EM-MMSE BASED CHANNEL ESTIMATION FOR OFDM SYSTEMS

A DISSERTATION

Submitted in partial fulfillment of the requirements for the award of the degree
of
MASTER OF TECHNOLOGY
in
ELECTRONICS AND COMMUNICATION ENGINEERING
(With Specialization in Communication Systems)

By

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CANDIDATE'S DECLARATION

I hereby declare that the work, which is being presented in this dissertation , entitled **"EM-MMSE BASED CHANNEL ESTIMATION FOR OFDM SYSTEMS"**, in the partial fulfillment of the requirements for the award of the degree of MASTER OF TECHNOLOGY with specialization in COMMUNICATION SYSTEMS, submitted in the Department of Electronics and Computer Engineering, Indian Institute of Technology, Roorkee is an authentic record of my own work carried out from July 2005 to June 2006, under the guidance and supervision of **Dr. D.K. MEHRA,** Professor, Department of Electronics and Computer Engineering, Indian Institute of Technology, Roorkee.

I have not submitted the matter embodied in this dissertation for the award of any other degree.

Date:

Place: **SACHIN JAIN**

CERTIFICATE

This is to certify that the above statement made by the candidate is correct to the best of my knowledge and belief.

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Abstract

Orthogonal frequency division multiplexing (OFDM) is an effective way to increase data rate and simplify the equalization problem in wireless communications. OFDM systems require an efficient channel estimation procedure to demodulate the received data coherently. Accurate estimation of channel parameters is a difficult task in OFDM systems.

Pilot based techniques have normally been employed for channel estimation in OFDM systems. The obvious drawback associated with pilot based techniques for channel estimation is bandwidth overhead. To reduce the bandwidth overhead, EM algorithm is an improved way to estimate the channel coefficients in OFDM systems. The EM algorithm is a semi-blind iterative estimation procedure with expectation and maximization steps and it requires initial estimation. An initial estimate, obtained using pilot based estimator, is improved iteratively with EM algorithm in OFDM systems. For time varying channels, pilot symbols need to be transmitted periodically to overcome the effect of fast fading. But EM-based techniques gives bandwidth advantage by using channel estimate for previous frame as the initial estimate for the next frame and avoid the need of periodic retransmission of pilot symbols.

Two main points are very important in the design of a good channel estimator, namely: number of pilot symbols must be small and computational complexity should be less. In this thesis, we propose an EM-MMSE based iterative procedure to estimate the channel coefficients. The proposed technique is computationally simpler than EM technique and also yields performance improvement over pilot based techniques.

CONTENTS

Chapter 1

Introduction

The use of radio waves to transmit information from one point to another was discovered over a century ago. Extensive penetration of the end user market is a direct result of advances in circuit design and chip manufacturing technologies that have enabled a complete wireless transmitter and receiver to be packaged in a pocket-sized device. The common feature of the next generation wireless technologies will be the convergence of multimedia services such as speech, audio, video, image and data. The rapid increase in the number of wireless mobile terminal subscribers highlights the importance of wireless communication in this new millennium.

First generation wireless systems (e.g., AMPS, TACS and NMT) use analog transmission and supports voice only. The adaptation of wireless technologies to the user's rapidly changing demands has been the main driver of this revolution [1]. These trends have accelerated since the beginning of the 1990 with the replacement of the first generation analog mobile networks by the current 2^{nd} generation(2G) systems(GSM, IS-95, D-AMPS and PDC),a fully digitized network. These networks cover services such as facsimile and low rate data (up to 9.6 kbps), in addition to voice. The enhanced version of 2G systems (e.g., GPRS, HDR) are referred as 2.5G systems and support more advance services such as medium data rates (up to 100 kbps) circuit and packet switch data.

Driven by growing demand for wideband services (e.g., High speed internet access, high quality voice and image transmission) and advances in telephony, display and semiconductor technology, development of 3^{rd} Generation (3G) systems (UMTS, IMT-2000 and CDMA 2000) is still continuing. The major driver for broadband wireless communication has been the need of high bit rate (144 kbps for high mobility user with wide area coverage and 2 Mbps for low mobility user with local coverage).While these systems are working, research community is focusing its activity towards the next generation beyond 3G i.e. fourth generation (4G) systems, with more ambitious technological challenges. The ultimate goal for broadband wireless communication is to provide: *"anytime, anywhere and anymedia, anydevice,"* broadband services.

4G systems should have the following requirements:

- Generic Architecture: enabling the integration of existing technologies.
- Higher Spectral Efficiency: offering higher data rates in a given spectrum.
- High Scalability: designing different cell configuration (hot spot, ad hoc) for better coverage.
- High Adaptability and Reconfigurability: supporting different standards and technologies.
- Low Cost: it has been proposed that 4G should only have a cost per bit (1/10 of 3G).
- Future Proof: opening the door for new technologies.

The main goal in developing the next generation of wireless communication systems is for delivering multimedia services such as voice, data and image in local coverage networks. These will be a complement to the existing wide area coverage systems, for example to the third generation of mobile communications. In order to provide these services, a high data rate and high quality digital communication system is required in a restricted bandwidth. The high data rate requirements motivate research efforts to develop efficient coding and modulation schemes along with sophisticated signal and information processing algorithm to improve the quality and spectral efficiency of wireless communication links. However, these developments must cope with several performance limiting challenges that include channel fading, multiuser interference (MUI), limitation of size/power especially at mobile units. Among these challenges, channel fading [2] degrades the performance of wireless transmission significantly, and becomes the bottleneck for increasing data rates.

Another limiting factor in the performance of mobile wireless communication systems is the inter symbol interference (ISI) which can lead to the bit-error-rate (BER) degradation, caused by the multipath. In single carrier systems the symbol duration (for

WAN/MAN standards should be capable of working efficiently in wide range of operating conditions, such as large range of mobile subscriber station (MSS) speeds, different carrier frequencies in licensed and licensed-exempt bands, and various delay spreads, asymmetric traffic loads and wide dynamic signal-to-noise ratio (SNR) ranges. IEEE 802.16 currently employs the most sophisticated technology solutions in the wireless world, and correspondingly it guarantees performance (in terms of covered area, bit-rate, QoS, etc) that are superior to all other alternative technologies. 802.16 provides specifications for both fixed Line of sight (LOS) communication in the range of 10-66GHz (802.16c), and fixed, portable, Non-LOS communication in the range of 2-11GHz (802.16a, 802.16d). It defines wireless communication for mobiles, moving at speed of 125 kmph, in the range of 2-6 GHz (802.16e). 802.16e is implemented with OFDMA as its physical layer scheme [7].

WiMax (Worldwide Inter-operability for Microwave Access) is a non-profit institution composed by the principal wireless device manufacturers. Its mission is: *"To spread the use of IEEE 802.16 technology solutions, to verify interoperability of 802.16 devices built by different manufacturers, and certify interoperable devices."*

IEEE 802.16e

IEEE 802.16e is an evolution of 802.16 that is aiming to fill the gap between wireless local area networks (WLAN) and cellular networks by adding mobility and increased data speeds. The 802.16e amendment covers "Physical and Medium Access Control Layers for Combined Fixed and Mobile Operation in Licensed Bands". The scope is "to provide enhancements to IEEE Std 802.16/802.16a to support subscriber stations moving at vehicular speeds and thereby specifies a system for combined fixed and mobile broadband wireless access. Functions to support higher layer handoff between base stations or sectors are specified" [6] [7].

An 802.16e mobile subscriber station (MSS) will be able to access mobile broadband data services, while handing over between base stations, adapting to the much harsher mobile channel, using an omni-directional antenna and adopting power saving methods to preserve battery life. In light of the increased security threats and concerns in a mobile environment, the 802.16e specification includes security enhancements. There are also proposals to address new features in future standardization work. These proposed new features include directed mesh extensions and point-to-point enhancements.

Technical Specifications of IEEE 802.16e

- Operation is limited to licensed bands suitable for mobility between 2 and 6 GHz.
- Support of both fixed and mobile services.
- Full support of the IEEE 802.16a air interface.
- Channel bandwidths from 1.25 MHz to 28 MHz (e.g., 3.5 & 7 MHz in the 3.5 GHz, and 5 &10 MHz in the mobile bands).
- Support vehicular terminals and battery operated devices (e.g., laptops, tablets, and to an extent, PDAs, mobile phones).
- Support for indoor pico-cell (target radius: 100m), for outdoor-to-indoor and pedestrian micro-cell (target radius: 100m – 1000m) and for vehicular, high BS antenna macro-cell (target radius: 1–15 km).
- Support for hierarchical cell operation (indoor, outdoor-to-indoor, vehicular) handover, synchronization, awareness of layers and for interference mitigation.
- The 802.16e standards use Orthogonal Frequency Division Multiplexing (OFDM) which uses many discrete tones (e.g. 256, 512, 1024, 2048) each carrying a separate data stream on the radio frequency carrier [7].
- Support handover to other standards, particularly 802.11.
- Incorporate power savings in active and standby modes.
- Ranging, tracking and power control for vehicular operation.

Need for Channel Estimation in OFDM Systems

For OFDM systems, an efficient and accurate channel estimation procedure is necessary to coherently demodulate received data. Although differential detection could be used to detect the transmitted signal in the absence of channel information, it would result in about 3dB loss in signal to noise ratio (SNR) compared to coherent detection [8].

Several channel estimation algorithms have been proposed in the literature. Classical method for channel estimation is based on the use of training sequence. A known sequence is transmitted for a limited period of time, during which a channel estimate is obtained. Pilot symbols (on pilot subcarriers) are embedded in between the data symbols (on data subcarriers), which provides the channel information at the receiver [9]. These estimated values are interpolated over the data subcarriers and the data symbols are decoded. The pilot spacing in both time and frequency domain plays a significant role as the channel characteristics should not change between pilot subcarriers. The estimated channel parameters are then used to decode the data symbols subsequently transmitted. In order to cope with the Doppler effect due to mobility of wireless systems, reference sequence must be repeated periodically and may result in a significant loss in the useful bit rate. The obvious drawback associated with pilot based techniques for channel estimation is bandwidth overhead [10] [11].

In, [12], minimum mean square error (MMSE) and least-squares (LS) channel estimator are proposed. The MMSE estimator has good performance but high complexity. The LS estimator has low complexity, but its performance is not as good as that of MMSE estimator. In [13], comb type pilot signals, uniformly spaced across subcarriers within each frame, have been used with interpolation for the remaining subcarriers. The effects of inter-carrier interference (ICI) and AWGN are reduced by designing a time domain low pass filter with dynamically determined cutoff. Hsieh and Wei [11] have modified the above method to use LS or MMSE based estimation, instead of low pass filtering, to reduce the effects of ICI and AWGN. These channel estimators are not very accurate but also computationally complex.

Diversity is an effective way to reduce the effect of fading [14]. Diversity techniques are based on the notion that if the receiver can be provided with several replicas of the same information signal transmitted over independently fading channels, than the probability that all the signal components will fade simultaneously is reduced considerably. Temporal and frequency types of diversity introduce redundancy in time or frequency domain, and therefore induce loss in bandwidth efficiency. Spatial diversities have redundancy in space only not in time or frequency. Space-time coding that relies on multiple antenna transmission and appropriate signal processing at the receiver are popular to provide diversity and coding gains over uncoded single antenna transmissions. Space-time coding has been recently adopted in third generation cellular standards (e.g., [15], [16]), and has been proposed for many wireless applications (e.g., [17]).

To increase the spectral efficiency of OFDM, the multiple-input-multiple-output (MIMO) techniques, which spatially multiplex data streams by multiple antennas, have been applied [18]. To achieve high data rates, OFDM schemes have been combined with transmitter and receiver diversity, among which space-time coded OFDM (STC-OFDM), is one of the most efficient transmitter diversity scheme [19]. In STC-OFDM systems, channel state information between each transmit and receive antenna pair is required for coherent decoding.

Several channel estimation techniques have been recently proposed for space time coded OFDM systems. However these estimation techniques suffer from various limitations. For example, a decision directed channel estimation [20] based on MMSE criteria has been proposed for MISO-OFDM systems by utilizing the information on channel delay profile. However its performance is severely degraded in decision directed mode and requires high computational complexity. In [21] channel parameter estimators based on the use of space-time block coded (STBC) training blocks have been proposed, by assuming that the channel frequency response is quasi-static over two consecutive OFDM symbols. The use of these training blocks increases overheads.

The expectation maximization (EM) algorithm provides maximum likelihood (ML) estimate of parameters when maximization of likelihood function may not be feasible directly as in many cases direct access of data is not available, some of the data is missing or the received data does not provide sufficient information to maximize the likelihood function. The EM algorithm consists of two major steps: an expectation step, followed by maximization step. The expectation step is performed with respect to unknown underlying parameters, using the current estimate of parameters, conditioned upon the incomplete observations. The maximization step then provides a new estimate of parameters that maximizes the expectation of log likelihood function defined over

Organization of Report

The report is organized in four chapters:

In *Chapter1,* we summarize problem statement of the dissertation work and also give an overview of channel estimation problem in OFDM systems.

In *Chapter 2,* we present the SISO OFDM and MISO STC-OFDM system models and review the pilot based channel estimation techniques for OFDM systems.

In *Chapter 3,* we study the EM based channel estimation technique for OFDM systems. We then introduce proposed EM-MMSE technique and then develop EM-MMSE based channel estimation for OFDM systems.

In *Chapter 4,* we present the implementation details for channel estimation in SISO-OFDM system, MISO-STC-OFDM system. We then present the simulation results for comparison of performance in terms of BER for EM-MMSE, EM and pilot based channel estimation techniques. This is followed by conclusions drawn from this work.

Chapter 2

Channel Estimation for OFDM Systems

In this chapter we first give the brief review of multi-carrier systems and evolution of OFDM. We then introduce OFDM system model for single-input single-output (SISO) and its extension for transmit diversity or multiple-input single-output (MISO). We finally present pilot based channel estimation techniques.

2.1 Single Transmitter and Single Receiver OFDM System

A single carrier system suffers from trivial ISI problem when data rate is extremely high, this situation can be compensated by using adaptive equalization techniques. Adaptive equalizers estimate the channel impulse response and multiply complex conjugate of the estimated impulse response with the received data signal at the receiver. However, there are some practical computational difficulties in performing these equalization techniques at tens of Mbps data rate with compact and low cost hardware. In fact, equalization procedures take bulk of receiver resources, costing high computation power and thus overall service and hardware cost becomes higher [3].

One way to achieve reasonable quality and solve the problems described above for broadband mobile communication is to use parallel transmission. The idea is to increase the symbol period of subchannels by reducing the data rate and thus reducing the effect of ISI. Reducing the effect of ISI yields an easier equalization, which in turn means simpler reception techniques. In principle we can say that parallel transmission is just the summation of a number of single carrier transmissions at the adjacent frequencies. The difference is that the channels have lower data transmission rate than the original single carrier system [23] [24].

OFDM was first presented in 1966, but the concept for the MCM system goes back to the early 1950s [23]. In 1971 the discrete-time OFDM was introduced using an efficient fast Fourier transform (FFT) technique at both transmitter and receiver [25]. The

ISI and ICI problems, due to a dispersive channel, were solved using the cyclic prefix (CP) extension of the OFDM symbols in 1980 [24]. OFDM is a high spectral efficiency type of MCM transmission system, where the available spectrum is divided into a number of narrow band subchannels. This allows for individually modulating each subcarrier and then transmitting the entire OFDM blocks at a significantly lower rate than in a single-carrier system. OFDM parameters are generally chosen such that each subchannel experiences flat fading, because the bandwidth of the modulated subcarrier becomes narrow compared with the coherence bandwidth of the dispersive channel.

The high spectral efficiency in OFDM is achieved using orthogonal signals, allowing spectrum in each subchannel to overlap another without interfering [3]. OFDM has an additional advantage of being computationally efficient because the fast Fourier transform (FFT) technique can be used to implement the modulation and demodulation functions. ISI i.e. inter-symbol interference between the consecutive OFDM symbols and ICI i.e. crosstalk between different subcarriers, can be eliminated by using the cyclic prefix (Cyclic prefix (CP) is a copy of the last part of OFDM symbol which is appended in front of the transmitted OFDM symbol) [25]. At the receiver side, CP is removed before any processing starts. The CP interval must be chosen larger than the length of the channel impulse response (CIR), such that multi path reflection from one symbol would not interfere with another [3].

Figure 2.1 shows a simplified diagram of conventional OFDM system. The input binary data is first fed into a serial to parallel (S/P) converter. Each data stream then modulates the corresponding sub-carrier by MPSK or MQAM. The serial to parallel converter collects the serial data symbols *X(m)*, with symbol interval T_s ,into a length *M* (number of subcarrier) vector $\underline{X} = [X(0),\cdots,X(M-1)]^T$. $X(0),\cdots,X(M-1)$ are then transformed by the inverse fast Fourier transform (IFFT) and denoted as $x(0),\cdots\cdots\cdots,x(M-1)$, where

$$x(m) = \frac{1}{\sqrt{N}} \sum_{k=0}^{M} X(k) \exp(j \frac{2\pi mk}{M}) \quad , \quad 0 \le m \le M-1 \tag{2.1.1}$$

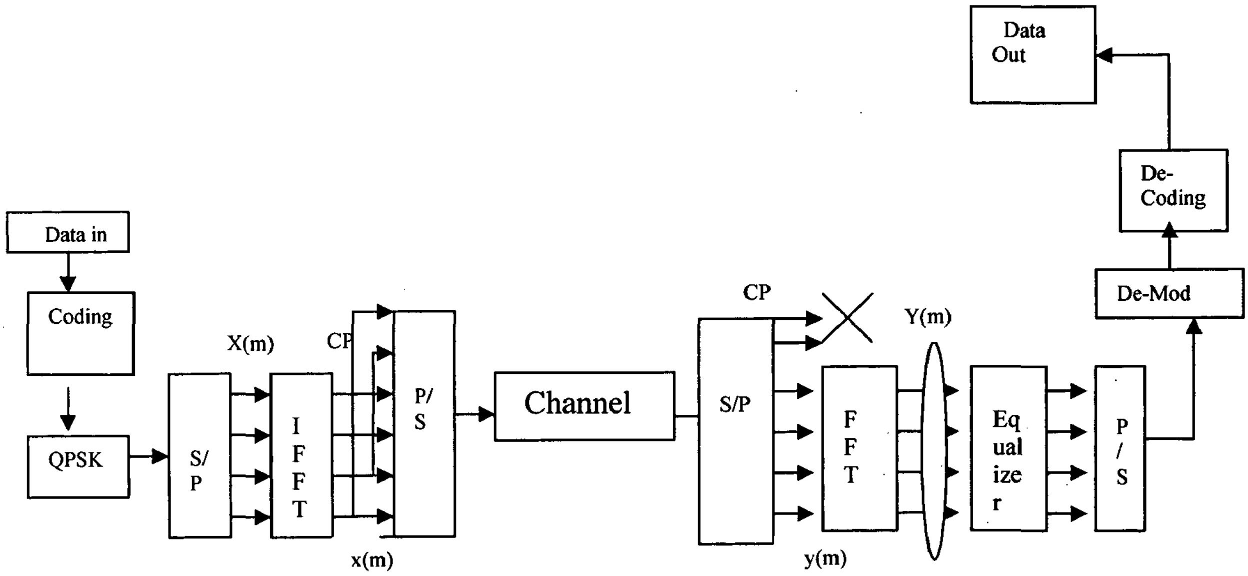


Fig. 2.1 A Simplified block schematic of SISO-OFDM system

After adding guard interval output $\underline{x}_f$ may be written as,

$$x_f(m) = \begin{cases} x(m+M-M_c) & 0 \le m \le M_c - 1 \\ x(m-M_c) & M_c \le m \le M + M_c - 1 \end{cases} \tag{2.1.2}$$

Cyclic prefix in each OFDM data frame is added, with its length M_C chosen to be longer than channel length (L), there will be no ISI between OFDM frames. Thus only one OFDM frame will be considered with M sub-carriers in analyzing the system performance. The parallel data are converted back to a serial data stream before being transmitted over the frequency selective channel.

The received sequence is corrupted by multipath fading and AWGN,

$$y_f(m) = \sum_{l=0}^{L-1} x_f(m-l)h(l) + n(m), \qquad 0 \le m \le M + M_C - 1 \tag{2.1.3}$$

Where $\underline{h} = [h(1), \cdots, h(L)]^T$ and $h(l)$ is the l^{th} tap of time domain CIR and $n(m)'s, 0 \le m \le M-1$ are independent complex-valued Gaussian random variables with zero mean and variance σ^2 for both real and imaginary components. L is the length of the time-domain CIR. Assuming that the channel impulse response remains constant during the entire frame interval.

At receiver, cyclic prefix is discarded and output *y(m)* may be written as,

$$y(m) = y_f(m + M_C), \qquad 0 \le m \le M-1$$

In terms of transmitted symbols and channel coefficient *y(m)* can be written as,

$$y(m) = \sum_{l=0}^{L-1} x(m-l)h(l) + n(m), \qquad 0 \le m \le M-1$$

M-point fast Fourier transform (FFT) is performed on $y(0),\cdots\cdots\cdots\cdots, y(M-1)$ and it is converted to $\underline{Y} = [Y(0)\cdots\cdots Y(m)]^T$. Inter-carrier interference (ICI) is also eliminated at the FFT output because of the orthogonallity between the sub-carriers.

$$Y(m) = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} y(k) e^{-j2\pi\frac{km}{M}}$$

A well known property of the FFT is that cyclic convolution in the time domain results in multiplication in the frequency domain. Thus the output *Y(m)* after FFT block can be expressed as,

$$Y(m) = X(m)H(m) + N(m), \qquad 0 \le m \le M-1 \tag{2.1.4}$$

where $H(m)$ and $N(m)$ are the frequency response of the channel at sub carrier m, and the set of the transformed noise variables respectively. $H(m)$ and $N(m)$ can be obtained by,

$$H(m) = \sum_{l=0}^{L-1} h(l) e^{-j2\pi\frac{ml}{M}}, \quad N(m) = \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} n(k) e^{-j2\pi\frac{mk}{M}}, \quad 0 \le m \le M-1$$

H(m) can be viewed as the complex gain of m^{th} sub-channel. The demodulated symbol *Y(m)* is just the product of the complex channel gain *H(m)*, data symbols *X(m)* and noise component *N(m)*.

Equation (2.1.4) can be represented in vector form for a frame of length *M*,

$$\underline{Y} = \underline{\mathbf{X}}\underline{H} + \underline{N} \tag{2.1.5}$$

where $\underline{\mathbf{X}}$ is a diagonal matrix with elements $X(0),\cdots,X(M-1)$.

Where $\underline{N} = [N(0),\cdots,N(M-1)]^T$ and $\underline{H} = \mathbf{W}\underline{h}$, $\mathbf{W}$ is a $M \times L$ matrix

$$\mathbf{W} = \begin{bmatrix} 1 & 1 & \cdots\cdots & 1 \\ 1 & e^{-j2\pi/M} & \cdots\cdots & e^{-j2\pi(L-1)/M} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 1 & e^{-j2\pi(M-1)/M} & \cdots\cdots & e^{-j2\pi(M-1)(L-1)/M} \end{bmatrix}_{M \times L} \tag{2.1.6}$$

2.2 Transmitter Diversity and Space-Time Coded Systems

Antenna diversity is a practical, effective and a widely applied technique for reducing the effect of multipath fading. The classical approach is to use multiple antennas at the receiver and perform combining or selection and switching in order to improve the quality of the received signal. The major problem with using the receive diversity approach is the cost, size, and power of the remote units. The use of multiple antennas makes the remote units larger and more expensive. As a result, diversity techniques have almost exclusively been applied to base stations to improve their reception quality. A base station often serves hundreds to thousands of remote units. Therefore, it is economical to add equipment to base stations rather than the remote units. For this reason, transmit diversity schemes are very attractive [26]. This is generally achieved by separating transmit antennas far enough so as to make zero or very low correlation between the transmission paths.

Recently, space–time coding (STC) has been proposed as an attractive solution for providing transmitter diversity and combating the adverse effects of fading [26] [27]. By using specially designed channel codes at the transmitter in combination with additional signal processing at the receiver, ST coding achieves bandwidth efficient transmit diversity, as well as excellent performance over flat fading channels. It has been shown in [28] that ST coded transmission over wireless channels is severely limited by intersymbol interference (ISI) caused by multipath fading.

In 1998, Alamouti proposed a simple transmitter diversity scheme which improves the signal quality at the receiver on one side of the link by simple processing across two transmit antenna at the opposite end [26]. The Alamouti block code succeeds in realizing the diversity gain, by arranging the symbols and their complex conjugate in a 2x2 matrix.

2.2.1 Alamouti's Scheme

At a given symbol period, two signals are simultaneously transmitted from the two antennas, namely c_1 from the first antenna, T_{x_1} and c_2 from the second T_{x_2}. In the next symbol period, signal $(-c_2^*)$ is transmitted from T_{x_1} and c_1^* from second antenna T_{x_2}, where * denotes the complex conjugation.

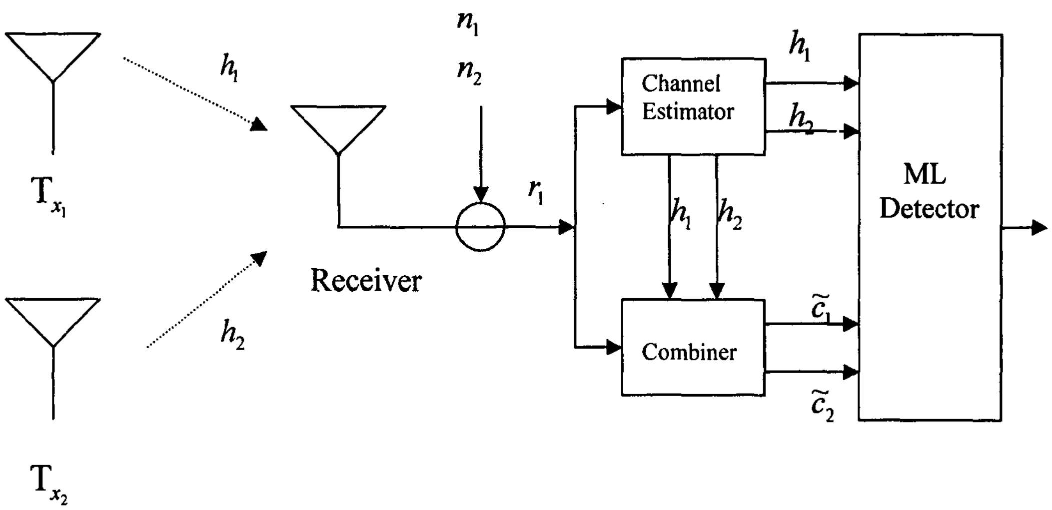


Fig 2.2 A Communication system using Alamouti's code for data

In accordance to our system model, Let $h_1(t)$ denote the path gain from T_{x_1} to the receiver and $h_2(t)$ is the path gain from T_{x_2} to receiver. If we assume that the fading is constant for two consecutive symbols, we can write

$h_1(t) = h_1(t+T) = h_1 = \alpha_1 \exp(j\theta_1)$

$h_2(t) = h_2(t+T) = h_2 = \alpha_2 \exp(j\theta_2)$, where T is the symbol period

The received signals are,

$$r_1 = r(t) = h_1 c_1 + h_2 c_2 + n_1 \tag{2.2.1}$$

$$r_2 = r(t+T) = -h_1 c_2^* + h_2 c_1^* + n_2 \tag{2.2.2}$$

Where r_1 and r_2 are received signals at time t and $t+T$. The combiner combines the received signal as follows,

$$\widetilde{c}_1 = h_1^* r_1 + h_2 r_2^* = (\alpha_1^2 + \alpha_2^2)c_1 + h_1^* n_1 + h_2 n_2^* \tag{2.2.3}$$

$$\widetilde{c}_2 = h_2^* r_1^* - h_1 r_2^* = (\alpha_1^2 + \alpha_2^2)c_2 + h_2^* n_1 - h_1 n_2^* \tag{2.2.4}$$

and sends them to the maximum likelihood detector, which minimize the following decision metric $\left\|r_1 - h_1 c_1 - h_2 c_2\right\|^2 + \left\|r_2 + h_1 c_2^* - h_2 c_1^*\right\|^2$ over all possible values of c_1 and c_2. Expanding this and deleting terms that are independent of code words, the above minimization reduces to separately minimizing,

$\left|r_1 h_1^* + r_2^* h_2 - c_1\right|^2 + (\alpha_1^2 + \alpha_2^2 - 1)\left|c_1\right|^2$, for detecting c_1 and

$\left|r_1 h_2^* - r_2^* h_1 - c_2\right|^2 + (\alpha_1^2 + \alpha_2^2 - 1)\left|c_2\right|^2$, for detecting c_2.

Equivalently using the notation

$$d^2(x,y) = (x-y)(x^* - y^*) = |x-y|^2$$

the decision rule for each combined signal c_j ;j=1;2 becomes:

Decision is c_i if and only if

$$(\alpha_1^2 + \alpha_2^2 - 1)|c_i|^2 + d^2(\widetilde{c}_j, c_i) \le (\alpha_1^2 + \alpha_2^2 - 1)|c_k|^2 + d^2(\widetilde{c}_j, c_k) \qquad \forall i \ne k$$

Alamouti further extended this scheme to the case of 2 transmit antennas and multiple receive antennas.

Characteristics of this scheme include:

- No feedback from receiver to transmitter is required.
- No bandwidth expansion (as redundancy is applied in space across multiple antennas not in time and or frequency).
- Low complexity decoder.

2.2.2 Two Transmitter Single Receiver STC-OFDM Systems

The combination of OFDM, space-time coding and transmitter diversity can further enhance the data rates in a frequency-selective fading environment. For OFDM systems with transmitter diversity using space-time coding, two or more different signals are transmitted from different antennas simultaneously. The received signal is the superposition of these signals, usually with equal average power. If the channel parameters corresponding to each transmitter and receiver antenna pair are estimated by the approach developed previously in [29] [30], the signals from other transmitter antenna(s) will become interference. The signal-to-interference ratio will always be less than 0 dB, and the MSE of the estimation will be very large. In ST-OFDM, channel state information between each transmit and receive antenna pair is required for coherent decoding.

An OFDM system with N_T transmit antennas and one receive antenna is shown in Fig 2.2. At time *n*, a data block $\{b(n,m), 0 \le m \le M-1\}$, where M is the number of sub-carrier (tones), is coded into N_T different symbol blocks, $X_i(n,m), 0 \le m \le M-1 \,\&\, 0 \le i \le N_T$. Each block is transmitted through different antennas over the same bandwidth using M OFDM tones. There is a communication link established by OFDM between each transmit antenna and the receiver.

The received signal after demodulation (performing a FFT), is the superposition of N_T distorted transmitted signals, which can be expressed in vector form as

$$\underline{r}_n = \underline{r}_{T_{x1},n} + \underline{r}_{T_{x2},n} \tag{2.2.5}$$

where $\underline{r}_n$ is received vector and $\underline{r}_{\mathrm{T}_{x_1},n}$ & $\underline{r}_{\mathrm{T}_{x_2},n}$ are the effective received vector by T_{x_1} and T_{x_2} respectively at time instant *n*.

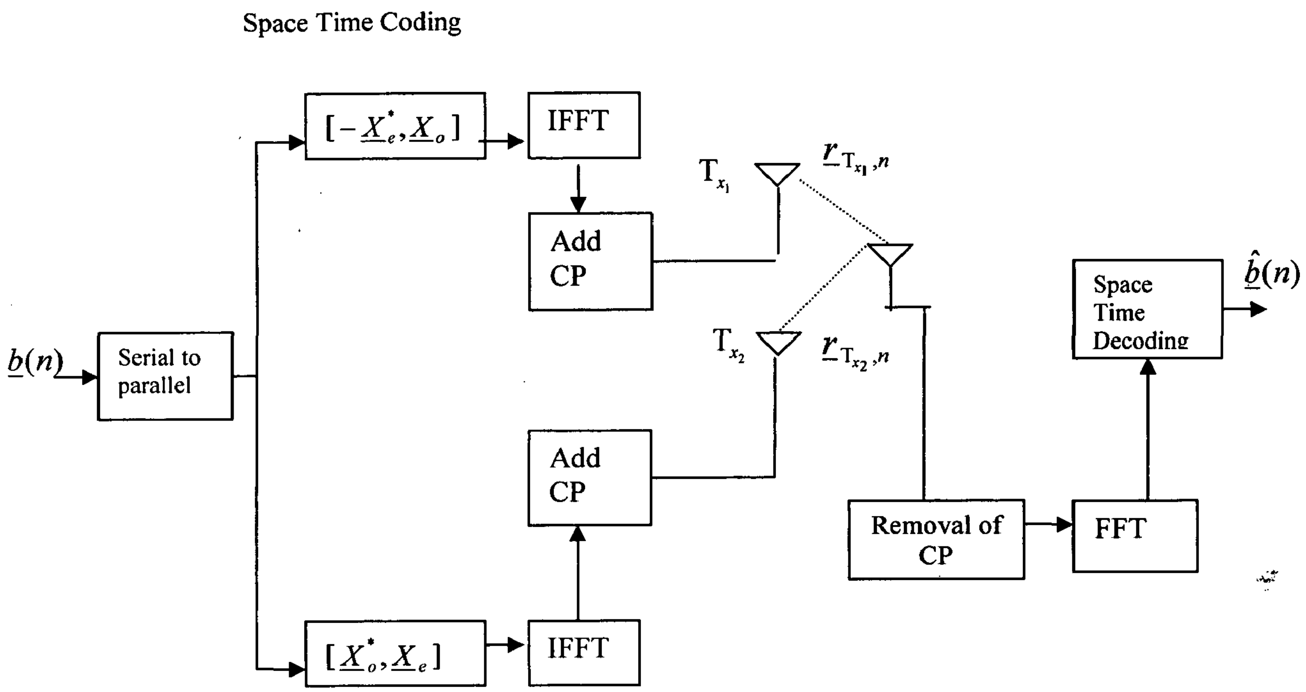


Figure. 2.2 A Simplified block schematic of two transmitter single receiver STC-OFDM system

$X_{1,n}(m)$ & $X_{2,n}(m)$ represents the symbol transmitted through the T_{x_1} and T_{x_2} antenna respectively over the m^{th} tone at time *n*. $\underline{H}_{i,n}$ is an $M \times 1$ vector with $H_{i,n}(m)$ denoting the channel frequency response at the m^{th} tone between the i^{th} antenna and the receiver at time *n*. Finally, $\underline{N}_n$ is a vector of AWGN noise in the *M* sub-carriers at time *n*, with mean zero variance σ_n^2. Assuming that channel impulse response is constant during the two time slots. In further analysis, we omit the suffix of *n*.

Let from first transmitter, $\underline{X}_o$ is transmitted during the first time slot *n* followed by $-\underline{X}_e^*$ in the second time slot *(n+1)*. From the second transmitter, $\underline{X}_e$ is transmitted in first time slot *n* followed by $\underline{X}_o^*$ in second time slot *(n+1)*. Where * stands for complex conjugate operation.

From Fig. 2.2 it can be shown that,

$\underline{X}_{1,n} = \underline{X}_o \;\&\; \underline{X}_{1,n+1} = -\underline{X}_e^*$ and $\underline{X}_{2,n} = \underline{X}_e \;\&\; \underline{X}_{2,n+1} = \underline{X}_o^*$

Received vector in first time slot *n*, will be represented as $\underline{r}_1$ and in time slot *(n+1)* will be represented as $\underline{r}_2$.The equivalent space-time block code transmission matrix is given by,

$G = \begin{pmatrix} \underline{X}_o & \underline{X}_e \\ -\underline{X}_e^* & \underline{X}_o^* \end{pmatrix}$ i.e. entries in the transmission matrix are the OFDM symbol vectors and their conjugates.

In OFDM space Time block Coding in a frame for each subcarrier:

$$\underline{r}_1 = \underline{H}_1 \bullet \underline{X}_o + \underline{H}_2 \bullet \underline{X}_e + \underline{N}_1 \quad (2.2.6)$$

$$\underline{r}_2 = -\underline{H}_1 \bullet \underline{X}_e^* + \underline{H}_2 \bullet \underline{X}_o^* + \underline{N}_2 \quad (2.2.7)$$

where $\bullet$ operator denotes Hadamard product and $(\underline{H}_1, \underline{H}_2)$ are the frequency domain impulse responses of channel.

Assuming the channel response are known or can be estimated accurately at the receiver, the decision variable are constructed by combining $\underline{r}_1, \underline{r}_2$ and channel impulse response as,

$$\hat{\underline{X}}_o = \underline{H}_1^* \bullet \underline{r}_1 + \underline{H}_2 \bullet \underline{r}_2^* \quad (2.2.8)$$

$$\hat{\underline{X}}_e = \underline{H}_2^* \bullet \underline{r}_1 - \underline{H}_1 \bullet \underline{r}_2^*, \quad (2.2.9)$$

Then maximum likelihood (ML) decoding is performed to estimate the transmitted sequence.

2.3 Channel Estimation Techniques

Channel estimation techniques for OFDM systems can be divided into two major categories as the supervised or trained techniques and the unsupervised or blind techniques. Blind techniques and semi-blind channel estimation techniques are considered more attractive than training based techniques but these techniques have large computational complexity. In this section we give the brief review of conventional supervised or pilot based technique for channel estimation in OFDM system.

In flat fading environment channel parameters can be estimated using pilot symbols inserted into the transmitted symbols. This technique is called pilot symbol assisted techniques and was introduced for single carrier system by Moher and Lodge [31]. Since each subchannel in OFDM is flat faded, pilot symbol assisted techniques can be used for channel estimation. The number of pilots to use is a trade-off between data rate and channel estimation performance.

There are two schemes in which the pilot based method can be used. In one scheme where one entire frame would be used periodically as transmit training data and the channel estimate would be obtained from this training data. This fixed estimate would be used until the next frame of pilot symbols is received. This scheme is called block-type pilot based channel estimation. This scheme has the advantage of a more accurate instantaneous channel estimate, and the disadvantage that the fixed estimate would degrade in quality over time, due to the channel variation [9]. In another scheme, it uses few pilot tones in each symbol. An instantaneous estimate of channel can be found in each symbol by using interpolation, and use it to update the average channel estimate. Another scheme uses the optimally-spaced tones set and is called Comb-type pilot based channel estimation [9].

2.3.1 Block-Type Pilot based Channel Estimators

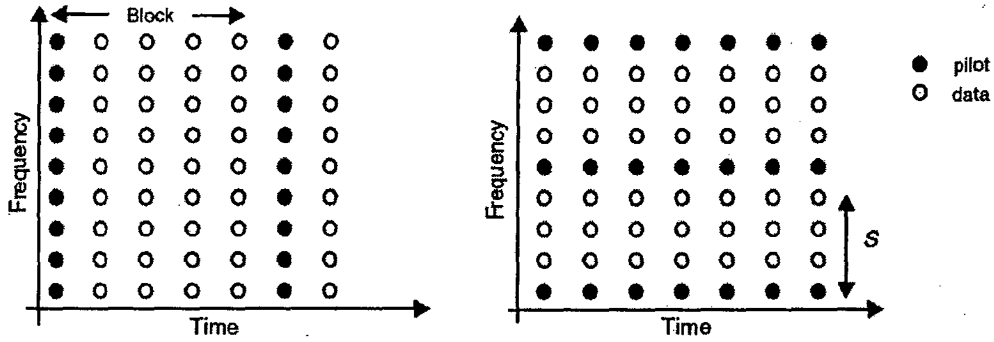
In block-type pilot-based channel estimation, as shown in Figure 2.3, reference frame is transmitted periodically, and all subcarriers are used as pilots. We have to estimate the channel parameters given the pilot signals and received signals. The receiver uses the estimated channel conditions to decode the received data inside the block until the next pilot symbol arrives. The estimation can be based on least square (LS), minimum mean-square error (MMSE) or modified MMSE.

LS Estimator

The LS estimator minimizes the parameter $\|\underline{Y} - \underline{\mathbf{X}}\underline{H}\|^2$, It is shown in [12] that LS estimator gives the estimated channel parameters as,

$$\begin{aligned}\underline{\hat{H}}_{LS} &= \mathbf{X}^{-1}\underline{Y} \\ &= [(X(m)/Y(m))]^T \qquad 0 \le m \le M-1,\end{aligned} \tag{2.3.1}$$

Without using any knowledge of the statistics of the channels, the LS estimators are calculated with very low complexity, but they suffer from a high mean-square error.



Block Type

Comb-type

Fig. 2.3 Pilot based Channel Estimation

MMSE Estimator

The MMSE estimator employs the second-order statistics of the channel conditions to minimize the mean-square error.

Denote by $\underline{R}_{\underline{hh}}, \underline{R}_{\underline{HH}}, \underline{R}_{\underline{YY}}$ and the autocovariance matrix of $\underline{h}, \underline{H}$ and $\underline{Y}$, respectively, and by $\underline{R}_{\underline{hY}}$ the cross covariance matrix between $\underline{h}$ and $\underline{Y}$. Also denote σ_N^2 by the noise variance $E\{|\underline{N}|^2\}$. Assume the channel vector $\underline{h}$ and the noise vector $\underline{N}$ are uncorrelated, Let,

$$\underline{R}_{\underline{HH}} = E\{\underline{H}\underline{H}^H\} = E\{(\mathbf{W}\underline{h})(\mathbf{W}\underline{h})^H\} = \mathbf{W}\underline{R}_{\underline{hh}}\mathbf{W}^H \tag{2.3.2}$$

$$\underline{R}_{\underline{hY}} = E\{\underline{h}\underline{Y}^H\} = E\{\underline{h}(\underline{X}\mathbf{W}\underline{h} + \underline{N})^H\} = \underline{R}_{\underline{hh}}\mathbf{W}^H\underline{\mathbf{X}}^H \tag{2.3.3}$$

$$\underline{R}_{\underline{YY}} = E\{\underline{Y}\underline{Y}^H\} = \underline{\mathbf{X}}\mathbf{W}\underline{R}_{\underline{hh}}\mathbf{W}\underline{\mathbf{X}}^H + \sigma_N^2\underline{I}_N \tag{2.3.4}$$

Assuming that $\underline{R}_{hh}, \underline{R}_{HH}$ and σ_N^2 are known at the receiver in advance, the MMSE estimator of $\underline{h}$ is given by $\underline{\hat{h}}_{MMSE} = \underline{R}_{hY} \underline{R}_{YY}^{-1} \underline{Y}^H$ [32].

We can write,

$$
\begin{aligned}
\underline{\hat{H}}_{MMSE} = \mathbf{W}\underline{\hat{h}}_{MMSE} &= \mathbf{W}[(\mathbf{W}^H \underline{\mathbf{X}}^H)^{-1} \underline{R}_{hh}^{-1} \sigma_N^2 + \underline{\mathbf{X}}\mathbf{W}]^{-1} \underline{Y} \\
&= \mathbf{W}\underline{R}_{hh}[(\mathbf{W}^H \underline{\mathbf{X}}^H \underline{\mathbf{X}}\mathbf{W})^{-1} \sigma_N^2 + \underline{R}_{hh}]^{-1} \mathbf{W}^{-1} \underline{\hat{H}}_{LS} \qquad (2.3.5) \\
&= \underline{R}_{HH}[\underline{R}_{HH} + \sigma_N^2 (\underline{\mathbf{X}\mathbf{X}}^H)^{-1}]^{-1} \underline{\hat{H}}_{LS}
\end{aligned}
$$

The MMSE estimator yields much better performance than LS estimators, especially under the low SNR values.

A major drawback of the MMSE estimator is its high computational complexity. Modified MMSE estimators are proposed in [32] to reduce complexity, where only the taps with significant energy are considered as most of the energy in $\underline{h}$ is contained in, or near, the first L taps. The elements in $\underline{R}_{hh}$ corresponding to low energy taps in $\underline{h}$ are approximated by zero. The effective size of matrix is reduced dramatically after the low-rank approximation is used.

Decision Feedback Estimator

In block-type pilot-based channels, the estimators are usually calculated once per block and are used until the next pilot symbol arrives. The channel estimation with decision feedback is proposed to improve the performance, where the estimators inside the block are updated using the decision feedback equalizer at each subcarrier. The receiver first estimates the channel conditions using the pilots and obtain $\underline{\hat{H}} = \{\hat{H}(m)\}(0 \le m \le M-1)$, which is based on LS, MMSE, or modified MMSE. Inside the block, for each symbol and for its each subcarrier, the estimated transmitted signal is found by the previous $\hat{H}(m)$ according to the equation $\hat{X}(m) = Y(m)/\hat{H}(m)$. $\{\hat{X}(m)\}$ is mapped to the binary data taking the decision and represented as $\{\tilde{X}(m)\}$. The estimated channel $\hat{H}(m)$ is updated by $\hat{H}(m) = Y(m)/\tilde{X}(m)$ and is used in the next symbol.

2.3.2 Comb-Type Pilot based Channel Estimators

In comb-type pilot based channel estimation, as shown in Figure 2.3, for each transmitted symbol, M_p pilot signals are uniformly inserted into $\underline{X}$ with S subcarriers apart from each other [11], where $S = M / M_p$

The receiver knows the pilots locations $\underline{P} = [P(m)]^T, 0 \le m \le M_p - 1$, the pilot values $\underline{X}_p = [X_p(m)]^T, 0 \le m \le M_p - 1$, and the received signal $\underline{Y}$. The LS estimates to the channel conditions at the pilot subcarriers $\underline{H}_p$ are calculated by

$$\underline{\hat{H}}_{p_{LS}} = [Y(P(m)) / X_p(m)]^T, 0 \le m \le M_p - 1 \tag{2.3.6}$$

The task here is to estimate the channel conditions $\underline{H}$ at the data subcarriers, given the LS estimates at pilot subcarriers, received signals $\underline{Y}$, and additional knowledge of the channel statistics. The solutions include LS estimator with 1D interpolation [32] and the maximum likelihood (ML) estimator [30].

LS Estimator with 1D Interpolation

1D interpolation is used to estimate the channel at data subcarriers, where $\hat{H}_{p_{LS}}$ the vector with length M_p is interpolated to the vector $\underline{H}$ with length M, without using additional knowledge of the channel statistics. The 1D interpolation methods are summarized below.

Linear Interpolation (LI)

The LI method performs better than the piecewise-constant interpolation, where the channel estimation at the data subcarrier between two pilots $\hat{H}_{p_{LS}}(m)$ & $\hat{H}_{p_{LS}}(m+1)$ is given by:

$$\hat{H}_{p_{LS}}(mS + b) = \hat{H}_{p_{LS}}(m) + (\hat{H}_{p_{LS}}(m+1) - \hat{H}_{p_{LS}}(m))b / S \qquad 0 \le b \le S \tag{2.3.7}$$

Second-Order Interpolation (SOI)

The SOI method performs better than the LI method, where the channel estimation at the data subcarrier is obtained by weighted linear combination of the three adjacent pilot estimates.

Low-Pass Interpolation (LPI)

The LPI method is performed by inserting zeros into the original $\underline{\hat{H}}_{p_{LS}}$ sequence and then applying a low-pass finite-length impulse response (FIR) filter (the *interp* function in MATLAB), which allows the original data to pass through unchanged. This method also interpolates so that the mean-square error between the interpolated points and their ideal values is minimized.

Spline Cubic Interpolation (SCI)

The SCI method produces a smooth and continuous polynomial fitted to given data points (the *spline* function in MATLAB).

Time Domain Interpolation (TDI)

The TDI method is a high-resolution interpolation based on zero-padding and FFT/IFFT. It first converts $\underline{\hat{H}}_{p_{LS}}$ to time domain by IFFT and then interpolate the time domain sequence to M points with simple piecewise-constant method [32]. Finally, the FFT converts the interpolated time domain sequence back to the frequency domain.

2.3.3 Other Pilot based Channel Estimators

Other channel estimation schemes include the simplified 2D channel estimators, the iterative channel estimators, and the channel estimators for the OFDM systems with multiple transmit-and-receive antennas.

In 2D channel estimation, the pilots are inserted in both the time and frequency domains, and the estimators are based on 2D filters. In general, 2D channel estimation yields better performance than the 1D scheme, at the expense of higher computational complexity and processing delay. The optimal solution in terms of mean-square error is based on 2D Wiener filter interpolation, which employs the second-order statistics of the channel conditions. However, such a 2D estimator structure suffers from a huge computational complexity, especially when the FFT size M is large. In [34], channel estimators based on 2D least square (LS) and 2D normalized least square (NLS) are proposed, and a parallel 2D (N) LS channel estimation scheme solves the realization problem due to the high computational complexity of 2D adaptive channel estimation.

Iterative channel estimators are proposed in [35]. To reduce complexity, the 2D transmission lattice is divided into 2D blocks, and the pilots are uniformly inserted inside each block. Channel estimation proceeds on a block-by-block basis. The first estimator is based on iterative filtering and decoding, which consists of two cascaded 1D Wiener filters to interpolate the unknown time-varying 2D frequency response between the known pilot symbols. The second estimator uses an a posteriori probability (APP) algorithm, in which the two APP estimators, one for the frequency and the other for the time direction, are embedded in an iterative loop similar to the turbo decoding principle. These iterative estimators yield robust performance even at low SNR scenarios, but with high computation complexity and large time delay.

For OFDM systems with multiple transmit antennas, each tone at receiver antenna is associated with multiple channel parameters, which makes channel estimation difficult. Channel parameters for different tones of each channel are correlated and the channel estimators are based on this correlation. Several channel estimation schemes have been proposed for the OFDM systems with multiple transmit-and-receive antennas for space diversity, or multiple input multiple output (MIMO) systems for high-rate wireless data access. For example, in [36], channel estimation is based on a 1D block-type pilot arrangement, and optimal training sequences are constructed not only to optimize, but also to simplify channel estimation during the training period.

In [37], channel estimation in 2Ds for OFDM systems with multiple transmit antennas is discussed. The approach estimates and separates N_T superimposed received signals, corresponding to N_T transmit antennas, by exploiting the correlation in 2D of the received signal. We can also say that it uses two 1D estimators instead of a true 2D estimator and divides the estimation and the separation task into two stages. The first stage separates a subset of the superimposed signals and estimates the channel response in the first dimension. The second stage further separates the signals of each subset in the second dimension, yielding an estimate for all transmit antennas. For space-time block-coded OFDM systems, the proposed estimator can track the channel variations even at high Doppler frequencies.

The LS estimator with low-pass interpolation (LPI) performs the best of all 1D interpolation methods, and it has a low computational complexity. In block-type pilot channel estimation, the modified MMSE estimator with decision feedback equalizer gives the best tradeoff between performance and complexity. Block-type pilot channel estimation is more suitable for the slow fading channel conditions, while the comb-type pilot channel estimation usually outperforms for the middle and fast fading channels.

An important problem associated with channel estimation is fast fading caused by movement of the mobile terminal. In current systems, the fading is so fast (order of 200 Hz) that channel estimation in a wireless digital receiver is often carried out on symbol-to-symbol basis. Therefore, channel estimation techniques have two major concerns. Firstly, the storage and complexity requirement of estimation schemes should be kept as minimum. Secondly, the pilot symbol overhead should be kept as small as possible.

Chapter 3

Channel Estimation for OFDM Systems using EM-MMSE Technique

In this chapter we first introduce expectation maximization algorithm and then consider EM-based channel estimation for single transmitter single receiver OFDM system. EM-based channel estimation for two transmitter single receiver STC-OFDM system is considered next. EM-MMSE based channel estimation for single transmitter single receiver OFDM system and EM-MMSE based channel estimation for two transmitter single receiver STC-OFDM system is presented finally.

3.1 Expectation Maximization Algorithm

Minimum mean squares error (MMSE) and maximum likelihood (ML) criterions are generally used in estimating deterministic unknown parameters. ML criterion is considered as an optimal detection method, when the transmitted symbols are equi-probable and unknown parameters are deterministic. However in many cases, the received data does not provide sufficient information to maximize the likelihood function. The expectation maximization (EM) algorithm provides ML estimation of parameters when maximization of likelihood function may not be feasible directly [22]. The EM algorithm is used, because a maximum likelihood (ML) estimate has no closed form solution [22].

For OFDM system model presented in section (2.1) CIR's can be calculated by defining a cost function. The log likelihood function (cost function) is defined as $f(\underline{Y}/\underline{H},\underline{X})$ where $f(\underline{Y}/\underline{H},\underline{X})=\frac{1}{2\pi\sigma^2}\exp\{-\|\underline{Y}-\underline{\mathbf{X}}\underline{H}\|^2\}$. Maximization of this cost function will give the estimated value of channel parameters. This is equivalent to minimizing the function, $\|\underline{Y}-\underline{\mathbf{X}}\underline{H}\|^2$ with respect to $\underline{\mathbf{X}}$. $\|\underline{Y}-\underline{\mathbf{X}}\underline{H}\|^2$. From Eq. (2.1.5) and (2.1.6) it can be written as $\|\underline{Y}-\underline{\mathbf{X}}\mathbf{W}\underline{h}\|^2$ and therefore, by minimizing this function,

$$\underline{\hat{h}} = (\mathbf{W}^H \underline{\mathbf{X}}^H \underline{\mathbf{X}}\mathbf{W})^{-1} \mathbf{W}^H \underline{\mathbf{X}}^H \underline{Y} \tag{3.1.1}$$

But at the receiver $\underline{\mathbf{X}}$ is unknown. However, there is some prior knowledge on the unknown $\underline{\mathbf{X}}$. For true ML channel estimation given a prior knowledge, one must resort to an exhaustive test on every hypothesis of $\underline{\mathbf{X}}$. This exhaustive search is too complex to be implemented. While the EM algorithm is suitable choice to exploit a prior knowledge and its result is believed to be the ML solution.

The EM algorithm is an iterative procedure with expectation and maximization steps. In the first step the conditional expectation of unobserved sufficient information (complete data) is taken under given observed insufficient information (incomplete data) and current estimation of parameters. The second step provides the new estimate of parameters by maximizing the conditional expectation over unknown parameters. EM algorithm is a batch approach which processes the whole received data [13]. Since the $\underline{\mathbf{X}}$ in the Eq. 3.1.1 is unknown, the EM algorithm maximizes the expectation of likelihood function over all possible values of $\underline{\mathbf{X}}$.

In other words, it can be said that the expectation-maximization (EM) algorithm provides an iterative approach to likelihood based parameter estimation when direct maximization of likelihood function may not be feasible. Although convergence of the algorithm to the maximum-likelihood estimator is not always guaranteed, EM does posses the intuitively pleasing property of producing estimate that monotonically increases in likelihood [8]. The generality of algorithm combined with its amenability for incomplete data problem has lead to its widespread use in such diverse applications as speech recognition, identification of impulsive noise channels, quantum limited imaging and numerous others in economics, medicine, psychology, etc.

General Statement of the EM Algorithm

Consider θ as a set of deterministic channel parameters to be estimated from the observed data *I*, ML estimation of θ is given by, $\hat{\theta} = \arg\max_{\theta}\{f(I/\theta)\}$. Where *I* is insufficient information (incomplete data), the maximization of $f(I/\theta)$ is not tractable and does not lead to an explicit expression [22]. Defining *D*, as the needed information to complete *I*, the ML estimation of θ becomes

$\hat{\theta} = \arg\max_{\theta}\{\int f(C/\theta)dD\}$, where *C={I,D}* is defined as the complete data.

Let , $l_y(\theta) = f(C/\theta)$ and the log likelihood function $L_y(\theta) = \log f(C/\theta)$.

The basic idea behind the EM algorithm is that we would like to find θ to maximize $\log f(C/\theta)$, but we do not have the D to compute the log-likelihood. So instead, we maximize the expectation of $\log f(C/\theta)$ given I and our current estimate of θ. This can be expressed in two steps. Let $\theta^{(p)}$ be the estimate of the parameters at p^{th} iteration.

$$\text{Let } Q(\theta/\theta^{(p)}) = E[\log(C/\theta)/I,\theta^{(p)}] \tag{3.1.2}$$

If $Q(\theta/\theta^{(p)}) > Q(\theta^{(p)}/\theta^{(p)})$, $L_y(\theta) > L_y(\theta^{(p)})$ is guaranteed.. Therefore by maximizing $Q(\theta/\theta^{(p)})$ with respect to θ causes the log-likelihood function $L_y(\theta)$ to monotonically increase .The idea of increasing the likelihood function iteratively is the main core of the EM algorithm, which has two steps; expectation and maximization. In the first step the conditional expectation of the complete data given the most recent estimation of the parameters , $Q(\theta/\theta^{(p)})$ is computed and in the second step the new estimation of the parameters, $\theta^{(p+1)}$ is chosen by maximizing $Q(\theta/\theta^{(p)})$.Steps Of EM algorithm are

E-step: $$Q(\theta/\theta^{(p)}) = E[\log f(C/\theta)/I,\theta^{(p)}] \tag{3.1.3}$$

The second argument $\theta^{(p)}$ is a conditioning argument to the expectation and known at every E-step. The first argument conditions the likelihood of the complete data.

For the M-step let $\theta^{(p+1)}$ be that value of θ which maximizes $Q(\theta/\theta^{(p)})$

M-Step:

$$\theta^{(p+1)} = \arg\max_{\theta} Q(\theta/\theta^{(p)}) \tag{3.1.4}$$

The maximization is with respect to the first argument θ of the Q function, the conditioner of the complete data likelihood. The EM algorithm consists of choosing an initial $\theta^{(p)}$, then performing the E-step and the M-step iterratiely until convergence.

For the EM algorithm, the convergence may be stated simply: at every iteration of the EM algorithm, a value of the parameter is computed so that the likelihood function does not decrease. Every iteration of the EM algorithm increases the likelihood function until a point of (local) maximum is reached. At this point the likelihood function cannot increase (but will not decrease) [22].

Convergence may be determined by examining when the parameters quit changing, i.e., stop when

$\left\|\theta^{(m)} - \theta^{(p+1)}\right\| < \varepsilon$ For some ε

For likelihood functions with multiple maxima, convergence will be to a local maximum, which depends on the initial starting point $\theta^{(0)}$.

The EM algorithm has the advantage of being simple in principle but computing the expectations and performing the maximizations may be computationally taxing.

3.1.1 Channel Estimation for SISO OFDM System

Channel estimation is usually performed using known training sequences periodically transmitted, implicitly assuming that the channel does not vary between two training sequences. Thus in order to enhance the mobility of wireless: systems and cope with the Doppler effects, reference sequences have to be repeated more often resulting in a significant loss in useful bit rate. But in EM based technique, channel variations are tracked by refining the channel coefficients blindly using the training sequences as initializations for the estimator [10].

We can see that the conventional EM algorithm can be applied to solve channel estimation problem for OFDM systems [8]. We assume that the transmitted signal is QPSK modulated with constellation size C (= 4). We denote the symbols in the constellation by $X_i(m)$, $1 \le i \le 4$.

From Eq. (2.1.4), assuming that each carrier is undergoing independent fading, we consider *H(m)* as deterministic parameter to be estimated from the observed data *Y(m)*. *Y(m)* is insufficient information, *X(m)*, as the unknown information.

Log likelihood function (cost function) for OFDM system can be defined as $\log f(Y(m),X(m)/H(m))$, where incomplete and complete data sets are $Y(m)$ and *(Y(m), X(m))*.

At $(p+1)^{th}$ iteration, in expectation step of EM technique, cost function is averaged over all possible values of transmitted data given the received symbol and channel estimate at p^{th} iteration.

E-Step:

$$Q(H(m)/H^{(p)}(m)) = \mathbf{E}_X[\log f(Y(m),X(m)/H(m))/Y(m),H^{(p)}(m)] \tag{3.1.5}$$

$$= \sum_{i=1}^{C} \log\left\{\frac{1}{C} f_i(Y(m)/H(m))\right\} \frac{f_i(Y(m)/H^{(p)}(m))}{Cf(Y(m)/H^{(p)}(m))} \tag{3.1.6}$$

Where,

$$f_i(Y(m)/H(m)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}\|Y(m) - H(m)X_i(m)\|^2\right\} \tag{3.1.7}$$

$$f_i(Y(m)/H^{(p)}(m)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}\|Y(m) - H^{(p)}(m)X_i(m)\|^2\right\} \tag{3.1.8}$$

$$f(Y(m)/H^{(p)}(m)) = \frac{1}{\sqrt{2\pi\sigma^2}C} \sum_{i=1}^{C} \exp\left\{-\frac{1}{2\sigma^2}\|Y(m) - H^{(p)}(m)X_i(m)\|^2\right\} \tag{3.1.9}$$

M-Step:

Putting the above values of conditional PDFs from equations (3.1.7), (3.1.8) and (3.1.9) in equation (3.1.6), and maximizing the Q function, the estimated channel parameter at $(p+1)^{th}$ iteration is obtained as,

$$\hat{H}^{(p+1)}(m) = \left[\sum_{i=1}^{C} X_i(m) X_i^*(m) \frac{f_i(Y(m)/H^{(p)}(m))}{f(Y(m)/H^{(p)}(m))}\right]^{-1} \times \left[\sum_{i=1}^{C} Y(m) X_i^*(m) \frac{f_i(Y(m)/H^{(p)}(m))}{f(Y(m)/H^{(p)}(m))}\right] \tag{3.1.10}$$

where * stands for complex conjugate operation.

These steps are performed till convergence.

Channel is estimated in frequency domain at each iteration and then IFFT is computed to convert it into time domain that has M paths. In those M paths only L paths are relevant, all other paths are made as zero and again FFT is computed as shown in fig. 3.1 [8].

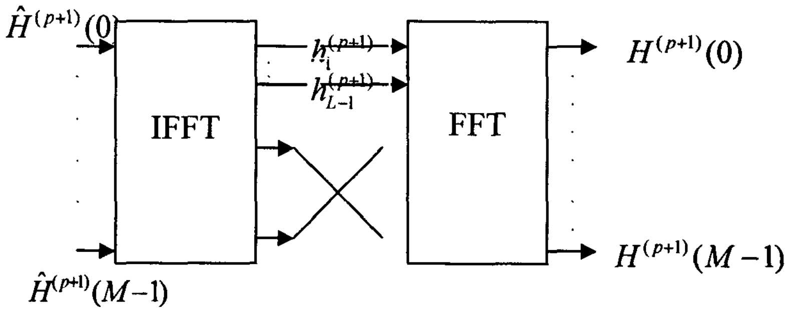


Figure 3.1 Filter to remove the effect of noise

From the general convergence property of the EM algorithm, there is no guarantee that the iterative steps converge to a global maximum. For a likelihood function with multiple local maxima the convergence point may be one of these local maxima, depending on the initial estimate $\hat{H}^{(0)}(m)$. The equally spaced pilot symbols with linear interpolation are used.

3.1.2 Channel Estimation for MISO-STC-OFDM System

In previous section, EM technique is applied for single transmitter and single receiver systems. We now consider EM technique for channel estimation in two transmitter-single receiver space-time coded OFDM system. We assume that the transmitted signal is QPSK modulated with constellation size C(= 4). We denote the symbols in the constellation by X_i , $1 \le i \le 4$.

Eq. (2.2.6) and Eq. (2.2.7) can be represented for single carrier of each frame,

$$r_1 = H_1 X_o + H_2 X_e + N_1$$

$$r_2 = -H_1 X_e^* + H_2 X_o^* + N_2$$

where received symbol for m^{th} subcarrier in first time slot *n* is r_1 and in time slot *(n+1)* is represented as r_2. H_1 and H_2 are the frequency domain impulse responses of channel for a subcarrier. For simplification the notation *m* has been omitted.

Taking each carrier independent, consider (H_1, H_2) as deterministic parameter to be estimated from the observed data (r_1, r_2). (r_1, r_2) is insufficient information and (X_o, X_e) is the unknown information. Log likelihood function (cost function) for this system can be defined as, $\log f(r_1, r_2, X_o, X_e / H_1, H_2)$, where incomplete and complete data sets are (r_1, r_2) and (r_1, r_2, X_o, X_e). $(H_1^{(p)}, H_2^{(p)})$ is the estimated parameter at p^{th} iteration.

E-Step:

At $(p+1)^{th}$ iteration, in expectation step of EM technique, cost function is averaged over all possible values of transmitted data X_o & X_e, given the received symbols r_1, r_2 and channel estimate $H_1^{(p)}, H_2^{(p)}$ at p^{th} iteration.

$$Q(H_1, H_2 / H_1^{(P)}, H_2^{(P)}) = \mathop{\mathbf{E}}_{X_o X_e} [\log f(r_1, r_2, X_o, X_e / H_1, H_2) / H_1^{(P)}, H_2^{(P)}, r_1, r_2] \tag{3.1.11}$$

$$Q(H_1, H_2 / H_1^{(P)}, H_2^{(P)}) =$$

$$\sum_{i=1}^{C}\sum_{j=1}^{C}[\ \log f(r_1,r_2 / H_1,H_2).f(X_i,X_j / H_1^{(P)},H_2^{(P)},r_1,r_2)\ ] \qquad (3.1.12)$$

Using Baye's theorem,

$$f(X_i,X_j / H_1^{(P)},H_2^{(P)},r_1,r_2) = \frac{f(r_1,r_2,X_i,X_j / H_1^{(P)},H_2^{(P)})}{f(r_1,r_2 / H_1^{(P)},H_2^{(P)})}$$

$$= \frac{1}{C^2}\frac{f_{i,j}(r_1,r_2 / H_1^{(P)},H_2^{(P)})}{f(r_1,r_2 / H_1^{(P)},H_2^{(P)})} \qquad (3.1.13)$$

where,

$$f_{i,j}(r_1,r_2 / H_1^{(P)},H_2^{(P)}) =$$

$$\frac{1}{2\pi\sigma^2}\exp\left(-\frac{\left\|r_1 - H_1^{(p)} * X_i - H_2^{(p)} * X_j\right\|^2 + \left\|r_2 + H_1^{(p)} * X_j^* - H_2^{(p)} * X_i^*\right\|^2}{2\sigma^2}\right)$$

$$(3.1.14)$$

and

$$f(r_1,r_2 / H_1^{(P)},H_2^{(P)}) =$$

$$\sum_{i=1}^{C}\sum_{j=1}^{C}\frac{1}{2\pi\sigma^2 C^2}\exp\left(-\frac{\left\|r_1 - H_1^{(p)} * X_i - H_2^{(p)} * X_j\right\|^2 + \left\|r_2 + H_1^{(p)} * X_j^* - H_2^{(p)} * X_i^*\right\|^2}{2\sigma^2}\right)$$

$$(3.1.15)$$

Taking log of Eq. (3.1.15)

$$\log f(r_1,r_2 / H_1^{(P)},H_2^{(P)}) =$$

$$\sum_{i=1}^{C}\sum_{j=1}^{C}\left(-\frac{\left\|r_1 - H_1^{(p)} * X_i - H_2^{(p)} * X_j\right\|^2 + \left\|r_2 + H_1^{(p)} * X_j^* - H_2^{(p)} * X_i^*\right\|^2}{2\sigma^2}\right) + \log K$$

$$(3.1.16)$$

using equations (3.1.13) (3.1.14) (3.1.15) and (3.1.16) into equation (3.1.12), we get

$$Q(H_1, H_2 / H_1^{(P)}, H_2^{(P)}) =$$

$$\left(\sum_{i=1}^{C} \sum_{j=1}^{C} \left[\left(-\frac{\left\| r_1 - H_1^{(p)} * X_i - H_2^{(p)} * X_j \right\|^2 + \left\| r_2 + H_1^{(p)} * X_j^* - H_2^{(p)} * X_i^* \right\|^2}{2\sigma^2} \right) + \log K \right] \times f(X_i, X_j / H_1^{(P)}, H_2^{(P)}, r_1, r_2) \right)$$

$$\text{Let, } K_{ij} = -\frac{1}{2\sigma^2} f(X_i, X_j / H_1^{(P)}, H_2^{(P)}, r_1, r_2),$$

$$Q(H_1, H_2 / H_1^{(P)}, H_2^{(P)}) =$$

$$\left[\sum_{i=1}^{C} \sum_{j=1}^{C} K_{ij} \left(-\frac{\left\| r_1 - H_1^{(p)} * X_i - H_2^{(p)} * X_j \right\|^2 + \left\| r_2 + H_1^{(p)} * X_j^* - H_2^{(p)} * X_i^* \right\|^2}{2\sigma^2} \right) + \log K \right] \tag{3.1.17}$$

M-step:

By maximizing the Q function, we can find the channel parameters at $(p+1)^{th}$ iteration,

Differentiating Eq. (3.1.17) with respect to H_1, H_2

$$\frac{dQ(H_1, H_2 / H_1^{(P)}, H_2^{(P)})}{dH_1} = 0, \qquad \frac{dQ(H_1, H_2 / H_1^{(P)}, H_2^{(P)})}{dH_2} = 0$$

when differentiating with H_1,

$$\left[\sum_{i=1}^{C} \sum_{j=1}^{C} K_{ij} \left[X_i . X_i^* + X_j . X_j^* \right] \right] . \hat{H}_1^{(p+1)} = \left[\sum_{i=1}^{C} \sum_{j=1}^{C} K_{ij} \left[r_1 . X_i^* - r_2 . X_j \right] \right],$$

$$\hat{H}_1^{(p+1)} = \left[\sum_{i=1}^{C} \sum_{j=1}^{C} K_{ij} \left[r_1 . X_i^* - r_2 . X_j \right] \right] \left[\sum_{i=1}^{C} \sum_{j=1}^{C} K_{ij} \left[X_i . X_i^* + X_j . X_j^* \right] \right]^{-1}$$

$$\hat{H}_1^{(p+1)} = \left[\sum_{i=1}^{C} \sum_{j=1}^{C} -\frac{1}{2\sigma^2} f(X_i, X_j / H_1^{(P)}, H_2^{(P)}, r_1, r_2) \left[r_1 . X_i^* - r_2 . X_j \right] \right] \times \left[\sum_{i=1}^{C} \sum_{j=1}^{C} -\frac{1}{2\sigma^2} f(X_i, X_j / H_1^{(P)}, H_2^{(P)}, r_1, r_2) \left[X_i . X_i^* + X_j . X_j^* \right] \right]^{-1}$$

From Eq. (3.1.13)

$$\hat{H}_1^{(p+1)} = \left[\sum_{i=1}^{C}\sum_{j=1}^{C} -\frac{1}{2\sigma^2}\frac{1}{C^2}\frac{f_{i,j}(r_1,r_2/H_1^{(P)},H_2^{(P)})}{f(r_1,r_2/H_1^{(P)},H_2^{(P)})}\left[r_1.X_i^* - r_2.X_j\right]\right] \times\left[\sum_{i=1}^{C}\sum_{j=1}^{C} -\frac{1}{2\sigma^2}\frac{1}{C^2}\frac{f_{i,j}(r_1,r_2/H_1^{(P)},H_2^{(P)})}{f(r_1,r_2/H_1^{(P)},H_2^{(P)})}\left[X_i.X_i^* + X_j.X_j^*\right]\right]^{-1} \tag{3.1.18}$$

$$\hat{H}_1^{(p+1)} = \left[\sum_{i=1}^{C}\sum_{j=1}^{C} f_{i,j}(r_1,r_2/H_1^{(P)},H_2^{(P)})\left[r_1.X_i^* - r_2.X_j\right]\right] \times\left[\sum_{i=1}^{C}\sum_{j=1}^{C} f_{i,j}(r_1,r_2/H_1^{(P)},H_2^{(P)})\left[X_i.X_i^* + X_j.X_j^*\right]\right]^{-1} \tag{3.1.19}$$

now differentiating with H_2,

$$\left[\sum_{i=1}^{C}\sum_{j=1}^{C} K_{ij}\left[X_i.X_i^* + X_j.X_j^*\right]\right].\hat{H}_2^{(p+1)} = \left[\sum_{i=1}^{C}\sum_{j=1}^{C} K_{ij}\left[r_1.X_j^* + r_2.X_i\right]\right],$$

$$\hat{H}_2^{(p+1)} = \left[\sum_{i=1}^{C}\sum_{j=1}^{C} K_{ij}\left[r_1.X_j^* + r_2.X_i\right]\right]\left[\sum_{i=1}^{C}\sum_{j=1}^{C} K_{ij}\left[X_i.X_i^* + X_j.X_j^*\right]\right]^{-1} \tag{3.1.20}$$

Solving Eq. (3.1.20) for $\hat{H}_2^{(p+1)}$ as Eq. (3.1.18) is solved to calculated $\hat{H}_1^{(p+1)}$.

$$\hat{H}_2^{(p+1)} = \left[\sum_{i=1}^{C}\sum_{j=1}^{C} f_{i,j}(r_1,r_2/H_1^{(P)},H_2^{(P)})\left[r_1.X_j^* + r_2.X_i\right]\right] \times\left[\sum_{i=1}^{C}\sum_{j=1}^{C} f_{i,j}(r_1,r_2/H_1^{(P)},H_2^{(P)})\left[X_i.X_i^* + X_j.X_j^*\right]\right]^{-1} \tag{3.1.21}$$

For initial estimation, $\hat{H}_1^{(0)},\hat{H}_2^{(0)}$ will be determined using pilot based techniques. The estimated channel parameters $\hat{H}_1^{(p)},\hat{H}_2^{(p)}$ are passed through the filter shown in fig. 3.1 to remove effect of noise.

3.2 EM-MMSE Technique for Channel Estimation

It may be been seen in Eq. (3.1.3) that expectation of cost function (Q) function is taken over all the possible values of unknown parameters. The EM technique has the advantage of being simple in principle but computing the expectations and performing the maximizations may be computationally taxing. To reduce this computational complexity, EMVA has been proposed in literature [38]. In this method instead of estimating the cost function for all possible values of transmitted vector, estimation is taken only for the main survivor paths. These survivor paths are calculated by the Viterbi algorithm.

In conventional EM-based techniques for estimating the channel parameters in OFDM systems, a cost function is defined in terms of received signal, channel information and transmitted signal. Transmitted signal and channel information is unknown at the receiver. In E-step of EM technique, averaging of cost function is done on all possible values of transmitted data. Then in M-step, the estimated cost function is maximized to estimate the channel parameters. This process is done iteratively until convergence.

In the proposed EM-MMSE technique, we propose an improved EM based technique wherein we first estimate the transmitted symbols by MMSE method, with the knowledge of channel estimate at particular iteration. In E-step, estimation of cost function is performed only for symbols estimated using MMSE method.

General statement of EM-MMSE Technique

From section (3.1) we can see that the basic idea behind the EM algorithm is that we would like to find θ to maximize $\log f(C/\theta)$, but we do not have the D (unknown parameter) to compute the log-likelihood. So instead, we maximize the expectation of $\log f(C/\theta)$ given I and our current estimate of θ. In EM-MMSE method this D is computed by the knowledge of incomplete information I and initial estimate $\theta^{(0)}$. EM-MMSE technique has three steps; calculation of D lets say it as step A, expectation step and maximization step.

Step A:

In the first step of the EM-MMSE technique D is calculated by the knowledge of knowledge of incomplete information I and initial estimate $\theta^{(0)}$, let this be denoted by $\widetilde{D}^{(p)}$.

E-step: From Eq. (3.1.3), Taking expectation of Q function over $\widetilde{D}^{(p)}$

$$Q(\theta / \theta^{(p)}) = \underset{\widetilde{D}^{(p)}}{E}[\log f(C / \theta) / I, \theta^{(p)}] \tag{3.2.1}$$

For the M-step let $\theta^{(p+1)}$ be that value of θ which maximizes $Q(\theta / \theta^{(p)})$

M-Step:

$$\theta^{(p+1)} = \arg\max_{\theta} Q(\theta / \theta^{(p)}) \tag{3.2.2}$$

The maximization is with respect to the first argument θ of the Q function, the conditioner of the complete data likelihood. The EM-MMSE technique consists of choosing an initial $\theta^{(0)}$, then performing Step A, the E-step and the M-step iteratively until convergence.

3.2.1 Channel Estimation for SISO OFDM System

In conventional EM technique, we estimate the cost function (Q from Eq. (3.1.5)) for all possible values of transmitted symbol. But in EM MMSE, at each iteration, we first find the transmitted sequence using the knowledge of channel at that iteration. The transmitted sequence is calculated using MMSE method in step A.

Equation (2.1.4) can be represented for each frame as,

$$\underline{Y} = \underline{H} \bullet \underline{X} + \underline{N} \tag{3.2.3}$$

where $\bullet$ operator denotes Hadamard product and $\underline{H}$ is the channel frequency response vector of dimension $1 \times M$.

At $(p+1)^{th}$ iteration,

Step A:

First $\left\| Y(m) - H^{(p)}(m) X_i(m) \right\|^2$ is calculated for $1 \le i \le 4$, for each carrier and the transmitted sequence is calculated at p^{th} iteration using MMSE method. This decision is taken for $0 \le m \le M-1$ and estimated transmitted vector at iteration p is represented by $\underline{\widetilde{X}}^{(p)}$.

E-step:

Log Likelihood (cost function) for OFDM system in vector form for a frame can be defined as $\log f(\underline{Y}, \underline{X} / \underline{H})$.

EM-MMSE can be directly applied for each frame,

Equation (3.1.5) can be written in vector form,

$$Q(\underline{H} / \underline{H}^{(p)})$$
$$= \mathbf{E}_{\underline{\widetilde{X}}^{(p)}} [\log f(\underline{Y}, \underline{X} / \underline{H}) / \underline{Y}, \underline{H}^{(p)}]$$

After taking expectation on estimated transmitted sequence $\underline{\widetilde{X}}^{(p)}$, given the received symbol $\underline{Y}$ and channel estimate $\underline{H}^{(p)}$ at p^{th} iteration.

$$Q(\underline{H} / \underline{H}^{(p)}) = [\log f(\underline{Y}, \underline{\widetilde{X}}^{(p)} / \underline{H}) / \underline{Y}, \underline{H}^{(p)}] \tag{3.2.4}$$

M-Step:

Maximizing the Q function of Eq. 3.2.4, we will get,

$$\underline{\hat{H}}^{(p+1)} = \left[\underline{\widetilde{X}}^{(p)} \bullet \underline{\widetilde{X}}^{(p)^*} \right]^{-1} \times \left[\underline{Y} \bullet \underline{\widetilde{X}}^{(p)^*} \right] \tag{3.2.5}$$

Using step A and (3.2.3), iteratively, channel is estimated in frequency domain. The estimated channel parameters $\hat{H}^{(p)}$ are passed through the filter shown in fig. 3.1 to remove effect of noise.

3.2.2 Channel Estimation for MISO-STC-OFDM System

For space-time coded OFDM system as shown in fig. 2.3, received vector in first time slot *n*, will be represented as $\underline{r}_1$ and in time slot *(n+1)* will be represented as $\underline{r}_2$ and from Eq. (2.2.6) and (2.2.7),

$$\underline{r}_1 = \underline{H}_1 \bullet \underline{X}_o + \underline{H}_2 \bullet \underline{X}_e + \underline{N}_1$$

$$\underline{r}_2 = -\underline{H}_1 \bullet \underline{X}_e^* + \underline{H}_2 \bullet \underline{X}_o^* + \underline{N}_2$$

Applying EM-MMSE channel estimation technique,

Step A:

First,

$$\left\| r_1(m) - H_1^{(p)}(m) \bullet X_i(m) - H_2^{(p)}(m) \bullet \underline{X}_j(m) \right\|^2 + \left\| r_2(m) + H_1^{(p)}(m) \bullet X_i^*(m) - H_2^{(p)}(m) \bullet X_j^*(m) \right\|^2$$

is calculated for $1 \le i \le 4$ and $1 \le j \le 4$, for m^{th} carrier, where $H_1^{(p)}(m)$ and $H_2^{(p)}(m)$ are estimated channel coefficients at p^{th} iteration for m^{th} carrier. The decision of transmitted symbol is taken by MMSE method at p^{th} iteration for m^{th} carrier. The decision is taken for $0 \le m \le M-1$ and estimated transmitted vector for each OFDM frame can be represented as $\underline{\widetilde{X}}_o^{(p)}$ & $\underline{\widetilde{X}}_e^{(p)}$.

E-step:

Log likelihood function (cost function) for OFDM system can be defined as, $\log f(\underline{r}_1, \underline{r}_2, \underline{X}_o, \underline{X}_e / \underline{H}_1, \underline{H}_2)$, where incomplete and complete data sets are $(\underline{r}_1, \underline{r}_2)$ and $(\underline{r}_1, \underline{r}_2, \underline{X}_o, \underline{X}_e)$. $(\underline{H}_1^{(p)}, \underline{H}_2^{(p)})$ is the estimated parameter at p^{th} iteration.

Equation (3.1.11) can be written in vector form,

$$Q(\underline{H}_1, \underline{H}_2 / \underline{H}_1^{(P)}, \underline{H}_2^{(P)}) = \mathop{\mathbf{E}}_{\underline{\widetilde{X}}_o^{(p)} \underline{\widetilde{X}}_e^{(p)}} [\log f(\underline{r}_1, \underline{r}_2, \underline{X}_o, \underline{X}_e / \underline{H}_1, \underline{H}_2) / \underline{H}_1^{(P)}, \underline{H}_2^{(P)}, \underline{r}_1, \underline{r}_2] \tag{3.2.6}$$

After taking expectation of cost function on estimated transmitted sequence $\underline{\tilde{X}}_o^{(p)}$ & $\underline{\tilde{X}}_e^{(p)}$, given the received symbol $\underline{r}_1, \underline{r}_2$ and channel estimate $\underline{H}_1^{(p)}$ & $\underline{H}_2^{(p)}$ at p^{th} iteration, Eq. (3.2.10) can be written as,

$$Q(\underline{H}_1, \underline{H}_2 / \underline{H}_1^{(P)}, \underline{H}_2^{(P)}) = [\log f(\underline{r}_1, \underline{r}_2, \underline{\tilde{X}}_o^{(P)}, \underline{\tilde{X}}_e^{(P)} / \underline{H}_1, \underline{H}_2) / \underline{H}_1^{(P)}, \underline{H}_2^{(P)}, \underline{r}_1, \underline{r}_2] \tag{3.2.7}$$

M-Step:

By maximizing this Q function and solving as in section (3.1.2), we can find the channel parameters at $(p+1)^{th}$ iteration as,

$$\begin{aligned}\underline{\hat{H}}_1^{(p+1)} &= \left[\underline{r}_1 \bullet \underline{\tilde{X}}_o^{*(p)} - \underline{r}_2 \bullet \underline{\tilde{X}}_e^{(p)}\right] \\ &\times \left[\underline{\tilde{X}}_o \bullet \underline{\tilde{X}}_o^{*(p)} + \underline{\tilde{X}}_e \bullet \underline{\tilde{X}}_o^{*(p)}\right]^{-1}\end{aligned} \tag{3.2.8}$$

$$\begin{aligned}\underline{\hat{H}}_2^{(p+1)} &= \left[\underline{r}_1 \bullet \underline{\tilde{X}}_e^{*(p)} + \underline{r}_2 \bullet \underline{\tilde{X}}_o^{(p)}\right] \\ &\times \left[\underline{\tilde{X}}_o \bullet \underline{\tilde{X}}_o^{*(p)} + \underline{\tilde{X}}_e \bullet \underline{\tilde{X}}_o^{*(p)}\right]^{-1}\end{aligned} \tag{3.2.9}$$

Using step A and Eq. (3.2.8) and (3.2.9), iteratively, channel is estimated in frequency domain. For the design of a good channel estimator, number of pilot symbols must be small and computational complexity must be less. EM-MMSE technique is computationally less complex and requires less number of iterations than EM technique, while giving better performance than pilot based methods.

Chapter 4

Simulation and Results

In this chapter, for SISO-OFDM system considered in this work, we first present channel models and then simulation techniques for channel estimation. We then present simulation techniques for channel estimation in MISO-STC-OFDM system. We finally present the simulation results in order to evaluate the performance of the proposed EM-MMSE based estimation techniques.

4.1 SISO-OFDM System Model

Figure 4.1.1 shows the implementation of the proposed SISO-OFDM system. The channel is assumed to be quasi-static and does not vary within each OFDM frame. For this proposed system equally spaced pilots are assumed to be known at the receiver to obtain an initial estimate over an OFDM frame. After calculating the initial estimate, channel estimation is done using EM-MMSE technique. Received array is decoded using the estimated channel parameters.

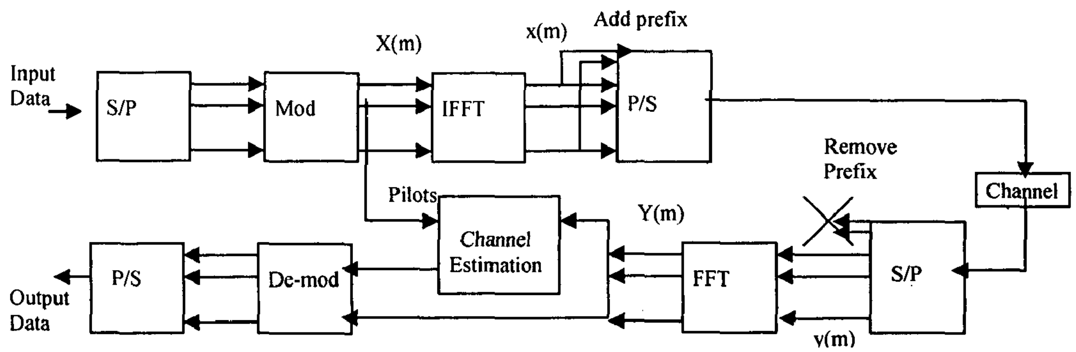


Figure 4.1.1 Block schematic of the proposed SISO-OFDM simulation structure

Simulation Environment and System parameters

In order to evaluate the performance of the proposed channel estimation technique, it is essential that transmission channel is satisfactorily characterized. The radio link performance in a mobile environment is primarily limited by Doppler and delay spread. In this work we have used either the Rayleigh stationary multipath fading envionment or time varying environment characterized by autoregressive process.

In this work an operating frequency of 2 GHz and a channel bandwidth of 2048 kHz have been assumed. Each OFDM frame is having 128 carriers with QPSK modulation. The subcarrier spacing is $\Delta_f = 16$ kHz and symbol duration T=1/ Δ_f=62.5 μs. We have assumed 4-tap multipath channel i.e. the length of CP is 3.

The channel is assumed to be quasi-static and does not vary within each frame. For Rayleigh stationary multipath fading, channel has been assumed independent for each OFDM frame and for Time selective multipath fading, channel varies for each frame according to Doppler values. We have taken moderate Doppler values for this work i.e. $f_d T = 0.01$ and $f_d T$=0.005 (Doppler frequencies are 160 Hz and 80 Hz respectively).

Simulation has been done in MATLAB environment. The system performance is evaluated for SNR values of 0-30 dB by averaging over 500 frames i.e. 1.28×10^5 bits.

4.1.1 Rayleigh Stationary Multipath Fading Channel Model

The impulse response *h(n)* of the stationary multipath fading channel between the transmit antenna and the receiver can be modeled as[4],

$$h(n) = \frac{1}{K}\sum_{k=0}^{L} e^{-k/2}\alpha_k \delta(n-k) \qquad (4.1.1)$$

where $K = \sqrt{\sum_{k=0}^{L} e^{-k}}$ is the normalization constant and $\alpha_k, 0 \le k \le L$ are independent complex-valued Gaussian distributed random variable with unit energy.

This is the conventional exponential decay multipath model. In this thesis, we have used, 4 tap stationary multipath channel model as in [5],

$$h(n) = .806\alpha_0\delta(n) + .486\alpha_1\delta(n-1) + .2952\alpha_2\delta(n-2) + .179\alpha_3\delta(n-3) \qquad (4.1.2)$$

Where, α_i for $0 \le i \le 3$ are independent complex valued Gaussian distributed random variables.

4.1.2 Time Varying Channel modeled using AR2 process

Time selective channel is approximated by an independent autoregressive process of order-2 (AR2). The channel tap vector for each OFDM frame is denoted by $h_n = [h(n,0)\ h(n,1) \ldots h(n,L-1)]$, where $h(n,l)$ is the l^{th} tap for the n^{th} frame and L is the length of time domain CIR.

Considering the AR2 model,

$$h(n,l) = a_1 h(n-1,l) + a_2 h(n-2,l) + v(n,l) \qquad (4.1.3)$$

Where a_1 and a_2 are the AR2 coefficients and $v(n,l)$ is the modeling noise for l^{th} tap at time frame *n*. The parameters a_1 and a_2 are closely related to the physical parameters of the underlying fading process. The values of AR2 coefficient can be obtained as,

$$a_1 = -2r_d \cos(2\pi f_p T) \text{ and } a_2 = r_d^2 \qquad (4.1.4)$$

where f_p is the spectral peak frequency, T is the symbol period, r_d is the pole radius that corresponds to the steepness of the peaks of the power spectrum

$$\text{i.e. } r_d = \left(1 - \frac{\omega_d}{\pi}\right) \qquad (4.1.5)$$

It has been observed in literature that when the spectral peak frequency $f_p = 0.8 \times f_d$ (f_d is the maximum Doppler frequency of the underlying fading channel), the autocorrelation function of AR2 process is close to the autocorrelation function of a fading process characterized by Bessel function.

The variance of the fading coefficient $h(n,l)$ is decided by the variance of $v(n,l)$, which is given by

$$\sigma_c^2 = \left(\frac{1+a_2}{1+a_1}\right)\frac{\sigma_w^2}{[(1+a_2)^2 - a_1^2]} \tag{4.1.6}$$

Channel Generation Algorithm

Figure 4.1.2 shows the steps involved for channel generation with the help of a flow chart. In order to generate the channel for K transmission blocks; we first calculate the channel taps L according to the value of delay spread τ_d and symbol period $T \approx 1/BW$ using the relation $L = 1 + \frac{\tau_d}{T}$. The value of Doppler frequency is obtained according to the operating frequency f_c and mobile speed **v** using the relation $f_d = \frac{\mathbf{v} f_c}{c}$.

Algorithm:

1. Obtain the values of Doppler frequency f_d, and symbol period T.
2. Calculate r_d using Eq. (4.1.5) and AR2 coefficients using Eq. (4.1.4).
3. Decide the variance σ_c^2 of the channel tap.
4. Calculate the variance σ_w^2 of the input noise to the AR2 process using Eq. (4.1.6) and generate the noise array $v(n,l)$.
5. Now generate the required channel taps using Eq. (4.1.3).

K values for each channel tap are generated in accordance with the aforementioned procedure. Note that the initial K+100 values are generated and than the first 100 values are discarded in order to make the process truly autoregressive in nature.

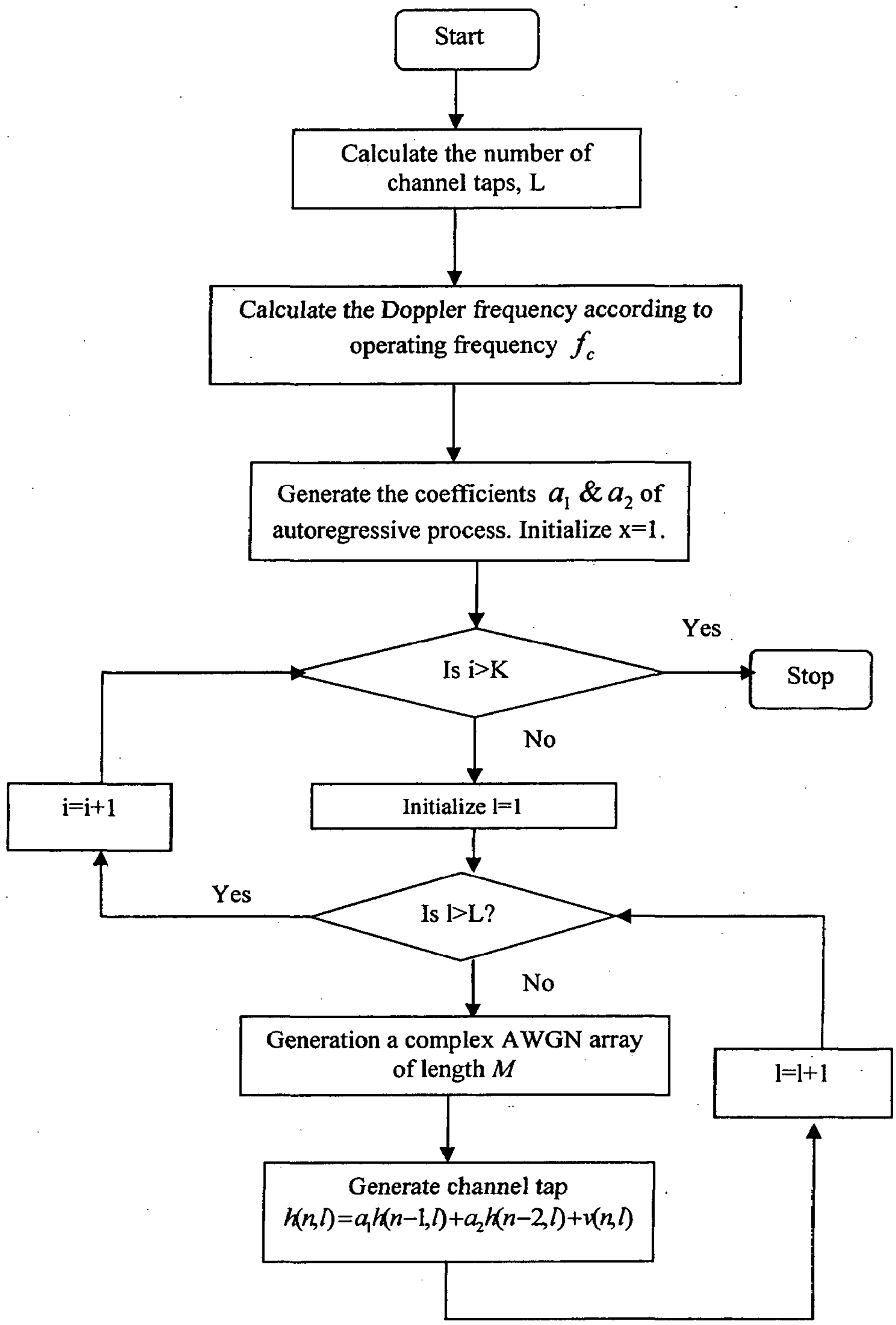


Figure 4.1.2 Flow Chart: Generation of Time varying channel

4.1.3 Simulation Techniques for Channel Estimation

At the receiver, the pilot symbols are extracted to provide a temporal estimate of the channel parameters at the pilot frequencies. From equation (2.14), at m^{th} subcarrier,

$$\hat{H}(m) = Y(m) / X_p(m) = H(m) + K \qquad (4.1.7)$$

where $\hat{H}(m)$ are estimated channel coefficient at m^{th} carrier and K is the effect of noise in the estimated parameter. $Y(m)$ and $X_p(m)$ are received symbol and pilot symbol at m^{th} carrier. When the noise is AWGN, then for the channel estimation, minimum number of pilots required is same as the channel length and these should be equally-placed in OFDM frame [9]. Channel coefficient is calculated first for equally spaced carriers and then linear interpolation is performed for estimating the channel coefficients for other sub-carriers. The method of linear interpolation is explained in section 2.3.2.

In EM and EM-MMSE based channel estimation, initial estimation of channel parameters is required. Figure 4.1.3 and fig 4.1.4 shows flow chart for implementation of EM and EM-MMSE techniques for channel estimation.

For EM technique, channel estimation is performed symbol-to-symbol basis. In EM, two steps are performed as shown in section 3.1 and channel parameters are estimated using Eq. 3.1.10 at each iteration.

For EM-MMSE technique, channel frequency response is estimated for each OFDM frame. In EM-MMSE, three steps are performed as shown in section 3.2. In step A, $\left\|Y(m) - H^{(p)}(m)X_i(m)\right\|^2$ is calculated for $1 \le i \le 4$, for each carrier and the transmitted sequence is calculated at p^{th} iteration using MMSE method. The decision is taken for $0 \le m \le M-1$ and estimated transmitted vector at iteration p is represented by $\underline{\tilde{X}}^{(p)}$ and channel parameters are estimated using step A and Eq. 3.2.5 at each iteration. These steps are computed till convergence is obtained.

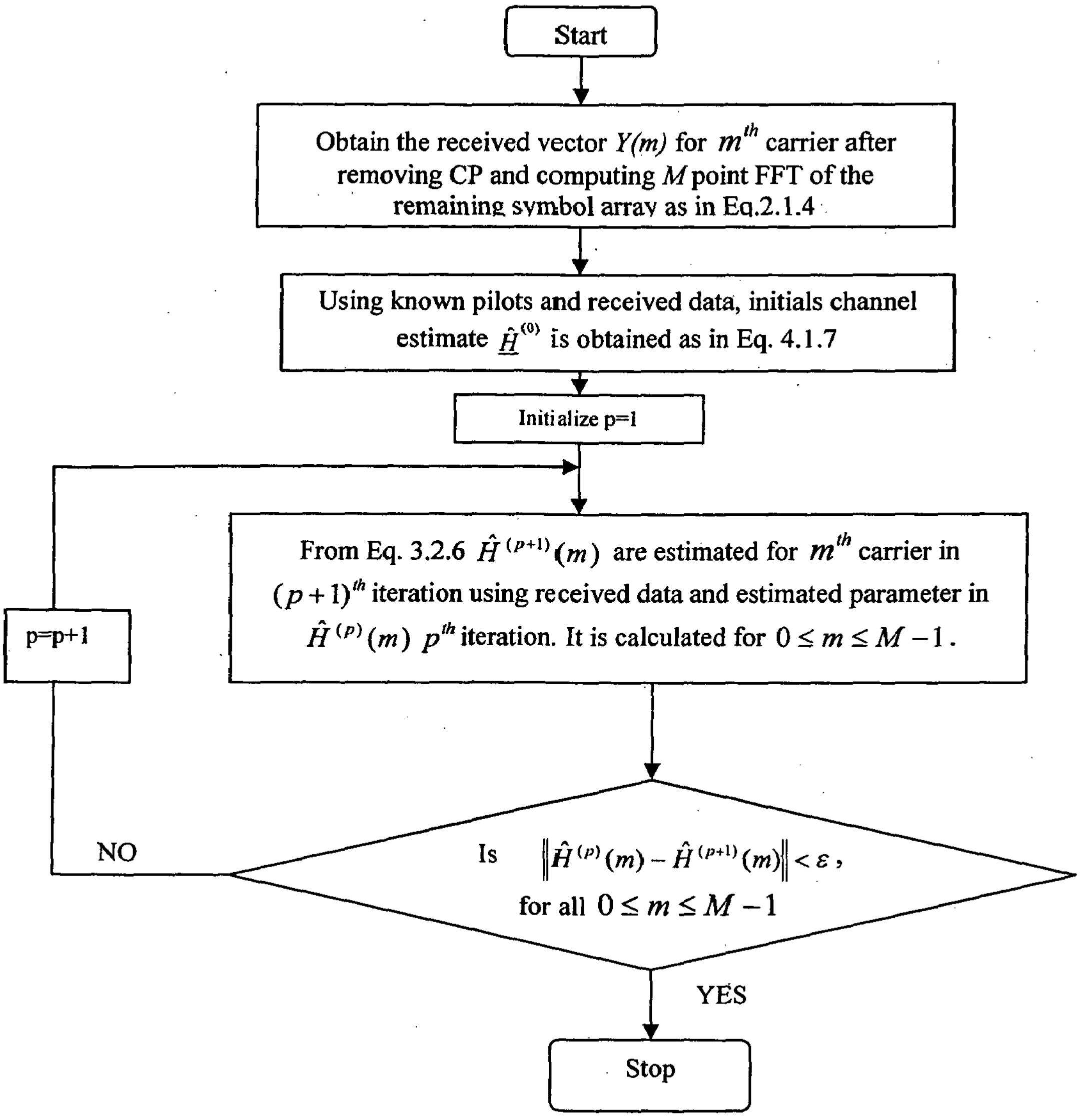


Figure 4.1.3 Flow Chart: EM based channel estimation Technique for SISO-OFDM systems

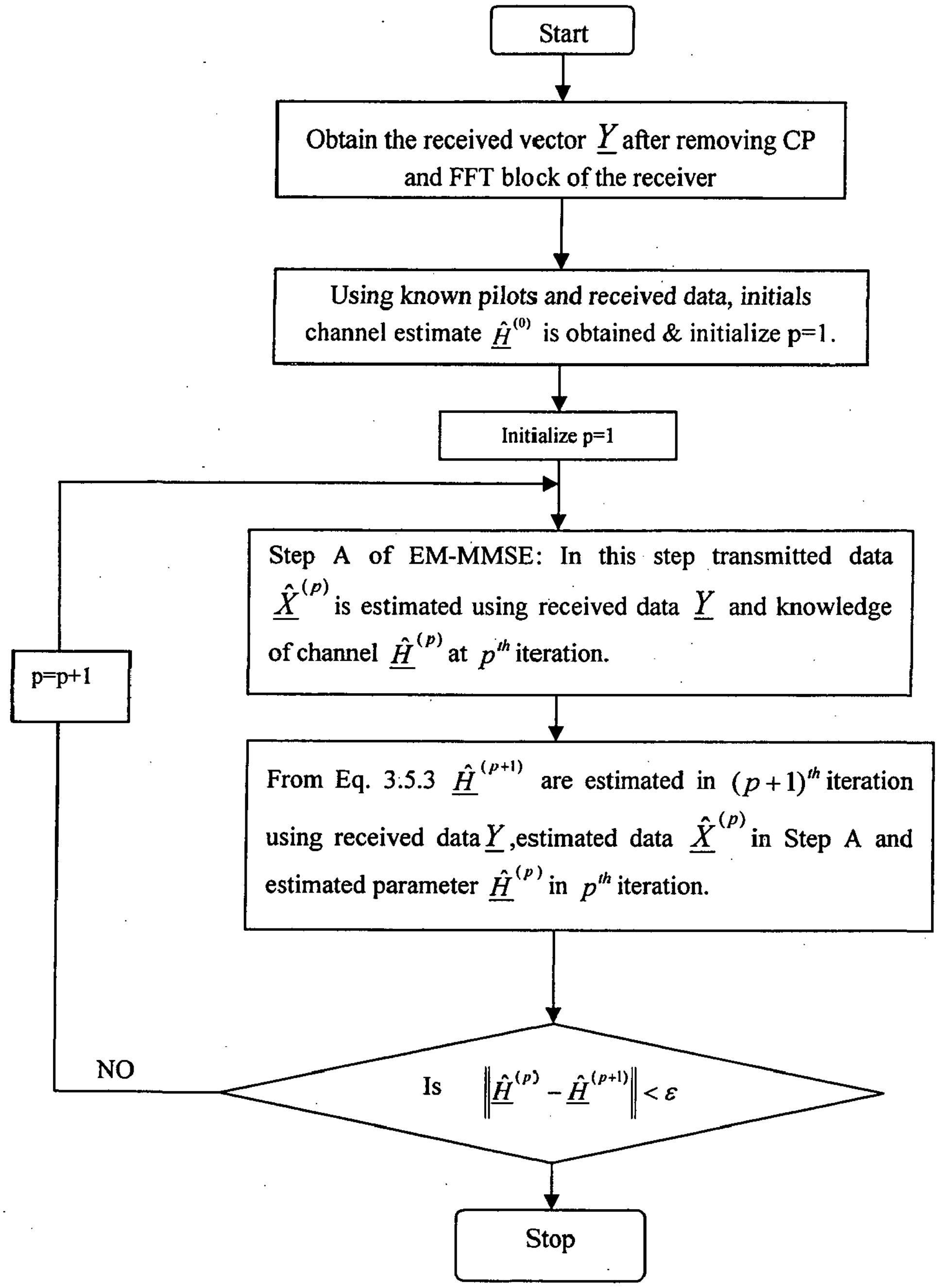


Figure 4.1.4 Flow Chart: EM-MMSE Technique to estimate the channel for SISO-OFDM system

4.1.4 System Simulation

Figure 4.1.5 shows flow chart for implementation of SISO-OFDM simulation structure. We first initialize system parameters. We then generate array of bits for i^{th} transmission block. These bits are then modulated using QPSK modulation to obtain symbol array $\underline{X}$. M point IFFT of each of these symbol arrays is obtained and CP of length M_c is appended to each block. Hence we obtain transmit OFDM frame $\underline{x}_f$. The transmitted array is corrupted by multipath fading and AWGN noise as shown in Eq. 2.1.3. This multipath fading channel is generated as explained in section 4.1.1 and 4.1.2.

The transmission of OFDM frame through the frequency selective channel is simulated with the help of overlap and save method. For i^{th} transmitted block, calculate the convolution of $\underline{x}_{i,f}$ and $\underline{h}_i$. Now store the last M_c values of these results in the buffer for the next iteration. To the first M_c values of these convolution results, add the buffer from previous iteration to obtain $y_{i,f}$.

After the reception of $y_{i,f}$, cyclic prefix is removed and y_i is obtained. Then M point FFT is performed to obtain $\underline{Y}$. At receiver initial estimate of the channel $\underline{\hat{H}}_i^{(0)}$ is obtained using known transmitted symbols i.e. pilots and received array $\underline{Y}_i$. Channel parameters are estimated iteratively by EM/EM-MMSE technique using $\underline{Y}_i$ and $\underline{\hat{H}}_i^{(0)}$ as shown in fig. 4.1.3 and 4.1.4. After estimating the channel parameters ML decoding is performed to obtain estimated transmitted array $\underline{\hat{X}}_i$.

These steps are performed for all K frames and then P/S is performed to obtain estimated transmitted bits. Now performance of system in terms of BER is calculated for different SNR values.

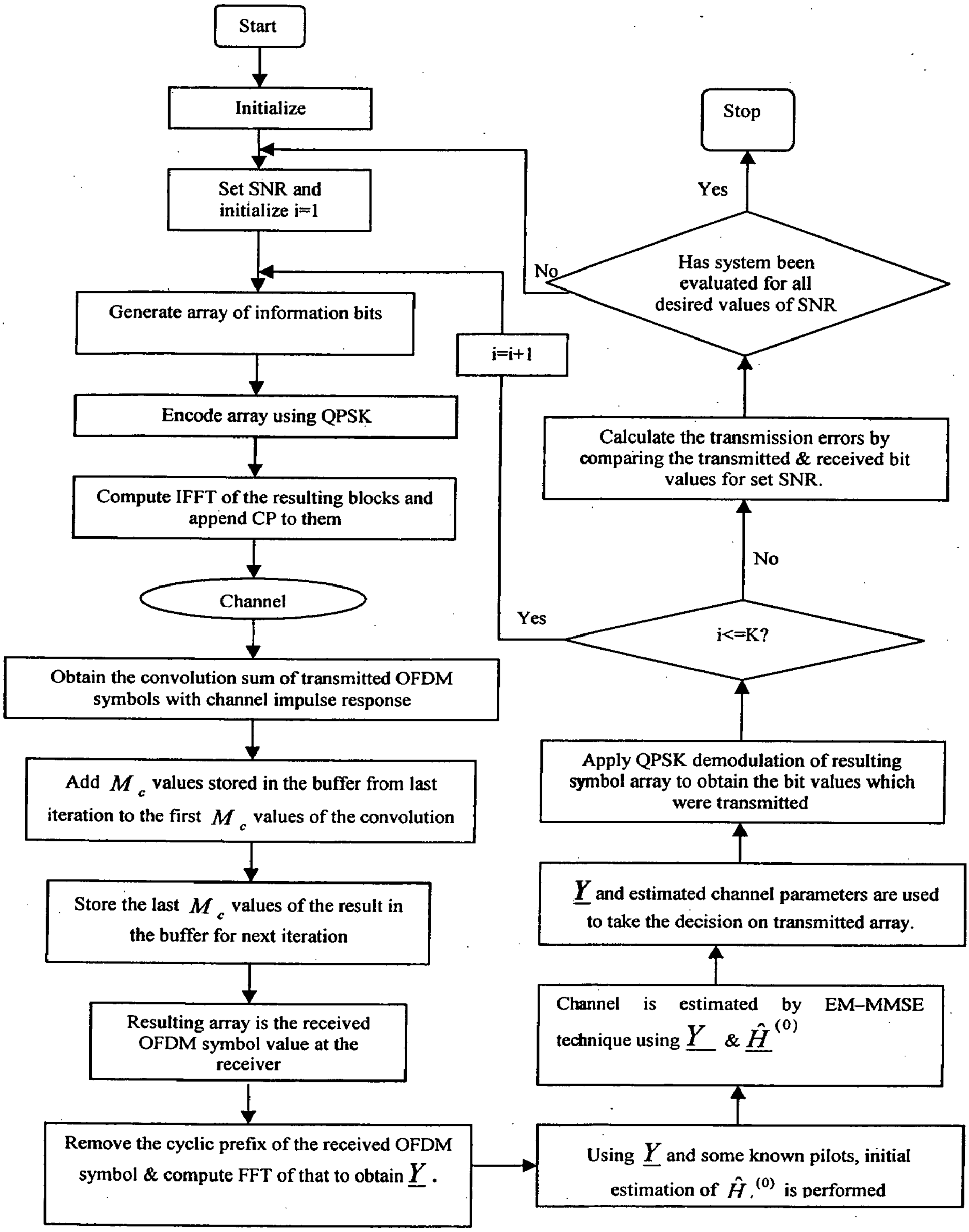


Figure 4.1.5 Flow Chart: System Simulation for SISO-OFDM

4.2 MISO-STC-OFDM System Model

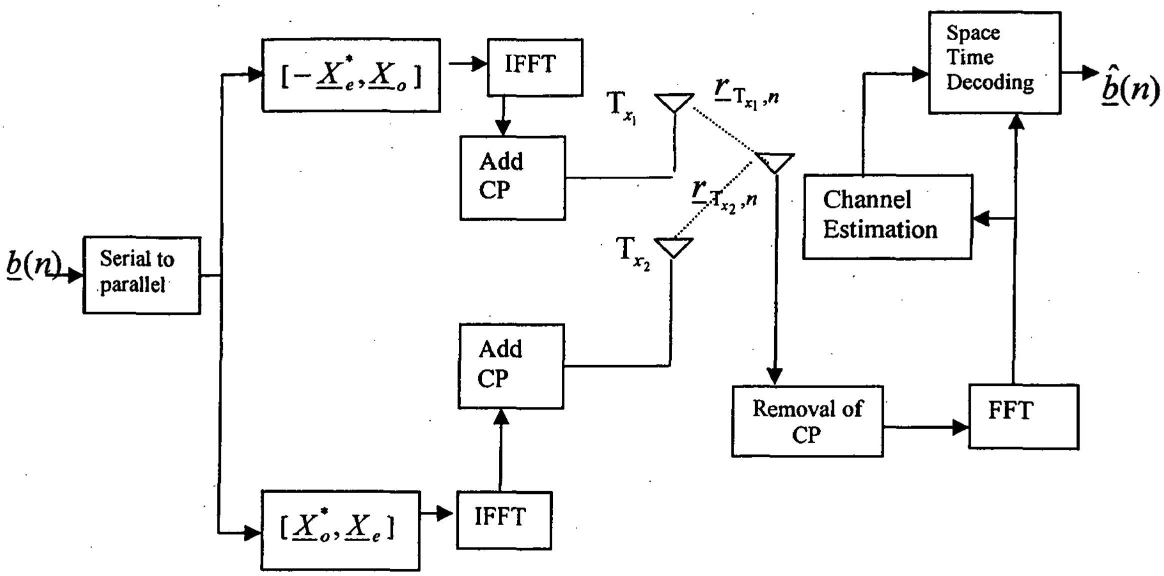


Figure. 4.2.1 A Block schematic of the proposed MISO-STC-OFDM simulation

For two transmitter single receiver space-time coded OFDM system has been implemented in this work as shown in fig. 4.2.1. There are two paths for transmitting symbols, one is from T_{x_1} to receiver and another is from T_{x_2} to receiver for MISO-STC-OFDM systems. We have considered both paths independent and channel models used for those paths are same as described in section 4.1. After two transmission one at time *n* and another at time *(n+1)*, we obtain two received OFDM frames at receiver. After removing cyclic prefix from these frames and taking *M* point FFT of remaining symbol array we obtain $\underline{r}_1$ & $\underline{r}_2$. For this proposed system, channel estimation is done using EM-MMSE technique.

4.2.1 Channel Estimation Techniques

For MISO-STC-OFDM system, we sent equally spaced pilot symbols in each frame. Let $X_{op}(m)$ & $X_{ep}(m)$ be the known pilot symbols for m^{th} carrier. Solving equation (2.3.2) and (2.3.3) to obtain $\hat{H}_1(m)$ & $\hat{H}_2(m)$,

$$\hat{H}_1(m) = \frac{-(X_{ep}(m).r_2(m) - r_1(m).X_{op}^*(m))}{(\|X_{op}(m)\|^2 + \|X_{ep}(m)\|^2)} = H_1(m) + K_1 \qquad (4.3.2)$$

$$\hat{H}_2(m) = \frac{(X_o(m).r_2(m) + r_1(m).X_e^*(m))}{(\|X_{op}(m)\|^2 + \|X_{ep}(m)\|^2)} = H_2(m) + K_2 \qquad (4.3.3)$$

where $\hat{H}_1(m)$ and $\hat{H}_2(m)$ are the estimated channel frequency response at m^{th} carrier respectively for T_{x_1} to receiver and T_{x_2} to receiver and K_1 and K_2 are the effect of noise in the estimated channel parameter.

To improve the estimated channel parameters, EM and EM-MMSE techniques have been used. Figure 4.2.2 and fig 4.2.3 shows flow chart for implementation of EM and EM-MMSE techniques for channel estimation.

For EM technique, channel estimation is performed symbol-to-symbol basis. In EM, two steps are performed as shown in section 3.1 and channel parameters are estimated using Eq. 3.1.19 and Eq. 3.1.21 at each iteration.

For EM-MMSE technique, channel estimation is performed frame-to-frame basis. In EM-MMSE, three steps are performed as shown in section 3.2. At step A,

$\|r_1(m) - H_1^{(p)}(m) \bullet X_i(m) - H_2^{(p)}(m) \bullet \underline{X}_j(m)\|^2 + \|r_2(m) + H_1^{(p)}(m) \bullet X_i^*(m) - H_2^{(p)}(m) \bullet X_j^*(m)\|^2$ is calculated for $1 \le i \le 4$ and $1 \le j \le 4$, for m^{th} carrier, where $H_1^{(p)}(m)$ and $H_2^{(p)}(m)$ are estimated channel coefficients at p^{th} iteration for m^{th} carrier. The decision of transmitted symbol is taken by MMSE method at p^{th} iteration for m^{th} carrier. The decision is taken for $0 \le m \le M-1$ and estimated transmitted vector for each OFDM frame can be represented as $\underline{\tilde{X}}_o^{(p)}$ & $\underline{\tilde{X}}_e^{(p)}$.

Channel parameters are estimated using step A and Eq. 3.2.8 and Eq. 3.2.9 at each iteration. These steps are computed till convergence is obtained.

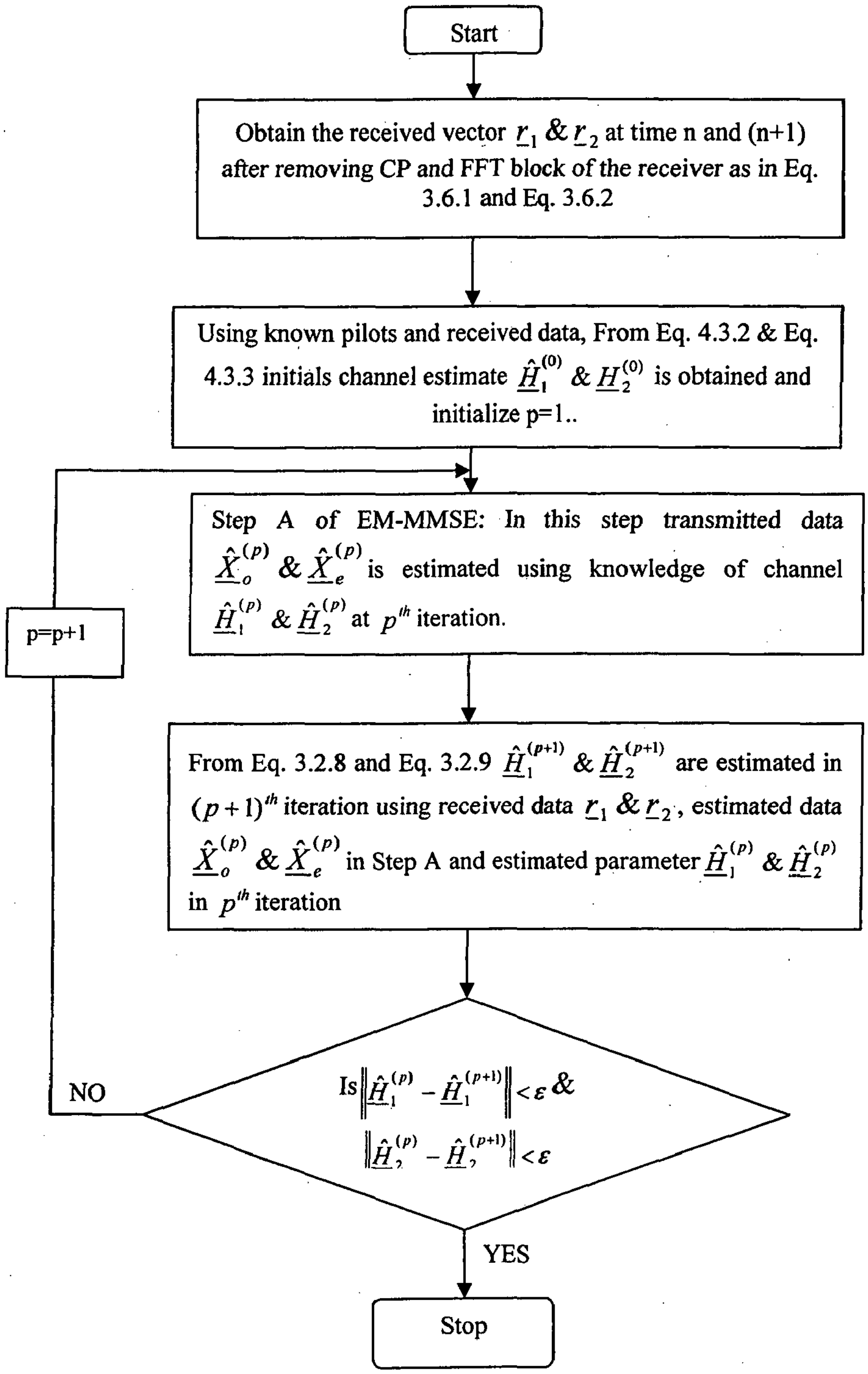
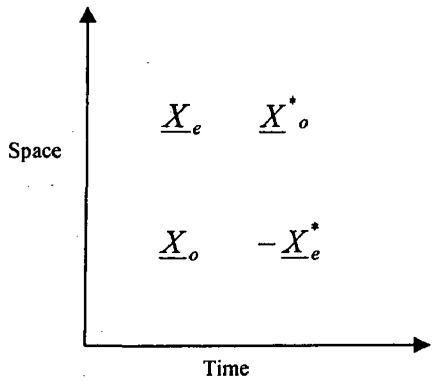


Figure 4.2.3 Flow Chart: EM-MMSE based channel estimation technique for MISO STC-OFDM Systems

4.2.2 System Simulation

Figure 4.2.4 shows the flow chart for implementation of MISO-STC-OFDM transmitter structure. Once the initialization is done, we obtain the values of channel tap coefficients $\underline{h}_1 \& \underline{h}_2$. The STC-OFDM transmission is simulated for K OFDM frame transmission from each of the two transmitters.

At the transmitter, an array of bits is generated and these bits are transmitted in parallel arrays. These arrays are modulated using QPSK modulation to obtain $\underline{X}_e \& \underline{X}_o$. Space time block coding is then performed on $\underline{X}_e \& \underline{X}_o$ as follows,



We first compute M point IFFT of $\underline{X}_e \& \underline{X}_o$ and add cyclic prefix of length M_c.Here we obtain the transmitted OFDM frame $\underline{x}_e \& \underline{x}_o$.

Channel implementation for STC-OFDM is shown in fig. 4.2.5. We transmit respective pair of STC-OFDM blocks at time instant *n* and *(n+1)*. The transmission of OFDM frame through the frequency selective channel is simulated with the help of overlap and save method. We then compute convolution of $\underline{x}_e \& \underline{x}_o$ with $\underline{h}_1 \& \underline{h}_2$ for a transmission block. Now store the last M_c values of these results in buffer for next transmitted frames. To the first M_c values of these convolution results add the buffer for previous frame. The received OFDM symbol is the sum of resulting arrays from channel $\underline{h}_1 \& \underline{h}_2$.

After two transmission one at time n and another at time $(n+1)$, we obtain two received OFDM frames at receiver. Figure 4.2.6 shows the implementation of receiver structure. After removing cyclic prefix from these frames and taking M point FFT of remaining symbol array we obtain $\underline{r}_1$ & $\underline{r}_2$. Also the frequency domain values of channel impulse response $\underline{\hat{H}}_1$ & $\underline{\hat{H}}_2$, are obtained using estimation techniques explained in section 4.2.1. These channel impulse respose are used to decode the symbol arrays by applying STBC decoding as shown in Eq. 2.2.8 and Eq. 2.2.9,

$$\underline{\hat{X}}_o = \underline{\hat{H}}_1^* \bullet \underline{r}_1 + \underline{\hat{H}}_2 \bullet \underline{r}_2^*$$

$$\underline{\hat{X}}_e = \underline{\hat{H}}_2^* \bullet \underline{r}_1 - \underline{\hat{H}}_1 \bullet \underline{r}_2^*,$$

Now QPSK demodulation is performed to estimate the transmitted bits using maximum likelihood decoding. All these steps are performed to all frames and then P/S is performed for obtain estimated transmitted bits. Now performance of system in terms of BER is calculated for different SNR values.

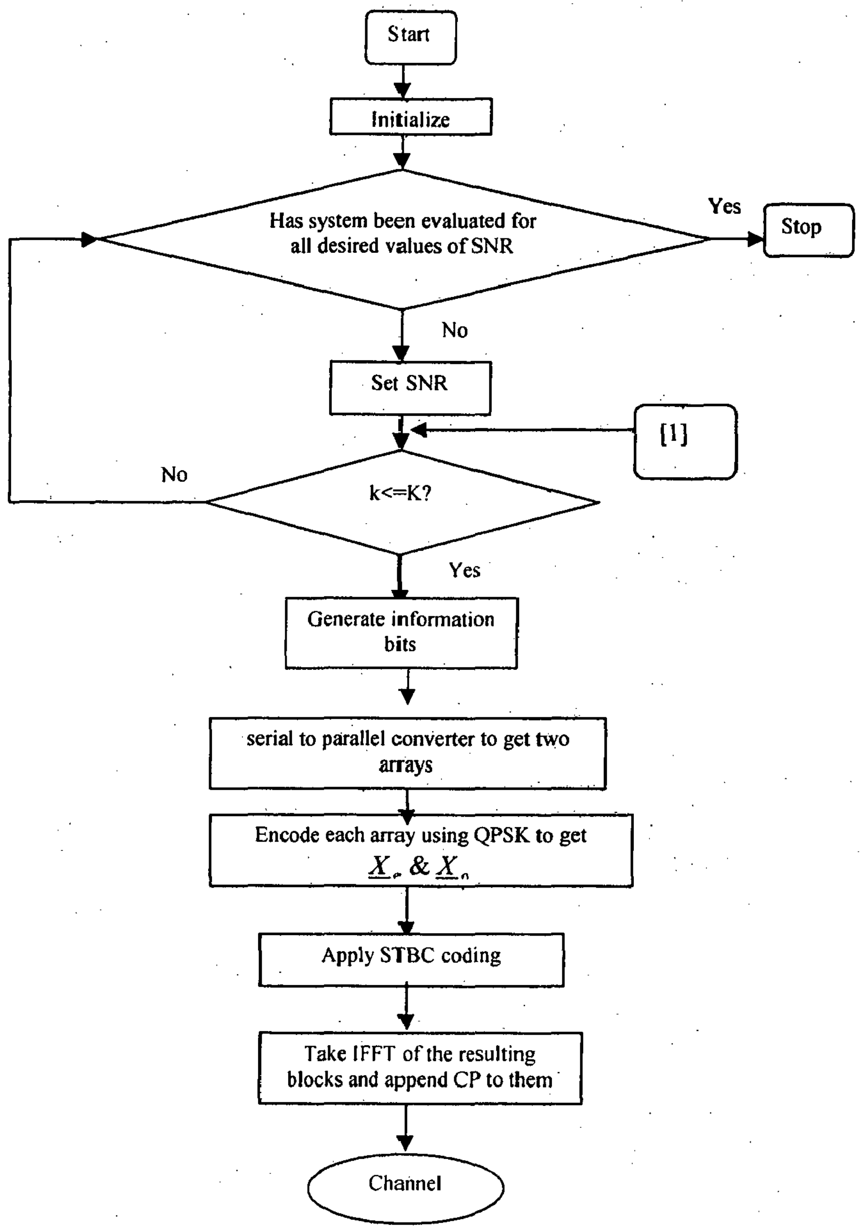


Figure 4.2.4 Flow Chart: Simulation of MISO STC-OFDM Transmitter

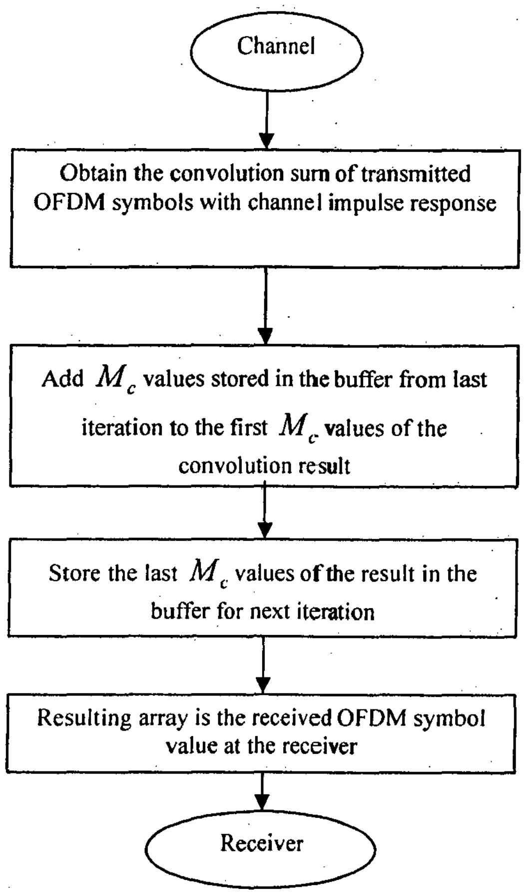


Figure 4.2.5 Simulation of Channel Implementation for MISO-STC-OFDM

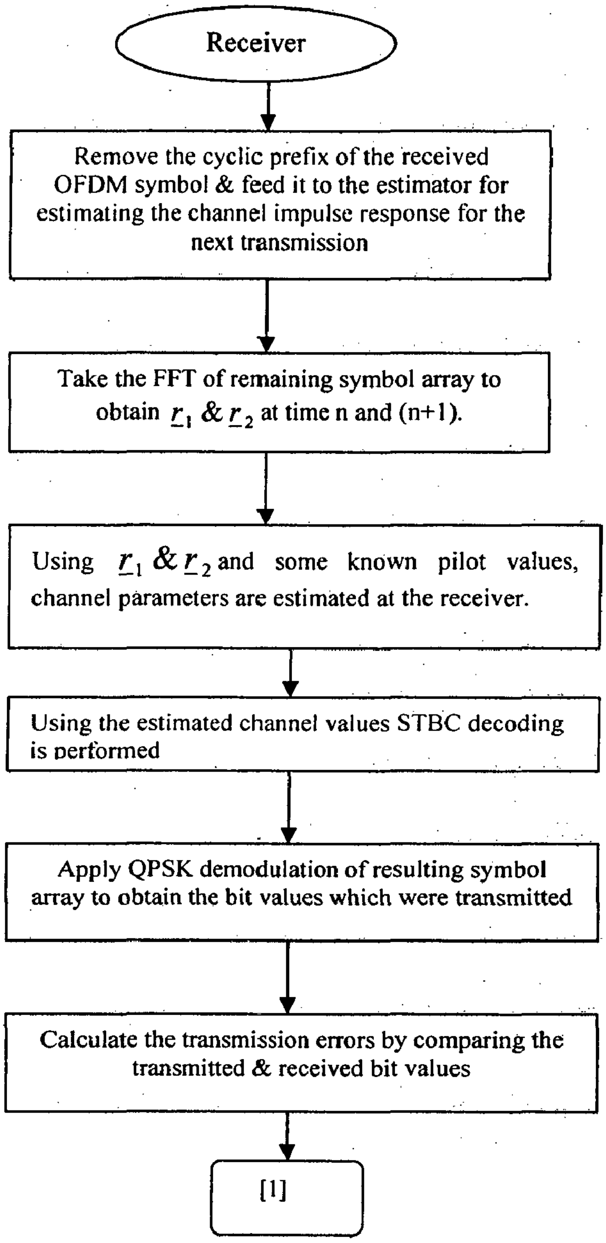


Figure 4.2.6 Flow Chart: Simulation of MISO-STC-OFDM Receiver

4.3 Simulation Results

The validity of the proposed algorithm is tested through Monte Carlo simulations. This section presents the results of the simulation for the aforementioned channel estimation techniques, carried out under MATLAB environment. The performance of the proposed channel estimator has been evaluated by comparing its BER performance with EM and pilot based techniques. The BER performance is evaluated by averaging over 500 frames i.e. 1.28×10^5 bits.

We compare the results for SISO-OFDM and MISO-STC-OFDM system model either in Rayleigh fading environment or in a time varying environment. The channel is assumed to be quasi-static and does not vary within each frame. To obtain an initial estimate, pilot based estimator with linear interpolation is used. In EM and EM-MMSE method, initial estimation is done only once and periodically refreshed after every multiframe in time varying environment.

4.3.1 BER Performance of SISO-OFDM System in Rayleigh Fading Environment

Figure 4.3.1a and fig.4.3.1b shows the performance curves of EM/EM-MMSE and pilot based estimators for SISO-OFDM systems in Rayleigh fading environment (when 4 pilots are used as initial estimation in each frame for EM and EM-MMSE techniques). As fig 4.3.1a shows, the BER curve, evaluated for pilot based estimator using 4 pilot symbols, saturates at BER 0.05. It can also be seen that using EM or EM-MMSE techniques, BER curves also saturates at BER .008 as initial estimation does not enable EM and EM-MMSE technique to perform satisfactorily. These performances are obviously undesirable for our purpose. It can be improved by pilot based estimator using 8 or 16 pilot symbols. Figure 4.3.1a shows that the performance of pilot based estimator using 16 pilots is close to actual channel coefficients.

Fig. 4.3.1b compares the average number of iterations for EM–MMSE and EM techniques for channel estimation in Rayleigh fading environment for SISO-OFDM Systems. It shows that the number of iterations required in EM-MMSE is reduced by a factor of almost 3 as compared with EM method.

Figure 4.3.2a and fig.4.3.2b shows the performance curves of EM/EM-MMSE and pilot based estimators for SISO-OFDM systems in Rayleigh fading environment (when 8 pilots are used as initial estimation in each frame for EM and EM-MMSE techniques). As fig 4.3.2a shows, the BER curve, evaluated for pilot based estimator using 8 pilot symbols, saturates at BER 0.001. It can be improved using EM or EM-MMSE techniques. It is evident from fig. 4.3.2a that EM and EM-MMSE technique performs almost identical to actual channel state information and outperform pilot based estimator using 16 pilots. For EM/EM-MMSE techniques, the SNR required to attain a BER of 0.02 is reduced by almost 3 dB as compared to pilot based estimator using 8 pilot symbols per OFDM frame. Figure 4.3.2b shows that the number of iterations required in EM-MMSE is reduced by a factor of almost 3 as compared with EM method.

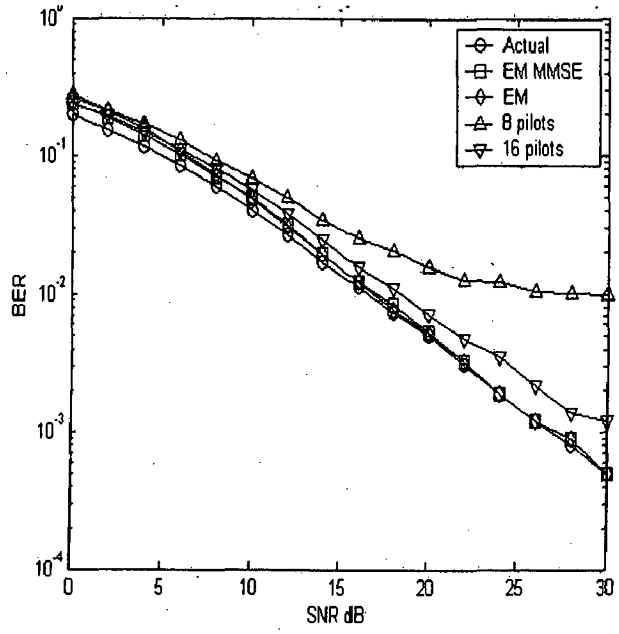


Fig.4.3.2a Comparison of BER performance for EM–MMSE, EM and pilot based channel estimation techniques in Rayleigh fading environment for SISO-OFDM system (8 pilots are used for initial estimation in each frame for EM and EM-MMSE techniques).

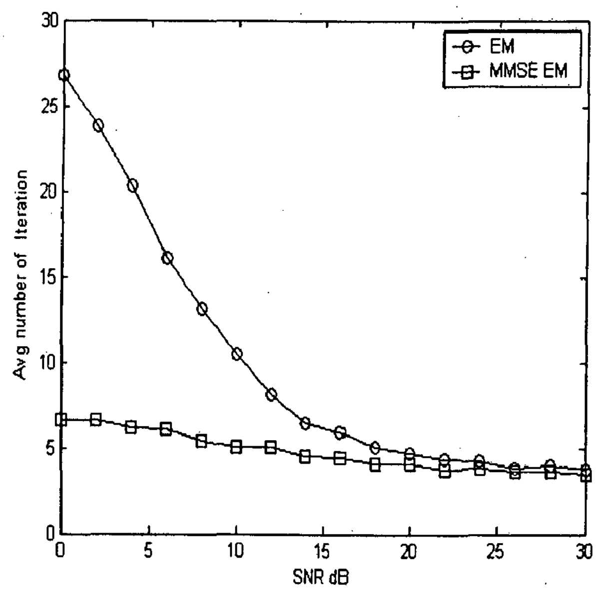


Fig. 4.3.2b Comparison of average number of iterations for EM–MMSE and EM techniques for channel estimation in Rayleigh fading environment for SISO-OFDM System (8 pilots are used as initial estimation in each frame for EM and EM-MMSE technique).

Figure 4.3.2

4.3.2 BER Performance of SISO-OFDM System in Time Varying Environment

Figure 4.3.3 and fig.4.3.4 shows the performance curves of EM/EM-MMSE and pilot based estimators for SISO-OFDM systems in time varying environment for moderately high Doppler i.e. for $f_d T = 0.005$ and at 0.1 for $f_d T = 0.01$ respectively (when 8 pilots are used as initial estimation in a multiframe of size 25 for EM and EM-MMSE techniques). The BER curve saturates at 0.04 for $f_d T = 0.005$ and at 0.1 for $f_d T = 0.01$ respectively for pilot based estimator using 8 and 16 pilots in a multiframe. This can be improved by EM/EM-MMSE technique or pilot estimators using 8 or 16 pilot symbols in each frame. As fig 4.3.3a shows, 5dB gain is achieved by EM/EM-MMSE technique over the pilot based estimator using 8 pilots per frame to attain a BER of 0.005 at $f_d T = 0.005$. While in fig 4.3.4a, 4-5dB gain is achieved by EM/EM-MMSE technique over the pilot based estimator using 8 pilots per frame to attain a BER of 0.02 at $f_d T = 0.01$. Results also show that EM and EM-MMSE techniques outperform pilot based estimator using 16 pilots in each frame. The proposed method provides a considerable saving in terms of bandwidth as it makes use of pilot symbols only once for initial estimation.

Figure 4.3.3b and fig. 4.3.4b shows that the number of iterations required in EM-MMSE is reduced by a factor of almost 4 as compared with EM method for $f_d T = 0.005$ and $f_d T = 0.01$.

Figure 4.3.5 shows the performance curves of EM/EM-MMSE and pilot based estimators for SISO-OFDM systems in time varying environment (when 8 pilots are used as initial estimation in a multiframe of size 50 for EM and EM-MMSE techniques). The BER curve shows that the performance of EM/EM-MMSE technique suffers at low SNR values when size of multiframe is large. But for SNR values greater than 12 dB, it shows satisfactory performance. Therefore, the size of multiframe should be kept small for high Doppler values i.e. initial estimation should be refreshed periodically.

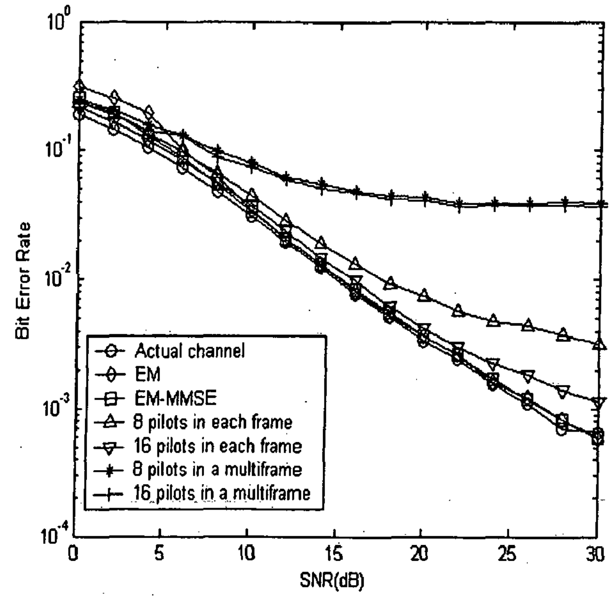


Fig.4.3.3a Comparison of BER performance for EM–MMSE, EM and pilot based channel estimation techniques in time varying environment for SISO-OFDM system (8 pilots are used for initial estimation in a multiframe of size 25 at $f_d T = 0.005$ for EM and EM-MMSE techniques).

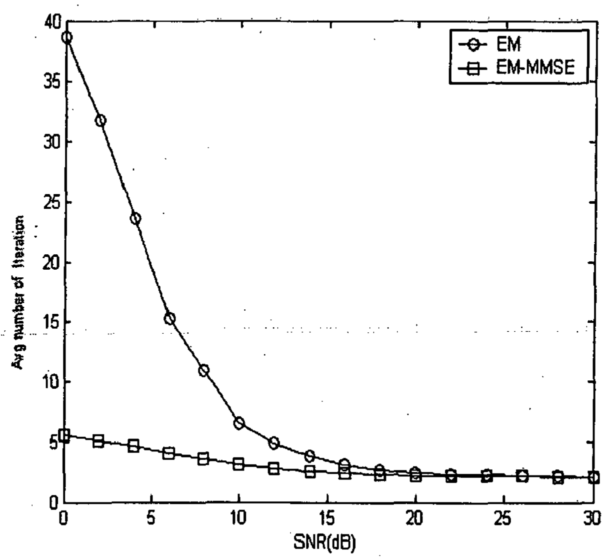


Fig. 4.3.3b Comparison of average number of iterations for EM–MMSE and EM techniques for channel estimation in time varying environment for SISO-OFDM System (8 pilots are used as initial estimation in a multiframe of size 25 at $f_d T = 0.005$ for EM and EM-MMSE techniques).

Figure 4.3.3

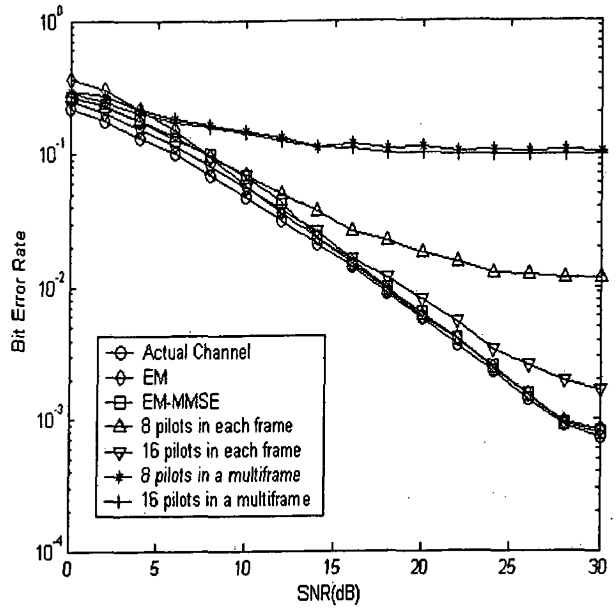


Fig.4.3.4a Comparison of BER performance for EM–MMSE, EM and pilot based channel estimation techniques in time varying environment for SISO-OFDM system (8 pilots are used for initial estimation in a multiframe of size 25 at $f_d T = 0.01$ for EM and EM-MMSE techniques).

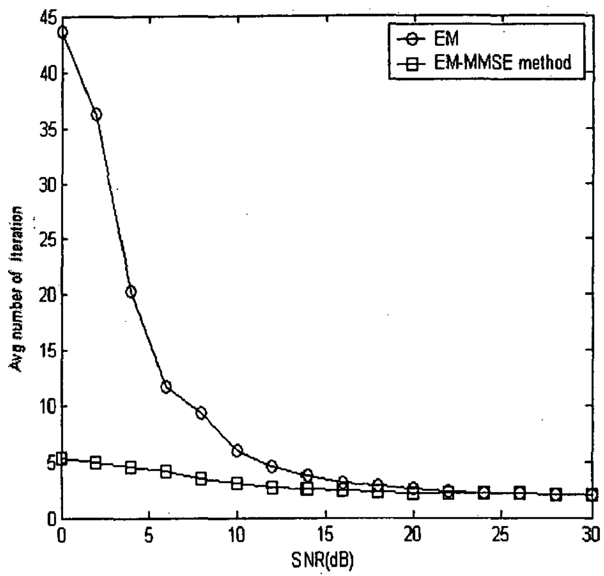


Fig. 4.3.4b Comparison of average number of iterations for EM–MMSE and EM techniques for channel estimation in time varying environment for SISO-OFDM System (8 pilots are used as initial estimation in a multiframe of size 25 at $f_d T = 0.01$ for EM and EM-MMSE techniques).

Figure 4.3.4

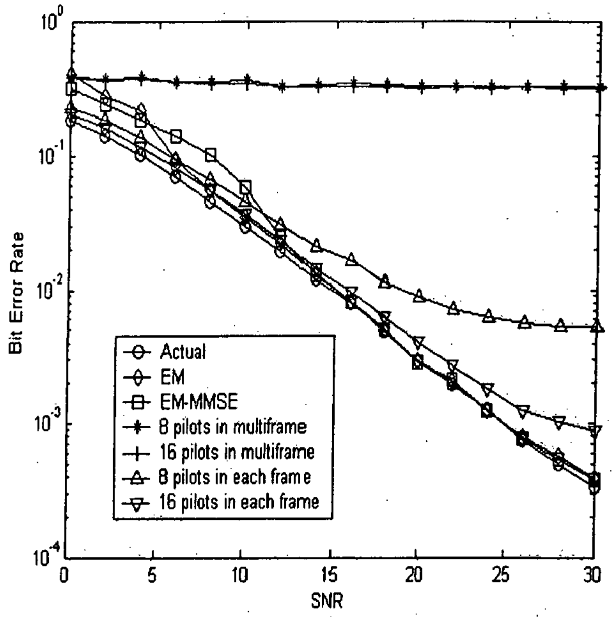


Fig.4.3.5a Comparison of BER performance for EM–MMSE, EM and pilot based channel estimation techniques in time varying environment for SISO-OFDM system (8 pilots are used for initial estimation in a multiframe of size 50 at $f_d T = 0.01$ for EM and EM-MMSE techniques).

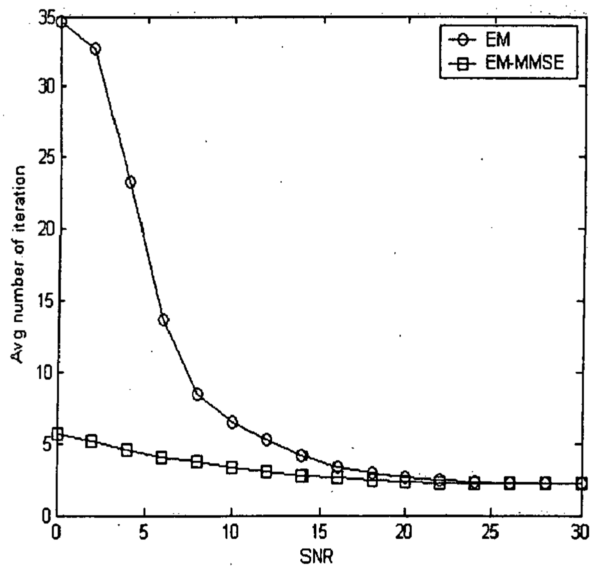


Fig. 4.3.5b Comparison of average number of iterations for EM–MMSE and EM techniques for channel estimation in time varying environment for SISO-OFDM System (8 pilots are used as initial estimation in a multiframe of size 50 at $f_d T = 0.01$ for EM and EM-MMSE techniques).

Figure 4.3.5

4.3.3 BER Performance of MISO-STC-OFDM System in Rayleigh Fading Environment

Figure 4.3.6 shows the performance curves of EM/EM-MMSE and pilot based estimators for MISO-STC-OFDM systems in Rayleigh fading environment (when 4 pilots are used as initial estimation in each frame for EM and EM-MMSE techniques). As fig 4.3.6a shows, the BER curve, evaluated for pilot based estimator using 4 pilot symbols in each frame, saturates at BER 0.04. This can be improved by EM/EM-MMSE based estimators or pilot based estimator using 8 pilot symbols. For EM/EM-MMSE techniques (initial estimation is computed by pilot based estimator using 4 pilots), the SNR required to attain a BER of 0.008 is reduced by almost 2 dB compared to pilot based estimator using 8 pilot symbols. Figure 4.3.6b shows that the number of iterations required in EM-MMSE is reduced by a factor of almost 3 as compared with EM method.

Figure 4.3.7 shows the performance curves of EM/EM-MMSE and pilot based estimators in Rayleigh fading environment for MISO-STC-OFDM systems (when 8 pilots are used as initial estimation in each frame for EM and EM-MMSE techniques). As fig 4.3.7a shows, the BER curve, evaluated for pilot based estimator using 8 pilot symbols in each frame, saturates at BER 0.002 while EM/EM-MMSE based techniques performs almost identical to actual channel state information. The SNR required to attain a BER of 0.01 by EM/EM-MMSE based estimators is reduced by almost 3 dB compared to pilot based estimator using 8 pilot symbols. Figure 4.3.6b shows that the number of iterations required in EM-MMSE is reduced by a factor of almost 3 as compared with EM method.

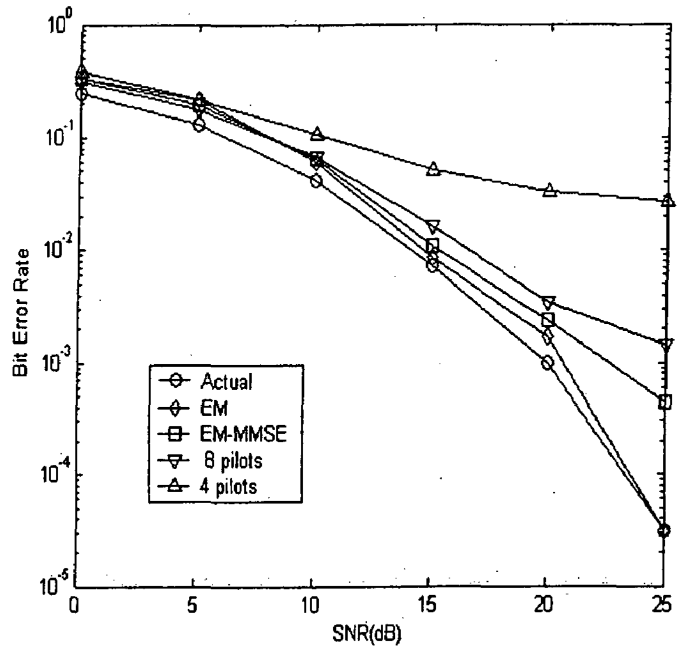


Fig. 4.3.6a Comparison of BER performance for EM–MMSE, EM and pilot based channel estimation techniques in Rayleigh fading environment for MISO-STC-OFDM system (4 pilots are used for initial estimation in each frame for EM and EM-MMSE techniques).

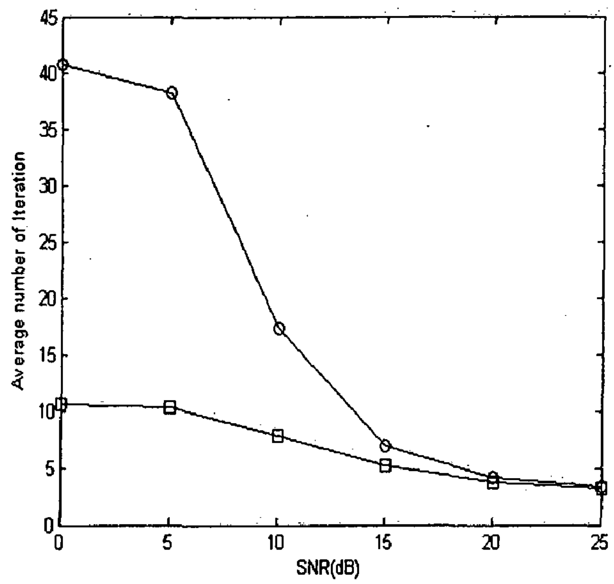


Fig. 4.3.6b Comparison of average number of iterations for EM–MMSE and EM techniques for channel estimation in Rayleigh fading environment for MISO-STC-OFDM System (4 pilots are used as initial estimation in each frame for EM and EM-MMSE technique).

Figure 4.3.6

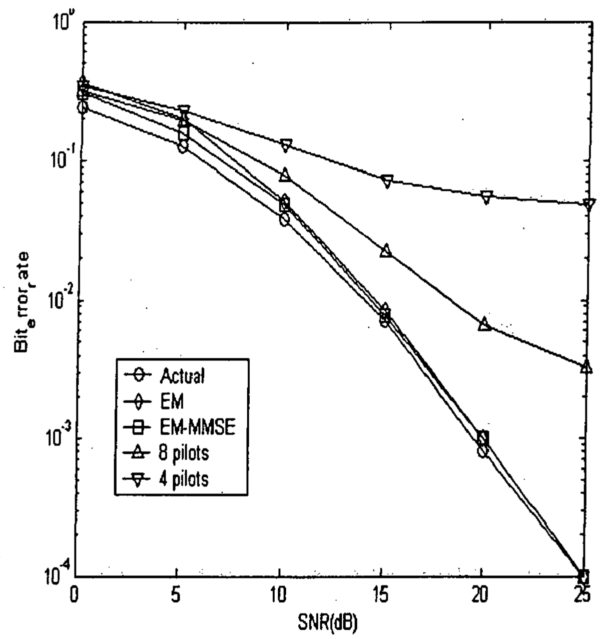


Fig. 4.3.7a Comparison of BER performance for EM–MMSE, EM and pilot based channel estimation techniques in Rayleigh fading environment for MISO-STC-OFDM system (8 pilots are used for initial estimation in each frame for EM and EM-MMSE techniques).

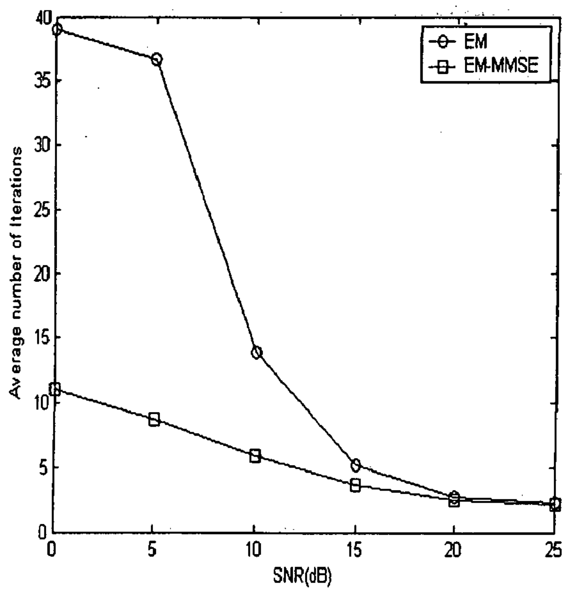


Fig. 4.3.7b Comparison of average number of iterations for EM–MMSE and EM techniques for channel estimation in Rayleigh fading environment for MISO-STC-OFDM System (8 pilots are used as initial estimation in each frame for EM and EM-MMSE technique).

Figure 4.3.7

4.3.4 BER Performance of MISO-STC-OFDM System in Time Varying Environment

Figure 4.3.8 and fig.4.3.9 shows the performance curves of EM/EM-MMSE and pilot based estimators MISO-STC-OFDM systems in time varying environment (when 8 pilots are used as initial estimation in a multiframe of size 50 for EM and EM-MMSE techniques). The BER curve saturates at 0.02 for $f_d T = 0.005$ and at 0.1 for $f_d T = 0.01$ respectively for pilot based estimator using 4 and 8 pilots in a multiframe. This can be improved by pilot estimators using 4 or 8 pilot symbols in each frame or by EM/EM-MMSE techniques. As fig 4.3.8a shows, the SNR required to attain a BER of 0.01 by EM and EM-MMSE technique is reduced almost 3 dB over pilot based estimator using 8 pilot symbols per OFDM frame. While in fig 4.3.9a, 2 dB gain is achieved by EM and EM-MMSE technique over the pilot based estimator using 8 pilots per frame at a BER of 0.02. It is evident that EM based technique performs almost identical to actual channel state information. Besides this, proposed method provides a considerable saving in terms of bandwidth as it makes use of pilot symbols only once for initial estimation.

Figure 4.3.8b and fig. 4.3.9b shows that the number of iterations required in EM-MMSE is reduced by a factor of almost 4 as compared with EM method for $f_d T = 0.005$ and $f_d T = 0.01$ simultaneously.

Figure 4.3.10 shows the performance curves of EM-MMSE and pilot based estimators for MISO-STC-OFDM systems in time varying environment (when 8 pilots are used as initial estimation in a multiframe of size 100 for EM and EM-MMSE techniques at $f_d T = 0.005$). The BER curve shows that the performance of EM/EM-MMSE technique suffers at low SNR values as size of multiframe is large. But for SNR values greater than 10 dB, it shows satisfactory performance.

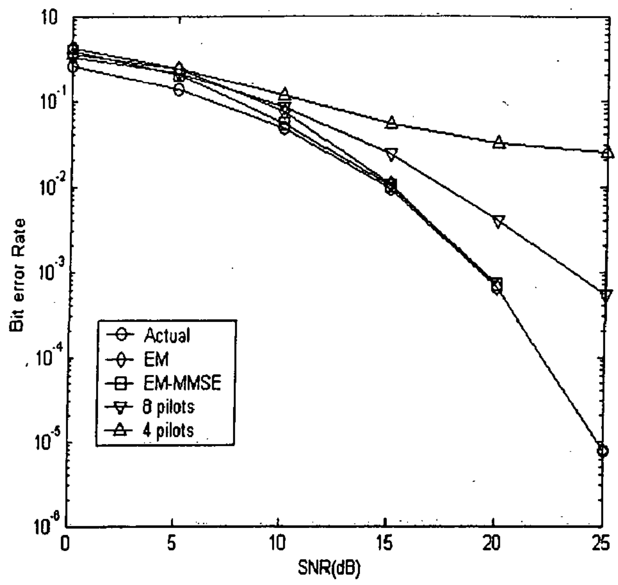


Fig.4.3.8a Comparison of BER performance for EM–MMSE, EM and pilot based (using 4 and 8 pilots in each frame) channel estimation techniques in time varying environment for MISO-STC-OFDM system (8 pilots are used for initial estimation in a multiframe of size 50 at $f_d T$ = 0.005 for EM and EM-MMSE techniques).

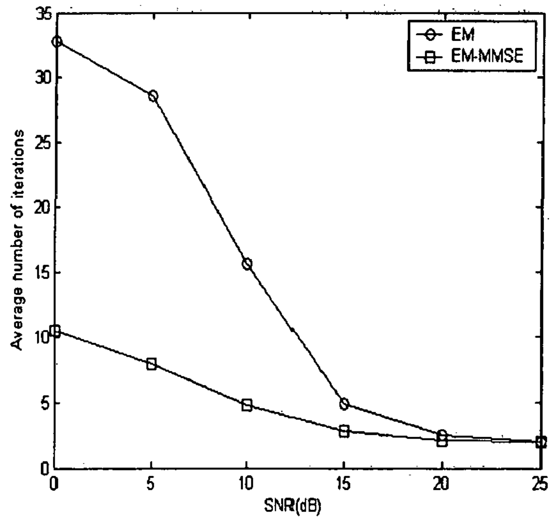


Fig. 4.3.8b Comparison of average number of iterations for EM–MMSE and EM techniques for channel estimation in time varying environment for MISO-STC-OFDM System (8 pilots are used as initial estimation in a multiframe of size 50 at $f_d T$ = 0.005 for EM and EM-MMSE techniques).

Figure 4.3.8

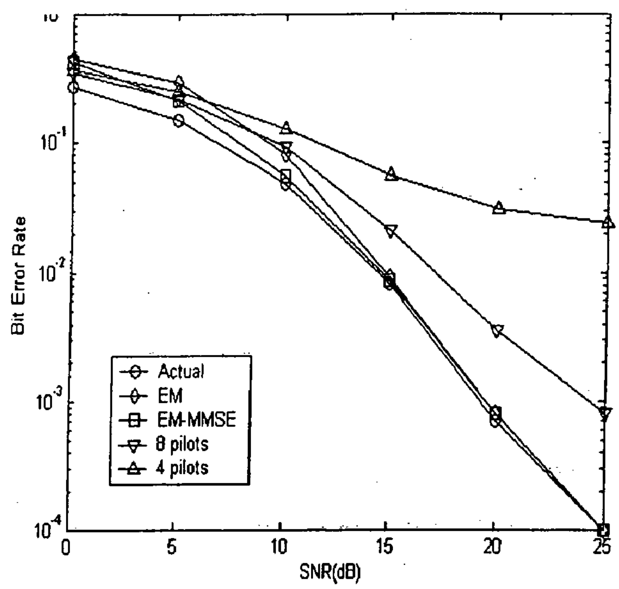


Fig.4.3.9a Comparison of BER performance for EM–MMSE, EM and pilot based (using 4 and 8 pilots in each frame) channel estimation techniques in time varying environment for MISO-STC-OFDM system (8 pilots are used for initial estimation in a multiframe of size 50 at $f_d T = 0.01$ for EM and EM-MMSE techniques).

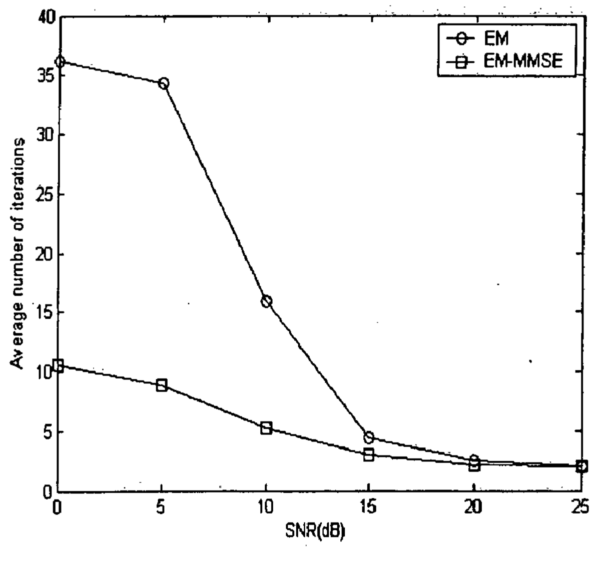


Fig. 4.3.9b Comparison of average number of iterations for EM–MMSE and EM techniques for channel estimation in time varying environment for MISO-STC-OFDM System (8 pilots are used as initial estimation in a multiframe of size 50 at $f_d T = 0.01$ for EM and EM-MMSE techniques).

Figure 4.3.9

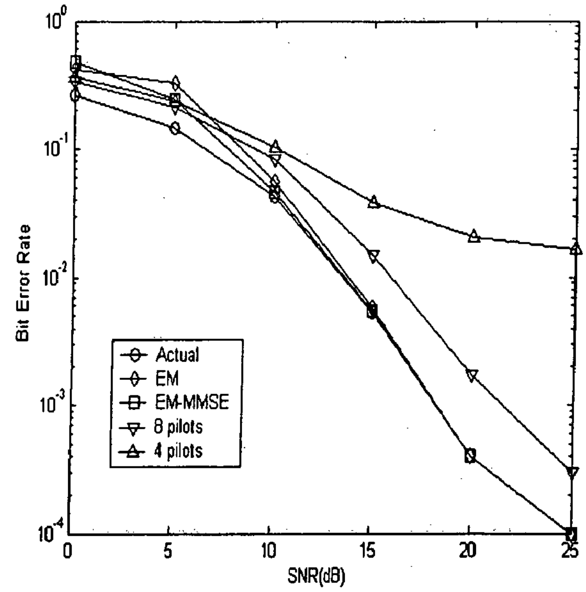


Fig.4.3.10a Comparison of BER performance for EM–MMSE, EM and pilot based (using 4 and 8 pilots in each frame) channel estimation techniques in time varying environment for MISO-STC-OFDM system (8 pilots are used for initial estimation in a multiframe of size 100 at $f_d T = 0.005$ for EM and EM-MMSE techniques).

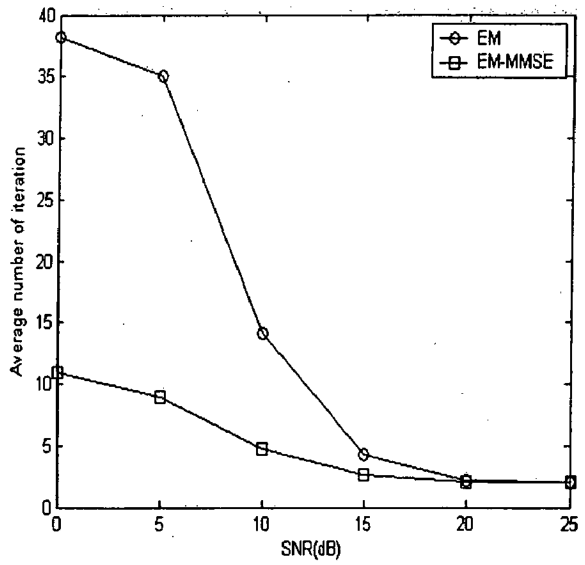


Fig. 4.3.10b Comparison of average number of iterations for EM–MMSE and EM techniques for channel estimation in time varying environment for MISO-STC-OFDM System (8 pilots are used as initial estimation in a multiframe of size 100 at $f_d T = 0.005$ for EM and EM-MMSE techniques).

Figure 4.3.10

4.4 Conclusion

In this work, EM-MMSE based iterative channel estimation technique for OFDM systems has been proposed. The conventional EM technique [8] requires high computational complexity and large number of iterations. The performance in terms of BER is compared for EM-MMSE, EM and pilot based estimation techniques for SISO-OFDM system and MISO-STC-OFDM system. The conclusions drawn based on simulation results are as follows:

- The performance of EM-MMSE technique is almost similar to that of EM while reducing computational complexity and the number of iterations to obtain convergence. The number of iteration required in EM-MMSE technique is reduced almost by a factor of 3 as compared with EM technique.
- The performance of EM/EM-MMSE technique when initial estimation is computed by pilot based estimator using 8 pilots is close to actual channel coefficients in stationary Rayleigh fading environment.
- For channel estimation in time varying environment using EM/EM-MMSE techniques, pilots are sent only once in a multiframe for initial estimation while using pilot based techniques, pilots have to be sent in each frame to attain comparable performance.
- For high Doppler values, initial estimation is refreshed periodically to enable EM/EM-MMSE techniques to perform satisfactorily.
- MISO-STC-OFDM system gives 3dB advantage over SISO-OFDM system at a BER of 0.01 for channel coefficients estimated by EM/EM-MMSE techniques.

Space-Alternating Generalized EM (SAGE) an improved EM based algorithm [39] has been proposed in literature. SAGE updates the parameters to be estimated sequentially by alternating between the several data spaces. SAGE algorithm converges faster than EM. The application of the SAGE technique may further reduce the required computational complexity for channel estimation in OFDM systems and may enhance the performance in terms of bit error rate. EM based techniques can be used to estimate frequency offset in OFDM systems.

Channel estimation techniques for faster fading channels need to be investigated further as future generation wireless OFDM systems may operate with much higher data rates.

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Appendix A

Software Listing

Channel estimation techniques for SISO-OFDM

```
close all
clear all

% SISO OFDM Systems Model

SN=0:2:30     % SNR values

avg_iteration_EM=zeros(1,length(SN));
avg_iteration_EM_MMSE=zeros(1,length(SN));
no_of_carriers=128;
symbol_size=2;
h_es_new1=zeros(length(SN),no_of_carriers);
h_es_new=zeros(length(SN),no_of_carriers);
blocks= 500; % number of OFDM frames generated
Mc=3; % Length of the cyclic prefix
channel_length=4;
GI=channel_length-1;% gaurd interval
m=no_of_carriers*symbol_size+GI;
N=no_of_carriers;

% Defining Modulation Sceme

s4=[3*pi/4 5*pi/4 pi/4 7*pi/4 ]


%-------------------------------------------------------------------
-
%                     Transmitter Section
%-------------------------------------------------------------------
-


generated_bits=symbol_size*symbol_length*no_of_carriers;
generated_symbol=generated_bits/symbol_size;

%Generate Data

data_bits=randint(1,generated_bits);

%Generation of symbols

for p=1:2:generated_bits
    phase((p+1)/2)=s4(data_bits(p:p+1)*[2;1]+1);
end

    [X,Y]=pol2cart(phase,ones(size(generated_bits/symbol_size)));
```

```
    carrcmplx=X+i*Y;

% Serial To parrallel and IIFT of each OFDM frame & Adding Cyclic
Prefix

    for k=1:1:blocks
        s=carrcmplx(((k-1)*no_of_carriers+1):(k*no_of_carriers));
        D(k,1:no_of_carriers)=s;
        X=D(k,:);
        x=ifft(X);
      d(k,1:GI)=x(N-GI+1:N);
      d(k,GI+1:N+GI)=x;
    end

 Tx_symbol=d;  %After IFFT &adding cyclic prefix each row has ofdm
frame


%--------------------------------------------------------------------
-
%                          Channel Tap Generation
%--------------------------------------------------------------------
-

h=zeros(blocks,channel_length);
for k=1:blocks
       h(k,1)=.86*reliegh(var);
    h(k,2)=.486*reliegh(var);
     h(k,3)=.295*reliegh(var);
      h(k,4)=.1790*reliegh(var);
end
%initialization of taps for complete frame......

Hbar=zeros(blocks,N);

for k=1:blocks
      Hbar(k,:)=fft(h(k,:),N);
end

for q1=1:length(SN)



%--------------------------------------------------------------------
-
%                        Channel  Implementation
%--------------------------------------------------------------------
-
noise_var=1/10^(SNR/10)
ch_noise=complex(sqrt(noise_var/2)*randn(blocks,m),sqrt(noise_var/2)
*randn(blocks,m));
Rc_symbol=zeros(blocks,m);
buff=zeros(blocks,m);
temp=zeros(blocks,m+channel_length-1);
for k=1:blocks
temp(k,:) = conv(Tx_symbol(k,:),h(k,:));
```

```
                l=0;

        while l==0 && p(j)<30

                p(j)=p(j)+1;
                    b=0;
                for k=1:N
                 H_es_new(j,k)=estimate(H_es(j,k),Y(j,k),noise_var);
                end
        % Filter to improve estimated channel parameters

         O1=ifft(H_es_new(j,1:N));
         h_es_new(1,1:channel_length)=O1(1,1:channel_length);
         H_es_new(j,1:N)=fft(h_es_new(1,1:channel_length),N);


            l=converge(H_es(j,:),H_es_new(j,:));
    end
   avg_iteration_EM(1,q1)=g1/( blocks);

Bit_error_rate_EM=BER(Y,H_es,X);


%--------------------------------------------------------------------
-
%                   Channel Estimation  by EM-MMSE method
%--------------------------------------------------------------------
-

% Channel estimation by EM-MMSE technique

H_es_new1=zeros(blocks,N);
p1=zeros(blocks,1);
O1=zeros(1,N);
h_es_new1=zeros(1,channel_length);
g1=0;
   for j=1:blocks


                L2=0;

        while L2==0 && p1(j)<30

                p1(j)=p1(j)+1;



H_es_new1(j,:)=estimate_ss(H_es1(j,:),Y(j,:),noise_var,N);


         O1=ifft(H_es_new1(j,1:N));
         h_es_new1(1,1:channel_length)=O1(1,1:channel_length);
         H_es_new1(j,1:N)=fft(h_es_new1(1,1:channel_length),N);

            L2=converge(H_es_new1(j,k)-H_es1(j,k));
```

```
            end
            p1(j)
            g1=g1+p1(j);

     end

      avg_iteration1_EM_MMSE(1,q1)=g1/blocks;
end

       Bit_error_rate_EM_MMSE=BER(Y,H_es1,X);
```

convert.m

```
function [data_es1,data_es2]=convert(carrcmplx_es)
d_bits=[0 0 0 1 1 0 1 1];
a1=real(carrcmplx_es);
a2=imag(carrcmplx_es);
if a1<0 && a2>0
 data_es1=d_bits(1);
 data_es2=d_bits(2);
end
if a1<0 && a2<0
 data_es1=d_bits(3);
 data_es2=d_bits(4);
end
if a1>0 && a2>0
 data_es1=d_bits(5);
 data_es2=d_bits(6);
end
if a1>0 && a2<0
 data_es1=d_bits(7);
 data_es2=d_bits(8);
end
```

converge.m

```
            function l=converge(H_es_new,H_es)
                  b=o;
      for k=1:N
                    a=(H_es_new(1,k)-H_es(1,k));

                   a1=real(a);
                   a2=imag(a);
                     if a1>=0
                       a1=a1;
                     else
                       a1=-a1;
                     end

                     if a2>=0
                       a2=a2;
                     else
                        a2=-a2;
                     end
```

estimate.m

```
% calculate the channel parameter estimation for EM Technique

function K=estimate(H,Y,noise_var);
X=[1/sqrt(2)+i*1/sqrt(2) 1/sqrt(2)-i*1/sqrt(2) -
1/sqrt(2)+i*1/sqrt(2) -1/sqrt(2)-i*1/sqrt(2)];
K1=0;
K2=0;
 F=pdf_Y_given_H(Y,H,noise_var);
for k=1:4

    S=pdf_YXgiven_H(Y,H,X(k),noise_var);

    K1=K1+X(k)*conj(X(k))*S/F;
    K2=K2+Y*conj(X(k))*S/F;
end
K=K2/K1;
```

pdf_Y_given_H.m

```
function F=pdf_Y_given_H(Y,H,noise_var);

X=[1/sqrt(2)+i*1/sqrt(2) 1/sqrt(2)-i*1/sqrt(2) -
1/sqrt(2)+i*1/sqrt(2) -1/sqrt(2)-i*1/sqrt(2)];
F=0;
 g=sqrt(2*pi*noise_var);
for j=1:4
    C=Y-H.*X(j);
C=real(C)*real(C)+imag(C)*imag(C);
    F=F+1/g*exp(50-C/2/noise_var);
end
F=F/4;
```

pdf_YXgiven_H.m

```
function S=pdf_YXgiven_H(Y,H,X,noise_var);

    C=(Y-H.*X);
    g=sqrt(2*pi*noise_var);
    C=real(C)*real(C)+imag(C)*imag(C);
    S=1/g*exp(-C/2/noise_var);

%-------------------------------------------------------------------
-
%           Channel Estimation using EM-MMSE technique
%-------------------------------------------------------------------
-
```

estimate_ss.m

```
% calculate the estimation for EM-MMSE Technique

function K=estimate_ss(H,Y,noise_var,N);
```

```
            H_int(j,k)=H_int(120)+(H_int(128)-H_int(120))/8*(k-
        120);
        end

end

%---------------------------------------------------------------------
-
%                    Generation of time varying channel
%---------------------------------------------------------------------
-


fd=100;
T=10^(-4);
[a1 , a2 , wd , variancear2n] = findar2coeff(fd , T);
L=channel_length;
Nr=5;

[hfinal] = ar2channel(variancear2n , Nr , L , a1 , a2 , Ni);
for m=1:Ni
 for k=1:numrealizations
      for j=1:L
     h((m-1)*Nr+k,j)=hfinal(k,(m-1)*L+j);
     end
 end
end
```

findar2coeff.m

```
function [a1 , a2 , wd , variancear2n] = findar2coeff (fd , T);
wd = 2 * pi * fd * T;
rd = 1 - wd / pi;
fp = (1 / sqrt(2)) * fd;
a1 = 2 * rd * cos(2 * pi * fp * T);
a2 = - (rd ^ 2);
variancear2n = (1 + a2) * ((( 1 - a2) ^ 2) - (a1 ^ 2)) / (1 - a2);
```

awgnterms.m

```
function [aw] = awgnterms (N , variance , realcomp);
for n = 1 : N
     x = rand(1);
     y = rand(1);
     z = rand(1);
     w = rand(1);
     if realcomp == 1
         aw1 = sqrt(-1 * variance * log(x)) * cos(2 * pi * y);
     elseif realcomp == 0
         aw1 = sqrt(-2 * variance * log(x)) * cos(2 * pi * y);
     end
     aw2 = sqrt(-1 * variance * log(z)) * cos(2 * pi * w);
     aw(n) = aw1 + j * realcomp * aw2;
realcomp * sin(2 * pi * y));
end;
```

ar2channel.m

```
function [hfinal] = ar2channel (variancear2n , Ni , L , a1 , a2 ,
numrealizations);
tot = numrealizations * L;
h = zeros(100 + Ni , tot);
for i1 = 1 : tot
    aw = awgnterms(Ni + 100 , variancear2n , 1);
    a = (1 / sqrt(2)) * randn(1);
    b = (1 / sqrt(2)) * randn(1);
    h(1 , i1) = a + j * b;
    h(2 , i1) = h(1 , i1) + aw(2);
    for i2 = 3 : Ni + 100
        h(i2 , i1) = a1 * h(i2 - 1 , i1) + a2 * h(i2 - 2 , i1) +
aw(i2);
    end;
end;
hfinal = h(101 : Ni + 100 , :);
```

Ml_decoding.m

```
function p=Ml_decoding (X);
d=zeros(1,4);
X1=[-1/sqrt(2)+i*1/sqrt(2) -1/sqrt(2)-i*1/sqrt(2)
1/sqrt(2)+i*1/sqrt(2) 1/sqrt(2)-i*1/sqrt(2)];
for j=1:4
    d(j)=abs(X-X1(j));
end
[Y I]=min(d);
p=X1(I);
```

Channel estimation techniques for MISO STC-OFDM System

```
%Two OFDM TRANCIVER
close all

clear all

SN=0:5:30   % SNR values
g5=zeros(1,length(SN));
g6=zeros(1,length(SN));
g5_MMSE=zeros(1,length(SN));
X_cons=[-1/sqrt(2)+i*1/sqrt(2) -1/sqrt(2)-i*1/sqrt(2)
            1/sqrt(2)+i*1/sqrt(2) 1/sqrt(2)-i*1/sqrt(2)];
N_Carr=128; % number of carriers in each OFDM frame
symbol_size=2;
blocks=500;  % number of OFDM frames to be generated

channel_length=4;

GI=channel_length-1;% gaurd interval

m=no_of_carriers+GI;
N=symbol_size*no_of_carriers;
```

```
% Defining Modulation Sceme

s4=[3*pi/4 5*pi/4 pi/4 7*pi/4 ]

%-------------------------------------------------------------------
-
%                    Transmitter Section
%-------------------------------------------------------------------
-

generated_bits=symbol_size*symbol_length*no_of_carriers;

generated_symbol=generated_bits/symbol_size;

%Generate Data

data_bits=randint(1,generated_bits);

%Generation of symbols

for p=1:2:generated_bits
    phase((p+1)/2)=s4(data_bits(p:p+1)*[2;1]+1);
end

    [X,Y]=pol2cart(phase,ones(size(generated_bits/symbol_size)));
    carrcmplx=X+i*Y;

% Serial To parrallel and IIFT of each OFDM frame & Adding Cyclic
Prefix

     for k=1:1: blocks/2
            for j=1:2:N
                  X_Odd(k,(j-1)/2+1)=carrcmplx((k-1)*N+j);
            end
            x_odd(k,:)=IFFT((X_Odd(k,:),N_carr);
            x_odd_cp(k,1:GI)=x_odd(k,N-GI+1:N);
            x_odd_cp(k,GI+1:N+GI)=x_odd(k,:);

            for j=2:2:N
                  X_Even(k,(j/2))=carrcmplx((k-1)*N+j);
            end
            x_even(k,:)=IFFT((X_Even(k,:)),N_carr);
            x_even_cp(k,1:GI)=x_even(k,N-GI+1:N);
            x_even_cp(k,GI+1:N+GI)=x_even(k,:);
    end


%-------------------------------------------------------------------
-
%                          Channel Tap Generation
%-------------------------------------------------------------------
-

total_taps=channel_length;
h1=zeros(blocks /2,channel_length);
h2=zeros(blocks /2,channel_length);
```

```
%initialization of taps for complete frame......
for k=1:2:no_of_iteration
    %for transmitter first
    h1((k+1)/2,1)=.86/2*rayleigh(1);
    h1((k+1)/2,2)=.486/2*rayleigh(1);
    h1((k+1)/2,3)=.2952/2*rayleigh(1);
    h1((k+1)/2,4)=.179/2*rayleigh(1);
    %for transmitter second
    h2((k+1)/2,1)=.86/2*rayleigh(1);
    h2((k+1)/2,2)=.486/2*rayleigh(1);
    h2((k+1)/2,3)=.2952/2*rayleigh(1);
    h2((k+1)/2,4)=.179/2*rayleigh(1);
end


for q1=1:length(SN)% calculated for different SNR values

H1bar=zeros(blocks /2,N_Carr);
H2bar=zeros(blocks /2,N_Carr);

for k=1: blocks/2
      H1bar(k,:)=fft(h1(k,:),N_Carr);
      H2bar(k,:)=fft(h2(k,:),N_Carr);
end

%------------------------------------------------------------------
          -                            Channel  Implementation
%------------------------------------------------------------------
-

noise_var=1/10^(SNR/10)
ch_noise=complex(sqrt(noise_var/2)*randn(blocks,m),sqrt(noise_var/2)
                                                                    *
                                                               randn(blocks,m));
Rc_symbol1=zeros(blocks,m);
buff1=zeros(blocks,m);
temp1=zeros(blocks,m+channel_length-1);

for k=1:2:blocks
temp1(k,:) = conv(x_odd_cp(k,:),h1(k,:));
Rc_symbol1(k,1:m)=temp1(k,1:m)+buff(k,1:m);
buff1(k,1:GI)=temp1(k,m+1:m+GI);
end

for k=2:2:blocks
temp1(k,:) = conv(-conj(x_even_cp(k,:),h1(k,:));
Rc_symbol1(k,1:m)=temp1(k,1:m)+buff(k,1:m);
buff1(k,1:GI)=temp1(k,m+1:m+GI);
end


Rc_symbol2=zeros(blocks,m);
buff2=zeros(blocks,m);
temp2=zeros(blocks,m+channel_length-1);

for k=1:2:blocks
```

```
temp2(k,:) = conv(x_odd_cp(k,:),h2(k,:));
Rc_symbol2(k,1:m)=temp2(k,1:m)+buff(k,1:m);
buff2(k,1:GI)=temp2(k,m+1:m+GI);
end

for k=2:2:blocks
temp2(k,:) = conv(conj(x_even_cp(k,:)),h2(k,:));
Rc_symbol2(k,1:m)=temp2(k,1:m)+buff(k,1:m);
buff2(k,1:GI)=temp2(k,m+1:m+GI);
end

for k=1:blocks
Rc_symbol(k,1:m)=
Rc_symbol1(k,1:m)+Rc_symbol2(k,1:m)+ch_noise(k,1:m);
end

%-------------------------------------------------------------------
-
%                        Receiver Side
%-------------------------------------------------------------------
-

% Removing Cyclic Prefix

y=zeros(blocks,N);      %Recived Signal after removing cyclic prefix
R=zeros(blocks,N_Carr); % received symbol after performing FFT

for k=1:blocks
  y(k,1:N)=Rc_symbol(k,GI+1:m);
  R(k,:)=FFT(y(k,:),N)   % performing fft
end

%-------------------------------------------------------------------
-
%  Calculation of BER when actual channel is known at the receiver
%-------------------------------------------------------------------
-

Bit_error_rate_act(1,q1)=BER_MISO(H1bar,H2bar,R,blocks,N_Carr);

%-------------------------------------------------------------------
-

%                              Initial Estimate Of Channel Taps
%-------------------------------------------------------------------
-

% Let we assume that  4 symbols equally spaced
% and know to reciever

for k=1:2:blocks
     t=(k+1)/2;

[H1_4pilots(t,:),H2_4pilots(t,:)]=interpolate_4pilots(R(k,1:N_Carr),
R((k+1),:),X_Odd(t,:),X_Even(t,:));
```

```
for k=1:2:blocks
     t=(k+1)/2;
[H1_int8(t,:),H2_int8(t,:)]=interpolate_8pilots(R(k,:),R((k+1),:),X_
                                                        Odd(t,:),X_Eve
                                                        n(t,:));
end
%-------------------------------------------------------------------
-

%  Calculation of BER for initial estimated channel
%-------------------------------------------------------------------
-


% when pilot based estimator using 8 pilots with linear interpolator
is %used

Bit_error_rate_8(1,q1)=BER_MISO(H1_8pilots,H2_8pilots,R,blocks,N_Car
r);


% when pilot based estimator using 4 pilots with linear interpolator
is %used

Bit_error_rate_4(1,q1)=BER_MISO(H1_4pilots,H2_4pilots,R,blocks,N_Car
r);

%-------------------------------------------------------------------
-

%                   Channel Estimation using EM technique
%-------------------------------------------------------------------
-

% First we estimate Tx Symbol by the estimated channel parameters
H1_es=H1_int8;
H2_es=H2_int8;
H1_es_new=zeros(blocks/2,N_Carr);
H2_es_new=zeros(blocks/2,N_Carr);
p=zeros(blocks/2,1);
%p2=zeros(blocks/2,1);
O1=zeros(1,N_Carr);
O2=zeros(1,N_Carr);
h1_es_new=zeros(1,channel_length);
h2_es_new=zeros(1,channel_length);
 g11=0;
 g21=zeros(1,N_Carr);
 %g12=0;
 g22=zeros(1,N_Carr);
   for k=1:2:blocks


                 l=0;

      while l==0 && p((k+1)/2)<=50

                 p((k+1)/2)=p((k+1)/2)+1;
```

```
                    b1=0;
                    b2=0;
              for j=1:N_Carr
                    K1=0;
                    K2=0;
                    K3=0;
                   for j1=1:4
                       for j2=1:4
   [K11,K21,K31]=estimate_em(R(k,j),R((k+1),j),H1_es((k+1)/2,j),H2_e
                        s((k+1)/2,j),X_cons(j1),X_cons(j2),noise_va
                        r);

                      K1=K1+K11;
                      K2=K2+K21;
                      K3=K3+K31;

                  end

                    H1_es_new((k+1)/2,j)=K2/K1/2;
                    H2_es_new((k+1)/2,j)=K3/K1/2;
              end
           O1=ifft(H1_es_new((k+1)/2,1:N_Carr));
           h1_es_new(1,1:channel_length)=O1(1,1:channel_length);

H1_es_new((k+1)/2,1:N_Carr)=fft(h1_es_new(1,1:channel_length),N_Carr
);
           O2=ifft(H2_es_new((k+1)/2,1:N_Carr));
           h2_es_new(1,1:channel_length)=O2(1,1:channel_length);

H2_es_new((k+1)/2,1:N_Carr)=fft(h2_es_new(1,1:channel_length),N_Carr
);


l=
converge(H1_es_new(k+1)/2,H1_es(k+1)/2,H2_es2(k+1)/2,H2_es_new(k+1)/
2);


H1_es((k+1)/2,1:N_Carr)=H1_es_new((k+1)/2,1:N_Carr);

H2_es((k+1)/2,1:N_Carr)=H2_es_new((k+1)/2,1:N_Carr);



           g11=g11+p((k+1)/2);

   end
   Avg_iteration_EM=g11/(blocks/2);


Bit_error_rate_EM(1,q1)=BER_MISO(H1_es,H2_es,R,blocks,N_Carr);


%-------------------------------------------------------------------
-

%                            Channel Estimation
```

```
%-----------------------------------------------------------------------
-

H1_es=H1_int8;
H2_es=H2_int8;

H1_es_new=zeros(blocks/2,N_Carr);
H2_es_new=zeros(blocks/2,N_Carr);
p=zeros(blocks/2,1);

O1=zeros(1,N_Carr);
O2=zeros(1,N_Carr);
h1_es_new=zeros(1,channel_length);
h2_es_new=zeros(1,channel_length);


for k=1:2:blocks


             l=0;

        while l==0 && p((k+1)/2)<=50

             p((k+1)/2)=p((k+1)/2)+1;
                 b1=0;
                 b2=0;


[X_Odd_es,X_Even_es]=Single_surv(H1_es((k+1)/2,:),H2_es((k+1)/2,:),R
(k,:),R((k+1),:),N_Carr);



[H1_es_new((k+1)/2,:),H2_es_new((k+1)/2,:)]=estimate1_MMSE(R(k,:),R(
(k+1),:),H1_es((k+1)/2,:),H2_es((k+1)/2,:),X_Odd_es,X_Even_es,noise_
var);


         O1=ifft(H1_es_new((k+1)/2,1:N_Carr));
         h1_es_new(1,1:channel_length)=O1(1,1:channel_length);

H1_es_new((k+1)/2,1:N_Carr)=fft(h1_es_new(1,1:channel_length),N_Carr
);
         O2=ifft(H2_es_new((k+1)/2,1:N_Carr));
         h2_es_new(1,1:channel_length)=O2(1,1:channel_length);

H2_es_new((k+1)/2,1:N_Carr)=fft(h2_es_new(1,1:channel_length),N_Carr
);


l=
converge(H1_es_new(k+1)/2,H1_es(k+1)/2,H2_es2(k+1)/2,H2_es_new(k+1)/
2);

H1_es((k+1)/2,1:N_Carr)=H1_es_new((k+1)/2,1:N_Carr);

H2_es((k+1)/2,1:N_Carr)=H2_es_new((k+1)/2,1:N_Carr);
```

```
            end

           g11=g11+p((k+1)/2);
end
   ad=blocks/2;
Avg_iteration_EM_MMSE(1,q1)=g11/(ad);

Bit_error_rate_EM_MMSE(1,q1)=BER_MISO(H1_es,H2_es,R,blocks,N_Carr);


end

%-------------------------------------------------------------------
-

%                Calculation of Bit error rate
%-------------------------------------------------------------------
-
```

BER_MISO.m

```
% Calculation of BER when true channel is  known at the
receiver.........

Bit_error_rate=function BER_MISO(H1bar,H2bar,R,blocks,N_Carr);

[X_Odd_es,X_Even_es]=receiver_two(H1bar,H2bar,R,blocks,N_Carr);

 data_es=zeros(1,generated_bits);
   for k=1:1:symbol_length/2
        for j=1:2:N
         carrcmplx_es(1,(k-1)*N+j)=X_Odd_es(k,(j-1)/2+1);
        end
        for j=2:2:N
         carrcmplx_es(1,(k-1)*N+j)=X_Even_es(k,(j/2));
        end

        for j=1:2:2*N
            [data_es(1,(k-1)*2*N+j),data_es(1,(k-1)*2*N+j+1)]
                              =convert(carrcmplx_es(1,(k-
                        1)*N+(j+1)/2));
        end
  end
  Bit_error=0;
 for j=1:generated_bits
  if(data_es(1,j)==data_bits(1,j))
      Bit_error=Bit_error;
  else
      Bit_error=Bit_error+1;
  end
end
```

```
 Bit_error_rate=Bit_error/generated_bits;


%---------------------------------------------------------------------
-
%          Channel Estimation using pilot based techniques
%---------------------------------------------------------------------
-
```

interpolate_8pilots.m

```
function [H_es1,H_es2]=interpolate_8pilots(R1,R2,X1,X2);

[H1_1 H2_1]=solve_eq(R1(1),R2(1),X1(1),X2(1));
[H1_15 H2_15]= solve_eq(R1(15),R2(15),X1(15),X2(15));
[H1_30 H2_30]= solve_eq(R1(30),R2(30),X1(30),X2(30));
[H1_50 H2_50]=solve_eq(R1(50),R2(50),X1(50),X2(50));
[H1_75 H2_75]= solve_eq(R1(75),R2(75),X1(75),X2(75));
[H1_90 H2_90]= solve_eq(R1(90),R2(90),X1(90),X2(90));
[H1_110 H2_110]=solve_eq(R1(110),R2(110),X1(110),X2(110));
[H1_128 H2_128]= solve_eq(R1(128),R2(128),X1(128),X2(128));
[H1_30 H2_30]= solve_eq(R1(30),R2(30),X1(30),X2(30));

for j=1:15
    H_es1(j)=H1_1+(H1_15-H1_1)/14*(j-1);
end
for j=15:30
    H_es1(j)=H1_15+(H1_30-H1_15)/15*(j-15);
end
for j=30:50
    H_es1(j)=H1_30+(H1_50-H1_30)/20*(j-30);
end

for j=50:75
    H_es1(j)=H1_50+(H1_75-H1_50)/25*(j-50);
end
for j=75:90
    H_es1(j)=H1_75+(H1_90-H1_75)/15*(j-75);
end
for j=90:110
    H_es1(j)=H1_90+(H1_110-H1_90)/20*(j-90);
end
for j=110:128
    H_es1(j)=H1_110+(H1_128-H1_110)/18*(j-110);
end

for j=1:15
    H_es2(j)=H2_1+(H2_15-H2_1)/14*(j-1);
end
for j=15:30
    H_es2(j)=H2_15+(H2_30-H2_15)/15*(j-15);
end
for j=30:50
    H_es2(j)=H2_30+(H2_50-H2_30)/20*(j-30);
end

for j=50:75
```

```
    H_es2(j)=H2_50+(H2_75-H2_50)/25*(j-50);
end
for j=75:90
    H_es2(j)=H2_75+(H2_90-H2_75)/15*(j-75);
end
for j=90:110
    H_es2(j)=H2_90+(H2_110-H2_90)/20*(j-90);
end
for j=110:128
    H_es2(j)=H2_110+(H2_128-H2_110)/18*(j-110);
end
```

interpolate_4pilots.m

```
function [H_es1,H_es2]=interpolate2_two(R1,R2,X1,X2);
[H1_1 H2_1]=solve_eq(R1(1),R2(1),X1(1),X2(1));
[H1_40 H2_40]= solve_eq(R1(40),R2(40),X1(40),X2(40));
[H1_80 H2_80]= solve_eq(R1(80),R2(80),X1(80),X2(80));
[H1_120 H2_120]=solve_eq(R1(120),R2(120),X1(120),X2(120));


for j=1:40
   H_es1(j)=H1_1+(H1_40-H1_1)/39*(j-1);
end
for j=40:80
    H_es1(j)=H1_40+(H1_80-H1_40)/40*(j-40);
end
for j=80:128
   H_es1(j)=H1_80+(H1_120-H1_80)/40*(j-80);
end
for j=1:40
   H_es2(j)=H2_1+(H2_40-H2_1)/39*(j-1);
end
for j=40:80
    H_es2(j)=H2_40+(H2_80-H2_40)/40*(j-40);
end
for j=80:128
   H_es2(j)=H2_80+(H2_120-H2_80)/40*(j-80);
End
```

solve_eq.m

```
function [H1 H2]= solve_eq(R1,R2,X1,X2);
H1=-(X2*R2-R1*conj(X1))/(conj(X2)*X2+conj(X1)*X1));
H2=(conj(X2*R1+R2*X1/(conj(X2)*X2+conj(X1)*X1));

%-----------------------------------------------------------------------
-
%                  is estimated channel converges
%-----------------------------------------------------------------------
-
```

converge.m

```
function l=converge(H1_es_new,H1_es,H2_es2,H2_es_new);
                  for j=1:N_Carr
```

```
                    a=(H1_es_new(1,j)-H1_es(1,j));

                   a1=real(a);
                   a2=imag(a);
                     if a1>=0
                       a1=a1;
                     else
                       a1=-a1;
                     end

                     if a2>=0
                       a2=a2;
                     else
                        a2=-a2;
                     end
                   a11=(H2_es_new((k+1)/2,j)-H2_es((k+1)/2,j));

                   a12=real(a11);
                   a21=imag(a11);
                     if a12>=0
                       a12=a12;
                     else
                       a12=-a12;
                     end

                     if a21>=0
                       a21=a21;
                     else
                        a21=-a21;
                     end
                     if a1<=1e-3 && a2<=1e-3
                         b1=b1+1;
                     else
                         b1=b1;
                     end
                     if a12<=1e-3 && a21<=1e-3
                         b2=b2+1;
                     else
                         b2=b2;
                     end
                 end
                  if b1>=120 && b2>=120
                   l=1;
                  end

          end
%--------------------------------------------------------------------
-
% Estimation of transmitted symbols using channel information and
% received symbols
%--------------------------------------------------------------------
-
```

receiver_two.m

```
function [X_Odd_es,X_Even_es]=
```

```
                              receiver_two(H1_es,H2_es,R,no_of_iteration,N
                              );

for k=1:2:no_of_iteration
    t=(k+1)/2;
    for j=1:N

X_Odd_es(t,j)=(conj(H1_es((k+1)/2,j))*R(k,j)+conj(R(k+1,j))*H2_es((k
+1)/2,j))/(H1_es((k+1)/2,j)*conj(H1_es((k+1)/2,j))+conj(H2_es((k+1)/
2,j))*H2_es((k+1)/2,j));

    X_Odd_es((k+1)/2,j)=Ml_decoding( X_Odd_es((k+1)/2,j));

    X_Even_es((k+1)/2,j) =-(H1_es((k+1)/2,j)*conj(R(k+1,j))-
R(k,j)*conj(H2_es((k+1)/2,j)))/(H1_es((k+1)/2,j)*conj(H1_es((k+1)/2,
j))+conj(H2_es((k+1)/2,j))*H2_es((k+1)/2,j));

    X_Even_es((k+1)/2,j)=Ml_decoding( X_Even_es((k+1)/2,j));
    end
end

%--------------------------------------------------------------------
-%             Channel Estimation using EM technique
%--------------------------------------------------------------------
-
```

estimate_em.m

```
function
[K11,K21,K31]=estimate1_two(R1,R2,H1_es,H2_es,X1,X2,noise_var);
X_cons=[-1/sqrt(2)+i*1/sqrt(2) -1/sqrt(2)-i*1/sqrt(2)
1/sqrt(2)+i*1/sqrt(2) 1/sqrt(2)-i*1/sqrt(2)];
K=0;
B=1/(2*pi*noise_var);
A=1/(2*noise_var);
V=100;
for j1=1:4
    for j2=1:4
        C1=R1-H1_es*X_cons(j1)-H2_es*X_cons(j2);
        C2=R2+H1_es*conj(X_cons(j2))-H2_es*conj(X_cons(j1));
        C=C1*conj(C1)+C2*conj(C2);
        K=K+B*exp(-C*A+V);
    end
end

C1=R1-H1_es*X1-H2_es*X2;
C2=R2+H1_es*conj(X2)-H2_es*conj(X1);
C=C1*conj(C1)+C2*conj(C2);
K11=B*exp(-C*A+V);
K21=K11*(R1*conj(X1)-R2*X2)/K;
K31=K11*(R1*conj(X2)+R2*X1)/K;
K11=K11/K;

%--------------------------------------------------------------------
-
%          Channel Estimation using EM-MMSE technique
```

%---
-

estimate1_MMSE.m

```
function
[H1_es_new,H2_es_new]=estimate1_MMSE(R1,R2,H1_es,H2_es,X1,X2,noise_v
ar);
X_cons=[-1/sqrt(2)+i*1/sqrt(2) -1/sqrt(2)-i*1/sqrt(2)
1/sqrt(2)+i*1/sqrt(2) 1/sqrt(2)-i*1/sqrt(2)];

H1_es_new=(R1.*conj(X1)-R2.*X2)/2;
H2_es_new=(R1.*conj(X2)+R2.*X1)/2;
```