SOME ASPECTS OF ANALYSIS AND SYNTHESIS OF MACHINE CONTROL PROBLEMS

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CERTIFICATE

CERTIFIED that the dissertation entitled 'SOME ASPECTS OF ANALYSIS AND SYNTHESIS OF MACHINE CONTROL PROBLEMS', which is being submitted by Sri K.C. Vijayasri in partial fulfilment for the award of the Degree of Master of Engineering in Advanced Electrical Machines of the University of Roorkee, is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of 7 months from Jan. to July for preparing this dissertation for Master of Engineering Degree of this University.

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LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

i	-	current in any winding
I	-	Laplace transform of the current 'i'
v	-	voltage applied or induced in any winding
V	-	Laplace transform of 'v'
s	-	Laplace transform operator
w	-	frequency in rad/sec
j	-	complex number $\sqrt{-1}$
τ, T	-	time constant of any winding or link
K	-	gain of the individual element
K_{ov}	-	overall gain
R	-	resistance in ohms
L	-	self-inductance in henries
M	-	mutual inductance in henries
C	-	capacitance in farads
A	-	amplitude of input signal
A_s	-	amplitude of input signal at which saturation occurs
F_N	-	A polynomial equation
$N(A)$	-	Describing function of a non-linear element and is a dependent on amplitude only
$N(A,w)$	-	Describing function of a non-linear element which is dependent on amplitude and frequency
$N_e(A,w)$	-	equivalent describing function
$G(s)$	-	transfer function of linear element(s)

Synopsis

Automatic control systems engineers have mostly relied on block diagram techniques and signal flow graph methods for analysing the performance of the system. These methods have a distinct advantage over the use of differential equations in that they serve as visual reminders of cause-and-effect relationships in the physical system.

In this work, the physical system has been represented by its structural diagram as proposed by Aizerman[(4)]. The advantage of such a representation is that the elements in any system may be represented as links.

Linear systems including that having time lags have been considered and analysed. The effect of a parameter (gain or time constant) on the system stability is found out using D-partition technique. For unstable systems stabilising devices have been used. The transfer functions of the stabilising devices are realised into physical circuits using conventional network synthesis methods. The transient response is plotted from the D-partition curve.

Stability analysis of systems having more than one non-linearity has been carried out using the describing function technique. A practical system-voltage regulation of a synchronous generator- is taken and studied with (a) neglecting all non-linear effects, (b) considering only the non-linearity (saturation type) of the synchronous generator, (c) considering the non-linearities of both the amplidyne and synchronous generator and (d) considering the non-linearities of amplidyne, main exciter and synchronous generator. The system is found to be unstable for cases (a) and (b) whereas stable for cases (c) and (d) upto a particular value of frequency. Thus, the stabilising effect of non-linearities has been brought out.

CHAPTER I

STABILITY OF LINEAR CONTROL SYSTEMS

1.1 GENERAL

In majority of the cases, the problem of control consists of establishing and maintaining over a period of time, the operating state of the controlled object. This problem gives rise to the requirement that the system of automatic control should possess a definite stability, even if it is acted upon by an external disturbance.

A.M. Lyapunoff[1] first formulated the definition and conception of 'stability'. He defined that a system will be called stable if, having been disturbed from a state of equilibrium and left to itself, it will, in the course of time tend to return to the earlier state of equilibrium. Let the controlled quantity has a certain value x_o in the steady state. If the system is disturbed by means of some external action so that x_o varies by δ (t). If the external disturbance is removed, then the system will be stable only if,

$$\lim_{t \to \infty} \delta(t) \to 0 \qquad \ldots \quad (1.1)$$

If equation (1.1) is not satisfied then the system is unstable.

In the general case of a system, the variation of the deviation $\delta(t)$ can be described by a nth order differential equation,

$$a_o \frac{d^n\delta(t)}{dt^n} + a_1 \frac{d^{n-1}\delta(t)}{dt^{n-1}} + \ldots + a_n\delta(t) = 0 \quad \ldots (1.2)$$

where $a_o, a_1 \ldots a_n$ are constant coefficients.

The solution of equation (1.2) is

$$\delta(t) = \sum_{i=1}^{n} A_i \, e^{Z_i t} \quad \ldots \quad (1.3)$$

where A_i are the integration constants and Z_i are the roots of the characteristic equation which is of the form

$$a_o Z^n + a_1 Z^{n-1} + \ldots + a_n = 0 \quad \ldots \quad (1.4)$$

If the system has to be stable it is necessary that $\delta(t) \to 0$ when $t \to \infty$ and this is possible only if Z_i is a negative real number, so that $A_i e^{Z_i t}$ will tend to zero. If Z_i is a complex quantity, then it should have a negative real part.

A general conclusion is made that, if a linear differential equation with constant coefficients has to be stable, then it is necessary and sufficient that all the real roots of the characteristic equation be negative and that the complex roots have negative real part.

If the order of the characteristic equation is large it is difficult to find out the roots of the equation and is often tedious. However, it is sufficient to determine whether all the roots lie to the left of the imaginary axis.

This problem leads to the two statements. Firstly, given certain parameters of the system it is necessary to find for which values of the remaining parameters the system is stable. Secondly, if all the parameters are given it is necessary to determine whether the system is stable for the initially chossen parameters' values.

The first problem is solved by constructing region of stability and the latter by using stability criteria.

1.2 STABILITY CRITERIA

The stability of the system can be determined by using (1) Routh-Hurwitz criterion (2) Amplitude-phase characteristic (Nyquist plot) (3) Root Locus method[2].

1.2.1 ROUTH-HURWITZ' CRITERION

This is an algebraic method which gives the solution to the absolute stability of the system. The Routh-Hurwitz criterion is concerned with statement of conditions which must be satisfied by the coefficients of the characteristic equation of any order to ensure that the real parts of the roots are negative.

Let the characteristic equation be

$$F(s) = a_o\ s^n + a_1\ s^{n-1} + \ \ldots + a_{n-1}s + a_n = 0 \qquad \ldots \quad (1.5)$$

The necessary and sufficient conditions which must be satisfied for the system having the characteristic equation (1.5) to be stable are

(1) All the coefficients of the polynomial have the same sign.

(2) None of the coefficients vanish.

The necessary and sufficient condition that all the roots of nth order polynomial lie in the left half plane of the s-plane is that Hurwitz' determinant

D_k (k = 1, 2, ... n) must all be positive.

The Hurwitz determinant of equation (1.5) are given by

$$D_1 = a_1, \quad D_2 = \begin{vmatrix} a_1 & a_3 \\ a_2 & a_4 \end{vmatrix} \ \ldots . \qquad (1.6)$$

Then Routh's criterion can be defined as 'The necessary and sufficient condition that all the roots of the polynomial F(s) = 0 lie in the left half of s-plane is that

$$a_O > 0; \quad D_1 > 0; \quad D_2 > 0 \ \ldots \ D_n > 0 \quad \ldots \quad (1.7)$$

The Routh-Hurwitz criterion though appears to be laborious is the simplest one in the analysis of control systems. The procedure can be further simplified as follows. The coefficients are arranged in a triangular array as shown

$$\begin{array}{llll} a_0 & a_2 & a_4 & a_6 \\ a_1 & a_3 & a_5 & a_7 \\ b_1 & b_3 & b_5 & \\ c_1 & c_3 & & \\ d_1 & & & \end{array} \qquad \ldots \quad (1.8)$$

where

$$b_1 = \frac{a_1\ a_2 - a_0\ a_3}{a_1}, \quad b_3 = \frac{a_1\ a_4 - a_0\ a_5}{a_1}, \ldots$$

$$c_1 = \frac{b_1\ a_3 - a_1\ b_3}{c_1}, \quad c_3 = \frac{b_1\ a_5 - a_1\ b_5}{c_1}, \ldots$$

$$d_1 = \frac{c_1\ b_3 - b_1 c_3}{c_1}$$

If all the coefficients in the first column of equation (1.8) are positive, then the system is stable. If there is a sign change, then such a root exist in right half plane and the system is unstable.

The advantage of the Routh-Hurwitz polynomial and the evaluation of its determinant gives a good indication about the stability of the system. However, the Routh-Hurwitz criterion is invalid for non-linear systems and time lag systems. Also, it indicates only about the absolute stability of the system and does not give any indication on how the system can be stabilised.

1.2.2 NYQUIST CRITERION (AMPLITUDE-PHASE CHARACTERISTIC)

If the order of the systm is high and if any of the roots are complex, Nyquist diagram can be effectively used to determine the stability of the system.

Nyquist criterion is based on 'Cauchy' principle of argument[2] and the manipulation involved is a conformal mapping of the imaginary axis of the s-plane on a polar plane defined by the loop transfer function. The procedure followed is

(1) The loop transfer function is found out and jw is substituted for s.

(2) The polar curve is plotted for the loop transfer function and 'w' is varied from zero to infinity.

(3) The stability of the system is determined by inspecting both the plots and poles of the loop transfer function.

The Nyquist criterion can now be defined as 'If the loop gain function is a stable function, the number of poles of F(s) that are in the right half of s-plane are zero, for a stable closed system, the Nyquist plot of the loop gain function must not enclose the critical point (-1,j0)[2].

The main advantage of the Nyquist criterion is that the method can be used both for linear and nonlinear system to predict the stability of the system.

1.2.3 D-PARTITION METHOD

It is well known that as the gain of the control system is increased, the system may become unstable. The Routh-Hurwitz criterion or frequency response method (Nyquist plot) do not indicate the optimum value of the gain of the system. In the case of Nyquist plot if it is desired to study the effect of any parameters of the system on its stability, a family of curves must be plotted, assuming in plotting each curve of the family a certain relevant value of the fixed parameter.

Yu.I.Neimark(3) proposed a method of stability analysis, by use of which it is possible to determine at once all the values of the parameters in question for which the system remains stable.

The characteristic equation (1.5) can be considered

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n = 0$$

The values of a_0, a_1, ... a_n may be interpreted geometrically as a point in the n-dimensional space, with axes for the values of the coefficients a_0, a_1 ... a_n. To each point of this space there corresponds a definite value of the coefficients a_0, a_1 .... a_n and consequently definite values of all the roots s_1, s_2 ... s_n of the characteristic equation.

If in this polynomial space there exists a region in which each point corresponds to a characteristic equation, all of whose roots lie to the left of the imaginary axis in the complex root plane, then the hyper surface bounding this region is called the 'boundary of the region of stability[4]'.

In practice it is sufficient to construct the region of stability on a numerical straight line (one parameter) or in a plane (two parameters).

Neimark proposed that if a polynomial of the nth degree has k roots to the left and (n-k) roots to the right of imaginary axis, as shown in Figure (1.a), and if all the value of the coefficients in equation (1.5) are known except a_o and a_n, then there always exist a curve in the plane of a_O and a_n, and bounds a region in which each point defines a polynomial having k roots to the left and (n-k) roots to the right of the imaginary axis (Figure 1.b). Such a type of distribution can be denoted by D (k, n-k), where k may be any value from O to n. If all the roots are to the left of the imaginary axis D (n, O), is a stable system.

The above principle is widely used in machine control and automatic regulating systems analysis. A specific example is considered[3].

Let the characteristic equation of the system be

$$Q(s) + \tau R(s) = 0 \qquad \ldots (1.9)$$

where Q (s) and R (s) are polynomials with constant coefficients and 'τ' is a parameter whose effect on stability is to be examined, τ can be either a time constant or gain of an individual link or a group of links. The limiting value of τ can be determined by using Neimark's method[3].

Substituting $s = jw$ in equation (1.9)

$$Q(jw) + \bar{\tau}R(jw) = 0$$

whence $$\bar{\tau} = - Q(jw)/R(jw) \qquad \ldots (1.10)$$

If w is varied from $-\infty$ to $+\infty$ then all the limiting values of $\bar{\tau}$ can be determined. The locus of points on the $\bar{\tau}$ surface from $-\infty$ to $+\infty$, divide the whole τ plane into regions where in all the polynomials have the same number of zeros to the left of the imaginary axis. Such a curve is called the 'D-partition' curve on $\bar{\tau}$ plane.

In order to determine the number of zeros to the left of the imaginary axis the 'hatching rule' proposed by Neimark is followed. In the D-partition curve from $w = -\infty$ to $w = 0$ and then to $w = +\infty$, the left hand side of the curve is hatched. When traversing from hatched side

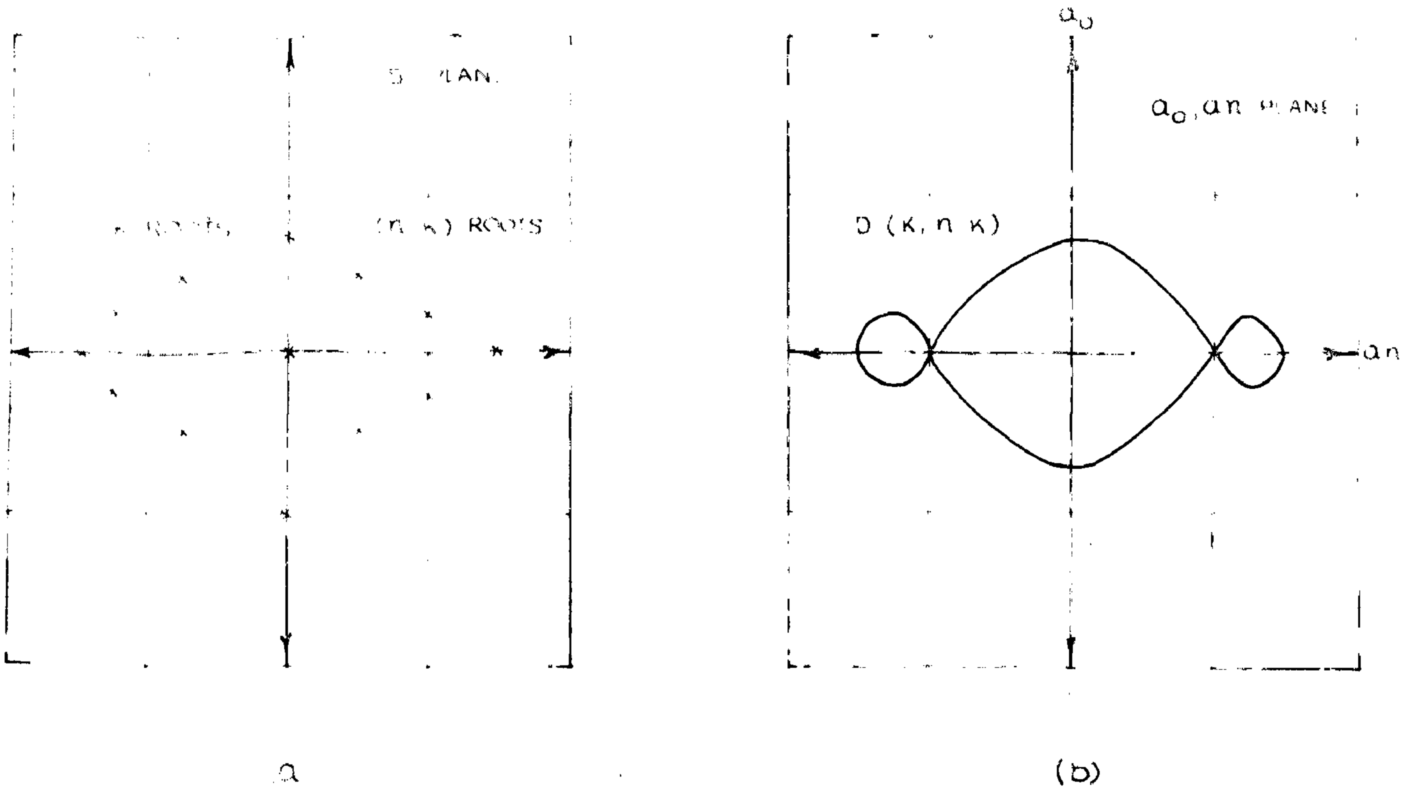


FIGURE 1
LOCATION OF ROOTS

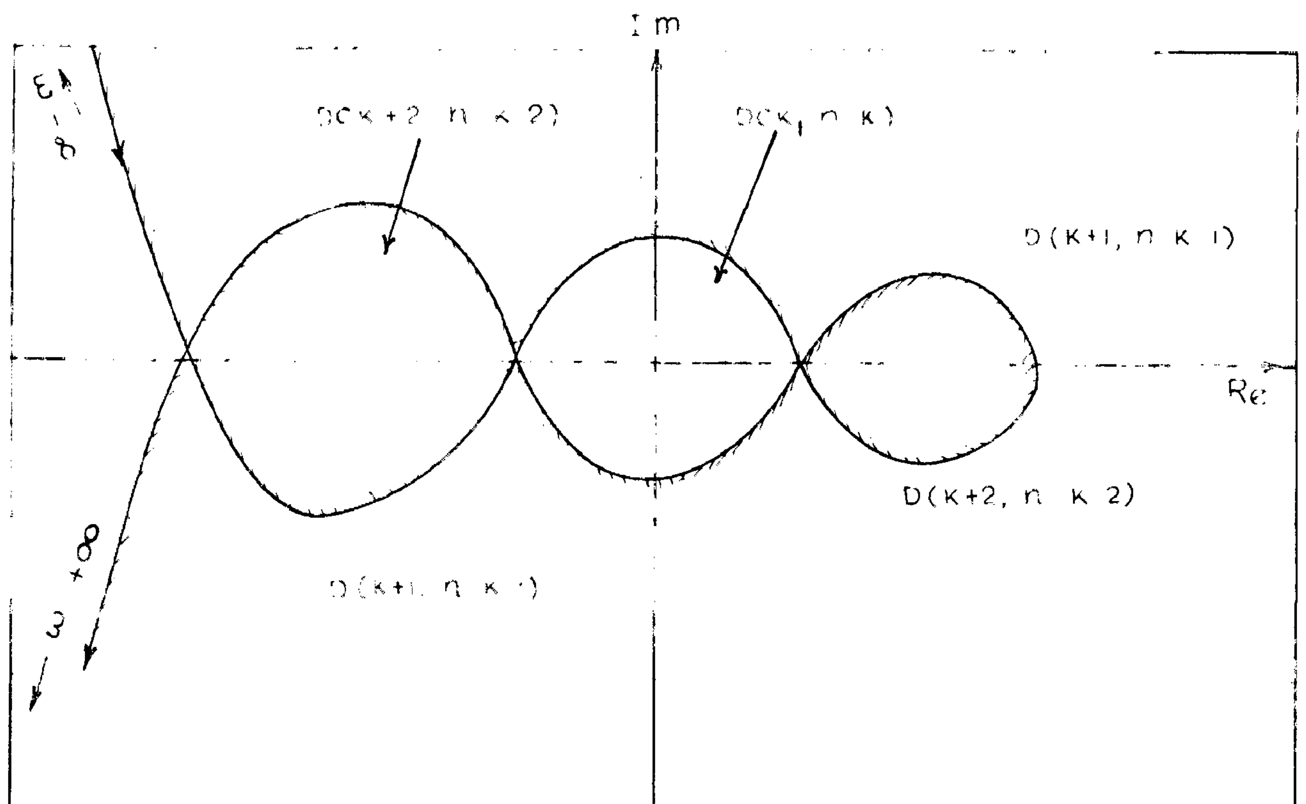


FIGURE 2
D PARTITION CURVE

to the unhatched side, one root to the left of the imaginary axis is lost. Conversely, when traversing from unhatched to the hatched side, one root to the right of the imaginary axis is lost (Figure 2). If the largest numbers of zeros are to the left of the imaginary axis then that region will be stable provided the number of zeroes to the left of the imaginary axis is equal to the order of the characteristic equation.

If the D-partition curve is plotted then the Amplitude-Phase Characteristic can be determined easily from the D-partition curve. It is also possible to determine the margin of stability of the system in terms of phase and the degree of oscillations which determine the magnitude of the peak on the APC of the closed-loop system[3].

If K is the overall gain of the system then considering it as a variable parameter, the D-partition curve is plotted in terms of the complex parameter $\overline{K}$. A possible form of D-partition curve in terms of $\overline{K}$ is shown in Figure (3). Let the closed-loop transfer function is of the form

$$K_s(jw) = \frac{K}{K + \frac{M(jw)}{N(jw)}} \quad \ldots \quad (1.11)$$

Then the characteristic equation is $K + \frac{M(jw)}{N(jw)} = 0$

$$\ldots \quad (1.12)$$

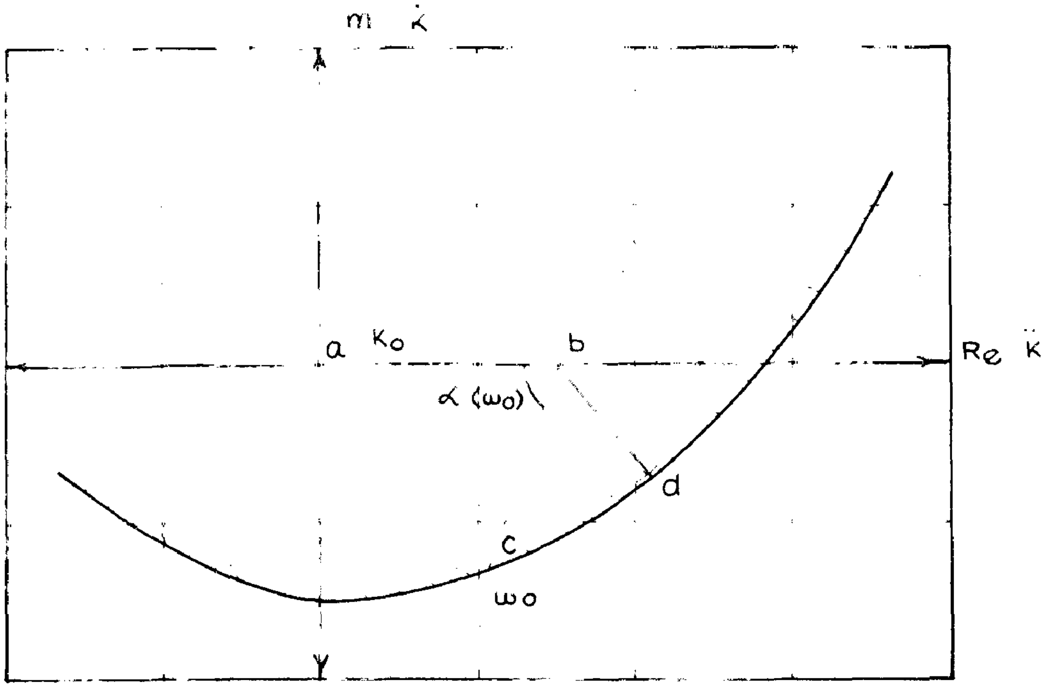


FIGURE- 3 (a)
DETERMINATION OF AMPLITUDE-PHASE CHARACTERISTIC

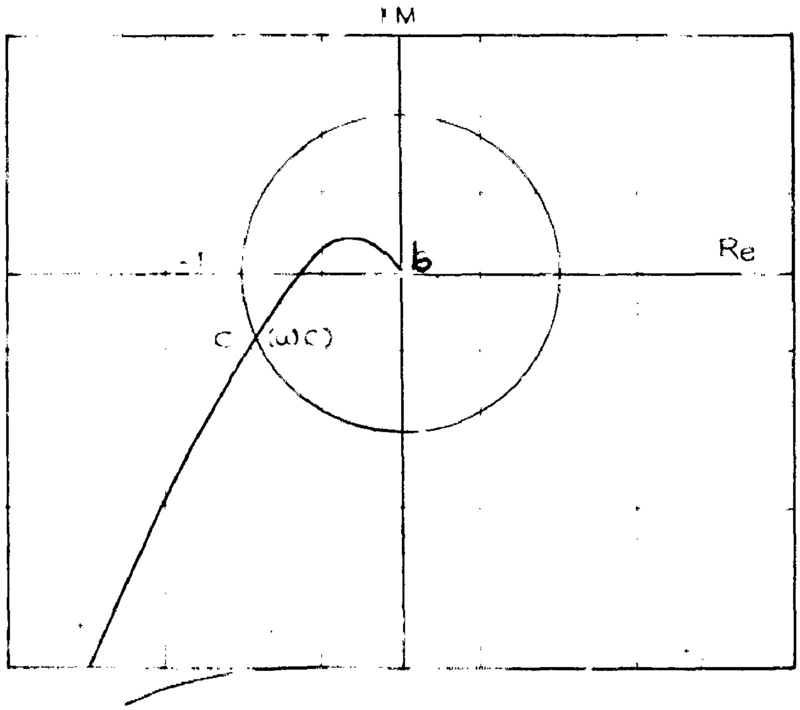


FIGURE 3 (b)
AMPLITUDE - PHASE CHARACTERISTIC OBTAINED FROM FIGURE - 3 (a)

whence $\bar{K} = - \frac{M(jw)}{N(jw)}$ ... (1.12a)

In Figure (3.a) K_0 is the value of gain at $w = 0$ and is denoted by vector 'ab'. The value of the denominator of the equation (1.11), for a given frequency w_0 and some selected value of K_0 is denoted by the vector 'bc'. Then the amplitude value of equation (1.11) is defined by the ratio ab/bc and its phase is $\alpha(w_0)$. In the same manner the amplitude characteristic can be determined for different band of frequencies. A possible type of Amplitude-Phase Characteristic (APC) is shown in Figure (3.b). The peak of the characteristic may be determined directly from Figure (3.a). A circle is drawn with centre as 'b' touching the D-partition curve. The ratio of ab/bd is the peak amplitude of the Amplitude-Phase Characteristic.

The critical frequency or margin of stability may be determined from Figure (3.b) in which the Amplitude-Phase Characteristic of the open-loop system is drawn. A unit circle is drawn with centre as 'O' and its inter-section with the curve gives the critical frequency (cut-off frequency). If this frequency is 'w_c', then for control period 't', the following inequality is obtained[3]

$$\frac{\pi}{w_c} < t < \frac{4\pi}{w_c} \quad \ldots (1.13)$$

It may sometimes be tedious to draw the D-partition curve of the variable parameter or it may be possible that the characteristic equation contain two or more variable parameters. If such a case exists, Meerov's and Aizerman's method of structural stability[3,4] are more suitable to determine at the outset, whether system is stable.

1.2.4 STRUCTURAL STABILITY

The system may be represented by its structural diagram. The structural diagram is represented in terms of links, the advantage of which is that the original structural diagram do not change even if the parameters are changed.

If x_{out} and x_{in} are the output and input quantities and the closed-loop transfer function may be of the form

$$d\,(s)\;L\,(x_{out}) \;=\; K(s)\;L\,(x_{in}) \qquad \ldots \quad (1.14)$$

where $d\,(s)$ and $K(s)$ may be polynomials. A specific example may be considered to explain equation (1.14). The linear differential equation of the system or the link may be of the form

$$T\;\frac{d\,x_{out}}{dt} + x_{out} = K\,x_{in} \qquad \ldots \quad (1.15)$$

If the initial conditions are assumed to be zero, the Laplace transform of equation (1.15) is

$$(Ts + 1)\;x_{out} \;=\; K\;x_{in} \qquad \ldots \quad (1.16)$$

In equation (1.16), $d(s)$ is $(Ts+1)$ and $K(s) = K$. Aizerman[4] proposed the name for $d(s)$ as 'the inherent operator' and $K(s)$ as 'the action operator', or 'the operator coefficient of amplification'.

The elements that are most frequently encountered in control systems are

$$(Ts+1),\ (T^2s^2 + Ts + 1),\ T^2s^2 + 1,\ Ts \text{ and } Ts-1$$

They have a special significance in control theory and separate names and agreed notation are given to them[4]. They are as follows:

1) An element for which $d(s) = (Ts+1)$ is called 'single capacitance' or 'aperiodic' and is denoted by a square (□)

2) An element for which $d(s) = T^2s^2 + Ts + 1$ is called 'oscillatory' and is designated by a rectangle (▯)

3) An element for which $d(s) = T^2s^2 + 1$ is called 'conservative' and is designated by a shaded rectangle (▨)

4) An element for which $d(s) = Ts$ is called 'astatic' or 'integrating' and is designated by a circle (○)

5) An element for which $d(s) = Ts-1$ is called 'unstable' and is designated by a triangle (▽).

The action operaters K(s) are often encountered in the following forms:

$$K; \quad K + \rho s, \quad K + \rho s + \gamma s^2$$

In the case $K(s) = K$ = constant, the actions on the element are called static. If $K(s) = K + \rho s$ it is called first derivative action and if $K(s) = K + \rho s + \gamma s^2$ it is called second derivative action.

With the use of the types of above notation any physical system can be represented easily by its structure consisting of links. In this work the above method of representing elements by their respective links is adopted.

1.2.5 CRITERIA OF STRUCTURAL STABILITY[4]

The criteria of stability may be used not only in order to determine the conditions of stability but also to study the general properties connected with stability for the whole class of control systems. If the properties of the system are known it is possible to make a number of inferences about the stability of a system from its scheme without using the criteria of stability.

Aizerman[4] in his classical work gave a number of criteria which can be used to predict the stability of the system. A brief description of the methods are given here.

a) The Conditions for the Structural Stability of Single-loop Systems without Derivative action

If 'q' is the number of astatic stages (having $d(s) = s$), 't' is the number of unstable stages (having $d(s) = Ts-1$) and 'r' is the number of conservative stages ($d(s) = T^2s^2+1$) in the system, and n is the degree of the polynomial $d(s)$, then the structure will be stable if (a) $q + t < 2$ and (b) $n < 4r$. The inequalities must be simultaneously satisfied.

b) The Conditions for Structural Stability in Single-loop Systems with Derivative action

If positive first derivative action is present in a single-loop system, the characteristic equation of the system is of the form

$$D(s) + Rs + K = 0 \qquad \ldots \qquad (1.17)$$

Equation (1.17) can be written in the form

$$D(s) + K(s) = 0 \qquad \ldots \qquad (1.17a)$$

where the degree of $K(s)$ may be n and is less than the degree of polynomial $D(s)$, m.

The necessary and sufficient condition for structural stability of the system having one first derivative at one point of the circuit is $q + t \leq 2$

(c) If in addition to the first derivative the second derivative also exists in the system then the characteristic equation may be of the form

$$D(s) + Ms^2 + Rs + K = 0 \qquad .... \qquad (1.18)$$

where M and R are positive numbers, equation (1.18) may be represented as equation (1.17a). Here $K(s)$ is the product of factors of the form $(R_i s+K)$ and $(M_i s^2 + R_i s + K)$ and R_i, M_i and K_i are positive non-zero numbers.

Assuming that $D(s)$ consists of factors of the form s, $Ts-1$; T^2s^2+1, $Ts+1$ and T^2s^2+Ts+1, $K(s)$ is a Hurwitz polynomial. If μ represents $q + t + 2r$ and ρ is the integral part of the fraction $\mu/2$ and $N = n + m$ then the conditions for the structural stability of system containing the derivative actions are determined by the following theorem[3].

In order that the system with characteristic equation (1.17a) in which $D(s)$ is the product of s, $Ts+1$, $T^2s^2 +Ts+1$, T^2s^2+1 and $Ts-1$ and $K(s)$ is a Hurwitz polynomial, shall be structurally stable, it is necessary and sufficient that the inequality

$$m \geqslant q + t - 1 \qquad ...$$

be satisfied and that one of the inequalities in Table 1. depending upon the values of m and n be satisfied.

TABLE 1

	m = 0	m = 0 and even	m = odd
μ even	$N > 4\rho$	$N > 4\rho - 1$	$N > 4\rho - 2$
μ odd	$N > 4\rho$	$N > 4\rho$	$N > 4\rho + 1$

If the system is a multi-loop system, it is possible to decompose the system into single-loop systems. Meerov's method of structural stability may be applied and is as follows.

The characteristic equation of any closed-loop system may be represented as

$$\prod_{i=1}^{n+1} (1+T_i s) + K_{\nu+1} \cdots K_{\nu+n}\Big[T_{N+1}s \prod_{j=1}^{\nu}(1+T_j s)\prod_{\eta=\nu+n-1}^{N}(1+T_\eta s)$$

$$+ K_1 K_2 \cdots K_\nu K_{\nu+n+1} K_N (1+T_{N+1}s)\Big] = 0 \qquad \ldots (1.19)$$

Substituting $K_{\nu+1}, K_{\nu+2} \cdots K_{\nu+n} = K$, equation (1.19) may be divided by K and using the notation $m = 1/K$ equation (1.19) becomes

$$\ldots (1.20)$$

Equation (1.20) may be written in the form

$$m \quad F_{N_2}(s) + F_{N_1}(s) = 0 \qquad \ldots (1.21)$$

If now $m \to 0$ ie., $K \to \infty$, the equation (1.21) degenerates as $F_{N_1}(s) = 0$ ... (1.22)

In order that the original system be stable the following conditions shall be satisfied.

(1) The degenerated equation must be a Hurwitz polynomial.

(2) If N_2 is the degree of polynomial $F_{N_2}(s)$ and N_1 is the degree of polynomial $F_{N_1}(s)$, then the necessary and sufficient condition for the structural stability of the system is $N_2 - N_1 = 2$.

In addition to the above the following conditions must also be fulfilled.

(a) If $N_2 - N_1 = 1$, the relation $\frac{Ao}{Bo} > o$ must hold good where B_o is the coefficient of highest power term in $F_{N_2}(s)$, and A_o is the coefficient of highest term in $F_{N_1}(s)$.

(b) If $N_2 - N_1 = 2$, then the condition

$\frac{B_1}{B_o} - \frac{A_1}{A_o} > 0$, must be satisfied.

Here B_o and B_1 are the coefficients of the first and second highest power terms of the polynomial $F_{N_2}(s)$ and A_o and A_1 are the coefficients of the first and second highest power terms of the polynomial $F_{N_1}(s)$ respectively.

(c) If $N_2-N_1 \geqslant 3$, the system is unstable.

The above methods are mainly used in this work.

CHAPTER II

STRUCTURAL SYNTHESIS OF LINEAR CONCTOL SYSTEMS

2.1 INTRODUCTION

The representation of a linear physical system by structural diagram is an useful tool in its design of such systems. The design of a control system mainly deals with the quality of the system. The problem of quality in turn deals with the numerical values of parameters of the linear system represented by its links. Also, a system has to be designed to meet certain requirements namely whether the system is stable in its given form and meet the required speed of response, steady-state offset, limited overshoot and damping. In most of the design problems certain data viz, speed of response, overshoot and steady-state offset are usually assumed and the parameters are so varied that the system remains stable with large values of overall gain.

However, if the system itself is large it becomes inadvisable to change the parameters. In such a case, the stability as well as the overall gain can be improved by providing certain types of stabilising links[3]. The provision of more or removal of certain links may or may not improve the performance of the system and the proper choice and connection of the links is purely a designer's task.

2.2 SPEED CONTROL OF D.C. MOTOR

The speed control of a d.c. motor is considered here as an illustrative example of structural synthesis.

The physical system of speed control of a d.c. motor is shown in Figure (4).

In the system considered it is desired to vary the speed of the d.c. motor within wide limits. The speed control is obtained by varying the armature voltage of the d.c. motor with a constant excitation flux. The amplidyne generator is chosen as the source for supplying the armature. The tacho-generator is used as a comparison unit. An amplifier is also connected so that the system possesses a large gain.

The working principle of the system is as follows. The voltage difference between the e.m.f. of the tacho-generator V_t and the reference voltage V_R is fed to the amplifier. The excitation winding is supplied by the amplified output voltage of the amplifier. The voltage developed across the d-axis of the amplidyne is applied to the armature terminals. During steady-state this ensures the desired speed of the motor. Due to external disturbance the speed of the motor will change causing a corresponding change in the e.m.f. developed by the tacho-generator and hence (V_R-V_t) also changes. The voltage developed by the amplidyne will also vary and hence the speed of the motor will change accordingly.

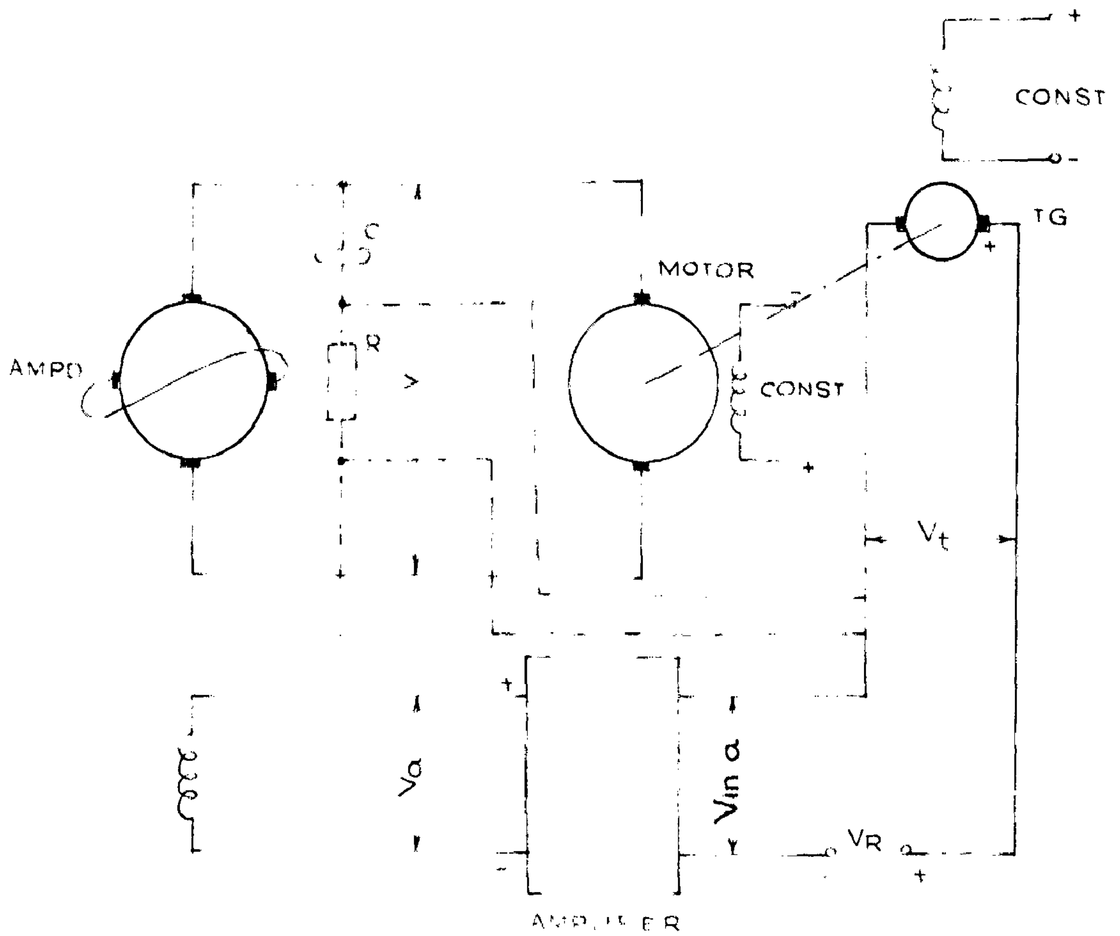


FIGURE- 4.
THE PHYSICAL REPRESENTATION OF A D.C. SPEED CONTROL SYSTEM

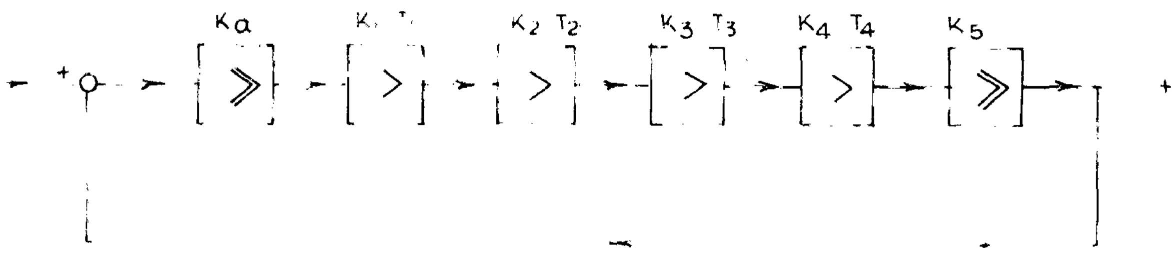


FIGURE- 5
STRUCTURAL DIAGRAM OF THE ABOVE SYSTEM

2.3 STRUCTURAL DIAGRAM

The structural diagram may be drawn if the transfer functions of the individual elements are known and can be derived as following:

(1) Assuming there is no time lag in the system,

$$V_a = K_a V_{in,a} \qquad \ldots \quad (2.1)$$

(2) The transfer function of the amplidyne may be determined as follows:

If L_1 and R_1 denote the inductance and ohmic resistance of the exciting (control) winding and i_1 is the current flowing in that winding, then the equation of electrical equilibrium is

$$R_1 i_1 + L_1 \frac{di_1}{dt} = V_a \qquad \ldots \quad (2.2)$$

Taking Laplace transform on both sides of equation (2.2) and denoting $T_1 = L_1/R_1$

$$I_1 (R_1 + L_1 s) = V_a$$

i.e.

$$I_1 = \frac{V_a}{R_1(1+T_1 s)} \qquad \ldots \quad (2.3)$$

The voltage induced in the q-axis of the amplidyne is

$$V_q = K_1' I_1 \qquad \ldots \quad (2.4)$$

where K_1' is a constant.

The equation of electrical equilibrium in the q-axis winding is

$$V_q = (R_2 + L_2 s)\ I_2 \qquad \ldots (2.5)$$

where i_2 is the current flowing in the q-axis of the winding. Substituting equations (2.3) and (2.4) in equation (2.5),

$$I_2 = \frac{K_1' V_a}{R_1 R_2 (1+T_1 s)(1+T_2 s)} \qquad \ldots (2.6)$$

The voltage induced in the d-axis of the amplidyne is

$$V = K_2'\ i_2 \qquad \ldots (2.7)$$

Substituting equation (2.6) in equation (2.7)

$$V = \frac{K_1' K_2'\ V_a}{R_1 R_2 (1+T_1 s)(1+T_2 s)}$$

$$= \frac{K_1 K_2 V_a}{(1+T_1 s)(1+T_2 s)} \qquad \ldots (2.7a)$$

where $K_1 = K_1'/R_1$ and $K_2 = K_2'/R_2$

The transfer function of the amplidyne may be obtained as

$$\frac{V}{V_a} = \frac{K_1 K_2}{(1+T_1 s)(1+T_2 s)} \qquad \ldots (2.8)$$

The amplidyne may be either represented by two aperiodic links or by one oscillatory link.

(3) The transfer function of the d.c. motor may be obtained as proposed by M.V. Meerov[3]. The behaviour of the motor is described by the following equations.

(a) The net torque developed in the motor can be written as

$$T_M - T_R = \frac{GD^2}{375}\frac{dn}{dt} \qquad \text{... (2.9)}$$

where GD^2 is the moment of gyration of the motor, n is the rotational speed in r.p.m. T_M is the motor torque and T_R is the load resistance torque.

(b) The equation of electrical equilibrium in the armature circuit is

$$V = e + i_A R + L\frac{di_A}{dt} \qquad \text{... (2.10)}$$

In equation (2.10) e is the back e.m.f. of the motor, R and L are the ohmic resistance and self-inductance of armature circuit respectively and i_A is the current in the armature circuit.

Since the excitation flux is assumed to be constant,

$$e = C_e \Phi\, n \qquad \text{... (2.11)}$$

where Φ is the flux, and C_e is the constant of proportionality.

The motor torque may be expressed in terms of the current and flux as

$$T_M = C_M \Phi i_A \qquad \ldots (2.12)$$

where C_M is the contant of proportionality.

Assuming $T_R = 0$, then

$$\frac{GD^2}{375} \frac{dn}{dt} = T_M = C_M \Phi i_A \qquad \ldots (2.12a)$$

or

$$i_A = \frac{1}{C_M \Phi} \frac{GD^2}{375} \frac{dn}{dt} \qquad \ldots (2.13)$$

and

$$\frac{di_A}{dt} = \frac{1}{C_M \Phi} \frac{GD^2}{375} \frac{d^2n}{dt^2} \qquad \ldots (2.13a)$$

Substituting equations (2.13) and (2.13a) in (2.10),

$$v = C_e \Phi n + \frac{RGD^2}{C_M \Phi} \frac{1}{375} \frac{dn}{dt} + \frac{L}{C_M \Phi} \frac{GD^2}{375} \frac{d^2n}{dt^2} \qquad \ldots (2.14)$$

Introducing the notation $T_M = \frac{RGD^2}{375 \, C_M \, C_e \, \Phi^2}$,

$T_A = L/R$ and $\frac{1}{C_e \Phi} = K$ and dividing the whole equation by $C_e\,\Phi$, equation (2.14) may be written, in the form

$$T_M\,T_A\,\frac{d^2n}{dt^2} + T_M\,\frac{dn}{dt} + n = K.v \qquad \ldots (2.15)$$

The Laplace transform of equation (2.15) is

$$(T_M\,T_A s^2 + T_M s + 1)\;N = K.V \qquad \ldots (2.15a)$$

Here T_M is usually called the electromechanical time constant and T_A, the armature time constant.

The equation (2.15a) may be represented by an oscillatory link or by two aperiodic links.

The system equations of Figure (4) may be summarized as follows:

$$V_a = K_a\,V_{in.a} \qquad \ldots (2.1)$$

The transfer function of the amplidyne is

$$\frac{V}{V_a} = \frac{K_1K_2}{(1+T_1s)(1+T_2s)} \qquad \ldots (2.16)$$

The transfer function of the motor is

$$\frac{N}{V} = \frac{K}{(T_MT_As^2 + T_Ms+1)} = \frac{K_3K_4}{(1+T_3s)(1+T_4s)} \qquad \ldots (2.17)$$

where $T_3 = -1/\alpha_1$ and $T_4 = -1/\alpha_2$ and α_1 and α_2 are the roots of the equation $T_M T_A s^2 + T_M s + 1 = 0$

The voltage developed at the output terminals of the tacho-generator is

$$V_T = K_5 N \qquad \ldots (2.18)$$

where K_5 is a constant.

Using equations (2.1),(2.16) to (2.18), the structural diagram may be drawn and the closed-loop transfer function may be determined.

The structural diagram is shown in Figure (5)

The transfer function of the closed-loop system is written in the form:

$$\frac{V_T}{V_R} = K(s) = \frac{K_A K_1 K_2 K_3 K_4 K_5}{(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)+K_A K_1 K_2 K_3 K_4 K_5} \qquad \ldots (2.19)$$

The characteristic equation of the single-loop is

$$(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s) + K_{ov} = 0 \qquad \ldots (2.20)$$

where $K_{ov} = K_A K_1 K_2 K_3 K_4 K_5$.

The following numerical values are chosen for synthesis purpose.

$$T_1 = 0.1 \text{ sec}, \; T_2 = 0.1 \text{ sec}, \; T_M = 0.5 \text{ sec},$$

$$T_{arm} = 0.01 \text{ sec}, \; K_1K_2 = 10, \; K_3K_4 = K = 1, \; K_A = 20$$

T_3 and T_4 may be found by determining the roots of the quadratic equation

$$T_M \, T_{arm} \, s^2 + T_M s + 1 = 0$$

$$0.5 \times 0.01 \, s^2 + 0.5s + 1 = 0$$

The roots are $\alpha_1 = -2$ and $\alpha_2 = -98$

$$T_3 = -1/\alpha_1 = 1/2 = 0.5 \text{ sec}$$

and $$T_4 = -1/\alpha_2 = 1/98 = 0.01 \text{ sec}$$

It may be assumed that the static-error of the system should not exceed 0.0012, duration of the regulating process must not exceed 0.3 sec and the overshoot has to be limited to 18 percent. Then the conditions governing the steady-state offset define the required overall gain of the order of

$$K_{ov} = \frac{1}{0.0012} = 800$$

Substituting the numerical values and K_{ov} in equation (2.20)

$$(1+0.1s)(1+0.1s)(1+0.5s)(1+0.01s)+800 = 0 \qquad \ldots (2.21)$$

$$0.00005s^4 + 0.0061s^3 + 0.117s^2 + 0.71s + 801 = 0$$

The Routh-Hurwitz polynomial may be used to determine the stability of the system. The Routh-Hurwitz polynomial is as follows

s^4	0.00005	0.117	801
s^3	0.0061	0.71	
s^2	0.1112	801	
s^1	-54.60		
s^0	801		

... (2.22)

Since there are two sign changes in the first column it may be deduced that there are two roots in the right half plane and hence the system is unstable.

The unstable system can be stabilised either by varying the parameter values or by providing a stabilising link, (Appendix A). The stabilising link is connected in the system as shown in Figure (4). The transfer function of the stabilising link is obtained as $\frac{Ts}{1+Ts}$ and is represented in the structural diagram as shown in Figure(6).

The transfer function of the closed-loop system with the inclusion of the stabilising device is

$$\frac{V_t}{V_R} = \frac{K_1K_2K_3K_4K_5K_A(1+\tau s)}{\prod_{i=1}^{4}(1+T_i s)(1+\tau s) + K_1K_2K_A\tau s(1+T_3 s)(1+T_4 s) + K_1K_2K_3K_4K_5K_A(1+\tau s)} \quad \ldots (2.23)$$

The characteristic equation is

$$\prod_{i=1}^{4}(1+T_i s)(1+\tau s) + K_1K_2K_A\tau s(1+T_3 s)(1+T_4 s) + K_{ov}(1+\tau s) = 0 \quad \ldots (2.24)$$

The stability of the system may be determined using Meerov's criteria.

Equation (2.24) is divided throughout by $K_1K_2K_A$ and denoting $m = \frac{1}{K_1K_2K_A}$, equation (2.24) becomes

$$m \prod_{i=1}^{4}(1+T_i s)(1+\tau s) + \tau s(1+T_3 s)(1+T_4 s) + K_3K_4K_5(1+\tau s) = 0 \quad \ldots (2.25)$$

The above equation is written in the form

$$m\, F_{N_2}(s) + F_{N_1}(s) = 0 \quad \ldots (2.26)$$

If $K_1K_2K_A$ tends to infinity, m tends to zero and the equation (2.26) degenerates to

$$\tau s\,(1+T_3 s)(1+T_4 s) + K_3K_4K_5(1+\tau s) = 0 \quad \ldots (2.27)$$

The above equation should satisfy Routh-Hurwitz conditions.

The time constant τ of the stabilising device is unknown and it can be so chosen that it will satisfy the conditions of stability at any value of gain $K_1K_2K_A$. This will be true only if[3]

$$\tau > \frac{T_3T_4}{T_3+T_4} \qquad \ldots \quad (2.28)$$

i.e., $$\tau > \frac{0.5 \times 0.01}{0.5 + 0.01} = \frac{0.005}{0.51} = 0.01 \qquad \ldots \quad (2.28a)$$

τ can be chosen to be 0.1 sec.

Substituting the numerical values in equation (2.27), the equation becomes

$$0.1s\ (1+0.5s)(1+0.01s)+4(1+0.1s) = 0$$

$$0.0005\ s^3+0.051s^2 + 0.5s + 4 = 0$$

s^3	0.0005	0.5
s^2	0.051	4
s^1	0.46	
s^0	4	

The degenerated equation (2.27) indicates the system as stable.

The difference in the degrees of the two polynomial $F_{N_1}(s)$ and $F_{N_2}(s)$ should not be greater than 2 (as discussed in chapter I).

In this case $N_2 - N_1 = 2$. Then the condition for a stable system is $\frac{B_1}{B_0} - \frac{A_1}{A_0} > 0$. In this example,

$$B_0 = T_1 T_2 T_3 T_4 \tau$$

$$B_1 = T_1 T_2 T_3 T_4 + T_1 T_2 T_4 \tau + T_1 T_3 T_4 \tau + T_2 T_3 T_4 \tau + T_1 T_2 T_3 \tau$$

$$A_o = T_3 T_4 \tau$$

$$A_1 = \tau(T_3 + T_4) = \tau\, T_3 + \tau T_4$$

$$\frac{B_1}{B_0} - \frac{A_1}{A_0} = \frac{1}{T_1} + \frac{1}{T_2} + \frac{1}{\tau}$$

$$= \frac{1}{0.1} + \frac{1}{0.1} + \frac{1}{0.01} > 0$$

With the introduction of stabilising link connected as shown in Figure (4) an unstable system tends to become stable.

The optimum value of the gain may be obtained by plotting D-partition curve in terms of K_{ov}. From the characteristic equation (2.24).

$$K_{ov} = -\frac{\prod_{i=1}^{4}(1+T_i s)(1+\tau s) + K_1 K_2 K_A \tau s\,(1+T_3 s)(1+T_4 s)}{(1+\tau s)} \quad \ldots (2.29)$$

$$K_{ov} = - \frac{(1+0.1jw)(1+0.1jw)(1+0.5jw)(1+0.01jw)(1+0.1jw) + 200 \times 0.1\ jw(1+0.5jw)(1+0.01jw)}{(1+0.1\ jw)} \qquad \ldots (2.30)$$

$$= - \frac{(1-10.388w^2 + 0.0065\ w^4)+j(20.81w - 0.1777w^3 + 0.000005w^5)}{(1 + 0.1\ jw)} \qquad \ldots (2.30a)$$

The D-partition curve is plotted by varying w from 0 to $+\infty$ and the curve is shown in Figure (7). From the D-partition curve it can be deduced that the system is stable in the region 'ab'.

The largest value of the gain with which the system will be stable is $K = 6350$. This value of gain is very large for the system to satisfy the required specifications. In order to obtain the optimum value of gain K, the real response frequency characteristic[3] may be drawn from the D-partition curve so as to obtain the lowest peak along the real response frequency characteristic. (Appendix B).

An initial value of gain is assumed to be nearer K i.e. 800, the real response frequency characteristics are drawn in Figure (8a) and Figure (8b) for $K = 1000$ and $K = 1500$. An observation of the two figures indicates

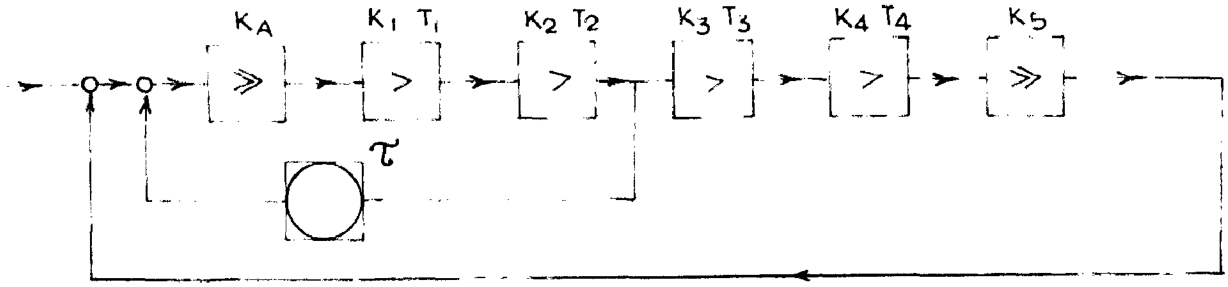


FIGURE - 6.

STRUCTURAL DIAGRAM WITH THE INCLUSION OF STABILISING DEVICE

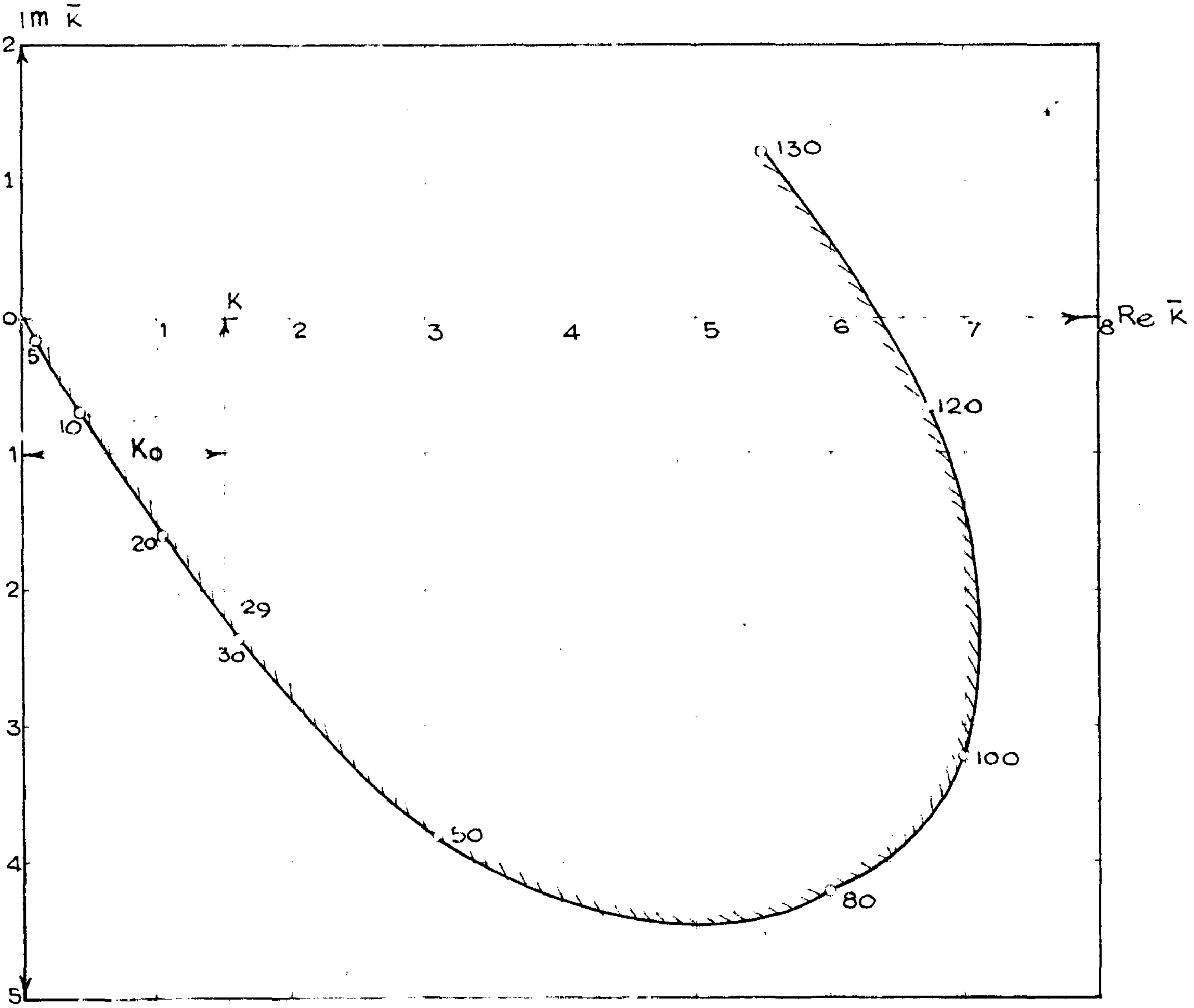


FIGURE 7.

D-PARTITION CURVE

that the gain $K = 1500$ is nearer to the optimum value since the peak occurs at a value of $R(w) = 0.923$.

In order to ensure that the overshoot is limited to 18 percent, a circle is drawn with a radius $\frac{K_o+1}{2}$ with its centre at $\frac{K_o-1}{2}$ (where K_o is the optimum value of the gain), in Figure (7).

The overshoot is limited to 18 percent provided the system gain does not exceed the length of the diameter K_o+1 of the circle.

In Figure (7) the circle is drawn with a radius of $\frac{1500 + 1}{2} = 750.5$ and with its centre at $\frac{1500-1}{2} = 749.5$. The circle passes through the point K which denotes the optimum value and hence the overshoot does not exceed 18 percent.

The critical frequency of the system may be obtained by dropping a perpendicular from the point K onto the D-partition curve. If w_c is the frequency at which the intersection occurs, then the duration of transient process is given by[3]

$$\frac{\pi}{w_c} < t < \frac{4\pi}{w_c} \qquad \ldots \quad (2.29)$$

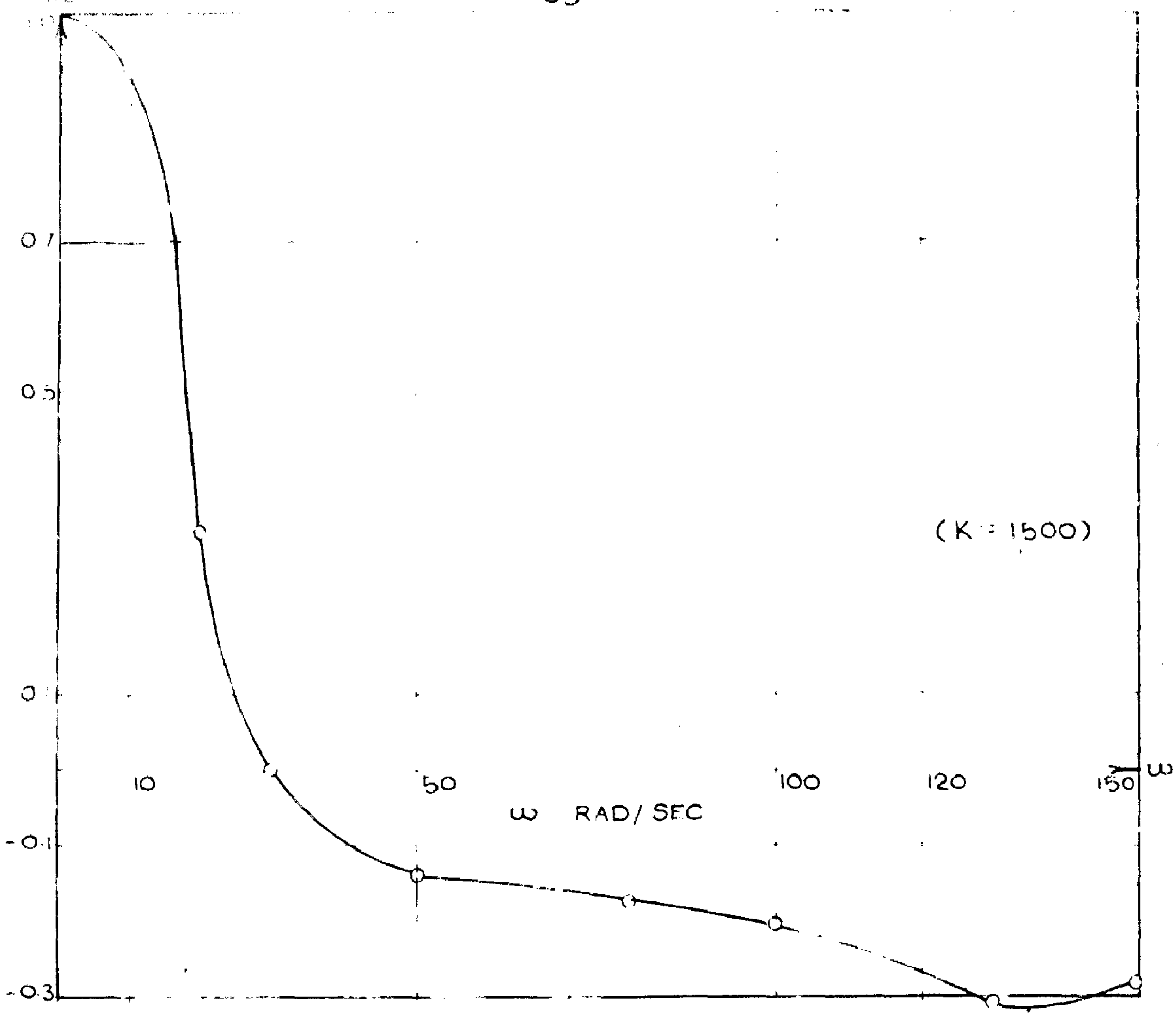
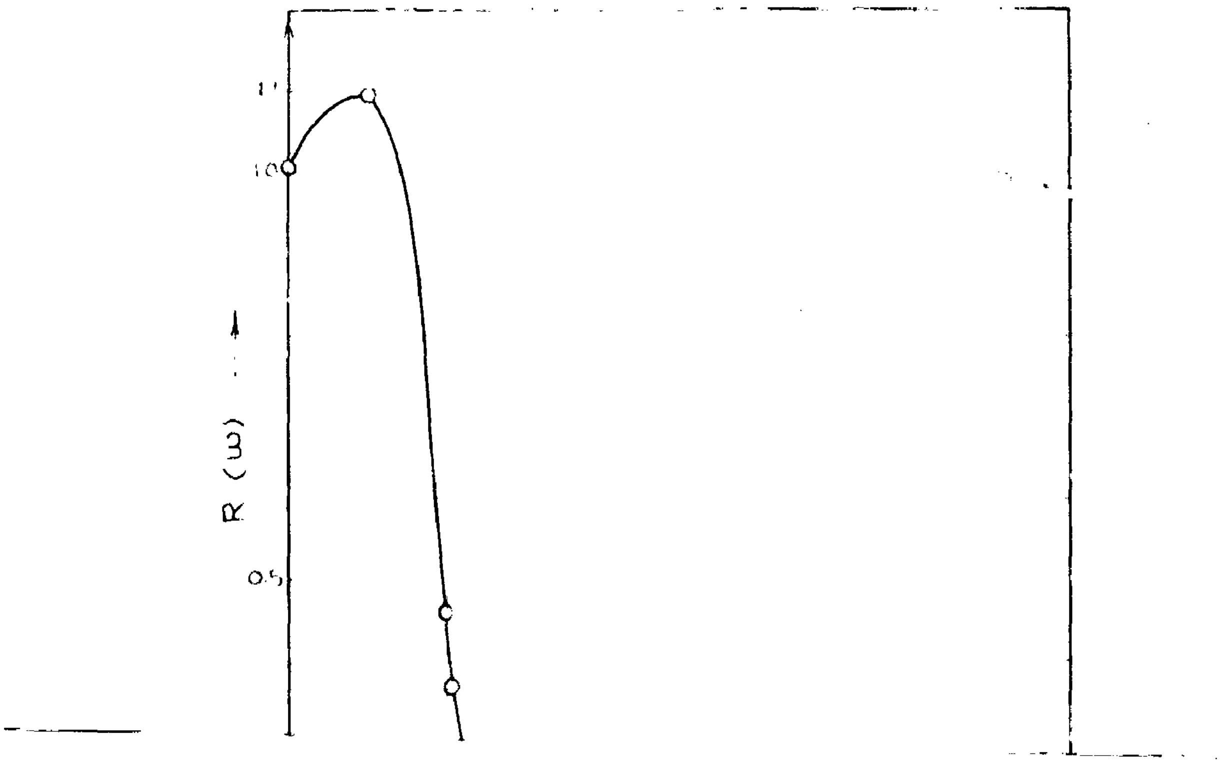


FIGURE 8(a)
REAL RESPONSE FREQUENCY CHARACTERISTIC FOR K = 1500



In Figure (7) w_c is obtained as 29 rad/sec.

Hence the duration of transient process is

$$\frac{\pi}{29} < t < \frac{4\pi}{29}$$

$$= 0.108 < t < 0.435$$

The initial assumed value of duration of transient process is whitin the limits of the equation (2.29). One of the requisites of a stable system is that the transients should die down in a short interval of time. The transient process may be obtained graphically from real response frequency characteristic using Floyd's method of Trapezoidal approximation (Appendix C).

In this example, the real response frequency characteristic shown in Figure (8a) is chosen to determine the transient process. The transient process obtained from the real response characteristic is shown in Fig (9).

A visual examination of the curve in Figure (9) indicates that the transients die down rapidly thereby ensuring a stable system.

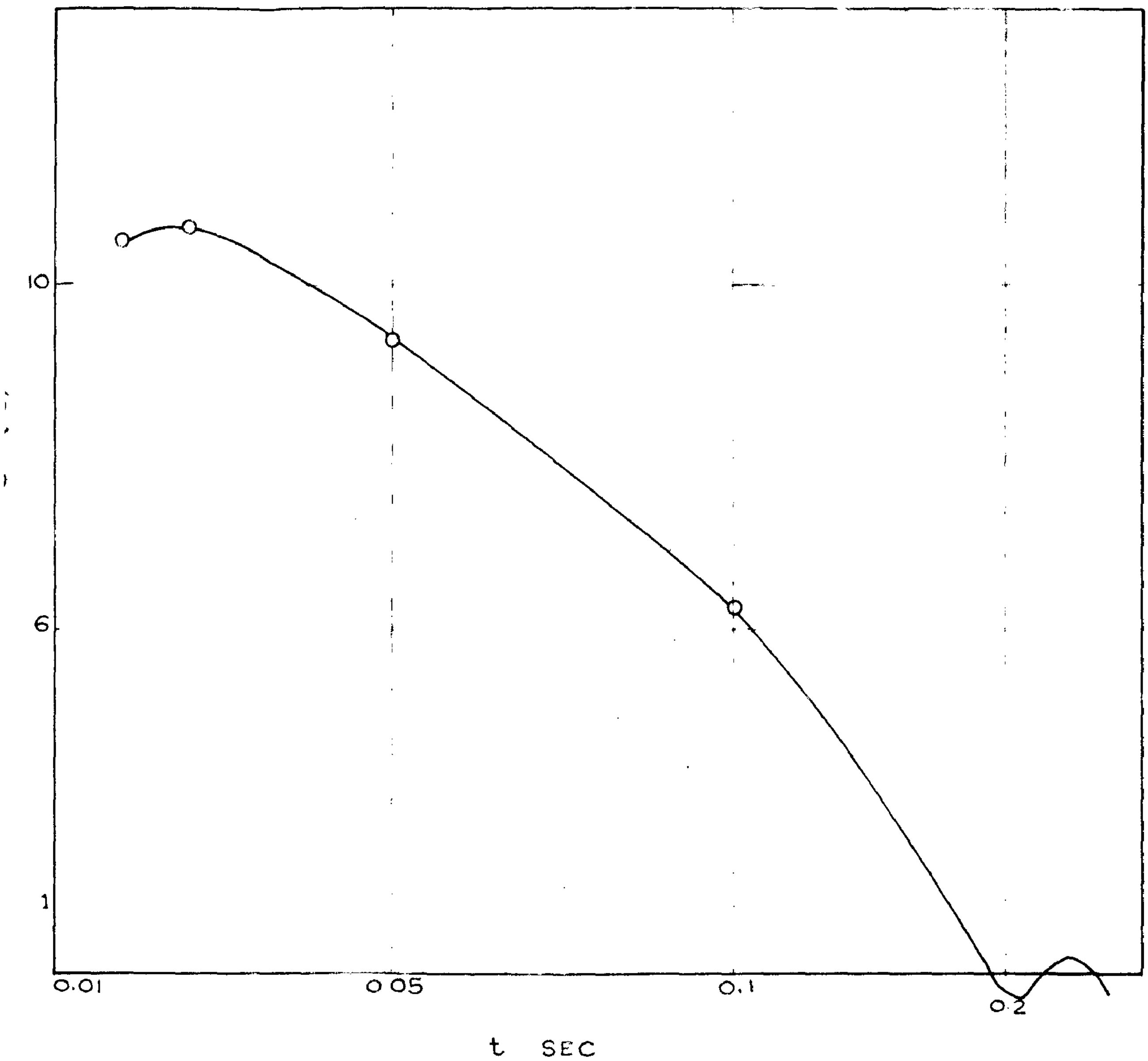


FIGURE - 9

TRANSIENT RESPONSE OF THE SYSTEM FROM REAL RESPONSE FREQUENCY CHARACTERISTIC

CHAPTER III

STRUCTURAL SYNTHESIS OF TIME LAG CONTROL SYSTEMS

3.1 INTRODUCTION

Control systems like hydraulic, pneumatic and mechanical process may be encountered with pure time lag, so that the output will not begin to respond to a transient input, until after a given time. Because of this time lag effect the transfer functions of these systems are no longer quotients of polynomial.

The time lag systems possess very low value of critical gain. The possibility of increasing the gain and constructing such systems are of very important practical significance. In general the system may contain one or many time lag elements in it.

3.2 TIME LAG SYSTEM

The synthesis of time lag systems plays a vital role in process control. An illustrative example is considered herein[3]. The automatic regulation of the strip thickness in the cold rolling mill is considered. The physical system is shown in Figure (10).

The distinguishing property of the system is the fact that the thickness of the strip is measured not at the point of rolls but at some distance 'ℓ' away from them. Hence the signal of thickness variation is received by the system after some time lag determined at the rolling speed v by

$$\tau = \ell / v \qquad \ldots (3.1)$$

and τ is known as the time lag or time delay in secs.

3.3 WORKING PRINCIPLE OF THE TIME LAG SYSTEM

In figure(10) the measuring element is an inductive thickness gauge. When the strip thickness deviates from the prescribed value a voltage appears at the output of the thickness meter which is fed to the input of the amplifier A. The output of the amplifier energises the control winding of the amplidyne generator. The output of the amplidyne is supplied to the armature of a d.c. motor which drives the roll feed screws. The amplified output voltage of the thickness gauge fed to the armature of the driving motor causes it to rotate so varying approximately the clearance between the rolls.

3.4 STRUCTURAL DIAGRAM

The structural diagram may be drawnwith the following assumptions:

(a) It may be assumed that the thickness indicator represents an aperiodic link. The deviation of thickness from the predetermined value is denoted by Φ and if the voltage at the output of the thickness indicator is V_t, then,

$$(1+T_1 \frac{d}{dt}) V_t = K_1 \Phi \qquad .. (3.2)$$

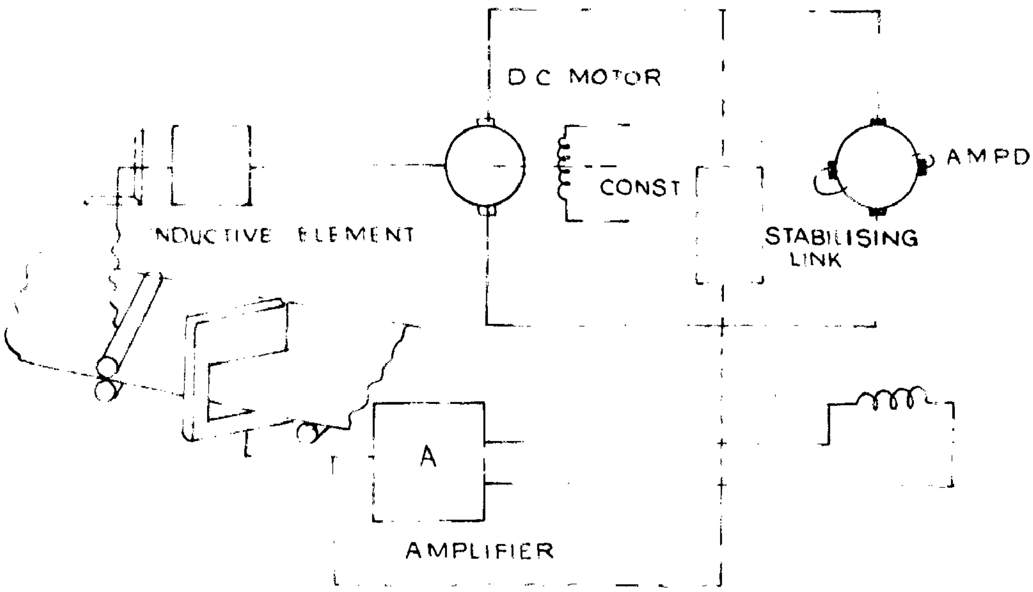


FIGURE- 10
PHYSICAL REPRESENTATION OF TIME LAG SYSTEM

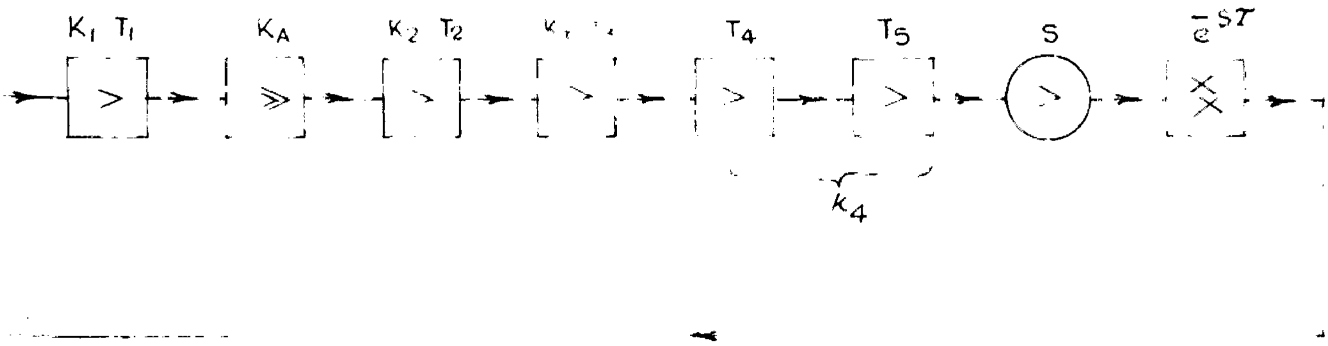


FIGURE 11
STRUCTURAL DIAGRAM OF THE ABOVE SYSTEM

where T_1 and K_1 are the time constant and gain of the indicator respectively.

(b) The equation of the amplifier may be written as

$$V_A = K_A V_T \qquad .. (3.3)$$

(c) The equation of the amplidyne is

$$(1+T_2\frac{d}{dt})(1+T_3\frac{d}{dt})V_{RA} = K_2K_3V_A \qquad .. (3.4)$$

(d) The equation of the d.c. motor is

$$(T_4T_5\frac{d^2}{dt^2} + T_4\frac{d}{dt} + 1)n = K_4V_A \qquad .. (3.5)$$

Since $n = \frac{d}{dt}\Phi_1$, .. (3.6)

where Φ_1 is the relative variation of the motor shaft angle of rotation. Equation (3.5) can be modified as

$$(T_4T_5\frac{d^2}{dt^2} + T_4\frac{d}{dt} + 1)\frac{d}{dt}\Phi_1 = K_4V_A \qquad .. (3.6a)$$

(e) The time lag equation of the link with delay is of the form

$$\Phi = \Phi_1(t-\tau) \qquad .. (3.7)$$

Substituting $s = d/dt$ and assuming initial conditions to be zero, the Laplace transform of the above equations are

$$\frac{V_T(s)}{\Phi} = \frac{K_1}{(1+T_1 s)} ,$$

$$\frac{V_A(s)}{V_T(s)} = K_A ,$$

$$\frac{V_{RA}(s)}{V_A(s)} = \frac{K_2 K_3}{(1+T_2 s)(1+T_3 s)}$$

$$\frac{\Phi_1(s)}{V_{RA}(s)} = \frac{K_4}{s(T_4 T_5 s^2 + T_4 s + 1)}$$

and $\Phi(s) = \Phi_1(s)\, e^{-s\tau}$ .. (3.8)

Using equation(3.8) the structural diagram is drawn as shown in Figure(11).

The characteristic equation of the system is

$$s(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s+T_4 T_5 s^2)+K_1 K_2 K_3 K_4 K_A e^{-s\tau} = 0 \quad .. (3.9)$$

The operator equation of the system in the open-loop condition is

$$W(s) = -\frac{K_{ov}\, e^{-s\tau}}{s(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s+T_4 T_5 s^2)} \quad .. (3.10)$$

where

$$K_{ov} = K_1 K_2 K_3 K_4 K_A$$

The numerical values are chosen to be as follows:

$$T_1 = 0.01 \text{ sec}, \quad T_2 = 0.05 \text{ sec}, \quad T_3 = 0.1 \text{ sec},$$

$$T_4 = 0.2 \text{ sec}, \quad T_5 = 0.05 \text{ sec},$$

$$K_1K_4 = 0.1 \quad , \quad K_2K_3K_A = 100,$$

$$\tau = 0.1 \text{ sec}$$

Assuming the limiting value of τ to be zero and substituting the numerical values of time constants and gains in equation (3.10), W(jw) (Amplitude-Phase characteristic for the limiting value of τ) is obtained as

$$W_{lim}(jw) = -\frac{10}{jw(1+0.01jw)(1+0.05jw)(1+0.1jw)(1+0.2jw+0.2x.05(jw)^2)}$$

The above equation on simplification becomes

$$= \frac{10}{(-0.36w^2+0.00295w^4-0.0000005w^6)+j(w-0.0485w^3+0.000075w^5)} \quad .. (3.11)$$

The Amplitude-Phase characteristic is obtained by substituting different values of w from 0 to + ∞
The APC is shown in Figure(12). The intersection of unit circle with the Attenuation-Phase characteristic occurs at w_o = 6 rad./sec. The maximum value of τ is given

by the relation,

$\tau_o = \theta(w_o)/w_o$ and for a stable system

the condition

$\tau < \tau_o$ should be satisfied.

In this case τ_o is very small, and almost zero. Hence the system with the given parameters is unstable.

In order to obtain a stable system with an infinitely large gain a stabilising device is introduced in the system and is connected as shown in Figure(13). The chosen stabilising link is a transformer whose transfer function is expressed as $\tau_1 s/1+\tau_1 s$, where τ_1 is the time constant of the device.

The characteristic equation of the modified system may be obtained as

$$(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s+T_4 T_5 s^2)(1+\tau_1 s)se^{sT}$$
$$+K_A K_2 K_3\left[\tau_1 s(1+T_1 s)(1+T_4 s+T_4 T_5 s^2)\right]se^{sT}$$
$$+K_A K_1 K_2 K_3 K_4(1+\tau_1 s) = 0 \quad ..(3.12)$$

Dividing equation(3.12) by $K_A K_2 K_3 e^{sT}$ and substituting $m = \frac{1}{K_A K_2 K_3}$, equation (3.12) may be written in the form

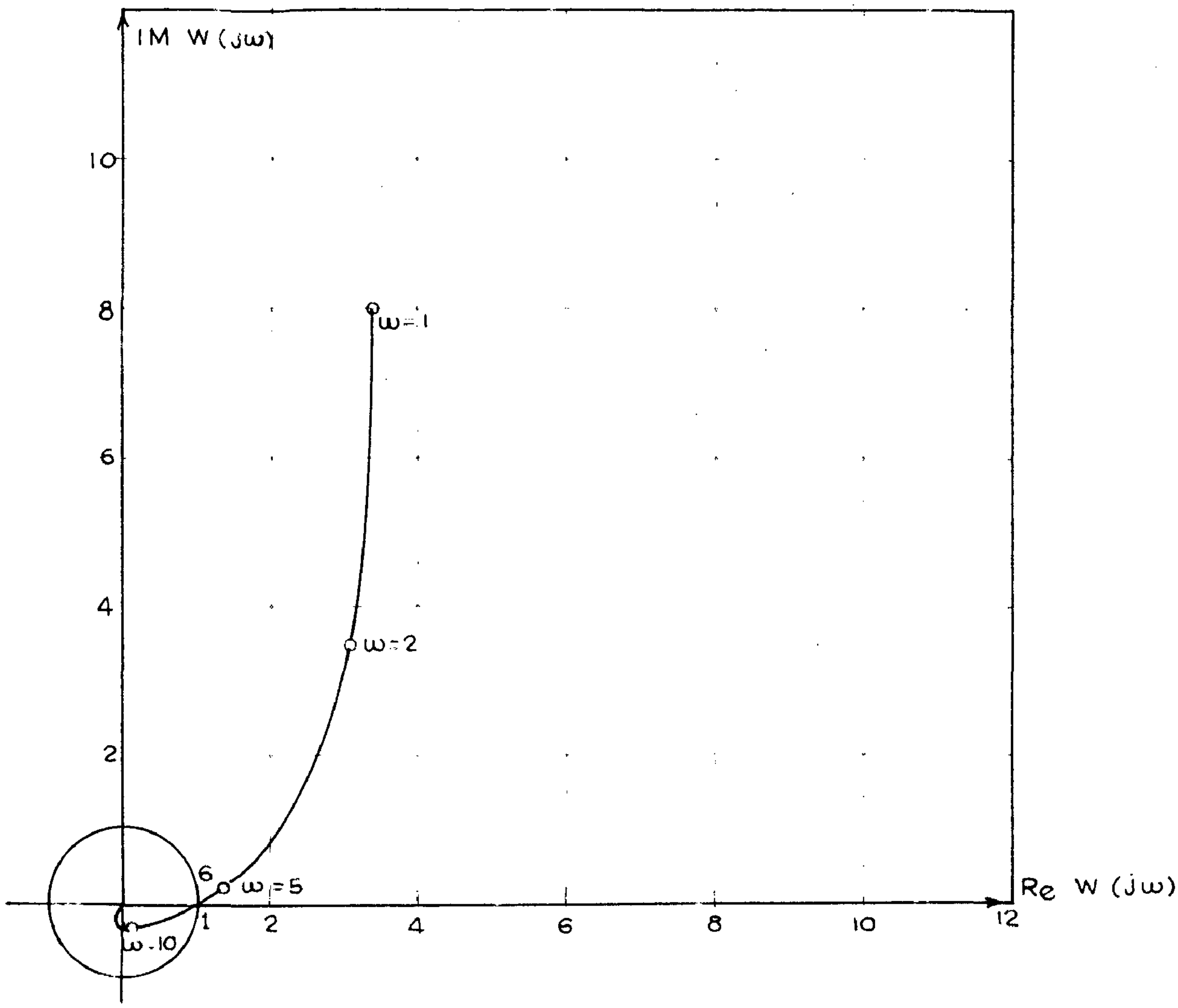


FIGURE - 12
A-p C. OF THE ORIGINAL UNSTABLE SYSTEM

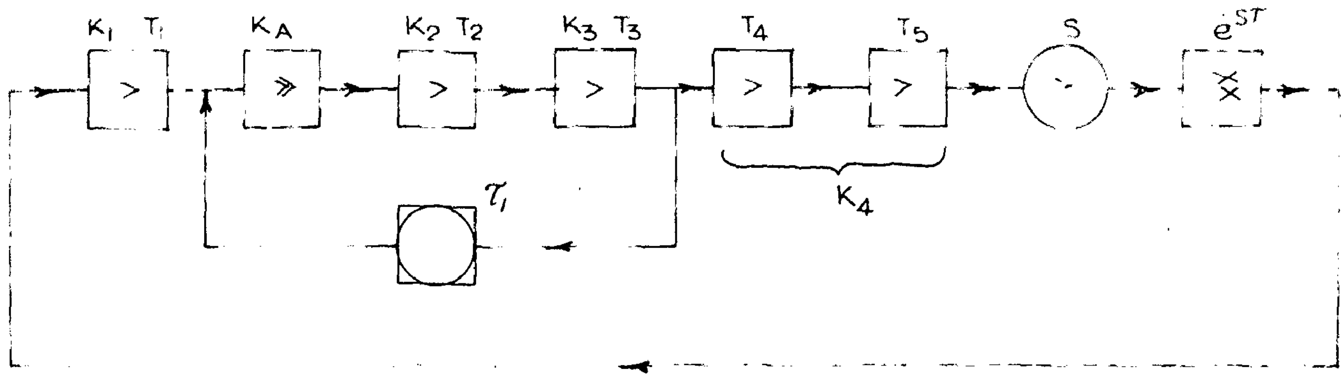


FIGURE- 13
STRUCTURAL DIAGRAM WITH THE INTRODUCTION OF STABILISING LINK

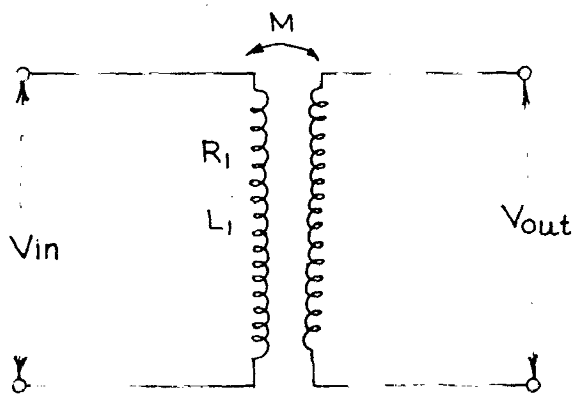


FIGURE- 14
STABILISING TRANSFORMER

$$m\left[(1+T_1s)(1+T_2s)(1+T_3s)(1+T_4s+T_4T_5s^2)(1+\tau_1s)s\right]$$
$$+ \tau_1 s(1+T_1s)(1+T_4s+T_4T_5s^2)s$$
$$+ K_1K_4(1+\tau_1s)\ e^{-s\tau} \qquad = 0 \quad .. (3.13)$$

If the gain $K_AK_2K_3$ is increased to infinity m tends to zero and the equation(3.13) degenerates to

$$\tau_1 s(1+T_1s)(1+T_4s+T_4T_5s^2)s + K_1K_4(1+\tau_1s)e^{-s\tau} = 0 \quad .. (3.14)$$

The limiting value of τ and the stability of the modified system may be determined from the Amplitude-Phase characteristic by chosing a proper value of τ_1. Also τ_1 must be so chosen that the system is stable. In this example, τ_1 is assumed to be 0.5 sec. Then the equation of the Amplitude-Phase Characteristic for the limiting value of τ=0, is obtained from equation (3.14)

$$W_{lim}(jw) = -\frac{K_1K_4(1+\tau_1 jw)}{\tau_1(jw)^2(1+T_1jw)(1+T_4jw+T_4T_5(jw)^2)} \quad .. (3.15)$$

Substituting the numerical values of T_1, T_4, T_5, K_1K_4 and τ_1, the equation (3.15) becomes

$$W_{lim}(jw) = -\frac{0.1(1+0.6jw)}{0.6(-w^2)(1+0.01jw)(1+0.2jw-0.01w^2)}$$
$$= +\frac{0.1+0.06jw)}{(0.6w^2-0.0072w^4)+j(0.126w^3-0.00006w^5)} \quad .. (3.16)$$

The Amplitude-Phase Characteristic is obtained by substituting different values for w from 0 to $+\infty$.

A unit circle is drawn with its centre at origin and the intersection of the circle with the curve gives w_o and angle of intersection is $\theta(w_o)$, as shown in Figure(15).

The maximum value of τ is determined by using the relationship

$$\tau_o = \frac{\theta(w_o)}{w_o} \qquad .. (3.17)$$

$$= \frac{0.166}{0.41} = 0.405 \text{ sec.}$$

Since the initial value of τ is greater than the limiting value τ_o, the system would be perfectly stable.

In order to determine whether the system will remain stable with increase in gain, the D-partition curve for the system is drawn.

D-partition curve may be drawn from equation (3.12). Rearranging equation (3.12),

$$\begin{aligned}(1+T_1s)(1+T_2s)(1+T_3s)(T_4T_5s^2+T_4s+1)(1+\tau_1s)se^{s\tau} \\ +K_AK_2K_3\{\tau_1s(1+T_1s)(T_4T_5s^2+T_4s+1)se^{s\tau} \\ +K_1K_4(1+\tau_1s)\} = 0 \qquad .. (3.18)\end{aligned}$$

The above equation may be simplified and expressed in

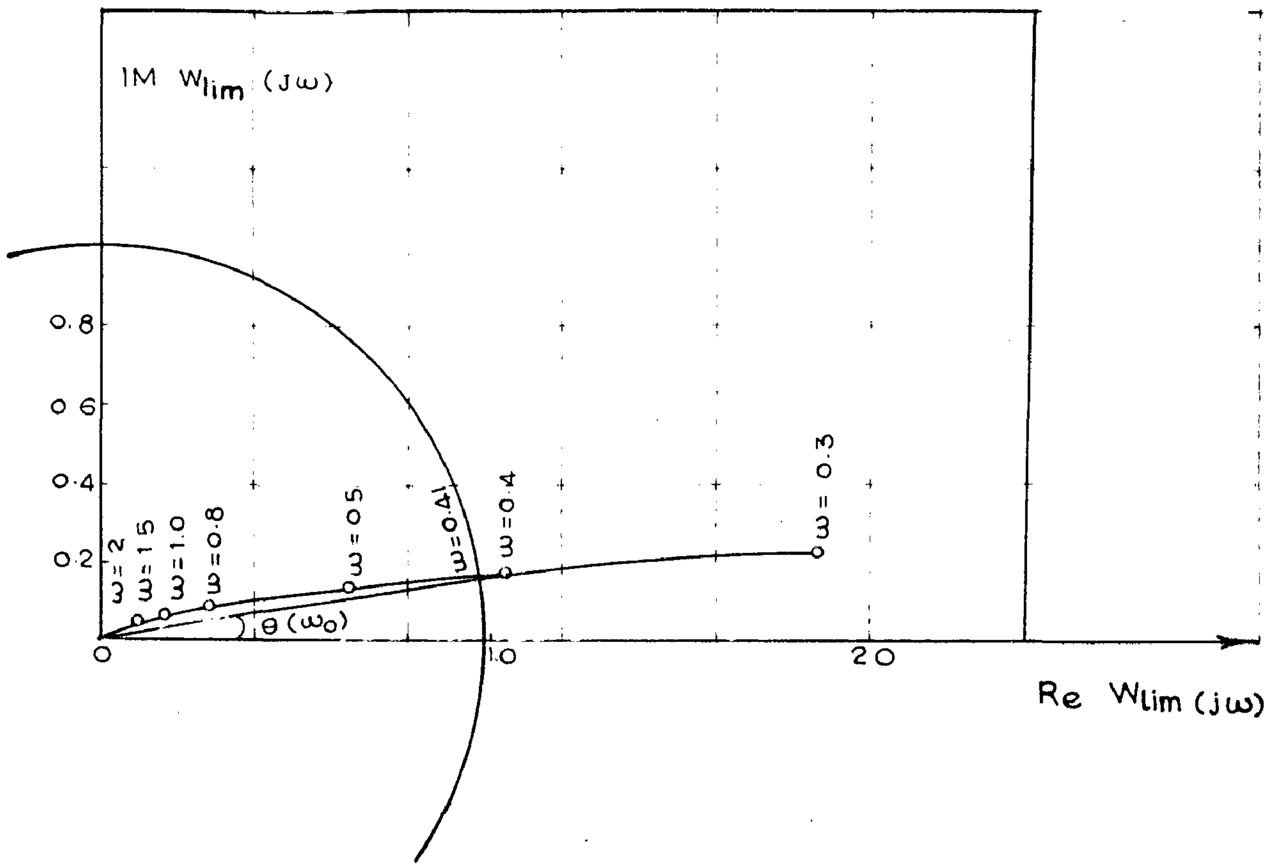


FIGURE - 15
A-P CHARACTERISTIC OF THE SYSTEM WITH THE STABILISING LINK

term of $K_AK_2K_3$.

$$\frac{1}{K_AK_2K_3} = \frac{\left\{(1+T_1jw)(1+T_2jw)(1+T_3jw)(T_4T_5(jw)^2 + T_4jw+1)\times jw(1+\tau_1jw)\right\}}{\tau_1jw(1+T_1jw)\left[T_4T_5(jw)^2+T_4jw+1\right] + K_1K_4(1+\tau_1jw)e^{-jw\tau}} \quad .. (3.19$$

Substituting the numerical values in equation(3.19) and simplifying

$$\frac{1}{K_AK_2K_3} = -\left(\frac{Nr}{Dr}\right) , \quad .. (3.20)$$

where,

$$Nr = (-9.6\times10^{-1}w^2+3.205\times10^{-2}w^4-4.55\times10^{-5}w^6) + j(w-2.645\times10^{-1}w^3+1.845\times10^{-3}w^5 - 3\times10^{-7}w^7)$$

$$Dr = \left[(-6\times10^{-1}w^2+7.2\times10^{-3}w^4)+j(-1.26\times10^{-1}w^3+6\times10^{-5}w^5)\right] + \left[(0.1+j0.06w)e^{-0.1jw}\right]$$

The D-partition curve is obtained by substituting different values for w, from 0 to ∞ . The curve obtained is as shown in Figure(16).

From D-partition curve obtained it can be deduced that the system with introduction of stabilising link is stable for all values of gain, $K_AK_2K_3$.

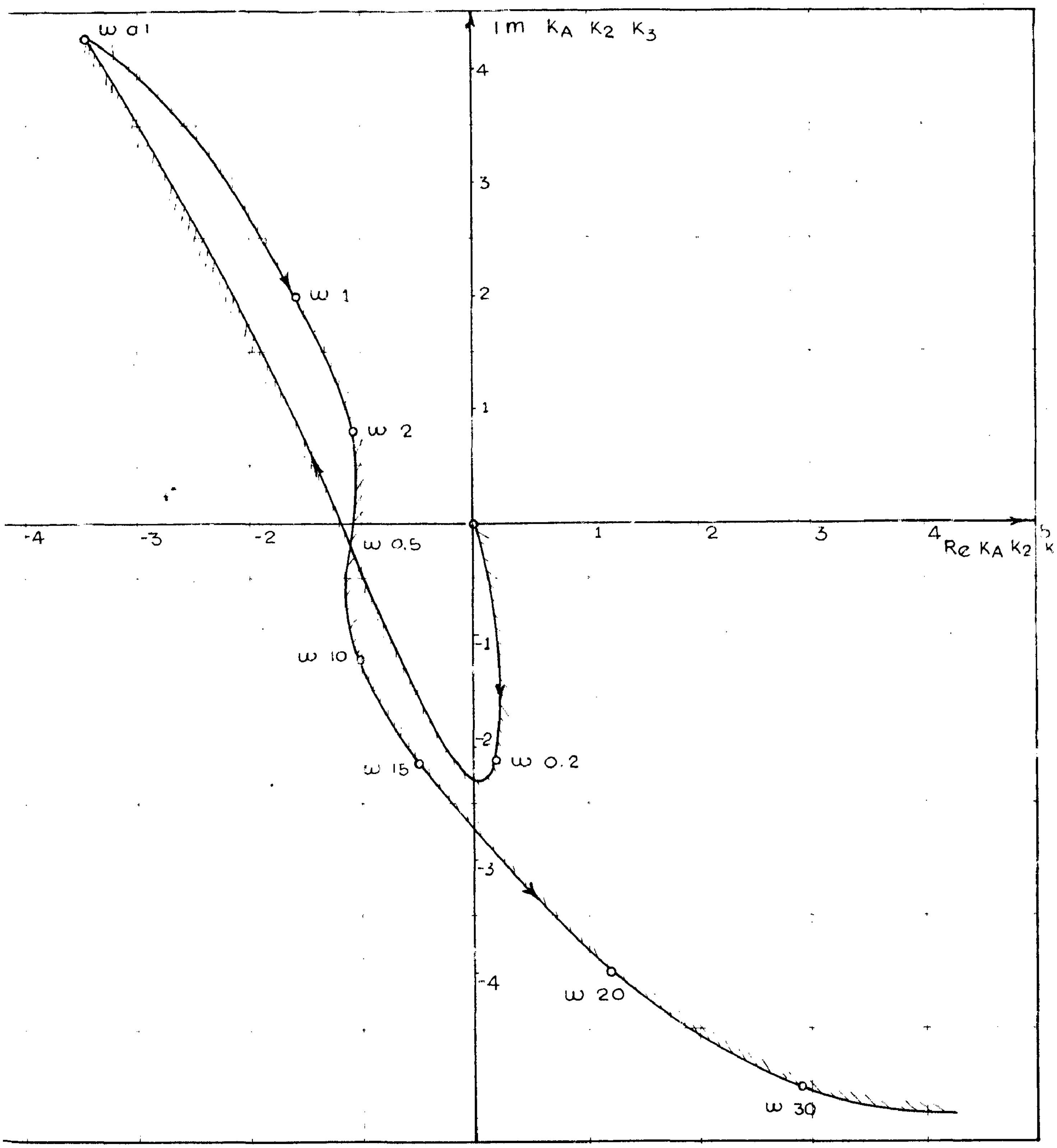


FIGURE- 16

D-PARTITION CURVE IN TERMS $K_A K_2 K_3$

3.5 PHYSICAL REALISATION OF THE STABILISING LINK

The stabilising device used in this example is a transformer. Its transfer function may be derived as follows. In Figure (14), the stabilising transformer under no-load conditions is shown.

If R_1 and L_1 are the resistance in ohms and inductance in henries respectively, then,

$$V_{in}(s) = (R_1+L_1s)I_1(s) \qquad \ldots (3.21)$$

The output of the transformer is

$$V_{out}(s) = MsI_1(s) \qquad \ldots (3.22)$$

where M is the mutual inductance between the two windings.

The transfer function of the stabilising transformer is

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{MsI_1}{(1_1+L_1s)I_1} \qquad \ldots (3.23)$$

Assuming $M = L_1$ and τ_1 represents the ratio L_1/R_1 then 1

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\tau_1 s}{(1+\tau_1 s}$$

IN the illustrative example, τ is 0.5 sec. A suitable combination of R_1 and L_1, within the practical limits is justifiable. For example, R_1 may be chosen to be 1 ohm, then L_1 is 0.5 henries.

CHAPTER IV

STRUCTURAL SYNTHESIS OF AUTOMATIC VOLTAGE REGULATION OF A SYNCHRONOUS GENERATOR

4.1 INTRODUCTION

The method of structural synthesis may be applied to the automatic voltage regulation of a synchronous generator to obtain a stable and high gain system. An unstable system can be stabilised by providing stabilising links. The proper choice of stabilising links and the connection of them play a vital role in the synthesis of such systems. If the original system is large (i.e. the degree of the polynomial of the characteristic equation is high) then the provision of the stabilising link complicates the system.

A new method is proposed below to stabilise a linear unstable system by considering the non-linear effect of the elements in the system without the inclusion of any stabilising device.

The automatic voltage regulation of a synchronous generator is considered as an illustrative example. The physical system is shown in Figure (17)[5].

4.2 WORKING PRINCIPLE OF AUTOMATIC VOLTAGE REGULATION

There are many types of voltage regulators that are used in practice. Voltage regulators are used to control the voltage of a synchronous generator at a desired value.

The physical system has in its forward path an amplidyne, a main exciter and the synchronous generator.

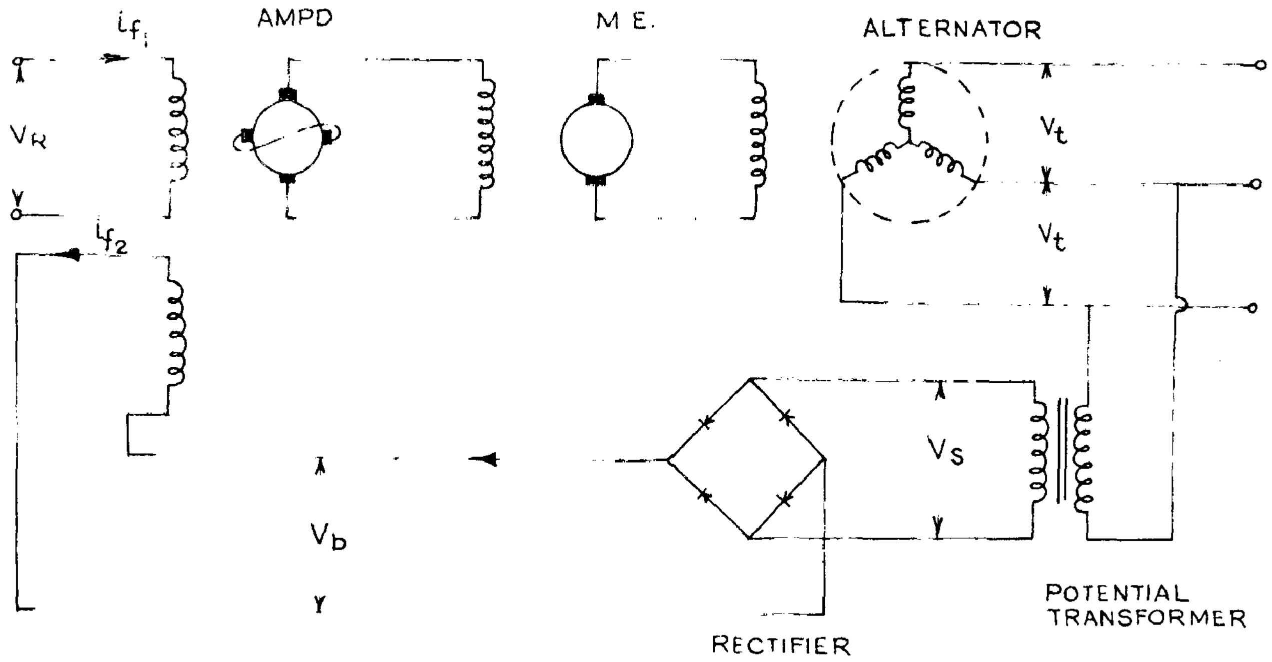


FIGURE - 17
PHYSICAL SYSTEM OF AVR.

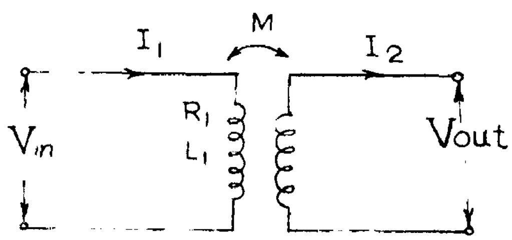


FIGURE - 18.
POTENTIAL TRANSFORMER

In its feedback path it has a potential transformer and a rectifier.

The working principle of the system is as follows. The main exciter is a shunt wound machine with amplidyne as its exciter. The amplidyne opts it self as an amplifier, since its large output can be controlled by a small input. The generator field, in turn, is excited by the main exciter. The potential transformer is connected to the output of the synchronous generator and the out put of the potential transformer is rectified and fed directly to the second exciting winding of the amplidyne. A current transformer and a magnetic amplifier are usually connected in the feed back path. The current transformer is used to take care of change in load disturbances and the magnetic amplifier is used to amplify the out put signal from the alternator. In this analysis an unloaded system is considered and the provision of current transformer and the magnetic amplifier do not serve any purpose. Hence they are omitted in the original physical system.

If the voltage output of the synchronous generator decreases the voltage at the secondary of the potential transformer also decreases and therefore the voltage applied to the exciting winding of the amplidyne is reduced. This brings in a decrease of m.m.f. produced by the feedback winding thereby causing a net increase in the resultant m.m.f. because the main m.m.f. being a greater one is opposed by the feedback winding m.m.f. The net increase in the field flux will cause an increase in the output voltage of the alternator trying to keep the voltage at a constant value.

4.3 STRUCTURAL DIAGRAM OF THE SYSTEM

The structural diagram may be obtained by deriving the linear differential equations of the system. With the help of Figure (17), the equations are derived as follows.

1. V_R is the applied reference voltage of the first exciting winding of the amplidyne and V_b is the feedback voltage applied to the second winding of the amplidyne. It may be assumed that the two windings are identical. The directions of the currents flowing in the two windings may be assumed to be as shown in Figure (17). Then,

$$V_R = (R_1+L_1s)\, I_{f_1} - Ms\, I_{f_2} \qquad \ldots (4.1)$$

$$V_b = (R_1+L_1s)\, I_{f_2} - Ms\, I_{f_1} \qquad \ldots (4.2)$$

Multiplying equation (4.1) by Ms and (4.2) by (R_1+L_1s),

$$I_{f_2} = \frac{V_R\ Ms+V_b(R_1+L_1s)}{(R_1+L_1s)^2-M^2s^2} \qquad \ldots (4.3)$$

similarly,

$$I_{f_1} = \frac{V_R(R_1+L_1s)+V_bMs}{(R_1+L_1s)^2-M^2s^2} \qquad \ldots (4.4)$$

The voltage induced in the q-axis of the amplidyne is proportional to the difference of the currents I_{f_1} and I_{f_2}. Assuming $L = M$

$$I_{f_1}-I_{f_2} = \frac{(V_R-V_b)}{R_1(1+2T_1s)} \quad \text{where} \quad T_1 = L_1/R_1$$

$$= \frac{K_1(V_R-V_b)}{(1+2T_1s)} \quad \text{where} \quad K_1 = 1/R_1 \qquad \ldots (4.5)$$

2. The transfer function of the amplidyne may be obtained as follows.

The voltage induced in the q-axis of the amplidyne

$$e_q = K_q\ \Phi_d \qquad \ldots (4.6)$$

In Laplace transform

$$E_q = K_2'(I_{f_1}-I_{f_2})(\text{since } \Phi_d = I_{f_1}-I_{f_2}), \qquad \ldots (4.7)$$

where K_2' is the number of volts induced in the q-axis per unit control field current in amps.

Also $E_q = I_2(R_2+L_2s)$ ... (4.8)

From equations (4.7) and (4.8),

$$\frac{I_2}{I_{f_1}-I_{f_2}} = \frac{K_2'}{R_2(1+T_2s)} \quad \left(\text{where } T_2 = \frac{L_2}{R_2}\right) \qquad \text{... (4.9)}$$

The voltage induced in the d-axis of the amplidyne is $e_d = K_3' i_2$ ... (4.10)

where K_3' is the number of volts induced in the d-axis circuit per ampere in the q-axis.

or $E_d = K_3' I_2$ ... (4.10a)

But $E_d = I_3(R_3+L_3s)$

$$I_3 = \frac{E_d}{(R_3+L_3s)} = \frac{K_3' I_2}{R_3(1+T_3s)} \qquad \text{... (4.11)}$$

$$= \frac{K_2K_3(I_{f_1}-I_{f_2})}{(1+T_2s)(1+T_2s)} \qquad \text{... (4.12)}$$

where ($K_2=K_2'/R_2$ and $K_3 = K_3'/R_3$)

3. The voltage in the main exciter is

$$V_{ME} = K_4' I_3 \qquad \text{... (4.13)}$$

4. The current flowing in the alternator field is

$$I_{fg} = \frac{V_{ME}}{R_4(1+T_4s)} \qquad \ldots (4.14)$$

substituting equation (4.13) in equation (4.14)

$$I_{fg} = \frac{K_4\ I_3}{(1+T_4s)}, \quad \text{where} \quad K_4 = K_4'/R_4 \qquad \ldots (4.15)$$

5. The voltage induced in the synchronous generator is

$$V_t = K_5 I_{fg} \qquad \ldots (4.16)$$

substituting equations, (4.5), (4.12), and (4.15) in equation (4.16), V_t is obtained as

$$V_t = \frac{K_1K_2K_3K_4K_5(V_R-V_b)}{(1+2T_1s)(1+T_2s)(1+T_3s)(1+T_4s)} \quad ..(4.17)$$

6. The transfer function of the potential transformer may be derived by assuming that the transformer is operating at no-load (Figure 18).

From Figure (18)

$$V_{in} = (R_1+L_1s)I_1$$

$$V_{out} = Ms\ I_1$$

Assuming $L_1 = M$

$$\frac{V_{out}}{V_{in}} = \frac{L_1s}{(R_1+L_1s)} = \frac{\tau s}{(1+\tau s)}$$

If V_{out} and V_{in} are denoted by V_t and V_s respectively then the transfer function of the potential transformer is

$$\frac{V_s}{V_t} = \frac{T_5 s}{1+T_5 s} \qquad \ldots (4.18)$$

7. The voltage applied to the rectifier is V_s and its corresponding output is V_b and is given by the relation

$$V_b = K_6 V_s \qquad \ldots (4.19)$$

By substituting equations (4.18) and (4.19) in equation (4.17), the closed-loop transfer function can be obtained as

$$\left[V_t + \frac{K_1 K_2 K_3 K_4 K_5 K_6 T_5 s}{(1+2T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)(1+T_5 s)} V_t\right] = \frac{K_1 K_2 K_3 K_4 K_5}{(1+2T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)} V_R \qquad \ldots (4.20)$$

on simplification equation (4.20) yields

$$\frac{V_t}{V_R} = \frac{K_1 K_2 K_3 K_4 K_5 (1+T_5 s)}{\prod_{i=1}^{5}(1+T_i s)+K_{ov} T_5 s} \qquad \ldots (4.21)$$

Equation (4.21) is the closed-loop transfer function of the system shown in Figure (17).

where $K_{ov} = K_1K_2K_3K_4K_5K_6$

The block diagram and the structural diagram may be represented as shown in Figure (19) and Figure (20) respectively.

In order to determine whether the linear system is stable, the following numerical values (6, 7) substituted in the equation (4.21). The numerical values are

$K_1 = 0.001$, $K_2 = 42.5$, $K_3 = 25$, $K_4 = 10$, $K_5 = 150$
$K_6 = 1.5$, $T_1 = 0.003$, $T_2 = 0.08$, $T_3 = + 0.03$,
$T_4 = 0.05$ and $T_5 = 0.3$.

Routh-Hurwitz criterion is applied primarily to determine the stability of the system. The characteristic equation of the system may be obtained from equation (4.21).

$$\prod_{i=1}^{5} (1+T_i s)+K_{ov}\, T_5 s = 0 \qquad \ldots \quad (4.22)$$

substituting the numerical values in (4.22), the characteristic equation may be written as follows.

$$(1+0.006s)(1+0.08s)(1+0.03s)(1+0.05s)(1+0.3s) + 2400\times 0.3s = 0 \qquad \ldots \quad (4.23)$$

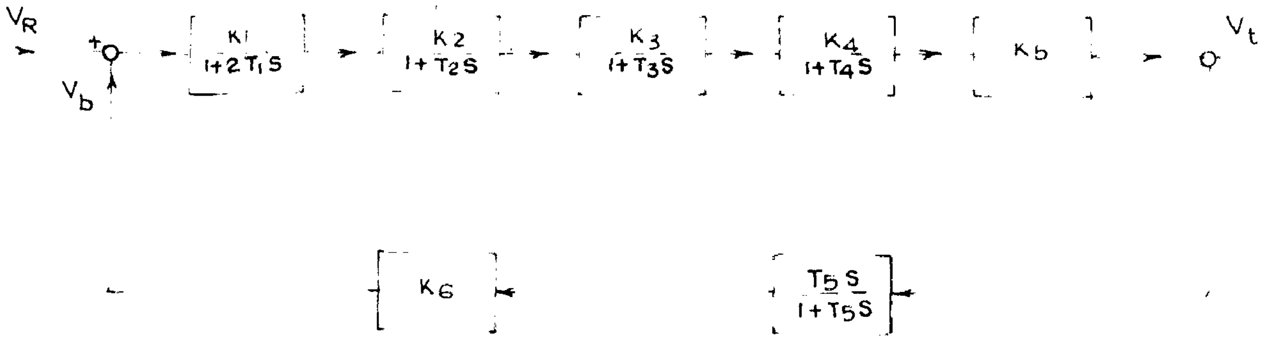


FIGURE - 19
BLOCK DIAGRAM OF THE PHYSICAL SYSTEM

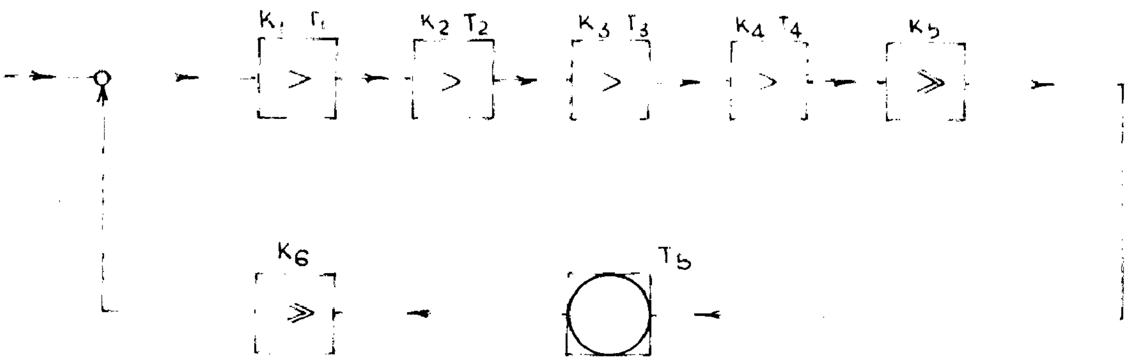


FIGURE - 20
STRUCTURAL DIAGRAM OF THE AUTOMATIC VOLTAGE REGULATING SYSTEM

simplifying,

$$2.17\times10^{-7}s^5+5.1\times10^{-5}s^4+2.86\times10^{-3}s^3+5.87\times10^{-2}s^2+720s+1 = 0 \quad \ldots (4.24)$$

The polynomial has no missing terms and the coefficients are all positive, hence it satisfies the necessary condition of stability. The absolute stability of the system may be found using Routh-Hurwitz criterion. The Routh's tabulation is

s^5	2.16×10^{-7}	2.86×10^{-3}	720
s^4	5.1×10^{-5}	5.87×10^{-2}	1
s^3	2.42×10^{-3}	720	
s^2	-14.7	1	
s^1	720		
s^0	1		

... (4.25)

There are two changes in the sign of the elements in the first column indicating that two roots of equation has positive real parts and hence the system is unstable.

The synthesis of the linear system to obtain a stable system depends upon the proper choice of the stabilising link and its connection. The method is essentially same as that discussed in Chapter I.

As a trial, a stabilising transformer whose transfer function is $\tau s/1+\tau s$ is introduced in the system across the terminals of the amplidyne. The corresponding structural diagram is shown in Figure (21). The transfer function of the closed-loop system is

$$\frac{V_t}{V_R} = \frac{K_1K_2K_3K_4K_5(1+T_5s)(1+\tau s)}{\prod_{i=1}^{5}(1+T_i s)(1+\tau s)+K_1K_2K_3\tau s(1+T_4s)(1+T_5s)+K_{ov}T_5s(1+\tau s)} \quad \ldots (4.26)$$

The equation of the closed-loop system is obtained by equating the denominator of the above expression to zero. That is

$$\prod_{i=1}^{5}(1+T_i s)(1+\tau s)+ K_1K_2K_3\tau s(1+T_4s)(1+T_5s)+K_{ov}T_5s(1+\tau s)=0 \quad \ldots (4.27)$$

substituting $m = 1/K_1K_2K_3$ and rearranging the terms, equation (4.27) can be written in the form

$$m \prod_{i=1}^{5} (1+T_i s)(1+\tau s)+\tau s(1+T_4s)(1+T_5s)+K_4K_5K_6T_5s(1+\tau s) = 0 \quad \ldots (4.28)$$

Equation (4.28) is of the form

$$m\, F_{N_2}(s)+F_{N_1}(s) = 0$$

Hence Meerov's stability criterion can be applied to determine

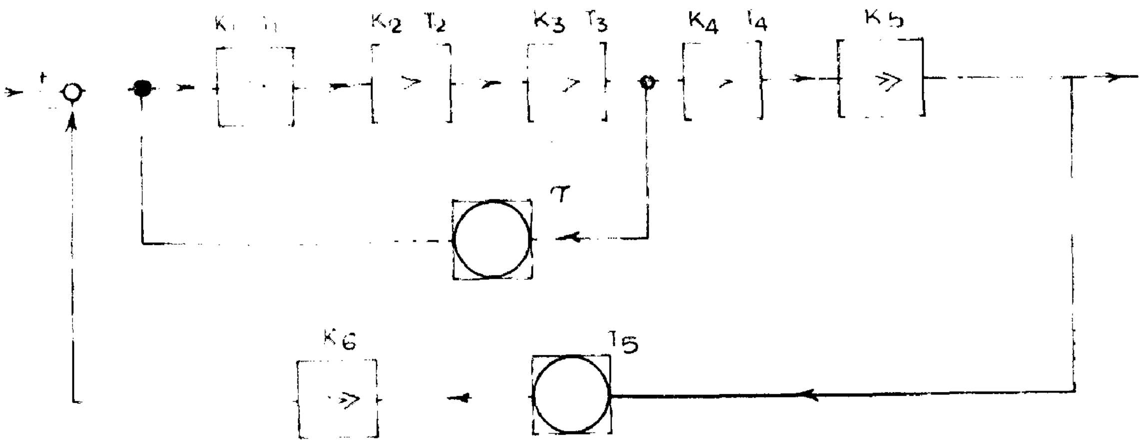


FIGURE - 21
SYSTEM WITH STABILISING LINK OF THE FORM $Ts/1+Ts$

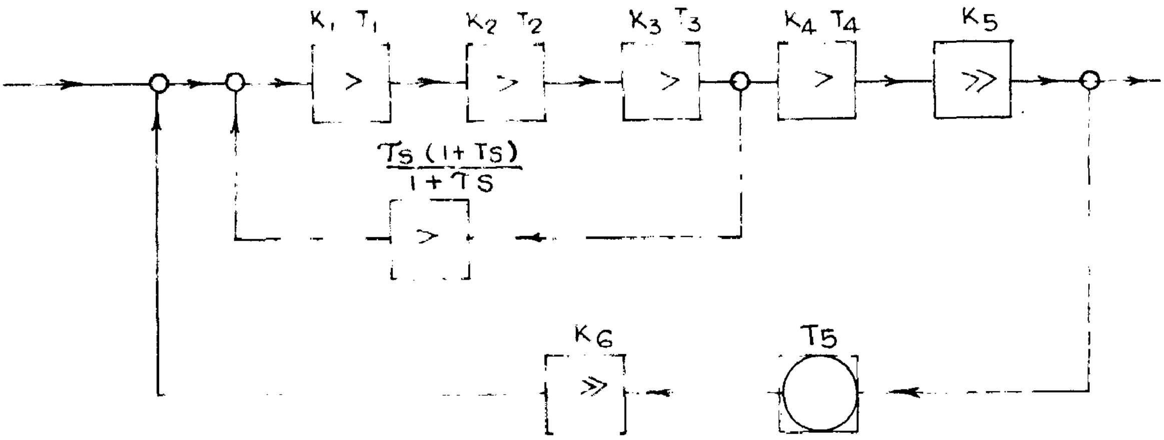


FIGURE - 22.
SYSTEM WITH STABILISING LINK OF THE FORM $\frac{Ts(1+Ts)}{1+Ts}$

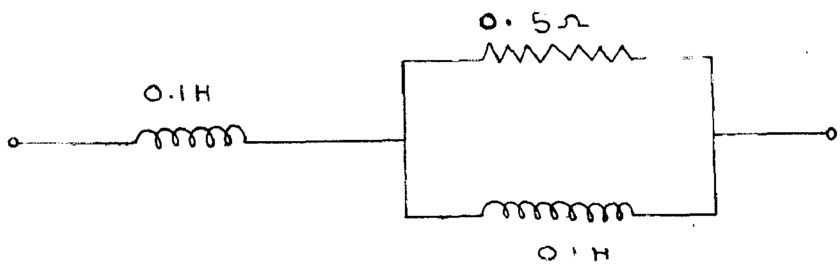


FIGURE - 23
STABILISING NETWORK REPRESENTING $\frac{Ts(1+Ts)}{(1+Ts)}$

whether the closed-loop system is stable with the introduction of the stabilising device.

In equation (4.28) the highest order of polynomial in F_{N_2} is 6 and the highest order of polynomial in F_{N_1} is 3. The difference of N_2 and N_1 is greater than 2 and hence the system is unstable. It can be deduced that system remains unstable even if the stabilising link of the form $\tau s/1+\tau s$ is introduced in the system.

A stabilising device whose transfer function is of the form $\frac{\tau s(1+Ts)}{1+\tau s}$ is connected across the amplidyne terminals. The corresponding structural diagram is shown in Figure (22). The characteristic equation of the closed-loop system is obtained as

$$\prod_{i=1}^{5}(1+T_i s)(1+\tau s)+K_1K_2K_3\tau s(1+Ts)(1+T_4 s)(1+T_5 s)+K_{ov}T_5 s(1+\tau s)=0 \quad \ldots (4.29)$$

Equation (4.29) can be written in the form

$$m \prod_{i=1}^{5}(1+T_i s)(1+\tau s)+\Big[\tau s(1+\tau s)(1+T_4 s)(1+T_5 s) +K_4K_5K_6T_5 s(1+\tau s)\Big] = 0 \quad ..(4.30)$$

where $m = 1/K_1K_2K_3$.

The system tends to be stable since $N_2-N_1 = 2$.

$$Z(s) = \frac{\tau s(1+Ts)}{(1+\tau s)}$$

$$= \frac{0.2s(1+0.1s)}{(1+0.2s)}$$

$$= \frac{0.1s(s+10)}{(s+5)} = \frac{0.1(s^2+10s)}{(s+5)} \qquad \ldots (4.31)$$

The following is derived with the help of second Cauer's method. Dividing the numerator of equation (4.31) by its denominator, Z(s) can be written in the form

$$= 0.1 \left[s + \frac{5s}{s+5}\right].$$

Simplifying the above equation

$$Z(s) = 0.1s + \frac{1}{.2 + \frac{1}{0.1s}}$$

The network realised is shown in Figure (23).

However, many practical systems are encountered with non-linearities. That is, the practical systems contain elements which are not linear in the sense of being describable by linear differential equations. It is proposed here to synthesize a system considering its non-linearities and it is justifiable in practice.

Expanding equation (4.29) and denoting

$$B_0 = T_1T_2T_3T_4T_5\tau$$

$$B_1 = T_1T_2T_3T_5\tau+T_1T_2T_4T_5\tau+T_1T_3T_4T_5\tau+T_2T_3T_4T_5\tau$$
$$+T_1T_2T_3T_4T_5+T_1T_2T_3T_4\tau$$

$$A_0 = \tau T_4T_5T$$

$$A_1 = T\ T_4\tau+T\ T_5\tau+T_4T_5\tau$$

If the system is to be stable then the condition

$$\frac{B_1}{B_0} - \frac{A_1}{A_0} > 0 \text{ must be satisfied.}$$

Choosing $T = 0.1$ sec and $\tau = 0.2$ sec and substituting the numerical values of all the time constants,

$$\frac{B_1}{B_0} - \frac{A_1}{A_0} = \frac{1}{0.003} + \frac{1}{0.08} + \frac{1}{0.03} - \frac{1}{0.1} - \frac{1}{0.2} > 0$$

Hence the system will approach stable operation.

4.4 PHYSICAL REALISATION OF THE STABILISING DEVICE

The stabilising device may be physically realised by using either Foster's method or Cauer's method of synthesis. Assuming that the transfer function of the stabilising device as an impedence function the stabilising device may be realised as follows.

4.5 NON-LINEARITIES ENCOUNTERED IN THE SYSTEM

As mentioned in the previous paragraph, the non-linearities may be expressed by non-linear differential equations (8) and the solution of such equations are tedious and time consuming. An approximate technique has been developed simultaneously by C.Goldfarb (U.S.S.R.)[9] Tustin A.[10](U.K.), and B. Kochenberger[11]. Such a technique is usually termed as Harmonic-balance method or frequency response method or describing function technique, all the names being synonymous.

The describing function method has attained great popularity because of ease in computation and also this method is useful if the order of the system is very large.

4.6 DESCRIBING FUNCTION TECHNIQUE

Describing functions are employed to represent the non-linear elements by their approximate linear amplitude sensitive transfer functions. The method is based on the assumptions that if a sinusoidal signal is applied to a non-linear element the resulting output can be represented by its fundamental Fourier component and the linear elements of the system attenuate all higher harmonics.

The describing function of the non-linear element which represents the ratio of fundamental Fourier component of the output to the sinusoidal input can be determined[(2)], when the describing function of the non-linear element is known.

The illustrative example shown in the block diagram Figure (24a), may be considered to study the stability analysis using describing function method. The Figure (24b) represents the system with its non-linear element replaced by their describing function N_e.

The closed-loop transfer function of the system is given by

$$\frac{C(jw)}{R(jw)} = \frac{G_1(jw)\ G_2(jw)\ N_e}{1+G_1(jw)G_2(jw)N_e} \qquad \ldots (4.31)$$

for the existance of self oscillation is R(jw) = 0

i.e., $1+G_1(jw)\ G_2(jw)\ N_e = 0 \qquad \ldots(4.32)$

The equation (4.32) may be written as

$$G\ (jw) = -1/N_e \text{ where } G(jw) = G_1(jw)G_2(jw) \quad \ldots(4.33)$$

Equation (4.33) may be represented as a single graphical interpretation. The functions G(jw) and $-1/N_e$ may be plotted in a complex plane and the intersection(s) of these two curves determine all possible frequencies and amplitudes of oscillation of the system. The stability analysis is usually done with the help of frequency response.

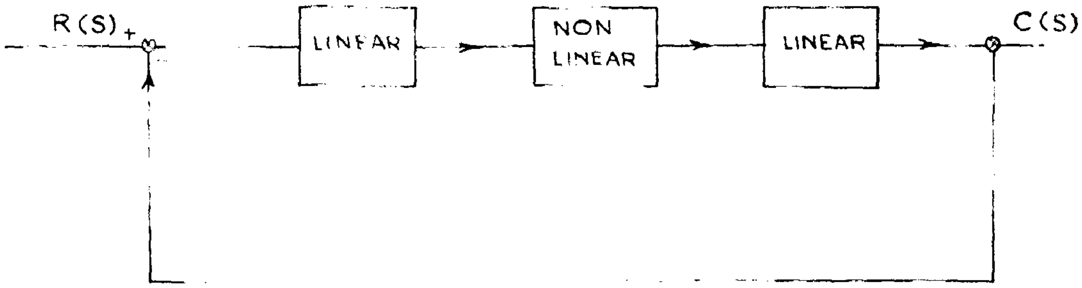


FIGURE- 24 (a)
DIVISION OF SYSTEM INTO LINEAR AND NON-LINEAR ELEMENTS

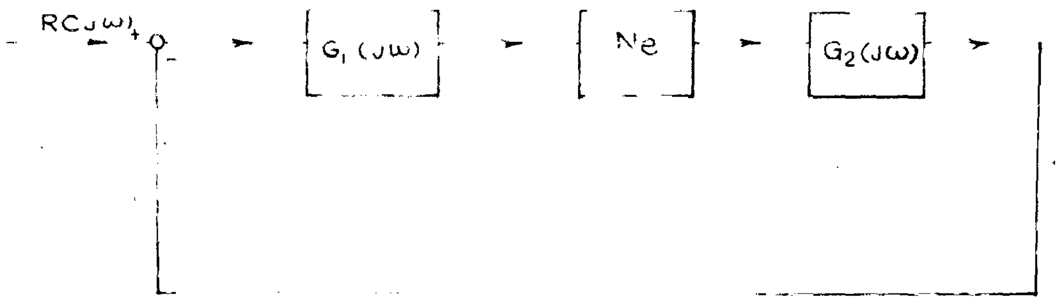


FIGURE- 24 (b)
NON-LINEAR ELEMENT REPLACED BY ITS DESCRIBING FUNCTION

4.7 STABILITY ANALYSIS BY FREQUENCY RESPONSE METHOD

In the Nyquist plot, the G(jw) curve may be considered as a linear frequency locus and $-1/N_e$ as the locus of the critical point corresponding to (-1,j0) point in the linear system theory[2]. The relative positions of G(jw) locus and $-1/N_e$ indicates the stability condition of non-linear system. The following criteria is usually applied to determine the stability of the non-linear system.

(a) When the $(-1/N_e)$ curve lies to the left of the G(jw) locus (with increasing value of w) or not enclosed by the G(jw) locus then the non-linear system is stable.

(b) When the $(-1/N_e)$ curve lies to the right of G(jw) locus, that is lies completely inside the G(jw) locus, then the system is unstable.

(c) If there are intersections between the two loci, sustained oscillations exist in the system depending upon the number of intersections.

The effect of the non-linearity on the dynamic properties of the system can be studied using Nyquist plot. The automatic voltage regulation may be chosen for such analysis.

4.8 EFFECT OF SATURATION OF THE SYNCHRONOUS GENERATOR

CASE 1

The synchronous generator may be assumed to operate under saturated conditions. The non-linearity may be represented by a blank square block in the structural diagram Figure (25).

The describing function for the saturation type of non-linearity may be derived as shown in Appendix- D .

The characteristic equation of the system with non-linearity in the synchronous generator is

$$1 + \frac{K_1K_2K_3K_4K_5K_6N_1(A_1)T_5s}{(1+2T_1s)(1+T_2s)(1+T_3s)(1+T_4s)(1+T_5s)} = 0$$

or

$$\prod_{i=1}^{5}(1+T_is)+K_{ov}N_1(A_1)T_5s = 0 \qquad \ldots (4.34)$$

Therefore,

$$-1/N_1(A_1) = \frac{K_{ov}T_5s}{\prod_{i=1}^{5}(1+T_is)} \qquad \ldots (4.35)$$

The describing function $N_1(A_1)$ for the saturation type of non-linearity is obtained as given in appendix.

$$N_1(A_1) = \frac{2K_s}{\pi}\left[\frac{A_s}{A_1}\sqrt{1-(\frac{A_s}{A_1})^2} + \sin^{-1}\frac{A_s}{A_1}\right] \qquad \ldots (4.36)$$

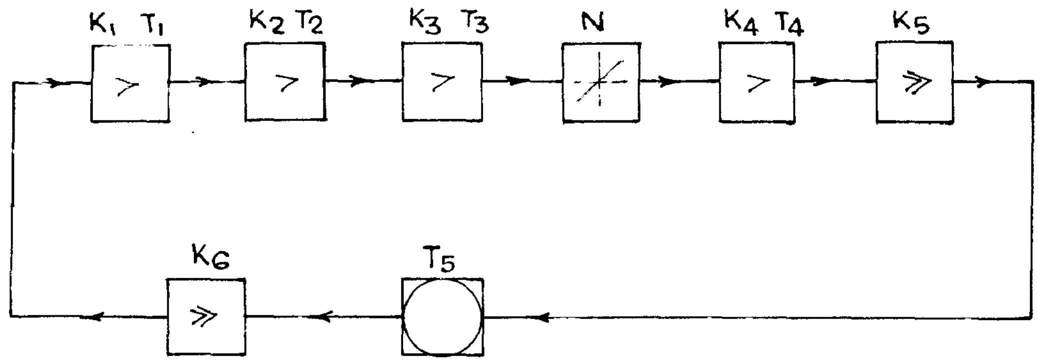


FIGURE - 25
STRUCTURAL DIAGRAM WITH A NON-LINEAR ELEMENT

Assuming $A_s = 1$ and separating K_s from the non-linear element, equation (4.36) may be written as

$$N_1(A_1) = \frac{2}{\pi}\left[\frac{1}{A_1}\sqrt{1-\left(\frac{1}{A_1}\right)^2} + \sin^{-1}\frac{1}{A_1}\right] \text{ if } A_1 > A_s$$

$$= 1 \text{ if } A_1 < A_s \qquad \ldots (4.37)$$

$N_1(A_1)$ may be evaluated for different values of A_1 and hence $\left[-1/N_1(A_1)\right]$ can be plotted on a polar graph. The values of A_1 and $N_1(A_1)$, $\left[-1/N_1(A_1)\right]$ are tabulated in Table 1.

The right hand side of equation (4.35) which represents $G(jw)$ locus, can be plotted in a Nyquist plot by substituting different values of w varying from 0 to $+\infty$. The numerical values of time constants and gain are chosen from section (4.3). The equation for $G(jw)$ locus may be obtained as

$$G(s) = \frac{K_{ov}T_5 s}{(1+2T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)(1+T_5 s)} \qquad \ldots (4.38)$$

In the complex plane putting $s = jw$ and substituting the numerical values,

$$G(jw) = \frac{720\ jw}{(1+0.006jw)(1+0.08jw)(1+0.03jw)(1+0.05jw)(1+0.3jw)} \quad \ldots (4.39)$$

The amplitude and phase of G(jw) for different values of w are calculated and the graph plotted. The locus G(jw) is drawn in Figure (26a).

The $(-1/N_1(A_1))$ plot is superimposed in G(jw) locus in Figure (26b).

It can be deduced from the graph that the system is unstable as the $(-1/N_1(A_1))$ curve is enclosed by G(jw) locus.

A stable system may now be obtained by either varying the paremeters of the system or the non-linearity itself. C.N. Shen[13] and A.K. Mahalanobis[14] provided a known non-linear element in the feedback path and stabilised the system.

However, to provide another non-linear element, most commonly adopted in practice it complicates the original system. It is proposed in this work to use one of the linear elements in the forward path as a non-linear element and its effect on the stability of system is analysed.

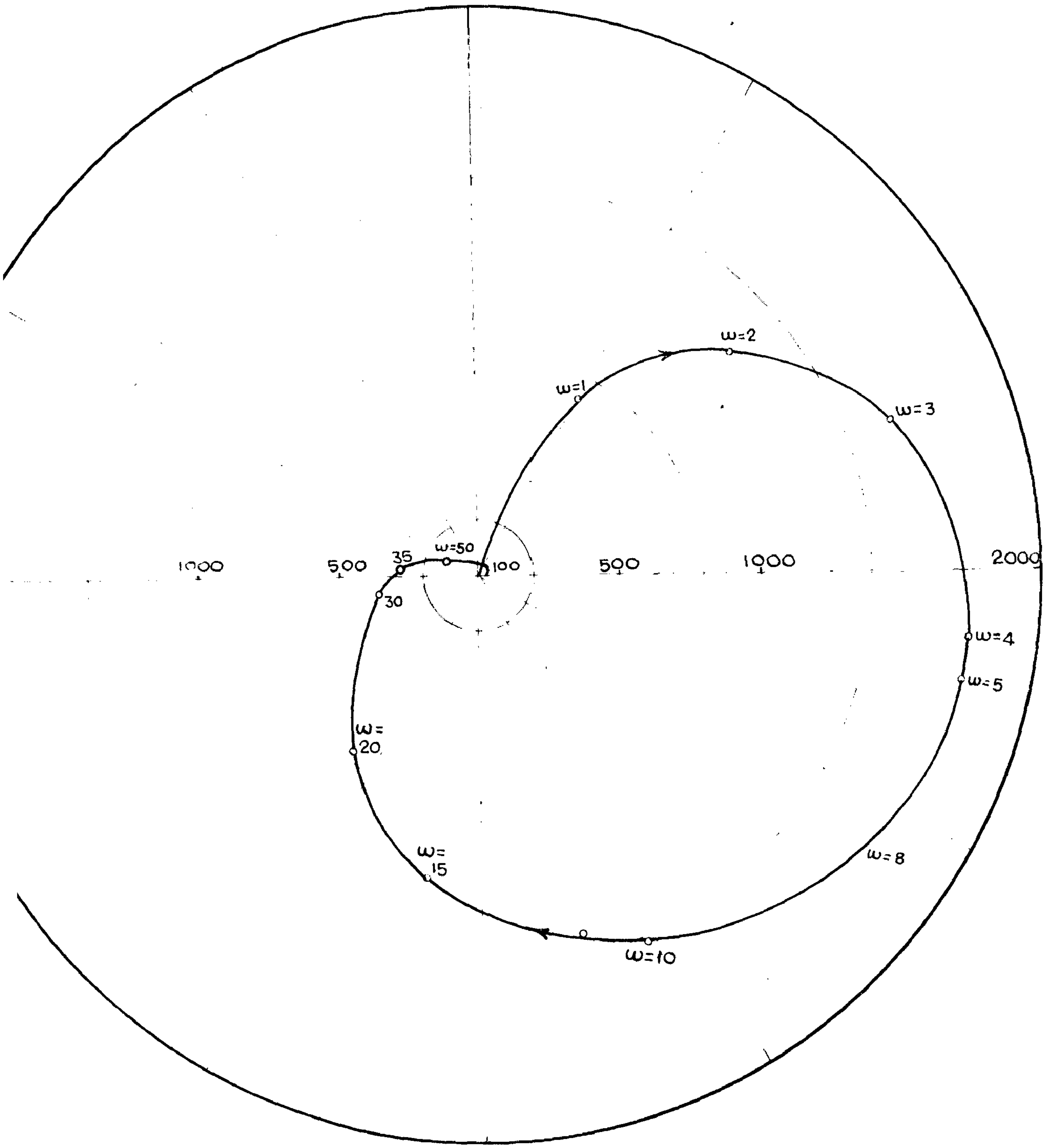


FIGURE- 26(a)

G (jω) LOCUS

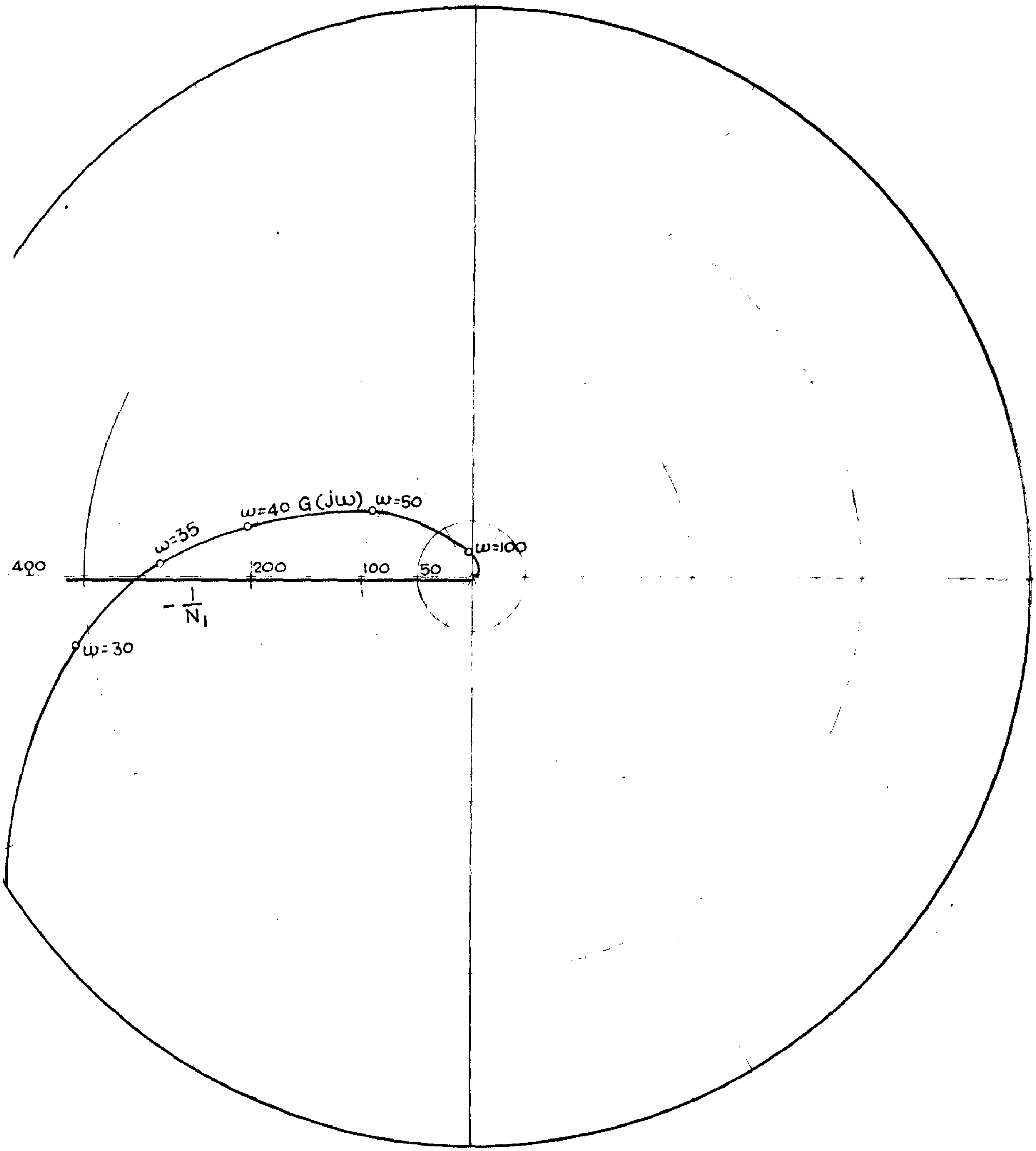


FIGURE- 26 (b)

G (jω) PLOT AND $-\frac{1}{N_1}$ PLOT

4.9 STABILITY ANALYSIS WITH TWO NON-LINEARITIES IN THE SYSTEM

The method of analysis of two non-linear elements existing in the system was proposed by E.A. Freeman. His method of analysis using describing function was illustrated by an example of torque limitation and backlash.

The describing function method applied to a system having two non-linear elements provides a criterion of stability. An illustrative example, is chosen to describe the method proposed by Freeman.

In Figure (27) g_1 to g_5 are linear elements which may or may not be frequency dependent and n_1 and n_2 are two non-linear elements with vector gains $N_1(A_1)$ and $N_2(A_2, w)$ respectively. However, in order that the input to second non-linear element is sinusoidal, g_2 and g_3 have to be frequency dependent so that the harmonics are attenuated by their low pass filter characteristics.

The transfer function of the closed-loop system of Figure (27) is,

$$\frac{C}{R} = \frac{G_1G_2G_3G_4N_1(A_1)\ N_2(A_2,w)}{1+G_1G_2G_3G_4G_5N_1(A_1)\ N_2(A_2,w)} \qquad \ldots(4.40)$$

The characteristic equation is

$$1+G_1G_2G_3G_4G_5N_1(A_1)N_2(A_2,w) = 0 \qquad \ldots(4.41)$$

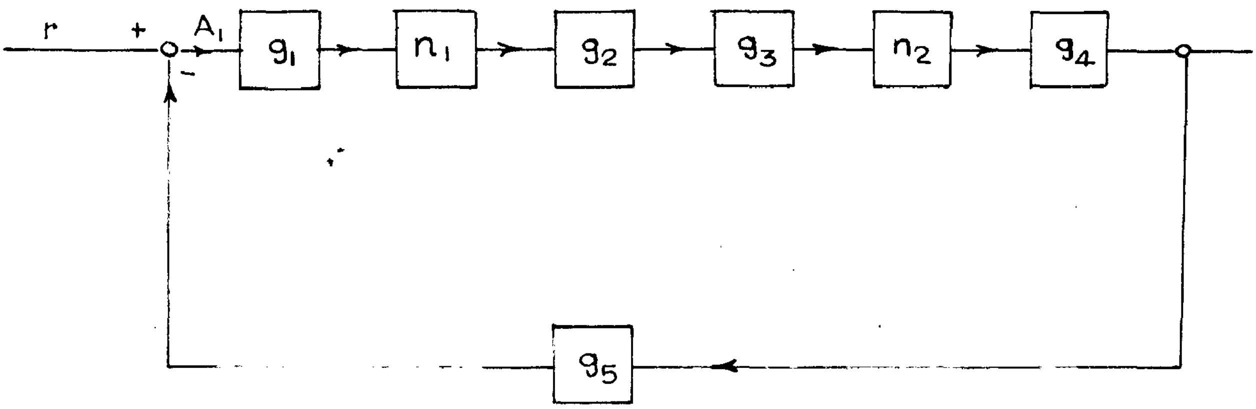


FIGURE - 27.
BLOCK DIAGRAM HAVING TWO NON-LINEARITIES.

The relationship between A_1 and A_2 may derived from Figure (27) as

$$A_2 = \left| N_1(A_1)G_2(jw)G_3(jw) \right| A_1 \qquad \dots (4.42)$$

the describing function for the second non-linearity is

$$N_2(A_2,w) = \frac{2}{\pi}\left[\sin^{-1} \frac{A_s}{A_2} + \frac{A_s}{A_2}\sqrt{1-\left(\frac{A_s}{A_2}\right)^2} \right] \dots (4.42a)$$

The describing function of the second non-linear element, $N_2(A_2,w)$, may be written as

$$N_2\left[\left| N_1(A_1)G_2(jw)G_3(jw) \right| A_1, w\right] \qquad \dots (4.43)$$

From equation (4.43), it can be deduced that as for as the system is concerned the two non-linear elements may be represented by a single non-linear element.

The equation (4.41) may be written as

$$1+G_1G_2G_3G_4G_5N_e(A,w) = 0 \qquad \dots (4.44)$$

where $N_e(A,w) = N_1(A_1) \times N_2(A_2,w)$

From equation (4.44)

$$-\frac{1}{N_e} = G_1G_2G_3G_4G_5 = G(jw) \qquad \dots (4.45)$$

The Nyquist plot of the right hand side may be plotted for different values of w varying from 0 to $+\infty$, to obtain $G(jw)$ locus. The describing function of $N_2(A_2,w)$ is same as that of $N_1(A_1)$ except that A_2 is to be calculated for various values of w and A_1. $N_1(A_1)$ and $N_2(A_2,w)$ are given by the equations,

$$N_1(A_1) = \frac{2}{\pi}\left[\sin^{-1}\frac{A_s}{A_1} + \frac{A_s}{A_1}\sqrt{1-(\frac{A_s}{A_1})^2}\right]$$

$$\text{and}\quad N_2(A_2,w) = \frac{2}{\pi}\left[\sin^{-1}\frac{A_s}{A_2} + \frac{A_s}{A_2}\sqrt{1-(\frac{A_s}{A_2})^2}\right] \qquad \ldots(4.46)$$

For different values of A_1, A_2 the describing functions of $N_1(A_1)$ and $N_2(A_2,w)$ may be obtained and their product may be determined. From the product $(-1/N_e)$ can be calculated. The plots of $(-1/N_e)$ and $G(jw)$ may be superimposed and the stability analysis can be carried out using Nyquist criterion.

4.10 SATURATION OF THE AMPLIDYNE CONSIDERED AS A SECOND NON-LINEARITY

In the example of automatic voltage regulation system, the amplidyne is assumed to operate with saturation besides the synchronous generator. The corresponding structural diagram is shown in Figure (28).

A_1 is the amplitude of sinusoidal input signal to the amplidyne and A_2 is the sinusoidal input signal to the

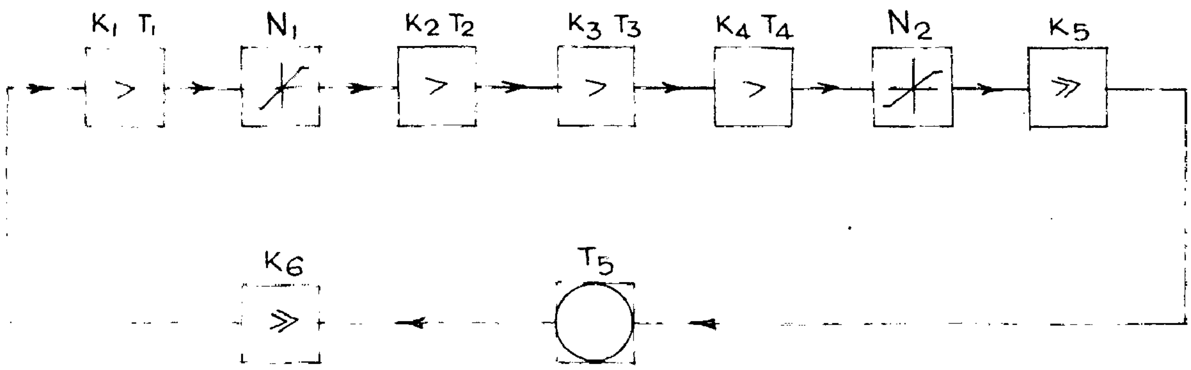


FIGURE - 28
STRUCTURAL DIAGRAM OF THE SYSTEM HAVING TWO NON-LINEAR ELEMENTS

synchronous generator. It is assumed that $A_s = 1$ p.u. in both cases. $N_1(A_1)$ is obtained for different values of A_1 from 0 to 100 p.u., using equation (4.36). Now A_2 is given by the relation

$$A_2 = \left| N_1(A_1) \frac{K_2K_3K_4}{(1+T_2jw)(1+T_3jw)(1+T_4jw)} \right| A_1 \qquad \ldots (4.47)$$

For different values of w and $N_1(A_1)$, A_2 is determined and is tabulated. $N_2(A_2,w)$ is calculated for a particular frequency and for different values of A_1, using equation (4.46). The values obtained are tabulated in Table 2.

The product of $1/N_1(A_1)$ and $1/N_2(A_2,w)$ is obtained and interpolated with $G(jw)$ locus in Figure (26). It is observed that if w is varied between 0 to 60 rad/sec the system tends to become stable and if w is above 60 rad/sec the system becomes unstable.

Therefore, the effect of the second non-linear element tends to stablise a basically unstable non-linear system upto a particulat value of w.

4.11 TRANSIENT RESPONSE OF THE SYSTEM

The transient response of the system may be obtained directly from the polar plots[16]. The method is as follows. A particular amplitude may be chosen on $(-1/N_e)$ locus and a straight line is drawn from O passing through G(jw) locus to C, Figure (29a,b and c). The point C represents a particular selected signal amplitude as (-1,jo) and the scale of the plot is defined by OC = 1.0. Using this scale and OC as the negative real axis, M circles can be added, locating a tangency with G(jw) locus and thus defining the peak amplitude M_p and the resonant frequency w_r. This procedure may be repeated for a number of signal amplitudes in the desired range of values.

A plot betwwen w_r and M_p indicates the transient response of the system, Figure (30). From Figure (30) it can be deduced that the transients die down indicating that the system satisfied the stability conditions.

4.12 SATURATION OF THE MAIN EXCITER CONSIDERED

CASE 3

In order to analyse the effect of third linearity on the system stability, the saturation of the main exciter is also considered. The structural diagram is shown in Figure (31).

The characteristic equation of the above system may be obtained as

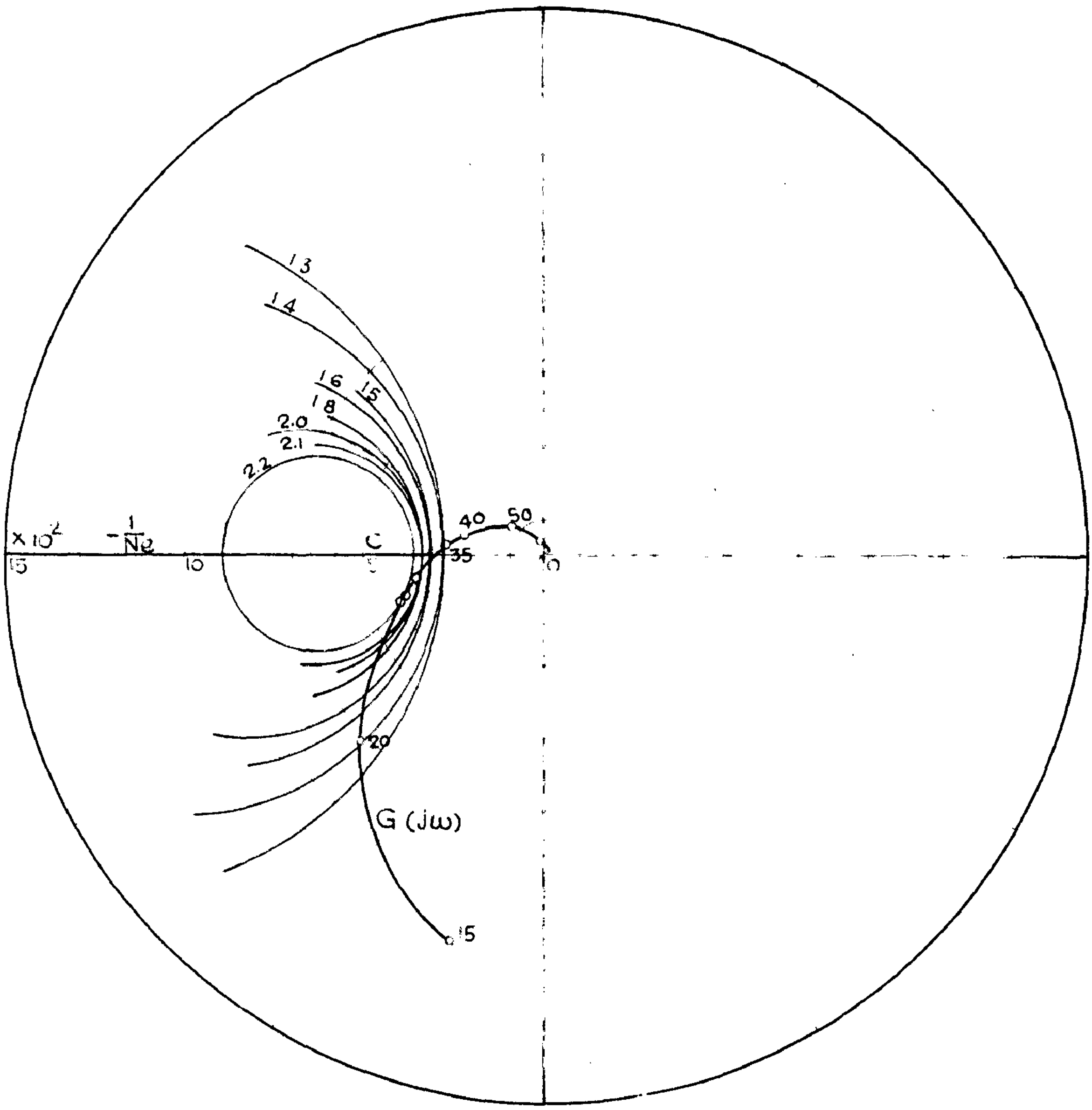


FIGURE - 29 (a)

DETERMINATION OF Mp AND ω_r ASSUMING OC = 1 UNIT HAVING TWO NON-LINEARITIES FOR THE SYSTEM

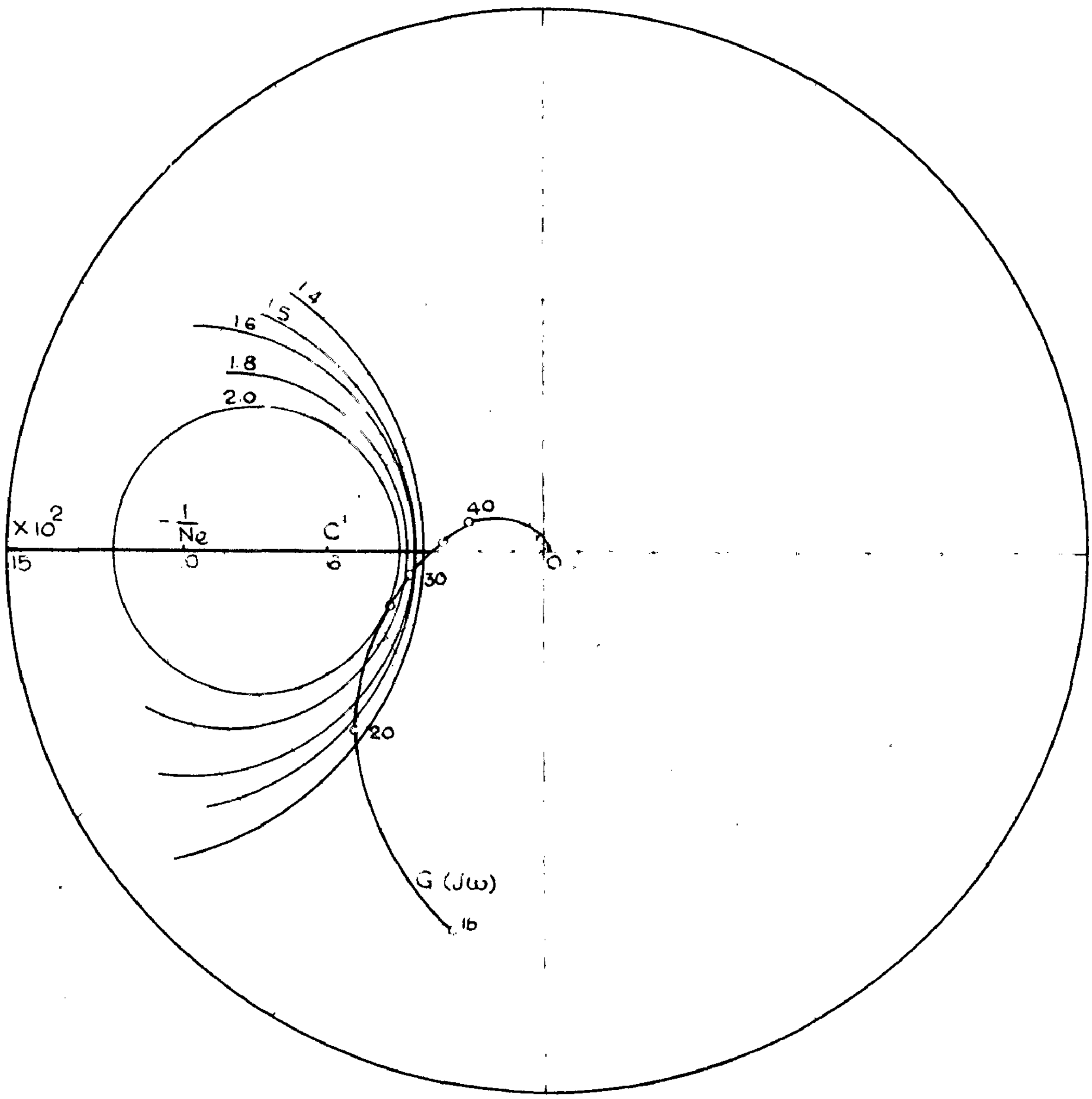


FIGURE - 29 (b)

DETERMINATION OF Mp AND ω_r ASSUMING $OC' = 1$ UNIT

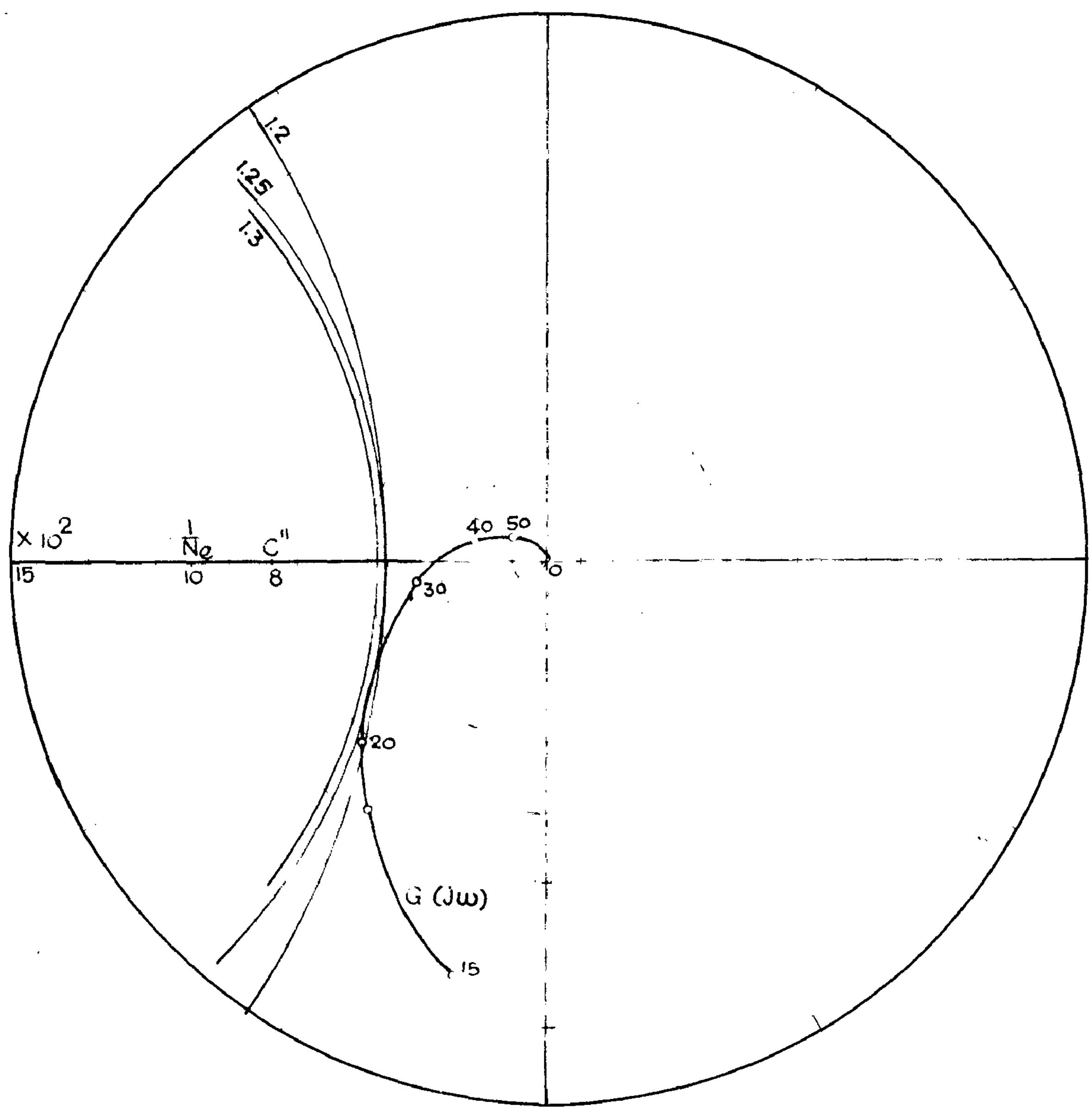


FIGURE - 29 (C)

DETERMINATION OF M_p AND ω_r ASSUMING $OC'' = 1$ UNIT

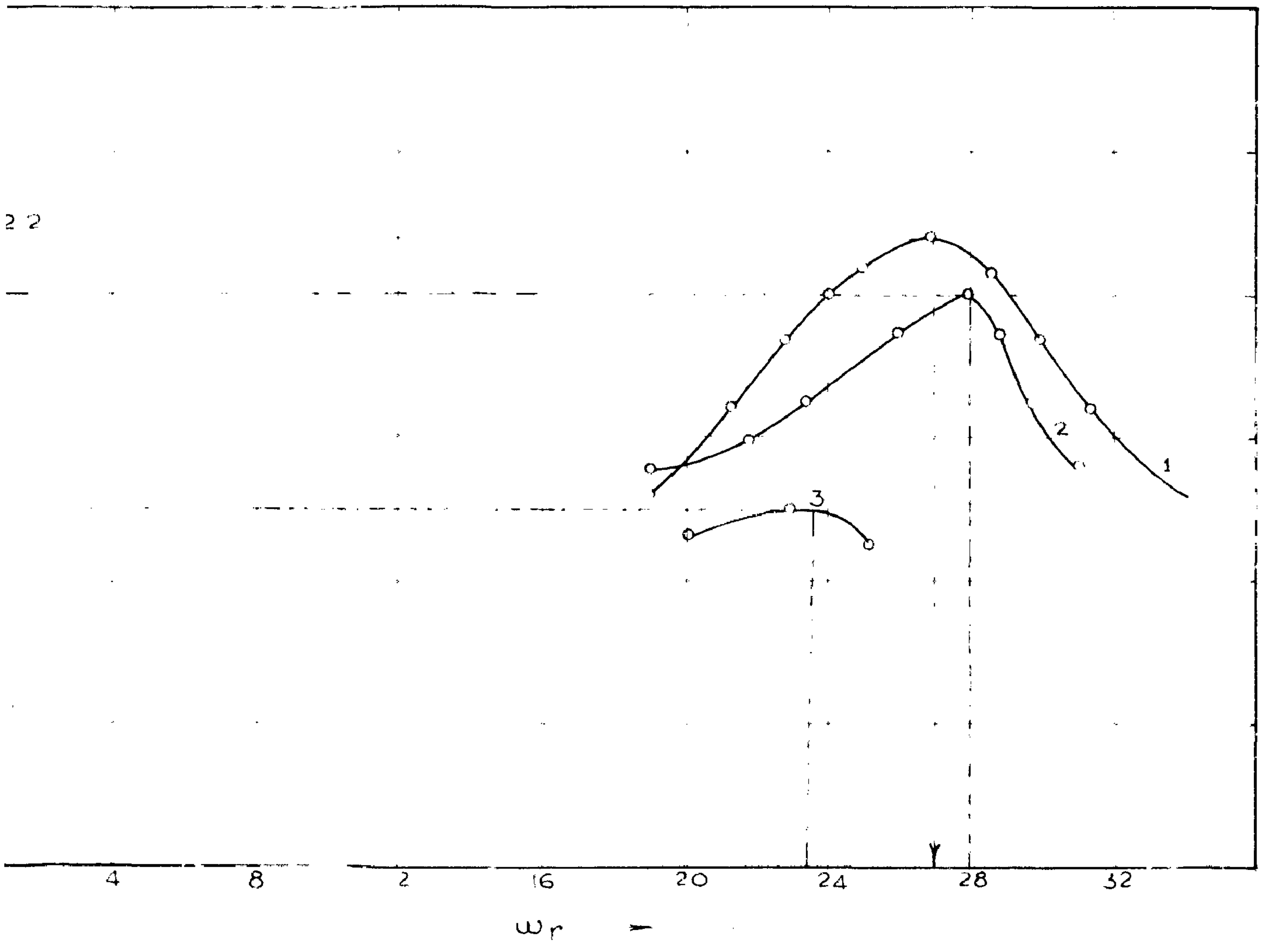


FIGURE - 30

ANSIENT RESPONSE OF THE SYSTEM IN TERMS OF ... AND Mp

$$\prod_{i=1}^{5}(1+T_i s)+K_{ov}T_5 s N_e = 0,$$

where $N_e = N_1(A_1)N_2(A_2,w)N_3(A_3,w)$ ... (4.48)

Therefore,

$$-1/N_e = \frac{K_{ov}T_5 s}{\prod_{i=1}^{5}(1+T_i s)} \qquad \text{... (4.49)}$$

$N_1(A_1)$, $N_2(A_2,w)$ and $N_3(A_3,w)$ may be obtained as follows.

For a particular input signal to the first non-linear element, the describing function for the non-linear element is obtained as

$$N_1(A_1) = \frac{2}{\pi}\left[\sin^{-1}\frac{A_s}{A_1} + \frac{A_s}{A_1}\sqrt{1-\left(\frac{A_s}{A_1}\right)^2}\right] \qquad \text{..(4.50)}$$

Assuming that the saturation of the amplidyne begins at $A_s = 1$ p.u., the above equation can be written in the form

$$N_1(A_1) = \frac{2}{\pi}\left[\sin^{-1}\frac{1}{A_1} + \frac{1}{A_1}\sqrt{1-\left(\frac{1}{A_1}\right)^2}\right] \qquad \text{..(4.50a)}$$

$$\text{if } A_1 > A_s$$

$$= 1 \quad \text{if } A_1 < A_s$$

$N_1(A_1)$ is obtained for different values of A_1.

The input to the second non-linear element is a frequency dependent. The amplitude of the input signal A_2 is

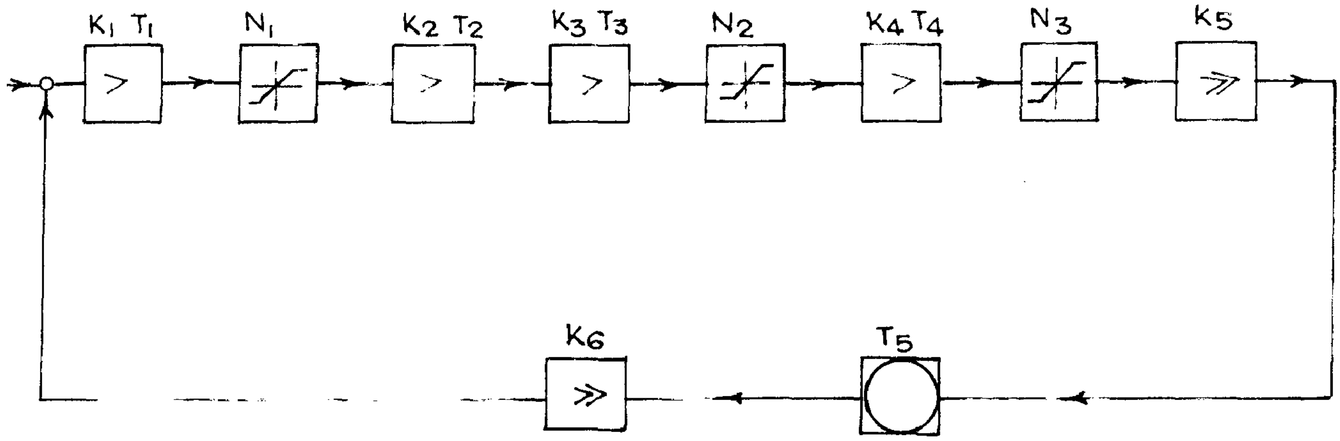


FIGURE - 31
STRUCTURAL DIAGRAM WITH NON-LINEAR ELEMENTS IN THE SYSTEM

$$A_2 = \left| N_1(A_1) \quad \frac{K_2K_3}{(1+T_2s)(1+T_3s)} \right| A_1 \quad \ldots (4.51)$$

substituting $s = jw$,

$$A_2 = \left| N_1(A_1) \quad \frac{K_2K_3}{(1+T_2jw)(1+T_3jw)} \right| \quad A_1 \quad \ldots (4.51a)$$

The input signal amplitude to the second non-linear element is obtained for various value of A_1 and w justifiable to the practical limits.

The describing function of the second non-linear element is obtained as

$$N_2(A_2,w) = \frac{2}{\pi}\left[\sin^{-1} \frac{A_s}{A_2} + \frac{A_s}{A_2} \sqrt{1-\left(\frac{A_s}{A_2} \right)^2} \right] \quad \ldots \quad (4.52)$$

Assuming that the saturation of the main exciter occurs at $A_s = 1$ p.u.,

$$N_2(A_2,w) = \frac{2}{\pi}\left[\sin^{-1} \frac{1}{A_2} + \frac{1}{A_2}\sqrt{1-\left(\frac{1}{A_2} \right)^2} \right] \text{ if } A_s < A_2$$

$$= 1 \text{ if } A_2 < A_s \quad \ldots \quad (4.52a)$$

The input signal to the third non-linear element A_3 is also a function of frequency. The signal amplitude A_3 is given by the equation

$$A_3 = \left| N_1(A_1) \frac{K_2K_3}{(1+T_2jw)(1+T_3jw)} N_2(A_2,w) \frac{K_4}{(1+T4jw)} \right| A_1$$

...(4.53)

The describing function for the third non-linear element is

$$N_3(A_3,w) = \frac{2}{\pi}\left[\text{Sin}^{-1} \frac{A_s}{A_3} + \frac{A_s}{A_3} \sqrt{1-(\frac{A_s}{A_3})^2} \right] \text{if } A_3 > A_s$$

$$= 1 \text{ if } A_3 < A_s$$

...(4.54)

Assuming that the saturation occurs at 1 p.u. in the synchronous generator,

$$N_3 (A_3,w) = \frac{2}{\pi}\left[\text{Sin}^{-1} \frac{1}{A_3} + \frac{1}{A_3} \sqrt{1-(\frac{1}{A_3})^2}\right]$$

...(4.54a)

The product of $N_1(A_1)$ and $N_2(A_2,w)$ and $N_3(A_3,w)$ is obtained and $-1/N_e$ is superimposed on the $G(jw)$ locus. The above calculations are done using Digital Computer.

It has been found that the $-1/N_e$ curve is enclosed by $G(jw)$ locus only at a very high value of frequency. Hence it may be deduced that the system is stable for all practical purposes.

TABLE - 1

CASE-1 DESCRIBING FUNCTION FOR ONE NON-LINEAR ELEMENT

A_1	$N_1(A_1)$	$1/N_1(A_1)$
1.2	0.9208	1.0859
6.2	0.2046	4.888
11.2	0.1136	8.8037
16.2	0.0785	12.725
21.2	0.0606	16.648
31.2	0.0408	24.49
...	...	...
51.2	0.0288	40.015
100.0	0.01192	83.87
200	0.003	333.33

TABLE - 2

DESCRIBING FUNCTION VALUES FOR TWO NON-LINEARITIES

A_1	$1/N_1(A_1)$	$1/N_2(A_2,w)$	$1/N_e$	w rad/sec
1	2	3	4	5
1.2	1.0859	0.9217×10^4	0.1008×10^5	0
11.2	8.8037	0.1061×10^5	0.9341×10^5	"
21.2	16.648	0.1062×10^5	0.1768×10^6	"
31.2	24.496	0.10623×10^5	0.2602×10^6	"
41.2	32.345	0.10623×10^5	0.3436×10^6	"
51.2	40.194	0.10624×10^5	0.4270×10^6	"
1.2	1.0859	0.2962×10^4	0.3216×10^4	20
11.2	8.8037	0.3409×10^4	0.3002×10^5	"
21.2	16.648	0.3413×10^4	0.5682×10^5	"
32.2	24.496	0.34138×10^4	0.8363×10^5	"
41.2	32.345	0.3414×10^4	0.1104×10^6	"
51.2	40.194	0.3414×10^4	0.1372×10^6	"
1.2	1.0859	0.78706×10^3	0.8547×10^3	40
11.2	8.8037	0.9061×10^3	0.7977×10^4	"
21.2	16.648	0.9069×10^3	0.1509×10^5	"
31.2	24.496	0.90717×10^3	0.2222×10^5	"
41.2	32.345	0.9072×10^3	0.2935×10^5	"
51.2	40.194	0.90727×10^3	0.3647×10^5	"

Contd ....

1	2	3	4	5
1.2	1.0859	0.2886×10^3	0.3134×10^3	60
11.2	8.8037	0.3324×10^3	0.2926×10^4	"
21.2	16.648	0.3326×10^3	0.5538×10^4	"
31.2	24.496	0.3327×10^3	0.8151×10^4	"
41.2	32.345	0.3328×10^3	0.1076×10^5	"
51.2	40.194	0.33288×10^3	0.1337×10^5	"
1.2	1.0859	0.1327×10^3	0.1447×10^3	80
11.2	8.8037	0.1528×10^3	0.1345×10^4	"
21.2	16.648	0.1529×10^3	0.2546×10^4	"
31.2	24.496	0.15298×10^3	0.3747×10^4	"
41.2	32.345	0.15299×10^3	0.4948×10^4	"
51.2	40.194	0.1530×10^3	0.6149×10^4	"

CONCLUSIONS

The machine control problems has been analysed and synthetised using structural diagram technique.

The speed control of a d.c. motor and also a system involving time lag were analysed from stability point of view. Both the systems with the chosen parameters were found to be unstable. In order to obtain a high gain stable system, a suitable stabilising device and the inter connection of it has been found out. The transient response is plotted using D-partition curve.

The effect of non-linearity on the system performance has been studied using describing function technique. The automatic voltage regulation of a synchronous generator is considered as a specific example. The above system has been found to be totally unstable in the linear case and also when one non-linearity is considered- However the system attains a stable operation upto a particular value of frequency, w, when two non-linearities are considered in the system. It has been further found that the system is absolutely stable when three non-linearities in the system are considered. It can be said that a totally unstable linear system becomes absolutely stable when all the non-linearities in the system are taken into account and the omission of non-linearities in practice is erraneous.

The study of automatic voltage regulation of a loaded synchronous generator with non-linearities encountered in the system can be taken up for further study.

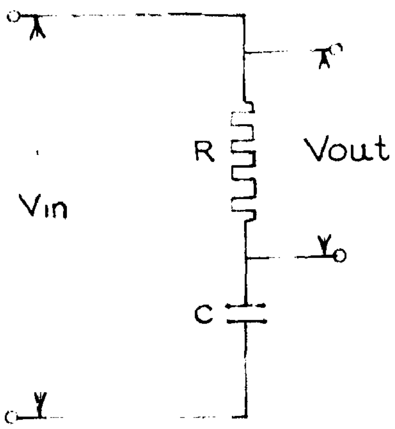


FIGURE - 32 (a)
Rc STABILISING DEVICE

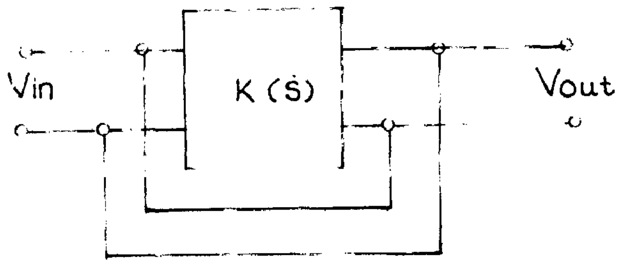


FIGURE - 32 (b)
CLOSED-LOOP REGULATING SYSTEM

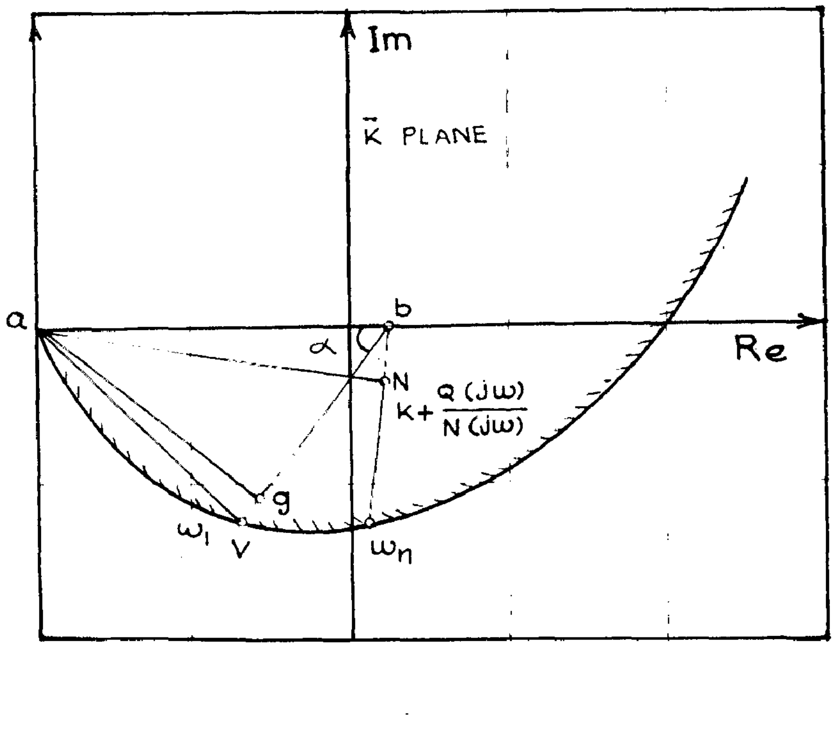


FIGURE - 32 (C)
D-PARTITION CURVE WITH RESPECT TO THE GAIN

APPENDIX - A

The transfer function of the RC circuit may be derived as follows.

In Figure (32a),

$$V_{in} = Ri + \frac{i}{Cs}$$

and

$$V_{out} = Ri$$

Eliminating i from the above equation

$$\frac{V_{out}}{V_{in}} = \frac{R}{(R+\frac{1}{Cs})}$$

$$= \frac{RCs}{1+RCs}$$

Denoting the time constant of the stabilising link by $RC = \tau$, then

$$\frac{V_{out}}{V_{in}} = \frac{\tau s}{1+\tau s}$$

APPENDIX - B

REAL RESPONSE FREQUENCY CHARACTERISTICS

The closed-loop transfer function of the regulating system shown in Figure (32.b) may be written as

$$K(s) = \frac{X_{out}}{X_{in}} = \frac{KW(s)}{1+KW(s)} \quad \dots (1)$$

The Attenuation-Phase Characteristic of the closed-loop system is

$$K(jw) = \frac{KW(jw)}{1+KW(jw)} \quad \dots (2)$$

where $KW(jw) = \frac{KN(jw)}{Q(jw)}$ ... (3)

is the equation of the Attenuation-Phase Characteristic of the open-loop system.

The characteristic equation of the closed-loop system is obtained as

$$1 + KW(s) = 0$$

$$= 1 + \frac{KN(s)}{Q(s)} = 0$$

Whence the equation of the D-partition boundary is

$$\bar{K} = - \frac{Q(jw)}{N(jw)}$$

The D-partition curve may be of the form as shown in Figure (32c).

The section ab equals K, the section $\overline{av}$ equals $-\frac{Q(jw_1)}{N(jw_1)}$ for the frequency w_1 and the section bv is the sum $K + \frac{Q(jw_1)}{N(jw_1)}$. The amplitude may now be obtained from the ratio ab/bv for the given frequency.

The phase may be determined by measuring the angle between the vector $\overline{bv}$ and the negative abscissal half-axis, denoted by α.

The real response frequency characteristic may be obtained as

$$R(w) = \frac{ab}{bv} \cos\alpha$$

If a perpendicular is drawn from point a to the vector $\overline{bv}$, then

$$bg = ab \cos\alpha$$

Hence $R(w) = bg/bv$.

APPENDIX - C

TRANSIENT RESPONSE FROM REAL RESPONSE

If $H(s)$, a function of s, and $h(t)$, a function of t, are related as

$$H(s) = \int_{0}^{\infty} h(t)\, e^{-st}\, dt$$

The inverse transform may be done with the help of a contour integration of Bromwitch as follows,

$$h(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} H(s)\, e^{st}\, ds.$$

where there is no singularity on the right half of the complex s-plane.

If $s = jw$, the above transformation becomes

$$h(t) = \frac{1}{2\pi} \int_{-\infty - jc}^{\infty - jc} H(jw)\, e^{jwt}\, dw$$

The contour integration may be done along the real axis as no singularity is encountered.

Thus,

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(jw)\, e^{jwt}\, dw$$

If the real part of the $H(jw)$ is an even and the imaginary part of $H(jw)$ is an odd function of w, then it may be shown that

$$h(t) = \frac{2}{\pi} \int_0^{\infty} Re\left[H(jw)\right] \cos wt \, dw$$

provided the following conditions are fulfilled,

(1) there is no poles on the right half of the plane,

(2) $\underset{w \to \infty}{Lt} H(jw) = 0$, or a finite quantity, for the integral to be finite,

(3) the overall transfer function $H(jw)$ is expressible as a quotient of polynomial in w, the order of the denominator being greater than the numerator at least by unity.

However, the above integral may be evaluated for arbitrary variation of $H(jw)$ by the method due to Floyd, as follows:

Considering the transfer function plotted in Figure (32d), and drawing straight lines to represent it approximately the time function may be shown equal to

$$h(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} A_n \frac{\sin w_n t}{w_n t} \quad \frac{\sin \Delta_n t}{\Delta_n t}$$

where

$$A_1 = r_1 w_1, \quad A_2 = r_2 w_2,$$

$$w_1 = (w_b + w_a)/2, \; w_2 = (w_c + w_b)/2,$$

$$\Delta_1 = (w_b - W_a)/2, \; \Delta_2 = (w_c - w_b)/2$$

The response due to an unit step function is obtained as

$$f(t') = \int_{0}^{t'} h(t)\, dt \,.$$

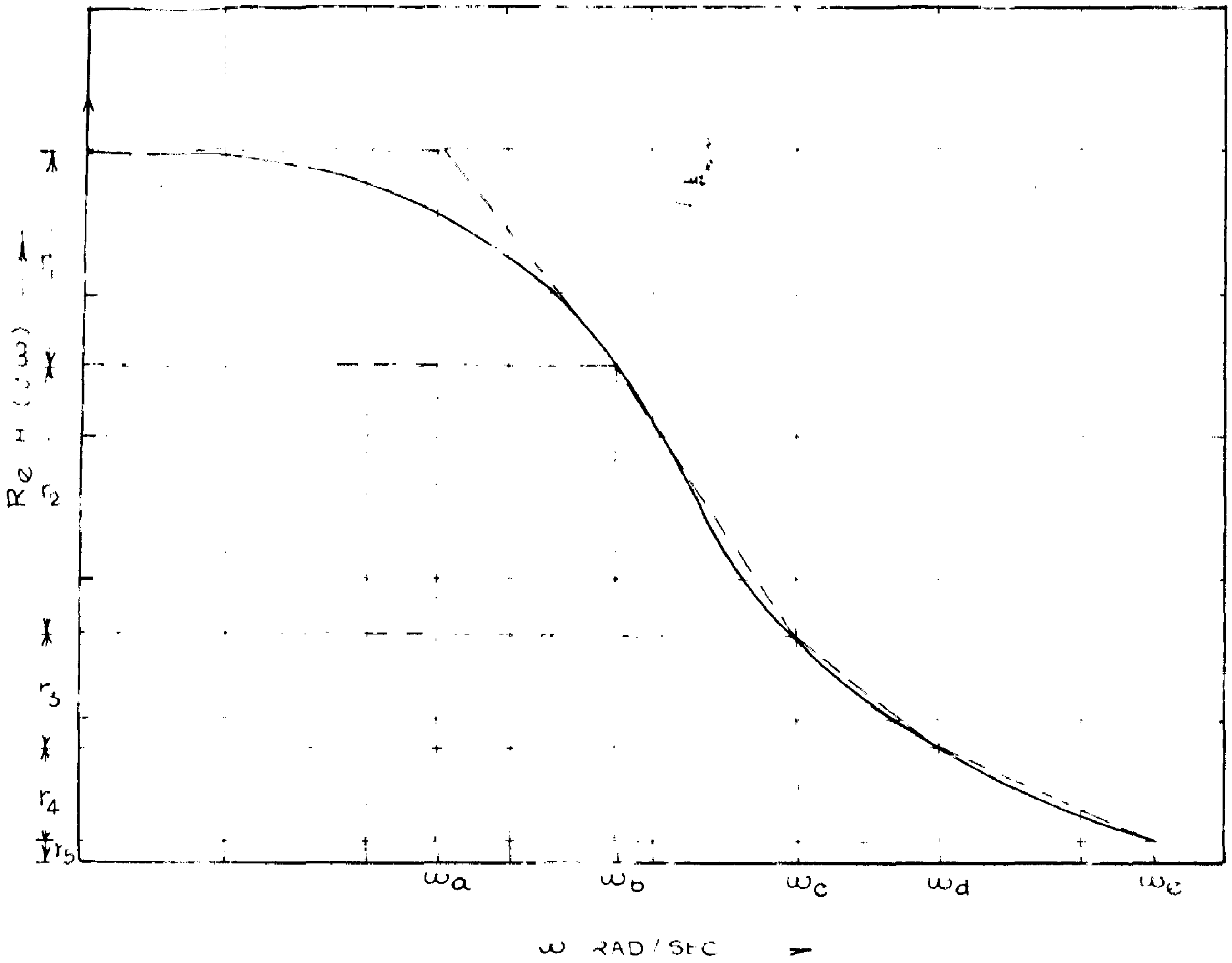


FIGURE - 32 (d)
METHOD OF CALCULATING TRANSIENT RESPONSE FROM REAL RESPONSE FREQUENCY CHARACTERISTIC

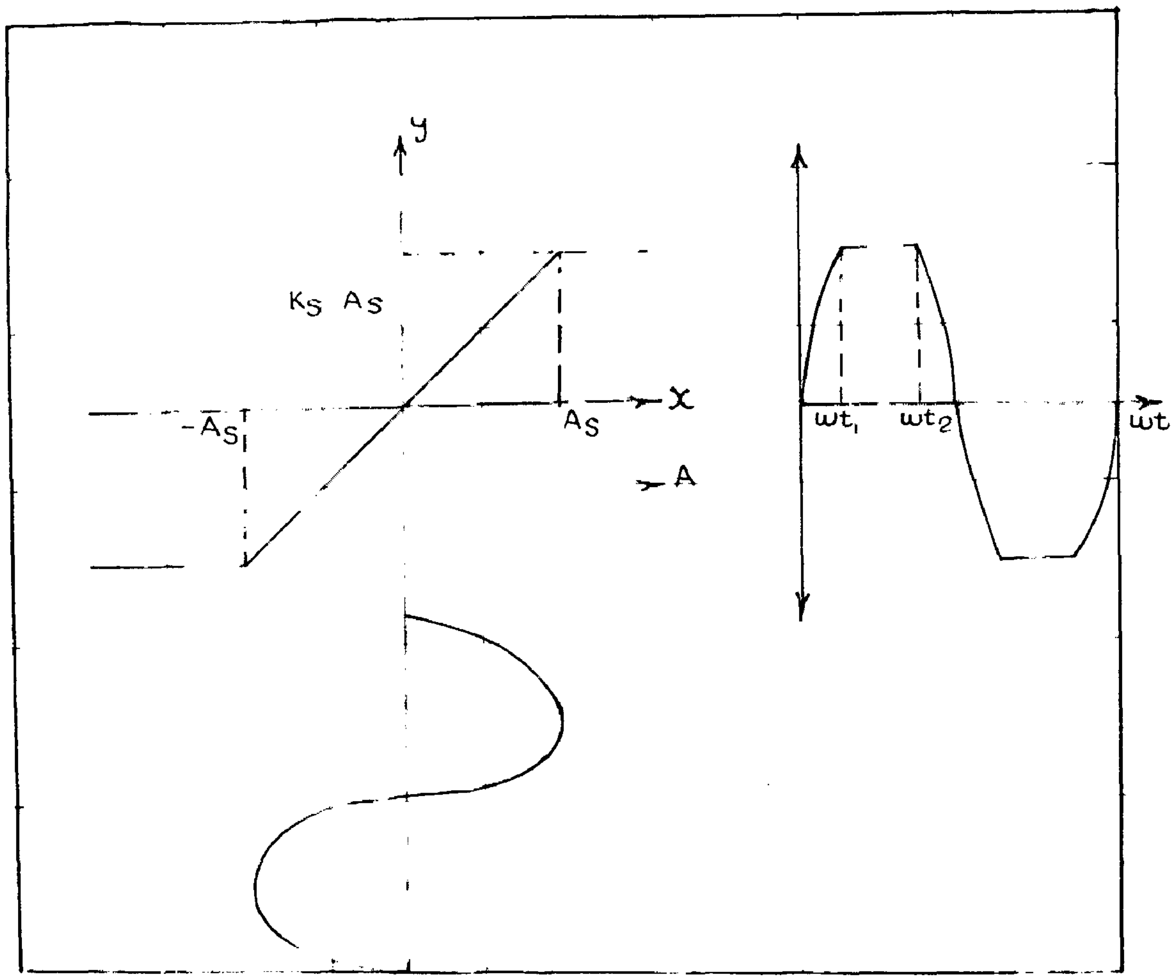


FIGURE- 32 (e)

DESCRIBING FUNCTION FOR SATURATION TYPE OF NON-LINEARITY

APPENDIX-D

The describing function for the saturation type of non-linearity may be obtained as follows.

In Figure (32c), for $X > A_s$, with $x = X \sin wt$ and $y = K_s X \sin nwt$, then the describing function is defined as

$$N = \frac{Y_1}{X} = K, \text{ where } Y_1 \text{ is the fundamental}$$

Fourier component of the output y. When X exceeds the saturation level A_s, the output signal y is distorted and the Fourier expression of y(t) is given by

$$y = Y_1 \sin wt + Y_3 \sin 3 wt + \ldots.$$

where
$$Y_1 = \frac{1}{\pi} \int_0^{2\pi} y \sin wt \, d(wt)$$
$$= \frac{4}{\pi} \int_0^{\pi/2} y \sin wt \, d(wt)$$

Therefore,

$$Y_1 = \frac{4}{\pi} \left[\int_0^{\omega t_1} K_s X \sin^2 wt \, d(wt) + \int_{\omega t_1}^{\pi/2} K_s A_s \sin wt \, d(wt) \right]$$

$$= \frac{4}{\pi} K_s X \left[\frac{wt_1}{2} - \frac{1}{4} \sin 2wt_1 \Big|_0 \left(- \frac{A_s}{X} \cos wt \right) \Big|_{\omega t_1}^{\pi/2} \right]$$

$$Y_1 = \frac{4}{\pi} K_s X \left[1/2 (wt_1 - 1/2 \sin 2wt_1) + \frac{A_s}{X} \cos wt_1 \right]$$

Now t_1 is defined by

$$X \text{Sin}\, wt_1 = A_s$$

$$\text{Sin}\, wt_1 = A_s/X$$

Hence $$\text{Coswt}_1 = \sqrt{1-\text{Sin}^2 wt_1} = \sqrt{1-(\frac{A_s}{X})^2}$$

$$\text{Sin}2wt_1 = 2\text{Sinwt}_1\ \text{Coswt}_1 = \frac{2A_s}{X}\sqrt{1-(\frac{A_s}{X})^2}$$

Thus

$$Y_1 = \frac{4K_s X}{\pi}\left\{\frac{1}{2}\left[\text{Sin}^{-1}\ \frac{A_s}{X} - \frac{A_s}{x}\sqrt{1-(\frac{A_s}{x})^2}\right.\right.$$
$$\left.\left. + \frac{A_s}{x}\sqrt{1-(\frac{A_s}{x})^2}\right\}\right.$$

$$N = \frac{Y_1}{x} = \frac{2K_s}{\pi}\left[\frac{A_s}{X}\sqrt{1-(\frac{A_s}{x})^2} + \text{Sin}^{-1}(\frac{A_s}{x})\right]$$

For a particular input of $X = A_1$

$$N = \frac{2K_s}{\pi}\left[\frac{A_s}{A_1}\sqrt{1-(\frac{A_s}{A_1})^2} + \text{Sin}^{-1}(\frac{A_s}{A_1})\right]$$

if $A_1 > A_s$

$= 1$ if $A_1 < A_s$.

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