

# SELECTIVE HARMONIC ELIMINATION IN VOLTAGE SOURCE INVERTER FED INDUCTION MOTOR DRIVE

A DISSERTATION

*Submitted in partial fulfillment of the  
requirements for the award of the degree*

*of*

**MASTER OF TECHNOLOGY**

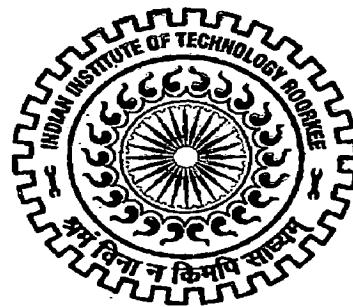
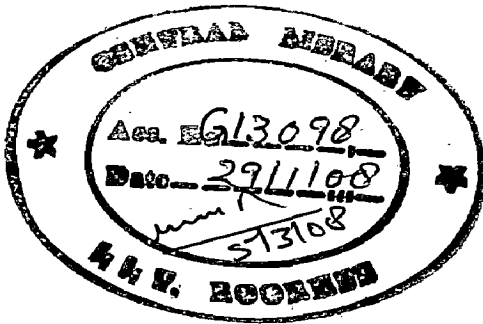
*in*

**ELECTRICAL ENGINEERING**

(With Specialization in Power Apparatus and Electric Drives)

By

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## CANDIDATE'S DECLARATION

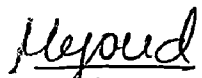
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I hereby declare that the work that is being presented in this dissertation report entitled "SELECTIVE HARMONIC ELIMINATION IN VOLTAGE SOURCE INVERTER FED INDUCTION MOTOR DRIVE" submitted in partial fulfillment of the requirements for the award of the degree of **Master Of Technology** with specialization in **Power Apparatus and Electric Drives**, to the **Department Of Electrical Engineering, Indian Institute Of Technology, Roorkee**, is an authentic record of my own work carried out, under the guidance of **Dr. S. P. Gupta**, Professor, Department of Electrical Engineering and **Dr. G. K. Singh**, Professor, Department of Electrical Engineering.

The matter embodied in this dissertation report has not been submitted by me for the Award of any other degree or diploma.

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**(Mahender Goud N)**

## ABSTRACT

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In 1973, H.S.Patel and R.G. Hoft for the first time introduced the selective harmonic elimination technique. Since then, the problem of eliminating harmonics in switching converters has been given attention. For systems where high switching efficiency is of importance, it is desirable to keep the switching frequency lower. For this an approach is to choose the switching times (angles) such that a desired fundamental output is generated and specifically chosen harmonics of the fundamental are suppressed.

Here, a computer program using NAG Library Routine E04UCF is presented which is used to solve the non-linear transcendental equations in order to get the switching angles. By using this routine, two solutions to the switching angles have been identified, one in 0-60 degree and other in 0-90 degree mode. The plots of these switching angles for varying modulation indices are presented. A digital logic switching circuit is deigned to generate the firing pulses to the pulse width modulated voltage source inverter. Also, the total harmonic distortion is analyzed for various cases. An attempt has been made to implement the closed loop speed control of the voltage source inverter (VSI) fed induction motor drive using the Selective Harmonic Elimination technique.

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## INTRODUCTION

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### 1.1 GENERAL

For many years, variable speed control using AC motors has been given more attention because of the advantages it has when compared to the speed control using DC motors. The main advantages are ruggedness of motor and superiority of its modified characteristics. The inverter which is the core element in the frequency conversion process is widely used in variable frequency speed regulation system. Therefore, the research of an inverter has been given importance to in the electric power system field. Inverters with variety of topologies and control strategies exist, but the appearance of pulse width modulation (PWM) technology is superior of all [1]. Several kinds of on-off control strategies of the switching devices have been developed to meet different applications and performance requests. The variable voltage variable frequency and the harmonic elimination are the most basic requests in a Pulse width modulated inverter.

Since the advent of the thyristor family, there has been an increase in the inverter technology, enabling the creation of most sophisticated inverter circuits for a wide variety of industrial applications. The availability of SCRs in the high power ratings with low turn-off times of the order of microseconds has increased the feasibility of achieving practically sinusoidal waveforms by employing complex switching patterns in inverter circuits. These optimal switching patterns can be obtained by the PWM methods.

Present-day available PWM schemes can be broadly classified as carrier-modulated PWM and pre-calculated programmed PWM schemes. Programmed PWM techniques [2] optimize a particular objective function so as to obtain minimum losses, reduced torque pulsations, selective elimination of harmonics, and therefore are the most effective means of obtaining good performance results. These objective functions are chosen to generate a particular programmed PWM technique which essentially constitutes the minimization of unwanted effects due to the harmonics present in the inverter output spectra. When significant numbers of low-order harmonics are eliminated, a little difference is observed between each one of the programmed techniques is used.

Each one of these programmed PWM techniques is associated with the difficult task of computing specific PWM switching instants to optimize a particular objective function. This difficulty is particularly encountered at lower-output frequency range due to the necessity of a large number of PWM switching instants. Also, in most cases only a local minimum can be obtained after considerable computational effort. In spite of these difficulties, programmed PWM's exhibit several advantages when compared to the conventional carrier-modulated sine PWM schemes as given below.

- 1) About 50% reduction in the inverter switching frequency is achieved when compared with the conventional carrier-modulated sine PWM scheme.
- 2) High utilization of the dc voltage is possible due to higher voltage gain which is possible through over modulation.
- 3) A reduction in the size of the dc link filter components can be achieved. This is due to reduced ripple in the dc link current because of the high quality of the output voltage and current.
- 4) The reduction in switching frequency contributes to the reduction in switching losses of the inverter and permits the use of gate-turn-off devices for high-power converters.
- 5) The elimination of lower-order harmonics causes no harmonic interference such as resonance with external line filtering networks typically employed in inverter power supplies.

The elimination of specific chosen harmonics from a given voltage waveform generated by a voltage-source inverter (VSI) using pulse-width modulation (PWM) has been discussed by many authors. In the technical literature, these methods are known as selective harmonic elimination (SHE) or programmed PWM techniques [3]. These methods provided solutions regarding the switching angles that eliminate a chosen number of harmonics [4], [5]. The main challenge associated with these techniques is to obtain the analytical solutions of nonlinear transcendental equations that contain trigonometric terms which naturally exhibit multiple solutions.

The Selective harmonic elimination has received attention due to the following reasons [6]. Firstly, digital implementation has become common. Secondly, many solutions to the problem have been identified which were unknown earlier [7]. Each of these solutions has different frequency spectrum, which provides the option for flattening



three-phase and for single phase and also depending on whether it is line to neutral or line to line PWM waveform. The techniques were also divided into those that seek solutions within  $60^\circ$  and  $90^\circ$ . He used the International mathematical and statistical library (IMSL) for solving the non-linear equations.

In 1995, Sidney R. Bowes and Paul R. Clarke developed the modified regular-sampling techniques to generate the PWM strategies online in real time using a simple microprocessor algorithm without resorting to the offline mainframe computer harmonic elimination numerical techniques. Using this technique, he achieved the voltage/frequency range from PWM to Quasi-square wave operation.

In 1996, A.R. Bakhshai, H. Jin and G. Joos proposed a technique for selective harmonic elimination in inverter fed induction machines to reduce harmonic concentration and acoustic noise. In 1999, Jose R. Espinoza, Geza Joos, J. Gunman presented a unified approach for generating the pulse-width modulated voltage source and current source inverters.

In 1999, T. Kato proposed an approach that makes it possible to solve the HE problem based on a homotopy method which finds multiple solutions for a specific degree of freedom  $N$  from those existing for  $N-1$  sequentially by the mathematical induction of varying a fundamental component value as the homotopy parameter. However, the method is long and cumbersome, and the paper makes no contribution towards showing which set of solutions from the multiple available ones is optimum as measured against overall harmonic performance.

In 2004, J. Chiasson, L. M. Tolbert and Z. Du explained the method of resultants to solve the harmonic elimination problem. In this, the transcendental equations that describe the HE problem he converted into an equivalent set of polynomial equations using trigonometric identities. The theory of resultants is used to compute the resultant polynomial and then work backwards to find all unknown switching angles. This method also finds all possible sets of solutions for the given HE equation. However, the problem in this method is the manipulation of high-order polynomials, whose order increases as the number of harmonics to be eliminated increases. Furthermore, the method has a limited chance to work with a high-order of harmonics, and it can only be easily applied when such a number is low.

In 2005, Wenyi Zhang, Shaoping Qu et al explained the harmonic elimination technique for a three-phase voltage source inverter. The Newton iterative method has been implemented to solve transcendental equations set. He gave four rules for choosing the initial values of the switching angles. Using this technique obtained switching angles of the order of 23 and 24. For these solutions; he plotted the total harmonic distortion curves with respect to modulation index.

In 2005, Nguyen Van Nho, Myung J. Youn proposed a simple online SHE pulse width modulation in two-level inverters method with linear control of fundamental voltage, including over modulation range using the principle control between limit angle trajectories. In 2005, Jason R. Wells, B. M. Nee, Patrick L Chapman gave the selective harmonic elimination schemes for the two-level and three-level case with all the harmonics controlled upto the 23rd.

In 2007, Jason R. Wells, Xin Greg, Patrick L. Chapman proposed a modulation-based method for generating pulse waveforms with selective harmonic elimination achieved. In this method, the desired output waveform through modulation rather than solving the transcendental equations has been obtained.

### **1.3 ORGANIZATION OF THE REPORT**

- Chapter 2 defines harmonics, the sources of harmonics, the need to eliminate the harmonics. Also, the comparison between the traditional PWM methods and the proposed selective harmonic elimination scheme is discussed
- Chapter 3 presents the basic operation of a Voltage source inverter fed induction motor drive and the harmonic elimination switching schemes.
- Chapter 4 discusses the methods available for solving non-linear transcendental equations and about the NAG routine E04ucf. Also the nature of switching trajectories for the variation of modulation index which are obtained through the NAG routine is plotted.
- Chapter 5 presents the simulation model of the voltage source inverter fed induction motor drive for the proposed selective harmonic elimination.
- Chapter 6 discusses the simulation results.
- Chapter 7 gives the conclusions and the scope for further work.

## **1.4 SCOPE OF THE WORK**

The present work describes the VSI fed induction motor drive using selective harmonic elimination. For this purpose three-phase induction motor is modeled and simulated in the stationary reference frame and a logic switching circuit has been developed to obtain the pulse patterns to the switching devices of the three-phase inverter. The switching angles needed to generate this pulse pattern are obtained by solving the non-linear transcendental equations using the nag routine E04UCF. These optimal switching angles which are computed offline through the computer program are stored in the look-up tables. Also, the speed control of the voltage source inverter fed induction motor drive has been obtained using the selective harmonic elimination technique.

## HARMONICS AND THE NEED OF ELIMINATION

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### 2.1 HARMONICS

A Harmonic is a component frequency of a harmonic motion of an electromagnetic wave that is an integral multiple of the fundamental frequency. When a signal passes through a non-linear device, additional content is added at the harmonics of the original frequencies. Total Harmonic distortion is used to measure the extent of that distortion.

In India, fundamental frequency is 50 Hertz

- 3rd Harmonic frequency is  $3 \times 50\text{Hz}$  or  $150\text{Hz}$
- 5th Harmonic frequency is  $5 \times 50\text{Hz}$  or  $250\text{Hz}$ , etc.

### 2.2 SOURCES OF HARMONICS

With the increased use of the static power converters, the problems of power system harmonics have increased considerably. Static power converters make use of power semiconductor devices as electronic switches to transfer and convert power from one form to another. The switching actions of the power devices result in a distorted current waveform. Harmonic pollution problems on the power system have long been bothering power companies, manufacturers and customers [8]. Non-linear loads generally do not cause reactive power to flow at the fundamental line frequency. They can, however, draw higher RMS currents and hence add to distribution system losses for a given load. The non-linear nature of these loads draws non-pure sine wave currents thus causing harmonics of the fundamental current to be present. Since harmonic distortion is caused by nonlinear elements connected to the power system, any device that has non-linear characteristics results in harmonic distortion. Examples of common sources of power system harmonics, some of which may not cause serious problems, are:

1. Solid State Electronic Devices which contain a poor power supply: Computers

(PCs/CPUs), Laser Printers, Copy Machines.

2. Solid State UPS Units
3. Solid State Devices (Fluorescent lighting ballasts)
4. Rectifiers (AC-DC Converters in Variable frequency drives)
5. Welding Units, Arc Furnaces
6. Inverters
7. MMF distribution in AC rotating machines.
8. Transformer saturation and inrush, Transformer neutral connections.

Harmonics cause distortion on the output voltage. Lower order harmonics (3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup> etc) are very difficult to filter, due to the filter size and high filter order. They can cause serious voltage distortion.

*Why need to consider harmonics?*

- Power Quality issue.
- Harmonics may cause degradation of equipment. Equipment need to be derated.

Harmonic distortion is:

- Typically generated within a facility, not a distribution issue
- Utilities does not protect for this condition

Total Harmonic Distortion (THD) is a measure to determine the “quality” of a given waveform.

### **2.3 NEED OF HARMONIC ELIMINATION**

The selective harmonic elimination switching scheme provides the opportunity to eliminate certain higher order harmonics by varying the times at which certain switches are turned “on” and turned “off” (i.e. varying the switching angles). Before doing so, one might ask: Why perform harmonic elimination?

One of the reason concerns the Electromagnetic Interference (EMI). Quite simply, harmonics are a source of EMI which creates voltage and current surges. Without harmonic elimination, designed circuits would need more protection in the form of snubbers and EMI filters resulting in excessive cost. EMI can also interfere with other

message signals, such as the control signals used to control power electronics devices and radio signals [9].

Harmonics can create losses in power equipment such as harmonic currents in an electrical induction motor will dissipate power in the motor stator and rotor windings. Additional core losses also occur due to harmonic frequency eddy currents [9].

Harmonics can also lower the power factor of a load. The power factor of a load is proportional to the ratio of the magnitude of the fundamental of the load current to the magnitude of the load current. Increased harmonic content may decrease the magnitude of the fundamental relative to the magnitude of the entire current. As a result, the power factor would decrease.

Eliminating harmonic content upto some order makes it easier to filter the remaining harmonic content. As a result, filters will be easier to design, build. And will be smaller and cheaper.

### **2.3.1 Effect of Harmonics on Rotating Machines**

Non-sinusoidal voltages containing harmonics when applied to electrical machines may cause overheating. So long as the harmonic distortion remains within the 5% normally recommended by the regulations, motors are not normally derated. Above that specified limit they will often experience excessive heating problems. On the positive side, motors contribute to the damping of the system harmonic content by virtue of the relatively high X/R ratio of their blocked rotor circuit.

Harmonic voltages or currents give rise to additional losses in the stator and rotor winding circuits, and stator and rotor laminations. The losses in the stator and rotor conductors are greater than those associated with the dc resistances because of eddy currents and skin effect.

Additional losses are produced because of the leakage fields set up by harmonic currents in the stator and rotor end-windings. Substantial iron loss is produced in the case of induction motors with skewed rotors because of the flux changes in both stator and rotor at high frequency. The magnitude of this loss depends upon the amount of skew, and the iron-loss characteristics of the laminations.

## 2.4 SELECTIVE HARMONIC ELIMINATION VS TRADITIONAL PWM

When considering the application of the selective harmonic elimination switching scheme, a question might arise: Why use this switching scheme when traditional PWM schemes can be used? [6]. An answer to this question refers to the switching frequencies employed by these schemes. Traditional PWM methods employ switching frequencies of the order of several kHz whereas the selective harmonic elimination switching scheme employs switching frequencies of the order of 50 Hz. One benefit of traditional PWM methods employing higher switching frequencies concerns harmonics. Undesirable harmonics occur at higher frequencies. Thus, filtering is easier and less expensive. Also, the generated harmonics might be above the bandwidth of some actual systems, which means there is less power dissipation due to these harmonics.

Switch conduction losses are approximately independent of switching frequency [9] for a specified switch duty ratio. Therefore, the selective harmonic elimination switching scheme will lead to switch conduction losses comparable to typical PWM schemes. However, switching losses increase as the switching frequency increases. As a result, it is desirable to make the switching frequency as low as possible. In this case, switching at the desired fundamental frequency seems to make the most sense. In other words, the selective harmonic elimination scheme will lead to lower switching losses. Therefore, using the selective harmonic elimination switching scheme will result in increased efficiency.

Traditional PWM schemes have the inherent problems of producing Electromagnetic Interference (EMI). Rapid changes in voltages ( $dv/dt$ ) are a source of EMI. The presence of a high  $dv/dt$  can cause damage to electrical motors. For example, a high  $dv/dt$  produces common-mode voltages across the motor windings. Furthermore, high switching frequencies can make this problem worse due to the increased number of times these common-mode voltages are applied to the motor during each fundamental cycle. Problems such as motor bearing failure and motor winding insulation breakdown can result due to circulating currents and voltage surges. Also, long current-carrying conductors connecting equipment can result in a considerable amount of EMI.

Therefore, switching at the fundamental frequency will result in decreasing the number of times these voltage changes occur per fundamental cycle.

the high frequency spectrum for noise reduction or optimizing efficiency. Third reason is some applications like high-speed motor drives used in electric vehicles for reducing mass require low switching-to-fundamental ratios. In SHE technique, the mathematical model of output waveform is constructed based on the characteristic of the output waveform of the man-made inverter, the amplitude of the fundamental wave of desired output voltage and the order and number of harmonics to be eliminated, and then solves the switching angles from the mathematical model to obtain the hoped output waveform. Thus the selected order and number of harmonics are not present in the output waveform of the inverter. The SHE technique is a PWM technology which has the advantages of lowering the switching frequency of the power device, eliminating the low order harmonic, etc.

The selective harmonic elimination technique is applied to obtain the output voltage control of a three-phase voltage-source SHE inverter to reduce the harmonic content of the output voltage in power electronic devices consisting of inverters, and improve the dynamic performance of variable voltage variable frequency drive.

## **1.2 LITERATURE REVIEW**

In 1973, H.S.Patel and R.G. Hoft for the first time introduced the selective harmonic elimination technique. In this, he discussed the harmonic elimination technique for the single phase half-bridge and full-bridge inverters with the generalized method for eliminating up to five harmonics. He used a Linearization technique of numerical method to solve the non-linear transcendental equations.

In 1987, P. N.Enjeti and J.F.Lindsay proposed an algorithm based on two straight lines with positive and negative slopes that closely approximate the exact solution pattern of the nonlinear equations. In this, the starting values for obtaining exact solutions using numerical techniques can be found even for large numbers of harmonics to be eliminated. Moreover, the close proximity of the starting values to the exact solutions ensures convergence.

In 1990, Prasad N. Enjeti, Phoivos D. Ziogas, F. Lindsay discussed the various programmed PWM techniques to eliminate the harmonics. He gave a general classification for various techniques depending on the inverter configuration that is



## BASIC CONFIGURATION OF A VSI FED INDUCTION MOTOR DRIVE

### 3.1 VSI FED INDUCTION MOTOR DRIVE

Inverter is a device which converts DC to AC power by switching the DC input voltage (or current) in a pre-determined sequence so as to generate AC voltage (or current) output.

General block diagram:

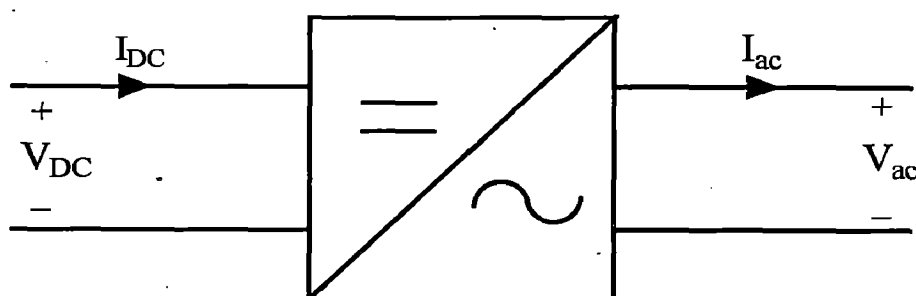


Fig. 3.1 Basic Inverter Circuit

Applications: Un-interruptible power supply (UPS), Industrial (induction motor) drives, Traction, HVDC.

The basic configuration of a VSI fed Induction Motor drive is shown in fig 3.2. Here the input is a constant voltage D.C. supply [1]. It consists of six identical switching devices with anti-parallel diodes connected across them. The voltage source inverter (VSI) produces a relatively well-defined switched voltage waveform at the input terminals of the A.C. motor. A switching device conducts when the firing pulse is given to it and the switch is forward biased. For the six-step inverter operation, the various waveforms are as shown in fig 3.3. The line voltage  $V_{AB}$ , the pole voltages and the six-step phase voltage  $V_{AN}$  are shown. The line voltages  $V_{BC}$  and  $V_{CA}$  will lag behind  $V_{AB}$  by  $120^\circ$  and  $240^\circ$  respectively. The pole voltage waveform for the phase A of the three phase inverter is a quasi-square wave consisting of one half-cycle with a magnitude of half the dc voltage and the second half cycle with a magnitude of half the dc voltage with

negative sign as shown in fig.3.3.

Three transistors remain on at any instant of time. Each switching device of the inverter is made to conduct for half cycle of the period, which is 180 degrees. Hence the pole voltage waveform of phase A of the inverter is a quasi-square wave. When S1 is turned on, terminal A is connected to the positive terminal of the D.C. supply source. When transistor S4 is switched on, the terminal A is connected to the negative side of the D. C. supply. The pole voltage waveform of phase A is obtained by switching the devices S1 and S4 in a complementary manner. The pole voltage waveforms of phase B and

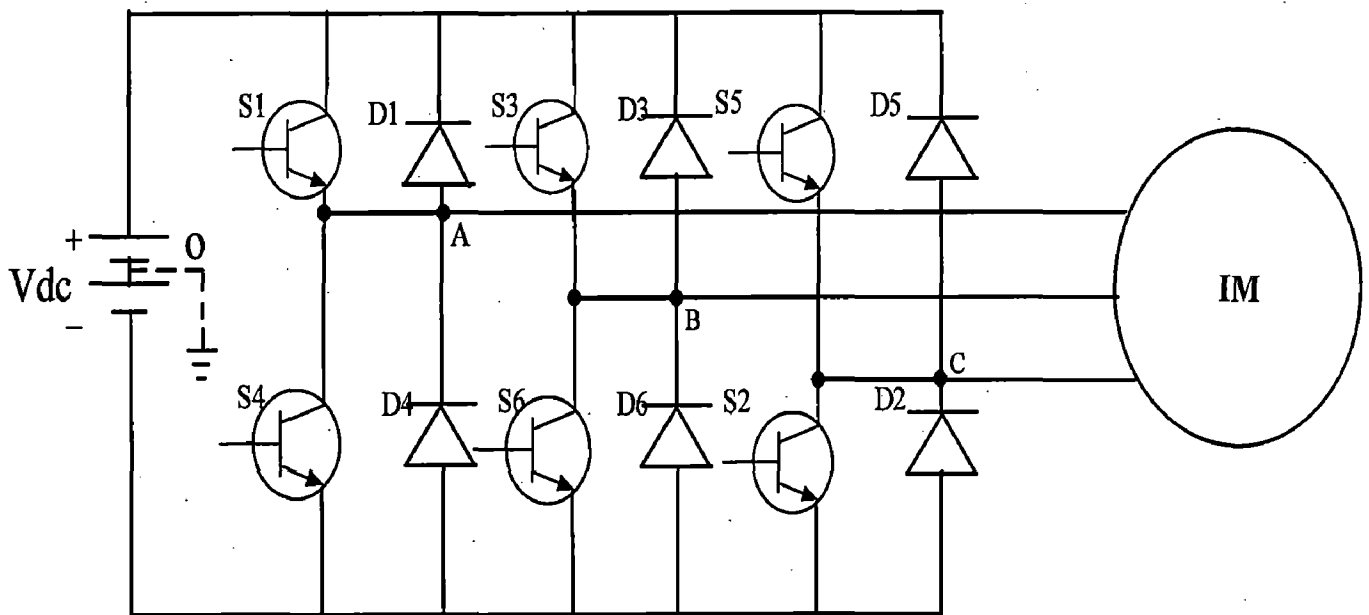


Fig. 3.2 Three Phase Voltage Source Inverter Fed Induction Motor Drive

phase C are obtained by phase-shifting them with respect to phase A by  $120^\circ$  and  $240^\circ$  respectively, which results in a balanced three-phase supply. For this, the six switching devices of the inverter S1 through S6 are fired sequentially one after the other in a  $60^\circ$  interval. The frequency of the pole voltage waveform depends on the switching frequency of the inverter devices S1 and S4 which are controlled by adjusting the firing pulses to the inverter. The voltage control can be achieved by changing the quasi-square wave operation to PWM operation [5]. The line voltage across the output terminals is obtained by the difference of the corresponding pole voltages.

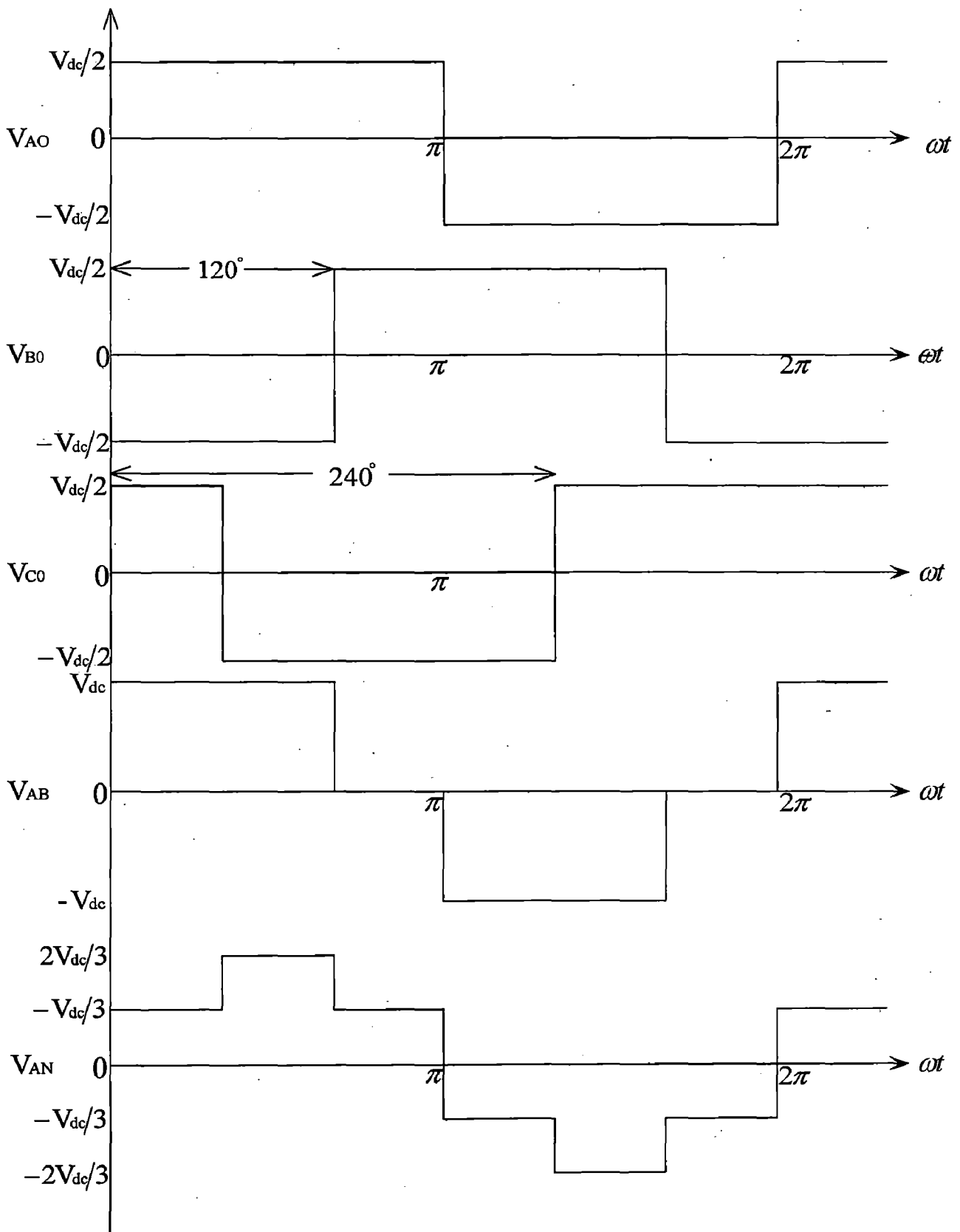


Fig. 3.3 Waveforms of a Three-Phase Six-step Voltage Source Inverter

As the firing instants of the switching devices coincide with the zero-crossing points of the quasi-square waveform, it can be said that the pole voltage waveform represents the firing pulses pattern of the inverter switches. The line voltage  $V_{AB}$  and the phase voltage  $V_{AN}$  are given by the fouries series expansion as:

$$V_{AB} = \frac{2\sqrt{3}}{\pi} V_{dc} \left[ \sin(\omega t + \pi/6) + \frac{1}{5} \sin(5\omega t - \pi/6) + \frac{1}{7} \sin(7\omega t + \pi/6) \dots \dots \right] \quad (3.1)$$

$$V_{AN} = \frac{2}{\pi} V_{dc} \left[ \sin \omega t + \frac{1}{5} \sin 5 \omega t + \frac{1}{7} \sin 7 \omega t \dots \dots \right] \quad (3.2)$$

From the above equations it can be seen that the triplen harmonics are absent. in both of them. And the harmonic content with respect to the fundamental is same in both phase and line voltages. The six-step inverter has six steps in the output line-to-neutral voltage (fig 3.3). In this configuration, the harmonics of the order  $6n \pm 1$  are present. From the Fourier analysis, it can be observed that the line-to-line and the line-to-neutral waveforms both contain  $1/5^{\text{th}}$  of the  $5^{\text{th}}$  harmonic,  $1/7^{\text{th}}$  of the  $7^{\text{th}}$  harmonic,  $1/11^{\text{th}}$  of the  $11^{\text{th}}$  harmonic and so on. That is 20% of  $5^{\text{th}}$  harmonic, 14.28% of  $7^{\text{th}}$  harmonic, 9.09% of  $11^{\text{th}}$  harmonic and so on which are quite high. The harmonic spectrum is as shown below.

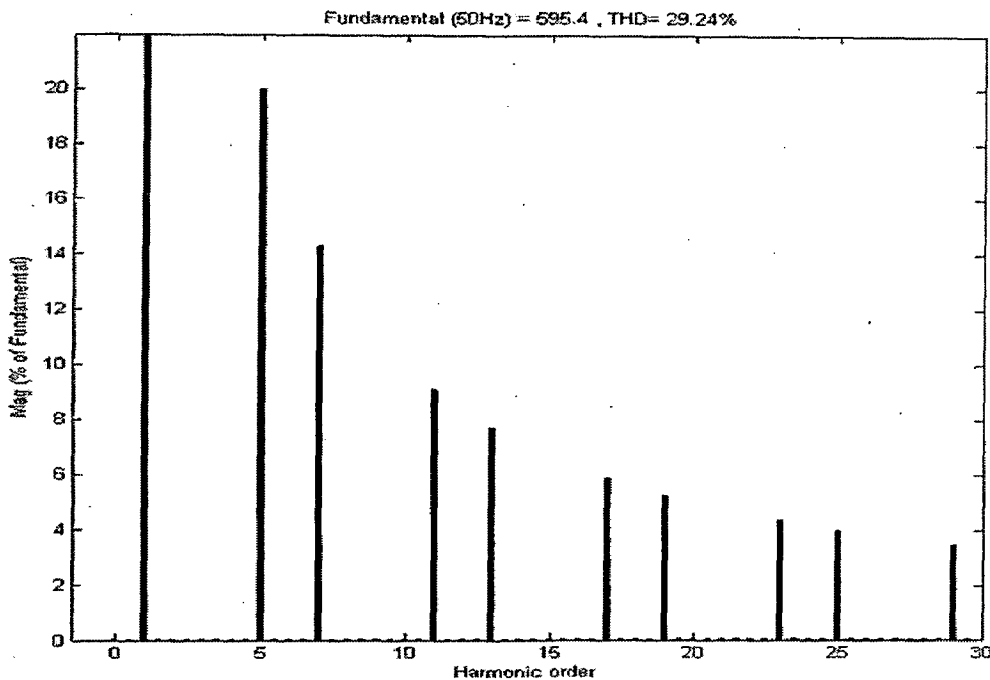


Fig 3.4 Harmonic Spectra of voltage of a six step Inverter.

The harmonic spectrum reveals that the Harmonic content decreases with a factor of  $1/n$ . Even and triplen harmonics are absent. Also due to the presence of harmonics near to the fundamental, the low-pass filter circuit design is difficult. So, the major concern is over the elimination of the odd non-triplen low-order harmonics which can be done with the elimination techniques mentioned in the below section.

### 3.2 HARMONIC ELIMINATION SWITCHING SCHEMES

This section presents some switching schemes employing harmonic elimination such as Bipolar Programmed PWM and Unipolar Programmed PWM.

#### 3.2.1 Bipolar Programmed PWM

Bipolar Programmed PWM is the switching scheme involving harmonic elimination that has been around for years. In this method, control of fundamental voltage with the simultaneous elimination of undesirable harmonics is obtained. The bipolar waveform of fig 3.5 can be obtained by firing the inverter switches  $S_1$  and  $S_4$  at the predetermined angles  $\alpha_1, \alpha_2, \dots, \alpha_n$  during the period  $0 - \pi/2$ . During the period  $\pi/2 - \pi$ , the switching devices are fired such that the waveform is symmetrical around  $\pi/2$ . For  $N-1$  harmonics to be eliminated the number of switching angles should be  $N/2$ , [3]. In Bipolar Programmed PWM, the output pole voltage is either  $+V_{dc}/2$  or  $-V_{dc}/2$ . Figure 3.4 illustrates the Bipolar Programmed PWM switching scheme using  $N$  switching angles and a dc voltage  $V_{dc}/2$ .

The Fourier series representation of the waveform is given by:

$$v(\omega t) = \sum_{n=1}^{\infty} [a_n \sin(n\omega t) + b_n \cos(n\omega t)] \quad (3.1)$$

where 
$$a_n = \frac{1}{\pi} \int_0^{2\pi} v(\omega t) \sin(n\omega t) d(\omega t)$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} v(\omega t) \cos(n\omega t) d(\omega t)$$

Substituting for  $v(\omega t)$  and using the half and quarter wave symmetry properties [10], we

have  $a_n = 0$  for even  $n$  and  $b_n = 0$  for all  $n$ . So, the Fourier series term on evaluating the integral becomes

$$a_n = \frac{2V_{dc}}{n\pi} \left[ 1 + 2 \sum_{k=1}^N (-1)^k \cos n\alpha_k \right] \quad (3.2)$$

$$\text{Also, } 0 < \alpha_1 < \alpha_2 < \dots < \alpha_n < \pi/2 \quad (3.3)$$

In the above equation,  $N$  represents the no of switching angles per quarter cycle to eliminate  $N-1$  harmonics. So, a total of  $N$  simultaneous equations are to be solved to get the values of  $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$  such that  $0 < \alpha_1 < \alpha_2 < \dots < \alpha_n$ . These equations are obtained by assigning the required value to the fundamental and zero value to the  $(N-1)$  harmonics to be eliminated [11]. So,  $N$  equations are obtained one is for the fundamental and the remaining for the elimination of  $N-1$  harmonics. This method of eliminating harmonics is called the Selective Harmonic Elimination.

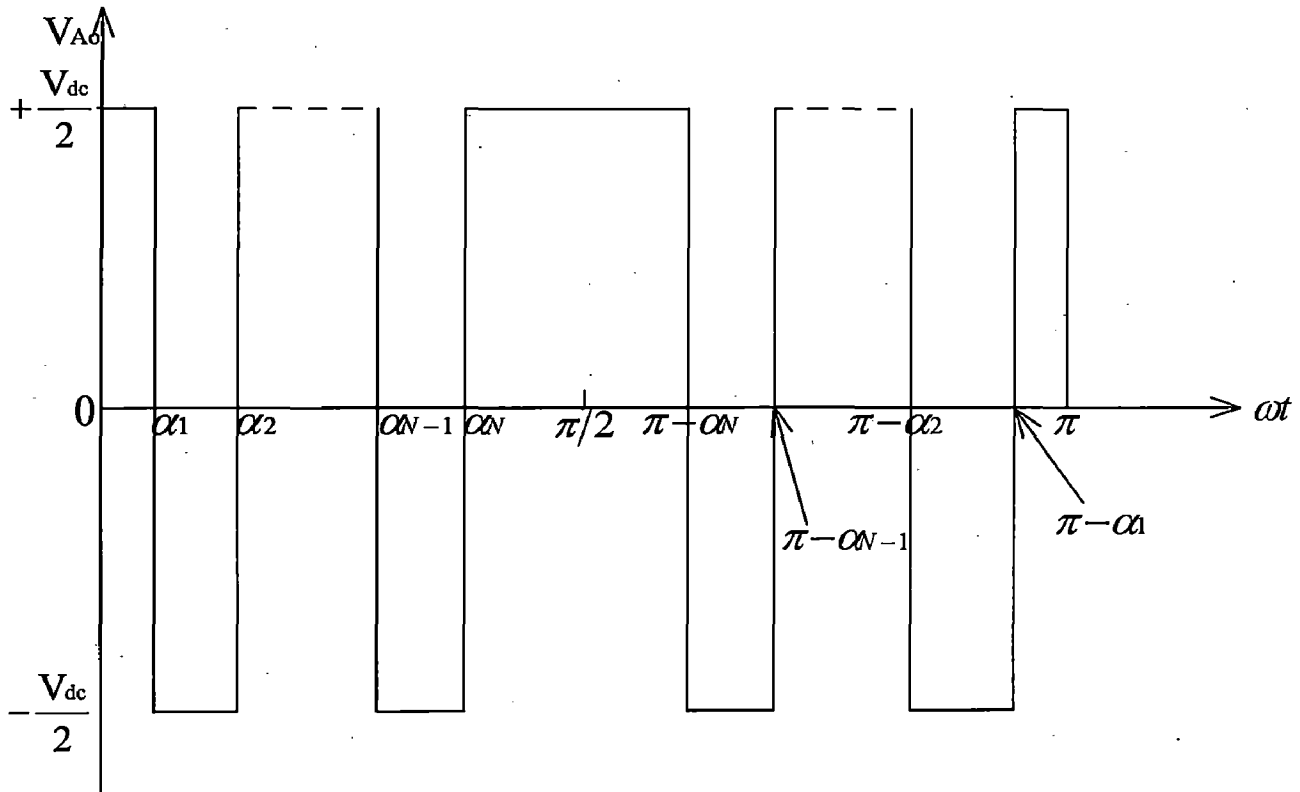


Fig. 3.5 Generalized Bipolar waveform for  $N$  switching angles

As can be seen from the figure, Bipolar Programmed PWM uses predetermined switching angles to cut notches into the square-wave output. These notches take the

voltage ranging between  $+V_{dc}/2$  to  $-V_{dc}/2$ . The number of notches cut per fundamental cycle is equal to twice the number of switching angles used [12].

For example, three switching angles can be used to eliminate the fifth and seventh order harmonics while at the same time controlling the value of the fundamental. One of the main advantages of using Bipolar Programmed PWM concerns its applicability when low modulation indices are used.

In section 3.1, it has been discussed about the quasi-square wave operation with its maximum voltage ranging between  $+V_{dc}/2$  to  $-V_{dc}/2$ . In that waveform; notches are introduced such that the effective pulse width is varied. This results in the control of the output voltage from the inverter. With the notches introduced as shown in fig 3.6, the maximum voltage is still  $\pm V_{dc}/2$ , but the effective width changes resulting in reduced voltage as compared to the quasi-square wave with the elimination of the selected harmonics.

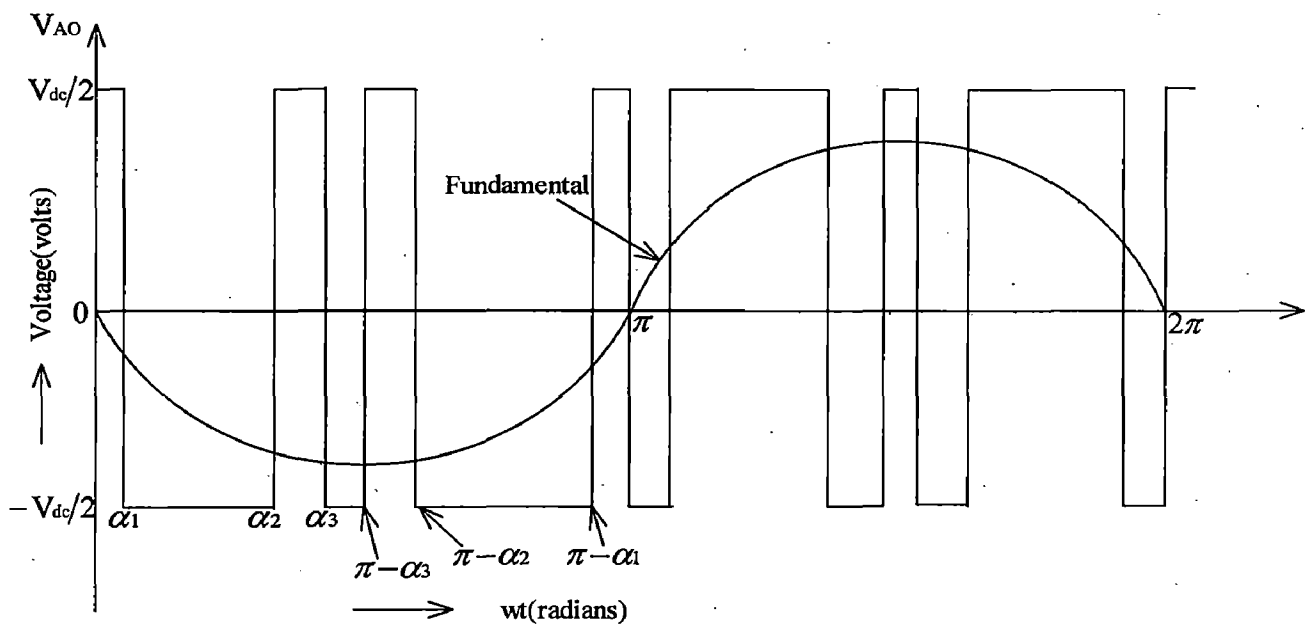


Fig. 3. 6 Bipolar Programmed PWM using three switching angles.

For the waveform, shown in fig 3.6, where the no of switching angles per quarter cycle are 3, the 3 transcendental equations for eliminating the fifth and seventh harmonics while obtaining the desired fundamental voltage can be written as

$$M = \frac{4}{\pi} [1 + 2(-\cos \alpha_1 + \cos \alpha_2 - \cos \alpha_3)] \quad (3.4)$$

$$0 = \frac{4}{\pi} [1 + 2(-\cos 5\alpha_1 + \cos 5\alpha_2 - \cos 5\alpha_3)] \quad (3.5)$$

$$0 = \frac{4}{\pi} [1 + 2(-\cos 7\alpha_1 + \cos 7\alpha_2 - \cos 7\alpha_3)] \quad (3.6)$$

In the above equations, the first equation is for the fundamental in which M is the modulation index defined as the ratio of the desired value of fundamental voltage to the d.c.voltage ( $V_{dc}/2$ ). The other two equations are for eliminating the two most dominant harmonics which are 5<sup>th</sup> and 7<sup>th</sup>. When the above three equations are solved simultaneously, they provide the three switching angles which results in the elimination of the 5<sup>th</sup> and 7<sup>th</sup> harmonics while retaining the desired value of fundamental voltage. For N Switching angles per quarter cycle, the highest harmonic that can be eliminated is (3N-2) when N is odd and (3N-1) when N is even.

By solving these non-linear equations for different values of fundamental voltages, the variation of switching angles with respect to fundamental can be achieved. The elimination of low-order harmonics is obtained at the expense of an increase in the next significant low-order harmonics [13].

Bipolar Programmed PWM when used with a multilevel inverter for a low modulation index, only one H-bridge is used. Therefore, one advantage is redundancy. If one H-bridge fails, another H-bridge can be used to provide the necessary voltage. Bipolar Programmed PWM can be used for a high range of modulation indices. Another advantage of Bipolar Programmed PWM is that control is not as complicated as some other switching schemes. Bipolar Programmed PWM also has some disadvantages. One disadvantage concerns EMI. The line voltage in a Bipolar Programmed PWM produces voltage changes equal to  $2V_{dc}$ . Therefore, a large  $V_{dc}$  can produce a considerable amount of EMI. Another disadvantage of Bipolar Programmed PWM concerns harmonic distortion at lower modulation indices. For low modulation indices, using Bipolar Programmed PWM leads to a high amount of harmonic content in the output. In fact, the Total Harmonic Distortion (THD) is over 100% for certain lower modulation indices.



### 3.2.2 Unipolar Programmed PWM

Unipolar Programmed PWM is another switching scheme involving harmonic elimination that has been in use for a long period. In Unipolar Programmed PWM, the output voltage is  $+V_{dc}/2$ ,  $-V_{dc}/2$  or 0. Furthermore, a voltage change is from  $\pm V_{dc}/2$  to 0 and vice versa. Fig 3.7 illustrates the Unipolar Programmed PWM switching scheme using three switching angles.

Unipolar Programmed PWM also uses predetermined switching angles to produce an output consisting of multiple pulses of varying widths. For the positive half-cycle of the fundamental period, these pulses have a voltage equal to  $+V_{dc}/2$ . For the negative half-cycle of the fundamental period, these pulses have a voltage equal to  $-V_{dc}/2$ . The number of pulses per fundamental cycle is equal to twice the number of switching angles used.

Similar to Bipolar Programmed PWM, Fourier series theory can be used to determine the switching angles such that certain harmonics are eliminated. In fact, these two switching schemes produce almost identical equations to solve. The only differences between the two sets of equations are that the Bipolar Programmed PWM equations contain a few extra numerical constants. In the similar way as with the case of bipolar waveform, by using the half and quarter wave symmetry properties [4], the Fourier series expansion gives

$$a_n = \frac{2V_{dc}}{n\pi} \left[ \sum_{k=1}^N (-1)^k \cos n\alpha_k \right] \quad (3.7)$$

Unipolar Programmed PWM shares many of the advantages of Bipolar Programmed PWM. For example, as mentioned earlier in the preceding section that Bipolar Programmed PWM can be used with low modulation indices. This statement holds true for Unipolar Programmed PWM as well.

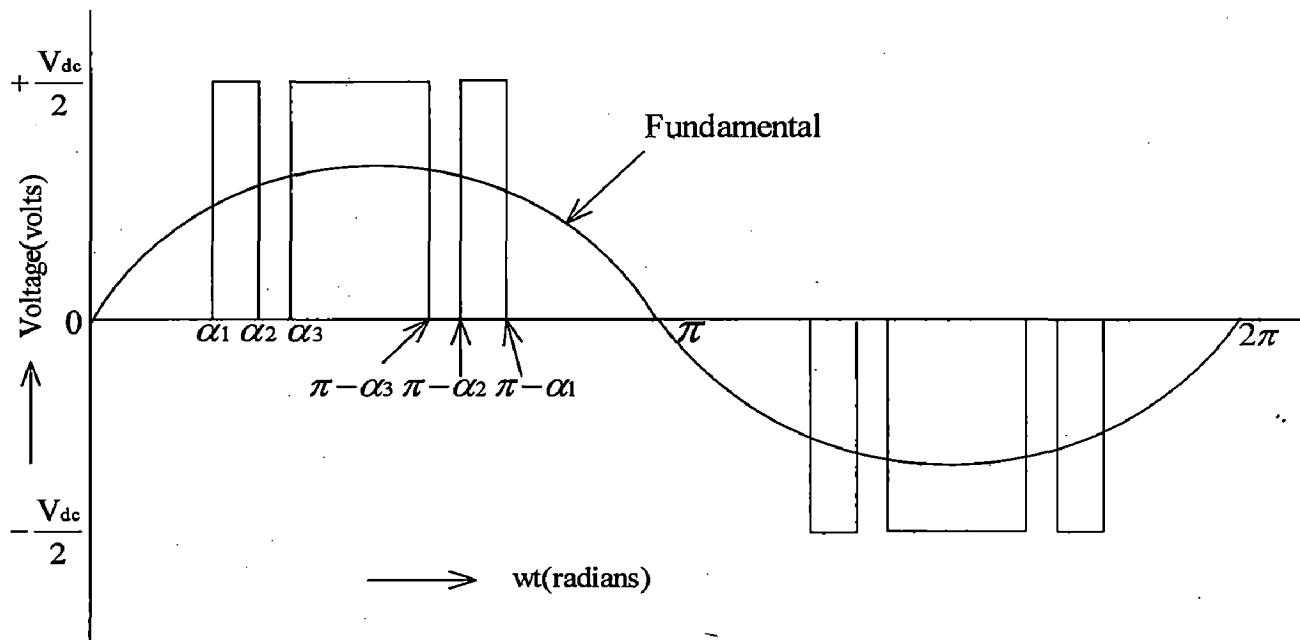


Fig. 3.7 Unipolar Programmed PWM using three switching angles.

Like Bipolar Programmed PWM, one disadvantage of Unipolar Programmed PWM concerns harmonic distortion for low modulation indices. For lower values, using Unipolar Programmed PWM leads to a high output THD. However, Unipolar Programmed PWM tends to produce a lower THD than Bipolar Programmed PWM. One possible explanation can be given by referring to Fig 3.6 and Fig 3.7. From these two figures, it can be seen that Unipolar Programmed PWM seems to provide a more natural approximation to a sinusoidal waveform.

## GENERATION OF OPTIMAL SWITCHING ANGLES

---

### 4.1 GENERAL

It has been discussed that when switching schemes involving harmonic elimination are used, the derived equations are nonlinear equations as discussed in chapter 3. As a result, many people have utilized numerical iterative techniques in order to solve these equations. For example, Cunyngham used the Newton-Raphson numerical technique in his research. Another numerical technique which can be used is Gauss-Seidel, although this particular numerical technique is not as robust as Newton-Raphson.

In [14], an algorithm based on two straight lines with positive and negative slopes that closely approximate the exact solution pattern of the nonlinear equations has been proposed. The starting values for obtaining exact solutions using numerical techniques can be found even for large number of harmonics to be eliminated. Moreover, the close proximity of the starting values to the exact solutions ensures convergence. However, although it reports the algorithm and discusses the case of multiple solutions for a three-phase case, it does not report the performance of the said technique against its ability to find all possible solutions. The algorithm seems to be a good approximation for a relatively large number of harmonics to be eliminated. The performance of these techniques was later analyzed in [4] where the techniques are studied with two systems in mind: the single-phase (line-to-neutral patterns) and three-phase (line-to-line patterns). Specifically, it reported techniques that seek switching angles within  $60^\circ$ .

A systematic method that makes it possible to solve the HE problem is proposed in [15]. This method is based on a homotopy method which finds multiple solutions for a specific degree of freedom  $N$  from those existing for  $N-1$  sequentially by the mathematical induction by varying a fundamental component value as the homotopy parameter. However, the method is long and cumbersome and does not about which set of solutions from the multiple available ones is optimum against overall harmonic performance and presents no experimental results to confirm the analysis.

Recently, a new method to the problem has been reported in [16], where the theory of resultants has been employed to get the solutions for the problem. Specifically, the transcendental equations that describe the harmonic elimination problem are converted into an equivalent set of polynomial equations using trigonometric identities. The theory of resultants is then used to compute the resultant polynomial and then work backward in order to find all unknown, i.e., switching angles. This method also finds all possible sets of solutions. However, the method introduces another step into the problem through the manipulation of high order polynomials that their order increases as the number of harmonics to be eliminated also increases. Furthermore, it has limited chance to work for a high order of harmonics and it is easy to apply only when such number is low. Moreover, it treats the bipolar waveform and reports the switching angles to minimize the fifth and seventh harmonics only.

Here, the objective is to eliminate the harmonics from the Voltage source inverter fed induction motor drive. That is given a desired fundamental output voltage; the problem is to find the switching times (angles) that produce the fundamental while not generating the specifically chosen harmonics. Unfortunately, Numerical iterative techniques have their drawbacks. One drawback is that these techniques require an initial guess in order to work. However, if the initial guess is not good enough, a solution will not be found. Another drawback is that numerical iterative techniques will only find one solution, if one exists. The obvious drawback here is that more than one solution might exist to the problem at hand. So, the Numerical Algorithms Group (NAG) Library Routine E04UCF [17] has been used to find the optimum switching angles such that the desired fundamental voltage is achieved by eliminating the harmonics.

E04UCF is designed to minimize an arbitrary smooth function subject to constraints (which may include simple bounds on the variables, linear constraints and smooth nonlinear constraints) using a sequential quadratic programming (SQP) method. As many first derivatives as possible should be supplied by you; any unspecified derivatives are approximated by finite differences.

## 4.2 SPECIFICATION FOR E04UCF

*SUBROUTINE E04UCF* (N, NCLIN, NCNLN, LDA, LDCJ, LDR, A, BL, BU, CONFUN, OBJFUN, ITER, ISTATE, C, CJAC, CLAMDA, OBJF, OBJGRD, R, X, IWORK, LIWORK, WORK, LWORK, IUSER, RUSER, IFAIL)

*INTEGER* N, NCLIN, NCNLN, LDA, LDCJ, LDR, ITER, 1 ISTATE (N+NCLIN+NCNLN), IWORK (LIWORK), LIWORK, LWORK, 2 IUSER (\*), IFAIL  
 Double precision A(LDA,\*),BL(N+NCLIN+NCNLN), BU (N+NCLIN+NCNLN), C (\*), CJAC (LDCJ,\*), CLAMDA (N+NCLIN+NCNLN), OBJF, OBJGRD (N), 2 R (LDR, N), X (N), WORK (LWORK), RUSER (\*)

*EXTERNAL* CONFUN, OBJFUN

E04UCF is designed to solve the nonlinear programming problem – the minimization of a smooth nonlinear function subject to a set of constraints on the variables. The problem is assumed to be stated in the following form:

$$\underset{x \in R^n}{\text{Minimize}} F(x) \quad \text{Subject to} \quad l \leq \begin{pmatrix} x \\ A_L x \\ c(x) \end{pmatrix} \leq u \quad (4.1)$$

Where  $F(x)$  (the objective function) is a nonlinear function,  $A_L$  is a  $n_L$  by  $n$  constant matrix, and  $c(x)$  is an  $n_N$  element vector of nonlinear constraint functions (The matrix  $A_L$  and the vector  $c(x)$  may be empty). The objective function and the constraint functions are assumed to be smooth, i.e., at least twice-continuously differentiable. (The method of E04UCF will usually solve (4.1) if there are only isolated discontinuities away from the solution). Note that although the bounds on the variables could be included in the definition of the linear constraints, we prefer to distinguish between them for reasons of computational efficiency. For the same reason, the linear constraints should not be included in the definition of the nonlinear constraints. Upper and lower bounds are specified for all the variables and for all the constraints. An equality constraint can be specified by setting  $l_i = u_i$ . If certain bounds are not present, the associated elements of  $l$  or  $u$  can be set to special values that will be treated as  $-\infty$  or  $+\infty$ .

An initial estimate of the solution to (4.1), together with subprograms that

define  $F(x)$ ,  $c(x)$  and as many first partial derivatives as possible must be supplied; any unspecified derivatives are approximated by finite differences. The objective function is defined by subprogram OBJFUN, and the nonlinear constraints are defined by subprogram CONFUN. On every call, these subprograms must return appropriate values of the objective and nonlinear constraints.

The  $N$  equations which are involved in the selective harmonic elimination are transcendental in nature which give rise to multiple (two) solutions. In one solution, the  $N$  switching angles lie in the  $0-60^\circ$  range such that

$$0 < \alpha_1 < \alpha_2 < \alpha_3 \dots < \alpha_n < 60^\circ \quad (4.2)$$

In the other solution the  $N$  switching angles lie in the interval  $0-90^\circ$  range such that

$$0 < \alpha_1 < \alpha_2 < \alpha_3 \dots < \alpha_n < 90^\circ \quad (4.3)$$

A Fortran program using the Nag routine E04UCF has been used to find the optimal switching angles. In that program, the input parameters are the number of switching angles in a quarter cycles, the desired value of fundamental voltage, the highest harmonic which can be optimized and the initial guess to the solution vector. The initial guess to the solution vector be entered such that the linear constraint (4.2) for  $0-60^\circ$  solution and (4.3) for the  $0-90^\circ$  solution be satisfied. This linear constraint determines the relationship between the  $n$  variables. Total number of such constraints is  $n_{clin}$ . The non-linear constraints whose total number given by  $n_{cnln}$  is specified in the subroutine CONFUN. These constraints are denoted by  $c(1)$ ,  $c(2)$ ,.....etc. Their partial derivatives with respect to each variable are also specified in the array  $cjac$ . Finally the objective function is specified in the subroutine OBJFUN. Here, the objective function which is the total harmonic distortion which is to be minimized has been specified. The partial derivatives for both the constraint functions and the objective function are optional. In the main program, while calling the routine E04ucf, these OBJFUN and CONFUN are supplied as the arguments to that routine.

In the above program, the value of modulation index has been varied from 0.1-1.1 and the optimal switching angles have been obtained for both odd and even values of  $N$  with  $n$  varying from 2 to 16. The nature of the switching angles with respect to Modulation index have plotted with  $M$  on the X-axis and Switching angles on the Y-axis.

Also, the solution for 0-90° case has been obtained by changing the upper bound of the constraint matrix to  $\pi/2$  in the computer program.

### 4.3 SWITCHING ANGLE TRAJECTORIES

#### Case 1: For Odd N

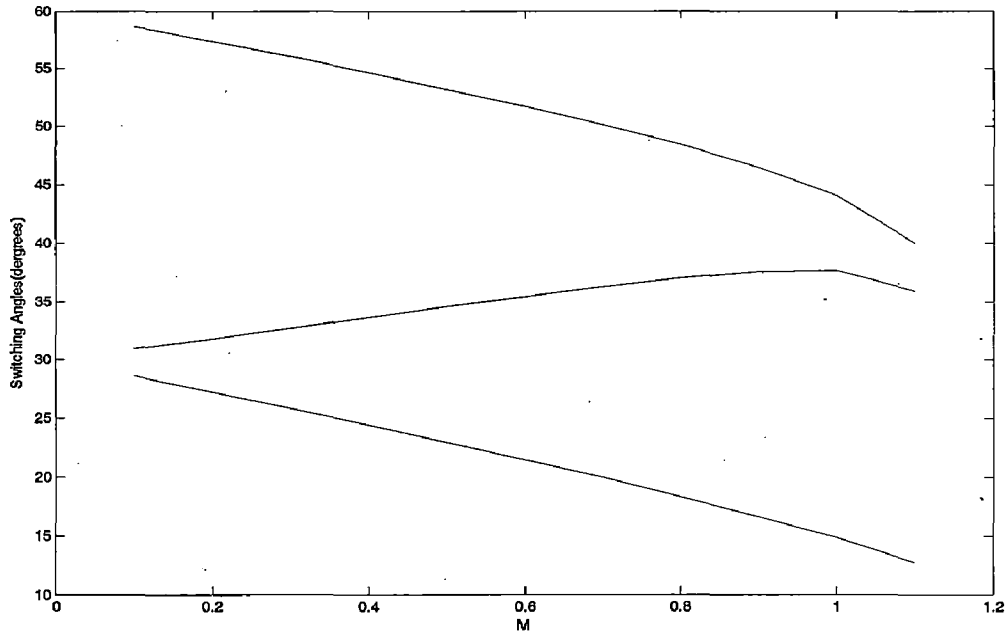


Fig. 4.1 Nature of switching angles for N=3

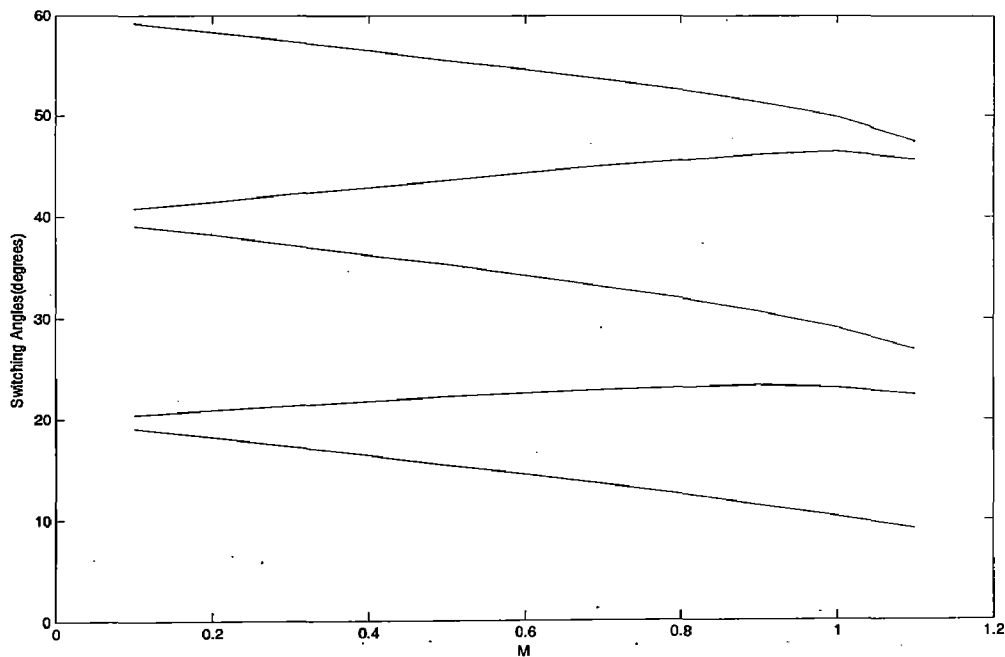


Fig. 4.2 Nature of switching angles for N=5

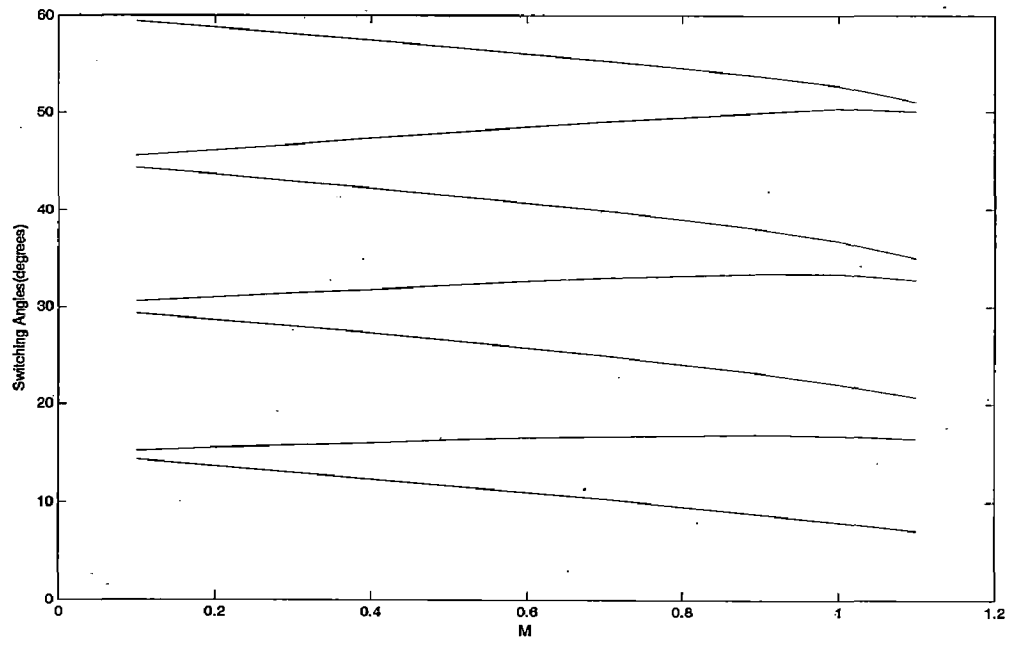


Fig. 4.3 Nature of switching angles for N=7

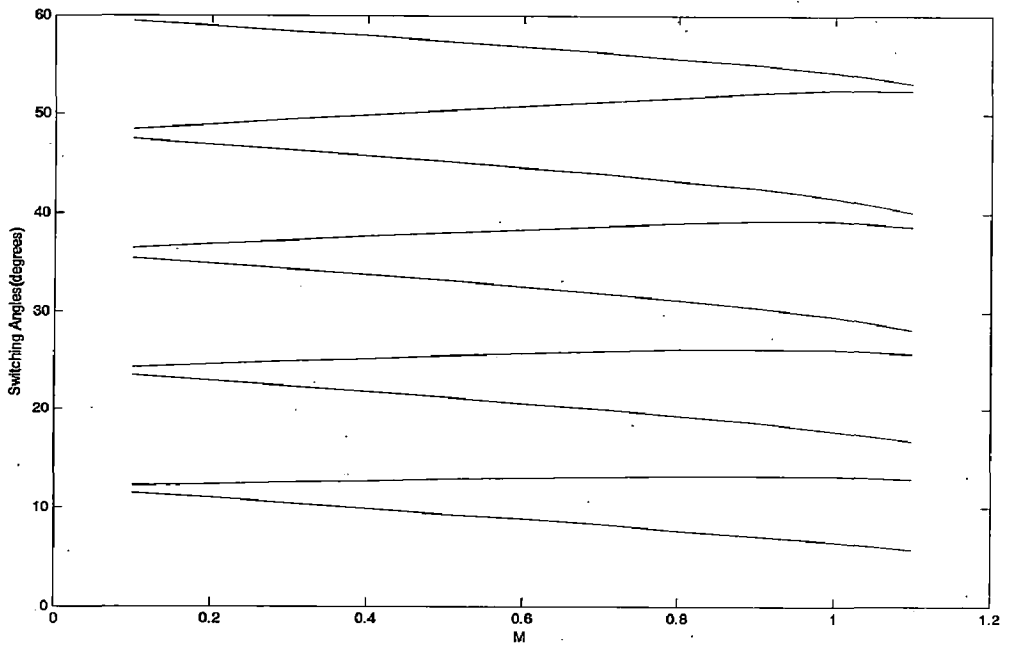


Fig. 4.4 Nature of switching angles for N=9



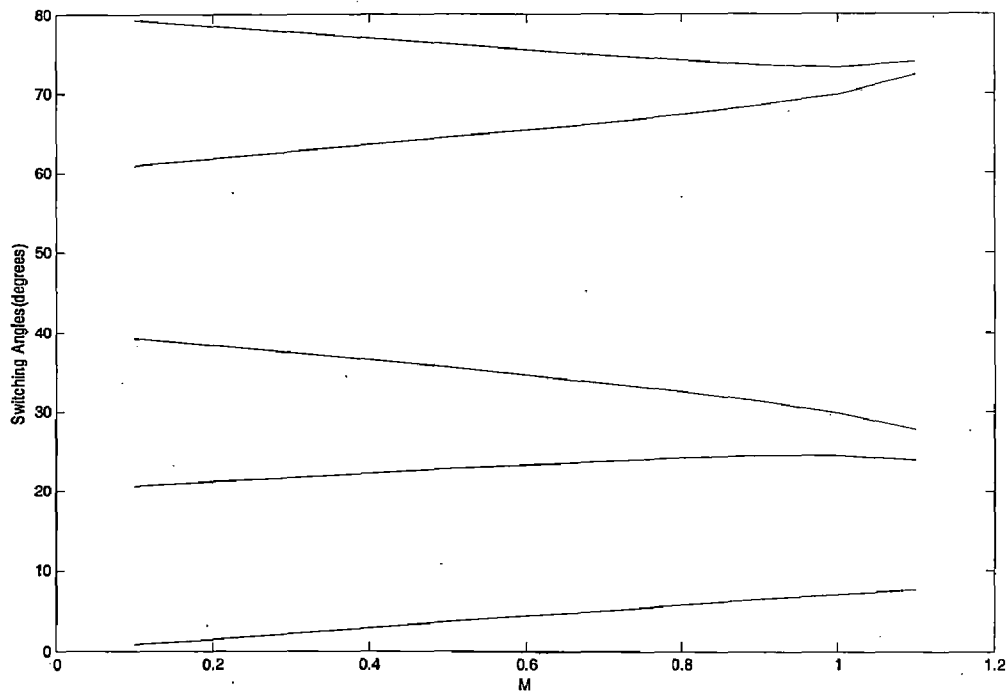


Fig. 4.5 Nature of switching angles (0- 90°) for N=5

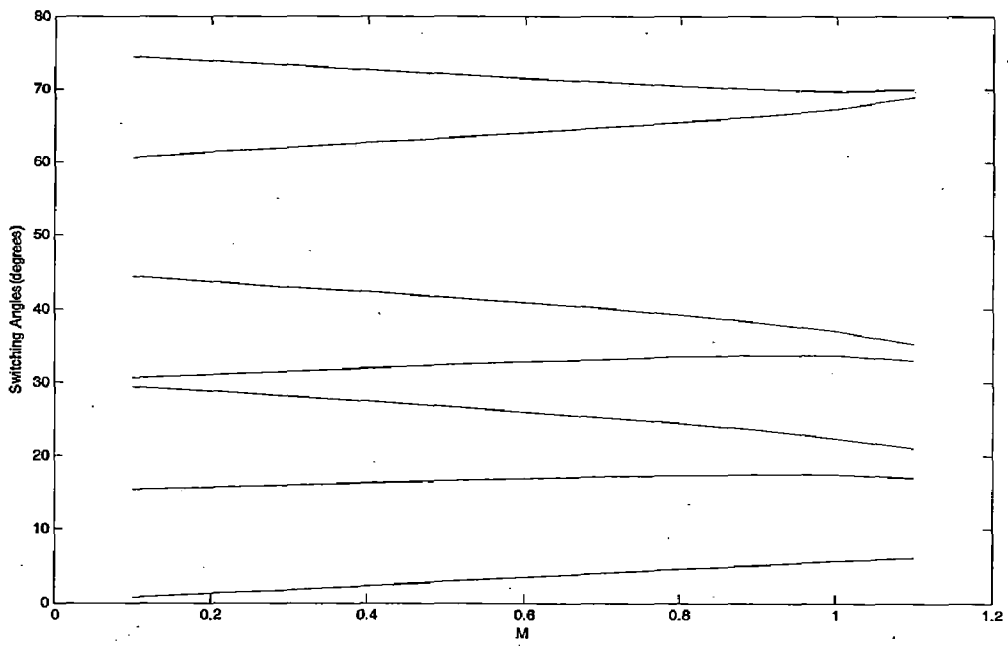


Fig. 4.6 Nature of switching angles (0- 90°) for N=7

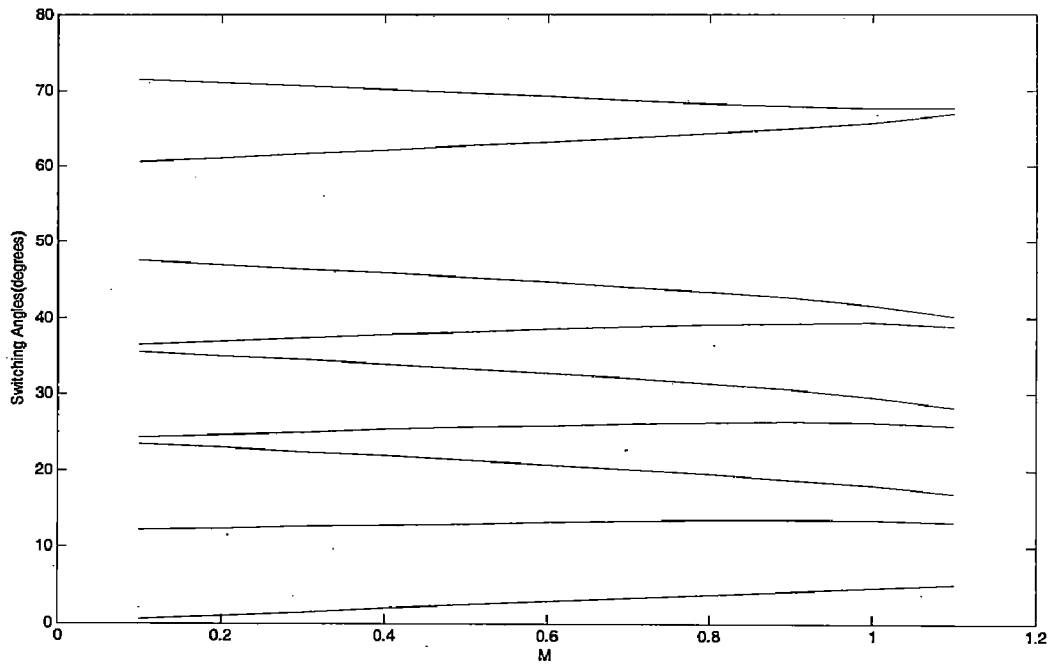


Fig. 4.7 Nature of switching angles (0- 90° ) for N=9

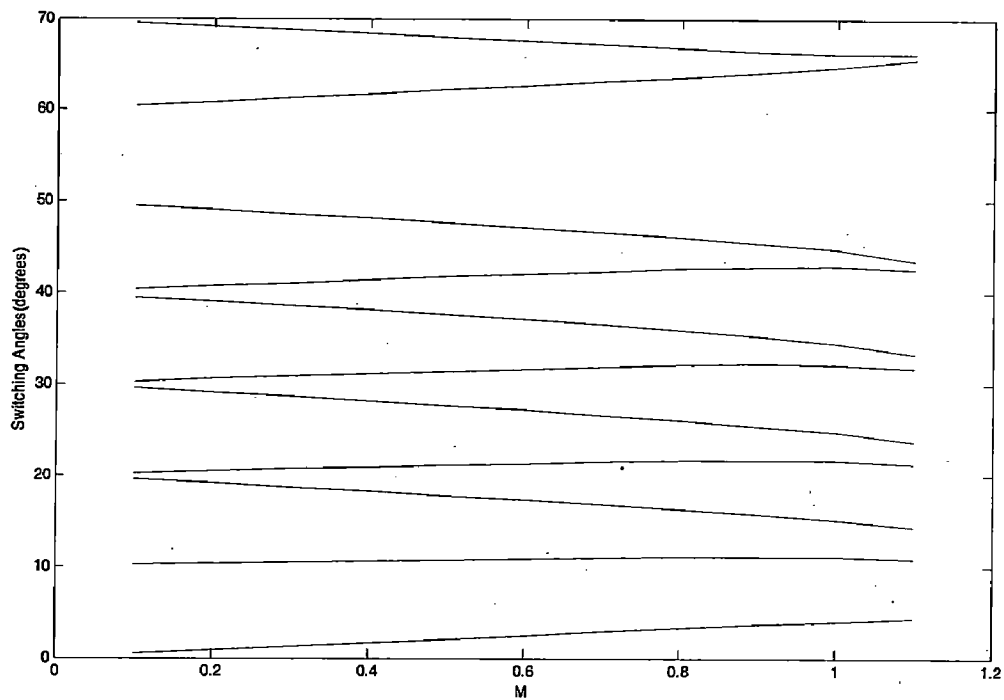


Fig. 4.8 Nature of switching angles (0- 90° ) for N=11

The following observations can be made from the switching angle characteristics for odd values of N.

1. At  $M=0$ , the angle  $\alpha_1$  becomes equal to  $\alpha_2$ ,  $\alpha_3$  becomes equal to  $\alpha_4$ , and so on for increasing n with  $\alpha_n$  becoming  $60^\circ$  and these equal pairs are separated by an angle of  $120/(N+1)$  at  $M=0$ . For  $0-90^\circ$  solution, this pattern is different with  $\alpha_1$  and  $\alpha_{n-1}$  taking the values  $0$  and  $60^\circ$  at  $M=0$ .
2. They are almost linear in the modulation index range of  $0.1$  to  $0.8$  and after that they deviate from the straight line approximation and become non-linear for both the solutions.
3. All the odd switching angles have negative slopes with their values decreasing with increasing M and even switching angles have positive slopes with their values increasing with increasing M. In the case of  $0-90^\circ$  solution, this characteristic is same except for the first angle  $\alpha_1$  which has a positive slope.

#### Case 2: For Even N

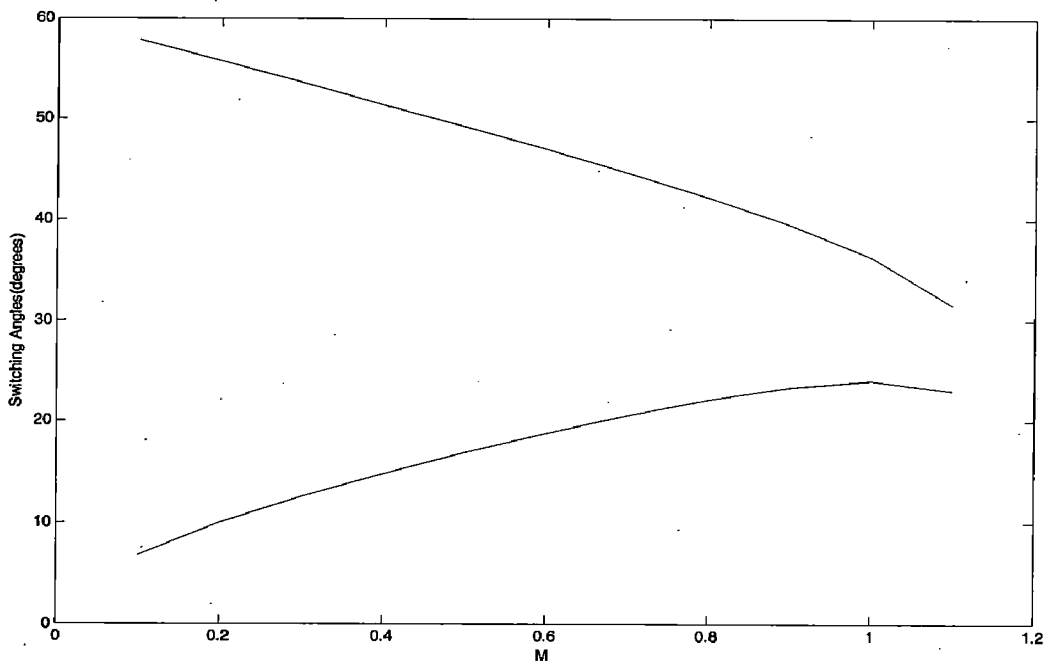


Fig. 4.9 Nature of switching angles for  $N=2$

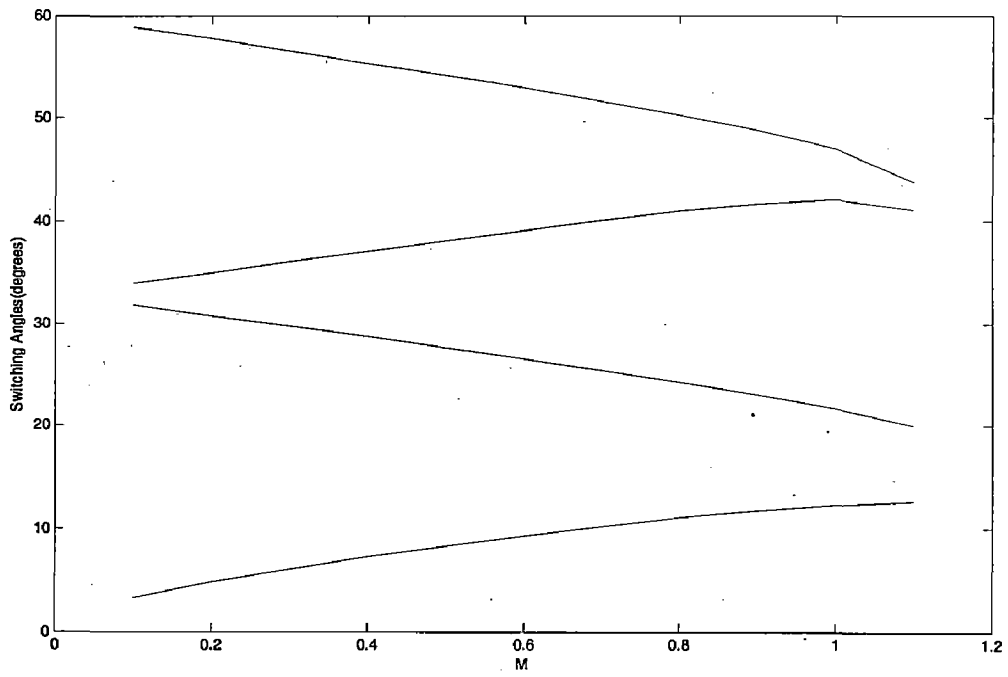


Fig. 4.10 Nature of switching angles for N=4

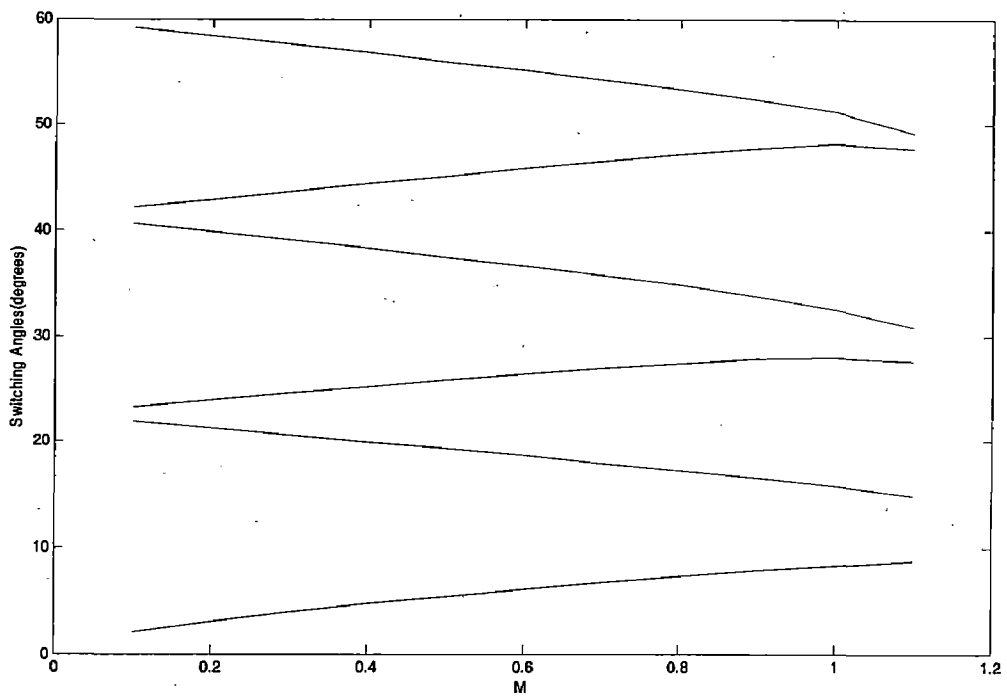


Fig. 4.11 Nature of switching angles for N=6

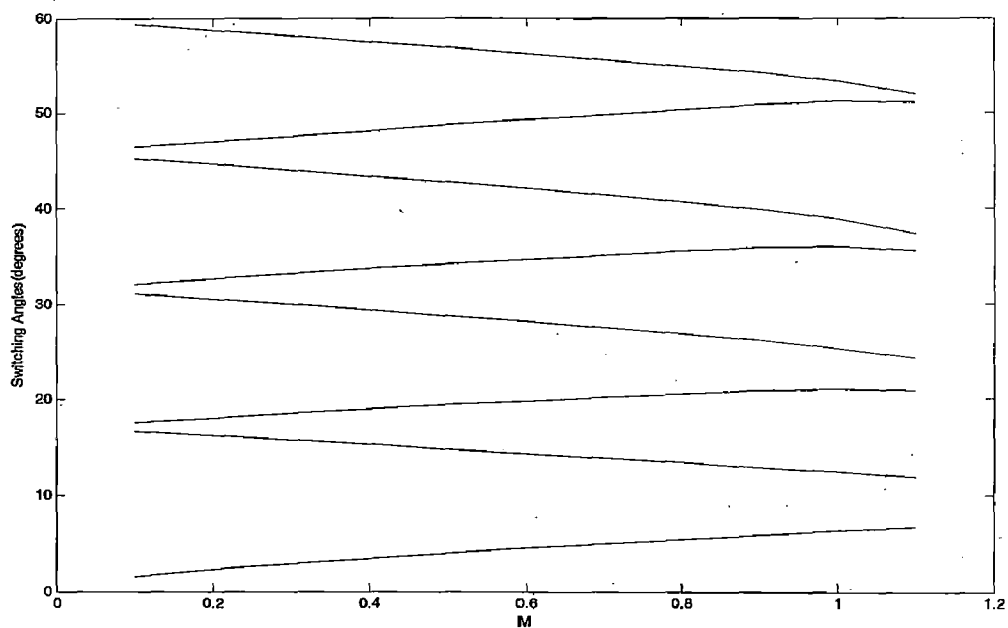


Fig. 4.12 Nature of switching angles for N=8

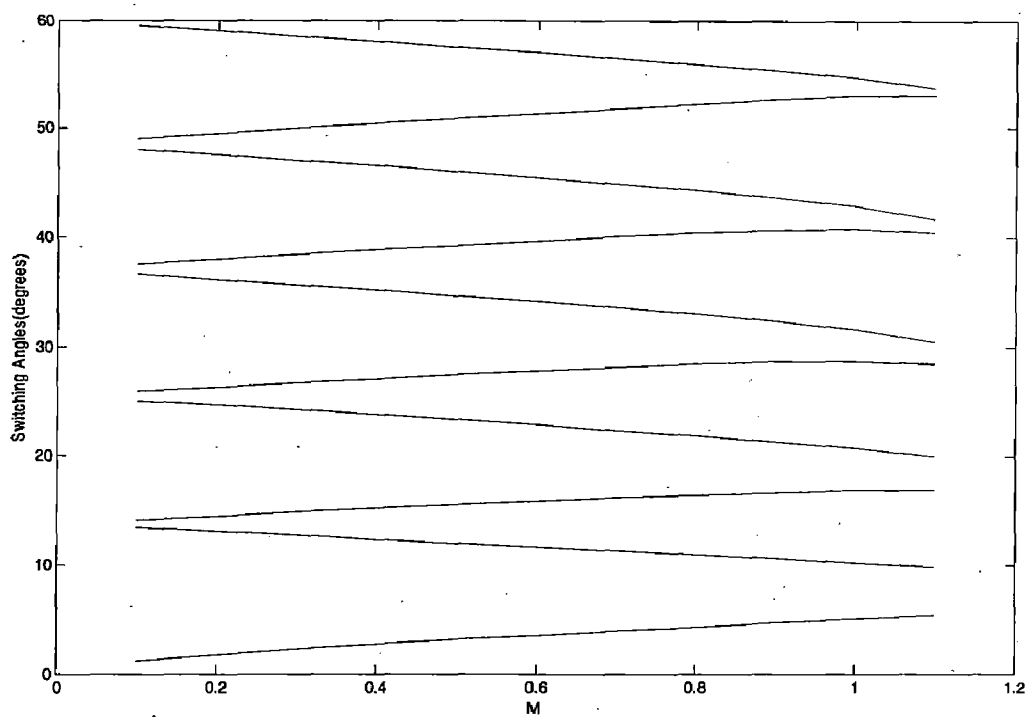


Fig. 4.13 Nature of switching angles for N=10

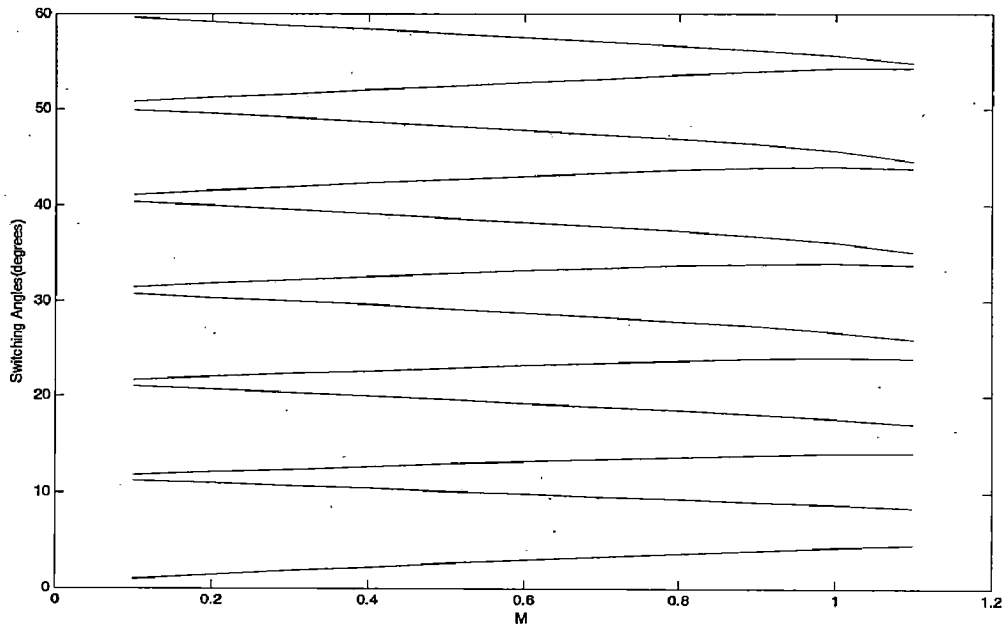


Fig. 4.14 Nature of switching angles for N=12

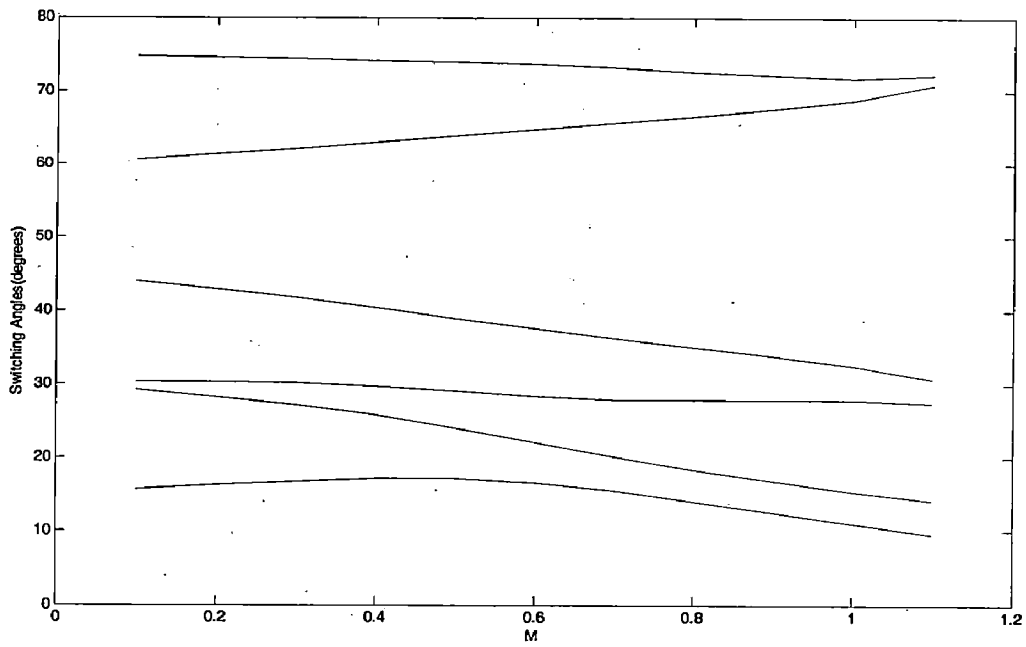


Fig. 4.15 Nature of switching angles (0- 90°) for N=6

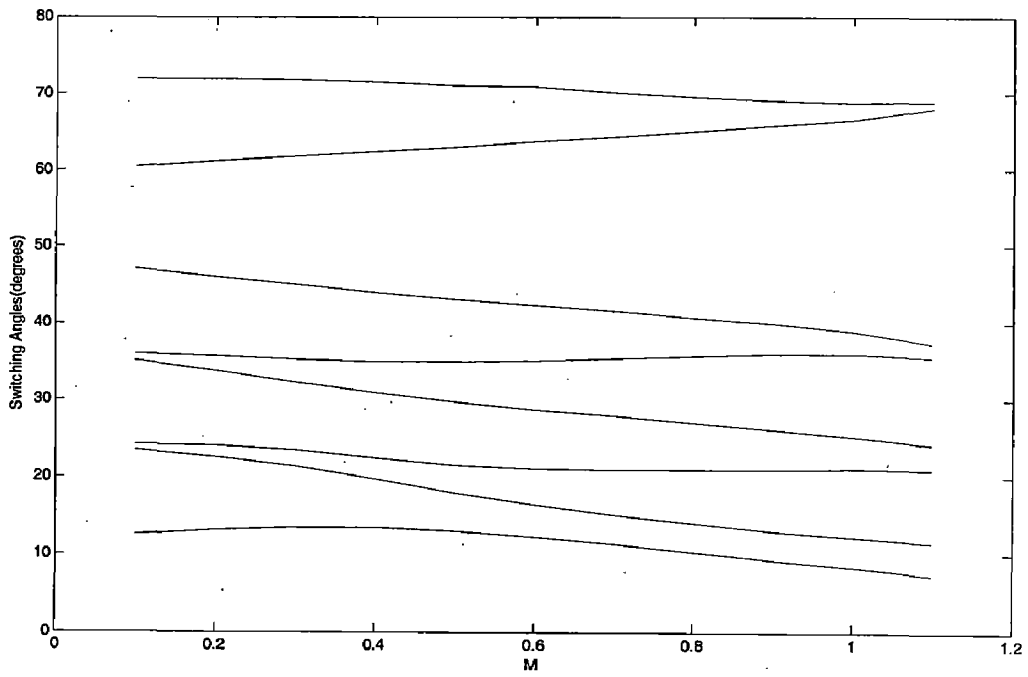


Fig. 4.16 Nature of switching angles (0- 90° ) for N=8

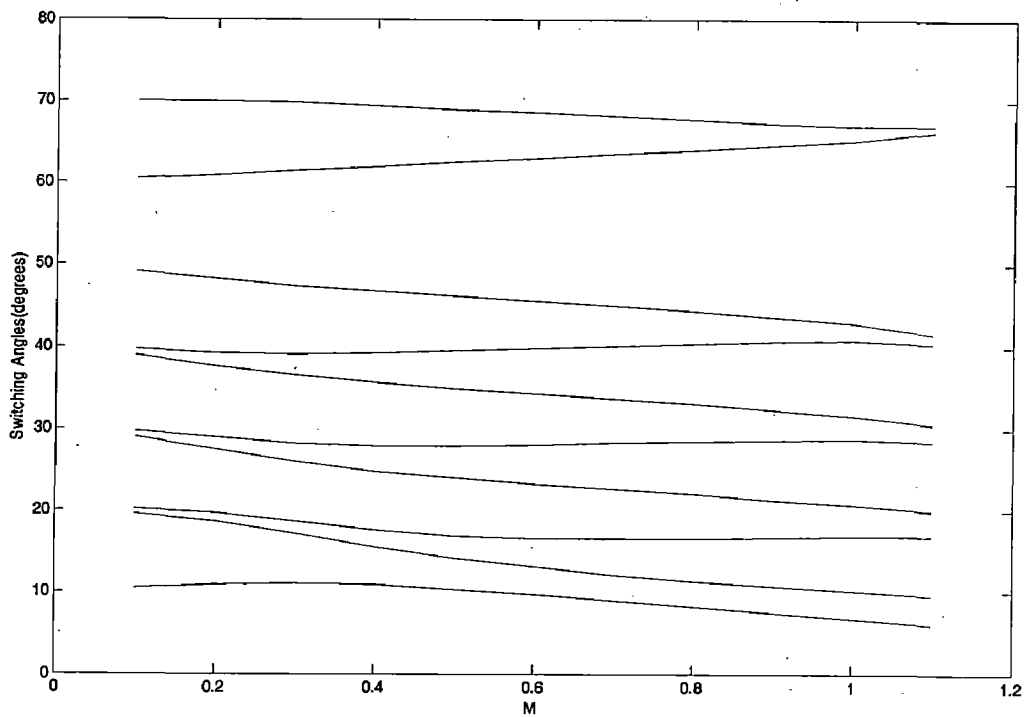


Fig. 4.17 Nature of switching angles (0- 90° ) for N=10

The following observations can be made from the switching angle characteristics for even value of N.

1. At  $M=0$ , the angle  $\alpha_2$  becomes equal to  $\alpha_3$ ,  $\alpha_4$  becomes equal to  $\alpha_5$ , and so on for increasing n with  $\alpha_n$  becoming  $60^\circ$  and these equal pairs are separated by an angle of  $120/N$  at  $M=0$ . This pattern is different for  $0-90^\circ$  solution.
2. They are almost linear in the modulation index range of 0.1 to approximately 0.9 and after that they deviate from the straight line approximation and become non-linear. This pattern is almost non-linear over the entire range of M for the  $90^\circ$  solution.
3. All the odd switching angles have positive slopes with their values increasing with increasing M and the even switching angles have negative slopes with their values decreasing with increasing M.



## IMPLEMENTATION OF THE VOLTAGE SOURCE INVERTER FED INDUCTION MOTOR DRIVE IN MATLAB

### 5.1 BLOCK DIAGRAM OF THE PROPOSED IMPLEMENTATION

The selective harmonic elimination in the voltage source inverter fed Induction Motor drive can be explained through the block diagram shown below. First the optimal switching angles can be obtained through the computer program using the NAG routine E04UCF. The inputs to this program are the number of switching angles, the desired value of the fundamental, and the initial guess to the solution vector such that the linear constraints defined in chapter 4 are satisfied. These optimal angles which are obtained through the program are stored in look-up tables for varying fundamental voltage. These angles are converted into the firing pulses to the inverter switches by a logic switching circuit. The inverter then produces the desired voltages free from the specific chosen eliminated harmonics, which are fed to the three phase induction motor.

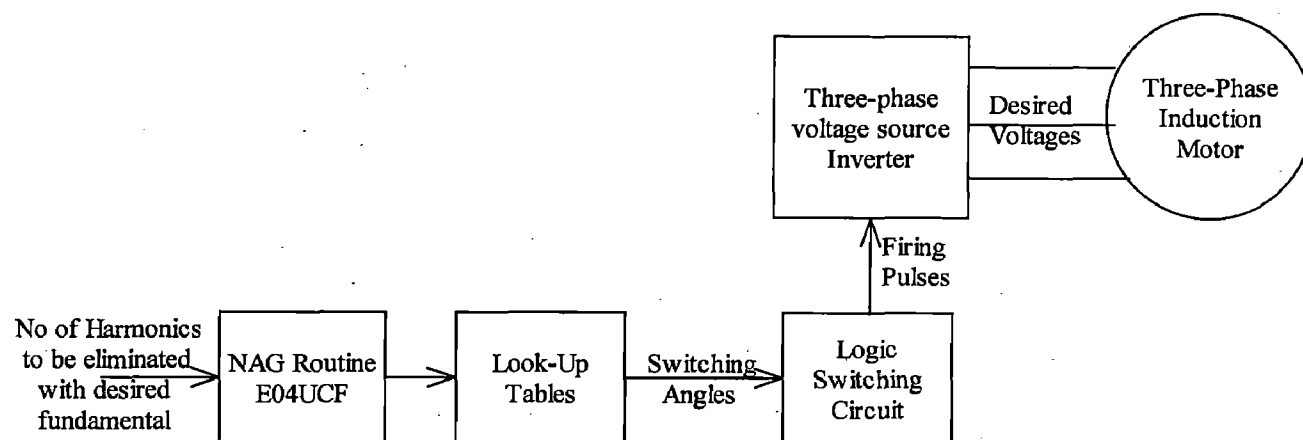


Fig. 5.1 Block Diagram of the Voltage Source Inverter Fed Induction Motor Drive  
with Selective Harmonic Elimination

## 5.2 INDUCTION MACHINE MODEL

The inputs to a squirrel cage induction machine are the three phase voltages, their fundamental frequency and the load torque. The outputs are the three-phase currents, the electrical torque, and the rotor speed. The Induction machine has been modeled in the d-q frame to reduce the complexity and for the ease of calculations. The d-q model requires that all the three-phase variables be converted to two-phase d-q frame. The fundamental equations needed for the development of these modeling equations are given in [18]. These modeling equations needed for developing the simulink model of the squirrel cage induction motor are given in the state space form as follows:

The stator and rotor flux linkage modeling equations are:

$$\frac{d\psi_{qs}}{dt} = \left[ v_{qs} - \frac{R_s}{L_{ls}} \left( \frac{L_{ml}}{L_{lr}} \psi_{qr} + \left( 1 - \frac{L_{ml}}{L_{ls}} \right) \psi_{qs} \right) \right] \quad (5.1)$$

$$\frac{d\psi_{ds}}{dt} = \left[ v_{ds} - \frac{R_s}{L_{ls}} \left( \frac{L_{ml}}{L_{lr}} \psi_{dr} + \left( 1 - \frac{L_{ml}}{L_{ls}} \right) \psi_{ds} \right) \right] \quad (5.2)$$

$$\frac{d\psi_{qr}}{dt} = \left[ -\omega_r \psi_{dr} - \frac{R_r}{L_{lr}} \left( \frac{L_{ml}}{L_{ls}} \psi_{qs} + \left( 1 - \frac{L_{ml}}{L_{lr}} \right) \psi_{qr} \right) \right] \quad (5.3)$$

$$\frac{d\psi_{dr}}{dt} = \left[ -\omega_r \psi_{qr} - \frac{R_r}{L_{lr}} \left( \frac{L_{ml}}{L_{ls}} \psi_{ds} + \left( 1 - \frac{L_{ml}}{L_{lr}} \right) \psi_{dr} \right) \right] \quad (5.4)$$

where

$\psi_{qs}, \psi_{ds}$  are the d-axis and q-axis stator flux-linkages

$\psi_{qr}, \psi_{dr}$  are the d-axis and q-axis rotor flux linkages

$v_{qs}, v_{ds}$  are the q and d-axis stator voltages

$L_{ls}, L_{lr}, L_m$  are the stator, rotor leakage inductances and Mutual inductance

$R_s, R_r$  are the stator and rotor resistances.

The reference currents in the stationary reference frame which are in the two-phase d-q frame are converted to three phase currents by using the transformation equations given by:

$$i_a = i_{ds} \quad (5.13)$$

$$i_b = (-1/2)i_{ds} + (\sqrt{3}/2)i_{qs} \quad (5.14)$$

$$i_c = (-1/2)i_{ds} - (\sqrt{3}/2)i_{qs} \quad (5.15)$$

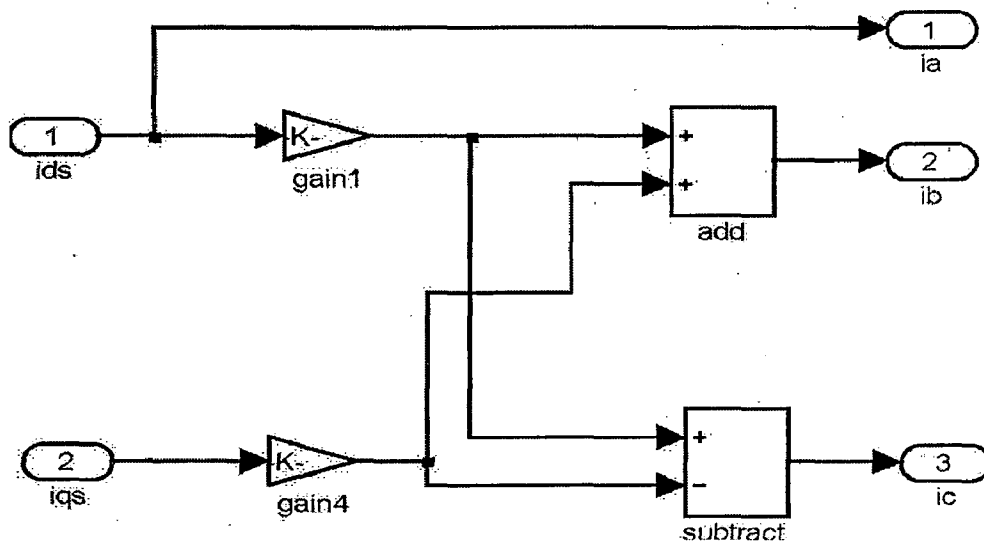


Fig. 5.2 dq-abc conversion

The Matlab Simulink model of the induction motor with the above modeling equations implemented is as shown in fig. 5.3



## 5.2 PWM INVERTER

The pulses to this inverter are obtained from the logic switching circuit as shown in the next section. The output voltage from the inverter is controlled by varying the width of the output pulses which is possible by the variation of the modulation index. The terminal voltages in terms of the switching pulses which are obtained from the logic switching circuit are given by:

$$V_a = (V_{dc}/3)(2S_a - S_b - S_c) \quad (5.16)$$

$$V_b = (V_{dc}/3)(S_a - 2S_b - S_c) \quad (5.17)$$

$$V_c = (V_{dc}/3)(S_a - S_b - 2S_c) \quad (5.18)$$

where  $S_a$ ,  $S_b$ ,  $S_c$  are the switching pulses to the upper switching devices of the inverter.

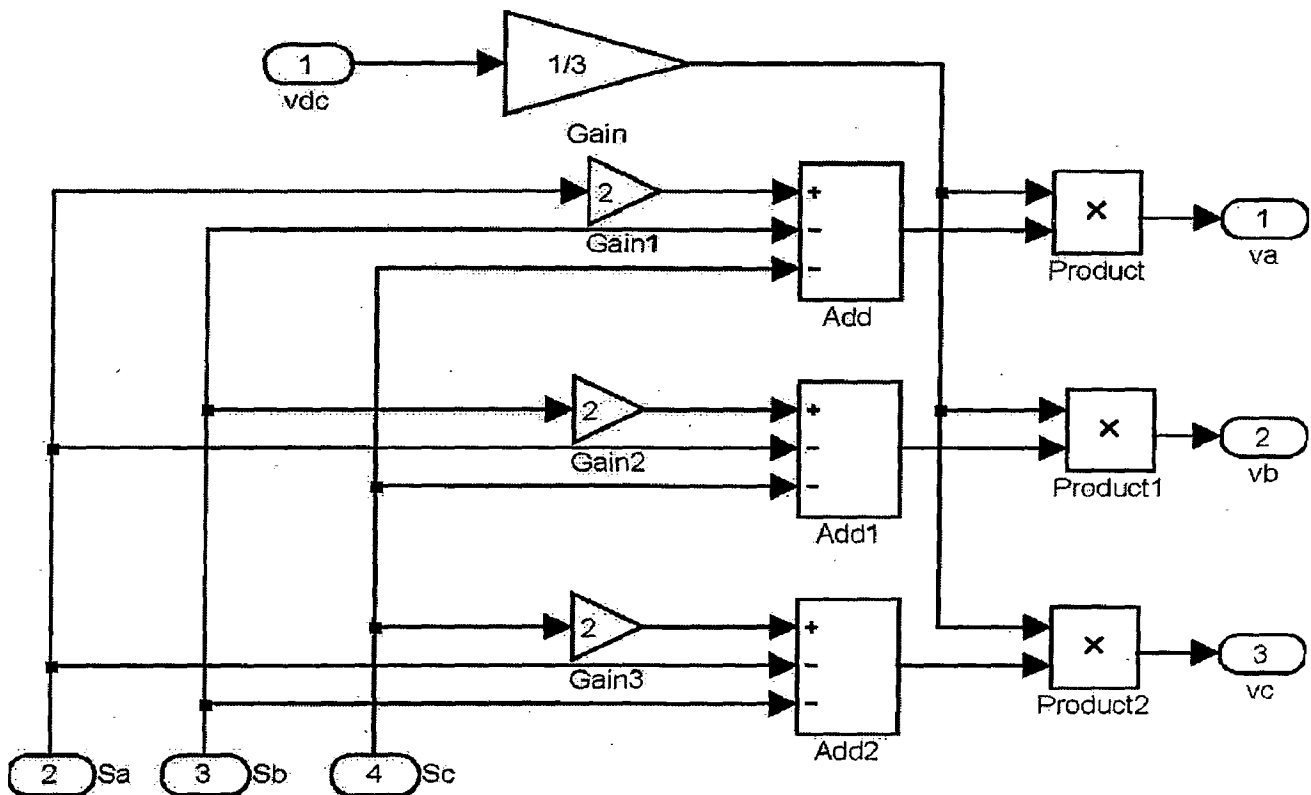


Fig. 5.4 PWM Inverter

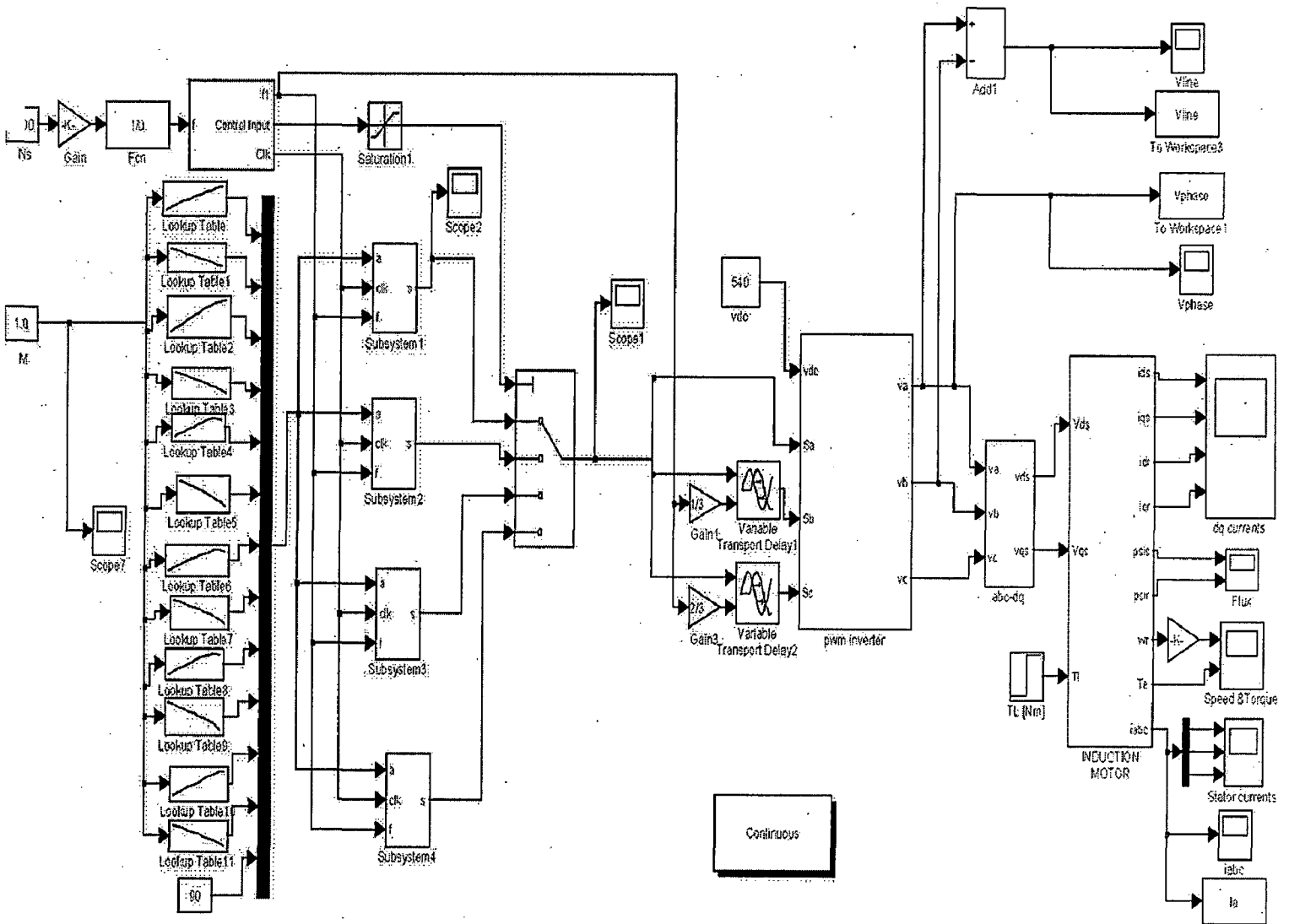


Fig. 5.7 Block diagram of the VSI fed Induction Motor Drive with Selective Harmonic Elimination

The switching angles which are obtained from the NAG routine E04UCF are stored in the look up tables which gives the appropriate switching angles for the respective modulation index value. These switching angles are then applied to the logic switching XOR gates as seen in the subsystem. It produces an output of the staircase type with each raising edge corresponding to the appropriate switching angle. These are generated for the full cycle by appropriate selection of the switching angles such that the half-wave and quarter wave symmetry are imbibed in the resulting output waveform. By applying this XOR gate output to the multiport switch and with proper switch selection, the resulting pulses to the inverter are obtained such that desired fundamental voltage is

obtained and the selected number of harmonics is eliminated. So, the output from the inverter is free from the selected number of harmonics which results in the three phase PWM voltages, which are then fed to the three-phase induction motor. The Matlab simulink model of the selective harmonic eliminated VSI fed Induction Motor with all these features incorporated is shown in fig. 5.7.

### **5.5 CLOSED LOOP SPEED CONTROL OF VOLTAGE SOURCE INVERTER FED INDUCTION MOTOR DRIVE USING SELECTIVE HARMONIC ELIMINATION TECHNIQUE**

The closed loop speed control of this drive has been done with the reference speed as its input. Here, the reference speed, which is the commanded value, is compared with the actual speed and the error is fed to a PI controller, which chooses the appropriate switching angles from the look-up table depending on the Modulation Index value such that the desired fundamental voltage is applied to the induction motor to reach the desired speed. The other PI controller generates the required frequency such that the voltage and frequency control is obtained. This process continues till the desired speed is obtained from the Induction Motor. The simulink model showing the various blocks used for this implementation is as shown in fig 5.8.

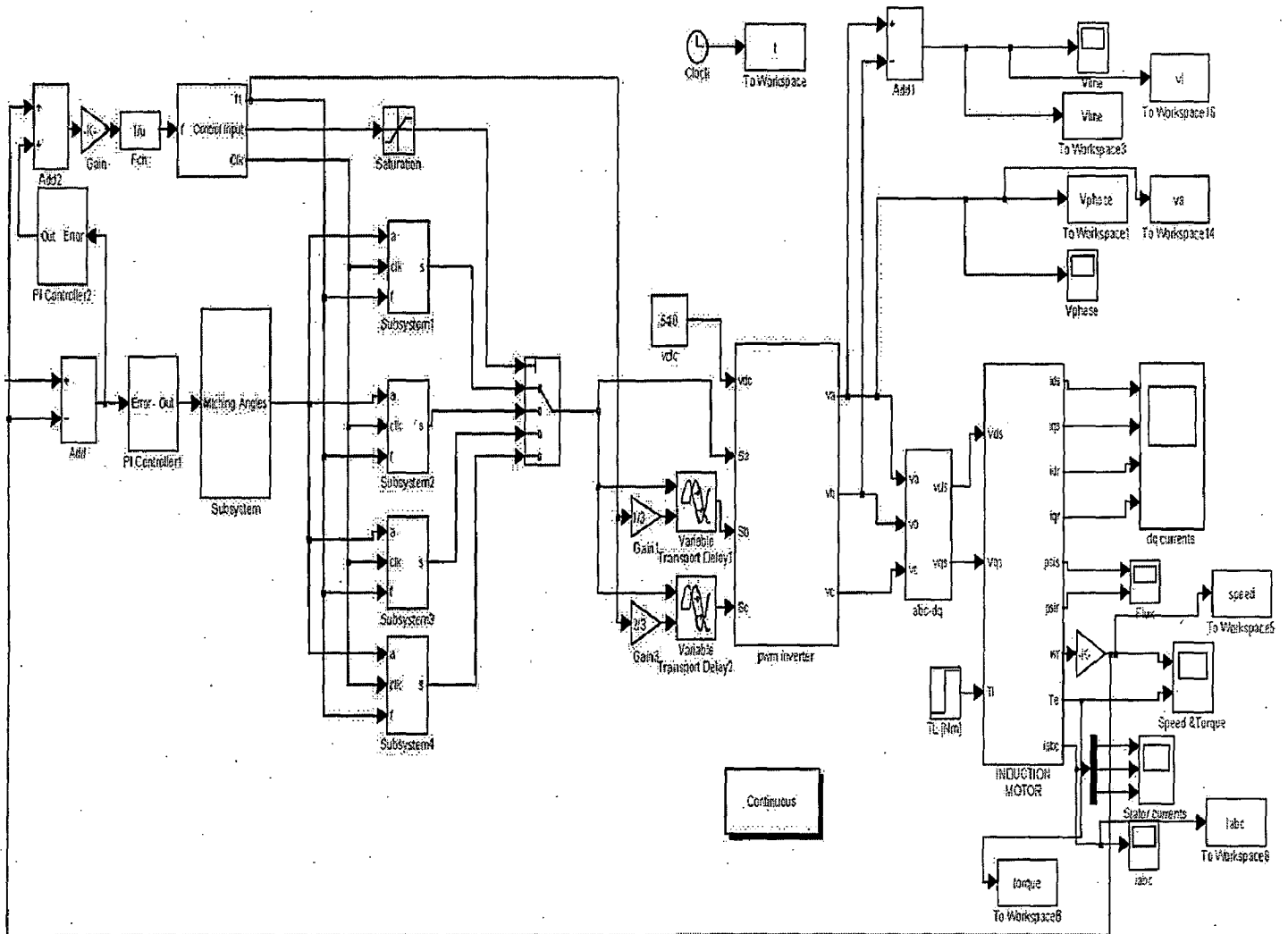


Fig. 5.8 Closed Loop speed Control of a VSI fed Induction Motor Drive Using the Selective Harmonic Elimination Technique.



RESULTS AND DISCUSSIONS

For the analysis of these results, an Induction Motor with the specifications as given in Appendix has been used. The selective harmonic elimination technique is implemented in Matlab/Simulink, which is the most popular and powerful tool for simulation and the various waveforms are plotted which are shown in this chapter. Figures 6.1 to 6.16 show the waveforms related to the total harmonic distortion. Here N is the number of switching angles.

Elimination of 5<sup>th</sup> and 7<sup>th</sup> Harmonics for N = 3:

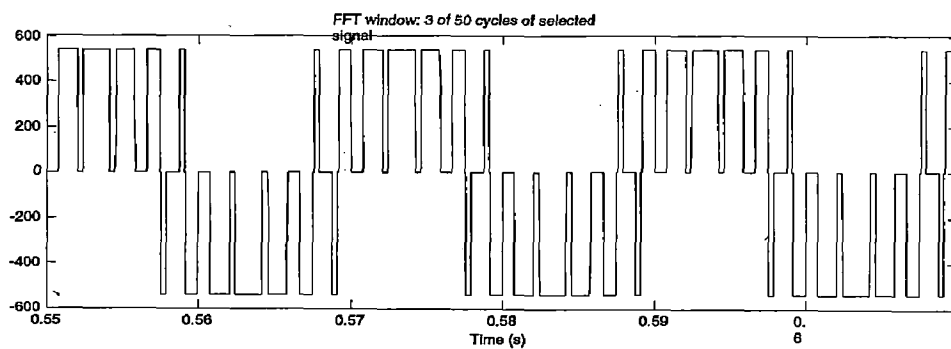


Fig. 6.1 Line voltage waveform

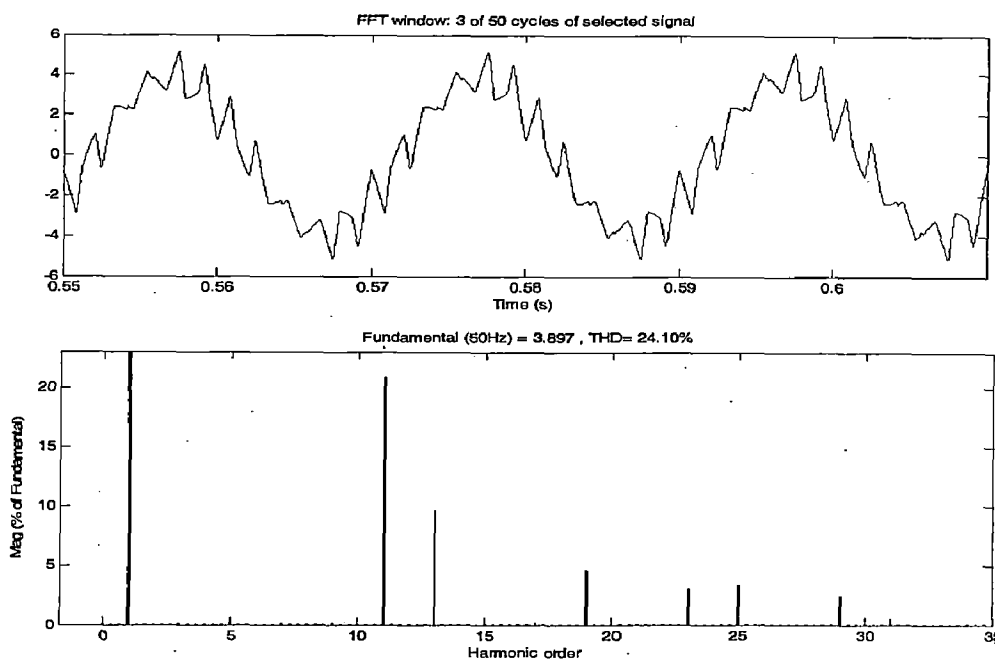


Fig. 6.2 Line current waveform and its harmonic spectra

### Elimination of 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup> Harmonics for N = 5:

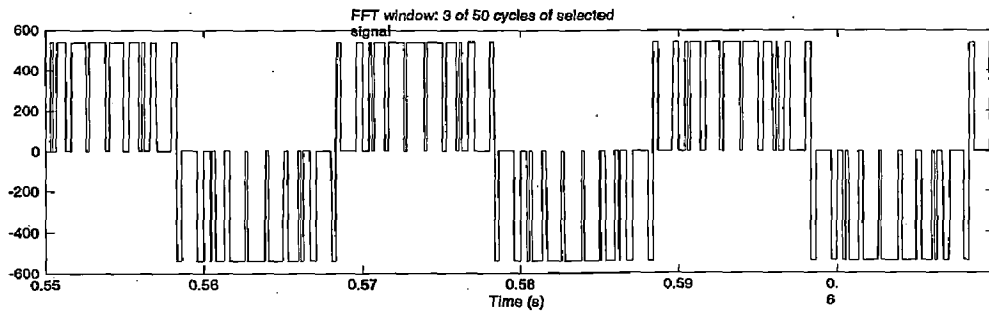


Fig. 6.3 Line voltage waveform

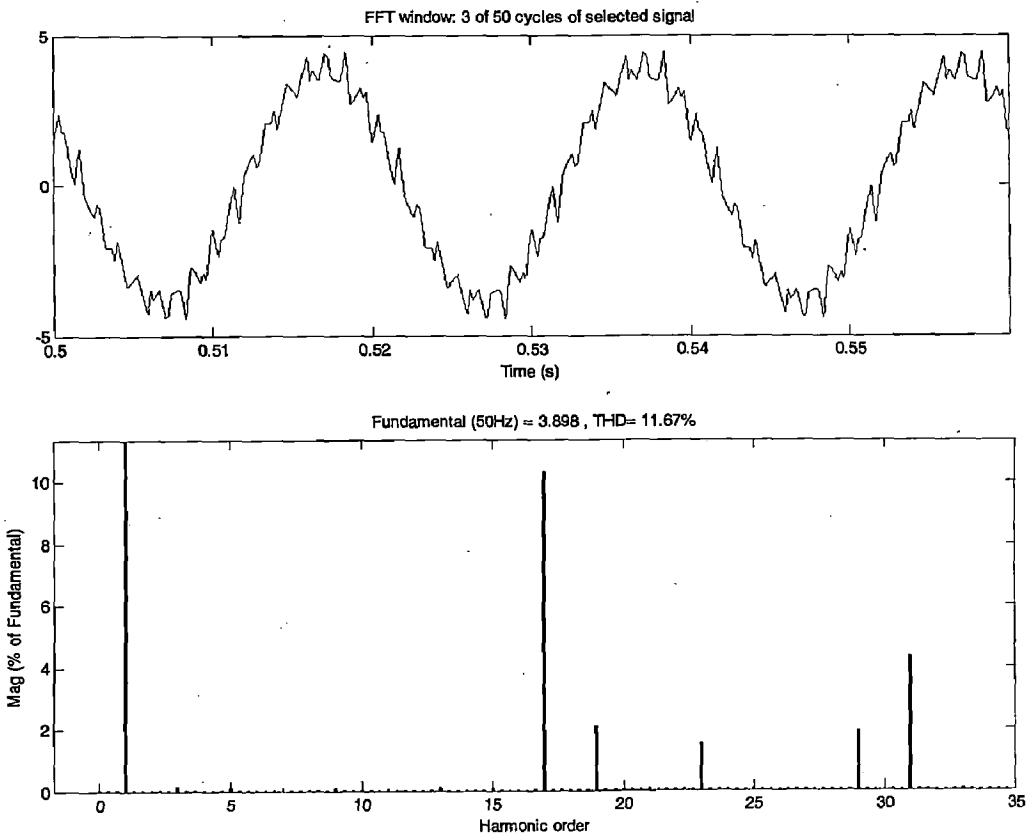


Fig. 6.4 Line current waveform and its harmonic spectra

Elimination of 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup>, 17<sup>th</sup>, 19<sup>th</sup>, 23<sup>rd</sup> Harmonics for N = 8:

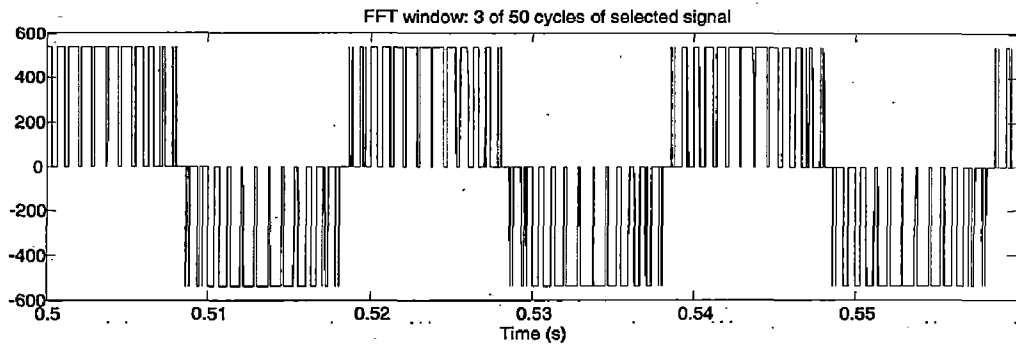


Fig. 6.5 Line voltage waveform

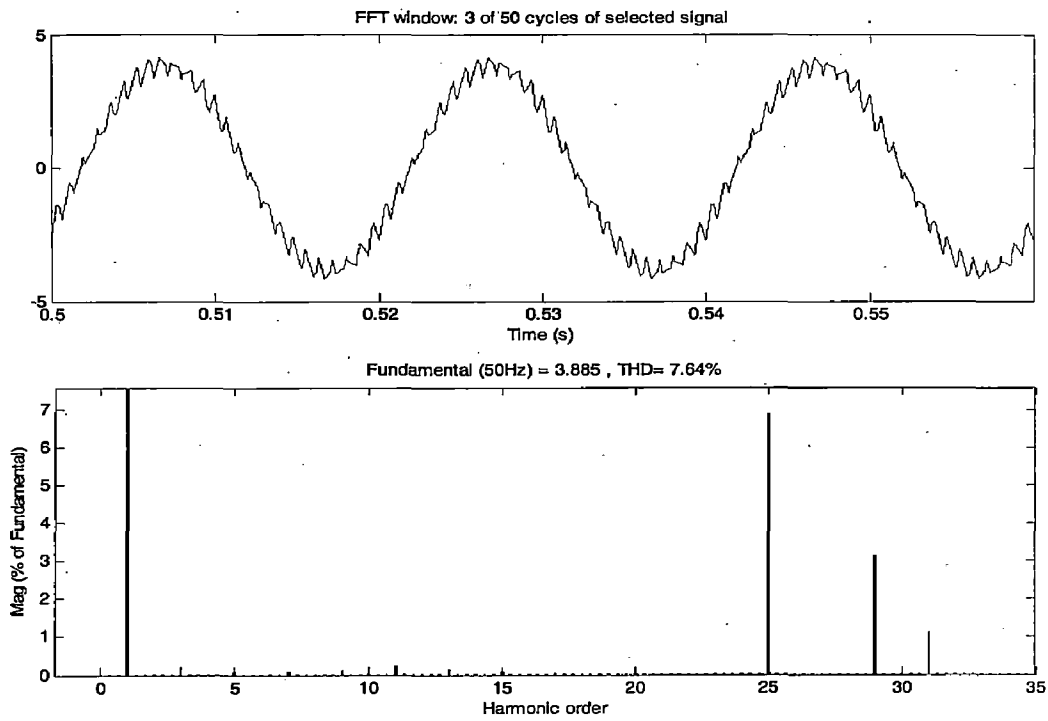


Fig. 6.6 Line current waveform and its harmonic spectra

Elimination of 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup>, 17<sup>th</sup>, 19<sup>th</sup>, 23<sup>rd</sup>, 25<sup>th</sup>, 29<sup>th</sup> Harmonics for N = 10:

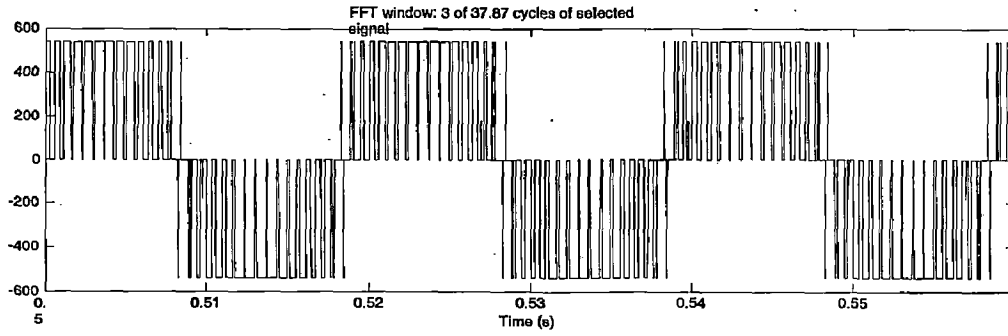


Fig. 6.7 Line voltage waveform

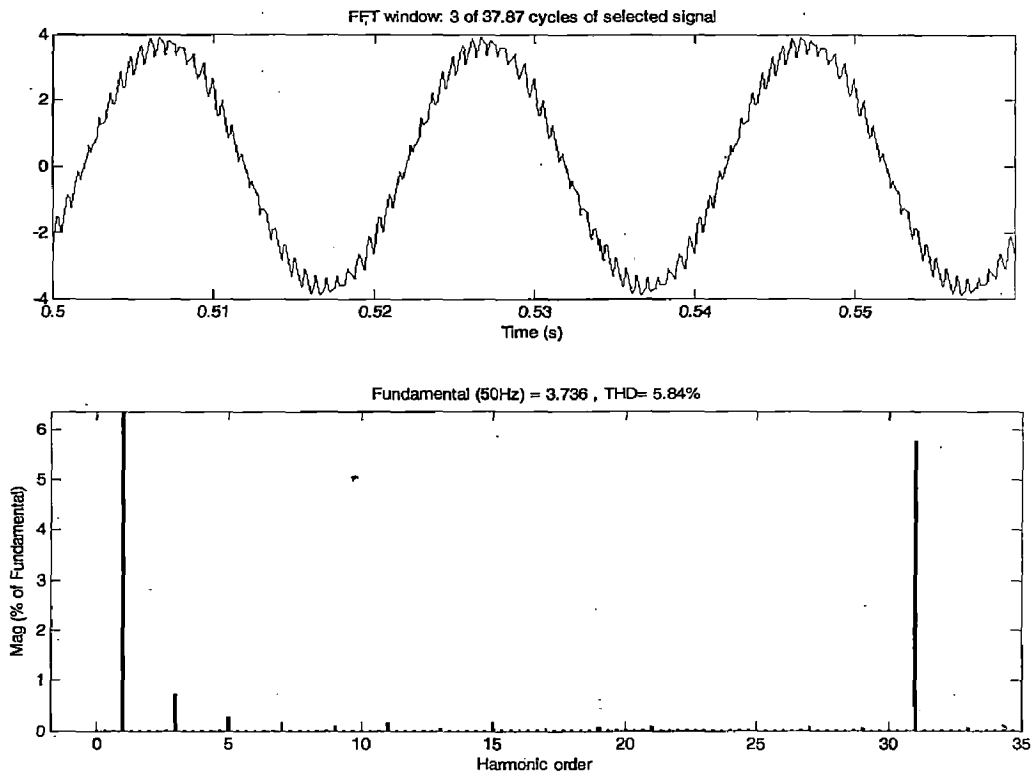


Fig. 6.8 Line current waveform and its harmonic spectra

Elimination of 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup>, 17<sup>th</sup>, 19<sup>th</sup>, 23<sup>rd</sup>, 25<sup>th</sup>, 29<sup>th</sup>, 31<sup>th</sup>, 35<sup>th</sup> Harmonics for N = 12:

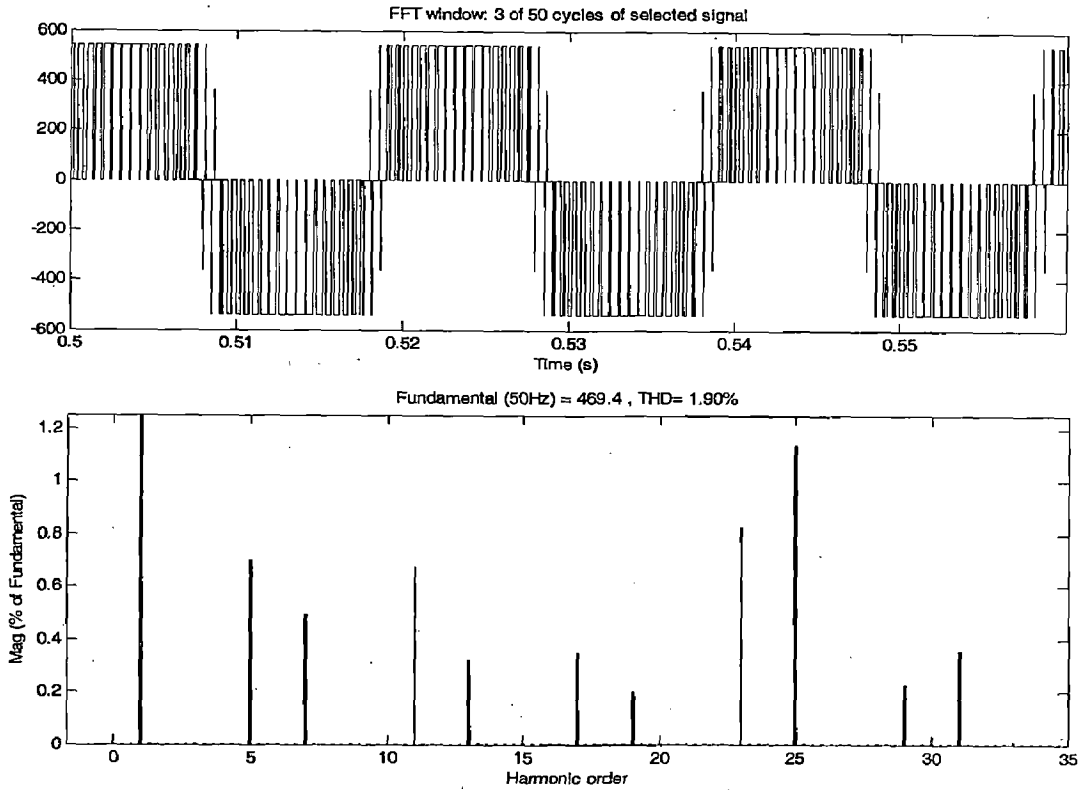


Fig. 6.9 Line voltage waveform and its harmonic spectra

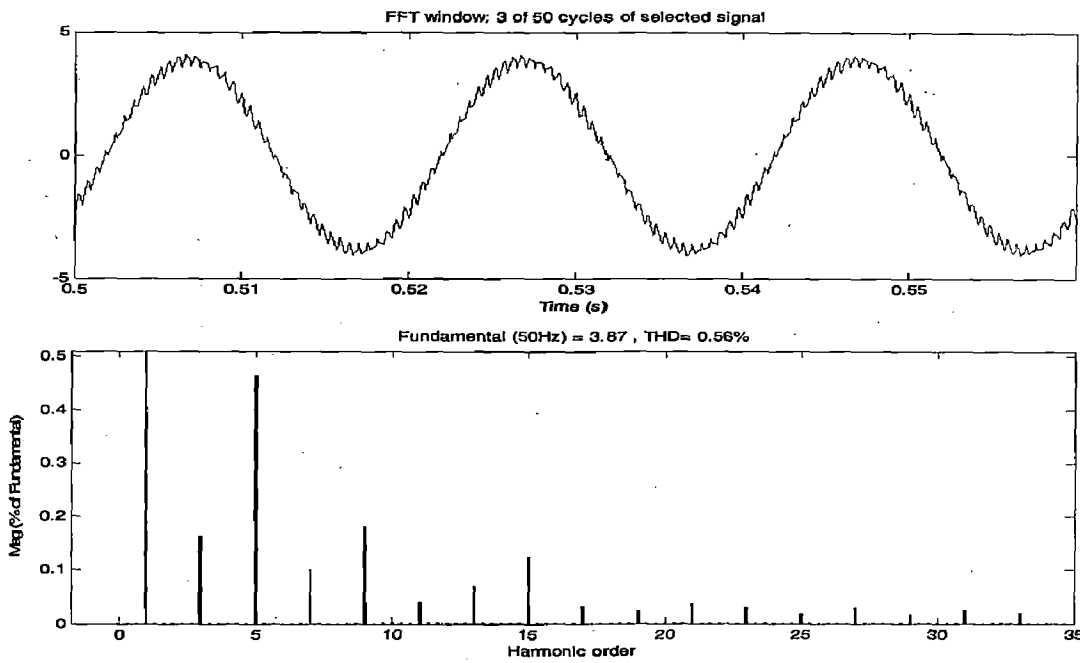


Fig. 6.10 Line current waveform and its harmonic spectra

From the plots of figures 6.1 to 6.10, it can be seen that the total current harmonic distortion reduces with the increase in the number of harmonics eliminated, which can be observed from the graph shown below (fig.6.11).

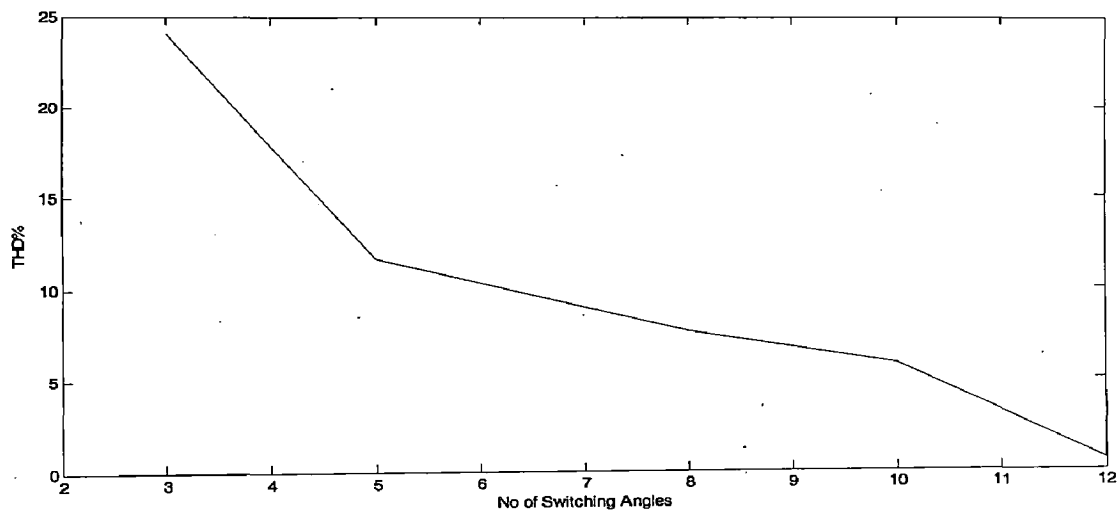


Fig. 6.11 Line Current THD variation with number of switching angles

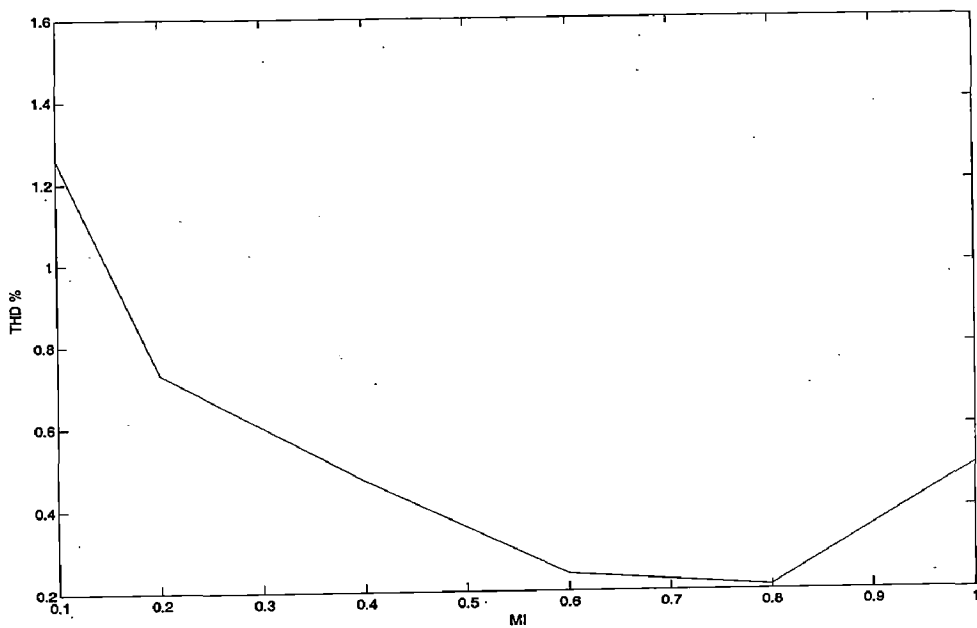


Fig. 6.12 Current harmonic distortion v/s M for N=8



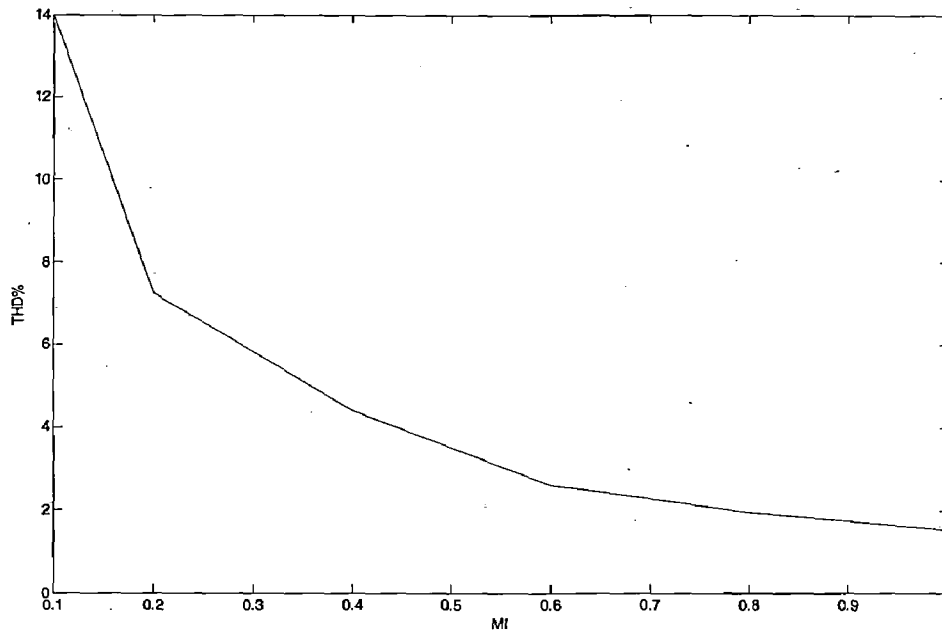


Fig. 6.13 Voltage harmonic distortion v/s M for N=8

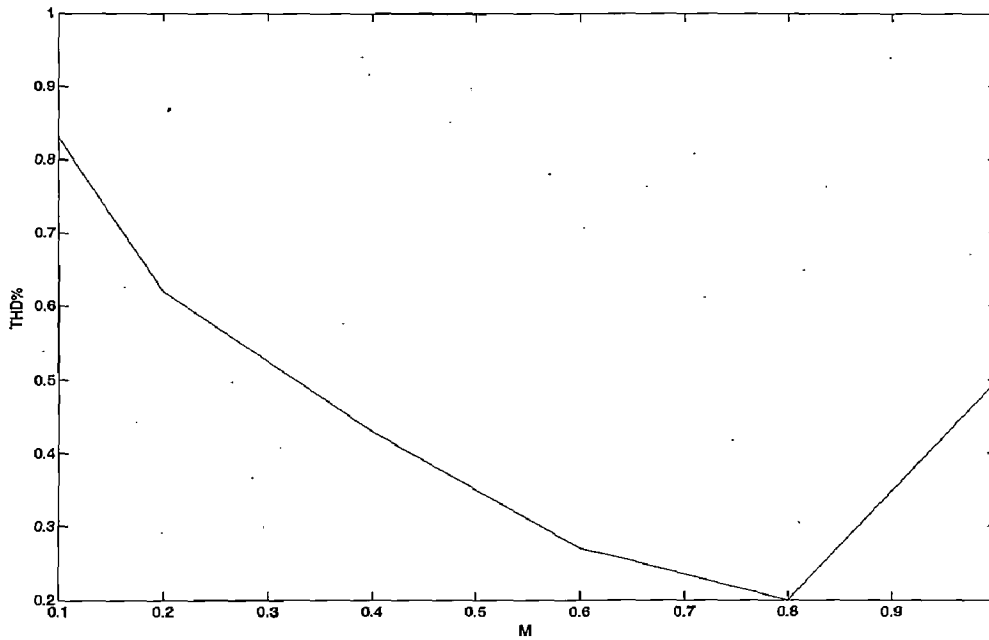


Fig. 6.14 Current harmonic distortion v/s M for N=12



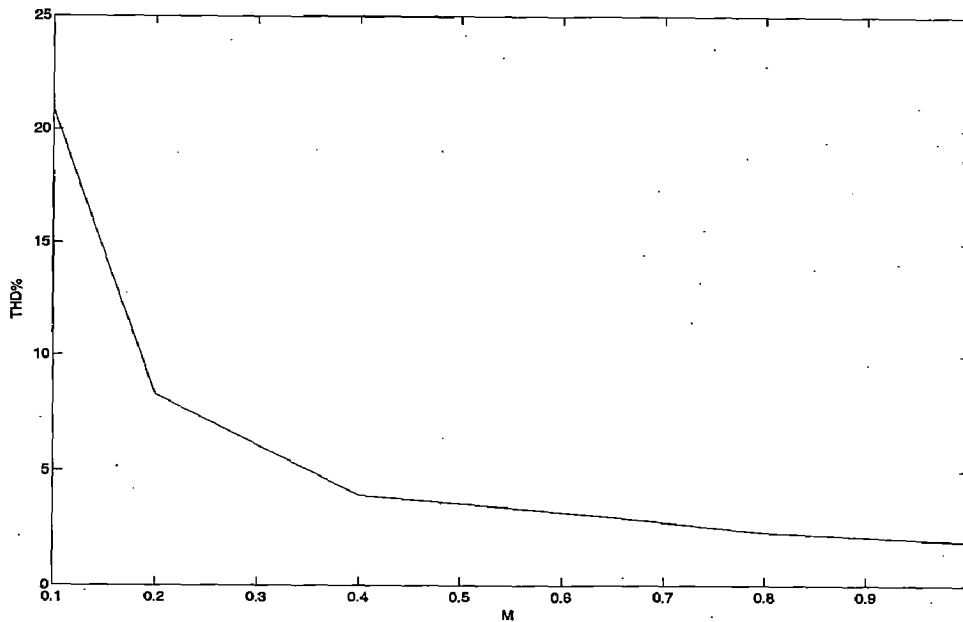


Fig. 6.15 Voltage harmonic distortion v/s M for N=12

From the graphs of figures 6.11 to 6.15, it can be observed that the total harmonic distortion increases for lower values of the modulation index.

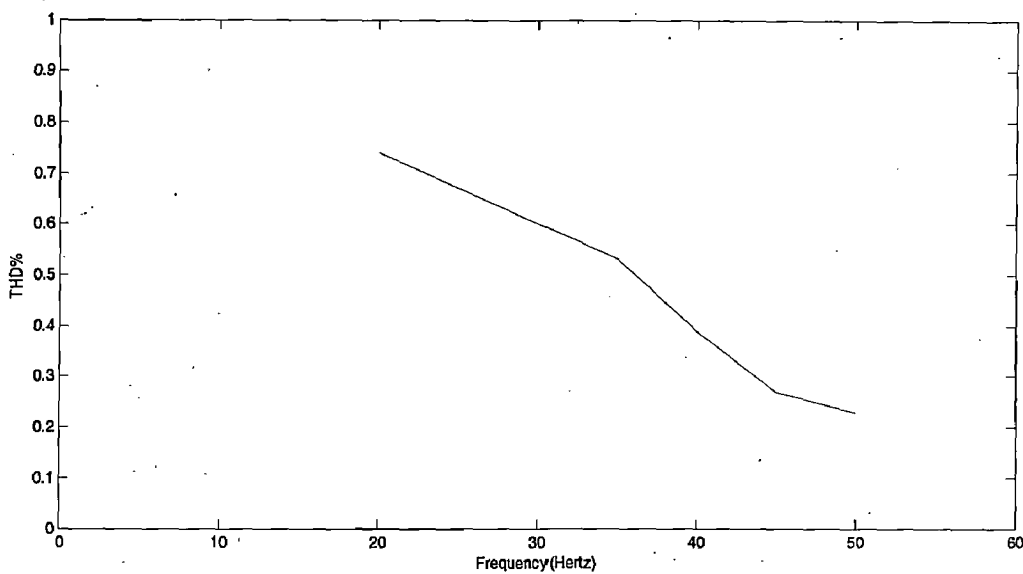


Fig. 6.16 Line Current THD variation with Frequency (N=12)

From the above graph (fig.6.16), which has been drawn for the total harmonic distortion with varying frequency, it can be seen that the THD increases for lower values of frequencies.

The waveforms shown below from figures 6.17 to 6.21 are for the closed loop speed control of the induction motor drive using selective harmonic elimination for the considered cases. Figures 6.17 to 6.20 are the various waveforms of the drive for a constant speed of 1200rpm and a change in load torque from 1 to 2 N-m applied at one second. Figure 6.17 shows the Speed and Torque responses for a change in load torque from 1N-m to 2N-m.

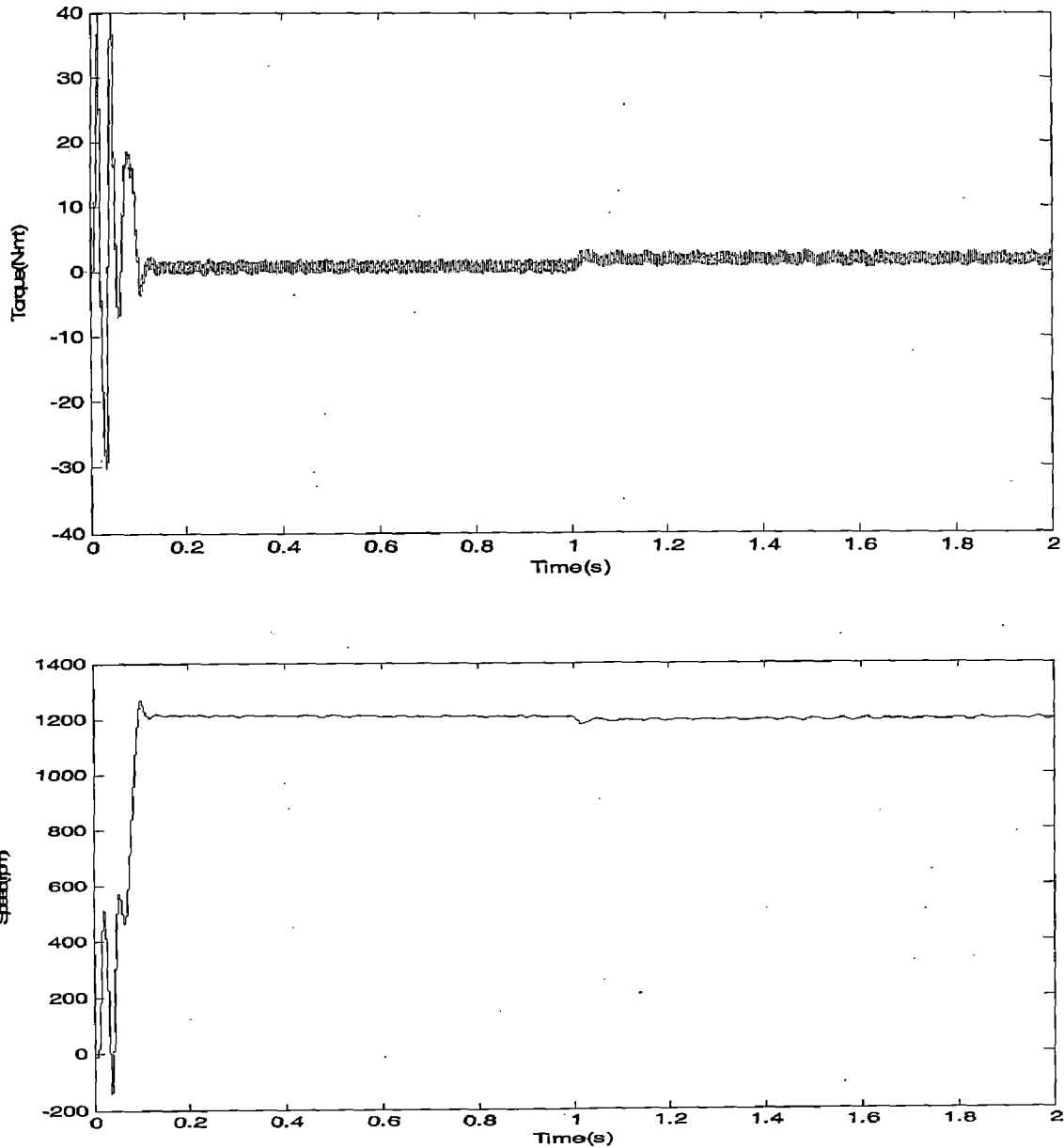


Fig.6.17 Speed and Torque Response of the induction motor

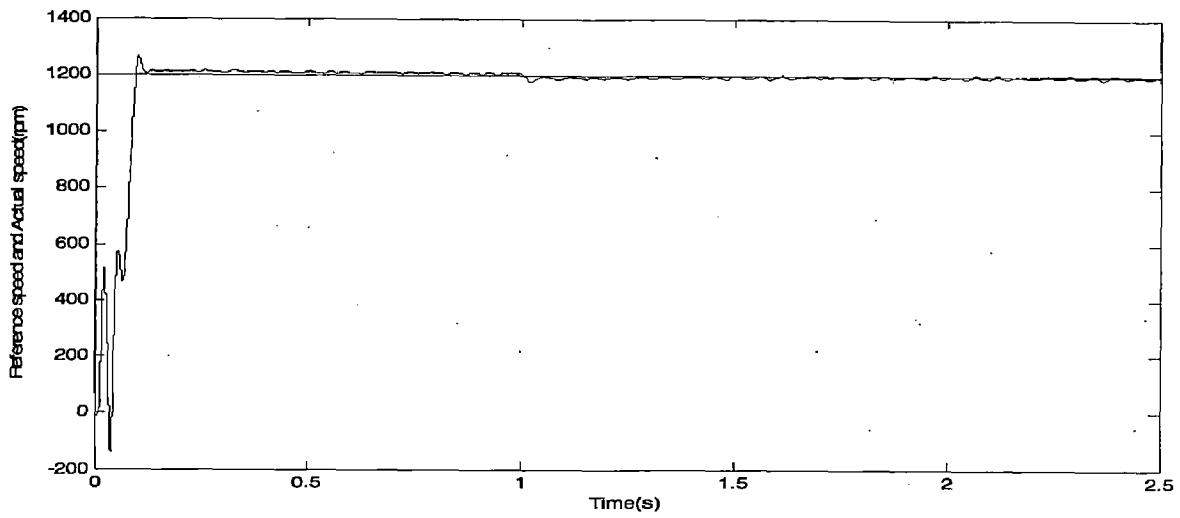


Fig.6.18 Comparison of Reference and Actual speed

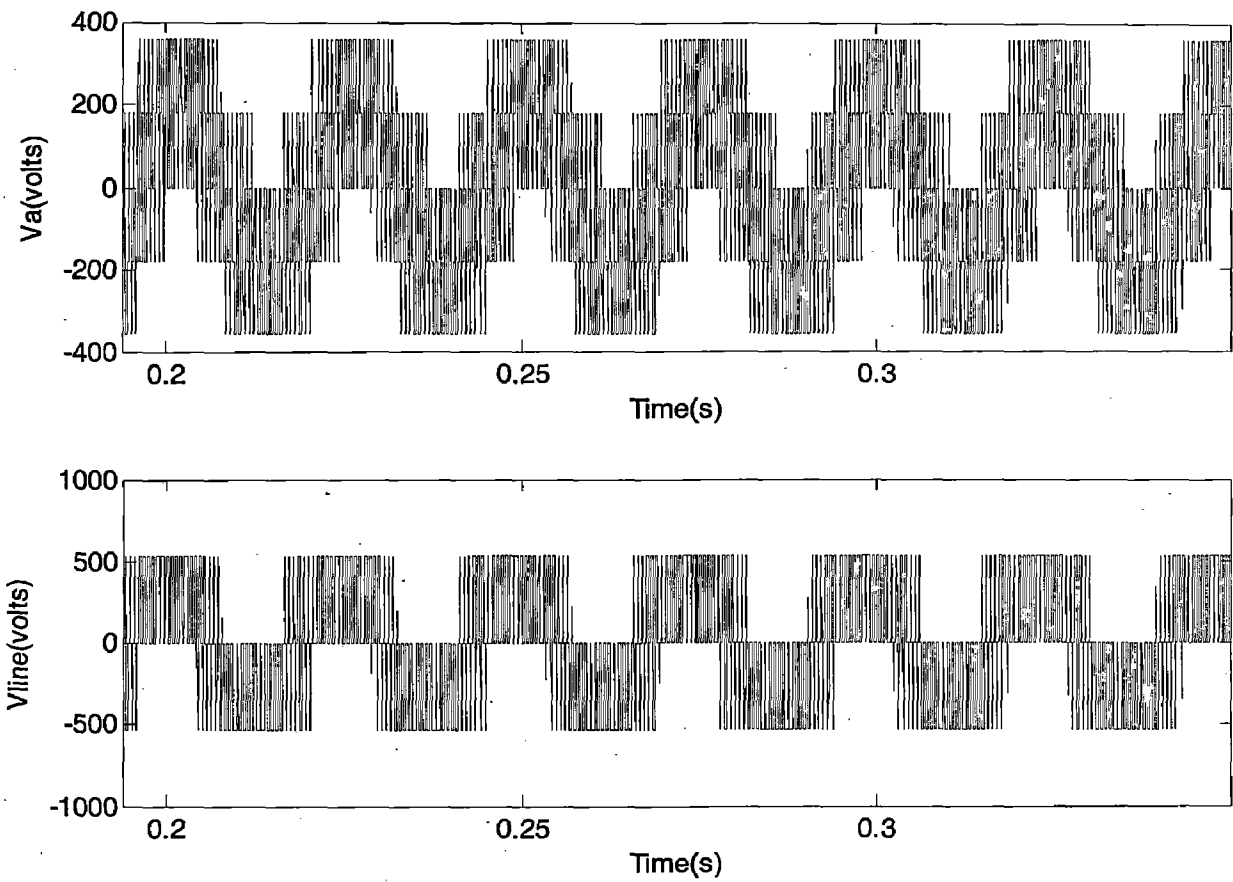


Fig.6.19 Phase and Line voltages waveforms of VSI

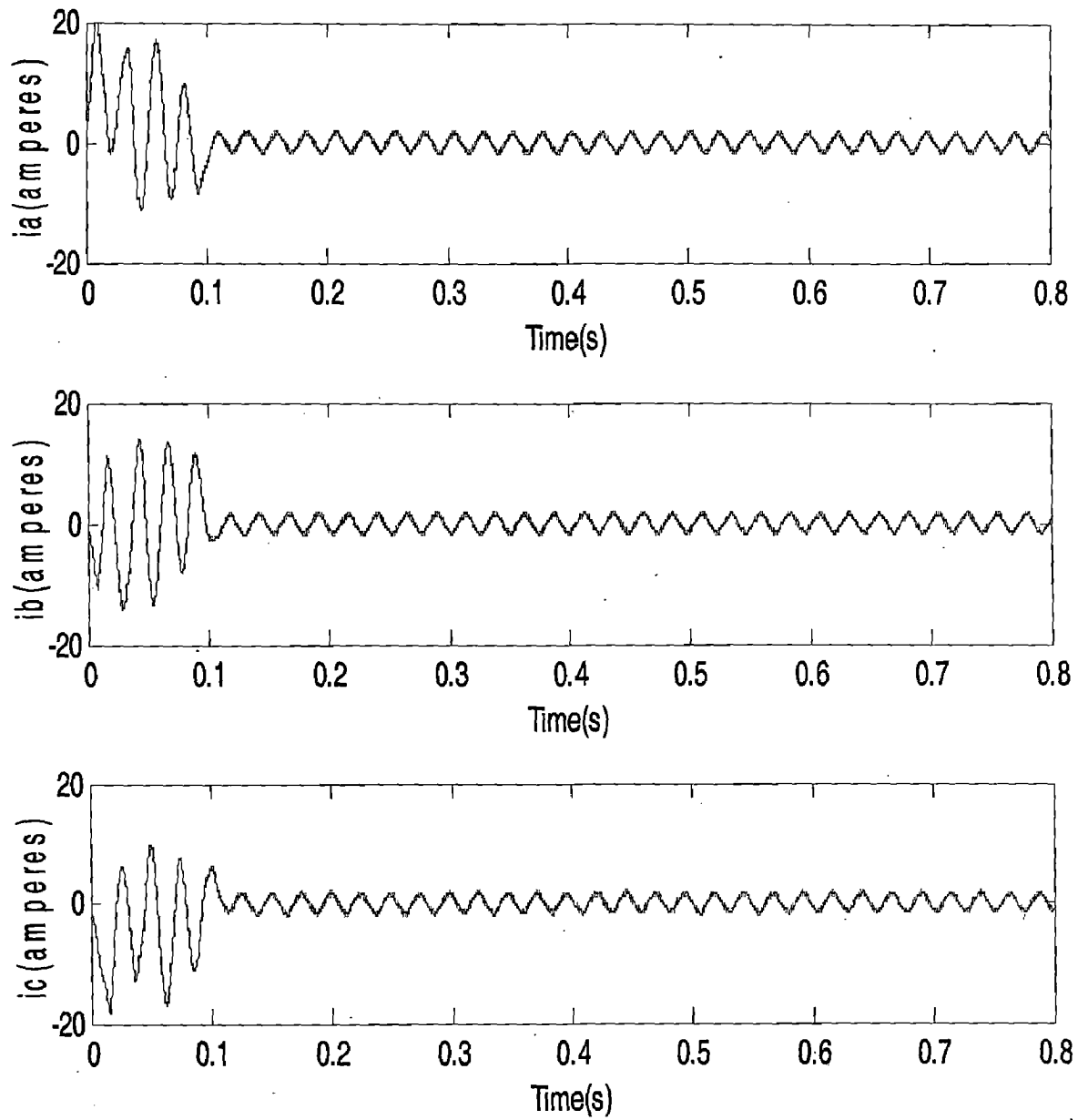


Fig.6.20 Three Phase Currents ( $i_a, i_b, i_c$ ) of the Induction Motor

Fig 6.21 shows the Speed and Torque Response of the induction motor for a change in speed from 1400 rpm to 1200rpm on no-load

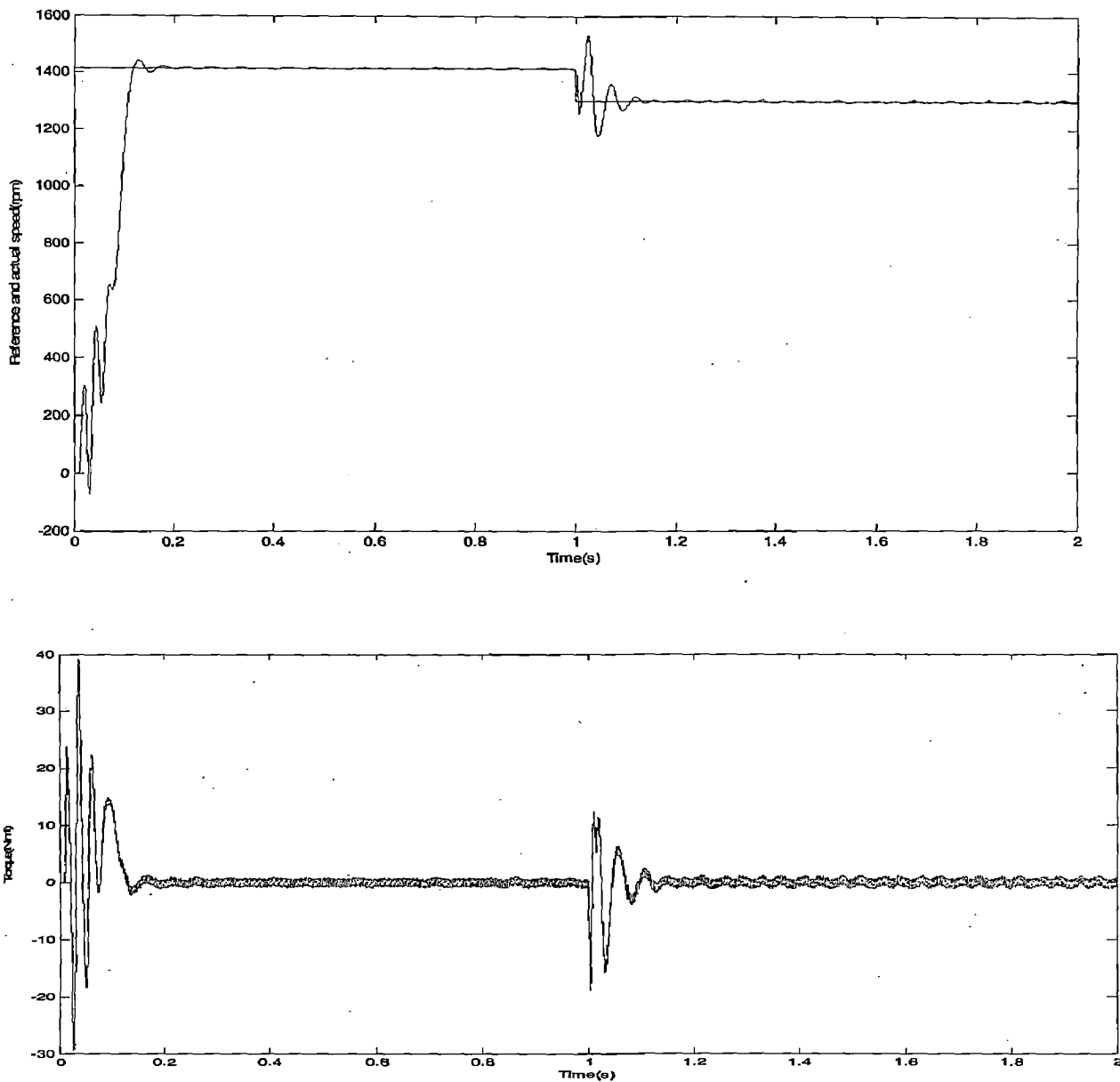


Fig.6.21 Speed and Torque Response for a sudden change in speed

It can be seen from the above figure that as the speed changes suddenly, there is an increase in the torque value which can be avoided by giving a gradual change in speed.

**CONCLUSIONS AND SCOPE FOR FURTHER WORK**

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In present work, the selective harmonic elimination for the voltage source inverter fed induction motor drive has been done. Programmed PWM techniques provide solutions regarding the switching angles that eliminate a chosen number of harmonics while maintaining the desired fundamental voltage. The main challenge associated with these techniques is to obtain the analytical solutions of nonlinear transcendental equations that contain trigonometric terms which exhibit multiple solutions.

Here, NAG Library Routine E04UCF has been used to solve the non-linear transcendental equations. A computer program in FORTRAN has been developed to solve these equations. It gives  $N$  switching angles needed to eliminate the  $N-1$  harmonics. The nature of these switching angles with varying modulation index has been plotted for both (0-60) and (0-90) solutions for even as well as odd  $N$ . These switching angles which are calculated offline for the bipolar PWM are stored in look-up tables. Using these look-up tables, a digital switching circuit has been developed which supplies the firing pulses to the inverter switching devices. So, the inverter produces the desired voltages with chosen harmonics eliminated which are fed to the three-phase induction motor drive.

By doing the FFT analysis, it has been observed that the specified harmonics are eliminated from the voltage and current waveforms. From the harmonic spectra of these waveforms, it has been observed that as the value of  $N$  increases the total current harmonic distortion decreases. The plot of THD with varying modulation index has been done which shows an increasing THD for lower values of modulation index. Also, by applying voltage to frequency control, it has been observed that the THD is increasing with decreasing frequency. The speed and torque responses for the closed loop control of the induction motor drive are obtained.

Here, the selective harmonic elimination technique has been applied using off-line solutions. Using the same technique, it is better to have the online control of the SHE, in line with the research work of S.R.Bowes [20] to reduce the storage space required for storing switching angles. The closed loop response of the drive has to be improved especially in terms of torque ripple which can be extended in the future scope of work. Further, Unipolar PWM generation should be explored for achieving better results.

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## APPENDIX

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### Induction Motor specifications

Type: 3 Phase Star-connected Squirrel cage Induction Motor

Rated Power Input = 1.1 KW

Rated Current = 2.77 Amps

Voltage ( $V_{L-L (rms)}$ ) = 415volts

Frequency = 50Hz

Rated Speed=1415 RPM

Pairs of pole pairs,  $p = 2$

Stator Resistance  $R_s = 6.03$  ohms

Stator Leakage Inductance  $L_{ls} = 29.9$ mH

Rotor Resistance  $R_r = 6.085$  ohms

Rotor Leakage Inductance  $L_{lr} = 29.9$ mH

Mutual Inductance  $L_m = 489.3$ mH

Moment of Inertia  $J = 0.00488$ Kg.m<sup>2</sup>.

## 5.4 Design of Firing Circuit

For the pulse generation the switching angles which are stored in the look-up tables are used which by implementing through the logic circuit produces the switching pulses to the three-phase inverter legs. The model below shows the pulse generation for the first quarter cycle of the output waveform in the fundamental period.

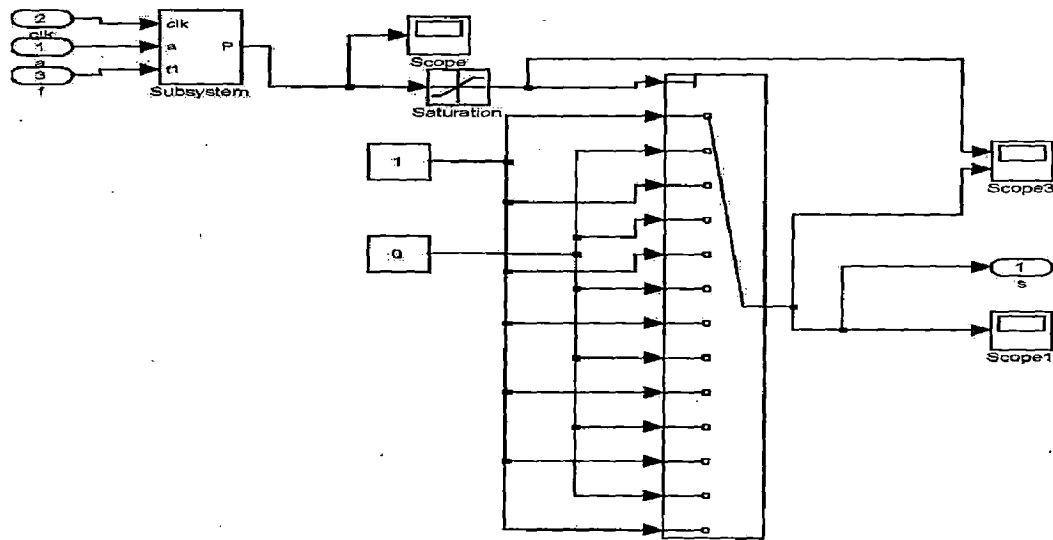


Fig. 5.5 Pulse generator for 0-90 degree interval

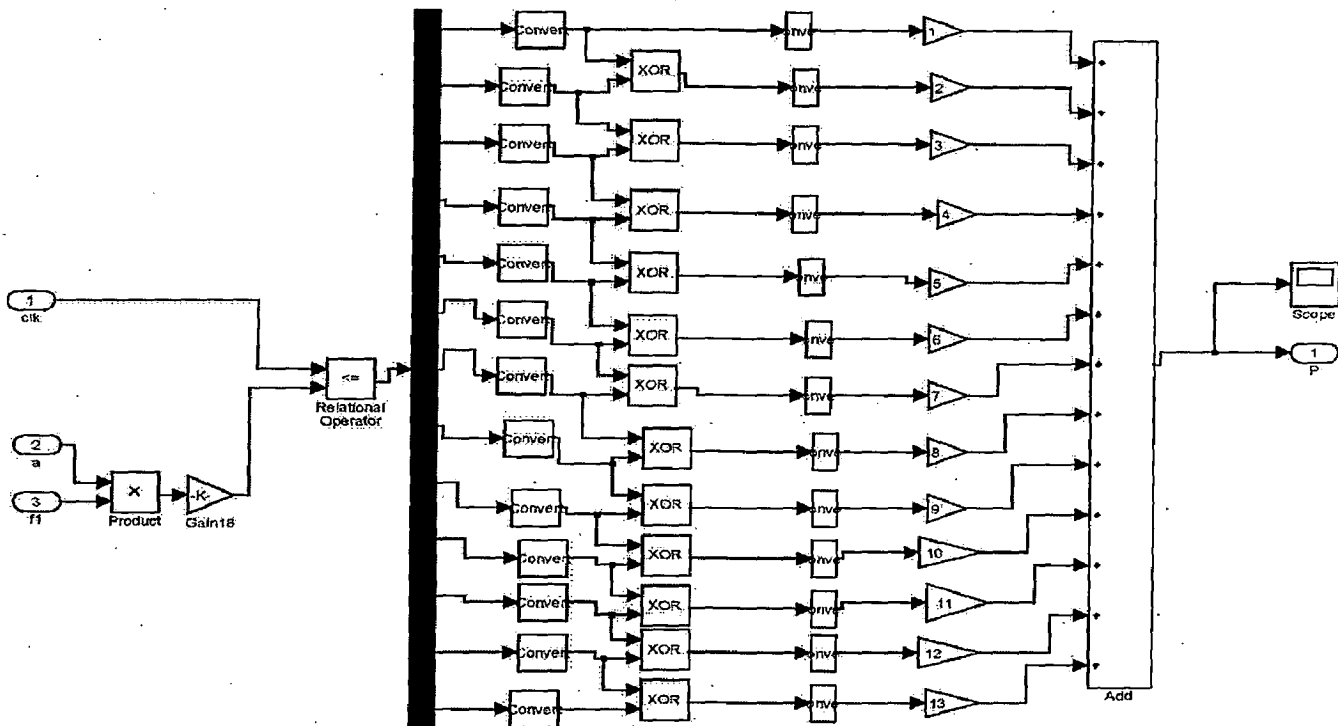


Fig. 5.6 Subsystem to generate Switching Pulses