OPERATION OF SINGLE PHASE INDUCTION GENERATOR

A Dissertation
submitted in partialfulfilment
of the requirements for the Degree
of
MASTER OF ENGINEERING
in
ADVANCED ELECTRICAL MACHINES

by
R.C. GUNE

DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE
U.P.
(INDIA)
1967

CERTIFICATE

Certified that the dissertation entitled "OPERATION OF SINGLE PHASE INDUCTION GENERATOR" which is being submitted by Sri R.C. Gune in partial fulfilment for the award of the Degree of Master of Engineering in Advanced Electrical Machines of University of Roorkee, Roorkee, is a record of candidate's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for 8 months from Jan. 1967 to Aug. 1967 for preparing dissertation for Master of Engineering Degree at the University.

S.K. Jain

(S.K. Jain)
Reader
Department of Elect. Engg.
University of Roorkee
ROORKEE

Dated,
August 30, 1967

ROORKEE

ACKNOWLEDGEMENTS

The author wishes to express his deep sense of gratitude to Mr.S.K. Jain, Reader in Electrical Engineering, University of Roorkee for his able guidance and affectionate encouragement to initiate this topic and for valuable suggestions and advices extended by him at every stage of preparation of this dissertation.

Sincere thanks are due to Professor C.S. Ghosh, Head of the Electrical Engineering Department, University of Roorkee, Roorkee for providing various facilities in connection with the work.

(R.C. Gune)

Dated-August 30th 1967

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CONTENTS

SYNOPSIS

The present work comprises of the study of a single phase induction machine run as a generator either at supersynchronous speed or as a self excited one. The different cases that have been analysed involve either the operation on one of the winding (namely main or auxiliary) or on both the windings.simultaneously.

The analysis has been carried out by assuming different cases as particular case of unbalanced operation of a balanced two phase machine. The expressions for the transient voltages available at the terminals of the stator winding have been deduced from the fundamental flux linkage equations in each case along with the time constant of voltage decay.

The steady state characteristics of the machine, employing various combinations of main and auxiliary winding resulting thereby either a single phase or a two phase operation, have been obtained.

The transient voltage induced and the time constant for different operations were determined experimentally to verify the results deduced.

It has been concluded that a single phase machine can be run successfully to generate power.

LIST OF SYMBOLS

(1) L_{11} = Self inductance of one stator phase i.e. either of the main winding or of auxiliary winding referred to the rotor side.

(2) L_{22} = Self inductance of one rotor phase. (Assuming a polyphase symmetrical rotor).

(3) M = Maximum value of mutual inductance between stator and rotor phases.

(4) r_1 = Resistance of main or auxiliary winding referred to rotor side.

(5) r_2 = Resistance of one rotor phase.

(6) I_m = Main winding current.

(7) I_a = Auxiliary winding current.

(8) I_{r1} = Rotor current along main winding axis.

(9) I_{r2} = Rotor current along auxiliary winding axis.

(10) I'_{r1}, I'_{r2} = Rotor currents after a sudden change along the main and auxiliary winding axes respectively.

(11) X_{22} = Self reactance of one rotor phase.

(12) X_{11} = Self reactance of one stator phase.

(13) X_m = Mutual reactance between stator and rotor phase.

(14) s = Fractional slip

(15) X_c = Capacitive reactance of the capacitor.

(16) λr_1, λr_2 = Rotor flux linkages.

(17) λs_1, λs_2 = Stator flux linkages.

(18) V = Applied voltage for motor operation.

(19) e = Open circuit voltage for a rotor current I_{r1} and I_{r2}.

(20) e' = Steady voltage that would exist at the terminals of the induction generator if the circuit breaker is opened suddenly while the generator is carrying load current.

(21) T_o' = Open circuit rotor time constant

CHAPTER- I

INTRODUCTION

When the stator winding of a polyphase induction machine is connected to supply, a synchronously rotating magnetic field is established in the air gap. The rotating field induces a voltage in the rotor conductors and the rotor if its circuit is closed, turns at a speed slightly less than that of the flux. The difference in the speed of rotation between the flux and the rotor, expressed as a percentage of synchronous speed, is called the slip. The slip is positive when the machine operates as a motor and the rotor speed is less than that of the flux.

As a generator, the induction machine is driven by a prime mover and as the speed of the rotor is increased to equal synchronous speed, there is no relative motion between the rotor conductors and the flux. Hence no voltage is induced in the rotor bars. A further increase in the speed causes a reversal in relative direction of rotation between the rotor bars and the flux, and the rotor voltage and current are correspondingly reversed. The slip under this condition is considered to be negative. The shaft torque supplied by the prime mover is transferred across the air-gap to the stator, from which it is delivered to the system as generated power. The net power output which is a function of slip equals the shaft input less the losses within the machine.

The driving of an induction machine faster than the synchronous speed causing it to generate alternating current power is quite well known. Some installations of induction machines operated in this manner have been made, but as yet the practice of using induction generators has not been adopted to any appreciable extent. Probably the main reason for this is that the induction machine must draw a lagging magnetizing current from the supply system to which it is connected. Moreover the voltage and frequency of the induction generator are dependent entirely upon that of the connected system. On the other hand the characteristic mechanical strength and ruggedness of an induction machine, its ability to run at high speeds, lesser station sustained short circuit risk and relatively low initial and upkeep cost are its advantages.

It may be shown that an induction machine can be made to operate with the exciting or magnetizing current obtained from static capacitance connected in shunt across the terminals of the machine, as an isolated or independent generator, incorporating all the foregoing advantages and most of the main disadvantages eliminated.

It can be seen that the only requirement for obtaining an output is a source of magnetizing Vars along with a suitable load below the power limit. The required Vars can be supplied by a synchronous machine or by shunt capacitors. Thus an output can be obtained from the induction generator when it is connected to a system consisting of only shunt capacitors and a load. Naturally some energy must be put (introduced) into the electrical system to start the build-up of voltage to the operating point. This can take the form of an initial charge on the capacitor, or more effectively, an initial current in the induction generator. Both of these sources are available when an induction generator, load and shunt capacitors are suddenly separated from a supply system.

If the induction generator is separated from a system with sufficient amount of capacitors connected to its terminals, the voltage rises or falls and the frequency changes until the net magnetizing Vars from the capacitor and load absorbed by the remaining system exactly match the requirements of the induction generator. In general an excess of capacitive Vars causes the voltage to rise. But if the number of Vars supplied by the capacitors exceed the value which can be absorbed by the induction generator, when the generator is delivering a given load, the load being such as to give a slip which is greater than the slip at the load determined from the power limit of the machine, then the voltage of the induction generator decays. The frequency of the system adjusts itself to a point where the generator slip gives the required load output.

It has been observed that the induction generator with static capacitance connected in shunt across its terminals will build up its voltage in a manner similar to the build up of the d.c. shunt generator[1]. Residual magnetism in the iron of the magnetic circuit sets up a small alternating voltage in the stator, this voltage applied to the capacitance causes a lagging magnetizing current to flow in the stator winding (machine applies leading quadrature current to the capacitance or draws a lagging quadrature current). If the capacitance is of the proper value, the current that can flow will be large enough to increase the flux existing in the air gap. An increase of the air gap flux will result in a higher voltage, larger exciting current drawn by the capacitance, more air gap flux and so on until the terminal voltage of the machine reaches its final build-up value. This value is determined by the saturation curve of the machine and by the capacitive reactance of the connected capacitance.

If both, the saturation curve (terminal voltage at no-load versus exciting current) and a straight line through origin, the slope of which is the capacitive reactance X_c are plotted to the same scales, the point where the straight line intersects the saturation curve is the final build up point. If the saturation curve of the induction generator is known, the final build up voltage for any particular capacitive reactance can be predetermined.

SINGLE PHASE INDUCTION MACHINE

It has been found that exactly same phenomena takes place in the case of a single phase induction motor and it can as well be made to operate as an induction generator by either rotating the machine at speeds greater than the synchronous speed, or by connecting capacitors across its stator terminals. In addition to this if the capacitor is connected in series with the machine stator winding, the machine constants can be well compensated to give fairly flat external characteristic.

G. ANGST[2] in his paper has presented a method for calculating the limits of self excitation in capacitor induction motor when the motors are disconnected from the power source and operating at constant speed. However, in this paper practical means are suggested to suppress the self excitation.

The self excitation of single phase motors was also studied by A. ISONO and K. OKUDA[3] and they have presented the analytical results of self excitation of single phase induction motor taking into account the external capacitor. The limit of the terminal voltage of the induction motor due to self excitation at the time of source disconnection was also obtained in this paper.

All this work was to eliminate or reduce the undesirable effects of self excitation at the instant of switching off a single phase capacitor motor connected to a load of high moment of inertia. No technical paper has come to the knowledge of the author where this phenomena is utilised to obtain single phase supply. In the present thesis an attempt has been made to this effect. Both the transient and steady state characteristics have been examined in detail with experimental verification.

CHAPTER - II

ANALYSIS AND EXPERIMENTAL VERIFICATION

2.1 General

2.2 Assumptions and Discussion

2.3 Expressions for transient voltage under different operating conditions of the main and auxiliary winding with experimental verifications.

CASE I :- Main winding connected to source and auxiliary winding open.

CASE II :- Main winding connected to source through capacitor and auxiliary winding open.

CASE III :- Main winding as well as auxiliary winding through capacitor connected to single phase supply.

CASE IV :- Main winding connected to source and auxiliary winding short circuited through capacitor.

2.1 GENERAL

When a three phase induction motor with external capacitor is disconnected from the power source, the terminal voltage of the motor may rise depending upon the circuit conditions. The terminal voltage of a three phase induction motor driven by a prime mover, will rise upto a certain value, if suitable capacitors are connected in its circuit. The voltage build-up is determined by the magnetization curve of the induction motor, capacitance of the capacitor and the number of rotations of the motor.

The same phenomenon also appears in the single phase capacitor motor. But the machine may develop a high transient voltage at its terminals even without a capacitor. The limit of excitation may be determined from Hurwitz criteria[3]. The analytical result of self excitation of single phase induction motor taking into account the external capacitor was obtained by A. ISONO and K. OKUDA by the use of method suggested by W.V. LYON[4]. The limit of the terminal voltage of the induction motor due to self excitation at the time of source disconnection was also obtained by them.

In the present analysis, equations for transient voltages across stator winding have been deduced following the method of analysis for three phase induction machine as given by A.E. FITZGERALD and C. KINGSLEY (JR.)[6]. For this analysis flux linkages of the rotor along any axis are required to be known. The flux linkages of rotor along any axis were obtained by FITZGERALD and KINGSLEY in the most generallised form for a three phase machine which was then extended to two phase balanced and single phase (special case of two phase balanced) machines.

Thus in the present work starting from the various voltage equations (obtained from the flux linkage equations) for the case of a balanced two phase machine, the current distribution in the various windings was obtained for varying operating conditions and from these currents, the flux linkages and subsequently the transient voltage obtainable at the stator winding terminals was found. The voltage so obtained was entirely dependent on the parameters of the machine, the parameters of the machine being evaluated by conducting the tests so as to simulate, as far as possible, all the conditions in the assumptions.

The following different combinations of the windings were studied for this purpose. These are

No.	MAIN WINDING	AUXILIARY WINDING
1.	Connected to source	Open
2.	Voltage V through capacitor	Open
3.	Voltage V	Voltage V through capacitor.
4.	Connected to source	Short circuited through capacitor.

Thus either, both the main and auxiliary windings were used, or only main (or auxiliary) winding was used, thereby resulting a two phase or a single phase induction generator operation.

2.2 ASSUMPTIONS

The present analysis is based on the following assumptions -

(1) The exciting conductance is negligible.

(2) The impedance is converted to the rotor side.

(3) The resistance and the self reactance of main and auxiliary winding referred to the rotor side are equal.

(4) The main winding and auxiliary winding are in space quadrature.

(5) The rotor is a balanced polyphase structure.

(6) The parameters of the motor used in theory are those of a balanced two phase motor.

(7) The resistance of any external capacitor is neglected.

(8) The ratio of the effective number of turns on the auxiliary winding to main winding is unity.

(9) Flux and M.M.F. distribution is sinusoidal.

Assumption (1) is equivalent to neglecting the core loss occuring in the machine. As is quite well known from the induction motor performance calculation, the core loss constitute only a small portion of the total losses in the machine, and therefore can be safely neglected.

Assumptions (2), (3) and (8) are equivalent to saying that the resistance and the self reactance of the main winding and the auxiliary winding referred to the rotor side, are equal. Assumption (3) was simulated by equalizing the impedance of the main winding and auxiliary winding by inserting the required impedance in either of the winding.

In order to simplify the analysis assumption (8) was made. However, if the ratio of the effective number of turns on the auxiliary winding to the effective number of turns on the main winding is not unity, the analysis can be carried out on the same lines, considering that the auxiliary winding is made up of two windings as in a transfomer, having the transformation ratio equal to the turns ratio of the main machine. Thus the turns ratio of the main winding to the fictitious auxiliary winding will be unity and the turns ratio between the fictious winding and the actual auxiliary winding will be the turns ratio of the actual machine. The resistance and reactance of the actual auxiliary winding can now easily be converted to the fictitious winding by ordinary transformer theory.

Assumption (6) was used to analyse a balanced two phase machine from which various other cases could be easily derived. This resulted in simplified analysis.

As both the main winding and the auxiliary winding were used in some operations, the parameters as calculated on the basis of either cross-field theory or double revolving field theory cannot be accepted. Therefore the parameters involved in the analysis must be the same as for balanced two phase operation.

2.3 EXPRESSIONS FOR TRANSIENT VOLTAGE UNDER DIFFERENT OPERATING CONDITIONS OF THE MAIN AND AUXILIARY WINDING WITH EXPERIMENTAL VERIFICATIONS.

CASE I :-

(i) Initial operation :- Single phase operation with main winding connected to source and auxiliary winding open.

(ii) Generator Operation :- Power developed from main winding connected to source with auxiliary winding open, the machine being run at super-synchronous speed.

If a two phase motor initially running on single phase supply with one winding open, is driven by a prime mover at a speed more than its synchronous speed, it will draw the exciting current from the synchronous machinery of the system to which it is connected i.e. the exciting current of the induction generator will be drawn from the source of supply.

For single phase operation of a two phase motor $I_a = 0$, therefore the flux linkage equations are given by (from Appendix 1).

$$\lambda s_1 = L_{11} \; I_m + M \; I_{r1}$$

Stator : .... (1)

$$\lambda s_2 = \pm \qquad M \; I_{r2}$$

$$\lambda r_1 = L_{22} \; I_{r1} + M \; I_m$$

Rotor : .... (2)

$$\lambda r_2 = L_{22} \; I_{r2}$$

and the voltage equations are given by

$$V = (r_1 + j \; X_{11}) \; I_m + j \; X_m \; I_{r1} - X_m \; I_{r2} \; .... \qquad (3 \; a$$

$$0 = (r_2 + j \; X_{22}) \; I_{r1} + j \; X_m \; I_m - s \; X_{22} \; I_{r2} \; .. \qquad (3 \; b$$

$$0 = (r_2 + j \; X_{22}) \; I_{r2} + s \; X_{22} \; I_{r1} + s \; X_m \; I_m \; .... \qquad (3$$

elimination of I_{r2} from (3 b) and (3 c) gives

$$\frac{(r_2 + j \; X_{22}) \; I_{r1} + j \; X_m \; I_m}{s \; X_{22} \; I_{r1} + s \; X_m \; I_m} = - \frac{s \; X_{22}}{r_2 + j \; X_{22}}$$

from which

$$I_m = - \left[\frac{(r_2 + j\, X_{22})^2 + s^2\, X_{22}^2}{j\, X_m\, r_2 + X_m\, X_{22}\, (s^2 - 1)} \right] I_{r1}$$

$$= \qquad K_{11} \quad I_{r1} \qquad \cdots\cdots\cdots\cdots \qquad (4)$$

Similarly, elimination of I_{r1} from equations (3 b) and (3 c) gives

$$I_m = - \left[\frac{(r_2 + j\, X_{22})^2 + s^2\, X^2_{22}}{s\, X_m\, r_2} \right] I_{r2}$$

$$= \qquad K_{21} \quad I_{r2} \qquad \cdots\cdots\cdots\cdots \qquad (5)$$

Now rotor flux linkages are $\lambda r_1 + j \lambda r_2$ and initially it has to remain constant. When the machine is disconnected from the supply the rotor currents should change to I'_{r1} and I'_{r2} such that

$$L_{22}\, I'_{r1} + j\, L_{22}\, I'_{r2} = L_{22}\, I_{r1} + M\, I_m + j\, L_{22}\, I_{r2}$$

which gives

$$I'_{r1} = I_{r1} + \frac{M}{L_{22}} \quad I_m$$

$$\text{and } I'_{r2} = I_{r2} \qquad \cdots\cdots\cdots\cdots \qquad (6)$$

Now the voltage induced is given by

$$e = j\,X_m\,I_{r1} - X_m\,I_{r2} \qquad \ldots\ldots \qquad (7)$$

When the supply is disconnected the induced voltage be e' say, then e' is given by

$$e' = j\,X_m\,I'_{r1} - X_m\,I'_{r2}$$

$$= j\,X_m\left(I_{r1} + \frac{M}{L_{22}}\,I_m\right) - X_m\,I_{r2} \;\ldots \qquad (8)$$

As in a synchronous machine or a three phase induction generator the voltage equation (instantaneous value) is given by

$$e + T'_o\,\frac{d}{dt}\,e' = 0 \qquad \ldots\ldots\ldots \qquad (9)$$

Where T'_o = open circuit rotor time constant.

The solution of above equation (9) is given by

$$e = e'\,\epsilon^{-t/T'_o} \qquad \ldots\ldots\ldots \qquad (10)$$

Where e' is given by (8) above.

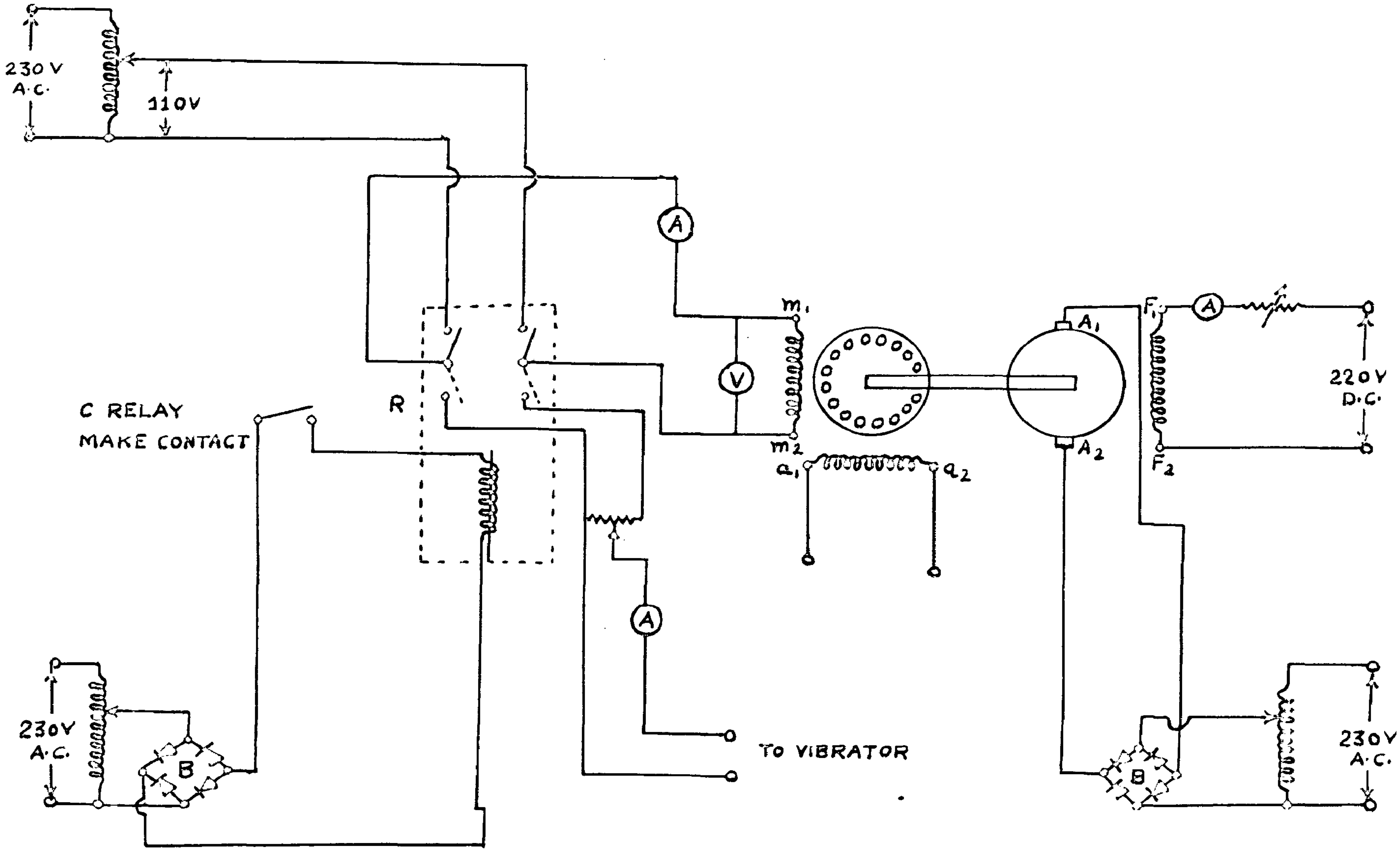
Now $e' = j\,X_m\left(I_{r1} + \frac{M}{L_{22}}\,I_m\right) - X_m\,I_{r2}$

$$= \left[j\,X_m\left(1 + \frac{X_m}{X_{22}}\cdot K_{11}\right) - \frac{X_m\,K_{11}}{K_{24}}\right] I_{r1} \qquad (8a)$$

Equation (10) gives the idea of the decay of the voltage of the machine from the initial value e' to any value e in the interval t. Also expression for e' shows that it is greater than e, this is due to the fact that [illegible] open circuit the drop in the winding becomes zero and more voltage is made available at the terminals of the machine.

<u>Experimental Verification</u> :- First the induction machine was run as a motor by connecting the main winding to the supply and keeping the auxiliary winding open and then gradually increasing the speed of the set by means of a coupled D.C. motor so as to run the set at a speed greater than the synchronous speed. The wattmeter connected in the main winding circuit during all these operations indicated the initial positive reading falling to zero and then to negative values. This ensures that the set is now operating as a generator. The generator output at the time when the source was disconnected, was fed through a rhehostat to one of the vibrator circuit of the Duddel Oscillograph. The relay being operated by a D.C. voltage of lower value. Thus the transient voltage induced in the winding was recorded with

EXPERIMENTAL SETUP FOR CASE I. (FIGURE 1).



the help of oscillograph, disconnecting the supply at the time of taking a photographic record by means of a relay.

Experimental set up is shown in fig. (1).

The result obtained is tabulated below.

Item	Theoretical	Experimental
Time Constant (in seconds)	0.126	0.123

The transient voltage wave form (oscillograph record) is shown in fig. (1a).

CASE II

(i) Initial Operation :- Single phase induction motor operation on one winding only.

(ii) Generator Operation:- Main winding disconnected from source and short circuited through a capacitor in parallel with load impedance.

In this case first the machine will draw its magnetizing current through the supply, but as the speed of the machine is made higher, the machine will start generating and now the supply phase is disconnected. The machine will act as a self excited induction generator, the exciting current now being supplied by the capacitors of the proper value. By slightly adjusting the speed of the machine the rated output voltage and frequency can be obtained.

For this operation the flux linkage equations are given by

$$\lambda s_1 = L_{11} I_m + M I_{r1}$$

Stator : .... (11)

$$\lambda s_2 = M I_{r2}$$

$$\lambda r_1 = L_{22} I_{r1} + M I_m$$

Rotor : .... (12)

$$\lambda r_2 = L_{22} I_{r2}$$

and the voltage equations are given by

$$V = r_1 + j(X_{11} - X_c) \; I_m + jX_m I_{r1} - X_m I_{r2} \quad .. \; (13a)$$

$$0 = (r_2 + jX_{22}) I_{r1} + j X_m I_m - s X_{22} I_{r2} \quad .... \; (13b)$$

$$0 = (r_2 + j X_{22}) I_{r2} + s X_{22} I_{r1} + s X_m I_m \quad .... \; (13c)$$

When the machine acts as a self excited generator then additional following equations are also satisfied namely,

$$X_c . I_c = V \qquad \ldots\ldots\ldots \qquad (14)$$

and $I_c = I_m$

As in CASE I, elimination of I_{r2} from (13 b) and (13 c) gives

$$I_m = -\left[\frac{(r_2 + j X_{22})^2 + s^2 X_{22}^2}{j X_m r_2 + X_m X_{22} (s^2 - 1)}\right] I_{r1}$$

$$= K_{12} \; I_{r1} \qquad \ldots\ldots \qquad (15)$$

and elimination of I_{r1} from (13 b) and (13 c) gives

$$I_m = -\left[\frac{(r_2 + j X_{22})^2 + s^2 X_{22}^2}{s X_m r_2}\right] I_{r2}$$

$$= K_{22} \; I_{r2} \qquad \ldots\ldots\ldots \qquad (16)$$

repeating the same procedure as in CASE I, gives

$$e' = j X_m \left(I_{r1} + \frac{M}{L_{22}} I_m\right) - X_m I_{r2}$$

$$= \left[j X_m \left(1 + \frac{X_m}{X_{22}} K_{12}\right) - \frac{X_m . K_{12}}{K_{22}}\right] I_{r1} \quad \ldots \quad (17)$$

The voltage equation is given by

$$e + T\,\frac{d\,e'}{d\,t} = 0 \quad \ldots\ldots\ldots \quad (18)$$

The solution of (18) is given by

$$e = e'\epsilon^{-t/T} \quad \ldots\ldots\ldots \quad (19)$$

Where $T = T_o' \times \frac{\text{flux linkages in this case}}{\text{flux linkages in the CASE I}}$

and T_o' = rotor open circuit time constant.

Now flux linkages in this case are given by

$$\lambda_{r1} + j\,\lambda_{r2} = L_{22}\,I_{r1} + M\,I_m + j\,L_{22}\,I_{r2} \ldots \quad (20)$$

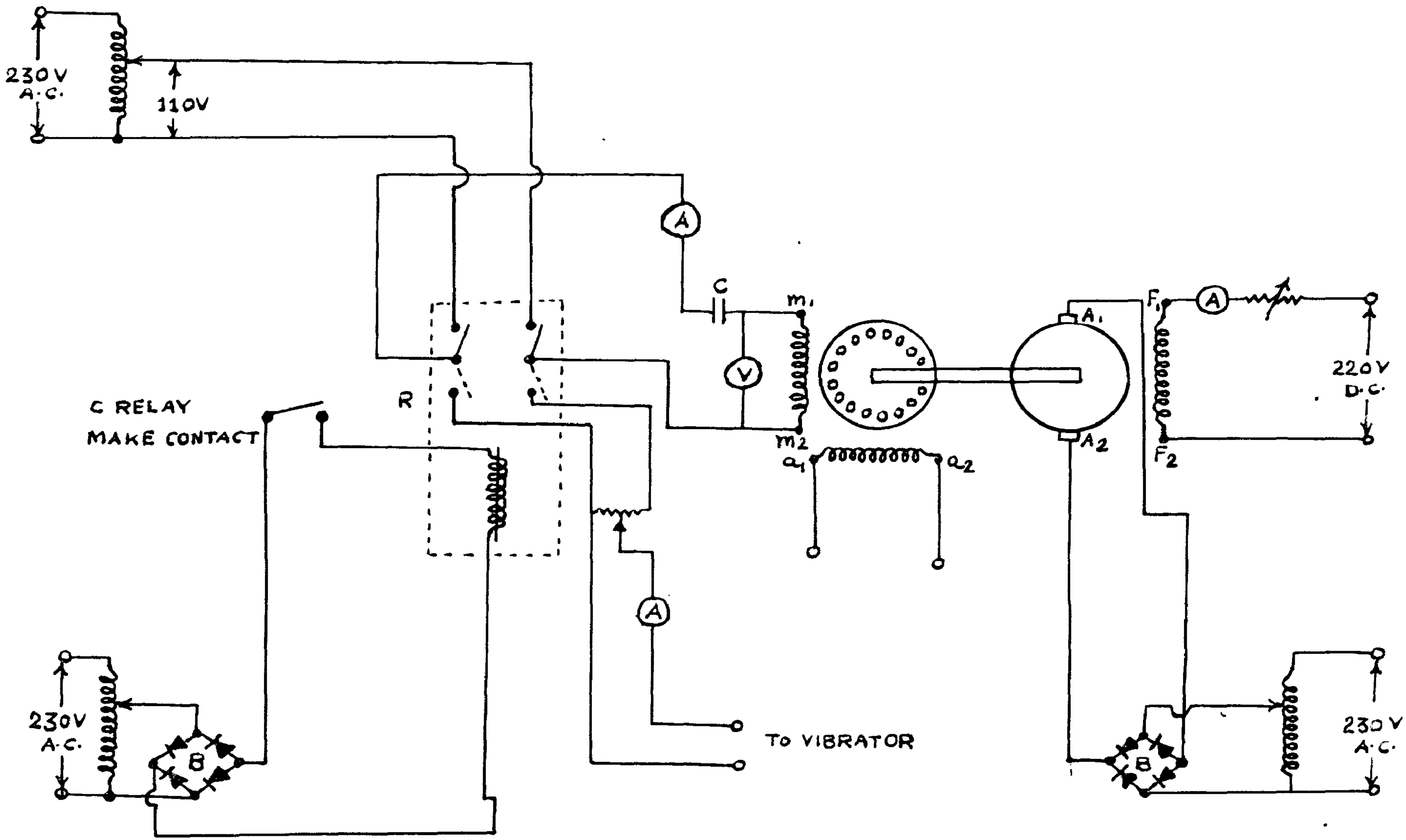
$$= \left[\frac{L_{22}}{K_{12}} + M + j\,\frac{L_{22}}{K_{22}}\right] I_m \quad \ldots \quad (20a)$$

Also I_m is given by (13a) after substituting for I_{r1} and I_{r2} from (15) and (16) in terms of I_m, thus

$$\left[r_1 + j(X_{11} - X_c) + \frac{j\,X_m}{K_{12}} - \frac{X_m}{K_{22}}\right] I_m = V$$

$$\text{or} \quad I_m = \frac{V}{\left[r_1 + j(X_{11} - X_c) + \frac{j\,X_m}{K_{12}} - \frac{X_m}{K_{22}}\right]} \quad (21)$$

EXPERIMENTAL SETUP FOR CASE II. (FIGURE 2).



Flux linkages in CASE I are given by (20a) similarly, but however I_m is given by

$$I_m \left[(r_1 + j\, X_{11}) + \frac{j\, X_m}{K_{11}} \quad \frac{X_m}{K_{21}} \right] = V \quad \text{For CASE I}$$

$$\text{or} \quad I_m = \frac{V}{\left[(r_1 + j\, X_{11}) + \frac{j\, X_m}{K_{11}} - \frac{X_m}{K_{21}} \right]} \quad \ldots\ldots \; (22)$$

on substituting the values from (20a), (21) and (22) yield

$$T = T_o' \, x \left\{ \frac{(r_1 + j\, X_{11}) + \frac{j\, X_m}{K_{11}} - \frac{X_m}{K_{21}}}{r_1 + j(X_{11} - X_c) \;+ \frac{j\, X_m}{K_{12}} \quad \frac{X_m}{K_{22}}} \right\} \;.. \; (23)$$

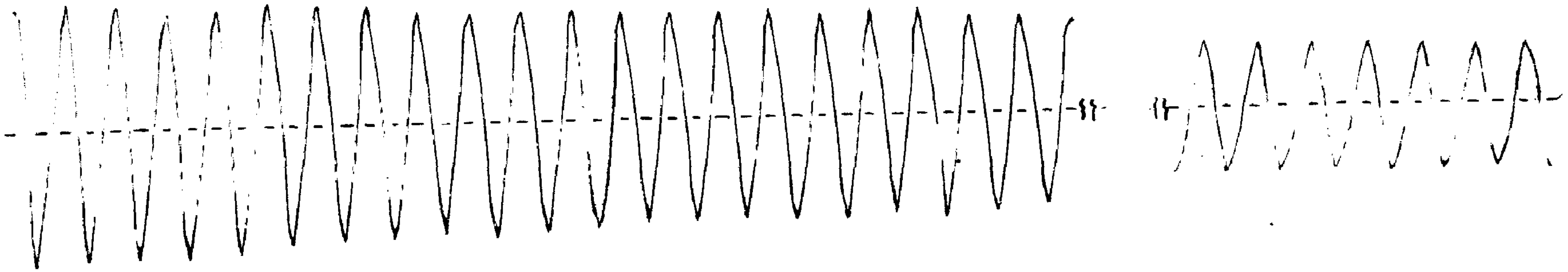
$$\text{Now since} \quad K_{11} = K_{12}$$

$$\text{and} \quad K_{21} = K_{22}$$

equation (23) reduces to

$$T = T_o' x \left\{ \frac{r_1 - \frac{X_m}{K_{21}} + j \, (X_{11} + \frac{X_m}{K_{11}} \,)}{r_1 - \frac{X_m}{K_{21}} + j \, (X_{11} - X_c + \frac{X_m}{K_{11}}} \right\} \ldots \; (23a)$$

If X_c is greater than $(X_{11} + \frac{X_m}{K_{11}})$ in



TRASIENT VOLTAGE WAVEFORM FOR CASE II. FIGURE (2a).

magnitude, then T becomes negative.

Hence the time constant T thus obtained is negative showing thereby that the machine is acting as a self excited generator.

Experimental Verification

In this case also the record of the transient voltage induced in the winding was taken by means of the arrangement described in CASE I. The machine was made a self excited generator, by adjusting the speed of the set slightly or by proper amount of capacitors in the circuit, the rated output voltage and frequency were adjusted. The experimental set up is as shown in fig.(2).

The result obtained is tabulated below.

Item	Theoretical	Experimental
Initial induced voltage e' in volts.	101.0	103

The transient voltage wave form is shown in fig. (2a).

CASE III

(i) Initial Operation :- Capacitor run single phase motor i.e. main winding as well as auxiliary winding through capacitor connected to single phase supply.

(ii) Generator Operation:- Electric connection unaltered, the machine being run at supersynchronous speed.

In this case the machine connected as a capacitor run motor to single phase supply (split phase motor), is made to run at a speed higher than the synchronous speed by a prime mover.

For this operation, the flux linkage equations are given by

$$\text{Stator :} \quad \begin{aligned} \lambda s_1 &= L_{11}\, I_m + M\, I_{r1} \\ \lambda s_2 &= L_{11}\, I_a + M\, I_{r2} \end{aligned} \qquad \text{.... (24)}$$

$$\text{Rotor :} \quad \begin{aligned} \lambda r_1 &= L_{22}\, I_{r_1} + M\, I_m \\ \lambda r_2 &= L_{22}\, I_{r2} + M\, I_a \end{aligned} \qquad \text{... (25)}$$

and the voltage equations are given by

$$\begin{bmatrix} (r_1 + j\,X_{11}) & -\,X_{11} & jX_m & -\,X_m \\ +\,X_{11} & \left[r_1+j(X_{11}-X_c)\right] & +X_m & jX_m \\ j\,X_m & -\,s\,X_m & (r_2+jX_{22}) & -sX_{22} \\ s\,X_m & j\,X_m & s\,X_{22} & (r_2+jX_{22}) \end{bmatrix} \begin{bmatrix} I_m \\ I_a \\ I_{r1} \\ I_{r2} \end{bmatrix} = \begin{bmatrix} V \\ V \\ 0 \\ 0 \end{bmatrix} \qquad (26)$$

$$\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} I \end{bmatrix} = \begin{bmatrix} V \end{bmatrix}$$

Solving these voltage equations for various currents will lead to.

$$I_m = \begin{vmatrix} V & -X_{11} & j\,X_m & -X_m \\ V & \left[r_1+j(X_{11}-X_c)\right] & +X_m & j\,X_m \\ 0 & -s\,X_m & (r_2+j\,X_{22}) & -s\,X_{22} \\ 0 & j\,X_m & s\,X_{22} & (r_2+j\,X_{22}) \end{vmatrix} \div |A|$$

$$= \frac{V}{|A|}\left\{\left[r_1+j(X_{11}-X_c)+X_{11}\right]\left[r_2^2 + 2jr_2\,X_{22} + X_{22}^2(s^2-1)\right]\right.$$

$$\left. + X_m(1+j)\left[-jX_m\,r_2-js\,X_m\,r_2+X_m\,X_{22}(1-s^2)\right]\right\}$$

Similarly

$$I_a = \begin{vmatrix} (r_1+j\,X_{11}) & V & j\,X_m & -X_m \\ +X_{11} & V & +X_m & j\,X_m \\ j\,X_m & 0 & (r_2+jX_{22}) & -s\,X_{22} \\ s\,X_m & 0 & s\,X_{22} & (r_2+j\,X_{22}) \end{vmatrix} \div |A|$$

$$= \frac{V}{|A|}\left\{(r_1-X_{11}+j\,X_{11})\left[r_2^2 + 2jr_2\,X_{22} + X_{22}^2(s^2-1)\right]\right.$$

$$+ X_m(1+j)\left[sX_m\,r_2+X_m\,r_2-j\,X_m\,X_{22}(s^2-1)\right]$$

$$I_{r1} = \begin{vmatrix} (r_1 + j\,X_{11}) & -X_{11} & V & -X_m \\ +X_{11} & \left[r_1+j(X_{11}-X_c)\right] & V & j\,X_m \\ j\,X_m & -s\,X_m & 0 & -s\,X_{22} \\ s\,X_m & j\,X_m & 0 & (r_2+j\,X_{22}) \end{vmatrix} \div |A|$$

$$= \frac{V}{|A|}\Big\{(r_1+ X_{11}+ jX_{11})\,(s\,X_m\,r_2) - \left[r_1+X_{11}+j(X_{11}-X_c)\right]$$

$$\times\left[j\,X_m\,r_2+ X_m\,X_{22}(s^2-1)\right] + j\,X_m^3\,(1+j)\,(s^2-1)\Big\}$$

and $I_{r2} = \begin{vmatrix} (r_1+j\,X_{11}) & -X_{11} & j\,X_m & V \\ +X_{11} & \left[r_1+j(X_{11}-X_c)\right] & +X_m & V \\ j\,X_m & -s\,X_m & (r_2+jX_{22}) & 0 \\ s\,X_m & j\,X_m & s\,X_{22} & 0 \end{vmatrix} \div |A|$

$$= \frac{V}{|A|}\Big\{r_1+X_{11}+j\,X_{11})\left[-jX_m r_2 + X_m\,X_{22}(1-s^2)\right]$$

$$+ \left[r_1-X_{11}+ j(X_{11}-X_c)\right]\,(+s\,X_m\,r_2)+ X_m^3(1+j)(s^2-1)\Big\}$$

from these equations (in terms of known parameters) I_{r1}, I_{r2} and I_a are found in terms of I_m

let
$$I_{r1} = K_{13} I_m$$
$$I_{r2} = K_{23} I_m \qquad \ldots\ldots \qquad (27)$$
and
$$I_a = K_{33} I_m$$

Now the rotor flux linkages are $\lambda r_1 + j \lambda r_2$ and initially it has to remain constant. When the machine is disconnected from the supply the rotor currents should change to I'_{r1} and I'_{r2} such that

$$L_{22} I'_{r1} + j L_{22} I'_{r2} = (L_{22} I_{r1} + M I_m) + j (L_{22} I_{r2} + M I_a)$$

this gives

$$I'_{r1} = I_{r1} + \frac{M}{L_{22}} I_m$$
$$\ldots \qquad (28)$$
and
$$I'_{r2} = I_{r2} + \frac{M}{L_{22}} I_a$$

The voltage induced is given by

$$e = j X_m I_{r1} - X_m I_{r2} \qquad \ldots\ldots\ldots\ldots \qquad (29)$$

When the supply is disconnected, let the induced voltage be e', which is given by

$$e' = j X_m I'_{r1} - X_m I'_{r2}$$

$$= j\, X_m \left(I_{r1} + \frac{M}{L_{22}}\, I_m\right) - X_m\left(I_{r2} + \frac{M}{L_{22}}\, I_a\right) \ldots \qquad (30)$$

The voltage equation is given by

$$e + T\,\frac{d\,e'}{d\,t} = 0 \quad \ldots\ldots\ldots\ldots\ldots \qquad (31)$$

Where $T = T_o' \times \dfrac{\text{flux linkages in this case}}{\text{flux linkages for balanced two phase operation.}}$

and $T_o' =$ open circuit rotor time constant

The solution of (31) is given by

$$e = e'\,\epsilon^{-t/T} \quad \ldots\ldots\ldots \qquad (32)$$

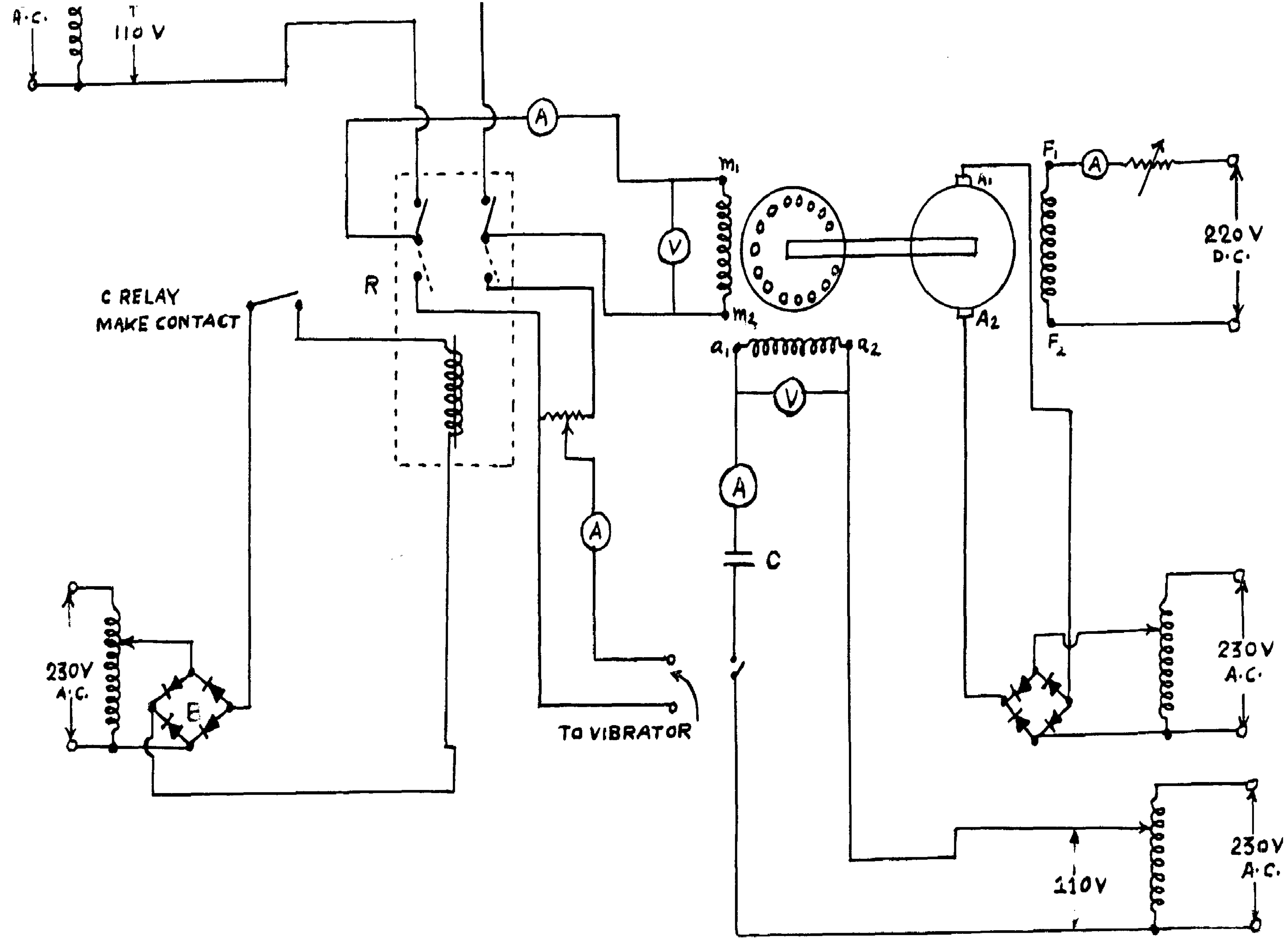
The initial value e' being given by (30). The value of e' can be further simplified by using equation (27).

Now $e' = j\, X_m\left(I_{r1} + \frac{M}{L_{22}}\, I_m\right) - X_m\left(I_{r2} + \frac{M}{L_{22}}\, I_a\right)$

expressing I_{r1}, I_{r2} and I_a in terms of I_m from (27) this gives

$$e' = \left[j\, X_m\left(K_{13} + \frac{X_m}{X_{22}}\right) - X_m\left(K_{23} + \frac{X_m}{X_{22}}\, K_{33}\right)\right] I_m \qquad (30a)$$

Also I_m is given by (26), after substituting for


A.C.
T
110 V
A
R
C RELAY
MAKE CONTACT
V
A
m1
m2
a1
a2
V
A
C
E
230V
A.C.
TO VIBRATOR
A1
A2
F1
F2
A
220 V
D.C.
230 V
A.C.
110V
230 V
A.C.

I_{r_1}, I_{r2} and I_a in terms of I_m, in this yields

$$\left[(r_1 + j\ X_{11}) + j\ X_m\ K_{13} - X_{11}\ K_{33} - X_m\ K_{23}\right] I_m = V$$

$$\text{or } I_m = \frac{V}{\left[(r_1 + j\ X_{11}) + j\ X_m\ K_{13} - X_{11}\ K_{33} - X_m\ K_{23}\right]} \quad ..(33)$$

Also flux linkages in this case are given by

$$\lambda_1 = \lambda_{r1} + j\ \lambda_{r2}$$

$$= (L_{22}\ I_{r1} + M\ I_m) + j\ (L_{22}\ I_{r2} + M\ I_a)$$

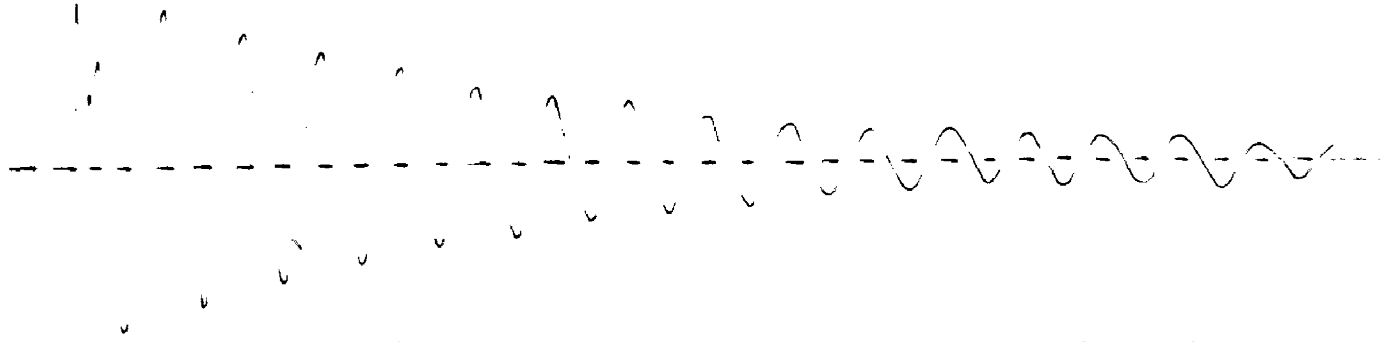
$$= \left[(L_{22}\ K_{13} + M) + j\ (L_{22}\ K_{23} + M\ K_{33})\right]\ I_m$$

$$= \frac{\left[(L_{22}\ K_{13} + M) + j\ (L_{22}\ K_{23} + M\ K_{33})\right]}{\left[(r_1 + j\ X_{11}) + j\ X_m\ K_{13} - X_{11}\ K_{33} - X_m\ K_{23}\right]}\ V \quad ...\ (34)$$

and the flux linkages for balanced two phase operation are given by

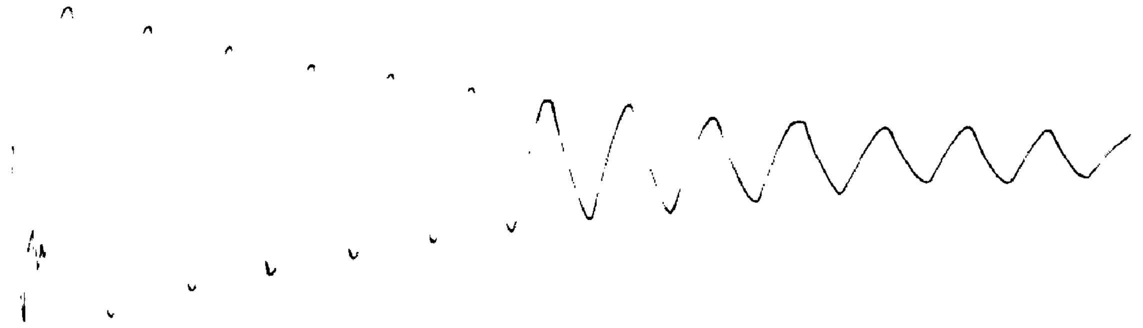
$$\lambda_2 = \frac{(L_{22}\ K_1 + M) + j(L_{22}\ K_2 + M\ K_3}{(r_1 + j\ X_{11}) + j\ X_m\ K_1 - X_{11}\ K_3 - X_m\ K_2} \cdot V$$

(from Appendix 1)



WAVE SHAPE OF TRANSIENT VOLTAGE FOR CASE III.

FIGURE (3a).



Therefore,

$$T = T_o' \frac{\left[(L_{22}K_{13}+M)+j(L_{22}K_{23}+MK_{33})\right]\left[r_1+jx_{11}+jx_mK_1-X_{11}K_3-X_mK_2\right]}{\left[(L_{22}K_1+M)+j(L_{22}K_2+MK_3)\right]\left[r_1+jx_{11}+jx_mK_{13}-X_{11}K_{33}-X_mK_{23}\right]}$$

In this case self excitation is not possible but if the machine is run at supersynchronous speed, generator action results. The load output of the machine being determined from the slip at which the machine works.

The experimental verification to the deduced analysis is rendered by determining the time constant experimentally.

The experimental set up is shown in fig.(3).

The result obtained is tabulated below.

Item	Theoretical	Experimental
Time constant in seconds T	0.122	0.120

The transient voltage waveform is shown in fig.(3a).

CASE IV

(i) Initial Operation :- Single phase motor operation with main winding connected to source and auxiliary winding short circuited through capacitor.

(ii) Generator Operation :- Main winding disconnected from source to supply load with auxiliary winding short circuited through capacitor.

The induction machine is run as a single phase motor with auxiliary winding short circuited through sufficient amount of capacitors and then the supply to main winding disconnected, the machine runs as a self excited induction generator. By adjusting the speed slightly the rated output voltage and frequency can be obtained from main winding.

The flux linkage equations are given by

Stator:
$$\begin{aligned} \lambda s_1 &= L_{11}\, I_m + M\, I_{r1} \\ \lambda s_2 &= L_{11}\, I_a + M\, I_{r2} \end{aligned} \qquad \ldots\ldots \quad (36)$$

Rotor:
$$\begin{aligned} \lambda r_1 &= L_{22}\, I_{r1} + M\, I_m \\ \lambda r_2 &= L_{22}\, I_{r2} + M\, I_a \end{aligned} \qquad \ldots\ldots \quad (37)$$

and the voltage equations for this operation are given by

$$\begin{bmatrix} (r_1 + j\,X_{11}) & -X_{11} & j\,X_m & -X_m \\ +X_{11} & \left[r_1 + j(X_{11} - X_c)\right] & +X_m & j\,X_m \\ j\,X_m & -s\,X_m & (r_2 + jX_{22}) & -s\,X_{22} \\ s\,X_m & j\,X_m & s\,X_{22} & (r_2 + jX_{22}) \end{bmatrix} \begin{bmatrix} I_m \\ I_a \\ I_{r1} \\ I_{r2} \end{bmatrix} = \begin{bmatrix} V \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (38)$$

$$[A] \quad [I] \quad = \quad [V]$$

Solving these for individual currents will lead to

$$I_m = \begin{vmatrix} V & -X_{11} & j\,X_m & -X_m \\ 0 & \left[r_1 + j(X_{11} - X_c)\right] & +X_m & j\,X_m \\ 0 & -s\,X_m & (r_2 + j\,X_{22}) & -s\,X_{22} \\ 0 & j\,X_m & s\,X_{22} & (r_2 + j\,X_{22}) \end{vmatrix} \div |A|$$

$$= \frac{V}{|A|} \Big\{ \left[r_1 + j(X_{11} - X_c)\right] \left[r_2^2 + 2\,j\,r_2\,X_{22} + X_{22}^2(s^2 - 1)\right]$$

$$+ X_m(+\,s\,X_m . r_2) + j\,X_m \left[-j\,X_m\,r_2 + X_m\,X_{22}(1 - s^2)\right] \Big\}$$

Similarly,

$$I_a = \begin{vmatrix} (r_1 + j\ X_{11} & V & j\ X_m & -\ X_m \\ +\ X_{11} & 0 & +\ X_m & j\ X_m \\ j\ X_m & 0 & (r_2 + j\ X_{22}) & -s\ X_{22} \\ s\ X_m & 0 & s\ X_{22} & (r_2 + j\ X_{22}) \end{vmatrix} \div |A|$$

$$= \frac{-V}{|A|} \left\{ X_{11} \left[r_2^2 + 2\ j\ r_2\ X_{22} + X_{22}^2 (s^2 - 1) \right] + j\ X_m(-s\ X_m\ r_2) \right.$$

$$\left. -\ X_m \left[j\ X_m\ r_2 + X_m\ X_{22}\ (s^2 - 1) \right] \right\}$$

$$I_{r_1} = \begin{vmatrix} (r_1 + j\ X_{11}) & -\ X_{11} & V & -\ X_m \\ +\ X_{11} & \left[r_1 + j(X_{11} - X_c) \right] & 0 & j\ X_m \\ j\ X_m & -\ s\ X_m & 0 & -\ s\ X_{22} \\ s\ X_m & j\ X_m & 0 & (r_2 + j\ X_{22}) \end{vmatrix} \div |A|$$

$$= \frac{V}{|A|} \left\{ -\ X_{11}(s\ X_m\ r_2) - \left[r_1 + j(X_{11} - X_c) \right] \right.$$

$$\left. \times \left[j\ X_m\ r_2 + X_m\ X_{22}\ (s^2 - 1) \right] + j\ X_m^3\ (s^2 - 1) \right\}$$

and

$$I_{r2} = \begin{vmatrix} (r_1 + j\,X_{11}) & -X_{11} & j\,X_m & V \\ +X_{11} & \left[r_1 + j(X_{11} - X_c)\right] & +X_m & 0 \\ j\,X_m & -s\,X_m & (r_2 + j\,X_{22}) & 0 \\ s\,X_m & j\,X_m & s\,X_{22} & 0 \end{vmatrix} \div |A|$$

$$= \frac{V}{|A|} \Big\{ X_{11} \left[-j\,r_2\,X_m + X_m\,X_{22}\,(1 - s^2) \right] + X_m^3\,(s^2 - 1) + \left[r_1 + j\,(X_{11} - X_c) \right] (+s\,X_m\,r_2) \Big\}$$

From above expressions I_{r1}, I_{r2} and I_a are found in terms of I_m

let $I_{r1} = K_{14}\,I_m$

$I_{r2} = K_{24}\,I_m$ .... (39)

and $I_a = K_{34}\,I_m$

Now the rotor flux linkages are $\lambda_{r1} + j\,\lambda_{r2}$ and initially it has to remain constant. When the machine is disconnected from the supply the rotor currents should change to I'_{r1} and I'_{r2} such that

$$L_{22}\,I'_{r1} + j\,L_{22}\,I'_{r2} = (L_{22}\,I_{r1} + M\,I_m) + j(L_{22}\,I_{r2} + M\,I_a)$$

this gives

$$I'_{r1} = I_{r1} + \frac{M}{L_{22}} I_m \qquad \ldots\ldots \qquad (40)$$

and $$I'_{r2} = I_{r2} + \frac{M}{L_{22}} I_a$$

The voltage induced is given by

$$e = j\, X_m\, I_{r1} - X_m\, I_{r2} \qquad \ldots\ldots \qquad (41)$$

When the supply is disconnected, let the induced voltage be e', which is given by

$$e' = j\, X_m\, I'_{r1} - X_m\, I'_{r2}$$

$$= j\, X_m(I_{r1} + \frac{M}{L_{22}} I_m) - X_m(I_{r2} + \frac{M}{L_{22}} I_a) \ldots \qquad (42)$$

The voltage equation is given by

$$e + T \frac{d\, e'}{d\, t} = 0 \ldots\ldots\ldots\ldots \qquad (43)$$

Where $T = T'_o \times \dfrac{\text{flux linkages in this case}}{\text{flux linkages for balanced two phase operation.}}$

and T'_o = open circuit rotor time constant

The solution of (43) is given by

$$e = e' \epsilon^{-t/T} \qquad \ldots\ldots \qquad (44)$$

The initial value of e' being given by (42)

The value of e' can be further modified by using (39).

Now $e' = j\,X_m(I_{r1} + \frac{M}{L_{22}} I_m) - X_m(I_{r2} + \frac{M}{L_{22}} I_a)$

expressing I_{r1}, I_{r2} and I_a in terms of I_m from (39) this gives

$$e' = \left[j\,X_m(K_{14} + \frac{X_m}{X_{22}}) - X_m\,(K_{24} + \frac{X_m}{X_{22}} K_{34}) \right] I_m \quad \ldots \quad (42a)$$

Also I_m is given by (38), after substituting for I_{r1}, I_{r2} and I_a in terms of I_m in this, yields

$$\left[(r_1 + j\,X_{11}) + j\,X_m\,K_{14} - X_{11}\,K_{24} - X_m\,K_{34} \right] I_m = V$$

$$\text{or} \quad I_m = \frac{V}{\left[(r_1 + j\,X_{11}) + j\,X_m K_{14} - X_{11}\,K_{24} - X_m\,K_{34} \right]} \qquad (45)$$

Also flux linkages in this case are given by

$$\lambda_1 = \lambda r_1 + j\ \lambda r_2$$

$$= (L_{22}\,I_{r1} + M\,I_m) + j\,(L_{22} I_{r2} + M\,I_a)$$

$$= \left[(L_{22}\,K_{14} + M) + j\,(L_{22}\,K_{24} + M\,K_{34}) \right] I_m$$

$$= \frac{L_{22} K_{14} + M) + j\,(L_{22} K_{24} + M K_{34})}{(r_1 + j X_{11}) + j X_m K_{14} - X_{11} K_{34} - X_m K_{24}} \cdot V \quad \ldots (46)$$

and the flux linkages for balanced two phase operation are given by

$$\lambda_2 = \frac{(L_{22} K_1 + M) + j\,(L_{22} K_2 + M K_3)}{(r_1 + j X_{11}) + j X_m K_1 - X_{11} K_3 - X_m K_2} \cdot V$$

(from Appendix 1)

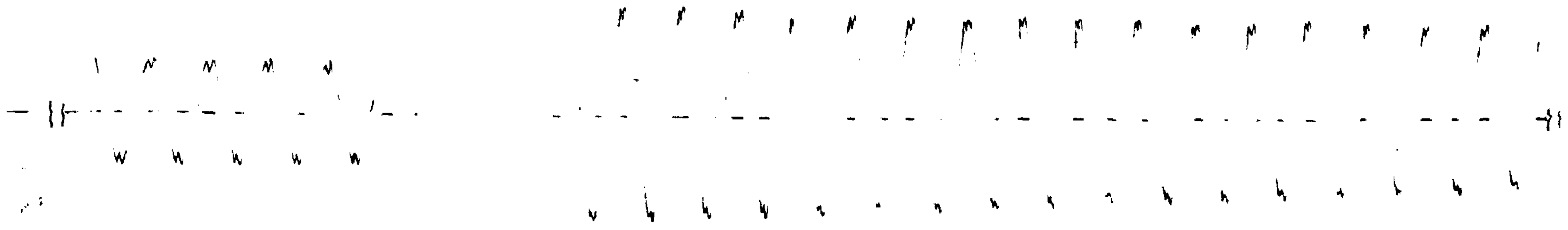
Therefore,

$$\boxed{T = T_0' \frac{\left[(L_{22} K_{14} + M) + j\left(L_{22} K_{24} + M K_{34}\right)\right]\left[r_1 + j x_{11} + j x_m K_1 - x_{11} K_3 - X_m K_2\right]}{\left[(L_{22} K_1 + M) + j\,(L_{22} K_2 + M K_3)\right]\left[r_1 + j x_{11} + j x_m K_{14} - x_{11} K_{34} - X_m K_{24}\right]}}$$

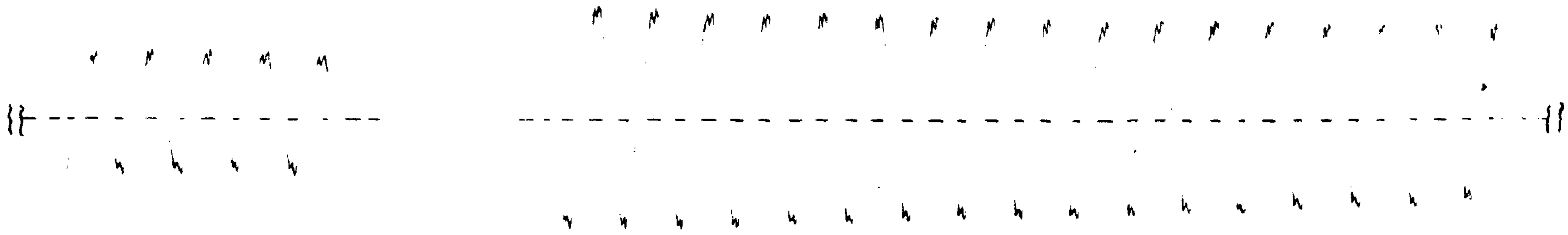
This time constant T can be shown to be negative indicating thereby that the machine is a self excited generator and will build up its voltage depending on the saturation curve of the machine and reactance of the excitation capacitance.

Experimental Verification :-

The machine under test was run as a motor by connecting its main winding to single phase supply and auxiliary winding short circuited through capacitor. The speed was adjusted and with proper amount of capacitors in the auxiliary winding circuit, the supply to the main winding was disconnected.



TRASIENT VOLTAGE WAVE FORM FOR CASE IV FIGURE (4a)



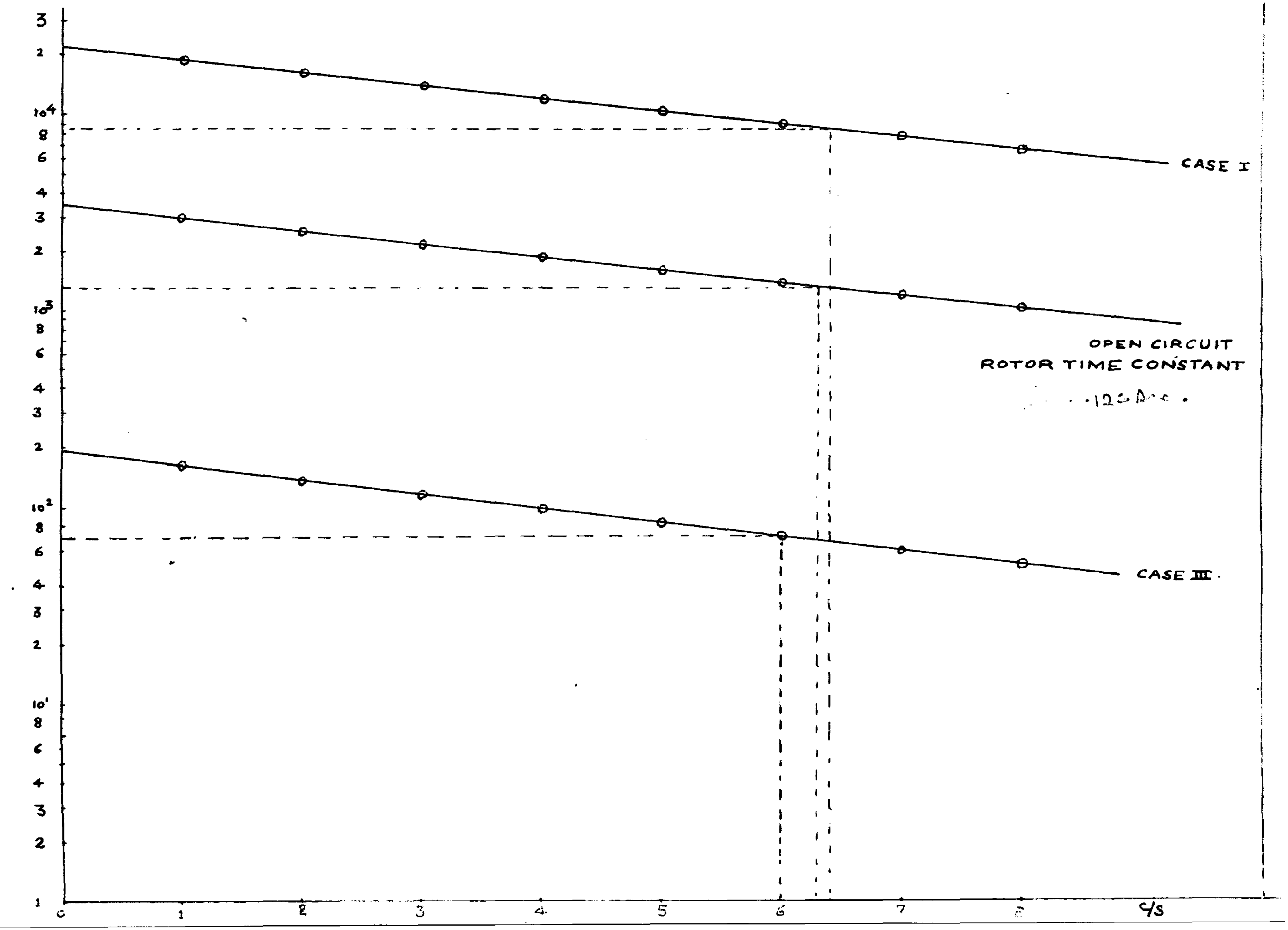
The machine now operates as a self excited generator, the exciting current being supplied by the capacitors of the auxiliary winding circuit. Again the oscillograph record was taken for the transient voltage induced in the main winding by means of a relay connected across the output of the machine, which in turn was connected to the vibrator circuit of the oscillograph.

The experimental set up is shown in fig.(4). The result obtained is tabulated below.

Item	Theoretical	Experimental
Initial induced voltage e' in volts.	102	105

The transient voltage waveform is shown in fig.(4a).

In the above analysis, equation for transient rise or decay of the voltage induced in the winding was obtained for different cases. The initial value of the voltage was also obtained. Thus at any instant the voltage of the machine can be predetermined for any of the cases discussed above.


3
2
10^4
8
6
4
3
2
10^3
8
6
4
3
2
10^2
8
6
4
3
2
10^1
8
6
4
3
2
1
0
1
2
3
4
5
6
7
8
c/s
CASE I
OPEN CIRCUIT
ROTOR TIME CONSTANT
·125 sec.
CASE III.

CHAPTER - III

STEADY STATE CHARACTERISTICS AND DISCUSSION OF THE RESULTS.

The following cases were studied in detail for the steady operation of the single phase induction generator.

CASE I :- Main winding connected to source and auxiliary winding open, the machine being run at supersynchronous speed.

CASE II :- Main winding connected to source and auxiliary winding loaded by resistive load, the machine running at supersynchronous speed. (The main winding kept floating).

CASE III:- Main winding as well as auxiliary winding through capacitor connected to single phase supply, the machine being run at supersynchronous speed.

CASE IV :- Main winding connected to source and auxiliary winding short circuited through capacitor and then main winding supply disconnected afterwards (self excitation case).

In this case following combinations were experimented :-

(i) With main winding unloaded.
(ii) Main winding loaded with constant impedance.
(iii) Main winding loaded with variable resistive load.

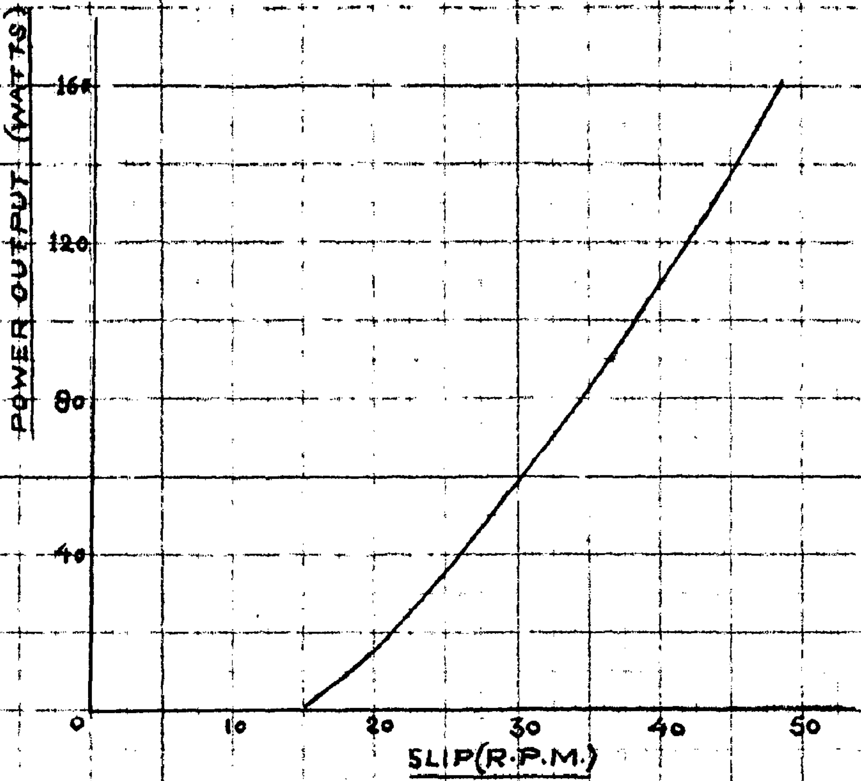


FIGURE (5b).

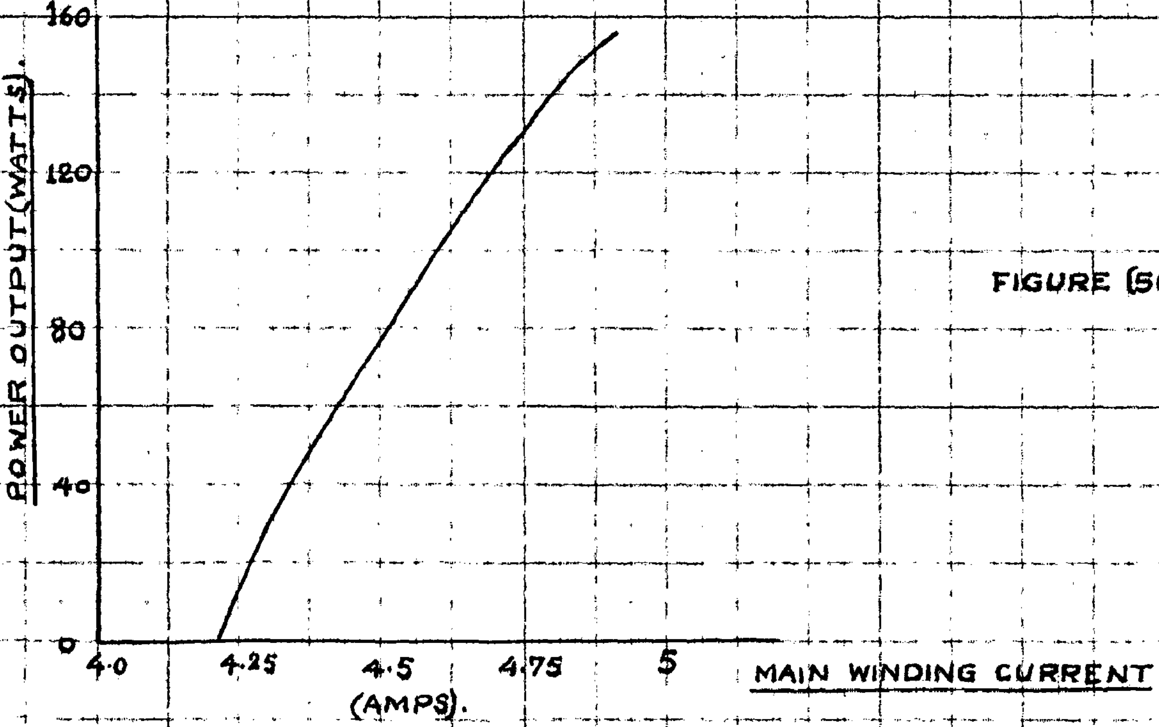


FIGURE (5a).

Considering first CASE I, the machine was started as a single phase induction motor with one winding only. Then the speed of the prime mover was increased gradually above the synchronous speed of the machine thereby operating it as an induction generator. The exciting current of the machine is drawn from the supply. The following curves plotted :-

(1) Main winding current versus power output (fig. 5 a).

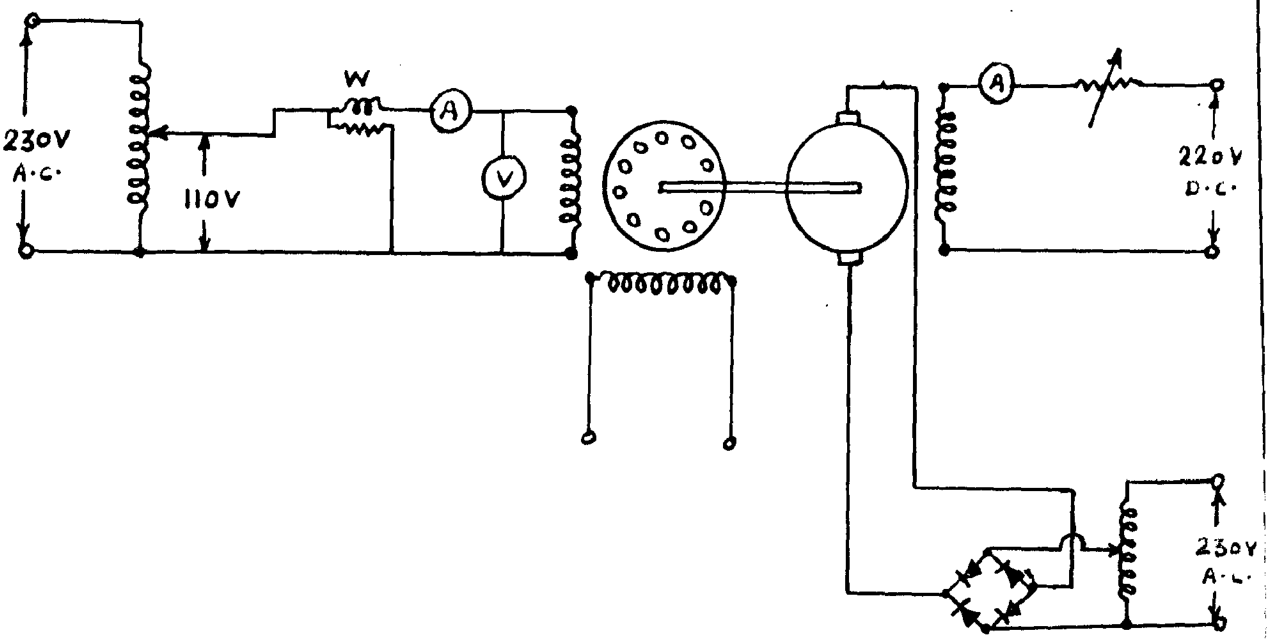
(2) Slip (in R.P.M.) versus power output (fig. 5 b).

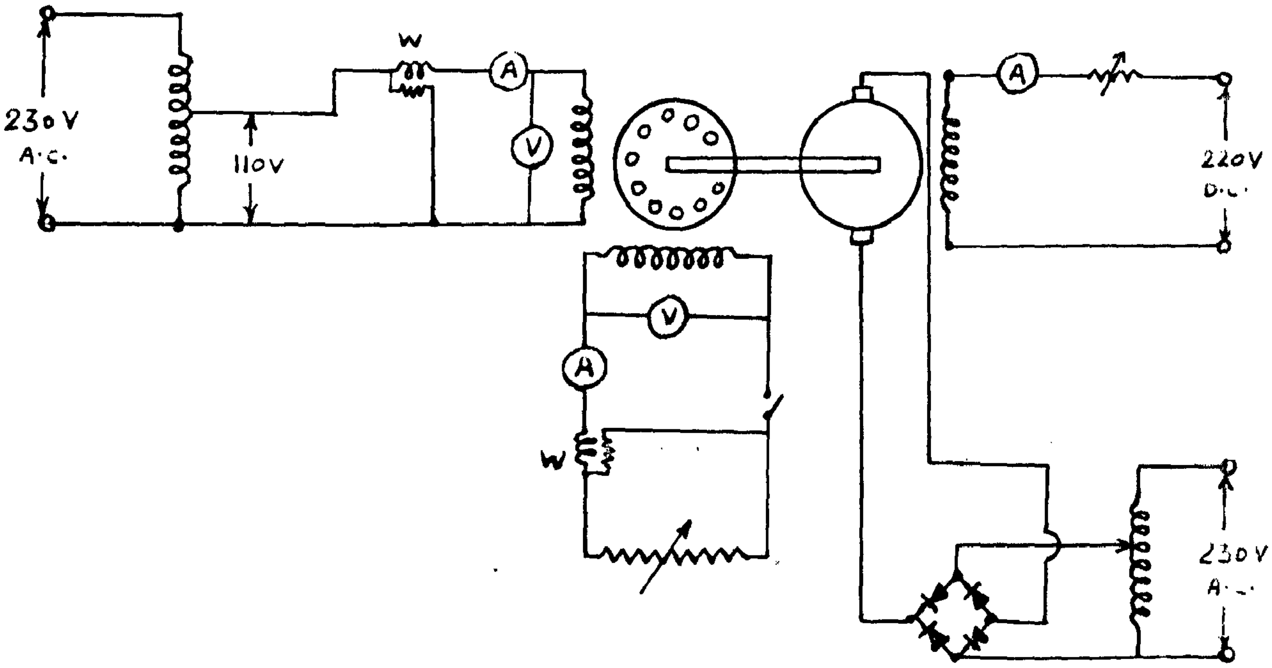
The experimental set up is shown in fig. (5).

The above curves show that the power output of such a generator can be increased by increasing the speed of the machine or the current delivered by the machine to supply. The limit of power output being decided by the current rating of the machine.

In CASE II, when the machine runs as generator at supersynchronous speed with main winding connected to source and auxiliary winding loaded with resistive load, the main supply can be made to furnish only the core

EXPERIMENTAL SETUP FOR CASE I. (FIGURE 5).





EXPERIMENTAL SETUP FOR CASE II. (FIGURE 6).

loss and small coper loss. The auxiliary winding load is delivered by the prime mover together with the mechanical losses of the set. The main winding was not used to take any load and it was kept just floating such that with a slight change in the speed, the main winding could be loaded. The variation in the speed of the set such that the wattmeter connected in the circuit of the main winding always brought to the same reading (indicating the core loss and a small copper loss) after change in the load on the auxiliary winding, was noted. Thus the main winding was not supplying any load, the load of the auxiliary winding together with the mechanical losses of the set being supplied by the prime mover. The machine was loaded by resistive load only as with inductive loads, terminal voltage drops excessively.

The test results are reported in the form of the following curves :-

(1) Slip (r.p.m.) versus output of the auxiliary winding (fig. 6 a).
(2) Main winding current versus auxiliary winding current. (fig. 6 b).

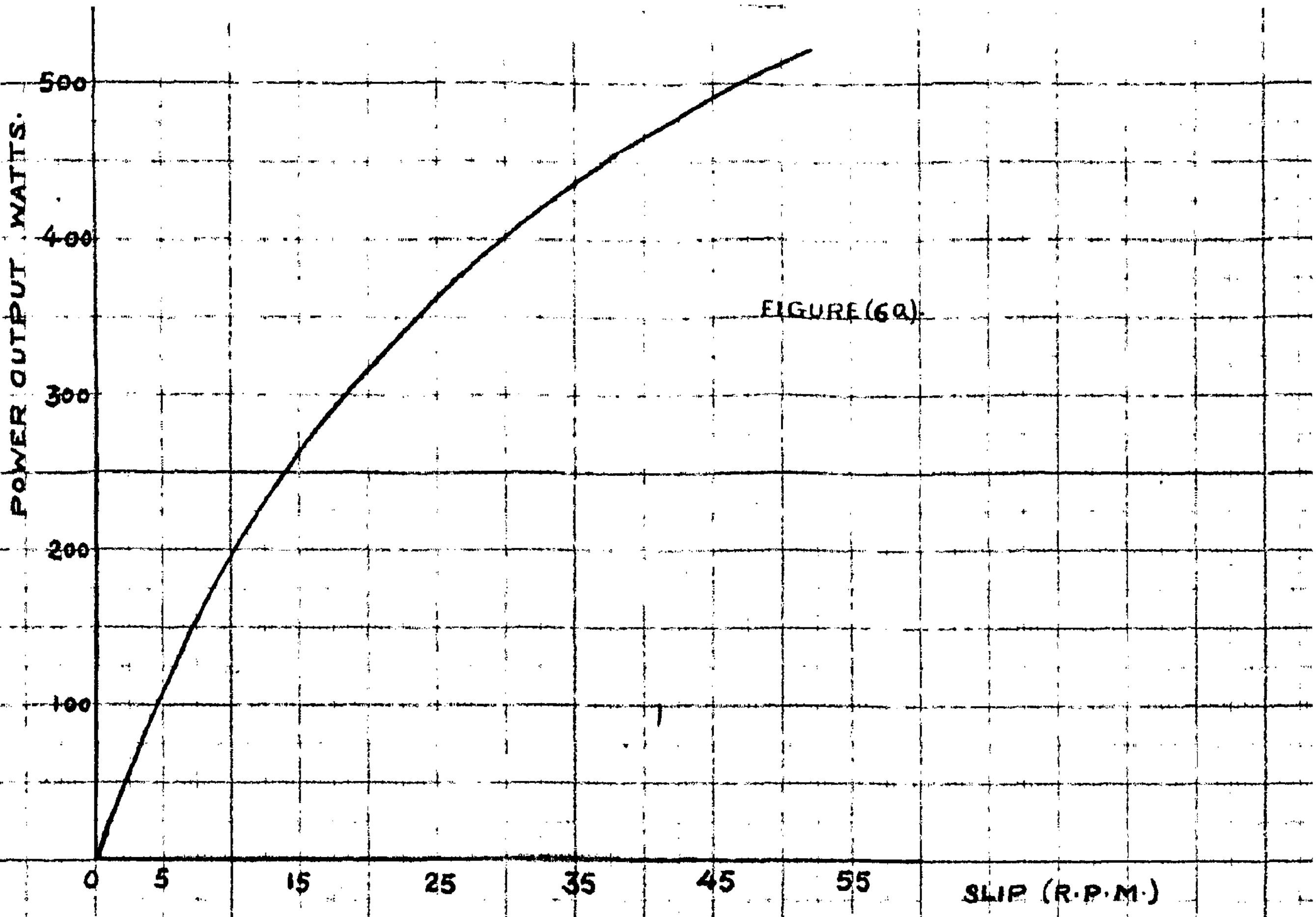


FIGURE (6a).

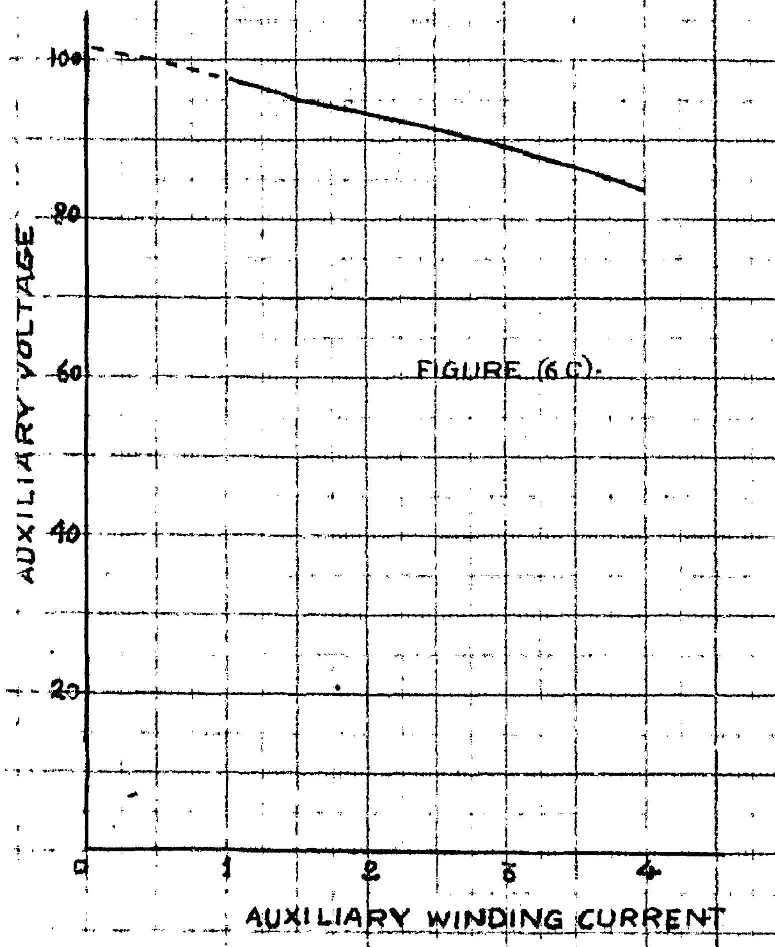


FIGURE (6c).

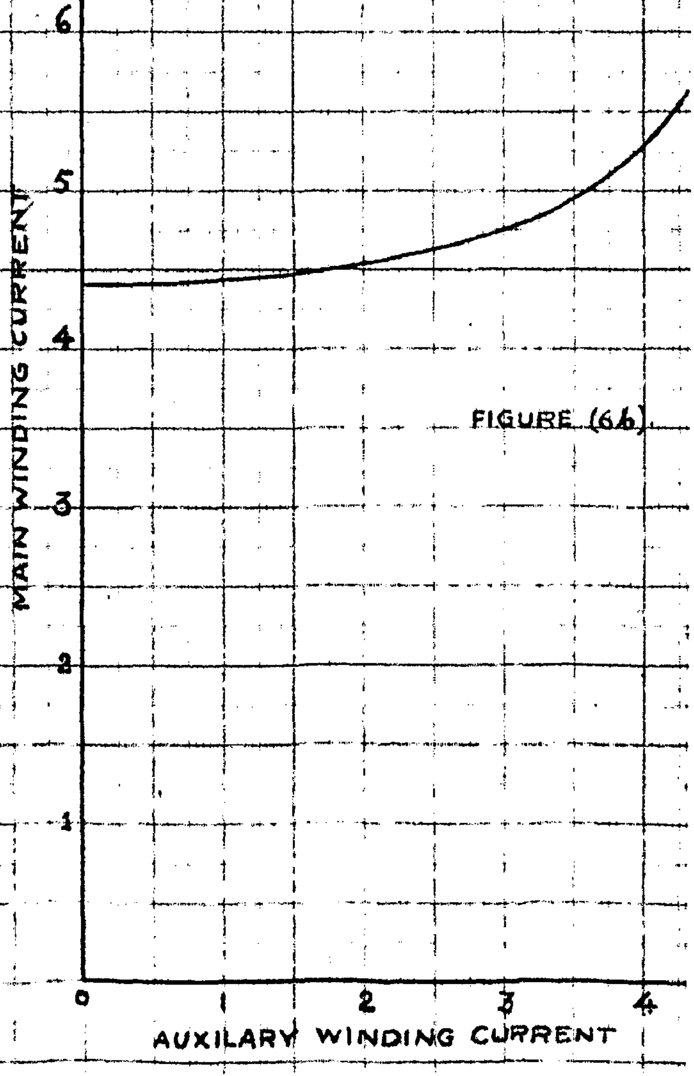


FIGURE (6b).

(3) Auxiliary winding voltage versus
auxiliary winding current (fig. 6 c).

The experimental set up is shown in fig.(6). The frequency of the output voltage was the same as that of the main winding exciting current.

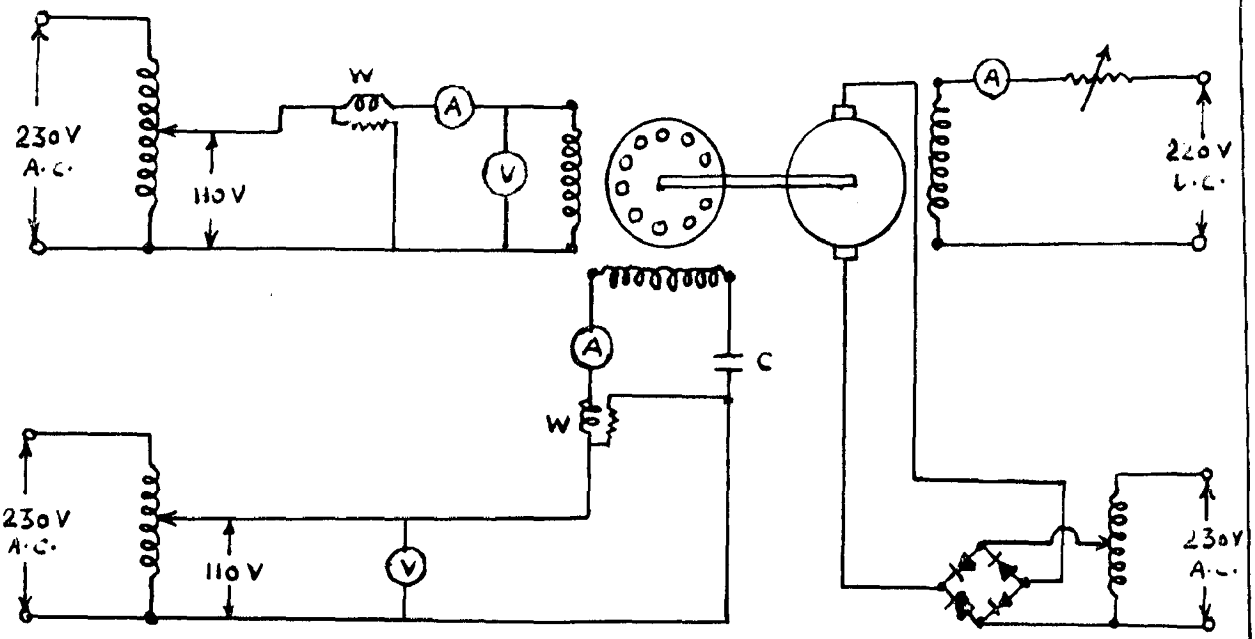
Fig. (6 a) shows that the output goes on increasing with the increased speed. Therefore, full load power can be delivered by the auxiliary winding by increasing the speed of the machine. The limit being decided by the current rating of the machine.

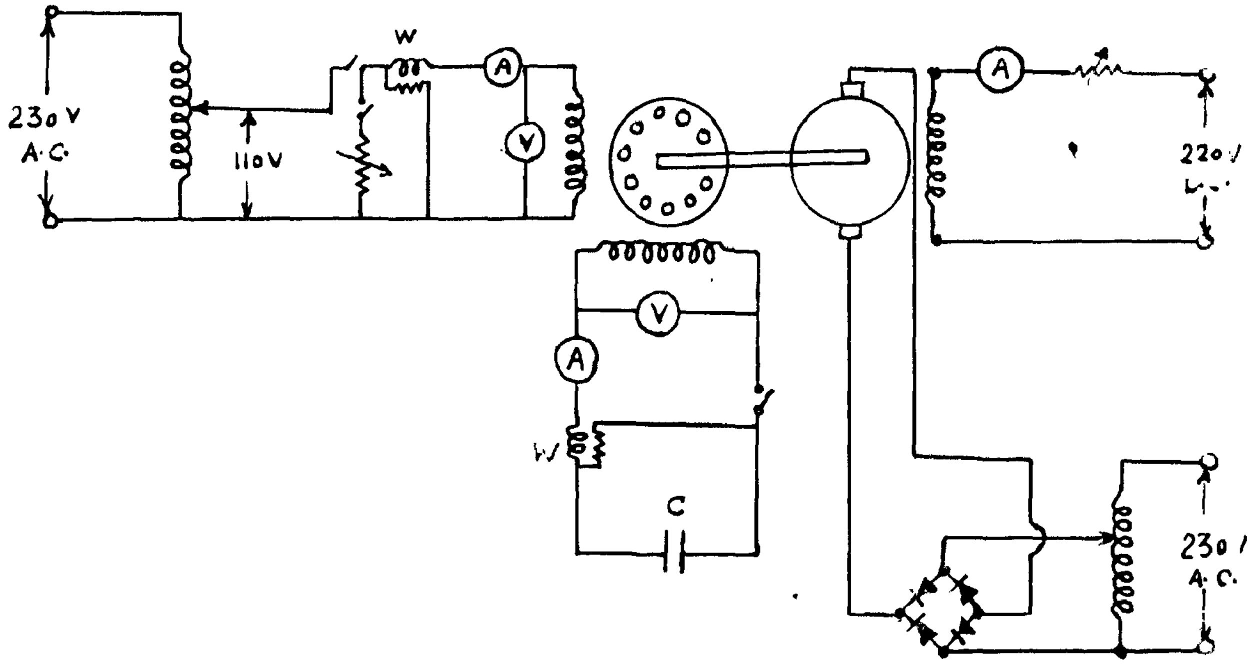
Fig. (6b) shows that the main winding current increases very slowly in comparison to the auxiliary winding current, this being true on account of the fact that with the increase of load on the auxiliary winding the excitation of the machine (drawn by the main winding from the supply) increases slightly.

Fig. (6 c) shows that the curve of auxiliary winding voltage versus auxiliary winding current is similar to the D.C. shunt generator characteristic.

In CASE III, with the set running at super synchronous speed the machine was made a generator. For different values of capacitors in the

EXPERIMENTAL SETUP FOR CASE III. (FIGURE 7).





EXPERIMENTAL SETUP FOR CASE IV. (FIGURE 8).

auxiliary winding circuit, the variation of load output of the machine with slip was observed and this was compared with the load output when only main winding is connected to source and the machine being made to act as a generator. For one particular operation the value of the capacitance choosen was kept constant and the slip varied.

The following curves are plotted on the same sheet :-

(1) Load output versus slip (in R.P.M.), when only main winding is connected to source and the set run at supersynchronous speed. (Curve I).

(2) Load output versus speed (in R.P.M.) when the main winding as well as the auxiliary winding through capacitor is connected to the single phase supply and the set being run at supersynchronous speed for (i) $c = 27$ F and (ii) $c = 37$ F. (Curves II and III).

These are shown in fig. (7 a). The experimental set up is shown in fig. (7).

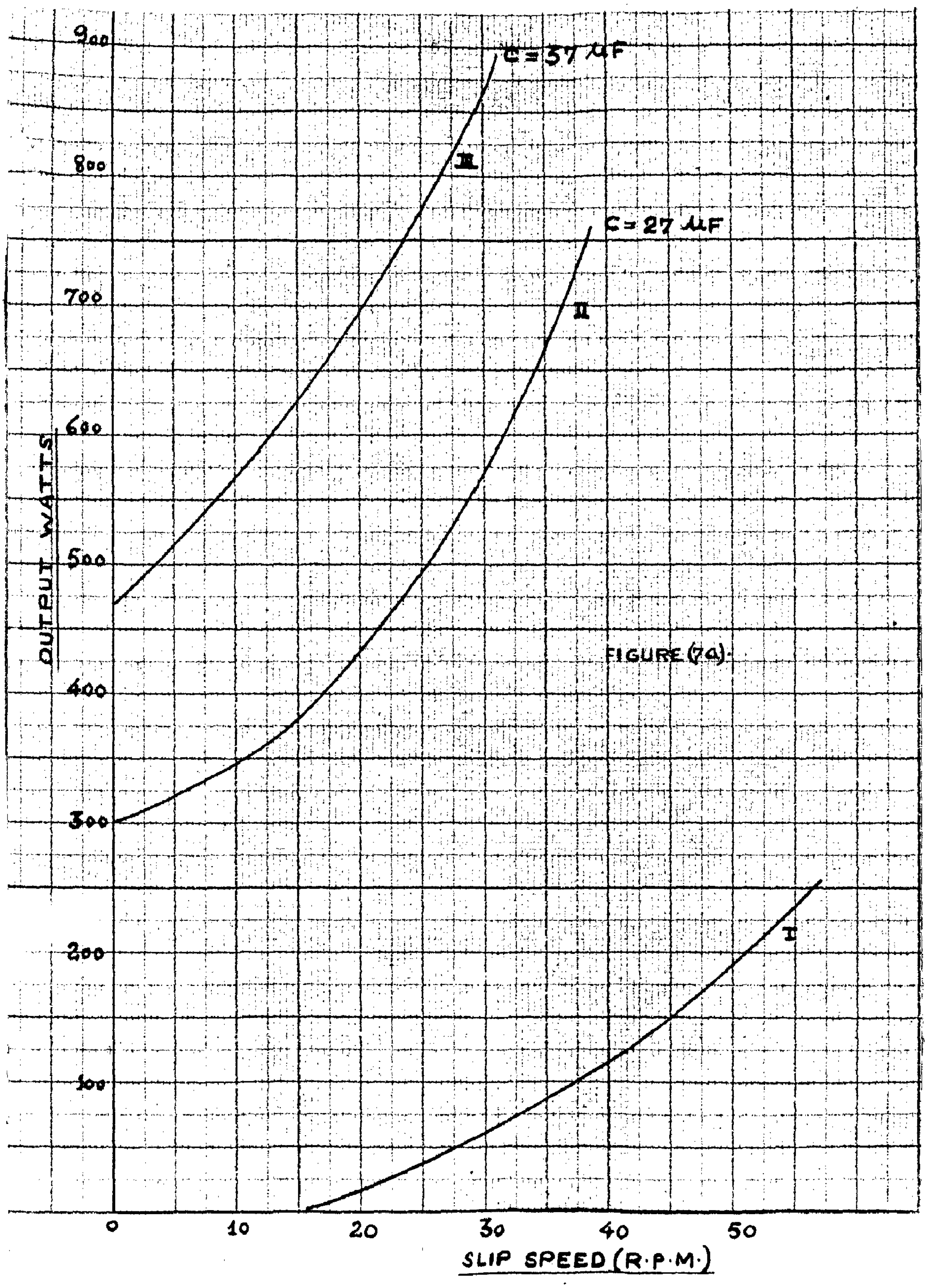

900
800
700
600
500
400
300
200
100
0
10
20
30
40
50
C = 37 μF
C = 27 μF
III
II
I
OUTPUT WATTS
SLIP SPEED (R.P.M.)

FIGURE (7a).

From the above curves it can be seen that the power output for a particular slip speed in the case of curves II and III is more than in case of curve I. Also with the value of the capacitor in the auxiliary winding circuit increased, the power output increases still further.

In CASE IV, the machine acts as a self excited generator. The rated voltage may be obtained by changing the speed or the capacitance in the auxiliary winding circuit.

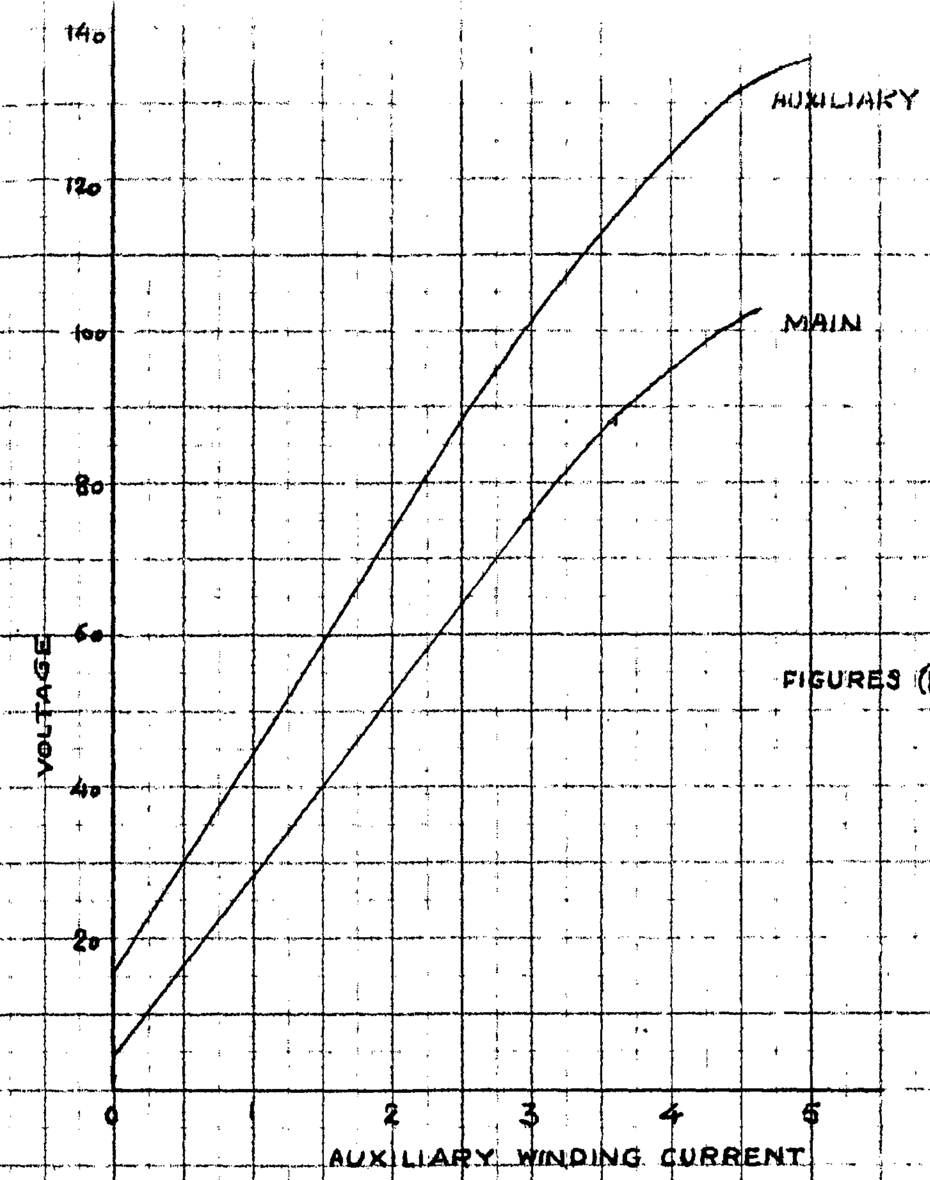
To determine the effect of exciting current of the auxiliary winding, on the performance of the machine, observations were recorded with main winding open, with a constant load impedance and with varying resistive load connected across the main winding, while the auxiliary winding current was varied by changing either external resistance or capacitance in the circuit.

The following curves are plotted :-

(i) With main winding unloaded.

  (1) Main winding voltage versus auxiliary winding current. (Fig. 8 a).

  (2) Auxiliary winding voltage versus auxiliary winding current. (Fig. 8 b).



FIGURES (8a) AND (8b)

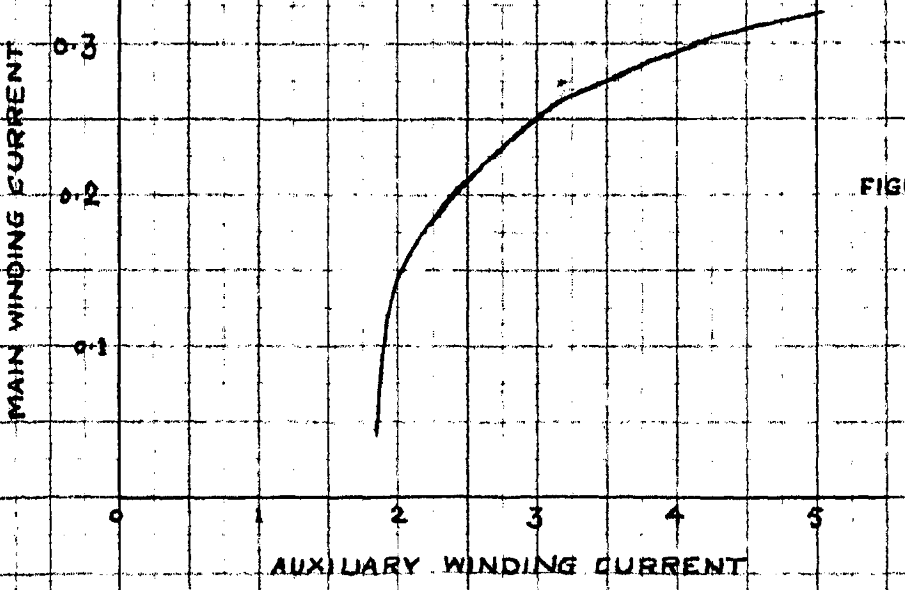


FIGURE (8c).

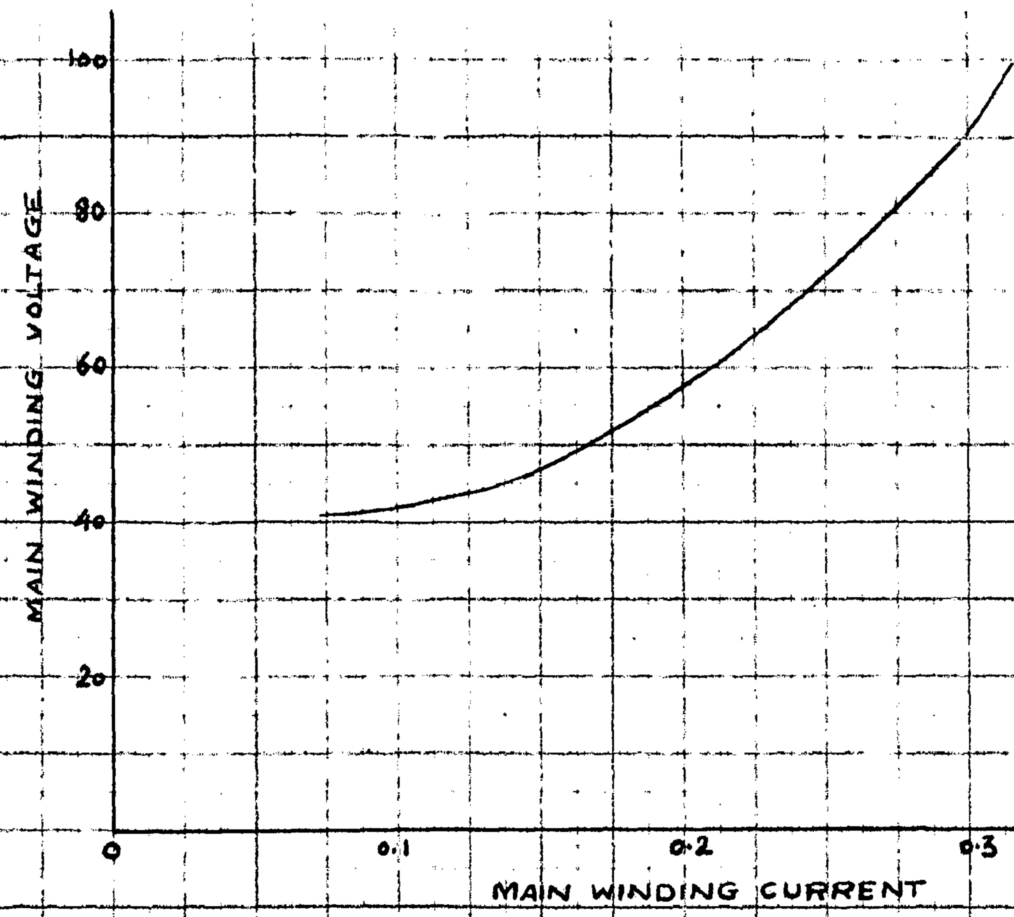


FIGURE (8c

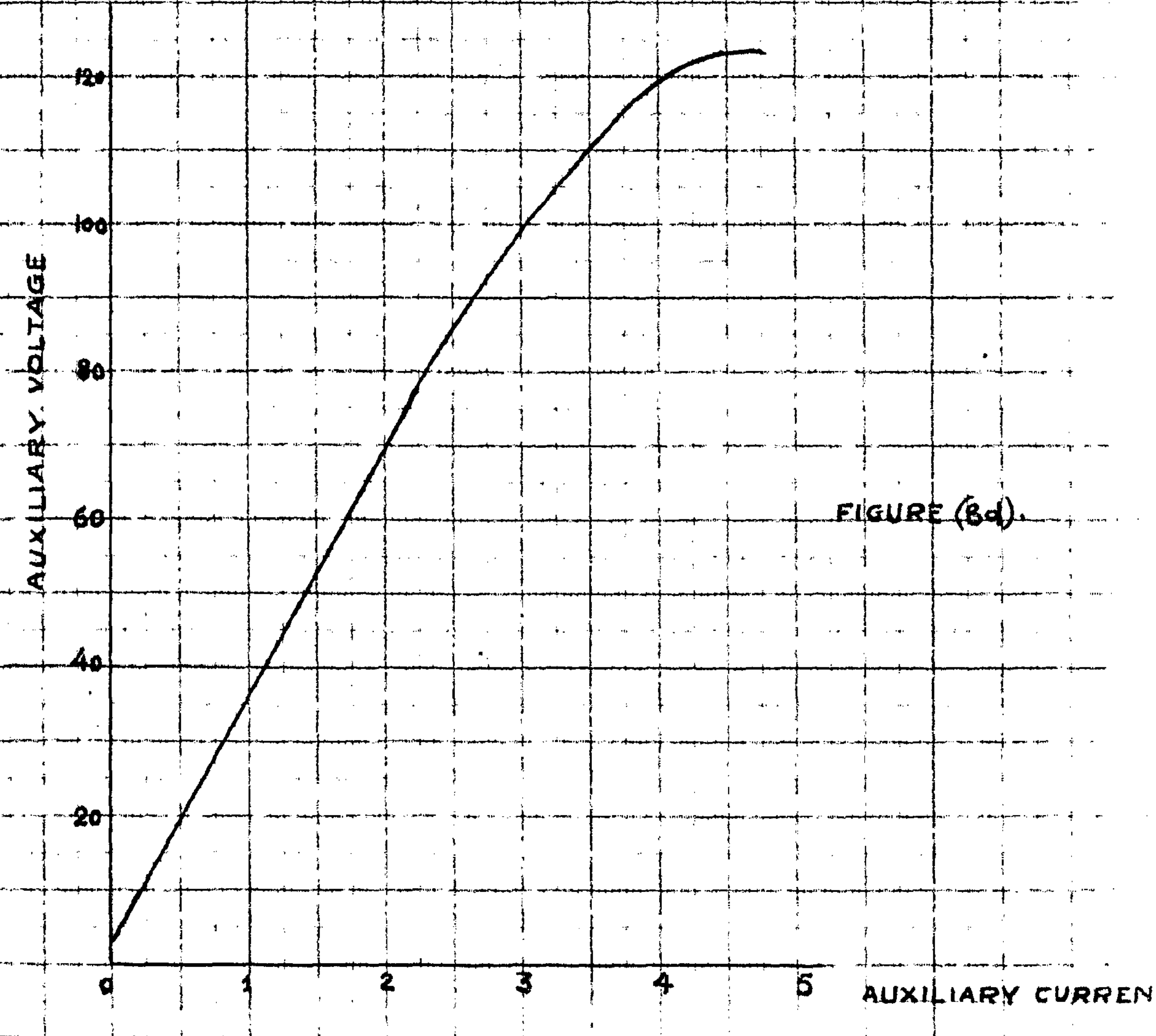


FIGURE (8d).

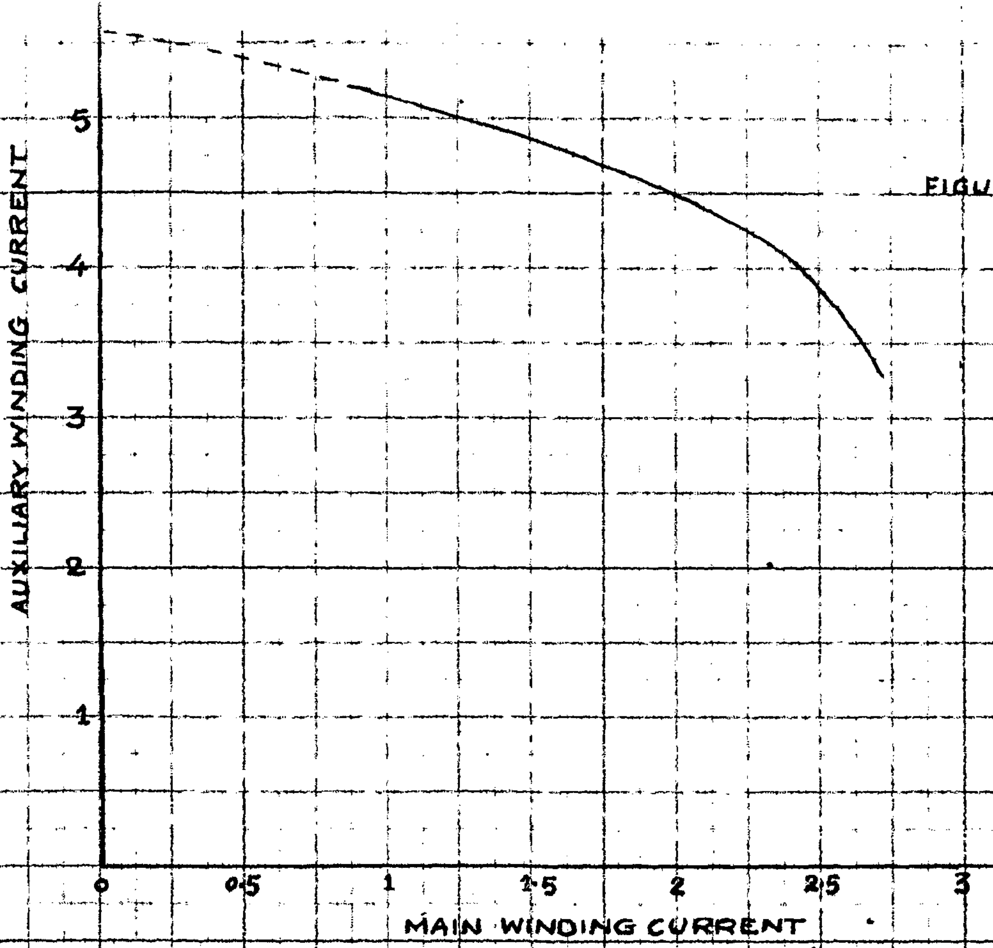


FIGURE (8h).

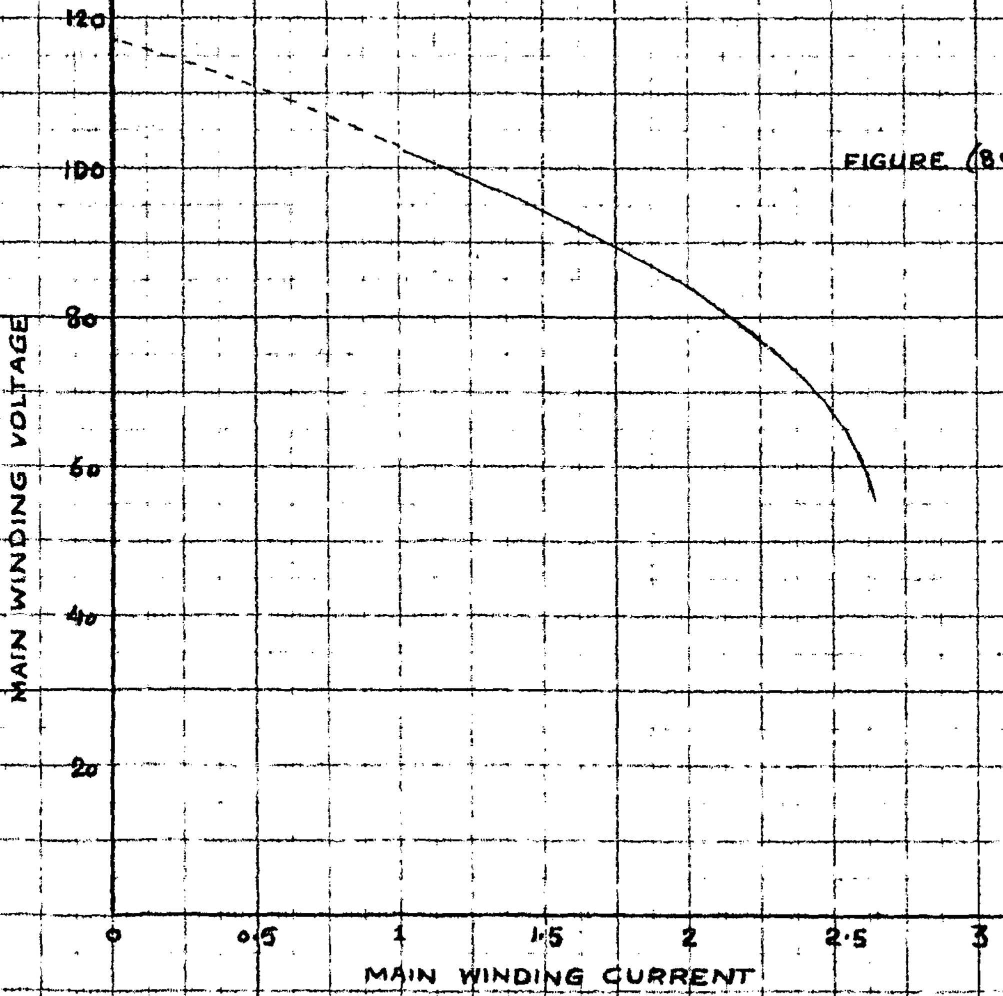


FIGURE (8g).

(ii) With main winding loaded with a constant load impedance.

   (1) Main winding voltage versus main winding current. (fig. 8 c).

   (2) Auxiliary winding voltage versus auxiliary winding current. (Fig. 8 d).

   (3) Main winding current versus auxiliary winding current. (fig. 8 e).

(iii) With main winding loaded with a variable resistive load.

   (1) Auxiliary winding voltage versus auxiliary winding current. (fig. 8 f).

   (2) Main winding voltage versus main winding current. (fig. 8 g).

   (3) Main winding current versus auxiliary winding current (fig. 8 h).

The experimental set up is shown in fig.(8).

Fig. (8 a) and (8 b) show that with the increase of current in the auxiliary winding the main and auxiliary winding voltage increases.

Fig. (8 c) and (8 d) show that with the increase of current, the voltage in the circuit (main or auxiliary) also increased.

Fig. (8 e) shows that for a greater change in the auxiliary winding current the main winding current changes very slowly.

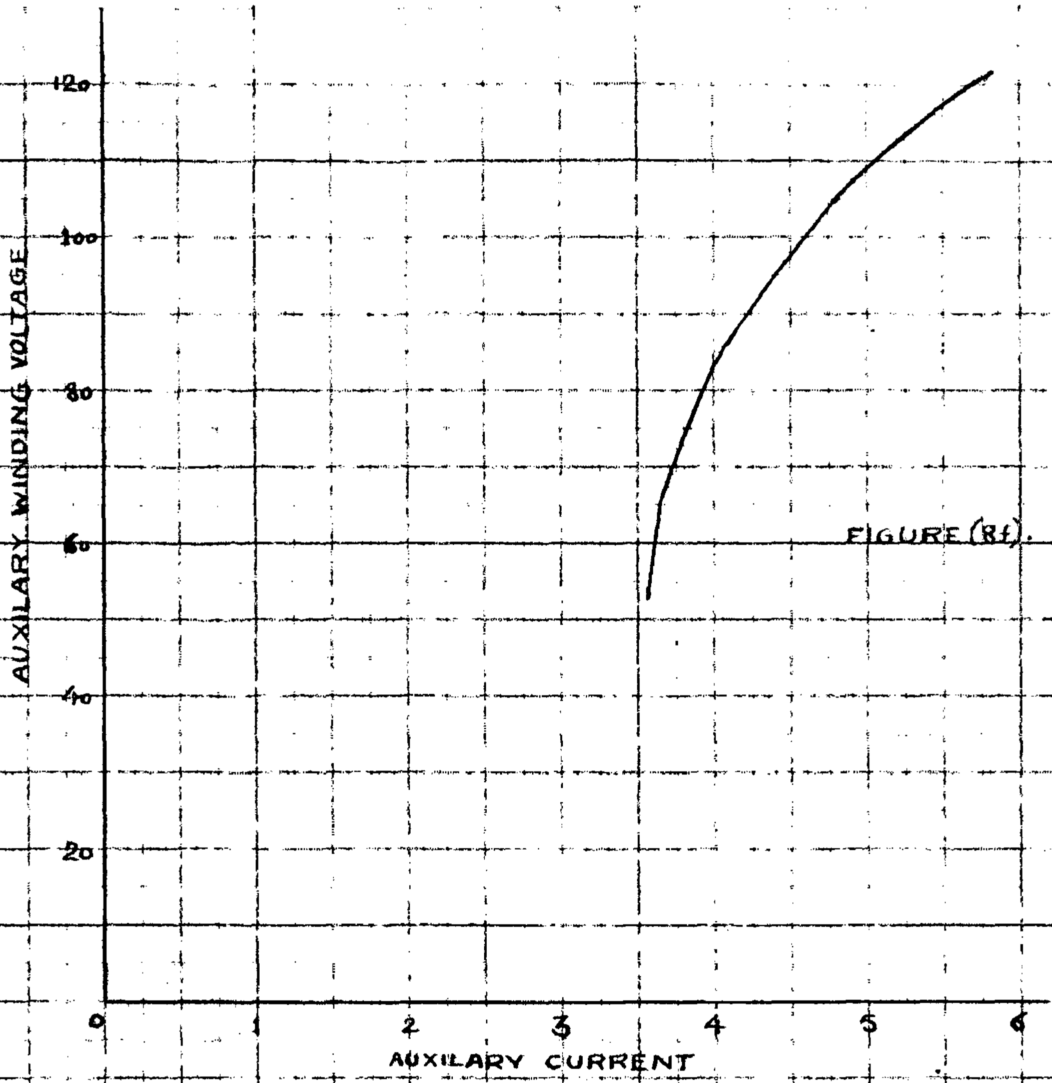


FIGURE (8f).

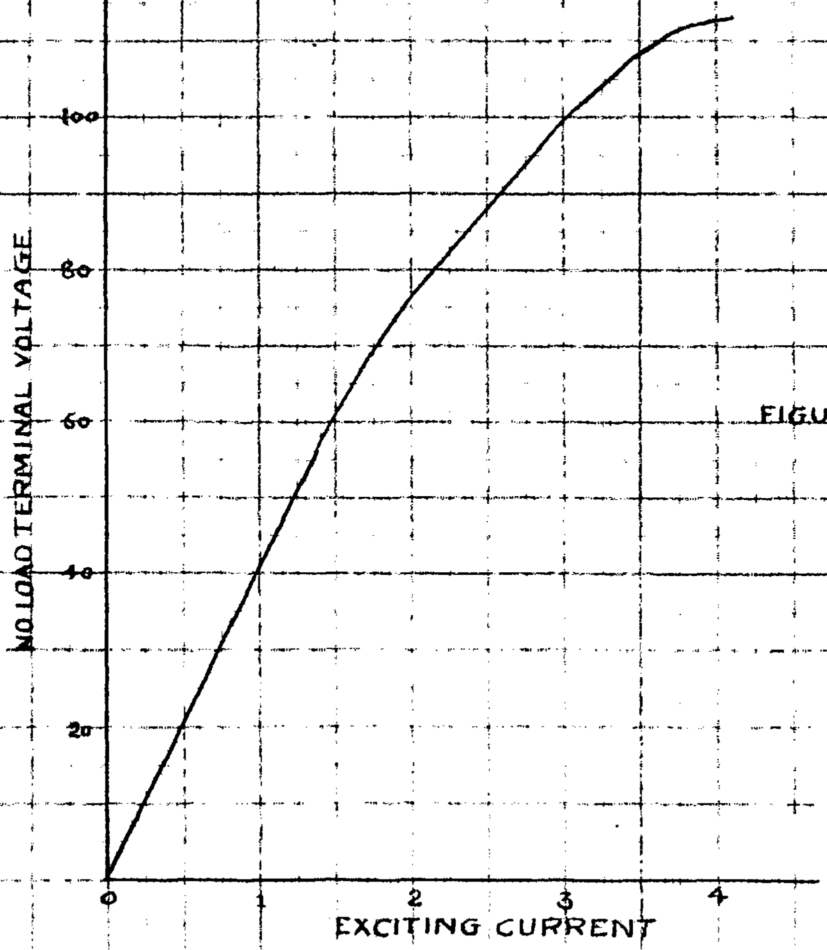


FIGURE (9).

Fig. (8 f) shows that with increase of current the voltage increases.

Fig. (8 f) shows that the characteristic is similar to that of a D.C. shunt generator.

Fig. (8 h) gives the variation of main winding current with auxiliary winding current.

The saturation curve of the machine was also obtained by putting the capacitors across the main winding of the machine and first connecting this to supply. The machine at first draws its magnetizing current from the supply but by properly adjusting the speed of the set and the value of the capacitors, the machine is made as a self excited generator and the supply being disconnected from the main winding. The exciting current now being supplied by the capacitors. By varying the capacitors across the machine terminals, the no-load terminal voltage was varied and a graph of no load terminal voltage versus exciting current was plotted. This gives the saturation curve or the magnetization characteristic of the machine. The saturation curve is shown in fig. (9).

CONCLUSION

The above experimental results also arrive at the following conclusions :-

1. The induction machine with capacitive excitation will build up its voltage exactly as does a d.c. shunt generator, the final build up value being determined by the saturation curve of the machine and by the value of the reactance of the excitation capacitance.
2. Wave shape of the induction generator with capacitive excitation is sinusoidal.
3. The induction generator can be made to handle almost any type of load, provided that the loads are compensated to present unity power factor characteristics to the generator.
4. Use of induction generator with capacitive excitation may be made (a) in laboratories where a source of sine wave power is desired and (b) in installations of small capacity where single phase power is required and where the cost of a synchronous generator and auxiliaries is prohibitive.
5. Low cost of capacitors makes possible comparatively cheap installations, especially those of small capacity.

DISADVANTAGES OF CAPACITOR EXCITATION

There are unfortunately, a number of disadvantages inherent in the capacitor excited systems. The voltage regulation is excessive and in addition there is a critical value for the magnetizing reactance determined by the slope of the air gap line. Thus the machine de-excites on a sustained short circuit. Secondly it is difficult to guarantee to any degree of accuracy the terminal voltage of any machine. This is due to the variations in magnetizing curves (for different flux densities) and the tolerance on capacitors which may be as much as 25 percent. Thirdly the generator as might be expected is very susceptible to inductive loads and finally the terminal voltage waveform is some what distorted due to the fact that the generator is being worked in a saturated condition.

While these disadvantages are formidable, a little thought will make it clear that most of the difficulties would be resolved if the machine is made to saturate suddenly instead of gradually. If the magnetizing curve were flat topped, then in fact changes in the slope of the reactance line would produce no change in terminal voltage and sufficient excitation capacitance could be initially provided to cope with the maximum load demand. However, if the magnetic circuit of the generator was made to saturate suddenly instead of gradually then

i) excessive iron losses would occur,

ii) excessive harmonic distortion, which would be very difficult to remove, would occur.

The alternative is to use an external system of saturable reactors whose characteristic can be closely controlled and whose function is to produce an effective magnetizing curve which minimizes the voltage regulation.

PRACTICAL APPLICATION

The induction generator system is likely to be some what heavier than a conventional synchronous machine for a given output and speed because of the bulk of the capacitors and reactors necessary for excitation. However, in terms of cost and robustness, the squirrel cage design has much to offer and if it is desired to use the machine as a motor as well as generator then it is difficulty to select another a.c. (motor) machine which is capable of performing both duties with high efficiency. These qualities together with the fact that it is brushless make the induction generator ideal for missile applications where it may be necessary to have a machine operating as a motor on the ground and as a generator during the flights.

APPENDIX - 1

BALANCED TWO PHASE MOTOR OPERATION

(flux Linkages)

In this case the supply voltage to the main and auxiliary winding is in quadrature so as to produce a balanced magnetic field in the air gap of the machine. The voltages fed to these windings are also balanced i.e. in the proportion of the turns ratio of the machine (unity in the present case).

The flux linkage equations are given by

$$\lambda s_1 = L_{11} I_m + M I_{r1}$$

Stator : ......(1)

$$\lambda s_2 = L_{11} I_a + M I_{r2}$$

$$\lambda r_1 = L_{22} I_{r1} + M I_m$$

Rotor : ......(2)

$$\lambda r_2 = L_{22} I_{r2} + M I_a$$

and the voltage equations are given by

$$\begin{bmatrix} (r_1 + j\,X_{11}) & -X_{11} & j\,X_m & -X_m \\ +X_{11} & (r_1 + j\,X_{11}) & +X_m & j\,X_m \\ j\,X_m & -s\,X_m & (r_2 + j\,X_{22}) & -s\,X_{22} \\ s\,X_m & j\,X_m & s\,X_{22} & (r_2 + j\,X_{22} \end{bmatrix} \begin{bmatrix} I_m \\ I_a \\ I_{r1} \\ I_{r2} \end{bmatrix} = \begin{bmatrix} V \\ j\,V \\ 0 \\ 0 \end{bmatrix} \cdots (3)$$

$$\left[A_1\right] \quad \left[I\right] \quad = \left[V\right]$$

Solving these equations for various currents gives

$$I_m = \begin{vmatrix} V & -X_{11} & j\,X_m & -X_m \\ j\,V & (r_1 + j\,X_{11}) & +X_m & j\,X_m \\ 0 & -s\,X_m & (r_2 + j\,X_{22}) & -s\,X_{22} \\ 0 & j\,X_m & s\,X_{22} & (r_2 + j\,X_{22}) \end{vmatrix} \div |A_1|$$

$$= \frac{V}{|A_1|} \Big\{ (r_1 + 2\,j\,X_{11}) \left[r_2^2 + 2j\,r_2\,X_{22} + (s^2 - 1) X_{22}^2 \right]$$

$$+ 2\,j\,X_m \left[\cancel{-j}\,X_m \;\; -j\,X_m\,r_2 + X_m\,X_{22}(1 - s^2) \right] + 2X_m (r_2\;s\;X_m) \Big\}$$

Similarly

$$I_a = \begin{vmatrix} (r_1 + j\,X_{11}) & V & j\,X_m & -X_m \\ +X_{11} & jV & +X_m & j\,X_m \\ j\,X_m & 0 & (r_2 + j\,X_{22}) & -s\,X_{22} \\ s\,X_m & 0 & s\,X_{22} & (r_2 + j\,X_{22} \end{vmatrix} \div |A_1|$$

$$= \frac{V}{|A_1|}\Big\{(j\,r_1 - 2\,X_{11})\left[r_2^2 + 2j\,r_2\,X_{22} + X_{22}^2(s^2-1)\right]$$

$$+ 2\,X_m\left[j\,X_m\,r_2 + X_m\,X_{22}\,(s^2-1)\right] + 2\,j\,X_m(s\,X_m\,r_2)\Big\}$$

$$I_{r1} = \begin{vmatrix} (r_1 + j\,X_{11}) & -X_{11} & V & -X_m \\ +X_{11} & (r_1 + j\,X_{11}) & jV & j\,X_m \\ j\,X_m & -s\,X_m & 0 & -s\,X_{22} \\ s\,X_m & j\,X_m & 0 & (r_2 + j\,X_{22}) \end{vmatrix} \div |A_1|$$

$$= \frac{V}{|A_1|}\Big\{(-j r_1 + 2\,X_{11})(-s\,X_m\,r_2) + 2j\,X_m^3\,(s^2-1)$$

$$-(r_1 + 2\,j\,X_{11})\left[j\,X_m\,r_2 + X_m\,X_{22}(s^2-1)\right]\Big\}$$

and $I_{r2} = \begin{vmatrix} (r_1+j\,X_{11}) & -X_{11} & j\,X_m & V \\ +X_{11} & (r_1+j\,X_{11}) & +X_m & jV \\ j\,X_m & -s\,X_m & (r_2+jX_{22}) & 0 \\ s\,X_m & j\,X_m & s\,X_{22} & 0 \end{vmatrix} \div |A_1|$

$$= \frac{V}{|A_1|}\left\{(j\,r_1)\left[-j\,X_m\,r_2 + X_m\,X_{22}(1-s^2)\right] + r_1(s\,X_m\,r_2)\right\}$$

from these expressions I_{r1}, I_{r2} and I_a are found in terms of I_m

Let $I_{r1} = K_1 \quad I_m$

$I_{r2} = K_2 \quad I_m$ ...... (4)

and $I_a = K_3 \quad I_m$

Then from the voltage equation

$$(r_1 + j\,X_{11})\,I_m - X_{11}\,I_a + j\,X_m\,I_{r_1} - X_m\,I_{r2} = V$$

Substitution of (4) in this gives

$$I_m\left[(r_1 + j\,X_{11}) - X_{11}.\,K_3 + j\,K_1\,X_m - X_m\,K_2\right] = V$$

$$\text{or } I_m = \frac{V}{\left[(r_1+j\,X_{11}) - X_{11}\,K_3 + j\,K_1\,X_m - X_m\,K_2\right]}$$

The rotor flux linkages under this condition are given by

$$\lambda_2 = (L_{22} I_{r1} + M I_m) + j (L_{22} I_{r2} + M I_a)$$

$$= \left[(L_{22} K_1 + M) + j (L_{22} K_2 + M K_3)\right] I_m$$

$$= \frac{(L_{22} K_1 + M) + j(L_{22} K_2 + M K_3)}{(r_1 + j X_{11}) - X_{11} K_3 + j K_1 X_m - X_m K_2} \times V$$

APPENDIX 2

NOTES ON THE MEASUREMENT OF PARAMETERS OF THE MACHINE UNDER TEST

Experimental set up :- The induction machine used is a fractional horse power motor, having two main windings, the connection can therefore be obtained for either 110 volts or 220 volts (by connecting these either in parallel or series), the former type (110 volts) being used, and another winding known as the auxiliary winding. The induction machine was coupled with a separately excited d.c. motor. The variable armature voltage was given to the d.c. motor aramture through a rectifier (bridge type) to control the speed of the set. The excitation was kept constant for the d.c. machine. This method of speed control is similar to the Ward-Leonard method and has the characteristic advantage of controlling the speed from zero to full speed and also in the negative direction.

Measurement :- The parameters of the induction machine were calculated by the method used for two phase motors and the single phase operation was treated as a special case of the two phase machine with one of the phase currents zero. For this purpose a two phase supply with one of the phase 90 degrees out of phase with respect to the other was given to the two windings namely the main and auxiliary and the machine was run by means of D.C. motor at synchronous speed. The open circuit test was thus conducted with slip equal to zero and the parameters thus obtained by this test were free from the effect of slip. The voltages fed to the two windings were adjusted so as to get a balanced field in the air gap of the machine i.e. the voltages applied to the auxiliary and main windings were in proportion of their turns ratio, the currents being equal under this condition. Thus exact conditions were simulated in performing the test, as made in the assumptions. The blocked rotor test was performed at a reduced voltage so as to get again a balanced condition. The two phase supply with one phase 90 degrees out with respect to the other was obtained from a Scott - connected transformer. The magnetizing reactance was calculated by a method suggested by D.R. KOHLI and S.K. JAIN[6] in which the losses of the d.c. machine at synchronous speed are calculated by decoupling the induction machine and also after coupling the two machines and exciting both of them, the input to the d.c. motor is found. From this X_m can be found as given below.

From this paper,

Torque is given by (at $s = 0$)

$$\text{Torque} = \frac{V^2}{r_2}\left(\frac{X_m}{X_{11}}\right)^2\left[\frac{1}{(T_o' + T_s')^2 + \left(\frac{2\,r_1}{r_2} + 1\right)^2}\right]$$

and the magnetizing current is given by,

$$I_{mo} = \frac{V}{X_{11}}\left[\frac{2\,T_o' - 1}{\left(\frac{2r_1}{r_2} + 1\right) + j(T_o' + T_s')}\right].$$

$$\therefore \frac{\text{Torque}}{|I_{mo}|^2} = \frac{X_m^2}{r_2\,(2T_o' - 1)^2}$$

$$\text{or } X_m^2 = \frac{\text{Torque}}{|I_{mo}|^2} \times r_2(2\,T_o' - 1)^2$$

where T_o' = open circuit rotor time constant

and T_s' = short circuit rotor time constant.

The parameters thus obtained by above tests are

r_1 = 1.995 ohms for both main and auxiliary winding.

$X_{11} = X_{22}$ = 27.0 ohms.

r_2 = 0.9 ohm.

and X_m = 24.8 ohms. ?

The impedance of the main or auxiliary winding was made equal by inserting extra impedance in either of the winding as required by assumption.

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