

# **UNIT COMMITMENT PROBLEM UNDER DEREGULATED ENVIRONMENT**

**A DISSERTATION**

**Submitted in partial fulfillment of the  
requirements for the award of the degree**

**of**

**MASTER OF TECHNOLOGY**

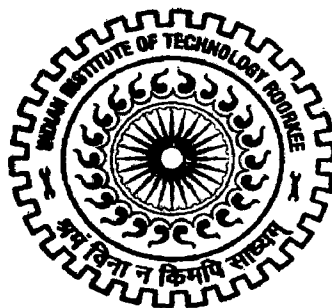
**in**

**ELECTRICAL ENGINEERING**

**(With Specialization in Power System Engineering)**

**By**

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**JUNE, 2004**

# **CANDIDATE'S DECLARATION**

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I hereby declare that the work presented in this dissertation entitled "**Unit Commitment Problem Under Deregulated Environment**" submitted in partial fulfillment of the requirements for the award of the degree of Master of Technology with specialization in Power System Engineering in the Department of Electrical Engineering, Indian Institute of Technology Roorkee, Roorkee is an authentic record of my own work carried out from July 2003 to June 2004 under the guidance of **Dr. N.P.Padhy**, Assistant Professor, Department of Electrical Engineering, Indian Institute of Technology Roorkee, Roorkee.

I have not submitted the matter embodied in this report for the award of any other degree or diploma.

**Date:** 29 June 2004

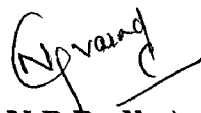
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## **CERTIFICATE**

This is to certify that the above statement made by the candidate is true to the best of my knowledge and belief.

  
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## ACKNOWLEDGEMENTS

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Last but not least, I'm highly indebted to my parents and family members, whose sincere prayers, best wishes, moral support and encouragement have a constant source of assurance, guidance, strength and inspiration to me.

(P.V.RAMA KRISHNA)

## ABSTRACT

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The unit commitment problem under deregulated environment involves determining the time intervals at which a particular generating unit should be online and available for generation, and the associated generation or dispatch, the aim being to maximize its total profits based on a given price profile. This dissertation describes how a lagrangian relaxation method and single unit dynamic programming algorithm is used to solve this complex optimization problem. All the usual unit constraints are considered, after which results for the chosen 26 generating units are presented, and discussed.

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## LIST OF SYMBOLS

<i>Symbol</i>	<i>Description</i>	<i>unit</i>	<i>page</i>
$\alpha^i$	Combined crew startup costs and the equipment maintenance cost.	\$/h	8
$\beta^i$	Cold-startup cost of unit i.	\$/h	8
$X_{i,off}^t$	Continuous off line time of unit i at time interval t.	h	8
$\tau_i$	Cooling time constant of unit i.	h	8
$M$	Total number of generating units.	—	12
$\lambda^t$	[Forecasted market price at Time interval t (Also used as The Lagrange multiplier for the power balance constraint at time interval t)]	\$/MWh	13
$T$	Total number of time intervals in the scheduling period.	h	13
$P_i^t$	Output power of unit i at time interval t.	MW	14
$F_i(P_i^t)$	Fuel cost of unit i when its output is $P_i^t$ .	\$/h	14
$F_{T,i}^t$	Total production cost of Unit i at time interval t.	\$/h	14
$a_i, b_i, c_i$	Parameters of the Second-order cost function of unit i.	\$/MWh <sup>2</sup> \$/MWh	14
$inc_i^k$	Incremental cost of segment $k$ of unit i. $k=1,2,3$	\$ \$/MWh	14
$nl_i^k$	No load cost of segment $k$ of unit i. $k=1,2,3$	\$/h	14

$P_i^{\min}$	Lower generation limit of unit i	MW	14
$P_i^{\max}$	Upper generation limit of unit i.	MW	14
$e_i^1$	First elbow point of the piece-wise linear cost function of unit i	MW	14
$e_i^2$	Second elbow point of the Piece-wise linear cost function of unit i	MW	14
$U_i^t$	Status of unit i at time interval t (0 or 1)	—	15
$X_i^t$	Cumulative up (down) time of unit i at time interval t.	h	15
$T_i^{up}$	Maximum up time constraint of unit i.	h	15
$T_i^{down}$	Minimum down time constraint of unit i	h	15
$R_i^{up}$	Ramp up constraint of unit i.	MW/h	15
$R_i^{down}$	Ramp down constraint of unit i	MW/h	15
$SU_i^t$	Start-up cost of unit i at time interval t.	\$/h	16
$t_{on}$	Time interval at which a unit is started up	h	17
$t_{off}$	Time interval prior to the one at which the unit is shut down.	h	17
$profit_i^t$	Profit attained by unit i over time t.	\$/h	17
$l_t$	Forecasted system load demand.	MW	24
$Y_i^{\max}$	Maximum number of hours required for unit i to ramp down from $P_i^{\max}$ to 0MW	h	31



$Y_i^t$	Number of hours until unit $i$ at time interval $t$ , is shut down.	h	31
$R_{i,limit}^{down}(X_i^t)$	Ramp-down limit of unit $i$ as a function of state $Y_i^t$	MW	32
$V_i^t$	Decision to shut down in $Y_i^{max} - 1$ hours. If ( $V_i^t = 1$ )	—	32
$R_{i,limit}^{up}(X_i^t)$	Ramp-up limit of unit $i$ as a function of state $X_i^t$	MW	33

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## INTRODUCTION AND LITERATURE REVIEW

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### 1.1 INTRODUCTION

Electric power consumption varies with time reflecting the predictably cyclical nature of human activities. The demand for electricity is higher during the day and early evening, weekdays and the summer or winter seasons as compared to the late night and early morning, weekends and fall or spring seasons. Also, electricity is a non-storable commodity and needs to be produced at the same rate at which it is consumed. In order to run the electric power generation system economically so as to reliably meet the demand, it is thus necessary to switch the generating units on and off at appropriate times. The generating units cannot however be turned on and off in a haphazard manner. Besides the start-up costs, one also needs to consider certain operating constraints that dictate how frequently and in what manner the units can be turned on and off. They are, for example: minimum up time, minimum down time, minimum capacity, maximum capacity, and ramping rate. The decision problem of optimally scheduling the operation of these machines is known as the unit commitment problem (UCP). [Wood and Wollenberg (1996)]. Before the restructuring of electric power systems it is the point of generation part of utility and after deregulation it is the Point of Generation Company wishing to optimize their operation, which is minimum production cost for the first case and maximize profit for the second case. The restructuring and deregulation of electric power systems have resulted in market-based competition by creating an open market environment. A restructured system allows the power supply to function competitively, as well as allowing consumers to choose suppliers of electric energy. In a regulated framework, an electric utility serves the customers of a certain region under tariffs calculated to guarantee the recovery of its costs. In this situation, a power generating utility solves the UCP to obtain an optimal production schedule of its units to meet customer demand. The optimal schedule is found by minimizing the production cost over a given time interval while satisfying the demand and the set of operating constraints. The minimization of the production costs assures maximum profits because the power generating utility has no option but to reliably supply the prevailing demand. The price of electricity over this period is predetermined. Therefore; the decisions on the operation of

individual units have no effect on the firms' revenues. Under deregulation the price of electricity is however no longer predetermined. The unit commitment decisions in this situation are based on the expected market price of electricity rather than on the demand although these variables are usually correlated. The UCP now requires the stochastic formulation that includes the representation of the electricity market. And a bottom up engineering economic approach is used to forecasting the market prices. Well-known mathematical programming techniques such as integer programming, dynamic programming, branch and bound, Benders decomposition, and Lagrangian relaxation method have been used to solve the UCP. For small problems, they can provide the optimal solution in a reasonable amount of time. However, for large problems, the computational time required to find the optimal solution becomes prohibitive. In such cases, the solution space is only partially searched and therefore there is no guarantee that the optimal solution can be found. Meta-heuristic methods such as simulated annealing tabu search, and genetic algorithms have also been used for solving these large and highly complex problems. Within a regulated framework, the unit commitment problem can be solved by minimizing the production cost over a given time interval while satisfying the demand and the set of operating constraints. The price of electricity over this period is predetermined. Under deregulation the price of electricity is however no longer predetermined. The unit commitment decisions in this situation are based on the expected market price of electricity rather than on the demand although these variables are usually correlated. In a deregulated environment if a power producer is having  $M$  number of generating then the unit commitment problem can be solved by considering each unit separately .in this situation the objective is to maximize expected profits, and the decisions are required to meet standard operating constraints. When the market price of electricity is considered exogenous to the unit commitment decisions and the demand constraints are the only coupling constraints, then the optimization problem (for a generation company acting as a price-taker in a Pool Co-type electricity market) decomposes in a straightforward manner into as many sub-problems as the number of generating units owned by the company. Therefore, the optimal solution of a UCP with  $M$  units can be found by solving  $M$  uncoupled sub-problems. The feature of decomposability into sub-problems for the deregulated market considerably reduces the computational burden.

## **1.2 MOTIVATION AND OBJECTIVE**

In order to stay in business, an electric utility company or an electric power producer must not only avoid making losses; they must also achieve sufficient revenue to meet a certain profit margin. Profit maximization is therefore the motivation for this dissertation project, the specific objective being to develop a software program to solve the unit commitment (UC) problem so that it maximizes a generator's total profits of a single power producer over a given scheduling period.

## **1.3 OUTLINE OF THE DISSERTATION**

The first chapter of this dissertation presents the introduction and motivation for this work and outlines the main objective that it seeks to achieve.

**Chapter 2** provides an idea of unit commitment problem, constraints and costs associated with the problem and solution techniques that are useful for solving conventional unit commitment problem.

**Chapter 3** provides an idea of unit commitment problem under deregulated environment and also gives the idea of structure of deregulated power system and calculation of marginal price or market clearing price.

**Chapter 4** suggests an idea of problem formulation for UCP and constraints associated with the main problem.

**Chapter 5** suggests some of classical and non classical solution techniques for solving the unit commitment problem under deregulation, which includes explanation about some of the classical methods like simulated annealing, branch and bound and some of the non classical methods like artificial intelligence methods and genetic algorithm.

**Chapter 6** provides the application of lagrangian relaxation method for decomposition of main problem into several sub problems and application of dynamic programming in solving these single units sub problems.

**Chapter 7** gives the results for the chosen 26 generating units for profit, Power and state.

## 1.4 LITERATURE REVIEW

A paper by "*Valenzuela, J; Majumdar, M*" suggests a formulation for the commitment of electric power generators under a deregulated electricity market in this the problem is expressed as a stochastic optimization problem in which the expected profits are maximized while meeting demand and standard operating constraints. They showed that when an electric power producer has the option of trading electricity at market prices an optimal unit commitment schedule could be obtained by considering each unit separately. Therefore they describe solution procedures for the self-commitment of one generating unit only. This description is given for three certainty equivalent formulations of the stochastic problem.

Another paper by "*Valenzuela, J; Majumdar, M*" suggests the computation of the bivariate probability distribution of the marginal unit. They suggested that for the calculation of the probability distribution of the marginal unit there is a need to consider the all-possible combinations of generating units.

Another paper by "*Cohen, A.I*" suggests how to consider the ramp limitation constraints in solving unit commitment problem.

Another paper by "*Wang, C; shahidehpour, S.M*" suggests the effects of ramp rate limits on unit commitment and economic dispatch.

Another paper by "*Valenzuela, J; Mazumdar, M*" highlights the need for considering the stochastic processes associated with the frequency and duration of generating unit outages for assessing the mean and variance of production costs under operating constraints. A numerical example based on a Markov model is given to show that Monte Carlo estimates of these quantities may be incorrect if only the forced outage rates are used in place of the stochastic parameters underlying the outage frequency and duration. Additionally it describes a variance reduction procedure whereby the Monte Carlo estimates can be obtained with a much smaller sample size than would be required.

Another paper described by "*kothari, D.P; Ahmad, A*" suggests hybrid expert system dynamic programming approach to the power system unit commitment problem. Here

supplementing it with the rule based expert system enhances the scheduling output of the usual dynamic programming.

Another paper by “*Sasaki, H; watanabe, M; Kubokawa, J; Yorino, N; Yokoyama, R*”. Suggests the possibility of applying the Hop field neural network to combinatorial optimization problems in power systems, in particular to unit commitment. They suggested that dedicated neural networks could handle a large number of inequality constraints included in unit commitment

Another paper by “*Kazarlis, S.A; Bakirtzis, A.G; Petridis, V*” presented a genetic algorithm (GA) solution to the unit commitment problem. They suggests that genetic algorithms are general purpose optimization techniques based on principles inspired from the biological evolution using metaphors of mechanism such as natural selection, genetic recombination and survival of the fittest. They implemented a simple GA algorithm using the standard crossover and mutation operators.

Another paper by “*Mantawy, A.H; Abdel-Magid, Y.L; Selim, S.Z*” suggests an application of the tabu search (TS) method to solve the unit commitment problem (UCP). The TS seeks to counter the danger of entrapment at a local optimum by incorporating a memory structure that forbids or penalizes certain moves that would return to recently visited solutions. New rules for randomly generating feasible solutions for the UCP are introduced. The problem is divided into two sub problems: a combinatorial optimization problem and a nonlinear programming problem. The former is solved using the tabu search algorithm (TSA) while the latter problem is solved via a quadratic programming routine.

Another paper by “*Baldick, R*” formulate a generalized version of the unit commitment problem that can treat minimum up and down time constraints, power flow constraints, line flow limits, voltage limits reserve constraints, ramp limits, and total fuel and energy limits on hydro and thermal power generating units.

Another paper by “*Sheble, G.B; Fahd, G.N*” suggests several optimization techniques that have been applied to the solution of the thermal unit commitment problem.



Another paper by “*Cohen, A.I; Brandwahjn, V; Show-kan Chang*” suggests that for many electric power systems the limitations of the transmission network strongly affects the system operation. In traditional regulated power systems, the operator follows operating rules and procedures to schedule generation so that the power system operates reliably. As the power industry moves towards open markets, it is necessary to develop power system schedules that produce high-quality non-discriminatory schedules that are consistent with secure operation. This paper describes the Security constrained unit commitment (SCUC) program that has been developed to meet this need.

Another paper by “*Bellman, R.E; Dreyfus, S.E*” suggests the application of dynamic programming in solving unit commitment problem.

Another paper by “*Kazarlis, S.A; bakirtzis, A, G; petridis, V*”: suggests the application of genetic algorithm solution in solving the unit commitment problem,’

Another paper by “*Chen, C.L. and Wang, S.C*” suggests an idea of branch and bound scheduling for thermal units. The details of branch and bound technique and simulated annealing and several other classical and non-classical techniques have been discussed in the coming chapter.

## UNIT COMMITMENT PROBLEM

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### 2.1 INTRODUCTION

Since human activities follow cycles, most systems supplying services to a large population will experience cycles. These include transportation systems, communication systems etc. The total loads on the system will generally be higher during day time and early evening when industrial loads are high, lights are on, and so forth, and lower during late evening and early morning when most of the population are asleep. In addition, the use of electric power has a weekly cycle, the load being lower over weekend than weekdays. Why is this a problem in the operation of an electric power system? Why not just simply commit enough units to cover the maximum system loads and leave them running? Note that to "commit" a generating unit is to "turn it on", i.e. to bring the unit up to speed synchronize it to the system, and connect it so that it can deliver power to the network. The problem with "commit enough units and leave them on line" is one of economics. It is quite expensive to run too many generating units. Turning units off when they are not needed can save a great deal of money. Hence, electricity generating companies and power systems has the problem of deciding how best to meet the varying demand for electricity.

### 2.2 UNIT COMMITMENT PROBLEM

The unit commitment problem is to schedule available generators (on or off) to meet the required loads at a minimum cost subject to system constraints.

#### 2.2.1 System Constraints

1. The total output of all the generating units must be equal to the forecast value of the system demand at each time-point.
2. The total spinning reserve from all the generating units must be greater than or equal to the spinning-reserve requirement of the system. This can be either a fixed requirement in MW or a specified percentage of the largest output of any generating unit. (The purpose of the spinning-reserve requirement is to ensure that there is enough spare capacity from the units on-load or 'spinning' at any

time to cover the accidental loss of any individual generating unit, or to meet higher than expected demands.)

3. Minimum up time: Once the unit is running, it should not be turned off immediately.
4. Minimum down time: Once the unit is decommitted (off), there is a minimum time before it can be recommitted.
5. The output power of the generating units must be greater or equal to the minimum power of the generating units.
6. The output power of the generating units must be smaller or equal to the Maximum power of the generating units.

### 2.2.2 Cost Calculation

Mainly, the total power production can be separated into two parts that is start-up cost and operating cost.

- 1) Start-up cost is warmth-dependent, corresponding to the hot, warm or cold condition of each generating unit, as determined by the time that the unit has been off-loaded. Its value depends on the shutdown time; alpha, beta and  $\tau$  which can be obtained from Unit Data.

$$SU_i^t = \alpha_i + \beta_i(1 - \exp(-X_{off,i}^t / \tau_i))$$

$SU_i^t$ : Start-up cost of unit i at time interval t.

$\alpha_i$ : Combined crew start-up costs and the equipment maintenance costs of unit i

$\beta_i$ : Cold start-up cost of unit i

$\tau_i$ : Cooling time constant of unit i

$X_{off,i}^t$ : Continuous offline time of unit i at time interval t.

- 2) Each generating unit has a 'no-load' or fixed operating cost and a number of incremental operating costs, which can define a non-linear profile of operating costs.

## **2.3 SOLUTION METHODS**

### **2.3.1 general background and concepts**

Various approaches have been developed to solve the optimal UC problem. These approaches have ranged from highly complex and theoretically complicated methods to simple rule-of thumb methods. The scope of operations scheduling problem will vary strongly from utility to utility depending on their mix of units and particular operating constraints. The economic consequences of operation scheduling are very important. Since fuel cost is a major cost component, reducing the fuel cost by little as 0.5% can result in savings of millions of dollars per year for large utilities. A very important task in the operation of a power system concerns the optimal UC considering technical and economical constraints over a long planning horizon up to one year. The solution of the exact long-term UC is not possible due to exorbitant computing time and, on the other hand, the extrapolation of short-term UC to long-term period is inadequate because too many constraints are neglected such as maintenance time and price increases, etc. Energy management systems have to perform more complicated and timely system control functions to operate a large power system reliably and efficiently.

### **2.3.2 Solution Methods For Conventional UCP**

Well-known mathematical programming techniques such as integer programming, dynamic programming, branch and bound, Benders decomposition, and Lagrangian relaxation have been used to solve the UCP. For small problems, they can provide the optimal solution in a reasonable amount of time. However, for large problems, the computational time required to find the optimal solution becomes prohibitive. In such cases, the solution space is only partially searched and therefore there is no guarantee that the optimal solution can be found. Meta-heuristic methods such as simulated annealing tabu search, and genetic algorithms have also been used for solving these large and highly complex problems.

## UCP UNDER DEREGULATION

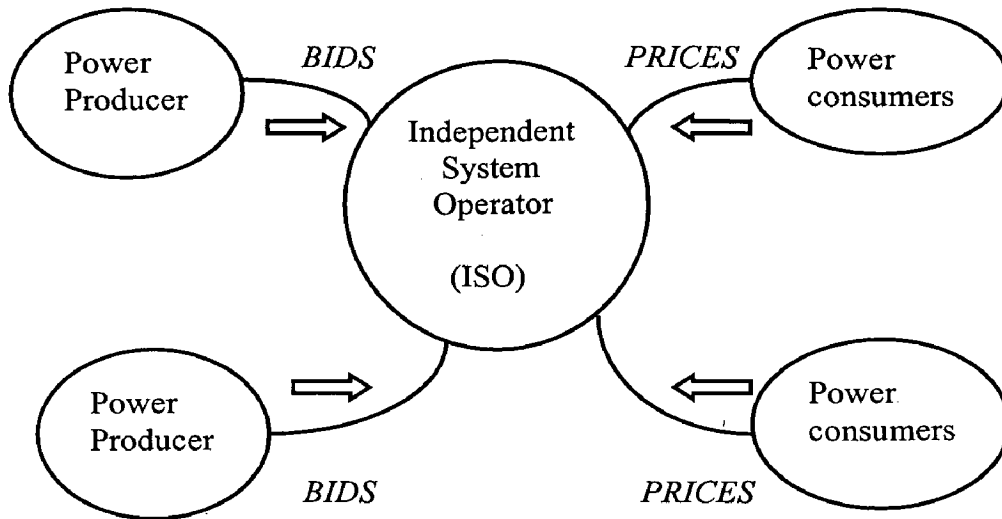
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### 3.1 INTRODUCTION

The unit commitment problem can be analyzed in two situations. The first one is the unit commitment before the restructuring of electric power systems, while the second one is based on the system after deregulation. Before the restructuring of electric power systems it is the point of generation part of utility and after deregulation it is the Point of Generation Company wishing to optimize their operation, which is minimum production cost for the first case and maximize profit for the second case. The restructuring and deregulation of electric power systems have resulted in market-based competition by creating an open market environment. A restructured system allows the power supply to function competitively, as well as allowing consumers to choose suppliers of electric energy. In a regulated framework, an electric utility serves the customers of a certain region under tariffs calculated to guarantee the recovery of its costs. In this situation, a power generating utility solves the UCP to obtain an optimal production schedule of its units to meet customer demand. The optimal schedule is found by minimizing the production cost over a given time interval while satisfying the demand and the set of operating constraints. The minimization of the production costs assures maximum profits because the power generating utility has no option but to reliably supply the prevailing demand. The price of electricity over this period is predetermined. Therefore; the decisions on the operation of individual units have no effect on the firms' revenues. Under deregulation the price of electricity is however no longer predetermined. The unit commitment decisions in this situation are based on the expected market price of electricity rather than on the demand although these variables are usually correlated. In a deregulated power system there will be mainly three components exist. One is power producer and second is power consumers and last one is independent system operator which acts as a mediator between power producers and power consumers, usually the independent system operator will forecast the load demand at each and every instant of our and accepts the bids and prices from producers and consumers to meet this load demand.

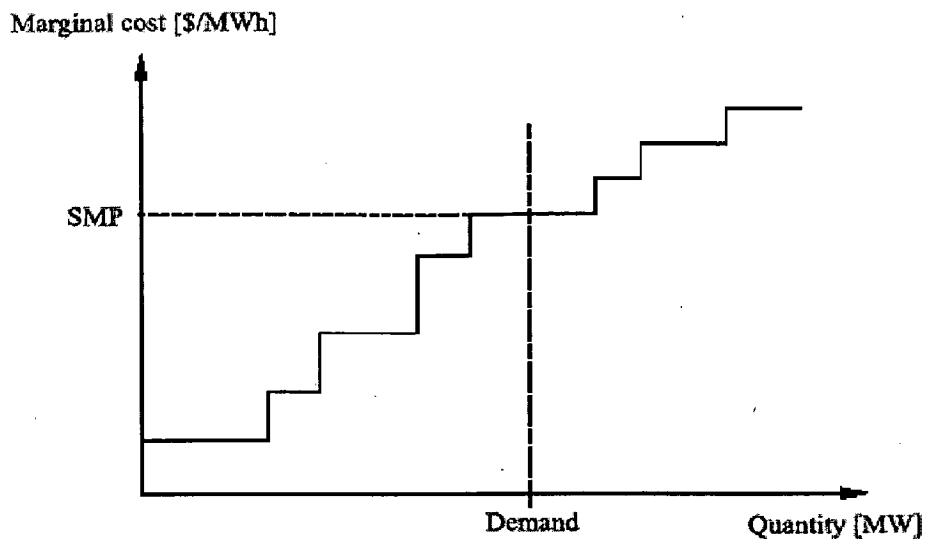
### 3.2 STRUCTURE OF DEREGULATED POWER SYSTEM



**Figure 3.1:**structure of deregulated power system

As shown in figure the central bidding and pricing system in a deregulated environment involves mainly a group of power producers and power consumers and an independent system operator, which acts as a mediator between producers and consumers. In a deregulated power system Scheduling and price-setting is done centrally by the system operator to determine which generating units to start up, when to connect them to the network, how much they should generate when they are online, the order in which they should be shut down, how the bids and prices should be accepted and forecasted load demand. A unit is said to be 'committed' when it is scheduled for connection to the system.

Generally power producers and power consumers will submit their bids and prices to the power pool. And the independent system operator forecasts the load demand at each and every instant of hour and it will accept the lowest bids to meet the load demand. In a pool-based competitive electricity market, bids offered by the generators are aggregated to form the supply curve. The prices are ranked and taken in an increasing order until the demand is satisfied. This is carried out every half-hour. The adjusted marginal price of the most expensive unit scheduled at each half hour is then used to set the market-clearing price for that half-hour. This price is known as the system marginal price (SMP), and every MWH traded during that period is sold at this price. **Figure 3.2** shows how the SMP is determined.



**Figure 3.2:** Determination of the system marginal price.

### 3.3 UNIT COMMITMENT PROBLEM UNDER DEREGULATION

Unit commitment problem under deregulation is particularly related with either power producer or generating company and it involves determining the optimum combination of available units (if a power producer is having  $M$  number of generating units) to serve the forecasted demand at minimum production cost and to obtain maximum profit, while observing all power system and unit constraints. The total production cost consists of fuel costs, transition costs (start-up/shut-down costs) and no-load costs (when a unit is idle or on standby).

## PROBLEM FORMULATION FOR UCP UNDER DEREGULATION

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### 4.1 UCP UNDER DEREGULATION

In a decentralized electricity market or in a deregulated environment, the aim is not to minimize the total production cost of a particular generating utility company or a power producer, but to maximize its total profit. This can be achieved by considering each generating unit separately and maximizing its profit independent of other units. The estimated values of the market clearing prices over the scheduling period is used as an input to the optimization of the schedule of this single unit. Once the optimal schedule is obtained, it can be used to assess which offers would be the most profitable based on a forecasted price profile. In practice, however, it may not be possible for a utility to make all the trades recommended at each trading period.

### 4.2 PROBLEM FORMULATION

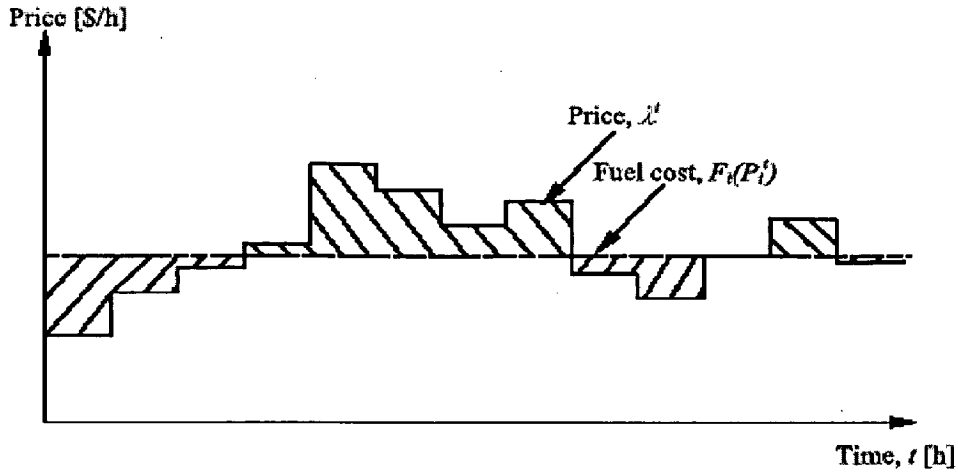
The unit commitment problem under deregulation can be stated as follows:

*“For an electric utility or a power producer with  $M$  generating units, and given a certain market price profile it is required to determine the start-up/shut-down times and the power output levels of all the generating units at each time interval  $t$  over a specified scheduling period  $T$ . so that the generator’s total profit is maximized, subject to the unit constraints.”*

The price can be the actual market prices or simply an estimate of how the prices will fluctuate during the scheduling period. This is illustrated in **figure 4.1** for an arbitrary price profile. The output of the unit and hence its fuel cost are assumed to be constant.

The areas above the fuel cost give the profit made during periods when  $\lambda^t$  is higher than the fuel cost, while the areas below the fuel cost give the loss incurred during periods when  $\lambda^t$  is lower than the fuel cost. A utility would try to maximize the gains and minimize the losses in order to make as large a net profit as possible.





**Figure 4.1:** profit and loss determination given an arbitrary price profile and  
Assuming a constant fuel cost.

In practice, the fuel cost for a unit is not constant. The fuel cost,  $F_i(P_i^t)$  of unit  $i$  in any given time interval  $t$  is a function of the power output,  $P_i^t$ , of that unit during that time interval. The cost function is usually modeled as a second-order polynomial, as given by equation ( $F_{T,i}^t(P_i^t) = a_i (P_i^t)^2 + b_i P_i^t + c_i$ ) Where  $a_i, b_i, \text{ and } c_i$  are constants Alternatively it can be represented by a piece-wise linear cost function as shown in figure 4.2 For piece wise linear characteristics the cost function will be given by the formula as below

$$F_{T,i}^t = [(inc_i^k * \sum_{ton}^{toff} P_i^t) + SU_i^{ton} + \sum_{ton}^{toff} nl_i^k] U_i^t \quad [4.1]$$

$inc_i^k$  : Incremental cost of segment  $k$  of unit  $i$  [\$/MWh],  $k=1,2$  and  $3$ ;

$nl_i^k$  : No-load cost of segment  $k$  of unit  $i$  [\$/h],  $k=1,2$  and  $3$ ;

$P_i^{\min}, P_i^{\max}$  : The lower and upper generation limits of unit  $i$ , respectively in [MW];

$e_i^1, e_i^2$  : The first and second elbow points of the piece-wise linear cost Function of unit  $i$ , respectively [MW].

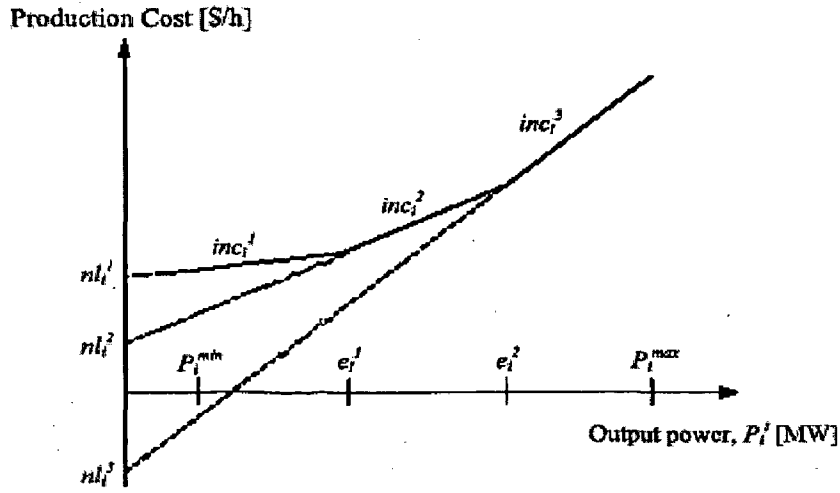


Figure 4.2: piece-wise linear cost function

The UCP under deregulation can be formulated by first defining the following:

- Let  $U_i^t = 0$  if unit  $i$  is offline during time interval  $t$ ;  
 $U_i^t = 1$  if unit  $i$  is online during time interval  $t$ ;  
 $X_i^t =$  Cumulative up time during time interval  $t$  if  $X_i^t > 0$ ;  
 $X_i^t =$  Cumulative down time during time interval  $t$  if  $X_i^t < 0$ ;

Thermal units are subject to a variety of constraints. The unit constraints that must be satisfied during the maximization process are:

1. Unit limits-units can only generate between given limits:

$$U_i^t P_i^{\min} \leq P_i^t \leq U_i^t P_i^{\max} \quad \text{For } i=1,2,3\dots N \text{ and } t=1,2,3\dots T \quad [4.2]$$

2. Unit minimum up time constraint:

$$(X_i^{t-1} - T_i^{up})(U_i^{t-1} - U_i^t) \geq 0 \quad \text{for } i=1,2,3\dots N \text{ and } t=1,2,3\dots T \quad [4.3]$$

Where  $T_i^{up}$  is the minimum up time constraint [h].

3. Unit minimum down time constraint:

$$(-X_i^{t-1} - T_i^{down})(U_i^t - U_i^{t-1}) \geq 0 \quad \text{for } i=1,2,3\dots N \text{ and } t=1,2,3\dots T \quad [4.4]$$

Where  $T_i^{down}$  is the minimum down time constraint [h]

4. Unit ramp-up constraint- the amount a unit's generation can increase in an Hour.

$$P_i^t - P_i^{t-1} \leq R_i^{up} \quad \text{For } i=1,2, \dots N \text{ and } t=1,2, \dots T \quad [4.5]$$

Where  $R_i^{up}$  is the ramp-up constraint [MW/h].

5. Unit ramp-down constraint-the amount a units generation can decrease in an Hour.

$$P_i^{t-1} - P_i^t \leq R_i^{down} \text{ For } i=1,2\dots N \text{ and } t=1,2\dots T \quad [4.6]$$

Where  $R_i^{down}$  is the ramp down constraint [MW/h].

Equations [4.5] and [4.6] apply to hours between start-up and shut down.

The limit at start-up is given by:

$$P_i^t \leq \text{Max} (R_i^{up}, P_i^{\min}) \text{ for } i=1,2\dots N \text{ and } t=1,2\dots T \quad [4.7]$$

The limit at shut down is given by:

$$P_i^t \leq \text{Max} (R_i^{down}, P_i^{\min}) \text{ for } i=1,2\dots N \text{ and } t=1,2\dots T \quad [4.8]$$

6. Unit status restrictions-certain units may be required to be online at certain time intervals (must run), or may become unavailable due to planned maintenance or forced outage (must not run), due to operating constraints, reliability requirements, or economic reasons.

7. The initial conditions of the units at the start of the scheduling period must be considered. Plant crew constraints were not considered (thermal plants may have limits on the number of units that can be committed or decommitted in a given time interval due to manpower limits). Also, units may be derated (i.e. have their generating limits reduced), or required to operate at pre-specified generation levels. These restrictions were also ignored. The start-up cost in any given time interval  $t$  depends on the number of hours a unit has been off prior to start-up. This can be modeled by an exponential function of the form:

$$SU_i^t = \alpha_i + \beta_i (1 - \exp(-X_{off,i}^t / \tau_i)) \quad [4.9]$$

Where

$\alpha_i$ : Combined crew start-up costs and equipment maintenance costs [\$];

$\beta_i$ : Cold start-up cost [\$];

$X_{off,i}^t$ : Number of hours the unit has been offline [h];

$\tau_i$ : Unit-cooling time constant [h].

The shutdown cost,  $SD_i^t$ , is usually given a constant value for each unit per shutdown and in this dissertation is assumed to be zero. The total production cost,  $F_{Ti}^t$ , for each unit at

each time interval is the sum of the running cost, start-up cost and shutdown cost during that interval.

$$F_{T,i}^t = [(inc_i^k \sum_{ton}^{toff} P_i^t) + SU_i^{ton} + \sum_{ton}^{toff} nl_i^k] U_i^t \quad [4.10]$$

The profit at each time interval is calculated by subtracting the total production cost during that interval from the revenue, as given by equation [4.11]. A negative profit indicates a loss

$$Profit_i^t = (\lambda^t * P_i^t) U_i^t - F_{T,i}^t \quad [4.11]$$

$$(\lambda^t * P_i^t) U_i^t = \text{Revenue at time interval } t.$$

As mentioned previously, the prices,  $\lambda^t$ , can be actual market prices or an estimate of how they would fluctuate, and are given in [\$/MWh]. The total profit for unit  $i$  is then given by

$$Total\ profit = \sum_{i=1}^T Profit_i^t \quad [4.12]$$

The main complication arises from the unit minimum up and down time constraints. When a unit is committed, it incurs a cost equal to its start-up cost. It then has to stay online until its minimum up time constraint has been satisfied before it can be shut down again. Similarly, once a unit is decommitted, it has to remain offline for as long as its minimum down time constraint requires before it can be recommitted.

Another difficulty lies in the time-dependent nature of the start-up cost. Although committing a unit at a particular point in the scheduling period may not be the most profitable choice at that instant, it may still yield a better solution over the entire study period compared to the case in which the unit remained offline at the aforesaid point in time. This option may have been totally lost during the optimization process through economic disqualification; i.e. if the feasible state associated with starting up the unit had been discarded because it incurred a loss during start-up.

Thus the utility has to decide whether to:

- a) Keep a unit committed even when the price is low during a particular period, incurring fuel cost during that period with the hope that the profit made in the following period when the price is high, together with the savings achieved from not having to start up the unit at the start of or during the high-price period would offset the losses; or

- b) Shut down the unit during the period of low price and incur a cost when starting up the unit for the following high-price period.

Note that in the second case, the unit might have to forfeit any profit it would otherwise have been able to attain in the period of high price if its minimum down time constraint required that it remained offline during that period (or part of it). Similar decisions must be considered when moving from a high-price period to a low-price period.

These considerations, compounded with the other unit constraints discussed above. Clearly render this a very complex problem, as making the wrong decisions could significantly reduce the profitability of the unit. Fortunately, there are several possible techniques that can be used to solve this problem. These will be discussed in the following chapter.

## SOLUTION TECHNIQUES FOR UCP UNDER DEREGULATION

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### 5.1 INTRODUCTION

The unit commitment problem under deregulation is a large, non-linear, mixed integer-programming problem. A number of techniques can be used to solve this problem. These can be divided into two categories.

- Classical methods, which include dynamic programming, simulated annealing, complete enumeration, lagrangian relaxation, and tabu search, interior point optimization.
- Non-classical methods like artificial neural networks, fuzzy systems, genetic algorithms, evolutionary programming.

### 5.2 CLASSICAL METHODS

These are traditional methods that require sufficient modeling of the problem. In other words, the system model must be formulated with the capabilities of the chosen algorithm in mind.

#### 5.2.1 complete enumeration

This ideal method involves an exhaustive trial of all the possible solutions and then choosing the best among them. All feasible solutions, i.e those that satisfy the unit constraints, are evaluated and stored. The solution with the highest total profit is taken as the optimal schedule. Given enough time this method is guaranteed to find the optimal solution. However, the length of time to complete this enumerative process and the amount of computer storage required usually render this method useless for its intended purpose.

#### 5.2.2 branch and bound

Branch and bound (B&B) is a general search algorithm, which finds the optimal solution by keeping the best solution found so far. A partial solution is abandoned if it cannot perform better than the best.

The method starts by considering the original problem (known as the root problem) with the complete feasible region. The lower bound and upper-bound procedures are then applied to the root problem if the bounds match, it means that an optimal solution has

been found and dividing it into two or more regions portions the procedure terminates. otherwise, the feasible region. Each being a strict sub-region of the original, which together cover the whole feasible region. These sub-problems become children of the root search node. This procedure is applied to each sub-problem recursively, thus generating a tree of sub-problems. An optimal solution found to a sub-problem will always be a feasible solution to the full problem, but not necessarily the global optimal. Since it is feasible, it can be used to 'prune' the rest of the tree: if the lower bound for a node is greater than the best known feasible solution, then that node can be eliminated since no globally optimal solution can exist in the sub-space of the feasible region represented by that node. In this way, the search proceeds until all nodes have been solved or pruned, or until some specified threshold is met between the best feasible solution found and the lower bounds on all unsolved sub-problems. In this latter case, the solution obtained will be sub-optimal.

### **5.2.3 simulated annealing**

Simulated annealing is a generalization of a Monte Carlo method for examining the equations of state and frozen states of N body systems. The concept is based on the manner in which metals recrystallise in the process of annealing. The generalization of this Monte Carlo approach to the profit maximization UC problem is straight forward, with the following analogies:

- 1) Current state of the thermodynamic system=current solution to the problem.
- 2) Energy equation for the thermodynamic system=objective function; and
- 3) Ground state=global optimum.

This method has the following advantages

- 1) It produces a reasonably good solution that does not strongly depend on the choice of the initial solution, and is therefore able to improve a sub-optimal solution obtained using another method.
- 2) It does not require complicated mathematical models of the UC problem, and it does not require excessive computer storage.

## **5.3 ARTIFICIAL INTELLIGENCE METHODS**

### **5.3.1 artificial neural networks**

An artificial neural network is a system that loosely models the human brain. it attempts to simulate the multiple layers of simple processing elements called neurons, each neuron receives inputs from many neighboring neurons, but produce only a single output, which

is communicated to other neurons. However, the neurons of one layer are always connected to the neurons of at least another layer. Communication between linked neurons is via varying coefficients of connectivity that represent the strengths or weights of these connections. The output of each neuron is simply the sum of the products of all the inputs and their respective weights. Learning is accomplished by adjusting the weights to cause the overall network to output appropriate results.

Designing an ANN consists of:

- 1) Arranging neurons in various layers.
- 2) Deciding the type of connections among neurons for different layers, as well as among the neurons within a layer.
- 3) Deciding the way a neuron receives input and produces output.
- 4) Determining the strength of connection within the network by allowing it to learn the appropriate values of connection weights by using a training data set.

### **5.3.2 genetic algorithm**

Optimization techniques based on some of the biological processes observed in nature are slowly finding widespread application as methods for solving complex problems, due to the robustness, flexibility and efficiency of biological systems. A good example of this is genetic algorithms (GA's). GA's represent a population of individuals by equal-size strings or matrices of bits. Each population of individuals, termed 'chromosomes' represent a candidate solution to a specific problem. Simple transformations based on the process of natural selection (survival of the fittest), mating and reproduction, and genetic information recombination within the population through a set of main genetic operators such as crossover and mutation, are performed on the existing chromosomes to produce new, improved 'offsprings', i.e. new problem solutions. These operators are very simple, involving nothing more complex than random number generation, string copying and partial string exchanging.

When applying GA's to real-life problems, it is necessary to encode the problem solution using a chromosome representation. An evaluation function is needed to provide a 'fitness' figure of merit for any chromosome in the context of the problem.

A typical genetic algorithm involves the following cycle:

- 1) initialize the generation counter to zero and randomly generate an initial population of chromosomes, each of which is called a 'member' of the population.
- 2) Evaluate the fitness of each member in the population.
- 3) Select pairs of members for fitness-proportionate mating and reproduction.



4) Perform crossover and mutation on the selected members to create new offspring.

5) Form a population for the next generation and delete the old one.

6) If the process has converged, or if the generation counter exceeds some pre-declared maximum. Stop the process and return the fittest member as the solution.

Other wise, go to step 2.

For the unit commitment problem under deregulation the fitness of each member is simply measured by its profit level, i.e.the higher the total profit the fitter the member.

## APPLICATION OF LAGRANGIAN RELAXATION METHOD AND DYNAMIC PROGRAMMING

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### 6.1 APPLICATION OF LAGRANGIAN RELAXATION METHOD

#### 6.1.1 modeling the operation of generating units

In determining an optimal commitment schedule, there are two decision variables  $P_i^t$  and  $U_i^t$  where  $P_i^t$  denotes the amount of power to be generated by unit  $i$  at time  $t$  and  $U_i^t$  is the control variable whose value is chosen to be “1” if the generating unit  $i$  is committed at hour  $t$  and “0” otherwise (of course if  $U_i^t = 0$ , then  $P_i^t = 0$ ) the cost of the power produced by the generating unit  $i$  depends on the amount of fuel consumed and is typically approximated by a quadratic cost function and later it was approximated by piece wise linear cost function. The start up cost can be calculated using equation 4.9. In addition to startup cost the generating unit must satisfy all the constraints (minimum up time, minimum down time, ramp up and ramp down, minimum power and maximum power generation) as discussed in the previous chapter.

#### 6.1.2 decomposition into sub-problems

The objective function is total profit, revenue minus cost over the interval  $[1, T]$ . The revenue during hour  $t$  is obtained from supplying the quantity stipulated under the long-term bilateral contracts and by selling surplus energy (if any) to the power pool at the market price,  $\lambda^t$  (\$/MWh). The cost includes those of producing the energy, buying short falls (if needed) from the power pool, and the start-up costs. Defining the amount to be sold under the bilateral contract by  $l_t$  (MWh), the contract price by  $R$  (\$/MWh), and the amount of energy bought or sold from the market by  $E_t$ , we solve the optimization problem by maximizing the expected profit over the period  $[1, T]$ . (A positive value of  $E_t$  indicates that  $E_t$  (MWh) is bought from the power poll and a negative value indicates that  $-E_t$  (MWh) is sold to the pool. The objective function is given by:

$$\text{Max } E \left\{ \sum_{t=1}^T \{ l_t R - \lambda^t E_t - \sum_{i=1}^M F_{T,i}^t \} \right\} \quad [6.1]$$

This is under the case that the cost function is represented by piece wise linear cost characteristics. If cost function is same as in the form of quadratic cost function then the objective function will become

$$\text{Max } E\left\{\sum_{t=1}^T \{l_t R - \lambda^t E_t - \sum_{i=1}^M [CF_i(P_i^t) + SU_i^t(X_i^{t-1})(1 - U_i^{t-1})]U_i^t\}\right\}$$

Where  $CF_i(P_i^t) = F_{T,i}^t(P_i^t) = a_i(P_i^t)^2 + b_i P_i^t + c_i$  but we usually consider that the cost function will be represented by piece wise linear cost characteristics because this representation will fetch us to take the values of cost function as discrete values. And hence the objective function will be given by equation (6.1)

Since the quantity  $l_t R$  is a constant, the optimization problem reduces to:

$$\text{Max } E\left\{\sum_{t=1}^T \{-\lambda^t E_t - \sum_{i=1}^M F_{T,i}^t\}\right\} \quad [6.2]$$

Where  $F_{T,i}^t = [(inc_i^k \sum_{ton}^{toff} P_i^t) + SU_i^{ton} + \sum_{ton}^{toff} nl_i^k]U_i^t$  as given by equation (4.10)

Subject to the following constraints (for  $t=1 \dots T, i=1 \dots M$ )

$$1) \text{ Load: } E_t + \sum_{i=1}^M P_i^t = l_t \quad [6.3]$$

$$2) \text{ Capacity limits: } U_i^t P_i^{\min} \leq P_i^t \leq U_i^t P_i^{\max} \quad [6.4]$$

$$3) \text{ Minimum up time: } U_i^t \geq I(1 \leq X_i^{t-1} \leq t_i^{UP} - 1) \quad [6.5]$$

$$4) \text{ Minimum down time: } U_i^t \leq 1 - I(-t_i^{down} + 1 \leq X_i^{t-1} \leq -1) \quad [6.6]$$

Where  $I(X) = 0$  if  $X$  is false,  
 $= 1$  if  $X$  is true.

And  $U_i^t = 1$  if  $X_i^t > 0$   
 $0$  if  $X_i^t < 0$

After substituting in the objective function  $E_t + \sum_{i=1}^M P_i^t = l_t$  obtained from equation

6.4 we can rewrite equation 6.3 as follows.

$$\text{Max } E\left\{\sum_{t=1}^T \{-\lambda^t (l_t - \sum_{i=1}^M P_i^t) - \sum_{i=1}^M F_{T,i}^t\}\right\} \quad [6.7]$$

Which after removing the constant term is equivalent to

$$\text{Max } E\left\{\sum_{t=1}^T \sum_{i=1}^M \lambda^t P_i^t - F_{T,i}^t\right\} \quad [6.8]$$

Subject to the operating constraints 6.5, 6.6, and 6.7 because these constraints refer to individual units only. Equation 6.8 shows that the optimization problem is now separable by individual units. The optimal solution can be found by solving M-decoupled sub problems. Thus the sub problem for the  $i$ th unit is

$$\text{Max } E\left\{\sum_{t=1}^T \lambda_t P_i^t - F_{T,i}^t\right\} \quad [6.9]$$

Subject to the operating constraints of the  $i^{\text{th}}$  unit. Equation (6.9) is similar to the sub-problem obtained in the standard version of the UCP using the Lagrangian relaxation method, except that the values of Lagrange multipliers are now replaced by the market price of electricity  $\lambda^t$  and the expected value is being maximized. When the optimization sub-problem is solved for a particular unit, we assume that the market consists of  $N$  generating units ( $N$  will be much larger than  $M$ ). The generating unit for which the sub-problem is solved is excluded from the market. Excluding a unit from the market does not influence the spot price because of the existence in all likelihood of a number of generating units with almost equal marginal costs, ready to produce if any of the infra-marginal (or marginal) units are unavailable.

## 6.2 THE PARAMETER $\lambda$

Usually the parameter  $\lambda$  is a Lagrangian multiplier, but in this deregulated case this parameter is considered to be the market price (whether it is observed price or a calculated one). Fig 6.1 shows how the parameter  $\lambda$  is adjusted in real life situation to solve the unit commitment problem. In this method usually the coordinator sends a set of Lagrange multipliers  $\lambda^t$ , to each generating unit. Each unit then tries to minimize its total production cost based on the fixed values of  $\lambda^t$ . If during the time interval  $t$ , the forecasted demand is  $L^t$  and if  $\sum_{i=1}^M P_i^t > L^t$ , then the coordinator decreases  $\lambda^t$ . Conversely

$\sum_{i=1}^M P_i^t < L^t$  then the coordinator increases  $\lambda^t$ . The values of  $\lambda^t$  are adjusted in this manner

until the balance between demand and supply is achieved during each time interval to get

the optimum schedule. The application of lagrangian relaxation method in solving unit commitment problem is described in greater detail in [3].

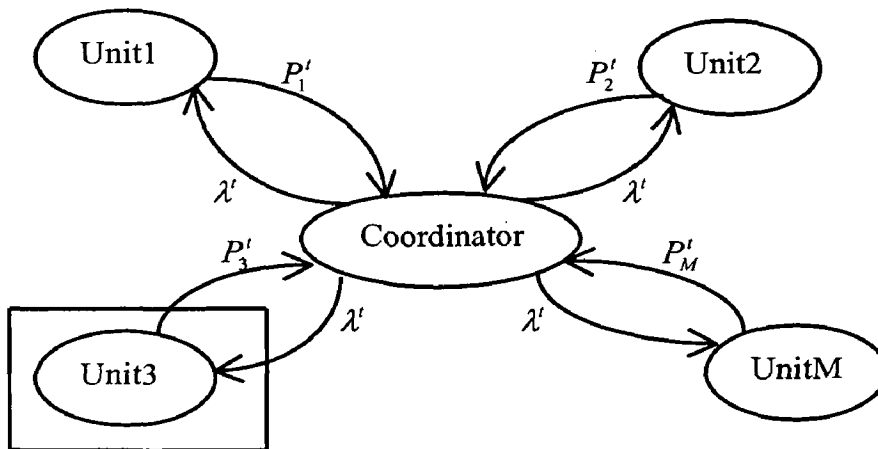


Fig 6.1: application of lagrangian relaxation method for single unit  
Considering M units

### 6.3 SINGLE UNIT DYNAMIC PROGRAMMING

Dynamic programming (DP) was the earliest optimization-based technique to be applied to the UC problem and is still used extensively all over the world. The DP technique employs a systematic searching algorithm that tries to achieve the optimal solution without having to access all the possible combinations. The unit commitment problem can be solved using a dynamic programming algorithm. This technique can be applied because:

- 1) The problem satisfies the principle of optimality if all parts of an optimal solution are themselves optimal solutions to sub-problems.
- 2) The number of relevant sub-problem depends on a limited number of smaller sub-problems.
- 3) The number of relevant sub-problems is limited by the unit constraints.

#### 6.3.1 the dynamic programming algorithm

The unit commitment problem can be solved in a bottom-up manner, whereby:

- 1) The smallest sub-problems are solved first. This corresponds to finding the feasible states (whether 0 or 1), the associated nominal generation or dispatch. And the profit that it would entail for each unit at each time interval.
- 2) These solutions are then combined to solve larger sub-problems. In this case the individual profits of each feasible solution path are added together to give the total

Profit over the scheduling period, after which the path with the highest total profit is determined. This gives the optimal schedule for that unit for a given price profile.

- 3) Finally, the individual maximum total profits are summed over all the units in the utility to give its maximum total profit for a given price profile.

The dynamic programming algorithm is given as follows:

- Specify the rule that relates large problems to small problems.
- Store the partial feasible solutions of each sub problem.
- Extract the final solution for main problem considering all solutions from sub problems.

### 6.3.2 Single Unit Dynamic Programming

The objective function for the dynamic programming algorithm would be the one that gives the total profits for a unit  $i$  over the scheduling period. This was defined in equation (4.12) and can be rewritten as:

$$\text{Total profit}_i = \sum_{t=1}^T [(\lambda^t * P_i^t U_i^t) - [inc_i^k * \sum_{ton}^{toff} P_i^t] + SU_i^{ton} + \sum_{ton}^{toff} nl_i^k] U_i^t \quad [6.10]$$

The aim is to find the maximum of this function. This is easily formulated as a single unit DP problem, as illustrated in figure 6.2

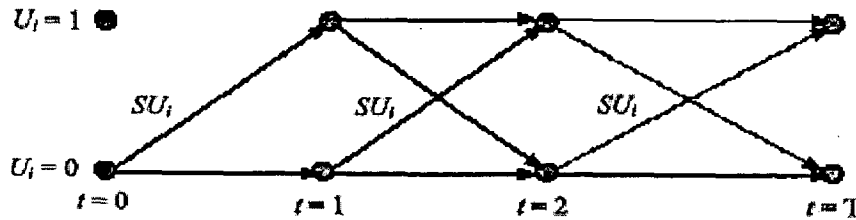


Figure 6.2: single unit dynamic programming problem

When  $U_i^t = 0$ , the value of the function to be maximized is trivial and equals to zero.

When  $U_i^t = 1$ , the function to be maximized is:

$$\text{Max} [(\lambda^t * P_i^t) - [inc_i^k * \sum_{ton}^{toff} P_i^t] + \sum_{ton}^{toff} nl_i^k] \quad [6.11]$$

The start-up cost,  $SU_i^{ton}$ , is neglected. As it is only added at instances when the unit is turned on. The maximum of this function is found by taking its first derivative:

$$d / dp_i^t [(\lambda^t p_i^t) - [inc_i^k * \sum_{ton}^{toff} P_i^t] + \sum_{ton}^{toff} nl_i^k] = \lambda^t - inc_i^k = 0 \quad [6.12]$$

Giving  $\lambda^t = inc_i^k$

However, the incremental costs of a unit can only take on discrete values if the unit characteristics are represented by a piece-wise linear cost function. Therefore, the output power of each unit,  $P_i^t$  is determined as follows:

- If  $\lambda^t \leq inc_i^1$  then  $P_i^t = p_i^{\min}$  [6.13]

- If  $inc_i^1 < \lambda^t \leq inc_i^2$  then  $P_i^t = e_i^1$  [6.14]

- If  $inc_i^2 < \lambda^t \leq inc_i^3$  then  $P_i^t = e_i^2$  [6.15]

- If  $\lambda^t > inc_i^3$  then  $P_i^t = p_i^{\max}$  [6.16]

Equations [6.13] to [6.16] only apply if the unit ramp rates are non-binding. *i.e.* in cases where the ramp-up and ramp-down rates are very large. Otherwise, the generation must comply strictly with the ramping constraints, regardless of the profit or loss level, to avoid shortening the life of the turbine due to excessive ramp rates.

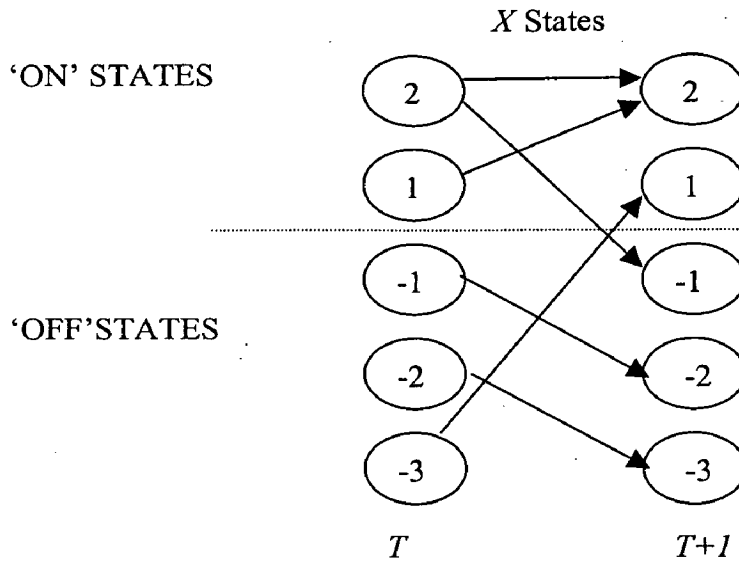
### 6.3.3 MODELLING THE EFFECTS OF $T_i^{up}$ and $T_i^{down}$

$X_i^t$  And  $U_i^t$ , are related by the following difference equation: where  $X_i^t$  is the number of hours the unit has been on or off line and  $U_i^t$  denotes the state of the unit either 0 or 1.

$$X_i^t = \begin{cases} X_i^{t-1} & \text{If } X_i^{t-1} = T_i^{up} \text{ and } U_i^t = 1 & [6.17] \\ X_i^{t-1} + 1 & \text{If } X_i^{t-1} \geq 1 \text{ and } U_i^t = 1 & [6.18] \\ 1, & \text{If } X_i^{t-1} = -T_i^{down} \text{ and } U_i^t = 1 & [6.19]^* \\ X_i^{t-1} & \text{If } X_i^{t-1} = -T_i^{down} \text{ and } U_i^t = 0 & [6.20] \\ X_i^{t-1} - 1 & \text{If } X_i^{t-1} \leq -1 \text{ and } U_i^t = 0 & [6.21] \\ -1 & \text{If } X_i^{t-1} = T_i^{up} \text{ and } U_i^t = 0 & [6.22] \end{cases}$$

Where  $X_i^{t-1} = T_i^{up} (-T_i^{down})$  corresponds to case in which the unit is on (off) for at least  $T_i^{up}$  hours. Figure 6.3 illustrates the state transition diagram for the case in which  $T_i^{up}$  is 2 hours and  $T_i^{down}$  is 3 hours. It is simply an expanded version of that given in figure 6.2 But shown here only for 2 time intervals, and is used to model the effects of the minimum up and down time constraints. However, because of the minimum up and down time constraints, not all states are feasible. Also if a state can be reached by more than one

path, then from the theorem of optimality [14] only the one with the highest cumulative profit needs to be preserved; the others can be eliminated. These not only reduce the size of the actual search space, but also the program run time.



**Figure 6.3:** state transition diagram for  $X$  states for  $T_i^{up} = 2$  hours and  $T_i^{down} = 3$  hours.

Here is an example of how we consider the up and down time constraints while solving unit commitment problem

**Example:** consider the total scheduling period to be 12 hours i.e.  $T=12$ ;

So for a particular unit there will be  $2^{T-1}$  combinations can be possible for the entire time period  $T$ . out of all these combination we will consider only the combinations which are satisfying the minimum up and down time constraints using equations [6.17] to [6.22] after considering minimum up and down time constraints we will get the feasible combinations which are satisfying up and down time constraints out of all  $2^{T-1}$  combinations. For example: considering some of the possible states for the above  $T_i^{up} = 2$  hours and  $T_i^{down} = 3$  hours and time period for 12 hours.

- 1) There will be  $2^{12-1} = 4095$  states are possible, out of all these states only some of the states are feasible states, feasible states in the sense that which satisfy up and down time constraints.

Considering some of the states: here only five states have shown for an idea



(TABLE-I)

Example For five combinations Considering Up And Down Time Constraints

1	1	1	1	1	0	0	0	0	0	1	1	0
2	0	0	0	0	1	1	1	1	0	0	0	1
3	0	0	1	1	1	1	0	0	1	1	1	0
4	1	1	1	1	1	0	0	0	0	1	1	1
5	1	1	1	1	0	0	0	1	1	1	0	0

Out of all these five states we have to see which combinations are feasible.

Considering the first state.  $U(1)$  If we calculate the  $X$  states for this combination assuming initial state at time zero is in off state of  $-3$  hours, then the  $X$  states will become  $X(1)=$

1	1	1	1	0	0	0	0	0	1	1	0
1	2	2	2	-1	-2	-3	-3	-3	1	2	-1

Considering the second state  $U(2)$  and  $X(2)=$

0	0	0	0	1	1	1	1	0	0	0	1
-3	-3	-3	-3	1	2	2	2	-1	-2	-3	1

Considering The Third State  $U(3)$  and  $X(3)$

0	0	1	1	1	1	0	0	1	1	1	0
-3	-3	1	2	2	2	-1	-2				

This is not a feasible state because it is not satisfying minimum down time constraint.

Considering The Fourth State  $U(4)$  and  $X(4)$

1	1	1	1	1	0	0	0	0	1	1	1
1	2	2	2	2	-1	-2	-3	-3	1	2	2

This is also feasible state and if we calculate  $X$  states for this it will be

Considering the Fifth State  $U(5)$  and  $X(5)$

1	1	1	1	0	0	0	1	1	1	0	0
1	2	2	2	-1	-2	-3	1	2	2	-1	-2

This is also feasible state and  $X$  values for this combination will be

So out of all five combinations one combination is not satisfying the up and down time constraints. Like this manner we will calculate all the feasible states for the entire time period. As mentioned already the up and down time constraints are useful for finding all feasible states from all the possible combinations.

### 6.3.4 MODELLING THE EFFECTS OF $R_i^{up}$ and $R_i^{down}$

To model the effects of the ramping constraints in the one-unit DP algorithm, the following approach can be used [4].

Let  $Y_i^t$  = number of hours until shut down.

$Y_i^{\max}$  = Maximum number of hours required to ramp down from  $P_i^{\max}$  to 0 MW.

To determine  $Y_i^{\max}$ , the following algorithm can be employed:

1) Start with  $Y_i^{\max} = 0$ ,  $R_{limit}^{down} = P_i^{\max}$

2)  $R_{limit}^{down} = R_{limit}^{down} - R^{down} * \Delta t$

$Y_i^{\max} = Y_i^{\max} + 1$ ;

3) Is  $R_{limit}^{down} > P_i^{\min}$

Yes  $\longrightarrow$  GO TO 2

NO  $\longrightarrow$  GO TO 4.

4) Is  $R_{limit}^{down} > 0$

YES  $\longrightarrow$   $Y_i^{\max} = Y_i^{\max} + 1$ ;

NO  $\longrightarrow$  END.

#### 6.3.4.1 calculation of y states and ramp down logic

$Y_i^t$ ,  $Y_i^{\max}$ ,  $U_i^t$  Are related by the five prong equations as below

$$Y_i^t = \begin{cases} Y_i^{\max} & \text{If } Y_i^{t-1} = 0 \text{ and } U_i^t = 1 \\ Y_i^{t-1} & \text{If } Y_i^{t-1} = Y_i^{\max} \text{ and } V_i^t = 0 \\ Y_i^{t-1} - 1 & \text{if } Y_i^{t-1} = Y_i^{\max} \text{ and } V_i^t = 1 \\ Y_i^{t-1} - 1 & \text{if } Y_i^{t-1} < Y_i^{\max} \\ 0 & \text{if } U_i^t = 0 \end{cases}$$

Where  $V_i^t = 1$  is the decision to shut down in  $(Y_i^{\max} - 1)$  hours. The  $Y$  states need only be considered if a state is within  $(Y_i^{\max} - 1)$  hours of meeting its minimum up time constraint.

Figure 6.4 illustrates the state transition diagram for the case in which  $T_i^{up} = 2$  hours and  $T_i^{down} = 4$  hours and  $Y_i^{max} = 2$  hours

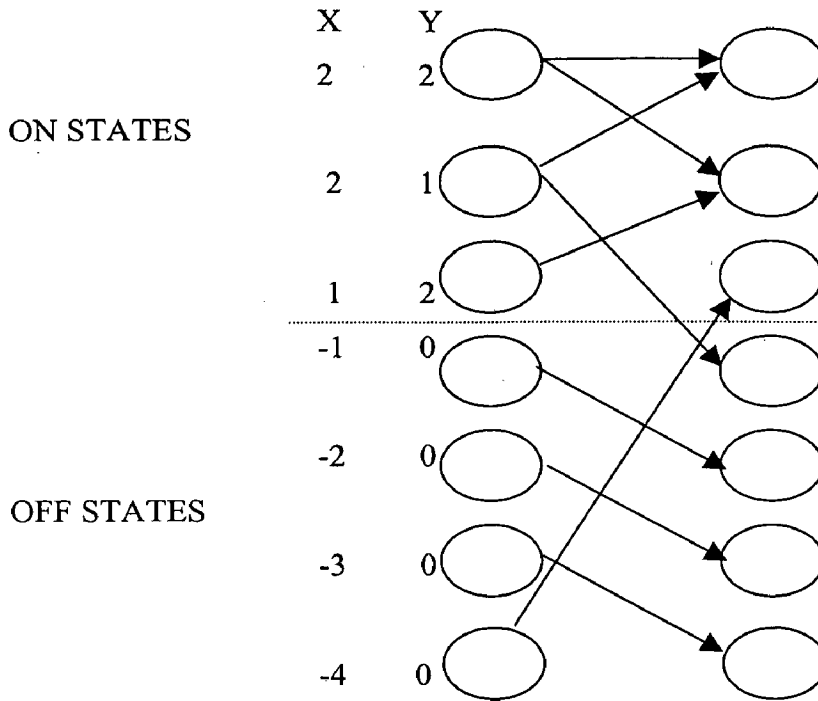


Fig 6.4: state transition diagram for  $X$  and  $Y$  states

If unit  $i$  is to be shut down in  $Y_i^t$  hours at time  $t$ , then:

$$R_{i\text{limit}}^{down}(Y_i^t) = \begin{cases} P_i^{\max} & \text{If } Y_i^t = Y_i^{\max} \\ \text{Max}(P_i^{\min}, R_{i\text{limit}}^{down}(Y_i^t + 1) - R_i^{down} * \Delta t) & \text{If } 0 < Y_i^t < Y_i^{\max} \\ 0 & \text{If } Y_i^t = 0 \end{cases}$$

Where  $R_{i\text{limit}}^{down}(Y_i^t)$  is the ramp-down limit as a function of state  $Y_i^t$ ,  $i=1,2,3\dots M$  and  $t=1,2,3\dots T$ . the generation  $P_i^t$  must satisfy the following:

$$\begin{aligned} P_i^t &\leq R_{i\text{limit}}^{up}(X_i^t) && \text{If } R_{i\text{limit}}^{up}(X_i^t) < R_{i\text{limit}}^{down}(Y_i^t) \\ R_{i\text{limit}}^{down}(Y_i^t) &\leq P_i^t \leq R_{i\text{limit}}^{up}(X_i^t) && \text{If } R_{i\text{limit}}^{up}(X_i^t) \geq R_{i\text{limit}}^{down}(Y_i^t) \end{aligned}$$

Figure 6.4 illustrates the state transition diagram for the case in which  $T_i^{up} = 2$  hours and  $T_i^{down} = 4$  hours and  $Y_i^{max} = 2$  hours

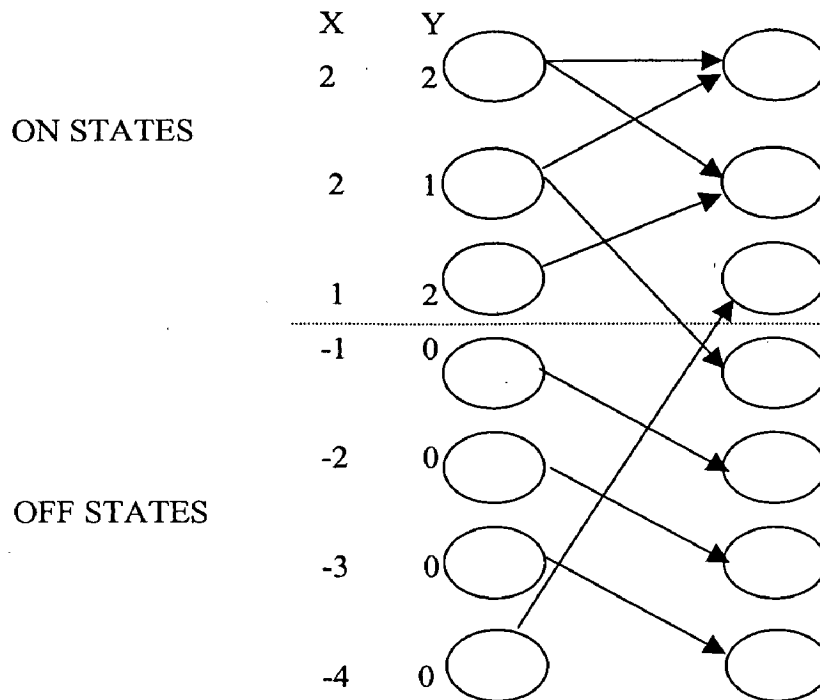


Fig 6.4: state transition diagram for X and Y states

If unit i is to be shut down in  $Y_i^t$  hours at time t, then:

$$R_{i \text{ limit}}^{down}(Y_i^t) = \begin{cases} P_i^{\max} & \text{If } Y_i^t = Y_i^{\max} \\ \text{Max}(P_i^{\min}, R_{i \text{ limit}}^{down}(Y_i^t + 1) - R_i^{down} * \Delta t) & \text{If } 0 < Y_i^t < Y_i^{\max} \\ 0 & \text{If } Y_i^t = 0 \end{cases}$$

Where  $R_{i \text{ limit}}^{down}(Y_i^t)$  is the ramp-down limit as a function of state  $Y_i^t$ ,  $i=1,2,3...M$  and  $t=1,2,3...T$ . the generation  $P_i^t$  must satisfy the following:

$$\begin{aligned} P_i^t &\leq R_{i \text{ limit}}^{up}(X_i^t) && \text{If } R_{i \text{ limit}}^{up}(X_i^t) < R_{i \text{ limit}}^{down}(Y_i^t) \\ R_{i \text{ limit}}^{down}(Y_i^t) &\leq P_i^t \leq R_{i \text{ limit}}^{up}(X_i^t) && \text{If } R_{i \text{ limit}}^{up}(X_i^t) \geq R_{i \text{ limit}}^{down}(Y_i^t) \end{aligned}$$

### 6.3.4.2 ramp-up logic:

If unit  $i$  has been on for  $X_i^t$  hours at time  $t$  then:

$$R_{i\text{limit}}^{up}(X_i^t + 1) = \begin{cases} \text{Max}(R_i^{up}, P_i^{\min}) & \text{if } X_i^t = 0 \\ \text{Min}(P_i^{\max}, R_{i\text{limit}}^{up}(X_i^t) + R_i^{up}), & \text{if } X_i^t \geq 1 \end{cases}$$

Where  $R_{i\text{limit}}^{up}(X_i^t)$  is the ramp-up limit as a function of state  $X_i^t$  for  $i=1,2,3\dots M$  and  $t=1,2,3\dots T$ . After considering all the ramp up and ramp down logic we can able to calculate the power generated by the unit at each time interval ( $P_i^t$ ). After finding  $P_i^t$  we have to calculate the profit associated with the unit for the time being considered. The profit will be calculated using the equation

$$\text{Total profit} = \sum_{t=1}^T [(\lambda^t * P_i^t U_i^t) - [(inc_i^k * \sum_{ton}^{toff} P_i^t) + S U_i^t + \sum_{ton}^{toff} nl_i^k] U_i^t] \quad [6.23]$$

While calculating the profit we have to consider the following important cases.

- If  $\lambda^t \leq inc_i^1$  then  $P_i^t = p_i^{\min}$
- If  $inc_i^1 < \lambda^t \leq inc_i^2$  then  $P_i^t = e_i^1$
- If  $inc_i^2 < \lambda^t \leq inc_i^3$  then  $P_i^t = e_i^2$
- If  $\lambda^t > inc_i^3$  then  $P_i^t = p_i^{\max}$

So after considering all the feasible states we can able to calculate the power and profit at each and every instant of hour and finally we can able to calculate the cumulative sum of the profit of all intervals for each and every unit and finally we will add all these profits of each unit to maximize the profit for entire system, and here each sub problem solution is optimum solution to them selves so the final solution after considering all the units will give the optimum profit. We have to consider the different segments of incremental cost functions for the profit calculation of equation (6.3) i.e.

- If  $\lambda^t \leq inc_i^1$  then  $inc_i^k = inc_i^1$
- If  $inc_i^1 < \lambda^t \leq inc_i^2$  then  $inc_i^k = inc_i^1$
- If  $inc_i^2 < \lambda^t \leq inc_i^3$  then  $inc_i^k = inc_i^2$
- If  $\lambda^t > inc_i^3$  then  $inc_i^k = inc_i^3$

### 6.4 FLOW CHART FOR THE PROGRAM ALGORITHM

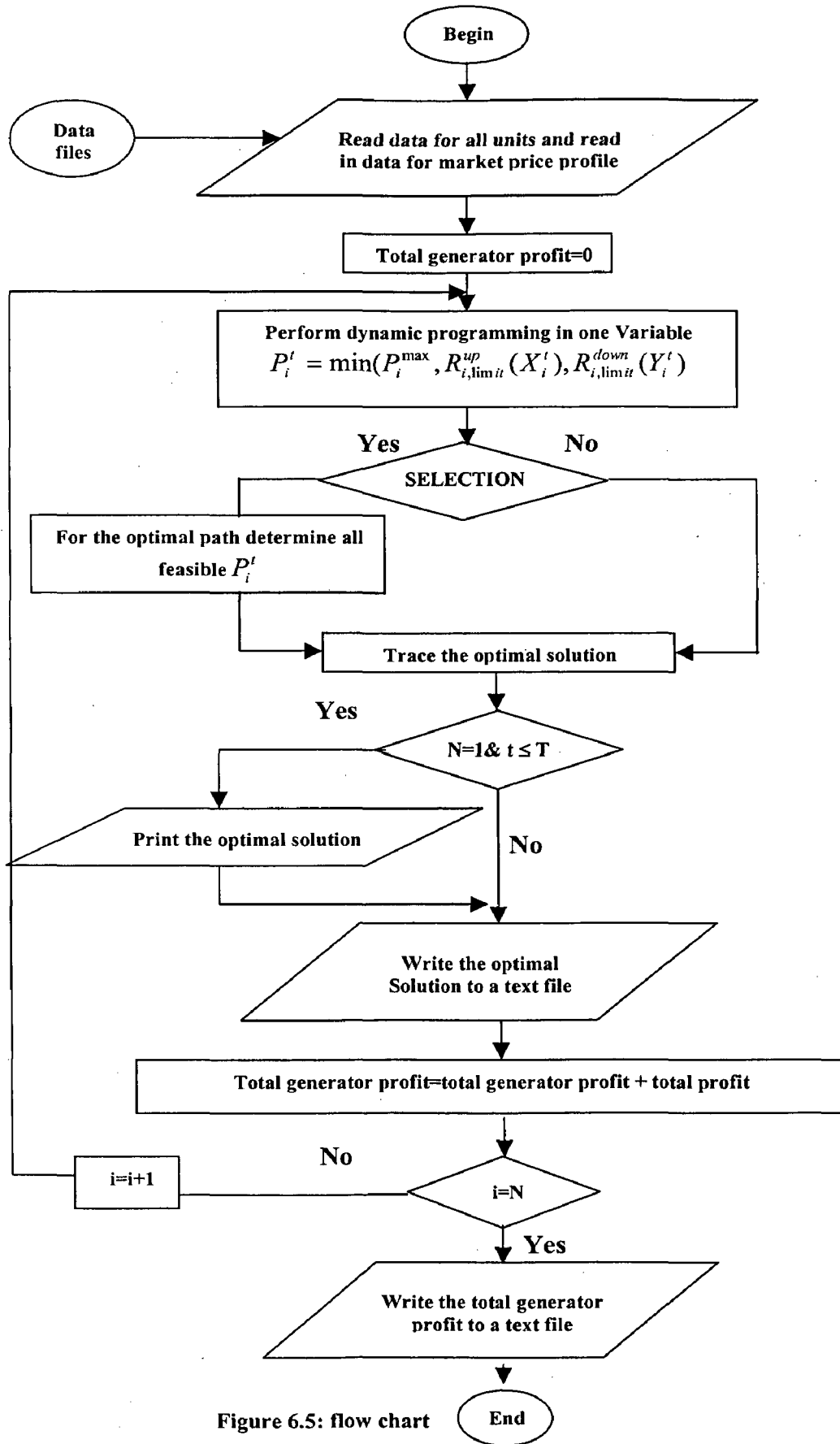


Figure 6.5: flow chart

## RESULTS AND DISCUSSION

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### 7.1 RESULTS FOR STATES (U), POWER (P), PROFIT FOR 26 GENERATING UNITS:

**TABLE II**  
**STATES (U) FOR 1-26 UNITS**

Unit	t=1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0	1	1	1	1	1	1	1	1	1	1	1
3	0	0	0	1	1	1	1	1	1	1	1	1	1	1
4	0	0	0	1	1	1	1	1	1	1	1	1	1	1
5	0	0	0	1	1	1	1	1	1	1	1	1	1	1
6	0	0	0	1	1	1	1	1	1	1	1	1	0	0
7	0	0	0	1	1	1	1	1	1	1	1	1	0	0
8	0	0	0	1	1	1	1	1	1	1	1	1	0	0
9	0	0	0	1	1	1	1	1	1	1	1	1	0	0
10	1	1	1	1	1	1	1	1	1	1	1	1	0	0
11	1	1	1	1	1	1	1	1	1	1	1	1	0	0
12	1	1	1	1	1	1	1	1	1	1	1	1	0	0
13	0	1	1	1	1	1	1	1	1	1	1	1	0	0
14	0	0	1	1	1	1	1	1	1	1	1	1	1	1
15	0	0	1	1	1	1	1	1	1	1	1	1	1	1
16	0	0	1	1	1	1	1	1	1	1	1	1	1	1
17	0	0	1	1	1	1	1	1	1	1	1	1	1	1
18	0	0	1	1	1	1	1	1	1	1	1	1	1	1
19	0	0	1	1	1	1	1	1	1	1	1	1	1	1
20	0	0	1	1	1	1	1	1	1	1	1	1	1	1
21	0	0	0	1	1	1	1	1	1	1	1	1	1	1
22	0	0	0	1	1	1	1	1	1	1	1	1	1	1
23	0	0	0	1	1	1	1	1	1	1	1	1	1	1
24	0	0	1	1	1	1	1	1	1	1	1	1	1	1
25	1	1	1	1	1	1	1	1	1	1	1	1	1	1
26	1	1	1	1	1	1	1	1	1	1	1	1	1	1

**TABLES-III, IV  
POWER (P<sup>h</sup>)**

Unit	T=1	2	3	4	5	6	7	8
1	0.0	0.0	0.0	4.0	8.0	12.0	12.0	12.0
2	0.0	0.0	0.0	4.0	8.0	12.0	12.0	12.0
3	0.0	0.0	0.0	4.0	8.0	12.0	12.0	12.0
4	0.0	0.0	0.0	4.0	8.0	12.0	12.0	12.0
5	0.0	0.0	0.0	4.0	8.0	12.0	12.0	12.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.0
10	15.2	30.2	45.2	60.2	75.2	76.0	76.0	76.0
11	15.2	30.2	45.2	60.2	75.2	76.0	76.0	76.0
12	15.2	30.2	45.2	60.2	75.2	76.0	76.0	76.0
13	0.0	20.0	40.0	60.0	76.0	76.0	76.0	76.0
14	0.0	0.0	25.0	50.0	75.0	100.0	100.0	100.0
15	0.0	0.0	30.0	60.0	90.0	100.0	100.0	100.0
16	0.0	0.0	30.0	60.0	90.0	100.0	100.0	100.0
17	0.0	0.0	100.0	155.0	155.0	155.0	155.0	155.0
18	0.0	0.0	150.0	155.0	155.0	155.0	155.0	155.0
19	0.0	0.0	150.0	155.0	155.0	155.0	155.0	155.0
20	0.0	0.0	150.0	155.0	155.0	155.0	155.0	155.0
21	0.0	0.0	0.0	197.0	197.0	197.0	197.0	197.0
22	0.0	0.0	0.0	197.0	197.0	197.0	197.0	197.0
23	0.0	0.0	0.0	197.0	197.0	197.0	197.0	197.0
24	0.0	0.0	200.00	350.00	350.00	350.00	350.00	350.00
25	250.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00
26	250.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00



**TABLE- IV**

<b>Unit1</b>	<b>T=9</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>
2	12.0	12.0	12.0	12.0	12.0	12.0
3	12.0	12.0	12.0	12.0	12.0	12.0
4	12.0	12.0	12.0	12.0	12.0	12.0
5	12.0	12.0	12.0	12.0	12.0	12.0
6	12.0	12.0	12.0	12.0	12.0	12.0
7	16.0	20.0	20.0	10.0	0.0	0.0
8	16.0	20.0	20.0	10.0	0.0	0.0
9	16.0	20.0	20.0	10.0	0.0	0.0
10	16.0	20.0	20.0	10.0	0.0	0.0
11	76.0	76.0	76.0	76.0	76.0	76.0
12	76.0	76.0	76.0	76.0	76.0	76.0
13	76.0	76.0	76.0	76.0	76.0	76.0
14	76.0	76.0	76.0	76.0	76.0	76.0
15	100.0	100.0	100.0	100.0	100.0	100.0
16	100.0	100.0	100.0	100.0	100.0	100.0
17	155.0	155.0	155.0	155.0	155.0	155.0
18	155.0	155.0	155.0	155.0	155.0	155.0
19	155.0	155.0	155.0	155.0	155.0	155.0
20	155.0	155.0	155.0	155.0	155.0	155.0
21	197.0	197.0	197.0	197.0	197.0	197.0
22	197.0	197.0	197.0	197.0	197.0	197.0
23	197.0	197.0	197.0	197.0	197.0	197.0
24	350.00	350.00	350.00	350.00	350.00	350.00
25	400.00	400.00	400.00	400.00	400.00	400.00
26	400.00	400.00	400.00	400.00	400.00	400.00

**TABLES V, VI, VII, VIII  
PROFIT**

<b>Time</b>	<b>Unit 1</b>	<b>Unit 2</b>	<b>Unit 3</b>	<b>Unit 4</b>	<b>Unit 5</b>	<b>Unit 6</b>	<b>Unit 7</b>
<b>1</b>	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<b>2</b>	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<b>3</b>	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<b>4</b>	-25.088	-25.644	-26.409	-27.056	-27.703	0.000	0.000
<b>5</b>	24.093	22.976	21.637	20.457	19.287	0.000	0.000
<b>6</b>	37.397	35.671	33.696	31.965	30.267	0.000	0.000
<b>7</b>	100.757	99.031	97.056	95.325	93.627	0.000	0.000
<b>8</b>	174.197	172.471	170.496	168.765	167.067	-118.404	-119.650
<b>9</b>	292.037	290.311	288.336	286.605	284.907	113.816	111.497
<b>10</b>	291.797	290.071	288.096	286.365	284.667	170.429	167.573
<b>11</b>	228.797	227.071	225.096	223.365	221.667	65.429	62.573
<b>12</b>	161.837	160.111	158.136	156.405	154.707	-80.204	-81.718
<b>13</b>	94.997	93.271	91.296	89.565	87.867	0.000	0.000
<b>14</b>	33.317	31.591	29.616	27.885	26.187	0.000	0.000

**TABLE VI**

<b>Time</b>	<b>Unit 8</b>	<b>Unit 9</b>	<b>Unit 10</b>	<b>Unit 11</b>	<b>Unit 12</b>	<b>Unit 13</b>	<b>Unit 14</b>
<b>1</b>	0.000	0.000	-230.540	-231.149	-231.756	0.000	0.000
<b>2</b>	0.000	0.000	-202.379	-203.532	-204.653	-249.012	0.000
<b>3</b>	0.000	0.000	-54.605	-56.376	-58.071	-60.295	-433.484
<b>4</b>	0.000	0.000	618.699	616.235	613.907	609.147	148.817
<b>5</b>	0.000	0.000	1258.344	1255.104	1252.080	1262.387	782.061
<b>6</b>	0.000	0.000	1211.399	1208.118	1205.057	1201.587	1019.805
<b>7</b>	0.000	0.000	1612.679	1609.398	1606.337	1602.867	1547.805
<b>8</b>	-120.890	-122.142	2077.799	2074.518	2071.457	2067.987	2159.805
<b>9</b>	109.079	106.721	2824.119	2820.838	2817.777	2814.307	3141.805
<b>10</b>	164.567	161.655	2822.599	2819.318	2816.257	2812.787	3139.805
<b>11</b>	59.567	56.655	2423.599	2420.318	2417.257	2413.787	2614.805
<b>12</b>	-83.252	-84.781	1999.519	1996.238	1993.177	1989.707	2056.805
<b>13</b>	0.000	0.000	1576.199	1572.918	1569.857	1566.387	1499.804
<b>14</b>	0.000	0.000	1185.559	1182.278	1179.217	1175.747	985.805

**TABLE-VII**

<b>Time</b>	<b>Unit 15</b>	<b>Unit 16</b>	<b>Unit 17</b>	<b>Unit 18</b>	<b>Unit 19</b>	<b>Unit 20</b>	<b>Unit 21</b>
<b>1</b>	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<b>2</b>	0.000	0.000	0.000	0.000	0.000	0.000	0.000
<b>3</b>	-457.500	-460.800	-73.271	38.770	33.475	28.597	0.000
<b>4</b>	206.705	200.685	2039.409	2033.395	2027.883	2022.814	-185.744
<b>5</b>	964.175	955.890	3017.459	3011.445	3005.933	3000.864	1373.953
<b>6</b>	1010.465	1001.425	2893.459	2887.445	2881.933	2876.864	1216.353
<b>7</b>	1538.465	1529.425	3711.859	3705.845	3700.333	3695.264	2256.513
<b>8</b>	2150.465	2141.425	4660.459	4654.445	4648.933	4643.864	3462.153
<b>9</b>	3132.465	3123.425	6182.559	6176.545	6171.033	6165.964	5396.693
<b>10</b>	3130.465	3121.425	6179.459	6173.445	6167.933	6162.864	5392.753
<b>11</b>	2605.465	2596.425	5365.709	5359.695	5354.183	5349.114	4358.503
<b>12</b>	2047.465	2038.425	4500.809	4494.795	4489.283	4484.214	3259.243
<b>13</b>	1490.465	1481.425	3637.459	3631.445	3625.933	3620.864	2161.953
<b>14</b>	976.465	967.425	2840.759	2834.745	2829.233	2824.164	1149.373

**TABLE-VIII**

<b>Time</b>	<b>Unit 22</b>	<b>Unit 23</b>	<b>Unit 24</b>	<b>Unit 25</b>	<b>Unit 26</b>
<b>1</b>	0.000	0.000	0.000	-896.475	-859.995
<b>2</b>	0.000	0.000	0.000	222.758	215.850
<b>3</b>	0.000	0.000	18.408	2114.758	2107.850
<b>4</b>	-206.396	-227.930	4755.457	6578.758	6571.850
<b>5</b>	1353.302	1331.768	6963.957	9102.758	9095.850
<b>6</b>	1195.702	1174.168	6683.957	8782.758	8775.850
<b>7</b>	2235.862	2214.328	8531.957	10894.758	10887.850
<b>8</b>	3441.502	3419.968	10673.957	13342.758	13335.850
<b>9</b>	5376.042	5354.508	14110.957	17270.758	17263.850
<b>10</b>	5372.102	5350.568	14103.957	17262.758	17255.850
<b>11</b>	4337.852	4316.318	12266.457	15162.758	15155.850
<b>12</b>	3238.592	3217.058	10313.457	12930.758	12923.850
<b>13</b>	2141.302	2119.768	8363.957	10702.758	10695.850
<b>14</b>	1128.722	1107.188	6564.957	8646.758	8639.850

**TABLE IX**  
**Final Solution (For All (26) Units)**

UNIT	TOTAL PROFIT FOR (T=14) HOURS
1	1414.134
2	1396.929
3	1377.049
4	1359.646
5	1342.546
6	151.066
7	140.274
8	129.071
9	118.107
10	19122.986
11	19084.224
12	19047.897
13	19207.385
14	18663.634
15	18795.565
16	18696.600
17	44956.134
18	45002.021
19	44936.085
20	44875.456
21	29841.747
22	29614.585
23	29377.709
24	103351.435
25	132119.379
26	132066.055
<b>TOTAL PROFIT</b>	<b>776190.00(\$/MWh)</b>

## 7.2 DISCUSSION ABOUT RESULTS

Based on the results obtained, the following general observations can be made:

- The unit constraints were respected in all cases.
- A negative net profit was obtained whenever the unit was brought online, due to the actual profit being amortized by the start up cost.

As shown above in the profit tables if a negative value of profit is there that means it is a loss incurred at that time due to that particular unit .if profit value is positive it is obviously a gain for the producer or a utility. So considering all the units we calculated the best profit for the whole system.

## CONCLUSIONS

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Unit commitment problem under deregulation was addressed in this dissertation. In solving this problem maximization of the profit for a single power producer was stated as the Motivation for this software project .the objectives that it sought to achieve were outlined from the outset, in the first chapter of the dissertation. The foundation for understanding unit commitment problem was laid in the introductory chapter.

The second chapter provided an idea of unit commitment problem constraints and costs associated with the problem and solution techniques that are useful for solving conventional unit commitment problem.

The following chapter presented an idea of unit commitment problem under deregulated environment and also gave the idea of structure of deregulated power system and calculation of marginal price or market clearing price.

The fourth chapter provided an idea of problem formulation for UCP and constraints associated with the main problem.

The fifth chapter suggested some of classical and non classical solution techniques for solving the unit commitment problem under deregulation, which includes explanation about some of the classical methods like simulated annealing, branch and bound and some of the non classical methods like artificial neural networks and genetic algorithm.

The sixth chapter focused on the application of lagrangian relaxation method for decomposition of main problem into several sub problems and application of dynamic programming in solving these single units sub problems.

The seventh chapter provided the test results for the chosen 26 generating units for Profit, Power and state.

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## APPENDIX

In this dissertation work I have considered that the power producer is having 26 generating units and the data for this 26 units had taken from [5]. And is given below.

**TABLE (X- XIII)**  
**DATA FOR 1-26 UNITS**

Data	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7
P (min)	2.40	2.40	2.40	2.40	2.40	4.00	4.00
P (max)	12.00	12.00	12.00	12.00	12.00	20.00	20.00
nlI	24.049	24.055	24.262	24.379	24.505	117.307	117.638
T (up)	0	0	0	0	0	0	0
T(down)	0	0	0	0	0	0	0
X (0)	-1	-1	-1	-1	-1	-1	-1
Alpha	0	0	0	0	0	20	20
Beta	0	0	0	0	0	20	20
Tao	1	1	1	1	1	2	2
Rup	4	4	4	4	4	8	8
Rdown	3	3	3	4	4	10	10
Y (0)	0	0	0	0	0	0	0
a	0.02533	0.02649	0.02801	0.02842	0.02855	0.01199	0.01261
b	25.5472	25.6753	25.8027	25.9318	26.0611	37.5510	37.6637
c	24.3891	24.4110	24.6382	24.7605	24.8882	117.7551	118.1083

**TABLE (XI)**

<b>Data</b>	<b>Unit 8</b>	<b>Unit 9</b>	<b>Unit 10</b>	<b>Unit 11</b>	<b>Unit 12</b>	<b>Unit 13</b>	<b>Unit 14</b>
<b>P (min)</b>	4.00	4.00	15.20	15.20	15.20	15.20	25.00
<b>P (max)</b>	20.00	20.00	76.00	76.00	76.00	76.00	100.00
<b>nll</b>	117.950	118.286	76.414	76.473	76.558	76.602	210.108
<b>T (up)</b>	0	0	3	3	3	3	4
<b>T (down)</b>	0	0	2	2	2	2	2
<b>X (0)</b>	-1	-1	-3	-3	-3	-3	-3
<b>Alpha</b>	20	20	50	50	50	50	70
<b>Beta</b>	20	20	50	50	50	50	70
<b>Tao</b>	2	2	3	3	3	3	4
<b>Rup</b>	8	8	15	15	15	20	25
<b>Rdown</b>	10	10	15	15	20	20	25
<b>Y (0)</b>	0	0	0	0	0	0	0
<b>a</b>	0.01359	0.01433	0.00876	0.00895	0.00910	0.00932	0.00623
<b>b</b>	37.7770	37.8896	13.3272	13.3538	13.3805	13.4073	18.0000
<b>c</b>	118.4576	118.8206	81.1364	81.2980	81.4641	81.6259	217.8952

**TABLE XII**

<b>Data</b>	<b>Unit 15</b>	<b>Unit 16</b>	<b>Unit 17</b>	<b>Unit 18</b>	<b>Unit 19</b>	<b>Unit 20</b>	<b>Unit 21</b>
<b>P (min)</b>	25.00	25.00	54.25	54.25	54.25	54.25	68.95
<b>P (max)</b>	100.00	100.00	155.00	155.00	155.00	155.00	197.00
<b>nl1</b>	210.685	211.300	120.673	120.491	120.399	120.392	239.196
<b>T (up)</b>	4	4	5	5	5	5	5
<b>T(down)</b>	2	2	3	3	3	3	4
<b>X (0)</b>	-3	-3	-5	-5	-5	-5	-4
<b>Alpha</b>	70	70	150	150	150	150	200
<b>Beta</b>	70	70	150	150	150	150	200
<b>Tao</b>	4	4	6	6	6	6	8
<b>Rup</b>	30	30	100	150	150	150	200
<b>Rdown</b>	30	30	100	150	150	150	250
<b>Y (0)</b>	0	0	0	0	0	0	0
<b>a</b>	0.00612	0.00598	0.00463	0.00473	0.00481	0.00487	0.00259
<b>b</b>	18.1000	18.2000	10.6940	10.7154	10.7367	10.7583	23.0000
<b>c</b>	218.3350	218.7752	142.7348	143.0288	143.3179	143.5972	259.1310

**TABLE XIII**

<b>Data</b>	<b>Unit 22</b>	<b>Unit 23</b>	<b>Unit 24</b>	<b>Unit 25</b>	<b>Unit 26</b>
<b>P (min)</b>	68.95	68.95	140.00	100.00	100.00
<b>P (max)</b>	197.00	197.00	350.00	400.00	400.00
<b>n11</b>	239.682	240.121	132.076	271.202	271.910
<b>T (up)</b>	5	5	8	8	8
<b>T(down)</b>	4	4	5	5	5
<b>X (0)</b>	-4	-4	-10	-10	-10
<b>Alpha</b>	200	200	300	500	500
<b>Beta</b>	200	200	200	500	500
<b>Tao</b>	8	8	8	8	10
<b>Rup</b>	200	200	200	250	250
<b>Rdown</b>	250	250	300	250	250
<b>Y (0)</b>	0	0	0	0	0
<b>a</b>	0.00260	0.00263	0.00153	0.00194	0.00195
<b>b</b>	23.1000	23.2000	10.8616	7.4921	7.5031
<b>c</b>	259.1310	260.1760	177.0575	310.0021	311.9102

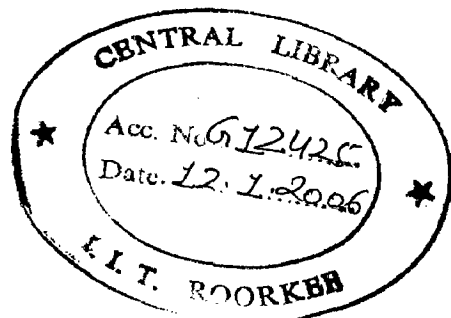
Total scheduling time period has been considered to be 14 hours i.e.  $T=14$   
 Market clearing price for the entire scheduling period is assumed to be volatile

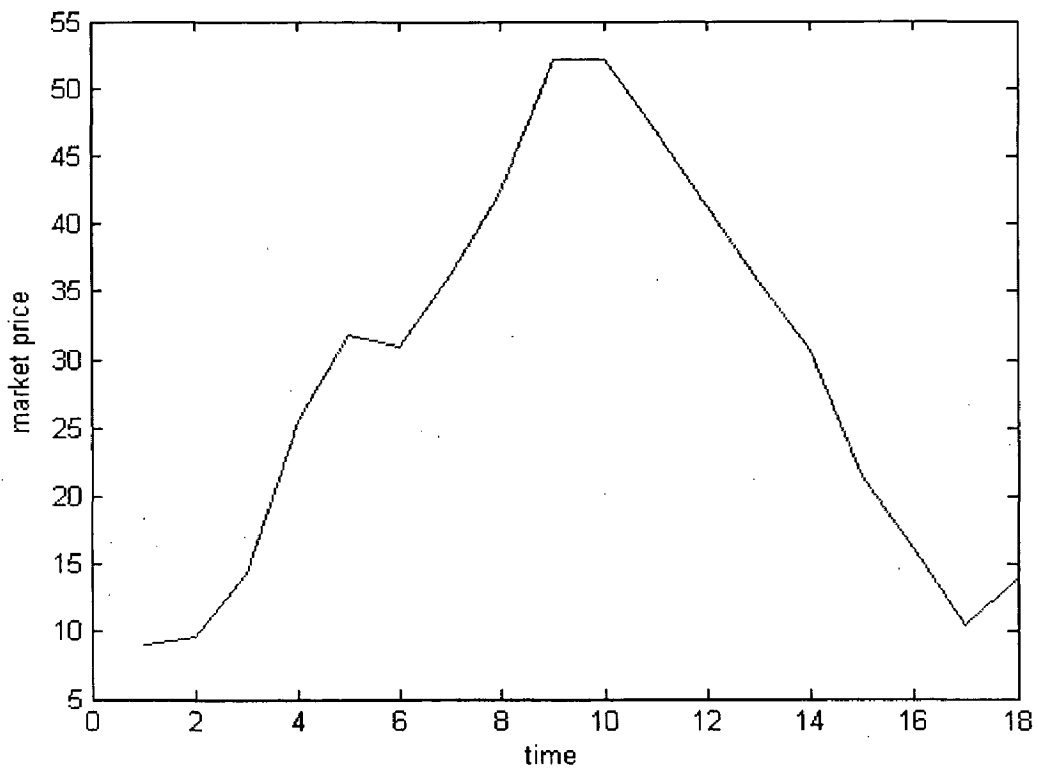
And is given here:  $\lambda^t$  =market clearing price.

**TABLE XIV**

**Time (vs.) market price profile**

<b>T=1</b>	<b><math>\lambda^t =9.00</math></b>
2	9.60
3	14.33
4	25.49
5	31.80
6	31.00
7	36.28
8	42.40
9	52.22
10	52.20
11	46.95
12	41.37
13	35.80
14	30.66
15	21.43
16	16.12
17	10.54
18	14.04





**Market Price Vs Time Graph**

**Some formulas used**

$$inc_i^1 = a_i[(e_i^1)^2 - (p_i^{\min})^2] + b_i[e_i^1 - p_i^{\min}] / (e_i^1 - p_i^{\min})$$

$$inc_i^2 = a_i[(e_i^2)^2 - (e_i^1)^2] + b_i[e_i^2 - e_i^1] / (e_i^2 - e_i^1)$$

$$inc_i^3 = a_i[(p_i^{\max})^2 - (e_i^2)^2] + b_i[p_i^{\max} - e_i^2] / (p_i^{\max} - e_i^2)$$

$$nl_i^1 = a_i(p_i^{\min})^2 + b_i(p_i^{\min}) + c_i - inc_i^1 * p_i^{\min}$$

$$nl_i^k = nl_i^{k-1} + e_i^{k-1} (inc_i^{k-1} - inc_i^k) \text{ For } k=2,3$$

$$R_{i \text{ limit}}^{\text{down}}(Y_i^t) = \begin{cases} P_i^{\max} & \text{If } Y_i^t = Y_i^{\max} \\ \text{Max}(P_i^{\min}, R_{i \text{ limit}}^{\text{down}}(Y_i^t + 1) - R_i^{\text{down}} * \Delta t) & \text{If } 0 < Y_i^t < Y_i^{\max} \\ 0 & \text{If } Y_i^t = 0 \end{cases}$$

$$R_{i \text{ limit}}^{\text{up}}(X_i^t + 1) = \begin{cases} \text{Max}(R_i^{\text{up}}, P_i^{\min}) & \text{if } X_i^t = 0 \\ \text{Min}(P_i^{\max}, R_{i \text{ limit}}^{\text{up}}(X_i^t) + R_i^{\text{up}}), & \text{if } X_i^t \geq 1 \end{cases}$$

$$\text{If } \lambda^t \leq inc_i^1 \quad \text{then } inc_i^k = inc_i^1$$

$$\text{If } inc_i^1 < \lambda^t \leq inc_i^2 \quad \text{then } inc_i^k = inc_i^1$$

$$\text{If } inc_i^2 < \lambda^t \leq inc_i^3 \quad \text{then } inc_i^k = inc_i^2$$

$$\text{If } \lambda^t > inc_i^3 \quad \text{then } inc_i^k = inc_i^3$$



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