UNBALANCED OPERATION OF POLYPHASE SOLID ROTOR INDUCTION MOTOR

A Dissertation
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By
K. SANKARAN

DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE

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CERTIFICATE

Certified that the dissertation entitled "UNBALANCED OPERATION OF POLYPHASE SOLID ROTOR INDUCTION MOTOR" which is being submitted by Sri K. SANKARAN in partial fulfilment for the award of the degree of MASTER OF ENGINEERING in ADVANCED ELECTRICAL MACHINES of the University of Roorkee, Roorkee, is a record of students own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of 7 months from Jan., 1968 to July, 1968 for preparing this dissertation for Master of Engineering at the University.

P. Mukhopadhyay
(P. MUKHOPADHYAY)
PROFESSOR,
Electrical Engg. Deptt.,
University of Roorkee
Roorkee.

Dated: Sept: 18 ,1968.

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(K. SANKARAN)

\+ Professor of Electrical Engineering Department, University of Roorkee, Roorkee.

++ Professor and Head of Electrical Engineering Department, University of Roorkee.

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PART A

UNBALANCED OPERATION OF POLYPHASE SOLID ROTOR INDUCTION MOTOR

-:SYNOPSIS:-

In the present dissertation work, the unbalanced operation of Polyphase Solid rotor Induction Motor has been studied utilizing electromagnetic theory and Symmetrical Component approach. As a preliminary step, starting from Maxwell's equations, the impedance of the solid rotor at various slips for both positive sequence equivalent circuit as well as negative sequence equivalent circuit are found out. A critical review, on unbalanced operation of Induction Motors, in general, has been presented. Later, using the derived rotor parameters of the equivalent circuits, the operation of a particular Solid rotor Induction Motor is studied especially with reference to the additional losses incurred due, to the negative sequence currents. It has also been shown how the different phases are loaded when various percentages of negative sequence voltages are present. The operation of the motor while one of stator phases is open, is also investigated. In the end, heating effects are studied and an attempt is made to determine approximately the variation of derating factor of Solid rotor Induction Motor.

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NOMENCLATURE

A,B,C,D	Arbitrary constants
B_x	Tangential component of flux density (Wb/m²)
B_y	Normal component of flux density (Wb/m²)
E	Volts/Phase
E_z	Axial Component
g	Air gap length
H	Magnetic field intensity
H_{x_I}, $H_{x_{II}}$	Tangential components of magnetic field intensity of air gap and rotor respectively.
I	Stator current per phase.
I_1	Positive sequence current
I_2	Negative sequence current
j	$\sqrt{-1}$
k	A ratio $\frac{I_2}{I_1}$
K_{dp1}	Pitch and breadth factor
L	Core length
m	Number of phases
p	Number of pairs of poles
P,Q	Ratios
R	Resistance
s	Fractional slip

t	Time in seconds
T'_{ph} or T_{eff}	Effective turns per phase
T	Torque
V_{a1}, V_{b1}, V_{c1}	Positive sequence system of voltages
V_{a2}, V_{b2}, V_{c2}	Negative sequence system of voltages
V_{ab}, V_{bc}, V_{ca}	Line voltages
X	Reactance
X_m	Magnetising reactance
Z	Air gap impedance
r	Used as a suffix for rotor parameters
s	Used as a suffix for stator parameters
1	Used as a suffix for positive sequence
2	Used as a suffix for negative sequence

GREEK SYMBOLS:

α	Angle between I_2 and I_1, as an abbreviation.
β	π/τ
γ	Abbreviation
θ_1 and θ_2	Arbitrary notation(real numbers)
	Resistivity of rotor material $=15X10^{-8}$ ohm-meter
τ	Pole pitch
w	Radian frequency (radians/sec.)
μ_o	Permeability of free space $-4\pi X10^{-7}$ Wb/amp-m
μ_r	Relative permeability
η	$\gamma/\beta\mu_r$ an abbreviation

-:INTRODUCTION:-

General Considerations and Scope of the Problem:

The problem of unbalance is not new to the power engineer. However, the study of an electrical machine on unbalanced conditions has gained its importance as the supply systems grew more and more complex and further with increase of single phase loadings. Hence, this necessitates the engineer to study the operation of the given motor on unbalanced conditions. We find that much study and investigation have been done in case of normal Induction Motors (wound type and cage rotor type). Out of the different types of Induction Motors Solid rotor Induction Motor is one. Though it is used rarely for continuous operation due to low torque and poor efficiency still, the solid rotor machine is used for special purposes such as " pony motors " for starting rotary Convertors and Synchronous motors. Also, the machine has its speciality with its high starting torque, as a servomotor. Further, recently it has also been proved that the machine has characteristics particularly suitable for solid-state power controls.

Probably due to the above mentioned reasons, much analysis is being attempted in this direction.

As a part of the problem, it is proposed here to study the unbalanced operation of the Solid rotor Induction Motor which, it is hoped, may lead to the better understanding of the usefulness of the machine.

Brief Description of the Problem:

The unbalanced applied voltages to stator terminals of Polyphase Solid rotor Induction Motor will adversely affect the operation due to the increase in motor losses associated with eddy losses and also due to the presence of negative sequence currents. However, at the outset, a facing problem in the analysis of the Induction Motor is to determine the phenomena connected with penetration of magnetic fluxes in the solid iron and distribution of cufrent. This has a peculiar characterestic arising from the fact that the rotor impedance values alter according to the slip and their relationship to the slip is to be determined before proceeding further. Once the impedance value is known, then a knowledge of variation of losses with varying unbalanced might prove useful. Further, it is also of interest, to know whether Solid rotor Induction Motor can continue to run if one of the phases is opened while running.

Other common and important aspect of the operation of Solid rotor Induction motor is from the view-point of its heating and rating. These, of course, depend upon various factors. The machine while producing its output must in general, meet its specified performance. Another basic requirement is that the life of machine shall not be unduly shortened due to overheating. Hence, to know all these factors, variation of losses, temperature-rise indication from these losses and the simultaneous loading of all the three phases under unbalanced conditions are the major factors to know the new rating of the machine.

So, in the present work, the following cases of unbalances on the operation of Polyphase Solid rotor Induction Motor have been attempted using electromagnetic theory and Symmetrical Component approach:

i) Unbalanced supply voltages to the stator,

ii) One phase of the supply voltage to the stator being open,

iii) Double unbalance,

iv) Heating and derating.

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CHAPTER-I

ELECTROMAGNETIC EQUATIONS AND SOLUTIONS:

1.1 Introduction:

In order to find the operation of Solid rotor Induction machine under unbalanced conditions, initially the parameters of the stator and rotor are to be known. Stator parameters can be known from the usual methods whereas it is not possible to know the parameters of the rotor which is simply a solid.

The object of this chapter is to find out the equivalent circuit parameters of the Solid rotor machine under consideration for both positive and negative sequence input voltages. However, the rotor being solid one it does not have clearly defined circuits. The resistance and reactance of the rotor are to be attributed to circuits spread over the entire volume of the rotor. In such a case, we can not follow the method of lumped circuit parameter approach since this fails to take into account the actual electromagnetic phenomena involved within the rotor medium of the Solid rotor Induction Motor. Obviously the most direct approach to such an eddy current type of machine is to formulate a boundary value problem based on Maxwell's electromagnetic equations and solve them to obtain flux densities

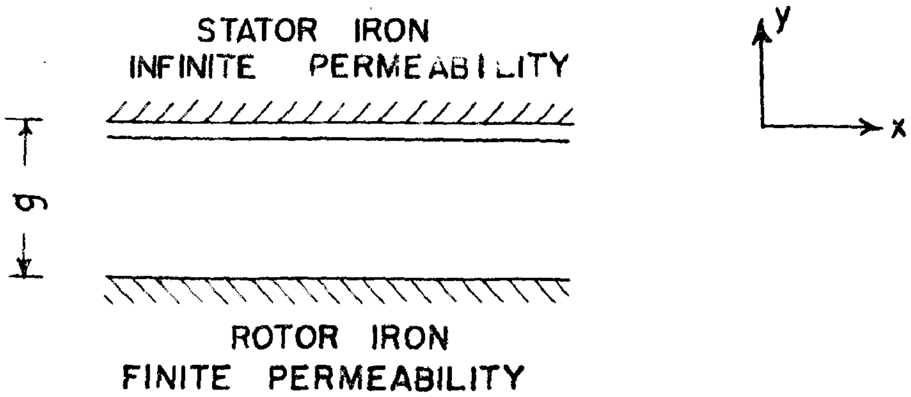

FIG.1. ELECTROMAGNETIC MACHINE MODEL
(RECTANGULAR CO-ORDINATES)

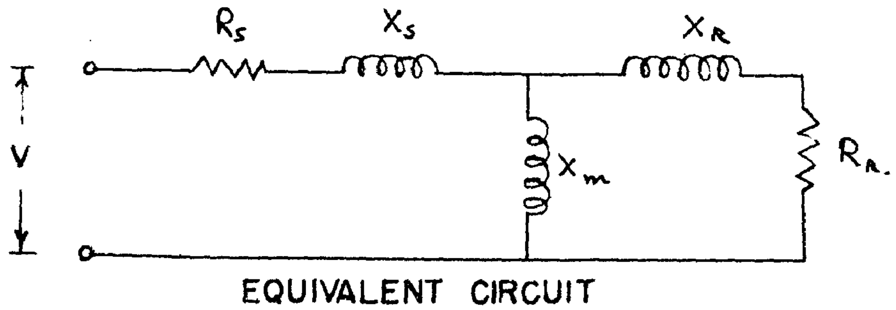

FIG.2.

and therefrom to get other parameters of the rotor. To analyse this physical condition of the problem it is to be viewed from the point of mathematics to enable us to apply the mathematical principles easily, simultaneously representing the actual machine model without loss of main physical features of the machine. Here, the analysis is attempted with two types of machine models under rectangular co-ordinates.

1.2 Assumptions:

i) The curvature of the airgap surface is negligible.

ii) Stator and rotor are of infinite length.

iii) Rotor has an infinite radial depth.

iv) The rotor region is considered to be composed of a linear, isotropic, homogeneous medium and a good conductor.

v) Infinite values for stator resistivity and permeability.

vi) Hysteresis effects are negligible.

vii) Displacement currents are negligible.

1.2.1 Analysis:

A machine model as shown in figure 1 is considered. Here, it is further assumed that the airgap is very small.

At the rotor surface,

$$H_y = Re\left\{(H_o \; e^{j\,(\beta x - w_o t)}\right\}$$

The electromagnetic equations which govern the field in a rotating machine are basically dependent upon Maxwell's Equations. For the rotor body, the Maxwell's Equations are:

$$\text{Curl } H = i \tag{1}$$

$$\text{Curl } E = -\frac{\partial B}{\partial t} \tag{2}$$

$$\text{Div } B = 0 \tag{3}$$

Also, as for the assumption (4) if the conductivity as well as the permeability of the rotor material are constant, then we have,

$$E = \rho i \tag{4}$$

$$B = \mu H \tag{5}$$

The above relations are sufficient to reduce the field equations to only one containing any one of the variables B, H, E, i.

From equation No.1 for two dimensional fields

H_x, H_y exist, $H_z = 0$

i_x and i_y do not exist, i_z exists.

Also, since H_x and H_y do not vary with respect to z, it can be found as:

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i_z \qquad (6)$$

and from Divergence Equation No.3

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad (7)$$

Since, electromotive force only exists in axial direction E_z is present and $E_x = E_y = 0$. So, from Curl Equation No.2 it can be obtained as :

$$\frac{\partial E_z}{\partial y} = - \frac{\partial B_x}{\partial t} \qquad (2)_a$$

$$\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t} \qquad (2)_b$$

Thus, the space variation of E_z has been correlated to time coordinates. Now from (6) and (7) with some manipulation it is obtained as :

$$\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} = \frac{\mu}{\rho} \cdot \frac{\partial B_y}{\partial t} \qquad (8)$$

The above is the differential equation of which left hand side shows the variation of flux density component B_y with respect to space coordinates being equal to variation of B_y with respect to time co-ordinate.

So B_y should be a f (x,y,t).

The complete solution of the differential Equation No.8 is as in Appendix No.1.

i.e. $$B_y = Re\left[\left\{\mu H_o e^{-\gamma y}\right\} e^{j(\beta x - sw_o t)}\right] \tag{10}$$

Also, from divergence Equation

$$B_x = Re\left[\left\{\frac{\mu H_o \gamma}{j\beta} e^{-\gamma y}\right\} e^{j(\beta x - sw_o t)}\right] \tag{11}$$

Now, to calculate the current density i_z Curl Equation $XE = -\frac{\partial B}{\partial t}$ is used.

that is, $\frac{\partial E_z}{\partial x} = \frac{\partial B_y}{\partial t}$

Therefore, $E_z = \int -\frac{\partial B_y}{\partial t} . \partial x$

which means that $i_z = \frac{1}{\rho} \int \frac{\partial B_y}{\partial t} . \partial x$

However, from equation No.10

$$B_y = Re\left[\left\{\mu H_o e^{-\gamma y}\right\} e^{j(\beta x - sw_o t)}\right]$$

or $$\frac{\partial B_y}{\partial t} = Re\left[\left\{-j\mu H_o sw_o e^{-\gamma y}\right\} e^{j(\beta x - sw_o t)}\right] = \frac{\partial E_z}{\partial x}$$

by equation$(2)_b$

Hence $E_z = Re\left[\left\{\frac{-j\mu H_o sw_o}{j\beta} e^{-\gamma y}\right\} e^{j(\beta x - sw_o t)}\right]$

Therefore, $i_z = Re\left[\left\{\frac{-\mu H_o sw_o}{\beta} e^{-\gamma y}\right\} e^{j(\beta x - sw_o t)}\right]$ (12)

Now, Ampere turns $= \frac{3}{2} \cdot \frac{I_{ph} \cdot T_{eff}}{2p}$

so that $I_{ph} = \frac{4}{3} \frac{p(AT)}{T_{eff}}$ (13)a

Also, one can write $I_{ph} = \int_0^{\infty}\int_0^{\chi} i_z \cdot dx \cdot dy$

Hence, current per phase

$$= \int_0^{\infty}\int_0^{\chi}\left\{- \frac{4p\ \mu H_o sw_o}{3\ T_{eff}\ \beta} e^{-\gamma y}\right\} e^{j(\beta x - sw_o t)}\ dx \cdot dy$$

Therefore,

$$I_{ph} = Re\left[\frac{4p\mu H_o sw_o}{3T_{eff}\ \beta^2\gamma} e^{j(\beta x - sw_o t - \frac{\pi}{2})}\right] \quad (13)$$

Now e.m.f induced per phase can be found out from Blv concept.

Therefore, e.m.f induced $= Re\left[-2\mu H_o L \frac{w}{\beta} T_{eff}\ e^{j(\beta x - wt)}\right]$

Impedance $= \frac{\text{Voltage}}{\text{Current}}$ both being peak values.

$$= \frac{-2\mu H_o \frac{Lw}{\beta} T_{eff}}{\frac{4p\mu H_o sw_o}{3T_{eff}\ \beta^2\gamma}}$$

Therefore,

$$Z = -j\ \frac{3\ L\ T_{eff}^2\ \rho\beta\gamma}{2\ p\ l\ \ \ s} \quad (14)$$

which represents the expression for rotor impedance.

Since $\gamma = \pm\, (\beta^2 - \frac{jsw_o\mu}{\rho})^{1/2}$ is a complex number and is of the form $-\theta_1 + j\theta_2$

then $-j(-\theta_1 + j\theta_2) = \theta_2 + j\theta_1$

So, the condition is that the real part of γ should be less than zero i.e. negative and the imaginary part of γ should be greater than zero i.e. positive.

For the purposes of finding the unbalanced operation of Solid rotor Induction Motor, the design data for a typical type of Solid rotor machine were as below:

$L = 0.14$ meter

$p = 6$

$T_{eff} = 408$

$\rho = 15 \times 10^{-8}$ ohm-meter

$\beta = 85.7$

s = Slip

$\mu = \mu_o\mu_r = 4\pi.10^{-7}.800 = 1.10^{-3}$

$\gamma = (\beta^2 - \frac{jsw_o\mu}{\rho})^{1/2} = (7380 - js\, 2.09 \times 10^6)^{1/2}$

From the expression (14) it was found that:

$Z_R = -j\, 0.075\ \gamma/s$

At higher slips i.e. s 0.1 this becomes $Z = K'/\sqrt{s}$

so that Torque = $K''\sqrt{s}$. Hence, Torque-speed characterestics approach a parabola.

1.2.2. Here another type of electromagnetic model as shown in figure No.1 has been attempted. All the assumptions in 1.2 will hold good here.

In this case, Maxwell's Equations are to be written:

(a) For the air gap which is a current free region

(b) For rotor whose permeability is finite and where eddy currents exist.

In the air gap,

$$\nabla \times H = 0 \qquad (15)$$

$$\nabla \cdot B = 0 \qquad (16)$$

$$B = \mu H \qquad (17)$$

In such a case after expanding the above equations, the final differential equation involving one variable, works out to be

$$\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} = 0 \qquad (18)$$

The above being a Laplace's Equation in two dimensions has a solution as in Appendix II

$$B_y = \mathrm{Re}\left[(A\epsilon^{\beta y} + B\epsilon^{-\beta y})\, \epsilon^{j(\beta x - w_o t)} \right] \qquad (19)$$

where A and B are constants yet to be evaluated.

Also, from the divergence Equation (16) the tangential component of flux density is given by

$$B_x = \mathrm{Re}\left[j\, (A\epsilon^{\beta y} - B\epsilon^{-\beta y})\, \epsilon^{j(\beta x - w_o t)} \right] \qquad (20)$$

Now, considering the rotor region, as in case (i) we have, as before

$$X\ H = I \qquad (1)$$

$$X\ E = -\frac{\partial B}{\partial t} \qquad (2)$$

$$B = 0 \qquad (3)$$

$$E = i \qquad (4)$$

$$B = \mu H \qquad (5)$$

which lead to the differential Equation in one variable B_y

$$\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} = \frac{\mu}{\rho} \cdot \frac{\partial B_y}{\partial t} \qquad (8)$$

In such a case, the solutions of flux density components in the rotor region are found to be

$$B_y = Re\left[\left\{D\varepsilon^{-Yy}\right\}\varepsilon^{j(\beta x - sw_o t)}\right] \qquad (10)$$

$$B_x = Re\left[\left\{\frac{DY}{j\beta}\varepsilon^{-Yy}\right\}\varepsilon^{j(\beta x - sw_o t)}\right] \qquad (11)$$

Now, the equations (10), (11), (19) and (20) are still general solutions since they have the constants A,B and D which are not yet known. However, these three constants A,B and D can be evaluated from the boundary conditions by matching the two fields at the rotor side of the air gap i.e. from the electromagnetic theory of reflection and refraction. The first boundary condition is "Normal Component of flux density is continuous at rotor surface"

i.e. $B_{yI(\text{air gap})} = B_{yII(\text{rotor surface})}$

$$\text{or,}\ \text{Re}\left[(A\,\epsilon^{\beta y}+B\,\epsilon^{-\beta y})\epsilon^{j(\beta x-wt)}\right]_{y=-g} = \text{Re}\left[D\,\epsilon^{-\gamma y}\ \epsilon^{j(\beta x-wt)}\right]_{y=-g} \qquad (21)_a$$

Second condition is obtained from the known value of the stator ampereturns at the infinitesimally thin current sheet,

$$\text{or,}\quad \left.\frac{\partial (AT)}{\partial x} = H_x\right]_{\text{at } y=0} \qquad (22)$$

Also since "Tangential component of intensity of magnetisation is continuous at the rotor surface" the third boundary condition will be

$$H_{xI}(\text{air gap}) = H_{xII}(\text{in rotor}) \text{ at } y = -g \qquad (21)_b$$

$$\text{or}\quad \text{Re}\left[\left\{\frac{j}{\mu_o}(A\,\epsilon^{-\beta g}-B\,\epsilon^{\beta g})\right\}\epsilon^{j(\beta x-w_o t)}\right] = \text{Re}\left[\left\{-j\,\frac{D\gamma}{\beta}\,\frac{1}{\mu_r\mu_o}\right\}\epsilon^{j(\beta x-w_o t)}\right]$$

So, the above expressed boundary conditions will lead to three simultaneous equations involving the three constants A, B, and D as below:

$$A\,\epsilon^{-\beta g}+B\,\epsilon^{\beta g} = D\,\epsilon^{\gamma g} \qquad (23)$$

$$A\,\epsilon^{-\beta g}-B\,\epsilon^{\beta g} = -\frac{\gamma D}{\beta\mu_r}\,\epsilon^{\gamma g} \qquad (24)$$

$$A-B = \mu_o M\,\beta \qquad (25)$$

solving the above simultaneous equations for A, B and D it is obtained as

$$A = \frac{\mu_o\,M\beta}{\epsilon^{(\gamma+\beta)g}(1-\eta) - \epsilon^{(\gamma-\beta)}(1+\eta)}\ \epsilon^{(\gamma+\beta)g}\,(1-\eta) \qquad (26)$$

$$B = \frac{\mu_o M\beta}{\epsilon^{(\gamma+\beta)g}(1-\eta) - \epsilon^{(\gamma-\beta)g}(1+\eta)} \epsilon^{(\gamma-p)g}(1+\eta) \qquad (27)$$

and,

$$D = \frac{2\mu_o M\beta}{\epsilon^{(\gamma+\beta)g}(1-\eta) - \epsilon^{(\gamma-\beta)}(1+\eta)} \qquad (28)$$

$$\text{where, } \eta = \frac{\gamma}{\beta\mu_r}$$

Thus, all the three constants A,B and D are expressed in terms of M, β, γ, η which are known.

Now, to evaluate the magnetising reactance and rotor impedance, the air gap induced e.m.f and current are found as in case(i).

The induced e.m.f in the stator is based on Blv concept where B_y is given by equation (19)

Therefore, induced e.m.f =

$$- Re\left[\left|(A+B)\right| \epsilon^{j(\beta x - wt)} \right] \frac{Lrw_o}{p} 2T'_{ph}$$

$$E_{max} = - \left|(A+B)\right| \frac{Lrw_o}{p} 2T'_{ph}$$

Also, current is found from (AT) relation

i.e. $(AT)_{inst} = Re\left[(M\,\epsilon^{j(\beta x - w_o t)}\right]$

$$\text{where } M = \frac{3}{2} \quad \frac{I_{max}\,T'_{ph}}{2p}$$

$$\text{Therefore, } I_{max} = \frac{4\,pM}{3\,T'_{ph}} \qquad (13)$$

$$\text{air gap Impedance} = \frac{\left|(A+B)\right| Lrw_o\,3\,T'^{2}_{ph}}{2p^2 M} \qquad (29)$$

Now, in the above general expression for air gap impedance magnetising reactance can be obtained by puting s=0 and utilizing suitable approximation.

If, s = 0 then $\gamma = \beta$ and $\eta = \frac{1}{\mu_r}$

Further, using suitable approximations Equations (26) and (27) reduce to the form as

$$A = \frac{\mu_o M\beta}{(\varepsilon^{2\beta g} - 1)} \varepsilon^{2\beta g} \tag{30}$$

$$B = \frac{\mu_o M\beta}{(\varepsilon^{2\beta g} - 1)} \tag{31}$$

Therefore,

$$(A+B) = \frac{\mu_o M}{g} (1+\beta g) \tag{32}$$

$$\text{Hence, Magnetising Reatance} = \frac{(A+B) \quad Lrw_o \; 3T'^2_{ph}}{2\; Mp^2} \tag{33}$$

Now, for the particular solid rotor Induction Motor under consideration from equation (33) the magnetising reactance is given by

$$= \frac{1.035X3}{2} \cdot \frac{\mu_o M}{g} \cdot \frac{Lrw_o \; T'^2_{ph}}{Mp^2}$$

$$= \frac{1.55 \; \mu_o \; Lrw_o \; T'^2_{ph}}{p^2 g}$$

$$= 69 \text{ ohm.}$$

As a rough check to the above expression from P.L. Alger's Book (Ref.13) Page 180

$$X_m = \frac{6.38 \; qfN^2 (K_{dp})^2 DL}{p^2 g \; 10^8}$$

$$= \frac{6.38X3X50X(408)^2 (5.51)^2}{0.0173X36X10^8}$$

$$= 77.8 \text{ ohm.}$$

Also, it is known that $Y = Y_R + Y_M$

Putting $\alpha_1 = \epsilon^{(\gamma+\beta)g}(1-\eta)$

$\alpha_2 = \epsilon^{(\gamma-\beta)g}(1+\eta)$

from (26) and (27) on simplification it can be obtained as:

$$A+B = \mu_o M\beta \frac{(\alpha_1+\alpha_2)}{(\alpha_1-\alpha_2)}$$

So, following the same procedure as outlined in case(i)

$$Z = \mu_o\beta\frac{(\alpha_1+\alpha_2)}{(\alpha_1-\alpha_2)} \quad \frac{Lrw_o \; 3T_{ph}'^2}{2p^2}$$

it can be shown that $\frac{\alpha_1+\alpha_2}{\alpha_1-\alpha_2} = \frac{1-\eta\beta g}{\beta g-\eta}$

so,

$$Z_R = \frac{Lrw_o \; \mu_o \; T_{ph}'^2}{p^2} \left[\frac{4.65\beta(\beta g-\eta)}{-3.1+6.1 \quad \beta g+3\beta^2 g^2} \right] \qquad (34)$$

for the Solid rotor motor under consideration.

Hence, it is found that Z_r is a function of fractional slip s since it contains slip term. Since all the quantities are known it can be evaluated numerically.

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-:CHAPTER II:-

A REVIEW ON UNBALANCED OPERATION OF INDUCTION MOTORS:

2.1 Introduction:

In general, it is observed that though the use of Induction Motor is prevalent since a long time, the operation of the Induction Motor on unbalanced conditions was not fully investigated till the middle of twentieth century. One of the reasons might be that probably the supply systems in those times were almost balanced as many of them were localized. Secondly, the theoretical analysis of the machine was not complete enough to explain the unbalanced conditions.

On these lines, a paper by Charter and Hilderbrand,(1) first in its kind, though preliminary in nature was published in 1909. It appears that not much significant work was done on this aspect for quite a long period after 1909. However, the subject gained its importance with the increase of single phase loading of the supply systems creating often unbalances in supply voltages.

With the theory of Symmetrical Components introduced by C.L.Fortescue (1919) the problem of attaching the analysis of Induction Motors on unbalanced conditions has become rather somewhat easier. Hence, utilizing the method of Symmetrical Components J.E.Williams(2) contributed a good work of analysis as well as experimental evidence to the unbalanced operation of Induction Motors and reiterated the importance of the investigation into the extra-losses incurred in such a case. Brown and Butler[3] have developed a few routine type of analytical methods to analyse the situation of different conditions causing unbalance.

Since the additional losses cause heating and temperature-rise which in turn reduce the over-all capacity of the machine ~~the~~ attention was drawn towards the "heating and temperature-rise of the Induction Motor under unbalanced voltages" and in this connection Duesterhoeft and others[4] tried to investigate the causes and effects of heating.

However, from the present trend of the recent papers[8,9,10] it appears that much importance is attached to the question of finding the allowable output from a three phase Induction Motor on unbalanced voltage supply, in other words, how far the machine is derated? On the same lines Roy,[8,9] Rama Rao and Jyothi Rao[10] tried to investigate the various methods to find the derating factors.

2.2 A Critical Review:

The state of the art on the theory as well as experimental methods of unbalanced operations of Induction Motor are reviewed here. A brief review regarding the same is included here because of its similarity with the theory of Solid rotor Induction Motor in many aspects. The exceptions are, however, indicated wherever necessary.

Charter and Hilderbrand[1] (1909) in their paper regarding the unbalanced operation of Induction Motor (both cage rotor and wound type) have encountered in a most elementary manner and presented their test data with some useful comments so as to bring about the importance. The authors[1] have realised that "The operating limit of a motor as determined by the temperature in any one phase is reached when the current in that phase has attained its normal full load value". It was experimentally shown by these authors that the performance is practically independent of the nature of voltage unbalance that is, whether this takes the form of two voltages equal and the other higher or lower of all the three at different values. They have further observed that "two high and one low voltage appears to be the worst condition". Another interesting feature is that the authors made an attempt to find out the extent to which the reduction in capacity is affected through voltage unbalance by comparing the performance of three commercial motors differing in type, voltage, and rating. The worst performance of all the machines was observed to be that of 2 H.P. 3 phase, 110 volts Induction Motor in which an unbalance of 10% caused a reduction in capacity to about 54%. The main conclusions drawn in this paper[1] are:

(i) Unbalancing of voltage and phase-shift leads to serious overheating in polyphase motors.

(ii) The phase-shift is found to be relatively of more importance where there is possibility to seperate the effects due to phase-shift and voltage unbalance.

(iii) The reduction in capacity (presently known as "derating") due to either phase-shift or magnitude unbalance causes a worse performance.

Among the above conclusions, no attempt has been made to assess the approximate degree of overheating in conclusion(i). Conclusion (ii) appears to be a hypothesis since no support by theoretical approach was given. However, conclusion (iii) attained more importance later and at present this aspect is often called as "Derating of Induction Motors". Thus, though the authors[1] did not enter into detailed analytical investigations they have emphasised the importance of study of unbalanced operation.

It was Williams[2] who possibly for the first time tried to investigate the main effects of unbalanced voltage on the operation of Induction Motor where he ascertains that the main effects are increased losses and unbalanced line currents. In his analysis, positive and negative sequence voltages are evaluated from a set of known line voltages by the method of Symmetrical Components and method of Superposition. Finally, the motor losses are estimated and the variation of the added losses in percentage of full load for various negative sequence voltages have been studied for motors of single cage rotor and double cage rotor.

The main conclusions of the author were:

(i) The additional losses due to operation on unbalanced voltages are larger than for motors with multiple cage rotors.

(ii) Unbalanced line voltages cause non-uniform distribution of stator copper loss.

(iii) Small unbalances in voltage cause larger unbalances in line currents.

However, the importance of this paper(2) is from the view-point that it gives a method of rapidly finding the positive and negative sequence components of set of unbalanced voltages and currents when the magnitudes of these line voltages are only known. Especially, the second conclusion is of utmost importance with reference to Solid rotor Induction Motor since multiple cage rotor machine is also basically an eddy current type of machine. Further, third conclusion gives an indication that the machine can not be operated at rated output continuously without overheating.

Brown and Butler(3) (1953) in their paper have analysed different conditions of unbalanced operation that can often occur such as

(i) Primary (Stator) unbalance,

(ii) Secondary (Rotor) unbalance.

With the assumptions of negligible saturation and validity of superposition they attempted to analyse the above cases of unbalance by the use of "Inspection Equations" through Symmetrical Component theory approach and also presented the variation of phase sequence parameters Z_1 and Z_2 by experiment. However, in this paper[3] the case of double unbalance, i.e. combination of cases(i) and (ii) have not been considered.

Later, in 1959, a paper by Gafford, Duesterhoeft, Mosher 111[4] surveys the effects of unbalanced voltage and presents the results of heat-run tests on a particular motor. They have analysed and shown the great detrimental effects of voltage unbalance on line currents and temperature rise of Polyphase machines, and made the following recommendations:

(i) Tests to determine the behaviour under unbalanced voltage operation should be on a wide range of classes and sizes of motors.

(ii) Data such as R_s, R_{r1}, Z_m, Z_{r1}, Z_{r2} should be listed for judicious derating to account for unbalance.

They concluded that eddy current machines such as deep bar or double cage type dissipate more power per ampere of negative sequence current than for an equal positive sequence current. They made clear that it is not the total copper loss that indicates the maximum temperature rise of the motor under unbalanced supply voltage conditions.

Berndt and Schmitz (1962) [5] in their paper presented the results which indicate the need for a severe reduction in the rating of the motors when operated with unbalanced line voltages. They have considered derating for varying amounts of positive and negative sequence applied voltages. An interesting feature is that the authors examined the most unfavourable conditions for unbalance, and discloses further in more detail just how serious these heating effects are going to be under the most adverse conditions.

Recent Works:

Nagrath and Sahni [6] (1967) have presented an unique method of analysing the problem of Induction Motor with double unbalance i.e. stator and rotor both having unbalance in their circuits. The assumptions are the same as before and basically utilizes the method of Symmetrical Components. They developed a method that can be called as "Reflection theory" which states that "When a positive sequence is applied to the stator terminals of an Induction Motor with unbalanced impedances both in stator and rotor circuits infinite number of reflections result". In such a case the positive and negative sequence currents of various orders of reflection create their own rotating fields in the air gap. Following these reflections, equivalent circuits are developed upto the required order of reflection for both positive and negative sequence applied voltages.

After Berndt's work (1962) on derating factor of Induction Motor, Roy[8,9] (1967) suggests a method to compute the relative magnitude of allowable load in terms of the rated load and further includes a generalised chart that can be used for any type of Induction Motor. However, this necessitates a knowledge of temperature coefficients that are to be determined experimentally. In his method, the transfer of heat from the different unevenly loaded phases has been taken into account by introducing coefficients of heat transfer. Expressions for the derating factor at any voltage unsymmetry have been developed with and without the determination of additional parameters such as heat transfer coefficients. In its companion paper Roy[9] analyses the situation of three phase Induction Motor on single phase circuits i.e. on one line open giving due consideration to temperature problem under such condition.

In a most recent paper (Jan.1968) Jyothi Rao and Rama Rao[10] went a step further and made an attempt to incorporate the effect of winding details and heat conduction in determining the re-rating factor of the Induction Motor on unbalanced voltages.

-:CHAPTER-III:-

OPERATION OF SOLID ROTOR INDUCTION MOTOR ON UNBALANCED VOLTAGE SUPPLY:

3.1 Introduction:

When the voltages applied to stator terminals of a three phase Solid rotor Induction Motor are unbalanced, the motor performance is modified. As usual, the approach to such a problem can be by the method of Symmetrical Components. The unbalanced voltages are split into two sets of balanced voltages of positive and negative sequence respectively. The behaviour of the machine to the positive sequence voltages will be normal. The negative sequence voltages, however, set up a reverse rotating field so that if the fractional slip of the rotor is 's' with respect to the positive sequence, then it will be (2-s) to the negative sequence. For moderate voltage unbalances, the machine is principally affected by I^2R losses due to the presence of both positive and negative sequence currents. In such a case, the positive and negative sequence currents are functions of their sequence voltages and the machine constants R_{r1}, X_m, X_{r1} and slip s. Depending upon the values of the Solid rotor circuit parameters the line currents are also unbalanced. Here, the rotor impedance, evaluated from the basic principle of electromagnetic theory, can be used for finding the performance.

In this chapter, an attempt is made to evaluate the losses of a Solid rotor machine on unbalanced voltages, thus analysing its performance.

3.2 Resolution of Unsymmetrical voltage phasors into Sequence Components:

From the basic theory of Symmetrical Components[11]

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = A \begin{bmatrix} V_{AB_0} \\ V_{AB_1} \\ V_{AB_2} \end{bmatrix} \quad (3.1)$$

$$\text{Where} \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

or it can be written as

$$\begin{bmatrix} V_{AB_0} \\ V_{AB_1} \\ V_{AB_2} \end{bmatrix} = A^{-1} \begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} \quad (3.2$$

$$\text{Where} \quad A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \quad (3.3$$

Now, by expansion of (3.2) the sequence voltages are expressed in terms of line voltages as

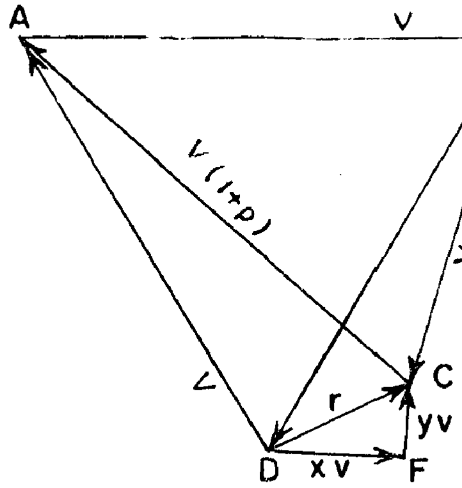


FIG. 3

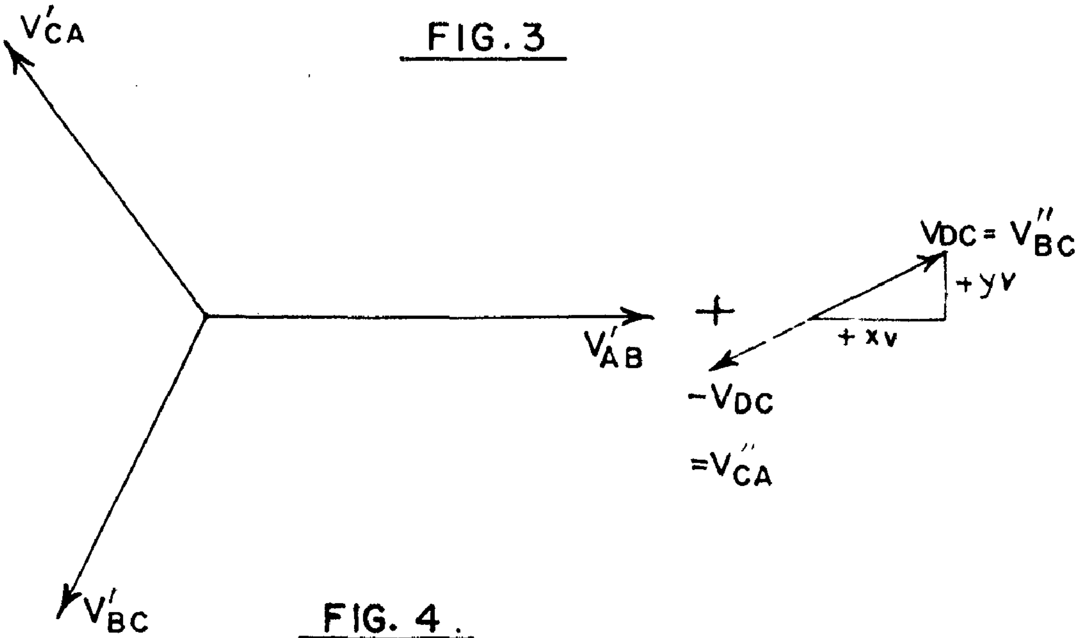


FIG. 4.

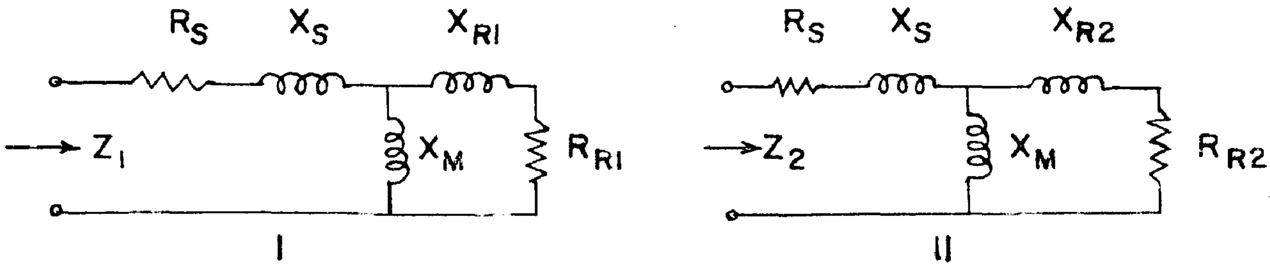


FIG. 5. FIG. 5.

EQUIVALENT CIRCUITS FOR DETERMINATION OF POSITIVE AND NEGATIVE SEQUENCE IMPEDANCE OF SOLID ROTOR INDUCTION MOTOR

$$\left.\begin{aligned} V_{AB_0} &= \frac{1}{3}\,(V_{AB}+V_{BC}+V_{CA}) \\ V_{AB_1} &= \frac{1}{3}\,(V_{AB}+a.V_{BC}+a^2.V_{CA}) \\ V_{AB_2} &= \frac{1}{3}\,(V_{AB}+a^2.V_{BC}+a.V_{CA}) \end{aligned}\right\} \quad (3.4)$$

Further, for a phase order ABC the other line voltages are given by

$$\left.\begin{aligned} V_{BC_1} &= a^2.V_{AB_1} \\ V_{CA_1} &= a.V_{AB_1} \end{aligned}\right\}(3.5) \qquad \left.\begin{aligned} V_{BC_2} &= a.V_{AB_2} \\ V_{CA_2} &= a^2.V_{AB_2} \end{aligned}\right\} \quad (3.6)$$

To find out V_{AB_1}, V_{AB_2} etc from the known unbalanced line voltages V_{AB}, V_{BC}, V_{CA} Equations (3.4) are to be used. However, another approach is from the utilization of assymmetry of the phasor diagrams and analysing graphically to account for all patterns of known line voltage unbalances.[2]

When line voltages are unbalanced, the line voltage phasor diagram can be represented as shown in figure 3 by the triangle ABC. It can be observed that due to assymmetry of unbalanced phasors the ΔABC is no more equilateral. Taking D as origin, let the point C be at a distance 'r' from D and further let the co-ordinates of C with respect to D be C (x.V,y.V) as in figure 4.

Then DF = x.V and CF = y.V

From the phasor diagram

$$\left.\begin{aligned} V_{AB} &= V_{AB}+0 \\ V_{BC} &= V_{BD}+V_{DC} \\ V_{CA} &= V_{DA}-V_{DC} \end{aligned}\right\} \quad (3.7)$$

Also V_{DC} can be written as (figure 4)

$$\left.\begin{aligned} V_{DC} &= V(x+jy) \\ -V_{DC} &= V(-x-jy) \end{aligned}\right\} \quad (3.8$$

Then, V'_{AB}, V'_{BC}, V'_{CA} are a set of positive sequence balanced voltages and

$$V''_{AB} = 0 \;;\; V''_{BC} = V_{DC} \;;\; V''_{CA} = V_{DA} \quad (3.9$$

The voltages V''_{AB}, V''_{BC}, V''_{CA} contain all the negative sequence and also a little contribution to positive sequence. On the basis of Equation(3.4) and (3.9)

$$V_{AB_2} = \frac{1}{3}\left(a^2 \cdot V''_{BC} + a \cdot V''_{CA}\right)$$

Also, from figure since $V''_{BC} = V(x+jy)$

$V''_{CA} = V(=x-jy)$

it can be written as

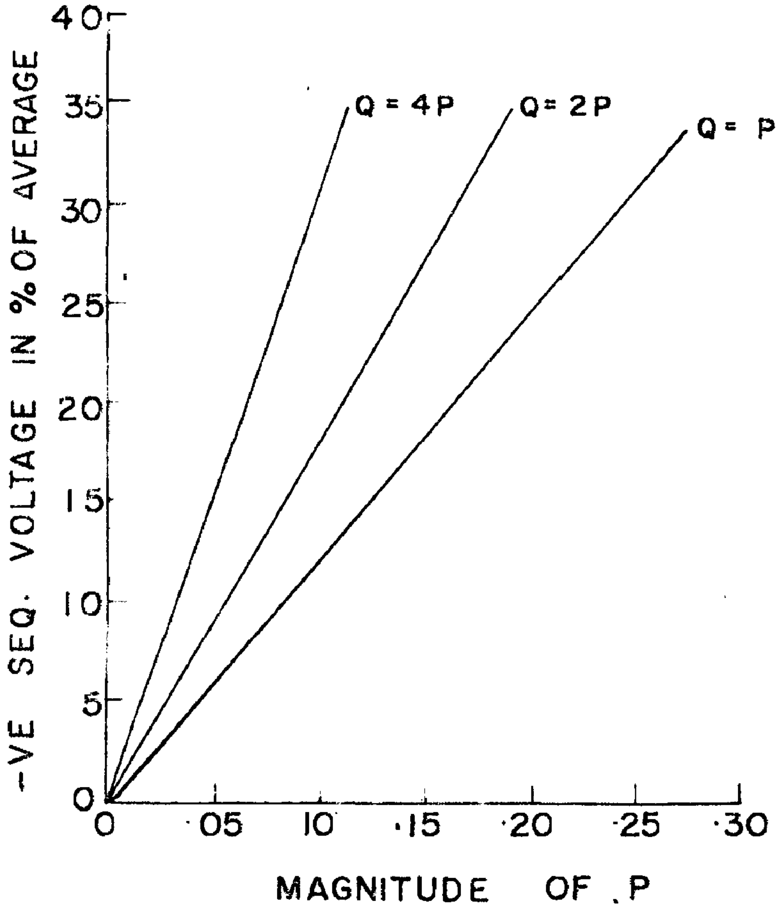
$$V_{AB_2} = \frac{1}{3}\left[a^2\, V(x+jy) \;\; +aV \;\; (-x-jy)\right]$$

$$= \frac{V}{3}\left[x(a^2-a)+y(a^2 \cdot j-a \cdot j)\right]$$

Therefore, $V_{AB_2} = \frac{V}{\sqrt{3}}(y-jx)$ (3.1

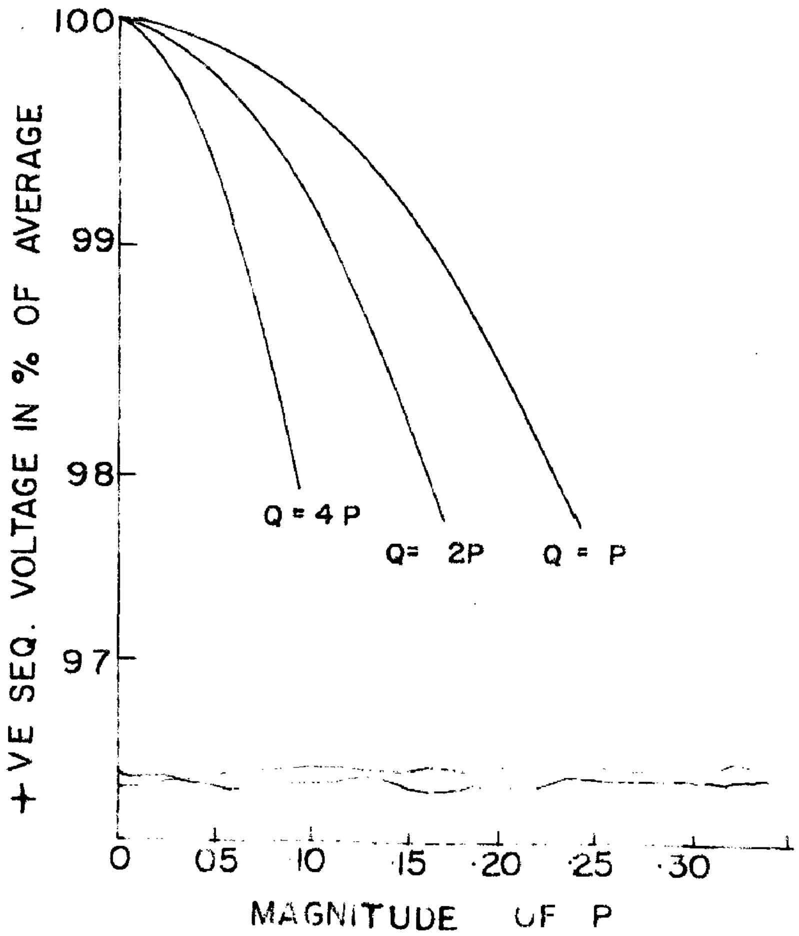
Also on the same lines $V_{AB_1} = \frac{V}{\sqrt{3}}\left[(1-y)+jx\right]$ (3.1

To express Sequence Components in terms of unbalance factors p and q using trigonometry.



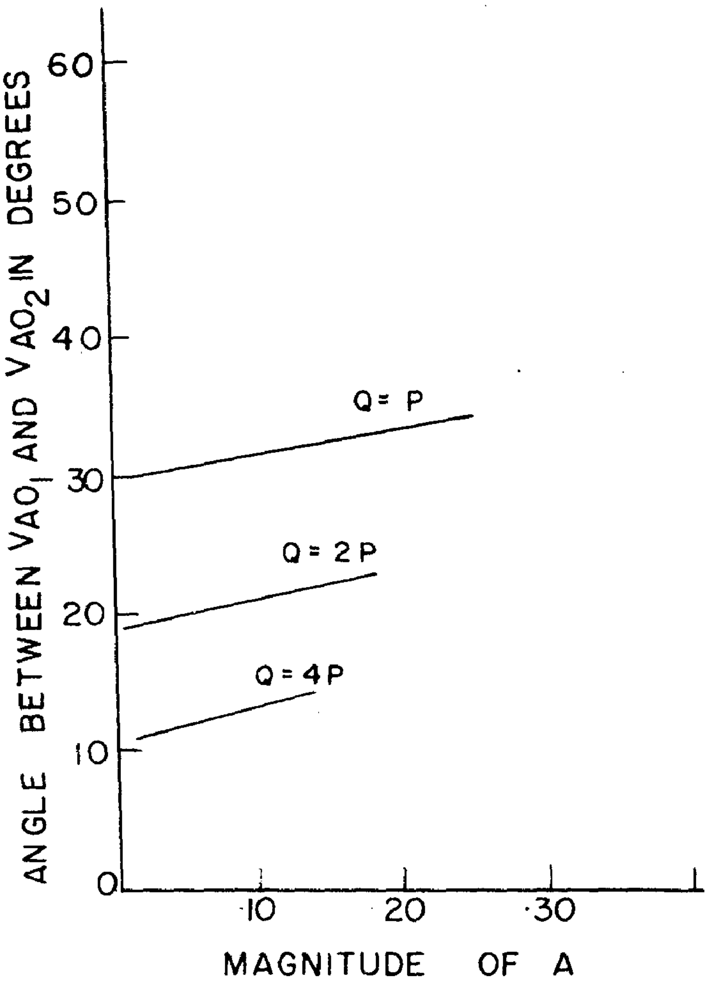
40
35
30
25
20
15
10
5
0
-VE SEQ. VOLTAGE IN % OF AVERAGE
Q = 4P
Q = 2P
Q = P
0
·05
·10
·15
·20
·25
·30
MAGNITUDE OF P

FIG. 6.



100
99
98
97
+VE SEQ. VOLTAGE IN % OF AVERAGE
Q = 4P
Q = 2P
Q = P
0
·05
·10
·15
·20
·25
·30
MAGNITUDE OF P

FIG. 7.



3. PHASE ANGLE BETWEEN THE POSITIVE AND NEGATIVE SEQUENCE VOLTAGES OF ONE PHASE FROM THE VALUES OF THE LINE TO LINE VOLTAGES

$$x = \frac{(p+q)+(p^2-q^2)}{2} \tag{3.12}$$

$$y = \frac{\sqrt{3}}{\sqrt{2}} - \sqrt{(1-q)^2-(1/2-x)^2} \tag{3.13}$$

However, the above derivation assumes a condition that $V_{CA} < V_{AB} < V_{BC}$. It can be noted that it is only for convenience, this condition is imposed.

Let
$$P = \frac{V_{CA} - V_{BC}}{V_{av}} = \frac{p}{1+\frac{p-q}{3}} \tag{3.14}$$

$$Q = \frac{V_{AB} - V_{BC}}{V_{av}} = \frac{q}{1+\frac{p-q}{3}} \tag{3.15}$$

It can also be seen that $P/Q = p/q$ (3.16)

Equations, 3.12, 3.13, 3.14, 3.15, 3.16 indicate that if V_{AB}, V_{BC}, V_{CA} are known p,q can be evaluated hence the sequence Components can be known.

It can be stated that the terms P and Q indicate the pattern of unbalance. This helps to find out percentage of positive sequence voltage, as well as negative sequence voltage for many patterns of Q/P. Graphs are developed as in figures 6,7,8[2] to directly find out the amount of positive sequence voltages as well as negative sequence voltages for a known value of Q/P.

So, instead of calculating the Sequence Components from the known magnitudes of line voltages, one can rapidly find the positive and negative Sequence Components of a known practical set of unbalanced voltages graphically.

3.3 Calculation of Sequence Impedances:

The input impedance to positive sequence voltages, Z_1, and the input impedance to the negative sequence voltages Z_2, are evaluated from the equivalent circuits shown in Figure 5 as below:

$$Z_1 = R_s + \frac{R_{r1} \cdot X_m^2}{(R_{r1})^2 + (X_{r1}+X_m)^2} + j \quad X_s + \frac{R_{r1}^2 X_m + X_{r1} X_m (X_m + X_{r1})}{(R_{r1})^2 + (X_{r1}+X_m)^2}$$

$$Z_2 = R_s + \frac{R_{r2} \cdot X_m^2}{(R_{r2})^2 + (X_{r2}+X_m)^2} + j \quad X_s + \frac{R_{r2}^2 X_m + X_{r2} X_m (X_m + X_{r2})}{(R_{r2})^2 + (X_{r2}+X_m)^2}$$

In the above expressions, R_s, X_s, and X_m are known. $Z_{r1} = R_{r1} + jX_{r1}$ is calculated when slip s is between 0 to 1. Similarly Z_{r2} is calculated when slip is between 1 to 2 i.e. s'=2-s.

Both of these values of rotor impedance are from electromagnetic theory approach based on Maxwell's Equations as given in chapter I.

Rotor impedance values are listed below:

s	Z_{r1}	Z_{r2}
0.04	365+j405	55.0+j55.0
0.08	218+j225	55.1+j55.1
0.10	242+j242	55.3+j55.3
0.20	168+j168	56.8+j56.8
0.30	139+j139	58.6+j58.6
0.40	121+j121	60.3+j60.3
0.50	108+j108	62.3+j62.3

s	Z_{r1}	Z_{r2}
0.60	98.9+j98.9	64.5+j64.5
0.70	90.1+j90.1	67.0+j67.0
0.80	85.3+j85.3	69.6+j69.6
0.90	80.5+j80.5	72.9+j72.9
1.00	76.4+j76.4	76.4+j76.4

3.4 Synthesis of Sequence Currents to obtain Line currents:

Once the sequence voltages and Sequence impedances of the Solid rotor motor are obtained the currents are given by

$$I_{a1} = \frac{V_{a1}}{Z_1} \qquad I_{a2} = \frac{V_{a2}}{Z_2}$$

$$I_{b1} = \frac{V_{b1}}{Z_1} = a^2 I_{a1} \qquad I_{b2} = \frac{V_{b2}}{Z_2} = a I_{a2}$$

$$I_{c1} = \frac{V_{c1}}{Z_1} = a I_{a1} \qquad I_{c2} = \frac{V_{c2}}{Z_2} = a^2 I_{a2}$$

The line currents are obtained by usual techniques of analysis and method of superposition. Then

$$I_a = I_{a1} + I_{a2}$$

or $I_a = I_{a1}\sqrt{1+k^2+2k\cos\alpha} = I_{a1}\cdot k_1$ similarly,

$$I_b = I_{a1}\sqrt{1+k^2+2k\cos(\alpha+120)} = I_{a1}\cdot k_2$$

$$I_c = I_{a1}\sqrt{1+k^2 2k\cos(\alpha-120)} = I_{a1}\cdot k_3$$

Thus, to find out the line currents only I_{a1}, k and α are to be known. Hence I_a, I_b and I_c can be calculated.

Now, Stator copper losses = $3I_{a1}^2 R_s + 3I_{a2}^2 R_s$.

The above equation shows that there is an additional loss term due to negative Sequence currents seperately. This is the second factor in which we are interested in the case of Solid rotor Induction Motor.

3.5 Effects of Unbalanced voltages on losses of Solid rotor Induction Motor:

Following the lines of the analysis mentioned above, the effects of unbalanced voltages on the operation of a particular Solid rotor Induction Motor of 2 H.P. 50 cps, 400V, designed and fabricated in the department of Electrical Engineering has been studied. Since the motor is basically a Solid rotor it has a high starting torque with a high rotor resistance. Now, for a slip of 0.04, if the line voltages are unbalanced so as to give 20% of negative sequence voltage,

then $Z_1 = 8.73+j\,102.4 = 103\,\underline{|85^\circ}$

$Z_2 = 17.7+j\,76.9 = 79.4\,\underline{|77^\circ}$

$V_{a2} = 20\%$ of $230 = 46$ Volts.

$$I_{a2} = \frac{V_{a2}}{Z_2} = \frac{23X2}{79.4} = 0.582 \text{ Amp.}$$

The added stator losses due to the negative sequence current are equal to $3\, I_{a_2}^2 \cdot R_s = 3(0.582)^2\,(4.1) = 4.2$ watts. Further, the negative sequence electric input to the rotor=

$$= 3\cdot(I_{a2})^2\,(R_{r_2}-R_s)$$

$$= 3\,(.582)^2(17.7-4.1)$$

$$= 13.9 \text{ watts.}$$

An additional amount of power subtracted from the mechanical output and converted to rotor loss

$$= (\overline{2-s}-1)3\cdot I_{a2}^2\,(R_{r_2}-R_s)$$

$$= 0.96\text{X}13.9$$

$$= 13.35 \text{ watts.}$$

Hence, total additional loss

$$= 4.2 + 13.9 + 13.35$$

$$= 31.6 \text{watts.}$$

So, it is found that total additional losses due to 20% negative sequence voltages are about 16.62% of normal losses of Solid rotor Induction Motor under consideration.

As a matter of interest, in the same manner, the additional losses due to the presence of various percentages of negative sequence voltages are calculated. Figure 9 is a curve showing the variation of additional losses with increase of percentage of negative sequence voltages on the abcissa. For a given value of slip the negative sequence input Impedance is constant, hence the additional losses due to negative sequence is proportional to the square of the negative sequence voltage. (see Fig.9)

3.6 Variation of line currents of Solid Rotor Induction for specified unbalanced voltages:

Since the impedance of rotor is indirectly reflected to stator side, it becomes important to consider as to how the line currents vary with different types of unbalanced conditions.

To determine the variation of line currents of Solid rotor Induction Motor under consideration, calculations are made for different sets of unbalanced voltages with various patterns and graphs are shown in figures

As an illustration, for a practical case, a set of unbalanced voltages are found to be

V_{AB} = 404 Volts

V_{BC} = 378 Volts

V_{CA} = 418 Volts

The unbalance factors P and Q are given by

$$P = \frac{V_{CA} - V_{AB}}{V_{av}} = \frac{418 - 404}{400} = 0.035$$

$$Q = \frac{V_{AB} - V_{BC}}{V_{av}} = \frac{404 - 378}{400} = 0.065$$

$$(Q/P) = \frac{0.065}{0.035} = 1.86 \quad \text{Therefore, } Q = 1.86P.$$

Now, with an idea of unbalance factors P, Q and their ratio the percentages of positive sequence as well as negative sequence voltages and the angle between them is found as explained in article 3.2 from figures 6,7 and 8.

Hence, V_{a1} = negative sequence volts present = 7.5%

V_{a2} = positive sequence volts present = 99.7%

Angle between V_{a1} and V_{a2} = 19°

where V_{a1} is the reference.

Accordingly, the positive and negative sequence currents are given by

$$I_{a1} = \frac{230 \times 0.977}{103 \angle 85.1} = 2.22 \angle -85.1^{\circ} \text{ Amp}$$

$$I_{a2} = \frac{230 \times 0.08 \angle -19^{\circ}}{79.4 \angle 77^{\circ}} = 0.23 \angle -96.2^{\circ} \text{ Amp}$$

Hence the ratio of negative sequence voltage to the positive seqeunce voltage

$$k = \frac{I_{a2}}{I_{a1}} = \frac{0.23}{2.22} = 0.105 \angle 11.1^{\circ}$$

From article 3.4,

$$k_1 = \sqrt{1+k^2+2k\cos\alpha} = 1.1$$

$$k_2 = \sqrt{1+k^2+2k\cos(\alpha+120)} = 0.93$$

$$k_3 = \sqrt{1+k^2+2k\cos(120-\alpha)} = 0.92$$

Hence, the line currents are

I_a = 2.22X1.10 = 2.44 Amp

I_b = 2.22X0.93 = 2.07 Amp

I_c = 2.22X0.92 = 2.04 Amp

The above results clearly show that the three phases are unevenly loaded. However, by increasing the

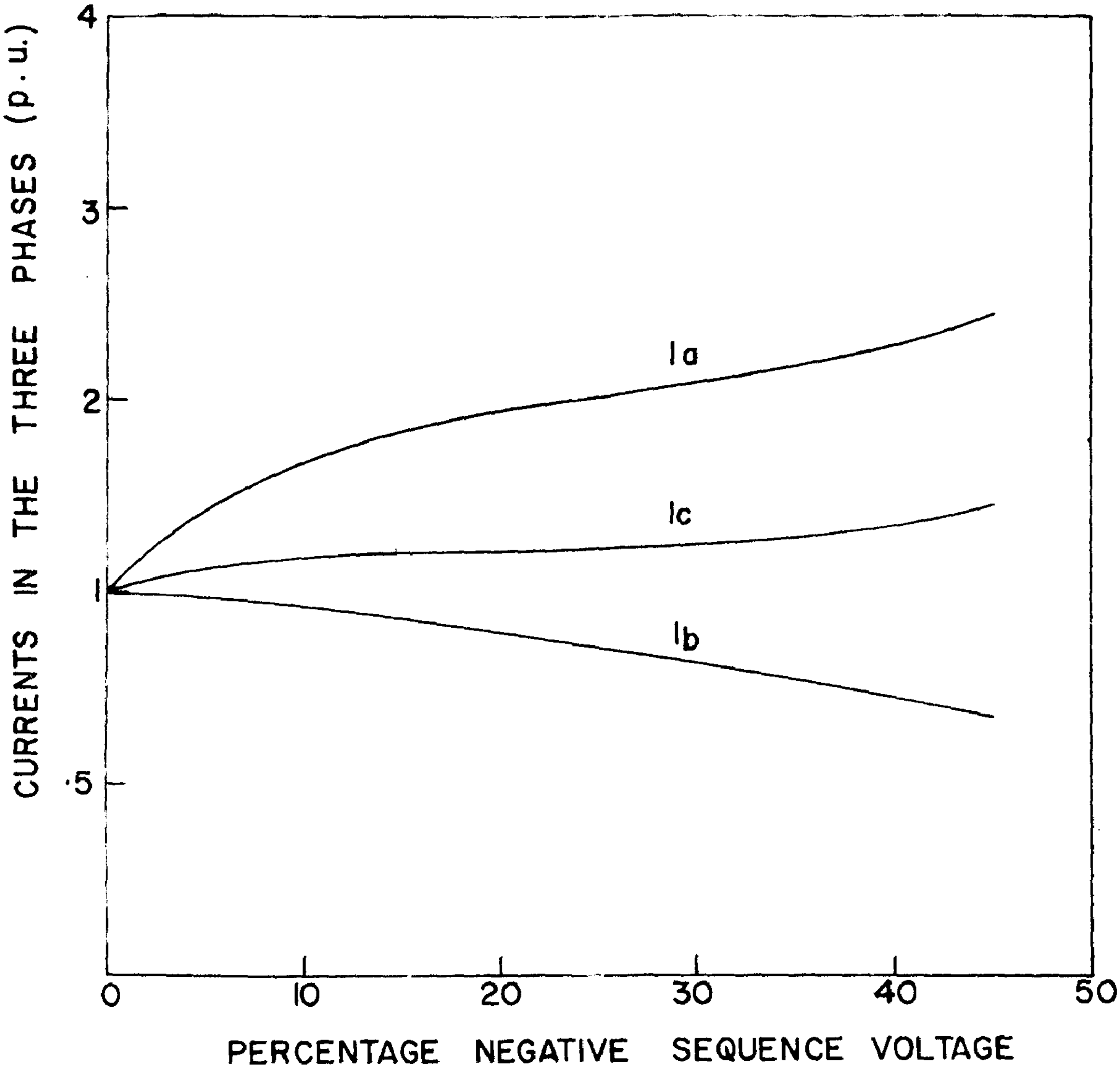


FIG.10. PHASE CURRENTS FOR DIFFERENT UNBALANCED CONDITIONS

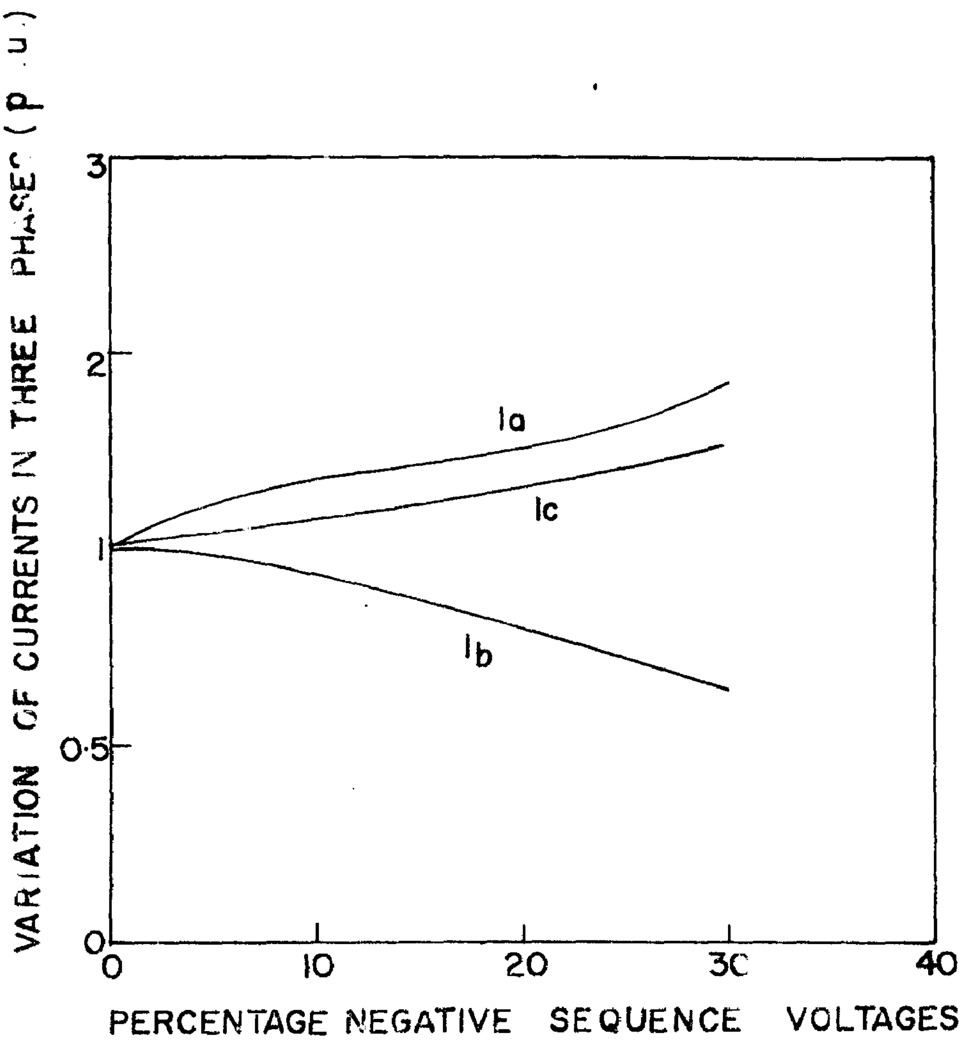


FIG.11. PHASE CURRENTS FOR DIFFERENT UNBALANCED CONDITIONS

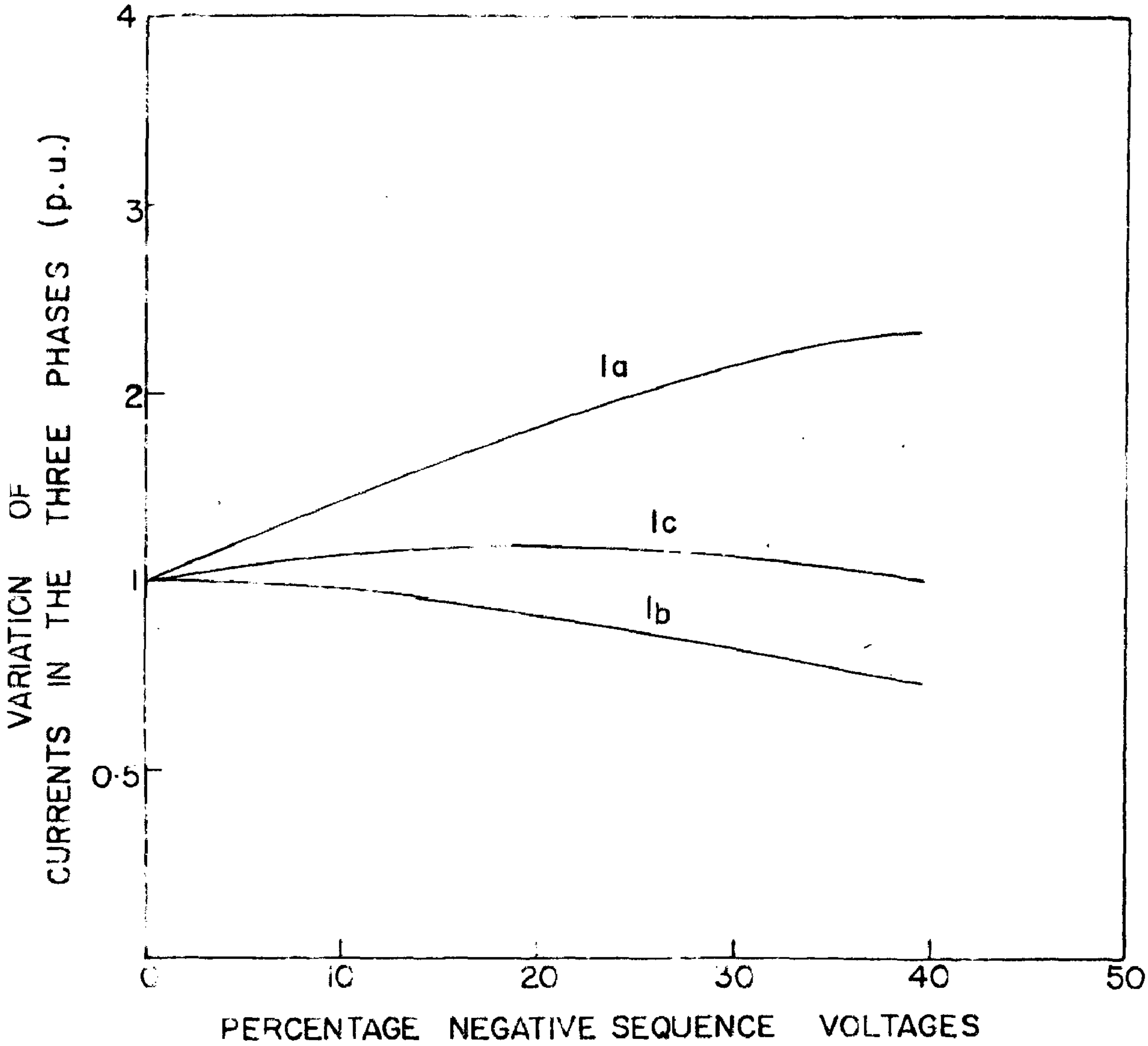


FIG.12. PHASE CURRENTS FOR DIFFERENT UNBALANCED CONDITIONS

value of P. in steps one can still maintain $\frac{Q}{P} = 1.86$ and further find out line currents for different values of increasing negative sequence voltage.

Now, if $\frac{Q}{P} = 1.86$, and $P = 0.1$ by repeating the above calculations the line currents were found to be as

I_a = 2.64 Amp

I_b = 1.914 Amp

I_c = 1.980 Amp

Thus, an idea of overloading of any phase is obtained for a given pattern of unbalanced supply voltage. As a typical study, the calculations were extended to two another practical sets of unbalanced voltages on Solid rotor machine as below:

Set No. II	Set No. III
V_{AB} = 410V	V_{AB} = 360V
V_{BC} = 370V	V_{BC} = 340V
V_{CA} = 430V	V_{CA} = 468V

and the curves of figures 10, 11, 12 show their variation of corresponding line currents indicating uneven loading of phases in each case.

-oOo-

-:CHAPTER IV:-

STATOR OPEN PHASE:

4.1.1 Introduction:

Much work appears to have been done on the performance of normal three phase Induction Motors with wound rotor or squirrel cage rotor while one of the phases are open. However, the author is unaware of any paper being published regarding the performance of the Solid rotor Induction Motor with one of the supply phases open. In any type of Induction Motor while considering various conditions of unbalance it becomes necessary to consider the operation of a three phase Induction Motor with one line open, since it can be recognised as the extreme case of unsymmetry that may occur. In such a case, it is a well established fact that a normal Induction Motor has no starting torque, but an already running motor continues to run even if one of the phases is suddenly opened. Obviously, there arises a question as to whether a Solid rotor Induction Motor already running, can continue to run if one of the phases of supply terminals is kept open suddenly. In this chapter an attempt has been made to give an answer to the above question.

4.1.2

Considering a Solid rotor Induction Motor under the normal conditions of operation it is known that the

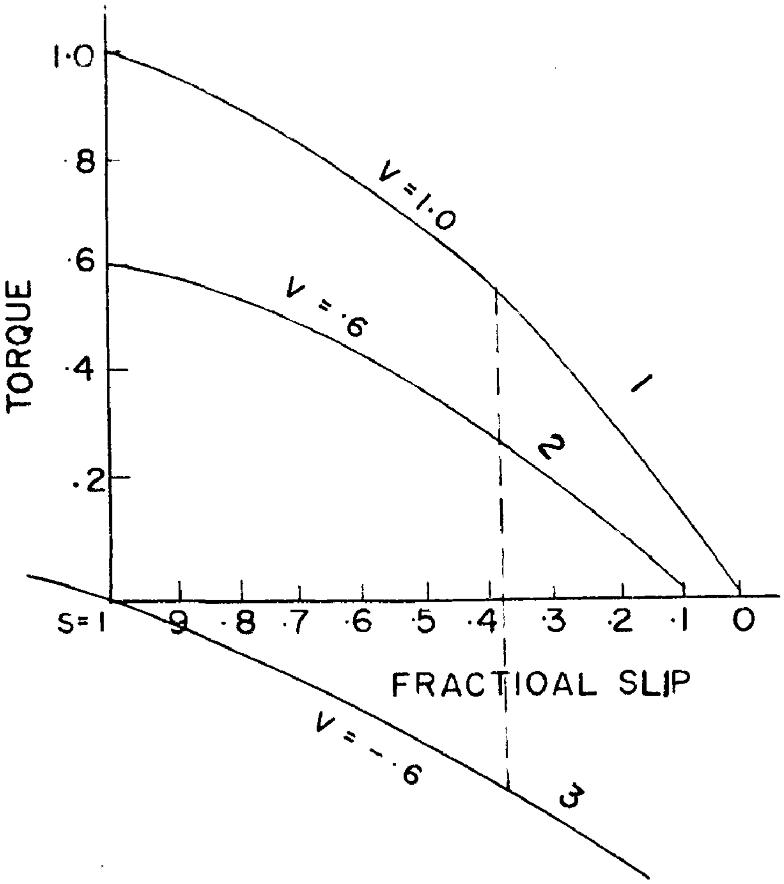


FIG.13. TYPICAL TORQUE - SLIP CURVES

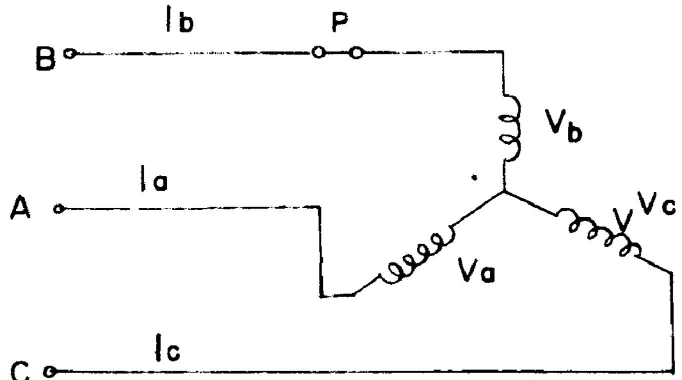


FIG.14 (a)

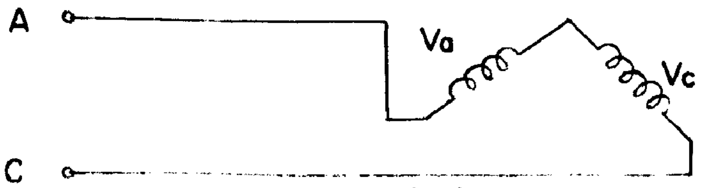


FIG.14 (b)

STATOR OPEN PHASE

torque is proportional to square root of the slip or in other words, Torque-slip characterestic approaches a parabola at a given supply voltage. The Torque-slip characterestics are shown in figure 13. It can be pointed out that as the supply voltages are reduced the nature of the Torque-slip characterestics remain approximately the same while magnetudes of torque get gradually reduced which satisfy the requirements of a servomotor. In such a condition the circuit diagram is shown to be as shown in figure 14.

Now, if one of the phases (say phase B) is opened by switch P, while the motor was running, then the circuit diagram reduces to that of figure 14(b).

On account of this, it is observed that, one of the phase voltages V_c has reversed in its direction. Under these conditions it can be pressumed that the point of operation gets momentarily shifted from characterestic number 1 onto the other characterestic No.(3) which shows the presence of a negative torque. This torque being in opposite direction tends to bring the rotor to a standstill.

Hence, from the above explanation it may be concluded that a Solid rotor Induction Motor running, ceases to run if one of the phases of supply terminals is opened.

4.1.8

An analytical proof to the extent of the above physical phenomena is attempted below:

From the basic approach through the electromagnetic theory and on the basis of Maxwell's equations the expression for the rotor impedance has been found to be

$Z_R = \frac{K'(1+j)}{\sqrt{s}}$ for a given Solid rotor Induction Motor,

$= \frac{K}{\sqrt{s}}$ for positive sequence

$= \frac{K}{\sqrt{2-s}}$ for negative sequence

If stator leakage impedance is neglected,

forward torque developed $T_1 = I^2_{a_1} R_{r_1}$

backward torque developed $T_2 = I^2_{a_2} R_{r_2}$

$T_1 = \frac{V^2_{a1}}{Z^2_1} \cdot \frac{R_r}{\sqrt{s}} = K_c \sqrt{s}$ where K_c is a constant

$T_2 = \frac{V^2_{a2}}{Z^2_2} \cdot \frac{R_r}{\sqrt{2-s}} = K_c \sqrt{2-s}$

Net torque $T = T_1 - T_2 = K_c (\sqrt{s} - \sqrt{2-s})$

The above relation indicates that

for $s = 1$ net torque is zero

and for $0 < s < 1$ net torque is negative, which mean that the the motor comes to a stand-still at slips less than 1.

4.2.1 Case of Double Unbalance:

Basically, the unbalanced operation of an Induction Motor can be classified into two categories:

(i) Single unbalance i.e. unbalance created either in stator or rotor.

(ii) double unbalance i.e. the unbalance created simultaneously in stator as well as in rotor.

The problem of double unbalance is generally of practical interest since it is noted that under double unbalance, excessive voltages may be present in stator and rotor circuits that may damage the insulation.

In the case of a solid rotor Induction Motor, double unbalance can not happen since the rotor is a fixed one in which no external impedance can be inserted. Also, internal rotor unbalance can not be used as a means of speed control since it has the solid rotor that has no accessibility.

Hence, it is felt that the problem of double unbalance does not arise in the case of solid rotor Induction Motor.

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-:CHAPTER V:-

HEATING AND DERATING OF SOLID ROTOR INDUCTION MOTOR

5.1 General Introduction:

It has been observed that excessive heating is generally experienced when Induction Motors are operated with unbalanced voltages. In such unbalanced conditions, there will be definitely temperature-rise above that of the temperature under normal balanced operating conditions. This is primarily due to increase in copper loss and also due to unbalanced spatial distribution of stator copper loss. Also, negative sequence current has an additional effect creating more heat.

5.2 Causes of Heating:

5.2.1 When a three phase Solid rotor Induction Motor operates on an unsymmetrical voltage supply, here also, the currents in different phases are unequal. Accordingly, there is likelihood of one of the phases being overloaded while the other two phases are being normally loaded or even less than the normal load(vide chapter III). Obviously the question will be at what conditions the worst situation can occur? Basically, since Solid rotor Induction Motor simulates a normal Induction Motor with respect to its stator side the analysis can be adopted in the following way:

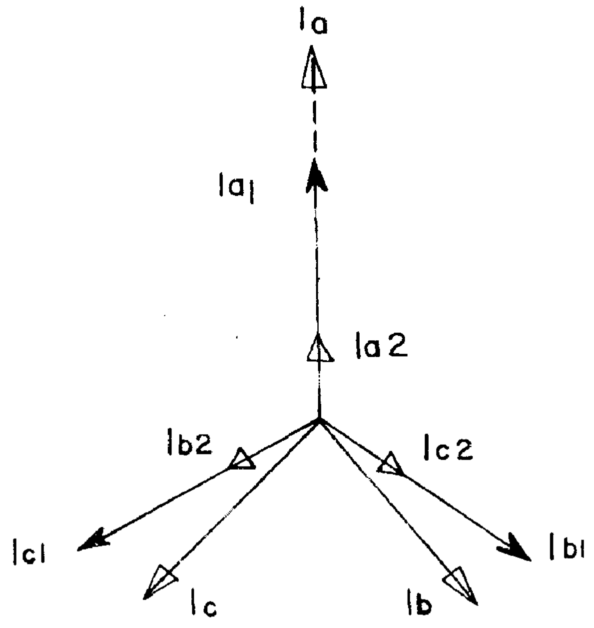

FIG. 15(a) CASE ⓐ WORST CONDITION OF UNBALANCE

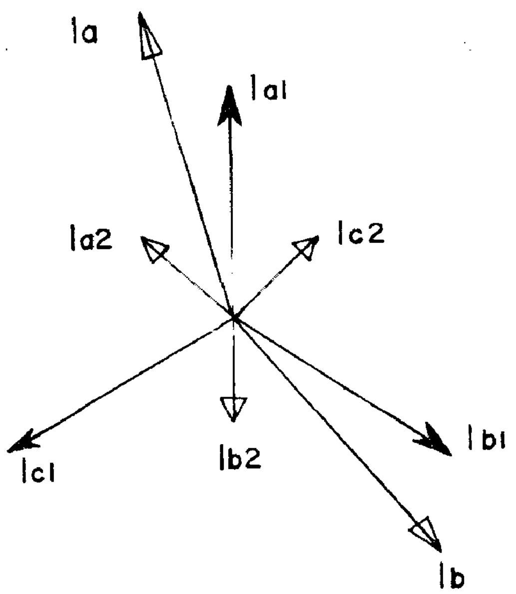

FIG. 15(b) CASE ⓑ WORST CONDITION LEADING TO MAXIMUM EQUAL LOADING OF TWO PHASES

If $$\frac{I_{a2}}{I_{a1}} = k \tag{5.1}$$

following the method of superposition from the phasor diagram,

$$I_a = I_{a1} + I_{a2} \tag{5.2}$$

from the law of resultant of vectors,

$$I_a^2 = I_{a1}^2\ (1+k^2+2k\cos\alpha) \tag{5.3}$$

$$I_b^2 = I_{a1}^2\ (1+k^2+2k\cos\ \overline{120+\alpha}) \tag{5.4}$$

$$I_c^2 = I_{a1}^2\ (1+k^2+2k\cos\ \overline{120-\alpha}) \tag{5.5}$$

In the above, if $k=0$, then $I_a = I_b = I_c$ which shows the case of balanced operation. Now, when the currents are unbalanced, the value of k and α will differ accordingly. Considering phase A alone, the worst condition of unbalance can occur when the current in phase A becomes a maximum. That is, the current in phase A will be maximum only when $\alpha = 0$ or in other words, when phasors I_{a1} and I_{a2} are collinear. (Figure No. 15a)

Therefore, $$I_a^2 = I_{a1}^2\ (1+k^2+2k) \tag{5.6}$$

or $$I_a = I_{a1}\ (1+k) \tag{5.6}$$

This expression gives an indication that the maximum copper loss is dissipated in phase A compared to that of the phases B and C when $\alpha = 0$ i.e. when positive sequence current I_{a1} and negative sequence current I_{a2} are in phase.

Under such conditions, it is in phase A in which one can expect maximum heat production that causes a maximum temperature rise.

5.2.2

However, from the phasor diagram, it is found that when $\alpha = 60^\circ$ (Equations 5.3, 5.4, and 5.5)

$$I_a^2 = I_1^2 \,(1+k^2+k) \quad (5.7)$$

$$I_b^2 = I_{a_1}^2 (1+k^2-2k) \quad (5.8)$$

$$I_c^2 = I_{a_1}^2 (1+k^2+k) \quad (5.9)$$

Hence, the above condition also leads to maximum overall loading since two phases A and C are equally overloaded depending upon the value of k while the third phase B is least loaded. This means that maximum heat is dissipated equally among the two phases A and C. (Fig 5.6)

5.2.3

For a given value of k it can be noticed that equation (5.6) gives a higher value than equation (5.7). Hence, it means that though both of the cases 5.2.1 and 5.2.2 lead to the worst condition from the point of view of overall temperature-rise, the maximum to which phase A can be loaded is from the consideration of case 5.2.1 where the phase A is extremely overloaded.

Considering case of 5.2.1, equation 5.6 shows that a negative sequence current of $0.5 I_{a_1}$ i.e. when $k = 0.5$, produces a 25% increase total stator copper loss and 125%

stator copper loss coupled with a some what greater increase in rotor loss which in turn causes an increase in the average temperature of the machine. However, the increased heat dissipation in phase A and decreased heat dissipation in the other two phases can cause only a small difference in overall average temperature-rise of the machine if the thermal conductivity of the material of iron between the phase belts tries to balance. However, if the thermal conductivity is poor in phase A then 125% increase in heat production in phase A obviously can result in a hot spot. Especially if the machine is of high voltage rating, then due to the increased conductor insulation hot-spot effects can be more disastrous.

With respect to the rotor, the negative sequence currents in the rotor at a per unit frequency of (2-s) presents a higher resistance to these eddy currents. Hence, it is expected that negative sequence currents produce more heat. Now, the heat developed within the solid rotor can be quickly dissipated since it basically is a homogeneous medium. Hence, heat of solid rotor is effective in further increasing the temperature of stator windings since there is a direct transfer of heat through the medium of the small air gap which already gets heated up due to increased temperature of the stator coils. Hence, it is expected that eddy current type of motor dissipates more power ampere. It becomes necessary that negative sequence currents should be given importance while determining the copper loss. Since it shows that total copper loss does not indicate the maximum temperature-rise. So unbalance factor should be known. However, roughly the relation $I_1^2+kI_{a_1}^2$ where $k < 1$ indicates the maximum temperature-rise conditions.

Thus, in the above paragraphs the detrimental effects of unbalance on line currents as well as temperature-rise of Polyphase Solid rotor Induction Machine have been indicated.

5.3.1 <u>Derating:</u>

Since it is established that under unbalanced voltage supply the Solid rotor machine is associated with increased losses, the machine can not be operated continuously at rated output without overheating. The only alternative to avoid overheating is to operate the machine somewhat at a lower current than the actual rated current. In such a case, the rating of the machine will be reduced below its specified rating by design, or in other words, with respect to its actual output we can say that the machine is "derated". The factor by which the machine is derated can be called as a derating factor.

Now, the derating factor of a Solid rotor machine can be said to be a function of the following:

(i) Negative sequence current or the ratio k which in turn depends upon the ratio of $\frac{Z_2}{Z_1}$.

(ii) Heat transfer coefficients of material of stator slot insulation, rotor etc.

In such conditions as explained above, it is of valuable interest to have an idea of allowable output under unbalanced voltage supply. Since the positive and negative sequence impedances have already been evaluated the same have been utilised to estimate approximately the allowable load of Solid rotor Induction Motor.

Now, $\frac{I_2}{I_1} = \frac{V_2}{V} \cdot \frac{Z_1}{Z_2}$

also, $I_a = I_1(1+k)$

Now, rated current $I_r = \frac{V}{Z_1} = \frac{\text{rated voltage}}{Z_1}$

Now, $I_1 = \frac{I_r}{1+k}$ (5.1

If the motor were not to be overloaded under conditions of unbalanced voltage supply the motor must run at such a load such that the load of the highest loaded phase may not exceed the safelimit, that is the rated value.

Hence, $I_1 = I_r - I_2$ for the worst condition, (5.1

Since, $I_2 = \frac{V_2}{V} \cdot \frac{Z_1}{Z_2} \quad I_r$

So, the positive sequence current should be

$$I_1 = I_r\left(1-\frac{V_2}{V} \cdot \frac{Z_1}{Z_2}\right) \quad (5.1$$

To avoid overheating, the positive sequence current should not exceed the value in (5.12) shown above. Now, to obtain an indication of variation of derating for all

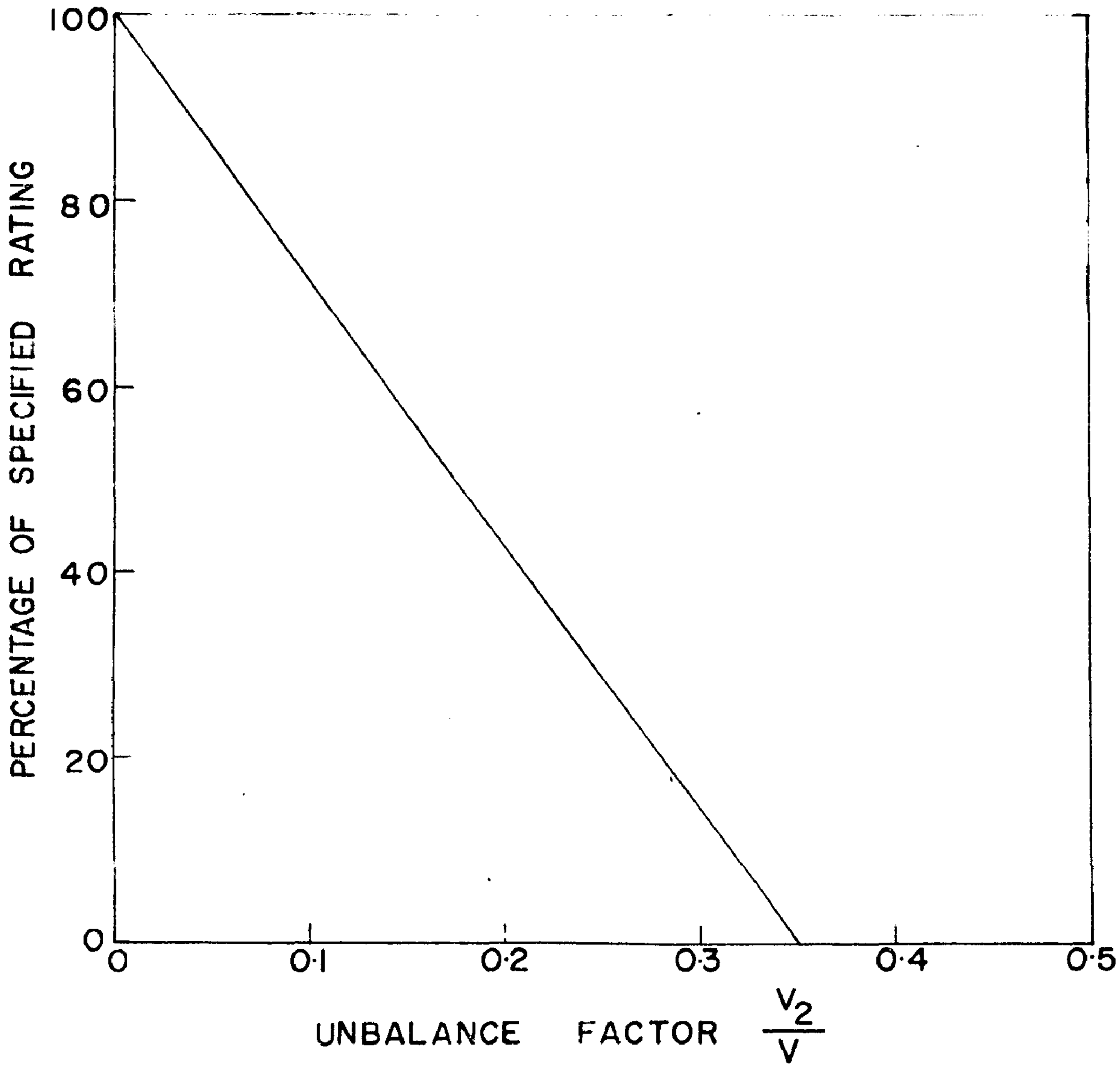

FIG. 16. ALLOWABLE OUTPUT FOR SOLID ROTOR INDUCTION MOTOR UNDER UNBALANCED VOLTAGES

practical purposes it is assumed that the output is directly proportional to the positive sequence current.

Thus, the maximum allowable output or Re-rating of the machine under unbalanced conditions is

$$P_u = \left(1 - \frac{V_2}{V} \cdot \frac{Z_1}{Z_2}\right) \text{ (specified rating)} \qquad (5.1$$

The factor in the brackets can be named as Re-rating factor since it is the the factor by which the new rating is determined under conditions of unbalanced operation. The variation of output for the Solid rotor Induction Motor under consideration has been determined and obtained as in figure 16 for a known value of $\frac{Z_1}{Z_2} = \frac{237}{82} = 2.88$. This obviously indicates roughly that when the unbalance is about 35% the output is practically zero for the worst condition of unbalance.

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-:SUMMARY AND CONCLUSIONS:-

On treating the Solid rotor Induction Motor the procedure followed here is a common one. A mathematical model of the device has been established and on the basis of the model typical analyses were carried out to find the performance.

The choice of the model, formulation of the problem, and analytical solution form the subject of the first chapter. The machine is idealized to the point where principles of linear electromagnetic theory are applicable. The nature of the rotor impedance is specified as a logical sequence of the machine geometry and idealized assumptions. Then basic Maxwell's equations are applied.

The limitations of the assumptions are worthy of comment. The first assumption 1.2 is admissible to some (24) extent since the error introduced due to this assumption is negligible. The second assumption, however, presents a highly distorted picture of the electromagnetic field phenomena in Solid rotor Induction Motor. The third assumption is permissible where as the fourth one does not at all represent the actual condition.

Utilizing the value of rotor impedance thus obtained by electromagnetic theory approach, the variation of losses of Solid rotor Induction Motor for different patterns of unbalanced voltages have been investigated. It shows that there is going to be loss as the unbalance increases. Further, it is found that one of the phases is overloaded.

In Chapter IV, it has been tried to establish a physical reasoning that a three phase Solid rotor Induction Motor ceases to run if one of the phases is opened while running. A very simple approximate picture of physical reasoning has been brought out to prove this phenomena.

Further, in an attempt to find out the derating of specified Solid rotor Induction Motor worst condition of unbalance has been considered. This shows a straight line relation of continuous output versus unbalance. This gives an idea of loading within the safe limits. However, the derating is not estimated from temperature-rise considerations which is expected to be some-what accurate.

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APPENDIX-I

Equation (8) in chapter I is a standard partial differential equation whose solution can be obtained by variable seperable method as indicated below:

The actual differential equation is

$$\frac{\partial^2 B_y}{\partial x^2}+\frac{\partial^2 B_y}{\partial y^2}=\frac{\mu}{\rho}\ \frac{\partial B_y}{\partial t} \qquad \text{(i)}$$

We begin by assuming a product solution for B_y

$$B_y = X(x)Y(y)T(t) \qquad \text{(ii)}$$

Then the differential equation (i) reduces to the form

$$\frac{X''}{X}+\frac{Y''}{Y}=\frac{\mu}{\rho}\frac{T'}{T}$$

Now, if $X = \epsilon^{+j\beta x}$; $T = \epsilon^{-jsw_o t}$

then, $\frac{X''}{X} = \beta^2$ (iii

$\frac{T'}{T} = -jsw_o$ (iv

so that $\frac{Y''}{Y} = \beta^2 - \frac{jsw_o\mu}{\rho}$

Putting $(\beta^2 - \frac{jsw_o\mu}{\rho}) = \gamma^2$

So $Y''+\gamma^2 Y=0$

$Y = C\epsilon^{\gamma y} + D\epsilon^{-\gamma y}$ is the general solution

Since $C = 0$ for physical reasons

$Y = D\epsilon^{-\gamma y}$

Therefore, total solution becomes

$$B_y = Re\left[\left\{D\,\epsilon^{-\gamma y}\right\}\epsilon^{j(\beta x - sw_o t)}\right]$$

Also,

$$\nabla \cdot B = 0 \text{ or } \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0$$

$$B_x = \int - \frac{\partial B_y}{\partial y} \cdot \partial x$$

$$= \quad Re \int \left\{ D\gamma\varepsilon^{-\gamma y} \right\} \varepsilon^{j(\beta x - sw_0 t)} \cdot \partial x$$

Therefore,

$$B_x = Re \left[\frac{D\gamma}{j\beta} \varepsilon^{-\gamma y} \varepsilon^{j(\beta x - sw_0 t)} \right]$$

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APPENDIX-II

Solution of the field equation in the air gap:

The differential equation obtained is a Laplace's equation in two dimensions.

So, $$\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} = 0 \qquad (1)$$

This equation can be solved by variable separable method, assuming a product solution if

$$B_y = X(x)\ Y(y)$$

Hence equation (1) becomes

$$\frac{X''}{X} + \frac{Y''}{Y} = 0$$

or $$\frac{Y''}{Y} = -\frac{X''}{X} = \beta^2 \text{ where } X = \mathrm{Re}\left[(\varepsilon^{j(\beta x - w_o t)}\right]$$

Therefore, $Y = A\,\varepsilon^{\beta Y} + B\,\varepsilon^{-\beta Y}$

A and B being arbitrary constants.

So that total solution becomes

$$B_y = XY = \mathrm{Re}\left[(A\,\varepsilon^{\beta Y} + B\,\varepsilon^{-\beta Y})\ \varepsilon^{j(\beta x - w_o t)}\right]$$

Further, from divergence equation

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0$$

or, $$B_x = -\frac{\beta}{j\beta}\ \mathrm{Re}\left[\left(A\,\varepsilon^{\beta Y} - B\,\varepsilon^{-\beta Y}\right)\varepsilon^{j(\beta x - w_o t)}\right]$$

Thus the required solutions of flux densities are obtained.

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PART B

AN APPROACH TO ELECTROSTATIC AND ELECTROMAGNETIC FIELDS SURROUNDING THE HEART

AN APPROACH TO ELECTROSTATIC AND ELECTROMAGNETIC FIELDS SURROUNDING THE HEART

1. Introduction:

In the era of technological revolution, the establishment of a sound difinition of Engineering is a fairly difficult task. No dèscrete line of demarcation can be drawn among different branches of Engineering. Especially, if one tries to see into the major advances made in the field of Electrical Engineering applied to medicine he can find how the engineering discipline is infused into the physiological problems. Much of the work appears to have been done in the last decade. Within a span of ten years, the subject gained its importance as one of the major branches of Electrical Engineering. At present, it goes with the name as Bio-engineering in general, and Bio-Medical Engineering in particular, where various principles of Electrical Engineering such as electrostatics, electromagnetics and control systems are being applied.

2.1 Bio-Medical Engineering - branches:

The main branches can be categorised as

i. Cardiography,

ii. Plethysmography,

iii. Electromyography,

iv. Electroencephalography.

Cardiography deals with activity of the heart. Plethysmography describes the subject of body volume and blood volume flow recording. Electromyography is concerned with the recording of muscle action potentials. Electroencephatography deals with the electrical activity of the brain.

2.2 In the present work, a few aspects of cardiology are dealt with. Cardiology from an engineer's point of view may be classified as

(i) Electrocardiography (ECG)
(ii) Magnetocardiography (MCG)
(iii) Vectorcardiography (VCG)
(iv) Phonocardiography (PCG)
(v) Ballistocardiography (BCG)

Electrocardiography (ECG) is with regard to the tracings of electrical potentials produced on the surface of the body by electrical activity of the heart. Vectorcardiography is a technique developed by the electrical engineer which facilitates the representation of the fluctuating heart vector at every instant of a cardiac cycle. Magnetocardiography deals with studies and record of magnetic fields developed in the medium immediately surrounding the heart. Phonocardiography is concerned with the record of heart sounds by obtaining a visual record which supplements the information obtained

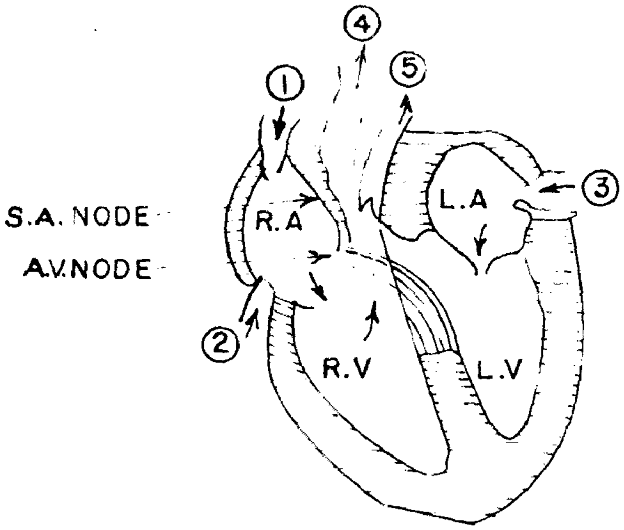


FIG. I. SCHEMATIC DIAGRAM OF THE HEART

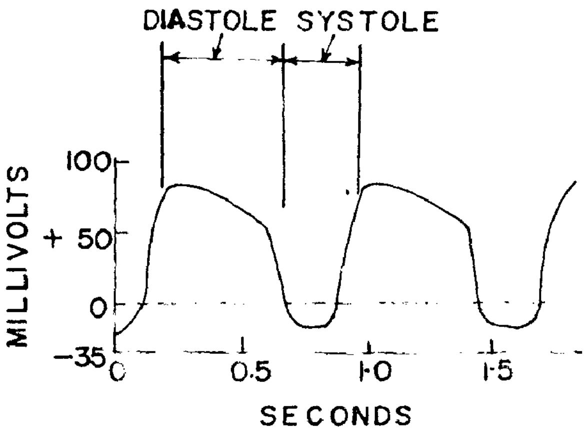


FIG. 2. RELATIONSHIP OF CARDIAC MEMBRANE POTENTIAL TO CARDIAC CONTRACTION (SYSTOLE) AND RELAXATION (DIASTOLE)

by an ordinary stethoscope. Ballistocardiography is a special case of transient oscillatory response to a pulseforce. In the following articles, attention is confined to the first two topics.

3.1 Mechanical action of the heart:

The healthy heart acts as a two stage pump. In the first stage, the action is performed on the right side of the heart. Here, Venous blood, collected from the body through two large veins (1) and (2) (as in the figure 1) is directed into a chamber known as the right auricle (R.A.). From there, it passes via an intake valve to the right Ventricle (R.V.) which is like a pumping chamber. The second stage of action takes place on the left side of the heart. The L.A serves as a reservoir that receives oxygenated blood. This oxygenated blood passes to the left Ventricle, a high pressure pumping chamber that forces the oxygenated blood through an exhaust valve to the aorta(4) and thence to the arterial system of the body.

3.2.1 The existence of Cardiac potential:

Associated with the above explained mechanical action of cardiac muscle fibres, there are other important simultaneous physiological occurrences. One among them with which we are concerned is the phenomena of electrical changes of potential that arise with all muscle and body cells engaged in cardiac activity. The electrogram is the result of record of such electrical potentials produced on body surface by cardiac electrical activity. Historically, in 1838 Maltenci proved that the muscle itself is a source of electrical potential and in 1856 Kollikin and Muller first showed that the heart muscle activity is accompanied by small changes of electrical potential. At this juncture,a question can be asked as to how the cardiac potentials manifest themselves on the body surface.

3.2.2 Production of cardiac potential:

All types of muscle and nerve tissues are composed of body cells which when engaged in activity have associated with themselves changes in electrical potential of the outside of the cell relative to the inside. The inside of a physiological cell is in the stationary state at about 85 mV negative with respect to the outside. On stimulation which may be provided in a variety of ways (through an electrical means or any other means) the cell potential alters and its polarity reversed for a short time the inside being positive with respect to outside by as much as 40 mV. From this polarised state, the cell soon reverts to the stationary state. All of these happenings occur in fraction of a second.

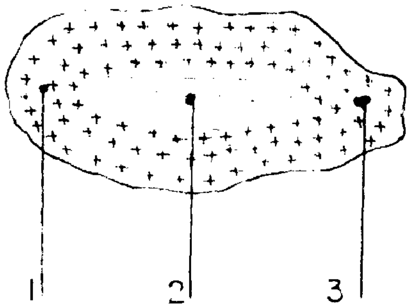


FIG. 3

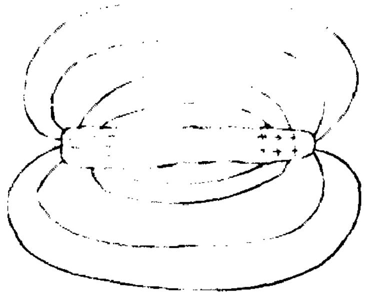
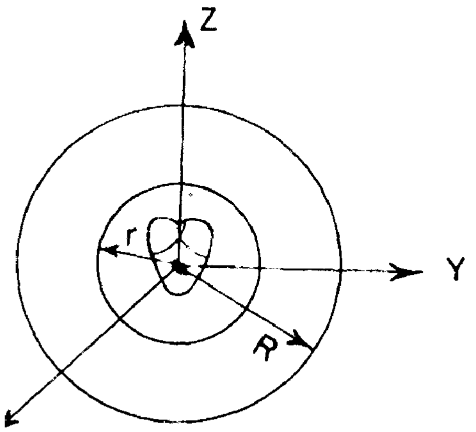


FIG. 4 FLOW OF CURRENTS THROUGH ELECTROLYTIC FLUID SURROUNDING A STRIP OF CARDIAC MUSCLE THAT HAS BEEN DEPOLARISED AT ONE END



_OCATION OF HEART IN RELATION TO SPHERE

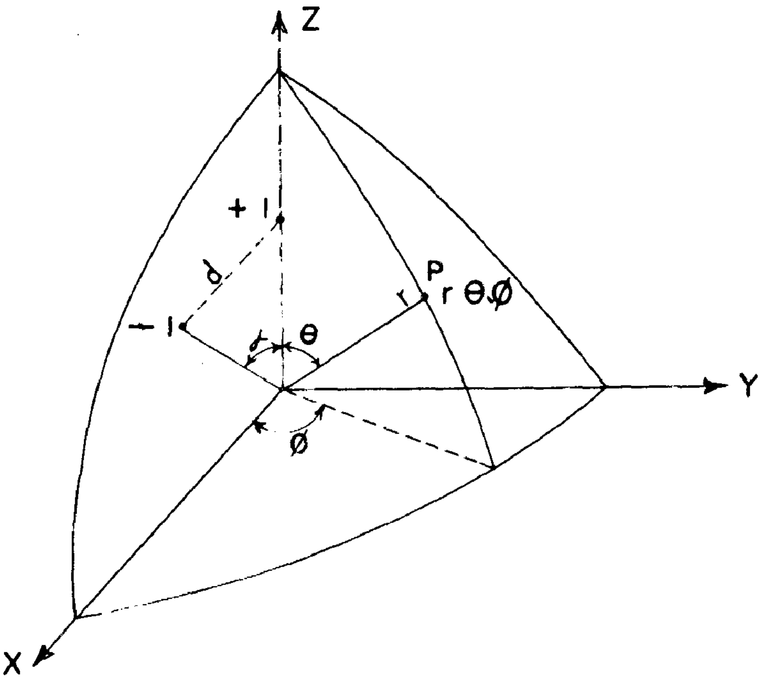
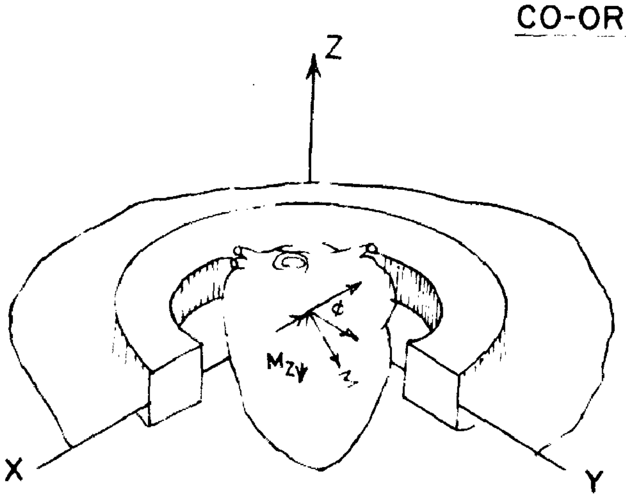


FIG. 6. ANALYSIS USING SPHERICAL CO-ORDINATES



G.7. GEOMETRICAL MODEL USED TO CORRELATE THE ELECTRIC AND MAGNETIC FIELDS IN THE VOLUME CONDUCTOR AROUND THE HEART

3.3.1 <u>Rhythmicity and altering cycle of cardiac muscle:</u>

Cardiac muscle contracts rhythmitically approximately once in every second. Figure 2 shows the variation of membrane potential during two complete cardiac cycles. It can be observed that toward the end of heart beat the membrane potential builds up to a normal resting value of +85 mV outside the membrane with respect to inside. Then during the diastole the membrane potential decays slowly at first until a critical level wherefrom it gets suddenly depolarised causing contraction to occur.

3.3.2 The figure (3) illustrates a syncytical mass of cardiac muscle which has been stimulated at its central-most point. Prior to the stimulation of the mass of cardiac muscle, all of the exterior muscle is positive and the interior is negative. As soon as one area of cardiac syncytium becomes depolarised, negative charges leak to the outside of the depolarised area making the particular surface area negative with respect of remaining surface area of heart which is still polarised. Once the cardiac muscle gets depolarised at one end it can be represented as shown in the figure (4). Since it is surrounded by the extracellular fluid the currents are expected to flow between the two areas of opposite potential in an electrolytic solution in large elliptical paths as shown in the figure 4.

3.4.1 Necessity for the determination of surface potentials:

The result of all the above phenomena is to produce electric potential at the surface of the skin. Hence, a knowledge of temporal variations of the electrical potentials and deviations from an accepted normal range can provide much diagnostic information. This necessitates to determine potentials on the surface either analytically or experimentally.

3.4.2 Concept of equivalent current generator:

To quantitate surface potential on the body, a proper form of internal current source is to be introduced; that is to say that, an exact theoritical model is needed based on which the analysis for potential distributions can be made. Such an assumed theoritical model of the generator within the heart is often called as "equivalent cardiac generator".

3.4.3 Analytical approach to find the electric potential produced by positive and negative current sources situated within the heart.[1]

Assumptions:

(i) The human body is assumed to be a homogeneous, isotropic, resistive medium of spherical shape.

(ii) The equivalent cardiac current generator is taken to have two current sources +I and -I seperated at a distance 'd'.

In brief, the problem consists of finding out an expression for the electric potential at the perfectly insulating wall of a homogeneous sphere of conductivity σ and radius R resulting from a current generator arbitrarily located within the sphere (Fig.5).

Mathematically, it means that the potential V should satisfy Laplace's Equation $\nabla^2 V=0$ as well as boundary condition. The boundary condition, here is that the normal derivative of the potential must be zero at the boundary $r = R$.

By using spherical co-ordinates, let O, the origin of the co-ordinate system be at the centre of the sphere. Let one of the sources +I pass through the Z-axis at a distance b ($b<R$ from the origin) oriented in ZX plane ($\Phi=0$). Let another source - I be at a distance d from +I such that

$$d = \sqrt{(a^2+b^2-2ab\cos\alpha)} \qquad (1$$

If a point P (r,θ,Φ) is chosen within the sphere such that it makes an angle θ with b and angle β with a then,

$$\cos\beta = \sin\alpha\,\sin\theta\,\cos\Phi + \cos\alpha\,\cos\theta$$

If the plane is unbounded (i.e. at infinite distance) the potential V_{∞} in an unbounded medium for a source at P is given by

$$V_{\alpha} = \frac{I}{4\pi\sigma}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) \qquad (2)$$

where $\frac{1}{r_b} = (r^2+b^2-2br\cos\theta)^{-1/2}$

$\frac{1}{r_a} = (r^2+a^2-2ar\cos\beta)^{-1/2}$

At the boundary for definite spherical shape it can be shown to be equal to

$$V_R = \frac{I}{4\pi\sigma}\left\{\frac{2}{r_b} - \frac{2}{r_a} + \frac{1}{R}\log_e \frac{r_a+R-a\cos\beta}{r_b+R-b\cos\theta}\right\} \qquad (3)$$

Now, as a special case if d = 0 in the above expression (3) the case of a general dipole is obtained.

3.5 However, the method outlined above is a grossly simplified one. The electrical model of the heart was considered to be electrically equivalent to a dipole situated in a linear, isotropic, homogeneous medium fixed in position with orientation and magnitude as variables. If non-linearity, anisotropy and heterogeneity effects are more then, the equivalent heart dipole may have the effects lumped in its dimensions. Hence, doubts regarding the validity of dipole hypothesis were raised by various authors. Many tried to find a more complicated equivalent cardiac generator. Two of them are:

(i) Multiple dipole generators

(ii) Multipole generators

Under the assumption of multipole generators alongwith dipole, it assumes that quadrupole etc. are also present. In this connection, Halvin and Plonsey[11] have shown mathematically and also experimentally that the potentials generated by a live turtle heart at the centre of a spherical electrolytic tank have a significant quadrupole term is present and also predicted the possibility of a higher order pole.

3.6 In 1966, Burr, Pikkington, Boineau and Spach[12] presented a method of determining the potentials over the surface of 3-dimensional volume due to internal current sources where the volume may be non-homogeneous and irregularly shaped. The method illustrates the determination of potentials using N-simultaneous which when solved produce the potentials at N different surface points. The N-simultaneous equations are solved by an iterative technique on an IBM computer.

Many biological tissues have some degree of directional organisation and would be expected to behave anisotropically to electrical current conduction. Hence, an analysis of current and potential distributions throughout the bulk may be required. In 1967, Stanley Rush[13] theoritically analysis the influence of the anisotropic heart which is particularly very difficult. In this the author expressed the basic differential equations of static fields and steady current flow which are arranged to stress the field and conductivity dependent change distributions that arise in an anisotropic media. The derived differential equations specify the change distributions which accumulate in anisotropic conductors as a result of current flow. This analysis showed that there are many anisotropic problems with simple geometry whose solutions can be found as those of isotropic problems.

It is normally impossible to construct a model and measure the required quantities since the model would require an anisotropically conducting medium which can/not be easily recognised in practice. However, in an interesting paper (1967) Nicholson[14] suggests a possibility of constructing a model by using another model with a suitable isotropic medium. The only difference is that one has to use the corresponding scale factors devised.

4.1.1 Magnetocardiology:

Earlier discussions were based upon static field theory. It is well known from basic electromagnetic theory that Maxwell's equations predict the existence of a magnetic field associated with any time varying electric field. However, the concept of a biologically produced magnetic field was suggested by Valentuzzi in 1958. Also, Seebel and Morrow reported the detection of magnetic field accompanying impulse conduction in 1960 which appears to be the first experiment on these lines.

4.1.2 In 1963, Stratbucker, Hyde, and Wixon (15) working on magnetocardiography demonstrated that the magnetic field associated with cardiac activation was of sufficient magnitude to be recorded by standard electronic techniques. They have tried to correlate the experimental data with analytical results.

In this experiment, the heart of a guinea pig was placed in the centre of a double-walled plastic sphere which contained six typical electrodes protruding into the inside surface. The bipolar electrodes were collinear with each of the axes XX', YY', ZZ'. The volume conductor medium surrounding the heart was maintained at a constant temperature. Cardiac electrograms were recorded from X, Y, Z axes. A toroidal solenoid containing N turns of suitable wire wound as a core was suspended in the centre of the sphere such that the solenoidal axis is coincident with the ZZ axis. The coil-output after suitable amplification is recorded.

4.1.3 The electric and magnetic fields in the volume conductor surrounding the heart are correlated as follows:

Assumption: The cardiac current generator is taken to be a dipole current generator M. The configuration is as shown in figure 7 . Since the volume conductor curfents induce a voltage into the sensing coil the rate of change of the total effective current is directly related to the voltage in the turns of the coil. Hence, MCG record bears a direct relationship to the time derivative of the cardiac dipole moment.

Development of this relationship is mainly based on Maxwell's equations. From Fig.7 magnetic field intensity H_Φ at a point P (at a distance R from a dipole) can be found out.[9]

$$H_\Phi = \frac{\sin\theta}{4\pi R^2} M \tag{4}$$

if r is much less than R then the flux density will almost be uniform throughout the cross-section of the core.

$$B_\Phi = \mu H_\Phi \tag{5}$$

where $\mu = \mu_r \mu_o$ and μ_r = relative permeability of the core material.

$$\Phi = B_\Phi \cdot A = \frac{\mu r^2}{\pi R^2} M \sin\theta \tag{6}$$

or,

$$\Phi = \frac{\mu}{\pi}\left(\frac{r}{R}\right)^2 M \sin\theta$$

Therefore, voltage induced in N turns on this core

$$V = N \frac{d\Phi}{dt}$$

$$= \frac{\mu N}{\pi}\left(\frac{r}{R}\right)^2 \frac{dM}{dt} \sin\theta \tag{7}$$

Thus, the authors were partly successful in ~~by~~ their experiment in proving that the above relation is valid approximately. This shows that it might eliminate the necessity of electrical contact to the patient as required in electrocardiography.

Eventhough the above expressions are derived for ideal cases, a rigorous mathematical approach is yet to be sought for a practical case of interest. However, the scope for further work, appears to be vast since many problems of electrophysiology are concerned with the distribution of current and potential due to sources and sinks inside a volume conductor.

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