DESIGN OF SOLID ROTOR SINGLE PHASE INDUCTION MOTORS

A Dissertation
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CERTIFICATE

Certified that the dissertation entitled " Design of Solid Rotor Single Phase Induction Motors" which is being submitted by Shri MADAN [illegible] DWIVEDI in partial fulfilment for the award of the Degree of Master of Engineering in Electrical Machine Design of University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma. This is further to certify that he has worked for about eight months from December 1966 to July 1967 in preparing this thesis for Master of Engineering.

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CONTENTS

SYNOPSIS

Single phase, capacitor run, induction motor with a solid rotor has been investigated in detail and an analysis is attempted on the basis of the rigorous solution of the pertinent electromagnetic field problem. The important performance parameters have been derived along with the equivalent circuit. The considerations for the design of the auxiliary winding capacitor and the choice of the flux density have been stated and the procedure broadly indicated.

1. INTRODUCTION

Polyphase induction motor with solid rotor has been analysed in sufficient detail in recent years[1-5]. Apart from the analysis and performance evaluation, certain design outlines have also been indicated. These investigations have shown that the solid rotor induction motor in the polyphase case, does have a performance comparable to that of the conventional induction motor. The efficiency is reasonably high though not quite as high as in the conventional case. Other characteristics are more or less the same though the rotor temperature rise is considerable, consequently its application is rather limited. The idea behind the use of solid rotor is largely to elliminate the laminated rotor construction and economise on the rotor cost. The ruggedness of the solid rotor can also be a consideration of secondary importance.

One important area, where the use of solid rotor may be of appreciable practical importance, is the field of single phase motors. Single phase induction motors are normally in the fractional horse power range and used largely for domestic appliances suchas fans, mixers etc. For such low output machines the efficiency of the device is not an overriding consideration, and the cost factor assumes appreciable importance. Further, the ruggedness has also to be considered. This possibility of the application of the solid rotor in single phase motors has received little attention so far and it is here that the cost reduction may evantually over weigh the lowered efficiency criterion and make the proposition considerably more feasible.

In the material to be presented here, a single phase capacitor run solid rotor induction motor has been considered. This has been chosen since by and large most domestic fans are of this type. The derivation of performance equations and the equivalent circuit is based on the rigorous solution of Maxwell's equations for the linear case with due consideration having been given to the hysteresis in the rotor material by assuming a complex permeability.

2. ANALYSIS

2.1 Boundary Conditions:

Consider the developed view of the induction motor as shown in Fig. 1. Both the rotor and the stator surfaces have been considered smooth with a uniform air gap in between.

Assume two co-ordinate systems x, y, z and x', y', z' fixed to the stator and rotor, respectively. The active length of the induction motor is very large compared to its air gap, and hence z and z' co-ordinates are ignorable and the problem reduces to a two dimensional one. Further, the air gap is very small when compared to its radius, and therefore the annular gap region can be transformed into a rectangular region without appreciable error.

If the rotor is rotating at an angular speed w_r, the relationship between the two co-ordinate systems is given by

$$\omega x = x' + {}_{r}\frac{T}{\pi} t \qquad (2.1)$$

and

$$y = y' \qquad (2.2)$$

where,

ω_r = Angular velocity of the rotor in radians per second.

T = Pole pitch in meters.

t = Time in seconds.

The air gap region, shown as region I in Fig. 1, is a current free region, and hence laplace Equation for the

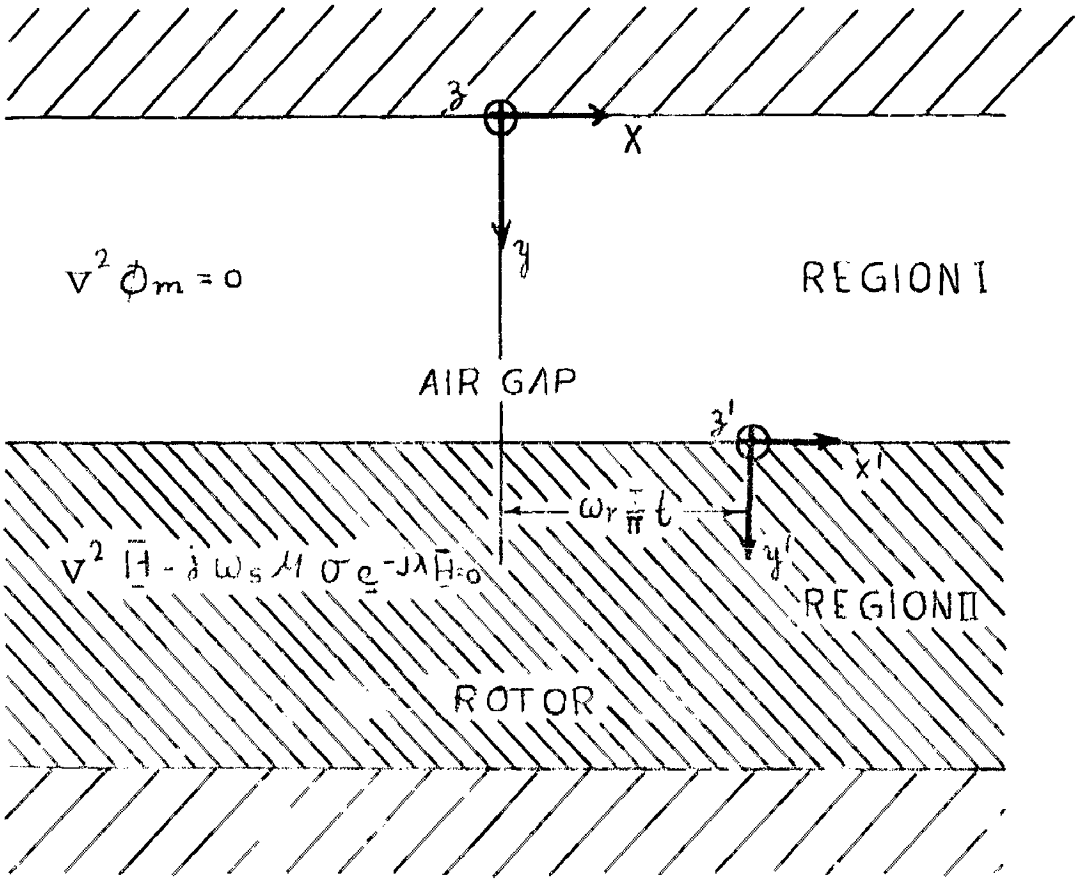

z
X
y
∇² φm = 0
REGION I
AIR GAP
z'
x'
ωr τ/π t
y'
∇² H - j ωs μ σ e^-jλ H = 0
REGION II
ROTOR

FIG 1

magnetic scalar potential is satisfied, which is given by

$$\nabla^2 \varphi_m = 0 \tag{2.3}$$

The rotor shown as region II in Fig. 1, is subjected to a harmonicaly time varying magnetic field, and hence satisfies the diffusion equation given by

$$\nabla^2 \bar{\underline{H}} - j\omega_s \mu \sigma \bar{\underline{H}} = 0 \tag{2.4}$$

where,

H = Magnetising force in amperes per meter.

ω_s = Slip frequency of the rotor in radians per second.

μ = Permeability of the rotor in heneries per meter.

σ = Conductivity of the rotor in mhos per meter.

A harmonically time varying magnetizing force gives rise to a magnetic field vector $\bar{\underline{B}}$ which also varies harmonically with time and is in time phase with the $\bar{\underline{H}}$ vector provided the magnetisation characteristic is linear. Since the rotor is made of magnetic material e.g. mild steel, due to the hysteresis effect the magnetisation characteristic would be non-linear and consequently the magnetising force would give rise to a magnetic field which contains harmonics apart from the fundamental component. Further the fundamental component of the magnetic field vector lags the magnetizing force by an angle λ , which is known as hysteresis angle. In what follows, the harmonics produced due to non-linear nature of the magnetisation characteristic would be ignored. This would amount to approximating the hysteresis loop by an ellipse and the problem would become a linear one. Under such conditions the diffusion equation (2.4) takes the form

$$\nabla^2 \bar{\underline{H}} - j\omega_s \mu \sigma \exp(-j\lambda)\, \bar{\underline{H}} = 0 \qquad (2.5)$$

Eqs.(2.3) and (2.5) now require a set of suitable boundary conditions for the complete solution.

Consider the auxiliary and main windings on the stator as sinusoidally distributed in space and this again amounts to ignoring the harmonics in the mmf waves produced by the two windings. The two windings are in space quadrature and the same voltage is applied to the two windings. The windings will carry sinusoidal currents of supply frequency, and evidently the current in the auxiliary winding will lead the current in the main winding by an angle θ due to the presence of capacitor in the auxiliary winding. Thus, the mmf at the stator core-air gap boundary can be expressed as

$$H_{x1}\Big|_{y=0} = 2\,A \cos\frac{\pi}{T}x \sin\omega t - 2\,A' \sin\frac{\pi}{T}x \sin(\omega t+\theta) \qquad (2.6)$$

where

2 A = Magnitude of the mmf produced by main winding.

and

2 A'= Magnitude of the mmf produced by auxiliary winding.

On expanding and rearranging the terms, the above equation yields,

$$\begin{aligned} H_{x1}\Big|_{y=0} = {} & (A + A'\sin\theta)\sin(\omega t - \tfrac{\pi}{T}x) - A'\cos\theta\cos(\omega t - \tfrac{\pi}{T}x) \\ & + (A - A'\sin\theta)\sin(\omega t + \tfrac{\pi}{T}x) \\ & + A'\cos\theta\cos(\omega t + \tfrac{\pi}{T}x) \end{aligned} \qquad (2.7)$$

Substituting,

$$\left.\begin{aligned} (A + A' \sin\theta) &= A_1 \cos\delta_1 \\ A' \cos\theta &= A_1 \sin\delta_1 \end{aligned}\right] \quad (2.8)$$

and

$$\left.\begin{aligned} (A - A' \sin\theta) &= A_2 \cos\delta_2 \\ A' \cos\theta &= A_2 \sin\delta_2 \end{aligned}\right] \quad (2.9)$$

in equation (2.7), one obtains

$$\begin{aligned} H_{x_1}\Big|_{y=0} &= A_1 \sin(\omega t - \tfrac{\pi}{T} x - \delta_1) \\ &\quad + A_2 \sin(\omega t + \tfrac{\pi}{T} x + \delta_2) \\ &= A_1 I_m \exp.\, j(\omega t - \tfrac{\pi}{T} x - \delta_1) \\ &\quad + A_2 I_m (\omega t + \tfrac{\pi}{T} x + \delta_2) \end{aligned} \quad (2.10)$$

where

$$\left.\begin{aligned} A_1 &= \left[A^2 + A'^2 + 2AA' \sin\theta\right]^{\frac{1}{2}} \\ \delta_1 &= \tan^{-1} \frac{A' \cos\theta}{(A+A' \sin\theta)} \end{aligned}\right] \quad (2.11)$$

and

$$\left.\begin{aligned} A_2 &= \left[A^2 + A'^2 - 2AA' \sin\theta\right]^{\frac{1}{2}} \\ \delta_2 &= \tan^{-1} \frac{A' \cos\theta}{(A - A' \sin\theta)} \end{aligned}\right] \quad (2.12)$$

It will be recognized that the first term of Equation (2.10) represents the forward field and the second term represents the backward field of the conventional, revolving field theory.

At the air gap-rotor boundary, the boundary conditions are :

$$H_{x_1}\Big|_{y=g} = H_{x_2}\Big|_{y=g} \qquad (2.13)$$

and

$$\mu_o H_{y1}\Big|_{y=g} = \mu H_{y_2}\Big|_{y=g} \qquad (2.14)$$

where,

g = Air gap length in meters.

Solutions of equations (2.3) and (2.5) will be obtained separately for forward and backward fields with the help of boundary conditions defined by equations (2.10), (2.13) and (2.14).

2.2 Forward Field:

The equation (2.3) in the differential form is written as,

$$\frac{\partial^2 \varphi_m}{\partial x^2} + \frac{\partial^2 \varphi_m}{\partial y^2} = 0 \qquad (2.15)$$

The solution of the equation (2.15) is obtained by the method of separation of variables, and is given by

$$\varphi_m = \exp.\ j(\beta_1 - \alpha_x) \left[c_1' \exp.(\alpha_y) + c_2' \exp.(-\alpha_y)\right] \qquad (2.16)$$

where c_1', c_2', β_1 and α are some constants.

Since,

$$\underline{H} = -\nabla \phi_m \tag{2.17}$$

$$\underline{H}_{x1f} = \frac{-\partial\phi_m}{\partial x}, \text{ and } H_{y1f} = \frac{-\partial\phi_m}{\partial y} \tag{2.18}$$

where the subscript f denotes the field quantities for the forward field.

From equations (2.16) and (2.18), one obtains,

$$H_{x1f} = j\alpha\left[\exp.\ j(\beta_1 - \alpha x)\right]\left[c_1'\ e^{\alpha y} + c_2'\ e^{-\alpha y}\right]$$

$$= \left[\exp.\ j(\beta_1 - \alpha_x)\right]\left[c_1\ e^{\alpha y} + c_2\ e^{-\alpha y}\right] \tag{2.19}$$

where $c_1 = j\alpha c_1'$ and $c_2 = j\alpha c_2'$

$$H_{y1f} = -\left[\exp.\ j(\beta_1 - \alpha x)\right]\left[\alpha\ c_1'\ e^{\alpha y} - \alpha\ c_2'\ e^{-\alpha y}\right]$$

$$= j\left[\exp.j(\beta_1 - \alpha x)\right]\left[c_1\ e^{\alpha y} - c_2\ e^{-\alpha y}\right] \tag{2.20}$$

Equation (2.19) at y = o yields,

$$H_{x1f}\Big|_{y=0} = \left[\exp.\ j\ (\beta_1 - \alpha x)\right]\left[c_1 + c_2\right] \tag{2.21}$$

The boundary condition at y = o is defined by equation (2.10), which reads

$$H_{x1f}\Big|_{y=0} = A_1 \exp.\left[j(wt - \frac{\pi}{T}x - \delta_1\right] \tag{2.22}$$

Comparing equations (2.21) and (2.22), one obtains,

$$\left.\begin{aligned} \beta_1 &= (wt - \delta_1) \\ \alpha &= \frac{\pi}{T} \\ \text{and} \quad c_1 + c_2 &= A_1 \end{aligned}\right] \tag{2.23}$$

Thus, the field distribution in the air gap region is given by,

$$H_{x1f} = \left[\exp. j(wt - \alpha x - \delta_1)\right]\left[c_1\, e^{\alpha y} + c_2\, e^{-\alpha y}\right] \tag{2.24}$$

$$H_{y1f} = j\left[\exp.\ j(wt - \alpha x - \delta_1)\right]\left[c_1\, e^{\alpha y} - c_2\, e^{-\alpha y}\right] \tag{2.25}$$

For the rotor Eqn.(2.5) can be rewritten as,

$$\nabla^2 \bar{H} - k^2 \bar{H} = 0 \tag{2.26}$$

where

$$k^2 = j\, w_1\, \mu\, \sigma\ \exp.(-j\lambda) \tag{2.27}$$

and $w_1 = w_s$ = Slip frequency of the rotor in radians per second for the forward field.

Expansion of Eqn.(2.26) in terms of its field components yields,

$$\frac{\partial^2 \bar{H}_{x'}}{\partial x'^2} + \frac{\partial^2 \bar{H}_{x'}}{\partial y'^2} - k^2\, \bar{H}_{x'} = 0 \tag{2.28}$$

and

$$\frac{\partial^2 \bar{H}_{y'}}{\partial x'^2} + \frac{\partial^2 \bar{H}_{y'}}{\partial y'^2} - k^2 \bar{H}_{y'} = 0 \qquad (2.29)$$

Let the magnetizing force in region II for the forward field be denoted by $H_{x'2f}$ and let this be denoted by,

$$\bar{H}_{x'2f} = \bar{H}_1(x') \bar{H}_2(y') \qquad (2.30)$$

where $H_1(x')$ and $H_2(y')$ are functions of only x', and y' co-ordinates, respectively.

Substituting the partial derivatives from Eqn.(2.30) in Eqn.(2.28) and rearranging the terms, one obtains,

$$\frac{1}{\bar{H}_1} \frac{d^2 \bar{H}_1}{d x'^2} + \frac{1}{\bar{H}_2} \frac{d^2 H_2}{d y'^2} - k^2 = 0 \qquad (2.31)$$

It is evident that the first and second terms in the above equation are constants, and let the constants be denoted by $-\alpha_1^2$ and α_2^2, respectively,

Thus,

$$\frac{1}{\bar{H}_1} \frac{d^2 \bar{H}_1}{d x'^2} = -\alpha_1^2 \qquad (2.32)$$

$$\frac{1}{H_2} \frac{d^2 \bar{H}_2}{d y'^2} = \alpha_2^2 \qquad (2.33)$$

and

$$\alpha_2^2 = \alpha_1^2 + k^2 \qquad (2.34)$$

Substitution of the solutions of Eqns.(2.32) and (2.33) in Eqn.(2.30) yields for the forward field

$$\bar{H}_{x'2f} = \left[D_1' \exp.(-j\,\alpha_1\,x') + D_2' \exp.(j\,\alpha_1 x')\right] \times \left[D_3' \exp.(\alpha_2 y) + D_4' \exp.(-\,\alpha_2 y)\right] \tag{2.35}$$

where D_1', D_2', D_3', and D_4' are some constants.

The skin depth for mold steel is considerably small compared to the radius of the rotor, and hence the field quantities attenuate to an insignificant value at $y = (r + g)$, where r is the radius of the rotor. In view of the above, the region II can be assumed to approach infinity without any appreciable error. In order that the fields may be bounded in region II, it is necessary that

$$D_3' = 0.$$

Since $D_3' = 0$, Eqn.(2.35) can be rewritten as,

$$\bar{H}_{x'2f} = \left[D_1 \exp.(-j\,\alpha_1 x') + D_2 \exp.(j\,\alpha_1 x')\right] \times \left[\exp.(-\,\alpha_2 y\,)\right] \tag{2.36}$$

where D_1 and D_2 are some other constants.

A sinusoidal time variation of the field quantities in region II has been assumed, and its inclusion in Eqn.(2.36) yields,

$$\bar{H}_{x'2f} = \left[D_1 \exp.j(w_1 t - \alpha_1 x') + D_2 \exp.j(w_1 t + \alpha_1 x')\right]\left[\exp.(-\alpha_2 y)\right] \qquad (2.37)$$

Substitution of $y = g$ in the Eqn.(2.37) results in,

$$H_{x'2f}\Big|_{y=g} = \left[D_1 \exp.j(w_1 t - \alpha_1 x') + D_2 \exp.j(w_1 t + \alpha_1 x')\right]\left[\exp.(-\alpha_2 g)\right] \qquad (2.38)$$

Further, substitution of $y = g$ in Eqn.(2.24) yields,

$$H_{x1f}\Big|_{y=g} = \left[\exp.j(wt - \alpha x - \delta_1)\right] \times \left[c_1 \exp.(\alpha g) + c_2 \exp.(-\alpha g)\right] \qquad (2.39)$$

The above equation when referred to rotor co-ordinates system with the help of Eqn.(2.1) becomes,

$$H_{x'1f}\Big|_{y=g} = \left[\exp.j\left[(w - w_r)\, t - \alpha_1 x' - \delta_1\right]\right] \times \left[c_1 \exp.(\alpha g) + c_2 \exp.(-\alpha g)\right] \qquad (2.40)$$

Equating Eqn.(2.38) to Eqn.(2.40), in order to satisfy the boundary condition (2.13), one obtains,

$$D_2 = 0,\ w_1 = (w - w_r),\ \alpha_1 = \alpha, \text{ and}$$

$$\left[c_1 \exp.(\alpha g) + c_2 \exp.(-\alpha g)\right]\left[\exp.(-j\delta_1)\right] = D_1 \left[\exp.(-\alpha_2 g)\right] \qquad (2.41)$$

Thus,

$$H_{x'2f} = D_1 \left[\exp.(-j\,\alpha x')\right]\left[\exp.(-\alpha_2 y')\right] \quad (2.42)$$

When the time variation is shown explicitly, one may rewrite the above as,

$$H_{x'2f} = D_1 \left[\exp.j(w_1 t - \alpha x')\right]\left[\exp.(-\alpha_2 y')\right] \quad (2.43)$$

Since,

$\nabla \cdot \underline{B} = 0$, and consequently $\nabla \cdot \underline{H} = 0$, one obtains,

$$H_{y'2f} = -\int \frac{\partial H_{x'2f}}{\partial x'}\, dy' \quad (2.44)$$

Substituting the partial derivative of $H_{x'2f}$ with respect to x' from Eqn.(2.42) in Eqn.(2.44) and integrating, one obtains,

$$H_{y'2f} = -j\,\frac{D_1 \alpha}{\alpha_2}\left[\exp.(-j\,\alpha x')\right]\left[\exp.(-\alpha_2 y')\right] \quad (2.45)$$

Showing the time variation explicitly in Eqn.(2.45) yields,

$$H_{y'2f} = \frac{-j\,D_1 \alpha}{\alpha_2}\left[\exp.j(w_1 t - \alpha x')\right] \times \left[\exp.(-\alpha_2 y')\right] \quad (2.46)$$

At y = g, Eqn.(2.46) yields,

$$H_{y'2f}\Big|_{y=g} = \frac{-j\,D_1 \alpha}{\alpha_2}\left[\exp.j(w_1 t - \alpha x')\right] \times \left[\exp.(-\alpha_2 g)\right] \quad (2.47)$$

Eqn.(2.25) for $y = g$ can be rewritten as,

$$H_{y1f}\Big|_{y=g} = j\left[\exp.j(wt - \alpha x - \delta_1)\right] \times \left[c_1 \exp.(\alpha g) - c_2 \exp.(-\alpha g)\right] \quad (2.48)$$

In terms of the rotor co-ordinate system, the above equation may further be express as,

$$H_{y'1f}\Big|_{y=g} = j\left[\exp.\left[(w - w_r)t - \alpha x' - \delta_1\right]\right] \times \left[c_1 \exp.(\alpha g) - c_2 \exp.(-\alpha g)\right] \quad (2.49)$$

The application of boundary condition (2.14) to Eqns.(2.47) and (2.49) yields,

$$j\mu_o\left[\exp.\left[(w - w_r)t - \alpha x' - \delta_1\right]\right] \times \left[c_1 \exp.(\alpha g) - c_2 \exp.(-\alpha g)\right] = -\frac{j\mu_o \mu_r D_1 \alpha}{\alpha_2}\left[\exp.j(w_1 t - \alpha x')\right] \times \left[\exp.(-\alpha_2 g)\right] \quad (2.50)$$

It follows from the above equation that,

$$\left[c_1 \exp.(\alpha g) - c_2 \exp.(-\alpha g)\right]\left[\exp.(-j\,\delta_1)\right] = -\frac{D_1 \alpha \mu_r}{\alpha_2}\left[\exp.\ (-\alpha_2 g)\right] \tag{2.51}$$

Constants c_1, c_2, and D_1 are related to each other and A_1 by the Eqs. (2.23), (2.41), and (2.51) and are rewritten here for convenience.

$$c_1 + c_2 = A_1 \tag{2.23}$$

$$\left[c_1 \exp.(\alpha g) + c_2 \exp.(-\alpha g)\right]\left[\exp.(-j\,\delta_1)\right] = D_1 \left[\exp.\ (-\alpha_2 g)\right] \tag{2.41}$$

$$\left[c_1 \exp.(\alpha g) - c_2 \exp.(-\alpha g)\right]\left[\exp.\ (-j\,\delta_1)\right] = -\frac{D_1 \alpha\ \mu_r}{\alpha_2}\left[\exp.\ (-\alpha_2 g)\right] \tag{2.51}$$

The solution of the above three simultaneous equations yields the values of the constants as,

$$c_1 = \frac{A_1}{2} \cdot \frac{\left[\alpha_2 - \alpha \mu_r\right]\left[\exp.(-\alpha g)\right]}{\alpha_2 \cosh \alpha g + \alpha \mu_r \sinh \alpha g} \tag{2.52}$$

$$c_2 = \frac{A_1}{2} \cdot \frac{\left[\alpha_2 + \alpha \mu_r\right]\left[\exp.\ (\alpha g)\right]}{\alpha_2 \cosh \alpha g + \alpha\ \mu_r \sinh \alpha g} \tag{2.53}$$

$$D_1 = A_1 \cdot \frac{\alpha_2}{\alpha_2 \cosh \alpha g + \alpha \mu_r \sinh \alpha g}$$

$$\times \left[\exp. (\alpha_2 g - j \delta_1)\right] \qquad (2.54)$$

Thus, the field quantities in the two regions are given by,

$$H_{x1f} = \left[\exp. j \ (wt - \alpha x - \delta_1)\right]$$

$$\times \left[c_1 \exp. (\alpha y) + c_2 \exp. (- \alpha y)\right] \qquad (2.55)$$

$$H_{y1f} = j \left[\exp. j (wt - \alpha x - \delta_1)\right]$$

$$\times \left[c_1 \exp. (\alpha y) - c_2 \exp. (- \alpha y)\right] \qquad (2.56)$$

$$H_{x'2f} = D_1 \left[\exp. \ j (w_1 t - \alpha x')\right]$$

$$\times \left[\exp. \ (- \alpha_2 \ y')\right] \qquad (2.57)$$

$$H_{y'2f} = \frac{-j \ D_1 \ \alpha}{\alpha_2} \left[\exp. \ j (w_1 t - \alpha x')\right]$$

$$\times \left[\exp. (- \alpha_2 y')\right] \qquad (2.58)$$

where the constants c_1 c_2 and D_1 are defined by Eqns.(2.52), (2.53) and (2.54) respectively, Further, the constant α_2 defined by Eqn.(2.34) is rewritten here for convenience,

$$\alpha_2^2 = \alpha^2 + k^2 \qquad (2.34)$$

But $k^2 = jw_1 \mu \sigma \; e^{-j\lambda}$, and hence,

$$\alpha_2 = \left[\alpha^2 + j\, w_1 \,\mu\, \sigma\, e^{-j\lambda}\right]^{\frac{1}{2}} \qquad (2.59)$$

Let the real and imaginary parts of the constant α_2 be denoted by a and b, respectively. Writing α_2 as (a+jb) in Eqn.(2.59), one obtains,

$$(a+jb) = \left[\alpha^2 + j\, w_1 \,\mu\, \sigma\, e^{-j\lambda}\right]^{\frac{1}{2}}$$

$$= \left[(\alpha^2 + w_1 \mu\, \sigma \sinh\lambda) + jw_1 \mu\, \sigma \cos\lambda\right]^{\frac{1}{2}}$$

(2.60)

Taking the square of the sides of the above equation and equating the real and imaginary parts, one obtains,

$$a^2 - b^2 = (\alpha^2 + w_1 \,\mu\, \sigma \sin\lambda) \qquad (2.61)$$

and $\qquad 2ab = w_1 \mu\, \sigma \cos\lambda \qquad (2.62)$

Solution of the Eqns. (2.61) and (2.62) for a and b yields,

$$a = \left[\frac{1}{2}\left[\alpha^4 + 2w_1 \,\mu\, \sigma \alpha^2 \sin\lambda + (w_1 \mu\, \sigma)^2\right]^{\frac{1}{2}} + \frac{1}{2}(\alpha^2 + w_1 \mu\, \sigma \sin\lambda)\right]^{\frac{1}{2}} \qquad (2.63)$$

and

$$(b) = \left[\frac{1}{2} \left[\alpha^4 + 2\, w_1 \mu \sigma \alpha^2 \sin \; + (w_1 \mu \sigma)^2 \right]^{\frac{1}{2}} - \frac{1}{2} (\alpha^2 + w_1 \mu \sigma \sin\lambda) \right]^{\frac{1}{2}} \qquad (2.64)$$

2.3 Backward Field:

Proceeding in a similar way, one obtains the field distribution in region I and II as,

$$H_{x1b} = \left[\exp.j(wt + \alpha x + \delta_2 \right] \times \left[c_1' \exp.(\alpha y) + c_2' \exp.(-\alpha y) \right] \qquad (2.65)$$

$$H_{y1b} = -j \left[\exp.\ j(wt + \alpha x + \delta_2 \right] \times \left[c_1' \exp.(\alpha y) - c_2' \exp.(-\alpha y) \right] \qquad (2.66)$$

$$H_{x'2b} = D_1' \left[\exp.j(w_2 t + \alpha x') \right] \times \left[\exp.(-\alpha_2' y') \right] \qquad (2.67)$$

$$H_{y'2b} = \frac{j\, \alpha D_1'}{\alpha_2'} \left[\exp.(w_2 t + \alpha x') \right] \left[\exp.(-\alpha_2' y') \right] \qquad (2.68)$$

where the subscript b means that the field quantities are partaining to the backward rotating field and w_2 is the slip frequency for the backward field.

The constants c_1', c_2', and D_1' are given by,

$$c_1' = \frac{A_2}{2} \cdot \frac{\left[\alpha_2' - \alpha/\mu_r\right]\left[\exp.(-\alpha g)\right]}{\left[\alpha_2' \cosh \alpha g + \alpha/\mu_r \sinh \alpha g\right]} \tag{2.69}$$

$$c_2' = \frac{A_2}{2} \cdot \frac{\left[\alpha_2' + \alpha/\mu_r\right]\left[\exp.(\alpha g)\right]}{\left[\alpha_2' \cosh \alpha g + \alpha/u_r \sinh \alpha g\right]} \tag{2.70}$$

$$D_1' = A_2 \cdot \frac{\alpha_2'}{\left[\alpha_2' \cosh \alpha g + \alpha/\mu_r \sinh \alpha g\right]} \times \left[\exp.\ (\alpha_2' g + j\,\delta_2)\right] \tag{2.71}$$

Further, the constant α_2' is given by,

$$\alpha_2' = \left[\alpha^2 + j\, w_2 \mu\, \sigma e^{-j\lambda}\right]^{\frac{1}{2}} \tag{2.72}$$

If the real and imaginary parts of the constants α_2' are denoted by a' and b' respectively, one obtains,

$$a'^2 - b'^2 = \alpha^2 + w_2 \mu\, \sigma \sin\lambda \tag{2.73}$$

$$2a'b' = w_2\, \mu\, \sigma \cos\lambda \tag{2.74}$$

$$a' = \left[\frac{1}{2} \left[\alpha^4 + 2 \, w_2 \, \mu \, \sigma \, \alpha^2 \sin\lambda + (w_2 \, \mu \, \sigma)^2 \right]^{\frac{1}{2}} + \frac{1}{2} \left(\alpha^2 + w_2 \, \mu \, \sigma \sin\lambda \right) \right]^{\frac{1}{2}} \qquad (2.75)$$

$$b' = \left[\frac{1}{2} \left[\alpha^4 + 2 \, w_2 \, \mu \, \sigma \, \alpha^2 \sin\lambda + (w_2 \, \mu \, \sigma)^2 \right]^{\frac{1}{2}} - \frac{1}{2} \left(\alpha^2 + w_2 \, \mu \, \sigma \sin\lambda \right) \right]^{\frac{1}{2}} \qquad (2.76)$$

3. PERFORMANCE PARAMETERS

3.1 Rotor Power :

The power flow out of a volume bounded by the surface S is given by

$$P = \oint_s \underline{N} . \underline{ds}$$

where $\underline{N}$ is the Poynting Vector, and for harmonically time varying electromagnetic fields is given by,

$$\underline{N} = \text{Re.} \left(\bar{\underline{E}} \times \bar{\underline{H}}^{*} \right)$$

$\underline{E}$ and $\underline{H}$ are the field vectors on the surface $\underline{ds}$.
The average power over a cycle is given by,

$$P = \text{Re} \frac{1}{2} \oint_s \left(\bar{\underline{E}} \times \bar{\underline{H}}^{*} \right) . \underline{ds} \qquad (3.1)$$

Consider the region I, use is made of Maxwell's equation,

$$\nabla \times \bar{\underline{E}} = - \frac{\partial \bar{\underline{B}}}{\partial t} \qquad (3.2)$$

to evaluate the $\underline{D}$ $\underline{E}$ field in the air gap region. Expanding Eqn.(3.2) into its components, and comparing the field quantities along the three axes, one obtains,

$$\left.\begin{aligned}
\frac{\partial \bar{E}_z}{\partial y} - \frac{\partial \bar{E}_y}{\partial z} &= - \mu_o \frac{\partial \bar{H}_x}{\partial t} \\
\frac{\partial \bar{E}_x}{\partial z} - \frac{\partial \bar{E}_z}{\partial x} &= - \mu_o \frac{\partial \bar{H}_y}{\partial t} \\
\frac{\partial \bar{E}_y}{\partial x} - \frac{\partial \bar{E}_x}{\partial t} &= - \mu_o \frac{\partial \bar{H}_z}{\partial t}
\end{aligned}\right] \qquad (3.3)$$

It follows directly from Eqn.(3.3) that,

$$\frac{\partial \bar{E}_{zlf}}{\partial y} = - \mu_o \frac{\partial \bar{H}_{xlf}}{\partial t} \quad (3.4)$$

H_{xlf} is given by Eqn.(2.55), and which is rewritten here as

$$\bar{H}_{xlf} = \left[\exp.j(wt - \alpha x - \delta_1)\right]$$

$$\times \left[c_1 \exp.(\alpha y) + c_2 \exp.(-\alpha y)\right] \quad (3.5)$$

From Eqn.(3.4), one obtains,

$$\bar{E}_{zlf} = - \mu_o \int \frac{\partial \bar{H}_{xlf}}{\partial t} dy \quad (3.6)$$

Substitutiön of the partial derivative of $\bar{H}_{xlf}$ with respect to time from Eqn.(3.5) in Eqn.(3.6) and integration results in,

$$\bar{E}_{zlf} = \frac{-j\, w \mu_o}{\alpha} \left[\exp.(-j)(\alpha x + \delta_1)\right]$$

$$\times \left[c_1 \exp.(\alpha y) - c_2 \exp.(-\alpha y)\right] \quad (3.7)$$

Considering per meter length of the rotor, $(\underline{\bar{E}} \times \underline{\bar{H}}) \cdot \underline{ds}$ is given by,

$$(\underline{\bar{E}} \times \underline{\bar{H}}.). \underline{ds} = - (\bar{E}_{zlf} H^*_{xlf}) dx \quad (3.8)$$

Hence,

$$(\underline{\bar{E}} \times \underline{\bar{H}}^*) \cdot \underline{ds} = - (\bar{E}_{zlf} \bar{H}_{xlf}) \quad dx \quad (3.10)$$

Considering a two pole machine, without any loss of generality, and substituting Eqn.(3.10) in Eqn.(3.1), one obtains the expression for power enering the rotor surface as,

$$P_f = \frac{1}{2} \text{ Re } \int_0^{2T} \bar{E}_{z1f} \Big|_{y=g} \bar{H}^*_{x1f} \Big|_{y=g} dx \tag{3.11}$$

$$\bar{E}_{z1f} \Big|_{y=g} = \frac{-j\, w \mu_o}{\alpha} \left[\exp. - j(\alpha x + \delta_1)\right]$$

$$x \left[c_1 \exp.(\alpha g) - c_2 \exp.(-\alpha g)\right] \tag{3.12}$$

Consider the term $\left[c_1 \exp.(\alpha g) - c_2 \exp.(-\alpha g)\right]$ in the above equation. Substitution of the values of c_1 and c_2 from Eqns.(2.52) and (2.53) gives,

$$\left[c_1 \exp.(\alpha g) - c_2 \exp.(-\alpha g)\right]$$

$$= \frac{A_1}{2} \cdot \frac{(\alpha_2 - \alpha/\mu_r) - (\alpha_2 + \alpha/\mu_r)}{\alpha_2 \cosh \alpha g + \alpha/\mu_r \sinh \alpha g}$$

$$= - \frac{A_1 \alpha/\mu_r}{(\alpha_2 \cosh \alpha g + \alpha/\mu_r \sinh \alpha g)} \tag{3.13}$$

As $\alpha g << 1$, approximation of $\cosh \alpha g = 1$ and $\sinh \alpha g = \alpha g$ and substitution of these values in Eqn.(3.13) results in,

$$\left[c_1 \exp.(\alpha g) - c_2 \exp.(-\alpha g)\right]$$

$$= \frac{- A_1 \alpha/\mu_r}{(a + \alpha^2/\mu_r g) + jb} \tag{3.14}$$

Hence,

$$\bar{E}_{zlf}\Big|_{y=g} = \frac{j\,A_1\,w\,\mu}{(a+\alpha^2/\mu_r\,g)+jb}\left[\exp.\ -j(\alpha x+\delta_1)\right] \tag{3.15}$$

Further,

$$\bar{H}_{xlf}\Big|_{y=g} = \left[\exp.(-j)(\alpha x+\delta_1)\right] \times \left[c_1\ \exp.(\alpha g) + c_2\ \exp.(-\alpha g)\right] \tag{3.16}$$

Substituting the values of c_1 and c_2 from Eqns.(2.52) and (2.53) in Eqn.(3.16), and after approximation one obtains,

$$\bar{H}_{xlf}\Big|_{y=g} = A_1 \cdot \frac{(a+jb)}{(a+\alpha^2/\mu_r\,g)+jb} \times \left[\exp.-j(\alpha x+\delta_1)\right] \tag{3.17}$$

It follows from Eqn.(3.17) that $\bar{H}^{*}_{xlf}\Big|_{y=g}$ is given by,

$$\bar{H}^{*}_{xlf}\Big|_{y=g} = A_1\ \frac{(a-jb)}{(a+\alpha^2/\mu_r\,g)-jb} \times \left[\exp.\ j\ (\alpha x+\delta_1)\right] \tag{3.18}$$

Thus,

$$\bar{E}_{zlf}\Big|_{y=g}\ \bar{H}^{*}_{xlf}\Big|_{y=g} = \frac{j\ A_1^2\ w\,\mu\ (a-jb)}{(a+\alpha^2/\mu_r\,g)^2+b^2} \tag{3.19a}$$

and
$$Re\left[\bar{E}_{z1f}\Big|_{y=g}\ \bar{H}^*_{x1f}\Big|_{y=g}\right] = \frac{A_1^2 .\ w/\mu\ b}{(a+\alpha^2/\mu_r\ g)^2+b^2} \tag{3.19b}$$

Substitution of Eqn.(3.19b) in Eqn.(3.11) yields the expression for rotor power pertaining to the forward field as,

$$P_f = \frac{A_1^2\ w\ /\mu\ \ b\ \ T}{(a+\alpha^2/\mu_r\ g)^2 + b^2} \tag{3.20a}$$

and
$$p.f = \frac{b}{\left[a^2 + b^2\right]^{\frac{1}{2}}} \tag{3.20b}$$

Proceeding in a similar way, one obtains the expression for power entering the rotor surface pertaining to backward field a

$$P_b = \frac{A_2^2\ \ w/\mu\ b'\ T}{(a'+\alpha^2/\mu_r\ g)^2 + b'^2} \tag{3.21a}$$

and
$$(p.f.) = \frac{b'}{\left[a'^2 + b'^2\right]^{\frac{1}{2}}} \tag{3.21b}$$

3.2 Rotor Losses:

The power loss in a conducting medium for harmonically varying fields is given by,

$$P_{1f} = \frac{1}{2\sigma} \int_v |\underline{J}|^2\, dv$$

Considering per metre length of the rotor, the above expression reduces to

$$P_{1f} = \frac{1}{2\sigma} \int_0^{2T} \int_g^{\infty} |\underline{J}|^2\, dx'\, dy' \tag{3.22}$$

where $\underline{J}$ is the current density in the rotor and σ is the conductivity of the rotor material.

Since, $\underline{J} = \sigma \underline{E}$ and $\underline{E}$ has only z component at the rotor surface, it follows that $\underline{J}$ has only z component. $\underline{J}$ is given by the relation,

$$\nabla \times \underline{H} = \underline{J}$$

Expansion of the above equation and after comparision of the co-efficients, one obtains,

$$J_{zf} = \frac{\partial H_{y'2f}}{\partial x'} - \frac{\partial H_{x'2f}}{\partial y'} \qquad (3.23)$$

The field distribution in the rotor given by Eqns.(2.57) and (2.58) is rewritten as,

$$H_{x'2f} = D_1 \left[\exp.(-j\,\alpha x')\right]\left[\exp.(-\,\alpha_2 y')\right] \qquad (3.24)$$

$$H_{y'2f} = \frac{-j\,D_1\,\alpha}{\alpha_2}\left[\exp.(-j\,\alpha x')\right]\left[\exp.(-\,\alpha_2 y')\right] \qquad (3.25)$$

Substitution of the partial derivaties of H'_{x2f} and $H_{y'2f}$ w.r.t y' and x', respectively, in Eqn.(3.23) results in,

$$J_{zf} = D_1 \left[\exp.(-j\,\alpha x')\right]\left[\exp.(-\,\alpha_2 y')\right] \left[\frac{\alpha_2^{\,2} - \alpha^2}{\alpha_2}\right] \qquad (3.26)$$

From Eqn.(2.59),

$$\alpha_2^2 - \alpha^2 = j\, w_1 \mu\, \sigma\, e^{-j\lambda}$$
$$= w_1 \mu\, \sigma \left[\exp.j\left(\frac{\pi}{2} - \lambda\right)\right] \qquad (3.27)$$

The constant D_1 defined by Eqn.(2.54) is reproduced as,

$$D_1 = A_1 \frac{\alpha_2}{\alpha_2 \cosh \alpha g + \alpha/\mu_r \sinh \alpha g} \times \left[\exp.(\alpha_2 g - j\,\delta_1)\right]$$

Substituting $\alpha_2 = (a + jb)$ and approximating $\cosh \alpha g$ and $\sinh \alpha g$ in the above equation, one obtains,

$$D_1 = A_1 \frac{(a + jb)}{(a + \alpha^2/\mu_r\, g) + jb} \left[\exp.(a + jb)g - j\,\delta_1\right] \qquad (3.28)$$

Substitution of $\alpha_2 = (a + jb)$, the value of $(\alpha_2^2 - \alpha^2)$ from Eqn.(3.26) and the value of D_1 from Eqn.(3.27) in Eqn.(3.25) yields,

$$J_{zf} = \left[j\, A_1\, (w_1 \mu\, \sigma)(a + jb)\, \exp.a(g - y')\right] \times \left[\exp.-j(bg + \delta_1 + \alpha x' + ay' - by')\right] \times \left[\frac{1}{\left[(a + \alpha^2/\mu_r\, g) + jb\right]\left[(a + jb)\right]}\right]$$

It follows from the above equation that,

$$\left|J_{zf}\right|^2 = \frac{A_1^2 \cdot \left[w_1 \mu \sigma\right]^2 \left[\exp.2a(g-y')\right]}{(a+\alpha^2 \mu_r g)^2 + b^2} \tag{3.29}$$

Substitution of Eqn.(3.28) in Eqn.(3.22) results in,

$$P_{1f} = \frac{1}{2\sigma} \cdot \frac{A_1^2 \cdot (w_1 \mu \sigma)^2}{(a+\alpha^2 \mu_r g)^2 + b^2} \int_0^{2T} \int_g^{\infty} \left[\exp.2a(g-y')\right] dx'\, dy'$$

$$= \frac{A_1^2 (w_1 \mu \sigma)\,(w_1 \mu T)}{2a\left[(a+\alpha^2 \mu_r g)^2 + b^2\right]} \tag{3.30}$$

Substituting the value of $(w_1 \mu \sigma) = 2ab \text{ Sec } \lambda$ from Eqn.(2.62) in the above equation, one obtains,

$$P_{1f} = \frac{A_1^2\, w_1 \mu\, b\, T \text{ Sec } \lambda}{(a+\alpha^2 \mu_r g)^2 + b^2} \tag{3.31}$$

The power loss given by Eqn.(3.30) includes hysteresis loss also. However, if it is assumed that $\lambda = 0$, the above equation reduces to

$$P_{1f} = \frac{A_1^2\, w_1\, \mu\, b\, T}{(a+\alpha^2 \mu_r g)^2 + b^2} \tag{3.32}$$

From Eqns.(3.20) and (3.31) one verifies the well known relation,

$$\text{Slip} = \frac{\text{Rotor copper loss}}{\text{Rotor Input}}$$

A similar analysis for the backward field shows that,

$$P_{1b} = \frac{A_2^2 \; w_2 \; \mu \; b' \; T}{(a' + \alpha^2 \mu_r \; g)^2 + b'^2} \qquad (3.33)$$

3.3 Torque Developed:

The general expression for the force is given by,

$$\underline{F} = \int_v (\underline{J} \times \underline{B}) \, dv$$

For harmonically varying fields, one obtains the time averaged force as,

$$\underline{F}_{av} = \frac{1}{2} \; \mathrm{Re} \int (\bar{\underline{J}} \times \bar{\underline{B}}^*) \, dv \qquad (3.34)$$

From Eqn.(3.26), the current density is given by,

$$\bar{J}_{zf} = \frac{(j \; w_1 \mu \; \sigma \; D_1) \left[\exp.(-j \; \alpha x' - \alpha_2 y' - j\lambda) \right]}{\alpha_2} \qquad (3.35)$$

Further,

$$\bar{B}_{x'2f} = \mu \; \bar{H}_{x'2f} \qquad (3.36)$$

and

$$\bar{B}_{y'2f} = \mu \; \bar{H}_{y'2f} \qquad (3.37)$$

only the tangential component of the force would contribute towards the production of the torque, and therefore only the x component of the force given by, Eqn.(3.34) need be considered. Expanding $(\bar{J} \times \bar{B}^*)$ into its components and considering only the x component of the final expression, one can write,

$$\left[\bar{J} \times \bar{B}^*\right]_{x\ \text{component}} = -(\mu \bar{J}_{zf} \bar{H}_{y'2f}) \qquad (3.38)$$

Substitution of Eqn.(3.38) in Eqn.(3.34) gives the tangential component of the force on the rotor per meter length of the rotor as,

$$F_{tf} = -\frac{1}{2}\mu \text{ Re} \int_0^{2T}\int_g^{\infty} (\bar{J}_{zf} \bar{H}_{y'2f}^*)\, dx'\, dy' \qquad (3.39)$$

From Eqn.(3.35), it follows that?

$$\bar{J}_{zf} = \frac{\left[j\, w_1 \mu\, \sigma D_1\right]\left[\exp.(-j\, \alpha x' - \alpha_2 y' - j\lambda)\right]}{\alpha_2} \qquad (3.40)$$

Substitution of $\alpha_2 = (a + jb)$ in Eqn.(3.40) results in,

$$\bar{J}_{zf} = \frac{\left[j\, w_1 \mu\, \sigma D_1\right]\left[\exp.-j(\alpha x' + by' + \lambda\right]\left[\exp.(-ay' j)\right]}{(a + jb)} \qquad (3.41)$$

From Eqn.(2.58),

$$\bar{H}_{y'2f} = \frac{-j\, D_1\, \alpha}{\alpha_2}\left[\exp.(-j\, \alpha x')\right]\left[\exp.(-\alpha_2 y')\right]$$

$$= \frac{-j\, D_1\, \alpha \left[\exp.-j(\alpha x' + by')\right]\left[\exp.(-ay')\right]}{(a + jb)} \qquad (3.42)$$

Taking the conjugate of both the sides of the above equation yields,

$$\bar{H}^{*}_{y'2f} = \frac{j\, D_1^{*}\, \alpha \left[\exp. j(\alpha x' + by')\right]\left[\exp.(-ay')\right]}{(a - jb)} \qquad (3.43)$$

It follows from the Eqns. (3.41) and (3.43) that,

$$\bar{J}_{zf}\ \bar{H}^{*}_{y'2f} = -\left[w_1 \mu\, \sigma\, D_1\, D_1^{*}\, \alpha\right] \times \frac{\left[\exp. -j(\alpha x' + by' + \lambda\right]\left[\exp.(-ay')\right] \times \left[\exp. j(\alpha x' + by')\right]\left[\exp.(-ay')\right]}{(a^2 + b^2)}$$

Noting that $D_1\, D_1^{*} = |D_1|^2$ and taking the real part of the above equation, one obtains,

$$\mathrm{Re}\left[\bar{J}_{zf}\ \bar{H}^{*}_{y'2f}\right] = -\frac{\left[(w_1 \mu\, \sigma \cos\lambda)\, \alpha\, |D_1|^2\right]\left[\exp. -(2ay')\right]}{(a^2 + b^2)} \qquad (3.44)$$

Substitution of Eqn.(3.44) in Eqn.(3.39) results in,

$$F_{tf} = \frac{1}{2} \cdot \frac{(w_1 \mu\, \sigma \cos\lambda)(\alpha/\mu)\, |D_1|^2}{(a^2 + b^2)} \int_0^{2T} \int_g^{\infty} \left[\exp.(-2ay')\right] d\ldots$$

$$= \frac{(w_1 \mu\, \sigma \cos\lambda)(\alpha/\mu\, T)\ |D_1|^2}{2a\ (a^2 + b^2)} \left[\exp. -(2ag)\right] \qquad (3.45)$$

The constant D_1 given by Eqn.(3.28), after approximation, is rewritten as,

$$D_1 = A_1 \cdot \frac{(a + jb)}{(a+ \alpha^2 \mu_r\ g)+jb} \left[\exp. \left[(a+jb)g-j\ \delta_1 \right] \right]$$

It follows from the above equation that,

$$|D_1|^2 = A_1^2\ \frac{(a^2 + b^2)\ \left[\exp.(2ag)\right]}{(a+ \alpha^2 \mu_r\ g)^2 + b^2} \qquad (3.46)$$

Combining Eqns.(3.45) and (3.46), one obtains,

$$F_{tf} = \frac{A_1^2\ \ (w_1\,\mu\ \sigma \cos\lambda)(\alpha\,\mu\ T)}{2a\ \left[(a + \alpha^2 \mu_r\ g)^2 + b^2\right]} \qquad (3.47)$$

Noting that radius of the rotor $r = \frac{T}{\pi}$, the expression for the torque is given by,

$$T_{dev.f} = \frac{A_1^2 . (w_1\,\mu\ \sigma \cos\lambda)(\alpha\,\mu\ T)}{2a\ \left[(a + \alpha^2 \mu_r\ g)^2 + b^2\right]} \cdot \frac{T}{\pi}$$

Since $\alpha = \frac{\pi}{T}$, the above equation results in, after the substitution of the value of $(w_1 \mu\ \sigma \cos \lambda)$ from Eqn.(2.62),

$$T_{dev.f} = \frac{A_1^2\ \mu\ b\ T}{(a + \alpha^2 \mu_r\ g)^2 + b^2} \qquad (3.48)$$

With the help of Eqns. (3.20) and (3.48), one again verifies the well known relation.

Torque = Rotor input in synchronous watts.

A similar analysis proves that the torque due to the backward field is given by,

$$T_{dev.b} = \frac{A_2^2 \mu \; b' \; T}{(a' + \alpha^2 u_r g)^2 + b'^2} \qquad (3.49)$$

4. EQUIVALENT CIRCUIT

The basic voltage equations for the main and auxiliary windings are,

$$V = r_{1m}\bar{I}_1 + jwx_{1m}\,\bar{I}_1 + \frac{d\,\bar{U}_m}{dt} \tag{4.1}$$

$$V = r_{1a}\,\bar{I}_2 + jwx_{1a}\,\bar{I}_2 + \frac{1}{jwc}\,\bar{I}_2 + \frac{d\,U_a}{dt} \tag{4.2}$$

where subscripts m and a refer to main and auxiliary windings, respectively,

and

V = Supply voltage in volts.

I = Current in a winding in amperes.

x_1= Leakage Reactance of a winding in ohms.

r = Resistance of a winding in ohms.

C = Capacitance in the auxiliary winding in farads.

U = Flux linkages of a winding in Webers.

The flux linkage U for both the windings will be a function of the currents I_1 and I_2. The total normal component of the magnetizing force in the air gap is given by the sum of Eqns.(2.56) and (2.66). At $y = 0$, the above mentioned Eqns. sum to give the total normal magnetizing force at the stator core-air gap boundary as,

$$H_y \Big|_{y=0} = j\left[\exp.j(wt - \alpha x - \delta_1)\right]\left[c_1 - c_2\right] - j\left[\exp.j(wt + \alpha x + \delta_2)\right]\left[c_1' - c_2'\right] \tag{4.3}$$

The constants c_1 and c_2 are given by Eqns.(2.52) and (2.53), respectively. Thus,

$$(c_1 - c_2) = \frac{A_1}{2}\left[\alpha_2\left[e^{-\alpha g} - e^{\alpha g}\right] - \alpha \mu_r\left[e^{-\alpha g} + e^{\alpha g}\right]\right] \times \left[\frac{1}{\alpha_2 \cosh \alpha g + \alpha \mu_r \sinh \alpha g}\right]$$

Substituting $\alpha_2 = (a + jb)$ and approximating one obtains,

$$(c_1 - c_2) = - A_1 \alpha. \frac{(a + jb)g + \mu_r}{(a + \alpha^2 u_r g) + jb} = - A_1 k_1 e^{j\theta_1} \tag{4.4}$$

where k_1 and θ_1 are some other constants.

Similarly,

$$(c_1' - c_2') = - A_2 \alpha \cdot \frac{(a' + jb')g + \mu_r}{(a' + \alpha^2 \mu_r g) + jb'} = - A_2 k_2 e^{j\theta_2} \tag{4.5}$$

Substitution of the value of $(c_1 - c_2)$ from Eqn.(4.4) and $(c_1' - c_2')$ from Eqn.(4.5) in Eqn.(4.3) yields,

$$H_y \Big|_{y=0} = - j\Big[A_1 k_1 \exp.j(wt - \alpha x - \delta_1 + \theta_1)$$

$$- A_2 k_2 \exp.j(wt + \alpha x + \delta_2 + \theta_2)\Big]$$

$$= - j\Big[k_1 \left[\exp.j(wt - \alpha x + \theta_1)\right]$$

$$x \left[A_1 \cos \delta_1 - j A_1 \sin \delta_1\right]$$

$$- k_2 \left[\exp.j(wt + \alpha x + \theta_2)\right]$$

$$x \left[A_2 \cos \delta_2 + j A_2 \sin \delta_2\right]\Big]$$

(4.6)

It follows from Eqns.(2.8), (2.9) and (4.6) that,

$$H_y \Big|_{y=0} = - j \Big[k_1 \left[\exp.j(wt - \alpha x + \theta_1)\right]$$

$$x \left[A - j A' e^{j\theta}\right]$$

$$- k_2 \left[\exp.j(wt + \alpha x + \theta_2)\right]$$

$$x \left[A + j A' e^{j\theta}\right]\Big] \qquad (4.7)$$

Let $A = k_{w1} \; I_1$ and $A' = k_{w2} \; I_2$

where k_w depends on the winding design. Replacing A and A' by $k_{w1} I_1$ and $k_{w2} I_2$, respectively, in Eqn.(4.7), one obtains,

$$H_y \Big|_{y=0} = -j \left[k_1 \left[\exp.j(wt - \alpha x + \theta_1) \right] \right.$$

$$\times \left[k_{w1} I_1 - j k_{w2} I_2 e^{j\theta} \right]$$

$$\left. - k_2 \left[\exp.j(wt + \alpha x + \theta_2 \right] \left[k_{w1} I_1 + j k_{w2} I_2 e^{+j\theta} \right] \right] \tag{4.8}$$

After additi^n and substraction of the term $- j k_2$ $\times \left[\exp.j(wt - \alpha x + \theta_2) \right] \left[k_{w1} I_1 + j k_{w2} I_2 e^{j\theta} \right]$, Eqn.(4.8) takes the form,

$$H_y \Big|_{y=0} = -j \left[\left[\exp.j(wt - \alpha x) \right] \left[k_{w1} I_1 (k_1 e^{j\theta_1} + k_2 e^{j\theta_2}) \right. \right.$$

$$\left. + j k_{w2} I_2 e^{j\theta} (k_2 e^{j\theta_2} - k_1 e^{j\theta_1}) \right]$$

$$- k_2 e^{j\theta_2} \left[\exp.j(wt + \alpha x) + \exp.j(wt - \alpha x) \right]$$

$$\left. \times \left[k_{w1} I_1 + j k_{w2} I_2 e^{j\theta} \right] \right] \tag{4.9}$$

Eqn.(4.9) expresses $H_y \Big|_{y=0}$ in two types of fields one of which is purely rotating and the other is purely pulsating. The rotating part of $H_y \Big|_{y=0}$ is given by,

$$H_{yr}\Big|_{y=0} = -j\left[\left[\exp.j(wt - \alpha x)\right]\left[k_{w1}\, I_1(k_1 e^{j\theta_1} + k_2 e^{j\theta_2})\right.\right.$$

$$\left.\left. + j\, k_{w2}\, I_2\, e^{j\theta}\, (k_2\, e^{j\theta_2} - k_1\, e^{j\theta_1})\right]\right] \quad (4.10)$$

and the pulsating part of $H_y\Big|_{y=0}$ is given by,

$$H_{yp}\Big|_{y=0} = 2j\, \cos \alpha x\, k_2\, e^{j\theta_2}\left[\cos wt + j\, \sin wt\right]$$

$$x\left[k_{w1}\, I_1 + jk_{w2}\, I_2\, e^{j\theta}\right] = 2\, jk_2\, \cos \alpha x$$

$$x\left[\exp.j(wt + \theta_2)\right]\left[k_{w1}\, I_1 + jk_{w2}\, I_2\, e^{j\theta}\right]$$

(4.11)

It is now unnecessary to carry on the term exp.j(wt), as $\frac{dU}{dt} = jw$ for the rotating field as well as for the pulsating field.

Thus, the rotating part of $H_y\Big|_{y=0}$ can be expressed as,

$$H_{yr}\Big|_{y=0} = -j\, e^{-j\alpha x}\left[k_{w1}\, I_1\, (k_1\, e^{j\theta_1} + k_2\, e^{j\theta_2})\right.$$

$$\left. + j\, k_{w2}\, I_2\, e^{j\theta}\, (k_2\, e^{j\theta_2} - k_1\, e^{j\theta_1})\right] \quad (4.12)$$

and the pulsating part of $H_y \Big|_{y=0}$ is given by,

$$H_{yp}\Big|_{y=0} = 2j\ k_2 \cos \alpha x\ e^{j\theta_2} \left[k_{w1}\ I_1 + jk_{w2}\ I_2\ e^{j\theta}\right] \tag{4.13}$$

Both the windings will have flux linkages due to rotating andpulsating parts of the magnetic field. Let the flux linkages due to rotating and pulsating components of the field be denoted by U_r and U_p respectively. Thus,

$$U_m = U_{mr} + U_{mp} \tag{4.14}$$

and $$U_a = U_{ar} + U_{ap} \tag{4.15}$$

The main and auxiliary windings are assumed to be sinusoidally distributed in space and are in space quadrature. It is also assumed, of course arbitrarily, that the distribution of the main winding is a cosine function and that of the auxiliary winding is a-vesine function. Thus, the winding distribution of the main winding is given by $k_{w1}\ e^{j\alpha x}$ and that of the auxiliary winding is given by $j\ k_{w2}\ e^{j\alpha x}$. Hence the flux linkage of the main winding due to rotating component of the magnetic field is given by,

$$U_{mr} = \mu_o \int_o^T \int_{-x}^{x} H_{yr}\ (\delta,y)\ \Big|_{y=0} k_{w1}\ e^{j\alpha x}\ d\ \delta\ dx \tag{4.16}$$

where δ is a dummy variable.

Substitution of $H_{yr}(\delta,y)\Big|_{y=0}$ from Eqn.(4.10) in Eqn.(4.16) results in,

$$U_{mr} = -j\mu_o \Big[k_{w1} I_1 (k_1 e^{j\theta_1} + k_2 e^{j\theta_2})$$

$$+ j k_{w2} I_2 e^{j\theta} (k_2 e^{j\theta_2} - k_1 e^{j\theta_1}) \Big]$$

$$x \int_0^T k_{w1} e^{j\alpha x} dx \int_{-x}^{+x} e^{-j\alpha\delta} d\delta$$

$$= \frac{k_{w1}\mu_o T}{\alpha} \Big[k_{w1} I_1 (k_1 e^{j\theta_1} + k_2 e^{j\theta_2})$$

$$+ j k_{w2} I_2 e^{j\theta} (k_2 e^{j\theta_2} - k_1 e^{j\theta_1}) \Big] \quad (4.17a)$$

U_{mp} is given by,

$$U_{mp} = \mu_o \int_0^T k_{w1} \cos\alpha x \, dx \int_{-x}^{x} H_{yp}(\delta,y)\Big|_{y=0} d\delta$$

Substitution of $H_{yp}(\delta,y)\Big|_{y=0}$ from Eqn.(4.13) in the above equation yields,

$$U_{mp} = \Big[2j k_2 \mu_o e^{j\theta_2} k_{w1} \Big] \Big[k_{w1} I_1 + j k_{w2} I_2 e^{j\theta} \Big]$$

$$x \int_0^T \cos\alpha x \, dx \int_{-x}^{+x} \cos\alpha\delta \, d\delta$$

$$= 0 \quad (4.17b)$$

Thus,

$$U_m = U_{mr}$$

$$= \frac{k_{w1} \mu_o T}{\alpha} \left[k_{w1} I_1 (k_1 e^{j\theta_1} + k_2 e^{j\theta_2}) \right.$$

$$\left. + j k_{w2} I_2 e^{j\theta} (k_2 e^{j\theta_2} - k_1 e^{j\theta_1}) \right] \qquad (4.18)$$

The distribution of the auxiliary winding is given by $j\ k_{w2}\ e^{+j \alpha x}$, and hence

$$U_{ar} = - j \mu_o \left[k_{w1} I_1 (k_1 e^{j\theta_1} + k_2 e^{j\theta_2}) \right.$$

$$\left. + j k_{w2} I_2 e^{j\theta} (k_2 e^{j\theta_2} - k_1 e^{j\theta_1}) \right]$$

$$\times \int_0^T j k_{w2} e^{j \alpha x} dx \int_{-x}^{x} e^{-j \alpha \delta} d\delta$$

$$= \frac{j k_{w2} \mu_o T}{\alpha} \left[k_{w1} I_1 (k_1 e^{j\theta_1} + k_2 e^{j\theta_2}) \right.$$

$$\left. + j k_{w2} I_2 e^{j\theta} (k_2 e^{j\theta_2} - k_1 e^{j\theta_1}) \right] \qquad (4.19)$$

U_{ap} is given by,

$$U_{ap} = - \mu_o \int_0^T \int_{-x}^{x} H_{yp} (\delta, y) \Big|_{y=0} k_{w2} \sin \alpha x \, d\delta \, dx \qquad (4.20)$$

Substituting the value of $H_{yp} \Big|_{y=o}$ from Eqn. (4.13) in Eqn. (4.20), one obtains,

$$U_{ap} = -\,2\,j\,k_{w2}\mu_o\,k_2\,e^{j\theta_2}(k_{w1}\,I_1 + j\,k_{w2}\,I_2\,d^{j\theta})$$

$$\times \int_0^T \sin\alpha x\,dx \int_{-x}^{+x} \cos\alpha\delta\,d\delta$$

$$= -\,\frac{2j\,k_{w2}\mu_o\,T\,k_2\,e^{j\theta_2}}{\alpha}\left[(k_{w1}\,I_1 + j\,k_{w2}\,I_2\,e^{j\theta}\right] \tag{4.21}$$

The total flux linkage of the auxiliary winding is given by the sum of Eqns.(4.19) and (4.21), which reads,

$$U_a = \frac{j\,k_{w2}\mu_o\,T}{\alpha}\Big[k_{w1}\,I_1\,(k_2\,e^{j\theta_2} + k_1\,e^{j\theta_1})$$

$$-\,j\,k_{w2}\,I_2\,e^{j\theta}\,(k_1\,e^{j\theta_1} + k_2\,e^{j\theta_2})\Big] \tag{4.22}$$

Substitution of Eqns.(4.18) and (4.22) after multiplication by (jw) in Eqns.(4.1) and (4.2), respectively, yields,

$$V = r_{1m}\,\bar{I}_1 + jw\,x_{1m}\,\bar{I}_1 + \frac{j\,k_{w1}^2\,w\,\mu_o\,T}{\alpha}\,\bar{I}_1(k_1 e^{j\theta_1} + k_2 e^{j\theta_2})$$

$$+ \frac{k_{w1}\,k_{w2}\,w\,\mu_o\,T}{\alpha}\,I_2\,e^{j\theta}(k_2\,e^{j\theta_2} + k_1\,e^{j\theta_1}) \tag{4.23}$$

$$V = r_{2a}\,\bar{I}_2 + jw\,x_{1a}\,\bar{I}_2 + \frac{\bar{I}_2}{jwc} + \frac{j\,k_{w2}^2\,w\,\mu_o\,T\,I_2\,e^{j\theta}}{\alpha}$$

$$\times\,(\,k_1\,e^{j\theta_1} + k_2\,e^{j\theta}\,)$$

$$-\,\frac{w\,k_{w1}\,k_{w2}\mu_o\,T}{\alpha}\,I_1\left[k_2\,e^{j\theta_2} + k_1\,e^{j\theta_1}\right] \tag{4.24}$$

In Eqns.(4.23) and (4.24) $e^{j\theta}$ indicates the phase of $\bar{I}_2$ w.r. to $\bar{I}_1$ and hence replacing $I_2\,e^{j\theta}$ by $\bar{I}_2$, one obtains,

$$V = r_{1m}\,\bar{I}_1 + jw\,x_{1m}\,\bar{I}_1 + \frac{jw\,k_{w1}^2\,\mu_o\,T}{\alpha}\,\bar{I}_1$$

$$\times\,(k_1\,e^{j\theta_1} + k_2\,e^{j\theta_2})$$

$$+\,\frac{w\,k_{w1}\,k_{w2}\,\mu_o\,T}{\alpha}\,\bar{I}_2\,(-k_2\,e^{j\theta_2} + k_1\,e^{j\theta_1}) \quad (4.25)$$

$$V = r_{1a}\,\bar{I}_2 + jw\,x_{1a}\,\bar{I}_2\;\frac{1}{jwc}\,\bar{I}_2 + \frac{jw\,k_{w2}^2\,\mu_o\,T}{\alpha}\,\bar{I}_2$$

$$\times\,(\,k_1\,e^{j\theta_1} + k_2\,e^{j\theta_2})$$

$$-\,\frac{w\,k_{w1}\,k_{w2}\,\mu_o\,T}{\alpha}\,I_1\,(-k_2\,e^{j\theta_2} + k_1\,e^{j\theta_1}) \quad (4.26)$$

Eqns.(4.25) and (4.26) can be rewritten as,

$$V = Z_{11}\,I_1 + Z_{12}\,I_2 \quad (4.27)$$

$$V = -\,Z_{12}\,I_1 + Z_{22}\,I_2 \quad (4.28)$$

where,

$$Z_{11} = r_{1m} + jw\,x_{1m} + \frac{jw\,k_{w1}^2\,\mu_o\,T}{\alpha}\left[k_1\,e^{j\theta_1} + k_2\,e^{j\theta_2}\right] \quad (4.29)$$

$$Z_{12} = +\,\frac{w\,k_{w1}\,k_{w2}\,\mu_o\,T}{\alpha}\left[-k_2\,e^{j\theta_2} + k_1\,e^{j\theta_1}\right] \quad (4.30)$$

$$Z_{22} = r_{1a} + jw\,x_{1a} + \frac{1}{jwc} + \frac{jw\,k_{w2}^2\,\mu_o\,T}{\alpha}$$

$$\times\left[k_1\,e^{j\theta_1} + k_2\,e^{j\theta_2}\right] \quad (4.31)$$

One concludes from Eqns.(4.27) and (4.28) that the impedance matrix of the single phase motor is given by,

$$\left[Z \right] = \begin{bmatrix} Z_{11} & Z_{12} \\ -Z_{12} & Z_{22} \end{bmatrix} \qquad (4.32)$$

and is non-reciprocal.

The non-reciprocal nature of the impedance matrix rules out the equivalent circuit representation in terms of purely passive circuit elements, such as resistances, inductances and capacitances. The most convenient representation, thus, is in terms of a network constrained by a gyrator as is shown in Fig.2.

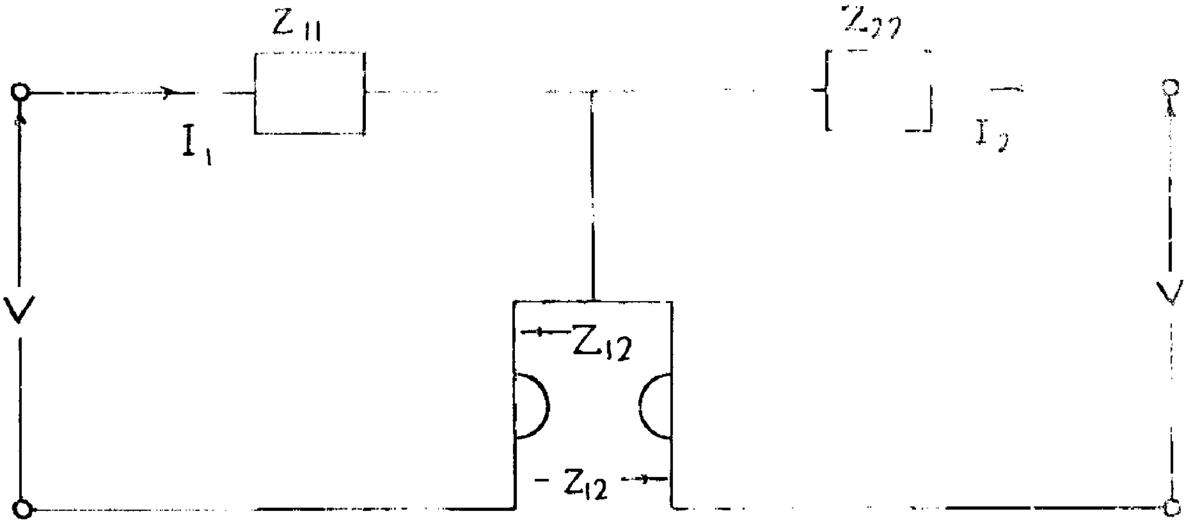


FIG. 2.

5. DESIGN CONSIDERATIONS

5.1 Choice of the Capacitor:

While designing a capacitor run single phase induction motor, one should aim at exploiting fully the presence of the capacitor in the auxiliary winding for the betterment of performance viz., better power factor, and a higher value of efficiency and torque. This can be done by a proper choice of capacitor value and the ratio of the effective main winding turns to that of the auxiliary winding so that the backward field is reduced to zero. However, this condition can be achieved at only one value of slip, and therefore the choice of capacitor value and turns ratio k_t should be such that the motor operates under balanced conditions at full load slip or at very near to the full load slip. The most suitable value of capacitor and turns ratio k_t can be obtained with the help of the equivalent circuit shown in Fig. 2, one can, thus, write,

$$V = Z_{11} I_1 + Z_{12} I_2 \tag{5.1}$$

and $$V = - Z_{12} I_1 + Z_{22} I_2 \tag{5.2}$$

The values of Z_{11}, Z_{12}, and Z_{22} are given by Eqns.(4.29), (4.30), and (4.31) and are rewritten herefor convenience :

$$Z_{11} = r_{1m} + j\omega x_{1m} + \frac{jw\, k_{w1}^2 \mu_o T}{\alpha} \cdot \left[k_1 e^{j\theta_1} + k_2 e^{j\theta_2}\right] \tag{5.3}$$

$$Z_{12} = \frac{w\, k_{w1}\, k_{w2} \mu_o T}{\alpha} \left[k_2 e^{j\theta_2} + k_1 e^{j\theta_1}\right] \tag{5.4}$$

$$Z_{22} = r_{1a} + jw\, x_{1a} + \frac{1}{jwc} + \frac{j\, w\, k_{w2}^2\, \mu_o T}{\alpha} \left[k_1 e^{j\theta_1} + k_2 e^{j\theta_2}\right] \tag{5.5}$$

After approximation, one obtains the values $k_1 e^{j\theta_1}$ and $k_2 e^{j\theta_2}$ from Eqns.(4.4) and (4.5) as,

$$k_1 e^{j\theta_1} = \frac{\alpha \mu_r}{(a + \alpha^2 \mu_r g) + jb} = \frac{\alpha \mu_r \left[(a + \alpha^2 \mu_r g) - jb\right]}{(a + \alpha^2 \mu_r g)^2 + b^2} \tag{5.6}$$

$$k_2 e^{j\theta_2} = \frac{\alpha \mu_r \left[(a' + \alpha^2 \mu_r g) - jb'\right]}{(a' + \alpha^2 \mu_r g)^2 + b'^2} \tag{5.7}$$

Substitution of Eqns.(5.6) and (5.7) in Eqn.(5.3) results in,

$$Z_{11} = r_{1m} + jw\, x_{1m} + \left[jw\, k_{w1}^2\, \mu_r\, T\right] \times \left[\frac{(a + \alpha^2 \mu_r g) - jb}{(a + \alpha^2 \mu_r g)^2 + b^2} + \frac{(a' + \alpha^2 \mu_r g) - jb'}{(a' + \alpha^2 \mu_r g)^2 + b'^2}\right]$$

After rearranging the above Eqn., one obtains,

$$Z_{11} = r_{1m} + jw\, x_{1m} + \left[w\, k_{w1}^2\, \mu\; T\right]$$

$$\times \left[\frac{b + j(a + \alpha^2 \mu_r\, g)}{(a + \alpha^2 \mu_r\, g)^2 + b^2} + \frac{b' + j(a' + \alpha^2 \mu_r\, g)}{(a' + \alpha^2 \mu_r\, g)^2 + b'^2}\right] \tag{5.8}$$

Let

$$(p + j\, q) = (w\, k_{w1}^2 \mu\, T)$$

$$\times \left[\frac{b + j\,(a + \alpha^2 \mu_r\, g)}{(a + \alpha^2 \mu_r\, g)^2 + b^2}\right] \tag{5.9}$$

and

$$(p' + j\, q') = (w\, k_{w_2}^2 \mu\, T)\;\; \frac{b' + j(a' + \alpha^2 \mu_r\, g)}{(a' + \alpha^2 \mu_r\, g)^2 + b'^2} \tag{5.10}$$

Now Eqn.(5.8) can be rewritten as,

$$Z_{11} = (r_{1m} + jw\, x_{1m} + p + jq + p' + jq') \tag{5.11}$$

Let the impedances Z_{12} and Z_{22} be referred to the main winding and the referred impedances be denoted by Z'_{12} and Z'_{22} respectively. Thus, one can write from Eqns.(5.4) and (5.5),

$$Z'_{12} = j\, k_t \left[(p' + jq') - (p + jq)\right] \tag{5.12}$$

$$Z'_{22} = k_t^2 \left[r_{1a} + jw\, x_{1a} + p + jq + p' + jq'\right] + \frac{1}{jwc}$$

$$= k_t^2 \left[(r_{1a} + p + p') + j(w\, x_{1a} + q + q')\right] - jx_c$$

where $x_c = \frac{1}{w\, c}$. (5.13)

Approximation and substitution of Eqns.(5.11), (5.12) and (5.13) in Eqns.(5.1) and (5.2) yields,

$$V = I_1 \left[(p + p') + j\,(q + q')\right] + j\,k_t\,I_2 \left[(p' + jq') - (p + jq)\right] \qquad (5.14)$$

$$V = -\,j\,k_t\,I_1 \left[(p' + jq') - (p + jq)\right] + I_2 \left[k_t^2 \left[(p + p') + j(q + q')\right] - j\,x_c\right] \qquad (5.15)$$

It follows from the Eqns.(5.14) and (5.15) that,

$$I_1 = V.\frac{k_t^2 \left[(p+p') + j(q+q')\right] - jx_c + j\,k_t\left[(p'+jq')-(p+jq)\right]}{D}$$

$$= V.\frac{\left[k_t^2(p+p') + k_t(q-q')\right] + j\left[k_t^2(q+q') - k_t(p-p') - x_c\right]}{D} \qquad (5.16)$$

$$I_2 = V.\frac{\left[(p+p') + j(q+q')\right] + j\,k_t\left[(p'+jq')-(p+jq)\right]}{D}$$

$$= V.\frac{\left[(p+p') + k_t(q-q')\right] + j\left[(q+q') - k_t(p-p')\right]}{D} \qquad (5.17)$$

where,

$$D = \left[(p+p') + j(q+q')\right]\left[k_t^2 \left[(p+p') + j(q+q')\right] - jx_c\right] - k_t^2 \left[(p'+jq') - (p+jq)\right]^2 \qquad (5.18)$$

For balanced operation at desired slip it is necessary that the magnitudes of I_1 and I_2 be equal and I_1 and I_2 be in phase quadrature, I_2 leading I_1. Equating the magnitudes of I_1 and I_2 from the Eqns.(5.16) and (5.17), one obtains,

$$\left[k_t^2(p+p')+k_t(q-q')\right]^2+\left[k_t^2(q+q')^2-k_t(p-p')-x_c\right]^2$$

$$=\left[(p+p')+k_t(q-q')\right]^2+\left[(q+q')-k_t(p-p')\right]^2$$

Simplification of the above equation results in,

$$(k_t^4-1)\left[(p+p')^2+(q+q')^2\right]-4\,k_t(k_t^2-1)\left[(pq'-p'q)\right]$$

$$=x_c\left[2\,k_t^2\,(q+q')-2\,k_t\,(p-p')-x_c\right] \qquad (5.19)$$

In order that I_2 leads I_1 in time phase by 90^o, it is necessary that,

$$\tan^{-1}\left[\frac{(q+q')-k_t\,(p-p')}{(p+p')+k_t\,(q-q')}\right]$$

$$-\tan^{-1}\left[\frac{k_t^2(q+q')-k_t\,(p-p')-x_c}{k_t^2(p+p')+k_t\,(q-q')}\right]$$

$$=\frac{\pi}{2}$$

Simplification of the equation yields,

$$x_c = 2k_t \cdot \left[\frac{k_t\left[p^2+q^2+p'^2+q'^2\right]+(1+k_t^2)(p'q-pq')}{(q + q') - k_t\,(p - p')} \right] \tag{5.20}$$

Thus, with the help of Eqns.(5.19) and (5.20) one can determine the values k_t and x_c for balanced operation at any desired slip.

The general solution for the above set, however, can not be given explicitly since it involves higher order equations and one has to resort to the usual numerical or graphical means.

5.2 Choice of the Flux Density:

The total rotor power is given by the sum of Eqns.(3.20a) and (3.21a). Under balanced operating conditions, the power corresponding to the backward field reduces to zero, and therefore the total power is given by,

$$P = \frac{A_1^2 \; w \; \mu \; b \; T}{\left[(a+ \alpha^2/\mu_r \; g)^2+b^2\right]} \tag{5.21}$$

The rotor efficiency under balanced conditions is (1 - S) and is maximum. As pointed out earlier, turns ratio k_t and the capacitor can be suitably chosen to obtain balanced operation at any desired slip, normally full load slip. Under these conditions, one can find, for a given mmf, a suitable value of the flux density so that the rotor input is maximum.

From the knowledge of proper value of the flux density in the air gap region at the given slip the number of turns in the main winding can be determined. The value of turns ratio k_t determines the no of turns in the auxiliary winding.

However, the determination of a suitable value of the flux density depends on a number of involved relationships e.g. the variations of the constants a and b. In order to obtain the general idea of these variations curves have been plotted indicating their nature. Fig.3 shows B V/S H, μ_r V/S H and λ V/S H curves for mild steel. Fig.4 has been drawn for a single phase induction motor of 5 cm rotor diameter. The conductivity of the rotor material has been taken as 0.6×10^7 mhos per meter. Fig. 4 shows the variation of a, which is defined by Eqn.(2.63) with slip for different values of flux density. Fig. 5 gives the variation of b with slip for different values of flux density.
Eqn.(5.21) can be rewritten as,

$$\frac{P}{A_1^2 \, w \, \mu_o} = \frac{b \, \mu_r}{\left[(a + \alpha^2 / \mu_r \, g)^2 + b^2 \right]} \qquad (5.22)$$

For constant mmf the right hand side of Eqn.(5.22) is proportional to the rotor input and takes values depending upon the flux density. With the help of the information

B V/S H, μ_r V/S H and λ V/S H

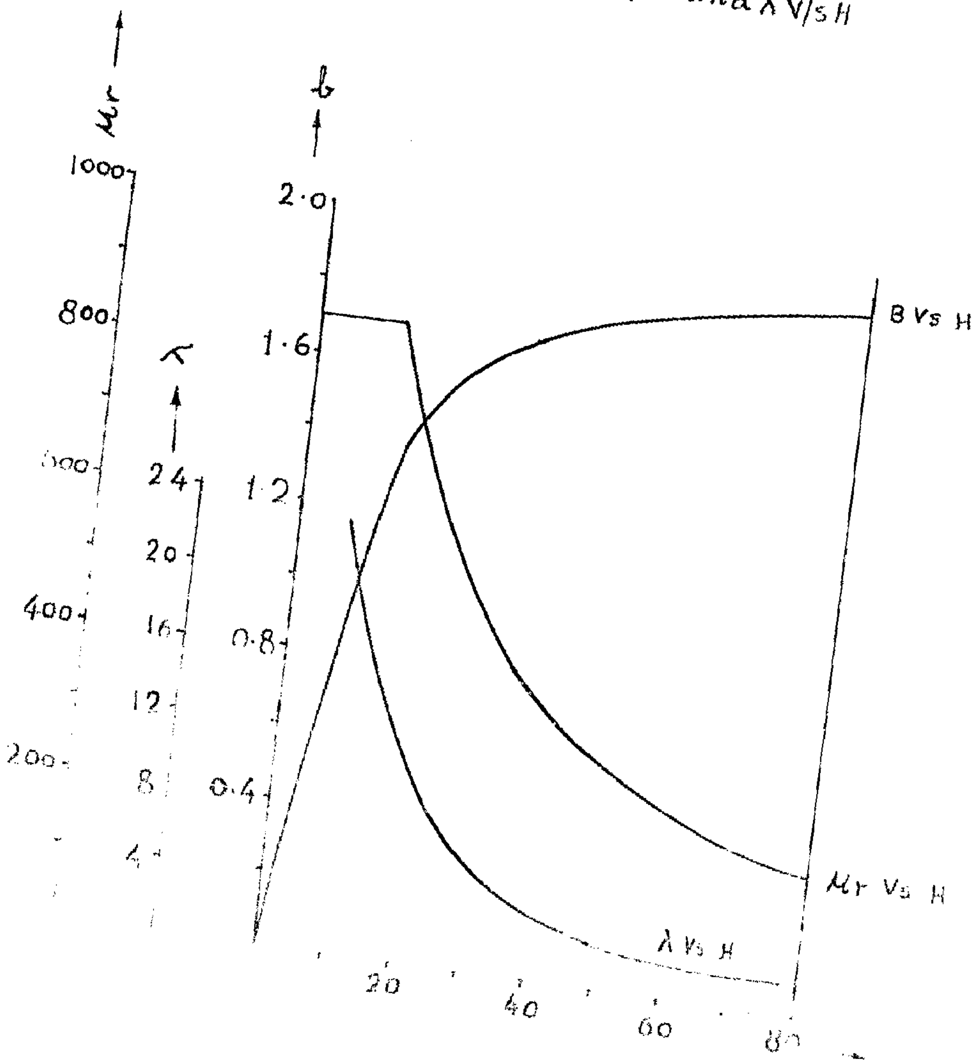
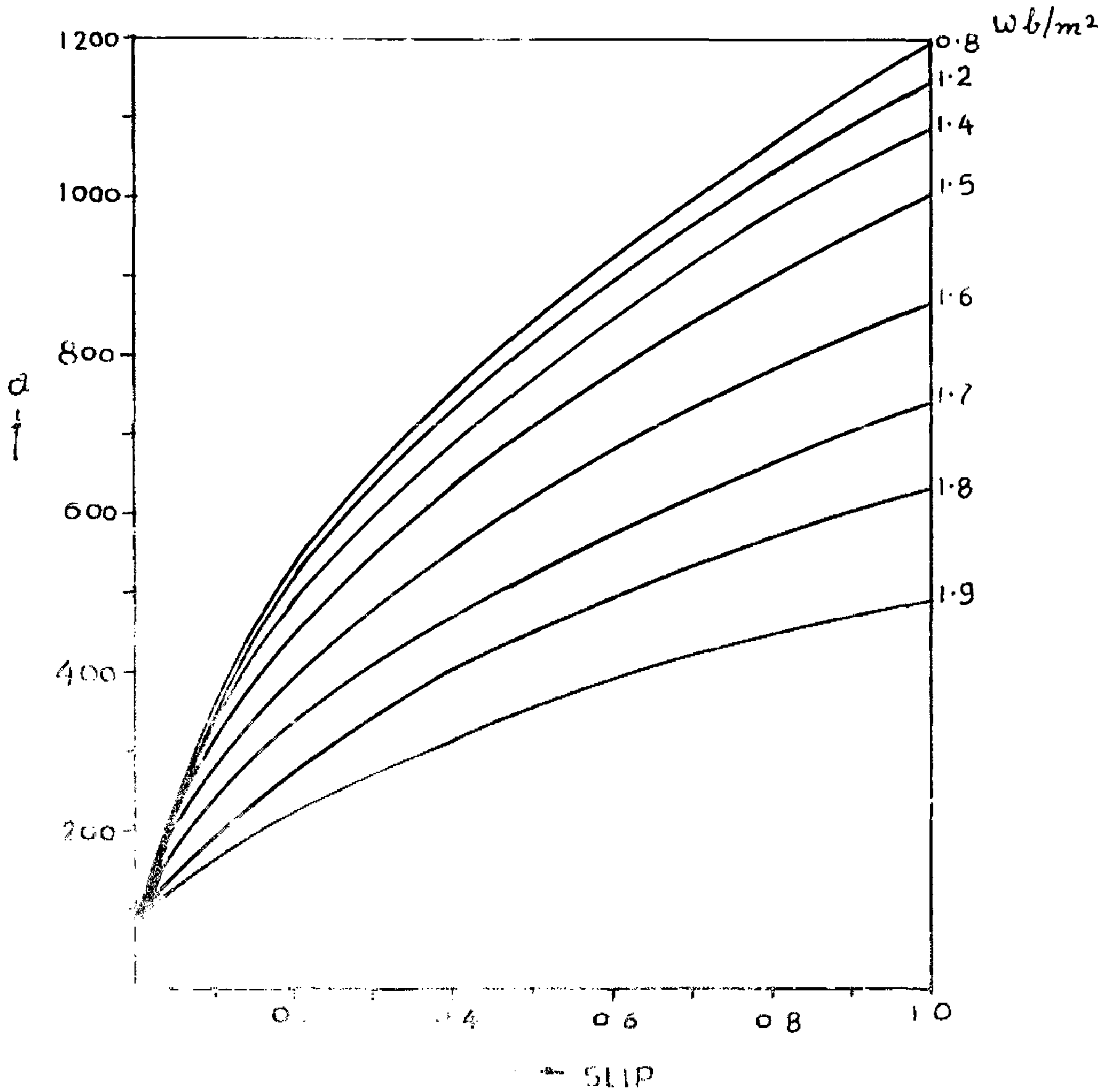
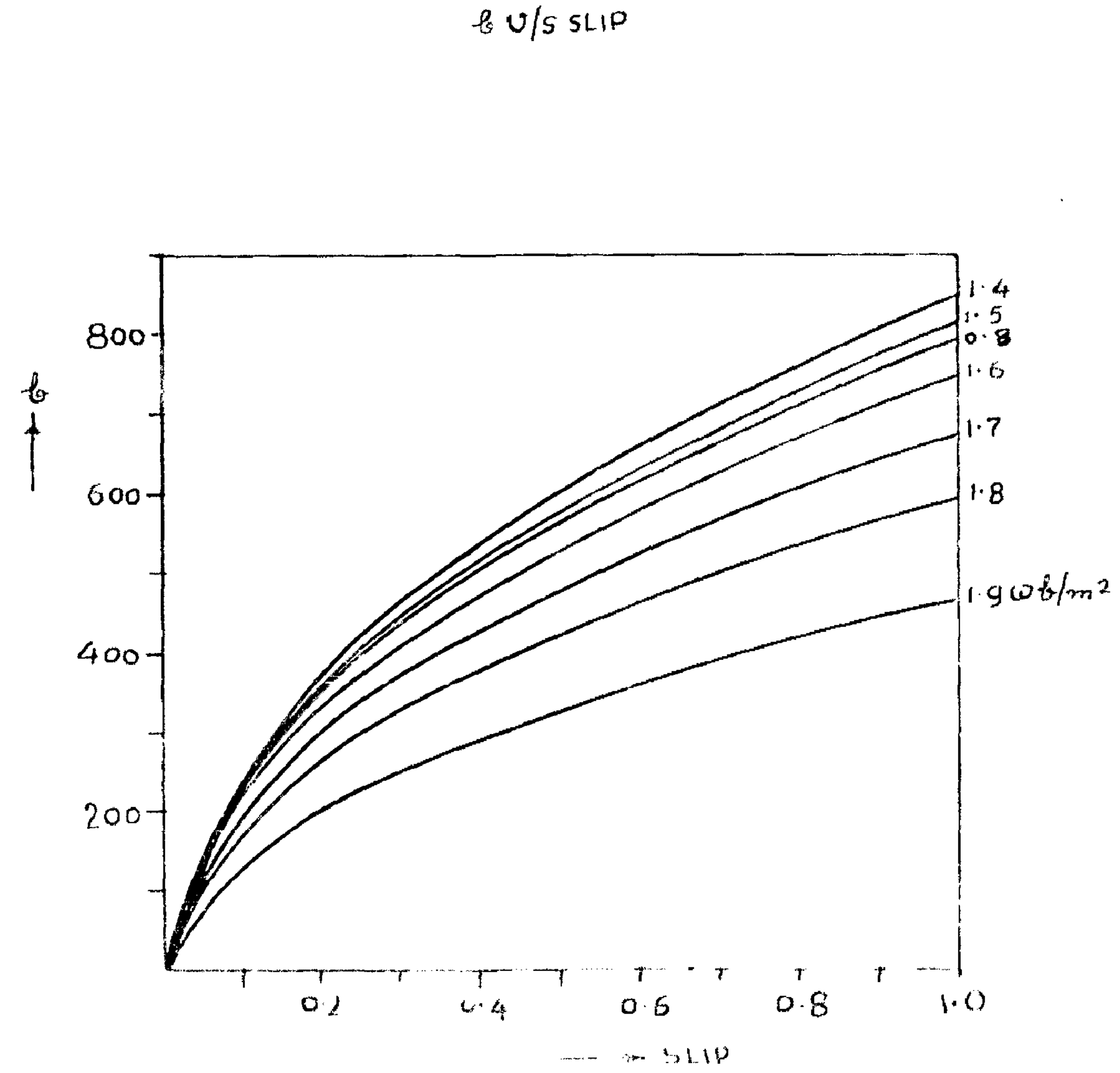


FIG 3.

α v/s SLIP





b V/S SLIP
b
800
600
400
200
0.2
0.4
0.6
0.8
1.0
SLIP
1·4
1·5
0·8
1·6
1·7
1·8
1·9 wb/m²

FIG. 5

available about the variations of μ_r, a and b with β in Figs. 3, 4 and 5 respectively, the variation of

$$\frac{P}{A_1^2 w \mu_o}$$

with respect to B has been plotted in Fig.6 at 10 per cent slip.

Another consideration would be the allowed power density in the rotor. This of course would depend on the permissible heat dissipation. It would be necessary to provide fins and ducts at suitable places since rotor heating would be appreciable.

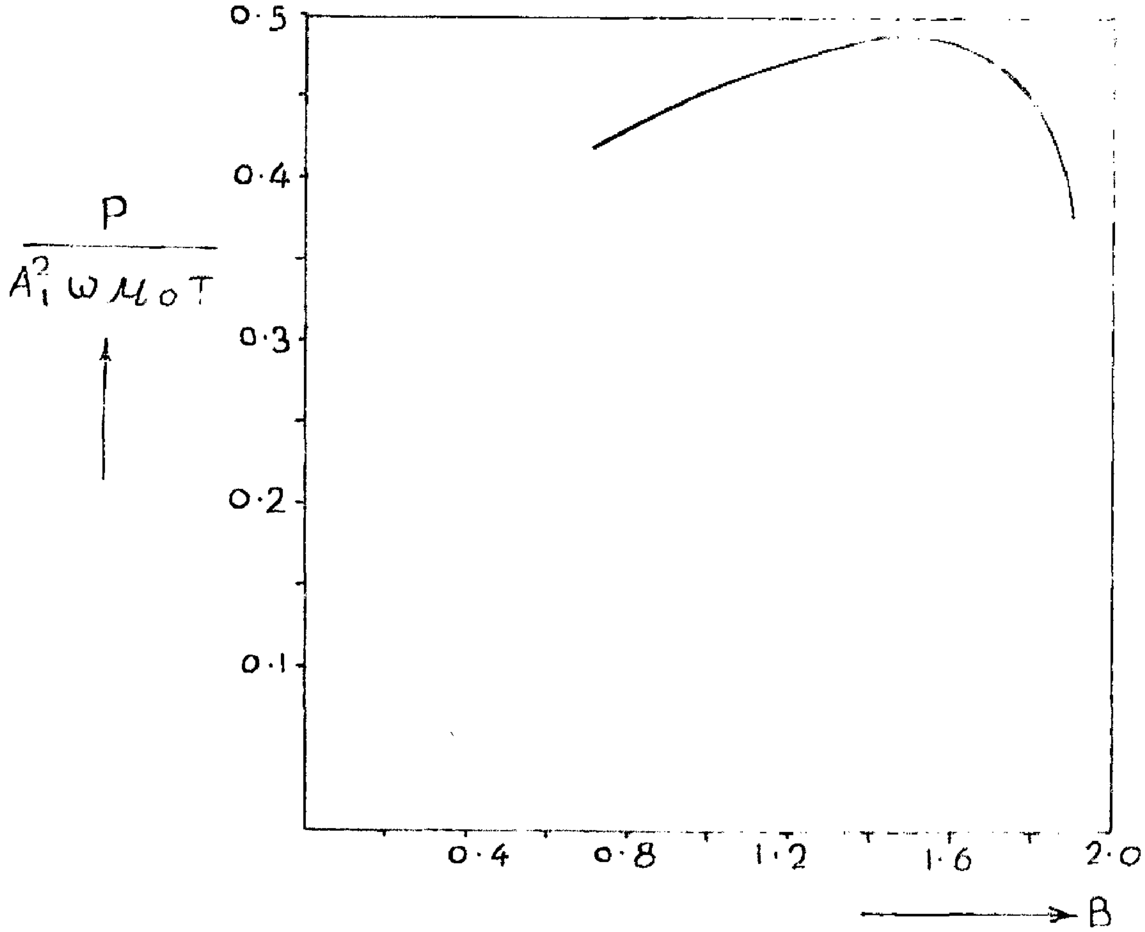


FIG 6.

6. CONCLUSIONS

In the foregoing study the solid rotor single phase capacitor run induction motor has been completely analysed. The analysis has been based on the formulation and the rigorous solution of the pertinent electromagnetic field problem.

Performance parameters such as rotor power, rotor losses and rotor torque, have been calculated for the general case of unbalance and the equivalent circuit derived. For the design of the auxiliary winding capacitor the required condition for the existence of the purely rotating field has been stated and the relevant expressions have been derived.

It can be seen from the material presented here that the equivalent circuit of the machine can not be represented by means of purely passive bilateral elements since it is non-reciprocal in nature. Here a gyrator has been utilized for the required circuit representation.

From the design angle it can be seen that the two important criteria are, the choice of the auxiliary winding capacitor and the operating flux density. As has been shown in detail these depend on certain involved relationships and one has to resort to numerical or graphical methods for their solution.

Due attention has not been given so far to the utility of the solid rotor for single phase operation. The investigations presented here indicate certain possibilities. Further work in this area can now establish the extent of its use and adaptability in the practical field.

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