EQUILIBRIUM STRUCTURE AND OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED GAS SPHERES

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the thesis entitled **"EQUILIBRIUM STRUCTURE AND OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED GAS SPHERES"** in fulfilment of the requirement for the award of the Degree of Doctor of Philosophy and submitted in the Department of Paper Technology of the Indian Institute of Technology Roorkee, Roorkee is an authentic record of my own work carried out during a period from July 1999 to July 2004 under the supervision of Dr. V. P. Singh and Dr. C. Mohan.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other Institute.

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ABSTRACT

In the present thesis we have primarily addressed ourselves to certain shortcomings noticed in the work of Mohan, Saxena and Agarwal (Astrophys. Space Sci., Vol.163, p.23,1990 and Vol.178, p.89, 1991) which relates to the problems of computing the equilibrium structure and periods of oscillations of rotating stars and star in binary system. In this work equipotentials surfaces of the rotationally and tidally distorted primary component of a binary star are approximated by Roche equipotential surfaces which are obtained by assuming the whole mass of the star to be concentrated at its centre which acts as a point mass surrounded by an evanescent envelope in which density varies as the square of distance from the centre. Since even in this approximation analytic solutions in closed form are not possible, following Kopal (1972) series expansions have been to represent the potential on an equipotentials surface. Convergence of this series expansion has not been possible. We have tried to analyze the effects of incorporating the effects of including mass variation inside a star on the computation of its equipotential surfaces while computing the equilibrium structures as well as periods of oscillations of rotating stars and stars in binary system. The problem of the validity of series expansions for certain parameters has also been considered.

The thesis consists of nine chapters. Chapter one we briefly discuss the astrophysical significance of the problems of determining the equilibrium structures, and the periods of oscillations of rotationally as well as tidally distorted stellar models. A brief survey of the literature available on the subject and summary of the work presented in the succeeding chapters of the thesis also appears in this chapter.

In chapter II we first present the concept of Roche equipotentials and Roche coordinates as introduced by Kopal (Astron. And Astrophys., Vol. 9, 1972) and how it has been used by Mohan and Singh (Astrophys Space Sci.,

Vol. 85, 1982) in Kippenhahn and Thomas technique to determine the equilibrium structures of rotationally and tidally distorted stars. The validity of the series expansions used in their work for certain Roche coordinates has been numerically checked. Our results show that these series expansions are reasonably valid under the assumptions under which these series are recommended to be used.

In chapter III we consider the problem of determining the equilibrium structures of rotationally and/or tidally distorted stars following Mohan, Saxena and Agarwal(Astrophys Space Sci., Vol. 163, p.23,1990) approach. This approach is modified by as modified by us to take into account the effect of mass variation inside the star on its equipotential surfaces inside the star. Mathematical expressions determining the equipotential surfaces, volume, surface area, etc are first derived and then used to obtain the system of differential equations governing equilibrium structure of a rotationally and tidally distorted star. This modified approach has then been used to numerically compute the equilibrium structures of rotationally and tidally distorted polytropic models. The results thus obtained have been compared with the results earlier computed by Mohan and Saxena (Astrophys Space Sci., Vol. 95, p. 369, 1983) for polytropic models of stars.

The methodology developed in chapter III is next used in chapter IV to determine the equilibrium structure of rotationally and/or tidally distorted Prasad model in which density ρ inside the star varies according to the law $\rho = \rho_c(1-x^2)$, ρ_c being density at the center and x a nondimensional measure of the distance of a fluid element from its center. This methodology has also been used to compute the equilibrium structures of a series of rotationally and/or tidally distorted composite models of the stars which have cores in which density varies as in Prasad model according to the law $\rho = \rho_c(1-x^2)$, and which are surrounded by envelopes in which density varies inversely as the square of the distance from the center as in Roche model. These composite models have

Prasad model at one extreme and Roche model at the other extreme and reasonably represent the effect of density variations inside the star on its structure. Analytical expressions for the density and the pressure at various points in the core and the envelope of these composite models have been obtained. The equilibrium structures and other physical parameters of the rotationally and tidally distorted composite models of stars have been computed for different models of this series by assuming the interface between the core and the envelope at the distance 0.3, 0.5, 0.7 and 0.9 of the total radius from the center. Results have been compared with earlier results obtained for such models in Roche approximation. Certain conclusions based on this study have also been drawn.

The problem of determining the equilibrium structures of certain differentially rotating and tidally distorted models computed so as to incorporate the effects of mass variation in the potential on its structure, has next been considered in chapter V. Boundary value problem governing the equilibrium structures of stars rotating differentially according the law $\omega = b_1 + b_2 s^2$, where ω is the angular velocity of rotation s is the distance of fluid element from axis of rotation and b_1, b_2 certain constants, is first formulated. It has next been used to numerically compute the equilibrium structures of differentially rotating Prasad model as well as certain polytropic models for polytropic indices 1.5, 3.0 and 4.0 for different numerical values of rotation parameters b_1 b_2. The results thus obtained have been compared with the earlier results obtained for these models by Mohan, Lal and Singh(1992)

In chapter VI we implement the approach developed in the earlier chapters to determine the equilibrium structures of various types of white dwarf models of the stars having solid body rotation as well as differential rotation assuming the law of rotation of the type $\omega = b_1 + b_2 s^2$. The explicit expressions that can be used to compute the shape, volumes, surfaces areas as well as other physical parameter of differentially rotating white dwarf models are also obtained. Computations have been performed to obtain the equilibrium structures of certain differentially rotating white dwarf models for the values of the parameter $1/\phi_0^2$ as 0.01, 0.05, 0.2, 0.4, 0.6, and 0.8. The results thus obtained have been compared with the results earlier computed by Mohan, Lal and Singh (16) for white dwarf models of the stars.

In chapter VII we consider the effect of mass variation in potential on the structures of rotationally and tidally distorted stars in which the angular velocity of rotation varies both along the axis of rotation, as well as in the direction perpendicular to the axis of rotation by assuming a general law of differential rotation of the type $\omega^2 = b_0 + b_1 s^2 + b_2 s^4 + b_3 z^2 + b_4 z^4 + b_5 z^2 s^2$, *s* being the distance of the fluid element from axis of rotation and z being the distance of the fluid element from the equatorial plane perpendicular to axis of rotation passing through the center of the star. By giving different values to constants b_0, b_1, b_2, b_3, b_4 and b_5 various types of differential rotations can be generated in which the angular velocity varies both along as well as perpendicular to the axis of rotation. In this chapter we have determined in particular the equilibrium structures of differentially rotating polytropic models of stars assuming this generalized law of differential rotation for polytropic models of indices 1.5, 3.0 and 4.0. Numerical results obtained in this chapter have also been compared with earlier results to draw some conclusions of practical significance.

In chapter VIII we next analyze the effect of mass variation in potential on the eigenfrequencies of small adiabatic barotropic modes of oscillations of rotating stars and stars in binary systems. The eigenvalued boundary value problems which determine the eigenfrequencies of small adiabatic pseudo-radial and nonradial modes of oscillations of differentially rotating stellar models obeying a law of differential rotation of the type $\omega^2 = b_1 + b_2 s^2$ have been formulated taking into account the effects of mass variation inside the star on its equipotentials surfaces. The method has been then used to determine the eigenfrequencies of various pesudo-radial and nonradial modes of oscillations of certain differentially rotating composite models as well as polytropic models of indices 1.5, 3.0 and 4.0. The eigenfrequencies of pseudo-radial modes of oscillations of certain rotationally and tidally distorted models have been also obtained.

Conclusions based on the present study are finally drawn in the concluding chapters IX. The astrophysical significance of the present work as well as the limitations and scope of the present work are also briefly discussed in this concluding chapter.

CONTENTS

CHAPTER – I

INTRODUCTION

This chapter is introductory in nature. In section 1.1 we first explain in brief the astrophysical significance carrying out of the theoretical study of the problem of determining the effects of rotation and/or tidal distortions on the equilibrium structure and the periods of small adiabatic oscillations of gaseous spheres. A brief survey of the literature available on the subject is presented in section 1.2. A brief summary of the work presented in the succeeding chapters of the thesis is finally presented in section 1.3.

1.1 ASTROPHYSICAL SIGNIFICANCE OF THE PROBLEM OF DETERMINING THE EFFECTS OF ROTATION AND TIDAL DISTORTIONS ON THE EQUILIBRIUM STRUCTURE AND THE PERIODS OF OSCILLATIONS OF GAS SPHERES

The theoretical model of a star is essentially a self gravitating gaseous sphere in hydrostatic and thermal equilibrium. Theoretical studies of the problems of the equilibrium structure of a gaseous sphere are often carried out to understand the nature of the internal structures responsible for various observed phenomena of the stars. Whereas some of the stars are observed as single stars others are observed in groups of two or more stars. Observations also show that some of the stars are rotating about their axes of rotation. This rotation may be a solid body rotation or a differential rotation. Many of the stars in binary and multiple systems are also known to be rotating about their axes as well as revolving around each other. Thus if we assume the equilibrium model of a single non rotating star as a gaseous sphere, the equilibrium model of a rotating star will be rotationally distorted gaseous sphere. Similarly, the equilibrium model of a star appearing in a binary system or a multiple system will be a tidally distorted gaseous sphere if it is not rotating and a rotationally and tidally distorted gaseous sphere if the star is rotating as well.

The brightness of certain observed stars varies with time. These stars are called variable stars. In some of these variable stars the variations in luminosity are periodic. In the case of such regular variable stars it is reasonable to assume that these stars are pulsating gaseous spheres in which the variation in luminosity are being caused by the periodic contraction and expansion of the gaseous mass. The regular variable stars gained importance in astrophysics when it was discovered that there exists a definite relation between the periods of pulsation and the luminosities of such stars. This relationship has often been used to estimate the distance of these stars. This important use of the regular variable stars motivated theoretical astrophysicists to investigate the problems of small oscillations of the equilibrium models of the variable stars so as to have a clear picture of the mechanism which could possibly be sustaining pulsations in these stars. Such investigations are also expected to help us in understanding the nature of the internal structure of the stars. In most of these theoretical studies, the variable star is represented by a gaseous sphere undergoing radial and nonradial oscillations.

Observations, however, show that some of the variable stars are rotating stars. The theoretical models of such rotating star can be regarded as rotationally distorted gaseous spheres performing small oscillations about their equilibrium configurations. Similarly some of the variable stars have also been observed in binary and multiple stellar systems. The theoretical models of such stars can be regarded as rotationally and tidally distorted gaseous spheres performing small oscillations about their equilibrium configurations.

It is thus evident that theoretical investigations determining the effects of rotation and tidal forces on the equilibrium structure and the periods of small radial and nonradial modes of oscillations of gaseous spheres may be of some help in better understanding the observed phenomena of the rotating stars and stars in binary and multiple systems. Such studies are also expected to help in a better understanding of the problems of stellar stability as well as the problems of stellar variability of rotating stars and stars in binary or multiple systems.

In the present thesis an attempt has, been made to investigate certain aspects of the problems of equilibrium structures and small oscillations of rotationally and tidally distorted gaseous spheres.

1.2 BRIEF REVIEW OF THE LITERATURE

Most of the theoretical studies about the equilibrium structures and oscillations of the stars have been carried out in literature by assuming the star as an undistorted gaseous sphere. Extensive literature is now available on this subject (See for instance Abhyankar (1), Bhatia (9), Bhatnagar (10), Bohm-Vitense (13), Clement (23), Chandrasekhar (19), Cox (27), Cox and Giuli (26), Dintransan and Rieutord (31), Deupree (30), Eddington (35), Hurley et al. (53), Kippenhahan and Thomas (62), Kippenhahn and Weigert (63), Kennedy and Bludman (61), Kopal (65), Lal (69), Ledoux and Walraven (72), Menzeil et al. (80), (131) Mohan and Singh (89), Mohan et al. (85, 91,92), Prasad (108), Rosseland (116), Schwarzschild (126), Singh Woodard (159), Sharma (130), Sperzum (141), Trehan (153).

There are some stars whose brightness varies with time. These are called variable stars. Whereas variation in brightness of some of the stars is regular and periodic in others it is not so. An important class of regular variable stars is Cepheid variables. The regular variable stars, gained importance in astrophysics in the year 1912, when Miss Leavitt discovered that there exists a definite relation between the periods of pulsation and the luminosities of such stars and the relationship could be utilized to determine the distance of these stars. In most of the theoretical studies of such stars, the variable star is represented by a gaseous sphere, both in hydrostatic and thermal equilibrium, undergoing small periodic oscillations. These oscillations can be radial as well as nonradial.

If the regular variable star is a nonrotating star which exists in isolation then it may be reasonable to represent it by a gaseous sphere performing radial or nonradial oscillations. However, if the star is a rotating or is a member of a binary or multiple systems then not only its equilibrium structure but also its modes of oscillations will also get affected by the rotational and or tidal forces. Mathematical models of such stars will obviously have to be rotationally and or tidally distorted gaseous spheres performing pseudo-radial or nonradial or some other types of modes of oscillations. As a result mathematical study of the problem of equilibrium structure and periods of small adiabatic oscillations of gaseous spheres gained importance in astrophysics. Ritter was perhaps the first to suggest in the year 1879 that the periodic variations in the luminosity of a variable star may be due to radial oscillations. Extensive studies have been made to the problems of small adiabatic radial modes of oscillations of gaseous spheres. (Buchler, Kollath and Marom (17), Cox (27), Das et. al (29), Goupil et al.(46), Gurm (47), Guzik and Cox (48), Ledoux and Walraven (72), Ledoux and Walraven Rosseland (116), Prasad (108), Prasad and Mohan (107), Saio and Jeffery (122), Tassoul et. al (147)) studied the effect of moderate rotation on stellar pulsations

In the case of regular variables, the high symmetry of their observed properties favors the hypothesis of purely radial oscillations. Even though now people generally seem to believe that Beta Cephei instability problem has been resolved with the advent of OPAL opacities, purely radial oscillations may not be able to explain many other phenomena observed in the case of certain variables stars. Ledoux and Walraven (72) pointed out that the dynamical instability leading to explosions in the stars might be easier to reach for some modes of nonradial oscillations. Chandrasekhar and Lebovitz (19) were of the view that it might be possible to explain variability of Beta Canis major types stars on the basis of resonance between the radial and nonradial modes of oscillations. Dalsgaard (28) suggested that certain observed phenomena in the outer layer of sun could be explained on the basis of certain modes of oscillations of the sun. Smith (137) studied zero-age main sequence B star and found that this star is pulsating nonradially.

Theoretical studies of the problem of nonradial oscillations commenced with Kelvin's investigation of the oscillatory modes of an incompressible gas sphere. But the proper formulation of the problem was given by Perkeris (105) who derived the forth order linear differential equation governing the adiabatic nonradial modes of oscillations of a compressible self- gravitating gaseous sphere. Since then the theoretical studies of the problem of nonradial oscillations of spherical models have been carried out by many investigators. Several authors such as Yojisoki (160), Mc Dermott et al. (79), Chandrasekhar and Ferrari (20) have made significant contributions to the studies of the problems of nonradial pulsations of stars. Cox and Cahn (25) calculated representative radial and nonradial pulsation modes of five Wolf- Rayet star models. Chandrasekhar and Ferrari (20) have proposed a complete theory of the nonradial oscillations of a static spherical symmetric distribution of matter described in terms of energy density and isotropic pressure on the premise that the oscillations are excited by incident gravitational waves. Bradely and Winget (16) computed the period and Kinetic distributions for nonradial g-modes of spherical harmonic indices from 1 to 3 in the adiabatic approximation. Rossenwald and Rabaey (117) have given an application of the continuous orthonormalization and adjoint methods to the computation of star eigenfrequencies and eigenfrequency sensitivities. This method integrates an eight-order nonlinear system of ordinary differential equations which define the linear adiabatic nonradial oscillatory modes of the sun. Telting and Schrijvers (151) used a model of a nonradially, adiabatically pulsating rotating star to generate time series of absorption line profiles. Clement (1998) also discussed normal modes of oscillations for rotating stars using a new numerical method for computing nonradial eigenfunctions. This technique for calculating the normal modes of spherical stellar models is generalized to two dimensions.

The theoretical investigations of problems of determining equilibrium structures and stability of rotating, self gravitating objects, possibly begun with the work of Newton. He was the first to realize the importance of the law of gravitation for explaining the figures of celestial bodies. Later on Maclaurin, Clairaut, Laplace, Legendre, Jacobi, Poincare etc. contributed ideas, necessary for the development of the general theory of rotating bodies. Maclaurin, Jacobi, Kelvin and Jeans investigated in detail the problem of structure and stability of rotating liquid masses assuming uniform rotation. Saxena (124) studied the structure of rotationally and tidally distorted polytropic models of stars.

In the year 1923, Edward Arthur Milne developed a technique for constructing the first detailed model for a slowly rotating star in pure radiative equilibrium. Later on in the year 1933, this technique of Milne was generalized and applied to slightly distorted polytrope by Chandrasekhar. Computation of the equilibrium structures of many of the rotating stellar models that do not greatly deviate from spherical symmetry often rely upon these two studies.

The effect of uniform rotation on slow rotating Cowling star obeying simple Kramer's opacity has been studied by Sweet and Roy (145), Sackmann and Anand (121), Chandrasekhar and Lebovitz (19), Roberts (113,114), Smith (136), Linnell (74), kopal (65), Mohan and Saxena (85), Geroyannis and Valvi (41), Roxburgh et al. (118) have also investigated the problems of equilibrium structures of rotating stars.. Much of the work on the effect of rotation on stellar interiors is summarized in the review article of Strittmatter (143). Later developments may be found in Tassoul (150), Durney (34), Kawaler (60), Soo and Kak (139). Mohan, Saxena and Aggarwal (92), Meynet and Meader (81) studied the effects of rotation on the equilibrium structure and evolution of massive stars. Whereas Antona et. al.(5) , investigated the theoretical models of low mass premain sequence rotating stars and Zeng (161) has developed more powerful evolutionary models for rotating stars.

The influence of uniform rotation on the global structure of white dwarf models has been considered by Anand (4), Bandyopadhyay (7), Chandrasekhar (21), Krishnan et al. (68), Suda (144) and. The most detailed models of uniformely rotating white dwarf are due to Anand (4), Anand et al. (3), Monaghan (84), Roxburgh (119). Some of the authors such as Roxburgh et al. (118), Ostriker and Tassoul (102), Shapiro and Teukolsky (129) have considered the stability analysis of uniformly rotating white dwarf stars. Ostriker and Bodenheimer (99), Smart and Monaghan (135), and Blinnikov (11) have analysed the models of zero-temperature white dwarfs in non- uniform rotation. Hachisu et al. (50) studied the fate of merging double white dwarfs. Bouvier (14), Cox (27), Durney (34), Kawaler (60), Lal et al. (70), Nelemans and Yungelson (95), Rudiger (120), Soo and Kak (139), Tassoul (149), and Vandervoord and Welty(156) have also made significant contributions in this directions.

Whereas many of the observed rotating stars may be having solid body rotation some of the stars are observed to be rotating differentially. In such type of stars different parts of the star are rotating about the axis of rotation with different angular velocities. Problems of differentially rotating stellar models have also been studied in literature. Stoeckly (142) obtained the numerical solution of the hydrostatic equilibrium equation for nonuniformly rotating stellar models having no meridional currents. With pressure density relation of the type $P \propto \rho^{3/2}$, Peraiah (104) showed that synchronism between orbital and rotational angular velocities of binary stars may not hold in many cases in the presence of differential rotation. Ireland (55) presented results for gravity darkening and limb darkening in a rapidly rotating Roche model of a star subject to nonuniform rotation and demonstrated that the effects of small uniform rotation are likely to be of greater significance than the actual values of rotational velocities themselves. Schmitz (125) studied the equilibrium structures and stability of differentially rotating self gravitating gaseous spheres. Komatsu et al. (64) applied the numerical method developed for Newtonian gravity models to general relativistic differentially rotating bodies including ring-like structures. He also obtained equilibrium structures for polytropes of indices 0.5 and 1.5. Goode et al. (45) also tried to analyze the nature of differential rotation in the interior of the sun for the study of its 5-min oscillations. Authors such as Bruning (15), Deupree (30), Durney (34), Endal and Sofia (36), Galli (37), Geroyannis and Hadjopolous (41), Glatzmaier et al. (43), Goldreich (44), Harris and Clement (51), Hoiland (52), Mohan and Singh (88), Pinsonneault et al. (106), Shapiro et al. (128), Solberg (138), Von Zeipal (157), Welty et al.(158), have also analysed the problems of differential rotation. Garud (38) worked on rotationally driven meridional flow in the stars.

Equilibrium structures of stars which appear in binary and multiple systems are likely to be effected by both the rotational effects as well as the tidal effects of the companion stars. Attempts have been made in literature to determine the effects of rotation and tidal distortions on the equilibrium structure and modes of oscillations of the stars in binary and multiple systems. In a series of papers Chandrasekhar developed a first order analysis which he applied to the study of the rotational problem, the tidal problem and the binary star problem. The method, however, was found unsuitable when the separation between the components is only a few times the undisturbed radius of the primary. Monaghan (93) modified it to get more accurate results near the surface.

The method of Monaghan and Roxburgh (119) to study the structure of the primary component of a synchronous close binary was further extended by Naylor and Anand (94). Goupil et al. (46) have analysed the effect of moderate rotation on stellar pulsations. Kippenhahn and Thomas (62) suggested a practical way of analyzing the effects of rotation and tidal distortions on the equilibrium structures of stars by approximating the actual equipotentials surfaces of the star by Roche equipotentials.

Kopal (65) introduced a system of coordinates, which he called Roche coordinates, to study the problems of rotating stars and stars in binary system. Kopal and Ali (67) studied the integrability of the Roche coordinates. Mohan and Saxena (85) used the Kippenhahn and Thomas (62) averaging technique in conjunction with Kopal''s results on Roche equipotentials to determine the combined effects of rotation and tidal distortions on the equilibrium structures of the theoretical models of the stars. This approach is presented in detail in Saxena (124). Later this approach was also used by Aggarwal (2), Manmohan and Singh (76) to study the effects of rotation and tidal distortions on the structure and periods of small adiabatic oscillations of composite models of stars. The technique was subsequently formalized by Mohan, Saxena and Agarwal (92) and used to study the problems of rotationally and tidally distorted main sequence stars. Seidov (127) derived the exact analytical formula for the potential and mass ratio as a function of Lagrangian points position, in the classical Roche model of the close binary stars.

Chan and Chau (18) developed a method which allows an efficient and accurate investigation of the structure and evolution of a rotationally and tidally distorted star in close binary systems. Nepon et al.(97) have discussed the evolution of rotationally and tidally distorted low-mass close binary systems. Iben (54) has studied the problem of evolution of binary components which first fill their Roche lobes after the exhaustion of central helium. Tassoul and Tassoul (148) considered the meridional circulation in rotating stars and mean steady motions in rotationally and tidally distorted stars. Tassoul (148) later extends the earlier work to study the reflection effects in close binaries when there is meridional circulation in rotating stars. Rocca (115) studied effect of slow uniform rotation on the tidal effects in close binary system. Paczynski & Kippenhahn (60) has discussed evolution process in close binary systems. The evolution of mass losing component of a close binary has been studied in literature without considering the dynamical effects of gas outflow from the star.

Avani and Schiller (6) studied the Roche potential systems where the stellar rotation axis is not aligned with the orbital revolution axis. Hachisu et al. (49) proposed a numerical method for constructing models of double white dwarf binary systems and central white dwarf heavy disk systems. He (50) also formulated a new-three dimensional method for obtaining structure of a rapidly rotating star and multiple stellar system including binaries. Rleutord (112) has shown that large scale flows driven by Ekman pumping in the spin up-down of a tidally distorted star is not efficient enough to reduce the synchronization time. Todaran (152) has used the time dependent potential function to study the equipotentials surfaces in close binary systems.

The simple hypothesis of the pulsating model of a regular variable star is made all the more complicated by the fact that some of the variable stars observed to be rotating stars or stars in binary or multiple systems. The eigenfrequencies of small oscillations of such stars are expected to be influenced by rotation and tidal effects of companion stars.

Most of the authors have studied pulsations of stars having solid body rotation. However, there are several variable stars which are suspected to be rotating differentially. Clement (24) has shown that by assuming a particular form of differential rotation the discrepancy that existed between observations and earlier calculations based on the assumption of uniform rotation could be removed. Woodard (159) considered the effect on eigenfrequencies and eigenfunctions of slow, axisymmetric differential rotation which is also mirror symmetric across the solar equatorial plane. Chandrasekhar and Ferrari (20) analysed the problem of nonradial oscillations of slowly rotating stars induced by the lense- thrinning effect. Urpin (155) studied the problem of rotation, circulation and turbulence in radiative zones of stars. Reyniers and Smeyers (111) have discussed tidal perturbation of linear, isentropic oscillations in components of circular orbit close binaries.

Trehan and Kochar (153), Sood and Singh (140) studied adiabatic pulsation and convective instability of uniformly rotating gaseous masses. Saio (123), Martin and Smeyers (78) investigated the problem of linear adiabatic oscillations of a uniformly and synchronously rotating component of a binary system. Mohan and Singh (89) considered the use of Roche coordinates in solving the problems of small adiabatic oscillations of rotationally and tidally distorted stellar models. They also considered the use of Kippenhahn and

Thomas averaging approach in conjunction with certain results on Roche equipotentials. Mohan and Saxena (85) considered the possibility of using this approach in general to determine the effects of rotation and tidal distortions on the eigenfrequencies of radial and nonradial modes of oscillations of stars and applied it on polytropic models of the stars. Based on these studies, Mohan, Saxena and Agarwal (92) proposed a method for computing the eigenfrequencies of small adiabatic barotropic modes of oscillations of rotationally and tidally distorted stars and applied it to the main sequence stars. Mohan, Lal, and Singh (69, 70) studied equilibrium structures and periods of oscillations of differentially rotating polytropic models of stars. Later on Singh and Sharma (133) also studied the oscillations of differentially rotating stars in binary system. Beech (8) presented a double polytropic model for low mass stars with $M < M_0$. Karino and Eriguchi (59) have considered the linear stability analysis of some differentially rotating polytropes.

Whereas the properties of equilibrium structures and periods of small adiabatic radial and nonradial modes of oscillations of undistorted gaseous sphere have been investigated in detail in literature, the effect of rotation and tidal distortions on the equilibrium structures and the modes of oscillations of gaseous sphere have still, not been fully understood. In the present work we have addressed ourselves to the analytic study of problems related to this field.

1.3 THE PRESENT WORK

The problem of determining the equilibrium structure and the periods of oscillations of the stars distorted by the effects of rotation and tidal forces has practical importance in astrophysics as it will help in better understanding the nature of the rotating stars and stars in binary and multiple systems. There is thus a need for an in-depth theoretical investigation of the effects of rotational and tidal forces on the equilibrium structures and the periods of small radial and nonradial modes of oscillations of gaseous spheres.

Analytic study of the problem of determining the equilibrium structures, periods of oscillations and stability of rotationally and tidally distorted stellar models is quite complex. The problem becomes still more complex if the rotation is differential. Therefore attempts have been often made in literature to investigate these problems in some approximate ways. In one such attempt Mohan, Saxena and Aggarwal (92) used Kippenhahn and Thomas (62) averaging technique in conjunction with Kopal's results (65) on Roche equipotentials, to determine the effects of rotation and tidal forces, on the equilibrium structure and the eigenfrequencies of small adiabatic barotropic radial and nonradial modes of oscillations of the theoretical models of the stars. They also demonstrated the use of this approach in the case of the polytropic models of the stars as well as certain realistic theoretical models of the main sequence stars. Lal (69) investigated the effectiveness of Mohan, Saxena and Agarwal (92) approach in computing the effects of differential rotation and tidal distortions on the equilibrium structures and the eigenfrequencies of radial and nonradial modes of oscillations of rotating stars.

In Mohan, Saxena and Agarwal approach the actual equipotentials surfaces of a rotationally and tidally distorted star are approximated by equipotentials surfaces obtained by assuming the entire mass of the star to be placed at the center of the star. This approximation is usually referred to as Roche approximation and the equipotentials surfaces thus generated are called Roche – equipotential surfaces. This approximation is reasonably valid for highly centrally condensed types of stars but not very much justified for less centrally condensed stars. It is, therefore, desirable to improve upon this approximation. For instance, instead of approximating the actual equipotentials surfaces inside the star by Roche equipotentials, these may be approximated by equipotentials surfaces which are obtained when the mass exterior to the equipotential surface is neglected and the mass interior to this equipotential surface is supposed to be concentrated at the center of the star. Such an approximation is motivated by the fact that in a self-gravitating spherical configuration the gravitational potential at a point inside the sphere depends only on the mass enclosed within the concentric spherical surface passing through that point.

Another shortcoming in the work of Mohan et. al is that for analysing properties of Roche equipotentials they have utilized the results of Kopal (65) on Roche coordinates which in the absence of the availability of mathematical expressions in closed form assume series expansions for some of these coordinates However the analytic proofs for the convergence of these series expansions are lacking.

In the present thesis we have primarily addressed ourselves to these two shortcomings in the work of Mohan et. al. We have investigated the validity of series expansions for certain parameters used in the system of Roche coordinates. We have also tried to analyse the effects of including the variation in mass inside a star on its equipotential surfaces while computing the equilibrium structure as well as periods of oscillations of rotating stars and stars in binary systems.

The thesis consists of nine chapters. Chapter one the present study only is introductory in nature. In this chapter we first briefly discuss the astrophysical significance of the problems of determining the equilibrium structures, and periods of oscillations of rotationally as well as tidally distorted stellar models. A brief survey of the literature available on the subject and summary of the work presented in the succeeding chapters of the thesis also appears in this chapter.

In chapter II we first present in brief the concept of Roche equipotentials and Roche coordinates and how it has been incorporated by Mohan et. al in Kippenhahn and Thomas technique to determine the equilibrium structures of rotationally and tidally distorted stars. The validity of the series expansions used in the system of Roche coordinates (for which analytic proofs of the series being convergent are not easily possible) has been checked numerically. Results show that these series expansions are reasonably valid under the assumptions under which these series are recommended to be used.

In chapter III we first consider the problem of determining the equilibrium structures of rotationally and/or tidally distorted stars using Mohan et.al approach as modified by us to take into account the effect of mass variation inside the star on its equipotential surfaces inside the star. Mathematical expressions determining the equipotential surfaces, volume, surface area, etc are first derived and then used to obtain the system of differential equations governing equilibrium structure of a rotationally and tidally distorted star. The modified approach has then been used to numerically compute the equilibrium structures of rotationally and tidally distorted polytropic models. The results thus obtained have been compared with the results earlier computed by Mohan and Saxena (85) for these polytropic models assuming whole mass to be concentrated at the centre while obtaining the equipotential surfaces.

The methodology developed in chapter III is next used in chapter IV to determine the equilibrium structure of rotationally and/or tidally distorted Prasad model in which density ρ inside the star varies according to the law $\rho = \rho_c(1-x^2)$, ρ_c being the density at the center and x a nondimensional measure of the distance of a fluid element from the center of the star. Methodology has also been used to compute the equilibrium structures of a series of rotationally and/or tidally distorted composite models of the stars which have cores in which density varies as in Prasad model according to the law $\rho = \rho_c(1-x^2)$, and which are surrounded by envelopes in which density varies inversely as the square of the distance from the center as in Roche model. These composite models have Prasad model at one extreme and Roche model at the other extreme and reasonably represent the effect of density variations inside the star on its structure. Analytical expressions for the density and the pressure at various points in the core and the envelope of these composite models have been obtained. The equilibrium structures and other physical parameters of the rotationally and tidally distorted composite models of stars have been computed for different models of this series by assuming the interface between the core and the envelope to be a distance 0.3, 0.5, 0.7 and 0.9 of the total radius from the center. Results have been compared with earlier results obtained for such models in Roche approximation. Certain conclusions based on this study have also been drawn.

The problem of determining the equilibrium structures of certain differentially rotating and tidally distorted models so as to incorporate the effects of mass variation in the potential on its structure, has been next considered in chapter V. Boundary value problem governing the equilibrium structures of stars rotating differentially according to the law $\omega = b_1 + b_2 s^2$, where ω is the angular velocity of rotation s is the distance of fluid element from axis of rotation and b_1, b_2 certain constants, is first formulated. It has next been used to numerically compute the equilibrium structures of differentially rotating Prasad model as well as certain polytropic models for polytropic indices 1.5, 3.0 and 4.0 for different numerical values of rotation parameters b_1 b_2. The results obtained have been compared with the results earlier obtained for these models in Roche approximation

In chapter VI we implement the approach developed in the earlier chapters to determine the equilibrium structures of various types of white dwarf models of the stars having solid body rotation as well as differential rotation assuming the law of rotation of the type $\omega = b_1 + b_2 s^2$. The explicit expressions that can be used to compute the shape, volumes, surfaces areas as well as other physical parameter of differentially rotating white dwarf models are also obtained. Computations have been performed to obtain the equilibrium structures of certain differentially rotating white dwarf models for the values of the parameter $1/\phi_0{}^2$ as 0.01, 0.05, 0.2, 0.4, 0.6, and 0.8. The results thus obtained have been compared with the results earlier computed by Mohan, Lal and Singh (91) for white dwarf models of the stars assuming Roche model for the star.

In chapter VII we consider the effect of mass variation in potential on the structures of rotationally and tidally distorted stars in which the angular velocity of rotation varies both along the axis of rotation, as well as in the direction perpendicular to the axis of rotation by assuming a general law of differential rotation of the type $\omega^2 = b_0 + b_1 s^2 + b_2 s^4 + b_3 z^2 + b_4 z^4 + b_5 z^2 s^2$, s being the distance of the fluid element from axis of rotation and z being the distance of the fluid element from the equatorial plane perpendicular to axis of rotation passing through the center of the star. By giving different values to constants b_0, b_1, b_2, b_3, b_4 and b_5 various types of differential rotations can be generated in which the angular velocity varies both along as well as perpendicular to the axis of rotation. In this chapter we have determined in particular the equilibrium structures of differentially rotating polytropic models of stars assuming this generalized law of rotation for polytropic models of indices 1.5, 3.0 and 4.0. Numerical results obtained in this chapter have also been compared with earlier results to draw some conclusions of practical significance.

In chapter VIII we next analyze the effect of mass variation in potential on the eigenfrequencies of small adiabatic barotropic modes of oscillations of rotating stars and stars in binary systems. The eigenvalued boundary value problems which determine the eigenfrequencies of small adiabatic pseudo-radial and nonradial modes of oscillations of differentially rotating stellar models obeying a law of differential rotation of the type $\omega = b_1 + b_2 s^2$ have been formulated taking into account the effects of mass variation inside the star on its equipotentials surfaces. The method has been then used to determine the eigenfrequencies of various pesudo-radial and nonradial modes of oscillations of certain differentially rotating composite models as well as polytropic models of indices 1.5, 3.0 and 4.0. The eigenfrequencies of pseudo-radial modes of oscillations of certain rotationally and tidally distorted models have been also obtained.

Conclusions based on the present study are finally drawn in the concluding chapters IX. The astrophysical significance of the present work as well as the limitations and scope of the present work are also briefly discussed in this concluding chapter.

CHAPTER - II

USE OF THE CONCEPT OF ROCHE EQUIPOTENTIALS IN DETERMINING THE EQUILIBRIUM STRUCTURES AND PERIODS OF OSCILLATIONS OF ROTATIONALLY AND/OR TIDALLY DISTORTED STARS AND STARS IN BINARY SYSTEMS

In this chapter we present the concept of Roche equipotentials and how it has been used by Kopal and subsequently Mohan et al. for determining the equilibrium structures of rotationally and tidally distorted stars. Since analytic expressions in closed form for all the three Roche coordinates were not possible, series expansions have been used in cases where analytic expressions in closed form were not possible. However, the convergence of these series expansions could not be analytically established. In the absence of this, one may doubt the correctness of analysis and subsequent results derived by using these series expansions. In this chapter we have tried to check the validity of these series expansions using numerical approach as we ourselves have not been able to establish analytically the convergence of these series expansions.

A brief discussion of the concept of Roche equipotentials and Roche coordinates is presented in sections 2.1 and 2.2, respectively. Certain results obtained by Kopal (65) and Mohan and Saxena (85) for Roche equipotentials are also presented in this section. In section 2.3 we show how Kippenhahn and Thomas (62) used an averaging technique for determining the equilibrium structures of rotationally and tidally distorted stars. In section 2.4 we next present how Mohan et al (92) used Kippenhahn and Thomas (62) approach in conjuction with certain results on Roche equipotentials to obtain the system of differential equations governing the equilibrium structures of rotationally and tidally distorted gaseous spheres. Section 2.5 is devoted to checking numerically the validity of series expansion used developed by Kopal for a Roche coordinate whose explicit expression in closed form was not possible. Whereas numerical approach adopted for this purpose is given in subsection 2.5.1, numerical computations carried out on this basis are given in subsection 2.5.2. Conclusions based on this numerical study are given in subsection 2.5.3.

2.1 ROCHE EQUIPOTENTIAL

In order to introduce the concept of Roche equipotential, we assume two components of a close binary system known as primary and secondary star. The primary star is supposed to be more massive than the secondary which acts as a point mass causing tidal effects on the more massive primary component. Both the component of binary system is assumed to be rotating about their axis as well as revolving about their common center of mass. Following Kopal (65), Mohan and Singh (87), Mohan Lal and Singh (90), certain results on Roche equipotential which are of practical interest to the present study, are summarized below:

Let us suppose M_0 and M_1 be the masses of the two components of a close binary system separated by a distance *D*. The primary component of this system of mass M_0 is much larger than its companion star of mass $M_1 (M_0 \geq M_1)$ which can be regarded as a point mass. Suppose that the position of the two components is referred to as a rectangular system of Cartesian coordinates with origin at the center of gravity of mass M_0 the $X-$ axis along the line joining the mass centers of two components, and $Z-$axis perpendicular to the plane of the orbit of the two components (See Fig. 1.1). Then the total potential ψ of the gravitational and disturbing force acting at an arbitrary point $P(x, y, z)$, which is not inside in any of this gaseous sphere is given by:

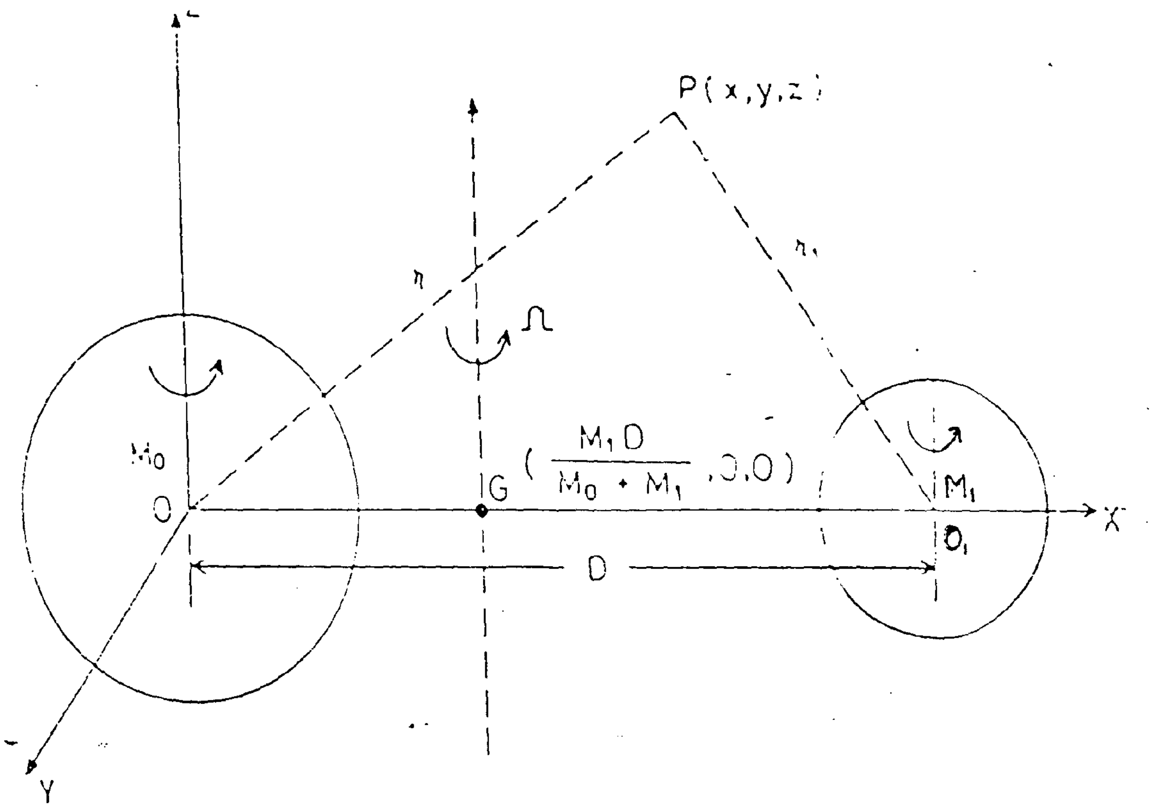

FIG.2.1 AXES OF REFERENCE

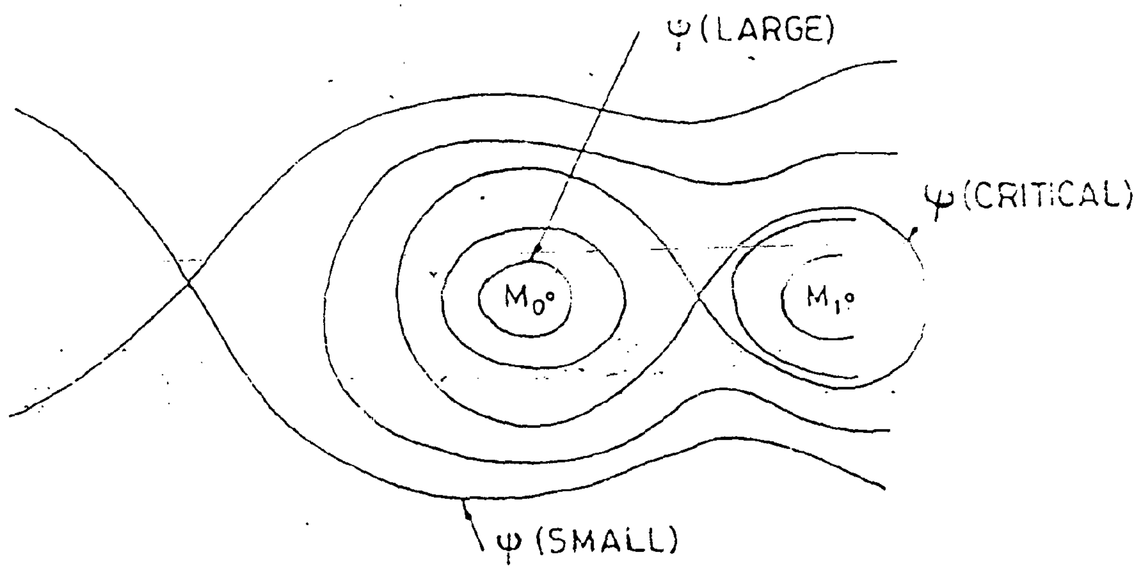

FIG.2.2 ROCHE EQUIPOTENTIAL SURFACES
(TWO DIMENSIONAL)

$$\psi = \frac{GM_0}{r} + \frac{GM_1}{r_1} + \frac{\Omega^2}{2}\left[\left(x - \frac{M_1 D}{M_0 + M_1}\right)^2 + y^2\right] \tag{2.1}$$

where $r^2 = x^2 + y^2 + z^2$ and $r_1^2 = (D-x)^2 + y^2 + z^2$ represent the squares of the distances of *P* from the center of gravity of the two components, Ω denotes the angular velocity of rotation of the system about an axis perpendicular to the *xy*- plane and passing through the center of gravity of the system and *G* the constant of gravitation. The first, second and third term on the right hand side of (2.1) represents the potential arises due to the mass of the primary component of mass M_0, the disturbing potential of its companion of mass M_1, and the potential arising from the centrifugal force, respectively. Equation (2.1) strictly holds at points which are the outside of both the components of binary system. In case we assume Roche model for the primary and a point mass for the secondary components, equation (2.1) holds everywhere.

In nondimensional form equation (2.1) can be expressed as

$$\psi^* = \frac{1}{r^*} + q\left[\frac{1}{\sqrt{1 - 2\lambda r^* + r^{*2}}} - \lambda r^*\right] + nr^{*2}(1 - \upsilon^2) \tag{2.2}$$

where $$\psi^* = \frac{D\psi}{GM_0} - \frac{M_1^{\,2}}{2M_0(M_0 + M_1)}$$

is the nondimensional form of total potential ψ and $r^* = r/D$ is nondimensional form of r, $\lambda = \sin\theta\cos\phi, \mu = \sin\theta\sin\phi, \upsilon = \cos\theta$ $(r, \theta, \phi$ being the polar spherical coordinate of the point *P*).

Also,

$$q=\frac{M_1}{M_0} \tag{2.3}$$

is a nondimensional parameter representing the ratio of mass of the secondary over primary and $2n$ represents the square of the normalized angular velocity Ω. The equation (2.1) reduces to the potential of a spherical model rotating with angular velocity only if $q=0$ and $n=0$, it reduces to the potential of a non-rotating spherical model distorted by the tidal effects of the companion. For a binary system in synchronous rotation, the angular velocity Ω is identical with Keplerian angular velocity so that

$$\Omega^2=G\frac{M_0+M_1}{D^3} \tag{2.4}$$

The relation expressed in terms of the nondimensional variable of equation (2.2) becomes

$$n=\frac{q+1}{2} \tag{2.5}$$

The surface generated by setting ψ=constant on the left hand side of (2.1) are referred to as Roche equipotentials. Roche equipotentials in nondimensional form may be represented by ψ^*= constant where ψ^* is same as defined in (2.2). The form of Roche-equipotential depends entirely upon the values of ψ. If ψ is large the corresponding equipotentials will consist of two separate ovals, closed around each of the two mass point (see Fig. 1.2). For specified values of M_0, M_1, Ω and D the right hand side of (2.1) can be large only if r and r_1 becomes small. Therefore, large value of ψ correspond to equipotentials which differ but little from spheres surrounding one of the two mass centers. With decreasing values of ψ of the ovals defined by (2.1) become increasingly elongated in the direction of the center of gravity of the system until for a certain critical value of ψ characterstic of each mass ratio. Both ovals will unite in a single point on the X-axis to form a dumbbell like configuration. These limiting values of ψ are called Roche limits. For certain mass ratios Kopal (65) computed the numerical values of Roche limits in the case of synchronous binary stars for a values of q ranging from zero to one. Defining a non-dimensional variable r_0 by the relation

$$r_0 = \frac{1}{\psi * - q} \tag{2.6}$$

Kopal has also shown that on the surface of Roche equipotentials (r, θ, ϕ) are connected through the relation

$$r^* = r_0\left[1 + C_3 r_0^3 + C_4 r_0^4 + C_5 r_0^5 + C_6 r_0^6 + C_7 r_0^7 + C_8 r_0^8 + C_9 r_0^9 + \ldots\right] \tag{2.7}$$

where

$$C_3 = q P_2 + n(1 - v^2), \quad C_4 = q P_3, \quad C_5 = q P_4$$

$$C_6 = q p_5 + 3C_3^2 \quad C_7 = q P_6 + 7qC_3^2 P_3$$

$$C_8 = q P_7 + 8q C_3 P_4 + 4q^2 P_3^2 \tag{2.8}$$

$$C_9 = q P_8 + 9q C_3 P_5 + 9q^2 P_3 P_4$$

And $P_j = P_j(\lambda)$ are Legendre polynomials and terms upto second order of smallness in n and q have been retained in (2.8). This relation helps to obtain the shape of a Roche equipotentials ψ=constant.

The volume enclosed by the equipotential surface ψ = constant is given by

$$V_\psi = \frac{2}{3}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^3}{\mu} d\lambda\, d\upsilon \tag{2.9}$$

Kopal has shown that the explicit expression of V_ψ in terms of r_0 defined by (2.6), can be represented as

$$V_\psi = \frac{4}{3}\pi D^3 r_0^3\left[1 + 2n r_0^3 + \left\{\frac{12}{5}q^2 + \frac{8}{5}nq + \frac{32}{5}n^2\right\}r_0^6 + \frac{15}{7}q^2 r_0^8 + 2q^2 r_0^{10} + \ldots\right] \tag{2.10}$$

where terms up to second order of smallness in n and q are retained.

Following the approach of Kopal (65), and Mohan and Singh (87), the explicit expressions for the surface area S_ψ and the values of averages or parameters $r_\psi, \overline{g}, \overline{g^{-1}}$ on the Roche equipotential ψ =constant are given as

$$S_\psi = 2\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}} \frac{r^2}{\mu} d\lambda\, d\upsilon$$

$$= 4\pi D^2 r_0^2\left[1 + \frac{4n}{3}r_0^3 + \left\{\frac{7}{5}q^2 + \frac{14}{15}nq + \frac{56}{15}n^2\right\}r_0^6 + \frac{9}{7}q^2 r_0^8 + \frac{11}{9}q^2 r_0^{10} + \ldots\right] \tag{2.11}$$

$$r_\psi = \left[\frac{3}{4\pi}V_\psi\right]^{1/3}$$

$$= D r_0\left[1+\frac{2n}{3}r_0^3+\left\{\frac{4}{5}q^2+\frac{8}{15}nq+\frac{76}{45}n^2\right\}r_0^6+\frac{5}{7}q^2 r_0^{\,8}+\frac{2}{3}q^2 r_0^{\,10}+\ldots\right]$$

(2.12)

$$\bar{g}=\frac{2}{S_\psi}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}}\left(\frac{d\psi}{dn}\right)\frac{r^2}{\mu}\,d\lambda\,d\upsilon$$

$$=\frac{GM_\psi}{D^2 r_0^2}\left[1-\frac{8n}{3}r_0^3-\left\{3q^2+2nq+\frac{40}{9}n^2\right\}r_0^6-\frac{51}{14}q^2 r_0^8-\frac{13}{3}q^2 r_0^{10}+\ldots\right]$$

(2.13)

$$\overline{g^{-1}}=\frac{2}{S_\psi}\int_{-1}^{1}\int_{-\sqrt{1-\lambda^2}}^{\sqrt{1-\lambda^2}}\left(\frac{d\psi}{dn}\right)^{-1}\frac{r^2}{\mu}\,d\lambda\,d\upsilon$$

$$=\frac{D^2 r_0^{\,2}}{GM_\psi}\left[1+\frac{8n}{3}r_0^3+\left\{\frac{31}{5}q^2+\frac{62}{15}nq+\frac{584}{45}n^2\right\}r_0^6+\frac{101}{14}q^2 r_0^{\,8}+\frac{75}{9}q^2 r_0^{\,10}+\ldots\right]$$

(2.14)

Inverting the relation (2.12) they also obtain

$$r_0=r_\psi^*\left[1-\frac{2n}{3}r_\psi^{*3}-\left\{\frac{4}{5}q^2+\frac{8}{15}nq-\frac{4}{45}n^2\right\}r_\psi^{*6}-\frac{5}{7}q^2 r_\psi^{*8}-\frac{2}{3}q^2 r_\psi^{*10}\ldots\right]$$

(2.15)

where $r_\psi^*=r_\psi/D$, r_ψ^* being the nondimensional form r_ψ. In all the above expressions terms upto second order of smallness in n and q are retained.

2.2 ROCHE COORDINATES

To study the problems of rotationally and tidally distorted stars, Kopal (65,66) introduced a system of coordinates, which he called Roche Coordinates. He and some of his co-workers (Kitumara (66), Ali (67) etc) investigated some of the mathematical properties of this system of coordinates. Kopal (65) also indicated how this system of coordinates could be used to study the problems of vibrations of rotationally and tidally distorted stellar models. In the system of Roche coordinates the equipotential surfaces of a distorted Roche model are chosen to represent the equipotential surfaces of an actual stellar model distorted by rotational and tidal forces. Choosing the equipotential as one coordinates, the other two coordinates are chosen to form a triply orthogonal system. In the system of Roche coordinates (ξ,η,ζ) , we take the ξ coordinate to be an equipotential surface of the form (2.1) and choose the other two coordinates η and ζ in such a way as to satisfy the conditions of mutual orthogonality with respect to ζ as well as each other.

Kopal (65) and his co-workers investigated the mathematical properties of this system of Roche coordinates. Their work shows that it is not possible in general to obtain expressions for η and ζ in closed analytic forms. Kopal investigated two particular cases of this problem in detail. In the one case q is taken to be zero, and in the other Ω^2 is taken to be zero. The first corresponds to the Roche coordinates of a star distorted by rotational forces alone and the second corresponds to the Roche coordinates of a nonrotating star distorted by the tidal effects of a companion star.

For the rotational case Kopal (65) has shown that whereas the first two Roche coordinates ζ and η are expressible in closed analytic form as

$$\varsigma = \frac{1}{r} + nr^2(1 - v^2), \qquad \eta = \psi \tag{2.16}$$

the expression for the third Roche coordinate ζ is not possible in closed analytic form. Kopal obtained an expression for it in the form of an infinite series in ascending powers of *n* as

$$\cos\zeta = v\sum_{J=0}^{\infty}(2n)^J r^{3J} x_J(v) \tag{2.17}$$

where $x_0(v) = 1,\ x_1(v) = -\frac{1}{3}(1 - v^2),\ \mathrm{n} = \frac{\Omega^2}{2}$ while for $j > 1$ all subsequent X_J (ζ)'s can be generated with the aid of recursion formula

$$3JX_J + (1 - v^2)\left[(\upsilon\, X_{j-1})' - 3(j-1)X_{j-1}\right] = 0 \tag{2.18}$$

where the prime denotes differentiation with respect to υ. Kopal also obtained the values of metric coefficients h_1, h_2, h_3 up to second order terms in *n*.

In case of tidally distorted Roche model Kopal (65) has shown that

$$\zeta = \frac{1}{r} + q\left[\frac{1}{\sqrt{(1 - 2\lambda r + r^2)}} - \lambda r\right] = \text{constant} \tag{2.19}$$

which represents the equipotential surface of a star distorted by the tidal forces of a nearby star Taking ζ as defined above Kopal (65) has shown that the second and third coordinates are given by

$$\eta = \cos^{-1}\lambda - \frac{q}{\sqrt{1 - \lambda^2}}\sum_{J=2}^{4}\frac{r^{J+1}}{J+1}P_j' \tag{2.20}$$

and

$$\xi = \cos^{-1}\frac{\upsilon}{\sqrt{1 - \lambda^2}} \tag{2.21}$$

respectively. In these relations a prime denotes differentiation with respect to λ.

Following Kopal's approach, Mohan and Singh (89) used the system of Roche coordinates to obtain explicit forms of equations of small radial oscillations of rotationally distorted and tidally distorted stars assuming Roche model for the star and used these to numerically compute certain eigenfrequencies of oscillation of such distorted models. Their work show that the system of Roche coordinates can be used with advantage to study the problem of small oscillations of rotating star as well as only tidally distorted stars. The main advantage of the technique of studying small oscillations of rotating stellar models through the use of Roche coordinates is that we were able to account for the effects of distortion caused by rotation or tidal effects automatically while studying the problem of small oscillations of these models in the usual way. One limitation of the present technique, however, is that it must be applied with care when studying the vibrations of stellar models which have unusually large angular velocities of rotation, because we do not get the expression for the third Roche coordinate in a closed analytic form and, instead, have to express it as an infinite series in ascending powers of the angular velocity of rotation.

However it was observed that use of this approach for determining the combined effects of rotation and tidal distortions on the equilibrium structure and periods of oscillations of binary stars, in which the rotational and tidal effects have to be considered jointly, is not convenient. Moreover the method could not be conveniently used when more realistic models in place of Roche model are to be used for the inner structure of the star.

2.3 AVERAGING TECHNIQUE OF KIPPENHAHN AND THOMAS

In order to study the effects of rotation and tidal distortions on the equilibrium structure of gaseous sphere, Kippenhahn and Thomas (62) developed the concept of topologically equivalent spherical surfaces corresponding to actual equipotential surfaces of a rotationally and tidally distorted model. They define on these equivalent spherical surfaces, quantities such as $\bar{f}, \bar{g}$ etc. which denote the certain averages of the quantities f, g, respectively on the actual equipotential surfaces. If ψ denotes the total potential (gravitation, rotation and tidal forces) arises of a rotationally and tidally distorted model at an arbitrary point $P(x,y,z)$ then $\psi\ (x,y,z)$ = constant is an equipotential surface. Let V_ψ be the volume enclosed by the equipotential surface ψ =constant and S_ψ is surface area of this equipotentials surfaces ψ = constant. For any function $f(x,y,z)$ they define $\bar{f}$ as its mean values over the equipotential surfaces ψ =constant by the relation

$$\bar{f} = \frac{1}{S_\psi} \int_{\psi = constant} f\, d\sigma \qquad (2.22)$$

Kippenhahn and Thomas define a variable r_ψ in analogy with a sphere by the relation

$$V_\psi = \frac{4}{3}\pi r_\psi^{\ 3} \qquad (2.23)$$

where $d\sigma$ denotes the surface element of the equipotential surface ψ =constant. Also $\bar{f}$ thus defined over the topologically equivalent surface is used to represent the value of f over the topologically equivalent spherical surfaces. Clearly if $\bar{f}$ is a function of equipotential surface ψ only and can be obtained as (2.22) for each equipotentials surface ψ =constant By definition

$$S_\psi = \int\limits_{\psi = constant} d\sigma \tag{2.24}$$

Obviously, S_ψ is in general not equal to $4\pi r_\psi^{\ 2}$. Kippenhahn and Thomas define a function $g(x, y, z)$ by the relation

$$g = \frac{d\psi}{dn} \tag{2.25}$$

This *g* corresponds to the force of gravity of a sphere. The distance dn in between two neighboring surface ψ=constant and $\psi+d\psi$=constant is in general not constant (i.e. not same at all points of the surface). From this equation (2.25) the mean values $\bar{g}$ and $\overline{g^{-1}}$ can be calculated with the help of relations

$$\left.\begin{aligned} \bar{g} &= \frac{1}{S_\psi} \int\limits_{\psi = constant} \frac{d\psi}{dn} d\sigma \\ \overline{g^{-1}} &= \frac{1}{S_\psi} \int\limits_{\psi = constant} \left(\frac{d\psi}{dn}\right)^{-1} d\sigma \end{aligned}\right\} \tag{2.26}$$

Both $\bar{g}$ and $\overline{g^{-1}}$ are functions of ψ alone and represent the value of g and g^{-1}, respectively over the topologically equivalent spherical surface. The volume dV_ψ between the surface $\psi =$ constant and $\psi+d\psi$=constant is given by

$$dV_\psi = \int\limits_{\psi = constant} dn\, d\sigma = \int\limits_{\psi = constant} \left(\frac{d\psi}{dn}\right)^{-1} dn = S_\psi \overline{g^{-1}}\, d\psi \tag{2.27}$$

Kippenhahn and Thomas also define nondimensional parameters u, v, w, as

$$u = \frac{S_\psi}{4\pi r_\psi^{\ 2}}, \quad v = \frac{\bar{g}\, r_\psi^2}{GM_\psi}, \quad w = \frac{\overline{g^{-1}}\, GM_\psi}{r_\psi^{\ 2}} \tag{2.28}$$

where M_ψ is the mass enclosed by equipotential surface ψ=constant.

We may thus regard the equipotential surface ψ=constant to be topologically equivalent to a sphere of radius r_ψ for which various functions are defined by the above relations. (It may be noticed that if ψ is the gravitational potential of a sphere then the surface ψ=constant are spherical surfaces with $r_\psi = r$ for which u =1 and $g = \frac{GM_\psi}{r_\psi^2}$ is constant on these spheres and therefore u and w are constants and equal to 1).

Equations (2.22) to (2.28) are purely mathematical definitions, which have been applied by Kippenhahn and Thomas to gravitational fields of gaseous spheres distorted by rotational and tidal forces. In hydrostatic equilibrium the equipotential surfaces are also surface of equipressure and equidensity. Therefore on an equipotential surface the pressure P_ψ and the density ρ_ψ are also constant. Using these concepts, Kippenhahn and Thomas obtain the equations governing the equilibrium structure of a rotationally and tidally distorted stellar model in the following manner

From equation (2.28) the mass dM_ψ between the equipotentials surfaces ψ=constant and $\psi + d\psi$=constant is given by

$$dM_\psi = dV_\psi \, \rho_\psi = 4\pi \, r_\psi^{\,2} \rho_\psi \, d\, r_\psi \qquad (2.29)$$

Thus we get

$$\frac{dM_\psi}{dr_\psi} = 4\pi r_\psi^{\,2} \rho_\psi \qquad (2.30)$$

From equation (2.27) and (2.29) we have

$$d\psi = \frac{d\psi}{dV_\psi} dV_\psi = (\frac{dV_\psi}{d\psi})^{-1} \frac{dM_\psi}{P_\psi} = \frac{dM_\psi}{S_\psi \, \overline{g^{-1}} \, P_\psi} \qquad (2.31)$$

Using relations (2.28) we get

$$d\psi = \frac{G M_\psi \, d M_\psi}{4\pi r_\psi^{\,4} P_\psi u w} \qquad (2.32)$$

The conditions for hydrostatic equilibrium, $dP_\psi \big/ d\psi = -P_\psi$, can now be written with equation (2.28) in the form

$$\frac{dP_\psi}{dM_\psi} = -\frac{G M_\psi}{4\pi r_\psi^{\,4}} f_p \qquad (2.33)$$

Where

$$f_P = \frac{1}{uw} = \frac{4\pi r_\psi^{\,4}}{G M_\psi} \frac{1}{S_\psi \, \overline{g^{-1}}}$$

The factor f_P is a function of ψ only. If ψ is known the equipotentials surfaces can be determined, and with them values of $S_\psi, r_\psi, \bar{g}$ and $\overline{g^{-1}}$ for each equipotentials surface simply from the geometry of the equipotentials. The mass M_ψ which depends on the density distribution ρ_ψ can be determined by integrating the equation (2.30). Similarly the other structure equations derived by Kippenhahn and Thomas (62), which includes the effects of rotation and tidal distortions on the equilibrium structure of gaseous spheres are as follows.

For chemically homogenous spheres, the nuclear energy generation rate ε depends only upon density ρ_ψ and the temperature T_ψ and are, therefore, constant on equipotentials surfaces. Thus if L_ψ is the energy which passes per second through the equipotential surface $\psi =$ constant, then

$$\frac{dL_\psi}{dM_\psi} = \varepsilon \qquad (2.34)$$

Using (2.30) it can be written as

$$\frac{dL_\psi}{dr_\psi} = 4\pi\, r_\psi^{\;2} \rho_\psi\, \varepsilon \tag{2.35}$$

If the energy is transported by radiation, then the energy transport equation is

$$F_\psi = -\frac{4acT_\psi^{\;3}}{3\kappa}\frac{d\psi}{dn}\frac{dT_\psi}{dM_\psi}\frac{4\pi\, r_\psi^{\;4}\, u\, w}{G\, M_\psi} \tag{2.36}$$

where F_ψ is the radiative flux on the equipotentials surface $\psi =$ constant by integrating F_ψ over the equipotentials surface $\psi =$ constant, we get

$$L_\psi = \int_{\psi = constant} F_\psi\, d\sigma$$

$$= -\frac{4acT_\psi^{\;3}}{3\kappa}\frac{dT_\psi^{\;3}}{dM_\psi} u\, w \frac{4\pi\, r_\psi^{\;4}}{G\, M_\psi} \int_{\psi = constant} (\frac{d\psi}{dn}) d\sigma$$

$$= -\frac{64\pi^2 acT_\psi^{\;3} r_\psi^{\;4}}{3\kappa} u^2\, v\, w \frac{dT_\psi}{dM_\psi} \tag{2.37}$$

so that

$$\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa\, L_\psi}{64\pi^2 acT_\psi^{\;3} r_\psi^{\;4}}\frac{1}{u^2 v\, w} \tag{2.38}$$

Using (2.29) this equation can be expressed as

$$\frac{dT_\psi}{dM_\psi} = -\frac{3\kappa\, \rho_\psi\, L_\psi}{16\pi\, acT_\psi^{\;3}\, r_\psi^{\;2}} f_T \tag{2.39}$$

where $$f_T = \frac{1}{u^2\, v.w}$$

Equations (2.30), (2.33), (2.34) and (2.38) which are the four basic equations governing the equilibrium structure of a gaseous sphere distorted by rotation and tidal forces and the boundary conditions to be satisfied are

$$\left.\begin{array}{l} M_\psi = 0,\ \ L_\psi = 0,\ at\ the\ \ centre\ \ r_\psi = 0 \\ M_\psi = M_0,\ L_\psi = L_{\psi S},\ P_\psi = 0,\ T_\psi = 0 \\ or\ \ P_\psi = P_{\psi S},\ T_\psi = T_{\psi S},\ at\ the\ \ free\ \ surface r_\psi = R_\psi \end{array}\right\} \quad (2.40)$$

2.4 MOHAN AND SAXENA APPROACH FOR DETERMINING THE EQUILIBRIUM STRUCTURE OF ROTATIONALLY AND TIDALLY DISTORTED STELLAR MODELS

In order to determine the inner structure of a rotationally and tidally distorted gaseous sphere the system of equations (2.30), (2.33), (2.34), (2.38) has to be integrated numerically subject to the boundary conditions (2.40) specified therein. Therefore the evaluation of the actual equipotential surface of a rotationally and tidally distorted gaseous sphere is complicated. Kippenhahn and Thomas (62) proposed that for evaluation of the distortion parameters u, v, w, f_P, f_T etc., the actual equipotentials surface may be replaced by Roche equipotentials surfaces.

Once the equipotential surfaces of a rotationally and tidally distorted star are approximated by the Roche equipotentials, the results obtained by Kopal (65) and Mohan and Singh (89) may be used to evaluate explicitly the values of the distortion parameters u, v, w, f_P, f_T appearing in stellar structure equations (2.33) and (2.39). Using (2.28), (2.33), (2.39) and (2.11- 2.14) the explicit expressions of the distortions parameters u, v, w, f_P, f_T on the equipotential surface as obtained by Mohan et al. are

$$
\left.
\begin{aligned}
u &= 1-\left\{\frac{q^2}{5}+\frac{2}{15}nq+\frac{4}{45}n^2\right\}r_\psi^{*6}-\frac{1}{7}q^2r_\psi^{*8}-\frac{1}{9}q^2r_\psi^{*10}+\ldots \\
v &= 1-\frac{4}{3}nr_\psi^{3}-\left\{\frac{7}{5}q^2+\frac{14}{15}nq+\frac{68}{45}n^2\right\}r_\psi^{*6}-\frac{31}{14}q^2r_\psi^{*8}-3q^2r_\psi^{*10} \\
.w &= 1+\frac{4}{3}nr_\psi^{*3}+\left\{\frac{23}{5}q^2+\frac{16}{15}nq+\frac{212}{45}n^2\right\}r_\psi^{*6}+\frac{81}{14}q^2r_\psi^{*8}+7q^2r_\psi^{*10} \\
f_P &= 1-\frac{4}{3}nr_\psi^{*3}-\left\{\frac{22}{5}q^2+\frac{44}{15}nq+\frac{128}{45}n^2\right\}r_\psi^{*6}-\frac{79}{14}q^2r_\psi^{*8}-\frac{62}{9}q^2r_\psi^{*10} \\
f_T &= 1-\left\{\frac{14}{5}q^2+\frac{28}{15}nq+\frac{56}{45}n^2\right\}r_\psi^{*6}-\frac{46}{14}q^2r_\psi^{*8}-\frac{34}{9}q^2r_\psi^{*10}
\end{aligned}
\right\} \quad (2.41)
$$

where $r_\psi^* = \dfrac{r_\psi}{D}$ is the nondimensional form of r_ψ and terms upto second order of smallness in n and q are retained.

The value of M_ψ, P_ψ, L_ψ etc. on the various equipotentials surfaces of a rotationally and tidally distorted gaseous sphere may now be obtained by solving the system of differential equations (2.30), (2.33), (2.34) with boundary condition (2.40) and using the values of the correction factors f_P and f_T.

It may be noted that approximating the equipotential surfaces of a rotationally and tidally distorted model by Roche equipotentials, the structure of the star is not approximated by the structure of a Roche model. In the case of no distortion *(n=q=0),* equation (2.41) gives $u = v = w = f_P = f_T = 1$ and the system of differential equations (2.30), (2.33), (2.34), (2.39) reduce to the equations governing the equilibrium structure of the original undistorted star but not of the Roche model.

Usual methods for stellar structure equations such as Henvey method can be used to integrate the system of differential equation (2.30), (2.33), (2.34), (2.39) governing the equilibrium structure of a rotationally and tidally distorted gaseous sphere. At every step the values of the parameters u, v, w, f_P *and* f_T must be taken from (2.41).

In case the thermal properties are not considered important and only hydrostatic equilibrium of a rotationally and tidally distorted gaseous spheres is to be investigated then we need only to integrate equation (2.30) and (2.33) subject to the boundary conditions

$$\left.\begin{array}{ll} M_\psi = 0,\ L_\psi = 0, & \textit{at the centre } r_\psi = 0, \\ M_\psi = M_0,\ L_\psi = L_{\psi S}, P_\psi = 0,\ T_\psi = 0,\ P_\psi = P_{\psi S}, & \textit{at the surface } r_\psi = R_\psi \end{array}\right\} \quad (2.42)$$

In case of star is being distorted by rotational forces alone (or tidal forces alone) we may set *q=0 (or n=0)* in (2.41) and still use the above approach to determine the equilibrium structure of its rotationally distorted or tidally distorted model. For obtaining the structure of the primary component synchronous binary system we should set $n = \frac{q+1}{2}$.

Mohan and Saxena (85) find it more convenient to work with r_0 in place of M_ψ or r_ψ as independent variable by introducing (2.6) which is connected with variable r_ψ through relations (2.10). Saxena (124) expressed the system of differential equations governing the equilibrium structure of a rotationally and tidally distorted model as

$$\frac{dM_\psi}{dr_0} = 4\pi D^3 \rho_\psi r_0^2 f_1 \quad (2.43a)$$

$$\frac{dP_\psi}{dr_0} = -\frac{GM_\psi}{Dr_0^2} \rho_\psi f_2 \quad (2.43b)$$

$$\frac{dL_\psi}{dr_0} = 4\pi D^3 \rho_\psi r_0^2 f_1 \quad (2.43c)$$

and

$$\frac{dT_\psi}{dr_0} = -\frac{3\kappa L_\psi}{16\pi D a c T_\psi^3} \frac{\rho_\psi}{r_0^2} \tag{2.43d}$$

where f_1, f_2 f_3 are certain functions of n, q and r_0 incorporating the effect of rotation and tidal distortions on the equilibrium structure equations of rotationally and tidally distorted models. The explicit expressions for these distortion parameters as given by Saxena (124) are

$$f_1 = 1 + 4n r_0^3 + \left\{\frac{36}{5}q^2 + \frac{24}{5}nq + \frac{96}{5}n^2\right\} r_0^6 + \frac{55}{7}q^2 r_0^8 + \frac{26}{3}q^2 r_0^{10} + \ldots \tag{2.44a}$$

$$f_2 = 1 - \left\{\frac{2}{5}q^2 + \frac{4}{15}nq + \frac{16}{15}n^2\right\} r_0^6 - \frac{9}{14}q^2 r_0^8 - \frac{8}{9}q^2 r_0^{10} + \ldots \tag{2.44b}$$

$$f_3 = 1 + \frac{4n r_0^3}{3} + \left\{\frac{6}{5}q^2 + \frac{4}{5}nq + \frac{224}{45}n^2\right\} r_0^6 + \frac{24}{14}q^2 r_0^8 + \frac{20}{9}q^2 r_0^{10} + \ldots \tag{2.44c}$$

In these above expressions terms upto second order of smallness in n and q are retained. The boundary conditions now become

$M_\psi = 0,\ L_\psi = 0$ at the center $r_0 = 0,$

$$M_\psi = M_0,\ L_\psi = L_{\psi s}\ \ P_\psi = 0,\ T_\psi = 0$$
$$or$$
$$P_\psi = P_{\psi s},\ T_\psi = T_{\psi s}$$

at the free surface $r_0 = r_{0s}$ (r_{0s} being the value of r_0 at the free surfaces.)

In fact

$$r_{0s} = \frac{1}{\psi_s^* - q} \tag{2.45}$$

where ψ_s^* is the nondimensional value of the total potential ψ on the outermost equipotential surface of the rotationally and tidally distorted stellar model.

In the case of no distortion $f_p = f_T = 1$ and the above equation reduce to the usual equations governing the equilibrium structure of an undistorted gaseous sphere. Kippenhahn and Thomas (62) advocated the use of these equations to determine the inner structure of stars distorted by rotation and tidal forces.

2.5 VALIDITY OF SERIES EXPANSION USED IN ROCHE COORDINATES AND MOHAN AND SAXENA APPROACH

The series expansion (2.2) of one of the of Roche coordinates was obtained by Kopal (65) for determining of equilibrium structure of rotating close binary stars in which the actual potential of a rotating dipole is replaced by the Roche equipotential. The system of Roche coordinates, using this series expansion (2.2) for one of the coordinate was then used by him to determine small oscillations of rotationally and tidally distorted stars. Mohan et al. (88) have also used this series expansion in their work. However as pointed out by Kopal in his work, the convergence of the series expansion has not been theoretically established. Mohan et al (92) also have not established convergence of the series expansions based on this approach and used in their work. Establishing validity of the convergence of these series is thus essential. Only then one can feel assured regarding the validity of these Roche equipotential based methods for computing equilibrium structures and oscillations of rotationally and tidally distorted stars.

Although it has not been possible for us so far to theoretically justify convergence of the series expansions, attempt has been made in this section to numerically verify the validity of the series expansions (2.7). The remainder of this chapter is devoted to checking the validity of series expansion (2.7) used by Kopal and also by Mohan and Saxena in their work, for which analytic proof of the convergence of the series expansion is not available. In 2.5.1 we discuss the numerical approach which has been adopted by us to check the validity of results obtained by using series expansions (2.7). In 2.5.2 we present the numerical computations based on this approach which have been carried out by us to check validity of numerical results obtained from (2.2) in different situations. Analyis of this numerical result is next carried out in 2.5.3 and conclusions drawn.

2.5.1 Numerical Approach for Checking Validity of Series Expansions

In order to establish validity of convergence of series (2.7) let us consider the equation (2.2) which is nondimensional form of the total potential ψ. Then equation (2.2) can be written as

$$\psi^* = \frac{1}{r^*} + q\sum_{j=2}^{\infty} r^{j+1} p_j(\lambda) + n(1-\upsilon^2)r^{*2} \tag{2.46}$$

Unfortunately, the expression (2.46) for ψ^* is such that r^* cannot be found explicitly in terms of ψ^*. Equation (2.46) of the Roche equipotentials represent an implicit function defining, for given values of ψ^*, q, and n, r^* as a function of λ, υ. When it has been rationalized and cleared of fractions, the results are an algebraic equation of eighth degree in r^*, which is very difficult to solve and whose analytical solution presents unsurmountable difficulties. In the case of pure rotational distortion i.e. (q=0), equation (2.46) can be reduced to a cubic equation and is solvable in terms of circular functions. In case of a purely tidal distortion i.e. $(n=0)$, equation (2.46) becomes a quadratic, which could also be solved for r^* in a closed form. However, in general case of rotational and tidal distortion interaction, any attempt at an exact solution of (2.46) for r^* ,becomes virtually hopeless. Therefore approximate solutions are sought by successive approximations. The desired approximate solution of equation (2.46) for r^* as a function of λ and υ in the form of series expansion as obtained by and written in the ascending power of r_0 is

$$\begin{aligned} r^* = r_0 \, [1 &+ (P_2\, q + n x)\, r_0^{\,3} + P_3\, q\, r_0^{\,4} + P_4\, q\, r_0^{\,5} + \left(P_5\, q + 3 q^2 P_2^{\,2} + 3 q P_2\, n x\right) r_0^{\,6} \\ &+ \left(P_6\, q + 7 q^2 P_2 P_3 + 7 q P_3\, n x\right) r_0^{\,7} + \left(P_2\, q + 8 q^2 P_2 P_4 + 8 q P_4\, n x + 4 q^2 P_3^{\,2}\right) r_0^{8} \\ &+ \left(P_8\, q + 9 q^2 P_3 P_4 + 9 q^2 P_2 P_5 + 9 q P_5 n x\right) r_0^{\,9} \\ &+ \left(P_9\, q + 10 q^2 P_2 P_6 + 10 q^2 P_3 P_5 + 10 q P_6\, n x + 5 q^2 P_4^{\,2}\right) r_0^{\,10} \\ &+ \left(P_{10}\, q + 11 q^2 P_2 P_7 + 11 q^2 P_3 P_{65} + 11 q^2\; P_4 P_5 + 11 q P_7\, n x\right) r_0^{\,11} + \ldots] \end{aligned} \tag{2.47}$$

where $x = 1 - \upsilon^2$ and $r_0 = \dfrac{1}{\psi^* - q}$

This is a basic relation determining the shape of a Roche equipotentials surface ψ^* = constant. Kopal (65), Mohan and Singh (87) used this relation to find various physical parameters V_ψ, S_ψ, $\bar{g}$, $\overline{g^{-1}}$ explicitly in terms of series expansion in powers of r_0. The series expansion for distortion parameters no doubt look appealing from the theoretical point of view.

In order to establish the convergence of series (2.47) numerically, we take the help of procedure, generally, adopted to test the convergence of any infinite series. A finite sequence $\{S_n\}$ of partial sums of series (2.47) is constructed by taking $D=1$. Four our convenience, only five sums have been considered which are given as

$$
\begin{aligned}
S_1 &= r_0 + (P_2\, q + n\, x) r_0^{\,4} \\
S_3 &= S_1 + P_3\, q\, r_0^{\,5}, \\
S_5 &= S_3 + P_4\, q\, r_0^{\,6} + \left(P_5\, q + 8\, P_3\, P_4\, q^2 + 8\, P_4\, q\, n\, x + 4\, P_3^{\,2}\, q^2\right) r_0^{\,7} \\
S_7 &= S_5 + \left(q\, P_5 + 7\, P_2\, P_3\, q^2 + 7\, P_7\, q\, n\, x\right) r_0^{\,8} \\
S_9 &= S_7 + \left(P_9\, q + 9\, P_3\, P_4\, q^2 + 9\, P_2\, P_5\, q^2 + 9\, P_5\, q\, n\, x\right) r_0^{\,9}\ etc
\end{aligned}
\tag{2.48}
$$

Thus the approximate value of r^* can be estimated either from each value of partial sums or computed by adding all these partial sums. To establish the validity of convergence of this series to r^* a numerical method is applied to equation (2.2) to find another approximate solution of r. This new approximate value of r helps us to estimate the growth of errors which arise in each computed value of the partial sums given in (2.48).

We apply fixed- point iteration method to compute r^* with specified error tolerance. An equation of the form

$$r = F(r) \tag{2.49}$$

can be derived from equation (2.2) so that any solution of (2.48) is the solution of (2.2). The iteration function $F(r)$ for solving (2.2) can be chosen as

$$F(r) = \frac{1}{\psi}\left[1 + q\, r \sum_{j=2}^{\infty} P_j(\lambda) r^{j+1} + n\, r^3 \left(1 - \upsilon^2\right)\right] \tag{2.50}$$

The iterative sequence generated by the recursion formula

$$r_{i+1} = F(r_i), \quad i = 0, 1, 2 \tag{2.51}$$

will converge to a point r for which equation (2.49) is satisfied. The value of r thus computed with desired accuracy is written as r_{exp} for avoiding the confusion between the two values of r.

2.5.2 Numerical Computations

The fixed- point iteration technique proposed in section (2.5.1) may be used to determine the value of r which gives us the shape of outermost surfaces of rotationally and tidally distorted gas spheres. Numerical computation for (2.51) have been performed with initial approximation $r_0 = \dfrac{1}{\psi_s^* - q}$, for some specified values of ψ_s^*, n, and q. This value of ψ_s^* and q were selected in such a manner that the distorted models are well within the respective Roche lobes. The values of r_{exp} is computed with desired accuracy of 0.000005.

In order to exhibit the convergence of the series r in (2.7) numerically, each partial sum $S_i\,(i,=1,3,5,7,9)$ is computed for the same specified values of r_0, ψ_s^*, n, q, θ and ϕ and is compared with the values of r_{exp}.Computed from equation (2.51) the percentage error needed to see the growth of errors in each partial sum relative to r_{exp} is given by

$$ES_i = \frac{|r_{\exp} - S_i|}{r_{\exp}} \times 100, \quad i = 1,3,5,7,9 \tag{2.52}$$

In order to exhibit the effect of rotation on the shape of rotationally distorted models, we present the values of r_{exp}, $S_i\,(i,=1,3,5,7,9)$ in Table 2.1 (a), 2.2 (a), 2.3 (a) . for various combinations of θ and ϕ on the outermost equipotential surfaces $\psi_s^* = 2.5, 5.0, 10.0$ of rotating models. The results written in parenthesis just below the values of S_i, indicate the percentage error in it with respect to r_{exp}. The results presented in Table 2.1 (b), 2.2 (b), 2.3 (b) show the effect of tidal forces on the shape of the outermost equipotential surfaces $\psi_s^* = 2.5, 5.0, 10.0$ of tidally distorted models. Finally the results shown in Tables 2.1 (c), 2.2 (c), 2.3 (c) and 2.1 (d), 2.2(d), 2.3 (d), 2.4 (d) are, respectively, the effect of nonsynchronously and synchronously rotating binary systems on the outermost surfaces of the equipotential surfaces $\psi_s^* = 2.5,\ 5.0,\ 10.0$

2.5.3 Analysis of the Numerical Results

Results presented in Tables 2.1, 2.2 , 2.3, 2.4 (a, b, c) and Table 2.5 essentially give the difference between the numerically computed value of r on the outer most surface at difference using exact equations (2.2) and the corresponding values of r as obtained from series expansion (2.7) when terms up to difference order are included. Where as Tables (2.1) correspond to purely rotating stars of different dimensions for $n = 0.2$ (ψ_s^* is value of ψ at the outer most surface, smaller the value of ψ_s^* more extended is the model), Table (2.2) corresponds to tidally distorted stars and Table 2.3 (a, b, c) to rotationally as well as tidally distorted stars. Tables 2.3 (d) represent a synchronously rotating binary system. Where as values of n ,q are reasonably small in these cases, in Table (2.5) we present a situation in which values of n ,q are large.

Our results show that where as in the case of purely rotating models Tables 2.1 (a) , 2.2 (a), 2.3 (a) the maximum percentage difference in the value of S_9 (when all terms in series expansion 2.7 are included) and corresponding values of r_{exp} computed numerically using (2.2) is only 0.402609 for most extended model given in Table 2.1(a) for ψ_s^*= 2.5, it is 0.418518 in Table 2.1 (b) for ψ_s^*=2.5 for tidally distorted models, 0.425011 in Table 2.1 (c) for ψ_s^*=

2.5 for rotationally and tidally distorted model and 1.440326% in case of Table 2.4 (d) for a synchronously rotating binary system. In all these cases values of n, q can be considered to be reasonably small. However in Table 2.5 when n, q are taken large for an extended rotationally and tidally distorted model, maximum value of this percentage difference is as high as 18.0044%.

Our numerical results in Tables 2.1, 2.2, 2.3 and 2.4 also show that the series expansion (2.7) shows a converging trend as the value of percentage difference between values computed from it and value of r_{exp} computed from (2.2), the value of percentage difference decreases (except on account of truncation errors in certain cases) as more and more terms are included in its expansion. It is expected that even this small percentage difference is expected to reduce further if higher terms are included in series expansion (2.7) (as has been done by Mohan et al. in certain cases in their series expansion and by us also in our subsequent studies).

It may be noted that for points inside the star $\psi > \psi_s^*$, the trend of results in Tables 2.1 (a, b, c), 2.2 (a, b, c), 2.3 (a, b, c) and 2.4 (a, b, c) therefore shows that points inside the star the difference between the value of r computed using series solutions (2.7) and its value as obtained numerically from (2.2) is expected to be less than difference in their values at surface $(\psi = \psi_s^*)$. However at points which are outside the star $(0 < \psi < \psi_s^*)$ the difference in the value of ψ as computed from series solutions (2.7) and as obtained through numerical solution of (2.2) is expected to be more than the difference in their values on the surface. Difference will increase as ψ decreases towards zero. However it may be noted that in problems regarding structure and pulsations of rotationally and tidally distorted stars analysis is usually carried out at points inside the star where series expansion (2.7) yields quite accurate values of r.

The numerical study thus shows that the sequence of partial sums contain more error for smaller values of ψ_s^* and larger values of n and q. As ψ_s^* decreases we are nearing Roche limit. As n increases star is rotating more rapidly and as q increases mass of accompanying star increases. Kopal had assumed that in his studies n and q are small and star is well within Roche limit. Our analysis thus shows that series solution (2.7) reasonably justified for rotationally and tidally distorted stars in which values of n and q are not large and star is well within Roche limit.

Table 2.1 (a) : Values of r_{exp} for given values of θ, ϕ and percentage errors in corresponding values of S_i $(i = 1,3,5,7,9)$ on the surface of primary components of a rotationally distorted model for $(\psi_s^* = 2.5,\ n = 0.1, q = 0.0)$

		Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star $(\phi = 0^0)$					Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star $(\phi = 90^0)$				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	S_9
-90	0.402610	0.402560 **0.012525**	0.402560 **0.012525**	0.402609 **0.000318**	0.402609 **0.000318**	0.402609 **0.000318**	0.402560 **0.012525**	0.402560 **0.012525**	0.402609 **0.000318**	0.402609 **0.000318**	0.402609 **0.000318**
-75	0.402432	0.402389 **0.010886**	0.402389 **0.010886**	0.402431 **0.000252**	0.402431 **0.000252**	0.402431 **0.000252**	0.402389 **0.010886**	0.402389 **0.010886**	0.402431 **0.000252**	0.402431 **0.000252**	0.402431 **0.000252**
-60	0.401948	0.401920 **0.007007**	0.401920 **0.007007**	0.401948 **0.000126**	0.401948 **0.000126**	0.401948 **0.000126**	0.401920 **0.007007**	0.401920 **0.007007**	0.401948 **0.000126**	0.401948 **0.000126**	0.401948 **0.000126**
-45	0.401292	0.401280 **0.003097**	0.401280 **0.003097**	0.401292 **0.000037**	0.401292 **0.000037**	0.401292 **0.000037**	0.401280 **0.003097**	0.401280 **0.003097**	0.401292 **0.000037**	0.401292 **0.000037**	0.401292 **0.000037**
-30	0.400643	0.400640 **0.000766**	0.400640 **0.000766**	0.400643 **0.00000**	0.400643 **0.00000**	0.400643 **0.00000**	0.400640 **0.000766**	0.400640 **0.000766**	0.400643 **0.00000**	0.400643 **0.00000**	0.400643 **0.00000**
-15	0.400172	0.400171 **0.000052**	0.400171 **0.000052**	0.400172 **0.00000**	0.400172 **0.00000**	0.400172 **0.00000**	0.400171 **0.000052**	0.400171 **0.000052**	0.400172 **0.00000**	0.400172 **0.00000**	0.400172 **0.00000**
0	0.400000	0.400000 **0.00000**	0.400000 **0.00000**	0.400000 **0.00000**	0.400000 **0.00000**	0.400000 **0.00000**	0.400000 **0.00000**	0.400000 **0.00000**	0.400000 **0.00000**	0.400000 **0.00000**	0.400000 **0.00000**
15	0.400172	0.400171 **0.000052**	0.400171 **0.000052**	0.400172 **0.00000**	0.400172 **0.00000**	0.400172 **0.00000**	0.400171 **0.000052**	0.400171 **0.000052**	0.400172 **0.00000**	0.400172 **0.00000**	0.400172 **0.00000**
30	0.400643	0.400640 **0.000766**	0.400640 **0.000766**	0.400643 **0.00000**	0.400643 **0.00000**	0.400643 **0.00000**	0.400640 **0.000766**	0.400640 **0.000766**	0.400643 **0.00000**	0.400643 **0.00000**	0.400643 **0.00000**
45	0.401292	0.401280 **0.003097**	0.401280 **0.003097**	0.401292 **0.000037**	0.401292 **0.000037**	0.401292 **0.000037**	0.401280 **0.003097**	0.401280 **0.003097**	0.401292 **0.000037**	0.401292 **0.000037**	0.401292 **0.000037**
60	0.401948	0.401920 **0.007007**	0.401920 **0.007007**	0.401948 **0.000126**	0.401948 **0.000126**	0.401948 **0.000126**	0.401920 **0.007007**	0.401920 **0.007007**	0.401948 **0.000126**	0.401948 **0.000126**	0.401948 **0.000126**
75	0.402432	0.402389 **0.010886**	0.402389 **0.010886**	0.402431 **0.000252**	0.402431 **0.000252**	0.402431 **0.000252**	0.402389 **0.010886**	0.402389 **0.010886**	0.402431 **0.000252**	0.402431 **0.000252**	0.402431 **0.000252**
90	0.402610	0.402560 **0.012525**	0.402560 **0.012525**	0.402609 **0.000318**	0.402609 **0.000318**	0.402609 **0.000318**	0.402560 **0.012525**	0.402560 **0.012525**	0.402609 **0.000318**	0.402609 **0.000318**	0.402609 **0.000318**

Note : Results shown in bold face are the values of percentage error of the corresponding partial sums relative to r_{exp}

Table 2.2(a) : Values of r_{exp} for given values of θ, ϕ and percentage errors in corresponding values of S_i $(i = 1,3,5,7,9)$ on the surface of primary components of a rotationally distorted model for $(\psi_c' = 5.0,\ n = 0.1, q = 0.0)$

	Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star $(\phi = 0^0)$						Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star $(\phi = 90^0)$				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	S_9
-90	0.200160	0.200160 **0.000194**	0.200160 **0.000194**	0.200160 **0.00000**	0.200160 **0.00000**	0.200160 **0.00000**	0.200160 **0.000194**	0.200160 **0.000194**	0.200160 **0.00000**	0.200160 **0.00000**	200160 **00000**
-75	0.200150	0.200149 **0.000164**	0.200149 **0.000164**	0.200150 **0.00000**	0.200150 **0.00000**	0.200150 **0.00000**	0.200149 **0.000164**	0.200149 **0.000164**	0.200150 **0.00000**	0.200150 **0.00000**	200150 **00000**
-60	0.200000	0.200120 **0.000104**	0.200120 **0.000104**	0.200120 **0.00000**	0.200120 **0.00000**	0.200120 **0.00000**	0.200120 **0.000104**	0.200120 **0.000104**	0.200120 **0.00000**	0.200120 **0.00000**	200120 **00000**
-45	0.200080	0.200080 **0.000045**	0.200080 **0.000045**	0.200080 **0.00000**	0.200080 **0.00000**	0.200080 **0.00000**	0.200080 **0.000045**	0.200080 **0.000045**	0.200080 **0.00000**	0.200080 **0.00000**	200080 **00000**
-30	0.200040	0.200040 **0.000015**	0.200040 **0.000015**	0.200040 **0.00000**	0.200040 **0.00000**	0.200040 **0.00000**	0.200040 **0.000015**	0.200040 **0.000015**	0.200040 **0.00000**	0.200040 **0.00000**	200040 **00000**
-15	0.200011	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	200011 **00000**
0	0.200000	0.200000 **0.00000**	0.200000 **0.00000**	0.200000 **0.00000**	0.200000 **0.00000**	0.200000 **0.00000**	0.200000 **0.00000**	0.200000 **0.00000**	0.200000 **0.00000**	0.200000 **0.00000**	200000 **00000**
15	0.200000	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	0.200011 **0.00000**	200011 **00000**
30	0.200040	0.200040 **0.000015**	0.200040 **0.000015**	0.200040 **0.00000**	0.200040 **0.00000**	0.200040 **0.00000**	0.200040 **0.000015**	0.200040 **0.000015**	0.200040 **0.00000**	0.200040 **0.00000**	200040 **00000**
45	0.200080	0.200080 **0.000045**	0.200080 **0.000045**	0.200080 **0.00000**	0.200080 **0.00000**	0.200080 **0.00000**	0.200080 **0.000045**	0.200080 **0.000045**	0.200080 **0.00000**	0.200080 **0.00000**	200080 **00000**
60	0.200120	0.200120 **0.000104**	0.200120 **0.000104**	0.200120 **0.00000**	0.200120 **0.00000**	0.200120 **0.00000**	0.200120 **0.000104**	0.200120 **0.000104**	0.200120 **0.00000**	0.200120 **0.00000**	200120 **00000**
75	0.200150	0.200149 **0.000164**	0.200149 **0.000164**	0.200150 **0.00000**	0.200150 **0.00000**	0.200150 **0.00000**	0.200149 **0.000164**	0.200149 **0.000164**	0.200150 **0.00000**	0.200150 **0.00000**	200150 **00000**
90	0.200160	0.200160 **0.000194**	0.200160 **0.000194**	0.200160 **0.00000**	0.200160 **0.00000**	200160 **00000**	0.200160 **0.000194**	0.200160 **0.000194**	0.200160 **0.00000**	0.200160 **0.00000**	200160 **00000**

Note : Results shown in bold face are the values of percentage error of the corresponding partial sums relative to r_{exp}

Table 2.3(a) : Values of r_{exp} for given values of θ, ϕ and percentage errors in corresponding values of S_i $(i = 1,3,5,7,9)$ on the surface of primary components of a rotationally distorted model for $(\psi_s^* = 10.0,\ n = 0.1, q = 0.0)$

	Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star $(\phi = 0^0)$						Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star $(\phi = 90^0)$				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	S_9
-0	100010	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**	0.10001 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**
-75	100009	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**
-60	100008	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**
-45	100005	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**
-30	100002	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**
-15	100001	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**
0	100000	0.100000 **0.00000**	0.100000 **0.00000**	0.100000 **0.00000**	0.100000 **0.00000**	0.100000 **0.00000**	0.100000 **0.00000**	0.100000 **0.00000**	0.100000 **0.00000**	0.100000 **0.00000**	0.100000 **0.00000**
15	100001	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**	0.100001 **0.00000**
30	100002	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**	0.100003 **0.000007**
45	100005	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**	0.100005 **0.00000**
60	100008	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**	0.100008 **0.00000**
75	100009	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**	0.100009 **0.00000**
90	100010	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**	0.100010 **0.00000**

Note : Results shown in bold face are the values of percentage error of the corresponding partial sums relative to r_{exp}

Table 2.1(b) : Values of r_{exp} for given values θ, ϕ and percentage error in corresponding values of $S_i \; (i = 1, 3, 5, 7, 9)$ on the surface of primary components of a tidally distorted model for $(\psi_s^* = 2.5,\ n = 0.0, q = 0.1)$

	Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star $(\phi = 0^0)$						Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star $(\phi = 90^0)$				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	S_9
-90	0.418824	0.419681 **0.204520**	0.418657 **0.039805**	0.418532 **0.069741**	0.418518 **0.073078**	0.418515 **0.073769**	0.419681 **0.204520**	0.418657 **0.039805**	0.418532 **0.069741**	0.418518 **0.073078**	0.418515 **0.073769**
-75	0.418661	0.419378 **0.171200**	0.418527 **0.031990**	0.418451 **0.050185**	0.418451 **0.050221**	0.418455 **0.049160**	0.419378 **0.171200**	0.418527 **0.031990**	0.418451 **0.050185**	0.418451 **0.050221**	0.418455 **0.049160**
-60	0.418196	0.418550 **0.084697**	0.418148 **0.011523**	0.418175 **0.004981**	0.418189 **0.001653**	0.418189 **0.001668**	0.418550 **0.084697**	0.418148 **0.011523**	0.418175 **0.004981**	0.418189 **0.001653**	0.418189 **0.001668**
-45	0.417501	0.417420 **0.019430**	0.417548 **0.011114**	0.417623 **0.029153**	0.417623 **0.029124**	0.417621 **0.028724**	0.417420 **0.019430**	0.417548 **0.011114**	0.417623 **0.029153**	0.417623 **0.029124**	0.417621 **0.028724**
-30	0.416689	0.416290 **0.095896**	0.416772 **0.019826**	0.416779 **0.021585**	0.416773 **0.019940**	0.416774 **0.020334**	0.416290 **0.095896**	0.416772 **0.019826**	0.416779 **0.021585**	0.416773 **0.019940**	0.416774 **0.020334**
-15	0.41591	0.415462 **0.107720**	0.415929 **0.004435**	0.415860 **0.012189**	0.415871 **0.009559**	0.415869 **0.010089**	0.415462 **0.107720**	0.415929 **0.004435**	0.415860 **0.012189**	0.415871 **0.009559**	0.415869 **0.010089**
0	0.415343	0.415160 **0.044150**	0.415247 **0.023155**	0.415235 **0.026054**	0.415237 **0.025623**	0.415238 **0.025351**	0.415160 **0.044150**	0.415247 **0.023155**	0.415235 **0.026054**	0.415237 **0.025623**	0.415238 **0.025351**
15	0.415188	0.415462 **0.066146**	0.415063 **0.030141**	0.415161 **0.006575**	0.415146 **0.010006**	0.415147 **0.009899**	0.415462 **0.066146**	0.415063 **0.030141**	0.415161 **0.006575**	0.415146 **0.010006**	0.415147 **0.009899**
30	0.415652	0.416290 **0.153359**	0.415673 **0.004997**	0.415727 **0.017860**	0.415738 **0.020578**	0.415735 **0.019954**	0.416290 **0.153359**	0.415673 **0.004997**	0.415727 **0.017860**	0.415738 **0.020578**	0.415735 **0.019954**
45	0.416902	0.417420 **0.124406**	0.417104 **0.048496**	0.417010 **0.025921**	0.417017 **0.027772**	0.417020 **0.028315**	0.417420 **0.124406**	0.417104 **0.048496**	0.417010 **0.025921**	0.417017 **0.027772**	0.417020 **0.028315**
60	0.418902	0.418550 **0.083907**	0.418964 **0.014755**	0.418920 **0.004205**	0.418899 **0.000811**	0.418895 **0.001757**	0.418550 **0.083907**	0.418964 **0.014755**	0.418920 **0.004205**	0.418899 **0.000811**	0.418895 **0.001757**
75	0.421072	0.419378 **0.402305**	0.420547 **0.124646**	0.420801 **0.064223**	0.420841 **0.054874**	0.420848 **0.053288**	0.419378 **0.402305**	0.420547 **0.124646**	0.420801 **0.064223**	0.420841 **0.054874**	0.420848 **0.053288**
90	0.422088	0.419681 **0.570341**	0.421169 **0.217702**	0.421607 **0.113960**	0.421710 **0.089621**	0.421740 **0.082405**	0.419681 **0.570341**	0.421169 **0.217702**	0.421607 **0.113960**	0.421710 **0.089621**	0.421740 **0.082405**

Note : Results shown in bold face are the values of percentage error of the corresponding partial sums relative to r_{exp}

Table 2.2(b) : Values of r_{exp} for given values θ, ϕ and percentage error in corresponding values of S_i $(i = 1,3,5,7,9)$ on the surface of primary components of a tidally distorted model for $(\psi_s^* = 5.0,\ n = 0.0, q = 0.1)$

	Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star $(\phi = 0^0)$						Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star $(\phi = 90^0)$				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	S_9
-9	0.204226	0.204255 **0.014294**	0.204223 **0.001474**	0.204222 **0.001934**	0.204222 **0.001941**	0.204222 **0.001941**	0.204255 **0.014294**	0.204223 **0.001474**	0.204222 **0.001934**	0.204222 **0.001941**	0.204222 **0.001941**
-75	0.204214	0.204238 **0.011770**	0.204211 **0.001087**	0.204211 **0.001321**	0.204211 **0.001313**	0.204211 **0.001313**	0.204238 **0.011770**	0.204211 **0.001087**	0.204211 **0.001321**	0.204211 **0.001313**	0.204211 **0.001313**
-60	0.204179	0.204190 **0.005393**	0.204179 **0.000204**	0.204179 **0.000029**	0.204179 **0.000015**	0.204179 **0.000015**	0.204190 **0.005393**	0.204179 **0.000204**	0.204179 **0.000029**	0.204179 **0.000015**	0.204179 **0.000015**
-45	0.204129	0.204125 **0.001883**	0.204130 **0.000540**	0.204131 **0.000810**	0.204130 **0.000803**	0.204130 **0.000803**	0.204125 **0.001883**	0.204130 **0.000540**	0.204131 **0.000810**	0.204130 **0.000803**	0.20413 **0.000803**
-30	0.204073	0.204060 **0.006542**	0.204075 **0.000591**	0.204074 **0.000577**	0.204074 **0.000570**	0.204074 **0.000570**	0.204060 **0.006542**	0.204075 **0.000591**	0.204074 **0.000577**	0.204074 **0.000570**	0.204074 **0.00057**
-15	0.204025	0.204012 **0.006288**	0.204025 **0.000080**	0.204025 **0.000299**	0.204025 **0.000292**	0.204025 **0.000292**	0.204012 **0.006288**	0.204025 **0.000080**	0.204025 **0.000299**	0.204025 **0.000292**	0.204025 **0.000292**
0	0.203998	0.203995 **0.001351**	0.203996 **0.000760**	0.203996 **0.000752**	0.203996 **0.000752**	0.203996 **0.000752**	0.203995 **0.001351**	0.203996 **0.000760**	0.203996 **0.000752**	0.203996 **0.000752**	0.203996 **0.000752**
15	0.204002	0.204012 **0.005172**	0.204001 **0.000584**	0.204001 **0.000278**	0.204001 **0.000285**	0.204001 **0.000285**	0.204012 **0.005172**	0.204001 **0.000584**	0.204001 **0.000278**	0.204001 **0.000285**	0.204001 **0.000285**
30	0.204043	0.204060 **0.008479**	0.204044 **0.000438**	0.204044 **0.000555**	0.204044 **0.000562**	0.204044 **0.000562**	0.204060 **0.008479**	0.204044 **0.000438**	0.204044 **0.000555**	0.204044 **0.000562**	0.204044 **0.000562**
45	0.204115	0.204125 **0.004789**	0.204117 **0.001080**	0.204117 **0.000796**	0.204117 **0.000796**	0.204117 **0.000796**	0.204125 **0.004789**	0.204117 **0.001080**	0.204117 **0.000796**	0.204117 **0.000796**	0.204117 **0.000796**
60	0.204201	0.204190 **0.005553**	0.204202 **0.000117**	0.204201 **0.000007**	0.204201 **0.00000**	0.204201 **0.00000**	0.204190 **0.005553**	0.204202 **0.000117**	0.204201 **0.000007**	0.204201 **0.00000**	0.204201 **0.00000**
75	0.204272	0.204238 **0.017048**	0.204268 **0.002035**	0.204270 **0.001335**	0.204270 **0.001306**	0.204270 **0.001306**	0.204238 **0.017048**	0.204268 **0.002035**	0.204270 **0.001335**	0.204270 **0.001306**	0.204270 **0.001306**
90	0.204300	0.204255 **0.022144**	0.204294 **0.003246**	0.204296 **0.002057**	0.204296 **0.001991**	0.204296 **0.001984**	0.204255 **0.022144**	0.204294 **0.003246**	0.204296 **0.002057**	0.204296 **0.001991**	0.204296 **0.001984**

Note : Results shown in parenthesis are the values of percentage error of the corresponding partial sums relative to r_{exp}

Table 2.3(b) : Values of r_{exp} for given values θ, ϕ and percentage error in corresponding values of S_i $(i = 1,3,5,7,9)$ on the surface of primary components of a tidally distorted model for $(\psi_s^* = 10.0,\ n = 0.0, q = 0.1)$

	Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star $(\phi = 0^0)$						Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star $(\phi = 90^0)$				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	S_9
-90	0.101020	0.101021 **0.000937**	0.10102 **0.000059**	0.101019 **0.000066**	0.101019 **0.000066**	0.101019 **0.000066**	0.101021 **0.000937**	0.10102 **0.000059**	0.101019 **0.000066**	0.101019 **0.000066**	0.101019 **0.000066**
-75	0.101019	0.101019 **0.000767**	0.101019 **0.000037**	0.101019 **0.000037**	0.101019 **0.000037**	0.101019 **0.000037**	0.101019 **0.000767**	0.101019 **0.000037**	0.101019 **0.000037**	0.101019 **0.000037**	0.101019 **0.000037**
-60	0.101016	0.101017 **0.000332**	0.101016 **0.000007**	0.101016 **0.000007**	0.101016 **0.000007**	0.101016 **0.000007**	0.101017 **0.000332**	0.101016 **0.000007**	0.101016 **0.000007**	0.101016 **0.000007**	0.101016 **0.000007**
-45	0.101013	0.101013 **0.000125**	0.101013 **0.000037**	0.101013 **0.000044**	0.101013 **0.000044**	0.101013 **0.000044**	0.101013 **0.000125**	0.101013 **0.000037**	0.101013 **0.000044**	0.101013 **0.000044**	0.101013 **0.000044**
-30	0.101009	0.101009 **0.000435**	0.101009 **0.000007**	0.101009 **0.000007**	0.101009 **0.000007**	0.101009 **0.000007**	0.101009 **0.000435**	0.101009 **0.000007**	0.101009 **0.000007**	0.101009 **0.000007**	0.101009 **0.000007**
-15	0.101006	0.101006 **0.000406**	0.101006 **0.000037**	0.101006 **0.000037**	0.101006 **0.000037**	0.101006 **0.000037**	0.101006 **0.000406**	0.101006 **0.000037**	0.101006 **0.000037**	0.101006 **0.000037**	0.101006 **0.000037**
0	0.101005	0.101005 **0.000044**	0.101005 **0.000030**	0.101005 **0.000030**	0.101005 **0.000030**	0.101005 **0.000030**	0.101005 **0.000044**	0.101005 **0.000030**	0.101005 **0.000030**	0.101005 **0.000030**	0.101005 **0.000030**
15	0.101006	0.101006 **0.000295**	0.101006 **0.000059**	0.101006 **0.000052**	0.101006 **0.000052**	0.101006 **0.000052**	0.101006 **0.000295**	0.101006 **0.000059**	0.101006 **0.000052**	0.101006 **0.000052**	0.101006 **0.000052**
30	0.101008	0.101009 **0.000465**	0.101008 **0.000007**	0.101008 **0.000007**	0.101008 **0.000007**	0.101008 **0.000007**	0.101009 **0.000465**	0.101008 **0.000007**	0.101008 **0.000007**	0.101008 **0.000007**	0.101008 **0.000007**
45	0.101012	0.101013 **0.000251**	0.101012 **0.000044**	0.101012 **0.000037**	0.101012 **0.000037**	0.101012 **0.000037**	0.101013 **0.000251**	0.101012 **0.000044**	0.101012 **0.000037**	0.101012 **0.000037**	0.101012 **0.000037**
60	0.101017	0.101017 **0.000339**	0.101017 **0.00000**	0.101017 **0.00000**	0.101017 **0.00000**	0.101017 **0.00000**	0.101017 **0.000339**	0.101017 **0.00000**	0.101017 **0.00000**	0.101017 **0.00000**	0.101017 **0.00000**
75	0.101020	0.101019 **0.000915**	0.101020 **0.000052**	0.101020 **0.000044**	0.101020 **0.000044**	0.101020 **0.000044**	0.101019 **0.000915**	0.101020 **0.000052**	0.101020 **0.000044**	0.101020 **0.000044**	0.101020 **0.000044**
90	0.101022	0.101021 **0.001165**	0.101022 **0.000081**	0.101022 **0.000066**	0.101022 **0.000066**	0.101022 **0.000066**	0.101021 **0.001165**	0.101022 **0.000081**	0.101022 **0.000066**	0.101022 **0.000066**	0.101022 **0.000066**

Note : Results shown in bold face are the values of percentage error of the corresponding partial sums relative to r_{exp}

Table 2.1(c) : Values of r_{exp} for given values θ, ϕ and percentage errors in corresponding values of $S_i\ (i = 1,3,5,7,9)$ on the surface of primary components of a nonsynchronously model for $(\psi_s^* = 2.5,\ n = 0.1, q = 0.1)$

	Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star $(\phi = 0^0)$						Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star $(\phi = 90^0)$				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	[illegible]
-90	0.422000	0.422695 **0.164683**	0.421672 **0.077804**	0.421679 **0.076081**	0.421664 **0.079569**	0.421668 **0.078694**	0.422695 **0.164683**	0.421672 **0.077804**	0.421679 **0.076081**	0.421664 **0.079569**	421668 **078694**
-75	0.421613	0.422190 **0.136856**	0.421339 **0.064911**	0.421382 **0.054747**	0.421383 **0.054464**	0.421390 **0.052831**	0.422190 **0.136856**	0.421339 **0.064911**	0.421382 **0.054747**	0.421383 **0.054464**	421390 **052831**
-60	0.420545	0.420811 **0.063326**	0.420409 **0.032357**	0.420519 **0.006208**	0.420535 **0.002282**	0.420533 **0.002735**	0.420811 **0.063326**	0.420409 **0.032357**	0.420519 **0.006208**	0.420535 **0.002282**	420533 **002735**
-45	0.419044	0.418927 **0.027758**	0.419055 **0.002674**	0.419168 **0.029792**	0.419168 **0.029749**	0.419166 **0.029237**	0.418927 **0.027758**	0.419055 **0.002674**	0.419168 **0.029792**	0.419168 **0.029749**	419166 **029237**
-30	0.417448	0.417043 **0.096879**	0.417526 **0.018633**	0.417540 **0.022053**	0.417532 **0.020104**	0.417534 **0.020625**	0.417043 **0.096879**	0.417526 **0.018633**	0.417540 **0.022053**	0.417532 **0.020104**	417534 **020625**
-15	0.416111	0.415664 **0.107253**	0.416131 **0.004849**	0.416060 **0.012183**	0.416071 **0.009597**	0.416069 **0.010120**	0.415664 **0.107253**	0.416131 **0.004849**	0.416060 **0.012183**	0.416071 **0.009597**	416069 **010120**
0	0.415343	0.415160 **0.044150**	0.415247 **0.023155**	0.415235 **0.026054**	0.415237 **0.025623**	0.415238 **0.025351**	0.415160 **0.044150**	0.415247 **0.023155**	0.415235 **0.026054**	0.415237 **0.025623**	415238 **025351**
15	0.415386	0.415664 **0.067083**	0.415265 **0.029158**	0.415358 **0.006730**	0.415344 **0.010052**	0.415344 **0.009944**	0.415664 **0.067083**	0.415265 **0.029158**	0.415358 **0.006730**	0.415344 **0.010052**	415344 **009944**
30	0.416398	0.417043 **0.155039**	0.416427 **0.006942**	0.416473 **0.018115**	0.416484 **0.020670**	0.416482 **0.020176**	0.417043 **0.155039**	0.416427 **0.006942**	0.416473 **0.018115**	0.416484 **0.020670**	416482 **020176**
45	0.418427	0.418927 **0.119586**	0.418611 **0.043953**	0.418544 **0.027934**	0.418546 **0.028497**	0.418548 **0.028924**	0.418927 **0.119586**	0.418611 **0.043953**	0.418544 **0.027934**	0.418546 **0.028497**	418548 **028924**
60	0.421277	0.420811 **0.110529**	0.421224 **0.012422**	0.421294 **0.004054**	0.421271 **0.001443**	0.421265 **0.002823**	0.420811 **0.110529**	0.421224 **0.012422**	0.421294 **0.004054**	0.421271 **0.001443**	421265 **002823**
75	0.424154	0.422190 **0.462998**	0.423359 **0.187356**	0.423828 **0.076819**	0.423883 **0.063806**	0.423892 **0.061656**	0.422190 **0.462998**	0.423359 **0.187356**	0.423828 **0.076819**	0.423883 **0.063806**	423892 **061656**
90	0.425461	0.422695 **0.650122**	0.424183 **0.300278**	0.424881 **0.136291**	0.425011 **0.105645**	0.425048 **0.096938**	0.422695 **0.650122**	0.424183 **0.300278**	0.424881 **0.136291**	0.425011 **0.105645**	425048 **096938**

Note : Results shown in bold face are the values of percentage error of the corresponding partial sums relative to r_{exp}

Table 2.2(c) : Values of r_{exp} for given values θ, ϕ and percentage errors in corresponding values of $S_i\,(i=1,3,5,7,9)$ on the surface of primary components of a nonsynchronously model for $(\psi_s^* = 5.0,\ n = 0.1, q = 0.1)$

	Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star $(\phi = 0^0)$						Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star $(\phi = 90^0)$				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	S_9
-90	0.204401	0.204429 **0.013676**	0.204396 **0.002078**	0.204397 **0.00199**	0.204397 **0.001998**	0.204397 **0.001998**	0.204429 **0.013676**	0.204396 **0.002078**	0.204397 **0.00199**	0.204397 **0.001998**	0.204397 **0.001998**
-75	0.204377	0.204400 **0.011250**	0.204373 **0.001597**	0.204374 **0.001356**	0.204374 **0.001349**	0.204374 **0.001349**	0.204400 **0.011250**	0.204373 **0.001597**	0.204374 **0.001356**	0.204374 **0.001349**	0.204374 **0.001349**
-60	0.204310	0.204320 **0.005069**	0.204309 **0.000525**	0.204310 **0.000058**	0.204310 **0.000044**	0.204310 **0.000044**	0.204320 **0.005069**	0.204309 **0.000525**	0.204310 **0.000058**	0.204310 **0.000044**	0.204310 **0.000044**
-45	0.204216	0.204212 **0.001970**	0.204217 **0.000452**	0.204217 **0.000839**	0.204217 **0.000832**	0.204217 **0.000832**	0.204212 **0.001970**	0.204217 **0.000452**	0.204217 **0.000839**	0.204217 **0.000832**	0.204217 **0.000832**
-30	0.204117	0.204103 **0.006541**	0.204118 **0.000591**	0.204118 **0.000584**	0.204118 **0.000577**	0.204118 **0.000577**	0.204103 **0.006541**	0.204118 **0.000591**	0.204118 **0.000584**	0.204118 **0.000577**	0.204118 **0.000577**
-15	0.204037	0.204024 **0.006273**	0.204037 **0.000066**	0.204036 **0.000299**	0.204036 **0.000292**	0.204036 **0.000292**	0.204024 **0.006273**	0.204037 **0.000066**	0.204036 **0.000299**	0.204036 **0.000292**	0.204036 **0.000292**
0	0.203998	0.203995 **0.001351**	0.203996 **0.000760**	0.203996 **0.000752**	0.203996 **0.000752**	0.203996 **0.000752**	0.203995 **0.001351**	0.203996 **0.000760**	0.203996 **0.000752**	0.203996 **0.000752**	0.203996 **0.000752**
15	0.204013	0.204024 **0.005186**	0.204012 **0.000570**	0.204013 **0.000285**	0.204013 **0.000292**	0.204013 **0.000292**	0.204024 **0.005186**	0.204012 **0.000570**	0.204013 **0.000285**	0.204013 **0.000292**	0.204013 **0.000292**
30	0.204086	0.204103 **0.008535**	0.204087 **0.000496**	0.204087 **0.000606**	0.204087 **0.000613**	0.204087 **0.000613**	0.204103 **0.008535**	0.204087 **0.000496**	0.204087 **0.000606**	0.204087 **0.000613**	0.204087 **0.000613**
45	0.204202	0.204212 **0.004721**	0.204204 **0.001014**	0.204204 **0.000832**	0.204204 **0.000832**	0.204204 **0.000832**	0.204212 **0.004721**	0.204204 **0.001014**	0.204204 **0.000832**	0.204204 **0.000832**	0.204204 **0.000832**
60	0.204332	0.204320 **0.005929**	0.204332 **0.000263**	0.204332 **0.000022**	0.204332 **0.000036**	0.204332 **0.000036**	0.204320 **0.005929**	0.204332 **0.000263**	0.204332 **0.000022**	0.204332 **0.000036**	0.204332 **0.000036**
75	0.204436	0.204400 **0.017734**	0.204430 **0.002733**	0.204433 **0.001399**	0.204433 **0.001363**	0.204433 **0.001363**	0.204400 **0.017734**	0.204430 **0.002733**	0.204433 **0.001399**	0.204433 **0.001363**	0.204433 **0.001363**
90	0.204475	0.204429 **0.022912**	0.204467 **0.004030**	0.204471 **0.002092**	0.204471 **0.002004**	0.204471 **0.001997**	0.204429 **0.022912**	0.204467 **0.004030**	0.204471 **0.002092**	0.204471 **0.002004**	0.204471 **0.001997**

Note : Results shown in bold face are the values of percentage error of the corresponding partial sums relative to r_{exp}

Table 2.3(c) : Values of r_{exp} for given values θ, ϕ and percentage errors in corresponding values of S_i $(i = 1, 3, 5, 7, 9)$ on the surface of primary components of a nonsynchronously model for $(\psi_s^* = 10.0,\ n = 0.1, q = 0.1)$

	Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star $(\phi = 0^0)$						Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star $(\phi = 90^0)$				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	S_9
-90	0.101030	0.101031 **0.000929**	0.101030 **0.000066**	0.101030 **0.000066**	0.101030 **0.000066**	0.101030 **0.000066**	0.101031 **0.000929**	0.101030 **0.000066**	0.101030 **0.000066**	0.101030 **0.000066**	0.101030 **0.000066**
-75	0.101028	0.101029 **0.000760**	0.101028 **0.000044**	0.101028 **0.000037**	0.101028 **0.000037**	0.101028 **0.000037**	0.101029 **0.000760**	0.101028 **0.000044**	0.101028 **0.000037**	0.101028 **0.000037**	0.101028 **0.000037**
-60	0.101024	0.101024 **0.000325**	0.101024 **0.000015**	0.101024 **0.000007**	0.101024 **0.000007**	0.101024 **0.000007**	0.101024 **0.000325**	0.101024 **0.000015**	0.101024 **0.000007**	0.101024 **0.000007**	0.101024 **0.000007**
-45	0.101018	0.101018 **0.000148**	0.101018 **0.000015**	0.101018 **0.000022**	0.101018 **0.000022**	0.101018 **0.000022**	0.101018 **0.000148**	0.101018 **0.000015**	0.101018 **0.000022**	0.101018 **0.000022**	0.101018 **0.000022**
-30	0.101012	0.101011 **0.000406**	0.101012 **0.000037**	0.101012 **0.000037**	0.101012 **0.000037**	0.101012 **0.000037**	0.101011 **0.000406**	0.101012 **0.000037**	0.101012 **0.000037**	0.101012 **0.000037**	0.101012 **0.000037**
-15	0.101007	0.101007 **0.000406**	0.101007 **0.000037**	0.101007 **0.000037**	0.101007 **0.000037**	0.101007 **0.000037**	0.101007 **0.000406**	0.101007 **0.000037**	0.101007 **0.000037**	0.101007 **0.000037**	0.101007 **0.000037**
0	0.101005	0.101005 **0.000044**	0.101005 **0.000030**	0.101005 **0.000030**	0.101005 **0.000030**	0.101005 **0.000030**	0.101005 **0.000044**	0.101005 **0.000030**	0.101005 **0.000030**	0.101005 **0.000030**	0.101005 **0.000030**
15	0.101006	0.101007 **0.000295**	0.101006 **0.000059**	0.101006 **0.000052**	0.101006 **0.000052**	0.101006 **0.000052**	0.101007 **0.000295**	0.101006 **0.000059**	0.101006 **0.000052**	0.101006 **0.000052**	0.101006 **0.000052**
30	0.101011	0.101011 **0.000494**	0.101011 **0.000022**	0.101011 **0.000022**	0.101011 **0.000022**	0.101011 **0.000022**	0.101011 **0.000494**	0.101011 **0.000022**	0.101011 **0.000022**	0.101011 **0.000022**	0.101011 **0.000022**
45	0.101018	0.101018 **0.000229**	0.101018 **0.000022**	0.101018 **0.000022**	0.101018 **0.000022**	0.101018 **0.000022**	0.101018 **0.000229**	0.101018 **0.000022**	0.101018 **0.000022**	0.101018 **0.000022**	0.101018 **0.000022**
60	0.101025	0.101024 **0.000347**	0.101025 **0.000007**	0.101025 **0.000007**	0.101025 **0.000007**	0.101025 **0.000007**	0.101024 **0.000347**	0.101025 **0.000007**	0.101025 **0.000007**	0.101025 **0.000007**	0.101025 **0.000007**
75	0.10103	0.101029 **0.000929**	0.101030 **0.000066**	0.101030 **0.000052**	0.101030 **0.000052**	0.101030 **0.000052**	0.101029 **0.000929**	0.101030 **0.000066**	0.101030 **0.000052**	0.101030 **0.000052**	0.101030 **0.000052**
90	0.101032	0.101031 **0.001173**	0.101032 **0.000088**	0.101032 **0.000066**	0.101032 **0.000066**	0.101032 **0.000066**	0.101031 **0.001173**	0.101032 **0.000088**	0.101032 **0.000066**	0.101032 **0.000066**	0.101032 **0.000066**

Note : Results shown in bold face are the values of percentage error of the corresponding partial sums relative to r_{exp}

Table 2.4(a) : Values of r_{exp} for given values θ, ϕ and percentage errors in corresponding values of $S_i\,(i=1,3,5,7,9)$ on the surface of primary components of a synchronously model for $(\psi_s^* = 2.5,\ n = 0.6, q = 0.2)$

	Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star $(\phi = 0^0)$						Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star $(\phi = 90^0)$				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	S_9
-90	0.467488	0.463370 **0.880877**	0.460863 **1.417128**	0.46474 **0.587838**	0.464696 **0.597317**	0.464831 **0.568470**	0.463370 **0.880877**	0.460863 **1.417128**	0.46474 **0.587838**	0.464696 **0.597317**	0.464831 **0.568470**
-75	0.464738	0.461216 **0.757817**	0.459128 **1.207040**	0.462666 **0.445742**	0.462704 **0.437636**	0.462775 **0.422278**	0.461216 **0.757817**	0.459128 **1.20704**	0.462666 **0.445742**	0.462704 **0.437636**	0.462775 **0.422278**
-60	0.457635	0.455330 **0.503761**	0.454335 **0.721198**	0.456950 **0.149795**	0.457049 **0.128083**	0.457011 **0.136392**	0.455330 **0.503761**	0.454335 **0.721198**	0.456950 **0.149795**	0.457049 **0.128083**	0.457011 **0.136392**
-45	0.448726	0.447290 **0.320109**	0.447595 **0.252053**	0.448968 **0.053856**	0.448969 **0.054142**	0.448954 **0.050794**	0.447290 **0.320109**	0.447595 **0.252053**	0.448968 **0.053856**	0.448969 **0.054142**	0.448954 **0.050794**
-30	0.440448	0.439249 **0.272035**	0.440435 **0.002788**	0.440701 **0.057433**	0.440665 **0.049401**	0.440681 **0.053001**	0.439249 **0.272035**	0.440435 **0.002788**	0.440701 **0.057433**	0.440665 **0.049401**	0.440681 **0.053001**
-15	0.43446	0.433364 **0.252263**	0.434521 **0.014234**	0.43432 **0.032007**	0.434359 **0.023206**	0.434348 **0.025655**	0.433364 **0.252263**	0.434521 **0.014234**	0.434320 **0.032007**	0.434359 **0.023206**	0.434348 **0.025655**
0	0.431708	0.431209 **0.115576**	0.431434 **0.063421**	0.431443 **0.061502**	0.431441 **0.061875**	0.431448 **0.060218**	0.431209 **0.115576**	0.431434 **0.063421**	0.431443 **0.061502**	0.431441 **0.061875**	0.431448 **0.060218**
15	0.432696	0.433364 **0.154330**	0.432378 **0.073442**	0.432629 **0.015504**	0.432591 **0.024293**	0.432591 **0.024251**	0.433364 **0.154330**	0.432378 **0.073442**	0.432629 **0.015504**	0.432591 **0.024293**	0.432591 **0.024251**
30	0.437696	0.439249 **0.354846**	0.437716 **0.004596**	0.437892 **0.044632**	0.437925 **0.052333**	0.437924 **0.051993**	0.439249 **0.354846**	0.437716 **0.004596**	0.437892 **0.044632**	0.437925 **0.052333**	0.437924 **0.051993**
45	0.446856	0.447290 **0.096999**	0.446496 **0.080519**	0.447207 **0.078405**	0.447127 **0.060678**	0.447128 **0.060711**	0.447290 **0.096999**	0.446496 **0.080519**	0.447207 **0.078405**	0.447127 **0.060678**	0.447128 **0.060711**
60	0.459993	0.455330 **1.013632**	0.456353 **0.791193**	0.459374 **0.134592**	0.459271 **0.156918**	0.459212 **0.169597**	0.455330 **1.013632**	0.456353 **0.791193**	0.459374 **0.134592**	0.459271 **0.156918**	0.459212 **0.169597**
75	0.475051	0.461216 **2.912280**	0.464126 **2.299722**	0.470434 **0.971929**	0.470920 **0.869533**	0.471013 **0.849928**	0.461216 **2.912280**	0.464126 **2.299722**	0.470434 **0.971929**	0.470920 **0.869533**	0.471013 **0.849928**
90	0.483193	0.463370 **4.102484**	0.467078 **3.335125**	0.474990 **1.697677**	0.475940 **1.501017**	0.476234 **1.440326**	0.463370 **4.102484**	0.467078 **3.335125**	0.474990 **1.697677**	0.475940 **1.501017**	0.476234 **1.440326**

Note : Results shown in bold face are the values of percentage error of the corresponding partial sums relative to r_{exp}

Table 2.4(b) : Values of r_{exp} for given values θ, ϕ and percentage errors in corresponding values of $S_i\ (i = 1,3,5,7,9)$ on the surface of primary components of a synchronously model for $(\psi_s^* = 5.0,\ n = 0.6, q = 0.2)$

	Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star ($\phi = 0^0$)						Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star ($\phi = 90^0$)				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	S_9
-90	0.209806	0.209840 **0.016620**	0.209769 **0.017323**	0.209795 **0.004915**	0.209795 **0.004830**	0.209795 **0.004808**	0.209840 **0.016620**	0.209769 **0.017323**	0.209795 **0.004915**	0.209795 **0.004830**	0.209795 **0.004808**
-75	0.209699	0.209727 **0.013146**	0.209669 **0.014581**	0.209692 **0.003411**	0.209692 **0.003318**	0.209692 **0.003304**	0.209727 **0.013146**	0.209669 **0.014581**	0.209692 **0.003411**	0.209692 **0.003318**	0.209692 **0.003304**
-60	0.209408	0.209417 **0.003999**	0.209391 **0.008098**	0.209408 **0.000235**	0.209408 **0.000185**	0.209408 **0.000192**	0.209417 **0.003999**	0.209391 **0.008098**	0.209408 **0.000235**	0.209408 **0.000185**	0.209408 **0.000192**
-45	0.209008	0.208993 **0.007101**	0.209004 **0.001875**	0.209011 **0.001832**	0.209011 **0.001811**	0.209011 **0.001811**	0.208993 **0.007101**	0.209004 **0.001875**	0.209011 **0.001832**	0.209011 **0.001811**	0.209011 **0.001811**
-30	0.208599	0.208569 **0.014644**	0.208601 **0.000814**	0.208602 **0.001293**	0.208602 **0.001272**	0.208602 **0.001272**	0.208569 **0.014644**	0.208601 **0.000814**	0.208602 **0.001293**	0.208602 **0.001272**	0.208602 **0.001272**
-15	0.208287	0.208259 **0.013643**	0.208287 **0.000143**	0.208286 **0.000687**	0.208286 **0.000665**	0.208286 **0.000665**	0.208259 **0.013643**	0.208287 **0.000143**	0.208286 **0.000687**	0.208286 **0.000665**	0.208286 **0.000665**
0	0.208151	0.208145 **0.003078**	0.208148 **0.001768**	0.208148 **0.001632**	0.208148 **0.001632**	0.208148 **0.001632**	0.208145 **0.003078**	0.208148 **0.001768**	0.208148 **0.001632**	0.208148 **0.001632**	0.208148 **0.001632**
15	0.208235	0.208259 **0.011178**	0.208232 **0.001324**	0.208234 **0.000673**	0.208234 **0.000694**	0.208234 **0.000694**	0.208259 **0.011178**	0.208232 **0.001324**	0.208234 **0.000673**	0.208234 **0.000694**	0.208234 **0.000694**
30	0.208531	0.208569 **0.018215**	0.208532 **0.000736**	0.208533 **0.001243**	0.208533 **0.001258**	0.208533 **0.001258**	0.208569 **0.018215**	0.208532 **0.000736**	0.208533 **0.001243**	0.208533 **0.001258**	0.208533 **0.001258**
45	0.208977	0.208993 **0.007715**	0.208976 **0.000335**	0.208980 **0.001847**	0.208980 **0.001804**	0.208980 **0.001804**	0.208993 **0.007715**	0.208976 **0.000335**	0.208980 **0.001847**	0.208980 **0.001804**	0.208980 **0.001804**
60	0.209459	0.209417 **0.020425**	0.209442 **0.008174**	0.209459 **0.000157**	0.209459 **0.000199**	0.209459 **0.000206**	0.209417 **0.020425**	0.209442 **0.008174**	0.209459 **0.000157**	0.209459 **0.000199**	0.209459 **0.000206**
75	0.209836	0.209727 **0.052046**	0.209795 **0.019593**	0.209828 **0.003835**	0.209828 **0.003593**	0.209828 **0.003579**	0.209727 **0.052046**	0.209795 **0.019593**	0.209828 **0.003835**	0.209828 **0.003593**	0.209828 **0.003579**
90	0.209979	0.209840 **0.065962**	0.209926 **0.025122**	0.209967 **0.005698**	0.209968 **0.005237**	0.209968 **0.005195**	0.209840 **0.065962**	0.209926 **0.025122**	0.209967 **0.005698**	0.209968 **0.005237**	0.209968 **0.005195**

Note : Results shown in bold face are the values of percentage error of the corresponding partial sums relative to r_{exp}

Table 2.4(c) : Values of r_{exp} for given values θ, ϕ and percentage errors in corresponding values of $S_i\ (i=1,3,5,7,9)$ on the surface of primary components of a synchronously model for $(\psi_s^* = 10.0,\ n = 0.6, q = 0.2)$

	Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star $(\phi = 0^0)$						Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star $(\phi = 90^0)$				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	S_9
-90	0.102126	0.102128 **0.001802**	0.102125 **0.000270**	0.102126 **0.000080**	0.102126 **0.000080**	0.102126 **0.000080**	0.102128 **0.001802**	0.102125 **0.000270**	0.102126 **0.000080**	0.102126 **0.000080**	0.102126 **0.000080**
-75	0.102120	0.102121 **0.001466**	0.102119 **0.000212**	0.102119 **0.000044**	0.102119 **0.000044**	0.102119 **0.000044**	0.102121 **0.001466**	0.102119 **0.000212**	0.102119 **0.000044**	0.102119 **0.000044**	0.102119 **0.000044**
-60	0.102103	0.102103 **0.000620**	0.102102 **0.000080**	0.102103 **0.000029**	0.102103 **0.000029**	0.102103 **0.000029**	0.102103 **0.000620**	0.102102 **0.000080**	0.102103 **0.000029**	0.102103 **0.000029**	0.102103 **0.000029**
-45	0.102079	0.102079 **0.000321**	0.102079 **0.000022**	0.102079 **0.000073**	0.102079 **0.000073**	0.102079 **0.000073**	0.102079 **0.000321**	0.102079 **0.000022**	0.102079 **0.000073**	0.102079 **0.000073**	0.102079 **0.000073**
-30	0.102055	0.102054 **0.000876**	0.102055 **0.000044**	0.102055 **0.000044**	0.102055 **0.000044**	0.102055 **0.000044**	0.102054 **0.000876**	0.102055 **0.000044**	0.102055 **0.000044**	0.102055 **0.000044**	0.102055 **0.000044**
-15	0.102037	0.102037 **0.000840**	0.102037 **0.000080**	0.102037 **0.000038**	0.102037 **0.000088**	0.102037 **0.000088**	0.102037 **0.000840**	0.102037 **0.000080**	0.102037 **0.000088**	0.102037 **0.000088**	0.102037 **0.000088**
0	0.102030	0.102030 **0.000088**	0.102030 **0.000051**	0.102030 **0.000051**	0.102030 **0.000051**	0.102030 **0.000051**	0.102030 **0.000088**	0.102030 **0.000051**	0.102030 **0.000051**	0.102030 **0.000051**	0.102030 **0.000051**
15	0.102036	0.102037 **0.000613**	0.102036 **0.000117**	0.102036 **0.000110**	0.102036 **0.000110**	0.102036 **0.000110**	0.102037 **0.000613**	0.102036 **0.000117**	0.102036 **0.000110**	0.102036 **0.000110**	0.102036 **0.000110**
30	0.102053	0.102054 **0.001015**	0.102053 **0.000037**	0.102053 **0.000044**	0.102053 **0.000044**	0.102053 **0.000044**	0.102054 **0.001015**	0.102053 **0.000037**	0.102053 **0.000044**	0.102053 **0.000044**	0.102053 **0.000044**
45	0.102078	0.102079 **0.000460**	0.102078 **0.000044**	0.102078 **0.000080**	0.102078 **0.000080**	0.102078 **0.000080**	0.102079 **0.000460**	0.102078 **0.000044**	0.102078 **0.000080**	0.102078 **0.000080**	0.102078 **0.000080**
60	0.102104	0.102103 **0.000788**	0.102104 **0.000088**	0.102104 **0.000022**	0.102104 **0.000022**	0.102104 **0.000020**	0.102103 **0.000788**	0.102104 **0.000088**	0.102104 **0.000022**	0.102104 **0.000022**	0.102104 **0.000022**
75	0.102123	0.102121 **0.002057**	0.102123 **0.000248**	0.102123 **0.000044**	0.102123 **0.000044**	0.102123 **0.000040**	0.102121 **0.002057**	0.102123 **0.000248**	0.102123 **0.000044**	0.102123 **0.000044**	0.102123 **0.000044**
90	0.102130	0.102128 **0.002597**	0.102130 **0.000336**	0.102130 **0.000080**	0.102130 **0.000080**	0.102130 **0.000080**	0.102128 **0.002597**	0.102130 **0.000336**	0.102130 **0.000080**	0.102130 **0.000080**	0.102130 **0.000080**

Note : Results shown in bold face are the values of percentage error of the corresponding partial sums relative to r_{exp}

Table 2.5 : Values of r_{exp} for given values θ, ϕ and percentage errors in corresponding values of S_i $(i = 1,3,5,7,9)$ on the surface of primary components of a synchronously model for $(\psi_s^* = 2.2327,\ n = 0.6, q = 0.2)$

	Section by plane through the axis of rotation and the line joining the mass centre of the primary and companion star ($\phi = 0^0$)						Section by plane through the axis of rotation and perpendicular to the lines joining the mass centre of the primary and companion star ($\phi = 90^0$)				
θ	r_{exp}	S_1	S_3	S_5	S_7	S_9	S_1	S_3	S_5	S_7	S_9
-90	0.552380	0.538806 **2.457336**	0.534303 **3.272422**	0.543141 **1.672452**	0.542871 **1.721342**	0.543366 **1.631721**	0.538806 **2.457336**	0.534303 **3.272422**	0.543141 **1.672452**	0.542871 **1.721342**	0.543366 **1.631721**
-75	0.546578	0.535257 **2.067966**	0.531503 **2.757971**	0.539603 **1.275523**	0.539641 **1.269118**	0.539913 **1.219404**	0.535257 **2.067966**	0.531503 **2.757971**	0.539603 **1.275523**	0.539641 **1.269118**	0.539913 **1.219404**
-60	0.532397	0.525627 **1.271512**	0.523785 **1.617448**	0.529860 **0.476504**	0.530178 **0.416832**	0.530033 **0.444037**	0.525627 **1.271512**	0.523785 **1.617448**	0.529860 **0.476504**	0.530178 **0.416832**	0.530033 **0.444037**
-45	0.515902	0.512449 **0.669247**	0.512956 **0.571008**	0.516238 **0.065150**	0.516269 **0.071239**	0.516210 **0.059766**	0.512449 **0.669247**	0.512956 **0.571008**	0.516238 **0.065150**	0.516269 **0.071239**	0.516210 **0.059766**
-30	0.501557	0.499271 **0.455864**	0.501428 **0.025812**	0.502114 **0.110963**	0.502001 **0.088846**	0.502064 **0.101073**	0.499271 **0.455864**	0.501428 **0.025812**	0.502114 **0.110963**	0.502001 **0.088846**	0.502064 **0.101073**
-15	0.491628	0.489626 **0.407752**	0.491792 **0.033274**	0.491310 **0.064608**	0.491426 **0.040991**	0.491324 **0.049623**	0.489626 **0.407752**	0.491792 **0.033274**	0.491310 **0.064608**	0.491426 **0.040991**	0.491324 **0.049623**
0	0.487109	0.486092 **0.208845**	0.486568 **0.111847**	0.486559 **0.112911**	0.486560 **0.112832**	0.486588 **0.107124**	0.486092 **0.208845**	0.486568 **0.111847**	0.486559 **0.112911**	0.486560 **0.112832**	0.486588 **0.107124**
15	0.488523	0.489623 **0.225285**	0.487817 **0.144557**	0.488423 **0.020443**	0.488301 **0.045424**	0.488303 **0.045508**	0.489623 **0.225285**	0.487817 **0.144557**	0.488423 **0.020443**	0.488301 **0.045424**	0.488303 **0.045508**
30	0.208531	0.499271 **0.576555**	0.496385 **0.004706**	0.496797 **0.078155**	0.496902 **0.099305**	0.496894 **0.097822**	0.499271 **0.576555**	0.496385 **0.004706**	0.496797 **0.078155**	0.496902 **0.099305**	0.496894 **0.097822**
45	0.511798	0.512449 **0.127246**	0.510918 **0.171815**	0.512558 **0.148581**	0.512302 **0.098421**	0.512303 **0.098678**	0.512449 **0.127246**	0.510918 **0.171815**	0.512558 **0.148581**	0.512302 **0.098421**	0.512303 **0.098678**
60	0.537127	0.525627 **2.140925**	0.527529 **1.786974**	0.534774 **0.438041**	0.534453 **0.501515**	0.534203 **0.544376**	0.525627 **2.140925**	0.527529 **1.786974**	0.534774 **0.438041**	0.534453 **0.501515**	0.534203 **0.544376**
75	0.578905	0.535275 **7.536746**	0.540772 **6.587193**	0.556112 **3.937235**	0.557671 **3.667193**	0.558040 **3.604279**	0.535275 **7.536746**	0.540772 **6.587193**	0.556112 **3.937235**	0.557671 **3.667193**	0.558040 **3.604279**
90	0.657116	0.538806 **18.00442**	0.545829 **16.93522**	0.565140 **13.99601**	0.568282 **13.53081**	0.569373 **13.35265**	0.538806 **18.00442**	0.545829 **16.93522**	0.565140 **13.99601**	0.568282 **13.53081**	0.569373 **13.35265**

Note : Results shown in bold face are the values of percentage error of the corresponding partial sums relative to r_{exp}

CHAPTER – III

EFFECT OF INCLUDING MASS VARIATION IN COMPUTATION OF THE POTENTIAL ON THE EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED POLYTROPIC MODELS

Investigators such as Kopal (65), Mohan and Singh (89), Mohan et al.(70, 85, 90,92) approximate the equipotentials surfaces of rotationally and tidally distorted models by equivalent rotationally and tidally distorted Roche equipotentials This approximation is valid for highly centrally condensed types of gaseous spheres. In the case of models in which the central condensation is not too large, this approximation is not justified. It will, therefore be useful to see if in such types of models which are not too centrally condensed, the approximation of actual equipotentials surfaces by the Roche equipotentials surfaces can be improved upon.

In the present chapter we study the equilibrium structures of rotationally and tidally distorted polytropic models by including in an approximate way the effect of mass variation inside the star on its equipotentials surfaces. The modified Roche equipotential surfaces of such rotationally and tidally distorted stars are presented in section 3.1. In section 3.2 the problem of determining the structures of rotationally and/ or tidally distorted stars using Mohan et al.(85) approach, as modified by us taking into account the effect of mass variation inside the star on its equipotentials surfaces, is then discussed. In this section mathematical expressions determining the equipotential surfaces, volumes, surface areas etc. are first derived and then used to obtain the system of differential equations governing the equilibrium structures of rotationally and tidally distorted stars. In section 3.3, the modified approach has been used to numerically determine the equilibrium structures of rotationally and tidally distorted polytropic models. Numerical results for the inner structure and shape and other physical parameters of certain rotating polytropic models with polytropic indices 1.5, 3.0, 4.0 are next obtained in section 3.4. Numerical results thus obtained have been compared in section 3.5 with the results earlier obtained by Mohan and Sexena (85) and other authors who while computing Roche equipotential surfaces considered the whole mass of star to be concentrated at its centre. Certain conclusions based on this study have finally been drawn in section 3.6

3.1 EQUATION OF MODIFIED ROCHE EQUIPOTENTIAL

In order to investigate the equilibrium structures and stability of binary stars, the concept of Roche equipotentials and Roche limits have often been used in literature. While computing Roche equipotential, the whole mass of the sphere is assumed to be concentrated at its centre. This approximation, though reasonably correct for highly centrally condensed stellar models, is not true for stars which are not very highly centrally condensed. The concept of Roche equipotentials therefore needs to be modified in case of stars which are not highly centrally condensed taking into account the effect of mass variation on its equipotential surfaces inside the star. The results on Roche equipotential based on this modification and which are of practical interest to the present study are summarized below.

Let M_0 and M_1 be the total masses of the primary and secondary components of a binary system which are assumed to be gaseous spheres. The primary is much larger than the secondary $(M_0 > M_1)$. Let $M_0(r)$ represent the mass interior to a sphere of radius r inside the primary component. Let D be the mutual separation between the centers of these two masses. Further suppose that the position of the two components of this binary system is referred to a rectangular system of cartesian coordinates having the origin at the center of gravity of mass M_0, the X-axis along the line joining the centers of the two components, and Z-axis perpendicular to the plane of the orbit of the two components (fig 3.1), then the total potential ψ due to the gravitational, rotational and other disturbing forces acting at an arbitrary point $P(x,y,z)$ may be expressed

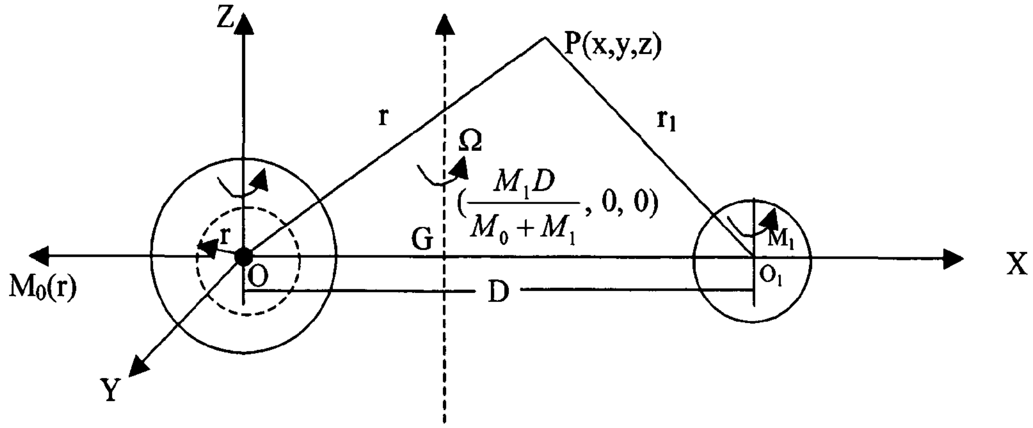

FIG. 3.1 AXES OF REFERENCE

$$\psi = G\frac{M_0(r)}{r} + G\frac{M_1}{r_1} + \frac{1}{2}\Omega^2\left((x - \frac{M_1 D}{M_0 + M_1})^2 + y^2\right) \tag{3.1}$$

where all other symbols have same meanings as assigned to them in chapter II. Thus, the three terms on the right hand side of (3.1) are, respectively, the potential arising from the mass M_0 of the primary, the disturbing potential of its companion of mass M_1 and the potential arising from the centrifugal force. In first term on the right hand side of (3.1) M_0 of (2.1) is replaced by $M_0(r)$ and is based on the fact that in a self-gravitating spherical configuration, the gravitational potential at a point inside the sphere depends only upon the mass enclosed within the concentric spherical surface passing through that point.

Equation (3.1) in nondimensional form can be expressed as

$$\psi^* = \frac{z}{r^*} + q\left[\frac{1}{\sqrt{1-2\lambda r^* + r^{*2}}} - \lambda r^*\right] + nr^{*2}(1-\upsilon^2) \tag{3.2}$$

where $\psi^{*} = \dfrac{D\psi}{GM_0} - \dfrac{M_1^{\,2}}{2M_0(M_0+M_1)}$, and $z = \dfrac{M_0(r)}{M_0}$

is the nondimensional form of the total potential ψ and z the ratio of mass $M_0(r)$ inside spherical surface of radius r of the primary to its total mass. Also $\lambda = \sin\theta \cos\phi, \mu = \sin\theta \sin\phi$ and $\upsilon = \cos\theta$, r,θ,ϕ are as earlier the spherical polar coordinates of the point P. Obviously z is a nondimensional parameter which becomes zero at center of primaries M_0 and one at the surface of the primary. Also as in chapter II,

$$q = \frac{M_1}{M_0}, \tag{3.3}$$

is a nondimensional parameter representing the ratio of mass of the secondary over primary and 2n represents the square of the normalized angular velocity Ω. In equation (3.2) if $q=0$. It reduces to the potential of a rotating spherical model rotating with angular velocity Ω and if $n=0$, then it reduces to the potential of a spherical model distorted only by the tidal effects of a companion. The angular velocity Ω and the nondimensional parameter n are defined in the similar way as defined by (2.4) and (2.5) in section 2.1 respectively.

Equipotential surfaces in nondimensional form represented by ψ^{*}=constant are the modified form of the Roche equipotentials of rotationally and tidally distorted spherical models when the effect of mass variation in computation of potential at points inside the primary is considered. On substituting $z = 1$, in (3.2) or $M_0(r) = M_0$ in (3.1) it reduces to the Roche equipotentials which were earlier obtained by Kopal.

Adopting an approach similar to the one adopted by Kopal (65) and Mohan and Singh (87), we define a nondimensional variable r_0 by the relation

$$r_0 = \frac{z}{\psi^* - q} \tag{3.4}$$

Then following Kopal (65), (r, θ, ϕ) on the surfaces of the modified Roche equipotentials given by (3.2) are connected through the relation

$$\begin{aligned} r^* = r^* = r_0[\,1 + r_0^{\,3} a_0 + \frac{q\,P_3}{z} r_0^{\,4} + \frac{qP_4}{z} r_0^{\,5} + r_0^{\,6}\{\frac{qP_5}{z} + \frac{3\,a_0^{\,2}}{z^2}\} + \\ + r_0^{\,7}\{\frac{q\,P_6}{z} + \frac{1}{z}(7\,q\,a_0\,P_3)\} + r_0^{\,8}\{\frac{qP_7}{z} + \frac{1}{z}(8q\,a_0\,P_4) + \frac{4\,q^2 P_3^{\,2}}{z^2}\} + \\ + r_0^{\,9}\{\frac{qP_8}{z} + \frac{1}{z}(9\,q\,a_0\,P_5) + \frac{9q^2\,P_3\,P_4}{z^2}\} \\ + r_0^{\,10}\{\frac{q\,P_9}{z} + \frac{1}{z}(10\,q\,a_0 P_6) + \frac{5\,q^2}{z^2}(P_4^{\,2} + 2P_3\,P_5)\} + \ldots] \end{aligned} \tag{3.5}$$

where $a_0 = \frac{q\,P_2}{z} + \frac{n(1-\upsilon^2)}{z}$, $P_j = P_j(\lambda)$ are Legendre polynomials and terms up to second order of smallness in n and q are retained. This relation can be used to obtain the modified shapes of Roche equipotentials ψ = constant inside the primary. (Outside primary where $z=1$, earlier approach and this approach become identical). Following Kopal (65) and Mohan, Saxena and Agarwal (92), the expressions for Volume V_ψ, Surface area S_ψ and r_ψ enclosed by the equipotentials surface ψ = constant inside the primary are given by

$$V_\psi = \frac{4}{3}\pi D^3 r_0^{\ 3}[1+\frac{2nr_0^{\ 3}}{z}+(\frac{12}{5z^2}q^2+\frac{8}{5z^2}nq+\frac{32}{5z^2}n^2)r_0^{\ 6}+\frac{15}{7z^2}q^2 r_0^{\ 8} + \frac{2}{z^2}q^2 r_0^{\ 10}+...] \quad (3.6)$$

$$S_\psi = 4\pi D^2 r_0^{\ 2}[1+\frac{4nr_0^{\ 3}}{3z}+(\frac{7q^2}{5z^2}+\frac{14nq}{15z^2}+\frac{56n^2}{15z^2})r_0^{\ 6}+\frac{9q^2 r_0^{\ 8}}{7z^2} + \frac{11q^2 r_0^{\ 10}}{9z^2}+...] \quad (3.7)$$

$$r_\psi = D r_0[1+\frac{2nr_0^{\ 3}}{3z}+(\frac{4}{5z^2}q^2+\frac{8}{15z^2}nq+\frac{76}{45z^2}n^2)r_0^{\ 6}+\frac{5}{7z^2}q^2 r_0^{\ 8} + \frac{2}{3z^2}q^2 r_0^{\ 10}+...] \quad (3.8)$$

Inverting the above relation we have

$$r_0 = r_\psi^{\ *}[1-\frac{2nr_\psi^{\ *3}}{3z}-(\frac{4}{5z^2}q^2+\frac{8}{15z^2}nq-\frac{4}{45z^2}n^2)r_\psi^{\ *6}-\frac{5q^2 r_\psi^{\ *8}}{7z^2}-\frac{2q^2 r_\psi^{\ *10}}{3z^2}+...] \quad (3.9)$$

where $r_\psi^{\ *} = r_\psi / D$, $r_\psi^{\ *}$ being the nondimensional form of r_ψ. Outside primary $z=1$, and these expressions are same as obtained by Mohan et al. (92). Similarly using equations (2.13), (2.14) we have obtained explicit expressions for the value of $\bar{g}$ and $\overline{g^{-1}}$ at points inside the primary as

$$\bar{g} = \frac{zGM_\psi}{D^2 r_0^2}[1 - \frac{8nr_0^3}{3z} - (\frac{3}{z^2}q^2 + \frac{2}{z^2}nq + \frac{40}{9z^2}n^2)r_0^6 - \frac{51}{14z^2}q^2 r_0^8 - \frac{13}{3z^2}q^2 r_0^{10} + ...] \quad (3.10)$$

$$\overline{g^{-1}} = \frac{D^2 r_0^2}{zGM_\psi}[1 + \frac{8nr_0^3}{3z} + (\frac{31}{5z^2}q^2 + \frac{62}{15z^2}nq + \frac{584}{45z^2}n^2)r_0^6 + \frac{101}{14z^2}q^2 r_0^8 + \frac{75}{9z^2}q^2 r_0^{10} + ...] \quad (3.11)$$

As in earlier studies in obtaining the above expressions terms up to second order of smallness in z, n and q are retained.

3.2 METHOD FOR DETERMINING THE EQUILIBRIUM STRUCTURE OF ROTATIONALLY AND TIDALLY DISTORTED STARS INCLUDING THE EFFECT OF MASS VARIATION IN COMPUTATION OF POTENTIAL INSIDE THE STAR

Once the equipotentials surfaces of a rotationally and tidally distorted star are approximated by the modified Roche equipotentials to take into account effects of mass variation inside the star on its equipotential surfaces, the approach followed by Kopal (65), Mohan and Singh (89), and Mohan and Saxena (85) may now be used to evaluate explicitly the values of modified distortion parameters u, v, w, f_P and f_T. Following the approach discussed in chapter II in section 2.3, these modified distortion parameters become

$$u = 1 - (\frac{1}{5z^2}q^{*2} + \frac{2}{15z^2}nq + \frac{4}{45z^2}n^2)r_\psi^{*6} - \frac{1}{7z^2}q^2 r_\psi^{*8} - \frac{1}{9z^2}q^2 r_\psi^{*10} + ... \quad (3.12a)$$

$$v = z[1 - \frac{4nr_\psi^{*3}}{3z} - (\frac{7}{5z^2}q^2 + \frac{14}{15z^2}nq + \frac{68}{45z^2}n^2)r_\psi^{*6} - \frac{31}{14z^2}q^2 r_\psi^{*8} - \frac{3}{z^2}q^2 r_\psi^{*10} + ...] \quad (3.12\ b)$$

$$w = \frac{1}{z}[1 + \frac{4nr_\psi^{*3}}{3z} + (\frac{23}{5z^2}q^2 + \frac{46}{15z^2}nq + \frac{212}{45z^2}n^2)r_\psi^{*6} + \frac{81}{14z^2}q^2 r_\psi^{*8} + \frac{7}{z^2}q^2 r_\psi^{*10} + ...] \qquad (3.12c)$$

$$f_P = z[1 - \frac{4nr_\psi^{*3}}{3z} - (\frac{22}{5z^2}q^2 + \frac{44}{15z^2}nq + \frac{128}{45z^2}n^2)r_\psi^{*6} - \frac{79}{14z^2}q^2 r_\psi^{*8} - \frac{62}{9z^2}q^2 r_\psi^{*10} + ...] \qquad (3.12d)$$

$$f_T = 1 - (\frac{14}{5z^2}q^2 + \frac{28}{15z^2}nq + \frac{56}{45z^2}n^2)r_\psi^{*6} - \frac{46}{14z^2}q^2 r_\psi^{*8} - \frac{34}{9z^2}q^2 r_\psi^{*10} + ... \qquad (3.12e)$$

where $r_\psi^* = r_\psi / D$ is the nondimensional form of r_ψ and terms up to second order of smallness in z, n and q are retained. For $z = 1$, the above expressions reduce to the expressions which were earlier obtained by Mohan and Saxena (85).

The values of P_ψ, ρ_ψ, L_ψ etc. on the various equipotentials surfaces of a rotationally and tidally distorted gaseous sphere may now be obtained by solving the system of differential equations (2.30), (2.33), (2.34) and (2.39) subject to the boundary conditions(2.40)and using the values of the correction factors f_P and f_T as given in (3.12).

It may be noted that approximating the equipotentials surfaces of a rotationally and tidally distorted model by Roche equipotentials, the structure of the star is not approximated by the structure of a Roche model. This is evident from the fact that in the case of no distortion ($n = q = 0$) and $z = 1$ equations (3.12) give $u = v = w = f_P = f_T = 1$ and the system of differential equations (2.30), (2.33), (2.34) and (2.39) reduce to the equations governing the equilibrium structure of the original undistorted star and not of the undistorted Roche model.

Usual numerical methods for solving the stellar structure equations can be used to integrate the system of differential equation (2.30 to 2.39) governing in the equilibrium structure of a rotationally and tidally distorted gaseous sphere. However, at each step the values of the distortion parameters u, v, w, f_P and f_T must be computed using (3.12).

In case a gaseous sphere is being distorted by rotational forces alone (or tidal alone) we may set $q = 0$ (or $n = 0$)in equation (3.12) and still use the above approach in determining the equilibrium structure of the distorted model. For the structure of the primary component of a synchronous binary system we should set $n = (q+1)/2$

If the thermal properties are not considered important and only hydrostatic equilibrium of a rotationally and tidally distorted gaseous sphere is to be investigated then we need only to integrate equations (2.30) and (2.33) subject to the boundary conditions

$$M_\psi = 0 \quad \text{at the center } r_\psi = 0,$$

and

$$M_\psi = M_0,\ P_\psi = 0 \ or\ P_{\psi z},\ \ \rho_\psi = 0 \ \ or\ \rho_{\psi z} \text{ at the free surface } r_\psi = R_\psi \qquad (3.13)$$

In expressions (3.15) we have only retained terms up to second order of smallness in n and q. Therefore the above analysis is valid for the rotationally and tidally distorted models in which the distorting forces causing rotational and tidal distortions are not too large.

The solution of the boundary value problem (2.40) with modified parameters as defined here determines the equilibrium structure of a rotationally and tidally distorted model. For computational work, we find it more convenient to work with r_0 in place of M_ψ or r_ψ as the independent variable. Variable r_0 defined in (3.4) is connected with variable r_ψ explicitly through relations (3.9). By using these relations in (2.30), (2.33), (2.34), (2.39) and (2.41), the system of equations governing the equilibrium structure of a rotationally and tidally distorted model can be expressed as:

$$\frac{dM_\psi}{d r_0} = 4\pi D^3 \rho_\psi r_0^2 f_1 \tag{3.14a}$$

$$\frac{dP_\psi}{dr_0} = -\frac{G M_\psi \rho_\psi f_2}{D r_0^2} \tag{3.14b}$$

$$\frac{dL_\psi}{dr_0} = 4\pi D^3 \rho_\psi r_0^2 f_1 \tag{3.14c}$$

$$\frac{dT_\psi}{dr_0} = -\frac{3\kappa L_\psi \rho_\psi P_\psi}{16\pi D a c T_\psi^3 r_0^2} f_3 \tag{3.14d}$$

where f_1, f_2, f_3 are certain functions of n, q, z and r_0 incorporating the effects of rotation and tidal distortions on the equilibrium structure equations of a distorted model. These can be expressed explicitly in terms of z, n, q and r_0 such as

$$f_1 = 1 + \frac{4nr_0^3}{z} + (\frac{36}{5z^2}q^2 + \frac{72}{15z^2}nq + \frac{864}{45z^2}n^2)r_0^6 + \frac{55}{7z^2}q^2r_0^8 + \frac{26}{3z^2}q^2r_0^{10} + ... \quad (3.15a)$$

$$f_2 = z[1 - (\frac{2}{5z^2}q^2 + \frac{4}{15z^2}nq + \frac{48}{45z^2}n^2)r_0^6 - \frac{9}{14z^2}q^2r_0^8 - \frac{8}{9z^2}q^2r_0^{10} + ...] \quad (3.15b)$$

$$f_3 = 1 + \frac{4nr_0^3}{3z} + (\frac{6}{5z^2}q^2 + \frac{12}{15z^2}nq + \frac{224}{45z^2}n^2)r_0^6 + \frac{24}{14z^2}q^2r_0^8 + + \frac{20}{9z^2}q^2r_0^{10} + ... \quad (3.15c)$$

In the above expressions again terms up to second order of smallness in z, n and q are retained. The boundary conditions given in (2.40) now become $M_\psi = 0$ at the center $r_0 = 0$ and $P_\psi = 0$ or $P_{\psi s}$, $\rho_\psi = 0$ or $\rho_{\psi s}$ at the free surface $r_0 = r_{0s}$ (r_{os} being the value of r_0 at the free surface). Also $z = 1$ at surface and value of r_0 at surface is given by

$$r_{0s} = \frac{1}{\psi_s^* - q} \quad (3.16)$$

where ψ_s^* is the nondimensional value of the total potential ψ on the outermost equipotentials surface of the rotationally and tidally distorted stellar models.

3.3 EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED POLYTROPIC MODELS TAKING INTO ACCOUNT OF MASS VARIATION IN COMPUTATION OF POTENTIAL INSIDE THE STAR

Polytropic models have frequently been used in literature to depict the inner structures of realistic stars. Chandrasekhar developed the theory of distorted polytropes. Since then the several investigators have discussed the structure of a rotating polytrope. However not much attention seems to have been paid to the problems of determining the effects of tidal distortions alone or the effects of tidal distortions in the presence of rotation on the equilibrium structures of polytropic models.

In this section we consider the feasibility of using the approach developed in section 3.1 and 3.2 of this chapter to determine the inner structures and equilibrium configurations of rotationally and tidally distorted polytropic models of stars.

Suppose a polytropic model is subject to rotation and tidal distortion then its structure will become a rotationally and tidally distorted polytropic model. Following the approach of section 3.2 we shall approximate the equipotentials surfaces of this distorted model by modified Roche equipotentials. Let P_ψ denote the pressure and ρ_ψ the density on the equipotentials surface ψ = constant of the distorted model. Then the value of the density and the pressure on the equivalent surface of the corresponding spherical model will also be ρ_ψ and P_ψ respectively. We shall assume that the distorted model also behave like a polytropic model so that ρ_ψ and P_ψ are connected through the polytropic type of relations.

$$P_\psi = P_{c\psi}\,\theta_\psi^{\;N+1} \text{ and } \rho_\psi = \rho_{c\psi}\,\theta_\psi^{\;N} \tag{3.17}$$

where $P_{c\psi}$ and $\rho_{c\psi}$ are the values of P_ψ and ρ_ψ at the center and θ_ψ is some average of θ on the equipotential surface $\psi =$ constant. In the case of polytropic models, the following equations

$$\frac{dM_\psi}{dr_\psi} = 4\,\pi\, r_\psi^{\;2}\rho_\psi \quad and \quad \frac{dP_\psi}{dr_\psi} = -\frac{GM_\psi}{r_\psi^{\;2}}\rho_\psi, \tag{3.18}$$

which govern the hydrostatic equilibrium structure of rotationally and tidally distorted gaseous spheres can be combined together with (3.17) to yield

$$\alpha^2 \frac{d}{dr_\psi}\left(r_\psi^2 \frac{d\theta_\psi}{dr_\psi}\right) = -\, r_\psi^2\,\theta_\psi^{\;N} \tag{3.19}$$

where $$\alpha^2 = \frac{(N+1)\,P_{c\psi}}{4\pi\,\rho^2{}_{c\psi}}$$

If we change the independent variable r_ψ into r_0 equations (3.19) is reduced to

$$\frac{d}{dr_0}\left[A\,(z,n,q,r_0)\frac{d\theta_\psi}{dr_0}\right] = -\frac{D^2}{\alpha^2}B(z,n,q,\,r_0)\,\theta_\psi^{\;N} \tag{3.20}$$

where $A(z,n,q,r_0) = r_\psi^2 \frac{dr_0}{dr_\psi}$ and $B(z,n,q,r_0) = r_\psi^2 \frac{dr_\psi}{dr_0}$. Explicit expressions for these can be written as

$$A(z,n,q,r_0) = r_0^{\;2}\left[1-\left(\frac{2}{5z^2}q^2 + \frac{4}{15z^2}nq + \frac{16}{15z^2}n^2\right)r_0^6 - \frac{6q^2}{7z^2}r_0^8 - \frac{10q^2}{9z^2}r_0^{10} + \cdots\right], \tag{3.21}$$

$$B(z,n,q,r_0)= r_0^{\,2}\left[1+\frac{4n}{z}r_0^{\,3}+\left(\frac{36}{5z^2}+\frac{24}{5z^2}nq+\frac{96}{5z^2}n^2\right)r_0^{\,6}+\frac{55}{7z^2}q^2 r_0^{\,8}+\frac{26}{3z^2}r_0^6\right]$$

(3.22)

$$\text{where } z=\frac{M_0(r)}{M_0}=\frac{r_0^2\dfrac{d\theta_\psi}{dr_0}}{\left(r_0^2\dfrac{d\theta_\psi}{dr_0}\right)_{r_0=r_{0s}}} \tag{3.23}$$

As regards the boundary conditions since ρ_ψ and P_ψ must be maximum at the center and zero at the free surface, these obviously lead to the conditions θ_ψ=1 and $\dfrac{d\theta_\psi}{dr_0}=0$ at the center and θ_ψ =0 at the free surface. Thus the boundary conditions which equation (3.20) must satisfy are

$$r_0=0, \theta_\psi=1, \frac{d\theta_\psi}{dr_0}=0, \text{ at the center}$$

$$\text{and } r_0=r_{0s}, \theta_\psi=0, \text{at the surface} \tag{3.24}$$

r_{0s} being the value of r_0 on the free surface.

The quantity α as defined in (3.19) is of the dimension of length. If we set $r_\psi=\alpha\xi$ then ξ will be nondimensional variable defined for the equivalent spherical model. It corresponds to the usual Emden variable ξ of the Lane – Emden equation for an undistorted spherical polytropic model when terms upto second order of smallness in distorting parameters $z, n,$ and q are retained.

It may be noted that the approximation of the Roche equipotentials surfaces by modified Roche equipotentials has not basically changed the structure of the polytropic model because in the absence of any distortion $z=1$, $(n=q=0)$, equation (2.6) reduces to the usual Lane- Emden equation of an undistorted polytropic model with index N in non dimensional form and not to the equation governing the equilibrium structure of undistorted Roche model.

In the case of a rotationally and tidally distorted polytropic model or a model which is a primary component of binary system, let K denote the ratio between the undistorted radius R_u of the primary and D the distance between the centers of the two components of the binary system. Then following Mohan et al.(85) we can write

$$\frac{D}{\alpha}=\frac{D\xi}{\alpha\xi_u}=\frac{D}{R_u}\xi_u=\frac{1}{k}\xi_u, \tag{3.25}$$

where ξ_u is the value of ξ at the outermost surface of the undistorted polytropic model. With this substitution equation (3.20) can be written as

$$\frac{d}{dr_0}\left[A(r_0,z,n,q)\frac{d\theta_\psi}{dr_0}\right]=-\frac{\xi_u^2}{K^2}\theta_\psi^N B(r_0,z,n,q) \tag{3.26}$$

Equation (3.26) subject to the boundary conditions (3.24) determines the equilibrium structure of a rotationally and tidally distorted polytropic model. On setting $q=0$ the equation (3.26) can be used to determine the equilibrium structure of a polytropic model distorted by rotation alone. If we set $n=0$ then the equation can be used to determine the equilibrium structures of polytropic models distorted by tidal forces alone. Also by setting $n=(q+1)/2$ this equation can be used to determine the equilibrium structure of the primary component of a synchronously rotating binary system.

In order to determine the numerical solution of the second-order nonlinear differential equation (3.26) subject to the boundary conditions (3.24), we can start integration of (3.26) (for certain specified values of N, ξ_u, k, n and q) from the centre using $\theta_\psi = 1$ and $\frac{d\theta_\psi}{dr_0}$ at the centre (r_0=0) as the initial conditions. The integration be continued till θ_ψ first becomes zero. In its computation we need the value of z at each step of integration to take into account the effect of mass variation inside the star on the shape of equipotential surfaces. These values can be calculated using equations (3.23). The value of r_0 (i.e. r_{0s}) when θ_ψ first becomes zero determines the outermost free surface of the model. Once the solutions of equation (3.26) are obtained, we know the values of θ_ψ for various values of the nondimensional independent variable r_0 varying from zero to r_{os}. The pressure P_ψ and the density ρ_ψ on various equipotentials of the distorted model may now be obtained through the relations (3.17) in the same manner as is done for undistorted polytropic models. Also, the radius r_ψ of the topologically equivalent spherical surface corresponding to the equipotential surface ψ = constant can be determined from (3.9) and written as

$$r=\left(\frac{\alpha\xi_u}{K}\right)r_0\left[1+\frac{2n}{3z}r_0^3+\left(\frac{4}{5z^2}q^2+\frac{8}{15z^2}nq+\frac{76}{45z^2}n^2\right)r_0^6+\frac{5}{7z^2}q^2r_0^8+\frac{2}{3z^2}q^2r_0^{10}+...\right] \tag{3.27}$$

3.4 VOLUMES, SURFACE AREAS AND OTHER PHYSICAL PARAMETERS OF POLYTROPIC MODELS

In this section we have developed explicit expressions to determine the volume, the surface area and the shape of a rationally and tidally distorted polytropic model. In addition to this we have also shown in this section how the surface area, the shape and the volume enclosed by an equipotential surface located in the interior of a rotationally and tidally distorted polytropic model may be determined. On using (3.25) with (3.6), the total volume enclosed by a rotationally and tidally distorted polytropic model is given by:

$$V_\psi = \frac{4\pi}{3}\left(\frac{\alpha\xi_u}{k}\right)^3 r_{0s}^3\left[1+\frac{2n}{z}r_{0s}^3+\left(\frac{12}{5z^2}q^2+\frac{8}{5z^2}nq+\frac{32}{5z^2}n^2\right)r_{0s}^6+\frac{15}{7z^2}q^2r_{0s}^8+\frac{2}{z^2}q^2r_{0s}^{10}\right] \tag{3.28}$$

Similarly on using (3.25) with (3.7) the total surface area covered by the free surface of a rotationally and tidally distorted polytropic model can be expressed as :

$$S_\psi = 4\pi\left(\frac{\alpha\xi_u}{k}\right)^2 r_{0s}^2\left[1+\frac{4n}{3z}r_{0s}^3+\left(\frac{7}{5z^2}q^2+\frac{14}{15z^2}nq+\frac{56}{15z^2}n^2\right)r_{0s}^6+\frac{9}{7z^2}q^2r_{0s}^8+\frac{11}{9}q^2r_{0s}^{10}\right] \tag{3.29}$$

Also the shape of the outermost equipotential surface of a rotationally and tidally distorted polytropic model may be obtained by using (3.25) with (3.5) to obtain

$$\begin{aligned} r=\frac{\alpha\xi_u}{K}r_{0s}[1&+\frac{1}{z}a_0r_{0s}{}^3+\frac{qP_3}{z}r_{0s}{}^4+\frac{qP_4}{z}r_{0s}{}^5+r_{0s}{}^6\{\frac{qP_5}{z}+\frac{3a_0{}^2}{z^2}\}+\\ &r_{0s}{}^7\{\frac{qP_6}{z}+\frac{1}{z}(7qa_0P_3)\}+r_{0s}{}^8\{\frac{qP_7}{z}+\frac{1}{z}(8qa_0P_4)+\frac{4}{z^2}q^2P_3{}^2\}+\\ &r_{0s}{}^9\{\frac{qP_8}{z}+\frac{1}{z}(9qa_0P_5)+\frac{1}{z^2}9q^2P_3P_4\}+\\ &r_{0s}{}^{10}\{\frac{qP_9}{z}+\frac{1}{z}(10qa_0P_6)+\frac{5q^2}{z^2}\{P_4{}^2+2P_3P_5\}] \end{aligned} \tag{3.30}$$

where $a_0=\frac{qP_2}{z}+\frac{n(1-\upsilon^2)}{z}$ and $P_j=P_j(\lambda)$ are Legendre's polynomials with

$\lambda = \sin\theta\cos\phi$, $\mu = \sin\theta\sin\phi$, $\upsilon = \cos\theta$ and (r,θ,ϕ) being the polar spherical coordinates with the pole at the centre of the primary (c.f.(3.5) and (3.1)). Also value of polar and equatorial radius R_P and R_e are given by

$$R_p = r_{0s} R \tag{3.31}$$

$$R_e = r_{0s} R\Big[1 + (\frac{q}{z} + \frac{n}{z}) r_{0s}^{\ 3} + \frac{q}{z} r_{0s}^{\ 4} + \frac{q}{z} r_{0s}^{\ 5} + (\frac{q}{z} + \frac{3q^2}{z^2}) r_{0s}^{\ 6} + + (\frac{q}{z} + \frac{7q^2}{z^2}) r_{0s}^{\ 7} + (\frac{q}{z} + \frac{12q^2}{z^2}) r_{0s}^{\ 8} + (\frac{q}{z} + \frac{18q^2}{z^2}) r_{0s}^{\ 9} + + (\frac{q}{z} + \frac{25q^2}{z^2}) r_{0s}^{\ 10} + \ldots\Big] \tag{3.32}$$

If we follow Geroyannis and Valvi (41) oblateness and ellipticity σ and ε which are used as measures of the departure of the shape of the star from spherical symmetry may be computed using

$$\sigma = \frac{R_e - R_p}{R_p} \tag{3.33}$$

$$\varepsilon = \frac{R_e - R_p}{R_e} \tag{3.34}$$

The values of gravitational force g_p at the pole and g_e at the equator are given by

$$g_p = \frac{GM_0}{R_p^{\ 2}} \text{ and} \tag{3.35}$$

$$g_e = \frac{GM_0}{R_e^{\ 2}}\left[\begin{array}{l} 1 - (\frac{2q}{z} + \frac{2n}{z}) r_{0s}^{\ 3} - \frac{3q}{z} r_{0s}^{\ 4} - \frac{4q}{z} r_{0s}^{\ 5} - (\frac{5q}{z} + \frac{6q^2}{z^2}) r_{0s}^{\ 6} \\ - \frac{6q}{z} r_{0s}^{\ 7} - (\frac{7q}{z} + \frac{2q^2}{z^2}) r_{0s}^{\ 8} - \frac{9q}{z} r_{0s}^{\ 9} - (\frac{9q}{z} + \frac{20q^2}{z^2}) r_{0s}^{\ 10} + \ldots \end{array}\right] \tag{3.36}$$

Polytropic model do not include energy conservation and therefore are not expected to be in thermal balance. However these models have been used in literature to compute the effect of rotation on variation in temperature and luminosity on stellar surface. Following Ireland (55) the effective temperature at any point on the surface of the star can be obtained as $\left(\frac{T}{T_p}\right)=\left(\frac{g}{g_p}\right)^{1/4}$ (3.37) where T_p is the polar temperature. Once temperature is known radiative flux L at any point on the surface may be estimated using $L=-\frac{4ac}{3\rho\chi}T^3 grad\,T$ (3.38) Where χ is the opacity, T the gas temperature, a the radiative constant, and c the velocity of light.

We have used relations (3.28-3.30) may be used to determine the volume, the surface area and the shape of a rotationally and tidally distorted polytropic model when terms upto second order of smallness in z, n and q are retained. In case we need the volume or the surface area or the shape of some inner equipotential surface of the distorted model then we need only replace r_{0s} by the appropriate value of r_0 for that surface in the above relations (3.28-3.30).Thus once the numerical solutions of the nonlinear differential equation (3.26) which governs the equilibrium structure of a rotationally and tidally distorted polytropic model has been obtained, the value of r_{0s} thus obtained may be used in the above formulae to determine the volume, the surface area and the shape of the outermost equipotential surface of the rotationally and tidally distorted polytropic model.

3.5 NUMERICAL COMPUTATIONS

To obtain the inner structure, the shape, the volume and the surface area of a rotationally and tidally distorted polytropic model, equation (3.26) has to be integrated numerically subject to the boundary conditions (3.24) for the specified values of the parameters N, ξ_u, n, q and K which denote respectively the polytropic index, the radius of the undistorted polytropic model, the nondimensional measure of angular velocity of rotation, the ratio of the mass of the companion to the mass of the primary and the ratio of the undistorted radius of the primary to the distance between the centres of the primary and secondary. The value of z required at each step has to be computed from the equation (3.23). For a polytropic model distorted by rotational forces alone we should take $K = 1$. In the case of the polytropic model being the primary component of a binary system the value of K must be chosen that the outermost surface of the primary component lies well within the Roche lobe otherwise the two stars will coalesce (cf. Kopal (65), page 11).

For obtaining the numerical solutions, equation (3.21) has been integrated by us numerically using fourth-order Runge-Kutta method for the specified values of the input parameters. A series solution similar to the one available for undistorted polytropic models (see Chandrasekhar (7) page 95) was developed to start integrations at points near the centre. This serious solution is given by

$$\theta_\psi = 1 - \frac{K^2}{6} r_0^2 + \frac{NK^4}{120} r_0^4 - \frac{2nK^2}{15z} r_0^5 - \frac{K^6 N(8N-5)}{3x5040} r_0^6 + \frac{K^4 N n}{70z} r_0^7 +$$
$$+ \left[\frac{K^8 N(122N^2 - 183N - 70)}{9x362880} - \frac{K^2}{36z^2}(3q^2 + 2nq + 8n^2) \right] r_0^8 + \qquad \dots (3.39)$$

Taking starting values from this series solution at $r_0 = 0.005$, numerical integration of equation (3.26) was then carried forward using Runge-Kutta method of order four. Using a step length of 0.005, numerical integration was continued till θ_ψ first became zero. Relations (3.20), (3.28) and (3.29) were then used to determine the shape of the distorted polytropic model, its volume and its surface area.

Results obtained for different values of the input parameters are tabulated in Tables 3.1 to 3.3. The value of the parameter has been taken as one for the rotationally distorted model and 0.5 for the tidally distorted or rotationally and tidally distorted models. (This value of K provides the outer-most surface of the model well within Roche lobe for each considered case). In Table 3.1 we present the values of θ_ψ for various types of distorted polytropic models of indices 1.5, 3.0 and 4.0. Following Chandrasekhar (21), Linnell (74) and James (56) we have also computed the results in Table 3.2 by taking $\alpha = 1$. The values of the volumes and the surface areas and other physical parameters obtained for each of these distorted models are then presented in Table 3.3. It will be interesting to compare the present results in which effects of mass variation in computation of potential has been includes with corresponding results earlier obtained by Mohan et al (85) in which entire mass of the model is supposed to be concentrated at centre in computation of equipotential surfaces. The Results shown in parenthesis for each models in Table 3.1 (a) (c) and Table 3.3 (a) - (c) represent the result earlier obtained by Mohan and Saxena (85).

3.6 ANALYSIS OF RESULTS

Results given in paranthesis in second rows of entries in Tables are reality the values of θ_ψ for respective polytropic models as obtained by Mohan et al. (85) who earlier carried out corresponding computations assuming whole mass to be concentrated at the centre. The results of the Table 3.1(a) show that for the polytropic model of index 1.5, the value of θ_ψ for each of the distorted model are larger compared to their corresponding values of the undistorted model (values tabulated in column I for $(n = q = 0)$ Our values for θ_ψ in the case of undistorted and rotationally distorted polytropic model for index 1.5 are smaller in comparison to the corresponding results obtained by Mohan and Saxena (85) and listed along side in parentheses. However, with the introductions of tidal effects, our values for θ_ψ increases in comparison to the corresponding results as obtained by Mohan and Saxena (1983). For the polytropic model of index 3.0 however, whereas the values of θ_ψ for the rotationally and tidally distorted models are larger than the corresponding values for undistorted model, the values of θ_ψ for tidally distorted models are marginally smaller than their corresponding values for undistorted polytropic model. A comparison of our results for the undistorted models of index 3.0 with the corresponding results obtained by Mohan and Saxena (85) shows that results obtained by us are in complete agreement with their results. However our values for θ_ψ for the tidally distorted models are marginally less in comparison to values for θ_ψ as obtained by Mohan and Saxena (85). However the values of θ_ψ obtained by us for rotationally and tidally distorted models are generally larger in comparison to the corresponding values obtained by them. As regards the comparison of our results for the polytropic model for index 4.0, with the corresponding results obtained by Mohan and Saxena (85) observed a trend similar to those in the case of results for polytrope.

The results presented in Table 3.3 (a), (b) and (c) exhibit the values of volumes, surface area, and other physical parameters for rotationally and tidally distorted polytropic models with polytropic indices 1.5, 3.0, 4.0, respectively. In each Table results shown in parenthesis are those obtained by Mohan and Saxena (85). A comparision of the results of volume V_ψ, and surface area S_ψ for rotationally and/ or tidally distorted model with undistorted model for polytropic indices 1.5, it indicates that our results are larger in comparison to undistorted model. However, for rotationally and tidally distorted models these are smaller. As regards the polytropic model of index 3.0, values of θ_ψ generally increased in comparison to the undistorted model with the introduction of distortion terms. A similar trend is noticed for the polytropic model of index 4.0. The values of the shape parameters σ and ε generally decreased in the presence of distortions. The values of T_e/T_P and L_e/L_P have generally increased with the introduction of rotational effects and decrease with the tidal and combined rotational and tidal effects. The results in a way indicate that the rotational forces partly restore the contraction in the equatorial plane caused by the tidal force.

Table 3.1(a) : Values of θ_{ψ} for rotationally and/ or tidally distorted polytropic models N= 1.5

$x=r_0/r_{0s}$	K=1.0 n=0.0 q=0.0	K=1.0 n=0.1 q=0.0	K=0.5 n=0.0 q=0.1	K=0.5 n=0.1 q=0.2	K=0.5 n=0.55 q=0.1	K= 0.5 n=0.6 q=0.2
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.1	0.97652 (0.97797)	0.97797 (0.97925)	0.97787 (0.97797)	0.97834 (0.97814)	0.98008 (0.97887)	0.98027 (0.9789)
0.2	0.90887 (0.91446)	0.91146 (0.91923)	0.91438 (0.91446)	0.91613 (0.91509)	0.92266 (0.91781)	0.92337 (0.9181)
03	0.80492 (0.81665)	0.81665 (0.82619)	0.81100 (0.81666)	0.82013 (0.81791)	0.83339 (0.82334)	0.83483 (0.8239)
0.4	0.67592 (0.69488)	0.69488 (0.70906)	0.69488 (0.69489)	0.70025 (0.69673)	0.72068 (0.70481)	0.72291 (0.7057)
0.5	0.56068 (0.56067	0.57797 (0.57797)	0.56068 (0.56068)	0.56291 (0.56291)	0.57276 (0.57276)	0.57384 (0.5738)
0.6	0.39210 (0.42490)	0.42490 (0.44279)	0.42497 (0.42491)	0.43282 (0.42720)	0.46345 (0.43736)	0.46687 (0.4384)
0.7	0.25855 (0.29638)	0.29638 (0.31202)	0.29647 (0.29639)	0.30456 (0.29837)	0.30725 (0.30725)	0.34027 (0.3082)
0.8	0.14018 (0.18112)	0.18112 (0.19219)	0.17587 (0.18112)	0.18890 (0.18250)	0.20175 (0.08592)	0.22347 (0.1894)
0.9	0.04028 (0.08218)	0.08218 (0.08756)	0.08228 (0.08219)	0.08910 (0.08283)	0.11712 (0.08592)	0.12040 (0.0862)
1.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Note: Results in paranthesis shown in these and subsequent tables are the corresponding results as obtained earlier by Mohan and Saxena (1983).

Table 3.1 (b) : Values of θ_ψ for rotationally and/ or tidally distorted polytropic models N= 3.0

$x=r_0/r_{0s}$	K=1.0 n=0.0 q=0.0	K=1.0 n=0.1 q=0.0	K=0.5 n=0.0 q=0.1	K=0.5 n=0.1 q=0.2	K=0.5 n=0.55 q=0.1	K= 0.5 n=0.6 q=0.2
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.1	0.92600 (0.92600)	0.92809 (0.92808)	0.92568 (0.92600)	0.92630 (0.92627)	0.92878 (0.92749)	0.92899 (0.92758)
0.2	0.75322 (0.75322)	0.75891 (0.75892)	0.75304 (0.75321)	0.75473 (0.75397)	0.76159 (0.75730)	0.76215 (0.75755)
03	0.56495 (0.56495)	0.57243 (0.57254)	0.56489 (0.56494)	0.56717 (0.56594)	0.57651 (0.57038)	0.57728 (0.57070)
0.4	0.40590 (0.40590)	0.41311 (0.41327)	0.40591 (0.40589)	0.40823 (0.40686)	0.41776 (0.41118)	0.41854 (0.41148)
0.5	0.28402 (0.28402)	0.28992 (0.29006)	0.28408 (0.28402)	0.28614 (0.28482)	0.29463 (0.28837)	0.29532 (0.28860)
0.6	0.19316 (0.19316)	0.19748 (0.19755)	0.19323 (0.19315)	0.19495 (0.19374)	0.20204 (0.19635)	0.20260 (0.19649)
0.7	0.12509 (0.12509)	0.12795 (0.12795)	0.12517 (0.12508)	0.12656 (0.12547)	0.13229 (0.12720)	0.13272 (0.12726)
0.8	0.07313 (0.07313)	0.07479 (0.07472)	0.07322 (0.07312)	0.07432 (0.07334)	0.07889 (0.07434)	0.07923 (0.07434)
0.9	0.03251 (0.03251)	0.03327 (0.03316)	0.03261 (0.03251)	0.03349 (0.03260)	0.03714 (0.03304)	0.037408 (0.03300)
1.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table 3.1 (c) : Values of θ_ψ for rotationally and/ or tidally distorted polytropic models N= 4.0

$x=r_0/r_{0s}$	K=1.0 n=0.0 q=0.0	K=1.0 n=0.1 q=0.0	K=0.5 n=0.0 q=0.1	K=0.5 n=0.1 q=0.2	K=0.5 n=0.55 q=0.1	K= 0.5 n=0.6 q=0.2
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.1	0.73999 (0.7399)	0.74184 (0.74168)	0.73949 (0.73993)	0.73949 (0.74020)	0.74171 (0.74123)	0.74162 (0.74128)
0.2	0.44089 (0.44089)	0.44304 (0.44287)	0.44238 (0.44083)	0.44613 (0.44114)	0.45266 (0.44234)	0.45337 (0.44240)
03	0.27382 (0.27382)	0.27540 (0.27528)	0.27430 (0.27377)	0.27476 (0.27400)	0.27634 (0.27490)	0.27623 (0.27493)
0.4	0.17893 (0.17893)	0.17999 (0.17991)	0.17941 (0.17889)	0.17976 (0.17906)	0.18090 (0.17966)	0.180801 (0.17967)
0.5	0.11984 (0.11984)	0.12050 (0.12045)	0.12030 (0.11981)	0.12056 (0.11992)	0.12138 (0.12030)	0.12128 (0.12030)
0.6	0.07999 (0.07999)	0.08038 (0.08034)	0.08044 (0.07997)	0.08063 (0.08004)	0.08123 (0.08027)	0.08113 (0.08026)
0.7	0.05144 (0.05144)	0.05163 (0.05160)	0.05187 (0.05142)	0.05201 (0.05146)	0.05244 (0.05158)	0.05235 (0.05157)
0.8	0.03001 (0.03001)	0.03008 (0.03006)	0.03042 (0.02999)	0.03053 (0.03001)	0.03084 (0.03006)	0.03076 (0.03004)
0.9	0.01334 (0.01334)	0.01333 (0.01333)	0.01322 (0.01322)	0.01322 (0.01322)	0.01333 (0.01333)	0.13922 (0.01332)
1.0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table 3.2 :A comparison of the volumes and surface areas of uniformly rotating polytropes as obtained by different investigators

$v=\dfrac{\Omega^2}{2\pi G\rho_c}$	Volume				Surface area			
	Present Value	Saxena's Value	Chandra-sekhar Value	Linnell's Value	Present value	Saxena's Values	Chanra-sekhar	Linnell's Value
Polytropic Index N=1.5								
0.0	2.0432	2.0432	2.0432	2.0432	1.6776	1.6776	1.6776	1.6776
0.04	2.0525	2.0854	2.0859	2.0880	1.6827	1.7008	1.7000	1.70280
0.08	2.0640	2.1300	2.1286	2.1377	1.6890	1.7250	1.7250	1.7286
0.012	2.0764	2.1777	2.1714	2.1933	1.6958	1.7509	1.7509	1.7577
0.016	2.0890	2.2275	2.2141	2.2560	1.7030	1.7778	1.7778	1.7900
0.020	2.1014	2.2789	2.2568	2.3276	1.7102	1.8058	1.8058	1.8260

0.024	2.1410	2.3315	2.2995	2.4102	1.7324	1.8345	1.8345	1.8667
0.028	2.1247	2.3850	2.3422	2.5067	1.7245	1.8639	1.8639	1.9130
0.032	2.1354	2.4388	2.3850	2.6203	1.7314	1.8936	1.8936	1.9666
0.036	2.1452	2.4930	2.4277	2.7534	1.7380	1.9237	1.9237	2.0298
0.040	2.1543	2.5470	2.4704	2.9052	1.7443	1.9539	1.9539	2.1059

Polytropic Index N=3.0

0.0	1.3741	1.3741	1.3741	1.3741	5.9774	5.9774	5.9774	5.9774
0.0004	1.4058	1.4099	1.4070	1.4086	6.0728	6.0814	6.0728	6.0770
0.0008	1.4405	1.4485	1.4399	1.4467	6.1682	6.1918	6.1682	6.1858
0.0012	1.4780	1.4902	1.4728	1.4890	6.2636	6.3106	6.2636	6.3057
0.0016	1.5053	1.5348	1.5057	1.5361	6.3590	6.4373	6.3590	6.4386
0.0020	1.5632	1.5825	1.5386	1.5891	6.4544	6.5721	6.4544	6.5874
0.0024	1.6078	1.6328	1.5715	1.6492	5.5498	6.7140	5.5498	6.7557
0.0028	1.6546	1.6856	1.6044	1.7178	6.6452	6.8628	6.6452	6.9486
0.0032	1.7063	1.7408	1.6373	1.7963	6.7406	7.0181	6.7406	7.0181
0.0036	1.7567	1.7984	1.6702	1.8844	6.8359	7.1803	6.8359	7.1803
0.0038	1.7826	1.8281	1.6866	1.9306	6.8836	2.2638	6.8836	7.2638

Table 3.3(a) : Volumes, Surface area and other physical parameters of rotationally and tidally distorted polytropic index 1.5

Model No.	n	q	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$	σ	ε	T_e/T_p	L_e/L_p
1	0.0	0.0	2.0431 (2.0432)	1.67760 (1.6776)	0.00000	0.00000	1.00000	1.00000
2	0.0	0.5	2.0493 (2.0664)	1.6794 (1.6890)	0.17395	0.14818	0.62261	0.37366
3	0.05	0.2	2.0248 (2.0621)	1.6673 (1.6879)	0.06374	0.05992	0.67388	0.69608
4	0.6	0.2	1.8397 (2.2437)	1.5643 (1.7862)	0.11087	0.09981	0.63402	0.55291
5	0.55	0.1	1.8544 (2.2214)	1.5727 (1.7744)	0.08023	0.07427	0.65009	0.65632
6	0.02	0.0	2.0543 (2.0903)	1.68370 (1.7035)	0.01931	0.01895	0.98448	0.90773

Table 3.3 (b) Volumes, surface area and other physical parameters of rotationally and tidally distorted polytropic index N =3.0

Model No.	n	q	$V_\psi \times 10^{-3}$	$S_\psi \times 10^{-2}$	σ	ε	T_e/T_p	L_e/L_p
1	0.0	0.0	1.3741 (1.3747)	5.9773 (5.9774)	0.00000	0.00000	1.00000	1.00000
2	0.0	0.5	1.3971 (1.3923)	6.0381 (6.0251)	0.19752	0.15101	0.98793	0.36484
3	0.05	0.2	1.3846 (1.3910)	6.0068 (6.0260)	0.17783	0.06129	0.67490	0.68970
4	0.6	0.2	1.4859 (1.5708)	6.2984 (6.5375)	0.06523	0.12004	0.63865	0.47871
5	0.55	0.1	1.4709 (1.5486)	6.2559 (6.4754)	0.13641	0.08699	0.65914	0.60454
6	0.02	0.0	1.4142 (1.4185)	6.0930 (6.1061)	0.01975	0.01937	0.98793	0.90574

Table 3.3(c) : Volumes, Surface area and other physical parameters of rotationally and tidally distorted polytropic index N =4.0

Model No.	n	q	$V_\psi \times 10^{-3}$	$S_\psi \times 10^{-3}$	σ	ε	T_e/T_p	L_e/L_p
1	0.0	0.0	14.0569 (14.062)	2.81672 (2.8175)	0.00000	0.00000	1.00000	1.00000
2	0.0	0.5	14.3202 (14.2581)	2.8491 (2.8412)	0.17846	0.15143	0.62170	0.36354
3	0.05	0.2	14.2430 (14.266)	2.8410 (2.8445)	0.06581	0.06175	0.67523	0.68755
4	0.6	0.2	16.3803 (16.539)	3.1199 (3.1406)	0.14858	0.12936	0.63931	0.44636
5	0.55	0.1	16.0872 (16.255)	3.0818 (3.1045)	0.10215	0.09267	0.66228	0.58209
6	0.02	0.0	14.5901 (14.604)	2.8874 (2.8897)	0.01991	0.01952	0.98918	0.90501

CHAPTER - IV

EQUILIBRIUM STRUCTURE OF ROTATIONALLY AND TIDALLY DISTORTED PRASAD MODEL AND A CLASS OF COMPOSITE MODELS

In the present chapter we use the methodology developed in chapter III to determine the equilibrium structures of rotationally and tidally distorted Prasad model as well as a class of composite models. These models are often used in astrophysics to represent the inner structure of certain types of stars. In Prasad model the density distribution follows the law $\rho=\rho_c(1-x^2)$, ρ_c being the value of density ρ at the centre and x the nondimensional measure of the distance from the centre. The series of composite models considered in this chapter consist of cores in which density decreases slowly from the centre to the interface between the envelope and the core according to the law $\rho=\rho_c(1-x^2)$.These cores are surrounded by envelopes in which density varies inversely as the square of the distance from the centre. These composite model have Prasad model at one extreme and Roche model at the other extreme and reasonably represent the effect of density variation inside the star on its structure. A series of such models can be constructed by varying the position of the interface between the core and the envelope. These models reasonably depict the inner structures of the stars which have developed cores of reasonable thickness in which density decreases slowly outwards from the centre to the interface and surrounding these cores have envelopes in which density falls of rapidly from the interface to the surface. Models of this series with extended cores can also be regarded as stars surrounded by their atmospheres. Prasad model and this series of semi-analytic composite models have often been used in literature to analyze the problems of stellar structure and stellar pulsations. Investigations carried on these models are thus expected to provide some insight into the problems associated with the structure of certain realistic models of the stars.

Aggarwal (2), Mohan et al. (85) determined the effects of rotation and tidal distortions on the equilibrium structures of Prasad model and this series of composite models of stars. In these studies the actual equipotential surfaces of rotationally and tidally distorted star were approximated by equipotential surfaces obtained by assuming the entire mass of the star to be placed at the center of the star. Here we have reinvestigated the equilibrium structures of these rotationally and tidally distorted models by approximating the actual equipotential surfaces by Roche equipotential surfaces which are determined by taking into account effect of mass variation in potential.

In section 4.1 we discuss the feasibility of using the approach developed in chapter III to determine the equilibrium structures of rotationally and tidally distorted Prasad model. In section 4.2 we have also used the methodology developed in chapter III to determine the equilibrium structure of rotationally and tidally distorted composite models of star. Results for numerical computation in case of Prasad model and composite models for different position of interfaces as 0.3, 0.5, and 0.7 of the radius are obtained in section 4.3. Certain conclusions based on this study have also been finally drawn in section 4.4.

4.1 EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED PRASAD MODEL

If we assume that the primary component of binary system behaves as Prasad model and rotating about its axis, then its equilibrium structure will be distorted by rotation as well as the tidal effects of the companion. In order to determine the equilibrium structure of this rotationally and tidally distorted stellar model we may follow the approach of Mohan and Saxena (85) as given in section 2.3 of Chapter II provided it is assumed that the rotational velocity and the mass of the secondary as compared to the primary are suitably small.

Let r_ψ denote the radius of the topologically equivalent spherical model which corresponds to an equipotential surface $\psi =$ constant of this rotationally and tidally distorted Prasad model. Also, let R_ψ be the value of r_ψ on the equipotentials surface $\psi =$ constant of this rotationally and tidally distorted model. Following the approach as discussed in chapter III, r_ψ and R_ψ are given as.

$$r_\psi = Dr_0\left[1+\frac{4nr_0^{\,3}}{3z}+\left(\frac{4q^2r_0^{\,6}}{4z^2}+\frac{8nq}{15z^2}+\frac{76n^2}{45z^2}\right)r_0^6+\frac{5q^2r_0^{\,8}}{7z^2}+\frac{2q^2r_0^{\,10}}{3z^2}+\ldots\right] \quad (4.1)$$

$$R_\psi = Dr_{0s}\left[1+\frac{4nr_{0s}^{\,3}}{3z}+\left(\frac{4q^2r_0^{\,6}}{4z^2}+\frac{8nq}{15z^2}+\frac{76n^2}{45z^2}\right)r_{0s}^6+\frac{5q^2r_{0s}^{\,8}}{7z^2}+\frac{2q^2r_{0s}^{\,10}}{3z^2}+\ldots\right]$$

and (4.2)

$$z=\frac{5}{2}\left(\frac{r_\psi}{R_\psi}\right)^3-\frac{3}{2}\left(\frac{r_\psi}{R_\psi}\right)^5$$

where $$r_0=\frac{z}{\psi-q} \quad (4.3)$$

Further let ρ_ψ denote the value of density on an equipotentials $\psi =$ constant. The density distribution law of rotationally and tidally distorted Prasad model is given as

$$\rho_\psi = \rho_c\left(1-\frac{r_\psi^2}{R_\psi^2}\right) \quad (4.4)$$

On substituting the value of r_ψ and R_ψ from equation (4.1) and (4.2) in equation (4.4) we get

$$\rho_\psi = \rho_c\left[1-\frac{D^2 r_0^{\ 2}}{R_\psi^{\ 2}}\left\{1+\frac{4nr_0^{\ 3}}{3z}+\left(\frac{8q^2r_0^{\ 6}}{5z^2}+\frac{16nq}{15z^2}+\frac{172n^2}{45\,z^2}\right)+\frac{10q^2r_0^{\ 8}}{7z^2}+\frac{4q^2r_0^{\ 10}}{3z^2}+\ldots\right\}\right] \tag{4.5}$$

On substituting value of ρ_ψ from (4.5) in (3.14a) of chapter III and integrating w.r.t. r_0 and using the fact that $M_\psi = 0$ at center $r_0 = 0$ we get

$$M_\psi = \frac{4\pi\rho_c D^3 r_0^{\ 3}}{3}[1-\frac{3D^2}{5R_\psi^{\ 2}}r_0^{\ 2}+\frac{2nr_0^{\ 3}}{z}-\frac{2nR^2r_0^{\ 5}}{zR_\psi^{\ 2}}+\left(\frac{12q^2r_0^{\ 6}}{5z^2}+\frac{8nq}{15z^2}+\frac{32n^2}{5z^2}\right)r_0^6$$
$$+\left\{\frac{15q^2}{7z^2}-\frac{12q^2D^2}{5\,z^2R_\psi^{\ 2}}-\frac{8nqD^2}{5\,z^2R_\psi^2}-\frac{116n^2}{15\,z^2}\right\}r_0^{\ 8}+(\frac{2q^2}{5z^2}-\frac{15q^2R^2}{7\,z^2R_\psi^{\ 2}})r_0^{\ 10}+\ldots] \tag{4.6}$$

Similarly on substituting ρ_ψ from (4.5) and M_ψ from (4.6) in (3.14 (b)) of chapter III and integrating with respect to r_0 we get

$$P_\psi = \frac{2\pi G\rho_c^2 D^2}{3}\left[K-r_0^2+\frac{4D^2r_0^{\ 4}}{5R_\psi^{\ 2}}-\frac{4nr_0^{\ 5}}{5z}-\frac{D^4r_0^2}{5R_\psi^{\ 2}}+\frac{32nD^2r_0^{\ 7}}{21zR_\psi^{\ 2}}\right.$$
$$-\left(\frac{q^2r_0^{\ 8}}{2z^2}+\frac{nq}{3z^2}+\frac{4n^2}{3z^2}\right)r_0^8.-\frac{28nD^4r_0^{\ 9}}{45z\ R_\psi^{\ 4}}+$$
$$\left.+\left(-\frac{3q^2r_0^{\ 10}}{10z^2}+\frac{144D^2q^2r_0^{\ 10}}{125z^2R_\psi^{\ 2}}+\frac{96D^2nq}{125\,z^2R_\psi^2}+\frac{4225n^2}{1125z^2}\right)r_0^{10}+\ldots\right] \tag{4.7}$$

where K is a constant of integration whose value may be calculated by using boundary condition say $P_\psi = 0$ at $r_0 = r_{0s}$. This yield

$$K = r_{0s}^{2} - \frac{4D^{2}r_{0s}^{4}}{5R_{\psi}^{2}} + \frac{4\,n\,r_{0s}^{5}}{5z} + \frac{D^{4}r_{0s}^{6}}{5R_{\psi}^{2}} - \frac{32\,n\,D^{2}r_{0s}^{7}}{21\,z\,R_{\psi}^{2}} + \left(\frac{q^{2}r_{0s}^{8}}{2z^{2}} + \frac{n\,q}{3z^{2}}\right.$$
$$\left. + \frac{28\,n\,D^{4}r_{0s}^{9}}{45\,z\,R_{\psi}^{4}} - \left(\frac{3q^{2}}{10\,z^{2}} + \frac{144\,D^{2}q^{2}}{125\,z^{2}R_{\psi}^{2}} + \frac{96\,D^{2}n\,q}{125\,z^{2}R_{\psi}^{2}} + \frac{4225\,n^{2}}{1125\,z^{2}}\right)r \qquad (4.8)$$

Similarly the volume V_{ψ}, surface area S_{ψ}, $\bar{g}$ and $\overline{g^{-1}}$ of rotationally and tidally distorted Prasad model are obtained as

$$V_{\psi} = \frac{4\,\pi D\,r_0^3}{3}[1 + \frac{2n\,r_0^3}{z} + \left(\frac{12q^2}{5z^2} + \frac{8n\,q}{5z} + \frac{32n^2}{5z^2}\right)r_0^6 + \frac{15q^2\,r_0^8}{7z^2} + \frac{2q^2\,r_0^{10}}{z^2} + ...] \qquad (4.9)$$

$$S_{\psi} = 4\pi\,r_0^2 D^2\,[1 + \frac{4n\,r_0^3}{3z} + \left(\frac{7q^2}{5z^2} + \frac{14nq}{15z^2} + \frac{56n^2}{15z^2}\right)r_0^6 + \frac{9q^2\,r_0^8}{7z^2} + \frac{11q^2\,r_0^{10}}{9z^2} + ...] \qquad (4.10)$$

$$\bar{g} = \frac{zGM_{\psi}}{r_0^2 D^2}[\,1 - \frac{8n\,r_0^3}{3z} - \left(\frac{3q^2}{z^2} + \frac{2nq}{z^2} + \frac{40n^2}{9z^2}\right)r_0^6 - \frac{51q^2 r_0^8}{14z^2} - \frac{13q^2\,r_0^{10}}{3z^2} + ...] \qquad (4.11)$$

$$\overline{g^{-1}} = \frac{r_0^2 D^2}{zGM_{\psi}}[1 + \frac{8n\,r_0^3}{3z} + \left(\frac{31q^2}{5z^2} + \frac{62nq}{15z^2} + \frac{584n^2}{45z^2}\right)r_0^6 + \frac{101q^2 r_0^8}{14z^2} + \frac{75q^2 r_0^{10}}{9z^2} + ...] \qquad (4.12)$$

4.2 EQUILIBRIUM STRUCTURE OF A CLASS OF ROTATIONALLY AND TIDALLY DISTORTED COMPOSITE MODELS

In this section we consider the problem of determining equilibrium structures of a class of rotationally and/or tidally distorted composite models with cores in which density varies according to the law $\rho = \rho_c(1 - x^2)$ and which are surrounded by envelope in which density follows the law $\rho = \rho_c / x^2$. Taking into account the effect of mass variation inside the star on its equipotential surfaces. Suppose the composite gaseous sphere is rotating about its axis and is also the primary component of a binary system.

Let r_ψ denote the radius of the topologically equivalent spherical model which corresponds to an equipotential surface ψ = constant of this rotationally and tidally distorted model. Also let R_ψ be the value of r_ψ on the outermost equipotential surface of the model and $b\,R_\psi$ the value of r_ψ for the equipotential surface of the interface between the envelope and the core of the model. Further let $\rho_{\psi c}$ denote the value of density on an equipotential surface of the core of the distorted model which corresponds to the radial distance r_ψ of the topologically equivalent spherical model. Corresponding to the density distribution law $\rho = \rho_c\left(1 - x^2\right)$ in the core of the original undistorted model we suppose that in the core $(0 \le r_\psi \le bR_\psi)$ of the distorted model density distribution on its equipotential surfaces is given by

$$\rho_{\psi c} = \rho_c\left(1 - \frac{r_\psi^2}{R_\psi^2}\right) \qquad 0 \le r_\psi \le b\,R_\psi \qquad (4.13)$$

$$z_c = \frac{M_{\psi c}}{(M_{\psi e})_{r_0 = r_{0s}}} \qquad (4.14)$$

where $M_{\psi c}$ represents the value of M_ψ in the core and is evaluated from equation (4.6) replacing z by z_c and appearing in the expression $(M_{\psi c})_{r_0 = r_{0s}}$ is computed from (4.17) on putting $r_0 = r_{0s}$ and $z_e = 1$.

Similarly for the envelope of the distorted model we shall assume that density distribution on its equipotential surfaces follows the law

$$\rho_{\psi e} = \rho_c\, b^2 (1 - b^2) \frac{R_\psi^{\,2}}{r_\psi^2} \qquad \mathrm{b}R_\psi \le r_\psi \le R_\psi \qquad (4.15)$$

where $\rho_{\psi e}$ denotes the density on an equipotential surface in the envelope of the distorted model which corresponds to a distance r_ψ from the centre of the equivalent spherical model.

Let n denote the nondimensional form the square of the angular velocity of rotation ω, q the ratio of the mass of the companion causing tidal distortions to the mass of the primary under investigations. Also let ψ_s^* denote the value of potential ψ^* (3.1) on the outermost equipotential surface. Then on the outermost surface the value of r_ψ denoted by R_ψ is given by (4.2)

Now for points inside the envelope $(bR_\psi \le r_\psi \le R_\psi)$ on substituting for r_ψ from (4.1) in (4.15) we get

$$\rho_{\psi e} = \frac{\rho_c b^2(1-b^2)R_\psi^2}{D^2 r_0^2}\left[1-\frac{4n}{3z_e}r_0^3-\left(\frac{8}{5z_e^2}q^2+\frac{16}{15z_e^2}nq+\frac{92}{45z_e^2}n^2\right)r_0^6-\frac{10}{7z_e^2}q^2r_0^8-\right.$$
$$\left.-\frac{4}{3z_e^2}q^2r_0^{10}+\ldots\right] \tag{4.16}$$

On substituting the value of $\rho_{\psi e}$ from (4.16) in (3.14a) and integrating w.r.t. r_0 we get

$$M_{\psi e}=4\pi D\rho_c b^2(1-b^2)R_\psi{}^2[M_{01}+r_0+\frac{2n}{3z_e}r_0{}^4+(\frac{4}{z_e^2}q^2+\frac{8}{15z_e^2}n\,q+$$
$$+\frac{76}{45z_e^2}n^2)r_0{}^7+\frac{5}{z_e^2}q^2r_0{}^9+\frac{2}{z_e^2}q^2r_0{}^{11}+\ldots] \tag{4.17}$$

where $M_{\psi e}$ is the mass contained within the equipotential surface $\psi =$ constant in the envelope and M_{01} is a constant of integration.

In order to ensure the continuity of mass across the interface we must have $M_{\psi c} = M_{\psi e}$ for $r_0 = r_{0i}$ Using this we have from (4.6) and (4.17) we have

$$M_{01} = \frac{D^2 r_{0i}^3}{3b^2(1-b^2)R_\psi^2}\Bigg[1 - \frac{3D^2}{5R_\psi^2} r_{0i}^2 + \frac{2n}{z_e} r_{0i}^3 - \frac{2nD^2}{z_e R_\psi^2} r_{0i}^5 + \left(\frac{12}{5z_e^2} q^2 + \frac{8}{5z_e^2} nq + \frac{32}{5z_e^2} n^2\right) r_{0i}^6$$

$$+ \left\{\frac{15}{7z_e^2} q^2 - \frac{D^2}{R_\psi^2}\left(\frac{12}{5z_e^2} q^2 + \frac{8}{5z_e^2} nq + \frac{116}{15z_e^2} n^2\right)\right\} r_{0i}^8 + \frac{2}{z_e^2} q^2 - \frac{15D^2}{7z_e^2 R_\psi^2} q^2 r_{0i}^{10} + \dots \Bigg] -$$

$$- r_{oi}\left[1 + \frac{2n}{3z_e} r_{oi}^3 + \left(\frac{4}{5z_e^2} q^2 + \frac{8}{15z_e^2} nq + \frac{76}{45z_e^2} n^2\right) r_{oi}^6 + \frac{5}{7z_e^2} q^2 r_{oi}^8 + \frac{2}{3z_e^2} q^2 r_{oi}^{10} + \dots\right]$$

(4.18)

where r_{0i} the value of r_0 at the interface between the envelope and the core (where $r_\psi = b R_\psi$) and is given by

$$r_{0i} = b_0\left[1 - \frac{2n}{3z_e} b_0^3 - \left(\frac{4}{5z_e^2} q^2 + \frac{8}{15z_e^2} nq - \frac{4}{5z_e^2} n^2\right) b_0^6 - \frac{5}{7z_e^2} q^2 b_0^8 - \frac{2}{3z_e^2} q^2 b_0^{10} \dots\right]$$

with $b_0 = \dfrac{bR_\psi}{D}$ (4.19)

and $z_e = \dfrac{M_{\psi e}}{(M_{\psi e})_{r_0 = r_{0s}}}$ (4.20)

Again substituting for $\rho_{\psi e}$ from (4.16) and $M_{\psi e}$ from (4.17) in (3.14(b)) and integrating we get

$$P_{\psi e} = \frac{4\pi G\rho_c^2 b^4 (1-b^2)^2 R_\psi^4 M_{01}}{3D^2 r_0^3}\Bigg[1 + \frac{3}{2M_{01}} r_0 + 3C_1 r_0^3 + \frac{4n}{z_e}(\log r_0) r_0^3 + \frac{2n}{z_e M_{01}} r_0^4$$

$$+ \left(\frac{2}{z_e^2} q^2 + \frac{4}{3z_e^2} nq + \frac{28}{9z_e^2} n^2\right) r_0^6 + \frac{3}{4M_{01}}\left(\frac{6}{5z_e^2} q^2 + \frac{4}{5z_e^2} nq + \frac{104}{45z_e^2} n^2\right) r_0^7 + \frac{87}{70z_e^2} q^2 r_0^8$$

$$+ \frac{19}{28z_e^2 M_{01}} q^2 r_0^9 \; \frac{20}{21z_e^2} q^2 r_0^{10} + \frac{7}{12z_e^2 M_{01}} q^2 r_0^{11} + \dots \Bigg]$$

(4.21)

where $P_{\psi e}$ is the value of pressure on the equipotential ψ = constant inside the envelope and C_1 is a constant of integration. At the free surface $(r_0 = r_{0s}), P_{\psi e} = 0.$ Therefore from (3.9) it becomes

$$C_1=-\frac{1}{3r_{0s}^3}\Bigg[1+\frac{3}{2M_{01}}r_{0s}+4n(\log r_{0s})r_{0s}^3+\frac{2n}{z_e M_{01}}r_{0s}^4$$
$$+\left(\frac{2q^2}{z_e^2}+\frac{4nq}{3z_e^2}+\frac{28n^2}{9z_e^2}\right)r_{0s}^6+\frac{3}{4M_{01}}\left(\frac{6q^2}{5z_e^2}+\frac{4nq}{5z_e^2}+\frac{104n^2}{45z_e^2}\right)r_{0s}^7+\frac{87q^2}{70z_e^2}r_{0s}^8$$
$$+\frac{19q^2}{28\,z_e^2M_{01}}r_{0s}^9+\frac{20q^2}{21z_e^2}r_{0s}^{10}+\frac{7q^2}{12\,z_e^2M_{01}}r_{0s}^{11}+\ldots\Bigg] \tag{4.22}$$

Also the pressure must be continuous across the interface. Hence from (4.7) and (4.21)

$$K_1^2=\frac{2b^4(1-b^2)^2R_\psi^4M_{01}}{}\Bigg[1+\frac{3}{2M_{01}}r_{0i}+3C_1r_{0i}^3+\frac{4n(\log r_{0i})r_{0i}^3}{z_e}$$
$$+\frac{2n}{z_eM_{01}}r_{0i}^4+\left(\frac{2q^2}{z_e^2}+\frac{4nq}{3z_e^2}+\frac{28n^2}{9z_e^2}\right)r_{0i}^6+\frac{3}{4M_{01}}\left(\frac{6q^2}{5z_e^2}+\frac{4nq}{5z_e^2}+\frac{104n^2}{45z_e^2}\right)r_{0i}^7+\frac{87q^2}{70z_e^2}r_{0i}^8$$
$$+\frac{19q^2}{28\,z_e^2M_{01}}r_{0i}^9+\frac{20q^2}{21z_e^2}r_{0i}^{10}+\frac{7q^2}{12z_e^2M_{01}}r_{0i}^{11}+\ldots\ \Bigg]+r_{0i}^2\Bigg[1-\frac{4D^2}{5R_\psi^2}r_{0i}^2+\frac{4n}{5z_c}r_{0i}^3+.$$
$$+\frac{D^4}{5R_\psi^4}r_{0i}^4-\frac{32nD^2}{21z_cR_\psi^2}r_{0i}^5+\left(\frac{q^2}{2z_c^2}+\frac{1nq}{3z_c^2}+\frac{4n^2}{3z_c^2}\right)r_{0i}^6+\frac{28nD^4}{45z_c\,R_\psi^4}r_{0i}^7-$$
$$-\left\{\frac{8D^2}{25R_\psi^2}\left(\frac{18q^2}{5z_c^2}+\frac{12nq}{5z_c^2}+\frac{532n^2}{45z_c^2}\right)-\frac{3q^2}{10z_c^2}\right\}r_{0i}^8-$$
$$-\left\{\frac{5q^2}{27z_c^2}-\frac{82q^2D^2}{105z_c^2R_\psi^2}+\frac{D^4}{10R_\psi^4}\left(\frac{26q^2}{5z_c^2}+\frac{52nq}{15z_c^2}+\frac{904n^2}{45z_c^2}\right)\right\}r_{0i}^{10}+..\ \Bigg].. \tag{4.23}$$

Thus for the composite model distorted by the combined effects of rotation and tidal forces, the value of ρ_ψ, M_ψ, P_ψ and on various equipotentials inside the core are given by (4.5), (4.6) and (4.7) respectively, and on the equipotential surfaces in the envelope, are given by (4.16), (4.17) and (4.21) respectively.

On setting $n=q=0$ we get the equilibrium structure of the original undistorted model. On setting $n=0$ or $q=0$ separately we get the equilibrium structure of the model which is distorted by the tidal effects alone or rotational effects alone. Also on setting $n=\frac{q+1}{2}$ we obtain the equilibrium structure of the rotationally and tidally distorted primary component of a synchronously rotating binary star system. By changing the value of $b\,(0 \le b \le 1)$ we can get equilibrium structures of models for different positions of the interface between the envelope and the core. On setting $b=0$ (no core) we get Roche model while on setting $b=1$ (no envelope) we get Prasad model.

4.3 NUMERICAL EVALUATION OF STRUCTURE FOR PRASAD MODEL AND COMPOSITE MODELS

For a better appreciation of the effects of rotation and tidal distortions on the values of density, mass and pressure at various points inside the star we have used equations (4.5), (4.6) and (4.7) to numerically compute the values of ρ_ψ, M_ψ and P_ψ at various points inside Prasad model. In the case of composite models values of these parameters computed in core again using (4.5), (4.6) and (4.7) and in envelope (4.16), (4.17) and (4.20) for values of interface $b=0.3$, 0.5 and 0.7 for different values of distortion parameters n and q. While evaluating various physical parameters of the composite model, we need the value of z_c in the core as well as z_e in the envelope. These two variables can be computed from (4.14) and (4.20). The results are presented in Tables 4.1. (a, b , c, d) and Table 4.2 (a, b, c, d) for Prasad model and composite models respectively.

4.4 ANALYSIS OF RESULTS

The results presented in Tables 4.1 (a ,b, c, d) and 4.2 (a, b, c, d) give the values of certain structures parameters and related observable quantities of undistorted, rotationally distorted, tidally distorted and rotationally and tidally distorted Prasad model as well as composite model for ψ=5.0. Results show that with the modification of expression for potential to account for mass variation inside the star on its equipotentials surface.our results show only marginal effects. No specific trend is observed.

The results presented in Tables 4.3, 4.4, 4.5 and 4.6 show the values of $M_\psi, P_\psi, V_\psi, S_\psi$ for various types of rotationally and or tidally distorted composite models with interface at b = 0.3 , b=0.5 and b=0.7 for ψ_s^*=5.0. The results shown in parenthesis are their corresponding value earlier obtained by Agarwal (2). Our results indicate no significant change in these values as well as in comparison to the result shown in parenthesis.

Table 4.1(a) : Structure Parameters of Undistorted Stars For Prasad Model

$\psi = 5, n = .0, q = 0$

X	V_ψ	S_ψ	ρ_ψ	M_ψ	P_ψ	σ	ε	T_e/T_P	L_e/L_P
0.1	0.00001	0.00041	0.99000	0.00248	0.00004	0.00000	0.00000	0.14142	1.00000
0.2	0.00006	0.00161	0.96000	0.01951	0.00028	0.00000	0.00000	0.20005	1.00000
0.3	0.00022	0.00362	0.91000	0.06384	0.00081	0.00000	0.00000	0.24497	1.00000
0.4	0.00052	0.00642	0.84000	0.14464	0.00150	0.00000	0.00000	0.28285	1.00000
0.5	0.00102	0.01040	0.75000	0.26562	0.00209	0.00000	0.00000	0.31626	1.00000
0.6	0.00171	0.01443	0.64000	0.42336	0.00228	0.00000	0.00000	0.34648	1.00000
0.7	0.00272	0.01967	0.51000	0.60539	0.00190	0.00000	0.00000	0.37417	1.00000
0.8	0.00402	0.02565	0.36000	0.78848	0.00111	0.00000	0.00000	0.40009	1.00000
0.9	0.00585	0.03246	0.19000	0.93676	0.00032	0.00000	0.00000	0.42425	1.00000
1.0	0.00800	0.04000	0.00000	1.00006	0.00000	0.00000	0.00000	0.44728	1.00000

Table 4.1(b) : Structure Parameters of Rotationally Distorted Stars For Prasad Model $\psi = 5, n = .1, q = 0$

X	V_ψ	S_ψ	ρ_ψ	M_ψ	P_ψ	σ	ε	T_e/T_P	L_e/L_P
0.1	0.00001	0.00040	0.99000	0.00242	0.00024	0.00032	0.00032	0.14139	0.99872
0.2	0.00006	0.00160	0.96002	0.01950	0.01404	0.00032	0.00032	0.19995	0.99865
0.3	0.00022	0.00360	0.91000	0.06379	0.01259	0.00033	0.00033	0.24490	0.99862
0.4	0.00051	0.0064 0	0.84009	0.14452	0.01037	0.00035	0.00035	0.28279	0.99855
0.5	0.00100	0.0104 0	0.75014	0.26543	0.00787	0.00037	0.00037	0.31616	0.99848
0.6	0.00173	0.0144 0	0.64018	0.42308	0.00537	0.00040	0.00040	0.34632	0.99830
0.7	0.00276	0.01960	0.5102	0.60509	0.00314	0.00045	0.00045	0.37408	0.99812
0.8	0.00403	0.02561	0.36002	0.78822	0.00141	0.00052	0.00051	0.39989	0.99796
0.9	0.00582	0.03242	0.19019	0.93664	0.00032	0.00062	0.00062	0.42413	0.99757
1.0	0.00800	0.04000	0.00000	1.0000	0.00000	0.0008	0.00079	0.44700	0.99607

Table 4.1(c) : Structure Parameters of Tidally Distorted Stars For Prasad Model $\psi = 5, n = 0, q = .1$

X	V_ψ	S_ψ	ρ_ψ	M_ψ	P_ψ	σ	ε	$T_e T_p$	$L_e L_p$
0.1	0.00001	0.00042	0.95000	0.00248	0.00003	0.00034	0.00034	0.14283	0.99823
0.2	0.00006	0.00166	0.96000	0.01952	0.00023	0.00036	0.00036	0.20195	0.99815
0.3	0.00022	0.00375	0.91000	0.06385	0.00084	0.00038	0.00038	0.24736	0.99806
0.4	0.00054	0.00664	0.84000	0.14464	0.00156	0.00040	0.00040	0.28566	0.99797
0.5	0.00105	0.01041	0.75000	0.26562	0.00218	0.00044	0.00044	0.31934	0.99772
0.6	0.00182	0.01497	0.64000	0.42336	0.00237	0.00048	0.00048	0.34988	0.99744
0.7	0.00291	0.02040	0.51000	0.60539	0.00198	0.00055	0.00055	0.37785	0.99710
0.8	0.00435	0.02665	0.36000	0.78848	0.00116	0.00064	0.00064	0.40392	0.99668
0.9	0.00618	0.03372	0.19000	0.93676	0.00033	0.00079	0.00078	0.42830	0.99586
1.0	0.00850	0.04165	0.00000	1.00000	0.00000	0.00103	0.0010	0.45141	0.99446

Table 4.1(d) : Structure Parameters of Rotationally and Tidally Distorted Stars for Prasad Model $\psi=5.0, n=0.1, q=0.1$

X	V_ψ	S_ψ	ρ_ψ	M_ψ	P_ψ	σ	ε	T_eT_p	L_eL_p
0.1	0.00001	0.00041	0.99000	0.00248	0.00005	0.00069	0.00069	0.14282	0.99687
0.2	0.00006	0.00166	0.96000	0.01950	0.00028	0.00071	0.00071	0.20193	0.99677
0.3	0.00023	0.00375	0.91000	0.06379	0.00083	0.00074	0.00074	0.24736	0.99662
0.4	0.00054	0.00666	0.84010	0.14451	0.00156	0.00078	0.00078	0.28556	0.99642
0.5	0.00106	0.01041	0.75015	0.26541	0.00218	0.00084	0.00084	0.31928	0.99613
0.6	0.00183	0.01499	0.64020	0.42307	0.00237	0.00092	0.00092	0.34972	0.99574
0.7	0.00292	0.02041	0.51024	0.60507	0.00198	0.00103	0.00103	0.37770	0.99520
0.8	0.00436	0.02667	0.36025	0.78820	0.00116	0.00120	0.00120	0.40387	0.99441
0.9	0.00620	0.03376	0.19020	0.93663	0.00032	0.00145	0.00145	0.42824	0.99319
1.0	0.008510	0.04166	0.00000	1.00000	0.00000	0.00188	0.00188	0.45121	0.99110

Table 4.2(a) : Density ρ_ψ **in Units of** ρ_c for Composite model

X	n=0.0, q=0.0 $\psi_s^\bullet=5.0$	n=0.0, q=0.1 $\psi_s^\bullet=5.0$	n=0.1, q=0.0 $\psi_s^\bullet=5.0$	n=0.1, q=0.1 $\psi_s^\bullet=5.0$	b=0.3 n=0.55, q=0.1 $\psi_s^\bullet=5.0$
1	2	3	4	5	6
0.1	0.99000 (0.99000)	0.990001 (0.99000)	0.99001 (0.99001)	0.99213 (0.99001)	0.99863 (0.99006)
0.2	0.96000 (0.96000)	0.960001 (0.96000)	0.96004 (0.96004)	0.96113 (0.96005)	0.96612 (0.96025)
0.3	0.91000 (0.91000)	0.910001 (0.91000)	0.91009 (0.91009)	0.91077 (0.91010)	0.91425 (0.91055)
0.4	0.51187 (0.51188)	0.51187 (0.51188)	0.51239 (0.51239)	0.51403 (0.51242)	0.52405 (0.51491)
0.5	0.32760 (0.32760)	0.32760 (0.32760)	0.32841 (0.32791)	0.32846 (0.32793)	0.33245 (0.32941)
0.6	0.22750 (0.22750)	0.22750 (0.22750)	0.22788 (0.22769)	0.22790 (0.22770)	0.22976 (0.22863)
0.7	0.16714 (0.16714)	0.16714 (0.16714)	0.16733 (0.16726)	0.16734 (0.16727)	0.16825 (0.16784)
0.8	0.12797 (0.12797)	0.12796 (0.12797)	0.12805 (0.12804)	0.128064 (0.12804)	0.12849 (0.12837)
0.9	0.1011 (0.10111)	0.10111 (0.10111)	0.10114 (0.10114)	0.10114 (0.10114)	0.10131 (0.10129)
1.0	0.08190 (0.08190)	0.08190 (0.08190)	0.08190 (0.08190)	0.08190 (0.08190)	0.08190 (0.08190)

Table 4.2(b) : Density ρ_ψ in Units of ρ_c for Composite model

b=0.5

X	N=0.0, q=0.0 $\psi_s^\bullet = 5.0$	n=0.0, q=0.1 $\psi_s^\bullet = 5.0$	n=0.1, q=0.0 $\psi_s^\bullet = 5.0$	n=0.1, q=0.1 $\psi_s^\bullet = 5.0$	n=0.55, q=0.1 $\psi_s^\bullet = 5.0$
1	2	3	4	5	6
0.1	0.99000	0.99000	0.99373	0.99394	0.99943
0.2	0.9600 0	0.96000	0.96208	0.96221	0.97154
0.3	0.91000	0.91000	0.91141	0.91150	0.91825
0.4	0.84000	0.84000	0.84107	0.84114	0.84631
0.5	0.75000	0.75000	0.75085	0.75090	0.75504
0.6	0.52080	0.52083	0.52187	0.52194	0.52707
0.7	0.38265	0.38265	0.38313	0.38316	0.38549
0.8	0.29296	0.29296	0.29318	0.29319	0.29425
0.9	0.23148	0.23148	0.23156	0.23156	0.23194
1.0	0.18750	0.18750	0.18750	0.18750	0.18750

Table 4.2(c) : Density ρ_ψ in Units of ρ_c for composite model

b=0. 7

X	n=0.0, q=0.0 $\psi_s^\bullet = 5.0$	n=0.0, q=0.1 $\psi_s^\bullet = 5.0$	n=0.1, q=0.0 $\psi_s^\bullet = 5.0$	n=0.1, q=0.1 $\psi_s^\bullet = 5.0$	n=0.55, q=0.1 $\psi_s^\bullet = 5.0$
1	2	3	4	5	6
0.1	0.99000	0.99000	0.99439	0.99463	0.99756
0.2	0.96000	0.96000	0.96250	0.96265	0.96367
0.3	0.91000	0.91000	0.91170	0.91181	0.91989
0.4	0.84000	0.84000	0.84129	0.84137	0.84759
0.5	0.75000	0.75000	0.75102	0.75109	0.75607
0.6	0.64000	0.64000	0.64083	0.64088	0.64493
0.7	0.51000	0.51000	0.51066	0.51070	0.51394
0.8	0.39040	0.39046	0.39076	0.39078	0.39225
0.9	0.30850	0.30851	0.30862	0.30863	0.30915
1.0	0.24990	0.24990	0.24990	0.24990	0.24990

Table 4.3(a) : Mass M_ψ in Units of $\frac{4}{3}\pi D^3\delta_c \times 10^{-3}$ for Composite model

b=0.3

X	n=0.0, q=0.0 $\psi_s^\bullet = 5.0$	n=0.0, q=0.1 $\psi_s^\bullet = 5.0$	n=0.1, q=0.0 $\psi_s^\bullet = 5.0$	n=0.1, q=0.1 $\psi_s^\bullet = 5.0$	n=0.55, q=0.1 $\psi_s^\bullet = 5.0$
1	2	3	4	5	6
0.1	0.00503 (0.00503)	0.00503 (0.00503)	0.00502 (0.00502)	0.00502 (0.00502)	0.00498 (0.00499)
0.2	0.03952 (0.03953)	0.03952 (0.03953)	0.03946 (0.03947)	0.03946 (0.03946)	0.03916 (0.03916)

0.3	0.12930 (0.12931)	0.12930 (0.12930)	0.12911 (0.12911)	0.12909 (0.12910)	0.12816 (0.12816)
0.4	0.25369 (0.25369)	0.25369 (0.25369)	0.25344 (0.25344)	0.25342 (0.25343)	0.25222 (0.25223)
0.5	0.37807 (0.37808)	0.37807 (0.37808)	0.37778 (0.37778)	0.37776 (0.37777)	0.37636 (0.37636)
0.6	0.50246 ().50246)	0.50246 (0.50246)	0.50214 (0.50214)	0.50212 (0.50213)	0.50061 (0.50061)
0.7	0.062684 (0.62685)	0.62684 (0.62684)	0.62654 (0.62654)	0.62652 (0.62652)	0.62503 (0.62504)
0.8	0.75123 (0.75123)	0.75123 (0.75123)	0.75097 (0.75097)	0.75095 (0.75095)	0.74969 (0.74970)
0.9	0.87561 (0.87562)	0.87561 (0.87562)	0.87545 (0.87545)	0.87544 (0.87544)	0.87465 (0.87466)
1.0	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)	1.0000 (1.0000)

Table 4.3(b) : Mass M_ψ in Units of $\frac{4}{3}\pi D^3 \rho_c \times 10^{-3}$ for composite model

b=0.5

X	n=0.0, q=0.0 $\psi_s^* = 5.0$	n=0.0, q=0.1 $\psi_s^* = 5.0$	n=0.1, q=0.0 $\psi_s^* = 5.0$	n=0.1, q=0.1 $\psi_s^* = 5.0$	n=0.55, q=0.1 $\psi_s^* = 5.0$
1	2	3	4	5	6
0.1	0.00256	0.00256	0.00256	0.00256	0.00254
0.2	0.02015	0.02015	0.02011	0.020116	0.01996
0.3	0.06591	0.06591	0.06581	0.06581	0.06533
0.4	0.14930	0.14930	0.14909	0.14908	0.14808
0.5	0.27419	0.27419	0.27385	0.27383	0.27219
0.6	0.41935	0.41935	0.41899	0.41896	0.41720
0.7	0.56451	0.56451	0.56415	0.56413	0.56240
0.8	0.70967	0.70967	0.70937	0.70935	0.70788
0.9	0.85483	0.85483	0.85464	0.85463	0.85371
1.0	1.0000	1.0000	1.0000	1.0000	1.0000

Table 4.3(c) : Mass M_ψ in Units of $\frac{4}{3}\pi D^3 \rho_c \times 10^{-3}$ for composite model

b=0.7

X	n=0.0, q=0.0 $\psi_s^* = 5.0$	n=0.0, q=0.1 $\psi_s^* = 5.0$	n=0.1, q=0.0 $\psi_s^* = 5.0$	n=0.1, q=0.1 $\psi_s^* = 5.0$	n=0.55, q=0.1 $\psi_s^* = 5.0$
1	2	3	4	5	6
0.1	0.00212	0.00212	0.00212	0.00212	0.00210
0.2	0.01671	0.01671	0.01669	0.01668	0.01656

0.3	0.05468	0.05468	0.05460	0.05459	0.05420
0.4	0.12387	0.12387	0.12369	0.12368	0.12285
0.5	0.22748	0.22748	0.22720	0.22718	0.22582
0.6	0.36256	0.36255	0.36219	0.36217	0.36037
0.7	0.51846	0.51846	0.51807	0.51804	0.51613
0.8	0.67897	0.67896	0.67864	0.67861	0.67699
0.9	0.83948	0.83948	0.83927	0.83926	0.83824
1.0	1.0000	1.0000	1.0000	1.0000	1.0000

Table 4.4(a) : Pressure P_ψ in Units of $\frac{2}{3}\pi GD^2\rho_c^2 \times 10^{-2}$ for composite model

b=0.3

X	n=0.0, q=0.0 $\psi_s^* = 5.0$	n=0.0, q=0.1 $\psi_s^* = 5.0$	n=0.1, q=0.0 $\psi_s^* = 5.0$	n=0.1, q=0.1 $\psi_s^* = 5.0$	n=0.55, q=0.1 $\psi_s^* = 5.0$
1	2	3	4	5	6
0.1	0.72973 (0.72973)	0.75999 (0.75982)	0.73056 (0.73056)	0.76206 (0.76074)	0.76492 (0.76492)
0.2	0.61448 (0.61448)	0.63984 (0.63982)	0.61531 (0.61531)	0.64073 (0.64073)	0.64488 (0.64488)
0.3	0.43475 (0.43475)	0.45268 (0.45268)	0.43555 (0.43555)	0.45356 (0.45356)	0.45758 (0.45758)
0.4	0.26872 (0.26873)	0.26631 (0.27981)	0.26929 (0.26929)	0.28043 (0.28043)	0.28327 (0.28327)
0.5	0.16783 (0.16783)	0.17012 (0.17476)	0.16822 (0.16822)	0.17519 (0.17519)	0.17716 (0.17716)
0.6	0.10491 (0.10491)	0.10722 (0.10924)	0.10519 (0.10519)	0.10954 (0.10954)	0.11094 (0.11094)
0.7	0.06362 (0.06363)	0.0652 (0.06625)	0.06383 (0.06383)	0.06647 (0.06647)	0.06749 (0.06749)
0.8	0.03525 (0.03525)	0.03613 (0.03670)	0.03540 (0.03540)	0.03687 (0.03687)	0.03763 (0.03763)
0.9	0.01497 (0.01497)	0.01523 (0.01559)	0.01508 (0.01508)	0.01571 (0.01571)	0.01629 (0.01629)
1.0	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)

Table 4.4(b) : Pressure P_ψ in Units of $\frac{2}{3}\pi GD^2\rho_c^2 \times 10^{-2}$ for composite model

b=0.5

X	n=0.0, q=0.0 $\psi_s^{\bullet}=5.0$	n=0.0, q=0.1 $\psi_s^{\bullet}=5.0$	n=0.1, q=0.0 $\psi_s^{\bullet}=5.0$	n=0.1, q=0.1 $\psi_s^{\bullet}=5.0$	n=0.55, q=0.1 $\psi_s^{\bullet}=5.0$
1	2	3	4	5	6
0.1	1.42594	1.4852	2.09033	2.2374 2	2.85234
0.2	1.31069	1.3648	1.37564	1.43687	1.84533
0.3	1.13096	1.1776	1.14997	1.19867	1.30128
0.4	0.90426	0.94156	0.912158	0.95030	0.99143
0.5	0.65312	0.68005	0.65692	0.68426	0.70382
0.6	0.43240	0.43020	0.43412	0.41835	0.36293
0.7	0.27149	0.27327	0.27230	0.26780	0.24237
0.8	0.15390	0.15523	0.15426	0.15225	0.13850
0.9	0.0664	0.06624	0.06655	0.0644	0.05604
1.0	0.0000	0.00182	0.00000	0.00304	0.0086

Table 4.4(c) : Pressure P_ψ in Units of $\frac{2}{3}\pi GD^2\rho_c^2 \times 10^{-2}$ for composite model

b =0.7

X	n=0.0, q=0.0 $\psi_s^{\bullet}=5.0$	n=0.0, q=0.1 $\psi_s^{\bullet}=5.0$	n=0.1, q=0.0 $\psi_s^{\bullet}=5.0$	n=0.1, q=0.1 $\psi_s^{\bullet}=5.0$	n=0.55, q=0.1 $\psi_s^{\bullet}=5.0$
1	2	3	4	5	6
0.1	1.66533	1.73464	2.62940	2.84362	2.8494
0.2	1.55008	1.61408	1.63747	1.71108	2.28869
0.3	1.37033	1.42688	1.39585	1.45513	1.59441
0.4	1.14366	1.19082	1.15429	1.20265	1.25825
0.5	0.89252	0.92932	0.89770	0.93506	0.96179
0.6	0.64241	0.66890	0.64507	0.67184	0.68545
0.7	0.41922	0.43650	0.42053	0.43795	0.44463
0.8	0.2420	0.24071	0.24260	0.23375	0.20156
0.9	0.10576	0.10372	0.10597	0.09963	0.08087
1.0	0.00000	0.00392	0.00000	0.00655	0.01859

Table 4.5 (a) : Volumes of Rotationally and Tidally Distorted Composite Models in Units of $\frac{4}{3}\pi D^3$

b=0.3

x	$n = 0.0,\ q = 0.0$, $\psi_s^* = 5.0$	$n = 0.0,\ q = 0.1$, $\psi_s^* = 5.0$	$n = 0.1,\ q = 0.0$, $\psi_s^* = 5.0$	$n = 0.1,\ q = 0.1$, $\psi_s^* = 5.0$	$n = 0.55,\ q = 0.01$, $\psi_s^* = 5.0$
1	2	3	4	5	6
0.1	0.008	0.00850	0.01055	0.01138	0.02469
0.2	0.008	0.00850	0.00832	0.00886	0.01056
0.3	0.008	0.00850	0.00809	0.00861	0.00913
0.4	0.008	0.00850	0.00805	0.00855	0.00882
0.5	0.008	0.00850	0.00803	0.00853	0.00871
0.6	0.008	0.00850	0.00802	0.00852	0.00866
0.7	0.008	0.00850	0.00801	0.00851	0.00862
0.8	0.008	0.00850	0.00801	0.00851	0.00860
0.9	0.008	0.00850	0.00801	0.00851	0.00859
1.0	0.008	0.00850	0.00801	0.00851	0.00858

Table 4.5 (b) : Volumes of Rotationally and Tidally Distorted Composite Models in Units of $\frac{4}{3}\pi D^3$

b=0.5

x	$n = 0.0,\ q = 0.0$, $\psi_s^* = 5.0$	$n = 0.0,\ q = 0.1$, $\psi_s^* = 5.0$	$n = 0.1,\ q = 0.0$, $\psi_s^* = 5.0$	$n = 0.1,\ q = 0.1$, $\psi_s^* = 5.0$	$n = 0.55,\ q = 0.01$, $\psi_s^* = 5.0$
1	2	3	4	5	6
0.1	0.00800	0.00850	0.01301	0.01416	0.04026
0.2	0.00800	0.00850	0.00863	0.00922	0.01254
0.3	0.00800	0.00850	0.00819	0.00872	0.00973
0.4	0.00800	0.00850	0.00808	0.00859	0.00904
0.5	0.00800	0.00850	0.00804	0.00855	0.00879
0.6	0.00800	0.00850	0.00803	0.00853	0.00869
0.7	0.00800	0.00850	0.00802	0.00852	0.00864
0.8	0.00800	0.00850	0.00801	0.00852	0.00861
0.9	0.00800	0.00850	0.00801	0.00851	0.00859
1.0	0.00800	0.00850	0.00801	0.00851	0.00858

Table 4.5(c): Volumes of Rotationally and Tidally Distorted Composite Models in Units of $\frac{4}{3}\pi D^3$

b=0.7

x	$n = 0.0, q = 0.0$ $\psi_s^* = 5.0$	$n = 0.0, q = 0.1,$ $\psi_s^* = 5.0$	$n = 0.1, q = 0.0$ $\psi_s^* = 5.0$	$n = 0.1, q = 0.1,$ $\psi_s^* = 5.0$	$n = 0.55, q = 0.01,$ $\psi_s^* = 5.0$
1	2	3	4	5	6
0.1	0.00800	0.00850	0.01404	0.01533	0.04679
0.2	0.00800	0.00850	0.00876	0.00937	0.01337
0.3	0.00800	0.00850	0.00823	0.00876	0.00998
0.4	0.00800	0.00850	0.00810	0.00861	0.00915
0.5	0.00800	0.00850	0.00805	0.00856	0.00885
0.6	0.00800	0.00850	0.00803	0.00854	0.00872
0.7	0.00800	0.00850	0.00802	0.00852	0.00865
0.8	0.00800	0.00850	0.00801	0.00852	0.00861
0.9	0.00800	0.00850	0.00801	0.00851	0.00859
1.0	0.00800	0.00850	0.00801	0.00851	0.00858

Table 4.6(a) : Surface Areas of Rotationally and Tidally Distorted Composite Models in Units of $4\pi D^2$

b=0.3

x	$n = 0.0, q = 0.0$ $\psi_s^* = 5.0$	$n = 0.0, q = 0.1,$ $\psi_s^* = 5.0$	$n = 0.1, q = 0.0$ $\psi_s^* = 5.0$	$n = 0.1, q = 0.1,$ $\psi_s^* = 5.0$	$n = 0.55, q = 0.01,$ $\psi_s^* = 5.0$
1	2	3	4	5	6
0.1	0.04000	0.04165	0.04851	0.05108	0.09395
0.2	0.04000	0. 04165	0.04108	0.04285	0.04830
0.3	0.04000	0. 04165	0.04033	0.04201	0.04368
0.4	0.04000	0. 04164	0.04016	0.04183	0.04268
0.5	0.04000	0. 04164	0.04011	0.04177	0.04234
0.6	0.04000	0. 04164	0.04008	0.04174	0.04217
0.7	0.04000	0. 04164	0.04006	0.04172	0.04206
0.8	0.04000	0. 04164	0.04005	0.04171	0.04200
0.9	0.04000	0. 04164	0.04004	0.04170	0.41950
1.0	0.04000	0. 04164	0.04004	0.04169	0.04191

Table 4.6 (b) : Surface Areas of Rotationally and Tidally Distorted Composite Models in Units of $4\pi D^2$

b=0.5

x	$n = 0.0, q = 0.0$, $\psi_s^* = 5.0$	$n = 0.0, q = 0.1$, $\psi_s^* = 5.0$	$n = 0.1, q = 0.0$, $\psi_s^* = 5.0$	$n = 0.1, q = 0.1$, $\psi_s^* = 5.0$	$n = 0.55, q = 0.01$, $\psi_s^* = 5.0$
1	2	3	4	5	6
0.1	0.04	0.04166	0.05669	0.06015	0.14522
0.2	0.04	0. 04165	0.04212	0.04400	0.05483
0.3	0.04	0. 04165	0.04068	0.04236	0.04567
0.4	0.04	0. 04165	0.04028	0.04196	0.04342
0.5	0.04	0. 04164	0.04015	0.04182	0.04261
0.6	0.04	0. 04164	0.04010	0.04176	0.04228
0.7	0.04	0. 04164	0.04007	0.04173	0.04211
0.8	0.04	0. 04164	0.04006	0.04171	0.042021
0.9	0.04	0. 04164	0.04005	0.04170	0.04195
1.0	0.04	0. 04164	0.04004	0.04169	0.04191

Table 4.6(c) : Surface Areas of Rotationally and Tidally Distorted Composite Models in Units of $4\pi D^2$

b=0.7

x	$n = 0.0, q = 0.0$, $\psi_s^* = 5.0$	$n = 0.0, q = 0.1$, $\psi_s^* = 5.0$	$n = 0.1, q = 0.0$, $\psi_s^* = 5.0$	$n = 0.1, q = 0.1$, $\psi_s^* = 5.0$	$n = 0.55, q = 0.01$, $\psi_s^* = 5.0$
1	2	3	4	5	6
0.1	0.04000	0.04166	0.06012	0.06395	0.01664
0.2	0.04000	0. 04165	0.04256	0.04448	0.05754
0.3	0.04000	0. 04165	0.04078	0.04251	0.04650
0.4	0.04000	0. 04165	0.04034	0.04203	0.04379
0.5	0.04000	0. 04165	0.04018	0.04185	0.04281
0.6	0.04000	0. 04164	0.04011	0.04178	0.04238
0.7	0.04000	0. 04164	0.04008	0.04174	0.04215
0.8	0.04000	0. 04164	0.04006	0.04171	0.04203
0.9	0.04000	0. 04164	0.04005	0.04170	0.04196
1.0	0.04000	0. 04164	0.04004	0.04169	0.04191

CHAPTER - V

EQUILIBRIUM STRUCTURE OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED POLYTROPIC MODELS AND PRASAD MODEL

Most of the stars in binary systems are known to be rotating about their axes as well as revolving around their common center of mass. Some of the stars in binary systems are also expected to be rotating differentially. Differential rotation is likely to influence the inner structure and equilibrium configurations of such differentially rotating stars. It is expected that equilibrium structures of such stars in binary systems are also influenced by the combined effects of differential rotation as well as the tidal forces of the companion star. In the present chapter we extend the analysis of chapter III to investigate the problem of determining the equilibrium structures of differentially rotating polytropic model as well as Prasad model of star following a law of differential rotation $\omega = b_1 + b_2 s^2$.

The law of differential rotation selected by us for present study is presented in section 5.1. In section 5.2 we use, the concept of Roche equipotentials which takes into account the effect of mass variation in potential to obtain results for differentially rotating stars in binary systems. In section 5.3 we use Kippenhahn and Thomas approach and the results on Roche equipotentials obtained in section 5.2 to derive the system of differential equations governing the equilibrium structures of differentially rotating and tidally distorted gaseous spheres. The technique is next used in section 5.4 to obtain the equilibrium structures of differentially rotating polytropic models, which are primary components of binary systems. The analysis of section 5.3 is also used in section 5.5 to obtained the equilibrium structures of differentially rotating Prasad model. Certain conclusions based on this study are finally drawn in section 5.6.

5.1 LAWS OF DIFFERENTIAL ROTATION

By differential rotation we mean rotation of a gaseous sphere in which all the fluid elements of the sphere do not have the same angular velocity. Different authors have used different laws of differential rotation to account for some of the observed features of differentially rotating stars. Theoretically the general form of a law of differential rotation for a star rotating about an axis of rotation passing through its centre should be of the type $\Omega=\Omega(s,z)$ in which the angular velocity Ω of rotation is a function of both distance s from the axis of rotation and the latitude z. In fact some of the authors such as Von Zeipel (157), Solberg (138), Hoiland (52), etc., used such types of laws. However, according to Tassoul (149, p. 175) it is perhaps not possible to build a chemically homogenous stellar model in radiative equilibrium with a rotation law of the type $\Omega=\Omega(s,z)$. According to him since in the zones of efficient convection the transport of energy is not by radiation so in such a case Von Zeipel s argument does not apply and therefore, in practice for such a differentially rotating star in equilibrium, law of differential rotation of the form $\Omega=\Omega(s)$ may well be used.

As early as 1865 Faye assumed a law of differential rotation of the type $\omega=b_1+b_2 s^2$ (where ω is the angular velocity of rotation of a fluid element at a distance s from the axis of rotation and b_1, b_2 are certain constants) to account for differential rotation of the Sun s surface. Stoeckly (142) constructed axisymmetric models of differentially rotating polytropes of index 1.5 with a law of a differential rotation.

$$\Omega(s)=\Omega_c\, e^{-\left(a s^2/\xi_e^2\right)} \qquad (5.1)$$

where s is measured from the axis of rotation, Ω_c denotes the angular velocity on the axis of rotation, ξ_e the equatorial radius of the polytropic model, and a is a suitably chosen constant. Ireland (55) calculated the results for gravity-darkening and limb- darkening in a rapidly rotating Roche model of a star subject to the non-uniform rotation assuming $\Omega = \Omega(s)$ where Ω is the angular velocity of the star and s is the distance of a fluid element from axis of rotation. Bodenheimer (12) calculated the structure of chemically homogenous main-sequence stars of mass $15\,M_0$, $30\,M_0$, and $60\,M_0\,(X = 0.70,\ Z = 0.03)$ by specifying a rotation law which gives the angular momentum per unit mass $J(m)$ as a function of m, the mass interior to a given cylinder about the axis of rotation. Haris and Clement (51) presented equilibrium models for slowly rotating stars of $16\,M_0$, $28\,M_0$, and $47\,M_0$ assuming the interior distribution of angular velocity is uniquely determined by the requirement that the azimuthal force near the surface vanishes and the steady state is free from meridian circulation. Geroyannis et al. (42) obtained a complete solution of the structural equation for differentially rotating polytropes by taking differential rotation law which is a function of position and time – dependent homoaxial rotation. Geroyannis and Antonakopoulos (40) studied the structural distortion on the polytropic stars by differential rotation using a law of differential rotation earlier proposed by Clement (24). According to this law , the angular velocity $\omega(s)$ of a fluid element is given by

$$\omega(s) = \left(\sum_{i=1}^{3} a_i\, e^{-b_i\, s^2} \right)^{1/2}, \qquad (5.2)$$

where s is a modified nondimensional cylindrical coordinate and a_i , b_i constants. Komatsu et al. (64) computed equilibrium structures of differentially rotating relativistic polytropes with indices 0.5 and 1.5 using a rotation law determined by specifying the angular momentum $J(\Omega)$. Although, theoretically choice of $J(\Omega)$ is arbitrary, stability criteria impose some constraints on its selection and thus

$$j(\Omega) = A^2(\Omega_c - \Omega), \tag{5.3}$$

where A is positive constant and Ω_c is the angular velocity at the centre of the coordinate system (Ω_c depends implicitly on the value of A which is called rotation parameter. For the Newtonian case this leads to the rotation law of the type

$$\Omega = \frac{\Omega_c A^2}{A^2 + s^2}, \tag{5.4}$$

where $s = r\sin\theta$. When $A \to \infty$, Ω approaches a rigid rotation. When $A \to \infty$, it becomes a J- constant rotation (i. e. the specifying angular momentum is constant in space). Woodard (159) considered a law of differential rotation of the type

$$\Omega(x) = B_0 + B_1 x^2 + B_2 x^4, \tag{5.5}$$

where Ω is an even function of latitude x.

For a differentially- rotating model to be Table against local perturbations, the assumed law of differential rotation should satisfy stability criteria against local perturbations such as one obtained by (Stoeckly, 142). According to this criteria a model rotating differentially according to the law $\omega = \omega(s)$ is Table if

$$\frac{d}{ds}[s^2 \omega(s)] > 0 \tag{5.6}$$

for all s from centre to surface.

In the present study we have preferred to use a law of differential rotation of the type.

$$\omega = b_1 + b_2 s^2 \tag{5.7}$$

where $s = r\sin\theta$. is a nondimension dimensionless measure of the distance of a fluid element from the axis of rotation passing through its centre, b_1, b_2 are suitably chosen arbitrary constants in units of ω^2.This law may be regarded as a Taylor series expansion of a general law of the form $\omega^2 = f(s^2)$ in which terms up to second- order of smallness in a Taylor series expansion of ω^2 are retained. This includes the law $\omega^2 = b_0 + b_1 s^2 + b_2 s^4$, used by Lal (69) as special case and ensures symmetry of ω^2 about the axis of rotation. It may also be considered as the truncated series expansion of the law (5.7) when terms beyond $s^4$are neglected. We have preferred this law of differential rotation in our present study. It not only generates a variety of differential rotation commonly expected in stars, but is also in a form which it can be convent ally subjected to the type of mathematical analysis which we proposed to carry out in the subsequent section of this chapter.

The nature of certain types of differential rotation which can be generated by the law (5.7) for giving different values of $b_1$and b_2 are shown in Table 5.1 For a star rotating differentially according to this law to be Table according to Stoeckly (142) criteria (5.6) must be non- negative for all values of s inside the star. The stability of each of the differential rotation considered by us in Table 5.1 has been analyzed are presented in the same Table.

5.2 ROCHE EQUIPOTENTIALS OF A DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED GAS SPHERES

A binary system of stars consists of a pair of stars in which one of the stars (called the primary) is usually much more massive and larger as compared to its companion star (called secondary) . Most of the binary stars are observed to be rotating about their axes as well as revolving around their common center of mass. Some of the stars in binary systems are also expected to be rotating differentially. Because of the differential rotation and the tidal effects of the companion, the equilibrium structures of stars in such binary systems get influenced by differential rotation as well as tidal effects of the companion stars.

Mohan and Saxena (85), Mohan, Saxena and Agarwal (92) proposed a method for determining the equilibrium structures of rotationally and tidally distorted primary components of stars in binary systems and applied it to main sequence stars. However in their work they consider the rotation of the star to be solid body rotation.

In the present chapter we have used the methodology of Mohan, Saxena and Agarwal (92) to determine the equilibrium structures of a primary components of stars in binary system by assuming that such a star is rotating differentially following a general law of differential rotation of the type $\omega = b_1 + b_2 s^2$ where b_1, b_2 are numerical constants and s, is the distance of rotating fluid element from the axis of rotation.

Following Kopal (65) and Mohan, Lal and Singh (70), we assume that the total mass M_0 of the differentially rotating star, which is primary component of a binary system, is much more massive than its companion star which is assumed to be point mass (i.e. $M_0 > M_1$ where M_1 is mass of the component star). Let D be the mutual separation between the centers of these two masses. Further suppose that the position of the two components of this binary system is referred to a rectangular system of cartesian coordinates having the origin at the center of gravity of mass M_0, the x axis along the line joining the centers of the components, the z axis perpendicular to the plane of the orbit of the two components, $M_0(r)$ is the interior mass of the primary component). The primary star is supposed to be differentially rotating and tidally distorted stellar model. For such a star in the binary system, following Kopal (65) the total potential Ω at a point $P(x, y, z)$ is given by

$$d\Omega = dV_0 + dV_1 + \frac{1}{2}\omega^2(s^2)d(s^2)$$

$$\Omega = V_0 + V_1 + \frac{1}{2}\int \omega^2(s^2)d(s^2) \tag{5.8}$$

where $s^2 = \left[(x - \frac{M_1 D}{M_0 + M_1})^2 + Y^2\right], \left[\frac{M_1 D}{M_0 + M_1}\right]$

being the position of the center of mass of the binary system. Also V_0 and V_1 are respectively the gravitational potential arising due to the primary and the secondary components of the binary system. Assuming Roche model for the primary and point mass for the secondary, the expression (5.8) can be written now as

$$\Omega = \frac{GM_0(r)}{r} + \frac{GM_1}{r_1} + \frac{1}{2}\int \omega^2(s^2)d(s^2) \tag{5.9}$$

for making (5.9) dimensionless, we multiply it throughout by $\frac{D}{GM_0}$ to obtain

$$\Omega = \frac{D\Omega}{GM_0} = \frac{z}{r} + \frac{q}{r_1} + \frac{1}{2}\int_0^{s^2} \omega^2(s^2.)d(s^2) \tag{5.10}$$

Here $r,\ r_1$ and s are in units of D , $q = M_1/M_0$,*and* ω^2 in units of $\frac{GM_0}{D^3}$ and

$z = \frac{M_0(r)}{M_0}$

writing $\omega(s^2) = b_1^2 . + 2b_1 b_2 s^2 + b_2^2 s^4$, (5.10) becomes

$$\psi = \frac{z}{r} + \frac{q}{r_1} + \frac{1}{2}\left[.b_1^2 s^2 + b_1 b_2 s^4 + \frac{1}{3} b_2^2 s^4 \right] \tag{5.11}$$

where b_1 ,and b_2 units of $\frac{GM_0}{D^3}$,

Also

$$s^2 = r^2\left(1-\upsilon^2\right) - \frac{2qr\lambda}{(1+q)} + \frac{q^2}{\left(1+q^2\right)^2} \tag{5.12}$$

using $\lambda = \sin\theta\cos\phi,\ \mu = \sin\theta\sin\phi,\ \upsilon = \cos\theta$ (r, θ, ϕ) being the polar spherical coordinates of the point with center of the star as the origin and θ being measured from the axis of rotation) in (5.12) r is non dimensional measure of the distance (r/D) from the center of the star. So that,

$$\psi = \frac{z}{r} + \frac{q}{r_1} + \frac{1}{2} b_1^2 \left\{ r^2(1-\upsilon^2) - \frac{2qr\lambda}{1+q} + \frac{q^2}{(1+q)^2} \right\} + \frac{b_1 b_2}{2} \left\{ r^2(1-\upsilon^2) - \frac{2qr\lambda}{1+q} + \frac{q^2}{(1+q)^2} \right\}^2$$
$$+ \frac{b_2^2}{6} \left\{ r^2(1-\upsilon^2) - \frac{2qr\lambda}{1+q} + \frac{q^2}{(1+q)^2} \right\}^3 \tag{5.13}$$

As the primary is considered to be much more massive than the secondary (i.e $M_0 >> M_1$) q is small. We also assume that ω^2 is small so that b_1, and b_2 are also small quantities. Therefore neglecting terms beyond second order of smallness in b_1, and b_2 with tidal distortions term q and cross effects of interaction between variations in angular velocity term b_1, and b_2 with tidal distortion term q and writing $r_1 = (1-2\lambda r + r^2)^{-\frac{1}{2}}$, we get after some simplifications.

$$\psi = \frac{z}{r} + q\sum_{J=2}^{\infty} r^j P_j(\lambda) + \frac{1}{2}(1-\upsilon^2)r^2\left[\ b_1^2 + b_1 b_2(1-\upsilon^2)r^2 + \frac{1}{3}b_2^2(1-\upsilon^2)^2 r^4\right] \tag{5.14}$$

on setting $q=0, z=1$ it reduces to the potential of a spherical model having differential rotation. On setting $z = 1$, $b_1 = b_2 = 0$. It reduces to the potential of a non-rotating model of a star distorted by the tidal effects of a companion. In case of synchronously rotating binary systems in which rotational velocity is synchronous with velocity of revolution, $\omega^2 = 1+q$. On setting $b_1^2 = 1+q$ and $b_2^2 = 0$ and $z=1$ in (5.14) it reduces to the expression of potential of a binary system as given in Kopal (65).

The surface generated by setting ψ=constant in (5.14) is usually referred to as a Roche equipotential. The Roche equipotential thus defined is a modification in the light of mass variation inside the primary component of binary system. Unfortunately, the expression (5.14) for ψ is such that r cannot be found explicitly in terms of ψ. To achieve these equations (5.14) has to be solved by successive approximations keeping in view that b_1, b_2, q are small quantities of first order.

Defining nondimensional variable $r_0 = \frac{z}{\psi - q}$ and following Kopal (65) and Lal (69) a relation connecting (r, θ, ϕ) on the surface of Roche equipotential is given as

$$r=r_0\,D\left[1+\left(\frac{q\,P_2}{z^2}+\frac{b_1^2}{2z}x\right)r_0^3+\frac{q\,P_3}{z^2}r_0^4+\left(\frac{q\,P_4}{z^2}+\frac{b_1\,b_2}{2z}x^2+\frac{5}{2z}\lambda\,b_1^2\,q\,x\right)r_0^5+\right.$$

$$+\left(\frac{q\,P_5}{z^2}+\frac{3q^2\,P_2^2}{z^2}+\frac{3qP_2\,b_1^2}{z^2}x\right)r_0^6+\left(\frac{qP_6}{z^2}+\frac{b_2^2}{6z}x^3+\frac{7}{4z^2}\lambda\,b_1^2\,q\,x^2+\frac{7q^2\,P_2\,P_3}{z^2}\right)r_0^7+$$

$$+\left(\frac{q\,P_7}{z^2}+\frac{8q^2\,P_2\,P_4}{z^2}+\frac{4q\,P_4\,b_1^2}{z^2}x+\frac{4q^2\,P_3^2}{z^2}\right)r_0^8+$$

$$+\left(\frac{q\,P_8}{z^2}+\frac{3\,\lambda\,b_2^2\,q}{2z^2}x^3+\frac{9q^2\,P_3\,P_4}{z^2}+\frac{9qP_3\,b_1^2}{4z^2}x^2+\frac{9q^2\,P_2\,P_5}{z^2}\right)r_0^9+$$

$$\left.+\left(\frac{q\,P_9}{z^2}+\frac{10q^2\,P_2\,P_6}{z^2}+\frac{10q^2P_3\,P_5}{z^2}+\frac{5q\,b_2^2\,P_2}{3z^2}x^3+\frac{5q^2\,P_4^2}{z^2}\right)r_0^{10}+\ldots\right]$$

(5.15)

where $x=\left(1-\upsilon^2\right)$and $p_j=p_j(\lambda)$ are legendre polynomial. In the above expression we have retained terms up to second order of smallness in b_1,b_2,q and up to order r_0^{12} in r_0.This relation can be used to obtain the shape of a Roche equipotentials $\psi =$ constant.

Following the approach of Kopal (65) Mohan, Lal and Singh (70), the explicit expressions for the volume V_ψ, surface area S_ψ the average gravitational force and its inverse $\overline{g},$ and $\overline{g^{-1}}$ of the equipotentials surface $\psi=$ constant can be

$$V_\psi=\frac{4\pi\;r_0^3\;D^3}{3}[1+\frac{b_1^2r_0{}^3}{z}+\frac{4b_1b_2r_0{}^5}{5z}+(\frac{12q^2}{5z^2}+\frac{4b_1^2\,q}{5\,z^2})r_0{}^6+\frac{8b_2^2r_0{}^7}{35z}+{}^{8}+$$

$$+(\frac{15q^2}{7z^2}+\frac{8b_1\,b_2q}{7z^2})r_0+(\frac{2q^2}{z^2}+\frac{16b_2\,q}{35z^2}+\frac{6b_1\,b_2\,q}{35z^2})r_0{}^{10}+\ldots]$$

(5.16)

$$S_\psi=4\pi\;r_0^2\;D^2\,[1+\frac{2b_1^2r_0{}^3}{3z}+\frac{8b_1b_2\,r_0{}^5}{15z}+(\frac{7q^2}{5z^2}+\frac{7b_1^2\,q}{15\,z^2})r_0{}^6+\frac{16b_2^2r_0{}^7}{35\,z}+$$

$$+(\frac{9q^2}{7z^2}+\frac{24b_1b_2\,q}{35z^2})r_0{}^8+(\frac{11q^2}{9z^2}+\frac{88\;b_2^2q}{315\,z^2}+\frac{22b_1b_2\,q}{210\,z^2})r_0{}^{10}+\ldots$$

(5.17)

$$\bar{g} = \frac{zGM_\psi}{D^2 r_0^2}[1 - \frac{4b_1^2 r_0^3}{3z} + \frac{8b_1b_2r_0^5}{5z} - (\frac{3q^2}{z^2} + \frac{b_1^2 q}{z^2})r_0^6 - \frac{64b_2^2 r_0^7}{105z} - (\frac{15q^2}{7z^2} + \frac{176b_1b_2q}{105z^2})r_0^8 - (\frac{21q^2}{9z^2} + \frac{56b_2^2 q}{63z^2} + \frac{1578b_1b_2q}{140z^2})r_0^{10} + ...] \quad (5.18)$$

$$\bar{g}^{-1} = \frac{D^2 r_0^2}{zGM_\psi}[1 + \frac{4b_1^2 r_0^3}{3z} + \frac{8b_1b_2r_0^5}{5z} + (\frac{31q^2}{5z^2} + \frac{26b_1^2 q}{15z^2})r_0^6 + \frac{64b_2^2 r_0^7}{105z} + (\frac{40q^2}{7z^2} + \frac{674b_1b_2q}{105z^2})r_0^8 - (\frac{57q^2}{9z^2} + \frac{184b_2^2 q}{385z^2} + \frac{2597b_1b_2q}{210z^2})r_0^{10} + ...] \quad (5.19)$$

Here M_ψ is mass contained within the equipotentials surface ψ = constant and terms

up to second order of smallness in $z, b_1, b_2,$ and q are retained. On setting $b_1^2 = 2n$, $b_2^2 = 0$ the results of this section reduce to the results of section 3.2 of chapter III for rotationally and tidally distorted stellar models.

5.3 EQUILIBRIUM STRUCTURE OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED GAS SPHERES

Following Kippenhan and Thomas approach given in section 2.3 of chapter II,_the equations governing the equilibrium structures of rotationally and tidally distorted gas spheres are given as

$$\frac{dM_\psi}{dr_0} = 4\pi D^3 \rho_\psi r_0^2 f_1 \quad 5.20(a)$$

$$\frac{dP_\psi}{dr_0} = -\frac{GM_\psi}{Dr_0^2}\rho_\psi f_2 \quad 5.20(b)$$

$$\frac{dL_\psi}{dr_0} = 4\pi\varepsilon D^3 \rho_\psi r_0^2 f_1 \quad 5.20(c)$$

$$\frac{dT\psi}{dr_0} = -\frac{3\kappa\, L_\psi \rho_\psi}{16\,\pi\, D\, a\, c T_\psi^{\,3} r_0^{\,2}} f_3 \qquad 5.20(d)$$

where $f_1 = \frac{r_\psi^{\,2}}{D^3}\frac{dr_\psi}{dr_0}$, $f_2 = \frac{Dr_0^{\,2}}{r_\psi^{\,2}}\frac{dr_\psi}{dr_0} f_P$ *and* $f_3 = \frac{Dr_0^{\,2}}{r_\psi^{\,2}}\frac{dr_\psi}{dr_0} f_T$ are distortion parameters on account of rotational and tidal effects.

In order to compute the values of these distortion parameters, r_ψ, u, v, w, f_p have to be computed following the approach as discussed in section 3.5 of chapter_III. Explicit expressions of these parameters which determine the value of r_ψ, u, v, w, f_p and f_T on the modified equipotentials surfaces of the primary component of a star in a binary system, rotating differentially according to the law $\omega = b_1 + b_2 s^2$ reduced to

$$r_\psi = r_0\, D[1 + \frac{b_1^2 r_0^{\,3}}{3z} + \frac{4b_1 b_2 r_0^{\,5}}{15z} + (\frac{4q^2}{5z^2} + \frac{4b_1^2 q}{15z^2})r_0^{\,6} + \frac{8b_2^2 r_0^{\,7}}{105z} + + (\frac{5q^2}{7z^2} + \frac{8b_1 b_2 q}{21z^2})r_0^{\,8} + (\frac{2q^2}{3z^2} + \frac{2b_1 b_2 q}{35z^2} + \frac{16q b_2^2}{105z^2})r_0^{\,10} + ...] \qquad (5.21)$$

$$u = 1 - (\frac{q^2}{5z^2} + \frac{b_1^2 q}{15z^2})r_0^{\,6} - (\frac{q^2}{7z^2} + \frac{8b_1 b_2 q}{105z^2}) - (\frac{q^2}{9z^2} + \frac{4b_1 b_2 q}{35z^2} + \frac{8b_2^2 q}{315z^2})r_0^{\,10} + ...$$

(5.22)

$$v = z\,[1 - \frac{2b_1^2 r_0^{\,3}}{3z} - \frac{16b_1 b_2 r_0^{\,5}}{15z} - (\frac{7q^2}{5z^2} + \frac{2b_1^2 q}{15z^2})r_0^{\,6} - \frac{16b_2^2 r_0^{\,7}}{35z} - (\frac{31q^2}{14z^2} + \frac{32b_1 b_2 q}{35z^2})r_0^{\,8} - (\frac{3q^2}{z^2} + \frac{1562b_1 b_2 q}{140z^2} + \frac{184b_2^2 q}{315z^2})r_0^{\,10} + ...]$$

(5.23)

$$w = \frac{1}{z}[1 + \frac{2b_1^2 r_0^{\,3}}{3z} + \frac{16b_1 b_2 r_0^{\,5}}{15z} + (\frac{23q^2}{5z^2} + \frac{23b_1^2 q}{15z^2})r_0^{\,6} + \frac{16b_2^2 r_0^{\,7}}{35z} + + (\frac{81q^2}{17z^2} + \frac{198b_1 b_2 q}{35z^2})r_0^{\,8} + (\frac{7q^2}{z^2} + \frac{5146b_1 b_2 q}{420z^2} + \frac{83b_2^2 q}{63z^2})r_0^{\,10} + ...]$$

(5.24)

$$f_P = z\,[1 - \frac{2b_1^2 r_0^3}{3z} - \frac{16b_1 b_2 r_0^5}{15z} - (\frac{22q^2}{5z^2} + \frac{22b_1^2 q}{15z^2})r_0^6 - \frac{16b_2^2 r_0^7}{35z} -$$

$$-(\frac{7q^2}{17z^2} + \frac{586b_1 b_2 q}{105z^2})r_0^8 - (\frac{62q^2}{9z^2} + \frac{3046b_1 b_2 q}{252z^2} + \frac{88b_2^2 q}{63z^2})r_0^{10} + ...]$$

(5.25)

$$f_T = 1 - (\frac{14q^2}{5z^2} + \frac{14\,b_1^2 q}{15z^2})r_0^6 - (\frac{46q^2}{14z^2} + \frac{482b_1 b_2 q}{105z^2})r_0^8 - (\frac{34\,q^2}{9z^2} + \frac{182b_1 b_2 q}{210z^2} + \frac{16b_2^2 q}{21z^2})r_0^{10} + ...$$

(5.26)

In the above expressions, terms upto second order of smallness in b_1, b_2, q and z and terms up to r_0^{12} in r_0 are retained.

Using these expressions the values of distortion parameters f_1, f_2, f_3 are obtained as

$$f_1 = 1 + \frac{2b_1^2 r_0^3}{z} + \frac{32b_1 b_2 r_0^5}{15z} + (\frac{36q^2}{5z^2} + \frac{36b_1^2 q}{5z^2})r_0^6 + \frac{16b_2^2 r_0^7}{21z} + (\frac{55q^2}{7z^2} + \frac{88b_1 b_2 q}{7z^2})r_0^8$$

$$+(\frac{26q^2}{3z^2} + \frac{26b_1 b_2 q}{35z^2} + \frac{208b_2^2 q}{105z^2})r_0^{10} + ...$$

(5.27)

$$f_2 = z[1 - (\frac{3q^2}{5z^2} + \frac{2b_1^2 q}{15z^2})r_0^6 + (\frac{6q^2}{7z^2} + \frac{306b_1 b_2 q}{105z^2})r_0^8 - (\frac{8q^2}{9z^2} - \frac{51037b_1 b_2 q}{4410z^2} - \frac{8b_2^2 q}{315z^2})r_0^{10} + ...]$$

(5.28)

$$f_3 = 1 + \frac{2b_1^2 r_0^3}{3z} + \frac{16b_1 b_2 r_0^5}{15z} + (\frac{6q^2}{5z^2} + \frac{6b_1^2 q}{5z^2})r_0^6 + \frac{16b_2^2 r_0^7}{35z} +$$

$$+(\frac{24q^2}{7z^2} - \frac{202b_1 b_2 q}{105z^2})r_0^8 + (\frac{26q^2}{9z^2} - \frac{74b_1 b_2 q}{210z^2} + \frac{64b_2^2 q}{105z^2})r_0^{10} + ...$$

(5.29)

The values of $P_\psi, \rho_\psi, L_\psi$ etc. on the various equipotentials surfaces of a differentially rotating stellar model may now be obtained by solving the system of differential equations (5.20a-5.20d) using the values of distortion parameters f_1, f_2, and f_3 as given in (5.27), (5.28) and (5.29) subject to the boundary conditions

$M_\psi = 0, L_\psi = 0$ at the center $r_0 = 0$ and at the free surface $r_0 = r_{0s}$

$$M_\psi = M_0,\ L_\psi = L_{\psi s},\ P_\psi = 0\ or\ P_{\psi s}\quad \rho_\psi = 0\ or\ \rho_{\psi s}\quad and\ T_\psi = 0\ or\ T_{\psi s} \tag{5.30}$$

(r_{0s} being the value of r_0 at the free surface).

Once the equilibrium structure of the primary component of a star in a binary system, rotating differentially according to the law $\omega = b_1 + b_2 s^2$, has been computed by solving the system of differential equations (5.20) subject to boundary conditions (5.30), its shapes, and values of various other observable physical parameters can be computed. Whereas its shape, volume and surface area can be computed using (5.15), (5.16), and (5.17) its oblateness σ and ellipticity ε may be computed using their definitions as given in section 3.3 of chapter III. The values of R_p, R_e, g_p, g_e needed for this purpose are to be calculated from

$$R_p = r_{0s} D, \tag{5.31}$$

$$R_e = r_{0s} D[1 + (\frac{q}{z} + \frac{b_1^2}{2z}) r_{0s}^{\ 3} + \frac{q r_{0s}^{\ 4}}{z} + (\frac{q}{z} + \frac{b_1 b_2}{2z}) r_{0s}^{\ 5} + (\frac{q}{z} + \frac{3b_1^2 q}{z^2} + \frac{3q^2}{z^2}) r_{0s}^{\ 6} +$$
$$(\frac{q}{z} + \frac{7b_1^2 q}{2z^2} + \frac{7q^2}{z^2} + \frac{b_2^2}{6z}) r_{0s}^{\ 7} + (\frac{q}{z} + \frac{4b_1^2 q}{z^2} + \frac{4b_1 b_2 q}{z^2} + \frac{12q^2}{z^2}) r_{0s}^{\ 8} +$$
$$(\frac{q}{z} + \frac{9b_1^2 q}{2z^2} + \frac{18 b_1 b_2 q}{4z^2} + \frac{18q^2}{z^2}) r_{0s}^{\ 9} + (\frac{q}{z} + \frac{5b_1^2 q}{z^2} + \frac{5b_1 b_2 q}{z^2} + \frac{5b_2^2 q}{3z^2} + \frac{25q^2}{z^2}) r_{0s}^{\ 10} + .. \tag{5.32}$$

where R_p and R_e are respectively , the polar and equatorial radial. The gravitational force g_p at the pole and g_e at the equator are given as below and the temperature and Luminosity may be calculated by using equations (3.34) and (3.35) of chapter III.

$$g_p = \frac{GM_0}{R_p^{\ 2}} \tag{5.33}$$

$$g_e = \frac{GM_0}{R_e^2}[1-(\frac{2q}{z}+\frac{b_1^2}{z})r_{0s}{}^3 - \frac{3q\,r_{0s}{}^4}{z} - (\frac{4q}{z}+\frac{2b_1 b_2}{z})r_{0s}{}^5 -$$

$$-(\frac{5q}{z}+\frac{6b_1^2 q}{z^2}+\frac{6q^2}{z^2})r_{0s}{}^6 - (\frac{6q}{z}+\frac{b_2^2}{z})r_{0s}{}^7 - (\frac{7q}{z}+\frac{13b_1b_2q}{z^2}+\frac{2q^2}{z^2})r_{0s}{}^8 -$$

$$-\frac{9qr_{0s}{}^9}{z} - (\frac{9q}{z}+\frac{20b_1b_2\,q}{z^2}+\frac{8\,b_2^2 q}{z^2}+\frac{20q^2}{z^2})r_{0s}{}^{10}+...] \tag{5.34}$$

A star rotating according to the present type of the law of differential rotation develops deformations in its shape but maintains spherical symmetry about the axis of rotation. If we follow Geroyannis and Valvi (42) oblateness σ *and* ε which are used as measures of the departure of the shape of the star from spherical symmetry may be computed by using

$$\sigma = \frac{R_e - R_p}{R_p} \quad \text{and} \quad \varepsilon = \frac{R_e - R_p}{R_e} \tag{5.35}$$

The polar angular velocity ω_p and equatorial angular velocity ω_e of the star can be computed as below in units of $\frac{GM}{R^3}$ using. The temperature and Luminosity may be calculated by using the equation (3.34) and (3.35) of chapter III.

$$\omega_p = \sqrt{b_1^2} \quad \text{and} \quad \omega_e = \sqrt{b_1^2 + 2b_1b_2\,R_e^2 + b_2^2\,R_e^4} \tag{5.36}$$

In the next two sections of the present chapter we apply this approach developed in this section to determine the equilibrium structures of differentially rotating and tidally distorted polytropic and Prasad models of stars.

5.4 EQUILIBRIUM STRUCTURES OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED POLYTROPIC MODELS OF GAS SPHERES

If we assure primary component of binary star as a polytropic model rotating differentially according to the law $\omega = b_1 + b_2\,s^2$ then the equilibrium structure of the primary component of a star will be differentially rotating and tidally distorted models.

Following the approach as given in section 3.3 of chapter III the differential equation governing the equilibrium structure of rotationally and tidally distorted model now become

$$\alpha^2 \frac{d}{dr_\psi}\left[r_\psi^2 \frac{d\theta_\psi}{dr_0}\right] = - r_\psi^2 \theta_\psi^N \tag{5.37}$$

where $\alpha^2 = \dfrac{(N+1)}{4\pi \rho_{c\psi}^2}$

On changing independent variable r_ψ in terms of r_0, equation (5.37) can be written as

$$\frac{d}{dr_0}\left[A(b_1, b_2, z, q, r_0)\frac{d\theta_\psi}{dr_0}\right] = -\frac{D^2}{\alpha^2} B(b_1, b_2, z, q, r_0) \tag{5.38}$$

$$A(,b_1, b_2, z, q, r_0) = r_0^2\,[1-\left(\frac{3q^2}{5z^2}+\frac{b_1^2 q}{5z^2}\right)r_0^6 - \frac{6q^2}{7z^2}r_0^8 - \frac{10q^2}{9z^2}r_0^{10} + ...],$$

and

$$B(b_1, b_2, z, q, r_0) = [1+\frac{2b_1^2 r_0}{z}^3 + \frac{32b_1 b_2 r_0}{15z}^5 + (\frac{36q^2}{5z^2}+\frac{12b_1^2 q}{5z^2})r_0^{\,6} + \frac{16b_2^2\, r_0}{21z}^7 +$$

$$+(\frac{55q^2}{7z^2}+\frac{88b_1 b_2 q}{21z})r_0^{\,8} + (\frac{26q^2}{3z^2}+\frac{208b^2 q}{105z^2}+\frac{124b_1 b_2 q}{35z})r_0^{\,10} + ...]$$

and

$$z = \frac{r_0^2 \dfrac{d\theta}{d\xi}}{\left(r_0^2 \dfrac{d\theta}{d\xi}\right)_{r_0 = r_{0s}}} \qquad \text{and} \qquad r_0 = \frac{z}{\psi - q}\ .$$

The terms up to second order of smallness in $z, b_1, b_2,$ and q up to $r_0^{\,12}$ are retained. The boundary conditions which equation (5.38) must satisfy are:

$$\theta_\psi = 1, \ \frac{d\theta_\psi}{dr_0} = 0, \text{ at the center }, \ r_0 = 0$$

$$\theta_\psi = 0 \quad \text{at the surface } r_0 = r_{0s} \tag{5.39}$$

In the expression $K = \frac{R_u}{D}$, R_u being the undistorted radius of the primary and D the distance between the centers of the primary and secondary stars of the binary system. We can write

$$\frac{D}{\alpha} = \frac{D\xi_u}{\alpha\xi_u} = \frac{D}{R_u}\xi_u = \frac{1}{K}\xi_u \tag{5.40}$$

With this substitution equation (5.38) can be written as

$$\frac{d}{dr_0}\left[A(b_1, b_2, r_0, z, q)\frac{d\theta_\psi}{dr_0}\right] = -\frac{\xi_u^2}{K^2}\theta_\psi^N B(b_1, b_2, r_0, z, q) \tag{5.38 a}$$

Where the quantity α is of the dimension of length defined in equation (3.19) of chapter III and ξ_u is the value of at the outer surface of the undistorted polytropic model.

Equation (5.38a) subject to the boundary conditions (5.39) determines the equilibrium structure of a differentially rotating and tidally distorted polytropic model. On setting $q = 0$ the above equation reduces to (3.20) which determine the equilibrium structure of a polytropic model of a star distorted by differential rotation alone under the mass variation inside the star. If we set $b_1 = b_2 = 0$ in (5.39), the equation reduces to an equation which determines the equilibrium structure of a polytropic model of a star distorted by the effects of the tidal forces of the companion.

To obtain the inner structure , the volume, the surface area and other physical parameters of certain differentially rotating and tidally distorted polytropic models, the equation(5.39) has to be integrated numerically with specified values of parameter N, b_1, b_2, q, and K which respectively denote the polytropic index, the radius of undistorted polytropic model , the values of constants (i.e b_1, and b_2) appearing in the law of differential rotation, the ratio of mass of the companion star to primary star in the binary system and the ratio of undistorted radius of the primary to the distance between the centers of the primary and the secondary. The value of $z = \frac{M_0(r)}{M_0}$ required at each point inside the star is computed from (3.2) of chapter III. The value of K must be such that the outermost surface of the primary component lies well within the Roche lobe otherwise the two star will coalesce (Kopal (65), page 11). For a single star distorted by differential rotation alone $K = 1$ and rotationally and/ or tidally distorted star, $K = 0.5$.

For obtaining the numerical solution, equation (5.38 a) was integrated using fourth-order Runge-Kutta method for the specified values of the input parameters. However the center and the surface of the star being singularities, series solution similar to the one available for undistorted polytropic model (cf. Chandrasekhar (21), page 85) was developed to start the numerical integration at points near the center. This series solution is given by

$$\theta_\psi = 1 - \frac{1}{6}\frac{\xi_u^2}{K^2} r_0^{\,2} + \frac{N\,\xi_U^{\,4}}{120\,K^4} r_0^{\,4} - \frac{b_1^2\,\xi_u^{\,2}}{15\,z\,K} r_0^{\,5} - \frac{N(8N-5)\,\xi_u^6}{15120\,K^6} r_0^6 + \left\{\frac{N\,b_1^2\xi_u^4}{140\,z\,K^4} - \frac{4\,b_1 b_2\,\xi_u^{\,2}}{105\,z\,K^2}\right\}$$
$$+ \left\{\frac{122N^3 - 183N^2 + 70N\xi_u^8}{3265920K^8} - \left(\frac{q^2}{8z^2} + \frac{b_1^2\,q}{24\,z^2}\right)\frac{\xi_u^{\,2}}{K^2}\right\} r_0^8 + \ldots \tag{5.41}$$

Taking starting values from this series solution at $r_0 = 0.005$, numerical integration of equation (5.37) was then carried forward using forth order Runge

–Kutta method with a step length of 0.005. Numerical integration was continued till θ_ψ first became zero.

Once we obtain r_{0s}, the value of r_0 where θ_ψ first becomes zero, relation (5.16) may be used determine its shape by replacing r_0 by r_{0s} and writing $z=1$ $D=\frac{\alpha\xi_u}{K}$ relations (5.16) and (5.17) can now be used to determine the volume and the surface area of the polytropic models of star. The other physical parameters of such models such as oblateness, ellipticity, polar angular velocity, equatorial angular velocity, temperature ratio $(\frac{T_e}{T_p})$ and luminosity ratio $\frac{L_e}{L_p}$ are computed using the relations (3.35) and (3.36) of chapter III respectively.

Numerical computations have been performed to compute the inner structures of certain differentially rotating and tidally distorted polytropic models of indices 1.5, 3.0, 4.0 with specified values of ξ_u, b_1, b_2, N, K, Z. The results on inner structures are presented in Tables 5.3(a), (b), (c). The results computed for various observable physical parameters are tabulated in Table 5.4 (a), (b), (c). The value of K has been taken one for the differentially rotating models and 0.5 for the tidally and/or differentially rotating and tidally distorted models.

5.5 EQUILIBRIUM STRUCTURE OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED PRASAD MODEL

In order to determine the equilibrium structures of differentially rotating and tidally distorted Prasad model in the influence of mass variation inside the model, primary component of binary star is assumed to be Prasad model rotating differentially according to the law (5.7). Thus the equilibrium structure of the primary component of a star will be rotationally and tidally distorted models. Let r_ψ denote the radius of the topologically equivalent spherical model which corresponds to an equipotentials surface $\psi =$ constant of this differentially and tidally distorted model and R_ψ be the value of r_ψ on the outermost equipotentials surface. Further, let ρ_ψ denote the value of density on an equipotentials surface $\psi =$ constant. The density distribution law of the differentially rotating and tidally distorted Prasad model is given by $\rho = \rho_c(1-x^2)$

$$\rho = \rho_c(1-x^2) \tag{5.42}$$

$$r_\psi = Dr_0\left[1+\frac{2b_1^2 r_0^3}{3z}+\frac{4b_1 b_2 r_0^5}{15z}+\frac{4q^2 r_0^6}{4z^2}+\frac{8b_2^2 r_0^7}{15z}+\frac{5q^2 r_0^8}{7z^2}+\frac{2q^2 r_0^{10}}{3z^2}+\ldots\right] \tag{5.43}$$

$$R_\psi = Dr_{0s}\left[1+\frac{2b_1^2 r_{0s}^3}{3z}+\frac{4b_1 b_2 r_{0s}^5}{15z}+\frac{4q^2 r_{0s}^6}{4z^2}+\frac{8b_2^2 r_{0s}^7}{15z}+\frac{5q^2 r_{0s}^8}{7z^2}+\frac{2q^2 r_{0s}^{10}}{3z^2}+\ldots\right] \tag{5.44}$$

with
$$r_{0s} = \frac{z}{\psi - q} \tag{5.45}$$

where z is same as defined in section 4.1 of chapter IV.

On substituting the value of r_ψ. and R_ψ from equation (5.43) and (5.44) in equation (5.42) we get

$$\rho_\psi = \rho_c\left[1-\frac{D^2 r_0^2}{R_\psi^2}\left\{1+\frac{2b_1^2 r_0^3}{3z}+\frac{8b_1 b_2 r_0^5}{15z}+\frac{8q^2 r_0^6}{5z^2}+\frac{6b_2^2 r_0^7}{105z}+\frac{10q^2 r_0^8}{7z^2}+\frac{4q^2 r_0^{10}}{3z^2}+\ldots\right\}\right] \tag{5.46}$$

On substituting value of ρ_ψ from (5.46) in (3.25a) and integrating w.r.t. r_0 and using the fact that $M_\psi = 0$ at center $r_0 = 0$ we get

$$M_\psi = \frac{4\pi\rho_c D^3 r_0^{\ 3}}{3}[1 - \frac{3D^2}{5R_\psi^{\ 2}} r_0^{\ 2} + \frac{b_1^{\ 2} r_0^{\ 3}}{z} + \frac{4b_1 b_2 r_0^{\ 5}}{5z^2} - \frac{b_1^{\ 2} R^2 r_0^{\ 5}}{zR_\psi^{\ 2}} + \frac{12q^2 r_0^{\ 6}}{5z^2}$$
$$+\frac{16b_2^{\ 2} r_0^{\ 7}}{70z} - \frac{4b_1 b_2 R^2 r_0^{\ 7}}{5zR_\psi^{\ 2}} + \left\{\frac{15q^2}{7z^2} - \frac{12q^2 R^2}{5z^2 R_\psi^{\ 2}}\right\} r_0^{\ 8} + (\frac{2q^2}{5z^2} - \frac{15q^2 R^2}{7R_\psi^{\ 2}}) r_0^{\ 10} + \ldots]$$

(5.47)

Similarly on substituting value of ρ_ψ from (5.46) and M_ψ from (5.47) in equation (3.25b) and integrating w.r.t. r_0 and

$$P_\psi = \frac{2\pi zG\rho_c^{\ 2} D^2}{3}[K - r_0^{\ 2} + \frac{4D^2 r_0^{\ 4}}{5R_\psi^{\ 2}} - \frac{2b_1^{\ 2} r_0^{\ 5}}{5z} - \frac{D^4 r_0^{\ 6}}{5R_\psi^{\ 2}} - \frac{8b_1 b_2 r_0^{\ 7}}{35z} + \frac{16D^2 b_1^{\ 2} r_0^{\ 7}}{21zR_\psi^{\ 2}} +$$
$$+\frac{q^2 r_0^{\ 8}}{2z^2} - \frac{16b_2^{\ 2} r_0^{\ 9}}{315z} + \frac{64b_1 b_2 D^2 r_0^{\ 9}}{135zR_\psi^{\ 2}} - \frac{14D^4 b_1^{\ 2} r_0^{\ 9}}{45zR_\psi^{\ 4}}$$
$$-\frac{3q^2 r_0^{\ 10}}{10z^2} + \frac{144D^2 q^2 r_0^{\ 10}}{125z^2 R_\psi^{\ 2}} + ..]$$

(5.48)

where K is a constant of integration whose value may be calculated by using boundary conditions $P_\psi = 0$ at $r_0 = r_{0s}$.

$$K = r_{0s}^{\ 2} - \frac{4D^2 r_{0s}^{\ 4}}{5R_\psi^{\ 2}} + \frac{2b_1^{\ 2} r_{0s}^{\ 5}}{5z} + \frac{D^4 r_{0s}^{\ 6}}{5R_\psi^{\ 2}} + \frac{8b_1 b_2 r_{0s}^{\ 7}}{35z} - \frac{16D^2 b_1^{\ 2} r_{0s}^{\ 7}}{21zR_\psi^{\ 2}} + \frac{q^2 r_{0s}^{\ 8}}{2z^2}$$
$$+\frac{16b_2^{\ 2} r_{0s}^{\ 9}}{315z} - \frac{64b_1 b_2 D^2 r_{0s}^{\ 9}}{135zR_\psi^{\ 2}} + \frac{14D^4 b_1^{\ 2} r_{0s}^{\ 9}}{45zR_\psi^{\ 4}} + \frac{3q^2 r_{0s}^{\ 10}}{10z^2} - \frac{144D^2 q^2 r_{0s}^{\ 10}}{125z^2 R_\psi^{\ 2}} + \ldots$$

The explicit expressions for Volume V_ψ, Surface area S_ψ, $\bar{g}$ and $\overline{g^{-1}}$ gravitational force are given as

$$V_\psi = \frac{4\pi\, r_0^3}{3}[1 + \frac{b_1^2\, r_0^3}{z} + \frac{4b_1\, b_2\, r_0^5}{5z} + \frac{12q^2}{5z^2} + \frac{8b_2^2\, r_0^7}{35z} + \frac{15q^2\, r_0^8}{7z^2} + \frac{2q^2\, r_0^{10}}{z^2} + \ldots]$$

(5.49)

$$S_{\psi}=4\pi r_0^2\,[1+\frac{2b_1^2 r_0^3}{3z}+\frac{8b_1 b_2 r_0^5}{15z}+\frac{7q^2 r_0^6}{5z^2}+\frac{16b_2^2 r_0^7}{105z}+\frac{9q^2 r_0^8}{7z^2}+\frac{11q^2 r_0^{10}}{9z^2}+\ldots]$$

(5.50)

$$\bar{g}=\frac{zGM_{\psi}}{r_0^2}[\,1-\frac{4b_1^2 r_0^3}{3z}-\frac{8b_1b_2 r_0^5}{5z}-\frac{3q^2 r_0^6}{z^2}-\frac{64b_2^2 r_0^7}{105z}-\frac{51q^2 r_0^8}{14z^2}-\frac{13q^2 r_0^{10}}{3z^2}+\ldots]$$

(5.51)

$$\bar{g}^{-1}=\frac{r_0^2}{zGM_{\psi}}[1+\frac{4b_1^2 r_0^3}{3z}+\frac{8b_1 b_2 r_0^5}{5z}+\frac{31q^2 r_0^6}{5z^2}+\frac{64b_2^2 r_0^7}{105z}+\frac{101q^2 r_0^8}{14z^2}+\frac{75q^2 r_0^{10}}{9z^2}+\ldots]$$

(5.52)

If we put $b_1^2=2n,\, b_2^2=0$ the results of this section reduce to the result of section 3.1 of chapter III.

Numerical computations have been performed to compute the inner structures of certain differentially rotating and tidally distorted Prasad model for two values of $\psi_s^{\prime}$ and taking four models we replaced r_0 by $x r_{0_s}$ to 5.43 and 5.44 and used x as the independent variable whose value is to zero at the center and one at the free surface. Results are presented in table 5.4(a), 5.4(b), 5.4(c) and 5.4(d).

5.6 ANALYSIS OF RESULTS

Results presented in Tables 5.1 show that the behavior of angular velocity in certain differentially rotating models. This Table also exhibits the stability of the models considered according to the Stoeckly criteria.

Table 5.2 shows the values of r_{0_s} for certain differentially rotating and/or tidally polytropic models with polytropic indices 1.5, 3.0, and 4.0. The results shown in Tables 5.3 (a), (b), and (c) present the values of certain structure parameters and related quantities of differentially rotating and tidally distorted polytropic models with polytropic indices 1.5, 3.0 and 4.0 respectively. The parameters and related quantities of differentially rotating and tidally distorted polytropic models with polytropic indices 1.5, 3.0 and 4.0 respectively. The results shown in paranthesis are the corresponding results earlier obtained by Mohan, Lal, and Singh (70). Compares on results in Table 5.3 (a) for volumes and surface areas with corresponding results shown in parenthesis indicate that our values obtained by us are smaller than the corresponding result shown in parenthesis. The decrease in these values is small in the case of polytropic model 3.0 and 4.0. However, the volumes and surface areas for tidally distorted models 4, 5, 6 are obtained by us are larger.

As regards the shape of the model represented by σ and ε for N=1.5, values obtained by us are smaller in comparison to the corresponding results shown in parenthesis. A similar trend is noticed in the case of models for polytropic indices N= 3.0 and 4.0. However, σ and ε obtained by us for tidally distorted models 4 and 6 are larger in comparison to the tidally distorted model, these are smaller in the case of model 5 for all the polytropic models with indices N = 1.5, 3.0 and 4.0.

It is also noticed that our results for $\frac{T_e}{T_P}$ and $\frac{L_e}{L_P}$ which give the temperature and luminosity at different points on the surface in comparision to their corresponding values at pole are larger in comparison to the corresponding results shown in parenthesis. It is also noticed that while the values of these parameters are smaller for the 4 and 6 in comparison to earlier obtained models these values are larger increase in the case of model 5.

Compared to the undistorted Prasad model the increase (decrease) in the values of structure parameter for differentially rotating and / or tidally distorted Prasad model is small for models with $\psi_s^* = 10.0$ compared to models with $\psi_s^* = 5.0$.

Table 5.1 : Behaviour of Angular Velocity in Certain Differentially Rotating Models

Model No.	Values of Various Parameters of differential rotation $\omega^2 = b_1^2 + 2b_1 b_2 s^2 + b_2^2 s^4$			Stability of the model according to Stoeckly criteria
	b_1	b_2	q	
1	0.0	0.0	0.1	Stable
2	0.3162	0.3162	0.1	Stable
3	0.0	0.3162	0.1	Stable
4	0.3162	0.3162	0.1	Stable

Table 5.2 : Values of r_{0s} for certain differentially rotating and tidally distorted models of stars indices 1.5, 3.0 , 4.0

Model No.	Values of distortion parameters			Polytropic indices of r_{0s}		
	b_1	b_2	q	1.5	3.0	4.0
1.0	0.0	0.0	0.1	0.499815	0.499935	0.499955
2.0	0.3162	0.0	0.1	0.496235	0.498620	0.499475
3.0	0.0	0.3162	0.1	0.499805	0.499935	0.499945
4.0	0.3162	0.3162	0.1	0.495895	0.498510	0.499430

Table 5.3 (a) : Values of certain structure parameters and related quantities of differentially rotating and tidally distorted polytropic models of index N= 1.5

Model No.	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$	σ	ε	ω_e	ω_P	T_e/T_P	L_e/L_p
1	2.04188 (2.0446)	1.67682 (1.67859)	0.02000 (0.0326)	0.0262 (0.3165)	0.0000 (0.0000)	0.0000 (0.000)	0.96769 (0.96471)	0.85390 (0.83871
2	2.02292 (2.05867)	1.66633 (1.68621)	0.03322 (0.03985)	0.03162 (0.03832)	0.3162 (0.3162)	0.3162 (0.3162)	0.96189 (0.95798)	0.82853 (0.80994)
3	2.04213 (2.04500)	1.67696 (1.67876)	0.02706 (0.03284)	0.02635 (0.3180)	0.08332 (0.08345)	0.0000 (0.0000)	0.96743 (0.96440)	0.85287 (0.83752)
4	2.02391	1.66684	0.03474	0.03358	0.39945	0.3162	0.95947	0.81902

Table 5.3(b) : Values of certain structure parameters and related quantities of differentially rotating and tidally distorted polytropic models of index N= 3.0

Model No.	$V_\psi \times 10^{-3}$	$S_\psi \times 10^{-2}$	σ	ε	ω_e	ω_P	T_e/T_P	L_e/L_p
1	1.37429 (1.3752)	5.97751 (5.98113)	0.0000 (0.03268)	0.0000 (0.03165)	0.0000 (0.000)	0.0000 (0.000)	0.96766 (0.96470)	0.85376 (0.83870)
2	1.38052 (1.3884)	5.99520 (6.01921)	0.03387 (0.0400)	0.03276 (0.03846)	0.3162 (0.3162)	0.3162 (0.3162)	0.96110 (0.95782)	0.82529 (0.80928)
3	1.36972 (1.37550)	5.96422 (5.98181)	0.02696 (0.03285)	0.02625 (0.03180)	0.083153 (0.08346)	0.0000 (0.0000)	0.96756 (0.96660)	0.85343 (0.83751)
4	1.38326	6.00292	0.03551	0.03429	0.40046	0.3162	0.95854	0.81523

Table 5.3 (c) : Values of certain structure parameters and related quantities of differentially rotating and tidally distorted polytropic models of index N= 4.0

Model No.	$V_\psi \times 10^{-3}$	$S_\psi \times 10^{-3}$	σ	ε	ω_e	ω_P	T_e/T_P	L_e/L_p
1	14.0598 (14.0688)	2.81700 (2.8185)	0.00000 (0.03269)	0.00000 (0.03165)	0.00000 (0.000)	0.00000 (0.0000)	0.96765 (0.96470)	0.85373 (0.83869)
2	14.1956 (14.2313)	2.83494 (2.84021)	0.03411 (0.04010)	0.03299 (0.03855)	0.3162 (0.3162)	0.3162 (0.3162)	0.96081 (0.95770)	0.82412 (0.80880)
3	14.0624 (14.0713)	2.81734 (2.81892)	0.02702 (0.03285)	0.02638 (0.03180)	0.08337 (0.08346)	0.0000 (0.0000)	0.96739 (0.96439)	0.85269 (0.83749)
4	14.2297	2.83941	0.03578	0.03455	0.40081	0.3162	0.95820	0.81388

Table 5.4(a) : Structure Parameters of Uniformly Distorted Prasad Model For $(\psi_s^* = 5, b_1 = 0, b_2 = 0, q = 0.1)$

x	V_ψ	S_ψ	ρ_ψ	M_ψ	P_ψ	σ	ε	T_e/T_P	L_e/L_P
0.1	0.00001	0.00042	0.99000	0.00248	0.01313	0.00035	0.00035	0.14283	0.99825
0.2	0.00006	0.00166	0.96000	0.01952	0.01499	0.00036	0.00036	0.20199	0.99817
0.3	0.00023	0.00374	0.91000	0.06385	0.01317	0.00038	0.00038	0.24738	0.99806
0.4	0.00054	0.00666	0.84000	0.14464	0.01081	0.00041	0.00041	0.28565	0.99791
0.5	0.00106	0.01041	0.75000	0.26563	0.00819	0.00044	0.00044	0.31936	0.99772
0.6	0.00183	0.01499	0.64000	0.42336	0.00559	0.00049	0.00049	0.34983	0.99745
0.7	0.00291	0.20408	0.51000	0.60536	0.00327	0.00056	0.00056	0.37784	0.99709
0.8	0.00435	0.02665	0.36000	0.78848	0.00149	0.00066	0.00066	0.40391	0.99657
0.9	0.00622	0.03373	0.18999	0.93676	0.00036	0.00081	0.00081	0.42837	0.99576
1.0	0.00800	0.41692	0.0000	1.0000	0.00000	0.00100	0.00100	0.45148	0.99438

Table 5.4(b) : Structure Parameters of Rotationally and Tidally Distorted Prasad Model $(\psi_s^{*} = 5, b_1 = 0.3162, b_2 = 0, q = 0.1)$

X	V_ψ	S_ψ	ρ_ψ	M_ψ	P_ψ	σ	ε	T_eT_p	L_eL_p
0.1	0.00001	0.00041	0.99000	0.00284	0.01305	0.00052	0.00052	0.14282	0.99733
0.2	0.00006	0.00166	0.96001	0.01951	0.01499	0.00053	0.00053	0.20197	0.99729
0.3	0.00023	0.00374	0.91002	0.06382	0.01317	0.00056	0.00056	0.24736	0.99715
0.4	0.00054	0.00666	0.84005	0.14457	0.01081	0.00059	0.00059	0.28562	0.99697
0.5	0.00106	0.01041	0.75007	0.26551	0.00820	0.00064	0.00064	0.31933	0.99671
0.6	0.00183	0.01499	0.64010	0.42321	0.00559	0.00071	0.00071	0.34979	0.99637
0.7	0.00291	0.02041	0.51012	0.60523	0.00327	0.00080	0.00080	0.37780	0.99589
0.8	0.00435	0.02666	0.36012	0.78834	0.00146	0.00093	0.00093	0.40385	0.99519
0.9	0.00620	0.03375	0.19010	0.93670	0.00036	0.00114	0.00114	0.42830	0.99411
1.0	0.00850	0.41673	0.00000	1.00000	0.00000	0.00149	0.00149	0.45138	0.99226

Table 5.4(c) : Structures Parameters of Rotationally and Tidally Distorted Prasad Model $(\psi_s^{*} = 5, b_1 = 0, b_2 = 0.3162, q = 0.1)$

X	V_ψ	S_ψ	ρ_ψ	M_ψ	P_ψ	σ	ε	T_e/T_P	L_e/L_P
0.1	0.00001	0.00041	0.99000	0.00249	0.01313	0.00035	0.00035	0.14832	0.998245
0.2	0.00006	0.00166	0.96000	0.01952	0.01499	0.00036	0.00036	0.20199	0.998168
0.3	0.00022	0.00374	0.91000	0.06386	0.01317	0.00038	0.00038	0.24738	0.998060
0.4	0.00054	0.00666	0.84000	0.14464	0.01081	0.00041	0.00041	0.28565	0.99915
0.5	0.00106	0.01041	0.75000	0.26562	0.00819	0.00044	0.00044	0.31936	0.997721
0.6	0.00183	0.01499	0.64000	0.42333	0.00559	0.00049	0.00049	0.34983	0.997459
0.7	0.00291	0.02040	0.51000	0.60539	0.00327	0.00056	0.00056	0.37784	0.997096
0.8	0.00435	0.02665	0.36000	0.78848	0.00146	0.00066	0.00066	0.40391	0.996572
0.9	0.00619	0.03373	0.18999	0.93676	0.00035	0.00081	0.00081	0.42837	0.995764
1.0	0.00850	0.41649	0.00000	1.00000	0.00000	0.00107	0.00107	0.45148	0.994383

Table 5.4(d): Structure Parameters of Differentially Rotating and Tidally Distorted Prasad Model $\left(\psi_s^* = 5, b_1 = .3162, b_2 = .32, q = 0.1\right)$

X	V_ψ	S_ψ	ρ_ψ	M_ψ	P_ψ	σ	ε	T_e/T_P	$L_e L_p$
0.1	0.00001	0.00042	0.99004	0.00248	0.01304	0.00053	0.00053	0.14282	0.99731
0.2	0.00006	0.00166	0.96001	0.01951	0.01489	0.00054	0.00054	0.20198	0.99726
0.3	0.00023	0.00375	0.91003	0.06385	0.01314	0.00056	0.00056	0.24736	0.99713
0.4	0.00054	0.00666	0.84005	0.14457	0.01087	0.00060	0.00060	0.28562	0.99694
0.5	0.00106	0.01041	0.75008	0.26551	0.00819	0.00065	0.00065	0.31933	0.99668
0.6	0.00184	0.01499	0.64001	0.42320	0.00559	0.00072	0.00072	0.34983	0.99632
0.7	0.00292	0.02041	0.51013	0.60522	0.00327	0.00082	0.00081	0.37782	0.99582
0.8	0.00435	0.02665	0.36014	0.78833	0.00146	0.00095	0.00095	0.40386	0.99512
0.9	0.00619	0.03378	0.19010	0.93669	0.00035	0.00116	0.00116	0.42830	0.99399
1.0	0.00850	0.04165	0.00000	1.00000	0.00000	0.00153	0.00153	0.45147	0.99203

CHAPTER - VI

EQUILIBRIUM STRUCTURE OF DIFFERENTIALLY ROTATING WHITE DWARF MODELS OF STARS

White dwarf star is largely supported against gravity by the pressure provided by the kinetic energy of the degenerate electrons. In contrast, its luminosity is almost entirely derived from the thermal energy of the nondegenerate ions, when nuclear process no longer comes into play and gravitational contraction has almost ceased. A completely degenerate white dwarf very much resemble a polytropic configuration, polytropic index ranging from the N=1.5 (in the limit $M \to 0$) to N=3.0 (in the limit $M \to M_3$, where M_3 is the mass of polytropic index 3.0). Such models have frequently been used in literature to depict the inner structures of realistic stars at the last stage of their evolution. The white dwarf stars of class DC, those which have no observable lines, are possible candidates for having differential rotation. To test this suggestion, Milton (82) calculated the emergent spectra of hydrogen rich, differentially rotating white dwarf models. By virtue of the poincare-Wavre theorem, a barotropic configuration in a state of permanent rotation must necessarily comply with the condition $\Omega = \Omega(\varpi)$, where ϖ the distance from the axis of rotation .The particular case of constant angular velocity of rotation has been considered by several authors such as James (56), Anand and Dubas (3), Roxburg (119), Ostriker and Hartwic (100) etc. Their results show that solid body rotation does not induce any substantial change in the global structure of degenerate dwarfs. However the intense study carried out by Hoyal and Roxburgh on some problems of differentially rotating white dwarf stars assuming an angular momentum distribution law of the type $J = J(m_s)$,where m_s is the mass fraction interior to the cylinder, pointed out completely different picture. Detailed models of massive white dwarfs in fast non-uniform rotation have been also constructed by Ostriker et al. (99).

In the present chapter we implement the approach developed in chapter III to determine the effects of differential rotation on the equilibrium structures of white dwarf models using a law of differential rotation of the type (5.7). Even though approximation of exact equipotentials surfaces of rotating white dwarfs by corresponding Roche equipotentials, used in the present method, may not be very much justified, in the absence of more accurate analysis. It will be of interest to see how the results obtained by the present approach compare with the earlier results and observations.

In section 6.1 we first briefly introduce white dwarf model. The boundary value problem determining the equilibrium structures of differentially rotating white dwarf models of stars based on Kippenhahn and Thomas averaging approach has next been set up in section 6.2. Expressions determining the volume, surface area and other physical parameters of a differentially rotating white dwarf model are obtained in section 6.3. Numerical results for the equilibrium structures of certain differentially rotating white dwarf models have been obtained in section 6.4. In section 6.5 numerical results have been analysed to draw some conclusions of practical significance.

6.1 INTRODUCTION

White dwarf models have been extensively studied in literature as representative models of low mass stars in their last stage of evolution (see for instance, Chanadrasekhar (21)). In the case of completely degenerate white dwarf model, the equation of state can be written as (cf. Chandrasekhar (21), eqns. (16), (17) and (18) chapter XI).

$$P = Af(x), \quad \rho = Bx^3 \tag{6.1}$$

where, $A = 6.01\times10^{22}$, $B = 9.82\times10^{5}\mu_e$, (6.2)

$$f(x) = x(2x^2-3)(x^2+1)^{1/2} + 3\sinh^{-1}x, \qquad (6.3)$$

$x = \frac{P_0}{mc}$ is a relativistic constants?

and μ_e denotes mean molecular weight per electron.

The equilibrium structure of a white dwarf model can be shown to be governed by the nonlinear differential equation

$$\frac{1}{\eta^2}\frac{d}{d\eta}\left(\eta^2\frac{d\phi}{d\eta}\right) = -\left(\phi^2 - \frac{1}{\phi_0{}^2}\right)^{3/2} \qquad (6.4)$$

which has to be solved subject to the boundary conditions

$$\phi = 1,\ \frac{d\phi}{d\eta} = 0 \quad \text{at the centre } \eta = 0,$$

$$\text{and} \quad \phi = \frac{1}{\phi_o} \quad \text{at the surface } \eta = \eta_1. \qquad (6.5)$$

The exact treatment of the differential equation (6.4) provides much more quantitative information. The boundary conditions (6.5) combined with a particular value of ϕ_0 determines ϕ completely and therefore the mass of the configuration as well. Once the solution to the differential equation (6.4) satisfying boundary conditions (6.5) is obtained, other physical parameters of the white dwarf model can be obtained.

Equation (6.4) does not admit of a homology constant, and hence each mass has a density distribution characteristic of itself, which cannot be inferred from the density distribution in a configuration of a different mass. This is most fundamental difference between the white dwarfs and the polytropic models. Chandrasekhar (21) and other investigators have numerically solved the equation (6.4) to satisfy the boundary conditions (6.5) for values of $1/\phi_0^{\,2}$ varying from 0 to 1 and used these to determine the values of various physical parameters of white dwarf stars.

6.2 EQUILIBRIUM STRUCTURES OF DIFFERENTIALLY ROTATING WHITE DWARF MODELS

In this section we use the method developed in chapter III to obtain the equilibrium structures of certain differentially rotating white dwarf models. In case a white dwarf model is rotating differentially then as a result of the rotational forces its equilibrium surfaces get distorted from there original form of spherical symmetry. Following the approach of chapter III, these distorted equipotentials surfaces due to mass variation may be approximated by the appropriate Roche equipotentials.

Let P_ψ and ρ_ψ denote the pressure and density respectively on the equipotentials surface $\psi =$ Constant of a differentially rotating white dwarf model. Then assuming that the distorted model is also a completely degenerate white dwarf model, P_ψ and ρ_ψ of such a configuration will be connected through the relations of the type

$$P_\psi = A\, f(x), \quad \text{and} \qquad \rho_\psi = B x^3. \tag{6.7}$$

where $f(x)$ is given by equations (6.3). Equations (3.18) and (3.19) which govern the hydrostatic equilibrium structure of a differentially rotating stellar model can be combined together to yield

$$\frac{1}{r_\psi^{\,2}} \frac{d}{d r_\psi} \left[\frac{r_\psi^{\,2}}{\rho_\psi} \frac{d\, p_\psi}{d\, r_\psi} \right] = -4\pi G \rho_\psi \tag{6.8}$$

and using relation (6.7) and substituting $\left(x^2+1\right) = \phi_0^{\,2}\phi_\psi^{\,2}$, it reduces to

$$\frac{\alpha^2}{r_\psi^{\ 2}}\frac{d}{dr_\psi}\left[r_\psi^{\ 2}\frac{d\phi_\psi}{dr_\psi}\right]=-\left(\phi_\psi\ -\frac{1}{\phi_0^{\ 2}}\right)^{3/2} \tag{6.9}$$

where $$\alpha^2=\frac{2A}{\pi GB^2\phi_0^2}$$

In case the white dwarf model is rotating differentially according to the law (5.7), the values of r_ψ needed in equation (6.9) is provided by equations (3.16) of chapter III. It may be noted that the approximation of equipotentials surfaces by Roche equipotentials does not basically alter the structures of white dwarf model because in the absence of any distortion equation (6.9) reduces to the usual structure of equation (6.4) of white dwarf given in the earlier section.

To obtain the equilibrium structure of a rotationally distorted model, (6.9) has to be integrated numerically subject to the boundary conditions

$$\phi_\psi=1,\ \frac{d\phi_\psi}{dr_\psi}=0 \text{ at the center } r_\psi=0,$$

and $$\phi_\psi=\frac{1}{\phi_0} \quad \text{at the surface } r_\psi=R_\psi. \tag{6.10}$$

The values of r_ψ on the outermost equipotentials surface of the distorted white dwarf is given by

$$r_\psi=\alpha\,\eta_u \tag{6.11}$$

where η_u is the value of η when ϕ equals $\frac{1}{\phi_0}$ for the undistorted model.

In case the white dwarf model is assumed to be rotating differentially according to the law (5.7), the value of r_ψ given by (5.22), of chapter V. On substituting expression for r_ψ and retaining terms up to second order of smallness in b_1 and b_2 and up to r_0^{10} in r_0 in equation (6.9), the differential equation governing the equilibrium structure of a differentially rotating white dwarf can be written explicitly in the nondimensional form as

$$\frac{d}{dr_0}\left[A(z,b_1,b_2,r_0)\frac{d\phi_\psi}{dr_0}\right] = -\eta_u^2 r_0^2 B(z,b_1,b_2,r_0)\left(\phi_\psi^2 - \frac{1}{\phi_0^2}\right)^{3/2} \qquad (6.12)$$

where

$$A(z,b_1,b_2,r_0) = r_0^2 + \text{Other terms containing higher powers of } r_0$$

$$B(z,b_1,b_2,r_0) = 1 + \frac{2b_1^2 r_0^3}{z} + \frac{32 b_1 b_2 r_0^5}{15z} + \frac{16 b_2^2 r_0^7}{21z} + \ldots$$

and
$$z = \frac{M_0(r)}{M_0} = \frac{r_0^2 \dfrac{d\phi_\psi}{dr_0} \times \eta_u}{\left(r_0^2 \dfrac{d\phi_\psi}{d r_0}\right)_{r_0 = r_{0s}}}$$

$$\text{where} \left(r_0^2 \frac{d\phi_\psi}{d r_0}\right)_{r_0 = r_{0s}} = \left(\eta^2 \frac{d\phi}{d\eta}\right)_{\eta=\eta_u} \qquad (6.12a)$$

The value $\left(\eta^2 \dfrac{d\phi}{dr_0}\right)_{\eta=\eta_u}$ are taken from Chandrasekhar (21) and the $r_0 = \dfrac{z}{\psi}$ is a nondimensional measure of the distance of the fluid element from the center. In the above expression terms up to second order of smallness in z, b_1 and b_2 and up to r_0^{10} in r_0 have been retained. Equation (6.12) has to be solved subject to the boundary conditions (6.5) which now become:

at the center: $r_0 = 0, \phi_\psi = 1, \dfrac{d\phi_\psi}{dr_0} = 0,$ and

at the surface $r_0 = r_{0s}, \phi_\psi = \dfrac{1}{\phi_0}$

(6.13)

r_{0s} being the value of r_0 at the outer surface (r_0 and r_{0s} are both nondimensional quantities).

Equation (6.12) subject to the boundary conditions (6.13) determines the equilibrium structure of a differentially rotating white dwarf model. On setting $b_1^{\,2} = 2n$ and $b_2 = 0$, the equation (6.12) can be used to determine the equilibrium structure of a white dwarf model distorted by solid body rotation alone.

In order to determine the numerical solution of the second-order nonlinear differential equation (6.12) subject to the boundary conditions (6.13), we can start integration of (6.13) from the center using $\phi_\psi = 1$ as the initial conditions. The integration is to be continued till ϕ_ψ equals to 1 and $\frac{d\phi_\psi}{dr_0} = 0$. However at each step of integration we need the value of z which can be computed using equation (6.13). The integration is to be continued till ϕ_ψ equals to $\frac{1}{\phi_0}$. The value r_{0s} of r_0 for which ϕ_ψ , becomes $\frac{1}{\phi_0}$, determines the outermost free surface of the model. Once the solution of equation (6.12) is obtained, we know the values of ϕ_ψ for various values of the nondimensional independent variable r_0 varying from 0 to r_{0s}. The values of pressure P_ψ and the density ρ_ψ on the various equipotentials surfaces of the distorted model may now be obtained through the relation (6.7) in the same manner as is done for the undistorted white dwarf models.

6.3 COMPUTATION OF VARIOUS PHYSICAL PARAMETERS

Following the approach of chapter V, the volume V_ψ, and the surface area S_ψ and the shapes of a differentially rotating white dwarf are given as respectively

$$V_\psi = \frac{4\pi}{3}(\alpha\eta_u)^3 {r_{0s}}^3\left[1+\frac{b_1^2 {r_{0s}}^3}{z}+\frac{4b_1b_2 {r_{0s}}^5}{5z}+\frac{8b_2^2 {r_{0s}}^7}{35z}+...\right] \tag{6.14}$$

and

$$S_\psi = 4\pi(\alpha\eta_u)^2 {r_{0s}}^2\left[1+\frac{2b_1^2 {r_{0s}}^3}{3z}+\frac{8b_1b_2 {r_{0s}}^5}{15z}+\frac{16b_2^2 {r_{0s}}^7}{105z}+...\right] \tag{6.15}$$

and its shape is determined by

$$r = (\alpha\eta_u)r_{0s}\left[1+\frac{b_1^2 x {r_{0s}}^3}{2z}+\frac{b_1b_2x^2 {r_{0s}}^5}{2z}+\frac{b_2^2x^3 {r_{0s}}^7}{6z}+...\right] \tag{6.16}$$

values of other parameters such as L_ψ, T_ψ, ω_e, ω_p, R_e, R_p, and ε etc. may now be determined as in section 5.3 of chapter V by assuming q = 0 and replacing D by simply $\alpha\eta_u$.

6.4 NUMERICAL RESULTS

To obtain inner structure, the shape, the volume and the surface area of a differentially rotating white dwarf model, equation (6.12) has to be integrated numerically subject to the boundary conditions (6.13) for the specified values of the parameters $1/{\phi_0}^2$, the radius of the undistorted white dwarf η_u, and the values of constants b_1 and b_2 appearing on the right hand side of the law of differential rotation (5.7). Numerical integration of this equation may be performed by the use of fourth order Runge-Kutta method. Since the center and the surface of the star are singularities of (6.12), for starting numerical integration, a series solution has been developed near the center. Such a series solution for the present case is given by

$$\phi_\psi = 1 - \frac{\eta_u^{\ 2}}{6} q^3 r_0^{\ 2} + \frac{\eta_u^{\ 4}}{40} q^4 r_0^{\ 4} - \frac{b_1^{\ 2} \eta_u^{\ 2}}{15z} q^3 r_0^{\ 5} - \frac{q^5 (5q^2 + 14)\, \eta_u^{\ 6}}{5040} r_0^{\ 6} + \frac{3 b_1^{\ 2} \eta_u^{\ 4}}{140z} q^4 r_0^{\ 7} + \ldots$$

(6.17)

where

$$q^2 = \frac{1}{\phi_0^2}$$

Numerical integrations have been performed to obtain the inner structures of certain differentially rotating white dwarf models taking the values of $1/\phi_0^{\ 2}$ *as* 0.01, 0.05, 0.2, 0.4, 0.6, and 0.8. After obtaining the starting value of ϕ_ψ from the series solution (6.17) at $r_0 = 0.005$, numerical integration of equation (6.12) was carried forward using Runge-Kutta method of fourth order using a step length of 0.005. It was continued till ϕ_ψ equaled to $1/\phi_0^2$. Whereas values of r_0 at surface are presented in Table 6.1, values for corresponding volumes, surface areas, shape and other physical parameters are presented in Table 6.2 (a-f).

6.5 ANALYSIS OF THE RESULTS

Values of r_{0s} for various types of differentially rotating white dwarf models are presented in Table 6.1. In this Table the value of b_1 and b_2 for first three models are same as taken by Mohan et al.(91). The results for the volumes and surfaces areas given in Tables 6.2 (a) to 6.2 (f) show similar trend as obtained by Mohan et al.(91). (For comparison the volumes and surface areas as obtained by Mohan et al. Lal (91) are shown in parenthesis). It is noticed that because of the modification in the formula for gravitational potential to account for mass variation at different points inside the stars, our values for volumes and surface areas are comparatively smaller as compared to the results obtained by Lal (91). However, the actual decreases in the volumes and surface areas differ from model to model. The maximum decrease has been noticed in the case of model 2 for all the values of parameters $\frac{1}{\phi_0^2}$ considered in our present study. For the model 5 and 6 which are unstable according to Stockely criteria, the values of volumes and surfaces area are still smaller compared to the corresponding values for the undistorted models.

The values of σ and ε presented in these Table give a reasonable idea of the distortion in the shape of the model. It is noticed that the values for these parameter are smaller compared to the corresponding values shown in parenthesis as obtained earlier by Lal (91). The maximum decrease in these values is for model 3. The values of $\frac{T_e}{T_P}$ and $\frac{L_e}{L_P}$ shown in these Tables also indicate that as in the case of earlier study of Lal (91), the values of luminosity and temperatures are less on equator as compared their values at the poles. The comparison of these results with the results of Mohan et al. (91) shown in parenthesis also shows that these values are larger than those obtained by Lal.

Table 6.1 : Values of r_{0s} for various types of differentially rotating white dwarf modelsfor different values of $\frac{1}{\phi_0^2}$

Model No.	Values of distortions parameters b_1	b_2	0.01	0.05	0.2	0.4	0.6	0.8
			Values of $\frac{1}{\phi_0^2}$					
			Values of r_{0s}					
1.0	0.0000	0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2.0	0.31623	0.0000	0.94151	0.96427	0.97217	0.97260	0.97259	0.97224
3.0	0.0000	0.31623	0.99947	0.99922	0.99891	0.99874	0.99864	0.99858
4.0	0.31623	0.31623	0.93837	0.95929	0.96542	0.96504	0.96457	0.96393
5.0	0.20000	-0.2000	0.97562	0.98691	0.99066	0.99130	0.99136	0.99134
6.0	0.10000	-.06000	0.99385	0.96499	0.99742	0.99750	0.99750	0.99749

Table 6.2(a): Values of certain structure parameters and related quantities for differentially rotating white dwarf models of stars for $\frac{1}{\phi_0^2} = 0.01$ *and* $\eta_u = 5.3571$

Model No.	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$	σ	ε	ω_e	ω_P	$\frac{T_e}{T_P}$	$\frac{L_e}{L_P}$
1.	6.43982 (6.44244	3.60636 (3.60780)	0.00000 (0.00000)	0.00000 (0.00000)	0.00000 (0.00000)	0.0000 (0.0000)	1.00000 (1.0000)	1.0000C (1.0000
2.	5.82334 (6.92549	3.37474 (3.78622)	0.04172 (0.05536)	0.04005 (0.05246)	0.316200 (0.31623)	0.31620 (0.31623	0.94941 (0.94528)	0.81077 (0.7565
3.	6.57604 (6.58056	3.65722 (3.65939)	0.016602 (0.01662)	0.01633 (0.01634)	0.326439 (0.32654)	0.00000 (0.0000)	0.97385 (0.96609)	0.85699 (0.8568
4.	6.14835	3.50467	0.88354	0.08118	0.31623	0.64599	0.88853	0.88909
5.	6.09629	3.47702	0.00650	0.006467	0.00714	0.2000	0.98817	0.98066
6.	6.35949	3.57630	0.00257	0.00256	0.04043	0.1000	0.99651	0.99071

Table 6.2(b): Values of certain structure parameters and related quantities for differentially rotating white dwarf models of stars for $\frac{1}{\phi_0^2} = 0.05$ and $\eta_u = 4.4601$

Model No.	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$	σ	ε	ω_e	ω_P	$\frac{T_e}{T_P}$	$\frac{L_e}{L_P}$
1.	3.71640 (3.71800	2.49976 (2.50082)	0.00000 (0.0000)	0.00000 (0.0000)	0.00000 (0.0000)	0.00000 (0.0000)	1.00000 (1.0000)	1.00000 (1.0000)
2.	3.63081 (3.97497	2.46323 (2.61493)	0.04482 (0.05505)	0.04289 (0.05219)	0.31620 (0.31623	0.31620 (0.31623)	0.94557 (0.95918)	0.79815 (0.75771)
3.	3.79203 (3.79536	2.5337 (2.53553)	0.01657 (0.01659)	0.01630 (0.01632)	0.32625 (0.32640	0.00000 (0.0000)	0.97377 (0.96614)	0.85722 (0.85706)
4.	3.83960	2.56166	0.09720	0.08859	0.66649	0.31623	0.88762	0.87068
5.	3.63252	2.46208	0.00657	0.00653	0.66649	0.20000	0.99342	0.98049
6.	3.69950	2.49219	0.00258	0.00257	0.04385	0.10000	0.98190	0.99094

Table 6.2(c): Values of certain structure parameters and related quantities for differentially rotating white dwarf models of stars for $\frac{1}{\phi_0^2} = 0.2$ and $\eta_u = 3.7871$

Model No.	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$	σ	ε	ω_e	ω_P	$\frac{T_e}{T_P}$	$\frac{L_e}{L_P}$
1.	2.16871 (2.16960	1.74563 (1.74634)	0.00000 (0.0000)	0.00000 (0.0000)	0.00000 (0.0000)	0.00000 (0.0000)	1.00000 (1.00000)	1.00000 (1.0000)
2.	2.17576 (2.30842	1.75089 (1.82018)	0.04593 (0.05479)	0.04391 (0.05195)	0.31620 (0.31623	0.31620 (0.31623)	0.96252 (0.94584)	0.79366 (0.75874)
3.	2.21071 (2.2132)	1.76819 (1.7698)	0.016538 (0.01657)	0.01626 (0.1630)	0.32603 (0.3262)	0.00000 (0.0000)	0.97368 (0.96619)	0.85751 (0.85727)
4.	2.29277	1.81672	0.09993	0.09085	0.67275	0.3162	0.88701	0.89909
5.	2.14421	1.73251	0.00660	0.00656	0.00111	0.2000	0.99531	0.98043
6.	2.16489	1.74359	0.00259	0.00258	0.03999	0.1000	0.99830	0.99067

Table 6.2(d):Values of certain structure parameters and related quantities for differentially rotating white dwarf models of stars for $\frac{1}{\phi_0^2} = 0.4$ and $\eta_u = 3.5245$

Model No.	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-}$	σ	ε	ω_e	ω_P	$\frac{T_e}{T_P}$	$\frac{L_e}{L_P}$
1.	1.83391 (1.83462)	1.55013 (1.56161	0.00000 (0.0000)	0.00000 (0.0000)	0.00000 (0.0000)	0.00000 (0.0000)	1.00000 (1.0000)	1.00000 (1.0000)
2.	1.84253 (1.94785)	1.56722 (1.62533)	0.04599 (0.05467)	0.04397 (0.05184)	0.31620 (0.3162)	0.31620 (0.3162)	0.96270 (0.94595)	0.79341 (0.75921)
3.	1.8684	1.5806	0.16510	0.01624	0.32590	0.00000	0.97363	0.85768

	(1.87095)	(1.58225)	(0.01656)	(0.01629)	(0.32616)	(0.0000)	(0.96622)	(0.85738
4.	1.93607	1.62302	0.09976	0.09071	0.67236	0.3162	0.88705	0.89981
5.	1.81677	1.55131	0.00661	0.00567	0.00085	0.2000	0.99564	0.98042
6.	1.83116	1.5594	0.00259	0.00258	0.03999	0.1000	0.99835	0.99067

Table 6.2(e):Values of certain structure parameters and related quantities for differentially rotating white dwarf models of stars for $\frac{1}{\phi_0^2} = 0.6$ *and* $\eta_u = 3.6038$

Model No.	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$	σ	ε	ω_e	ω_P	$\frac{T_e}{T_P}$	$\frac{L_e}{L_P}$
1.	1.96052 (1.96123)	1.63204 (1.63265)	0.00000 (0.0000)	0.00000 (0.0000)	0.00000 (0.0000)	0.00000 (0.0000)	1.00000 (1.0000)	1.00000 (1.0000)
2.	1.969695 (2.07993)	1.63850 (1.69799)	0.04599 (0.05460)	0.04397 (0.05184)	0.31620 (0.31623)	0.31620 (0.31623)	0.96269 (0.94001)	0.79342 (0.75944)
3.	1.99678 (1.99969)	1.65220 (1.65402)	0.01650 (0.01654)	0.01623 (0.16279)	0.32583 (0.32611)	0.00000 (0.0000)	0.97360 (0.96623)	0.85777 (0.85743)
4.	2.0660	1.6948	0.09955	0.09053	0.67187	0.31620	0.88710	0.90070
5.	1.94254	1.62210	0.00661	0.00657	0.00083	0.2000	0.99567	0.98042
6.	1.95754	1.63039	0.00259	0.00258	0.03999	0.1000	0.99834	0.99067

Table 6.2(f):Values of certain structure parameters and related quantities for differentially rotating white dwarf models of stars for $\frac{1}{\phi_0^2} = 0.8$ *and* $\eta_u = 4.0446$

Mode l No.	$V_\psi \times 10^{-2}$	$S_\psi \times 10^{-2}$	σ	ε	ω_e	ω_P	$\frac{T_e}{T_P}$	$\frac{L_e}{L_P}$
1.	2.77150 (2.7725)	2.05571 (2.0564)	0.00000 (0.0000)	0.00000 (0.0000)	0.00000 (0.0000)	0.00000 (0.0000)	1.00000 (1.0000)	1.00000 (1.0000)
2.	2.78116 (2.9381)	2.06223 (2.1377)	0.04594 (0.05456)	0.04392 (0.05174)	0.31620 (0.3162)	0.31620 (0.3162)	0.96255 (0.94605)	0.79362 (0.75960)
3.	2.82224 (2.8265)	2.08083 (2.0832)	0.01649 (0.01654)	0.01623 (0.1627)	0.32579 (0.32608)	0.00000 (0.0000)	0.97358 (0.96624)	0.85783 (0.85747)
4.	2.91367	2.13138	0.09926	0.09029	0.67122	0.31620	0.88717	0.89193
5.	2.74554	2.04291	0.00661	0.00657	0.00084	0.2000	0.99564	0.98042
6.	2.7672	2.05361	0.00259	0.00258	0.03991	0.1000	0.99834	0.99067

CHAPTER VII

EQUILIBRIUM STRUCTURE OF DIFFERENTIALLY ROTATING GAS SPHERES FOLLOWING A MORE GENERALIZED LAW OF DIFFERENTIAL ROTATION

In the chapters V and VI we considered the problem of determining equilibrium structures of differentially rotating gas spheres obeying a law of differential rotation of the form $\omega^2 = \omega\,(s^2)$. In this chapter we consider the problem of determining the equilibrium structures of differentially rotating gas spheres assuming a more general law of differential rotation of the type $\omega^2 = \omega(s^2, z^2)$ (where ω is a nondimensional measures of the angular velocity of rotation, s and z nondimensional measures of the distance of the fluid element from and along the axis of rotation) which accounts for variations in angular velocity along the axial direction as well as in a direction perpendicular to it.

We assume a law of differential rotation of the type $\omega^2 = b_0 + b_1 z^2 + b_2 z^4 + b_3 z^2 + b_4\, z^4 + b_5\, z^2 s^2$ which accounts for variations in angular velocity along s as well as z directions. As in the earlier chapters, Kippenhahn and Thomas averaging approach has been used to obtain the equilibrium structures of such types of differentially rotating models following the approach explained in chapter III which accounts for the effect of mass variation on the potential. The technique has been then used to obtain the equilibrium structures of differentially rotating polytropic models of indices 1.5, 3.0 and 4.0 for various choices of the values of rotation parameters b_0, b_1, b_2, b_3, b_4 and b_5.

The law of differential rotation selected by us for our present study is presented in section 7.1. The nature of some of the important types of differential rotations is considered in this section. In section 7.2 we consider the problem of determining the Roche equipotentials of differentially rotating stars taking effect of mass variation inside the star on its equipotential surface. The methodologies is next used in section 7.3 to derive the system of differential equations which govern the equilibrium structures of such types of differentially rotating gas spheres. The methodology is next applied in section 7.4 to determine the equilibrium structures of differentially rotating polytropic models of stars. Numerical solutions have been next obtained in section 7.5 to determine the equilibrium structures of certain differentially rotating polytropic models of indices 1.5 and 3.0. Certain conclusions based on the present study are finally drawn in section 7.6.

7.1 PROPOSED GENERALIZED LAW OF DIFFERENTIAL ROTATION

As far back as 1932, Giao and Wehrle suggested a differential rotation law of the type

$$\Omega(s, z) = \Omega_0 \cos m z \sum_{n=0}^{\infty} \frac{m s}{2^{2n}(n+1)(n!)^2} \tag{7.1}$$

where Ω_0 and m are two constants of integration. In the case of a quasi spherical system (such as the sun), it is convenient to use spherical coordinates $(r, \phi = 90 - \theta, \psi)$. On substituting $s = r\cos\phi$ and $z = r\cos\phi$ in equation (7.1) we obtain

$$\Omega(R, \phi) = \Omega_0 \cos\,(m R \sin\phi) \sum_{n=0}^{\infty} \frac{m R \cos\phi}{2^{2n}(n+1)(n!)^2} \tag{7.2}$$

where R gives mean boundary of the configuration with mean radius $r = R$. The remarkable feature of equation (7.2) is that it reproduces with good accuracy the solar rotation law, when Ω_0 and mR are fitted at two different heliocentric latitudes. Another approach, which also takes into account the viscous forces in a restricted form, was suggested by Schwarzschild in 1942. The method consists of the derivation of the function $\Omega(s,t)$, which is a solution of the usual equations for an inviscid fluid. Practical means to determine the function $\Omega(s,t)$ and various application of these ideas were suggested by several authors such as Jeans (57,58), Clement (24), Marks and Clement (77) etc. In order to be able to compute the inner structure of a differentially rotating star it will be helpful if the law of rotation is assumed in a form which takes into account the true nature of the differential rotation in the star as well as is convenient to use. In case we assume symmetry of rotation along and perpendicular to the axis of rotation, the law of differential rotation which can account for variations in angular velocity both along s and z directions will be of the type $\omega = \omega\ (s^2, z^2)$. Following the commonly assumed law of differential rotation $\omega = b_1 + b_2 s^2$ for stars in which there is no variation in angular velocity along z axis we may assume

$$\omega = c_0 + c_1 s^2 + c_2 z^2 \tag{7.3}$$

as a law which accounts for variations in s, as well as z directions. Here c_0, c_1, c_2 are suitably chosen arbitrary constants which account for variations in ω along s and z directions. By squaring (7.3) we get,

$$\omega^2 = c_0{}^2 + c_1{}^2 s^4 + c_2{}^2 z^4 + 2c_0 c_1 s^2 + 2c_0 c_2 z^2 + 2c_1 c_2 s^2 z^2 \tag{7.4}$$

In analogy with our earlier assumption of a law of differential rotation of the type $\omega^2 = b_1 + b_2 s^2$ for stars in which there is variation in the angular velocity perpendicular to the axis of rotation alone we may therefore assume

$$\omega^2 = b_0 + b_1 s^2 + b_2 s^4 + b_3 z^2 + b_4 z^4 + b_5 z^2 s^2 \tag{7.5}$$

as a law of differential rotation which accounts for variations in the angular velocity along s and t directions. The law (7.5) is more general than (7.4) and reduces to (7.4) for $b_0 = c_0^2\, b_1 = 2c_0 c_1$, $b_2 = c_1{}^2$, $b_3 = 2\, c_0\, c_2$, $b_4 = c_2^2$ and $b_5 = 2c_1\, c_2$.

For a gaseous sphere rotating differentially according to law (7.5) b_0, b_1, b_2, b_3, b_4 and b_5 must be so chosen such that ω^2 is non-negative everywhere inside the star. For $b_3 = b_4 = b_5 = 0$ it reduces to the law $\omega^2 = b_0 + b_1 s^2 + b_2 s^4$ used in the earlier chapters. According to the law (7.5), the value of the angular velocity ω_c at the center, ω_p at the pole and ω_e at the equator are given by

$$\omega_c = \sqrt{b_0} \tag{7.6}$$

$$\omega_p = \sqrt{b_0 + b_3 R_p{}^2 + b_4 R_p{}^4} \tag{7.7}$$

and

$$\omega_e = \sqrt{b_0} + b_1 R_e^2 + b_2 R_e^4 \tag{7.8}$$

where R_p is the polar and R_e the equatorial radius of the star. For suitable choice of the values of b_0, b_1, b_2, b_3, b_4 and, b_5 we can generate a variety of differentials rotations. Some of which may correspond to the differential rotations actually occurring in the case of certain differentially rotating stars Lal (91). The nature of certain types of differentials rotations which can be generated by (7.5) by giving different values to b_0, b_1, b_2, b_3, b_4 and b_5 are shown in Table 7.1 (cf. Lal (91)).

As far as dynamical stability of such differentially rotating stars is concerned, stable density stratifications permit certain rotation laws that depend on both s and z and are not in conflict with Von Zeipal paradox. As discussed in chapter II, a baroclinic star in permanent rotation rotating according to the law $\Omega = \Omega(s, z)$ will be dynamically stable with respect to axisymmetric motions if and only if the condition (5.6) given in chapter V is satisfied.

For a star rotating differentially according to the law (7.5) to be locally stable according to Hoiland (52) criteria, $b_0 + b_1 s^2 + b_2 s^4 + b_3 z^2 + b_4 z^4 + b_5 z^2 s^2$ must be non-negative for all values of s and z. The stability of each of the differential rotations considered by us in Table (7.1) was analyzed by Lal (91) according to these criteria and the results of this stability analysis are presented in the same Table for ready reference.

7.2 THE ROCHE EQUIPOTENTIALS OF DIFFERENTIALLY ROTATING GAS SPHERE INCORPORATING THE EFFECT OF MASS VARIATION ON THE POTENTIAL

For a star rotating differentially according to the law (7.5) the total potential Ω of the fluid element is given by

$$\Omega = \int dV + \int \omega^2 s \, ds$$

$$= V + \frac{1}{2} \int \omega^2 d(s^2)$$

$$= \frac{GM_0(r)}{r} + \frac{1}{2} \int \omega^2 d(s^2) \qquad (7.9)$$

Assuming Roche model for a differentially rotating gas sphere, $V = \frac{GM_0(r)}{r}$ at a point distant r from the center. $M_0(r)$ is mass interior to sphere of radius r and M_0 the total mass of the rotating gas sphere. Substituting these in (7.9) and multiplying throughout by $\frac{R}{GM_0}$ we get

$$\psi = \frac{t}{(r/R)} + \frac{1}{2} \frac{R}{GM_0} \int \omega^2 \, d(s^2). \qquad (7.10)$$

where 1* $t = \frac{M_0(r)}{M_0}$

Since dimension of s is same as that of R, assuming ω^2 to have a dimension of $\frac{GM_0}{R^3}$, the nondimensional form of (7.10) can be represented as

$$\psi = \frac{t}{r} + \frac{1}{2}\int \omega^2 d(s^2) \tag{7.11}$$

Substituting $\omega^2 = b_0 + b_1 s^2 + b_2 s^4 + b_3 z^2 + b_4 z^4 + b_5 z^2 s^2$, we get

$$\psi = \frac{t}{r} + \frac{1}{2}\int (b_0 + b_1 s^2 + b_2 s^4 + b_3 z^2 + b_4 z^4 + b_5 z^2 s^2)\, d(s^2)$$

$$= \frac{t}{r} + b_0 s^2 + \frac{1}{2} b_1 s^4 + \frac{1}{3} b_2 s^6 + b_3 t^2 s^2 + b_4 t^4 s^2 + \frac{1}{2} b_5 t^2 s^4$$

Writing $s^2 = r^2(1-\upsilon^2)$ and $z^2 = r^2 \upsilon^2$ we get

$$\psi = \frac{t}{r} + \frac{1}{2} r^2 (1-\upsilon^2)\Big[b_0 + \Big\{\frac{1}{2} b_1 (1-\upsilon^2) r^2 + b_3 \upsilon^2 (1-\upsilon^2)\Big\} r^2 + \\ + \Big\{\frac{1}{3} b_2 (1-\upsilon^2)^2 + b_4 \upsilon^4 (1-\upsilon^2) + \frac{1}{2} b_5 \upsilon^2 (1-\upsilon^2)\Big\} r^4 \Big] \tag{7.12}$$

Here ψ is now the nondimensional form of the total potential $\Omega(\psi = \frac{R\Omega}{GM})$, $\lambda = \sin\theta\cos\phi$, $\mu = \sin\theta\sin\phi, \upsilon = \cos\theta, (r,\theta,\phi$ being the polar spherical coordinates of the point with center of the star as the origin, X - axis in the equatorial plane, θ being measured from the axis taken as Z -axis).

(**Note 1*: Normally we have been using symbol for $z = \frac{M_0(r)}{M_0}$ and are also taking z one of the variable in the law of differential rotation $\omega = \omega(s, z)$ .To avoid confusion the variable z generally used for in the mass ratio is taken as t in this chapter**).

Also in (7.12), r is a nondimensional measure (r/R) of the distance of the fluid element from the center of the star and b_0, b_1, b_2, b_3, b_4 and b_5 are numerical constants in units of GM_0/R^3 . In our present study, we shall assume the law of differential rotation (7.5) in which $\omega^2, b_0, b_1, b_2, b_3, b_4$ and b_5 are in units of GM_0/R^3 , s is a nondimensional measure of the distance of the fluid element from the axis of rotation and z is nondimensional measure of its distance from the equatorial plane.

Following the approach used in section 3.4 of chapter III it can be shown that the coordinates (r,θ,ϕ) of an element on a Roche equipotentials $\psi =$ constant of a star rotating differentially according to the law (7.5) are connected through the relation

$$r=r_0\, R\left[1+\frac{1}{2t}b_0 x r_0^{\,3}+\left\{\frac{1}{2t}b_1 b_2 x^2+\frac{1}{2t}b_3 x(1-x)\right\}r_0^{\,5}+\frac{3}{4t^2}b_0^{\,2}x^2 r_0^{\,6}\right.$$
$$+\left\{\frac{1}{6t}b_2 x^3+\frac{1}{2t}b_4 x(1-x)+\frac{1}{2t}b_5 x^2(1-x)\right\}r_0^7+\left\{\frac{b_0 b_1 x^3}{t^2}+\frac{2b_0 b_3 x^2}{t^2}(1-x)\right\}r_0^8$$
$$\left.+\left\{\frac{5}{6t^2}b_0 b_2 x^4+\frac{5}{2t^2}b_0 b_4 x^2(1-x)+\frac{5}{2t^2}b_0 b_5 x^3(1-x)+\frac{5}{4t^2}b_1 b_3 x^3(1-x)\right\}r_0^{10}+\ldots\right]$$

(7.13)

where $x=(1-\upsilon^2)$, $r_0=\frac{t}{\psi}$ and terms upto second order of smallness in t, b_0, b_1, b_2, b_3, b_4 and b_5 and upto order $r_0^{\,10}$ in r_0 are retained. We may obtain the shapes of various equipotential surfaces of the differentially rotating gas sphere by setting $r=$ constant. In (7.13) R denotes the radius of the undistorted model.

Following the approach discussed in chapter V and VI the volume V_ψ, surface area S_ψ, average value $\overline{g}$, of acceleration due to gravity and its inverse $\overline{g^{-1}}$ are given by the explicit relations

$$V_\psi=\frac{4\pi}{3}R^3 r_0^{\,3}\left[1+\frac{b_0}{t}r_0^{\,3}+(\frac{2b_1}{5t}+\frac{b_3}{5z})r_0^{\,5}+\frac{8b_0^{\,2}r_0^{\,6}}{5t^2}+(\frac{8b_2}{35t}+\frac{b_4}{5t}+\frac{4b_5}{35t})r_0^{\,7}\right.$$
$$+(\frac{12}{7t}b_0 b_1+\frac{4}{7t^2}b_0 b_3)r_0^{\,8}+(\frac{128}{105t^2}b_0 b_2+\frac{24}{35t^2}b_0 b_4+\frac{16}{35t^2}b_0 b_5$$
$$\left.+\frac{8}{35t^2}b_1 b_3+\frac{12b_3^{\,2}}{35t^2})r_0^{\,10}+\ldots\right]$$

(7.14)

$$S_\psi = 4\pi\, R^2 r_0^{\,2}\Big[1+\frac{2}{3t}b_0 r_0^{\,3} + (\frac{4}{15t}b_1 + \frac{2}{15t}b_3)r_0^{\,5} + \frac{14}{15t^2}b_0^{\,2} r_0^{\,6} + (\frac{16}{105t}b_2$$

$$+\frac{2}{15t}b_4 + \frac{8}{105t}b_5)r_0^{\,7} + (\frac{36}{35t^2}b_0 b_1 + \frac{12}{35t^2}b_0 b_3)r_0^{\,8} + (\frac{704}{945t^2}b_0 b_2 +$$

$$+\frac{44}{105t^2}b_0 b_4 + \frac{88}{315t^2}b_0 b_5 + \frac{44}{315t^2}b_1 b_3 + \frac{22}{315t^2}b_3^{\,2})r_0^{\,10} + \ldots\Big]$$

(7.15)

$$\bar{g} = \frac{zGM_\psi}{R^2 r_0^{\,2}}\Big[1-\frac{4}{3t}b_0 r_0^{\,3} - (\frac{4}{5t}b_1 + \frac{2}{5t}b_3)r_0^{\,5} - \frac{7}{9t^2}b_0^{\,2} r_0^{\,6} - (\frac{64}{105t}b_2 + \frac{32}{105t}b_4 + \frac{32}{105t}b_5)r_0^{\,7}$$

$$-(\frac{488}{315t^2}b_0 b_1 + \frac{214}{315t^2}b_0 b_3)r_0^{\,8} - (\frac{1616}{945t^2}b_0 b_2 + \frac{46}{105t^2}b_0 b_4 + \frac{64}{105t^2}b_0 b_5$$

$$+\frac{36}{175t^2}b_1 b_3 + \frac{67}{525t^2}b_3^{\,2}+)r_0^{\,10} + \ldots\Big]$$

(7.16)

$$\overline{g^{-1}} = \frac{R^2 r_0^{\,2}}{zGM_\psi}\Big[1+\frac{4}{3t}b_0 r_0^{\,3} + (\frac{4b_1}{5t} + \frac{2b_3}{5t})r_0^{\,5} + \frac{131}{45t^2}b_0^{\,2} r_0^{\,6} + (\frac{64}{105t}b_2 + \frac{32}{105t}b_4$$

$$+\frac{32}{105t}b_5)r_0^{\,7} + (\frac{1352}{315t^2}b_0 b_2 + \frac{418}{315t^2}b_0 b_3)r_0^{\,8} + (\frac{3664}{945t^2}b_0 b_2 + \frac{22}{21t^2}b_0 b_4$$

$$+\frac{448}{315t^2}b_0 b_5 + \frac{116}{175t^2}b_1 b_3 + \frac{187}{525t^2}b_3^{\,2})r_0^{\,10} + \ldots\Big]$$

(7.17)

In the above expression M_ψ is the same masses contained within the equipotential surface $\psi =$ constant and terms up to second order of smallness in b_0, b_1, b_2, b_3, b_4 and b_5 are retained. On setting $b_0 = b_1^2 = 2n$, $b_1 = b_2 = b_3 = b_4 = b_5$ the above expressions reduce to the corresponding expressions for solid body rotation as obtained by Mohan, Saxena and Aggarwal (92). Again on setting $b_3 = b_4 = b_5 = 0$ these expressions reduce to corresponding expressions for the differential rotation obtained in section 5.2 of chapter V.

7.3 EQUILIBRIUM STRUCTURE OF DIFFERENTIALLY ROTATING MODEL FOLLOWING GENERALIZED LAW OF DIFFERENTIAL ROTATION

In case of rotating models in which rotation depends upon distance from axis of rotation as well as distance from equatorial axis surfaces of constant density and pressure may not coincide. Therefore strictly speaking the approach being adopted in the present work which assumes that surfaces of equipressure are same as equidensity is not strictly applicable. However in the absence of a more realistic approach which will be more complicated, we have analysed here this problem also adopting the earlier approach.

The equations governing the equilibrium structures of rotating gas sphere which are rotating differentially according to the law (7.5) are same as (5.21) of chapter V. Following the approach adopted in section 5.3 of chapter V, the results of the last section may be used to explicitly evaluate the values of the distorting parameters r_ψ, u, v, w, f_P and f_T for such types of differentially rotating stars. The explicit expressions of the parameters determining the value of r_ψ, u, v, w, f_P and f_T on the various equipotential surfaces of a differentially rotating star according to the law (7.5) are obtained as:

$$r_\psi = R r_0 \left[1 + \frac{1}{3t} b_0 r_0^3 + (\frac{2}{15t} b_1 + \frac{1}{15t} b_3) r_0^5 + \frac{19}{45t^2} b_0^2 r_0^6 + (\frac{8}{105t} b_2 + \frac{1}{15t} b_4 + \frac{4}{105t} b_5) r_0^7 \right.$$
$$+ \left(\frac{152}{315t^2} b_0 b_1 + \frac{46}{315t^2} b_0 b_3 \right) r_0^8 + (\frac{112}{315t^2} b_0 b_2 + \frac{58}{315t^2} b_0 b_4 + \frac{8}{63t^2} b_0 b_5$$
$$\left. + \frac{92}{1575t^2} b_1 b_3 + \frac{173}{1575t^2} b_3^2) r_0^{10} + \ldots \right]$$

(7.18)

$$u = 1 - \frac{1}{45t^2} b_0^2 r_0^6 - (\frac{8b_0 b_1}{315t^2} - \frac{2b_0 b_3}{315t^2}) r_0^8 - (\frac{16}{945t^2} b_0 b_2 - \frac{2}{315t^2} b_0 b_4$$
$$- \frac{8}{1575t^2} b_1 b_3 + \frac{27}{175t^2} b_3^2) r_0^{10} + \ldots$$

(7.19)

$$v = t\left[1-\frac{2}{3t}b_0 r_0^{\,3}-(\frac{8}{15t}b_1+\frac{4}{15t}b_3)r_0^{\,5}-\frac{32}{45t^2}b_0^{\,2}r_0^{\,6}-(\frac{16}{35t}b_2+\frac{18}{105t}b_4\right.$$
$$+\frac{24}{105t}b_5)r_0^{\,7}-(\frac{436}{315t^2}b_0b_1+\frac{248}{315t^2}b_0b_3)r_0^{\,8}-(\frac{1472}{945t^2}b_0b_2+\frac{128}{315t^2}b_0b_4$$
$$\left.+\frac{40}{63t^2}b_0b_5+\frac{448}{1575t^2}b_1b_3-\frac{68}{1575t^2}b_3^{\,2})r_0^{\,10}+\dots\right]$$

(7.20)

$$w=\frac{1}{t}\left[1+\frac{2}{3t}b_0 r_0^{\,3}+(\frac{8}{15t}b_1+\frac{4}{15t}b_3)r_0^{\,5}+\frac{68}{45t^2}b_0^2r_0^{\,6}+(\frac{16}{35t}b_2+\frac{6}{35t}b_4\right.$$
$$+\frac{8}{35t}b_5)r_0^{\,7}+(\frac{284}{105t^2}b_0\,b_1+\frac{76}{105t^2}b_0b_3)r_0^8+(\frac{152}{189t^2}b_0b_2+\frac{136}{315t^2}b_0b_4$$
$$\left.+\frac{296}{315t^2}b_0b_5+\frac{608}{1575t^2}b_1b_3+\frac{152}{1575t^2}b_3^2)r_0^{10}+\dots\right]$$

(7.21)

$$f_P=t\left[1-\frac{2}{3t}b_0 r_0^{\,3}-(\frac{8}{15t}b_1+\frac{4}{15t}b_3)r_0^{\,5}-\frac{47}{45t^2}b_0^{\,2}r_0^{\,6}-(\frac{16}{35t}b_2+\frac{6}{35t}b_4\right.$$
$$+\frac{8}{35t}b_5)r_0^{\,7}-(\frac{124}{63t^2}b_0b_1+\frac{118}{315t^2}b_0b_3)r_0^{\,8}-(\frac{656}{315t^2}b_0b_2+\frac{22}{105t^2}b_0b_4$$
$$\left.+\frac{40}{63\,t^2}b_0b_5+\frac{56}{525t^2}b_1b_3-\frac{203}{1575t^2}b_3^{\,2})r_0^{\,10}+\dots\right]$$

(7.22)

and

$$f_T=1-\frac{14}{45t^2}b_0^{\,2}r_0^{\,6}-(\frac{176}{315t^2}b_0b_1-\frac{128}{315t^2}b_0b_3)r_0^{\,8}-(\frac{32}{63t^2}b_0b_2-\frac{4}{21t^2}b_0b_4$$
$$-\frac{272}{1575\,t^2}b_1b_3+\frac{42}{175t^2}b_3^{\,2})r_0^{\,10}+\dots$$

(7.23)

where r_0 is nondimensional variable defined by $r_0=\frac{t}{\psi}$. The above expressions contain terms upto second order of smallness in b_0, b_1, b_2, b_3 b_4 and b_5 and terms up to order r_0^{10} in r_0. Variable r_0 is connected to r_ψ through the relation (7.18).

Following the approach adopted in section 5.3 of chapter V, the equations governing the equilibrium structure of a star rotating differentially according to the law (7.5) may finally be written in terms of independent variable r_0 as

$$\frac{dM_\psi}{dr_0} = 4\pi\,\rho_\psi r_0^{\;2} R^3 f_1$$

$$\frac{dP_\psi}{dr_0} = -\frac{GM_\psi}{R r_0^{\;2}}\rho_\psi\, f_2$$

$$\frac{dL_\psi}{dr_0} = 4\pi\,\varepsilon R^3 r_0^{\;2}\,\rho_\psi f_1, \tag{7.24}$$

and

$$\frac{dT_\psi}{dr_0} = -\frac{3KL_\psi}{16\pi acRT_\psi^{\;3}}\frac{\rho_\psi}{r_0^{\;2}} f_3,$$

where,

$$f_1 = 1 + \frac{2b_0}{t}r_0^{\;3} + (\frac{16}{15t}b_1 + \frac{8}{15t}b_3)r_0^{\;5} + \frac{24}{5t^2}b_0^{\;2}r_0^{\;6} + (\frac{16}{21t}b_2 + \frac{2}{3t}b_4 + \frac{8}{21\,t}b_5)r_0^{\;7}$$
$$+ (\frac{44}{7t^2}b_0b_1 + \frac{44}{21t^2}b_0b_3)r_0^{\;8} + (\frac{1664}{315t^2}b_0b_2 + \frac{104}{35t^2}b_0b_4 + \frac{208}{105t^2}b_0b_5 +$$
$$+ \frac{104}{105t^2}b_1b_3 + \frac{52b_3^{\;2}}{35t^2})r_0^{\;10} + \ldots$$

$$f_2 = t\left[1 + \frac{1}{15t^2}b_0^{\;2}r_0^{\;6} + \frac{8}{35t}b_4r_0^{\;7} + (\frac{8}{105t^2}b_0b_1 + \frac{2}{105t^2}b_0b_3)r_0^{\;8} + (\frac{16}{315t^2}b_0b_2\,.\right.$$
$$\left. + \frac{2}{3t^2}b_0b_4 - \frac{8}{315t^2}b_0b_5 - \frac{8}{315t^2}b_1b_3 + \frac{317}{315t^2}b_3^{\;2})r_0^{\;10} + \ldots\right]$$

and

$$f_3 = 1 + \frac{2}{3t}b_0r_0^{\;3} + (\frac{8}{15t}b_1 + \frac{4}{15t}b_3)r_0^{\;5} + \frac{28}{45t^2}b_0^{\;2}r_0^{\;6} + (\frac{16}{35t}b_2 + \frac{2}{5t}b_4 + \frac{8}{35t}b_5)r_0^{\;7}$$
$$+ (\frac{692}{315t^2}b_0b_1 + \frac{352}{315t^2}b_0b_3)r_0^{\;8} + (\frac{704}{315t^2}b_0b_2 + \frac{152}{105t^2}b_0b_4 + \frac{32}{35\,t^2}b_0b_5 +$$
$$+ \frac{848}{1575\,t^2}b_1b_3 + \frac{182}{175t^2}b_3^{\;2})r_0^{\;10} + \ldots$$

In the above expressions, terms up to second order of smallness in $z, b_0, b_1, b_2, b_3, b_4$ and b_5 and up to order $r_0^{\;10}$ in r_0 are retained. On setting

$q = b_3 = b_4 = b_5 = 0$ and neglecting the fourth order terms as b_1^4 in the expressions for f_1, f_2, f_3, these reduce to their corresponding forms which is obtained in section 5.3 of chapter V for differentially rotating model with differential rotation law (5.7). And also if we take $b_1^2 = 2n,\ b_2 = 0$ then these terms reduce to equation (3.20) of chapter III for rotationally and tidally distorted models.

The values of $P_\psi, \rho_\psi, L_\psi$, *etc.* on the various equipotentials surfaces of a differentially rotating gas sphere may be obtained by solving the system of differential equation(7.24) using the values of distortion parameters f_1, f_2, and f_3 subject to boundary conditions (5.31).

7.4 EQUILIBRIUM STRUCTURE OF DIFFERENTIALLY ROTATING POLYTROPIC MODELS FOLLOWING GENERALIZED LAW OF DIFFERENTIAL ROTATION

Following the approach adopted in chapter V, the equation governing the equilibrium structure of a rotating polytropic model, rotating differentially according to the law (7.5) can be written in nondimensional form as

$$\frac{d}{dr_0}\left[\left[A(r_0,t,b_0,b_1,b_2,b_3,b_4,b_5)\frac{d\theta_\psi}{dr_0}\right] = -\theta_\psi{}^N \xi_u{}^2 r_0{}^2 B\left(r_0,t,\, b_0,b_1,b_2,b_3,b_4,b_5\right) \tag{7.25}$$

where

$$A(r_0) = r_0{}^2\left[1 - \frac{1}{15t^2}b_0{}^2 r_0{}^6 - \frac{8}{35t}b_4 r_0{}^7 - \left(\frac{8}{105t^2}b_0b_1 - \frac{2}{105t^2}b_0b_3\right)r_0{}^8 - \left(\frac{16}{315t^2}b_0b_2 + \frac{2}{3t^2}b_0b_4 - \frac{8}{105t^2}b_0b_5 - \frac{8}{315t^2}b_1b_3 + \frac{1585}{1575t^2}b_3{}^2\right)r_0{}^{10} + \ldots\right]$$

and

$$B(r_0) = [\ 1 + \frac{2b_0}{t} r_0^{\ 3} + (\frac{16}{35t} b_1 + \frac{8}{15t} b_3) r_0^{\ 5} + \frac{24}{5t^2} b_0^{\ 2} r_0^{\ 6} + (\frac{16}{21t} b_2 + \frac{2}{3t} b_4 + \frac{8}{21t} b_5) r_0^{\ 7}$$

$$+ (\frac{44}{7t^2} b_0 b_1 + \frac{44}{21t^2} b_0 b_3) r_0^{\ 8} + (\frac{1664}{315t^2} b_0 b_2 + \frac{104}{35t^2} b_0 b_4 + \frac{208}{105t^2} b_0 b_5 +$$

$$+ \frac{104}{105t^2} b_1 b_3 + \frac{52}{35t^2} b_3^{\ 2}) r_0^{\ 10} + ..\ \Bigg]$$

$$t = \frac{M_0(r)}{M_0} = \frac{r_0^2 \dfrac{d\theta_\psi}{d r_0} \times \xi_u}{\left(r_0^2 \dfrac{d\theta_\psi}{d r_0} \right)_{r_0 = r_{0s}}} \qquad 7.25(a)$$

The equation (7.25) may be integrated numerically following approach explained in section 5.3 of chapter V.

The central angular velocity ω_c, the polar angular velocity ω_p and the equatorial angular velocity ω_e can be obtained using (7.6), (7.7) and (7.8), respectively. The effective temperature and luminosity at any point on the surface can also be computed using the method given in section 5.2.

7.5 NUMERICAL COMPUTATIONS

To obtain the inner structure, the shape, the volume and the surface area of a rotating polytropic model, rotating differentially according to the law (7.5), equation (7.25) has to be integrated numerically subject to the boundary conditions (7.26) for the specified values of the polytropic index N, the radius of the undistorted polytropic ξ_u and the values of numerical constants b_0, b_1, b_2, b_3, b_4 and b_5 appearing in the right hand side of expression (7.5). As explained in chapter V, numerical integration of (7.25) can be performed using fourth order Runge-Kutta method. Since, we need the value of t at each step of the integration to account the effect of mass variation inside the models. For this, equation 7.25 (a) is used to calculate t at each interior point. Since the center and the surface of the model are singularities of (7.25). Series solution may be used for starting numerical integration. A series solutions valid near the center and which has been used by us in our present computations is given by.

$$\theta_\psi = 1 - \frac{1}{6}\xi_u^{\,2} r_0^{\,2} + \frac{N}{120}\xi_u^{\,4} r_0^{\,4} - \frac{b_0}{15t}\xi_u^{\,2} r_0^{\,5} - \frac{N(8N-5)}{15120}\xi_u^{\,6} r_0^{\,6} + \left\{\frac{b_0 N}{140t}\xi_u^{\,4} - \left(\frac{4}{105t}b_1 + \frac{1}{105t}b_3\right)\xi_u^{\,2}\right\} r_0^{\,7} + \left\{\frac{122N^3 - 183N^2 + 70N}{3265920} - \frac{5}{72t^2}b_0^{\,2}\xi_u^{\,2}\right\} r_0^{\,8} + \ldots \tag{7.26}$$

Numerical integrations have been performed to obtain the inner structures of certain rotating polytropes of indices 1.5, 3.0 and 4.0 rotating differentially according to the law (7.5) for values of constants b_0, b_1, b_2, b_3, b_4 and b_5 listed in Table 7.1. Values of the volume V_ψ, and the surface area S_ψ of the distorted polytropic models were then computed using (7.14), (7.15) taking α equal to one. The results are presented in Tables 7.2 (a), 7.2 (b), 7.2 (c) for models with polytropic indices 1.5, 3.0 and 4.0, respectively. We also present in these Tables values of distortion parameters σ and ε. Values ω_c, ω_p, and ω_e the angular velocities of rotation at the center, pole and equator are also given in these Tables. Relations (3.43) and (3.44) of chapter III have been used to compute values of T_e/T_p and L_e/L_p to get an insight into the effects of differential rotation on the values of surface temperatures and luminosities of such differentially rotating stars. values of T_e/T_p and L_e/L_p values of R_e, R_P, g_e and g_P, the equatorial radius, the polar radius, the equatorial gravitational force and the polar gravitational force respectively are required were computed from the relations (7.13), and (7.16) after substituting $\theta=0^\circ$ and $\theta=90^\circ$ respectively

7.6 ANALYSIS OF RESULTS

Table (7.1) presents different laws of differential rotation of the type (7.5) which have been considered by us. These laws were earlier used by Mohan, Lal, and Singh (91) to determine the equilibrium structure of differentially rotating polytropic models. Our results in Tables 7.2(a, b, c) give the values of various structure parameters as obtained by us for certain differentially rotating polytropes of indices 1.5, 3.0 and 4.0

In the case of polytropic models 1.5 and 3.0 presented in Table 7.2 (a) and 7.2 (b) our results for volumes and surface areas are smaller in comparison to the corresponding results shown in parenthesis. Our results presented in Table 7.2 (c) regarding the effects of such types of differential rotation on the volumes and the surface area of the polytropic models with index 4.0 show that because of differential rotation , volume and surface areas in general increases compared to the results earlier obtained by Mohan et al (91). However in the case of models 9 and 10 (which are rotationally unstable) the inclusion of this differential rotation reduces these values for the polytropic stars with indices 1.5, 3.0 and 4.0.

As regards the shape parameter σ and ε model 1, 2, 3 and 4 show these to be of undistorted type as their oblations and ellipticity are zero. However their volumes and surfaces areas are larger than the undistorted model. Also our values of σ and ε are smaller than the corresponding values shown in parenthesis.

Our results in these Tables also depict the effects of such types of differential rotation on the values of temperatures and luminosities at various points on the surface of such types of differentially rotating stars. Our results for

$\frac{T_e}{T_P}$ are smaller for model 1, 2, 3, 4, 9, and 10 and larger for model 5, 6, 7, 8 in comparison to the corresponding results shown in parenthesis. This behaviour is common to all the polytropic models of indices 1.5 and 3.0. However, while the values of $\frac{L_e}{L_P}$ remains unchanged for model 1, 2, 3, 4, it increases for models 5, 6, 7, 8 and decreases for models 9 and 10 in comparison to the corresponding results earlier obtained by Mohan et al. (91) and shown in parenthesis.

Table 7.1 : Behaivour of Angular Velocity of Certain Differentially Rotating Models

Model No.	**Values of various parameters in the low of differential rotation** $\omega^2 = b_0 + b_1 s^2 + b_2 s^4 + b_3 z^2 + b_4 z^4 + b_5 z^2 s^2$						Stability of the model according to Hoiland's criterion
	b_0	b_1	b_2	b_3	b_4	b_5	
1.	0.0	0.0	0.0	0.1	0.0	0.0	Stable
2.	0.0	0.0	0.0	0.1	0.0	0.1	Stable
3.	0.0	0.0	0.0	0.1	0.1	0.0	Stable
4.	0.0	0.0	0.0	0.1	0.1	0.1	Stable
5.	0.1	0.1	0.0	0.5	0.0	0.0	Stable
6.	0.1	0.1	0.0	0.1	0.0	0.0	Stable
7.	0.1	0.1	0.0	-0.05	0.0	0.0	Stable
8.	0.1	0.1	0.0	0.05	0.0	0.1	Stable
9.	0.4	-0.16	0.16	0.04	0.0	0.0	Unstable
10.	0.04	-0.16	0.16	0.04	0.0	0.16	Unstable

Note: This Table has been taken from (cf. Table 5.1, Lal (69)) and is given here for ready reference

Table 7.2(a) : Values of Certain Structure Parameters and Related Quantities for Differentially rotating polytropic models of polytropic index N = 1.5

Model No.	r_{OS}	$V_{\Psi} \times 10^{-3}$	$S_{\Psi} \times 10^{-2}$	σ	ε	ω_e	ω_P	ω_e	T_e/T_p	L_e/L_p
1.	0.996329	2.06043 (2.07406)	1.68710 (1.69221)	0.000000 (0.00000)	0.000000 (0.00000)	0.000000 (0.00000)	0.315066 (0.31623)	0.000000 (0.00000)	0.99816 (1.00000)	1.000000 (1.00000)
2.	0.995634	2.07833 (2.06477)	1.69696 (1.68732)	0.000000 (0.00000)	0.000000 (0.00000)	0.000000 (0.00000)	0.314847 (0.31623)	0.000000 (0.00000)	0.99781 (1.00000)	1.000000 (1.00000)
3.	0.991549	2.06752 (2.69704)	1.69116 (2.01347)	0.000000 (0.00000)	0.000000 (0.00000)	0.000000 (0.00000)	0.441564 (0.44721)	0.000000 (0.00000)	0.99576 (1.00000)	1.000000 (1.00000)
4.	0.990889	2.08441 (2.57094)	1.70051 (1.95143)	0.000000 (0.00000)	0.000000 (0.00000)	0.000000 (0.00000)	0.441124 (0.44721)	0.000000 (0.00000)	0.99543 (1.00000)	1.000000 (1.00000)
5.	0.970024	2.16928 (2.26002)	1.74689 (1.79500)	0.08119 (0.08726)	0.07509 (0.08026)	0.3162 (0.31623)	0.383467 (0.38730)	0.458252 (0.44721)	0.927023 (0.84969)	0.62099 (0.45942)
6.	0.968430	2.17640 (2.27556)	1.75102 (1.80213)	0.08063 (0.08670)	0.074614 (0.07979)	0.3162 (0.31623)	0.440210 (0.44721)	0.457733 (0.44721)	0.92673 (0.85096)	0.62323 (0.48254)
7.	0.971914	2.14346 (2.22495)	1.73241 (1.77577)	0.08186 (0.08788)	0.07567 (0.08078)	0.31623 (0.31623)	0.229715 (0.22361)	0.458869 (0.44721)	0.92735 (0.84829)	0.61831 (0.47598)
8.	0.969559	2.18268 (2.26824)	1.75455 (1.79973)	0.08103 (0.08628)	0.07495 (0.07942)	0.31623 (0.31623)	0.383408 (0.38730)	0.457445 (0.44721)	0.92694 (0.85193)	0.62164 (0.48492)
9.	0.981534	1.99270 (2.05443)	1.65055 (1.68426)	0.001442 (0.00206)	0.001400 (0.00205)	0.200000 (0.20000)	0.281085 (0.28284)	0.1912163 (0.20000)	0.989192 (1.00574)	0.977553 (1.02105)
10	0.986569	2.01944 (206751)	1.66540 (1.69105)	0.001440 (0.00211)	0.001438 (0.00211)	0.200000 (0.20000)	0.280949 (0.28284)	0.1904501 (0.20000)	0.988770 (1.00486)	0.977802 (1.01742)

Table 7.2(b) : Values of Certain Structure Parameters and Related Quantities for Differentially rotating polytropic models of polytropic index N = 3.0

Model No.	r_{OS}	$V_\Psi \times 10^{-3}$	$S_\Psi \times 10^{-2}$	σ	ε	ω_e	ω_P	ω_e	T_e/T_p	L_e/L_p
1.	0.998304	1.39430 (1.39981)	6.03588 (6.04336)	0.000000 (0.00000)	0.000000 (0.00000)	0.000000 (0.00000)	0.315691 (0.31623)	0.000000 (0.00000)	0.999152 (1.00000)	1.000000 (1.00000)
2.	0.998199	1.40927 (1.39505)	6.0793 (6.03022)	0.000000 (0.00000)	0.000000 (0.00000)	0.000000 (0.00000)	0.315658 (0.31623)	0.000000 (0.00000)	0.999099 (1.00000)	1.000000 (1.00000)
3.	0.994490	1.40388 (1.90770)	6.06420 (7.41641)	0.000000 (0.00000)	0.000000 (0.00000)	0.000000 (0.00000)	0.443525 (0.44721)	0.000000 (0.00000)	0.997241 (1.00000)	1.000000 (1.00000)
4.	0.994385	1.41825 (1.79572)	6.10612 (7.12903)	0.000000 (0.00000)	0.000000 (0.00000)	0.000000 (0.00000)	0.443529 (0.44721)	0.000000 (0.00000)	0.997241 (1.00000)	1.000000 (1.00000)
5.	0.988795	1.56385 (1.57791)	6.52000 (6.55743)	0.088116 (0.09130)	0.080980 (0.08366)	0.316227 (0.31623)	0.385857 (0.38730)	0.464501 (0.44721)	0.929995 (0.84039)	0.593860 (0.45706)
6.	0.987815	1.56385 (1.59528)	6.54000 (6.559423	0.088116 (0.09148)	0.080980 (0.08382)	0.31622 (0.31623)	0.444497 (0.44721)	0.464501 (0.44721)	0.929859 (0.83996)	0.593860 (0.45606)
7.	0.989290	1.53452 (1.54528)	6.43570 (6.46423)	0.088305 (0.09148)	0.081140) (0.08382)	0.316227 (0.31623)	0.225976 (0.22361)	0.464669 (0.44721)	0.930062 (0.83996)	0.593128 (0.45606)
8.	0.988769	1.557773 (1.58601)	6.56076 (6.58150)	0.088106 (0.09032)	0.080972 (0.08284)	0.316227 (0.31623)	0.385853 (0.38730)	0.464492 (0.44721)	0.929991 (0.84268)	0.593860 (0.46249)
9.	0.995634	1.37380 (1.38646)	5.9790 (6.01283)	0.001456 (0.00204)	0.001454 (0.00204)	0.200000 (0.20000)	0.282225 (0.28284)	0.197670 (0.20000)	0.992694 (1.00600)	0.975365 (1.02213)
10	0.995519	1.39734 (1.39748)	6.04760 (6.04535)	0.001458 (0.00209)	0.001453 (0.00208)	0.200000 (0.20000)	0.282209 (0.28284)	0.197578 (0.20000)	0.992645 (1.00526)	0.975398 (1.01910)

Table 7.2(c) : Values of Certain Structure Parameters and Related Quantities for Differentially rotating polytropic models of polytropic index N=4.0

Model No.	r_{OS}	$V_\Psi \times 10^{-3}$	$S_\Psi \times 10^{-2}$	σ	ε	ω_e	ω_P	ω_e	T_e/T_p	L_e/L_p
1.	0.998794	14.2845 (1.39981)	2.8471 (6.04336)	0.000000 (0.00000)	0.000000 (0.00000)	0.000000 (0.00000)	0.315846 (0.31623)	0.000000 (0.00000)	0.99939 (1.00000)	1.000000 (1.00000)
2.	0.998784	14.4427 (1.39505)	2.8683 (6.03022)	0.000000 (0.00000)	0.000000 (0.00000)	0.000000 (0.00000)	0.31584 (0.31623)	0.000000 (0.00000)	0.99939 (1.00000)	1.000000 (1.00000)
3.	0.995114	14.3898 (1.90770)	2.8614 (7.41641)	0.000000 (0.00000)	0.000000 (0.00000)	0.000000 (0.00000)	0.44394 (0.44721)	0.000000 (0.00000)	0.997755 (1.00000)	1.000000 (1.00000)
4.	0.995104	14.5423 (1.79572)	2.8819 (7.12903)	0.000000 (0.00000)	0.000000 (0.00000)	0.000000 (0.00000)	0.44393 (0.44721)	0.000000 (0.00000)	0.99754 (1.00000)	1.000000 (1.00000)
5.	0.99574	16.4104 (1.57791)	3.1253 (6.55743)	0.09080 (0.09130)	0.08324 (0.08366)	0.31622 (0.31623)	0.38674 (0.38730)	0.46686 (0.44721)	0.93088 (0.84039)	0.58355 (0.45706)
6.	0.994905	16.5345 (1.59528)	3.14178 (6.559423	0.09048 (0.09148)	0.08297 (0.08382)	0.31622 (0.31623)	0.44607 (0.44721)	0.46659 (0.44721)	0.93078 (0.83996)	0.58476 (0.45606)
7.	0.995759	16.0648 (1.54528)	3.0799 (6.46423)	0.090817 (0.09148)	0.083256 (0.08382)	0.31622 (0.31623)	0.224551 (0.22361)	0.466884 (0.44721)	0.93089 (0.83996)	0.58348 (0.45606)
8.	0.995784	16.5687 (1.58601)	3.1465 (6.58150)	0.09082 (0.09032)	0.08326 (0.08284)	0.316227 (0.31623)	0.386754 (0.38730)	0.466893 (0.44721)	0.930894 (0.84268)	0.583449 (0.46249)
9.	0.998344	14.16932 (1.38646)	2.83303 (6.01283)	0.001462 (0.00204)	0.001460 (0.00204)	0.200000 (0.20000)	0.282608 (0.28284)	0.199843 (0.20000)	0.993852 (1.00600)	0.974591 (1.02213)
10	0.998379	14.4237 (1.39748)	2.86707 (6.04535)	0.001462 (0.00209)	0.001460 (0.00208)	0.200000 (0.20000)	0.282613 (0.28284)	0.199674 (0.20000)	0.993867 (1.00526)	0.974581 (1.01910)

CHAPTER – VIII

EIGENFREQUENCIES OF SMALL ADIABATIC BAROTROPIC MODES OF OSCILLATIONS OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED GAS SPHERES

In the present Chapter we consider the use of averaging approach developed in Section 3.4 and 3.5 of Chapter III and section 5.3 and 5.4 of chapter V to study the effect of including mass variation inside the star on the eigenfrequencies of small adiabatic pseudo radial and nonradial modes of oscillations of differentially rotating and tidally distorted gas spheres.

Using the approach discussed in Section 3.4 and 3.5 of Chapter III and using a law of differential rotation of the type (5.7), an eigenvalued boundary value problems determining the eigenfrequencies of small adiabatic pseudo-radial modes oscillations of differentially rotating and tidally distorted gas spheres have been formulated in Section 8.1. An eigenvalued boundary value problem which determines the effect of differential rotation and tidal distortion on the eigenfrequencies of nonradial modes of oscillations of the gas spheres has next been formulated in Section 8.2. The formulations of these eigenvalue problems are based on the analysis developed earlier by Mohan, Saxena and Aggarwal (92). In Section 8.3 analysis of sections 8.1 has been used to formulate the eigenvalue problems to determine the pseudo radial modes of oscillations of rotationally and tidally distorted composite models. The analysis of section 8.1 and 8.2 have been used in section 8.4 and 8.5, respectively to formulate the eigenvalue problems which determine pseudo radial and nonradial modes of oscillations of a differentially rotating and tidally distorted polytropic models with polytropic indices 1.5, 3.0 and 4.0 The eigenvalue problems developed in Section 8.3 of rotationally and tidally distorted composite model and 8.4 and 8.5 for differentially rotating and tidally distorted polytropic models have been solved numerically in Section 8.5 whose inner structures was earlier obtained in Chapter III, IV and V. Analysis of the numerical results has finally been carried out in Section 8.6 to draw certain conclusions.

8.1 EIGENVALUED BOUNDARY VALUE PROBLEM TO DETERMINE THE EIGENFREQUENCIES OF SMALL ADIABATIC PSEUDO-RADIAL MODES OF OSCILLATIONS OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED GAS SPHERES

The problem of determining the effect of differential rotation on the eigen-frequencies of differentially rotating stars in quite complex. Mohan and Singh (87) formulated an eigenvalued boundary value problem to determine the periods of small adiabatic pseudo-radial modes of oscillations of rotationally and tidally distorted Roche model. Mohan, Saxena and Aggarwal (92) used this approach to formulate eigenvalue problems which determine the eigenfrequencies of small adiabatic pseudo-radial and nonradial modes of oscillation of rotationally and tidally distorted gaseous spheres in general. The approach adopted by them was also be used by Lal (69) to set up the eigenvalue problems which determine the eigenfrequencies of small adiabatic pseudo-radial and nonradial modes of oscillations of differentially rotating and tidally distorted stars.

Assuming that during the oscillations the fluid elements on an equipotential surface oscillate in unison, the eigenfrquencies of small adiabatic pseudo-radial modes of oscillations of the actual rotating star rotating differentially according to the law (5.7) can be obtained from its topologically equivalent spherical model developed on the basis of the averaging technique of Kippenhahn and Thomas. Following the approach of Mohan et al (92), the equation determining the eigenfrequencies of pseudo-radial modes of oscillations of a differentially rotating and tidally distorted stellar models which correspond to the eigenvalue problem determining the eigenfrequencies of radial modes of oscillations of the topologically equivalent spherical model may be expressed as :

$$\frac{d^2\eta}{dr_{0\psi}^2}+\frac{4-\mu}{r_{0\psi}}\frac{d\eta}{dr_{0\psi}^2}+\left[\frac{\rho_{0\psi}}{rP_{0\psi}}\sigma^2-\left(3-\frac{4}{\gamma}\right)\frac{\mu}{r_{0\psi}^2}\right]\eta=0$$

where $\mu=-\frac{r_{0\psi}}{P_{0\psi}}\frac{dP_{0\psi}}{dr_{0\psi}}$ (8.1)

Here $r_{0\psi}$, $\rho_{0\psi}$ and $P_{0\psi}$ are the values of r_ψ, ρ_ψ and P_ψ on the equipotential ψ = const. in its equilibrium position, σ the eigenfrequency of oscillation and η some average of the relative amplitudes of pulsation of the fluid elements on the equipotential surface ψ = constant. Using r_ψ, ρ_ψ and P_ψ in place of $r_{0\psi}$, $\rho_{0\psi}$ and $P_{0\psi}$ to denote the equilibrium values on the equipotentials surfaces, taking $r_0=\frac{z}{\psi-q}$ in place of r_ψ as the independent variable, and assuming $\omega^2=b_1^2+2b_1b_2s^2+b_2^2s^4$ as the law of differential rotation, the equation (8.1) governing the small adiabatic pseudo-radial modes of oscillations of a differentially rotating and tidally distorted gas sphere may be expressed as:

$$A(z,b_1,b_2,q)\frac{d^2\eta}{dr_0^2}+\left[\frac{4-\mu}{r_0}B(z,b_1,b_2,q)-C(z,b_1,b_2,q)\right]\frac{d\eta}{dr_0}$$
$$+\left[\frac{R^2\sigma^2\rho_\psi}{rP_\psi}-\left(3-\frac{4}{r}\right)\frac{\mu}{r_0^2}E(z,b_1,b_2,q\right]\eta=0 \qquad (8.2)$$

$$A(z,b_1,b_2,q) = \left[1-\frac{8b_1^2r_0^3}{3z}-\frac{16b_1b_2r_0^5}{15z}-\frac{28q^2r_0^6}{5z^2}-\frac{128b_2^2r_0^7}{105z}-\frac{90q^2r_0^8}{7z^2}-\frac{44q^2r_0^{10}}{3z^2}+\ldots\right]$$

$$B(z,b_1,b_2,q)=\left[1-\frac{5.b_1^2r_0^3}{3z}-\frac{28b_1b_2r_0^5}{15z}-\frac{32q^2r_0^6}{5z^2}-\frac{24b_2^2r_0^7}{35z}-\frac{50q^2r_0^8}{7z^2}-\frac{2q^2r_0^{10}}{3z^2}+\ldots\right]$$

$$C(z,b_1,b_2,q)=\frac{1}{r_0}\left[\frac{4.b_1^2r_0^3}{z}+\frac{8b_1b_2r_0^5}{5z}+\frac{168q^2r_0^6}{5z^2}+\frac{64b_2^2r_0^7}{z}+\frac{360q^2r_0^8}{7z^2}+\frac{220q^2r_0^{10}}{3z^2}+\ldots\right]$$

$$E(z,b_1,b_2,q)=\left[1-\frac{2.b_1^2r_0^3}{3z}-\frac{8b_1b_2r_0^5}{15z}-\frac{8q^2r_0^6}{5z^2}-\frac{16b_2^2r_0^7}{35z}-\frac{10q^2r_0^8}{7z^2}-\frac{4q^2r_0^{10}}{3z^2}+\ldots\right]$$

Also $$\mu=-\frac{r_\psi}{P_\psi}\frac{dP_\psi}{dr_0}\frac{dr_0}{dr_\psi}=-F(z,b_1,b_2,q)\frac{r_0}{P_\psi}\frac{dP_\psi}{dr_0}$$

Where

$$F(z,b_1,b_2,q)=\left[1-\frac{.b_1^2r_0^3}{z}-\frac{4b_1b_2r_0^5}{15z}-\frac{24q^2r_0^6}{5z^2}-\frac{56b_2^2r_0^7}{105z}-\frac{40q^2r_0^8}{7z^2}-\frac{20q^2r_0^{10}}{3z^2}+\ldots\right]$$

In the absence of any distortion i.e. $z=1,\ b_1=b_2=0,\rho_\psi=\rho,\mathrm{P}_\psi=P,r_0=x$, the above equation reduces to

$$\frac{d^2\eta}{dx^2}+\frac{4-\mu}{x}\frac{d\eta}{dx}+\left[\frac{R^2\sigma^2\rho}{r\rho}-\left(3-\frac{4}{r}\right)\frac{\mu}{x^2}\right]\eta=0$$

$$\text{with } \mu=-\frac{x}{p}\frac{dP}{dx}$$

which is the usual equation determining the eigenfrequencies of small adiabatic radial modes of oscillations of a gaseous sphere (cf. Rosseland (105) p.30 with $(\gamma=0)$.

Equation (8.2) forms an eigenvalue problem in the eigenfrequency of oscillation σ. As usual, this eigenvalue problem is of Sturm-Liouville type having singularities both at the centre and the surface of the model. It has to be solved subject to the boundary conditions which require η to be finite at the centre as well as at the free surface.

In reality equation (8.2) determines the periods of small adiabatic radial modes of oscillations of the topologically equivalent spherical model. However, since equipotential surfaces of the actual differentially rotating distorted model are also the surfaces of equipressure and equidensity, the values of pressure and density on the equipotential surfaces of the differentially rotating star are same as on the corresponding equipotential surfaces of the equivalent spherical model. Hence the eigenfrequencies of the radial modes of oscillations determined by solving the eigenvalue problem for the topologically equivalent spherical model are indeed the eigenfrequenices of the radial modes of oscillation of the undistorted model which have got influenced by the rotational effects of the star. However, the values of the eigenfunction η obtained on solving (8.2) for the equivalent spherical model are not the actual values of amplitudes of pulsation η for the distorted model but rather some averages of the true values of eigen functions η on the differentially rotating model.

We may thus use equation (8.2) to determine the effects of differential rotation and the tidal distortions on the periods of small adiabatic radial modes of oscillations of a stellar model. The effects of differential rotation and tidal distortions have been incorporated through introduction of terms $A(z,b_1,b_2,q)$, $B(z,b_1,b_2,q)$, $C(z,b_1,b_2,q)$, $E(z,b_1,b_2,q)$, and $F(z,b_1,b_2,q)$, and dependence of ρ_ψ and P_ψ on ψ. The present method in fact incorporates the effects of distortional forces both while computing the equilibrium structure (in computing the values of P_ψ, ρ_ψ etc.) as well as in the coefficients A, B and C of the equation (8.2) which determines the periods of adiabatic small radial modes of oscillations.

The eigenvalue problem (8.2) together with the boundary conditions which require η to be finite both at the centre as well as the free surface of the star may be solved numerically in the usual manner as is done in the case of undistorted models. For convenience in numerical work it is sometimes found convenient to set

$$\eta = \frac{\zeta}{r_0} \text{ and } r_0 = x r_{OS} \tag{8.3}$$

(r_{OS} being the value of r_0 on the outermost surface) in equation (8.2) and treat x as the independent variable and ζ as the dependent variable. With these substitutions x is now zero at the centre and one at the free surface. The boundary condition η=finite at the centre now gets replaced by ζ =0 at the centre. The boundary condition η= finite at the free surface now becomes ζ finite at $x=1$. Using (8.3) equation (8.2) gets transformed in terms of the variables ζ and x and as

$$A^*(z,b_1,b_2,q,x)\frac{d^2\zeta}{dx^2} + B^*(z,b_1,b_2,q,x)\frac{d\zeta}{dx} + C^*(z,b_1,b_2,q,x)\zeta = 0 \tag{8.4}$$

where

$$A^*(z,b_1,b_2,q,x) = A(z,b_1,b_2,xr_{OS}),$$

$$B^*(z,b_1,b_2,q,x) = \frac{4-\mu}{x}B(z,b_1,b_2,q,xr_{OS}) - r_{OS}C(z,b_1,b_2,q,xr_{OS}) - \frac{2}{x}A(z,b_1,b_2,q,xr_{OS}),$$

and

$$C^*(z,b_1,b_2,q,x) = \frac{r_{OS}^2 R^2 \rho_\psi}{\gamma P_\psi}\sigma^2 - (3-\frac{4}{\gamma})\frac{\mu}{x^2}E(z,b_1,b_2,q,xr_{OS}) - \frac{1}{x}B^*(z,b_1,b_2,q,xr_{OS})$$

The boundary conditions now are :

$$\left.\begin{array}{l}\zeta = 0 \text{ at the centre } x = 0 \\ \textit{and} \\ \zeta = \text{finite at the surface } x = 1\end{array}\right] \tag{8.5}$$

For computing an eigenvalue σ (8.4) has to be solved numerically subject to the specified boundary conditions (8.5). Centre and the free surface of the star being singularities of this differential equitation it may be advisable to write the series solutions of (8.4) near the singularities to start numerical integrations. If we assume ζ to be normalized to have value one at the free surface, we can assume a series solutions of the type

$$\zeta = \sum_{J=0}^{\infty} a_J x^{J+\lambda} \tag{8.6}$$

near the centre x = 0 and

$$\zeta = 1 + \sum_{J=0}^{\infty} b_J (1-x)^{J+\lambda} \tag{8.7}$$

near the surface x=1, to start the integration of (8.4) near these two singularities.

For obtaining an eigenfrequency of pseudo-radial mode of oscillation, the equation (8.4) has to be integrated numerically for trial values of σ till a value of σ is obtained for which both the boundary conditions are satisfied. One way to achieve this objective could be to integrate equation (8.4) numerically from the surface towards the centre using say fourth-order Runge-Kutta method. Starting values near the surface may be obtained from series solution (8.7). Similarly we can integrate equation (8.4) numerically outwards from the centre starting from a point near the centre. The starting values near the centre may be obtained from the series solution (8.6). Trials with different values of σ may be continued till a value of σ is found for which the value $\zeta \Big/ \frac{d\zeta}{dx}$ from the inward and outward integrations match to desired accuracy at some suitably selected point inside the model.

The quantities ρ_ψ, P_ψ and the eigenfrequencies σ are still in dimensional form. For determining the eigenfrequencies it is recommended that these be first converted into suitable nondimensinal forms keeping in view the physical nature of the model under investigation.

It may be noted that the eigenvalued boundary value problem set up in this section determines the eigenfrequencies of the pseudo-radial modes of oscillations of a differentially rotating and tidally distorted gas spheres rotating differentially according to the law $\omega^2 = b_1^2 + 2b_1b_2s^2 + b_2^2s^4$. For pseudo-radial oscillations of a rotating model having solid body rotation we may set $b_1^2 = 2n,\ b_2^2 = 0,\ z=1$ ($2n$ being the square of the angular velocity of rotation in equation (8.4)).

8.2 EIGENVALUED BOUNDARY VALUE PROBLEM TO DETERMINE THE EIGENFREQUENCIES OF SMALL ADIABATIC NONRADIAL MODES OF OSCILLATIONS OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED GAS SPHERES

Mohan, Saxena and Agarwal (92) also formulated an eigenvalued boundary value problem to determine the eigenfrequencies of the nonradial modes of oscillations of rotationally and tidally distorted gaseous spheres. As in the earlier case values of the physical parameters ρ_ψ and P_ψ on the equipotential surfaces of the distorted model being same as those on the corresponding equipotential surfaces of the topologically equivalent spherical model, we may use this topological equivalent spherical model to determine the eigenfrequencies of nonradial modes of oscillations of the differentially rotating and tidally distorted gaseous spheres Following Saxena (124), the eigenvalue problem determining the eigenfrequencies of nonradial modes of oscillations of a differentially rotating and tidally distorted gas spheres can be expressed in an explicit form convenient for computational work as

$$\left.\begin{aligned}
&\frac{d\zeta}{dx} + B_1\xi + \left(B_2 + \frac{1}{\sigma^2}B_3\right)\eta + \frac{1}{\sigma^2}B_3\phi = 0,\\
&\frac{d\eta}{dx} + \left(E_1\sigma^2 + E_2\right)\xi + E_3\eta + E_4\phi + \frac{d\phi}{dx} = 0,\\
&\text{and}\\
&\frac{d^2\phi}{dx^2} + F_1\frac{d\phi}{dx} + F_2\xi + F_3\eta + F_4\phi = 0
\end{aligned}\right] \tag{8.8}$$

where

$$B_1 = \frac{l+1}{x} + \frac{1}{\gamma P_\psi}\frac{dP\psi}{dx},$$

$$B_2 = \frac{2\pi G\rho_c}{Rx}\frac{\rho_\psi}{\gamma P_\psi}r_\psi^2\frac{dr_\psi}{dx}$$

$$= \frac{2\ \pi\ G\rho_c}{\gamma\ P_\psi}D^2\ \rho_\psi\ r_{0s}{}^3\ x\left[1 + \frac{2\ b_1{}^2 r_{0s}{}^3 x^3}{z} + \frac{32\ b_1\ b_2\ r_{0s}{}^5\ x^5}{15z} + \left(\frac{36\ q^2}{5\ z^2} + \frac{12 b_1^2 q}{5z^2}\right) r_{0s}^6 + \frac{16}{21}\frac{b_2{}^2\ r_{0s}{}^7 x^7}{z}\right.$$
$$\left. + \frac{55\ q^2 x^8 r_{0s}{}^8}{7\ z^2} + \frac{26 q^2}{3 z^2} + \frac{62\ b_1^2\ q}{35 z^2})x^{10} r_{0s}{}^{10} + \dots\ \right],$$

$$B_3 = -\frac{l(l+1)}{Rx}\frac{d\,r_\psi}{d\,x}2\pi G\rho_c$$

$$B_3 = -\frac{l(l+1)}{Rx}2\pi G\,\rho_c r_{0s}\left[1 + \frac{4 b_1^2 r_{0s}^3 x^3}{3z} + \frac{8 b_1 b_2 r_{0s}^5\ x^5}{5z} + \left(\frac{28 q^2}{5 z^2} + \frac{28 b_1^2\ q}{15 z^2}\right) r_{0s}^6 + \right.$$
$$\left. + \frac{64 b_2^2 r_{0s}^7\ x^7}{105 z} + \left(\frac{45}{7 z^2}q^2 + \frac{24 b_1 b_2\ q}{7 z^2}\right) r_{0s}^8\ x^8 + \left(\frac{22 q^2}{3 z^2} + \frac{22 b_1\ b_2\ q}{35 z^2}\right) r_{0s}^{10}\ x^{10} + \dots\ \right]$$

$$E_1 = -\frac{1}{2\pi G\rho_c}\frac{Rx}{r_\psi^2}\frac{dr_\psi}{dx}$$

$$E_1 = -\frac{1}{2\pi G\rho_c r_{0s}x}\left[1+\frac{2b_1{}^2x^3r_{0s}{}^3}{3z}+\frac{16b_1b_2x^5r_{0s}{}^5}{15z}+\left(\frac{4q^2}{z^2}+\frac{4b_1{}^2q}{3z^2}\right)x^6r_{0s}{}^6+\frac{16b_2{}^2x^7r_{0s}{}^7}{35z}+\right.$$
$$\left.+\left(\frac{5q^2}{z^2}+\frac{8b_1b_2q}{3z^2}\right)x^8r_{0s}{}^8+\left(\frac{6q^2}{z^2}+\frac{18b_1b_2q}{35z^2}\right)x^{10}r_{0s}{}^{10}+\ldots\right]$$

$$E_2 = \frac{1}{2\pi G\rho_c}\frac{A_\psi}{\rho_\psi}\frac{dP_\psi}{dx}\frac{Rx}{r_\psi^2}$$

$$E_2 = \frac{1}{2\pi G\rho_c D^2}\frac{1}{\rho_\psi}\left(\frac{1}{\rho_\psi}\frac{d\rho_\psi}{dx}-\frac{1}{\gamma P_\psi}\frac{dP_\psi}{dx}\right)\frac{dP_\psi}{dx}\frac{1}{xr_{0s}^2}\left[1-\frac{2b_1{}^2x^3r_{0s}{}^3}{z}-\frac{32b_1b_2x^5r_{0s}{}^5}{15z}-\right.$$
$$-\left(\frac{36q^2}{5z^2}+\frac{12b_1{}^2q}{5z^2}\right)x^6r_{0s}{}^6-\frac{16}{21z}b_2{}^2x^7r_{0s}{}^7-\left(\frac{55q^2}{7z^2}+\frac{88b_1b_2q}{21z^2}\right)x^8r_{0s}{}^8-$$
$$\left.-\left(\frac{26q^2}{3z^2}+\frac{26b_1b_2q}{35z^2}\right)x^{10}r_{0s}{}^{10}+\right]$$

$$E_3 = \frac{l}{x}+A_\psi\frac{dr_\psi}{dx}$$

$$E_3 = \frac{l}{x}+\left(\frac{1}{\rho_\psi}\frac{d\rho_\psi}{dx}-\frac{1}{\gamma P_\psi}\frac{dP_\psi}{dx}\right)$$

$$E_4 = \frac{l}{x},$$

$$F_1 = \frac{2l}{x}-\frac{d^2r_\psi/dx^2}{dr_\psi/dx}+\frac{2}{r_\psi}\frac{dr_\psi}{dx}$$

$$= \frac{2(l+1)}{x}-\frac{1}{x}\left[\frac{2b_1{}^2x^3r_{0s}{}^3}{z}+\frac{16b_1b_2x^5r_{0s}{}^5}{3z}+\left(\frac{24q^2}{z^2}+\frac{8b_1{}^2q}{z^2}\right)x^6r_{0s}{}^6+\right.$$
$$\left.+\frac{112b_2{}^2x^7r_{0s}{}^7}{35z}+\left(\frac{40q^2}{z^2}+\frac{448b_1b_2q}{21z^2}\right)x^8r_{0s}{}^8+\left(\frac{60q^2}{z^2}+\frac{36b_1b_2q}{7z^2}\right)x^{10}r_{0s}{}^{10}+.\right]$$

$$F_2 = 2\frac{\rho_\psi}{\rho_c}\frac{A_\psi\, R x}{r_\psi^2}\left(\frac{d\, r_\psi}{d\, x}\right)^2$$

$$= 2\frac{\rho_\psi}{\rho_c}\left(\frac{1}{\rho_\psi}\frac{d\, P_\psi}{d\, x} - \frac{1}{\gamma\, P_\psi}\frac{d\, P_\psi}{dx}\right)\frac{1}{x\, r_{0s}}\Bigg[1 + \frac{2b_1{}^2 x^3 r_{0s}{}^3}{3z} + \frac{16 b_1 b_2 x^5 r_{0s}{}^5}{15z} + \left(\frac{4q^2}{z^2} + \frac{4b_1{}^2 q}{3z^2}\right)x^6 r_{0s}^6$$
$$+ \frac{16 b_2{}^2 x^7\, r_{0s}{}^7}{35\, z} + \left(\frac{5q^2}{z^2} + \frac{8 b_1 b_2 q}{3z^2}\right)x^8 r_{0s}{}^8 + \left(\frac{6q^2}{z^2} + \frac{18 b_1 b_2\, q}{35\, z^2}\right)x^{10} r_{0s}{}^{10}\Bigg]$$

$$F_3 = -\frac{4\pi G\rho_\psi^2}{\gamma\, P_\psi}\left(\frac{d\, r_\psi}{d\, x}\right)^2$$

$$F_3 = \frac{4\pi G\rho_\psi^2}{\gamma\, P_\psi} r_{0s}^2\, R^2\Bigg[1 + \frac{8b_1{}^2 x^3 r_{0s}{}^3}{3z} + \frac{16 b_1 b_2 x^5 r_{0s}{}^5}{5z} + \left(\frac{56q^2}{5z^2} + \frac{56 b_1{}^2 q}{15z^2}\right)x^6 r_{0s}^6 +$$
$$+ \frac{128 b_2{}^2 x^7\, r_{0s}{}^7}{105\, z} + \left(\frac{90 q^2}{7\, z^2} + \frac{48 b_1 b_2 q}{7z^2}\right)x^8 r_{0s}{}^8 + \left(\frac{44 q^2}{3z^2} + \frac{44 b_1 b_2\, q}{35\, z^2}\right)x^{10} r_{0s}{}^{10} + \ldots\Bigg]$$

$$F_4 = \frac{l\,(l+1)}{x^2} - \frac{l}{x}\left(\frac{d^2\, r_\psi}{d\, x^2}\right)/\left(\frac{d\, r_\psi}{d\, x}\right) + \frac{2l}{x}\left(\frac{1}{r_\psi}\frac{d\, r_\psi}{d\, x}\right) - \frac{l\,(l+1)}{r_\psi^2}\left(\frac{d\, r_\psi}{d\, x}\right)^2$$

$$F_4 = -\frac{l}{x^2}\Bigg[\frac{4 b_1{}^2 x^3\, r_{0s}{}^3}{z} + \frac{8 b_1 b_2\, x^5 r_{0s}{}^5}{z} + \left(\frac{168 q^2}{5 z^2} + \frac{56 b_1{}^2 q}{5 z^2}\right)x^6 r_{0s}{}^6 + \frac{448 b_2{}^2 x^7 r_{0s}{}^7}{105 z}$$
$$+ \left(\frac{360 q^2}{7 z^2} + \frac{576 b_1 b_2 q}{21 z^2}\right)x^8\, r_{0s}{}^8 + \left(\frac{220 q^2}{3z^2} + \frac{352 b_2{}^2 q}{21 z^2}\right)x^{10}\, r_{0s}{}^{10} + l\,\Bigg\{\frac{2b_1^2\, r_{0s}^3\, x^3}{z} + \frac{8 b_1\, b_2\, r_{0s}^5\, x^5}{3z} +$$
$$+ \left(\frac{48 q^2}{5z^2} + \frac{16 b_1{}^2 q}{5z^2}\right)x^6 r_{0s}{}^6 + \frac{112}{105}\frac{b_2{}^2 x^7 r_{0s}{}^7}{z} + \left(\frac{80 q^2}{7z^2} + \frac{128 b_1 b_2\, q}{21 z^2}\right)x^8 r_{0s}{}^8$$
$$+ \left(\frac{40 q^2}{3z^2} + \frac{8 b_1 b_2 q}{7 z^2}\right)x^{10} r_{0s}{}^{10} + \ldots\Bigg\}\Bigg]$$

Also σ is the eigenfrequency of oscillations, $x = r_0/r_{0s}$ and

$$\xi = \frac{r_\psi^2\,\delta r_\psi}{R^3 x^{l+1}}, \quad \eta = \frac{P'_\psi}{2\,\pi\, G\,\rho_c\, R^2 x^l \rho_\psi}, \text{ and } \phi = \frac{\psi'_g}{2\pi\, G\rho_c\, R^2 x^l}$$

δr_ψ being an average of the amplitudes of Lagrangian variations in the radial direction and P'_ψ, ψ'_g the amplitudes of Lagrangian variation in pressure and gravitational potential on the equipotential on ψ =constant.

In the above expressions terms upto second order of smallness in z ,b_1 and b_2 and upto order r_0^{10} in r_0 have been retained. On setting z =1 ,$b_1^2 = 2n$, and $b_2^2 = 0$ the above expressions reduce to the corresponding ones obtained by Mohan, Saxena and Agarwal (92) for a stellar model having solid body rotation.

The eigenvalue problem (8.8) determining the eigenfrequencies of nonradial modes of oscillations of a differentially rotating and tidally distorted gas spheres is to be solved subject to the boundary conditions at the centre and the free surface. Boundary conditions at the centre require $\delta r_\psi = 0, P'_\psi / \rho_\psi = 0$ and $\psi'_g = 0$ for $r_\psi = 0$. These requirements lead to the analytic conditions

$$\begin{aligned} \eta + \phi &= \frac{\sigma^2}{2\pi G \rho_c \, l \, r_{OS}} \xi, \\ \frac{d\phi}{dx} &= 0 \end{aligned} \tag{8.9}$$

at the centre x = 0.

If the pressure P_ψ on the free surface $(r_\psi = R_\psi)$ is taken to be zero, then δP_ψ, the Lagrangian variation in pressure, should be zero at the outer surface. This leads to the condition

$$2\pi G \rho_c r_\psi^2 \rho_\psi \frac{d r_\psi}{d x} \eta + R \frac{d P_\psi}{d x} \xi = 0$$

$$2\pi G \rho_c \rho_\psi R^2 r_{0s}^3 \left[1 + \frac{2b_1^2 r_{0s}^3}{z} + \frac{32 b_1 b_2 r_{0s}^5}{15z} + \left(\frac{36q^2}{5z^2} + \frac{12 b_1^2 q}{5z^2} \right) r_{0s}^{6} + \frac{16 b_2^2 r_{0s}^7}{21z} + \right.$$

$$\left. + \left(\frac{55q^2}{7z^2} + \frac{88 b_1 b_2 q}{21z^2} \right) r_{0s}^8 + \left(\frac{26q^2}{3z^2} + \frac{124 b_1 b_2 q}{35z^2} \right) r_{0s}^{10} + \dots \right] \eta + \frac{dP_\psi}{dx} \xi = 0.$$

(8.10a)

However if the pressure P_ψ does not totally vanish on the outermost surface then following Cox ((27), p.232), the boundary condition (8.10a) is to be replaced by

$$\frac{2\pi G \rho_c R^2 \rho_\psi}{P_\psi} \eta + \frac{1}{P_\psi} \frac{dP_\psi}{dx} \xi \frac{1}{r_{0s}^3} \left[1 - \frac{2b_1^2 r_{0s}^3}{z} - \frac{32 b_1 b_2 r_{0s}^5}{15z} - \left(\frac{36q^2}{5z^2} + \frac{12b_1^2 q}{5z^2} \right) r_{0s}^6 \right.$$

$$\left. - \frac{16 b_2^2 r_{0s}^7}{21z} - \left(\frac{55q^2}{7z^2} + \frac{88 b_1 b_2 q}{21z^2} \right) r_{0s}^8 - \left(\frac{26q^2}{3z^2} + \frac{26 b_1 b_2 q}{35z^2} \right) r_{0s}^{10} + \dots \right]$$

$$= \left[\sigma^2 \rho_\psi \left(\frac{dP_\psi}{dx}\right)^{-1} - 4\pi G \rho_\psi^2 \left(\frac{dP_\psi}{dx}\right)^{-1} \frac{D^2}{r} \left\{ 1 + \frac{2b_1^2 r_{0s}^3}{3z} + \frac{16 b_1 b_2 r_{0s}^5}{15z} + \left(\frac{4q^2}{z^2} + \frac{4b_1^2 q}{3z^2} \right) r_{0s}^6 + \frac{16 b_2^2 r_{0s}^7}{35z} \right. \right.$$

$$\left. + \left(\frac{5q^2}{z^2} + \frac{8b_1 b_2 q}{3z^2} \right) r_{0s}^8 + \left(\frac{6q^2}{z^2} + \frac{18 b_1 b_2 q}{35z} \right) r_{0s}^{10} + \dots \right\} - \frac{4}{r_{0s}^3} \left\{ 1 - \frac{b_1^2 r_{0s}^3}{z} - \frac{4 b_1 b_2 r_{0s}^5}{5z} - \left(\frac{12q^2}{5z^2} + \frac{4b_1^2 q}{5z^2} \right) r_{0s}^6 \right.$$

$$\left. \left. - \frac{8 b_2^2 r_{0s}^7}{35z} - \left(\frac{15q^2}{7z^2} + \frac{8 b_1 b_2 q}{7z^2} \right) r_{0s}^8 - \left(\frac{2q^2}{z^2} + \frac{6 b_1 b_2 q}{35z} \right) r_{0s}^{10} + \dots \right\} \right] \xi + \frac{2\pi G \rho_c l(l+1)}{\sigma^2 r_{0s}^2} (\eta + \phi)$$

$$\left[1 - \frac{2b_1^2 r_{0s}^3}{3z} - \frac{8 b_1 b_2 r_{0s}^5}{15z} - \left(\frac{8q^2}{5z^2} + \frac{8b_1^2 q}{15z^2} \right) r_{0s}^6 - \frac{16 b_2^2 r_{0s}^7}{105z} - \left(\frac{10q^2}{7z^2} + \frac{16 b_1 b_2 q}{21z^2} \right) r_{0s}^8 - \left(\frac{4q^2}{3z^2} + \frac{4 b_1 b_2 q}{35z^2} \right) r_{0s}^{10} \right.$$

$$+ 2\pi G \rho_\psi \left(\frac{dP_\psi}{dx}\right)^{-1} \left[\frac{2\rho_\psi D^2}{r_{0s}} \xi \left\{ 1 + \frac{2b_1^2 r_{0s}^3}{3z} + \frac{16 b_1 b_2 r_{0s}^5}{15z} + \left(\frac{4q^2}{z^2} + \frac{4b_1^2 q}{3z^2} \right) r_{0s}^6 + \frac{16 b_2^2 r_{0s}^7}{35z} + \right. \right.$$

$$\left. + \left(\frac{5q^2}{z^2} + \frac{8 b_1 b_2 q}{3z^2} \right) r_{0s}^8 + \left(\frac{6q^2}{z^2} + \frac{18 b_1 b_2 q}{35z^2} \right) r_{0s}^{10} + \dots \right\} + (l+1) \rho_c D^2 \phi \left\{ 1 + \frac{b_1^2 r_{0s}^3}{z} + \frac{2b_1^2 r_{0s}^5}{z} + \right.$$

$$\left. \left. + \left(\frac{24q^2}{5z^2} + \frac{8b_1^2 q}{5z^2} \right) r_{0s}^6 + \frac{56 b_2^2 r_{0s}^7}{105z} + \left(\frac{40q^2}{7z^2} + \frac{64 b_1 b_2 q}{21z^2} \right) r_{0s}^8 + \left(\frac{20q^2}{3z^2} + \frac{4 b_1 b_2 q}{7z^2} \right) r_{0s}^{10} + \dots \right\} \right]$$

(8.10b)

The condition requiring gravitational potential to be continuous across the free surface gives

$$\frac{d\phi}{dx} + \left[1 + \frac{(l+1)}{r_\psi} \frac{dr_\psi}{dx} \right] \phi + \frac{2R\rho_\psi}{\rho_c r_\psi^2} \frac{dr_\psi}{dx} \xi = 0$$

or

$$\frac{d\phi}{dx}+\left[l+(l+1)\left\{1+\frac{b_1^2\, r_{0s}^3}{z}+\frac{4b_1\, b_2\, r_{0s}^5}{3z}+\left(\frac{24q^2}{5z^2}+\frac{8b_1^2\, q}{5z^2}\right)r_{0s}^6+\frac{56b_2^2\, r_{0s}^7}{105z}+\right.\right.$$

$$\left.\left.\left(\frac{40q^2}{7z^2}+\frac{64b_1\, b_2\, q}{21z^2}\right)r_{0s}^8+\left(\frac{20q^2}{3z^2}+\frac{4b_1\, b_2\, q}{7z^2}\right)r_{0s}^{10}+\ldots\right\}\right]\phi$$

$$+\frac{2\rho_\psi}{\rho_c\, r_{0s}}\left[1+\frac{2b_1^2\, r_{0s}^3}{3z}+\frac{16b_1\, b_2\, r_{0s}^5}{15z}+\left(\frac{4q^2}{z^2}+\frac{4b_1^2\, q}{3z^2}\right)r_{0s}^6+\frac{16b_2^2\, r_{0s}^7}{35z}+\right.$$

$$\left.+\left(\frac{5q^2}{z^2}+\frac{8b_1\, b_2\, q}{3z^2}\right)r_{0s}^8+\left(\frac{6q^2}{z^2}+\frac{18b_1\, b_2\, q}{35z^2}\right)r_{0s}^{10}+\ldots\right]\xi=0$$

(8.10 c)

at the surface $x=1$

Thus in terms of the nondimensional eigenfunctions ξ,η and ϕ the problem determining the eigenfrequencies of nonradial modes of oscillation of a differentially rotating and tidally distorted gas spheres reduces to solving the system of differential equation (8.8) subject to the boundary conditions (8.9) at the centre and the boundary conditions (8.10) at the free surface.

8.3 EIGENVALUED BOUNDARY VALUE PROBLEM TO DETERMINE THE EIGENFREQUENCIES OF SMALL ADIABATIC PSEUDO-RADIAL MODES OF OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED COMPOSITE MODSELS

The eigenvalued boundary value problem governing the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of a rotationally and tidally distorted gaseous sphere has been formulated in section 8.1. In order to use this formulation to determine the eigenfrequencies of small adiabatic pseudo radial modes of oscillations of rotationally and tidally distorted composite models, we have to use in this eigenvalue problem the values of P_ψ and ρ_ψ for the appropriate rotationally and tidally distorted composite model.

The boundary conditions require ς to be finite at the centre and the outermost surface of the gaseous sphere. We can use this equation to determine the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of rotationally and tidally distorted composite models whose equilibrium structures were investigated in Chapter IV. For this we have to first obtain explicit expressions for μ_c (the value of μ inside the core) and μ_e (the value of μ inside the envelope) by substituting the appropriate values of r_ψ and P_ψ for the core and envelope.

On using the value of P_ψ for points inside the core from equation (4.7) and the value of r_ψ from (3.8) in the expression for μ as given in equation (8.2), we get after simplifications.

$$\mu_c = \frac{2r_0^{\,2}Y_1}{X_1}\Bigg[1+(\frac{2n}{zY_1}-\frac{2n}{z})r_0^{\,3}+(\frac{4n}{5zX_1}-\frac{16nD^2}{3Y_1zR_\psi^{\,2}})r_0^{\,5}+\{\frac{1}{Y_1}(\frac{2q^2}{z^2}+\frac{4nq}{3z^2}+\frac{4n^2}{3z^2})-.$$
$$-(\frac{24q^2}{5z^2}+\frac{16nq}{5z^2}+\frac{72n^2}{15z^2})\}r_0^6+(\frac{14nD^4}{5Y_1zR_\psi^4}-\frac{32nD^2}{21zX_1R_\psi^{\,2}})r_0^7+\{\frac{1}{X_1}(\frac{q^2}{2z^2}+\frac{nq}{3z^2}-\frac{20n^2}{3z^2})-$$
$$-\frac{8D^2}{5Y_1R_\psi^2}(\frac{18q^2}{5z^2}+\frac{12nq}{5z^2}+\frac{232n^2}{45z^2})+\frac{3q^2}{2z^2Y_1}+\frac{8n^2}{5X_1z^2Y_1}-\frac{40q^2}{7z^2}\}r_0^8+\frac{28nD^4}{45X_1zR_\psi^4}+\{\frac{10q^2}{9z^2Y_1}-$$
$$-\frac{328q^2D^2}{70Y_1z^2R_\psi^2}+\frac{3D^4}{15Y_1R_\psi^4}(\frac{26q^2}{5z^2}+\frac{52nq}{15z^2}+\frac{484n^2}{45z^2})-\frac{8D^2}{25X_1R_\psi^2}(\frac{18q^2}{5z^2}+\frac{12nq}{5z^2}+\frac{724n^2}{45z^2})+$$
$$+\frac{3q^2}{10z^2X_1}++\frac{16n^2}{25z^2X_1^2}-\frac{768n^2D^2}{105X_1z^2Y_1R_\psi^2}-\frac{20q^2}{3z^2}\}r_0^{10}+\ldots\Bigg] \qquad (8.11)$$

where

$$X_1 = K_1^2 - r_0^2 + \frac{4}{5}\frac{D^2}{R_\psi^2}r_0^4 + \frac{1}{5}\frac{D^4}{R_\psi^4}r_0^6$$

and

$$Y_1 = 1 - \frac{8}{5}\frac{D^2}{R_\psi^2}r_0^2 + \frac{3}{5}\frac{D^4}{R_\psi^4}r_0^4$$

Similarly on substituting for P_ψ from equation (4.21) for points in the envelope and for r_ψ from (3.8) in the expressions for μ as given in equation (8.2) we get

$$\mu_e = \frac{3Y}{X}\left[1 - \frac{2n}{z}\{1 + \frac{2}{3Y} + \frac{2(\log r_0)}{X}\} r_0^3 - \frac{2n}{M_{01}}(\frac{1}{3Yz} + \frac{1}{zY}) r_0^4 - \right.$$

$$-\{(\frac{24q^2}{5z^2} + \frac{16nq}{5z^2} + \frac{72n^2}{45z^2}) + \frac{1}{Y}(\frac{2q^2}{z^2} + \frac{4nq}{3z^2} + \frac{4n^2}{9z^2}) + \frac{1}{X}(\frac{2q^2}{z^2} + \frac{4nq}{3z^2} + \frac{28n^2}{9z^2} - \frac{8n^2}{z^2}(\log r_0))$$

$$-\frac{16n^2(\log r_0)}{3XYz^2} - \frac{16n^2(\log r_0)^2}{z^2X^2}\} r_0^6 - \frac{1}{M_{01}}\{\frac{1}{Y}(\frac{6q^2}{5z^2} + \frac{4nq}{5z^2} + \frac{44n^2}{45z^2}) + \frac{1}{X}(\frac{9q^2}{10z^2} + \frac{3nq}{5z^2} - \frac{34n^2}{45z^2}) -$$

$$-\frac{8n^2}{3XYz}(1 + (\log r_0)) - \frac{16n^2(\log r_0)}{zX^2}\} r_0^7 - \{\frac{40q^2}{7z^2} + \frac{29q^2}{14Yz} + \frac{87q^2}{70Xz} - \frac{4n^2}{3XYzM_{01}^2} - \frac{4n^2}{X^2zM_{01}^2}\} r_0^8$$

$$\left. -\frac{19q^2}{14zM_{01}}\{\frac{1}{Yz} + \frac{1}{2Xz}) r_0^9 - \frac{20q^2}{3}\{1 + \frac{1}{3Yz} + \frac{1}{7Xz}\} r_0^{10} - \ldots \right]$$

(8.12)

where

$$X = 1 + \frac{3}{2Mo_1} r_0 + 3C_1 r_0^3$$

and

$$Y = 1 + \frac{1}{Mo_1} r_0$$

Now substituting the values of $P_{\psi c}$, $\rho_{\psi c}$ and μ_c from (4.7), (4.5) and (8.11) in (8.2) the nondimensional form of the pulsation equation inside the core becomes

$$H_1 \frac{d^2\zeta}{dr_0^2} + H_2 \frac{d\zeta}{dr_0} + \left[H_3\omega^2 - H_4\right]\zeta = 0 \tag{8.13}$$

where

$$H_1 = 1 - \frac{16n}{3z} r_0^3 - (\frac{56q^2}{5z^2} + \frac{112nq}{15z^2} + \frac{104n^2}{45z^2}) r_0^6 - \frac{90q^2 r_0^8}{7z^2} - \frac{44q^2 r_0^{10}}{3z^2} + \ldots$$

$$H_2 = \frac{2r_0 Y_1}{X_1}\left[\frac{4X_1}{2r_0^2 Y_1} - 1 - (\frac{64nX_1}{6r_0^2 zY_1} + \frac{2n}{zY_1} - \frac{16n}{3z})r_0^3 - (\frac{4n}{5zX_1} - \frac{16nD^2}{3Y_1 zR_\psi^2})r_0^5\right.$$

$$-\{\frac{X_1}{2r_0^2 Y_1}(\frac{296q^2}{5z^2} + \frac{592nq}{15z^2} + \frac{1064n^2}{45z^2}) + \frac{1}{Y_1}(\frac{2q^2}{z^2} + \frac{4nq}{z^2} - \frac{16n^2}{3z^2}) -$$

$$-(\frac{56q^2}{5z^2} + \frac{112nq}{15z^2} + \frac{104n^2}{45z^2})\}r_0^6 - (\frac{14nD^4}{5Y_1 zR_\psi^4} - \frac{32nD^2}{21zX_1R_\psi^2})r_0^7 -$$

$$-\{\frac{560q^2X_1}{14z^2r_0^2Y_1} + \frac{q^2}{2X_1z^2} + \frac{nq}{3X_1z^2} - \frac{44n^2}{15X_1z^2} - \frac{144q^2D^2}{25z^2Y_1R_\psi^2} + \frac{3q^2}{2Y_1z^2} - \frac{90q^2}{7z^2}\}r_0^8 -$$

$$-\frac{28nD^4}{45zX_1R_\psi^4}r_0^9 - \{\frac{316q^2X_1}{6z^2r_0^2Y_1} - \frac{10q^2}{9Y_1z^2} - \frac{328q^2D^2}{70z^2Y_1R_\psi^4} + \frac{78q^2D^4}{25z^2Y_1R_\psi^4} - \frac{144q^2D^2}{125z^2X_1R_\psi^2} +$$

$$\left. + \frac{3q^2}{10X_1z^2} + \frac{16n^2}{25X_1^2} - \frac{44q^2}{3z^2}\}r_0^{10} + \dots\right]$$

$$H_3 = \frac{3}{X_1\gamma}\left[1 - \frac{D^2}{R_\psi^2}r_0^2 + (\frac{4n}{5X_1z} - \frac{4nD^2}{3zR_\psi^2})r_0^5 - \frac{224nD^2}{105zX_1R_\psi^2})r_0^7 + \{\frac{1}{X_1}(\frac{q^2}{2z^2} +\right.$$

$$+\frac{nq}{3z^2} + \frac{4n^2}{3z^2}) - \frac{D^2}{R_\psi^2}(\frac{8q^2}{5z^2} + \frac{16nq}{15z^2} + \frac{172n^2}{45z^2})\}r_0^8 + \frac{676nD^4}{315X_1R_\psi^4}r_0^9 +$$

$$\left. + \{-\frac{8D^2}{25X_1R_\psi^2}(\frac{413q^2}{80z^2} + \frac{413nq}{120z^2} + \frac{6956n^2}{360z^2}) + \frac{3q^2}{10zX_1} + \frac{16n^2}{25z^2X_1^2} - \frac{10q^2}{7z^2}\frac{D^2}{R_\psi^2}\}r_0^{10} + \dots\right]$$

$$H_4 = (3 - \frac{4}{\gamma})\frac{2Y_1}{X_1}\left[1 + (\frac{2n}{Y_1z} - \frac{10n}{3z})r_0^3 + (\frac{4n}{5zX_1} - \frac{16nD^2}{3Y_1zR_\psi^2}{}_2)r_0^5 + \{(\frac{2q^2}{z^2Y_1} + \frac{4nq}{3z^2Y_1} - \frac{4n^2}{3z^2}) -\right.$$

$$-(\frac{32q^2}{5z^2} + \frac{64nq}{15z^2} + \frac{188n^2}{3z^2})\}r_0^6 + \{\frac{14nD^4}{5Y_1zR_\psi^4} - \frac{32nD^2}{21X_1zR_\psi^2}\}r_0^7 + (\frac{q^2}{2z^2X_1} + \frac{nq}{3zX_1} - \frac{4n^2}{3z^2})$$

$$+\frac{3q^2}{2z^2Y_1} - \frac{144q^2D^2}{5Y_1z^2R_\psi^2} - \frac{96nqD^2}{25z^2Y_1R_\psi^2} + \frac{8n^2}{X_1Y_1z^2} - \frac{50q^2}{7z^2}\}r_0^8 + \frac{28nD^4}{45zX_1R_\psi^4} + \{\frac{10q^2}{9Y_1z^2} - \frac{328q^2D^2}{70z^2Y_1R_\psi^2} +$$

$$+\frac{78q^2D^4}{25Y_1z^2R_\psi^4} - \frac{156nqD^4}{75zY_1R_\psi^4} + \frac{3q^2}{10X_1z^2} - \frac{144q^2D^2}{125z^2X_1R_\psi^2} - \frac{96nqD^2}{125z^2X_1R_\psi^2} + \frac{16n^2}{25zX_1^2} -$$

$$\left. -\frac{768n^2D^2}{105X_1Y_1z^2R_\psi^2} - \frac{8q^2}{z^2}\}r_0^{10} + \dots\right]$$

and

$$\omega^2 = \frac{\sigma^2}{2\pi G\rho_c}$$

ω being the nondimensional form of the eigenfrequency σ. Also ζ is the value of the relative amplitude of pulsation at r_0 in the equivalent spherical model and thus denotes a suitable average of the amplitudes of pulsations of the fluid elements on the equipotential surface ψ = constant of the distorted model.

Similarly on substituting the values of $P_{\psi e}$, $\rho_{\psi e}$ and μ_e from equations (4.21), (4.16) and (8.12) in (8.2), the nondimensional form of the pulsation equation inside the envelope is same as (8.13). However, now

$$H_1 = 1 - \frac{16n}{3z} r_0^{\ 3} - \left(\frac{56q^2}{5z^2} + \frac{112nq}{15z^2} + \frac{104n^2}{45z^2}\right) r_0^{\ 6} - \frac{90q^2 r_0^{\ 8}}{7z^2} - \frac{44q^2 r_0^{\ 10}}{3z^2} + \ldots$$

$$\begin{aligned}
H_2 = \frac{3Y}{Xr_0}\Bigg[& \frac{4X}{3Y} - 1 - \left(\frac{64nX}{9zY} - \frac{4n}{3zY} - \frac{4n(\log r_0)}{Xz}\right) r_0^{\ 3} + \frac{2n}{M_{01}}\left\{\frac{1}{Xz} + \frac{1}{3Yz}\right\} r_0^4 - \\
& -\left\{\frac{X}{3Y}\left(\frac{296q^2}{5z^2} + \frac{592nq}{15z^2} + \frac{1064n^2}{45z^2}\right) - \left(\frac{56q^2}{5z^2} + \frac{122nq}{15z^2} + \frac{104n^2}{45z^2}\right) - \frac{1}{Y}\left(\frac{2q^2}{z^2} + \frac{4nq}{z^2} - \frac{4n^2}{z^2}\right)\right. \\
& \left. -\frac{1}{X}\left(\frac{2q^2}{z^2} + \frac{4nq}{z^2} + \frac{28n^2}{9z^2} - \frac{64n^2(\log r_0)}{3z^2} + \frac{16n^2(\log r_0)}{3\,XY\,z^2} + \frac{16n^2(\log r_0)^2}{3\,X^2 z^2}\right\} r_0^6 \\
& + \frac{1}{M_{01}}\left\{\frac{1}{Y}\left(\frac{6q^2}{5z^2} + \frac{4nq}{5z^2} - \frac{56n^2}{45z^2}\right) + \frac{1}{X}\left(\frac{9q^2}{10z^2} + \frac{3nq}{5z^2} - \frac{134n^2}{15z^2}\right) - \frac{8n^2}{3XYz^2}(1 + \log(r_0)) - \right. \\
& \left. -\frac{16n^2\log(r_0)}{X^2 z^2}\right\} r_0^7 - \left\{\frac{560q^2X}{21zY} - \frac{29q^2}{14Yz^2} - \frac{87q^2}{70Xz^2} + \frac{4n^2}{3M_{01}^2 z^2 X^2} - \frac{90q^2}{7z^2}\right\} r_0^{\ 8} \\
& + \frac{19q^2}{14M_{01}}\left\{\frac{1}{Yz} + \frac{1}{2Xz}\right\} r_0^9 - \left\{\frac{316q^2X}{9Yz^2} - \frac{44q^2}{9z^2} - \frac{20q^2}{9Yz^2} - \frac{20q^2}{21Xz}\right\} r_0^{\ 10} + \ldots \Bigg]
\end{aligned}$$

$$\begin{aligned}
H_3 = \frac{3D^2 r_0}{2b^2\gamma(1-b^2)R_\psi^2 X_1}\Bigg[& 1 - \left(\frac{4n\log(r_0)}{Xz} + \frac{4n}{3z}\right) r_0^{\ 5} - \frac{2n}{XzM_{01}} r_0^4 - \\
& -\left\{\left(\frac{8q^2}{5z^2} + \frac{16nq}{15z^2} + \frac{92n^2}{45z^2}\right) + \frac{1}{X}\left(\frac{2q^2}{z^2} + \frac{4nq}{3z^2} + \frac{28n^2}{9z^2} - \frac{16n^2(\log r_0)}{3z^2} - \frac{16n^2(\log r_0)}{z^2 X^2}\right\} r_0^6 - \right. \\
& -\frac{1}{M_{01}}\left\{\frac{1}{X}\left(\frac{9q^2}{10z^2} + \frac{3nq}{5z^2} - \frac{14n^2}{15z^2}\right) - \frac{16n^2(\log r_0)}{zX^2}\right\} r_0^7 - \left\{\frac{10q^2}{7z^2} + \frac{87q^2}{70z^2X} - \frac{4n^2}{M_{01}^2\ z^2X^2}\right\} r_0^8 \\
& - \frac{19q^2}{28Xz^2M_{01}} r_0^9 - \left\{\frac{4q^2}{3z^2} + \frac{20q^2}{21z^2X}\right\} r_0^{10} + \ldots \Bigg]
\end{aligned}$$

$$
\begin{aligned}
H_4 = (3-\frac{4}{\gamma})\frac{3Y}{X r_0^2}\Bigg[& 1-(\frac{4n}{3Y_1 z}-\frac{10n}{3\,Y z}+\frac{4n(\log r_0)}{X z})r_0^{\,3}-\frac{2n}{M_{01}}\{\frac{1}{3Y z}+\frac{1}{X z})r_0^{\,4}- \\
& -\{(\frac{32q^2}{5z^2}+\frac{64nq}{15z^2}+\frac{188\,n^2}{3z^2})\}r_0^{\,6}+\frac{1}{Y}(\frac{2q^2}{z^2}+\frac{4nq}{3z^2}-\frac{4\,n^2}{3z^2})+ \\
& +\frac{1}{X}(\frac{2q^2}{z^2}+\frac{4nq}{3z^2}+\frac{28\,n^2}{9z^2}-\frac{40n^2(\log r_0)}{3z^2})-\frac{16n^2(\log r_0)}{3XYz^2}-\frac{16n^2(\log r_0)^2}{z^2X^2}\}\,r_0^6- \\
& -\frac{1}{M_{01}}\{\frac{1}{Y}(\frac{6q^2}{5z^2}+\frac{4nq}{5z^2}+\frac{4\,n^2}{45z^2})+\frac{1}{X}(\frac{9q^2}{10z^2}+\frac{3nq}{5z^2}-\frac{74\,n^2}{15z^2}) \\
& -\frac{8n^2}{3XYz^2}(1+(\log r_0))-\frac{16n^2(\log r_0)}{z^2}\}\,r_0^7-\{\frac{50q^2}{7z^2}+\frac{29\,q^2}{14Yz^2}+\frac{87\,n^2}{70Xz^2}-\frac{4n^2}{3\,zM_{01}^2\,XY}\}\,r_0^8 \\
& -\frac{19q^2}{14M_{01}}\{\frac{1}{z^2Y}+\frac{1}{2Xz^2}\}\,r_0^9-\{\frac{8q^2}{z^2}+\frac{20q^2}{9Yz^2}+\frac{20q^2}{21Xz^2}\}r_0^{10}+\ldots\Bigg]
\end{aligned}
$$

Equation (8.13) together with the boundary condition which require ζ to be finite at centre and free surface and continuous across the interface between the core and the envelope, constitute an eigenvalued boundary problem which determines the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of rotationally and tidally distorted composite models consisting of cores in which density varies according to the law $\rho_\psi = \rho_c(1-\frac{r_\psi^2}{R_\psi^2})$ and envelopes in which density varies as $\rho_\psi = \rho_c\, b^2(1-\frac{R_\psi^2}{r_\psi^2})$. In the formulation of this eigenvalue problem terms upto second order of smallness in distortion parameter n and q have been retained.

It can be easily verified that if we set $n = q = 0$ in these equations,

Then we obtain the usual equation determining the eigenfrequencies of small adiabatic radial modes of oscillations of an undistorted model of the above series of composite models. On setting n = (q+1)/2 in the above formulation we can determine the effects of rotation and tidal distortions on the periods of small adiabatic pseudo-radial oscillations of the primary component of a synchronously rotating binary system. Also by setting q = 0 or n=0 separately, we may study the effects of rotation alone or tidal distortions alone on the periods of small adiabatic pseudo-radial oscillations of the models of the series of composite models.

8.4 EIGENVALUE PROBLEM DETERMINING THE EIGENFREQUENCIES OF SMALL ADIABATIC PSEUDO-RADIAL MODES OF OSCILLATIONS OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED POLYTROPIC MODELS

The eigenvalued boundary value problem governing the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of a differentially rotating and tidally distorted gas sphere has been formulated in section 8.1. In order to use this formulation to determine the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of a differentially rotating and tidally distorted polytropic model, we have to use in this eigenvalue problem the values of ρ_ψ and P_ψ for the appropriate differentially rotating polytropic model.

On substituting in equation (8.2) the values of P_ψ, and ρ_ψ as defined by relations 3.17 of chapter III for a differentially rotating and tidally distorted polytropic models, we get after some simplifications.

$$H_1 \frac{d^2\eta}{dr_0^2} + H_2 \frac{d\eta}{dr_0} + \left(H_3\omega^2 - H_4\right)\eta = 0 \tag{8.14}$$

where

$$H_1(z,b_1,b_2,q)=1-\frac{8b_1^2r_0^3}{3z}-\frac{16b_1b_2r_0^5}{5z}-\frac{28q^2r_0^6}{5z^2}-\frac{128b_2^2r_0^7}{105z}-\frac{90q^2r_0^8}{15z}-\frac{44q^2r_0^{10}}{3z^2}+..$$

$$H_2=\frac{1}{r_0}\left[4-\frac{32b_1^2r_0^3}{3z}-\frac{232b_1b_2r_0^5}{15z}-\frac{296q^2r_0^6}{5z^2}-\frac{736b_2^2r_0^7}{35z}-\frac{560q^2r_0^8}{7z^2}-\frac{76q^2r_0^{10}}{z^2}+.. \right.$$
$$\left. +(N+1)(\frac{1}{\theta_\psi}\frac{d\theta_\psi}{dr_0})r_0\{1-\frac{8b_1^2r_0^3}{3z}-\frac{16b_1b_2r_0^5}{5z}-\frac{128b_2^2r_0^7}{105z}-\frac{56q^2r_0^6}{5z^2}-\frac{90q^2r_0^8}{7z^2}-\frac{44q^2r_0^{10}}{3z^2}\}\right]$$

$$H_3=\frac{(N+1)\xi_u^2k}{3\gamma r_{0s}^3}(\frac{\bar{\rho}}{\rho_c})\frac{1}{\theta_\psi}$$

$$H_4=-(3-\frac{4}{\gamma})(N+1)(\frac{1}{\theta_\psi}\frac{d\theta_\psi}{dr_0})\frac{1}{r_0}\left[1-\frac{5b_1^2r_0^3}{3z}-\frac{28b_1b_2r_0^5}{15z}-\frac{32q^2r_0^6}{5z^2}-\frac{24b_2^2r_0^7}{5z}\right.\cdot$$
$$\left.-\frac{50q^2r_0^8}{7z^2}-\frac{8q^2r_0^{10}}{z^2}+...\right],$$

where $$\omega^2=\frac{D^3r_{OS}^3\sigma^2}{GM_0},$$

ω being the nondimensional form of the eigenfrequency σ. In the above expressions values of the parameters ξ_u, ρ_c and $\bar{\rho}$ are to be taken for the original undistorted polytropic model.

Equation (8.14) is the general equation in nondimensional form which determines the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of a differentially rotating and tidally distorted polytropic model when terms upto second order of smallness z, b_1 , b_2 and q are retained. For numerical evaluation of the eigenfrequencies, the second order differential equation (8.14) is to be solved numerically subject to the boundary conditions which require η to be finite at points corresponding to the centre $(r_0=0)$ and the free surface $(r_0=r_{OS})$ of the model.

On setting $z=1$, $b_1=b_2=0$ (i.e. in the absence of any distortion, equation (8.14) reduces to the usual equation which determines the eigenfrequencies of small adiabatic radial modes oscillations of an undistorted polytropic model. By setting $z=1$, $b_1^2=2n, b_2^2=0$ we can study the effects of solid body rotation on the eigenfrequencies.

8.5 EIGENVALUED BOUNDARY VALUE PROBLEM DETERMINING THE EIGENFREQUENCIES OF SMALL ADIABATIC NONRADIAL MODES OF OSCILLATIONS OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED POLYTROPIC MODELS.

System of equation (8.8) with the boundary conditions (8.9-8.10) constitutes the eigenvalued boundary value problem which determines the effects of differential rotation and tidal distortions on the eigenfrequencies of nonradial modes of oscillations of a differentially rotating and tidally distorted gas spheres. In order to use this eigenvalue problem to determine the effects of differential rotation and tidal distortions on the eigenfrequencies of nonradial modes of oscillations of polytropic models the values of P_ψ, and ρ_ψ etc. appearing in these equations are to be taken from relations (3.17) of Chapter III, the system of differential equations (8.8) governing the nonradial modes of oscillations of a differentially rotating and tidally distorted polytropic model, can be expressed as

$$\left.\begin{aligned}
&\frac{d\zeta}{dx}+B_1\xi+\left(B_2+\frac{B_3}{\omega^2}\right)\eta+\frac{B_3}{\omega^2}\phi=0,\\
&\frac{d\eta}{dx}+\left(E_1\omega^2+E_2\right)\xi+E_3\eta+E_4\phi+\frac{d\phi}{dx}=0,\\
&\text{and}\\
&\frac{\mathrm{d}^2\phi}{dx^2}+F_1\frac{d\phi}{dx}+F_2\xi+F_3\eta+F_4\phi=0
\end{aligned}\right] \tag{8.15}$$

where

$$B_1 = \frac{l+1}{x} + \frac{N+1}{\gamma}\left(\frac{1}{\theta_\psi}\frac{d\theta_\psi}{dx}\right).$$

$$B_2 = \frac{(N+1)\xi_u^{\ 2} r_{0s}^3 x}{2\gamma k^2 \theta_\psi}\left[1 + \frac{2b_1^{\ 2} r_{0s}^{\ 3} x^3}{z} + \frac{32 b_1 b_2 r_{0s}^{\ 5} x^5}{15z} + \left(\frac{36q^2}{5z^2} + \frac{12 b_1^2 q}{5z^2}\right) + \frac{16}{21}\frac{b_2^{\ 2} r_{0s}^{\ 7} x^7}{z}\right. .$$
$$\left. + \left(\frac{55q^2}{7z^2} + \frac{88 b_1 b_2 q}{21 z^2}\right) x^8 r_{0s}^{\ 8} + \left(\frac{26q^2}{3z^2} + \frac{124 b_1 b_2 q}{35 z^2}\right) x^{10} r_{0s}^{\ 10} + \ldots\right],$$

$$B_3 = -\frac{3l(l+1) r_{0s}^{\ 4}}{2k^3 x}\left(\frac{\rho_c}{\rho}\right)\left[1 + \frac{4b_1^{\ 2} r_{0s}^{\ 3} x^3}{3z} + \frac{8 b_1 b_2 x^5 r_{0s}^{\ 5}}{5z} + \left(\frac{28q^2}{5z^2} + \frac{28 b_1^{\ 2} q}{15z^2}\right) r_{0s}^{\ 6} + .\right.$$
$$\left. + \frac{64 b_2^{\ 2} x^7 r_{0s}^{\ 7}}{105z} + \left(\frac{45q^2}{7z^2} + \frac{24 b_1 b_2 q}{7z^2}\right) x^8 r_{0s}^{\ 8} + \left(\frac{22q^2}{3z^2} + \frac{22 b_1 b_2 q}{35 z^2}\right) x^{10} r_{0s}^{\ 10} + \ldots\right]$$

$$E_1 = -\frac{2k^3}{3 r_{0s}^{\ 4} x}\left(\frac{\bar{\rho}}{\rho_c}\right)\left[1 + \frac{2b_1^{\ 2} x^3 r_{0s}^{\ 3}}{3z} + \frac{16 b_1 b_2 x^5 r_{0s}^{\ 5}}{15z} + \left(\frac{4q^2}{z^2} + \frac{4b_1^{\ 2} q}{3z^2}\right) x^6 r_{0s}^{\ 6} + \frac{16 b_2^{\ 2} x^7 r_{0s}^{\ 7}}{35z} +\right. .$$
$$\left. + \left(\frac{5q^2}{z^2} + \frac{8 b_1 b_2 q}{3z^2}\right) x^8 r_{0s}^{\ 8} + \left(\frac{6q^2}{z^2} + \frac{18 b_1 b_2 q}{35 z^2}\right) x^{10} r_{0s}^{\ 10} + \ldots\right]$$

$$E_2 = \frac{2k^2}{\xi_u^{\ 2}}\left(N - \frac{N+1}{\gamma}\right)\frac{1}{\theta_\psi}\left(\frac{d\theta_\psi}{dx}\right)^2 \frac{1}{r_{0s}^{\ 3} x}\left[1 - \frac{2b_1^{\ 2} x^3 r_{0s}^{\ 3}}{z} - \frac{32 b_1 b_2 x^5 r_{0s}^{\ 5}}{15z} - \left(\frac{36q^2}{5z^2} + \frac{12 b_1^{\ 2} q}{5z^2}\right) x^6 r_{0s}^{\ 6} + .\right.$$
$$\left. - \frac{16}{21z} b_2^{\ 2} x^7 r_{0s}^{\ 7} - \left(\frac{55q^2}{7z^2} + \frac{88 b_1 b_2 q}{21 z^2}\right) x^8 r_{0s}^{\ 8} - \left(\frac{26q^2}{3z^2} + \frac{26 b_1 b_2 q}{35 z^2}\right) x^{10} r_{0s}^{\ 10} + ..\right]$$

$$E_3 = \frac{l}{x} + \left(N - \frac{N+1}{\gamma}\right)\frac{1}{\theta_\psi}\frac{d\theta_\psi}{dx,},$$

$$E_4 = \frac{l}{x},$$

$$F_1 = \frac{l}{x}\left[2\,(l+1) - \frac{2b_1^{\ 2} x^3 r_{0s}^{\ 3}}{z} + \frac{16 b_1 b_2 x^5 r_{0s}^{\ 5}}{3z} + \left(\frac{24q^2}{z^2} + \frac{8 b_1^{\ 2} q}{z^2}\right) x^6 r_{0s}^{\ 6} +\right.$$
$$\left. + \frac{112 b_2^{\ 2} x^7 r_{0s}^{\ 7}}{35z} + \left(\frac{40q^2}{z^2} + \frac{448 b_1 b_2 q}{21 z^2}\right) x^8 r_{0s}^{\ 8} + \left(\frac{60q^2}{z^2} + \frac{36 b_1 b_2 q}{7z^2}\right) x^{10} r_{0s}^{\ 10} + \ldots\right]$$

$$F_2 = \frac{2}{r_{0s}x}(N - \frac{N+1}{\gamma})\theta_\psi^{N-1}\frac{d\theta_\psi}{dx}\left[1 + \frac{2b_1^2x^3r_{0s}^3}{3z} + \frac{16b_1b_2x^5r_{0s}^5}{15z} + (\frac{4q^2}{z^2} + \frac{4b_1^2q}{3z^2})x^6r_{0s}^6\right.$$

$$\left. + \frac{16\,b_2^2x^7r_{0s}^7}{35z} + (\frac{5q^2}{z^2} + \frac{8b_1b_2\,q}{3z^2})x^8r_{0s}^8 + (\frac{6q^2}{z^2} + \frac{18b_1b_2q}{35z^2})x^{10}r_{0s}^{10}) + \ldots\right]$$

$$F_3 = -\frac{(N+1)}{\gamma}\frac{\xi_u^2}{k^2}\theta_\psi^{N-1}r_{0s}^2\left[1 + \frac{8b_1^2x^3r_{0s}^3}{3z} + \frac{16b_1b_2x^5r_{0s}^5}{5z} + (\frac{56q^2}{5z^2} + \frac{56b_1^2q}{15z^2})x^6r_{0s}^6\right.$$

$$\left. + \frac{128b_2^2x^7r_{0s}^7}{105z} + (\frac{90q^2}{7z^2} + \frac{48b_1b_2q}{7z^2})x^8r_{0s}^8 + (\frac{44q^2}{3z^2} + \frac{44b_1b_2q}{35z^2})x^{10}r_{0s}^{10} + \ldots\right]$$

$$F_4 = -\frac{1}{x^2}\left[l\left\{\frac{4b_1^2x^3r_{0s}^3}{z} + \frac{8b_1b_2x^5r_{0s}^5}{z} + (\frac{168q^2}{5z^2} + \frac{56b_1^2q}{5z^2})x^6r_{0s}^6 + \frac{448b_2^2x^7r_{0s}^7}{105z}\right.\right.$$

$$\left. + (\frac{360q^2}{7z^2} + \frac{576b_1b_2q}{21z^2})x^8r_{0s}^8 + (\frac{220q^2}{3z^2} + \frac{44b_1b_2q}{7z^2})x^{10}r_{0s}^{10}\right\} + l\left\{\frac{2b_1^2x^3r_{0s}^3}{z} + \frac{8b_1b_2x^5r_{0s}^5}{3z}\right.$$

$$+ (\frac{48q^2}{5z^2} + \frac{16b_1^2q}{5z^2})x^6r_{0s}^6 + \frac{112}{105}\frac{b_2^2x^7r_{0s}^7}{z} + (\frac{80q^2}{7z^2} + \frac{128b_1b_2q}{21z^2})x^8r_{0s}^8$$

$$\left.\left. + (\frac{40q^2}{3z^2} + \frac{8b_1b_2q}{7z^2})x^{10}r_{0s}^{10} + \ldots\right\}\right]$$

and

$$\omega^2 = \frac{D^3\,r_{0S}^3\,\sigma^2}{G\,M_0},$$

ω being the nondimensional form of the eigenfrequency σ. As mentioned in the radial case values of the parameters ξ_u, ρ_c and $\overline{\rho}$ are to be taken from the original undistorted polytropic model.

The boundary conditions (8.9) at the centre ($x=0$) for the case of distorted polytropic models become

$$\eta + \phi = \frac{2\omega^2}{3l\,r_{0s}^4}\left(\frac{\overline{\rho}}{\rho_c}\right)\xi, \qquad (8.16)$$

and

$$\frac{d\phi}{dx}=0 \tag{8.17}$$

On substituting the values of P_ψ, and ρ_ψ from (3.17) in the boundary conditions (8.10) at the free surface ($x=1$), the boundary conditions at the free surface in the case of polytropic models become

$$\eta r_{0s}^{\ 3}\left[1+\frac{2b_1^{\ 2}r_{0s}^{\ 3}}{z}+\frac{32b_1b_2r_{0s}^{\ 5}}{15z}+\frac{36q^2r_{0s}^{\ 6}}{5z^2}+\frac{16b_2^{\ 2}r_{0s}^{\ 7}}{21z}+\frac{55q^2r_{0s}^{\ 8}}{7z^2}+\frac{26q^2r_{0s}^{\ 10}}{3z^2}+..\right]$$
$$+2\frac{k^2}{\xi_u^{\ 2}}\frac{d\theta_\psi}{dx}\zeta=0, \tag{8.18}$$

$$\frac{d\phi}{dx}+\phi\left[(l+1)\left\{1+\frac{b_1^{\ 2}r_{0s}^{\ 3}}{z}+\frac{4b_1b_2r_{0s}^{\ 5}}{3z}+\frac{24q^2r_{0s}^{\ 6}}{5z^2}+\frac{56b_2^{\ 2}r_{0s}^{\ 7}}{105z}+\frac{40q^2r_{0s}^{\ 8}}{7z^2}+\right.\right.$$
$$\left.\left.+\frac{20q^2r_{0s}^{\ 10}}{3z^2}.\right\}+l\right]=0, \tag{8.19}$$

The system of differential equations (8.15) together with the boundary conditions (8.16-8.19) constitutes the eigenvalued boundary value problem determining the effects of differential rotation and tidal distortions on the eigenfrequencies of nonradial modes of oscillations of polytropic models.

In the absence of any distortion (i.e. $z=1$, $b_1=b_2=0$), the system of differential equations (8.15) along with the boundary conditions (8.16-8.19) reduce to the usual eigenvalued boundary value problem determining the eigenfrequencies of nonradial modes of oscillations of undistorted polytropic models. By setting $z=1, b_1^2=2n, b_2^2=0$ we can also study the effects of solid body rotation on the eigenfrequencies of nonradial modes oscillations of the polytropic models.

8.6 NUMERICAL EVALUATION AND ANALYSIS OF RESULTS

In order to determine eigenfrequencies of pseudo-radial modes of oscillations of rotationally and tidally distorted composite models, computations were started for some trial value of σ^2. For this chosen value of σ^2 starting from points near the centre (x=0.02), outward integration was performed right up to the interface for the pulsation equation (8.13) using the difference method earlier used by Aggarwal (2) (The details of this method are given in Aggarwal (2)) with a step length h=0.02. Again using the same value of σ^2, inward integration of this equation was performed up to the interface starting from points near the surface (x=0.98) using a step length 0.2 and the same difference formula. In outward in inward integrations we need the value of z_c and z_e, respectively. These values were earlier obtained in chapter IV. The value of $\zeta/(d\zeta/dx)$ obtained from the core integrations and the envelope integrations were matched at the interface. Trials with different values of σ^2 were continued till the value of $\zeta/(d\zeta/dx)$ at the interface from the core and envelope integrations agreed to the desired accuracy (the difference in the values of this ratio obtained from two solutions was required not to exceed 0.0001).

Computations have been performed to determine the eigenfrequencies of pseudo-radial modes of oscillations of the fundamental, the first and the second mode for each distorted composite models, for different sets of the values of the input parameters $n, q, \psi_s^{\bullet}$. and $\gamma = \frac{5}{3}$. The results are presented in Table 8.2 (a, b, c). Results presented in the first row of this Table for $n=0.1$, $q=0$ depict the eigenvalues for the fundamental, the first and the second mode of radial oscillations of the corresponding rotationally distorted model. We also present in this table results for the eigenfrequencies of the fundamental, the first and the second mode of pseudo-radial oscillations of the primary components of certain synchronously rotating binary systems obtained by setting $n=\frac{(q+1)}{2}$.

The eigenvalue problem of section 8.4 is of Sturm-Liouville type. For determining the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of differentially rotating and tidally distorted polytropic models, equation (8.14) is to be integrated numerically subject to the boundary conditions which require η being finite at points corresponding to the centre and the free surface of the model. The numerical integration can be performed using the approach suggested in Section 8.1. The values of z, θ_ψ and $\frac{d\theta_\psi}{dx}$ needed at various points are to be taken from the numerical solution of the equation (5.38a) obtained in Chapter V.

In order to determine the eigenfrequencies of pseudo-radial modes of oscillations of differentially rotating and tidally distorted polytropic models, computations are started with some trial value of ω^2. For this chosen value of ω^2 at the points very near the centre series solution is first developed and this solution is then used to carry outward integration of the pulsation equation (8.14) using fourth order Runge-Kutta method. Again using the same value of ω^2, series solution is first developed at points near the surface and this solution is then used to carry inward integration of the equation (8.14). Again, we need the value of z at each point in the outward in the inward integrations these values were earlier obtained in chapter V. The value of

$\zeta/(\frac{d\zeta}{dx})$ obtained from the outward integration and the inward integration of (8.14), is matched at some preselected point in the interior of the model. To start integrations from points near the centre and the surface series solutions were developed at x=0.01 and x=0.99. Outward and inward integrations were performed using Runge-Kutta method of order four using a step length x=0.01. Trials with different values of ω^2 were continued till the absolute difference in the value $\zeta/(\frac{d\zeta}{dx})$ at the preselected point in the interior of the model from the outward and inward integrations was found to be less than 0.0005.

Computations have been performed to compute the fundamental and the first mode of pseudo-radial oscillations of differentially rotating and tidally distorted polytropic models of indices 1.5, 3.0 and 4.0 for those values of distortion parameters z, b_1, b_2, q for which equilibrium structures were earlier obtained in Chapter V. The results are presented in Table 8.3

The eigenfrequencies of the nonradial modes of oscillations of some of these differentially rotating and tidally distorted polytropic models have also been computed using Chebyshev polynomial expansion technique earlier used by Mohan, Saxena and Agarwal (92). The essential details of the method are given in Saxena (124).The boundary condition (8.18) was used as the discriminant condition and $\xi = 1$ at the centre was used as the normalization condition. The values of z, θ_ψ and $\frac{d\theta_\psi}{dx_e}$ needed at various points in the interior of the model were obtained from the solutions of the structure equation (5.38 a) of these models earlier obtained in Chapter V. For polytropic indices 1.5 and 3.0 we ordinarily used 10 and 15 collocation points, respectively. However, for determining the eigenfrequencies of certain higher modes of nonradial oscillations, the number of collocation points was increased to achieve the desired accuracy of 0.0001 for the polytropic models of indices 1.5 and 3.0 in getting the discriminant condition satisfied. The number of collocation points used in determining a specific mode of nonradial oscillation of a distorted polytropic model was same as used in determining this mode for the corresponding undistorted model. The numerical results are presented in Tables 8.4 (a), to (b) for polytropic indices 1.5 and 3.0, respectively. The number of nodes appearing in the eigenfunctions ξ and η are also shown in parenthesis in these tables.

Table 8.1 shows the values of r_{0_s} for certain differentially rotating and tidally distorted model with polytropic indices 1.5, 3.0 and 4.0 The results in tables 8.2 (a) -(c) present the eigenfrequencies of the fundamental, the first and the second pseudo radial modes of oscillations of rotationally and/ or tidally distorted composite models for $\gamma = \frac{5}{3}$ with interfaces b = 0.3, 0.5, and 0.7. The results shown in parenthesis in these tables are the corresponding results earlier obtained by Aggarwal (2). A comparison of the results for b= 0.3 with the corresponding results shown in parenthesis shows that values obtained by us are generally smaller in comparison to the respective values results shown in parenthesis. However, for the fundamental modes of models $(\psi = 5.0,\ n = 0.0, q = 0.2,\ \psi = 10.0,\ n = 0.1, q = 0,\ \psi = 10.0,\ n = 0,\ q = .1, .2,\ n = .1, q = .5)$ and first mode of models $(\psi = 2,\ n = 0.2, q = 0.0)$ the values obtained by us are larger than earlier obtained values. The amount of increase or decrease in the value varies from model to model. For distorted composite models with interfaces b= 0.5, it is noticed that eigenvalues for the fundamental mode increase. The first mode decrease and for second mode again increases in comparison to the results shown in brackets. For the rotationally or tidally distorted composite model with interfaces b=0.7, our results for various models are larger compared to the earlier obtained value in some cases and smaller in other cases. No specific trend of increase or decrease in the values of eigenfrequencies of the fundamental, the first and the second modes of oscillations of such type of composite models has been noticed.

The results presented in table 8.3 show that eigenfrequencies for the fundamental and the first mode of pseudo-radial oscillations of differentially rotating and tidally distorted polytropic models with indices 1.5, 3.0 and 4.0. The value ω_0^2 and ω_1^2 shown in parenthesis are corresponding values earlier obtained by Lal (69). A comparison of our results with the corresponding results presented in parenthesis for models 1 and 2 shows that values obtained by us are generally smaller in comparison to earlier obtained values while these are larger for model 3 for all the polytropic models with indices 1.5, 3.0 and 4.0. The model 4 represents differentially rotating and tidally distorted model which is rotationally stable. However, Model 5 and 6 represent differentially rotating and tidally distorted models which are rotationally unstable. Our results show that eigenfrequencies for the fundamental and the first modes for model 4 are larger for models 5 and 6 these are smaller in comparison to tidally distorted models.

The results presented in table 8.4 (a) show the eigenfrequencies of nonradial modes of oscillations of various types of differentially rotating and tidally distorted polytroipic models with index 1.5. On comparing our results for f, p1, p2, p3 modes with the corresponding results shown in parenthesis earlier obtained by Lal (69), it is noticed that values of eigenfrequencies obtained by us are smaller. These are also smaller for differentially rotating and tidally distorted polytropic models in comparison to tidally distorted polytropic models. The eigenfrequencies of different modes of non radial oscillations of differentially rotating and tidally distorted polytropic models of stars with index 3.0 are presented in table 8.4(b). On comparing the results for different modes of differentially rotating and tidally distorted polytropic models with the corresponding values depicted in parenthesis it is observed that the values obtained by us are smaller in comparison to the values earlier obtained by Lal (69) and shown in parenthesis. While eigenfrequencies of g_1, g_2, g_3 modes of differentially rotating and tidally distorted polytropic models 4 and 5 are smaller in comparison to corresponding eigenvalues of tidally distorted polytropic model eigenfrequencies f, P_1, P_2, P_3 modes of these models increase. It is also noticed that the eigenfrequencies of g_1, g_2, g_3 modes of differentially rotating and tidally distorted models increase and those of f, P_1, P_2, P_3 modes decrease in comparison to these of tidally distorted polytropic models. However amount of increase or decrease in these eigenvalues varies from model to model. No specific trend in variation of the values of eigenfrequencies of these modes of differentially rotating and tidally distorted polytropic models has been noticed.

Table 8.2(a) : Eigenfrequencies $\omega^2 \left(= \dfrac{\sigma^2}{2\pi G \rho_c} \right)$ of the fundamental, the first and the second pseudo-radial modes of oscillations of rotationally and/or tidally distorted models of the composite series for $\gamma = 5/3$

b= 0.3

ψ	η	q	ω_0^2	ω_1^2	ω_2^2
2	0.1	0.0	- (0.2259)	- (1.3979)	2.5782 ((3.4179)
2	0.2	0.0	0.16230 (0.2232)	1.88131 (1.3965)	1.62278 (3.4833)
2	0.0	0.1	0.192492 (0.2301)	0.6151851 (1.4076)	2.38009 (3.3831)
2	0.0	0.2	0.180213 (0.2299)	- (1.4065)	2.06986 (3.3839)
2	0.1	0.1	[illegible] (0.2252)	0.187757 (1.3971)	1.31467 (3.4194)
2	0.1	0.5	- (0.1992)	- (1.2950)	2.31878 (3.4245)
5	0.1	0.0	0.185471 (0.2298)	0.736950 (1.4068)	2.20825 (3.3832)
5	0.2	0.0	- (0.2296)	0.791353 (1.4058)	- (3.3842)
5	0.0	0.1	0.220214 (0.2301)	0.631686 (1.4078)	1.7848 (3.3823)
5	0.0	0.2	.232320 (0.2301)	0.651950 (1.4076)	1.87023 (3.3823)
5	0.1	0.1	0.181136 (0.2298)	0.594009 (1.4078)	1.9085 (3.3847)
5	0.1	0.5	- (0.22)	0.778640 (1.4064)	2.31878 (3.3835)
10	0.1	0.0	0.238511 (0.2301)	0.739072 (1.4076)	1.72344 (3.3824)
10	0.2	0.0	0.22852 (0.2300)	0.655952 (1.4075)	1.48745 (3.3825)
10	0.0	0.1	0.240369 (0.2301)	0.6615078 (1.4078)	1.36303 (3.3823)
10	0.0	0.2	0.249330 (0.2301)	0.662567 (1.4078)	1.39183 (3.3823)
10	0.1	0.1	0.229939 (0.2301)	0.816056 (1.4076)	1.9959 (3.3824)
10	0.1	0.5	0.230548 (0.2297)	0.762350 (1.4064)	2.43559 (3.3835)

Table 8.2(b) : Eigenfrequencies $\omega^2\left(=\dfrac{\sigma^2}{2\pi G\rho_c}\right)$ of the fundamental, the first and the second pseudo-radial modes of oscillations of rotationally and/or tidally distorted models of the composite series for $\gamma = 5/3$

b= 0.5

ψ	η	q	ω_0^2	ω_1^2	ω_2^2
2	0.1	0.0	- (0.2259)	2.6679 (2.7001)	- (3.4119)
2	0.2	0.0	- (0.2232)	- (2.6976)	- (3.4373)
2	0.0	0.1	0.262870 (0.2301)	2.48996 (1.4076)	9.4085 (3.5831)
2	0.0	0.2	- (0.2299)	2.4896 (1.4065)	- (3.3839)
2	0.1	0.1	- (0.2252)	0.815780 (1.3971)	2.6505 (3.4194)
2	0.1	0.5	- (0.1992)	0.815780 (1.2950)	- (3.4245)
5	0.1	0.0	0.31713 (0.2298)	1.3039 (1.4068)	- (3.3832)
5	0.2	0.0	0.313886 (0.2296)	1.3894 (1.4058)	4.09301 (3.3842)
5	0.0	0.1	0.306709 (0.2301)	1.24598 (1.4078)	3.313195 (3.3823)
5	0.0	0.2	0.311214 (0.2301)	1.25157 (1.4078)	3.8991 (3.3824)
5	0.1	0.1	0.31339 (0.2298)	1.31382 (1.4067)	- (3.3833)
5	0.1	0.5	0.313395 (0.2297)	1.34296 (1.4064)	3.1726 (3.3835)
10	0.1	0.0	0.324357 (0.2301)	0.685063 (1.4076)	- (3.3824)
10	0.2	0.0	0.363307 (0.2300)	2.26040 (1.4075)	6.3373 (3.3825)
10	0.0	0.1	0.393512 (0.2301)	0.75337 (1.4078)	6.4035 (3.3823)
10	0.0	0.2	0.415197 (0.2301)	0.793491 (1.4078)	6.7683 (3.3823)
10	0.1	0.1	0.323089 (0.2301)	0.65563 (1.4076)	6.6146 (3.3824)
10	0.1	0.5	0.505068 (0.2297)	0.605637 (1.4076)	4.91818 (3.3835)

Table 8.2(c) : Eigenfrequencies $\omega^2\left(=\frac{\sigma^2}{2\pi G\rho_c}\right)$ of the fundamental, the first and the second pseudo-radial modes of oscillations of rotationally and/or tidally distorted models of the composite series for $\gamma = 5/3$

b=0.7

ψ	η	q	ω_0^2	ω_1^2	ω_2^2
2	0.1	0.0	0.35437 (0.4125)	0.930719 (3.3845)	11.5460 (8.8415)
2	0.2	0.0	0.4201 (0.4106)	4.91374 (3.3729)	12.5913 (9.4412)
2	0.0	0.1	0.23240 (0.4198)	- (3.3920)	3.7147 (8.6392)
2	0.0	0.2	0.22726 (0.4192)	1.02482 (3.3891)	10.339 (8.6397)
2	0.1	0.1	0.3114 (0.4112)	0.963990 (3.3864)	11.6018 (8.8936)
2	0.1	0.5	- (0.3751)	- (3.3850)	- (9.2865)
5	0.1	0.0	0.292576 (0.4194)	3.5343 (3.3911)	9.7739 (8.6374)
5	0.2	0.0	0.288906 (0.4189)	8.9778 (3.3897)	4.3692 (8.6454)
5	0.0	0.1	0.267831 (0.4199)	1.06443 (3.3926)	- (8.6301)
5	0.0	0.2	0.258978 (0.4199)	- (3.3926)	2.85601 (8.6301)
5	0.1	0.1	0.266041 (0.4193)	0.826112 (3.3910)	3.5444 (8.6379)
5	0.1	0.5	- (0.4192)	0.669890 (3.3904)	- (8.6402)
10	0.1	0.0	0.515265 (0.4198)	1.14959 (3.3924)	9.132115 (8.6309)
10	0.2	0.0	0.491009 (0.4197)	3.8537 (3.3922)	- (8.6310)
10	0.0	0.1	- (0.4199)	0.857630 (3.3926)	- (8.6301)
10	0.0	0.2	0.612730 (0.4199)	0.957635 (3.3926)	1.27005 (8.6301)
10	0.1	0.1	0.65800 (0.4198)	2.6499 (6.3924)	1.15065 (8.6310)
10	0.1	0.5	1.30089 (0.4198)	1.30089 (3.3924)	0.69424 (8.6311)

Table 8.3 : Eigenfrequencies $\omega^2 = \left(\frac{r_{OS}^3 R^3 \sigma^2}{GM_0}\right)$ for the fundamental (ω_0^2) and the first (ω_1^2) the pseudo-radial modes of oscillations of differentially rotating and tidally distorted polytropic models

Model No.	N=1.5		N=3.0		N=4.0	
	(ω_0^2)	(ω_1^2)	(ω_0^2)	(ω_1^2)	(ω_0^2)	(ω_1^2)
1.	2.64622 (2.69269)	12.50035 (12.51098)	9.24115 (9.26008)	16.98440 (16.95728)	15.00133	24.73604
2	2.63913 (2.66545)	12.178095 (12.32704)	9.03280 (9.17380)	16.45697 (16.72728)	14.77951	24.33086
3.	2.70309 (2.69226)	12.54682 (12.50688)	9.240501 (9.25933)	16.98125 (16.94656)	14.99890	24.73044
4.	2.63136	12.12323	9.09602	16.61996	14.74029	24.24683
5.	2.68289	12.42983	9.19938	16.88695	14.93174	24.61716
6.	2.69350	12.49290	9.22428	16.94273	14.97307	24.68189

Table 8.4 (a): Eigenfrequencies $\omega^2 = \left(\frac{r_{OS}^3 D^3 \sigma^2}{GM_0}\right)$ of nonradial modes of oscillations of various types of differential rotating and tidally distorted polytropic models of stars with polytropic index 1.5.

Model No.	g_3	g_2	g_1	f	P_1	P_2	P_3
1.	-	-	-	2.01669 2.4558 (0-0)	10.26798 (10.2812) (1-1)	23.46957 (23.4920) (2-2)	41.02976 (41.2378) (3-3)
2.	-	-	-	2.120807 (2.3856)	9.64289 (10.1135)	21.08376 (23.1136)	36.96368 (40.5845)
3.	-	-	-	2.11610 (2.4198)	10.26426 (10.2779)	23.46076 (23.4854)	41.00848 (41.2321)
4.	-	-	-	1.99002	9.51642	21.780768	38.3473
5.	-	-	-	2.04887	9.86694	22.5644	39.6534
6.	-	-	-	2.0773	10.00394	22.95472	40.3310

Table 8.4(b): Eigenfrequencies $\omega^2 = \left(\frac{r_{OS}^3 D^3 \sigma^2}{GM_0}\right)$ of different modes of nonradial oscillations of differential rotating and tidally distorted polytropic models of N = 3.0

Model No.	g_3	g_2	g_1	f	P_1	P_2	P_3
1.	1.8497 (1.8700) (3-3)	2.8296 (2.8500) (2-2)	4.8399 (4.8932) (1-1)	8.0070 (8.2487) (0-0)	14.2672 (15.2517) (1-1)	24.6246 (26.6736) (2-2)	37.98749 (41.3569) (3-3)
2.	1.8296 (1.8636)	2.8370 (2.8400)	4.7898 (4.8729)	7.9408 (8.1748)	14.4533 (15.6271)	25.1679 (26.2351)	38.91084 (40.6576)
3.	1.8475 (1.8700)	2.8383 (2.8600)	4.8538 (4.8936)	8.1910 (8.2480)	15.2319 (15.2533)	26.6520 (26.6655)	41.3223 (41.3435)
4.	1.8400	2.7999	4.8130	7.9724	14.4803	25.2145	39.0588
5.	1.8430	2.8199	4.8227	8.0509	14.7805	25.7928	39.9317
6.	1.8565	2.8499	4.8378	8.1158	14.9817	26.1723	40.5416

CHAPTER - IX

CONCLUDING OBSERVATIONS

In the present thesis we have primarily investigated the effectiveness of the use of the concepts of Roche equipotentials in determining the equilibrium structure and periods of oscillations of rotationally and tidally distorted gas spheres which have relevance in problems of stellar structure. In this chapter we critically review in brief the work done in the earlier chapters and outline the scope for further work in this direction.

9.1 VALIDITY OF SERIES SOLUTION USED IN A ROCHE COORDINATE

In chapter II we have tried to check numerically the validity of series expansion used by Kopal (65) in one of the Roche coordinates which has been used by him and subsequently by Mohan et al. (70, 85, 89, 92) for determining the equilibrium structures of rotationally and tidally distorted stars. Since the analytic expressions are not possible in closed form for all the three Roche Coordinates, series expansions were used in cases where analytic expressions in closed form are not possible. However the convergence of these series expansion could not be analytically established.

Our numerical results presented in Chapter II show that the series expansion (2.7) shows a converging trend as the value of percentage difference between the value computed from (2.2) decreases (except on account of truncation errors in certain cases) as more and more terms are included in its expansion. Even this small percentage difference is expected to reduce further if higher terms are included in the series expansions 2.7 (as has been done by Mohan et al. and us also in certain cases).

9.2 EFFECT OF INCLUDING MASS VARIATION INSIDE THE STARS ON ITS EQUIPOTENTIAL SURFACES IN DETERMINING THE EQUILIBRIUM STRUCTURE OF ROTATIONALLY AND TIDALLY DISTORTED GAS SPHERES

In chapter III we modified approach of Mohan et al. to include effects of mass variation inside the star on its Roche equipotentials to determine more accurately the equilibrium structure of rotationally and tidally distorted gas spheres. Subsequently, we used this approach in chapters IV to VII to determine the equilibrium structures of rotationally and tidally distorted gas spheres of different varieties such as Prasad model, composite models, polytropic models and white dwarf models. Our results have shown that this improvement in analysis modifies to different extents the value of various structure parameters in different cases. However, no specific trend has been observed which could summarize the effects in general except that as expected, the changes are more in case of gaseous spheres which are less centrally condensed compared to gaseous spheres which are more centrally condensed and in whose case Roche approximation is more justified.

9.3 EFFECT OF INCLUDING MASS VARIATON INSIDE THE GAS SPHERE ON ITS EQUIPOTENTIAL SURFACES ON THE PERIODS OF OSCILLATIONS

In chapter VIII we have developed a method for determining eigenfrequencies of radial and nonradial modes of oscillations of rotationally and tidally distorted gas spheres when effect of mass variation inside the star on its equipotential surfaces is included in the analysis. We have also applied this methodology in this chapter to determine the eigenfrequencies of radial modes of composite models with interfaces b=0.3, 0.5 and 0.7 and radial and nonradial modes of oscillations of polytropic models with polytropic indices 1.5 and 3.0. Our results show that even though the values of eigenfrequencies get modified and are now expected to be more accurate, no specific trend is observed in these changes.

9.4 SCOPE FOR FUTURE WORK

In the present thesis our effort has been to develop a methodology with which equilibrium structures and periods of oscillations of rotationally and tidally distorted gas spheres could be determined more accurately. However our results show that with these modifications the analysis becomes too unwieldy. Even though efforts have been made to develop in series form analytic expressions where closed from solutions have not been possible, even these series expansions do not provide any analytic expression which could provide some result of physical significance. It, may, therefore, be of interest to see if instead of developing detailed series expansions of distortion parameters u, v, w, f_P and f_T etc. required in determining the equilibrium structures and periods of oscillations of rotationally and tidally distorted stars as discussed in sections 2.4 of Chapter II and section 8.1 and section 8.2 of chapter VIII direct numerical evaluations of these distortion parameter be done numerically during computations is done. This does not now seem to be a difficult proposition in view of availability of fast computing machines.

While investigating the problems of differentially rotating stars in binary system in chapter V, the companion star has been assumed to be a point mass star whose mass is much smaller than the mass of the primary star. It is also assumed that the axis of rotation is perpendicular to the line joining the mass centers of the two stars. It may be of interest to analyze the problems in which the axis of rotation is not perpendicular but inclined at some angle to the line joining the mass centres and companion star is not assumed to be a point mass.

From the astrophysical view point, it will be worth while to incorporate the present methodology into certain available computer codes for stellar structure and stellar pulsations and apply it to determine the equilibrium models and trace the evolutionary tracks of certain realistic models of differentially rotating stars and stars in binary system.

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