ANALYSIS OF PROTECTIVE RELAY PERFORMANCE BY MEANS OF RELIABILITY THEORY

A Dissertation
submitted in partial fulfilment
of the requirements for the Degree
of
MASTER OF ENGINEERING
in
POWER SYSTEM ENGINEERING

By
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CERTIFICATE

Certified that the dissertation entitled "Analysis of protective relay performance by means of reliability theory" which is submitted by Sri JUGAL KISHORE in partial fulfilment for the award of the Degree of Master of Engineering in Power System Engineering of University of Roorkee is a record of student's own work carried out by him under our supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for a period of 8 months from December 1967 to July 1968 for preparing the dissertation for Master of Engineering at the University.

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(JUGAL KISHORE)

CONTENTS

S_Y_N_O_P_S_I_S

The statistical information about the failures in the working of devices can be used for improving the construction of existing ones and developing new ones as well as in the analysis of their production. This calls for development of the general methods of Reliability Theory to make it suitable for the failure analysis of protective schemes of large Power Systems. Failure of a protective scheme may lead to great financial loss and adverse consequences. Therefore the failure analysis of a protective scheme needs thorough investigation. The present work discusses mainly the following:

1. General methods of Reliability Theory, including the discussion of different failure distributions.

2. Failure analysis of maintained and non-maintained systems for different mathematical models usually encountered in Reliability analysis.

3. Component failures, their reasons and the effect of their failure on the system containing these components.

4. Protective relays whether conventional or otherwise operate under different conditions and this calls for modification of reliability principles for application to their failure analysis.

5. Detailed analysis of Selective and non-selective operation of relays.

6. Different techniques of reliability evaluation of a complex system and their usefulness in reliability assessment.

LIST OF SYMBOLS

M_o	..	Fixed number of components
M_f	..	Number of components failed out of M_o
M_s	..	Number of components survived out of M_o
$\frac{d}{dt}$	..	Differential
R(t)	..	Reliability function (R)
F(t)	..	Failure function (F)
f(t)	..	Failure density function (f)
A	..	Failure rate
B	..	Time between two consecutive failures (1/A).
h(t)	..	Failure rate function.
a,b	..	Emperical derived parameters or constants
t	..	Time
m	..	Number of trials
n	..	Number of occurance out of m trials
p	..	Probability of success occurance
q	..	Probability of failure occurance (1-p)
k	..	Average number of failures
s	..	Laplace operator
dt	..	Small time interval
v	..	mean wearout life
u	..	Standard deviation of the mean life time from the mean life v.
N	..	Number of events
C	..	Constant
w(t)	..	Deterioration function
d(t)	..	Instantaneous damage function.
G	..	Constant of integration
T	..	Total life time
x	..	Number of equipments or components.

P	..	Probability of surviving
A_o	..	Failure rate probability of on line equipment
A_f	..	Failure rate probability of off line equipment
A_s	..	Failure rate probability of switch
T_w	..	Time at which wearout starts.
T_b	..	Debugging time
G(t)	..	Repair distribution function
r	..	Repair rate
A(t)	..	Availability function
MTTF	..	Mean time to failure
m,	..	Number of series elements
n,	..	Number of parallel elements
q	..	Probability of appearing the undesired output signal of one element (probability of non-selective action).
q'	..	Probability of non-selective action for scheme.
q_1	..	Probability of failure for an element.
q_1'	..	Probability of failure for scheme.
q_a	..	Actual probability of non-selective action.
Y_c	..	Gain in selectivity
Y_o	..	Reduction in probability of failure.
$1 \cup 2$	..	Elements 1 and 2 are in parallel (or gate)
$1 \cap 2$	..	Elements 1 and 2 are in series (and gate).

INTRODUCTION

In the last few years reliability has become of prime concern to an Engineer or Technologist as the complexity of his system grows more and more. With the advancement of space technology, reliability has taken an important place, as each constituent part or system of a space vehicle has to function properly during its operative life for a successful experiment. The cost of failure to operate successfully may be quite high.

Generally the cost of unreliability is not only the cost of the failed item, but of the associated equipment as a whole, which is damaged or destroyed as a result of failure, due to inter-dependancy between components in a complex system. To a power Engineer the reliability of protective scheme is equally important, since the failure of a protective scheme whose function primarily is to protect the equipment under abnormal conditions, may lead to heavy loss of money, time and also human life.

Recently, there have been efforts [22,27] in the direction of making reliable operation of protective schemes. Studies have been carried out to analyse the underlying theory of the reliable operation of protective relays.

In the present work, the author has tried to analyse the successful operation of relays through the existing concepts of Reliability theory as such reliability is a new and rapidly developing field. New ideas and methods are appearing constantly and in this context one can safely say that reliability is a field in which theory of today may become the fact of tomorrow or other-wise.

In the first chapter the author has explained the mathematical concepts of reliability and different factors on which it is dependent or depends.

In the second chapter of this dissertation the author elaborately discusses the different statistical distributions necessary to represent the failure phenomenon of any component or equipment. Some of the distributions have been deduced for a combination of two failure functions and with the assumption of different failure rates. Also discussed in Chapter-2 is the common life curve and possible, appropriate distribution functions describing a particular zone of this curve. Applications of different distributions have been indicated clearly.

Depending on system maintenance the failure analysis has been done in Chapter 3 of the dissertation. Different system configurations have been considered for the analysis of maintained and non-maintained systems using proper mathematical models. Importance of maintenance has been stressed with the help of proper graphs.

As the relays differ in respect of their operation from other conventional components which are constantly in use during their life time, the reliability theory, before it could be applied to relays, needs certain modifications. The relay must operate when desired but should not when not required. This discriminative requirement is called selectivity of the relay and must be present for reliable operation. The selective and non-selective features of protective relays have been analysed thoroughly in Chapter-4. A general study of component configuration has been studied with the help of the computer (IBM 1620) and optimum values of probability of failure and probability of non-selective action have been found out for different configurations. One can select from the charts of Chapter-4, the optimum values of these two varients as per requirements during the design stage.

The critical review has been carried out for component failure analysis in Electrical and Electronic circuits in Chapter-5. The mode of these failures of different electronic components have also been discussed. The author performed some experiments on carbon composition resistors and found results consistant with the theory as per detail given in Chapter-5.

Chapter 6 discusses the causes of common failures in conventional and unconventional relays with particular emphasis on reasons of change in relay contact resistance. Certain precautions needed to ensure a reliable operation of relays have been described.

In the last Chapter the author has given different techniques for the evaluation of reliability of a system. Techniques for reliability evaluation of complex systems such as non-series parallel etc., have been studied through the Bayesian theorem. Out of tolerance failures can be analysed effectively by the Monte-Carlo method. The method of approach and related flow chart have been given for the use of the Monte-Carlo method.

In short the effort has been to present the modified reliability approach for the failure analysis of protective schemes using conventional and unconventional (static) relay circuits.

CHAPTER - 1

CONCEPTS OF RELIABILITY THEORY

1.1. Definition of Reliability:

In its general form "Reliability is a probability of success". A widely accepted definition reads "Reliability is the probability of a device performing its purpose adequately for the period of time desired under the operating conditions encountered in practice". More specifically reliability expresses the number of chance of an equipment to operate without failure for a given length of time in an environemtn for which it is designed.

1.2. Mathematical Concepts:(1,2)

If M_o number of components (same type) are repeatedly tested and out of which M_f the number of component fail and M_s number of components which survive then,

$$M_o = M_s + M_f \quad \ldots \quad \ldots \quad (1.1)$$

from equation (1.1) it is clear that if number of surivival components increases then the number of failures decreases exactly by the same amount, M_s increases. So reliability of components surviving at any time t is defined as-

$$R(t) = \frac{M_s}{M_o} = \frac{M_s}{M_s+M_f} \quad \ldots \quad (1.2)$$

and similarly,

$$F(t) = \frac{M_f}{M_o} = \frac{M_f}{M_s+M_f} \quad \ldots \quad (1.3)$$

$R(t) + F(t) = 1.$ (which is evident also)

$$R(t) = \frac{M_o-M_f}{M_o} = 1 - \frac{M_f}{M_o} \quad \ldots \quad (1.4)$$

differentiating (1.4),

$$\frac{d}{dt} R(t) = \frac{d}{dt} (1 - \frac{M_f}{M_o})$$

$$= - \frac{1}{M_o} \frac{d}{dt} M_f \qquad \ldots \qquad (1.5)$$

as M_o is constant-

from (1.5),

$$\frac{d}{dt} M_f = - M_o \frac{d}{dt} R(t) \qquad \ldots \qquad (1.6)$$

which is the rate at which component fails-

But $M_f = M_o - M_s$

therefore,

$$\frac{d}{dt} M_f = - \frac{d}{dt} M_s$$

dividing (1.6) by M_s

$$\frac{1}{M_s} (\frac{d}{dt} M_f) = - \frac{M_o}{M_s} \cdot \frac{d}{dt} R(t)$$

The left hand side is defined as the probability of failure per survival component and will be called the failure rate A.

$$A = - \frac{M_o}{M_s} \frac{d}{dt} R(t)$$

or

$$A = - \frac{1}{R(t)} \cdot \frac{d}{dt} R(t)$$

or

$$A.dt = - \frac{dR(t)}{R(t)}$$

Integrating

$$\int_0^t A.dt = - \int_0^R \frac{dR(t)}{R(t)} = - \log_e R(t) + \text{Constant}$$

but when $t = 0$ $R(t) = 1$

hence-

$$R(t) = e^{-\int_0^t A dt}$$

$$= \exp\left(-\int_0^t A\,dt\right) \quad \ldots \quad \ldots \qquad (1.7)$$

$$B = \frac{1}{A} \quad \text{or} \quad A = \frac{1}{B}$$

where,

B is the time between two consecutive failures.

In the above derivation no assumptions have been made about the failure rate A, and therefore A may be a constant, any variable, a differential or a integral function of time t. Therefore equation (1.7) can be represented as reliability function in most general way. The equation (1.7) can be applied to all possible kinds of failure distribution functions.

From equation (1.5) it is clear that $\frac{d}{dt}R(t)$ represents the slope of R(t) at any point t. This slope is always negative right from t = 0 to when t = infinity. In equation (1.5) $\frac{d}{dt} M_f$ which represents the frequency at which failures take place at any time t provided none of the components is replaced. If $\frac{d}{dt}M_f$ is plotted against t, the time distribution of the failures of all the original M_o components is obtained. If $\frac{1}{M_o} \frac{d}{dt} M_f$ is plotted against time, then failure frequency curve per component is obtained. It is thus a unit frequency curve, called the failure density curve f(t) or generally denoted by f.

or

$$f = -\frac{d}{dt} R(t) = \frac{1}{M_o} \frac{d}{dt} M_f \quad \ldots \qquad (1.8)$$

$$F(t) = \frac{M_f}{M_o} \qquad \text{(from equation 1.3)}$$

or

$$\frac{d}{dt}F(t) = \frac{1}{M_o} \frac{d}{dt}M_f$$

substituting this in equation (1.8)

$$f(t) = \frac{d}{dt} F(t)$$

or

$$F(t) = \int_0^t f(t)\, dt \quad \cdots \quad (1.9)$$

equation (1.9) shows that the probability of failure at any time t is the area under the failure density curve taken from t=0 to t.

$$R(t) = 1-F(t)$$

As the area under the density curve is always unity.

i.e. $\int_0^\infty f(t)\, dt = 1.$

or

$$\int_0^t f(t)dt + \int_t^\infty f(t)\, dt = 1.$$

or

$$R(t) = \int_t^\infty f(t)dt. \quad \cdots \quad (1.10)$$

1.3. Properties of Conditional failure rate A:

The failure rate A, which has been defined earlier in general, is a function of time t, i.e.

$$A = h(t) \quad \cdots \quad (1.11)$$

The conditional failure rate of a life or failure distribution plays an important role in reliability analysis. The correct knowledge of this failure rate h(t) uniquely determines the failure density function, failure function and reliability function or some other related function.

$$H(t) = \int_0^t h(t)\, dt \quad \cdots \quad (1.12)$$

or

$$H(t) = \int_0^t h(x)\, dx$$

and the failure density function can be defined as-

$$f(t) = h(t).e^{-H(t)} \quad \cdots \quad (1.13)$$

The failure distribution is-

$$F(t) = \int_0^t f(t)\, dt = 1- e^{-H(t)} \quad \cdots \quad (1.14)$$

and the reliability function $R(t)= e^{-H(t)}$ ... (1.15)

CHAPTER - 2

STATISTICAL DISTRIBUTIONS

To analyse any statistical data available, the knowledge of different distribution function is essential. All events behave according to some law or the other although being random in nature. It is possible that a particular distribution function may not exactly represent the history of events, but it may still be done with some tolerance on the closeness.

2.1. Classification of various distributions:

2.1.1. Conditional distribution functions:

If the probability of failure of a component in time interval t to (t + dt), provided it has survived to time t is independent of t. This means that component does not age or wearout but fails due to some severe adverse sudden conditions for example, sudden over voltages, short circuit or severe shock. This corrosponds to

$$h(t) = A \quad \text{(constant failure rate)}$$

hence-

$$H(t) = \int_0^t h(t)\,dt$$

$$= A.t$$

$$\text{or } R(t) = e^{-A.t} \qquad \ldots \qquad \ldots \qquad (2.1)$$

equation (2.1) is called an Exponential Distribution.

If the probability of failure in the time interval t to (t+dt) is not constant (as assumed above) but varies linearly with time-

$$\text{i.e., } h(t) = a.t$$

$$H(t) = \int_0^t at\,dt$$

$$= \tfrac{1}{2}\,at^2$$

$$R(t) = \exp\left(-\tfrac{1}{2}\, a.t^2\right) \quad \ldots \quad \ldots \quad (2.2)$$

Now considering a more general case i.e. probability of faiłure in time interval t to (t+dt) is given by-

$$h(t) = \frac{b}{a} t^{b-1} \qquad a > 0$$

$$H(t) = \frac{b}{a} \int_0^t t^{b-1}\, dt = \frac{t^b}{a} \qquad b > 0$$

$$t \geq 0$$

$$R(t) = e^{-t^b/a} \quad \ldots \quad \ldots \quad (2.3)$$

$$f(t) = \frac{bt^{b-1}}{a} e^{-t^b/a} \quad \ldots \quad (2.4)$$

equation (2.3) is known as Weibull Distribution.

If the probability of failure in t to (t+dt) is taken as $ab.e^{bt}$ i.e. it varies exponentially,

$$h(t) = ab.e^{bt} \qquad a > 0$$

$$b > 0$$

then,

$$R(t) = \exp\left[-a(e^{bt} - 1)\right] \qquad t \geq 0 \quad \ldots \quad (2.5)$$

equation (2.5) is known as Extreme Value Distribution.

The modified extreme value function can be derived if-

$$h(t) = \frac{e^t}{b} \qquad b > 0$$

$$t \geq 0$$

and

$$R(t) = \exp\left[-\frac{1}{b}(e^t - 1)\right] \quad \ldots \quad (2.6)$$

which is known as Modified Extreme Value Function.

The fundamental difference between the Weibull and Exponential distribution is that an exponential law admits only one parameter model while the Weibull's model(3) consists of a class of two parameter models of statistical reliability function. In Weibull's distribution

$$R(t) = e^{-t^b/a} \quad \ldots \quad \ldots \quad (2.7)$$

where a and b are the empirically derived parameters. From equation (2.7), it is clear that the two parameter[4] model permits greater flexibility in curve fitting. As a specific example if b = 1 the Weibull distribution turns out to be an Exponential distribution and if b = 2 the general form of Gaussian error curve is obtained as a special case. A better fit to empirical data can be obtained with two parameter model than with a single parameter model. However, if a better fit is the sole criterion of acceptability, the two parameter models can be used and further improvements can be made by adding third and even a fourth parameter also.

for Weibull Distribution-

$$f = \frac{b}{a}\, t^{b-1}. \quad e^{-t^b/a} \quad \text{and } F(t) = 1-e^{-t^b/a}$$

Then except for the degenerate Exponential law case (b=1) failure rate is not constant with time. This checks out with immediate idea about failure mechanism, particularly wearout phenomenon. If this function is to be fitted to the wearout end of an empirical distribution then b must be greater than one, whereupon F(t) becomes initially very large and later tends to zero as time increases to infinity, which is in direct contradiction with any known and reasonable aging process.

If the Weibull distribution is justified then some additional provision analogous to the protocols are adopted.

Typically the wearout end of the failure rate curve is of more interest and hence b must be greater than one. The burn-in period is then defined irrelevant by the assumption that relay contacts are sufficiently aged before the test begin to reduce burn-in type failure.

To fit the entire range of the classical failure rate curve by assuming that observed failure rate really aggregate of the failures due to different causes and subjected to different laws. This argument[5] provides a failure density function of the form;

$$f(t) = a_3 t^{b_1 - 1} e^{-t^{b_1}/a} + b_3 t^{b_2 - 1} e^{-t^{b_2}/a} \quad \ldots \quad (2.8)$$

where $b_1 \angle 1$ and $b_2 > 1$

and a_3, b_3, a_1 and a_2 all are greater than one.

An other type of the failure distribution can be derived from the bionomial theory of distribution which is the probability of exactly n occurance out of m trials is given by,

$$^mC_n p^m q^n = \frac{(m)!}{(n)!(m-n)!} p^{m-n} q^n \quad \ldots \quad (2.9)$$

in which p represents the probability of non occurance of events (failure) and q -(1-p), the probability of occurance. It is assumed that p thereby q also remains constant throughout an independent trial.

In case of reasonably good equipment the failures will be small in number and the probability of failure will be small. As q approahces to zero and m approaches to infinity in such a way that the product mq=k i.e. the expected or average number of failures remains finite.

From the definition of m, $q = \frac{k}{m}$ and $p=(1- \frac{k}{m})$. Now substituting these in equation (2.9) and substituting the limits as m approaches to infinity and simplifying the equation (2.9). The equation (2.9) can be rewritten as-

$$\lim_{m\to\infty} \left[\frac{(m)!}{(n)!(m-n)!} \left(\frac{k}{m}\right)^n \left(1-\frac{k}{m}\right)^{m-n} \right]$$

or $\lim_{m\to\infty} \left[{}^mC_n \, p^m q^n \right] = \frac{k^n}{(n)!} e^{-k}$ (Appendix-1) ... (2.10)

or $P(m,n) = \frac{k^n}{(n)!} e^{-k}$ ... ... (2.11)

k = average number of failures during time interval t which can be replaced by (A.t), if A is the average failure rate.

Then,

$$P_n(t) = \frac{(A.t)^n}{(n)!} e^{-A.t} \quad \ldots \quad \ldots \quad (2.12)$$

The equation (2.12) is known as Poisson's density function.

It is interesting to note that Poisson process directly leads to another well-known distribution known as Gamma distribution. If instead of obtaining the probability of n failures upto a time t, it is desired that the probability of failure at a specified time should be exactly n, one can deduce the Gamma distribution. For the Poisson distribution the random variable is the number of failures while for the Gamma distribution the time is a random variable. Thus Gamma distribution(6) can be obtained by differentiating the Poisson distribution and is of the form,

$$P_n(s) = \frac{A^n}{(s+A)^n} \quad \text{in Laplace transform}$$

The inverse of this expression is,

$$P_n(t) = \frac{A^n \, t^{n-1}}{(n-1)!} e^{-A.t}$$

$$\text{or } f(t) = \frac{A^n \, t^{n-1}}{(n-1)!} e^{-A.t} \quad \ldots \quad \ldots \quad (2.13)$$

equation (2.13) is known as Gamma failure density function.

$$R(t) = \int_t^{\infty} \frac{A^n t^{n-1}}{(n-1)!} e^{-A.t} \, dt \quad \ldots \quad \ldots \quad (2.14)$$

$$F(t) = \int_0^t \frac{A^n \, t^{n-1}}{(n-1)!} . e^{-A.t} \, dt \quad \ldots \quad \ldots \quad (2.15)$$

Besides, the distributions described above there exists an important distribution known as Normal or Gaussian distribution.

Failure density function f(t) is defined as-

$$f(t) = \frac{1}{u\sqrt{2\pi}} \, e^{-\left[\frac{(t-v)^2}{2u^2}\right]} \quad \ldots \quad \ldots \quad (2.16)$$

$$u > 0$$

$$-\infty < v < \infty$$

$$0 \leq t \leq \infty$$

where v = mean wearout life

u = standard deviation of the life time from the mean life v.

$$R(t) = \int_t^{\infty} \frac{1}{u\sqrt{2\pi}} \, e^{-\left[\frac{(t-v)^2}{2u^2}\right]} dt. \quad \ldots \quad \ldots \quad (2.17)$$

$$F(t) = \int_0^t \frac{1}{u\sqrt{2\pi}} \, e^{-\left[\frac{(t-v)^2}{2u^2}\right]} dt. \quad \ldots \quad (2.18)$$

$$u = \frac{\sqrt{\Sigma (t-v)^2}}{N}$$

The value of N in the expression of u is the number of events over which $\Sigma (t-v)^2$ is made. The plots for density function, reliability function and failure function for different cases are given in Figs.(2.1 to 2.5).

2.1.2. Unconditional Distribution Functions:

In the previous analysis it is assumed that failure rate

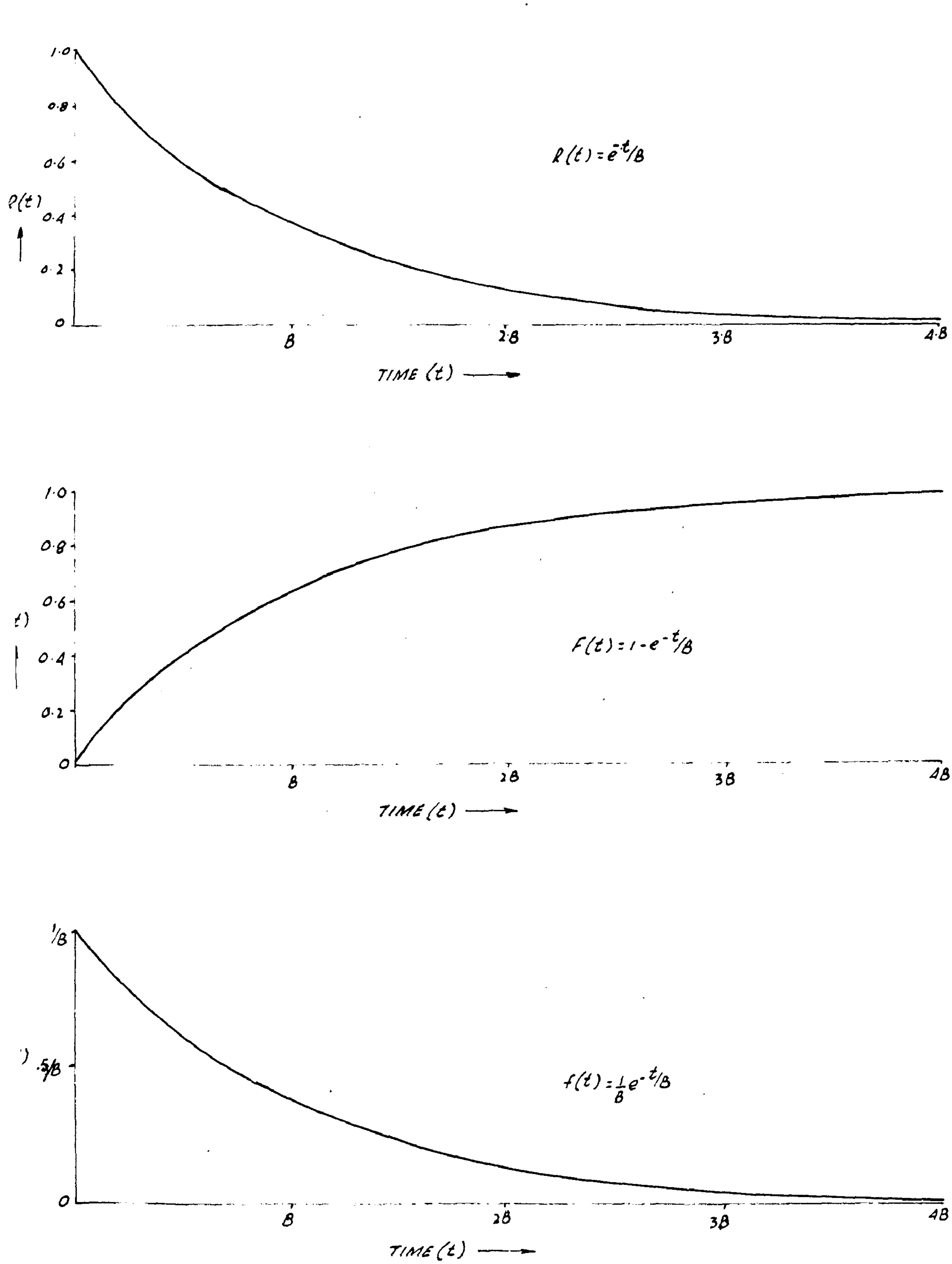


FIG. 2·1 EXPONENTIAL DISTRIBUTION

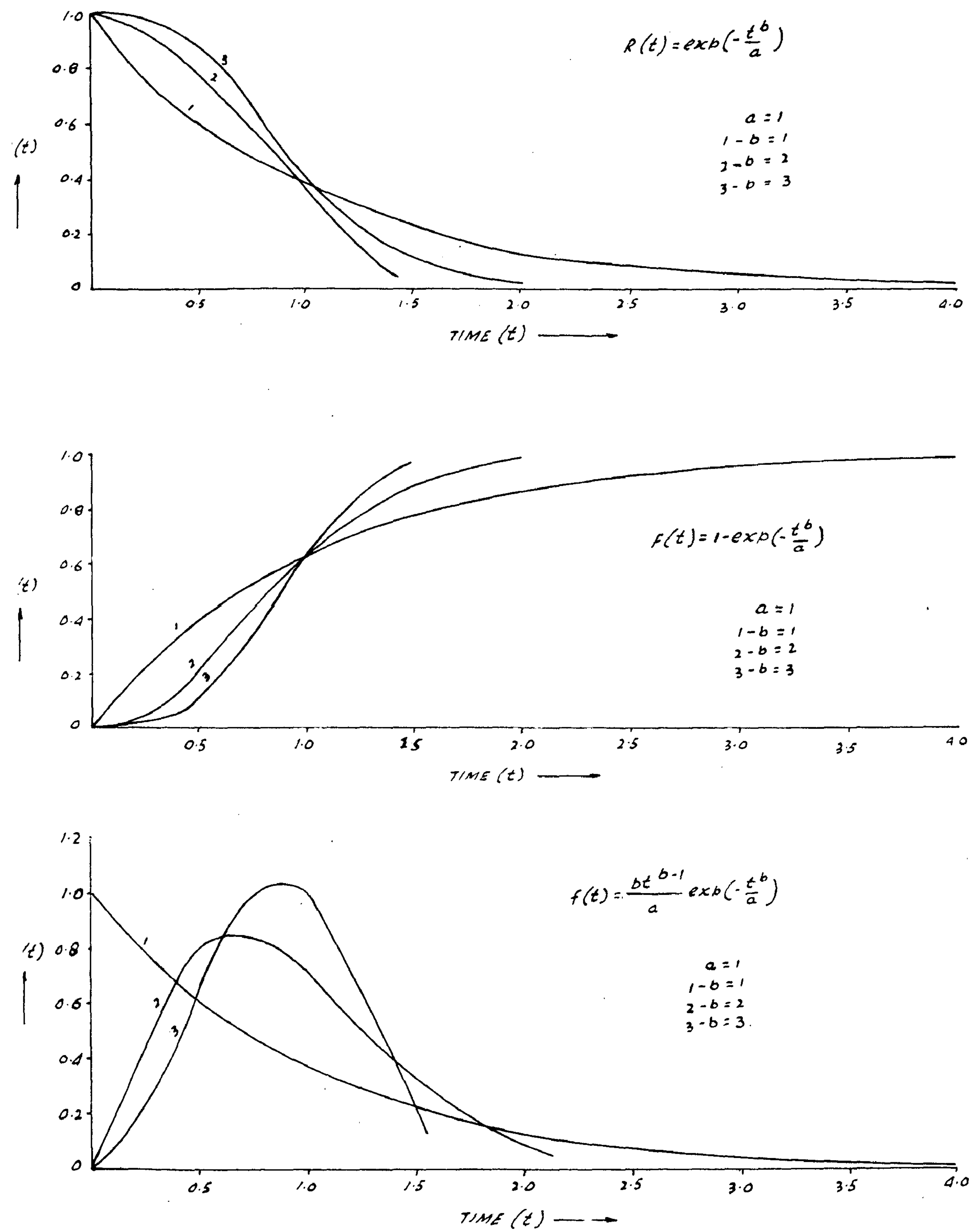


FIG. 2.2 WEIBULL DISTRIBUTIONS

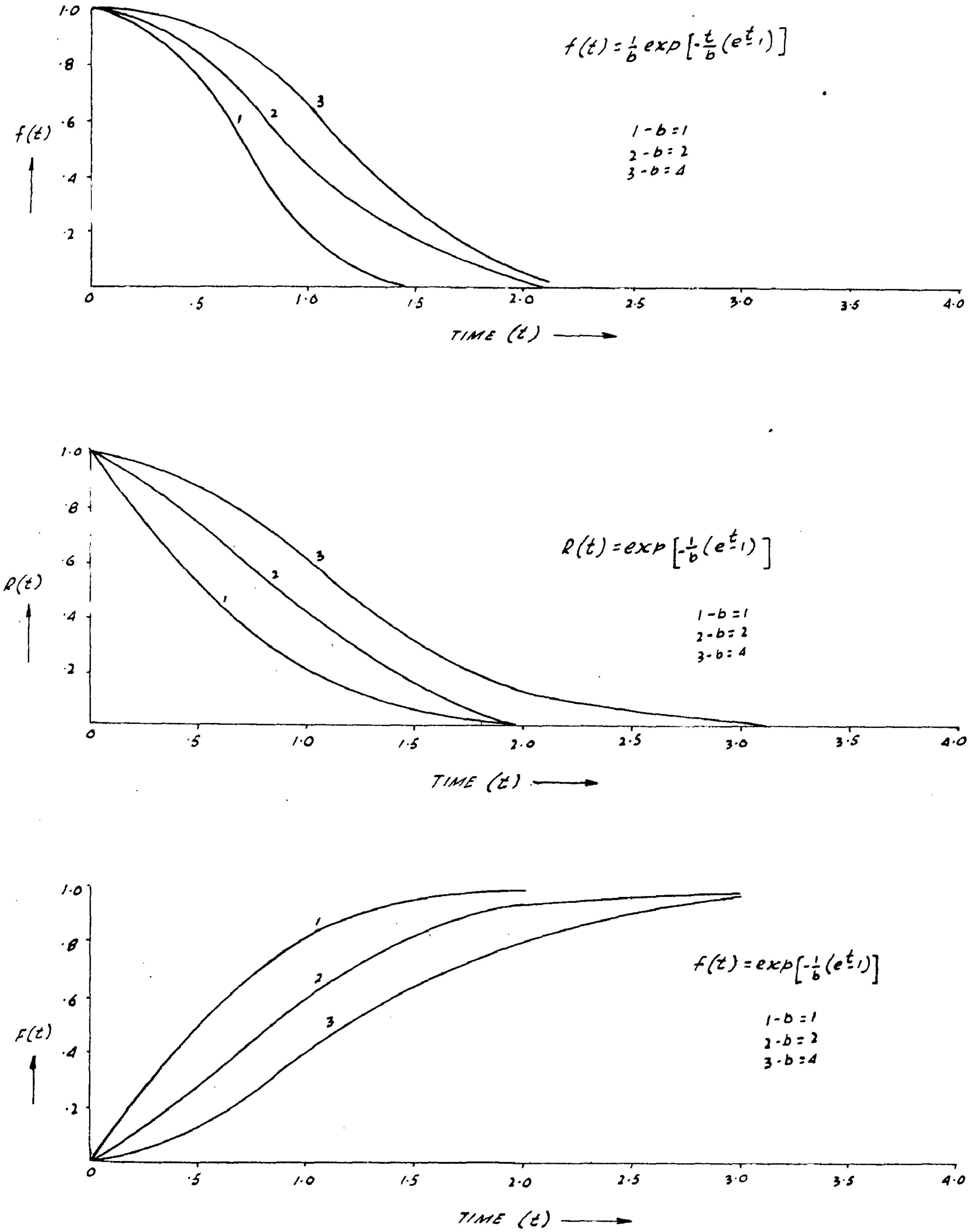


FIG. 2·3 MODIFIED EXTREME VALUE DISTRIBUTION

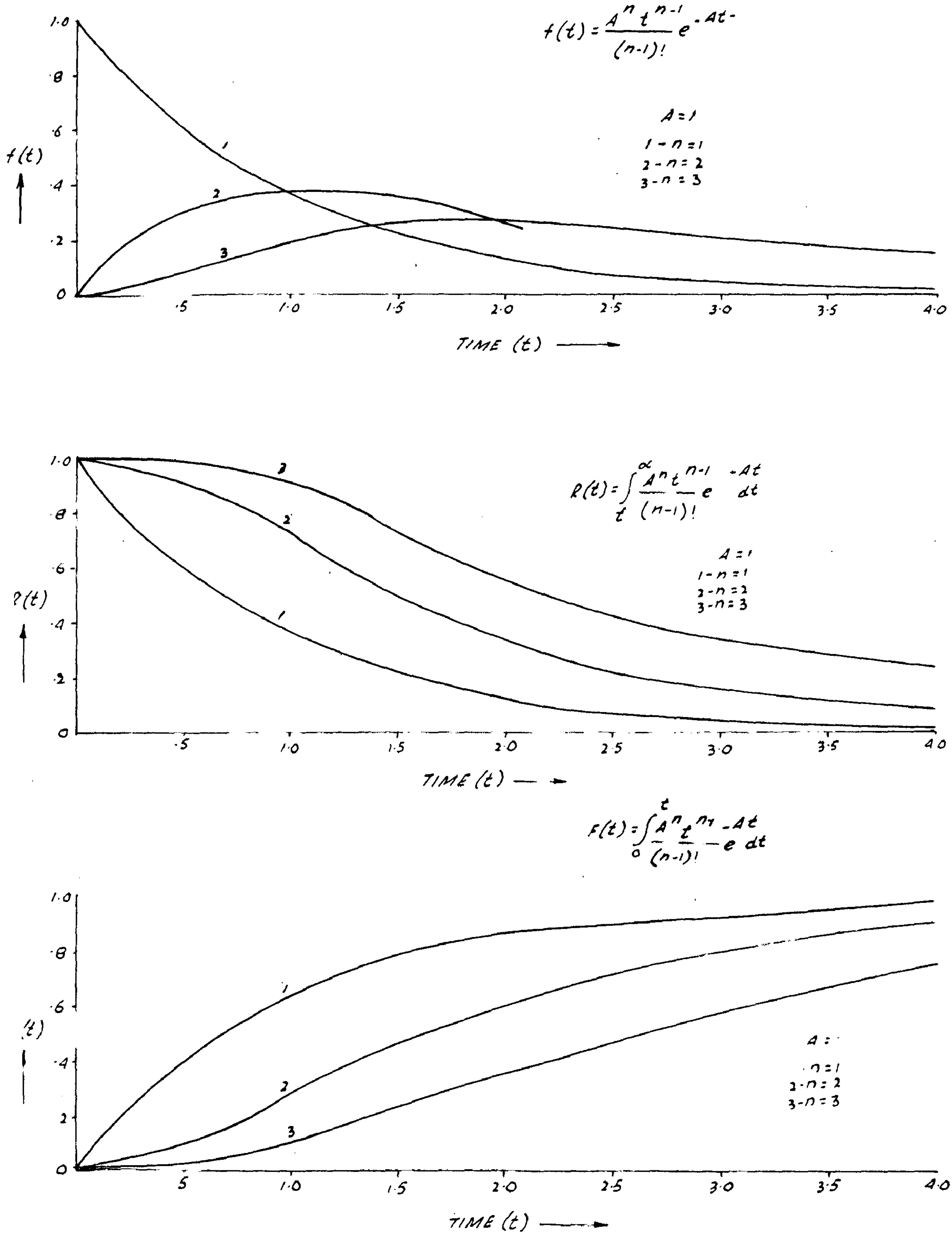


FIG. 2·4 GAMMA DISTRIBUTIONS

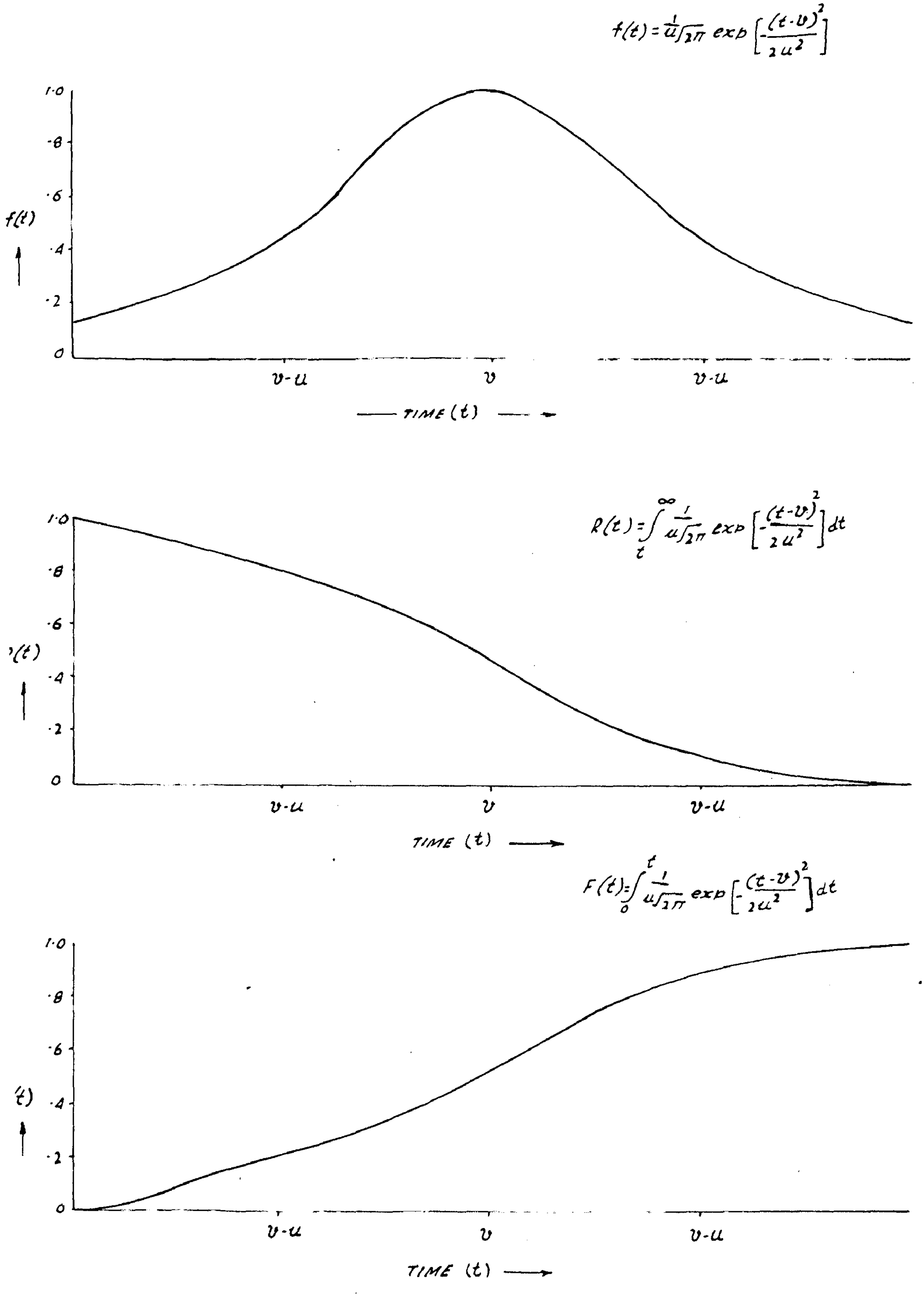


FIG. 2·5 NORMAL DISTRIBUTIONS

does not depend upon the past history of the operating conditions i.e. only conditional probability, of failure has been considered. But the above idea has been further extended[7] in the following manner.

If an item is made up of i independent components. In the course of time one component after another fails, and there is a critical number of failures d, such that entire system fails when d of its components have failed. Thus number d depends in general on the items under the assumptions.

(i) The conditional failure rate depends upon the elapse of time t and instantaneous damage at time t in the following manner,

$$h(t) = w(t).\ d(t)$$

where w(t) is the deterioration function.

and d(t) is instantaneous damage at time t.

(ii) If at time t an item has sustained damage and j of its components have failed, at that time, then each of the remaining (i-j) components, is exposed to damage by $\frac{d(t)}{i-j}$.

From the first assumption, which clearly states that conditional failure rate can be divided into two parts, wearout and severe shock, overloads, over voltage or short circuit which may cause damage at any time.

Taking the case where-

$$\frac{d}{dt} w(t) = 0 \quad \text{and } d(t) = C$$

the exponential distribution is derived.

If $\frac{d}{dt} w(t) = a$

$$w(t) = a.t + G \quad \text{where } G = \text{Constant of integration}$$

Also when $t = 0$ $w(t) = 0$

therefore $G = 0$

hence $w(t) = a.t.$

and $d(t) = C$

Then $h(t) = a.t.C.$

$$H(t) = \int_0^t a.t.C\, dt.$$

$$= \frac{a.C\, t^2}{2}$$

and $$R(t) = \exp.\left[-\frac{a.C\, t^2}{2}\right] \qquad \ldots \qquad \ldots \qquad (2.19)$$

$$f(t) = a.C\, t \exp.\left[-\frac{a.C\, t^2}{2}\right] \qquad \ldots \qquad \ldots \qquad (2.20)$$

Taking another general form-

$$\frac{d}{dt} w(t) = a \text{ and } d(t) = b\, e^{bt}$$

$$w(t) = a.t$$

Then $h(t) = a.t\, b\, e^{bt}$

and therefore $H(t) = a\, e^{bt} (t - \frac{1}{b}) + 1$

$$R(t) = e^{-H(t)}$$

$$f(t) = \frac{-dR(t)}{dt}$$

The second assumption states that irreversible and cumulative damage occur in the item in the course of time, as the components fail one by one, remaining components are exposed to an increased share of total damage. However the deterioration of component will remain unchanged, regardless how many of the components have failed. Assuming that critical number of failed components is d, then the density function f(d,t) is given by-

$$f(d,t)= \frac{1}{(d-1)!2^d} \left[2 \int_0^t w(t)d(t)\ dt\right]^{d-1} \exp.\left[-2 \int_0^t w(t).d(t)dt\right] \quad \ldots (2.21)$$

As a special case when component does not deteriorate $[w(t)=A]$ and that the damage is assumed to be constant than above equation can be written as-

$$f(d,t) = \frac{A^d\ t^{d-1}}{(d-1)!}\ e^{-A.t} \quad \ldots \quad \ldots \quad (2.22)$$

which is density function of Gamma distribution.

2.2. Application of Different Distribution Functions:

The exponential distribution has constant failure rate and this property limits its use in many of the reliability models. Since failure rate is constant it imparts an impression that a component at any time during the life span T was new and placed in operation just then. The probability of failure of a component at any time t after it is put to use is same as for the remaining life (T-t) such as electric fuses etc. This does not very much simulate the actual conditions encountered in practice. However in many cases, this model may be used effectively with reasonable tolerance. The Weibull and Gamma functions with parameter 'a' greater than one have increasing failure rate as time increases. The modified extreme value and Normal distributions also have an increasing rates. These items under close control of both the manufacturing process and the condition of test, a Normal theory of failure seems to be consistant with the data. However many life length distribution occuring in practical applications are not Normal because they are markedly skewed where the normal distribution is symmetrical.

The Gamma distribution[8] is extremely useful in fatigue and wearout studies, Weibull family of distributions have increasing failure rate for a is greater than one, but in this case failure rate is unbounded. This type of distribution is also useful to describe fatique failures, vacuum tube failure and ball bearing failures etc.

2.2.1. Life Distribution:

Failure rate vs. time curve if plotted from empirical data on actual system, resembles the theoretical curve closely enough to make a consideration of the later worthwhile from a practical standpoint.

A theoretical failure rate vs. time curve is shown in Fig.2.6. This curve can be divided into three distinct regions viz. early failure, chance or random failure and wearout failure zone.

2.2.1.1. Region of Early Failure:

In this region the failure rate is initially very high but shows a tendency to decrease shappely with time. These failures are due to defects in manufacturing and poor quality control techniques or during the assembly of an equipment, a poor connection may go through unnoticed. These failures can be eliminated by 'debugging' or 'burn-in' processes[5].

The debugging process consists of operating an equipment for a number of hours under conditions of actual field use. The weak or substandard component fails in the early hours of the operation. These failed components are replaced by good components and only then the equipment is released for service. The burn-in

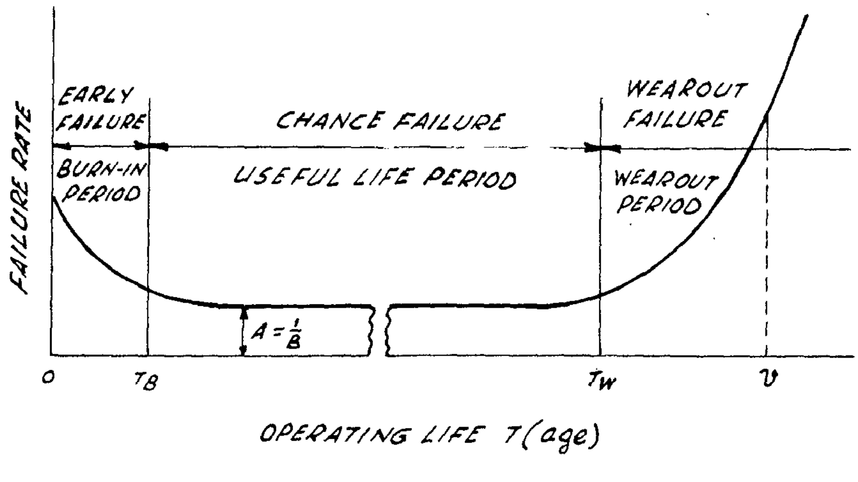


FIG. 2·6 IDEAL FAILURE RATE Vs. TIME CURVE process consists of operating at lot of components under actual working conditions for a number of hours and then using only those components which survive, for the assembly of the equipment. Generally early failure follows an exponential law of failure distribution with reasonable tolerance.

2.2.1.2. Region of Chance or Random Failure:

These failures can not be eliminated either from good debugging techniques or even the best maintenance practices. These failures are caused by sudden stress accumulation beyond the designed strength of the component. The chance failures occur at random intervals irregularly and unknowingly. It is difficult to predict chance failures, however they obey certain rules of collective behaviour so that the frequency of their occurance during sufficietntly long period is approximately constant.

2.2.1.3. Region of Wear-out Failure:

In general the failure increases slowly as the item reaches the end of its useful life. These failures occur when the equipment is either not properly maintained or not at all maintained. In most of the cases wear-out failures[9] can be prevented by periodic inspection and replacing the equipment or component before wear-out takes place. This region obeys closely Normal failure distribution law.

CHAPTER - 3

NON-MAINTAINED AND MAINTAINED SYSTEMS

In general maintenance policies can be formulated into two categories viz.

Non-maintained and Maintained systems.

3.1. Non-maintained Systems:

The non-maintained systems are those systems where the maintenance action is not at all taken during the useful life of an equipment. While describing the reliability of a given system it is necessary to specify:

(i) The equipment failure process.

(ii) The system configuration which describes how the equipment or the component is connected and the mode of their operation.

(iii) The state in which the syste m is to be defined as having failed.

The simplest hypothesis, from mathematical point of view, is to assume that equipment fails in accordance with negative exponential distribution. This assumption helps to use Markor-process[10,11] which gives simple homogeneous linear differential equations with constant coefficients.

There is plenty of experimental and operational information available to justify the use of exponential failure law. Cahrat[12] is one of the earliest investigators to show the statistical nature and further studies have been done by others[13,14], who clearly indicate that exponential distribution adequately tallies with the statistically determined failure distribution. Although certain components within an equipment may not exibit the exponential failure pattern, the equipment will generally behave so, provided the components are replaced as and when they fail, so that their ages become mixed after some time. This phenomenon is demonstrated in reference (15).

Many types of failure distribution functions have been described in Chapter 2, but infact the failure pattern of complex electronic circuits is much more complicated, to be thoroughly described by a simple statistical failure model. This does not mean however that no reasonable statement can be made about equipment failure distribution but the correct approach is to select proper failure distribution function which will be very close to any standard distribution.

For non-maintained systems, the reliability function R(t) gives the probability that an equipment will not fail in the given interval of operating time (0,t). From this other functions can also be derived very easily. For non-maintained systems the following configurations have been considered as shown in Fig.(3.1), which are-

(i) Series, (ii) Parallel Standby (iii) Parallel redundant
(iv) State Dependancy (v) Redundant with imperfect switching.

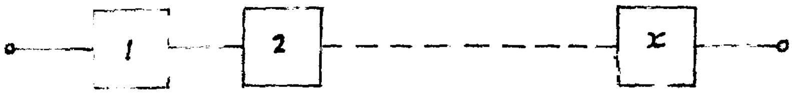
3.1.1. Reliability Models for Series Configuration:

In the derivation of reliability function R(t) for series configuration the following important assumptions will be made:

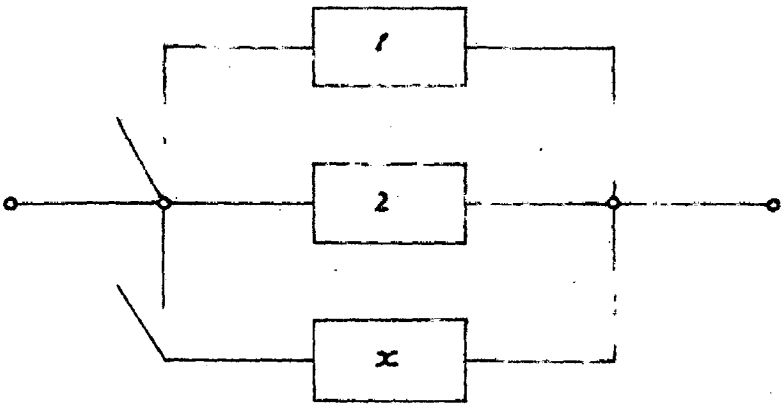
(i) The system will be completely inoperative when any one of the x equipments or components will fail.

(ii) The probability of failure of each equipment is independent of the remaining (x-1) equipments.

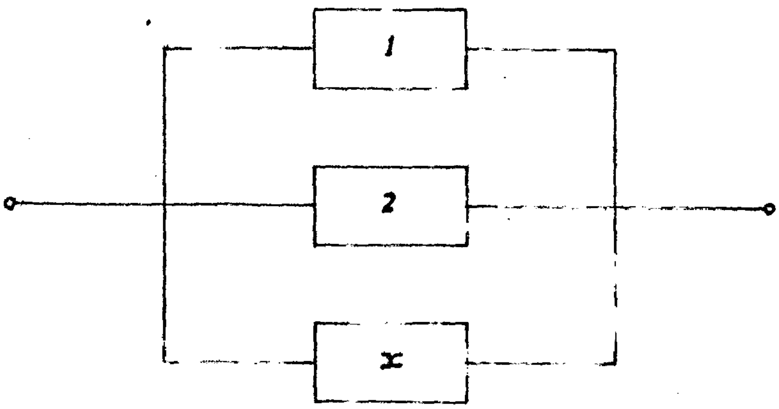
(iii) The probability that any one equipment will fail in the time interval t, (t+dt) is A.dt, provided it has survived upto



SERIES CONFIGURATION



STANDBY RUDUNDANT CONFIGURATION



PARALLEL RUDUNDANT CONFIGURATION

FIG 3.1

time t.

(iv) Cumulative probability of failure of x equipments in series will be xA.

The transition matrix is prepared as follows:

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & \quad 1 \end{array} \\ \begin{bmatrix} 1-xA & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$

for initial conditions, when t = 0 all equipments are operative.

i.e. $P_o(t) = 0$ if $t \neq 0$

$= 1$ if $t = 0$

The above transition matrix gives,

$$P_o(t+dt) = P_o(t) \left[1-xA\,dt \right] + O(dt) \quad \ldots \quad (3.1)$$

or $$\frac{P_o(t+dt) - P_o(t)}{dt} = P_o'(t) = -xA\,P_o(t)$$

or $$P_o'(t) = -xA\,P_o(t) \quad \ldots \quad \ldots \quad (3.2)$$

The solution of equation (3.2) will be of the form-

$$P_o(t) = e^{-xAt} \quad \ldots \quad \ldots \quad (3.3)$$

or $R(t) = P_o(t) = e^{-x.At}$

if $P = e^{-At}$

The reliability of series configuration can easily be written as-

$$R(t) = P^x \quad \ldots \quad \ldots \quad (3.4)$$

3.1.2. Reliability Models for Parallel Standby Configuration:

In this type of configuration only one equipment is operating but when it fails a standby equipment is switched on to the line and the failed equipment is taken off the line.

The process continues until all (x-1) equipments have failed. The following assumptions are made:

(i) The system will fail when all the x equipments have failed.

(ii) The failure probability of each equipment is independent of the remaining (x-1) equipments.

(iii) Switching is perfect.

(iv) The equipment can only fail while in the operating positions with conditional probability A.dt. Thus off line equipment can not fail until switched on to the line.

The transition matrix is formed as below:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \vdots \\ x \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & \cdots & x \end{array} \\ \begin{bmatrix} (1-A) & A & 0 & 0 & \cdots & 0 \\ 0 & (1-A) & A & 0 & \cdots & 0 \\ 0 & 0 & (1-A) & A & \cdots & 0 \\ \vdots & & \vdots & & & \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \end{array}$$

For simplicity only three equipment redundant system is considered.

The transisition matrix gives the following equations.

$$\left.\begin{aligned} P_0(t+dt) &= P_0(t)\,[1-A\,dt]+P_1(t)\,A\,dt+O(dt) \\ P_1(t+dt) &= P_1(t)\,[1-A\,dt]+P_2(t)\,A\,dt+O(dt) \\ P_2(t+dt) &= P_2(t)\,[1-Adt]+P_3(t)\,A\,dt.+O(dt) \end{aligned}\right\} \quad \ldots (3.5)$$

$$\text{Reliability function } R(t) = P_0(t) + P_1(t) + P_2(t) \quad \ldots (3.6)$$

The following Linear differential equations are obtained from the set of equations (3.5)

$$\left.\begin{aligned} P_0'(t) &= -A\,P_0(t) \\ P_1'(t) &= -A\,P_0(t) - AP_1(t) \\ P_2'(t) &= -A\,P_1(t) - A\,P_2(t) \end{aligned}\right\} \quad \ldots \quad (3.7)$$

The solution of set of equations (3.7) is as follows:

$$\left.\begin{aligned} P_0(t) &= e^{-At} \\ P_1(t) &= \frac{A.t}{(1)!}\,e^{-At} \\ P_2(t) &= \frac{(At)^2}{(2)!}\,e^{-At} \end{aligned}\right\} \quad \ldots \quad (3.8)$$

Hence $R(t) = P_0(t) + P_1(t) + P_2(t)$

$$= e^{-At}\left(1+ \frac{A.t}{(1)!} + \frac{(At)^2}{(2)!} \right) \quad \ldots \quad (3.9)$$

The reliability function for x redundant standby systems can be generalised by induction method from equation (3.9)-

$$R(t)_x = e^{-At} \left\{ \sum_{w=0}^{w=x-1} \frac{(At)^w}{(w)!} \right\} \quad \ldots \quad \ldots \quad (3.10)$$

Fig.(3.2) shows a plot of two parallel standby redundant configuration reliability function as compared with reliability function for a single equipment system.

3.1.2.1. Reliability Model of Parallel Standby with off line Equipment failure:

In the foregoing analysis it was assumed that the off line equipment does not fail but if it is assumed that off-line equipment also has some probability of failure $A_f dt$, but switching is still perfect. The other assumptions are the same.

The transition matrix is formed only for two redundant systems for simplicity.

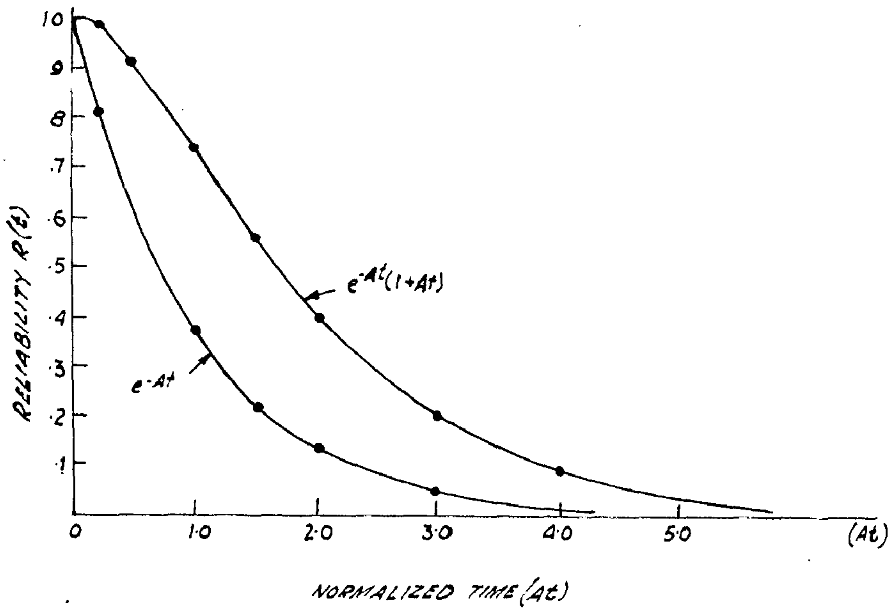


FIG.3.2 RELIABILITY FUNCTIONS FOR A TWO-EQUIPMENT STANDBY REDUNDANT SYSTEM AND SINGLE-EQUIPMENT SYSTEM.

$$
\begin{array}{c|ccc}
 & 0 & 1 & 2 \\
\hline
0 & 1-(A_o+A_f) & (A_o+A_f) & 0 \\
1 & 0 & 1-A_o & A_o \\
2 & 0 & 0 & 1
\end{array}
$$

The following linear equations are formed from the above matrix.

$$
\left.
\begin{aligned}
P_o(t+dt) &= P_o(t)\,[1-(A_o+A_f)dt] + 0(dt) \\
P_o(t+dt) &= P_1(t)\,[(A_o+A_f)dt] + P_1(t)[1-A_o dt] + 0(dt)
\end{aligned}
\right\} \qquad (3.11)
$$

The set of equations (3.11) gives the following differential equations.

$$
\left.
\begin{aligned}
P_o'(t) &= -(A_o + A_f)\,P_o(t) \\
P_1'(t) &= (A_o+A_f)P_o(t) - A_o P_1(t)
\end{aligned}
\right\} \qquad \ldots \qquad \ldots \qquad (3.12)
$$

but

$$R(t) = P_o(t) + P_1(t)$$

The solution of equations is-

$$
\left.
\begin{aligned}
P_o(t) &= \exp\,[-(A_o + A_f)t] \\
P_1(t) &= \frac{A_o+A_f}{A_o} e^{-A_o t} - \frac{A_o+A_f}{A_o} e^{-(A_o+A_f)t}
\end{aligned}
\right\} \qquad (3.13)
$$

therefore-

$$R(t) = e^{-(A_o+A_f)t} + \frac{A_o+A_f}{A_o} e^{-A_o t} - \frac{A_o+A_f}{A_o} e^{-(A_o+A_f)t} \qquad \ldots \quad (3.14)$$

3.1.3. Reliability Models of Parallel Redundant Systems[16,17,18,19

In this type of configuration all the x equipments are sharing the total load equally (operating simultaneously). The reliability function is derived with the following assumptions.

(i) The system will be inoperative when all the x components or equipments have failed.

(ii) The probability of failure of each equipment is independent of remaining (x-1) equipments.

(iii) Each equipment is having same failure rate.

(iv) The probability of failure in the time interval t,(t+dt) is Adt, provided it is operative at time t. The transition matrix is formed as below:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \vdots \\ x \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & & x \end{array} \\ \begin{bmatrix} 1-xA & xA & 0 & 0 & \cdots & 0 \\ 0 & 1-(x-1)A & (x-1)A & 0 & \cdots & 0 \\ 0 & 0 & 1-(x-2)A & (x-2)A & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & & 0 \end{bmatrix} \end{array}$$

For simplicity, three parallel redundant systems have been considered, and generalised reliability function for x equipment has been formulated.

The following equations are formed from the above matrix-

$$\left.\begin{aligned} P_0(t+dt) &= P_0(t)[1-3Adt]+0(dt) \\ P_1(t+dt) &= P_0(t).3Adt+P(t)[1-2Adt]+0(dt) \\ P_2(t+dt) &= P_1(t)2Adt + P_2(t)[1-A\,dt]+0(dt) \end{aligned}\right] \quad \ldots (3.15)$$

The set of equations (3.15) give the following linear differential equations.

$$\left.\begin{aligned} P_0'(t) &= -3A\,P_0(t) \\ P_1'(t) &= 3AP_0(t) - 2AP_1(t) \\ P_2'(t) &= 2AP_1(t) - AP_2(t) \end{aligned}\right\} \quad \ldots \quad \ldots \quad (3.16)$$

The solution of set of equations (3.16) is -

$$\left.\begin{aligned} P_0(t) &= e^{-3At} \\ P_1(t) &= 3e^{-2At} - 3e^{-3At} \\ P_2(t) &= 3e^{-At} - 6e^{-2At} + 3e^{-3At} \end{aligned}\right\} \quad \ldots \quad (3.17)$$

But -

$$R(t) = P_0(t) + P_1(t) + P_2(t)$$

$$= 3e^{-At} - 3e^{-2At} + e^{-3At} \quad .. \quad \ldots \quad (3.18)$$

For two parallel system-

$$R(t) = 2e^{-At} - e^{-2At}$$

If $P = e^{-At}$

Then R(t) for general x parallel system will be given by-

$$R(t)_x = 1-(1-P)^x \quad \ldots \quad \ldots \quad (3.19)$$

If each equipment has different failure rate, then-

$$R(t)_x = 1-(1-P_1)\ (1-P_2)\ \ldots\ldots(1-P_x) \quad \ldots \quad (3.20)$$

Fig.(3.3) compares the reliability functions for both stand-by and parallel redundancy of a two equipment system against normalised time.

3.14. Reliability function for State Dependency Models:

In the previous models it was assumed that the probability of failure during the time interval t, (t+dt) is constant, which in general is not true. For example two

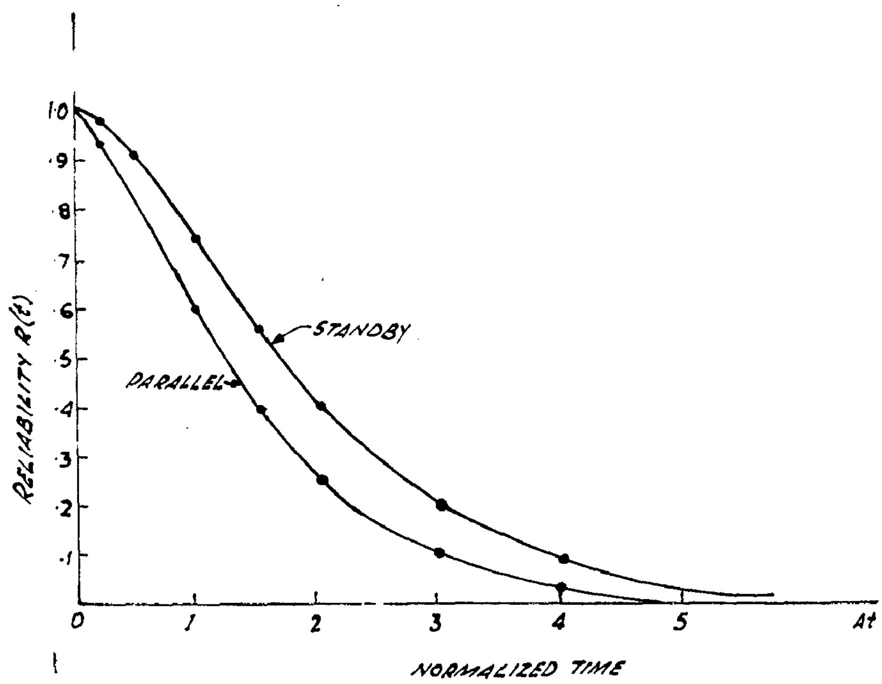


FIG.3.3 RELIABILITY FUNCTIONS FOR STANDBY AND PARALLEL TWO-EQUIPMENT REDUNDANT SYSTEMS.

equipments are operating in parallel and sharing an equal amount of load. If one of the equipments has failed then the entire load is transferred on to the other equipment. This will definitely increase the probability of failure, assuming that it was operating with failure rate A. If the above fact is kept in view then it is necessary to develop such models in which failure rate changes with the state of the system. Assuming that transition probabilities are linearly related with the state of the system. i.e.,

$$A_x = A(x+1) \quad \text{(Linear Markov-process)}$$

$$\text{or} \quad A_x dt = A(x+1)\, dt \qquad \ldots \qquad \ldots \qquad (2.21)$$

The transition matrix is formed as follows:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \\ x \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & & x \end{array} \\ \begin{bmatrix} 1-A & A & 0 & 0 & \ldots & 0 \\ 0 & 1-2A & 2A & 0 & \ldots & 0 \\ 0 & 0 & 1-3A & 3A & \ldots & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & 0 & & 1 \end{bmatrix} \end{array}$$

To simplify, three parallel state dependency models have been considered.

The following linear equations are formed from the above matrix for the case under consideration.

$$\left.\begin{aligned} P_0(t+dt) &= P_0(t)\,[1-Adt] + O(dt) \\ P_1(t+dt) &= P_1(t)\,Adt + P_1(t)[1-2Adt] + O(dt) \\ P_2(t+dt) &= P_1(t)\,2Adt + P_2(t)\,[1-3Adt] + O(dt) \end{aligned}\right\} \qquad (3.22)$$

The above set of equations (3.22) gives the following differential equations.

$$\left.\begin{aligned} P_0'(t) &= -A\, P_0(t) \\ P_1'(t) &= AP_0(t) - 2AP_1(t) \\ P_2'(t) &= 2AP_1(t) - 3AP_2(t) \end{aligned}\right\} \quad \ldots \quad \ldots \quad (3.23)$$

The solution for equations (3.23) will be-

$$\left.\begin{aligned} P_0(t) &= e^{-At} \\ P_1(t) &= e^{-At} - e^{-2At} \\ P_2(t) &= e^{-At} - 2e^{-2At} + e^{-3At} \end{aligned}\right\} \quad \ldots \quad (3.24)$$

$$\text{But } R(t) = P_0(t) + P_1(t) + P_2(t)$$

$$= 3e^{-At} - 3e^{-2At} + e^{-3At} \quad \ldots \quad \ldots \quad (3.24)$$

Fig.(3.4) illustrates this phenomena for x=1, 2,3 equipment systems.

3.1.5. Reliability Models of Imperfect Switching:

In the previous analysis of standby redundant systems it was assumed that switching is perfect i.e. swithh can not fail. But switch can also sometimes fail and has a failure rate of As. If this is a redundant standby system with equipments X and Y with a switch S. It is assumed that X is on to line and has failure rate A and Y is off to line which can not fail. However switch S can fail at any time. The acceptable stages are as follows:

(i) Only X or Y fails ($\overline{X}$) - $\overline{X}SY$ or $XS\overline{Y}$

(ii) Only S fails ($\overline{S}$) - $X\overline{S}Y$

(iii) X,Y and S are operating - XSY.

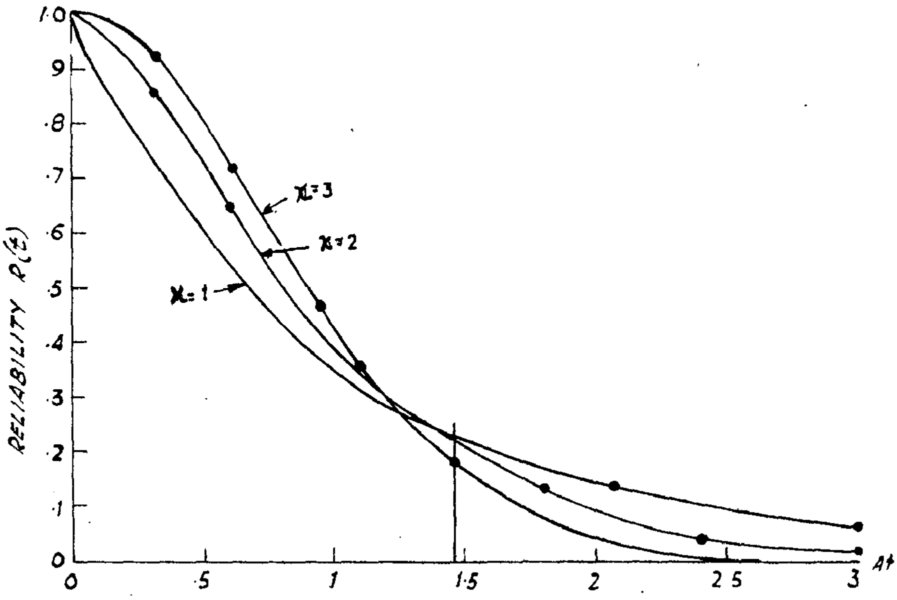


FIG. 3.4 RELIABILITY FUNCTIONS n = 1, 2, AND 3 PARALLEL REDUNDANT EQUIPMENT SYSTEMS.

The transisition matrix has been formed as below:

$$
\begin{array}{ll} & \begin{array}{cccccc} \overset{0}{XSY} & \overset{1}{X\bar{S}Y} & \overset{2}{\bar{X}SY} & \overset{3}{\bar{X}\bar{S}Y} & \overset{4}{\bar{X}\,S\,\bar{Y}} & \overset{5}{\bar{X}\,\bar{S}\,\bar{Y}} \end{array} \\
P = \begin{array}{r} 0\ \bar{X}SY \\ 1\ X\bar{S}Y \\ 2\ \bar{X}SY \\ 3\ \bar{X}\bar{S}Y \\ 4\ \bar{X}S\bar{Y} \\ 5\ \overline{XSY} \end{array} & \begin{bmatrix} 1-(A+A_s) & A_s & A & 0 & 0 & 0 \\ 0 & 1-A & 0 & A & 0 & 0 \\ 0 & 0 & 1-(A+A_s) & A_s & A & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}
$$

The transition matrix gives the following linear equations.

$$
\left.\begin{aligned}
P_o(t+dt) &= P_o(t)\,[1-(A+A_s)dt] + 0(dt) \\
P_1(t+dt) &= P_o(t)A_s dt + P_1(t)[1-Adt] + 0(dt) \\
P_2(t+dt) &= P_o(t)Adt + P_2(t)[1-(A_s+A)dt\,] + 0(dt)
\end{aligned}\right\} \qquad (3.25)
$$

The above set of equations gives the following differential equations.

$$
\left.\begin{aligned}
P_o'(t) &= -(A+A_s)\,P_o(t) \\
P_1'(t) &= A_s P_o(t) - AP_1(t) \\
P_2'(t) &= AP_o(t) - (A_s + A)\,P_2(t)
\end{aligned}\right\} \qquad \ldots \qquad \ldots \qquad (3.26)
$$

The solution of these differential equations (3.26) is

$$P_o(t) = e^{-(A+A_s)t}$$

$$P_1(t) = e^{-At} - e^{-(A+A_s)t}$$

$$P_2(t) = At\, e^{-(A+A_s)t}$$

But

$$R(t) = P_o(t) + P_1(t) + P_2(t)$$

$$= e^{-At} + At\, e^{-(A+A_s)t} \quad \ldots \quad \ldots \quad \ldots \quad (3.27)$$

This can be seen that if $A_s = 0$ i.e. switching is perfect then equation (3.27) becomes-

$$R(t) = e^{-At}\,(1+At)$$

This function for reliability is same as Reliability function for two parallel redundant standby system.

3.1.6. Comparision of Parallel Standby and Parallel Redundant Systems:

Examination of Reliability models in non-maintained systems for standby redundant and parallel redundant reveals that reliability of standby redundant system is greater than parallel redundant system for the same number of equipments as is observed from Fig.(3.3). If the same is considered for imperfect switching conditions, equating the reliability functions of two equipment parallel redundant and two equipment standby with imperfect switching.

$$2\, e^{-At} - e^{-2At} = e^{-At} + At\, e^{-(A+A_s)t}$$

which gives-

$$\frac{A_s}{A} = -\log\left(\frac{1-e^{-At}}{At}\right)/At \quad \ldots \quad \ldots \quad \ldots \quad (3.28)$$

If the above ratio holds good both system will have the same reliability. But if A_s/A is less than right hand side of equation (3.28), the standby system with imperfect switching will be preferred.

3.2. Reliability Models for Maintained Systems:

In the case of non-maintained systems it was assumed that failure distribution function can be represented by negative exponential distribution i.e.

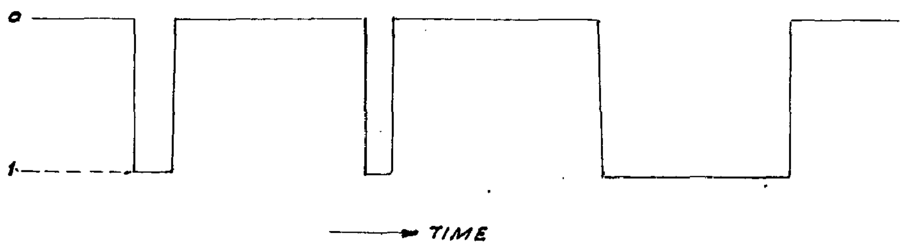
$$F(t) = 1-e^{-At}$$

so that the probability of conditional failure in time interval t,(t+dt) is Adt.

Similarly in case of maintained systems the same type of assumptions hold good. It is assumed that most of the failures can be repaired in a short time; while the items that fail infrequently take a long time for repair. Therefore the equipment repair distribution is exponentially distributed as $G(t)=1-e^{-rt}$, and in the same way it can be shown that probability of completeing a repair in time interval t,(t+dt) is rdt, provided it was not completed at time t.

In case of maintained systems one has to develop forward and backward differential equations which describe the transitions, back and forth from state to state. But in the case of non-maintained systems only forward differentials equations were required because the state can not reset back once it has passed. In maintained systems when an equipment fails it is immediately detected, the repair is started and time to failure and time to repair each is independently exponentially distributed.

In the case of non-maintained systems one is primarily concerned with two figures of merit, the first is reliability function and the second is mean time to failure. But for maintained system there are two more additional figures of merits which are usually of interest. The proportion of time in which the system will be in acceptable state is some times referred as to system availability. Another figure of merit is reoccurance time. The length of time for an equipment to return to an acceptable state from a failed state is sometimes referred as mean single down-time. Considering the single equipment operating continuously, if a record is kept as to when the equipment is operating or down for a period of time it is possible to describe its availability as a random vatiable defined by a distribution function H(A).



The availability is simply the average value of the function over all possible intervals.

3.2.1. Single Equipment System:

In the case of non-maintained systems the Markov process was used to define the reliability function, but for maintained systems one will be more concerned with the Availability function.

3.2.2. Availability Function:

The method of expressing the probability is same as in the case of non-maintained systems, except that in maintained system transition is possible in forward and backward instead of forward direction due to possibility of system to be repaired. Single equipment will have two states; State zero the system is operating, State one when system is failed and under repair.

The transition matrix can be formed as follows:

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{bmatrix} 1-A & A \\ r & 1-r \end{bmatrix} \end{array}$$

The above matrix gives the following equations-

$$\left. \begin{aligned} P_0(t+dt) &= P_0(t)\ (1-Adt) + P_1(t) r.dt + 0(dt) \\ P_1(t+dt) &= P_0(t).A\ dt + P_1(t)(1-r.dt) + 0(dt) \end{aligned} \right\} \quad \ldots \quad (3.29)$$

The term 0(dt) in both the equations represents the probability of two events taking place in t,(t+dt) which is negligible.

The set of equations (3.29) gives the following differential equations-

$$\left. \begin{aligned} P_0'(t) &= -AP_0(t) + rP_1(t) \\ P_1'(t) &= A\ P_0(t) - rP_1(t) \end{aligned} \right\} \quad \ldots \quad \ldots \quad (3.30)$$

At t = 0 the system was operative with the initial conditions, $P_0(t) = 1$ $P_1(0) = 0$, but taking into consideration the case when repair just started the system is down i.e.

$$P_0(0) = 0 \qquad P_1(0) = 1$$

Taking Laplace Transform of equation (3.30) and simplifying-

$$P_0(s) = \frac{S + r}{S(S + A + r)}$$

Availability function is designated by A(t), which is the inverse of $P_0(s)$-

or

$$\left. \begin{aligned} A(t) &= \mathcal{L}^{-1}\{P_0(s)\} = P_0(t) = \frac{r}{r+A} + \frac{A}{r+A} e^{-(r+A)t} \\ \text{and } 1-A(t) &= P_1(t) = \frac{A}{r+A} - \frac{A}{r+A} e^{-(r+A)t} \end{aligned} \right\} \quad (3.31)$$

If the system was initially failed-

i.e.

$$P_o(0) = 0 \quad P_o(1) = 1$$

$$A(t) = P_o(t) = \frac{r}{r+A} + \frac{A}{r+A} e^{-(r+A)t} \qquad \ldots \quad (3.32)$$

and

$$1-A(t) = P_1(t) = \frac{A}{r+A} + \frac{A}{r+A} e^{-(r+A)t}$$

If t becomes large, the equation (3.31) and equation (3.32) become the same. This indicates that after the system has been operating for some time, its behaviour becomes independent of starting point.

As described earlier that availability function is an average value of the function over all intervals -

i.e.

$$A(t) = \frac{1}{T}\int_0^T A(t)dt$$

In this case for instance-

$$A(t) = \frac{r}{r+A} + \frac{A}{(r+A)^2} - \frac{A}{(r+A)^2 T} e^{-(r+A)T} \qquad \ldots \quad \ldots \quad (3.33)$$

when $t \longrightarrow \infty$

$$A(\infty) = \frac{r}{r+A} \qquad \ldots \quad \ldots \quad \ldots \quad (3.34)$$

This condition is sometimes referred as steady state availability.

3.2.3. Steady-State Behaviour:

For all cases, where it is possible to go from one state to another over a long period of time, then-

$$P_i = \lim_{t \to \infty} P_i(t) \quad \text{always exists.}$$

This means that steady state solution can be found by setting the derivative, $P_i'(t) = 0$. Then system of differential equations reduces to an ordinary algebriac equations to solve as $\sum_{w=0}^{x} P_1 = 1$

The equation (3.30) turns out to be-

$$0 = -A\,P_o(t) + r\,P_1(t)$$

$$0 = A\,P_o(t) - rP_1(t)$$

and $P_o(t) + P_1(t) = 1.$

This gives- $P_o(t) = \frac{r}{r+A}$ and $P_1(t) = \frac{A}{r+A}$

With the above results it is clear that many complex problems can be solved in the steady-state. Now considering the problem, where the equipment is subjected to two types of repair, when the equipment fails for the first time a partial repair is performed which restores the system to operation, however it increases the probability of failure. After it has failed second time through repair is performed which brings the equipment to good as new condition. If A_1 is the failure rate when equipment has been through a complete repair and A_2 when it is between through and partial repair ($A_2 > A_1$). Similarly r_1 is the repair rate for partial repair and r_2 the repair rate for a complete repair ($r_2 < r_1$). This gives four states in which the system remains at any time t. -

(i) (State - 0)- The system is operating after a complete repair has been performed.

(ii) (State - 1)- The system has failed and a partial repair is being performed.

(iii) (State - 2)- The system is operating after the completion of partial repair.

(iv) (State - 3)- The system is failed and a complete repair is being performed so that it can again come to state zero.

Thus only state zero and two are acceptable.

The transition matrix is formed as below:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1-A_1 & A_1 & 0 & 0 \\ 0 & 1-r_1 & r_1 & 0 \\ 0 & 0 & 1-A_2 & A_2 \\ r_2 & 0 & 0 & 1-r_2 \end{bmatrix} \end{array}$$

The matrix gives the following equations-

$$\left.\begin{aligned} P_o(t+dt) &= P_o(t)\ (1-A_1dt) + r_2\ P_3(t) + O(dt) \\ P_1(t+dt) &= P_o(t)\ A_1dt + P_1(t)\ (1-r_1\ dt) + O(dt) \\ P_2(t+dt) &= P_1(t)\ r_1dt + P_2(t)\ (1-A_2dt) + O(dt) \\ P_3(t+dt) &= P_2(t)\ A_2dt + P_3(t)\ (1-r_2dt) + O(dt) \end{aligned}\right\} \quad \ldots \quad \ldots \quad (3.36)$$

The set of equations (3.35) gives the following differential equations-

$$\left.\begin{aligned} P_o'(t) &= -A_1\ P_o(t) + r_2P_3(t) \\ P_o'(t) &= A_1\ P_o(t) - r_1P_1(t) \\ P_2'(t) &= r_1P_1(t) - r_2P_2(t) \\ P_3'(t) &= A_2P_2(t) - r_2P_3(t) \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3.36)$$

Equating these equations (3.36) equal to zero and simplifying keeping in view that-

$$P_o(t) + P_1(t) + P_2(t) + P_3(t) = 1$$

But the acceptable states are $P_o(t)$ and $P_2(t)$-

Hence

$$A(\infty) = P_o(t) + P_2(t)$$

$$= \frac{2A_1 r_1 r_2}{A_1 A_2 r_1 + A_2 r_1 r_2 + A_1 A_2 r_1 + r_1 r_2 A_1}$$

If $A_1 = A_2 = A$

and $r_1 = r_2 = r$

then $A(\infty) = \dfrac{r}{r + A}$

This is the same as equation (3.34).

3.2.4. Reliability Functions:

In the previous sections steady-state or long term behaviour of the system has been considered. The steady-state solution of differential equation gives an idea about the proportion of the time the system remains in the failed and repair state, from which it is easy to determine the system availability. In many cases one may be interested in examining the time dependent behaviour of the failure and repair process in order to make some statement about the probability that system will not reach to failed state within the time (0,t). This is the reliability function.

In order to find an expression for reliability function of maintained system one can employ the same transition matrix as above with the exception that it is so defined that transition can not be made out ot state x, the state of system failure.

Considering the most simple case of two equipment standby redundant system with one repair man transition matrix can be formed as follows:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 1-A & A & 0 \\ r & 1-(r+A) & A \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

This matrix gives-

$$\left.\begin{aligned} P_o(t+dt) &= P_o(t)\ (1-Adt)+P_1(t).rdt + O(dt) \\ P_1(t+dt) &= P_o(t)\ Adt + P_1(t)\ [1-(r+A)dt]+O(dt) \\ P_2(t+dt) &= P_1(t).Adt + P_2(t)dt + O(dt) \end{aligned}\right\} \quad \ldots (3.37)$$

The set of equations (3.37) gives the following differential equations.

$$\left.\begin{aligned} P_o'(t) &= -A\ P_o(t) + rP_1(t) \\ P_1'(t) &= A\ P_o(t) - (A+r)P_1(t) \\ P_2'(t) &= AP_1(t) \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots (3.38)$$

initial conditions are-

$$P_o(0) = 1, \quad P_1(0) = 0, \quad P_2(0) = 0$$

and $\quad R(t) = P_o(t) + P_1(t)$

Solving the set of equations (3.38) by Laplace Transform

$$P_o(s) = \frac{-A(A+r+s)}{s^2 + (2A+r) + A^2} = \frac{-(A+r+s)}{(s-s_1)\ (s-s_2)}$$

$$P_1(s) = \frac{A}{(s_1-s_2)} \left[\frac{1}{s-s_1} - \frac{1}{s-s_2} \right]$$

where $\quad s_1 = \dfrac{-(2A+r) + \sqrt{r^2 + 4Ar}}{2}$

$$s_2 = \frac{-(2A+r) - \sqrt{r^2 + 4Ar}}{2}$$

$$P_o(t) = \frac{[(A+r)-s_1]\ e^{-s_1 t} + [(A+r)-s_2]e^{-s_2 t}}{s_1-s_2}$$

$$P_1(t) = \frac{A}{(s_1-s_2)} (e^{-s_1 t} - e^{-s_2 t})$$

$$R(t) = P_o(t) + P_1(t)$$

$$= \frac{s_2 e^{s_1 t} - s_1 \, e^{-s_2 t}}{s_1 - s_2} \qquad \ldots \qquad \ldots \quad (3.39)$$

The difference between the reliability functions for maintained and non-maintained systems is shown in fig.(3.5).

3.25. Non-Markov Process:

In the previous analysis it is assumed that failure distribution is exponential but if it is assumed that-

$$F(t) = 1 - e^{-At} - At \, e^{-At}$$

and repair distribution $G(t) = 1 - e^{-rt}$

From failure distribution it is clear that it goes through exponential phases each of average length 1/A.

Therefore one has to designate three states instead of two where the equipment goes through one exponential distribution. The repair process is commenced when the system reaches state 2, since state 0, and 1 are operating states.

The transistion matrix will be of the form-

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 1-A & A & 0 \\ 0 & 1-A & A \\ r & 0 & 1-r \end{bmatrix} \end{array}$$

Steady-state availability function $A(\infty) = P_0(t) + P_1(t)$ forming the differential equations from the matrix and solving by Laplace Transform.

$$A(\infty) = \frac{2r}{2r + A} \qquad \ldots \qquad \ldots \quad (3.40)$$

on the otherhand if-

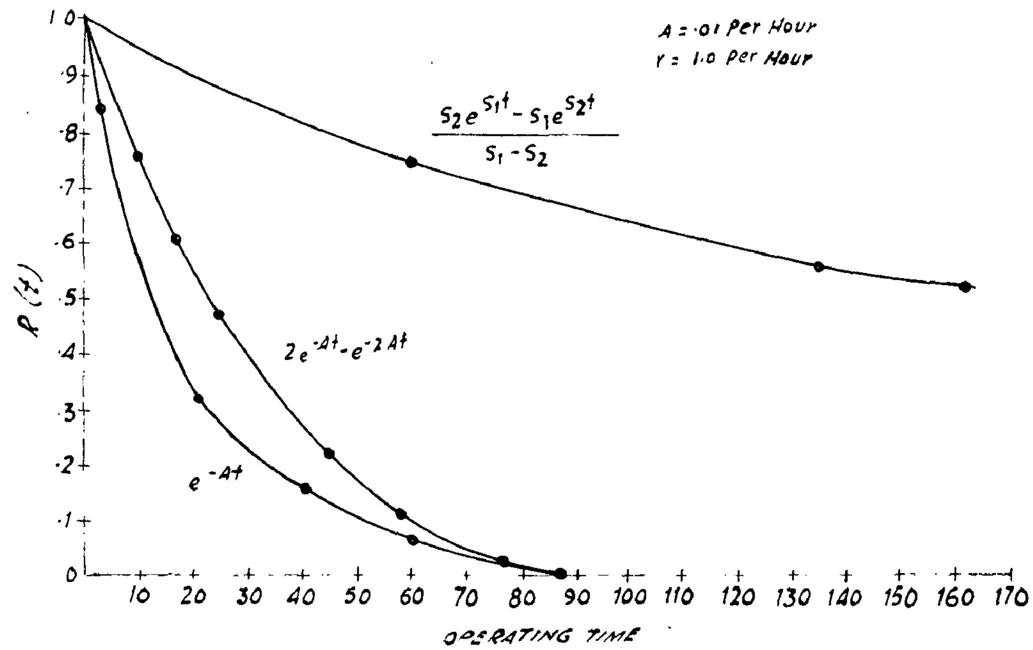


FIG. 3·5 COMPARISON OF RELIABILITY FUNCTIONS. TWO-EQUIPMENT MAINTAINED AND NON-MAINTAINED REDUNDANT SYSTEMS.

$$G(t) = 1 - e^{-rt} - rt\, e^{-rt}$$

and $F(t) = 1 - e^{-At}$

Then transisition matrix will be-

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 1-A & 0 & A \\ r & 1-r & 0 \\ 0 & r & 1-r \end{bmatrix} \end{array}$$

System is only available in zero state.

After solving the differential equations formed, from the above matrix.

which gives-

$$A(\propto) = P_0(\propto) = \frac{r}{r+2A} \qquad \ldots \quad (3.41)$$

C H A P T E R - 4

SELECTIVE AND NON-SELECTIVE OPERATION OF RELAYS

Every protective relay must meet two main requirements:[20]

(i) It should not operate when it is not required to do so. This discriminative requirement is known as selectivity of the relay.

(ii) It must operate when it is required.

To meet the first requirement there arise two different cases of disturbance of selectivity. Faults may take place on the particular component which may give rise to some operative signal, which disturbs the selectivity i.e. the signal is present when unnecessary. Another cause of disturbance of selectivity is; no signal is emitted, when necessary. A good relay must distinguish these two cases. It is simply wrong to add up all the instances of the disturbance independent of the particular class of fault because,

(i) Back-up arrangements which raise the reliability in respect of selectivity, increase the probability of failure or vice-versa.

(ii) With a disturbance of selectivity of one or the other class, the probability of non-selective action is not the same as the probability of failure, and their economic consequences differ as well.

It is assumed that fault on any component may lead to unwanted signal and disturbs the selectivity. Thus the selectivity can be increased by duplicating the component by using the same type of components, and arranged in such a way that output signal only appears when all the components arranged as above are operative. If the probability of appearing of the undesired signal in the output of one of the element is denoted by q ($q < 1$) the probability of false operation in Fig.(4.1.1.) is $q' = q^{m,}$ because $q^{m,} < q$, hence selectivity is increased.

However if the element fails, this increases the probability of failure. Denoting the probability of failure of one element by q_1, the probability of the output signal of the same element appearing is $p_1 = 1 - q_1$, the probability of the output signal through m, elements is $p_1^{m,}$, while the probability of failure is $q_1' = 1 - p_1^{m,}$

or $\quad q_1' = [\, 1 - (1-q_1)^{m,} \,]$

since $[\, 1 - (1 - q_1)^{m,} \,] > q_1$, the probability of failure is increased.

If the elements are duplicated in parallel as in Fig.(4.1.2.) then probability of signal output is $q_1' = q_1^{n,}$

since $q_1^{n,} < q_1$ hence probability of failure decreases, but the probability of non selective action is given by-

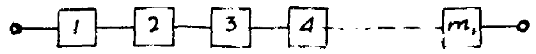
$$q' = 1 - (1-q)^{n,}$$

since q' q, the probability of non selective action is increased.

Keeping in view the above statement, some compromise has to be made by arranging some series-parallel elements in such a way that probability of non selective action is decreased as well as probability of failure is also decreased.

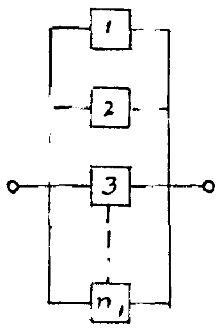
The circuit as shown in Fig.(4.1.3.), has been analysed as a general case, (the circuit (4.1.4) is converse of it), in which there are m, elements in series with n, elements in parallel. Algebric equations have been framed for different values of m, and n,.

SCHEME	PROBABILITY	
	NON-SELECTIVE OPERATION q	FAILURE q_1



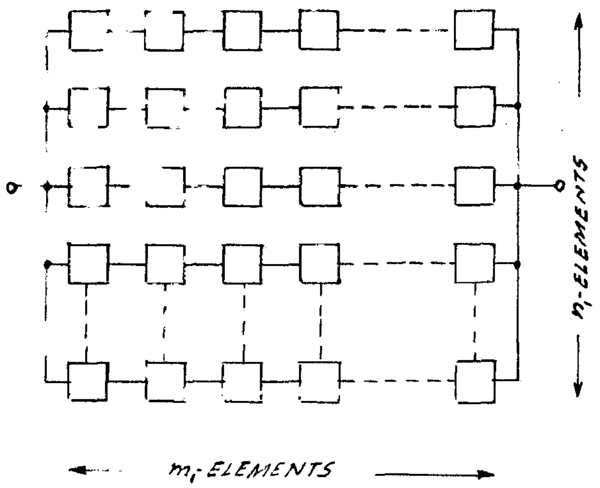
4.1.1

Non-selective operation: q^m — Failure: $1-(1-q_1)^m$



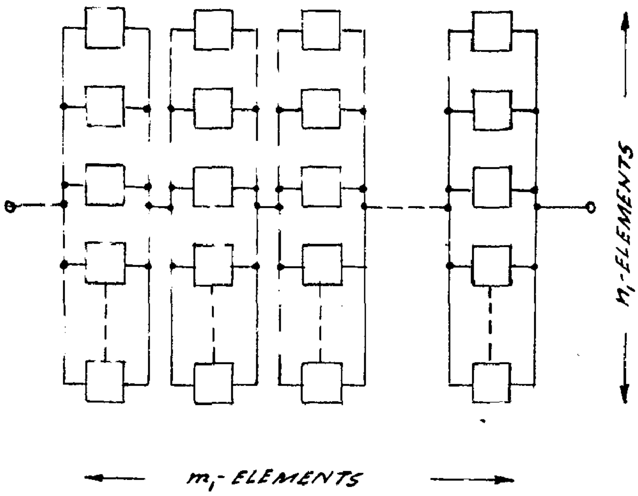
4.1.2

Non-selective operation: $1-(1-q)^{n_1}$ — Failure: $q_1^{n_1}$



4.1.3

Non-selective operation: $[1-(1-q^{m_1})^{n_1}]$ — Failure: $[1-(1-q_1)^{m_1}]^{n_1}$



4.1.4

Non-selective operation: $[1-(1-q)^{m_1}]^{n_1}$ — Failure: $[1-(1-q_1^{m_1})^{n_1}]$

FIG. 4.1

4.1. Analysis for Relay Selectivity:

Case 1 for $n_1 = 2$, $m_1 = 2, 3, 4, 5, 6$.

The following inequalities should be satisfied.

$1 - (1-q^2)^2 < q$ or $q-2q^2 + q^4 \geq 0$ ... (4.1)

$1 - (1-q^3)^2 < q$ or $q-2q^3 + q^6 \geq 0$ ... (4.2)

$1 - (1-q^4)^2 < q$ or $q-2q^4 + q^8 \geq 0$ ... (4.3)

$1 - (1-q^5)^2 < q$ or $q-2q^5 + q^{10} \geq 0$ ... (4.4)

$1 - (1-q^6)^2 < q$ or $q-2q^6 + q^{12} \geq 0$ ... (4.5)

Case 2:

when $n_1 = 3$, $m_1 = 2,3,4,5,6$

The following inequalities should be satisfied.

$1 - (1-q^2)^3 < q$ or $q-3q^2 + 3q^4 - q^6 \geq 0$ ... (4.6)

$1 - (1-q^3)^3 < q$ or $q-3q^3 + 3q^6 - q^9 \geq 0$ ... (4.7)

$1 - (1-q^4)^3 < q$ or $q-3q^4 + 3q^8 - q^{12} \geq 0$... (4.8)

$1 - (1-q^5)^3 < q$ or $q-3q^5 + 3q^{10} - q^{15} \geq 0$... (4.9)

$1 - (1-q^6)^3 < q$ or $q-3q^6 + 3q^{12} - q^{18} \geq 0$... (4.10)

Case 3:

when $n_1 = 4$, $m_1 = 2, 3, 4, 5, 6$.

The following inequalities should be satisfied.

$1 - (1-q^2)^4 < q$ or $q-4q^2+6q^4- 4q^6+q^8 \geq 0$ (4.11)

$1 - (1-q^3)^4 < q$ or $q-4q^3+6q^6- 4q^9+q^{12} \geq 0$ (4.12)

$1 - (1-q^4)^4 < q$ or $q-4q^4+6q^8- 4q^{12}+q^{16} \geq 0$ (4.13)

$1 - (1-q^5)^4 < q$ or $q-4q^5+6q^{10}-4q^{15}+q^{20} \geq 0$ (4.14)

$1 - (1-q^6)^4 < q$ or $q-4q^6+6q^{12}-4q^{18}+q^{24} \geq 0$ (4.15)

Case 4:

when $n_1 = 5$, $m_1 = 2, 3, 4, 5, 6$

The following inequalities should be satisfied.

$1 - (1-q^2)^5 < q$ or $q-5q^2+10q^4 - 10q^6+ 5q^8-q^{10} \geq 0$ (4.16)

$1 - (1-q^3)^5 < q$ or $q-5q^3+10q^6 - 10q^9+ 5q^{12}-q^{15} \geq 0$ (4.17)

$1 - (1-q^4)^5 < q$ or $q-5q^4+10q^8 - 10q^{12}+5q^{16}-q^{20} \geq 0$ (4.18)

$1 - (1-q^5)^5 < q$ or $q-5q^5+10q^{10}- 10q^{15}+5q^{20}-q^{25} \geq 0$ (4.19)

$1 - (1-q^6)^5 < q$ or $q-5q^6+10q^{12}- 10q^{18}+5q^{24}-q^{30} \geq 0$ (4.20)

Case 5:

when $n_1 = 6$ $m_1 = 2, 3, 4, 5, 6.$

The following inequalities should be satisifed.

$1 - (1-q^2)^6 < q$ or $q-6q^2+15q^4-20q^6+15q^8-6q^{10}+q^{12} \geq 0$ (4.21)

$1 - (1-q^3)^6 < q$ or $q-6q^3+15q^6-20q^9+15q^{12}-6q^{15}+q^{18} \geq 0$ (4.22)

$1 - (1-q^4)^6 < q$ or $q-6q^4+15q^8-20q^{12}+15q^{16}-6q^{20}+q^{24} \geq 0$ (4.23)

$1 - (1-q^5)^6 < q$ or $q-6q^5+15q^{10}-20q^{15}+15q^{20}-6q^{25}+q^{30} \geq 0$ (4.24)

$1 - (1-q^6)^6 < q$ or $q-6q^6+15q^{12}-20q^{18}+15q^{24}-6q^{30}+q^{36} \geq 0$ (4.25)

Since

$q > 0$

and $(q-1) < 0$

i.e. $0 < q < 1$

From the above 25 inequalities the optimum acceptable value of q is found out and tabulated as shown below:

Table 4.1

Case 1	q_{n_1,m_1}	q_{22}	q_{23}	q_{24}	q_{25}	q_{26}
	q	0.618	0.84	0.92	0.95	0.96
Case 2	q_{n_1,m_1}	q_{32}	q_{33}	q_{34}	q_{35}	q_{36}
	q	0.388	0.68	0.81	0.86	0.905
Case 3	q_{n_1,m_1}	q_{42}	q_{43}	q_{44}	q_{45}	q_{46}
	q	0.29	0.58	0.725	0.808	0.855
Case 4	q_{n_1,m_1}	q_{52}	q_{53}	q_{54}	q_{55}	q_{56}
	q	0.22	0.52	0.66	0.755	0.81
Case 5	q_{n_1,m_1}	q_{62}	q_{63}	q_{64}	q_{65}	q_{66}
	q	0.18	0.46	0.62	0.72	0.78

4.1.1. Gain in Selectivity:

The gain $Yc = f(q)$ in selectivity is obtained as the ratio of the probabilities of non selective action of one element to the probabilities of non-selective action of the circuit of Fig.(4.1.3) for different values of n_1 and m_1.

Case 1:

for $n_1 = 2$, $m_1 = 2, 3, 4, 5, 6$.

$$Yc(2,2) = \frac{q}{2q^2 - q^4} = \frac{1}{2q - q^3} \qquad \ldots \qquad \ldots (4.26)$$

$$Yc(2,3) = \frac{q}{2q^3 - q^6} = \frac{1}{2q^2 - q^5} \qquad \ldots \qquad \ldots (4.27)$$

$$Yc(2,4) = \frac{q}{2q^4 - q^8} = \frac{1}{2q^3 - q^7} \qquad \ldots \qquad \ldots (4.28)$$

$$Yc(2,5) = \frac{q}{2q^5 - q^{10}} = \frac{1}{2q^4 - q^9} \qquad \ldots \qquad \ldots (4.29)$$

$$Yc(2,6) = \frac{q}{2q^6 - q^{12}} = \frac{1}{2q^5 - q^{11}} \qquad \ldots \qquad \ldots (4.30)$$

Case - 2:

for $n_1 = 3$, $m_1 = 2, 3, 4, 5, 6$

$$Yc(3,2) = \frac{q}{3q^2-3q^4+q^6} = \frac{1}{3q-3q^3+q^5} \quad \ldots \quad \ldots (4.31)$$

$$Yc(3,3) = \frac{q}{3q^3-3q^6+q^9} = \frac{1}{3q^2-3q^5+q^8} \quad \ldots \quad \ldots (4.32)$$

$$Yc(3,4) = \frac{q}{3q^4-3q^8+q^{12}} = \frac{1}{3q^3-3q^7+q^{11}} \quad \ldots \quad \ldots (4.33)$$

$$Yc(3,5) = \frac{q}{3q^5-3q^{10}+q^{15}} = \frac{1}{3q^4-3q^9+q^{14}} \quad \ldots \quad \ldots (4.34)$$

$$Yc(3,6) = \frac{q}{3q^6-3q^{12}+q^{18}} = \frac{1}{3q^5-3q^{11}+q^{17}} \quad \ldots \quad \ldots (4.35)$$

Case - 3:

for $n_1 = 4$, $m_1 = 2, 3, 4, 5, 6$

$$Yc(4,2) = \frac{q}{4q^2-6q^4+4q^6-q^8} = \frac{1}{4q-6q^3+4q^5-q^7} \quad \ldots \quad \ldots (4.36)$$

$$Yc(4,3) = \frac{q}{4q^3-6q^6+4q^9-q^{12}} = \frac{1}{4q^2-6q^5+4q^8-q^{11}} \quad \ldots \quad \ldots (4.37)$$

$$Yc(4,4) = \frac{q}{4q^4-6q^8+4q^{12}-q^{16}} = \frac{1}{4q^3-6q^7+4q^{11}-q^{15}} \quad \ldots (4.38)$$

$$Yc(4,5) = \frac{q}{4q^5-6q^{10}+4q^{15}-q^{20}} = \frac{1}{4q^4-6q^9+4q^{14}-q^{19}} \quad \ldots (4.39)$$

$$Yc(4,6) = \frac{q}{4q^6-6q^{12}+4q^{18}-q^{24}} = \frac{1}{4q^5-6q^{11}+4q^{17}-q^{23}} \quad \ldots (4.40)$$

Case - 4:

for $n_1 = 5$, $m_1 = 2, 3, 4, 5, 6.$

$$Yc(5,2) = \frac{q}{5q^2-10q^4+10q^6-5q^8+q^{10}} = \frac{1}{5q-10q^3+10q^5-5q^7+q^9} \quad \ldots (4.41)$$

$$Yc(5,3) = \frac{q}{5q^3-10q^6+10q^9-5q^{12}+q^{15}} = \frac{1}{5q^2-10q^5+10q^8+5q^{14}-5q^{11}} \quad \ldots (4.42)$$

$$Yc(5,4) = \frac{q}{5q^4-10q^8+10q^{12}-5q^{16}+q^{20}} = \frac{1}{5q^3-10q^7+10q^{11}-5q^{15}+q^{19}} \quad (4.43)$$

$$Yc(5,5) = \frac{q}{5q^5-10q^{10}+10q^{15}-5q^{20}+q^{25}} = \frac{1}{5q^4-10q^9+10q^{14}-5q^{19}+q^{24}} \quad (4.44)$$

$$Yc(5,6) = \frac{q}{5q^6-10q^{12}+10q^{18}-5q^{24}+q^{30}} = \frac{1}{5q^5-10q^{11}+10q^{17}-5q^{23}+q^{29}} \quad (4.45)$$

Case - 5:

for $n_1 = 6$, $m_1 = 2, 3, 4, 5, 6.$

$$Yc(6,2) = \frac{q}{6q^2-15q^4+20q^6-15q^8+6q^{10}-q^{12}} = \frac{1}{6q-15q^3+20q^5-15q^7+6q^9-q^{11}} \quad \ldots (4.46)$$

$$Yc(6,3) = \frac{q}{6q^3-15q^6+20q^9+15q^{12}+6q^{15}-q^{18}} = \frac{1}{6q^2-15q^5+20q^8+6q^{14}-15q^{11}-q^{17}} \quad \ldots (4.47)$$

$$Yc(6,4) = \frac{q}{6q^4-15q^8+20q^{12}-15q^{16}+6q^{20}-q^{24}} = \frac{1}{6q^3-15q^7+20q^{11}-15q^{15}+6q^{19}-q^{23}} \quad \ldots (4.48)$$

$$Yc(6,5) = \frac{q}{6q^5-15q^{10}+20q^{15}-15q^{20}+6q^{25}-q^{30}} = \frac{1}{6q^4-15q^9+20q^{14}-15q^{19}+6q^{24}-q^{29}} \quad \ldots (4.49)$$

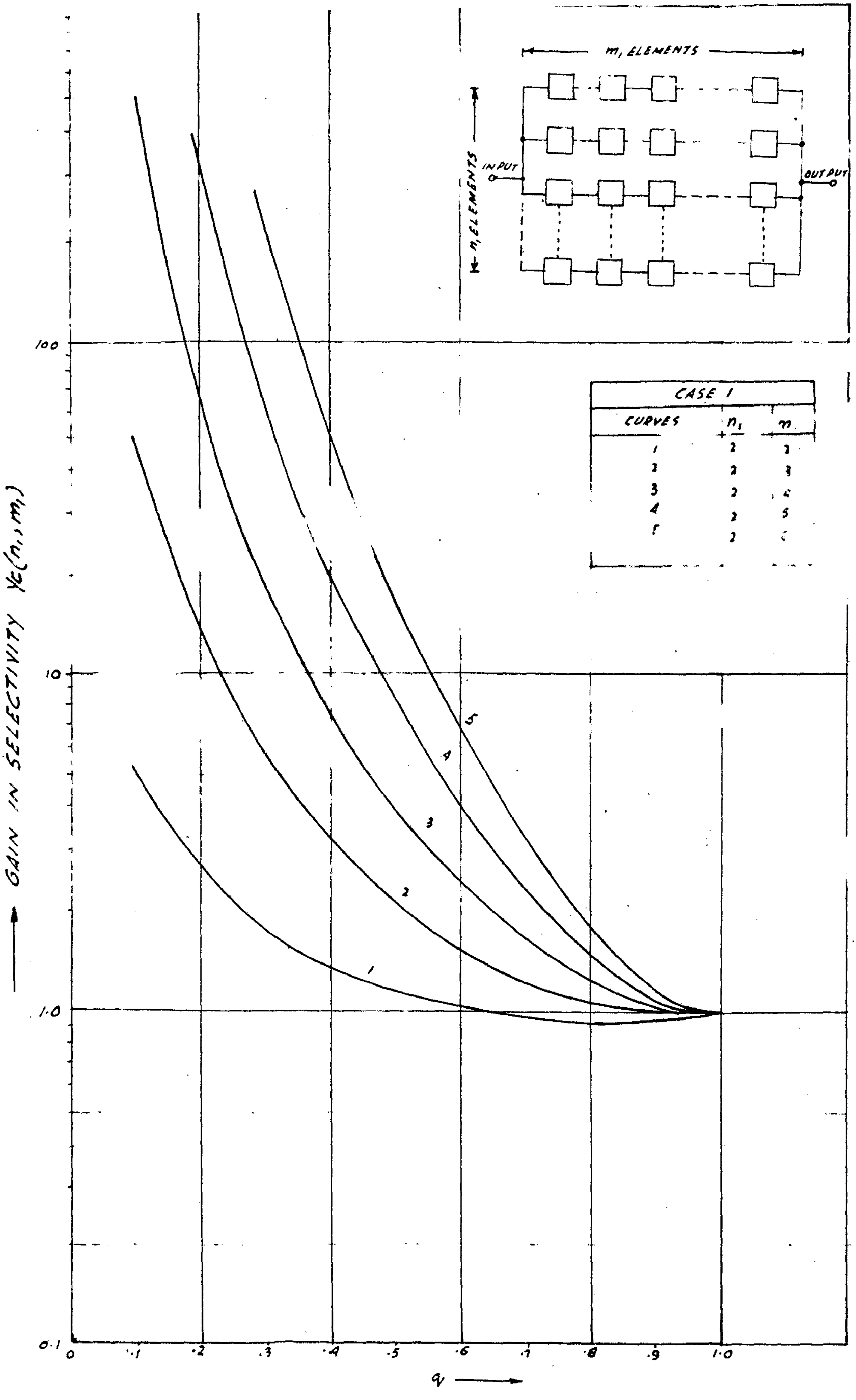


FIG.4.2 CURVES FOR $Y_C(n_1, m_1)$ Vs q

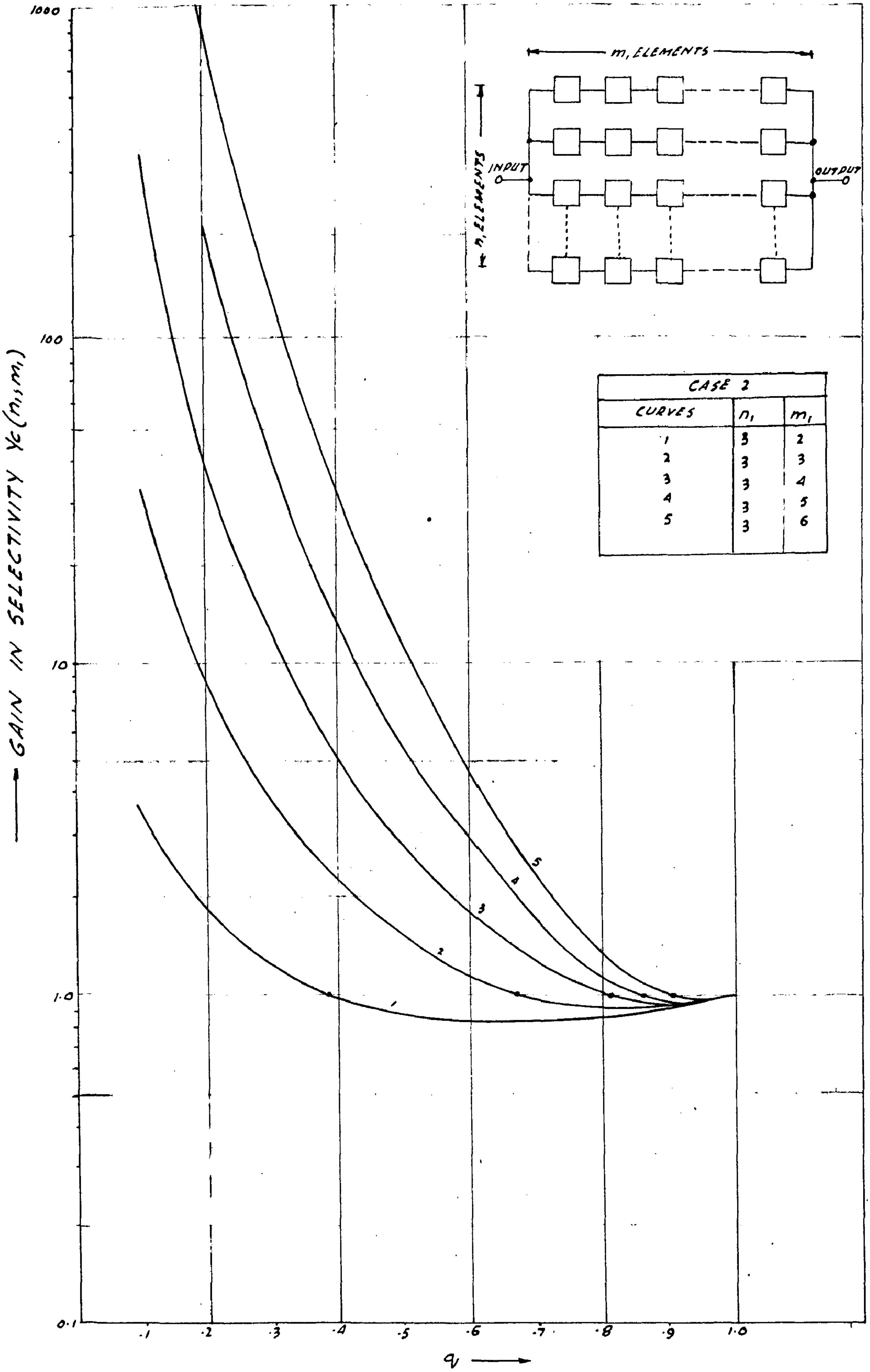
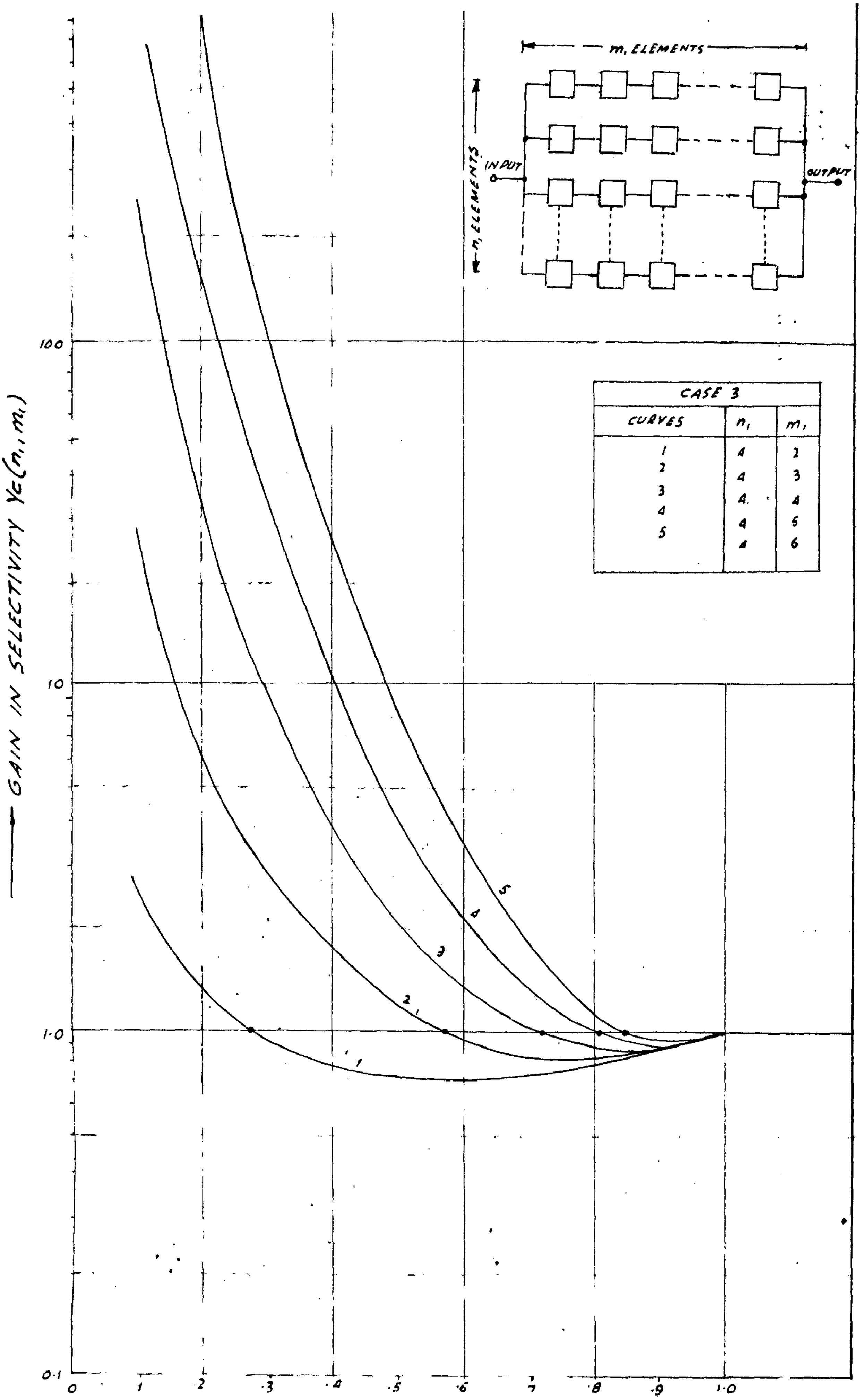


FIG. 4.3 CURVES FOR $Y_c(n_1, m_1)$ Vs q



CASE 3		
CURVES	n_1	m_1
1	4	2
2	4	3
3	4	4
4	4	5
5	4	6
	4	6

F.G 4·4 CURVES FOR $Y_c(n_1, m_1)$ Vs q

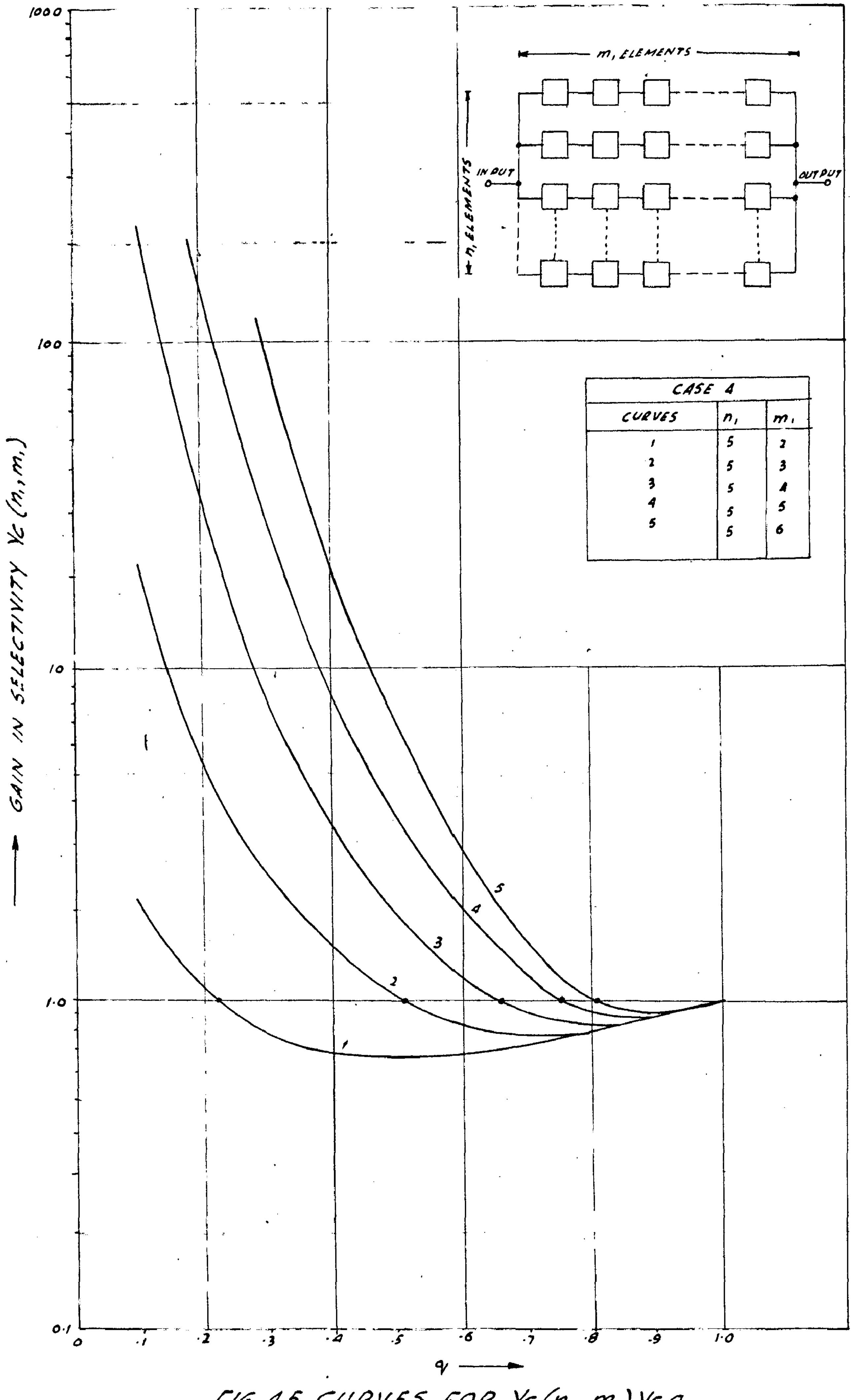


FIG. 4·5 CURVES FOR Yc(n, , m,) Vs q

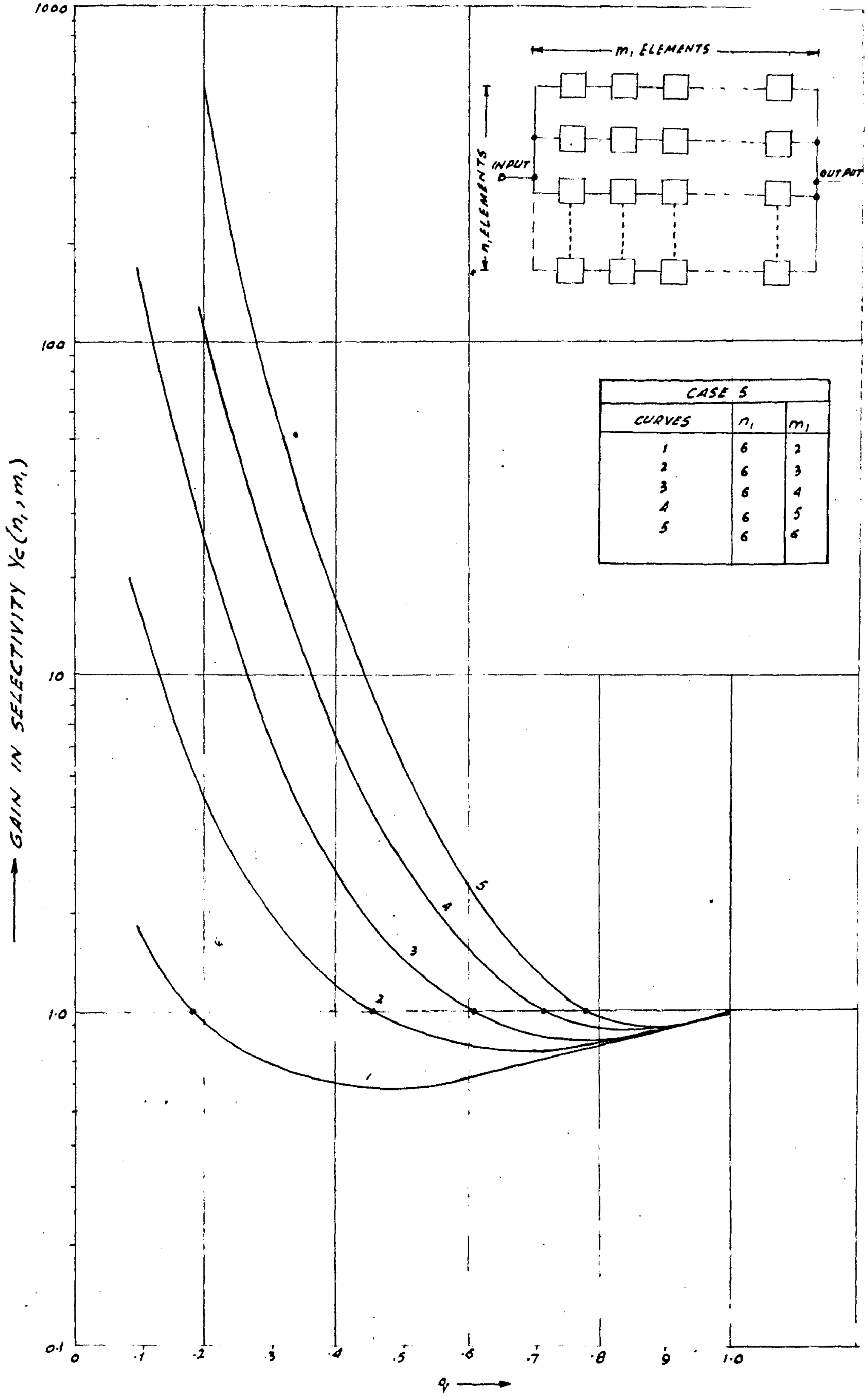


FIG. 4·6 CURVES FOR Yc(n,, m,) Vs q

$$Yc(6,6)=\frac{q}{6q^{6}-15q^{12}+20q^{18}-15q^{24}+6q^{30}-q^{36}}=\frac{1}{6q^{5}-15q^{11}+20q^{17}-15q^{23}+6q^{29}-q^{35}} \quad \ldots (4.50)$$

The function Yc=f(q) for different cases are plotted in Figs. (4.2 to 4.6). The computer programme for the plotting of the curve [Yc=f(q)] for the case n_1=6, m_1 = 2,3,4,5 and 6, has been given in Appendix 2.

From the curves it is clear that gain appears (Yc 1), if the different values of q are less than that the values shown in table (4.1) for different configurations and increases as q decreases, i.e. increasing the reliability of the element.

4.2. Analysis of Relay Failure:

Case - 1:

when n_1 = 2, m_1 = 2, 3, 4, 5, 6

The following inequalities must be satisfied:

$$[1-(1-q_1)^2]^2 < q_1 \quad \text{or} \quad (2q_1 - q_1^2)^2-q_1 \leq 0 \quad \ldots \quad \ldots (4.51)$$

$$[1-(1-q_1)^3]^2 < q_1 \quad \text{or} \quad (3q_1 - 3q_1^2 +q_1^3)^2 -q_1 \leq 0 \quad \ldots (4.52)$$

$$[1-(1-q_1)^4]^2 < q_1 \quad \text{or} \quad (4q_1-6q_1^2 +4q_1^3-q_1^4)^2- q_1 \leq 0 \quad \ldots (4.53)$$

$$[1-(1-q_1)^5]^2 < q_1 \quad \text{or} \quad (5q_1-10q_1^2+10q_1^3-5q_1^4+q_1^5)-q_1 \leq 0 \quad \ldots (5.54)$$

$$[1-(1-q_1)^6]^2 < q_1 \quad \text{or} \quad (6q_1-15q_1^2+20q_1^3-15q_1^4+6q_1^5-q_1^6)^2-q_1 \leq 0 \quad \ldots (5.55)$$

Case - 2:

when n_1 = 3, m_1 = 2, 3, 4, 5, 6.

The following inequalities must be satisfied.

$$[1-(1-q_1)^2]^3 < q_1 \text{ or } (2q_1-q_1^2)^3-q_1 \leq 0 \quad \ldots \quad (4.56)$$

$$[1-(1-q_1)^3]^3 < q_1 \text{ or } (3q_1-3q_1^2+q_1^3)^3 - q_1 \leq 0 \quad \ldots \quad (4.57)$$

$$[1-(1-q_1)^4]^3 < q_1 \text{ or } (4q_1-6q_1^2+4q_1^3-q_1^4)^3 - q_1 \leq 0 \quad \ldots \quad (4.58)$$

$$[1-(1-q_1)^5]^3 < q_1 \text{ or } (5q_1-10q_1^2+10q_1^3-5q_1^4+q_1^5)^3-q_1 \leq 0 \quad (4.59)$$

$$[1-(1-q_1)^6]^3 < q_1 \text{ or } (6q_1-15q_1^2+20q_1^3-15q_1^4+6q_1^5-q_1^6)^3-q_1 \leq 0 \quad (4.60)$$

Case - 3:

for $n_1 = 4$, $m_1 = 2, 3, 4, 5, 6$.

The following inequalities must be satisifed.

$$[1-(1-q_1)^2]^4 < q_1 \text{ or } (2q_1-q_1^2)^4 - q_1 \leq 0 \quad \ldots \quad (4.61)$$

$$[1-(1-q_1)^3]^4 \leq q_1 \text{ or } (3q_1-3q_1^2+q_1^3)^4-q_1 \leq 0 \quad \ldots \quad (4.62)$$

$$[1-(1-q_1)^4]^4 < q_1 \text{ or } (4q_1-6q_1^2+4q_1^3-q_1^4)^4 - q_1 \leq 0 \quad \ldots \quad (4.63)$$

$$[1-(1-q_1)^5]^4 < q_1 \text{ or } (5q_1-10q_1^2+10q_1^3-5q_1^4+q_1^5)^4-q_1 \leq 0 \quad (4.64)$$

$$[1-(1-q_1)^6]^4 < q_1 \text{ or } (6q_1-15q_1^2+20q_1^3-15q_1^4+6q_1^5-q_1^6)^4-q_1 \leq 0 \quad (4.65)$$

Case - 4:

$n_1 = 5$, $m_1 = 2, 3, 4, 5, 6$.

The following inequalities must be satisfied.

$$[1-(1-q_1)^2]^5 < q_1 \text{ or } (2q_1-q_1^2)^5-q_1 \leq 0 \quad \ldots \quad (4.66)$$

$$[1-(1-q_1)^3]^5 < q_1 \text{ or } (3q_1-3q_1^2+q_1^3)^5-q_1 \leq 0 \quad \ldots \quad (4.67)$$

$$[1-(1-q_1)^4]^5 < q_1 \text{ or } (4q_1-6q_1^2+4q_1^3-q_1^4)^5-q_1 \leq 0 \quad \ldots \quad (4.68)$$

$$[1-(1-q_1)^5]^5 < q_1 \text{ or } (5q_1-10q_1^2+10q_1^3-5q_1^4+q_1^5)^5-q_1 \leq 0 \quad (4.69)$$

$$[1-(1-q_1)^6]^6 \leq q_1 \text{ or } (6q_1-15q_1^2+20q_1^3-15q_1^4+6q_1^5-q_1^6)^5-q_1 \leq 0 \quad (4.70)$$

Case - 5:

for $n_1 = 6$, $m_1 = 2,3,4,5,6$.

The inequalities given below must be satisfied.

$$[1-(1-q_1)^2]^6 < q_1 \text{ or } (2q_1-q_1^2)^6 - q_1 \leq 0 \quad \ldots \ldots \quad (4.71)$$

$$[1-(1-q_1)^3]^6 < q_1 \text{ or } (3q_1-3q_1^2+q_1^3)^6 - q_1 \leq 0 \quad \ldots \quad (4.72)$$

$$[1-(1-q_1)^4]^6 < q_1 \text{ or } (4q_1-6q_1^2+4q_1^3-q_1^4)^6 - q_1 \leq 0 \quad (4.73)$$

$$[1-(1-q_1)^5]^6 < q_1 \text{ or } (5q_1-10q_1^2+10q_1^3-5q_1^4+q_1^5)^6 - q_1 \leq 0 \quad (4.74)$$

$$[1-(1-q_1)^6]^6 < q_1 \text{ or } (6q_1-15q_1^2+20q_1^3-15q_1^4+6q_1^5-q_1^6)^6-q_1 \leq 0 \quad (4.75)$$

From the above 25 inequalities the optimum acceptable value is calculated and tabulated as shown below:

Table 4.2

Case-1	$q_1(n_1,m_1)$	$q_{1(22)}$	$q_{1(23)}$	$q_{1(24)}$	$q_{1(25)}$	$q_{1(26)}$
	q_1	0.382	0.16	0.08	0.05	0.04
Case-2	$q_1(n_1,m_1)$	$q_{1(32)}$	$q_{1(33)}$	$q_{1(34)}$	$q_{1(35)}$	$q_{1(36)}$
	q_1	0.612	0.320	0.19	0.14	0.095
Case-3	$q_1(n_1,m_1)$	$q_{1(42)}$	$q_{1(43)}$	$q_{1(44)}$	$q_{1(45)}$	$q_{1(46)}$
	q_1	0.71	0.42	0.275	0.192	0.145
Case-4	$q_1(n_1,m_1)$	$q_{1(52)}$	$q_{1(53)}$	$q_{1(54)}$	$q_{1(55)}$	$q_{1(56)}$
	q_1	0.72	0.48	0.34	0.245	0.19
Case-5	$q_1(n_1,m_1)$	$q_{1(62)}$	$q_{1(63)}$	$q_{1(64)}$	$q_{1(65)}$	$q_{1(66)}$
	q_1	0.82	0.54	0.38	0.282	0.22

4.2.1. Analysis of Reduction in probability of failure:

The reduction in probability of failure $Yo(n_1, m_1)=f(q_1)$ is defined as the ratio of probability of failure of one element to the probability of failure of the scheme under consideration for different values of m_1 and n_1.

Case - 1: for $n_1 = 2$, $m_1 = 2,3,4,5,6$.

$$Yo(2,2) = \frac{q_1}{(2q_1 - q_1^2)^2} = \frac{1}{q_1(2-q_1)^2} \quad \ldots \quad (4.76)$$

$$Yo(2,3) = \frac{q_1}{(3q_1-3q_1^2+q_1^3)^2} = \frac{1}{q_1(3-3q_1+q_1^2)^2} \quad \ldots \quad (4.77)$$

$$Yo(2,4) = \frac{q_1}{(4q_1-6q_1^2+4q_1^3-q_1^4)^2} = \frac{1}{q_1(4-6q_1+4q_1^2-q_1^3)^2} \quad (4.78)$$

$$Yo(2,5) = \frac{q_1}{(5q_1-10q_1^2+10q_1^3-5q_1^4+q_1^5)^2} = \frac{1}{q_1(5-10q_1+10q_1^2-5q_1^3+q_1^4)^2} \quad (4.79)$$

$$Yo(2,6) = \frac{q_1}{(6q_1-15q_1^2+20q_1^3-15q_1^4+6q_1^5-q_1^6)^2} = \frac{1}{q_1(6-15q_1+20q_1^2-15q_1^3+6q_1^4-q_1^5)^2} \quad \ldots \quad (4.80)$$

Case - 2:

when $n_1 = 3$, $m_1 = 2,3,4,5,6$

$$Yo(3,2) = \frac{q_1}{(2q_1-q_1^2)^3} = \frac{1}{q_1^2(2-q_1)^3} \quad \ldots \quad (4.81)$$

$$Yo(3,3) = \frac{q_1}{(3q_1-3q_1^2+q_1^3)^3} = \frac{1}{q_1^2(3-3q_1+q_1^2)^3} \quad \ldots \quad (4.82)$$

$$Yo(3,4) = \frac{q_1}{(4q_1-6q_1^2+4q_1^3-q_1^4)^3} = \frac{1}{q_1^2(4-6q_1+4q_1^2-q_1^3)^3} \quad (4.83)$$

$$Yo(3,5)= \frac{q_1}{(5q_1-10q_1^2+10q_1^3-5q_1^4+q_1^5)^3} = \frac{1}{q_1^2(5-10q_1+10q_1^2-5q_1^3+q_1^4)^3} \quad (4.84)$$

$$Yo(3,6)= \frac{q_1}{(6q_1-15q_1^2+20q_1^3-15q_1^4+6q_1^5-q_1^6)^3} = \frac{1}{q_1^2(6-15q_1+20q_1^2-15q_1^3+6q_1^4-q_1^5)^3}$$

$$\ldots \quad (4.85)$$

Case-3:

when $n_1 = 4$ $m_1 = 2,3,4,5,6.$

$$Yo(4,2)= \frac{q_1}{(2q_1-q_1^2)^4} = \frac{1}{q_1^3(2-q_1)^4} \quad \ldots \quad (4.86)$$

$$Yo(4,3)= \frac{q_1}{(3q_1-3q_1^2+q_1^3)^4} = \frac{1}{q_1^3(3-3q_1+q_1^2)^4} \quad \ldots \quad (4.87)$$

$$Yo(4,4)= \frac{q_1}{(4q_1-6q_1^2+4q_1^3-q_1^4)^4} = \frac{1}{q_1^3(4-6q_1+4q_1^2-q_1^3)^4} \quad (4.88)$$

$$Yo(4,5)= \frac{q_1}{(5q_1-10q_1^2+10q_1^3-5q_1^4+q_1^5)^4}$$

$$= \frac{1}{q_1^3(5-10q_1+10q_1^2-5q_1^3+q_1^4)^4} \quad \ldots \quad (4.89)$$

$$Yo(4,6)= \frac{q_1}{(6q_1-15q_1^2+20q_1^3-15q_1^4+6q_1^5-q_1^6)^4}$$

$$= \frac{1}{q_1^3(6-15q_1+20q_1^2-15q_1^3+6q_1^4-q_1^5)^4} \quad \ldots \quad (4.90)$$

Case - 4:

When $n_1 = 5$, $m_1 = 2,3,4,5,6.$

$$Yo(5,2) = \frac{q_1}{(2q_1-q_1^2)^5} = \frac{1}{q_1^4(2-q_1)^5} \quad \ldots \quad (4.91)$$

$$Yo(5,3) = \frac{q_1}{(3q_1-3q_1^2 + q_1^3)^5} = \frac{1}{q_1^4(3-3q_1 + q_1^2)^5} \quad (4.92)$$

$$Yo(5,4) = \frac{q_1}{(4q_1-6q_1^2+4q_1^3 -q_1^4)^5} = \frac{1}{q_1^4(4-6q_1 +4q_1^2 -q_1^3)^5} \quad (4.93)$$

$$Yo(5,5) = \frac{q_1}{(5q_1-10q_1^2 + 10q_1^3-5q_1^4 +q_1^5)^5}$$

$$= \frac{1}{q_1^4(5-10q_1 + 10q_1^2 -5q_1^3 +q_1^4)^5} \quad \ldots \quad (4.94)$$

$$Yo(5,6) = \frac{q_1}{(6q_1-15q_1^2 + 20q_1^3-15q_1^4 + 6q_1^5- q_1^6)^5}$$

$$= \frac{1}{q_1^4(6-15q_1 +20q_1^2-15q_1^3 +6q_1^4-q_1^5)^5} \quad \ldots \quad (4.95)$$

Case - 5:

when $n_1 = 6$, $m_1 = 2,3,4,5,6$.

$$Yo(6,2) = \frac{q_1}{(2q_1 -q_1^2)^6} = \frac{1}{q_1^5 (2 - q_1)^6} \quad \ldots \quad (4.96)$$

$$Yo(6,3) = \frac{q_1}{(3q_1-3q_1^2+ q_1^3)^6} = \frac{1}{q_1^5 (3-3q_1 +q_1^2)^6} \ldots \quad (4.97)$$

$$Yo(6,4) = \frac{q_1}{(4q_1-6q_1^2+4q_1^3-q_1^4)^6} = \frac{1}{q_1^5(4-6q_1+4q_1^2 - q_1^3)^6} \quad (4.98)$$

$$Yo(6,5) = \frac{q_1}{(5q_1-10q_1^2 +10q_1^3 - 5q_1^4 +q_1^5)^6}$$

$$= \frac{q_1}{q_1^5(5-10q_1 +10q_1^2 -5q_1^3 + q_1^4)^6} \quad \ldots \quad (4.99)$$

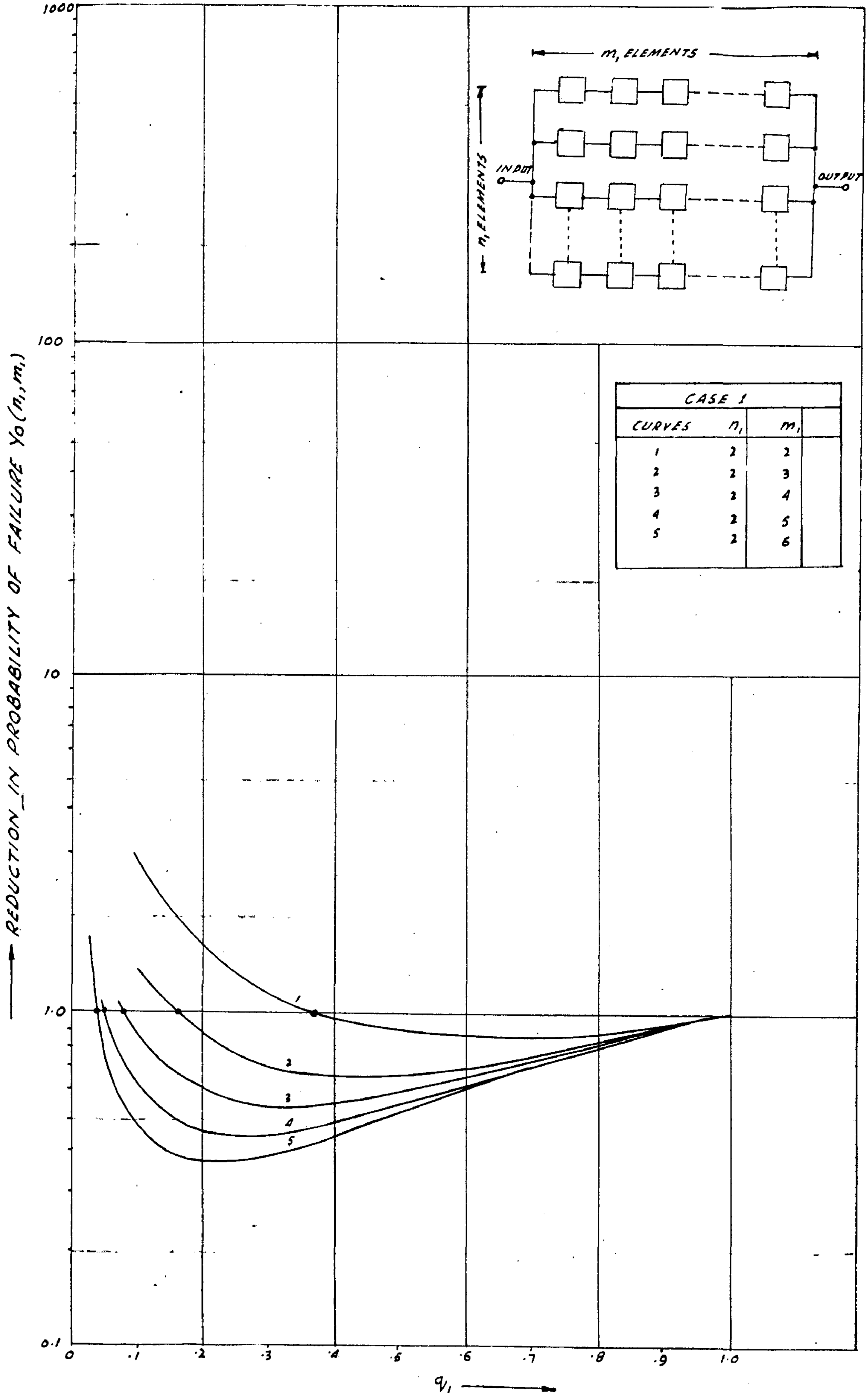


FIG. 4.7 CURVES FOR $Y_0(n_1, m_1)$ Vs. q_1

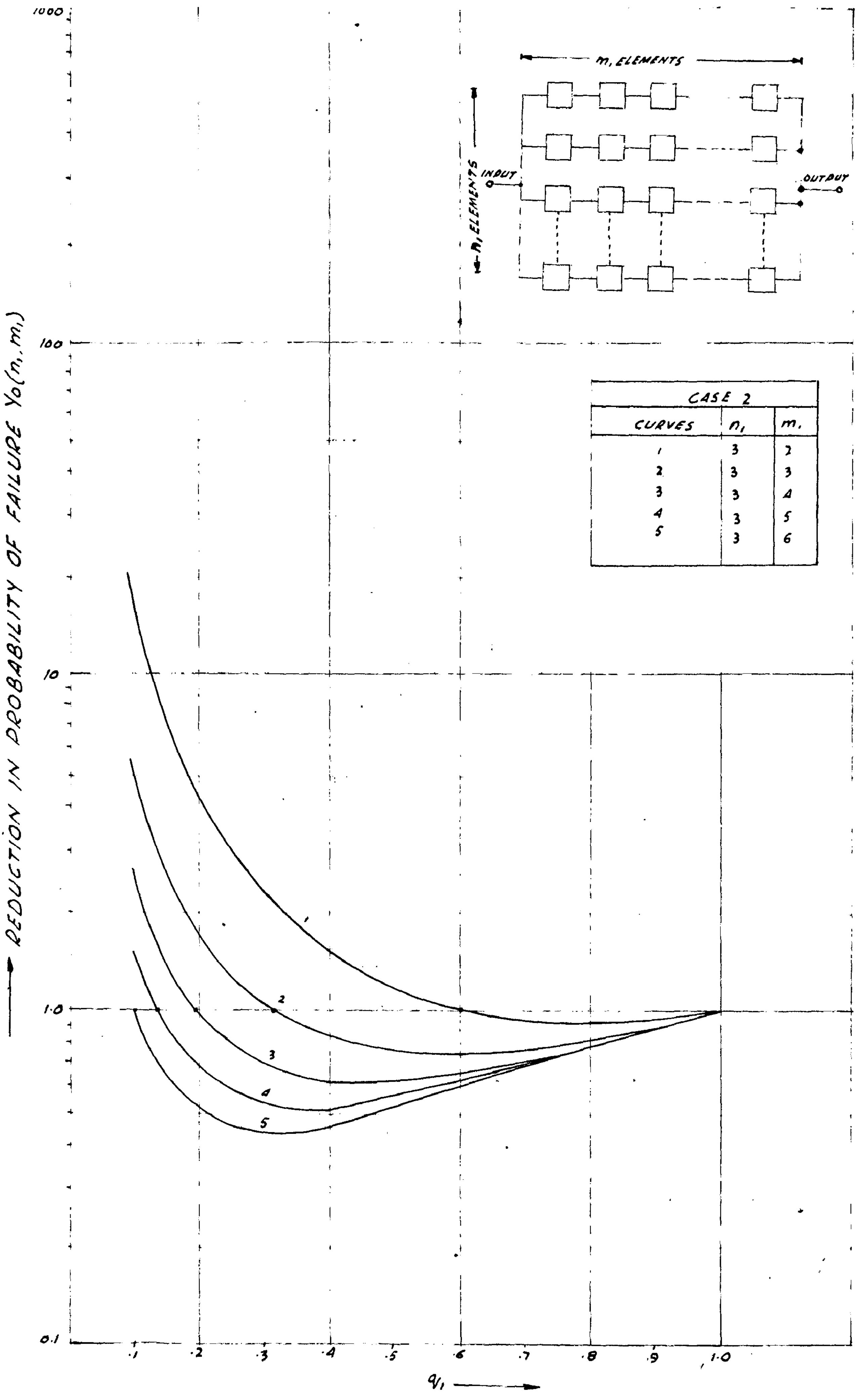


FIG. 4.8 CURVES FOR $Y_0(n_1, m_1)$ Vs. q_1

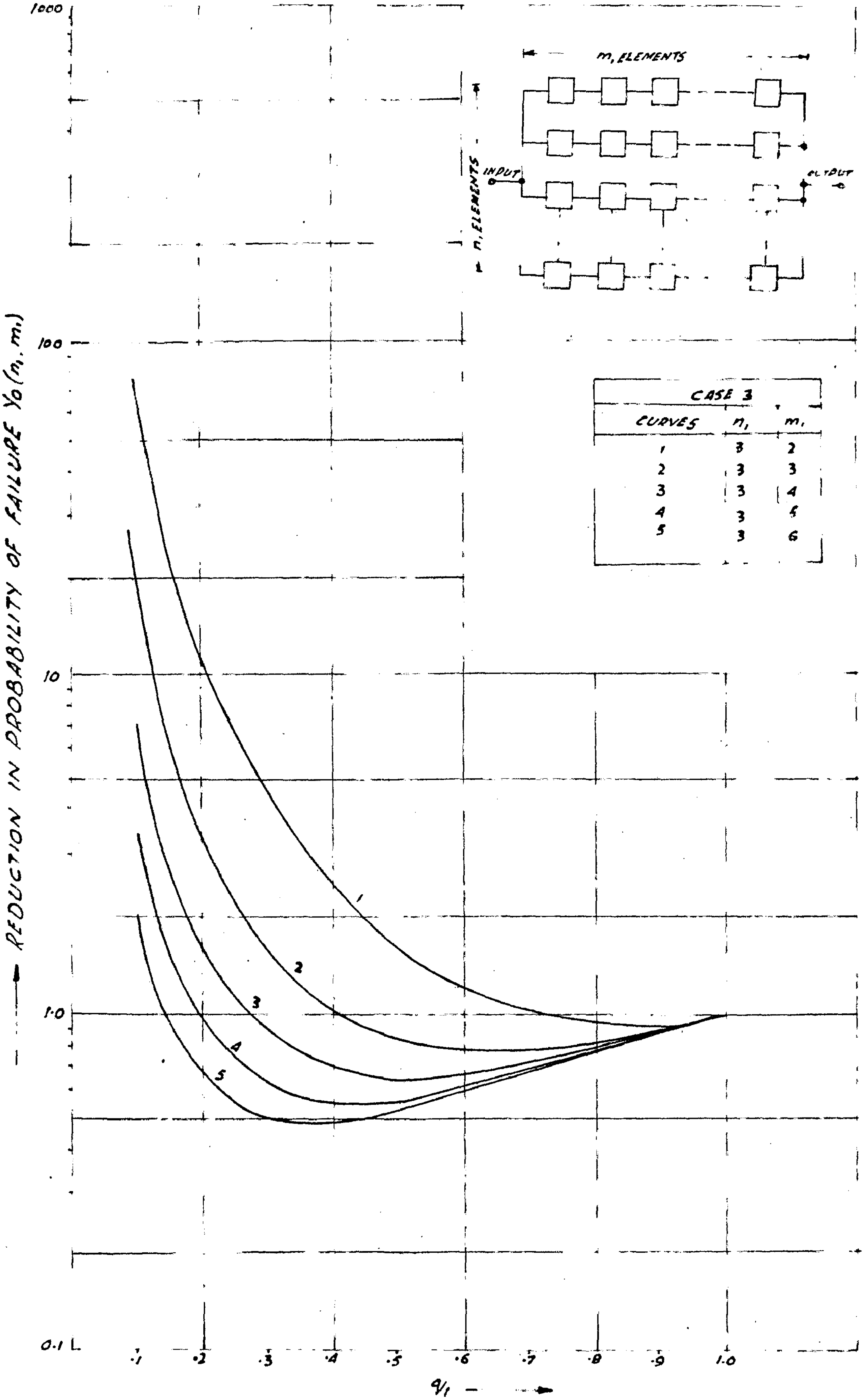


FIG. 4·9 CURVES FOR Yo(n,, m,) Vs. q,

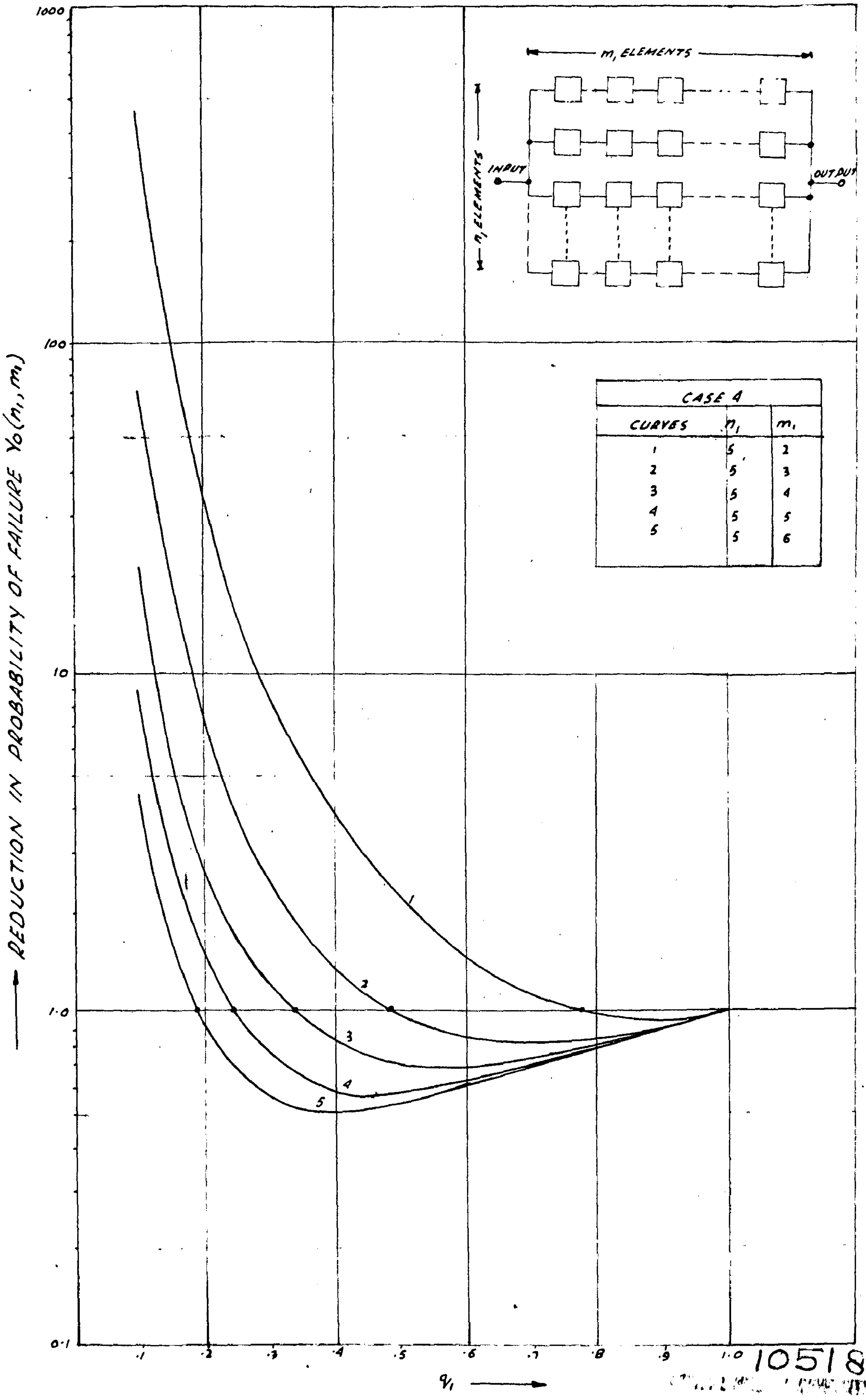


FIG. 4·10 CURVES FOR Yo(n,, m,) Vs. q,

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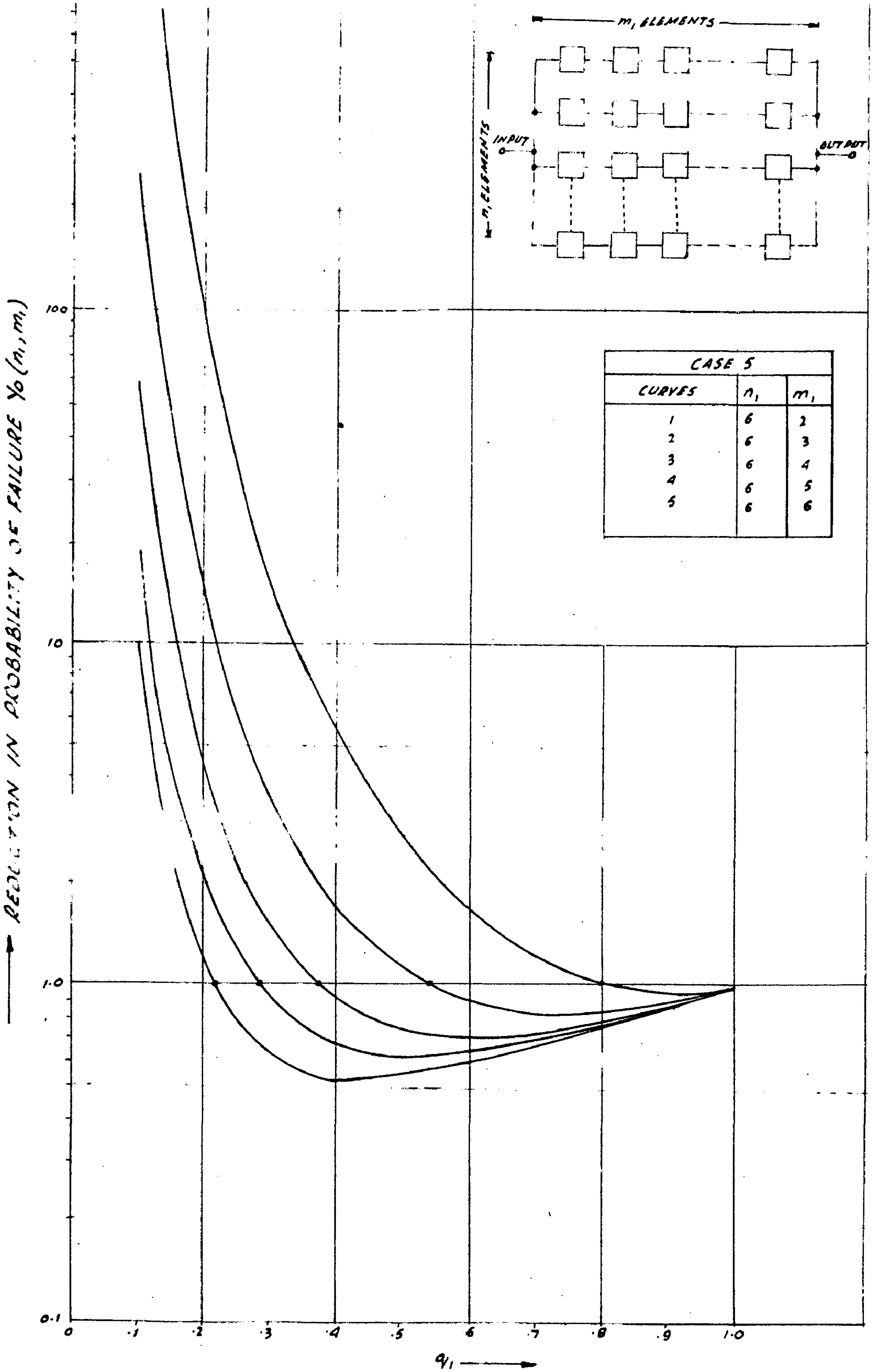

CASE 5		
CURVES	n_1	m_1
1	6	2
2	6	3
3	6	4
4	6	5
5	6	6

FIG. 4.11 CURVES FOR $Y_0(n_1, m_1)$ Vs. q_1

$$Yo(6,6) = \frac{q_1}{(6q_1-15q_1^2+20q_1^3-15q_1^4+6q_1^5-q_1^6)^6}$$

$$= \frac{1}{q_1^5(6-15q_1+20q_1^2-15q_1^3+6q_1^4-q_1^5)^6} \quad \ldots \quad (4.100)$$

The function $Yo=f(q_1)$ for different cases has been plotted in Figs.(4.7 to 4.11). From the curves it is clear that gain appears ($Yo > 1$) if q_1 is less than the values indicated in Table (4.2) for that particular scheme, and increases with decreasing values of q_1, thus increasing the reliability of the element. Here the probabilities q & q_1 are regarded as referring not a particular specimen, but as requirements which must be satisfied by any specimen. Thus the actual probability q_a of each specimen must satisfy the condition.

$$0 \leq q_a \leq q$$

with an understanding of the probability q, even if q 0.5 it is impossible to replace the element by the opposite element as assumed in reference (21) only for $n_1=2$ and $m_1=2$.

The computer programming for the plotting of $Yo=f(q_1)$ for the case $n_1 = 6$ and $m_1 = 2,3,4,5,6$ has been given in Appendix 3.

The chart given below provides a complete idea about the probability of non-selective action and probability of failure of relay circuits, with different configurations.

→ m, (Series elements)

n, (parallel elements)

n \ m	*0	*1	*2	*3	*4	*5	*6
*1		*	*	*	*	*	*
*2		*	.382 (.618)	.612 (.388)	.71 (.29)	.78 (.22)	.82 (.18)
*3		*	.16 (.84)	.32 (.68)	.42 (.58)	.48 (.52)	.54 (.46)
*4		*	.08 (.92)	.19 (.81)	.275 (.725)	.34 (.66)	.38 (.62)
*5		*	.05 (.95)	.14 (.86)	.192 (.808)	.245 (.755)	.28 (.72)
*6		*	.04 (.96)	.095 (.905)	.145 (.855)	.19 (.81)	.22 (.78)

The values entered with in brackets denoted the probability of non-selectivity, and those without are probability of failure. It is evident, if the probability of failure is decreased, the probability of non-selectivity increases. One has to get a compromise between these two, to have a relay scheme with minimum probability of failure and also the probability of non-selective action. If the components in relay assembly are properly debugged and reasonably reliable, then the criterion of selection for a particular configuration mainly depends on probability of non-selectivity. From the chart it is also evident that one will be tempted to use configurations as (3,4), (3,5) and (3,6) for almost equal values of probability of non-selective action and probability of failure.

4.3. Working Conditions of Protective Relays:

Relay protection works under different conditions, because if a fault occur on a relay protection device which can lead to failure of the protection, this still does not mean that failure has taken place. For failure to occur it is necessary that conditions must be produced before rectification or replacement of the damaged element, in which the particular device would have to operate, i.e. the element of the electrical system being protected by the device must be damaged.

Thus the probability of failure is basically dependent on the coincidence of two events, failure on the protection device and fault on the element of the system being protected. hence,

$$q_o = q_1' q_1'' \quad \ldots \quad (4.101)$$

where q_1' is the probability of such a fault on the relay device which can produce a missing operation, and q_1'' is the probability of fault on the protected element for which the particular relay device must operate.

From this it is clear that faults are possible on the relay protection which may cause non-selective operation is also possible with a fault on the output device which are usually very reliable. Non selective operation on relay protective gear is much probable under conditions when some of the main organs on which the action of the relay depends, must operate and selectivity depends only on some of them. In this case a fault on this element may lend to a non-selective action of the device as a whole.

Experiences have shown[22] that most of the non-selective actions of the relay protective gear take place at the time of short circuit outside the action zone of the protection. Now assuming q_2' as the probability of faults of protection devices which can cause slow non-selective action and q_3', $q_4' \ldots q_{n'}'$ are the probabilities of these which can cause non-selective action under some external conditions or other. While the probability of these external conditions are denoted by q_3'', $q_4'' \ldots q_{n'}''$. The total probability of non-selective action is-

$$q_{non-s} = 1-(1-q_2')\ (1-q_3'\ q_3'')\ (1-q_4'\ q_4'')\ldots(1-q_{n'}'\ q_{n'}'') \quad (4.102)$$

for small values of q_2', $q_3' \ldots q_{n'}'$

$$q_{non-s} = q_2' + q_3'\ q_3'' + q_4'\ q_4'' \ldots q_{n'}' q_{n'}'' \quad \ldots \quad (4.103)$$

$$= \sum_{a_1=2}^{n'} q_{a_1}'\ q_{a_1}'' \quad \text{where } q_2'' = 1 \quad \ldots \quad (4.104)$$

If it is assumed that $q_2' = 0$ and $q_3'' = q_4'' \ldots q_{n'}'' = q''$

then

$$q_{non-s} = q'' \sum_{a_1=3}^{n'} q_{a_1}' = q' q'' \quad \ldots \quad (4.105)$$

where $$q' = \sum_{a_1=3}^{n'} q_{a_1}'$$

This indicates that probability of non-selective action and failure of protection not only depends on the probability of faults on the corresponding element, but also on external conditions. The values of q_o and q_{non-s} are determined from statistical data.

The probability of non-selective action or of failure during the interval of time T is-

$$q_{non-s(t)} = 1-e^{-a_{non-s} \cdot t} \quad \ldots \quad (4.106)$$

where a_{non-s} is the rate of the non-selective action.

If a_{non-s} is very very small, then equation (4.106) can be expanded and neglecting the higher powers-

$$q_{nones} = a_{non-s} \cdot t \quad \ldots \quad \ldots \quad (4.107)$$

The value of q_{non-s} can be defined approximately as the ratio of the number of protective operation which acts non-selective during time t, to the total number of protective operation.

CHAPTER - 5

FAILURES IN CONVENTIONAL AND UNCONVENTIONAL RELAYS:

The failure or defectiveness is caused by sudden or random failures or by deterioration of the characteristic of the components used in the relay circuits, such as slackening of springs, increasing contact resistances etc. The failure of relay is defined as either total breakdown or an unsatisfactory operation beyond the permissible limits. These may be due to dirty contacts, open circuits in fine wire coils (d.c.), wrong setting or incorrect adjustment. In the case of total breakdown the damaged or failed part is completely replaced by a new part of the same characteristic, while unsatisfactory operation means a fine adjustment is required such as (greater senstivity, tensioning of springs, reduction of contact resistance etc.). It has been estimated[23, 24,25] that line contacts have about half the resistance of square flat contacts of the same length. The cylindrical contacts at right angle provide the most reliable arrangement of relay contact without concentrating the current at an actual point which would tend to burn and erode away.

Silver is the most widely used metal for relay contacts since it has the lowest resistance, copper circuits are not used in relays because the resistance of clean, new copper contact is eleven times that of silver ones and oxidation raises the resistance of copper contacts several hundred thousand times. The resistance of the contacts is partly that of the contacts themselves, which depend upon their material and dimensions, partly of the actual contacting surface as explained above. For clean dry silver contacts $R = \frac{X}{Y^c}$, where R is resistance in ohms Y is the contact pressure in Grams. For silver c=0.8 and X depends upon contact shape and dimensions.

The resistance of a clean contact has also[25] been expressed as $\rho/2x$ where ρ is the resistivity of the metal and x is the radius of the contact area, x =1.11 Y.z/E where Y= contact pressure in Grams, z =radius of two cylindrical rods in contact at right angle (in cms.) and E is the elastic modulas of the metal used.

A special problem exists in relays with poor ventilation especially in sealed units. High resistance polimers can appear on the contacts due to organic emanations from coil insulation, specially where traces of iron or copper are rubbed into the surface during manufacture[26]. Contacts containing palladium are the most affected and gold plated the least.

In general the failures in the relay can be divided into two main categories: gradual failure and sudden failures.

5.1. Gradual Failures:

The gradual failures depend upon the duration of operation because the variation with time of the relay parameter to some extent, depends upon the aging of the elements which may be due to physico-chemical change of structure, The most affected parts are damaged due to large number of operations of breaking excessive currents for which it is not designed. Due to the pitting of the contacts the contact resistance is increased. So the gradual failures are associated with slow random variation of one or several characteristics of the elements, These changes in characteristics may be determined by environmental factors, the nature of their work etc. Usually in the first instance most rapidly changing parameter of the relay should be considered which is said to be decisive. This parameter may be regarded as random variable which can have any value with some tolerance (previously specified). Hence the probability of fault free operation of a relay during time t is given by-

$$P(t) = P(R_t \leqslant R_{t\ perms}) \quad \ldots \quad \ldots \quad (5.1)$$

where $R_{t\ perms}$ is the permissible limit of the most rapidly changing parameter.

Because the varying parameters are monotonically increasing and decreasing function of time t. The parameter R_t varies during time t under the action of a number of factors (climatic, mechanical etc.). In many cases it can be assumed that these factors are independent of each other. According to Ljaounov's theorem the distribution law of R_t at any time t is almost normal. This fact has also been confirmed by N.M. Zul and F.A. Kuliev[27] through experiments on double auto reclose circuit. It has also been established, for instance, that even if packing of the best quality is used 10% of the packed apparatus is damaged[28].

5.2. Sudden Failures:

These types of failure arise due to sudden changes in the values of one or several parameters of the relay circuit, examples are burnout of valves, breakdown of capacitors, short-circuit in reactor windings etc. The failures may be dependent or independent of each other, but for simplicity it is assumed that these are independent of each other. In this type of failure the parameter value passes abruptly beyond the permissible limits, tending to zero or infinity. These failures are random in nature and follow the exponential type behaviour. Hence under these conditions the reliability R(t) is given by the expression-

$$R_e(t_1, t_2) = \exp - \{A\ (t)\ dt\} \quad \ldots \quad (5.2)$$

where A(t) is the failure rate during the time interval t_1 and t_2.

As has been seen, gradual failures in relay systems behave like normal distribution, where as sudden failures behave as exponential distribution. If it is assumed that these two types of failures are independent of each other the probability of successful operation is given by-

$$R(t) = R_f(t) . R_e(t_1, t_2) \quad \ldots \quad \ldots \quad (5.3)$$

where $R_f(t)$ is the reliability function of normal distribution (for gradual failures).

5.3. General Considerations:

The relay performance may be affected due to friction developed in the bearings because most protective relay bearings run dry. The most common type of bearing for precision relays such as in the induction type is a pivot and jewel bearings. For special application, requiring high sensitivity and low friction a single ball bearing(29) running between two cup shaped sapphire jewels has been used. The moving coil type relays are most effected by mechanical vibrations than an induction cup type because the distance travelled by the cup type relay is too small. The static relays are least effected. Most of the coils which are made of fine wires are liable to subsequent failure on open circuit, usually near one of the leads but some times at the kink or crossed turns, due to fine wires having been eaten through by corrosion. Failures from this cause are much more common when coils are connected to the positive end of the d.c. circuit, because the coil became the electrode to which the acid ions are attracted. There is a statistical evidence that coils wound with 0.006 in. wire are no less liable to failure than those wound with 0.002 in. wire, although they may take somewhat longer time to fail.

5.4. Precautions for Maximum Reliability:

The relay should be designed for high contact pressure under all operating conditions. If necessary, it should be augmented as the contacts are approaching and almost closed. This is done in certain modern relays[30], for instance, by a notch in the induction disc.

The relay case should be made dust proof and provided with a filter breather to equalise the pressure inside and outside the case without allowing the dust to get in.

Fine wire relay coils and trip coils should have well braced junction between the coil wire and the outside lead so that stress on the latter will not cause an open circuit. Acid fluxes or acid providing insulation should be avoided. Mechanical removal of enamel from the wire should be avoided. In general a.c. coils should use wire not less than 0.05mm, dia.and d.c. coil not less than 0.1 mm dia. Coils should not be connected directly to the positive side of d.c. supply unless all these precautions have been taken.

Maintenance testing should be done without disturbing the switch board wiring, and infrequently except by the conditions of severe humidity, new untried components etc. Infrequent maintenance eliminates the risk of relay failure due to improper adjustment by an unexpert personnel, which is one of the commonest causes.

Adequate maintenance can often anticipate failures due to a.c. wiring faults, including multicore cables and current transformers. Failure to trip due to loss of a.c. potential can be prevented by an overvoltage alarm relay connected across secondary potential fuses.

Where devices are used which are too recent for comprehensive reliability statistics to be available, they should be connected so that their failure or deterioration does not cause undesirable tripping or failure to trip. For instance, transistors should be protected not only against voltage surges but also preferably should be protected so that the selectivity of the relay does not depend upon the drift in the transistor characteristics.

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CHAPTER - 6

ANALYSIS OF COMPONENT FAILURE IN ELECTRONIC AND ELECTRICAL CIRCUITS:

The failures in electronic and electrical components can cost time, effort, and even life. Most of the reliability programmes concentrate upon failure prevention. Reliability principles have established certain concepts of failure rate acceleration. These may be due to increasing stress of temperature, voltage or power. However, all failures do not have similar effects, few of them are catastrophic, while others have negiligib effects. But to have smooth operation, the failures should be minimised. Environmental conditions also play an important role while considering the component failures under actual working conditions. The evaluation of extremely low failure rate in Laboratory is practically impossible within reasonable length of time and with limited number of components. The reduction of time to failure can be obtained by artificial methods like accelerated test on capacitors, which some times present many problems in actual life testing. The text also gives an idea about the remedies to different types of failure.

6.1. Failure Classification:

In general the failures of electronic and electrical components may be classified into four major groups, viz; Catastrophic, Intermittent, Out of tolerance and Mal-adjustment.

6.1.1. Catastrophic failure[31] takes place when either a part is completely damaged or shows a gross change in its characteristic; examples that may be quoted are; shorted vacuum tubes, open or short-circuited resistors and capacitors, a leaky valve, a stuck relay or a broken switch. This type of failure sometimes can be minimized during the design of a component, but some catastrophic failures are random in nature. Thus the designer can not be expected to eliminate all such failures.

6.1.2. Intermittent failures are also unpredictable, and the designer can do very little to reduce them. This type of failure is periodic in nature and takes place within the piece part itself, hence this must be corrected during the design of a component itself rather than in the design of an equipment or a system.

6.1.3. Out of tolerance failure results from degradation, deterioration, drift and wear-out. The examples are the drifting of resistance and capacitor's values, wearing out of relay contacts bearing etc. and solenoid valves etc. These changes may take place due to time, temperature, humidity or altitude. When the gradual changes are considered collectively the characteristic of the components, reaches a point where these are not acceptable, that is to say there is a gross change in the parameter itself which inturn changes the performances beyond permissible limits and the component is said to have failed.

Some times random selection of a component out of a manufactured lot may lead to unacceptable parameter value or it is also possible that the component had proper value at the time of assembly just nearer to the tolerance value and there happens a major drift in the parameter as it is put into operation and thus leads to failure of the circuit, fabricated.

As an example several lots of resistances and capacitors in lower and higher ranges were tested and their values measured quite accurately, commensurate with their values and what the author has observed is reported below:

In the lower range a large number of resistance having face values as 68 ohms, carbon composition with 10% tolerance were tested and it was observed that only 7% of the lot had value exactly as 68 ohms and about 11% had lower than that and the remaining 82% had values higher than 68 ohms. It may be noted that the minimum value of resistance measured was 63 ohms and maximum as 76 ohms whereas according to the tolerance specified the lower and higher limits should be 61.2 ohms and 74.8 ohms. A curve showing the parameter value and the probability of incidence of the value equal or less than that is shown in fig.(6.1) for the resistance value of 68 ohms, and for other components in figs.(6.2 to 6.4).

Similarly, in higher range side the resistance chosen was 3.3 Kohms. Also capacitors of 200 pf and .04 micro-farad, (paper insulated) were tested and the results observed for all these components are listed in table 1.

Table 1

Component	Value according to tolerance.		Observed values		Probability of occurance of exactly the nominal value.	Probability of values occurance greater than nominal	Probability of values occurance less than the nominal values.
	Min.	Max.	Min.	Max.			
Resistance 68 ohms	61.2	74.8	63	76	7%	82%	11%
Resistance 3.3K ohms	2.97	3.63	3.0	3.45	18%	37%	45%
Paper Capacitor 200 PF	-	-	230	350	0.0%	100%	0.0%
Paper Capacitor .04Micro-farad	-	-	.036	.0435	18%	56%	26%

6.1.4. Maladjustment failures are generally due to human errors. These failures take place by improper adjustment of the equipment or component as well as the abuse of adjusting device due to lack

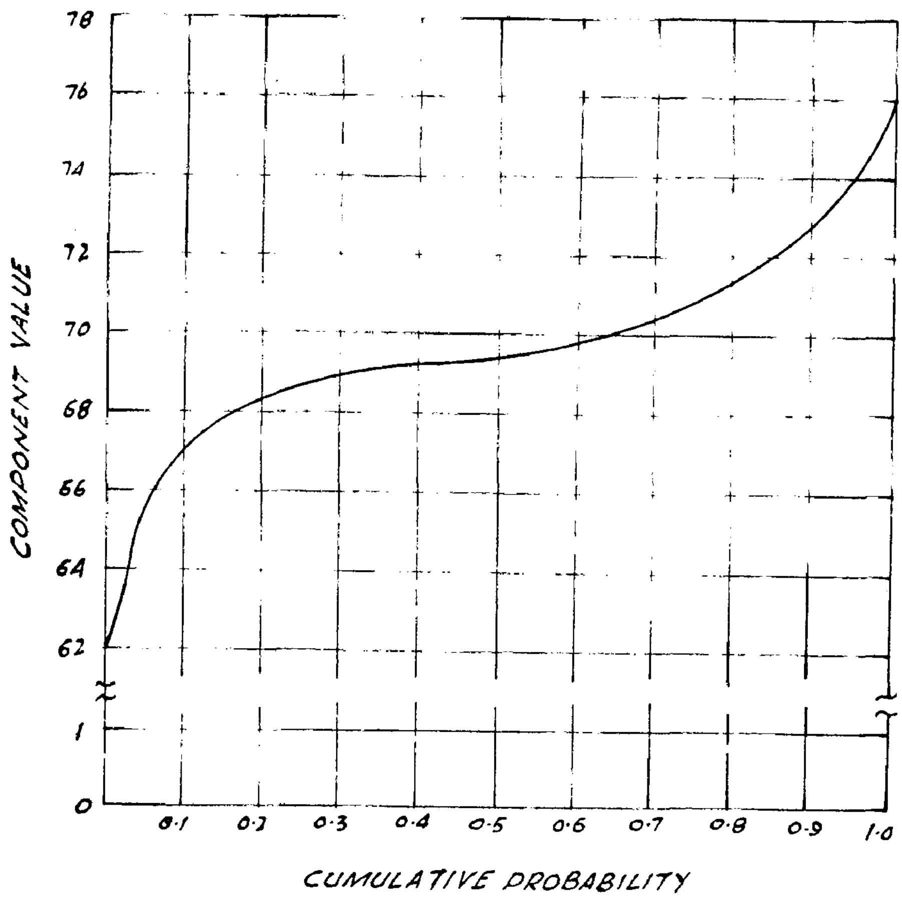

FIG. 6·1 RESISTANCE 68 OHMS. ½ WATTS, 10% TOLERANCE

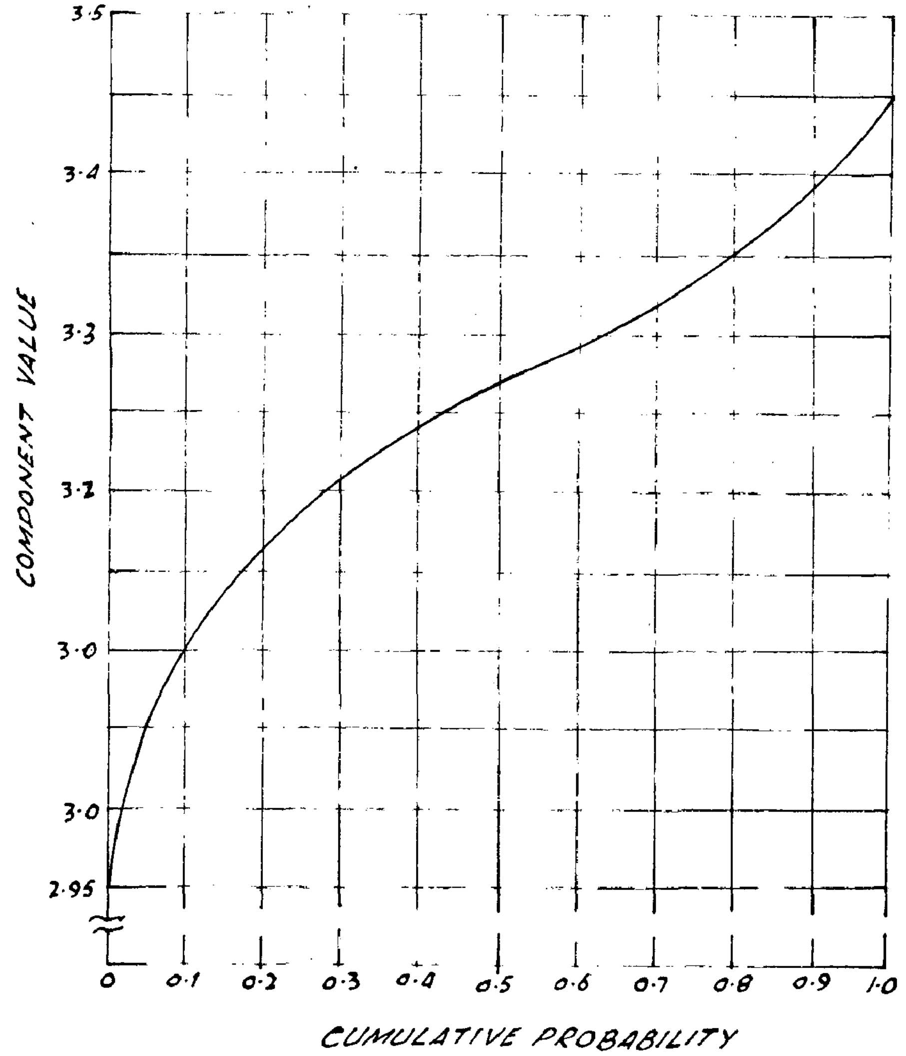

FIG. 6·2 RESISTANCES 3. 3K. ½ WATT, 10% TOLERANCE.

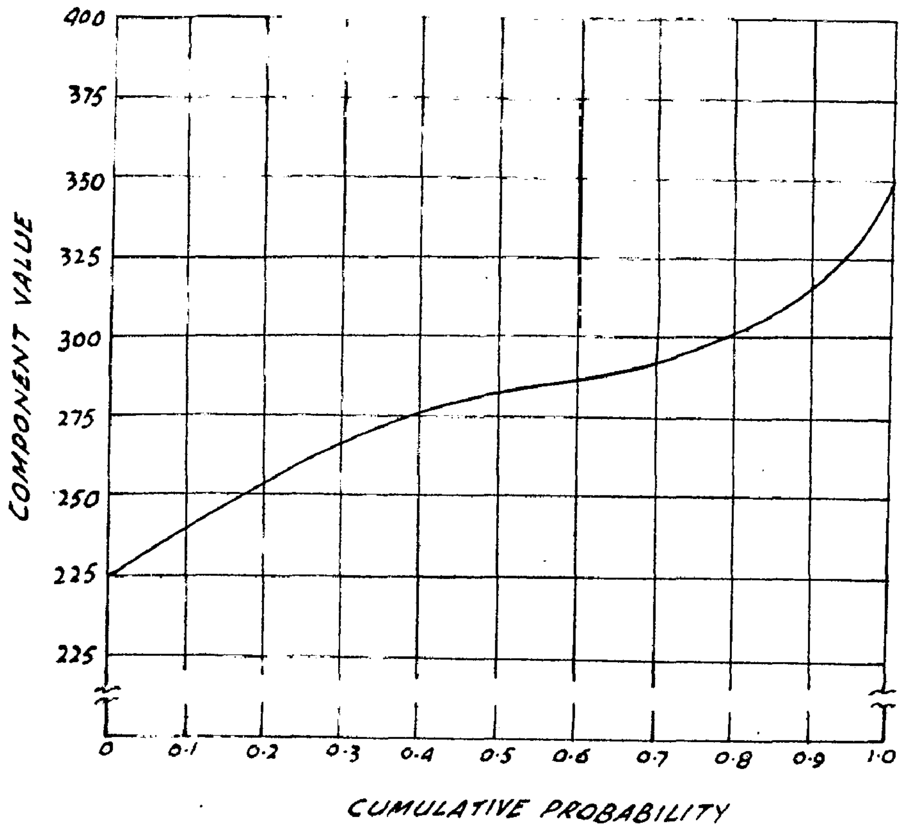

FIG. 6·3 CAPACITOR 200 Pf. 600 V. d.c.

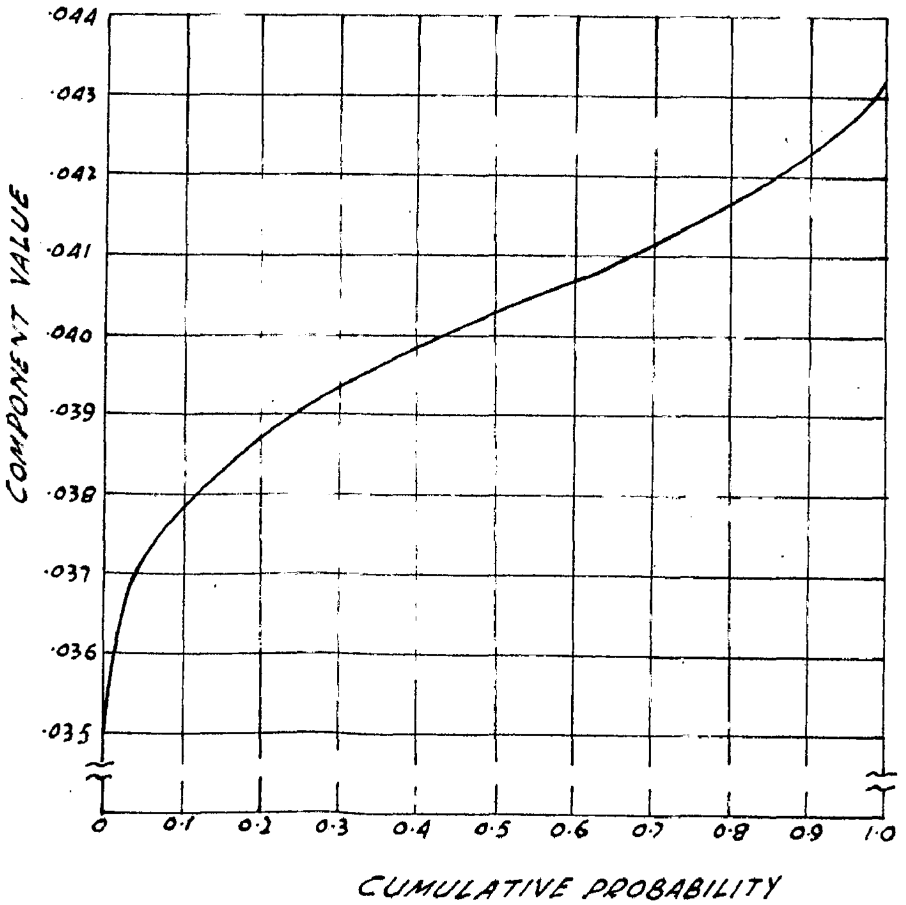

FIG. 6·4 CAPACITOR 0·04 uF 600 V. d.c.

of understanding of the adjustments and the capabilities of the component. The examples are increasing and decreasing the sensitivity of a relay through hair springs. These failures are difficult to evaluate and to avoid, but their effects must be considered duri assessment of the reliability of an equipment or component.

6.2. Environmental Considerations[32,33]:

While describing the nature of component failures, it is very necessary to take into consideration the environmental effects which play an important role in describing the behaviour of components under actual working conditions. These can broadly be classified under the following headings; Shock & Vibration, Heat Transfer, Corrosion & Biological growth and Chemical action.

6.2.1. Shock and Vibration:

These are probably the most controversial areas in the environmental testing of electronic and electrical components like electronic tubes, relays and other parts. One should simulate the actual environmental stresses that the item will encounter in the actual field use. The characteristic of any part under the above test should be stable during the test period. The most frequent failures due to vibration are:

(1) Flexing of electrical leads which support resistors and capacitors.

(2) Damaging the vacuum tubes, electric bulbs etc.

It is advisable that the equipment should be mounted on shock resistant material like synthetic rubber to reduce the effect of vibration. Special instruments which can provide menitored shock & vibration are used to detect the presence of foreign particles in transistors and diodes. The same can be used to determine their structural rigidity.

6.2.2. Heat Transfer:

Poor heat transfer is a major problem in electrical and electronic circuits. The heat generated may be due to I^2R losses, hysteresis losses or eddy current losses. This results in physical damages or help in accelerating the chemical reaction rates. The semi-conductor devices are much sensitive to temperature and most effected.

The common methods of heat transfer in electrical & electroni equipments are (i) free convection (ii) forced air cooling (iii) conduction (iv) radiation and (v) vaporisation cooling.

Convection being slow even when sufficient air space is provided, the technique is only applicable when dissipation is less than 0.25 watts per square inch under normal atmospheric conditions. Forced cooling is used when the dissiptation is upto 2.0 watts per square inch. Radiation is the most effective method of heat transfer. Vaporisation cooling is used when dissipation is more than 7 watts per square inch. The conventional way of vaporisation cooling is through refrigeration. The chart, given in Appendix-4, gives an idea about the temperature precautions, which should be taken into consideration in designing an equipment.

6.2.3. Corrosion and Biological growth:

Because, the environment contains many deteriorators like oxygen, Carbon dioxide, dust, chemicals etc. Numerious types of parts like vacuum tubes, batteries and capacitors are susceptible to chemical action and biological growth. To cope up with this difficulty, the specification of the component should withstand the specified levels of temperature, humidity, fungus, rain, dust etc.

6.2.4. Chemical Action:

The material of electronic components can undergo change, in a number of ways. Some of them are chemical interaction with other materials and modification in the material itself (recrystillisation, phase change of changes induced by irradiation). To avoid this, corrosion resistant materials should be used as far as possible.

6.3. Failure Analysis:

To have a complete idea about the mode of failure of electronic and electrical components, an analysis of different types of components is given below:

6.3.1. Resistors

6.3.1.1. Carbon Composition:

A survey of the existing literature indicates that the frequent types of failures in carbon composition resistors are due to resistance drift[34], a decrease or an increase in resistance. The former may occur due to change in moisture contents, and the latter due to carbonisation which causes curing of binders and the third type resulting from curing and cracking of the resistance element. The cracks will normally occur at the hottest point or at the centre of the resistance element. Also excessive external heat such as manual soldering operation or excessive temperature cycling will cause failure in the vicinity of lead termination where the mismatch of thermal coefficient is the greatest. Experimental results obtained in the Laboratory on the pattern of failure of these resistances are in agreement with the above analysis. Figs.(6.5) and (6.6) show the nature of failure of some of the resistors put under test. Figs. are quite explainatory and stands as evidence for the mode of failures discussed above.

6.3.1.2. Metal Film:

In metal film resistances, the drift characteristic is

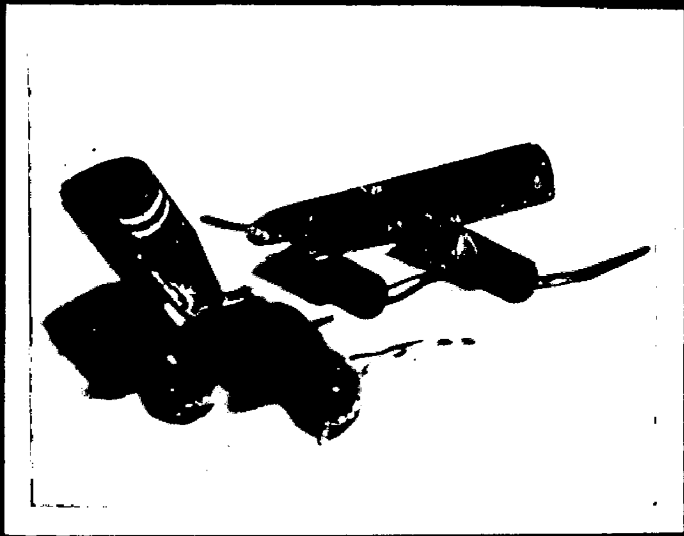

Fig.6.5

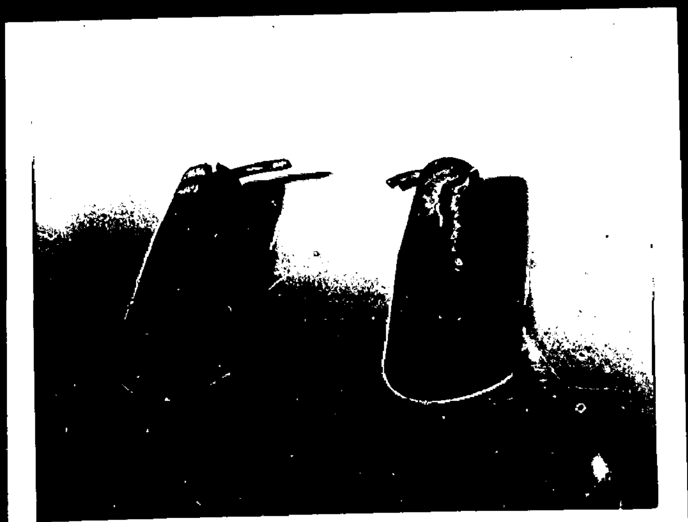

Fig.6.6

very much important if the long term reliability of the part is taken into consideration. Generally, there are four obvious reasons of failures of these metal film resistors. One may be scratched film which ofcourse does not cause an immediate failure but may cause a premature failure. Another is due to cracking of the substrate where the fracture finally interrupts the film continuity and the other two are due to lifting of the metal film from the substrate.

6.3.1.3. Wire Wound:

Very low resistance units may fail due to short circuiting of the turns. Twisting or pulling of the leads may create an open circuit due to poor holding power of the leads. The failures may also be due to uneven distribution of winding, compound and other irregularities.

6.3.2. Capacitors:

6.3.2.1. Paper and Mayler Dielectric Capacitors:

Basically, a capacitor consists of two metal film foils separated by a dielectric material. A large number of capacitors get damaged due to external causes such as the piercing of a few layers of the foil which results in short-circuit. The other two of failures are due to severe vibration in which the capacitor may lose its leads resulting in an open circuit. But some of the faults may be due to the deformation of internal sleeves if it is not inserted carefully. The life of capacitors under consideration varies inversely to some power (p) of the applied voltage[35] which mathematically can be represented as follows:

$$\left(\frac{L_1}{L_2}\right) = \left(\frac{V_2}{V_1}\right)^p \qquad \ldots \qquad \ldots \qquad (6.1)$$

where L_1 and L_2 being the mean life of mayler capacitor under test corresponding to the applied D.C. voltages V_1 and V_2. The best exponent for the inverse power rule ranges from 2 to 7[36], but different investigators suggest different values for the exponent 'p' however, several authorities have shown that a factor of 4 to 6 for most of the capacitor applications[37, 38] is reasonable in practical ranges of voltage and temperature.

The effect of temperature on the life of capacitors can also be established by the following relationship,

$$\log_{10}\left(\frac{L_2}{L_1}\right) = C\left(\frac{1}{t_{1k}} - \frac{1}{t_{2k}}\right) \qquad \ldots \qquad (6.2)$$

where C is any constant.

The above equation relates the mean life ratio to a difference of the above inverse of the temperature in degrees on Kelvin scale. From test results for a particular group the value of constant C can be determined.

6.3.2.2. Tantalum:

The most frequent type of failure mode in tantalum foil capacitors is the shorted units, which may result due to the application of reverse bias; generally such units will display a colour corresponding to specified test voltages.

6.3.2.3. Solid Tantalum:

The failure mode is very high leakage or nearly shorted units. The most probable cause of these failures may be the trapped impurities, some of the units may be found fractured which might be resulted from shock damage.

6.3.3. Semiconductors:

6.3.3. Semiconductors:

The most frequent failures in semiconductor devices are due to thermal, electrical and mechanical abuses. It is needless to mention the tremendous difference between passive and semiconductor elements, A new 500 volt paper capacitor may withstand 10,000 volts momentarily without failure, a power resistor may be able to take a power overload pulse of 100000 or more times the normal power rating for a micro-second; while a diode or transistor however has no such over-load or over voltage capability. The semiconductor devices may fail or show a change in operating characteristic in so many ways, few of them are described as below:

6.3.3.1. Electrical Abuse:

Thermal considerations are of utmost importance in the reliability of semi-conductors devices. These points may be where electrical stresses or the current density is the highest or a point where the thermal resistivity is the highest. A power transistor[39] cannot dissipate heat into its surroundings because of its small area. Heat radiations from the transistor case is inadequate because of the Stefan's law. The major effects of time and temperature on the transistor are to reduce the value of current gain and to increase the value of collector cut-off current. The effect of higher temperatures, is also to hasten the aging process. Zenor diodes may fail due to loose soldered balls within the device. This may be due to over load conditions which create enough temperature to melt the eutictic and whisker pressure which will force the material out in ball shaped particle. The effect of temperature[40] on the failure of transistor is shown in Fig.(6.7).

6.3.3.2. Mechanical Defects:

Some packages are inherently more susceptible to mechanical damage then others. The improperly wetted crystal.

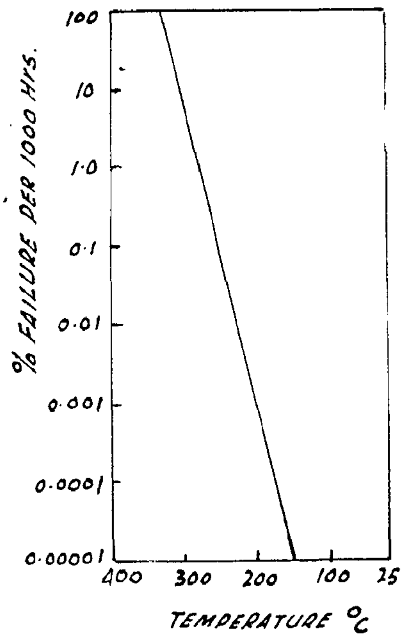

FIG. 6.7b TRANSISTOR FAILURE RATE Vs. TEMPERATURE

becomes more a mechanical problem in the extreme cases. Cracked crystals are another form of mechanical defects. The failure in the transistor may be of two types; an open circuit due to loss of internal solder connection to collector base or emitter and a short between collector and emitter. The open circuit may be caused due to poor construction. Other types of mechanical defects may be responsible for some of the alloyed- through shorted devices, for example mettalic deposits bringing the glass feed through on the inside of transistor case. The most common failure of Zenor diodes, seems to be a rounding of the breakdown knee of sufficient severety to finally affect the operating region.

6.3.3.3. Surface Conditions:

Most of the failures in semi-conductors are the change in surface conditions. The changes are due to the effect of time and temperature because the effect of higher temperature is to hasten the aging process. Thermal conditions may give reproducible and reversible cyclical changes or irreversible or permanent changes.

The surface conditions during the test or use can also result in the component becoming in-operative due to the variation of the true characteristic beyond permissible limits.

The purpose of failure analysis is to emphasize the importance of an accurate failure analysis in any test programme. Accurate failure analysis is truely the cornerstone of the corrective action that will be taken to improve the reliability and performance of electronic and electrical equipments.

C H A P T E R - 7

RELIABILITY EVALUATION

Techniques useful in the analysis and prediction of equipment reliability have developed rapidly during the recent past. Concurrently with this development, emphasis has been placed on the accumulation of failure rate data on parts and the measurement of reliability of existing equipments in order to provide numerical significance to the various mathematical expressions used in describing the reliability. These efforts are being accelerated by an increeasing recognition of the value of applying an analysis and prediction tachniques during the design phases itself.

The techniques available currently can be classified depending[41] on application as follows:

(i) Prediction of circuit or module reliability when part reliability, system configuration and internal and external stresses are given (as will be discussed in section 7.1.).

(ii) Prediction of system or equipment reliability when module reliability, equipment diagram and operational requirements are available (as will be discussed in section 7.2).

(iii) Advanced mathematical or statistical techniques which supplement the preceeding methods under certain prescribed conditions (as will be discussed in section 7.3).

7.1. Prediction of Module Reliability:

Basic technique of obtaining the module reliability is, the summing of the failure rates of the constituent parts. Appropriate formulas are apolied to account for the series and parallel configuration of the parts which compose the module. It has been determined that there are several ways in which the component can fail, viz. (i) frequently repeated failures (ii) randomly occuring failures (iii) degradation failures of various parts. The methods of rectifying the first and third type of failures are explained in Chapter 2 in detail, but the methods to reduce the random failures are not known. The mathematical model giving the failure rate of the module as the summation of the failure rates of the constituent parts has received wide acceptance. The reliability of the module, in turn, is computed from $R(t) = e^{-At}$, where A is the failure rate of the module. The use of this formula will justify the acceptance of certain basic assumptions.

(i) All theparts are operating independently, i.e. the probability of one part is independent of remaining parts.

(ii) The successful functioning of each and every part is required forthe successful operation of the module.

(iii) Failure rates of various parts are known.

(iv) The parts experience constant failure rate during the period of operation.

In situations where the reliability of large numbers of modules are to be predicted, sampling procedure is used to advantage. The use of such technique does not affect the basic techniques used in prediction, but rather leads to the stipulation that a detailed prediction of reliability will be made for only the selected sample modules where, merely a quick analysis if any at all, of the estimated reliability will be made for the other modules.

7.2. Prediction of Equipment Reliability:

The old technique for the numerical prediction of equipment reliability is based on the application of product rule and simple redundancy consideration as explained in detail, while considering the reliability models of non-maintained systems. This technique is valid and extremely useful where the modules comprising an equipment, operate in a simple series, parallel or redundant configuration with respect to reliability. One of the more mathematical treatment of reliability analysis techniques(42) discusses the product rule and shows that actually it can be applied with reasonable validity to a variety of situations.

7.2.1. Use of Switching Circuit Analogy:

Since the switch is a two state device, either open or close, it is evident that a switching circuit can be considered an analogue of any group of interconnected elements where the operation of such element is described as either a success or failure. Three important steps are necessary to use switching circuit analogy as the reliability prediction technique-

(i) Preparation of circuit diagram where each component is represented by a switch, the open position being analogous to failure and closed position analogous to success.

(ii) Derivation of formula (Transfer function) for transmission through the circuit showing all combination of switch closures which can lend to success.

(iii) Interpretation of formula for successful transmission in terms of probability of success.

Extension of this technique to complex multi-element series parallel networks have been discussed in reference(43).

7.3. Advanced Mathematical and Statistical Techniques:

Besides the techniques discussed in the earlier chapters and above, there exist some techniques to obtain means for obtaining valid prediction of reliability. These new techniques are derived from the application of advanced mathematical and statistical procedures based on, Boolean Algebra, Baye's theorem, Monte Carlo methods and various distribution theories such as Exponential, Gamma, Weibull, Normal, Extreme value and Poission etc.

The reliability models for statistical have been explained in detail in Chapter 2. Other mathematical techniques will be discussed in brief.

7.3.1. Application of Boolean Algebra:

Boolean Algebra(44, 45) expressions can be used to describe how element operating state must combine simultaneously to make the output of the signal available.

$1 \cup 2$ denotes all states where elements 1 and 2 are operating in parallel and $1 \cap 2$ corresponds to the elements operating in series. The following posulates are used.

$$X \cup \bar{X} = 1$$

$$X \cap \bar{X} = 0$$

$$X \cup X = X$$

$$X \cap X = X$$

The following configuration are analysed with the help of Boolean Algebra.

7.3.1.1. Series Systems:

If the reliability of each sub-system is R_x, R_y and R_z (the probability of the signal to be present at output terminal of each subsystem). Then the reliability of the complete system will be given as-

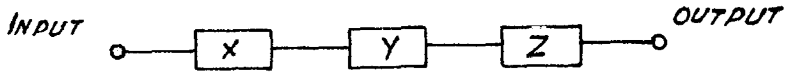

FIG. 7·1

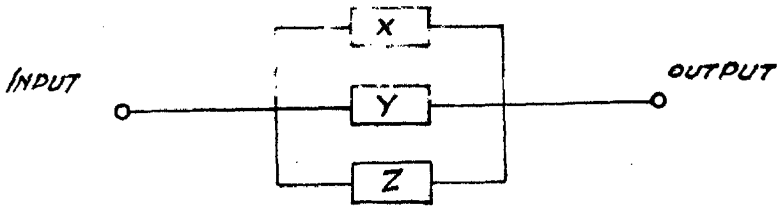

FIG. 7·2

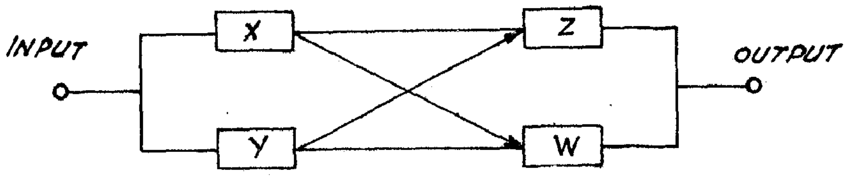

FIG. 7·3

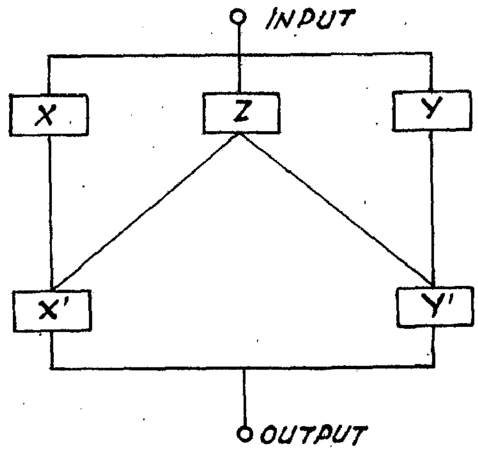

FIG. 7·4 EXAMPLE OF BAYES THEOREM IN RELIABILITY

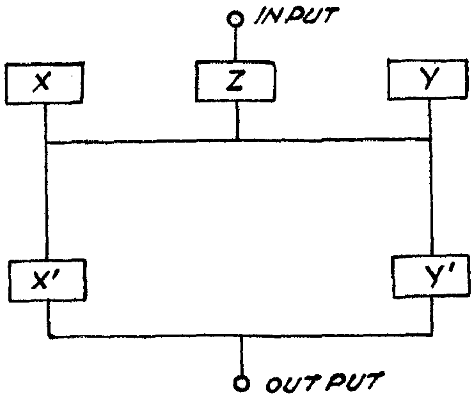

FIG. 7·5

$$R_{system} = R_x \cap R_y \cap R_z$$

$$= R_x . R_y . R_z$$

if the reliability of all the subsystems are equal = R,

then- $R_{system} = R^3$

if $R = e^{-At}$

$$R_{system} = e^{-3At} \quad \ldots \quad \ldots \quad (7.1)$$

7.3.1.2. Parallel Systems:

The total probability of the signal to be present at the output terminals is-

$$P_{system} = P(X \cap \overline{Y} \cap \overline{Z}) \cup P(Y \cap \overline{X} \cap \overline{Z}) \cup P(Z \cap \overline{X} \cap \overline{Y}) \cup P(X \cap Y \cap Z)$$

$$= P[X \cap (1-Y) \cap (1-Z)] \cup P[Y \cap (1-X) \cap (1-Z)] \cup P[Z \cap (1-X) \cap (1-Y)] \cup P(X \cap Y \cap Z).$$

on solving-

$$P_{system} = P(X)+P(Y)+P(Z) - P(X).P(Y) - P(Y).P(Z) - P(X).P(Z) + P(X).P(Y).P(Z).$$

if $P(X) = P(Y) = P(Z) = P = e^{-At}$

$$P_{system} = 3P - 3P^2 + P^3$$

$$R_{system} = 3e^{-At} - 3e^{-2At} + e^{-3At} \quad \ldots \quad \ldots \quad (7.2)$$

7.3.1.3. Multistage System:

The application of Boolean Algebra is very helpful when analysing the multistage systems for demonstration taking a simple example of the circuit in fig.(7.3).

The total probability of the signal to be present at the output is given by the equation-

$$P_{system} = P(X \cap Z \cap \overline{Y} \cap \overline{W}) \cup P(Y \cap Z \cap \overline{X} \cap \overline{W}) \cup P(Y \cap W \cap \overline{X} \cap \overline{Z}) \cup$$
$$P(X \cap W \cap \overline{Y} \cap \overline{Z}) \cup P(X \cap Y \cap Z \cap W)$$

Simplifying-

$$P_{system} = P(X).P(Z) + P(Y).P(Z) + P(X)P(W) + P(Y)P(W) - P(X)P(Y)P(Z) -$$
$$P(X)P(Y)P(W) - P(X)P(Z)P(W) - P(Y)P(Z)P(W) +$$
$$P(X) \cdot P(Y) \cdot P(Z) \cdot P(W).$$

if,

$$P(X) = (P(Y) = P(Z) = P(W) = P = e^{-At}$$

then,

$$P_{system} = 4P^2 - 4P^3 + P^4$$

or $R_{system} = 4\, e^{-2At} - 4\, e^{-3At} + e^{-4At}$ ... (7.3)

7.3.2. Bayes' Theorem as Applied to Reliability Evaluation:

As all the systems can not be reduced to either series or parallel or any combination of series parallel arrangement, there exist systems which are under the class of non-series parallel systems. To find the reliability function of such systems, the Baye's(1, 46) theorem may be very useful.

Considering the schematic reliability block diagram in Fig.(7.4).

Two equal paths X-X' and Y-Y' operate in parallel, so that at least one of them is good and the output is assured. If X and Y are not reliable a third similar unit Z is introduced so that it can provide the output signal through X' and Y'. Therefore the following possible operations are possible; X-X', Y-Y', Z-X' and Z-Y'. Now the problem is to find out the reliability of this arrangement.

If the element are connected as shown in Fig.(7.5) the reliability evaluations becomes very easy.

To findout the reliability model of circuit shown in Fig.(7.4), the Bayes' theorem can be used which states, If X is an event which depends on one or two mutually exclusive events Y_i and Y_j of which one must necessarily occur, then the probability of occurance of X is given by-

$$P(X) = P(X, \text{given } Y_i) P(Y_i) + P(X, \text{ given } Y_j) . P(Y_j) \quad \ldots \quad (7.4)$$

or it can be stated as, the probability of system failure equals the probability of system failure that a specific component in the system is good, times the probability that the component is good, plus the probability of system failure given that said component is bad, times the probability that the component is bad or

$$P_{\text{system failure}} = P(\text{System failure if component Z is good}) . P(\text{Z is good}) + P(\text{System failure if component Z is bad}) . P(\text{Z is bad}) \quad \ldots \quad (7.5)$$

Z is a component or unit upon which the system reliability depends.

if,

F_s = probability of system failure

R_z = Probability of that the component Z is good

F_z = Probability of that the component Z is bad.

From equation (7.5),

$$F_s = F_s(\text{if Z is good})\, R_z + F_s(\text{if Z is bad}) F_z \quad \ldots \quad (7.6)$$

and $R_s = 1 - F_s$

The equation (7.6) is most powerful tool to solve the arrangement as shown in Fig.(7.4).

If Z is good the system can fail only if both X' and Y' fail, and because X' and Y' are in parallel the system unreliability if Z is good amounts to -

$$F_s(\text{ if Z is good}) = (1- R_{x'}) (1- R_{y'})$$

if Z is bad the system can fail only if both X-X' and Y-Y' fail, and the system unreliability, if Z is bad amounts to-

$$F_s(\text{if Z is bad}) = (1- R_x R_{x'}) (1- R_y R_{y'})$$

Then the unreliability of the whole system is- ...

$$F_s = (1-R_{x'})(1-R_{y'}) R_z + (1-R_x R_{x'})(1-R_y R_{y'}).F_z$$

if components X, Y and Z are similar and having failure rate A_1 and components X' and Y' are also similar with failure rate A_2, then-

$$F_s = (1-e^{-A_2t}) (1-e^{-A_2t}) e^{-A_1t} + [1-e^{-(A_1+A_2)t}]^2 (1-e^{-A_1t})$$

and $$R_s = 1-(1-e^{-A_2t})^2 e^{-A_1t} + [1-e^{-(A_1+A_2)t}]^2 (1-e^{-A_1t}). \qquad \ldots \quad (7.7)$$

7.3.3. Monte-Carlo Techniques[47,48,49]:

The application of statistical methods to the design and analysis of reliable circuits has been possible recently with the advent of high speed computers. Monte-Carlo method for reliability analysis mainly concerns the drift caused by out of tolerance. The problem of estimating how deviation from nominal component values make deviation in the performance of a circuit and can be best analysed by using Monte-Carlo techniques. The method has simplicity and infact closeness to reality by direct simulation. The method can provide information as regards the behaviour of a circuit under different simulated conditions.

Invariably, Monte-Carlo techniques constitute the process of working with a model of a system, imposing specific inputs and using some random process to select values of component parameters, combining them according to some rule to obtain the system output. This process is repeated for several runs and samples of output are obtained, from which the reliability information or data may be deduced. Thus Monte-Carlo technique consists of the following steps.

(i) A suitable random process is selected.

(ii) Enough component information is obtained so that the component response distribution is estimated with reasonable accuracy. This shows how to weigh the various probabilities of occurance of parameter values.

(iii) A formula is obtained for the output of the system as a function of systemm input and component parameters.

(iv) Several runs are made and output is evaluated taking the random parametric values.

Mainly, two processes are involved in the simulation techniques viz. random component simulation and performance simulation.

7.3.3.1. Random Component Simulation:

This involves a derivation and plotting of the parameter value in terms of the cumulative probability of incidence of that particular value or less than that parametric value. These $\{x=p(z)\}$ have been plotted in Figs.(6.1),(6.2), (6.3) and (6.4) for resistors and capacitors of different nominal values as given in Chapter 6. This information is needed to simulate the component value for Monte-Carlo analysis. Next the relationship of that parametric value and cumulative probability or random number between 0 and 1, is given some mathematic formula by fitting a curve which closely follows the pattern.

These curve fitting methods for different mathematical relations, have been shown on a comparative basis in Tables 1 and 2.

TABLE 1- Curve Fitting for 68 ohms resistor:

	p	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
Actual value 68ohms with 10% tolerance	R (ohms)	66.6	68.2	69.0	69.35	69.6	69.9	70.6	71.5	72.8	76.0
$R=63e^{.186p}$	R (ohms)	65.0	66.0	67.8	68.4	70.0	71.0	72.0	73.8	74.2	76.0
$R=63+13p$	R (ohms)	64.3	65.6	66.9	68.2	69.5	70.8	72.1	73.4	74.7	76.0
$R=63+9p+5p^2$	R (ohms)	64.0	65.0	66.2	67.4	68.8	70.2	71.8	73.4	75.1	76.0

TABLE 2- Curve Fitting for 3.3K ohm resistor:

	p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Actual values 3.3Kohms with 10% tolerance	R (Kohms)	3.10	3.175	3.2	3.22	3.25	3.29	3.32	3.345	3.39	3.4[illegible]
$R=3e^{0.15p}$	R (Kohms)	3.092	3.12	3.18	3.20	3.25	3.3	3.32	3.38	3.40	3.4[illegible]
R=3+.45p	R (Kohms)	3.045	3.090	3.135	3.18	3.225	3.270	3.315	3.36	3.405	3.4[illegible]
$R=3+.65p-.25p^2$	R (Kohms)	3.03	3.12	3.1725	3.22	3.26	3.3	3.333	3.36	3.382	3.4[illegible]

7.3.3.2. Performance Simulation:

Simulation of performance is made by means of a computer programme for-

$$y = f(x_1 \ldots . x_n) \qquad \ldots \qquad \ldots \qquad (7.8)$$

giving the dependance of the performance parameter y on circuit component values $x_1 \ldots . x_n$. Usually the performance function f can be found from the mesh and model equations for the circuit. If y is associated with the transient behaviour of the circuit, such as propogation delay the function f is likely to be extremely complex. This however is theoretically no hinderance to the application of the method, so long as f can be evaluated by the computer.

A flow chart of Monte-Carlo method for reliability analysis is shown in Fig.(7.6). For each component x_i we have as input a set of pairs of coordinate points on $x_i = p_i(z)$.

$$\{(Z_i, x_{ij}): j=1, \ldots\ldots m, \quad i=1, \ldots . n\}$$

If several components have the same distribution, only distinct distribution may be approximated.

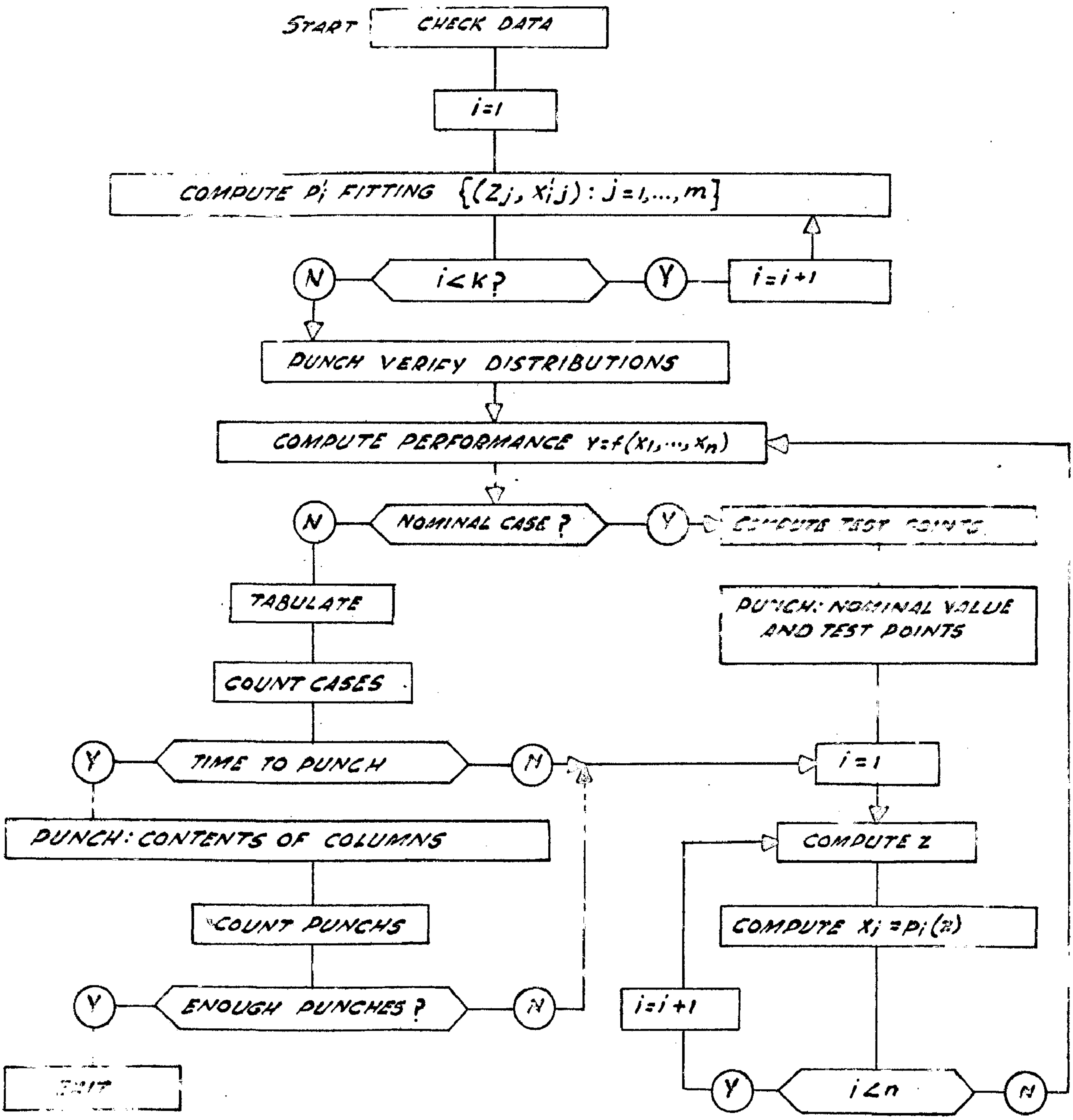

FIG.7.6 FLOWCHART FOR A MONTE CARLO ANALYSIS PROGRAM.

CONCLUSIONS

Reliability is based on probability and statistics. As in any statistical and probability calculations, assumptions have to be made, time and again, about the distribution of various kinds of failures which affect reliability. It should be realised that the realiability equations approach reality only to the extent that the actual distributions approach the assumed models. Whenever a distribution is assumed it always remains just a model, whether it is exponential, normal, Weibull, Gamma, Extreme value or Poisson distribution etc. Distributions of actual samples never fit exactly. Therefore while evaluating actual available statistical data one can never be sure that the calculated value represents the true population. However they can be regarded as good estimated and pridictions, as long as they are not disproved. When a component exhibits a certain probability of survival or failure rate under certain environmental and operating stress conditions, the parameters change immediately even with the slightest changes in stress conditions. Changes in environmental are quite often, drastic in the operation of one and the same system. Therefore the important factor in reliability prediction is about the laws governing the changes of failure rate with temperature, voltage, current and many other stresses. Correct failure rate can only be obtained by testing the component or equipment at several stress levels and then fitting the curves with the derived and available model.

Three types of failures have been generally recognised as having time characteristics viz. initial failure chance failure, and wearout failure; each has associated with it a period of operation of the device. The initial type of failure occuring at time zero or shortly there after, the chance failure occuring during useful life of the device and finally the wearout failure after the useful life. Exponential distribution has been found more appropriate for change failure. Gamma distribution is extremely useful if fatigue and wearout studies. Weibull family of distributions are useful to describe the fatigue failure, vacuum tube failure and ball bearing failures etc. With reference to relay problems it has been seen that most affected parts of a conventional relay are the relay contacts, which are damaged due to large number of operations or breaking of excessive currents for which it is not designed; the distribution law for reliability analysis is almost normal, and the other types of failures which are due to sudden stresses behave as exponential law.

In general the failures of electrical and electronic components fall under four major groups viz., Catastrophic, Intermittent out of tolerance and maladjustment. Catastrophic failure occurs when there is a gross change in characteristic, they can however be eliminated during the design of the component itself. Intermittent failures are unpredictable, so a designer can do very little to reduce them. Out of tolerance failures result from degradation, drifting and wearout. Drifting of resistor or capacitor values, wearout of relay contacts and wearout of ball bearings are the example of this type of failure. Maladjustment failures are due to human error and the effect of this might lead to error in relay sensitivity. The author has conducted some experiments on carbon resistances; their mode of failure is consistent with the investigations made in different references. In some cases the parameter drift phenomenon of certain components such as resistors, capacitors etc. does not affect the realibility of the equipment for this drift necessarily reduces the strength of the component involved but it does affect the performance of the electronic equipment, causing a gradual degradation of the performance. When components are not reliable enough, derating techniques must be used i.e., operating the component at half or even less of their rated values of voltage, wattage, temperature etc.

Instances where even extreme derating does not help and the component failure rate is too high redundancy techniques must be used in the form of parallel or standby components. It has been found that the reliability of standby redundant system is higher than the reliability of parallel redundant system. Maintenance action canngive the higher reliability. Details have been explained in Chapter-3.

Every protective relay scheme must meet two main requirement that it should not operate when not desired. This action is called the selectivity and the second requirement is, that it should operate when desired. Fault on any particular element may lead to non-selective operation. Redundancy in general increases the reliability undoubtedly but at the same time decreases the selectivity. Back up arrangements can increase the selectivity but decreases the reliability. It is thus difficult to have both of them higher. A compromise is usually recommended, depending on the requirement of the relay circuit. To study the selective and non-selective behaviour of a relay scheme a most general configuration of m_1 components in series and n_1 in parallel have been considered, for different configurations arising from $m_1 = 2, 3, 4, 5, 6$ and $n_1 = 2, 3, 4, 5, 6$. The different inequalities have been derived to find the optimum values of probability of failure and probability of non-selectivity for the respective configurations. The plots for gain in selectivity and reduction in probability of failure have been obtained with the help of digital computer (IBM 1620). As is obvious from the charts of Chapter 4 the author recommends the comprosing configurations as $n_1 = 3$ and $m_1 = 4,5$ and 6 respectively for all practical purposes. However, one has to make a choice depending upon other factors also, such as cost, availability and reliability requirements.

Before the reliability evaluation of a system can be done the information about the components behaviour and as to how they are connected in the circuit must be obtained. Usually the arrangement of the components may be either series, parallel, series-parallel or non-series-parallel. The reliability evaluation of series, parallel and series-parallel configuration can effectively be done with the help of Boolean Algebra Techniques, specially in case of multistage systems. The non-series parallel systems can be analysed through Baye's theorem. Monte-Carlo method for predicting the performance of the system is the best technique available when the components are subject to drift in their characteristics and their parameter values. This method can also take into account different environmental effects through proper simulation.

APPENDIX 1

$$\lim_{m \to \infty} \left[\frac{(m)!}{(n)!(m-n)!} p^{m-n} q^n \right] = \frac{k^n}{(n)!} e^{-k}$$

as,

$$q = \frac{k}{m}$$

and

$$p = \left(1 - \frac{k}{m} \right)$$

The problem is to find

$$\lim_{m \to \infty} \left[\frac{(m)!}{(n)!\,(m-n)!} \left(\frac{k}{m} \right)^n \left(1 - \frac{k}{m} \right)^{m-n} \right]$$

or

$$= \lim_{m \to \infty} \left[\frac{(m)!}{(n)!(m-n)!} \left(\frac{k}{m} \right)^n \left(1 - \frac{k}{m}\right)^m \left(1 - \frac{k}{m}\right)^{-n} \right]$$

or

$$= \lim_{m \to \infty} \left\{ \left[\frac{k^n}{(n)!} \left(1 - \frac{k}{m}\right)^m \right] \left[\frac{(m)!}{(m-n)!\,m^n \left(1 - \frac{k}{m}\right)^n} \right] \right\} \qquad \ldots \quad (1.1)$$

Now solving-

$$\lim_{m \to \infty} \left[\frac{k^n}{(n)!} \left(1 - \frac{k}{m}\right)^m \right]$$

$$= \frac{k^n}{(n)!} \lim_{m \to \infty} \left(1 - \frac{k}{m}\right)^m$$

$$= \frac{k^n}{(n)!} \lim_{m \to \infty} \left[1 - \frac{k}{m} \cdot m + \frac{m(m-1)}{(2)!} \frac{k^2}{m^2} - \frac{m(m-1)(m-2)}{(3)!} \cdot \frac{k^3}{m^3} + \ldots\ldots \text{neglecting higher powers.} \right]$$

$$= \frac{k^n}{(n)!} \lim_{m \to \infty} \left[1 - k + \frac{k^2}{(2)!} - \frac{1}{n} \frac{k^2}{(2)!} - \frac{2k^2}{(3)!\,m^2} - \frac{k^3}{(3)!} + \frac{3k^3}{(3)!\,m} + \ldots \right]$$

$$= \frac{k^n}{(n)!} \left[1 - k + \frac{k^2}{(2)!} - \frac{k^3}{(3)!} + \ldots\ldots \right]$$

$$= \frac{k^n}{(n)!} e^{-k}$$

Now taking

$$\lim_{m \to \infty} \left[\frac{(m)!}{(m-n)!\; m^n \left(1 - \frac{k}{m}\right)^n} \right] \qquad \ldots \quad (1.2)$$

From sterling's approximation-

$$(m)! = \sqrt{2\pi m} \quad m^m \quad e^{-m}$$

Putting this value in Equation (1.2)

$$\lim_{m \to \infty} \left[\frac{\sqrt{2\pi m} \quad m^m \quad e^{-m}}{\sqrt{2\pi(m-n)} \quad (m-n)^{m-n} \; e^{-(m-n)} m^n \; (1 - \frac{k}{m})^n} \right]$$

Now $\sqrt{m-n} = (m-n)^{\frac{1}{2}} = m^{\frac{1}{2}} \; (1 - \frac{n}{m})^{\frac{1}{2}}$

and $(m-n)^{m-n} = m^{(m-n)} \quad (1 - \frac{n}{m})^{m-n}$

which gives-

$$\lim_{m \to \infty} \left[\frac{m^m \; e^{-m} \quad m^{\frac{1}{2}}}{m^{\frac{1}{2}}(1 - \frac{n}{m})^{\frac{1}{2}} \; m^{m-n} \quad (1 - \frac{n}{m})^{m-n} \; e^{-m} \; e^n \; m^n (1 - \frac{k}{m})^n} \right]$$

$$= \lim_{m \to \infty} \left[\frac{1}{(1 - \frac{n}{m})^{m-n+\frac{1}{2}} \quad e^n \; (1 - \frac{k}{m})^n} \right]$$

$$= \lim_{m \to \infty} \left[\frac{1}{(1 - \frac{n}{m})^m \; (1 - \frac{n}{m})^{-n} \; (1 - \frac{k}{m})^{\frac{1}{2}} \; e^n \; (1 - \frac{k}{m})^n} \right] \quad \ldots (1.3)$$

as $m \to \infty \quad (1 - \frac{n}{m})^n = e^{-n}$ (as before)

and $m \to \infty \quad (1 - \frac{n}{m})^{-m} = 1$

$m \to \infty \quad (1 - \frac{n}{m})^{\frac{1}{2}} = 1$

$m \to \infty \quad (1 - \frac{k}{m})^n = 1$

Therefore equation (1.3) is equal to -

$$\frac{1}{e^n \; e^{-n}} = 1$$

or finally-

$$\lim_{m \to \infty} \left[{}^mC_n \; p^{m-n} \; q^n \right] = \frac{k^n}{(n)!} \quad e^{-k}$$

APPENDIX 2

```
C  C   JUGAL KISHORE PLOTTING OF GAIN IN SELECTIVITY CASE 5
       DIMENSIONX(110),Y(110),Z(110),W(110),V(110)
       DIMENSIONA(110),B(110),C(110),D(110),R(110)
       Q=.01
       DO1I=1,105
       Q2=Q*Q
       Q3=Q*Q2
       Q4=Q*Q3
       Q5=Q*Q4
       Q7=Q2*Q5
       Q8=Q*Q7
       Q9=Q2*Q7
       Q11=Q2*Q9
       Q14=Q3*q11
       Q15=Q*Q14
       Q17=Q3*Q14
       Q19=Q2*Q17
       Q23=Q4*Q19
       Q24=Q*Q23
       Q29=Q5*Q24
       Q35=Q24*Q11
       X(I)=6.+Q-15.*Q3+20.*Q5-15.*Q7+6.*Q9-Q11
       Y(I)=6.*Q2-15.*Q5+20.*Q8-15.*Q11+6.*Q14-Q17
       Z(I)=6.*Q3-15.*Q7+20.*Q11-15.*Q15+6.*Q19-Q23
       W(I)=6.*Q4-15.*Q9+20.*Q14-15.*Q19+6.*Q24-Q29
       V(I)=6.*Q5-15.*Q11+20.*Q17-15.*Q23+6.*Q29-Q35
       A(I)=1./X(I)
       B(I)=1./Y(I)
       C(I)=1./Z(I)
       D(I)=1./W(I)
       R(I)=1./V(I)
       IF(Q-1.)2,3,3
     2 Q=Q+.01
       GOTO1
     3 Q=Q+.05
     1 CONTINUE
       PUNCH100,(A(I),B(I),C(I),D(I),R(I),I=1,105)
   100 FORMAT (5E14.4)
       STOP
       END
```

APPENDIX 3

```
C C      JUGAL KISHORE PLOTTING OF REDUCTION IN PROBAB.OF FAUL.CASE 5
         DIMENSIONX(110),Y(110),Z(110),W(110),V(110)
         DIMENSIONA(110),B(110),C(110),D(110),R(110)
         Q=01
         DO1I=1,105
         Q2=Q*Q
         Q3=Q*Q2
         Q4=Q*Q3
         Q5=Q*Q4
         F=2.-Q
         F2=F*F
         F3=F*F2
         F4=F*F3
         F5=F*F4
         F6=F*F5
         G=3,23,*Q+Q2
         G2=G*G
         G3=G*G2
         G4=G*G3
         G5=G*G4
         G6=G*G5
         H=4.-6.*Q+4.*Q2-Q3
         H2=H*H
         H3=H*H2
         H4=H*H3
         H5=H*H4
         H6=H*H5
         P=5.-10.*Q+10.*Q2-5.*Q3+Q4
         P2=P*P
         P3=P*P2
         P4=P*P3
         P5=P*P4
         P6=P*P5
         S=6.-15.*Q+20.*Q2-15.*Q3+6.*Q4-Q5
         S2=S*S
         S3=S*S2
         S4=S*S3
         S5=S*S4
         S6=S*S5
         X(I)=Q5*F6
         Y(I)=Q5*G6
         Z(I)=Q5*H6
         W(I)=Q5*P6
         V(I)=Q5*S6
         A(I)=1./X(I)
         B(I)=1./Y(I)
         C(I)=1./Z(I)
         D(I)=1./W(I)
         R(I)=1./V(I)
         IF(Q-1.)2,3,3
       2 Q=Q+.01
         GOT01
       3 Q=Q+.05
       1 CONTINUE
         PUNCH100,(A(I),B(I),C(I),D(I),R(I),I=1,105)
     100 FORMAT(5E14,4)
         STOP
         END
```

APPENDIX - 4

Thumb rules as regards precautions to prevent over heating.

Capacitors:

(i) Mount away from the hot parts.

(ii) Use of radiation shields if required.

Electron tubes:

(i) Use heat shields designed to aid heat radiation.

(ii) Measure bulb temperature.

Resistors:

(i) Use short leads if they go to cooler parts.

(ii) Power Resistors should be mounted vertically.

(iii) Prevent power resistors from radiating to temperature sensitive parts like diodes, transistors etc.

Semiconductor Devices:

(i) Minimise thermal resistance to chesis.

(ii) Locate away from heat sensitive parts.

Transformers:

(i) Minimise thermal contact resistance to chesis.

(ii) Provide heat radiating fins where possible.

REFERENCES

1. I. Bayovsky, "Reliability theory and practice", Book (Prentice Hall), New York 1961.

2. R.F. Drenic, "Mathematical aspects of the Reliability Problems", Bell Telephone System Monograph No.3592, 1962.

3. J.H.K. Kao, "A new life quality measure for electron tubes" IRE.Trans. on Reliability and Quality Control, 1956, Vol.PGRQC-7, pp.1-11.

4. B.J. Flehimger, and P.A. Lewis, "Two parameter life time distribution for Reliability studies of renewal process", IBMJ, Res.and Dev.Vol.3, Jan.1959, pp.58-73.

5. Richard, S. Robinson, "On models for Reliability Prediction", IRE Trans. on Reliability and Quality Control, May 1962, pp.33-43.

6. G.H. Sandler, "System Engineering Reliability", (Book), (Prentice Hall), New York, 1963.

7. Z.W. Birnhaum and S.C. Saunders, "A Statistical model for life length of material", J.Am.Stat. Association Vol.53, 1958, pp.151-160.

8. John,E. Nylander, "Statistical Distribution in Reliability", IRE Trans. on Reliability and Quality Control July 1962, pp.43-53.

9. J.H. Corner, "Calculations of the risk of component application in Electronic System", IRE Trans. on Reliability and Quality Control, Jan. 1957, pp.30-44.

10. J.G. Kemeny and J.L. Snell, "Finite Markov Chain", Book (Van Nostrand), New York, 1960.

11. Enrico Dollzza, "System State Analysis and Flow graph Diagram in Reliability", IEEE Trans. on Reliability, Dec.1966, Vol.R-45, No.3, pp.85-94.

12. Chart, R.R., "A survey of the current status of the Reliability Problems" Rand Corp. Research Memo, RM-1131, Aug.14,1953.

13. D.J. Davis, "An analysis of some failure data", Journal of the American Statistical Assoc.Vol.47, No.258, June 1952.

14. D.M. Boodman, "The Reliability of Airborne Radar Equipments", Journal of the Operational Research Society of America Vol.1 No.2, Feb.1953.

15. D.R. Cox, and W.L. Smith, "On the superposition of Renewal Process".

16. F. Moskowitz and J.B. Melean, "Some Reliability aspects in system design",
IRE Trans. On Reliability and Quality Control, Sept.1956, pp.7-35.

17. F. Moskowitz,"The Analysis of Redundant Networks",
Communication and Electronics No.37, Nov.1958,

18. C.J. Creveling, "Increasing the Reliability of electronic components by using the redundant circuits",
Proc.IRE Vol.44, April 1956, pp.509-515.

19. H.W. Price, "Reliability of parallel electronic components",
IRE TRans.On Reliability and Quality Control Vol.RQC-9, April 1960, pp.35-39.

20. A.M. Fedoseev "Bases of Relay protection (osnovyreleinoi zashchity) Gosenergoizdat, 1961.

21. E.F. Moore and J.Shannon "Reliable circuits using less reliable relays"
J. Franklin Sep.1956, Vol.262, Pt.I,pp.191-208, and Pt.II pp.281-297.

22. V.L. Fabrikant "Applying reliability theory to appriasal of the performance of Relay protection gear",
Elekrichestvo No.9, 1965, pp.36-40.

23. A.C. Keller, H.N. Weger, R.L. Peek and M.A. Logan, "Design of Relays",
Bell Telephone System Technical Publication, Monograph No.2180, 1954.

24. S.L. Goldsborough, and A.V. Hill, "Relays and Breaking for High Speed Singh Pole Tripping and Reclosing",
Trans.A.I.E.E. 61, 1942, pp.77-81. Disc.p.429.

25. R.L. Peck, and Wager, H.N. Switching Relay Design,
Van Nostrand, Book, New York.

26. S.W. Chaikin, "Mechanics of Electrical Failure Caused by Surface Contamination",
Electro- Technology, Aug.1961.

27. N.M. Zul and F.A. Kuliev- "On the Dependibility of Relay Control gear in Electrical Networks",
Elektrichestov No.9, 40-44, 1965, Vol.3.

28. N.M. Shulekin, "Reliability of terrestrial radio electronic equipment (Nadezhost nazemnogo radioelektronnogo oborudovaniya). Iz.Sov.Radio 1957.

29. A.R. Van C. Warrington- "Protective Relays (Their theory and Practice)",
Vol.I, Chapman & Hall 1962.

30. A.R. Van C. Warrington, "Back up protection", C.I.G.R.E. Paris May 12th-22nd, 1954, Report No.330.

31. C.H. Zierdt Jr. "Success Story-Transistor reliability 1956", IRE Trans.on RQC June 1957, pp.57-67.

32. ARINC Research Corp. "Reliability Engineering" (Book), Prentice Hall, 1964.

33. E. Pieruschka, "Principle of Reliability", (Book), Prentice Hall, 1953.

34. David C. Porter, "Failure analysis of Electronic Parts", IEEE Trans. on Reliability 1965, pp.56-65.

35. G.J. Leverback, "Accelerated life testing of capacitors", IRE Trans.June 1957, pp.9-20.

36. J.B. Whitehead, "Impregnated Paper Insulation", (Book), Johnwiley, 1935.

37. D.A. Mclean and L. Egerton "Paper Capacitors Containing Chlorinated Impregnants", Ind.& Eng.Chemistry 37, 1945 pp.73-79.

38. L.J. Berberich, C.V. Fields & R.E. Morbury, "Chlorinated Impregnants", AIEE Trans.1944, 63-1179.

39. Andrew B. Jacobsen, "Factors in Reliability of Germanium Power Transistors", IRE Trans.on RQC June 1957, pp.43-48.

40. E.B. Hakim and B. Reich "U.S. Advancement in Transistor Reliability through manufacturing Process improvement", IEEE, Trans.on Reliability Oct. 1965, pp.94-99.

41. W.H.E. Blanton and R.M. Jacobs "A survey of Techniques for Analysis and Prediction of equipment reliability", IRE Trans.on Reliability and Quality Control July 1962 Vol.RQC-11, pp.18-35.

42. W.R. Allen, L.J. Tick and M.A. Woodbury- "Some mathematical and Statistical techniques useful in Reliability Analysis", Proc.4th Natl. Symp. on Reliability and Quality Control, Jan.1958 pp.63-68.

43. J.P. Lipp, "Topology of switching elements Vs Reliability", IRE Trans. on Reliability and Quality Control Vol.RQC-10, Jan.1957, pp.21-33.

44. C.F. Desieno, "A probability method for determining the Reliability of Electrical Power Systems", IEEE Trans. on Reliability, March, 1965, pp.30-35.

45. S.H. Caldwell, "Switching circuits and Logical Design", (Book)(John Wiley), New York, 1958.

46. A.M. Breipohl, R.R. Prairic, W.J. Zimmer, "A consideration of the Bayesian Approach in Reliability Evaluation", IEEE Trans. on Reliability Oct. 1965, pp.107-113.

47. Leo Hellerman, "A computer Application to Reliable Circuit Design", IRE Trans. on Reliability and Quality Control, May 1962, Vol.RQC-11, pp.9-18.

48. J.M. Hammersley and D.C. Handscomb, "Monte Carlo Methods", Book (Methnew & Co. Ltd.) London, 1964.

49. Norman, H. Roberts "Mathematical methods in Reliability Engineering", Book (McGraw Hill), New York- 1964.