OPERATION OF INDUCTION MACHINE WITH SINGLE AND DOUBLE UNBALANCES

By

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CERTIFICATE

CERTIFIED that the dissertation entitled "OPERATION OF INDUCTION MACHINE WITH SINGLE AND DOUBLE UNBALANCES" which is being submitted by Mr. TILAK RAJ SAHNI in partial fulfilment for the award of the Degree of MASTER OF ENGINEERING in ELECTRICAL MACHINE DESIGN is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

It is further to certify that he has worked for a period of 1½ years from February, 1963 to July, 1964, for preparing this dissertation for the degree of Master of Engineering of the University of Roorkee, Roorkee.

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CHAPTER I

INTRODUCTION

Problem of unbalanced operation of induction machine has been of great interest to a large number of authors (1, 3, 4). Methods of symmetrical components have been the chief tool for studying this problem. The unbalanced operation can be divided into two categories:

(i) Single Unbalance i.e. impedance unbalance on either side of air-gap.

(ii) Double Unbalance i.e. impedance unbalance on both sides of air-gap.

The principle of super-position always makes it possible to extend the analysis of the above two categories of unbalance to the still more general case where stator voltage unbalance is also present simultaneously. For this reason the voltage unbalance has not been considered in separate category of unbalance.

The single unbalance on stator side has been analysed thoroughly by many authors. The single unbalance on rotor side may be caused accidently by any of two possibilities given below:-

(i) The resistance introduced in rotor circuit for starting purpose might not be short-circuited properly during acceleration and thus leaving unbalanced resistance in the rotor circuit.

(ii) Lead from a slip-ring might be open.

Intentional rotor unbalance has been also suggested as a means of speed control.(4) While a machine is operating with unbalanced rotor impedance due to either accidentally or intentionally, it is possible that stator may simultaneously become unbalanced due to any of the following reasons:-

(i) One of the links on stator side might be weak.

(ii) Fuse of one line might blow-off.

(iii) The resistance introduced in stator circuit for limiting voltage across stator might be unbalanced.

The problem of double unbalance is thus of practical interest. Its importance is further enhanced by the fact that it is expected that under double unbalance excessive voltages may be present in stator and rotor circuits, which may harm the insulation and moreover saturate the magnetic circuit to a high degree.

Double unbalance has been dealt with by a few authors only (1 abd 7). In the present thesis analysis has been carried out for double unbalance with the help of symmetrical components. Expressions for steady torque, r.m.s. currents and voltages, and peak voltage have been derived. No solution is possible in case of general impedance unbalance. However, a solution can be obtained for the case of symmetrical unbalance i.e. when impedance is symmetrical about one of lines in both stator and rotor circuits. The equivalent circuits have been presented for symmetrical double unbalance. A Table is given from which equivalent circuit for any specific case can be derived.

A study has been carried out for variation of steady torque, r.m.s. currents and voltages, and peak voltages with motor speed for specific practical cases of double unbalance. Stator and rotor resistances have been taken into account in the study, while the other authors (7) have neglected these; but of-course iron losses have not been considered. Since both stator and rotor contain currents having infinite series of harmonic components, the effect of variation of resistance with frequency has been taken into account.

Because of a number of assumption made in the analysis, experimental verification of theoretical analysis is presented. The analysis of single unbalances follows simply from the general analysis of double unbalance by introducing the necessary circuit constraints. Expressions for peak pulsating torques for single stator or rotor unbalance have been derived. Their dependence on the motor speed has been studied. The low frequency pulsating torque in case of single rotor unbalance presents a serious practical problem on account of possible resonance with mechanical parts of motor-load system.

CHAPTER II

INDUCTION MACHINE WITH SIMULTANEOUS STATOR AND ROTOR IMPEDANCE UNBALANCE

2.1. A general unbalanced operation of an induction machine occurs when unbalanced impedances are connected in stator as well as rotor circuits as shown in Fig. 2.1. The voltage applied across the stator terminals may also be unbalanced. The analysis of this general unbalance is carried out under the following assumptions:-

(i) The machine itself has balanced impedances in both stator and rotor circuits. An unbalance is caused by the addition of external impedances in these circuits.

(ii) The resistance representing iron loss component in the equivalent circuit is considered to be much greater than magnetizing reactance and has therefore been omitted. More over the iron loss varies with the frequency and it is difficult to calculate accurately for the wide range of frequencies, which are present in the machine.

(iii) Magnetic saturation does not exist.

(iv) All impedances are referred to stator side, and equivalent circuits are reduced to the supply frequency.

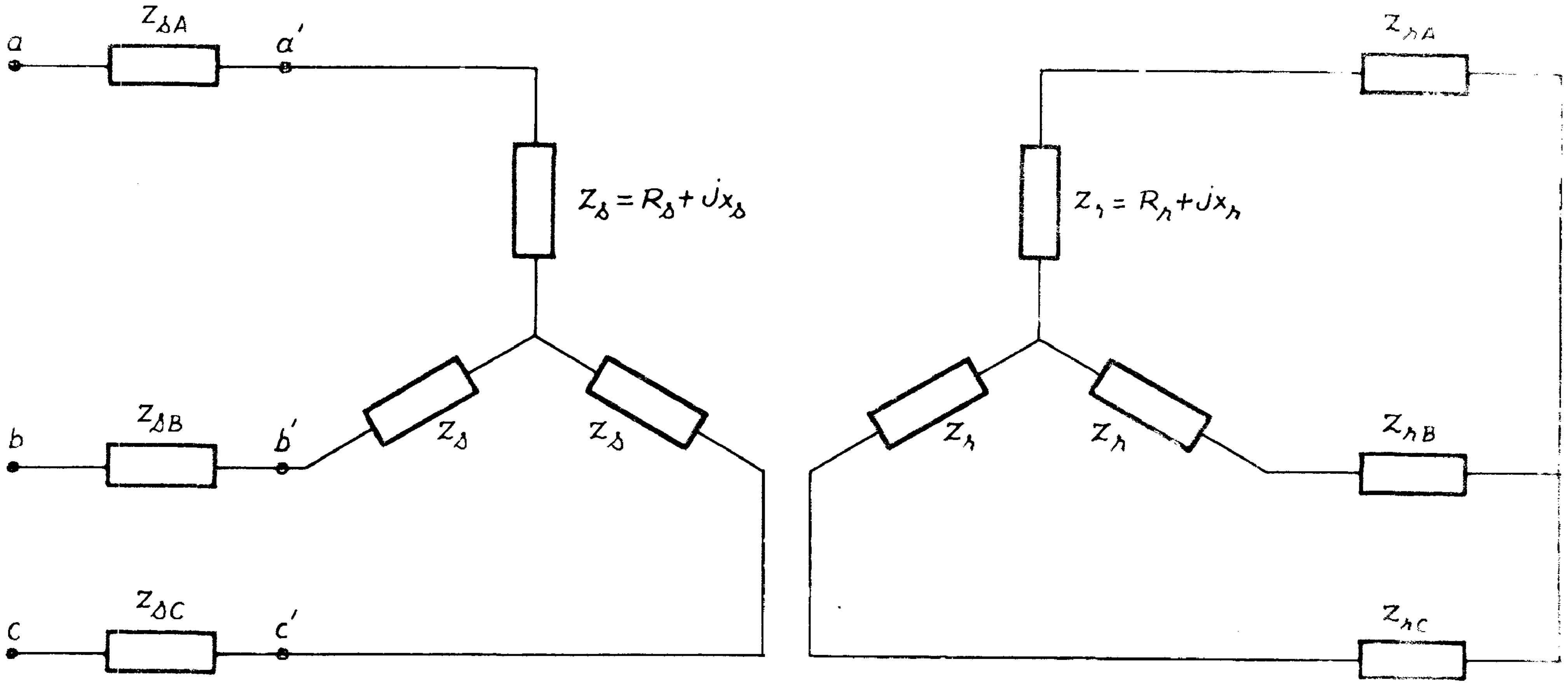

Fig. 2.1 — Induction Machine with Unbalanced Impedances in Stator and Rotor Circuits.

From the theory of symmetrical components it is understood that if balanced line voltages are applied to a set of three unbalanced impedances connected in star, the currents which flow in the impedances are unbalanced. These unbalanced currents can be resolved into positive and negative sequence currents. Thus due to positive and negative sequence currents in the impedances, the positive and negative sequence voltages will appear across the impedances. Therefore it can be said that the unbalanced impedances are a source of negative sequence voltage and current when positive sequence voltage and current are applied. The reverse is equally true. (Refer Appendix II)

When a balanced voltage is applied to an Induction Machine with unbalanced impedances connected in both stator and rotor circuits, both positive and negative sequence voltages would exist at stator terminals as explained above. For the purpose of analysis it can be considered that first positive sequence voltage alone and secondly negative sequence voltage alone is applied to stator terminals of the machine i.e. a', b', c' terminals shown in Fig. 2.1. When both positive and negative sequence voltages are applied simultaneously, the net results are obtained by super position principle.

2.2. With Positive Sequence Voltage Alone Applied To Stator Terminals

Let a positive sequence voltage V_{s1}^{OP} of frequency f be applied to the stator terminals of the machine. The positive sequence current in stator produces a rotating magnetic field, which has an angular velocity $\omega = 2\pi f$. Its direction of rotation depends upon the phase sequence of stator currents. The rotor is rotating with an angular velocity with respect to the stator. If the direction of rotation of rotor is same as the direction of rotating magnetic field, the relative angular velocity of rotor with respect to the rotating magnetic field is $(\omega - n)$. A positive sequence e.m.f. of slip frequency is therefore induced in the rotor, where the slip is defined as

$$s = \frac{\omega - n}{n} \qquad \ldots \qquad 2.1$$

As the unbalanced impedances are connected in the rotor circuit, the rotor currents would be unbalanced and can be resolved into positive and negative sequence currents I'^{p}_{r1} and I'^{p}_{r2} respectively. These positive and negative sequence currents flowing through unbalanced impedances connected externally to the rotor circuit, would cause a set of positive sequence voltage V'^{p}_{r1} and a set of negative sequence voltage V'^{p}_{r2} to appear across the slip rings. First considering the positive sequence rotor current I'^{p}_{r1} which produces a magnetic field rotating at an angular velocity of $(\omega - n) = s\omega$ with respect to rotor or with angular velocity of ω with respect to stator in the positive direction. The equivalent circuit of the machine showing only positive sequence currents and voltages in stator and rotor would thus be as given in Fig. 2.2. The frequencies of rotor and stator are sf and f respectively. The circuit of Fig. 2.2 is reduced to the supply frequency f, by dividing voltages and impedances of the rotor circuits by s, so that the rotor current remains unchanged.

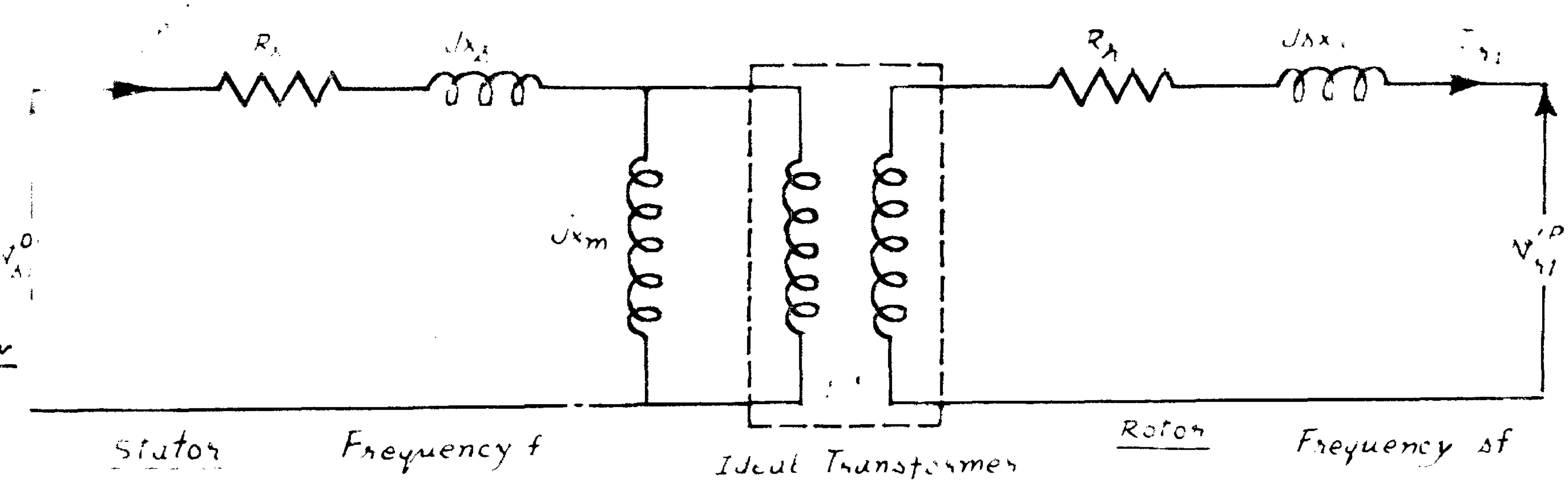

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Fig 2.2 — Network for Positive Sequence Voltage (Stator and Rotor Impedance Unbalanced.)

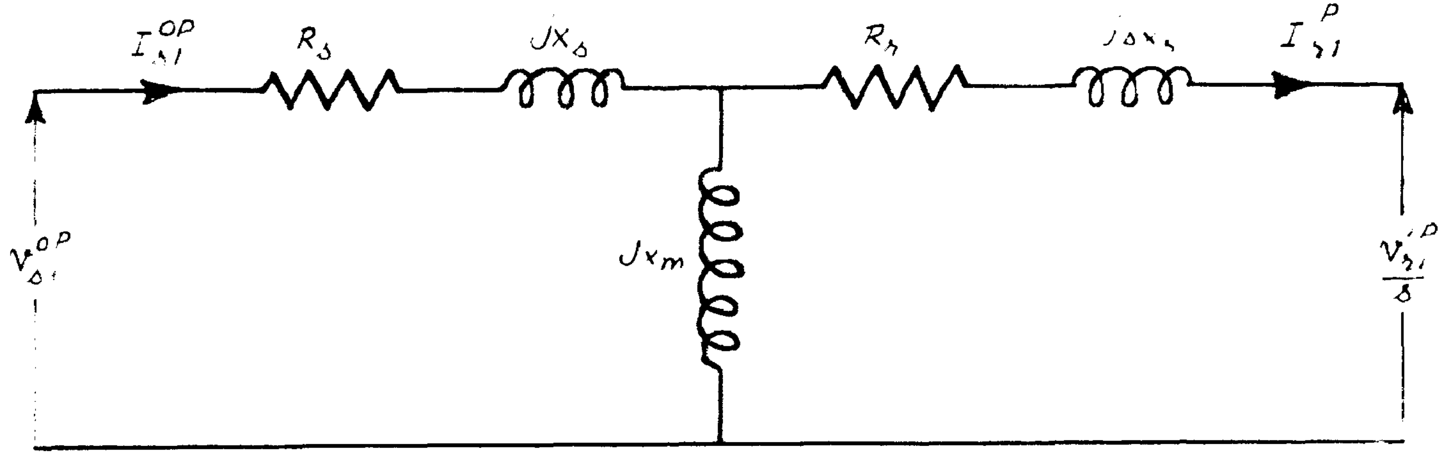


Fig 2.3 — Network of 'Fig. 2.2' reduced at Supply Frequency.

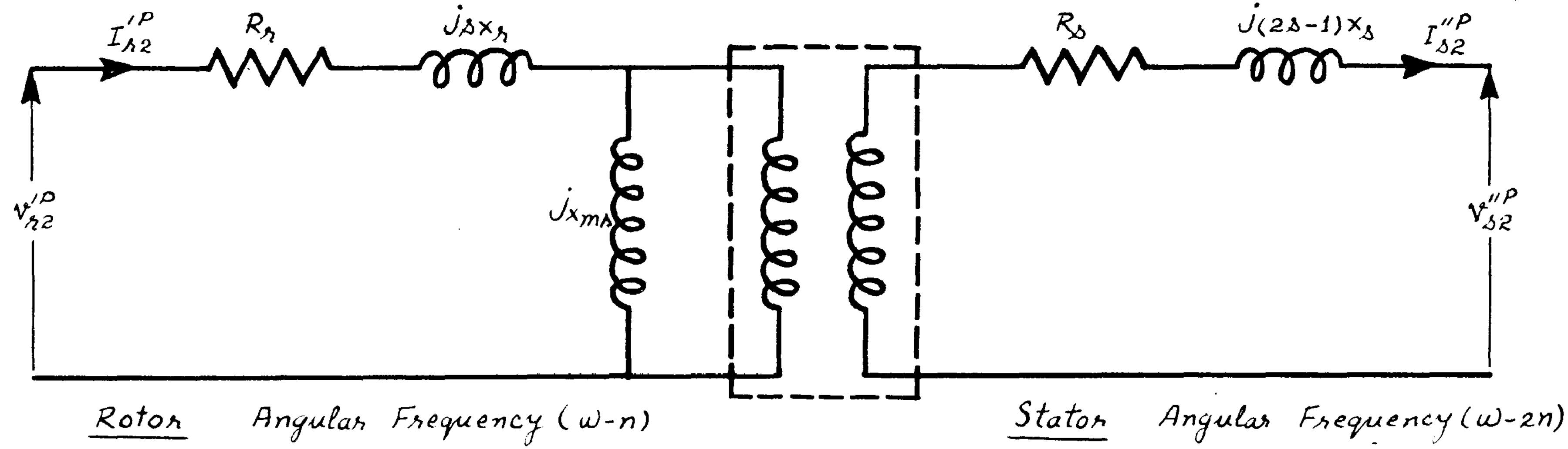


Fig. 2·4 – Network when Negative Sequence Voltage (due to First Reflection) is Applied at the Rotor with Unbalance Stator Impedances.

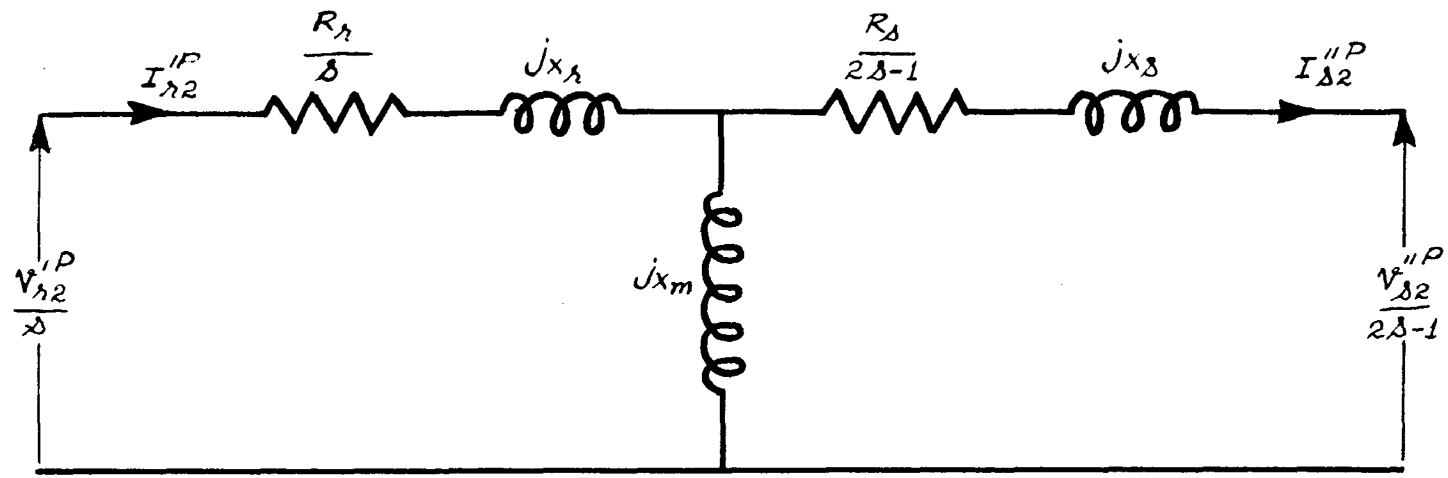


Fig. 2·5 — Network of 'Fig 2·4' reduced at the Supply Frequency.

The circuit thus reduced is shown in Fig. 2.3.

Secondly, the negative sequence voltage $V_{r2}^{'p}$ of frequency sf is created at the rotor terminals on account of unbalanced impedances connected in slip rings. It can be imagined that this voltage is generated by the reflection at the unbalanced impedance and is fed at the rotor terminals. This reflection would thus be numbered as one. Let the positive direction of negative sequence rotor current $I_{r2}^{'p}$ (due to first reflection) be taken opposite to that of the positive sequence rotor current $I_{r1}^{'p}$. The negative sequence rotor current $I_{r2}^{'p}$ of frequency sf produces a magnetic field, which rotates with an angular velocity of $(\omega - n) = s\omega$ with respect to rotor in the negative direction or with an angular velocity of $(\omega - 2n) = (2s - 1)\omega$ with respect to stator in the negative direction. Due to this rotating field, negative sequence e.m.fs. of frequency (2s - 1)f are induced in the stator. The stator impedances being unbalanced, the negative and positive sequence currents $I_{s2}^{''p}$ and $I_{s1}^{''p}$ would flow in the stator circuit, and negative and positive sequence voltages of frequency (2s - 1)f would exist at the stator terminals.

The negative sequence stator current $I_{s2}^{''p}$ produces a field which rotates with same velocity and direction as of the field created by negative sequence rotor current $I_{r2}^{'p}$. The machine equivalent circuit for these negative sequence voltages and currents caused by the first reflection (at rotor terminals) is shown in Fig. 2.4. In the equivalent circuit rotor and stator are at frequencies sf and (2s - 1)f respectively. To reduce the equivalent circuit to supply frequency f, divide voltages and impedances of rotor by s to keep I'^{p}_{r2} unchanged, and voltages and impedances of stator by (2s - 1) to keep I''^{p}_{s2} unchanged. This results with equivalent circuit shown in Fig. 2.5.

Now a relationship between positive and negative sequence voltages V'^{p}_{r1} and V'^{p}_{r2} created due to first reflection at rotor terminals can be established with the help of constraint equations from the theory of symmetrical components. (Refer Appendix II)

$$\left.\begin{aligned} V'^{p}_{r1} &= Z_{r0} I'^{p}_{r1} - Z_{r2} I'^{p}_{r2} \\ V'^{p}_{r2} &= Z_{r1} I'^{p}_{r1} - Z_{r0} I'^{p}_{r2} \end{aligned}\right\} \quad \ldots \quad 2.2$$

where Z_{r0} , Z_{r1} and Z_{r2} are zero, positive and negative sequence impedances respectively of the unbalanced impedances connected externally in rotor circuit. Their values are

$$\left.\begin{aligned} Z_{r0} &= \tfrac{1}{3} (Z_{rA} + Z_{rB} + Z_{rC}) \\ Z_{r1} &= \tfrac{1}{3} (Z_{rA} + aZ_{rB} + a^2 Z_{rC}) \\ Z_{r2} &= \tfrac{1}{3} (Z_{rA} + a^2 Z_{rB} + aZ_{rC}) \end{aligned}\right\} \quad \ldots \quad 2.3$$

A similar relationship between negative and positive sequence voltages V''^{p}_{s2} and V''^{p}_{s1} created due to second reflection at stator terminals, can be written

$$V''^{p}_{s2} = Z_{s0} I''^{p}_{s2} - Z_{s1} I''^{p}_{s1}$$

$$V''^{p}_{s1} = Z_{s2} I''^{p}_{s2} - Z_{s0} I''^{p}_{s1} \qquad \ldots \quad 2.4$$

where Z_{s0}, Z_{s1} and Z_{s2} are zero, positive and negative sequence impedances respectively of the unbalanced impedances connected externally in the stator circuits. Their values are

$$Z_{s0} = \frac{1}{3}(Z_{sA} + Z_{sB} + Z_{sC})$$

$$Z_{s1} = \frac{1}{3}(Z_{sA} + aZ_{sB} + a^2 Z_{sC}) \qquad \ldots \quad 2.5$$

$$Z_{s2} = \frac{1}{3}(Z_{sA} + a^2 Z_{sB} + aZ_{sC})$$

The constraint equation at a reflection cannot be represented by an equivalent circuit for a general case where zero, positive and negative sequence impedances have different values. Therefore equivalent circuits given in Fig. 2.3 and Fig. 2.5 cannot be connected together by a network realizable by passive elements.

Now the positive sequence voltage V''^{p}_{s1} and the current I''^{p}_{s1} of frequency $(2s - 1)f$, created by the unbalanced external stator impedances, can be said to be caused by the second reflection at stator terminals. The current I''^{p}_{s1} creates a field, which rotates in the positive direction with an angular velocity of $(\omega - 2n) = (2s - 1)\omega$ with respect to the stator and $(\omega - 3n) = (3s - 2)\omega$ with respect to the rotor. A third reflection would now occur at rotor terminals creating negative sequence voltage and current V'''^{p}_{r2} and I'''^{p}_{r2} respectively of

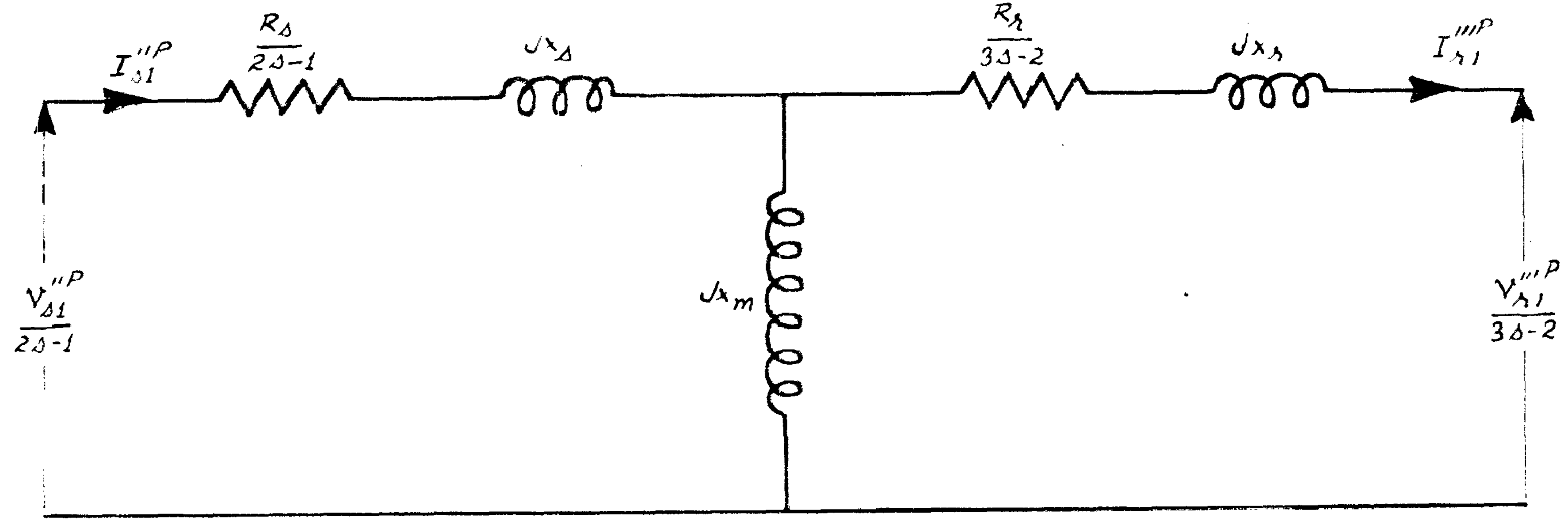


Fig 26 — Network when Positive Sequence Voltage (due to Second Reflection) applied at the Stator with Unbalanced Rotor Impedances

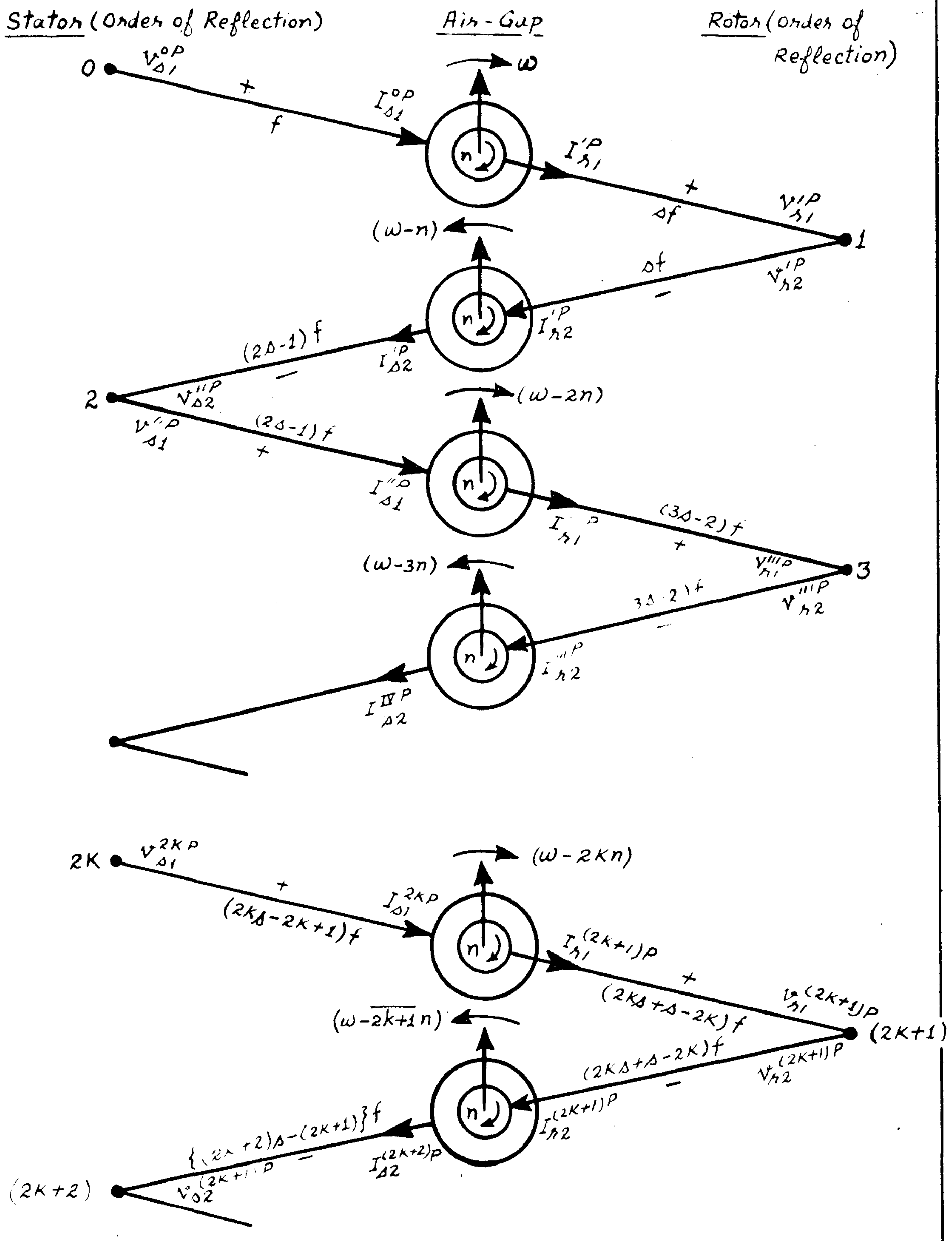


Fig. 2.7 – Flow Chart No. 1 – Positive Sequence Voltage at Stator.

frequency (3s - 2)f. The machine equivalent circuit for the positive sequence voltages and currents caused by the second reflection at the stator when reduced to supply frequency f, is given by Fig. 2.6.

It follows from the above discussions that when a positive sequence voltage is applied to the stator terminals of an Induction Machine with unbalanced impedances in both stator and rotor circuits, an infinite number of reflections result. The first reflection takes place at unbalanced impedances connected at rotor terminals, creating negative sequence voltages and currents. These first reflection negative sequence currents are fed into the rotor circuit and causes an inverted operation of the machine. The corresponding negative sequence stator currents are re-reflected at stator terminals (it is a second reflection) due to unbalanced stator impedances. The second reflection thus results in creating positive sequence voltages and currents at the stator terminals. This process of reflections at stator and rotor terminals continues for infinite number of times. The positive and negative sequence currents of various order of reflection create their own rotating fields in the air-gap. The order of reflection, the frequency of rotor and stator currents and the velocity of rotating fields with direction are given in flow chart No. 1 of Fig. 2.7.

It is to be seen from the chart that a reflection at the rotor always takes with an in-coming positive sequence and results in an out-going negative sequence, and reverse is the case at the stator terminals. Odd numbered reflections always take place at the rotor and even numbered at the stator. The positive sequence currents of any even numbered reflection are fed into the stator terminals and the negative sequence currents of any odd reflection order are fed into the rotor terminals.

The equivalent circuit of machine for the positive sequence currents only created by even numbered i.e. 2K th reflection, referred to the supply frequency f is given in Fig. 2.8. And the equivalent circuit of machine for the negative sequence currents only created by odd numbered i.e. (2K + 1)th reflection, referred to the supply frequency f is given in Fig. 2.9.

The constraint equations between positive and negative sequence voltages and currents at odd order of reflection (taking place at rotor) is given by equation 2.6

$$\left.\begin{aligned} V_{r1}^{(2K+1)P} &= Z_{r0} I_{r1}^{(2K+1)P} - Z_{r2} I_{r2}^{(2K+1)P} \\ V_{r2}^{(2K+1)P} &= Z_{r1} I_{r1}^{(2K+1)P} - Z_{r0} I_{r2}^{(2K+1)P} \end{aligned}\right\} \quad 2.6$$

And the constraint relationship between negative and positive sequence voltages and currents at even numbered reflection (taking place at stator) is given by equation 2.7

$$\left.\begin{aligned} V_{s2}^{2KP} &= -Z_{s1} I_{s1}^{2KP} + Z_{s0} I_{s2}^{2KP} \\ V_{s1}^{2KP} &= -Z_{s0} I_{s1}^{2KP} + Z_{s2} I_{s2}^{2KP} \end{aligned}\right\} \quad \ldots \quad 2.7$$

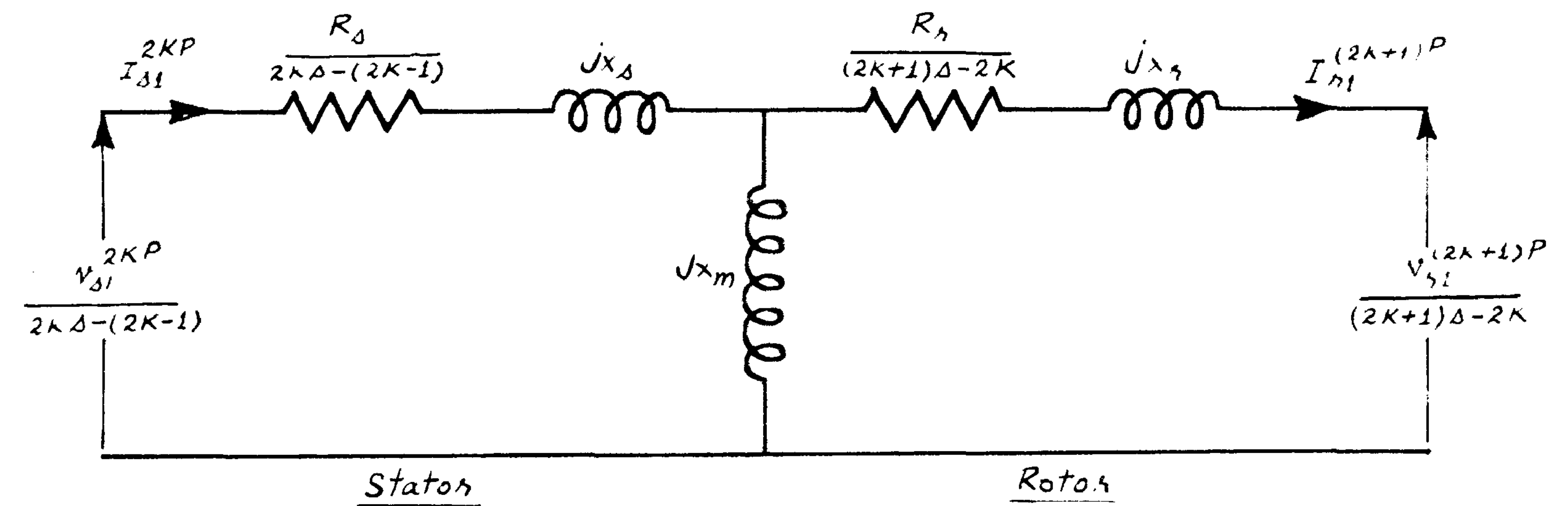


Fig. 2.8 — Network-Positive Sequence Voltage (due to $2K^{th}$ Reflection) at Stator Terminals for Positive Sequence Supply Voltage.

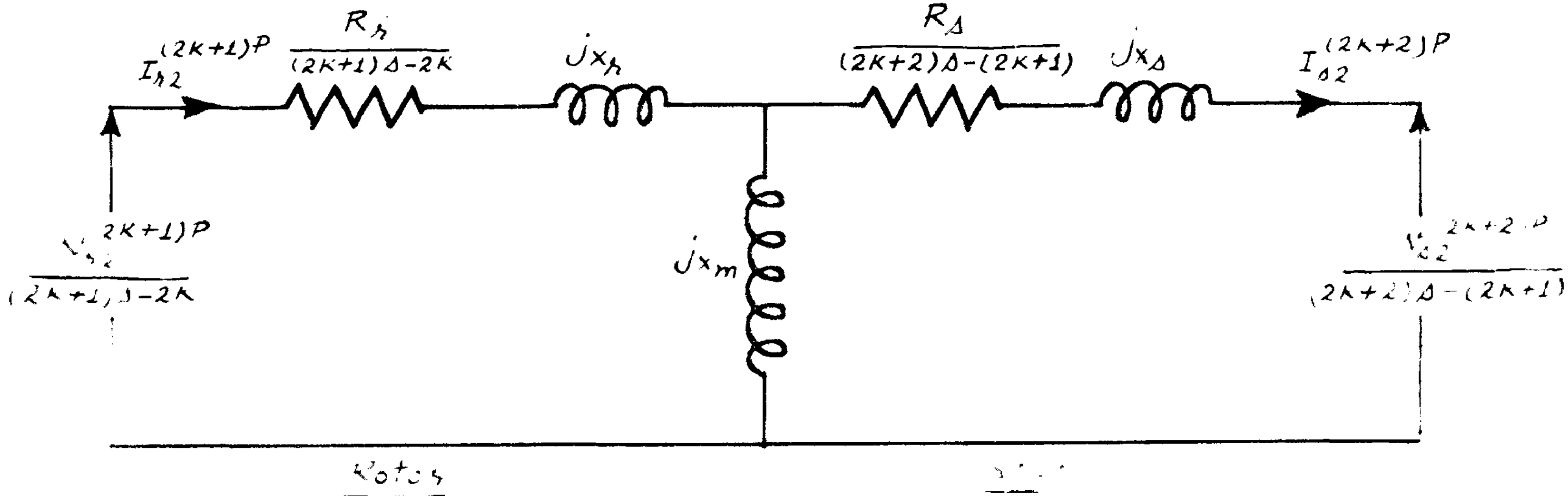


Fig. 2.9 — Network - Negative Sequence Voltage {due to $(2K+1)^{th}$ Reflection} at Rotor Terminals for Positive Sequence Supply Voltage

2.3. With Negative Sequence Voltage Alone Applied to Stator Terminals

When negative sequence voltage alone applied at stator terminals, the analysis proceeds exactly on the same line as explained in the positive sequence case. Though the velocity of various sequence fields will be different. A brief out line of analysis is given below:-

When a negative sequence voltage V_{s2}^{ON} of frequency f is applied to the stator, a magnetic field is produced which rotates at an angular velocity ω with respect to stator in the negative direction and at an angular velocity of $(\omega + n) = (2 - s)\omega$ with respect to rotor. A set of negative sequence e.m.f. of frequency (2 - s)f are induced in the rotor. Due to unbalanced impedances connected in the rotor circuit, both negative and positive sequence currents and voltages of frequency (2 - s)f appear at rotor terminals. Thus the first reflection occuring at the rotor terminals creates positive sequence voltage $V_{r1}^{'N}$ of frequency (2 - s)f, which is applied to rotor terminals, inducing a set of positive sequence e.m.fs. in the stator. Due to unbalanced impedances connected in the stator, both positive and negative sequence voltages $V_{s1}^{''N}$ and $V_{s2}^{''N}$ and currents $I_{s1}^{''N}$ and $I_{s2}^{''N}$ of frequency (3 - 2s)f appear at stator terminals. Thus the second reflection occuring at the stator terminals causes negative sequence voltage $V_{s2}^{''N}$ of frequency (3 - 2s)f. Again this voltage being applied to the stator terminals of the machine results in a third reflection occuring at rotor terminals. This process of reflections alternately occuring at rotor and stator terminals would continue for infinite number of times.

As discussed in the previous case (when only positive sequence voltage is applied to the stator), the negative and positive sequence currents of various order of reflections create their own rotating fields in the air-gap. The order of reflection, the frequency of rotor and stator currents and the velocity and direction of rotating fields are given in flow chart No.2 of Fig. 2.10.

It is to be seen from the chart that a reflection at the rotor always takes place with an in-coming negative sequence to an out-going positive sequence, and the reverse is the case at the stator terminals. Odd-numbered reflections always occur at the rotor and even-numbered at the stator terminals. The negative sequence currents of any even-numbered reflections are fed into stator terminals and positive sequence currents of odd-numbered reflections are fed into the rotor terminals.

Following from the discussions given in the previous case, the equivalent circuit of the machine for the negative sequence currents created by 2K th reflection referred to the supply frequency f, is given in the Fig. 2.11.

Similarly the equivalent circuit of the machine for the positive sequence currents created by the odd numbered (2K + 1)th reflection at the rotor, referred to the supply frequency f, is given in the Fig. 2.12.

The constraint equations between the negative and positive sequence voltages and currents at rotor terminals for

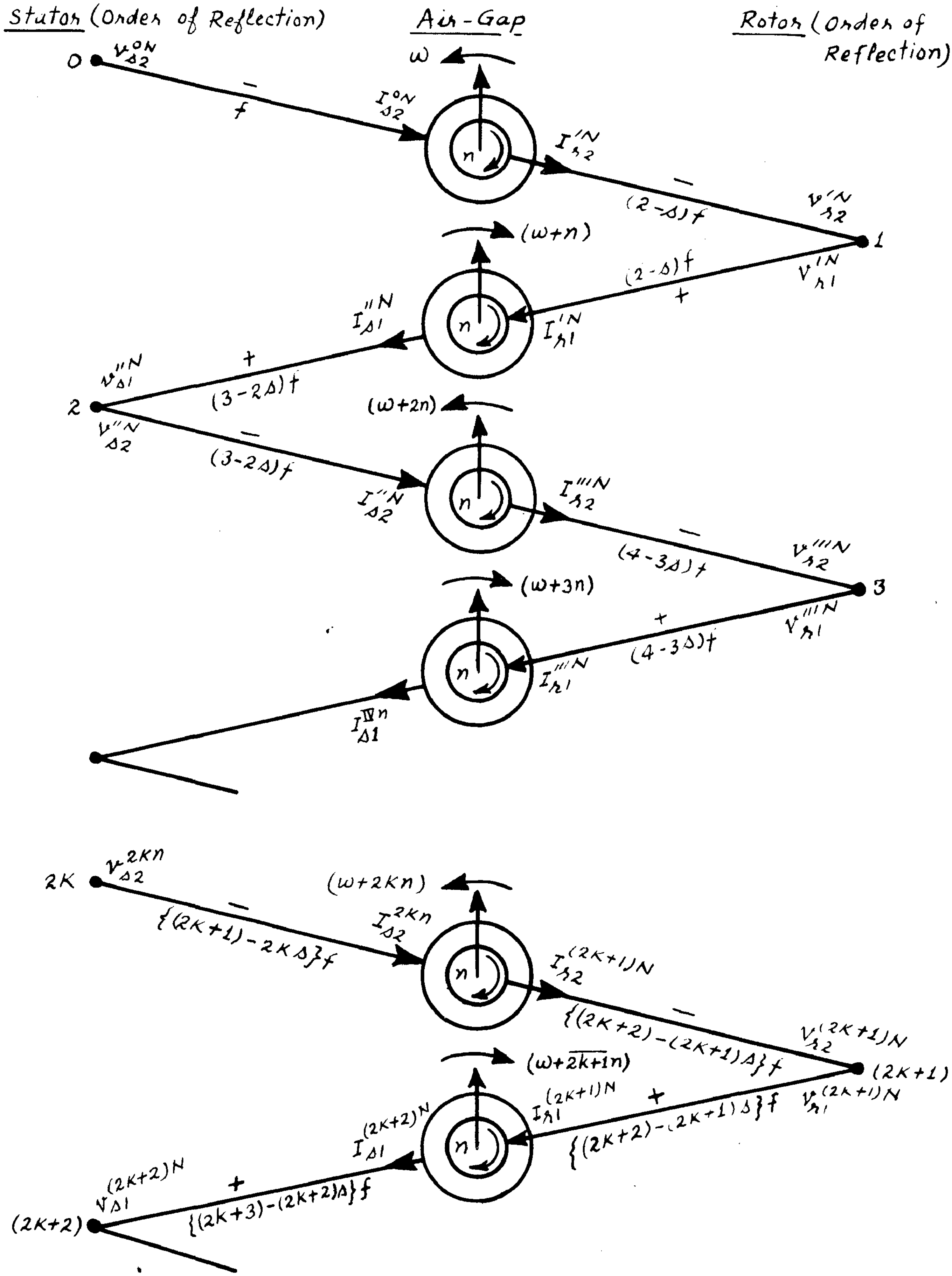


Fig. 2.10 – Flow Chart No. 2 – Negative Sequence Voltage at Stator.

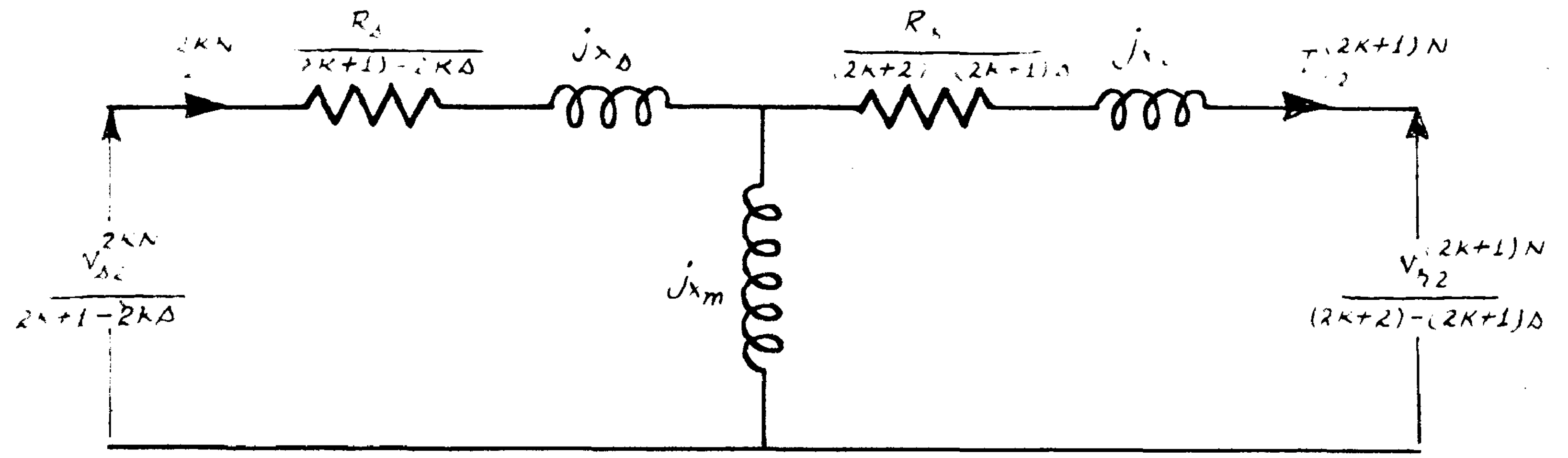


Fig. 11 — Network — Negative Sequence Voltage (due to 2Kth Reflection) at Stator Terminals for Negative Sequence Supply Voltage.

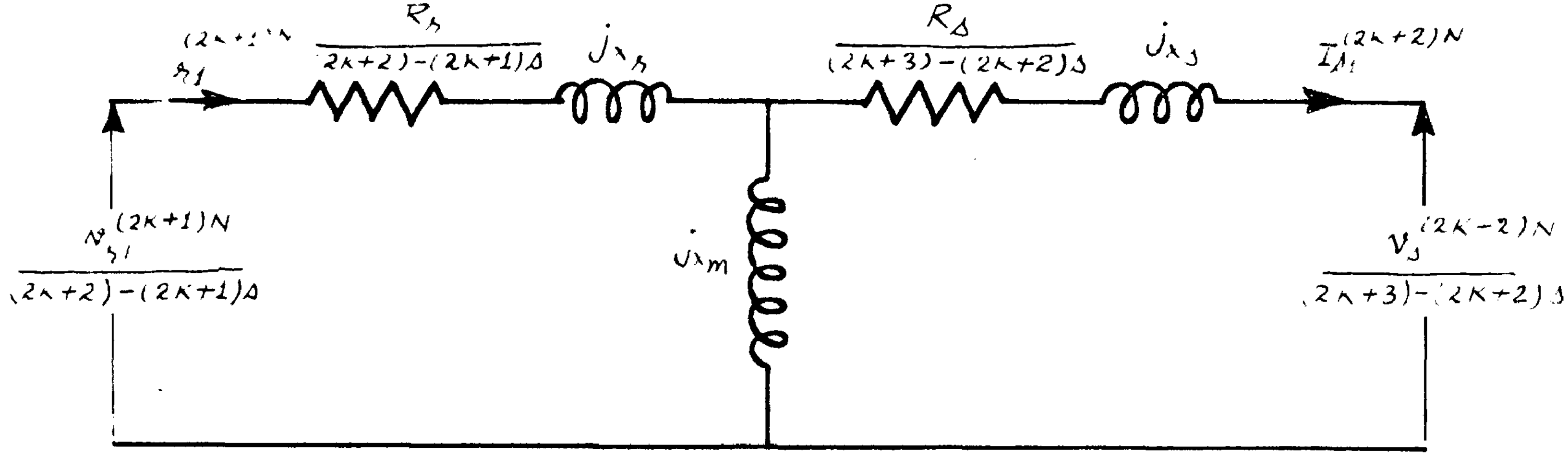


Fig. 12 — Network — Positive Sequence Voltage {due to (2K+1)th Reflection} at Rotor Terminals for Negative Sequence Supply Voltage.

the (2K + 1)th reflection, is given by equation 2.8.

$$\left.\begin{aligned} V_{r2}^{(2K+1)N} &= Z_{r0} I_{r2}^{(2K+1)N} - Z_{r1} I_{r1}^{(2K+1)N} \\ V_{r1}^{(2K+1)N} &= Z_{r2} I_{r2}^{(2K+1)N} - Z_{r0} I_{r1}^{(2K+1)N} \end{aligned}\right\} \quad 2.8$$

Similarly the constraint relationship between positive and negative, sequence voltages and currents at stator terminals for 2K th reflection is given in equation 2.9.

$$\left.\begin{aligned} V_{s1}^{2KN} &= Z_{s0} I_{s1}^{2KN} - Z_{s2} I_{s2}^{2KN} \\ V_{s2}^{2KN} &= Z_{s1} I_{s1}^{2KN} - Z_{s0} I_{s2}^{2KN} \end{aligned}\right\} \quad \ldots \quad 2.9$$

2.4. With Positive and Negative Sequence Voltage Applied Simultaneously At Stator Terminals

In the above discussion the machine analysis with double unbalance has been carried out when only positive or negative sequence voltage i.e. V_{s1}^{OP} or V_{s2}^{ON} of frequency f is applied at stator terminals. The analysis can now be extended to the case when both positive and negative sequence voltages (V_{s1}^{OP} and V_{s2}^{ON} respectively) of frequency f are simultaneously applied to stator terminals of the machine. By the principle of super position all the currents and voltages of different frequencies, which are created by considering positive and negative sequence voltage separately applied to stator terminals, would now exist simultaneously in stator and rotor circuits. To obtain phase currents and voltages of stator and rotor, it is not possible to add vectorially the sequence currents and voltages of different frequencies. A time expression can, however, be written by adding the instantaneous currents and voltages. The magnitude and frequency of various currents and voltages, existing in stator and rotor due to different reflections, are given in two flow charts presented earlier.

The instantaneous stator current for phase-A is

$$
\begin{aligned}
i_{sA} = \sqrt{2} \Bigg[& \left|I_{s1}^{0P}\right| \operatorname{Cos}(\omega t + \alpha_0) \\
& + \sum_{K=1}^{\infty} \Bigg\{ \left|I_{s2}^{2KP}\right| \operatorname{Cos}(\overline{2Ks - 2K + 1}\ \omega t + \beta_{2K}) \\
& - \left|I_{s1}^{2KP}\right| \operatorname{Cos}(\overline{2Ks - 2K + 1}\ \omega t + \alpha_{2K}) \Bigg\} \\
& + \left|I_{s2}^{0N}\right| \operatorname{Cos}(\omega t + \gamma_0) \\
& + \sum_{K=1}^{\infty} \Bigg\{ \left|I_{s1}^{2KN}\right| \operatorname{Cos}(\overline{2K + 1 - 2Ks}\ \omega t + \delta_{2K}) \\
& - \left|I_{s2}^{2KN}\right| \operatorname{Cos}(\overline{2K + 1 - 2Ks}\ \omega t + \gamma_{2K}) \Bigg\} \Bigg]
\end{aligned}
\qquad 2.10
$$

The currents I_{s1}^{2KP} and I_{s2}^{2KP} are of the same frequency and can be added vectorially. Therefore the addition can be replaced by I_{sA}^{2KP}. Similarly replacing the quantities of same frequency by their vector addition, the equation 2.10 can be written in simplified form.

$$i_{sA} = \sqrt{2}\Big[\left|I_{sA}^{0}\right| \text{Cos}\,(\omega t + \psi_0)$$

$$+ \sum_{K=1}^{\infty} \left|I_{sA}^{2KP}\right| \text{Cos}\,(\overline{2Ks - 2K + 1}\;\omega t + \varepsilon_{2K})$$

$$+ \sum_{K=1}^{\infty} \left|I_{sA}^{2KN}\right| \text{Cos}\,(\overline{2K + 1 - 2Ks}\;\omega t + \eta_{2K})\Big] \quad 2.11$$

Now the r.m.s. value of current for stator A-phase is

$$I_{sA} = \sqrt{\left|I_{sA}^{0}\right|^2 + \sum_{K=1}^{\infty} \left(\left|I_{sA}^{2KP}\right|^2 + \left|I_{sA}^{2KN}\right|^2 \right)} \quad 2.12$$

Similarly the time expression for the rotor currents of phase-A can be written.

$$i_{rA} = \sqrt{2}\Big[\sum_{K=0}^{\infty}\Big\{\left|I_{r1}^{(2K+1)P}\right| \text{Cos}(\overline{2Ks + s - 2K}\;\omega t + \theta_{2K+1})$$

$$- \left|I_{r2}^{(2K+1)P}\right| \text{Cos}(\overline{2Ks + s - 2K}\;\omega t + \phi_{2K+1})$$

$$+ \sum_{K=0}^{\infty}\Big\{\left|I_{r2}^{(2K+1)N}\right| \text{Cos}(\overline{2K + 2 - 2Ks - s}\;\omega t + \sigma_{2K+1})$$

$$- \left|I_{r1}^{(2K+1)N}\right| \text{Cos}(\overline{2K + 2 - 2Ks - s}\;\omega t + \mu_{2K+1})\Big\}\Big] \quad 2.13$$

Replacing the quantities of same frequencies by their vector addition, the equation can be simplified.

$$i_{rA} = \sqrt{2}\left[\sum_{K=0}^{\infty}\left\{\left|I_{rA}^{(2K+1)P}\right| \operatorname{Cos}(\overline{2Ks+s-2K}\,\omega t + \lambda_{2K+1})\right.\right.$$

$$\left.\left. + \left|I_{rA}^{(2K+1)N}\right| \operatorname{Cos}(\overline{2K+2-2Ks-s}\,\omega t + \gamma_{2K+1})\right\}\right] \qquad 2.14$$

The r.m.s. value of current for rotor A-phase is

$$I_{rA} = \sqrt{\sum_{K=0}^{\infty}\left(\left|I_{rA}^{(2K+1)P}\right|^2 + \left|I_{rA}^{(2K+1)N}\right|^2\right)} \qquad 2.15$$

Similarly stator and rotor voltages can be found.

Stator A-phase voltage

$$V_{sA} = \sqrt{\left|V_{sA}^{0}\right|^2 + \sum_{K=1}^{\infty}\left(\left|V_{sA}^{2KP}\right|^2 + \left|V_{sA}^{2KN}\right|^2\right)} \qquad 2.16$$

Rotor A-phase voltage

$$V_{rA} = \sqrt{\sum_{K=0}^{\infty}\left(\left|V_{rA}^{(2K+1)P}\right|^2 + \left|V_{rA}^{(2K+1)N}\right|^2\right)} \qquad 2.17$$

Peak Voltage:

After finding the maximum value of component voltages, the resultant peak voltage can be computed by making the assumptions that the peaks of all these voltages occur at the same time and the sum of all the peak voltages equals the peak of the composite wave.

Thus

$$V_{sA}(\text{peak}) = \sqrt{2}\left[\left|V_{sA}^{0}\right| + \sum_{K=1}^{\infty}\left(\left|V_{sA}^{2KP}\right| + \left|V_{sA}^{2KN}\right|\right)\right] \qquad 2.18$$

and

$$V_{rA}\ (\text{peak}) = \sqrt{2}\left[\sum_{K=0}^{\infty}\left(\left|V_{rA}^{(2K+1)P}\right| + \left|V_{rA}^{(2K+1)N}\right|\right)\right] \qquad 2.19$$

These will give higher values than the actual peak value in the machine.

2.5. Relationship Between Supply Voltage And Sequence Voltages At Stator Terminals

The above analysis is carried out when both positive and negative sequence voltages V_{s1}^{OP} and V_{s2}^{ON} respectively of frequency f are simulteneously applied at the stator terminals. However, the supply voltage of fundamental frequency f which may in general be unbalanced, is connected not at stator terminals of the machine but at the terminals a, b, c shown in the Fig. 2.1. The supply voltage can be resolved into a positive sequence voltage V_1 and a negative sequence voltage V_2 of frequency f. The sequence voltage V_{s1}^{OP} and V_{s2}^{OP} applied at stator terminals can be related to the positive and negative sequence voltages V_1 and V_2 of the supply by the constraint equations given below:-

$$\left.\begin{aligned} V_1 &= Z_{s0}\, I_{s1}^{OP} + Z_{s2}\, I_{s2}^{ON} + V_{s1}^{OP} \\ V_2 &= Z_{s1}\, I_{s1}^{OP} + Z_{s0}\, I_{s2}^{ON} + V_{s2}^{ON} \end{aligned}\right\} \quad \ldots \quad 2.20$$

The other voltages and currents in the stator circuit are at different frequencies other than fundamental, and can not be considered in the equation 2.20, which applies only to voltages and currents of fundamental frequency.

Now if the supply voltage is balanced, $V_2 = 0$

I_{s1}^{OP} and I_{s2}^{ON} as obtained from equation 2.20 are

$$I_{s1}^{OP} = \frac{Z_{s0}(V_1 - V_{s1}^{OP}) + Z_{s2} V_{s2}^{ON}}{Z_{s0}^2 - Z_{s1} Z_{s2}}$$

$$I_{s2}^{ON} = -\frac{Z_{s1} (V_1 - V_{s1}^{OP}) + Z_{s0} V_{s2}^{ON}}{Z_{s0}^2 - Z_{s1} Z_{s2}} \qquad \ldots \; 2.21$$

From the equations 2.20, V_{s1}^{OP} and V_{s2}^{ON} can be obtained from the known values of V_1 and V_2 only if I_{s1}^{OP} and I_{s2}^{ON} are known. This, however, not been the case, analysis cannot proceed at all. Even if a trial and error solution is attempted by assuming the values of V_{s1}^{OP} and V_{s2}^{ON}, the currents I_{s1}^{OP} and I_{s2}^{ON} cannot be obtained, because of infinite number of reflections with the intervening constraint equations at each reflection being such that these equations cannot be represented by an equivalent circuit. The solution is, however, possible for some restricted cases which are dealt-with in succeeding chapters.

CHAPTER III

INDUCTION MACHINE WITH SYMMETRICAL DOUBLE UNBALANCE

3.1. In the previous chapter, it is found that in case of general unbalance, three impedances connected externally in the stator circuit are of different values and so are the impedances in the rotor circuit. At the points of unbalance, constraint equations have been given but it is not possible to solve the constraint equations and represent them by an equivalent circuit. But, however, the equivalent circuit can be drawn in case of restriction in unbalance with equal positive and negative sequence components of external impedances. This condition is satisfied where the impedances connected in external circuit are such that two out of three impedances are equal i.e. unbalance is symmetrical about one phase.

Under the restriction mentioned above, Z_{sA} , Z_{sB} and $Z_{sC} = Z_{sB}$ are the impedances connected in the stator circuit, and Z_{rA} , Z_{rB} and $Z_{rC} = Z_{rB}$ in the rotor circuit as shown in Fig. 3.1. The zero, positive and negative sequence impedances obtained from equations 2.3 and 2.5 are as follows:-

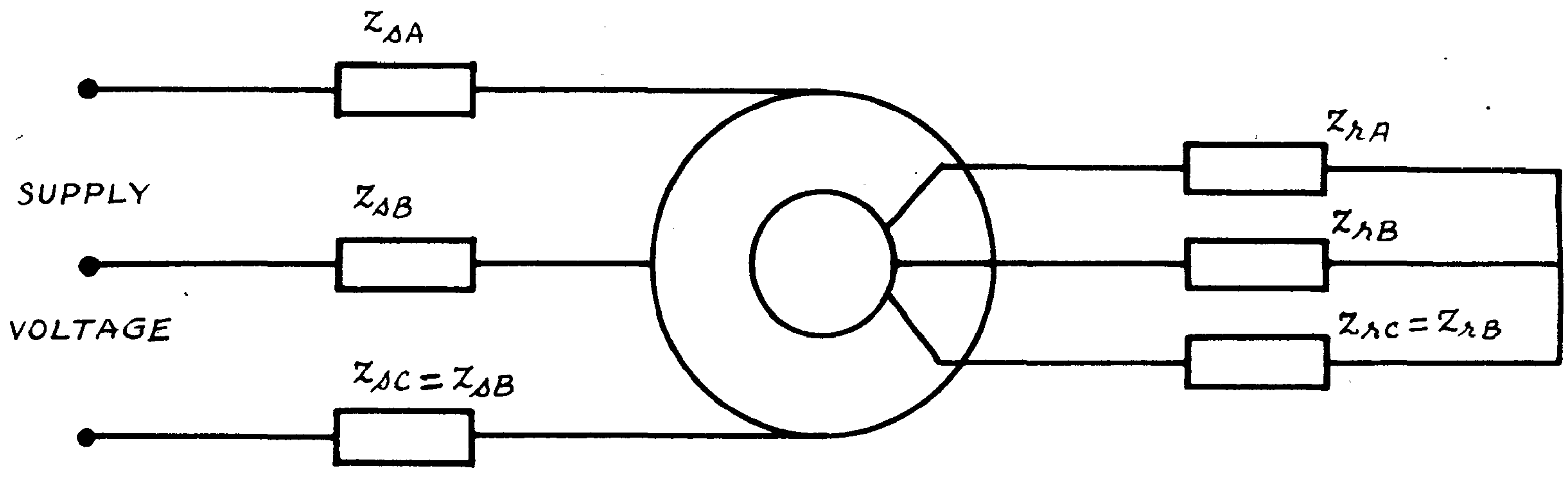


Fig. 3·1 — Machine with Symmetrical Double Unbalanced Impedances.

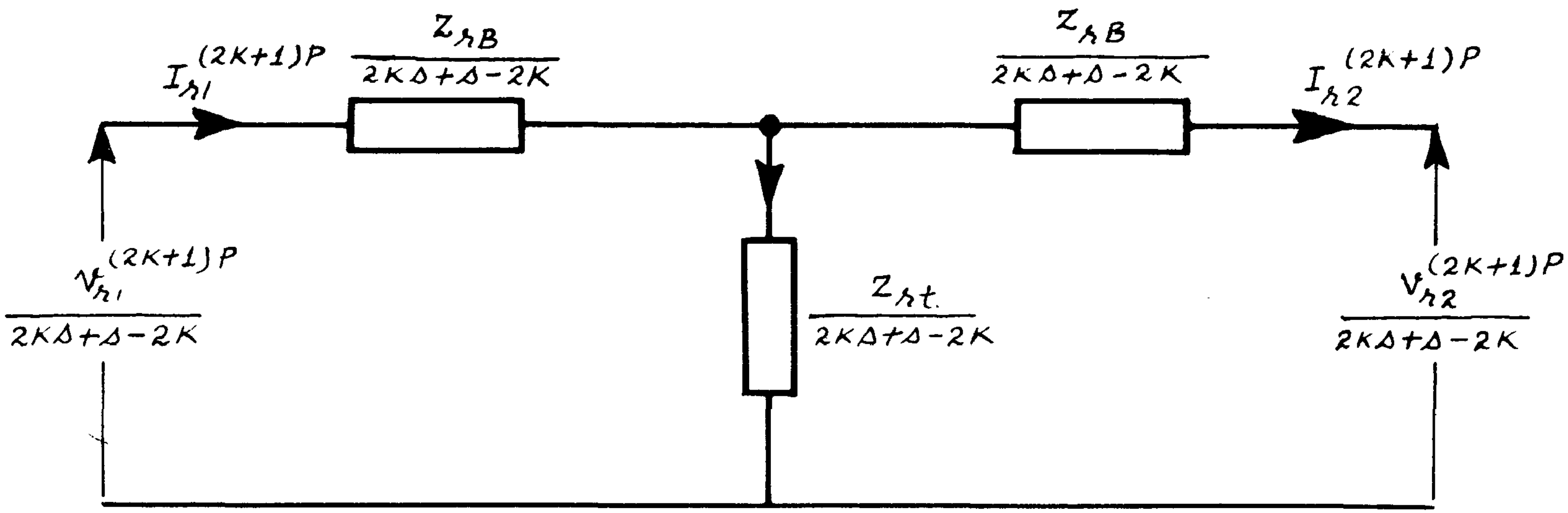


Fig. 3·2 — Equivalent Circuit for Constraint Relationship for $(2K+1)^{th}$ Reflection at Rotor when Positive Sequence Voltage only is Applied to Stator Terminals.

Stator

Zero sequence impedance $Z_{s0} = \frac{1}{3}(Z_{sA} + 2Z_{sB})$

Positive sequence impedance $Z_{s1} = \frac{1}{3}(Z_{sA} - Z_{sB})$ ... 3.1

Negative sequence impedance $Z_{s2} = \frac{1}{3}(Z_{sA} - Z_{sB})$

Hence $Z_{s1} = Z_{s2} = Z_{st}$ (say) ... 3.2

and $Z_{s0} - Z_{s1} = Z_{sB}$ ... 3.3

Rotor

Zero sequence impedance $Z_{r0} = \frac{1}{3}(Z_{rA} + 2Z_{rB})$

Positive sequence impedance $Z_{r1} = \frac{1}{3}(Z_{rA} - Z_{rB})$ ... 3.4

Negative sequence impedance $Z_{r2} = \frac{1}{3}(Z_{rA} - Z_{rB})$

Hence $Z_{r1} = Z_{r2} = Z_{rt}$ (say) ... 3.5

and $Z_{r0} - Z_{r1} = Z_{rB}$ ... 3.6

The constraint equations of chapter 2 are now modified by substituting these values of sequence impedances. Therefore equation 2.6 for (2K + 1)th reflection at rotor when positive sequence voltage alone is applied at stator, is modified to

$$\begin{aligned} V_{r1}^{(2K+1)P} &= Z_{rB}\, I_{r1}^{(2K+1)P} + Z_{rt}\left\{ I_{r1}^{(2K+1)P} - I_{r2}^{(2K+1)P} \right\} \\ V_{r2}^{(2K+1)P} &= -Z_{rB}\, I_{r2}^{(2K+1)P} + Z_{rt}\left\{ I_{r1}^{(2K+1)P} - I_{r2}^{(2K+1)P} \right\} \end{aligned} \qquad 3.7$$

The equation 3.7 remains unchanged if voltages and impedances are divided by (2Ks + s - 2K). Thus

$$\left.\begin{aligned}
\frac{V_{r1}^{(2K+1)P}}{2Ks+s-2K} &= \left[\frac{Z_{rB}}{2Ks+s-2K} I_{r1}^{(2K+1)P} + \frac{Z_{rt}}{2Ks+s-2K}\left\{ I_{r1}^{(2K+1)P} - I_{r2}^{(2K+1)P}\right\}\right] \\
\frac{V_{r2}^{(2K+1)P}}{2Ks+s-2K} &= \left[-\frac{Z_{rB}}{2Ks+s-2K} I_{r2}^{(2K+1)P} + \frac{Z_{rt}}{2Ks+s-2K}\left\{ I_{r1}^{(2K+1)P} - I_{r2}^{(2K+1)P}\right\}\right]
\end{aligned}\right\} \quad 3.8$$

The constraint equation 3.8 for (2K + 1)th reflection at rotor terminals, can now be represented by the equivalent circuit given in Fig. 3.2. This circuit representing the constraint equation interlinks two consecutive machine equivalent circuits i.e. the machine equivalent circuit between 2K th reflection at stator and (2K + 1)th reflection at rotor; and the machine equivalent circuit between (2K + 1)th reflection at rotor and (2K + 2)th reflection at stator. The overall circuit is drawn in Fig. 3.3.

Similarly the constraint equation for 2K th reflection at stator terminals are modified as

$$\left.\begin{aligned}
\frac{V_{s2}^{2KP}}{2Ks-2K+1} &= \left[\frac{Z_{sB}}{2Ks-2K+1} I_{s2}^{2KP} + \frac{Z_{st}}{2Ks-2K+1}\left(I_{s2}^{2KP} - I_{s1}^{2KP}\right)\right] \\
\frac{V_{s1}^{2KP}}{2Ks-2K+1} &= \left[-\frac{Z_{sB}}{2Ks-2K+1} I_{s1}^{2KP} + \frac{Z_{st}}{2Ks-2K+1}\left(I_{s2}^{2KP} - I_{s1}^{2KP}\right)\right]
\end{aligned}\right\} \quad 3.9$$

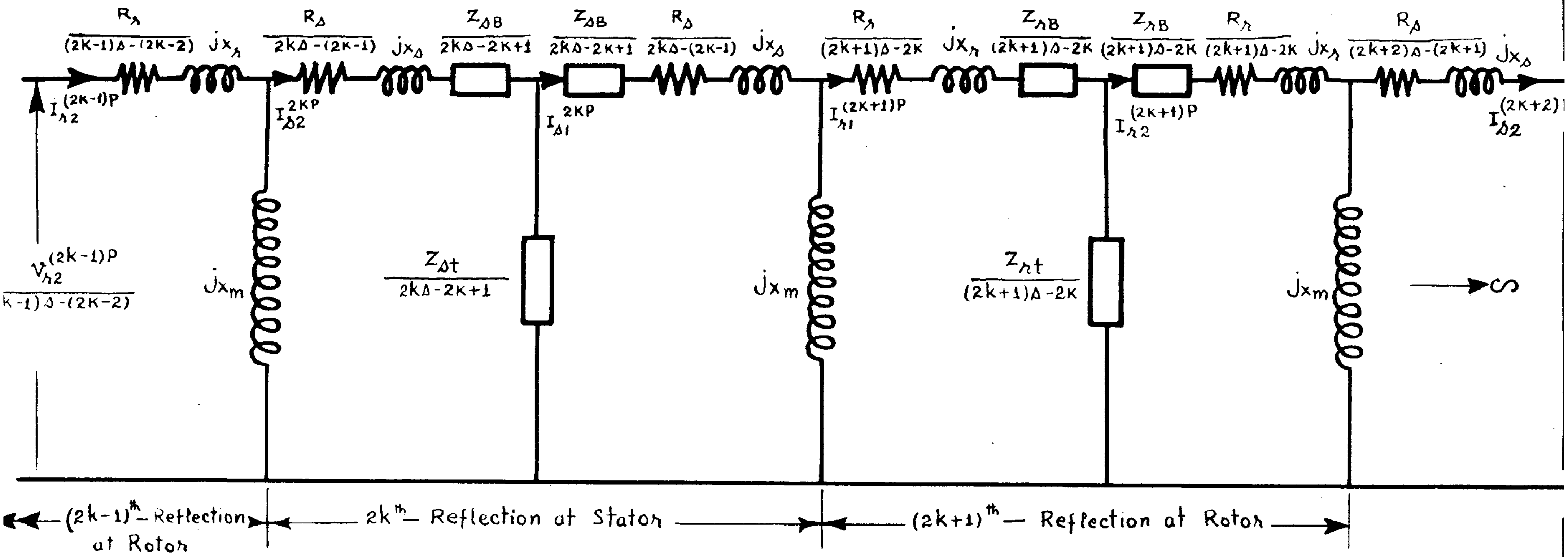


Fig. 3·3 – *Equivalent Circuit — Positive Sequence Voltage at Stator.*

'Infinite Network No. 1'

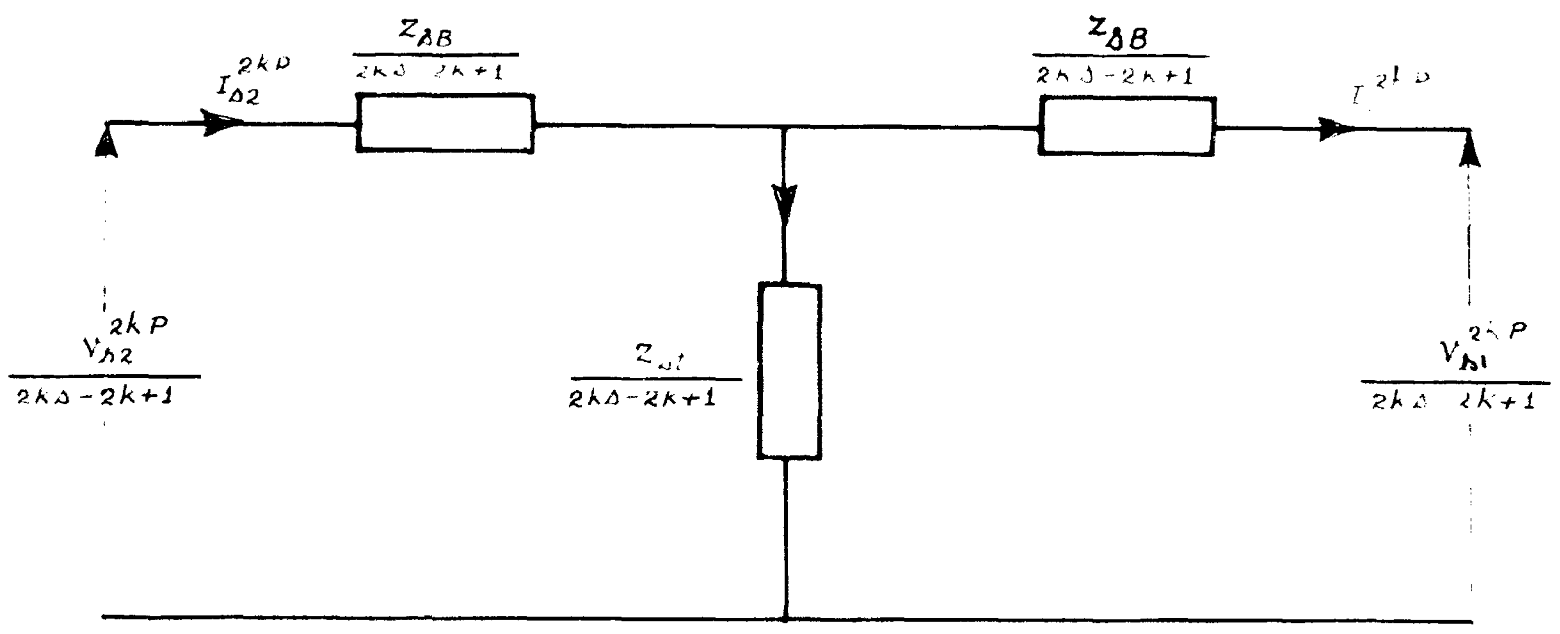


Fig. 3.4 – Equivalent Circuit from Constraint Relationship for $2k^{th}$ Reflection at Stator when Positive Sequence Voltage Only is Applied to Stator Terminals

This modified constraint equation can be represented by an equivalent circuit shown in Fig. 3.4. With the help of this circuit, the machine equivalent circuit between (2K - 1)th rotor reflection and 2K th stator reflection can be interlinked with the machine equivalent circuit between 2K th stator reflection and (2K + 1)th rotor reflection. This connection has also been shown in Fig. 3.3.

Similarly with the negative sequence voltage alone applied to stator terminals, the equivalent circuits can be drawn from the modified constraint equations for a particular order of reflection occuring at stator (or rotor) terminals. The constraint equations for 2K th reflection at stator and for (2K + 1)th reflection at rotor would be

Stator

$$\left.\begin{aligned}
\frac{V_{s1}^{2KN}}{2K+1-2Ks} &= \left[\frac{Z_{sB}}{2K+1-2Ks} I_{s1}^{2KN} + \frac{Z_{st}}{2K+1-2Ks}\left(I_{s1}^{2KN} - I_{s2}^{2KN}\right)\right] \\
\frac{V_{s2}}{2K+1-2Ks} &= \left[-\frac{Z_{sB}}{2K+1-2Ks} I_{s2}^{2KN} + \frac{Z_{st}}{2K+1-2Ks}\left(I_{s1}^{2KN} - I_{s2}^{2KN}\right)\right]
\end{aligned}\right\} \quad 3.10$$

Rotor

$$\frac{V_{r2}^{(2K+1)N}}{2K+2-2Ks-s} = \left[\frac{Z_{rB}}{2K+2-2Ks-s} I_{r2}^{(2K+1)N} + \frac{Z_{rt}}{2K+2-2Ks-s}\left\{I_{r2}^{(2K+1)N} - I_{r1}^{(2K+1)N}\right\}\right] \quad \ldots\ldots 3.11$$

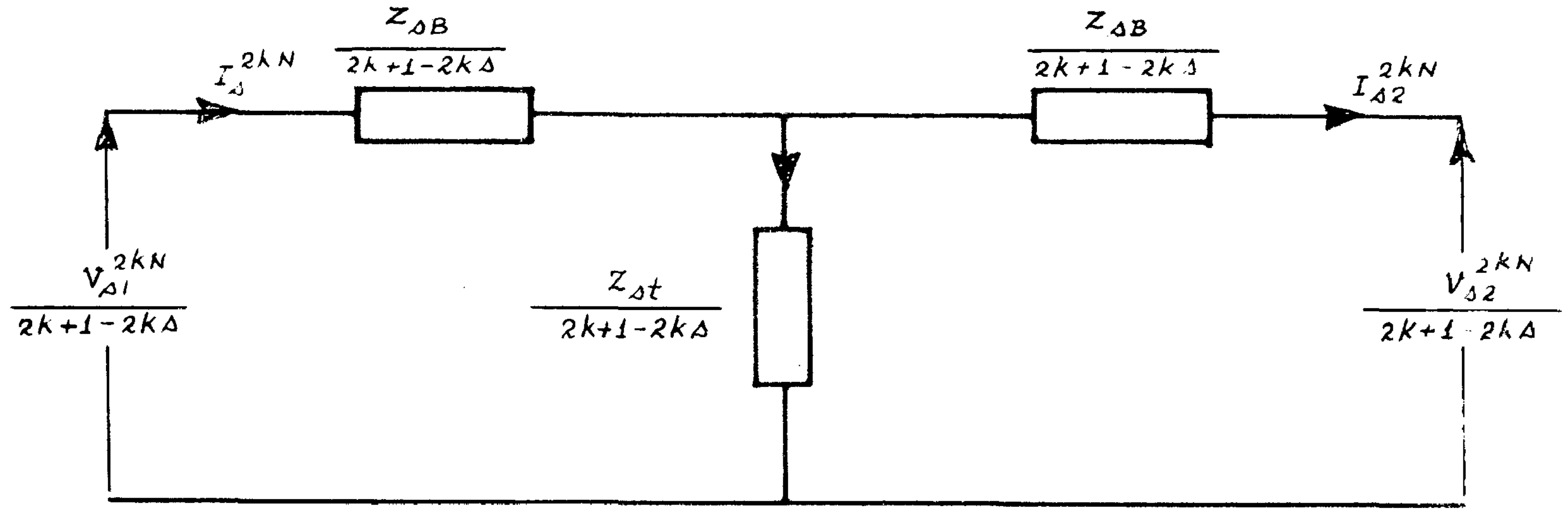


Fig. 3·5 – Equivalent Circuit from Constraint Relationship for $2k^{th}$ Reflection at Stator when Negative Sequence Voltage only is applied to Stator.

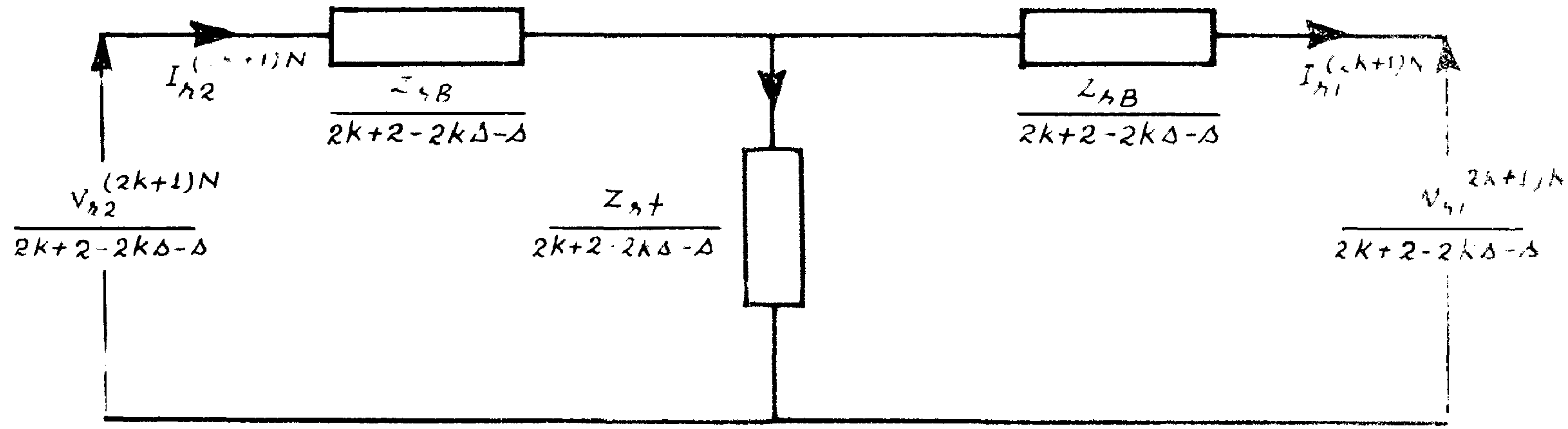


Fig. 3·6 – Equivalent Circuit from Constraint Relationship for $(2k+1)^{th}$ Reflection at Rotor when Negative Sequence Voltage Only is applied to Stator.

$$\frac{V_{r1}^{(2K+1)N}}{2K+2-2Ks-s} = \left[-\frac{Z_{rB}}{2K+2-2Ks-s} I_{r1}^{(2K+1)N} + \frac{Z_{rt}}{2K+2-2Ks-s} \left\{ I_{r2}^{(2K+1)N} - I_{r1}^{(2K+1)N} \right\} \right] \quad 3.11$$

The equivalent circuit corresponding to these equations are given in Figs. 3.5 and 3.6. With these circuits the machine equivalent circuit between (2K - 1)th rotor reflection and 2K th stator reflection can be connected to the machine equivalent between 2K th stator reflection and (2K + 1)th rotor reflection. Similarly the machine equivalent circuit between 2K th stator reflection and (2K + 1)th rotor reflection can be connected to the machine equivalent circuit between (2K + 1)th rotor reflection and (2K + 2)th stator reflection as shown in Fig. 3.7.

Now the complete equivalent circuit in each case can be generated by putting different values of K in the circuits of Fig. 3.3 and 3.7.

When both positive and negative sequence voltages exist simultaneously at the stator terminals, the currents and voltages caused by each of the sequence voltage would exist simultaneously in both rotor and stator circuits and can be super-posed as explained earlier. These positive and negative sequence voltages (V_{s1}^{OP} and V_{s2}^{ON}) applied to stator terminals are related to sequence components of supply voltage through stator sequence impedances by the constraint equation 2.20, which when modified

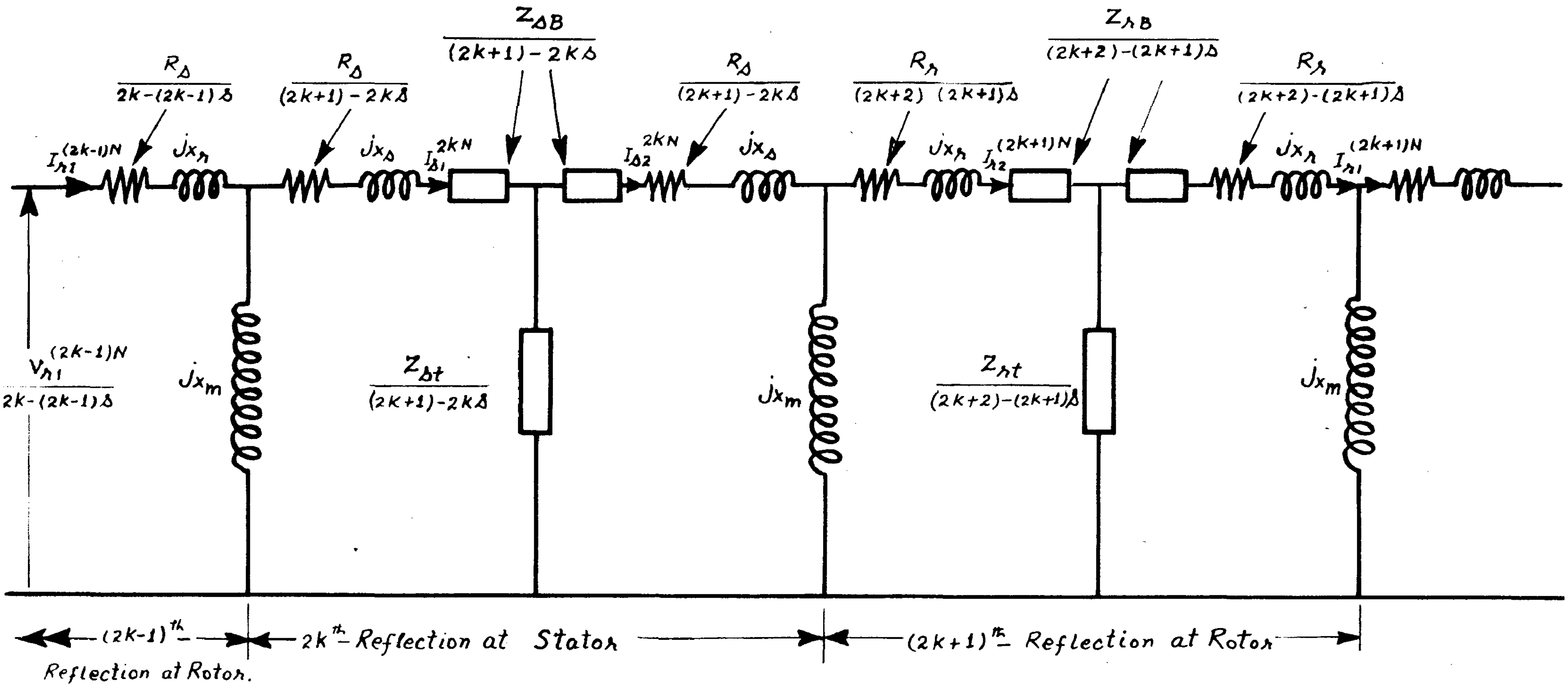


Fig. 3·7 – Equivalent Circuit – Negative Sequence Voltage at Stator.

'Infinite Network No. 2'

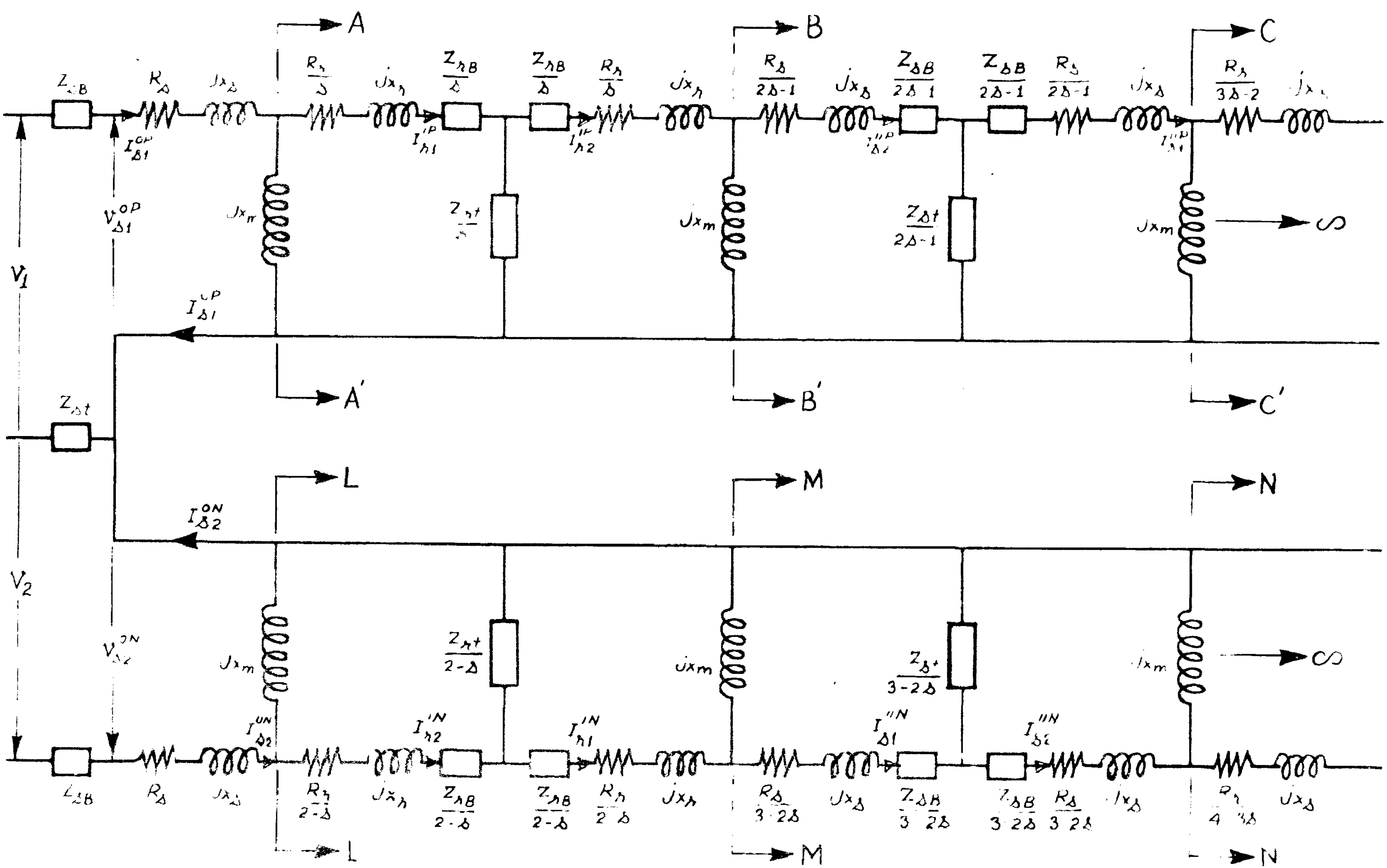


Fig. 3·8 — Complete Equivalent Circuit of Machine with Double Symmetrical Unbalance.

for the case of symmetrical unbalance becomes

$$\left.\begin{aligned} V_1 &= Z_{sB}\, I_{s1}^{OP} + Z_{st}\,(I_{s1}^{OP} + I_{s2}^{ON}) + V_{s1}^{OP} \\ V_2 &= Z_{sB}\, I_{s2}^{ON} + Z_{st}\,(I_{s1}^{OP} + I_{s2}^{ON}) + V_{s2}^{OP} \end{aligned}\right| \quad \ldots \quad 3.12$$

The equation 3.12 links the equivalent circuits for positive sequence voltage alone applied to stator terminals and the corresponding circuit for negative sequence voltage applied alone. The complete equivalent circuit of the machine with symmetrical unbalance impedances in both stator and rotor circuits is given in Fig. 3.8.

3.2. Solution of Equivalent Circuit

(i) Currents and Voltages

The discussions carried out in the previous sections show that the machine equivalent circuit consists of an infinite number of cascaded circuits when a positive or negative sequence voltage alone is applied at its stator terminals. These two infinite-networks get inter connected at their input ends, giving the overall equivalent circuit for the case of symmetrical unbalance being discussed. For the sake of convenience and clarity of expressions, the infinite-network presented by the machine when positive sequence voltage alone is applied, would be designated as 'infinite-network 1' and the network presented by the machine to negative sequence voltage alone would be designated as 'infinite-network 2'.

It is seen that in both the infinite networks the resistances throughout decrease monotonically with the increasing order of reflection and can be regarded as zero after a finite number of reflections for any specific case. The order of reflection at which this happens would not be in general be the same for the two infinite networks. Beyond this order of reflection, each of the infinite network becomes a reactive iterative circuit, and can be terminated by the characteristic impedance of one of its sections. The input impedance of each of these networks can now be computed. Let Z_{M1} and Z_{M2} be the input impedances of the 'infinite network 1' and 'infinite network 2' respectively. The sequence voltages and currents at the input of these two networks would be related as below:-

$$\left.\begin{aligned} V_{s1}^{OP} &= Z_{M1}\, I_{s1}^{OP} \\ V_{s2}^{ON} &= Z_{M2}\, I_{s2}^{ON} \end{aligned}\right\} \quad \ldots \quad 3.13$$

Substituting these values of V_{s1}^{OP} and V_{s2}^{ON} in equation 3.12, the positive and negative sequence components of the supply voltage can be expressed as

$$\left.\begin{aligned} V_1 &= (Z_{sB} + Z_{st} + Z_{M1})\, I_{s1}^{OP} + Z_{st}\, I_{s2}^{ON} \\ V_2 &= (Z_{sB} + Z_{st} + Z_{M2})\, I_{s2}^{ON} + Z_{st}\, I_{s1}^{OP} \end{aligned}\right\} \quad 3.14(a)$$

or therefrom

$$\left.\begin{aligned} I_{s1}^{OP} &= \frac{V_1 (Z_{sB} + Z_{st} + Z_{M2}) - Z_{st} V_2}{(Z_{sB} + Z_{st} + Z_{M1})(Z_{sB} + Z_{st} + Z_{M2}) - Z_{st}^2} \\ I_{s2}^{ON} &= \frac{V_2 (Z_{sB} + Z_{st} + Z_{M1}) - Z_{st} V_1}{(Z_{sB} + Z_{st} + Z_{M1})(Z_{sB} + Z_{st} + Z_{M2}) - Z_{st}^2} \end{aligned}\right\} \quad 3.14(b)$$

Knowing I_{s1}^{OP} and I_{s2}^{ON} from equation 3.14(b), current and voltage in each section of the two infinite networks of the equivalent circuit can now be computed. The stator and rotor phase currents and voltages can be calculated from equations 2.12 to 2.17 given in the previous chapter.

(ii) Torque

From the theory of induction machine it is known that torque in synchronous watts is equal to power crossing the air-gap.

$$\text{or} \quad T \text{ (Synchronous watts)} = I_2^2 \, Re(Z_2) \qquad \ldots \quad 3.15$$

where Z_2 is impedance of the equivalent secondary circuit and $Re(Z_2)$ is its real part (effective resistance $\leftrightarrow R_2$).

In the case of double unbalance under discussion, the power drawn from either positive or negative sequence voltage applied at stator terminals, crosses the air-gap for infinite number of times. A torque is developed at each crossing. The net machine torque would thus be sum of infinite series - one contributed by the initial positive sequence voltage and the other by the initial negative sequence voltage applied at the stator terminals. Each individual component of these torques will follow the law of equation 3.15.

Let us consider first, the torques created by initial positive sequence voltage at the stator terminals. The 'infinite network 1' in Fig. 3.8 is sectioned at A, B, C, D, ... which are locations of air-gap on the equivalent circuit. Let Z_A, Z_B, Z_C, ... denote the equivalent impedances to the right of these sections. Referring to the flow chart No. 1 of chapter 2, it is to be noted that power is transferred across the air-gap grom stator to rotor with a magnetic field revolving in the positive direction, while the power across the air-gap from rotor to stator is transferred by a field revolving in the negative direction, such that all these torques will have the same algebraic sign. Thus the total torque produced by the positive sequence voltage is

$$T_1(\text{Syn. watts}) = (I_{r1}^{'P})^2 \, Re(Z_A) + (I_{s2}^{''P})^2 \, Re(Z_B) + (I_{r1}^{'''P})^2 \, Re(Z_C) + (I_{s2}^{'vP})^2 \, Re(Z_D) + \ldots \quad 3.16$$

Similarly the 'infinite network 2' with the negative sequence voltage applied at the stator terminals, can be sectioned at L, M, N, P, ... and the corresponding impedances be Z_L, Z_M, Z_N, Z_P, ... The torques acting on the rotor would be in the negative sequence direction and can be algebraically added up. Therefore

$$T_2(\text{Syn. watts}) = (I_{r2}^{'N})^2 \, Re(Z_L) + (I_{s1}^{''N})^2 \, Re(Z_M) + (I_{r2}^{'''N})^2 \, Re(Z_N) + (I_{s1}^{'vN})^2 \, Re(Z_P) + \ldots \quad 3.17$$

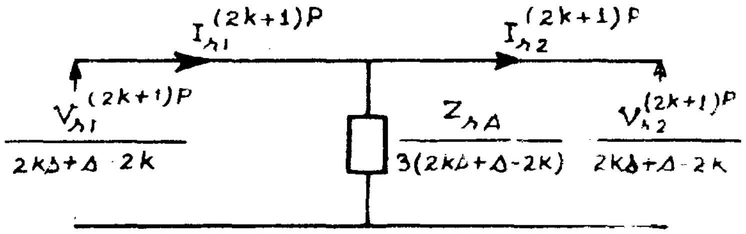
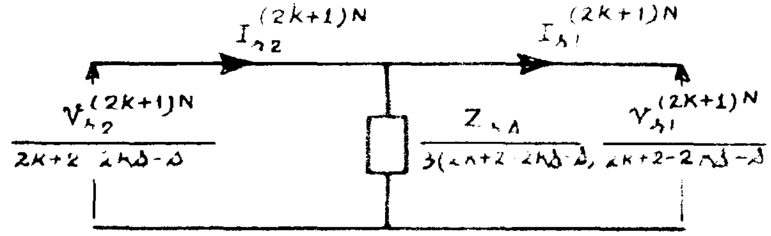
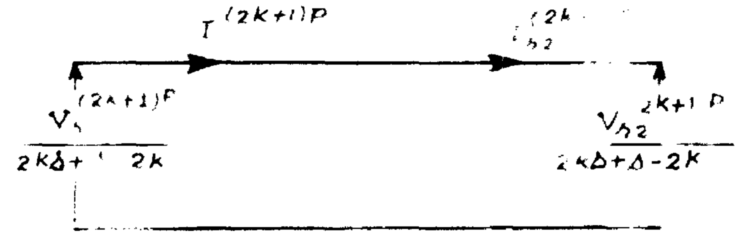
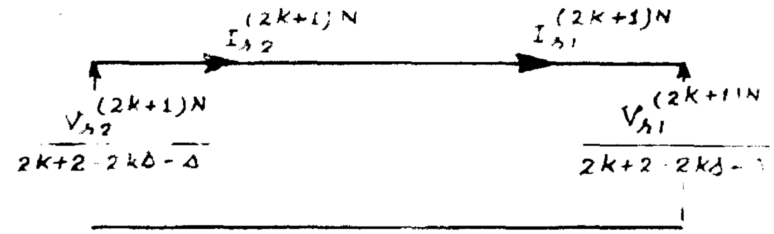
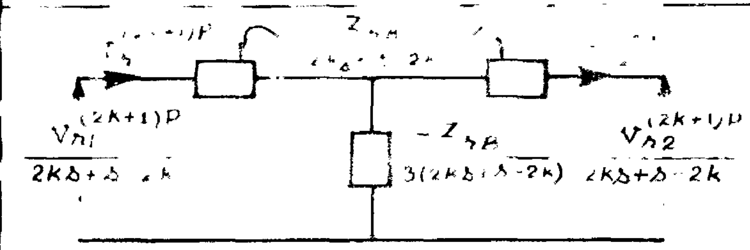
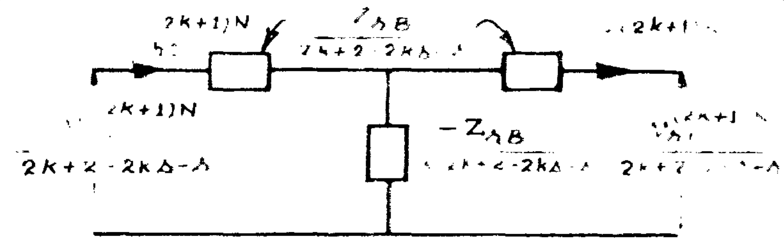
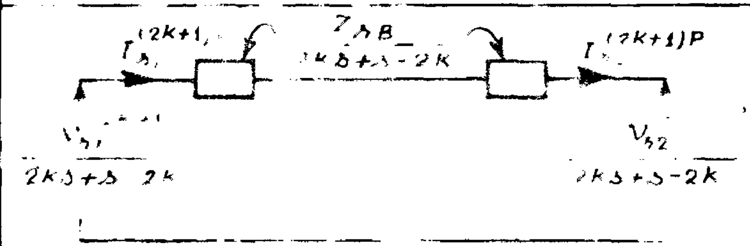
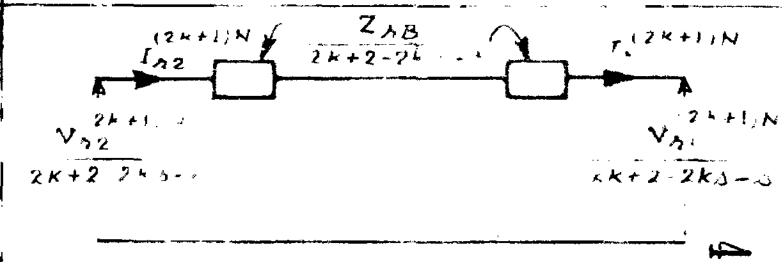
The net torque acting on the rotor in positive sequence direction is

$$T_{net} = (T_1 - T_2) \quad \ldots \quad 3.18$$

3.3. <u>Specific Cases of Double Unbalance</u>

A slip-ring induction motor is usually started with external resistances connected in rotor circuit. These resistances are short-circuited during acceleration. It may happen

TABLE - 3·1

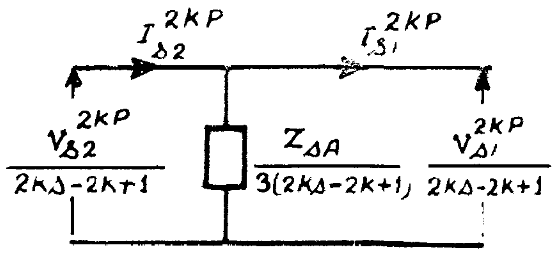
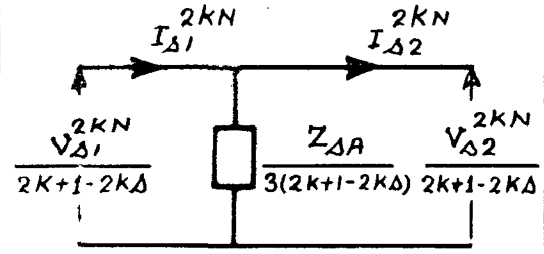
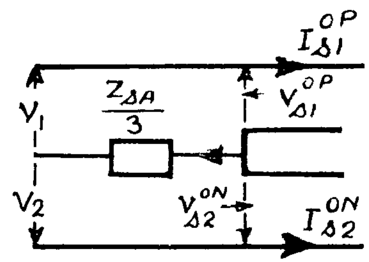
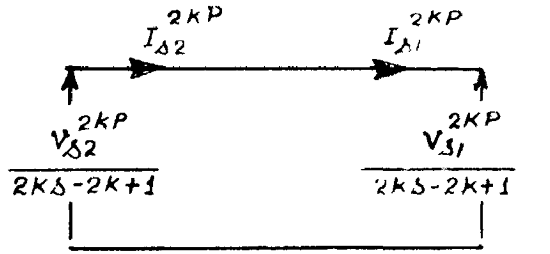
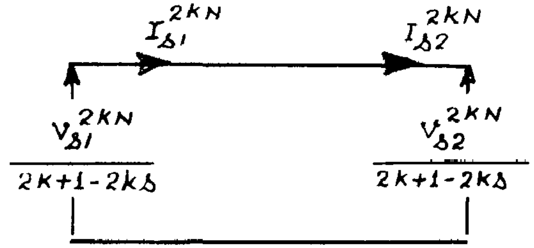
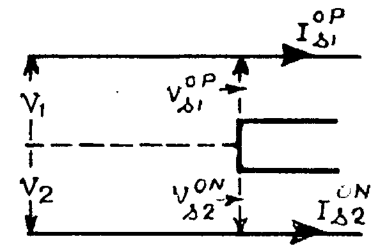
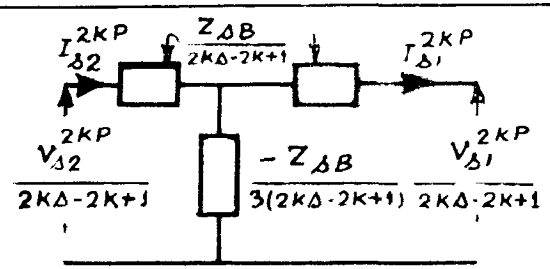
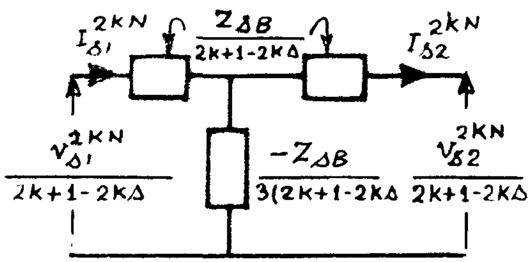
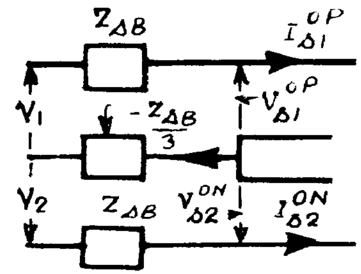
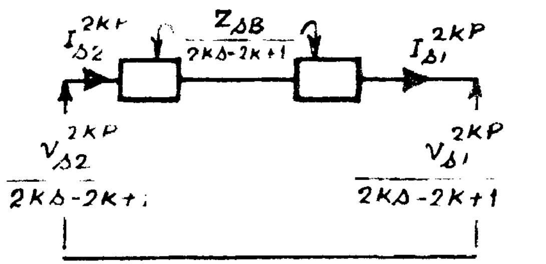
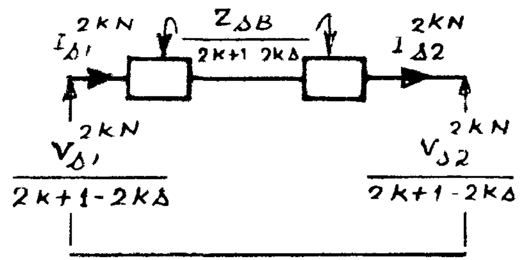
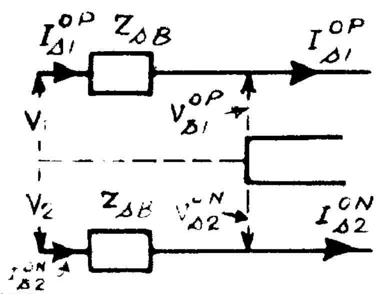
S. No.	Rotor Impedances	Sequence Impedances	Connecting Circuit at $(2k+1)^{th}$ Reflection at Rotor Terminals for 'Infinite Network-No. 1'	Connecting Circuit at $(2k+1)^{th}$ Reflection at Rotor Terminals for 'Infinite Network-No. 2'
1	$Z_{rA}, 0, 0$	$Z_{r0}=Z_{r1}=Z_{r2}=\frac{1}{3}Z_{rA}$ $Z_{rt}=\frac{Z_{rA}}{3}$ $Z_{rB}=0$		
2	$\infty, 0, 0$	$Z_{r0}=Z_{r1}=Z_{r2}=\infty$ $Z_{rt}=\infty$ $Z_{rB}=0$		
3	$0, Z_{rB}, Z_{rB}$	$Z_{r0}=\frac{2Z_{rB}}{3}$ $Z_{r1}=Z_{r2}=Z_{rt}=-Z_{rB}$ [illegible] $Z_{rB}=Z_{r}$[illegible]		
4	∞, Z_{rB}, Z_{rB}	Z_{r0} [illegible] $Z_{r1}=Z_{r2}=Z_{rt}=\infty$ [illegible]		

sometimes that some of the short-circuiting switches fail to operate and thus leaving unbalanced resistances in the rotor circuit. Often additional resistances are intentionally introduced in the rotor circuit in order to obtain speek-control with a desired speed-torque characteristic. The majority of the practical problems of speed-control and mal-function of short-circuit switch fall in the category of symmetrical rotor unbalance, a detailed study of which is made in the next chapter. In addition to the unbalance in rotor impedances, it may also happen that on the stator side a fuse blows off, a contact-link fails or a link is weak. Such an eventuality would create a stator impedance unbalance simultaneously with rotor impedance unbalanced. When one fuse blows off with other two intact, it causes a symmetrical stator impedance unbalance while when two fuses blow off, it is a trivial case and the machine stops. The stator impedance unbalance caused by weak contact-link is symmetrical, and in majority of cases of all probability one link would be faulty at a time.

Sometime the stator impedance unbalance may also be introduced by the resistances employed to limit the supply voltage. Such a stator unbalance is also usually symmetrical since one line resistance would be faulty at a time.

As discussed above the great majority of cases of double unbalance of an induction machine lie in the category of symmetrical double unbalances. Various symmetrical stator and rotor unbalances along with their relevant connecting circuits are given in Tables3.1 and 3.2. There can be various possible combinations of such double unbalances. From these tables, the

TABLE 3·2

S. No.	Stator Impedances	Sequence Impedances	Connecting-Circuit at $2k^{th}$ Reflection at Stator Terminals for "Infinite Network No. 1"	Connecting-Circuit at $2k^{th}$ Reflection at Stator Terminals for "Infinite Network No. 2"	Inter-Connecting Circuit for Two Infinite Networks at Input Terminals.
1	$Z_{\Delta A}$, 0, 0	$Z_{\Delta 0} = Z_{\Delta 1} = Z_{\Delta 2} = \frac{1}{3} Z_{\Delta A}$ $Z_{\Delta 2} = \frac{Z_{\Delta A}}{3}$ $Z_{\Delta B} = 0$			
2	∞, 0, 0	$Z_{\Delta 0} = Z_{\Delta 1} = Z_{\Delta 2} = \infty$ $Z_{\Delta t} = \infty$ $Z_{\Delta B} = 0$			
3	0, $Z_{\Delta B}, Z_{\Delta B}$	$Z_{\Delta 0} = \frac{2}{3} Z_{\Delta B}$ $Z_{\Delta 1} = Z_{\Delta 2} = Z_{\Delta t} = \frac{-Z_{\Delta B}}{3}$ $Z_{\Delta B} = Z_{\Delta B}$			
4	∞, $Z_{\Delta B}, Z_{\Delta B}$	$Z_{\Delta 0} = Z_{\Delta 1} = Z_{\Delta 2} = \infty$ $Z_{\Delta t} = \infty$ $Z_{\Delta B} = Z_{\Delta B}$			

equivalent circuit for any symmetrical double unbalance can be derived.

3.4. Experimental Verification

The machine used for experimental work has the following data. These data have been found experimentally as given in Appendix I.

Machine specifications

6.5 KW , 8.8 CV , 400-440 V , 13.2 A ,
Cos ϕ = 0.83 , 1380 T/min. , 50 c/s ,
3 phase , Star connected, Rotor 102 V,
45.5 A , slip-ring, No. ? 84991 ,
Type AV 2524-1 V
A.C.E.C. Welco Charleroi, Belgium.

The machine is coupled with a D.C. dynemometer.

Machine constants

R_s = 0.1715 p.u. ; X_s = 0.286 p.u. ; X_m = 8.32 p.u.
R_r = 0.269 p.u. ; X_r = 0.286 p.u.
Base Voltage = 115.5 V ; Base Impedance = 8.75ohms.
Base Current = 13.2 A ; Base Torque = 22.4 ft. lbs.

The external impedances in stator and rotor circuits used for experimental work are limited to resistances only, which is the normal practical case.

In all experimental work reduced balanced voltage of 200 Volts is applied to the stator for the reasons given below:

It has been found that positive and negative sequence magnetic fields, which are proportional to the sequence voltages, exist simultaneously in the machine. These fields rotate with different velocities and in opposite directions. At each instant some fields are aiding and others are opposing. The sum of various components of fields may exceed the normal maximum flux density in the machine, though the individual components may be of normal magnitude only. It may result in a gross saturation and therefore considerable increase in magnetizing current and deterioration of wave form. This happens when the sum of sequence voltages is more than the normal phase voltage. It is also expected that the high voltages are induced in the unbalanced operation of the machine, which may cause break down of the insulation. More-over for unbalanced operation over the complete speed range with normal applied voltage a very high current would flow at large slips, which would cause excessive temperature-rise and insulation may be damaged. Therefore the machine cannot be operated at normal voltage and hence it is essential to test the machine at reduced voltage.

The various combinations of symmetrical double unbalance of stator and rotor impedances are virtually unlimited. Therefore, it is decided to study the operation in the cases given in Table 3.3.

Table No. 3.3

Number	Z_{sA}	Z_{sB}	Z_{sC}	Z_{rA}	Z_{rB}	Z_{rC}
Case No. 1	0.332 pu	0	0	0.66	0	0
Case No. 2	0.07	0	0	∞	0	0
Case No. 3	∞	0	0	0.66	0	0
Case No. 4	∞	0	0	∞	0	0

Speed-Torque Characteristics

Double Unbalance $R_{\Delta A} = 0{\cdot}332$, $R_{\lambda A} = 0{\cdot}66$ p.u.

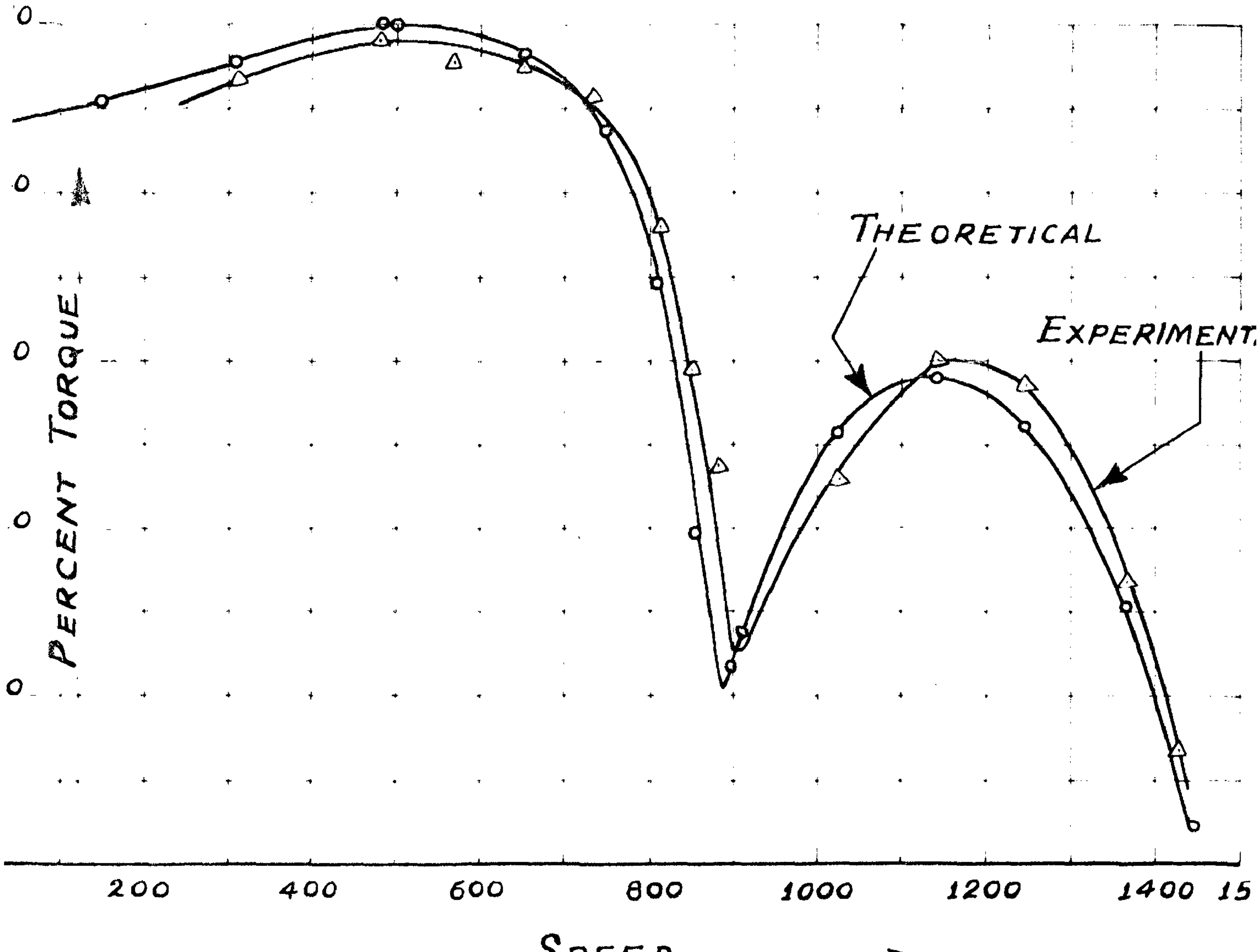


Fig. 3·9

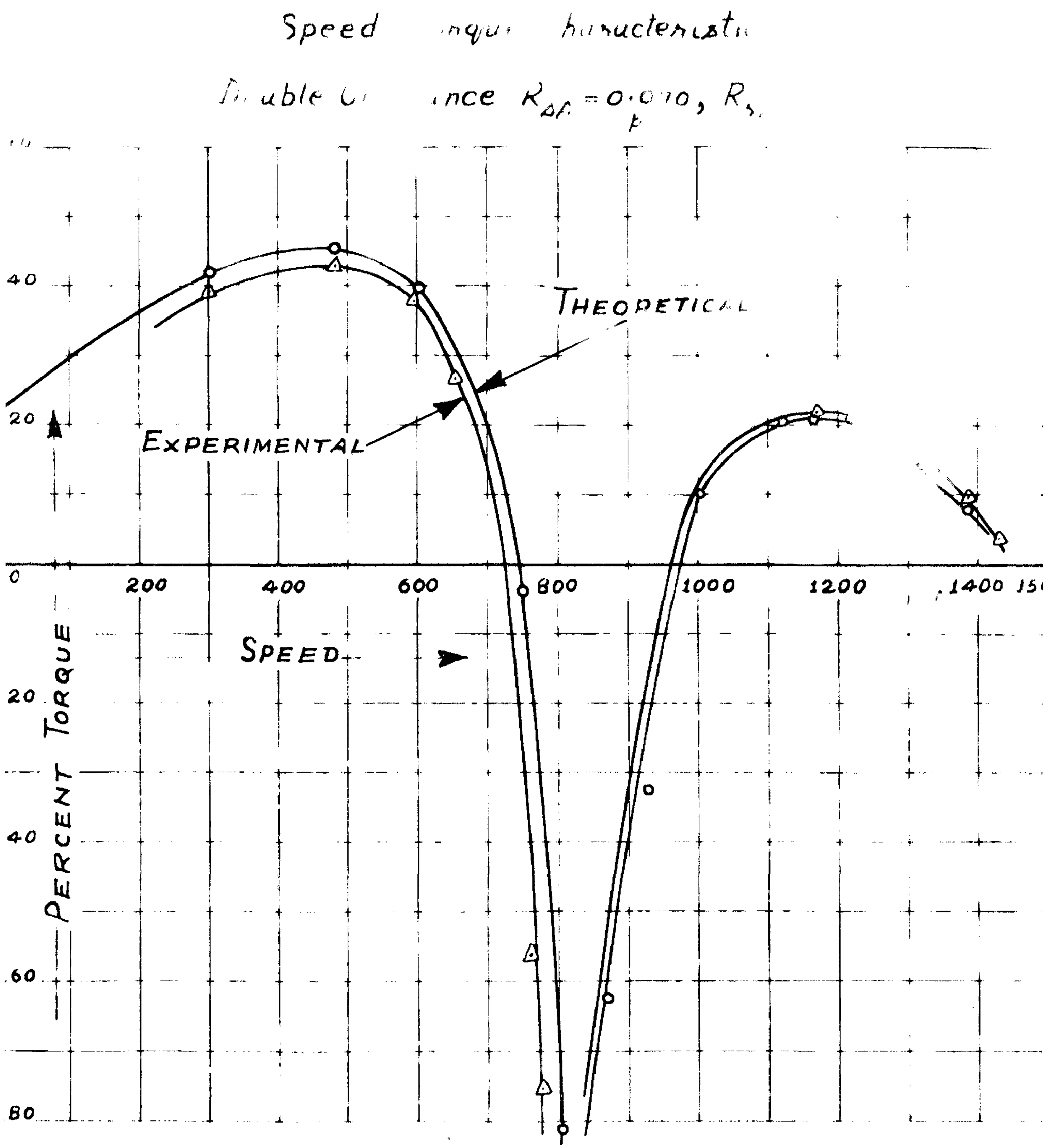

Speed
ance $R_{AR} = 0.010$, R
THEORETICAL
EXPERIMENTAL
SPEED
PERCENT TORQUE
0
200
400
600
800
1000
1200
1400
40
20
20
40
60
80

Fig. 3·10

Speed-Torque (Components) Characteristics

Double Unbalance $R_{sA} = \infty$, $R_{rA} = 0.66$ p.u.

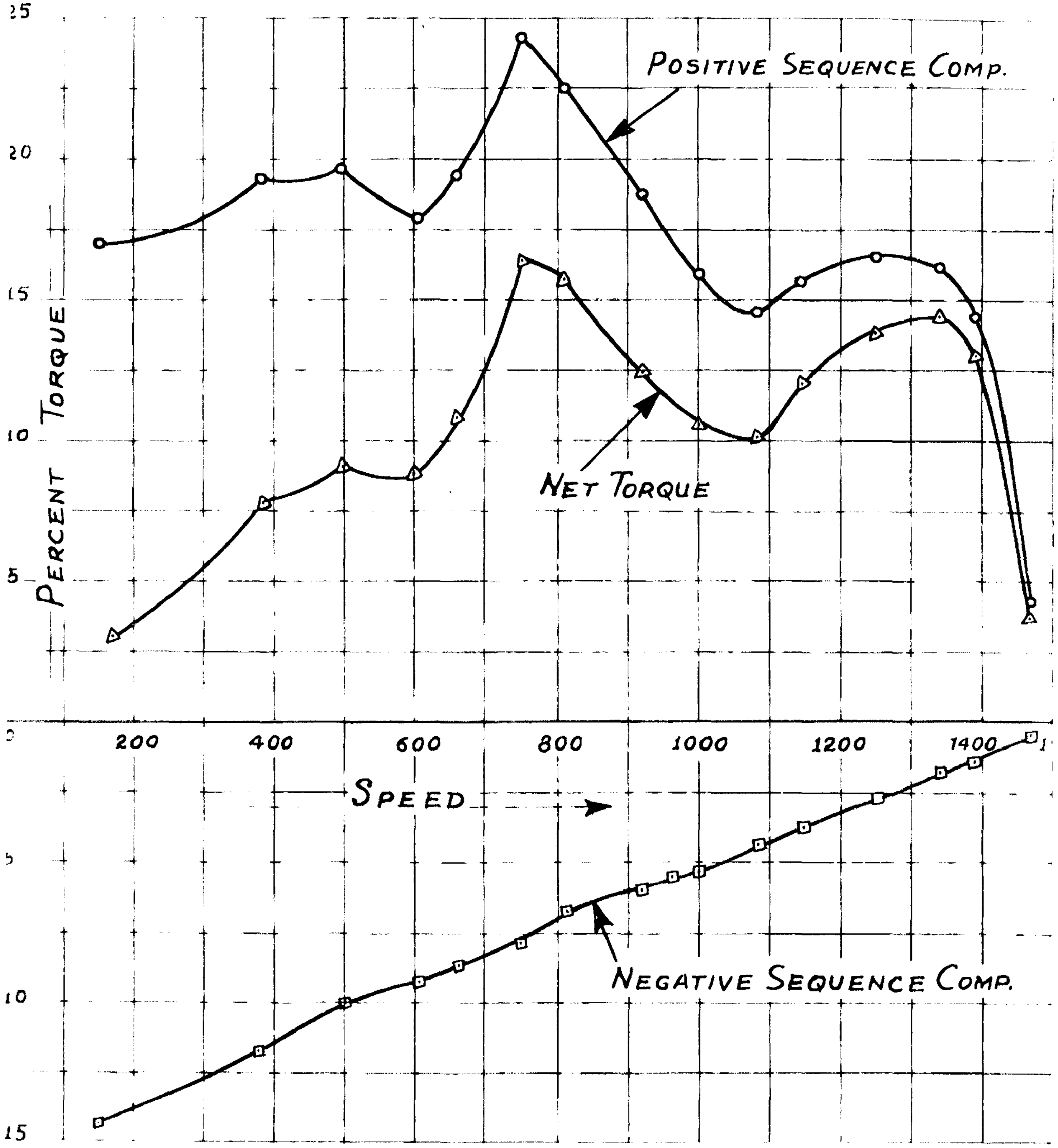


Fig. 3·11 (a)

Speed–Torque (Net Torques) Characteristics

Double Unbalance $R_{sA} = \infty$, $R_{rA} = 0.66$ p.u.

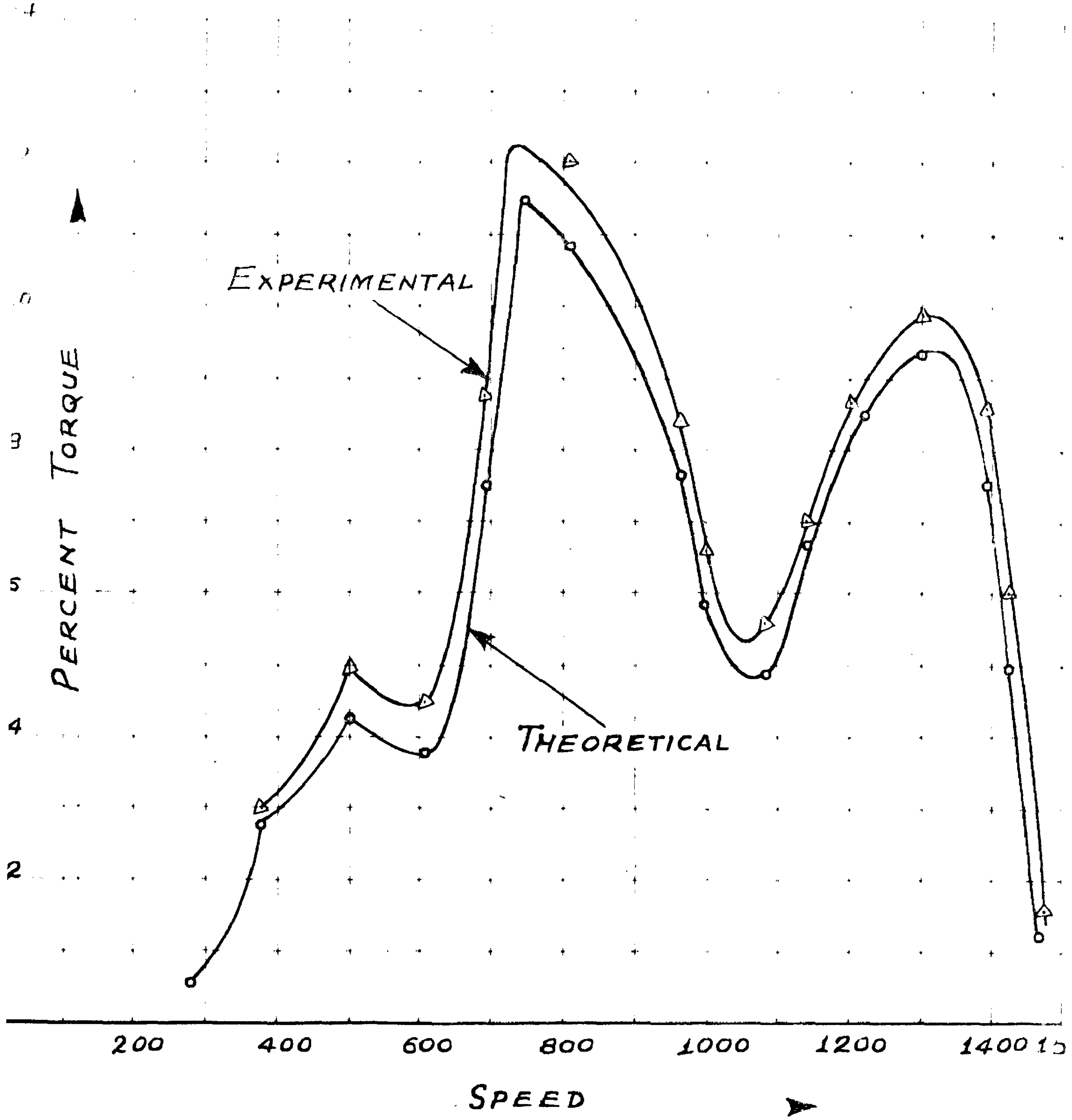


Fig. 3.11(b)

Speed-Torque Characteristics

Double Unbalance $R_{\Delta A} = \infty$, $R_{rA} = \infty$

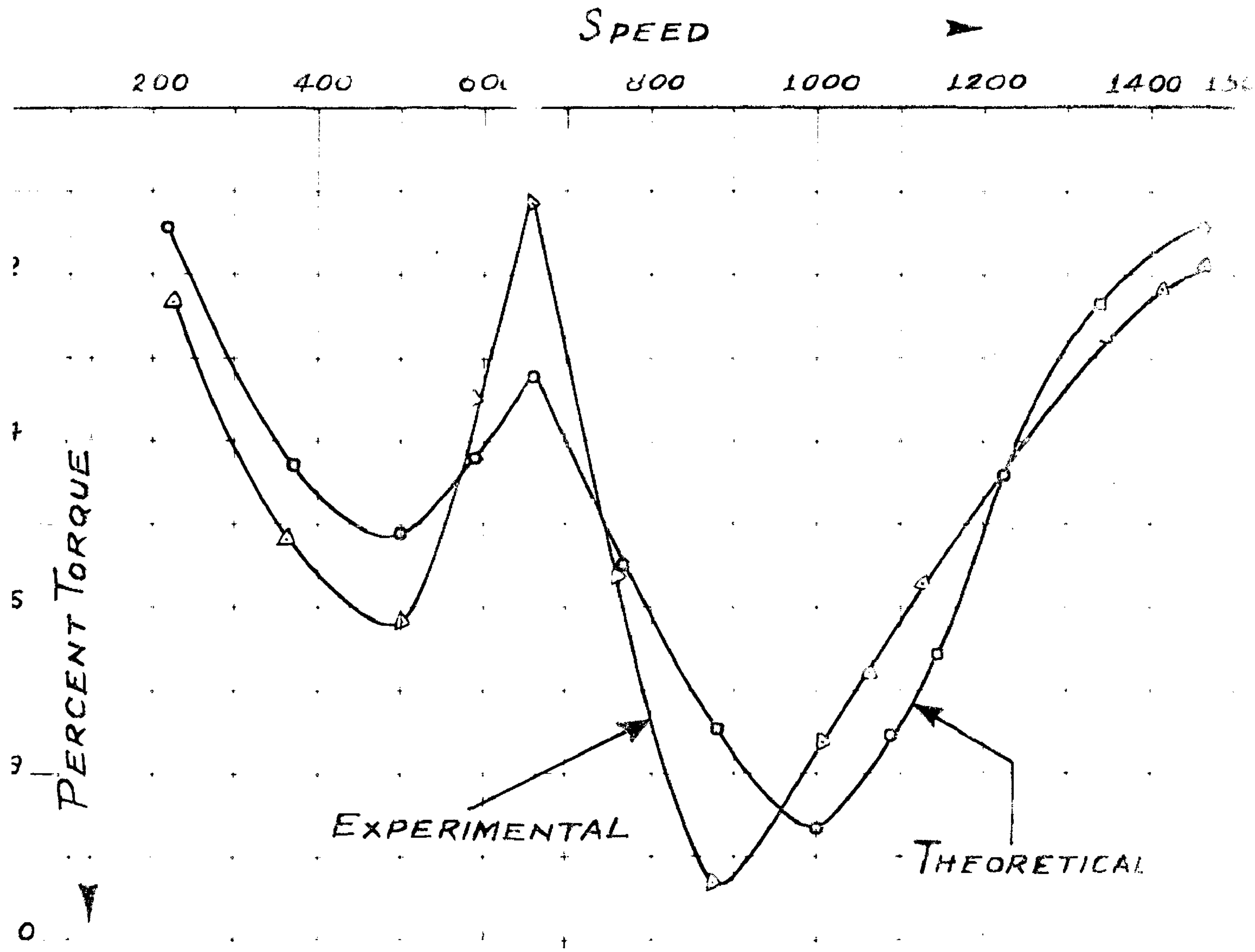


Fig. 3·12

Speed – Stator Current Characteristics

Double Unbalance — $R_{\Delta A} = 0 \cdot 332$ p.u.

$R_{rA} = 0 \cdot 66$ p.u.

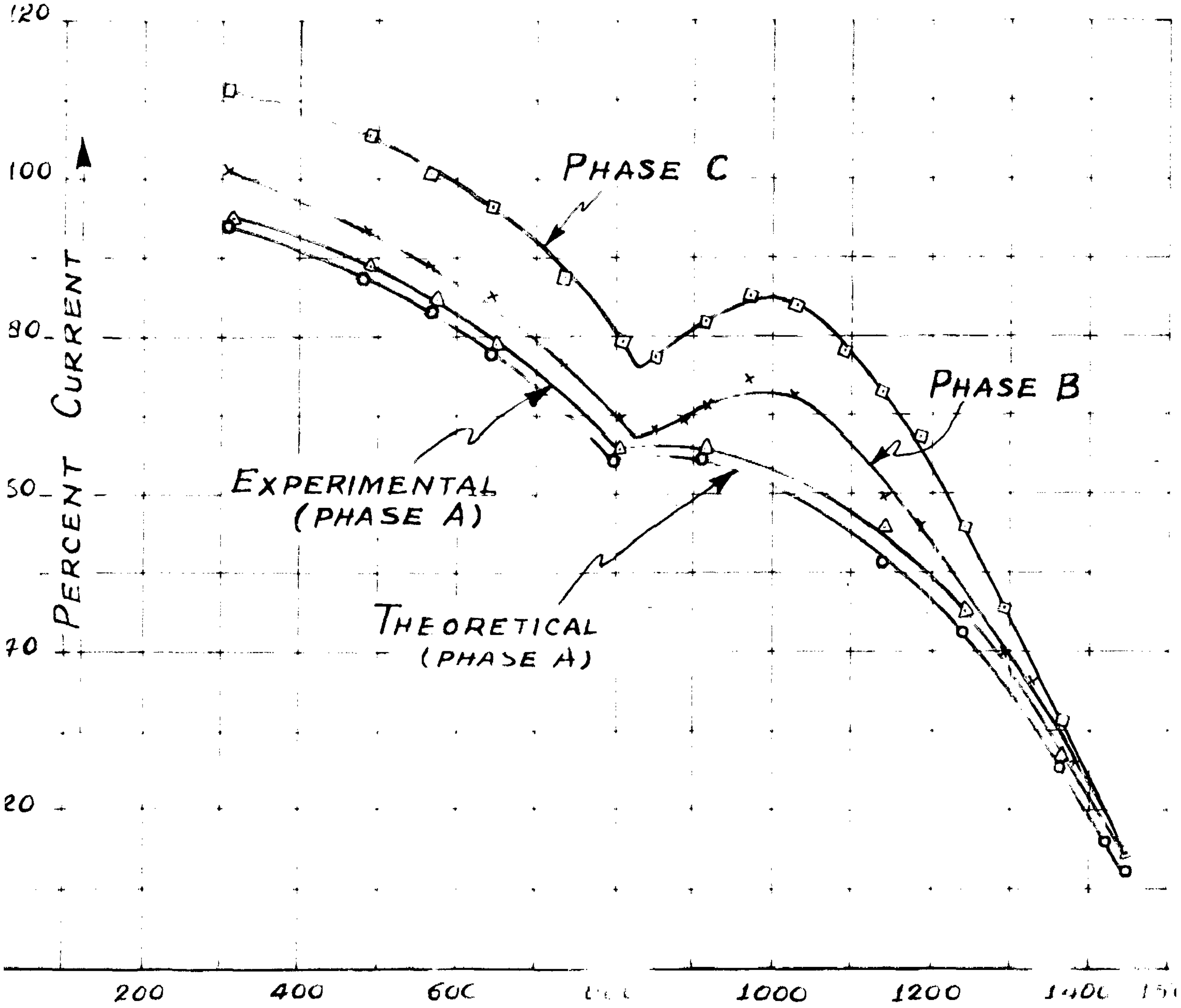


Fig. 3.13

Speed-Current (Stator) Characteristics

Double Unbalance $R_{sA} = 0.332$ p.u. &

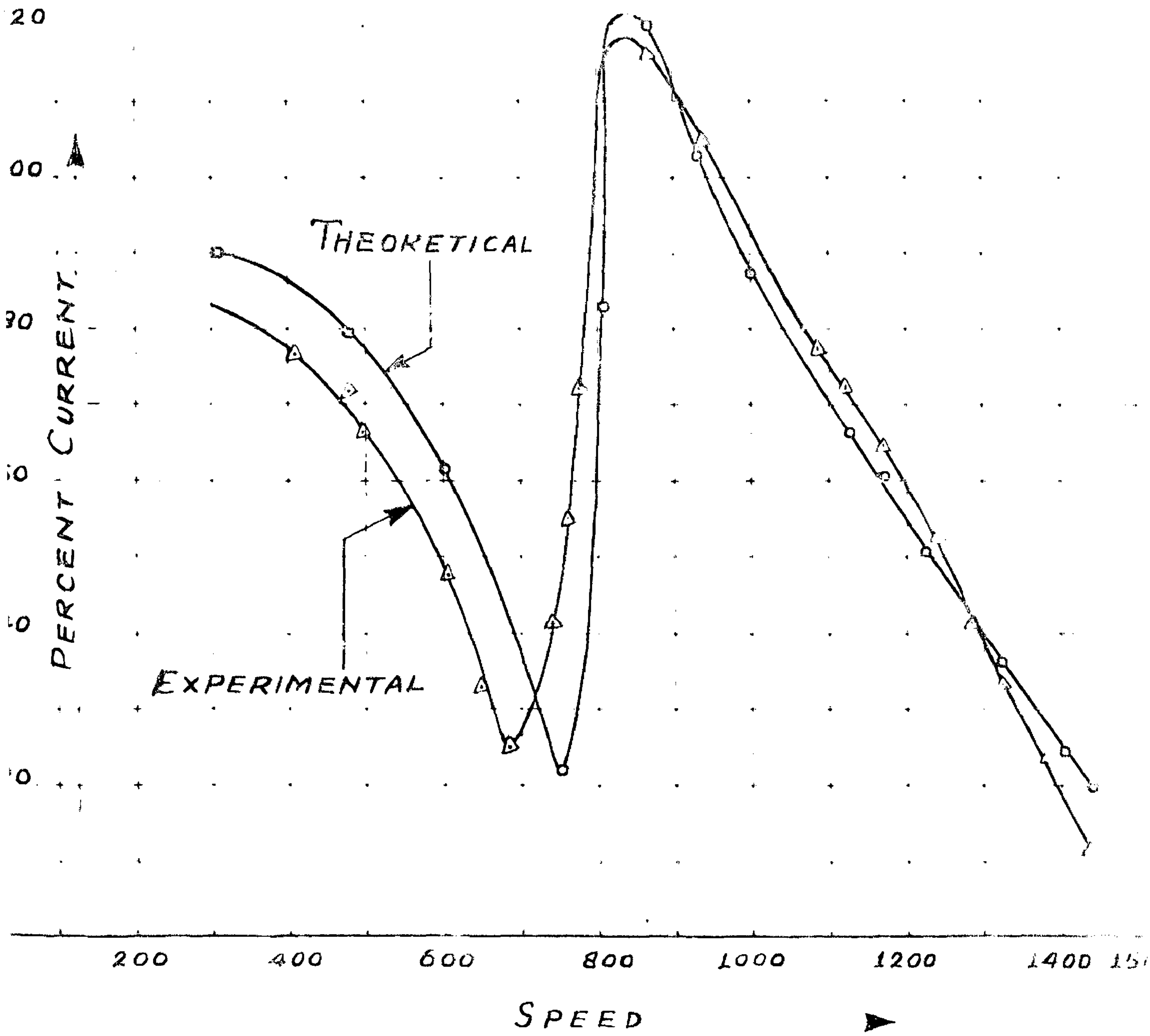


Fig. 3.14

Speed-Current (Stator) Characteristics

Double Unbalance $R_{sA} = \infty$, $R_{rA} = 0{\cdot}66$ p.u.

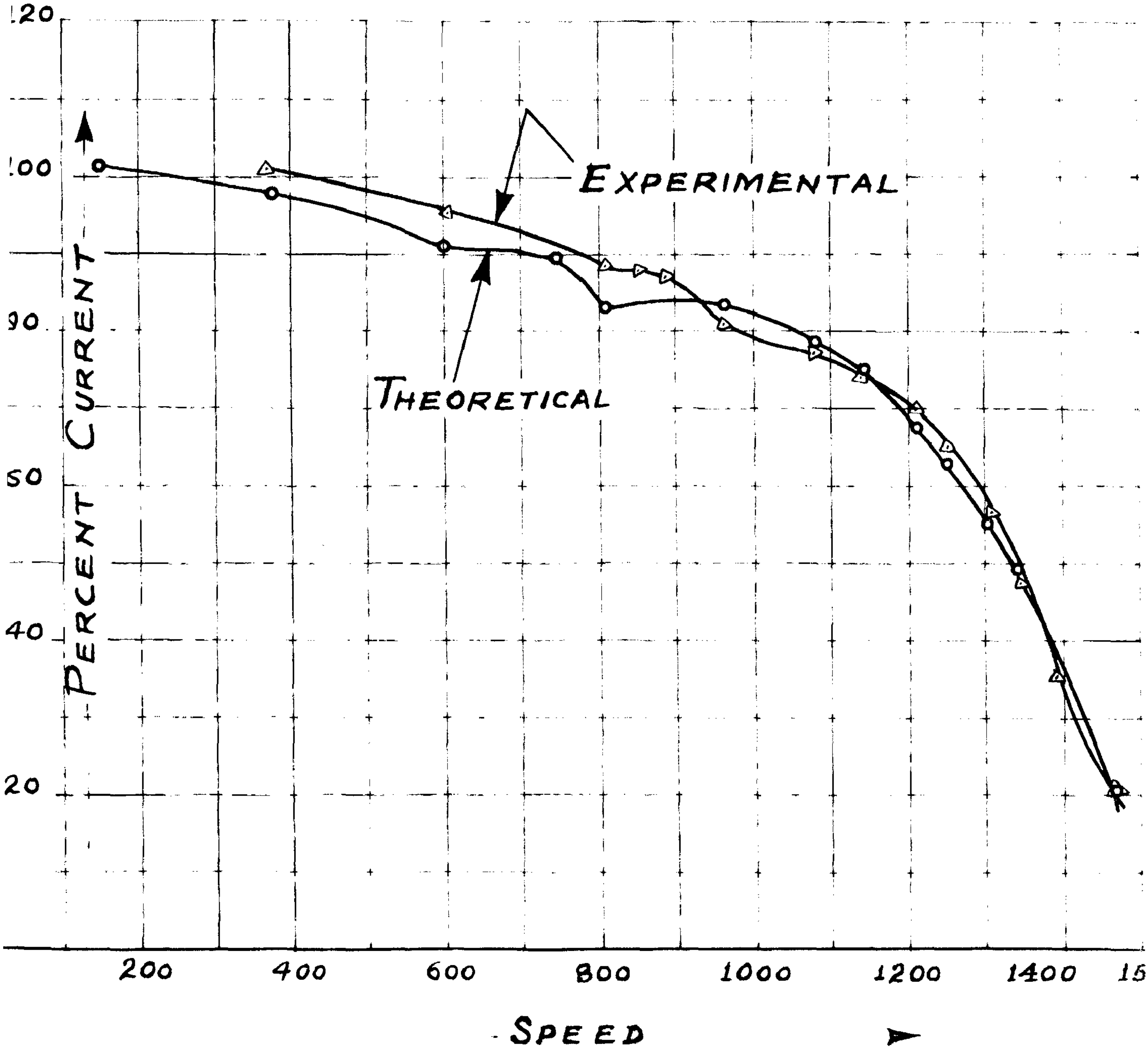


Fig. 3·15

Speed-Stator Current Characteristics

Double Unbalance $R_{sA} = \infty, R_{rA} = \infty$

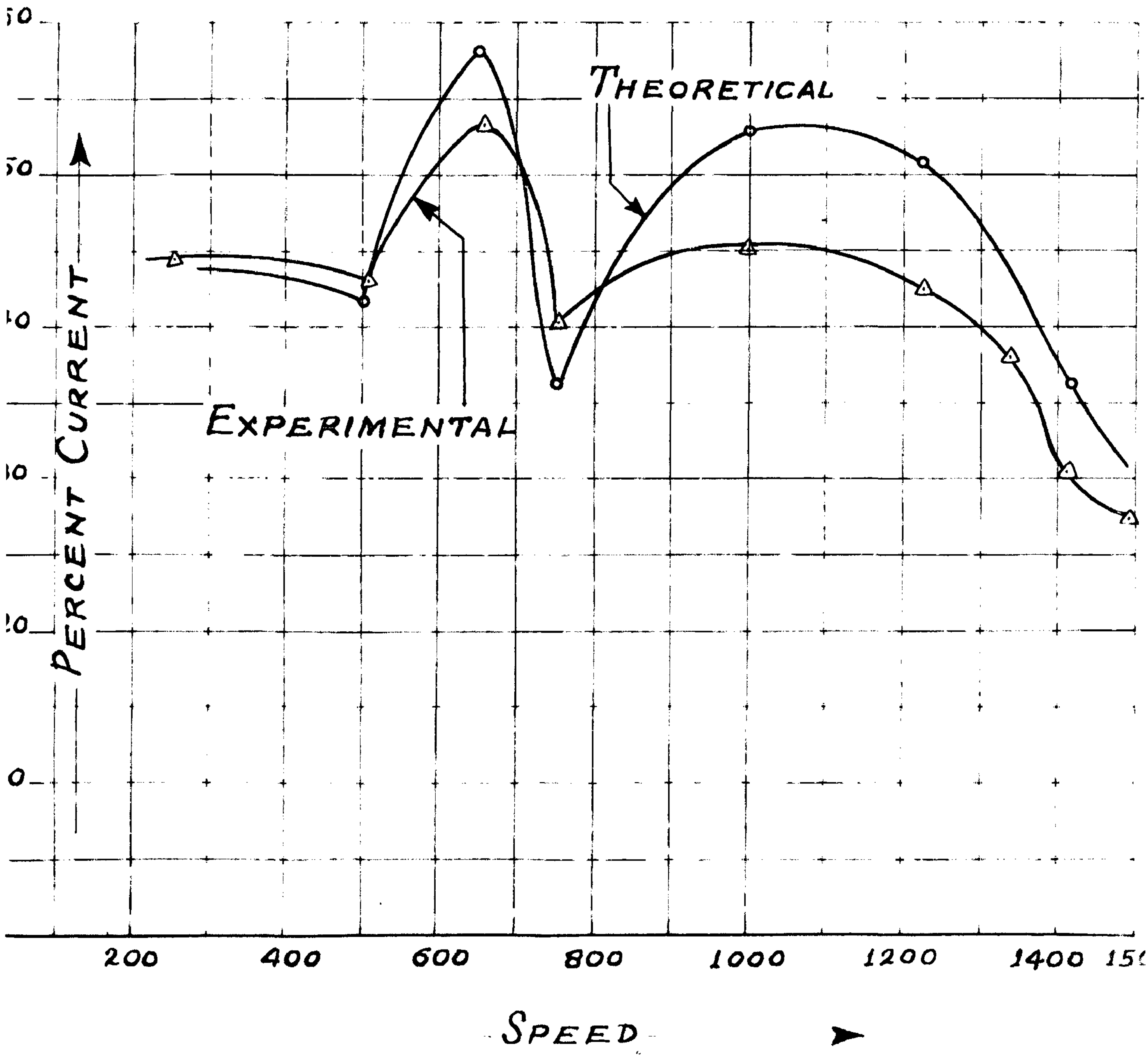


Fig. 3.16

Rotor - R.M.S. Voltage

Double Unbalance $R_{sA} = \infty$, $R_{rA} = \infty$

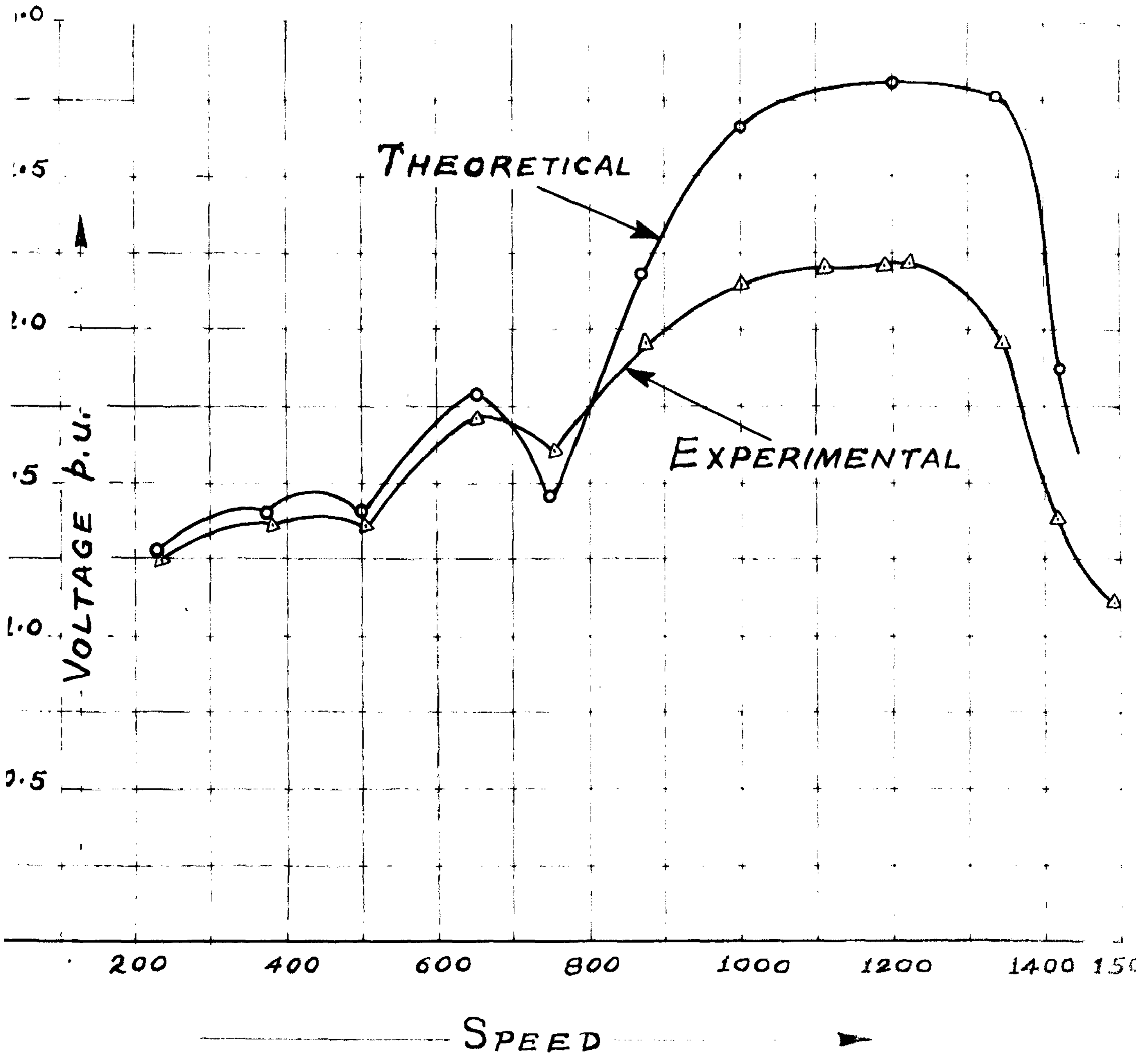


Fig. 3·17

Double Unbalance $R_{bA} = \infty, R_{rA} = \infty$

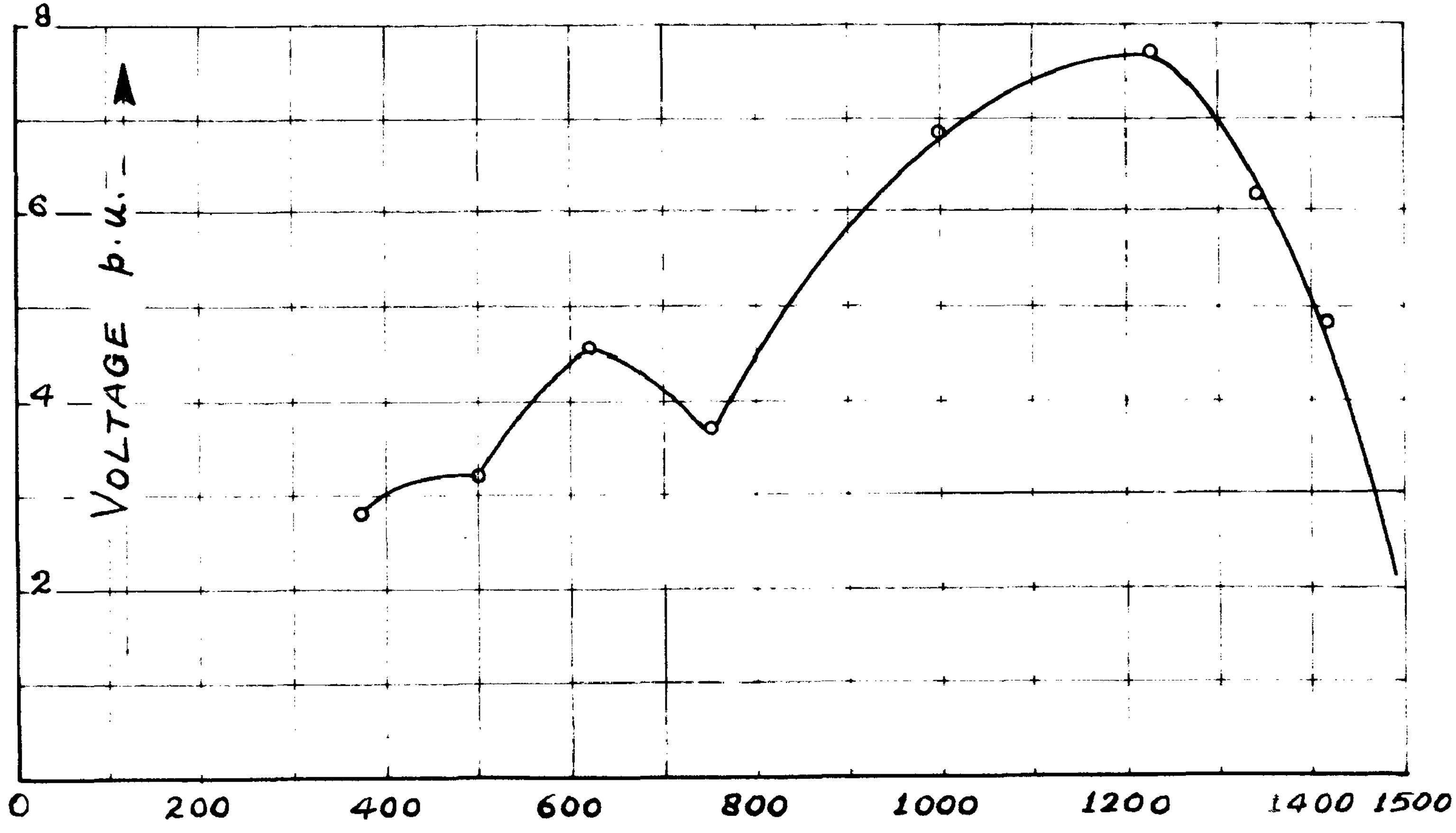


Fig. 3.18

The various currents, voltages and torque are measured over the complete speed range as given in Appendix IV for the different sets of unbalances. The results are predetermined also by solving the equivalent circuits. While calculating it is observed that a wide range of frequencies is present in the machine. A suitable linear approximation has been found experimentally and is used for finding the resistance offered by stator and rotor circuits at different frequencies (Appendix I). It has also been assumed that the presence of other frequency currents does not affect the resistance seen for any particular frequency current. Speed torque and speed current curves have been calculated for all the cases. For case No. 4 peak and r.m.s. voltages of stator and rotor have also been calculated.

The experimental and calculated results are plotted in Figs. 3.9 to 3.18 for all these cases. It is found that the results agree with in the difference of 8-12%. It has been noted that it is difficult to calculate the torque and current accurately at slips close to $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, ... where the equivalent circuit becomes open at a particular reflection. The maximum error occurs at unstable and negative torque regions. The various reasons for difference in calculated and experimental results are attributed to the following factors:-

(1) The effect of various frequencies on the external resistances is not taken into account.

(2) It is not possible to estimate iron losses to a reasonable degree of accuracy for such a wide range of frequencies.

(3) The winding resistance may be affected by the presence of frequencies other than the one for which resistance

is considered.

(4) The magnetic circuit may not remain unsaturated.

(5) The change of contact resistance between slip ring and brush is not taken into account.

A special note is made for case No. 4 in which both stator and rotor circuits have maximum unbalance i.e. one line is open in both the circuits. In this case the torque is negative at all slips. The voltages in rotor and stator circuits are more compared to other cases. From calculations it is found that the maximum value of ratio of peak voltage to normal voltage is 7.7 at a slip of 0.18 and the maximum value of ratio of r.m.s. voltage to normal voltage is 2.8 at a slip of 0.20. The r.m.s. voltage is measured with the help of moving iron voltmeter. While measuring r.m.s. voltage in the rotor circuit, it is observed that needle oscillates and the average value is recorded. The oscillations are due to the presence of low frequencies voltage in the voltage wave. Thus the experimental rotor voltage is always less than the calculated value because the low frequency voltage is not read by voltmeter.

The speed-torque characteristics may be explained qualitatively with the help of equivalent circuits and the torque equations 3.16 to 3.18.

Case I Resistance of 0.332 p.u. and 0.66 p.u. have been connected in one line of stator and rotor respectively. The results calculated from the equivalent circuits are tabulated in Table 3.4. It is found that the impedance of 'infinite network 2' i.e. Z_{M2} is fairly constant throughout and its value is sufficiently greater than $\frac{Z_{sA}}{3}$ = 0.11 p.u. Therefore 'infinite network 2' draws comparatively very low current and produces negligible torques. Thus the net torque is produced soley by 'infinite network 1' of the equivalent circuit. For 'infinite network 1', the torque contributed by the first term of equation 3.16 is always positive and all other terms contribute negative torque upto 0.5 slip. The second term of torque equation contributes small torque upto $s = 0.1$ and for $s > 0.25$ it contributes sufficient torque which is always in negative direction. Thus the net torque increases first and then decreases. At $s = 0.5$, the network at second reflection becomes open. At this point all torque components except first term vanish. For $s > 0.5$, the second term also becomes additive and other terms are negative. Similarly at $s = \frac{2}{3}$ and $\frac{3}{4}$, the network becomes open at third and fourth reflection and torque contribution by third and fourth term becomes positive after these slips respectively. But it is observed that for $s > 0.666$, the input impedance is not affected much whatever be the impedance of circuit after third reflection. Thus third and fourth terms do not contribute much in net torque.

Case II A resistance of 0.07 p.u. is connected in one line of stator and one line of rotor is open. In this case Z_{M1} and Z_{M2} are larger because shunt impedance of connecting-circuit at rotor is infinite. Z_{M2} is very large compared to $\frac{Z_{sA}}{3} = 0.0233$ p.u. Therefore parallel combination of Z_{M2} and $\frac{Z_{sA}}{3}$ can be replaced by $\frac{Z_{sA}}{3}$. Thus the torque is contributed by 'infinite network 1' only. The torque contributed by second term equation 3.16 predominates and is negative upto $s = 0.5$, which makes the net torque negative at slips close to 0.5. For $s > 0.5$, the second term is positive and thus net torque becomes positive. The operation is unstable between the slip 0.5 and 0.32 (speed 970 to 750 r.p.m.)

Case III One line of stator is open and a resistance of 0.66 p.u. is in one line of rotor. The shunt resistance of connecting-circuit at stator is infinite. In this case Z_{M1} and Z_{M2} become connected in series and stator positive sequence current and negative sequence current are equal but opposite in direction. Z_{M1} decreases with the increase of slip while Z_{M2} remains fairly constant. At $s = 1$, Z_{M1} and Z_{M2} are equal, and the voltage across 'infinite networks 1 and 2' are equal. The torques produced by both the networks are equal and opposite. Thus the starting torque is zero. At other slips, the torque contributed by 'infinite network 1' has various components as discussed earlier in cases I and II. The torque contributed by first term is always positive and the other terms give negative torque upto $s = 0.5$. At $s = 0.5$, the network opens at second reflection. Thus there is no component which gives negative torque and therefore positive peak occurs at this slip. For $s > 0.5$, the second term also contributes positive torque while other terms give negative torque. Similarly at $s = \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$, etc. the network opens at third, fourth and fifth reflections respectively and contains all positive torque terms and no negative term. Thus torque peaks occur at these slips.

Unlike previous case, the torque contributed by the 'infinite network 2' is not negligible but increases with slip. This torque is always negative. Therefore net torque reduces, but net speed-torque characteristic has also peaks and humps as discussed above.

Comparing this case with the previous one where stator circuit contains low impedance, the 'infinite network 2' draws very small current and gives negligible torque. If the external impedance chosen is such that 'infinite network 2' draws sufficient current, this contributes appreciable torque which is in the negative direction. The net torque reduces. Therefore if the external impedance is varied from zero to infinite, a family of curves can be obtained.

It is observed that Gorges' Phenomenon or stable half speed operation occurs with double unbalance but it always occurs at more than half speed and not at half speed as in the case of rotor unbalance only.

Voltage Speed Characteristics:- These could also be discussed qualitatively. At $s = 0$, the rotor impedance of 'infinite network 1' is infinite at first reflection and circuit becomes open. Thus there is only stator fundamental voltage and other voltages are zero. The voltages in sections of 'infinite network 2' are small in magnitude. Thus resultant voltage is relatively low at the start. As s increases, the components of voltages in all the sections of two infinite networks increase and thus the resultant voltage increases. At $s = 0.5$, stator branch of 'infinite network 1' at second reflection opens and all component voltages vanish after that point and consequently lowering the resultant voltage. Similar reasoning would show dips in the voltage speed curve at $s = \frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$ etc. It is noted that the component voltages have sufficiently large values at the points where the circuits are terminated and voltages converge slowly. To calculate voltage to a good degree of accuracy further sections must be considered using the current solution obtained by the termination.

Table 3.4

Double Unbalance (Case I) $R_{sA} = 0.332$, $R_{rA} = 0.66$ p.u.

S.No.	1	2	3	4	5
Speed	1450	1138	915	855	810
Slip	0.0333	0.208	0.39	0.43	0.46
Z_{M1}	4.27 + j5.4	1.628 + j1.01	1.087 + j1.03	1.225 + j0.954	1.16 + j0.78
Z_{M2}	0.415 + j0.576	0.422 + j0.585	0.449 + j0.582	0.45 + j0.61	0.47 + j0.749
R_A	10.62	1.685	1.082	1.224	1.11
R_B	- 0.285	- 0.42	- 1.08	- 1.65	- 2.82
R_L	0.252	0.26	0.288	0.291	0.309
I'^{P}_{r1}	0.0868	0.465	0.592	0.565	0.636
I''^{P}_{s2}	-	0.249	0.415	0.296	0.1432
I'^{N}_{r2}	0.0252	0.0664	0.0828	0.0831	0.085
T_A %	8.04	36.5	38.0	39.1	45.0
T_B %	-	- 2.5	- 19.6	- 14.4	- 5.8
T_L %	-	-	-	-	- 0.2
T_{FW} %	5.70	5.28	5.14	5.0	5.0
T %	2.34	28.72	13.26	19.7	34.0

S.No.	6	7	8	9	10
Speed	750	652	500	310	150
Slip	0.5	0.566	0.666	0.793	0.9
Z_{M1}	1.08 + j0.582	0.935 + j0.593	0.91 + j0.582	0.706 + j0.593	0.65 + j0.592
Z_{M2}	0.48 + j0.75	0.483 + j0.586	0.483 + j0.586	0.555 + j0.59	0.57 + j0.59
R_A	0.94	0.792	0.654	0.556	0.496
R_B	0	1.49	0.8	0.444	0.343
R_L	0.312	0.331	0.331	0.396	0.416
I'^{P}_{r1}	0.736	0.804	0.897	0.96	1.0
I''^{P}_{s2}	0	0.12	0.1675	0.205	0.233
I'^{N}_{r2}	0.090	0.096	0.098	0.1195	0.124
T_A %	49.0	51.2	52.8	51.3	49.6
T_B %	0	2.2	2.25	1.87	1.86
T_L %	- 0.26	- 0.305	- 0.318	- 0.568	- 0.64
T_{FW} %	4.94	4.94	4.94	4.94	4.94
T %	43.8	48.16	49.76	47.66	45.87

Table 3.5

Double Unbalance (Case III) $R_{sA} = \infty$, $R_{sA} = 0.66$ p.u.

S.No.	1	2	3	4	5
Speed	1390	1340	1145	965	810
Slip	0.0734	0.1067	0.237	0.357	0.46
Z_{M1}	2.42 + j2.31	2.68 + j1.765	1.358 + j0.84	1.262 + j0.87	1.16 + j0.74
Z_{M2}	0.425 + j0.574	0.425 + j0.573	0.425 + j0.573	0.449 + j0.574	0.484 + j0.575
R_A	4.11	3.28	1.61	1.23	1.084
R_B	- 0.675	- 0.712	- 0.993	- 0.574	- 4.52
R_C	-	- 0.297	- 0.394	- 0.745	- 0.895
R_L	0.262	0.262	0.262	0.287	0.323
I'^{P}_{r1}	0.192	0.23	0.355	0.42	0.454
I''^{P}_{s2}	0.0896	0.1106	0.184	0.1828	0.0557
I'''^{P}_{r1}	-	0.1078	0.178	0.175	0.0534
I'^{N}_{r2}	0.246	0.254	0.373	0.482	0.460
T_A %	15.10	17.4	20.3	21.7	22.3
T_B %	- 0.546	- 0.87	- 3.37	- 1.99	- 1.40
T_C %	-	- 0.30	- 1.25	- 2.28	- 2.55
T_L %	1.58	1.69	3.65	5.36	6.82
T_{FW} %	5.48	5.37	5.21	5.1	4.98
T %	7.45	9.17	7.82	6.46	9.1

S.No.	6	7	8	9	10
Speed	750	605	500	377	150
Slip	0.5	0.596	0.666	0.75	0.9
Z_{M1}	1.1 + j0.565	0.871 + j0.692	0.86 + j0.584	0.75 + j0.624	0.7 + j0.562
Z_{M2}	0.466 + j0.562	0.504 + j0.58	0.52 + j0.578	0.545 + j0.58	0.606 + j0.562
R_A	0.962	0.766	0.716	0.625	0.545
R_B	0	0.0652	1.055	0.229	0.1150
R_C	-	1.905	-	1.86	-
R_L	0.306	0.343	0.362	0.385	0.450
I'^{P}_{r1}	0.501	0.506	0.521	0.545	0.556
I''^{P}_{s2}	0	0.1	0.037	0.0475	-
I'''^{P}_{r1}	0	0.0924	0	0.0445	-
I'^{N}_{r2}	-	0.514	0.523	0.545	0.556
T_A %	24.2	19.60	19.4	18.6	16.8
T_B %	0	-	-	0.516	-
T_C %	0	- 1.63	0	0.37	-
T_L %	7.60	9.0	9.86	11.4	13.0
T_{FW} %	4.94	4.94	4.94	4.94	4.94
T %	11.62	7.25	4.66	3.1	-

CHAPTER IV

INDUCTION MACHINE WITH SINGLE UNBALANCE.

4.1. In the previous two chapters the analysis of double unbalances in an induction machine has been presented. But, however, in practice a single unbalance is most likely to occur. The various single unbalances can be classified as below:-

(i) Stator impedance unbalance with balanced or unbalanced supply voltage.

(ii) Rotor impedance unbalance with balanced or unbalanced supply voltage.

A single unbalance can be treated as a special case of double unbalance where circuit on one side of the air-gap remains balanced. The supply voltage may as well be unbalanced. Figs. 4.1 and 4.2 show the machine with single impedance unbalance in stator and rotor circuits respectively.

The analysis of single unbalance can be followed from the analysis of double unbalance by introducing proper circuit constraints at the terminals having balanced impedances. As has been discussed in chapter 2, both positive and negative sequence voltages at the stator terminals initiate an infinite number of reflections at points of unbalanced impedances in

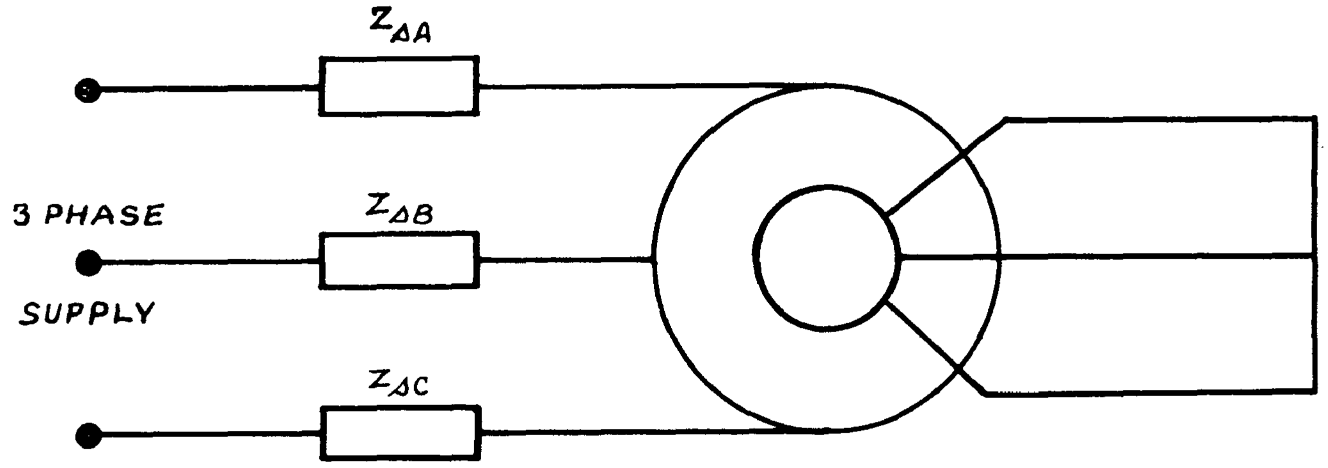


Fig. 4·1 – Machine with Stator Unbalanced Impedances.

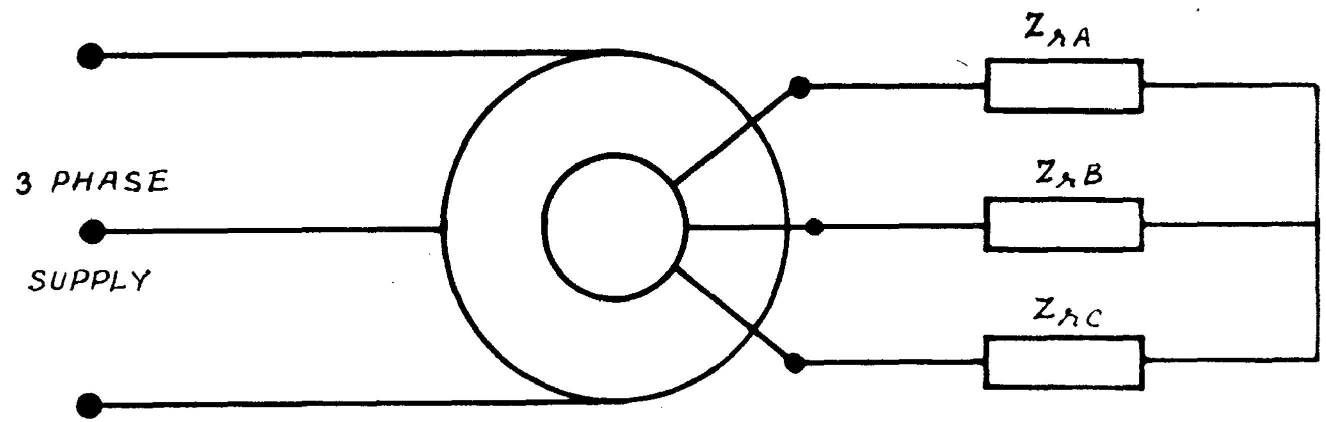


Fig 4·2— Machine with Rotor Unbalanced Impedances.

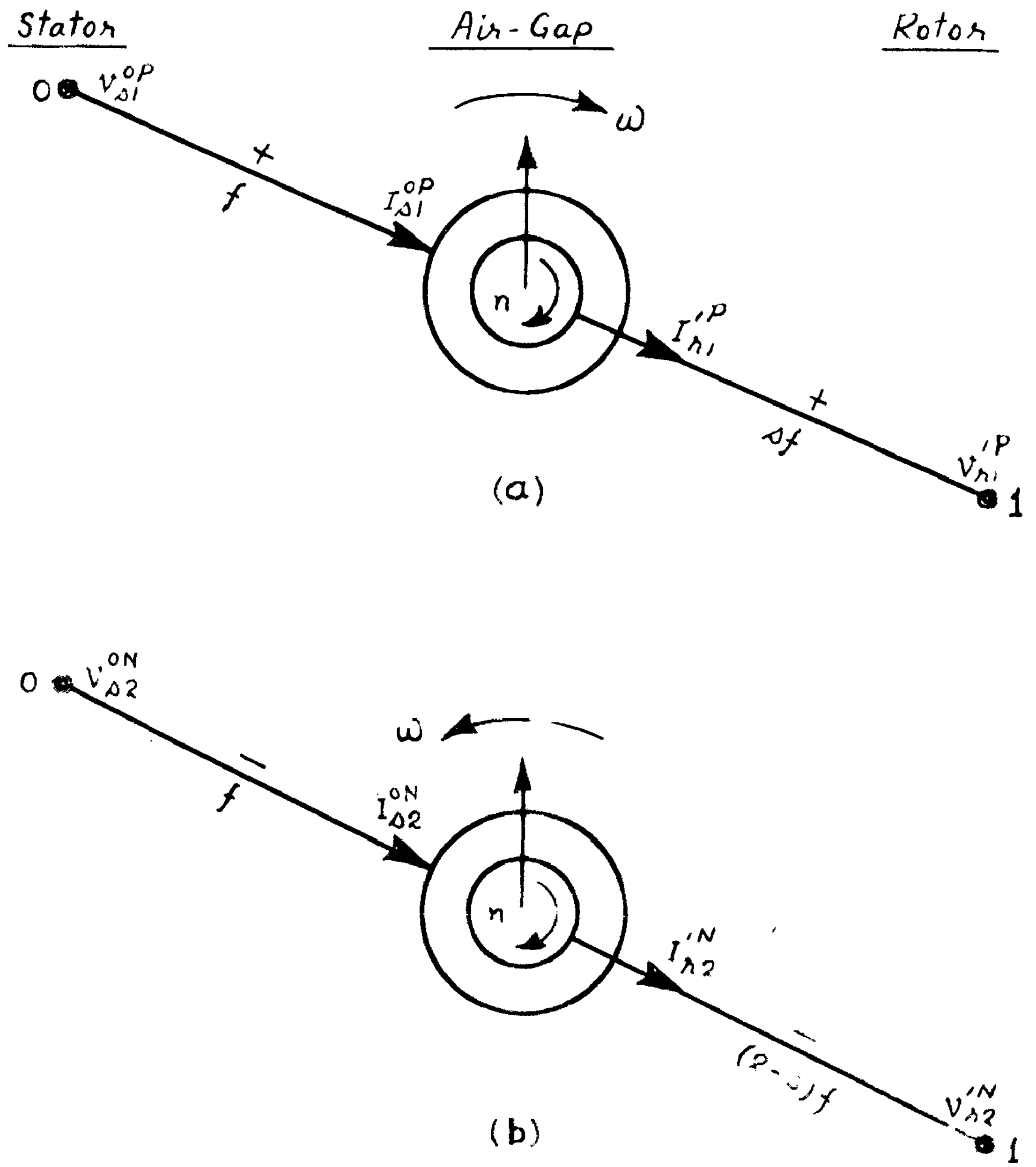


Fig. 4·3— Flow Chart for Stator Impedance Unbalance
(a)- Positive Sequence Voltage at Stator Terminals.
(b)- Negative Sequence Voltage at Stator Terminals.

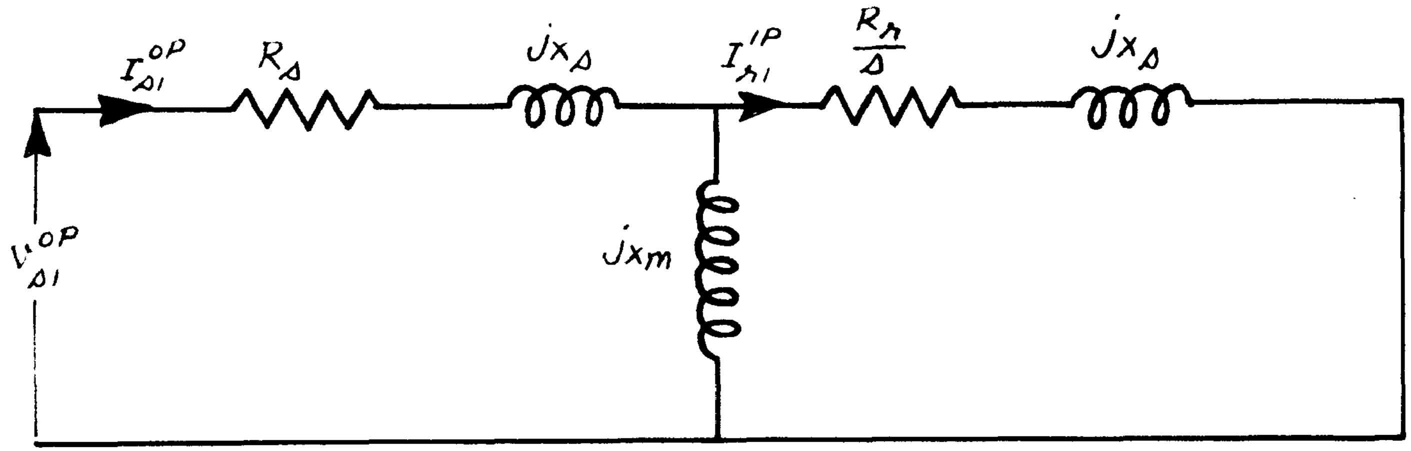


Fig. 4.4 — 'Network No. 1' for Positive Sequence Voltage.

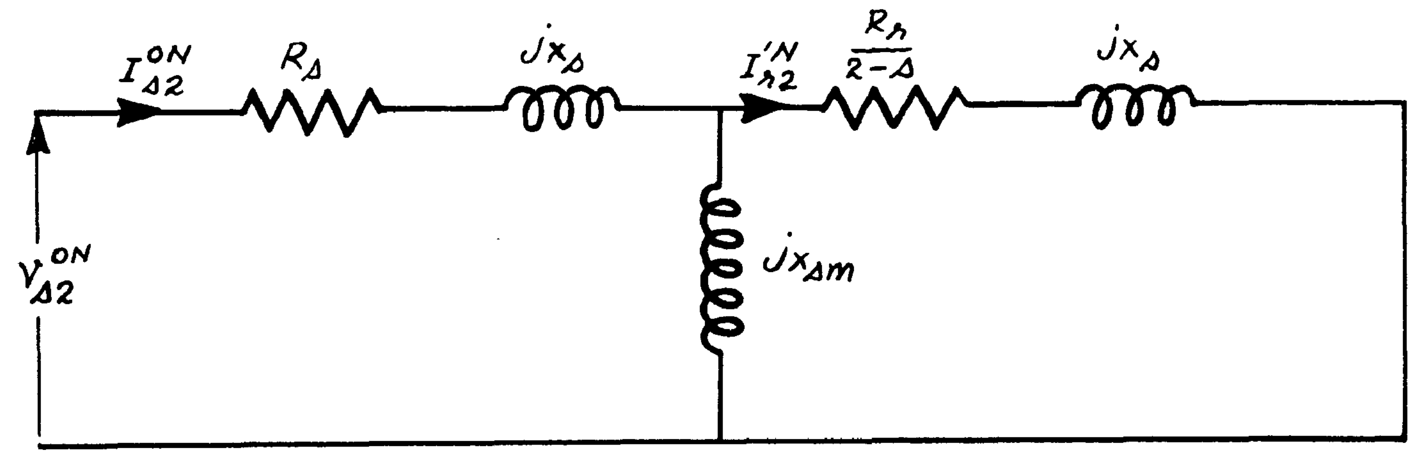


Fig. 4.5 — 'Network No. 2' for Negative Sequence Voltage.

stator and rotor circuits. However, if the impedance in one of the circuits on either side of air-gap is balanced, no reflection would occur there. Thus if the unbalance is on the stator side, there will be no reflection at all, while there will be only one reflection if the unbalance is on the rotor side.

4.2. Stator Impedance Unbalance

4.2.1. When stator impedances are unbalanced, both positive and negative sequence voltages would exist at stator terminals even though a balanced voltage is applied. Therefore the analysis can be made with positive or negative sequence voltage alone applied at the stator terminals, and the results are super-posed. Thus the machine having balanced rotor impedance has positive and negative sequence voltage applied at stator terminals. The rotor will have same set of sequence voltages and currents as that applied at stator terminals. The rotor impedance being balanced, no reflection occurs at rotor terminals. Therefore flow-charts No.1 and 2 of Figs. 2.7 and 2.10 get simplified to those shown in Fig. 4.3. Between the points 0 and 1, lies the balanced induction machine. The two networks No. 1 and 2 are drawn in Figs. 4.4 and 4.5 when positive sequence voltage and negative sequence voltage are applied respectively at stator terminals.

Let the two networks No. 1 and 2 have the input impedance Z_{M1} and Z_{M2} respectively. A relation between sequence voltage and current at input terminals is give below:-

$$\left.\begin{aligned} V_{s1}^{OP} &= Z_{M1}\, I_{s1}^{OP} \\ V_{s2}^{ON} &= Z_{M2}\, I_{s2}^{ON} \end{aligned}\right\} \qquad \ldots \qquad 4.1$$

At the point O, the stator positive sequence voltage V_{s1}^{OP} and negative sequence voltage V_{s2}^{ON} are related to the sequence voltages of supply through the sequence impedances of stator external circuit as given in the equation 2.2. Rewriting it

$$\left.\begin{aligned} V_1 &= Z_{s0}\, I_{s1}^{OP} + Z_{s2}\, I_{s2}^{ON} + V_{s1}^{OP} \\ V_2 &= Z_{s1}\, I_{s1}^{OP} + Z_{s0}\, I_{s2}^{ON} + V_{s2}^{ON} \end{aligned}\right\} \quad \ldots \quad 4.2$$

From equations 4.1 and 4.2,

$$\left.\begin{aligned} I_{s1}^{OP} &= \frac{(Z_{s0} + Z_{M2})V_1 - Z_{s2}\, V_2}{(Z_{s0} + Z_{M1})(Z_{s0} + Z_{M2}) - Z_{s1}\, Z_{s2}} \\ I_{s2}^{ON} &= \frac{(Z_{s0} + Z_{M1})V_2 - Z_{s1}\, V_1}{(Z_{s0} + Z_{M1})(Z_{s0} + Z_{M2}) - Z_{s1}\, Z_{s2}} \end{aligned}\right\} \quad \ldots \quad 4.3$$

Knowing positive and negative sequence components V_1 and V_2 of the supply voltage, I_{s1}^{OP} and I_{s2}^{ON} can be calculated. If supply voltage is balanced, V_2 is zero.

From the networks of Figs. 4.4 and 4.5, rotor sequence currents can be written as

$$\left.\begin{aligned} I_{r1}^{'P} &= I_{s1}^{OP} \frac{J\, X_m}{Z_{r1} + J\, X_m} \\ I_{r2}^{'N} &= I_{s2}^{ON} \frac{J\, X_m}{Z_{r2} + J\, X_m} \end{aligned}\right\} \quad \ldots \quad 4.4$$

where $Z_{r1} = \left(\frac{R_r}{s} + J\, X_r \right)$ = Positive sequence impedance of rotor and $Z_{r2} = (\frac{R_r}{2 - s} + j X_r)$ = Negative sequence impedance of rotor

The steady torques produced by positive and negative sequence rotor currents are

$$\left.\begin{aligned} T_1 &= (I'^{P}_{r1})^2 \frac{R_r}{s} \quad \text{Syn. Watts} \\ T_2 &= (I'^{N}_{r2})^2 \frac{R_r}{2 - s} \quad \text{Syn. Watts} \end{aligned}\right\} \quad \ldots \quad 4.5$$

T_2 is in the negative direction,

$$\therefore \quad T_{net} = (T_1 - T_2) \quad \ldots \quad 4.6$$

4.2.2. Symmetrical Stator Impedance Unbalance

For the case of general unbalanced stator impedances, an equivalent circuit cannot be drawn. But, however, sequence currents in stator and rotor circuits required are to be calculated from equations 4.3 and 4.4. As discussed in the previous chapter, an equivalent circuit is possible only for the case of symmetrical unbalance i.e. any two lines contain equal impedances. The constraint equation 4.2 gets modified to equation 4.7 for the symmetrical unbalance.

$$\left.\begin{aligned} V_1 &= (Z_{sB} + Z_{M1}) I^{OP}_{s1} + (I^{OP}_{s1} + I^{ON}_{s2}) Z_{st} \\ V_2 &= (Z_{sB} + Z_{M2}) I^{ON}_{s2} + (I^{OP}_{s1} + I^{ON}_{s2}) Z_{st} \end{aligned}\right\} \quad \ldots \quad 4.7$$

With the help of constraint equation 4.7, the two networks No. 1 and 2 can be interconnected and thus the complete equivalent circuit is drawn in Fig. 4.6. The sequence currents

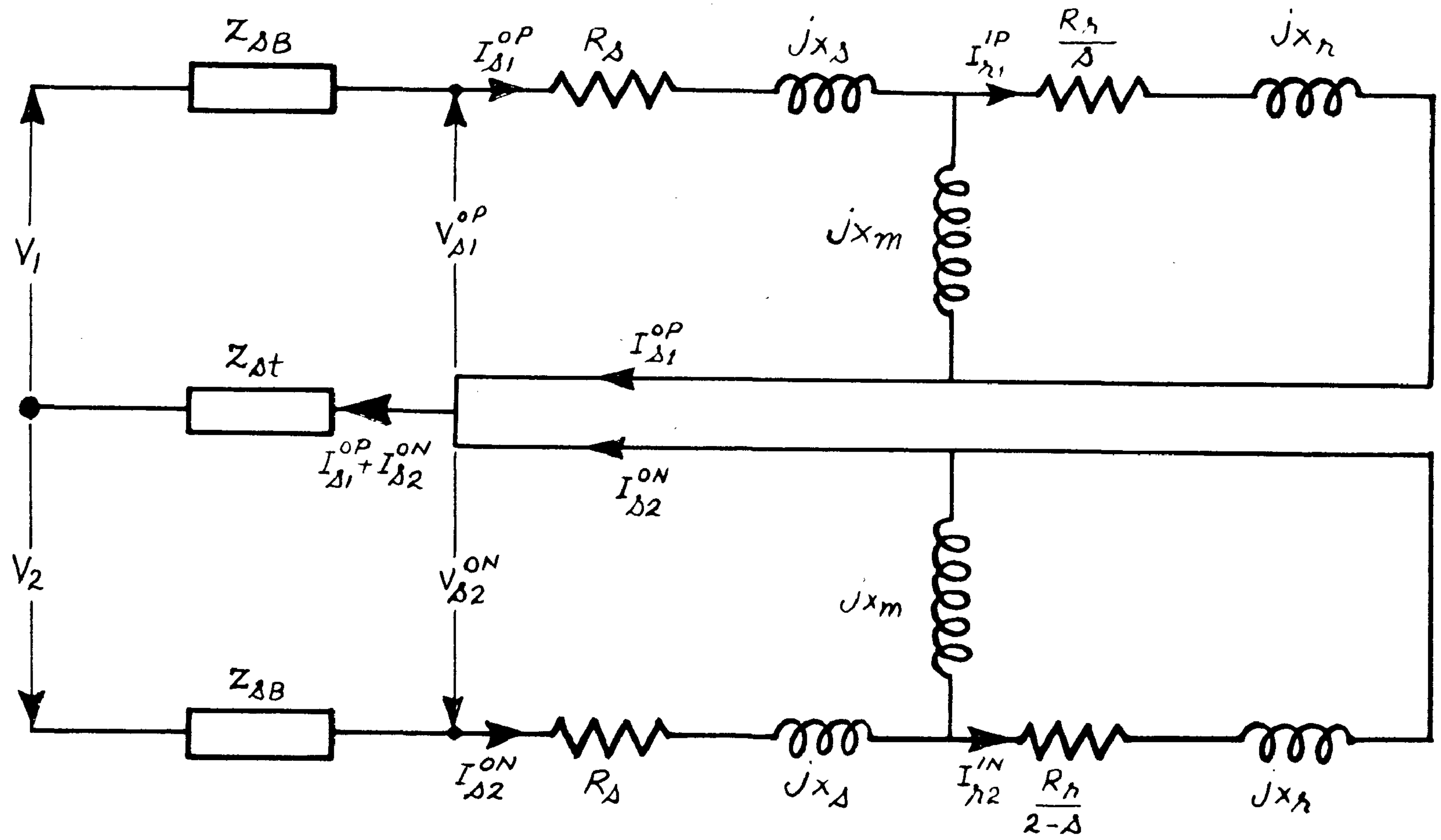


Fig. 4·6 — Equivalent Circuit — Stator Symmetrical Unbalance Impedances can now be computed easily from the equivalent circuits. The equivalent circuit for any specific case of symmetrical unbalance can be obtained by substituting the corresponding sequence impedances of the external circuit in the Fig. 4.6.

4.2.3. Pulsating Torque

From Fig. 4.3 it is evident that both positive and negative sequence currents and fluxes are present simultaneously. These are classified as given below:-

(i) Positive sequence flux ϕ_{s1}^{OP} due to positive sequence voltage V_{s1}^{OP} of frequency f. It rotates in the positive direction with a velocity of ω with respect to stator and $s\omega$ with respect to rotor.

(ii) Negative sequence flux ϕ_{s2}^{ON} due to negative sequence voltage V_{s2}^{ON} of frequency f. It rotates in the negative direction with a velocity of ω with respect to stator and $(2 - s)\,\omega$ with respect to rotor.

(iii) Positive sequence rotor current $I_{r1}^{'P}$ of frequency sf. MMF due to this current rotates in the positive direction with a velocity of $s\omega$ with respect to rotor and ω with respect to stator.

(iv) Negative sequence rotor current $I_{r2}^{'N}$ of frequency $(2 - s)f$. MMF due to this current rotates in the negative direction with a velocity of $(2 - s)\omega$ with respect to rotor and ω with respect to stator.

The inter action of flux caused by primary voltage and mmf produced by secondary current, results in a torque. It can be express as

$$T \propto \phi_m \, \mathcal{F} \cos\delta \qquad \ldots \qquad 4.8a$$

where ϕ_m is maximum air-gap flux which is proportional to the induced e.m.f. E divided by frequency;

$\mathcal{F}$ is mmf which is proportional to secondary current;

and δ is angle between e.m.f. and current.

$$\therefore \quad T = K_T \left(\frac{E}{f}\right) I \ \mathrm{Cos}\,\delta \qquad \ldots \qquad 4.8\text{ b}$$

where K_T is a torque constant.

It develops steady torque if angle δ between e.m.f. and current is independent of time i.e. when both e.m.f. and current waves travel with the same velocity in the same direction. However, if e.m.f. and current waves travel with different velocities or direction, the angle δ is a function of time and accordingly the torque would also vary sinusoidally whose amplitude is

$$T_p \text{ (peak)} = K_T \left(\frac{E}{f}\right) I \qquad \ldots \qquad 4.8\text{ c}$$

This is known as pulsating torque.

The components of torque developed by the interaction of two flux and two mmf waves are as follows:-

(i) Positive sequence flux ϕ^{OP}_{s1} which causes induced e.m.f. E^{OP}_{s1} of frequency f in the stator and positive sequence rotor current I'^{P}_{r1} rotate with the same velocity in the same direction and hence produce a steady torque given by

$$T_1 = K_T \left(\frac{E^{OP}_{s1}}{f}\right) I'^{P}_{r1} \ \mathrm{Cos}\,\delta_1 \qquad \ldots \qquad 4.9$$

The angle δ_1 between E^{OP}_{s1} and I'^{P}_{r1} can be calculated by the network No. 1 and its value is

$$\delta_1 = \tan^{-1}\left(\frac{s\ x\ r}{R_r}\right) = \theta_1 \qquad \ldots \qquad 4.9\text{ a}$$

(ii) Negative sequence flux ϕ_{s2}^{ON} which causes induced e.m.f. E_{s2}^{ON} of frequency f in the stator and negative sequence rotor current $I_{r2}^{'N}$ rotate with the same velocity in the same direction and hence produce a steady torque given by

$$T_2 = K_T \left(\frac{E_{s2}^{ON}}{f}\right) I_{r2}^{'N} \cos \delta_2 \quad \ldots \quad 4.10$$

From the network No. 2,

$$\delta_2 = \tan^{-1} \frac{(2 - s) X_r}{R_r} = \theta_2 \quad \ldots \quad 4.10\,a$$

(iii) Positive sequence flux ϕ_{s1}^{OP} which causes induced e.m.f. E_{s1}^{OP} of frequency f in the stator and negative sequence rotor current $I_{r2}^{'N}$ rotate with a relative velocity of 2ω in the positive direction and inter-act to produce a pulsating torque,

$$T_{P1} = K_T \left(\frac{E_{s1}^{OP}}{f}\right) I_{r2}^{'N} \cos \delta_{14} \quad \ldots \quad 4.11$$

$$\text{where} \quad \delta_{14} = 2\omega t + \alpha - \theta_2 \quad \ldots \quad 4.11\,a$$

and α is the angular difference between the two waves E_{s1}^{OP} and E_{s2}^{ON} at $t = 0$

$$\therefore \quad T_{P1} = T_{P1}(\text{peak}) \cos (2\omega t + \alpha - \theta_2) \quad \ldots \quad 4.11\,b$$

$$\text{where} \quad T_{P1}(\text{peak}) = K_T \left(\frac{E_{s1}^{OP}}{f}\right) I_{r2}^{'N} \quad \ldots \quad 4.11\,c$$

(iv) Negative sequence flux ϕ_{s2}^{ON} causes induced e.m.f. E_{s2}^{ON} of frequency f in the stator and positive sequence rotor current $I_{r1}^{'P}$ rotate with a relative velocity 2ω in the negative direction and interact to produce a pulsating torque,

$$T_{P2} = K_T \left(\frac{E_{s2}^{ON}}{f}\right) I_{r1}^{'P} \cos \delta_{23} \quad \ldots \quad 4.12$$

where $\delta_{23} = 2\omega t + \alpha - \theta_1$ ... 4.12 a

$$\therefore \quad T_{P2} = T_{P2}(peak) \cos (2\omega t + \alpha - \theta_1) \qquad 4.12\,b$$

where $T_{P2}(peak) = K_T \left(\frac{E_{s2}^{ON}}{f}\right) I'^{p}_{r1}$ ... 4.12 c

The items (i) and (ii) give steady torques and the net steady torque is

$$T_{net} = (T_1 - T_2)$$

The expressions for steady torque have been derived already from power transfer concept.

The items (iii) and (iv) give pulsating torque which varies sinusoidally with a frequency of 2f. The instantaneous pulsating torque is

$$T_P(inst) = (T_{P1} - T_{P2}) \qquad 4.13$$

$$= T_{P1}(peak) \cos (2\omega t + \alpha - \theta_2)$$

$$- T_{P2}(peak) \cos (2\omega t + \alpha - \theta_1) \qquad 4.13\,a$$

$$T_P(peak) = \sqrt{T_{P1}(peak)^2 + T_{P2}(peak)^2 - 2\,T_{P1}(peak)\,T_{P2}(peak) \times \cos(\theta_1 - \theta_2)} \qquad 4.14$$

4.2.4. Experimental Verification

The experiments were performed on the machine whose constants are given in Appendix I. A reduced supply voltage was used and impedance unbalance was limited to resistance only as in case of impedance double unbalance. The experiments were

Steady Torque

Stator Impedance Unbalanced

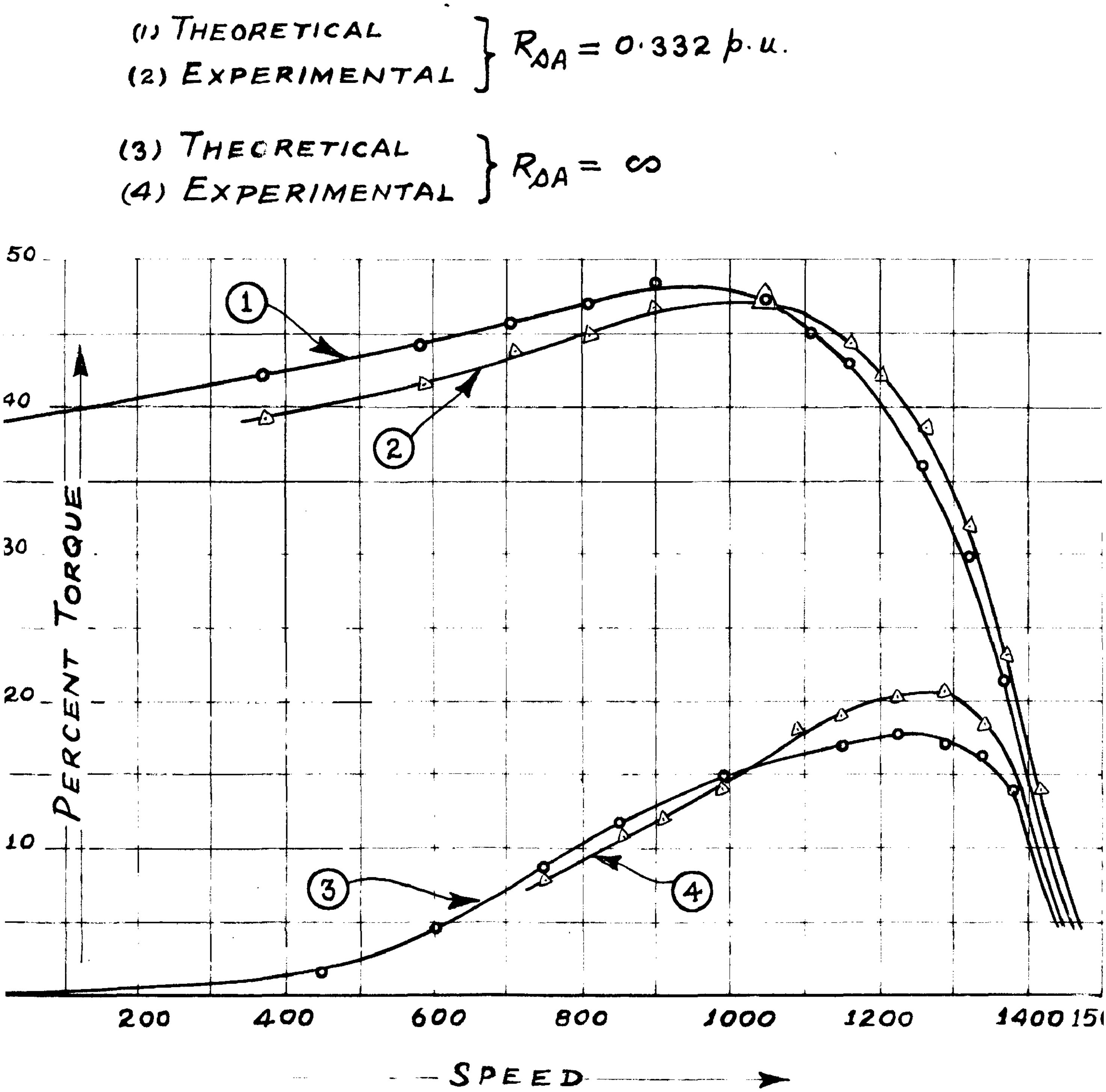


Fig. 4·7

Stator Current

Stator Impedance Unbalanced

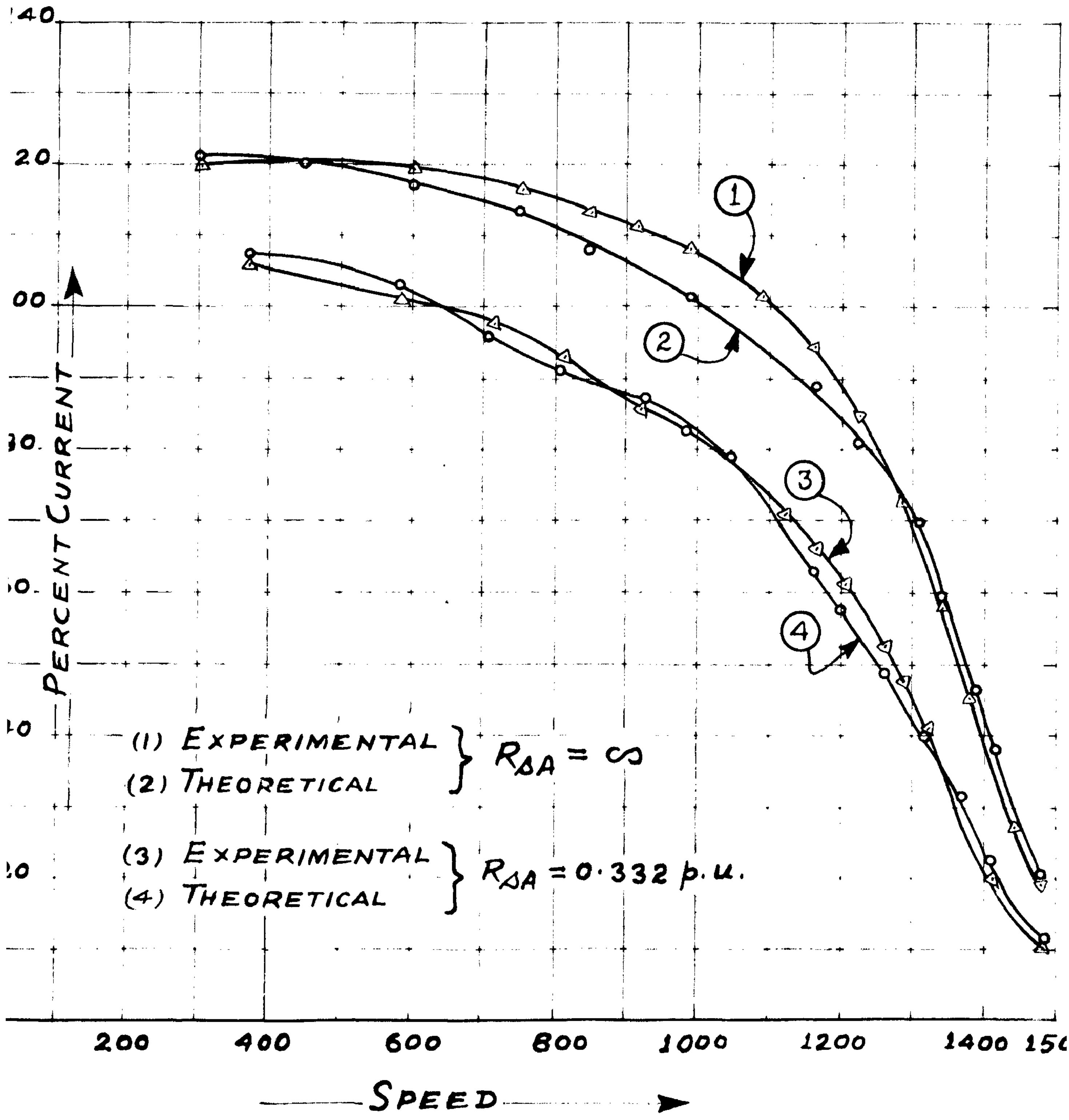


Fig. 4.8

Steady Torque

Stator Voltage Unbalance

(1) $|V_1| = 0.875$ $|V_2| = 0.143$ p.u.

(2) $|V_1| = 0.762$ $|V_2| = 0.286$ p.u.

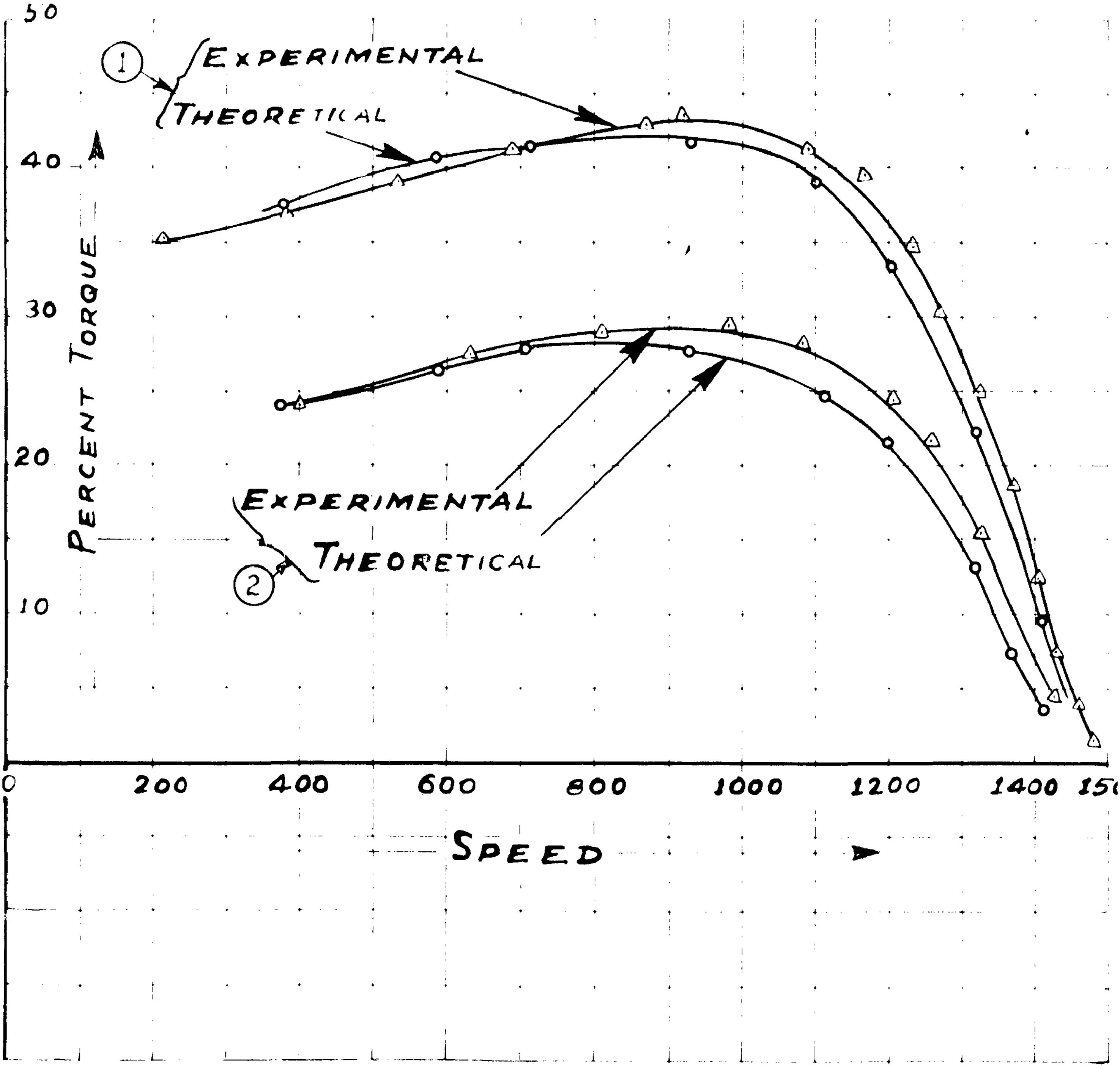


Fig. 4·9

Peak Pulsating Torque
Stator Unbalance

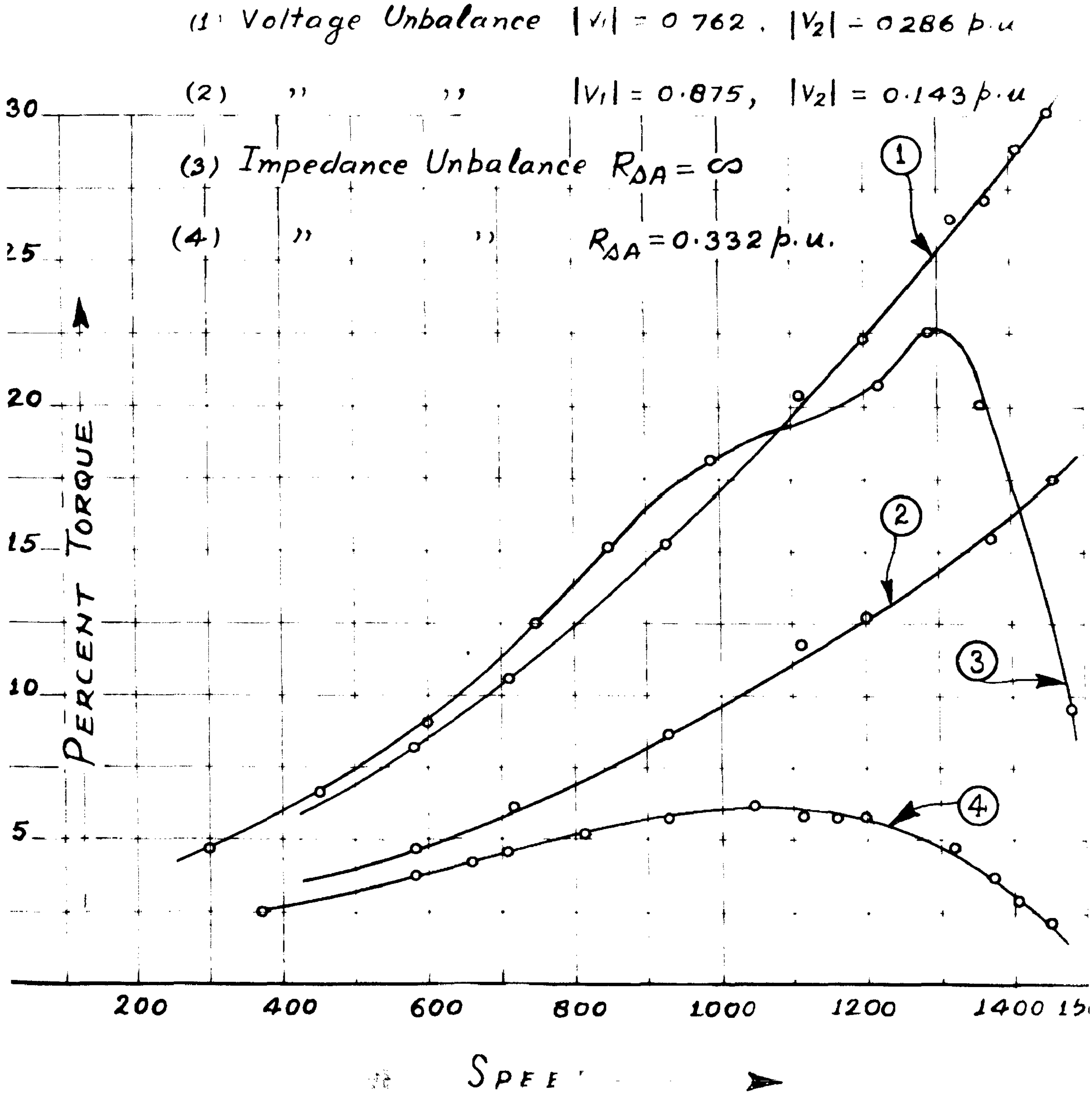


Fig 4.17

performed for the types of stator unbalance usually occur in practice and are given below:-

(i) Addition of resistance in one line due to weak link.

(ii) Fuse of one line blown off.

The supply being balanced one.

(iii) Unbalanced supply voltage and no impedance unbalance.

A number of sets were taken with different values of R_{sA} = 0.332 p.u. and infinite; and voltage unbalance with $/V_1/$ = 0.875, $/V_2/$ = 0.143, and $/V_1/$ = 0.762, $/V_2/$ = 0.286 p.u. The steady torque and current were measured over the complete speed range. The results are plotted in Figs. 4.7 ~~and~~ to 4.10.

The results are also computed from the equivalent circuits and are compared with the experimental results. These are found to be in agreement with the maximum difference of 7%.

A qualitative explanation can be given below:-

Case I - R_{sA} in series with line A. For low values of R_{sA}, $\frac{R_{sA}}{3}$ is very small compared to Z_{M2} at all slips, and therefore the network No. 2 takes negligible current and contributes insignificant torque. Hence network No. 2 can be omitted and the equivalent circuit reduces to network No. 1 in series with a resistance $\frac{R_{sA}}{3}$. Thus it behaves like a balanced induction machine with an additional resistance $\frac{R_{sA}}{3}$ in series with the stator. The torque produced would be less than that of a balanced machine. The stator currents are unbalanced with phase C carrying largest current. Therefore phase C will be hotter than the other two phases.

Case II - One line open i.e. $Z_{sA} = \infty$. In this case two networks No. 1 and 2 become connected in series; and positive and negative sequence stator currents are equal and opposite. At $s = 1$, both Z_{M1} and Z_{M2} are equal, therefore the voltages across them are equal and so are the rotor sequence currents. The torque produced by both the networks is equal and opposite and resulting in zero starting torque. At lower values of slip, Z_{M1} increases and Z_{M2} decreases but, however, Z_{M2} does not change appreciably. The network No. 1 contributes more torque than by network No. 2 and therefore results in a net positive torque. At $s = 0$, the rotor circuit of network No. 1 becomes open, and only the network No. 2 contributes torque which is in negative direction and of small magnitude since the sequence currents are very small.

Comparing the torque in the two cases, it is clear that network No. 2 contributes negligible torque in the Case I while it is appreciable in the Case II in the negative direction. Therefore net torque is less in Case II. Thus if a resistance in one line of external stator circuit is increased from zero to infinite, the net torque would decrease for all the values of slip.

Similarly in Case III, the net torque reduces with the increase of amount of unbalance in voltage.

Pulsating torque

The amplitude of pulsating torque is computed and plotted in Fig. 4.10 for various values of slips. No experimental verification could be carried out for pulsating torque. It is found that the pulsating torque is larger in Case II compared to that in Case I. This can be explained by the fact that the negative sequence voltage and current in the rotor of network No. 2 are negligible in Case I while those are appreciable in Case II. Since pulsating torques are contributed by the product of positive sequence voltage and negative sequence current and vice-versa, both components of the pulsating torque and therefore net pulsating torque is less in Case I. It is observed that maximum value of peak pulsating torque is 22.6% of normal steady torque at a slip of 0.2 in Case II. The peak pulsating torque increases with the amount of unbalance. Similarly in case of voltage unbalance, the peak pulsating torque increases with the increase of unbalance.

4.3. Rotor Impedance Unbalance

4.3.1. A second type of single unbalance occurs where stator has balanced impedances and unbalanced impedances are connected in rotor circuit. This is also a special case of double impedance unbalance and analysis can be followed similarly. On application of positive sequence voltage to the machine, positive sequence voltage and current of frequency sf are induced in rotor. The unbalanced rotor impedance causes a reflection which results in negative sequence voltage and current also appearing at rotor terminals, and so negative sequence voltage of frequency (2s - 1)f is induced in the stator. The stator impedances being balanced, no further reflection would occur. Therefore flow chart No. 1 of Fig. 2.7 gets simplified as in Fig. 4.11 a. Similarly for negative sequence voltage applied at stator, the flow chart No. 2 is modified to Fig. 4.11 b.

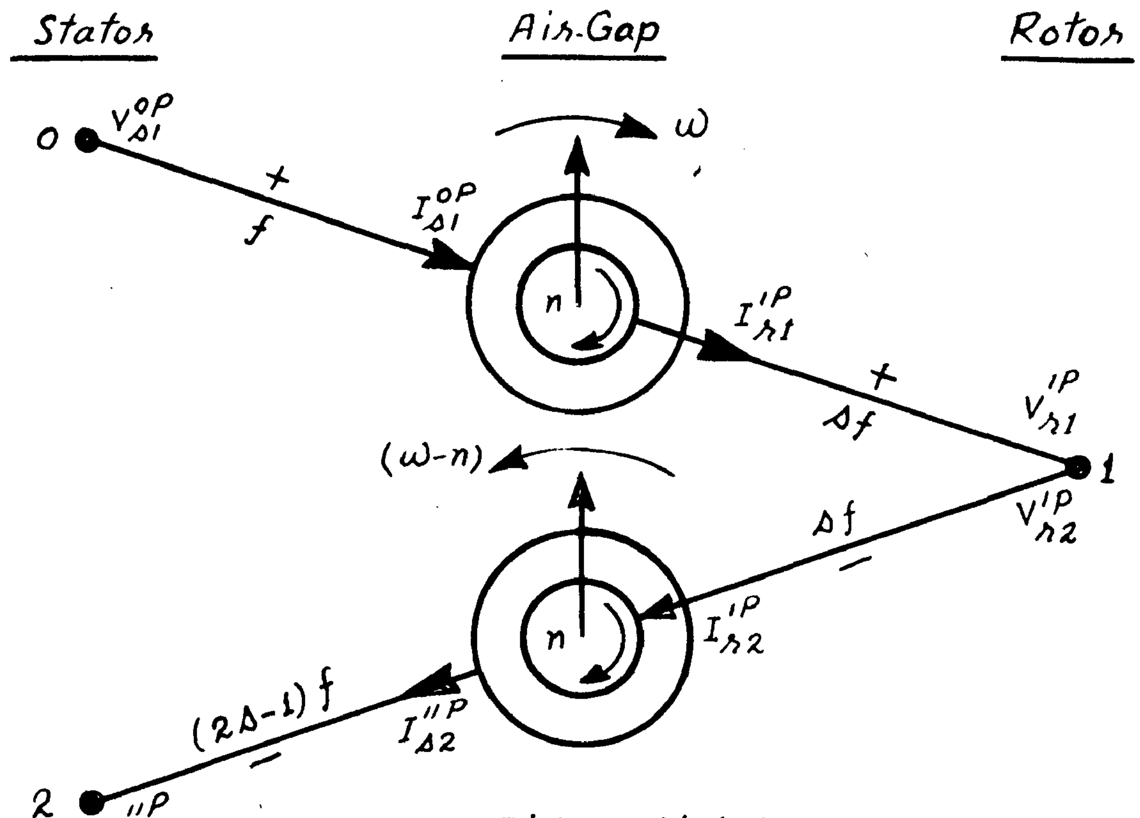


Fig. 4.11 (a)

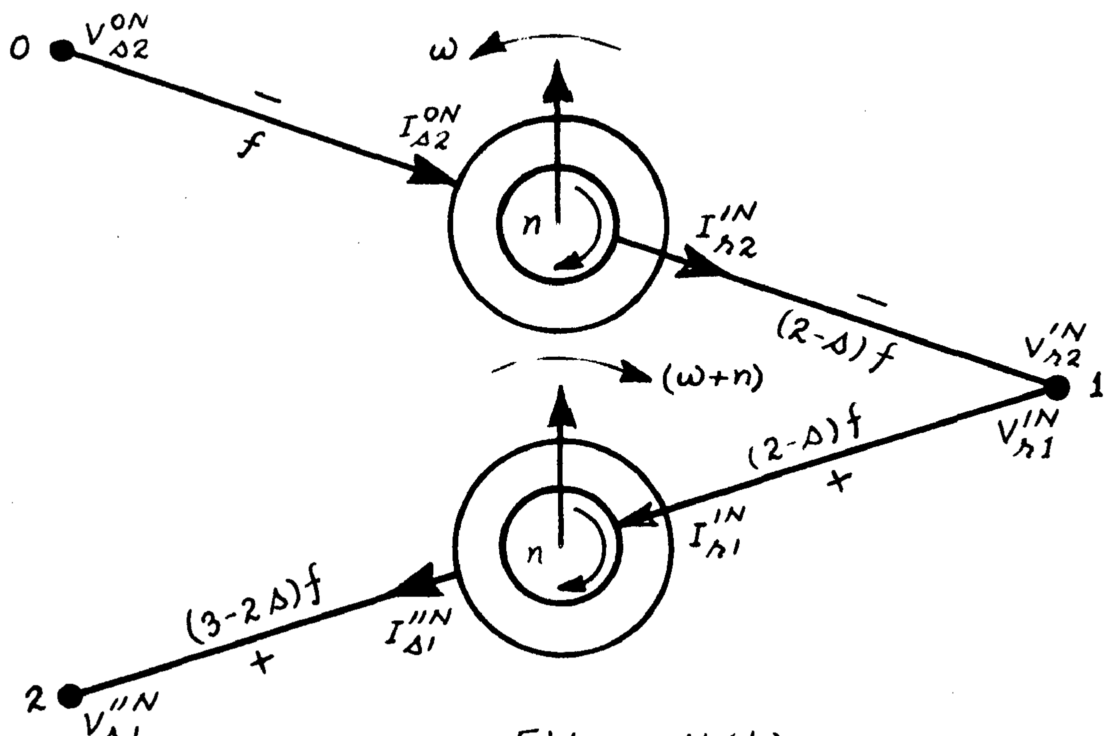


Fig. 4.11 (b)

Flow Charts— Stator Balanced and Rotor Unbalanced Impedances.

(a) Positive Sequence Voltage at Stator Terminals.

(b) Negative Sequence Voltage at Stator Terminals

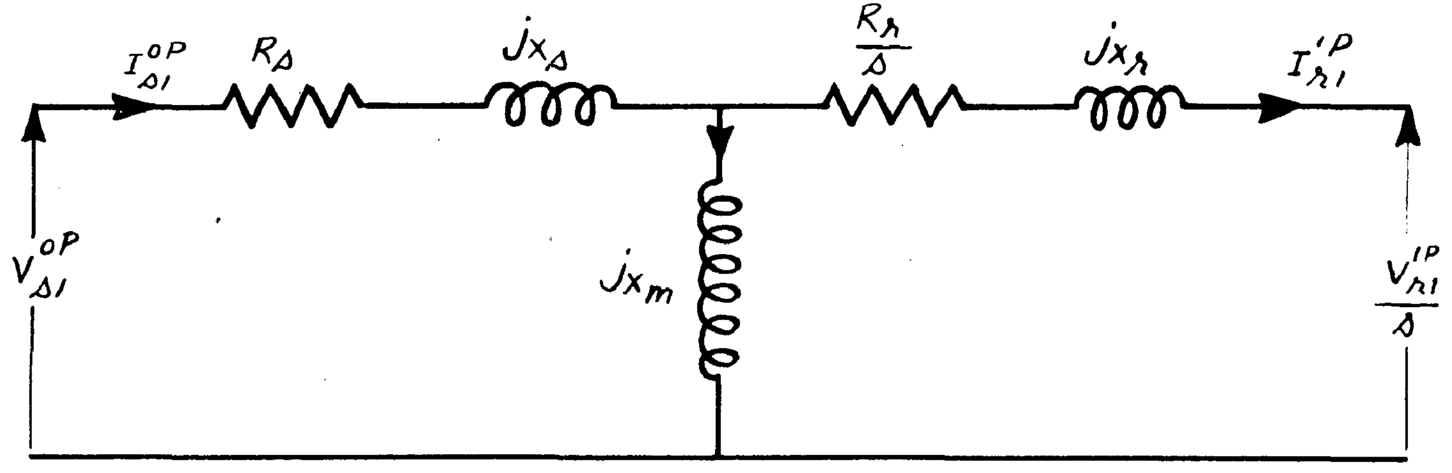


Fig. 4·12 (a)

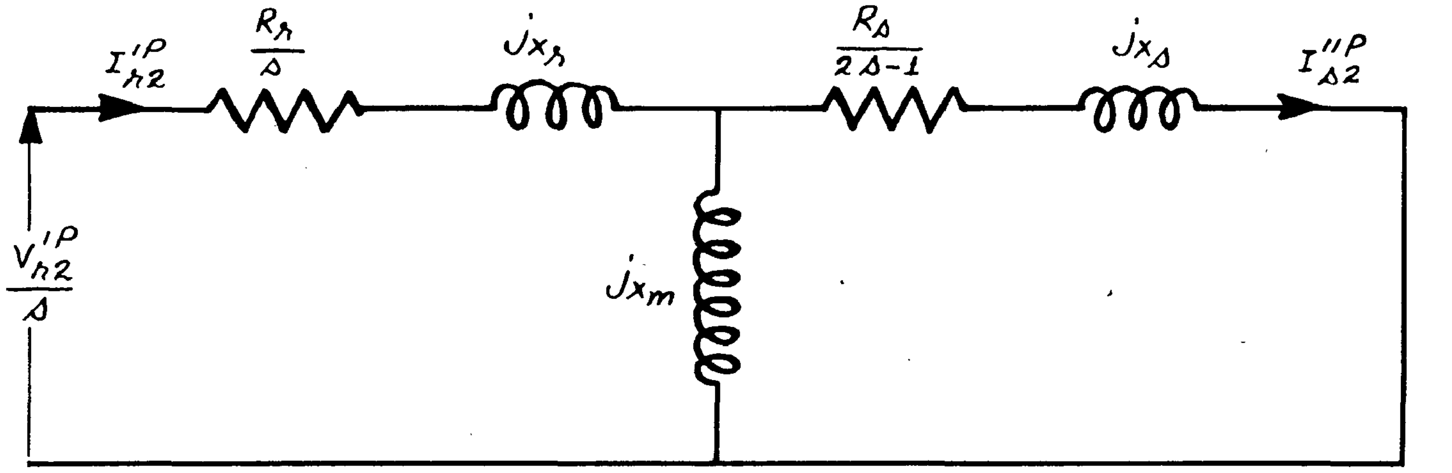


Fig. 4·12 (b)

Networks for Rotor Unbalanced Impedances when Positive Sequence Voltage is applied to Stator.

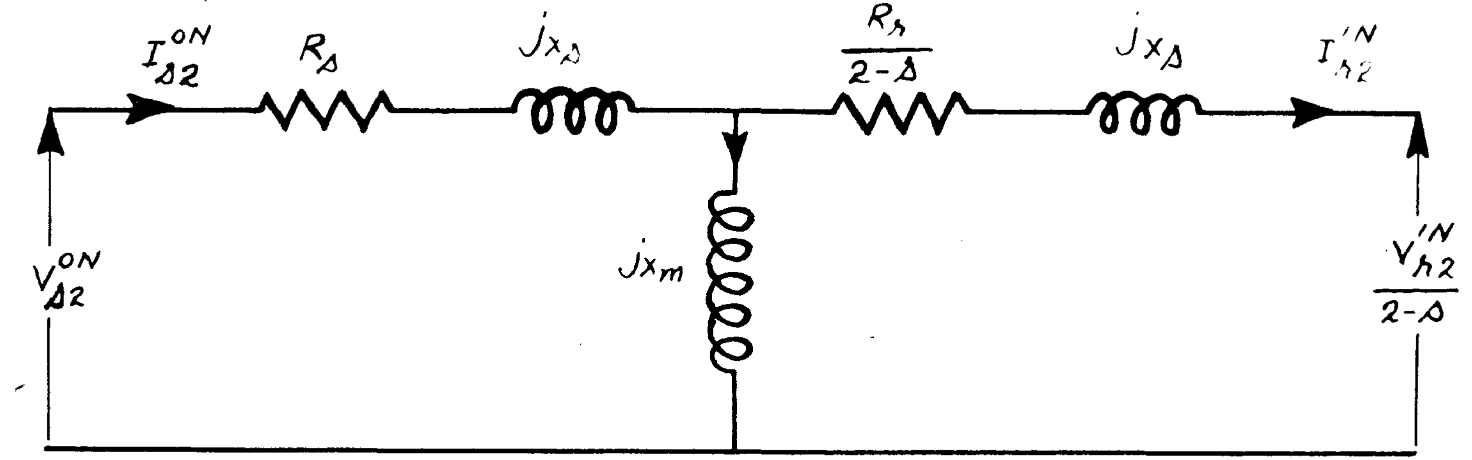


Fig. 4·13(a)

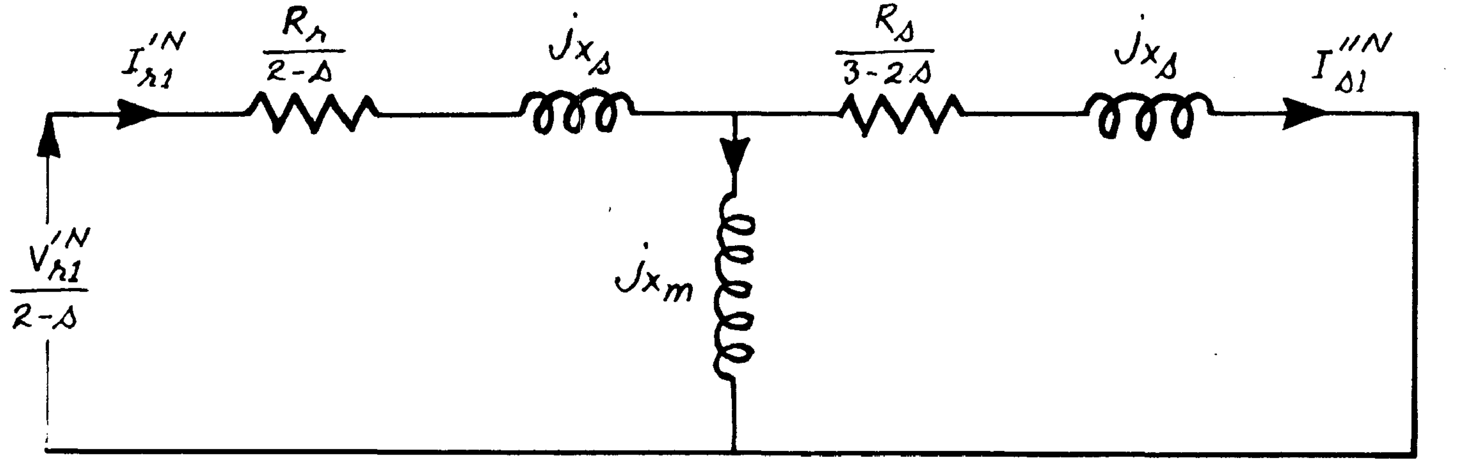


Fig. 4·13 (b)

Networks for Rotor Unbalanced Impedances when Negative Sequence Voltage is applied to Stator.

The networks between points 0 and 1, and points 1 and 2 of Figs. 4.11 (a and b) with positive or negative sequence voltage alone applied at stator terminals are drawn in Figs. 4.12 (a and b) and 4.13 (a and b). The constraint equations at point 1 (from equations 2.2 and 2.8) with positive or negative sequence voltage alone applied at stator are given below:-

With positive sequence voltage only

$$\left.\begin{aligned} V'^{P}_{r1} &= Z_{r0}\, I'^{P}_{r1} - Z_{r2}\, I'^{P}_{r2} \\ V'^{P}_{r2} &= Z_{r1}\, I'^{P}_{r1} - Z_{r0}\, I'^{P}_{r2} \end{aligned}\right\} \quad \ldots \quad 4.15\text{ a}$$

With negative sequence voltage only

$$\left.\begin{aligned} V'^{N}_{r2} &= Z_{r0}\, I'^{N}_{r2} - Z_{r1}\, I'^{N}_{r1} \\ V'^{N}_{r1} &= Z_{r2}\, I'^{N}_{r2} - Z_{r0}\, I'^{N}_{r1} \end{aligned}\right\} \quad \ldots \quad 4.15\text{ b}$$

It is found that the equivalent circuits and constraint equations are exactly similar in form for the two cases i.e. with positive sequence voltage and negative sequence voltage alone applied at stator terminals. The only difference lies in the fact that the slip which is s for the positive sequence voltage case changes to (2 - s) for the negative sequence case. Thus any analysis which will be carried out for voltages, currents and steady torques with positive sequence voltage alone applied at stator terminals, can be used when negative sequence voltage is applied by replacing s by (2 - s). Therefore analysis below is carried out when only positive sequence voltage is applied to stator.

With the help of networks 4.12 (a and b) and equation

4.15 a, the stator and rotor sequence currents when positive sequence voltage alone is applied, are given below. (Derivation is given in Appendix III).

$$I'^{P}_{r1} = \frac{Z_{22} + Z_{r0}}{(Z_{11} + Z_{r0})(Z_{22} + Z_{r0}) - Z_{r1} Z_{r2}} \; sKV^{OP}_{s1} \qquad \ldots \quad 4.16$$

or

$$I'^{P}_{r1} = \frac{KV^{OP}_{s1}}{\left(\frac{Z_{11}}{s} + Z_L\right)} \qquad \ldots \quad 4.17$$

where Z_L can be considered as load impedance, and its value is

$$Z_L = \frac{Z_{r0}}{s} - \frac{\frac{Z_{r1}}{s} \cdot \frac{Z_{r2}}{s}}{\frac{Z_{22}}{s} + \frac{Z_{r0}}{s}} = R_L + jX_L \qquad \ldots \quad 4.17\text{ a}$$

$$Z_{11} = R_r + js(X_m + X_r) + \frac{sX_m^2}{R_s + jX_m + jX_s} \qquad \ldots \quad 4.17\text{ b}$$

$$Z_{22} = R_r + j(X_m + X_r)s + \frac{sX_m^2(2s - 1)}{R_s + j(X_m + X_s)(2s - 1)} \qquad 4.17\text{ c}$$

and

$$K = \frac{jX_m}{R_s + jX_s + jX_m} \quad \text{a constant} \qquad \ldots \quad 4.17\text{ d}$$

$$I'^{P}_{r2} = \frac{Z_{r1}}{(Z_{11} + Z_{r0})(Z_{22} + Z_{r0}) - Z_{r1} Z_{r2}} \; sKV^{OP}_{s1} \qquad \ldots \quad 4.18$$

$$I^{OP}_{s1} = \frac{V^{OP}_{s1}}{R_s + jX_s + jX_m} + \frac{jX_m}{R_s + jX_s + jX_m} I'^{P}_{r1} \qquad 4.19$$

$$I''^{P}_{s2} = \frac{jX_m}{\frac{R_s}{2s - 1} + jX_m + jX_s} I'^{P}_{r2} \qquad \ldots \quad 4.20$$

Knowing the sequence currents, the phase currents, voltages and steady torque can be computed. As the air-gap is being crossed twice, the steady torque has two components as given below:-

$$T_1 = (I'^{p}_{r1})^2 \left(\frac{R_r}{s} + R_L \right) \text{ Syn. Watts} \quad \ldots \quad 4.21$$

where R_L is the load resistance as defined earlier.

and $$T_2 = (I''^{p}_{s2})^2 \left(\frac{R_s}{2s - 1} \right) \quad \ldots. \quad 4.22$$

The second torque component T_2 acts on the stator in the negative direction and its reaction on rotor will be in the positive direction. Therefore net torque acting on rotor is the algebraic addition of these two components.

$$\therefore \quad T_{net} = (T_1 + T_2) \quad \ldots \quad 4.23$$

The direction of T_2 is given by the factor $(2s - 1)$. It is negative for $s < 0.5$ and positive for $s > 0.5$.

4.3.2. Symmetrical Rotor Impedance Unbalance

The currents and torques for any degree of unbalance can be calculated by solvihg the equations 4.16 to 4.23 without drawing the equivalent circuits. But, as discussed in other types of unbalances, it is possible to draw an equivalent circuit only when the unbalance is symmetrical i.e. two of the external impedances in the rotor should be equal or $Z_{rB} = Z_{rC}$. For such type of unbalance, inter-connecting circuits are drawn in Fig. 4.14 for interconnecting networks of Figs. 4.12 a and 4.12 b for positive sequence supply voltage, and in Fig. 4.15 for inter-

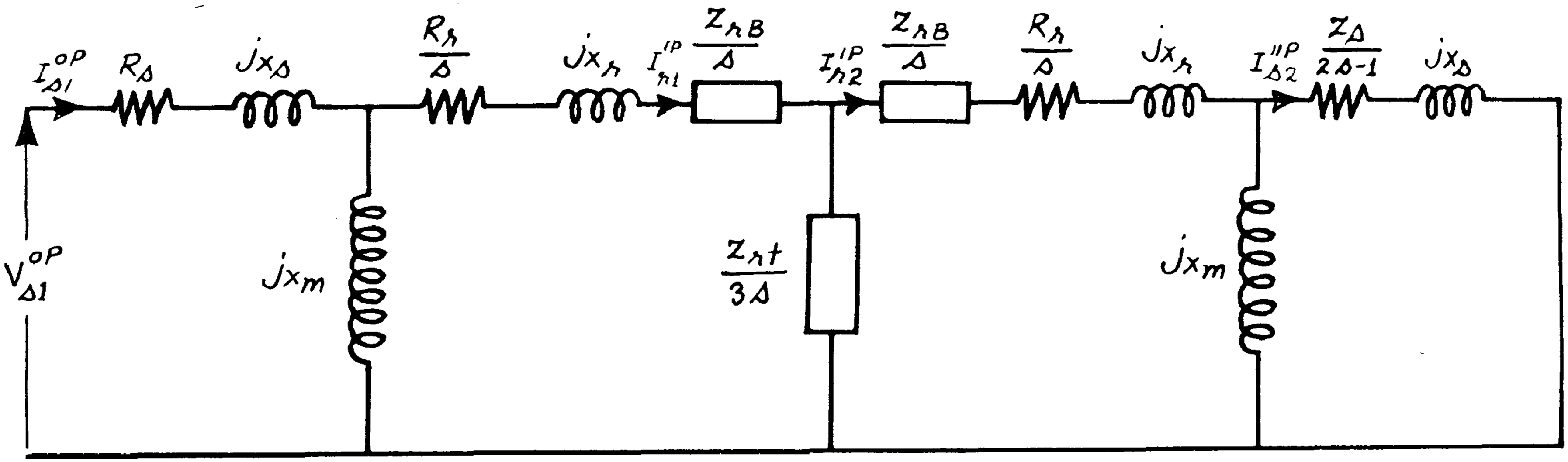


Fig. 4.14 — Equivalent Circuit for Rotor Symmetrical Unbalanced Impedances for Positive Sequence Voltage at Stator Terminals

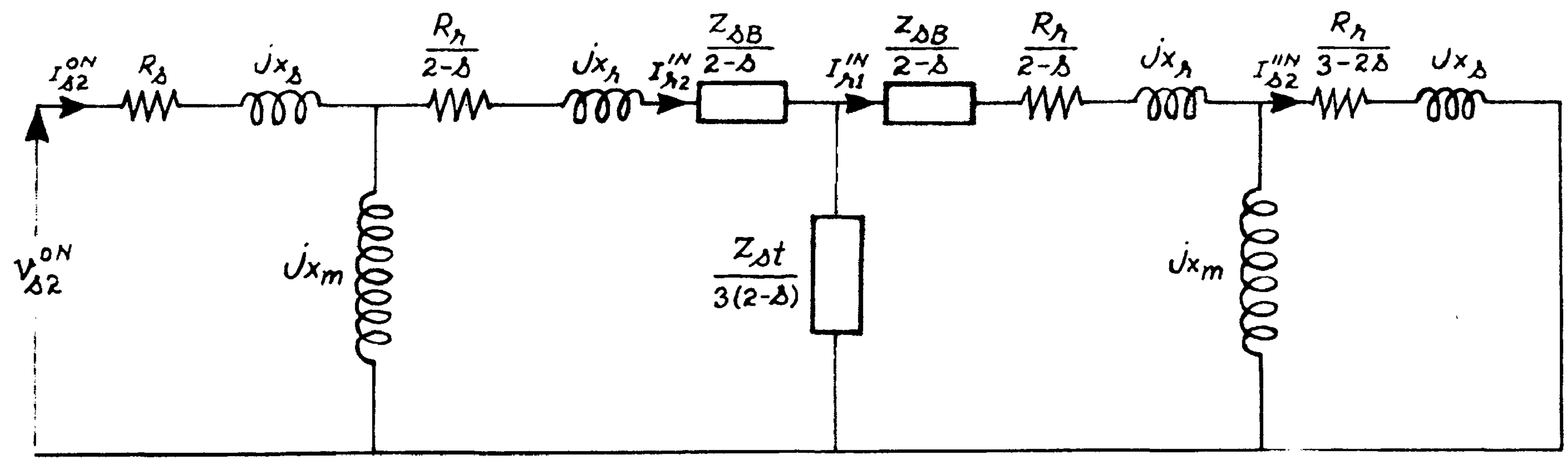


Fig. 4·15 – Equivalent Circuit for Rotor Symmetrical Unbalanced Impedances for Negative Sequence Supply at Stator Terminals connecting networks of Figs. 4.13 a and 4.13 b for negative sequence supply voltage. Now the solution for sequence currents becomes simpler and can be carried out with the holp of equivalent circuits. For any specific case of symmetrical unbalance, the equivalent circuits can be obtained by substituting the corresponding values of sequence impedances of external circuit in Figs. 4.14 and 4.15.

4.3.3. Pulsating Torque

When unbalanced voltage is applied to the maching having unbalanced impedances in rotor, the various components of flux in the air-gap and sequence currents in stator and rotor circuits exist simultaneously and are listed in Table 4.1. Their frequency and relative velocity with respect to stator are also given in the Table.

Table 4.1

S.No.	Item	Frequency	Velocity w.r. to stator
1.	Positive sequence flux ϕ_{s1}^{OP} causing E_{s1}^{OP}	f	ω
2.	Negative sequence flux $\phi_{r2}^{'P}$ causing $E_{r2}^{'P}$	sf	$-(\omega - 2n)$
3.	Negative sequence flux ϕ_{s2}^{ON} causing E_{s2}^{ON}	f	$-\omega$
4.	Positive sequence flux $\phi_{r1}^{'N}$ causing $E_{r1}^{'N}$	(2 - s)f	$\omega + 2n$
5.	Positive sequence stator current I_{s1}^{OP}	f	ω

(Contd.)

S.No.	Item	Frequency	Velocity w.r. to stator
6.	Positive sequence rotor current I'^{P}_{r1}	sf	ω
7.	Negative sequence rotor current I'^{P}_{r2}	sf	$-(\omega - 2n)$
8.	Negative sequence stator current I''^{P}_{s2}	(2s - 1)f	$-(\omega - 2n)$
9.	Negative sequence stator current I^{ON}_{s2}	f	$-\omega$
10.	Negative sequence rotor current I'^{N}_{r2}	(2 - s)f	$-\omega$
11.	Positive sequence rotor current I'^{N}_{r1}	(2 - s)f	$\omega + 2n$
12.	Positive sequence stator current I''^{N}_{s1}	(3 - 2s)f	$\omega + 2n$

The flux caused by primary voltage and m.m.f. due to secondary current interact and produce torque. The reverse is also true. The torque is expressed as

$$T = K_T \left(\frac{E}{f}\right) I \cos\delta$$

where δ is angle between e.m.f. and current and it is a function of time when both e.m.f. and current vectors do not rotate with the same velocity and in same direction. From the Table 4.1, it is clear that all e.m.f. and current vectors are not rotating with the same velocity. Therefore angle δ is not independent of time for all torques. If δ is a function of time, a pulsating torque would result in otherwise steady torque where δ is independent of time.

The components of e.m.f. and current producing torque along with their relative velocity are given in Table 4.2.

Table 4.2

S.No.	E.M.F. and Current producing torque	Relative velocity
1.	E_{s1}^{OP} and $I_{r1}^{'P}$	0
2.	E_{s1}^{OP} and $I_{r2}^{'P}$	$2s\omega$
3.	E_{s1}^{OP} and $I_{r2}^{'N}$	2ω
4.	E_{s1}^{OP} and $I_{r1}^{'N}$	$-2(1-s)\omega$
5.	$E_{r2}^{'P}$ and $I_{s2}^{''P}$	0
6.	$E_{r2}^{'P}$ and I_{s1}^{OP}	$-2s\omega$
7.	$E_{r2}^{'P}$ and I_{s2}^{ON}	$2(1-s)\omega$
8.	$E_{r2}^{'P}$ and $I_{s1}^{''N}$	-2ω
9.	E_{s2}^{ON} and $I_{r2}^{'N}$	0
10.	E_{s2}^{ON} and $I_{r1}^{'N}$	$-(2-s)\omega$
11.	E_{s2}^{ON} and $I_{r1}^{'P}$	-2ω
12.	E_{s2}^{ON} and $I_{r2}^{'P}$	$-2(1-s)\omega$
13.	$E_{r1}^{'N}$ and $I_{s1}^{''N}$	0
14.	$E_{r1}^{'N}$ and I_{s2}^{ON}	$(2-s)\omega$
15.	$E_{r1}^{'N}$ and I_{s1}^{OP}	$2(1-s)\omega$
16.	$E_{r1}^{'N}$ and $I_{s2}^{''P}$	2ω

The items 1, 5, 9 and 13 give steady torque whose values have been discussed earlier and all other items give pulsating torque. The frequencies of pulsating torque components are 2sf, 2(1 - s)f, 2f and (2 - s)f. The oscillations are set up due to pulsating torque of such frequencies. The oscillations caused by high frequency torques are sufficiently damped out by motor and load inertia while pulsating torque of low frequency presents a serious problems which are discussed later. Thus pulsating torque of low frequency (i.e. 2 sf) only is of interest.

There are only two low frequency torque components which are given by items 2 and 6 of Table 4.2 and their values are given below:

$$T_{P1} = K_T\left(\frac{E_{s1}^{OP}}{f}\right) I_{r2}^{'P} \text{ Cos } (2\, s\omega t + \alpha - \gamma_2) \quad \ldots \quad 4.24$$

$$T_{P2} = K_T\left(\frac{E_{r2}^{'P}}{sf}\right) I_{s1}^{OP} \text{ Cos } (2s\omega t + \alpha - \gamma_1) \quad \ldots \quad 4.25$$

where α is angular displacement between E_{s1}^{OP} and $E_{r2}^{'P}$ at $t = 0$

γ_1 is angle by which I_{s1}^{OP} lags E_{s1}^{OP}

and γ_2 is angle by which $I_{r2}^{'P}$ lags $E_{r2}^{'P}$

The component T_{P2} acts on the stator in the negative direction. Thus its reaction on rotor will be in the positive direction. Therefore instantaneous pulsating torque is sum of two components

$$T_P \text{ (inst.)} = (T_{P1} + T_{P2}) \quad \ldots \quad 4.26$$

And the peak pulsating torque can be given by equation

4.27

$$T_P(\text{peak}) = \sqrt{T_{P1}^2(\text{peak}) + T_{P2}^2(\text{peak}) + 2T_{P1}(\text{peak})\, T_{P2}(\text{peak}) \cos(\gamma_1 - \gamma_2)} \qquad 4.27$$

From equations 4.24 and 4.25,

$$T_{P1}(\text{peak}) = K_T \left(\frac{E_{s1}^{OP}}{f}\right) I_{r2}^{'P} \quad \ldots \quad 4.28$$

and

$$T_{P2}(\text{peak}) = K_T \left(\frac{E_{r2}^{'P}}{sf}\right) I_{s1}^{OP} \quad \ldots \quad 4.29$$

4.3.4. Experimental Verification

The experiments were performed on the same machine and under the same conditions as in the case of double impedance unbalance. These conditions were

(i) Reduced balanced voltage applied to stator

(ii) Unbalance limited to addition of resistance only.

The experiments were performed for the types of unbalances which usually occur in practice and are given below :-

(1) One line open i.e. ∞ , 0 , 0

(2) Resistance in one line i.e. R_{rA} , 0 , 0

(3) Equal resistances in two lines i.e. 0 , R_{rB} , R_{rB}

A number of sets with various values of R_{rA} = 1.48 , 0.282 , 0.151 and R_{rB} = 1.48 and 0.205 p.u. were taken.

Another set of experiments were performed with unbalanced supply voltage and unbalanced rotor impedances of types give above. The sequence components of supply voltage were $|V_1|$ = 0.875 , $|V_2|$ = 0.143 and $|V_1|$ = 0.762 , $|V_2|$ = 0.286 p.u. In each case

Steady Torque (Experimental)

Rotor Unbalance

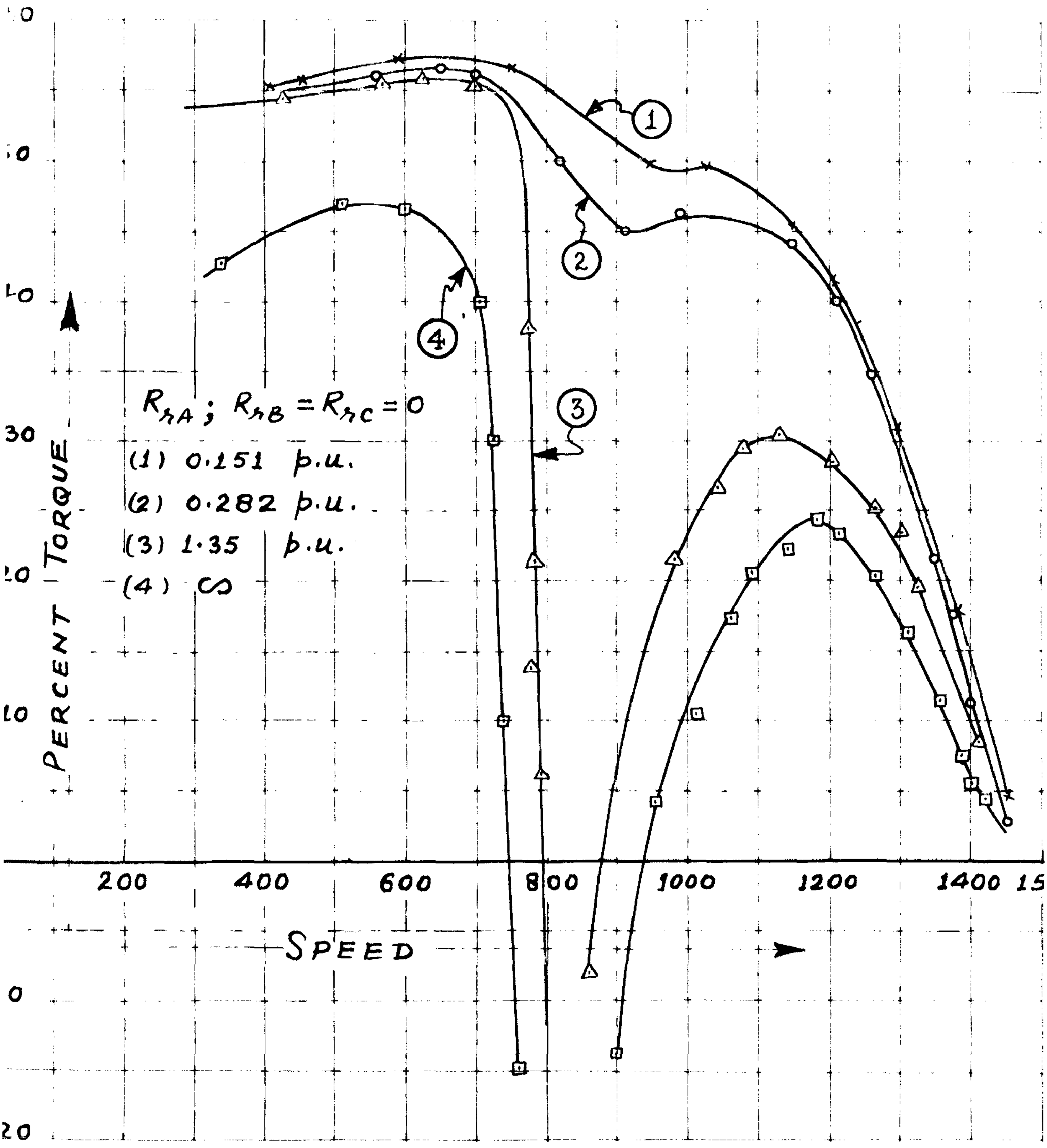


Fig. 4.16

Steady Torque (Theoretical)
Rotor Unbalance

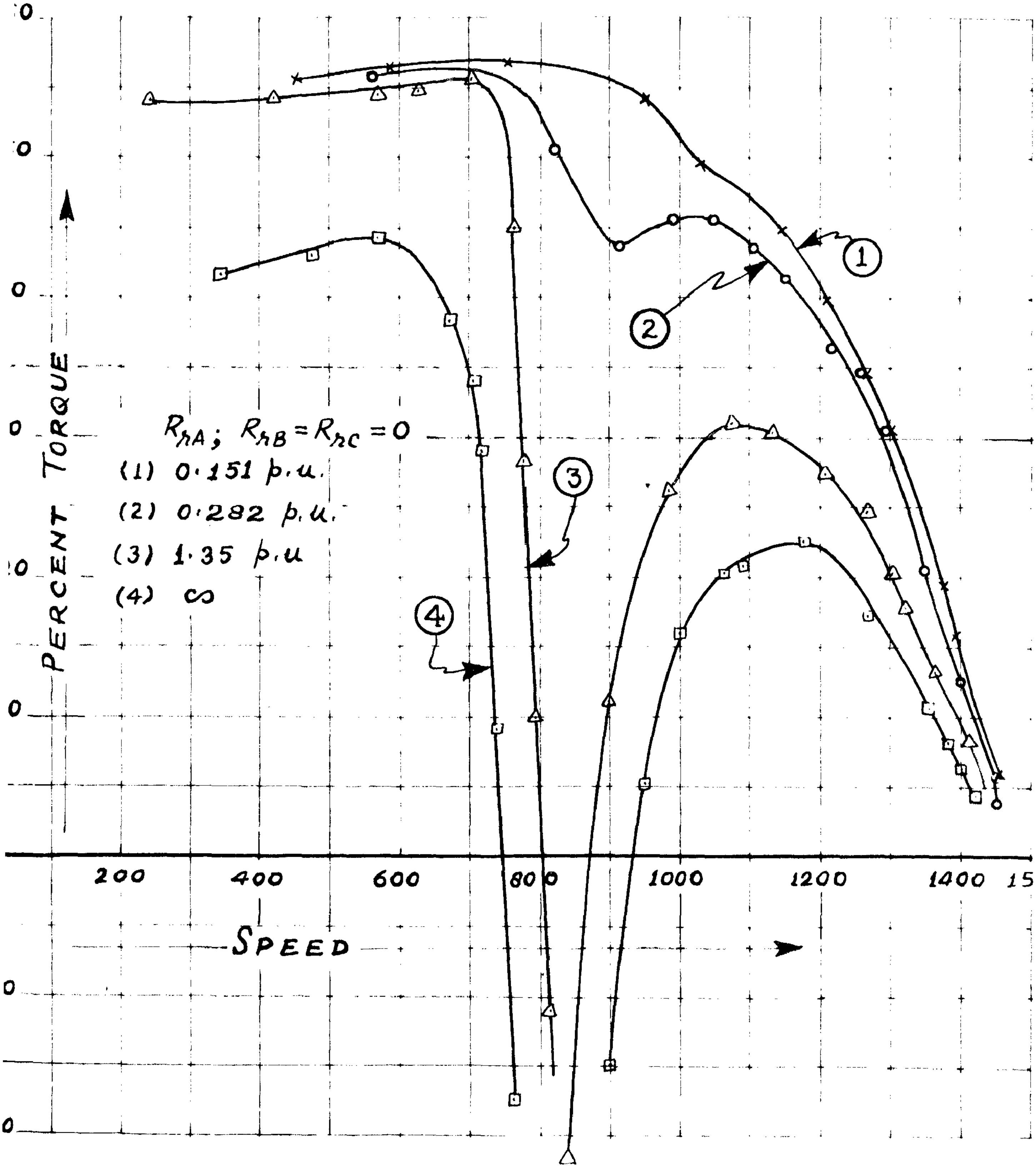


Fig. 4·17

Steady Torque

Rotor Unbalance

$R_{rA} = 0,\ R_{rB} = R_{rC} = 0{\cdot}205\,p.u.$

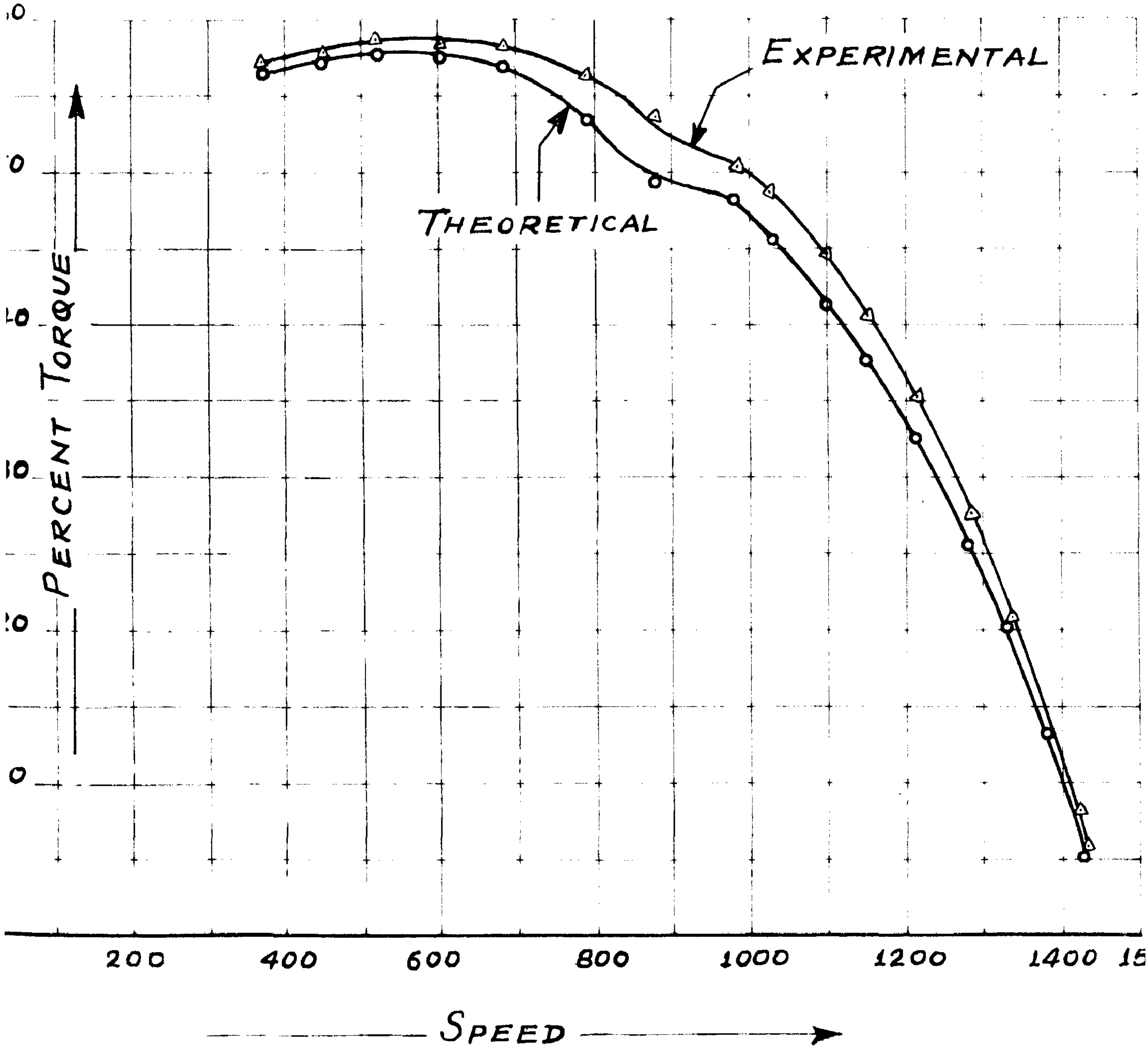


Fig. 4.18

Steady Torque

Rotor Unbalance with Unbalanced Supply

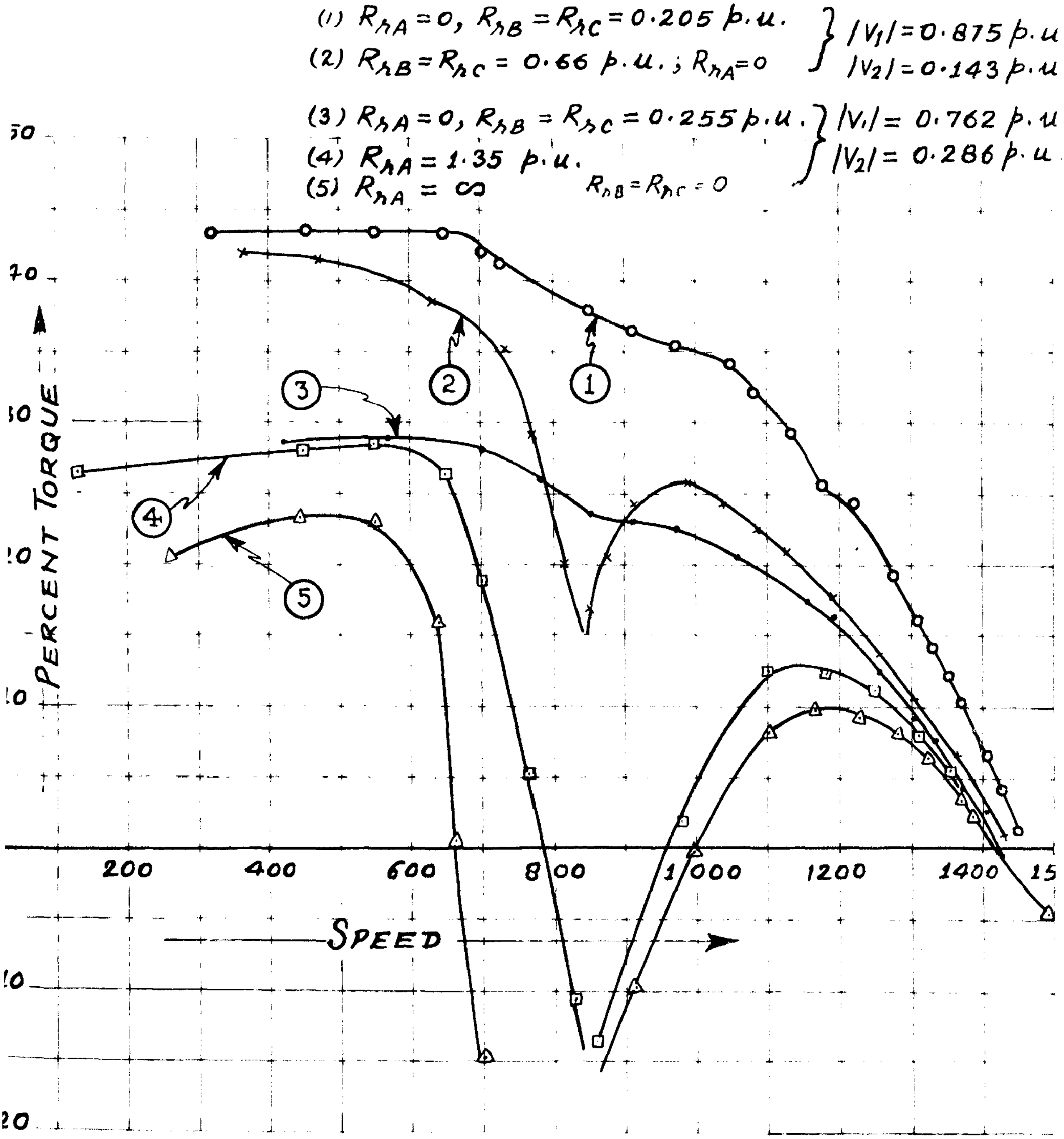


Fig. 4.19

Stator Current

Rotor Unbalance

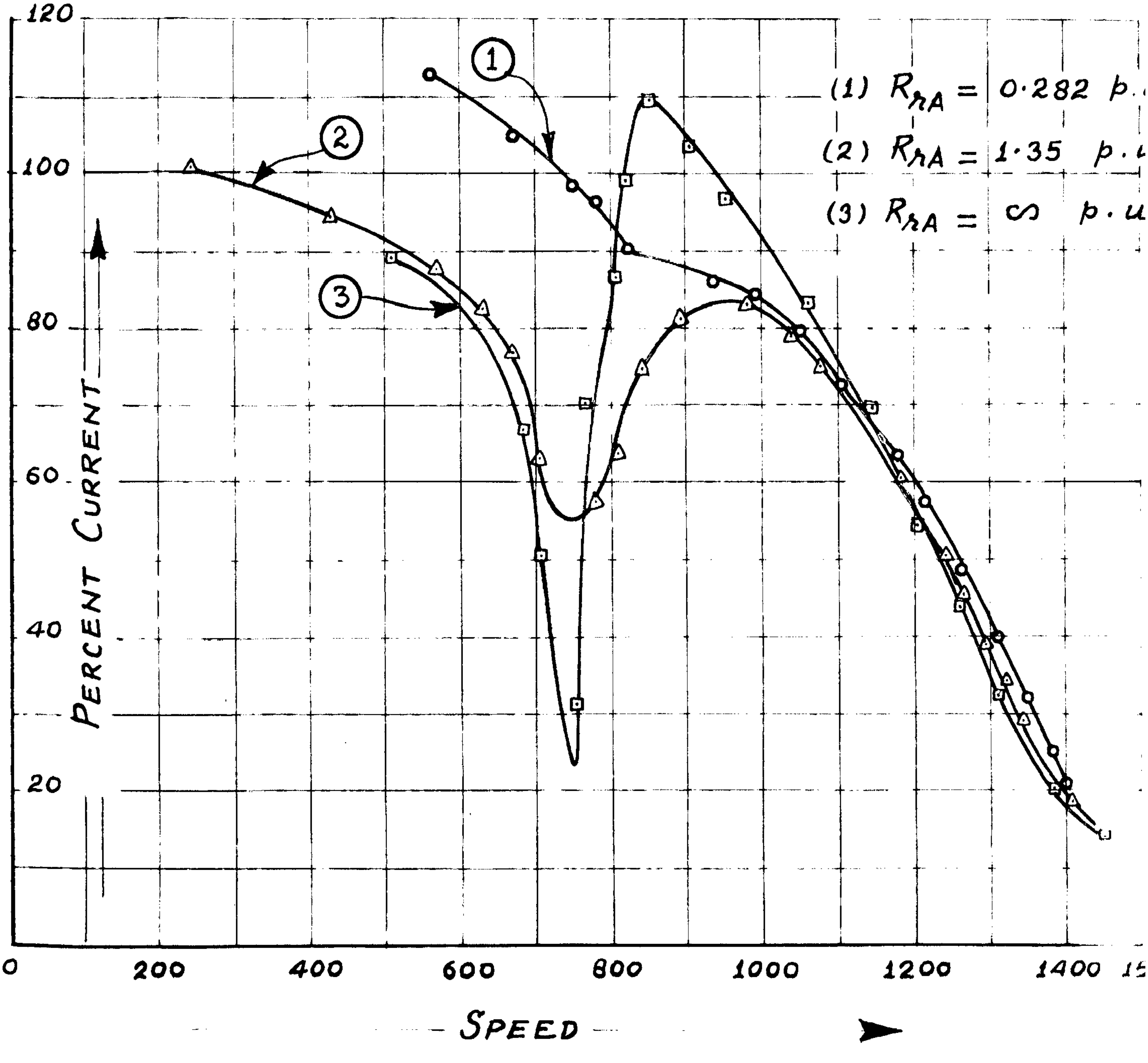


Fig. 4·20

Peak Pulsating Torque

Rotor Unbalance

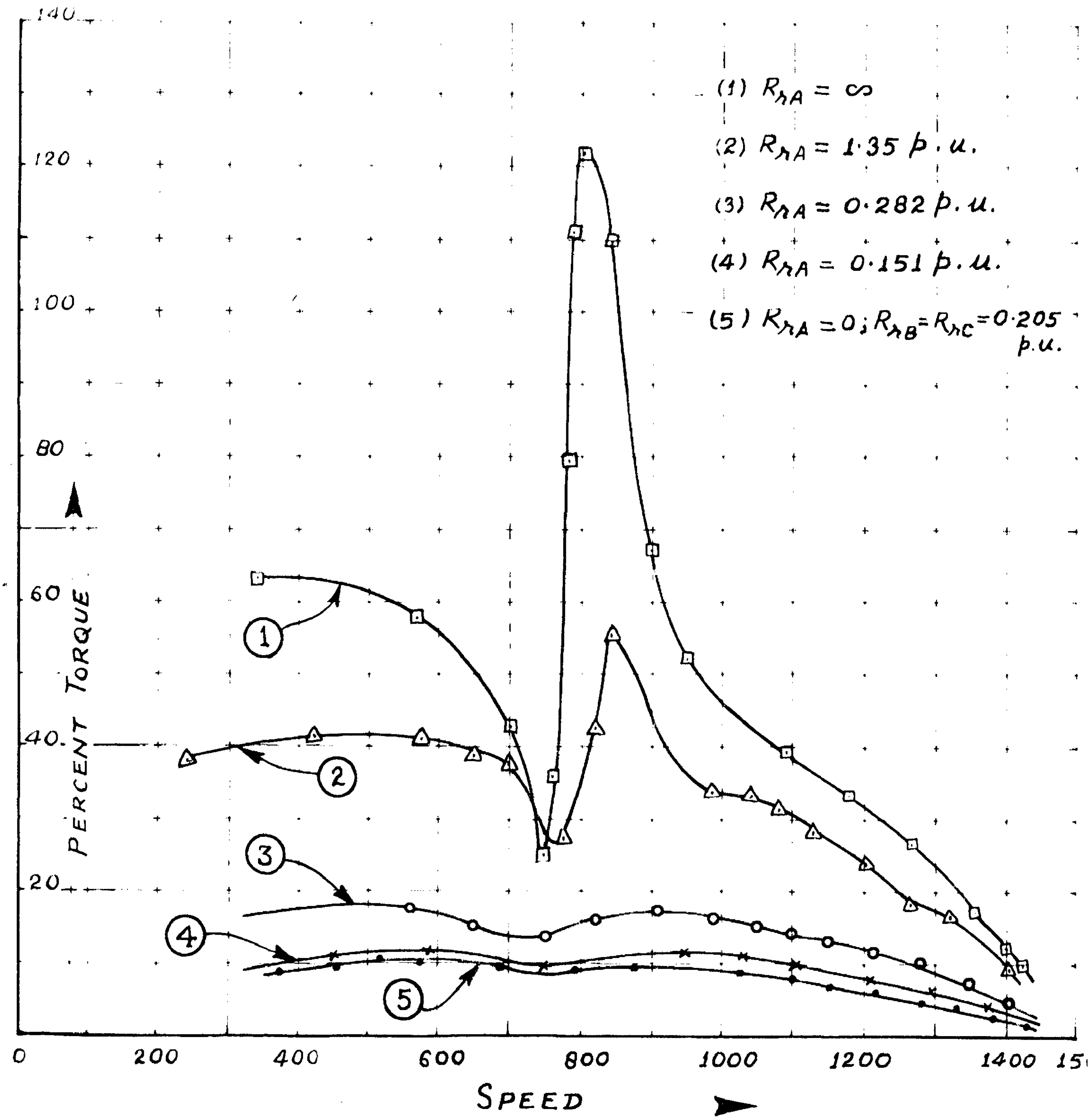


Fig. 4·21

currents and steady torques were measured over the complete speed range and results are plotted in Figs. 4.16 to 4.21. The steady torques are also computed from the equivalent circuits and compared with experimental values. It is found that the difference is well within the limit of 6%.

The following points are observed from the speed-torque characteristics with the increase of amount of unbalance:-

(1) Steady torque decreases at all slips

(2) Dip goes down

(3) Width of dip region increases.

If the amount of unbalance is increased sufficiently, the steady torque becomes negative over a sufficient range of speed. With the maximum unbalance where one line is open, the steady torque is negative between the speed of 930 and 740. At $s = 0.5$, the stator resistance $\frac{R_s}{2s - 1}$ in the equivalent circuit of Fig. 4.14 becomes infinite and the circuit opens there. Therefore this component does not contribute any torque. For $s < 0.5$ it gives negative torque and for $s > 0.5$ it contributes positive torque. Thus for $s \geqslant 0.5$, the net steady torque is positive. Hence the machine can run stable at half speed which is called half-speed stable operation and commonly known as 'Gorges Phenomenon'.

The above stated phenomenon occurs even when the supply voltage is unbalanced with unbalanced rotor. The steady torque also reduces with the increase of unbalance in supply voltage.

<u>Pulsating Torque</u>

The peak pulsating torque for the specific cases of unbalance mentioned earlier have been computed over the complete speed-range and plotted in Fig. 4.21. It is observed that the peak pulsating torque increases with the amount of unbalance. It is maximum with the value of 1.22 times base torque with the maximum unbalance i.e. one line is open. It is observed that the peak pulsating torque dips to a low value at s nearly equal to 0.5. This can be explained by the fact that in the equivalent circuit of Fig. 4.14 the negative sequence stator impedance $\frac{R_s}{2s - 1}$ becomes infinite reducing the current I'^{p}_{r2} to a low value. So that one component of pulsating torque becomes nearly equal to zero. The slip at which pulsating torque is actually minimum would depend on the value of unbalance resistances. It is to be noted from the graphs of Fig. 4.20 that the slip for minimum peak pulsating torque shifts slightly to s less than 0.5 and again comes back to s = 0.5 when one line is open.

The low frequency pulsating torque in case of rotor unbalance, presents a serious problem on account of possible resonance with the natural frequency of oscillation of the mechanical system of motor-load. At resonance severe stress would be developed in the shaft and which may cause break down of the mechanical parts. Such conditions of resonance must be avoided.

CHAPTER V

CONCLUSION.

The analysis of an induction machine with unbalanced impedances connected on both or either side of air-gap has been studied with the help of symmetrical components. The theoretical and experimental results agree within close degree and prove the validity of the theory presented within the limits of assumptions made.

A general analysis of induction machine with double unbalance is advanced. It is found that either positive or negative sequence voltage applied at stator results in an infinite number of reflections occuring alternately at rotor and stator terminals where unbalanced impedances are connected. Two flow-charts have been devised, with the help of which it is possible to trace these reflections. The machine equivalent circuits for positive and negative sequence currents of any order of reflection can be obtained from the general equivalent circuits given in the text. The general constraint equations linking these circuits at any order of reflections are also given. The infinite number of reflections result in an infinite number of frequency components in currents and voltages, where the frequency is a function of rotor speed. The angular frequency for stator currents is $(\omega \mp 2Kn)$ and for rotor current referred to stator is $(\omega \mp \overline{2K - 1}\, n)$, where K is any positive integer, negative sign for positive sequence voltage and positive sign for negative sequence voltage applied to the stator terminals. The general equations for r.m.s. currents and voltages, and also peak voltages in rotor and stator are derived. A relationship is obtained between supply voltage and stator sequence voltages. It is found that the general solution cannot be obtained because of infinite number of reflections with the intervening constraint equations being such that these cannot be represented by an equivalent circuit.

A solution and an overall equivalent circuit, however, are possible for the case of symmetrical unbalance, where the impedances in stator and rotor external circuits are symmetrical about one line. This in fact is a more practical case and is likely to occur accidentally when the machine is operating with unbalanced rotor either accidentally or intentionally. A table is given from which, the interlinking circuit can be easily looked up for any specific case of symmetrical double unbalance. A complete equivalent circuit is given for symmetrical double unbalance, for which the voltage applied at stator may also be unbalanced. It contains two infinite cascaded networks. At any given slip, the series and shunt resistances of the two infinite networks decrease monotonically with the order of reflection. After certain number of reflections, each infinite network becomes iterative circuit with no series resistance and can be terminated by the characteristic impedance of one of its sections. The order of reflection at which the series resistance becomes zero may not be same for the two infinite networks. Other authors have neglected this series resistance while computing currents in each section.

Each section of the two infinite networks carries current of a different frequency from those of other sections. Therefore, a frequency coxxection must be applied in calculating the stator and rotor resistances for each of these sections. It is assumed that the presence of other frequency currents ~~with~~ in the windings does not affect the resistance seen for any particular frequency, and also the effects of various frequencies on the external resistances are considered negligible. The iron-loss is neglected throughout the equivalent circuit because the iron-loss branch-current is very small and its effect is negligible in computing currents, voltages and torques. Because of the number of assumptions involved, a practical verification of results obtained from equivalent circuits has been carried out and it is found to be in agreement within an error of 8-12%. To avoid saturation effects and to limit the excessive voltage in stator and rotor which are expected, the experiments were conducted at half rated voltage.

With double impedance unbalance, various types of speed-torque characteristics are possible with different amount of unbalance on either side. The nature of characteristics is mainly controlled by the amount of unbalance in rotor and it is somewhat similar to that of single unbalance on rotor side. However, the point of stable operation which occurs at $s \approx 0.5$ for rotor unbalance (Gorges Phenomenon) shifts towards higher speed.

It is observed that peaks appear in speed-torque characteristics at $s = \frac{1}{2}, \frac{1}{4}, \frac{4}{5}$ .... These peaks are clearly shown in case III where one stator line is open and rotor has a resistance of 0.66 p.u. in one line. It is also noted that torque decreases with increase of unbalance. If it is increased to its maximum value i.e. both stator and rotor have one line open, the torque becomes negative throughout and the machine cannot run on its own. Further it is also found that voltages in both stator and rotor increase with the increase of unbalance. The maximum value of peak voltage is found to be 7.7 times normal voltage at $s = 0.18$ and maximum value of r.m.s. voltage is 2.8 times normal value at $s = 0.2$.

The analysis of single unbalances follows from the general unbalance analysis by introducing the relevant circuit constraints. The reflections stop at the terminals having balanced impedance. Therefore no reflection at all occurs in case of unbalanced impedance in stator and only one reflection at rotor takes place in the case of unbalanced impedance in rotor circuit. In case of stator unbalance, the stator current has two sequence components each of fundamental frequency and rotor current has sequence components of frequencies sf and $(2 - s)f$; while in case of rotor unbalance, the stator current has two sequence components of frequencies f and $(2s - 1)f$ and rotor current has two sequence components each of frequency sf. In both the cases of single unbalance, the theoretical and experimental results closely agree with each other and difference being 8-10%. It is observed that torque decreases with increase of unbalance. In case of unbalanced rotor, the width of dip in speed-torque characteristic increases with the amount of unbalance. At the dip the torque goes down and even becomes negative with sufficiently high unbalance.

Expressions for peak pulsating torque have been derived in both the cases of single unbalance. It is found that the pulsating torque varies with a frequency of 2f in case of stator unbalance and with a frequency of 2sf in case of rotor unbalance. The low frequency pulsating torque, in case of rotor unbalance, presents a serious problem on account of possible resonance with the natural frequency of oscillation of the mechanical system of motor-load. This peak pulsating torque increases with the amount of unbalance and dhows a dip at $s \approx 0.5$. As the amount of unbalance is gradually increased, the exact slip for minimum pulsating torque shifts to some what lower value of slip, but comes back to $s \approx 0.5$ at the maximum unbalance of one line open.

For limitation of time and space no practical verification of pulsating torque could be carried out. However, a low frequency oscillation for the case of rotor unbalance was observed with the help of a strobotac. The low frequency torque oscillations cause corresponding frequency speed variation of rotor coupled to the mechanical system. It is suggested that if the speed oscillations could be picked up by a suitable transducer, it should be possible to measure the magnitude of low frequency pulsating torques experimentally. It would, however, be necessary to know the time constant of the mechanical system. A homopolar generator may be used as transducer for this purpose provided the brush noise could be made negligible compared to the useful signal generated.

The high frequency pulsating torque caused by the stator unbalance is much more difficult to measure because these oscillations are thoroughly damped by the inertia of mechanical system and the resulting speed oscillation would be negligible. It may be possible to measure high frequency pulsating torque by introducing a pressure transducer in the torque transmission mechanism.

APPENDIX I

Machine Constants

1. Specifications:

No. r 84991 , Type AV 2521-1 V

Star 400-440 V , 13.2 A

6.5 KW , 8.8 CV , Cos ϕ = 0.83

1380 T/min. , 50 cycles per sec.

Rotor - 102 V , 45.5 A , Slip ring

Charleroi - Belgium

A.C.E.C. Welco Charleroi Belgium

Induction Machine is coupled to a D.C. Dynamometer.

2. Determination of Constants:

The following tests were performed to determine the various constants of the machine.

(1) Voltage ratio (Open Circuit) test:

The stator and rotor were connected in star (normal connections), slip-rings were open and the machine was stationary. First the voltage V_s' was applied to stator terminals and the voltage V_r' was recorded at the rotor terminals. Secondly, V_r'' was applied to the rotor terminals and the voltage V_s'' was recorded at the stator terminals.

$$\therefore \text{ turns ratio } = \frac{V_s'}{V_r'} \sqrt{\frac{V_s'}{V_s''} \cdot \frac{V_r''}{V_r'}} \qquad \text{I - 1}$$

The voltage V_r' and V_r'' were taken equal.

(ii) No-load Test:

The machine with rotor open circuit was made to run at synchronous speed using D.C. machine as a drive. The input current and power recorded would give the magnetizing and iron loss components and dynamometer readings would give the friction and windage loss at synchronous speed. The same experiment was repeated at different speeds to determine frictional and windage loss at various speeds. Thus the friction loss is subtracted from the output of machine calculated at various slips from the equivalent circuits to get output at shaft.

Both experiments (i) and (ii) were performed at the reduced voltage of 200 V, because load tests were performed at this reduced voltage.

(iii) Short circuit test:

As the short circuit current depends upon the position at which the rotor is locked, the rotor was rotated at very slow speed instead of making it stationary. Thus average short circuit current was recorded by applying low voltage at stator terminals. Before taking the readings, the machine was to run for sufficient period to bring to normal temperature. Thus hot resistances of stator and rotor were found.

The frequency of supply was varied and the same experiment was repeated for different frequencies. The curves giving the relationship of frequency and resistances of stator and rotor were obtained as shown in Fig. I-1.

Immediately after the short circuit test when the machine was hot, D.C. resistance of stator and rotor were measured.

Results:

All constants are calculated on per unit basis

Base voltage = 115.5 V , Base current = 13.2 A

Base impedance = 8.75 ohms , Base torque = 21.4 Lb.ft.

R_s = 0.1715 ; X_s = 0.286 ; X_m = 8.32 p.u.

R_r = 0.269 ; X_r = 0.286 ; R_m = 102 p.u.

Voltage ratio = 4.56

The variation of R_s and R_r with frequency is given in Fig. I-1, which follows nearly the linear law given below:

For stator $R_s = 0.153 \; (3.65 \times 10^{-4})f$

For rotor $R_r = 0.240 \; (5.66 \times 10^{-4})f$

These equations have been used for calculating the stator and rotor resistances at various frequencies.

Frequency Vs. Resistance

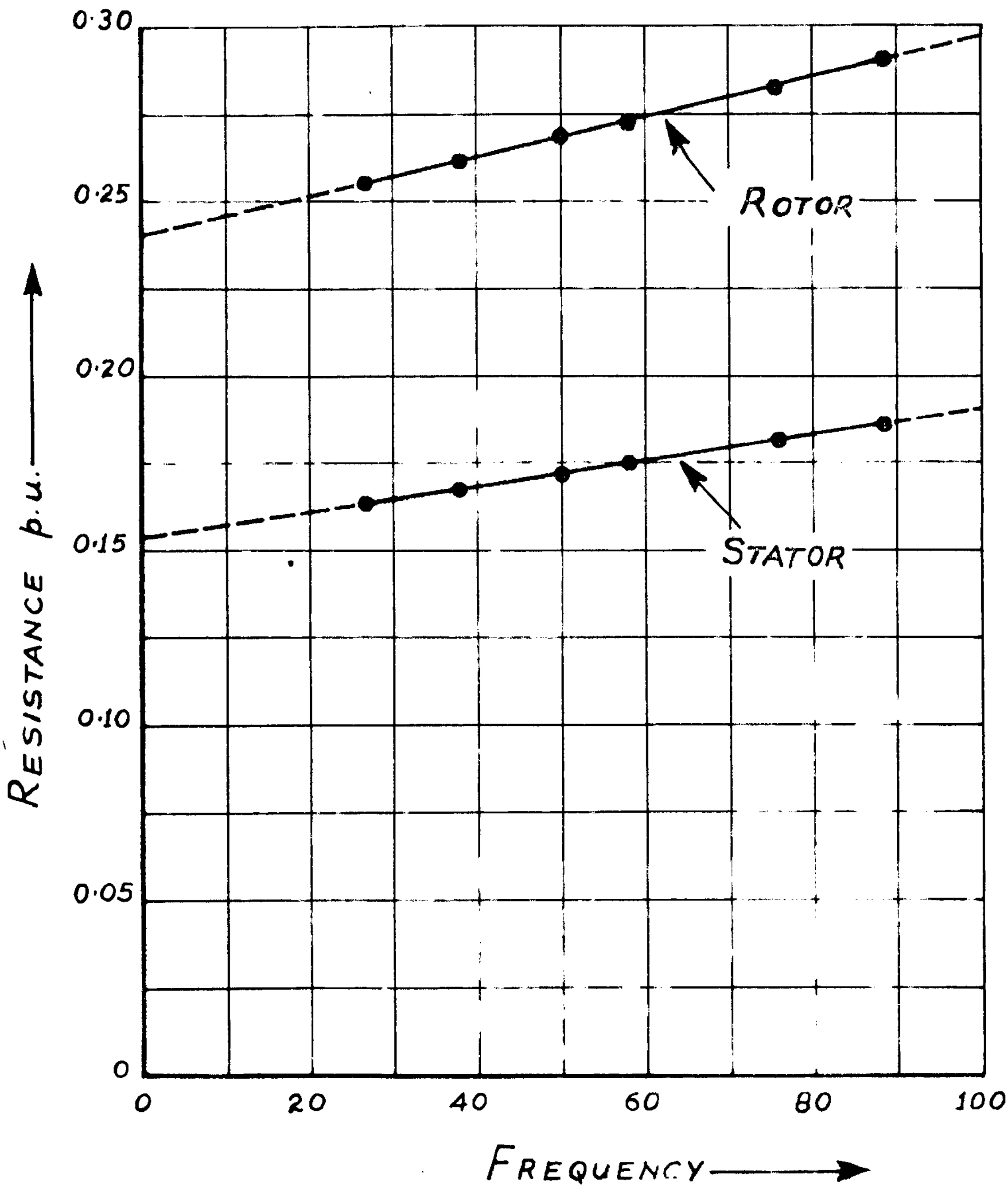


Fig. I·1

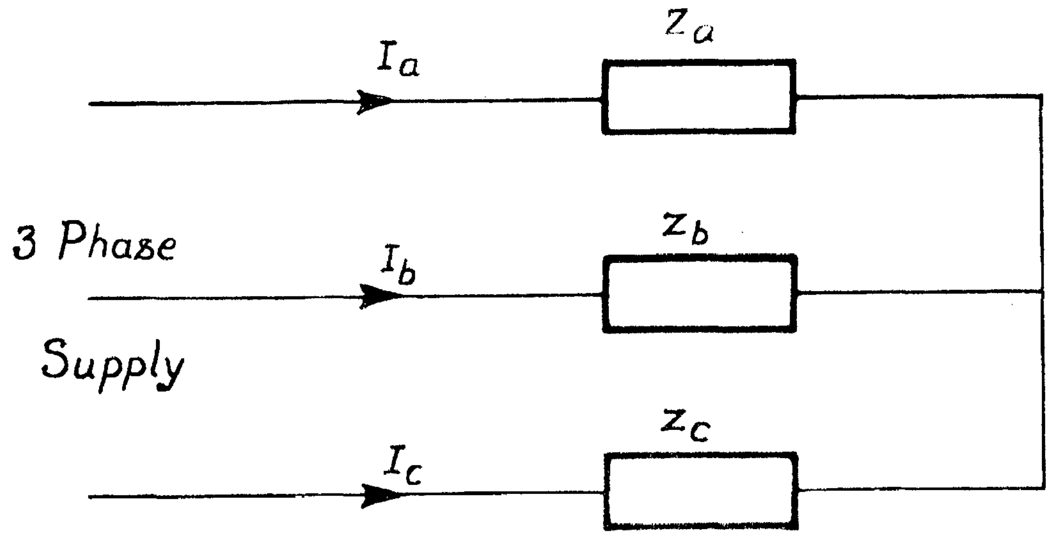


Fig. II-1 — Unbalanced Impedances Connected in Star.

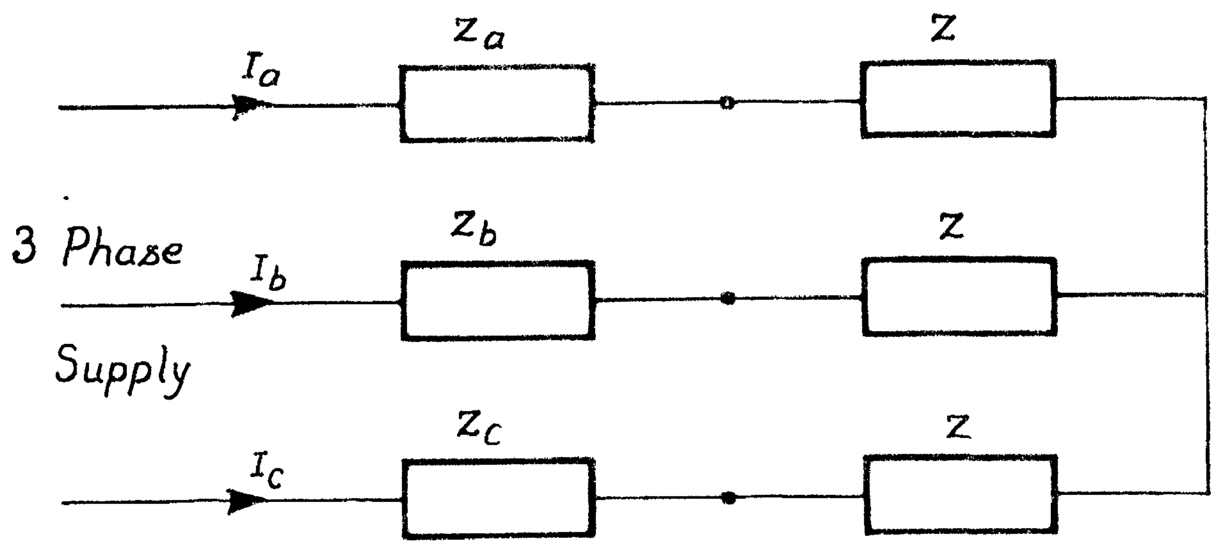


Fig. II-2 — Unbalanced Impedances in Series with Balanced Impedances Connected in Star.

APPENDIX II

Symmetrical Components

II.1 A set of three unbalanced voltages V_A , V_B and V_C (similarly currents also) can be resolved into positive, negative and zero sequence components as given below:

$$\left.\begin{aligned} V_A &= V_1 + V_2 + V_0 \\ V_B &= a^2V_1 + aV_2 + V_0 \\ V_C &= aV_1 + a^2V_2 + V_0 \end{aligned}\right\} \quad \ldots \quad \text{II.1}$$

Solving these equations, the sequence components can be found out. Therefore

$$\left.\begin{aligned} &\text{Positive Sequence Component, } V_1 = \frac{1}{3}(V_A + aV_B + a^2V_C) \\ &\text{Negative Sequence Component, } V_2 = \frac{1}{3}(V_A + a^2V_B + aV_C) \\ &\text{Zero Sequence Component, } \quad V_0 = \frac{1}{3}(V_A + V_B + V_C) \end{aligned}\right\} \quad \text{II.2}$$

where a is an operator of value

$$a = |1| e^{j\frac{2\pi}{3}} \quad \ldots \quad \text{II.3}$$

II.2 Determination of sequence voltages from unbalanced line voltages:

Let V_{AB} , V_{BC} and V_{CA} be the line voltages, and

$$\left.\begin{aligned} V_{AB} &= V_A - V_B \\ V_{BC} &= V_B - V_C \\ V_{CA} &= V_C - V_A \end{aligned}\right\} \quad \ldots \quad \text{II.4}$$

Substituting the values of V_A , V_B and V_C from equation II.1 in equation II.4, one gets

$$V_{AB} = (1 - a^2)V_1 + (1 - a)V_2 \quad \ldots \quad II.5$$

$$V_{BC} = (a^2 - a)V_1 + (a - a^2)V_2 \quad \ldots \quad II.6$$

From equations II.5 and II.6, positive and negative sequence components can be found out,

$$V_1 = \frac{1}{3} V_{AB} + \frac{1}{3} \angle 60 \; V_{BC} \quad \ldots \quad II.7$$

$$V_2 = \frac{1}{3} V_{AB} + \frac{1}{3} \angle -60 \; V_{BC} \quad \ldots \quad II.8$$

II.3 Determination of Sequence Impedances of three Unbalanced Impedances Connected in Star:

If three impedances Z_a , Z_b and Z_c are connected in star and I_a , I_b and I_c are line currents as shown in Fig. II.1, the voltage drop across each impedance is given by

$$\left.\begin{aligned} V_a &= I_a Z_a = (I_1 + I_2)Z_a \\ V_b &= I_b Z_b = (a^2 I_1 + aI_2)Z_b \\ V_c &= I_c Z_c = (aI_1 + a^2 I_2)Z_c \end{aligned}\right\} \quad \ldots \quad II.9$$

From equation II.2, positive and negative sequence voltage components can be derived.

$$\left.\begin{aligned} V_1 &= \tfrac{1}{3}(Z_a + Z_b + Z_c)I_1 + \tfrac{1}{3}(Z_a + a^2 Z_b + aZ_c)I_2 \\ V_2 &= \tfrac{1}{3}(Z_a + Z_b + Z_c)I_2 + \tfrac{1}{3}(Z_a + aZ_b + a^2 Z_c)I_1 \end{aligned}\right\} \quad \ldots \quad II.10$$

Let $$\frac{1}{3}(Z_a + Z_b + Z_c) = Z_0 \quad \text{Zero Sequence Impedance}$$
$$\frac{1}{3}(Z_a + a^2Z_b + aZ_c) = Z_1 \quad \text{Positive Sequence Impedance} \qquad \text{II.11}$$
$$\frac{1}{3}(Z_a + aZ_b + a^2Z_c) = Z_2 \quad \text{Negative Sequence Impedance}$$

Therefore substituting equation II.11 in equation II.10,

$$\begin{aligned} V_1 &= Z_0I_1 + Z_2I_2 \\ V_2 &= Z_0I_2 + Z_1I_1 \end{aligned} \qquad \ldots \qquad \text{II.12}$$

Thus the phase voltage contains both positive and negative sequence components even the line voltage may or may not be balanced.

If balanced impedances in series with unbalanced impedances, are connected in star, as shown in Fig. II.2, the sequence impedances can be determined as follows:

$$\begin{aligned} V_a &= (Z_a + Z)I_a = (Z_a + Z)(I_1 + I_2) \\ V_b &= (Z_b + Z)I_b = (Z_b + Z)(a^2I_1 + aI_2) \\ V_c &= (Z_c + Z)I_c = (Z_c + Z)(aI_1 + a^2I_2) \end{aligned} \qquad \ldots \qquad \text{II.13}$$

Manipulating these equations, positive and negative voltages can be derived

$$\begin{aligned} V_1 &= (Z_0 + Z)I_1 + Z_2I_2 \\ V_2 &= (Z_0 + Z)I_2 + Z_1I_1 \end{aligned} \qquad \ldots \qquad \text{II.14}$$

APPENDIX III

Derivation of Formulae when Rotor Impedances are unbalanced

The analysis is made for positive sequence only with the help of Figs. 4.12 and 4.14 and constraints equation 4.15 a at the rotor terminals.

From the network of Fig. 4.12(a)

$$V_{s1}^{OP} = (R_s + jX_s)I_{s1}^{OP} + jX_m\,(I_{s1}^{OP} - I_{r1}^{'P}) \quad \ldots \quad \text{III.1}$$

$$\text{or } V_{s1}^{OP} = (R_s + jX_s + jX_m)I_{s1}^{OP} - jX_m I_{r1}^{'P} \quad \ldots \quad \text{III.1(b)}$$

$$\text{and } -\frac{V_{r1}^{'P}}{s} = (\frac{R_r}{s} + jX_r)I_{r1}^{'P} + jX_m\,(I_{r1}^{'P} - I_{s1}^{OP}) \quad \ldots \quad \text{III.2}$$

$$\text{or } -\frac{V_{r1}^{'P}}{s} = (\frac{R_r}{s} + jX_r + jX_m)I_{r1}^{'P} - jX_m\,I_{s1}^{OP} \quad \ldots \quad \text{III.2(b)}$$

Solving equations III.1(b) and III.2(b), one gets

$$V_{r1}^{'P} = s\,V_{s1}^{OP}\frac{jX_m}{R_s + jX_s + jX_m} - \left[R_r + j(X_r + X_m)s + \frac{sX_m^2}{R_s + jX_s + jX_m}\right]I_{r1}^{'P}$$

III.3

$$\text{Let } \quad \frac{jX_m}{R_s + jX_s + jX_m} = K \quad \text{a constant}$$

$$\text{and } \quad R_r + j(X_r + X_m)\cdot s + \frac{sX_m^2}{R_s + jX_s + jX_m} = Z_{11}$$

$$\text{Therefore } \quad V_{r1}^{'P} = sK\,V_{s1}^{OP} - Z_{11}I_{r1}^{'P} \quad \ldots \quad \text{III.4}$$

Now from the network 4.12(b)

$$\frac{V'^{P}_{r2}}{s} = (I'^{P}_{r2} - I''^{P}_{s2})\, jX_m + \left(\frac{R_r}{s} + jX_r\right) I'^{P}_{r2} \quad \ldots \quad \text{III.5}$$

and

$$0 = \left(\frac{R_s}{2s-1} + jX_s\right) I''^{P}_{s2} + (I''^{P}_{s2} - I'^{P}_{r2})\, jX_m \ldots \quad \text{III.6}$$

Solving these two equations III.5 and III.6

$$V'^{P}_{r2} = (I'^{P}_{r2}) \left[R_r + j(X_r + X_m)\cdot s + \frac{sX_m^2\,(2s-1)}{R_s + j(X_m + X_s)(2s-1)} \right]$$

$$= (I'^{P}_{r2})\,(Z_{22}) \quad \text{III.7}$$

where

$$Z_{22} = R_r + j(X_r + X_m)\cdot s + \frac{sX_m^2\,(2s-1)}{R_s + j(X_m + X_s)(2s-1)} \quad \text{III.8}$$

But V'^{P}_{r1} and V'^{P}_{r2} are related through sequence impedances of external rotor impedances. Rewriting it

$$\left.\begin{aligned} V'^{P}_{r1} &= Z_{r0}\, I'^{P}_{r1} - Z_{r2}\, I'^{P}_{r2} \\ V'^{P}_{r2} &= Z_{r1}\, I'^{P}_{r1} - Z_{r0}\, I'^{P}_{r2} \end{aligned}\right\} \quad \ldots \quad \text{III.9}$$

Comparing the equations III.4 , III.7 and III.9

$$V'^{P}_{r1} = Z_{r0} I'^{P}_{r1} - Z_{r2} I'^{P}_{r2} = sK\, V^{0P}_{s1} - Z_{11} I'^{P}_{r1} \quad \text{III.10}$$

$$V'^{P}_{r2} = Z_{r1} I'^{P}_{r1} - Z_{r0} I'^{P}_{r2} = I'^{P}_{r2}\, Z_{22} \quad \ldots \quad \text{III.11}$$

Solving equation III.10 and III.11, one gets

$$I'^{P}_{r1} = \frac{Z_{22} + Z_{r0}}{(Z_{11} + Z_{r0})\,(Z_{22} + Z_{r0}) - Z_{r1}\, Z_{r2}}\; sK\, V^{0P}_{s1}$$

$$= \frac{1}{\frac{Z_{11}}{s} + \frac{Z_{r0}}{s} - \frac{\frac{Z_{r1}}{s} \cdot \frac{Z_{r2}}{s}}{\frac{Z_{22}}{s} + \frac{Z_{r0}}{s}}} K\, V_{s1}^{OP}$$

$$= \frac{1}{\frac{Z_{11}}{s} + Z_L} K\, V_{s1}^{OP} \quad \ldots \quad III.12$$

$$\text{where } Z_L = \frac{Z_{r0}}{s} - \frac{\frac{Z_{r1}}{s} \cdot \frac{Z_{r2}}{s}}{\frac{Z_{22}}{s} + \frac{Z_{r0}}{s}} = R_L + jX_L \quad \ldots \quad III.13$$

This can be called as load impedance.

Now $K\, V_{s1}^{OP}$ is Thevenin equivalent positive sequence open circuit voltage of network 4.12(a) and $\frac{Z_{11}}{s} = Z_e$ Thevenin equivalent impedance of network 4.12(a).

A network can be represented from the equation III.12 as shown in Fig. III.1.

$$\text{Now} \quad I_{r2}^{'P} = \frac{Z_{r1}}{(Z_{11} + Z_{r0})(Z_{22} + Z_{r0}) - Z_{r1}\, Z_{r2}} sK\, V_{s1}^{OP} \quad III.14$$

Substituting the values of $I_{r1}^{'P}$ and $I_{r2}^{'P}$ in equations III.1(b) and III.6 respectively, the values of I_{s1}^{OP} and $I_{s2}^{''P}$ can be written as follows:

$$I_{s1}^{OP} = \frac{V_{s1}^{OP}}{R_s + jX_s + jX_m} + \frac{jX_m}{R_s + jX_m + js_m} I_{r1}^{'P} \quad III.15$$

$$\text{and} \quad I_{s2}^{''P} = \frac{jX_m}{\frac{R_s}{2s-1} + jX_m + jX_s} I_{r2}^{'P} \quad \ldots \quad III.16$$

Similarly these equations for negative sequence supply voltage at stator terminals can be derived by replacing s by (2 - s)

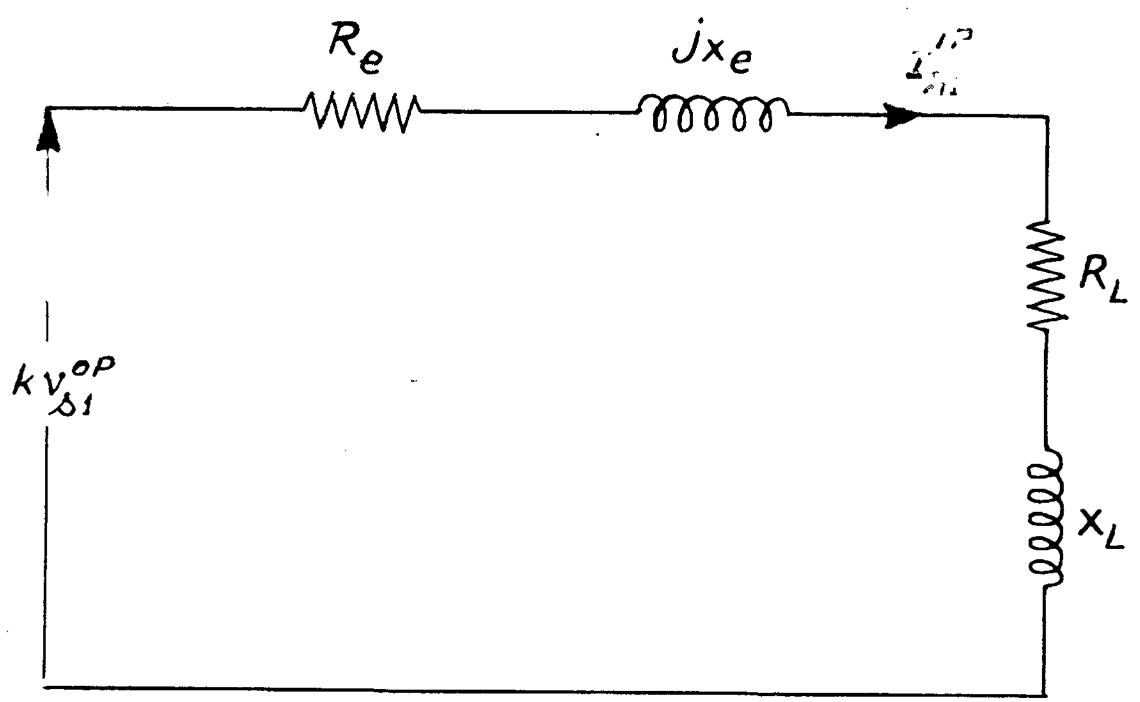


Fig. III-1 — Reduced Equivalent Circuit of Machine with Rotor Impedances Unbalanced.

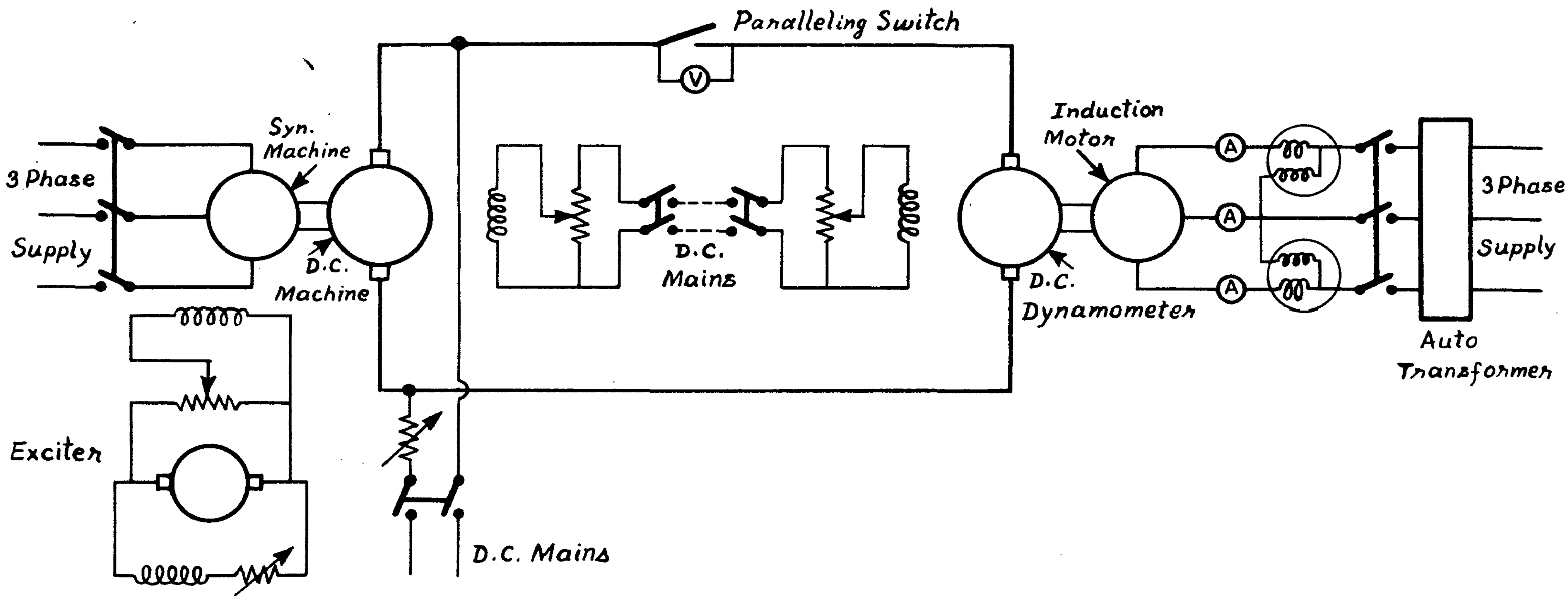


Fig. IV-1 — Connection Diagram for Experimental Set-up for Load Test

APPENDIX IV

Technique for Load Test over Complete Speed Range.

In a simple load test on an induction machine using belt or a loading machine, load test cannot be conducted beyond the slip at which the torque developed is maximum. In order to conduct the test over the complete slip range from 0 to 1, the Ward-Leonard arrangement shown in Fig. IV.1 was used. In this a d.c. dynamometer was coupled to the induction machine under test and another d.c. machine was coupled to a synchronous machine. Both the d.c. machines were separately excited and paralleled. By adjusting the field current of d.c. machine, voltage across the armature of the dynamometer was adjusted to get the desired speed. The inherent speed torque characteristic of the dynamometer should be such that its slope is greater than that of speed torque curve of induction machine at the point of intersection of two speed torque characteristics, so that machine runs stable at all speeds.

NOTATIONS

$\omega = 2\pi f$ Synchronous velocity

n Angular velocity of rotor

f Fundamental frequency

s Slip

R_s Stator resistance

R_r Rotor resistance

X_s Stator reactance at supply frequency

X_r Rotor reactance at supply frequency

X_m Magnetizing reactance at supply frequency

Z_{sA}, Z_{sB}, Z_{sC} Stator external impedances in lines A, B, C respectively

Z_{rA}, Z_{rB}, Z_{rC} Rotor external impedance in lines A, B, C respectively

Z_{s0}, Z_{s1}, Z_{s2} Zero, positive & negative sequence impedances of external unbalanced impedances connected in stator

Z_{r0}, Z_{r1}, Z_{r2} Zero, positive and negative sequence impedances of external unbalanced impedances connected in rotor

V and I R.M.S. voltage and current

Two sub-scripts and two super-scripts are attached to each voltage and current

The first sub-script (s or r) denotes whether the quantity exists in stator or rotor

The second sub-script (1 or 2) denotes whether it is positive or negative sequence quantity

The first super-script (dash like $'$, $''$, $'''$ ...) denotes order of reflection.

The second super-script (P or N) denotes whether the quantity is generated from positive or negative sequence supply voltage.

V_1	Positive sequence component of supply voltage
V_2	Negative sequence component of supply voltage
Z_{M1}	Input impedance of machine equivalent circuit for positive sequence supply voltage
Z_{M2}	Input impedance of machine equivalent circuit for negative sequence supply voltage
T_1	Average Torque when positive sequence voltage is applied
T_2	Average Torque when negative sequence voltage is applied
T_{net}	Net average torque
T_p	Pulsating torque
T_p(peak)	Peak Pulsating torque

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