LIMITATIONS IN THE DESIGN OF SYNCHORONOUS MACHINES COUPLED TO DIESEL ENGINES

THESIS SUBMITTED IN PARTIAL FULFILMENT

FOR THE AWARD OF THE DEGREE OF

MASTER OF ENGINEERING

IN

ELECTRICAL MACHINE DESIGN

UNIVERSITY OF ROORKEE
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September 1961

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No less obligations are due to Prof. C. S. Ghosh, Head of the Electrical Engineering Department, University of Roorkee, Roorkee, for giving me a part of his precious time whenever I needed it.

It is the author's personal feeling that the work would have been incomplete without the accurate guidance and personal interests taken by my learned teachers.

CONTENTS

LIST OF SYMBOLS USED

X_d - Synchronous Reactance in Direct Axis

X_{ad} - Armature Reaction Reactance in Direct Axis

X_{al} - Armature Leakage Reactance

X_q - Synchronous Reactance in Quadrature Axis

X_{aq} - Armature Reaction Reactance in Quadrature Axis.

X_d' - Transient Reactance in Direct Axis

X_d'' - Sub-transient Reactance in Direct Axis

X_q'' - Sub-transient Reactance in Quadrature Axis

r_f - Resistance of Field Winding

X_{fl} - Leakage Reactance of Field Winding

r_{Dd} - Damper winding Resistance in Direct Axis

X_{Dd} - Damper Winding Reactance in Direct Axis

r_{Dq} - Damper Winding Resistance in Quadrature Axis

X_{Dq} - Damper Winding Reactance in Quadrature Axis

T_d'' - Short Circuit Sub-transient Time Constant in Direct Axis

T_q'' - Short Circuit Sub-transient Time Constant in Quadrature Axis

T_d - Damper Time Constant

T_I - Inertia Time Constant

P_s - Synchronising Power

P_e - Electrical Power developed

P_a - Accelerating or Retarding Power

P_D - Damping Power

ω - Angular Velocity

CHAPTER I

INTRODUCTION

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INTRODUCTION

With the increasing use of the internal combustion engine in driving electrical machinery and of synchronous motor in connection with the compressors, many engineers have been diverted to the problem of oscillation and resonance in systems of parallel connected synchronous machines. The problem of hunting is not serious in generators when the machines are driven by steam turbines or water-wheels in which the driving torque remains constant during a revolution. In the case of generators driven by reciprocating engines, (steam, gas or oil) the turning moment varies periodically at a fixed frequency during any revolution. The angular magnitude of this (as a result of the imparted nonuniform torque) depends upon the total moment of inertia of rotating parts, and it may be limited by supplying the proper flywheel. If, however, the period of the applied torque is equal to or near the natural frequency of oscillation of the system, then the resulting oscillations become greater and greater; it would become infinitely great if no damping were present. In such a case the magnitude of the imparted torque is of no consequence whatsoever, and the small-

-ent torque suffices to produce violent oscillations. Such pulsations are greatest at heavy loads. When gas or oil engine driven generating sets are operated in parallel, the surge of the energy component of power between the machines may become serious, unless the units are properly designed to reduce this angular shift to as small a amount as feasible. Slow speed units having a large number of poles are more liable to give trouble when operated in parallel, since they have greater number of poles.

In the case of synchronous motors, a reciprocating load such as an air compressor may cause periodic angular changes in the required torque, which may cause trouble with the motor. Also, in the case of synchronous motors driving loads having a uniform rotational torque, hunting may be experienced due to periodic oscillations in the system frequency itself, either by reciprocating prime mover or reciprocating loads on other motors operating through line and transformer impedance. In such cases, a synchronous motor which has been operating properly may suddenly start hunting owing to some new load on the system or to some new reciprocating prime-mover being connected which has the same periodicity in its oscillations as the natural frequency of this particular synchronous motor.

However, here only the case of a synchronous generator coupled to prime-mover whose torque is irregular, will be considered. The results can be easily interpreted in terms of a synchronous motor driving an irregular load.

In this paper an easy method of calculating the size of flywheel necessary for the successful parallel operation of engine driven generator is given. The calculations are derived directly from a knowledge of the resonance frequencies involved.

In general the dissertation is a mathematical treatment of the phenomenon which occur in connection with hunting of a synchronous machine electrically in parallel to an infinite bus and direct connected to reciprocating apparatus. Specific example has been treated which make quite clear not only the nature and causes of hunting, but also the methods of overcoming it.

Most important contribution in this connection, that can be cited, is a paper by R. L. Doherty and E. F. Franklin.[6] This paper, however, deals only with the problem of a synchronous motor direct connected to a compressor and operating from an infinite bus system. The differential equation on which the solution depends has been given by Dr. A. J. Borg[3] and other writers of standard textbooks which deal with the problem of hunting. Theo Schou[9] has published another paper on the same subject which deals with many of the practical problems occuring in connection with the application of synchronous motors to compressors and presents short methods and sets of curves from which the correct size of flywheel can be chosen without much calculation. Among so many other works, papers due to R. L. Doherty[7] and Late J. Mischen-Hinnen[10] are worthy of mention in this connection.

Dr. G. C. Jain[4] (Under whose able guidance this dissertation has been done) has dealt this subject to quite a good extent. The present work is fundamentally based on his lines. Thus the following theory of a single machine operating on an infinite bus and driven by a reciprocating engine is not completely original with me. However, the point of view used in ascertaining the various means and calculating size of flywheel for satisfactory parallel operation is new.

CHAPTER II

ASSUMPTIONS

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ASSUMPTIONS

Before actually taking up the problem, it will be worthwile to state the assumptions and their validity which have been used during the dealing of this problem.

1. The machine is connected to a relatively large power system i.e. only the case of a generator having irregular prime-mover torque operating in parallel with the grid has been dealt with.

2. The effect of armature resistance is negligible. The truthness of this assumption is so well known to electrical engineers that it needs no explanation.

3. The currents are polyphase, balanced sine waves (in time). They can be resolved, therefore, into two complementary polyphase current systems, one in which the current in each individual phase reaches maximum at the instant the axis of the field pole coincides with the axis of the magnetization of the phase under consideration - this is termed the direct component of the current and another in which the current in the same phase reaches maximum one quarter

cycle later, that is, in time quadrature. This is termed the quadrature component.

4. The machine has salient poles.- Cylindrical rotor thus becomes a special case of salient poles in which the synchronous reactance in the direct and quadrature axis become equal.

5. The machine has a short-circuited winding in the quadrature axis, as well as the main field winding in the direct axis. The effect of an amortisseur wind-ing may thus be taken into account.

6. Saturation is negligible - while the results apply strictly only to machines in which magnetic saturation is negligible, nevertheless this does not mean that practical calculations, within practical accuracy, cannot be thus made when saturation is present. Indeed, they are made, just as many other similar calculations are made, by exercising the constants of the machine with respect to the degree and distribution of saturation.

7. The non-uniform torque of the prime-mover consists of, in general, of a steady component with superimposed alternating components. Mathematically it can be expressed as :-

$$a_o + \Sigma (a_n \mathrm{Sin}\ nt + b_n \mathrm{Cos}\ nt) .$$

i.e. the periodically varying part of the torque which produces the hunting of the machine consists of a fundamental and harmonics, these later harmonics having frequencies 2, 3 etc. times that of fundamental. In the present problem only the fundamental will be taken into consideration. Each harmonic may be treated independently and the effect can be added. However, the harmonics of lower order must be watched carefully even when their amplitude is not high, because the oscillations produced by them are in comparison larger to those produced by the higher harmonics. Because, for an instant, assume that the fundamental has the same amplitude as the second harmonic. Then the amplitude of variation of w for the fundamental will be two times as large as for the second harmonic, because the duration of oscillation is two times as long as for the latter. The amplitude of the oscillation of pole-wheel will be four times as large as for second harmonic since the variation of w is two times as large and the time is two times as long.

CHAPTER III

CHARACTERISTICS OF SYNCHRONOUS MACHINES

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CHARACTERISTICS OF SYNCHRONOUS MACHINES

A few characteristics and constants will be reviewed in this chapter which will be further used in subsequent treatment.

Synchronizing Power:-

It is a well known fact that when an alternator is loaded its field structure moves forward with reference to its terminal voltage or the voltage of the bus to which it is connected. Similarly the rotor of a synchronous motor moves backward with reference to its terminal voltage when the rotor is loaded mechanically. In either case the angle by which the rotor is displaced from the terminal voltage is called the angular displacement of the machine or the load angle and is denoted here by the symbol δ. Equation 3.1, shows the variation of power with load angle for a given excitation in the case of a salient pole synchronous machine.

$$P_p = \frac{EV}{X_d}\sin\delta + \frac{V^2}{2}\cdot\frac{X_d - X_q}{X_d X_q}\sin 2\delta \quad \text{-------} 3.1$$

If the output of an alternator connected to a constant potential bus and operating under constant excitation (as given by equation 3.1), is plotted as a function of load angle θ, a curve similar to that shown in fig. 3.1. is obtained.

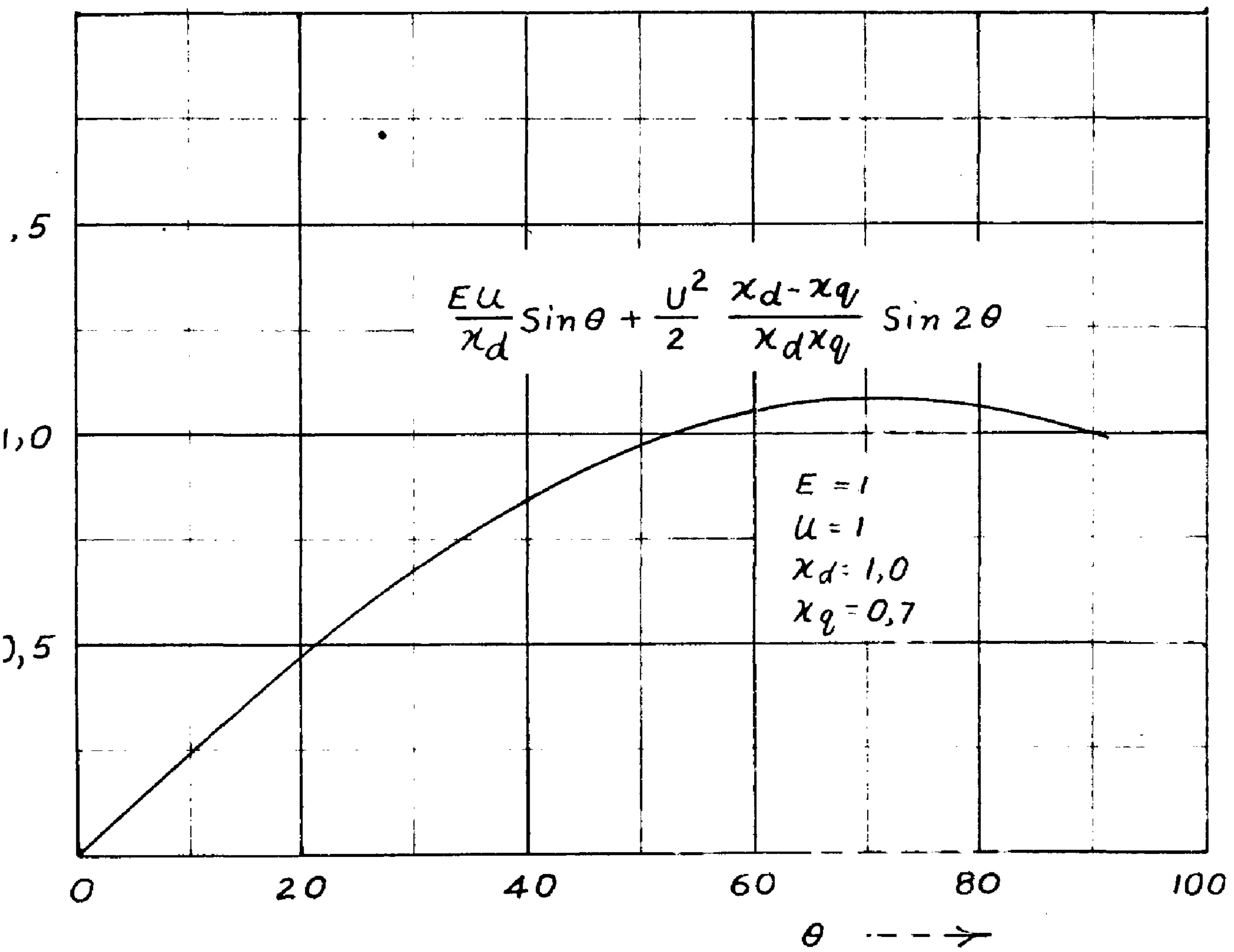

FIG. 3.1.

If due to some external influence the load angle changes, the excitation, however, remaining constant the balance between the prime-mover power and the electrical power is disturbed. Since with an increase in load angle the machine can supply more electrical power and if the prime-mover input remains constant, the machine gets a braking action due to the increased power output and the increase in load angle is checked. This feedback power is equal to the rate of change of electrical power with load angle. It can be seen from the curve that if the rotor oscillates about it's mean position, the instantaneous power changes with the change of rotor position, but the output per degree displacement remains practically constant as long as the oscillations are small. This factor, output per unit displacement or the rate of change of electrical power with load angle is known as the synchronising power and the corresponding torque is called the synchronising torque. It (synchronising power) is denoted by P_{ps}. Thus $P_{ps} = \frac{dP_p}{d\delta}$. This power tries to keep the machine in step with the grid. It should be noted that P_{ps} is different at different loads since the curve cannot be considered a straight line over the whole range. P_{ps} also differs with excitation, terminal voltage, degree of saturation and other factors. The synchronising power is different in steady state and transient operations.

Synchronising Power in Steady State:-

The synchronising power for gradual changes of the load angle can be obtained for a constant excitation by differentiating equation 3.1, with respect to θ.

$$P_p = \frac{EU}{X_d}\sin\theta + \frac{U^2}{2}\,\frac{X_d - X_q}{X_d\,X_q}\sin 2\theta \quad \text{--------.3.1}$$

$$P_{ps} = \frac{dP_p}{d\theta} = \frac{d}{d\theta}\left[\frac{UE}{X_d}\sin\theta + \frac{U^2}{2}\,\frac{(X_d - X_q)}{X_d\,X_q}\sin 2\theta\right]$$

$$\text{Or,}\quad P_{ps} = \frac{EU}{X_d}\cos\theta + U^2\,\frac{X_d - X_q}{X_d\,X_q}\cos 2\theta \quad \text{-------.3.2}$$

If $X_d = X_q$, case of cylindrical rotor

$$P_{ps} = \frac{EU}{X_d}\cos\theta \quad \text{-------- -------.3.3}$$

Figure 3.2 and 3.3 show the variation of synchronising power in turbo-alternators and salient pole machine respectively.

The synchronising power is maximum at $\theta = 0^{\circ}$ and decreases with increase in Load angle.

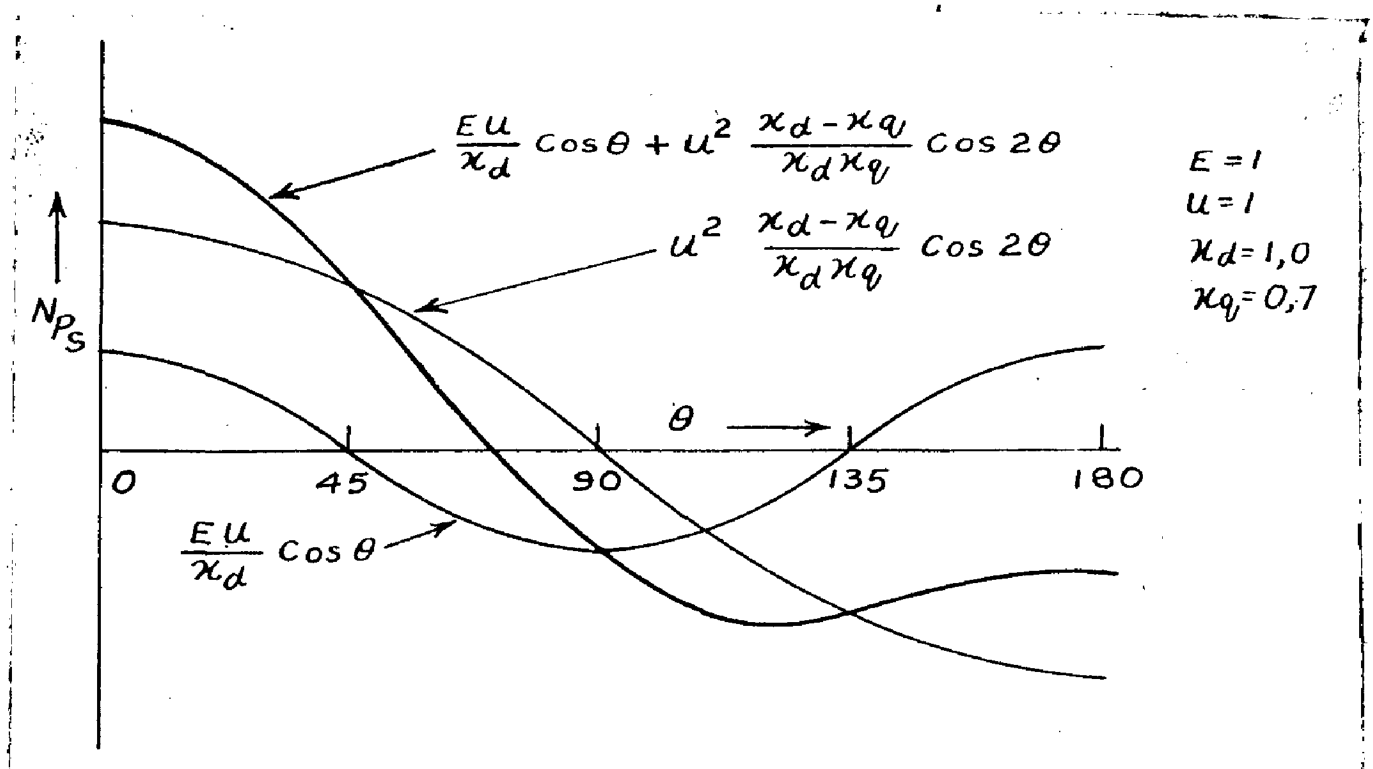

FIG. 3.2 & 3.3.

Knowing N_{ps}, the synchronizing torque T_s, which is defined as the change in the torque per degree change in the displacement of the Ø rotor, can be calculated since the speed is practically constant. Thus

$$T_s = 975 \frac{N_{ps}}{n_s} \text{ in m-Kg.}$$

if N_{ps} is in KW.

and n_s is the synchronous speed in R.P.M.

Damping Power of a Salient-Pole Synchronous Machine at Small slips.

Practically all synchronous motors and engine-driven generators have amortisseur windings or squirrel cages in the pole faces. One of the main purpose of damper windings is to provide positive phase sequence damping which results from the torque caused by the interaction of the damper currents(which are induced whenever the rotor tends to speed up or slow down) with the positive phase sequence i.e. forward rotating magnetic field in the air-gap. The amount of positive phase sequence is greatly increased by reducing the resistance of the damper windings. The following assumptions are made in derivation of the expression for damping power:-

(i) The armature circuit resistance of the synchronous machine is negligible.

(ii) The field winding which is provided only in direct axis has negligibly small resistance as compared to the damper winding in the direct axis.

(iii) Small values of slips are considered.

(iv) Damping power is caused only by the damper winding on the rotor. No other short-circuited iron paths are considered in the derivation of the expression.

In order to derive expressions for the damping power, equivalent circuit diagrams of synchronous machine in asynchronous run are drawn as shown in figures 3.4, and 3.5.

If the applied voltage or the voltage at terminals of the machine be U then the voltage causing flow of current in the direct and quadrature axes U_d and U_q, are respectively U Sin θ and U Cos θ.

(a) Damping Power in direct axis P_{Dd}

The plan of attack is that knowing the component of voltage in the direct axis and the equivalent circuit of a synchronous machine in direct

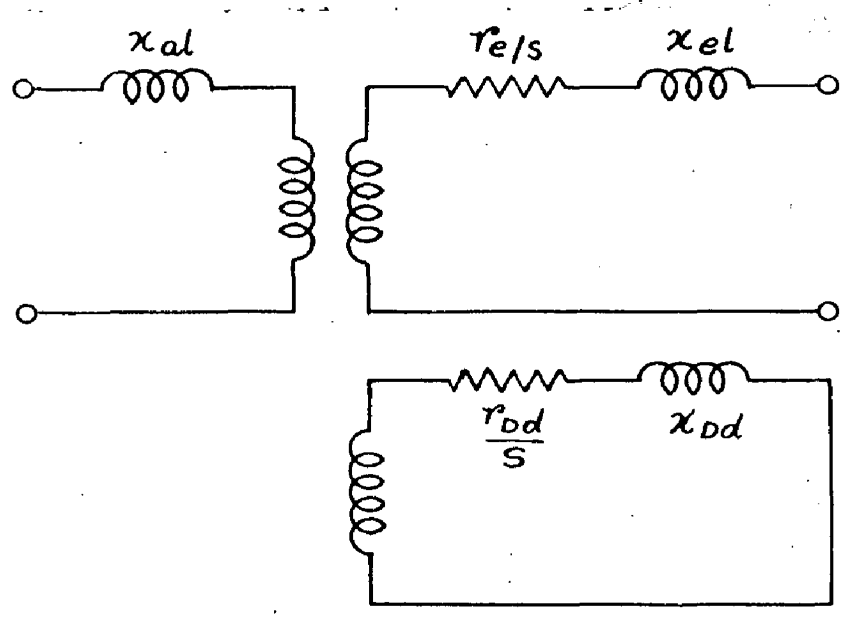

FIG. 3.4.. EQUIVALENT CIRCUIT OF A SYCHRONOUS MACHINE IN DIRECT AXIS UNDER ASYNCHRONOUS RUN.

axis at a slip s, the current flowing in the damper winding in the direct axis is calculated. Then the damping power is calculated as usual according to the theory of the asynchronous machines.

The effective impedance at any slip s at the armature terminals in direct axis is

$$\dot{Z}_{d(s)} = jX_{al} + \cfrac{jX_{ad}\,\cfrac{(r_{Dd}/s + jX_{Ddl})\,jX_{ol}}{r_{Dd}/s + j(X_{Ddl} + X_{ol})}}{jX_{ad} + \cfrac{(r_{Dd}/s + jX_{Ddl})\,jX_{ol}}{r_{Dd}/s + j(X_{Ddl} + X_{ol})}}$$

$$= jX_{al} + \frac{j^2 X_{ad} X_{ol}(r_{Dd}/s + jX_{Ddl})}{jX_{ad}\, r_{Dd}/s + j^2(X_{Ddl} + X_{ol}) X_{ad} + jX_{ol}(r_{Dd}/s + jX_{Ddl})}$$

$$= jX_{al} + \frac{j^2 X_{ad}X_{ol}\left(\frac{r_{Dd}}{s} + j X_{Ddl}\right)}{j\frac{r_{Dd}}{s}(X_{ad} + X_{ol}) + j^2(X_{Ddl}X_{ad} + X_{ol}X_{ad} + X_{ol}X_{Ddl})}$$

$$= \frac{j^2 \frac{r_{Dd}}{s}(X_{al}X_{ad} + X_{al}X_{ol} + X_{ad}X_{ol}) + j^3(X_{Ddl}X_{ad}X_{al} + X_{Ddl}X_{ol}X_{al} + X_{ol}X_{ad}X_{al} + X_{ad}X_{ol}X_{Ddl})}{j\frac{r_{Dd}}{s}(X_{ol} + X_{ad}) + j^2(X_{ad}X_{Ddl} + X_{ad}X_{ol} + X_{ol}X_{Ddl})}$$

This calculation assumes that the value of r_c/s at very small value of slip is comparatively very small.

The armature current in the direct axis at any particular slip s is given by

$$\dot{I}_{d(s)} = \frac{U_d}{\dot{Z}_{d(s)}}$$

$$= U_d \cdot \frac{j\,\frac{r_{Dd}}{s}(X_{ad}+X_{ol}) + j^2(X_{ad}X_{Dd1}+X_{ad}X_{ol}+X_{Dd1}X_{ol})}{j^2\frac{r_{Dd}}{s}(X_{ad}X_{al}+X_{al}X_{ol}+X_{ad}X_{ol}) + j^3(X_{ad}X_{ol}X_{Dd1} + X_{al}X_{ad}X_{Dd1} + X_{Dd1}X_{ol}X_{al} + X_{al}X_{ad}X_{Dd1})}$$

The portion of this total armature current in the rotor circuit

$$i_{r(s)} = I_{d(s)} \cdot \frac{jX_{ad}}{jX_{ad} + \dfrac{(r_{Dd}/s + jX_{Dd1})\,jX_{ol}}{r_{Dd}/s + j(X_{Dd1}+X_{ol})}}$$

or

$$i_r(s) = i_d(s)\frac{jX_{ad}\left[r_{Dd}/s + j(X_{Dd1}+X_{ol})\right]}{j\,\frac{r_{Dd}}{s}\left[X_{ad}+X_{ol}\right] + j^2\left[X_{ad}X_{Dd1}+X_{ad}X_{ol}+X_{Dd1}X_{ol}\right]}$$

Substituting the value of $I_{d(s)}$ in the above equation:-

$$i_{r(s)} = \frac{U_d}{D}\, jX_{ad}\left[\frac{r_{Dd}}{s} + j\,(X_{Dd1}+X_{ol})\right]$$

where

$$\therefore D = j^2 \frac{r_{Dd}}{s} (X_{al}X_{ad} + X_{al}X_{ol} + X_{ad}X_{ol}) + j^3 (X_{al}X_{ad}X_{Ddl} + X_{Ddl}X_{ol}X_{al} + X_{al}X_{ad}X_{ol} + X_{ad}X_{ol}X_{Ddl})$$

The portion of the rotor current flowing in the direct axis damper winding

$$i_{Dd(s)} = i_{r(s)} \frac{j X_{ol}}{r_{Dd}/s + j(X_{Ddl} + X_{ol})}$$

Substituting the value of $i_{r(s)}$ in the above equation

$$i_{Dd(s)} = U_d \cdot \frac{1}{D} \cdot j^2 X_{ol} X_{ad}$$

Substituting the value of D in this equation

$$i_{Dd(s)} = U_d \frac{j^2 X_{ol} X_{ad}}{j^2 \frac{r_{Dd}}{s}(X_{al}X_{ad} + X_{al}X_{ol} + X_{ad}X_{ol}) + j^3 (X_{al}X_{ad}X_{Ddl} + X_{Ddl}X_{ol}X_{al} + X_{al}X_{ad}X_{ol} + X_{ad}X_{ol}X_{Ddl})}$$

Using the definition of T_d'' (Short-circuit Sub-transient time constant in the direct axis)

$$T_d'' = \frac{X_{al}X_{ad}X_{Ddl} + X_{Ddl}X_{ol}X_{al} + X_{al}X_{ol}X_{ad} + X_{ad}X_{ol}X_{Ddl}}{r_{Dd}(X_{al}X_{ad} + X_{al}X_{ol} + X_{ad}X_{ol})}$$

$$i_{Dd(s)} = U_d \cdot \frac{j^2 X_{ol} X_{ad}}{j^2 \frac{r_{Dd}}{s}(X_{al}X_{ad} + X_{al}X_{ol} + X_{ad}X_{ol})(1 + j s T_d'')}$$

According to the theory of the asynchronous machines, the damping power due to the direct axis damper winding:-

$$- P_{Dd} = i_{Dd}^2(s) \; \frac{r_{Dd}}{s} \cdot (1 - s)$$

Since s is very small (1 - s) can be taken as equal to 1.

Therefore,

$$P_{Dd} = i_{Dd}^2(s) \cdot \frac{r_{Dd}}{s}$$

Substituting the value of $i_{Dd(s)}$ in the above equation-

$$P_{Dd} = U_d^2 \frac{X_{ol}^2 \, X_{ad}^2}{(\frac{r_{Dd}}{s})^2 (X_{al}X_{ad}+X_{al}X_{ol}+ X_{ad}X_{ol})^2 (1+s^2T_d''^2)} \cdot \frac{r_{Dd}}{s}$$

Further,

$$\frac{X_{ol}^2 \; X_{ad}^2}{\frac{r_{Dd}}{s} \, (X_{al}X_{ad}+X_{al}X_{ol} + X_{ad}X_{ol})^2} = (\frac{1}{X_d''} - \frac{1}{X_d'}) sT_d''$$

Hence,

$$P_{Dd} = U_d^2 \; (\frac{1}{X_d''} - \frac{1}{X_d'}) \frac{sT_d''}{1 + s^2 \, T_d''^2} .$$

or, $$P_{Dd} = U^2 \sin^2\theta (\frac{1}{X_d''} - \frac{1}{X_d'}) \frac{sT_d''}{1 + s^2 \, T_d''^2} .$$

Since s is small,

$$s^2 \; T_d''^2 \rightarrow 0$$

and therefore

$$P_{Dd} = U^2 \sin^2 \theta \left(\frac{1}{X_d''} - \frac{1}{X_d'}\right) sT_d'' \text{ ------3.4.}$$

(b) **Damping Power in the Quadrature Axis**

Following the same procedure as in finding the damping power in direct axis, damping power in the quadrature axis can be found.

The effective impedance at any slip s in the quadrature axis at the armature terminals is:-

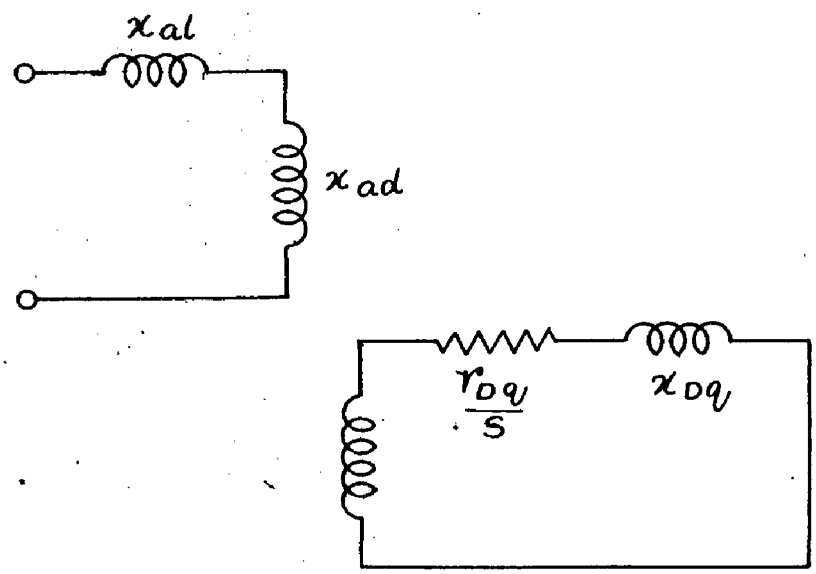

FIG.3.5. EQUIVALENT CIRCUIT OF A SYCHRONOUS MACHINE IN QUDRATURE AXIS UNDER ASYNCHRONOUS RUN.

$$\dot{z}_q(s) = jX_{al} + \frac{jX_{aq}\left(\frac{r_{Dq}}{s} + jX_{Dql}\right)}{\underline{r_{Dq}} + j(X_{aq} + X_{Dql})}$$

Therefore the armature current flowing in the quadrature axis at slip s is,

$$\dot{I}_{q(s)} = \frac{U_q}{\dot{Z}_{q(s)}}$$

or

$$\dot{I}_{q(s)} = U_q \cdot \frac{\frac{r_{Dq}}{s} + j(X_{aq} + X_{Dql})}{jX_q \cdot \frac{r_{Dq}}{s} + j^2(X_{al}X_{aq} + X_{al}X_{Dql} + X_{aq}X_{Dql})}$$

The portion of this armature current flowing in the quadrature axis damper winding

$$i_{Dq(s)} = \dot{I}_{q(s)} \frac{j\,X_{aq}}{\frac{r_{Dq}}{s} + j(X_{aq} + X_{Dql})}$$

Substituting the value of $\dot{I}_{q(s)}$ in the above equation

$$i_{Dq(s)} = U_q \cdot \frac{j\,X_{aq}}{jX_q \cdot \frac{r_{Dq}}{s} + j^2(X_{al}X_{aq} + X_{al}X_{Dql} + X_{aq}X_{Dql})}$$

Using the definition of T_q''

$$T_q'' = \frac{X_{al}X_{aq} + X_{al}X_{Dql} + X_{aq}X_{Dql}}{r_{Dq}}$$

Therefore,

$$i_{Dq(s)} = U_q \quad \frac{X_{aq}}{X_q(1 + j\,sT_q'')\, r_{Dq}/s}.$$

The damping power due to quadrature axis damper winding is, according to the theory of asynchronous machine :-

$$W_{Dq} = i_{Dq(s)}^2 . \frac{r_{Dq}}{s} (1 - s)$$

If s is very small $(1 - s)$ is approximately equal to 1 and hence,

$$W_{Dq} = i_{Dq(s)}^2 . \frac{r_{Dq}}{s}$$

or, $$W_{Dq} = U_q^2 \frac{X_{aq}^2}{X_q^2 (1 + s^2 T_q''^2)} . \frac{1}{(\frac{r_{Dq}}{s})^2} . \frac{r_{Dq}}{s}$$

Further,

$$\frac{X_{aq}^2}{X_q^2} . \frac{1}{r_{Dq}/s} = (\frac{1}{X_q''} - \frac{1}{X_q}) sT_q''$$

Hence,

$$W_{Dq} = U_q^2 (\frac{1}{X_q''} - \frac{1}{X_q}) \frac{sT_q''}{1 + s^2 T_q''^2}$$

Since s is very small, $s^2 T_q'' \to 0$

and therefore,

$$W_{Dq} = U_q^2 (\frac{1}{X_q''} - \frac{1}{X_q}) sT_q''$$

or, $$W_{Dq} = U^2 \cos^2 \theta \; (\frac{1}{X_q''} - \frac{1}{X_q}) sT_q'' \qquad \text{........3.5.}$$

The total damping power,

$$W_D = W_{Dd} + W_{Dq}$$

Substituting the values of P_{Dd} and P_{Dq},

$$P_D = V^2 \left[\left(\frac{1}{X_d''} - \frac{1}{X_d'}\right) T_d'' \sin^2\delta + \left(\frac{1}{X_q''} - \frac{1}{X_q}\right) T_q'' \cos^2\delta \right] s$$

--------..3.6.

If the various constants and also the voltage are expressed in P.U., P_D is obtained in P. U. of rated KVA.

$$s = \frac{1}{\omega} \cdot \frac{d\delta}{dt} \quad \text{where} \quad \omega = 2\pi f$$

when $s = 0$, $P_D = 0$, the damping power is zero when the machine runs at synchronous speed.

when $\delta = 0$ and $\delta = 180^\circ$

$$P_D = V^2 \left(\frac{1}{X_q''} - \frac{1}{X_q}\right) T_q'' s$$

when $\delta = 90^\circ$, and $\delta = 270^\circ$

$$P_D = V^2 \left(\frac{1}{X_d''} - \frac{1}{X_d'}\right) T_d'' s$$

Figure 3.6 shows the variation of damping power with the load angle, the slip s being the parameter. It can be seen that these curves have approximately V-shape and minimum damping power is obtained at $\delta = 90^\circ$. It is obvious from the curves that the major contribution is that of the quadrature axis damper winding and therefore a machine having full damper winding will have relatively high values of damping power.

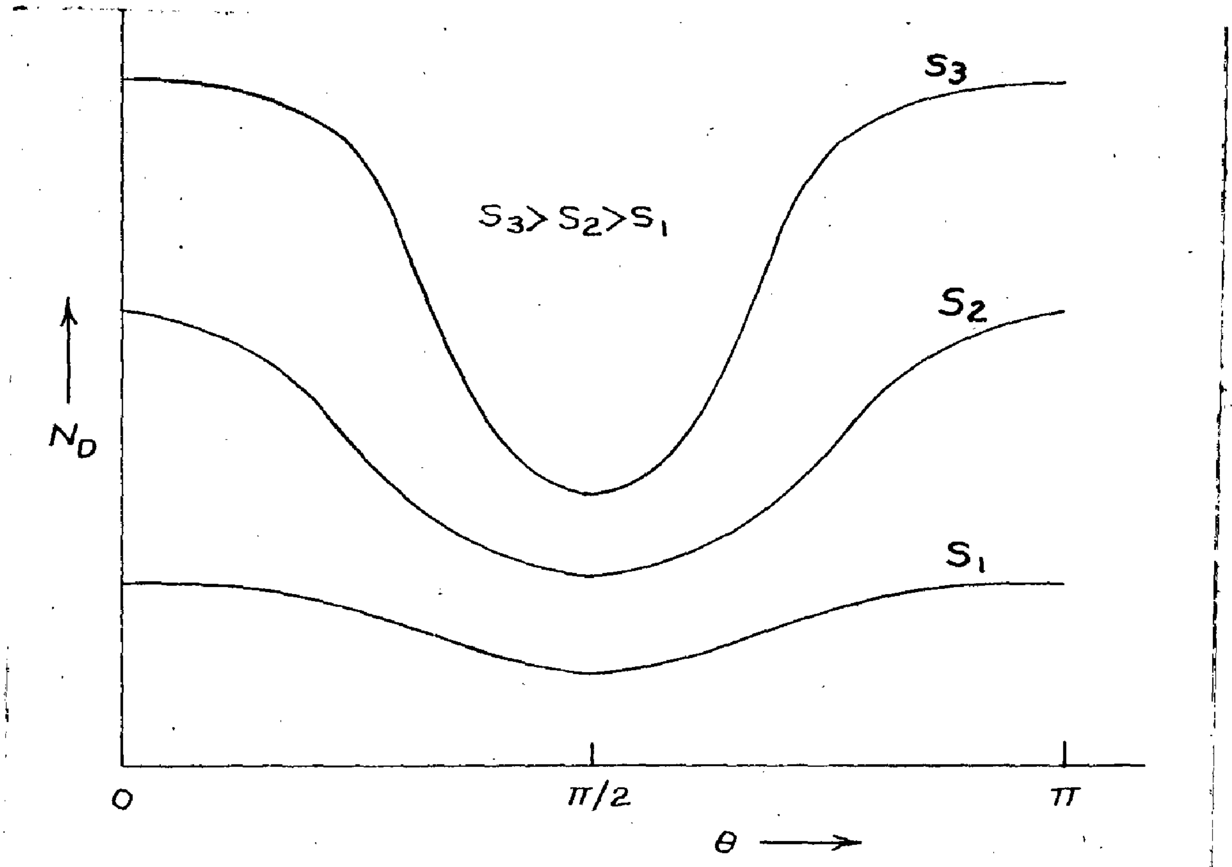

FIG. 3.6.

(c) Polar Mass Moment of Inertia of Rotor ($GD^2/4$)

To evaluate this the rotor can be conside-red to be made up of two parts:-

(i) the circular mass such as the pole yoke of flywheel rim, and

(ii) the radial parts such as spider legs or spokes and the poles with their copper etc,.

(i) To determine mass moment of inertia of a ring:-

Let

l = axial length of the ring in cms.

g	=	981 cm. per second per second
W	=	weight of ring material in grammes per cubic cms.
w	=	2 X R. P. S.
t	=	time in seconds
R_1 and R_2	=	Radii in cms. as shown in figure 3.7.

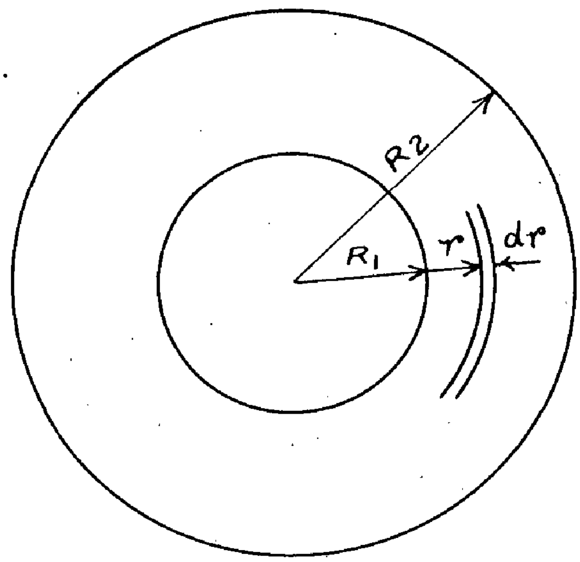

FIG. 3.7.

The moment at any point r is then

$$r^2 \left[\frac{2 \pi r l w (dr)}{g} \right] \quad gm - cm^2$$

The moment for all points of a ring __is__.

$$= \frac{2 \pi l W}{g} \int_{R_1}^{R_2} r^3 dr = \frac{2 \pi l W}{4g} (R_2^2 - R_1^2)$$

To this may be added $\Sigma \frac{W_2}{g} r_1^2$ for the poles or other concentrated masses, where

W_2 = the total weight of the mass considered in gms.

r_1 = the distance from the centre of gravity of the mass to the centre of the shaft, in cms.

The polar mass moment of inertia for a rotor of several rings and other parts can be expressed then as,

$$\frac{GD^2}{4} = \Sigma \frac{2 \pi l w}{4g} (R_2^2 - R_1^2) \Sigma \frac{W_2}{g} r_1^2 \qquad \text{-------.3.7.}$$

This is in gm-cm.^{2} To convert it into tons-meter^2 it should be multiplied by $\frac{1}{4 \times 981 \times 2240 \times 100}$ factor.

CHAPTER IV

OPERATION OF A SINGLE GENERATOR HAVING PRIME-MOVER TORQUE IRREGULAR

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OPERATION OF A SINGLE GENERATOR HAVING PRIME-MOVER TORQUE IRREGULAR

The irregular torque of the prime-mover acting upon a synchronous generator operating independently i.e. supplying its own load along, causes the velocity of the pole-wheel to vary between a maximum and a minimum.

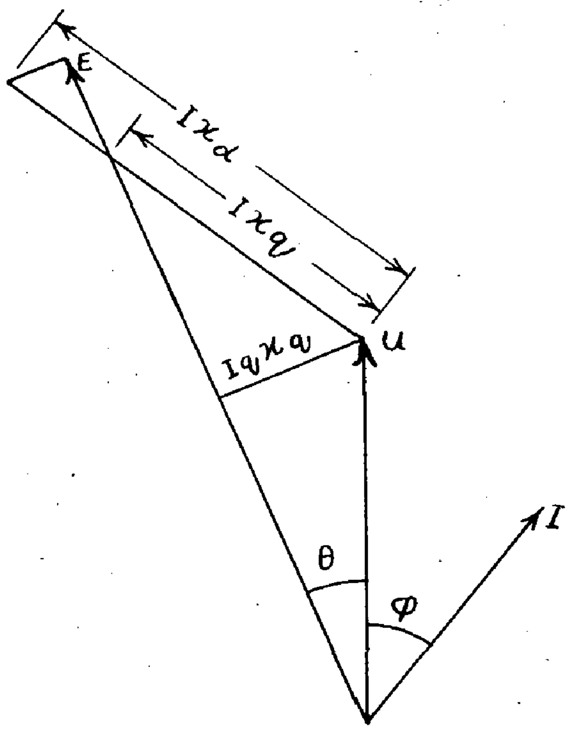


FIG. 4.1. VECTOR DIAGRAM OF A SALIENT POLE SYNCHRONOUS MACHINE.

A uniformly driven machine has a constant speed of the rotor and hence a fixed position of the vector E representing the induced E.M.F. in the armature circuit. In relation to the pole structure operating with the constant mean velocity of the irre-

-gularily driven structure, the later is either leading or lagging in relation to the vector E of the uniformly driven machine ; the vector E of the irregularily driven machine hunts to the right and to the left through a fixed angle. The voltage vector U will hunt together with the e.m.f. vector E, because in case of generator supplying it's load directly the voltage vector U is fixed by the induced vector E. Thus the load angle θ will remain constant. Since the synchronizing power

$$N_{P_s} = \frac{dN_p}{d\theta}$$

no synchronizing power exists if θ remains constant. It is otherwise also true that a synchronous generator operating individually or separately possesses no synchronising power.

If the influence of the change in speed of the machine on the line load is ignored, the surplus torque accelerates the machine and the deficiency torque retards it; both produce a change in it's velocity relative to it's mean velocity.

The power equation true for all instant is

$$N_A = N_G + N_p \qquad \text{--------..4.1.}$$

where

N_A = Prime-mover Power

N_P = Electrical power drawn from the generator.

N_θ = Accelerating or Retarding power

$$N_\theta = \frac{T_I}{w} \cdot \frac{d^2\theta}{dt^2}$$

where T_I = Inertia Time Constant(See Appendix)

The surplus power resulting in N_θ is

$= N_A - N_P$

If $N_A - N_P$ can be represented by a pulsating power with the angular frequency w_1 and having an amplitude N_1,

$$N_A - N_P = N_1 \text{ Sin } w_1 t,$$

where N_1 is in P.U. of N_n

Therefore,

$$\frac{T_I}{w} \cdot \frac{d^2\theta}{dt^2} = N_1 \text{ Sin } w_1 t$$

or, $$\frac{d^2\theta}{dt^2} = \frac{N_1 w}{T_I} \text{ Sin } w_1 t \quad \text{------------..4.2.}$$

Integrating the expression 4.2. with respect to time

$$\frac{d\theta}{dt} = \frac{N_1 w}{T_I w_1} \text{ Cos } w_1 t \quad \text{------------..4.3.}$$

is the variation in velocity or speed. The total variation between highest and lowest speed is given by:-

$$2 \triangle w = \frac{2M_1}{T_I} \cdot \frac{w}{w_1}$$

The ratio of the total variation in speed to the mean value of the speed is known as the deg--ree of angular irregularity or the co-efficient of cyclic irregularity.

$$\delta = \frac{2 \triangle w}{w} = \frac{2M_1}{T_I} \cdot \frac{w}{w_1} \cdot \frac{1}{w} .$$

$$\delta = \frac{2M_1}{T_I} \cdot \frac{1}{w_1} \qquad \text{---------.. 4.4.}$$

Satisfactory parallel operation requires a co-efficient of cyclic irregularity that does not exceed a certain fixed value. This may be larger in a machine which operates alone than in one which operates in parallel with other synchronous machines.

Integrating again the equation 4.3, with respect to time, we obtain,

$$\theta = \frac{M_1 w}{T_I \, w_1^2} \cdot \sin w_1 t$$

or, $$\theta_{max} = \frac{M_1 w}{T_I \, w_1^2} \qquad \text{---------.. 4.5.}$$

Substituting the value of δ from equation 4.4, in the maximum value equation,

$$\theta_{max.} = \frac{\delta}{2} \cdot \frac{v}{v_1} = \frac{\delta f}{2 \alpha_1}$$

where α_1 is the number of impulses per sec.

Substituting

$$f = \frac{np}{120} \quad \text{where } P \text{ is number of poles}$$

$$\text{and} \quad \alpha_1 = \frac{ni}{60}$$

where i is the number of impulses per revolution.

$$\theta_{max.} = \frac{\delta}{2} \cdot \frac{np}{120} \cdot \frac{60}{ni}$$

$$= \frac{\delta}{4} \cdot \frac{P}{i} \cdot \text{radians}$$

or, θ_{max} (measured in degrees)

$$= 57.3 \; \frac{\delta}{4} \cdot \frac{P}{i}$$

CHAPTER V

OPERATION OF A GENERATOR IN PARALLEL TO THE GRID WHEN THE PRIME-MOVER TORQUE OF THE GENERATOR IS IRREGULAR

CHAPTER V.

OPERATION OF A GENERATOR IN PARALLEL TO THE GRID WHEN THE PRIME-MOVER TORQUE OF THE GENERATOR IS IRREGULAR

The case of a synchronous generator operating in parallel with a constant voltage constant frequency bus is entirely different from that of the singly operated generator supplying its own load. In this case the terminal voltage of the grid and the machine is common, and therefore hunting of the machine i.e. the to and fro motion of the pole structure due to forced oscillations by the prime-mover, results in a change in the load angle since the voltage vector remains constant. The variation in the load angle calls for a synchronizing torque. This

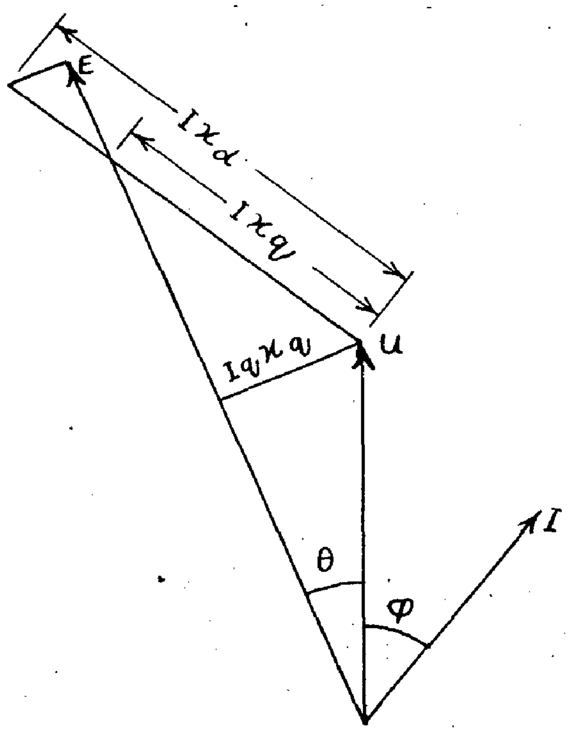


FIG. 5.1. VECTOR DIAGRAM OF A SALIENT POLE SYNCHRONOUS MACHINE.

synchronising torque is defined as the machine torque per unit angle. For a change in the load angle from θ_m to $\theta_m + \theta$ where θ_m corresponds to the mean po-sition of the pole structure, the change in machine power is $M_{p_s}\theta$. The surplus or the deficient powe-r supplied by the prime mover is used in accelarating or retarding the polewheel, in balancing the synchronising and damping powers. Therefore,

$$M_e + M_D + M_P = M_O + \Sigma\,(M_1 \mathrm{Sin}\; nw_1t + M_2 \mathrm{Cos}\; nw_1t)$$

$$M_e = \frac{T_I}{w} \cdot \frac{d^2\theta}{dt^2}$$

$$M_D = \frac{1}{S_n \cdot w} \cdot \frac{d\theta}{dt} \quad \text{(See Appendix)}$$

$$M_P = M_{p_s} \cdot \theta.$$

Expressing M_1 in per unit, M_D and M_P being already in per unit,

$$\frac{T_I}{w} \cdot \frac{d^2\theta}{dt^2} + \frac{1}{S_n w} \cdot \frac{d\theta}{dt} + M_{p_s}\theta = M_O + \Sigma(M_1 \mathrm{Sin}\; nw_1t + M_2 \mathrm{Cos}\; nw_1t)$$

or

$$\frac{d^2\theta}{dt^2} + \frac{1}{T_I S_n}\,\frac{d\theta}{dt} + M_{p_s}\theta \cdot \frac{w}{T_I} \cdot \theta = \frac{w}{T_I}\left[M_O + \Sigma(M_1 \mathrm{Sin}\; nw_1t + M_2 \mathrm{Sin}\; nw_1t)\right].$$

Neglecting the transient, the solution for θ consists of two parts, one due to constant term N_0 and the other due to series

$$\Sigma \left(N_1 \operatorname{Sin} n w_1 t + N_2 \operatorname{Cos} n w_1 t \right)$$

The first part of this is the average displacement of the rotor due to the load under which the machine is operating. The second part represents the hunting or the variations in the angular displacement due to the harmonics in it.

Only the second part is of practical interest as it is this part upon which the power pulsation depends and consequently it is this part which determined whether satisfactory operation can be obtained in any particular case.

However, as has been stated in chapter II, assumption no. 7, only the fundamental will be taken here and therefore the power equation reduces to (ignoring the constant term also)

$$\frac{d^2\theta}{dt^2} \quad \frac{1}{T_I\, S_n}\,\frac{d\theta}{dt} + N_p P_s \cdot \frac{w}{T_I} \cdot \theta = \frac{N_1 w}{T_I} \operatorname{Sin} w_1 t \quad \text{..........2.1.}$$

Substituting the definitions of T_D and w_e (See Appendix)

T_D = Damper Time constant

= $2S_n T_I$

w_e = Natural Angular frequency of Oscillation

= $2\pi f_e = \sqrt{\dfrac{wN_{ps}}{T_I}}$ (See Appendix)

$$\frac{d^2\theta}{dt^2} + \frac{2}{T_D}\frac{d\theta}{dt} + w_e^2\theta = \frac{N_1 w}{T_I}\operatorname{Sin} w_1 t$$

or,

$$\left(P^2 + \frac{2}{T_D}P + w_e^2\right)\theta = \frac{N_1 w}{T_I}\operatorname{Sin} w_1 t \text{ --------5.3.}$$

:Solution of equation 5.3:

The transient part of the solution which corresponds to the oscillations of the machine when it is first switched-on to the bus, is of little importance, as these oscillations quickly die out. It can, however, be calculated quite easily in a simple case such as this.

The permanent hunting is easily obtained by putting $p = jw$.

In order to show the effect of the damper winding the solution of the above equation will be carried out in two stages. In the first stage it shall be assumed that there is no damper winding provided on the machine and in the second stage the effects of the damper windings will be considered.

Solution of equation 5.3, without damper-winding:

Ignoring any other damping paths, the damping power is zero when the damper winding is absent i.e.

$$\frac{1}{S_n} \frac{1}{w} \frac{d\theta}{dt} = 0$$

The equation 5.3, then becomes:-

$$(p^2 + w_e^2)\,\theta = \frac{N_1 w}{T_I} \text{Sin } w_1 t \quad \text{---------..5.4}$$

Putting $p = jw_1$

$$(-w_1^2 + w_e^2)\,\theta = \frac{N_1 w}{T_I} \text{Sin } w_1 t$$

or,
$$\theta = \frac{N_1 w}{T_I} \frac{\text{Sin } w_1 t}{w_e^2 - w_1^2}$$

The maximum value of θ is,

$$\theta_{max} = \frac{N_1 w}{T_I} \cdot \frac{1}{w_e^2 - w_1^2}$$

$$= \frac{N_1 w}{T_I w_1^2} \frac{1}{\left[-1 + \left(\frac{w_e}{w_1}\right)^2\right]} \quad \text{-----. 5.5.}$$

It should be noted that the maximum load angle obtained by operating the synchronous machine without damper winding in parallel with the grid is different from the maximum load angle obtained in the individual operation; the prime mover being pul-sating in both the cases, which is simplified by assuming a sinusoidal variation of the prime-mover power.

If θ_{max_1} be the maximum load angle in individual operation and θ_{max_2} be the maximum load angle in grid operation of a machine having no damp-er winding, then with the help of equations 4.5 and 5.5, we obtain :-

$$\epsilon = \frac{\theta_{max_2}}{\theta_{max_1}} = \frac{1}{1-\left(\frac{w_e}{w_1}\right)^2} \quad \text{--------..5.6.}$$

The factor ϵ is called the amplification factor or the modulus of resonance. It gives the ratio of the maximum load angle in a machine contain-ing mass or inertia and the synchronizing force to the maximum load angle in a machine in which only mass is present. The amplication factor ϵ depends upon the ratio $\frac{w_e}{w_1}$ where,

$$w_e = 2\pi\alpha_e$$

and $w_1 = 2\pi\alpha_1$

Since θ_{max_1} given by equation 4.5 has a value depending upon $\frac{M_{1V}}{T_1 \, w_1^2}$ it has a constant fin--ite magnitude.

Therefore,

$$\theta_{max_2} = \theta_{max_1} \cdot \frac{1}{1 - (\frac{w_e}{w_1})^2}$$

$$= \theta_{max_1} \cdot \epsilon.$$

Since θ_{max_1} is a constant, the amplica-tion factor ϵ is therefore directly a measure of magnitude of oscillations which appear in parallel operation.

From equation 5.6 it is clear that as the natural frequency of the machine and the frequency of the forced oscillations of the prime-mover approa--ch each other, the amplication factor becomes great, till if $w_e = w_1$, resonance occurs and the ampli-cation factor, theoretically, becomes infinite.

Therefore to avoid the effects of resonance very large values of maximum load angle, the natural frequmncy of the machine and the frequency of the forced oscillations of the prime mover must be diffe--rent.

Further,

$$\frac{w_e}{w_1} = \sqrt{\frac{P_s}{T_I}} \cdot \frac{1}{w_1}$$

and

$$\left(\frac{w_e}{w_i}\right)^2 = \frac{wN_{p_s}}{T_I} \cdot \frac{1}{w_i^2} \quad \text{--------..5.7}$$

Thus $\left(\frac{w_e}{w_i}\right)$ can be easily varied by varying T_I since the synchronizing power N_{p_s} can be varied only within small limits.

Solution of Equation 5.3 with damper windings

Equation 5.3, is

$$\left(p^2 + \frac{2}{T_D} \cdot p + w_e^2 \right) \theta = \frac{N_i w}{T_I} \operatorname{Sin} w_i t$$

Putting $p = jw_i$, gives

$$\left(-w_i^2 + j\frac{2}{T_D} w_i + w_e^2\right) \theta = \frac{N_i w}{T_I} \operatorname{Sin} w_i t$$

or,

$$\theta = \frac{\frac{N_i w}{T_I} \operatorname{Sin} w_i t}{\left(-w_i^2 + j\frac{2}{T_D} \cdot w_i + w_e^2\right)}$$

$$= \frac{\frac{N_i w}{T_I} \operatorname{Sin} w_i t}{\left(w_e^2 - w_i^2\right) + j\frac{2}{T_D} \cdot w_i}$$

The maximum value is

$$\theta_{max} = \frac{\frac{N_i w}{T_I}}{\sqrt{\left[\left(w_e^2 - w_i^2\right)^2 + \left(\frac{2 w_i}{T_D}\right)^2\right]}} .$$

Or,

$$\theta_{max} = \frac{N_1}{T_I}\,\frac{v}{v_1^{\,2}}\,\frac{1}{\sqrt{\left[1-\left(\frac{v_e}{v_1}\right)^2\right]^2+\left[\frac{2}{T_D v_1}\right]^2}}$$

Taking the maximum load angle in this case to be θ_{max_3}

$$\theta_{max_3} = \theta_{max_1}\,\frac{1}{\sqrt{\left[1-\left(\frac{v_e}{v_1}\right)^2\right]^2+\left[\frac{2}{T_D\,v_1}\right]^2}}$$

If $\varepsilon_D = \theta_{max_3}/\theta_{max_1}$

where ε_D is called the amplification fact--or or modulus of resonance of a synchronous machine with damper winding. Thus ε_D gives the ratio of the maximum load angle in a machine containing mass or inertia, the damping power and the synchronizing force, to the maximum load angle in a machine in which only mass is provided.

Therefore,

$$\varepsilon_D = \frac{1}{\sqrt{\left[1-\left(\frac{v_e}{v_1}\right)^2\right]^2+\left[\frac{2}{T_D\,v_1}\right]^2}} \quad \text{----.5.8.}$$

For a given value of v_e and v_1, $\varepsilon_D < \varepsilon$ i.e. the amplication factor with damper winding is lower than the one without damper winding. Therefore damper winding results in a reduction of hunting.

CHAPTER VI

DESIGN LIMITATIONS

CHAPTER VI

DESIGN LIMITATIONS

Hence it has been seen that when an alt--ernator is driven by a reciprocating engine which dev--elops, inherently, a periodically varying turning eff--ort or driving force, there exists as a natural result a possibility of unstable operation of alternator - a possibility that the natural frequency at which the rotor tends to oscilate may be equal to or very near the periodic variations of the driving force. This condition of resonance will cause swinging, or'Hunting' of the rotor. The amplitude of such rotor oscillations, as measured by the maximum displacement of the rotor from it's stable position (the position of uniform rotation) is determined by two factors, namely,

(i) the magnitude of the variation in angular velo-city itself(the result of periodically varying driving force working against the practically constant resisting force of load and friction), and

(ii) the proximity to a condition of resonance.

A large amplitude of swing might be produced by a small periodic variation in angular velocity, if the natural frequency of the swing is very near the frequency of the variation. As to the question of design of new units, it is possible to determine N_{p_s} from the design of alternator. This makes it possible to design the flywheel by the natural frequency consideration (The natural frequency should not be in neighbourhood of frequency of the engine variations or the impulses). However, a complete investigation will be made using equation 6.8, which involves damper winding also as one of the factor to control the amplication factor. But, as will be seen later, the flywheel design could be more convenient and enough accurate from the natural frequency consideration alone.

We had in the last chapter, the amplication factor with damper winding as,

$$e_D = \frac{1}{\sqrt{\left[1 - \left(\frac{w_e}{w_1}\right)\right] + \left[\frac{2}{T_D\, w_1}\right]}} \quad \text{----6.8.}$$

The angular velocity of the prime-mover is given by:-

$$w_1 = 2\pi C_e = \frac{2\pi}{t_1} = \frac{2\pi na}{60}$$

Where, n = speed in revolutions per minute

a = number of impulses per revolution

t_1 = time between two impulses

'a' is given in the following table:-

Single Cylinder, 4-cycle gas engine	a = 1/2
Two-Cylinder, 4-cycle gas engine (Cranks to-gather)	a = 1
Four-Cylinder, 4-cycle gas engine (Cranks at 180°)	a = 2
Single Cylinder and tandem steam engines	a = 2
Twin Engines (Cranks at 90°)	a = 4
Triple expansion engines (Cranks at 120°)	a = 6

The question arises as to the curve for hunt-ing in a synchronous machine.

Examining the various terms in equation 5.8, it is easy to see that $w_1 = 2\pi n_1$ is constant for a particular type of driving reciprocating engine.

$$w_e = \sqrt{\frac{w\, E_{p_s}}{T_I}} \qquad \text{(See Appendix)}$$

Here $E_{p_s} = \dfrac{dE_p}{d\theta}$ is the slope of the (E_p, θ) curve. For the turbogenerator, this slope is inversely proportional to the direct axis synchronous reactance X_d of the machine and is nearly so for salient pole machine.

$$E_{p_s} = \frac{dE_p}{d\theta} = \frac{UE}{X_d} \cos\theta$$

Thus a decrease in the value of X_d has the effect of increasing the value of M_{p_0}. Now,

$$X_d = \frac{1}{S.C.R.}$$

S.C.R. = Short circuit Ratio

$$\therefore \quad M_{p_0} = U.B.\ (S.C.R.).\ \cos\theta$$

Therefore M_{p_0} can be increased by increasing the short circuit ratio of the machine i. e. by increasing the length of the air gap. But since this will make the machine costly, M_{p_0} can be varied within small limits.

The inertia time constant T_I is

$$T_I = \left(\frac{v}{60}\right)^2 \frac{n^2\ GD^2\ (tm^2)}{M_n\ (KVA)}\ \text{Secs.}$$

Therefore, variation in T_I is possible by changing the moment of inertia of the moving parts. With a proper choice of the moment of inertia, n_0 is so chosen that it does not lie in the neighbourhood of n_I.

Also

$$T_D = 2 S_n T_I \qquad \text{(See Appendix)}$$

where S_n is the slip at which the damping power corresponds to the rated K.V.A. of the machine.

Damping power can be increased by lowering the effective resistance of the amortisseur bars, or by providing them in a machine where no damping wind--ing is present.

Thus the remedies of hunting lie in,

(i) Addition of the damper winding or cha--nge in the resistance of any damper which is already provided.

(ii) Change in the air gap of the machine.

(iii) Change in the moment of inertia of the rotating parts by providing a suitable flywheel is the most common method of attacking the problem.

In the subsequent discussion a neumerical example will be taken to show how the moment of inertia of the flywheel can be estimated for stable operation.

Consider a machine with following specifications:-

300 K.V.A., Cos ϕ = 0.8, 3150 - Volts

55 A, 50 c/s, 375 RPM.

Moment of inertia = 3.5 Tons meter^2

Equation 5.8, gives the amplication factor with damper windings as,

$$\delta_D = \frac{1}{\sqrt{\left[1 - \left(\frac{v_e}{v_1}\right)^2\right]^2 + \left[\frac{2}{T_D\, v_1}\right]^2}}$$

where,

$$v_e = \sqrt{\frac{w\, N_{P_s}\ (KVA)}{T_I}}$$

$$N_{P_s} \quad \frac{UE}{X_d} \operatorname{Cos} \phi$$

$$T_I = \left(\frac{w}{60}\right)^2 \frac{n^2 \ GD^2\ (tm^2)}{N_n\ (KVA)} \text{ Secs}$$

and $T_D = 2\, \delta_n\, T_I$

Taking $N_{P_s} = 1$ P.U.

$$v_e = \sqrt{\frac{w N_{P_s}}{T_I}} = \sqrt{\frac{2\pi \times 50 \times 300}{T_I}}$$

$$T_I = \left(\frac{w}{60}\right)^2 \frac{(375)^2 \times 3.6 \times 4}{300}$$

$$= 18 \text{ Secs.}$$

$$v_e = \sqrt{\frac{314 \times 300}{18}}$$

$$= 72.3$$

Taking

$$S_n = 0.025$$

$$T_D = 2 \times 0.025 \times 18$$

$$= 0.9 \text{ Sec.}$$

Assuming a Four-cylinder, 4 cycle gas engine (Cranks at 180°)

$$a = 2.$$

Therefore,

$$w_i = \frac{2\pi\ na}{60}$$

$$= 2\pi . \times \left(\frac{2 \times 375}{60}\right)$$

$$= 78.5.$$

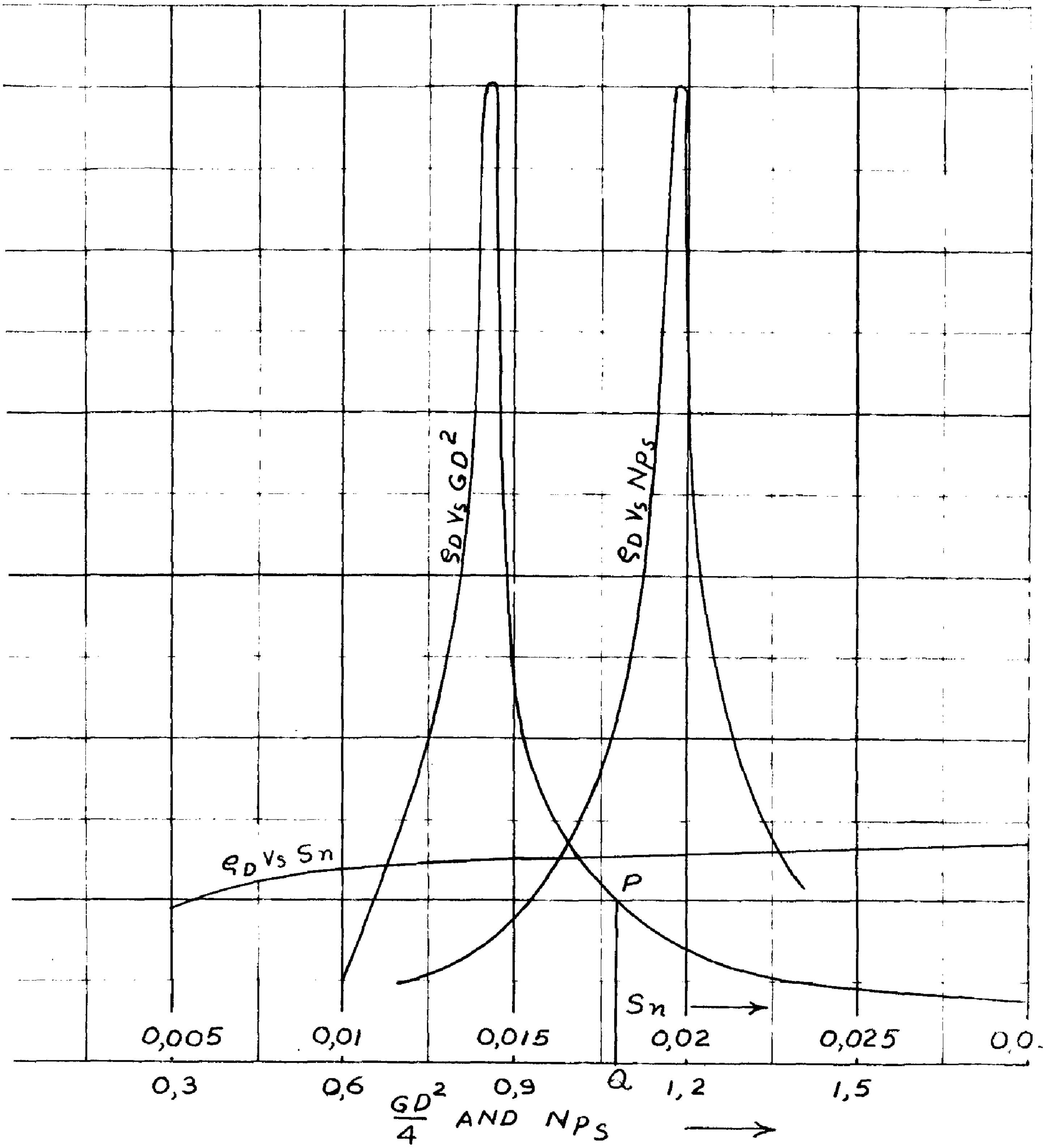
$$\theta_D = \frac{1}{\sqrt{\left[1 - \left(\frac{72.3}{78.5}\right)^2\right]^2 + \left[\frac{2}{0.9 \times 78.5}\right]^2}}$$

$$= \frac{1}{\sqrt{\left[0.0225 + (0.0283)^2\right.}}$$

$$= \frac{1}{\sqrt{0.0225 + 0.0008}}$$

$$= \frac{1}{\sqrt{0.0233}} = 6.55$$

Now the curves showing the variations of θ_D with variation in GD^2, N_{ps} and S_n separately are



ρ_D Vs GD^2
ρ_D Vs N_{PS}
ρ_D Vs S_n
P
S_n →
0,005
0,01
0,015
0,02
0,025
0,0
0,3
0,6
0,9
Q
1,2
1,5
$\frac{GD^2}{4}$ AND N_{PS} →

FIG. 6.1.

(i) Variation of e_D as a function of $\frac{GD^2}{4}$ Taking the given values of moment of inertia as unity.

Table I.

$\frac{GD^2}{4}$	T_I	T_D	v_e	e_D
0.6	10.8	0.54	93.2	2.42
0.8	14.4	0.72	80.8	14. 4
0.85	15.3	0.765	78.5	30. 0
1.0	18.0	0.9.	72.3.	6.55
1.2	21.6	1.08	66.0	3.44
1.4	25.2	1.26	61.0	2. 5
1.6	28.8	1.44	57.1	2.13
1.8	32.4	1.62	53.9	1.89

(ii) Variation of e_D as a function of N_{ps}

Table II.

N_{ps}	v_e	e_D
0.7	60.5	2.42
0.8	64.7	3.12
0.9	68.6	4.23
1.0	72.3	6.55
1.1	75.8	13. 2
1.2	79.3	29. 8
1.3	82.5	9.42
1.4	85.6	5. 2

(iii) Variation of ε_D as a function of δ_n

Table III

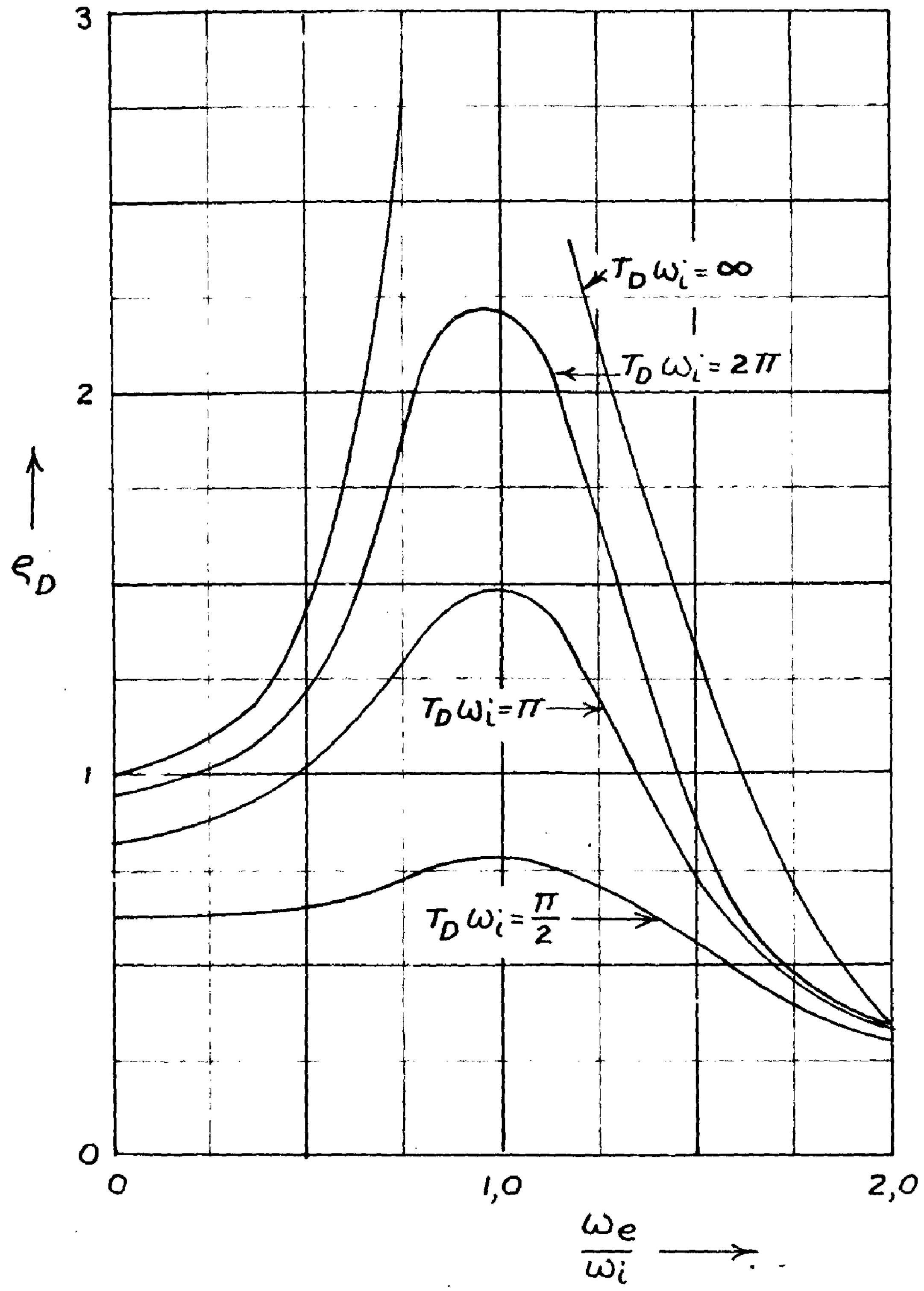
δ_n	T_D	ε_D
0.0	0.0	0.0
0.005	0.18	4.85
0.01	0.36	6.03
0.015	0.54	6.39
0.02	0.72	6.49
0.025	0.9	6.55
0.03	1.08	6.63

The three curves are shown in figure 6.1.

It can be seen from the curves of ε_D Vs., δ_n that the variation of values of δ_n effects ε_D, to a very small extent.

In practice the variation of N_{p_s} is feasi--ble only within small limits. Therefore, the moment of inertia is only left to adjust the value of the ampli--fication factor.

Normally, the amplification should not be more than five in any case. With such a limitation in



3
2
1
0
ϱ_D
0
1,0
2,0
$\frac{\omega_e}{\omega_i}$
$T_D \omega_i = \infty$
$T_D \omega_i = 2\pi$
$T_D \omega_i = \pi$
$T_D \omega_i = \frac{\pi}{2}$

FIG. 6. 2.

the value of the amplification factor a line such as PQ can be always drawn which limits the minimum value of moment of inertia of rotating parts which should be used for satisfactory operation. Knowing the moment of inertia of the rotor, the flywheel can be choosen.

It should be noted that as the value of moment of inertia goes below the point Q, the amplification factor rises greatly.

Fig. 6.2 shows variations in G_D as a function of $\frac{v_o}{v_1}$, T_D v_1 being the parameter. It should be noted that the amplification factor is greatest for $\frac{v_o}{v_1} = 1$. Experience has shown that if the natural frequency of the unit is at least 20 percent different from the frequency of the periodic impulses of the engine, there will be no trouble from resonant hunting. The critical frequencies to be avoided are:-

For a four cycle engine ; particularly one-half the revolution of the crank, but also the revolution of the crank.

For a two cycle engine ; particularly the revolutions of the crank but also twice the revolutions of the crank.

It will be noted that the lowest critical frequency in either case is the cam shaft revolutions.

As an illustration, the danger zones of natural frequency for a generator to be driven by a twin tandem, double acting, four cycles, 200 R. P.M. diesel engine is 80 to 120, and 160 to 240, periods per minute. For a two-cycle engine running at the same speed, the danger zones would be 160 to 240, and 320 to 480.

Turning now to the permissible periodic displacement of the rotor due to variations in angular velocity, it has been seen that, regardless of how a displacement is produced, it will cause a proportional flow of energy current. If a current I_o corresponding to the power W_{p_o}, flows at a displacement of one electrical radian, then for one degree the current will be:-

$$I_o/57.3$$

Since the permissible value of such a pulsating current is properly based on normal rated current of the alternator, the permissible number of degrees displacement is related to W_{p_o}. For older type of the steam - engine driven units which were put in service before the days of the voltage regulators, and which, therefore, had close voltage regulation (Large value of W_{p_o} as compared to the normal rating), the limit was set at ± 2.5 electri-

-cal degrees. But for modern units, designed for use with regulators and especially the single (Maximum) rated generators for use with internal combustion engines, N_{p_s} is much smaller - of the order of 1.4 to 2.0 times the normal rating. For these machines the permissible angle has been increased to ± 3 degrees, which would give, in the case of N_{p_s} = 1.5 normal rating, a pulsating current,

$$\frac{3 \times 1.5 \cdot I_n}{57.3} = 0.079\ I_n$$

where I_n = normal current. If the generator was operating at a power factor lower than unity (and most generators are operated under that condition)the pulsations would be some what reduced, because there will be very little variation in the wattless component.

Hence the flywheel must fulfill two conditions; it must limit the periodic variation in angular velocity so that the resulting displacement will not exceed ± 3 electrical degrees, and at the same time must give a natural frequency 20 percent different from the frequency of the engine variations. That is, if it works out that the flywheel effect which is required to limit the displacement to ± 3 degrees(electrical), gives a dangerous natural frequency, then the flywheel must be increased to remove the natural frequency from

Based on the fact that the natural frequency should be about 20 percent different from the frequency of the engine variations, the amplification factor, for the worst case when neglecting the effect of damper windings, becomes

$$G_D = \frac{1}{\sqrt{\left[1 - \left(\frac{w_n}{w_1}\right)^2\right]^2}}$$

$$= \frac{1}{\left[1 - \left(\frac{0.8}{1}\right)^2\right]} = \frac{1}{0.36}$$

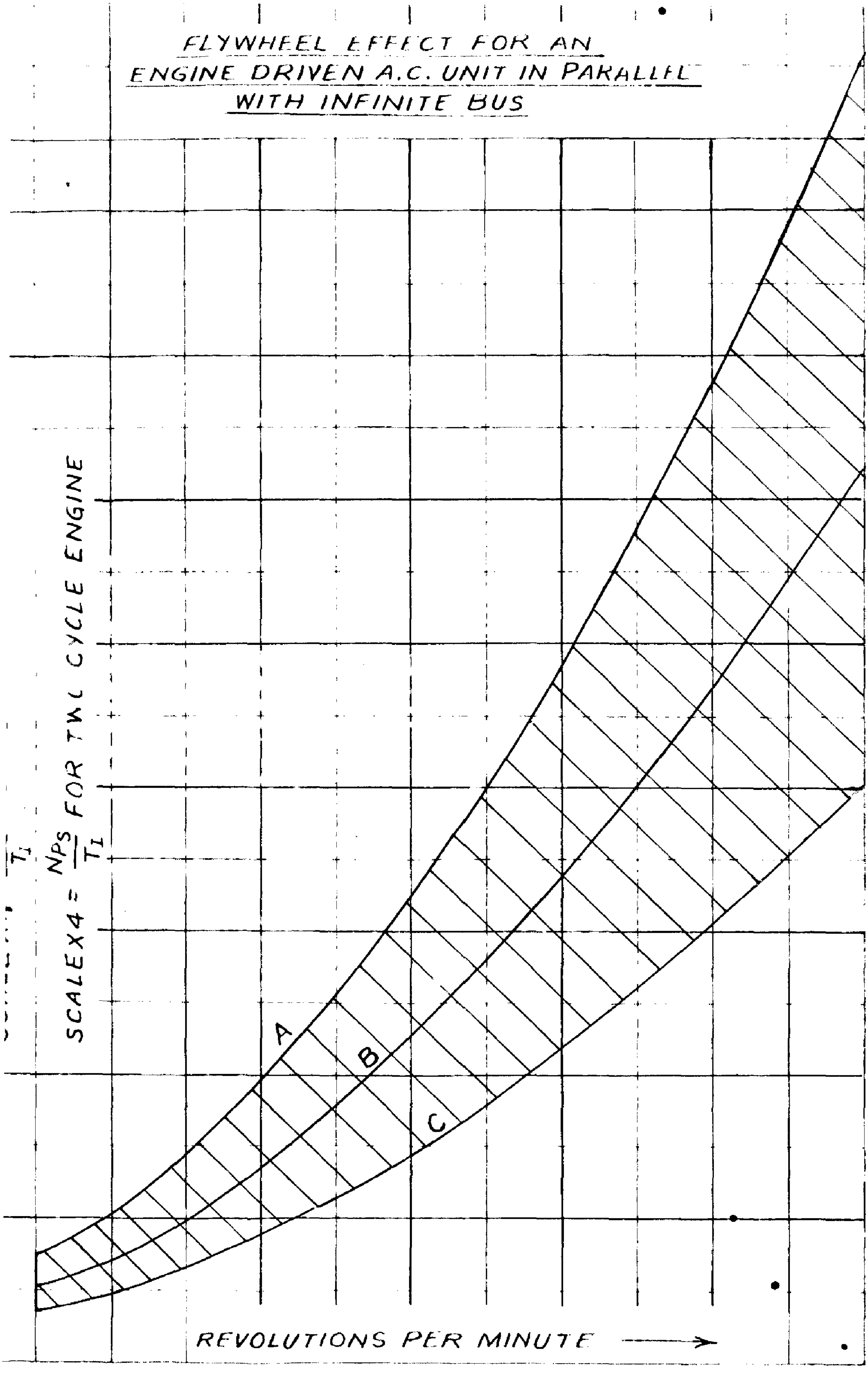
$$= 2.8$$

Curves have been drawn in fig. 6.3 showing the values of M_{p_0}/T_I for natural frequencies 20 percent above the lowest engine impulse frequency which is the cam-shaft revolutions, for 60 c/s alternator driven by either a two or four-cycle engine. We had the natural frequency given by,

$$w_o = \sqrt{\frac{wM_{p_0}}{T_I}}$$

Or, $$\frac{M_{p_0}}{T_I} = \frac{w_o^2}{w} = \frac{w_o^2}{2\pi f} = \frac{w_o^2}{2\pi \cdot 60}$$

$$= \frac{w_o^2}{377}$$


FLYWHEEL EFFECT FOR AN
ENGINE DRIVEN A.C. UNIT IN PARALLEL
WITH INFINITE BUS
SCALE X 4 = NPS/TI FOR TWO CYCLE ENGINE
A
B
C
REVOLUTIONS PER MINUTE →

$$w_i = \frac{2\ v.\ na}{60}$$

where n = revolutions per minute.

a = 1, for four cycle engine, and

= 2, for two cycle engine.

Hence the scale for two cycle engine on N_{p_s}/T_I axis will be 4 times that of for the 4-cycle engine; the same curves being valid for both.

Table IV

R. P. M n	w_i a = 1 (4-cycle engine)	6 N_{p_s}/T_I $w_e = 1.2 w_i$	$w_e = w_i$	$w_e = 0.8 w_i$
0	0	0	0	0
80	8.4	0.384	0.267	0.171
100	10.5	0.504	0.35	0.224
120	12.6	0.72	0.5	0.32
140	14.7	0.994	0.69	0.442
160	16.75	1.28	0.89	0.57
180	18.7	1.6	1.11	0.71
200	21.0	2.02	1.4	0.895
220	23.0	2.44	1.69	1.08
240	25.1	2.9	2.01	1.29
260	[illegible].2	3.4	2.36	1.51
280	29.3	3.94	2.74	1.76
300	31.4	4.52	3.14	2.01

The upper and lower curves give respectively the required value of E_{p_D}/Z_X for a natural frequency 20 percent above and 20 percent below the lowest engine impulse frequency, which is the cam-shaft revolutions. The middle curve gives the critical value which will make the natural frequency equal the cam shaft revolutions.

However, the factor of flywheel effect is not all that must be considered for satisfactory parallel operation. It is essential for good parallel operation that the adjustments of feeding and igniting mechanisms, when once made properly, do not change. Poor operation would naturally be expected if the adjustments were bad to the extent of giving an enormous difference in work of the different cylinders.

With the above considerations in the design of new units, the parallel operation would be quite satisfactory, without having to take any additional precaution in the design of the generator; however, use of rotors with low resistance amortisseur winding, which dampens any tendency to oscillate by consuming as loss the energy of oscillation, will be further advisable in view of any remaining unfavourable condition.

APPENDICES

APPENDIX I

INERTIA TIME CONSTANT

From the laws of Mechanics we know that,

$$T = I \frac{d\omega}{dt}$$

where,

T = Torque

I = Moment of Inertia

ω = Angular velocity

The power required to accelerate the rotor from a speed zero to a speed corresponding to synchr--onous speed n_0 of the machine is given by

$$\int \Delta W = \frac{I\omega_0}{t} \int_0^{\omega_0} d\omega$$

where ω_0 is the angular velocity corr--esponding to the synchronous speed n_0.

Therefore,

$$W = \frac{I \omega_0^2}{t}$$

where,

$$I = \frac{GD^2}{4g}$$

Therefore,

$$W = \frac{GD^2}{4g} \cdot \frac{\omega_0^2}{t}$$

Or, $$P = \frac{2.73 \times 10^{-3} \times GD^2(tm^2)\ n_s^2\ (rpm)^2}{t\ (sec.)}$$

Or, $$t_{(sec)} = \frac{2.73 \times 10^{-3} .\ GD^2(tm^2)\ n_s^2\ (rpm)^2}{P_{KW}}$$

If $P_{KW} = P_n$ K.V.A.

$t = T_I$, where T_I is the inertia time constant.

The inertia time constant of the synchronous machine is defined as the time in seconds required by the rotor to accelerate from stand-still to synchronous speed, if an accelerating power P_c K.W. = P_n K.V.A., is applied to the rotor.

$$T_I = \frac{2.73 \times 10^{-3} .\ GD^2(tm^2).n_s^2(rpm)^2}{P_n(KVA)} \text{sec.}$$

APPENDIX IX

FREQUENCY OF NATURAL OR FREE OSCILLATIONS

Any sudden change in load is accompanied by oscillations. Since in such a case there is no impressed force the frequency with which the system oscillates under such conditions is known as natural frequency of oscillations. This corresponds to a mechanical system consisting of a flywheel and a spring, where the flywheel is twisted and then allowed to oscillate freely. The equation of power oscillations can be written as

$$P_a + P_D + P_p = 0 \quad \text{..............(1)}$$

when load is thrown - off.

Now,

$$P_a = \frac{T_J}{\omega} \cdot \frac{d^2\theta}{dt^2}$$

and

$$P_D = \left[U^2 \sin^2\theta \left(\frac{1}{X_d''} - \frac{1}{X_d'}\right) T_d'' + U^2 \cos^2\theta \left(\frac{1}{X_q''} - \frac{1}{X_q'}\right) T_q''\right] s$$

Taking the mean values of $\sin^2\theta$ and $\cos^2\theta$ we have

$$\sin^2\theta = \cos^2\theta = 1/2$$

$$N_D = \frac{U^2}{2}\left[\left(\frac{1}{X_d''} - \frac{1}{X_d'}\right) T_d'' + \left(\frac{1}{X_q''} - \frac{1}{X_q'}\right) T_q''\right] S$$

If S_n is the value of the slip resulting in a damping power corresponding to rated power $N_n = 1$ PU

Then,

$$\frac{N_D}{N_n} = \frac{S}{S_n}$$

Or, $N_D = N_n \frac{S}{S_n} = \frac{S}{S_n}$ in P.U.

Therefore,

$$N_D = \frac{S}{S_n} = \frac{1}{S_n\, w} \quad \frac{d\theta}{dt}$$

Further for small gradual changes,

$$N_p = \frac{EU}{X_d} \cdot \sin\theta + \frac{U^2}{2} \cdot \frac{X_d - X_q}{X_d \; X_q} \sin 2\theta$$

and the synchronising power,

$$N_{p_s} = \frac{dN}{d\theta} \quad \frac{EU}{X_d} \cdot \cos\theta + U^2 \frac{X_d - X_q}{X_d X_q} \cos 2\theta$$

Or, $dN_p = N_{p_s} \cdot d\theta$

For small changes in load angles N_{p_s} is constant and hence,

$$N_p = N_{p_s} \times \theta$$

Therefore, substituting the values of N_θ, N_D and N_a in equation (1),

$$-\frac{T_j}{w}\frac{d^2\theta}{dt^2} - \frac{1}{S_n w}\frac{d\theta}{dt} + P_{s0}\,\theta = 0 \text{ ---- (2)}$$

This is a homogeneous linear differential equation with constant coefficients.

$$\frac{d^2\theta}{dt^2} + \frac{1}{S_n T_j}\frac{d\theta}{dt} + P_{s0}\cdot\frac{w}{T_j}\cdot\theta = 0 \text{ ------ (3)}$$

Equation (3) represents simple oscillations. If the total change in load angle be θ_0 then the solution of equation (3) for variation of load with time is,

$$\theta(t) = \theta_0\, e^{-t/T_D}\cos(w_0 t + r) \text{ ------ (4)}$$

Which shows that there are damped oscilla- tions and where the value of T_D is the damping time constant

$$T_D = 2S_n T_j$$

and w_0 the angular velocity of free oscillations or the number of oscillations in 2π seconds.

$$w_0 = 2\pi f_0$$

where f_0 is the frequency of free osci- llations

$$w_0 = 2\pi f_0 = \sqrt{\frac{w P_{s0}}{T_j}} \text{ ---- (5)}$$

If T_0 be the time for one free oscillation,

$$T_0 = 2\pi/\omega_0$$

Equation (6) shows that the natural frequency of oscillations solely depends upon the synchronising power and the moment of inertia.

If in equation (4) $\varepsilon = \varepsilon_0 e^{-T_0/T_D}$ gives the ratio of the amplitudes of two waves or oscillations, following each other,

$e^{-T_0/T_D} = e^{-L}$ where L is the logarithmic decrement of the damped oscillations.

$$L = \frac{T_0}{T_D} = \frac{2\pi}{\omega_0} \cdot \frac{1}{2S_n T_I}$$

$$= 2\pi\sqrt{\frac{T_I}{\omega P_s}} \cdot \frac{1}{2S_n T_I}$$

$$\text{Or,} \quad L = \frac{\pi}{S_n \sqrt{\omega T_I P_s}}$$

In diesel generators it is customary to give give the number of oscillations per minute,

$$\omega_0/\text{minute} = \frac{60}{2\pi}\sqrt{\frac{\omega P_s}{T_I}}$$

$$\omega = 2\pi f$$

and, $T_1 = \left(\frac{\pi}{60}\right)^2 \cdot \frac{n^2 \cdot GD^2}{N_n} \, 8000.$

Therefore,

$$\omega_o/min = \frac{60}{2\pi}\sqrt{\frac{2\pi f \; N p_o}{\left(\frac{\pi}{60}\right)^2 \frac{n^2 \; GD^2}{N_n}}}$$

Or,

$$\omega_o/min = \frac{60}{2\pi}\sqrt{\frac{60^2 \times 2\pi f \times N p_o \times N_n}{\pi \; GD^2 \; n^2}}$$

Or,

$$= \frac{60 . 60}{2\pi}\sqrt{\frac{2}{\pi}} \cdot \frac{1}{n}\sqrt{\frac{f N p_o . N_n}{GD^2}}$$

Or,

$$= \frac{457}{n}\sqrt{\frac{f N p_o . N_n}{GD^2}}$$

In the case of large synchronous machines the number of free oscillations per minute lies between 60 to 120.

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