12	0.08	0.08	0.008	0.08	0.008
13	0.08	0.08	0.008	0.08	0.008
14	0.3201	0.32	0.032	0.32	0.032
15	0.08	0.08	0.008	0.08	0.008
16	0.08	0.08	0.008	0.08	0.008
17	0.08	0.08	0.008	0.08	0.008
18	0.08	0.08	0.008	0.08	0.008
19	0.6415	0.64	0.064	0.64	0.064
20	0.08	0.08	0.008	0.08	0.008
21	0.08	0.08	0.008	0.08	0.008
22	0.4805	0.48	0.048	0.48	0.048
23	0.08	0.08	0.008	0.08	0.008
24	0.08	0.08	0.008	0.08	0.008
25	0.3201	0.32	0.032	0.32	0.032
26	0.08	0.08	0.008	0.08	0.008
27	0.08	0.08	0.008	0.08	0.008
28	0.16	0.16	0.016	0.16	0.016
29	0.08	0.08	0.008	0.08	0.008
30	0.08	0.08	0.008	0.08	0.008

From Table 5.10 to 5.12 It can be seen that, in all cases, the standard deviation of voltages, active and reactive power were greater when the nodal powers were to be dependent than when the load loads are independent and normal (Table 5.1 to 5.3). The standard deviation increases or decreases, is dependent upon the impact between the linear dependence of the nodal powers as well as sign of the sensitivity coefficients. In this particular case it is seen that there is increase in the standard deviation of voltages and active power, when nodal powers were assumed dependent with positive correlation.

Fig 5.14 shows the probability distribution function of voltage, when the loads are correlated with positive linear dependency



Probabilistic (LT) Gram Charlier

Figure 5.14: Probability Distribution function of voltage plot at node 28

The parameters for the input probability density function are the same as before. As it can be seen that there is significant increase in the deviation of the fig 5.14 probability distribution, as compared to the Fig. 5.1

5.2 Simulation Results for Complex Random Variable analysis:

The probabilistic load flow using complex random variable analysis has been applied to IEEE 13 bus system and 30 bus test system is shown in Fig.5.15 and 5.1. The failure data of each distribution line and bus are assumed shown in Table B3, B4 and slack bus is 1 in this test system.

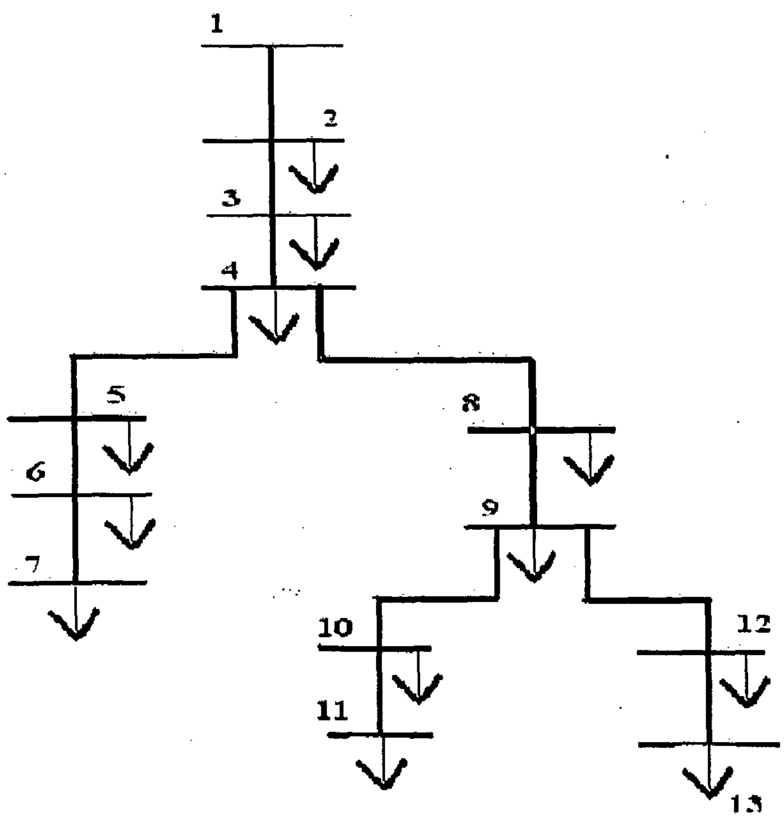


Figure 5.15: 13 bus test system

The moments of 13 bus and 30bus system, voltages are shown in table 5.13 and 5.14.

Table 5.13: Moments of bus voltage using complex random variable for 13 bus system

Bus no	First order moments	Second order moments	Third order moments	Fourth order moments
1	1.0500	1.1025	1.1576	1.2155
2	1.0017 - 0.0603i	0.9999 - 0.1214i	0.9945 - 0.1830i	0.9852 - 0.2448i
3	0.9892 - 0.0818i	0.9720 - 0.1624i	0.9484 - 0.2413i	0.9186 - 0.3180i
4	0.9810 - 0.0958i	0.9534 - 0.1886i	0.9174 - 0.2776i	0.8734 - 0.3620i
5	0.9759 - 0.1040i	0.9418 - 0.2037i	0.8981 - 0.2979i	0.8455 - 0.3860i
6	0.9748 - 0.1058i	0.9393 - 0.2070i	0.8939 - 0.3024i	0.8394 - 0.3912i
7	0.9747 - 0.1062i	0.9389 - 0.2075i	0.8932 - 0.3032i	0.8384 - 0.3921i
8	0.9558 - 0.1290i	0.8972 - 0.2475i	0.8258 - 0.3540i	0.7435 - 0.4473i
9	0.9525 - 0.1351i	0.8892 - 0.2583i	0.8123 - 0.3678i	0.7238 - 0.4624i
10	0.9491 - 0.1414i	0.8810 - 0.2692i	0.7982 - 0.3817i	0.7034 - 0.4775i
11	0.9456 - 0.1477i	0.8725 - 0.2802i	0.7838 - 0.3955i	0.6825 - 0.4921i
12	0.9513 - 0.1372i	0.8864 - 0.2619i	0.8075 - 0.3725i	0.7169 - 0.4675i
13	0.9511 - 0.1376i	0.8860 - 0.2625i	0.8067 - 0.3732i	0.7158 - 0.4683i

Table 5.14: Moments of bus voltage using Complex random variable for 30 bus system

Bus no	First order moments	Second order moments	Third order moments	Fourth order moments
1	1.0500	1.1025	1.1576	1.2155
2	0.9419 - 0.0405i	0.8856 - 0.0763i	0.8312 - 0.1077i	0.7787 - 0.1350i
3	0.9408 - 0.0408i	0.8835 - 0.0767i	0.8281 - 0.1081i	0.7748 - 0.1354i
4	0.9399 - 0.0405i	0.8819 - 0.0761i	0.8259 - 0.1072i	0.7721 - 0.1341i
5	0.9398 - 0.0405i	0.8816 - 0.0760i	0.8256 - 0.1070i	0.7717 - 0.1339i
6	0.9349 - 0.0403i	0.8725 - 0.0754i	0.8128 - 0.1056i	0.7558 - 0.1315i
7	0.9344 - 0.0402i	0.8716 - 0.0750i	0.8115 - 0.1051i	0.7542 - 0.1307i
8	0.9344 - 0.0402i	0.8715 - 0.0750i	0.8114 - 0.1050i	0.7541 - 0.1307i
9	0.9337 - 0.0399i	0.8702 - 0.0745i	0.8096 - 0.1042i	0.7518 - 0.1296i
10	0.9332 - 0.0406i	0.8692 - 0.0757i	0.8081 - 0.1059i	0.7499 - 0.1315i
11	0.9326 - 0.0404i	0.8682 - 0.0754i	0.8068 - 0.1053i	0.7483 - 0.1307i
12	0.9323 - 0.0403i	0.8677 - 0.0752i	0.8060 - 0.1050i	0.7474 - 0.1303i
13	0.9327 - 0.0404i	0.8683 - 0.0754i	0.8068 - 0.1053i	0.7484 - 0.1308i
14	0.9325 - 0.0407i	0.8680 - 0.0758i	0.8065 - 0.1060i	0.7479 - 0.1315i
15	0.9322 - 0.0406i	0.8673 - 0.0756i	0.8055 - 0.1055i	0.7467 - 0.1310i
16	0.9321 - 0.0406i	0.8673 - 0.0756i	0.8054 - 0.1055i	0.7466 - 0.1309i
17	0.9314 - 0.0403i	0.8660 - 0.0751i	0.8036 - 0.1048i	0.7444 - 0.1299i
18	0.9319 - 0.0405i	0.8669 - 0.0754i	0.8049 - 0.1053i	0.7460 - 0.1306i
19	0.9389 - 0.0411i	0.8798 - 0.0771i	0.8230 - 0.1084i	0.7684 - 0.1355i
20	0.9387 - 0.0410i	0.8795 - 0.0769i	0.8225 - 0.1082i	0.7678 - 0.1352i
21	0.9378 - 0.0407i	0.8779 - 0.0763i	0.8204 - 0.1073i	0.7651 - 0.1339i
22	0.9374 - 0.0413i	0.8771 - 0.0773i	0.8191 - 0.1086i	0.7635 - 0.1355i
23	0.9373 - 0.0413i	0.8768 - 0.0773i	0.8187 - 0.1085i	0.7630 - 0.1354i
24	0.9368 - 0.0411i	0.8760 - 0.0769i	0.8176 - 0.1080i	0.7617 - 0.1346i
25	0.9367 - 0.0414i	0.8757 - 0.0774i	0.8171 - 0.1087i	0.7610 - 0.1355i
26	0.9362 - 0.0412i	0.8748 - 0.0771i	0.8159 - 0.1082i	0.7595 - 0.1348i
27	0.9362 - 0.0412i	0.8747 - 0.0771i	0.8158 - 0.1081i	0.7594 - 0.1347i
28	0.9363 - 0.0414i	0.8749 - 0.0775i	0.8160 - 0.1087i	0.7596 - 0.1355i
29	0.9352 - 0.0411i	0.8729 - 0.0767i	0.8132 - 0.1075i	0.7562 - 0.1339i
30	0.9360 - 0.0413i	0.8744 - 0.0773i	0.8153 - 0.1084i	0.7588 - 0.1351i

The cumulants are calculated from the moments of each bus voltages and they can be converted into normal distribution using the Edge worth type of Gram-Charlier expansion. The PDF of each bus voltage is shown in Fig.5.16and 5.17.

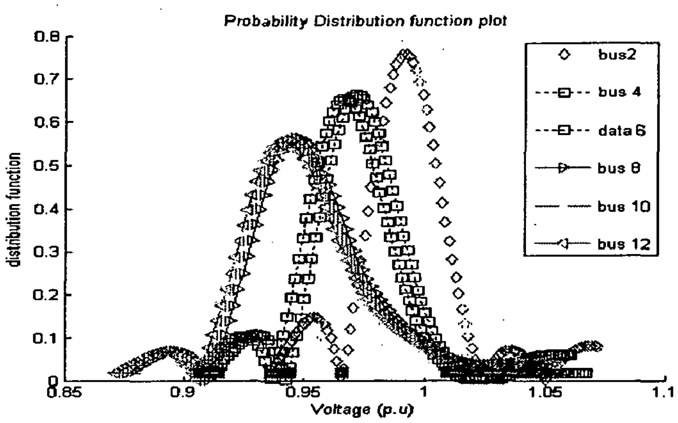


Figure 5.16: the PDF of each bus voltage using Gram-Edgeworth

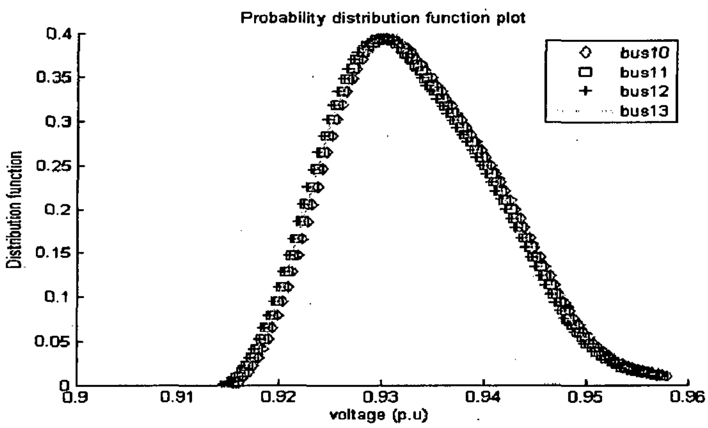


Figure 5.17: PDF of bus voltage using Gram-Edgeworth

The realization of PDF using Probabilistic Load Flow with method of moments measures the probability of bus voltages instability through determining bus indices. The bus voltage levels depend mainly on the reactive power production of the generators and failure rate by unplanned outage, this method can provides a measure of the severity of abnormal voltages.

The advantage of the method using Complex Random Variable analysis is that that network uncertainties is directly incorporated in this method. This would greatly simplify contingency analysis as there is no need of repeated load flows for different contingencies.

Conclusion and Future Scope

The application of Probabilistic load flow method using Gram-Charlier Series Expansion for distribution system has been presented. The probability distribution function of bus voltages, bus active powers and bus reactive powers have been calculated and compared with Probabilistic Load flow and Monte Carlo simulation method. The probabilistic load flow using Gram-Charlier series expansion method adopted in this thesis provides a new way of computing probabilistic distribution function of line flows for distribution system. Correlations between the nodal loads of the system were taken in to account using a simple expression based on their standard deviations.

In Comparison with Monte Carlo with 5000 iterations, this method is 10-20 times faster and significant reduction in memory requirement in comparison to probabilistic load flow using Laplace transform this method has reducing the complexity of calculation and significant improvement in reduction of calculation time.

This thesis also presents a method of solution of the stochastic load flow problem based on the method of moments of complex random variables for radial distribution system. In comparison to the probabilistic load flow computation using Gram-Charlier expansion, this method can easily incorporate network uncertainty.

The probabilistic load flow study gives qualitatively more information about the system analyzed as compared to the conventional deterministic method. In this thesis the loads are modeled as statistical uncertainties that always exist in the process of planning and operation of practical systems. The various conditions, situations and constrains in a much more flexible way by weighting them with appropriate probabilities. This should result in a more realistic picture about the observed system.

Gram-Charlier series is valid for (-∞, +∞), some negative probabilistic values can occur as the random variable is in power system varies between 0 to ∞. Laguerre and Legendre series are valid for random variable between 0 to ∞ and their application to the Probabilistic Load Flow can be explored.

Consider a Complex Random Variable with real and imaginary components X and Y, also random variables. Assume that the moments and cumulants of X are known and that Y is completely known i.e. it has a probability density function with only one impulse with probability one. The first four moments are

1. $\alpha_1(Z) = \alpha_1(X) + j\alpha_1(Y)$

Where

$$\alpha_1(Y) = Y \; (value\ of\ the\ random\ variable\ Y)$$

2. $\alpha_2(Z) = \alpha_2(X) - \alpha_2(Y) + 2jE\{XY\}$

Where

$$\alpha_2(Y) = Y^2$$

3. $\alpha_3(Z) = \alpha_3(X) - 3E\{XY^2 + j[3E\{X^2Y\} - \alpha_3(Y)$

Where, if Y is a constant one has

$$\alpha_3(Z) = \alpha_3(X) - 3Y^2\alpha_1(X) + j[3Y\alpha_2(X) - Y^3]$$

4. $\alpha_4(Z) = \alpha_4(X) - \alpha_4(Y) - 6E\{X^2Y^2\} + j[4E(X^3Y) - 4E\{XY^3]$

Where, if Y is a constant, one has

$$\alpha_4(Z) = \alpha_4(X) - Y^4 - 6Y^2\alpha^2(X) + j[4Y\alpha_3(X) - 4Y^3\alpha_1(X)]$$

Gram-Charlier type A series expansion: Consider a random variable ξ with a distribution of a continuous type and denote the mean value as μ and the standard deviation as σ. For the standardized variable $(\xi - \alpha)(\sigma)$, its density function is denoted as f(x). According to Gram-Charlier expansion, the Probability density function is

$$f(x) = \sum_{j=0}^{\infty} c_j H_j(x)\phi(x)$$

Where

$$C_j = \frac{1}{j!} \sum_{k=0}^{j/2} \left(\frac{-1}{2}\right)^k \frac{j!\beta_{j-2k}}{k!(j-2k)} \left(\frac{1}{\sigma^{j-2k}}\right)$$

$$H_j(x) = \sum_{k=0}^{j/2} \left(\frac{-1}{2}\right)^k \frac{j!x^{j-2k}}{k!(j-2k)}$$

$$\phi(x) = \frac{1}{\sqrt{2\Pi}} e^{\frac{-x^2}{2}}$$

[n/2] denotes the largest integer<=n/2

The expression $H_j(x)$ are known as Hermite Polynomials

Some of these expressions are

$H_0(x) = 1$

$H_1(x) = x$

$H_2(x) = x^2 - 1$

$H_3(x) = x^3 - 3x$

$H_4(x) = x^4 - 6x^2 + 3$

$H_5(x) = x^5 - 10x^3 + 15x$

$H_6(x) = x^6 - 15x^4 + 45x^2 - 15$

The expression C is known as Gram-Charlier coefficients.

Some of those expressions are

$C_0 = 1$

$C_1 = C_2 = 0$

$$C_3 = -\frac{\beta_3}{\sigma^3}$$

$$C_4 = -\frac{\beta_4}{\sigma^4} - 3$$

$$C_5 = -\frac{\beta_5}{\sigma^5} + 10\frac{\beta_3}{\sigma^3}$$

$$C_6 = -\frac{\beta_6}{\sigma^6} - 15\frac{\beta_4}{\sigma^4} + 30$$

Test data for 30 bus distribution system

Base MVA=1.0MVA

BASEKV=11KV

Table B1: Network data in per units

Line no	From node	TO node	R(p.u)	X(p.u)
1	1	2	0.0236	0.0233
2	2	3	0.0003	0.0002
3	3	4	0.0051	0.0005
4	3	5	0.0062	0.0006
5	3	6	0.0032	0.0011
6	6	7	0.003	0.0003
7	6	8	0.003	0.0003
8	6	9	0.0079	0.0008
9	6	10	0.0013	0.0008
10	10	11	0.0033	0.0003
11	10	12	0.0050	0.0005
12	10	13	0.0027	0.0003
13	10	14	0.0008	0.0005
14	14	15	0.0025	0.0003
15	14	16	0.0026	0.0003
16	14	17	0.0065	0.0007
17	14	18	0.0041	0.0004
18	3	19	0.0012	0.0007
19	19	20	0.0011	0.0001
20	19	21	0.0061	0.0006
21	19	22	0.0012	0.0008
22	22	23	0.0008	0.0003
23	22	24	0.0034	0.0003
24	22	25	0.0009	0.0006
25	25	26	0.003	0.0003
26	25	27	0.0032	0.0003
27	25	28	0.0009	0.0006
28	28	29	0.006	0.0006
29	28	30	0.0016	0.0002

TABLE B2: Load data in per units

BUS NO	MEAN (P)	SIGMA(P)	MEAN(Q)	SIGMA(Q)
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0.16	0.016	0.08	0.008
5	0.16	0.016	0.08	0.008
6	0	0	0	0
7	0.16	0.016	0.08	0.008
8	0.16	0.016	0.08	0.008
9	0.16	0.016	0.08	0.008
10	0	0	0	0
11	0.16	0.016	0.08	0.008
12	0.16	0.016	0.08	0.008
13	0.16	0.016	0.08	0.008
14	0	0	0	0
15	0.16	0.016	0.08	0.008
16	0.16	0.016	0.08	0.008
17	0.16	0.016	0.08	0.008
18	0.16	0.016	0.08	0.008
19	0	0	0	0
20	0.16	0.016	0.08	0.008
21	0.16	0.016	0.08	0.008
22	0	0	0	0
23	0.16	0.016	0.08	0.008
24	0.16	0.016	0.08	0.008
25	0	0	0	0
26	0.16	0.016	0.08	0.008
27	0.16	0.016	0.08	0.008
28	0	0	0	0
29	0.16	0.016	0.08	0.008
30	0.16	0.016	0.08	0.008

1EEE 13 BUS RADIAL SYSTEM :

BASE KV=11KV
BASE MVA=1.0MVA

TABLE B3: NETWORK UNCERTAINTY DATA in (p.u)

Line no	from node	to node	Resistance (p.u)	Reactance (p.u)	availability
1	1	2	0.00148	0.00287	0.91
2	2	3	0.00044	0.00124	0.86
3	3	4	0.00028	0.00078	0.95
4	4	5	0.0006	0.00167	0.83
5	5	6	0.00034	0.00097	0.87
6	6	7	0.00032	0.00092	0.93
7	4	8	0.0016	0.0031	0.87
8	8	9	0.00029	0.00083	0.92
9	9	10	0.00053	0.00151	0.89
10	10	11	0.00059	0.00166	0.88
11	9	12	0.00038	0.00107	0.93
12	12	13	0.00037	0.00104	0.94

TABLE B4: LOAD UNCERTAINITY DATA

bus no	active power (MW)	reactive power (MW)	availability
1	0	0	0.91
2	4.73	1.55	0.88
3	1.27	0.41	0.96
4	0.35	0.11	1
5	4.38	1.44	0.89
6	2.11	0.69	0.87
7	0.42	0.13	0.89
8	4.73	1.55	1
9	1.27	0.41	0.92
10	0.35	0.11	0.88
11	4.38	1.44	0.97
12	2.11	0.69	0.83
13	0.42	0.13	0.81

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