STUDY OF INDUCTION MOTOR WITH ASYMMETRIC ROTOR

A Dissertation
submitted in partial fulfilment
of the requirements for the Degree
of

MASTER OF ENGINEERING
in
ADVANCED ELECTRICAL MACHINES

By

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July, 1968

CERTIFICATE

Certified that the dissertation entitled 'STUDY OF INDUCTION MOTOR WITH ASYMMETRIC ROTOR' which is being submitted by Sri T.K. Chatterjee in partial fulfilment for the award of the degree of Master of Engineering in Advanced Electrical Machines of the University of Roorkee, Roorkee, is a record of students' own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of seven months from January 1968 to July 1968, for preparing this dissertation for Master of Engineering at this University.

Dated, July 31 1968.

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(T.K. Chatterjee)

A B S T R A C T.

In the present work an attempt has been made to start the single phase induction motor without the starting winding, by introducing saliency in the rotor. This magnetic asymmetry in the rotor circuit has been investigated in detail and an analysis is made on the basis of two-axis theory. The parameters of importance have been derived and experimentally verified. Further the case of mechanical transients has been studied with a due consideration to non-linearity of torque-speed curve with the help of computer.

An induction motor with asymmetrical rotor has been designed, fabricated and tested for the above purpose.

C_O_N_T_E_N_T_S:

<u>LIST OF SYMBOLS</u>

The symbols defined below have been used without defining them. They have the meaning below:

v_o = Instantaneous applied voltage

v_d^s = Instantaneous d-axis voltage

v_q^s = Instantaneous q-axis voltage.

V_m = Maximum value of instantaneous applied voltage.

v_{d1}^s = Instantaneous d-axis slip frequency voltage.

v_{d2}^s = Instantaneous d-axis (2-s) frequency voltage.

v_{q1}^s = Instantaneous q-axis slip frequency voltage

v_{q2}^s = Instantaneous q-axis (2-s) frequency voltage.

s = Slip in p.u.

p = $\frac{d}{dt}$ operator.

ν = Speed of the rotor.

ψ_{d1} = Flux linkage in the d-axis of the stator circuit for slip frequency voltage.

ψ_{d2} = Flux linkage in the d-axis of the stator circuit for (2-s) frequency voltage.

ψ_{q1} = Flux linkage in the q-axis of the stator circuit for slip frequency voltage.

ψ_{q2} = Flux linkage in the q-axis of the stator circuit for (2-s) frequency voltage.

L_o = Steady-component of self-inductance of stator circuit.

L_m = Double frequency component of self-inductance of stator circuit

v_{d1}^r = Voltage across the rotor circuit in the d-axis at slip frequency.

v_{d2}^r = Voltage across the rotor circuit in the d-axis at (2-s) frequency.

v^{2}_{q1} = Voltage across the rotor circuit in the q-axis at slip frequency.

v^{F}_{q2} = Voltage across the rotor circuit in the q-axis at (2-s) frequency.

ψ_{rd1} = Flux linkage of the rotor circuit in the d-axis for slip frequency voltage.

ψ_{rd2} = Flux linkage of the rotor circuit in the d-axis for (2-s) frequency voltage.

ψ_{rq1} = Flux linkage of the rotor circuit in the q-axis for slip frequency voltage.

ψ_{rq2} = Flux-linkage of the rotor circuit in the q-axis for (2-s) frequency voltage.

L_{ad} = Mutual inductance between the stator and the rotor in the d-axis.

L_{aq} = Mutual inductance between the stator and the rotor in the q-axis.

L_{rd} = Self-inductance of the rotor circuit in the d-axis.

L_{rq} = Self-inductance of the rotor circuit in the q-axis.

L_d = Self-inductance of the d-axis stator circuit.

L_q = Self-inductance of the q-axis stator cirduit.

$L_d(p)$ = Operational inductance of the d-axis stator circuit.

$L_q(p)$ = Operational inductance of the q-axis stator circuit.

V_1 = R.M.S. value of slip frequency voltage.

V_2 = R.M.S. value of (2-s) frequency voltage.

$X_d(p)$ = Operational recetance in the d-axis stator circuit.

$X_q(p)$ = Operational reactance in the q-axis stator circuit.

r_s = Resistance of the stator circuit.

E_{d1} = R.M.S. value of d-axis stator circuit at slip frequency.

I

I_{d2} = R.M.S. value of d-axis stator current at (2-s) frequency.

I_{q1} = R.M.S. value of q-axis stator current at slip frequency.

I_{q2} = R.M.S. value of q-axis stator current at (2-s) frequency.

I_d = R.M.S. value of the q-axis stator current

I_q = R.M.S. value of the q-axis stator current.

I_s = R.M.S. value of stator current.

i_s = Instantaneous value of stator current.

i_{m1} = Instantaneous maximum value of d-axis stator current at slip frequency.

i_{m2} = Instantaneous maximum value of q-axis stator current at slip frequency.

i_{m3} = Instantaneous maximum value of d-axis stator current at (2-s) frequency.

i_{m4} = Instantaneous maximum value of q-axis stator current at (2-s) frequency.

F_e = Electro-magnetic developed torque

F_m = Average motor-torque.

f = Instantaneous value of torque.

x_1 = Leakage reactance of stator circuit

r_{rd} = Resistance of the rotor circuit in the d-axis.

x_{rd} = Leakage reactance of the rotor circuit in the d-axis.

r_{rq} = Resistance of the rotor circuit in the q-axis.

x_{rq} = Leakage reactance of the rotor circuit in the q-axis.

CHAPTER - I

INTRODUCTION:

Many-fold applications of single phase induction motors in domestic and industrial fields have drawn the attention of several authors. But all of them have been confronted with the well-known difficulty of single phase induction motor that it lacks inherent starting torque. This is obvious from the fig.1.1 which shows a normal cage rotor excited by a single stator winding. The current flowing in each of cage bar reacts with the gap flux producing a torque. The net torque averaged over the entire periphery is zero. In other words the machine is symmetric about the neutral axis NN'. To start the motor, this symmetry should be disturbed. This can be done by introducing asymmetry either in the electric or in the magnetic circuit of the motor. The single phase induction motors in use to the method of starting, are

i) Split phase motors

ii) Capacitor motor

iii) Shaded pole motor.

In the first two cases, the electrical asymmetry is introduced by connecting an auxiliary winding in the stator in the stator besides the main winding. The two stator windings are of different impedances and hence they cary currents of different magnitudes and phases. The auxiliary winding is normally placed in space quadrature to the main winding. The time and space phase differences between the two winding m.m.fs. give rise to a resultant field having a rotating component. The squirrel cage rotor reacts with the rotating field to produce starting torque.

In the split phase type, the auxiliary winding is of finer wire gauge and hence a higher resistance than the main winding. This results in a time phase difference between the currents in the two windings. In the capacitor motor, a capacitor is connected in series with the auxiliary winding. The time phase spliting is more in the case of capacitor motor than the split phase one and hence the starting torque.

In the third case, the magnetic asymmetry is introduced by imparting saliency in the stator structure. Around one portion of each pole is wrapped a copper strip, forming a closed circuit. This 'shading coil' delays the flux passing through it, so that the flux lags in phase behind that in the unshaded part. This gives rise to a sweeping action, magnetically, across the face of the pole, resulting a revolving flux and thereby the starting torque.

Besides the above three types, single phase repulsion motors are also used. The repulsion motor has a rotor winding like a d.c. machine armature winding and is connected to a commutator. The stator has a single winding. Brushes are kept short circuited and the brush axis is at a space angle to the stator winding axis. When the stator winding is excited, induced currents flow in the short circuited rotor winding. The rotor winding m.m.f. is at a space angle to the stator winding m.m.f. The two winding m.m.f's react with each other to produce a torque.

A radically different method of starting was suggested by Baum[2]. He has stated "A single phase induction motor (Fig.1.2) may be given starting torque by varying the rotor constants over the pole face either by asymmetrical iron structure or winding". Starting torque arises from the difference in leakage reactances of the rotor bars on both sides of the pole structure. It causes a different current to flow on each side producing a resultant torque. The difference in the mutual reactances on two sides of the poles makes

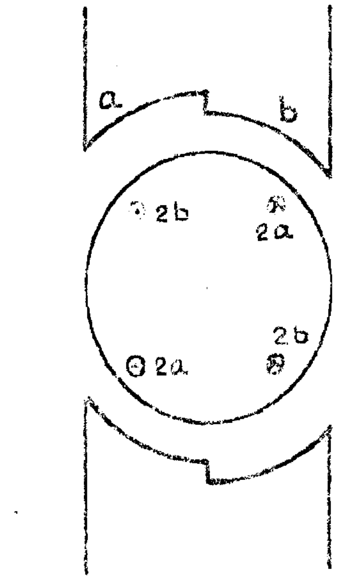


Fig.1.5

Asymmetrical Stator Motor.

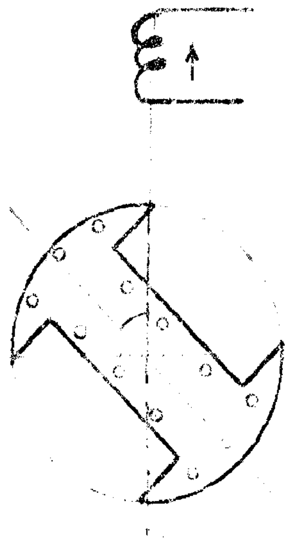
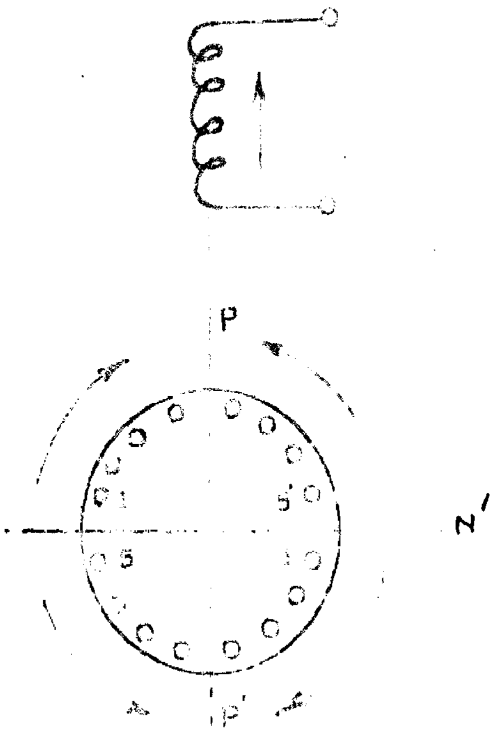


Fig.1.6

Asymmetric [illegible] motor



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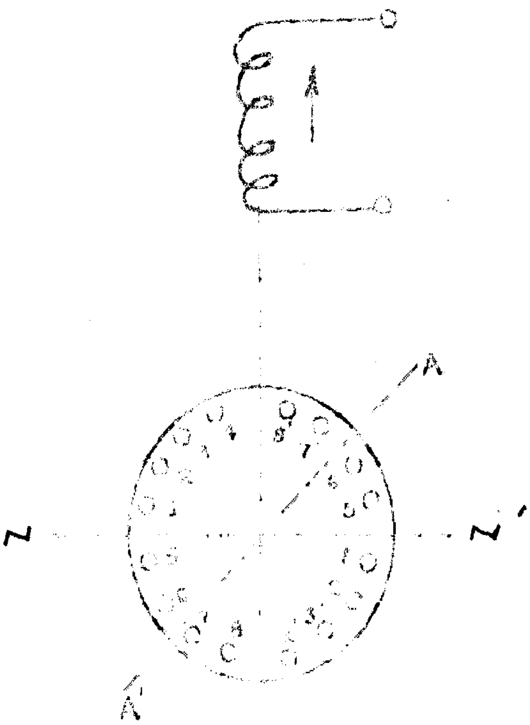


[illegible]

a phase difference between the currents flowing in both sides. This also attributes to the resultant torque.

The methods suggested by him to get the asymmetry are:

i) To wind more turns about one side of pole face than about the other side thus causing difference in the permeability of both the rotor and stator iron.

ii) To suitably shape the stator making the airgap under each pole non-uniform. Neither this method has been taken advantage of in practice nor the exact implications underlying this principle seem to have been thoroughly studied.

The constant loss in the shaded ring has attributed to the lower efficiency of shaded-pole motor. The short-circuited squirrel cage winding on the rotor itself has the effect of shading. Considering these points Deshpande and Desai[3] have suggested a type of single phase induction motor with asymmetrical rotor which they name as 'a single phase induction motor with partially open cage rotor'. Fig.(1.3) shows a cage where some slots have been left open without cage bars. The end-rings are complete. Now, refering to Fig.(1.1) it can be shown that the clockwise torque due to bars 1-1' is being cancelled by the counterclockwise torque due to similarly situated bars 6-6' and so on, producing a net zero torque. But in the case of Fig.(1.3) there are no bars in the slots 6-6' and the clockwise torque due to barks 2-2' is left uncompensated and the rotor will experience a clockwise torque even when excited by a single stator winding. The axis AA' of the open slots will try to align with the neutral axis NN'. The magnitude and direction of the torque depend on the position of AA'. However, as the rotor starts rotating under the action of this torque, the torque will also change in magnitude and direction. When the axis AA' turns past the axis NN', it is obvious that the direction of the torque is reversed. But the rotor in the meantime will develop speed voltage and will have a tendency to maintain rotation. Also the rotor will have an angular momentum and store mechanical energy and hence it can not be brought to rest instantaneously by the reversing torque. If the point A is able to go past P', it will get again an accelerating torque.

The absence of bars has introduced an asymmetry in the electrical circuit of the rotor. It may however be noted that the magnetic circuit of the rotor is absolutely symmetrical. Mosai has suggested many ways of creating the asymmetry namely use of different bar materials, use of different bar sections etc.

Another asymmetrical rotor motor has been developed by Subba Rao.[4] He has suggested a double cage rotor with all the inner slots and some of the outer-slots filled with suitable bars connected to the same end-ring ~~Fig.(1.4)~~. This is evidently a case of electrical asymmetry of the rotor.

But in the present case, an attempt has been made to exploit the magnetic property (reluctance) of the material to create the asymmetry and thereby to start the single phase induction motor with only one stator coil. The construction of the machine is simplest one. It may seem paradoxical that the motor though the simplest in structure presents great difficulties when analysed quantitatively. This reluctance-start induction motor is different from reluctance motor. It can be explained from the Fig.(1.4). Since the flux passing through the inter-polar zone traverses less iron path in the magnetic circuit compared to the air-gap path. The damping of flux due to eddy currents in the iron is also less. Therefore, the flux passing through the inter-polar portion leads the flux passing through the pole section. This in turn shifts the resultant flux from the wide to the narrow gap, and the motor follows the flux movement. In other words the rotor-axis will always try to align to the minimum reluctance path. The position of rotor with its polar-axis shifted from the stator axis will always produce some torque to align its polar-axis with the stator axis. Hence it can be started in any direction depending upon its rotor position.

CHAPTER - II

2.1. GEOMETRY OF THE MACHINE:

The machine fabricated by the author has a stator of laminated sheet steel with slots. The rotor is dumbell-shaped and hence a salient pole type one. An air-gap separates the rotor from the stator.

The electrical circuits of the machine can be differentiated into two classes:

i) the stator circuits, and

ii) the rotor circuits.

In the stator, each winding may be regarded as constituting an electric circuit. The rotor circuit consists of a no.of copper bars arranged in the form of a squirrel cage with the ends shorted by copper rings. This arrangement of conductors becomes a network and the current distribution can be determined either by the node-pair method or preferably, by the cyclic-current method.

2.2. REFERENCE FRAME AND ASSUMPTIONS:

The rotating machine analysis demands the selection of some reference frame. This reference frame may be attached either to the stator or to the rotor or a reference frame stationary for stator, and rotating for the rotor may be chosen. Besides, it is also possible to select reference frames which rotate at an arbitrary speed not connected with the rotor speed. In the present case, the reference frame is fixed on the rotor and is moving at the rotor speed, ν rad/sec.

The machine can be represented by Fig.(2.1). With the reference frame chosen, the machine analysis has been done with the following idealised assumptions.

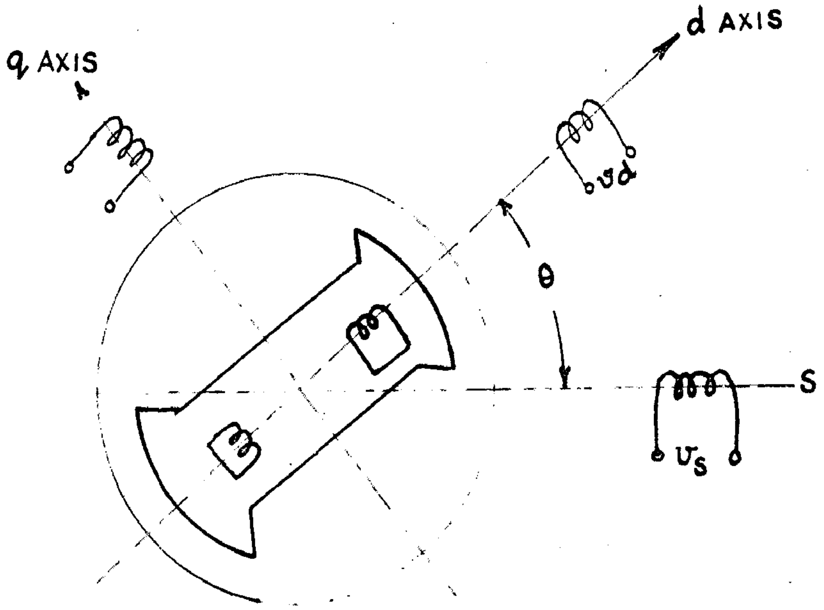


Fig. 2.1

Representation of the asymmetrical [illegible]

i) The magnetic paths and all electric circuits are symmetrical about the pole and interpole axis.

ii) Hysteresis saturation and other non-linear effects are neglected.

iii) The effect of space harmonics in the flux wave is also neglected.

2.3. <u>ANALYSIS</u>:

The stator winding is splitted into two fictitious axis namely d-axis and q-axis.

$$v_d = v_s \cos\theta$$

$$v_q = -v_s \sin\theta$$

where $\theta = (1-s)\,wt$.

and $v_s = v_d \cos\theta - v_q \sin\theta$

Assuming $v_s = V_m \cos wt$.

$$\therefore \quad v_d = V_m \left[\cos wt . \cos (1-s)\,wt \right]$$

$$= \frac{V_m}{2} \left[\cos s\,wt + \cos (2-s)\,wt \right]$$

$$= v_{d1} + v_{d2}$$

when $v_{d1} = \frac{V_m}{2} \cos s\,wt$.

$$v_{d2} = \frac{V_m}{2} \cos (2-s)\,wt.$$

Similarly,

$$v_q = -V_m \left[\cos wt . \sin (1-s)\,wt \right]$$

$$= \frac{V_m}{2} \left[\sin s\,wt - \sin (2-s)\,wt \right]$$

$$= v_{q1} + v_{q2}$$

$$= V_m/2 \sin s\,wt$$

$$v_{q2} = -\frac{V_m}{2} \sin (2-s)\, wt.$$

Now, using Park's equation for each frequency voltage, it can be written,

$$\left.\begin{aligned} v_{d1} &= p\psi_{d1} + \nu\psi_{q1} + r_s\, i^s_{d1} \\ v_{q1} &= p\psi_{q1} - \nu\psi_{d1} + r_s\, i^s_{q1} \end{aligned}\right\} \quad \ldots (1)$$

where $p = jsw$, $\nu = (1-s)w$ for the motor running with a constant slip s, i.e. at steady state.

and for (2-s)w rad/sec. frequency voltage-

$$\left.\begin{aligned} v_{d2} &= p\psi_{d2} + \nu\psi_{q2} + r_s\, i^s_{d2} \\ v_{q2} &= p\psi_{q2} - \nu\psi_{d2} + r_s\, i_{q2} \end{aligned}\right\} \quad \ldots (2)$$

when $p = j(2-s)w$ and $\nu = (1-s)w$ for steady state operation.

Now, it can be shown that-

$$\psi_{d1} = i^s_{d1} \left(L_o + \frac{L_2}{2} \right) + L_{ad}\, i^r_{d1}$$

$$\psi_{q1} = i^s_{q1} \left(L_o - \frac{L_2}{2} \right) + L_{aq}\, i^r_{q1}$$

and $$\psi_{d2} = i^s_{d2} \left(L_o + \frac{L_2}{2} \right) + L_{ad}\, i^r_{d2}$$

$$\psi_{q2} = i^v_{q2} \left(L_o - \frac{L_2}{2} \right) + L_{aq}\, i^r_{q2}$$

In the same way, the Park's Equation for the rotor circuits.

$$\left.\begin{aligned} v^r_{d1} &= p\, \psi^r_{d1} + r_{rd}\, i_{rd1} \\ v^r_{q1} &= p\, \psi^r_{q1} + r_{rq}\, i_{rq1} \end{aligned}\right\} \quad \ldots (3)$$

where $p = jsw$ for steady-state operation.

$$v^r_{d2} = p\ \psi^r_{rd2} + r_{rd}\ i_{rd2}$$

$$v^r_{q2} = p\ \psi^r_{rq2} + r_{rq}\ i_{rq2} \qquad \ldots (4)$$

when $p = j(2-s)w$ for steady-state operation.

Again, the flux linkages for the rotor circuits can be calculated as follows:

$$\psi^r_{rd1} = L_{ad}\ i^s_{d1} + L_{rd}\ i^r_{d1}$$

$$\psi^r_{rq1} = L_{aq}\ i^s_{q1} + L_{rq}\ i^r_{q1}$$

$$\psi^r_{rd2} = L_{ad}\ i^s_{d2} + L_{rd}\ i^r_{d2}$$

$$\psi^r_{rq2} = L_{ad}\ i^s_{q2} + L_{rq}\ i^r_{q2}$$

Putting the values of $\psi_{d1}, \psi_{q1}, \psi^r_{rd1}, \psi^r_{rq1}$, in (1) and (3)-

$$v^s_{d1} = p\ [\ i^s_{d1}\ (L_o + \frac{L_2}{2}) + L_{ad}\ i^r_{d1}] + \nu [i^s_{q1}(L_o - \frac{L_2}{2}) + L_{aq}\ i^r_{q1}] + r_s i^s_{d1}$$

$$v^s_{q1} = p\ [i^s_{q1}\ (L_o - \frac{L_2}{2}) + L_{aq} i^r_{q1}\ ] - \nu [i^s_{d1}\ (L_o + \frac{L_2}{2}) + L_{ad} i^r_{d1}\ ] + r_s\ i^s_{q1}$$

$$v^r_{d1} = p\ [i^s_{d1}\ L_{ad} + K_{rd}\ i^r_{d1}\ ] + r_{rd}\ i^r_{d1}$$

$$v^r_{q1} = p\ [i^s_{q1}\ L_{aq} + L_{rq}\ i^r_{q1}\ ] + r_{rq}\ i^r_{q1} \qquad \ldots (5)$$

Writing $L_d = (L_o + L_2/2)$ and $L_q = (L_o - L_2/2)$ and arranging in the matrix form, the following matrix (6) can be developed.

$$\begin{bmatrix} v^s_{d1} \\ v^s_{q1} \\ v^r_{d1} \\ v^r_{q1} \end{bmatrix} = \begin{bmatrix} (pL_d + r_s) & \nu L_q & pL_{ad} & \nu L_{aq} \\ -\nu L_d & (pL_q + r_s) & -\nu L_{ad} & pL_{aq} \\ pL_{ad} & & (pL_{rd}+r_{rd}) & \\ & pL_{aq} & & (pL_{rq}+r_{rq}) \end{bmatrix} \begin{bmatrix} i^s_{d1} \\ i^s_{q1} \\ i^r_{d1} \\ i^r_{q1} \end{bmatrix}$$

...(5)

In the same way, writing value of $\psi_{d2}, \psi_{q2}, \psi_{dr2}, \psi_{qr2}$ in (2) and (4),

$$v^s_{d2} = p[\,i^s_{d2}\,(L_o + \frac{L_2}{2}) + L_{ad} i^r_{d2}] + \nu[i^s_{q2}(L_o - \frac{L_2}{2}) L_{aq}\, i^r_{q2}] + r_s i^s_{d2}$$

$$v^s_{q2} = p[\,i^s_{q2}\,(L_o - \frac{L_2}{2}) + L_{aq} i^r_{q2}] - \nu[i^s_{d2}\,(L_o + \frac{L_2}{2}) + L_{ad} i^r_{d2}\,] + r_s i^s_{q2}$$

$$v^r_{d2} = p[i^s_{d2}\,L_{ad} + L_{rd}\,i^r_{d2}\,] + r_{rd}\,i^r_{d2}$$

$$v^r_{q2} = p[i^s_{q2}\,L_{aq} + L_{rq}\,i^r_{q2}\,] + r_{rq}\,i^r_{q2}$$

... (6)

Now writing $L_d = (L_o + L_2/2)$ & $L_q = (L_o - L_2/2)$ the following matrix(6) can be formed:

$$\begin{bmatrix} v^s_{d2} \\ v^s_{q2} \\ v^r_{d2} \\ v^r_{q2} \end{bmatrix} = \begin{bmatrix} (pL_d + r_s) & \nu L_q & pL_{ad} & \nu L_{aq} \\ -\nu L_d & (pL_q + r_s) & -\nu L_{ad} & pL_{aq} \\ pL_{ad} & & (pL_{rd}+r_{rd}) & \\ & pL_{aq} & & (pL_{rq}+r_{rq}) \end{bmatrix} \begin{bmatrix} i^s_{d2} \\ i^s_{q2} \\ i^r_{d2} \\ i^r_{q2} \end{bmatrix}$$

where p = [illegible] ....(6)

Now taking the values of $v^r_{d1} = v^r_{q1} = v^r_{d2} = v^r_{q2} = 0$ and reducing the matrices (5) and (6) to the (2 x 2) matrix form and representing the impedance matrix in its operational form (vide Appendix I), the matrices (5) and (6) can be written as (7) and (8).

The matrix (5) reduces to-

$$\begin{bmatrix} v^s_{d1} \\ v^s_{q1} \end{bmatrix} = \begin{bmatrix} r_s + pL_d(p) & \nu L_q(p) \\ -\nu L_d(p) & r_s + pL_q(p) \end{bmatrix} \begin{bmatrix} i^s_{d1} \\ i^s_{q1} \end{bmatrix} \qquad \ldots(7)$$

where $p = jsw$, $\nu = (1-s)w$

In the same way the matrix (6) reduces to-

$$\begin{bmatrix} v^s_{d2} \\ v^s_{q2} \end{bmatrix} = \begin{bmatrix} r_s + pL_d(p) & \nu L_q(p) \\ -\nu L_d(p) & r_s + p\,L_q(p) \end{bmatrix} \begin{bmatrix} i^s_{d2} \\ i^s_{q2} \end{bmatrix} \qquad \ldots(8)$$

where $p = j(2-s)w$, $\nu = (1-s)w$

Now, Let-

$$v^s_{d1} = V_m/2 \ \cos swt = V_1$$

$$\therefore v^s_{q1} = V_m/2 \ \sin swt = -jV_1$$

$$\& \ v^s_{d2} = V_m/2 \ \cos(2-s)wt = V_2$$

$$\therefore v^s_{q2} = -V_m/2 \ \sin(2-s)wt = jV_2$$

Hence for the steady-state operation can be represented by-

$$\begin{bmatrix} V_1 \\ -jV_1 \end{bmatrix} = \begin{bmatrix} r_s + jsX_d(jsw) & (1-s)X_q(jsw) \\ -(1-s)X_d(jsw) & r_s + jsX_q(jsw) \end{bmatrix} \begin{bmatrix} I_{d1} \\ I_{q1} \end{bmatrix} \qquad \ldots(9)$$

$$\begin{bmatrix} V_2 \\ jV_2 \end{bmatrix} \quad \begin{bmatrix} r_s + j(2-s)X_d[j(2-s)w] & (1-s)X_q[j(2-s)w] \\ -(1-s)X_d[j(2-s)w] & r_s + j(2-s)X_q[j(2-s)w] \end{bmatrix} \quad \begin{bmatrix} I_{d2} \\ I_{q2} \end{bmatrix} \qquad \ldots(10)$$

Now from (9)-

$$I_{d1} = \frac{V_1 [r_s + jX_q(jsw)]}{[r_s^2 + (1-2s)X_d(jsw)X_q(jsw)] + jsr_s [X_d(jsw) + X_q(jsw)]} \qquad \ldots(11)$$

and

$$I_{q1} = \frac{-jV_1 [r_s + jX_d(jsw)]}{[r_s^2 + (1-2s)X_d(jsw)\, X_q(jsw)] + jsr_s [X_d(jsw) + X_q(jsw)]} \qquad \ldots(12)$$

Again from 10,

$$I_{d2} = \frac{V_2 [r_s + jX_q[j(2-s)w]]}{r_s^2 - (3-2s)X_d[j(2-s)w]X_q[j(2-s)w] + jr_s(2-s)\left(X_d[j(2-s)w] + X_q[j(2-s)w]\right)} \qquad \ldots(13)$$

and

$$I_{q2} = \frac{jV_2 \; r_s + jX_d[j(2-s)w]}{r_s^2 - (3-2s)X_d[j(2-s)w]X_q[j(2-s)w] + jr_s(2-s)\left(X_d[j(2-sw] + X_q[j(2-s)w]\right)} \qquad \ldots(14)$$

Now, the currents I_{d1} and I_{q1} alternate at a frequency sw rad/secs. While the currents I_{d2} and I_{q2} alternate at a frequency (2-s)w rad/sec

Since $\quad I_d = I_{d1} + I_{d2}$

$\qquad\quad I_q = I_{q1} + I_{q2}$

and $\quad I_s = I_d \cos\theta - I_q \sin\theta$

Therefore, all the quantities are now again transformed to their instantaneous value.

$$\therefore\ i_s = i^s_{d1}\cos\theta + i^s_{d2}\cos\theta - i^s_{q1}\sin\theta - i^s_{q2}\sin\theta$$

$$\therefore\ i^s_{d1}\cos\theta = i_{m1}\cos(swt - \alpha_1)\cos(\overline{1-s}\ wt)$$

where α_1 dependents upon d-axis stator impedance.

$$= i_{m1}/2\ [\cos(wt-\alpha_1)+\cos(\overline{1-2s}\ wt + \alpha_1)]$$

Similarly,

$$i^s_{d2}\cos\theta = i_{m3}\cos[\overline{2-s}\ wt - \alpha_3]\cos(\overline{1-s}\ wt)$$

$$= i_{m3}/2\ [\cos(\overline{3-2s}\ wt - \alpha_3) + \cos(wt - \alpha_3)]$$

$$-i^s_{q1}\sin\theta = -i_{m2}\sin(swt - \alpha_2)\sin(\overline{1-s}\ wt)$$

$$= i_{m2}/2\ [\cos(wt - \alpha_2) - \cos(\overline{1-2s}\ wt + \alpha_2)$$

$$-i^s_{q2}\sin\theta = -i_{m4}\sin(\overline{2-s}\ wt - \alpha_4)\sin(\overline{1-s}\ wt)$$

$$= i_{m4}/2\ [\cos(wt - \alpha_4) - \cos(\overline{3-2s}\ wt - \alpha_4)]$$

$$\therefore\ i_s = [i_{m1}/2\cos(wt-\alpha_1) + i_{m2}/2\cos(wt-\alpha_2) + i_{m3}/2\cos(wt-\alpha_3) +$$

$$i_{m4}/2\cos(wt-\alpha_4)] + [i_{m1}/2\cos(\overline{1-2s}\ wt + \alpha_1) - i_{m2}/2\cos(\overline{1-2s}\,wt+$$

$$+[i_{m3}/2\cos(\overline{3-2s}\ wt - \alpha_3) - i_{m4}/2\cos(\overline{3-2s}\ wt - \alpha_4)] \qquad \ldots(15)$$

Obviously the expression (15) shows that the stator current contains not only the fundamental frequency component but also the traces of the (1-2s)w and (3-2s)w frequency components.

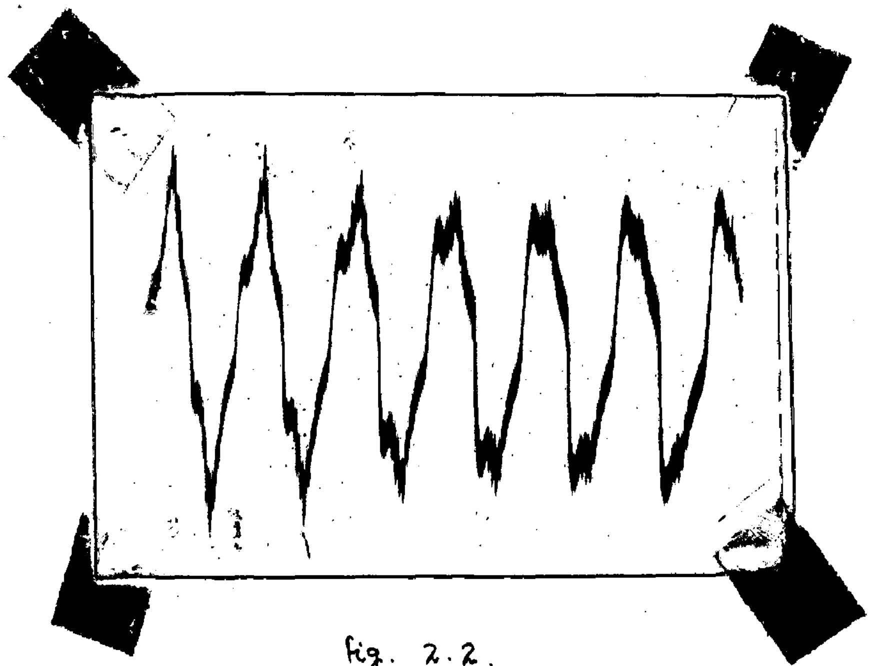


fig. 2.2.
Oscillogram of Current.

2.4. EXPRESSION FOR TORQUE:

In terms of the real currents and flux-linkages, the electrical torque is given by-

$$F_e = \frac{w}{2} \operatorname{Re} (\Psi_d I_q^* - \Psi_q I_d^*)$$

The mean motoring torque F_m will be equal and opposite to this torque F_e and is given by

$$F_m = \frac{w}{2} \operatorname{Re} (\Psi_q I_d^* - \Psi_d I_q^*) \qquad \ldots \qquad \ldots(16)$$

Hence the expression for torque with the two frequency components, can be calculated as follows:

$$F_m = \frac{1}{2} \operatorname{Re} [w \Psi_{q1} I_{d1}^* - w \Psi_{d1} I_{q1}^*] - \operatorname{Re} [w \Psi_{q2} I_{d2}^* - w \Psi_{d2} I_{q2}^*] \qquad \ldots \qquad \ldots(17)$$

Now from (7),

$$V_m \cos swt = p \Psi_{d1} + (1-s) w \Psi_{q1} + r_s i_{d1}^s$$

$$V_m \sin swt = -(1-s) w \Psi_{d1} + p \Psi_{q1} + r_s i_{q1}^s$$

where $\Psi_{d1} = \frac{x_d(p)}{w} i_{d1}^s$ and $\Psi_{q1} = \frac{x_q(p)}{w} i_{q1}^s$

Considering the r.m.s. quantity, the vector equations are obtained by replacing p = jsw and writing capital letters for the small letters and putting/over flux linkage terms. (bar)

$$\therefore \quad \left. \begin{aligned} V_1 &= jsw \bar{\Psi}_{d1} + (1-s) w \bar{\Psi}_{q1} + r_s I_{d1} \\ -jV_1 &= -(1-s) w \bar{\Psi}_{d1} + jsw \bar{\Psi}_{q1} + r_s I_{q1} \end{aligned} \right] \qquad \ldots(18)$$

where, $w \bar{\Psi}_{d1} = x_d(jsw) I_{d1}$

$$w \bar{\Psi}_{q1} = x_q(jsw) I_{q1}$$

From (18) it can be solved for $w\bar{\Psi}_{d1}$ and $w\bar{\Psi}_{q1}$

$$w\bar{\Psi}_{d1} = \frac{V_1 \dfrac{r_s}{x_q(jsw)} + jV_1}{(1-2s) + \dfrac{r_s^2}{x_d(jsw)x_q(jsw)} + jsr_s\left[\dfrac{1}{x_d(jsw)} + \dfrac{1}{x_q(jsw)}\right]}$$

$$w\bar{\Psi}_{q1} = \frac{V_1 \dfrac{r_s}{x_q(jsw)} + jV_1}{(1-2s) + \dfrac{r_s^2}{x_d(jsw)\,x_q(jsw)} + jsr_s\left[\dfrac{1}{x_d(jsw)} + \dfrac{1}{x_q(jsw)}\right]} \qquad \ldots(19)$$

From (8)-

$$\frac{V_m}{2}\cos(2-s)wt = p\Psi_{d2} + (1-s)w\Psi_{q2} + r_s\, i^s_{d2}$$

$$-\frac{V_m}{2}\sin(2-s)wt = -(1-s)w\Psi_{d2} + p\Psi_{q2} + r_s\, i^s_{q2} \qquad \ldots(20)$$

Writing in terms of r.m.s. quantity, the vector equations are-

$$V_2 = j(2-s)w\bar{\Psi}_{d2} + (1-s)\,w\bar{\Psi}_{q2} + r_s I_{d2}$$

$$-jV_2 = -(1-s)w\bar{\Psi}_{d2} + j(2-s)w\bar{\Psi}_{q2} + r_s I_{q2}$$

where $w\bar{\Psi}_{d2} = x_d\left[j(2-s)w\right] I_{d2}$

$$w\bar{\Psi}_{q2} = x_q\left[j(2-s)w\right] I_{q2}$$

From (20) also the values of $w\bar{\Psi}_{d2}$ and $w\bar{\Psi}_{q2}$ can be calculated as

$$w\bar{\psi}_{d2} = \frac{\left[V_2 \dfrac{r_s}{r_q[j(2-s)w]} + jV_2(3-2s)\right]}{-(3-2s) + \dfrac{r_s^2}{s_d[j(2-s)w]x_q[j(2-sw]} + j(2-s)r_s\left[\dfrac{1}{x_d[j(2-s)w]} + \dfrac{1}{x_q[j(2-s)w]}\right]}$$

$$w\bar{\psi}_{q2} = \frac{j\left[V_2 \dfrac{r_s}{x_d[j(2-s)w]} + jV_2(3-2s)\right]}{-(3-2s) + \dfrac{r_s^2}{x_d[j(2-s)w]x_q[j(2-s)w]} + j(2-s)r_s\left[\dfrac{1}{x_d[j(2-s)w]} + \dfrac{1}{x_q[j(2-s)w]}\right]}$$

... ...(21)

Now, taking the conjgate of the expressions (11), (12), (13) and (14) and using the expression for torque in (17) we can get the actual expression for torque.

This expression (16) for torque may be checked by direct substitution as follows:

Let the complex quantities be-

$I_d = a + jb$

$I_q = c + jd$

$w_d = e + jf$

$w_q = g + jh.$

$\therefore F_m = \frac{1}{2}(ag + bh - ce + df)$

On the other hand, the real currents and flux linkages which alternate at slip frequency are $i^s_{d1} = \text{Re}\,(a_1 + jb_1)e^{jst}$

$= a_1 \cos st - b_1 \sin st.$ etc.

when the instantaneous torque is-

$$f_1 = (a_1 \cos st - b_1 \sin st)(g_1 \cos st - h_1 \sin st)$$
$$-(C_1 \cos st - d_1 \sin st)(e_1 \cos st - f_1 \sin st)$$

$$= a_1g_1\cos^2 st + b_1h_1\sin^2 st - (b_1g_1+a_1h_1)\sin st.\cos st -$$

$$c_1e_1\cos^2 st - d_1f_1\sin^2 st + (d_1e_1+c_1f_1)\sin st.\ \cos st.$$

$$= \tfrac{1}{2}\ (a_1g_1+b_1h_1)+(a_1g_1-b_1h_1)\cos 2\,st-(b_1g_1+a_1h_1)\sin 2\,st-$$

$$(c_1e_1+d_1f_1)-(c_1e_1-d_1f_1)\cos 2st + (d_1e_1+c_1f_1)\sin 2\,st$$

In the 2nd case when the currents and flux linkages all will vary as $e^{j(2-s)t}$, $p = j(2-s)t$.

$$\therefore\ i^s_{d2} = \mathrm{Re}\ (a_2 + jb_2)\ e^{j(2-s)t}$$

$$= a_2\cos\ (2-s)t - b_2\sin\ (2-s)t.\ \text{etc.}$$

When the instantaneous torque in this frequency,

$$f_2 = [\ a_2\cos(2-s)t - b_2\sin(2-s)t\ ]\ [g_2\cos(2-s)t-h_2\sin(2-s)t]-$$

$$[c_2\cos(2-s)t- d_2\sin(2-s)t][e_2\cos(2-s)t-f_2\sin(2-s)t]$$

$$=a_2g_2\cos^2(2-s)t + b_2h_2\sin^2(2-s)t - (b_2g_2+a_2h_2)\sin(2-s)t\cos(2-s)t.$$

$$c_2e_2\cos^2(2-s)t - d_2f_2\sin^2(2-s)t+(d_2e_2+c_2f_2)\sin\ (2-s)t\ \cos(2-s)t.$$

$$= \tfrac{1}{2}\ \ (a_2g_2+b_2h_2) + (a_2g_2-b_2h_2)\ \cos(4-2s)t-(b_2g_2+a_2h_2)\sin\ (4-2s)t-$$

$$(c_2e_2+d_2f_2)-\ (c_2e_2-d_2f_2)\cos\ (4-2s)t+\ (d_2e_2+c_2f_2)\sin\ (4-2s)t$$

$\therefore$ Instantaneous resultant torque-

$$f = (f_1 - f_2)$$

$$= \tfrac{1}{2}\ \ [(a_1g_1+b_1h_1)-(c_1e_1+d_1f_1)-(a_2g_2+b_2h_2)+(c_2e_2+d_2f_2)]+[(a_1g_1-b_1h_1)$$

$$(c_1e_1-d_1f_1)]\cos 2st+[(c_2e_2-d_2f_2)-(a_2g_2-b_2h_2)\cos(4-2s)t+$$

$$[(d_1e_1+c_1f_1)-b_1g_1+a_1h_1)]\sin 2st+[(b_2g_2+a_2h_2)-(d_2e_2+c_2f_2)$$

$$\sin(4-2s)t] \qquad (22)$$

Thus an expression for torque has been developed directly in terms of the components of the compex number form of the d-axis and q-axis quantities. But it is very difficult to get the exact expression for the torque. However, from the above expression (22) one can get an idea about the torque-slip characteristic of the motor.

CHAPTER - III

3.1. EQUIVALENT CIRCUIT:

All the rotating machines behave in accord with immutable laws of physics. In them electrical, magnetic, and mechanical energies co-exist and interact. Two methods have been followed to analyse these electromagnetic and mechanical systems. One method is to represent the system by differential equations, carried to the required degree of exactness and to get the numerical solution of these equations. The second method is to represent the system or its differential equations by an equivalent circuit, consisting of stationary electric-circuit elements, which can be calculated theoretically and verified experimentally.

In order to calculate the torque-speed curve of the machine, it is easier to use the equivalent circuits instead of the expression (17)(Chapter II, page17). The method, similar to that commonly used for induction motor, has been applied to the two fictitious axes circuits, namely d-axis and q-axis circuits. Following the operational eqns.(7) and (8) (Chapter II, page13) and reducing them to the r.m.s. quantities of A.C. systems, the following equivalent circuits as shown in fig.3.1 can be developed. Thus each of the component torques can be determined from the appropriate equivalent circuit.

From the equivalent circuits of Fig.(3.1) the values of I_{d1}, I_{q1}, I_{d2} and I_{q2} can be formed out as follows:

$$I_{d1} = \frac{V_1}{(r_s + jX_1) + \dfrac{j\,x_{md}(r_{rd} + j\,\frac{x_{rd}}{T})}{r_{rd} + j\,(x_{md} + \frac{x_{rd}}{s})}}$$

$$I_{q1} = \frac{jV_1}{(r_s + jX_1) + \dfrac{jx_{mq}(r_{rq} + j\,\frac{x_{rq}}{T})}{r_{rq} + j\,(x_{mq} + \frac{x_{rq}}{s})}}$$

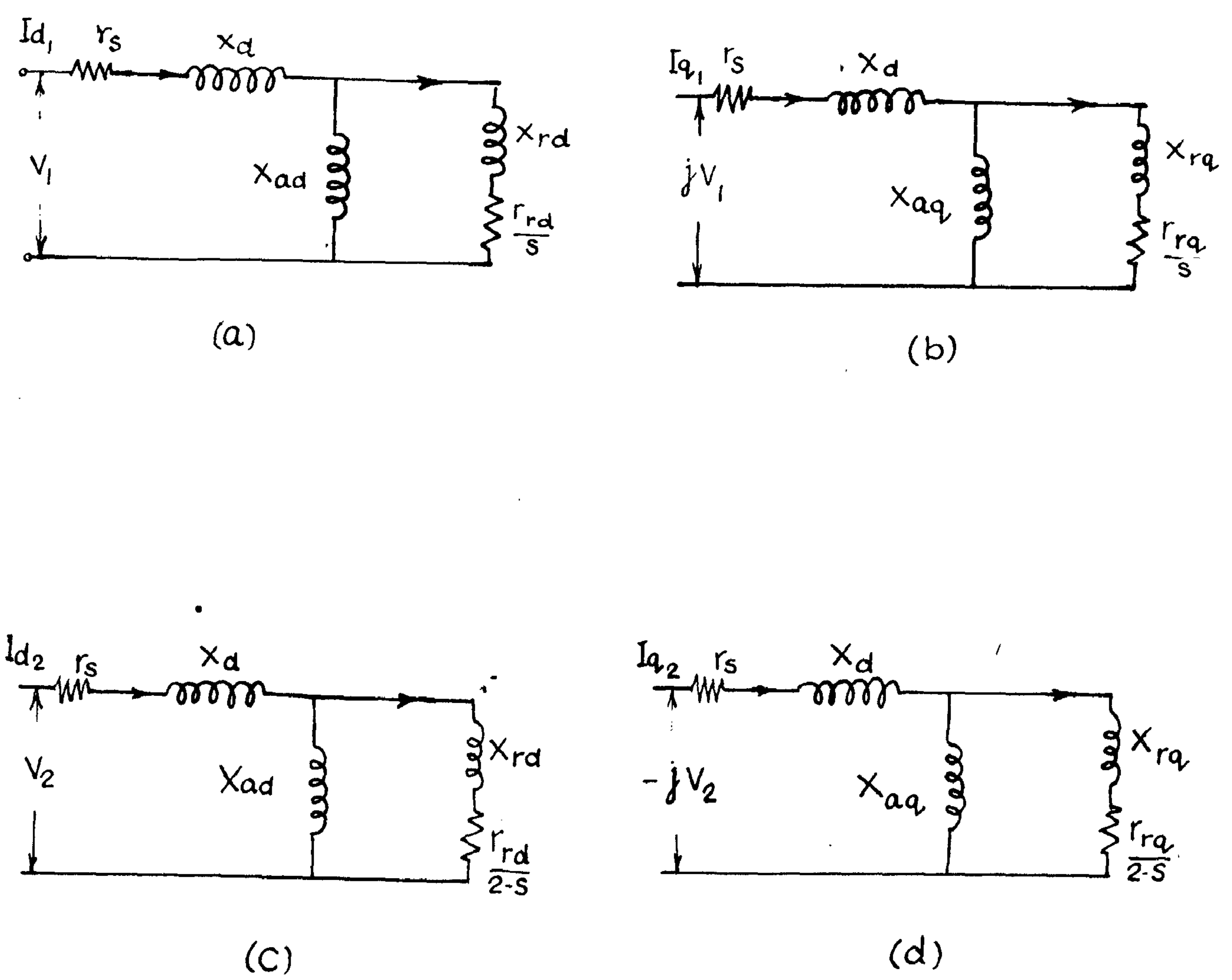


Fig.3.1.

Equivalent circuits

$$I_{d2} = \frac{V_2}{(r_s+jX_1)+\dfrac{jx_{ad}\left(r_{rd}+j\dfrac{x_{rd}}{(2-s)}\right)}{r_{rd}+j\left(x_{ad}+\dfrac{x_{rd}}{2-s}\right)}}$$

$$I_{q2} = \frac{-jV_2}{(r_s+jX_1)+\dfrac{jx_{aq}\left(r_{rq}+j\dfrac{x_{rq}}{2-s}\right)}{r_{rq}+j\left(x_{aq}+\dfrac{x_{rq}}{2-s}\right)}}$$

Using these values of I_{d1}, I_{q1}, I_{d2} and I_{q2} the expression for the torque can be developed.

$$F_{d1} = I_{d1}^2 \frac{r_{rd}}{s}$$

$$F_{d2} = I_{d2}^2 \frac{r_{rd}}{2-s}$$

$$F_{q1} = I_{q1}^2 \frac{r_{rq}}{s}$$

$$F_{q2} = I_{q2}^2 \frac{r_{rq}}{2-s}$$

$$\therefore F_d = F_{d1} + F_{d2}$$

and $$F_q = F_{q1} + F_{q2}$$

$$\therefore \text{The net torque } F = F_d + F_q.$$

3.2. CALCULATION OF MACHINE CONSTANT:

In an ordinary induction motor the air-gap is uniform, and hence the magnetic circuit is symmetric about any axis. But due to saliency the magnetic circuit will have two axes of symmetry. The presence of saliency in the magnetic circuit of an electrical machine affects the reluctance of the magnetic circuit by the so called 'fringing of flux' affects. The flux lines are ratified in the interpolar region. The reluctance of the air-path is increased and thus some flux is lost. This effect can be considered as due to the slot-opening in the turbo generator rotor. The increased reluctance is attributed to an increase in the effective air-gap under polar and inter-polar region. A method with the help of Fourier series representation of air-gap flux is outlined, from which gap co-efficients in the two axes are obtained.

Now, putting the values from Fig.(A_1), we can calculate,

$C_d = 1.04$

and $C_q = 1.90$

So long we have considered only the saliency of the rotor, but the slotting in stator will also attribute something in 'flux fringing phenomenon. This can be taken into account as a further elongation of the effective gap-length in both polar and inter-polar region. This factor can be calculated by formula in Say's book.

$$K_g = \frac{y_s}{y_s - K\ \ w}$$

K_g is calculated as 1.10.

Calculation of X_{md} and M_{mq}:

The formula given by Veinott to calculate the magnetising reactance has been used. Only the air-gap length in the proper axis has been taken into consideration by multiplying the gap-length by the corresponding gap co-efficient.

Formula given by Veinott,

$$X_m = 2\pi f\ (CK_v)^2 \times \frac{0.411}{D} \times \frac{A_g}{g_0} \times 10^{-8} \text{ ohms.}$$

Hence this has been modified as-

$$X_{md} = 2\pi f\ (CK_v)^2 \times \frac{0.411}{D} \times \frac{A_g}{g_0 \times C_d} \times 10^{-8} \text{ ohms}$$

and $X_{mq} = 2 \; f(CK_v)^2 \times \frac{0.411}{p} \times \frac{A_g}{g_o \times c_q} \times 10^{-8}$ ohms.

where, CK_v = Effective series conductors.

f = frequency in cycles/sec.

p = no. of poles.

A_g = Air-gap area $(in)^2$

g_o = Equivalent air-gap length (in).

Putting the values of the proper constants we have calculated-

$X_{md} = 550$ ohms

$X_{mq} = 294$ ohms.

Stator and Rotor leakage reactances:

The stator leakage is calculated following the approach of Veinott.[8] The total leakage of the stator winding is taken as the sum of stator slot leakage, half the zigzag, belt and the end leakage. The skew leakage is neglected since the machine fabricated is not having the skew. The leakage reactance of the d-axis and q-axis rotor windings are calculated following the approach of Talhat.[9] Only the formulae in their final form are presented here. For detailed derivation the original papers may be considered. As far as possible, symbols used by the above authors have been used. To keep the sequence a few symbols used by Talaat has been changed. Both the above authors used inch as the unit of dimension and the same has been retained here.

Stator Leakage:

Stator leakage reactance $X_1 = K_x \times$ (stator leakage constant)

where $K_x = 2\pi f(CK_v)^2 \times 10^{-8}$

Stator leakage constant = stator slot $+\frac{1}{2}$ (Belt+zigzag +End).

$$\text{Stator slot} = \frac{3.19\, m\, K_{s1}\, C_x L_1}{s_1}$$

where, m = no. of phases to be taken as 2 for single-phase motor.

K_{s1} = Stator slot permeance (see fig.17.10,p.329 Veinott).

C_x = Corrector factor.

L_1 = Stator stack length (in).

s_1 = No.of stator slot.

'Belt' = $0.00118\, m.K_m K_B$

where $K_m = \dfrac{A_g}{g_e SF_m\, p.}$

SF_m = Saturation factor

$$= \frac{\text{Air-gap AT + Iron AT}}{\text{Air-gap AT}}$$

K_B is belt leakage constant to be read from Fig.17.12 of Veinott's book.

$$\text{'Zigzag'} = \frac{1.065\, mL_1}{S_1 g_e} K_{zz}$$

when $K_{zz} = \dfrac{(t_{10} + t_{20})^2}{4(\lambda_1 + \lambda_2)}$

where, t_{10} = stator tooth face (in)

t_{20} = Rotor tooth face (in)

λ_1 = Stator tooth pitch (in)

λ_2 = Rotor tooth pitch (in)

$$\text{'End'} = \frac{1.57m\, D_c\, (ACT)}{s_1\, p}$$

where D_c = Diameter at the centre of stator slots

ACT = Average Coil throw.

for the experimental machine

$K_x = 2.512$

Stator slot=1.27

'Belt' = 1.25

'Zigzag' = 4.16

'End' = 2.82

$\therefore$ Stator leakage constant = 1.27 + ½(1.25 + 4.16 + 2.82)

=1.27 + 4.11 = 5.38

$\therefore$ Stator leakage reactance

$X_1 = K_x$ x (Stator leakage constant)

= 2.512 x 5.38 = 13.5 ohms.

Direct-Axis Rotor winding Leakage Reactance:

The formula given by Talaat[9] for this reactance is re-arranged as follows:

$$X_{2d} = 2\pi f(CK_w)^2 \times \frac{2}{p} \times \frac{L_2 \times \lambda_{bed}}{n_b\,(1-K_b)} \times 10^{-8}$$

where,

n_b = No. of bars per pole.

$K_b = \dfrac{\sin n_b \alpha_b}{n_b \sin \alpha_b}$, α_b = Angular pitch of rotor slot.

λ_{bed}, the permeance constant as defined by Talaat consists of four components.

λ_{bed} = 'Rotor slot' + 'Tooth Top'+'Harmonics'+'End'

These components are calculated by the following expression-

'Rotor slot'= 3.19x K_{s2} when K_{s2} is rotor slot permeance constant (See Figs.17.10 & 17.11 of Veinott's book)

'Tooth Top' = 3.19 x F_{tb} when F_{tb} is to be read from Fig.2 of Talâat's paper.

(Neglect this for closed rotor slots)

$$\text{'Harmonics'} = 3.19 \times \frac{\tau_b}{12\, g_c}$$

where, τ_b = Rotor slot pitch (in)

$$\text{'End'} = 3.19 \times \frac{0.12}{m} \times \frac{\tau_r}{\tau_b} \times \frac{\tau_{ring}}{L_2}\left(2 + \frac{\cos n_b \alpha_b - K_b \cos \alpha_b}{1-K_b}\right)$$

where τ_r = Pole pitch at rotor surface (in)

τ_{ring} = Pole pitch at average ring diameter (in)

Other symbols have been defined earlier.

For the experimental machine, these values of constants are calculated as follows:

'Rotor slot'	= 31.9	n_b	= 15
'Harmonics'	= 4.5	K_b	= 0.205
'End'	= 3.4		

$$\therefore \quad \lambda_{bed} = 31.9 + 4.5 + 3.4 = 39.8$$

$$\therefore \quad X_{rd} = 2\pi f \times L_2 \times (CK_w)^2 \times \frac{2}{p} \times \frac{\lambda_{bed}}{n_b(1-K_b)} \times 10^{-8} \text{ ohms.}$$

= 25 ohms.

Quadrature Axis Rotor Winding Leakage Reactance:

The formula for this reactance as given by Talaat is as follows:

$$X_{rq} = 2\pi f (CK_w)^2 \times L_2 \times \frac{2}{p} \times \frac{\lambda_{beq}}{n_b(1+K_b)} \times 10^{-8} \text{ ohms.}$$

where λ_{beq} is the permeance constant for the quadrature axis winding. Other symbols have been already defined earlier.

Like λ_{bed}, λ_{beq} consists of four components.

λ_{beq} = 'Rotor slot' + 'Tooth Top' +'Harmonic' +'End'

For the experimental machine, these values of constants are calculated as follows:

'Rotor slot' = 41.5 n_b = 15

'Harmonics' = 4.5 K_b = 0.205

'End' = 3.4

$\therefore \lambda_{beq} = 41.5 + 4.5 + 3.4 = 49.4$

$\therefore X_{xq}$ = 20.4 ohms.

Stator and Rotor Resistance Calculation:

Stator Resistance r_s : The stator resistance can be calculated by knowing the length of the mean conductor. This will depend to a certain extent, on the method of manufacture.

The formula given by Veinott is as follows:

Length of the mean conductor = LMC = $L_1 + \dfrac{\gamma (ACD) \pi D_s}{s_1}$

where γ = Emperical constant

= 1.30 for two pole design

= 1.50 for four pole design

= 1.70 for six or more pole design.

Other symbols have already been defined earlier for the experimental machine.

LMC = 10.32 in

$\therefore$ The total length of the winding wire

= No.of conductors x LMC

= 296 yards.

The wire size used is 22 S.W.G. and from the table given by the manufacturer, the resistance for this length at 25'C is 12.1ohms.

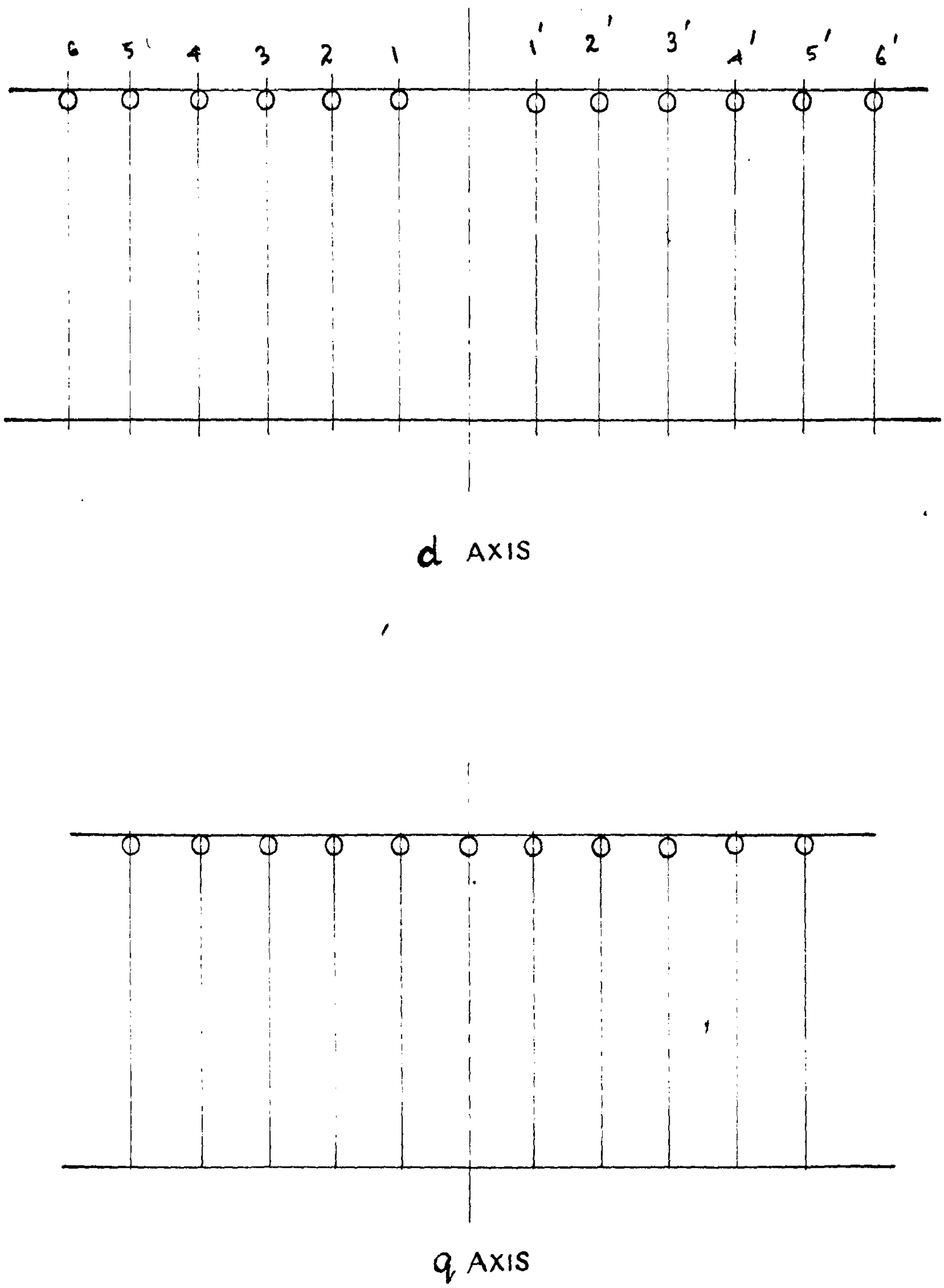


Fig.3.2

Representation of the rotor circuit

$\therefore r_q = 1211$ ohms.

Direct Axis Rotor Winding Resistance, r_{2d}:

The formula given by Talaat is as follows:

$$r_{2d} = (CK_v)^2 \times \frac{2}{p} \frac{r_{bed}}{n_b\,(1-K_b)}$$

r_{bed} is resistance constant to be evaluated. Talaat has not given any formula for r_{bed}. It would be equal to the resistance of a bar if the end-ring resistance can be neglected. This may be case in a synchronous machine with a large no. of poles. But for the fractional motors with very small no. of poles, the end-ring resistance can not be neglected. In deriving at the expression for r_{bed}, the following assumptions as made by Talaat has been retained.

1) The space distribution of bar currents are sinusoidal.
2) A sinusoidal pulsating flux density is assumed to be entered on the direct-axis.

According to the assumptions made, the current in bars (1-1') would be $i_{Dd} \sin \frac{\theta_1}{2}$ where θ_1 is the electrical space angle between bars (1-1') and I_{Dd} is the current that would be flowing in a bar situated at 90° (electrical) from the d-axis.

Similarly, the currents flowing in bars (2-2'), (3-3'), would be $I_{Dd} \sin \frac{\theta_2}{2}$, $I_{Dd} \sin \frac{\theta_3}{2}$ etc.

If r_b is the resistance of each bar, the losses taking place in bars in one pole pitch would be given by the following.

Losses in bars (per pole pitch)

$$= 2\, r_b \left(I_{Dd}^2 \sin^2 \frac{\theta_1}{2} + I_{Dd}^2 \sin^2 \frac{\theta_2}{2} + \ldots\right)$$

The terms in the bracket have been summed up by Talaat as

$$I_{Dd}^2 \frac{n_b(1-K_b)}{4}$$

$\therefore$ Total loss taking place in all the bars-

$$P_{bar} = \frac{r_b \times p \times n_b\ (1-K_b)\ I_{Dd}^2}{2}$$

The losses in the end-rings can be calculated as follows:

The current flowing in sector n and (n-1) of the end-ring is I_{Dd} sin $\frac{\theta_{d1}}{2}$). The total losses in the ring can be found out by finding out losses in the individual sections.

Let the total losses in end-rings be denoted by P_{ring},

when, $P_{ring} = I_{Dd}^2 \times r_{end}$

The total losses in the rotor are ($P_{bar} + P_{ring}$). Equating the total losses to expression (79) of Talaat's paper, which gives the total losses in terms of r_{bed} we get,

$$P_{bar} + P_{ring} = \frac{1}{2}\ r_{bed}\ I_{Dd}^2\ n_b\ (1-K_b)p.$$

from this, substituting the values of P_{bar} and P_{ring},

$$r_{bed} = r_b + \frac{r_{end}(d)}{\frac{1}{2}\ n_b(1-K_b)\ p}$$

For the experimental machine the values has been calculated

as $r_b = 1.23 \times 10^{-4}$ ohms

$r_{sector} = 5.21 \times 10^{-6}$ ohms.

$r_{end}(d) = 2.51 \times 10^{-4}$ ohms.

$\therefore$ $r_{bed} = 1.46 \times 10^{-4}$ ohms.

and $r_{Yd} = 10.5$ ohms.

Quadrature Axis Rotor Winding Resistance:

The formula given by Talaat for this resistance is-

$$r_{Yq} = (CK_w)^2 \times \frac{2}{p} \times \frac{r_{beq}}{n_b\ (1+K_b)}$$

Talaat has not given expression for r_{boq}, but it can be found in exactly the same fashion as the expression for r_{bod}.

$$\therefore r_{boq} = r_b + \frac{r_{end}(q)}{\frac{1}{8} n_b (1 + K_b)p}$$

$r_{end}(q)$ is found in the same manner as $r_{end}(d)$.

For the experimental machine the following values have been computed:

$r_b = 1.23 \times 10^{-4}$ ohms

$r_{sector} = 6.11 \times 10^{-6}$ ohms

$\therefore r_{end}(q) = 1.10 \times 10^{-4}$ ohms

$\therefore r_{boq} = 1.83 \times 10^{-4}$ ohms.

$\therefore r_{rq} = 8$ ohms.

Moment of Inertia:

The moment of inertia of the shaft and the rotor can be easily calculated following the well-known formula for Hollow cylinder.

Moment of inertia for hollow cylinder-

$$= \frac{M (R^2 + r^2)}{2}$$

where M = Mass of the cylinder

R = External radius

r = Internal radius

For the solid cylinder 'r' would be zero. The moment of inertia of the shaft of the experimental machine was calculated as 0.144 in^2-lb.

After making suitable corrections for the presence of saliency and slots, end rings etc. the moment of inertia of the rotor is found to be 11.743 in^2- lb. The total inertia for the shaft and the rotor is thus found to be 11.743 in^2.lb. which is equivalent to 35 x 10^{-4} Kg.m^2.

CHAPTER - IV

4.1. STUDY OF ELECTRO MECHANICAL TRANSIENT:

The application of induction motor in control systems has led to the study of dynamic behaviour of induction motor in which non-linear equations must be solved. It involves the separate solution of the equations for all the inputs of interest. Digital computer is now-a-days gaining popularity, because it does all the mathematical manipulation of numerical solution in a shorter time. But analog computer is also a powerful tool, which helps simulating the actual system with electrical or electromechanical components. Other methods of investigating the dynamic behaviours are phase-plane techniques, describing functions, quasi-linearizations and statistical methods. These methods are mainly concerned with the consideration of stability in non-linear systems.

4.2. ANALOGUE COMPUTER:

When a machine is viewed from the standpoint of dynamic circuit theory, a set of integro-differential equations of greater or less complexity depending on the machine predict the whole transient performance of the machine. The solution of these non-linear equations are frequently a formidable task. A very convenient method for studying simultaneous ordinary introgro-differential equations (both linear and non-linear) is the electronic analogue computer. The analogue computer consists of components of various types that can be interconnected to simulate the operation of the system in time domain.

Considering the Fig.(2.1) and with the help of dynamic circuit theory the following equations can be developed.

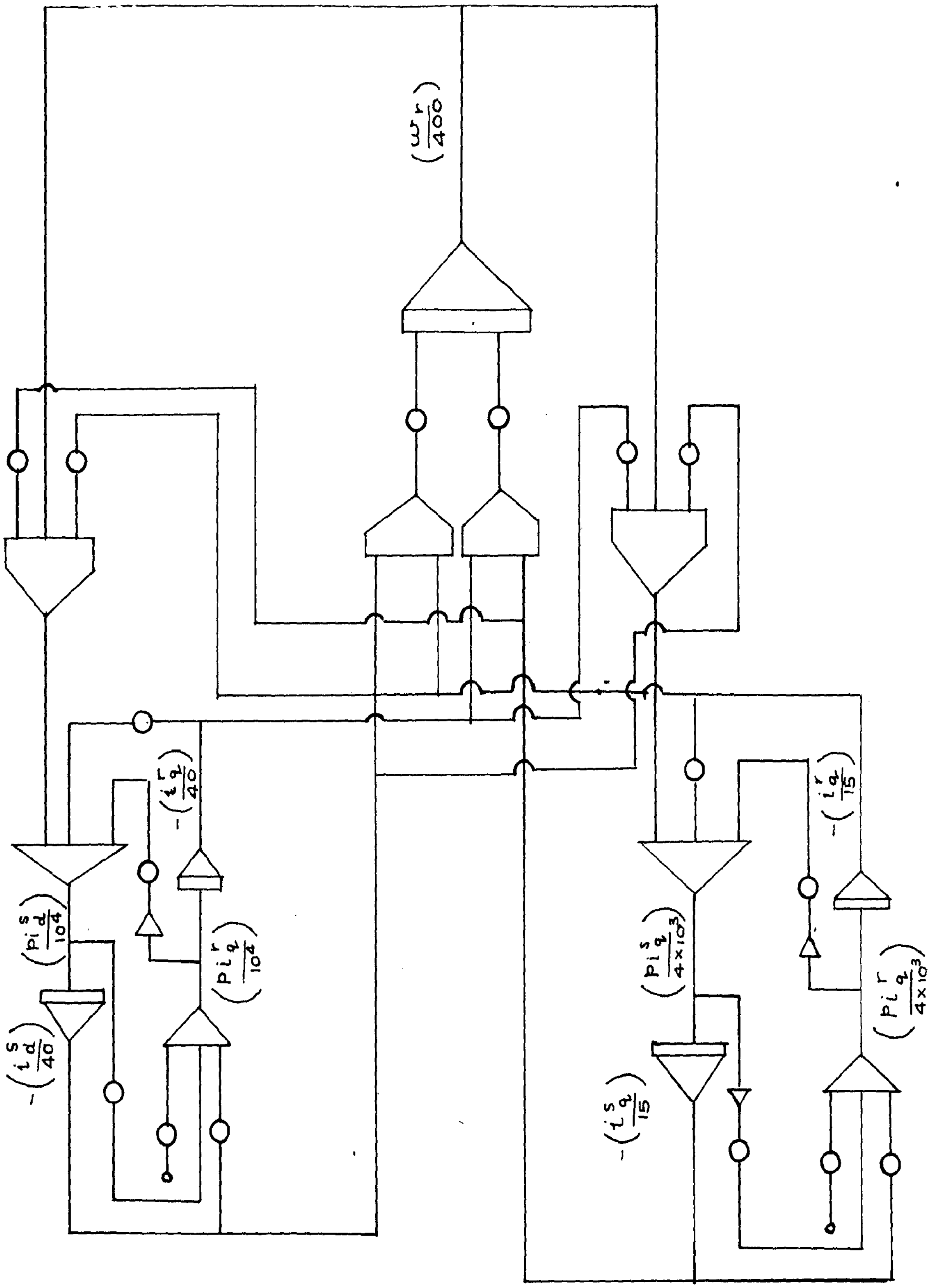


Fig.4.1

Analogue Representation of the transient study

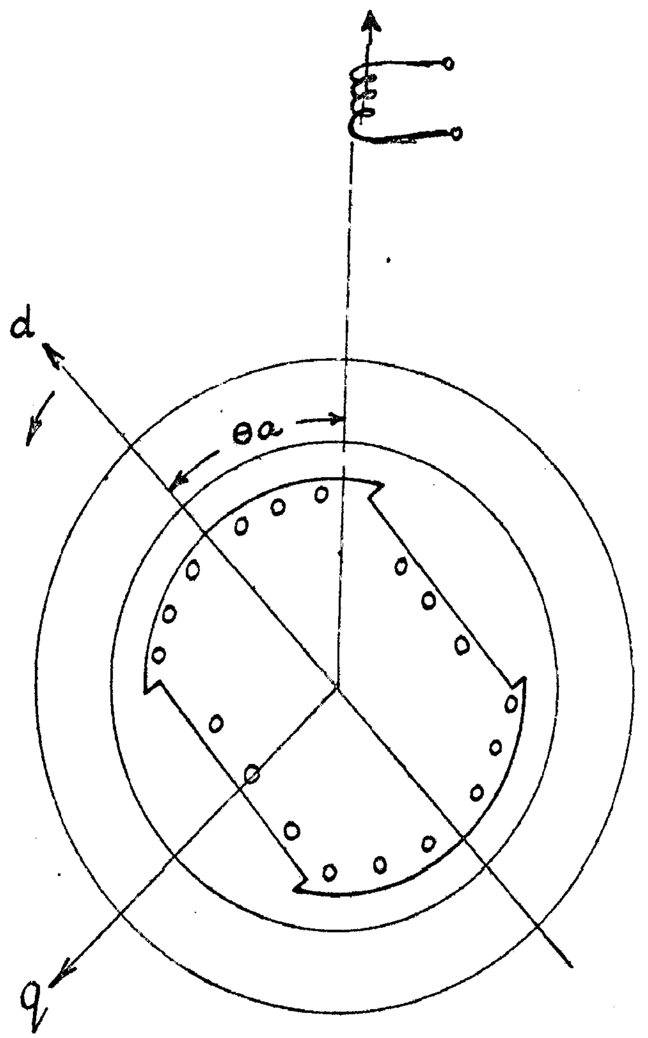


Fig.4.2

Representation of Experimental machine

$$r_s i_d^s + L_d \frac{di_d^s}{dt} + L_{ad} \frac{di_d^r}{dt} = v_d(t)$$

$$L_{ad}\frac{di_d^s}{dt} + L_{rd}\frac{di_d^r}{dt} + r_{rd} i_d^r + w_r \,(L_{rq} i_q^r + L_{aq} i_q^s) = 0$$

$$r_s i_q^s + L_q \frac{di_q^s}{dt} + L_{aq} \frac{di_q^r}{dt} = v_q(t)$$

$$L_{aq}\frac{di_q^s}{dt} + L_{rq}\frac{di_q^r}{dt} + r_{rq}\, i_q^r - w_r \,(L_{rd} i_d^r + L_{ad}\, i_d^s) = 0 \qquad \ldots(1)$$

$$T_D = (i_d^s\, i_q^r\, L_{aq} - i_q^s\, i_q^r \;\; L_{ad})$$

$$= J.\frac{dw_r}{dt} + T_L$$

if $T_L = 0$ and $\theta = 45^o$

The analogue set-up given in Fig.(4.2) can be developed.

4.3. <u>DIGITAL COMPUTER STUDY:</u>

Following the Fig.(4.1), the voltage equations can be written in terms of flux-linkage also.

$$r_s i_s + \frac{d\psi_s}{dt} = V_m \sin\,(wt + \phi)$$

$$r_{rd}\, i_d^r + \frac{d\,\psi_{rd}}{dt} = 0$$

$$r_{rq} i_q^r + \frac{d\,\psi_{rq}}{dt} = 0 \qquad \ldots(2)$$

$$T_D = J\,\frac{d^2\theta}{dt^2} + T_L$$

The flux-linkages can be written as

$$\psi_s = L_s i_s + L_{ad} \cos\theta\, i_d^r - L_{aq} \sin\theta\, i_q^r$$

$$\psi_{rd} = L_{ad} \cos\theta\, i_s + L_{rd}\, i_{rd}$$

$$\psi_{rq} = - L_{aq} \sin\theta \, i_s + L_{rq} i_q^r .$$

Now, the electro-dynamic developed torque-

$$T_D = \frac{W_{field}}{\theta}$$

and the energy stored in the electromagnetic field-

$$W_{field} = \tfrac{1}{2} L_s i_s^2 + \tfrac{1}{2} L_{rd} i_d^{r2} + \tfrac{1}{2} L_{rq} i_q^{r^2} + L_{ad} \cos\theta \, i_s i_d^r - L_{aq} \sin\theta \, i_s i_q^r$$

$$\therefore T_D = - L_{ad} i_s i_d^r \sin\theta - L_{aq} i_s i_q^r \cos\theta$$

Now, substituting the values for $\frac{d\psi_s}{dt}$, $\frac{d\psi_{rd}}{dt}$ and $\frac{d\psi_{rq}}{dt}$ and writing $\frac{d\theta}{dt} = w_r$ & $\frac{d}{dt} = p$ the expression (2) can be constrained to a set of first-order non-linear differential equations in the form.

$$\begin{bmatrix} L_s & L_{ad}\cos\theta & -L_{aq}\sin\theta \\ L_{ad}\cos\theta & L_{rd} & 0 \\ -L_{aq}\sin\theta & 0 & L_{rq} \end{bmatrix} \begin{bmatrix} pi_s \\ pi_d^r \\ pi_q^r \end{bmatrix} = \begin{bmatrix} V\sin(vt+\phi) - r_s i_s + L_{ad} i_d^r w_r \sin\theta + L_{aq} i_q^r w_r \cos\theta \\ -r_{rd} i_d^r + L_{ad} i_s w_r \sin\theta \\ -r_{rq} i_q^r + L_{aq} i_s w_r \cos\theta \end{bmatrix}$$

$$pw_r = \frac{1}{J} (T_L + L_{ad} i_s i_d^r \sin\theta + L_{aq} i_s i_q^r \cos\theta)$$

$$p\theta = w_r$$

These non-linear differential equations are solved in digital computer by Runge-Kutta-Gill method. The computer-program has been incorporated in Appendix III.

CHAPTER - V

5.1. EXPERIMENTAL DETERMINATION OF EQUIVALENT CIRCUIT CONSTANTS:

The method of experimentally determining the machine constants of 3-phase induction motor is quite well known. In case of single phase induction motors, the problem of determining the motor constants is not so simple. It may be noted that for a single phase motor under no-load conditions, the rotor slip with respect to the forward field is nearly zero, but is nearly double with respect to the backward field. Hence there are substantial rotor cu loss even at no load. It may be also noted that in the usual revolving field equivalent circuit, half of the magnetising reactance is nearly short circuited at the time of no-load test. Veinott has given a test method which takes care of the above points and his books may be referred for the details of this method.

The separation of stator and rotor leakage reactance for squirrel cage induction motor is quite a difficult problem. No simple method is available for this separation. Hence the comparison between the theory and the experiment is also very difficult. It is well known that the running performance calculation is of less importance for the normal motor. The starting performance is the prime important factor. In our machine also starting performance is very interesting. The starting torque arises due to different impedances of the two rotor circuits. The difference between winding impedances must be known as accurately as possible.

For the present machine, it is considered desirable that some method may be approximate one, shall be used to determine the leakage separately. The stator leakage reactance is measured separately by a test called the 'rotor out' test. Besides this test, the 'synchronous reactance' of the machine was measured by 'maximum reluctance' test. Magnetising reactances for both the axis was determined by substracting leakage reactance from the 'synchronous reactances'.

The rotor impedance is calculated from the locked rotor test. The rotor impedance is different in the two axis. Hence the locked-rotor test is done twice, once with the d-axis of the rotor aligned with the stator winding axis and the other is with q-axis of the rotor aligned with the stator winding operational impedance is calculated from frequency response method. The moment of inertia is measured by 'decceleration test'.

'Rotor Out' Test:

In this method, the rotor with shaft is removed from the motor. End shields are kept fitted. The reactance of the stator winding is measured under this condition. This reactance is the sum of the stator leakage and the magnetising reactance. The magnetising reactance is due to the small amount of mutual flux going through the very large air-gap resulting from the removal of the rotor. It is necessary to find the reactance due to this flux. This can be calculated from Veinott's formula for the magnetising reactance with the modification as 'g'= the air-gap length as the ratio of the stator bore and no. of poles. The modified formula becomes-

$$X_m(\text{rotor out}) = 2\pi f(CK_w)^2 \times 0.6468 \times \frac{L}{P} \times 10^{-8} \text{ ohms.}$$

where, f = frequency in c/s.

L = Length of the stator in inch.

CK_w = Effective conductors in series.

P = no.of poles.

The resistance and reactance are calculated from the readings taken. The value of X_m(rotor out) is subtracted from the measured total reactance to get the loakage reactance x_1.

Test Readings:

Voltage : 37 volts.

Current : 1.5 Amps

Power : 30 watts.

X_{Total} : 20.8 ohms.

X_m(rotor out) : 5.05 ohm.

$\therefore X_1 = X_{total} - X_m$ (rotor out) = 15.75 ohms.

The d.c. stator resistance is found. The a.c. resistance measured in this test includes a small iron loss component. After allowi ng for this loss also the resistance is found to be greater than d.c. value which can be accounted for the skin effect.

Maximum Reluctance Test:

The present machine when driven at synchronous speed represents the reluctance motor. The load is gradually increased and at the break point from synchronous to induction run, the power input, the current and the voltage are noted.

For maximum power condition-

$$P_m = \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_q}\right)$$

$$\text{and } I = \sqrt{I_d^2 + I_q^2} = \frac{V}{\sqrt{2}}\sqrt{\frac{1}{X_d^2} + \frac{1}{X_q^2}}$$

Substituting $X_d/X_q = K$, there are,

$$P_m = \left(\frac{V^2}{2X_q}\right)\left(1 - \frac{1}{K}\right) \qquad \ldots \qquad \ldots(1)$$

$$I = (V/\sqrt{2}\, X_q K)\,(\sqrt{1+K^2}) \qquad \ldots \qquad \ldots(2)$$

From (1) & (2),

$$\frac{K-1}{\text{[illegible]}} = \sqrt{2}\, P_m/VI = a$$

$$\therefore \; K = \frac{1 \pm a\sqrt{2-a^2}}{(1-a^2)}$$

That value of K which is greater than one is retained and X_q is calculated from either (1) or (2).

Then X_d is calculated, as, $X_d = KX_q$

Test Results:

Power input in watts = 33

Voltage in volts = 295

Current in Amperes = 0·70

K is calculated = 1·39

and X_q is calculated as 319 ohms.

$\therefore X_d = KX_q =$ 514 ohms

Locked Rotor Test:

The locked rotor tests are done twice once with d-axis of rotor aligned with stator axis and the other with q-axis of rotor aligned with stator axis. Keeping the rotor in locked position, the rotor is turned gradually to get the maximum deflection in an ammeter showing the alignment of q-axis with stator axis and minimum for d-axis respectively.

The test results for the experimental machine are given below:

A) Test results with minimum deflection

Power in watts = 62

Voltage in volts = 85

Current in Amps. = 1·5

B) Test results with maximum deflection.

Power in watts = 132

Voltage in volts = 96

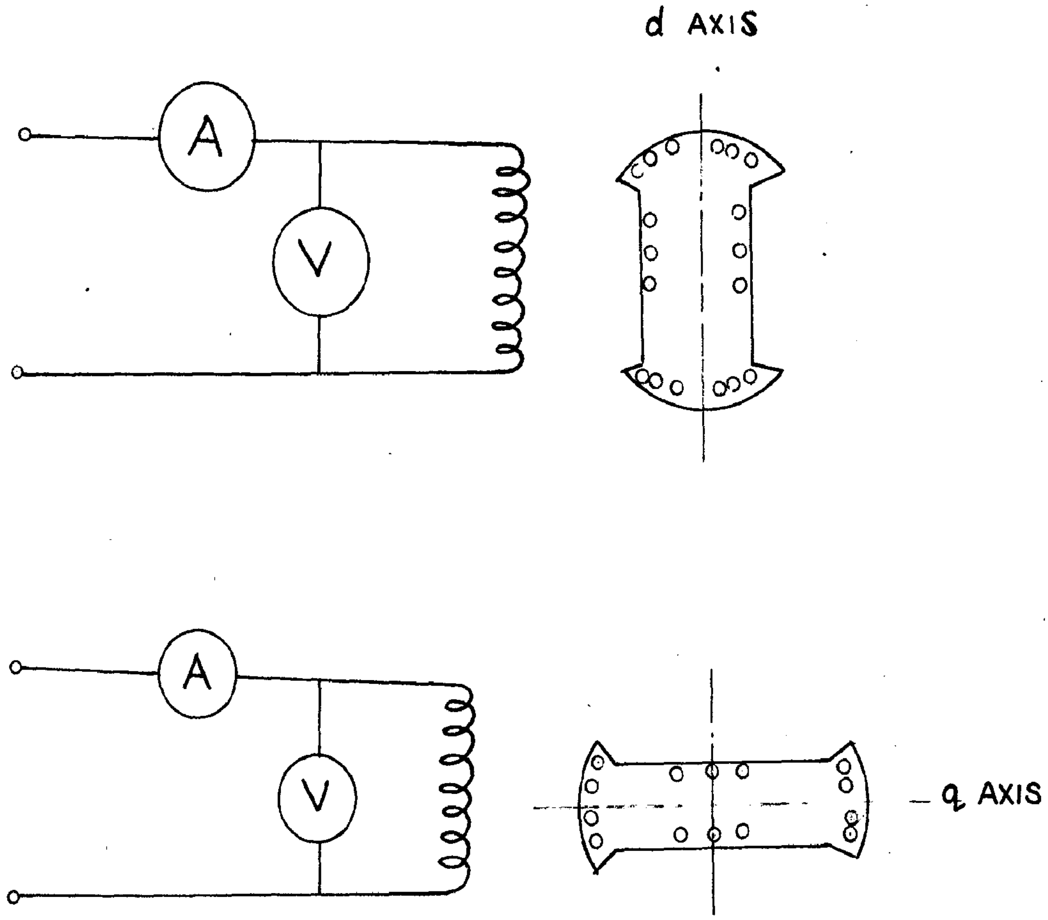


Fig.5.1

Circuit diagram for determination of operational impedances

From the locked rotor tests (rated current) the values of r_{rd}, x_{rd}, r_{rq}, x_{rq} are obtained. These are obtained by calculating the total locked rotor impedances and subtracting the stator impedance value from them. The effect of magnetising reactance is neglected.

From the d-axis test,

$$Z_d = \frac{V}{I}$$

$$R_d = \frac{W}{I^2}$$

$$\therefore X_{d12} = \sqrt{Z_d^2 - R_d^2}$$

The stator impedance from the 'rotor-out' test

$$= r_s + jx_1$$

$$\therefore r_{rd} = R_d - r_s$$

$$x_{rd} = X_{d12} - x_1$$

Similarly, the values for r_{rq} and x_{rq} is calculated

$\therefore r_{rd}$ = 14.3 ohms $\qquad r_{rq}$ = 10.6 ohm.

x_{rd} = 33.5 ohm. $\qquad x_{rq}$ = 18.5 ohm.

6.2. DETERMINATION OF OPERATIONAL IMPEDANCES:

The torque expression (17) of Chapter I has been developed in term of operational impedances. So the calculation of torque-slip characteristic demands the determination of operational impedances. The method presented here is a slight modification of that already presented by Sen & Adkins[19]. If the input impedance at a frequency sw rad/sec. with rotor standstill is Z_d for direct axis in line with the stator m.m.f. axis (position being determined by indication of

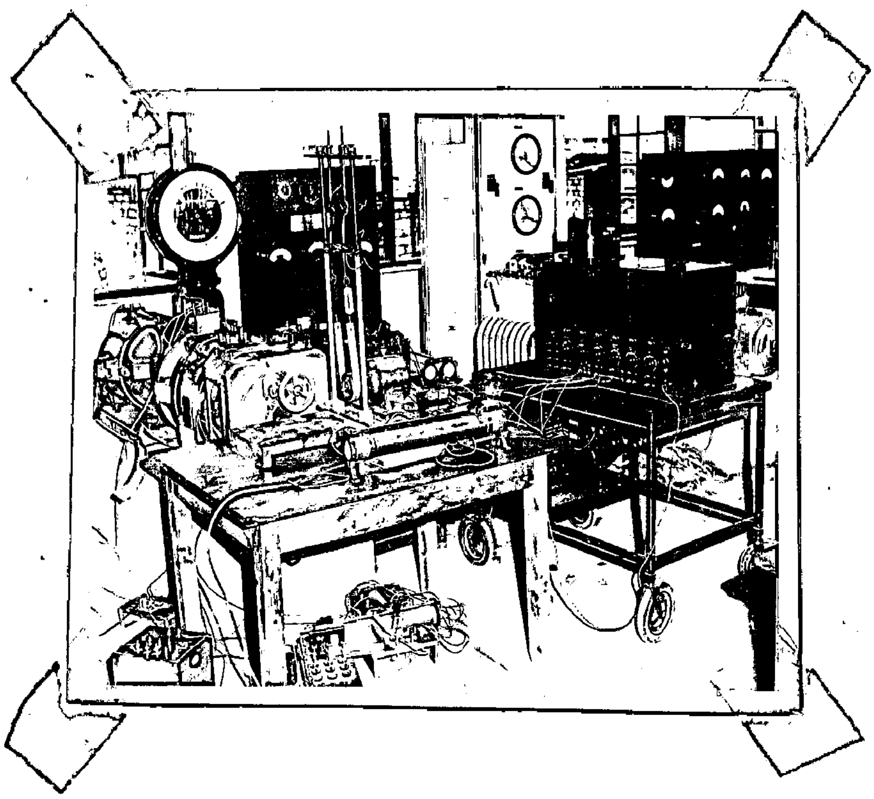


Fig 5.1 (a)

Experimental Set-up for operational Impedance Measurement.

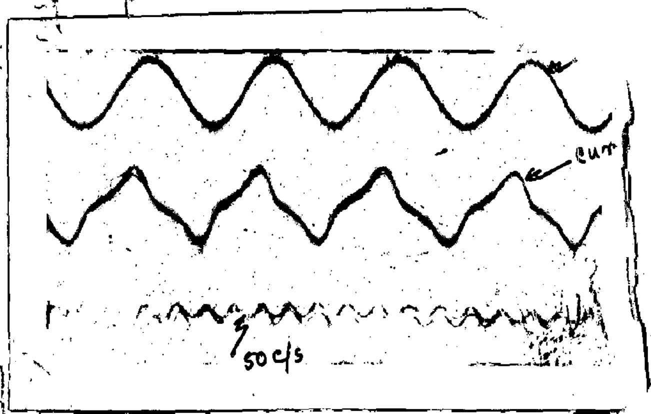


fig 5.2

Scale :

1 cm = 7.5 volt.

1 cm = 175 mA.

minimum deflection in the ammeter), then,

$$Z_d = r_s + j s\, X_d\,(js\omega)$$

$$\text{where } X_d(js\omega) = X_d \frac{1 + js\omega T_d'}{1 + js\omega T_{do}'}$$

Similarly, the operational impedance $X_{q1}(js\omega)$ is obtained from the following,

$$Z_q = r_s + j s\, X_q\,(js\omega)$$

where Z_q is the input impedance at a frequency $s\omega$ rad/sec. with rotor standstill for quadrature axis in line with the m.m.f. axis (position being determined by the maximum deflection of ammeter),

$$X_q\,(js\omega) = X_q \frac{1 + js\omega T_q'}{1 + js\omega T_{qo}'}$$

The connections are shown in Fig.(5.1). The frequency is varied from very low value to rated value, and the input impedances are measured. The magnitude and phase difference between the voltage and the current is measured with the help of a six element Cambridge Duddel Oscillograph. Variable frequency supply is obtained from a separately driven three phase commutator machines. A typical oscillogram of current and voltage is given in Fig.(5.2). The amplitudes of $X_d(js\omega)$ and $X_q(js\omega)$ are plotted against $\log_{10}(s\omega)$ Fig.(5.3) & (5.4). The inverse of corner frequencies obtained from their straight line approximation will yield the four time constants T_{do}', T_d', T_{qo}', T_q', respectively.

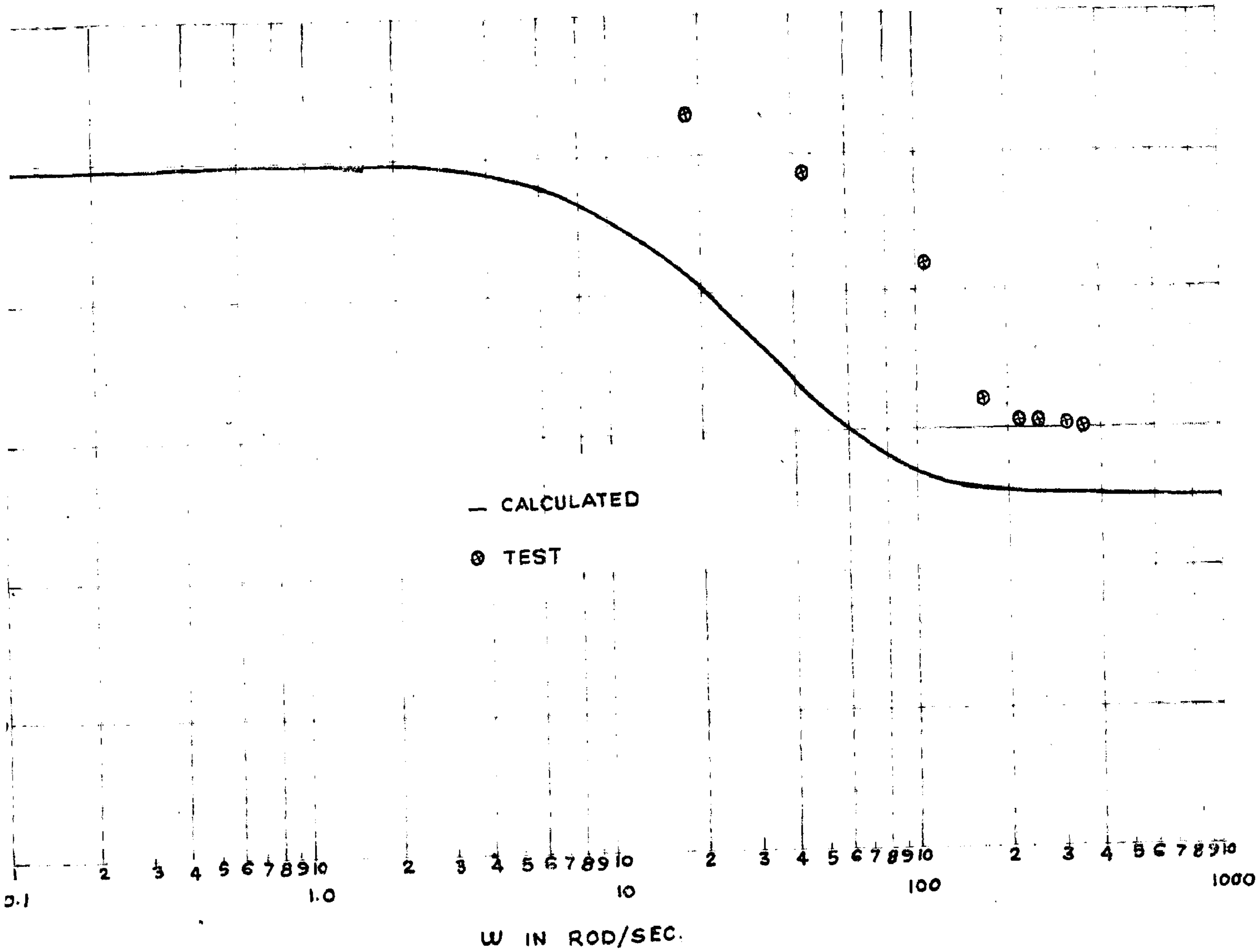
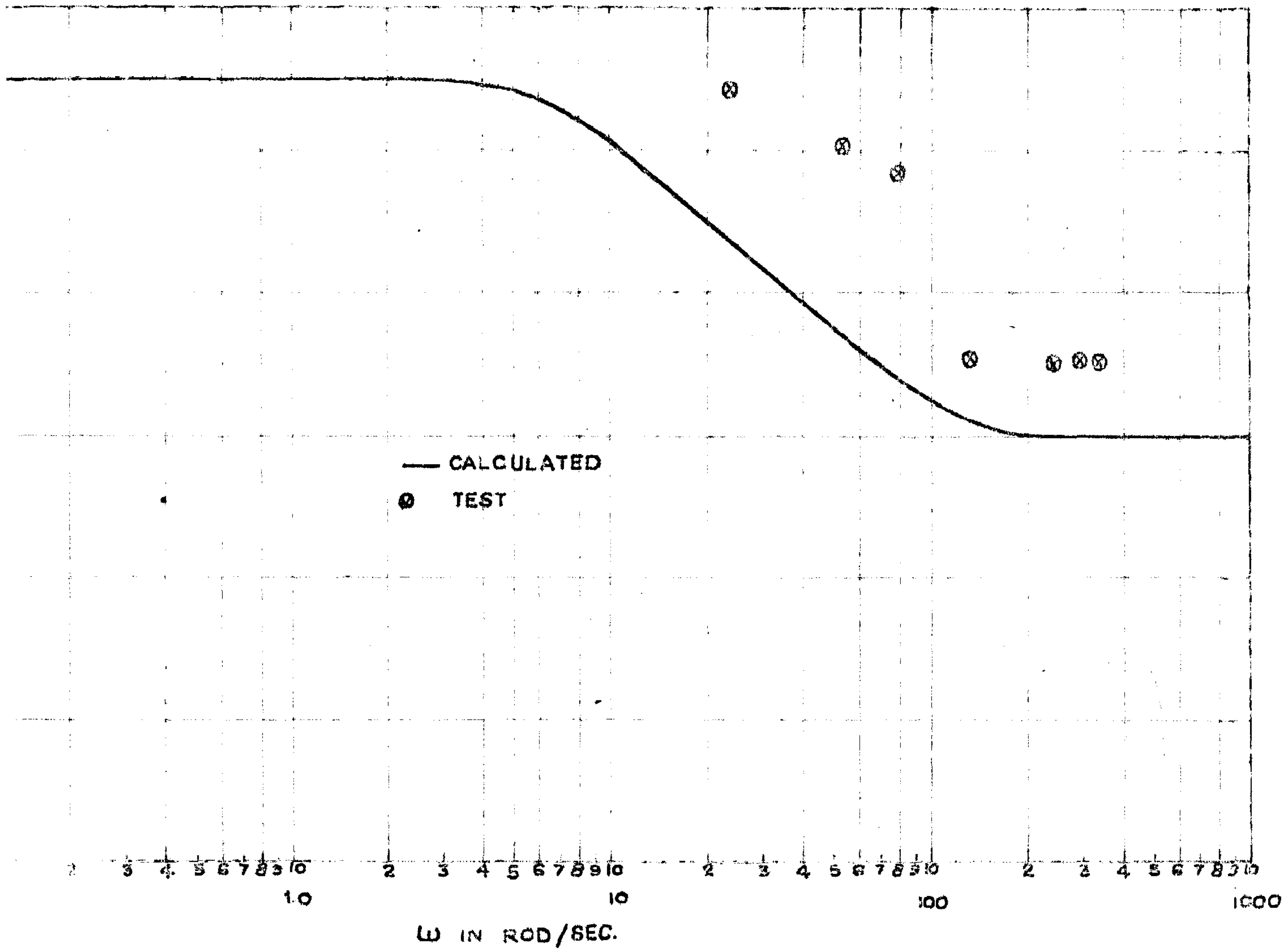


Fig. 3

Plot of amplitude $x_{f}(p)$ in terms of log-frequency



— CALCULATED
⊗ TEST
2 3 4 5 6 7 8 9 10
1.0
2 3 4 5 6 7 8 9 10
10
2 3 4 5 6 7 8 9 10
100
2 3 4 5 6 7 8 9 10
1000
ω IN ROD/SEC.

5.3. DETERMINATION OF MOMENT OF INERTIA BY DECCELERATION TEST:

'Decceleration Test':

The moment of inertia has been measured by the decceleration test. The rotor is allowed to slow down from no-load to lower-speeds due to friction. Knowing the friction torque, the moment of inertia can be found out. This can not be very accurate but has been used as a check on the theoretically calculated value. The theoretical value can be calculated accurately for the rotor and shaft.

Experimentally the value has been found out as 3.1×10^{-3}. Kg.-m^2.

5.4. LOAD TEST:

The dynamometer required for the test machine is not available. Hence the load test has been carried out by the 'pulley-belt' method. The belt-ends are connected to the two spring balances and the motor is loaded by the usual method. The test results are shown in a tabular form below. From the results, curves shown in figs. (5.5), (5.6), & (5.7) are drawn.

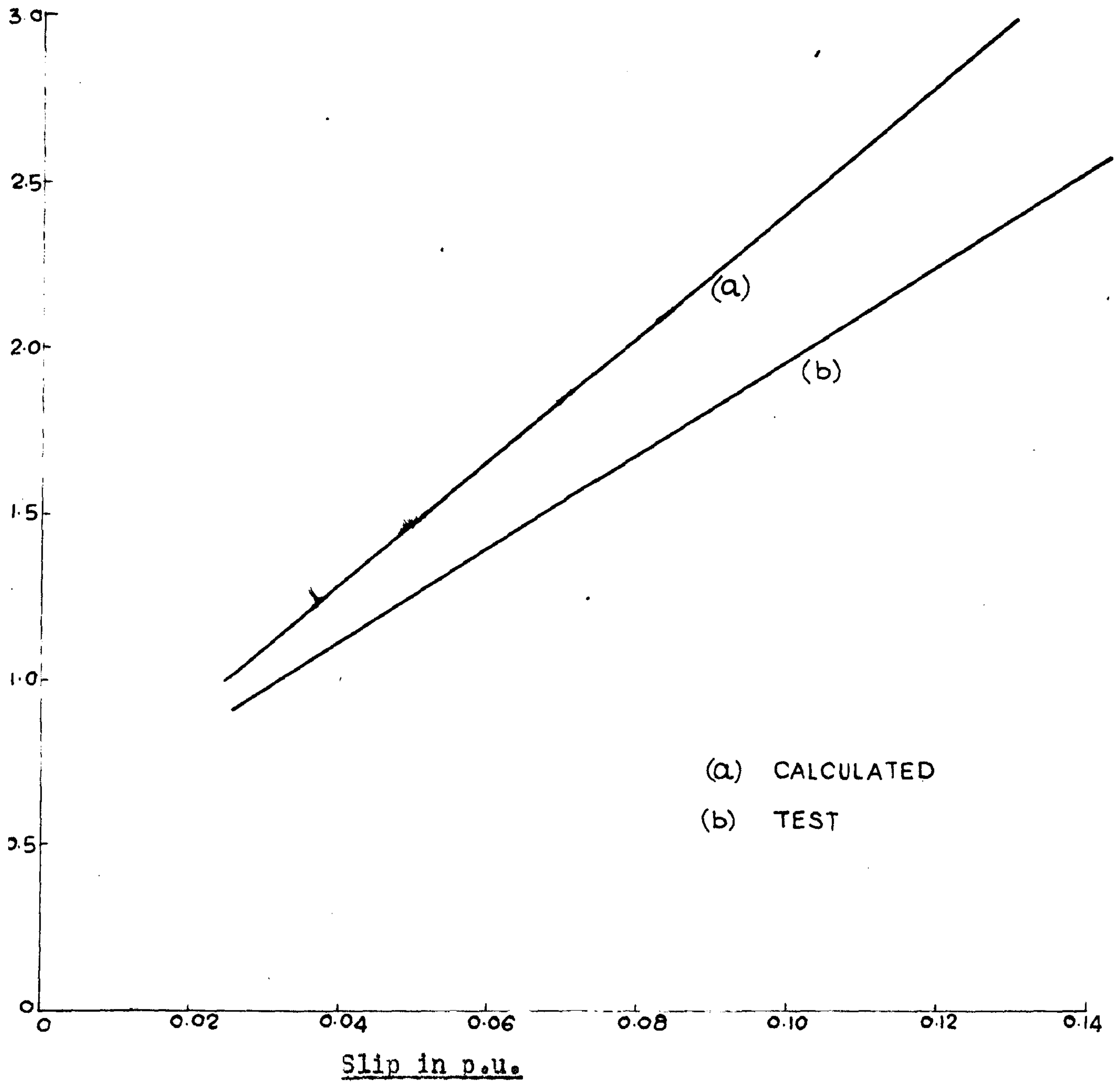

3.0
2.5
2.0
1.5
1.0
0.5
0
0
0.02
0.04
0.06
0.08
0.10
0.12
0.14
(a)
(b)
(a) CALCULATED
(b) TEST
Slip in p.u.

Fig.5.5.

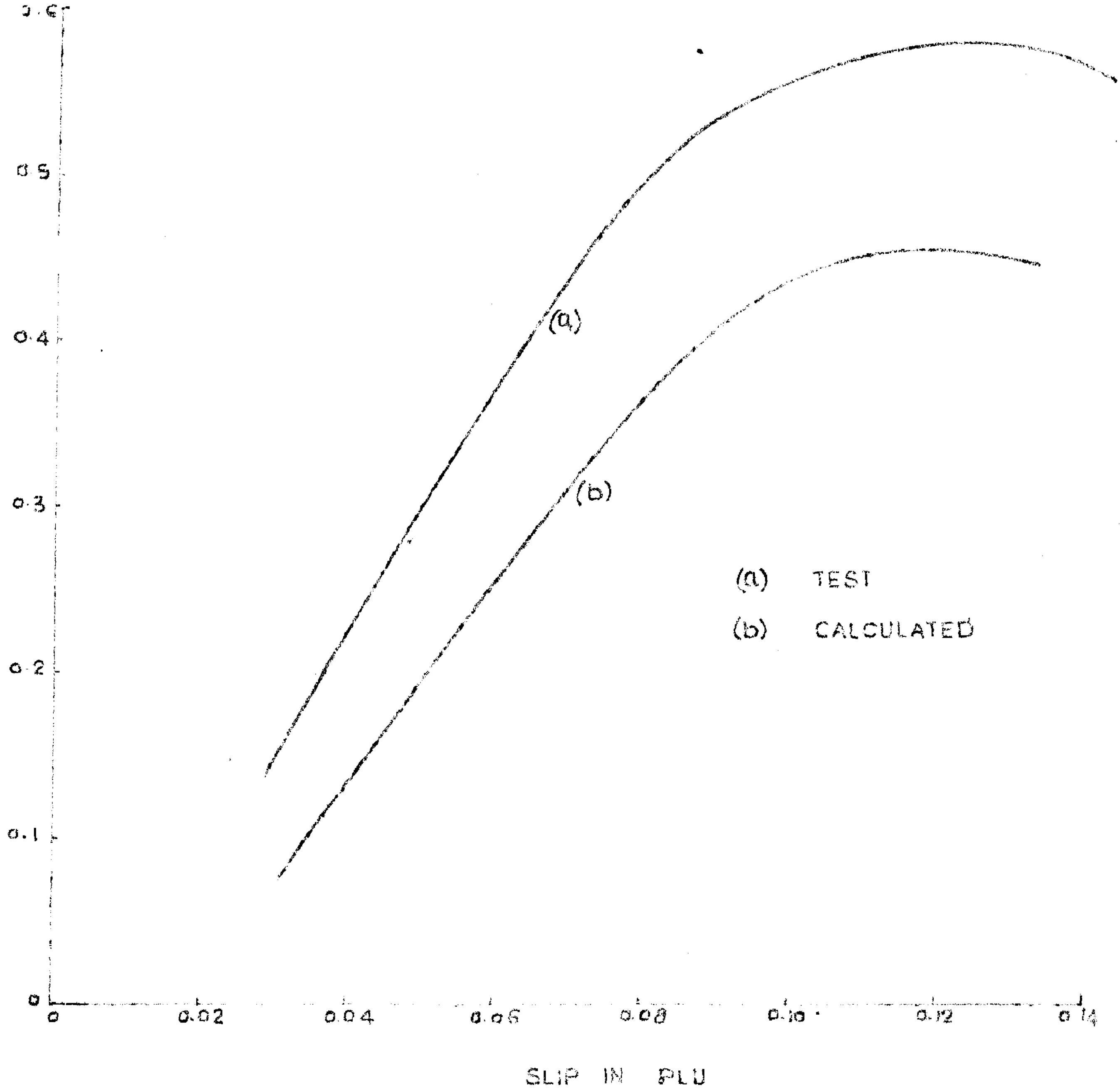

0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.02
0.04
0.06
0.08
0.10
0.12
0.14
(a)
(b)
(a) TEST
(b) CALCULATED
SLIP IN PU

Fig.3.6

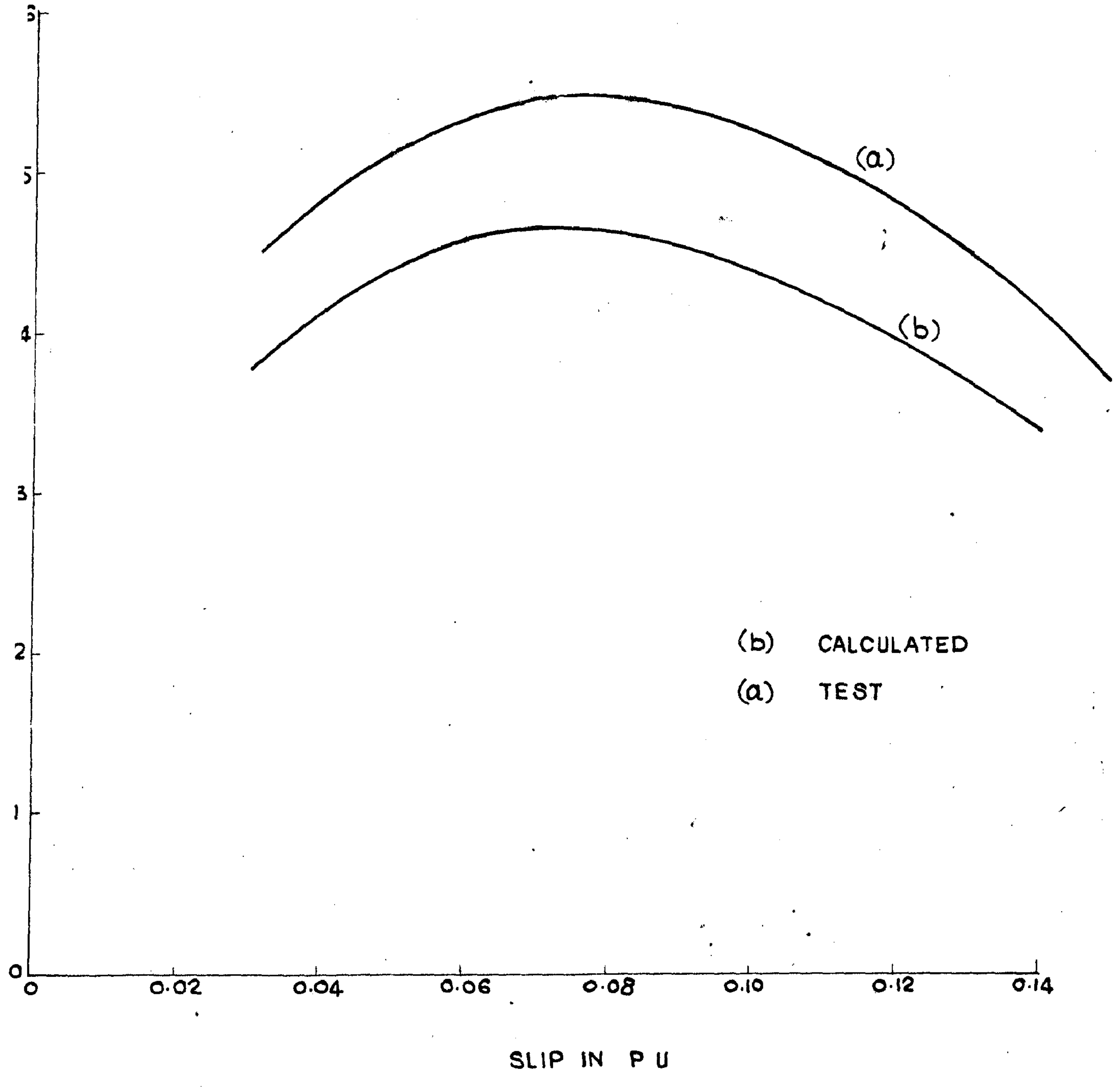

6
5
4
3
2
1
0
0
0.02
0.04
0.06
0.08
0.10
0.12
0.14
(a)
(b)
(b) CALCULATED
(a) TEST
SLIP IN P U

Fig.5.7

TEST RESULTS OF THE EXPERIMENTAL MACHINE:

Pulley diameter = 3½"

V in Volts	I in Amps.	W in watts	N in r.p.m.	T_1 in lbs.	T_2 in lbs.
a) 220	1.0	108	2920	0	0
b) 220	1.1	136	2910	0	1
c) 220	1.25	180	2890	0	1.5
d) 220	1.5	220	2850	0	2
e) 220	1.85	316	2790	0	3
f) 220	2.1	364	2750	0	3.5
g) 220	3.0	520	2610	0	4.0

CONCLUSION

MAIN FEATURES OF THE MOTOR DEVELOPED:

The principle of using asymmetry of the rotor for starting a single-phase induction motor is developed and has been found to be successful.

The method of construction is easier and eliminates the use of auxiliary windings and shaded rings.

The efficiency of the machine is quite good, compared to conventional single phase motors.

The motor can be run in either direction depending upon the initial position of the rotor with respect to the stator winding.

From the above discussion one can suggest some fields of application. The motor will be particularly suitable in the fractional and sub-fractional range, for applications involving light starting duty.

Before the motor can be a commercially successful one, the main two draw-backs should be over-come:

i) The motor can start only for some particular positions of the rotor with respect to stator winding. This demands for the positioning device to set the rotor in position. The author believes that it is also possible to construct such positioning device without much difficulty. Perhaps, the method suggested by Desai[29] may be adopted.

ii) The motor has a poor starting and accelerating torque.

SCOPE FOR FURTHER STUDY:

It may be noted that the forgoing results are for the experimental machine built from available materials and without experience on the design of this type of machine. The author believes it will be possible to improve the performance of the machine which needs a number of machines to be designed and studied. Also the experimental machine used a stamping which is normally meant for a four-pole design. The core was heavily saturated because of this and the no-load current is higher.

Some other problems which need further study are the effects of harmonics and their suppression. The stator current contains (1-2s) and (3-2s) frequency components in addition to the fundamental. Attempt may be made to reduce these harmonics. Further with the help of the digital computer, the various transients can be studied, and the machine parameters can also be optimised.

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R E F E R E N C E S

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APPENDIX-I

REDUCTION OF MATRIX:

The work involved by the routine calculation in elimination of arrays of a matrix can be avoided and the reduced array can be written from the full array only by inspection of the following rules are applied:

1) The denominator of the fraction for each element consists of the determinant of those rows and columns which are to be eliminated which are also common to both.

2) The numerator of the fraction for each element consists of the determinant formed by the rows and columns to be eliminated which are common to the element being considered, together with the rows and columns of the dinominator.

This rule becomes very simple when only one row and columns are eliminated at a time, especially when several elements in the array are zero.

If the last row and column are required to be eliminated from.

a_{11}	a_{12}	a_{13}	a_{14}
a_{21}	a_{22}	a_{23}	a_{24}
a_{31}	a_{32}	a_{33}	a_{34}
a_{41}	a_{42}	a_{43}	a_{44}

The result is-

$$
\begin{bmatrix}
a_{11} - \dfrac{a_{14} \cdot a_{41}}{a_{44}} & a_{12} - \dfrac{a_{14} \cdot a_{42}}{a_{44}} & a_{13} - \dfrac{a_{14} \cdot a_{43}}{a_{44}} \\
a_{21} - \dfrac{a_{24} \cdot a_{41}}{a_{44}} & a_{22} - \dfrac{a_{24} \cdot a_{42}}{a_{44}} & a_{23} - \dfrac{a_{24} \cdot a_{43}}{a_{44}} \\
a_{31} - \dfrac{a_{34} \cdot a_{41}}{a_{44}} & a_{32} - \dfrac{a_{34} \cdot a_{42}}{a_{44}} & a_{33} - \dfrac{a_{34} \cdot a_{43}}{a_{44}}
\end{bmatrix}
$$

Applying the above inspection rule the matrix (1-5) can be reduced first to a 3 x 3 matrix and then to a 2 x 2 matrix.

Thus the result is-

By 1st stage of reduction

$$
\begin{bmatrix} v^s_{d1} \\ v^s_{q1} \\ v^r_{q1} \end{bmatrix}
=
\begin{bmatrix}
pL_d + r_s & L_q - \dfrac{p\ L^2_{aq}}{pL_{rq} + r_{rq}} & pL_{ad} \\
- L_d & (pL_q + r_s) - \dfrac{p^2 L^2_{aq}}{pL_{rq} + r_{rq}} & - L_{ad} \\
pL_{ad} & & pL_{rd} + r_{rd}
\end{bmatrix}
\begin{bmatrix} i^s_{d1} \\ i^s_{d2} \\ i^r_{d1} \end{bmatrix}
$$

and by 2nd stage of reduction-

$$
\begin{bmatrix} v^s_{d1} \\ v^s_{q1} \end{bmatrix}
=
\begin{bmatrix}
(pL_d + r_s) - \dfrac{p^2 L^2_{ad}}{(pL_{rd} + r_{rd})} & L_q - \dfrac{p\ L^2_{aq}}{pL_{rq} + r_{rq}} \\
- L_d + \dfrac{p\ L^2_{ad}}{(pL_{rd} + r_{rd})} & (pL_q + r_s) - \dfrac{p^2 L^2_{aq}}{pL_{rq} + r_{rq}}
\end{bmatrix}
\begin{bmatrix} i^s_{d1} \\ i^s_{q1} \end{bmatrix}
$$

Now

Now putting $(L_1 + L_{ad}) = L_d$ & $(L_1 + L_{aq}) = L_q$

and $(L_{r1} + L_{ad}) = L_{rd}$ & $(L_{r1} + L_{aq}) = L_{rq}$

$$(pL_d + r_s) - \frac{p^2 L_{ad}^2}{(pL_{rd} + r_{rd})}$$

$$= r_s + p\,(L_1 + L_{ad}) - \frac{p^2 L_{ad}^2}{p(L_{r1} + L_{ad}) + r_{rd}}$$

$$= r_s + \frac{p(L_1 + L_{ad})\left[p(L_{r1} + L_{ad}) + r_{rd} - \frac{pL_{ad}^2}{(L_1 + L_{ad})}\right]}{p(L_{r1} + L_{ad}) + r_{rd}}$$

$$= r_s + \frac{\left[pL_d\right]\left[1 + \frac{1}{r_{rd}}\ p\left(L_{r1} + \frac{L_1\ L_{ad}}{L_1 + L_{ad}}\right)\right]}{\left[1 + \frac{1}{r_{rd}}\ p(L_{r1} + L_{ad})\right]}$$

$$= r_s + pL_d \frac{1 + T'_d\ p}{1 + T'_{do}\ p}$$

$$= r_s + pL_d(p)$$

where,

$$T'_d = \frac{1}{r_{rd}}\left(L_{r1} + \frac{L_1\ L_{ad}}{L_1 + L_{ad}}\right)$$

$$T'_{do} = \frac{1}{r_{rd}}\left(L_{r1} + L_{ad}\right)$$

Again ,

$$L_q - \frac{pL_{aq}^2}{pL_{rq} + r_{rq}}$$

$$= (L_1 + L_{aq}) - \frac{p^2 L_{aq}^2}{p(L_{r1} + L_{aq}) + r_{rq}}$$

$$= \frac{(L_1 + L_{aq})r_{rq} + p(L_1 + L_{aq})(L_{r1} + L_{aq}) - pL_{aq}^2}{p(L_{r1} + L_{aq}) + r_{rq}}$$

$$= \frac{(L_1 + L_{aq}) \left[1 + \frac{1}{r_{rq}} p \left(L_{r1} + \frac{L_1 \; L_{aq}}{L_1 + L_{aq}} \right) \right]}{\left[1 + \frac{1}{r_{rq}} p' \left(L_{r1} + L_{aq} \right) \right].}$$

$$= L_q \frac{1 + T_q'}{1 + T_{qo}'}$$

$$= L_q(p)$$

where,

$$T_q' = \frac{1}{r_{rq}} \left(L_{r1} + \frac{L_1 \; L_{aq}}{L_1 + L_{aq}} \right)$$

$$T_{qo}' = \frac{1}{r_{rq}} \left(L_{r1} + L_{aq} \right)$$

APPENDIX - II

CALCULATION OF GAP CO-EFFICIENT:

In calculation of gap co-efficients the following assumptions are made:

1) The air-gap 'g' under the salient -pole is assumed to be uniform.

2) The air-gap 'g_o' under the inter-pole is also assumed to be uniform.

3) With uniform air-gap the spatial flux distribution in the air-gap is taken to be sinusoidal.

4) The flux enters the rotor radially even over the portions occupied by the inter-pole, but the magnitude is reduced in proportion.

With the assumptions made the flux density wave in the air-gap portion can be represented as in Fig.1.

$$B(\theta) = B_1 \sin\theta \qquad 0 < \theta < \alpha$$

$$= B_2 \sin\theta \qquad \alpha < \theta < \pi/2$$

where,

$$B_1 = \frac{\mu_o M}{g_o} \quad \text{and} \quad B_2 = \frac{\mu_o M}{g}$$

The fundamental component of the wave can be analysed by Fourier Series analysis.

$$b_1 = \frac{4}{\pi} \left[\int_0^{\pi/2} B(\theta) \sin\theta \, d\theta. \right]$$

$$= \frac{4}{\pi}. \left[\int_0^{\alpha} B_1 \sin^2\theta \, d\theta + \int_{\alpha}^{\pi/2} B_2 \sin^2\theta \, d\theta \right]$$

$$= \frac{4}{\pi} \mu_o M \left[\int_0^{\alpha} \frac{1}{g_o} \sin^2\theta \, d\theta + \int_{\alpha}^{\pi/2} \frac{1}{g} \sin^2\theta \, d\theta \right]$$

$$= \frac{4}{\pi} \mu_o M \times \frac{1}{2} \left[\frac{1}{g_o} \int_0^{\alpha} (1-\cos 2\theta) d\theta + \frac{1}{g} \int_{\alpha}^{\pi/2} (1-\cos 2\theta) d\theta \right]$$

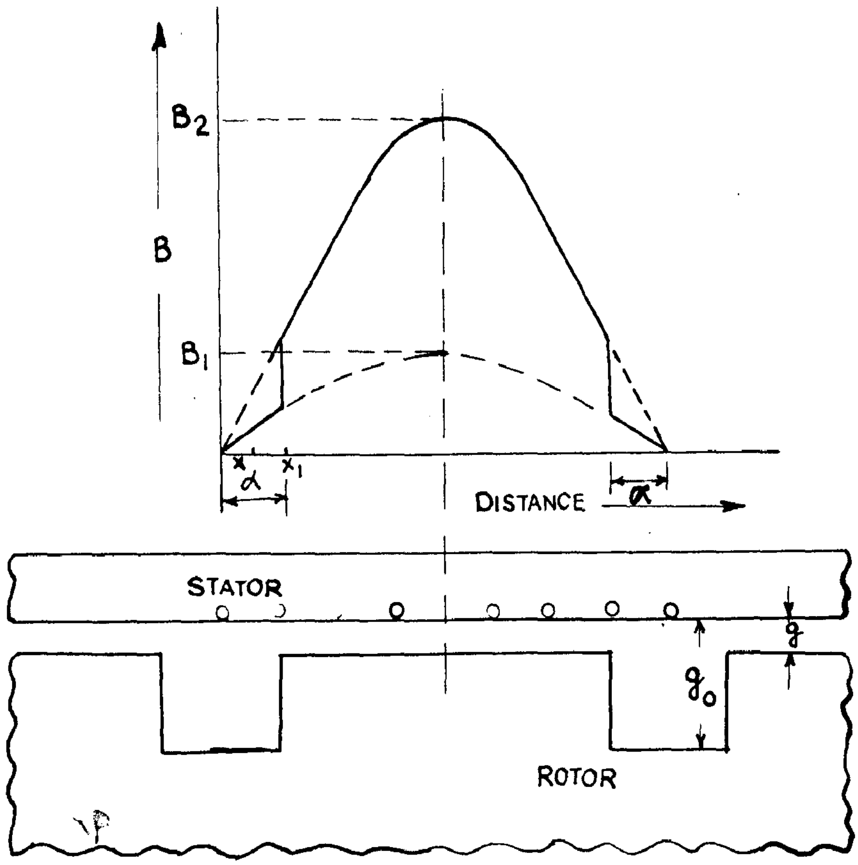


Fig. A.

Flux distribution pattern

$$= \frac{2}{\pi} \mu_o M \left[\frac{1}{g_o} \left(\alpha - \tfrac{1}{2}\sin 2\alpha\right) + \frac{1}{g} \left\{\left(\pi/2 - \alpha\right) + \tfrac{1}{2}\sin 2\alpha\right\}\right]$$

∴ The peak flux-density at any point on the salient polar region

$$B_d = \frac{2}{\pi} \mu_o M \left[\frac{1}{g_o}\left(\alpha - \tfrac{1}{2}\sin 2\alpha\right) + \frac{1}{g}\left\{\left(\pi/2 - \alpha\right) + \tfrac{1}{2}\sin 2\alpha\right\}\right]$$

∴ To calculate C_d:

$$C_d = \frac{\text{Smooth rotor flux}}{\text{Direct axis flux}} = \frac{\phi_m}{\phi_d} = \frac{B_m}{B_d}$$

$$\text{Now, } B_m = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} B(\theta) \sin D \; d\theta$$

$$= \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \mu_o \frac{M}{g} \sin^2\theta \; d\theta$$

$$= \frac{2}{\pi} \mu_o M/g \times \tfrac{1}{2} \int_0^{\frac{\pi}{2}} (1 - \cos 2\theta)\, d\theta$$

$$= \mu_o M/g$$

$$\therefore C_d = \frac{\frac{1}{g}}{\frac{2}{\pi}\left[\frac{1}{g_o}\left(\alpha - \tfrac{1}{2}\sin 2\alpha\right) + \frac{1}{g}\left\{\left(\pi/2 - \alpha\right) + \tfrac{1}{2}\sin 2\alpha\right\}\right]}$$

To calculate C_q:

The peak flux-density at any point on the inter-polar region:

$$B_q = \frac{4}{\pi} \left[\int_0^{\frac{\pi}{2}-\alpha} B_2 \sin^2\theta\, d\theta + \int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}} B_1 \sin^2\theta\, d\theta \right]$$

$$= \frac{4}{\pi} \mu_o M \left[\frac{1}{g} \int_0^{\frac{\pi}{2}-\alpha} \sin^2\theta\, d\theta + \frac{1}{g_o} \int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}} \sin^2\theta\, d\theta \right]$$

$$= \frac{4}{\pi} \mu_o M \times \tfrac{1}{2} \left[\frac{1}{g} \int_0^{\frac{\pi}{2}-\alpha} (1 - \mathrm{soc}\, 2\theta)\, d\theta + \frac{1}{g_o} \int_{\frac{\pi}{2}-\alpha}^{\pi/2} (1 - \cos 2\theta)\, d\theta \right]$$

$$= \frac{2}{\pi} \mu_o M \left[\frac{1}{g} \left\{\left(\pi/2 - \alpha\right) - \tfrac{1}{2}\sin(\pi - 2\alpha)\right\} + \frac{1}{g_o}\left(\alpha + \tfrac{1}{2}\sin 2\alpha\right) \right]$$

$$\therefore C_q = \frac{\text{Smooth rotor flux}}{\text{Quadrature axis flux}} = \frac{\phi_m}{\phi_q} = \frac{B_m}{B_q}$$

$$\therefore C_q = \frac{\frac{1}{g}}{\frac{2}{\pi}\left[\frac{1}{g_o}\left(\alpha + \frac{1}{2}\sin 2\alpha\right) + \frac{1}{g}\left\{\left(\pi/2 - \alpha\right) - \frac{1}{2}\sin 2\alpha\right\}\right]}$$

Now, putting the values from fig.(A_1), we can calculate-

$C_d = 1.04$ when $2\alpha = 49^\circ.5$

$g = 0.016$ inch.

$g_o = 0.312$ in

and $C_q = 1.90$

APPENDIX - III

COMPUTER PROGRAMMING FOR TRANSIENT STUDY:

```
C C SOLN OF DIFF EQN FOR TRANS COND TK CHATTERJEE EED UOR
DIMENSION Z(6), PD(6),Q(6),F(5,5)
DIMENSION AX(4), CX(4),H(6), BX(4)
PUNCH 10
10 FORMAT (5HTHETA,12X,2HWR,10X,2HI1,10X,3HID2,9X,3HIQ2, 9X,4HTIME)
PI=22.0/7.0
W =2.0*PI*50.0
TL=0
TB=0.
VI= 230.0*1.414
PU=3.5*(10.**(-3))
RI=12.1
RD2=10.5
RQ2=8.0
AL11=1.38
ALD2=1.42
ALQ2=1.41
HMD=520.0/W
HMQ=370.0/W
READ1, THETA
1 FORMAT(F10.2)
DO 2 I=1,6
Z(I)=0.0
PD(I)=0.0
Q(I)=0.0
2 CONTINUE
Z(1)=THETA*PI/180.0
T=0.0
TS=0.0005
```

```
PD(6)=1.0
AN=1.
HMT=1.0*Z(1)
F(1,1)=AL11
F(2,2)=ALD2
F(3,3)=ALQ2
5 F(1,2)=HMD*COSF(HMT)
F(2,1)=F(1,2)
F(1,3)=-HMQ*SINF(HMT)
F(3,1)=F(1,3)
F(2,3)=0.0
F(3,2)=0.0
FF=VI*SINF(W*T)+F(1,3)*Z(2)*Z(4)
F(1,4)=FF-F(1,2)*Z(2)*Z(5)-R1*Z(3)
F(2,4)=F(1,3)*Z(2)*Z(3)-RD2*Z(4)
F(3,4)=F(1,2)*Z(2)*Z(3)-RQ2*Z(5)
CALL SOLEQN(F)(3,4))
DO 3 I=1,3
3 PD(I+2)=F(I,4)
PD(1)=Z(2)
TE=-Z(3)*Z(4)*F(1,3)+Z(3)*Z(5)*F(1,2)
PD(2)=TE/PJ
AX(1)=0.5
AX(2)=1.0-SQRTF(0.5)
AX(3)=1.0+SQRTF(0,5)
AX(4)=1.0/6.0
BX(1)=2.0
BX(2)=1.0
BX(3)=1.0
```

```
BX(4)=2.0
CX(1)=0.5
CX(2)=AX(2)
CX(3)=AX(3)
CX(4)=0.5
DO 4 I=1,6
DO 4 J=1,4
H(I)=TS*PD(I)
R=AX(J)*(H(I)-BX(J)*Q(I))
Z(I)=Z(I)+R
Q(I)=Q(I)+3.0*R-CX(J)*H(I)
4 CONTINUE
PUNCH6,(Z(I),I=1,6)
6 FORMAT(6E12.6)
GO TO 5
STOP
END
```

APPENDIX - IV

DESIGN CONSIDERATION:

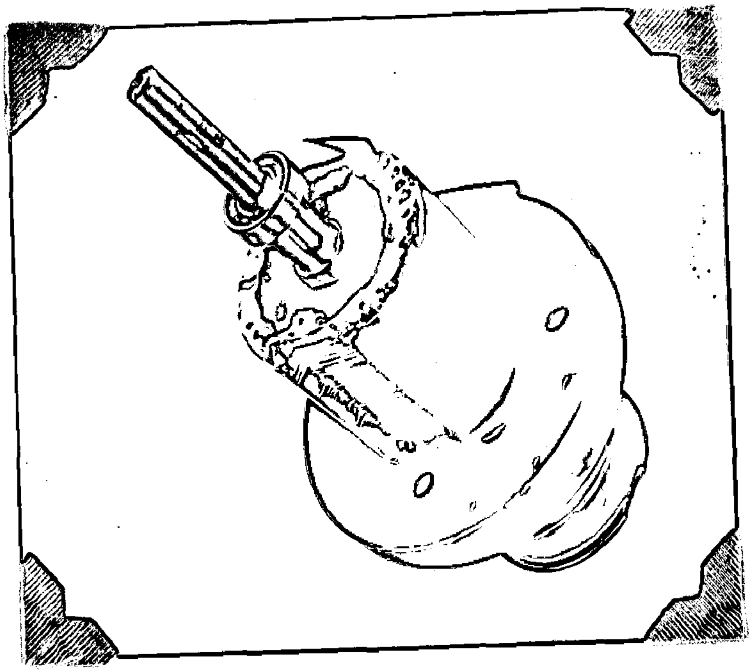
The problem of design of any rotating machine can be considered in two parts. The first part to calculate the performance of the machine from the knowledge of the machine geometry and winding details. This is normally done by calculating certain basic machine constants and then applying the methods of machine analysis to calculate the performance. The second part of the design problem is to determine the machine geometry and winding details from the performance specifications. This has to be more or less always tackled by trial and error method. Since a large number of solutions are possible in this case, a certain amount of experience is necessary to arrive at a satisfactory solution without spending excessive amount of time.

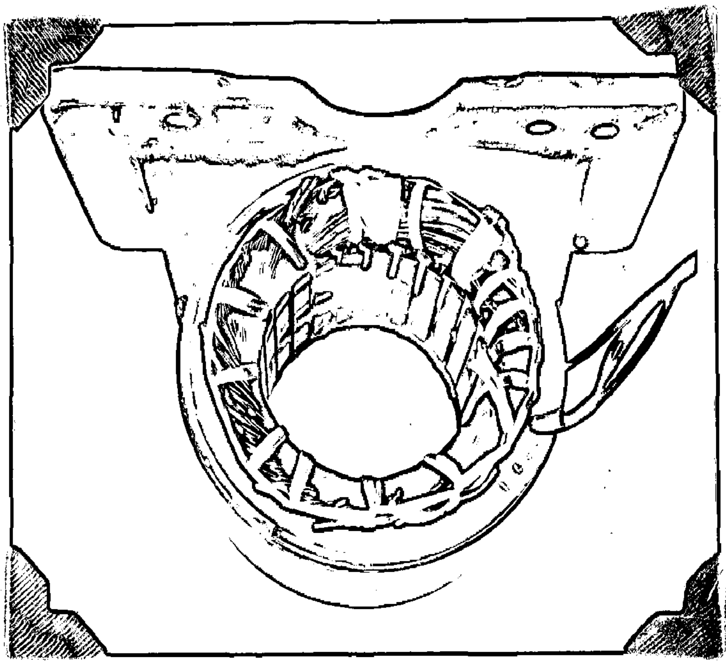
It may be added that this machine has not been commercially manufactured uptil now and hence large amount of experimental data is not available to verify the methods presented here. This study is presented here simply as a begining and many assumptions, formulae etc. can be modified later in the light of experimental results. In fact, the experimental method of determining machine constants itself can be improved.

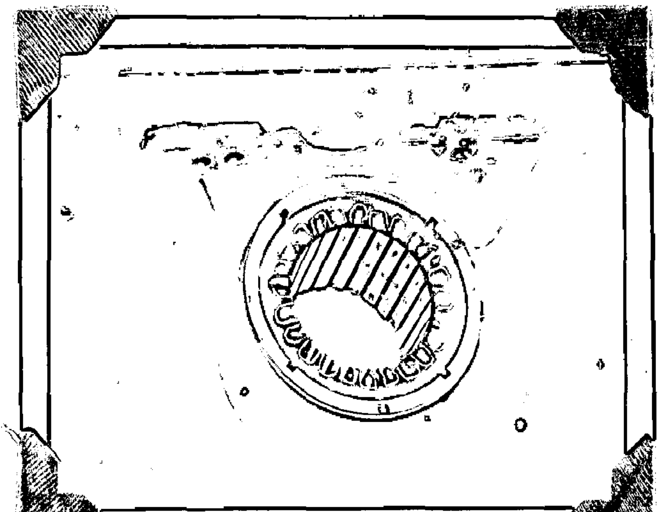
The following procedure as followed by the author may be suggested tentatively, but will be modified in light of further experience. Familiarity with design of normal types of single phase motors is assumed.

1) The frame size and main dimensions can be kept same as far a split-phase machine.

2) The stator stamping can be selectedas for a normal motor. Since no auxiliary winding is needed, it would be







desirable to omit slots normally used for auxiliary winding and fill the slot spaces.

3) The stator winding can be designed as for normal motors to secure a good m.m.f. wave shape.

4) The air-gap should be kept as large as possible to reduce undesirable effects of harmonics.

5) The rotor stamping can be selected as for normal motors. The end ring section can be selected to give about 10 to 15 percent of rotor resistance. Closed slots are desirable in reducing troubles due to permeance harmonics, although it will contribute to higher leakage.

6) Skewing is a considered absolutely essential.

7) The amount of rotor dissymmetry shall be kept as low as 10 percent or so.

The design can be completed following the usual trial and error method, to achieve the desired performance.

Principal Design data: $\frac{1}{2}$ h.p. 230 V. single phase, 50 c/s

Stator frame:

No. of slots = 24

Tooth pitch = $\frac{23}{32}$ in

Pole pitch = $5\frac{31}{32}$ in

Outside dia = 6 in.

Bore dia = $3\frac{11}{16}$ in.

Stack length = 3 in.

Material - Special Lohys (0.020" thick) steel manufactured by M/s. Sankey Electrical Stampings Ltd.

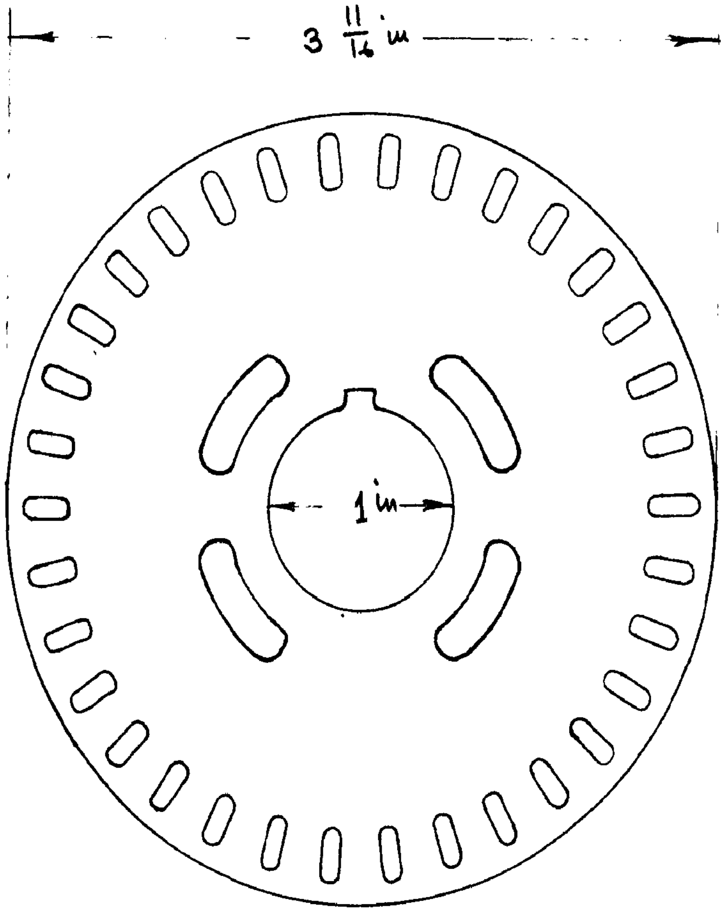


Fig. A2

Original Rotor Stampings

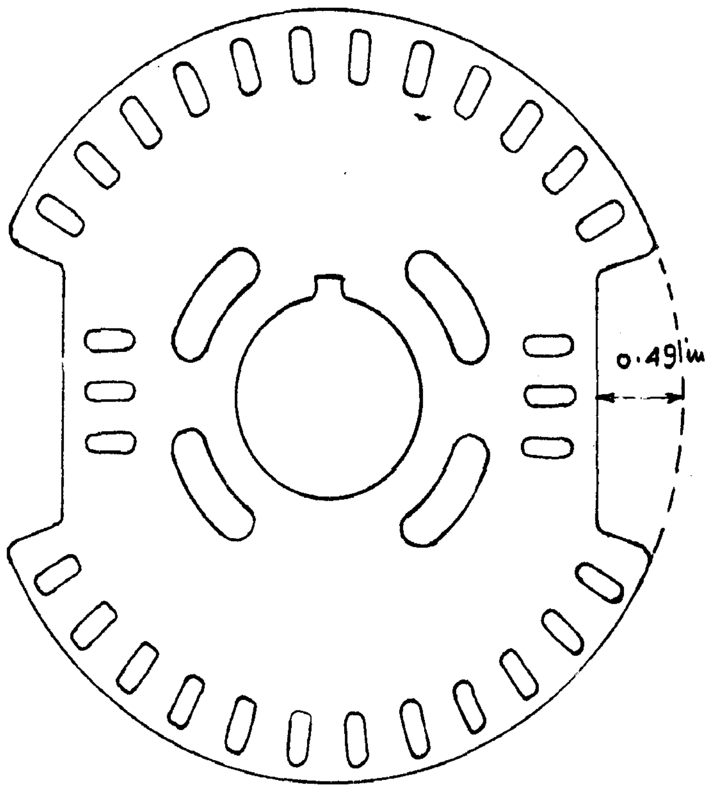


Fig. A_3

Rotor Stamping designed & Used

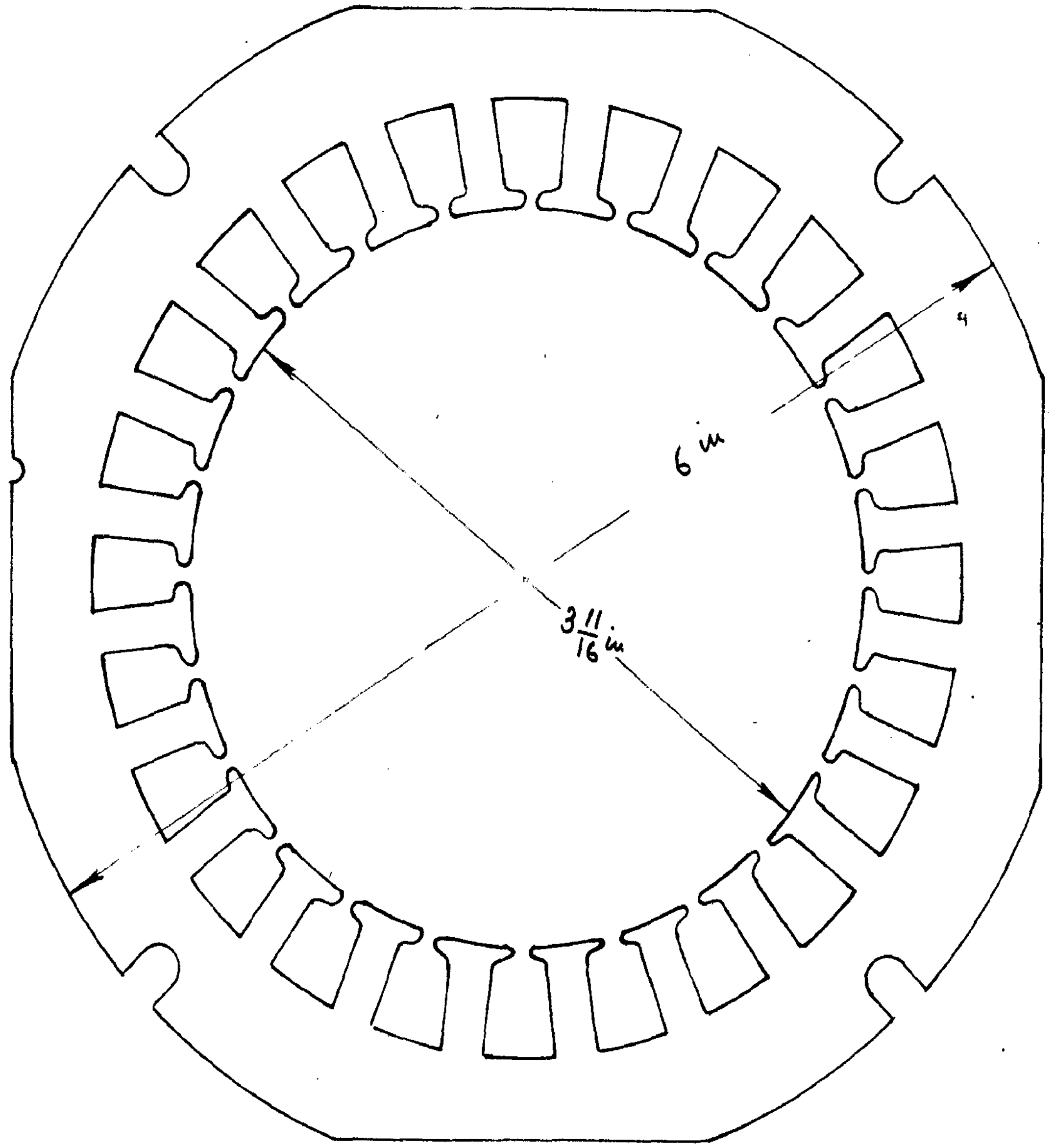


Fig. A_4

. Stator Stamping used

Stator Windings:

No. of coils =24

frequency = 50 c/s.

Coil pitch = 9

No.of poles = 2

No.of turns per coil = 22

No.of turns in series= 528

Each coil of 2 strands of 22 S.W.G. super enamelled copper wire.

Double layer resistance at 25°C = 12.1 ohms.

Stator leakage reactance = 15.4 ohms.

Rotor Stamping:

Out dia = 3 $\frac{11}{16}$ in. (unfinished)

Air-gap = 0.016 in.

Bore dia = 1 in

No.of slots (unshaped) = 36

No.of slots (After shaping) = 30

Stack length = 2 $\frac{63}{64}$ in.

Material - Special Lohy's (0.020" thick) steel manufactured by M/s. Sankey Electrical Stampings Ltd.