DESIGN OF ROBUST POWER SYSTEM CONTROLLERS

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree

of

MASTER OF TECHNOLOGY

in

ELECTRICAL ENGINEERING

(With Specialization in Power System Engineering)

By

BASAM VENKATA RAO

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE-247 667 (INDIA)

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CANDIDATE'S DECLARATION

I hereby declare that the work, which is presented in this dissertation report, entitled **"DESIGN OF ROBUST POWER SYSTEM CONTROLLERS"**, being submitted in partial fulfillment of the requirements for the award of the degree of **MASTER OF TECHNOLOGY** with specialization in **POWER SYSTEM ENGINEERING,** in the Department of Electrical Engineering, Indian Institute of Technology, Roorkee is an authentic record of my own work carried out from July 2006 to June 2007, under the valuable guidance and supervision of Dr. C. P. Gupta, Assistant Professor, Department of Electrical Engineering, Indian Institute of Technology, Roorkee.

The results embodied in this dissertation have not been submitted for the award of any other Degree or Diploma.

B. Venkata Rao

Date: 29-06-2007 **BASAM VENKATA RAO**

Place: Roorkee **Roll. No: 054001**

CERTIFICATE

This is to certify that the statement made by the candidate is correct to the best of my knowledge and belief.

C.P. Gupta 29/6/07

Date: 29-06-2007 **Dr. C. P. GUPTA**

Place: Roorkee

Assistant Professor

Department of Electrical Engineering

Indian Institute of Technology Roorkee

Roorkee – 247667, (INDIA)

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BASAM. VENKATA RAO

ABSTRACT

Power system is such a complex structure that it is not always easy to derive an exact model for it. These complexities in modeling leads to model errors which are also called as 'model uncertainties'. If a controller is designed for a particular control problem in the power system, without considering these model uncertainties, that controller may fail in real application. So a design methodology which can take these model uncertainties into account is necessary specifically to the power system control problems. Structured singular value (μ) synthesis is the design method which is having the ability to deal with these model uncertainties. So a controller that is designed using this method will be robust enough to work on the real power system.

In this dissertation work, a deregulated power system load frequency control problem is considered for explaining the development of the μ-synthesis from the H_∞-synthesis. Three types of controllers are designed for the deregulated power system load frequency control problem, namely

- H_∞-controller
- Weighted H_∞-controller
- μ-controller

'Bounded complex uncertainty' models are developed for the Damping coefficient uncertainties in the deregulated power system model. Along with these two uncertainties, the neglected high frequency dynamics uncertainty due to the first order approximation of turbines and governors is also considered. A μ-controller is designed after taking these model uncertainties into account. The robustness properties of this μ-controller are compared with the weighted H_∞-controller with the help of time response simulations. Controller order reduction technique is applied on this μ-controller to reduce it to a 3rd order controller from 25th order. The robustness of this third order controller is also checked using time response simulations. For this work, the various algorithms available in the Robust Control Toolbox of MATLAB® are used. μ-controller is designed using the DKITGUI tool available in this Robust Control Toolbox of MATLAB®.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

Robust control design is different from the ordinary linear control design in the sense that this design process considers a family of plants instead of a single nominal plant. If a designer is asked to design a controller for a system, first he needs to derive the model for that system. But no nominal model should be considered complete without some assessment of its errors. These errors in the derived model for the plant may be having sources such as uncertainties in the parameters, neglected high frequency dynamics and simplifications to the actual model which are jointly called as 'model uncertainties'. Because of these uncertainties in the model, the designer comes across the tough task of thinking in terms of a family of plants as shown in Fig.1.1 instead of a single nominal plant while designing the controller.

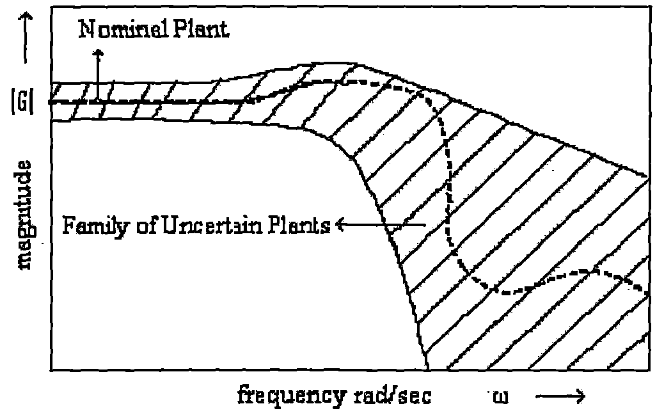


Fig.1.1: Family of plants due to uncertainties in modeling [1]

John Doyle [1] proposed first that these model uncertainties can be described effectively in terms of norm-bounded perturbations. In order to deal this bounded perturbation representation and the H-infinity performance objective at a single throw, he developed a powerful tool called *structured singular value* (μ). This 'μ' can be used for testing "robust stability" and "robust performance". "Robust stability" means stability in the presence of model uncertainty and "robust performance" means performance in the presence of model uncertainty. Before the introduction of structured singular value, the era was of H-infinity optimal control in which performance objectives can be formulated in terms of minimization of the H-infinity norm. But it doesn't suit well to consider model uncertainty. So the introduction of structured singular value concept brought a new era in control design called "robust control".

An important point to be noted here is, even though H-infinity optimal control is not directly useful for robust control design, it is 'a part and has to be used repeatedly, in the D-K iterations of μ-synthesis robust control design algorithm. This point will become clear by the discussion on μ-synthesis in chapter-5. Hence it is must to understand first about the H-infinity synthesis procedure before trying to understand the μ-synthesis design procedure.

1.1 Bounded Uncertainty [1]: It can be observed from Fig.1.1 that any uncertain system will have bounds for its uncertainty from the nominal plant at each frequency. This maximum bound can be represented by a frequency domain function $W_a(s)$ (called as 'maximum uncertainty bound' function), whose magnitude $|W_a(s)|$ will represent the maximum deviation from the nominal plant at each frequency. A new term '$\Delta(s)$' called as bounded uncertainty is introduced here. If '$\Delta(s)$' is a single scalar uncertainty, then it is a unit circle in the complex plane as shown in Fig.1.2. If '$\Delta(s)$' is a matrix, then the matrix norm of this '$\Delta(s)$' is unity at each frequency. In the robust control theory this matrix norm in general will be the H_∞-norm.

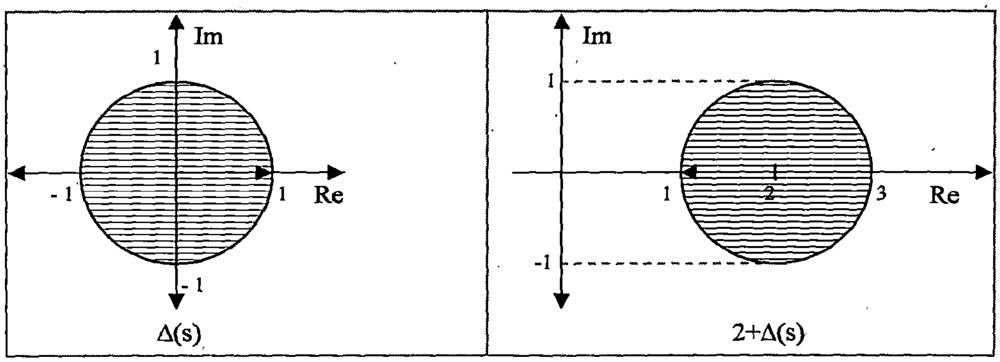


Fig.1.2: Meaning of scalar (not a matrix) complex uncertainty '$\Delta(s)$' in the complex plane

Any uncertain plant '$G_u(s)$' can be written as $G_u(s) = G(s) + W_a(s)\,\Delta(s)$ as an additive uncertainty. The maximum amount of deviation from the nominal plant G(s), is bounded by $|W_a(s)|$. The meaning of this bounded additive uncertainty representation is shown below in Fig.1.3. This $|W_a(s)|$ at any frequency can be said, as the radius of a circle in the Nyquist plot, enclosing all the possible points of the uncertain system at that frequency.

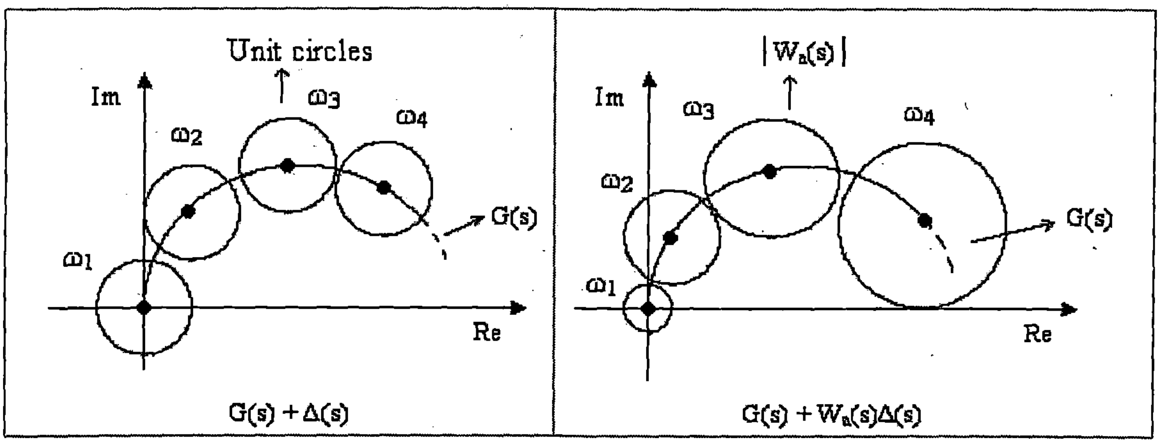


Fig. 1.3: Representation of an uncertain system with '$\Delta(s)$'

Fig.1.3 explains the 'bounded additive uncertainty' representation of an uncertain system '$G_u(s)$'. Where, ω_1, $\omega 2$, $\omega 3$ and ω_4 are various frequency points in the whole frequency range. The Fig.1.4 given below shows this 'bounded additive uncertainty' representation of '$G_u(s)$' using transfer function blocks.

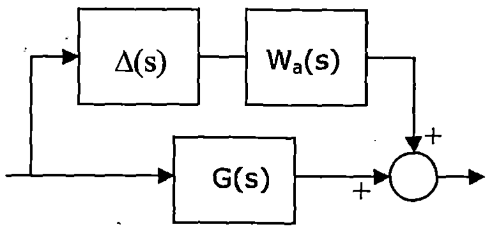


Fig. 1.4: Bounded additive uncertainty representation

The above additive uncertainty representation is modified slightly as below, to derive another form of representation called 'bounded multiplicative uncertainty' representation.

$$G(s) = G_o(s)[1 + (W_a(s)/G_o(s))\ \Delta(s)] \quad (1.1)$$

$$W_m(s) = W_a(s)/G_o(s) \quad (1.2)$$

Where, $W_m(s)$ is called the 'maximum multiplicative uncertainty' bound function. This form of uncertainty representation is shown in the Fig.1.5 below, using transfer function blocks.

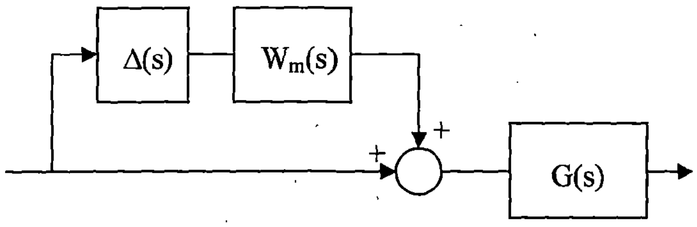


Fig. 1.5: Bounded multiplicative uncertainty representation

1.2 Linear Fractional Transformations: Linear Fractional Transformation (LFT) representation of systems is frequently used in robust control design methodology [2]. Before using many of the robust control algorithms, one must have Linear Fractional Transformation representation of the systems. Mainly for the application of μ-analysis and μ-synthesis algorithms, we need to separate the uncertainty block ($\Delta(s)$), controller block ($K(s)$), and plant model ($G(s)$) in Linear Fractional Transformation form as shown in Fig.1.6. This is because, these algorithms internally assumes this representation. For example in case of *μ-analysis*, we need to obtain: $\Delta(s)$ (called as uncertainty block structure data), $K(s)$ (controller in SYSTEM variable form), and $G(s)$ (plant model in SYSTEM variable form), before calling the μ-analysis algorithm. Similar is the case of using μ-synthesis algorithm, but this time $K(s)$ will be the output of the algorithm. So this Linear Fractional Transformation representation is important in robust control design.

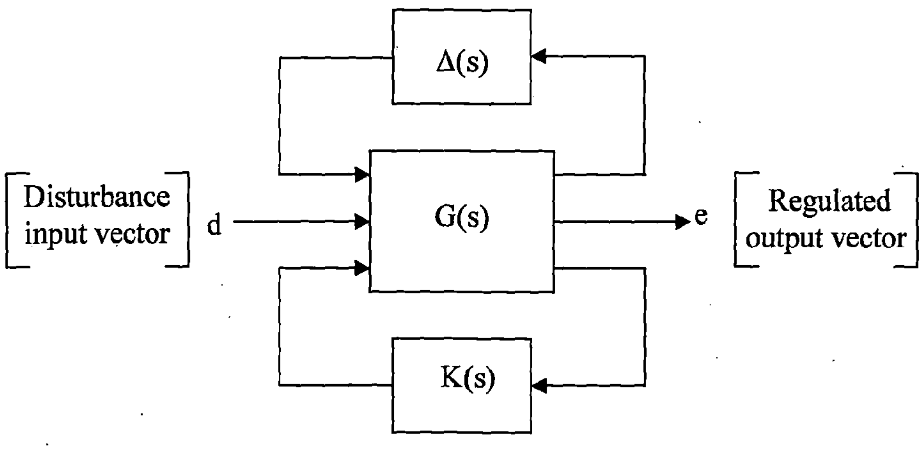


Fig.1.6: Linear Fractional Transformation (LFT) representation [2]

1.3 H_∞-synthesis: The H_∞-optimal control design involves the steps given in the flow chart below in its synthesis procedure. An in depth discussion of these steps is given in chapter-3 on H_∞-synthesis. Briefly, the objective is to obtain an optimal controller that can minimize the cost function 'γ', where, 'γ' is the H_∞-norm of the closed loop system from disturbance inputs to regulated outputs ('d' to 'e' in Fig.1.6 above). The flow chart given in Fig.1.7 below hopefully may help in providing the first and simple insight into the H-infinity synthesis procedure.

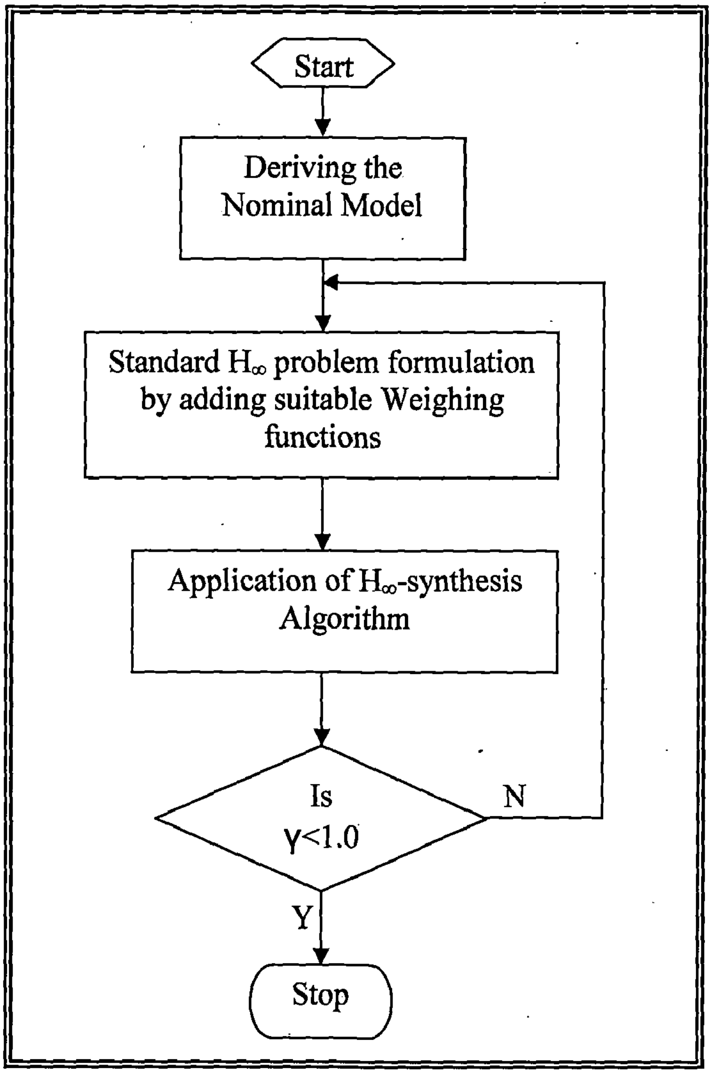


Fig. 1.7: Flow chart of H_∞-synthesis procedure

A point that is of worth noting at this stage is that the above design steps results in a H_∞-optimal controller for the nominal model of the system. This design does not take the model uncertainties into account. Hence this is not a robust control design.

1.4 μ-synthesis: The structured singular value synthesis (μ-synthesis) is different from the H_∞-optimal controller synthesis in the sense that the former is having the ability to take model uncertainties into account in its synthesis process.

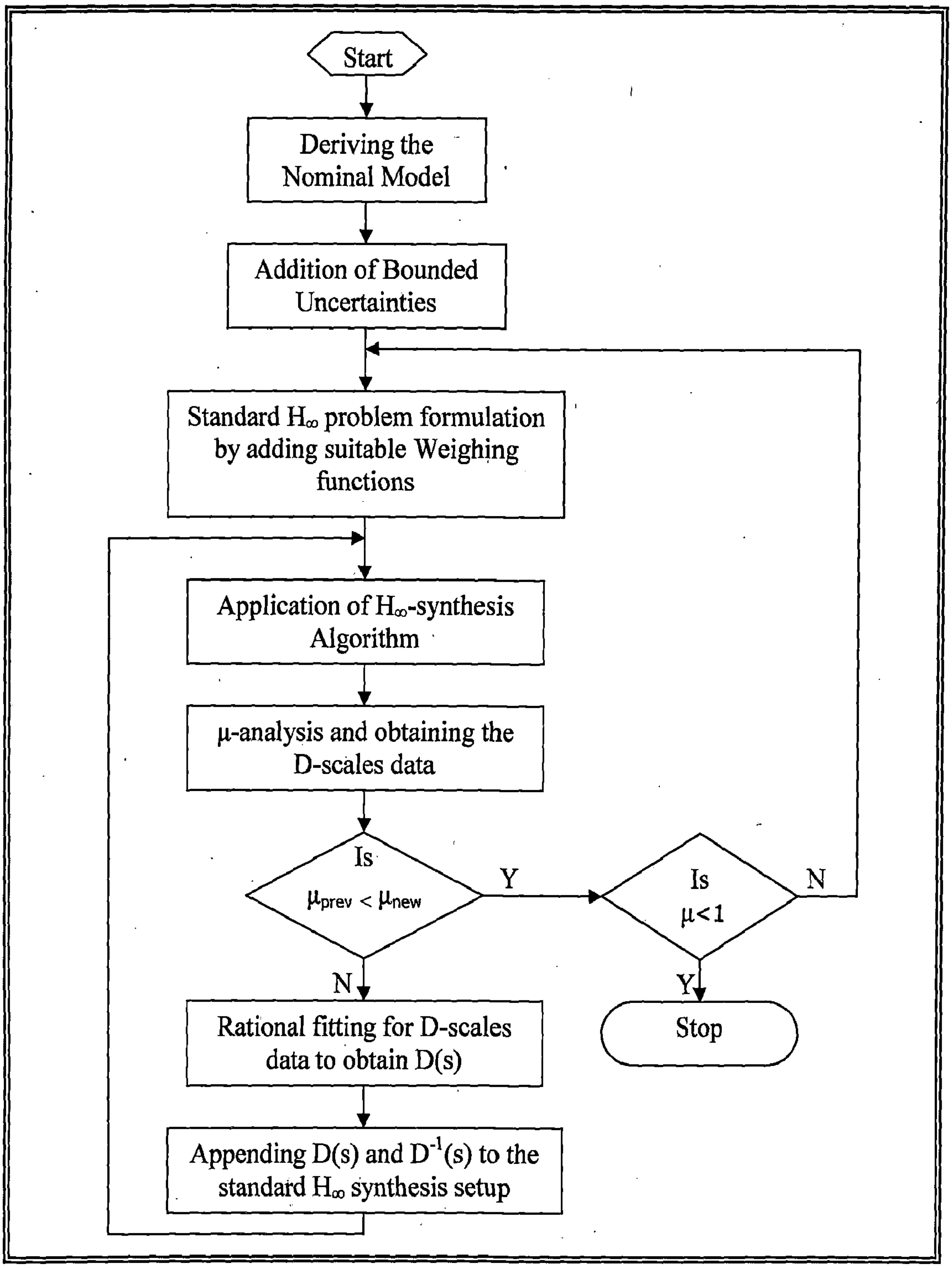


Fig. 1.8: Flow chart of μ-synthesis procedure

The μ-synthesis algorithm in its synthesis steps calculates the worst case uncertainties in terms of D-scales (refer section-5.2) and modifies the design setup (by appending these D-scales) before calling the H_∞-synthesis algorithm. It will do this repeatedly until a minimum value of 'μ' is achieved. Here 'μ' is the structured singular value of the closed loop uncertain system from the disturbance inputs to the regulated outputs (from'd' to 'e' in Fig.1.6). The flow chart given in the Fig.1.8 above will help in getting the first and simple insight into the μ-synthesis procedure.

1.5 Design Evaluation: Once the controller synthesis is completed, it is necessary to evaluate our design by checking the robustness properties of the controller in terms of stability and performance. This design evaluation can be done with the help of frequency domain methods or by running time response simulations. The frequency domain method of design evaluation is called as structured singular value (μ) analysis.

1.5.1 μ-analysis: This is a frequency domain method for checking the robustness of a designed controller. This concept can be explained with the help of a simple example in our power systems. Consider a single-machine infinite-bus (SMIB) system with a TCSC in the transmission line. If a designer is asked to design a controller for this TCSC, first he will obtain a linear model of this SMIB system at a particular power flow condition in the transmission line and will design a controller at this operating condition. Now the problem is to check-whether the designed controller is robust for different flow conditions in the line!! For this purpose the μ-analysis can be used.

For different power flow conditions in the transmission line, certain parameters in the transfer function of the SMIB system linear model varies. These parameter variations can be modeled as bounded complex uncertainties by using the method discussed in chapter-4 and the closed-loop uncertain (parameter uncertainties) system can be represented in LFT form as in Fig.1.6. Now μ-analysis can be applied on this LFT form. If 'μ' is less than or equal to 1, the controller is robust in terms of stability and performance for the considered power flow variations in the transmission line. In this way the robustness of any designed controller can be checked by using the frequency domain μ-analysis technique. An in-depth discussion on μ-analysis is given in chapter-5.

1.5.2 Time Response Simulations [2]: Even though the frequency domain μ-analysis technique can be used for analyzing the robustness properties of any designed controller, simulations in the time domain helps in directly visualizing the superior qualities of one controller, when compared with another controller. For example, in this report, a *H_∞-controller* and a *μ-controller* are designed for a Deregulated power system Load Frequency Control (LFC) case. These two designs can be compared for robustness properties using μ-analysis technique. But, time response simulations help in directly visualizing the *superior qualities of μ-controller* in terms of robustness when compared with the *H_∞-controller*. Three types of time response simulations can be run. They are response of:

- Open-loop nominal system
- Closed-loop nominal system
- Closed-loop perturbed system

The basic structures of the above three types of systems on which time response simulations are to be run are shown in Fig.1.9 below.

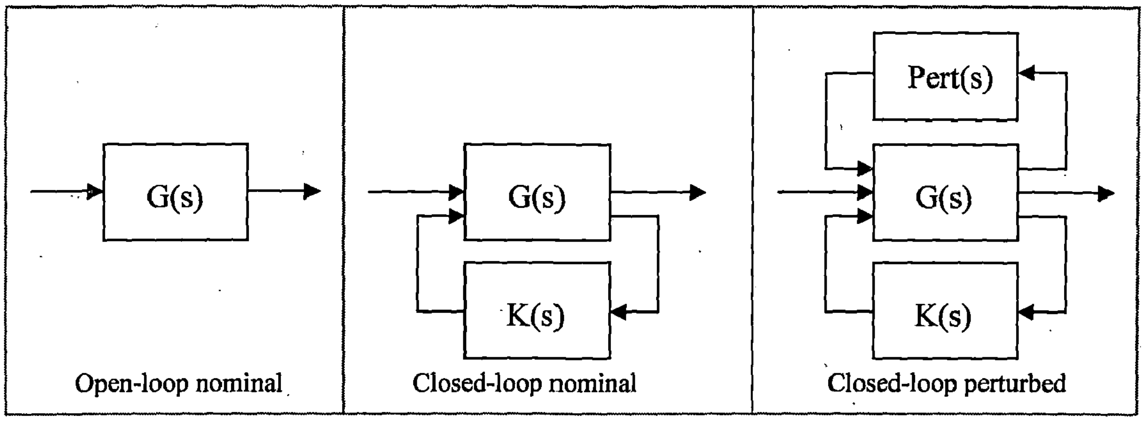


Fig.1.9: Basic structures for time response simulations

Note that 'Pert(s)' is different from 'Δ(s)' of Fig.1.6. As explained in section-1.2, 'Δ(s)' is a unity-norm bounded uncertainty where as 'Pert(s)' is a worst case in this norm bounded 'Δ(s)'. 'Δ(s)' along with the nominal plant represents a family of plants of an uncertain system as in Fig.1.3. Where as 'Pert(s)' along with the nominal plant represents the worst case in this family of plants which will bring the close-loop system to the verge of instability. This difference can be stated simply as below:

$$G(s) + \Delta(s) \longrightarrow \text{Family of plants}$$
$$G(s) + Pert(s) \longrightarrow \text{Worst case in this family of plants}$$

A planned procedure must be followed so as to make an effective comparison between the robustness properties of the designed controllers. In this report, the robustness properties of the H_∞-controller and the μ-controller (designed for Deregulated power system LFC problem) are compared using time response simulations by following a particular plan. This planning for the time response simulations is summarized in steps below.

- Choose a suitable input signal, for example a 'Step signal'.
- Do time response simulations on: open-loop nominal system, Closed-loop nominal system with the H_∞-controller (Fig.6.4), and Closed-loop nominal system with the μ-controller (Fig.6.13).
- Apply μ-analysis algorithm on the closed-loop uncertain system (like Fig.1.6) with H_∞-controller.
- Obtain 'Pert(s)' from this μ-analysis (Table 6.3). This 'Pert(s)' is the perturbation which will bring the closed-loop system with the H_∞-controller to the verge of instability.
- Replace 'Δ(s)' (in Fig.1.6) with this 'Pert(s)' to form a closed-loop perturbed system (like Fig.1.9), and run the time response simulations with the H_∞-controller (Fig.6.15) and with the μ-controller (Fig.6.16). It can be observed that the closed-loop system with the H_∞-controller will go to the verge of instability for this perturbation 'Pert(s)', where as the *closed-loop system with the μ-controller will remain stable and meets the performance requirements*.
- Now apply μ-analysis on the uncertain closed-loop system (like Fig.1.6) with the μ-controller.
- Obtain 'Pert(s)' from this μ-analysis (Table 6.4). This 'Pert(s)' is the perturbation which will bring the closed-loop system with the μ-controller to the verge of instability.

- Replace 'Δ(s)' (in Fig.1.6) with this 'Pert(s)' to form a closed-loop perturbed system (like Fig.1.9), and run the time response simulations with the μ-controller (Fig.6.18) and with the H_∞-controller (Fig.6.19). It can be observed that the closed-loop system with the μ-controller will go to the verge of instability for this perturbation 'Pert(s)', where as the *closed-loop system with the H_∞-controller will become unstable.*

1.6 Robust Control in Power Systems: Even though the robust control theory was developed in mid 1980's, this technique was initially applied mostly to mechanical systems, structural systems etc. From the last decade, there was an increased research in applying this robust control theory to electrical power systems. This robust control technique is being applied in power systems for applications like: design of power system stabilizers, design of load frequency controllers, design of FACTS device's controllers etc.

A power system in general is such a complex structure that it is not possible for a designer to simply derive a model for it. Even if a model is derived with difficulty after simplifications and neglected high frequency dynamics, the parameters in the transfer function blocks of that model vary due to changes in operating conditions. With this many types of errors in the model-if a controller is designed, it will fail when applied to the real system. So a design which doesn't take these errors in the model may become a waste when applied on real power systems.

Before the invention of this robust control theory by *John Doyle*, the era was of H_∞-optimal control design which does not take the model uncertainties (errors) into account. The robust control design (structured singular value synthesis) brought a new era in the control design. The robust control design takes these model uncertainties into account in its synthesis steps (see section-6.3). Since power system is the case where these modeling errors are common because of its complexity and continuously varying operating conditions, it is necessary to utilize this robust control theory for power system control design applications. In this report, a H_∞-controller and a μ-controller are designed for a Deregulated Power system Load Frequency Control case. The *Robust Control Toolbox* of MATLAB® has been used for the design purpose in which various algorithms necessary for robust control design are available. A comparison is made between the H_∞-controller and the μ-controller (robust controller) using: frequency domain μ-analysis technique (Figs.6.14 & 6.17) and time response simulations. Results show the superior quality of the μ-controller in terms of robustness as compared to the H_∞-controller.

Robust control is a development from the H_∞-optimal control with the additional quality of being able to take modeling uncertainties into account in the design process. During the 1980's H_∞-control was famous. The introduction of the concepts of Bounded uncertainty and structured singular value has leaded to a new control design technique namely the Robust Control Design.

John C. Doyle [1] has given the definitions of the various fundamental concepts in the present robust control theory like Singular values of a matrix, Maximum singular value, 2-norm, H_∞-norm etc. He also explained clearly the meaning of the concept 'bounded uncertainty' and discussed the use of these bounded uncertainties for designing a controller for an uncertain model of a real plant. These definitions are necessary to understand the control design theory further.

Doyle et al., [3] has discussed the mathematical concepts of H_∞-optimal feedback control design. The necessary conditions that a standard H_∞ problem setup need to satisfy (pp.834-835) for the H_∞-synthesis algorithm to have a solution are discussed clearly. As discussed in section-3.4 of this report, if these necessary conditions are not satisfied, the H_∞-synthesis algorithm of MATLAB® returns an error.

Sigurd Skogestad and Manfred Morrari [5] has nicely explained the definitions of the Structured singular value (μ), and the procedure for analyzing the Robust stability (RS) and Robust performance (RP) using this Structured singular value concept. All the theoretical concepts discussed in chapter-5 are taken from this paper and [2].

[1], [3], [4] and [5] explains the basic mathematical concepts in the robust control theory. [9-12] has discussed the application of these robust control theory concepts on different example problems. These references are very useful for understanding the application of

the theoretical concepts on real problems. These are useful to bridge the gap between the theoretical understanding and the problem level understanding.

S. Chen and O.P. Malik in [14] has discussed the application of the H_∞-optimal control design technique to a particular power system control design problem namely the design of a robust Power System Stabilizer for a single-machine infinite-bus (SMIB) system. In [23], they had discussed a development to the above design by considering the parameter uncertainties in the SMIB model. They discussed the design of a robust controller which is robust to these parameter uncertainties using the μ-synthesis design technique.

Ali Feliachi [17] has discussed about a deregulated power system load frequency control problem. He discussed about the design of a H_∞-optimal controller for this deregulated power system load frequency control problem. In this paper he had not used the concept of weighing functions for tuning the performance requirements. This means the design in this paper is a simple H_∞-optimal controller but not a weighted H_∞-optimal controller.

Vidal et al., [18] has discussed very nicely about the method of deriving bounded complex uncertainty models from the uncertainties data of the plant model. The concepts in this paper are used for obtaining the bounded uncertainty models, for the uncertain transfer functions present in the deregulated power system model for which we are designing a robust controller. It is necessary to obtain these bounded complex uncertainty models for the DKITGUI tool of robust control design to work.

The concept of μ-analysis is a part of μ-synthesis (refer chapter-5) and at the same time this is a measure for evaluating the designed controllers for the robustness properties (refer section-1.5). [19-22] has discussed the application of this μ-analysis on various power system problems for testing the robust stability and robust performance.

Hassan Bervani [25] has discussed the application of the μ-synthesis technique on a deregulated load frequency control problem. The system considered in this paper is same as that in [17]. In this paper Bervani has considered only the neglected high frequency dynamics uncertainty. In dissertation work extensions are applied to both [17], [25] by considering,

- The application of weighing functions for performance tuning
- Adding more uncertainties possible in the power system model.

Consideration of these additional uncertainties will lead to a controller design which will work better on real time systems.

[28-31] discusses on the application of the μ-synthesis on power electronic systems and are highly descriptive for understanding the robust control design in MATLAB®.

This chapter covers the basic concepts of H_∞-optimal control design. Consider Fig.3.1 given below. "G(s)" is the plant to be controlled, "K(s)" is the controller to be designed, "d" is the vector of external disturbances (set point changes, noise signals etc.), and "e" is the vector of output signals to be regulated (tracking errors, controller outputs etc.).

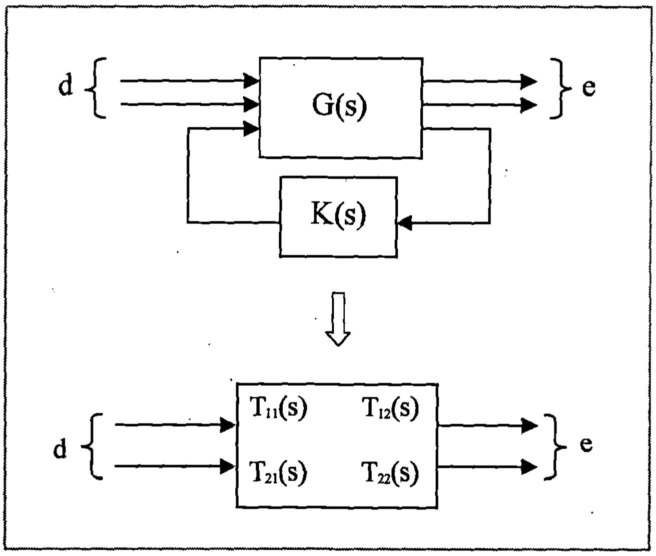


Fig.3.1: A simple closed-loop system

$$T(s)=\begin{bmatrix} T_{11}(s) & T_{12}(s) \\ T_{21}(s) & T_{22}(s) \end{bmatrix} \tag{3.1}$$

The modern approach in control design is to characterize the performance objectives using various matrix norms on the closed-loop system transfer function matrices such as 'T(s)' above. Obtaining a controller 'K(s)' which will minimize this closed-loop norm is the objective in the modern optimal control theory. The performance objective in simple terms can be said as-minimize the regulated output variable values for different classes disturbance inputs. In modern control theory, this performance objective is expressed in terms of certain closed-loop transfer function matrix norm. So synthesizing a stabilizing controller 'K(s)' using some algorithm to optimize (minimize) these closed-loop transfer function norms is the concept of modern control theory. One of these norms being used frequently in modern control theory is the H_∞-norm. Synthesizing a controller 'K(s)' to minimize this H_∞-norm of the closed-loop system 'T(s)' is called the H_∞-optimal control synthesis.

3.1 H_∞-norm [1] [2] [4]: The 2-norm of a scalar signal, e(t) is defined as

$$\|e\|_2 = \left[\int_{-\infty}^{+\infty} e(t)^2 dt \right]^{1/2} \tag{3.2}$$

If e(t) is a vector of time domain signals, then the 2-norm of e(t) is defined as

$$\|e\|_2 = \left[\int_{-\infty}^{+\infty} e(t)^T e(t) dt \right]^{1/2} \tag{3.3}$$

Now the H_∞-norm of the closed-loop transfer function 'T(s)' from disturbance inputs to regulated outputs is defined as follows:

$$\| T \|_\infty \stackrel{\Delta}{=} \sup_\omega \bar{\sigma} \left(T(j\omega) \right) \tag{3.4}$$

Where,

ω: Frequency rad/sec.

$\sup_\omega$: Maximum over the entire 'ω' range.

$\bar{\sigma}$: Maximum singular value of the transfer function matrix $T(j\omega)$ at each frequency point over the entire 'ω' range.

Consider the closed-loop transfer function matrix 'T(s)' as a function of frequency as

$$T(j\omega) = \begin{bmatrix} T_{11}(j\omega) & T_{12}(j\omega) \\ T_{21}(j\omega) & T_{22}(j\omega) \end{bmatrix} \tag{3.5}$$

At each frequency in the range of 'ω', $T(j\omega)$ will be a complex valued matrix of size ($n_e \times n_d$), where 'n_e' is the number of regulated output variables and 'n_d' is the number of disturbance inputs. At some frequency 'ω_1' in the 'ω' range, the singular values of the matrix $[T(j\omega_1)]_{n_e \times n_d}$, denoted as '$\sigma_i$', are the non-negative square roots of the eigen values of $[T^H T]$ ordered such that $\sigma_1 \geq \sigma_2 \geq ... \sigma_p \geq 0$. Here p=min {$n_e$, n_d}. If 'r' is the rank of the matrix $[T(j\omega_1)]_{n_e \times n_d}$, then

$$\sigma_{r+1} = \sigma_{r+2} = \cdots = \sigma_p = 0 \tag{3.6}$$

The greatest of these singular values σ_1, is denoted as $\bar{\sigma}[\,T(j\omega_1)\,]$ at frequency ω_1. These maximum singular values can be calculated at each frequency point in the 'ω' range. Now the supremum(maximum) of these maximum singular values over the entire 'ω' range denoted as $\sup\limits_{\omega}\bar{\sigma}[T(j\omega)]$ is called as the H_∞-norm of 'T(s)' and is denoted by, $\|\,T(s)\,\|_\infty$. For a closed-loop system as in Fig.3.1,

$$\text{If, } \|\,T(s)\,\|_\infty \le 1 \text{ and } \|d\|_2 \le 1\text{, then}$$

$$\|e\|_2 \le 1 \tag{3.7}$$

This means, if the H_∞-norm of the closed-loop system from the disturbance inputs to the regulated outputs is less then '1', then for any input vector of disturbances 'd' whose 2-norm $\|d\|_2$ is less than '1', the 2-norm of the regulated output variables $\|e\|_2$ will also be less than '1'. $\|T(s)\|_\infty$, can also be interpreted in terms of the maximum RMS gain from input to output for different classes of input signals as given below.

$$\|\,T(s)\,\|_\infty = \max\frac{\|e\|_2}{\|d\|_2} \tag{3.8}$$

In the H_∞-optimal control design that we are discussing in this chapter, the objective is to obtain a controller 'K(s)' which will minimize this H_∞-norm of the closed-loop transfer function T(s). There can be many classes of disturbance input signals whose 2-norm is less than '1'. Similarly there can be many classes of regulated output signals whose 2-norm is less than '1'. So it will be a good idea to add some frequency weighing functions to the problem setup G(s), in the disturbance input and regulated error output channels to give importance to certain frequency components.

For example if the designer wants that the steady state value of a regulated output variable must become less than 0.01, then before applying the H_∞-synthesis algorithm, the designer has to add a weight in that particular regulated variable output channel a weighing function whose magnitude at low frequencies and at zero frequency will be greater than or equal to 100. Similarly if the designer wants to put information about a disturbance input such as-any frequency component in that signal will have amplitude not more than 0.5, then a weight of 0.5 is needed to be kept in that particular disturbance input channel. Hence it is to be noted that for characterizing the realistic MIMO system performance objectives in terms of a single H_∞-norm on the closed-loop transfer function, it is necessary to incorporate additional scalings (weighing functions) into the problem setup.

3.2 Theory of Weighing Functions [2] [6]: The interpretation of H_∞-norm of closed-loop transfer function as the maximum of RMS gain from input disturbances to the output regulated variables may not be helpful for a designer in understanding the concept of addition of weighing functions to the problem setup. The following sinusoidal steady state interpretation helps the designer a lot in understanding the concept of weighing functions in H_∞-optimal control design. Let,

$$d(t) = \begin{bmatrix} a_1\sin(\omega_1 t+\varphi_1) \\ \vdots \\ a_{nd}\sin(\omega_1 t+\varphi_{nd}) \end{bmatrix} \tag{3.9}$$

Where,

ω_1 : A point in ω range $\in R$

a_i: Amplitude $\in R_{nd}$

φ_i : Phase angle $\in R_{nd}$

Also assume,

$$\|a\|_2 = \left[\sum_{i=1}^{nd} a_i^2\right]^{1/2} \leq 1 \tag{3.10}$$

Applying this vector of disturbance inputs to the closed-loop system with transfer function 'T(s)' with $\| T(s) \|_\infty \leq 1$, will result in an output regulated variable vector

$$e(t) = \begin{bmatrix} b_1\sin(\omega_1 t+\psi_1) \\ \vdots \\ b_{ne}\sin(\omega_1 t+\psi_{ne}) \end{bmatrix} \tag{3.11}$$

Where,

ω_1 : A point in ω range $\in R$

b_i: Amplitude $\in R_{ne}$

ψ_i : Phase angle $\in R_{ne}$

And,

$$\| b \|_2 = \left[\sum_{i=1}^{ne} b_i^2 \right]^{1/2} \leq 1 \tag{3.12}$$

Note that this interpretation of H_∞-norm is totally different from that given earlier. The earlier interpretation is not putting any stress on the individual frequency components of the input and output signals. Since the systems that we are dealing here are linear, and any signal can be represented as a summation of sinusoidal signals (using Fourier transformation), the interpretation given for one frequency component ω_1 above will hold for a general signal according to superposition theorem. This new interpretation of H_∞-norm is specifically helpful in understanding the concept of weighing functions. This is further explained below. Let,

$$w_d = \begin{bmatrix} w_{d1} & 0 \\ 0 & w_{d2} \end{bmatrix} \tag{3.13}$$

$$w_e = \begin{bmatrix} w_{e1} & 0 \\ 0 & w_{e2} \end{bmatrix} \tag{3.14}$$

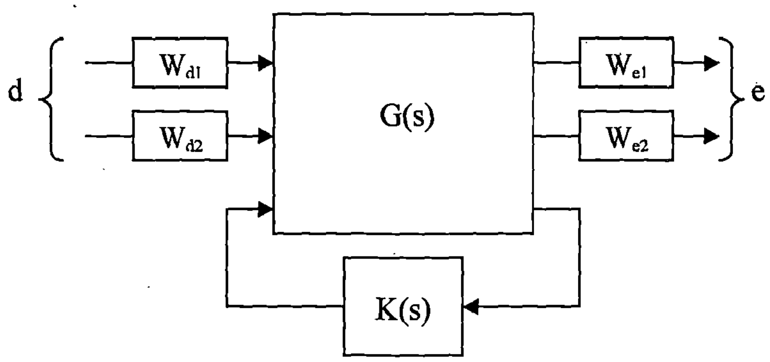


Fig.3.2: Addition of weighing functions to the problem setup

If,

$$\| w_e T w_d \|_\infty \leq 1, \tag{3.15}$$

$$\left[\sum_{i=1}^{nd} \left| \frac{a_i}{w_{di}(j\omega)} \right|^2 \right]^{1/2} \leq 1 \tag{3.16}$$

Then,

$$\left[\sum_{i=1}^{ne}\left|b_i w_{ei}(j\omega)\right|^2\right]^{1/2} \leq 1 \tag{3.17}$$

This is approximately same as, if

$$\left\|w_e T w_d\right\|_\infty \leq 1, \tag{3.18}$$

$$a_i \leq \left|w_{di}(j\omega)\right|, \text{ for all '}a_i\text{' in vector'}d\text{'} \tag{3.19}$$

Then,

$$b_i \leq \frac{1}{\left|w_{ei}(j\omega)\right|}, \text{ for all '}b_i\text{' in vector '}e\text{'} \tag{3.20}$$

The above expressions for one frequency component ω_1 holds good for any other frequencies. This interpretation will help a designer in choosing appropriate weighing functions, so that he can concentrate more on certain frequency components. In Fig.3.3 below, the above explanation is given diagrammatically with a simple example.

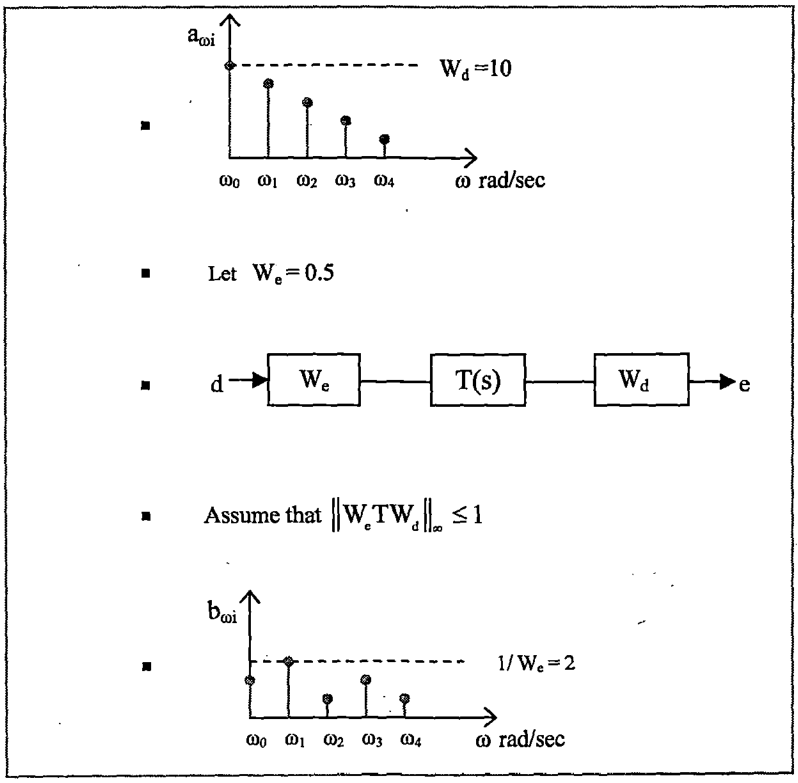


Fig.3.3: An example showing the meaning of H_∞-norm with weighing functions

In the above example, W_d=10 means that each of the frequency components (ω_o-ω_4) in d(t) will have amplitude less than or equal to 10. If the weighted closed-loop H_∞-norm is less than 1, then in the regulated output variable e(t) each of the frequency component will have a magnitude not more than $1/W_e$=2. In the above example constant weights are chosen for all the frequency components. But in a real problem these weights will be chosen as a function of frequency so that the designer can concentrate more on certain frequency components of interest. For example if the designer wants that the steady state value of the regulated output variable must become less than 0.01, then the designer may choose the weighing function 'W_e' to have a magnitude greater than or equal to 100 at low frequencies. Similarly the weighing function 'W_d' has to be chosen such that it will give information about the maximum amplitudes of various frequency components in the input disturbance signals.

3.3 H_∞-optimal Controller Synthesis [2]: This H_∞-optimal controller design problem can be solved using the H_∞-synthesis algorithm available in MATLAB®. Given a linear system model 'G(s)', before applying the H_∞-synthesis algorithm, the system 'G(s)' has to be represented with in a single matlab SYSTEM variable. For this, the SYSIC (system inter-connections) routine of MATLAB® has to be used. The following are the steps in converting 'G(s)' into a single matlab SYSTEM variable.

- Represent individual transfer blocks of 'G(s)' with different variables. They are called as 'subsystems'.
- Specify the external inputs to 'G(s)' in the order of exogenous disturbances and control inputs as shown in Fig.3.4.
- Specify the inter-connections between different subsystems and the subsystems to which the external inputs are connected.
- Specify the outputs of 'G(s)' in terms of the outputs of different subsystems in the order of regulated output variables and outputs which will go as inputs to the controller as shown in Fig.3.4.
- Specify a name for the SYSTEM variable with which the total system 'G(s)' will be represented.
- Run the SYSIC routine. This returns a SYSTEM variable which represents 'G(s)'.

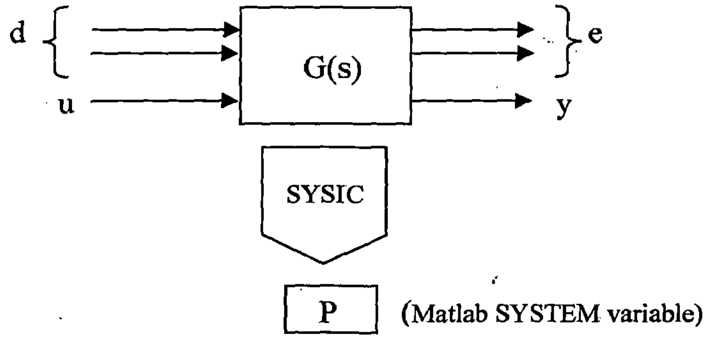


Fig.3.4: From 'G(s)' to 'P' using SYSIC

But as mentioned before, for characterizing the realistic MIMO system performance objectives in terms of a single H_∞-norm on the closed-loop transfer function T(s), weighing functions must be incorporated into the above problem setup. The following are the additional steps required for this purpose.

- Represent each of the weighing functions with different variables in a manner similar that for that for each of the individual transfer blocks in 'G(s)'.
- Add these variables to the set of sub-systems.
- Specify again the inter-connections between the sub-systems.
- Specify the external inputs and outputs maintaining the same order as before.
- Run the SYSIC routine again to represent the system 'G(s)' along with the added frequency scalings (weighing functions) with a single matlab SYSTEM variable.

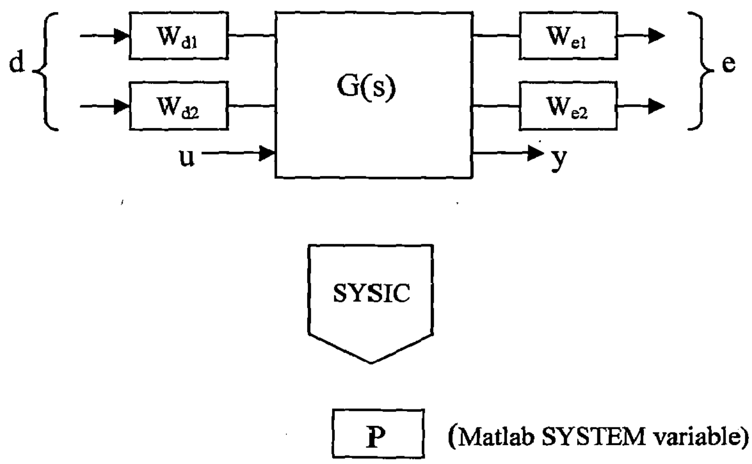


Fig.3.5: From 'G(s)' with weighing functions to 'P' using SYSIC

The H_∞-optimal control design problem is: find a stabilizing controller 'K(s)' such that the closed-loop system T(s) is stable and the H_∞-norm T(s) is less than 'γ', where 'γ' is called as H_∞-cost. The standard state-space technique to find the H_∞-optimal controller is called 'γ-iteration' in which a 'γ' value will be selected and 'K(s)' is derived such that $\|T\|_\infty \leq \gamma$. Next, this 'γ' is modified using the modified bi-section algorithm and 'K(s)' is derived again using the 'γ-iteration'. This iterative procedure continues until a tolerance condition is met with. This iterative procedure is explained in Fig.3.6 with the help of a flow chart.

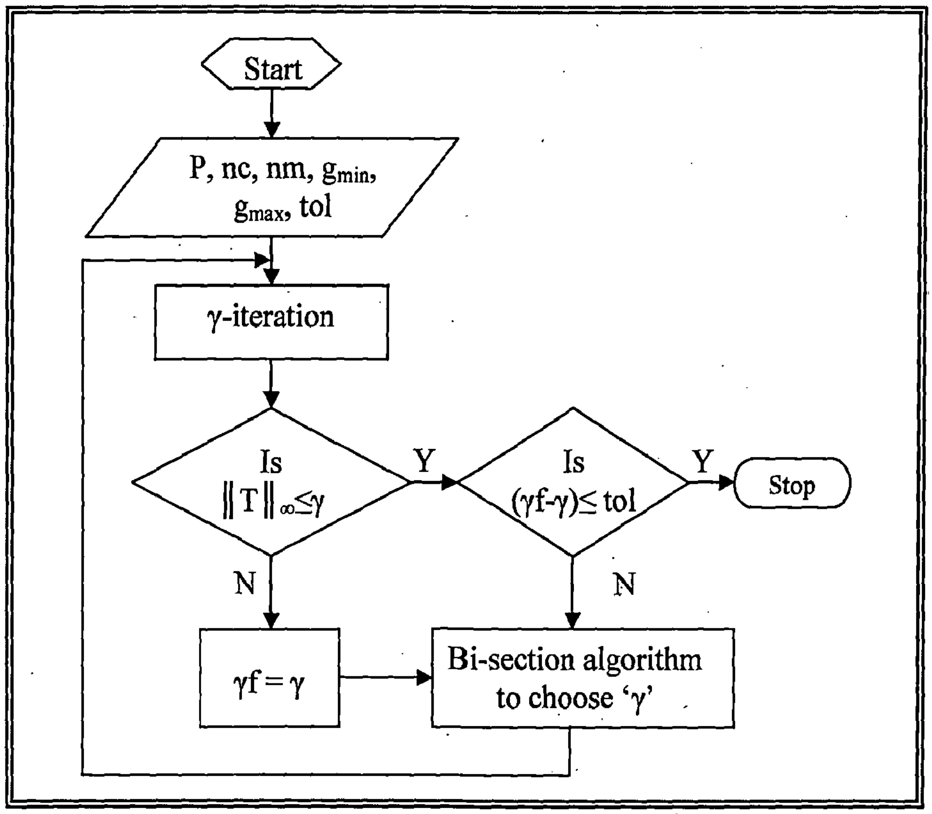


Fig.3.6: Flow chart of H_∞-synthesis algorithm in MATLAB®

Where,

γ = H_∞-cost

nc = number of control signals

nm = number of measurements which are inputs to the controller

g_{min}, g_{max} = minimum and maximum values for 'γ'

γf = value of 'γ' in the iteration in which γ-iteration fails

3.4 Necessary Conditions for H_∞-synthesis[3][6]: There are certain necessary conditions to be satisfied by the system inter-connection structure 'P' for the H_∞-synthesis algorithm to provide a solution. These conditions are listed below. If these conditions are not satisfied before the application of the H_∞-synthesis algorithm, the algorithm will return an error. So it is necessary for a designer to check these conditions and, adjust the feedback connections, if any of these conditions were not satisfied. The system inter-connection structure 'P' can be represented in packed state space form as below.

$$P = \left[\begin{array}{c|cc} A & B1 & B2 \\ \hline C1 & D11 & D12 \\ C2 & D21 & D22 \end{array}\right] \tag{3.21}$$

This can be written in terms of state-space equations as below

$$\begin{aligned} \dot{X} &= A\,X + B1d + B2\,u \\ e &= C1\;X + D11d + D12\,u \\ y &= C2\;X + D21d + D22\,u \end{aligned} \tag{3.22}$$

Where,

X= state variable vector

e = vector of regulated output variables (error signals to be minimized)

y = vector of outputs which will be fed to the controller

d = vector of disturbance input signals

u = vector of control signals which are outputs from the controller

The necessary conditions to be satisfied before applying the H_∞-synthesis algorithm are listed below.

- (A,B_2) *is stabilizable.*

  This can be checked by obtaining the controllability matrix 'CO' and testing whether it is of full row rank or not. Where,

  $$CO = [B\ AB\ldots A^{n-1}B] \tag{3.23}$$

  $$n = \text{size of } A$$

- (C2,A) *is detectable.*

  This is checked by obtaining the observability matrix 'OB' and testing whether it is of full column rank or not.

$$\mathrm{OB} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{3.24}$$

The above two conditions are necessary in order to obtain a stabilizing controller.

- D12 *must have full column rank.*

  This condition checks whether all the control inputs in the vector 'u' have influence on the regulated output signals 'e'.

- D21 *must have full row rank.*

  This condition checks whether the input disturbances can be observed in all the out put signals in the vector 'y'.

The last two conditions are required to be satisfied for avoiding the singular control problems. Once the above conditions are satisfied, the designer can apply the H_∞-synthesis algorithm of MATLAB® on the inter-connection structure 'P'. The algorithm works as shown in the flow chart above in Fig.3.6 and returns the H_∞-optimal controller 'K(s)'. If the final value of the H_∞-cost, γ not less than '1', then modify the weighing functions which in turn means modifying the performance characterization and apply the H_∞-synthesis algorithm again. This procedure has to be continued until γ less than '1'condition is achieved. Note that, the condition on γ, to be less than '1'is kept for the reason that the initial characterization of the performance specifications in terms of weighing functions, is done keeping in mind that $\|T(s)\|_\infty$ will be less than or equal to'1'.

In the previous chapter on H_∞-synthesis, the discussion was on the design of an optimal controller 'K(s)', for the nominal model 'G(s)' of an actual plant. For most of the real systems, this design work is not sufficient. Because, any linear model like 'G(s)', derived for an actual plant, can not be considered as precise. In fact it is not possible to derive an exact model for most of the real systems. This statement is true at least in the case of power systems because, it involves many components with high degree of complexity for modeling. Hence, the controller designed for 'G(s)' with the synthesis method of the previous chapter may fail when applied to real systems for the simple reason that our model 'G(s)' of the actual plant is not 100% correct. This means that our design may not be *robust* in terms of stability and performance.

Assume that a designer is asked to design a controller for a plant. As a first step, he will try to derive a transfer function model for plant. But, after some striving, he may realize that the transfer function is becoming very complex or he may feel that it is difficult to get the transfer function for the plant at high frequencies. This will force the designer to simplify the model, and neglect the high frequency dynamics by concentrating over a specific frequency range of interest. The model obtained with these simplifications is called the nominal model 'G(s)'.But in robust control design, the designer has to consider these errors with a family of plants as shown in Fig.4.1.

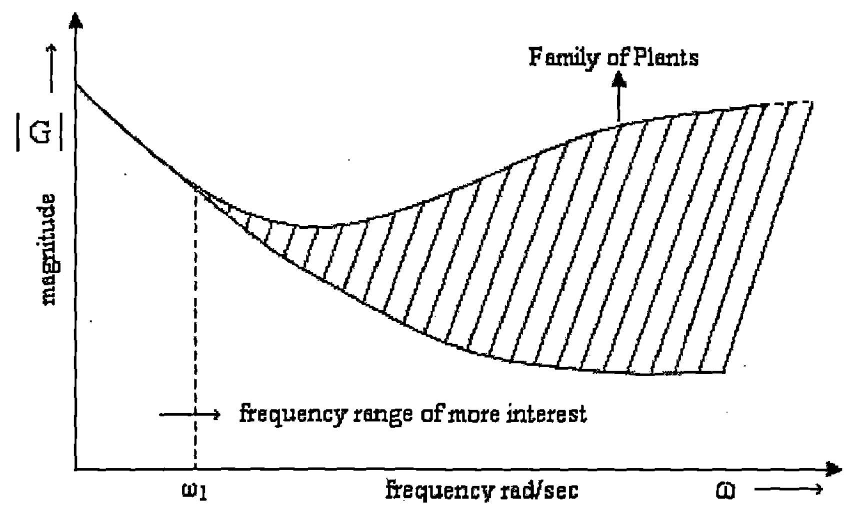


Fig.4.1: Uncertain model of the designer for a plant

Along with the above two types of modeling errors, namely, simplifications to the actual model and neglected high frequency dynamics, there is also another type of modeling error. This error is uncertainties in the parameters. As shown in Fig.4.1, the model of the actual plant is assumed to be 100% correct in the frequency grid of (0-ω_1). But in fact even in this frequency range, the parameters of the transfer function model will be uncertain for the reason that these parameters may vary with time. For example, in the frequency range of (0-ω_1), let the transfer function model of the plant is

$$G(s) = \frac{K}{sT+D} \tag{4.1}$$

After further assessment of errors, the designer may realize that even in this frequency range, the parameter 'D' is not constant and varies with time i.e. 'D' is uncertain. From the above discussion the sources for modeling errors can be divided primarily into 3 types. They are

- Simplifications to the actual model
- Neglected high frequency dynamics
- Parametric uncertainties

All these errors are called as 'model uncertainties'. Because of these model uncertainties, the designer comes across the tough task of thinking in terms of a family of plants instead of a single nominal plant while designing the controller. Now the problem is, How to deal with these model uncertainties in mathematical terms? *John Doyle* proposed first that these model uncertainties can be described mathematically in terms of bounded uncertainties.

4.1 Bounded Complex Uncertainties [18]: This concept is very important in the understanding of robust control theory. With out understanding this, it will not be possible to step further in robust control design. The meaning of bounded uncertainty is as follows. For a real plant, if we have an exact transfer function model, then the Nyquist plot of this plant model will have one point at each frequency. But, due to the errors or uncertainties in the model for the plant, at each frequency in the Nyquist plot, we will have more than one point as shown in Fig.4.2.

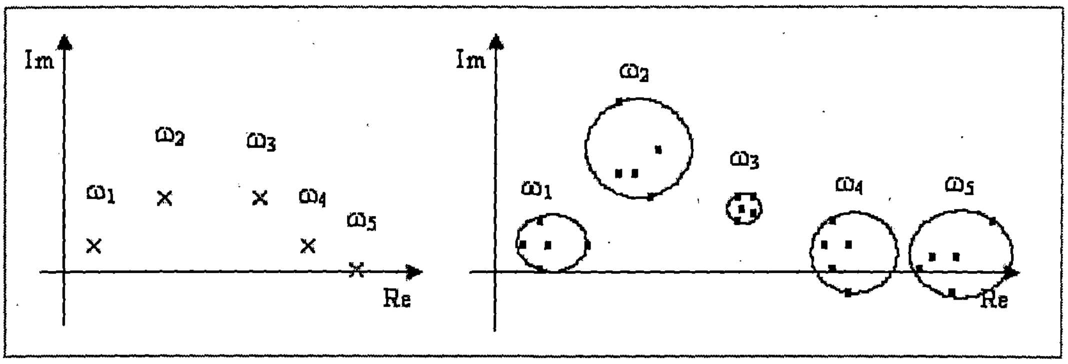


Fig.4.2: Nyquist plots for plant without uncertainty and with uncertainty

These uncertainties may be due to any or all of the three reasons mentioned above. In Fig.4.2, only 5 points of the entire frequency grid are shown. But in fact, this uncertainty will be there in the entire frequency grid. Note that the uncertainty at each frequency point has a bound shown by a circle. This means at each frequency point, the uncertainty has a maximum bound. If we can find the center and radius of each of the circle at all the frequencies, then we can represent this model uncertainty mathematically. The following example illustrates the steps in obtaining the bounded complex uncertainty model for any of the above three types of uncertainties in the model for a plant. Assume that we are given a transfer function,

$$G(s) = \frac{K}{sT+D} \tag{4.1}$$

Where, K=2, T=0.05 and D = ($0.5 \leq 1 \leq 1.5$) i.e. 50% uncertainty.

If we draw the Nyquist plot for this uncertain transfer function using Manto_Carlo samplings for 'D', we obtain a family of Nyquist plots as shown in Fig.4.3. In this example the parametric uncertainty is considered for the purpose of illustration. Even for the other types of uncertainties like neglected dynamics, if the uncertainty bounds in Nyquist plot are provided, the following procedure will give the bounded complex uncertainty model mathematically. An important point to be noted here is that, the present DKITGUI of MATLAB® which is the algorithm for the robust controller synthesis can deal only the bounded complex uncertainties. Hence the real parametric uncertainties can not be considered in that algorithm and it is necessary to convert these real parametric uncertainties in the transfer functions into bounded complex uncertainties.

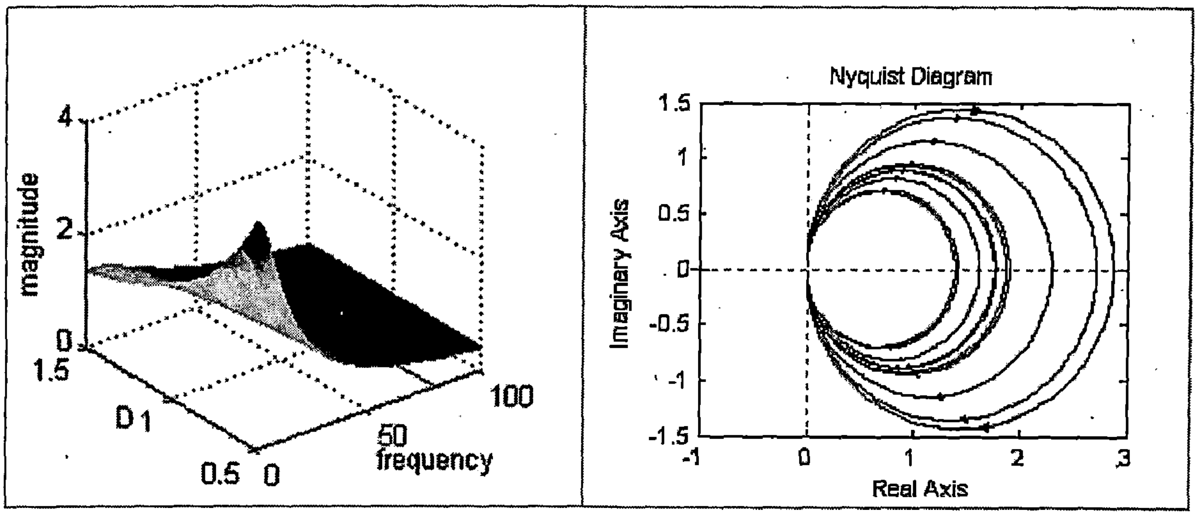


Fig.4.3: Uncertain transfer function for 10 randomly chosen samples of 'D'

Now the problem is, How to represent mathematically this uncertain transfer function? At each frequency, we can get 10 points in the Nyquist plot for 10 random samples of 'D'. As mentioned earlier, if we can find the center and radius of each of the bounding circle of these 10 points in the Nyquist plot at all the frequencies, we can represent this model uncertainty mathematically. The steps to be followed for this are:

- At each frequency, obtain the real and imaginary parts of each of the 10 points in the Nyquist plot.
- At each frequency, find the averages of the real and imaginary parts of all the 10 points individually. This gives the centers of the bounding circles.
- At each frequency, find the maximum distance from the circle center to the any of the 10 points. This gives the radii of the bounding circles.
- Do curve fitting for the circle centers to obtain the *nominal plant*.
- Do curve fitting for the circle radii to obtain the '*uncertainty maximum bound*' function as a function of frequency.

This curve fitting can be done using the 'FITMAG' function available in MATLAB®. The following Table 4.1 lists the centers and radii of the bounding circles at some of the frequency points, for the uncertain transfer function that we are discussing. This data can be used for obtaining the *nominal plant model* and the *maximum uncertainty bound* function.

Table 4.1: Values of centers and radii of circles in the Nyquist plot for the uncertain transfer function

ω rad/sec	Center of the Bounding circle	Radius of the Bounding circle
0.001	2.1847+0.0604i	1.7390
0.1	2.1846+0.0473i	1.7398
1	2.1754-0.0713i	1.7377
2	2.1483-0.1993i	1.7167
⋮	⋮	⋮
10	1.5882-0.8449i	1.1895
⋮	⋮	⋮
98	0.0808-0.3705i	0.0511
99	0.0792-0.3672i	0.0501
100	0.0777-0.3638i	0.0492

Now curve fitting can be done, on the data of centers of circles given in Table 4.1 above, to obtain a nominal plant model $W_{an}(s)$. The order of the curve fit can be chosen appropriately to get as close fit as possible. Fig.4.4 shows the curve fitting process with a chosen order of fit of '1'.

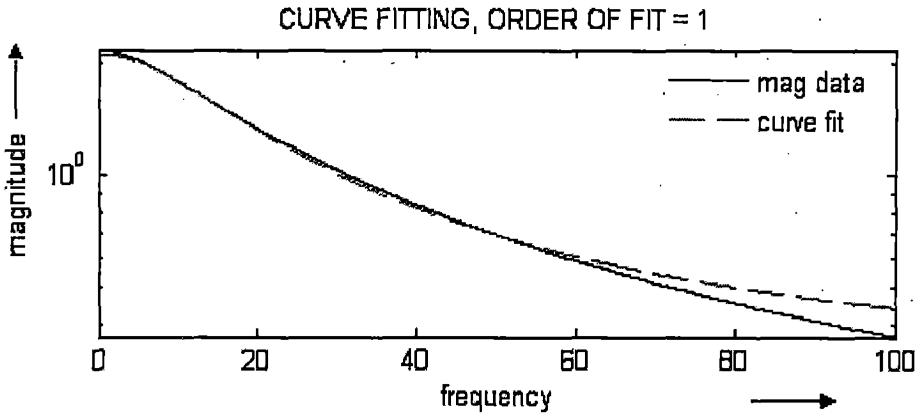


Fig.4.4: Curve fitting for the nominal plant model '$W_{an}(s)$'

Similarly, curve fitting can be done, on the data of radii of circles in Table 4.1above, to obtain the 'maximum uncertainty bound' function '$W_a(s)$' as a function of frequency. Because of the improper fit with order'1', the second order curve fit is chosen for this as shown in the Fig.4.5.

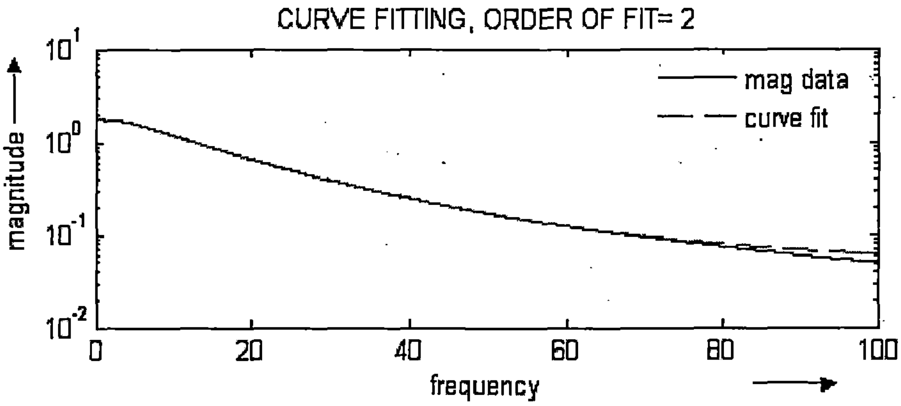


Fig.4.5: Curve fitting for the 'maximum uncertainty bound' function '$W_a(s)$'

The 'nominal plant model' (first order) and the 'maximum uncertainty bound' function (second order) are given below in the transfer function form.

$$W_{an}(s)=\frac{0.3s+33.2}{s+15.6} \qquad W_a(s)=\frac{0.058s^2+6.42s+534.7}{s^2+39.9s+309.6}$$

Once the above two transfer functions $W_{an}(s)$ and $W_a(s)$ are obtained, the uncertain transfer function can be represented as shown in the Fig. below as an additive uncertainty.

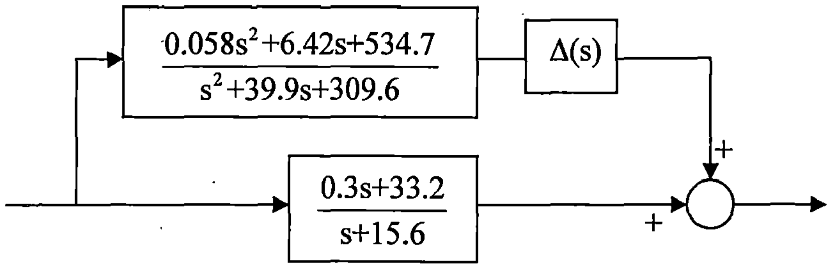


Fig.4.6: Additive uncertainty representation

$\Delta(s)$ is the unity norm bounded complex uncertainty (see section-1.1) i.e. the H_∞-norm of $\Delta(s)$ is less than or equal to '1'. All the uncertainties in the model for an actual plant can be replaced with representations of the type shown above. This completes the first step in robust controller synthesis. Before concluding this chapter, it is necessary to check whether the above procedure is correct or not. This is done by drawing the bode plots for the actual uncertain transfer function and for its bounded complex uncertainty model.

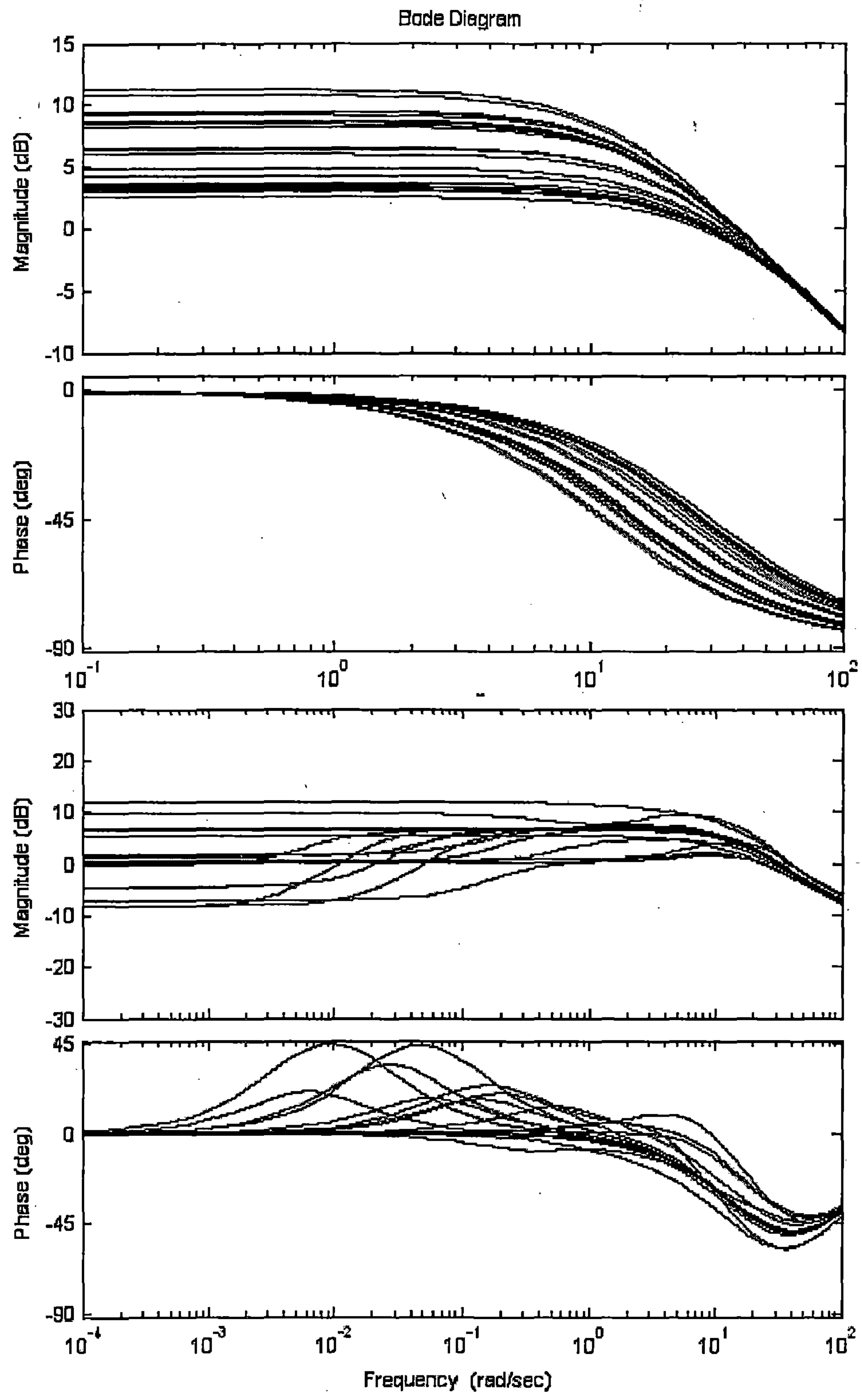


Fig.4.7: Bode plots of uncertain transfer function and its 'bounded complex uncertainty' model

The Bounded complex uncertainty model (bode plot-2) covers all the uncertainties, but it is adding little conservativeness. This means that it is covering more uncertainty than that in bode plot-1. But it is not missing any of the plant in bode plot-1 of Fig.4.7.

Assume that a controller 'K(s)' has been designed for a system. A general curiosity after this is: how robust this control 'K(s)' is? So the immediate step after the controller design must be checking its robustness. There are two types of robustness checks.

- Stability robustness
- Performance robustness

As already discussed in chapter-1, robust stability means stability in the presence of model uncertainties. Similarly robust performance means performance in the presence of model uncertainties. There must be some method for checking the robustness of any designed controller 'K(s)' from the view point of stability and performance. The concept of *structured singular value* (μ) introduced by *John Doyle* [1] can be used as a measure for checking the robust stability and robust performance properties of any uncertain closed-loop system controlled with a controller 'K(s)'. An important point to be noted at this stage is that the structured singular value basically tests the robust stability of an uncertain closed-loop system. In order to use the same concept for robust performance test, the closed-loop H_∞-norm condition for performance is reformulated as a robust stability problem. This reformulation of the H_∞ performance condition into a robust stability problem is discussed in the following sections. This structured singular value concept is very powerful and is the base for the whole 'Robust control design' concept.

5.1 Structured Singular Value (μ) Analysis [2] [5]: It will be good to explain this concept with reference to a diagram of an uncertain closed-loop system as shown in Fig.5.1. As already been discussed in the chapter-4, any model of a real system will have uncertainties. These uncertainties may exist in different transfer function blocks ($\Delta_1(s)$, $\Delta_2(s)$) or as un-modeled dynamic uncertainties ($\Delta_3(s)$) on the whole model as shown in Fig.5.1. Before applying the μ-analysis, it is necessary to transform the basic uncertain closed-loop system model to a standard (M(s)-Δ(s)) structure as shown in the Fig. below. This transformation is done by separating the unity-norm bounded uncertainty blocks (Δ's).

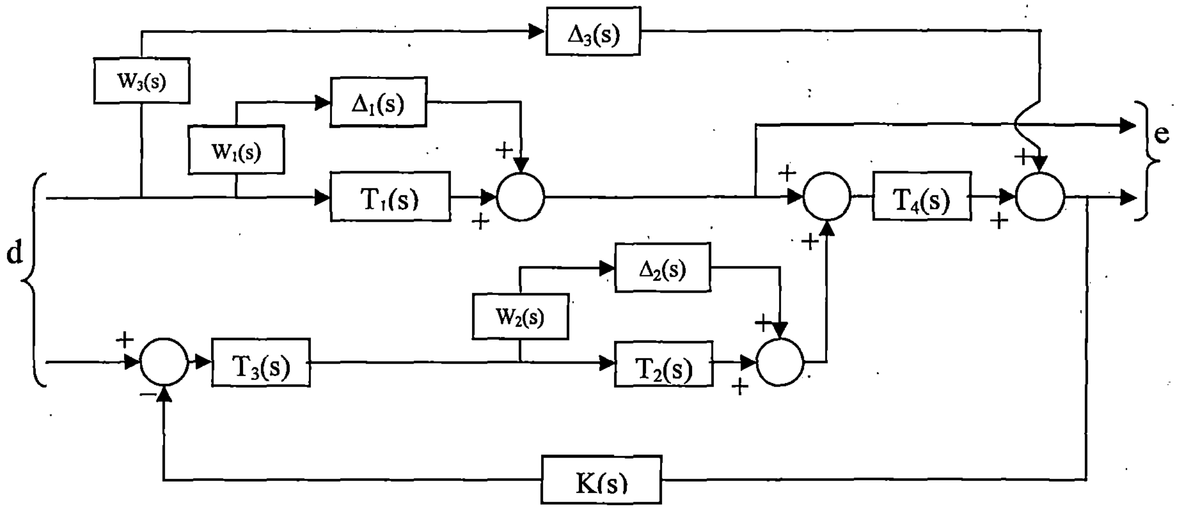


Fig.5.1: An uncertain closed-loop system

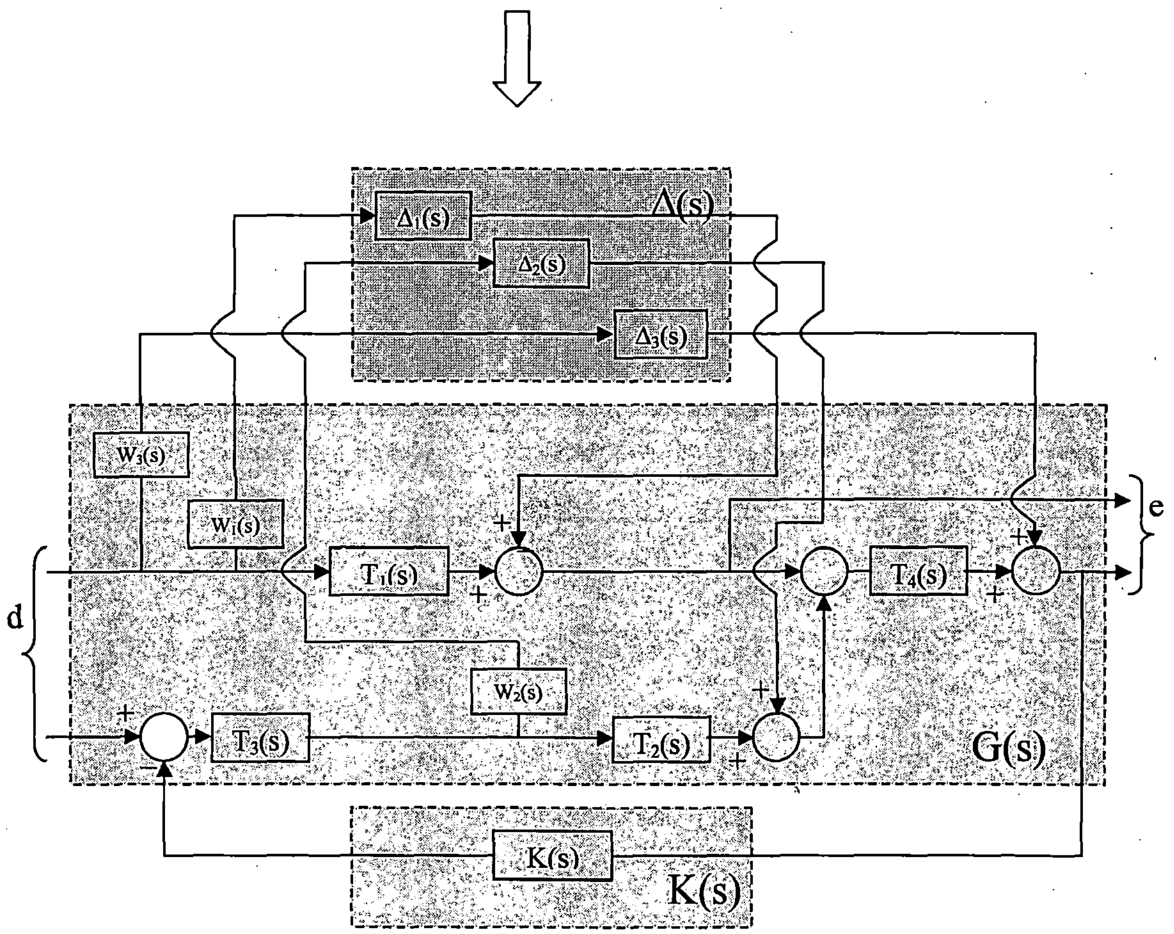


Fig.5.2: Separation of $\Delta(s)$ and $K(s)$

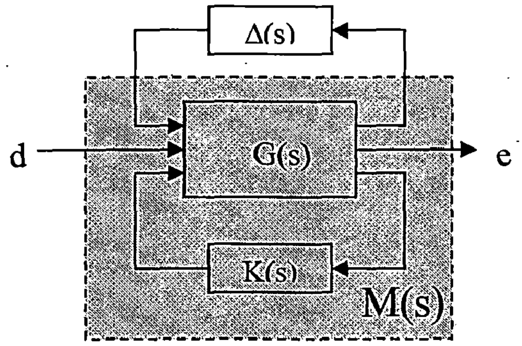


Fig.5.3: Δ-G-K structure to M(s)-Δ(s) structure

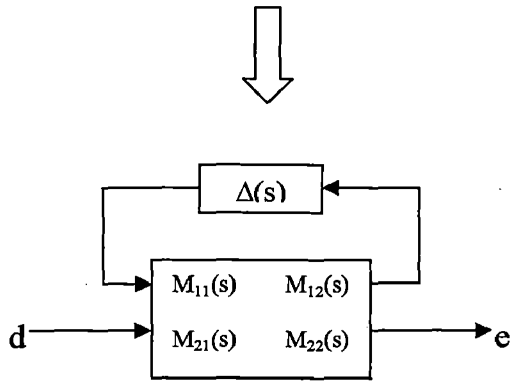


Fig.5.4: Final M(s)-Δ(s) structure.

But, for analyzing the robust stability of an uncertain closed-loop system by μ-analysis, it is necessary to separate a part of 'M(s)' which will interact with 'Δ(s)'. From the Fig.5.4, it can be seen that $M_{11}(s)$ is the one which is interacting with 'Δ(s)'. So the final structure on which μ-analysis has to be applied for robust stability analysis becomes as shown in the Fig.5.5.

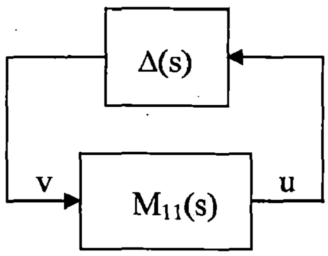


Fig.5.5: Structure for robust stability analysis

Note: In the Following discussions a variable with out the Laplacian variable's' represents that variable, at a particular frequency. For ex: M, Δ are matrices with complex values as elements at a particular frequency.

5.1.1 Robust Stability Analysis: The structured singular value $\mu_\Delta(.)$ of any complex matrix $M \in C^{n\times n}$ with respect to a norm bounded uncertain block structure 'Δ' is defined as,

$$\mu_\Delta(M) = \frac{1}{\min_\Delta\{\ \bar{\sigma}(\Delta): \det(I_n - \Delta M) = 0\ \}} \tag{5.1}$$

and $\mu_\Delta(M) = 0$ if there is no Δ solves $\det(I_n - \Delta M) = 0$. $\bar{\sigma}(\Delta)$ means the maximum singular value of $\Delta \in C^{n\times n}$.

In the case of the uncertain closed-loop system above, at each frequency $M_{11}(s)$ will be a complex matrix of size $C^{3\times3}$. To find the structured singular value, we need to find a Δ, which is a complex matrix of size $C^{3\times3}$ at that frequency and solves $\det(I - \Delta M_{11})$ to zero. The *inverse of the maximum singular value* of this complex Δ matrix is called the structured singular value of M_{11} ($C^{3\times3}$) at that particular frequency. Like this the μ has to be found at each frequency over the entire ω (frequency) range.

In the $M_{11}(s)$-Δ(s) structure shown in the Fig. above, if $\det(I_3 - \Delta_{3\times3} M_{3\times3}) \neq 0$, then the only solution for 'u' and 'v' are, u = v = 0. On the other hand if this determinant is zero, then 'u' and 'v' can have infinite number of solutions. This later condition can be said as instability.

$$\det(I_3 - \Delta_{3\times3} M_{3\times3}) \neq 0 \text{ (Stable condition)} \tag{5.2}$$

$$\det(I_3 - \Delta_{3\times3} M_{3\times3}) = 0 \text{ (Unstable condition)} \tag{5.3}$$

Since the structured singular value finds the $\min\{\bar{\sigma}(\Delta_{3\times3})\}$ which makes the system to reach the above unstable condition, this can be used as a measure of robust stability. If the $\bar{\sigma}(\Delta_{3\times3})$ which makes $\det(I_3 - \Delta_{3\times3} M_{3\times3}) = 0$, is greater than '1' over the entire frequency range, then the 'μ' value will be less than '1' over the entire frequency range. Now the robust stability condition is stated as follows:

A system will be robustly stable if its structured singular value (μ) is less than '1' over the entire frequency range.

The statement $\mu \leq 1$ implies $\bar{\sigma}(\Delta_{3\times3}) \geq 1$. But, in the bounded complex uncertainty representation that was discussed in chapter-4, $\Delta(s)$ is an uncertainty whose $\max\{\bar{\sigma}(\Delta)\}$ is less than or equals to '1' over the entire frequency range. This implies that if $\mu \leq 1$ for an uncertain closed loop system over the entire frequency range, then that system is robustly stable. The 'MU' function of MATLAB® can be used for doing this robust stability analysis. The steps for robust stability analysis can be summarized as follows.

- Recast the uncertain closed-loop system into the standard M(s)-Δ(s) structure.
- Separate the $M_{11}(s)$ from this M(s)-Δ(s) structure.
- Apply the 'MU' function of MATLAB® on this $M_{11}(s)$-Δ(s) structure.
- Plot the structured singular value bounds as a function of frequency and find the peak value.
- The closed -loop uncertain system will be stable if the peak value of μ is less than '1'.

A point to be noted is that the 'MU' function can not compute the μ exactly. It computes upper and lower bounds for 'μ'. For testing the robust stability the peak value of the upper bound has to be considered.

5.1.2 Robust Performance Analysis: As already discussed in the chapter-3, the performance of a MIMO closed-loop system is characterized using the H_∞-norm from the disturbance inputs to the regulated output. This was expressed mathematically as $\|T(s)\|_\infty \leq 1$. Similarly, an uncertain closed-loop system can be said to achieve performance robustness, if the H_∞-norm of '$T_u(s)$' shown in the Fig.5.6 is less than or equal to '1'.

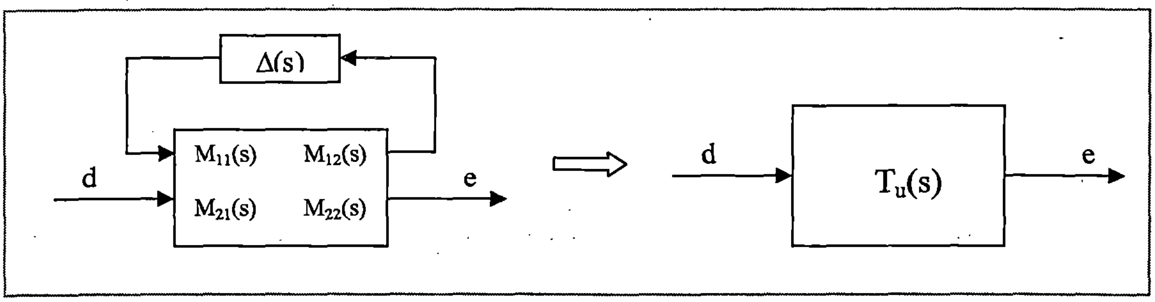


Fig.5.6: Uncertain closed-loop system

But, the problem now is, how the structured singular value concept can be used for the evaluation of the performance robustness of '$T_u(s)$'. For this the problem of $\|T(s)\|_\infty \leq 1$ has to be re-casted into a robust stability problem. According to the small gain theorem, $\|T(s)\|_\infty \leq 1$, iff the feedback loop shown in Fig.5.7 is stable.

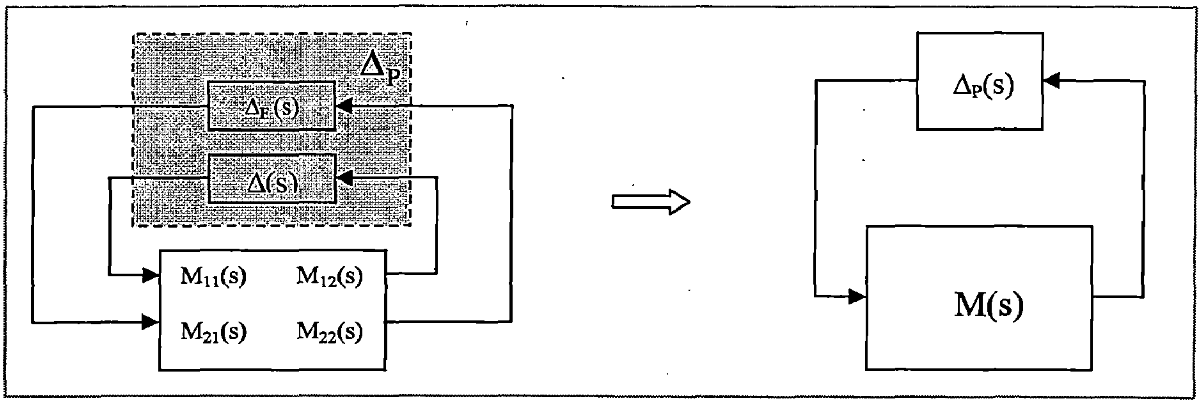


Fig.5.7: Setup for robust performance evaluation

This means that, by appending a $\Delta_F(s)$ block of size $(n_d \times n_e)$, the robust performance problem can be re-casted into a robust stability problem. This implies that for an uncertain closed-loop system, robust stability can be checked by applying μ-analysis on the $M_{11}(s)$-$\Delta(s)$ structure, and robust performance can be checked by applying μ-analysis on the $M(s)$-$\Delta_P(s)$ structure. The steps in the robust performance analysis can be summarized as follows.

- Recast the uncertain closed-loop system into the standard $M(s)$-$\Delta(s)$ structure.
- Append $\Delta_F(s)$ of size $(n_d \times n_e)$ to $\Delta(s)$ to obtain $M(s)$-$\Delta_P(s)$ structure.
- Apply the 'MU' function of MATLAB® on this $M(s)$-$\Delta_P(s)$ structure.
- Plot the structured singular value bounds as a function of frequency and find the peak value.
- The closed-loop uncertain system is robust with respect to performance if the peak value of 'μ' is less than '1'.

Since the 'MU' function of MATLAB® computes the upper and lower bounds for 'μ', in this case also, the peak value of the upper bound has to be taken as a measure. Fig.5.8 shows the steps in the structured singular value analysis diagrammatically.

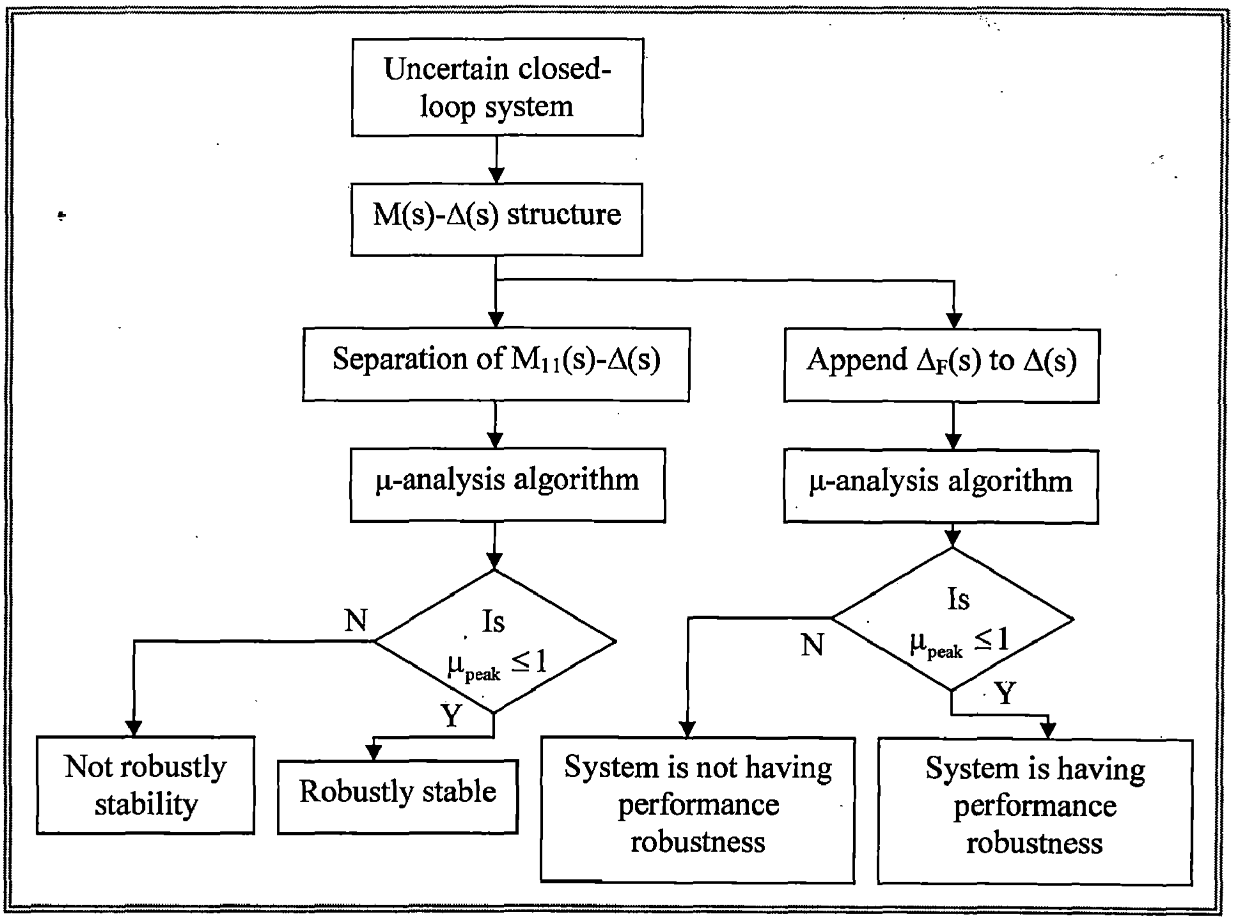


Fig.5.8: Steps in the structured singular value analysis of a closed-loop uncertain system

5.2 Structured Singular Value Synthesis [2] [5] [8]: The H_∞-synthesis that we discussed in chapter-3 was not a robust control design, because, it is not taking the model uncertainties into account. But, as we discussed in chapter-4, any real system model will have errors which are collectively called as 'model uncertainties'. So it is necessary to go for a design process which takes these model uncertainties into account and synthesizes a controller which will work properly on a real system. The μ-synthesis algorithm which will be discussed in this section is a robust control design process. Note that the μ-synthesis process is not totally different from the H_∞-synthesis. In fact, the μ-synthesis algorithm repeatedly uses the H_∞-synthesis algorithm in its design process. Hence the H_∞-synthesis must always be the concept to be understood first before stepping to the more powerful μ-synthesis. The 'DKITGUI' tool available in the Robust Control Toolbox of MATLAB® can be used for doing this μ-synthesis. Here 'DKIT' means D-K iterations.

5.2.1 The μ-upper Bound: This topic is very important for understanding the meaning of D-K iterations and hence the μ-synthesis algorithm. Refer to the $M_{11}(s)$-Δ(s) structure in Fig.5.5. All the variables below are matrices at a particular frequency in the frequency grid 'ω'.

$$\mu_{\Delta}(M) \le \bar{\sigma}(M) \tag{5.4}$$

$$\Delta \in \{\chi : \bar{\sigma}(\Delta) \le 1\} \tag{5.5}$$

Let 'D' belongs to a set of real positive diagonal invertible matrices with same structure as 'Δ'. For example, for the uncertain system case shown in Fig.5.1, the structure of 'Δ(s)' and 'D' are:

$$\Delta(s) = \begin{bmatrix} \Delta_1(s) & 0 & 0 \\ 0 & \Delta_2(s) & 0 \\ 0 & 0 & \Delta_3(s) \end{bmatrix} \tag{5.6}$$

$$\Delta_1 : C^{1\times1};\ \Delta_2 : C^{1\times1};\ \Delta_3 : C^{1\times1} \tag{5.7}$$

$$D = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \tag{5.8}$$

Where,

a_1, a2 and a_3 are real positive numbers.

With this property of 'D', $\quad D\Delta D^{-1} \in \{\chi : \bar{\sigma}(\Delta) \le 1\}$ (5.9)

This implies $\quad \mu(DMD^{-1}) = \mu(M)$ (5.10)

From equation (5.4), $\quad \mu(M) = \mu(DMD^{-1}) \le \bar{\sigma}(DMD^{-1})$ (5.11)

So this becomes an optimization problem to find a 'D' which belongs to a set of real positive diagonal invertible matrices with the structure same as (5.8), and minimizes the $\bar{\sigma}(DMD^{-1})$. This can be written mathematically as

$$\mu_{\Delta}(M) = \min_{D} \bar{\sigma}(DMD^{-1}) \tag{5.12}$$

The above optimization problem of finding, a 'D' which will minimize the $\bar{\sigma}(DMD^{-1})$ as in (5.12), gives the upper bound for 'μ' at each frequency.

The 'D' matrix has one real diagonal element corresponding to each scalar complex unity-norm bounded uncertainty as in Fig. 5.1 or as in (5.6) at each frequency in the frequency grid 'ω'. Since there are '3' complex unity-norm bounded uncertainty blocks in Fig. 5.1, for that example case, the 'D' matrix will have '3' real diagonal elements. For finding the 'μ' upper bound, this 'D' matrix is found at each frequency by solving the minimization problem of (5.11). The values of each of these 'D' matrix diagonal elements as a function of frequency are called as 'D-scales'. In the D-K iteration process of μ-synthesis algorithm, 'D' stands for these 'D-scales'. The 'MU' function of MATLAB® which calculates the μ-bounds also returns the above D-scales. In fact, the 'MU' function of MATLAB® follows the above procedure in calculating the μ-upper bound.

The above discussion on 'μ' upper bound is for the robust stability analysis ($M_{11}(s)$-$\Delta(s)$ structure) case. For the case of robust performance analysis ($M(s)$-$\Delta_P(s)$ structure), the 'μ' upper calculation problem will be modified due to the extra $\Delta_F(s)$ block that is appended to the $\Delta(s)$. In this case the structure of '$\Delta_P(s)$' and 'D' are:

$$\Delta_P(s) = \begin{bmatrix} \Delta_1(s) & 0 & 0 & 0 \\ 0 & \Delta_2(s) & 0 & 0 \\ 0 & 0 & \Delta_3(s) & 0 \\ 0 & 0 & 0 & \Delta_F(s) \end{bmatrix} \tag{5.13}$$

$$\Delta_1{:}C^{1\times1};\ \Delta_2{:}C^{1\times1};\ \Delta_3{:}C^{1\times1};\ \Delta_F{:}C^{n_d\times n_e} \tag{5.14}$$

$$D = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & a_4 I_{n_e} \end{bmatrix} \tag{5.15}$$

$$D^{-1} = \begin{bmatrix} \frac{1}{a_1} & 0 & 0 & 0 \\ 0 & \frac{1}{a_2} & 0 & 0 \\ 0 & 0 & \frac{1}{a_3} & 0 \\ 0 & 0 & 0 & \frac{1}{a_4} I_{n_d} \end{bmatrix} \tag{5.16}$$

Now also, a same minimization problem as (5.12) has to be solved to obtain the 'D' and 'D^{-1}'. This gives the 'μ' upper bound which can be used for checking the robust performance.

5.2.2 D-K iterations: Once we input the standard problem setup which can be derived by following the procedures of chapter-3 and chapter-4, to the 'DKITGUI' tool, the following iterations will occur. Refer to Fig.5.9 below in which the D(s) and $D^{-1}(s)$ blocks are added to the uncertain closed-loop system in Fig.5.3.

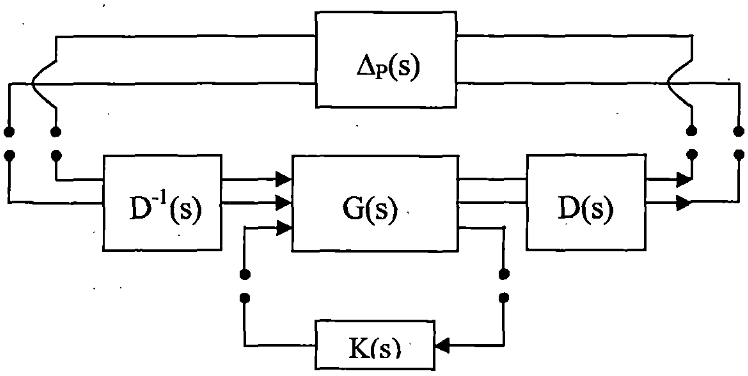


Fig.5.9: Uncertain closed-loop system during D-K iterations

- **K-iteration:** In this first iteration, separate $\Delta_P(s)$ and K(s) as shown in Fig.5.9. Now D(s) and $D^{-1}(s)$ are taken as unity matrices. These unity matrices are added to G(s) as shown above and the H_∞-synthesis algorithm is applied. This gives a controller $K_1(s)$.
- **D-iteration:** During this iteration, the D(s) and $D^{-1}(s)$ blocks are to be taken out. After this the $\Delta_P(s)$ and $K_1(s)$ blocks are connected to G(s) and μ-analysis is applied for testing the robust performance. The 'MU' function used for this returns the $D(\omega)$

and $D^{-1}(\omega)$ matrices for the entire frequency range. Curve fitting is applied on these $D_1(\omega)$ and $D_1^{-1}(\omega)$ to obtain $D_1(s)$ and $D_1^{-1}(s)$. The order of the fit can be chosen by the designer.

- **K-iteration:** Now, $\Delta_P(s)$ and $K_1(s)$ are to be separated from G(s) and the new D-scales $D_1(s)$ and $D_1^{-1}(s)$ are added to G(s) as in Fig.5.9. Now, the H_∞-synthesis algorithm will be applied on this modified G(s), to obtain $K_2(s)$.

This iterative procedure is continued until a minimum value of 'μ' is reached. Note that in some cases 'μ' may not converge globally. In that case the iterations are to be continued up to the point where the present 'μ' is greater than the 'μ' of previous iteration.

5.2.3 μ-synthesis [2]: From Fig.5.2, remove the Δ's and 'K(s)' and add appropriate weighing functions at the disturbance inputs and regulated output variables similar to that in the case of H_∞-synthesis. Represent this transfer function model in terms of a Matlab SYSTEM variable, say 'P' using 'SYSIC'. 'P' will have the interconnection structure as shown in Fig.5.10. This 'P' is different from that in Fig. 3.5 for the reason that this includes the 'uncertainty maximum bound' functions also.

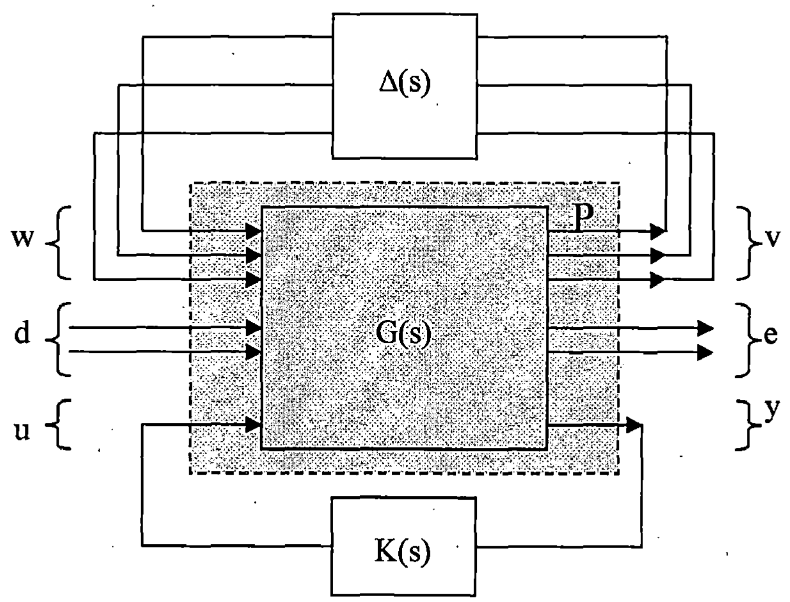


Fig.5.10: The sequence of inputs and outputs of 'P' during μ-analysis

Where,

w: outputs from '$\Delta(s)$' block

v: Inputs to '$\Delta(s)$' block

d: vector of disturbance input signals

e: vector of regulated output signals

u: control input

y: input to the controller '$K(s)$'

After this, the synthesis procedure proceeds as follows.

- Call the DKITGUI
- Inputs to the 'DKITGUI' are- P, dimensions of $\Delta(s)$ (called as uncertainty block structure), no. of input disturbance signals (d), no. of output regulated variables (e), no. of control inputs (u), and no. of inputs to the controller (y).
- The D-K iterations as explained in section-5.2.2 will follow. During each iteration choose an appropriate order for $D(s)$ so that it fits better for the $D(\omega)$.
- In each of the D-k iterations, check whether 'μ' has reached the minimum.
- If the final 'μ' achieved is less than '1', choose the final controller as the output of the design process.
- If the final 'μ' is not less than '1', then modify the weighing functions and do the μ-synthesis again.

The μ-synthesis procedure is given as a flow chart in chapter-1. The value of 'μ' less than '1' means, the designed controller can make the system robust in terms of stability and performance. The robustness of the designed H_∞-controller and μ- controller can be checked by using μ-analysis or with the help of time domain simulations.

CHAPTER-6 LOAD FREQUENCY CONTROL PROBLEM IN DEREGULATED ENVIRONMENT

Currently, electric power industry is in a transition from large, vertically integrated utilities providing power at regulated rates to an industry that will incorporate *competitive companies* selling unbundled power at lower rates. Ultimately consumers will benefit from lower rates as a result of serious competitive bulk power markets. With this new structure, that will include separate-generation (GENCOs), distribution (DISCOs) and transmission companies (TRANSCOs) with an open access policy, comes a need for the novel control design methods.

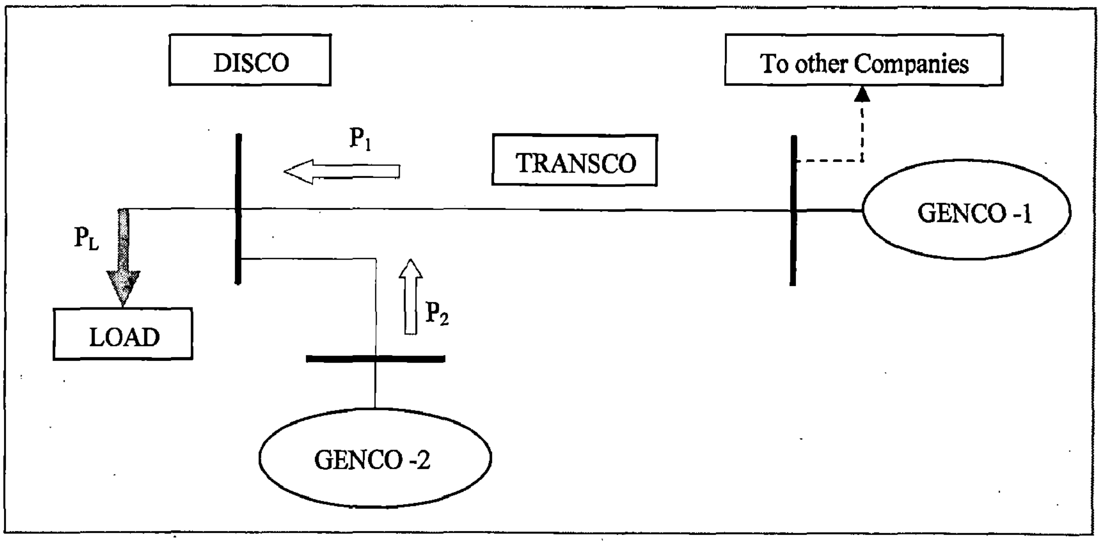


Fig.6.1: A simple deregulated power system structure [17] [25]

The power system structure shown in Fig.6.1 is having one DISCO, two GENCOs and a TRANSCO. Distribution Company buys power from GENCOs and distributes it to its customers, directly or through TRANSCOs. DISCO is the one which will be tracking the loads of its customers (P_L) continuously and hence is responsible for performing the Load Frequency Control (LFC) task by securing as much power as needed from the GENCOs. DISCO buys firm-power (P_2) from GENCO-2 and enough power (P_1) from GENCO-1 to supply its customers load. TRANSCO transmits power from GENCO -1. GENCO-1 is also connected to other companies, which are treated as disturbances. GENCO-1 and GENCO-2 are assumed to have one generator each for simplicity of the problem.

In this work, design of the load frequency controller is considered for the deregulated power system structure described above, using the 'H_∞-synthesis' and the 'μ-synthesis' approaches of control design. The basic control requirements are as follows:

- The frequency deviations of both GENCO-1 and GENCO-2 must come to zero after a load or other disturbances.
- The firm-power contract with GENCO-2 must be maintained by bringing the power supply deviation (ΔP_2) in the line from GENCO-2, to zero after a load or other disturbance.
- There are also some standard performance requirements like: steady-state frequency deviations of both GENCO-1 and GENCO-2 should not be more than 2 Hz, governor control input rate should not be too high (rate limit), power swings in the lines must be as minimum as possible etc. Along with these basic control requirements, robustness is also another important requirement in control design. The control design method must be able to take into account the model uncertainties.

Table 5.1 gives the data of various components present in the deregulated power system shown in Fig.6.1 for which the load frequency controller has to be designed.

Table 6.1: Data of the deregulated power system [17] [25]

Name	Quantity	Genco-1(1000MW)	Genco-2(750MW)
T	Synchronizing power coefficient of transmission line	0.2	0.1
H	Constant of inertia	5	5
D	Damping constant	0.02	0.015
f_o	Nominal frequency	50	50
T_m	Turbine time constant	0.5	0.5
T_h	Governor time constant	0.2	0.1
K_m	Gain of turbine	1	1
K_h	Gain of governor	1	1
R	Droop characteristic	4	5
T_P	Generator time constant	0.167	0.167

The linear model of this deregulated power system structure is shown in Fig. 6.2 below.

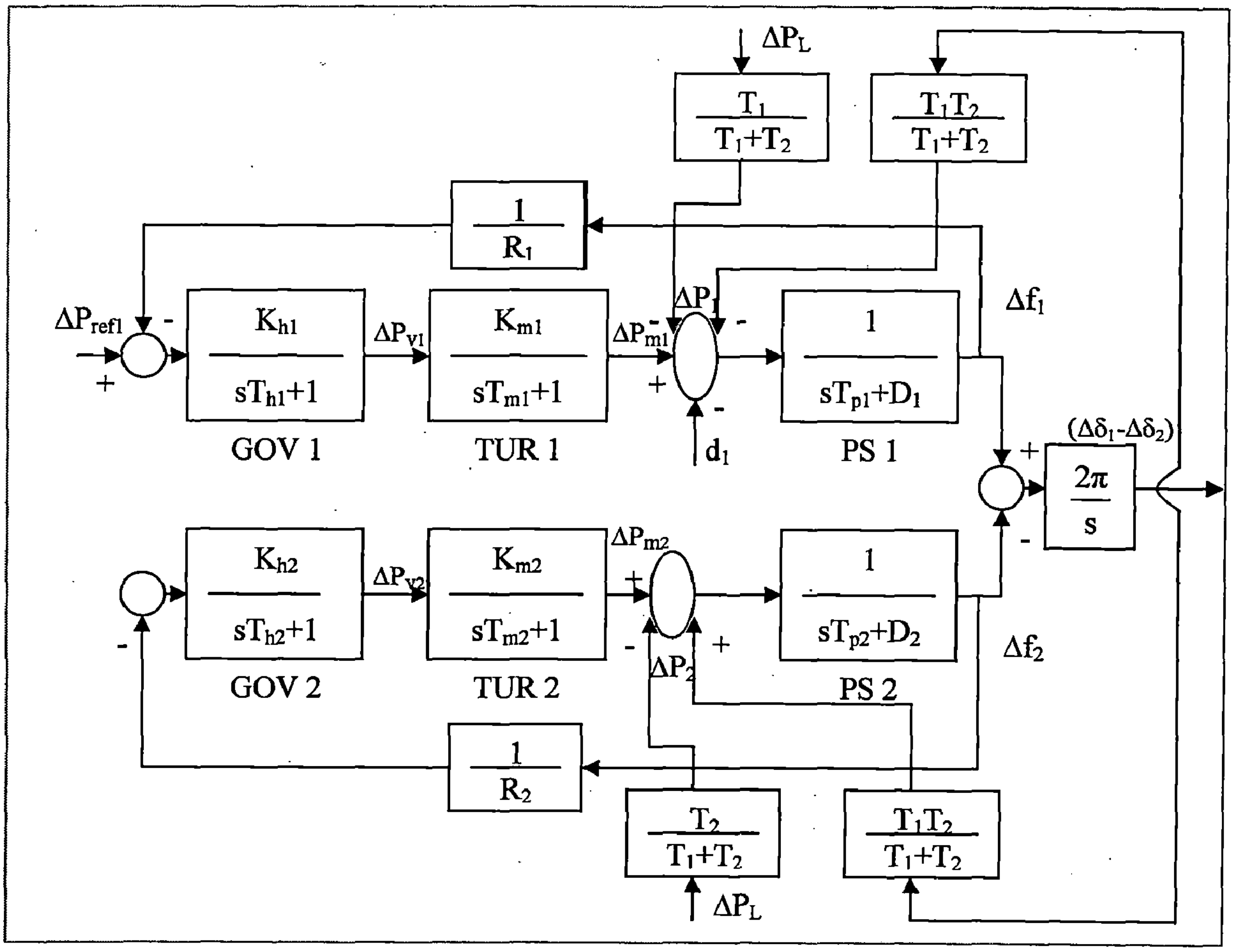


Fig. 6.2: Linear-model of the deregulated power system structure

Table 6.2 given below gives the descriptions for some of the variables and transfer function block names used in the above linear model of the deregulated power system.

Table 6.2: Description of the terms in the linear model

Δ	Deviation from the nominal value	GOV1	Governor of Genco-1
δ	Rotor angle: $\int \Delta f.dt$	TUR1	Turbine of Genco-1
f	Frequency of Gencos	PS1	Generator of Genco-1
P_m	Turbine mechanical power output	GOV2	Governor of Genco-2
d_1	Disturbance (power quantity)	TUR2	Turbine of Genco-2
P_v	Steam valve power	PS2	Generator of Genco-2
$P_{ref} = u$	Set point reference (control input	P_L	Load on Disco

6.1 H_∞-controller: For synthesizing a controller, the above linear model of the power system is changed into a standard H_∞-problem setup which satisfies the *necessary conditions* given in section-3.4. The standard problem setup for this deregulated power system problem involves the following input and output signals shown in Fig.6.3.

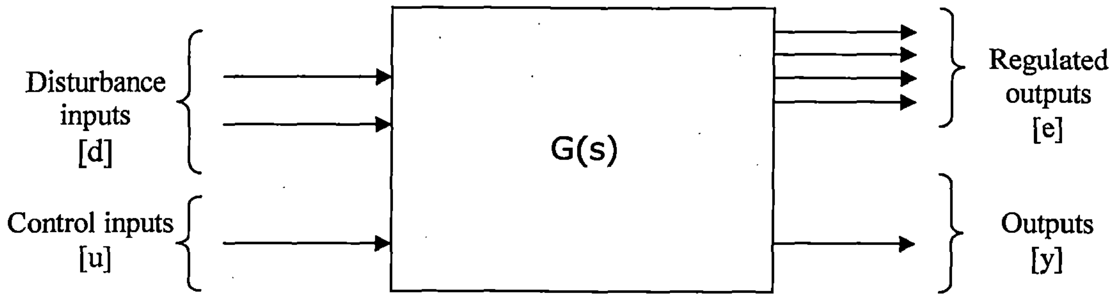


Fig.6.3: Inputs and outputs of the H_∞-problem setup

$$[d] = [\Delta P_L \; d_1]^T \tag{6.1}$$

$$[e] = [\Delta f_1 \; \Delta\delta_1 \text{-} \Delta\delta_2 \; \Delta\delta_1 \; u]^T \tag{6.2}$$

$$[u] = [\Delta P_{ref1}] \tag{6.3}$$

$$[y] = [\beta_1 \Delta f_1 + \beta_2 \Delta f_2 + (\Delta\delta_1 - \Delta\delta_2) + \Delta\delta_1 + \Delta P_1 + \Delta P_2] \tag{6.4}$$

Where,

$\beta_1 = D_1 + 1/R_1 = 0.27$ and $\beta_2 = D_2 + 1/R_2 = 0.27$

Equation (6.4) is called as Distribution Company Error (DCE) similar to the traditional Area Control Error (ACE) [ ]. The above problem setup is converted into a single matlab SYSTEM variable 'P' using the 'SYSIC' routine of MATLAB®. The necessary conditions for the H_∞-synthesis are checked. Then the H_∞-synthesis algorithm of MATLAB® is applied on this 'P'. This has resulted the following H_∞-optimal controller '$K_H(s)$'.

$$K_H(s) = \frac{-0.023s^7 - 0.459s^6 - 3.110s^5 - 10.533s^4 - 25.920s^3 - 42.021s^2 - 43.348s - 21.761}{0.09s^6 + 1.51s^5 + 14.79s^4 + 69.33s^3 + 96.85s^2 + 154.39s + 58.96} \tag{6.5}$$

The H_∞-cost 'γ' achieved for the closed loop system is 10.9510. The time response plots in Fig.6.4 are the response of the closed-loop nominal power system to a step load disturbance (ΔP_L) of 0.15 Pu.

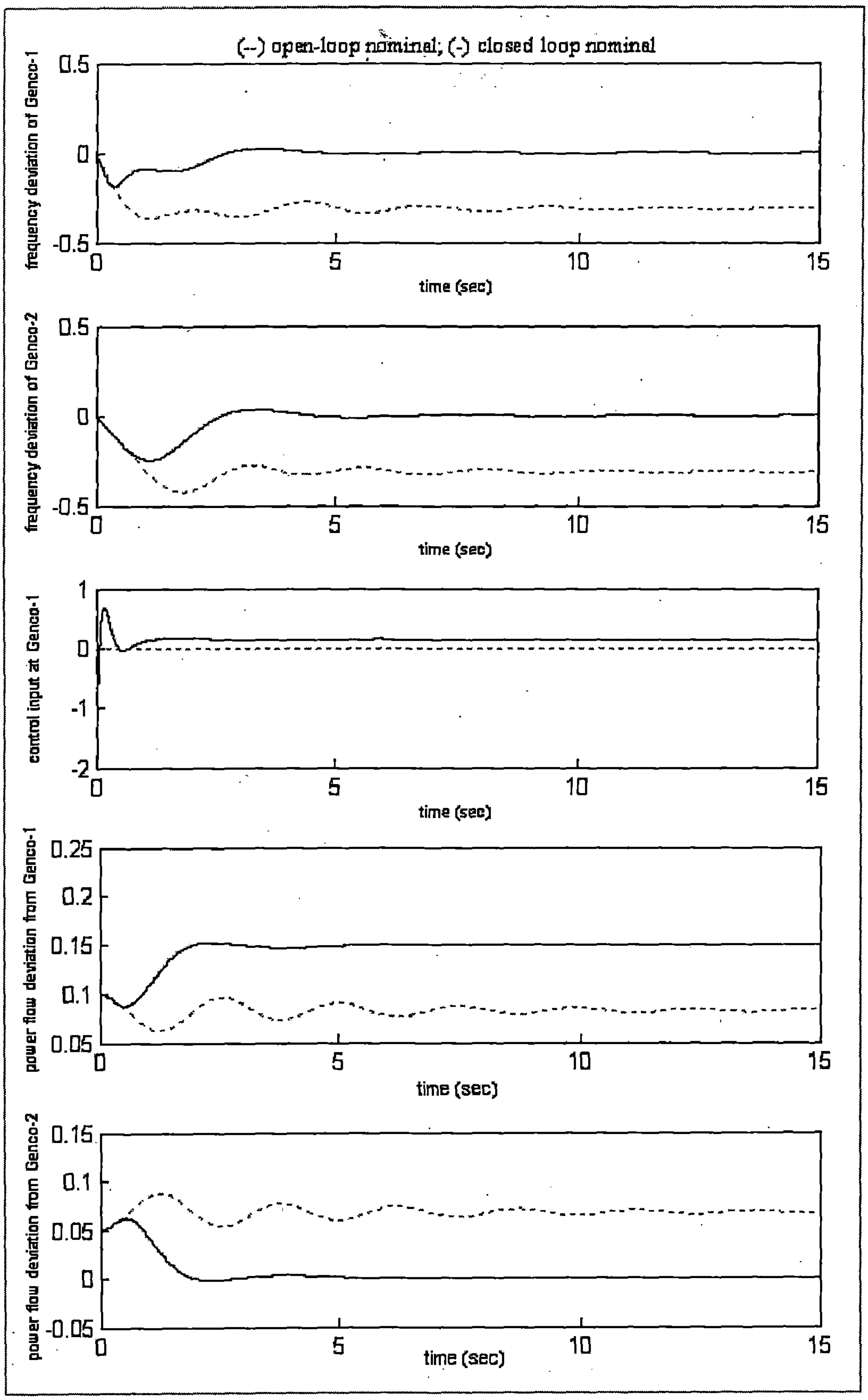
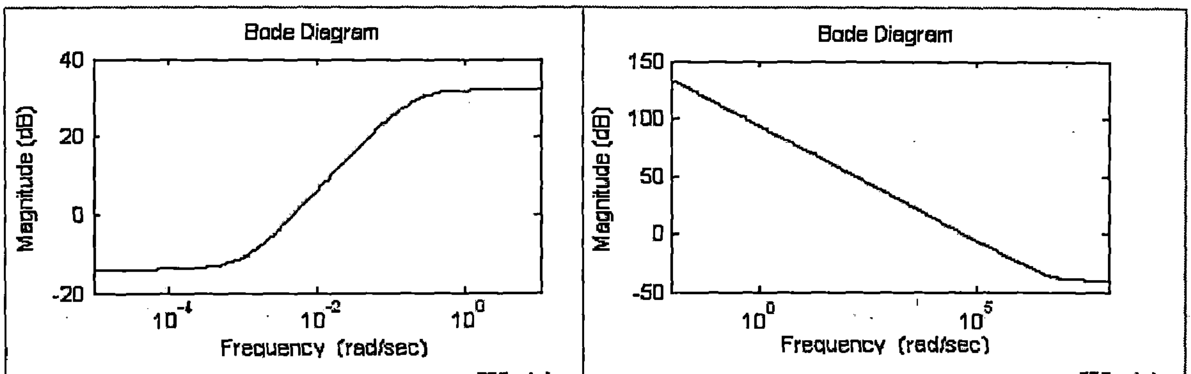


Fig. 6.4: Response of the deregulated power system to a step load disturbance of 0.15 Pu with and without the H_∞-optimal controller $K_H(s)$

6.2 Weighted H_∞-controller: As already discussed in section 3.2, the addition of appropriate weighing functions to the problem setup (Fig.6.3), which will resemble the performance requirements on the regulated outputs and data of the possible disturbance inputs, modifies the problem to designer's requirement. It can be observed from Fig.6.4:

- The frequency response (Δf_1) of GENCO-1 is taking almost '5 sec.' to settle down. To reduce this settling time, a weighing function '$W_{f1}(s)$' which stresses on the reduction of the amplitudes of low frequency components is selected for modifying the design problem.
- It can also be observed from Fig.6.4 that, the control input ($u = \Delta P_{ref1}$) at GENCO-1 is having very steep rate of change in the very starting. The rate limit in the governor action will not allow this. For reducing this high rate of change of the control input, a weighing function '$W_{u1}(s)$' which stresses the reduction of the amplitudes of the high frequency components is selected to modify the design problem accordingly.
- For bringing the H_∞-cost (γ) to a value less than '1', the data of the disturbance inputs is also modified with the help of weights. In this problem, the maximum amplitude of any frequency component of the disturbance inputs is assumed to be not more than 0.5. Hence, the weights on the disturbance inputs $W_{P_L}(s)$, $W_{d1}(s)$ are chosen as 0.5.

Fig.6.5 shows the inverse magnitude plots of the weighing functions chosen for modifying the performance requirements of GENCO-1 frequency (Δf_1) and the control input ($u = \Delta P_{ref1}$) at GENCO-1.



Inverse of frequency weighing function $W_{f1}(s)$ | Inverse of control input weighing function $W_{u1}(s)$

Fig. 6.5: Inverse magnitude plots of performance weighing functions [25]

It was already discussed section-3.2 that the performance weights are to be selected based on their inverse of magnitude plots. Hence in Fig.6.5, the inverse magnitude plots are shown for the performance weighing functions. The transfer functions of these weighing functions are:

$$W_{f1}(s) = \frac{s+0.2}{40(s+0.001)} \tag{6.6}$$

$$W_{u1}(s) = \frac{2\times10^{-5}s}{2\times10^{-7}s+1} \tag{6.7}$$

The weights for the load disturbance (ΔP_L) and disturbance (d_1) are chosen as 0.5 with the assumption that no frequency component amplitude in these signals crosses 0.5.

$$W_{P_L} = 0.5 \tag{6.8}$$

$$W_{d1}(s) = 0.5 \tag{6.9}$$

These weights resemble the frequency domain data of the disturbance inputs. Equations (6.7) and (6.8) say that any of the frequency components in these disturbance inputs will have amplitude not more than 0.5. With the addition of the above weights the standard H_∞-problem setup will get modified as shown in Fig.6.6.

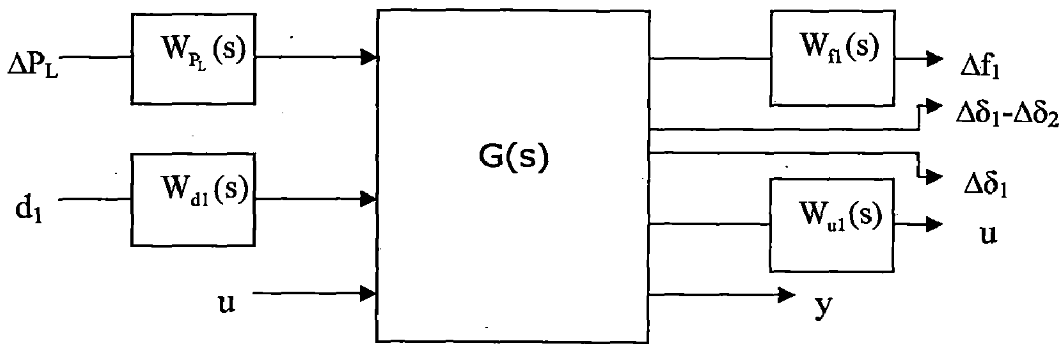


Fig.6.6: H_∞-problem setup after the addition of weighing functions

The above problem setup is converted into a single matlab SYSTEM variable 'P' using the 'SYSIC' routine of MATLAB®. Application of H_∞-synthesis algorithm on this 'P' has given the following controller with the achieved H_∞-cost (γ) of '0.8275'.

$$K_{HW}(s) = \frac{-0.04s^8-0.76s^7-5.48s^6-22.2s^5-66.6s^4-113.8s^3-115.2s^2-60.2s-0.06}{0.1s^6+1.39s^5-11.9s^4-198.4s^3-113.6s^2-161.5s-0.16} \tag{6.10}$$

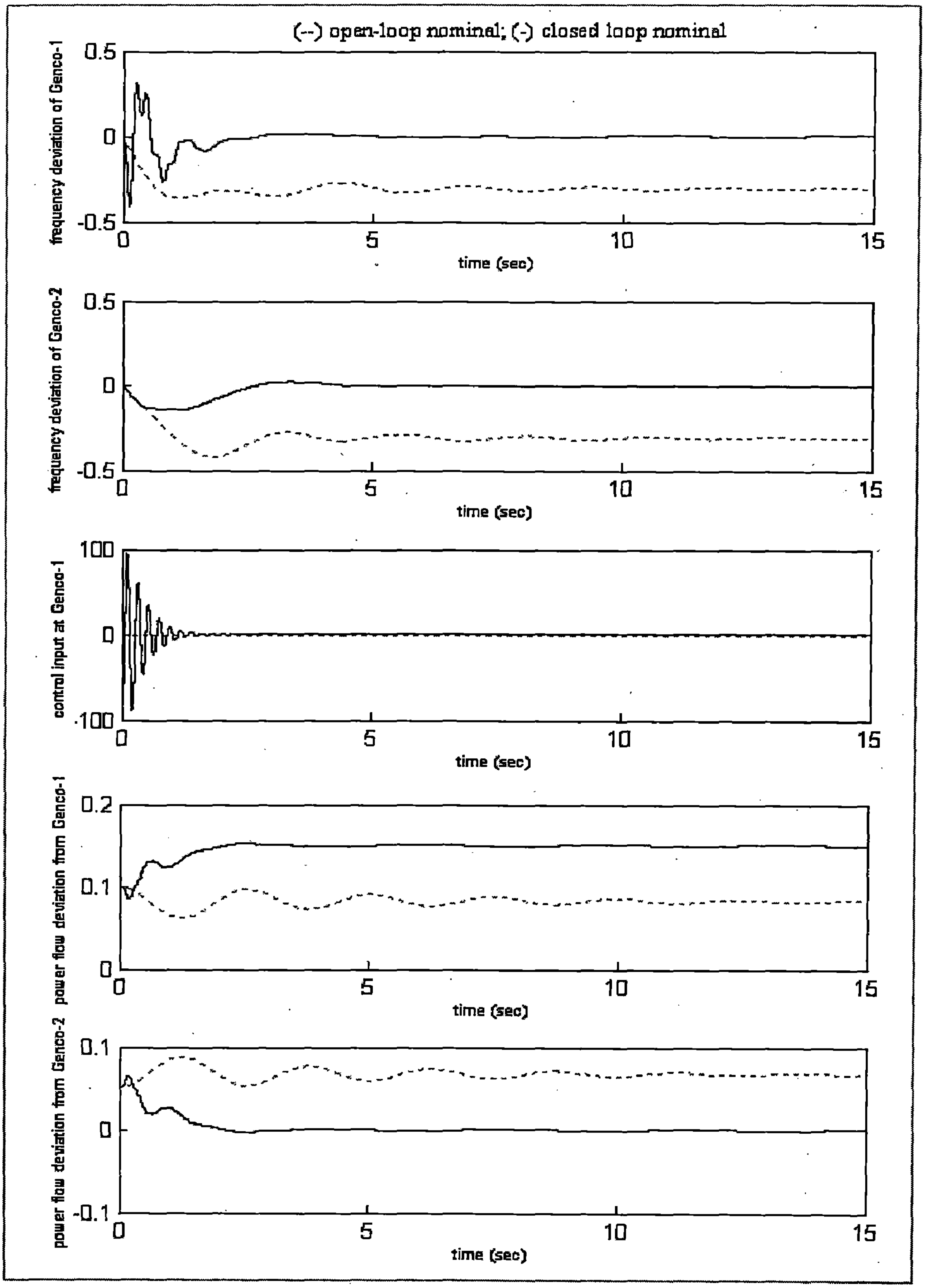


Fig.6.7: Response of the deregulated power system to a step load disturbance of 0.15 Pu with and without the weighted H_∞-optimal controller $K_{HW}(s)$

6.3 μ-controller: As already discussed earlier, the greatness of the μ-synthesis is in its ability to deal with modeling uncertainties. In the above H_∞-optimal controller designs (sections-6.1&6.2), no modeling uncertainties are taken into account. For the deregulated power system in Fig.6.1, three types of modeling uncertainties are considered. They are:

- Un-modeled high frequency dynamics uncertainty [25].

  The magnitude plot of the 'maximum uncertainty bound' function '$W_{mu}(s)$', for this uncertainty is given in Fig.6.8. It shows that at low frequencies the uncertainty is negligible but at high frequencies, there is a considerable amount of uncertainty.

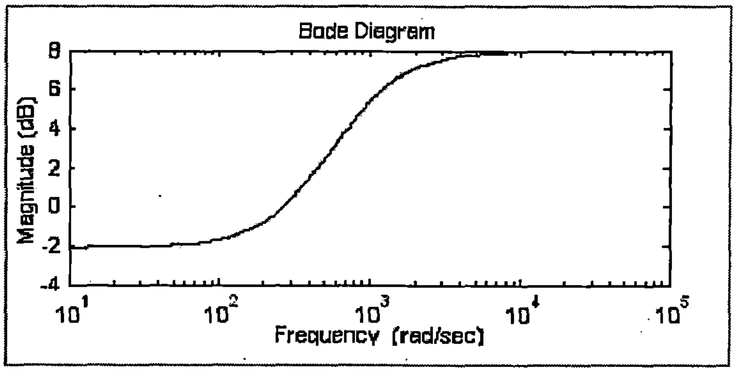


Fig. 6.8: Magnitude plot of the 'multiplicative uncertainty maximum bound' function $W_{mu}(s)$ [25]

$$W_{mu}(s) = \frac{2.5(s+315)}{s+1000} \tag{6.11}$$

- Uncertainty in the damping coefficient 'D_1' of GENCO-1 (Refer Fig.6.2).

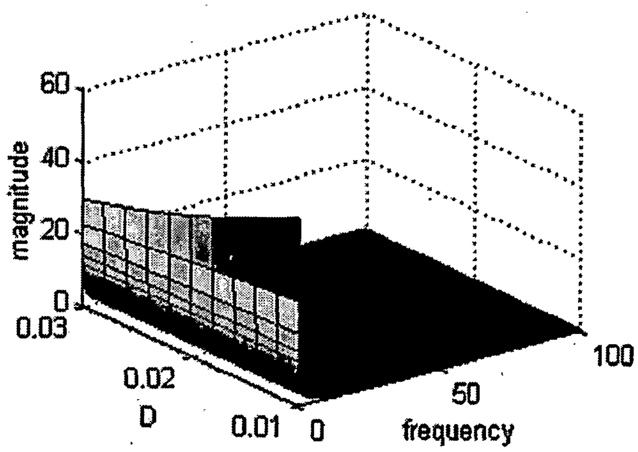


Fig.6.9: Effect of uncertainty on the transfer function 'PS1' of GENCO-1

The damping co-efficient is a relation between the change in the load and the frequency deviation. This damping tells-by how much the load on GENCO-1 decreases for a small decrement in the frequency of GENCO-1. A 50% uncertainty is assumed in this damping co-efficient D_1, here. This uncertainty in D_1 causes uncertainty in the transfer function block 'PS1' of GENCO-1 as shown in Fig.6.9. By using the method discussed in chapter-4, this uncertainty in the transfer function 'PS1' can be represented as a *bounded complex uncertainty.* The corresponding nominal model '$W_{PS1}(s)$' and the 'maximum additive uncertainty bound' function '$W_{auD1}(s)$' are given below.

$$W_{PS1}(s)=\frac{0.0473s+5.5067}{s+0.1045};\ W_{auD1}(s)=\frac{0.0016s^2+0.2556s+0.4302}{s^2+0.2184s+0.0098} \qquad (6.12)$$

- Uncertainty in the damping co-efficient 'D_2' of GENCO-2 (Refer Fig.6.2).

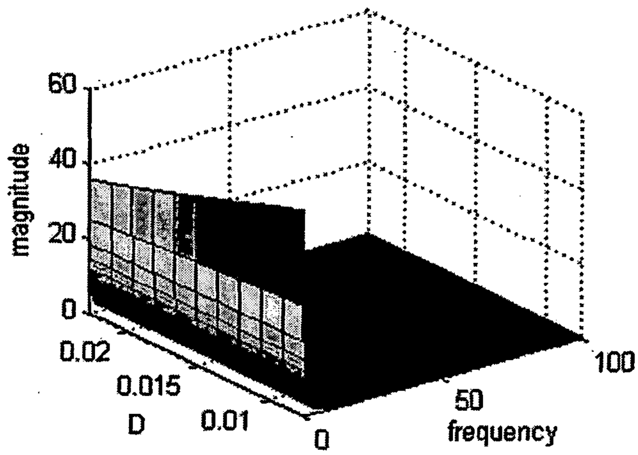


Fig. 6.10: Effect of uncertainty on the transfer function 'PS2' of GENCO-2

Similar to that in GENCO-1 an uncertainty is assumed in the damping coefficient 'D_2' of GENCO-2. This uncertainty of in the damping co-efficient 'D_2' causes uncertainty in the transfer function block 'PS2' of GENCO-2 as shown in Fig.6.10. This uncertainty in the transfer function 'PS2' can be represented as a *bounded complex uncertainty.* The corresponding nominal model '$W_{PS2}(s)$' and the 'maximum additive uncertainty bound' function '$W_{auD2}(s)$' are given below.

$$W_{Ps2}(s)=\frac{0.0465s+5.5652}{s+0.0807};\ W_{auD2}(s)=\frac{0.0016s^2+0.2532s+0.3080}{s^2+0.1566s+0.0053} \qquad (6.13)$$

Once the model uncertainties are represented as *bounded complex uncertainties* as in (6.11),(6.12) and (6.13), the μ-synthesis problem setup can be formulated as shown in Fig.6.11.

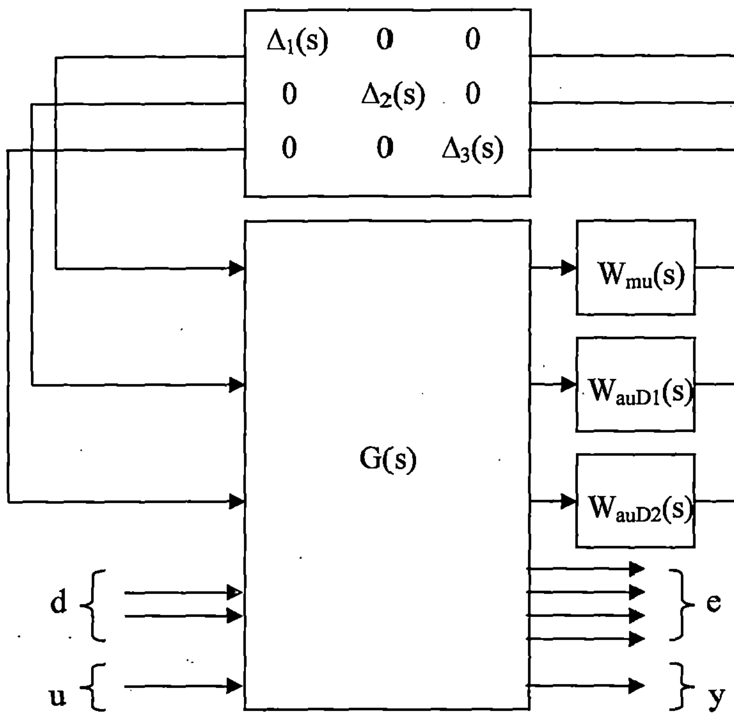


Fig.6.11: Standard μ-synthesis problem setup

Where, $\Delta_1(s)$, $\Delta_2(s)$ and $\Delta_3(s)$ are the unit-norm bounded complex scalar (non matrix) uncertainty as discussed in section-1.1. This problem setup (without $\Delta(s)$) is converted into a single matlab SYSTEM variable 'P' using the 'SYSIC' routine of MATLAB®. Then, the D-K iteration algorithm is applied on to this problem setup with the help of the 'DKITGUI' tool of MATLAB®. The D-K iteration algorithm has taken '4' iterations and has achieved a final μ value of 0.907 peak. During each of the D-K iteration, appropriate rational fittings are chosen for the three D-scales magnitude data corresponding to the three uncertainties $\Delta_1(s)$, $\Delta_2(s)$ and $\Delta_3(s)$ as discussed in section-5.2.

Before the starting of the first D-K iteration, the D-scales matrix D(s) (and hence $D^{-1}(s)$) is chosen as an identity matrix. At the end of each of the D-K iteration, μ-analysis on the closed-loop system of that iteration gives the D-scales magnitude data for which a rational fit is chosen as shown in Figs.6.12, 6.13 and 6.14 to obtain D(s) and $D^{-1}(s)$.

Iteration-1: μ-value achieved is 1.8188. The D-scales magnitude data and their rational fittings are shown by solid and dashed curves respectively.

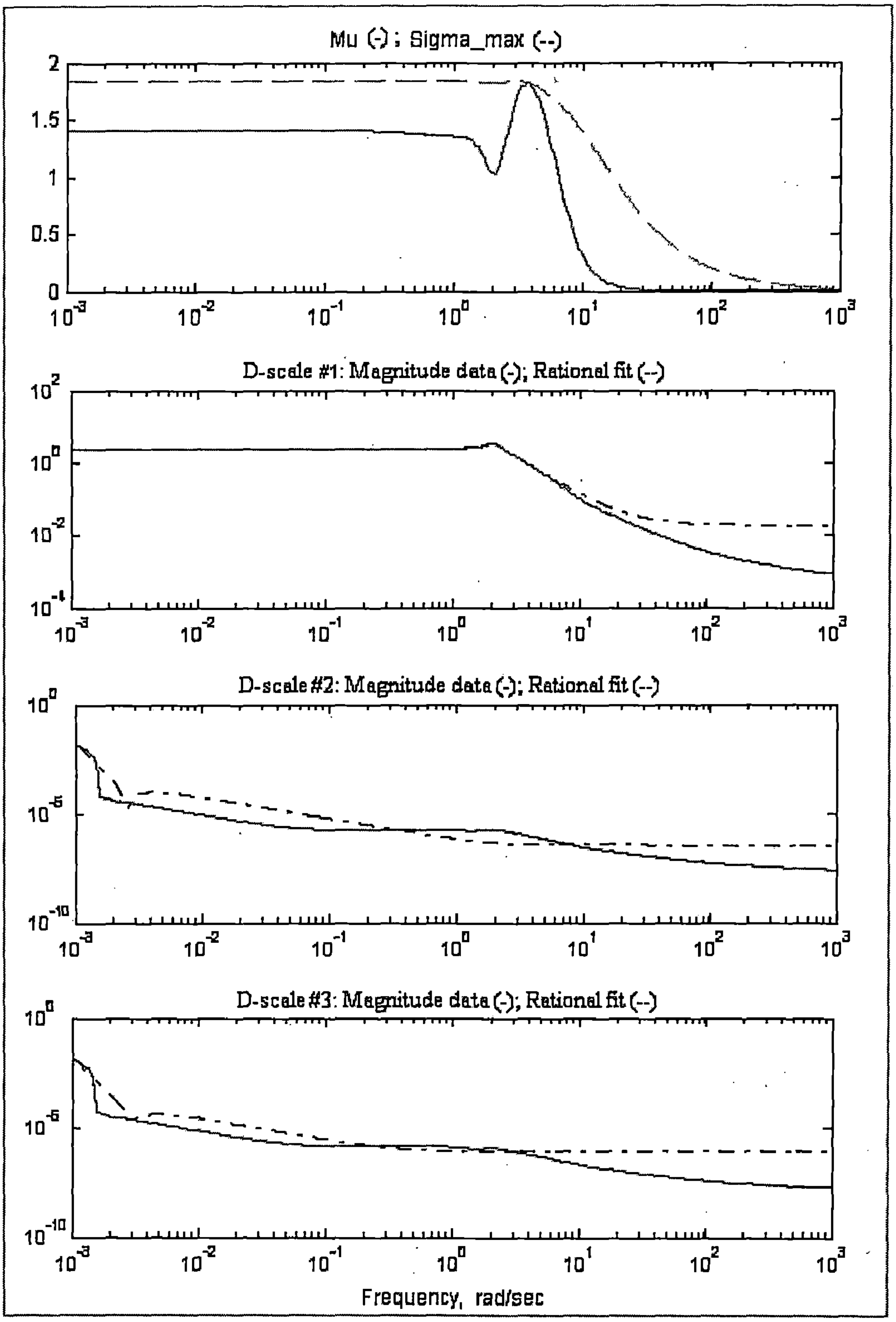


Fig.6.12: D-scale fittings of iteration-1

Iteration-2: μ-value achieved is 1.0252. The D-scales magnitude data and their rational fittings are shown by solid and dashed curves respectively.

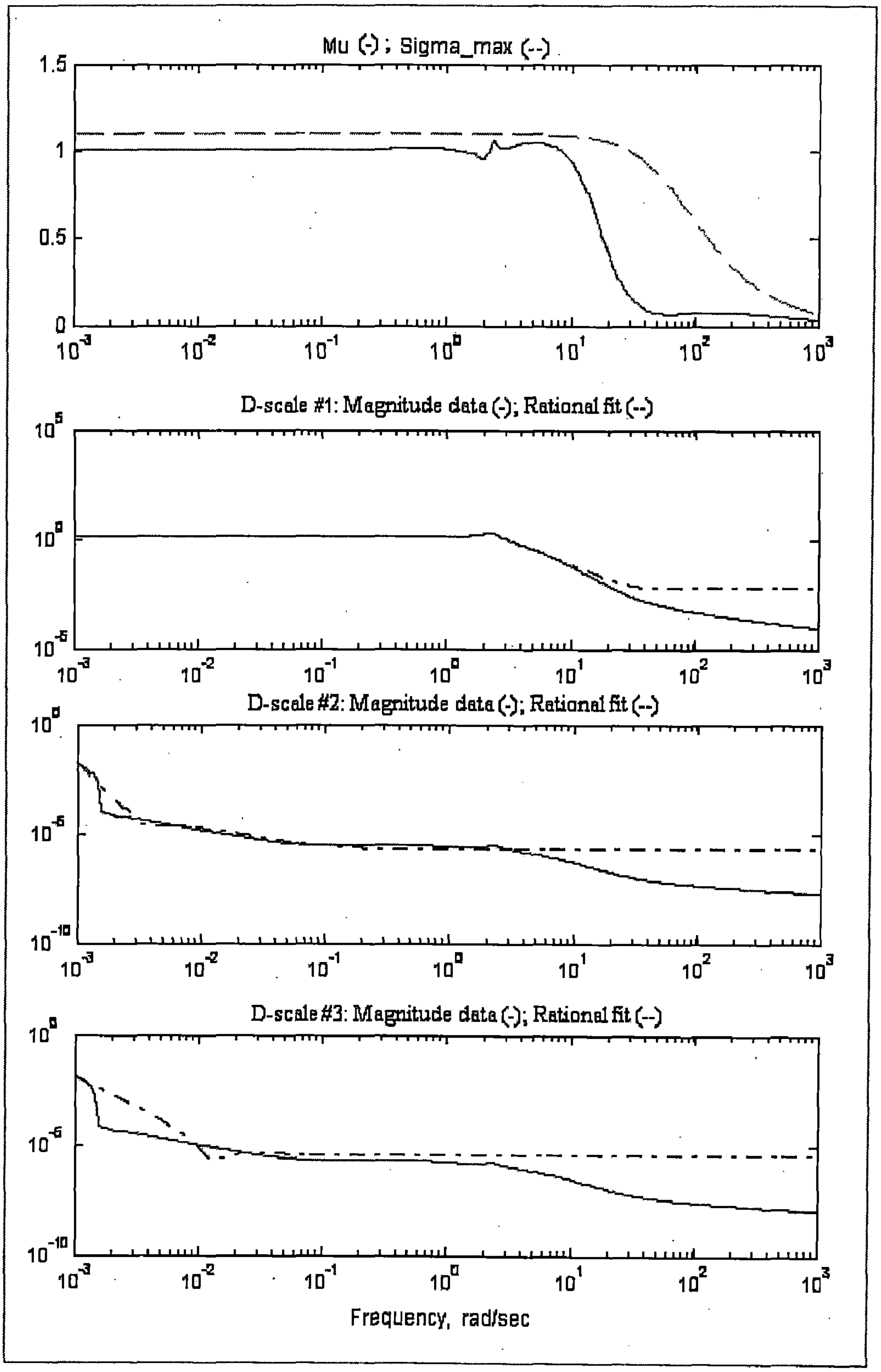


Fig.6.13: D-scale fittings of iteration-2

Iteration-3: μ-value achieved is 0.92. The D-scales magnitude data and their rational fittings are shown by solid and dashed curves respectively.

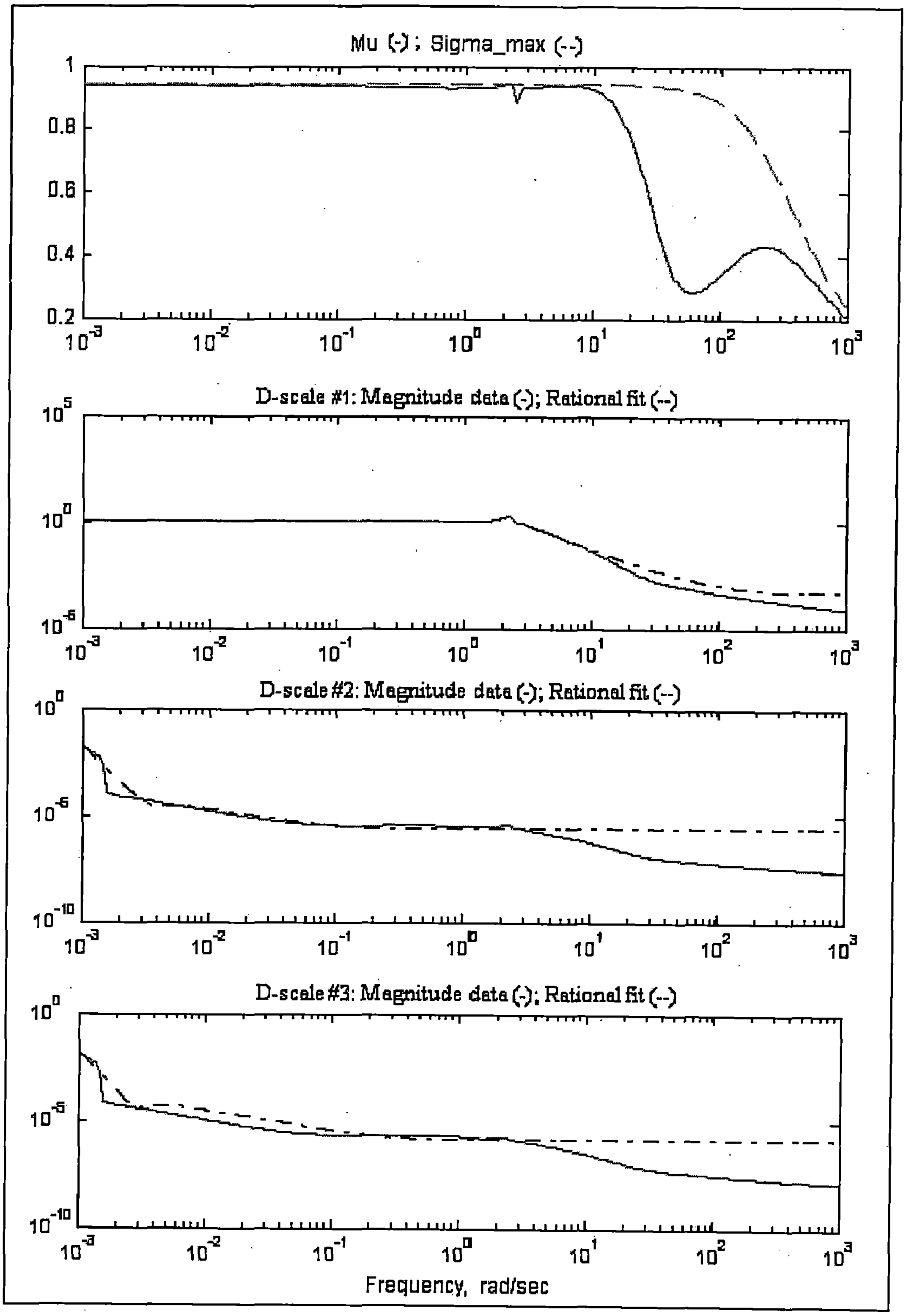


Fig.6.14: D-scale fittings of iteration-3

These D-scale matrices D(s) and $D^{-1}(s)$ calculated at the end of each of the iteration are appended to the problem setup in Fig.6.11 and a modified controller is designed. At the end of the 4^{th} iteration, a peak 'μ' value of 0.907 is achieved as shown in Fig.6.15. A try for the next iteration has resulted in a peak 'μ' value of 0.94. Hence the controller resulted from the 4^{th} iteration is taken as the final controller.

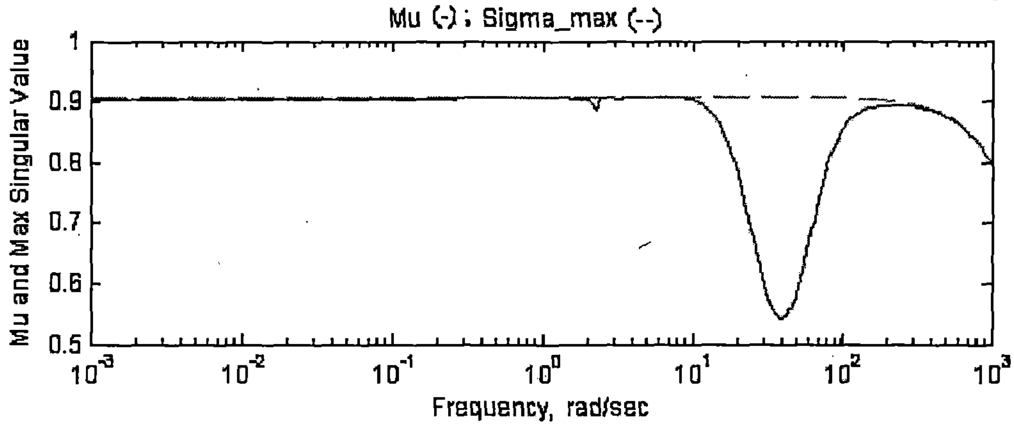


Fig.6.15: μ-plot after the 4^{th} iteration with peak value '0.907'

The controller obtained in the 4^{th} iteration is of order 25. The total order of the controller depends on the order chosen for the D-scales fittings. This is a general result in μ-synthesis. For making the designed controller realizable, controller order reduction technique is to be applied necessary. The order reduction of this 25^{th} order controller is done in section-6.5. This higher order μ-controller is given in (6.14) below.

$$K_{\mu}(s) = \frac{-0.001s^{25} - 0.006s^{24} - 0.027s^{23} - 0.097s^{22} - 0.31s^{21} - 0.83s^{20} - 1.95s^{19}}{0.0001s^{25} + 0.0006s^{24} + 0.004s^{23} + 0.0195s^{22} + 0.076s^{21} + 0.25s^{20} + 0.68s^{19}}$$

$$\frac{-3.99s^{18} - 7.13s^{17} - 11.03s^{16} - 14.68s^{15} - 16.59s^{14} - 15.65s^{13} - 11.98s^{12} - 7.16s^{11}}{1.608s^{18} + 3.22s^{17} + 5.50s^{16} + 7.98s^{15} + 9.73s^{14} + 9.78s^{13} + 7.9140s^{12} + 4.94s^{11}}$$

$$\frac{-3.148s^{10} - 0.939s^{9} - 0.175s^{8} - 0.020s^{7} - 0.001s^{6}}{2.24s^{10} + 0.67s^{9} + 0.125s^{8} + 0.014s^{7} + 0.0009s^{6}} \qquad (6.14)$$

Time response simulations can be run on the closed-loop system with this 25^{th} order μ-controller, by applying a step load disturbance of size 0.15 Pu. The responses of the deregulated power system for this step load disturbance are shown in Fig.6.16.

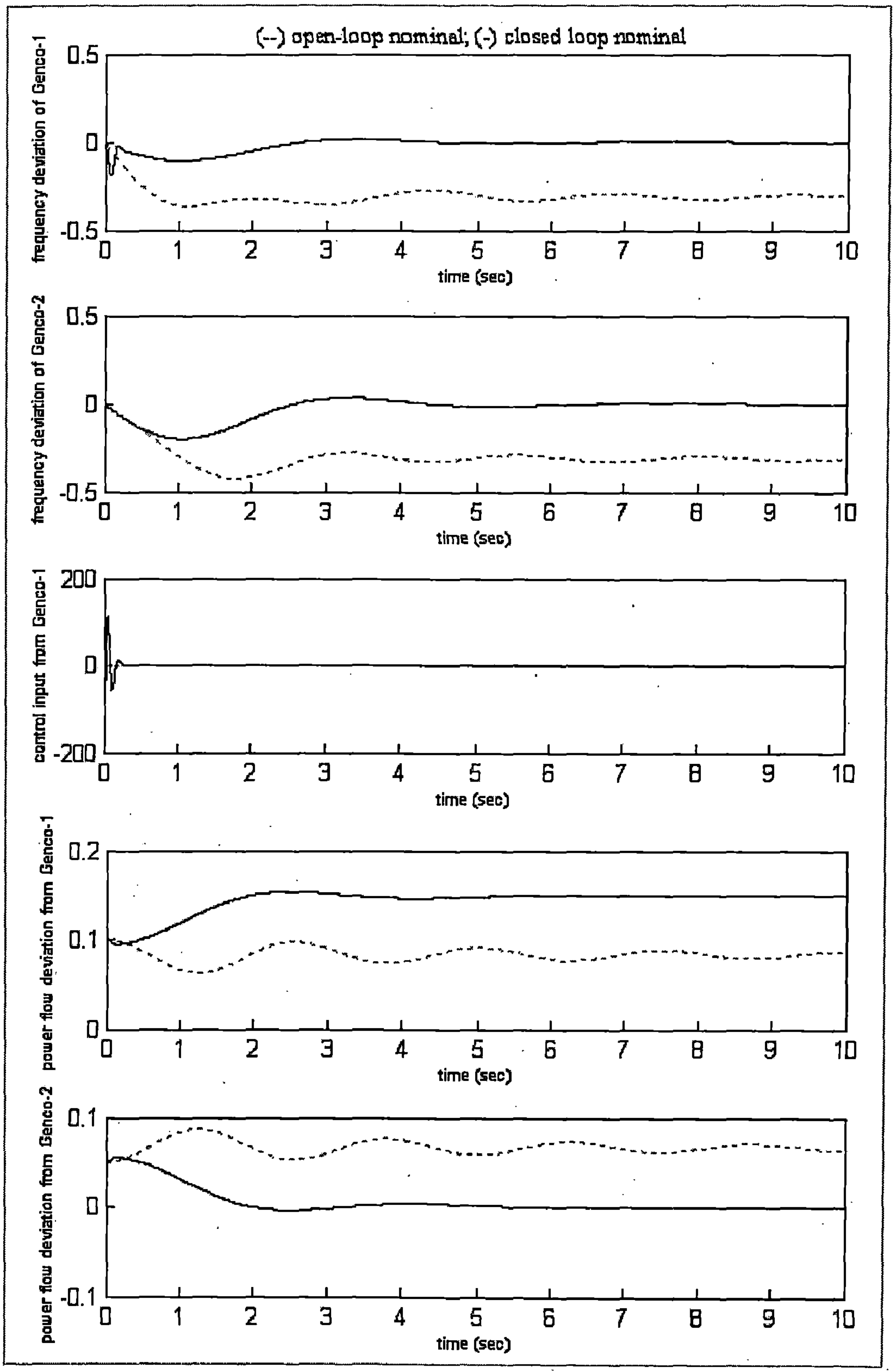


Fig.6.16: Response of the deregulated power system to a step load disturbance of 0.15 pu with and with out the μ-controller $K_{\mu}(s)$

6.4 Comparison of Robustness between Weighted H_∞-controller and μ-controller: It was discussed in section-1.5, that a planned procedure must be followed for doing this robustness comparison. According to that procedure, μ-analysis of the uncertain closed-loop system with the weighted H_∞-controller yielded a μ-plot and a perturbation matrix as below in Fig.6.14 and table 6.3. This perturbation is the one which will cause the system with the weighted H_∞-controller to go to the verge of instability.

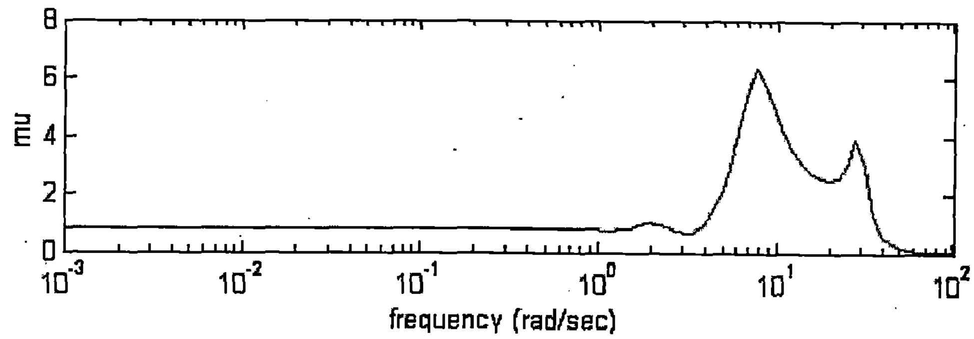


Fig.6.17: μ-plot of closed-loop uncertain system with the weighted H_∞-optimal controller

Table 6.3: Perturbation matrix returned from μ-analysis on the system with $K_{HW}(s)$

Pert1(ω)/Δ_1	**Pert2(ω)/Δ_2**	**Pert3(ω)/Δ_3**	**ω(rad/sec)**
-1.2698 + 0.000264i	1.2698 - 6.6085e-018i	1.2698 + 4.4056e-018i	0.001
-1.2698 + 0.000297i	1.2698 - 4.4056e-018i	1.2698	0.0011233
-1.2698 + 0.00033i	1.2698 - 4.4056e-018i	1.2698	0.0012619
-1.2698 + 0.000375i	1.2698 - 8.8113e-018i	1.2698	0.0014175
-1.2698 + 0.000421i	1.2698 - 4.4056e-018i	1.2698	0.0015923
⋮	⋮	⋮	⋮
-0.17955 + 0.06142i	0.18977	0.18977	6.8926
-0.16647 - 0.023586i	0.16813	0.16813 - 9.333e-018i	7.7426
-0.14835 - 0.098354i	0.17799 + 1.4821e-017i	0.17799 + 1.4821e-017i	8.6975
-0.12395 - 0.16543i	0.20671 - 3.4424e-017i	0.20671 - 1.1475e-017i	9.7701
-0.092366 - 0.22563i	0.24381 + 1.3534e-017i	0.24381 + 5.4136e-017i	10.975
-0.052904 - 0.27836i	0.28335 + 1.5729e-017i	0.28335 - 1.5729e-017i	12.328
-0.005183 - 0.3215i	0.32159	0.32159	13.849
⋮	⋮	⋮	⋮
-9.4288 + 5.0775i	10.709 + 1.1889e-015i	0	55.908
-17.402 + 7.1954i	18.831	0	62.803
-31.186 + 10.025i	32.757 + 1.8184e-015i	0	70.548
-54.756 + 13.941i	56.503	0	79.248
-94.662 + 19.73i	96.696 - 1.0735e-014i	0	89.022
-161.56 + 29.059i	164.15 + 9.1122e-015i	0	100

This perturbation is applied to the deregulated power system with the weighted H_∞-controller and μ-controller respectively and the responses to a step load of 0.15 Pu are as shown in Figs.6.18 and 6.19. This is the first step in the robustness comparison plan.

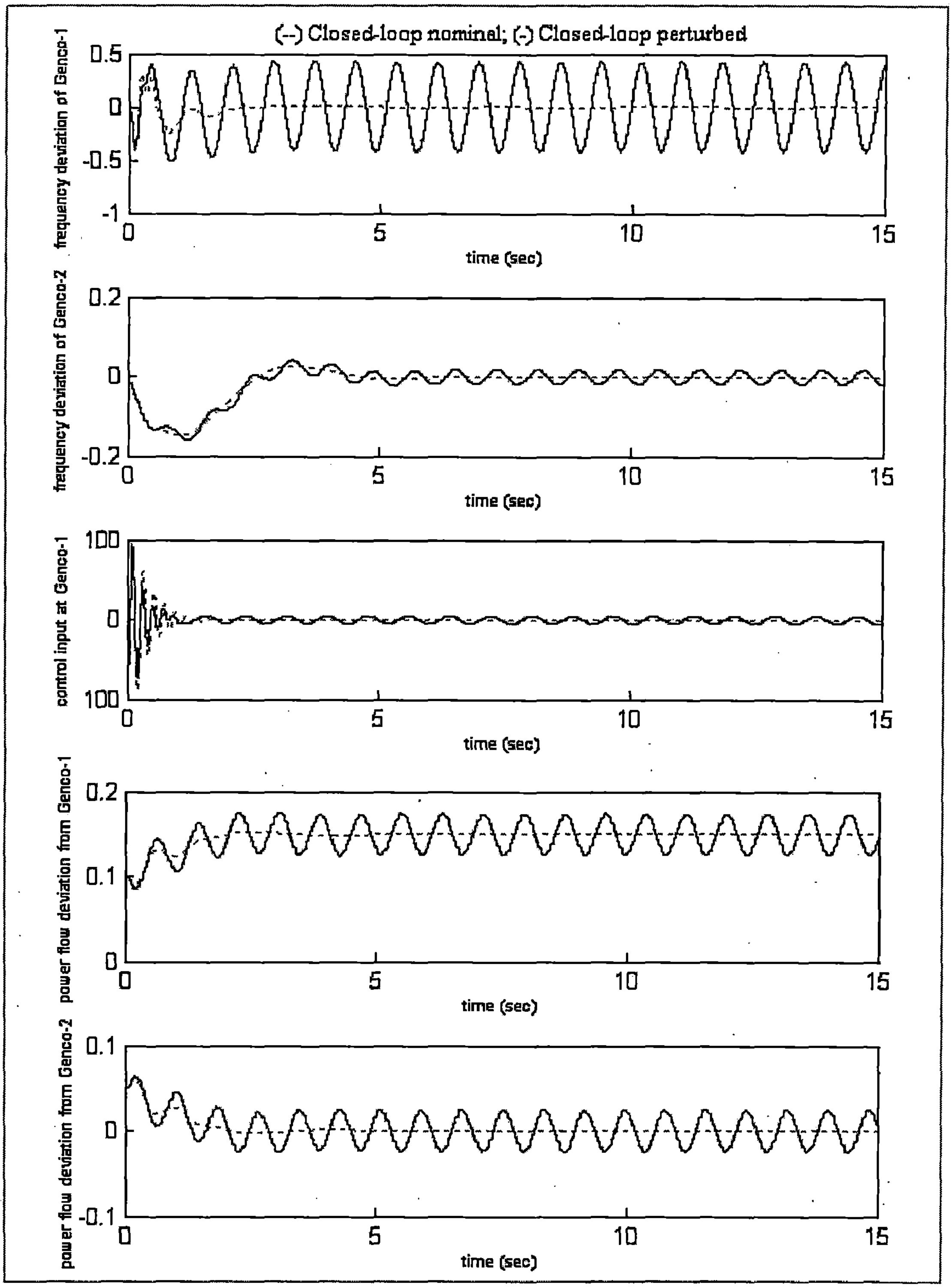


Fig.6.18: Responses of the perturbed and un-perturbed deregulated power system to a step load disturbance of 0.15 pu with weighted H_∞-weighted optimal controller $K_{HW}(s)$

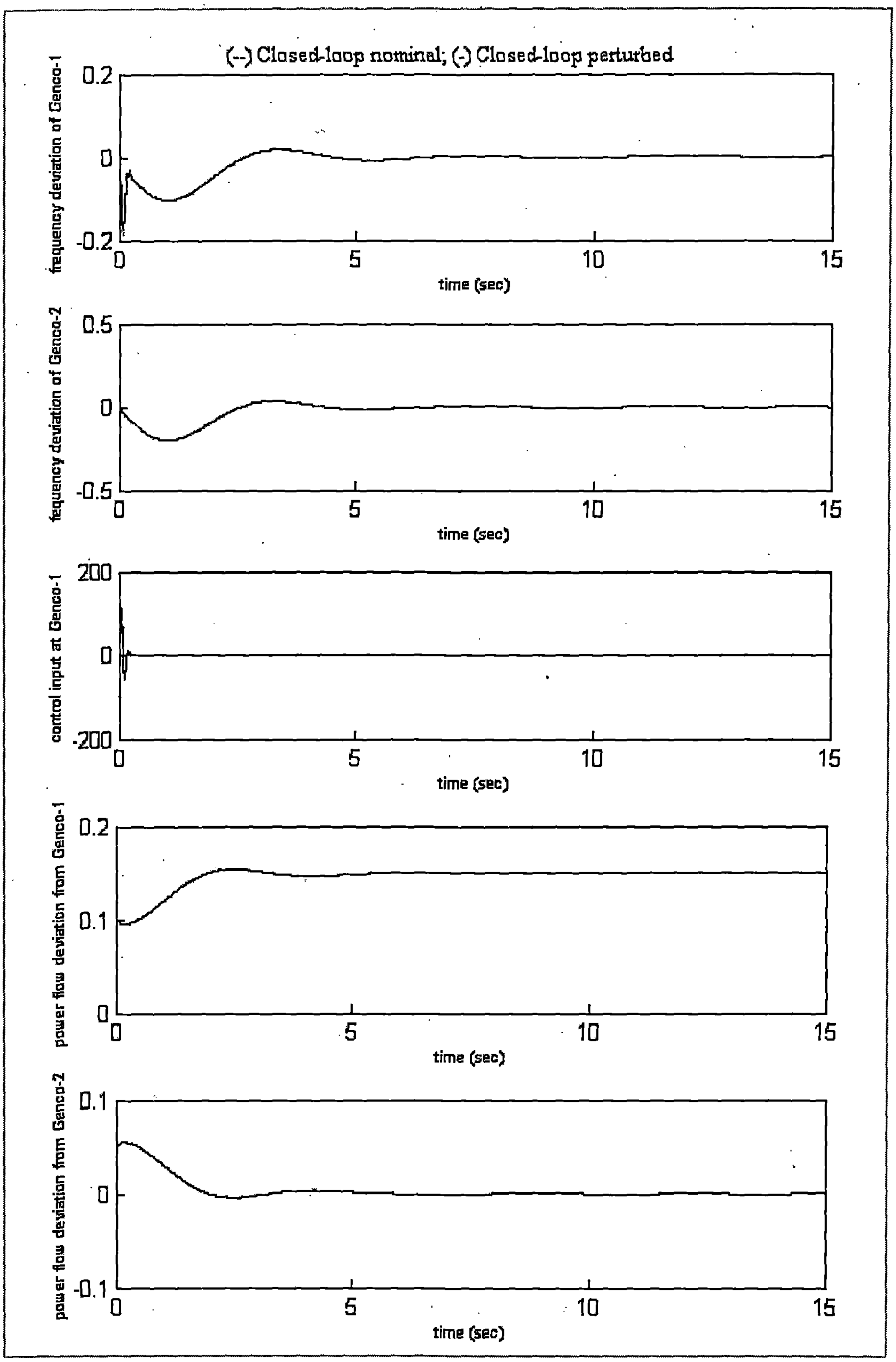


Fig.6.19: Response of the perturbed and un-perturbed deregulated power systems to a step load disturbance of 0.15 pu with the μ-controller $K_\mu(s)$

It can be observed from Figs.6.18 and 6.19 that, the perturbation which has caused the closed-loop perturbed system with the weighted H_∞-optimal controller to go to the verge of instability has no effect on the closed-loop perturbed system with μ-controller. Following again the robustness comparison plan of section-1.5, μ-analysis of the uncertain closed-loop system with the μ-controller yielded a μ-plot and a perturbation matrix as below in Fig.6.20 and Table 6.4 respectively. Note that this is the perturbation which will cause the system with the μ-controller to go to the verge of instability. This perturbation is applied to the deregulated power system to compare the controllers.

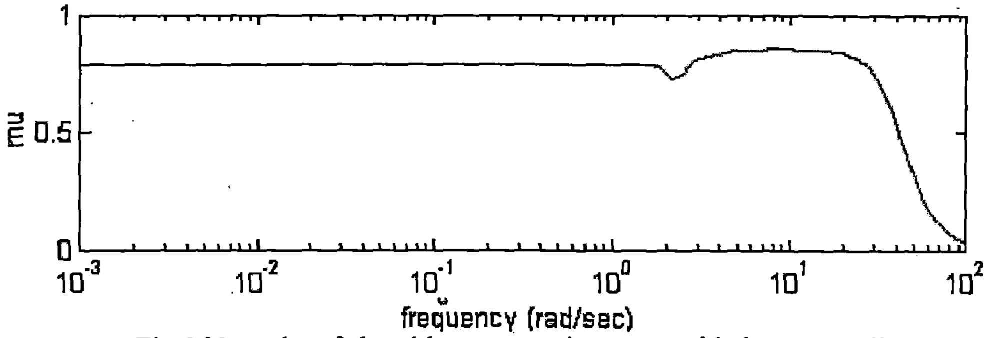


Fig.6.20: μ-plot of closed-loop uncertain system with the μ-controller

Table 6.4: Perturbation matrix returned from μ-analysis on the system with $K_\mu(s)$

Pert1(ω)/Δ_1	Pert2(ω)/Δ_2	Pert3(ω)/Δ_3	ω(rad/sec)
-1.2698 - 6.6632e-05i	1.2698 - 4.4056e-018i	1.2698	0.001
-1.2698 - 7.4849e-05i	1.2698 - 6.6085e-018i	1.2698	0.0011233
-1.2698 - 8.4079e-05i	1.2698	1.2698	0.0012619
-1.2698 - 9.4448e-05i	1.2698 - 8.8113e-018i	1.2698 + 8.8113e-018i	0.0014175
-1.2698 - 0.0001061i	1.2698 - 4.4056e-018i	1.2698 + 4.4056e-018i	0.0015923
⋮	⋮	⋮	⋮
-1.088 - 0.42969i	1.17 + 6.4947e-017i	1.17 + 6.4947e-017i	6.8926
-1.060 - 0.49146i	1.1689 - 6.4886e-017i	1.1689	7.7426
-1.027 - 0.55823i	1.1689 - 6.4886e-017i	1.1689 + 6.4886e-017i	8.6975
-0.98574 - 0.6301i	1.1699 + 6.4944e-017i	1.1699	9.7701
-0.93464 - 0.7069i	1.1719 - 1.3011e-016i	1.1719 - 6.5053e-017i	10.975
-0.87089 - 0.7882i	1.1746 - 9.7806e-017i	1.1746	12.328
-0.79098 - 0.8727i	1.1779 - 8.1731e-017i	1.1779 - 1.3077e-016i	13.849
⋮	⋮	⋮	⋮
5.6668 + 4.4951i	7.2332 - 8.0304e-016i	0	62.803
7.1529 + 7.8762i	10.639 - 5.9061e-016i	0	70.548
8.9102 + 12.978i	15.742 + 2.6217e-015i	0	79.248
11.009 + 20.618i	23.373 - 1.2975e-015i	0	89.022
13.576 + 31.969i	34.733 + 3.8561e-015i	0	100

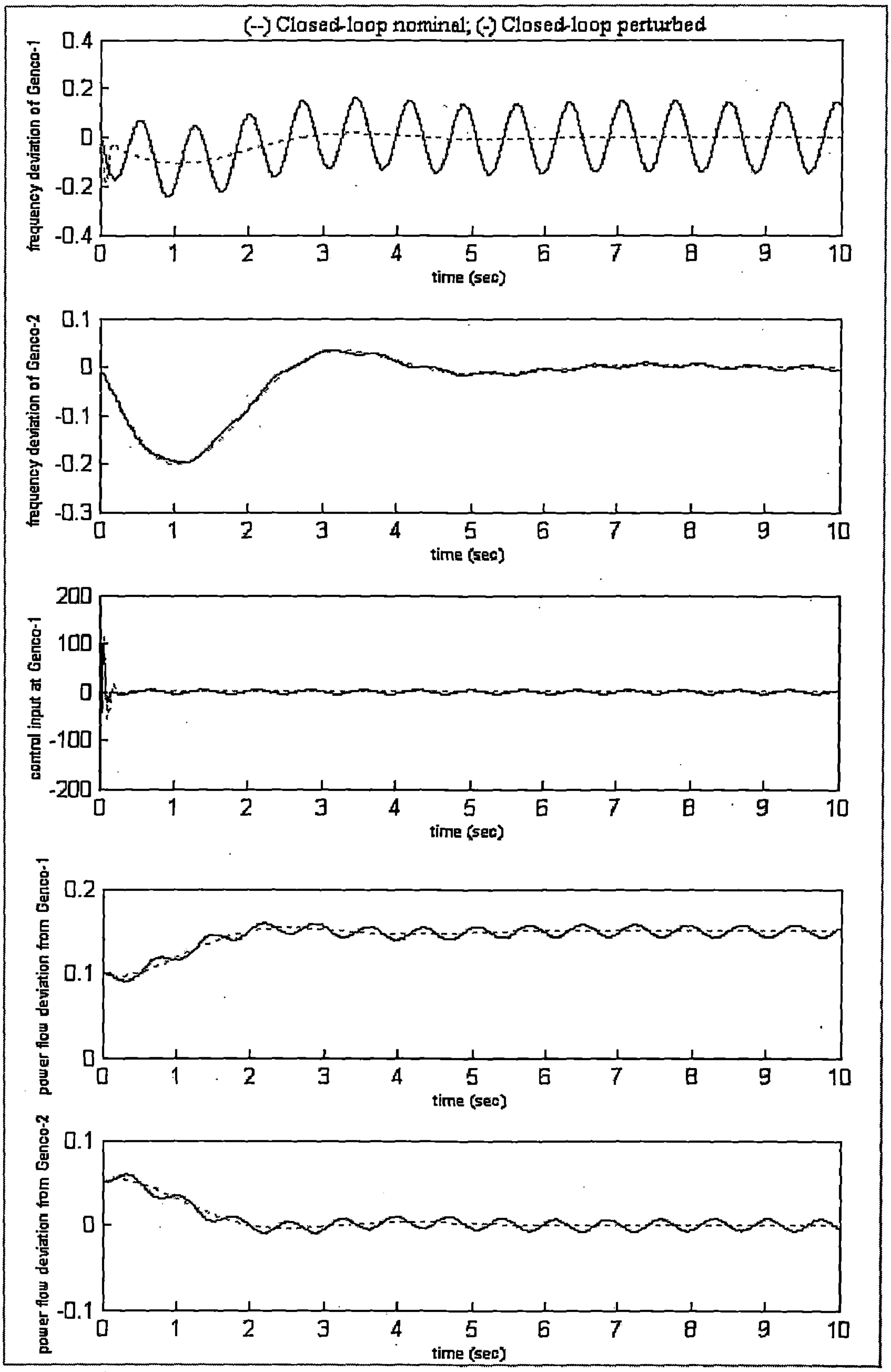


Fig.6.21: Response of the perturbed and un-perturbed deregulated power systems to a step load disturbance of 0.15 pu with μ-controller $K_\mu(s)$

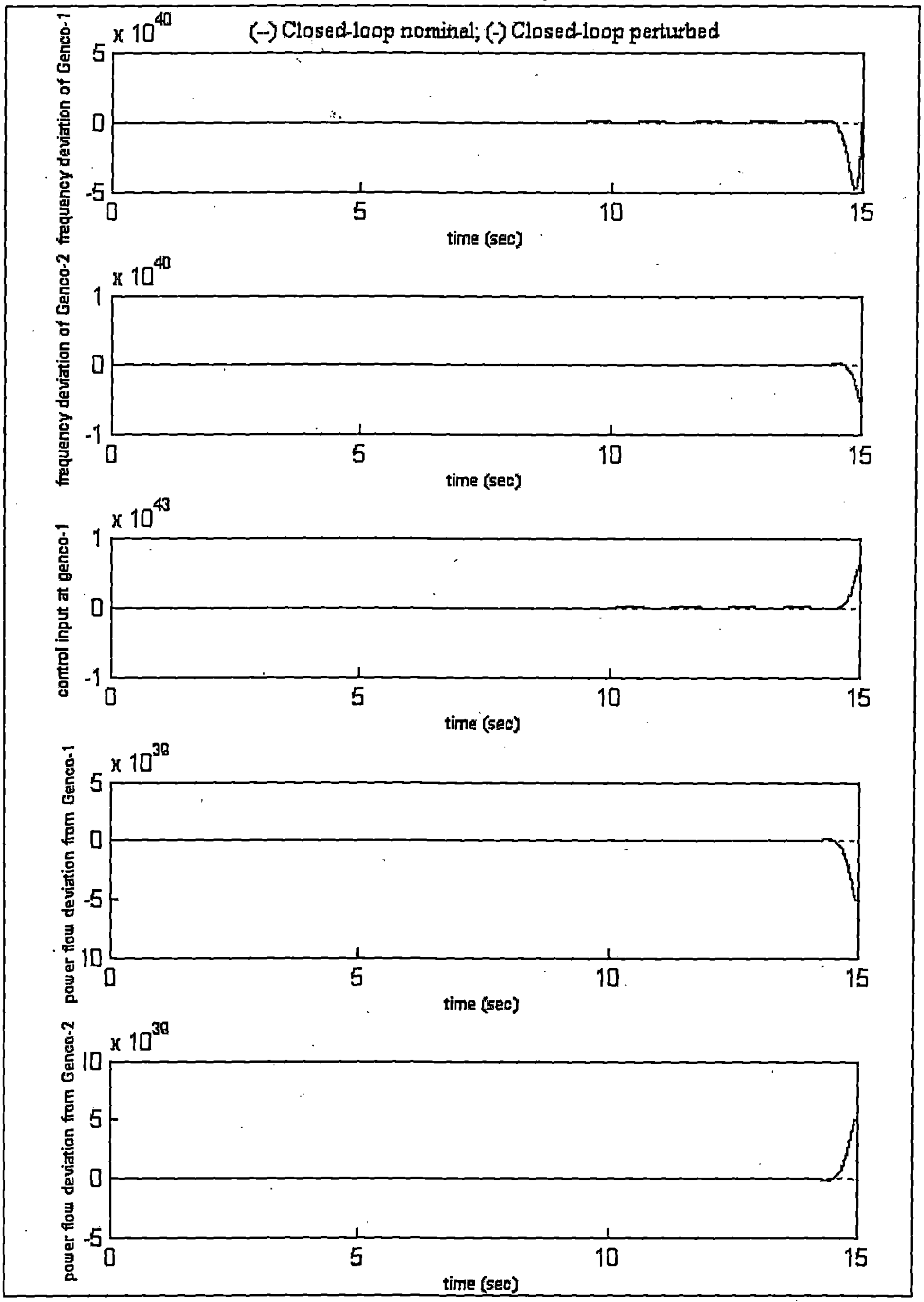


Fig.6.22: Response of the perturbed and un-perturbed deregulated power systems to a step load of 0.15 pu with weighted H_∞-optimal controller $K_{HW}(s)$

Figs.6.21 and 6.22 shows that, the perturbation which has brought the system with μ-controller to the verge of instability has caused the system with the H_∞-controller to go to instability.

Table 6.3 which has send the power system with the weighted H_∞-controller to the verge of instability is applied to the system with reduced order μ-controller.

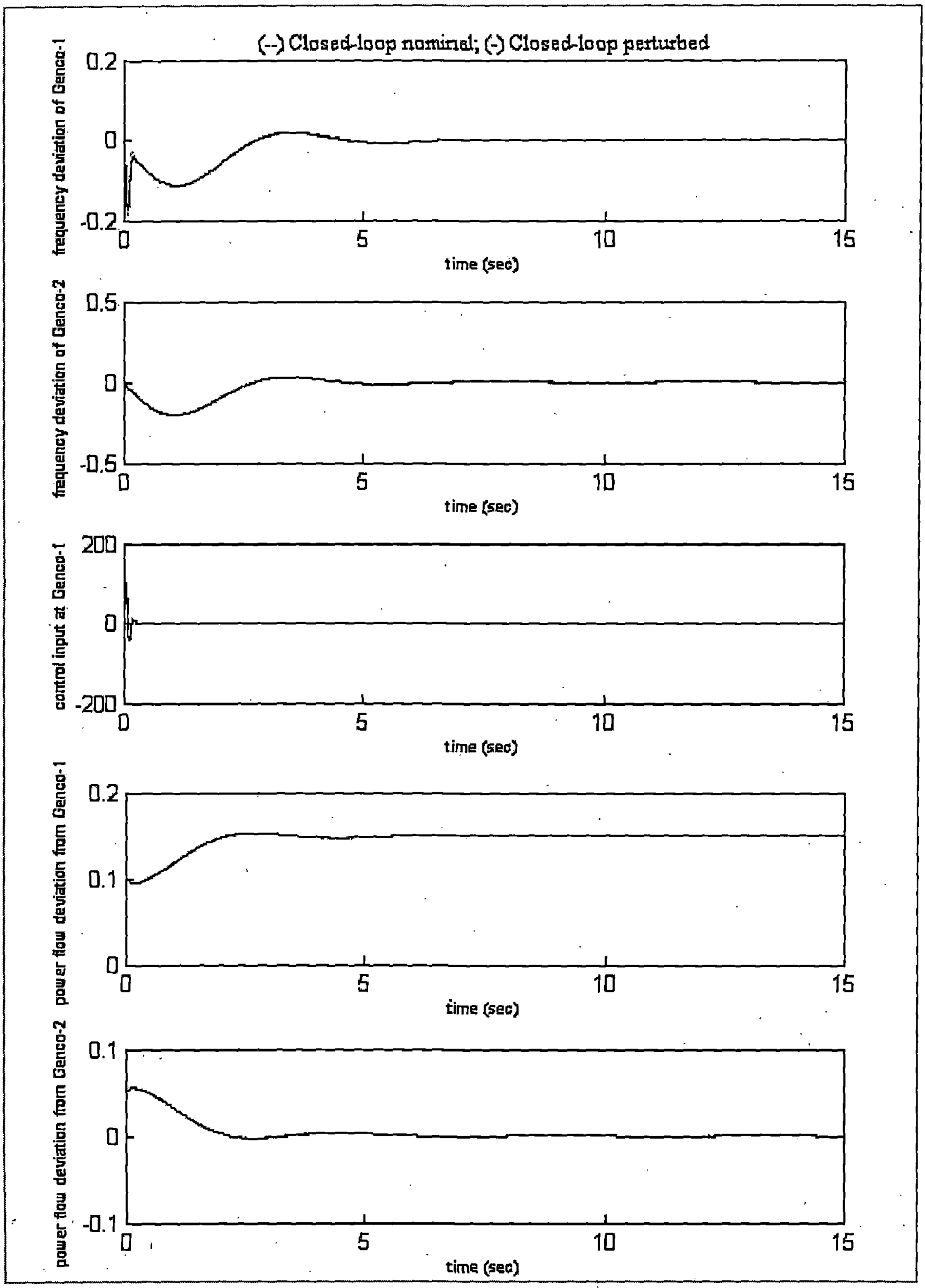


Fig.6.25: Response of the perturbed and un-perturbed deregulated power systems to a step load of 0.15 pu with the reduced 3rd order μ-controller '$K_{r\mu}(s)$'

The time response simulations in Fig.6.25 confirms that the 3rd order μ-controller is robust enough just similar to the 25th order μ-controller. This implies that our controller order reduction process is successful. This reduced order robust controller is realizable because of its lesser order and robust enough for performing satisfactorily on real power systems.

Conceptual framework required for the 'Design of Robust Power System Controllers' is developed and is validated by designing a robust load frequency controller for a deregulated power system problem. First a H_∞-optimal controller is designed and is improved to a weighted H_∞-optimal controller by using the concept of weighing functions in order to concentrate on particular frequency components in the output responses. The weighing functions are chosen to resemble the performance requirements and the data of input disturbance frequency components. This modified H_∞-controller is meeting the targeted performance requirements. Then, bounded uncertainty models are obtained for some of the uncertainties in the deregulated power system model. These uncertainties are added to the power system nominal model to resemble the real power system as closely as possible. μ-synthesis is applied on this uncertain power system model and a robust load frequency controller is designed which is called as μ-controller. This μ-controller is an extension to the weighted H_∞-controller which takes the model uncertainties also into account.

The weighted H_∞-controller and the μ-controller are compared for robustness properties using the frequency domain μ-analysis technique. A planned procedure is developed for comparing the robustness properties of the two controllers in terms of time response simulations. Two perturbation matrices are obtained with the help of μ-analysis and these perturbations are applied to the power system with the weighted H_∞-controller and the power system with μ-controller. Time response simulations for a step load disturbance are showing the superior qualities of the μ-controller in terms of robustness.

The μ-controller obtained is of 25^{th} order. This order is reduced to 3^{rd} order for making the μ-controller is realizable for actual applications. The order reduction process is checked by testing the robustness properties of this 3^{rd} controller. μ-analysis and time response simulations are showing that the order reduction has not changed the robustness qualities of the μ-controller. Results say that the μ-controller is robust enough to model uncertainties which mean that this controller works better when kept in the actual power system.

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Deregulated Power System State-space matrices: The state-space equations of the deregulated power system in Fig. 6.1 are given below.

$$\dot{X} = AX + Bu + Dw \tag{A-1}$$

Where, $X^T = [\Delta f_1\ \Delta P_{M1}\ \Delta P_{V1}\ (\Delta\delta_1 - \Delta\delta_2)\ \Delta P_{M2}\ \Delta\delta_1]$ (A-2)

$$u = \Delta P_{ref1} \tag{A-3}$$

$$W^T = [\Delta P_L\ d_1] \tag{A-4}$$

$$A = \begin{bmatrix} -\frac{D_1}{T_{p1}} & \frac{1}{T_{P1}} & 0 & -\frac{\propto}{T_{P1}} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{T_{M1}} & \frac{K_{M1}}{T_{M1}} & 0 & 0 & 0 & 0 & 0 \\ -\frac{K_{H1}}{R_1 T_{H1}} & 0 & -\frac{1}{T_{H1}} & 0 & 0 & 0 & 0 & 0 \\ 2\pi & 0 & 0 & 0 & -2\pi & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\propto}{T_{P2}} & -\frac{D_2}{T_{P2}} & \frac{1}{T_{P2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{M2}} & \frac{K_{M2}}{T_{M2}} & 0 \\ 0 & 0 & 0 & 0 & -\frac{K_{H2}}{R_2 T_{H2}} & 0 & -\frac{1}{T_{H2}} & 0 \\ 2\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{A-5}$$

$$B^T = \begin{bmatrix} 0 & 0 & \frac{K_{H1}}{T_{H1}} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{A-6}$$

$$D^T = \begin{bmatrix} -\frac{T_1}{(T_1+T_2)T_{P1}} & 0 & 0 & 0 & -\frac{T_2}{(T_1+T_2)} & 0 & 0 & 0 \\ -\frac{1}{T_{P1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{A-7}$$

Where,

$$\propto = \frac{T_1 T_2}{(T_1 + T_2)} \tag{A-8}$$

These state-space equations can be divided into transfer blocks as in Fig.6.2 and can be written as subsystems before calling the SYSIC routine.