SHORT-RANGE ECONOMIC OPERATION OF A COMBINED HYDRO-THERMAL SYSTEM

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By

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Roorkee.

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C_E_R_T_I_F_I_C_A_T_E

Certified that the dissertation entitled "SHORT-RANGE ECONOMIC OPERATION OF A COMBINED HYDRO-THERMAL POWER SYSTEM" which is being submitted by Sri R.Thakur in partial fulfilment forthe award of the Degree of Master of Engineering in "Power System Engineering" of University of Roorkee, Roorkee, is a record of candidate's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for..8..... months from January 1968 to..Aug. 1968...1968 for preparing dissertation for the Master of Engineering Degree at this University.

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	PAGE
Synopsis	1
CHAPTER I: INTRODUCTION	2
CHAPTER II: MAXIMUM PRINCIPLE APPROACH TO THE DISPATCH PROBLEMS	3
2.1. Statement of the problem	10
2.2. Application of the Maximum Principle	14
2.3. Solution for specific cases	15
2.3.1. Constant head hydro plant	
2.3.2. Constant head with linear discharge	
2.4. General results for the optimum load allocation	17
2.4.1. Variable head hydro plants with negligible line losses	
2.4.2. Variable head hydro plants with line losses	
2.5. Simplified model system	21
CHAPTER III: MATHEMATICAL TREATMENT OF THE DISPATCH PROBLEM	
3.1. Two point boundary value problem	24
3.2. Methods of Solution	26
3.3. Gradient Technique	29
3.4. General equations of the simplified model	33
3.4.1. Constants characterizing the model	38
3.5. Numerical solution	39
CONCLUSION	45
APPENDIX	
i) Computer Programme	
ii) Lists of Results	
BIBLIOGRAPHY	

S_Y_N_O_P_S_I_S

L.S. Pontryagin's Maximum principle is applied to the problem of determining the operation of a combined hydro-steam generating system for the minimum generating costs. The dispatch formulae have been derived for power systems using various operating characteristics governing the dynamics of the systems. A gradient technique has been described for the determination of the optimum control Trajectory using the maximum principle.

The numerical treatment is given to the economical operation of a simplified model system. Variable hydraulic head is considered at the hydro station. The system operation is assumed for short-range. The merits of the Maximum principle have been discussed as a new technique for obtaining solution of economic problem.

INTRODUCTION:-

Recent trends towards economical aspects of the integrated Power System Operation has given much importance for investigation of the methods for the Optimum load scheduling between individual unit at stations and between stations in the Power System. A State of Activity has characterised the past decade in the economic power dispatch field. Various papers have been written, incorporating different combinations of the enumerated features in formulating the problem. Still some new methods for solution of the problem are to be introduced for the efficient and satisfactory results at the same time reducing the complexity in computation.

The long range problem involves a prediction of the probable flows, for at least a year in advance, of rivers and their tributaries above the dam sites. After the amount of water that is to be available at each dam site for use within the specified period of say a day or a week is determined from long range study, the use of this water in conjunction with steam to supply the load at a minimum cost of delivered power is the short-range study, which is the main concern of the present one at hand. Stated briefly the short-range problem resolves itself into the determination of the optimum load allocation among hydro and thermal stations, all of which have peak capabilities, while the hydro stations are constrained to use a specified amount of water. The reservoir storage capacities are small so that over the span of 24 hrs the variation in withdrawals caused significant variations in head. The permissible variations of withdrawals and discharge are confined to boundary intervals. The reservoir is assumed to be located in close proximity so that the transmission losses are relatively small factor in an optimum schedule for the hydro plants.

In the combined system operation the problem appears to be a variational one and thus the main objective is to minimize the integrated fuel cost of thermal plants. This has to be achieved only by the well-planned usage of water to attain maximum economy. The complexity of this problem is due to the variety of constraints to which they are subjected. The method of solution must be able to produce schedules fast enough so that they may be used on daily basis.

1.2. GENERAL REVIEW:-

Since last many years different techniques have been developed for the solution of the problem and also have been applied successfully to several existing systems in operation. Pierre Mass'e formerly of the Electricitede France, is among the originators of these methods. He is concerned with operations over long period of time. The complexity of this decisive problem is due to its stochastic nature.

Upto 1955 very little had been published relative to optimum loading of combined hydrothermal system. The major step in the solution of this problem was the development of classical calculus of variations [14], [25], [26], [27]; gradient methods [2], [24]; Dynamic programming [6], [7], [26] and the maximum principles of L.S. Pontryagiri's [15], [19]. The real difficulty in adapting the calculus of variation approach to any existing physical system is that all variables must be made time dependent. For a typical. hydrocubic plant, this leads to complicated expressions, subject to many non-linear constraints. Though in some cases the Lagrangian multipliers and adjoint variables have identical appearance and also have the similar treatment, but the correspondence between them breaks down when the magnitudes of forcing functions are limited;

such cases cannot then be handled by classical calculus of variation.

Prof R.J.Cypser [24] has derived a relation for the load division among hydro and steam stations, the hydro being subject to water restraints. Cypser's relation is non-linear, and thus does not lend itself to a solution either by Numerical iteration or by means of an analog computer of Network Analyzer type. John J. Carey [11] has tried to linearize Cypser's relations so that it may be solved by algebraic methods etc. He has ignored minimum loading or peacking capability of the individual plant in the methematical formulation of the optimum loading problem. In the problem a total generation curve is assumed instead of a total load curve. In the analysis the consideration of all operating restraints with the exception of that on water usage has been omitted to simplify the mathematical formulation.

V.S.Shakhanov(Moscow) [4] has argued that the method of relative increments should be adopted as the theoretical basis for computer programming. For the past fourty years ever since the work of some authors, the mathematically substantiated variational principle of equal differential consumption of fuel (or water) has been used for determining the economical load distribution between individual units at stations and between stations in the Power systems. Without exception, this principle is the foundation of all methods and means of load distribution proposed in every part of the world; the works of Bolotov, Kirchmayer etc., including the "method of relative increments" and the methods which employ the operation of "gradient descent".

L.K. Kirchmayer in his paper [14] alongwith several other authors have used variational methods to develop coordination equations for use in the digital computer solution with series plant multiple chains of plants, and intermediate reservoirs. These techniques have been applied in a program which integrates the hydraulic and steam resources upto a week's period. The output consists of hourly plant loading which will provide onerations at minimum fuel costs. The computer program developed for the solution of this problem requires as input: (1) gas fuel availability at each steam plant hourly, (2) gas and oil fuel prices at each steam plant (3) incremental heat-rate curves for each steam unit, (4) Hydro and steam unit availability and minimum and maximum hourly capability. (5) estimated system hourly load demand, (6) incremental water rate curves for each hydro plants, (7) desired storage water releases, an (8)Transmission loss coefficients. A mathematical model is construct where by means of conversion coefficients, the hydro curves are effectively converted to incremental plant cost curve. The computer proceeds to simulate economic system operation in hourly steps in accordance with the theory of equal incremental costs. At the end of the specified time interval, the water withdrawn is compared with that scheduled for withdrawal. The conversion coefficients are adjusted and the search repeated untill the computed amount of water withdrawn are equal to that scheduled.

A.Arismunandar and F.N.Noakes, have also derived [13] time dependent functional equations using calculus of variations. Necessary and sufficient conditions are given to establish the facts The paper proves that several previously developed equations for short-range optimization are equivalent, and that these formulae are simplified forms of the general equations developed here.

While other variational methods solve problem in a point by point manner, his approach solves for whole interval to be optimized as integral units. It formulates four necessary and three sufficient conditions to guarantee the attainment of the required optimum solution. The first necessary condition is given in the form of general equations for the thermal and hydro plants, while the other six conditions are actually tests to establish this true optimum. The third necessary condition is given in two identical forms to allow flexibility in whatever computational method may be employed. Due to variable end point problem to be complex, only fixed optimizing periods are considered.

The more successful study of the problem is extended by several authors including B.Bernhotlz, using dynamic programming. Anstine in his article [3] has used Dynamic programming with successive approximation to determine optimized dispatches for the operation of two series connected variable head reservoirs. B. Bernhotlz published several papers [6],[7],[8],[9], [10] concerning the present problem and deal with the equally important but mathematically neglected problem of economic operation of an electric power system over short-period of time, say 24 hours. An iterative procedure for determining economic daily schedules is presented, in which successive system schedules are determined each yielding a greater profit than the preceding. A realistic model of part of the system operated by Hydro-electric power Commission of Ontario contains 16 sources of generation. Because of the constraints on water usage, each hour's operation cannot be considered separately so that the problem involves 16 x 24 = 384 variables.

The author has extended his work in six parts. While in previous two papers the author describes how to obtain the minimum minimorium by using an iterative procedure in which successive hydro schedules are determined with the property that the corresponding minimum costs of thermal generation decreases monotonically, in these papers, the iterative procedure is explained in terms of examples and restricted to system with one thermal station and a number of hydro stations. In his 3rd part of scheduling the thermal sub-system using constrained steepest Descent governing equations of some complexity have been derived using classical variational method, and employing Lagrange multipliers, but a completely rigorous solution had yet to be given because of the omission, in forming the problem, of inequality constraints fixing the ranges of station outputs. To include these he has used an alternative approach, called "Constrained steepest descent" or the "gradient projection method". The problem of determining which unit to operate is not treated here. It is assumed that the unit to be operated, and the times they are brought on and taken off the time, are known. Second part of this paper shows how to utilize the result of first paper in scheduling a system consisting of any number of fixed head hydro stations and any number of steam stations. In the ninth article the maximum station output is approximated by a concave differential function of discharge. In the last publication of his paper the criterian of an optimum schedule is maximum system profit rather than maximum fuel cost on assumption that the energy may be brought and sold across inter-connection.

Recently the computational approach to the problem of most economical operation has been made by a few authors employing L.S. Pontryagin's maximum principle. E.B. Dahlin and D.W.C.Shen have done remarkable work in the field and have considered all possible aspects coming into system operation. They have also considered river transport delay and wave phenomena in this dispatch formulae. Hano etc. in their paper [19] have presented the solution for economical operation of a simplified model system and the long term operation of a multi-reservoir system. The relaxation method is successfully applied to the solution of optimal water usage policy. He has also made comparison between the maximum principle approach and dynamic programming.

From the computational analysis of the problem it has become evident that the maximum principle is a powerful tool, in comparison to other computational procedures, not only for the treatment of engineering problems, such as time-minimal control of servo-mechanisms, but also as a new technique of mathematical programming for the treatment of problem in mathematical economics.

The study underlying here at hand is to utilize the maximum principle by Pontryagin [1] . The most outstanding advantage over several others is its great generality with respect to permissible system characteristics. Only dynamic programming has a similar range of application. The two-point boundary problem always arises in the problem of optimal control, whether one uses classical calculus of variations or Pontryagin's equations. It is the major difficulty of the method. Pontryagin's principle have been greatly effective fort the successful results in the problem of terminal control of Servomechanism.

Uptill now a very few applications of the maximum principle as a new approach to mathematical programming, have been made. Everyone finds difficulty in determining the initial values of the adjoint variables to a system. As will be shown later that the

Maximum Principle in Association with a Gradient Technique for the determination of optimum control and Trajectories has been found most rapid and accurate method to overcome these difficulties in comparison to other procedures. All the additional Complexities arising due to the movement of state and central variable within the permitted boundary have been effectively anticipated. Actually these are the obstacles in easy application of this method. Some modifications are made by changing the hard constraints into soft constraints and also making use of some inherent properties of the problem.

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CHAPTER - II

THE MAXIMUM PRINCIPLE APPROACH TO THE DISPATCH PROBLEMS

2.1. STATEMENT OF THE PROBLEM:-

A combined hydro-thermal system can be operated optimally with respect to fuel cost. This leads to an integral type of cost function which can be minimized with methods like Pontryagin's maximum principle. A degree of complexity is introduced by consideration of variable heads at the hydro station. The object is to obtain an optimum allocation of generated power to meet load requirements which act as a constraint on the problem.

Daily operation may be considered as a deterministic process. Those elements, such as future load demand and water head, which are stochastic when viewed in the long term can be assumed to be known 24 hours in advance. The complexity of this problem is due to the large number of variables and the variety of constraints to which they are subjected. The method of solution should be such that it can allocate the optimum load schedules fast enough so that they may be used on a daily basis.

Theoperation of the combined system is analyzed for different operating characteristic governing on the mode of operation of the system. For rigorous mathematical analysis a simplified model of the system consisting of one hydro station and one thermal station as shown in fig. 1. is considered. It is assumed that the stations are jointly supplying electric power to a load centre, through a system of lossless transmission lines. A prediction of future load as a function of time is assumed available and is shown in fig. 1A on a daily load demand basis. Likewise, all necessary information with regard to future water availability at the hydro

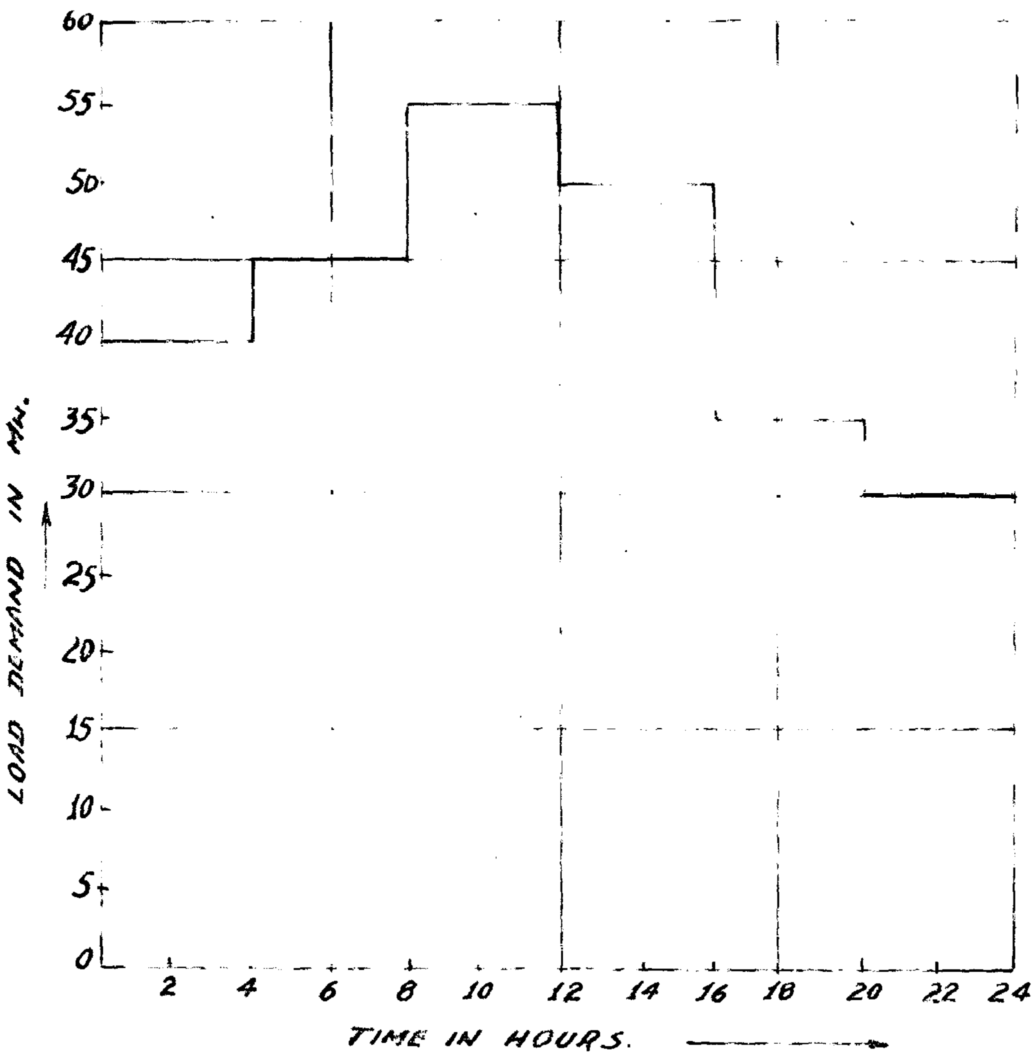


FIG. 1 A. DAILY LOAD CURVE.

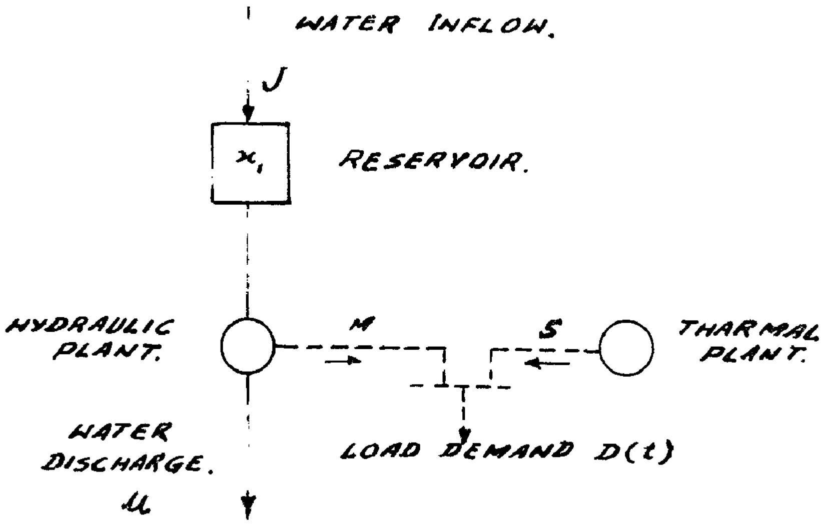


FIG. 1 SIMPLIFIED MODEL SYSTEM.

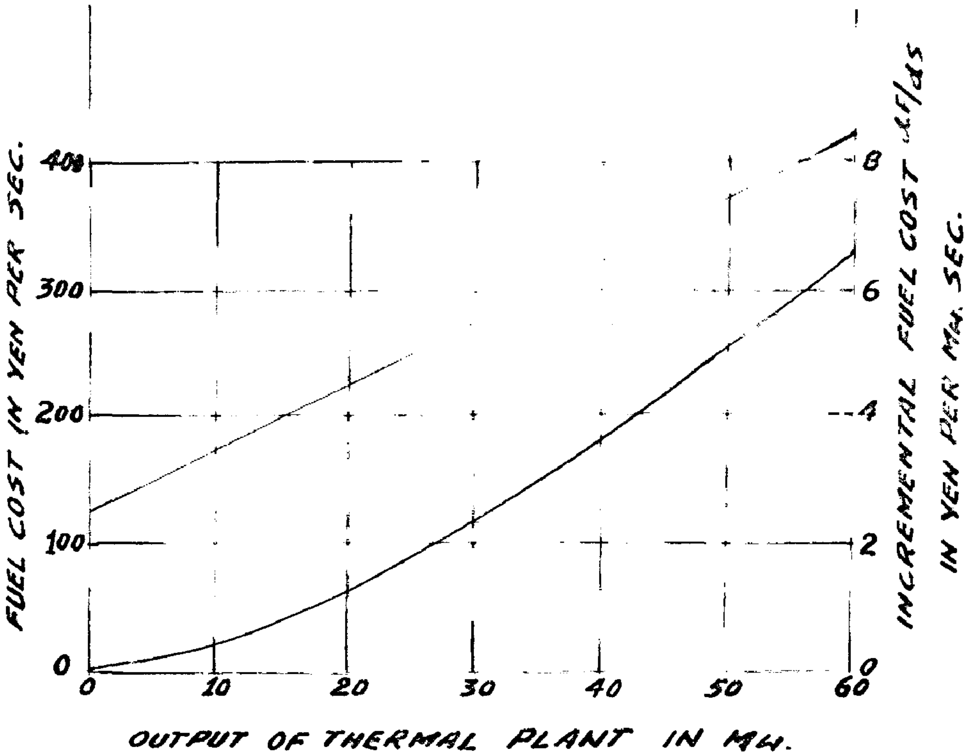


FIG. 2 FUEL COST CHARATERISTICS.

power plants is assumed known. The hydro plant efficiency and the incremental cost of the hydro have been neglected. The starting cost of the steam plant may be neglected if it is in the operation all the time.

The water storage in the reservoir, the rate of water inflow and the rate of water discharge is expressed by the following relation, neglecting the effect of overflow etc.

$$\frac{dx_i}{dt} = Ji - u_i \qquad (1)$$

where:

x_i = water storage in the reservoir of i^{th} plant (m^3)

J_i = water inflow into the reservoir of i^{th} plant (m^3/sec.)

u_i = water discharge from the reservoir of i^{th} plant (m^3/sec.)

The choice of u_i is restricted by its upper and lower limit, for the hydraulic turbine has its limited capacity specified by its rating and therefore u_i is bounded as follows:-

$$u_{i\ max} \geqslant u_i \geqslant u_{i\ min}$$

Where $u_{i\ max}$ and $u_{i\ min}$ are the upper and lower limit of the discharge, respectively.

The hydraulic head is dependent upon the reservoir capacity and thus, the power output of the hydro plant may be expressed as

$$W_i = f\ (x_i, u_i) \qquad (2)$$

where $i = 1,2 \ldots\ldots r$

W_i= hydro power developed at the i^{th} plant (megawatts)

Since the total generation of the combined system equals the total load demand on the system at that instant, the power balance equation

$$\sum_{i=1}^{N} S_i + \sum_{i=1}^{T} W_i = D(t) \qquad (3)$$

Where:

N = total no of steam plants

S_i = Steam power developed of the i^{th} steam plant(Mws)

$D(t)$ = Total load demand at time t(megawatts)

It is also specified that the steam unit operates with both maximum and minimum generating limit i.e.

$$S_{i\ max} \geqslant S_i \geqslant S_{i\ min}$$

Where:

$S_{i\ man}$ and $S_{i\ min}$ are the maximum and minimum limit of the steam unit for operation.

The problem is to determine the drawdown at the hydro stations and the generation of the steam units over the optimization interval under the condition that the total fuel cost over the optimization interval is minimized and the total generation of the systems equals the total load demand.

The objective can be stated in mathematical terms as the minimization of a functional

$$f_0 = \int_0^T \sum_{i=1}^{N} F_i(S_i)dt \qquad (4)$$

Where:

T = length of optimization interval (time)

$F_i(S_i)$ = Cost per unit time of operation of steam unit No. i (Rs/time)

The minimization of the functional has to be achieved for steam generation and the water discharge as control variables. The choice or control u_i is to be determined for a given load demand D(t) and the initial and final storages of water as $x_i(0)$ and $x_i(T)$, respectively.

It is convenient to define a new variable

$$x_0(T) = \int_0^T \sum_{i=1}^{N} F_i(S_i)dt \qquad (5)$$

Clearly, $x_0(T) = f_0$ and $x_0(0) = 0$

The problem can now be restated for employing the maximum principle in the following terms:

Define a vector $X = \{X_0, X_i, X_{x+1}\}$ whose initial and final states are $X(0) = \{x_0(0), x_i(0), 0\}$ and $X(T) = \{x_0(T), x_i(T), T\}$, respectively, and the dynamics of the systems are governed by the following differential equations for a given time-dependent function D(t) as

$$\frac{dx_0}{dt} = F_i(S_i) \qquad (6)$$

$$\frac{dx_i}{dt} = J_i - u_i \qquad (7)$$

$$\frac{dx_{r+1}}{dt} = 1 \qquad (8)$$

where the new state variables x_{r+1} is clearly specified as

$$x_{r+1}(0) = 0$$

$$x_{r+1}(T) = T$$

Therefore, the dynamic system to be controlled can also be described by the state equations

$$\frac{dX}{dt} = f\,(X,u,t) \qquad (9)$$

Where X and u are vectors of dimension "n" and "r" respectively, with $r \leqslant n$. The all initial and m terminal states are specified as

$$X_i(0) = x_0(0)\ ,\ i = 1,2\ \ldots\ldots n$$

$$X_i(T) = x_i(T),\ \ i = 1,2\ \ldots\ldots m \leqslant n \qquad (10)$$

Given the system (9) with boundary conditions (10), the problem is now to determine the admissible control u which minimizes the performance index

$$x_0(T) = \int_0^T L(x,u,t)dt \qquad (11)$$

2.2 APPLICATION OF MAXIMUM PRINCIPLE:

With reference to the optimizing condition of the maximum principle, which states that optimal control here is the one which minimizes the corresponding Hamiltonian function:

$$H = p_0 \sum_{i=1}^{N} F_i(S_i) + \sum_{i=1}^{T} p_i(J_i - u_i) + p_{r+1} \qquad (12)$$

where:

p_i are the auxiliary or adjoint variables.

From the maximum principle $p_0 \leqslant 0$ and it is equal to -1 for a homogenous adjoint equations which is admissible in the present problem. It is also marked that p_{r+1} is a constant and does not influence the maximization of H. The above relation (12) is solved further and the problem can, therefore, be simplified by considering the maximization of

$$H' = -\sum_{i=1}^{N} F_i(S_i) + \sum_{i=1}^{T} p_i(J_i - u_i) \qquad (13)$$

The components of u_i are restricted within or on the boundary in u_i space. Within the boundary, because of minimization.

$$\frac{\partial H}{\partial u_i} = 0 \qquad (14)$$

Similarly the components of S_i are restricted within or on the boundary in S_i space. Within the boundary because of minimization.

$$\frac{\partial H}{\partial S_i} = 0 \qquad (15)$$

The adjoint variables, $p(t)$ are defined by the Hamiltonian system

$$\frac{dx_i}{dt} = \frac{\partial H}{\partial p_i} \qquad (16)$$

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial x_i} \qquad (17)$$

The applications of the maximum principle are analyzed for the following specific cases:

2.3. SOLUTION FOR SPECIFIC CASES

The system can be characterised as follow:

2.3.1. CONSTANT HEAD HYDRO PLANT

The computation time and the difficulty of converging to the solution is strongly dependent upon the number of differential equations and boundary conditions for the state variables at $t = T$

The hydro power developed is expressed in equation (2) as a function of water storage and the water discharge, can be approximated by the following relations:

$$W_i = HO\ (1 + cx_i)u_i \qquad (18)$$

Where

HO = the basic water head (in meters)

C is the correction factor for the change of water head because the hydraulic head will vary with the change of reservoir capacity. When the cross-section area of the reservoir is very large, then the water head correction factor for the change of water storage for the simplified model system will be negligibly small.

The cost function F(S) can be approximated by the following quadractic function of power developed by the steam unit i.e.

$$F_i(S_i) = a_i S_i + a_i + 1\ S_i^2 \qquad (19)$$

Since $\frac{dp_i}{dt} = -\frac{\partial H}{\partial x_i}$

With reference to equation (18)

$$\frac{dp_i}{dt} = -\frac{dF_i}{ds_i}\ HO\ Cu_i$$

for C=0 we find that $\frac{dp_i}{dt} = 0$ and hence the adjoint variable p_i is a constant. The effect of this can be seen on the incremental cost of the thermal plant.

Maximizing Hamiltonian function w.r. to u_i

$$\frac{\partial H}{\partial u_i} = -\frac{dF_i}{dS_i} \cdot \frac{\partial S_i}{\partial W_i}\ \frac{\partial W_i}{\partial u_i} - p_i = 0$$

we find, $\frac{dF_i}{dS_i}$ = the incremental rate of the thermal plant No.i.

$$= p_i / HO(1+Cx_i)$$

Therefore, we observe that for C =0 and p_i to be a constant, the incremental rate is also constant.

2.3.2. CONSTANT HEAD WITH LINEAR DISCHARGE:

Sometimes for simplicity to a great extent, in addition to the previous assumption of constant heads hydro plants, it can also be assumed that the hydroplants have linear operating characteristics, with which the following equations apply:

$$\frac{\partial (J_i - u_i)}{\partial W_i} = m_i \qquad (20)$$

for i = 1,2 ......r

where

m_i = a constant for each hydro plant integrating equation (18).

$$(J_i - u_i) = m_i W_i + m_i' \qquad (21)$$

where

m_i' = constant determined by the known function $W_i(u_i)$

Making use of the power balance equation (3) together with (13).

$$H\& = -F_i \left(D(t) - \sum_{i=1}^{r} W_i\right) + \sum_{i=1}^{r} p_i \left(m_i W_i + m_i'\right) \qquad (22)$$

It is seen in the above relation (21) that the first term is always constant for any variation of W_i as long as ΣW_i is constant. The last term will have the same property if

$$p_i m_i = \text{constant} = K \qquad (23)$$

The usefulness of the above relation can be experienced when desired for the numerical solution of the problem.

2.4. GENERAL RESULTS FOR THE OPTIMUM LOAD ALLOCATION:-

2.4.1. Variable head hydro plant with negligible lines losses.

The usefulness of the maximum principle is realised when it is utilized here for allocation of optimum load among the power generating plants.

For convenience let us assume that S_i for $i = 2, 3 \; N$ and W_i for $i = 1,2 \; \ldots\ldots r$ are independent variables. Thus, the power balance equation can be written in the modified form

$$S_1 + \sum_{i=2}^{N} S_i + \sum_{i=1}^{r} W_i = D(t) \qquad (24)$$

Therefore, from equation (24)

$$\frac{\partial S_1}{\partial S_i} = -1 \qquad (25)$$

$$\frac{\partial S_1}{\partial W_i} = -1 \qquad (26)$$

The Hamiltonian function is also written in the form

$$H' = -F_1(S_1) - \sum_{i=2}^{N} F_i(S_i) + \sum_{i=1}^{r} p_i(J_i - u_i) \qquad (27)$$

From equation (27) utilizing relation (14)

$$\frac{\partial H}{\partial u_i} = \frac{dF_1(S_1)}{dS_1} \; \frac{\partial S_1}{\partial W_i} \; \frac{\partial W_i}{\partial u_i} - p_i$$

$$= 0$$

or

$$\frac{dF_1(S_1)}{dS_1} = - \frac{p_i}{\frac{\partial S_1}{\partial W_i} \; \frac{\partial W_i}{\partial u_i}} \qquad (28)$$

The partial derivative of equation (18) w.r. to u_i is given by

$$\frac{\partial W_i}{\partial u_i} = HO\,(1 + Cx_i) \qquad (29)$$

Substituting (26) and (29) in (28) we have

$$\frac{dF_1(S_1)}{dS_1} = \frac{p_i}{HO(1+Cx_i)} \qquad (30)$$

where

$\frac{dF_1(S_1)}{dS_1}$ is defined as the incremental rate of the steam plant No. 1.

With reference to equation (19)

$$\frac{dF_1(S_1)}{dS_1} = a_1 + 2a_2 S_1 \qquad (31)$$

Substituting (31) in (30) we have

$$a_1 + 2a_2S_1 = \frac{p_i}{HO(1+Cx_i)}$$

$$\therefore S_1 = \left\{ \frac{p_i}{HO(1+Cx_i)} - a_1 \right\} \frac{1}{2a_2} \qquad (32)$$

Now with S_i, for $i = 2,3 \ldots N$, as another control variables we can write from equation (15) as

$$\frac{\partial H}{\partial S_i} = 0$$

$$\text{or} \quad -\frac{dF_1(S_1)}{dS_1} \frac{\partial S_1}{\partial S_i} - \frac{dF_i(S_i)}{dS_i} = 0 \qquad (33)$$

With reference to equation (25)

$$\frac{dF_1(S_1)}{dS_1} = \frac{dF_i(S_i)}{dS_i} \qquad (34)$$

The relation obtained above in equation (34) establishes an important result determining the optimum load sharing between the steam plants. This can be stated: For economical loading, the incremental rate of the input cost of each thermal station must be equal.

On the other hand this also shows the advantage of using maximum principle in the light of [20], where the rigorous treatment of Lagrangian multiplier is given while arriving at the same result.

2.4.2. Variable Head hydro plant with Line Losses:

The losses will be function of power transmitted through the transmission lines and therefore, the power balance relation in this case modified to

$$\sum_{i=1}^{N} S_i + \sum_{i=1}^{Y} W_i = D(t) + L(S,W) \qquad (35)$$

where

S = N - dimensional vector defining the generation of all steam units(megawatts)

W = Vector defining the generation of all hydro stations (megawatts)

With the above assumption of one steam unit as dependent variable we have from equation (35)

$$\frac{\partial S_1}{\partial S_i} + 1 = \frac{\partial L}{\partial S_i} + \frac{\partial L}{\partial S_1} \cdot \frac{\partial S_1}{\partial S_i}$$

or

$$\frac{\partial S_1}{\partial S_i} = -(1 - \frac{\partial L}{\partial S_i}) / (1 - \frac{\partial L}{\partial S_1}) \qquad (36)$$

With reference to equation (33) together with (36)

$$\frac{dF_1(S_1)}{dS_1} \cdot \frac{(1 - \frac{\partial L}{\partial S_i})}{(1 - \frac{\partial L}{\partial S_1})} = \frac{dF_i(S_i)}{dS_i}$$

or

$$\frac{dF_1(S_1)}{dS_1} / (1 - \frac{\partial L}{\partial S_1}) = \frac{dF_i}{dS_i} / (1 - \frac{\partial L}{\partial S_i}) \qquad (37)$$

The result obtained in equation (37) can now be compared with equation (34) where the losses were ignored. We find that one extra factor in the denominator of equation (37) appears on both sides. This factor has been called in [20] as PENALTY FACTOR. Thus, the statement of economical load allocation changes with a slight modification of penalty factor in the denominator of the incremental rate of the fuel cost.

2.5. SIMPLIFIED MODEL SYSTEM:-

The problem is simplified considering only one thermal plant and one hydro palnt jointly supplying a load centre throggh a system of lossless transmission lines. The idea behind the consideration of this simplified model system is to extend it for further investigation of mathematical treatment for satisfactory and rapid solution.

With reference to equation (3) for the existing system

$$S_1 + W_1 = D(t) \qquad (38)$$

The power developes by the hydroplant

$$W_1 = HO\,(1 + Cx_1)u_1 \qquad (39)$$

Equations (38) and (39) combined together

$$u_1 = \frac{(D - S_1)}{HO(1+Cx_1)} \quad \text{if} \quad u_{min} \leqslant u_1 \leqslant u_{1\,max}$$

$$= u_{1\,max} \qquad \text{if } u_1 > u_{1\,max}$$

$$= u_{1\,min} \qquad \text{if } u_1 < u_{1\,min} \tag{40}$$

With the above constraints on the choice or control u_1 it should be understood that the state variable x_1 is also restricted between its boundary values. Thus, if the overflow and emptying of reservoir is not permitted the state variable must lie between the two end conditions. Where the state variable is defined in the present case as

$$\frac{d_{x1}}{dt} = J_1 - u_1 \tag{41}$$

The power developed from steam plant for optimum condition is defined from equation (32)

$$S_1 = \left\{ \frac{p_1}{HO(1+Cx_1)} - a_1 \right\} \frac{1}{2a_2} \tag{42}$$

The value of steam generation S_1 obtained from equation (42) should always lie within the steam station generation limits. Supposing at any instant the steam generation crosses either of the maximum or minimum limit of generation then at that time the water discharge or the choice of u_1 has to be modified so that it again comes in the operating region.

The adjoint equation which defined in equation (17) can be expressed here for one hydro plant as

$$\frac{dp_1}{dt} = - \frac{\partial H}{\partial x_1}$$

or

$$\frac{dp_1}{dt} = - \frac{\partial H}{\partial S_1} \cdot \frac{\partial S_1}{\partial W_1} \cdot \frac{\partial W_1}{\partial x_1} \tag{43}$$

With the above references of equations we can also write as

$$\frac{dp_1}{dt} = (a_1 + 2a_2\ S_1)\ (\quad -1)\ (HO\ u_1 C)$$

$$= -\ HO\ C\ U_1\ (a_1 + 2a_2\ S_1) \qquad (44)$$

Ultimately we have a set of two differential equations (41) and (44) which have to be solved for the optimization interval. As it is specified that the state variable x_1 has been defined by the relation (10) and therefore, at the end of the optimization interval it has to be satisfied and converged to its initial value.

C H A P T E R III

MATHEMATICAL TREATMENT OF THE DISPATCH PROBLEM

3.1. TWO POINT BOUNDARY VALUE PROBLEMS:

Pontryagin's equations defined in equations (16) and (17) give rise to the general forms of system equations. For the present case of simplified model system which have been expressed as

$$\frac{dx_1}{dt} = \frac{\partial H}{\partial p_1} = J_1 - u_1 \qquad \text{(i)}$$

and

$$\frac{dp_1}{dt} = -\frac{\partial H}{\partial x_1} = -\ HoC\ u_1 \frac{dF_1}{dS_1} \qquad \text{(ii)}$$

where the state variable is defined as

$$X_1(0) = \bar{x}_1(0)$$
$$X_1(T) = x_1(T)$$

Therefore, it is a set of simultaneous differential equations with an incomplete set of prescribed conditions. The solution for adjoint variable p_1 has to be adjusted such that it finally satisfies the prescribed boundary conditions.

For the purpose of numerical solution itis convenient to separate ordinary differential equations into two classes, according to the position of the associated prescribed conditions. If all such necessary conditions are given at one point in the range of the independent variable they are usually called initial conditions, and the differential system is of "initial value" type. When more than one value of the independent variable is involved the conditions are called boundary conditions, and the differential system is of "Boundary value" type.

Here for the state variable x_1, we have one condition at each end or boundary of the range of integration. A problem of this kind we call a "boundary-value" problem, and this definition is extended to all problems in which two separate points are involved in the prescribed conditions, here called "boundary conditions".

Two-point boundary conditions arise in all sorts of physical problems. In the light of the above introduction to the 2-point boundary problem we find that the load dispatch problem analysis with the application of maximum principle is always a case of two-point boundary value type. This situation always appears in variational problems.

In the above forms of general equations it is eliminated by the minimization of H subject to any constraints. Such generalized constraints cannot be included in the treatment by classical calculus of variations.

The solution of (ii) is not complete without a specification of the boundary conditions. We have to minimize the performance index defined in (11) by choice of $u_1(t)$. This minimized form of integral (11) is defined to be the cost function.

$$V\,[X_1(0),\,0] = \underset{u_1(t)}{\text{Min}} \int_0^T L\,(X_1, u_1, t)\,dt \qquad \text{(iii)}$$

By definition of the cost function V in equation (iii),

$$V\,(x_1(T),\,T) = 0 \qquad \text{(iv)}$$

In other words, the minimized performance integral must be zero when the lower limit of integration equals the upper limit. It follows from (iv) that

$$\left[-\frac{V}{x_1} \quad dx_1 \right]_{t=T} = 0 \qquad (v)$$

or

$$\left[p_1 \; dx_1 \right]_{t=T} = 0 \qquad (vi)$$

Two cases arise:

(a) Fixed end-points

$x_1(T)$ is specified, hence, because $dx_1 = 0$ equation (vi) is satisfied.

(b) Free end-points

x_1 is subject to variations, dx_1 is not zero, and therefore to satisfy equation (vi)

$$p_1(T) = 0 \qquad (vii)$$

From the above classifications of the fixed and free end points we find that our present general forms of equations representing the simplified model is a fixed end point two point boundary value problem.

3.2. METHODS OF SOLUTION:

In the following paragraph we examine some of the proposed ways of attacking this problem.

The methods of finite Differences probably the most common approach adapted in the past has been the application of the calculus of finite differences, for example Fox [29]. He takes full account of the difference correction but it becomes a process ofsuccessive approximation. Now, in applying the method of finite differences to nonlinear equations, an iterative process is required in any case. It is probably preferable therefore to avoid an iterative cycle inside another such cycle. The alternative approach is the so-called deferred approach to the limit.

Furthermore, one must anticipate that, because the solution may not be unique, an approximate solution may be required to ensure that convergence occurs towards the right solution. The method of relaxations could be employed but group relaxations would be essential. The generalized Newton process is perhaps the convenient one and was employed.

The methods of Linearized solution, steepest Ascent (Descent) of the Hamiltonian and Boundary iterations have been used for the solution of two point boundary problems but their applications are limited for the load dispatch problems.

Some new methods e.g. Binary Search Technique developed in [19] and Relaxation methods for Computer solution disscussed in [15] have been applied effectively for the power systems dispatch problems using the maximum principle.

In [19] the authors have mentioned the Binary Search Technique to obtain the optimal Control Variable. The Control variable first assumes its largest admissible value, and the Hamiltonian is calculated. A slightly smaller value is then assigned to it, and the corresponding Hamiltonian is computed. If the former, is greater, the former control variable is optimal one. If not, the smallest value is then assigned to ie, and the corresponding Hamiltonian is computed. Similarly, the Hamiltonian for a slightly larger value, say $U_{min} + \Delta u$, is computed. If the former is greater, the optimal control is u_{min}. If not, the arithmatic mean of the upper and lower limits$(u_{max} + u_{min})/2$ is assigned and the corresponding Hamiltonian is computed. If $H\,[(u_{max} + u_{min})/2] >$ $H\,[(u_{max} + u_{min})/2 - \Delta u]$, the optimal control should lie somewhere between u_{max} and $(u_{max} + u_{min})/2$. If $H\,[(u_{max} + u_{min})/2] <$ $H\,[(u_{max} + u_{min})/2 - \Delta u]$, the optimal control should be between u_{min} and $(u_{max} + u_{min})/2$. This perturbation procedure is repeated, Until the optimal control is obtained with an error of permissible order. They have also shown the analytical method to obtain the optimal control in the case of the simplified model system.

Dahlin and Shen have also solved dispatch problems using maximum principle. They have developed easy relations which are very conveniently tackled for computer solution. Hamiltonian function for variable head is finally expressed in the same form as in the problem with negligible head variations. In that case, all the adjoint variable were constants. Therefore, the probleme is simplified further.

3.2.4. Proposed Gradient Method:

It is purely a new technique to be applied forthe solution of load dispatch problem of power systems. The effectivenes of the method is yet to be tested for a practical problems. A means alleviating the difficulty of the determination of a suitable initial choice of unknown boundary values is presented. Therefore, this removes the major difficulty of the problem because it was the main drawback associated with all method employed so far. The complete explanation of the method is given in the following paragraph.

3.$ GRADIENT TECHNIQUES

Gradient methods establish only certain necessary conditions which must be satisfied by an optimum solution although in some cases the conditions also are sufficient. The term optimum indicates a solution satisfying these necessary conditions. Some where it is also known as an Indirect method. We have seen that the Pontryagin's maximum principle results in a set of differential equations which lack a complete set of boundary conditions at each end of the solution interval. This gradient technique is employed to complete either set of boundary conditions and thereby provide the optimum solution.

Briefly it can be understood as follows:
The technique uses the Pontryagin's maximum principle to eliminate the control variables fromthe system equations. The result is a set of simultaneous first-order differential equations which have an incomplete set of boundary conditions. In order to complete the boundary conditions at the initial or terminal points, an auxiliary cost function is introduced. This cost function is designed so that its minimization will lead to a specification of the unknown boundary conditions. The gradient method [23] is used for minimization of the auxiliary cost function.

The mathematical application of the gradient method is explained in the following steps:
The Hamiltonian defined in equation(12) can also be written

$$H = \sum_{i=1}^{n} p_i f_i - f_o$$

$$H = \sum_{i=0}^{n} p_i f_o \qquad \text{for } p_o = -1 \qquad (45)$$

The auxiliary variables, p(t), are defined by Hamiltonian system in equation (16) and (17)

where,

$$x_i = f_i \qquad \text{for } i = 0,\ 1 \ldots\ldots n$$

Application of maximum principle provides relations, frequently explicit, from which each component of Control Variable (u) may be determined, given t, x and p. In general, it is not possible that u can be solved explicitly in terms of X(0) and p(0) satisfying equation (10) . It is, therefore, proposed to determine p(0) by gradient technique. As stated above in order to accomplish this technique, a secondary cost function involving the specified terminal conditions is introduced.

If Vectors X and p are combined to form the 2n + 2 dimensional Vector

$$Y = \begin{bmatrix} X \\ p \end{bmatrix},$$

then this cost function may be defined as

$$\mathcal{E} = \sum_{i=0}^{2n+1} A_i E_i \left[y_i(T) - y_2^{(f)} \right] \qquad (46)$$

The A_i's are constants which are zero for those final states which are unspecified. Typical E_i are

$$E_i = \left[y_i(T) - y_i^{(f)} \right]^{2\nu},$$

$$E_i = \left| y_i\ (T) - y_2^{(f)} \right|^{n} \ ; \ n,\ \nu \text{ positive integers.}$$

$y_i(T)$ and $y^{(f)}$ are the terminal value of y at the end of the optimization interval i.e. at t = T and the final specified value, respectively.

Provided that the terminal condition (10) can be achieved, it is clear that $\mathcal{E}_{min} = 0$. If the y(o) producing this minimum is found, the corresponding y(t) is the optimum trajectory. In attempting to find the proper y(o) by an iterative procedure, supposing that at the j^{th} step, some estimate, namely, $y^j(o)$, does not minimize (46), i.e. $\mathcal{E}^j \neq 0$. A change in $y^j(o)$ designated $\Delta y^i(o)$, is then desired such that $\Delta \mathcal{E}^j = \mathcal{E}^{j+1} - \mathcal{E}^j < 0$

Expanding $\mathcal{E}^{j+1}$ about $y^j(o)$,

$$\Delta \mathcal{E}^j = \sum_{i=0}^{2n+1} \left[\frac{\partial \mathcal{E}}{\partial y_i(o)} \right]^j \Delta y_i^j(o) + \text{higher order terms.} \quad \ldots\ldots \quad (47)$$

The heigher order terms may be neglected provided the vecotor length $\|\Delta y^j(o)\|$ is sufficiently small. The condition $\Delta \mathcal{E}^j < 0$ is then satisfied if

$$\Delta y_i^j(o) = -K_1' \left[\frac{\partial \mathcal{E}}{\partial y_{i(o)}} \right]^j, \qquad i=1,2 \ldots 2n+1 \quad \ldots \quad (48a)$$

or

$$\Delta y^j(o) = -K_1' \underset{\{y(o)\}}{\text{GRAD}} \mathcal{E}^j \qquad (48b)$$

The notation GRAD $\{y(o)\} \mathcal{E}^j$ signifies the gradient of $\mathcal{E}^j$ with respect to y(o). Equation (48) expresses a suitable choice for $\Delta y^j(o)$, provided K_1' is sufficiently small. The choice of K_1', A_i and E_i have got remarkable effect on the convergence properties.

GRAD $\mathcal{E}^i$ is now determined as follows:

Let us assume first that the maximum principle yields explicit functions u, $u = u(x, p, t)$. These relations may be usedto eliminate u from 16 & 17 with the result that

$$\dot{Y} = \begin{bmatrix} \dot{X} \\ \dot{p} \end{bmatrix} = g\,(y,\ t) \tag{49}$$

It is proved that [28]

$$\underset{\{Y(o)\}}{\text{GRAD}}\ \mathcal{E}^j = d^i(o) \tag{50}$$

where

$$\dot{d}^i\ (t) = -F^i\ (t)\ d^i(t) \tag{51}$$

$$d_k^j(T) = \left[\frac{\partial \mathcal{E}}{\partial y_k(T)}\right]^j, \qquad k = 0,\ 1,\ \ldots 2n + 1 \tag{52}$$

and

$$\left[F_{ki}^j\right] = \left[\frac{\partial g_i}{\partial y_k}\right] = F^j(t) \tag{53}$$

Equation (51) represents the adjoint system for the linearized form of (49). The term of (53) are evaluated from knowledge of $Y^i(t)$. Starting with the boundary condition (52), (51) is numerically solved backward from T to 0 yielding $d^i(o)$ which together with (48) and (50) yields the desired increment $\Delta Y^j(o)$.

Commencing with an initial choice of p(o), the p components of Y(o) are successively adjusted using (48) until the auxiliary error function is arbitrarily close to zero. Alternatively iterations may be continued until successive $p^i(o)$ are arbitrarily close.

3.4. GENERAL EQUATIONS OF THE SIMPLIFIED MODEL

The boundary conditions prescribed for the variables are "hard" constraints and the solution is not feasible with the application of the gradient technique. It is, therefore, desired to modify these "hard" constraints into "soft" constraints.

Introducing a new control variable

$$u_1' = u_1 - (u_{max} + u_{min})/2 \qquad (54)$$

and state variable x_1 is modified to $\left(\frac{x_1}{\bar{x}_1}\right)^{2m}$

where

$\bar{x}_1$ = the upper limit of the state variable x_1

m = a constraint > 1

It is now intended to employ the arctan function to simulate saturation of the forcing function, instead of using the constraint $|u_1| \leqq 1$ on the simple term u_1

For the existing system there is a set of simultaneous differential equations expressed in equations (41) and (44) which lac a complete set of boundary conditions at each end of the solution interval. In order to complete the boundary conditions at the initial or terminal points, an auxiliary cost function is introduced. Thus, defining cost function.

$$\frac{dx_o}{dt} = \left(\frac{x_1}{x_1}\right)^{2m} + F_1(S_1) \qquad (55)$$

The cost function is designed so that its minimization will lead to a specification of the unknown boundary conditions.

From power balance equation

$$KH_o\,(1 + Cx_1)u_1 + S_1 = D \qquad (56)$$

where K is the conversion factor from Kg.m. to Mw.

Therefore,

$$S_1 = D - KHo\ (1+Cx_1)\ u_1 \tag{57}$$

The state equation (41) is now modified to

$$\frac{dx_1}{dt} = J_1 - \frac{u_{max}-u_{min}}{\pi} \tan^{-1} u^1 - \frac{u_{max}+u_{min}}{2} \quad \ldots\ldots \tag{58}$$

If the cost function $F_1(S_1)$ in equation (55) is replaced by its quadratic function of thermal power developed, it becomes

$$\frac{dx_o}{dt} = \left(\frac{x_1}{x_1}\right)^{2m} + a_1 S_1 + a_2 S_1^2 \tag{59}$$

Substituting for S_1 from equation (57)

$$\frac{dx_o}{dt} = \left(\frac{x_1}{x_1}\right)^{2m} + a_1 \left\{ D - KHo\ (1 + Cx_1)u_1 \right\} + a_2 \left\{ D - KHo\ (1 + Cs_1)\ u_1 \right\}^2 \tag{60}$$

Eliminating u_1 from above for its assumed function of arctan

$$\frac{dx_o}{dt} = \left(\frac{x_1}{x_1}\right)^{2m} + a_1 \left\{ D\text{-}KHo\ (1+Cx_1) \left(\frac{u_{mx}-u_{min}}{\pi} \tan^{-1} u^1 + \frac{u_{max}+u_{min}}{2} \right) \right\}$$

$$+ a_2 \left\{ D\ \text{-}KHo\ (1+Cx_1) \left(\frac{u_{max}-u_{min}}{\pi} \tan^{-1} u^1 + \frac{u_{max}+u_{min}}{2} \right) \right\}^2 \quad \ldots\ldots \tag{61}$$

For simplicity let $D\text{-}KHo(1Cx_1) \left(\frac{u_{max}-u_{min}}{\pi} \tan^{-1} u^1 + \frac{u_{max}+u_{min}}{2} \right) = (A_1)$

Using the Maximum principle the Hamiltonian is

$$H = -\left\{\left(\frac{x_1}{\bar{x}_1}\right)^{2m} + a_1(A_1) + a_2(A_1)^2\right\}$$

$$+ p_1 \left(J - \frac{u_{max}+u_{min}}{2} - \frac{u_{max} - u_{min}}{\pi} \tan^{-1} u^1\right) \quad \ldots\ldots\ldots \quad (62)$$

Now u' to maximize H

$$\frac{\partial H}{\partial u'} = 0$$

$$\text{or } \theta = -a_1 \left\{ KHo\ (1 + Cx_1)\ K_1 \ \frac{1}{u'^2 + 1} \right\}$$

$$- a_2\left[(A_1)2\right.$$

$$\left.\left\{-KHo\ (1 + Cx_1)\ K_1 \ \frac{1}{u'^2 + 1}\right\}\right] - p_1 K_1 \ \frac{1}{u'^2+1}$$

where,

$$K_1 = \frac{u_{max} - u_{min}}{\pi}$$

$$K_2 = \frac{u_{max}+u_{min}}{2}$$

$$\text{or } a_1 + 2a_2\ (A_1) = \frac{p_1}{KHo(1+Cx_1)}$$

or

$$(A_1) = \frac{1}{2a_2} \left\{ \frac{p_1}{KHo(1+Cx_1)} - a_1 \right\}$$

Substitute $p_1/KHo\ (1 + Cx_1) = (B_1)$

or

$$KHo\ (1 + Cx_1)\ (K_1 \tan^{-1} u' + K_2)$$

$$= D - \left\{ (B_1) - a_1 \right\} \frac{1}{2a_2}$$

or

$$K_1 \tan^{-1} u' + K_2 = \frac{D}{KHo(1+Cx_1)} - \left\{ (B_1) - a_1 \right\} \frac{1}{2a_2 KHo(1+Cx_1)}$$

or

$$\tan^{-1} u' = \frac{D}{KK_1 Ho(1+Cx_1)} - \left\{ (B_1) - a_1 \right\} \frac{1}{2a_2 KK_1 Ho(1+Cx_1)} - \frac{K_2}{K_1} \qquad \ldots\ldots \quad (63)$$

Substituting for u' in(61)

$$\frac{dx_o}{dt} = \left(\frac{x_1}{\bar{x}_1}\right)^{2m} + a_1 \left\{ (B_1) - a_1 \right\} \frac{1}{2a_2}$$

$$+ a_2 \left[\left\{ (B_1) - a_1 \right\} \frac{1}{2a_2} \right]^2$$

or

$$\frac{dx_o}{dt} = \left(\frac{x_1}{\bar{x}_1}\right)^{2m} + \frac{1}{4a_2} \left\{ (B_1)^2 - a_1^2 \right\} \qquad (64)$$

Equation(58) after eliminating u' becomes

$$\frac{dx_1}{dt} = J - K_2 - \frac{D}{KHo(1+Cx_1)} + \left\{ (B_1) - a_1 \right\} \frac{1}{2a_2 KHo(1+Cx_1)} + K_2$$

or

$$\frac{dx_1}{dt} = J - \frac{1}{KHo(1+Cx_1)} \left[D - \{(B_1) - a_1\} \frac{1}{2a_2} \right] \quad (65)$$

Since $\dfrac{dp_1}{dt} = -\dfrac{\partial H}{\partial x_1}$

or

$$\frac{dp_1}{dt} = \frac{2m}{\bar{x}_1} \left(\frac{x_1}{\bar{x}_1}\right)^{2m-1} + a_1 \left\{ -KHo\,(K_1 \tan^{-1} u' + K_2)C \right\}$$

$$+ a_2\, 2(A_1) \left\{ -KHo\,(K_1 \tan^{-1} u' + K_2)C \right\}$$

or

$$\frac{dp_1}{dt} = \frac{2m}{\bar{x}_1} \left(\frac{x_1}{\bar{x}_1}\right)^{2m-1} - \left[\frac{CD}{(1+Cx_1)} - \left\{ \frac{p_1}{KHo(1+Cx_1)} - a_1 \right\} \frac{C}{2a_2(1+Cx_1)} \right]$$

$$\left[a_1 + 2a_2 \left\{ (B_1) - a_1 \right\} \frac{1}{2a_2} \right]$$

or

$$\frac{dp_1}{dt} = \frac{2m}{\bar{x}_1} \left(\frac{x_1}{\bar{x}_1}\right)^{2m-1} - (B_1) \left[\frac{CD}{(1+Cx_1)} - \left\{ (B_1) - a_1 \right\} \right.$$

$$\left. \frac{C}{2a_2(1 + Cx_1)} \right] \quad (66)$$

$$\frac{dp_o}{dt} = 0 \quad (67)$$

(Since $p_o = -1$)

It is observed that the above set of differential equations represents a non-linear system, not only because of arctan (u'), but because of the performance integrand when $m > 1$.

These four differential equations 11,12,13, and 14 are, in general, characterising the simplified model of the system. The actual numerical solution is accomplished in the following steps.

3.4. CONSTANTS CHARACTERIZING THE SIMPLIFIED MODEL SYSTEM

Constants	Nomenclature & Unit	Value
Maximum water storage	$\bar{x}_1 (m^3)$	115.2×10^4
Minimum water storage	$\underline{x}_1 (m^3)$	0
Basic Water head	Ho (m)	20.00
Maximum water head	$\bar{H}o$ (m)	28.00
Maximum water discharge	$u_{max} (m^3/s)$	130.00
Minimum water discharge	$u_{min} (m^3/s)$	2.00
Maximum hydraulic output	W (Mw)	35.85
Maximum thermal output	S_{max} (Mw)	30.00
Minimum thermal output	S_{min} (Mw)	0
Water inflow	$J (m^3/s)$	100.00
a_1	a_1 (Rs/Mw.Sec.)	2.75
a_2	a_2 (Rs/Mw^2.Sec)	0.052

3.5. NUMERICAL SOLUTION

The set of differential equations representing the model have to be solved simultaneously. The solution obtained will give the optimum operating point at a certain instant.

After eliminating u, from (16) and (17) making use of the Maximum principle, for its explicit function, we have obtained:

$$\begin{bmatrix} \frac{dy_o}{dt} \\ \frac{dy_1}{dt} \\ \frac{dy_2}{dt} \\ \frac{dy_3}{dt} \end{bmatrix} = \begin{bmatrix} \frac{dx_o}{dt} \\ \frac{dx_1}{dt} \\ \frac{dp_o}{dt} \\ \frac{dp_1}{dt} \end{bmatrix} = g\ (y,t)$$

$(B_1) = y_3 /\ KHo\ (1+Cy_1)$ for $x_1 = y_1$ and $p_1 = y_3$

or

$$\begin{bmatrix} \frac{dy_o}{dt} \\ \frac{dy_1}{dt} \\ \frac{dy_2}{dt} \\ \frac{dy_3}{dt} \end{bmatrix} = \begin{bmatrix} \left(\frac{y_1}{\bar{y}_1}\right)^{2m} + \frac{1}{4a_2}\left\{(B_1)^2 - a_1^2\right\} \\ J - \frac{1}{KHo(1+cy_1)}\left[D - \left\{(B_1) - a_1\right\}\frac{1}{2a_2}\right] \\ 0 \\ \frac{2m}{\bar{y}_1}\left(\frac{y_1}{\bar{y}_1}\right)^{2m-1} - (B_1)\left[\frac{CD}{(1+Cy_1)} - \left\{(B_1) - a_1\right\}\frac{C}{2a_2(1+Cy_1)}\right] \end{bmatrix} \qquad (68)$$

The numerical solution of (68) is accomplished simultaneously in a step-by-step method of integration

$$\text{i.e. } y_{n+1} = y_n + \dot{y}_n \, \Delta t \qquad (69)$$

Thus, an initial guess value of y_3 is assumed which is in the present case $y_3 = 0.01$ and a nominal trajectory is obtained for y_1 for the optimization interval of 24 hours or a day. It is observed that the terminal value obtained for y_1 does not converge to its fixed boundary value. A complete computer programme on an IBM 1620 has been written and shown in Appendix for the forward numerical integration of the above set of simultaneous differential equations.

Since the boundary condition is not satisfied an iterative procedure is followed for the initial correction of the guess value of y_3. This procedure of initial guess value modificatio is followed in the following equations:

With reference to equation (53)

$$F^j(t) = \left[\frac{\partial g_i}{\partial y_k}\right]^j$$

$$\text{or } F^j(t) = \begin{bmatrix} \frac{\partial g_0}{y_0} & \frac{\partial g_0}{\partial y_1} & \frac{\partial g_0}{\partial y_2} & \frac{\partial g_0}{\partial y_3} \\ \frac{\partial g_1}{\partial y_0} & \frac{\partial g_1}{\partial y_1} & \frac{\partial g_1}{\partial y_2} & \frac{\partial g_1}{\partial y_3} \\ \frac{\partial g_2}{\partial y_0} & \frac{\partial g_2}{\partial y_1} & \frac{\partial g_2}{\partial y_2} & \frac{\partial g_2}{\partial y_3} \\ \frac{\partial g_3}{\partial y_0} & \frac{\partial g_3}{\partial y_1} & \frac{\partial g_3}{\partial y_2} & \frac{\partial g_3}{\partial y_3} \end{bmatrix} \qquad (70)$$

Equation (70) when treated with (68) yields

$$F^{j}(t) = \begin{bmatrix} 0 & \dfrac{\partial g_o}{\partial y_o} & 0 & \dfrac{\partial g_o}{\partial y_3} \\ 0 & \dfrac{\partial g_1}{\partial y_1} & 0 & \dfrac{\partial g_1}{\partial y_3} \\ 0 & \underline{0} & 0 & 0 \\ 0 & \dfrac{\partial g_3}{\partial y_1} & 0 & \dfrac{\partial g_3}{\partial y_3} \end{bmatrix}$$

Denote $(B_2) = \dfrac{C}{KHo(1+Cy_1)^2}$

$$\frac{\partial g_o}{\partial y_1} = \frac{2m}{\bar{y}_1} \left(\frac{y_1}{y_1} \right)^{2m-1} - \frac{(B_1)(B_2)}{2a_2}$$

$$\frac{\partial g_o}{\partial y_3} = \frac{y_3(B_2)}{2a_2 KHoC}$$

$$\frac{\partial g_1}{\partial y_1} = (B_2) \left\{ D - (B_1)/a_2 + a_1/2a_2 \right\}$$

$$\frac{\partial g_3}{\partial y_1} = \frac{2m(2m-1)}{(\bar{y}_1)^2} \left(\frac{y_1}{\bar{y}_1} \right)^{2m-2} + \frac{(B_2)\, y_3 C}{(1+Cy_1)}$$

$$\left\{ 2D + a_1/a_2 - 3\,(B_1)/2a_1 \right\}$$

$$\frac{\partial g_3}{\partial y_3} = (B_2) \left\{ -D - a_1/2a_2 + (B_1)/a_2 \right\} \qquad (72)$$

From equation (51) we have

$$\begin{bmatrix} \dot{d}_0 \\ \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix}^j = - \begin{bmatrix} 0 & \frac{\partial g_0}{\partial y_0} & 0 & \frac{\partial g_0}{\partial y_3} \\ 0 & \frac{\partial g_1}{\partial y_1} & 0 & \frac{\partial g_1}{\partial y_3} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{\partial g_3}{\partial y_1} & 0 & \frac{\partial g_3}{\partial y_3} \end{bmatrix}^j \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix}^j \quad \ldots \quad \ldots \quad (73)$$

where

$\dot{d}^j$ represent the first derivative with respect to t at j^{th} step. The initial values of d for backward solution i.e. to say that at time T, d is determined from equation (52) which gives

$$d_k^j\,(T) = \left[\frac{\partial \mathcal{E}}{\partial y_k(T)}\right]^j, \qquad k = 0, 1 \,\ldots\ldots 2n+1$$

where

the auxiliary error function $\mathcal{E}$ is assumed as

$$\mathcal{E} = (y_1(T) - 10^6)^2$$

Therefore, we find from above relation that

$$d_0(T) = 0$$

$$d_1(T) = 2(y_1(T) - 10^6)$$

$$d_2(T) = 0$$

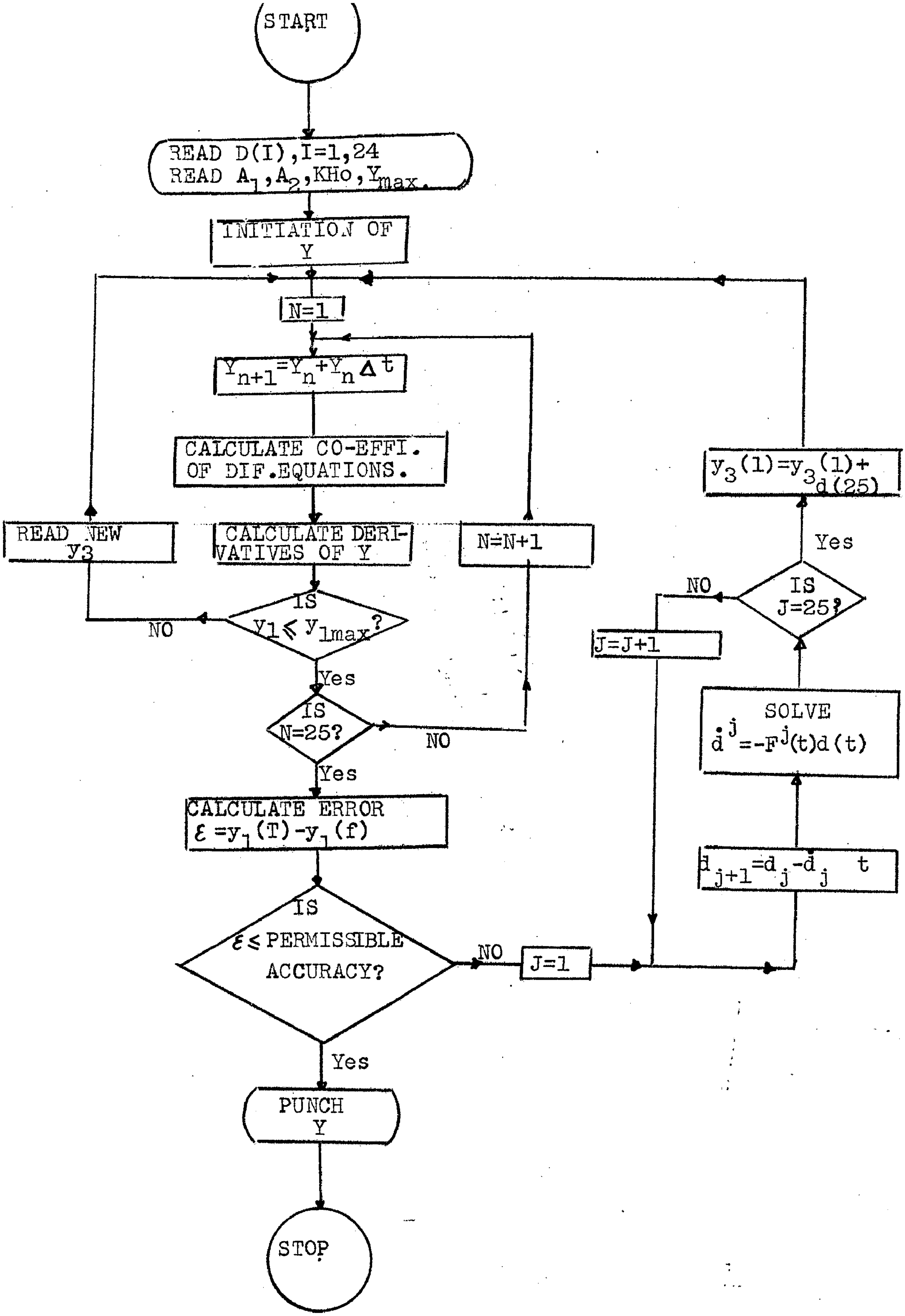
$$d_3(T) = 0$$

Equation (73) which has to be solved backward can now be written in the following modified form

$$\begin{bmatrix} \dot{d}_o \\ \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \end{bmatrix} = - \begin{bmatrix} 0 & \frac{2m}{\bar{y}_1}\left(\frac{y_1}{\bar{y}_1}\right)^{2m-1} - \frac{(B_1)(B_2)}{2a_2} & 0 & \frac{y_3(B_2)}{2a_2 KHoC} \\ 0 & (B_2)\left\{D-(B_1)/a_2+a_1/2a_2\right\} & 0 & \frac{1}{2a_2\ K^2Ho^2(1+Cy_1)^2} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{2m(2m-1)}{(\bar{y}_1)^2}\left(\frac{y_1}{\bar{y}_1}\right)^{2m-2} + \frac{(B_2)y_3C}{(1+Cy_1)}\left\{2D+a_1/a_2 -3(B_1)/2a_1\right\} & 0 & (B_2)\left\{-D -a_1/2a_2+(B_1)/a_2\right\} \end{bmatrix} \begin{bmatrix} d \\ d. \\ d \\ d \end{bmatrix}$$

Thus, the equation (75) is solved backward on the nominal trajector obtained from equation (68) with the initial conditions at $t = T$ given in equation (21). Therefore, from the value of d obtained at $t = 0$, the relation (50) is established because $d^j(0)$ is now known.

Once GRAD $\{y(0)\}\mathcal{E}^j$ is known, $\Delta y^j(0)$ is calculated. Where in the present problem k_i' is assumed to be one.



(FLOW DIAGRAM FOR NUMERICAL CALCULATION) ON COMPUTER

The complete computer programming is shown in Appendix. After making increment of $\Delta y^{j}(0)$ in the initial guess value of y_3, the forward solution is again started. This process of iteration continues till it converges to its optimum value. The optimum trajectory for y_1^j which is shown in fig. (3).

From this optimum trajectory obtained for x_1, the control variable u_1 is determined and the optimal control is plotted in fig. (4). The optimum load sharing of each plant on hourly basis have been shown in fig. (5) and (6).

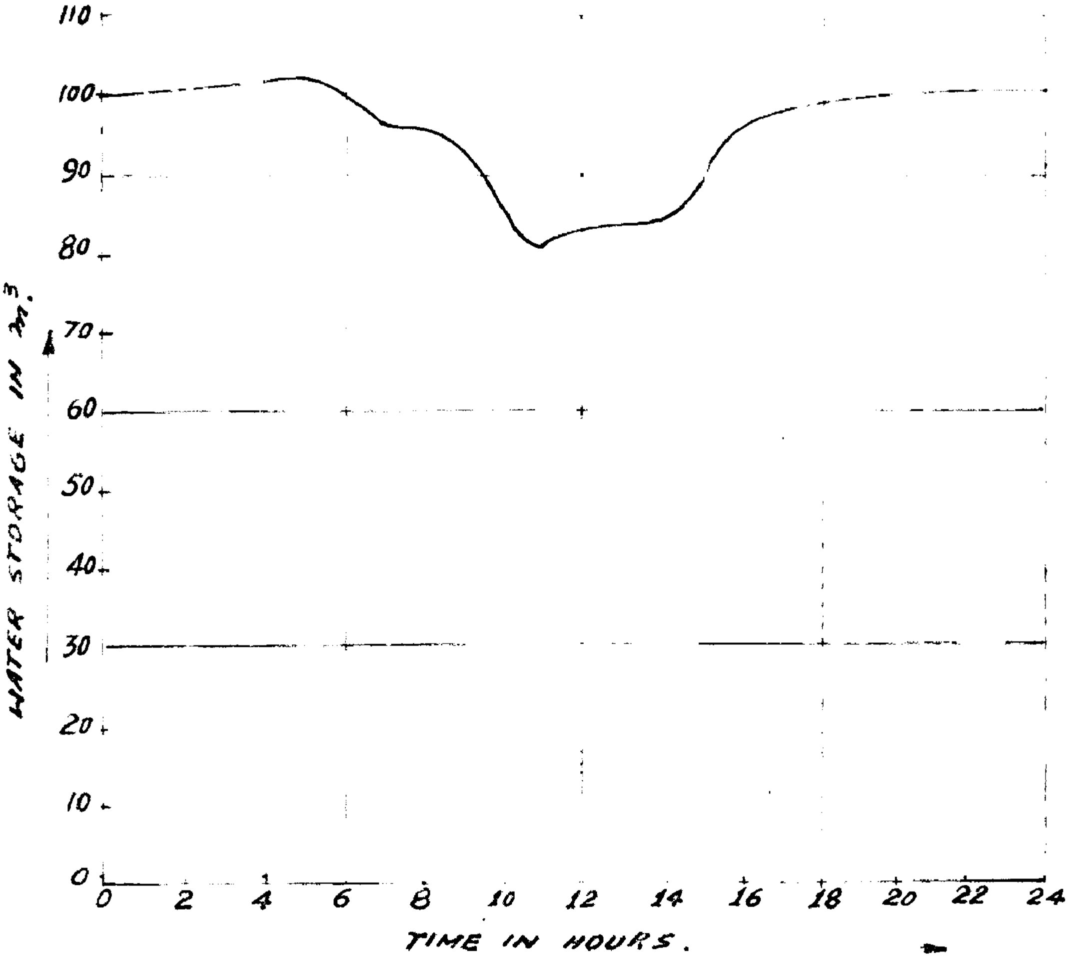


FIG. 3. OPTIMAL WATER STORAGE FOR SIMPLIFIED MODEL SYSTEM.

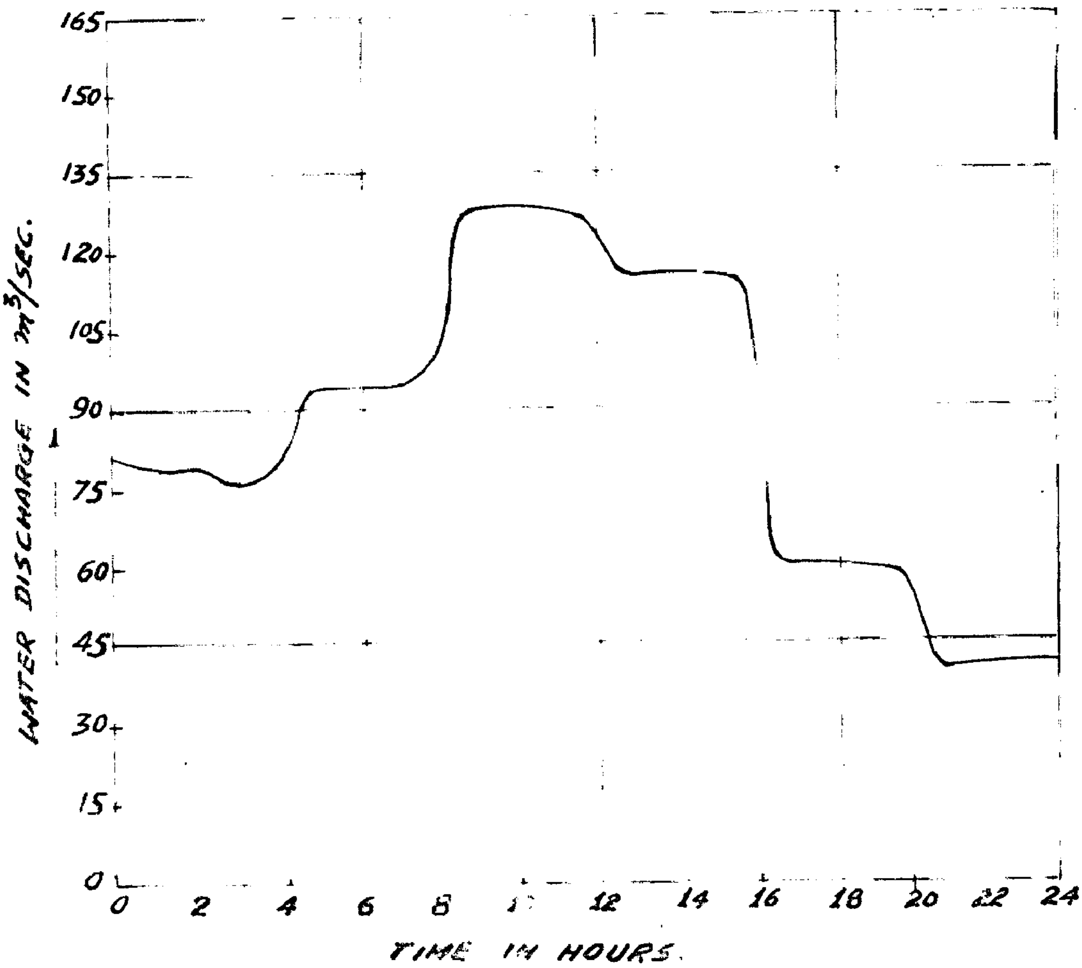


FIG. 4

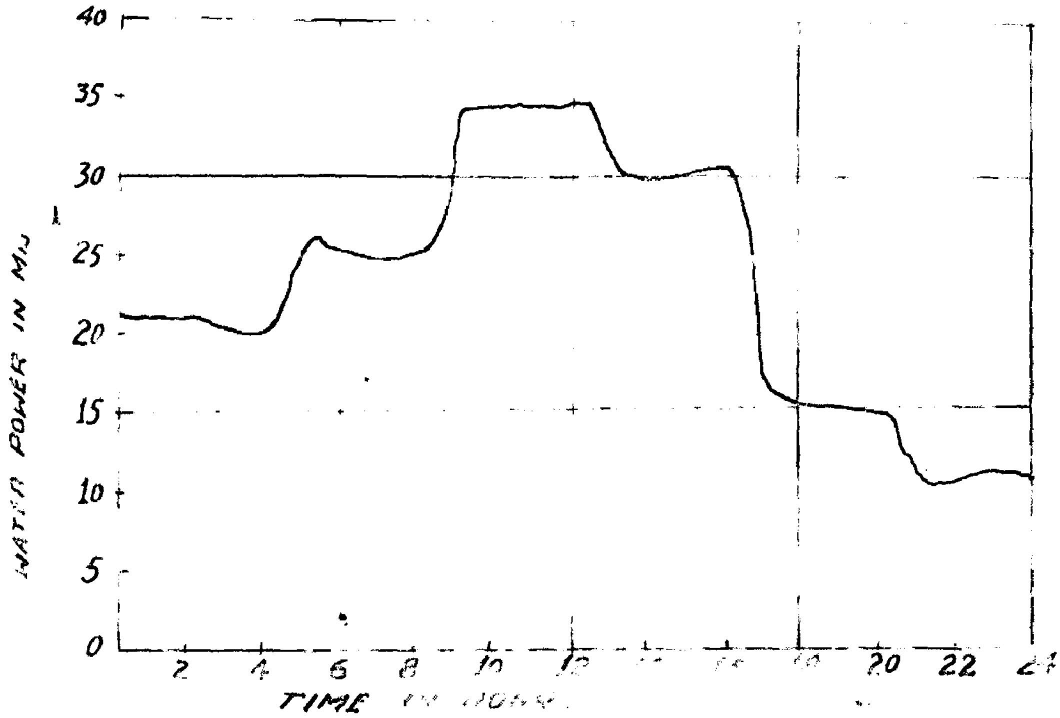


FIG. 5 OPTIMAL HYDRAULIC POWER OUTPUT.

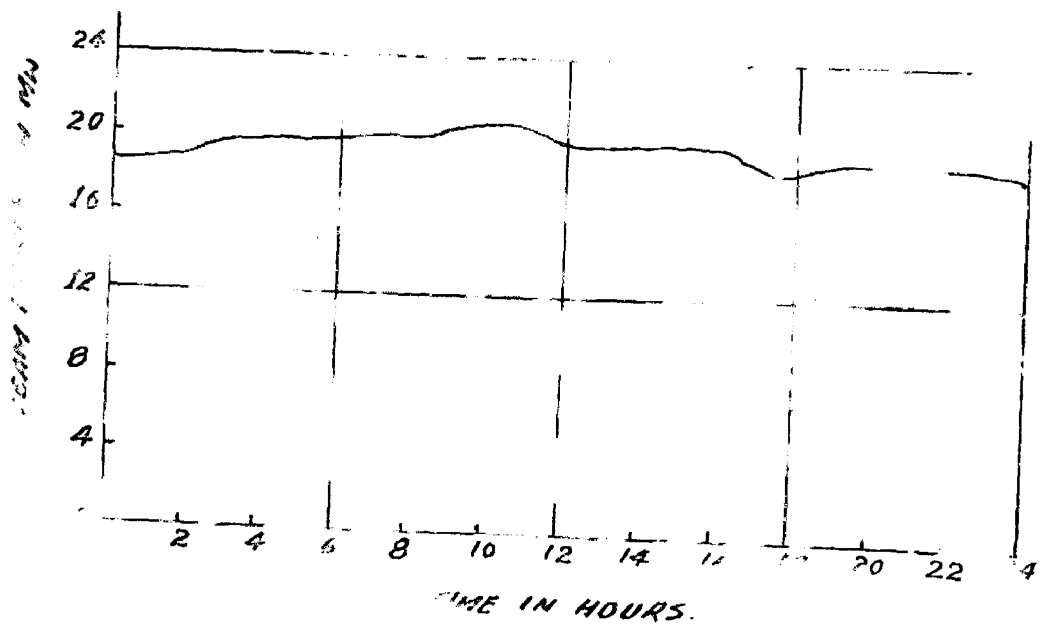


FIG. 6 OPTIMAL STEAM GENERATION

CONCLUSION

C_O_N_C_L_U_S_I_O_N

The previous Chapters have presented a brief discussion of the problems involved in optimizing the load dispatching of a combined hydro thermal power system. A new techniqu utilizing Pontryagin's maximum principle, has been suggested for the numerical calculations of the solution.

One of the main advantages of the suggested gradient technique in association with the maximum principle was found to be the ease with which the iterative procedure would be started. The difficulty actually experienced with previous methods was thus slow rate of convergence if the initial choice of the starting values were not suitable. Thus, for example, the method of finite differences requires the selection ofa priming trajectory and two separate computations in order to carry out λ^2-extrapolation. Similarly the binary search technique presents a laborious and time consuming method. The gradient technique, on the other hand, was able to converge quite rapidly to the desired solution even when the initial choice of the boundary values was very different from the final value. Thus it look only four iterations to arrive from an initial choice $p = 0.01$ to a final value of $p=56.3862$

However, a disadvantage with the present technique is its inability to handle "hard" constraints directly instead of changing them to "soft" constraints. Considerable computational difficulties were experienced in trying to make the "soft" constraints approximate the given actual constraints to a reasonable degree. In a numerical problem, for example, the computer overflowed for $m > 4$. It is felt, however, that the gradient techniques offer a considerable improvement in ease of computation over the techniques being used so far and that further investigation along these will prove to be useful, especially if the present technique can be modified to handle"hard"constraints directly.

The usefulness of the maximum principle is well realised i physical study of the system. For instance, some important features of mathematical analysis are directly giving the behaviour of the systems operation. The incremental rate of the fuel input to the thermal plant was found to be constant for the water head correction factor C =0. This shows that the thermal output will remain constant as a function of time when it is in the combined operation of hydro plant whose water head variations are negligible.

In numerical solution it was observed that the value of C was very small and therefore, it produces less effect to the rate of change of thermal output. Another important point to be noted with the reference of the relation of incremental cost of thermal plant is that with the decrease of water level the incremental cost increases or the thermal output. The system load fluctuations for the short period is met by the hydraulic plant. The optimal load sharing of the steam plant is depicted in fig. 6 which shows a small increment in loa sharing when there is decrease in water level and, therefore, the total load fluctuations have been absorbed by the hydro power plant as shown in fig. (5).

Although the computational analysis is limited to a simplified model system, this procedure may be extended to any number of systems in the combined operation. Naturally, the complexity of the problem will increase due to more number of variables and varieties of constraints.

B̲I̲B̲L̲I̲O̲G̲R̲A̲P̲H̲Y̲

B I B L I O G R A P H Y

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A P P E N D I X

COMPUTER PROGRAMME ON IBM 1620

```
      THAKUR EED UOR NUM SOLUN.
      DIMENSION D(24) Yo (25), Y1 (25), Y3(25)
      DIMENSION D5(25), D1(25), D3(25)
      READ 3, (D(I), I = 1, 24)
      READ  1, A1, A2, PH, YM1
5     FORMAT (6 F 10.0)
      FORMAT (4 F 16.4)
      C = 0.4/1152000.
      DELT = 3600
      Yo(1)= 0.
      Y1(1)= 1000000.
      Y3(1)= 0.01
      Y25  = 1000000.
3     DO 2 I = 1,24
      F12  = (Y1(I)/YM1)
      F1   = F12 * F12
      X1   = F1 * F1
      F2   = PH * (1 +C * Y1(I))
      FF2  = 1./(1. +C * Y1(I))
      F3   = (Y3(1)/F2)
      FF3  = F3 - A1
      F4   = (0.25/A2) *(F3 * F3 F3 - A1 * A1)
      F5   = D(I) - FF3/(2.0 * A2)
      F7   = 4./Y1(I) * X1
      F8   = C * D(1) * FF2
      F9   = (FF3 * FF2 * C * 0.5)/A2
      F10  = (F8 - F9) * F3
      C1   = 128./3.14159
```

```
        C2    = 66.
        DELYo = X1 + F4
        DELY1 = 100.0 - F6
        DELY3 = F7 - F10
        F     = (D(I)/(C1*F2)) - FF3/(2.*A2*C1*F2)-C2/C1
        JF    = F* 0.159195
        AJF   = JF
        AF    = AJF *2.* 3.14159
        F     = F - AF
        I1    = I + 1
        Yo(I1)= Yo(I) + DEL Yo * DELT
        Y1(I1) = Y1(I) + DEL Y1 * DEL T
        Y3(I)  = Y3(I) + DEL Y3* DEL T
        U(I1) = SIN F(F) / (COS F(F))
        PUNCH 100, I1, Yo(I1) , Y1(I1), Y3(I1),U(I1)
100 FORMAT (I3, 4 E15.6)
2       CONTINUE
        DIF = Y1(25) - Y25
        IF (ABSF(DIF) - 50000.) 5,5,6
6       DEL T1     = - DEL T
        D5 (1)     = 0
        D1(*)      = 2.*D1F
        D3(I)      = 0.
        DO (I      = 1, 24
        J          = 26 - I
        E2         = F2 *F2
        EE2        = Y3(J) * C/E2
        EEE2       = EE2 * FF2
```

```
      E3         = EEE2/2. * A2
      EE3        = Y3(J)/2. * A2*A2*E2
      E4         = C/F2
      EE4        = E4 * FF2
      EEE4       = F3/A2
      EF2        = 1./2 * A2 * E2
      EF3        = C * Y3(J) * EE4 * FF2
      EA1        = 2. * EA
      EF4        = 3. * F3/2. * A1
      EE5        = EF3 * (2. * D(I) + EA1 - EF4)
      EE6        = (D(I) - EEE4 + EA)
      E6         = EE4 * (-EE6)
      E7         = EE4 * (D(I) - EEE4 + EA)
      E8         = F7 - E3
      E5         = (12./(Y1(J) * Y1(J))) * X1 + EE5
      DEL D5     = - E8 * D1(I) - EE3 * D3(I)
      DEL D1     = -E7 * D1(I) - EF2 * D3 (I)
      DEL D3     = -E5 * D1(F) - E6 * D3(I)
      D5(I1)     = D5(I) + DEL D5* DEL T1
      D1(I1)     = D1(I) + DEL D1 * DEL T1
      D3(I1)     = D3(I) + DEL D3 * DEL T1
      PUNCH 300, I1, D5(I1), D1(F1), D3(I1)
00 FORMAT (' I3,3 E 20.8)
      CONTINUE
      Y3(1)      = Y3(1) + D3(25)
      GO TO 8
      STOP
      END
```

Input Data

40.	40.	40.	40.	45.	45.
45.	45.	55.	55.	55.	55.
50.	50.	50.	50.	35.	35.
35.	35.	30.	30.	30.	30.

120 2.5 0.196 1152000.

LIST OF THE FINAL RESULTS OBTAINED:

Time in Hour	Optimum Water storage $y_1 = x_1$ (In multiple of 10^4)	Adjoint Variable $y_3 = p_1$
0	100.0000	56.3862
1	100.3770	56.4173
2	100.9230	56.7056
3	101.8250	57.9142
4	102.2860	58.2068
5	102.8380	58.3172
6	100.8720	58.0683
7	97.9235	57.7464
8	96.2123	57.2933
9	94.2866	56.8263
10	87.6523	56.5828
11	81.0189	56.4256
12	83.1576	56.4662
13	83.3842	56.4782
14.	83.9852	56.4886
15.	87.6287	56.5069
16.	95.1824	56.5292
17.	98.2101	56.5998
18.	98.8921	56.6086
19.	99.7658	56.6288
20.	100.0860	56.6980
21.	100.1220	56.7128
22.	100.3680	56.7338
23.	100.6820	56.7421
24.	100.8680	56.7928

LIST OF THE FINAL RESULTS OBTAINED (Contd...)

Time in Hour	Optimum Discharge LL_1	Optimum hydro power developed W in Mw	Optimum Steam power developed S in Mw.
0	81.15	21.40	18.60
1	80.40	21.20	18.80
2	80.20	21.00	19.00
3	77.00	20.40	19.60
4	76.60	20.20	19.80
5	94.00	25.18	19.82
6	95.00	25.05	19.95
7	95.50	25.00	20.00
8	96.50	25.00	20.00
9	128.50	33.60	21.40
10	129.20	33.58	21.42
11	129.30	33.52	21.48
12	128.20	33.25	21.75
13	116.80	30.25	19.75
14	116.80	30.25	19.75
15	116.78	30.24	19.76
16	117.10	30.60	19.40
17	61.80	16.40	18.60
18	61.80	16.40	18.60
19	60.20	16.20	18.80
20	60.21	16.20	18.80
21	41.70	11.00	19.00
22	42.30	11.12	18.88
23	42.20	11.12	18.88
24	42.80	11.40	18.60