"TRANSIENT ANALYSIS OF SYNCHRONOUS MACHINE UNDER UNBALANCED LOAD AND FAULT CONDITIONS"

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CERTIFICATE

Certified that the dissertation entitled "TRANSIENT ANALYSIS OF SYNCHRONOUS MACHINE UNDER UNBALANCED LOAD AND FAULT CONDITIONS" which is being submitted by Sri Prem Singh Bimbhra, in partial fulfilment for the award of the Degree of Master of Engineering in Advanced Electrical Machines of University of Roorkee, Roorkee, is a record of candidate's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for six months from Jan. 1968 to June 1968 for preparing dissertation for Master of Engineering Degree at the University.

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CONTENTS

S_Y_N_O_P_S_I_S

The transient behaviour of synchronous machines has been studied by various authors, by using different concepts, to arrive at the results. The present thesis aims at finding out a unified and comprehensive method for the analysis of the transients of a synchronous generator. The analysis has been carried out for

a) sudden switching on or off of any type of balanced loads.

b) sudden switching on or off of unbalanced loads or faults.

Another method has been presented in this thesis for the analysis of single phase loads and faults only. Experiments are done to compare the results obtained from tests and calculations.

LIST OF SYMBOLS

a, b, c,	= The armature phases lettered along the direction of rotation of the field poles.
d, q	= the direct axis and quadrature axis fixed to the field poles of the rotor, the quadrature axis leading the direct axis.
θ	= the angle of the direct axis with respect to stator
δ	= the power angle.
s	= any odd integer. When s = 1, the s[th] harmonic corresponds to the fundamental frequency.
e_a, e_b, e_c	= the terminal voltages of the phases a, b and c.
e_a^s, e_b^s, e_c^s	= the s[th] harmonic terminal voltages of phases a, b and c
e_1^s, e_2^s, e_0^s	= the s[th] harmonic positive sequence, negative sequence and zero sequence components of the terminal voltage of the phase a.
e_d, e_q, e_0	= the direct axis, quadrature axis and zero sequence armature voltages.
e_{d0}, e_{q0}	= the direct axis and quadrature axis armature voltages for mean position of oscillation.
E_0, E_0'	= the open circuit induced voltage.
E'	= the voltage behind the quadrature axis impedance.
E_d, E_q	= the flux linkages of the armature direct axis and quadrature axis due to air-gap flux.
E_d^0, E_q^0; ψ_{dg}^0; ψ_{qg}^0	= The flux linkages of the armature direct axis and quadrature axis due to the air-gap flux of zero frequency during the transients.

E_1^s, E_2^s = the s^{th} harmonic positive sequence and negative sequence components of the induced voltages of the phase a due to the armature linkage of the air gap flux.

$\psi_{dg}^{s+1}, \psi_{qg}^{s+1}$, ψ_1^s, ψ_2^s = The air gap flux linkages.

E_{d0}, E_{q0} = the flux linkages of the armature direct axis and quadrature axis due to the air gap flux during the balanced steady state operation before the transients.

E_{d0}, E_{q0} = the flux linkages of the armature direct axis due to the air-gap flux for the mean position of oscillation.

E_{d0}^0, E_{q0}^0 = the initial values of E_d^0 and E_q^0

$E_{d0}^{0'}, E_{q0}^{0'}$ = the value of E_d^0 and E_q^0 at the dying away of the amortisseur transients.

E_{ds}^0 = the steady state value of E_d^0 after the transients.

$e_{aL}^s, e_{bL}^s, e_{cL}^s$ = the s^{th} harmonic terminal voltages of the phases a, b and c of the balanced load in an unbalanced system.

$e_{1L}^s, e_{2L}^s, e_{0L}^s$ = the s^{th} harmonic positive sequence, negative sequence and zero sequence components of the terminal voltage of the phase a of the balanced load in an unbalanced system.

i = any current

i_a, i_b, i_c = the armature currents of the phases a, b and c.

i_a^s, i_b^s, i_c^s = the s^{th} harmonic armature currents of the phases a, b and c.

i_1^s, i_2^s, i_0^s = the s^{th} harmonic positive sequence, negative sequence and zero sequence components of the armature current of phase a.

$i_{1F}^s, i_{2F}^s, i_{0F}^s$; $i_{1L}^s, i_{2L}^s, i_{0L}^s$ = the s^{th} harmonic positive sequence, negative sequence and zero sequence components of fault current and balanced load currents, respectively.

i_d, i_q, i_0 = the direct axis, quadrature axis and zero sequence armature currents.

i_{d0}, i_{q0} = the direct axis and quadrature axis armature currents during the balanced steady state operation before the transients.

i_{d0}, i_{q0} = the direct axis and quadrature axis armature currents for the mean position of oscillation.

i_d^{s-1}, i_q^{s-1} = the $(s-1)^{th}$ harmonic direct axis and quadrature axis armature currents during the transients.

i_{d0}^0, i_{q0}^0 = the initial values of i_d^0 and i_q^0

$i_{d0}^{0'}$ = the value of i_d^0 at the dying away of the amortisseur transients.

ψ_a, ψ_b, ψ_c = the flux linkages of the phases a, b and c of the armature.

ψ_d, ψ_q, ψ_0 = the direct axis, quadrature axis and zero sequence

= armature flux linkages.

ψ_{d0}, ψ_{q0} = the direct axis, and quadrature axis armature flux linkages for the mean position of oscillation.

e_{fd}, e_{kd}, e_{kq} = the terminal voltages of the field, direct axis amortisseur and quadrature axis amortisseur.

i_{fd}, i_{kd}, i_{kq} = the field, direct axis amortisseur and quadrature axis amortisseur currents.

i_{fd0} = the field current during the balanced steady-state operation before the transients.

i_{fd0} = the field current for the mean position of oscillation.

$i^{s-1}_{fd}, i^{s-1}_{kd}, i^{s-1}_{kq}$ = the $(s-1)^{th}$ harmonic field, direct axis amortisseur and quadrature axis amortisseur currents during the transients.

$i^{0}_{fd0}, i^{0}_{kd0}, i^{0}_{kq0}$ = the initial values of i^{0}_{fd}, i^{0}_{kd} and i^{0}_{kq}.

$i^{0'}_{fd0}$ = the value of i^{0}_{fd} at the dying away of the amortisseur transients.

$\psi_{fd}, \psi_{kd}, \psi_{kq}$ = the field, direct axis amortisseur and quadrature axis amortisseur flux linkages.

$\psi_{fd0}, \psi_{kd0}, \psi_{kq0}$ = the field, direct axis amortisseur and quadrature axis amortisseur flux linkages during the balanced steady-state operation before the transients.

$\psi^{s-1}_{fd}, \psi^{s-1}_{kd}, \psi^{s-1}_{kq}$ = the $(s-1)^{th}$ harmonic field, direct axis amortisseur and quadrature axis amortisseur flux linkages during the transients.

$\psi^{0}_{fd0}, \psi^{0}_{kd0}, \psi^{0}_{kq0}$ = the initial values of $^{0}_{fd}$, $^{0}_{kd}$ and $^{0}_{kq}$

$\psi^{0\prime}_{fd0}$ = the value of $^{0}_{fd}$ at the dying away of the amortisseur transients.

r = the armature resistance per phase of the synchronous machine.

x_l = the leakage reactance per phase of the synchronous machine.

x_{ad} = the magnetising reactance of the direct axis circuits.

x_{afd} = the magnetising reactance between direct axis armature and the field.

x_{akd} = the magnetising ractance between the direct axis armature and the direct axis amortisseur.

x_{fkd} = the magnetising reactance between the field and the direct axis amortisseur.

x_d = the direct axis self-reactance.

r_{fd}, r_{kd} = the resistance of the field and the direct axis amortisseur.

x_f, x_{kd} = the leakage reactances of the field and the direct axis amortisseur.

x_d', x_d'' = the direct axis transient and sub-transient reactances.

$r'+jx'$, $r''+jx''$ = the direct axis impedances in blocked rotor tests.

x_{aq} = the magnetising reactance of the quadrature axis circuits.

x_{akq} = the magnetising reactance between the quadrature axis armature and the quadrature axis amortisseur.

x_q = the quadrature axis self-reactance.

r_{kq} = the resistance of the quadrature axis amortisseur.

x_{kq} = the leakage reactance of the quadrature axis amortisseur.

x_q'' = the quadrature axis subtransient reactance.

T_{d0}'', T_{q0}'' = the direct axis and quadrature axis subtransient open circuit time constants.

T_{d0}' = the direct axis transient open circuit time constant.

x_0 = the zero sequence reactance per phase of the synchronous machine.

$R+j(x_L-x_c)=R+jx$ = the total balanced load per phase of a balanced system.

$R+jx$ = the equivalent balanced load per phase of an unbalanced system.

r_n, x_n = the resistance and inductance of the neutral impedance of the synchronous machine.

Z_{nt}^s, Z_n^s = the s^{th} harmonic transferred transient and steady-state neutral impedance of the synchronous machine.

$r_L, x_L, \frac{1}{x_{cL}}$ = the resistance, inductance and capacitance per phase of the balanced load in an unbalanced system.

Z_{Lt}^s, Z_L^s = the s^{th} harmonic transferred transient and steady state impedance of the balanced load in an unbalanced system.

r_{nL}, x_{nL} = the neutral resistance and inductance of the neutral impedance of the balanced load in an unbalanced system.

Z_{nLt}^s, Z_{nL}^s = the s^{th} harmonic transferred transient and steady-state neutral impedance of the balanced load in an unbalanced system.

$r_F, x_F, \frac{1}{x_{cF}}$ = the resistance, inductance and capacitance of the fault or unbalanced load.

Z_{Ft}^s, Z_F^s = the s^{th} harmonic transferred transient and steady-state impedance of the fault or unbalanced load.

Z_3^s, Z_4^s = $r_3^s + jx_3^s$; $r_4^s + jx_4^s$; Z_2 = the s^{th} harmonic equivalent steady-state load impedances; the equivalent negative sequence impedance of the synchronous machine.

θ^s, θ^{s+1} = zero time phase angles of the s^{th} and $(s+1)^{th}$ harmonic quantities.

ϕ = the power factor angle.

λ = the phase angle between the quadrature axis and the current vector in the vector diagram of a synchronous machine on balanced loads.

t = time

w = the angular frequency

$|\;|$ = indicates the rms values of a sinusoidally time varying quantity.

Re = the abbreviation for "real part of"

p = d/dt.

P, Q = Power, reactive volt-amperes.

Im(p) = Maximum value of current, expressed as a function of the derivative operator p.

γ = any angle

λ_1 = the angle between the direct axis and the axis of phase a at time t=0.

$x_d(p), x_q(p)$	= the direct axis and quadrature axis operational impedances of the synchronous machine.
E_m	= Maximum value of the voltage before the sudden loading or short circuit.
x_2	= Negative sequence reactance of the synchronous machine.

$A, B, C, D, C_1, C_2, K_1, K_2, K_3, K_4$

are all constants.

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C H A P T E R - I

1.1 INTRODUCTION:-

In normal operation, the synchronous generator, may be subjected to transient conditions, which may result from different possible unbalanced loads and fault conditions or from sudden short circuits. The nature of the behaviour of the synchronous machine under all such abnormal operations, is very much necessary, particularly for large machines, in order to apply proper relaying and for study of stability. Also, the synchronous machine behaviour under sudden inductive loading and capacitive loading is important, especially for interconnected systems, with series and shunt capacitors as found now a days to increase the stability of power system and for voltage regulation purposes.

1.2 REVIEW OF THE PREVIOUS WORK

Short circuits on synchronous machines have been treated, as early as, 1912. Boucherot presented his paper in 1912, after following the early work done by Steinmetz and Berg. He dealt with alternators of the laminated cylindrical rotor type. Diamant reviewed the work of Berg and Boucherot in 1915, and gave expressions for the envelope of short circuit current. In 1918 he presented an analysis of sustained short circuits, dealing principally with the nature of flux distribution under that condition. In the same year Doherty proposed the use of the constant flux linkage theorem in dealing with short circuits, and its use was illustrated in 1921 and 1923. Franklin applied this method to a large number of cases of single phase and 3-phase short circuits in 1923. Additional cases were solved by Laffoon in 1924, by using the same fundamental principles. In 1925 Karapetoff analysed the same cases as those of Franklin and Laffoon, but he started from a somewhat different theoretical basis i.e. from Kirchhoff's Laws. But, he like Franklin and Lafoon neglected resistance, which made the fundamental premises the same, since the Constant Flux Linkage theorem is merely a corollary of Kirchhoff's Second Law. That is, his equations involved voltages, instead of magnetic linkages, and were therefore the first derivatives of the corresponding equations in Franklin's work. Involving the same assumptions regarding the circuits, the results naturally agreed.

In 1923, Lyon outlived a method according to which certain low frequency transient conditions in electric machinery may be analysed by a vector method. From the vector point of view, first proposed by Dreyfus in 1912, the transient voltage or current may shrink exponentially as they rotate. All of the foregoing investigations have dealt with cylindrical rotor machines. Granting the simplifying assumptions regarding resistance and saturation, it may be said that a practical solution of short circuits for the cylindrical rotor type has been attained.

The important case of salient pole synchronous machines was not considered till then. In 1928, Doherty and Nickle were the first to develope the expressions for the transient currents in armature and field circuits for single phase short circuits and partial short circuits, which were applicable to both kinds of synchronous machines. They had considered only one field winding. In 1930, they presented another paper for the study of 3-phase short circuits, applicable again to both types of synchronous machines; though 3-phase short circuit studies were already carried out for cylindrical rotor type only. They had applied constant flux linkage theorem to arrive at the results. They had not made another assumption that the alternating quantities change slowly and this had led to a solution containing an asymmetrical component. Here also they had considered only one field winding.

In 1936 Miller and Weil extended the operational solution given by Park in 1929. Park had neglected the resistance and at the same time the alternator was reduced to an equivalent circuit, resulting in simplicity in its application to electrical circuit theory and in its solution. Miller and Weil had extended the solution by using the constants of the machine and circuits arranged for the particular solution, showing how these constants are measured, converted and applied. In 1937, they used the equations developed in the previous year, to obtain the solution of the currents in the armature and field circuits under unsymmetrical fault conditions. They had applied the symmetrical components of currents and included the resistance in both the armature and field circuits. In 1936, they had made use of two rotor circuits, but in 1937, only one rotor circuit. Smith and Weygandt presented a paper in 1937, in which the starting point was Park's formulae for armature and field flux linkages of an ideal synchronous machine, simplified further by assuming all resistances to be neglegible but not in decrement factors. They used 2 field circuits in the direct axis.

Charles Concordia, gave a full theoretical treatment of the various types of short circuits on a synchronous generator in 1951. For each case, the expressions for voltages and currents are given. He had analysed some of the cases by employing "$\alpha - \beta$" components. The method used by him is to derive the initial values of the components of the currents by approximate methods and to estimate a time constant appropriate to each component.

A more rigorous analysis of unbalanced conditions on synchronous machines was given by Ching Y.K. and Adkin B. in 1954 by using the Laplace transform method. General formulation of the transient problem are given and harmonics during steady unbalanced operation can be calculated.

The purpose of the present thesis is to analyse the transient of a synchronous machine under any type of unbalanced loads and faults by the use of equivalent circuits. Equivalent circuit approach is better since they show more clearly how the results are obtained. All the fundamental frequency currents and voltages and their harmonic components can be obtained from the equivalent circuits.

In Chapter II, mathematical description has been given for a synchronous machine, including the vector diagrams.

In Chapter III, the performance equations for solving the transients of a synchronous generator, including the transient and subtransient stages, have been developed for balanced loads and faults, disregarding the voltage regulator action. Armature transients are neglected. Tests are done with a salient pole synchronous machine and current and voltage oscillograms are taken on sudden switching on and on sudden throw off of capacitive loads. Calculated and tests results are compared.

In Chapter IV, first the transient equivalent circuits for infinite time harmonics for the synchronous machine are developed, neglecting the free transients. Sinusoidal components of currents and voltages are assumed. Then the transient equivalent circuits for the load involving different frequencies are derived and connected with the alternator. Their simplification gives the ladder networks.

In Chapter V, the infinite ladder networks, are simplified for certain types of unbalanced loads or faults.

In Chapter VI, single phase loads and faults are solved in terms of operational impedances, where Park's equations are made use of. Sinusoidal time varying current forms sthe starting point of this method. Then the transients for the unbalanced loads and faults are solved with the help of ladder networks derived in the last chapter and with the method presented in this chapter. Verification with the test results obtained from oscillogram is done.

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C_H_A_P_T_E_R - II

A SYNCHRONOUS MACHINE

2.1 INTRODUCTION:-

This chapter presents a mathematical description of a synchronous machine, its equivalent circuits and steady state vector diagrams. Certain reasonable assumptions are made as follows to do away with the troublesome equations.[1]

The first assumption is that stator windings are all sinusoidally distributed along the air gap as far as all the mutual effects with the rotor circuits are concerned. This assumption is justified, since the windings are usually arranged so as to minimise al the harmonics.

Secondly, it is assumed that all electrical and magnetic circuits in the rotor are symmetrical about both the pole axis or direct axis and interpole or quadrature axis. Though there are damper bars, the iron in the rotor gives rise to infinite number of electrical circuits and therefore exact analysis is difficult to achieve. Usually all the synchronous machines are studied with one amortisseur kd in the direct axis and another kq in the quadrature axis and the same is done here. These amortisseurs in the direct and quadrative axis may be thought of as equivalent to all the amortisseur circuits in the rotor.

The third assumption is that the stator slots cause no appreciable variation of any of the rotor inductances with rotor angle.

Fourthly, the effect of saturation and hysterilisis is neglected.

Lastly, the effect of the presence of eddy currents in conductors is neglected.

2.2 SYNCHRONOUS MACHINE CIRCUITS

A synchronous machine with all three phases a,b,c lettered in the direction of rotation, the field fd and the amortisseurs kd and kq is shown in fig. 2.1. Generator action is taken while writing down the equations.

(a) Armature

$$\begin{aligned} e_a &= p\psi a - r\, i_a \\ e_b &= p\psi b - r\, i_b \\ e_c &= p\psi c - r\, i_c \end{aligned} \qquad (2.1)$$

(b) Field

$$e_{fd} = p\psi_{fd} + r_{fd} \cdot i_{fd} \qquad (2.2)$$

(c) Amortisseurs

$$e_{kd} = 0 = p\psi_{kd} + r_{kd} \cdot i_{kd} \qquad (2.3)$$

$$e_{kq} = 0 = p\psi_{kq} + r_{kq} \cdot i_{kq} \qquad (2.4)$$

where all e's, i's, ψ's are instantaneous values.

The armature voltage equations(2.1) can be transformed to Park's equations by the following transformation equations.

$$\begin{bmatrix} e_d \\ e_q \\ e_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos & \cos(\quad -120) & \cos(\quad +120) \\ -\sin & -\sin(\quad -120) & -\sin(\quad +120) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \qquad (2.5)$$

Exactly similar transformation equations for the flux linkages and armature currents i can be written, once with ψ replacing e and then with i replacing e.

∴ The Part's equations are

$$e_d = p\psi_d - p\theta\,\psi_q - r i_d$$
$$e_q = p\theta\psi_d + p\,\psi_q - r i_q$$
$$e_0 = p\psi_0 - r i_0$$

The reverse transformation of equation(2.5) gives

$$\begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} = \begin{bmatrix} \cos & -\sin & 1 \\ \cos(-120) & -\sin(-120) & 1 \\ \cos(-120) & -\sin(+120) & 1 \end{bmatrix} \begin{bmatrix} e_d \\ e_q \\ e_0 \end{bmatrix} \qquad (2.7)$$

and ψ's and i's transform similarly.

Flux linkages and inductance relations:-

The relations between flux linkages and inductances become much simpler when expressed in dq0 variables.

$$\begin{aligned} \psi_d &= -x_d . i_d + x_{afd} + x_{akd}\, i_d \\ \psi_q &= -x_q\, i_q + x_{akq}\, i_q \\ \psi_0 &= -x_0\, i_0 \\ \psi_{fd} &= -3/2\, x_{afd}.\, i_d + x_{ffd}.i_{fd} + x_{fkd}.\, i_{kd}. \\ \psi_{kd} &= -3/2\, x_{akd}.i_d + x_{fkd}.i_{fd} + x_{kkd}.i_{kd} \\ \psi_{kq} &= -3/2\, x_{akq}.i_q + x_{kkq}.\, i_{kq} \end{aligned} \qquad (2.8)$$

2.3. PER UNIT REPRESENTATION & EQUIVALENT CIRCUITS:

The relations (2.8) can be written as

Direct axis

$$\begin{aligned} \psi_d &= -x_d . i_d + x'_{afd}\, i'_{fd} + x'_{akd}\, i'_{kd} \\ \psi_{fd} &= -x'_{afd}\, i_d + x'_{ffd}.\, i'_{fd} + x'_{fkd}\, i'_{kd} \\ \psi_{kd} &= -x'_{akd}\, i_d + x'_{fkd}\, i'_{fd} + x'_{kkd}\, i'_{kd} \end{aligned} \qquad (2.9)$$

Quadrature axis

$$\psi_q = -x_q . i_q + x'_{akq} . i'_{kq}$$

$$\psi_{kq} = -x'_{akq} . i_q + x'_{kkq} . i'_{kq} \qquad (2.10)$$

Zero sequence axis

$$\psi_0 = -x_0 . i_0 \qquad (2.11)$$

where the primed inductances are $\frac{3}{2}$ times the unprimed inductances in henries, and the currents are $\frac{2}{3}$ times unprimed currents in amperes. (2.12)

Equations (2.9) suggest as if there are three magnetically interlinked coils with reciprocal mutual inductances carrying currents i_d, i_{fd} and i_{kd} in the direct axis. Similarly equations(2.10) suggest the existance of two magnetically coupled coils with current i_q and i_{kq} in the quadrature axis.

The rotor quantities of equations(2.9) and (2.10) can be transferred to the stator or armature side, as is done in transformers, that is, all the quantities may be expressed in per unit values by dividing them with the corresponding base values. The base values for currents are inversely proportional to the number of turns, the base values for voltages are directly proportional to the number of turns and the base values for impedances are directly proportional to the square of the turns.

When all the quantities are expressed in per unit, then the equation (2.9) and (2.10) can be used with primes dropped while the unprimed represents the values in per unit. Assuming the following relations:

$x_{afd} = x_{akd} = x_{ad} = x_{fkd}$ = direct axis magnetising reactance.

$x_\ell = x_d - x_{ad}$ = armature leakage reactance.

$x_f = x_{ffd} - x_{ad}$ = field leakage reactance.

$x_{kd} = x_{kkd} - x_{ad}$ = direct axis amortisseur leakage reactance.

$x_{aq} = x_{akq}$ = quadrature axis magnetising reactance

$x_\ell = x_q - x_{aq}$ (2.13)

$x_{kq} = x_{kkq} - x_{aq}$ = quadrature axis amortisseur leakage reactance.

the equations (2.9) and (2.10) can be written as; Direct axis.

$$\begin{aligned} \psi_d &= -x_l \cdot i_d + x_{ad}(i_{fd}+i_{kd}-i_d) \\ \psi_{fd} &= x_f \cdot i_{fd} + x_{ad}(i_{fd}+i_{kd}-i_d) \\ \psi_{kd} &= x_{kd} \cdot i_{kd} + x_{ad}(i_{fd}+i_{kd}-i_d) \end{aligned} \tag{2.14}$$

Quadrature axis

$$\begin{aligned} \psi_q &= -x_\ell i_q + x_{aq}(i_{kq} - i_q) \\ \psi_{kq} &= x_{kq} i_{kq} + x_{aq}(i_{kq} - i_q) \end{aligned} \tag{2.15}$$

Zero sequence axis

$$\psi_0 = -x_0 \cdot i_0 \tag{2.16}$$

The equivalent circuits of the figures 2.2(a), b & (c) can be drawn from the above equations(2.14) to (2.16) and the voltage relations(2.2) to (2.4).

Reciprocal per unit System:- A system, with similar mutual terms as described above, is known as the reciprocal system. The base values of armature current i_a and voltage e_a are ordinarily determined by the machine rating. The base values for i_d, i_q and e_d, e_q are the same as those of i_a and e_a as the transformation equation(2.5) suggests. If e_a and i_a are expressed in r.m.s. values, they are to be multiplied by $\sqrt{2}$ to get the base values for e_a and i_a.

The base currents for the rotor circuits, can be obtained from the equivalent circuit 2.2(a). The field current i_{fd} in the field produces armature linkages

$\psi_d = x_{ad} . i_{fd}$. On open circuit and at rated speed ($p\theta=1$)

$e_q = \psi_d = x_{ad} . i_{fd}$

On normal armature terminal voltage $e_q=1.0$ when the actual field current corresponding to rated voltage is i_{f0}, then

The required base field current $i_{fd}(\text{base}) = x_{ad}\, i_{f0}$ amps...(2.

Let the absolute value of field resistance r_{fd} in ohms be converted to per unit system. From the air gap line of the open circuit characteristics, $i_{fd}(\text{base})$ can be obtained. Then in the equation (2.9);

$$i'_{fd}\,(\text{base}) = \frac{2}{3}\, i_{fd}\,(\text{base})$$

Effective turns ratio between stator and field turns

$$= \frac{i'_{fd(\text{base})}}{i_{a(\text{base})}}$$

$r'_{fd} = \frac{3}{2}\, r_{fd}$, when referred to stator, will become as in transformers,

$$= r'_{fd} \times (\text{turns ratio})^2$$

$$= r'_{fd} \times \left[\frac{i'_{fd(\text{base})}}{i_a(\text{base})}\right]^2$$

$$= \frac{3}{2}\, r_{fd} \left[\frac{\frac{2}{3}\, i_{fd}\,(\text{base})}{i_a(\text{base})}\right]^2$$

$$= \frac{2}{3}\, r_{fd} \left[\frac{i_{fd(\text{base})}}{i_a(\text{base})}\right]^2$$

When referred to per unit system, this becomes

$$r_{fd}\ (p.u.) = \frac{2}{3}\, r_{fd} \left[\frac{i_{fd(base)}}{i_{a(base)}} \right]^2 \times \frac{i_a(base)}{e_a\ (base)} \qquad (2.18)$$

The base value of $p\,\theta$ = the rated value of $p\,\theta = 2\,\pi f$
where f is the rated frequency.
The base value of p or $\frac{d}{dt}$ = base value of $p\,\theta = 2\,\pi f$ and the base value of time = $\frac{1}{2\,\pi f}$

Thus the inductance in p.u. is the same as the reactance in p.u. at rated frequency, with the above definition of base values.

Henceforth the reciprocal system of units and per unit values will be used, unless otherwise specified.

Frequency Loci[2]: From the equivalent circuit of fig. 2.2(a), it is noted that ψ_d is a function of i_d and e_{fd}. Thus a relation may be obtained as

$$\psi_d = -\,x_d(p)\, i_d + G(p)\, e_{fd}$$

when $e_{fd} = 0$ and i_d is varying sinusoidally at a frequency of w, then p can be replaced by jw.

Therefore, ψ_d can be obtained from the equivalent circuit 2.2(a) by putting $e_{fd} = 0$ and by dividing all the parameters by $w = p/j$. The modified equivalent circuit showing $j\psi_d = -j\, x_d(jw) i_d$ and the corresponding frequency locus are shown in fig. 2.5(a) and (b) respectively. In the figures,

$$x_d'' = x_d(jw) = x_1 + \frac{x_{ad}.x_f.x_{kd}}{x_{ad}.x_f + x_f.x_{kd} + x_{kd}.x_{ad}} \qquad (2.19)$$

In the equivalent circuit 2.2(b), $\psi_q = -x_q(p)\, i_q$

When i_q is an alternating quantity of frequency w, p=jw and ψ_q can be obtained from the equivalent circuit 2.2(b) after dividing all the parameters by w = p/j. The modified equivalent circuit and the corresponding frequency locus are shown in fig. 2.4 (a) and (b) respectively. Also,

$$x_q'' = x_q(j\infty) = x_1 + \frac{x_{aq}.x_{kq}}{x_{aq} + x_{kq}} \qquad (2.20)$$

2.4 STEADY STATE VECTOR DIAGRAM:

For balanced three phase steady state operation

$e_0=0$, $i_0=0$, p=0 in equation in (2.8)

and $p\theta=1$ (at rated frequency). Therefore, the equation (2.8) can be written as

$$e_d = -\psi_q - ri_d \qquad (2.21)$$

$$e_q = \psi_d - ri_q$$

In equations (2.3) and (2.4) also p=0, $i_{kd}=0$; $i_{kq}=0$

∴ Equations (2.14) and (2.15) show that

$$\psi_d = -(x_l + x_{ad})\, i_d + x_{ad}.i_{fd}$$

$$\psi_q = -(x_l + x_{aq})\, i_q$$

Now $i_{fd} = \dfrac{e_{fd}}{r_{fd}}$ is the steady field current, therefore $x_{ad}.i_{fd}$ is the open circuit voltage E_0. So from the equations (2.21) and (2.22),

$$e_d = x_q\, i_q - ri_d$$

$$e_q = E_0 - x_d i_d - ri_q \qquad (2.23)$$

$$= E_0 - (x_d - x_q)\, i_d - x_q i_d - r i_q$$

Putting $E' = E_0 - (x_d - x_q) i_d$

$$e_q \quad = E' - x_q i_d - r i_q$$

It is evident from equations(2.21) and (2.22) that all the quantities referred have steady magnitudes of zero frequency and therefore the armature phase voltages and currents are of fundamental frequency as is obvious from the transformation equations(2.7). Hereafter, the superscripts are employed for the frequencies. Therefore

$$e_a \quad = e_d^o \cos \quad - e_q^o \sin$$

$$i_a \quad = i_d^o \cos \quad - i_q^o \sin$$

where $\quad = t + \quad$ being the angle by which the direct axis leads the axis of phase a at t=0. The vectors e_a and i_a may be expressed in complex form as

$$e_a \quad = e_1{}' = e_d^o + j\, e_q^o$$

$$\text{and } i_a \quad = i_1' = i_d^o + j\, i_q^o \qquad (2.24)$$

The equation (2.23) can be written in terms of these complex values, as

$$e_1' = e_d^o + j\, e_q^o = jE' - (r + j\, x_q) i_1{}' \qquad (2.25)$$

It should be remembered that E_0 and E' are only the magnitudes and not in complex notation.

The phase displacement between $e_1{}'$ and $i_1{}'$ is the same as that between e_a and i_a, since the transformation equations for both voltage and current are of the same form. Thus, the phase currents, phase voltages and the angle between them being known, the vector diagram can be drawn. Add the $i_1{}'(r+j\, x_q)$ voltage drop to the terminal or $P_m = - P_g$

and the power angle δ is the angle by which e_1^1 leads jE_0. So from equation (2.26)

$$P_m = \frac{E_0 \, |e_1^1| \sin\delta}{x_d} + \frac{|e_1^1|^2}{2} \left(\frac{1}{x_q} - \frac{1}{x_d} \right) \sin 2\delta \quad (2.27)$$

Reactive volt-amperes for a generator,

$$Q_g = e_1^1 \,(j\, i_1^1) = e_q^o \, i_d^o - e_d^o \, i_q^o$$

On similar substitution

$$Q_g = \frac{E_0 \, |e_1^1| \cos\delta}{x_d} - \left(\frac{1}{x_q} + \frac{1}{x_d} \right) \frac{|e_1^1|^2}{2} + \left(\frac{1}{x_q} - \frac{1}{x_d} \right) \frac{|e_1^1|^2}{2} \cos 2\delta \quad (2.28)$$

On similar considerations, the reactive volt-amperes for the motor

$$Q_m = \frac{|e_1^1|^2}{2} \left(\frac{1}{x_q} + \frac{1}{x_d} \right) - \left(\frac{1}{x_q} - \frac{1}{x_d} \right) \frac{|e_1^1|^2}{2} \cos 2\delta - \frac{E_0 \, |e_1^1|}{x_d} \cos\delta \quad (2.29)$$

The generator vector diagram of fig. 2.5 can be re-drawn as shown in fig. 2.6 with the new vectors E_d^o and E_q^o introduced[8,9,10]. It is clear from the figure,

$$E_d^o = e_q^o + i_q^o\, r + i_d^o\, x_1$$

$$= \psi_d^o - i_q^o r + i_q^o r + i_d^o\, x_1$$

$$= \psi_d^o + i_d^o\, x_1$$

= air gap flux linkage in the direct axis.

$$= x_{ad}\,(i_{fd}^o - i_d^o) \quad (2.30)$$

Now $E_q^o = e_d^o + i_d^o r - i_q^o \; x_\ell$

$$= - \;{}_{q}^{o} - i_d^o \, r + i_d^o \, r - i_q^o \; x_\ell$$

$$= - (\;{}_{q}^{o} + i_q^o \; x_\ell)$$

= air gap flux linkage in quadrature axis (2.31)

$$= x_{aq} \, i_q^o$$

During steady state, when $i_{kq} = 0$, the line ab = $x_q \, |i_1^1|$

—

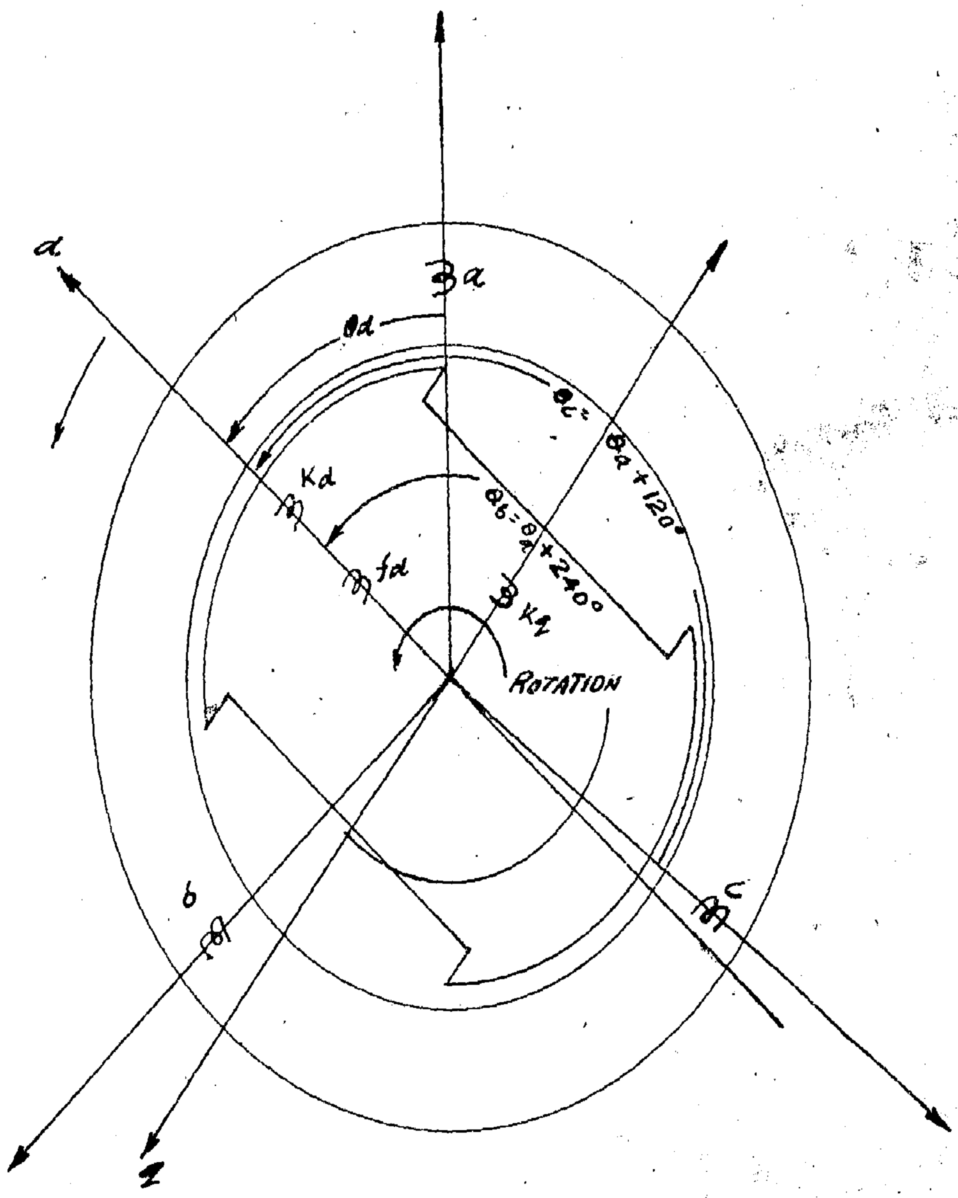
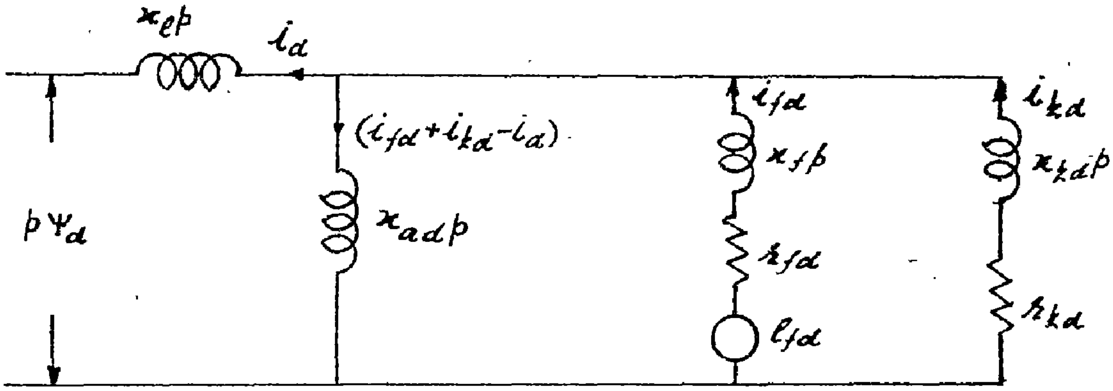
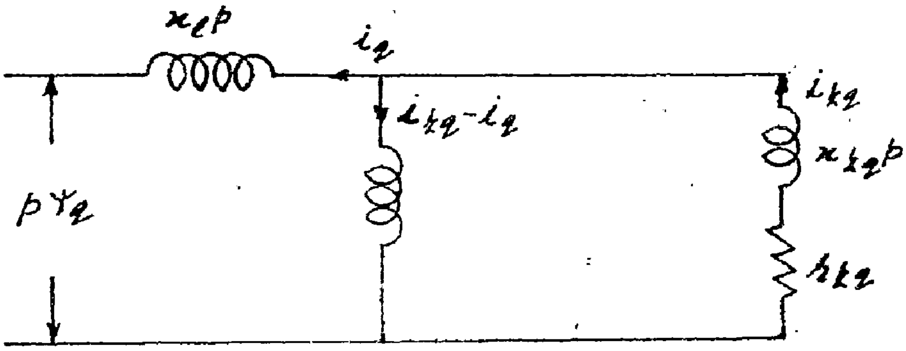


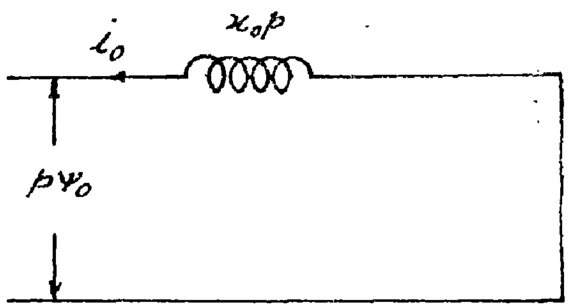
FIG.2-1 DIAGRAM OF A SYNCHRONOUS MACHINE.



(a) DIRECT AXIS.

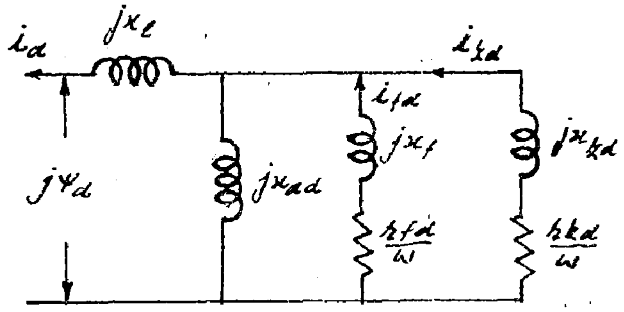


(b) QUADRATURE AXIS.

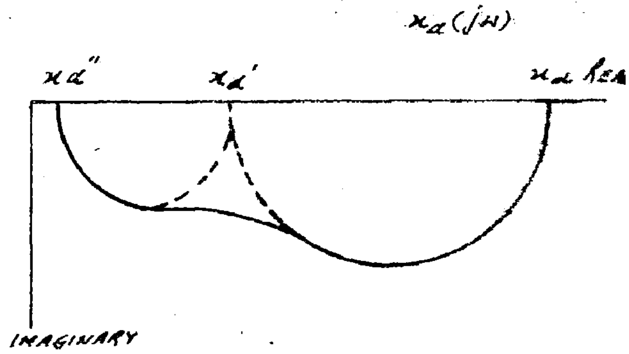


(C) ZERO SEQUENCE AXIS.

FIG. 2-2 EQUIVALENT CIRCUIT OF A SYNCHRONOUS MACHINE.

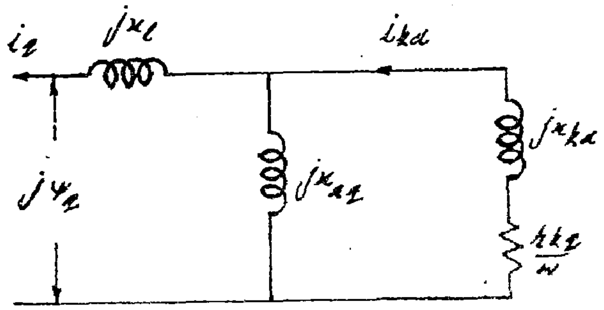


(a) EQUIVALENT CIRCUIT

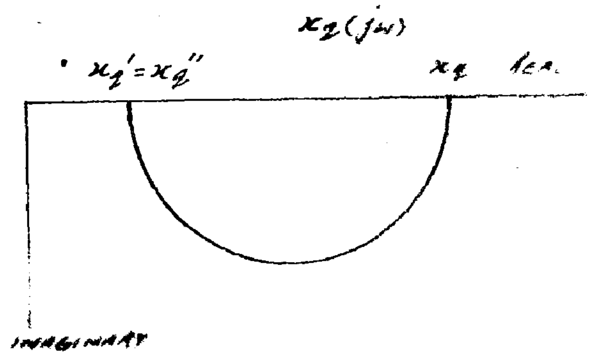


(b) FREQUENCY LOCUS.

FIG. 2-3 DIRECT AXIS.



(a) EQUIVALENT CIRCUIT.



(b) FREQUE...

FIG. 2-4 QUADRATURE AXIS.

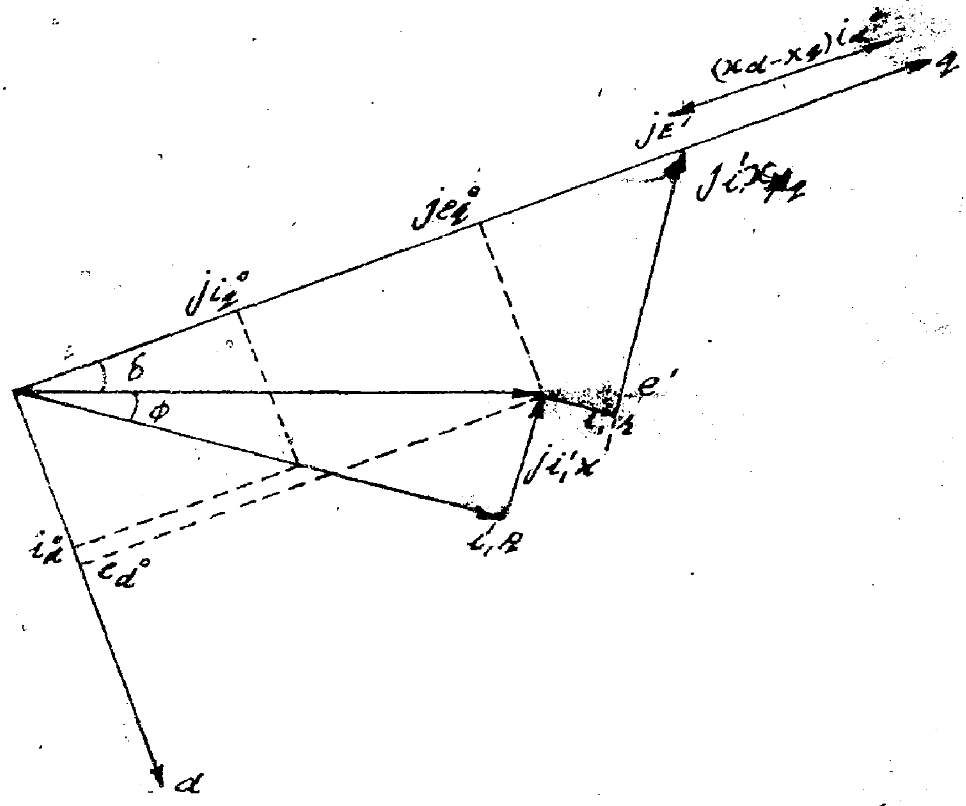


FIG. 2-5 STEADY STATE VECTOR DIAGRAM ON BALANCED LOADS.

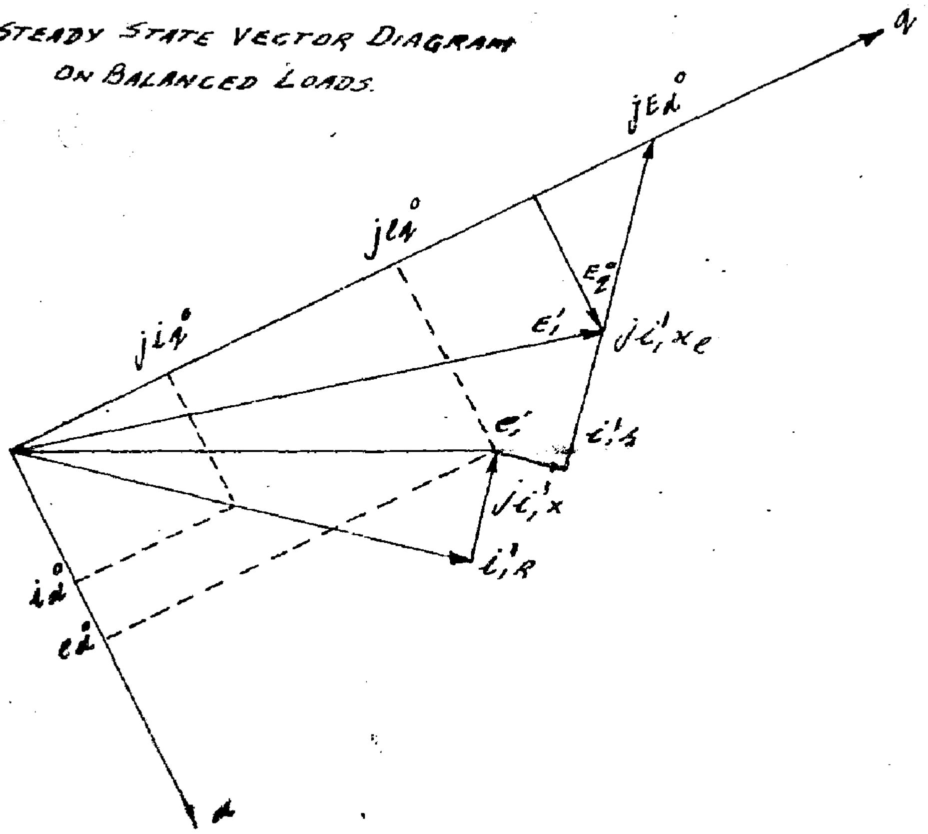


FIG. 2-6 STEADY STATE VECTOR DIAGRAM FOR BALANCED LOADS.

CHAPTER - III

TRANSIENT PERFORMANCE OF A SYNCHRONOUS MACHINE ON BALANCED LOADS OR FAULTS.

3.1. INTRODUCTION:

In this chapter, the transients of a synchronous generator without any voltage regulator, have been analytically solved in two stages - subtransient and post-subtransient, the former being of very short duration compared to the latter.

In addition to the assumptions made in the article(2.1), th following assumptions are made here for the solution of the transient

The solution of transients is divided in two parts(as in a linear static system) - free transient or complementary function, whi is obtained by equating all voltages in the different circuits to zer and particular solution or particular integral which is obtained by solving the simplified differential equations of the circuits with their respective voltages. The actual initial armature current is equal to the sum of the initial value of the free transient of the armature current and the initial value of the particular solution.

Since the rotor circuits have larger time constants, the ra of variation of the d-q axis currents in the particular solution is v slow compared to $p\,\theta$ which is unity.

So, it has been assumed that the amplitude variation of the symmetrical components of the armature voltages and currents is small compared to their frequency variation. This assumption simplifies th particular solution and the whole operation during the subtransient and post-subtransient stages may be referred to steady state vector diagram as deduced for a synchronous machine.

The free transient$\not$ dies away very rapidly i.e., it lasts for a few cycles only - a very short duration as compared to the same of the particular solution, so it can be neglected easily.

It is assumed that the frequency or the speed does not change appreciably during the transients.

The voltage regulator action is disregarded that is, the field voltage is assumed to be constant.

3.2. DIFFERENTIAL EQUATIONS OF THE MACHINE CIRCUITS:-

Let the total impedance per phase of the loads or a fault be $R+j\ (x_L - x_c) = R+jx$ in per unit referred to the machine bases. Th the voltage equation of all the phases can be written as

$$e_a = p\psi_a - ri_a = (R + x_L p + \frac{x_c}{p})i_a$$

$$e_b = p\psi_b - ri_b = (R + x_L p + \frac{x_c}{p})\ i_b \qquad (3.1)$$

$$e_c = p\psi_c - ri_c = (R + x_L p + \frac{x_c}{p})i_c$$

$$p^2\psi_a = \left\{(r+R)\ p + x_L.p^2 + x_c\right\}\ i_a$$

$$p^2\psi_b = \left\{(r+R)\ p + x_L\ p^2 + x_c\right\}\ i_b \qquad (3.2)$$

$$p^2\psi_c = \left\{(r+R)\ p + x_L\ p^2 + x_c\right\}\ i_c$$

The above relations when transformed to the d - q - 0 axes by means of equation (2.5) and similar to it for ψ and i, become

$$p\left[-p\psi_d + x_L p\ i_d + (r+R)i_d + p\theta\psi_q - p\theta\, x_L\ i_q\right] + x_c\ i_d$$

$$= p\theta\left[-p\,\psi_q + x_L p\ i_q + (r+R)i_q - p\theta\psi_d + p\theta x_L\ i_d\right] \qquad (3.3)$$

and

$$p\left[-p\psi_q + X_L p\, i_q + (r+R)i_q - p\theta\psi_d + p\theta X_L\, i_d\right] + X_c\, i_q$$

$$= -p\theta\left[-p\psi_d + X_L p\, i_d + (r+R)i_d + p\theta\psi_q - p\theta X_L\, i_q\right]$$

In balanced loads, the zero sequence currents and voltages are absen

In the above equations $p\theta = 1$

For an inductive load only, $X_c = 0$ and equation (3.1) becomes

$$p\psi_a - \left\{(r+R) + X_L p\right\} i_a$$
$$p\psi_b - \left\{(r+R) + X_L\, p\right\} i_b \qquad (3.1\text{ a})$$
$$p\psi_c - \left\{(r+R) + X_L\, p\right\} i_c$$

The above relations can also be transformed to the d-q-0 axes as is do in obtaining equation (3.3); The result is

$$p\left[\psi_d - X\, i_d - (\psi_q - X i_q) - (r+R)\, i_d = 0\right. \qquad (3.4)$$

and

$$p\left[\psi_q - X i_q\right] + (\psi_d - X i_d) - (r+R)i_q = 0$$

where $X_L = X$

The equation (3.4) can also be obtained from equation (3.3).
The solution of this equation and the equations (2.2), (2.3) and (2.4) determine the performance of the synchronous machine completely.

3.3 THE SOLUTION OF TRANSIENTS:

Since the free transient is neglected, p is assumed to be zero in the particular solution. The equations (3.3) or (3.4) can therefore be written as

$$-\psi_q - r\, i_d = R\, i_d - X\, i_q = e_d$$

and

$$\psi_d - r\, i_q = R\, i_q + X\, i_d = e_q$$

which is similar to equation (2.21)

Thus the whole operation can be referred to the steady state vector diagram, as derived in the article 2.4. So, the above equation is valid even though the load does not consist of series impedances onl; $R + j\,(X_L - X_c) = R + j\,X$ stands for the equivalent total load impedance.

During transients, in the vector diagram 2.6, air gap flux linkage in the direct axis $E_d^o = x_{ad}\,(i_{fd}^o + i_{kd}^o - i_d^o)$, and air gap flux linkage in the quadrature axis,

$$-E_q^o = x_{aq}\,(i_{kq}^o - i_q^o)$$

As explained earlier, the analysis of transients for the armature anc field circuits is divided in two parts - subtransient stage, where tl amortisseur bars are assumed to carry appreciable currents and the post-subtransient stage, where, they have died down to negligible values.

All sinusoidally varying quantities are represented by the complex notation, unless otherwise stated.

3.3.1. ARMATURE CIRCUIT TRANSIENTS :

(i) SUBTRANSIENT STAGE

At this stage, the solution of the transients for an unsaturated synchronous machine is not appreciably different from the same with non-linear magnetising characteristic.

The following relations may be obtained from the vector diagram of fig. 2.6.

$$|e_1^1| = \sqrt{\frac{(E_d^o)^2 + (E_q^o)^2 (R^2 + x^2)}{(R + r)^2 + (X + x_1)^2}} \quad (3.6)$$

$$|i_1^1| = \frac{|e_1^1|}{\sqrt{R^2 + X^2}} \quad (3.7)$$

$$\phi = \tan^{-1} \frac{X}{R} \quad (3.8)$$

$$\lambda = \tan^{-1} \frac{i_d^o}{i_q^o} \quad (3.9)$$

$$\left.\begin{aligned} E_q^o &= (R + r)\, i_d^o - (X + x_1) i_q^o \\ E_d^o &= (R + r)\, i_q + (X + x_1)\, i_d^o \end{aligned}\right\} \quad (3.10)$$

The equation (3.10) can be written as

$$\begin{aligned} i_d^o &= A\, E_d^o + B\, E_q^o \\ i_q^o &= B\, E_d^o - A\, E_q^o \end{aligned} \quad (3.11)$$

where

$$A = \frac{X + x_1}{(R + r)^2 + (X + x_1)^2}$$

$$B = \frac{R + r}{(R + r)^2 + (X + x_1)^2}$$

Determination of the initial values (E_{d0}^o, E_{q0}^o) of E_d^o and E_q^o :-

The flux linkages of rotor can't change suddenly and so they will remain at the same value initially as they were before the sudden switching operation.

Therefore

$$\psi_{k_{d0}} = \psi_{k_{d0}}^o$$

or $$E_{d0} = E_{d0}^{o} + x_{kd}\, i_{kd0}^{o} \qquad (3.12)$$

and $$\psi_{fd0} = \psi_{fd0}^{o}$$

$$E_{do} + x_f\, i_{fd0} = E_{d0}^{o} + x_f\, i_{fd0}^{o} \qquad (3.13)$$

Also $$E_{d0} = x_{ad}\,(i_{fd0} - i_{d0}) \qquad (3.14)$$

$$E_{d0}^{o} = x_{ad}\,(i_{fd0}^{o} + i_{kd0}^{o} - i_{d0}^{o}) \qquad (3.15)$$

Eliminating i_{fd0}^{o} and i_{kd0}^{o} from (3.15) by means of relations (3.12), (3.13) and (3.14).

$$E_{d0} + (x_{d''} - x_{\ell})\, i_{d0} = E_{d0}^{o} + (x_d'' - x_{\ell})\, i_{d0}^{o} \qquad (3.16)$$

where, as defined earlier, the direct axis subtransient reactance

$$x_d'' = x_{\ell} + \frac{x_{ad}\, x_f\, x_{ad}}{x_{ad}\, x_f + x_f\, x_{kd} + x_{kd}\, x_{ad}} \qquad (2.19)$$

Now eliminating i_{d0}^{o} from (3.16) by the equations (3.11) and (3.16),

$$E_{d0} + (x_d'' - x_{\ell})\, i_{d0} = E_{d0}^{o} + (x_d'' - x_{\ell}) \left[A\, E_{d0}^{o} + B\, E_{q0}^{o} \right] \qquad (3.17)$$

Similarly for the quadrature axis,

$$\psi_{kq0} = \psi_{kq0}^{o}$$

or $$-E_{q0} = -E_{q0}^{o} + x_{kq}\, i_{kq0}^{o} \qquad (3.18)$$

Also $$-E_{q0} = -x_{aq}\, i_{q0} \qquad (3.19)$$

$$-E_{q0}^{o} = x_{aq}\,(i_{kq0}^{o} - i_{q0}^{o}) \qquad (3.20)$$

Eliminating i_{kq0}^{o} from (3.20) by the relations(3.18) & (3.19)

$$E_{q0} - (x_q'' - x_\ell)\, i_{q0} = E_{q0}^{o} - (x_q'' - x_\ell)\, i_{q0}^{o} \qquad (3.2$$

where, as defined earlier, the quadrature axis subtransient rea

$$x_q'' = x_\ell + \frac{x_{aq}\; x_{kq}}{x_{aq} + x_{kq}} \qquad (2.2$$

Eliminating i_{q0}^{o} by the equation (3.11)

$$E_{q0} - (x_q'' - x_\ell) i_{q0} = E_{q0}^{o} - (x_q'' - x_\ell)\left[B\, E_{d0}^{o} - A\, E_{q0}^{o}\right] \qquad (3.2$$

Solving (3.17) and (3.22) for E_{d0}^{o} & E_{q0}^{o}, the following relatio are obtained.

$$E_{d0}^{o} = \frac{\left[1 + A\,(x_q - x_\ell)\right]\left[E_{d0} + (x_d - x_\ell) i_{d0}\right] - B(x_d'' - x_\ell)\left[E_{q0} - (x_q''\right.}{\left[1 + A\,(x_q'' - x_\ell)\right]\left[1 + A\,(x_d'' - x_\ell)\right] + B^2 (x_q'' - x_\ell)\,(x_d'' - x_\ell)}$$

$$E_{q0}^{o} = \frac{\left[1 + A\,(x_d - x_\ell)\right]\left[E_{q0} - (x_q'' - x_\ell) i_{q0}\right] + B(x_q'' - x_\ell)\left[E_{d0} + (x_d'' -\right.}{\left[1 + A\,(x_q'' - x_\ell)\right]\left[1 + A\,(x_d'' - x_\ell)\right] + B^2\,(x_q'' - x_\ell)\,(x_d'' - x}$$

..... (3.2

Solution of the amortisseur voltage equations: The amortisseur equations must be expressed in terms of E_d^{o} and E_q^{o}. The soluti these equations will then determine E_d^{o} and E_q^{o} as functions of

Assuming constant field flux linkage in the subtransi stage, which is of very short duration

$$\overset{o}{\psi}_{fd} = \psi_{fd0}$$

or

$$E_d^{o} + x_f\, i_{fd}^{o} = E_{d0} + x_f\, i_{fd0}$$

With the help of equation (3.14)

$$E_d^o + x_f\, i_{fd}^o = E_{d0} + x_f \left(\frac{E_{d0}}{x_{ad}} + i_{d0} \right)$$

$$= E_{d0}\left(1 + \frac{x_f}{x_{ad}}\right) + x_f\, i_{d0} \qquad (3.24)$$

Also $$E_d^o = x_{ad}\,(i_{fd}^o + i_{kd}^o - i_d^o) \qquad (3.25)$$

Eliminating i_{fd}^o and i_d^o from (3.25) by equations (3.24) and (3.11)

$$i_{kd}^o = CE_d^o + BE_q^o - \left[E_{d0}\,(C-A) + i_{d0}\right] \qquad (3.26)$$

where $C = \frac{1}{x_d' - x_l} + A$

and the direct axis transient reactance

$$x_d' = x_l + \frac{x_f\ x_{ad}}{x_f + x_{ad}}$$

Now voltage equation (2.3) for the direct axis amortisseur can be written as,

$$r_{kd}\left[E_{d0}\,(C-A) + i_{d0}\right] = E_d^o\left[(1 + x_{kd}C)p + Cr_{kd} + E_q^o\,(x_{kd}\,B\,p + r_{kd}B)\right] \quad ..(3.2$$

Again

$$-E_q^o = x_{aq}\,(i_{kq}^o - i_q^o) \qquad (3.28)$$

Eliminating i_q^o by equation (3.11)

$$i_{kq}^o = B\,E_d^o - D\,E_q^o \qquad (3.29)$$

where $D = A + \frac{1}{x_{aq}}$

Again the voltage equation (2.4) for the quadrature axis amortisseur can be written as

$$0 = (x_{kq}\,Bp + r_{kq}B)\,E_d^o - \left[(1 + x_{kq}\,D)p + r_{kq}\,D\right]\,E_q^o \qquad (3.30)$$

Even if saturation is considered, the change in values of x_d' and x_a''

not appreciable, since x_f and x_{kd} are very small

3.30) yield the result as

$$\frac{\ldots] \left[E_{d0}(C-A) + i_{d0}\right] r_{kd}}{\ldots_{d}Bp + r_{kd}B) + \left[(1 + x_{kq}D)\, p + r_{kq}D\right] \times \left[(1+x_{kd}C)\, p + r_{kd}C\right]} \qquad (3.31)$$

and

$$E_q^o = \frac{\left[x_{kq}\, Bp + r_{kq}B\right] \left[r_{kd}\left\{E_{d0}(C-A) + i_{d0}\right\}\right]}{(1 + x_{kd}C)p + r_{kd}C \quad (1 + x_{kq}D)\, p + r_{kq}D + \left[x_{kq}\, Bp + r_{kq}B\right] \times \left[x_{kd}Bp + r_{kd}B\right]}$$

Obviously the solution of the above equation is

$$E_d^o = C_1 e^{-\alpha_1 t} + C_2 e^{-\alpha_2 t} + E_{d0}^{o\prime} \qquad (3.32a)$$

where $E_{d0}^{o\prime} = \left[E_{d0}\, (C-A) + i_{d0}\right] \times \dfrac{D}{B^2 + CD}$

and $E_q^o = C_1 e^{-\alpha_1 t} + C_2 e^{-\alpha_2 t} + E_{q0}^{o\prime}$ (3.32 b)

where $E_{q0}^{o\prime} = \left[E_{d0}\, (C-A) + i_{d0}\right] \times \dfrac{B}{B^2 + CD}$

Here α_1 and α_2 are the roots of the equation in p obtained by putting the denominator equal to zero.

The constants C_1 and C_2 can be calculated from the initial values E_{d0}^o and E_{q0}^o as already obtained in eqn. (3.23)

When E_d^o and E_q^o are completely known at any time t, then from the vector diagram fig. 2.6, through the relations (3.6) to (3.10), $|e_1'|$, $|i_1'|$, i_d^o, i_q^o, λ etc. can be determined.

As a special case, consider the sudden throw off of all loads. In this case

$$A = 0 \qquad B = 0 \tag{3.33}$$

S_0, E_{d0}^o & E_{q0}^o from equation (3.25) are

$$\begin{aligned} E_{d0}^o &= E_{d0} + (x_d'' - x_1)\, i_{d0} \\ E_{q0}^o &= E_{q0} - (x_q'' - x_1)\, i_{q0} \end{aligned} \tag{3.34}$$

Also $$C = \frac{1}{x_d' - x_1} \quad \text{and } D = \frac{1}{x_{aq}} \tag{3.35}$$

Therefore, the equations (3.27) and (3.30) take the form

$$r_{kd}\left[C\, E_{d0} + i_{d0}\right] = \left[(1 + x_{kd}\, C)p + r_{kd}C\right] E_d^o \tag{3.36}$$

and

$$\left[(1 + x_{kq}\, D)\, p + r_{kq}D\right] E_q^o = 0 \tag{3.37}$$

The solution of the above equations, gives

$$E_d^o = E_{d0}^{o\prime} - (E_{d0}^{o\prime} - E_{d0}^o)\, e^{-(t/T_{d0}'')}$$

$$E_q^o = E_{q0}^o \quad e^{-(t/T_{q0}'')} \tag{3.38}$$

where

$$E_{d0}^{o\prime} = E_{d0} + (x_d' - x_1)\, i_{d0}$$

and the open circuit direct axis subtransient time constant $T_{d0}'' = \frac{1 + x_{kd}C}{r_{kd}.C} = \frac{x_{aq} + x_{kq}}{r_{kq}}$

Thus it is obvious that if the machine does not contain amortisseur winding, the subtransients do not occur.

ii) <u>POST- SUBTRANSIENT STAGE</u>

When the amortisseur currents have disappeared, in the vector diagram of fig. 2.6, the line ab = $x_q \, |i_1^1|$. Thus the following relations can be determined from fig. 2.6

$$i_d^o = |i_1^1| \sin\lambda = k_1 \, |i_1^1|$$
$$i_q^o = |i_1^1| \cos\lambda = k_2 \, i_1^1 \qquad (3.39)$$

where

$$k_1 = \frac{X + x_q}{\sqrt{(R + r)^2 + (X + x_q)^2}}$$

$$k_2 = \frac{R + r}{\sqrt{(R + r)^2 + (X + x_q)^2}}$$

Also

$$E_d^o = (R+r)\, i_q + (X + x_1)\, i_d^o + k_3\, i_d^o$$
$$E_q^o = (x_q - x_1)\, i_q^o \quad ; \; |i_1^1| = k_4 E_d^o \qquad (3.40)$$
$$|e_1^1| = \sqrt{R^2 + X^2} \, . \, K_4 \, E_d^o$$

where

$$k_3 = \frac{(R + r)k_2}{k_1} + (X + x_1) \quad , \; k_4 = 1/(k_3 k_1)$$

Therefore, the transients can be determined in terms of only one unknown E_d^o

Determination of the initial value $E_{d0}^{o'}$ - The initial value of E_d^o for the post-subtransient stage, ($E_{d0}^{o'}$) has been determined already in the equation 3.32 as

$$E_{d0}^{o'} = \left[E_{d0}\ (C-A) + i_{d0}\right] \times \frac{D}{B^2 + CD}$$

The initial value of E_d^o ($E_{d0}^{o'}$) can also be determined in terms of the constants of the equation (3.40).

Assuming that the field flux linkage has remained constant as it was before the sudden switching operation;

Therefore $\psi_{fd0} = \psi_{fd0}^{o'}$

or

$$E_{d0} + x_f\ i_{fd0} = E_{d0}^{o'} + x_f\ i_{fd0}^{o'} \qquad (3.41)$$

Also $$E_{d0} = x_{ad}\ (i_{fd0} - i_{d0}) \qquad (3.14)$$

$$E_{d0}^{o'} = x_{ad}\ (i_{fd0}^{o'} - i_{d0}^{o'}) \qquad (3.42)$$

From the equation (3.40); $i_{d0}^o = \frac{E_{d0}^{o'}}{K_3}$

Solving the above equations for $E_{d0}^{o'}$

$$E_{d0}^{o'} = \frac{E_{d0} + (x_d' - x_\ell)\, i_{d0}}{1 + (x_d' - x_\ell)/k_3} \qquad (3.43)$$

As already explained, the value of x_d' is not appreciably affected due to saturation, so the above equation (3.43) is applicable even when the saturation in the pole-axis is considered.

After the subtransient period i.e. during the post-subtransient period, the air gap flux E_d^o is given by

$$E_d^o = x_{ad}(i_{fd}^o - i_d^o \;) \tag{3.44}$$

or

$$i_{fd}^o = \frac{E_d^o\left(1 + \frac{x_{ad}}{k_3}\right)}{x_{ad}} \tag{3.45}$$

The voltage equation of the field, from equation (2.2) is

$$e_{fd} = p\,\psi_{fd}^o \; + r_{fd}\; i_{fd}^o \tag{3.46}$$

Now ψ_{fd}^o = Total flux linked by the field coil.

= air gap flux E_d^o + field leakage flux

= $E_d^o + x_f\; i_{fd}^o$

From equation (3.45)

$$\psi_{fd}^o = E_d^o \left[1 + \frac{x_f}{x_{ad}} + \frac{x_f}{k_3} \right] \tag{3.47}$$

Substitution of the values of i_{fd}^o and ψ_{fd}^o from equations (3.45) and (3.47) in the equation (3.46) results.

$$e_{fd} = E_d^o \left[\left(1 + \frac{x_f}{x_{ad}} + \frac{x_f}{k_3}\right) p + \frac{r_{fd}}{x_{ad}} + \frac{r_{fd}}{k_3} \right]$$

$$\therefore E_d^o = \left\{ \frac{e_{fd}}{p\left[1 + \frac{x_f}{x_{ad}} + \frac{x_f}{k_3}\right] + \frac{r_{fd}}{x_{ad}} + \frac{r_{fd}}{k_3}} \right\} \tag{3.48}$$

Clearly, the solution of the above equation is

$$E_d^o = C_1 e^{-\beta t} + E_{ds}^o \tag{3.49}$$

where

$$|e_1^1| = \left[|e_1^1| \text{ subtransient } -k_4 \sqrt{R^2 + X^2}\, E_{d0}^{o'} \right] + |e_1^1| \text{ post-subtransient} \qquad (3.53)$$

The current $|i_1^1|$ may be obtained from the relation (3.7)

3.3.2 FIELD TRANSIENTS

Since the field flux linkage is constant, the field currnet for the subtransient stage may be written as

$$i_{fd}^{o} = \frac{E_{d0} + x_f\, i_{fd0} - E_d^o}{x_f} = \frac{E_{d0}^{o'} + x_f\, i_{fd0}^{'o} - E_d^o}{x_f} \qquad (3.54)$$

But during the subtransient period

$$E_d^o = C_1 e^{-\alpha_1 t} + C_2 e^{-\alpha_2 t} + E_{d0}^{o'}$$

So equation (3.54) can be written as

$$i_{fd}^{o} = i_{fd0}^{o'} - \frac{1}{x_f} \left(C_1 e^{-\alpha_1 t} + C_2 e^{-\alpha_2 t} \right) \qquad (3.55)$$

where

$$i_{fd0}^{o'} = \frac{E_{d0} + x_f\, i_{fd0} - E_{d0}^{o'}}{x_f}$$

For the post-subtransient period, the expression for field current has already been obtained in equation (3.45).

Since the time constants for the amortisseur transient are small, the equations(3.45) and (3.55) may be combined into a single equation such as

$$i_{fd}^{o} = \frac{E_d^o \left(1 + \frac{x_{ad}}{k_3}\right)}{x_{ad}} - \frac{1}{x_f} \left(C_1 e^{-\alpha_1 t} + C_2 e^{-\alpha_2 t} \right) \qquad (3.56)$$

For sudden throw-off of all loads, according to the equations(3.38) and (3.55) the field current during the subtransient stage is,

$$i_{fd}^{o} = i_{fd0}^{o'} - \frac{1}{x_f}(E_{d0}^{o} - E_{d0}^{o'})\ e^{-t/T_{d0}''} \qquad (3.57)$$

During the post-subtransient stage, the field current is determined by equation (3.45) with $\frac{1}{k_3} = 0$ $\quad i_{fd}^{0} = \frac{E_d^{o}}{x_{ad}}$ So the combined equation for the field transient is

$$i_{fd}^{o} = \frac{E_d^{o}}{x_{ad}} - \frac{1}{x_f}(E_{d0}^{o} - E_{d0}^{o'})\ e^{-t/T_{d0}''} \qquad (3.58)$$

3.4 TESTS AND VERIFICATIONS:

A salient pole synchronous machine (G.E.) rated at 110 volts, 21 amperes, 4 KVA, 50 c/s, 1000 r.p.m. star was selected for tests and verifications. Voltages and currents oscillograms for sudden capacitive loading and sudden throw-off of capacitive loads have been taken on this machine. The quadrature axis amortisseur bars are incomplete. The oscillograms are compared with the values calculated by the method described in the last article. The following tests have been conducted to determine the parameters of the machine, which are necessary for calculation purposes. They are all in p.u. unless stated otherwise.

3.4.1. TESTS

Test No. 1 The resistance per phase of the armature has been measured by ordinary voltmeter ammeter method with a d.c. supply.

The corrected a.c. resistance = $r = 0.0333$ (3.59)

Test No.2 The d.c. field resistance has been measured similarly.

$$r_{fd(ohmic)} = 29 \text{ ohms.} \qquad (3.60)$$

Test No.3 Block rotor tests have been done on (i) direct axis (ii) and the quadrature axis. The values obtained are

direct axis - $0.0445 + j\,0.1122 = r' + jx'$

Quadrature axis - $0.0605 + j\,0.364$ (3.61)

Test No.4 Blocked rotor test on the direct axis has been carried out with the field circuit open. The value obtained is

$$r'' + jx'' = 0.0740 + j\,0.148 \qquad (3.62)$$

Test No.5 The open circuit and short circuit tests have been performed. The curves are shown in fig. 3.1 where the induced voltage is the line to line voltage and the short circuit current is the phase current of the star-connected armature. The value of x_d is obtained from the air gap line and the short circuit characteristic.

$$x_d = 0.900 \qquad (3.63)$$

Test No.6 The maximum-lagging-current test has been done on unloaded machine with a d.c. motor coupled with it supplying the no load losses. The negative excitation of the machine has been gradually increased till the rotor slips by one pol pitch accompanied by a sudden drop in current. The armature and the terminal voltage of the machine have been recorded at this point, that is when the armature current was maximum. Then the Quadrature axis synchronous is given by

$$x_q = \frac{|e_1^1|}{i_m} \qquad (3.64)$$

where i_m is the maximum armature current.

This test was carried out and the result was $x_q = 0.432$ (3.65)

Test No. 7 Reluctance motor test: The synchronous machine is run as a reluntance motor and the load is increased till it steps out of synchronism. At this juncture i.e. at maximum power conditions, current, volts and power are recorded.

Now reluctance power $P = \frac{V^2}{2} \left(\frac{1}{x_q} - \frac{1}{x_d} \right) \sin 2\delta$ (3.66)

For maximum reluctance power $\delta = 45^0$

$$\therefore P_{max} = \frac{V^2}{2} \left(\frac{1}{x_q} - \frac{1}{x_d}\right) \qquad (3.67)$$

Assume $\frac{x_d}{x_q} = k$

$$\therefore P_{max.} = \frac{V^2}{2x_q} \left(1 - \frac{1}{k}\right) \quad \ldots(3.68)$$

From the vector diagram

$V \sin \quad = x_q I_q$

$$\therefore I_q = \frac{V}{\sqrt{2}\, x_q} \quad \ldots(3.69)$$

Also

$$E_0 - V \cos\delta = I_d x_d$$

$\therefore I_d = - \frac{V}{\sqrt{2}\, x_d}$ Here $E_0 = 0$, because of no field excitation. (3.70)

Now $$I = \sqrt{I_d^2 + I_q^2} = \frac{V}{\sqrt{2}} \sqrt{\frac{1}{x_d^2} + \frac{1}{x_q^2}}$$

$$= \frac{V}{\sqrt{2}\, x_q} \frac{\sqrt{k^2 + 1}}{k} \qquad (3.71)$$

From relation (3.68) and (3.71)

$$\frac{P_{max}}{I} = \frac{V}{\sqrt{2}} \left(\frac{k-1}{\sqrt{k^2+1}} \right) \qquad (3.72)$$

P_{max}, V and I being known, k can be determined.

The value of k which is greater than one is retained. Thus the value of x_d and x_q can be obtained. An advantage of this method is that the values of x_d and x_q can be found under approximately normal operating conditions.

This test gave the following results:

$$x_d = 0.897 \text{ p.u.} \qquad x_q = 0.431 \text{ p.u.} \qquad (3.73)$$

Test No.8 The slip test has been carried out for the verification of the values of x_d and x_q obtained earlier. The values obtained are

$$x_d = 0.86 \qquad x_q = 0.393 \qquad (3.74)$$

These values are in close proximity with the values already obtained. However, the values chosen for x_d and x_q are the following, since slip test has some inherent errors.

$$x_d = 0.897 \qquad x_q = 0.431 \qquad (3.73)$$

The values of r_{fd}, x_f and x_l can be calculated, from the above tests as explained below. In the equation (3.61), the direct axis value $r' + j\,x' = 0.0445 + j\,0.1122$ is the value of the impedance of the circuit consisting of $j\,x_{ad}$, $r_{fd} + j\,x_f$ and $r_{kd} + j\,x_{kd}$ as is evident, from the equivalent circuit of fig. 2.3(a). Again $r'' + jx'' = 0.0740 + j\,0.1480$ of the equation (3.62) is the value of the impedance of the circuit consisting of $r + j\,x_l$ in series with the parallel combination of $j\,x_{ad}$ and $r_{kd} + j\,x_{kd}$. Therefore

$$\frac{1}{(r'-r) + j(x' - x_l)} - \frac{1}{(r'' - r) + j(x'' - x_l)} = \frac{1}{r_{fd} + j\,x_f} \qquad (3.75)$$

In this expression, only x_l is unknown in the left hand side. Thus the values of r_{fd} and x_f have been calculated for different values of x_l and they have been plotted on the base of x_l as shown in fig. 3.2. Also the values of r_{fd} determined from the ohmic value r_{fd} obtained from test No. 2 for different values of x_l are plotted on the same abscissa of x_l. The intersecting point between these two curves gives the values of x_f, r_{fd} and x_l.

As obtained in the relation (2.18), the per unit value of field resistance

$$r_{fd} = \frac{2}{3}\, r_{fd}(\text{ohmic}) \left[\frac{i_{fd(base)}}{i_{a(base)}}\right]^2 \times \frac{i_{a(base)}}{e_{a(base)}} \qquad (2.18)$$

where $i_{fd(base)} = x_{ad}\, i_{f0}$ (2.17)

i_{f0} being the actual field current corresponding to the rated voltage.

Now i_{f0} from the air gap line of the open circuit characteristic corresponds to 1.497 ampers. Therefore

$$i_{fd(base)} = (x_d - x_1) \times 1.497 \quad \text{amperes}$$

$$i_{a(base)} = 21 \times \sqrt{2} \quad \text{amperes} \qquad (3.76)$$

$$e_{a(base)} = (110 \times \sqrt{2}) / \sqrt{3} \quad \text{volts}$$

The value of r_{fd} are calculated for different values of x_1 by the relations (3.76) and (2.18) and they have been plotted in fig. 3.2.

The intersecting points between these two curves give

$$r_{fd} = 0.01082$$

$$x_1 = 0.0803 \qquad (3.77)$$

$$x_f = 0.0565$$

r_{kq} and x_{kd} are obtained from equation (3.62). Thus, the entire parameters of the synchronous machine as determined to a fair approximation from the above tests, are listed below:

$$x_{ad} = x_d - x_1 = 0.897 - 0.0803 = 0.8167$$

$$x_{aq} = x_q - x_1 = 0.431 - 0.0803 = 0.3507$$

$$x_d' = 0.133$$

$$x_d'' = 0.1122$$

$$x_q' = x_q'' = 0.364 \qquad (3.78)$$

$$r_{kd} = 0.0482$$

$$x_{kd} = 0.0710$$

Thus

$$i_{fd(base)} = (x_d - x_1) \times 1.497 \text{ amperes}$$

$$= 1.222 \text{ amperes} \qquad (3.78)$$

3.4.2. TRANSIENT LOAD TEST

(i) SUDDEN CAPACITIVE LOADING:-

The machine initially generating rated frequency and rated voltage on no load, has been suddenly loaded with a capacitive reactance of $R + jX = 0-j5.26$. The d.c. supply given to the field is assumed to be taken from a constant large capacity generator. The field has been connected in series with a resistance to the above d.c. supply and the steady field current i_{fd0} is 1.58 amperes.

After the machine parameters from tests have been obtained, the particular solution of the transient can be determined. The operation during the subtransient stage is determined approximately since the amortisseur has a very small time constant. The free transients, as discussed earlier, is disregarded.

The parameters obtained from all equations(3.59),(3.64),(3.73),(3.77) and (3.78) are

r	$= 0.0333$	x_{kd}	$= 0.0710$
x_1	$= 0.0803$	x_d	$= 0.897$
x_f	$= 0.0565$	x_d'	$= 0.133$
r_{fd}	$= 0.01082$	x_d''	$= 0.1122$
r_{kd}	$= 0.0482$	x_q'	$= 0.364$

(a) Subtransient Stage:-

The resistance in the armature, being very small, is neglected for the solution of the subtransient period

$$A = \frac{1}{X + x_1} = -0.1871 \qquad C = A + \frac{1}{x_d' - x_1} = 18.763$$

$B = 0$

$$X = -5.26 \qquad D = A + \frac{1}{x_{qq}} = 2.134$$

So

$$E_d^o = \frac{E_{d0}}{1 + (x_d'' - x_1)\,A} = 1.006$$

$$E_{q0}^o = E_q^o = 0$$

$$E_{d0}^o = \frac{E_{d0}\,(C-A)}{C} = \frac{E_{d0}}{1 + \frac{x_d' - x_1}{k_3}} = 1.011$$

There is a single time constant

$$\frac{1}{\alpha_1} = \frac{1}{\alpha_2} = \frac{1 + x_{kd}C}{r_{kd}C} = 2.575$$

Therefore, for the subtransient stage

$$E_d^o = 1.011 - 0.005e^{-t/2.575}$$

As discussed, the term $0.005e^{-t/2.575}$ subtracted from the time function of E_d^o in the post-subtransient stage, gives the total transients.

(b) Post-Subtransient Stage:-

$$R + jX = 0 - j5.26$$

From relations (3.39) & (3.40) and fig. (2.6)

$$i_d^o = k_1 \left| i_1^1 \right|$$

$$i_q^o = k_2 \left| i_1^1 \right|$$

where

$$k_1 = \sin\lambda = \frac{X + x_q}{\sqrt{(R + r)^2 + (X + x_q)^2}} = -0.999 \qquad (3.79)$$

$$k_2 = \cos\lambda = 0.0446$$

and

$$E_d^o = k_3\, i_d^o$$

$$e_1^1 = \sqrt{R^2 + X^2}\,.\; k_4\, E_d^o$$

where

$$k_3 = \frac{(R + r)\, k_2}{k_1} + (X + x_1) = -5.181 \qquad (3.80)$$

$$\frac{1}{k_3} = -0.193$$

$$\sqrt{R^2 + X^2}\,.\, k_4 = 1.015$$

The base field current = $i_{fd(base)}$ = 1.222 amperes

Therefore

$$i_{fd0} = \frac{1.58}{1.222} = 1.293$$

The total resistance in the field circuit

$$r_{fd}(\text{ohmic}) = \frac{75}{1.58} = 47.4 \text{ ohms} \qquad (3.81)$$

$$= \frac{47.4}{29} \times 0.01082 = 0.0177 \text{ p.u.}$$

the alternator field resistance being 29 ohms as in equation (3.60)

Therefore the field voltage

$$e_{fd} = r_{fd} \times i_{fd0} \qquad (3.82)$$

$$= 0.0177 \times 1.293 = 0.0229$$

From the relations (3.48) and (3.49)

$$E_d^o = C_1 e^{-\beta t} + E_{ds}^o \qquad (3.49)$$

Also

$$E_d^o = \frac{e_{fd}}{p\left[1 + \frac{x_f}{x_{ad}} + \frac{x_f}{k_3}\right] + \frac{r_{fd}}{x_{ad}} + \frac{r_{fd}}{k_3}} \qquad (3.48)$$

where β is the root of the equation (3.48) in p, obtained by putting the denominator =0

Therefore $$\beta = \frac{r_{fd}\,(k_3 + x_{ad})}{x_{ad}\,k_3 + x_f(k_3 + x_{ad})} = 0.01728$$

Also $$E_{ds}^o = \frac{e_{fd}}{r_{fd}\left(\frac{1}{x_{ad}} + \frac{1}{k_3}\right)} = 1.257$$

So

$$E_d^o = C_1\, e^{-0.01728t} + 1.257$$

or

$$E_d^o = 1.257 - 0.246\, e^{-0.01728\, t}$$

Thus the total transients are given by

$$E_d^o = 1.257 - 0.246\, e^{-0.01728t} - e^{-.3985t} \qquad (3.83)$$

The values of the terminal voltage

$$\left|e_1^1\right| = 1.015\, E_d^o \qquad (3.80)$$

are plotted on a base of time t in the figure 3.3 incorporating sub-transient stage also as given in equation (3.83). The current $\left|i_1^1\right| = \frac{e_1}{\sqrt{R^2 + X^2}}$ is also plotted on the base of time t in the figure 3.4.

Oscillogram: The current and voltage oscillograms are shown in figure

3.5. The curves-voltage Vs. time and the amplitude of symmetrical component of current Vs. time are also shown in fig. 3.3 and 3.4 respectively.

(ii) SUDDEN THROW OFF LOADS

The machine initially supplying current to an impedance $R + jX = 0 - j5.26$ at the rated terminal voltage and at the rated speed is suddenly unloaded. The steady value of the field current is 1.415 amperes taken from a large source of voltage of 50.5 volts in series with a resistance.

(a) Subtransient Stage

The subtransient stage is determined approximately since the amortisseur has a very small time constant.

$$A = 0 \qquad C = A + \frac{1}{x_d' - x_1} = 18.97$$

$$B = 0 \qquad D = A + \frac{1}{x_{aq}} = 2.858$$

Load resistance is zero and the armature resistance being small, is neglected. Therefore, the Quadrature axis components are all absent.

$$E_{d0} = \left[\frac{|e_1^1|}{X}(X + x_1)\right]_{\text{initial}} = 0.985$$

$$i_{d0} = \left[\frac{|e_1^1|}{X}\right]_{\text{initial}} = -0.1901$$

So

$$E_{d0}^{o} = E_{d0} + (x_d'' - x_1)\, i_{d0} = 0.979$$

$$E_{d0}^{o} = E_{d0} + \frac{i_{d0}}{C} = E_{d0} + (x_d' - x_1)\, i_{d0} = 0.975$$

$$T_{d0}'' = \frac{1 + x_{kd}\, C}{r_{kd} C} = 2.56$$

Therefore, the equation (2.38) gives

$$E_d^o = 0.975 + 0.01\, e^{-t/2.56}$$

(b) Post-Subtransient Stage:-

$$\left.\begin{aligned} i_{fd} &= \frac{1.415}{1.222} = 1.159 \\ r_{fd(ohmic)} &= \frac{50.5}{1.415} = 35.7 \text{ ohms.} \\ r_{fd} &= \frac{35.7}{29} \times 0.01082 = 0.01331 \end{aligned}\right\} \quad (3.84)$$

Therefore, the field voltage

$$e_{fd} = r_{fd} \times i_{fd} = 0.01331 \times 1.159 = 0.01542 \quad (3.85)$$

Now

$$E_d^o = C_1 e^{-\beta t} + E_{ds}^o$$

where

$$\beta = \frac{r_{fd}\,(k_3 + x_{ad})}{x_{ad}\, k_3 + x_f\, k_3 + x_f\, x_{ad}}$$

Here $\frac{1}{k_3} = 0$

Therefore

$$\beta = \frac{r_{fd}}{x_{ad} + x_f} = 0.01523$$

The initial value

$$E_{d0}^{o\prime} = \frac{E_{d0} + (x_{d'} - x_1)\, i_{d0}}{1 + \frac{x_d' - x_1}{k_3}}$$

where $\frac{1}{k_3} = 0$

$$\left.\begin{aligned} E_{d0} &= \left[\frac{e_1^1}{\phi\sqrt{R^2 + X^2 \cdot k_4}}\right]_{initial} = \frac{1}{1.015} = 0.985 \\ i_{d0} &= \left[\frac{E_d^o}{k_3}\right]_{initial} = -\frac{0.193}{1.015} = 0.19 \end{aligned}\right\} \quad (3.80)$$

Therefore, the value of

$$E_{d0}^{o} = 0.975$$

$$E_{ds}^{o} = \frac{e_{fd}}{r_{fd}\left(\frac{1}{x_{ad}} + \frac{1}{k_3}\right)} = \frac{e_{fd}}{r_{fd}} x_{ad}$$

$$= 1.159 \times 0.8167 = 0.946$$

$$\therefore E_d^o = 0.029\, e^{-0.01523\, t} + 0.946$$

Therefore the total transients are given by

$$E_d^o = 0.946 + 0.01e^{-t/2.56} + 0.029\, e^{-0.01523\, t} \tag{3.86}$$

The value of terminal voltage $e_1^1 = E_d^o$ Vs. time t incorporating subtransient stage has been plotted in fig. 3.6

Oscillogram: The oscillogram for voltage is shown in Fig. 3.7. The curve of the voltage Vs. time obtained from the oscillogram is drawn in fig. 3.6 to compare with the calculated values.

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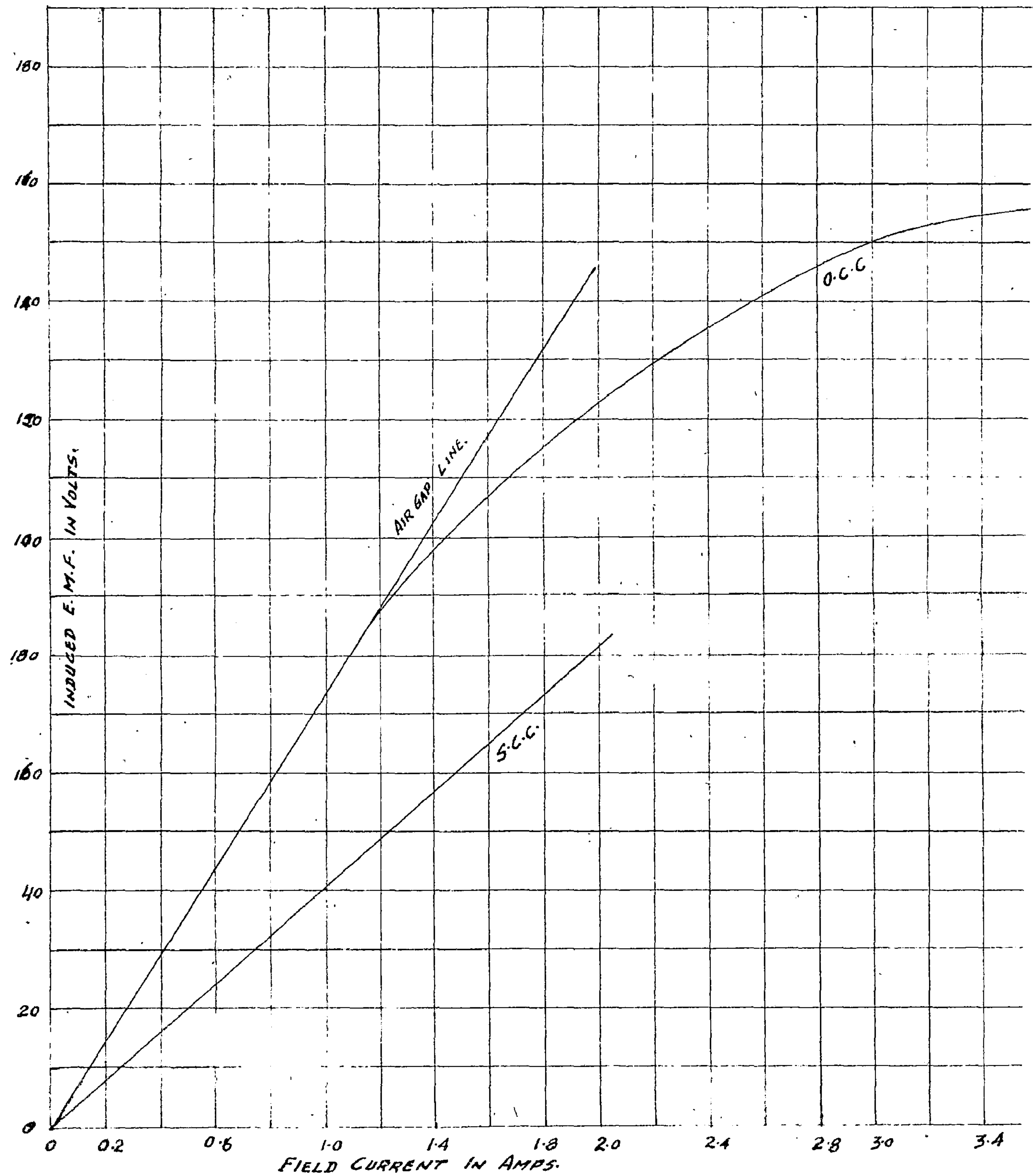
FIG. 3-1 OPEN CIRCUIT AND SHORT CIRCUIT CHARACTERISTICS.



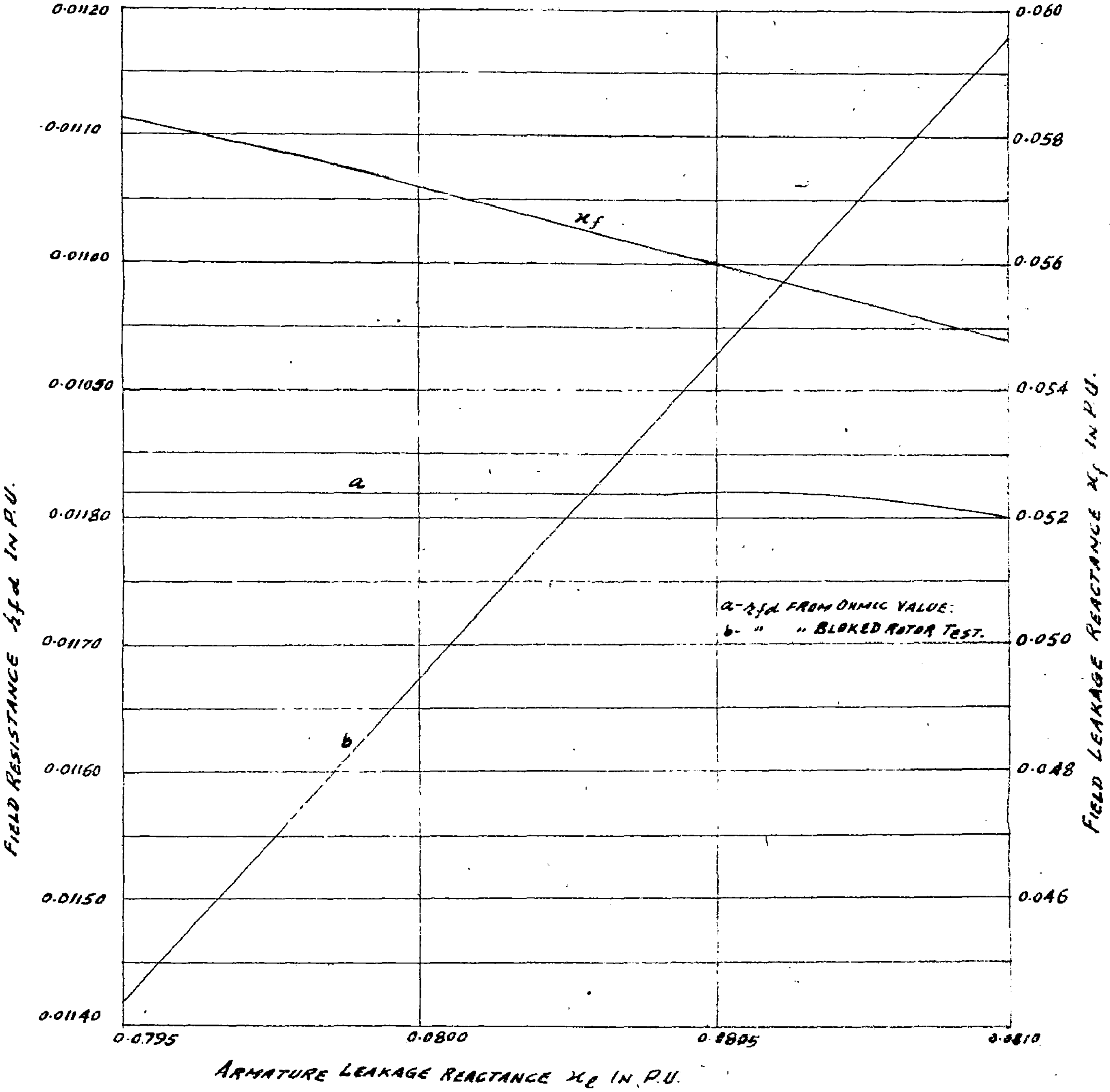


FIG. 3 2 FIELD RESISTANCE AND LEAKAGE REACTANCES.

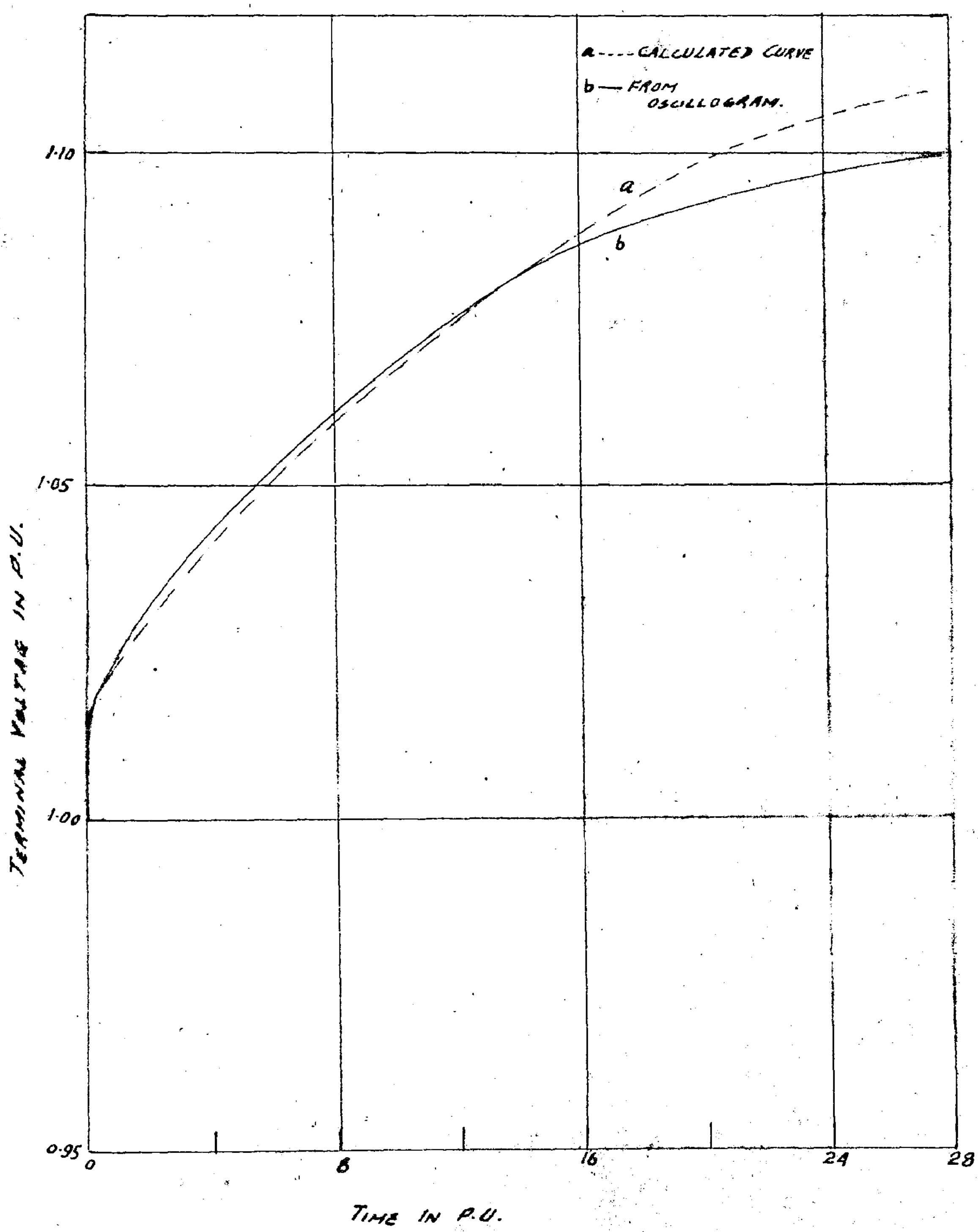


FIG. 3·3 SUDDEN CAPACITIVE LOADING.

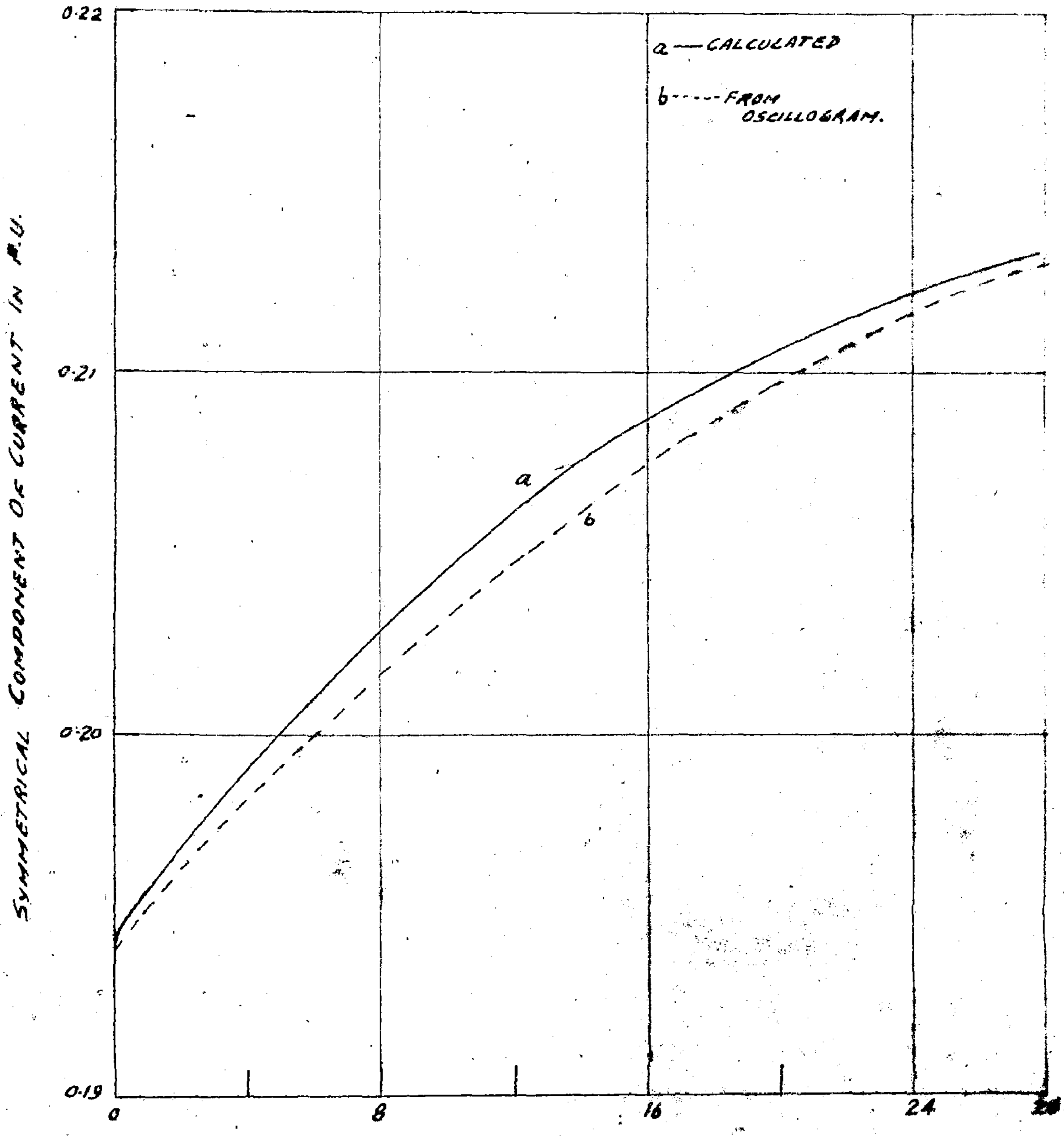
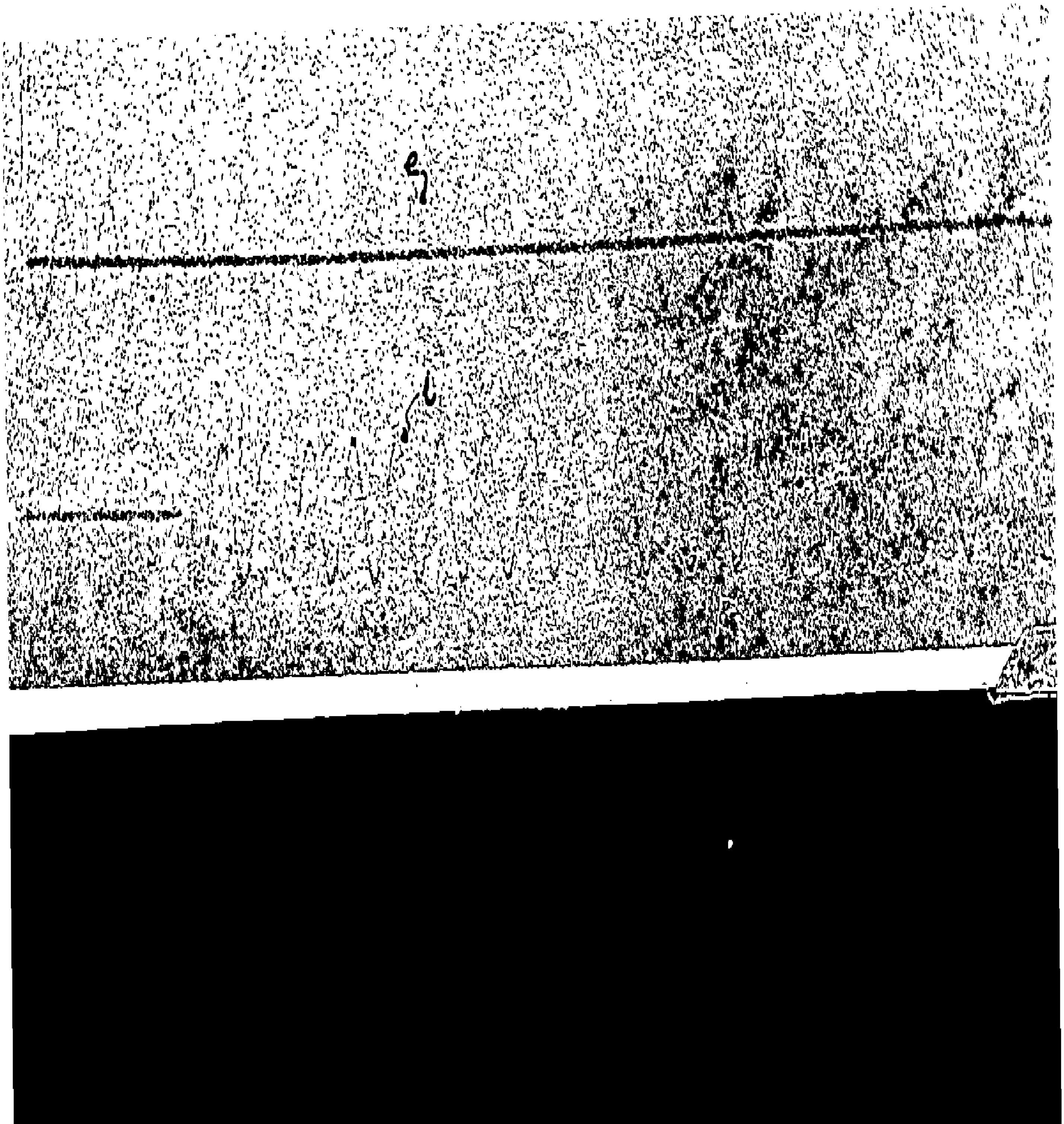


FIG. 3-4 SUDDEN CAPACITIVE LOADING.



-5 OSCILLOGRAM OF SUDDEN CAPACITIVE LOADING

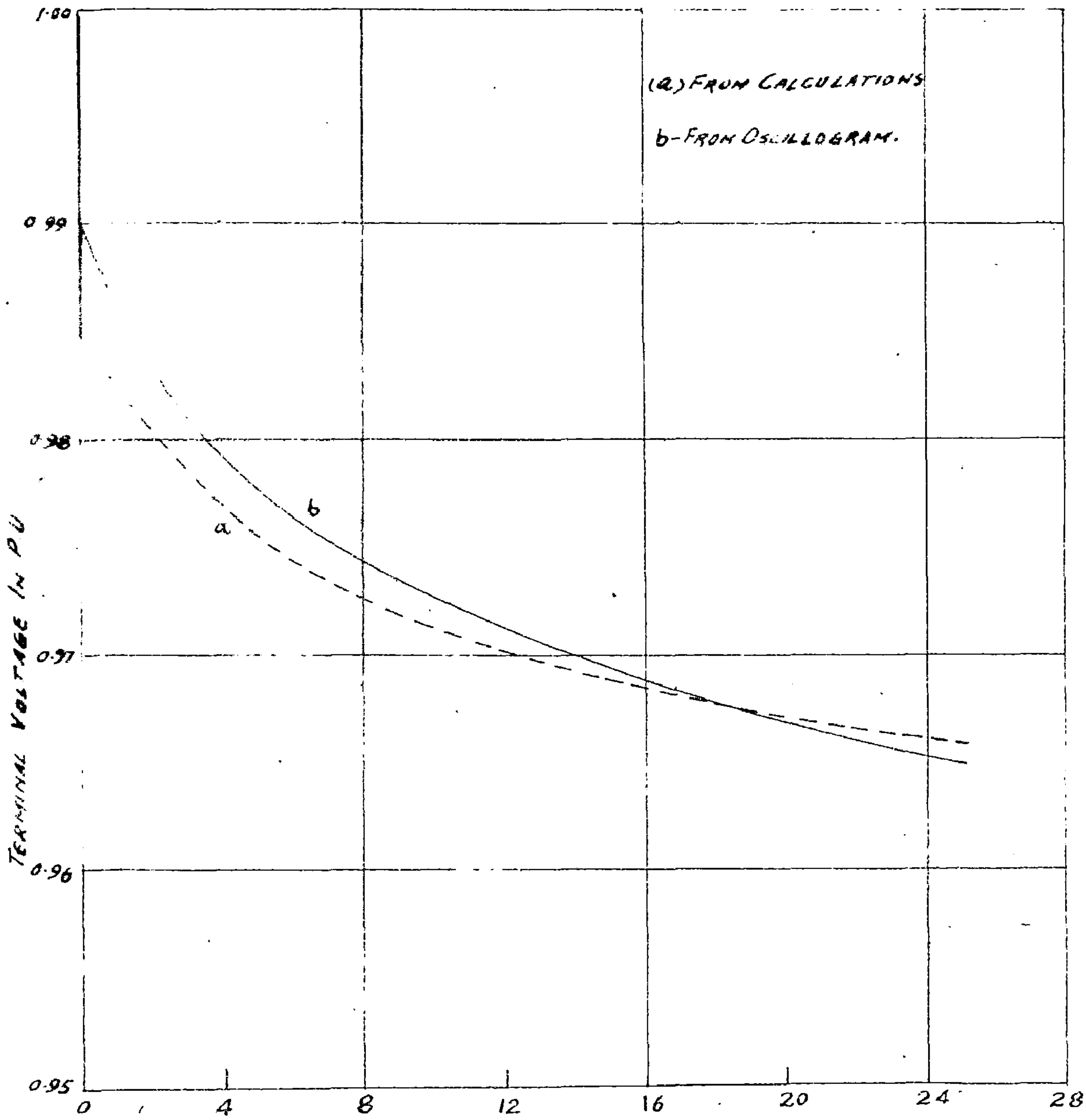
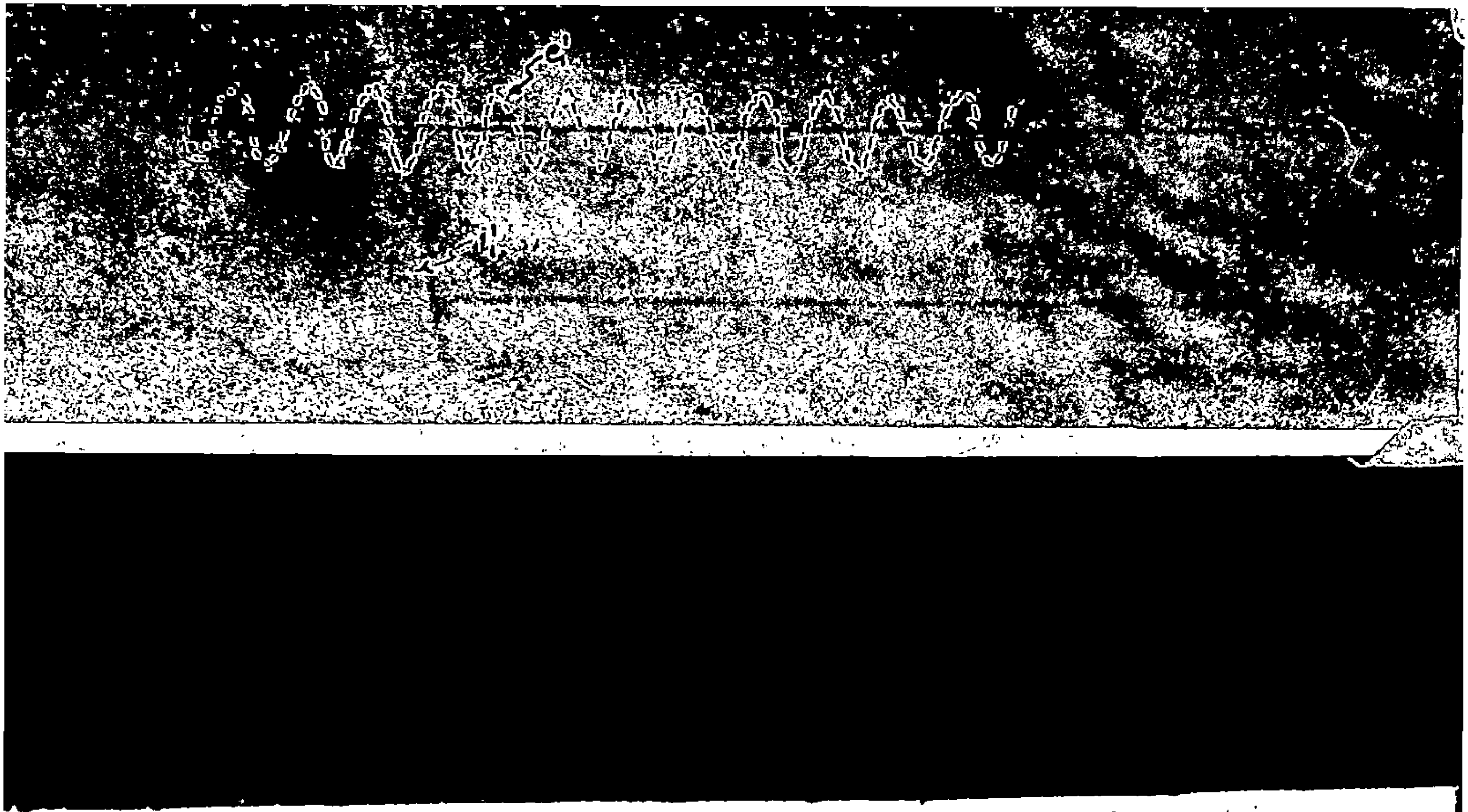


FIG. 3-6 SUDDEN THROW-OFF OF LOADS.



3-7 OSCILLOGRAM OF SUDDEN THROW-OFF OF LOADS

C_H_A_P_T_E_R - IV

THE EQUIVALENT CIRCUITS OF A SYNCHRONOUS MACHINE UNDER UNBALANCED LOAD AND FAULT CONDITIONS

4.1. INTRODUCTION :

In this chapter, the equivalent circuits of a synchronous machine, for usual type of faults and unbalanced loads, have been developed, making use of the conventional symmetrical components.

First of all, the transient equivalent circuits of the synchronous machine, involving positive sequence and negative sequence components of currents, voltages and flux linkages of infinite time harmonics have been developed, neglecting the zero sequence components, which have been considered along with the load circuits. Then the transient equivalent circuits of the load, for different types of faults etc. involging again the infinite time harmonics, have been developed and connected with the alternator equivalent circuits. The scheme of interconnection is same for different frequencies depending upon the type of fault or loading. Further, it has been possible to employ complex notation for the sinusoidally varying currents, voltages and flux linkages, since the armature transients have been neglected.

Assumptions made in the article 2.1 for the derivation of Park's equations are also applicable here. Further, the following assumptions are made here:

(a) The saturation in pole axis, quadrature axis and leakage paths is neglected.

(b) At second, or higher harmonics frequencies, $x_d(jw)$ and $x_q(jw)$ are assumed to be equal to the subtransient reactances x_d'' and x_q'' respectively.

((c) The speed of the machine or its frequency is assumed to remain constant at its initial rated value during the transients.

The amplitude variation of the symmetrical components of the armature voltages, currents and flux linkages has been assumed to be small compared to their frequency variation. This assumption is justified that the rotor circuits have larger time constants and thus the rate of variation of the d-q axis currents in the particular solution is very slow compared to $p\theta$ which is unity. On this assumption, the transient equivalent circuits developed are transferred to equivalent circuits, similar to the steady-state equivalent circuits, developed by Kron. It should be noted that the rms values of currents, voltages and flux linkages are slowly varying with time. After simplification of these circuits, the ladder networks are obtained showing the distribution of alternator currents of different harmonics at any instant.

4.2. DETERMINATION OF THE TRANSIENT EQUIVALENT CIRCUITS

4.2.1. ALTERNATOR EQUIVALENT CIRCUITS

The linkages, due to the zero frequency air-gap flux in the direct axis and quadrature axis can be expressed as

$$\psi_{dg}^{o} = x_{ad}\,(i_{fd}^{o} + i_{kd}^{o} - i_{d}^{o}) = E_{d}^{o} \qquad (4.1)$$

and

$$\psi_{qg}^{o} = -x_{aq}\,(i_{q}^{o} - i_{kq}^{o}) = -E_{q}^{o} \qquad (4.2)$$

The positive sequence voltage of the fundamental frequency induced in the armature due to air gap flux linkage is given by

$$p\left[\psi_{dg}^{o}\cos(t+\theta') - \psi_{qg}^{o}\sin(t+\theta')\right]$$

$$= p\psi_{dg}^{o} \cos(t+\theta') - \psi_{dg}^{o} \sin(t+\theta') - p\psi_{qg}^{o} \sin(t+\theta')$$

$$- \psi_{qg}^{o} \cos(t+\theta')$$

$$= p\psi_{dg}^{o} \cos(t+\theta') + \psi_{dg}^{o} \cos(t+\theta'+90^{o})$$

$$-p \; {}_{qg}^{o} \sin(t+\theta') - \psi_{qg}^{o} \sin(t+\theta'+90^{o})$$

$$= (p+j1)\,\psi_{dg}^{o} \cos(t+\theta') - (p+j1)\,\psi_{qg}^{o} \sin(t+\theta')$$

$$= (p+j1)\left[\psi_{dg}^{o} \cos(t+\theta') + {}_{qg}^{o} \cos(t+\theta'+90^{o})\right]$$

Here θ^1 is the angle by which the direct axis leads the phase a at the instant of the starting of the transients, when time t is assumed to be equal to zero.

The above can be written in complexor form as

$$E_1^1 = (p+j1)\,(\psi_{dg}^{o} + j\psi_{qg}^{o})$$

or

$$\frac{jE_1^1}{p+j1} = j\,(\psi_{dg}^{o} + j\psi_{qg}^{o}) = x_{aq}\,(i_k^{o} - i_{kq}^{o})$$

$$+ j\,x_{ad}\,(i_{fd}^{o} + i_{kd}^{o} - i_d^{o}) \qquad (4.3)$$

The positive sequence component of i_a^1 due to i_d^o and i_q^o

$$= i_d^{o} \cos(t+\theta') - i_q^{o} \sin(t+\theta')$$
$$= i_d^{o} \cos(t+\theta') + i_q^{o} \cos(t+\theta'+90^{o})$$

or, in complex notation

$$i_1^1 = i_d^{o} + j\,i_q^{o} \qquad (4.4)$$

At any other higher even harmonic of (s+1)th order, for the d,q axis

$$\psi_{dg}^{s+1} = -(x_d'' - x_1)\, i_d^{s+1}$$

$$\psi_{qg}^{s+1} = -(x_q'' - x_1)\, i_q^{s+1} \qquad (4.5)$$

The $(s+1)^{th}$ harmonic currents in d,q axis can be split into two parts, such as

$$i_d^{s+1} = \tfrac{1}{2}\,(i_d^{s+1} + j\, i_q^{s+1}) + \tfrac{1}{2}\,(i_d^{s+1} - j\, i_q^{s+1})$$

and

$$i_q^{s+1} = -\tfrac{1}{2}j\,(i_d^{s+1} + j\, i_q^{s+1}) + \tfrac{1}{2}j\,(i_d^{s+1} - j\, i_q^{s+1})$$

The first part of each of the above expressions corresponds to positive sequence components of i_a of frequency (s+2) times the fundamental, that is,

$$i_a^{s+2} = \tfrac{1}{2}\,(i_d^{s+1} + j\, i_q^{s+1}) \qquad (4.6a)$$

as

$$\mathrm{Re}\Big[\tfrac{1}{2}\,(i_d^{s+1} + j\, i_q^{s+1})\, e^{j(\overline{s+1}\, t + {}^{s+1})} \times \cos(t + \theta^1)$$

$$+ \tfrac{1}{2}j\,(i_d^{s+1} + j\, i_q^{s+1})\, e^{j(\overline{s+1}\, t + {}^{s+1})} \sin(t + \theta^1)\Big]$$

$$= \mathrm{Re}\Big[\tfrac{1}{2}\,(i_d^{s+1} + j\, i_q^{s+1})\, e^{j(\overline{s+2}\, t + \theta^{s+1} + \theta^1)}\Big]$$

The other components of i_d^{s+1} and i_q^{s+1} correspond to the negative sequênce component of i_a of s times the fundamental, that is,

$$i_2^{s} = \tfrac{1}{2}\,(i_d^{s+1} - j\, i_q^{s+1}) \qquad (4.6\text{ b})$$

as

$$\text{Re}\ \tfrac{1}{2}\,(i_d^{s+1} - j\,i_q^{s+1})\,e^{j(s+1\,t\,+\,\theta^{s+1})}\cos(t+\theta^1)$$

$$-\tfrac{1}{2}\,j(i_d^{s+1} - j\,i_q^{s+1})\,e^{j(s+1\,t\,+\,\theta^{s+1})}\sin(t+\theta^1)$$

$$= \text{Re}\ \tfrac{1}{2}\,(i_d^{s+1} - j\,i_q^{s+1})\,e^{j(s\,t\,+\,\theta^{s+1} - \theta^1)}$$

It is observed from the above expressions that the positive sequence component of i_a of fundamental frequency is introduced by the direct and quadrature axis currents of zero frequency and vice versa. Also, the fundamental frequency negative sequence component of i_a is due to the direct and quadrature axis currents of double the fundamenta frequency and vice versa. Thus only the even harmonics are present in the field and amortisseur windings and odd harmonics in the particular solution of the armature. It is also seen from the above two expressions, that

$$\theta^{s+1} + \theta^1 = \theta^{s+2}$$

and

$$\theta^s = \theta^{s+1} - \theta^1$$

Since the above two equations are valid for any odd value of s,

$$(s+2)\theta^1 = \theta^{s+2}$$

and

$$s\theta^1 = \theta^s \tag{4.7}$$

Now the symmetrical components for the flux linkages may also be introduced, as

$$\psi_1^{s+2} = \tfrac{1}{2}\left[\psi_{dg}^{s+1} + j\psi_{qg}^{s+1}\right]$$
$$\psi_2^{s} = \tfrac{1}{2}\left[\psi_{dg}^{s+1} - j\psi_{qg}^{s+1}\right] \tag{4.8}$$

Relations (4.5), (4.6a), (4.6b) and (4.8) result in

$$j\psi_1^{s+2} = -\left[j\,(x_d'' - x_1)\, i_1^{s+2} + j\,\frac{x_q'' - x_d''}{2}\,(i_1^{s+2} - i_2^{s}) \right] \quad \ldots (4.9)$$

or

$$j\psi_1^{s+2} = -\left[j\,(x_q'' - x_1)\, i_1^{s+2} + j\,\frac{x_d'' - x_q''}{2}\,(i_1^{s+2} + i_2^{s}) \right] \quad \ldots (4.9a)$$

and

$$j\psi_2^{s} = -\left[j\,(x_d'' - x_1)\, i_2^{s} - j\,\frac{x_q'' - x_d''}{2}\,(i_1^{s+2} - i_2^{s}) \right] \quad \ldots (4.10)$$

or

$$j\psi_2^{s} = -\left[j\,(x_q'' - x_1)\, i_2^{s} + j\,\frac{x_d'' - x_q''}{2}\,(i_1^{s+2} + i_2^{s}) \right] \quad \ldots (4.1$$

Now the $(s+2)^{th}$ harmonic positive sequence voltage induced in phase a due to the air gap flux is

$$= p\left[\psi_1^{s+2}\; e^{j(\overline{s+2}\, t + \overline{s+2}\, \theta^1)}\right]$$

$$= (p + j\; s+2) \left[\psi_1^{s+2}\; e^{j(\overline{s+2}t + \overline{s+2}\theta^1)}\right]$$

Expressing it in complex notation,

$$\frac{jE_1^{s+2}}{p + j^{s+2}} = j\,\psi_1^{s+2} \qquad (4.11)$$

Similarly for the negative sequence component of s^{th} harmonic

$$\frac{j\,E_2^{s}}{p + j^{s}} = j\,\psi_2^{s} \qquad (4.12)$$

Now E_1^s and E_2^s are the induced positive and negative sequence components of s^{th} harmonic of phase a. So the terminal voltage with respect to either generator star-point or ground (assuming the generator to be earthed through an impedance) satisfy the following relations:

$$e_1^s = E_1^s - (r + x_1 \overline{p + j^s})\ i_1^s$$

$$e_2^s = E_2^s - (r + x_\ell \overline{p + j^s})\ i_2^s$$

since

$$p\,(i\epsilon^{j\overline{st + s\theta}^1}) = (p + js)\ i.\epsilon^{j\overline{st + s\theta}^1}$$

The above equations can be written as

$$\frac{je_1^s}{p+js} = \frac{j\,E_1^s}{p+js} - \left(\frac{jr}{p+js} + j\,x_1\right) i_1^s$$

(4.13)

$$\frac{j\,e_2^s}{p+js} = \frac{j\,E_2^s}{p+js} - \left(\frac{jr}{p+js} + j\,x_\ell\right) i_2^s$$

Now the transient equivalent circuits for the synchronous machine are developed in two different ways as shown in figures 4.1(a) and 4.1(b). The fundamental positive sequence equivalent circuit is derived from the equations (4.1), to (4.4) and (4.13), in both the diagrams. The harmonic equivalent circuits of fig. 4.1(a) have been derived from the equations (4.5), (4.6a), (4.6b), (4.8) to (4.13), wher $(i_1^{s+2} - i_2^s)$ flows through the common branch of reactance $j\left(\frac{x_q'' - x_d''}{2}\right)$. The harmonic equivalent circuits of fig. 4.1(b) have been derived from the equations (4.5),(4.6a),(4.6b),(4.8),(4.9a), (4.10a), (4.11) to (4.13), where $i_1^{s+2} + i_2^s$ flows through the common branch of reactance $j\,\frac{x_d'' - x_q''}{2}$. The zero sequence components will be considered along with the load equivalent circuits.

4.2.2. LOAD EQUIVALENT CIRCUITS:

The synchronous machine, grounded through a resistance r_n and an inductance x_n in series, has been supplying a balanced equivalent star - connected load, each phase having a resistance r_L, inductance x_L and a capacitance $1/x_{cL}$ all in series, the star-point of the load being grounded through a resistance r_{nL} and an inductance x_{nL} in series. The transient equivalent circuits are developed for the following cases of unbalanced loads or faults on the above machine.

(a) A single phase load or fault between one line and generator star-point.

(b) A single phase load or fault between two lines.

(c) A load or fault between two lines shorted and generator star point.

(d) A load or fault between two lines shorted and third line.

(e) One line or phase open.

(f) Simultaneous opening and connecting of one phase to the generator star-point at the alternator end.

(a) A single phase load or fault between one line and generator star-point.

Let the load or fault has a resistance of r_F an inductance of x_F and a capacitance of $\frac{1}{x_{cF}}$ all in series as shown in figure 4.2(a).

From the above figure, the following current relations may be written as:

$$\left.\begin{aligned} i_1^s &= i_{1L}^s + i_{1F}^s \\ i_2^s &= i_{2L} + i_{2F}^s \\ i_0^s &= i_{0L}^s + i_{0F}^s \end{aligned}\right\} \qquad (4.14)$$

$$i_{bF}^s = i_{eF}^s = 0$$

or

$$i_{1F}^{s} = i_{2F}^{s} = i_{0F}^{s}$$

and

$$i_{1F}^{s} = 1/3\; i_{aF}^{s} \qquad (4.15)$$

Assuming the generator star-point as the reference potential, the following voltage equations may be written as

$$e_1^s = e_{1L}^s,\quad e_2^s = e_{2L}^s,\quad e_0^s = e_{0L}^s$$

$$\frac{je_1^s}{p+js} = Z_{Lt}^{s} \cdot i_{1L}^{s}$$

$$\frac{j\,e_2^s}{p+js} = Z_{Lt}^{s}\; i_{2L}^{s} \qquad (4.16)$$

$$\frac{je_0^s}{p+js} = \left[Z_{Lt}^{s} + 3\,(Z_{nLt}^{s} + Z_{nt}^{s})\right] \quad i_{0L}^{s}$$

where the transferred transient impedances are defined as

$$Z_{Lt}^{s} = \frac{jr_L}{p+js} + jx_L + \frac{jx_{cL}}{(p+js)^2}$$

$$Z_{nLt}^{s} = \frac{jr_{nL}}{p+js} + j\,x_{nL}$$

$$Z_{nt}^{s} = \frac{jr_n}{p+js} + jx_n$$

In the equation (4.16) the following identity has been considered.

$$p\left[i.e^{\,j\overline{st+s\theta}^{1}}\right] = (p+js)\left[i.e^{\,j\overline{st+s\theta}^{1}}\right]$$

and $$1/p\left[i.e^{\,j\overline{st+s\theta}^{1}}\right] = \frac{1}{(p+js)}\left[i.e^{\,j\overline{st+s\theta}^{1}}\right]$$

Also

$$\frac{je_a^s}{p+js} = Z_{Ft}^s \; i_{aF}^s$$

With the equation (4.15) the above can be written as

$$\frac{je_1^s}{p+js} + \frac{je_2^s}{p+js} + \frac{je^s}{p+js} = Z_{Ft}^s \; 3\, i_{1F}^s \qquad (4.17)$$

The zero sequence voltage equation from the alternator side may be written as:

$$\frac{je_0^s}{p+js} = - Z_{0t}^s \; i_0^s \qquad (4.18)$$

where the transient transferred impedance is defined as

$$Z_{0t}^s = \frac{jr}{p+js} + jx_0$$

Now considering the equations (4.14), (4.15),(4.17); and (4.18), the transient load equivalent circuit can be derived as shown in figure 4.2(b) with reference to the alternator equivalent circuit of figure 4.1(a)

(b) A single phase load or fault between two lines-

Let the fault or load of resistance r_f inductance x_f and capacitance $1/x_{cf}$ be suddenly switched across the phases b and c as shown in figure 4.3 (a)

The current equation (4.14) are also valid here.

Further

$$i_{bF}^s + i_{cF}^s = 0$$

and

$$i_{aF}^s = 0$$

Expressing in symmetrical components,

$$i_{0F}^s = 0$$

$$i_{1F}^s + i_{2F}^s = 0 \qquad (4.19)$$

The voltage equations, assuming the ground as the reference potential may be written as

$$e_1^s = e_{1L}^s , \; e_2^s = e_{2L}^s \quad e_0^s = e_{0L}^s$$

$$\frac{je_1^s}{p+js} = Z_{Lt}^s \, i_{1L}^s \qquad \frac{je_2^s}{p+js} = Z_{Lt}^s \, i_{2L}^s \tag{4.20}$$

$$\frac{je_0^s}{p+js} = (Z_{Lt}^s + 3\, Z_{nLt}^s)\, i_{0L}^s$$

Also

$$\frac{je_b^s - je_c^s}{p+js} = Z_{Ft}^s \; i_{bF}^s$$

Expressing in terms of symmetrical components

$$\frac{je_1^s - je_2^s}{p+js} = Z_{Ft}^s \, i_{1F}^s \tag{4.21}$$

Considering the equations (4.14),(4.19) to (4.21), the equivalent circuits of the loads may be derived as shown in figure 4.3(b), with reference to the alternator equivalent circuit of figure 4.1(b)

A load or fault between two lines shorted and generator star point.

Let the fault or load impedance across the lines b,c shorted and the ground star-point have a resistance r_F an inductance x_F and a capacitance $1/x_{cF}$ as shown in fig. 4.4(a).

The current relations (4.14) are also valid here.

Also $\quad i_{aF}^s = 0 \qquad$ or $\quad i_{1F}^s + i_{2F}^s + i_{0F}^s = 0 \qquad (4.22)$

The voltage relations (4.16) taking generator-star point as the reference potential are also applicable here.

Further

$$\frac{je_b^s}{p+js} = \frac{je_c^s}{p+js} = Z_{Ft}^s \,(i_{bF}^s + i_{cF}^s)$$

Expressing in terms of symmetrical components and simplifying,

$$\frac{je_1^s}{p+js} = \frac{je_2^s}{p+js} = \frac{je_0^s}{p+js} - 3\, Z_{Ft}^s \; i_{0F}^s \tag{4.23}$$

From the considerations of equations(4.14),(4.16),(4.22) and (4.23) the load equivalent circuits can be derived as shown in fig. 4.4(b), with reference to the alternator equivalent circuit of fig. 4.1(b)

(d) A load or fault between two lines shorted and third line.

Let the load or fault has a resistance of r_F, inductance of x_F and a capacitance of $1/x_{cF}$ all in series as shown in figure 4.5(a)

The current equations(4.14) are also valid here

Further

$$i_{aF}^s = -(i_{bF}^s + i_{cF}^s)$$

$$\therefore \quad i_{0F}^s = 0 \qquad (4.24)$$

The voltage equation(4.20), taking ground as the reference potential are also applicable here.

Also

$$e_b^s = e_c^s$$

and

$$\frac{je_b^s - je_a^s}{p+js} = (i_{bF}^s + i_{cF}^s)\, z_{Ft}^s$$

After simplification and introducing symmetrical components.

$$\frac{je_1^s}{p+js} = \frac{je_2^s}{p+js} = (i_{1F}^s + i_{2F}^s)\,\frac{z_{Ft}^s}{3} \qquad (4.25)$$

Considering the equations(4.14), (4.20)(4.24) and (4.25) the load equivalent circuits can be derived as shown in figure 4.5(b) w reference to the alternator equivalent circuits of fig. 4.1(b).

(e) One line or phase open -

Let the phase a be open at any point. The following current equations can be written:

$$i_a^s = i_{aL}^s = 0$$

$$i_b^s = i_{bL}^s$$

$$i_c^s = i_{cL}^s$$

or

$$i_1^s + i_2^s + i_0^s = 0$$

and

$$i_1^s = i_{1L}^s \, , \; i_2^s = i_{2L}^s , \; i_0^s = i_{0L}^s \tag{4.26}$$

The voltage equations, taking generator-star as the reference potential, may be written as:

$$e_b^s - e_{bL}^s = 0$$

$$e_c^s - e_{cL}^s = 0$$

or

$$\frac{e_1^s - e_{1L}^s}{p+js} = \frac{e_2^s - e_{2L}^s}{p+js} = \frac{e_0^s - e_{0L}^s}{p+js} \tag{4.27}$$

Consideration of the equations (4.16)(4.18)(4.26) and (4.27) lead to the load equivalent circuits as shown in figure 4.5(b), with reference to the alternator equivalent circuit of figure 4.1(b).

(f) Simultaneous opening and connecting of one phase to the generator star-point at the alternator end.

Let the phase a be opened and connected to the generator star-point through an impedance of resistance r_f inductance x_f and capacitance $1/x_{cf}$ as shown in figure 4.7(a).

The load equivalent circuit will, obviously be the combination of figs. 4.2(b) and 4.5(b) as derived earlier. However, to avoid intermixing, an insulating 1:1 transformer is employed. Thus the combined equivalent circuit is shown in fig. 4.7(b). This is shown with reference to the alternator equivalent circuit of the fig. 4.1(b)

4.3. SIMPLIFIED EQUIVALENT CIRCUITS

The complete equivalent circuits are obtained by the combination of the alternator and load equivalent circuits. The simplified equivalent circuits are obtained by putting p=0, Since the amplitude variation of sinusoidal currents, voltages and flux linkages is assumed to be very small as compared to their frequency variation. Thus the transferred transient impedance now becomes the transferred steady-state impedance, as defined below:

$$Z_0^s = \frac{r}{s} + jx_0$$

$$Z_n^s = \frac{r_n}{s} + jx_n$$

$$Z_L^s = \frac{r_L}{s} + j\left(x_L - \frac{x_{cL}}{s^2}\right) \qquad (4.28)$$

$$Z_F^s = r_f/s + j\left(x_f - \frac{x_{cF}}{s^2}\right)$$

$$Z_{nL}^s = \frac{r_{nL}}{s} + j\,x_{nL}$$

All the impedances in the above equation (4.28) are 1/s times the equivalent impedances of the branch in question at s times the fundamental frequency, even when the branch consists of a number of sersies parallel elements.

After eliminating p and then simplifying the load equivalent circuit of fig. 4.2(b) in conjuction with the alternator equivalent circuit of the fig. 4.1(a), the complete equivalent circuits or ladder networks may be drawn as shown in fig. 4.8(a). Similarly the other load equivalent circuits of figure 4.3(b), 4.4(b), 4.5(b),4.6(b) and 4.7(b) in conjunction with the alternator equivalent circuit of fig. 4.1(b), are simplified to fig. 4.8(b). However, the impedances Z_3^s and Z_4^s are different for different cases as shown below:

(a) On delta-star conversion of the load impedances of fig. 4.2(b);

$$Z_3^s = \frac{(Z_L^s)^2 (Z_0^s + Z_L^s + 3 Z_n^s + 3 Z_{nL}^s)}{(2Z_L^s + 3 Z_F^s)(Z_0^s + Z_L^s + 3 Z_n^s + 3 Z_{nL}^s) + (Z_0^s)(Z_L^s + 3Z_n^s + 3Z_{nL}^s)}$$

$$Z_4^s = \frac{3 Z_L^s Z_F^s (Z_0^s + Z_L^s + 3Z_n^s + 3Z_{nL}^s) + Z_0^s Z_L^s (Z_L^s + 3Z_n^s + 3 Z_{nL}^s)}{(2Z_L^s + 3Z_F^s)(Z_0^s + Z_L^s + 3Z_n^s + 3Z_{nL}^s) + Z_0^s (Z_L^s + 3Z_n^s + 3Z_{nL}^s)}$$

.... (4.29)

as shown in figure 4.8 (a).

(b) On delta-star conversion of the load impedance of figure 4.3(b),

$$Z_3^s = \frac{(Z_L^s)^2}{(2Z_L^s + Z_F^s)}$$

and

$$Z_4^s = \frac{Z_L^s Z_F^s}{2Z_L^s + Z_F^s} \qquad (4.30)$$

as shown in fig. 4.8(b)

(c) By replacing the three parallel impedances of fig. 4.4(b) by an equivalent impedance and then using the delta-star conversion:

$$Z_3^s = \frac{3Z_L^s Z_F^s (Z_0^s + Z_L^s + 3Z_n^s + 3Z_{nL}^s) + (Z_L^s)(Z_0^s)(Z_L^s + 3Z_n^s + 3Z_{nL}^s)}{(Z_L^s + 6Z_F^s)(Z_0^s + Z_L^s + 3 Z_n^s + 3 Z_{nL}^s) + 2 (Z_0^s)(Z_L^s + 3 Z_n^s + 3Z_{nL}^s)}$$

.... (4.31)

and $Z_4^s = 0$

as shown in the fig. 4.8(b)

(d) From fig. 4.5(b),

$$Z_3^s = \frac{Z_F^s}{3} \qquad (4.32)$$

and $Z_4^s = Z_L^s$

as shown in figure 4.8(b)

(e) From fig. 4.6(b),

$$Z_3^s = Z_0^s + Z_L^s + 3\ Z_n^s + 3Z_{nL}^s \qquad (4.33)$$

and

$$Z_4^s = Z_L^s$$

as shown in fig. 4.8 (b)

(f) Simplifying the circuits of the figure 4.7(b) and eliminating 1:1 transformer as shown in appendix (4.4.1), Z_3^s and Z_4^s of the figure 4.8(b) can be obtained as:

$$Z_3^s = (Z_0^s + Z_L^s + 3Z_n^s + 3Z_{nL}^s) - \frac{3(Z_0^s + Z_L^s + 2Z_n^s + 2Z_{nL}^s)^2}{3Z_0^s + 2Z_L^s + 4\ Z_n^s + 4\ Z_{nL}^s + Z_F^s} \quad \ldots\ldots\ldots \quad (4.34)$$

and

$$Z_4^s = Z_L^s$$

4.4 Appendix

4.4.1. THE SIMPLIFICATION OF THE EQUIVALENT CIRCUIT OF FIGURE 4.6(b)

The figure 4.6(b) can be redrawn as shown in figure 4.9

The voltage equations may be written as,

$$\frac{e_1^s}{s} = (Z_0^s)\ (i_1^s + i_2^s - 3i_{0F}^s) + (Z_L^s + 3Z_n^s + 3Z_{nL}^s)(i_1^s + i_2^s - 2\ i_{0F}^s) + Z_L^s\ (i_1^s - i_{0F}^s)$$

$$\frac{e_2^s}{s} = (Z_0^s)\ (i_1^s + i_2^s - 3\ i_{0F}^s) + (Z_L^s + 3Z_n^s + 3Z_{nL}^s)\ (i_1^s + i_2^s - Zi_{0F}^s) + Z_L^s\ (i_2^s - i_{0F}^s)$$

$$\frac{e_0^s}{s} = Z^s\ (i_1^s + i_2^s - 3\ i_{0F}^s) \qquad (4.35)$$

Also

$$\frac{e_1^s + e_2^s + e_0^s}{s} = 3\ Z_F^s\ \ i_{0F}^s$$

Solving the above equations:

$$i_{0F}^{s} = \frac{Z_0^s + Z_L^s + 2Z_n^s + 2Z_{nL}^s}{3Z_0^s + 2Z_L^s + 4Z_n^s + 4\,Z_{nL}^s + Z_F^s}\,(i_1^s + i_2^s) \qquad (4.33)$$

Now eliminating i_{0F}^{s} in (4.35) by the equation (4.36)

$$\frac{e_1^s}{s} = \left[(Z_0^s + Z_L^s + 3Z_n^s + 3Z_{nL}^s) - \frac{3(Z_0^s + Z_L^s + 2Z_n^s + 2Z_{nL}^s)^2}{3Z_0^s + 2Z_L^s + 4Z_n^s + 4Z_{nL}^s + Z_F^s}\right](i_1^s + i_2^s) + Z_L^s\, i_1^s$$

$$\frac{e_2^s}{s} = \left[(Z_0^s + Z_L^s + 3Z_n^s + 3Z_{nL}^s) - \frac{3(Z_0^s + Z_L^s + 2Z_n^s + 2Z_{nL}^s)}{3Z_0^s + 2Z_L^s + 4Z_n^s + 4Z_{nL}^s + Z_F^s}\right](i_1^s + i_2^s) + Z_L^s\, i_2^s \qquad (4.37)$$

In the above equation (4.37), clearly the coefficient of $(i_1^s + i_2^s)$ is Z_3^s and the other coefficient is Z_4^s as shown in the equation (4.34).

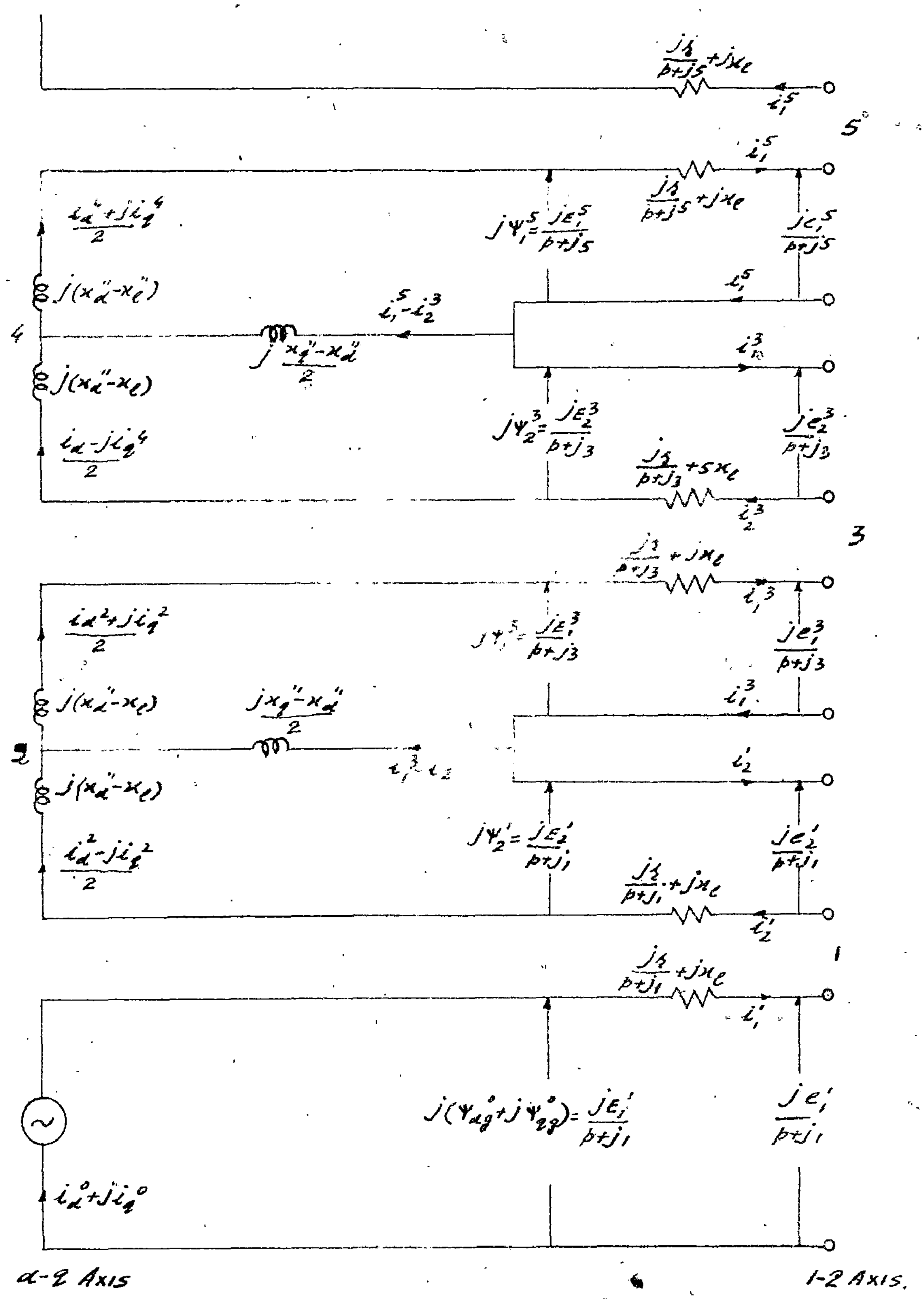


FIG. 4-1 (a) THE TRANSIENT EQUIVALENT CIRCUIT OF AN ALTERNATOR.

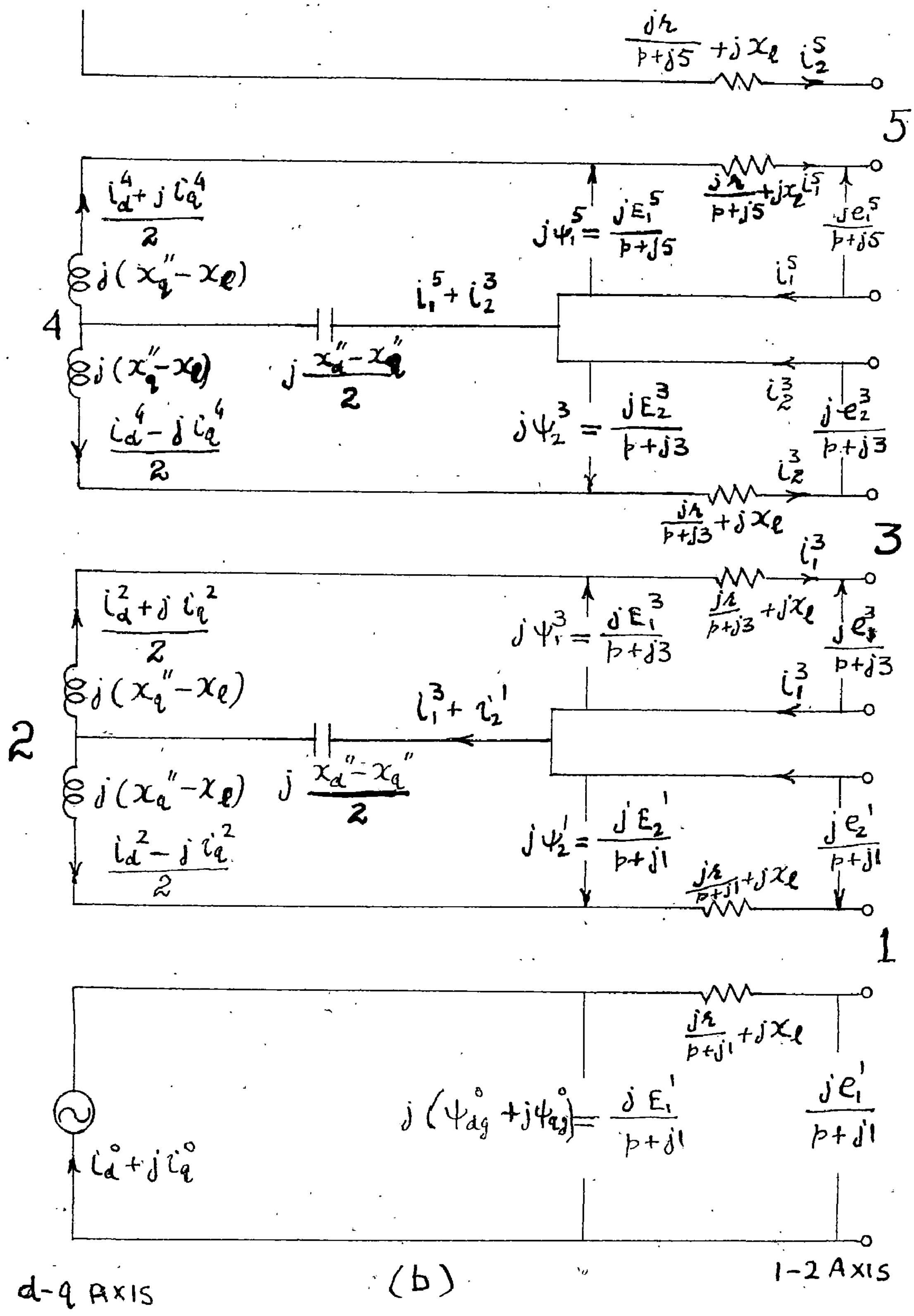


FIG 4-1 (b) THE TRANSIENT EQUIVALENTS OF AN ALTERNATOR

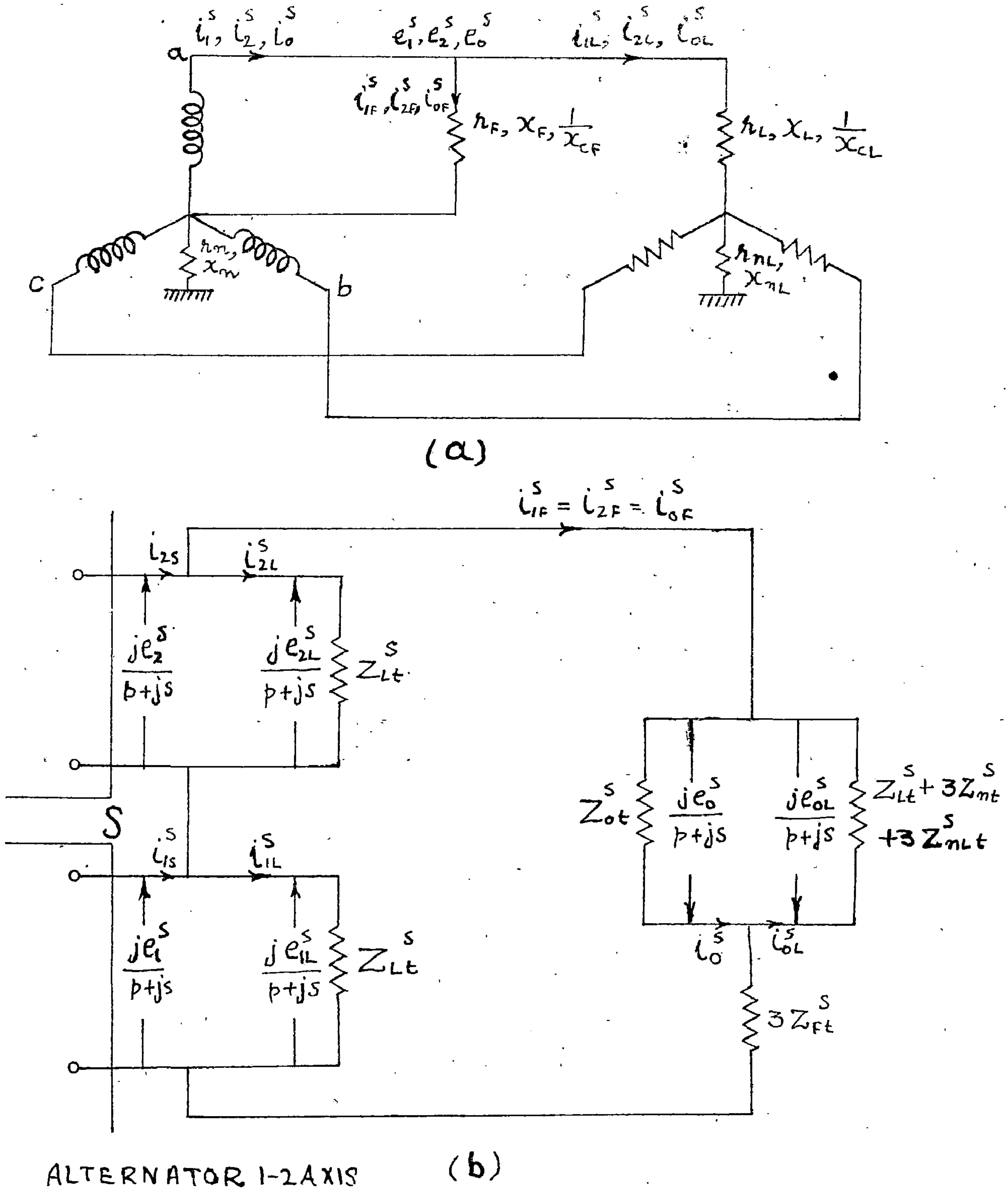
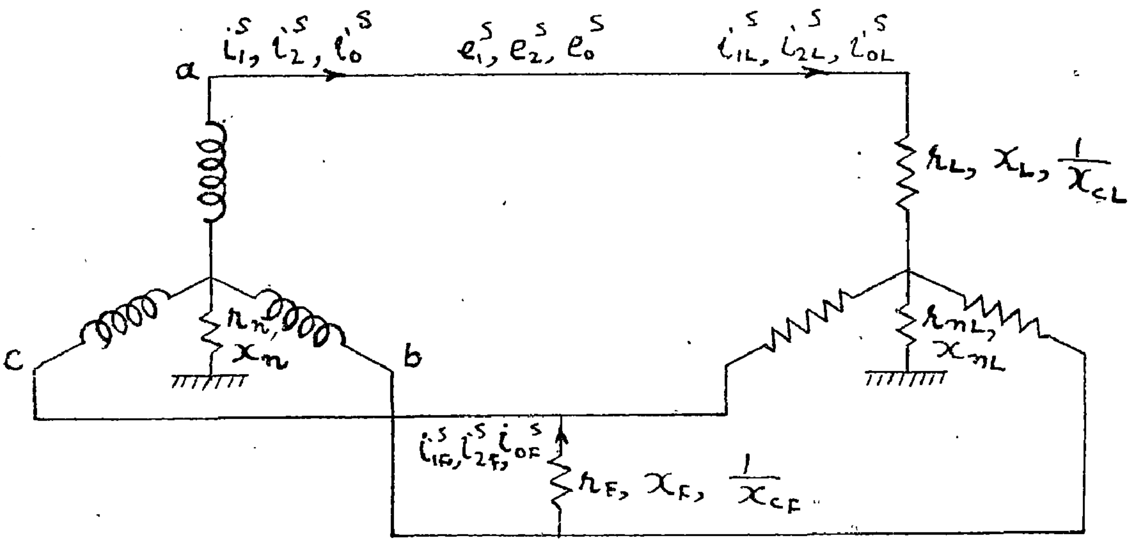
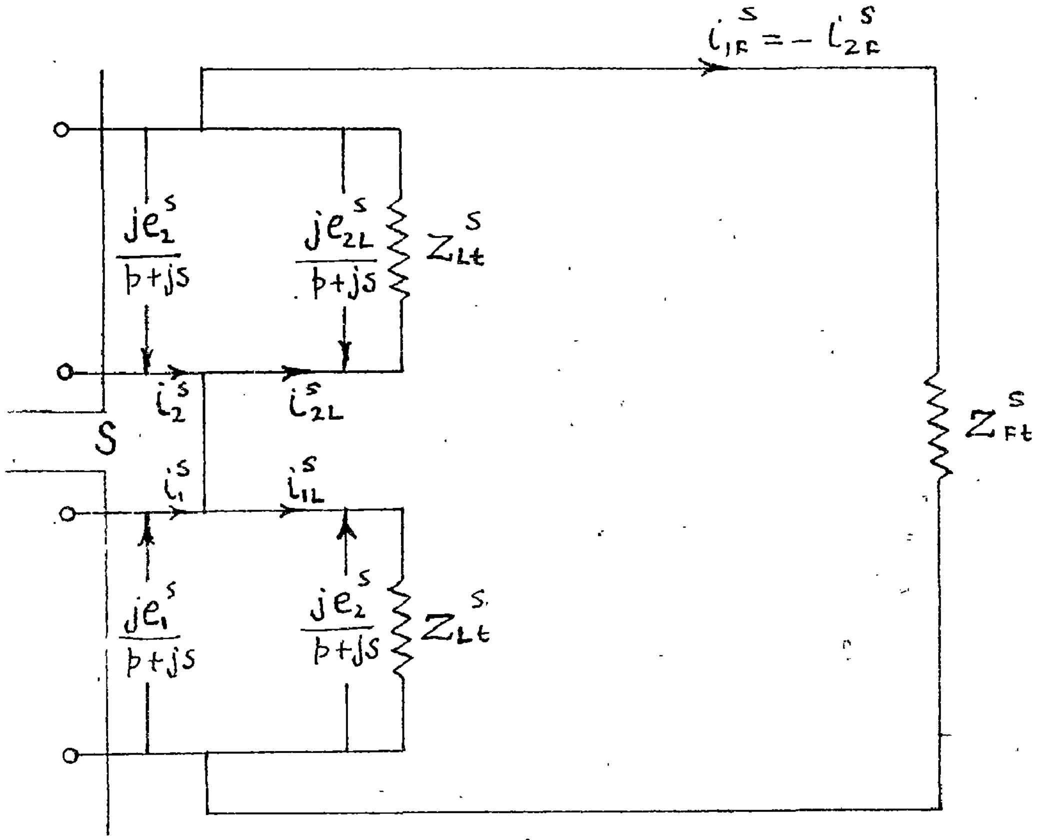


FIG 4-8 THE CIRCUITS FOR A SINGLE PHASE LOAD OR FAULT BETWEEN A LINE AND GENERATOR STAR POINT



(a)



(b)

ALTERNATOR 1-2 AXIS

FIG 4-3 THE CIRCUITS FOR A SINGLE PHASE LOAD OR FAULT BETWEEN TWO LINES

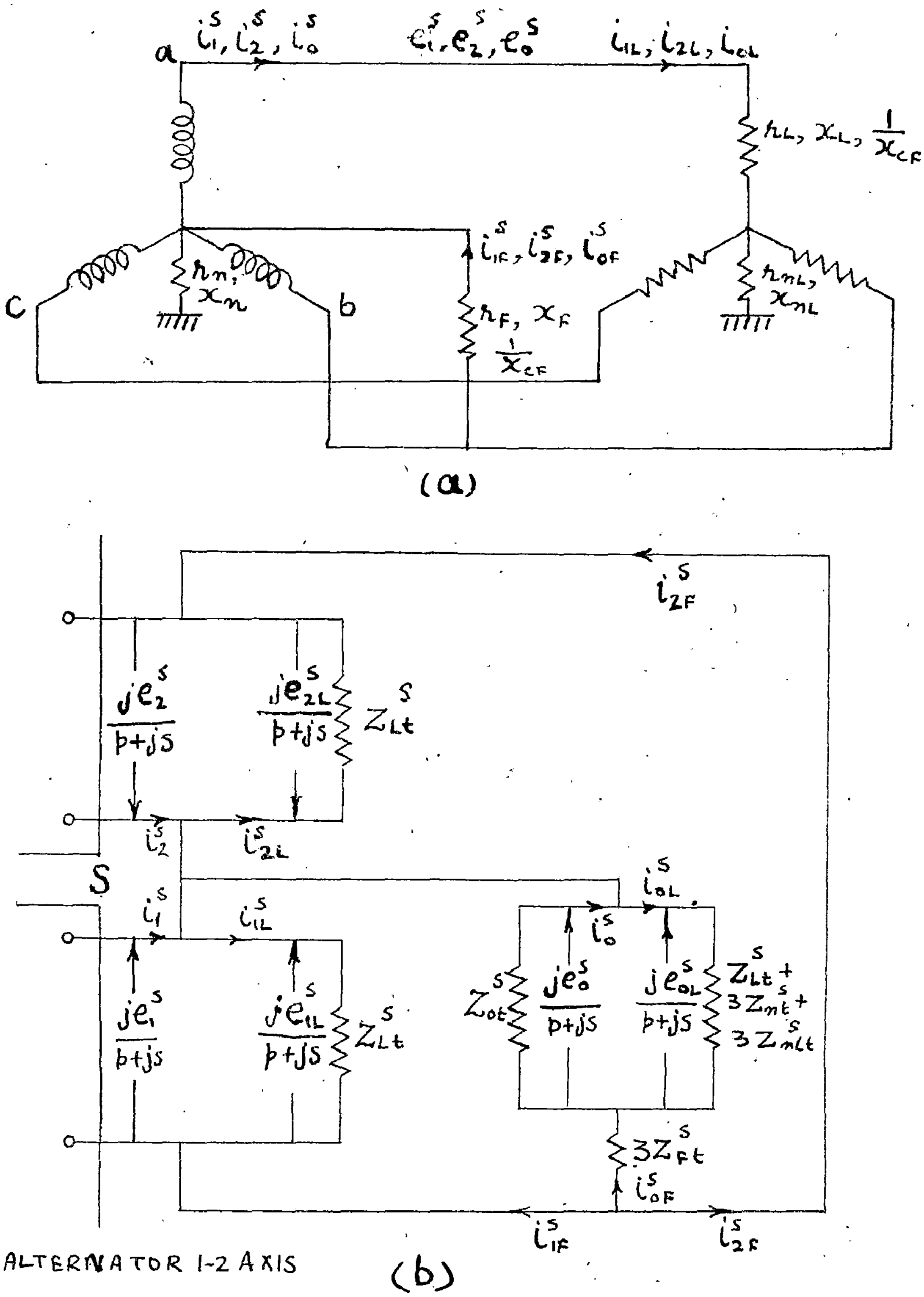
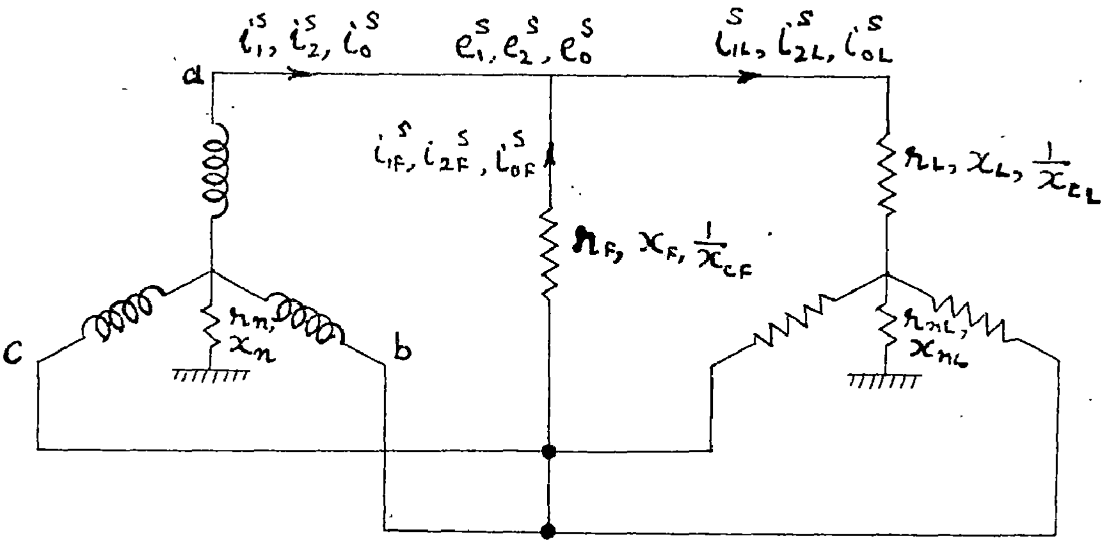
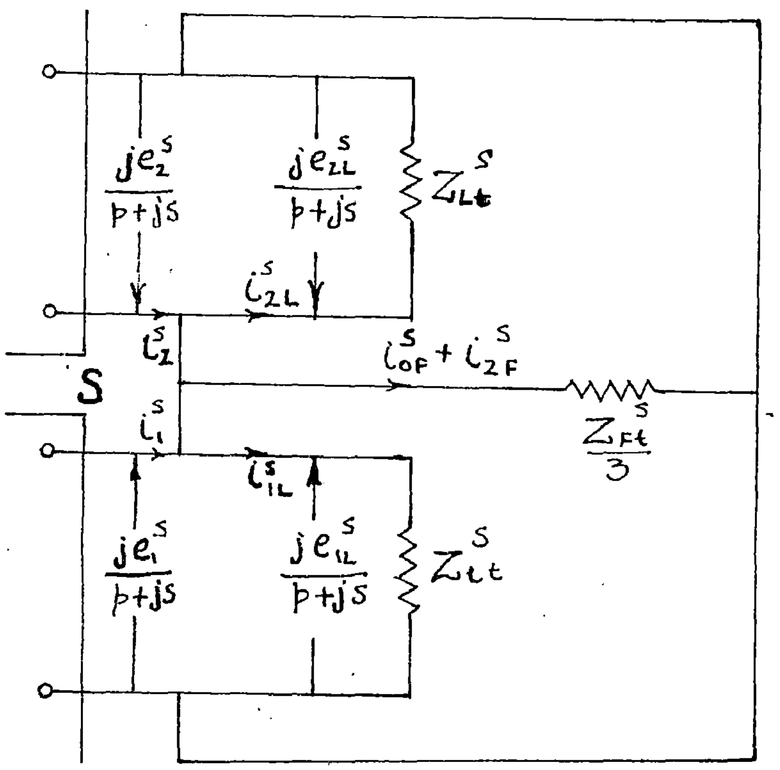


FIG 4-4 THE CIRCUITS FOR A LOAD OR FAULT BETWEEN TWO LINES SHORTED AND GENERATOR STAR POINT

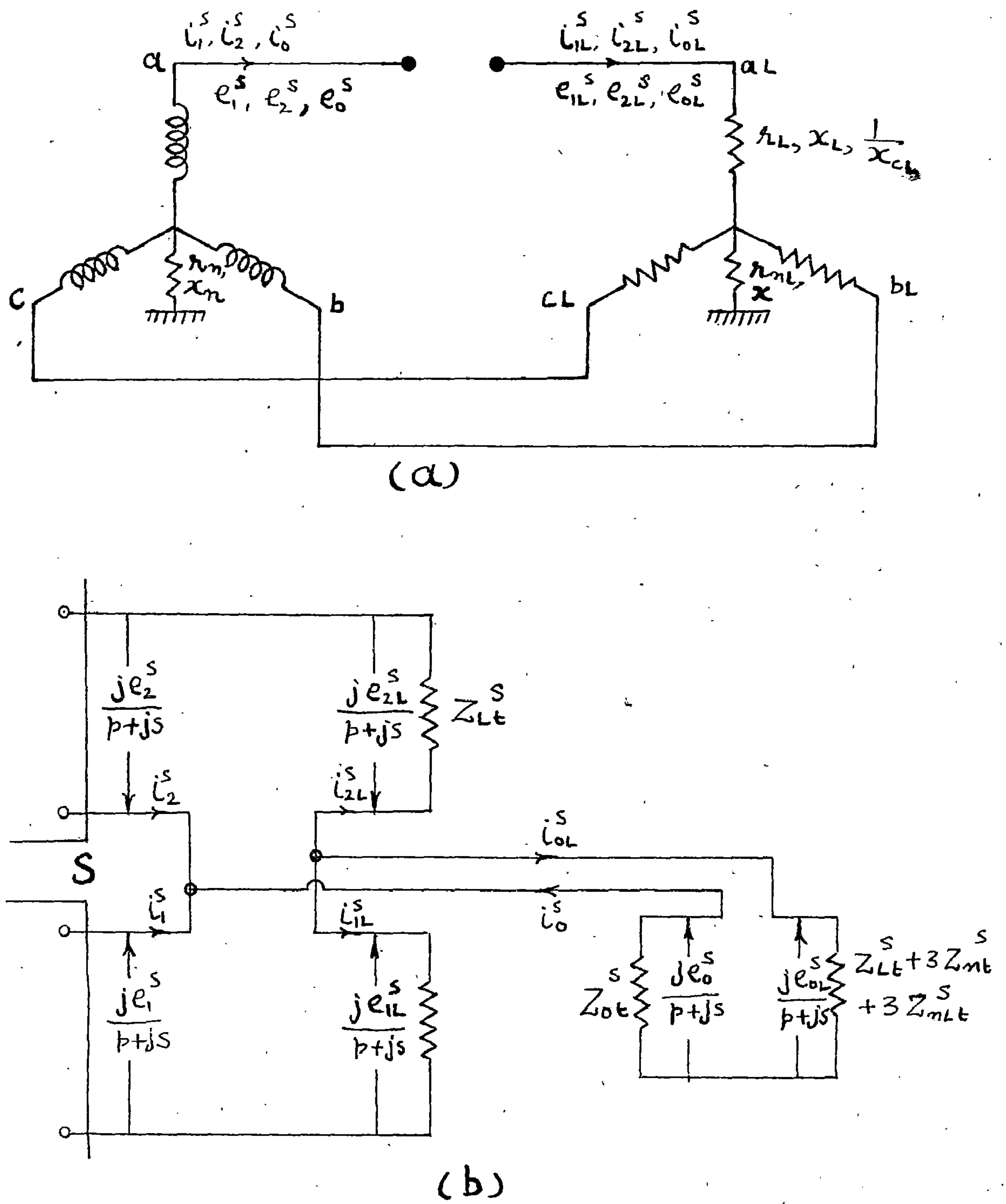


(a)



ALTERNATOR:1-2 AXIS (b)

FIG 4-5 THE CIRCUITS FOR A LOAD OR FAULT BETWEEN TWO LINES SHORTED AND THIRD LINE.



ALTERNATOR 1-2 AXIS

FIG 4-6 THE CIRCUITS FOR ONE LINE OR PHASE OPEN.

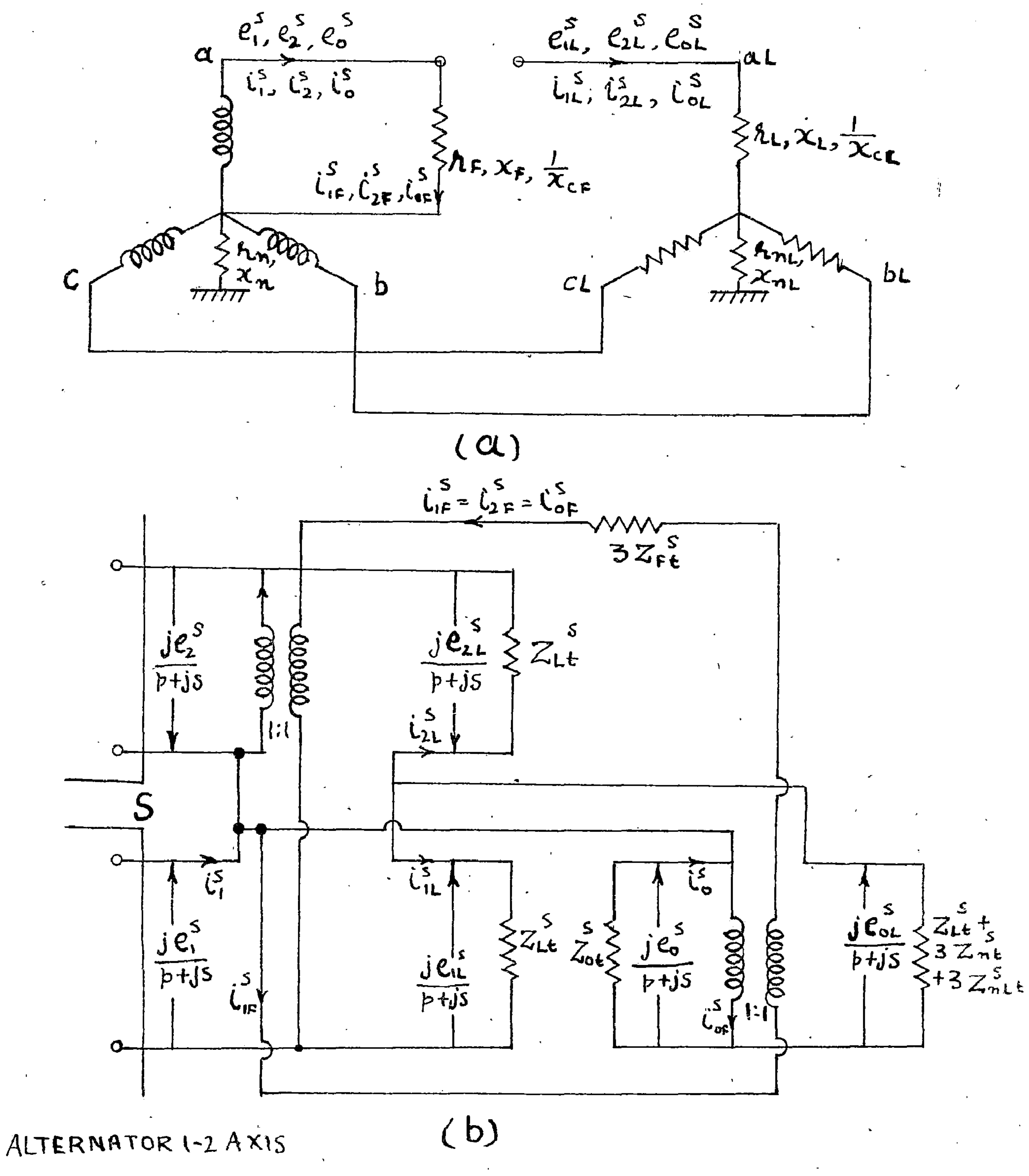
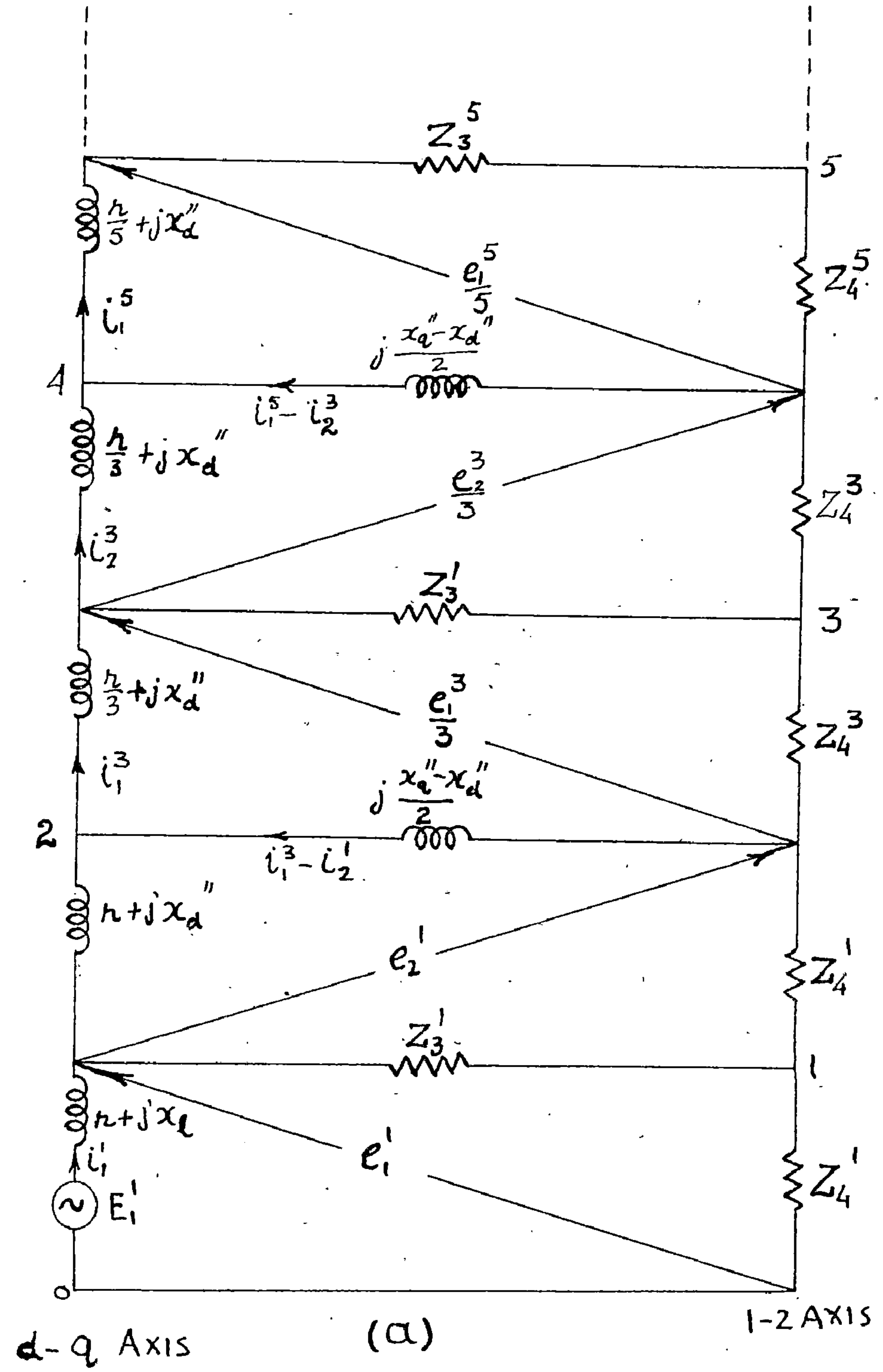
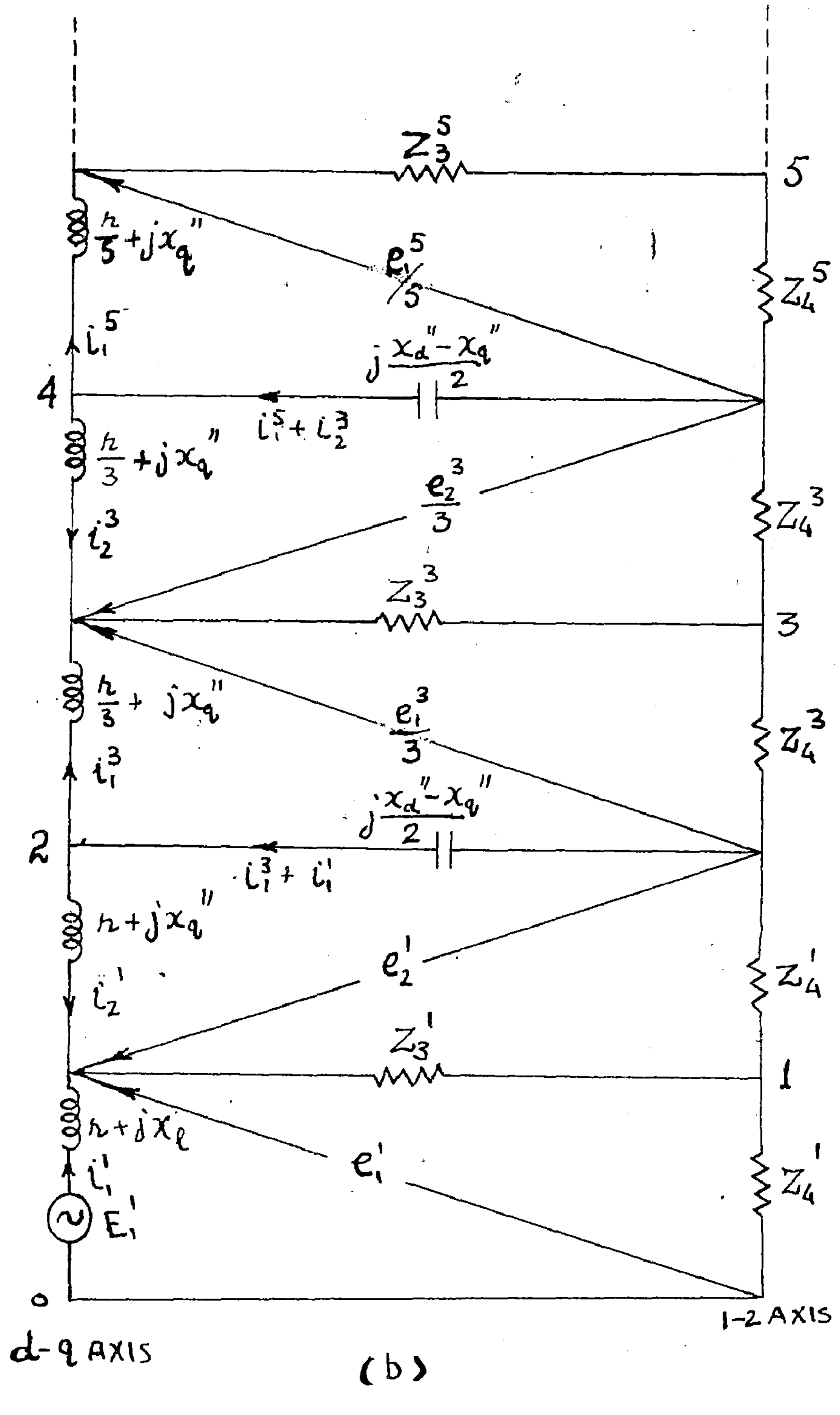


FIG 4-7 THE CIRCUITS FOR SIMUTANEOUS OPENING AND CONNECTING OF ONE PHASE TO THE GENERATOR STAR-POINT AT THE ALTERNATOR END



(a)

FIG 4-8 THE EQUIVALENT LADDER NETWORKS.



(b)

FIG 4-8 THE EQUIVALENT LADDER NETWORKS.

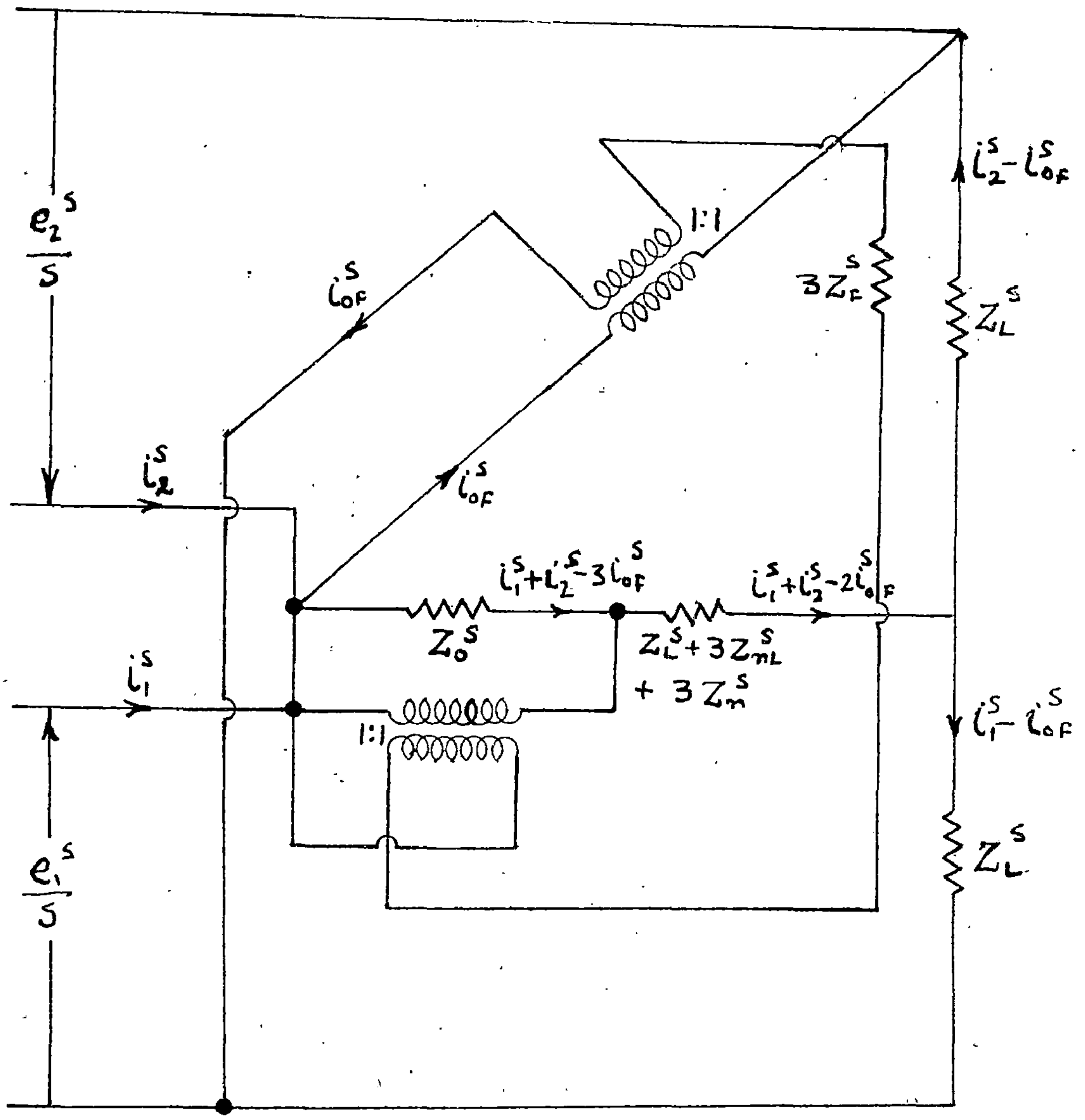


FIG 4-9 THE SIMPLIFIED EQUIVALENT CIRCUIT FOR SIMULTANEOUS OPENING AND CONNECTING OF ONE PHASE TO THE GENERATOR STAR POINT AT THE ALTERNATOR END.

C_H_A_P_T_E_R - V

A SYNCHRONOUS MACHINE UNDER UNBALANCED LOAD AND FAULT CONDITIONS

5.1. INTRODUCTION:

In the last chapter, the transient equivalent circuits of the synchronous machine, along with the common types of faults and unbalanced loads have been developed and therefrom the equivalent ladder networks for infinite time harmonics have been deduced on neglecting armature transients. The currents, voltages, flux linkages etc are expressed in complex notation in the equivalent circuits, but their magnitudes vary with time.

The harmonic currents, voltages and flux-linkages are small in their magnitudes and their effects on the r.m.s. value is negligible for all practical purposes. So only the fundamental frequency quantities are considered in terms of their rms magnitudes, as functions of time, after reducing the ladder networks, that is, after considering the effects of other harmonics on the fundamental frequency performance.

5.2. REDUCTION OF LADDER NETWORKS:

The equivalent ladder networks as deduced in the last chapter are shown in fig. 5.1(a) and 5.1(b) respectively. The values of Z_3^4 and Z_4^s have been calculated in terms of the transferred steady state impedances of the machine. Now the definition of any transferred steady-state impedance, corresponding to s^{th} harmonic, for example,

$$Z_L^s = \frac{r_L}{s} + j\left(x_L - \frac{x_{cL}}{s^2}\right)$$

shows that with increasing s, the resistive and capacitive part of the impedance decreases. Therefore a value of s is so chosen that the portion of the network at frequencies of s or higher is assumed to consist of reactances only as shown in fig. 5.1(a) and 5.1(b).

$$Z_3^s = j\,x_3^s = j\,x_3^{s+2} = j\,x_s^{s+4} = \ldots\ldots = jx_3$$

$$Z_4^s = j\,x_4^s = j\,x_4^{s+2} = j\,x_4^{s+4} = \ldots\ldots = jx_4 \qquad (5.1)$$

In figure 5.1(a), the u^{th} fraction of the total current entering at the branch O_1 is diverted through the branch of $j\,\dfrac{x_q'' - x_d''}{2}$ and the same fraction of the total current is diverted through reactances at other junctions such as O_3, O_5 etc. as the network is spread to infinity following the same pattern. Similarly the v^{th} fraction is diverted through the branch $j\,x_3$. Now writing the voltage equation for the loop $O_1\ O_2\ O_2'\ O_1'\ O_1$

$$u\left(\frac{x_q'' - x_d''}{2}\right) = (1-u)\,(x_d'' + x_u) + v(1-u)\,x_3$$

which gives
$$v = \frac{u}{1-u}\ \frac{x_q'' - x_d''}{2\,x_3} - \frac{x_d'' + x_4}{x_3} \qquad (5.2)$$

Equation for the loop $O_1\ O_2\ O_3\ O_3\ O_2\ O_1\ O_1$ is

$$u\left(\frac{x_q'' - x_d''}{2}\right) = (1-u)(x_d'' + x_4) + (1-v)(1-u)(x_d'' + x_4)$$
$$+ u(1-v)(1-u)\left(\frac{x_q'' - x_d''}{2}\right) \qquad (5.3)$$

Eliminating v by the relation (5.2) and putting

$$w = \frac{(x_d'' + x_4)\ (x_d'' + x_4 + 2x_3)}{\frac{x_d'' + x_q''}{2} + x_3 + x_4} \tag{5.4}$$

The equation (5.3) is reduced to

$$u^2 \left(\frac{x_q'' - x_d''}{2} \right) + u\,w - w = 0$$

which gives

$$u = \frac{-w/2 + \sqrt{\frac{w^2}{4} + \frac{x_q'' - x_d''}{2}\, w}}{\frac{x_q'' - x_d''}{2}} \tag{5.5}$$

The other value of u is negative which means increasing currents for higher harmonics, which is impossible and thus is neglected.

Similarly, solving for figure 5.1(b)

$$u = \frac{-\frac{w}{2} + \sqrt{\frac{w^2}{u} + \frac{x_d'' - x_q''}{2}\, w}}{\frac{x_d'' - x_q''}{2}} \tag{5.6}$$

where

$$w = \frac{(x_q'' + x_4)\ (x_q'' + x_4 + 2x_3)}{\frac{x_d'' + x_q''}{2} + x_3 + x_4} \tag{5.7}$$

The network for the frequency ranges upto (s-2) will remain undisturbed if the impedance $ju \frac{x_q'' - x_d''}{2}$ replace the branch $j \frac{x_q'' - x_d''}{2}$ across $0_1 0_1$ in figure 5.1(a) and $ju \frac{x_d'' - x_q''}{2}$ across $0_1 0_1$ in figure 5.1(b) and then the rest of the networks for the frequency s and higher is removed. The above process, reduces the networks to a finite number of harmonics. The networks 5.1(a) and 5.1(b) are further reduced to the fundamental frequency as shown in fig. 5.2(a) and 5.2(b) respectively by simplifying the parallel circuits of higher harmonics, successively, since the solution is needed for fundamental frequency only. In the figures Z_2 may be termed as the equivalent negative sequence impedance of the synchronous machine.

The equivalent negative sequence impedance has been calculated for simplified cases, in the following article.

5.3 DETERMINATION OF EQUIVALENT NEGATIVE SEQUENCE IMPEDANCES OF A SYNCHRONOUS MACHINE

5.3.1. LINE TO NEUTRAL INDUCTIVE LOADS OR FAULTS

Let the synchronous machine having a small armature resistance is initially on no load, when an inductance x_F is switched across the phase a and the neutral, Since r is small, (r/3), (r/5) may be neglected and the equivalent ladder network may be drawn as shown in fig. 5.3. The voltage equation for the loop $0_1\ 0_3\ 0_3'\ 0_1'\ 0_1$ is

$$u\ \frac{x_q'' - x_d''}{2} = 2(1-u)x_d'' + u(1-u)\ \frac{x_q'' - x_d''}{2} + (1-u)(x_0+3x_F)$$

On solving this,

$$u\ \frac{x_q'' - x_d''}{2} = -\left[x_d'' + \frac{x_0+3x_F}{2}\right] + \sqrt{\left(x_d'' + \frac{x_0+3x_F}{2}\right)\left(x_q'' + \frac{x_0+3x_F}{2}\right)}$$

Therefore the equivalent negative sequence impedance

$$Z_2 = r + j\left[-\frac{x_0+3x_F}{2} + \sqrt{\left(x_d'' + \frac{x_0+3x_F}{2}\right)\left(x_q'' + \frac{x_0+3x_F}{2}\right)}\right] \quad (5.8)$$

5.3.2. TWO SHORTED LINES TO THIRD LINE INDUCTIVE LOADS OR FAULTS:

Let an inductance x_F be connected across two shorted phases b,c and the phase a of a synchronous machine on no load. As r is small, $\left(\frac{r}{3}\right)$ $\left(\frac{r}{5}\right)$ etc. can be neglected and the equivalent circuit of the synchronous machine is drawn as shown in figure 5.4. The voltage equation for the loop $0_1\ 0_2\ 0_2'\ 0_1'\ 0_1$ is

$$u\left(\frac{x_d'' - x_q''}{2}\right) = (1-u)\, x_q'' + v\,(1-u)\, \frac{x_3}{3}$$

$$\therefore\ v = \frac{u}{1-u}\left[\frac{3(x_d'' - x_q'')}{2\,x_3}\right] - \frac{3\,x_q''}{x_3} \qquad (5.9)$$

and the equation for the loop $0_1\ 0_2 0_3 0_3 0_2 0_1 0_1$

$$u\left(\frac{x_d'' - x_q''}{2}\right) = (1-u)\, x_q'' + (1-v)(1-u)\, x_q'' + u(1-v)(1-u)\, \frac{x_d'' - x_q''}{2}$$

Eliminating v by the relation (5.9)

$$u^2\left(\frac{x_d'' - x_q''}{2}\right) + u\, \frac{2(2x_q''\, x_3 + 3x_q''^2)}{2x_3 + 3x_d'' + 3x_q''} = \frac{2\,(2x_q'' x_3 + 3x_q''^2)}{2x_3 + 3x_d'' + 3x_q''}$$

$$\therefore\ u\left(\frac{x_d'' - x_q''}{2}\right) = -\frac{w'}{2} + \sqrt{\frac{w'^2}{4} + \frac{x_d'' - x_q''}{2}\, w'}$$

where
$$w' = \frac{2\,(2\,x_q''\, x_3 + 3x_q''^2)}{2x_3 + 3x_d'' + 3\,x_q''}$$

So the negative sequence impedance is

$$r + jx_q'' - j\,\frac{w'}{2} + j\sqrt{\frac{w'^2}{4} + \frac{x_d'' - x_q''}{2}\, w'} \qquad (5.10)$$

5.3.3. LINE TO LINE INDUCTIVE LOAD OR FAULT

Let an inductance x_F be suddenly switched across the phases b and c of a synchronous machine on no load. r being small, $\left(\frac{r}{3}\right)$, $\left(\frac{r}{5}\right)$ etc. may be neglected and the equivalent circuit of the synchronous machine is drawn as shown in figure 5.5.

The voltage equation for the loop $0_1 0_3 0_3 0_1 0_1$ is

$$u\left(\frac{x_d'' - x_q''}{2}\right) = 2(1-u)x_q'' + u(1-u)\, \frac{x_d'' - x_q''}{2} + (1-u)x_F$$

From which

$$u\ \frac{x_d'' - x_q''}{2} = -\left[x_q'' + \frac{x_F}{2}\right] + \sqrt{\left(x_q'' + \frac{x_F}{2}\right)\left(x_d'' + \frac{x_F}{2}\right)}$$

So, the negative sequence impedance is,

$$r + j\left[-\frac{x_F}{2} + \sqrt{\left(x_q'' + \frac{x_F}{2}\right)\left(x_d'' + \frac{x_F}{2}\right)}\right] \qquad (5.11)$$

5.4. SOLUTION OF TRANSIENTS:

Fig. 5.2(a) gives

$$i_2^1 = \frac{(r + j\,x_1 + Z_3 + Z_4)\,i_1^1 - E_1^1}{Z_3^1}$$

$$e_2^1 = Z_2\,i_2^1 \qquad (5.12)$$

and figure 5.2 (b) gives

$$i_2^1 = \frac{E_1^1 - (r + j\,x_1 + Z_3^1 + Z_4^1)\,i_1^1}{Z_3^1}$$

$$e_2^1 = -Z_2 i_2^1$$

e_0^1 and i_0^1 in terms of E_1^1 and i_1^1 can be obtained from the load equivalent circuit as derived in the last chapter. Also, the currents in the faults or unbalanced load can be determined from the same equivalent circuits.

Thus all the fundamental frequency currents and voltages and their harmonic components can be determined from the equivalent circuits deduced earlier.

Figures 5.2(a) and 5.2(b) can be reduced to the equivalent circuit of figure 5.6, which is similar to that for balanced loads.

$$Z = R + jX$$

is the equivalent load impedance. Figure 5.6 gives the vector diagram exactly similar to that shown in figure 2.6.

Thus the whole operation can be referred to the equivalent balanced operation and the solution for the subtransient and post-subtransient stages, so far as E_d^0, $|E_1^1|$, $|e_1^1|$, $|i_1^1|$ and i_{fd}^0 are concerned, will be similar to that done in Chapter III. Thus, all the quantities can be determined from the ladder networks.

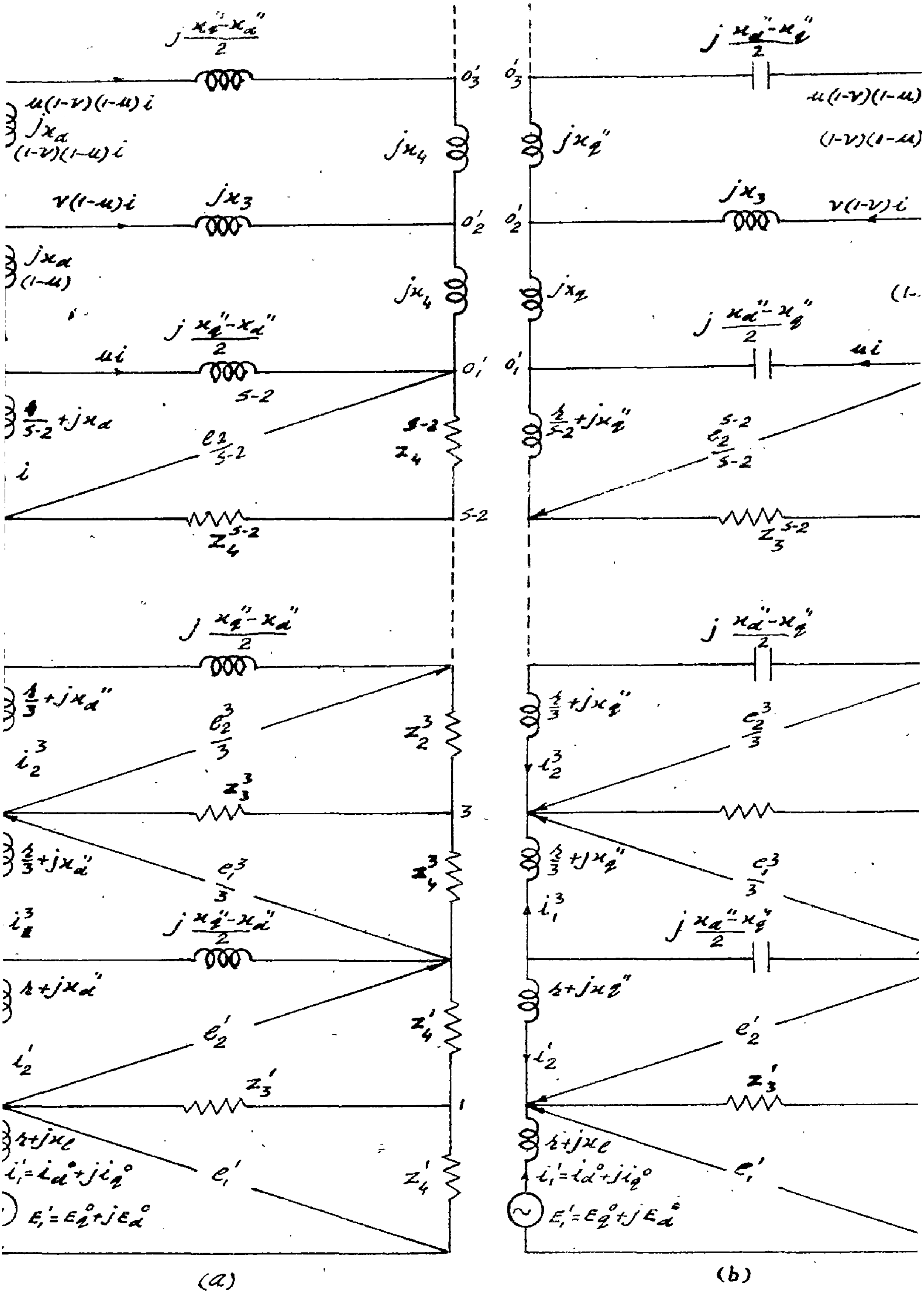


FIG. 5-1 THE EQUIVALENT LADDER NETWORK.

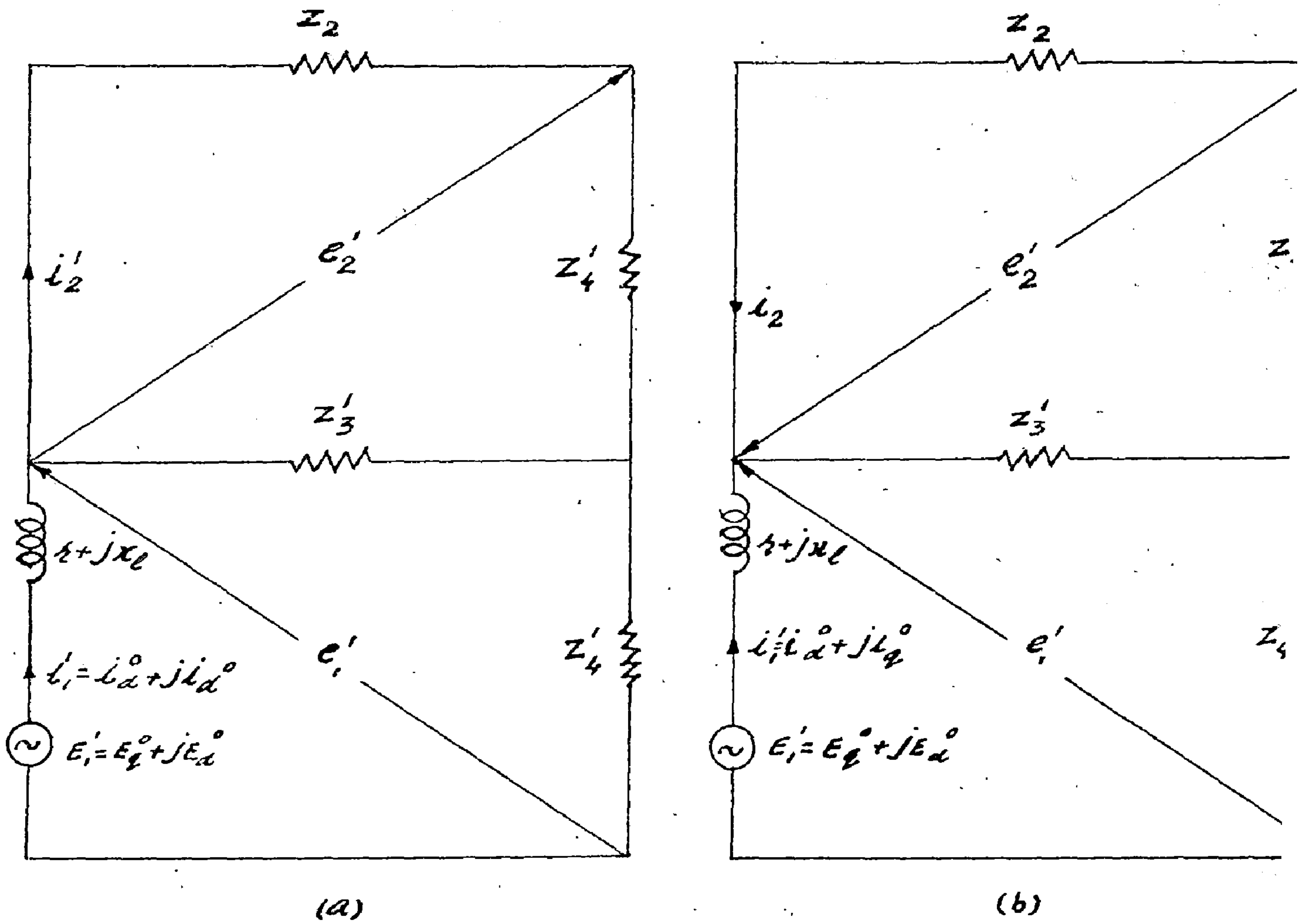


FIG. 5-2 REDUCED NETWORKS.

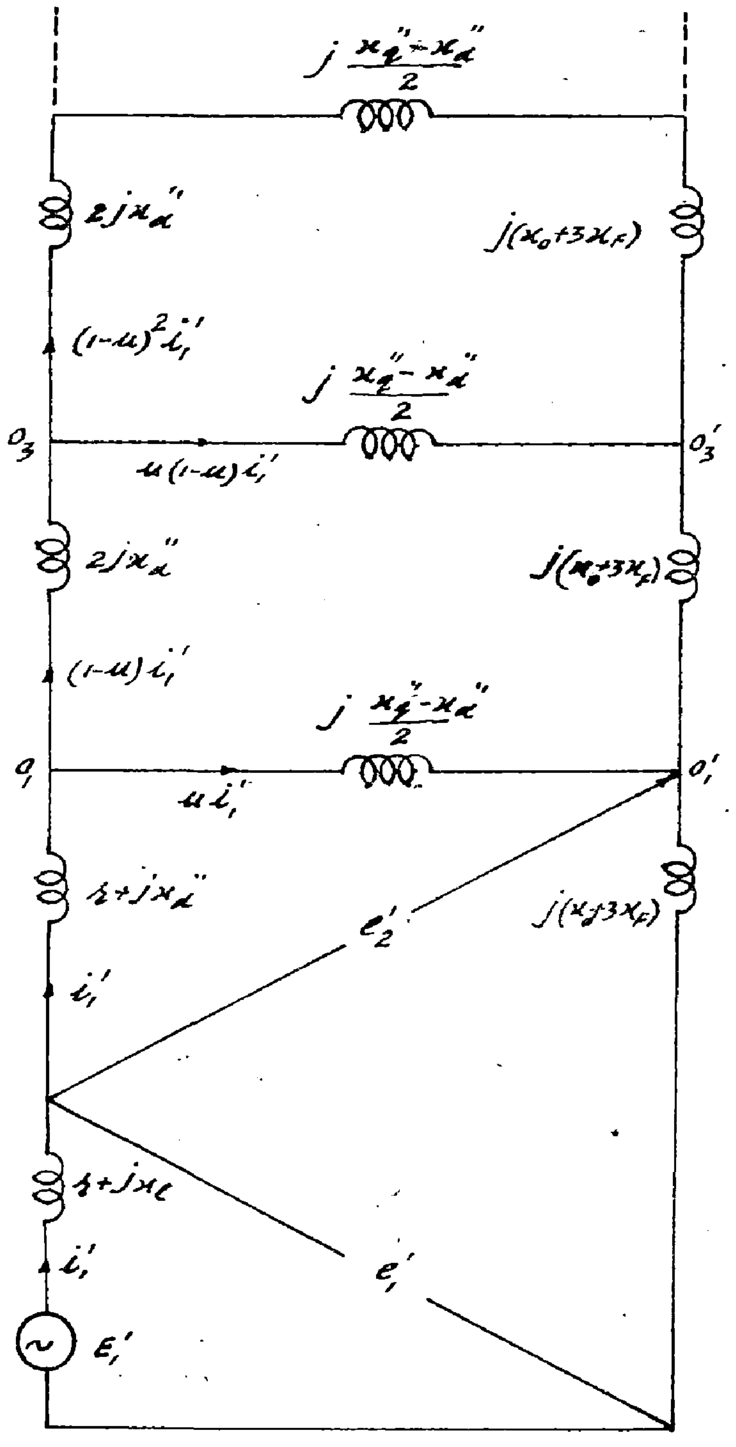


FIG. 5-3 SIMPLIFICATION OF LADDER NETWORKS FOR LINE TO NEUTRAL INDUCTIVE LOAD OR FOULT.

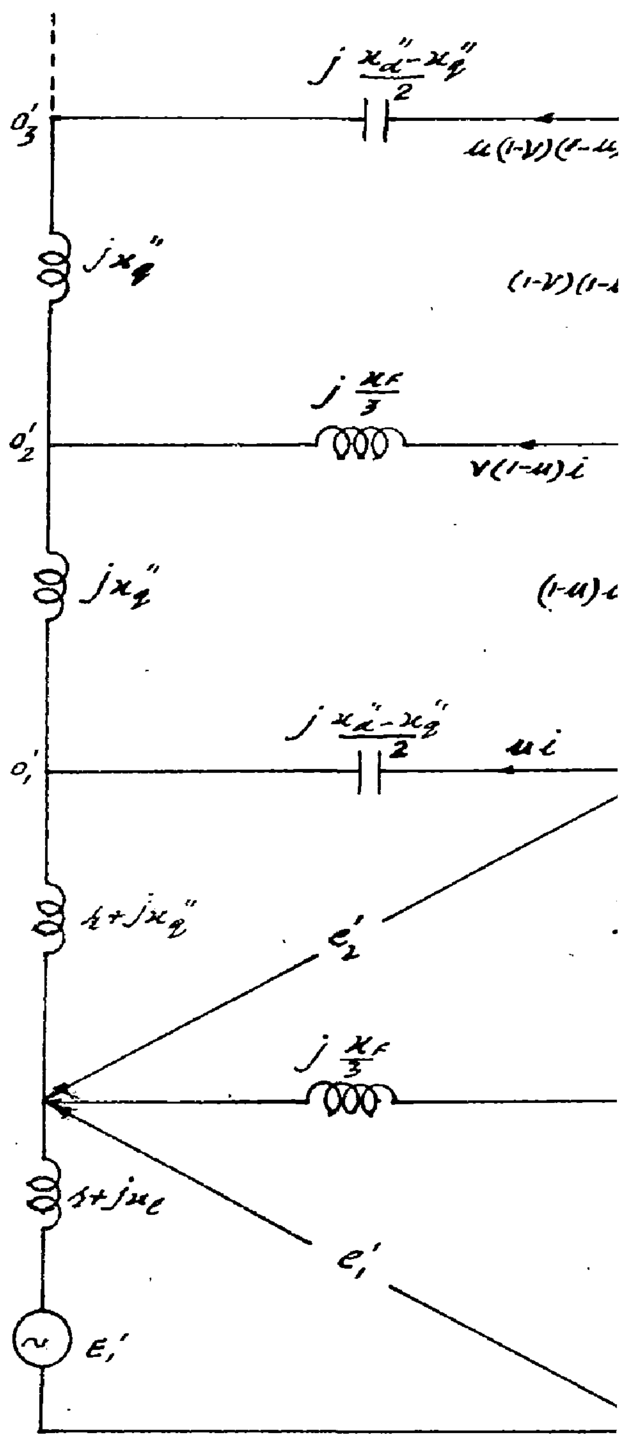


FIG. 5-4 SIMPLIFICATION OF LADD NETWORKS FOR INDUCTIVE OR FAULT BETWEEN TWO SH LINES AND THIRD LINE.

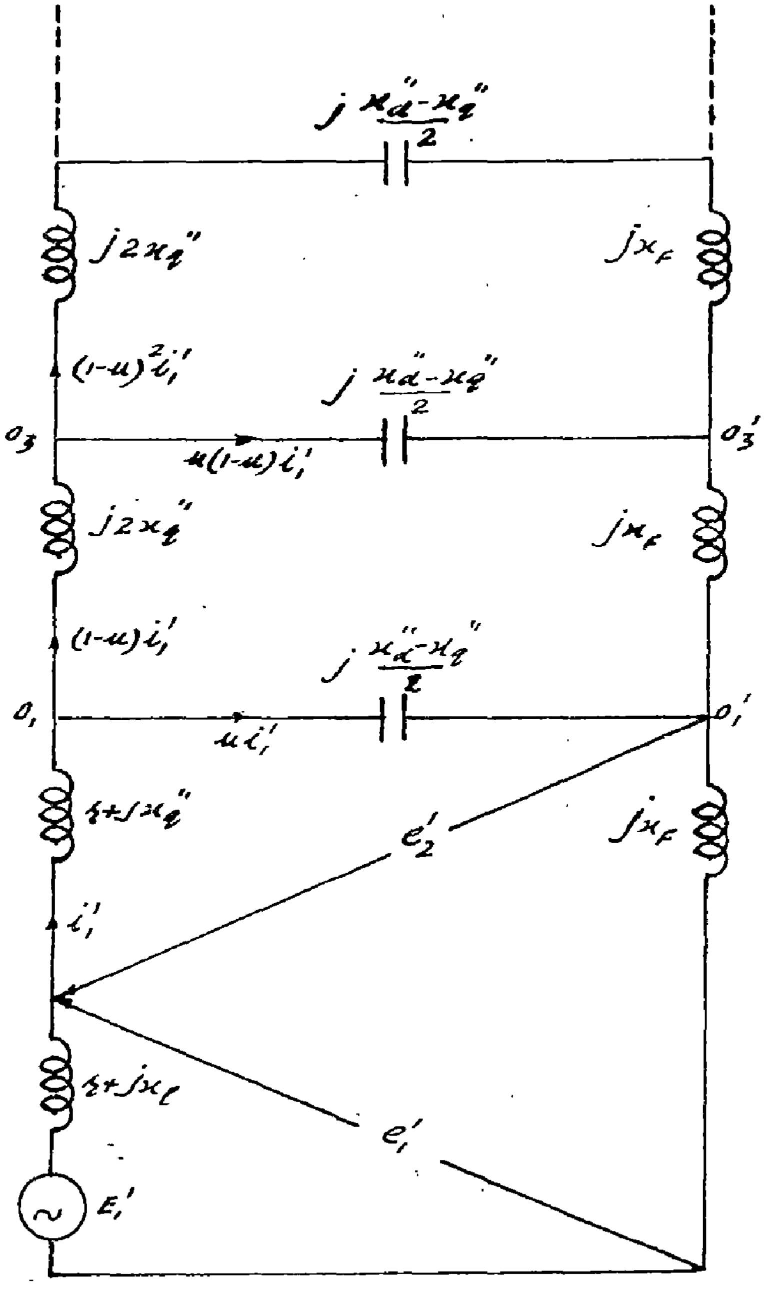


FIG. 5-5 SIMPLIFICATION OF LADDER NETWORKS FOR LINE TO LINE INDUCTIVE LOAD OR FAULT.

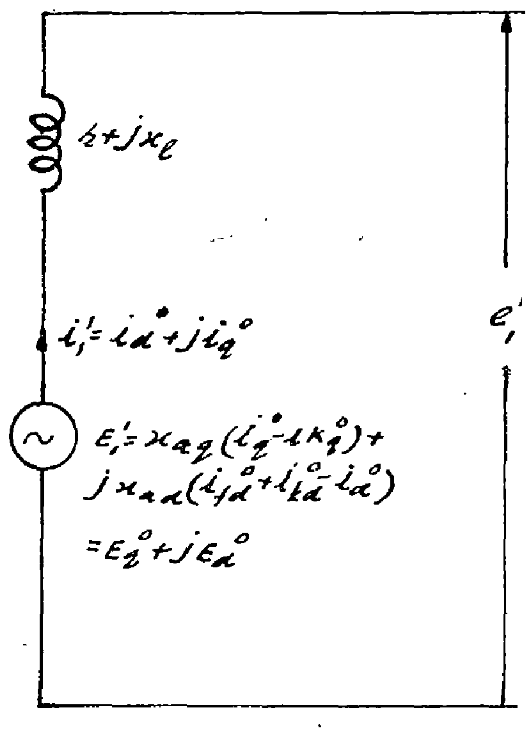


FIG. 5-6 THE SIMPLIFIED EC CIRCUIT.

CHAPTER VI

SINGLE PHASE LOADS AND FAULT CONDITIONS

6.1. INTRODUCTION:

This chapter deals with single phase loads and fault conditions (both line to line and line to neutral) of a synchronous machine, in terms of operational impedances. General equations of Park form the starting point of the present analysis and then the short circuit analysis is carried out.

All the assumptions made in article 4.1 for the derioation of the transient equivalent circuits, are also made here. Usually, many machine analyses are carried out with the transformer voltages omitted. The advantage is that the resulting equations are much simpler. Neglect of the transformer voltages amounts to dropping out the d.c. and second harmonic components in the phase currents or in other words the armature transients or free transients are not considered, since they have no significant effect on the currents, voltages and performance of the mach: Sinusoidally time varying current forms the starting point of the present analysis.[11]

6.2. LINE TO LINE SHORT CIRCUIT

Let the current in phases 'b' and 'c' having only fundamental component and with a varying amplitude,(the nature of variation being unknown at this stage) be given by

$$i_b = I_m(p) \cos (t + \gamma) \qquad (6.1)$$

" γ " having any value

Let "θ" the electrical angle between the direct axis and t] axis of phase "a" be given by

$$\theta = t +$$

Also

$i_b = I_m(p) \cos(t + \lambda_1 - \pi/2)$ as the short circuit current is a zero lagging one

$$\therefore \quad \gamma = \lambda_1 - \pi/2 \quad \text{or} \quad \lambda_1 - \gamma = \pi/2 \qquad (6.2)$$

When '$\lambda_1 = 0$' the linkage in phase 'a' is a maximum and hence in the common axes of phases "b" and "c" a minimum. Thus, by the constant linkage theorem, the currents in phases 'b' and 'c' are zero, which is satisfied if '$\gamma = \pi/2$'. Similarly when '$\lambda_1 = \pi/2$' the currents in phases 'b' and 'c' must be maximum which is satisfied if $\gamma = 0$. Thus the relation (6.2) is true.

Thus, on applying Park's transformation, there are

$$i_d = -\frac{2}{3}\left[i_b \cos(\theta - 120^\circ) + i_c \cos(\theta + 120^\circ)\right]$$

$$= \frac{2}{3} I_m(p) \cos(t + \gamma)\left[\cos(t + \lambda_1 - 120^\circ) - \cos(t + \lambda_1 + 120^\circ)\right]$$

$$= \frac{1}{\sqrt{3}} I_m(p)\left[\operatorname{Sin}(2t + \lambda_1 + \gamma) + \operatorname{Sin}(\lambda_1 - \gamma)\right]$$

$$= \frac{1}{\sqrt{3}} I_m(p)\left[1 + \operatorname{Sin}(2t + \lambda_1 + r)\right]$$

$$i_q = -\frac{2}{3}\left[i_b \sin(\theta - 120^\circ) + i_c \sin(\theta + 120^\circ)\right]$$

$$= \frac{2}{3} I_m(p) \cos(t + \gamma) \cos(t + \lambda_1)$$

$$= \frac{1}{\sqrt{3}} I_m(p)\left[\operatorname{Cos}(2t + \lambda_1 + \gamma) + \cos(\gamma - \lambda_1)\right]$$

$$= \frac{1}{\sqrt{3}} I_m(p) \cos(2t + \lambda_1 + \gamma)$$

The linkages in the direct and quadrature axis are obtained from the currents and the corresponding operational impedances. Constant excitation is assumed.

$$\therefore \psi_d = - x_d(p)\, i_d$$

$$= - \frac{1}{3} I_m(p) \left[x_d(p) + x_d'' \sin (2t + \gamma + \lambda_1) \right]$$

$$\psi_q = - x_q(p)\, i_q$$

$$= - \frac{1}{3} I_m(p) \left[x_q'' \cos (2t + \gamma + \lambda_1) \right]$$

The linkages of phases b and c may be calculated from expressions of "ψ_d" and "ψ_q" as

$$\psi_b = \psi_d \cos (\theta - 120^\circ) - \psi_q \sin (\theta - 120^\circ)$$

$$\psi_c = \psi_d \cos (\theta + 120^\circ) - \psi_q \sin (\theta + 120^\circ)$$

$\therefore \psi_b - \psi_c$ = Net linkages.

$$= \sqrt{3} \left[\psi_d \sin \theta + \psi_q \cos \theta \right]$$

$$= -I_m (p) \left[x_d(p) + x_d'' \sin (2t + \gamma + \lambda_1) \right] \sin (t + \lambda_1)$$

$$- I_m(p)\, x_q'' \cos (2t + \gamma + \lambda_1) \cos (t + \lambda_1)$$

$$= - I_m(p) \left\{ \left[x_d(p) + \frac{x_d'' + x_q''}{2} \right] \sin (t + \lambda_1) + \right.$$

$$\left. + \left[\frac{x_q'' - x_d''}{2} \right] \cos (3t + \gamma + 2 \lambda_1) \right\}$$

Neglecting the third harmonic component of linkage, the voltage between phases 'b' and 'c' may be obtained as,

$$e_b - e_c = - I_m(p) \left[x_d(p) + x_2 \right] \cos (t + \lambda_1)$$

$$= - E_m \cos (t + \lambda_1)$$

where

$$x_2 = \frac{x_d'' + x_q''}{2}$$

Therefore, the response of the current is

$$I_m(p) = \frac{E_m}{x_d(p) + x_2} \quad 1 \qquad (6.3)$$

Here 'E_m' is the maximum of line to line voltage before short circuit.

6.3. LINE TO NEUTRAL SHORT CIRCUIT

Let the current in phase 'a' having only fundamental component with its varying amplitude be given by

$$i_a = I_m(p) \cos (t + \gamma) \qquad (6.4)$$

Similarly let $\theta = t + \lambda_1$

$$\therefore i_a = I_m(p) \cos (t + \lambda_1)$$

$$\therefore \lambda_1 = \gamma \qquad (6.5)$$

When $\lambda = 0$, γ should be equal to zero for the linkage in the phase 'a' to be maximum.

Since there is zero sequence current also, the transformation equation is

$$i_a = i_d \cos \theta - i_q \sin \theta + i_0$$

where

$$i_d = \frac{2}{3} \quad i_a \cos \theta$$

$$= \frac{2}{3} \quad I_m(p) \cos^2 (t + \lambda_1)$$

$$= \frac{1}{3} \quad I_m(p) \left[1 + \cos (2t + 2\lambda_1)\right]$$

$$i_q = -2/3 \; i_a \sin \theta$$

$$= -2/3 \; I_m(p) \cos (t + \lambda_1) \sin (t + \lambda_1)$$

$$= - 1/3\ I_m(p)\ \sin (2t + 2\lambda_1)$$

$$i_0 = 1/3\ i_a = 1/3\ I_m(p) \cos (t + \lambda_1)$$

As in the last section, the linkages are

$$\psi_d = -1/3\ I_m(p)\ x_d(p) + x_d(j2w) \cos (2t+2\lambda_1)$$

$$\psi_q = 1/3\ I_m(p)\ x_q\ (j2w) \sin\ (2t + 2\lambda_1)$$

$$\psi_0 = - 1/3\ I_m(p)\ x_0 \cos (t + \lambda_1)$$

Thus

$$\psi_a = \psi_d \cos \theta - \psi_q \sin \theta + \psi_0$$

$$= -\frac{I_m(p)}{3} \left[x_d(p) + x_d'' \cos (2t + 2\lambda_1) \right] \cos(t + \lambda_1)$$

$$- \frac{I_m(p)}{3}\ x_q'' \sin (2t + 2\lambda_1) \sin (t + \lambda_1)$$

$$- \frac{I_m(p)}{3}\ x_0 \cos (t + \lambda_1)$$

$$= - \frac{I_m(p)}{3} \left[x_d(p) + \frac{x_d'' + x_q''}{2} + x_0 \right] \cos (t + \lambda_1)$$

$$- \frac{I_m(p)}{3} \left[\frac{x_d'' - x_q''}{2} \right] \cos (3t + \gamma + 2\lambda_1)$$

On putting $\frac{x_d'' + x_q''}{2} = x_2$ and neglecting the third harmonic component of linkage, the voltage of the phase 'a' before short circuit is

$$e_a = \frac{I_m(p)}{3} \left[x_d(p) + x_2 + x_0 \right] \sin (t + \lambda_1)$$

$$= E_m \sin (t + \lambda_1)$$

Thus, the current response is given by,

$$I_m(p) = \frac{3E_m}{x_d(p) + x_2 + x_0}\ 1 \qquad (6.6)$$

Here 'E_m' is the maximum of line to neutral voltage before short circuit.

6.4. TESTS AND VERIFICATIONS:

Tests have been conducted on three phase star-connected salient pole synchronous machine (G.E.), rated at 110 volts, 21 amperes, 4 kvA, 50 c/s 1000 r.p.m. having the following parameters; all in p.u.

r	= 0.0333	x_{kd}	= 0.0710
x_1	= 0.0803	x_d	= 0.897
x_0	= 0.0565	x_d'	= 0.133
x_f	= 0.01082	x_d''	= 0.1122
r_{kd}	= 0.0482	x_q	= 0.431
		x_q''	= 0.364

The quadrature axis amortisseur bars are incomplete. This machine is the same as that in Chapter III.

6.4.1. CAPACITIVE LOADING

(i) ARMATURE TRANSIENTS

The machine is running on no load and generating 1.022 p.u. voltage at rated speed, when a load $-j3.51$, across the line a and the neutral of the machine, is suddenly applied. The field voltage e_{fd} =96 V and the field current is 1.68 amperes.

Neglecting $(r/3)$ $(r/5)$ etc. the equivalent circuit may be drawn as shown in fig. 6.1(a). Neglecting the effects of capacitance on 9th harmonic and higher that is, neglecting $\left(\frac{3x_{cF}}{81}\right)$, $\left(\frac{3x_{cF}}{121}\right)$ etc. the equivalent reactance across $0_1\ 0_1'$ is,

$$= -\left(x_d'' + \frac{x_0}{2}\right) + \sqrt{\left(x_d'' + \frac{x_0}{2}\right)\left(x_q'' + \frac{x_0}{2}\right)}$$

$$= 0.0948$$

as deduced in the article 5.3.1. Simplifying the circuits of figure 6.1(a) step by step, the equivalent circuit of figure 6.1(b) is obtained

(a) Sub-transient Stage:

Armature resistance, being small, is neglected and the following values are obtained.

$$A = \frac{1}{X + x_1} = -0.09725 \qquad B=0$$

$$C = A + \frac{1}{x_d' - x_1} = 18.85275; \quad D = A + \frac{1}{x_{aq}} = 2.75275$$

So

$$E_{d0}^{o} = \frac{E_{d0}}{1 + (x_d'' - x_1)A} = 1.026$$

$$E_{q0}^{o} = E_q^{o} = 0$$

$$E_{d0}^{o'} = \frac{E_{d0}(C-A)}{C} = \frac{E_{d0}}{1 + \frac{x_d' - x_1}{k_3}} = 1.028$$

$$1/\alpha_1 = 1/\alpha_2 = \frac{1 + x_{kd}C}{r_{kd}C} = 2.57$$

So

$$E_{d0}^{o} = 1.026 - 0.002e^{-t/2.57} \qquad (6.7)$$

Therefore, the voltage of phase a

$$|e_a^1| = 3x_{cF}\,|i_1^1| = 3x_{cF}\, i_d^o = 3\, x_{cF}\, AE_d^o = \frac{3x_{cF}}{k_3} E_d^o$$

$$= 1.038\, E_d^o \qquad (6.8)$$

(b) Post -Subtransient Stage

$k_1 = -1.0 \qquad k_3 = -10.13$

$k_2 = 0.0102 \qquad k_4 = -\frac{1}{k_3} = 0.0986$

$$i_{fd0} = \frac{1.68}{1.222} = 1.375$$

$$r_{fd}(\text{ohmic}) = \frac{96}{1.68} = 57.12 \text{ ohms.}$$

$$r_{fd} = \frac{57.12}{29} \times 0.01082 = 0.0213$$

So the field voltage

$$e_{fd} = r_{fd} \times i_{fd0} = 0.0213 \times 1.375 = 0.0293$$

Now

$$E_d^o = C_1 e^{-\beta t} + E_{ds}^o$$

where

$$\beta = \frac{k_3 r_{fd} + x_{ad} r_{fd}}{k_3(x_{ad}+x_f) + x_f x_{ad}} = 0.0226$$

The initial value

$$E_{d0}^o = \frac{E_{d0}}{1+\frac{x_d' - x_1}{k_3}} = 1.028$$

$$E_{ds}^o = \frac{e_{fd}}{r_{fd}\left(\frac{1}{x_{ad}} + \frac{1}{k_3}\right)} = 1.222$$

Therefore $E_d^o = 1.222 - 0.194\, e^{-0.0226t}$ (6.9)

The voltage of phase a

$$|e_a^1| = 3\, x_{cF}\, |i_1^1| = 3\, x_{cF}\, i_d^o = \frac{3\, x_{cF}}{k_3} E_d^o = 1.038 E_d^o \qquad (6.8)$$

The values of the terminal voltage of phase a have been obtained from the equations (6.7) (6.8) and (6.9) and compared with the values as obtained from oscillogram, in figure 6.3. The oscillogram is shown in figure 6.2.

(ii) FIELD TRANSIENTS

$$i'^{o}_{fd0} = \frac{E_{d0} + x_f\, i_{fd0} - E'^{o}_{d0}}{x_f} = 1.270$$

So the solution for the subtransient stage, as obtained from the relations (3.55) and (6.7)

$$i^{o}_{fd} = 1.27 + 0.0347\, e^{-t/2.57} \qquad (6.10)$$

For the post-subtransient stage

$$i^{o}_{fd} = \frac{E^{o}_{d}\left(1 + \frac{x_{ad}}{k_3}\right)}{x_{ad}} - \frac{1}{x_f}\left(C_1 e^{-\alpha_1 t} + C_2 e^{-\alpha_2 t} \ldots\right) \qquad (3.56)$$

$$= 1.1254\, E^{o}_{d} + 0.0354\, e^{-t/2.57}$$

$$= 1.375 - 0.2185\, e^{-0.0226\, t} + 0.0354\, e^{-t/2.57} \qquad (6.11)$$

The field currents are compared with the test in figure 6.4 The field current oscillogram is shown in fig. 5.2.

6.4.2. INDUCTIVE LOADING

(i) ARMATURE TRANSIENTS

The machine initially running on no load, is generating 1.01 p.u. voltage at rated frequency. A load of impedance 0.208+ j3.225 is suddenly switched across phases b and c. The field voltage e_{fd}=97 volts and the steady field current is 1.62 amperes.

METHOD No. 1

The equivalent circuit neglecting (r/3), (r/5) etc. may be drawn as shown in fig. 6.5(a). The equivalent reactance across $0_1 0_1'$ is

$$= -\left(x_q'' + \frac{x_F}{2}\right) + \sqrt{\left(x_q'' + \frac{x_F}{2}\right)\left(x_d'' + \frac{x_F}{2}\right)}$$

$$= -0.1305$$

as deduced in the article 5.3.3.

So, after eliminating higher harmonics, the equivalent circuit is reduced to that shown in fig. 6.5(b)

(a) Sub-transient Stage:-

The armature and load resistances are neglected, as they are small, thus the following values are obtained.

$$A = \frac{1}{X + x_1} = 0.282 \qquad B = 0$$

$$C = A + \frac{1}{x_d' - x_1} = 19.232 \qquad D = A + \frac{1}{x_{aq}} = 3.132$$

So

$$E_{d0}^{o} = \frac{E_{d0}}{1 + (x_d'' - x_1)\,A} = 1.00$$

$$E_{q0}^{o} = E_q^{o} = 0$$

$$E_{d0}^{o'} = \frac{E_{d0}\,(C-A)}{C} = \frac{E_{d0}}{1 + \frac{x_d' - x_1}{k_3}} = 0.995$$

$$1/\alpha_1 = 1/\alpha_2 = \frac{1 + x_{kd}C}{r_{kd}C} = 2.545$$

Therefore for the subtransient stage

$$E_d^{o} = 0.995 + 0.005\,e^{-t/2.545} \qquad (6.12)$$

The voltage between the lines b and c (in p.u. of base line voltage).

$$|e_b^1 - e_c^1| = \sqrt{r_F^2 + x_F^2}\ |i_1^1| = \sqrt{r_F^2 + x_F^2}\ i_d^o = \sqrt{r_F^2 + x_F^2}\ .AE_d^o$$

$$= \sqrt{r_F^2 + x_F^2}\ \frac{E_d^o}{k_3} = 0.906\ E_d^o \qquad (6.13)$$

(b) Post-Subtransient Stage

$k_1 = 1.0$ $k_3 = 3.56$

$k_2 = 0.717$ $k_4 = \frac{1}{k_3} = 0.281$

$$i_{fd0} = \frac{1.62}{1.222} = 1.328$$

$$r_{fd}\ \text{(ohmic)} = \frac{97}{1.62} = 59.8 \text{ ohms}$$

$$r_{fd} = \frac{59.8}{29} \times 0.01082 = 0.02235$$

So, the field voltage

$$e_{fd} = r_{fd} \times i_{fd0} = 0.0297$$

$$E_d^o = C_1 e^{-\beta t} + E_{ds}^o$$

where

$$\beta = \frac{k_3 r_{fd} + x_{ad} r_{fd}}{k_3(x_{ad} + x_f) + x_f x_{ad}} = 0.0308$$

The initial value

$$E_{d0}^{o\prime} = \frac{e_{fd}}{1 + \frac{x_d' - x_1}{k_3}} = 0.995$$

$$E_{ds}^o = \frac{e_{fd}}{r_{fd}\left(\frac{1}{x_{ad}} + \frac{1}{k_3}\right)} = 0.882$$

Therefore, $E_d^o = 0.882 + 0.113\ e^{-0.0308\ t}$ (6.14)

The voltage between lines b and c is

$$\left| e_b^1 - e_c^1 \right| = \sqrt{r_F^2 + x_F^2} \;\; \left| i_1^1 \right| = \sqrt{r_F^2 + x_F^2} \;\; i_d^o = \frac{\sqrt{r_F^2 + x_F^2}}{k_3} E_d^o$$

$$= 0.906 \; E_d^o \qquad (6.13)$$

The equations (6.12), (6.13) and (6.14) give the line voltage between b and c. The calculated values are compared with those obtained from the oscillogram in fig. 6.7. The oscillogram is shown in fig. 6.8

(ii) FIELD TRANSIENTS

$$i_{fd0}^{o'} = \frac{E_{d0} + x_f i_{fd0} - E_{d0}^{o'}}{x_f} = 1.573$$

So the solution for the subtransient, as obtained from the relations (3.55) and (6.12)

$$i_{fd}^o = 1.573 - 0.0874 \; e^{-t/2.545} \qquad (6.15)$$

For the post-subtransient stage,

$$i_{fd}^o = \frac{E_d^o \left(1 + \frac{x_{ad}}{kl}\right)}{x_{ad}} - \frac{1}{x_f} \left[C_1 e^{-\alpha_1 t} + C_2 e^{-\alpha_2 t} \right]$$

$$= 1.503 \; E_d^o - 0.0885 \; e^{-t/2.545}$$

$$= 1.327 + 0.170 e^{-0.0308t} - 0.0874 e^{-t/2.545} \qquad (6.16)$$

The field currents are compared with the test in figure 6.6 The field current oscillogram is shown in figure 6.8.

METHOD No. 2

The article 6.2 for obtaining the line to line short circuit current can also be used to determine armature transients for the curren under line to line inductive loading.

$$I_m(p) = \frac{E_m}{x_d(p) + x_2} \; 1 \qquad (6.3)$$

Now $$x_d(p) = x_d \frac{(1+T_d'p)(1+T_d''p)}{(1+T_{d0}'p)(1+T_{d0}''p)}$$

As the amortisseur has a very small time constant, T_d'' and T_{d0}'' may be neglected.

Therefore

$$x_d(p) = x_d \frac{(1 + T_d' p)}{(1 + T_{d0}' p)}$$

When, instead of direct short circuit, there is inductive loading between line to line, the expression for $x_d(p)$ may be written as

$$x_d(p) = (x_d + x_F) \quad \frac{1 + T_d' p}{1 + T_d' p}$$

Therefore

$$I_m(p) = \frac{E_m (1 + p\, T_{d0}')}{p\left[(x_d + x_F)(1 + pT_d') + (x_2(1 + p\, T_{d0}')\right]}$$

which, after further simplification, may be written as

$$i(t) = \frac{E_m}{x_d + x_F + x_2} \quad \left[1 - (1 - \alpha\, T_{d0}')\; e^{-\alpha t} \right] \tag{6.17}$$

where

$$\alpha = \frac{x_d + x_F + x_2}{(x_d + x_F) T_d' + x_2 T_{d0}'} \tag{6.18}$$

Now

$$T_{d0}' = \frac{x_{ad} + x_f}{r_{fd}} = 39.1$$

$$T_d' = \frac{x_d' + x_F}{x_d + x_F} \; T_{d0}' = 31.85$$

$$= \frac{x_d + x_F + x_2}{T_d' (x_d + x_F) + T_{d0}' \; x_2} = 0.031$$

Therefore from (6.17) expression,

$$i(t) = \frac{1.01 \sqrt{2}}{4.3601} \quad \left[1 + 0.212\; e^{-0.031t} \right] \tag{6.19}$$

The voltage between phases b and c

$$\left| e_b^1 - e_c^1 \right| = \sqrt{r_F^2 + X_F^2} \; \left| i(t) \right| = 3.225 \left| i(t) \right| \tag{6.20}$$

The equations (6.19) and (6.20) give the line voltage between b and c The calculated values are compared with those obtained from the oscillogram in fig. 6.7.

6.4.3. LINE TO LINE SHORT CIRCUIT

The machine initially running on no load and generating 0.1773 p.u. voltage at rated speed or frequency, is suddenly short circuited across the phases b and c. The field voltage is 107 volts and initial field current is 0.255 amperes.

METHOD NO. 1

As the armature reistanceis small, neglecting its fractions, the equivalent circuit may be deduced as shown in figure 6.9(a).

As deduced in the article 5.33 the equivalent reactance across O, O'

$$= \sqrt{(x_q'' + \frac{x_F}{2})\ (x_d'' + \frac{x_F}{2})} - (x_q'' + \frac{x_F}{2})$$

$$= \sqrt{x_q''\ x_d''} - x_q'' = -0.162$$

Simplifying the circuits of fig. 6.9(a), the equivalent circuit of figure 6.9(b) is obtained.

(a) Subtransient Stage:-

As the armature resistance is not small, the following values are obtained on taking it into consideration.

$$A = \frac{X + x_1}{(R + r)^2 + (X + x_1)^2} = 3.29$$

$$B = \frac{R + r}{(R + r)^2 + (X + x_1)^2} = 0.76$$

$$C = A + \frac{1}{x_d - x_1} = 22.24 \qquad D = A + \frac{1}{x_{aq}} = 6.14$$

So

$$E_{d0}^{o} = \frac{E_{d0}}{1 + (x_d'' - x_1)\,A} = 0.1605$$

$$E_{q0}^{o} = E_q^{o} = 0$$

$$E_{d0}^{o} = \frac{E_{d0}(C-A)D}{B^2 + CD} = \frac{E_{d0}}{1 + \frac{x_d - x_1}{k_3}} = 0.1505$$

There is a single time constant

$$\frac{1}{\alpha_1} = \frac{1}{\alpha_2} = \frac{1 + x_{kd}C}{r_{kd}C} = 2.38$$

Therefore, for the subtransient stage,

$$E_d^{o} = 0.1505 + 0.01\ e^{-t/2.38}$$

The short circuit current

$$\left| i_1^1 \right| = k_4\ E_d^{o} = 0.51 + 0.0339\ e^{-t/2.38} \qquad (6.21)$$

(b) Post-Subtransient Stage:-

$$k_1 = \frac{X + x_q}{\sqrt{(R + r)^2 + (X + x_q)^2}} = 0.994; \quad k_2 = \frac{R + r}{\sqrt{(R+r)^2 + (X + x_q)^2}} = 0.116$$

$$k_3 = \frac{(R+r)k_2}{k_1} + (X + x_1) = 0.2964$$

$$k_4 = \frac{1}{k_3 k_1} = 3.39$$

$$i_{fd0} = \frac{0.255}{1.222} = 0.209$$

$$r_{fd}\ (\text{ohmic}) = \frac{107}{0.255} = 419 \text{ ohms}$$

$$r_{fd} = \frac{419}{29} \times 0.01082 = 0.1565$$

So, the field voltage

$$e_{fd} = r_{fd} \times i_{fd0} = 0.1565 \times 0.209 = 0.0327$$

$$E_d^o = C_1 e^{-\beta t} + E_{ds}^o$$

where $$\beta = \frac{r_{fd}\,(k_3 + x_{ad})}{k_3\,(x_{ad} + x_f) + x_f x_{ad}} = 0.562$$

The initial value

$$E_{d0}^{o'} = \frac{E_{d0}}{1 + \dfrac{x_d - x_1}{k_3}} = 0.1505$$

$$E_{ds}^{o} = \frac{e_{fd}\ \&}{r_{fd}\left(\dfrac{1}{x_{ad}} + \dfrac{1}{k_3}\right)} = 0.0455$$

So $$E_d^o = 0.105\, e^{-0.562t} + 0.0455$$

The short circuit current

$$\left|i_1^1\right| = k_4\, E_d^o = 0.356\, e^{-0.562t} + 0.1542 \qquad (6.22)$$

With the equation (6.21) and (6.22) the short circuit current is obtained. The calculated values are compared with the values obtained from the oscillogram, in fig. (6.14). The oscillogram is shown in figure (6.10).

<u>METHOD No. 2</u>: The equation (6.17) may be modified for line to line short circuit as

$$i(t) = \frac{E_m}{x_d + x_2}\left[1 - (1 - \alpha T_{d0}')\, e^{-\alpha t}\right] \qquad (6.17\ a)$$

where $$\alpha = \frac{x_d + x_2}{x_d\, T_d' + x_2\, T_{d0}'} \qquad (6.18a)$$

Now

$$T_{d0}' = \frac{x_{ad} + x_f}{r_{fd}} = 5.58$$

$$T_d' = \frac{x_d'}{x_d} \times T_{d0}' = 0.827$$

$$\alpha = \frac{x_d + x_2}{T_d' \; x_d + T_{d0}' \; x_2} = 0.548 = 0.548$$

From equation (6.17a);

$$i(t) = \frac{.1773 \sqrt{2}}{1.1351} \left[1 + 2.06 \; e^{-0.548t} \right]$$

$$= \sqrt{2} \; (0.1560) \left[1 + 2.06 \; e^{-0.548 \, t} \right] \qquad (6.23)$$

The variation of the short circuit current during sudden line to line short circuit can be obtained from equation (6.23) The calculated values are compared with the test values in figure (6.14).

6.4.4. LINE TO NEUTRAL SHORT CIRCUIT:

The machine, initially running on no load and generating 0.1432 p.u. voltage at rated frequency, is suddenly short-circuited across the line a and the neutral of the machine. The field voltage $e_{fd} = 0.331$ p.u. and the field current $i_{fd0} = 0.1595$ p.u.

METHOD NO.1

The equivalent circuit, neglecting fractions of armature resistance as it is small, is shown in fig. 5.11(a). The equivalent reactance across $O_1 \; O_1'$ is

$$= -\left(x_d'' + \frac{x_0}{2}\right) + \sqrt{\left(x_d'' + \frac{x_0}{2}\right)\left(x_q'' + \frac{x_0}{2}\right)}$$

$$= 0.0948.$$

as deduced in the article 5.3.1. Then simplifying the circuits of fig. 6.11(a) step by step, the equivalent circuit of fig. 6.11(b) is obtained.

(a) <u>Subtransient Stage:</u>

As the armature resistance is not small, the following values are obtained, taking into consideration the armature resistance

$$A = \frac{X + x_1}{(R + r)^2 + (X + x_1)^2} = 2.65$$

$$B = \frac{R + r}{(R + r)^2 + (X + x_1)^2} = 0.735$$

$$C = A + \frac{1}{x_d' - x_1} = 21.60 \qquad D = A + \frac{1}{x_{aq}} = 5.50$$

So

$$E_{d0}^{o} = \frac{E_{d0}}{1 + (x_d'' - x_1)A} = 0.1320$$

$$E_{q0}^{o} = E_q^{o} = 0$$

$$E_{d0}^{o\prime} = \frac{E_{d0}(C - A)\,D}{B^2 + CD} = \frac{E_{d0}}{1 + \frac{x_d' - x_1}{k_3}} = 0.1250$$

There is only single time constant

$$\frac{1}{\alpha_1} = \frac{1}{\alpha_2} = \frac{1 + x_{kd}C}{r_{kd}C} = 2.43$$

Therefore, for the subtransient stage

$$E_d^{o} = 0.1250 + 0.007\, e^{-t/2.43}$$

The short circuit current

$$|i_a^1| = 3\,|i_1^1| = 3\, k_4 E_d^{o} = 8.22\, E_d^{o} = 1.028 + 0.05754\, e^{-t/2} \qquad (6.24)$$

(b) Post-Subtransient Stage

$$k_1 = \frac{X + x_q}{\sqrt{(R + r)^2 + (X + x_q)^2}} = 0.989 \quad k_2 = \frac{R + r}{\sqrt{(R+r)^2 + (X + x_q)^2}} = 0.1449$$

$$k_3 = \left[\frac{(R+r)k_2}{k_1} + (X + x_1) \right] = 0.369 \qquad k_4 = \frac{1}{k_3 k_1} = 2.74$$

$$E_d^o = C_1 e^{-\beta t} + E_{ds}^o$$

where

$$\beta = \frac{r_{fd}\,(k_3 + x_{ad})}{k_3(x_{ad} + x_f) + x_f\, x_{ad}} = 0.667$$

The initial value

$$E_{d0}^{o'} = \frac{E_{d0}}{1 + \frac{x_d' - x_1}{k_3}} = 0.1250$$

$$E_{ds}^o = \frac{e_{fd}}{r_{fd}\left(\frac{1}{x_{ad}} + \frac{1}{k_3} \right)} = 0.0406$$

Therefore,

$$E_d^o = 0.0844 e^{-0.667t} + 0.0406$$

The short circuit current

$$|i_a^1| = 3|i_1^1| = 3\, k_4 E_d^o = 8.22\, E_d^o$$

$$= 0.694\, e^{-0.667\, t} + 0.3340 \qquad (6.25)$$

The equations (6.24) and (6.25) give the short circuit current for line to neutral short circuit between the line a and the neutral of the machine. The calculated values are compared in figure (6.13). The oscillogram is shown in figure.(6.12)

METHOD No.2

From the equation (6.6)

$$I_m(p) = \frac{3E_m}{x_d(p) + x_2 + x_0} \, 1 \qquad (6.6)$$

Taking

$$x_d(p) = x_d \frac{1+pT_d'}{1+p\,T_{d0}'}$$ and following the procedure, step by step, as adopted for the derivation of equation (6.17)

$$i(t) = \frac{3E_m}{x_d + x_2 + x_0}\left[1 - (1 - \alpha T_{d0}')\, e^{-\alpha t}\right] \qquad (6.26)$$

where

$$\alpha = \frac{x_d + x_2 + x_0}{x_d\, T_d' + (x_2 + x_0) T_{d0}'}$$

$$= 0.649$$

$$T_{d0}' = \frac{x_{ad} + x_f}{r_{fd}} = 4.21$$

$$T_d' = \frac{x_d'}{x_d} \times T_{d0}' = 0.623$$

From the expression (6.26);

$$i(t) = \frac{3E_m}{x_d + x_2 + x_0}\left[1 - (1 - \alpha T_{d0}')\, e^{-\alpha t}\right]$$

$$= \frac{3E_m}{1.2051}\left[1 - (1 - .649 \times 4.21) e^{-0.849\, t}\right]$$

$$= 0.3565\sqrt{2}\left[1 + 1.73\, e^{-0.649\, t}\right] \qquad (6.27)$$

The short circuit current for line to neutral short circuit can be obtained from equation (6.27) The calculated values are compare in fig. (6.13)

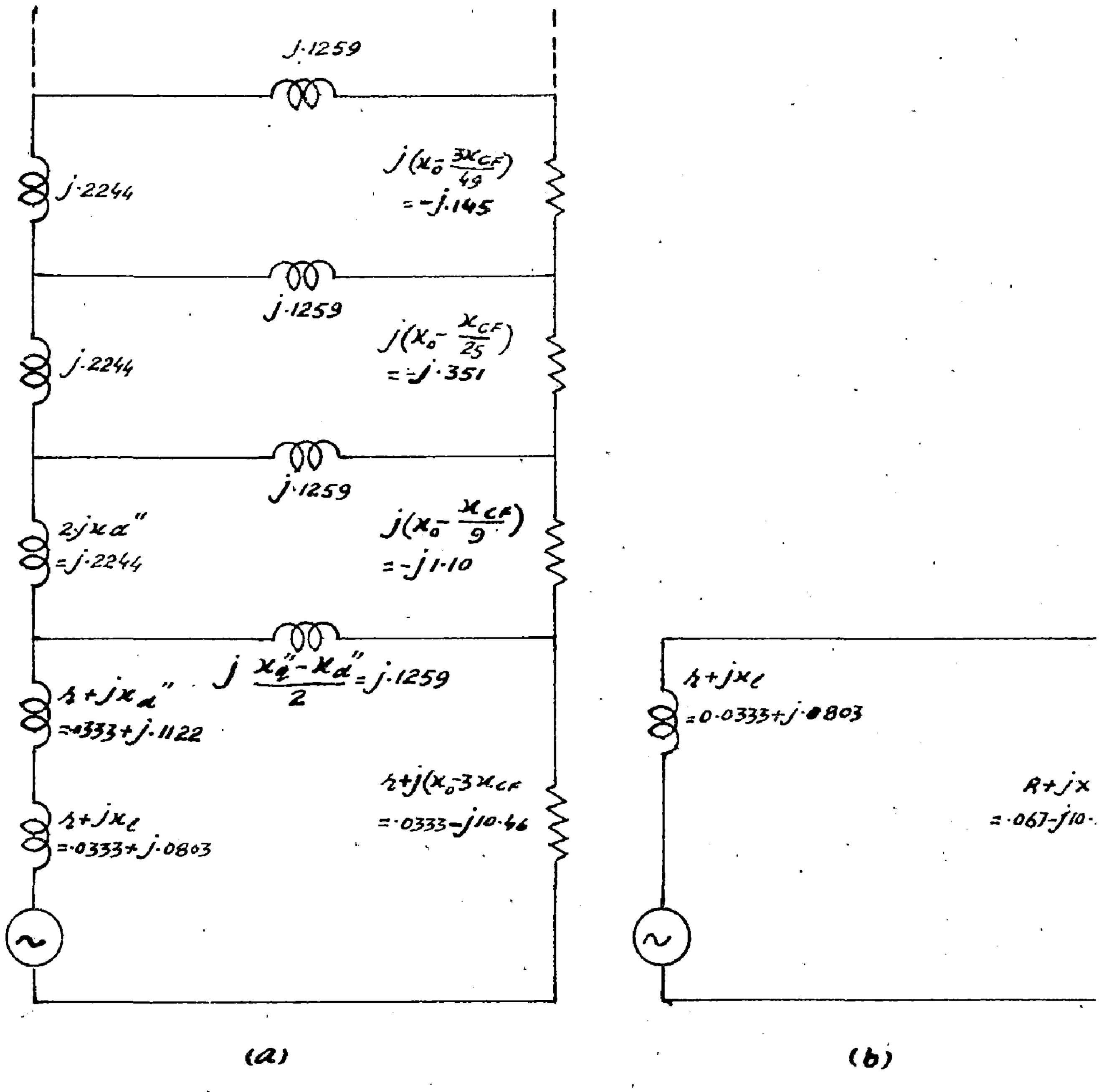


FIG. 6-1 THE CIRCUIT FOR SINGLE PHASE CAPACITOR LOADING.

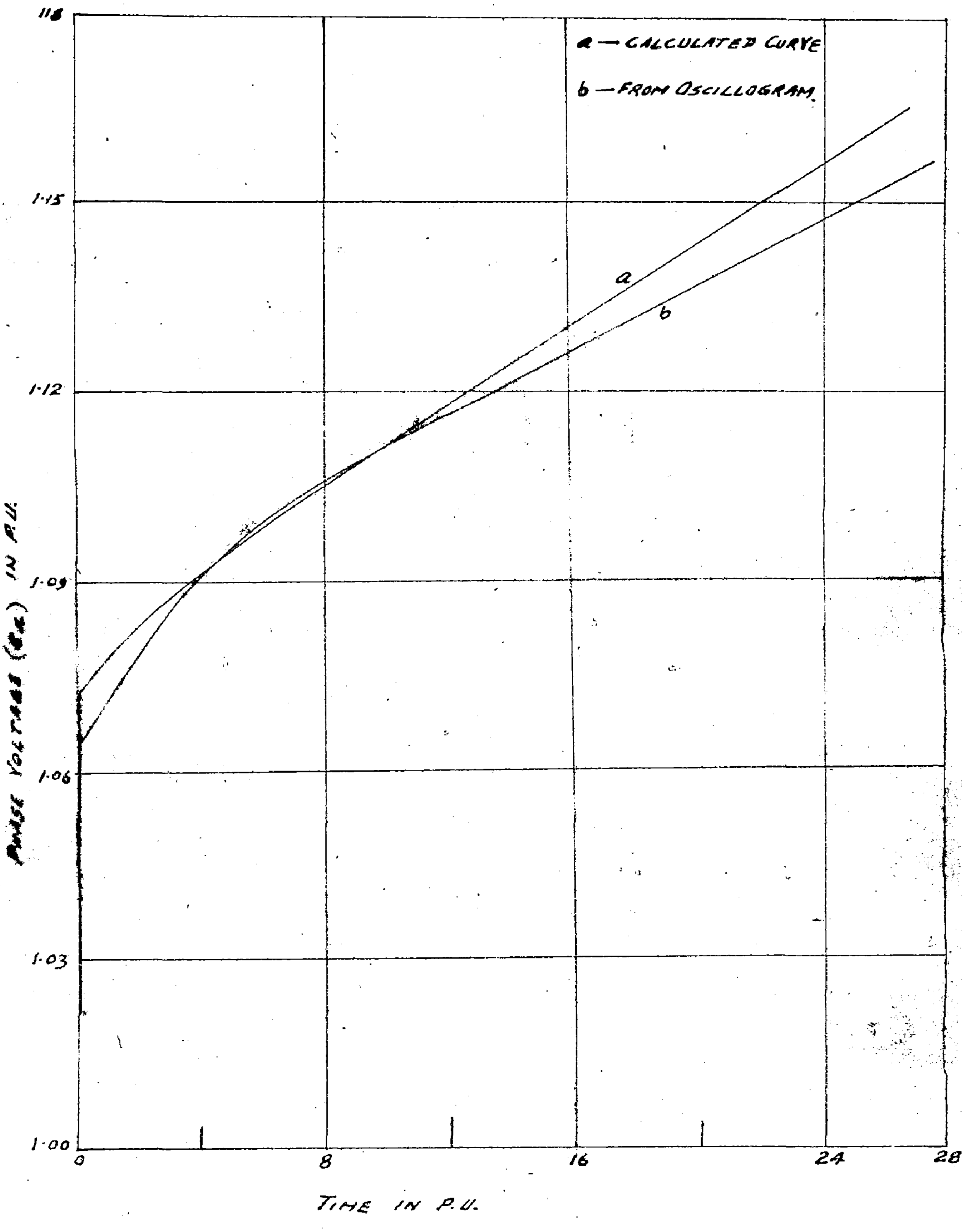


FIG. 6-3 LINE TO NEUTRAL CAPACITIVE LOADING.

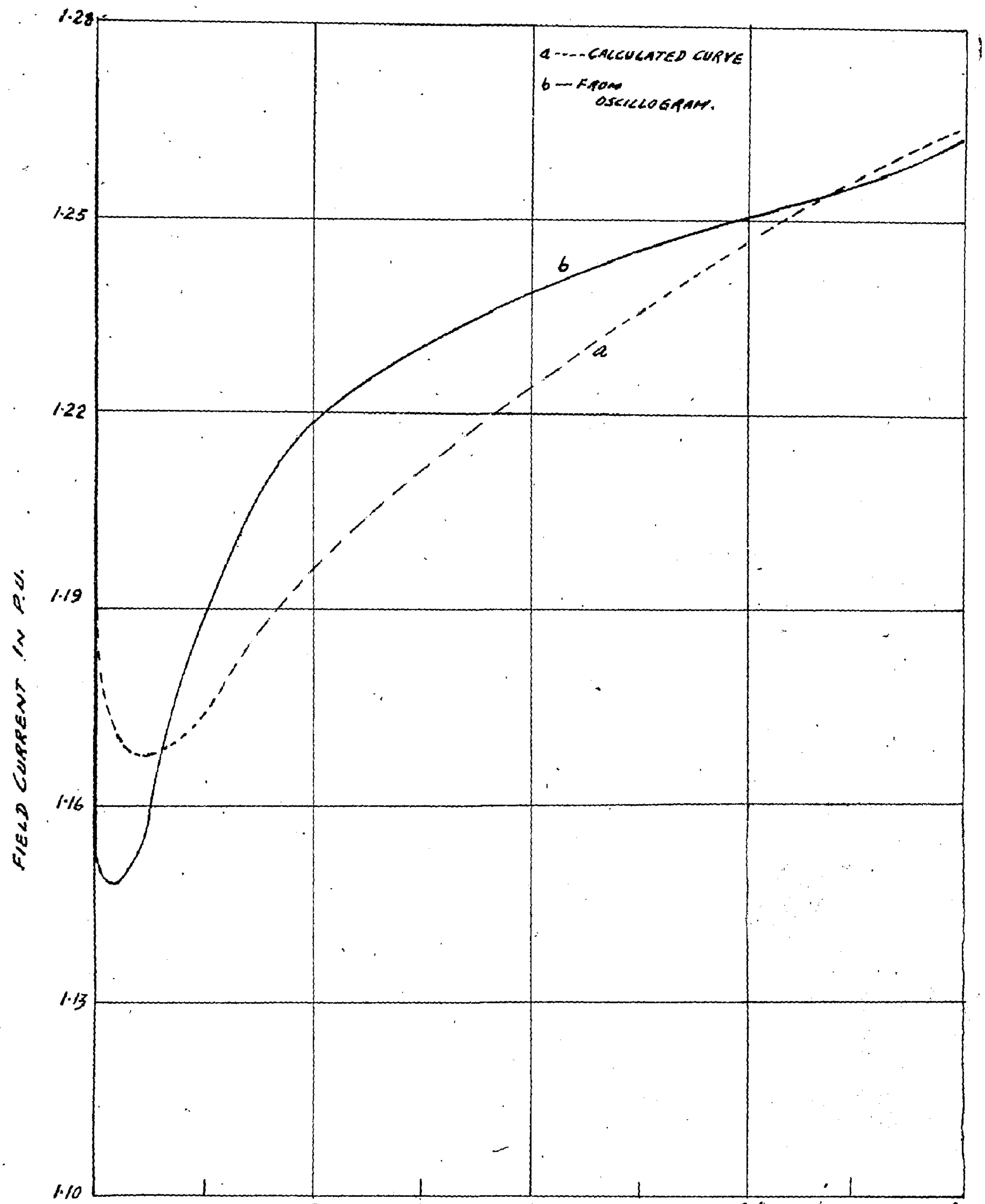


FIG.6-4 LINE TO NEUTRAL CAPACITIVE LOADING.

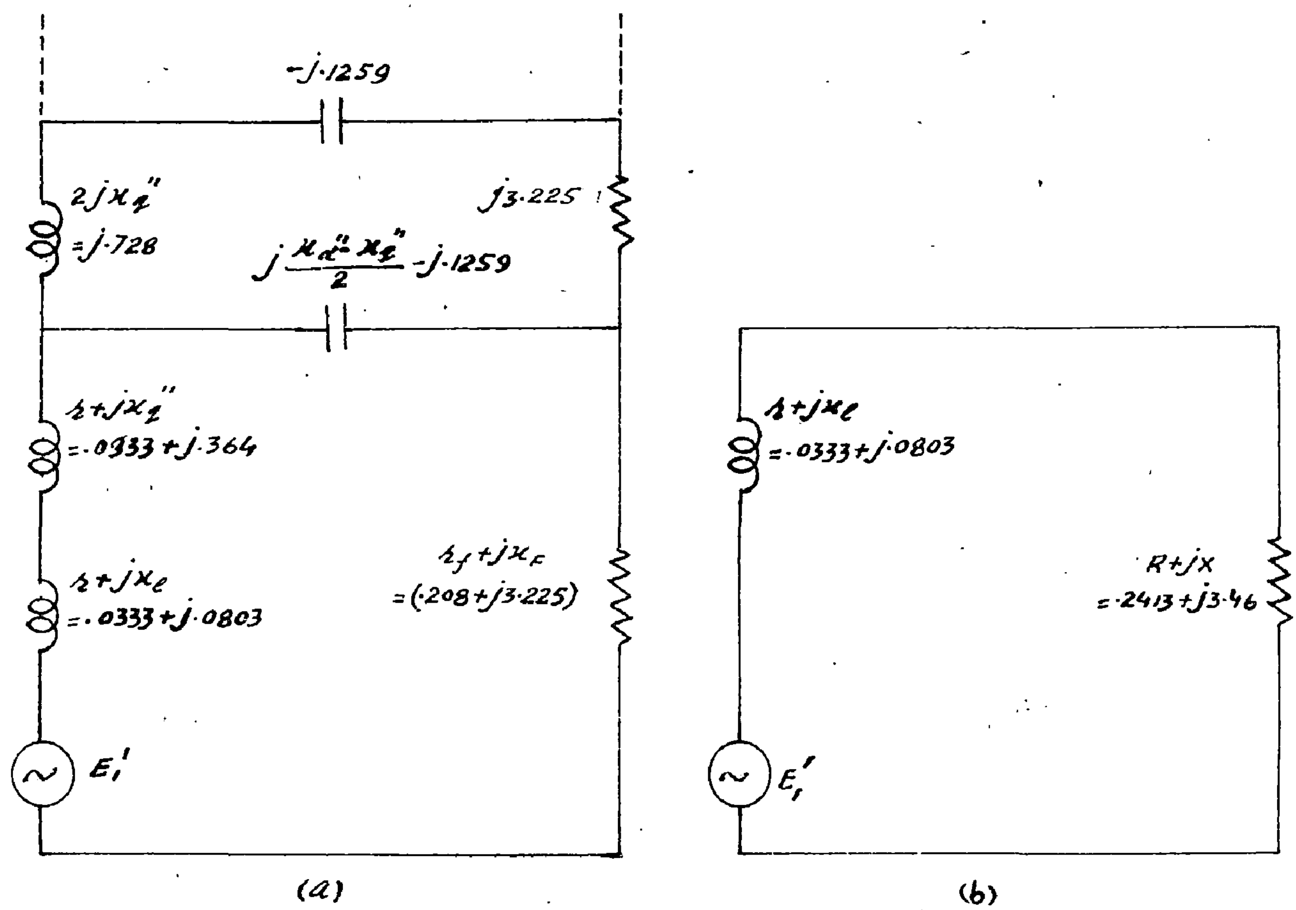


FIG. 6-5 THE CIRCUITS FOR LINE TO LINE INDUCTIVE LOADING.

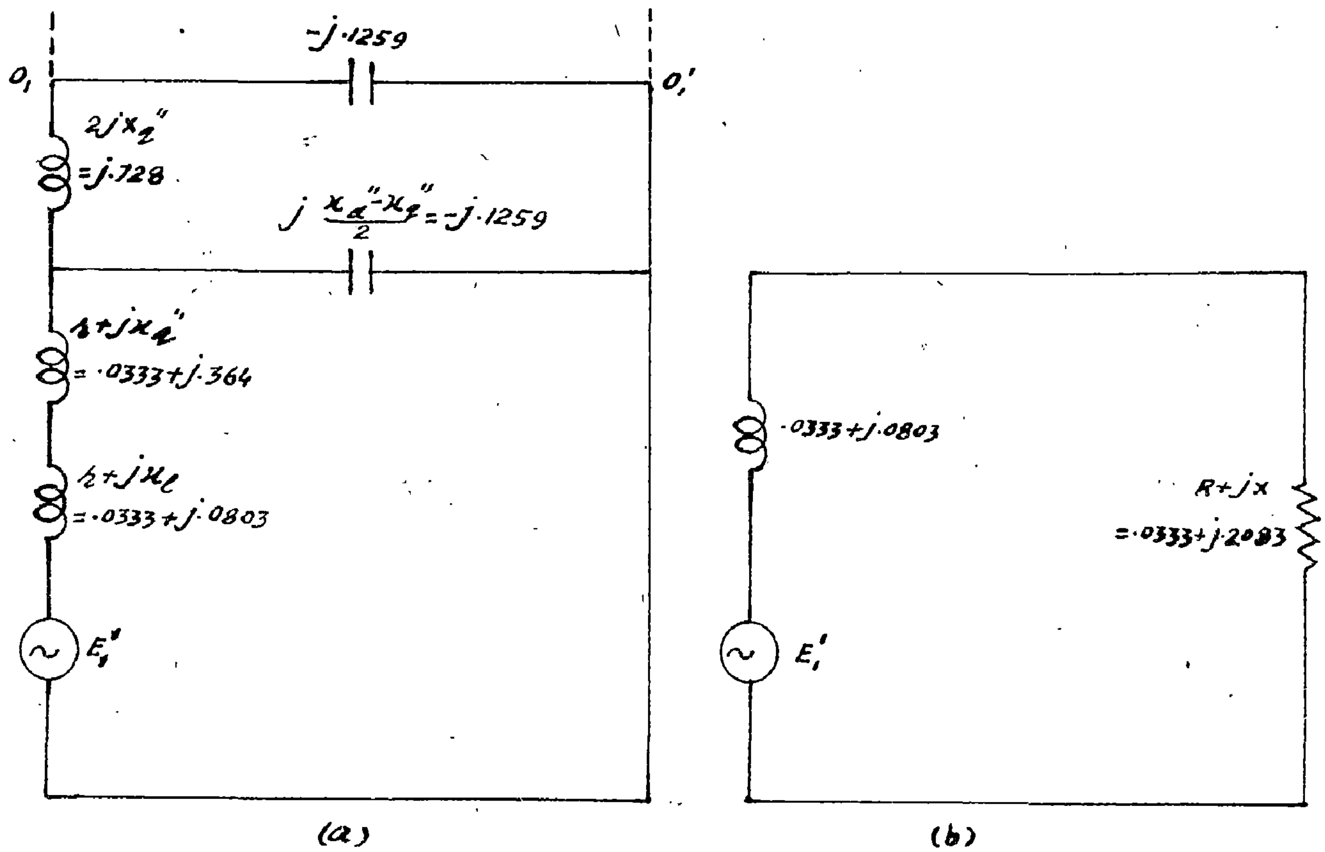


FIG. 6-9 THE CIRCUIT FOR LINE TO LINE SHORT CIRCUIT.

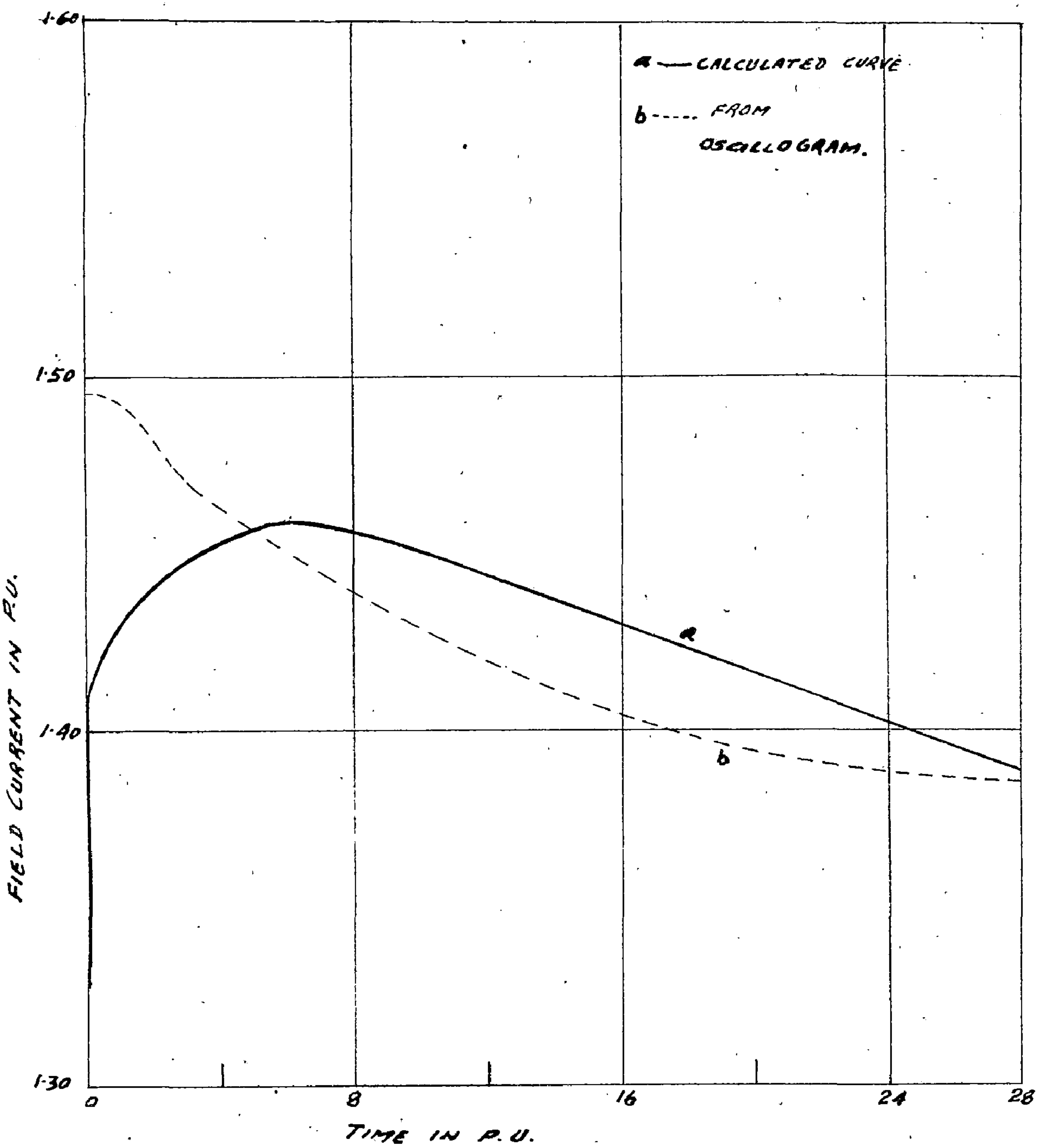


FIG. 6-6 LINE TO LINE INDUCTIVE LOADING.

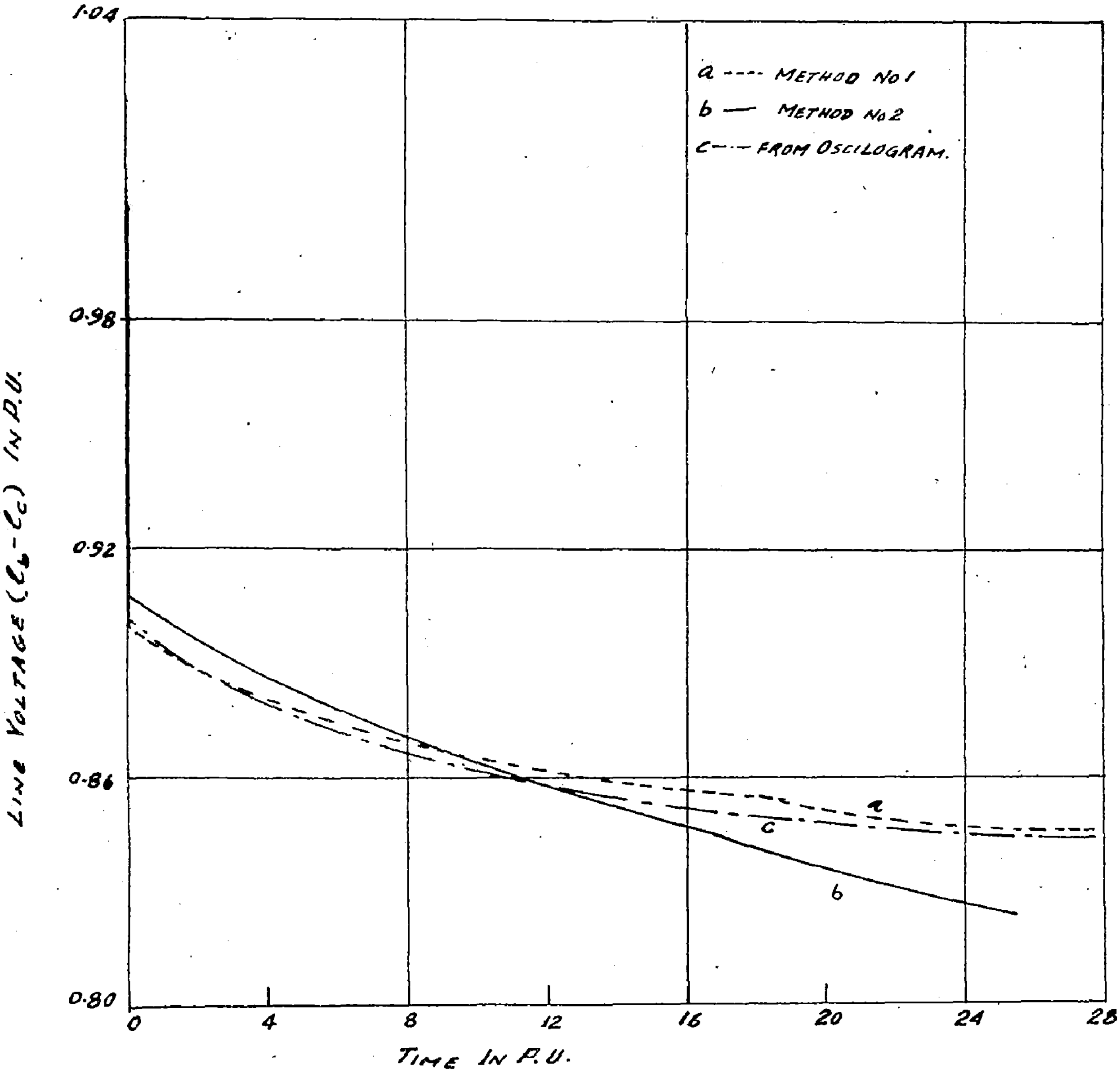


FIG. 6-7 LINE TO LINE INDUCTIVE LOADING.

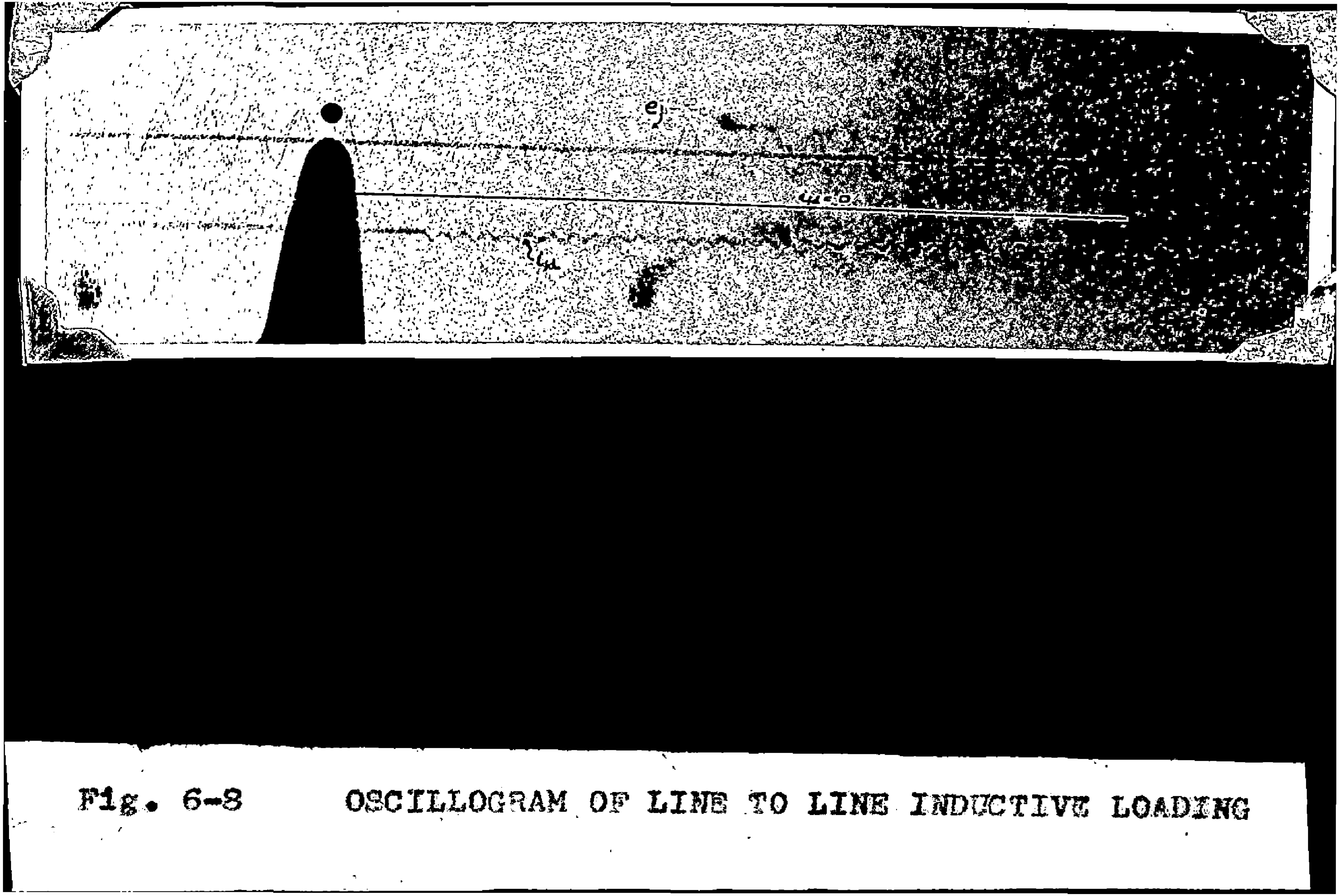


Fig. 6-8 OSCILLOGRAM OF LINE TO LINE INDUCTIVE LOADING

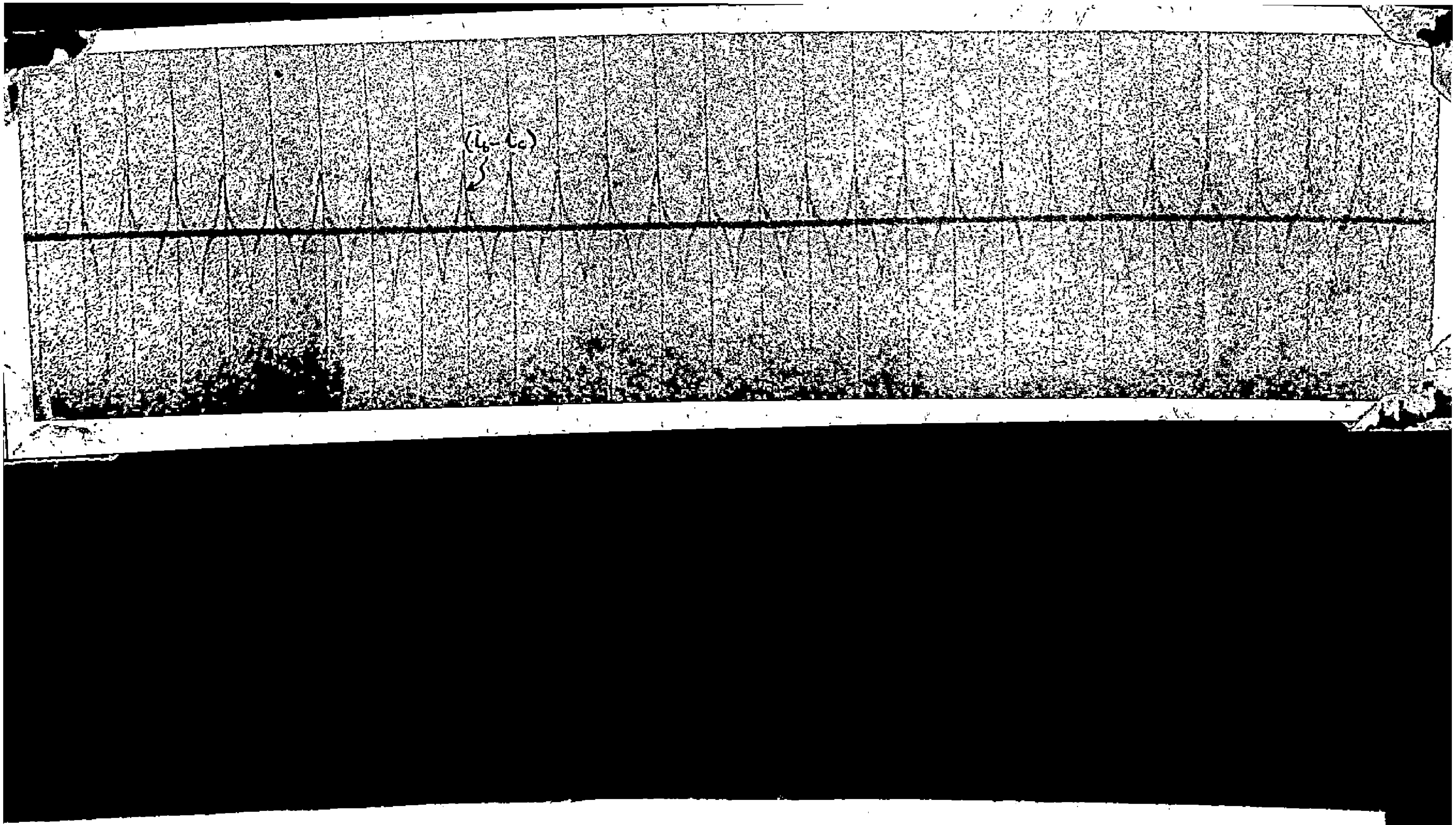


Fig. 6-10 OSCILLOGRAM OF LINE TO LINE SHORT CIRCUIT

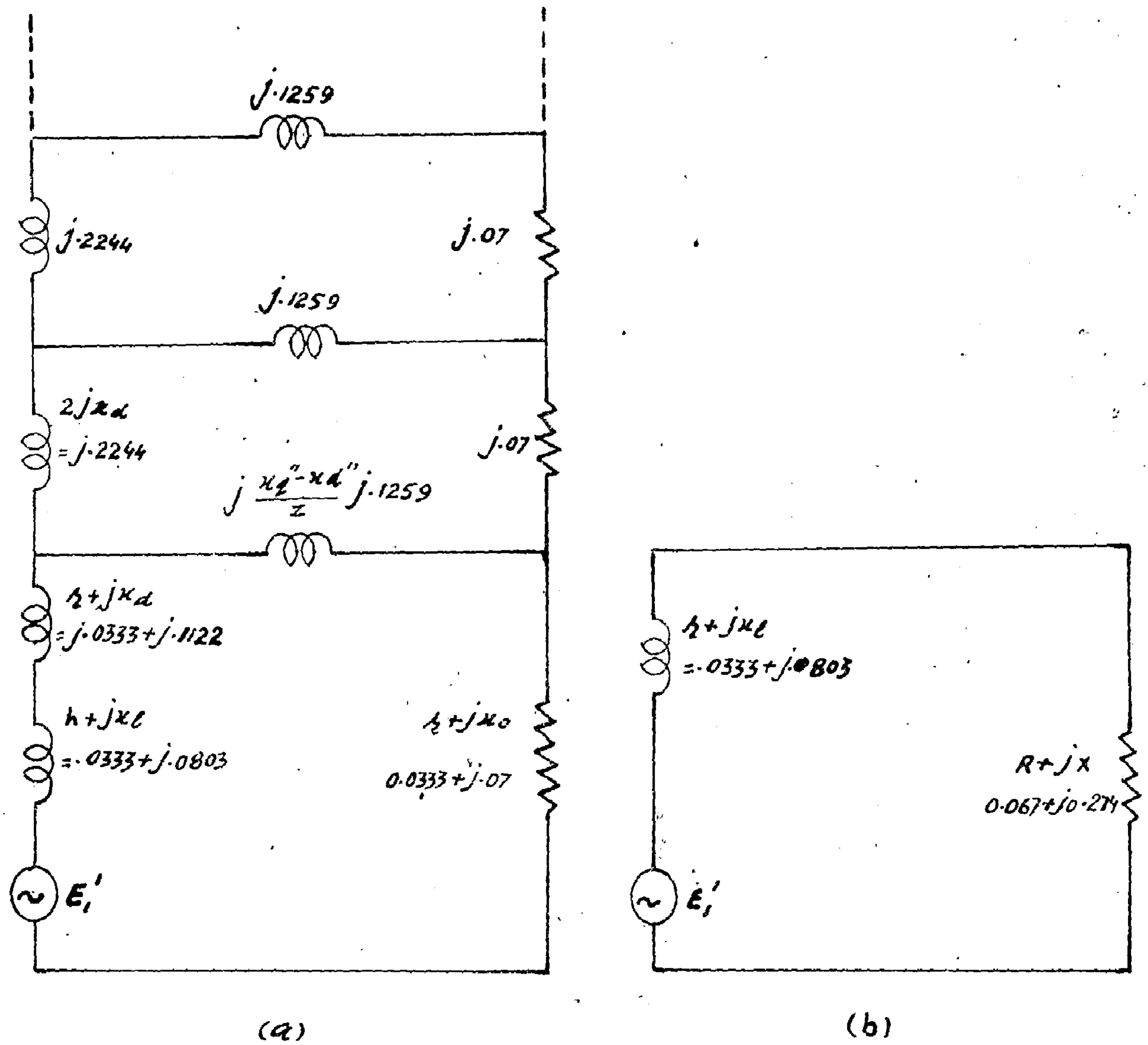


FIG. 6·11 THE CIRCUIT FOR LINE TO NEUTRAL CIRCUIT.

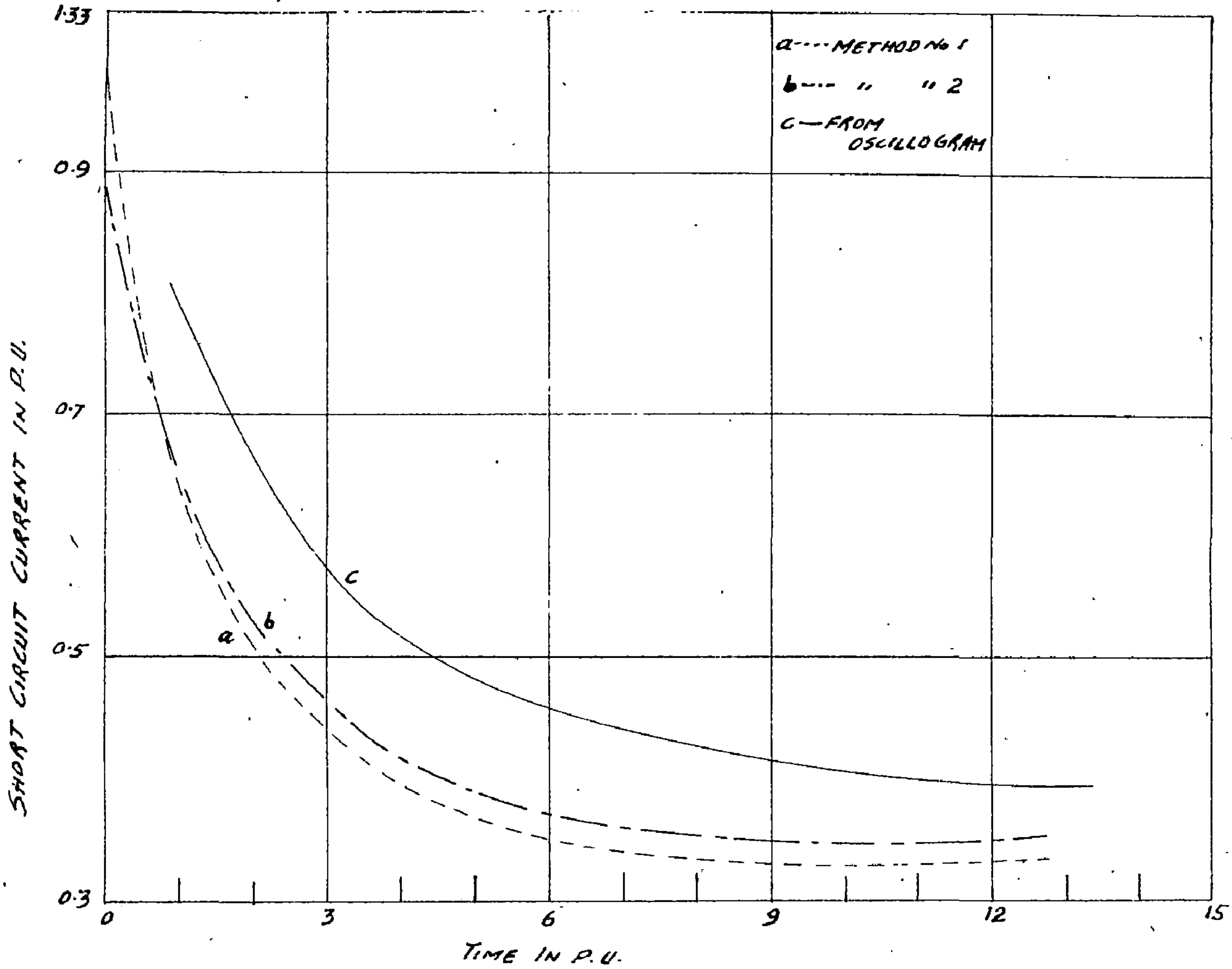


FIG. 6-13 LINE TO NEUTRAL SHORT CIRCUIT.

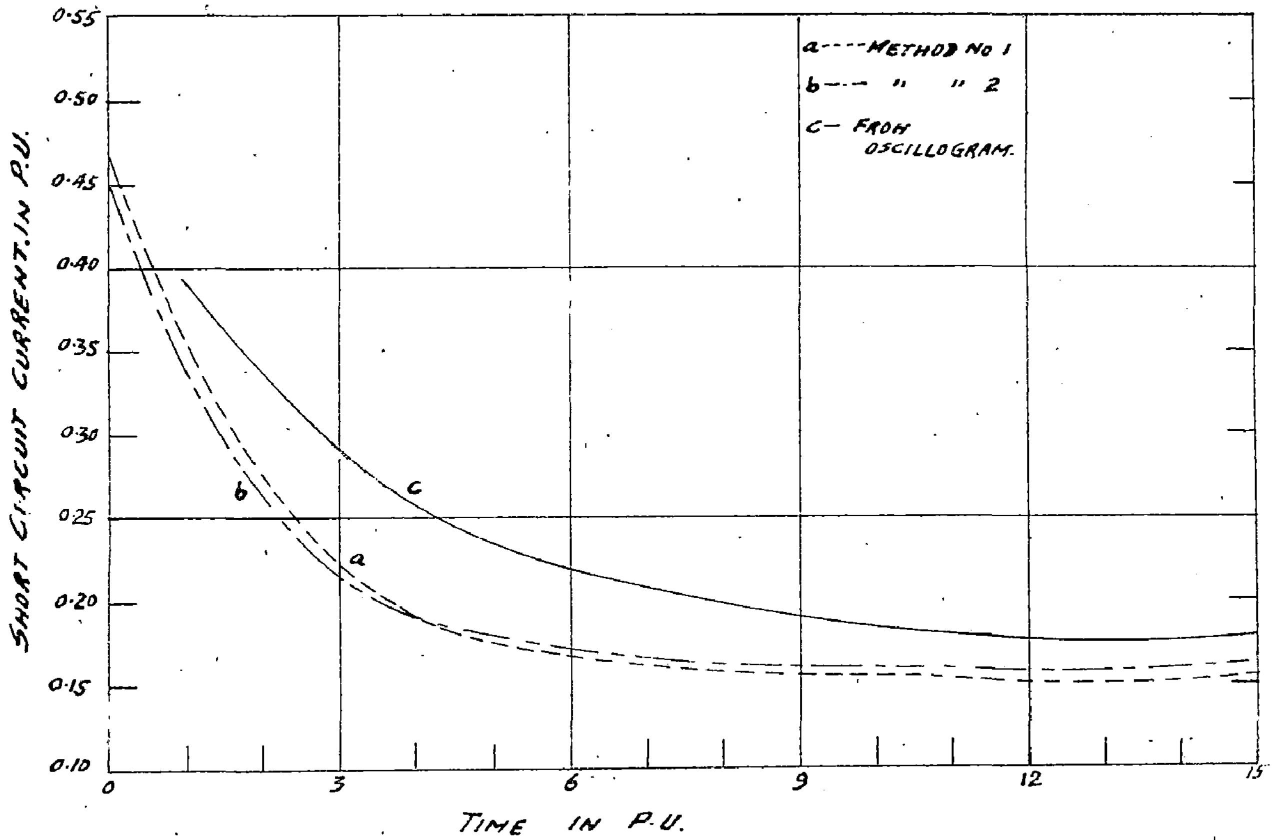


FIG. 6-14 LINE TO LINE SHORT CIRCUIT.

C O N C L U S I O N S

The transients of a synchronous generator have been analystically described on sudden switching on or sudden throwing off of balanced loads in Chapter III. Calculated and test results are compared and there is a satisfactory agreement between them. Any discripancy may be due to the approximation of the open circuit characteristic as a straight line.

The transient equivalent circuits are developed in Chapter IV and from them the equivalent ladder networks are derived in Chapter IV and V. They are further used to determine the transient on unbalanced and faults in Chapter VI. Analysis of the single phase loads and faults have also been carried out by another method presented in Chapter VI. The results obtained by these two methods are compared with the test results. The discrepancy is due to the ignoring of the effect of higher harmonics, in addition to what has been stated earlier.

In the present thesis, though the saturation has been neglected, but it can be easily taken into consideration by approximating the open circuit characteristic of the synchronous machine.[8,9,10]

By the application of equivalent circuit approach to synchronous machines, it has been possible to shorten the numerical calculations. Equivalent circuit show in a better way how the results are obtained. This method has also greater flexibility, since the same process can handle all possible types of unbalanced loads and faults.

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