On Stability of Synchronous Machine Using Phase-Plane and Liapunov's Methods

A Dissertation
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By
P. NARASIMHULU NAIDU

DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE
(INDIA)
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CERTIFICATE

Certified that the dissertation entitled 'ON STABILITY OF SYNCHRONOUS MACHINES USING PHASE-PLANE AND LIAPUNOV'S METHODS' which is being submitted by Shri P. Narasimhulu Naidu, in partial fulfilment for the award of the degree of Master of Engineering in Advanced Electrical Machines at University of Roorkee, is a record of candidate's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is to certify that he has worked for a period of 7 months from Jan. to July for preparing dissertation for Master of Engineering Degree at the University.

(P. MUKHOPADHYAY)
PROFESSOR
ELECTRICAL ENGINEERING DEPARTMENT
UNIVERSITY OF ROORKEE
ROORKEE, UP
(INDIA)

P. Mukhopadhyay

Dated 3.10.68
ROORKEE

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P. [illegible]

CONTENTS

SYNOPSIS

The electromechanical power equations of a synchronous machine connected to an infinite bus are derived, starting from the basic voltage and flux linkage equations, considering, (1) saliency and field damping and (2) saliency, field damping and regulator actions. The Phase-plane technique is used to determine the stability of both the systems from the nature of its phase trajectories. Further, to determine the region of asymptotic stability and critical switching time, the second method of Liapunov is applied.

Later, the mathematical model for the multimachine system is developed and the second method of Liapunov is extended to the multimachine system without regulator actions, to determine stability and critical switching time.

NOTATIONS

e_f, e_d, e_q = Field, Direct Axis, Quadrature axis voltage

i_f, i_d, i_q = Field, Direct Axis, Quadrature axis current

ϕ_f, ϕ_d, ϕ_q = Field, Direct Axis, Quadrature axis flux linkage

L_f, L_d, L_q = Field, Direct Axis, Quadrature axis winding self inductances

L_{ad} = Mutual inductance of field and D-axis winding

x_d', x_q' = Direct axis, Quadrature axis transient reactance including external reactance if any.

x_d, x_q = D-axis, Q-axis reactance

Y_d, Y_q = Direct axis and Quadrature axis equivalent admittances

δ = Angle between generator voltage and bus

E = Voltage behind transient reactance or bus voltage

ν = Electric Speed

f = Frequency c/s

w = Electric Speed, rad/sec or P.U.

P = Power

T_{do}' = Direct axis transient open circuit time constant

T_{do}'' = Direct axis subtransient open circuit time constan

CHAPTER 1

INTRODUCTION AND STATEMENT OF THE PROBLEM

1.1 INTRODUCTION

The transient stability limit is the maximum power that the machine can supply under sudden disturbance without falling out of synchronism. Transient stability studies are important, since an actual power system is always associated with large or small size disturbances which do not cause the system to be truly in the steady state. The study of transient stability shows the performance of a system when subjected to sudden disturbances.

The stability information obtained for a linear system from the roots of the characteristics equation implies the stability in the entire space. Such an inference in a nonlinear system is invalid[1], because its stability depends on the parameters and the initial conditions. Therefore, it is essential to determine the initial conditions defining the region of stability. The trajectory of the system which has an initial condition within this region, will asymptotically tend to the stable equilibrium state as time tends to infinity. This indicates asymptotic stability of the system. The present methods of analysing the stability of a power system under transient conditions involve the explicit solution of the nonlinear differential equations describing the power system dynamics, to observe whether the various machines tend to maintain or lose synchronism, are quite tedious.

The direct or second method of Liapunov[2],[3],[4] is used to determine the stability or instability of the equilibrium states without the actual solution of the system differential equations. The method of Liapunov functions has the advantage of being applicable to multimachine system and establish the region of asymptotic stability. Also this method determines the critical switching time with the help of numerical solution to the differential equations of the system. The Phase-plane method also determines the stability or instability of the system from the nature of its phase trajectory.

1.2 STATEMENT OF THE PROBLEM

The problem is to determine the stability of a synchronous machine using phase plane and Liapunov's methods. In transient stability studies, it is necessary to know to what extent the system is stable. It is also necessary to determine the exact critical switching time, by which the system can be restored to its normal operation on switching at the critical time.

In achieving this, the mathematical model of the system, with lesser number of assumptions so as to represent the actual system to a nearer approximation, is developed. The phase plane method is used to determine the stability or instability of the single machine system from the nature of its phase trajectories. To determine the region of stability and critical switching time the second method of Liapunov is used. The critical switching time for the multimachine system without regulator action, is determined by extending the second method of Liapunov.

CHAPTER 2

A REVIEW ON THE TRANSIENT STABILITY OF SYNCHRONOUS MACHINES

2.1 GENERAL CONSIDERATIONS AND SURVEY OF LITERATURE

The problems associated with the maintenance of stability of synchronous machines in Power systems have received considerable attention in recent years. This has led to continous investigations in the subject and several authors have contributed towards the study of power system stability and methods to solve stability problems. The important aspects of stability problems are to determine the transient stability regions and critical switching time of power systems.

However, on these lines, Kimbark[5] and Crary[6] discussed the application of equal area criterion, point-by-point computation of swing curves, graphical integration of swing curves, and pre-calculated swing curves to determine transient stability and critical switching angle and critical switching time of power systems. Their discussions gained importance since these methods give atleast an approximate solution to the stability problem.

Later in 1958, Aylett[16] probably for the first time, used an entirely different approach namely, the energy integral criterion to study the transient stability limits of power systems. He devised the methods for identifying the nature of the phase trajectories in the study of nonlinear second order differential equations without finding the solutions to the equations and derived the formulae for the critical switching time. However, the importance of this paper is that it gives a method of finding the critical switching time, though approximate and aids in determining the transient stability limits with usual assumptions without finding the solutions to the system differential equations.

After Aylett's work, Dharma Rao[17] in 1962 presented a new approach to the transient stability problem. By energy methods he found out a rapid way of determing the critical switching time, while Ramachandra Rao and Dharma Rao[18] gave a simple graphical method to determine the critical switchigg time for a simple system. The importance of these papers is that they give a method of rapidly determining the critical switching time of a conservative system.

Later in 1966, Gless[19] and EL-Abiad and Nagappan[20] investigated successfully the transient stability of multimachine system using second method of Liapunov. Though they used many assumptions, their investigations are of at most importance since they have provided an approach for automatic determination of stability.

In 1967, Schuler[21] investigated the stability of various systems of synchronous machines using linearised system of differential equations. He solved the problem of stability in critical cases such as when the characteristic equation of the linearised system has roots with real parts equal to zero without having any other roots with positive real parts, by setting up Liapunov functions.

Just in the same year 1967, Natesan and Thanikasalam[22] went a step further and studied some aspects of the transient stability domain of integrated power systems including governor action using Liapunov's method.

In the present dissertation, transient stability of a single and multimachine system is investigated, taking into account, saliency, damping and regulator action so as to represent the actual system to a nearer approximation, using the general and powerful method of Liapunov.

2.2 A CRITICAL REVIEW

The problems associated with the maintenance of stability of synchronous machines in power systems and the methods of analyse the transient stability of power systems is reviewed in this article.

Kimbark[5] and Crary[6] haved discussed the application of equal area criterion, point-by-point computation of swing curves, graphical integration of swing curves, and pre-calculated swing curves to determine the transient stability and critical clearing angle and critcal clearing time. Both the authors in their stability studies discussed the effects of (1) flux decay (2) voltage regulators (3) amortisseur and prime mover damping and ex-citation systems. They emphasised that the transient stability limit can be increased by reducing the time of fault closing, by reducing the generator reactance and by using automatic voltage regulators. It is worth noting that the authors have given approximate methods to determine transient stability of power systems and critical switching angle and critical switching time.

Aldred and Doyle[7] have dealt with the subject at length and placed an emphasis on the time varying field flux linkage during transient disturbances. They gave the solutions of transient stability problems with varying field flux linkage and constant flux linkage to show the effect of the former (1) in response to a step function of the mechanical power input (2) in response to a disturbance caused by a transmission line fault without clearing by circuit breakers and (3) in response to similar disturbance with clearing by circuit breakers.

Aldred and Shackshaft[8] in their paper showed the effect of voltage regulators on the steady state stability and transient stability of a synchronous generator. The effect of the main regulator loop parameters, such as gain, exciter and main field time constants etc., on the stability of the system were examined. They concluded that while steady state stability increases considerably by the use of voltage regulators, the transient stability remains unaffected because of the relatively long delays involved in the regulator action owing to the main field time constant. The authors also concluded that the addition of damper windings may or may not have an effect on stability when a voltage regulator is used, since the damping created by the voltage regulator action and subsidiary feed backs may well outweigh any damping introduced by damper windings.

Ainsley and Kennedy[9] described the effects of various governors, namely, velocity governor, acceleration governor, position governor and combined governor on the stability of synchronous generator connected to a large system, by a single faulted transmission line fitted with autoreclosing switch. The stability of power systems ultimately depends on the accurate matching of the input and output powers of each individual machine in all conditions of operation. Hence, it is important that the input to a machine should be made to respond to the difference between the input and output powers to reduce the difference to zero as soon as possible. Therefore, the authors investigated the effect of various governors controlling the input power, which maintain the input output power relations.

The authors concluded that too high a gain of velocity governor ($G_g = \Delta P_g/\Delta\omega_g$) leads to self induced oscillations, especially with low inertia constants and small viscous damping. They noted that at low inertia constant self induced oscillations never occurred with gains of 50 or less and increasing the governor gain leads to reduce the positive damping. To keep the system free from the extraneous effects of self oscillations the authors preferred lower velocity governor gains.

It was found that the improvement in the transient performance of the synchronous generator might be produced by replacing the velocity actuating signal by one proportional to the rotor acceleration, since the acceleration is directly related to the power difference $(P_i - P_o)$, the accelerating power, on which stability ultimately depends. It was also found that the combined governor offers the possibility of greater assistance, not only to transient stability but also to eliminate the self induced oscillations arising from large gains in velocity governors. Rotor-angle governor or position governor was found to be inferior to other governors.

[illegible][10] studied some transient stability problems connected with displacement governors. Displacement governor is one which control the input power of a machine as a function of relative angular displacement between the machine shaft and the standard reference vector, that is, $P_i = K\alpha$, where α is the angular displacement.

Jacovides and Adkins[11] reported a detailed study of the effect of voltage regulators on the steady state stability of an alternator connected through a reactance to an infinite bus. They analysed the stability by Nyquist loci, and concluded that (1) proportional regulator can extend the stability limit upto the point corresponding to the peak of the transient power angle curve (2) an integrator regulator gives much better accuracy but gives a less satisfactory response and has less effect in extending the stability and (3) a derivative regulator can extend the stability limit beyond the transient power angle peak and gives rapid response.

A paper, published by Surana and Hariharan[12], presents a case of correlating transient response with transient stability limit of power systems. The effect of varying regulator gains on transient response was investigated with the help of gain locus diagram on a gain plane (D- Partition Method). The gains which gave good transient response were compared with those which gave high stability.

Jones[13], in his paper concludes that bang-bang excitation scheduling applied to synchronous machine returning from load rejection increases the generator's degree of transient stability and terminates its mechanical oscillations.

Morgan[14], in his paper on power system stability criteria for design, described (1) the effect of switching station (2) the effect of sending generator inertia and loading (3) the effect of type and location of fault and (4) the effect of fault clearing times. He evaluated the stability improvement and concluded as follows:

(1) The intermediate switching station has increased the stability power limit of a single interconnecting circuit by about 40% for higher plant reactance and about 15% for lower plant reactance.

(2) The type of fault and its location on the line have little effect on the stability power limit of a line when power flow is from a system with large inertia to one of small inertia. With faster relaying and circuit breaker times, 3 phase fault could be used as the criterion for design.

(3) In the case of relatively small inertia plant supplying power to a large inertia system, attention should be given to

the loading assumed on the generators prior to the stability conditions, as well as to the fault type and location, time of clearing and reclosing etc.

(4) The fast reclosing of circuit breakers in a time of about 20 cycles will give a stability power limit without interruption to service for all faults due to lightning.

(5) The transient power limits can be improved by reduced generators and transformer reactance.

A paper, published by Cooper and Kaualla[15], presents a valuable information on the calculation of power system stability. The authors noted that the low inertia constants allow much greater acceleration when load is dropped, and increased reactance, which is matched by a similar increase in generator transformer reactance tends to make the stability problem more acute. They found that this effect is to some extent counterbalanced by the decrease in fault clearance times. The authors also surveyed the effects of flux decay, amortisseur damping, voltage regulators and governors.

So far several authors have discussed about the various factors namely (1) the flux decay, which reduces stability (2) the amortisseur damping in the rotor body which increases stability (3) the governors, which reduce machine swing (4) the voltage regulators, which increase synchronising power and thus reduce machine swing (5) the very high response excitation systems, which improve the stability of the machines and many other factors in relation to the design and improvement of power system stability.

Recently ten years before (1958), Aylett[16], probably for the first time, used the energy integral criterion to study the transient stability limits of power systems. He devised the methods for identifying the nature of the phase trajectories in the study of nonlinear second order differential equations, without having to find the solutions to the equations. The author derived the formulae for the critical switching time taking resistance into account and generalised these methods for multimachine systems. The energy integrals in conjunction with step by step integration were used by the author to find the critical switching time for a faulted system.

The author has done quite a good amount of work on the stability and the importance of his paper is that it gives a method of finding the critical switching time, though approximate, and aids in determing the transient stability limits with usual assumptions without finding the solutions to the system differential equations.

After Aylett's work on the transient stability of power systems, Sharma Rao[17] in 1962 gave an approximate analytical solution to the swing equation with zero damping by an operational method and analysed the swing equation in the phase plane. By energy methods, he found out a rapid way of determining the critical switching time. Ramachandra Rao and Sharma Rao[18] in the same year presented a graphical method of determining the critical clearing angle making use of the fundamental stability theorem of Lagrange and Liapunov for a conservative system. The authors determined the critical closing angle as the abscissa of the intersection point between the separatrix curve and the locus of the initial conditions (swing curve of the faulted system). The critical switching time corresponding to the critical clearing angle was found by graphical method where the increment in time is given by $\Delta t = (\Delta\delta/\Delta\omega)$.

However, the importance of these papers is, they give an approximate method of rapidly determining the critical switching time of a conservative system using the energy integrals.

Later in 1966, Gless[19] demonstrated the application of the Liapunov functions to the transient stability of power systems in a direct method of solution. The author also demonstrated the close relationship of the equal area criterion, the phase plane method, the energy integral criterion and the method of Liapunov and applied the second method of Liapunov to the three machine system. In the same year El-Abiad and Nagappan[20] analysed the transient stability of multimachine system using Liapunov's method. The authors obtained a region of asymptotic stability of post fault system and determined the critical switching time of 4-machine system.

Though the authors, Gless and El-Abiad and Nagappan, used many assumptions, it can be said that their investigations opened the door for automatic determination of the stability of power systems.

In 1967, Schuler[21] investigated the stability of various systems of synchronous machines (two round rotor machines without damping tied over a lossless two port network, round rotor machine with damper winding connected to an infinite bus, salient pole machine without damping on an infinite bus) using linearized system of differential equations. As is well known, an investigation of this kind is not sufficient, if the characteristics equation of the linearized system has roots with real parts equal to zero without having any other roots with positive real parts. He solved the problem of stability in such critical cases by setting Liapunov's functions. This paper is also of considerable importance, since it gives an entirely different approach just to investigate the stability of synchronous machines.

Just in the same year 1967, Nelson and Thanikachalam[22] went a step further and studied some aspects of the transient stability domain of integrated power systems including governor action using Liapunov's method. It was observed that low velocity governor gains have no significant effect on the switching time. Critical switching time increases with further increase of gain. The authors found that for smaller values of H of a machine which is tightly connected reduces critical switching time, while a machine which is loosely connected, has no effect on critical switching time at all. Therefore, the authors concluded that critical switching time changes with change of H if the machine is tightly connected, otherwise not.

At this juncture, it can be said that it is an [illegible] of reference (20) and describes further in more detail the inclusion of governor action.

It is also necessary to review some of the existing techniques to generate the Liapunov's functions. Since no general method exists, the investigators have to try with various techniques to construct a suitable Liapunov function.

The usual choice of Liapunov function[3] is a positive definite quadratic form since it is mathematically convenient, that is, $V = x'Dx$, where x's are dependent variables and D are the elements of a square matrix.

Schultz and Gibson[23],[24] described the variable gradient method of generating Liapunov functions based on the assumption of a vector ∇V, with n undetermined components. The authors have found $\dot{V}$, the time derivative of Liapunov function and V, the Liapunov function from ∇V as

$$\dot{V} = \nabla V' \dot{X}$$

$$V = \int_0^X \nabla V \cdot dX$$

where ∇V is a new vector, $\nabla V'$ is the transpose of ∇V, and $\dot{X}$ is a column vector in state variables given by $\dot{X} = B(X)\,X$, where X is an column vector, B(X) is a square matrix.

The author assumed ∇V to be an arbitrary column vector whose coefficients are allowed to be functions of the state variables as

$$W = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{bmatrix} = \begin{bmatrix} \alpha_{11}x_1 + \alpha_{12}x_2 + \cdots + \alpha_{1n}x_n \\ \alpha_{21}x_1 + \alpha_{22}x_2 + \cdots + \alpha_{2n}x_n \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \alpha_{n1}x_1 + \alpha_{n2}x_2 + \cdots + \alpha_{nn}x_n \end{bmatrix}$$

whose coefficients α_{11}, α_{12} ....etc can be found from curl equations

$$\frac{\partial W_i}{\partial x_j} = \frac{\partial W_j}{\partial x_i}$$

The whole process can be summarised as follows:

1. Determination of $\dot{V}$ from W, the coefficients are yet undetermined.
2. Constraining $\dot{V}$ to be atleast semidefinite.
3. Determination of the remaining coefficients of W using the $(n-1)\,n/2$ curl equations.
4. Rechecking $\dot{V}$, since the addition of terms due to step 3 may have altered it.
5. Determination of V and checking for closedness.

Puri[23], in his paper on the global stability of a class of nonlinear time varying systems, developed two different methods to generate Liapunov functions. In the first method, he formulated a Liapunov function by multiplying the system differential equation with various state variables and integrating by parts and illustrated this method for three different systems namely, linear, nonlinear and nonlinear and nonautonomous systems. In the second method the author assumed a quadratic Liapunov function $V = X'S(X,t)\,X$, where the elements s_{ij} of $S(X,t)$ involve nonlinear and time varying functions. The author illustrated this method with three different systems namely, (1) systems with time varying parameters only (2) systems with nonlinearities and (3) systems with nonlinearities and time varying functions.

The final and perhaps the more apt choice of Liapunov's function is the total energy in the system.

The second method of Liapunov is named after its inventor A.M. Liapunov (1857-1918)[3]. Roughly speaking, he was a contemporary of Routh and Hurwitz. The powerful and general second method, first published in Russian in 1892, was buried in obscurity until it was exhumed and employed with considerable success, to investigate the stability of nonlinear automatic controls. This practical result was achieved by the Russian applied mathematician Lur'e and his associate Postnikov in 1944.

CHAPTER 3

STABILITY OF ONE MACHINE SYSTEM

3.1 DERIVATION OF POWER ANGLE EQUATIONS

When the power system to be studied consists of a group of machines connected through a transmission line, the system may be replaced as a single equivalent machine connected to an infinite bus through a tie line.

The following assumptions are made in the derivation of power angle equations:

1. The synchronous machine is ideal.
2. The input power remains constant during the entire transient period.
3. The armature and line resistances are negligible.
4. The amortisseur damping is negligible.

3.1.1 Synchronous Machine Without Regulators

The line diagram of a system considered is shown in Fig 2. The voltage and flux linkage equations[23] of a synchronous machine (shown in Fig 1) are,

$$\begin{bmatrix} e_f \\ e_d \\ e_q \end{bmatrix} = \begin{bmatrix} r_f + L_f p & L_{fd} p & 0 \\ L_{fd} p & L_d p & L_q \\ -L_{fd} & -L_d & L_q p \end{bmatrix} \cdot \begin{bmatrix} i_f \\ i_d \\ i_q \end{bmatrix} \qquad (3.1)$$

$$\left.\begin{aligned} \psi_f &= L_f\, i_f + L_{md}\, i_d \\ \psi_d &= L_{md}\, i_f + L_d\, i_d \\ \psi_q &= L_q\, i_q \end{aligned}\right] \qquad (3.2)$$

From (3.1) and (3.2), the voltage equations in terms of field, direct axis and quadrature axis flux linkages are

$$e_f = r_f\, i_f + p\,\psi_f \qquad (3.3)$$

$$e_d = p\,\psi_d + \nu\,\psi_q \qquad (3.4)$$

$$e_q = p\,\psi_q - \nu\,\psi_d \qquad (3.5)$$

Final solutions for ψ_d and ψ_q from equations (3.4) and (3.5) can be obtained as

$$\psi_d = -\,e_q/\omega \qquad (3.6)$$

$$\psi_q = +\,e_d/\omega \qquad (3.7)$$

Since, neglecting transformer voltages and substituting $\nu = \omega$, the flux linkages ψ_d and ψ_q from (3.4) and (3.5) are $\psi_d = -(e_q/\omega)$ and $\psi_q = (e_d/\omega)$.

Using the relations $e_d = E_2 \sin\delta$ and $e_q = E_2 \cos\delta$ in (3.6) and (3.7), the values of ψ_d and ψ_q are

$$\psi_d = -\,E_2 \cos\delta\,/\,\omega \qquad (3.8)$$

$$\psi_q = +\,E_2 \sin\delta\,/\,\omega \qquad (3.9)$$

where E_2 is the terminal or bus voltage.

From first of the equations (3.1)

$$i_f = \frac{e_f - L_{md}\, p\, i_d}{r_f + L_f\, p} \qquad (3.10)$$

Substituting i_f in second of the equations (3.2)

$$\psi_d = L_d i_d + L_{ad} \cdot \frac{e_f - L_{ad} p i_d}{r_f + L_f p} \qquad (3.11)$$

or

$$\psi_d = i_d \cdot \frac{L_d r_f + p(L_f L_d - L_{ad}^2)}{r_f + L_f p} + \frac{L_{ad} e_f}{r_f + L_f p} \qquad (3.12)$$

Neglecting the term $(L_f L_d - L_{ad}^2)$, ψ_d becomes

$$\psi_d = i_d \frac{L_d r_f}{r_f + L_f p} + \frac{L_{ad} e_f}{r_f + L_f p} \qquad (3.13)$$

or

$$i_d = \frac{\psi_d}{L_d} + p \psi_d \frac{L_f}{L_d r_f} - \frac{L_{ad} e_f}{L_d r_f} \qquad (3.14)$$

or

$$i_d = -\frac{E_2 \cos \delta}{x_d} + E_2 \sin \delta . (p\delta) . \frac{T_{do}'}{x_d} + \frac{E_1}{x_d} \qquad (3.15)$$

where $E_1 = \frac{w L_{ad}}{r_f} e_f$, is the quadrature axis transient internal voltage.

From third of the equations(3.2)

$$i_q = \frac{\psi_q}{L_q} = \frac{E_2 \sin \delta}{x_q} \qquad (3.16)$$

The electrical power is given by

$$P_e = w(i_d \psi_q - i_q \psi_d) \qquad (3.17)$$

Substituting ψ_d, ψ_q, i_d and i_q from (3.8), (3.9), (3.15) and

(3.16) in (3.17), the electrical power is

$$P_e = \frac{E_1 E_2}{X_d} \sin \delta + \frac{E_2^2}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta + \frac{E_2^2 T \omega'}{2 X_d} (1 - \cos 2\delta) \frac{d\delta}{dt} \qquad (3.18)$$

The electromechanical power equation of a synchronous machine might be written as

$$M(d^2\delta / dt^2) = P_a = P_i - P_e \qquad (3.19)$$

where P_a is the accelerating power corrected for losses.

or

$$M \frac{d^2\delta}{dt^2} + \frac{E_2^2 T \omega'}{2 X_d} (1 - \cos 2\delta) \frac{d\delta}{dt} + \frac{E_1 E_2}{X_d} \sin \delta + \frac{E_2^2}{2} \left(\frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta = P_i \qquad (3.20)$$

Thus, the electromechanical power equation of a salient pole synchronous machine is a second order nonlinear differential equation. The second and fourth terms on the left hand side of the equation (3.20) are, the damping power developed by the change of field flux linkage and the power developed due to saliency respectively.

3.1.2 Synchronous Machine With Regulators

In this case, as shown in Fig 3, the voltage proportional to the power angle and the voltage proportional to the time derivative of the power angle are applied to the field in addition to the steady state voltage of the field winding. Therefore the voltage equation of the field is

$$e_f = e_{fo} + k_1'\delta + K_2'(p\delta) = r_f i_f + p\Psi_f \qquad (3.21)$$

Substituting (3.21) in (3.14)

$$i_d = -\frac{E_2 \cos\delta}{x_d} + E_2 \sin\delta \, .(p\delta) . \frac{T_{do}'}{x_d} + \frac{E_1 + k_1\delta + k_2 p\delta}{x_d} \qquad (3.22)$$

where

$$k_1 = \frac{x_{ad}}{r_f} k_1' , \quad k_2 = \frac{x_{ad}}{r_f} k_2' \text{ and } E_1 = \frac{w L_{ad}}{r_f} e_{fo}$$

The electrical power is given by

$$P_e = w(i_d \Psi_q - i_q \Psi_d) \qquad (3.23)$$

Substituting Ψ_d, Ψ_q, i_d and i_q from (3.8), (3.9), (3.22) and (3.16) in (3.23), the electrical power is

$$P_e = \frac{E_1 E_2}{x_d} \sin\delta + \frac{E_2^2}{2}\left(\frac{1}{x_q} - \frac{1}{x_d}\right) \sin 2\delta + \frac{E_2 k_1}{x_d} .\delta . \sin\delta + \frac{E_2 k_2}{x_d} \sin\delta .(p\delta) + \frac{E_2^2 T_{do}'}{2 x_d}(1 - \cos 2\delta)(p\delta) \qquad (3.24)$$

The electromechanical power might be written as

$$M\frac{d^2\delta}{dt^2} + \frac{E_2^2 T_{do}'}{2 x_d}(1 - \cos 2\delta)\frac{d\delta}{dt} + \frac{E_2 k_2}{x_d}\sin\delta . \frac{d\delta}{dt} + \frac{E_1 E_2}{x_d}\sin\delta + \frac{E_2 k_1}{x_d} .\delta . \sin\delta + \frac{E_2^2}{2}\left(\frac{1}{x_q} - \frac{1}{x_d}\right)\sin 2\delta = P_i \qquad (3.25)$$

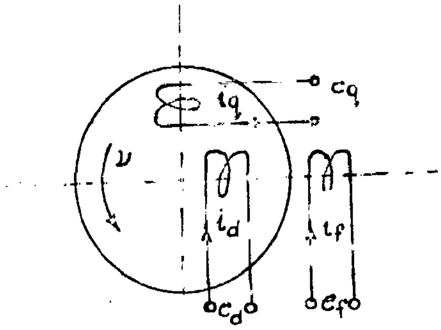


FIG.1. DIAGRAM OF SYNCHRONOUS MACHINE.

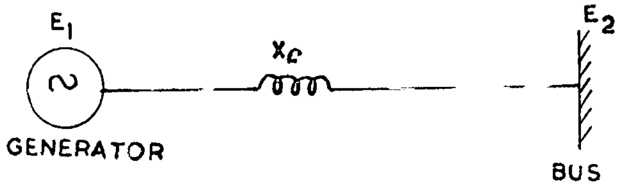


FIG.2. ONE MACHINE SYSTEM.

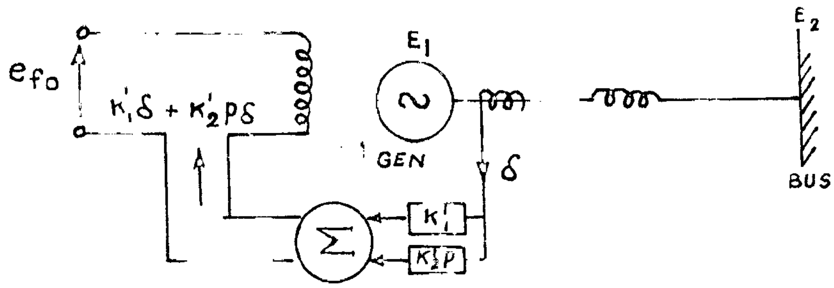


FIG.3. ONE MACHINE SYSTEM WITH REGULATORS.

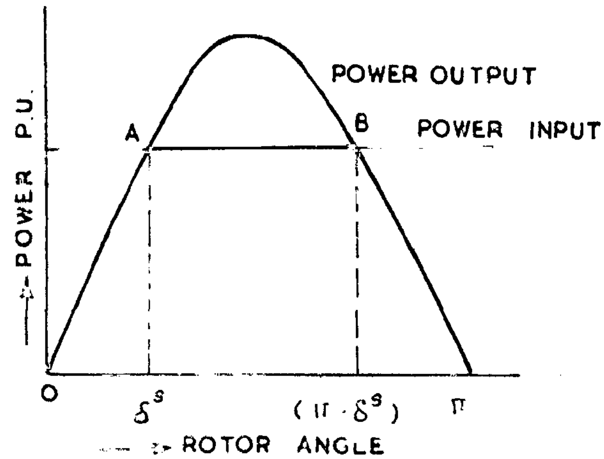


FIG.4 POWER V_S ROTOR ANGLE

A = STABLE EQUILIBRIUM STATE

B = UNSTABLE EQUILIBRIUM STATE

Thus, the electromechanical power equation of a salient pole synchronous machine with regulators is a second order non-linear differential equation. The second, third, fifth and sixth terms on the left hand side of the equation (3.29) are, the damping power developed by the change of field flux linkages, the damping power developed due to the time derivative-angle regulator, the power developed due to the angle regulator and the power developed due to saliency respectively.

3.2 APPLICATION OF THE PHASE-PLANE TECHNIQUE

The phase-plane technique is used in the transient stability studies to find out whether the system is totally stable or unstable. The system is stable if and only if the phase trajectory comes to a stable operating point, otherwise unstable.

The phase trajectory of a second order nonlinear differenti equation may be obtained either by graphical methods or by numerical methods. A detailed description of the graphical method is found in (3), (27), (28). Rungekutta method[(29),(30),(31)] of numerical solution is superior to graphical methods, because, it gives more accurate results, the computations can be done on digital computer, and the time solution can simultaneously be obtained.

As an example, a nonlinear second order differential equation (3.20) is solved by Rungekutta fourth order method as follows:

The equation (3.20) can be written as

$$\frac{d^2\delta}{dt^2} + c(1-\cos 2\delta)\frac{d\delta}{dt} + a\sin\delta + b\sin 2\delta = P \qquad (3.26$$

where

$$a = \frac{E_1 E_2}{x_d \cdot M}, \quad b = \frac{E_2^2}{2 \cdot M}\left(\frac{1}{x_q} - \frac{1}{x_d}\right) \text{ and } c = \frac{E_2^2 T_{do}'}{2\, x_d \cdot M}$$

The equation (3.26) is converted into two first order differential equations, as

$$\frac{d\delta}{dt} = v \qquad (3.27$$

$$\frac{dv}{dt} = P - a \operatorname{Sin}\delta - b \operatorname{Sin} 2\delta - c\,(1 - \operatorname{Cos} 2\delta)\, v \qquad (3.28)$$

If the initial conditions are,

$\delta = \delta_{n-1}$

$v = v_{n-1}$

$t = t_{n-1}$

then the ordinate at the nth interval can be obtained by Rungekutta formulae.

$$d\delta_1 = H\,(v_{n-1})$$

$$dv_1 = H\left[P - a \operatorname{Sin}\delta_{n-1} - b \operatorname{Sin} 2\delta_{n-1} - c\,(1 - \operatorname{Cos} 2\delta_{n-1})v_{n-1}\right]$$

where H is the increment in time.

$$d\delta_2 = H\left[v_{n-1} + \frac{dv_1}{2}\right]$$

$$dv_2 = H\left\{P - a \operatorname{Sin}\left(\delta_{n-1} + \frac{d\delta_1}{2}\right) - b \operatorname{Sin} 2\left(\delta_{n-1} + \frac{d\delta_1}{2}\right) - c\left[1 - \operatorname{Cos} 2\left(\delta_{n-1} + \frac{d\delta_1}{2}\right)\right]\left(v_{n-1} + \frac{dv_1}{2}\right)\right\}$$

$$d\delta_3 = H\left[v_{n-1} + \frac{dv_2}{2}\right]$$

$$dv_3 = H\left\{P - a \operatorname{Sin}\left(\delta_{n-1} + \frac{d\delta_2}{2}\right) - b \operatorname{Sin} 2\left(\delta_{n-1} + \frac{d\delta_2}{2}\right) - c\left[1 - \operatorname{Cos} 2\left(\delta_{n-1} + \frac{d\delta_2}{2}\right)\right]\left(v_{n-1} + \frac{dv_2}{2}\right)\right\}$$

$$d\delta_4 = H\left[v_{n-1} + dv_3\right]$$

$$dv_4 = H\left\{P - a \operatorname{Sin}(\delta_{n-1} + d\delta_3) - b \operatorname{Sin} 2(\delta_{n-1} + d\delta_3) - c\left[1 - \operatorname{Cos} 2(\delta_{n-1} + d\delta_3)\right](v_{n-1} + dv_3)\right\}$$

The values of δ, v and t at the n th interval are,

$$\delta_n = \delta_{n-1} + \frac{1}{6}(d\delta_1 + 2d\delta_2 + 2d\delta_3 + d\delta_4)$$

$$v_n = v_{n-1} + \frac{1}{6}(dv_1 + 2dv_2 + 2dv_3 + dv_4)$$

$$t_n = t_{n-1} + H$$

The processis repeated as many times as necessary. The entire procedure given above, has been programmed for an IBM 1620, and the program is given in Appendix I.

3.3 THEORY OF THE SECOND METHOD OF LIAPUNOV

Liapunov's Theorem[3] states: If a positive definite function $V(x_1, x_2, \ldots, x_n, t)$ exists for a system of the n th order, described by the ordinary differential equations

$$\dot{x}_i = F_i(x_1, x_2, \ldots, x_n), \quad i = 1, 2 \ldots, n$$

so that its derivative with respect to time along the trajectories of the system $\dot{V} = W(x_1, x_2, \ldots, x_n)$ is negative definite, the system is asymptotically stable.

The function V, called Liapunov function will yield a strong sufficient condition[1],[4]. To study the stability of the system, the Liapunov function is constructed satisfying the following conditions:

1. The function should be a continous scalar function of the state variables.
2. Its value at the equilibrium state is zero and is positive inside the bounded region R.

3. Its time derivative should exist and be continuous in a region R defined by $V < D$ where D is a positive constant quantity.

4. Its time derivative should be negative in R except at the equilibrium state where it vanishes.

There are no general rules to construct Liapunov functions, although special methods have been developed for certain classes of functions. The usual methods of generating Liapunov functions are a quadratic form[3], a variable gradient method[23], [24], and a quadratic form plus an appropriate integral[23].

The simplest example of an actual Liapunov function is the total energy in the system. In a system with differential equation

$$\ddot{x} + g_1(x)\,\dot{x} + g_2(x) = 0 \qquad (3.29)$$

the Liapunov function might be written, using the total energy, that is, the kinetic energy of the system plus the potential energy stored in the system, as

$$V = \frac{(\dot{x})^2}{2} + \int_0^x g_2(x)\,dx \qquad (3.30)$$

This function is positive definite over the whole phase plane. The time derivative of V is

$$\dot{V} = \dot{x}\,\ddot{x} + \dot{x}\,g_2(x) \qquad (3.31)$$

$$= \dot{x}\,(\ddot{x} + g_2(x)) \qquad (3.32)$$

Substituting for $\ddot{x}$ from (3.29) in (3.32) gives

$$\dot{V} = \dot{x}\,(-g_1(x)\,\dot{x} - g_2(x) + g_2(x)) \qquad (3.33)$$

$$\dot{V} = -g_1(x)\,\dot{x}^2 \qquad (3.34)$$

which is always negative or zero for damping parameter $g_1(x)$ positive. If damping factor $g_1(x)$ is zero, $\dot{V}$ is zero and V is constant and the system travels along a contour of constant energy and is stable. If $g_1(x)$ is positive, $\dot{V}$ is negative and therefore $\dot{V}$ and the system energy decrease with time. Such a system is asymptotically stable as it comes to rest at a stable equilibrium. If $g_1(x)$ is negative, the energy would continually increase and the system would be unstable.

3.4 APPLICATION OF THE SECOND METHOD OF LIAPUNOV

The method of application of the second Liapunov process for establishing the region of stability and determining the critical switching time consists of the following main steps:

1. Construction of a suitable Liapunov function.
2. Determination of the equilibrium states of the system before the disturbance.
3. Determination of the limiting value of V
4. Forward integration of the disturbed system to find the critical switching time.

The above steps are explained in detail as under.

3.4.1 Construction of Liapunov Functions

In view of the many choices available, 'what passes for competence is a knowledge of previous fortunate experience'[3] is still not inappropriate. Various Liapunov functions were tried, before the Liapunov function based on energy concept is found suitable in this stability study.

The Liapunov function for a synchronous machine without regulator can be constructed using the total energy in the system, that is, the kinetic energy of the system plus the potential energy stored in the system, as

$$V = \frac{1}{2} M v^2 + \int_{\delta^s}^{\delta} -\left(P_i - \frac{E_1 E_2}{x_d} \sin\delta - \frac{E_2^2}{2}\left(\frac{1}{x_q} - \frac{1}{x_d}\right) \sin 2\delta\right) d\delta \quad (3.35)$$

or

$$V = \frac{1}{2} M v^2 - P_i(\delta - \delta^s) - \frac{E_1 E_2}{x_d}(\cos\delta - \cos\delta^s)$$

$$- \frac{E_2^2}{4}\left(\frac{1}{x_q} - \frac{1}{x_d}\right)(\cos 2\delta - \cos 2\delta^s) \quad (3.36)$$

This function is positive definite over the whole phase plane. The time derivative of V is

$$\dot{V} = M v \dot{v} + \left(- P_i + \frac{E_1 E_2}{x_d} \sin\delta + \frac{E_2^2}{2}\left(\frac{1}{x_q} - \frac{1}{x_d}\right) \sin 2\delta\right) v \quad (3.37)$$

Substituting the value of $\dot{v}$ from (3.20) in (3.37)

$$\dot{V} = - \frac{E_2^2 T_{do}'}{2x_d} (1 - \cos 2\delta)\, v^2 \quad (3.38)$$

which is always negative definite except at the equilibrium where it vanishes and the equilibrium is asymptotically stable.

The Liapunov function can similarly be constructed for a synchronous machine with regulators using the total energy in the system, as

$$V = \frac{1}{2} M v^2 + \int_{\delta^s}^{\delta} -\left(P_i - \frac{E_1 E_2}{x_d} \sin\delta - \frac{E_2^2}{2}\left(\frac{1}{x_q} - \frac{1}{x_d}\right) \sin 2\delta - \frac{E_2 k_1}{x_d} \cdot \delta \cdot \sin\delta\right) d\delta \quad (3.39)$$

or

$$V = \frac{1}{2} Mv^2 - P_i(\delta - \delta^s) - \frac{E_1 E_2}{x_d} (\text{Cos } \delta - \text{Cos } \delta^s)$$

$$- \frac{E_2^2}{4} \left(\frac{1}{x_q} - \frac{1}{x_d} \right) (\text{Cos } 2\delta - \text{Cos } 2\delta^s)$$

$$- \frac{E_2 k_1}{x_d} (\text{Sin } \delta - \text{Sin } \delta^s) + \frac{E_2 k_1}{x_d} (\delta - \delta^s)(\text{Cos}\delta - \text{Cos}\delta^s) \qquad (3.40)$$

This function is positive definite in the whole phase plane. The time derivative of V is

$$\dot{V} = Mv\dot{v} + \left(- P_i + \frac{E_1 E_2}{x_d} \text{Sin } \delta + \frac{E_2^2}{2} \left(\frac{1}{x_q} - \frac{1}{x_d} \right) \text{Sin } 2\delta \right.$$

$$\left. + \frac{E_2 k_1}{x_d} \cdot \delta \cdot \text{Sin } \delta \right) v \qquad (3.41)$$

Substituting for $\dot{v}$ from (3.25) in (3.41)

$$\dot{V} = - \frac{E_2 k_2}{x_d} \text{Sin } \delta \cdot v^2 - \frac{E_2^2 T_{do}'}{2x_d} (1 - \text{Cos } 2\delta) \cdot v^2 \qquad (3.42)$$

which is always negative definite and the equilibrium is asymptotically stable.

3.4.2 Equilibrium States

The stable equilibrium state of the pre-disturbance system is found by Newton Raphson method[30] with the velocities and accelerations made equal to zero, since, at the equilibrium state velocities and accelerations are zero.

The unstable equilibrium state closest to the stable equilibrium state is found in the same way as in the case of stable equilibrium, but the initial value of δ is chosen as π radians, since the stable equilibrium value δ^s is approximately away from unstable equilibrium value δ^u by $(\pi-\delta^s)$, as shown in Fig 4.

The equilibrium states can also be found from the potential energy of the Liapunov function. The value of δ which gives the minimum value of the potential energy is the stable equilibrium sta-te and the value of δ which gives the maximum value of the potential energy is the unstable equilibrium state.

3.4.3 Limiting Value of V

The limiting or the maximum value of V in the closed region R is given by

$$B = V(\delta^u , v^u) \tag{3.43}$$

$$v^u = 0 \tag{3.44}$$

The surface given by V=B passes through the unstable equilibrium state. The region R includes only the equilibrium state under the investigation. At this equilibrium state dV/dt is zero.

3.4.4 Critical Switching Time

During the forward (swing curve) integration of the disturbed system, the state at every instant of time is tested to determine whether or not it is inside the region R, by checking the corresponding value of V against B which serves as an index for the region R. The time at which the value of V is just equal to B gives the critical switching angle and the cirtical switching time.

3.5 NUMERICAL EXAMPLES

The synchronous machine connected to an infinite bus will be considered. Assuming average values for the machine constants, the power angle equations for both the systems will be obtained, and stabilities of the systems are determined for sudden input loads using the phase plane and Liapunov methods described earlier.

The synchronous machine under study has the following parameters:

$R = 0.02$ P.U.

$x_d = 1.2$ P.U.

$x_q = 0.8$ P.U.

$x_{ad} = 1.0$ P.U.

$T_{do}' = 5$ secs

$\omega = 2\pi f$ rad /sec. or 1.0 P.U.

$f = 50$ c/s

$x_d' = 0.3$ P.U.

The synchronous machine is assumed to be operating initially at no load, that is, $P_i=0$ and $\delta^0=0$ and hence the internal and terminal voltages are $E_1=E_2=1.0$ P.U. The regulator gains are set at $k_1=k_2=1$. A sudden input load of 2.4 P.U. is applied.

The power angle equations, without and with regulators are therefore respectively given by

$$\frac{d^2\delta}{dt^2} + 104(1-\cos 2\delta)\frac{d\delta}{dt} + 41.6\sin\delta + 10.5\sin 2\delta = P \qquad (3.45)$$

$$\frac{d^2\delta}{dt^2} + 104(1-\cos 2\delta)\frac{d\delta}{dt} + 41.6\sin\delta\cdot\frac{d\delta}{dt} + 41.6\sin\delta\cdot\delta + 41.6\sin\delta + 10.5\sin 2\delta = P \qquad (3.46)$$

To determine the stability, the differential equations of initial and final systems are solved, first by phase plane method and secondly by Liapunov's method.

Phase-Plane Method

The problem is to find whether the machine is stable or unstable before and after the disturbance. The pre-disturbance systems, that is, when input, power $P_i=0$ found to be stable since the phase trajectory comes to a stable operating point. The phase trajectories for both the systems are shown in Fig 5(a) and 5(b). When a sudden input load of 2.4 P.U. is applied both the systems are found to be unstable as shown in Fig 6(a) and 6(b). It can be seen from Fig 6(a) and 6(b) that the phase trajectories are oscillatory due to the damping powers only, otherwise the phase trajectories would have followed the dotted lines without damping powers. When sudden loads equal in magnitude to the value obtained by equal area criterion, that is, $P_i=0.6$ P.U. and $P_i = 1.2$ P.U. are applied to the systems without and with regulators respectively the systems are found to be stable as shown in Fig 7(a) and 7(b). It can be observed that the voltage regulators increase the stability limit.

Liapunov's Method

The Liapunov's functions for both the systems with initial loads of $P_i = 0$ are

without regulators:

$$V = \frac{1}{2} M\omega^2 + \int_{\delta_0}^{\delta} (+\, 0.032 \sin\delta + 0.289 \sin 2\delta)\, d\delta \qquad (3.47)$$

or

$$V = \frac{1}{2} M\omega^2 - 0.032 \cos\delta + 0.032 \cos\delta^0 - 0.145 \cos 2\delta + 0.145 \cos 2\delta^0 \qquad (3.48)$$

With regulators:

$$V = \frac{1}{2} Mv^2 + \int_{\delta^s}^{\delta} (+0.832 \, \mathrm{Sin}\delta + 0.832\delta \, \mathrm{Sin}\delta + 0.210 \, \mathrm{Sin}\, 2\delta) \, d\delta \qquad (3.49)$$

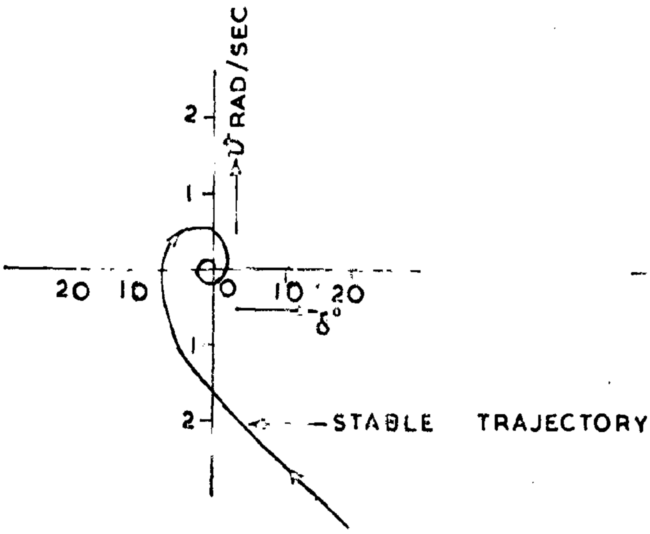
$$V = \frac{1}{2} Mv^2 - 0.832 \, \mathrm{Cos}\delta + 0.832 \, \mathrm{Cos}\delta^s + 0.832 \, \mathrm{Sin}\delta - 0.832 \, \mathrm{Sin}\delta^s$$
$$- 0.832 . \delta . \mathrm{Cos}\delta + 0.832 . \delta^s . \mathrm{Cos}\delta^s - 0.105 \, \mathrm{Cos}2\delta + 0.105 \, \mathrm{Cos}2\delta^s \qquad (3.50)$$

The steady state stable equilibrium and unstable equilibrium values for the systems with $P_i = 0$ are found to be $\delta^s = 0^\circ$ and $\delta^u = 180^\circ$. The maximum trajectory which is the stable boundary for the particular system may be found by setting $v = 0$ and $\delta = \delta^u$, since at the unstable equilibrium point the velocities are zero. The result is a value for V which defines precisely the region of stability for the equilibrium $\delta = \delta^s$. The maximum values of V for the machine without and with regulators are V=B= 1.6640 and V=B= 4.2778 respectively. The equation to the separatrix or the maximum phase trajectory in the v-δ plane (which is not necessary to determine stability) is

$$v = \sqrt{\frac{2}{M} \left[V_{max} - P.E(\delta) \right]} \qquad (3.51)$$

where P.E(δ) is the potential energy. The details of the procedure for constructing the maximum trajectory are shown in Fig 8(a) and 8(b).

The critical switching angle and critical switching time are determined as per the procedure outlined earlier. The critical clearing angle and critical clearing time for the systems without and with regulators for an input load of 2.4 P.U. are $\delta_c = 62.4^\circ$ and $t_c = 0.97$ sec and $\delta_c = 51.2^\circ$ and $t_c = 1.03$ secs respectively.



:3·5.a FREE OSCILLATIONS OF THE SYSTEM WITHOUT REGULATORS.

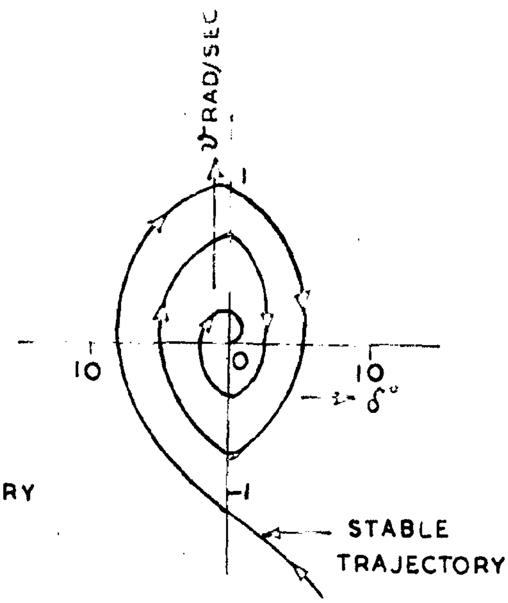
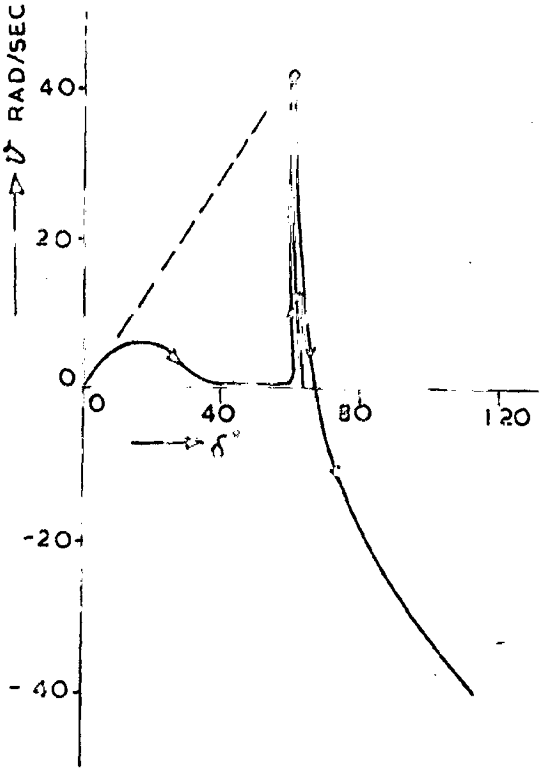


FIG. 5.b. FREE OSCILLATIONS OF THE SYSTEM WITH REGULATORS.



6 a. PHASE TRAJECTORY OF SYNCHRONOUS MACHINE WITH INPUT LOAD OF $P_L = 2{\cdot}4$ P.U.

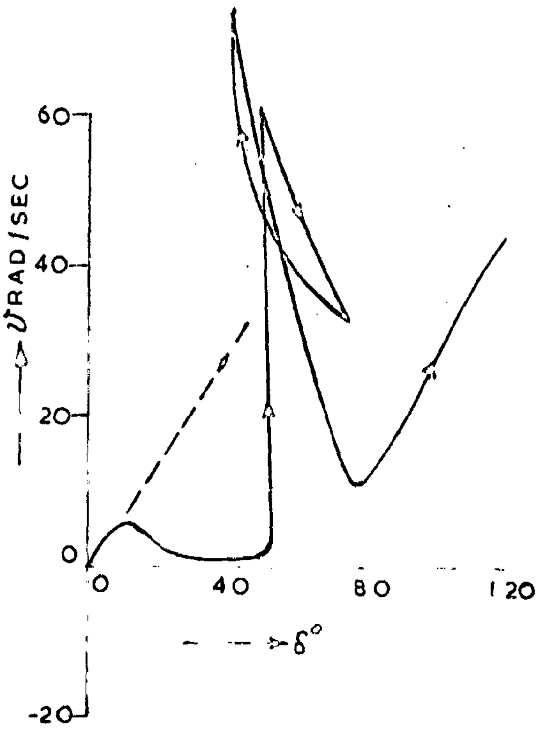


FIG 6.b. PHASE TRAJECTORY OF THE SYSTEM WITH REGULATORS AND WITH AN INPUT LOAD OF P_L 2·4 P.U.

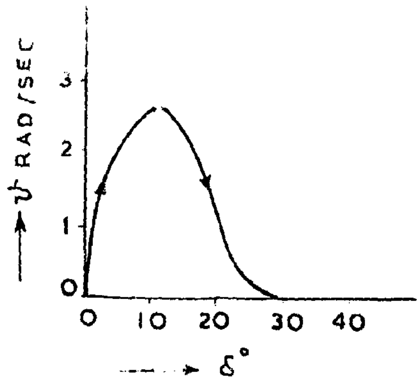


FIG_7.a. STABLE PHASE TRAJECTORY OF SYSTEM WITHOUT REGULATORS AND WITH INPUT LOAD OF 0.6 P.U.

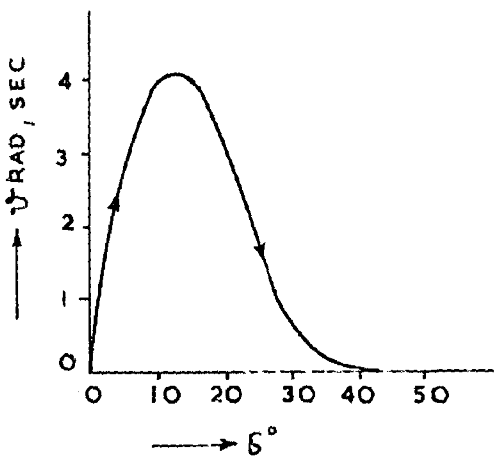


FIG 7.b. STABLE PHASE TRAJECTORY OF THE SYSTEM WITH VOLTAGE REGULATORS AND WITH INPUT LOAD OF 1.2 P.U.

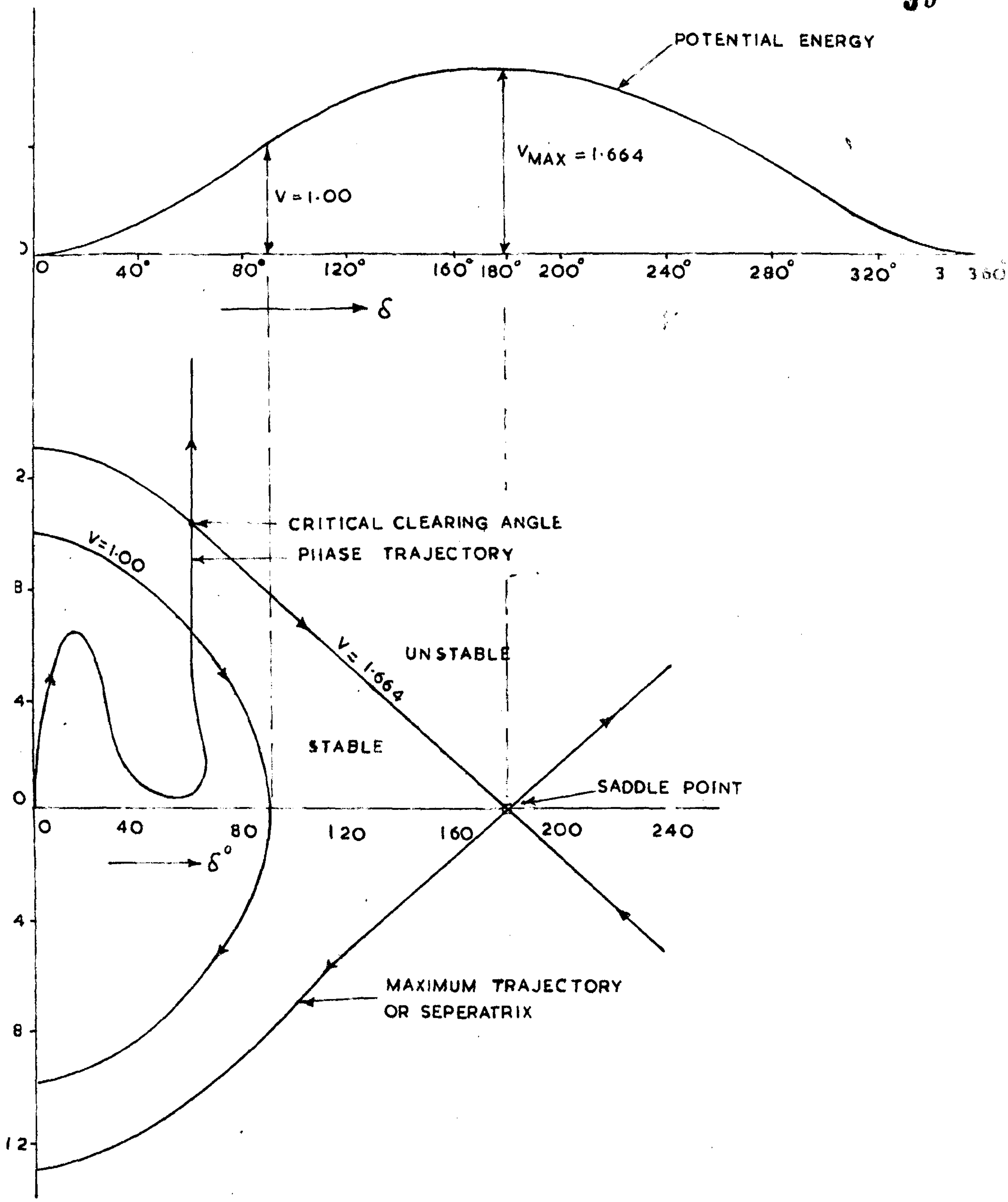
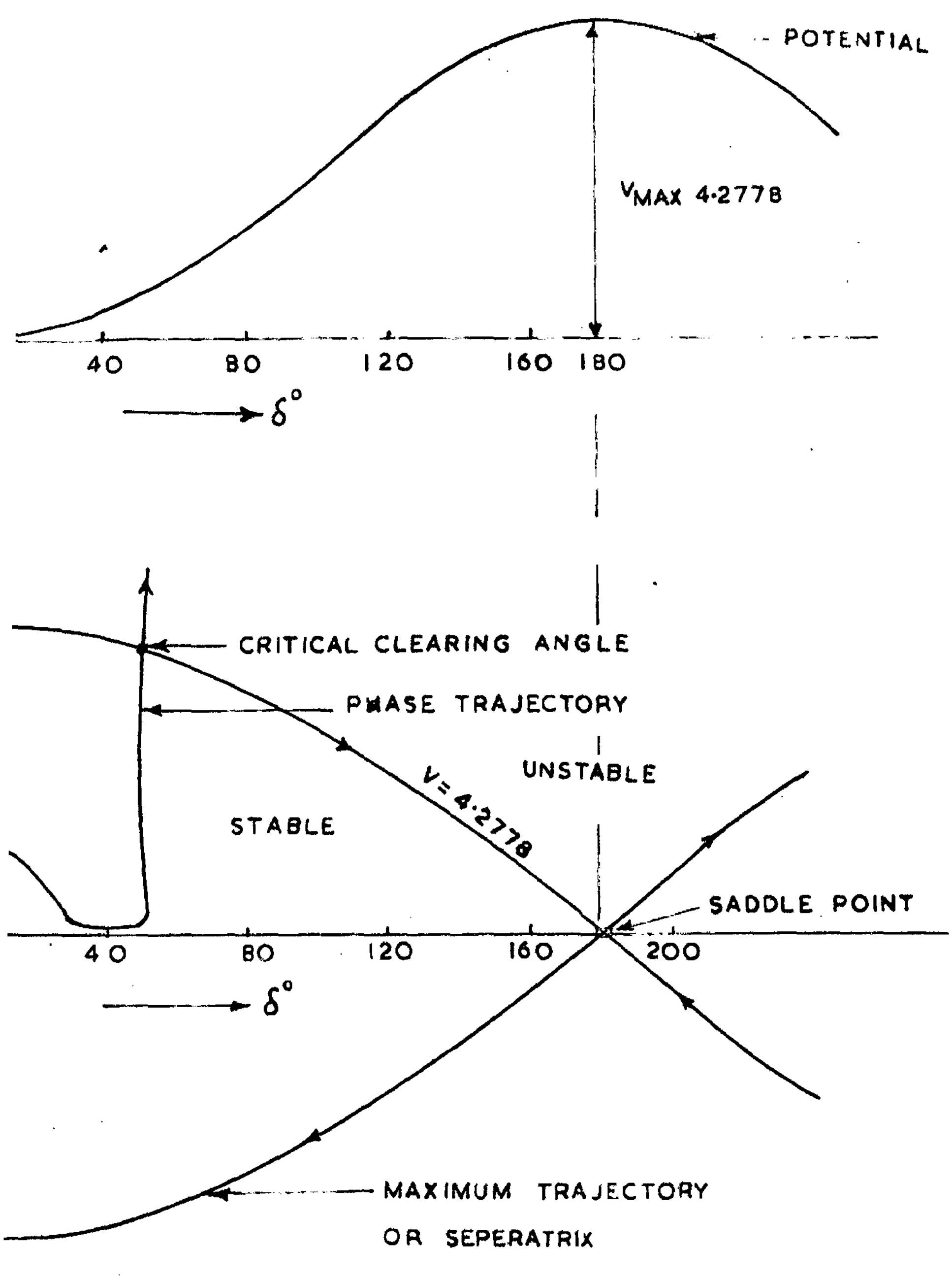


FIG. 8(a) PHASE TRAJECTORY FOR THE SYSTEM WITH OUT REGULATORS.



G 8(b)PHASE TRAJECTORY FOR THE SYSTEM WITH REGULATORS.

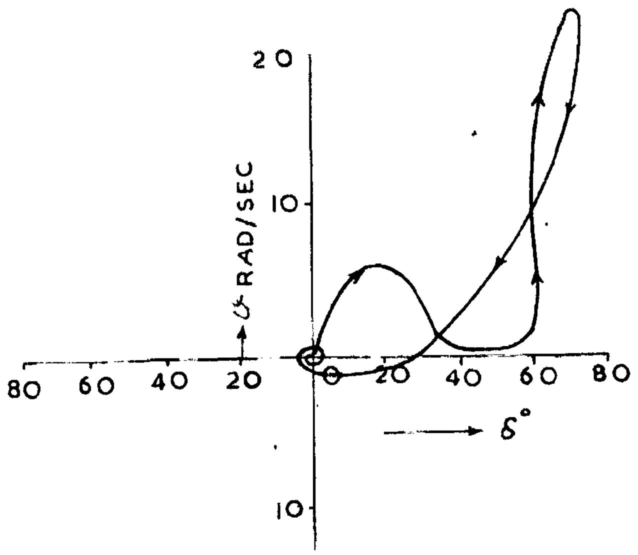


FIG 9a STABLE DISTURBANCE CLEARED AT 0·97 SEC, FOR THE SYSTEM WITHOUT REGULATORS.

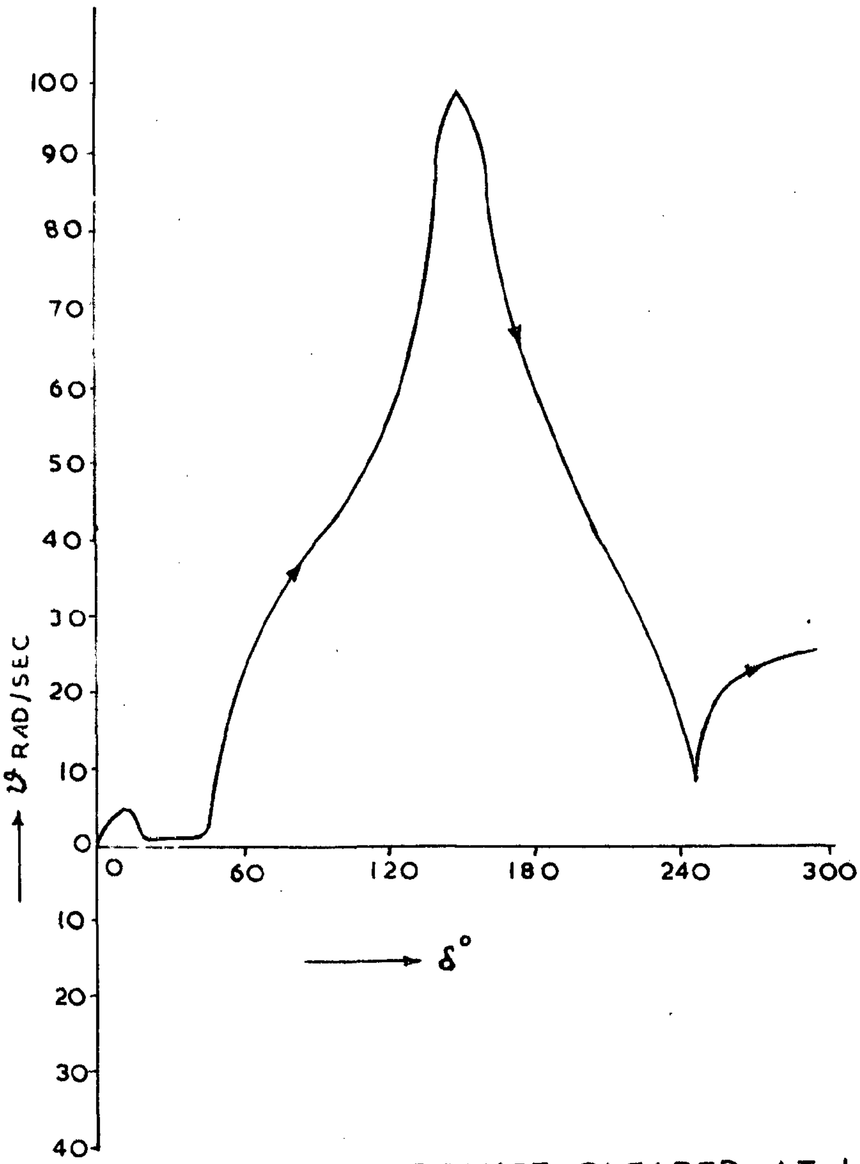


FIG 9b UNSTABLE DISTURBANCE CLEARED AT 1·05 SECS FOR THE SYSTEM WITH REGULATORS.

TABLE-1

RESULTS OF SINGLE MACHINE SYSTEM WITH AN INPUT LOAD OF 2.4 P.U. BY RUNGE-KUTTA FOURTH ORDER METHOD

SYSTEM WITHOUT REGULATORS-

TIME	DELTA	VELOCITY	VFUNCTION
0.00	0.000	0.000	0.000
.04	5.436	4.661	.223
.08	19.318	6.411	.481
.12	30.318	3.055	.261
.16	35.093	1.503	.243
.20	38.022	1.135	.269
.24	40.427	.980	.297
...	...	...	...
...	...	...	...
.80	60.746	.490	.588
.90	63.244	.886	.633
.92	63.584	1.415	.650
.94	63.697	2.727	.706
.96	63.275	6.258	1.017
.98	61.684	16.818	3.428
1.00	58.848	42.487	18.607

SYSTEM WITH REGULATORS-

TIME	DELTA	VELOCITY	VFUNCTION
0.00	0.000	0.000	0.000
.04	5.351	4.480	.206
.08	17.530	5.083	.324
.12	25.849	2.319	.202
.16	29.688	1.285	.215
.20	32.236	.991	.246
...	...	...	...
...	...	...	...
.84	50.882	.324	.614
.96	52.731	.828	.666
1.00	52.721	3.042	.752
1.02	52.066	7.380	1.187
1.04	50.452	21.675	5.300
1.08	74.022	32.975	12.174
1.10	42.993	79.511	63.651

A pictorial explanation of determining critical clearing angle and critical clearing time are shown in Fig 8(a) and 8(b), and the results are given in Table I. The effect of critical switching times are shown in Fig 9(a) and 9(b). The system with voltage regulators is tested with different input loads and found to be stable for P_i 1.2 P.U. Similarly the system without regulators is tested and found to be stable for P_i 0.6 P.U.

3.6 CONCLUSIONS

Phase plane and Liapunov's methods have been successfully applied to determine the stability of a single machine system without and with regulators. The damping powers and voltage regulators increase the stability limits to a considerable extent. The phase plane method is useful only to know whether the system is entirely stable or not. The advantages of Liapunov's second method over the phase plane and other methods are

1. Easy determination of stability or instability of the system.
2. Exact determination of critical switching time
3. Determination of the index of the region of stability
4. Extension of the method to multimachine system.

CHAPTER 4

TRANSIENT STABILITY OF MULTIMACHINE SYSTEM

4.1 MATHEMATICAL MODEL

The direct method of Liapunov may be extended to any dynamical system, if the system can be represented as a mathematical model. The set of differential equations describing the multi-machine system (without regulators) may be developed from the basic synchronous machine equation (3.20), using the same assumptions as in Chapter 3.1. The equations corresponding to the three stages namely pre-fault system, faulted system and post-fault system will be same as given by (4.1), except for the difference in parameters from one stage to the other.

$$\sum_{K=1}^{N} M_K \frac{d^2\delta_K}{dt^2} + \sum_{K=1}^{N} \sum_{\substack{J=1 \\ J\neq K}}^{N} \left[\frac{E_J^{2}(T_{do}'')_K}{2(x_d')_{KJ}}(1 - \mathrm{Cos}\ 2\delta_K)\frac{d\delta_K}{dt} + \frac{E_K E_J}{(x_d')_{KJ}}\sin(\delta_K-\delta_J) + \frac{E_J^{2}}{2}\left(\frac{1}{(x_q')_{KJ}} - \frac{1}{(x_d')_{KJ}}\right)\sin 2(\delta_K - \delta_J)\right] = \sum_{K=1}^{N} P_K \qquad (4.1)$$

It is of primary importance to determine the system stabilit during and after clearing the disturbance.

4.2 APPLICATION OF THE LIAPUNOV'S METHOD

The procedure for establishing the region of stability and determining the critical switching time using the second method of Liapunov consists of the following main steps:

1. Construction of a suitable scalar function

2. Load flow for the pre-fault system and determination of driving point and transfer admittances between the internal busses of the machines (Y_{oq}) for pre-fault, and faulted and post-fault systems.

3. Determination of the stable and unstable equilibrium states for the post-fault system.

4. Determination of the limiting value of V.

5. Forward integration of the faulted system to find the critical switching time.

Computer flow chart for establishing the region of stability and determining the critical switching time is shown in Fig 10.

A detailed description of the second step is found in (8), (32). The other steps are explained in detail in the following pages.

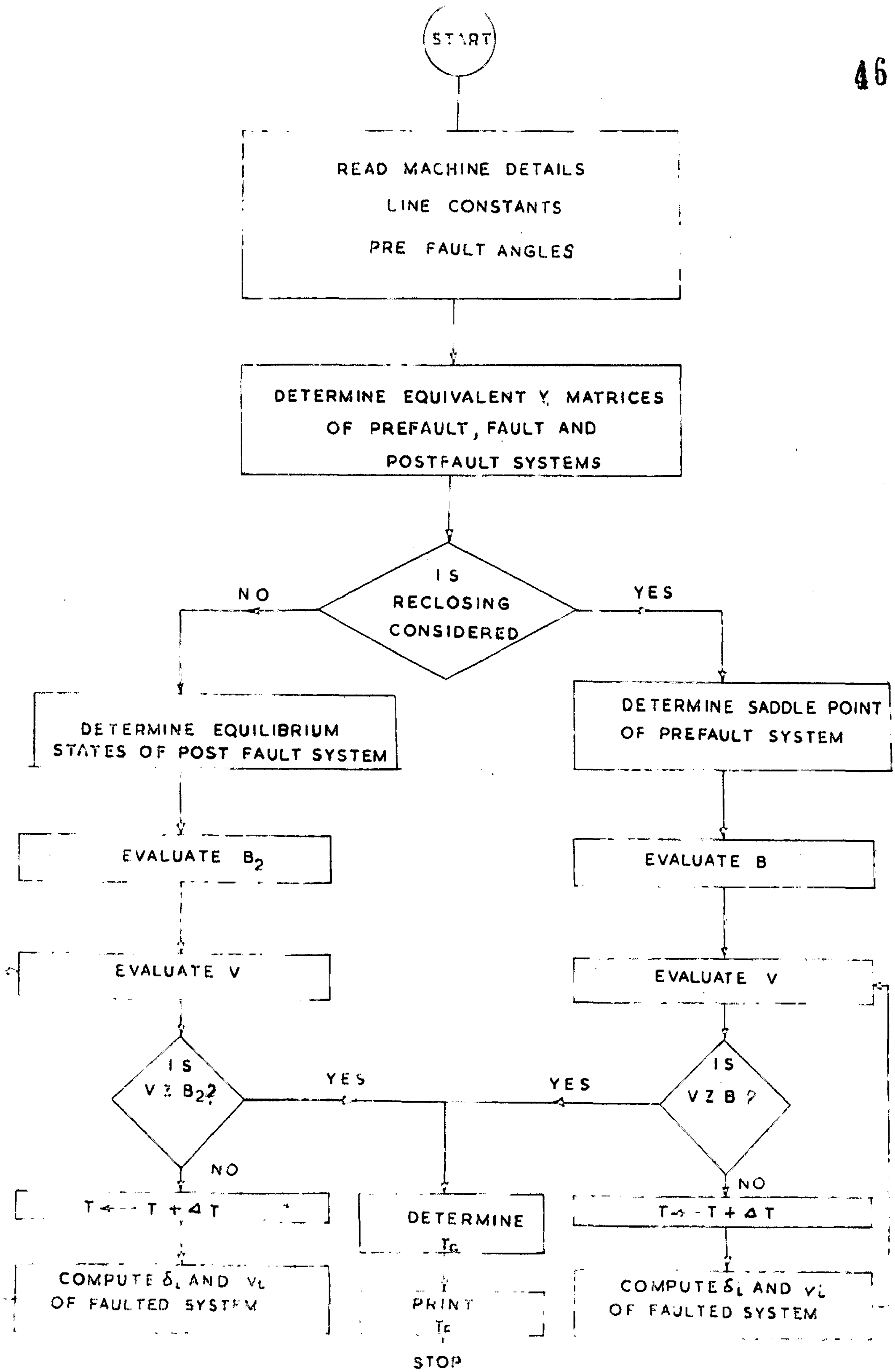


FIG 10. COMPUTER FLOW CHART

4.2.1 Construction of Liapunov Function

The Liapunov function may be written from the mathematical model of the multimachine system (4.1), using the total energy in the system, as

$$V = \sum_{K=1}^{N} \frac{1}{2} M_K v_K^2 - \sum_{K=1}^{N} P_K(\delta_K - \delta_K^s)$$
$$+ \sum_{K=1}^{N-1} \sum_{J=K+1}^{N} \left\{ - \frac{E_K E_J}{(x_d')_{KJ}} \left[\mathrm{Cos}(\delta_K - \delta_J) - \mathrm{Cos}(\delta^s_K - \delta_J^s) \right] \right.$$
$$\left. - \frac{E_J^2}{4} \left(\frac{1}{(x_q')_{KJ}} - \frac{1}{(x_d')_{KJ}} \right) \left[\mathrm{Cos}\,2(\delta_K - \delta_J) - \mathrm{Cos}\; 2(\delta_K^s - \delta_J^s) \right] \right\} \qquad (4.2)$$

The first summation term on the right hand side of (4.2) is the kinetic energy of the system, and the last three terms are equal to potential energy of the system.

This function is positive definite in the whole phase space and the value of V at the equilibrium state (δ^s, v^s) is

$$V(\delta^s, 0) = 0 \qquad (4.3)$$

The time derivative of V is given by

$$\frac{dV}{dt} = \sum_{K=1}^{N} \left(\frac{\partial V}{\partial \delta_K} \frac{d\delta_K}{dt} + \frac{\partial V}{\partial v_K} \frac{dv_K}{dt} \right) \qquad (4.4)$$

Hence, the time derivative of (4.2), after substituting for v_K from (4.1) is

$$\frac{dV}{dt} = \sum_{K=1}^{N} \sum_{\substack{J=1 \\ J \neq K}}^{N} - \frac{E_J^2 (T_{do}'')_K}{2 (x_d')_{KJ}} (1 - \mathrm{Cos}\; 2\delta_K) \cdot v_K^2 \qquad (4.5)$$

$\dot{V}$ is negative definite for all values of v_K except at the equilibrium where it vanishes. Therefore the equilibrium of the system is asymptotically stable.

4.2.2 Equilibrium States

The stable equilibrium state of the post-fault system is found by solving the following nonlinear algebraic equations (4.6) by the method of steepest Descent to a minimum[33].

$$\sum_{K=1}^{N} \sum_{\substack{J=1 \\ J\neq K}}^{N} \left[\frac{E_K E_J}{(x_d')_{KJ}} \sin(\delta_K-\delta_J) + \frac{E_J^2}{2} \left(\frac{1}{(x_q')_{KJ}} \;\; \frac{1}{(x_d')_{KJ}} \right) \sin 2(\delta_K-\delta_J) \right] - \sum_{K=1}^{N} P_K = 0 \qquad (4.6)$$

The equations (4.6) can also be represented in simple form, as

$$F_i(\delta_1,\delta_2,\ldots\ldots,\delta_n) - P_i = 0, \quad i = 1,2,\ldots,n \qquad (4.7)$$

Defining the function

$$\Phi = \sum_{i=1}^{n} (F_i - P_i)^2 \qquad (4.8)$$

which has a minimum at the solution of (4.6) and the minimum value of Φ is zero.

The Process of minimising is done by changing all the coordinates (δ_r) to $(\delta_r+d\delta_r)$ where $d\delta_r$ are given by

$$d\delta_r = - \frac{\sum_{r=1}^{n} \left[(\Phi_r)^2 \right] \Phi_r}{\sum_{r,s=1}^{n} \left[\Phi_{r,s} \Phi_r \Phi_s \right]} \qquad (4.9)$$

in which $\Phi_r = \frac{\partial\Phi}{\partial\delta_r}$, $\quad \Phi_{r,s} = \frac{\partial^2\Phi}{\partial\delta_r\partial\delta_s}$

and repeating the process starting with these coordinates as the origin until Φ is minimised.

As an example, the solutions of nonlinear algebraic equations

$$\sin(\delta_1-\delta_2) - 1 = 0$$

$$\sin(\delta_2-\delta_1) + 1 = 0$$

can be found with the aid of steepest Descent to a minimum method as follows.

As per definition

$$\Phi = \left[\sin(\delta_1-\delta_2)-1\right]^2 + \left[\sin(\delta_2-\delta_1)+1\right]^2$$

The partial derivatives of Φ are

$$\Phi_1 = \sin 2(\delta_1-\delta_2) - 2\,\mathrm{Cs}(\delta_1-\delta_2) - \sin 2(\delta_2-\delta_1) - 2\cos(\delta_2-\delta_1)$$

$$\Phi_{1,2} = 2\cos 2(\delta_1-\delta_2) + 2\sin(\delta_1-\delta_2) + 2\cos 2(\delta_2-\delta_1) - 2\sin(\delta_2-\delta_1)$$

$$\Phi_{1,2} = -2\cos 2(\delta_1-\delta_2) - 2\sin(\delta_1-\delta_2) - 2\cos 2(\delta_2-\delta_1) + 2\sin(\delta_2-\delta_1)$$

$$\Phi_2 = -\Phi_1$$

$$\Phi_{2,1} = \Phi_{1,2}$$

$$\Phi_{2,2} = \Phi_{1,1}$$

The initial values of δ_1 and δ_2 are assumed to be zero and substituted in $\Phi_1, \Phi_{1,1}, \Phi_{1,2}, \Phi_2, \Phi_{2,1}$ and $\Phi_{2,2}$ to get $-4, +4, -4, +4, -4$ and $+4$ respectively. The increments in δ_1 and δ_2 are given by

$$d\delta_1 = -\frac{(\Phi_1^2 + \Phi_2^2)\,\Phi_1}{\Phi_1\Phi_1\Phi_{1,1} + \Phi_1\Phi_2\Phi_{1,2} + \Phi_2\Phi_1\Phi_{2,1} + \Phi_2\Phi_2\Phi_{2,2}} \qquad (4.10)$$

$$d\delta_2 = -\frac{(\Phi_1^2 + \Phi_2^2)\,\Phi_2}{\Phi_1\Phi_1\Phi_{1,1} + \Phi_1\Phi_2\Phi_{1,2} + \Phi_2\Phi_1\Phi_{2,1} + \Phi_2\Phi_2\Phi_{2,2}} \qquad (4.11)$$

Substituting the numerical values in (4.10) and(4.11), the increments of the angles $d\delta_1$ and $d\delta_2$ are

$$d\delta_1 = + 0.5$$
$$d\delta_2 = - 0.5$$

The change in the coordinates are

δ_1 = starting value of δ_1 + increment in δ_1

$\delta_1 = \delta_1 + d\delta_1 = 0 + 0.5 = + 0.5$ rad $= + 28.648^\circ$

δ_2 = starting value of δ_2 + increment in δ_2

$\delta_2 = \delta_2 + 2\delta_2 = 0 - 0.5 = - 0.5$ rad $= - 28.648^\circ$

The value of Φ with these new values of δ_1 and δ_2 is

$\Phi = + 0.050263$

The process is repeated until Φ becomes zero taking the preceding ordinate as the origin. The values of δ's and Φ for each process are shown in Table I. After ten steepest descent, the values of δ_1, δ_2 and Φ are

$\delta_1 = + 45^\circ$

$\delta_2 = - 45^\circ$

$\Phi = 0$

TABLE II

The values of δ's and Φ at each steepest descent

δ_1 (degrees)	δ_2 (degrees)	Φ
0.000	-0.000	2.0000
28.648	-28.648	5.0263×10^{-2}
34.417	-34.417	9.1018×10^{-3}
38.027	-38.027	1.7373×10^{-3}
40.375	-40.375	3.3823×10^{-4}
41.923	-41.923	6.6394×10^{-5}
42.951	-42.951	1.3080×10^{-5}
43.634	-43.634	2.5818×10^{-6}
44.090	-44.090	5.1025×10^{-7}
44.530	-44.530	1.1030×10^{-7}
45.000	-45.000	0.0

The post-fault unstable equilibrium angles δ_i^u are found in the same way as that for the stable equilibrium. But, for the minimisation of Φ, the initial guess of δ_i for the machine, which is likely to go out of step first (most probably the machine which is connected to the faulted bus) may be assumed to be π radians and for the other machines, their respective pre-fault values.
The computer program for an IBM 1620 is given in Appendix II.

4.2.3 Limiting Value of V

The limiting value of V in the closed region R is given by

$$B = V(\delta_i^u, v_i^u)$$

$$v_i^u = 0 \quad \text{for } i = 1,2,\ldots,n$$

The region R is defined by V B . It can be shown that the surface given by (4.2) is closed for V B and open for V B. These surfaces completely span the region R. Hence, V is greater than zero in the region R, except at the equilibrium state (δ_i^s, v_i^s) where it vanishes. The surface given by V=B passes through the unstable equilibrium state closest to the steady state stable equilibrium state of the post-fault system. Thus, the region R includes only the equilibrium state under investigation. At this equilibrium sate time derivative of V is zero. This state is the only invariant set in R, and, hence, the largest.

4.2.4 Critical Switching Time

As stated, before, the region R defines all the initial conditions of the post-fault system for which it is asymplotically stable. In power system transient stability studies, the possible initial conditions for the post-fault system are along the trajectory of the faulted system.

Therefore, during the forward swing curve integration of the faulted system, the state at every instant of time is tested to determine whether or not it is inside the region R_s by checking the corresponding value of V against B which serves as an index for the region R. The time at which the value of V is just equal to B gives the critical switching angle and critical switching time.

The Rungekutta method of obtaining swing curves, for multimachine system is explained as follows.

The equations (4.1) can be written in a simplified form as

$$M_K \frac{d^2\delta_K}{dt^2} + \sum_{\substack{J=1 \\ J\neq K}}^{N} F(\delta_K, \delta_J, v_K) = P_K \qquad (4.12)$$

The equations (4.12) are converted into two first order differential equations as

$$\frac{d\delta_K}{dt} = v_K \qquad (4.13)$$

$$\frac{dv_K}{dt} = P_K - \sum_{\substack{J=1 \\ J\neq K}}^{N} F(\delta_K, \delta_J, v_K) \qquad (4.14)$$

The ordinate at the next interval can be obtained by Rungekutta formulae

$d\delta_{1K} = H(v_K)$, where H is the increment in time.

$$dv_{1K} = H\left[P_K - \sum_{\substack{J=1 \\ J\neq K}}^{N} F(\delta_K, \delta_J, v_K)\right]$$

$$v_{1K} = (v_K + \frac{1}{2} dv_{1K})$$

$$\delta_{1K} = (\delta_K + \frac{1}{2} d\,\delta_{1K})$$

$$d\delta_{2K} = H\,(v_{1K})$$

$$dv_{2K} = H\left[P_K - \sum_{\substack{J=1 \\ J\neq K}}^{N} F(\delta_{1K}, \delta_{1J}, v_{1K})\right]$$

$$v_{2K} = (v_K + \frac{1}{2} d\,v_{2K})$$

$$\delta_{2K} = (\delta_K + \frac{1}{2} d\,\delta_{2K})$$

$$d\delta_{3K} = H(v_{2K})$$

$$d\,v_{3K} = H\left[P_K - \sum_{\substack{J=1 \\ J\neq K}}^{N} F(\delta_{2K}, \delta_{2J}, v_{2K})\right]$$

$$v_{3K} = (v_K + d\,v_{3K})$$

$$\delta_{3K} = (\delta_K + d\,\delta_{3K})$$

$$d\delta_{4K} = H\,(v_{3K})$$

$$d\,v_{4K} = H\left[P_K - \sum_{\substack{J=1 \\ J\neq K}}^{N} F(\delta_{3K}, \delta_{3J}, v_{3K})\right]$$

The values at the next interval are

$$t = t + H$$

$$\delta_K = \delta_K + \frac{1}{6}\,(d\delta_{1K} + 2d\,\delta_{2K} + 2d\,\delta_{3K} + d\,\delta_{4K})$$

$$v_K = v_K + \frac{1}{6}\,(dv_{1K} + 2d\,v_{2K} + 2d\,v_{3K} + d\,v_{4K})$$

The process is repeated as many times as necessary. The entire procedure given above, has been programmed for an IBM 1620, and the program is given in Appendix III.

4.3 NUMERICAL EXAMPLES

In the following numerical examples, firstly an interesting symmetrical 3-machine system whose behaviour can be compared with that of a 2-machine system is considered, and secondly, a more general 3-machine system is studied.

Symmetrical 3-machine system:

The following are the constants assumed for a symmetrical 3-machine system shown in Fig 11.

$$M_1 = M_2 = M_3 = 1$$

$$E_1 = E_2 = E_3 = 1$$

$$P_1 = -P_3 = 1, \quad P_2 = 0$$

$$(Y_d)_{12} = (Y_d)_{21} = (Y_d)_{13} = (Y_d)_{31} = (Y_d)_{23} = (Y_d)_{32} = 1$$

$$(Y_q)_{12} = (Y_q)_{21} = (Y_q)_{13} = (Y_q)_{31} = (Y_q)_{23} = (Y_q)_{32} = 1/2$$

$$(T_{do}'')_1 = (T_{do}'')_2 = (T_{do}'') = 4 \text{ sec}$$

If the disturbances are introduced by changing only Y_{13}, the symmetry will be maintained, and $\delta_1 = -\delta_3$, $\delta_2 = 0$, $d\delta_2/dt = d^2\delta_2/dt^2 = 0$. Under these conditions equations (4.1) become

$$\left.\begin{aligned}
\frac{d^2\delta_1}{dt^2} &= 1 - \sin\delta_1 - \sin 2\delta_1 + \tfrac{1}{4}(\sin 2\delta_1 + \sin 4\delta_1) \\
&\qquad - 2(1-\cos 2\delta_1)\frac{d\delta_1}{dt} \\
0 &= 0 - \sin(-\delta_1) - \sin\delta_1 + \tfrac{1}{4}(\sin(-2\delta_1) + \sin 2\delta_1) \\
-\frac{d^2\delta_1}{dt^2} &= -1 - \sin(-\delta_1) - \sin(-2\delta_1) + \tfrac{1}{4}(\sin(-2\delta_1) + \sin(-4\delta_1)) \\
&\qquad - 2(1-\cos(-2\delta_1))\left(-\frac{d\delta_1}{dt}\right)
\end{aligned}\right\} \quad (4.16)$$

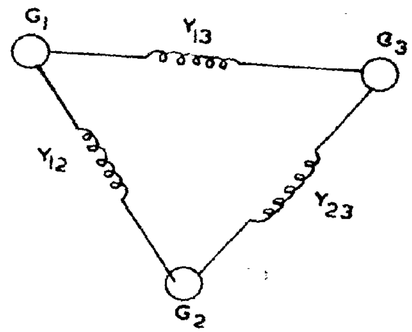


FIG. 11. THREE MACHINE SYSTEM

Using the relation Sin x = - Sin(-x), (4.16) becomes

$$\left.\begin{aligned} \frac{d^2\delta_1}{dt^2} &= 1-\sin\delta_1-\sin 2\delta_1+\tfrac{1}{4}(\sin 2\delta_1+\sin 4\delta_1)-2(1-\cos 2\delta_1)\frac{d\delta_1}{dt} \\ \frac{d^2\delta_1}{dt^2} &= 1-\sin\delta_1-\sin 2\delta_1+\tfrac{1}{4}(\sin 2\delta_1+\sin 4\delta_1)-2(1-\cos 2\delta_1)\frac{d\delta_1}{dt} \end{aligned}\right\} \quad (4.17)$$

The Liapunov function for the system might be written as

$$V = \frac{y^2}{2} + \int_{\delta^s}^{\delta^u} -(1-\sin\delta_1 - \sin 2\delta_1 + \tfrac{1}{4}(\sin 2\delta_1 + \sin 4\delta_1))\, d\delta \qquad (4.18)$$

The equation (4.18) should be compared with (3.47) for the 1-machine system. The presence of the quadraple angle term is caused by the nature of the system.

Now the faulted and post-fault systems are set up. In this case, the post-fault system happens to be the same as the pre-fault system, since the faulted line is first switched out and then reconnected after an interval of time. The faulted system is set up by reducing $(Y_d)_{13}$ from 1 to 1/4 and $(Y_q)_{13}$ from 1/2 to 1/8.

The stable and unstable equilibrium states of the post-fault system are $\delta^s = 31.8^\circ$ and $\delta^u = 90^\circ$, as obtained by setting up $d\delta_1/dt = d^2\delta_1/dt^2 = 0$ in (4.17). Substitution of these values of δ^s and δ^u in (4.18) gives the value of V = 0.476 for this ~~[illegible]~~ system. The critical switching time obtained by this method is 1.465 It is evident that the symmetrical 3-machine system is much like that of the 1-machine system.

3-Machine System:

The general 3-machine system to be studied is shown in Fig 11. The machine details, internal bus voltages for pre-fault condition, equivalent admittances (subseptances) for pre-fault, faulted and post-fault conditions are given in Tables III-VI. In this case also the post-fault system happens to be the same as the pre-fault system, since the faulted line is first switched out and then reconnected after an interval of time. The stable and unstable equilibrium states for the post-fault system are given in Table VII.

The critical switching time ob ained by this method of Liapunov is t_c = 0.40 sec. When the fault was cleared at 0.40 sec the system was stable, when cleared at 0.42 sec, the system was found to be unstable as shown in Fig 12(a) and 12(b).

Table III

Machine Details

Generator	M P.U.	x_d' P.U.	x_q' P.U.	T_{do}'' P.U.
1	0.02	1.000	2.000	1.000
2	0.002	0.500	1.000	0.500
3	0.03	0.400	0.800	0.040

Table IV

Internal Bus Voltages for Pre-fault System

Gen	E P.U.	δ Radians	P_i (input) P.U.
1	1.0410	0.48852	0.500
2	1.1900	-0.074018	0.0
3	1.0710	-0.41450	-0.500

Table V

Matrix for Pre-fault and post-fault systems

	Y_d Matrix			Y_d Matrix		
	1	2	3	1	2	3
1	1.000	0.500	0.455	0.500	0.286	0.278
2	0.500	2.000	0.835	0.286	1.000	0.476
3	0.455	0.835	2.500	0.278	0.476	1.250

Table VI

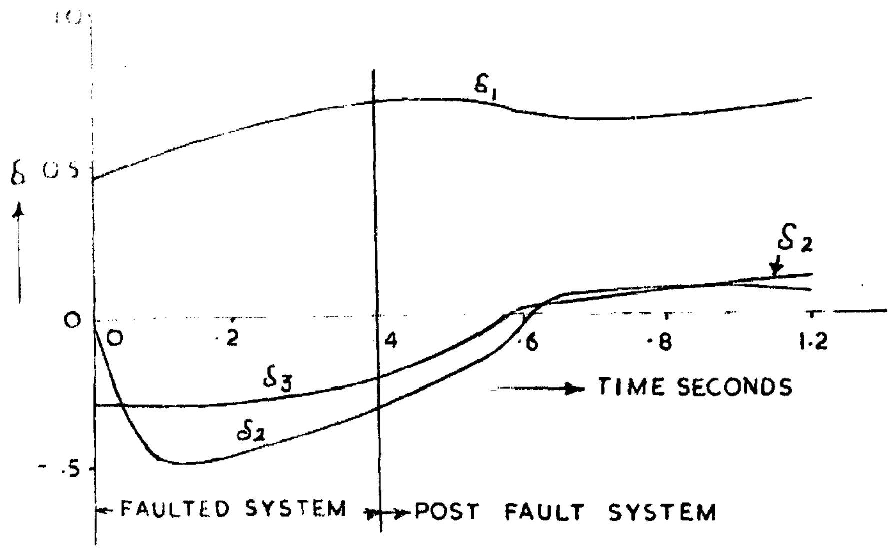
Matrix for faulted system

	Y_d Matrix			Y_q Matrix		
	1	2	3	1	2	3
1	1.000	0.500	0.00	0.500	0.286	0.00
2	0.500	2.000	0.00	0.286	1.000	0.00
3	0.000	0.000	2.50	0.000	0.000	1.25

Table VII

Stable and Unstable Equilibrium Values

Angles	Stable Equilibrium values in radians	Unstable Equilibrium values in radians
δ_1	0.48852	2.737394
δ_2	-0.074018	0.343136
δ_3	-0.4145	-0.46904



G. 12a STABLE FAULT CLEARED AT 0.4 SEC FOR THE 3-MACHINE SYSTEM.

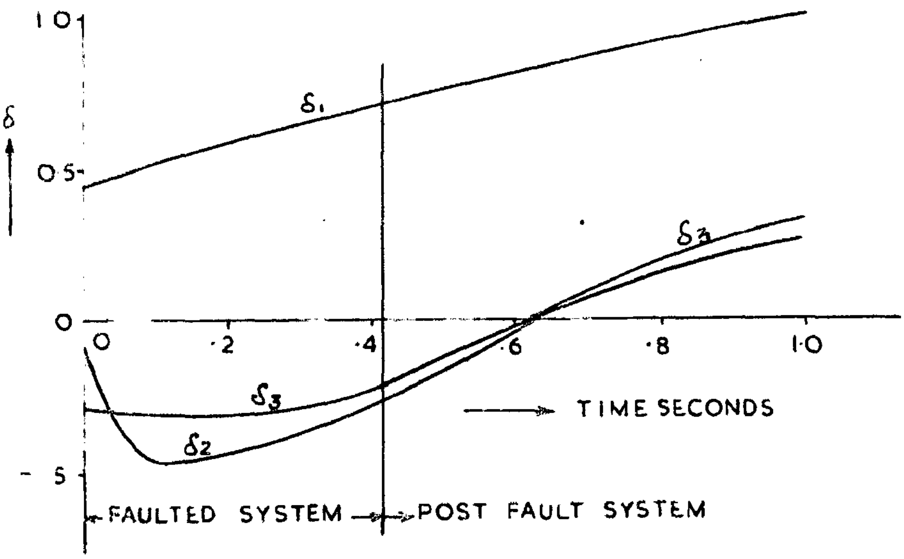


FIG.12.b. UNSTABLE FAULT CLEARED AT 0.42 SEC FOR THE 3 MACHINE SYSTEM.

As an extension to the above problem the four machine system was also attempted. However, in this study, much difficulty was experienced in finding out the equilibrium states with the method of steepest descent. It was found that this method is not convergent for a four machine system. If one can find out the equilibrium values it appears that the four and multi-machine systems can also be studied on the above similar lines.

4.4 CONCLUSIONS

Liapunov's second method has been used to determine the stability of 3-machine system. The advantages of this method over other methods are:

1. Exact determination of stability or instability.
2. Determination of critical switching time.

There is a vast scope to study the power system stability by Liapunov's method including saturation, governor action, regulator action, armature and line resistances, amortisseur damping etc. Further investigation is necessary to develop mathematical equations describi g the system with all these factors taken into account, and to determine the equilibrium values of the system.

CHAPTER 5

RESUMÉ

5.1 SUMMARY

A review has been presented in Chapter 2, on stability of synchronous Machines in Power systems. To provide a unified picture of the whole, various techniques to determine the transient stability and critical switching time have been surveyed. A basic problem in power system stability is the exact determination of the critical switching time by which the system can be restored to its normal operation by switching at the critical time.

In contribution towards determining the actual critical switching time by mathematical means, the Liapunov's second method has been used. The Phase plane technique has also been used, in comparison with Liapunov's method to determine the stability of a single machine system.

In Chapter 3, the power angle equations of a synchronous machine connected to an infinite bus have been derived, starting from voltage and flux linkage equations considering (1) saliency and field damping and (2) saliency, field damping and regulator action. Lesser number of assumptions, namely, (1) constant input power, (2) omission of amortisseur damping and (3) omission of armature and line resistances have been made in deriving the equations so as to represent the actual system to a nearer approximation.

Prior to the Liapunov's method, the phase plane technique to determine stability from the nature of its phase trajectories and the numerical method of obtaining phase trajectories by Rungekutta method have been explained.

Next, Liapunov's method has been used to establish the region of asymptotic stability and to determine the critical switching time. Theory and method of application of Liapunov's process and method of construction of Liapunov functions have been explained. The application of the Second Liapunov process consists of (1) consturction of Liapunov function based on energy concept (2) determination of stable and unstable equilibrium values (3) determination of the limiting value of V and (4) forward step by step integration (swing curves) of the disturbed system to find the critical switching time.

Finally, numerical examples have been illustrated.

In Chapter 4, a mathematical model for a multimachine system has been developed and the Liapunov function has been constructed. The method of application of Second Liapunov's process, the method of determining equilibrium values by steepest descent method, determination of limiting value of Liapunov function and forward integration (swing curve) of the faulted system by Rungekutta method to obtain time solution and critical switching time have clearly been explained.

Finally, a numerical example has been illustrated.

5.2 CONCLUSIONS

Phase plane and Liapunov's methods have been successfully applied to determine the stability of a single machine system. An endeavour has been made to extend the Liapunov's method to multimachine system. The advantages of Liapunov's direct method over the phase plane and other methods are

1. Easy determination of stability or instability.
2. Exact determination of the index of the region of stability and critical switching time.
3. Extension of the method to multimachine system.

There is an immense scope to analyse the stability problem by the Second Liapunov's process including saturation, governor action, amortisseur damping and armature and line resistances. A thorough and successful investigation is necessary to develop a mathematical model describing the system dynamics to include all the above factors, to evaluate equilibrium values easily and to construct better and easier Liapunov functions.

APPENDIX -1

SOURCE PROGRAM FOR AN IBM 1620

```
C  RUNGE-KUTTA FOURTHORDER METHOD FOR THE SYSTEM WITHOUT REGULATORS Z
   READ5,H,TLAST,E,A,B,C,T,XX,VV
   DD=XX*3.1415926/180.
   PUNCH 4
   AKE=+0.5*0.02*VV*VV
   PE=+0.937-0.832*COSF(DD)-0.105*COSF(2.*DD)
   VF=AKE+PE
 1 PUNCH3,T,XX,VV,VF
   F=E-A*SINF(DD)-B*SINF(2.*DD)-C*(1.-COSF(2.*DD))*VV
   DELD1=H*VV
   DELV1=H*F
   D=DD+DELD1/2.
   V=VV+DELV1/2.
   F=E-A*SINF(D)-B*SINF(2.*D)-C*(1.-COSF(2.*D))*V
   DELD2=H*V
   DELV2=H*F
   D=DD+DELD2/2.
   V=VV+DELV2/2.
   F=E-A*SINF(D)-B*SINF(2.*D)-C*(1.-COSF(2.*D))*V
   DELD3=H*V
   DELV3=H*F
   D=DD+DELD3
   V=VV+DELV3
   F=E-A*SINF(D)-B*SINF(2.*D)-C*(1.-COSF(2.*D))*V
   DELD4=H*V
   DELV4=H*F
   T=T+H
   DD=DD+(DELD1+2.*DELD2+2.*DELD3+DELD4)/6.
   VV=VV+(DELV1+2.*DELV2+2.*DELV3+DELV4)/6.
   XX=DD*180./3.1415926
   AKE=+0.5*0.02*VV*VV
   PE=+0.937-0.832*COSF(DD)-0.105*COSF(2.*DD)
   VF=AKE+PE
   IF(T-TLAST) 1,1,2
 2 STOP
 3 FORMAT (F7.2,4F14.3)
 4 FORMAT(3X,4HTIME,8X,5HDELTA,7X,8HVELOCITY,6X,9HVFUNCTION//)
 5 FORMAT (6F12.6)
   END
```

APPENDIX -2

SOURCE PROGRAM FOR AN IBM 1620

NOMENCLATURE

X= δ_1, Y= δ_2, Z= δ_3, W= δ_4 , G = ϕ , F1 = ϕ_1 , F11 = $\phi_{1,1}$ ....

P1,P2,P3,P4 =INPUT POWERS

```
C  C  STEEPEST DESCENT TO A MINIMUM METHOD FOR FOUR MACHINE SYSTEM Z
      READ4,P1,P2,P3,P4
      READ5,A1,A2,A3,A4,A5,A6
      READ5,B1,B2,B3,B4,B5,B6
      READ5,C1,C2,C3,C4,C5,C6
      READ5,D1,D2,D3,D4,D5,D6
    1 READ4,X,Y,Z,W
    2 E1=P1-A1*SINF(X-Y)-A2*SINF(X-Z)-A3*SINF(X-W)
      E2=-A4*SINF(2.*(X-Y))-A5*SINF(2.*(X-Z))-A6*SINF(2.*(X-W))
      G1=E1+E2
      E3=P2-B1*SINF(Y-X)-B2*SINF(Y-Z)-B3*SINF(Y-W)
      E4=-B4*SINF(2.*(Y-X))-B5*SINF(2.*(Y-Z))-B6*SINF(2.*(Y-W))
      G2=E3+E4
      E5=P3-C1*SINF(Z-X)-C2*SINF(Z-Y)-C3*SINF(Z-W)
      E6=-C4*SINF(2.*(Z-X))-C5*SINF(2.*(Z-Y))-C6*SINF(2.*(Z-W))
      G3=E5+E6
      E7=P4-D1*SINF(W-X)-D2*SINF(W-Y)-D3*SINF(W-Z)
      E8=-D4*SINF(2.*(W-X))-D5*SINF(2.*(W-Y))-D6*SINF(2.*(W-Z))
      G4=E7+E8
      G=G1*G1+G2*G2+G3*G3+G4*G4
      PUNCH6,X,Y,Z,W,G
      IF(ABSF(G)-0.001)1,1,3
    3 H1=-A1*COSF(X-Y)-A2*COSF(X-Z)-A3*COSF(X-W)
      H2=2.*(-A4*COSF(2.*(X-Y))-A5*COSF(2.*(X-Z))-A6*COSF(2.*(X-W)))
      H3=+B1*COSF(Y-X)+2.*B4*COSF(2.*(Y-X))
      H4=+C1*COSF(Z-X)+2.*C4*COSF(2.*(Z-X))
      H5=+D1*COSF(W-X)+2.*D4*COSF(2.*(W-X))
  100 F1=2.*(G1*(H1+H2)+G2*H3+G3*H4+G4*H5)
      H6=+A1*COSF(X-Y)+2.*A4*COSF(2.*(X-Y))
      H7=-B1*COSF(Y-X)-B2*COSF(Y-Z)-B3*COSF(Y-W)
      H8=2.*(-B4*COSF(2.*(Y-X))-B5*COSF(2.*(Y-Z))-B6*COSF(2.*(Y-W)))
      H9=+C2*COSF(Z-Y)+2.*C5*COSF(2.*(Z-Y))
      H10=+D2*COSF(W-Y)+2.*D5*COSF(2.*(W-Y))
  110 F2=2.*(G1*H6+G2*(H7+H8)+G3*H9+G4*H10)
      H11=+A2*COSF(X-Z)+2.*A5*COSF(2.*(X-Z))
      H12=+B2*COSF(Y-Z)+2.*B5*COSF(2.*(Y-Z))
      H13=-C1*COSF(Z-X)-C2*COSF(Z-Y)-C3*COSF(Z-W)
```

```
      H14=2.*(-C4*COSF(2.*(Z-X))-C5*COSF(2.*(Z-Y))-C6*COSF(2.*(Z-W)))
      H15=+D3*COSF(W-Z)+2.*D6*COSF(2.*(W-Z))
  120 F3=2.*(G1*H11+G2*H12+G3*(H13+H14)+G4*H15)
      H16=+A3*COSF(X-W)+2.*A6*COSF(2.*(X-W))
      H17=+B3*COSF(Y-W)+2.*B6*COSF(2.*(Y-W))
      H18=+C3*COSF(Z-W)+2.*C6*COSF(2.*(Z-W))
      H19=-D1*COSF(W-X)-D2*COSF(W-Y)-D3*COSF(W-Z)
      H20=2.*(-D4*COSF(2.*(W-X))-D5*COSF(2.*(W-Y))-D6*COSF(2.*(W-Z)))
  130 F4=2.*(G1*H16+G2*H17+G3*H18+G4*(H19+H20))
      Q1=+A1*SINF(X-Y)+A2*SINF(X-Z)+A3*SINF(X-W)
      Q8=4.*(+A4*SINF(2.*(X-Y))+A5*SINF(2.*(X-Z))+A6*SINF(2.*(X-W)))
      Q9=Q1+Q8
      Q2=+B1*SINF(Y-X)+4.*B4*SINF(2.*(Y-X))
      Q3=+C1*SINF(Z-X)+4.*C4*SINF(2.*(Z-X))
      Q4=+D1*SINF(W-X)+4.*D4*SINF(2.*(W-X))
      H50=(H1+H2)*(H1+H2)
  140 F11=2.*(G1*Q9+H50+G2*Q2+H3*H3+G3*Q3+H4*H4+G4*Q4+H5*H5)
      Q5=-A1*SINF(X-Y)-4.*A4*SINF(2.*(X-Y))
  150 F12=2.*(G1*Q5+(H1+H2)*H6-G2*Q2+H3*(H7+H8)+H4*H9+H5*H10)
      R1=-A2*SINF(X-Z)-4.*A5*SINF(2.*(X-Z))
  160 F13=2.*(G1*R1+(H1+H2)*H11+H3*H12-G3*Q3+H4*(H13+H14)+H5*H15)
      R8=-A3*SINF(X-W)-4.*A6*SINF(2.*(X-W))
  170 F14=2.*(G1*R8+(H1+H2)*H16+H3*H17+H4*H18-G4*Q4+H5*(H19+H20))
      F21=F12
      S1=+B1*SINF(Y-X)+B2*SINF(Y-Z)+B3*SINF(Y-W)
      S2=4.*(+B4*SINF(2.*(Y-X))+B5*SINF(2.*(Y-Z))+B6*SINF(2.*(Y-W)))
      S3=S1+S2
      S4=+C2*SINF(Z-Y)+4.*C5*SINF(2.*(Z-Y))
      S5=(H7+H8)*(H7+H8)
      S6=+D2*SINF(W-Y)+4.*D5*SINF(2.*(W-Y))
  180 F22=2.*(-G1*Q5+H6*H6+G2*S3+S5+G3*S4+H9*H9+G4*S6+H10*H10)
      S7=-B2*SINF(Y-Z)-4.*B5*SINF(2.*(Y-Z))
  190 F23=2.*(H6*H11+G2*S7+(H7+H8)*H12-G3*S4+H9*(H13+H14)+H10*H15)
      S9=-B3*SINF(Y-W)-4.*B6*SINF(2.*(Y-W))
  200 F24=2.*(+H6*H16+G2*S9+(H7+H8)*H17+H9*H18-G4*S6+H10*(H19+H20))
      F31=F13
  210 F32=F23
      T1=+C1*SINF(Z-X)+C2*SINF(Z-Y)+C3*SINF(Z-W)
      T2=4.*(C4*SINF(2.*(Z-X))+C5*SINF(2.*(Z-Y))+C6*SINF(2.*(Z-W)))
      T3=T1+T2
      T4=+D3*SINF(W-Z)+4.*D6*SINF(2.*(W-Z))
      T5=(H13+H14)*(H13+H14)
```

```
220 F33=2.*(-G1*R1+H11*H11-G2*S7+H12*H12+G3*T3+T5+G4*T4+H15*H15)
    T6=-C3*SINF(Z-W)-4.*C6*SINF(2.*(Z-W))
    F34=2.*(+H11*H16+H12*H17+G3*T6+(H13+H14)*H18-G4*T4+H15*(H19+H20))
    F41=F14
    F42=F24
230 F43=F34
    T7=+D1*SINF(W-X)+D2*SINF(W-Y)+D3*SINF(W-Z)
    T8=4.*(+D4*SINF(2.*(W-X))+D5*SINF(2.*(W-Y))+D6*SINF(2.*(W-Z)))
    T9=T8+T7
    T10=(H19+H20)*(H19+H20)
240 F44=2.*(-G1*R8+H16*H16-G2*S9+H17*H17-G3*T6+H18*H18+G4*T9+T10)
    F=F1*F1+F2*F2+F3*F3+F4*F4
    U1=F1*F1*F11+2.*F1*F2*F12+2.*F1*F3*F13+2.*F1*F4*F14+F2*F2*F22
    U2=2.*F2*F3*F23+2.*F2*F4*F24+F3*F3*F33+2.*F3*F4*F34+F4*F4*F44
    U=U1+U2
250 S=F/U
    EX=-S*F1
    EY=-S*F2
    EZ=-S*F3
    EW=-S*F4
    X=X+EX
    Y=Y+EY
    Z=Z+EZ
    W=W+EW
    GOTO 2
  4 FORMAT (4F12.6)
  5 FORMAT (6F12.6)
  6 FORMAT (5F12.6)
    STOP
    END
```

APPENDIX -3
SOURCE PROGRAM FOR AN IBM 1620

NOMENCLATURE

T	= t
D(K)	= δ_K
V(K)	= v_K
DS(K)	= δ^s_K
P(K)	= P_K
E(K)	= E_K
A(K)	= M_K
TD(K)	= T''_{d_0}
Y1	= (Y_d)
Y2	= (Y_q)
Y3	= (Y_d)
Y4	= (Y_q)

```
C  C  PROGRAM FOR MULTIMACHINE SYSTEM BY RUNGE-KUTTA METHOD Z
  600 DIMENSION COEF(4)
      DIMENSION D(4)
      DIMENSION V(4)
      DIMENSION DS(4)
      DIMENSION P(4)
      DIMENSION E(4)
      DIMENSION A(4)
      DIMENSION TD(4)
      DIMENSION Y1(4,4)
      DIMENSION Y2(4,4)
      DIMENSION Y3(4,4)
      DIMENSION Y4(4,4)
      DIMENSION DB(4)
      DIMENSION VB(4)
      DIMENSION SUMD(4)
      DIMENSION SUMV(4)
      DIMENSION DD(4)
      DIMENSION DV(4)
  700 COEF(1)=1./6.
      COEF(2)=2./6.
      COEF(3)=COEF(2)
      COEF(4)=COEF(1)
      READ1,N,H,TLAST
      READ2,T,(D(K),K=1,N),(V(K),K=1,N)
      READ3,(DS(K),K=1,N)
      READ3,(P(K),K=1,N)
      READ3,(E(K),K=1,N)
      READ3,(A(K),K=1,N)
      READ3,(TD(K),K=1,N)
      READ3,((Y1(K,J),J=1,N),K=1,N)
      READ3,((Y2(K,J),J=1,N),K=1,N)
      READ3,((Y3(K,J),J=1,N),K=1,N)
      READ3,((Y4(K,J),J=1,N),K=1,N)
```

```
 50 SUM=0.
    DO 200 K=1,N
200 SUM=SUM+0.5*A(K)*V(K)*V(K)-P(K)*(D(K)-DS(K))
    M=N-1
    DO 210 K=1,M
    I=K+1
    DO 210 J=I,N
    G1=+(E(K)*E(J)*Y3(K,J))*(COSF(D(K)-D(J))-COSF(DS(K)-DS(J)))
    G2=E(J)*E(J)*0.25*(Y4(K,J)-Y3(K,J))
    G3=G2*(COSF(2.*(D(K)-D(J)))-COSF(2.*(DS(K)-DS(J))))
    SUM=SUM-(G1+G3)
210 CONTINUE
    DO 110K=1,N
    DB(K)=D(K)
    SUMD(K)=D(K)
    SUMV(K)=V(K)
110 VB(K)=V(K)
    DO120KK=1,4
    DO100K=1,N
    DD(K)=H*V(K)
    SUMA=0.
    DO 90 J=1,N
    IF(J-K)400,90,400
400 G4=(E(K)*E(J)*Y1(K,J))*SINF(DB(K)-DB(J))
    G5=E(J)*E(J)*0.25*(Y2(K,J)-Y1(K,J))
    G6=2.*G5*SINF(2.*(DB(K)-DB(J)))
    G7=(E(J)*E(J)*TD(K)*0.5*Y1(K,J))*(1.-COSF(2.*DB(K)))*VB(K)
    SUMA=SUMA+(G4+G6+G7)
 90 CONTINUE
    DV(K)=H*(P(K)-SUMA)/A(K)
    SUMD(K)=SUMD(K)+COEF(KK)*DD(K)
100 SUMV(K)=SUMV(K)+COEF(KK)*DV(K)
    IF(KK-4)70,120,120
 70 DO 80 K=1,N
    IF(KK-3)30,60,30
 60 VB(K)=V(K)+DV(K)
    DB(K)=D(K)+DD(K)
    GO TO 80
 30 VB(K)=V(K)+0.5*DV(K)
    DB(K)=D(K)+0.5*DD(K)
 80 CONTINUE
120 CONTINUE
    PUNCH2,T,(SUMD(K),K=1,N),(SUMV(K),K=1,N),SUM
    T=T+H
    DO 40 K=1,N
    DB(K)=SUMD(K)
    D(K)=SUMD(K)
    VB(K)=SUMV(K)
 40 V(K)=SUMV(K)
    GO TO 50
  1 FORMAT(I2,2F10.2)
  2 FORMAT(10F7.3)
  3 FORMAT(4F12.4)
    STOP
    END
```

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