EFFECT OF DISTRIBUTED GENERATION ON DISTRIBUTION SYSTEM PERFORMANCE

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

ELECTRICAL ENGINEERING

(With Specialization in Power System Engineering)

By

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CANDIDATE'S DECLARATION

As partial fulfillment of the requirements for the Master of Technology (Electrical) Degree (Honours), I hereby submit for your considerations this thesis entitled:

"Effect of Distributed Generation on Distribution System Performance"

I declare that the work submitted in this thesis is to the best of my knowledge and ability and is an authentic record of my own work carried out during the period from July 2006 to June 2007 under the supervision of Dr. Vinay Pant, Assistant Professor and Dr. Biswarup Das, Associate Professor, Electrical Engineering Department, Indian Institute of Technology Roorkee.

This work has not been previously submitted for a degree at the Indian Institute of Technology Roorkee or any other institutes.

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CERTIFICATE

This is to certify that the above statements made by the student are correct to the best of my knowledge.

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Abstract

These days the technology employed for power generation, transmission and utilization in a power system is changing continuously from day to day. Similarly the load demand in the distribution system also varies continuously from time to time. Many new technologies have been employed to meet all the requirements of the distribution system quantities like voltage profile, distribution power loss etc. One of the emerging new technologies is generation of power within the distribution system. This is termed as "Distributed Generation" (DG).

Even though distributed generation helps in improving voltage profile and reduction in distribution losses, it may affect some of the distribution system quantities such as distribution protection scheme, power quality etc. In this work, the effect of DG on distribution system voltage profile, power loss and distribution protection scheme has been considered. Placement of DG also plays a vital role in affecting distribution power loss and voltage profile. For this purpose, a sensitivity analysis method has been used for DG location based on the power loss sensitivity obtained from the sensitivity analysis.

Distribution Load Flow (DLF) plays an important role in analyzing the behavior of the distribution system. In this work, a DLF has been developed for the distribution system with distributed generation. To incorporate the DG into the DLF, the DG has to be modeled as "P-V" or "P-Q" source. In this work, Wind Turbine Generating Units (WTGU) employing Induction Generators for power generation has been considered as DG and modeled as "P-Q" source.

As a part of thesis work, a new DLF has been developed with input parameters being intervals instead of crisp values, termed as "Interval Distribution Load Flow" (IDLF). This IDLF is helpful during Power System Planning. This IDLF has been developed to incorporate the variation in distribution line parameters, load demand and power generated by DG. The solution obtained from the IDLF gives the lower and upper limits for the distribution system quantities (like voltage, power loss). The accuracy of the solutions depends upon the number of input interval quantities and size of the input intervals. In this work, a multi state interval analysis is used so as to improve the accuracy of the solution obtained from IDLF for large size input intervals and is verified with the results obtained from repeated load flow solution.

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Nomenclature

R_1: Stator resistance of the Induction Generator

R_2: Rotor resistance of the Induction Generator

X_{l1}: Stator leakage reactance of Induction Generator

X_{l2}: Rotor leakage reactance of Induction Generator

X_m: Magnetizing reactance of Induction Generator

V: Terminal Voltage of the Induction Generator

Pe: Active Power produced by the WTGU

Qe: Reactive power consumed by the WTGU

S: Slip of the induction generator

I_{2d}: Rotor d-axis current of the induction generator

I_{2q}: Rotor q-axis current of the induction generator

I_2: Rotor current of the induction generator

S_i: Complex power at a particular 'i'

Introduction

At present the power system industry is undergoing a new era in both technology and business structure so as to meet the customer load demands. With centralized power generating stations, it is not economical to transfer power from the generating stations to meet the increased load demands in the distribution system, which leads to increased transmission and distribution system losses. Reliability of service and power quality is enhanced by the proximity of the generators to the customers and efficiency is often boosted in on-site applications by using the heat from power generation. So with the emerging new technologies in power generation in distribution system called "Distributed Generation" (DG) avoids the aforementioned problems. This distributed generation can also be termed as "Dispersed Generation" or "Embedded Generation".

Thomas et.al [1] has defined distributed generation as *"an electric power source connected directly to the distribution network or on the customer site of the meter"*. The terms "Distributed" or "Dispersed" can be used interchangeably. With the advancement in technology and the growing demand for electricity, distributed generation is being used as viable alternative to meet the growing demand. Properly interconnected distributed generation is a viable alternative for the electric power industry's future. The capacity of DG varies from few hundreds of KW to tens of MW. i.e., 500 KW to 10 MW. A typical network with DG is shown in Fig. 1.1.

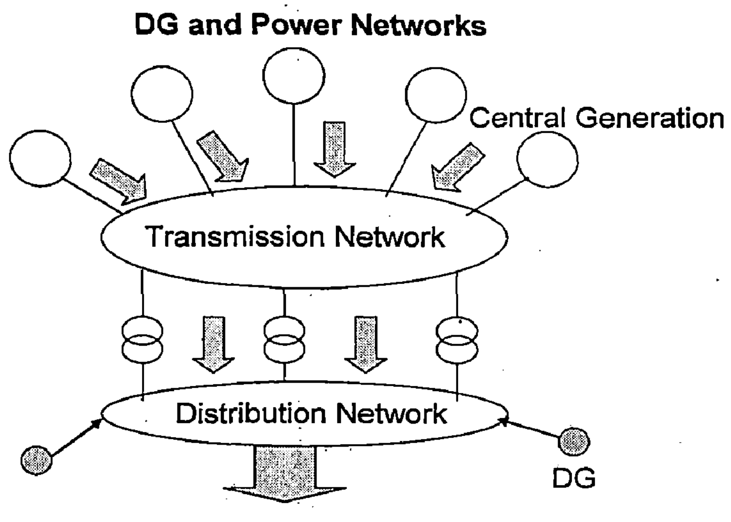

Fig. 1.1 Typical distribution network with DG

Relative Definitions:

Embedded Generation: It is appropriate to describe that the power output of the distributed source is only used locally.

Distributed Resources: These are "demand and supply side resources that can be deployed throughout an electric distribution system to meet the energy and reliability needs of the customers served by that system". Distributed Resources can be installed on either the customer side or the utility side of the meter.

Distributed Capacity: This includes all aspects of distributed generation and distributed resources, plus the requirements for transmission and distribution capacity (reserve capacity). However, distributed generation does not include any reserve capacity. Hence the transmission and distribution network has to be able to cover some part of generation usually supplied by DG.

Distributed Utility: Distributed utility stands for a future network and utility architecture, based on distributed generation, distributed resources and distributed capacity.

Island operation: An "*Island*" is a condition where a portion of the grid is energized solely by distributed generation while that portion of the grid is electrically separated from the rest of the power system [7].

Characteristics of DG:

1. High level of reliability
2. Can be sized to operate independently of the grid
3. Has excellent load following ability
4. Uses available fuels
5. Small in size
6. Noise less operation and can be controlled
7. Environmentally friendly

Distribution system with Distributed Generation:

Most types of the distributed generation utilize traditional power generation paradigms – Diesel, Combustion Engines, and Low Head Hydro Plants etc. But in addition, DG includes Fuel cells, Renewable power generation methods such as Wind, Solar, Geothermal etc. These renewable generators are often lumped into the DG category because their small size makes them very convenient to connect to the lower voltage (Distribution parts) of the electric utility grid. DG works with high efficiency as the traditional power generation system. But the increase in DG penetration is not because of its efficiency, it is due to reduction in the transmission and distribution costs as it is located at the customer site. At time the nearness of the source to the load requires more importance than the efficiency. A high quality DG unit has a service availability of about 95% (including time out of service for both scheduled maintenance and forced outages and subsequent repairs).

Considering all the above advantages, at present most of the distribution systems are equipped with DG. But with the increase in the penetration level of DG in distribution system, several impacts of DG affecting the distribution system performance have been observed. Some of the issues related to DG mainly deals with distribution losses, voltage profile, voltage regulation, protection schemes (relaying operation), instantaneous reclosing and power quality.

In this Thesis work, the effect of DG on Distribution losses, Voltage profile, and Voltage regulation are mainly considered. In this work, Distribution Load Flow (DLF) with DG has been developed for observing the above effects of DG. In order to incorporate DG in load flow studies, the distributed generation has to be modeled as 'P-V' source or 'P-Q' source. In this work, small Wind Turbine Generating Unit (WTGU) of approximately 1 MW is considered as DG and is modeled as -P and +Q sources, as explained in section 4.2.

Further, an Interval Distribution Load Flow (IDLF) with DG has been developed to incorporate the variations in the distribution loads, parameters of distribution lines and the variations in the active and reactive power generation of distributed generation. For this, INTLAB, a separate toolbox in MATLAB is used for developing DLF with intervals as input variables. This interval distribution load flow will give the output in terms of intervals having lower and upper limits, so as to bind the actual output with a marginal error.

Overview of the Thesis report: This thesis is ordered as follows:

Chapter-1: as stated above gives a brief idea on Distributed Generation.

Chapter-2: provides a review of the research work done in the area of distribution system with distributed generation.

Chapter-3: provides the information regarding the effects of distributed generation on distribution system performance, mainly on voltage profile, distribution losses, protection schemes etc.

Chapter-4: describes about the modeling of distributed generation in distribution load flow analysis. In this work, the wind turbine generating units are modeled as DG's.

Chapter-5: gives an idea of distribution load flow with inputs as intervals. In this chapter, a new interval distribution load flow is developed for the radial distribution system with distributed generation.

Chapter-6: discussion about the results obtained in the work.

Chapter-7: Concludes the overall thesis work.

With the increased load demand in distribution system, there is necessity for new generating plants. But this takes more time and transmitting that much of power for a long distance will result in increased transmission losses and is not economical. Now the power systems has progressed a step ahead by installing small generating units at the load centers, called "Distributed Generation". Thomas et.al [1] defined exactly what 'DG' is. They considered various factors for defining DG. Distributed generation is defined as "electric power generation connected directly to the distribution network or on the customer side of the network". They also defined different terms such as 'Distributed Resources', 'Distributed Capacity' and 'Distributed Utility'.

Distributed generation employs many new technologies so as to suit the situation in distribution network. These technologies are classified into traditional and non-traditional technologies [2]. Ann Chambers [3] has given in detail the applications of DG such as peak shaving, stand alone, standby power, grid support etc which represents the necessity of DG in distribution system. Distributed generation is capable of providing excess power required by the load and the new technology employed in distributed generation made the distribution planners to implement DG as a new approach in Distribution System Planning (DSP) [4]. DSP mainly deals with the planning benefits of distributed generation, problems faced due to and the solution to solve those problems.

Placing distributed generation in distribution system may cause variations in distribution system performance in terms of Distribution Losses, Voltage Profile, Protection schemes and Reliability. Barker and de Mello [5] have given the details of the impacts of distributed generation on power system in radial distribution network by considering protection system coordination, harmonics, losses. Placing distributed generation in radial distribution network causes changes in the over current protection scheme, instantaneous reclosing period [6]. The distributed generation reduces the reach of the relays in distribution protection system and also the distributed generation should be disconnected at the first sign of the fault for proper working of relay and reclosing of protective devices.

Khoan [7] has given in detail some more effects of distributed generation on distribution system, such as harmonic distortion, voltage flickers, voltage regulation. He stated that distributed generation introduces some harmonics in the supply due to some inverter technology employed to convert voltage/current in one form to another (ac-dc, dc-dc, and dc-ac) according to the requirement at the load centre. Voltage regulation is a major task in distribution system with distributed generation. Kojovic [8] gives the impact of DG on voltage regulation with in the distribution network with voltage regulators and capacitor banks.

Effect of DG on protective device coordination in distribution system mainly deals with the coordination between different protective devices such as fuse-fuse, recloser-fuse and relay-relay [9, 10]. Without proper coordination, during temporary faults there may be misunderstanding between the protective devices and may lead to permanent fault. During temporary faults, reclosure will try to instantaneously reclose after certain time interval. During first reclosure, if distributed generation is still connected to distribution system, there may be an increase in the fault clearing time leading to equipment damage. Lauri et.al [11] has simulated the reclosing coordination with distributed generation and reported that.

Menon and Nehrir [12] have given some issues regarding the use of distributed generation and its effect on distribution system reliability. To observe the effect of distributed generation on distribution system losses, voltage profile and voltage regulation, DLF is necessary. Brain Stott [13] has given a brief review of load flow calculations methods. He mentioned that load flow for distribution system is different from transmission system due to variations in distribution network structure and parameter values. Glamocanin [14] has given a simple load flow algorithm for radial distribution network, based on nodal currents.

In this work, Wind Turbine Generating Units (WTGU) has been considered as DG's. Divya and Nagendra Rao [15] have modeled WTGU as sources that inject active power and consume reactive power from the supply. They have modeled different types of WTGU as distributed generation and given mathematical equations for obtaining injected active and reactive power. In this work, a 33 bus distribution network [16] is considered. [17] had given in detail about the WTGU operation in power system, considering DG's as –P and +Q loads clearly mentioning the variations in power requirement and current directions in terms of mathematical equations and mentioned the WTGU parameters for modeling purpose.

As a part of Thesis work, an Interval Distribution Load Flow (IDLF) with distributed generation is developed considering the variations in load power, distribution line parameters and generating power of DG. Zian Wang and F.L.Alvarado [18] developed the load flow analysis using interval arithmetic. B.Das [19] developed a DLF for a radial network using interval arithmetic considering the load power variations with in certain possible limits. A brief idea on interval computation and its uses are given by [20].

In my work, INTLAB a separate toolbox in MATLAB is used for interval computations. The application of Intlab in Matlab is given by Department of Mathematics, The University of Manchester, in Numerical Analysis Report No: 416 [21]. This software is capable of carrying out basic arithmetic operations on real interval numbers. Since the DLF requires the use of complex numbers, functions have been developed in this work for arithmetic operations on complex interval numbers.

Finally, the results for a 33 bus radial distribution system are presented and discussed.

Effect of DG on Distribution System Performance

3.1. General:

Normally all distribution systems are designed to operate satisfactorily with in specified limits (Voltage variations, Power loss, etc). But with the increase in the penetration level of distributed generation in distribution system, several impacts of DG affecting the distribution system have been observed. DG has positive impacts on distribution voltage support, distribution system reliability and distribution system losses but also have negative impacts on distribution protection schemes, voltage regulations and short circuit levels [23].

Most of the distribution systems are in radial configuration, so it is necessary to consider some of the network considerations in order to properly coordinate distribution system with distributed generation. With the inclusion of distributed generation in distribution system, various power system parameters may vary. Some of them are:

i. Distribution system voltage profile
ii. Distribution system power losses
iii. Distribution system voltage regulation
iv. Distribution protection schemes
v. Instantaneous reclosing
vi. Power quality

In this chapter, the above mentioned effects of DG are briefly explained.

3.2. Distribution System Voltage Profile:

Now a day, most of the distribution systems are equipped with distributed generation due to the reason that it will improve the distribution system voltage profile and maintain that in acceptable limits. Here the DG has to be modeled as P-V source or P-Q source. In a radial distribution network as the loads in the network are increasing, there will be a reduction in the voltage profile at each bus. In order to improve the bus voltage, extra amount of power is to be send by the nearest substation, so as to compensate the increased load, thereby maintaining constant voltage profile. While transferring that large amount of power from substation, the currents in each branch from the substation increases and thereby the voltage drop increases and again a reduction in bus voltage.

The improvement in voltage profile can be achieved with the help of distributed generation. With increased load in distribution system, addition of DG into that network will meet the extra load demand and improve the voltage profile. This is practically shown in the results of chapter 6. The advantage in adding DG to improve voltage profile is "only some branches in the distribution network will carry the increased amount of power", thereby reducing a large amount of voltage drop in the distribution system and therefore improvement in distribution system voltage profile. Consider an example, a simple 10 bus radial distribution system with loads at each bus. The improvement in the voltage profile after adding a DG at bus number 10 is shown in Fig. 3.1.

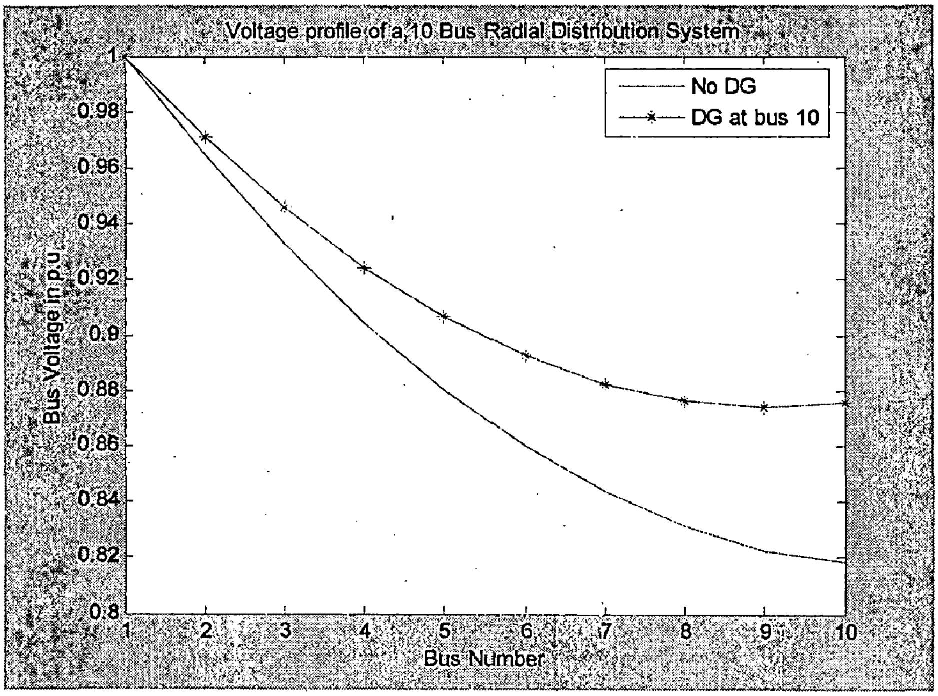

Fig. 3.1 Voltage profile of a 10 bus radial distribution system with DG

The distribution network with good voltage profile, will improve the efficiency of the electrical loads connected to the distribution network because of reduction in the load losses and at the same time, the damages occurring due to lower voltages can be minimized, so that the customers connected to the distribution network is well satisfied.

3.3. Distribution System Power Loss:

Distributed generation in a distribution system may also contribute in minimizing the power losses in the distribution system. In order to meet load demand in distribution system, the power has to be transferred from the main substation to the load point. Since most of the distribution network is radial, the power required by the end load point is to be transferred through several branches ahead of the load point. This will increase the distribution system power loss. So in order to reduce these power losses, the required load power has to be transferred through a minimum number of branches. This can be achieved with the help of distributed generation.

The distributed generation is to be allocated in such a way; the power loss in the distribution system should be minimum. In this work, for finding optimum DG location, a "Sensitivity Analysis" is carried out on the distribution network by placing DG one by one at proper places where the power loss sensitivity is more. This analysis will give the optimal and exact location of DG in a distribution system based on power loss. When a DG is placed in the distribution network, the DG will supply power to the loads which are nearer to that bus where DG is connected. So that the power supplied by the main substation is reduced and at the same time the power losses in the distribution system is also reduced. The power loss in the distribution system depends upon the location of DG also.

A graph is shown in chapter 6, showing that the power loss in the distribution system is a function of DG location and hence an optimal DG location has to be chosen to minimize the power loss. The power loss in the distribution system will reduce as the number of DG's in the distribution system is increased. This is clearly explained in chapter 6.

3.4. Distribution System Voltage Regulation:

Under normal conditions, the required regulation is $\pm$ 5% on a 120 volt base and -2.5% to +5% for voltage greater than 600v services [7]. Under unusual conditions, the allowable range is -8.3% to +5.8%. Traditionally, voltage regulation is based on a single source of power and a single path. Under this practice, the voltage is assumed to always drop on the primary feeder as the distance from the substation increases.

With DG running in the distribution system, the voltage of the bus where DG is connected is improved to a better value as explained earlier in section [3.2]. The above statement holds good only when the system is running under healthy condition. Under faulty conditions, the DG has a negative impact on the voltage regulation. Consider the distribution system under peak loading condition, with DG running in parallel with the utility system, if a fault on the distribution network occurs, the DG has to be disconnected on the first detection of the fault. It will remain disconnected until the utility voltage is stabilized after the fault is clear (takes few minutes).

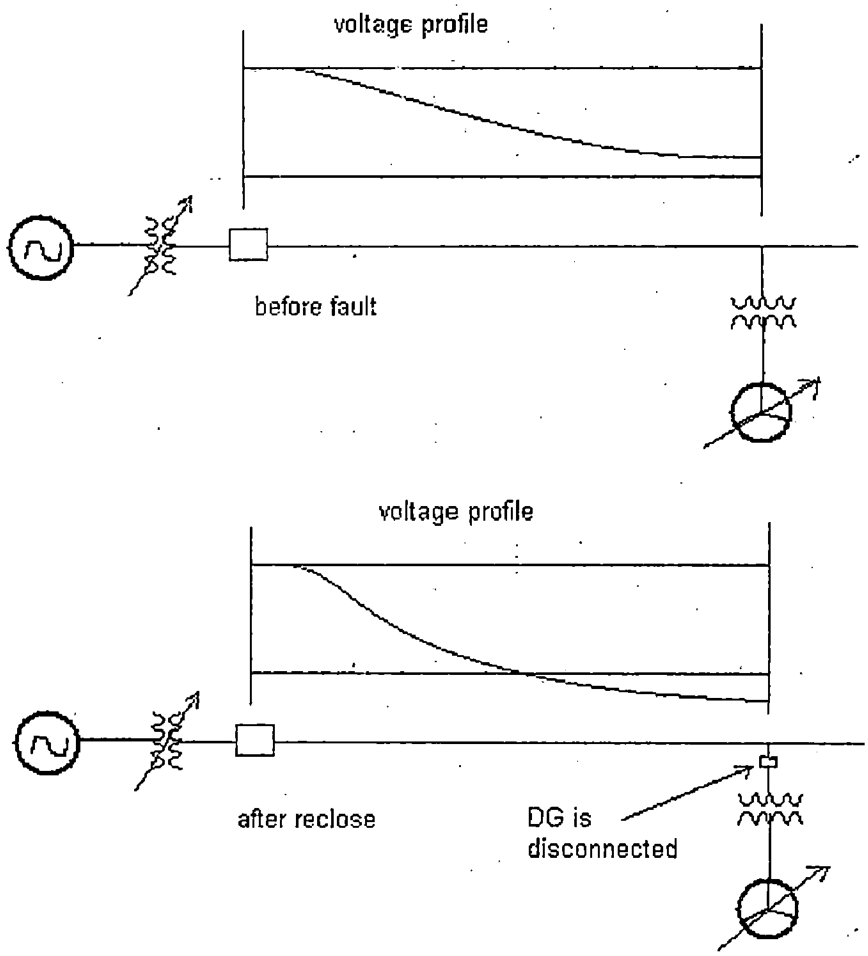

Fig. 3.2 Voltage sags too low after generators are disconnected

Fig. 3.2 shows the voltage profile of a distribution system with a DG running in parallel with it under peak loading conditions. During this period the voltage is supported by the DG. If a fault occurs during this period, the DG has to be disconnected first and then auto reclosing takes place. At the first reclosure, the utility system alone is not able to meet the load because already the distribution system is under peak loading condition and this will result in poor voltage profile at the load end. In order to avoid these conditions, some load has to be disconnected but this will increase the interruption duration of the customers and worsen the reliability also.

With these situations, some standards have been laid for DG power generation, which have to be followed while operating under peak loading conditions. The standards determine how much power the DG should be generating in order to avoid the low voltage in the generating system after occurrence of a forward situation.

3.5. Distribution Protection Scheme:

All distribution system has designed to deliver power in one direction only, but with the placement of DG, there may be chance of power reversal in distribution system and at the same time distribution system protection scheme typically are designed to rapidly isolate faults occurring either at load locations or on the line itself. The DG has an observable effect on distribution system protection scheme. This is explained as below:

The DG in a distribution system affects distribution protection scheme mainly in two ways. First one is flow of power with in the distribution system even under the main substation is isolated by the main circuit breaker during temporary faults. Most of the faults are temporary faults. i.e., if the fault arc is interrupted, the fault will automatically heal itself and power can be restored immediately by reclosing the interrupting device. In radial configuration, fault clearing is simple because only one device can clear the fault unlike in mesh system, where more than one device is necessary. This is the case with DG also. When DG is present, it is similar to a radial network with multiple sources. So opening the utility breaker does not guarantee that the fault will clear promptly. One way is to reconstruct the protection scheme that can work properly for mesh connected system which adds to the cost. So it is better to disconnect the DG when the fault is detected and the protection scheme is now applicable as it was earlier for radial distribution network configuration.

Second impact of distributed generation regarding utility protection relaying is as follows: For every protective device there is certain distance down the radial feeder. This is also known as "reach" of the device. [Reach is determined by the minimum fault current that the device can detect].

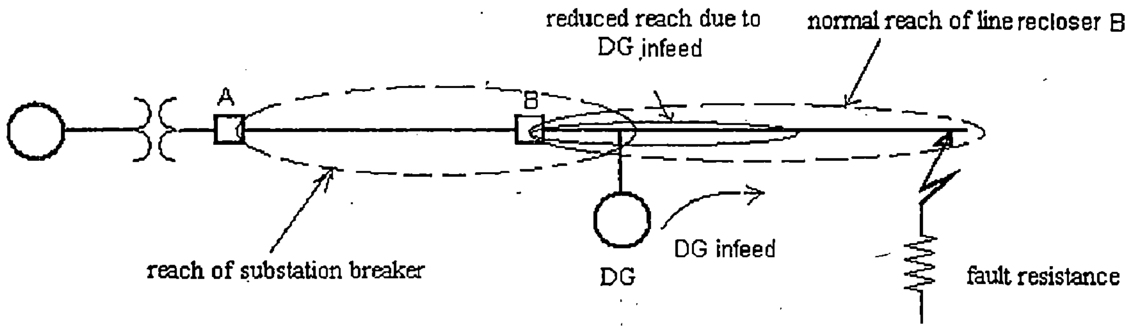

Fig. 3.3 Generator infeed reduces the reach of utility relaying

Consider a radial distribution feeder with two relays A and B. The relay has certain normal reach as shown in Fig. 3.3. During peak loading conditions, the DG is interconnected to the feeder after relay B, the relaying is fairly sensitive i.e., reach is large. Under such conditions, if any fault occurs on the distribution system within the zone covered by the relay B, the relay may not operate. Because the DG also feeds the fault current along with the utility system, therefore the fault current flowing through relay B is reduced and thereby the reach seen by the relay B is reduced. The result is that there will be more damage to utility than without DG. If we want to connect DG under such conditions (interruption period), the DG should be used a back up generation, after the fault is cleared by the utility breaker.

Some other effects on DG on distribution system are explained below:

- **Instantaneous Reclosing (Restoration):**

  Many faults are temporary faults, therefore instantaneous reclosing is necessary. But in distribution system with distributed generation, instantaneous reclosing faces some problems. For the reclosure to be successful there must be sufficient time between shots for the fault arc to dissipate and clear. For this purpose distributed generation has to be disconnected as quickly as possible as the fault occurs as shown in Fig. 3.4. The distributed generation should be disconnected within the reclosing interval; otherwise it leads to continuation of fault.

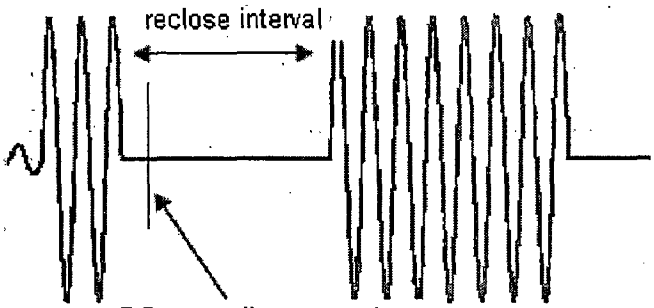

Fig. 3.4 Distribution system reclosing sequence

In order to overcome the above problem, a reclosure interval of 1 sec or more would be preferable. But this will dramatically reduce the chances that the DG will fail to separate in time, although it will also result in reduced power quality to some customers. The reclosure interval is normally 0.5 sec but can be as short as 0.2 sec. The utilities use this short interval; but this will increase the probability that DG will not disconnect in time.

- **Power Quality:**

Some of the technologies used in distributed generation employ inverter based units essential for converting one form to another (dc to ac or vice versa). These units are viewed as non linear loads injecting harmonic current into the distribution feeder. This could result in an unacceptable level of Total Harmonic Distortion (THD). THD is defined as ratio of the rms value of harmonics and rms value of the fundamental.

$$V^2 = V_1^2 + V_H^2 \qquad (3.1)$$

Where

V is the rms value of a voltage waveform

V_1 and V_H are the fundamental and harmonic components

$$THD = \frac{V_H}{V_1} * 100 \qquad (3.2)$$

Harmonics, unlike some transient events such as lightning or voltage sags that can last from a few to several cycles are steady state periodic phenomena that produce continuous distortion of voltage and current waveforms. These distorted voltage and current can significantly affect "True Power Factor" of the system. TPF is the average power divided by the product of rms voltage, rms current and since harmonic quantities increase rms values of voltage and current, they can have significant impact on the value of TPF.

Table 3.1

Current THD (%)	Maximum True PF
20	0.98
50	0.89
100	0.71

From Table 3.1, it is clear that as the harmonic content (THD) in current is increased there is a significant decrease in True Power Factor.

The ill effects of harmonics in the system are:

1. Over heating of the equipments in distribution system.
2. Degradation of meter accuracy.
3. Overloading of neutral conductors serving single phase loads.

Some other technical impacts of DG on distribution system are:

- If the DG has no power export agreement with the grid, under such condition, if a large amount of load connected to DG is suddenly goes off, the DG may not be able to reduce its generation quickly; thereby there is a chance of sending power into the grid. This will cause circuit breaker tripping referred to as "*Nuisance Tripping*".
- If the DG is importing power from grid, if an islanding happens, and the same time if the micro grid is not able to meet the loads connected to DG, then there will be droop in the load voltage so that the power quality supplied to the loads by DG is worsened. Similarly if DG is exporting power to grid, there will be a chance of rise in voltage across the loads after islanding.

Modeling of DG in Distribution Load Flow Analysis

4.1. General:

Most of DG technologies employ renewable energy sources such as wind, solar, small hydro etc. But these renewable energy sources are not continuous in nature; they depend upon the atmospheric conditions. So therefore there is need for proper modeling of DG and at the same time, the DG has to be incorporated into the distribution load flow. Hence the DG has to be modeled considering the atmospheric and distribution network conditions.

Normally a DG can be modeled in two ways, one way is to model it as a source generating active power at constant voltage (P-V source), second way is to model DG as a source that generates active power and consumes reactive power (P-Q model). Here in the second case, the bus voltage where DG is connected is not maintained constant. This bus voltage depends upon the amount of reactive power consumed by the DG. *The reactive power consumption by the DG depends upon the technology employed in generating active power.* In this thesis work, Wind Turbine Generating Units (WTGU) is used as DG. These WTGU's are modeled as P-Q source [15] depending upon the type of WTGU.

4.2. Modeling of WTGU [15]:

Now a days, use of wind energy for electrical power generation is gaining importance, at both the transmission and distribution voltage levels. Modeling of WTGU connected to distribution system is different from modeling of WTGU connected to transmission system. Because the primary distribution system are traditionally designed to operate with power flowing in one direction only. However, the power output of the WTGU depends on the turbine characteristics as well as the control characteristics of the power controllers. These turbine characteristics will depend on the type of wind turbine and the generator employed. Normally, WTGU's employ induction generators to generate electric power. The induction generator equivalent model is given in appendix [A]. So these induction generators consume reactive power from the main distribution system, during active power generation.

Therefore, the WTGU has to be modeled as electric sources that generates active power and consumes reactive power. The modeling of different types of WTGU is explained in the next section.

The WTGU's are classified into 3 categories - Fixed, Semi variable and Variable speed. The models developed here for the WTGU are intended to obtain the power output of the WTGU for a given terminal voltage and wind speed.

4.3. Fixed speed WTGU:

This type of WTGU has a squirrel cage induction generator (SCIG) which is driven by a wind turbine either having a fixed turbine blade angle (Stall regulated fixed speed WTGU) or having a pitch controller to regulate the blade angle (Pitch regulated fixed speed WTGU). In both these types of WTGU, the induction generator is directly connected to the grid. In the operating range the rotor speed varies within a very small range (± 5% of nominal voltage), hence these are reckoned as fixed speed WTGU. A fixed shunt capacitor is used to provide reactive power consumption for these WTGU.

In this work, pitch regulated fixed speed WTGU has been used. According to the wind speed, the pitch angle controller regulates the wind turbine blade angle. The power output of this class of WTGU depends on the characteristics of the pitch controller in addition to the turbine and generator characteristics. In this type, the power output for any wind turbine is maintained constant irrespective of the bus voltage. The designed power output Pe of the WTGU with wind speed is provided by the manufacturer in the form of a power curve. Thereby the power output by the WTGU is obtained from the power curve. Since induction generator is used for generating power, this will consume reactive power from the distribution system and this consumption of reactive power will depend on the bus voltage where DG is connected. The power curve for the pitch regulated fixed speed WTGU is shown in Fig. 4.1 [15].

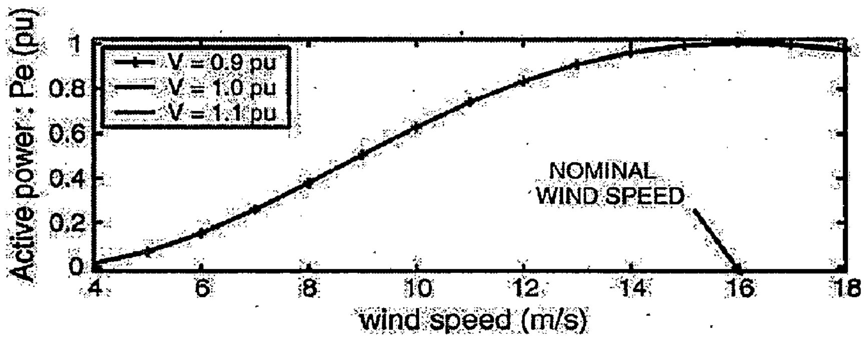

Fig. 4.1 Power curve of Pitch regulated fixed speed WTGU

This WTGU is modeled in such a way, the active power Pe is obtained from the power curve and the reactive power Qe consumed is obtained with the help of the quadratic equation, given in the algorithm below. This equation is solved to get the slip value. With the known slip, the reactive power output Qe is calculated from the induction generator equivalent circuit. These WTGU require fixed shunt capacitor banks to provide reactive power compensation. The algorithm for modeling pitch regulated fixed speed WTGU is described below [15]:

Algorithm: Given wind speed u_w and terminal voltage V

1. For the given u_w obtain Pe from the power curve of the WTGU.
2. Pe of the induction generator is given by

$$Pe=\frac{\left[R_1(R_2^2+s^2(Xm+X_{l2})^2)+sR_2X_m^2\right]*|V|^2}{\left[R_2R_1+s(X_m^2-(X_m+X_{l2})(X_m+X_{l1}))\right]^2+\left[R_2(X_m+X_{l1})+sR_1(X_m+X_{l2})\right]^2} \tag{4.1}$$

Knowing Pe and all the other parameters of the induction generator, equation (4.1) can be written as a quadratic equation in 's' as follows

$$a\, s^2 + b\, s + c = 0 \tag{4.2}$$

where,

$$a = PeR_1^2(X_{l2}+X_m)^2 + Pe(X_mX_{l2}+X_{l1}(X_{l2}+X_m))^2 - |V|^2 R_1(X_{l2}+X_m)^2$$

$$b = 2PeR_1R_2X_m^2 - |V|^2 R_2X_m^2$$

$$c = PeR_2^2(X_{l1}+X_m)^2 + Pe(R_2R_1)^2 - |V|^2 R_1R_2^2$$

Then the slip is given by

$$s = \min\left|\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right| \tag{4.3}$$

3. Knowing 's' compute Qe as

$$Qe = \frac{\left[X_m X_{l2} s^2 (X_m + X_{l2}) + X_{l1} s^2 (X_m + X_{l2})^2 + R_2^2 (X_m + X_{l1})\right] * |V|^2}{\left[R_2 R_1 + s(X_m^2 - (X_m + X_{l2})(X_m + X_{l1}))\right]^2 + \left[R_2 (X_m + X_{l1}) + s R_1 (X_m + X_{l2})\right]^2} \tag{4.4}$$

4.4. Semi variable speed WTGU:

This WTGU consists of pitch controlled wind turbine and a wound rotor induction generator. An external variable resistance is connected to the rotor circuit of the WTGU. With the help of power electronic circuits, the external resistance is varied to control its active power and reactive power output. Here also the active power output Pe is obtained by the power curve which is provided by the manufacturer. These WTGU requires fixed shunt capacitor banks to provide reactive power compensation. The power curve for semi variable speed WTGU is shown in Fig.4.2 [15].

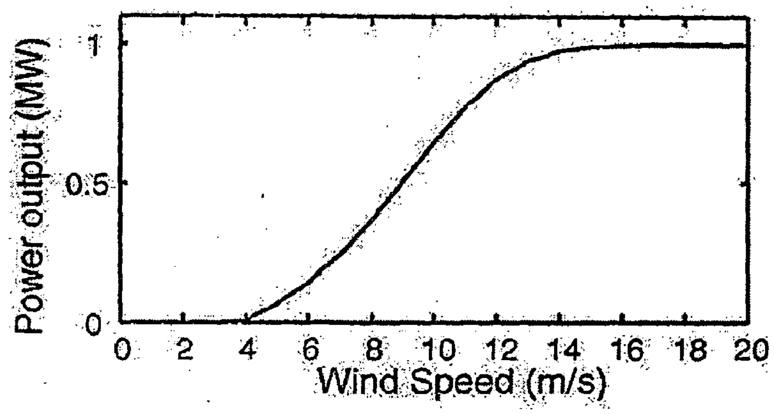

Fig. 4.2 Power curve of Semi variable speed WTGU

Similar to the pitch controlled WTGU, calculation of reactive power Qe of this type is obtained from the quadratic equation with slip as a variable. But in this WTGU an additional control parameter is the external variable resistance. So this control has to be taken into account. In order to consider both the slip and variable resistance into account, the quadratic expression is rearranged with R_2/S as variable. Solving the quadratic equation for R_{eq} (R_2/S) involving Pe, gives the reactive power output Qe. The quadratic equation for R_{eq} is derived from equation 4.1 is as follows [15]:

$$a\,R_{eq}^2 + b\,R_{eq} + c = 0 \qquad (4.5)$$

where,

$$a = Pe(R_1^2 + (X_{l1} + X_m)^2) - |V|^2 R_1^2$$

$$b = 2R_1 Pe X_m^2 - X_m^2 |V|^2$$

$$c = PeR_1^2 (X_{l2} + X_m)^2 + Pe(X_m^2 - (X_m + X_{l2})(X_m + X_{l1}))^2 - R_1 (X_m + X_{l2})^2 |V|^2$$

Algorithm: Given wind speed u_w and the terminal voltage V

1. For the given u_w obtain Pe from the power curve of the WTGU.
2. Compute R_{eq} by solving equation 4.5.

$$R_{eq} = \min\left|\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right| \qquad (4.6)$$

3. Knowing R_{eq}, compute Q_e as

$$Qe = \frac{\left[R_{eq}^2 (X_m + X_{l1}) - (X_m + X_{l2}) + (X_m^2 - (X_m + X_{l2})(X_m + X_{l1}))\right] * |V|^2}{\left[R_{eq} R_1 + (X_m^2 - (X_m + X_{l2})(X_m + X_{l1}))\right]^2 + \left[R_{eq}(X_m + X_{l1}) + R_1 (X_m + x_{L2})\right]^2} \qquad (4.7)$$

4.5. Variable speed WTGU:

These WTGU are classified in to two types according to the type o connection of the induction generator. One is WTGU having double fed inductioı generator (DFIG). This consists of pitch controlled wind turbine and inductioı generator whose stator winding is directly connected to the grid but the rotor circuit i connected to the grid through a back to back voltage source converter. The voltag source controller applies voltage across the rotor which is regulated by two roto current controllers.

The second one is WTGU having generator (synchronous/induction) with front end converter (GFEC). This consists of pitch controlled wind turbine and a variable frequency synchronous or induction generator connected to the grid through a power electronic converter. The VSC output applied to the stator is varied by the control signals obtained from the current controllers.

In both the schemes in addition to the two current controllers (d- and q-axis) there are two power controllers (active and reactive). The current controllers are very fast and they regulate the q and d components of machine current to reference values that are generated by the relatively slow power controllers. The reference value specified by the active power controller ensures that maximum power is extracted below nominal wind speed. However, above nominal wind speed the pitch angle controller operates so that rated power output is maintained. Unlike in fixed and semi variable speed WTGU, these WTGU's doesn't require any reactive power compensation equipment. Some of the WTGU's are equipped with power factor controller. This controller maintains a constant power factor at all wind speeds by appropriately varying the reference setting of the reactive power. The power curves for the DFIG and GFEC variable speed WTGU are shown in Fig. 4.3.

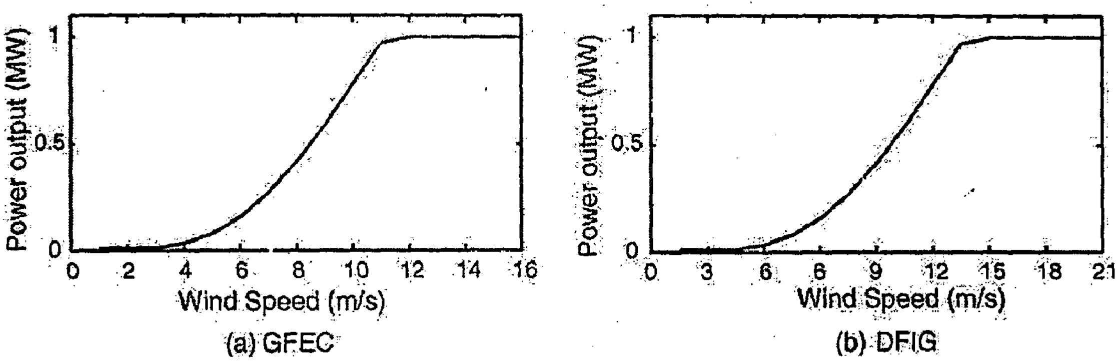

Fig. 4.3 Power curves of the variable speed WTGU

While modeling this type of WTGU, the two current controllers and the power controllers are also taken into account. The active power output Pe and reactive power output Qe are calculated as mentioned in the algorithm given below. In the algorithm, if there are any violations in the currents (d- and q-axis), the corresponding currents are reset to their appropriate limit values and Pe as well as Qe is recomputed. The algorithm for modeling variable speed WTGU is given below [15].

Algorithm: For a given wind speed u_w and the terminal voltage V.

1. For the given wind speed, obtain Pe from the power curve.
2. If Qe is specified

   (a) set Qe = $Qe_{specified}$

   Else, if power factor ($\cos\phi$) is specified,

   (b) $$Qe = Pe\frac{\sqrt{1-\cos^2\phi}}{\cos\phi} \quad (4.8)$$

3. For this Qe compute I_{2d} using

$$I_{2d} = \frac{|V|}{X_m} - \frac{2Qe(X_{l1}+X_m)}{3|V|X_m} \quad (4.9)$$

4. If I_{2d} is greater than its maximum limit

   a) Set I_{2d} to its maximum limit

   b) Recompute Qe as

$$Qe = \frac{3}{2}|V|\frac{X_m}{X_{l1}+X_m}\left(\frac{|V|}{X_m} - I_{2d}\right) \quad (4.10)$$

   Else, if I_{2d} is less than its minimum limit

   a) Set I_{2d} to its minimum limit

   b) Recompute Qe using equation 4.10.

5. If it is Type-1 variable speed WTGU then

   (a) Compute I_{2d} using equation 4.9, I_{2q} using equation 4.11 and I_2 using equation 4.12.

$$I_{2q} = \frac{2Pe(X_{l1}+X_m)}{3|V|X_m} \quad (4.11)$$

$$I_2 = \sqrt{I_{2q}^2 + I_{2d}^2} \quad (4.12)$$

   (b) If I_2 is greater than the maximum current limit

   i. Set it to its maximum value.

   ii. For this value of I_2, with I_{2d} unchanged, recalculate I_{2q} as

$$I_{2q} = \sqrt{I_2^2 - I_{2d}^2}$$

   iii. using this value of I_{2q}, recompute Pe

$$Pe = -\frac{3}{2}|V|\frac{X_m}{X_{l1}+X_m}I_{2q} \quad (4.13)$$

Else if it is Type-2 variable speed WTGU then

(a) Compute I_{2q} using equation 4.11.

(b) If I_{2q} is greater than its maximum limit

Then set I_{2q} to its maximum value and for this value of I_{2q}, recompute Pe using equation 4.13.

- ✓ In the above algorithm, Type-1 refer to GE variable speed DFIG WTGU. In this type, when the specified Pe and Qe cannot be met due to the maximum limit violation of the total current, maintaining the Qe output is given a higher priority. Thus Pe is reduced so as to maintain the specified Qe. Hence, in this case it is necessary to check for d-axis as well as total current limit violations. While in Type-2, the limits for d-axis as well as q-axis current automatically take care of the total maximum current limit. Thus in this case it is sufficient to check for d- as well as q-axis current limit violations.

- ✓ In this work, only Pitch Regulated fixed speed WTGU and Semi Variable speed WTGU has been considered for representing DG's in the distribution system.

Interval Distribution Load Flow with Distributed Generation

5.1. General:

Power flow analysis is used as a basic tool to study the behavior of a power system whether it is transmission system or distribution system. It is different for transmission system as compared to distribution system due to the wide range of resistances, reactances and its radial structure of the latter. Most popular methods like Newton Raphson and Fast Decoupled methods cannot be used to analyze distribution system.

Power flow analysis essentially gives the steady state operating point of a distribution system corresponding to specified, fixed loading conditions [19]. But in reality, it is not the case. The input parameters for the load flow analysis are quite uncertain. The uncertainties are mainly [18]:

i. Errors in the calculated or measured parameters (Resistances and Reactances) of the various lines and transformers in the system.

ii. Errors in the magnitude of the demand assumed for the system load buses.

The results obtained in a normal load flow are valid only for single specific system configuration and operating condition. Here in this chapter, a distribution load flow with inputs as intervals instead of crisp values has been developed, taking into account the variations in load demand and variations in distribution system parameters using complex interval arithmetic calculations.

In this thesis work, an Interval Distribution Load Flow (IDLF) with distributed generation has been developed. Here the interval inputs are load power, distribution line parameters and variations in the power generated by the distributed generation. The variations in load power and distribution line parameters are taken as ±5% and ±5% respectively. Similarly, the power generated by distributed generation is also taken as an interval varying in between [0.8 – 0.9] MW for pitch regulated fixed speed WTGU and [0.3 – 0.9] MW for semi variable speed WTGU.

5.2. Interval Arithmetic [21]:

In this section, the basic concepts of interval arithmetic are reviewed. Interval arithmetic is an arithmetic defined on sets of intervals, rather than sets of real numbers. The concept of interval analysis is to compute with intervals of real numbers in place of real numbers. While floating point arithmetic is affected by rounding errors, and can produce inaccurate results, interval arithmetic has the advantage of giving rigorous bounds for the exact solution. An application is when some parameters are not known exactly but are known to lie within a certain interval; algorithms may be implemented using interval arithmetic with uncertain parameters as intervals to produce an interval that bounds all the possible results. If the lower and upper bounds of the interval can be rounded down and rounded up respectively then finite precision calculations can be performed using intervals, to give an enclosure of the exact solution.

5.2.1. Notation:

Intervals have been represented by boldface, with the brackets "[.]" has been used for intervals defined by an upper bound (supremum) and a lower bound (infimum). For intervals defined by a mid point and a radius the brackets "$< . >$" has been used.

5.2.2. Real Interval Arithmetic:

A real interval **x** is a non empty set of real numbers

$$\mathbf{x} = [x_1, x_2] = \{x \varepsilon R : x_1 \leq x \leq x_2\},$$

where x_1 is called the infimum and x_2 is called the supremum. The set of all intervals over R is denoted by IR where

$$\mathbf{IR} = \{[x_1, x_2] : x_1, x_2 \varepsilon R, x_1 \leq x_2\}.$$

The *midpoint* of **x**,

$$a = \text{mid}(\mathbf{x}) = (x_1 + x_2)/2. \quad (5.1)$$

and the *radius* of **x**

$$r = \text{rad}(\mathbf{x}) = (x_1 - x_2)/2. \quad (5.2)$$

may also be used to define an interval $\mathbf{x} \ \varepsilon\ \mathbf{IR}$. An interval with midpoint 'a' and radius 'r' will be denoted by $< a, r >$. If an interval has zero radius it is called a *point interval* or *thin interval,* and contains a single point represented by [x , x] = = x. A *thick interval* has a radius greater than zero.

The *absolute value* or *magnitude* of an interval **x** is defined as

$$|\mathbf{x}| = \text{mag}(\mathbf{x}) = \max\{|x| : x \varepsilon \mathbf{x}\},$$

and the *mignitude* of **x** is defined as

$$\text{mig}(\mathbf{x}) = \min\{|x| : x \varepsilon \mathbf{x}\}.$$

These can both be calculated using the end points of **x** by

$$\text{mag}(\mathbf{x}) = \max\{|x_1|, |x_2|\},$$

$$\text{mig}(\mathbf{x}) = \min(|x_1|, |x_2|) \quad \text{if } 0 \notin \mathbf{x}$$

$$= 0 \qquad \text{otherwise.}$$

An interval **x** is a *subset* of an interval **y**, denoted by $\mathbf{x} \subseteq \mathbf{y}$, if and only if $y_1 \leq x_1$ and $y_2 \geq x_2$. The relation $\mathbf{x} < \mathbf{y}$ means that $x_2 < y_1$ and other inequalities are defined in a similar way. Interval arithmetic operations are defined on **IR** such that the interval result encloses all possible real results. Given $\mathbf{x} = [x_1, x_2]$ and $\mathbf{y} = [y_1, y_2]$, the four elementary operations are defined by

$$\mathbf{x} \text{ op } \mathbf{y} = \{x \text{ op } y : x \varepsilon \mathbf{x}, y \varepsilon \mathbf{y}\} \text{ for op } \varepsilon \{+, -, *, /\}.$$

Therefore, the elementary operations with intervals as inputs are

$$\mathbf{x} + \mathbf{y} = [x_1+y_1, x_2+y_2] \tag{5.3}$$

$$\mathbf{x} - \mathbf{y} = [x_1-y_2, x_2-y_1] \tag{5.4}$$

$$\mathbf{x} * \mathbf{y} = [\min\{x_1y_1, x_1y_2, x_2y_1, x_2y_2\}, \max\{x_1y_1, x_1y_2, x_2y_1, x_2y_2\}] \tag{5.5}$$

$$1/\mathbf{x} = [1/x_2, 1/x_1] \quad \text{if } x_1 > 0 \text{ or } x_2 < 0 \tag{5.6}$$

$$\mathbf{x} / \mathbf{y} = \mathbf{x} * 1/\mathbf{y} \tag{5.7}$$

5.2.3. Complex Interval Arithmetic [19]:

Any complex number Z = X + iY, where 'i' is the complex operator, is said to be a complex interval number if both of its real part (X) and the imaginary part(Y) are interval numbers. Hence, X can be represented as $X = [x_1, x_2]$ and $Y = [y_1, y_2]$, where x_1, y_1 are the lower limits and x_2, y_2 are the upper limits, respectively. The conjugate of a complex interval number is given by Z* = X - iY. Let $Z_1 = A_1 + iB_1$ and $Z_2 = A_2 + iB_2$ be two complex interval numbers. Then the addition, subtraction, multiplication and division of these complex interval numbers are defined as:

$$Z_1 + Z_2 = (A_1+A_2) + i(B_1+B_2) \tag{5.8}$$

$$Z_1 - Z_2 = (A_1-A_2) + i(B_1-B_2) \tag{5.9}$$

$$Z_1 * Z_2 = (A_1*A_2-B_1*B_2) + i(A_1*B_2+A_2*B_1) \tag{5.10}$$

$$Z_1 / Z_2 = C + iD \tag{5.11}$$

where $C = (A_1*A_2 - B_1*B_2) / (A_2^2 + B_2^2)$ and

$D = (A_2*B_1 - A_1*B_2) / (A_2^2 + B_2^2)$

The distance between these two interval numbers is

$$q(X, Y) = \max(|x_1 - y_1|, |x_2 - y_2|) \tag{5.12}$$

The power of interval arithmetic lies in its implementation on computers. In particular, outwardly rounded interval arithmetic allows rigorous enclosures for the ranges of operations and functions.

5.3. Interval Distribution Load Flow with Distributed Generation:

To study the behavior of a distribution system, load flow is necessary. Normally, distribution load flow is developed with the help of backward and forward sweep algorithm. This algorithm basically comprises of basic nodal and mesh equations only. Usually, in this algorithm first the branches currents starting from the end nodes are calculated and traversed back to the main substation branch using nodal current equations. Then the bus voltages starting from main station upto end nodes are updated with the branch currents obtained in the backward sweep. This is known as forward sweep. Calculating the error mismatch of the bus voltages between the updated voltages and previous iteration voltages and checking whether the error is less than the convergence criteria (0.0001 p.u) to stop the load flow analysis. Now using these bus voltages and branches currents, the complex power at each bus and the corresponding line losses are obtained.

In this work, the distributed generation has been incorporated in the distribution system, so that the distribution load flow is made capable to run with DG. To incorporate DG in distribution load flow, the DG has to be properly modeled. Here in this work, wind turbine generating units (WTGU) are taken as DG's and these are modeled as sources that generate active power and consume reactive power. The consumption of reactive power is due to the inductive property of the generator used in WTGU (Induction Generators). So in this way, all the loads and the distributed generators are modeled in terms of active power and reactive power loads, thereby reducing the complexity in developing distribution load flow with DG. The modeling of WTGU has already been discussed in section 4.2.

Till now, the distribution load flow discussed above is meant for real values only. But in practical situation, the input parameters to the distribution load flow may vary as per the reason explained earlier. In this thesis work, a distribution load flow with distributed generation with interval parameters as input parameters has been developed. This is known as Interval Distribution Load Flow with Distributed Generation. Here in this work, the variation in the distribution line parameters, load parameters and the power generated by the DG are considered. This interval load flow with give the exact bounds for the load flow output values (Bus voltages, Power loss etc), within acceptable limits and will be an asset for Distribution System Planners.

The interval distribution load flow developed here is implemented with the help of INTLAB [21], a separate tool box in MATLAB. The basic operation with Intlab in Matlab has been described in appendix [B]. The algorithm for interval distribution load flow with distributed generation comprises of several modules. These modules are given below:

- ✓ Tracing the distribution network using Linked list.
- ✓ Identifying the buses with DG's.
- ✓ Carrying out the backward/forward sweep algorithm for load buses.
- ✓ If a DG is connected to a bus, then first model the DG as per our requirement.
- ✓ Calculate the error mismatch
- ✓ Calculating the final bus voltages, complex power and losses in the lines.

Here in the work, the input parameters (distribution line parameters, load demand and power generated by DG) are taken as interval values instead of real values. The flowchart for the Linked list and the main Interval distribution load flow with DG are shown in Fig. 5.1 and Fig. 5.2 respectively.

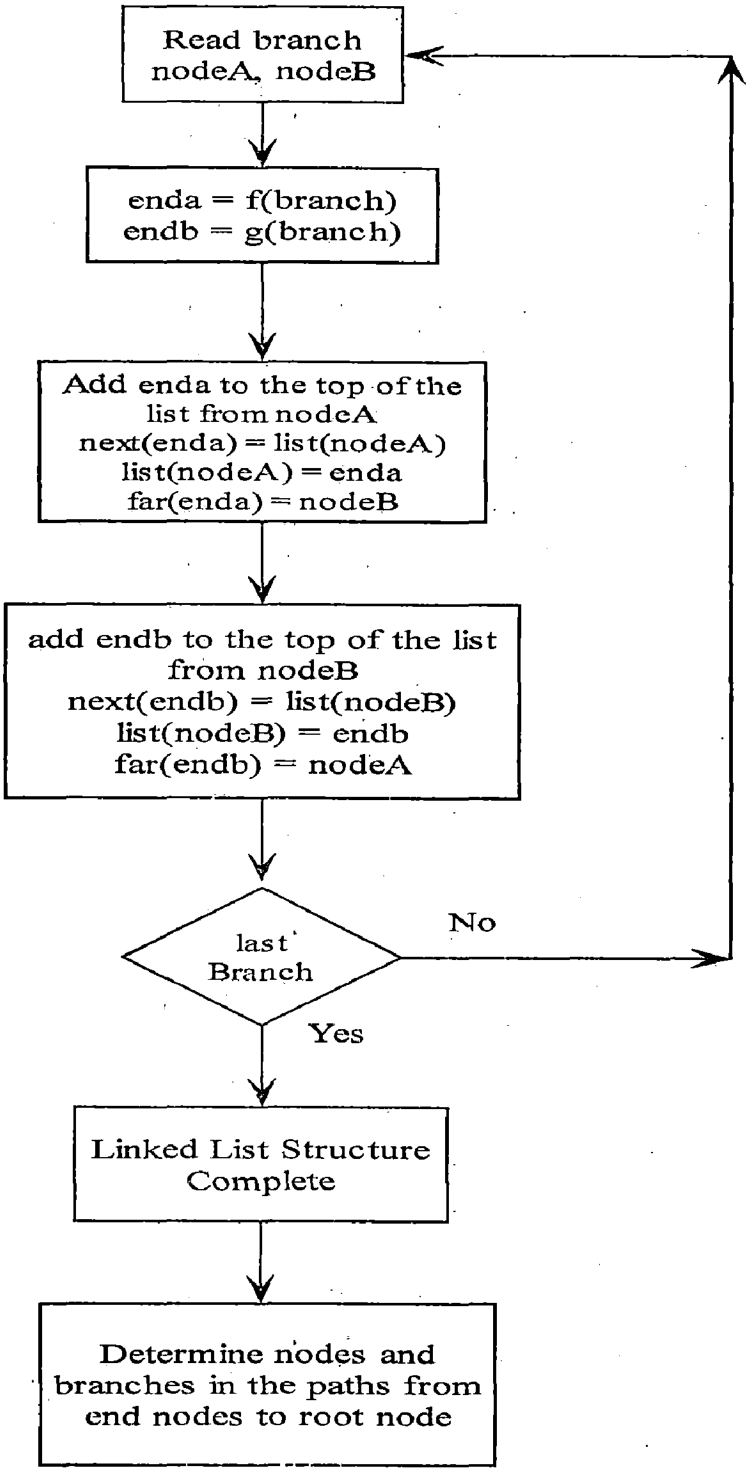

Fig. 5.1 Flowchart of Linked List Program

This program helps in finding out the branches and nodes from the end nodes to the root nodes and also gives related information regarding the nodes connected to each node and branches connected to each node etc. This program also helps in random search of the branches and nodes with in the network.

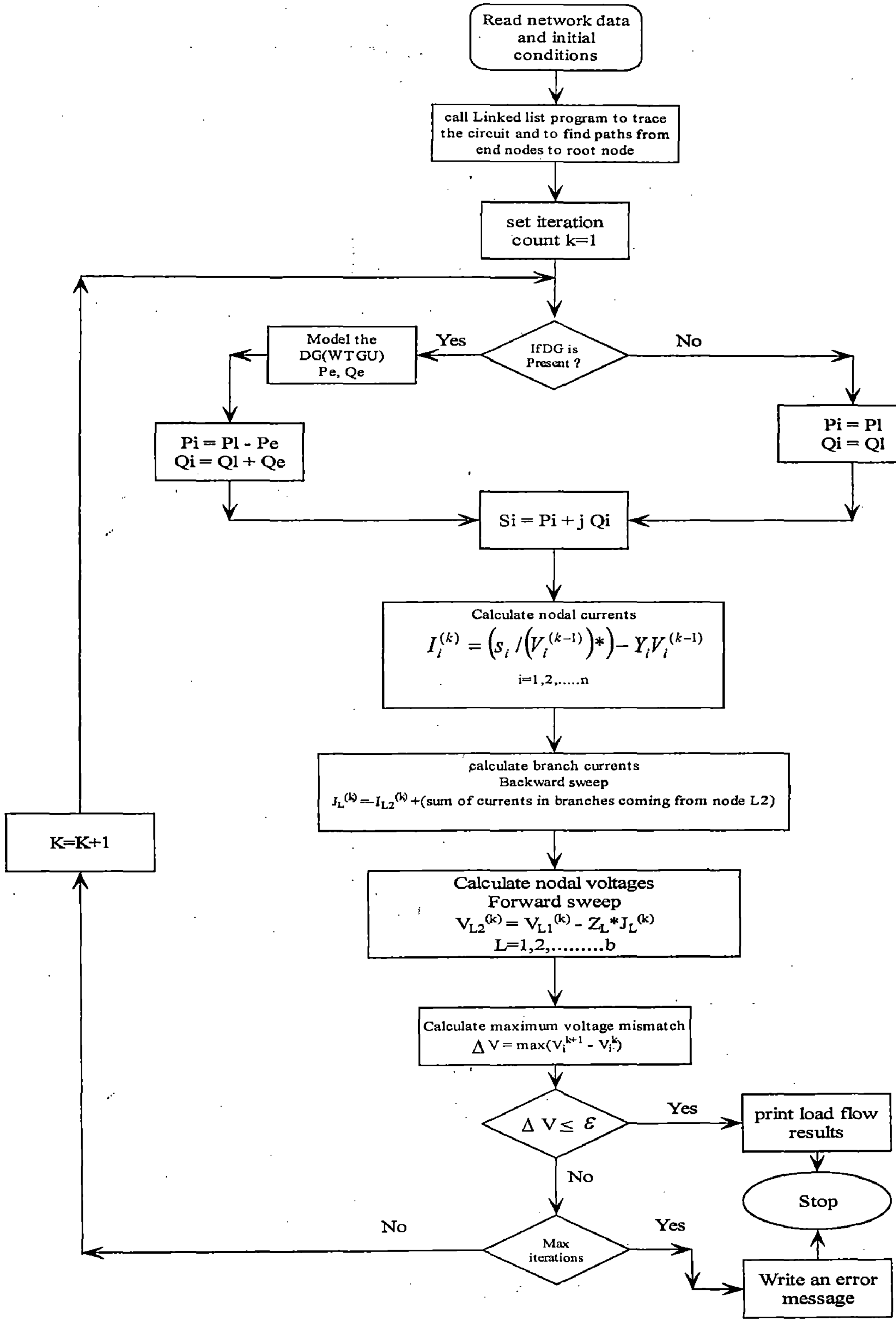

Fig. 5.2 Flowchart of Distribution Load flow with Distributed Generation

- **Convergence criteria in IDLF:**

  The convergence criterion in IDLF is different from the normal DLF. The convergence criterion in IDLF is calculated as follows:

  Consider $V_{i-1} = [x_1, x_2]$ be the bus voltage in $(i-1)^{th}$ iteration

  $V_i = [y_1, y_2]$ be the bus voltage in i^{th} iteration

  Then the convergence is given by

  $\mathbf{Err}_i = \max\{\,|x_1 - y_1|, |x_2 - y_2|\,\}$.

  Convergence = $\mathrm{Err}_i < 0.0001$

5.4. Advantages and Disadvantages of Interval Arithmetic [20]:

Some of the advantages of using interval arithmetic in engineering applications are given below:

- ✓ Interval arithmetic gives the bounds of the actual results; even though the exact inputs are not known (Intervals of the inputs are known).
- ✓ While floating point arithmetic is affected by rounding errors, and can produce inaccurate results, interval arithmetic has the advantage of giving rigorous bounds for the exact solution.
- ✓ Bounds from interval arithmetic often are sharper and are simpler to derive than bounds from other techniques.
- ✓ The power of interval arithmetic lies in its implementation on computers. In particular, outwardly rounded interval arithmetic allows rigorous enclosures for the ranges of operations and functions.
- ✓ Interval arithmetic makes a qualitative difference in scientific computations, since the results are now intervals in which the exact result must lie.
- ✓ It also enables use of computations for automated theorem proving.

Some of the disadvantages of using Interval Arithmetic are given below:

- ✓ Since interval arithmetic is only sub distributive, expressions that are equivalent in real arithmetic differ in interval arithmetic.
- ✓ As the number of interval inputs increased, the size of the output limits also increases and results in inaccurate solution.
- ✓ Also, the size of the output limits increases with the size of the intervals given at the input.

6.1. General:

In this thesis work, to observe the effect of distributed generation on distribution system performance, a 33 bus radial distribution network [16] has been considered. The distribution network comprises of both active and reactive power loads at each bus except at the main substation (bus no. 1). The network data and load data is given in appendix [A]. In this work, the DG (1 MVA of WTGU type) is incorporated in the distributed system. The modeling of DG is already discussed in chapter 4. The distributed generation considered here are pitch regulated fixed speed WTGU and semi variable speed WTGU. The design parameters for modeling of distributed generation are given in appendix [A]. In this primary analysis of distribution network with distributed generation, all the loads and power generated by the DG are kept constant. With this configuration, the effect of DG on distribution system performance is observed. Mainly, the effect on distribution voltage profile, distribution losses, and distribution protection scheme are observed.

As an extension to this work, an Interval Distribution Load Flow is developed for distribution network with distributed generation. This analysis helps in *Power System Planning*. Here the input parameters for the load flow are interval values instead of real values. The variations in distribution line parameters, load demand and power generated by distributed generation have been considered. The effect of DG on distribution system performance is verified and analyzed as given below.

6.2. Normal DLF with Distributed Generation:

A Distribution Load Flow (DLF) has been developed to observe the behavior of distribution system with distributed generation. In this section, the effect of DG on distribution voltage profile, distribution power losses, and distribution protection schemes are observed with the help of obtained results. The input parameters are considered to be crisp values.

6.2.1 DG placement in Distribution System:

Proper placement of distributed generation leads to improvement in voltage profile and minimization of distribution losses. For proper location of DG, a sensitivity analysis method has been used and DG's are located based on the power loss sensitivity obtained in sensitivity analysis [22]. In this sensitivity analysis, the active power loss per unit MW is calculated at each bus. Now with the obtained results (active power loss sensitivity), the first DG has to be placed at the bus where the power loss sensitivity is more (negative maxima). [i.e., If 1 unit DG power is supplied at that bus; the power loss is reduced by that value shown in the Table 6.1. Now again this sensitivity analysis is executed on the modified distribution network configuration (after placing DG). This will give the power loss sensitivity for the modified network and the second DG has to be placed at the bus where the power loss sensitivity is maximum. In this way, the sensitivity analysis program is executed after placing DG one at a time.

In this work, after executing sensitivity analysis on the 33 bus radial distribution system with no DG in the network, the power loss sensitivity is observed to be highest at bus number 18. Therefore, one DG has to be kept at bus number 18. Again the sensitivity analysis is executed on the distribution system with DG at bus 18. Now the power loss sensitivity is more at bus number 33. Therefore, another DG has to be placed at bus number 33. In this way, after executing sensitivity analysis on the distribution network, the DG locations obtained are at bus numbers 18, 33, 25, 8 and so on. These bus numbers are obtained in a sequential manner after executing sensitivity analysis program placing DG one at a time. Table 6.1 shows the power loss sensitivity obtained from the sensitivity analysis program. From the table, the bold face values indicate the bus where maximum power loss is occurring. Based on that value, the DG has to be placed. With proper placement of DG obtained from the sensitivity analysis, the DLF is executed on the distribution system with modified network configuration (after DG placement). From table 6.1, after adding DG at bus 25, the power loss sensitivity is observed to be very small. If another DG is added at bus 8, the reduction in power loss is very low. Considering the cost factor in to account, the DG placement is restricted to buses 18, 25 and 33 only.

Table 6.1 Power loss sensitivity obtained from sensitivity analysis

Bus No.	No DG	DG 18	DG 33	DG 25	DG 8	DG 22
2	-0.00479	-0.00357	-0.0024	-0.00132	-0.00027	0.000751
3	-0.02791	-0.01999	-0.0124	-0.00547	0.001201	0.002219
4	-0.04029	-0.0273	-0.01492	-0.00794	0.002919	0.003934
5	-0.05272	-0.03434	-0.01692	-0.00991	0.005321	0.006331
6	-0.07975	-0.04914	-0.02052	-0.01345	0.01129	0.012286
7	-0.08341	-0.04977	-0.0211	-0.01402	0.013075	0.014067
8	-0.09344	-0.04985	-0.02116	-0.01408	0.02098	0.021955
9	-0.10512	-0.04692	-0.01841	-0.01136	0.023509	0.024478
10	-0.11609	-0.04304	-0.01474	-0.00775	0.026859	0.027822
11	-0.11792	-0.04216	-0.01391	-0.00693	0.027625	0.028586
12	-0.12115	-0.04024	-0.01209	-0.00514	0.029288	0.030246
13	-0.13278	-0.03126	-0.00363	0.003191	0.037008	0.037949
14	-0.13667	-0.02738	5.62E-06	0.006773	0.040317	0.04125
15	-0.13955	-0.02226	0.004831	0.011525	0.044716	0.04564
16	-0.14236	-0.01525	0.011427	0.018022	0.050738	0.051648
17	-0.146	-0.00179	0.024067	0.030463	0.062231	0.063115
18	-0.14719	0.006121	0.031527	0.037811	0.069046	0.069915
19	-0.00554	-0.00432	-0.00314	-0.00206	-0.00101	0.001824
20	-0.01075	-0.00951	-0.00832	-0.00723	-0.00617	0.013277
21	-0.0117	-0.01046	-0.00927	-0.00817	-0.00711	0.016819
22	-0.01253	-0.01129	-0.01009	-0.00899	-0.00793	0.023678
23	-0.03368	-0.02567	-0.01799	-0.00566	0.00101	0.002028
24	-0.04422	-0.03603	-0.02818	-0.005	0.001659	0.002676
25	-0.04956	-0.04127	-0.03333	0.0005	0.007081	0.008086
26	-0.08282	-0.05202	-0.02066	-0.01358	0.011167	0.012164
27	-0.08686	-0.05581	-0.0206	-0.01353	0.011219	0.012216
28	-0.10138	-0.06939	-0.01944	-0.01238	0.012314	0.013309
29	-0.11179	-0.07912	-0.01778	-0.01075	0.013863	0.014854
30	-0.11721	-0.08419	-0.01587	-0.00887	0.015644	0.016632
31	-0.1246	-0.09109	-0.00928	-0.00238	0.021775	0.022748
32	-0.12615	-0.09253	-0.00649	0.000362	0.024363	0.02533
33	-0.12654	-0.0929	-0.00232	0.004465	0.02823	0.029188

6.2.2 Effect on Distribution Power Loss:

In the above section, it is cleared that placing DG will reduce the power transfer from main substation. Thereby the branch currents will be reduced and therefore the power loss in the distribution system is reduced. Reducing power loss in the distribution network with the help of DG is possible only when the DG is properly placed in the distribution system. The losses in the distribution system vary along with the DG location. This is shown in the graph below.

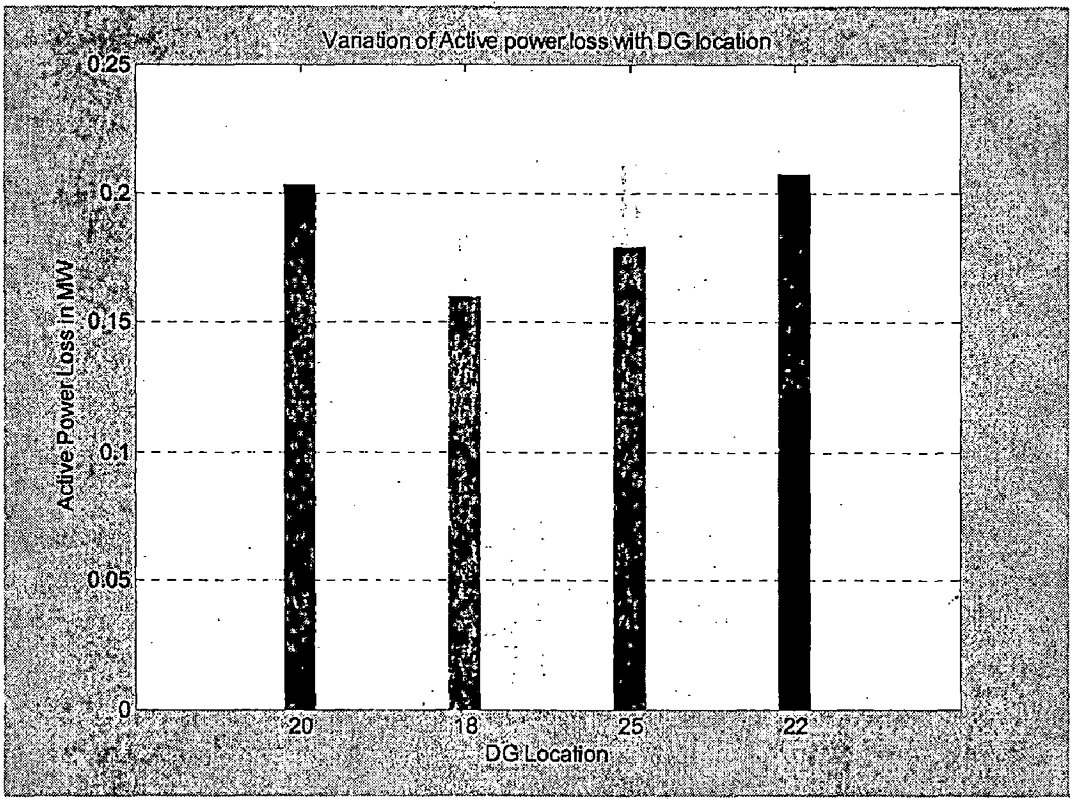

Fig. 6.1 Variation of active power loss with DG location

From Fig. 6.1, it is clear that the distribution power loss is a function of DG location and at the same time, the distribution power loss is a function of number of DG's in the distribution network. But this is not always true. Because, the distribution power loss will reduce to a minimum extent and then it will increase again because the DG will try to feed back power to the other loads connected to main distribution system on the other side. Thereby, an increase in the distribution power loss. The variation in power loss with the number of DG's in the distribution system is shown in Fig. 6.2.

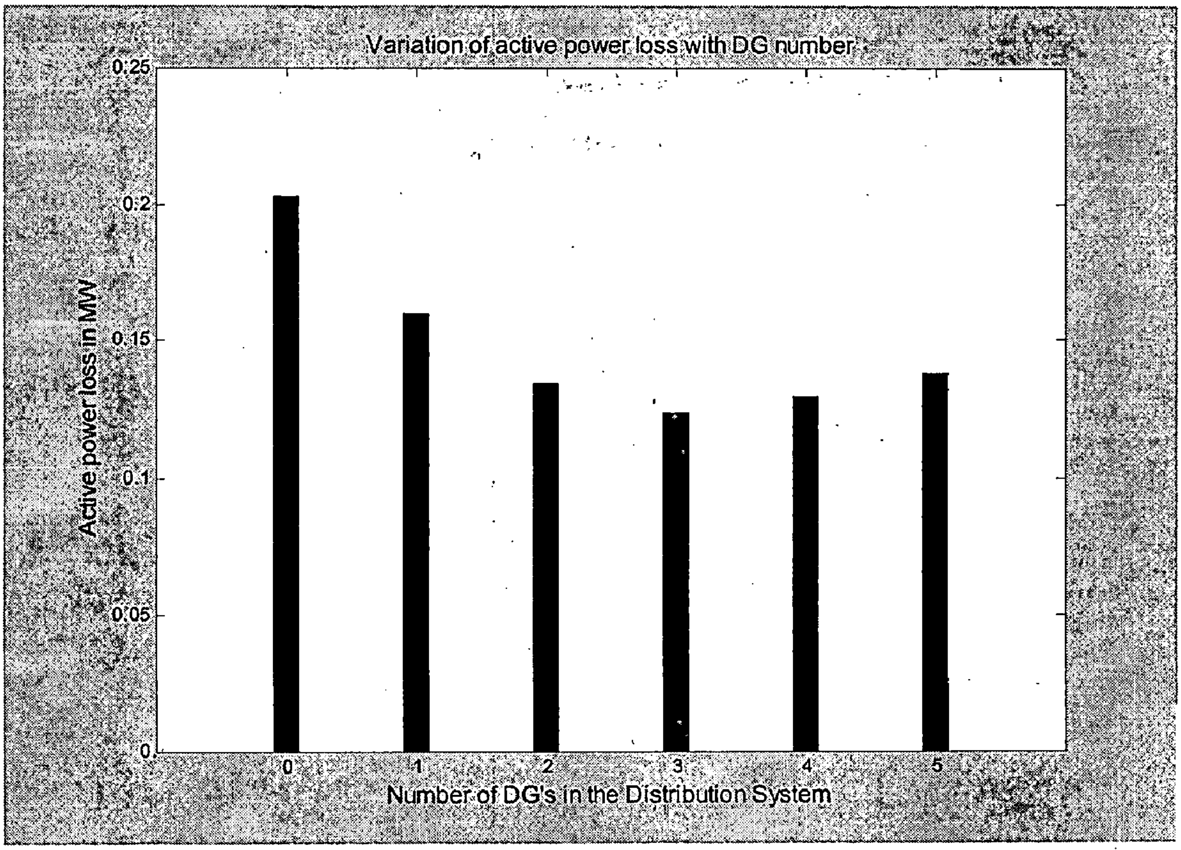

Fig. 6.2 Variation of active power loss with number of DG's

From the above figure, it is clear that the distribution power loss will reduce as the number of DG's in the distribution system is increased; it reaches minima and then increases again. Therefore, a proper coordination is required between the DG location and the number of DG's in the distribution system so as to reduce the distribution power loss and at the same time the voltage profile has to be maintained within the permissible limits.

In view of all above facts, only 3 DG's are considered in this work, to provide good voltage profile and reduced distribution power loss. If 4^{th} DG is added to the distribution network, the voltages will improve but with an increment in distribution loss. Taking into account, the cost of DG installation in the distribution system, putting 3 DG's at bus numbers 18, 25, and 33 is advisable to maintain good voltage profile with reduced power loss.

6.2.3 Effect on Distribution Voltage Profile

As discussed in the earlier sections, a distributed generation can meet the load demand in the distribution system. This will reduce the amount of power transferred from the main substation and thereby the branch current will reduce to some extent and therefore the voltage drop is reduced and finally, the voltage at all the buses is improved.

A graph representing the improvement in the voltage profile after placing DG's at the specified location is shown in Fig.6.3.

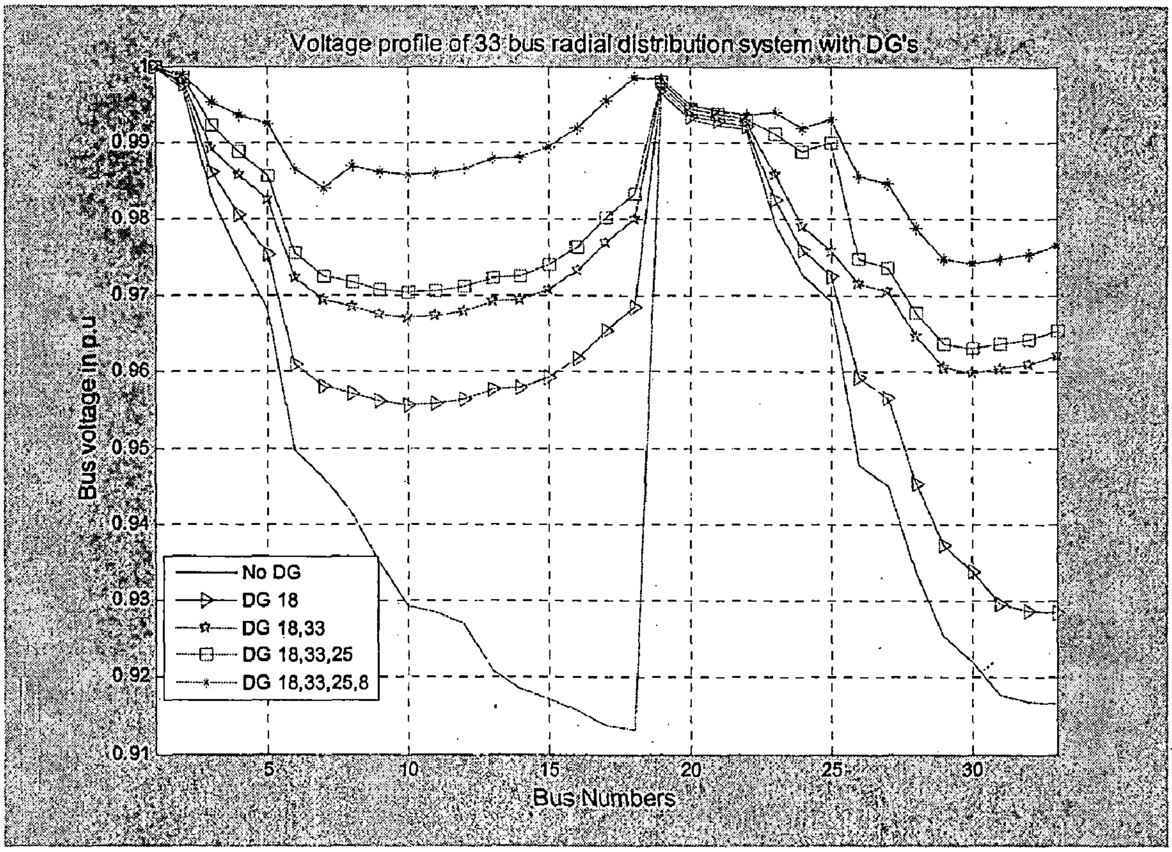

Fig. 6.3 Voltage Profile of 33 bus radial distribution system with DG's

From the Fig. 6.3, it is clearly observed that the bus voltages are substantially improved as compared to the bus voltages before the addition of DG's. From this graph, it is also clear that the improvement in the voltage profile depends on the number of DG's present in the distribution system. From the graph, it is observed that the bus voltage where DG is connected is not maintained at 1 p.u. This is because of the WTGU employed for DG uses induction generator for generating active power. These induction generators consume reactive power from the main distribution system and thereby the voltage will be less than 1 p.u. The reactive power consumed by the DG, depends upon the corresponding bus voltage. The distribution load flow results are given in table 6.2 (DLF with out any DG) and table 6.3 (DLF with DG's).

Table 6.2 DLF results of 33 bus radial distribution system with no DG

Bus No.	Voltage (p.u)	Angle (degrees)	Complex Injected Power (p.u)
1	1.0000	0	3.9176 + 2.4351i
2	0.9970	0.0145	3.9054 + 2.4288i
3	0.9829	0.0960	3.3924 + 2.1814i
4	0.9755	0.1616	2.3429 + 1.6740i
5	0.9681	0.2283	2.2042 + 1.5845i
6	0.9497	0.1338	2.1060 + 1.5215i
7	0.9462	-0.0965	1.0933 + 0.5215i
8	0.9413	-0.0604	0.8885 + 0.4199i
9	0.9351	-0.1335	0.6843 + 0.3169i
10	0.9292	-0.1960	0.6208 + 0.2944i
11	0.9284	-0.1888	0.5602 + 0.2742i
12	0.9269	-0.1773	0.5143 + 0.2439i
13	0.9208	-0.2686	0.4517 + 0.2069i
14	0.9185	-0.3473	0.3909 + 0.1709i
15	0.9171	-0.3849	0.2706 + 0.0906i
16	0.9157	-0.4082	0.2103 + 0.0804i
17	0.9137	-0.4855	0.1500 + 0.0600i
18	0.9131	-0.4950	0.0900 + 0.0400i
19	0.9965	0.0037	0.3610 + 0.1609i
20	0.9929	-0.0633	0.2701 + 0.1202i
21	0.9922	-0.0827	0.1800 + 0.0801i
22	0.9916	-0.1030	0.0900 + 0.0400i
23	0.9794	0.0651	0.9364 + 0.4551i
24	0.9727	-0.0237	0.8413 + 0.4010i
25	0.9694	-0.0674	0.4200 + 0.2000i
26	0.9477	0.1733	0.9481 + 0.9723i
27	0.9452	0.2295	0.8848 + 0.9456i
28	0.9337	0.3124	0.8135 + 0.9106i
29	0.9255	0.3903	0.7457 + 0.8838i
30	0.9220	0.4956	0.6218 + 0.8118i
31	0.9178	0.4112	0.4202 + 0.2103i
32	0.9169	0.3881	0.2700 + 0.1400i
33	0.9166	0.3804	0.0600 + 0.0400i

Active power loss = 0.2027 MW.

Table 6.3 DLF results with DG at 18, 25, 33 in the distribution system

Bus No.	Voltage (p.u)	Angle (degrees)	Complex Injected Power (p.u)
1	1.0000	0	1.1385 + 2.8643i
2	0.9985	0.0754	1.1330 + 2.8615i
3	0.9923	0.4842	0.6490 + 2.6287i
4	0.9888	0.7128	0.5144 + 1.9764i
5	0.9855	0.9498	0.3853 + 1.8917i
6	0.9756	1.4312	0.3065 + 1.8455i
7	0.9726	1.4334	0.1988 + 0.6870i
8	0.9717	1.5914	-0.0028 + 0.5865i
9	0.9707	1.8382	-0.2047 + 0.4852i
10	0.9703	2.0968	-0.2667 + 0.4637i
11	0.9705	2.1380	-0.3271 + 0.4436i
12	0.9711	2.2142	-0.3728 + 0.4134i
13	0.9724	2.6137	-0.4360 + 0.3758i
14	0.9726	2.8172	-0.4973 + 0.3391i
15	0.9741	2.9975	-0.6191 + 0.2576i
16	0.9764	3.2060	-0.6816 + 0.2457i
17	0.9801	3.7915	-0.7467 + 0.2189i
18	0.9832	4.0173	-0.8100 + 0.1964i
19	0.9980	0.0546	0.3610 + 0.1609i
20	0.9944	-0.0022	0.2701 + 0.1202i
21	0.9937	-0.0215	0.1800 + 0.0801i
22	0.9931	-0.0418	0.0900 + 0.0400i
23	0.9910	0.5799	0.0338 + 0.6067i
24	0.9889	0.7768	-0.0580 + 0.5553i
25	0.9900	1.0154	-0.4800 + 0.3537i
26	0.9747	1.5161	0.0453 + 1.1355i
27	0.9737	1.6358	-0.0170 + 1.1093i
28	0.9678	2.0987	-0.0852 + 1.0771i
29	0.9638	2.4635	-0.1513 + 1.0517i
30	0.9631	2.6824	-0.2749 + 0.9799i
31	0.9637	3.0011	-0.4773 + 0.3776i
32	0.9642	3.1252	-0.6283 + 0.3064i
33	0.9654	3.3229	-0.8400 + 0.2037i

Active power loss = 0.1235 MW.

6.2.4 Effect on Distribution Protection Scheme:

As discussed in section 3.5, distribution protection scheme is affected in two ways. One is flow of current in opposite direction from DG to the main substation and second one is maloperation of relay even under fault conditions. Table 6.4, shows the changes in the current directions and current magnitude before and after placing DG's.

From the table 6.4, it is clear that the direction of current from bus 18, 25, and 33 is reversed towards the main substation and this will lead to reversal of power flow as already explained in section 3.5. In view of this effect, the distribution system will require re-coordination of the protective devices due to the changes in the current in the distribution system.

From the same table, it is also clear that the magnitude of branch currents towards the substation from the DG buses is reduced compared to the previous values. This reduction in branch current will leads to maloperation of relays during faulty conditions. Normally, the distribution system has been already designed to operate for particular condition (without DG). But with this penetration of DG's in the distribution system, the amount of power transferred by the main substation is reduced and thereby the current in the branches from the main substation also reduced. Therefore, the previously equipped relays may not operate for the faults with in its region and will require readjustment of their setting.

Table 6.4 Branch currents in the distribution system with/without DG's

Branch Number	Branch currents With No DG	Branch currents With DG 's at 18, 25 & 33
1	3.9176 - 2.4351i	1.1385 - 2.8643i
2	3.4550 - 2.2135i	0.6764 - 2.6435i
3	2.4067 - 1.7093i	0.5450 - 1.9921i
4	2.2835 - 1.6277i	0.4227 - 1.9127i
5	2.2214 - 1.5969i	0.3613 - 1.8833i
6	1.1546 - 0.5532i	0.2220 - 0.7011i
7	0.9434 - 0.4471i	0.0139 - 0.6034i
8	0.7310 - 0.3407i	**-0.1947 - 0.5063i**
9	0.6669 - 0.3191i	**-0.2571 - 0.4877i**
10	0.6024 - 0.2974i	**-0.3197 - 0.4693i**
11	0.5541 - 0.2649i	**-0.3672 - 0.4402i**
12	0.4895 - 0.2269i	**-0.4303 - 0.4065i**
13	0.4245 - 0.1886i	**-0.4936 - 0.3734i**
14	0.2944 - 0.1007i	**-0.6209 - 0.2973i**
15	0.2290 - 0.0894i	**-0.6829 - 0.2903i**
16	0.1637 - 0.0671i	**-0.7454 - 0.2733i**
17	0.0982 - 0.0447i	**-0.8078 - 0.2570i**
18	0.3623 - 0.1615i	0.3619 - 0.1608i
19	0.2719 - 0.1213i	0.2717 - 0.1209i
20	0.1813 - 0.0809i	0.1812 - 0.0806i
21	0.0907 - 0.0405i	0.0906 - 0.0403i
22	0.9567 - 0.4636i	0.0403 - 0.6118i
23	0.8647 - 0.4126i	**-0.0510 - 0.5623i**
24	0.4330 - 0.2068i	**-0.4784 - 0.3658i**
25	1.0035 - 1.0229i	0.0773 - 1.1633i
26	0.9401 - 0.9967i	0.0151 - 1.1393i
27	0.8766 - 0.9705i	**-0.0472 - 1.1154i**
28	0.8122 - 0.9494i	**-0.1100 - 1.0970i**
29	0.6820 - 0.8747i	**-0.2375 - 1.0298i**
30	0.4595 - 0.2258i	**-0.4740 - 0.4172i**
31	0.2955 - 0.1507i	**-0.6333 - 0.3528i**
32	0.0657 - 0.0432i	**-0.8564 - 0.2611i**

6.3. Interval DLF with Distributed Generation:

In this section, a new distribution load flow has been developed for distribution system with distributed generation, where the input parameters are interval values instead of real values. In this the variation in resistance and reactance of the distribution lines are taken as $\pm$ 5% and similarly the variation in the load demand is taken as $\pm$ 5%. Along with these variations, the power generated by the distributed generation has also been varied.

Here the DG's are located at bus number 18, 25, and 33. At bus 18 the DG is Pitch regulated fixed speed WTGU and at 25, 33, the DG's are Semi variable speed WTGU. Three cases has been considered in this section, first one is running the interval distribution load flow with distributed generation keeping the power generated by all DG's fixed at 0.9 MW, secondly the power generated by the DG is varied in between [0.8 – 0.9 MW] for pitch regulated fixed speed WTGU and [0.3 – 0.9 MW] for semi variable speed WTGU. From the second case, it is to be observed that the upper and lower limits obtained by the IDLF are very large. In case 3, this large bounded output of the IDLF has been reduced to acceptable limits by dividing the large interval input [0.3 – 0.9 MW] into multi intervals of smaller interval value. All these cases are tested on the 33 bus radial distribution system. The results obtained from the Interval Distribution Load Flow are verified with the Repeated Load Flow (RLF) executed for 100,000 iterations satisfying all the above conditions. In RLF, the variations in the distribution line parameters, load demand and DG generation are generated randomly within the limits specified above and are given as inputs for the RLF. In each case, again three sub cases have been considered. First one is varying the distribution line parameters only; second one is varying the load demand only and finally, varying both line parameters and load demand simultaneously. From the results it is observed that, the bounds obtained from the IDLF increase as the interval input increases and also observed that the interval bounds from the IDLF depends upon the size of the input intervals. i.e, larger the size of the input interval, larger will be the output bounds. This is clearly observed from the results shown below.

Case 1: Fixed DG power output (0.9 MW):

In this case, the active power output (Pe) of the DG's are fixed to 0.9 MW and the reactive power consumed by the DG's depends upon the bus voltage where the

DG is connected. The graph shown in Fig. 6.4 represents the voltage profile of the test distribution system with fixed DG power output at 0.9 MW and the distribution line parameters R, X are taken as interval inputs to the load flow. The results obtained from the interval load flow are verified with the repeated load flow executed for 100,000 iterations. The maximum error observed between the IDLF and RLF is 0.76%.

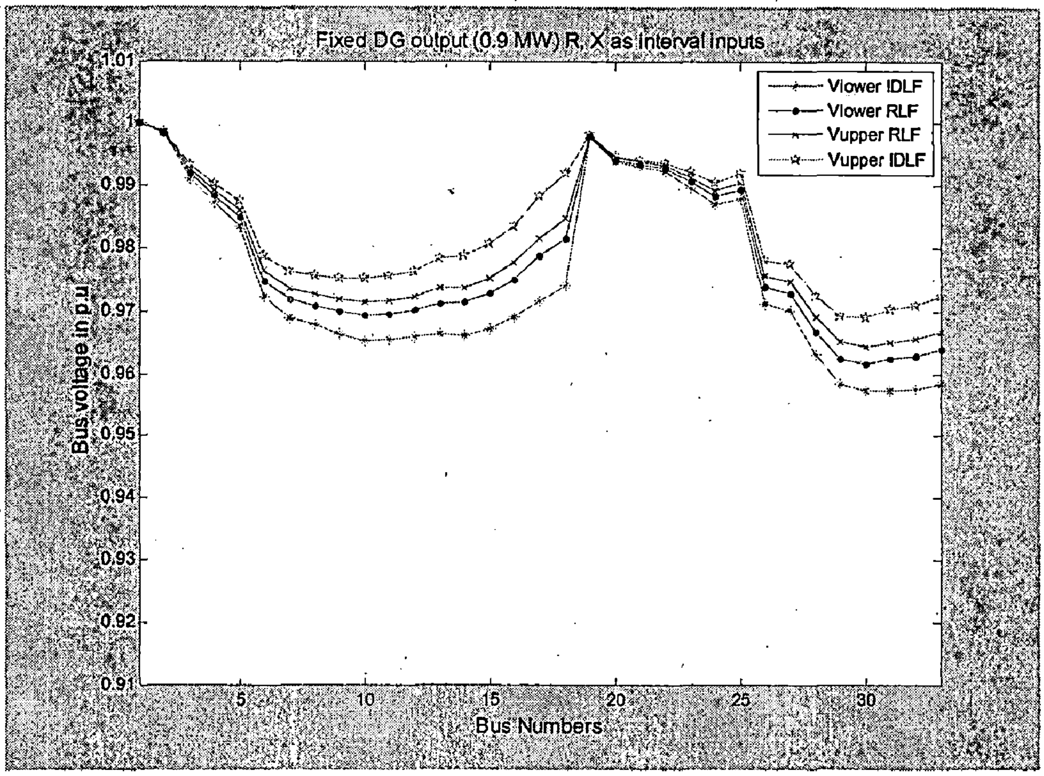

Fig. 6.4 Voltage profile of DS with Fixed DG output, R,X as interval inputs

In the sub case 2, the distribution line parameters are kept as crisp values and the load demand in the distribution system are taken as interval inputs varying in between ± 5%. Figure 6.5 shows the voltage profile of the distribution system with fixed DG output and the load demand as interval inputs. In this case, the maximum error obtained is 0.75%. Therefore the results obtained from the IDLF are almost same as the actual values.

In the sub case 3, both the distribution line parameters and load demand are taken as interval inputs, keeping the DG output fixed at 0.9 MW. Here the maximum error obtained between IDLF and RLF is 1.48%. This is due to the increase in the number of interval inputs given to the IDLF. By this, an observation has been made that the IDLF error will increase with the number of interval inputs given to the IDLF. The graph representing the voltage profile of the 33 bus radial distribution system, with fixed DG output and both R, X & Pl, Ql as interval inputs are shown in Fig.6.6.

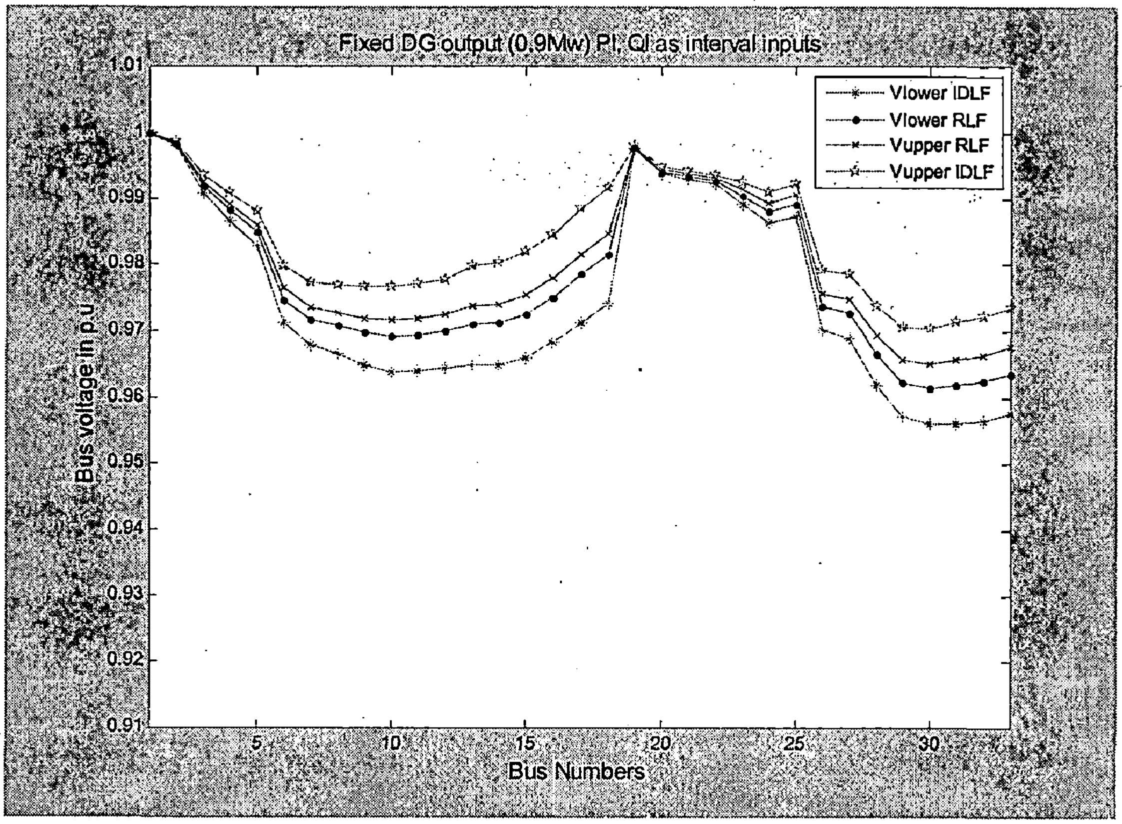

Fig. 6.5 Voltage profile of DS with Fixed DG output, P_L, Q_L as interval inputs

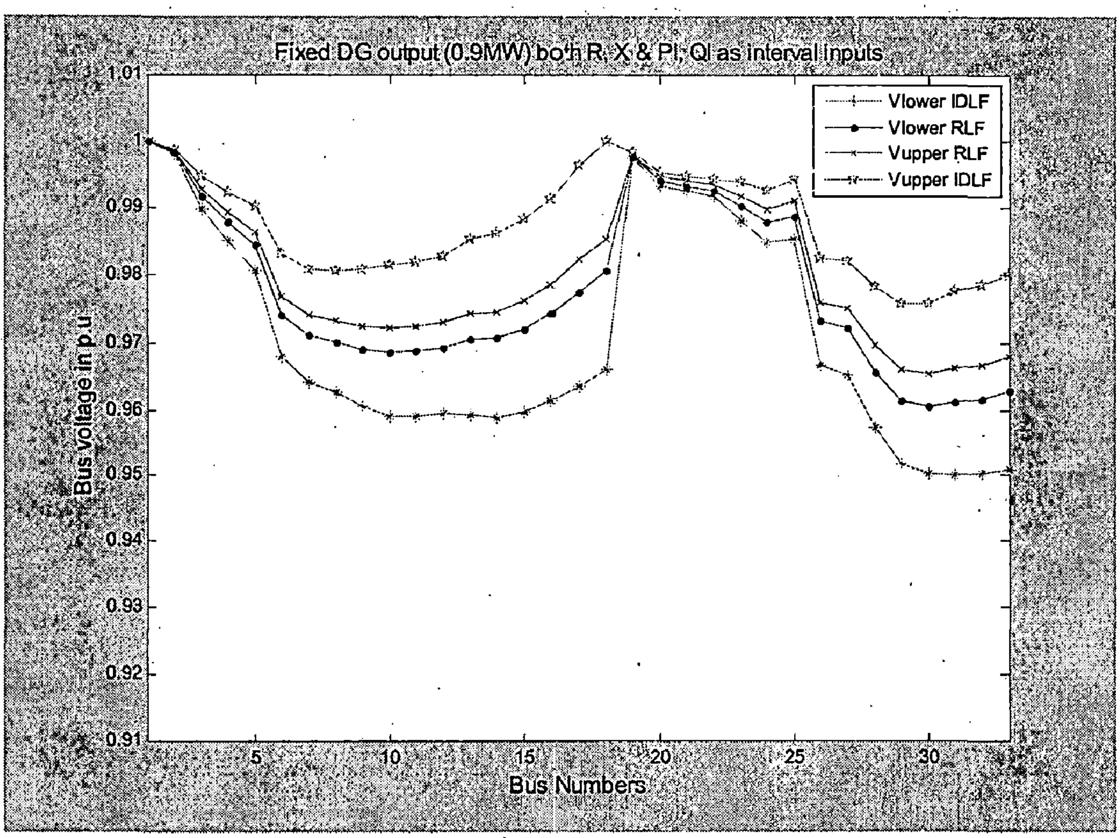

Fig. 6.6 Voltage profile of DS with Fixed DG, both R, X & P_L, Q_L as interval inputs

Case 2: Varying DG power output:

In this case, the DG output has been considered to be varying with in its specified limits. For pitch regulated fixed speed WTGU, the power output is varied between [0.8 – 0.9 MW]. For semi variable speed WTGU, the power output is varied between [0.3 – 0.9 MW]. Here also three sub cases have been considered. But mainly the voltage profile of the distribution system with varying DG outputs and both distribution line parameters (R, X) and load demand (P_L, Q_L) as interval inputs, is shown in Fig. 6.7.

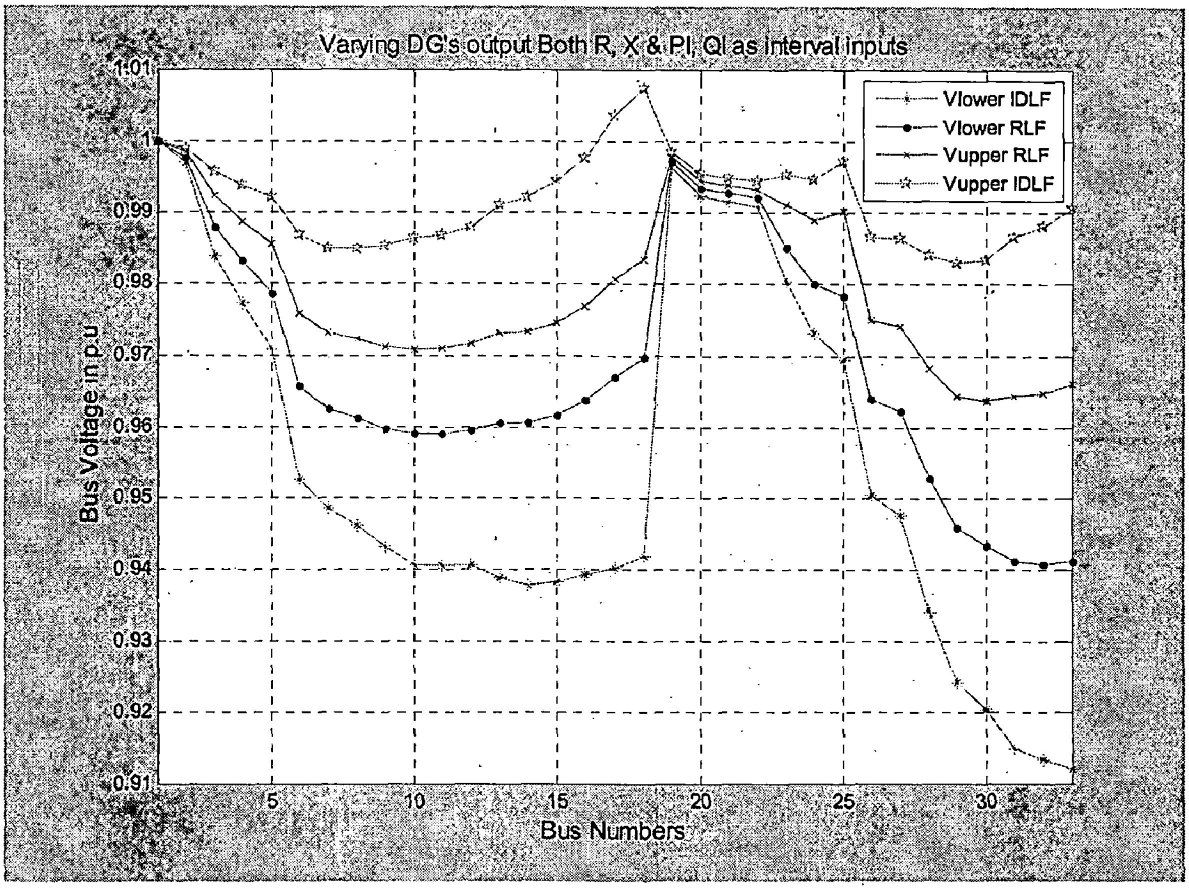

Fig. 6.7 Voltage profile obtained using IDLF with varying DG output

The above figure shows the voltage profile obtained with IDLF and RLF for the distribution system with varying DG outputs and both distribution line parameters and load demand as interval inputs. The maximum error obtained in this case is 3 %. This increase in error is due to larger size of interval inputs (DG output [0.3 – 0.9]).

Case 3: Multi State DG power output:

In the above case 2, due to larger size of the DG power variation, the result obtained from IDLF gives larger bounds. In order to reduce the output interval size and to improve the accuracy of the IDLF results, the large DG output variation is divided into multiple intervals with small interval values. Here the larger interval [0.3 – 0.9] is divided into six smaller intervals [0.3 – 0.4], [0.4 – 0.5], [0.5 – 0.6], [0.6 – 0.7], [0.7 – 0.8], [0.8 – 0.9]. These multiple intervals are given as inputs for the IDLF one at a time and all possible combinations of the intervals has to be given to the IDLF. For example, the possible combinations obtained for 2 DG's with six states are 36 combinations. i.e., S^N , where S is the number of states of DG and N is the number of DG's in the distribution system. In this work, 3 DG's are placed in the distribution system; therefore a total of 216 combinations of DG power output have been given as input to the IDLF. After executing 216 combinations, the lower limit is the lowest of all the 216 combinations and the upper limit is the highest of all the 216 combinations.

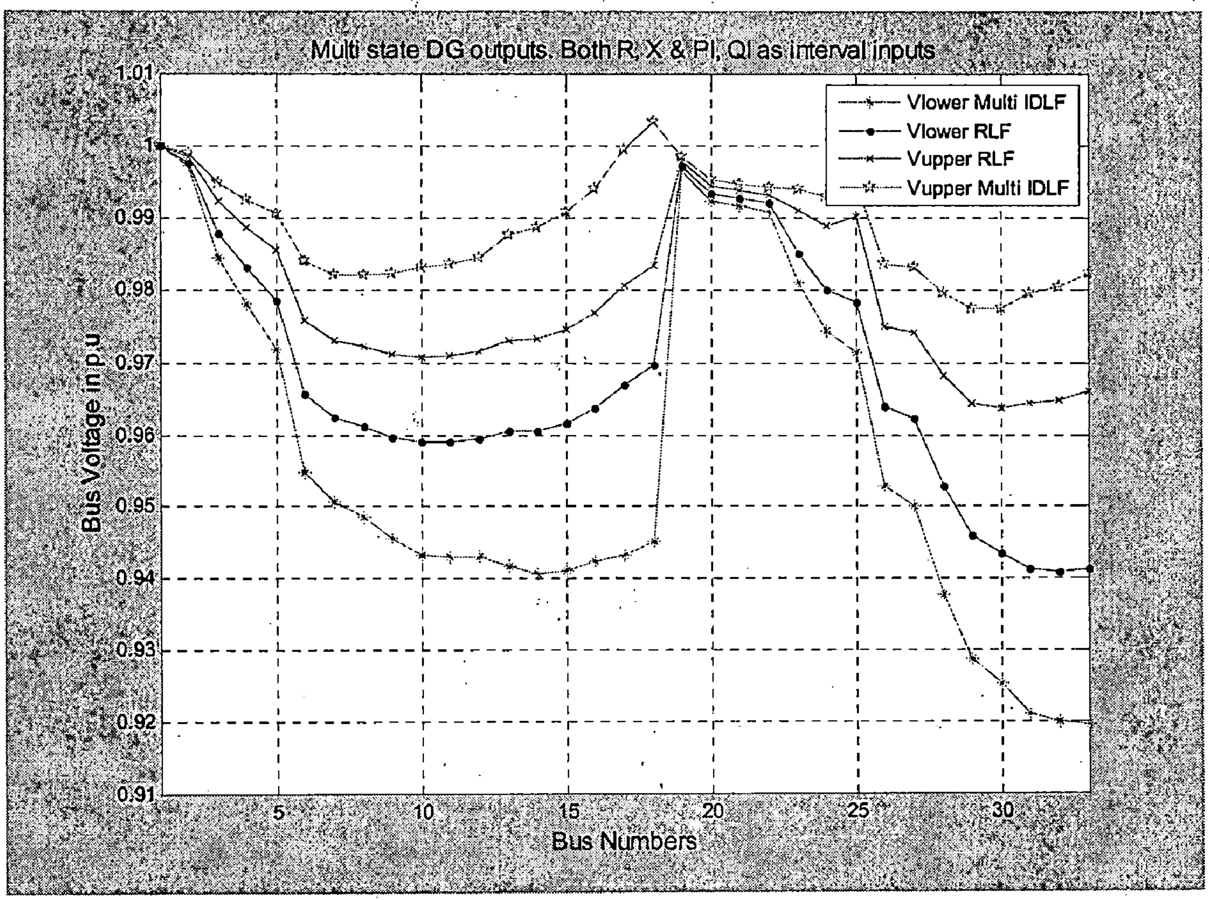

Fig. 6.8 Voltage profile obtained using IDLF with Multi state DG power output

Fig 6.8 shows the voltage profile of the distribution system obtained by IDLF with multiple state DG power output along with R, X & P_L, Q_L as interval inputs and is verified with the RLF. The maximum error obtained in this case is 2.4%, which is less than the error obtained in case 2. This shows that the IDLF is made more accurate by splitting the larger intervals in to smaller intervals. The IDLF and RLF results for the above 3 cases with 3 sub cases each is given in appendix [C].

The computational time required for RLF for this 33 bus radial distribution system to run 100,000 iterations is approximately 7 hours. Whereas the IDLF will give the results with a marginal acceptable error in 3 seconds on a 2.88 GHz, Intel P4 configured system. A large amount of computational time is reduced, by using IDLF. This will prove to be helpful in the Power System Planning.

Conclusion

From the results, it is observed that by adding DG to the distribution network, the distribution losses are reduced and at the same time voltage profile is improved. The distribution losses are a function of DG location and number of DG's in the distribution system. Therefore, proper DG location will reduce the distribution loss to a larger amount. For this, a sensitivity based method has been employed in this work. From the sensitivity analysis, DG locations and number are obtained. They are located at buses 18, 25 and 33 giving least power loss and improved voltage profile. But with the placement of DG in distribution system, the distribution protection scheme is affected due to the reversal in flow of power in the distribution network. This will lead to the rearrangement of relay settings. All the above observations are made with the help of distribution load flow. A distribution load flow with DG's has been developed. The wind turbine generating units are used as DG and these are modeled as 'P-Q' sources. These WTGU will generate active power and consume reactive power from the main distribution system.

As a part of dissertation work, an Interval Distribution Load Flow for Distribution system with DG has been developed, so as to consider the variations in the distribution line parameters, load demand and power generated by the distributed generation. This IDLF provides the results in terms of intervals with in which exact solution must lie. From the IDLF, it is observed that the results obtained gives larger bounds if the number of interval inputs is more and at the same time the output will depend upon the size of the input interval. i.e., larger the size of the input interval, larger will be the size of the output interval. This shows that the accuracy of the IDLF is reduced with large size interval inputs. The accuracy of the results obtained from IDLF has been improved by dividing the large interval into a set of smaller intervals. The results obtained from the IDLF are verified with the results obtained from repeated load flow. The IDLF will reduce the computational time to a large extent when compared with the computational time of the repeated load flow.

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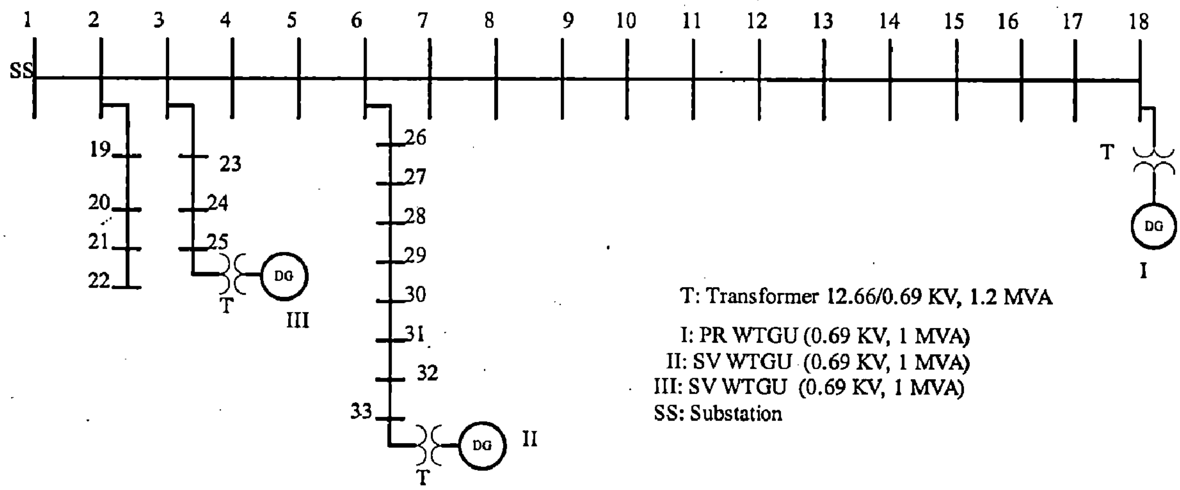
[24] "Wind Turbine Grid Connection and Interaction", Deutsches Windenergie-Institute, Tech-wise A/S, DM Energy.

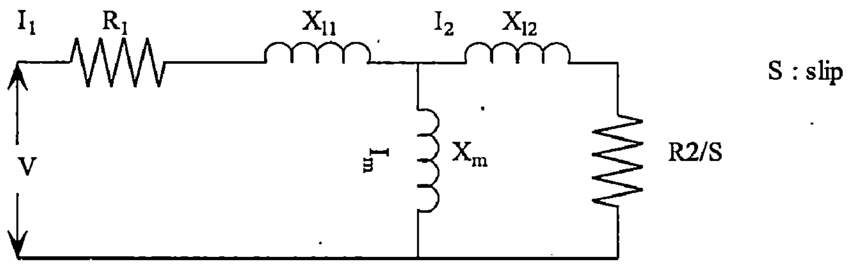
- **33 bus radial distribution network diagram with DG:**



- **WTGU Induction Generator Parameters [15]:**

Type of WTGU	Pitch Regulated WTGU	Semi variable WTGU
Rating (MW)	1.0	1.0
Rated (KV)	0.69	0.69
R1 (p.u)	0.005986	0.005671
R2 (p.u)	0.01690	0.00462
X_{l1} (p.u)	0.08212	0.05250
X_{l2} (p.u)	0.107225	0.096618
X_m (p.u)	2.5561	2.8985
X_c (p.u)	2.5561	2.8985

- **Induction Generator Equivalent Diagram:**



➢ **Distribution Network Line parameters:**

Branch No.	From Bus	To Bus	Line R (Ohms)	Line X (Ohms)
1	1	2	0.0922	0.0470
2	2	3	0.4930	0.2511
3	3	4	0.3660	0.1864
4	4	5	0.3811	0.1941
5	5	6	0.8190	0.7070
6	6	7	0.1872	0.6188
7	7	8	0.7114	0.2351
8	8	9	1.0300	0.7400
9	9	10	1.0440	0.7400
10	10	11	0.1966	0.0650
11	11	12	0.3744	0.1238
12	12	13	1.4680	1.1550
13	13	14	0.5416	0.7129
14	14	15	0.5910	0.5260
15	15	16	0.7463	0.5450
16	16	17	1.2890	1.7210
17	17	18	0.7320	0.5740
18	2	19	0.1640	0.1565
19	19	20	1.5042	1.3554
20	20	21	0.4095	0.4784
21	21	22	0.7089	0.9373
22	3	23	0.4512	0.3083
23	23	24	0.8980	0.7091
24	24	25	0.8960	0.7011
25	6	26	0.2030	0.1034
26	26	27	0.2842	0.1447
27	27	28	1.0590	0.9337
28	28	29	0.8042	0.7006
29	29	30	0.5075	0.2585
30	30	31	0.9744	0.9630
31	31	32	0.3105	0.3619
32	32	33	0.3410	0.5302

Base voltage: 12.66 KV

Base MVA: 1 MVA

- **Distribution Network Load parameters:**

Bus No.	Initial bus voltage (p.u)	Initial Voltage angle (degrees)	Active power Demand (MW)	Reactive power Demand (MW)	DG Type
1	1	0	0	0	-
2	1	0	0.1000	0.0600	-
3	1	0	0.0900	0.0400	-
4	1	0	0.1200	0.0800	-
5	1	0	0.0600	0.0300	-
6	1	0	0.0600	0.0200	-
7	1	0	0.2000	0.1000	-
8	1	0	0.2000	0.1000	-
9	1	0	0.0600	0.0200	-
10	1	0	0.0600	0.0200	-
11	1	0	0.0450	0.0300	-
12	1	0	0.0600	0.0350	-
13	1	0	0.0600	0.0350	-
14	1	0	0.1200	0.0800	-
15	1	0	0.0600	0.0100	-
16	1	0	0.0600	0.0200	-
17	1	0	0.0600	0.0200	-
18	1	0	0.0900	0.0400	PRWTGU
19	1	0	0.0900	0.0400	-
20	1	0	0.0900	0.0400	-
21	1	0	0.0900	0.0400	-
22	1	0	0.0900	0.0400	-
23	1	0	0.0900	0.0500	-
24	1	0	0.4200	0.2000	-
25	1	0	0.4200	0.2000	SVWTGU
26	1	0	0.0600	0.0250	-
27	1	0	0.0600	0.0250	-
28	1	0	0.0600	0.0200	-
29	1	0	0.1200	0.0700	-
30	1	0	0.2000	0.6000	-
31	1	0	0.1500	0.0700	-
32	1	0	0.2100	0.1000	-
33	1	0	0.0600	0.0400	SVWTGU

➢ Interval Arithmetic using INTLAB in Matlab:

Real intervals in INTLAB are stored by the infimum and supremum, whereas complex intervals are stored by the midpoint and radius. For example the interval **x** = [-1 ,1] is entered using infimum and supremum as

```
>> x = infsup(-1,1);
```

But the same interval could be entered using the midpoint and radius as

```
>>x = midrad(0,1);
```

Since complex intervals are stored as a circular region using the midpoint and radius is more accurate to input such intervals in this way. For example the circular region with midpoint at 1 + i and radius 1 is entered using

```
>>y = midrad(1+i, 1);
```

If a rectangular region is entered using the infimum and supremum then the region is stored with an overestimation as the smallest circular region enclosing it. The region with a infimum of 1+i and a supremum of 2+2i is entered as

```
>>z = infsup(1+i, 2+2i);
```

However it is stored by the midpoint and radius notation as

```
>> midrad(z)
Intval z =
< 1.50000000000000 + 1.50000000000000i , 0.70710678118655 >
```

The infimum, supremum, midpoint and radius of a interval **x** can be obtained with the commands inf(x), sup(x), mid(x), and rad(x) respectively.

The basic operation Addition, Subtraction, Multiplication and Division in INTLAB is evaluated as follows:

Consider two interval inputs x = [2, 5]; y = [1, 4];

Addition:

```
>> x = infsup(2,5);
>> y = infsup(1,4);
>> z = x + y
intval z =
1_.____
>>infsup(z)
intval z =
[   3.0000,   9.0000]
```

Subtraction:

```
>> x=infsup(2,5);
>> y=infsup(1,4);
>> z=x-y
intval z =
  0_.____
>> infsup(z)
intval z =
[  -2.0000,   4.000C
```

Multiplication:

```
>> x=infsup(2,5);
>> y=infsup(1,4);
>> z=x*y
intval z =
  1_.____
>> infsup(z)
intval z =
[   2.0000,  20.0000]
```

Division:

```
>> x=infsup(2,5);
>> y=infsup(1,4);
>> z=x/y
intval z =
  1_.____
>> infsup(z)
intval z =
[   0.5000,   5.0000]
```

Table C.1 Comparison table of IDLF & RLF with R, X as intervals and fixed DG output

Bus No.	V_{lower} with IDLF	V_{lower} with RLF	V_{upper} with IDLF	V_{upper} with RLF
1	1	1	1	1
2	0.998327183	0.998431384	0.998580747	0.998680784
3	0.99125997	0.991973529	0.992675668	0.993352815
4	0.987240436	0.98834344	0.98926537	0.990332666
5	0.983474385	0.984987881	0.986130172	0.987575186
6	0.972181331	0.974699878	0.976488639	0.978904355
7	0.968907249	0.97170733	0.973556243	0.976217328
8	0.967716273	0.970832032	0.972670881	0.975645065
9	0.966221243	0.969819298	0.971729722	0.97518339
10	0.965250174	0.969292218	0.971385095	0.975268762
11	0.965394921	0.969519353	0.971605478	0.975576891
12	0.965803928	0.970093995	0.972158899	0.976297362
13	0.966269305	0.971242991	0.973631795	0.97837938
14	0.966025136	0.971406132	0.973776584	0.978934709
15	0.967108427	0.972850653	0.975273686	0.980772156
16	0.969026968	0.975060818	0.977719301	0.983609938
17	0.971531806	0.978731461	0.981613425	0.988392592
18	0.974046158	0.981647559	0.984734503	0.991938277
19	0.997759729	0.997879443	0.998067068	0.998193097
20	0.993920395	0.994174063	0.994654478	0.994888082
21	0.993161755	0.993470815	0.993952168	0.994239872
22	0.992474149	0.992811074	0.993330139	0.993654599
23	0.989776607	0.990656909	0.991445835	0.992261221
24	0.987196678	0.988423554	0.989340898	0.990460886
25	0.987904922	0.98948295	0.990535873	0.991975808
26	0.971215704	0.973874977	0.975678168	0.978253394
27	0.970003606	0.972835691	0.974699174	0.977474982
28	0.963114761	0.96666069	0.968971246	0.972551262
29	0.958389603	0.962468235	0.965125883	0.96931331
30	0.957278701	0.961688695	0.964338612	0.968951223
31	0.957330543	0.962352126	0.965015263	0.970235372
32	0.957649002	0.962895578	0.965599525	0.971021609
33	0.958458738	0.964024175	0.966712955	0.97253518

The maximum error obtained in this case is 0.76 %

Table C.2 Comparison table of IDLF & RLF with P_L, Q_L as intervals and fixed DG output

Bus No.	V_{lower} with IDLF	V_{lower} with RLF	V_{upper} with IDLF	V_{upper} with RLF
1	1	1	1	1
2	0.998261249	0.998455594	0.998562255	0.998746706
3	0.99087512	0.992001849	0.992654498	0.993738349
4	0.986698127	0.988355432	0.989255799	0.99087648
5	0.982775238	0.98496956	0.98611351	0.98827698
6	0.971139924	0.974628516	0.976452233	0.979952326
7	0.96771885	0.971626761	0.973535298	0.977414109
8	0.966392387	0.970672214	0.972753314	0.9769791
9	0.964794153	0.969592801	0.971896504	0.976622627
10	0.963757321	0.969057453	0.971559986	0.976775486
11	0.963893378	0.969265571	0.97180052	0.977092554
12	0.96429673	0.969796529	0.97239337	0.977819035
13	0.964861451	0.970948954	0.973800031	0.979801916
14	0.964704455	0.971072545	0.974010637	0.980269587
15	0.965946095	0.972528228	0.975498257	0.981947134
16	0.968092351	0.974909236	0.977917604	0.984554694
17	0.971243599	0.978571123	0.981632616	0.98868288
18	0.974090545	0.981605989	0.984686449	0.991891469
19	0.997696414	0.997914504	0.998044551	0.998256404
20	0.993874343	0.994232834	0.99457765	0.994934142
21	0.993119192	0.993506548	0.993903029	0.994282443
22	0.99243451	0.992843757	0.993292995	0.993694247
23	0.989299503	0.990633675	0.991494101	0.992739445
24	0.986555695	0.988191242	0.989565642	0.991104007
25	0.987345262	0.989179464	0.990837565	0.992537654
26	0.97014348	0.973772367	0.975676089	0.979332678
27	0.968893503	0.972712969	0.974739263	0.97859281
28	0.96189753	0.96643916	0.96922917	0.97377885
29	0.957112378	0.962166201	0.965540495	0.970602788
30	0.955980838	0.961313307	0.964942965	0.970262184
31	0.956131012	0.96185391	0.965722295	0.971447741
32	0.956540134	0.962373556	0.966294109	0.972142472
33	0.957581749	0.963509715	0.967448191	0.97342156

The maximum error obtained in this case is 0.75%

Table C.3 Comparison table of IDLF & RLF with both (R,X & P_L, Q_L) as intervals and fixed DG output

Bus No.	V_{lower} with IDLF	V_{lower} with RLF	V_{upper} with IDLF	V_{upper} with RLF
1	1	1	1	1
2	0.998090195	0.998395196	0.998603447	0.998917811
3	0.989875091	0.991767481	0.992865991	0.994741778
4	0.985216647	0.988048151	0.98946743	0.992365474
5	0.980811228	0.984645274	0.986434408	0.990254194
6	0.967920978	0.974061132	0.976925783	0.983205458
7	0.96423029	0.971118034	0.974074833	0.980943242
8	0.9626274	0.970167997	0.973289488	0.980792092
9	0.960573655	0.969056424	0.972519259	0.980903222
10	0.959073906	0.968549911	0.972201874	0.981532029
11	0.959139786	0.968748768	0.972426652	0.981921403
12	0.959410036	0.969325174	0.972992008	0.982785008
13	0.959275496	0.970574168	0.974346635	0.985487219
14	0.958773879	0.970653493	0.974617778	0.986309273
15	0.959691644	0.972019036	0.976216016	0.988319658
16	0.961442794	0.974271581	0.978618672	0.991333299
17	0.963612292	0.977630979	0.982375046	0.996468925
18	0.966014375	0.980827294	0.985411768	1.000134448
19	0.997482837	0.997829305	0.998084965	0.998470083
20	0.993376014	0.994054999	0.994720198	0.995433182
21	0.992562037	0.993331313	0.994068821	0.994840494
22	0.991822848	0.992670212	0.99345927	0.994306994
23	0.988122681	0.990345005	0.991707764	0.993921155
24	0.985032784	0.987949147	0.989868036	0.992635382
25	0.985479346	0.988881727	0.99123707	0.994415615
26	0.966773257	0.97321977	0.976161286	0.982740143
27	0.965315635	0.972202998	0.975198316	0.982212276
28	0.957378982	0.96563311	0.969757815	0.978360735
29	0.951885753	0.961488364	0.966116926	0.975911581
30	0.950402993	0.960610715	0.965520452	0.975932191
31	0.950015726	0.961185976	0.966198881	0.977670606
32	0.950221548	0.961642754	0.96679514	0.978574087
33	0.950957479	0.962842641	0.968033303	0.980166039

The maximum error obtained in this case is 1.48%

Table C.4 Comparison table of IDLF & RLF with R,X as intervals and varying DG output

Bus No.	V_{lower} with IDLF	V_{lower} with RLF	V_{upper} with IDLF	V_{upper} with RLF
1	1	1	1	1
2	0.997410952	0.997790473	0.998531402	0.998843197
3	0.985442218	0.988119321	0.992405122	0.994386932
4	0.979349537	0.983192398	0.988862356	0.991825101
5	0.973421507	0.978607987	0.985550128	0.989549705
6	0.956952735	0.965606437	0.975493848	0.982432166
7	0.953274003	0.962582671	0.972519316	0.980025987
8	0.951458026	0.961395815	0.97160899	0.979649711
9	0.948993656	0.959980075	0.970568608	0.979551805
10	0.947048394	0.958989763	0.970173063	0.980002248
11	0.947029602	0.959120031	0.970397419	0.980360903
12	0.947130575	0.959495801	0.970986495	0.981176042
13	0.946227015	0.960114524	0.972171631	0.983795563
14	0.945375889	0.960055567	0.972415118	0.984653803
15	0.945906363	0.961224939	0.973816009	0.986722764
16	0.947169517	0.963402161	0.976047466	0.989813972
17	0.948256238	0.966811002	0.979774052	0.995324795
18	0.950119568	0.969527138	0.982866267	0.999133085
19	0.996843085	0.997254346	0.997997842	0.998355563
20	0.993000914	0.993560363	0.994522986	0.995050947
21	0.992241753	0.992865609	0.993840779	0.994402781
22	0.99155369	0.992243038	0.99321031	0.993817536
23	0.981924603	0.985368772	0.991139194	0.993632757
24	0.975189857	0.980443591	0.988993576	0.992615093
25	0.971909431	0.978883667	0.990083526	0.994782411
26	0.955018268	0.964161398	0.974636648	0.981981047
27	0.952451668	0.962239	0.973623049	0.98148187
28	0.939939478	0.953023573	0.967605499	0.978166188
29	0.930942287	0.946547999	0.963513392	0.976147236
30	0.927410135	0.944135623	0.962793681	0.976292625
31	0.922416817	0.941999685	0.96333488	0.979032104
32	0.921056286	0.941729434	0.963929393	0.980366078
33	0.919808015	0.942093829	0.965149529	0.982722563

The maximum error obtained in this case is 2.2 %

Table C.5 Comparison table of IDLF & RLF with P_L, Q_L as intervals and varying DG output

Bus No.	V_{lower} with IDLF	V_{lower} with RLF	V_{upper} with IDLF	V_{upper} with RLF
1	1	1	1	1
2	0.997391737	0.997846838	0.998523361	0.998862429
3	0.985340492	0.988128957	0.992444793	0.994489486
4	0.979193577	0.983182677	0.988954468	0.991982845
5	0.973214218	0.978419923	0.985736295	0.989760062
6	0.956680081	0.965445288	0.975878583	0.982711437
7	0.9528829	0.962438166	0.972943434	0.980427322
8	0.95096168	0.961203507	0.972156324	0.980159312
9	0.948436559	0.959798215	0.971281962	0.980124529
10	0.946464937	0.958934791	0.970929214	0.980602785
11	0.946442905	0.95907239	0.971167897	0.980964838
12	0.946547761	0.959468017	0.971751909	0.981776069
13	0.945787434	0.960193798	0.973126202	0.984248286
14	0.945042643	0.960208409	0.973313136	0.984996823
15	0.945745099	0.961472581	0.974793382	0.986887711
16	0.947250486	0.963576417	0.977182665	0.989727669
17	0.949015973	0.966919849	0.980853087	0.994533468
18	0.951224023	0.969688889	0.983878152	0.997982883
19	0.996826508	0.99731929	0.997996209	0.998372154
20	0.993001735	0.99371306	0.994507555	0.995050116
21	0.992246086	0.992992983	0.99382187	0.994398431
22	0.991560967	0.992333377	0.99320362	0.993810235
23	0.981826664	0.985429586	0.991240068	0.993731763
24	0.975113112	0.980417201	0.98921152	0.992693108
25	0.971972333	0.97876413	0.990399147	0.994718066
26	0.954758058	0.963927433	0.975057464	0.982247951
27	0.952212114	0.961943092	0.97404596	0.981728071
28	0.939853287	0.952626264	0.968113699	0.978255858
29	0.930985303	0.946036782	0.964120795	0.976103075
30	0.927520177	0.943528583	0.963453987	0.97617836
31	0.922731571	0.941341356	0.964098966	0.978701974
32	0.921484135	0.941049463	0.964627775	0.979916145
33	0.920483288	0.941432948	0.96577976	0.98201022

The maximum error obtained in this case is 2.0 %

Table C.6 Comparison table of IDLF & RLF with both (R,X & P_L, Q_L) as intervals and varying DG output

Bus No.	V_{lower} with IDLF	V_{lower} with RLF	V_{upper} with IDLF	V_{upper} with RLF
1	1	1	1	1
2	0.997167894	0.997846781	0.998533091	0.999086109
3	0.984024158	0.98806019	0.99237262	0.995816014
4	0.977277883	0.983258627	0.988907124	0.993920778
5	0.970697367	0.978528374	0.985674752	0.992316183
6	0.952609584	0.965533897	0.975906627	0.986885326
7	0.948511672	0.962508665	0.973023963	0.984917105
8	0.946280477	0.961269037	0.972135257	0.984976762
9	0.943253464	0.95975135	0.971256558	0.985472828
10	0.940776351	0.958953317	0.970793217	0.986487453
11	0.940678208	0.959109413	0.971031372	0.986930678
12	0.94063961	0.959527566	0.971624093	0.987894974
13	0.939133602	0.96051162	0.973000803	0.991160349
14	0.938024408	0.960449109	0.973204248	0.992296291
15	0.938389193	0.961669114	0.974613103	0.994546259
16	0.939484923	0.963707565	0.976953949	0.997822121
17	0.940236459	0.966971333	0.980796391	1.003703037
18	0.941987847	0.969617132	0.983580681	1.007638016
19	0.996560112	0.997313466	0.998018181	0.998638489
20	0.992450381	0.99359557	0.994551658	0.995602355
21	0.991635869	0.992874612	0.993866766	0.995009787
22	0.99089621	0.992222794	0.993252059	0.994476386
23	0.980236627	0.985227794	0.991184535	0.995339357
24	0.972993336	0.980091481	0.989080669	0.994850002
25	0.969455788	0.978442517	0.990359971	0.997292629
26	0.950491017	0.964027931	0.975085948	0.986629566
27	0.947676541	0.962129024	0.97410185	0.986394495
28	0.93411537	0.952856036	0.96821648	0.984209123
29	0.924356106	0.946048232	0.964365112	0.983024019
30	0.92045597	0.94348203	0.96366549	0.98357664
31	0.915046366	0.941295894	0.964382934	0.986804374
32	0.913582452	0.940964261	0.964841112	0.988264945
33	0.912273756	0.941360377	0.966018245	0.990707188

The maximum error obtained in this case is 3.0 %

Table C.7 Comparison table of IDLF & RLF with R, X as intervals with Multi State intervals of DG power output

Bus No.	V_{lower} with IDLF	V_{lower} with RLF	V_{upper} with IDLF	V_{upper} with RLF
1	1	1	1	1
2	0.997502773	0.997790473	0.998531402	0.998720925
3	0.986037227	0.988119321	0.992405122	0.993607472
4	0.980208032	0.983192398	0.988862356	0.990716755
5	0.974557973	0.978607987	0.985550128	0.98809515
6	0.959021478	0.965606437	0.975493848	0.97986397
7	0.95538424	0.962582671	0.972519316	0.977400774
8	0.953638673	0.961395815	0.97160899	0.976935972
9	0.951280973	0.959980075	0.970568608	0.976702371
10	0.94944372	0.958989763	0.970173063	0.977015422
11	0.949443165	0.959120031	0.970397419	0.977351139
12	0.949578239	0.959495801	0.970986495	0.978123324
13	0.948828144	0.960114524	0.972171631	0.980547154
14	0.948047105	0.960055567	0.972415118	0.981315284
15	0.94864003	0.961224939	0.973816009	0.98330408
16	0.949976277	0.963402161	0.976047466	0.986301341
17	0.951230074	0.966811002	0.979774052	0.991596633
18	0.953166401	0.969527138	0.982866267	0.99531063
19	0.996934889	0.997254346	0.997997842	0.998233251
20	0.993092626	0.993560363	0.994522986	0.994928335
21	0.992333419	0.992865609	0.993840779	0.994280136
22	0.991645303	0.992243038	0.99321031	0.99369487
23	0.982726068	0.985368772	0.991139194	0.992570256
24	0.976503185	0.980443591	0.988993576	0.990895447
25	0.973812707	0.978883667	0.990083526	0.992516356
26	0.957217643	0.964161398	0.974636648	0.979249372
27	0.954835637	0.962239	0.973623049	0.978521683
28	0.943461327	0.953023573	0.967605499	0.973882378
29	0.935327656	0.946547999	0.963513392	0.97085987
30	0.932143341	0.944135623	0.962793681	0.970589134
31	0.928347437	0.941999685	0.96333488	0.972125666
32	0.927472315	0.941729434	0.963929393	0.973007373
33	0.927002223	0.942093829	0.965149529	0.974667547

The maximum error obtained in this case is 1.64%

Table C.8 Comparison table of IDLF & RLF with P_L, Q_L as intervals with Multi State intervals of DG power output

Bus No.	V_{lower} with IDLF	V_{lower} with RLF	V_{upper} with IDLF	V_{upper} with RLF
1	1	1	1	1
2	0.997469551	0.997846838	0.998523361	0.998776711
3	0.985838179	0.988128957	0.992444793	0.993932386
4	0.979913509	0.983182677	0.988954468	0.991173278
5	0.974167555	0.978419923	0.985736295	0.988682558
6	0.958445984	0.965445288	0.975878583	0.980726373
7	0.954684344	0.962438166	0.972943434	0.978392299
8	0.952826483	0.961203507	0.972156324	0.978045322
9	0.950397155	0.959798215	0.971281962	0.977891127
10	0.948522996	0.958934791	0.970929214	0.978248503
11	0.948517641	0.95907239	0.971167897	0.978590156
12	0.948653777	0.959468017	0.971751909	0.979363241
13	0.948033815	0.960193798	0.973126202	0.981664163
14	0.947353226	0.960208409	0.973313136	0.982334388
15	0.948113977	0.961472581	0.974793382	0.984155751
16	0.949688228	0.963576417	0.977182665	0.986914246
17	0.951611611	0.966919849	0.980853087	0.991534771
18	0.953889518	0.969688889	0.983878152	0.994902968
19	0.996904298	0.99731929	0.997996209	0.998286417
20	0.99307939	0.99371306	0.994507555	0.994964217
21	0.992323688	0.992992983	0.99382187	0.994312523
22	0.991638509	0.992333377	0.99320362	0.993724329
23	0.982503949	0.985429586	0.991240068	0.992974253
24	0.976238895	0.980417201	0.98921152	0.991439297
25	0.973539657	0.97876413	0.990399147	0.992972205
26	0.956632282	0.963927433	0.975057464	0.980136283
27	0.954238445	0.961943092	0.97404596	0.979437964
28	0.94286194	0.952626264	0.968113699	0.974867446
29	0.93473428	0.946036782	0.964120795	0.971876899
30	0.93154749	0.943528583	0.963453987	0.971613837
31	0.927744149	0.941341356	0.964098966	0.973035909
32	0.926886889	0.941049463	0.964627775	0.973821775
33	0.926517406	0.941432948	0.96577976	0.975242485

The maximum error obtained in this case is 1.6 %

Table C.9 Comparison table of IDLF & RLF with both (R,X & P_L, Q_L) as intervals with Multi State intervals of DG power output

Bus No.	V_{lower} with IDLF	V_{lower} with RLF	V_{upper} with IDLF	V_{upper} with RLF
1	1	1	1	1
2	0.997264019	0.997846781	0.998533091	0.998959362
3	0.984644681	0.98806019	0.99237262	0.995006015
4	0.978174624	0.983258627	0.988907124	0.992764878
5	0.971884609	0.978528374	0.985674752	0.990795695
6	0.954759297	0.965533897	0.975906627	0.984202954
7	0.950702638	0.962508665	0.973023963	0.982169066
8	0.948541729	0.961269037	0.972135257	0.982131106
9	0.945621305	0.95975135	0.971256558	0.982478655
10	0.943252088	0.958953317	0.970793217	0.983343508
11	0.943172147	0.959109413	0.971031372	0.98376189
12	0.943167589	0.959527566	0.971624093	0.984679821
13	0.941815021	0.96051162	0.973000803	0.987734602
14	0.940775995	0.960449109	0.973204248	0.988773943
15	0.94120333	0.961669114	0.974613103	0.990939153
16	0.942372268	0.963707565	0.976953949	0.994116183
17	0.943291643	0.966971333	0.980796391	0.999771592
18	0.945116259	0.969617132	0.983580681	1.003608499
19	0.996656215	0.997313466	0.998018181	0.99851166
20	0.992546365	0.99359557	0.994551658	0.995474942
21	0.9917318	0.992874612	0.993866766	0.994882282
22	0.990992083	0.992222794	0.993252059	0.994348806
23	0.981065672	0.985227794	0.991184535	0.994240923
24	0.974338073	0.980091481	0.989080669	0.993083059
25	0.971393745	0.978442517	0.990359971	0.994970921
26	0.952776914	0.964027931	0.975085948	0.98377594
27	0.950154643	0.962129024	0.97410185	0.983301485
28	0.937765846	0.952856036	0.96821648	0.97974585
29	0.928894264	0.946048232	0.964365112	0.977520967
30	0.925352943	0.94348203	0.96366549	0.977637653
31	0.921170174	0.941295894	0.964382934	0.979634775
32	0.920202054	0.940964261	0.964841112	0.980635447
33	0.919686097	0.941360377	0.966018245	0.98237531

The maximum error obtained in this case is 2.4 %