SIMULATION AND MODELLING OF AE SENSORS FOR CHARACTERIZATION OF ELECTRICAL PARAMETERS

A DISSERTATION

Submitted in partial fulfilment of the
requirements for the award of the degree
of
MASTER OF TECHNOLOGY
in
ELECTRICAL ENGINEERING
(With Specialization in Measurement & Instrumentation)

By

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CANDIDATE'S DECLARATION

I hereby declare that the work that is being presented in this dissertation report entitled "**SIMULATION AND MODELLING OF AE SENSORS FOR CHARACTERIZATION OF ELECTRICAL PARAMETERS**" submitted in partial fulfillment of the requirements for the award of the degree of **MASTER OF TECHNOLOGY** in **ELECTRICAL ENGINEERING** with specialization in **MEASUREMENT AND INSTRUMENTATION**. submitted in the Department of Electrical Engineering, Indian Institute of Technology, Roorkee, is an authentic record of my own work carried out, under the guidance of Dr. R.S. Anand, Asst. Professor, Department of Electrical Engineering and Shri. H.R. Mehta, Head, IIS, Control Instrumentation Division, Bhabha Atomic Research Centre., Mumbai.

The matter embodied in this dissertation report has not been submitted by me for the award of any other degree or diploma.

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ABSTRACT

Acoustic Emission Analysis (AEA) is becoming increasingly popular among various Nondestructive Testing (NDT) techniques. Piezoelectric type sensors are widely used for Acoustic Emission (AE) measurements. A piezoelectric type AE sensor mainly consists of a piezoelectric element and a non-piezoelectric backing layer. The backing layer presents acoustical loading to the piezoelectric element. The material properties and physical properties of these elements decide the performance of the sensor and hence the suitability of a particular sensor for a particular application. A comprehensive model for AE sensors is presented which combines Mason's model for piezoelectric element in thickness mode and acoustic transmission line model for backing element This AE sensor model is analyzed to derive expressions for electrical impedance, pressure sensitivity, velocity sensitivity and displacement sensitivity of an AE sensor. A simulation program using MATLAB has been developed to generate impedance and sensitivity characteristics of the sensor. Further, a finite element model of AE sensor has been developed using commercial finite element software ANSYS to generate same performance characteristics of AE sensor. The characteristics obtained as the results of the two programs were compared and found to be matching with each other. This theoretically verified that the model and simulation results obtained using MATLAB program are accurate. Such a program can be useful for predicting performance characteristics of an AE sensor without actually fabricating the sensor and thus simplifying the sensor design process.

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NOMENCLATURE

h_{33}	h-coefficient of piezoelectric material
ε_{33}^{S}	Clamped permittivity of piezoelectric element
A	Area of cross-section of piezoelectric element
v_s	Sound velocity in piezoelectric element
t	Thickness of piezoelectric element
ρ	Density of piezoelectric element
ω	Radian frequency
ρ_B	Density of backing layer
v_B	Sound velocity in backing layer
Z_E	Electrical input impedance of piezoelectric element
Z_B	Equivalent impedance of backing layer
Z_{B0}	Acoustic impedance of backing layer
S_P	Pressure sensitivity of AE sensor
S_v	Velocity sensitivity of AE sensor
S_X	Displacement sensitivity of AE sensor
j	Complex unit, $j = \sqrt{-1}$
{T}	Stress vector
{D}	Electric flux density vector
{S}	Strain vector
$[c]$	elasticity matrix (evaluated at constant electric field) for piezoelectric element
$[e]$	Piezoelectric stress matrix
$[\varepsilon]$	Dielectric matrix (clamped) for piezoelectric element

Chapter 1

INTRODUCTION

1.1 Background

The acoustic emission Analysis (AEA) is considered quite unique among the non-destructive testing (NDT) methods. Today AEA is widely used for non-destructive testing of pressure vessels, aerospace structures, concrete structures, bridges, real time leakage detection etc.

An essential component of an AE measurement system is the sensor used to sense the acoustic emission. Various characteristics such as sensitivity, bandwidth, impedance characteristics, signal to noise ratio, the loading on the structure due to the sensor etc. decide the suitability of a particular sensor for a particular application. Therefore it is important to make a proper choice of sensor for an AE testing system. For proper selection of AE sensor, it is important to know the requirements of the application and the various sensor characteristics *a priori* so that it can be decided whether an available sensor is suitable for the application.

The various characteristics of a sensor depend on the physical and material properties of the various components of the sensor. The sensor characteristics can be found in two ways: (1) by some experimental method, (2) by developing a model of the sensor which can be used to determine the characteristics of interest with known physical and material properties of the sensor elements. With the availability of high speed computers, the second approach is much more economical both in time and money as compared to the first one and hence is used most widely.

1.2 Introduction to Acoustic Emission Analysis (AEA)

AE Analysis is an extremely powerful technology that can be deployed within a wide range of usable applications of non-destructive testing such as testing of metal pressure vessels, piping systems, reactors, and similar. AEA provides comprehensive information on the origin of faults in a loaded object and the development of the fault when subjected to continuous or repetitive stress.

Acoustic emission is the elastic energy that is spontaneously released by materials when they undergo deformation. AE is the term used when defects in metals, plastics and other materials release energy when subjected to mechanical loading. The energy propagates in form of high frequency, in the range of ultrasound usually between 20 kHz and 1 MHz, stress waves. Acoustic emission is non-directional. Most AE sources appear to function as point sources that radiate energy in spherical wavefronts. Often a sensor located on the surface of the specimen detects acoustic emission and the electrical signal obtained from the sensor is recorded as the AE signal.

Formally defined, acoustic emission is, *"the class of phenomena where transient elastic waves are generated by the rapid release of energy from localized sources within a material, or the transient elastic waves so generated"* ASTM (1982), [1, 2]. This is a definition embracing both the process of wave generation and the wave itself.

The AE analysis is the characterization of the signals according to intensity and frequency content, and also entails locating the release mechanisms by comparing signal travel times in a measurement procedure using several sensors. Fig. 1 shows a schematic diagram in which a sensor, mounted on a loaded test structure, is interfaced with an AE instrument used for performing AE analysis.

AE testing is a *passive, receptive* technique analyzing the ultrasound pulses emitted by a defect *right in the moment of its occurrence.* In contrast to the ultrasound technique one does not measure the response to an artificial and repeatable acoustic excitation of the test object. Instead, the sound signals produced by defects are evaluated; every growth of a defect is a unique event and can't be exactly reproduced again.

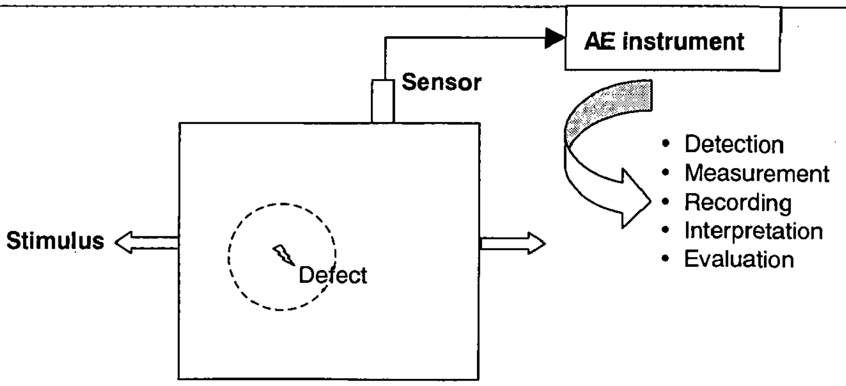


Fig.1. Schematic diagram of AEA system

AE is a *dynamic* inspection method in that it provides a response to discontinuity growth under an imposed structural stress; static discontinuities will not generate AE signals. This makes AE method particularly suitable for in service inspection of structures for defects. Another advantage of AE over conventional ultrasonic and radiographic testing is that large areas or volumes can be inspected with limited number of transducers.

Also corrosion, e.g. at the bottom of oil tanks, produce burst AE that propagates through the liquid oil to the tank wall, where it can be detected. With leakages, AE is produced by turbulent flow in the leak itself or by particles rebounding from the tank support. Burst AE from leakages will occur mainly at high pressure. Small pressure differences mainly cause laminar flow that emits continuous AE with low amplitudes and small propagation distances.

Also, if due to mechanical loading composites de-laminate, glue joints detach, fiber reinforcements break, etc. AE is produced, which can be analyzed for testing or monitoring these structures.

Thus, the important features of AE Analysis method can be stated as follows:

- The AE testing detects defects, which are growing under load during the test but does not recognize defects that are not moving.
- Because of its capability to detect defects right at the moment of their growth, the AE testing method may also be used as a real-time monitoring and warning system to avoid a failure of the structure under test with possibly disastrous consequences.
- The high level of modern computer technology, measurement techniques, and software has generated an enormous increase of the range of applications, the reliability, and the significance of the AE testing method. Especially, the real-time location calculation provides a big advantage.
- AE testing requires limited access of the test structure i.e. access only at the places where the sensors are mounted in contrast to other NDT methods which require access at all regions inspected. This fact makes AE testing more suitable for structures whose all parts are not easily accessible.

1.3 Applications of AEA

AEA is widely used in different engineering fields and industries for non-destructive testing of structures. A brief overview of applications of AEA is presented here.

- Periodic or continuous monitoring of pressure vessels and other pressure containment systems to detect active discontinuities.
- Monitoring fusion or resistance weldments during welding or during the cooling period.
- Monitoring acoustic emission response during stress corrosion cracking and hydrogen embrittlement susceptibility tests.
- Non-destructive testing of heavily mechanically stressed components or complete structures of fiber-reinforced plastics or composites, as used e.g. in the aerospace industry.
- Material research (e.g. investigation of material properties, breakdown mechanisms, and damage behavior)
- Inspection and quality assurance, e.g. monitoring of welding and wood drying processes, series inspection of ceramic components, scratch tests and more.
- Real-time leakage test and location within various components ranging from a small valve up to a tank bottom with diameter of 100 m.
- Geological and micro-seismic research.
- Detection and location of high-voltage partial discharges in large transformers.

1.4 Introduction to AE Sensors

The first step in the AE measurement process is the conversion, or transduction, of the mechanical energy of the elastic wave into electrical energy. When an acoustic emission wave-front impinges on the surface of a test object, very minute movements of the surface molecules occur. A sensor's function is to detect this mechanical movement and convert it into a specific, usable electrical signal. A wide range of transduction mechanisms can be used to achieve sensor's function: the detection of surface motion and the subsequent generation of an electrical signal. The various types of sensors that can be used for AE testing are discussed in the following.

1.4.1 Standard AE Sensors

Standard piezoelectric transducers have been found to be very suitable for acoustic emission testing. Acoustic emission testing is nearly always performed with sensors that use piezoelectric elements for transduction. The element is usually a special ceramic such as lead zirconate titanate (PZT) and is acoustically coupled to the test item so that the dynamic surface motion propagates into the piezoelectric element. The dynamic strain in the element produces a voltage-time signal as the sensor output. Since virtually all acoustic emission testing is performed with a piezoelectric sensor, the term *acoustic emission sensor* (AE sensor) will be used for a sensor with piezoelectric transduction element throughout this report unless otherwise specified.

This work is an attempt to model such a sensor and develop a simulation program for generating various performance curves for a given sensor or designing a sensor with desired performance characteristics. Practical issues related with AE sensor design will be discussed in detail in Chapter 2.

1.4.2 Capacitive Transducers

Capacitive transducers are non-contacting type transducers which directly measure the surface displacement of the test object. A stationary electrode is suspended above the material surface and a bias voltage (Vs) is applied to it. The material surface must be electrical conductor and electrically earthed. The capacitance of the transducer can be related to the electrode gap using the following equation,

$$C=\varepsilon A/x \tag{1.1}$$

Where C is the capacitance, A is the area of the electrode; x is the electrode gap and ε is the permittivity of the dielectric medium.

The voltage output from the charge amplifier is then,

$$V(t) = \eta Q(t) = \eta V_s C(t) = \eta V_s \varepsilon A/x(t) \tag{1.2}$$

Where η is the amplifier gain and Q is the charge on the electrode.

The electrode gap is typically 10μm with electrode areas of being 1cm^2 being commonplace; in order to maintain such a small gap accurately, both the stationary electrode and the material surface must be polished optically flat and be free from contaminants. Equation (1.2) clearly shows that the sensitivity of the transducer is directly proportional to the bias voltage, the electrode area and the permittivity; the dielectric material is usually air which fixes the permittivity. Therefore, for higher sensitivity the area and bias voltage must be as high as possible. The area is constrained by both the physical size of the transducer and the aperture effects and the bias voltage is limited by the electrical breakdown of the dielectric, which for air has been measured to be 35V/μm.Typically, bias voltages of around 50 to 100V have been used in literature.

1.4.3 Laser Interferometers

Laser interferometry is an alternative non-contact method for AE sensing. Interferometers directly measure the surface motion (displacement or velocity, depending on the interferometer layout) and have flat frequency response over bandwidths of 20MHz or more. The incident laser beam can be focused onto the material surface making the measurement area small, thus reducing the aperture effects and, being non-contacting, no damping is added to the structure and the coupling repeatability is excellent.

Laser interferometers generally have similar sensitivities to capacitive transducers making them relatively insensitive (as compared to piezoelectric devices) and, like capacitive transducers, require extremely careful surface preparation. Laser systems are also delicate and expensive making them poorly suited for AEmeasurements.

1.4.4 Conical Piezoelectric Transducers

This is a contacting piezoelectric transducer with a conical element with a very small measurement area which exhibits many of the required transducer characteristics. The small transducer area has been found to reduce the damping introduced by the transducer to the structure and increase the coupling repeatability while also reducing the aperture effects (compared to standard AE transducers). It has also been found that the conical shape of the element increases the bandwidth of the device as compared to the response of a standard disk element.

1.5 Literature Review

Extensive research has been carried out for modeling piezoelectric transducers. In this section, a brief review of published works in the area of modeling and simulation of piezoelectric transducers is presented.

Mason (1948) [3, 4, 5] was the first to introduce an exact electrical equivalent circuit for piezoelectric element operating in thickness mode with and without acoustical loading. This model will be discussed in detail in Section 3.1.

One of the perceived problems with Mason's model is that it required a negative capacitance at the electrical port. Although Redwood (1961) [6] showed that this capacitance could be transformed to the acoustic side of the transformer and treated like a length of the acoustic line it was still thought to be "un-physical". In an effort to remove circuit elements between the top of the transformer and the node of the acoustic transmission line Krimholtz, Leedom and Matthae (1970) [7] published an alternative equivalent circuit as shown in Fig. 2. The parameters of this model are listed in Table 1. The model is commonly referred to as the KLM model and has been used extensively in the medical imaging community in an effort to design high frequency transducers, multilayers, and arrays.

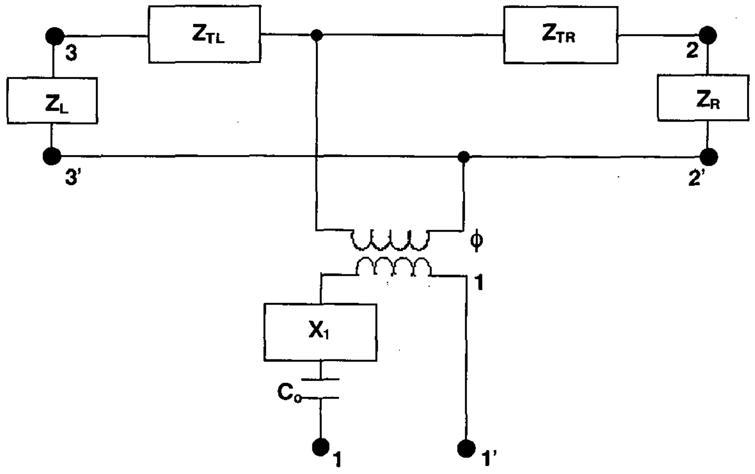


Fig.2. KLM equivalent circuit for piezoelectric element

Table 1.

KLM model parameters

h_{33} = h-coefficient of piezoelectric material,
A=area of cross-section of piezoelectric element,
ω = radian frequency,
v_s=longitudinal velocity in piezoelectric material ,
t=thickness of piezoelectric element ,
Z_L, Z_R = Acoustic loads on two faces of piezoelectric element,
ρ = density of piezoelectric material

$$Z_0 = \rho A v_s,\ \alpha = \frac{\omega}{v_s},\ M = \frac{h_{33}}{\omega Z_0},\ X_1 = jZ_0 M^2 \sin(\alpha t),\phi = \frac{1}{2M}\csc(\alpha t/2)$$

$$Z_{TL} = Z_0 \left[\frac{Z_L + jZ_0 \tan(\alpha t/2)}{Z_0 + jZ_L \tan(\alpha t/2)}\right]$$

$$Z_{TR} = Z_0 \left[\frac{Z_R + jZ_0 \tan(\alpha t/2)}{Z_0 + jZ_R \tan(\alpha t/2)}\right]$$

S. Sherrit, et al. (2002) [3] presented Mason's and KLM models to include dielectric elastic and piezoelectric loss. They also have compared the two models with different boundary conditions and found equivalence between the two models.

In order to remove the center-fed nature (in that, the electrical port is connected to the center node of the two acoustic ports) of Mason's model (and other models) and also to remove the negative capacity in Mason's model, Jean-Luc Dion (1993) [8] presented a new model for piezoelectric element vibrating in extensional and shear mode.

With the advent of high speed computers and developments in computational techniques, researchers have adopted different approaches for modeling piezoelectric transducers. Finite Element method, which is relatively new and effective computational method, was first used for piezoelectric media in 1960s.

The first computational examples of applications of the FE method to the modeling of piezoelectric media were given by Allik, where natural frequencies of a piezoelectric disk and a simple piezoelectric structure were given. In 1972 Hunt, Smith and Barach analyzed and redesigned an axisymmetric sonar piezoelectric projector using the FE method. In 1974, Allik, Webman and Hunt used the FE method to analyze a complicated three-dimensional transducer.

N.N. Abboud et al. (1998) [9] made a comparative study of different FE methods available for modeling piezoelectric ultrasonic transducers.

Also, Novak et al. (2002) [10] demonstrated the application of FEM in modeling of piezoelectric resonators for resonant characteristics.

In the following years the FE method got increasingly popular for the modeling of piezoelectric media, but the method was not widely used until the 1980s. In 1986 piezoelectric elements were included in the commercial finite element programANSYS, which has been used by many groups for FE modeling of piezoelectric media. Later piezoelectric finite elements have also been included in other commercially available finite element programs like e.g. ATILA, PZFlex, ABAQUS, MODULEF and PHOEBE.

Recently in 2003, Mercedes C. Reaves and Lucas G. Horta [11] used finite element program MSC/NASTRAN and MATLAB to model piezoelectric actuators.

In AE analysis, the piezoelectric transducer essentially operates in receive mode. The literature review reveals that very less work has been done for modeling piezoelectric transducers operating in receive mode. Present work focuses on modeling piezoelectric type AE transducers and thus emphasizes on modeling the behavior of piezoelectric transducer in receive mode

.

1.6 Outline of the Work

While designing AE sensor for a particular AE Analysis system, a number of factors are required to be considered. Since piezoelectric type AE sensors are mostly resonant type sensors and AE sources are essentially wideband, the choice of operating frequency range for AE sensors is a crucial factor. Secondly acoustic emission signals are usually very week, the sensitivity of sensor in operating frequency region should be very high. These factors, i.e. frequency range and sensitivity, depend on the physical and material properties of piezoelectric element, backing material and other constitutive elements of AE sensor.

If one can formulate this dependence of performance characteristics of AE sensors on properties of its constitutive elements, the task of sensor design would be greatly simplified. The present work addresses this problem by developing a model for AE sensor and using it for developing a computer program to generate desired performance characteristics for AE Sensors.

The dissertation mainly involves following,

- **Development of a model for AE sensors:** The performance of an AE sensor is mainly dependent on the physical properties and material properties of piezoelectric element and backing layer (please refer Chapter 2 for detailed information on backing) which are shown in Fig. 3.Using Mason's model [3, 4, 5] for the piezoelectric element and acoustic transmission line model [4] for the backing, a comprehensive model for AE sensor has been developed.
- **Development of simulation program:** Using the above model, an interactive computer program has been developed using MATLAB which can generate performance characteristics viz. impedance/admittance characteristics, displacement, pressure and velocity sensitivity characteristics of an AE sensor whose piezoelectric element and backing element properties are provided to the program as input. Similarly, the program can be used to maximize sensitivity or bandwidth within a particular frequency range by providing suitable ranges for every design parameter of the sensor. The output of the program, in this case, will be a set of design parameters for AE sensor corresponding to maximum sensitivity or bandwidth.
- **Simulation using ANSYS:** Finally, a finite element model of AE sensor has been developed using the finite element software package ANSYS. Using this model, same performance characteristics which were generated by using MATLAB simulation program for AE sensor, have been generated. These characteristics were compared with those generated using MATLAB simulation program. There was a close agreement between the two sets of characteristics.

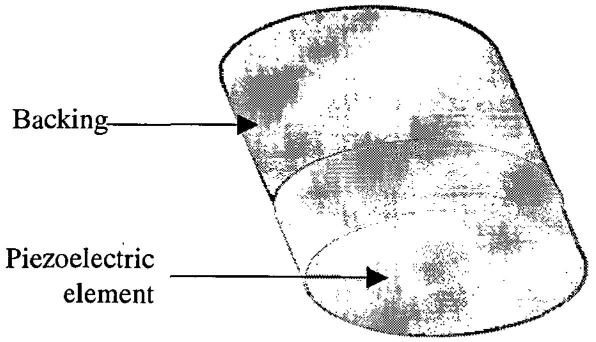


Fig.3. Important parts of AE sensor

Chapter 2

AE SENSORS: CHARACTERISTICS AND DESIGN

2.1 Practical AE Sensor

As stated in Section 1.4, the function of an AE sensor is to detect mechanical movement of the surface of the test object due to acoustic emission and convert it into a specific, usable electrical signal. An ideal transducer would provide a voltage output that was directly proportional to the surface displacement, velocity or acceleration at the measurement point. This ideal sensor would operate over the entire frequency spectrum, without introducing any noise to the signal, and without disturbing the motion of the elastic wave. In reality, there is no such device. All real transducers have limited bandwidth, over which the sensitivity may vary. In addition, real transducers have finite area over which the received signal is integrated. Since they usually contact the surface, real sensors may also disturb or alter the motion of the wave being detected. Furthermore, the transducer may introduce noise to the measured signal. Thus selecting AE transducer, like any other physical measurement device, requires that compromises and tradeoffs be considered to determine the best suitable transducer for the measurement situation.

2.2 Frequency Range of AE Sensors

Majority of acoustic emission testing is based on the processing of signals with frequency in the range from 30 kHz to about 1 MHz. In special applications, detection of acoustic emission at frequencies below 20 kHz or near audio frequencies can improve testing and for that, conventional microphones and accelerometers are used. However, the single most common frequency range for AE testing is 100 kHz to 300 kHz.

Attenuation of the wave motion increases rapidly with frequency and for materials with higher attenuation (such as fiber reinforced plastic composites), it is necessary to sense lower frequencies in order to detect acoustic emission events. At higher frequencies, background noise is lower and for materials with low attenuation, acoustic emission events tend to be easier to detect at higher frequencies.

Acoustic emission sensors can be designed to sense a portion of the whole frequency range of interest by choosing the appropriate dimensions of piezoelectric element and backing element (to be discussed in subsequent sections). This, along with its high sensitivity, accounts for the popularity of this transduction mechanism.

AE sensors, depending on their operating frequency band, are categorized as,

1) Resonant sensors and
2) Wideband sensors.

Resonant sensor: A sensor that uses the mechanical amplification due to a resonant frequency (or several close resonant frequencies) to give high sensitivity in a narrow band, typically ±10 percent of the principal resonant frequency at the -3 dB points is called as a *resonant sensor* [1].

Wideband sensor: A sensor which uses mechanical amplification due to the superposition of multiple resonances to give high sensitivity in several narrow bands within a specified wide band is called a *wideband sensor* [1].

2.3 Direction of Sensitivity to Motion

Surface motion of a point on a test piece may be the result of acoustic emission. Such motion contains a component normal to the surface and two orthogonal components tangential to the surface. Acoustic emission sensors can be designed to respond principally to any component of motion but virtually all commercial acoustic emission sensors are designed to respond to the component normal to the surface of the structure. Since waves traveling at the longitudinal, shear and Rayleigh wave speeds; all typically have a component of motion normal to the surface and, acoustic emission sensors can often detect the various wave arrivals.

2.4 Typical Acoustic Emission Sensor Design

Fig. 4 shows the schematic diagram of a typical acoustic emission sensor. The sensor is attached to the surface of a test object and a thin intervening layer is of couplant is usually used. The couplant facilitates the transmission of the acoustic waves from the test object to the sensor. Attachment of sensor may also be achieved with an adhesive bond designed to act as an acoustic couplant.

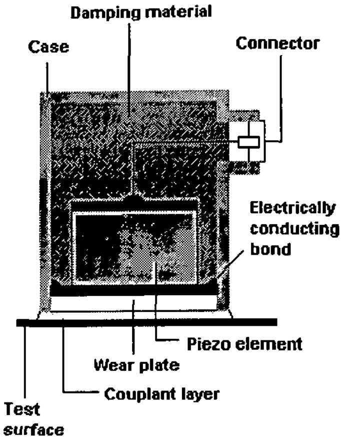


Fig.4. Schematic diagram of a typical AE sensor mounted on a test object

An acoustic emission sensor normally consists of several parts. The active element is a piezoelectric ceramic with electrodes on each face. One electrode is connected to electrical ground and other is connected to a signal lead. A wear plate protects the active element.

Behind the active element is usually a backing material and this is often made by curing epoxy containing high density particles. The backing is usually designed so that acoustic waves easily propagate into it with little reflection back to the active element. In the backing, much of the wave energy is attenuated by scattering and absorption. The backing also serves to load the active element so that it is less resonant or more broadband (however, in some applications, a resonant transducer is desirable). Less resonance helps the sensor respond more evenly over a wider range of frequencies.

A typical AE sensor also has a case with a connector for signal cable attachment. The case provides an integrated mechanical package for the sensor components and may also serve as a shield to minimize electromagnetic interference.

2.5 Factors Associated with Sensor Design

2.5.1 Thickness of Sensor Element

Primarily, the element thickness controls the frequencies at which the transducer has the highest electrical output for a given input surface velocity (highest sensitivity). The half-wave resonant frequencies of the sensor define the approximate frequencies where the sensor will have maximum output. These are the frequencies for which: $t = \lambda/2$, $3\lambda/2$, $5\lambda/2$, and so on, where λ is the wavelength of the wave in the element. The wavelength can be defined as the speed of sound in the piezoelectric element divided by the acoustic frequency.

Thus, the resonant frequency, f_0, for an element with thickness t is given by, [2]

$$f_0 = \frac{v_s}{2t} \tag{2.1}$$

where v_s is the velocity of sound in the element.

2.5.2 Active Element Diameter

The other principal characteristic of an AE sensor is the active element diameter. Sensors have been designed with element diameters as small as 1mm. But, larger diameters are more common.

The element diameter defines the area over which the sensor averages the surface motion. For waves resulting in uniform motion under the transducer (as in case of longitudinal wave propagating in a direction perpendicular to the surface), the diameter of the element has little or no effect. However, for waves traveling along the surface, the element diameter strongly influences the sensor sensitivity as a function of wave frequency.

If the displaced surface of the test object is a spatial sine wave, then there are occasions when one or more full wavelengths (in the object item) will match the diameter of the sensor element. When this occurs, the sensor averages the positive and negative motions to give zero output. This is called as the aperture effect.

For sensors larger than the wavelengths of interest in the test object, the sensitivity will vary with the properties of the test material, depending strongly on frequency and direction of wave propagation.

Because of these complications, it is recommended that the sensor diameter be as small as other constraints allow.

2.5.3 Transducer Backing

The primary purpose of transducer backing is to provide damping and loading to the piezoelectric element. Transducers are damped to minimize the effect of mechanical resonances present in the piezoelectric element thus broadening the overall bandwidth of the transducer. An effective backing material must extract and absorb ultrasonic energy from the piezoelectric element; this is inevitably linked with some loss of sensitivity at the resonance frequencies. The transducer backing, being large in comparison with the piezoelectric element, also provides convenient attachment for wires and guides for the transducer.

Selection of the proper backing material for a piezoelectric transducer demands careful consideration of such parameters as operating frequency, bandwidth, insertion loss, and operational environment. In general , the acoustic requirements for a non-reflective type backing (an absorber), are (1) it must have an acoustic impedance which is close to that of the piezoelectric element, and (2) it must have sufficient acoustic attenuation to prevent unwanted acoustic reverberations (i.e., back wall reflections). Other important requirements for the backing material include (1) must be able to adhere the piezoelectric material to it, and (2) the material must be available in sufficiently thick substrate form and have a high surface quality (e.g, polished).

The specific acoustic impedance (Z_B) of the backing material must be similar to that of the piezoelectric element for it to be effective in removing energy. If the materials are poorly matched (dissimilar Z), very little ultrasonic energy crosses the boundary into the backing and the energy is trapped in the piezoelectric element. The transmission coefficient across a boundary (say material 1 to material 2) can be defined as the proportion of the intensity (energy) of the incident ultrasonic signal which is transmitted through the interface. This can be written in terms of acoustic impedances as follows,

$$T_{12} = \frac{4m}{(m+1)^2} \tag{2.2}$$

where m is the ratio of the impedances and is given by $m=Z_1/Z_2$ and T_{12} is the transmission coefficient from material 1 to 2. Also the acoustic impedance is given by,

$$Z=\rho v_s \tag{2.3}$$

where ρ is the material density and v_s is the bulk longitudinal velocity.

Complete transmission occurs when the materials are matched i.e. when $Z_1 = Z_2$.

Therefore an ideal backing material matches the piezoelectric element.

The unit of specific acoustic impedance (Z) is kgm^{-2}s^{-1} or Rayl.

Chapter 3

AE SENSOR MODELING USING MATLAB

The function of AE sensor is to convert mechanical energy received from AE source into a usable electrical signal. To model the behavior of the sensor and to predict its performance characteristics, it is necessary to develop mathematical expressions which relate the electrical output of the sensor to the mechanical input of the sensor.

As discussed in Section 2.3, most of the acoustic emission sensors are designed to respond to the component of motion normal to the surface of the structure. Thus the piezoelectric element has to operate in thickness extensional mode for sensing acoustic emission. Though analytical solutions to the wave equation in piezoelectric materials can be quite cumbersome to derive from first principles in all but a few cases, Mason [3] was able to show that for one-dimensional analysis most of the difficulties in deriving the solutions could be overcome by borrowing from network theory. Thus by developing an electrical equivalent circuit of AE sensor, the required expressions for the sensor can be derived. In present work, this approach has been used to generate various performance characteristics of the AE sensor.

As mentioned in Chapter 2, there exist several electrical equivalent circuit representations for AE piezoelectric elements operating in thickness mode. In present work, the widely accepted Mason's model is used to develop electrical equivalent circuit for AE sensor.

3.1 Mason's Equivalent Circuit

The Mason's model is shown in Fig. 5 for the thickness mode. In Mason's equivalent circuit, an electrical port (1-1′) is connected to the center node of the two acoustic ports (2-2′ and 3-3′) representing the front and back face of the transducer. On the electrical port of the transformer all circuit elements are standard electrical elements and the voltage is related to the current via $V = ZI$ where Z is an electrical impedance. On the acoustical side of the transformer the force F and the velocity v are related through $F = Z_a v$ where Z_a is the acoustic impedance. $Z_a = \rho v_s A$, where ρ is the density, v_s is the

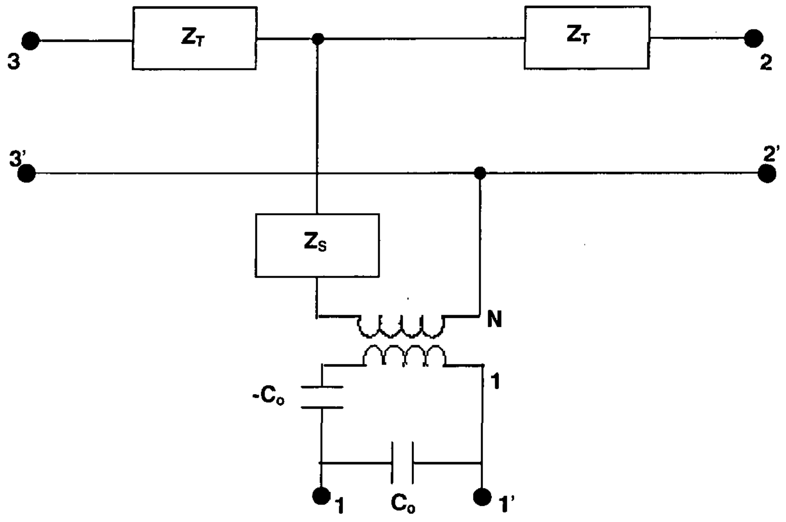


Fig.5. Mason's equivalent circuit for piezoelectric element

longitudinal velocity of the piezoelectric material and A is the area of ceramic (a detailed discussion on acoustic impedance can be found in [2]). It should be noted that the v is a variable of the circuit model while v_s is a constant of the material. The transformer is an ideal electromechanical transformer that conserves power during the transformation.

If the acoustic ports are shorted the model reduces to the free resonator equation derived from the linear piezoelectric equations and the wave equation which has been adopted by the IEEE Standard on Piezoelectricity for determination of the thickness material constants.

3.1.1 Mason's Model Parameters

The capacitance C_0 is given by,

$$C_0 = \frac{\varepsilon_{33}^{S} A}{t} \tag{3.1}$$

where, ε_{33}^{S} is clamped permittivity, t is thickness and A is the cross-sectional area of piezoelectric element.

The transformer ratio, N, is given by,

$$N = C_0 h_{33} \tag{3.2}$$

where h_{33} is the h-coefficient of the element.

The acoustic impedance of piezoelectric element, Z_0, is given by,

$$Z_0 = \rho A v_s \tag{3.3}$$

where, ρ is the density and v_s is the sound velocity for piezoelectric element.

The impedances Z_T and Z_S are given by,

$$Z_T = jZ_0 \tan(\alpha t/2) \tag{3.4}$$

$$Z_S = -jZ_0 \csc(\alpha t) \tag{3.5}$$

where, α is given by,

$$\alpha = \frac{\omega}{v_s} \tag{3.6}$$

3.2 Model for Backing Layer

The network representation described in [4] will be used for the non-piezoelectric backing layer attached to one face of piezoelectric element. This network representation is shown in Fig. 6. Like Mason's model, this network representation is the solution to one dimensional wave equation with open boundary conditions. The similarity can be observed between acoustic side of Mason's model and the equivalent circuit of backing element.

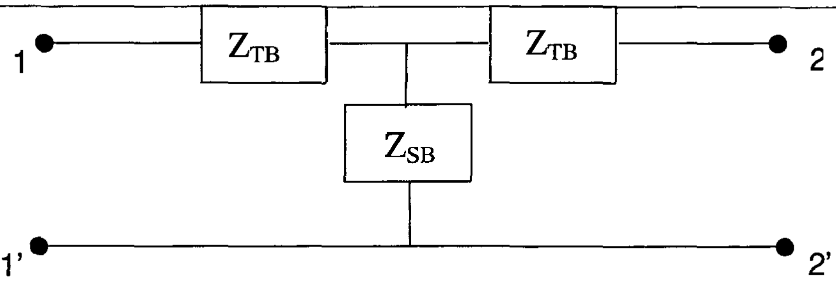


Fig.6.Equivalent circuit for backing element

3.2.1 Backing Model Parameters

Let's define,

$$Z_{B0} = \rho_B A v_B \tag{3.7}$$

$$\alpha_B = \frac{\omega}{v_B} \tag{3.8}$$

Then the parameters of backing equivalent circuit are,

$$Z_{TB} = jZ_{B0}\tan(\alpha_B t_B / 2) \tag{3.9}$$

$$Z_{SB} = -jZ_{B0}\csc(\alpha t_B) \tag{3.10}$$

where, t_B is the thickness, ρ_B is the density of backing element and v_B is the sound velocity in the backing element. The cross-sectional area of backing element is assumed to be same as that of piezoelectric element i.e.A.

3.2.2 Equivalent Impedance of Backing

As backing is attached to one face of piezoelectric element, in the equivalent circuit of AE sensor, one of the ports of backing equivalent circuit (say, 1-1′ in Fig. 6) will be connected to one of the acoustic ports (say, 2-2′ in Fig. 5) of Mason's equivalent circuit. To reduce complexity of the model, backing equivalent circuit can be replaced by its equivalent impedance observed from the port connected to the Mason's equivalent circuit. If the backing face which is not attached with piezoelectric element, corresponding to port 2-2′ of backing equivalent circuit, is free to vibrate i.e. no load is applied, then port 2-2′ will be shorted. On the other hand, if the backing face is rigidly fixed so that its motion in any direction is restricted then port 2-2′ will be open. Thus, the two boundary conditions will result in different expressions for the equivalent impedance of backing element. The two boundary conditions are discussed separately in the following.

Condition 1: Port 2-2′ is open

When port 2-2′ is open, the equivalent impedance,Z_B, of backing circuit is given by,

$$Z_B = Z_{TB} + Z_{SB} \tag{3.11}$$

After substituting values of Z_{TB} and Z_{SB} from Equations (3.9) and (3.10) and simplifying the expression,

$$Z_B = -jZ_{B0}\cot\left(\frac{\omega t_B}{v_B}\right) \tag{3.12}$$

Condition 2: Port 2-2′ is shorted

When port 2-2′ is shorted, the equivalent impedance,Z_B, of backing circuit is given by,

$$Z_B = Z_{TB} + Z_{SB} \| Z_{TB} \tag{3.13}$$

After substituting values of Z_{TB} and Z_{SB} from Equations (3.9) and (3.10) and simplifying the expression,

$$Z_B = jZ_{B0} \tan\left(\frac{\omega t_B}{v_B}\right) \tag{3.14}$$

3.3 Equivalent Circuit of Overall AE Sensor

After combining Mason's equivalent circuit for piezoelectric element and equivalent circuit for backing element, the equivalent circuit for AE sensor can be drawn as shown in Fig. 7.

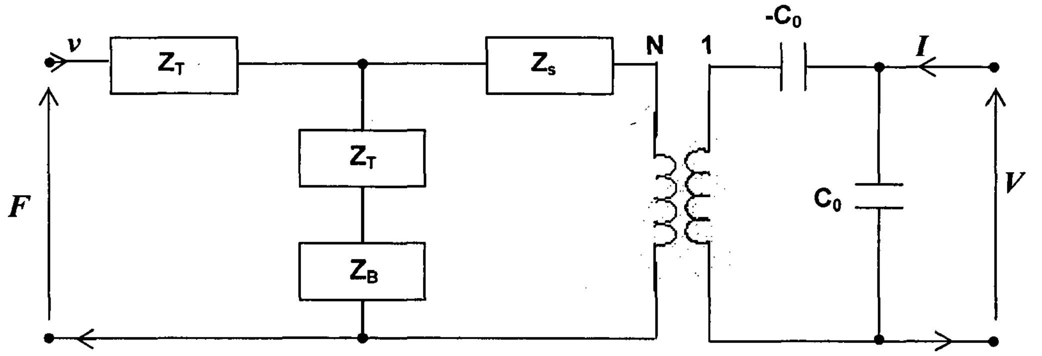


Fig.7. Electrical equivalent of AE sensor with backing

The value of Z_B should be chosen properly based on the boundary conditions i.e. whether the free face of the backing element is free to vibrate or rigidly clamped.

It can be seen that, this network is a two port network with one acoustic port and other electrical port. As AE sensor operates in receive mode, the acoustic port is the input port while electrical port acts as output port.

3.4 Electrical Input Impedance of the AE Sensor

Electrical Impedance Plots provide important information about the design and construction of a transducer and can allow users to obtain electrically similar transducers from multiple sources.

The impedance viewed from electrical port (Z_E) can be obtained by shorting the acoustic port and finding out the ratio of applied voltage V to the current I.

i.e. $$Z_E = \frac{V}{I}\Big|_{F=0} \tag{3.6}$$

$$Z_1 = \frac{j}{C_0\omega} + \frac{Z_s}{N^2} + \frac{1}{N^2}\frac{Z_T Z_C}{Z_T + Z_C} \tag{3.7}$$

where, $$Z_C = Z_T + Z_B \tag{3.8}$$

$$Z_E = \frac{Z_1}{1 + jZ_1 C_0 \omega} \tag{3.9}$$

3.5 Sensitivity Functions of AE Sensors

The sensitivity of AE sensor relates the acoustic wave motion at the point of measurement and the acoustic emission sensor electrical output [1].

Since there are many types of sensors in use, and since they may be called on to detect waves of different kinds in different materials, it is not possible to have a universal method for specifying sensitivity of the sensor. There are several fundamental problems encountered for specifying sensitivity of AE sensor, as listed below.

1. The displacement of a point on the surface of a test block is a three dimensional vector but the output of the sensor is a scalar.
2. Displacement is altered by the presence of the sensor.
3. The face of the sensor covers an area on the test surface of the test block and displacement is a function not only of time but of the position within this area.

Because of these problems some simplifying assumptions are necessary for determining sensitivity of AE sensors. These assumptions are discussed below.

Regarding the vectoral nature of displacement (problem-1), it is usually assumed that the sensor is sensitive only to normal displacement. Naturally, errors will be induced if the sensor is sensitive to tangential displacement.

The loading effect that the sensor has on the surface motion of a test block (problem 2) is significant but is not subject to any simple analysis. In general, the test block may be considered as having a mechanical impedance (source impedance) at the location of the sensor. However, quantifying this mechanical impedance is complex task and no method exists to measure it. The usual solution for this problem is to define the input to the sensor as the unloaded (free) displacement of the test block with no sensor attached. This solution is practical because it is the displacement of the test block (and not the interactive effects between sensor and test block surfaces) that is of interest.

The finite size of sensor's face (problem-3) is often ignored. This is equivalent to,

- assuming that the diameter of the face is small compared to all wavelengths of interest in the test block; or
- assuming that all motion is in phase over the face.

The later assumption is true in the case of plane waves impinging on the sensor from a direction perpendicular to its face.

3.5.1 Pressure Sensitivity

Pressure sensitivity relates the pressure signal that appears at the face of transducer due to acoustic emission and the voltage output of the sensor.

If a uniform pressure P is assumed to be appearing at the face of transducer, the force input to the transducer can be calculated. The relation between this force input and the voltage output of the transducer can be derived by analyzing equivalent circuit for AE sensor shown in Fig. 7.

$$S_F(j\omega) \equiv \frac{V(j\omega)}{F(j\omega)} \tag{3.10}$$

By analyzing the circuit shown in Fig. 7,

$$S_F(j\omega) = \frac{NZ_C}{jC_0\omega(Z_T(Z_S + Z_C) + Z_S Z_C)} \tag{3.11}$$

where Z_C is defined in Equation (3.8).

Since, $F = P \cdot A$ (3.12)

and as pressure sensitivity can be defined as,

$$S_P(j\omega) \equiv \frac{V(j\omega)}{P(j\omega)} \tag{3.13}$$

$$S_P = A \cdot S_F \tag{3.14}$$

hence, from Equation (3.11) the expression for pressure sensitivity is,

$$S_P(j\omega) = \frac{ANZ_C}{jC_0\omega(Z_T(Z_S + Z_C) + Z_SZ_C)} \tag{3.15}$$

3.5.2 Velocity Sensitivity

Velocity sensitivity relates the velocity of a point on transducer face due to acoustic emission and the voltage output of the sensor.

Velocity sensitivity can be defined as,

$$S_v(j\omega) \equiv \frac{V(j\omega)}{v(j\omega)} \tag{3.16}$$

Noting that the equivalent quantity for velocity at acoustic port of circuit shown in Fig. 7 is current, the expression for velocity can be derived easily by analyzing the circuit. The expression for velocity sensitivity is,

$$S_v = \frac{NZ_C}{jC_0\omega(Z_s + Z_C)} \tag{3.17}$$

3.5.3 Displacement Sensitivity

Displacement sensitivity relates the displacement of a point on transducer face due to acoustic emission and the voltage output of the sensor.

Displacement sensitivity can be defined as,

$$S_X(j\omega) \equiv \frac{V(j\omega)}{X(j\omega)} \tag{3.18}$$

The velocity of a point can be obtained by differentiating the displacement of the point, which is a function of time, with respect to time. The differentiation in time domain corresponds to multiplication by $j\omega$ in frequency domain.

hence, $v(j\omega) = j\omega X(j\omega)$

Thus the relation between displacement sensitivity and velocity sensitivity becomes,

$$S_X(j\omega) = j\omega S_v(j\omega) \tag{3.19}$$

from Equations 3.17 and 3.19, $S_X = \dfrac{NZ_C}{C_0(Z_s + Z_C)}$ (3.20)

3.6 Development of Programs Using MATLAB

After deriving necessary mathematical expressions for the quantities of interest viz. impedance, pressure sensitivity, velocity sensitivity and displacement sensitivity; the 'Simulation Program' for generating performance characteristics can be developed.

In present work, two programs using MATLAB have been developed for performing following functions

1. Simulation of performance characteristics of AE sensor i.e. *impedance/admittance characteristics* and *sensitivity characteristics.* Here onwards this program will be referred as *'Simulation Program'.*
2. Getting the set of design parameters (physical and material properties of piezoelectric element and backing element) for achieving maximum sensitivity or bandwidth. Here onwards this program will be referred as *'Design Program'.*

3.7 The Simulation Program

An interactive program has been developed using MATLAB for generating various characteristics of AE sensors. The program takes various physical and material properties of sensor elements, viz. piezoelectric element and backing as input, and generates the performance characteristics specified by the user. The various properties required by the program are listed in Table 2.The program interacts with user using a GUI (Graphical User Interface) designed using MATLAB. Following are the tasks which a user can perform on the GUI.

Table 2. Input properties for the Simulation Program

Piezoelectric Element Properties	Backing Properties
• h_{33} coefficient, (V/m) • dielectric constant ($\varepsilon_{33}^{S} / \varepsilon_0$) • density, (kg/m^3) • velocity of sound in the material (m/s) • element thickness (mm) • element diameter (mm)	• density, (kg/m^3) • velocity of sound in the material (m/s) • element thickness (mm) Note: Diameter of backing is assumed same as that of piezoelectric element

Instead of entering values for all the properties manually, the user can select piezoelectric material and backing material from the lists provided on GUI. The values for the properties are automatically loaded from the program's database. Further, the user can change one or more property values by editing corresponding fields on GUI.

As discussed in Section 3.2.2, there are two boundary conditions for backing element i.e. the free face of backing element can be free to expand or can be rigidly clamped. A checkbox has been provided on the GUI which allows user to select one of these two conditions.

A list of plots which the program can draw as output has been provided on the GUI. The list includes sensitivity plots (pressure, velocity and displacement) impedance and admittance plots. The user can specify the plots to be drawn by selecting the plots from the list. For all the plots, the X-axis is frequency and Y-axis is the quantity being plotted. For certain plots, the program also lists the points corresponding to maxima and minima of the quantity.

The frequency range over which the quantities are plotted can be program chosen or can be entered manually on the GUI. The user can also specify the number of frequency points at which the quantities should be evaluated. The frequency points are uniformly distributed within the frequency range of interest.

After clicking the RUN button on GUI, the program plots the various quantities and lists the maxima and minima points for the various plots in command window of MATLAB.

3.7.1 Steps Involved in Simulation Program

Instead of listing whole script for the Simulation Program, the steps involved in the development of the program are briefly discussed here. The steps are as follows.

1. The parameters and options provided by user on GUI are passed to a MATLAB function *'simall'* written in the file *'simall.m'* The further steps are executed by this function.
2. Using the physical and material properties of piezoelectric and backing element, the parameters of Mason's model (discussed in Section 3.1.1) and Backing model parameters (discussed in Section 3.2.1) are calculated. The parameters, which are

frequency dependent are calculated at all desired frequency points (specified by user by frequency range and number of frequency points.)

3. Using these parameters, the input impedance, input admittance, pressure sensitivity, velocity sensitivity and displacement sensitivity are calculated at all the desired frequency points.
4. During calculation, whenever a maximum or minimum is detected, its value and corresponding frequency are stored in the matrix of maxima or minima.
5. The required plots (specified by user) are drawn.
6. The list of maxima and minima is displayed in command window of MATLAB.

3.8 Design Program

While designing a sensor, the designer may want to know the values for various design parameters of the sensor i.e. physical and material properties of piezoelectric element and backing element for which sensitivity of the sensor or usable frequency range (bandwidth) of the sensor is maximized over a desired frequency range. The Design Program can be used for this purpose.

The Design Program accepts range of values for each of the design parameters and frequency range of interest and maximizes pressure sensitivity or bandwidth (for pressure sensitivity curve) as specified by the designer and outputs the corresponding set of design parameters.

This program also, like Simulation Program, interacts with user using a GUI designed using MATLAB. The GUI includes editable fields for inputting range values for each parameter. The user can specify whether pressure sensitivity or bandwidth is to be maximized by checking the corresponding check box.

After clicking RUN button on GUI, the program calculates the set of parameters and plots the sensitivity curve for this set of parameters over the specified frequency range.

The Design Program has been developed by using Genetic Algorithm for maximizing sensitivity or bandwidth. Two important parameters which control the performance of genetic algorithm are cross-over rate and mutation rate. The user can also specify values for these parameters on GUI. The default values for these parameters are

0.7 and 0.001. (Abundant literature, both in printed and electronic formats, is available on Genetic Algorithms or GAs. Appendix C provides basic information on GAs in order to understand the terms such as crossover rate, mutation rate etc. and the basic operations involved in GA).

3.8.1 Why Genetic Algorithm?

Equation (3.15) gives mathematical expression for pressure sensitivity. If the values of various quantities i.e. N, Z_C, C_0, Z_T and Z_S are substituted in the expression in terms of material properties and physical properties of the sensor then the resulting expression for pressure sensitivity will be,

$$S_P = \frac{h_{33}}{\omega}\left(\frac{\rho v_S \tan(\omega t/2v_S) + \rho_B v_B \tan(\omega t_B/v_B)}{\rho^2 v_S^2 + \rho v_S \rho_B v_B \cot(\omega t/v_S)\tan(\omega t_B/v_B)}\right) \tag{3.21}$$

Equation (3.21) shows that pressure sensitivity is a function of eight parameters viz. $h_{33}, \rho, v_S, \rho_B, v_B, t$ and t_B along with frequency ω. Thus it would be quite difficult to maximize sensitivity function with respect to all parameters using conventional methods such as partial differentiation.

Secondly, an explicit mathematical expression for bandwidth of transducer cannot be derived easily; hence it is impossible to use any conventional method for maximizing bandwidth which relies on mathematical expression for a quantity for maximizing it.

The approach of using genetic algorithm solves both of the problems discussed above. Secondly genetic algorithm is a proven effective computational technique for maximization problems and can be very easily implemented using a programming tool such as MATLAB.

3.8.2 Steps Involved in Design Program

Again, to avoid listing whole script for the Design Program, the steps involved in the development of the program are briefly discussed here. The steps are as follows.

1. The various parameters provided by user on GUI are passed to a MATLAB function *'mainGA'* written in the file 'mainGA.*m*' The further steps are executed by this function.

2. An initial population of 100 chromosomes is generated. The length of chromosome is 80 bits. For each variable out of the eight variables ($h_{33}, \rho, v_S, \rho_B, v_B, t, t_B$ and .), 10 bits are reserved in the chromosome. The population is generated on random basis. This population acts as parent generation. After this step, the iterative procedure of GA begins.
3. For all the chromosomes in the parent generation, fitness values are calculated. As either sensitivity or bandwidth is to be maximized, the objective function, i.e. sensitivity or bandwidth is used as fitness function.
4. Based on the fitness values of chromosomes in the parent generation, new generation containing same number of chromosomes i.e. 100 is created. The selection method used for creating new generation is *'roulette wheel'* method. This generation is an intermediate new generation.
5. The next step is *'cross-over'*. Two chromosomes are selected at random from the new generation (of step 4). Depending upon the cross-over rate (cross-over probability specified by the user), the two chromosomes undergo cross-over operation at a randomly selected point along the length of chromosomes. The newly generated chromosomes are inserted into again a new generation. This step is repeated until the number of chromosomes in this new generation become 100. This generation is also an intermediate new generation.
6. From this newly created population, again a new population is created by mutation operation. Every bit in the population generated after step 5 undergoes mutation depending upon mutation rate (mutation probability specified by the user). The population so generated is the final new generation. This new generation replaces the previous generation (parent generation in step 3). If this generation meets the stopping criteria, then the iterative procedure stops otherwise steps from step 3 onwards are repeated.
7. After the iterative procedure stops, the parent generation contains best chromosomes. From this population, one chromosome which corresponds to highest fitness value is selected. This chromosome corresponds to the values of design parameters required to maximize objective function i.e. sensitivity or bandwidth. The output of the program is this set of design parameters of the AE sensor.

Chapter 4

AE SENSOR MODELING USING ANSYS

4.1 What is ANSYS?

ANSYS is a commercial finite element analysis software with the capability to analyze a wide range of engineering problems. Like any finite element software, ANSYS solves governing differential equations by breaking the problem into small elements. The governing equations of elasticity, fluid flow, heat transfer, and electromagnetism can all be solved by the finite element method in ANSYS. ANSYS can solve transient problems as well as nonlinear problems. In general, ANSYS enables engineers to perform following tasks:

- Build computer models
- Apply operating loads or other design performance conditions.
- Study physical responses, such as stress levels, temperature distributions, or electromagnetic fields.
- Optimize a design early in the development process to reduce production costs.
- Do prototype testing in environments where it otherwise would be undesirable or impossible (for example, biomedical applications).

The ANSYS program has a comprehensive graphical user interface (GUI) that gives users easy, interactive access to program functions, commands, documentation, and reference material. An intuitive menu system helps users navigate through the ANSYS program. Users can input data using a mouse, a keyboard, or a combination of both.

In present work, ANSYS 7.0 has been used to develop finite element model for AE sensor and to perform required analysis. In this chapter, development of finite element model for AE sensor using ANSYS will be discussed in detail.

As ANSYS uses Finite Element Method (FEM) for solving modeling problems, a brief discussion on finite element method is presented in the next section.

4.2 Introduction to FEM

Virtually every phenomenon in nature, whether biological, geological, mechanical or electrical, can be described with the aid of laws of physics, in terms of algebraic, differential, or integral equations relating various quantities of interest.

While studying and analyzing a physical phenomenon, one has to perform following two major tasks:

1. Mathematical formulation of the physical process
2. Numerical analysis of the mathematical model.

The limitations of the human mind are such that it cannot grasp the behavior of its complex surroundings and creations in one operation. Thus the process of subdividing all systems into their individual components or 'elements', whose behavior is readily understood, and then rebuilding the original system from such components to study its behavior is a natural way in which the engineer, the scientist, or even the economist proceeds.

In many situations an adequate model is obtained using a finite number of well defined components. Such problems are termed as *discrete.* In others the subdivision is continued indefinitely and the problem can only be defined using the mathematical fiction of an infinitesimal. This leads to differential equations or equivalent statements which imply an infinite number of elements. Such systems are termed as *continuous.*

With the advent of digital computers, *discrete* problems can generally be solved readily even if the number of elements is very large. As the capacity of all computers is finite, *continuous* problems can only be solved exactly by mathematical manipulation. Here, the available mathematical techniques usually limit the possibilities to oversimplified situations.

To overcome the intractability of realistic types of continuum problems, various methods of *discretization* have from time to time been proposed both by engineers and mathematicians. All involve an *approximation* which, hopefully, approaches in the limit the true continuum solution as the number of discrete variables increases.

Finite element method is one such method which involves discretization of domain of problem into simple subdomains, called finite elements. The method is characterized by three features:

1. The domain of the problem is represented by a collection of simple subdomains, called finite elements. The collection of finite elements is called finite element mesh.
2. Over each finite element, the physical process is approximated by functions of desired type (polynomials or otherwise), and algebraic equations relating physical quantities at selective points, called nodes, of the element are developed.
3. The element equations are assembled using continuity and/or "balance" of physical quantities.

In the finite element method, in general, we seek an approximate solution u to a differential equation in the form,

$$u \approx \sum_{j=1}^{n} u_j \psi_j + \sum_{j=1}^{m} c_j \phi_j \tag{4.1}$$

where u_j are the values of u at the element nodes, ψ_j are the interpolation functions, c_j are the nodeless coefficients, and ϕ_j are the associated approximation functions. Direct substitution of such approximations into the governing differential equation does not always result, for an arbitrary choice of the data of the problem, in a necessary and sufficient number of equations for the undetermined coefficients u_j and c_j. Therefore, a procedure whereby a necessary and sufficient number of equations can be obtained is needed. Many methods have been proposed for this purpose. One such method is using weighted-integral form of the governing differential equations.

It should be noted that, there can be more than one *finite element model* of the same problem. The type of model depends on the differential equations and methods used to derive the algebraic equations for the undetermined coefficients over an element.

4.3 Equations of Piezoelectricity

As ANSYS uses variational principles to develop the finite element equations which incorporate the piezoelectric effect, it would be imperative here to discuss the basic equations of piezoelectricity which are used in solving finite element models of piezoelectric structures. The electromechanical constitutive equations of piezoelectricity for linear material behavior can be expressed as follows.

$$\{T\}=[c]\{S\}-[e]\{E\} \tag{4.2}$$

$$\{D\}=[e]^T\{S\}+[\varepsilon]\{E\} \tag{4.3}$$

or equivalently,

$$\begin{Bmatrix}\{T\}\\ \{D\}\end{Bmatrix}=\begin{bmatrix}[c] & [e]\\ [e]^T & -[\varepsilon]\end{bmatrix}\begin{Bmatrix}\{S\}\\ -\{E\}\end{Bmatrix} \tag{4.4}$$

where:

$\{T\}$ = stress vector

$\{D\}$ = electric flux density vector

$\{S\}$ = strain vector

$\{E\}$ = electric field vector

$[c]_{6x6}$ = elasticity matrix (evaluated at constant electric field), symmetric matrix.

$[e]_{6x3}$ = piezoelectric stress matrix

$[\varepsilon]_{3x3}$ = dielectric matrix (evaluated at constant mechanical strain), diagonal matrix.

4.4 Development of Finite Element Model for AE Sensor

A typical ANSYS analysis has three distinct steps:

1. Build the model.
2. Apply loads and obtain the solution.
3. Review the results.

In this chapter, the first step will be discussed in detail and the rest two steps will be left for next chapter. The important steps in building finite element model are as follows.

1. Defining element types
2. Defining material types
3. Creating model geometry.

These three steps will be discussed in the following sections of this chapter.

It should be remembered that, ANSYS has vast capabilities and generalized procedures for building and analyzing finite element models. In this report, however, only those procedural steps and features of ANSYS will be discussed which are specifically required by the problem of modeling **piezoelectric type AE sensors.** This discussion should not be treated as a generalized procedure of developing and analyzing finite element models using ANSYS.

4.5 Defining Element Types

In case of modeling AE sensors using MATLAB, two main parts of AE sensor were considered viz. piezoelectric element and backing element. While modeling AE sensor using ANSYS, also, these two parts have been considered as the main parts of AE sensor. Thus, it is required to specify types of elements for piezoelectric element and backing.

4.5.1 Element Type for Piezoelectric Element

In ANSYS, for performing piezoelectric analysis, one of the following three elements can be used

1. PLANE13 -coupled-field quadrilateral solid
2. SOLID5 -coupled-field brick
3. SOLID98 -coupled-field tetrahedron

As the AE sensor is an axisymmetric structure, in present work it is analyzed as a 2-D axisymmetric model. For details, please refer Section 4.7.1. As among the three elements mentioned above, only PLANE13 can be used for 2-D axisymmetric models, this element type is used for modeling piezoelectric element.

4.5.2 Element Type for Backing

As PLANE13 element can also be used for modeling non-piezoelectric structures, the same element type is used for modeling backing

4.5.3 The PLANE13 Element

PLANE13 has a 2-D magnetic, thermal, electrical, piezoelectric and structural

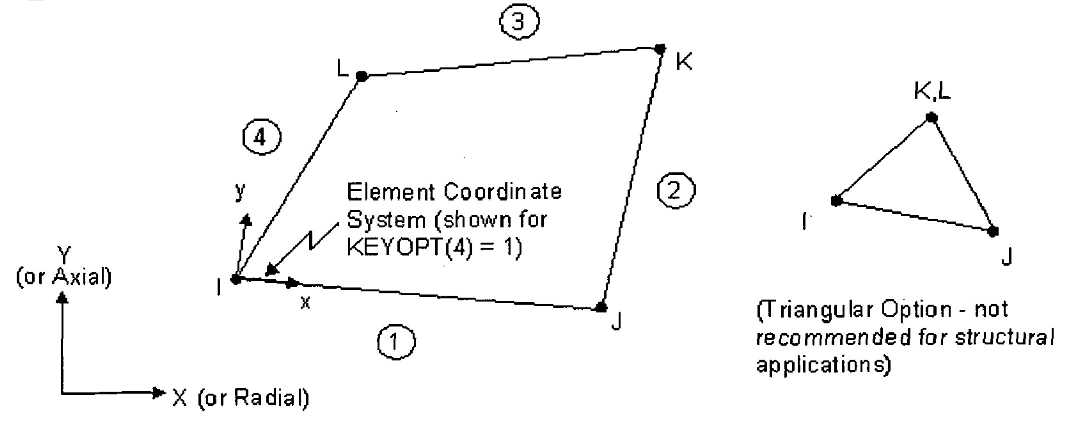


Fig.8. PLANE13, 2-D Coupled-Field solid

field capability with limited coupling between the fields. PLANE13 is defined by four nodes with up to four degrees of freedom per node. PLANE13 has large deflection and stress stiffening capabilities. When used in purely structural analyses, PLANE13 also has large strain capabilities. The geometry, node locations, and the coordinate system for this element are shown in Fig. 8. For more detailed information on PLANE13 element, refer ANSYS 7.0 documentation [12].

KEYOPTs (or key options) are switches, used to turn various element options on or off. KEYOPT options include stiffness formulation choices, printout controls, element coordinate system choices, etc. KEYOPTs are identified by number, such as KEYOPT(1), KEYOPT(2), etc., with each numbered KEYOPT able to be set to a specific value. Different KEYOPT values for PLANE13 element must be used when the element is used for piezoelectric element and backing element. Table 3and Table 4 give various KEOPTs for PLANE13 when it is used for piezoelectric element and backing respectively.

TABLE 3. KEOPTs for PLANE13 when used for piezoelectric element

KEYOPT	Value	Meaning
KEYOPT(1)	7	UX, UY, VOLT degrees of freedom
KEYOPT(2)	0	Include extra shapes (default)
KEYOPT(3)	1	Axisymmetric
KEYOPT(4)	0	Element coordinate system is parallel to the global coordinate system
KEYOPT(5)	0	Basic element printout (default)

TABLE 4. KEOPTs for PLANE13 when used for piezoelectric element

KEYOPT	Value	Meaning
KEYOPT(1)	3	UX, UY degrees of freedom
KEYOPT(2)	0	Include extra shapes (default)
KEYOPT(3)	1	Axisymmetric
KEYOPT(4)	0	Element coordinate system is parallel to the global coordinate system
KEYOPT(5)	0	Basic element printout (default)

It can be observed from Tables 3 and 4 that when PLANE13 is used for piezoelectric element, UX, UY and VOLT degrees of freedom are activated whereas only UX and UY degrees of freedom are activated when it is used for backing.

4.6 Defining Material Types

While developing finite element model for a structure in ANSYS, the user has to specify material properties for various materials used in the model. As for developing model of AE sensor, two material types are used viz. PZT for piezoelectric element and a non-piezoelectric material for backing; the material properties for these two types of materials are required to be specified. Every set of material properties corresponds to one particular material type and is referred to as a material model. Thus, by defining set of material properties for PZT material, a material model is created in ANSYS terminology.

4.6.1 Material Properties for Piezoelectric Element

While supplying material properties for their PZT materials, manufacturers usually follow IEEE standards. However the format for material properties required by ANSYS is different from that of IEEE standards. Thus, suitable conversion is necessary while entering manufacturer-supplied data into ANSYS.

The material properties which are required to be specified for a piezoelectric material are density, piezoelectric matrix, permittivity matrix and stiffness/compliance matrix.

A. Piezoelectric Matrix

One can define the piezoelectric matrix in [e] form (piezoelectric stress matrix) or in [d] form (piezoelectric strain matrix). The [e] matrix is typically associated with the input of the anisotropic elasticity in the form of the stiffness matrix [c], while the [d] matrix is associated with the compliance matrix [s].

In present work, [e] matrix is used to define piezoelectric matrix. For most published piezoelectric materials, the order used for the piezoelectric matrix is x, y, z, yz, xz, xy, based on IEEE standards, while the ANSYS input order is x, y, z, xy, yz, xz. This means that it is required to transform the matrix to the ANSYS input order by switching row data for the shear terms as shown below:

Standard form of [e] matrix:

$$[e]=\begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \\ e_{41} & e_{42} & e_{43} \\ e_{51} & e_{52} & e_{53} \\ e_{61} & e_{62} & e_{63} \end{bmatrix}$$

ANSYS form of [e] matrix:

$$[e]=\begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \\ e_{61} & e_{62} & e_{63} \\ e_{41} & e_{42} & e_{43} \\ e_{51} & e_{52} & e_{53} \end{bmatrix}$$

As it will be explained in Section 4.7.1, 2-D axisymmetric model has been used while developing model for AE sensor consisting of piezoelectric element and backing. Since, for axisymmetric models in ANSYS, the axis of symmetry must coincide with global Cartesian Y-axis, the direction of polarization of PZT material must be Y-direction. Assuming polarization in the 2-direction (Y-direction) and symmetry in the unpolarized directions (1 and 3), so that $e_{23}= e_{21}$ and $e_{34}= e_{16}$ the [e] matrix becomes,

$$[e]^T = \begin{bmatrix} 0 & 0 & 0 & e_{16} & 0 & 0 \\ e_{21} & e_{22} & e_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{16} & 0 \end{bmatrix}$$

B. Permittivity Matrix

The permittivity matrix evaluated at constant strain is input into ANSYS.

The permittivity matrix has only diagonal terms. Again, assuming polarization in the 2-direction (Y-direction) and symmetry in the unpolarized directions (1 and 3), the permittivity matrix becomes,

$$\left[\varepsilon^S\right] = \begin{bmatrix} \varepsilon_{11}^S & 0 & 0 \\ 0 & \varepsilon_{22}^S & 0 \\ 0 & 0 & \varepsilon_{11}^S \end{bmatrix} = \varepsilon_0 \begin{bmatrix} K_{11}^S & 0 & 0 \\ 0 & K_{22}^S & 0 \\ 0 & 0 & K_{11}^S \end{bmatrix}$$

C. Stiffness/Compliance Matrix

In ANSYS, the stiffness matrix can be entered by entering values for elastic moduli (EX, EY and EZ), Poisson's ratios (PRXZ, PRXY and PRYZ) and shear moduli (GXY, GXZ and GYZ).

Assuming polarization in 2-direction, EX=EZ, PRXY =PRYZ and GXY=GYZ.

4.6.2 Material Properties for Backing Material

As the backing material is non-piezoelectric in nature, the only material properties required to be entered are density, Young's modulus and Poisson's ratio. Further, as there is no polarization, the material properties are isotropic (independent of direction).

4.7 Creating Model Geometry

After defining element types and material models, the final step of building the model is model generation or creating model geometry. *Model generation* in this discussion will mean the process of *defining the geometric configuration of the model's nodes and elements.*

There are two methods available in ANSYS to create the finite element model:

1. **Solid modeling:** With *solid modeling,* the geometric shape of the model is described and then ANSYS program is instructed to automatically *mesh* the geometry with nodes and elements. The size and shape of the elements created by ANSYS can be controlled.

2. **Direct generation:** With *direct generation*, the location of each node and the connectivity of each element are defined "manually". Several convenience operations, such as copying patterns of existing nodes and elements, symmetry reflection, etc. are available.

Any of the two approaches can be used for model generation. In present work, the solid modeling is used for generating the finite element model of AE sensor as it is easier for present problem. In the following, the various steps involved in model generation are discussed.

4.7.1 Choosing Model Type

A finite element model in ANSYS may be categorized as being 2-dimensional or 3-dimensional, and as being composed of point elements, line elements, area elements, or solid elements. Of course, one can intermix different kinds of elements as required (taking care to maintain the appropriate compatibility among degrees of freedom).

In present work, a 2-D axisymmetric analysis model is used for generating model for AE sensor.

Why 2-D axisymmetric analysis model?

Any structure that displays geometric symmetry about a central axis (such as a shell or solid of revolution) is an *axisymmetric structure.* Examples would include straight pipes, cones, circular plates, domes, and so forth.

Models of axisymmetric 3-D structures may be represented in equivalent 2-D form. One may expect that results from a 2-D axisymmetric analysis will be more accurate than those from an equivalent 3-D analysis. Moreover, creating 2-D equivalent of a 3-D structure is easier than creating actual 3-D model.

As an AE sensor is cylindrical in shape (circular cross-section with thickness equal to thickness of piezoelectric element plus thickness of backing element), it is an axisymmetric structure. Because of the advantages of 2-D axisymmetric analysis described in previous paragraph, the of 2-D axisymmetric analysis approach has been used for generating model of AE sensor.

Some Special Requirements for Axisymmetric Models

ANSYS imposes some special requirements for axisymmetric models, which are,

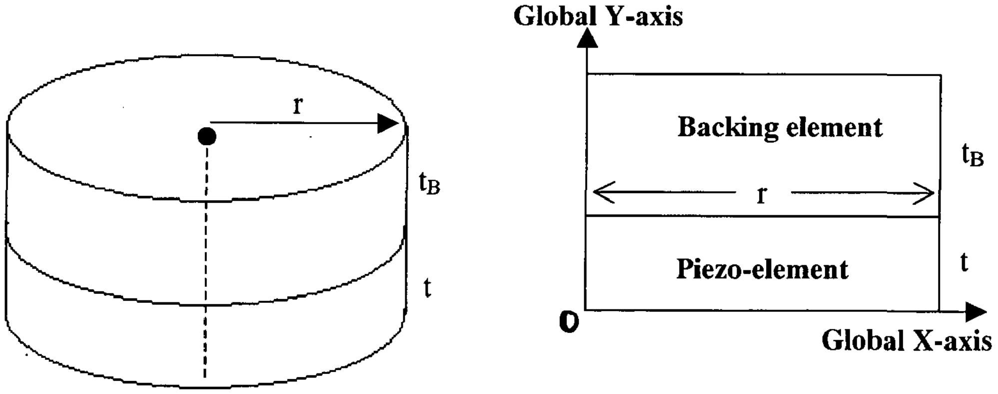
- The axis of symmetry *must* coincide with the global Cartesian Y-axis.
- Negative nodal X-coordinates are not permitted.
- The global Cartesian Y-direction represents the axial direction, the global Cartesian X-direction represents the radial direction, and the global Cartesian Z-direction corresponds to the circumferential direction

4.7.2 Developing Geometry

ANSYS provides many convenient operations for creating model geometry. Fig. 9 shows the basic geometry of the AE sensor elements. Fig. 9(a) shows the 3-D geometry of the sensor elements while Fig. 9(b) shows the 2-D axisymmetric model geometry. It can be seen that, the axis of elements is along the Cartesian global Y-axis while X-axis coincides the radial direction as required for axisymmetric models.

The steps involved in creating geometry are discussed below.

1. Two rectangles with dimensions and positions as shown in Fig. 9(b) are created. ANSYS doesn't allow variable lengths; hence values for r, t and t_B should be used while creating the rectangle. In Chapter 5, the results of analysis for particular problems with predefined values for various geometric parameters will be discussed.



(a) 3-D geometry

(b) 2-D axisymmetric model geometry

Note: Dimensions in (a) and (b) are not proportional

Fig.9. Basic geometry of AE sensor elements

2. In case of AE sensor, the piezoelectric element and backing are perfectly attached so that the waves can pass across the boundary. In Fig. 9(b), the two rectangles seem to be perfectly attached with each other; however, ANSYS treats them as separate. Therefore, it is necessary to attach them so that interaction between them is possible, maintaining their individuality. To achieve this purpose, the Boolean operation "glue" is performed on the two rectangles. The solid model creation completes with this step.
3. The next step, after creating solid model, is meshing. The procedure for generating a mesh of nodes and elements consists of three main steps:
    a) **Set the element attributes:** In present problem, setting element attributes means specifying (i) element type and (ii) material property set (material model) for the two rectangular areas of the model. For the rectangle corresponding to piezoelectric element, the element type is specified as PLANE13 with KETOPTs listed in Table 3 and material model number as 1 which corresponds to material property set defined for piezoelectric element. Similarly, for the rectangle corresponding to piezoelectric element, the element type is specified as PLANE13 with KETOPTs listed in Table 4 and material model number as 2 which corresponds to material property set defined for backing element.
    b) **Set mesh controls (optional):** In present problem, the mesh controls will be used for setting element size. Thus, if we select one of the two horizontal lines of piezo-element rectangle and specify the number of element divisions as 10, there will be 10 elements along that line.
    c) **Generate the mesh:** The model can be meshed as a free mesh or mapped mesh. A *free* mesh has no restrictions in terms of element shapes, and has no specified pattern applied to it. Compared to a free mesh, a *mapped* mesh is restricted in terms of the element shape it contains and the pattern of the mesh. A mapped area mesh contains either only quadrilateral or only triangular elements. As our model contains only rectangular areas, it has been observed that, even free meshing generates rectangular elements.

Therefore the default meshing option, *free mesh*, has been used for meshing the model.

After meshing the model, the task of creating finite element model of the AE sensor completes which, as stated in Section 4.4, completes the first step of finite element analysis of AE sensors using ANSYS.

Fig. 10 shows 2-D axisymmetric model for AE sensor with geometric parameters listed in Table 5. Fig. 11 shows same model after performing 'symmetry expansion' for the 2-D axisymmetric model.

Table 5. Geometric parameters for AE sensor model

Piezoelectric Element	**Backing**
Thickness (t) = 0.5mm Radius (r) = 6mm	Thickness (t_B) = 0.5mm Radius (r) = 6mm
Element Size Control	
Horizontal lines: 2 elements/mm	Vertical lines: 4 elements/mm

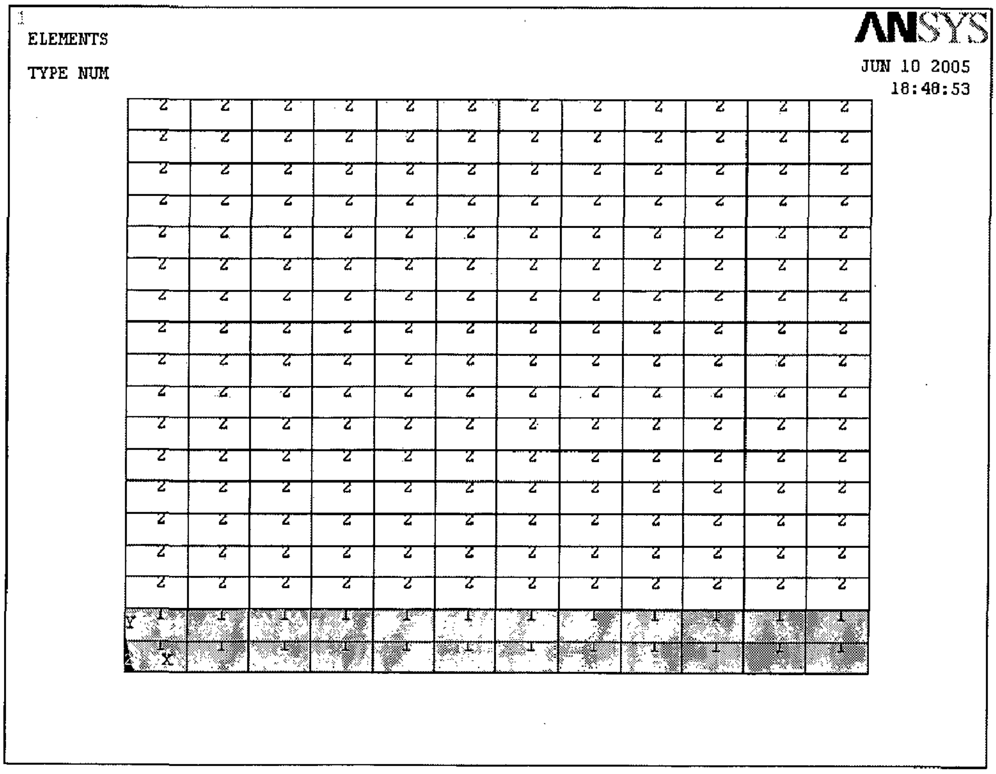


Fig.10. 2-D axisymmetric model for AE sensor

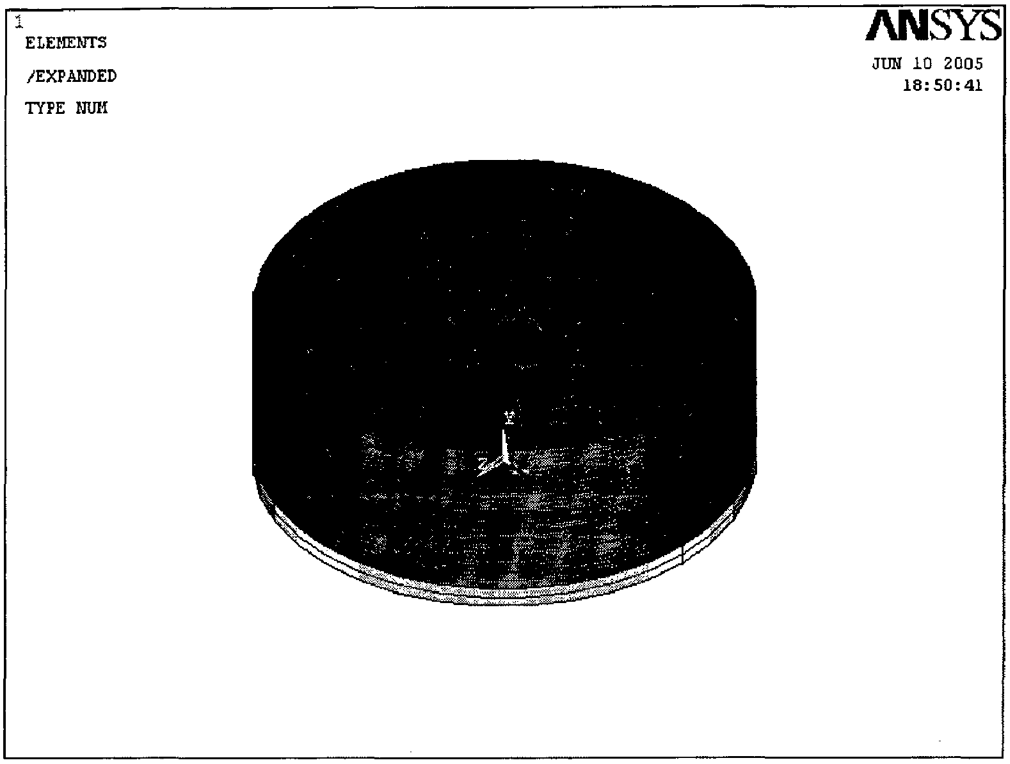


Fig.11. 'Symmetry Expanded' model for the 2-D axisymmetric model of Fig. 10

In Fig. 10, the number (1 or 2) inside each element corresponds to the element type number of the element. The element type 1 is used for piezoelectric element and type 2 is used for backing.

Chapter 5

RESULTS AND DISCUSSIONS

The purpose of developing a simulation program for generating various performance characteristics of AE sensors is to develop a tool for designing AE sensors. Also, by knowing the performance characteristics of the AE sensor, one can decide the suitability of a particular sensor for a particular application. The MATLAB programs (the Simulation Program and Design Program discussed in Chapter 3) can be used for this purpose.

The purpose of developing model of AE sensor in ANSYS and generating same performance characteristics as those generated by the MATLAB program, is to verify the characteristics generated by the MATLAB program. As discussed in Chapter 4, ANSYS is a commercial software with the capability to analyze a wide range of engineering problems and it is widely used and accepted by engineers and scientists for verifying or predicting behavior of practical systems. Secondly the method used by ANSYS to solve the problems is finite element method whereas the MATLAB programs are developed using an equivalent circuit model for AE sensor (please refer Chapter 3 for details). Thus the two approaches used for generating performance characteristics are completely different and if the results obtained by these methods match, it can be said that the performance characteristics are accurate, at least theoretically. However, validation of the performance curves by performing experiments on actual AE sensors is must; though it is not covered in the scope of present work.

The advantages of using MATLAB programs for generating performance characteristics of AE sensors are,

- User friendly GUIs are provided for interaction with the programs,
- For generating performance characteristics using MATLAB programs for various sensors, the user has to enter only material and physical properties of sensor elements viz. piezoelectric element and backing. On the other hand, in case of ANSYS, one has to develop and analyze separate models for sensors with different geometries, piezoelectric elements and backing materials.

- The execution time for the MATLAB programs is very less as the program involves finding values for various quantities using simple mathematical expressions discussed in Chapter 3. ANSYS takes more time to find the solution because of the nature of finite element method. Further, one must consider the time required for developing and analyzing models for different sensors using ANSYS.

In Section 4.4, three main steps involved in performing a typical ANSYS analysis were mentioned. The first step i.e. building the model of AE sensor was discussed in detail in Chapter 4. The discussion on later two steps i.e. applying loads and obtaining the solution and reviewing the results was postponed till the present chapter. Therefore, in the next section, these steps will be discussed. After that two cases of AE sensors having different geometries and/or piezoelectric materials and/or backing materials will be taken and; for each sensor, will be presented the performance characteristics viz. pressure sensitivity, velocity sensitivity, displacement sensitivity and impedance curves generated using MATLAB program (Simulation Program discussed in Section 3.7) and ANSYS in order to make a comparative study. Finally, the results of Design Program (MATLAB Design Program discussed in Section 3.8) will be presented.

5.1 Application of Loads and Generation of Characteristics in ANSYS

The word *loads* in ANSYS terminology includes boundary conditions *and* externally or internally applied forcing functions such as pressure, displacement, voltage etc.

After applying suitable loads, one has to obtain the solution. In case of all the performance characteristics, it is desired to plot the variation of the quantity of interest (sensitivity or impedance) with frequency; therefore the type of analysis used in ANSYS is **'harmonic analysis'**. When harmonic analysis is performed, ANSYS treats all the loads harmonically (sinusoidally) varying. While performing harmonic analysis, one has to specify the frequency range for the loads and the number of substeps i.e. the number of points within the specified frequency range at which the solution is to be obtained. After specifying these options, the solution is obtained by issuing 'SOLVE' command to ANSYS program.

After the solution is done, one can obtain the various characteristics of interest by **postprocessing**. For obtaining frequency response curves, ANSYS postprocessor, POST26 is used. Using this postprocessor, one can specify the output quantity of interest and the location (node or element) at which it should be measured. The values of quantity (magnitude and phase or real and imaginary parts) corresponding to all the frequency points are stored in a vector. The contents of this vector can be viewed by list options or can be used to calculate some other quantity of interest. The quantity can also be plotted against some other quantity such as frequency.

For more details on applying loads, specifying analysis options (such as harmonic analysis, static analysis etc.), obtaining solution and postprocessing, please refer ANSYS 7.0 documentation [12].

Fig. 12 shows the model of AE sensor developed using ANSYS. The same model which was presented in Section 4.7.2 is shown in Fig. 12. For different sensor geometries and different element sizes, the model will be different; however the method of applying loads and getting solution remains same.

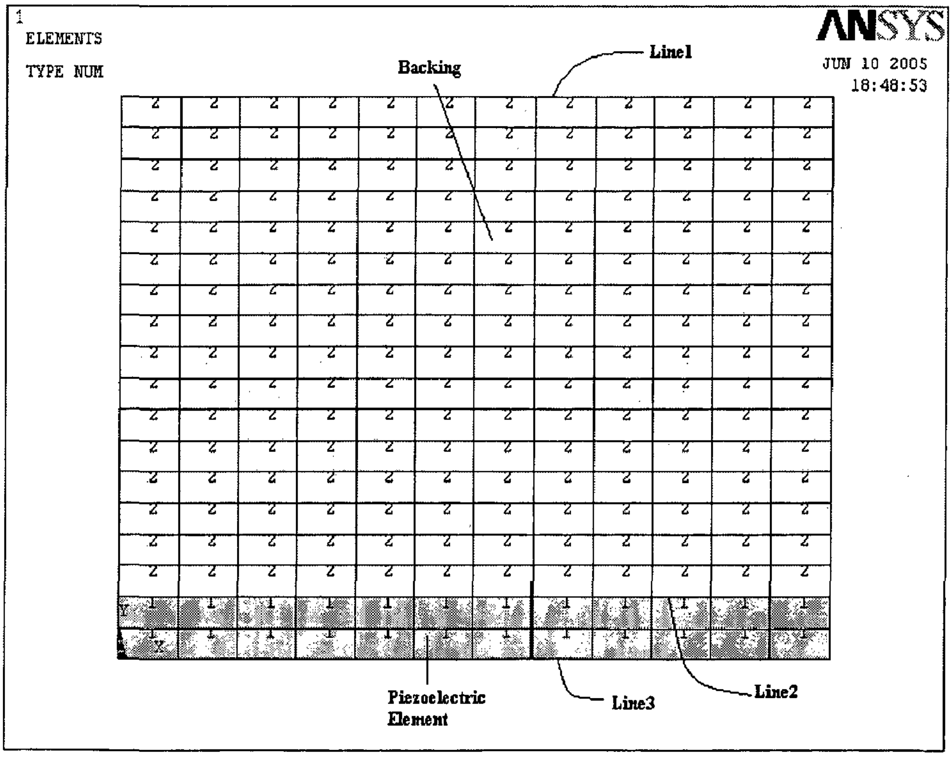


Fig.12. Finite element model of AE sensor

5.1.1 Boundary Conditions

For generation of all types of characteristics i.e. various sensitivity characteristics and impedance characteristics, the boundary conditions will remain same.

In Section 3.2.2, two possible boundary conditions were discussed for the upper backing face which is represented by Line1 in Fig. 12. This face of backing can freely vibrate or can be rigidly held so that its movement in any direction is restricted. In present analysis the later condition is assumed. For specifying this boundary condition in ANSYS, the Line1 is selected and the degrees of freedom for this line are set to zero.

While measuring voltage output of the sensor, the lower face of piezoelectric element acts as electrical ground and the voltage generated at the electrode attached to the face common between backing and piezoelectric element is taken as output voltage. Therefore, the lower face of piezoelectric element corresponding to Line3 in Fig. 12 is grounded. For specifying this boundary condition in ANSYS, the Line3 is selected and the voltage degree of freedom for this line is set to zero.

5.1.2 Generating Pressure Sensitivity Characteristics in ANSYS

The pressure sensitivity of AE sensors is usually specified in the units of volts/microbar. Therefore a pressure of 1 microbar (0.1 Pa) is applied to the lower face of sensor corresponding to Line3. For specifying this load, Line3 is selected and the pressure is applied. The generated voltage is measured at Line2. For measuring output voltage, using pstprocessor POST26, values of voltages are calculated at all the nodes on Line2 and the average value of these voltages is considered as the output voltage. As this voltage corresponds to 1 microbar pressure input, it directly gives the pressure sensitivity of AE sensor. The pressure sensitivity is converted into decibels and then plotted against frequency.

5.1.3 Generating Displacement Sensitivity Characteristics in ANSYS

The displacement sensitivity of AE sensors is usually specified in the units of volts/micrometer. Therefore a displacement of 1 μm is applied to the lower face of sensor corresponding to Line3. For specifying this load, Line3 is selected and the displacement is applied. The generated voltage is measured at Line2. The method used for calculating output voltage is same as that used in case of pressure sensitivity characteristics. Again, as this voltage corresponds to 1 . m displacement input, it directly gives the displacement sensitivity of AE sensor. The displacement sensitivity is converted into decibels and then plotted against frequency.

5.1.4 A comment on Velocity Sensitivity Characteristics

In ANSYS, velocity is derived from displacement by differentiating it. Thus, the velocity sensitivity contains no more information than displacement sensitivity. It should be recalled that, a similar approach has been used while deriving expression for displacement sensitivity in Section 3.5.3. Therefore, velocity sensitivity curves have not been generated in ANSYS. However, to give an idea about the nature of velocity sensitivity curves, these curves generated using MATLAB Simulation Program will be presented.

5.1.5 Generating Impedance Characteristics in ANSYS

For generating impedance characteristics, a sinusoidal voltage of 1 V is applied at Line2 and the current flowing through the piezoelectric element is measured. In ANSYS, current flow is an element solution as opposed to voltage which is a nodal solution. For calculating current flow, all the elements in the uppermost row of piezoelectric element are selected and average current flow through these elements is calculated. As this current flow corresponds to an applied voltage of 1 V, it directly gives the admittance of the sensor. The impedance is obtained by taking inverse of admittance. The log value of magnitude of impedance is plotted against frequency.

5.2 Results for MATLAB Simulation Program and ANSYS Model

As stated earlier in this chapter, two cases of AE sensors will be considered which are having different geometries and/or piezoelectric materials and/or backing materials. The results for these models will be presented separately. Table 6 gives the piezoelectric element type, backing material and the geometry of sensor for each case.

The material properties of piezoelectric elements and backing materials for both the cases are presented in Tables 7 and 8 respectively. For PZT materials, the polarization is assumed in direction 2 (Y-direction).

Table 6. Sensor specifications for the two cases

	Piezoelectric Element		Backing Material		Diameter (mm)
	Material	Thickness (mm)	Material	Thickness (mm)	
Case I	PZT2	1.5	Aluminium	5	12
Case II	PZT4	1	Copper	5	12

Table 7. Properties of piezoelectric materials (all quantities in SI units)

Property	Notation	PZT 2	PZT 4
Density	ρ	7600	7500
Dielectric constant	PERX=PERZ	504	730
	PERY	260	635
Piezoelectric matrix [e]	$e_{23}= e_{21}$	-1.86	-5.2
	e_{22}	9.0	15.1
	$e_{34}= e_{16}$	9.8	12.7
Stiffness Matrix	EX=EZ	86.207e9	81.3e9
	EY	67.5675e9	64.5161e9
	PRXZ	0.287	0.3292
	PRXY =PRYZ	0.3358	0.3426
	GXZ	33.4448e9	30.581e9
	GXY=GYZ	22.2222 e9	25.6410 e9
h-coefficient	h	3.92e9	2.68e9
Sound velocity	v_s	4410	4600

Table 8. Properties of backing materials (all quantities in SI units)

Material	Young's Modulus (E)	Density (ρ)	Poisson's Ratio (ν)	Sound Velocity (v_s)
Aluminium	68.95e9	2700	0.335	6208.7
Copper	117.2e9	8960	0.365	4747.3

5.2.1 AE Sensor Characteristics for Case I

Pressure Sensitivity Curves

Fig. 13(a) shows pressure sensitivity curve for present case AE sensor generated using ANSYS and Fig. 13(b) shows the same curve generated using MATLAB Simulation Program. The similarity between the curves is quite clear. For both the curves, maximum peak occurs near 180 kHz frequency and minimum peak occurs near 220 kHz frequency. Similarly, the values of pressure sensitivity at the peaks are also very close.

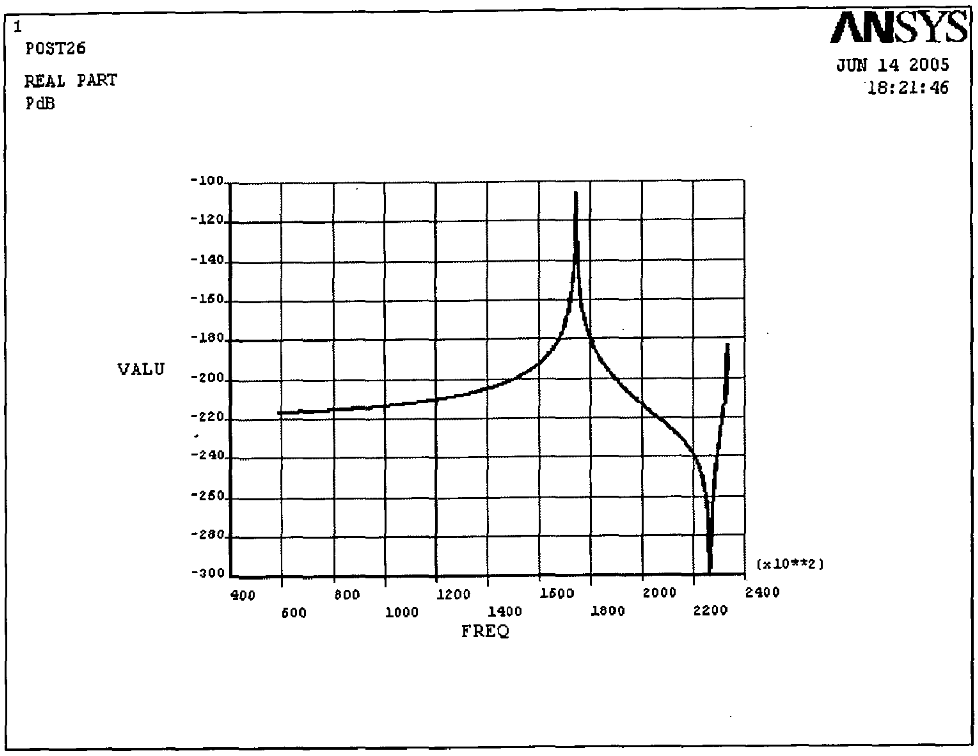


Fig. 13 (a) Pressure sensitivity curve for Case I (ANSYS)

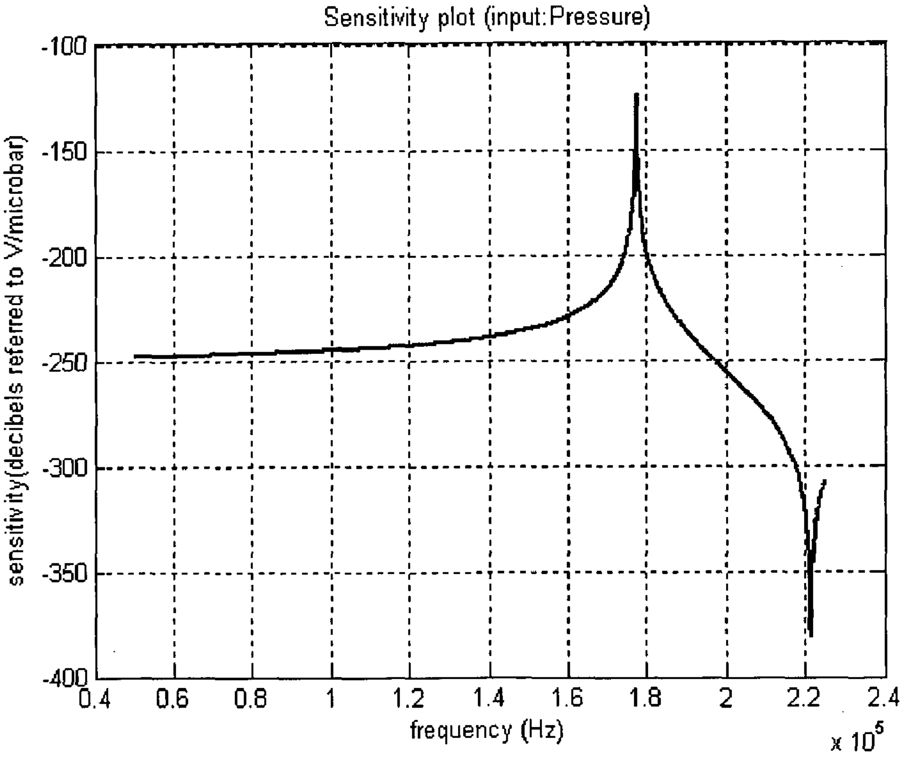


Fig. 13 (b) Pressure sensitivity curve for Case I (MATLAB)

Displacement Sensitivity Curves

Fig. 14(a) shows displacement sensitivity curve for present case AE sensor generated using ANSYS and Fig. 14 (b) shows the same curve generated using MATLAB. The curves are quite similar. For both the curves, the frequencies corresponding to maximum peaks and minimum peaks can be observed to be matching. Similarly, the values of displacement sensitivity at the peaks are also very close.

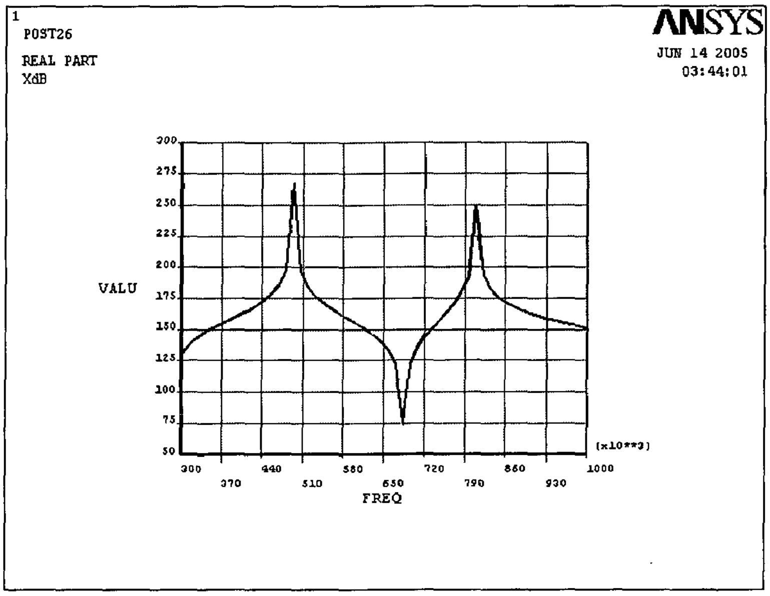


Fig.14 (a) Displacement sensitivity curve for Case I (ANSYS)

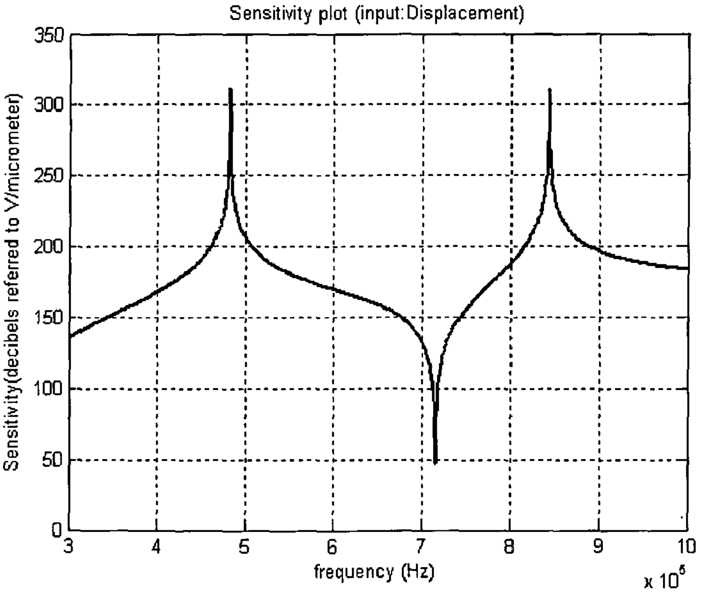


Fig.14 (b) Displacement sensitivity curve for Case I (MATLAB)

Impedance Curves

Fig. 15(a) shows impedance curve for present case AE sensor generated using ANSYS and Fig. 15(b) shows the same curve generated using MATLAB. The similarity between the curves is quite clear. For both the curves, the frequencies corresponding to maximum peaks and minimum peaks can be observed to be closely matching. Similarly, the impedance values at the peaks are also very close.

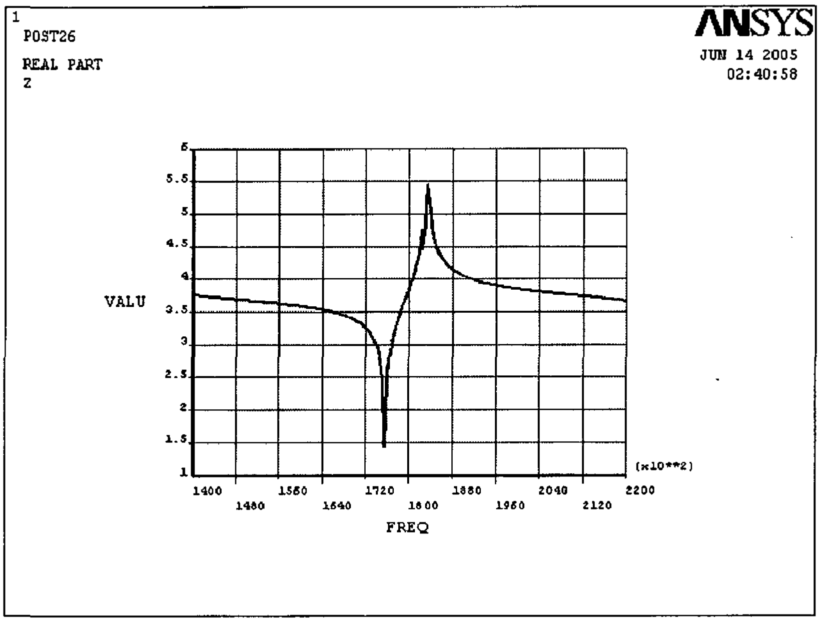


Fig.15 (a) Impedance curve for Case I (ANSYS)

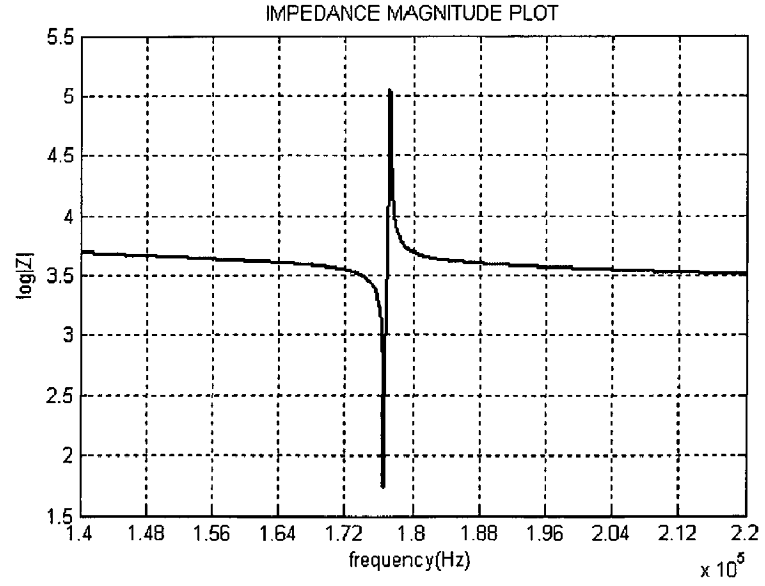


Fig.15 (b) Impedance curve for Case I (MATLAB)

5.2.2 AE Sensor Characteristics for Case II

Pressure Sensitivity Curves

Fig. 16(a) shows pressure sensitivity curve for present case AE sensor generated using ANSYS and Fig. 16(b) shows the same curve generated using MATLAB. It can be observed that though the minimum peaks of the two curves are matching, the maximum peaks are found to be slightly at different frequencies. However the two curves are similar in nature.

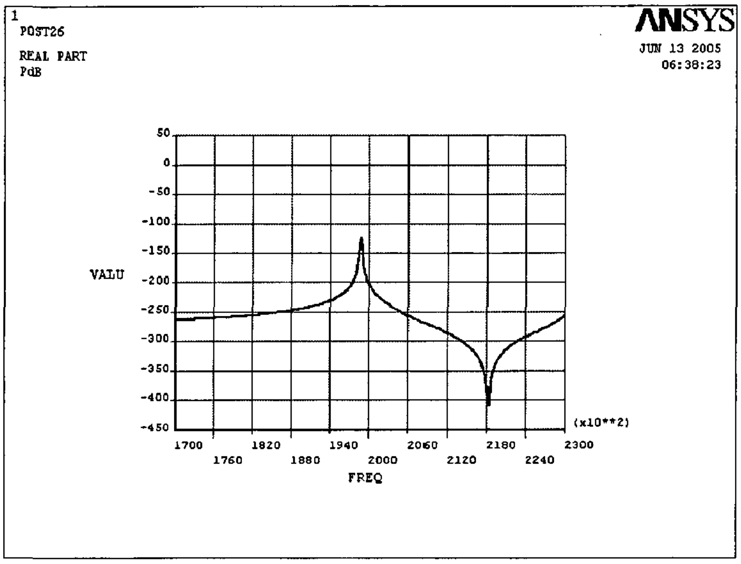


Fig.16 (a). Pressure sensitivity curve for Case II (ANSYS)

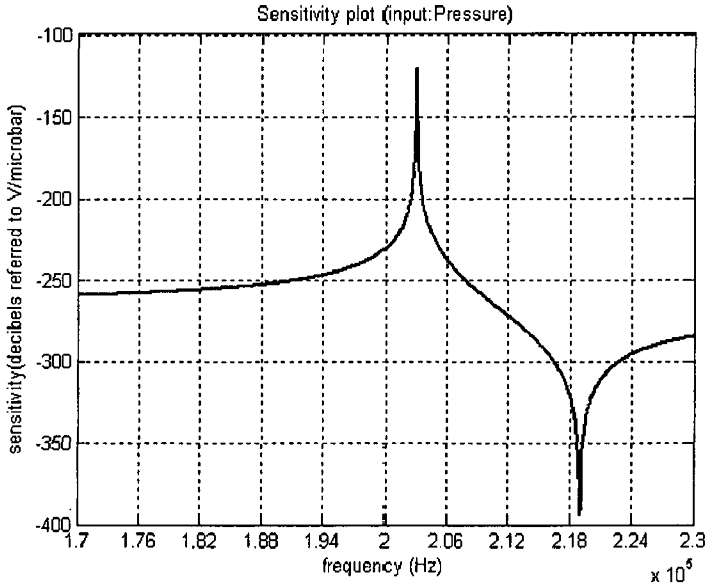


Fig.16 (b). Pressure sensitivity curve for Case II (MATLAB)

Displacement Sensitivity Curves

Fig. 17(a) shows displacement sensitivity curve for present case AE sensor generated using ANSYS and Fig. 17(b) shows the same curve generated using MATLAB. It can be observed that, in Fig. 17(a), extra maximum-minimum peaks are present whereas the curve in Fig. 17(b) is smooth on right side. However, the major dip in the two curves and the left parts of the curves are matching and the overall curves are similar.

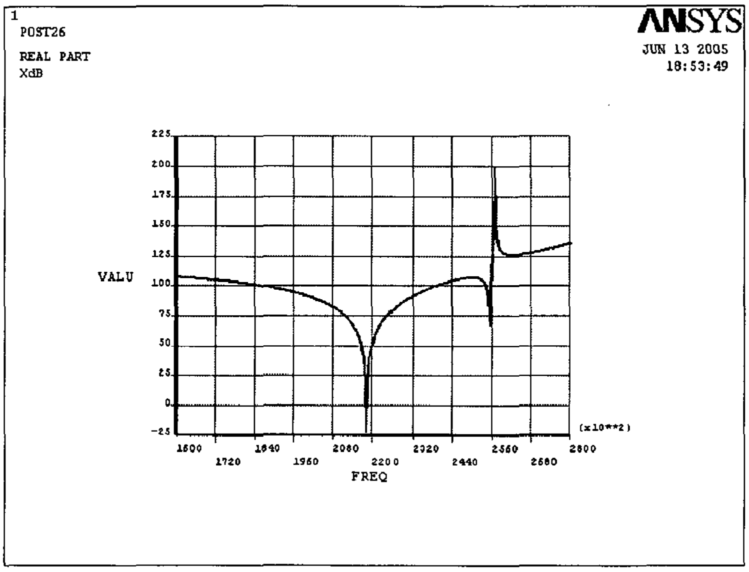


Fig.17 (a) Displacement sensitivity curve for Case II(ANSYS)

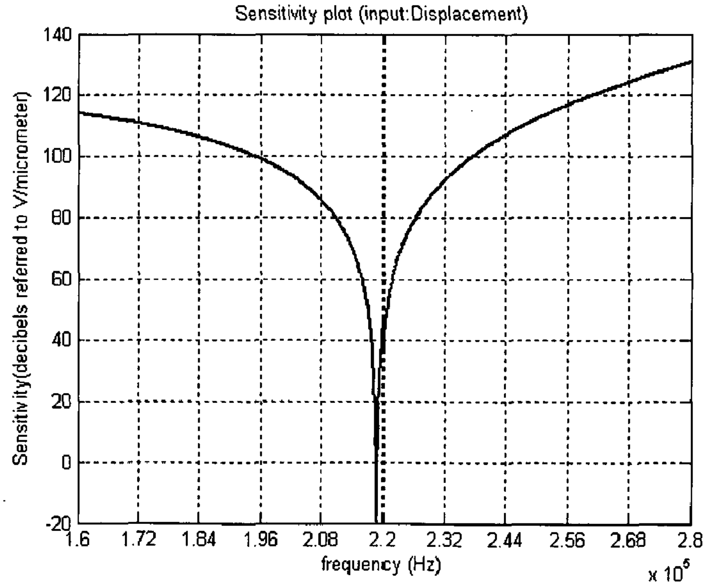


Fig.17 (b) Displacement sensitivity curve for Case II(MATLAB)

Impedance Curves

Fig. 18(a) shows impedance curve for present case AE sensor generated using ANSYS and Fig. 18(b). shows the same curve generated using MATLAB. It can be observed that for the curve in Fig. 18(a), the difference in frequencies corresponding maxima and minima is more than that of the curve in Fig. 18(b). However, the average values for the curve and the frequencies at which the peaks occur are close. Henceit can be said that the two curves are similar.

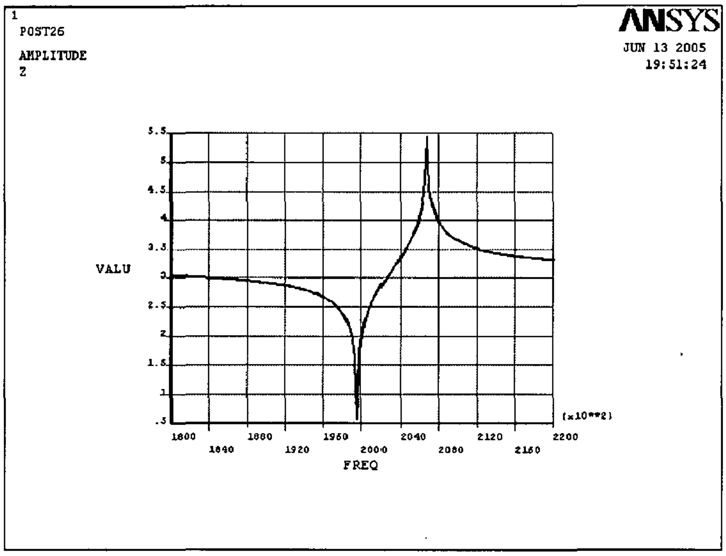


Fig.18 (a) Impedance curve for Case II (ANSYS)

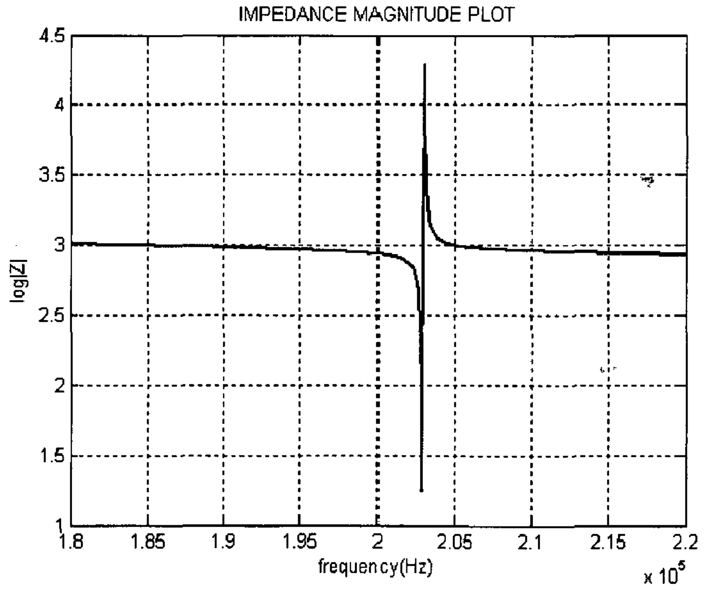


Fig.18 (b) Impedance curve for Case II (MATLAB)

5.2.3 Velocity Sensitivity Curves

As stated in Section 5.1.4, the velocity sensitivity curves for the two cases are presented in Fig. 19 for the two cases of sensors discussed previously. These curves are generated using MATLAB Simulation Program.

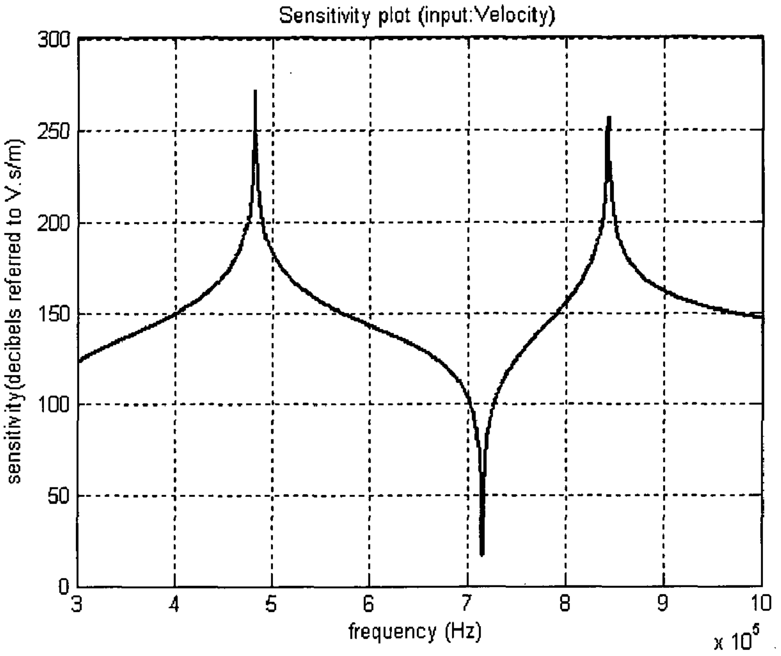


Fig.19 (a) Velocity sensitivity curve for Case I (MATLAB)

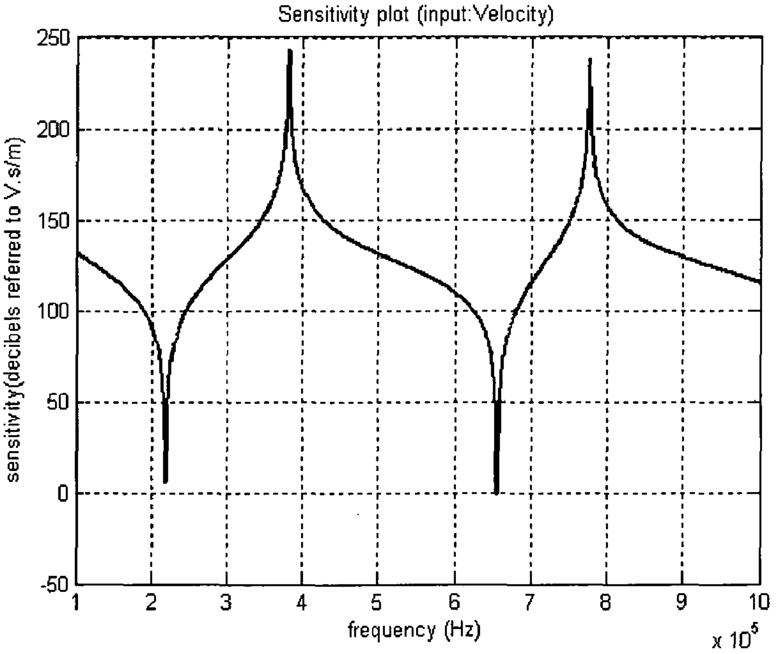


Fig.19 (b) Velocity sensitivity curve for Case II (MATLAB)

5.3 Results for MATLAB Design Program

The MATLAB Design Program (discussed in Section 3.8) can be used to maximize pressure sensitivity or bandwidth (for pressure sensitivity curve). The Design Program accepts range of values for each of the design parameters and frequency range of interest; and outputs the design parameters (material properties and dimensions of sensor elements) and the corresponding frequency response curve. The results of this program are listed in Table 9 for a particular set of ranges. Figures 20 and 21 show the pressure sensitivity versus frequency plots for the optimum set of parameters for the two cases i.e. maximizing sensitivity and maximum bandwidth.

The values of cross-over rate and mutation rate used for genetic algorithm were 0.7 and 0.01 respectively. The input frequency range was 1-1.2 MHz.

Table 9. Input ranges and output values for AE sensor Design Program

Parameter	Input Range	Value for Maximizing Sensitivity	Value for Maximizing Bandwidth
h-coefficient (x 10^9V/m)	2.65- 2.7	2.6996	2.6760
Density of piezoelectric material (ρ, kg/m^3)	7450 - 7550	7478.739	7513.6363
Sound velocity in piezoelectric material	4550-4650	4559.188	4590.6647
Thickness of piezo-electric element (mm)	0.9-1.1	1.05	0.9598
Density of backing material (ρ, kg/m^3)	8550-8650	8584.213	8613.63
Sound velocity in backing material (m/s)	4650-4750	4693.69	4706.989
Thickness of backing element (mm)	0.9-1.1	0.968	1.016

In Fig. 20, which corresponds to the set of parameter values for maximizing sensitivity, the peak value of sensitivity is -99.98 dB and it occurs at 1.1388 MHz with bandwidth 2 kHz whereas for Fig. 21, which corresponds to the set of parameter values for maximizing bandwidth, the peak value of sensitivity is -108.79 dB and it occurs at 1.1764 MHz with bandwidth 5 kHz. It can be noticed that, the peak sensitivity has been reduced while increasing bandwidth. Also, it can be observed form Table 9 that the specific acoustic impedance (ρ x v_s) of backing for maximizing sensitivity is 40.29 MRayl whereas it is 40.55MRayl for maximizing bandwidth which is higher in value.

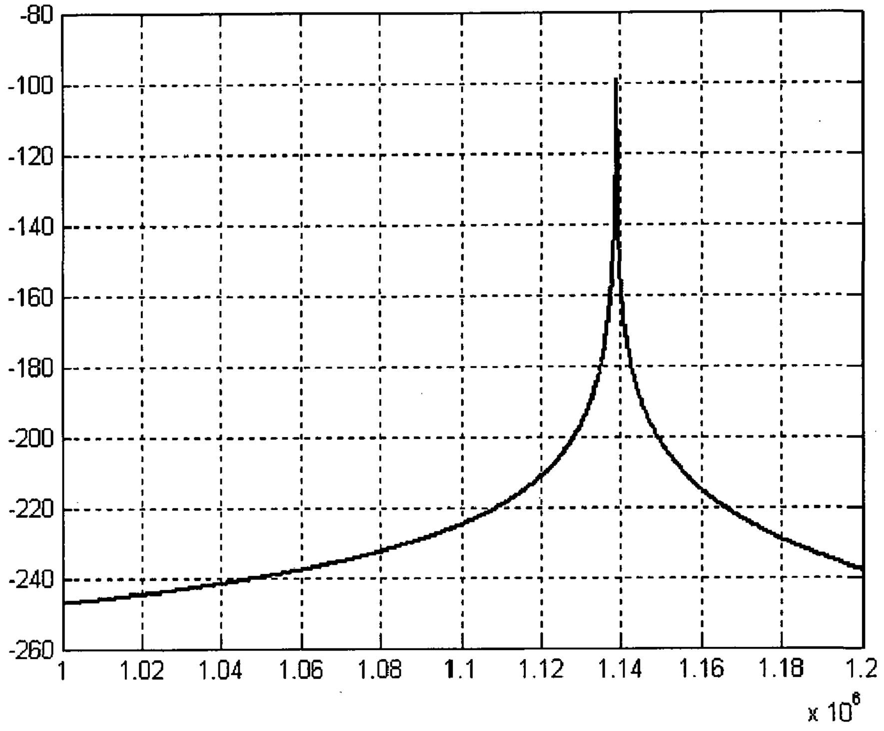


Fig.20. Pressure sensitivity curve corresponding to maximum sensitivity

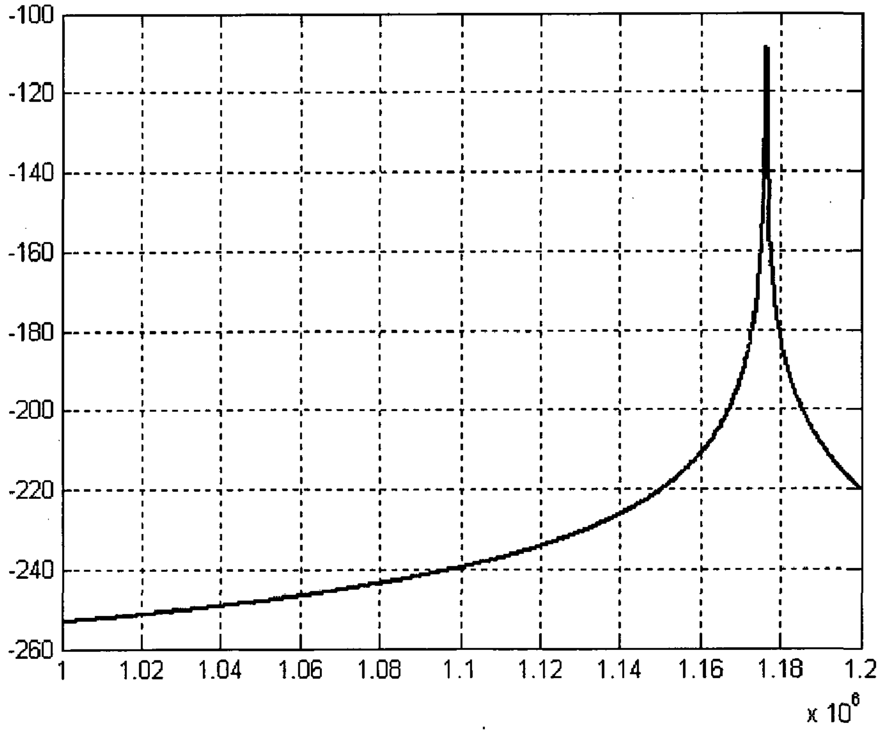


Fig.21. Pressure sensitivity curve corresponding to maximum bandwidth

Chapter 6

CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

An acoustic emission sensor consists of several parts such as piezoelectric element, backing material, electrodes for taking electrical output signal from the sensor, the adhesive layers (used to develop contacts between piezoelectric element, backing and electrodes) wear plate to protect piezoelectric element from direct exposure to external surface etc. Among all these parts, the important parts of AE sensor which mainly decide the performance of the sensor are piezoelectric element and backing material.

In present dissertation work, an electrical equivalent circuit model has been developed using Mason's equivalent circuit for piezoelectric element in thickness extensional mode and acoustic transmission line circuit model for backing layer. Mathematical expressions for pressure sensitivity, velocity sensitivity, displacement sensitivity and electrical impedance of sensor have been derived by analyzing the equivalent circuit model of AE sensor. Using these expressions, a simulation program has been developed using MATLAB which can be used to generate the performance characteristics of AE sensor which exhibit the variation of sensitivity and impedance of sensor with frequency. To verify the results of this simulation program, a Finite Element Model of AE sensor has been developed using the commercial finite element analysis software ANSYS (version 7.0). The same performance curves have been generated using ANSYS and a comparative study of the results obtained by MATLAB simulation program and ANSYS model have been carried out. The results were observed to be confirming with each other. Thus it can be concluded that the equivalent circuit model approach can be used to develop a comprehensive model for AEsensors and can be used to simulate various performance characteristics of AE sensors. Further, the simulation programs such as one developed in this work using MATLAB can be used effectively as tools for designing AE sensors; as using such programs, one can predict the dependence of characteristics of AE sensors on various material and physical properties of constituent elements of the sensor.

6.2 Future Work

As mentioned in Section 1.5, very less work has been done previously in the area modeling acoustic emission sensors. The present work can be considered as a foundation work for future developments in this field of research.

The present work attempts to model the effect of various material and physical properties of piezoelectric element and backing layer on the performance of AE sensors. However, as stated in Section 6.1, an AE sensor consists of many other parts such as wear plate, electrodes, adhesive layers etc. Thus a more comprehensive model can be developed to include effects of properties of these elements of AE sensors.

Secondly, in present work, the verification of model has been done by developing a model in a standard FE analysis software ANSYS. Thus the method of verification is theoretical and it is necessary to verify the results of simulation programs by performing experiments on actual AE sensors.

Further, in present work, the input signals i.e. pressure, displacement or velocity are assumed to be applied directly to the sensor face; however in practice these signals are result of acoustic wave fields generated due to phenomenon of acoustic emission. The source of acoustic emission is present in the test structure. Thus, an AE sensor is essentially mounted on a test structure and its output must be affected by the properties of the structure on which it is mounted. To study the effect of interaction between test surface and AE sensor on the performance characteristics of AE sensor, a model must be developed for AE source in a test structure and a combine study of this model and AE sensor model must be carried out. However, this work is not a subject of simple analysis and would require considerable time and resources for its successful completion.

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Appendix A

PIEZOELECTRICITY

As in this report, we have frequently used terms specifically related with piezoelectric effect; this appendix has been provided to give some basic information about piezoelectric phenomenon.

Piezoelectricity (from: *pressure electricity*) is a property of certain classes of crystalline materials including natural crystals of Quartz, Rochelle Salt and Tourmaline plus manufactured ceramics such as Barium Titanate and Lead Zirconate Titanates (PZT).

When mechanical pressure is applied to a piezoelectric material, the crystalline structure produces a voltage proportional to the pressure. Conversely, when an electric field is applied, the structure changes shape producing dimensional changes in the material.

The piezoelectric effect for a given item depends on the type of piezoelectric material and its mechanical and electrical axes of operation. These axes are set during "poling"; the process that induces piezoelectric properties in the ceramic. The orientation of the dc poling field determines the orientation of the mechanical and electrical axes.

These axes are identified by numerals: (refer Figure A-1)

1 corresponds to X axis,

2 corresponds to Y axis,

3 corresponds to Z axis.

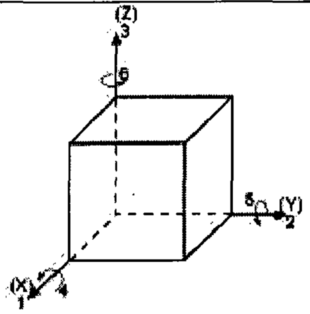

Figure A-1. Axes of piezoceramic

The direction of the poling field is usually identified as the 3 direction. However, while developing model of AE sensor in ANSYS, the axis of symmetry of piezoelectric element was Y axis, hence the poling direction was assumed to be direction 2 or Y direction. 4, 5 and 6 refer to shear strains along 1, 2 and 3 directions respectively.

A.1. Piezoelectric Constants

A set of constants is defined for piezoelectric materials which relate the externally applied electrical field and the mechanical stress/strain produced in the piezoelectric crystal or the electrical polarization produced due to external application of stress/strain to the crystal. These constants are d-constant, e-constant, g-constant and h-constant. The definitions of these constants are summarized in Table A-1.

Table A-1. Definitions of piezoelectric constants

Constant	Symbol	Definition	Unit
d-constant	d_{ij}	$\frac{\text{Charge density developed in direction i}}{\text{Stress applied in direction j}}$	C/N
		$\frac{\text{Mechanical strain developed in direction i}}{\text{applied electric field in direction j}}$	m/V
e-constant	e_{ij}	$\frac{\text{Charge density developed in direction i}}{\text{Strain applied in direction j}}$	C/m^2
		$\frac{\text{Mechanical stress developed in direction i}}{\text{Applied electric field in direction j}}$	V·N/m
g-constant	g_{ij}	$\frac{\text{Electric field developed in direction i}}{\text{Applied mechanical stress in direction j}}$	V·m/N
		$\frac{\text{Mechanical strain developed in direction i}}{\text{Applied charge density in direction j}}$	m^2/C
h-constant	h_{ij}	$\frac{\text{Electric field developed in direction i}}{\text{Strain applied in direction j}}$	V/m
		$\frac{\text{Mechanical stress developed in direction i}}{\text{Applied charge density in direction j}}$	N/C

A.2 Permittivity

In electromagnetism, the **permittivity** . of a medium is the ratio **D** / **E** where **D** is the electric displacement in coulombs per square meter (C/m^2) and **E** is the electric field strength in volts per meter (V/m). In the common case of an *isotropic* medium, **D** and **E** are parallel and ε is a scalar, but in more general *anisotropic* media this is not the case and ε is a rank-2 tensor (causing birefringence).

Permittivity is specified in farads per meter (F/m). It can also be defined as a dimensionless **relative permittivity**, or **dielectric constant (K)**, normalized to the absolute vacuum **permittivity** $\varepsilon_0 = 8.854\ 10^{-12}$F/m.

ε_{33}^{T}	permittivity for dielectric displacement and electric field in direction 3 (parallel to the direction in which ceramic element is polarized), under constant stress. It is also known as free permittivity.
ε_{33}^{S}	permittivity for dielectric displacement and electric field in direction 3 (parallel to the direction in which ceramic element is polarized), under constant strain. It is also known as clamped permittivity.

A.3 Elastic Compliance

Elastic compliance, s, is the strain produced in a piezoelectric material per unit of stress applied and, for the 11 and 33 directions, is the reciprocal of the modulus of elasticity (Young's modulus, Y). s^D is the compliance under a constant electric displacement; s^E is the compliance under a constant electric field. The first subscript indicates the direction of strain; the second is the direction of stress

A.4 Young's Modulus

Young's modulus, Y, is an indicator of the stiffness (elasticity) of a ceramic material. Y is determined from the value for the stress applied to the material divided by the value for the resulting strain in the same direction.

Appendix B

MATERIAL PROPERTIES FOR PIEZOELECTRIC AND BACKING MATERIALS

In Section 3.7, it was mentioned that two lists have been provided on GUI of MATLAB Simulation Program for selecting piezoelectric material and backing material. This appendix provides material properties of the various materials available in the lists.

B-1. Properties of PZT Materials

Table B-1 provides the various piezoelectric materials available in the list along with their material properties. For piezoelectric materials (PZT ceramics), the direction of polarization is assumed to be direction-2 (Y-axis).

Table B-1. Properties of PZT materials

Property	Notation	PZT 2	PZT 4	PZT 5A	PZT 5H
Density	.	7600	7500	7750	7500
Dielectric constant	PERX=PERZ	504	730	916	1700
	PERY	260	635	830	1470
Piezoelectric matrix [e]	$e_{23}= e_{21}$	-1.86	-5.2	-5.4	-6.55
	e_{22}	9.0	15.1	15.8	23.3
	$e_{34}= e_{16}$	9.8	12.7	12.3	17.0
Stiffness Matrix	EX=EZ	86.207e9	81.3e9	60.975e9	60.6061e9
	EY	67.5675e9	64.5161e9	53.1914e9	48.3091e9
	PRXZ	0.287	0.3292	0.35	0.2897
	PRXY =PRYZ	0.3358	0.3426	0.3840	0.4082
	GXZ	33.4448e9	30.581e9	22.5733 e9	23.4741 e9
	GXY=GYZ	22.2222 e9	25.6410 e9	21.0526 e9	22.9885 e9

In Table B-1, the meaning of various notations is as follows:

PERX, PERY, PERZ = Dielectric constants in X, Y and Z directions respectively.

EX, EY, EZ = Young's moduli in X, Y and Z directions respectively.

PRXY, PRYZ, PRXZ = Poisson's ratios in XY, YZ and XZ directions respectively.

GXY, GYZ, GXZ = Shear moduli XY, YZ and XZ directions respectively.

From above table, only e_{23}, e_{21}, e_{22}, e_{34} and e_{16} are nonzero and rests of the elements of e-matrix are **zero** hence, ANSYS e-coefficient matrix is given by,

$$[e]=\begin{bmatrix} e_{11} & e_{21} & e_{31} \\ e_{12} & e_{22} & e_{32} \\ e_{13} & e_{23} & e_{33} \\ e_{16} & e_{26} & e_{36} \\ e_{14} & e_{24} & e_{34} \\ e_{15} & e_{25} & e_{35} \end{bmatrix}=\begin{bmatrix} 0 & e_{21} & 0 \\ 0 & e_{22} & 0 \\ 0 & e_{23} & 0 \\ e_{16} & 0 & 0 \\ 0 & 0 & e_{34} \\ 0 & 0 & 0 \end{bmatrix}$$

B-2. Properties of Backing Materials

Usually metals and their alloys are used as backing materials. Table B-2 provides the various backing materials available in the list along with their material properties. While developing model in ANSYS, the required properties of backing material are, density (ρ, kg/m^3), Young's modulus (E, Pa) and Poisson's ratio (ν). While entering input to MATLAB Simulation Program, the properties for backing material required to be entered are density (ρ, kg/m^3) and sound velocity or longitudinal velocity (V_l, m/s). The relation between V_l, E and ν is,

$$V_l=\sqrt{\frac{E}{\rho}\left(\frac{1-\nu}{(1+\nu)(1-2\nu)}\right)}$$

Table B-2. Properties of backing materials

Material	E(GPa)	ρ(kg/m^3)	ν	V_l (m/s)
Aluminium	68.95	2700	0.335	6208.7
Copper	117.2	8960	0.365	4747.3
Iron	196.5	7850	0.32	5985
Nickel	213.7	8900	0.31	5769.1
Silver	72.39	10500	0.37	3491.9
Tungsten	344.7	19300	0.28	4778.3
Lead	13.79	11350	0.43	1859.9

Appendix C

GENETIC ALGORITHMS

In Section 3.8, it was mentioned that Genetic Algorithm (or GA) has been used to develop the MATLAB Design program which aims to maximize pressure sensitivity or usable frequency range of pressure sensitivity curve.

The basic purpose of genetic algorithms (GAs) is optimization. Since optimization problems arise frequently, this makes GAs quite useful for a great variety of tasks. This appendix provides an introductory material on GA to understand the various terms related with GAs and the steps involved in a basic form of a genetic algorithm.

C-1 Introduction to GA

In a maximization problem, we generally seek for some solution which will be the best among others. The space of all feasible solutions (the set of solutions among which the desired solution resides) is called **search space** (also state space). Each point in the search space represents one possible solution. Each possible solution can be "marked" by its value (or fitness) for the problem. With GA we look for the best solution among number of possible solutions - represented by one point in the search space.

Genetic algorithms are inspired by Darwin's theory of evolution. Solution to a problem solved by genetic algorithms uses an evolutionary process (it is evolved).

Algorithm begins with a set of solutions (represented by chromosomes) called population. Solutions from one population are taken and used to form a new population. This is motivated by a hope, that the new population will be better than the old one. Solutions which are then selected to form new solutions (offspring) are selected according to their fitness - the more suitable they are the more chances they have to reproduce.

This is repeated until some condition (for example number of populations or improvement of the best solution) is satisfied. The next section describes these steps in more systematic manner.

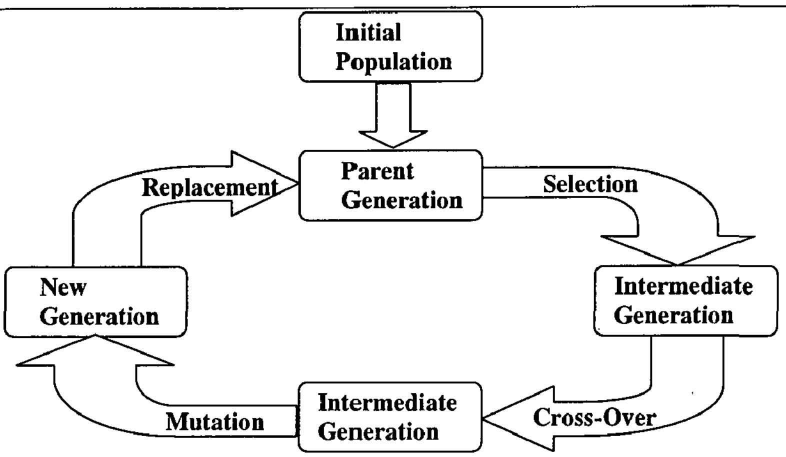

Fig. C-1. Basic evolution cycle of GA

C-2 Outline of the Basic Genetic Algorithm

Fig. C-1 shows the important steps involved in a basic GA. The steps can be listed as follows:

1. **[Start]** Generate random population of *n* chromosomes (suitable solutions for the problem)
2. **[Fitness]** Evaluate the fitness $f(x)$ of each chromosome x in the population
3. **[Intermediate Population]** Create an intermediate population by some suitable selection method.
4. **[New population]** Create a new population by repeating following steps until the new population is complete
    i. Select two chromosomes (parents) randomly from intermediate generation.
    ii. **[Crossover]** With a crossover probability cross over the parents to form new offspring (children). If no crossover was performed, offspring is the exact copy of parents.
    iii. **[Mutation]** With a mutation probability mutate new offspring at each locus (position in chromosome).
    iv. **[Accepting]** Place new offspring in the new population
5. **[Replace]** Use new generated population for a further run of the algorithm
6. **[Test]** If the end condition is satisfied, **stop**, and return the best solution in current population
7. **[Loop]** Go to step **2.**

As it can be seen that the outline of the basic GA is very general. There are many parameters and settings that can be implemented differently in various problems.

While implementing Genetic Algorithm one has to decide following things.

1. **Encoding:** A chromosome should in some way contain information about solution that it represents. The most used way of encoding is a binary string. Usually, every variable of the problem is represented by a binary string and all the strings are combined in a particular order to form one chromosome. For example, in a two variable (say x_1, x_2) problem, if we represent each variable by 4 bits then the chromosome will be of 8 bits. Further, we can choose first 4 bits to represent x_1 and next 4 bits to represent x_2.
2. **Selection Scheme:** The next question is how to select parents for crossover. This can be done in many ways, but the main idea is to select the better parents (best survivors) in the hope that the better parents will produce better offspring.
3. **Crossover Rate and Mutation Rate:** The crossover rate and mutation rate greatly decide the performance of the GA. Before discussing these, the two main operators of GA, crossover and mutation will be discussed.

After deciding on these issues, one can implement GA to develop a computer program.

C-3 Operations Involved in GA

C-3.1 Selection

As it can be observed from basic outline of GA, the first operation is selecting a pair of chromosomes for mating (performing crossover and mutation operations).There many methods suggested in literature for selection. All the methods maximize the chances of best chromosomes for getting selected for mating. Examples of selection methods are roulette wheel selection, Boltzman selection, tournament selection, rank selection, steady state selection and some others. While developing ‘design program’, roulette wheel selection method was used hence it will be discussed here.

Parents are selected according to their fitness. The better the chromosomes are, the more chances to be selected they have. Imagine a roulette wheel where all the chromosomes in the population are placed. The size of the section in the roulette wheel is proportional to the value of the fitness function of every chromosome - the bigger the value is, the larger the section is. A marble is thrown in the roulette wheel and the chromosome where it stops is selected. Clearly, the chromosomes with bigger fitness value will be selected more times. This process can be described by the following algorithm.

1. For every i^{th} chromosome in the population, calculate its fitness value f_i.
2. Evaluate the sum of fitness values of all the chromosomes and divide the fitness value of each chromosome by this sum. i.e. for i^{th} chromosome, evaluate,

   $$p_i = \frac{f_i}{sum}$$

   This number, p_i, is the probability of i^{th} chromosome being selected for the intermediate population.
3. For every i^{th} chromosome, evaluate the cumulative sum,

   $$c_i = \sum_{j=1}^{i} p_j$$

4. Generate a random number, m in the range [0, 1]. If $c_{i-1} < m < c_i$, i^{th} chromosome will be passed to the intermediate generation. If m is less than c_1, first chromosome will be passed to the intermediate generation.
5. Repeat step 4 till the intermediate generation contains desired number of chromosomes.

C-3.1 Crossover

Crossover operates on selected genes from parent chromosomes and creates new offspring. The simplest way how to do that is to choose randomly some crossover point and copy everything before this point from the first parent and then copy everything after the crossover point from the other parent. For example, consider two parent chromosomes as shown below. '|' is the crossover point.

Parent1: **010010**|1011001001

Parent2: 110110|**1010010101**

Offspring1: **0100101010010101**

Offspring2: 1101101011001001

The bold-typed parts of the two parents are combined to form offspring1 while the other parts are combined to form offspring2.

There are other ways how to make crossover, for example we can choose more crossover points. Crossover can be quite complicated and depends mainly on the encoding of chromosomes. Specific crossover made for a specific problem can improve performance of the genetic algorithm.

C-3.2 Mutation

After a crossover is performed, mutation takes place. Mutation is intended to prevent falling of all solutions in the population into a local optimum of the solved problem. Mutation operation randomly changes the offspring resulted from crossover. In case of binary encoding we can switch a few randomly chosen bits from 1 to 0 or from 0 to 1.

C-4 Parameters of GA

C-4.1 Crossover Rate (Crossover Probability)

It decides, how often crossover will be performed. If there is no crossover, offspring are exact copies of parents. If there is crossover, offspring are made from parts of both parent's chromosome. If crossover probability is **100%**, then all offspring are made by crossover. If it is **0%**, whole new generation is made from exact copies of chromosomes from old population (but this does not mean that the new generation is the same!).

Crossover is made in hope that new chromosomes will contain good parts of old chromosomes and therefore the new chromosomes will be better. Crossover rate should be high generally, about **80%-95%**. (However some results show that for some problems crossover rate about 60% is the best.)

C-4.2 Mutation Rate (Mutation Probability)

It decides, how often parts of chromosome will be mutated. If there is no mutation, offspring are generated immediately after crossover (or directly copied) without any change. If mutation is performed, one or more parts of a chromosome are changed. If mutation probability is **100%**, whole chromosome is changed, if it is **0%**, nothing is changed.

Mutation generally prevents the GA from falling into local extremes. Mutation should not occur very often, because then GA will in fact change to **random search**. Mutation rate should be very low. Best rates seem to be about**0.5%-1%**.

C-4.3 Population Size

Population size means how many chromosomes are in population (in one generation). If there are too few chromosomes, GA have few possibilities to perform crossover and only a small part of search space is explored. On the other hand, if there are too many chromosomes, GA slows down. Research shows that after some limit (which depends mainly on encoding and the problem) it is not useful to use very large populations because it does not solve the problem faster than moderate sized populations.

It may be surprising, that very big population size usually does not improve performance of GA (in the sense of speed of finding solution). Good population size is about **20-30**, however sometimes sizes 50-100 are reported as the best.