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Speed Control of Induction Motors with the Help of Saturable-Core Reactors

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C O N T E N T S

SUMMARY

The induction motor compares favourably with d.c. motors as regards cost, size, maintenance, and wear and tear, but suffers from the limitations that its speed of operation remains more or less constant, and can not be varied as desired. A number of methods for its speed control are known but the variation of speed is neither smooth nor suitable for certain drives, especially hoist crane etc.

A standard torque-speed characteristic of an induction m motor is obtained when a positive-sequence system of voltages is impressed on its terminals. Under negative-sequence operation the characteristic is the inverse image of that of the previous one. When a zero-sequence system of voltages is applied, the motor behaviour could be determined by recourse to single-phase motor theory since the resultant field caused by cophasal voltages is pulsating.

If the induction motor is subjected to the positive and negative-sequence systems of voltages simultaneously, the resultant torque-speed characteristics will be the algebric sum of the two separate characteristics, and this resultant characteristic could be modified, as desired, by varying the relative magnitude of the two systems. This suggests one of the ways of smooth speed control of the induction motor.

A simultaneous production of positive and negative-sequence system of voltages could be achieved by a number of ways. One of the easier method is an unbalanced operation of the induction motor. Under unbalanced operation, zero-sequence system, if not eliminated, is also present alongwith the positive and negative-sequence systems. The zero-sequence system could be easily

eliminated by providing a winding arrangement such that the algebric sum of the phase vectors is zero.

The unbalancing could be produced, simply by introducing an impedance in one of the lines, or using an auto-transformer.

The degree of unbalance produced depends to some extent on the load conditions on the motor, but in general a variable impedance is required to control the degree of unbalance introduced.

A saturable core reactor (S C R) can be used as a circuit element having a variable impedance. The S C R consists of two windings - a load winding connected to the alternating current supply, and a control winding connected to a direct current source, and providing a variable bias. The S C R is driven electrically by the line voltage and the bias, and the flux density in the core reaches a saturation value Bs sometimes during one of the half cycle of the line voltage. If the core material is rectangular, the transition into saturation is sharp, the impedance of the coil drops abruptly and most of the line voltage appears across the load, and hence provides use ful power. The reversal of line voltage drives the flux out of saturation, load conduction ceases, and the direction of flux change is opposite to that of the previous half cycle. Load power, and conduction angle are controlled by moving the initial flux density 'B' towards positive Bs (direct current). In general, the above technique controls inductance by applying an adjustable d.c. magnetization.

The speed of an induction motor can be controlled by connecting the a.c. (load) winding of the SCR in one of the supply lines, and obtaining d.c. for the control winding either with the help of rectifiers or directly from a d.c. source available.

By varying the control current of the S C R the relative magnitude of negative-sequence voltage can be varied as desired. As such, this method is not effective, because the motor operation is stable only in a very limited range of speed. However, with a motor having a high rotor resistance, the speed control is possible in all the four quadrants. The high rotor resistance also limits the high input current caused by the unbalanced operation. In case of a slip-ring induction motor, the secondary resistance can easily be increased by adding an additional resistor in the rotor circuit, and a wide range of speed control can be obtained by using a capacitor in addition to the resister, or by using a combination of external resistance, capacitance, and inductance elements in the secondary circuit. The speed control can also be made automatic and responsive to speed as suggested by Wickerham[24] in the case of slip-ring induction motors. In case of squirrel cage induction motor the current in the control winding of the S C R is automatically varied by the d.c. tachometer generator (fixed on the shaft of the induction motor) and comparing it with a reference direct voltage. Also by using a capacitor in parallel with the S C R , the external reactance may be varied upto a greater extent alongwith the facility of reversing its sign too, the direction of rotation of the motor can be reversed without interrupting the mains circuit, and a dynamic braking is possible without an extra equipment. This form of speed control of induction motor, however, causes problems of magnetic vibrations, noise, overheating, and high input line current. However, all ~~the~~ these difficulties could be alleviated to some extent.

In the laboratory standard tests on a slip-ring induction motor, and a squirrel-cage induction motor were performed to obtain the parameters of the motor windings. The possibility of speed control of the induction motors, by mixed-sequence operation, was studied by measuring the torque-speed, and torque-input current characteristics for the above motors under the following different conditions:

(a) with balanced applied voltages

(b) with various unbalanced voltages obtained with the S C R.

(c) with various unbalanced voltages obtained with the single phase variac.

and (d) in the case of slip-ring induction motor (a), (b), and (c) with the different external resistances in the rotor circuit.

LIST OF PRINCIPAL SYMBOLS

a = unit complex operator $e^{j2\pi/3}$

B, B_r, B = flux density, Retentive, and Saturation flux densities

f = supply frequency

F_n' = for zero-sequence excitation, amplitude of the n^{th} space harmonic m.m.f. of each phase, for concentrated, and full pitch winding.

$F_{n\theta}$ = for zero sequences excitation, amplitude of the n^{th} space harmonic m.m.f. of each phase, at a point θ elec. degrees, at any instant of time t.

H, H_c = Magnetizing, and coercive forces.

I_a, I_b, I_c = phase currents

I_1, I_2, I_0 = positive, and negative and zero-sequence stator currents per phase

I_{2p}, I_{2n} = Positive and negative-sequence rotor currents per phase

I_{2f}, I_{2b} = Zero-sequence rotor currents per phase for forward, and bacward rotating fields.

K = open circuit voltage ratio for each tapping utilized in auto-transformer.

K_n = for zero-sequence excitation, n^{th} space harmonic winding factor for each phase for distributed, and fractional pitch winding.

N_c, N_L = number of turns in the control, and load windings of the SCR.

r_1, r_2 = stator, and rotor resistances per phase

r_{2f}, r_{2b} = zero-sequence rotor resistances per phase for forward, and backward rotating fields.

R_c , R_L = resistances of the control and load circuits of the SCR

s , s' , s'' , s''' = fractional slips for positive, negative, and zero-sequence (forward, and backward rotating fields) systems.

T_1 , T_2 , T_o = positive, negative, and zero-sequence torques.

V_1 , V_2 , V_o = positive, negative, and zero-sequence voltages per phase.

Y_1 , Y_2 , Y_o = admittances for positive, negative, and zero-sequence systems.

Y = external admittance, inserted in one of the lines.

Y_s = equivalent source impedence of the auto-transformer.

SECTION I

1. INTRODUCTION

In the field of utilisation of electric energy the major role is played by the electric motor. The ploy-phase induction motor has widespread industrial used, but alongwith very valuable advantages it is known to suffer from the inherent disadvantage of very poor speed regulation characteristics. The use of induction motor has been, as a result, ignored in most variable speed drives, especially with wide range speed regulation. In recent years there has been a great deal of research activity to devise means of obtaining a variable speed operation of induction motors.

The d.c. motor which is very widely used in variable speed drives has the great advantage of a giving a smooth speed control but this is obtained at the cost of additional significant expenses on installing convertors, and energy losses in the conversion of a.c. into d.c. The d.c. motor itself costs considerably more, requires a greater care, and wears out quicker than a.c. motor. Therefore, the common interest in induction motor speed control has increased steadily in last few years.

The problems of speed control for crane hoists, and other similar drives are more involved than those for general drives. For speed control of electric drives in general, we have to consider load changes ranging from zero to full load. In the case of crane hoist, the motor load might change from zero to full load during hoisting and from +10% for lowering the empty hook to -64% for lowering the full load.

Another difference between general drives and hoist drives is the fact that in the case of general speed control, failure of the control would merely stop the operation, but usually nothing more would happen. In the case of a crane hoist, failure of the control might cause the abrupt dropping of the load, an extremely dangerous situation.

The requirements of a satisfactory hoist control are as follows:-

(1) Safety in operation, Ruggedness, and Simplicity of apparatus.

(2) To hoist all normal loads at slow, Medium and high speed.

(3) To lower all loads at low, medium and high speed.

(4) To provide accurate positioning of all loads, overhauling or non-overhauling (inching).

(5) To avoid too high currents in the windings and thus reduce heating of motor and apparatus.

Speed control of slip-ring induction motors, when the torque developed by the motor is appreciable and in the same direction as the mechanical rotation, may be obtained merely by adjusting resistance in the rotor circuit. However, this simple method is inefficient at low speed settings, and the no load speed is always synchronous speed. In the case when the torque is in the opposite direction to the rotation, as in crane hoist lowering, the problem of speed control at less than synchronous speed is not easy. A.C.motors have been used to obtain speed- torque characteristics similar to that of hoist control by means of auxiliary machines as in Kramer or Scherbius drive. Another type of a.c. drive employs two slip-ring induction motors so that their torques oppose each other in a manner as to permit stable operation at speed less than synchronus speed. All of these schemes are unsuitable due to the problems regarding either power supply, cost, size of equipment, or complications of control. The latter method, however, suggests a means whereby the previously outlined features may be obtained with only one slip-ring induction motor.

It has been shown[19] that the application of unbalanced Voltages to the stator of an induction motor produces mixed-sequence systems of voltages, and if zero sequence system of voltages is eliminated, motor operates only under positive and negative sequence systems of voltages. The operation of induction motor simultaneously under positive and negative-sequence systems of voltages is equivalent to that of two identical motors operating with balanced primary voltages and so connected that their torques oppose each other. The use of a single induction motor with unbalanced voltages applied in a definite predetermined manner will yield characteristics similar to those of the two motors.

There are a number of ways of producing unbalanced voltages, but with the use of saturable core reactors, the unbalancing can be easily produced and controlled. Also saturable core reactors provide a means of making the primary unbalanced voltages a function of the motor speed and consequently an automatic speed control of the motor speed and consequently an automatic speed control of the induction motor can be obtained.

This dissertation gives a review of work on the speed control of induction motors by mixed-sequence operation with the use of saturable core reactors, and the results of an investigation carried out to demonstrate the principles involved.

SECTION 2

2. GENERAL PRINCIPLES OF MIXED - SEQUENCE OPERATION

The operation of an induction motor depends upon the successful production of a rotating magnetic field in the air gap of the motor. When balanced poly-phase voltages are applied to the stator of an induction motor, the rotating field is produced by the combined action of the balanced poly-phase currents flowing in the poly-phase primary windings of the motor. The rotating field rotates at synchronous speed and follows magnetic paths through the stator iron, the air-gap and the iron of the rotor. The magnitude of this rotating field remains more or less constant, and hence can be represented by a rotating vector of constant amplitude, whose locus is a circle. If the poly-phase voltages applied are unbalanced, the magnitude of the rotating field is no longer constant thereby making the locus of the rotating field vector an ellipse. The shape of the ellipse depends upon the type of unbalance and the speed of the rotor. However, any unbalanced or unsymmetrical 3-phase system can be resolved according to symmetrical component theory into three separate symmetrical systems of positive, negative, and zero-sequence respectively. If saturation effects were small, the effect of the unbalanced system is the synthesis of the separate effects of the 3 component symmetrical systems.

2.1 Operation of Induction Motor under Positive-Sequence.

The positive-sequence system of voltages in a three-phase system consists of three voltages of equal magnitude having a progressive phase displacement of 120 between them in a given cycle order (abc). When a positive sequence system of voltages, of amplitude V_1 and frequency f , is applied to the symmetrical 3-phase stator winding of an induction motor, it produces a rotating magnetic field in its air-gap, which rotates, in the positive or forward direction relative to the primary circuit, at synchronous speed Ns. This field, at least the fundamental component has a constant amplitude, and induces e.m.f's. causing the flow of the currents in the rotor or secondary. Due to the resultant electromagnetic interaction of the magnetomotive-force and field, an unidirectional torque is developed, tending to rotate the rotor in the direction of the rotating field. If the torque developed is sufficient, the rotor follows the rotating field at a speed N, somewhat lesser than the synchronous speed. Under running conditions, the field rotates forwards relative to the secondary circuit with a velocity sN, where s is slip equal to $(N_s-N)/Ns$, and induces in its windings a positive-sequence system of voltages of amplitude, sV, and frequency sf, which causes positive-sequence currents to flow in the secondary. The primary and secondary positive-sequence currents satisfy the m.m.f. balance equation, so that the difference of their m.m.fs. just produces the positive-sequence flux.

The equivalent circuit of the induction motor for the positive-sequence system of voltages is shown in Fig. 2.1.

The positive-sequence torque at an slip s is given in synchonous watts, by

$$T_1 = 3\ I_{2p}^2\ r_2 / s \qquad \ldots \quad (2.1)$$

where I_{2p} = positive-sequence rotor phase current.

r_2 = rotor resistance per phase

Fig. 2.2 represents the torque speed characteristics of a standard induction motor under positive-sequence operation.

2.2 Operation of Induction-Motor under Negative-Sequence.

The negative-sequence system of voltages, consists of three equal voltages, each making an angle of 120° with the other, buthaving the phase-sequence in the reverse order to that of the positive-sequenc system. When a negative-sequence system of voltages, of amplitude V_2, and frequency f, is applied to the symmetrical stator winding of an induction motor, it produces a rotating magnetic field in its air-gap, which rotates, in the negative or backward direction (i.e. in the oppositedirection to the field due to the positive-sequence system) relative to the primary circuit at synchronous speed, and at least the fundamental of this field has a constant amplitude.

If, the rotor is rotating in the direction of the positive-sequence field with slips (the negative-sequence field rotating at synchronous speed in the opposite direction), the slip of the rotor with respect to the negative-sequence field is

$$s' = -N_s - N/-N_s = 2-s \quad .. \quad .. \quad .. \quad (2.2)$$

The equivalent circuit of the induction motor for the negative-sequence operation is shown in Fig. 2.3

The negative-sequence torque, in synchronous watts, at slip s', is given by

$$T_2 = 3 \; I_{2n}^2 \; r_2 / s' = 3 \; I_{2n}^2 \, r_2 / 2-s \quad .. \quad .. \quad (2.3)$$

where I_{2n} = negative-sequence rotor phase current.

r_2 = rotor resistance phase

rotor resistance increases with the increase of slip

Fig. 2.4 represents the torque-speed characteristics of a standard induction motor, under negative-sequence operation.

2.3 Operation of Induction Motor Under Zero-Sequence

It may be shown that when a balanced 3-phase stator winding of an induction motor is excited with zero-sequence or co-phasal sinusoidal currents, the n^{th} space harmonic m.m.f. at a point α elec. degrees along the periphery of the armature from the axis of any one phase winding, at any instant of time t, is

$$F_{n\alpha} = K_n \, F_n' \, (1 + 2 \cos \tfrac{2n\Pi}{3}) \cos (n\alpha + \tfrac{2n\Pi}{3}) \sin 2 \pi f t$$

(2.4) where F_n' = amplitude of the n^{th} space harmonic m.m.f. of each phase for concentrated & full pitch winding

K_m = m^{th} space harmonic winding factor of each phase, when each phase winding is of the distributed and fractional-pitch form.

It will be seen that F_{md} is zero for all values of m, except m= 3 and multiples of 3. In practice, the higher multiples of the third space harmonic are sufficiently small, and may be neglected. Thus the m.m.f. waveform, due to zero-sequence currents, produces a magneticfield in space which has three times the number of poles for which the machine is actually wound. The resultant field is stationary in space and pulsates at the fundamental frequency f. This stationary and pulsating field may be resolved into two rotating components of equal magnitude but rotating in opposite directions at a synchronous speed equal to one-third of that of normal 3-phase operation.

It follows that the equivalent circuit of the machine, for zero-sequence operation, will be similar to the well known equivalent circuit of a single phase induction motor. This circuit is modified,[26] as shown in Fig. 2.5 by the introduction of the link L and switch M in order that the effect of physical arrangement of both the primary and secondary windings may be considered.

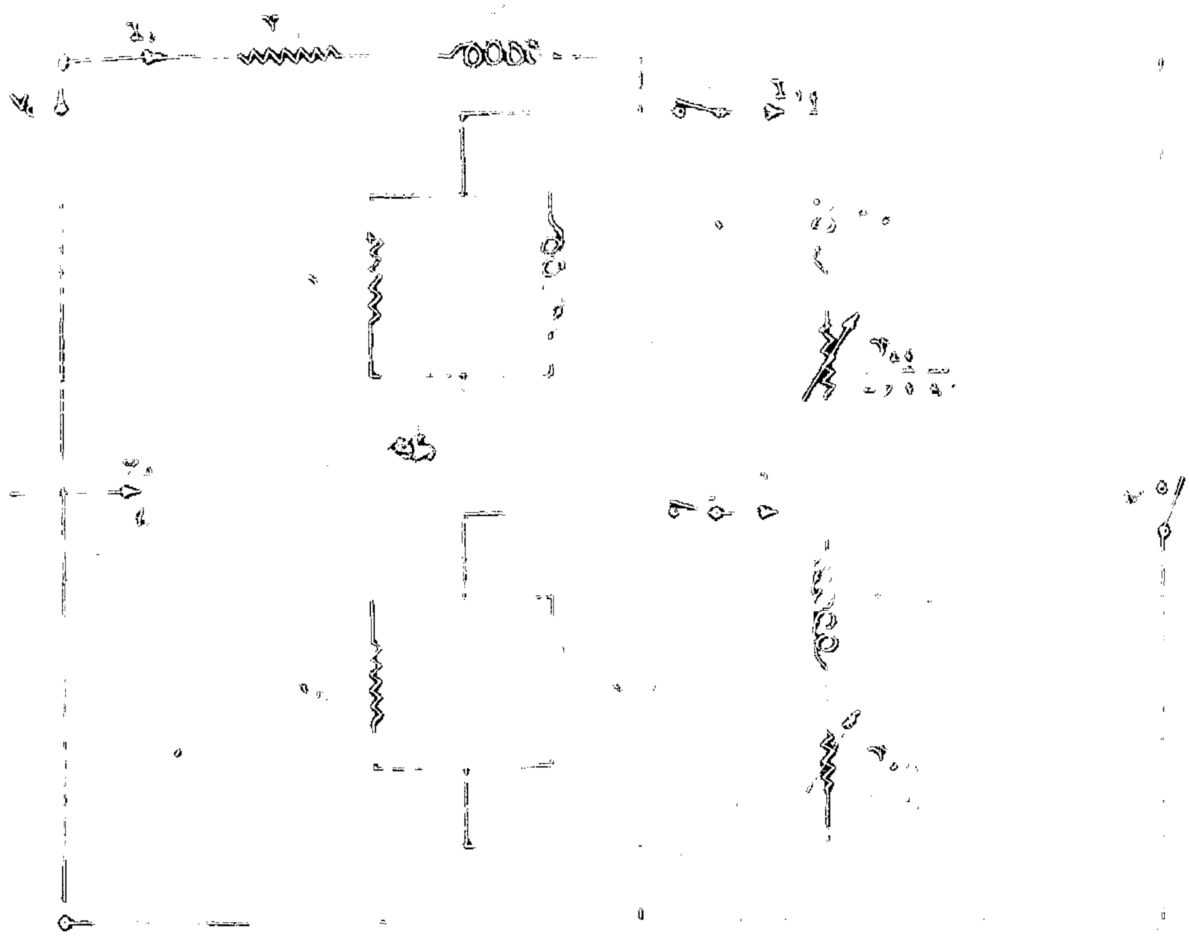
The slip of the rotor with respect to the third harmonic field, rotating in the positive or forward direction, is given by

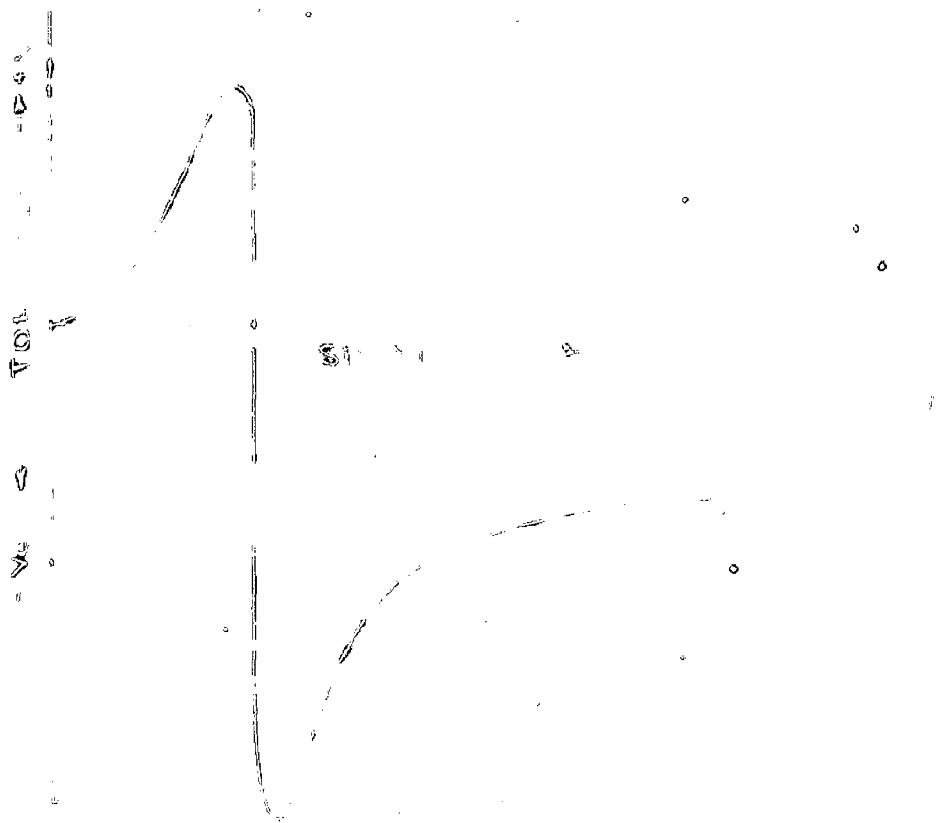
$$s'' = \frac{(Ns/3) - N}{Ns/3} = -2 + 3S$$

and the slip of the rotor with respect to the third harmonic field, rotating in the negative or backward direction, is given by

$$s'' = \frac{(-Ns/3) - N}{-Ns/3} = -4 - 3S$$

When the primary winding has a coil span of two-third the pole pitch or each phase winding is distributed in two-thirds of the pole





pitch, the value of winding factor K is zero. With either of these physical arrangements of the primary winding, the third harmonics are always eliminated, and there is no interaction between the primary and secondary windings. The equivalent circuit of Fig. 2.5 reduces to the leakage impedence ($r_1 + jx_1$) only, and therefore, the switch M must be closed.

When the phases of the secondary are connected in star, and the neutral point is isolated, there is no circulating path for zero-sequence currents. Thus, with this physical arrangement of the secondary windings, the load connecting links L must be opened. Again, when the secondary winding has a coil span of two-thirds the pole pitch, or each phase winding is distributed in two-thirds of the pole pitch, no zero-sequence e.m.fs can be induced in the windings, irrespective of whether it is star or delta connected, and thus the links L must be opened.

The zero-sequence torque, in synchronous watts, at any slip s, is given by

$$T_0 = 3\left[\left\{I_{2f}^2\, r_{2f} / (-2+3s)\right\} - \left\{I_{2b}^2\, r_{2b} / (4-3s)\right\}\right]$$

where r_{2f} and I_{2f} = Secondary resistance and currnt per phase for forward rotating field.

r_{2b} and I_{2b} = Secondary resistance and current per phase for backward rotating field.

The determination of paramaters, and performance calculations, for zero-seuence operation, are completely discussed by Brown and Butler[18].

Fig. 2.6 represents the torque-speed characteristics of a standard induction motor, under zero-sequence operation.

2.4 Operation of Induction-Motor under Mixed-Sequence.

The zero-sequence system will be absent from all types of

3-phase operation in which the algebric sum of the phase vectors is zero. This condition is satisfied in many types of unbalanced operations of 3-phase induction machines. Hence in the discussion of operation of induction motor under mixed-sequence the zero-sequence will be ignored and only the positive and negative-sequences will be considered.

When an induction motor operates under both positive and negative-sequence systems, both the positive and negative-sequence torques are present together, and the net output torque of the motor is the algebraic sum of the two. Also the relative magnitude of the negative-sequence torque depends upon the degree of unbalance.

2.4.1 Mixed-Sequence Operation-Normal Motor.

Typical torque-speed curves for a normal induction motor, under different degrees of unbalance, are shown in Fig. 2.7,(neglecting the effect of stray load losses). Curve 1 is for normal positive-sequence operation at rated voltage (or 1 per unit). Curve 2, for negative-sequence operation at rated voltage is therefore the inverse mirror image of curve 1.

Curve 3 is for the condition in which both the positive and negative-sequence voltages are of the full rated value. It is the algebraic sum of the ordinates of curves 1and 2. Curve 3 thus is applicable for the condition $|V_1| = |V_2|$ and the torques would be reduced in proportion to the square of voltages when the values of $|V_1|$ and $|V_2|$ were other than the rated value. Curves 4 and 5 apply respectively tothe conditions of operation in which $|V_1| > |V_2|$ and $|V_1| < |V_2|$

2.4.2. Mixed-Sequence Operation -- High Resistance Rotor.

Typical torque-speed curves for a machine with high

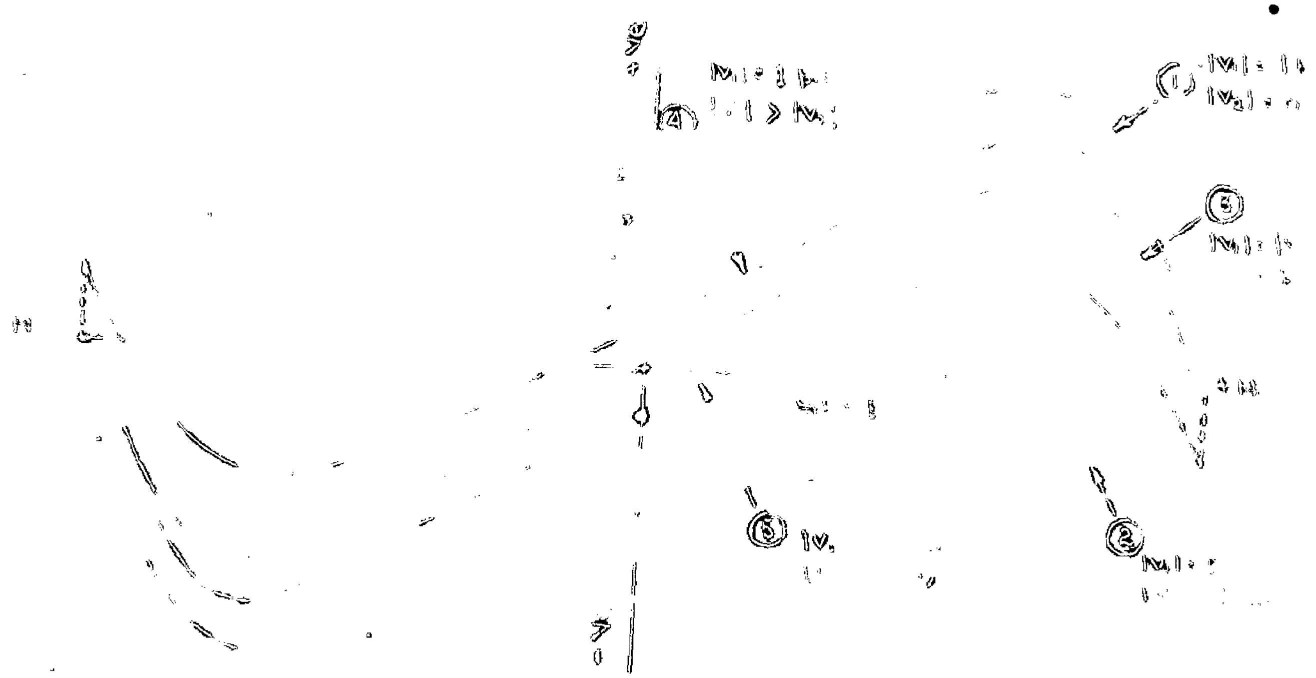
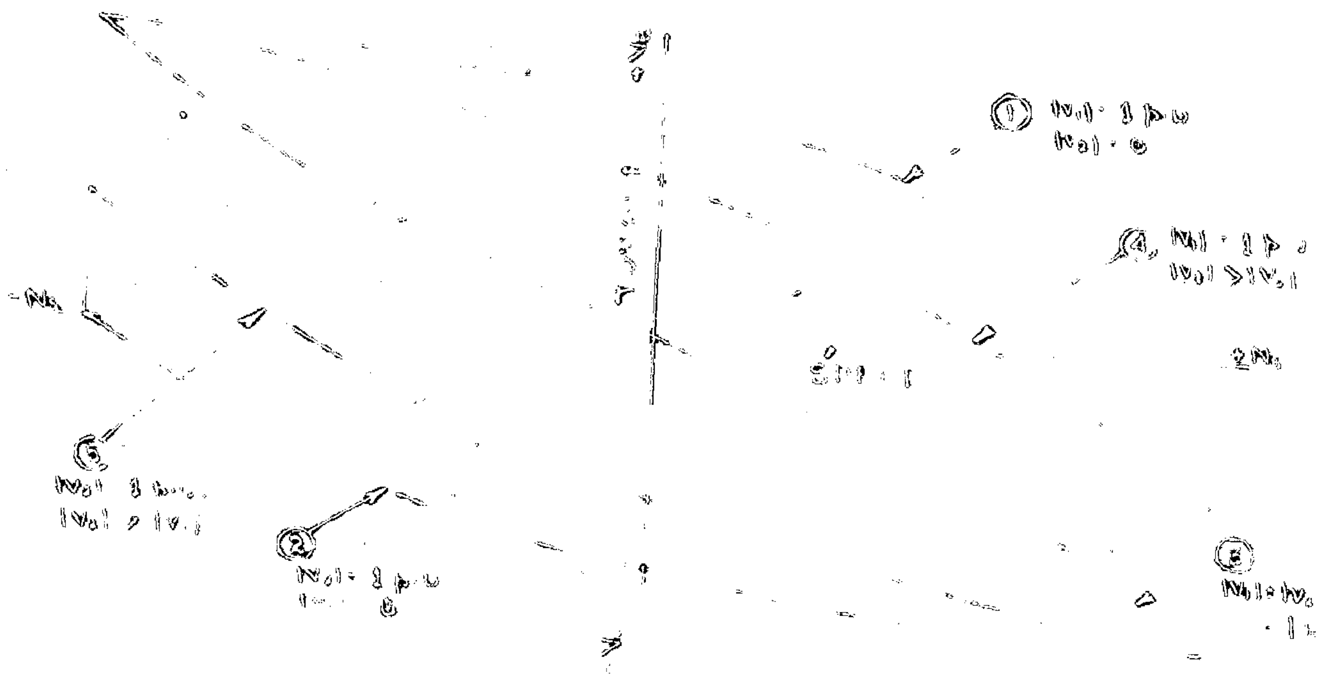


FIG. 2 [illegible]

[illegible]



[illegible]

resistance rotor are shown in Fig.2.8 Curves for various values of $|V_1|$ and $|V_2|$ are included as in section 2.4.1. Curve 3 shows that when $|V_1| = |V_2|$ dynamic braking is obtained over the whole speed range.

2.4.3. Speed Control of Induction-Motor with Mixed-Sequence.

From Fig. 2.7, it is obvious that the possibility of effective speed control by varying the ratio of $|V_1| / |V_2|$ is very limited in a normal motor but from Fig. 2.8 (applicable to motor having a high secondary resistance) the speed control of induction motor in all the four quadrants is possible. The speed control of induction motor with mixed sequence is discussed in greater detail in section 4.

SECTION 3

3. PRODUCTION OF MIXED SEQUENCE VOLTAGES

3.1. From Two Generators

The direction of rotation of an induction motor connected to the supply depends upon the phase sequence of the applied voltages, and can easily be changed by simply interchanging any two of its terminal connections. A mixed-sequence opration of the motor can be achieved if the primary windings of the induction motor are connected simultaneously to two generators in such a way that the motor connected to individual generator separately runs in opposite directions. The generator rotating the motor in the forward direction supplies the positive-sequence system, and the other rotating it is the opposite direction, the negative-sequence system of voltages. The magnitude of the positive and negative-sequence voltages can be varied by changing the excitations of the generators. The connections are shown in Fig. 3.1

3.2 Various Forms of Easily Produced Unbalanced Operations.

3.2.1 Unsymmetrical Impedences in the Stator

The primary winding of the motor itself is connected symmetrically in star or delta. The insertion of a variable impedence, resistance, inductance, and capacitance, either singly or combined, as shown in Fig. 3.2, produces unbalancing, and the degree of unbalance can be varied by varying the impedence.

The phase voltages, and currents are shown in the figure. Applying Kirchoff's Law, the inspection equations are

$$V_b + V - I_a Z - V_a = 0 \quad \ldots\ldots\ldots\ldots\ldots (3.1)$$

$$V_b - a^2 V - V_c = 0 \quad \ldots\ldots\ldots\ldots\ldots (3.2)$$

$$I_a + I_b + I_c = 0 \quad \ldots\ldots\ldots\ldots\ldots (3.3)$$

where V_a , V_b , and V_c are phase voltages

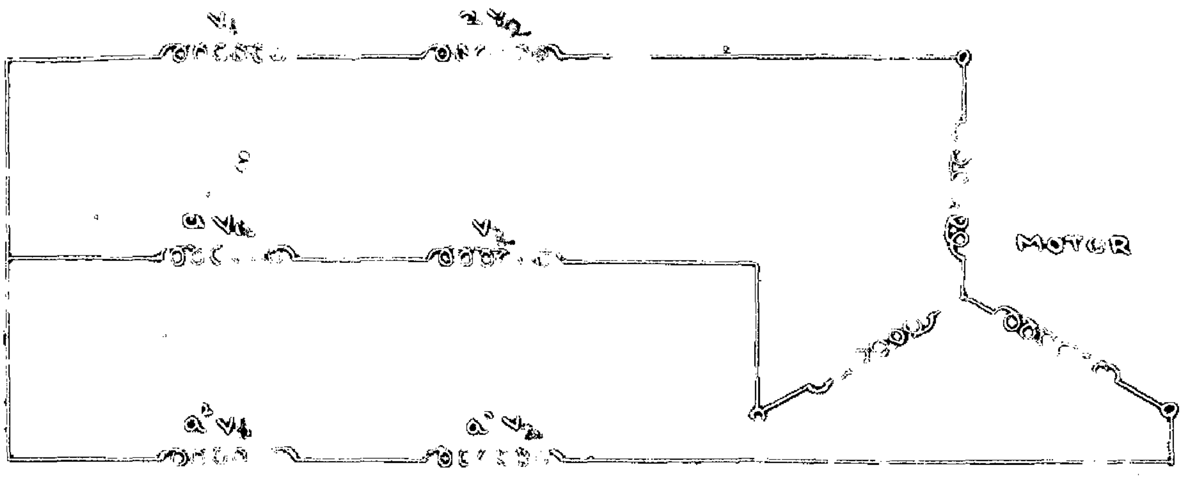


FIG. 3·1

MIXED-SEQUENCE VOLTAGES — FROM TWO GENERATORS

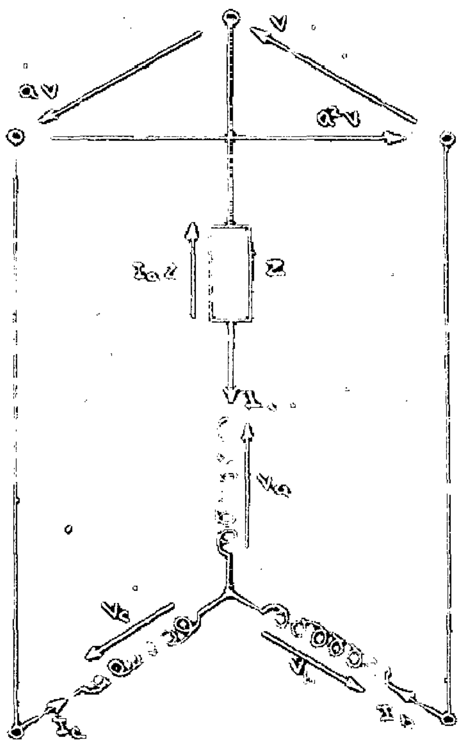


FIG 3·2

MIXED-SEQUENCE VOLTAGES FROM UNSYMMETRICAL IMPEDENCES IN THE STATOR

I_a , I_b , and I_c are phase currents

and "a" is operator = $e^{j2\pi/3}$

The solution of equations (3.1) , (3.2) , and (3.3) , with the use of symmetrical component theory gives

$$V_1 = \text{Positive-sequence voltage} = -aV\left[\frac{(3 + Y_2/Y)}{(1-a)\,(3+Y_1/Y + Y_2/Y)}\right]$$

and

$$V_2 = \text{negative-sequence voltage} = V\left[\frac{(3 + Y_1/Y)}{(1-a)\,(3+Y_1/Y + Y_2/Y)}\right] \quad \ldots \ldots \quad (3.4)$$

where Y_1 , and Y_2 are admittances of the phase windings for currents of positive, and negative-sequence systems respectively, and Y the inserted external admittance.

The ratio $|V_2|/|V_1|$ gives the degree of unbalance and is obtained from eqns. (3.4)

as

$$\left|\frac{V_2}{V_1}\right| = \left|\frac{3Y + Y_1}{3Y + Y_2}\right| \quad \ldots\ldots\ldots \quad \ldots\ldots \quad (3.5)$$

3.2.2 Using a Line Auto-Transformer

The unbalancing can also be obtained from the insertion of a single phase auto-transformer between the machine and the supply system, and a number of voltage tappings on the auto-transformer are provided for variable control of the machine (Fig. 3.3). Since, the auto-transformer may be regarded as a four terminal network, as indicated by Morris, and included in Fig. 3.4. It is necessary to determine initially the equivalent source impedance Z_s , the magnetising impedance Z_m , and the open circuit voltage ratio K for each tapping utilised.

The inspection equations are

$$K V - Z_s I_a - V_a + V_b = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.6)$$

$$a^2 V - V_b + V_c = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.7)$$

$$I_a + I_b + I_c = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.8)$$

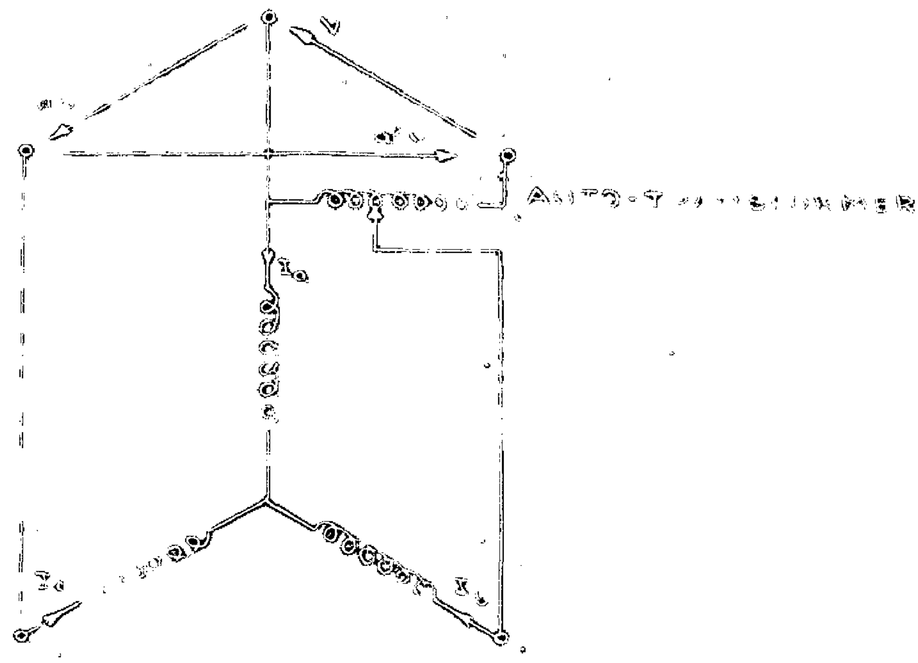


FIG. 3.3

MIXED-SEQUENCE VOLTAGES — USING LINE AUTO-TRANSFORMER —

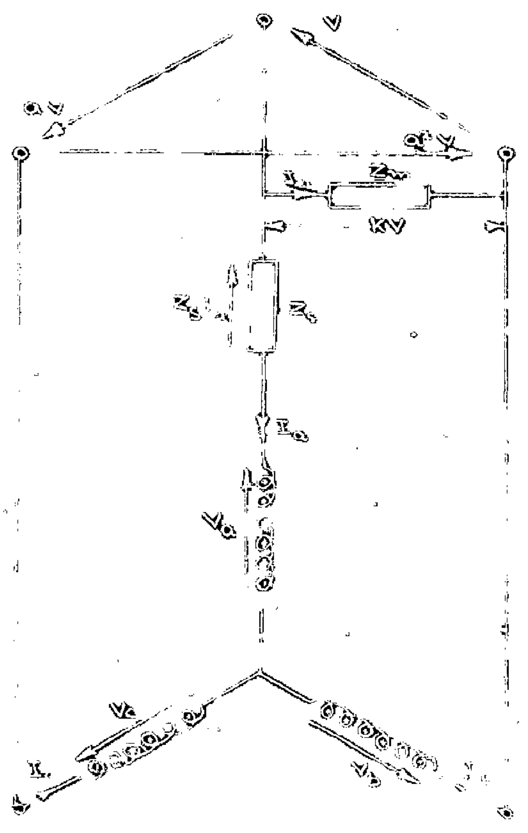


FIG. 3.4

EQUIVALENT CIRCUIT FOR FIG. 3.3

The solutions of equations (3.6), (3.7), and (3.8), with the use of symmetrical component theory, gives

$$\left.\begin{aligned} V_1 &= V \left[\frac{(K - 1) - a\,(K + 2) - a\,Y_2 / Y_0}{(1 - a)\,(3 + Y_1 / Y_0 + Y_2 / Y_0)} \right] \\ \text{and } V_2 &= V \left[\frac{(2 + K) + a\,(1 - K) + Y_1 / Y_0}{(1 - a)\,(3 + Y_1 / Y_0 + Y_2 / Y_0)} \right] \end{aligned}\right\} \quad \ldots\ldots (3.9)$$

The ratio $|V_2|/|V_1|$ gives the degree of unbalance and is obtained from equations (3.9) as

$$\left|\frac{V_2}{V_1}\right| = \left|\frac{(2 + K) + a\,(1 - K) + Y_1 / Y_0}{(K + 1) - a\,(K + 2) - a\,Y_2 / Y_0}\right| \quad \ldots\ldots\ldots (3.10)$$

3.3 <u>The use of Saturable Core Reactor (SCR)</u>

The saturable core reactor is a variable a.c. impedance device. Its impedance can be varied by the control of the core flux with direct current. The SCR has a three limbed core. Two identical main windings (a.c. coils) are placed on the outer limbs, and the control winding (d.c. coil) is placed on the central limb. The application of direct current to the control winding introduces d.c. flux which reduces capacity for alternating flux and accordingly the reactance of the a.c. coils.

The SCR is also used as a magnetic amplifier, and has been applied to various purposes in a wide range of power output.

3.3.1 <u>Core Material</u>

The ideal cores are non-existent and the actual core materials will increase their suitability of cores of magnetic amplifier as they approach the following conditions.

(i) Minimum hysteresis and eddy current losses

(ii) High differential permeability ($\mu_d = \frac{dB}{dh}$) below the knee of the hysteresis loop and extremely low differential permeability in the saturation branches.

(iii) Abrupt change between zones of high and low different-
al permeability.

(iv) High saturation flux-density Bs.

Only three elements, Cobalt, Nickel and Iron exhibit high B-H ratios at normal temperatures. Most ferromagnetic materials are composed of alloys of these three elements with small but controlled amounts of other substances.

The ferromagnetic alloys, mostly used as core materials for linear inductors and transformers, however, in themselves are not suitable for SCR, and additional properties must be designed into SCR cores.

The manufacture of the ferromagnetic alloys, suitable for magnetic amplifier core, is a detailed process to enhance the desireable magnetic characteristics. The chemical composition is very important. Nickel and iron are principal constituents. Controlled traces of copper, chromium, molybdenum, and silicon may be introduced into the alloys or used in its manufacture. It is not the magnetic properties of the trace substances, but their effect on the crystalline and metallurgical structure of the ferromagnetic alloy which enhances certain characteristics.

The atoms of the ferromagnetic elements--cobalt, nickel and iron have excess electrons in their outer rings. Each single atom may be considered a small electromagnet in which the magnetizing force is set up by the outer shell electrons spinning in their orbits. However, for the material itself to be ferromagnetic, this effect must be comulative. This requires a specific arrangements of these atoms in a crystal lattice such that the individual magnetic field produced by each atom will not cancel each other.

Crystal or grain orientation is effective in enhancing certain characteristics. Two methods are used : Cold rolling, and annealing in the presence of magnetic field. The amount of certain trace elements must be controlled. Annealing in dry hydrogen and reduction with strong deoxidizing agents are effective techniques. Alloys with most identical properties are vailable commercially from several different manufacturers.

3.3.2 Core Fabrication

SCR cores may be

(i) fabricated from flat laminations of thickness ranging upward from 0.002", stacked together as in transformer practice (Fig. 3.7)

(ii) wound from continuous tape, of thickness ranging upward from 0.001" into a toroidal spiral (Fig.3.8), or

(iii) stacked into a toroid from flat stamped rings of thickness greater than 0.002" (Fig. 3.9)

of the above three, the tape wound core is the most widely used due to the following reasons:

(a) It is available in all the principal alloys from several manufacturers.

(b) It has an excellent magnetic characteristics.

(c) It can be would to any dimensions without requiring new dies.

(d) The standard core sizes available simplify the magnetic design.

(e) The leakage flux is lower.

A stacking factor, defined as the ratio of effective cross-sectional area to geometrical cross-sectional area, may be expressed as a percentage.

Fig. [illegible]

FLAT LAMINATION AND [illegible] CORE

[illegible]

[illegible]

[illegible]

[illegible]

S.F. = [effective area/ Geometrical area] x 100

It varies with material thickness. Typical figures for tape wound cores are:

70% for 0.001" tape, 80% for 0.002", and 85% for 0.004".

3.3.3 B-H or Flux-Current Loop

The B-H loops, showing the relation between the two inter-dependent magnetic variables, flux-density B and magnetizing force H, are obtained experimentally under carefully controlled conditions of driving electrical variables, frequency, sample size, and core configuration. Fig- 3.10. shows the major B-H loop for a typical magnetic amplifier material. This is called major loop, because it represents the limiting magnetic states and a further increase in the applied voltage does not change the shape appreciably. Fig. 3.11 shows comparatively the B-H loops for transformer silicon steel and a magnetic amplifier material. The significant parameters of the B-H loops are given below:

(1) Saturation flux-density, Bs - This is the practical maximum flux-density the material is capable of. Beyond this point B increases very slowly with H, the relative permeability becomes constant, approximately equal to unity.

(2) Retentive flux-density or Retentivity, Br- If H is made zero when the flux-density is = Bs , the flux decays to $\pm B_r$ instead of zero. B_r is a measure of residual magnetization in the material, and is maintained indefinitely until additional excitation is applied. Any flux level between $\pm B_r$ can be stored.

(3) Squareness ratio or Rectangularity, B_r/B_s - A value close to unity is desirable for the magnetic amplifier reactors.

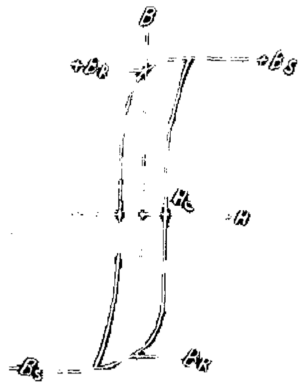


FIG. 8·10

B-H LOOP OF TYPICAL MAGNETIC AMPLIFIER

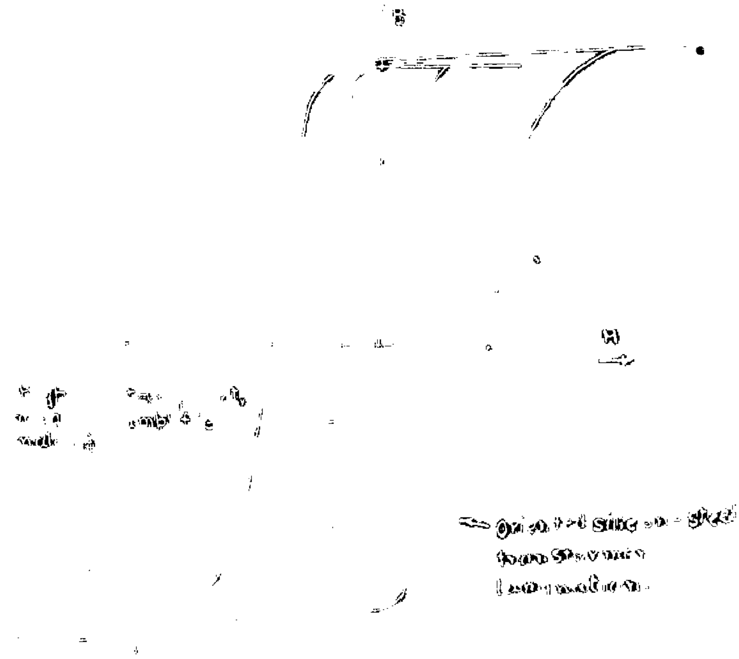


FIG. 8·11

B-H [illegible] OF [illegible] FORMER GRADE A[illegible] MAGNETIC [illegible] OF [illegible] A TOR CORE [illegible]RIALS

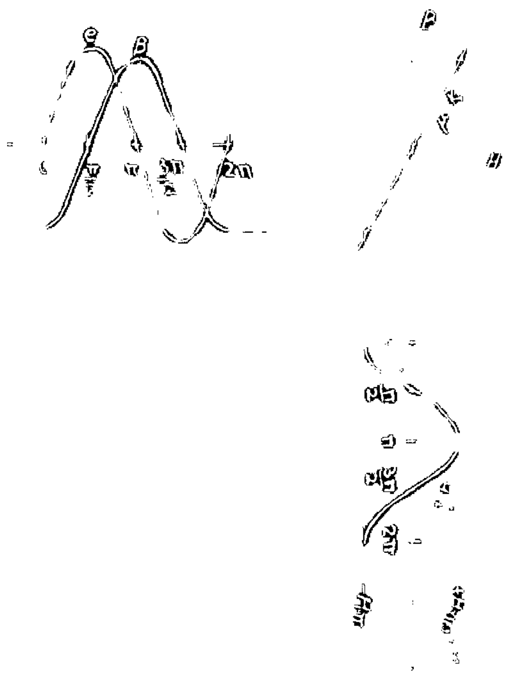


FIG. 8·12

LINEAR INDUCTOR

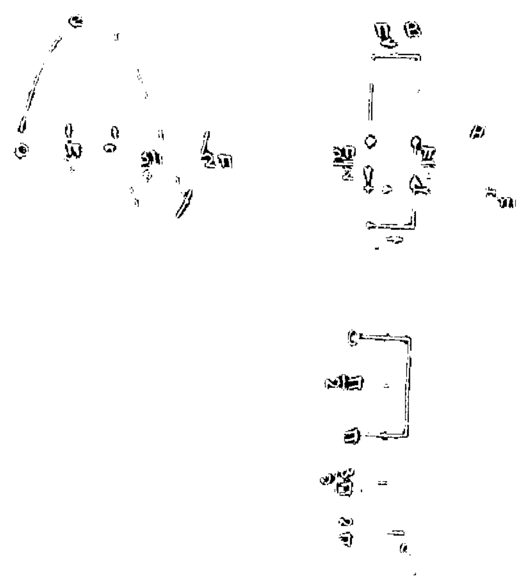


FIG. 8·13

SQUARE LOOP INDUCTOR

(4) Coercive force. H_c is the magnetizing force necessary to bring the flux-density to zero from B_r. In cores of good rectangularity, H_c determines the peak value of the magnetizing current.

3.3.4 Hysteresis and Eddy-Current Effects

Fig. 3.12 and Fig. 3.13 show the relation between the exciting voltage and exciting currents for the materials having linear and non linear (Square) B-H loops respectively.

For linear B-H loop, the exciting current is in quadrature with the exciting voltage and no power is absorbed from the source. As the B-H loop opens up, the inphase component of current appears and in the extreme cases the exciting current is in phase with the exciting voltage. The powere is dissipated in the core material and is assigned to the hysteresis and eddy current losses.

For a given flux-density showing, the eddy current losses increase with the frequency and the B-H loop widens alongwith the increase of H_c (Figs. 3.14 & 3.15).

The shape of the B-H loop also changes if the flux-waveshape is not sinusoidal. If the core is excited by the sinusoidal current, the flux no longer varies sinusoidally, and the loops of Figs.3.16 & 3.17-result. The loop measured with sinusoidal exciting current have power rectangularity and larger H_c than those taken with sine-flux.

Ideally, to compute the electric characteristics of the reactor, B should be known for every value of H and vice-versa. With sine-flux drive, the flux-swing is set by Faraday's Law, and is independent of either hysteresis or eddy current effects in the core. But with sin-current excitation, H is known, since the excitation current is independent variable. But B cannot be computed, because the rate

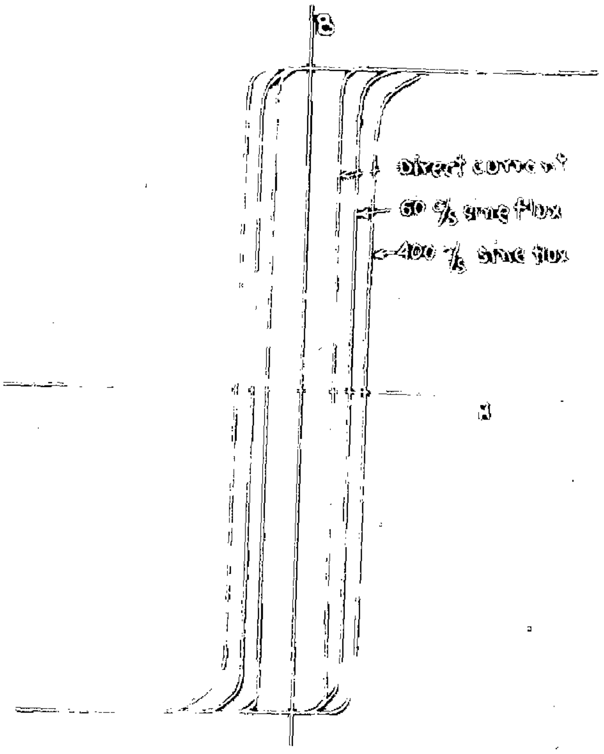


FIG. 3.14

D.C. AND SINE-FLUX LOOPS

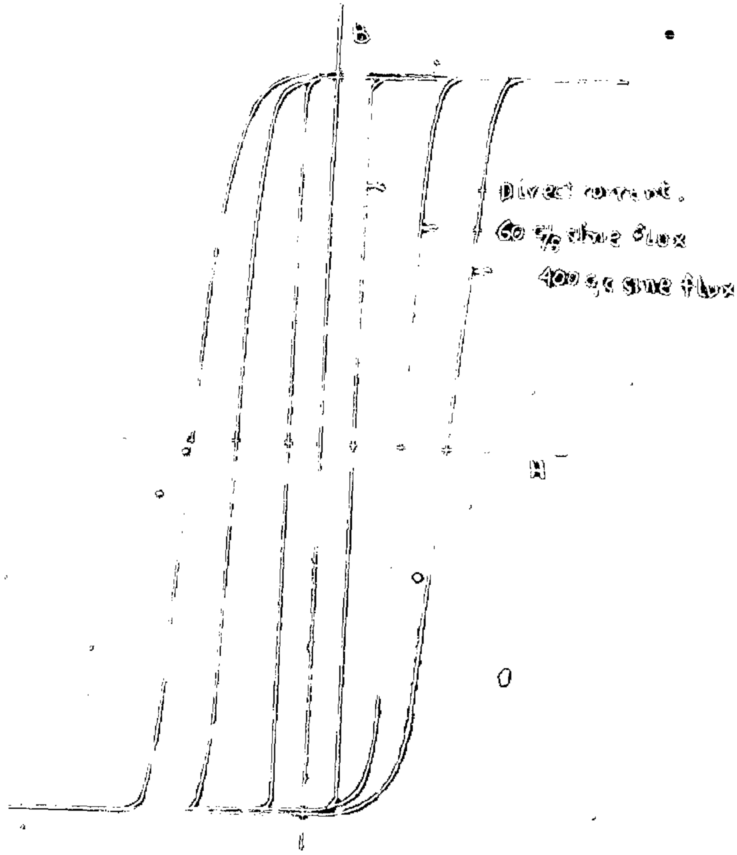


FIG. 3.15

D.C. AND SINE CURRENT LOOPS

FOR DELTAMAX

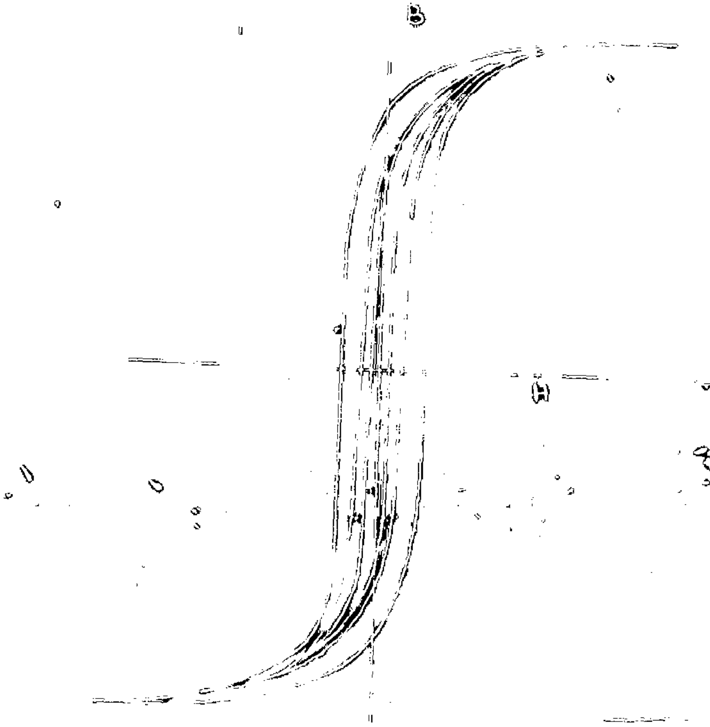


FIG. 3.16

D.C. AND SINE-FLUX LOOPS

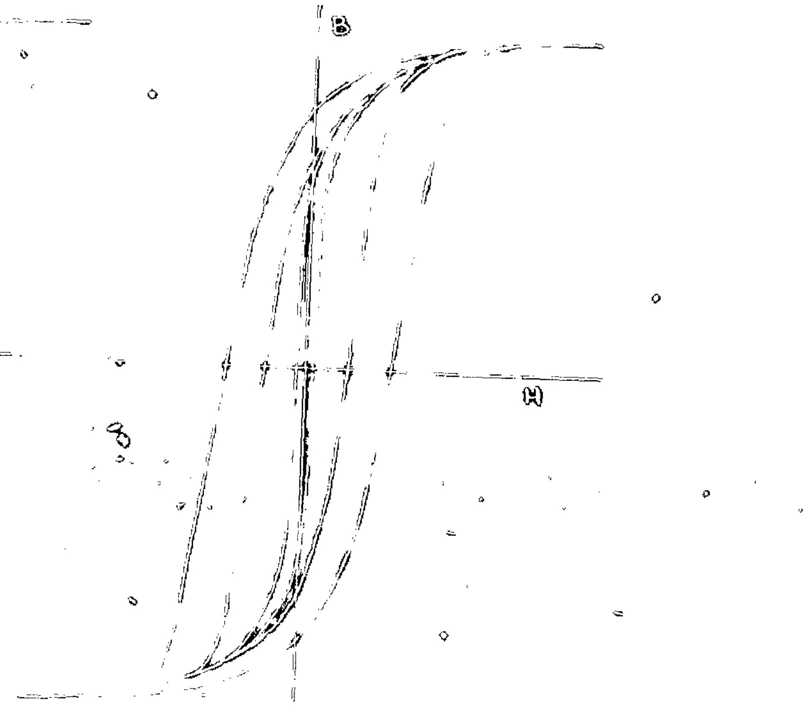


FIG. 3.17

D.C. AND SINE-CURRENT LOOPS

FOR SUPERMALLOY

of change of flux itself sets the eddy current losses, and hence determines how much of the magnetizing current goes to change flux and how much is necessary to account for the eddy-current losses. Therefore, the problem is some-what indeterminate and only the experimental datas are relied upon.

3.3.5 Temperature Effects

The B-H loop is sensitive to temperature changes. One or more of the parameters, saturation flux-density, B_S-, rectangularity B_r/B_s, and coercive force H_c, may be affected. The resistivity of the material increases with the temperature thereby decreassing the eddy-currents losses. Typical effects are shown in Figs. 3.18 & 3.19.

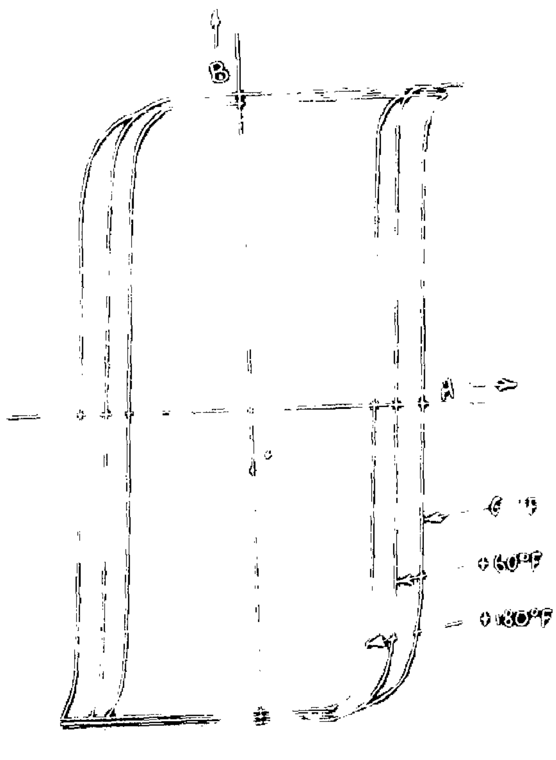
As the temperature is raised, a value (the Curie point) is reached at which the material lose their ferromagnetic properties and become paramagnetic. The Curie point is different for each alloy and well above normal operating temperatures. The change is reversible, because magnetic properties reappear as the temperature decreases.

3.3.6 Minor Loops

The B-H loops considered so far are the major loops, because they indicate the limiting magnetic states, Bs and H_c . The lower excitation on the reactor coil produce minor loops with flux swings smaller than 2Bs. The eddy-current losses are reduced, because the rate of change of flux are lower, and the width of the loop (2 H_c) decreases.

As with major loops, the shape of the minorloops depends on the waveshape and type of electrical drive. The sine-flux families for Deltamax and Supermalloy are shown in Figs. 3.20 & 3.21.

The effective permeability is defined as the slope of the line between opposite tips of the major or the minor loop. The slope of



Temperature eff[illegible]cts on the [illegible] loop (s[illegible] current)

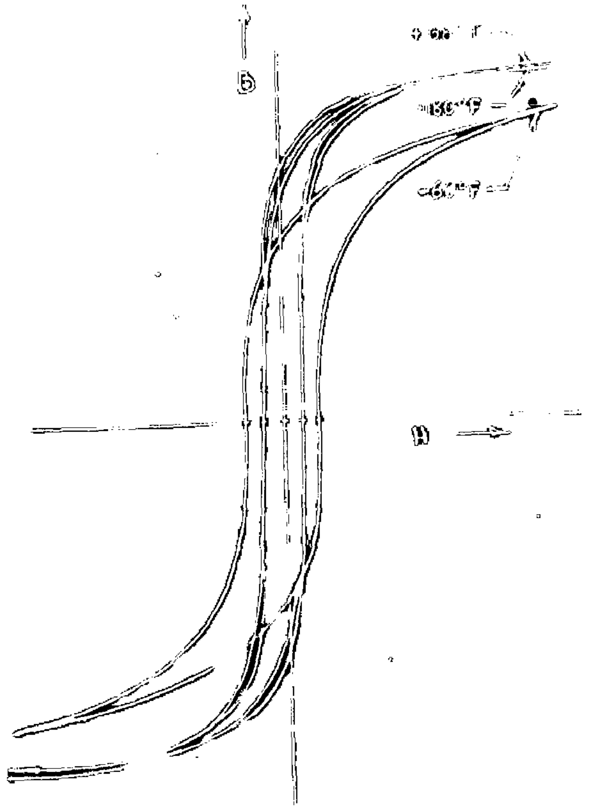
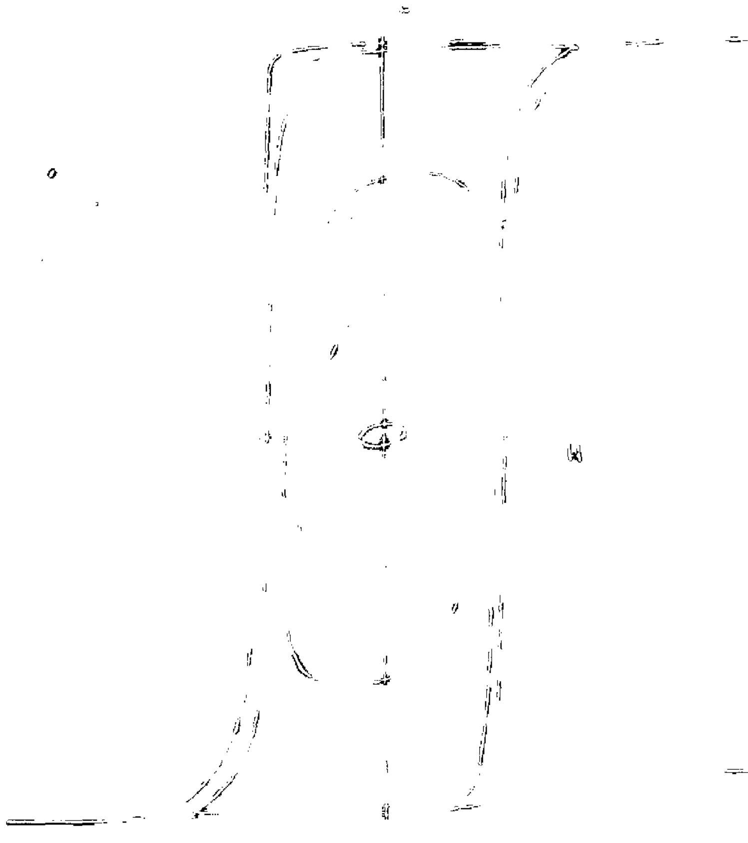
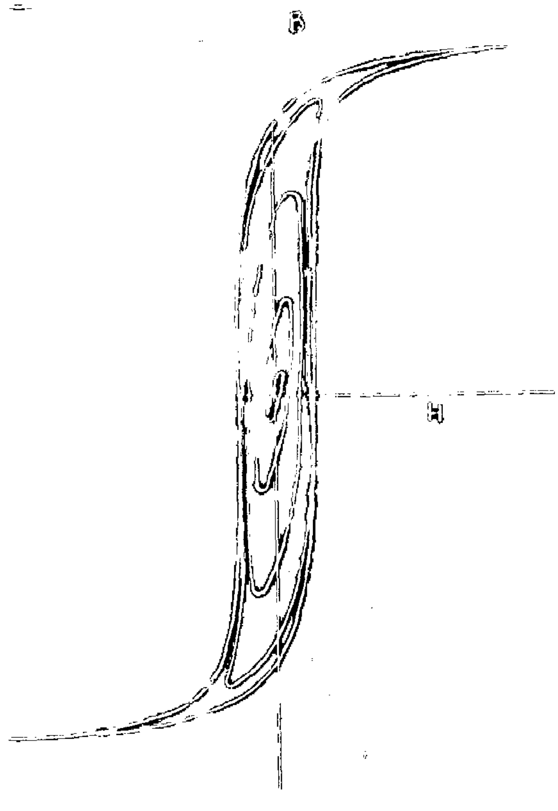


FIG. [illegible]

Temperature effects on the Supermalloy loop (sine-flu[illegible]).





[illegible]

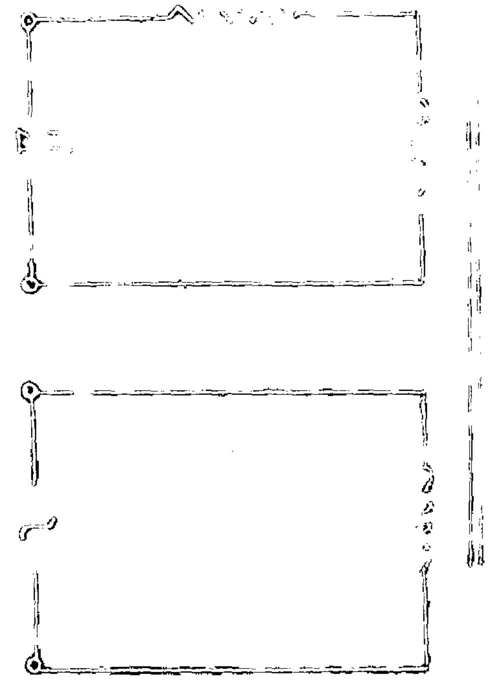
Supermalloy is fairly constant, because the shape of the loop does not change as the excitation is reduced. However, the minor loops of Deltamax are relatively wider than the major loop, because the width of the loops tends to remain constant. Supermalloy is excellent for high grade transformer or inductors, because the magnetizing impedence remains high over a wide range of applied voltages. Its magnetic characteristic may be assumed linear. The effective permeability of Deltamax drops off rapidly with B, indicating collapse of impedence at low voltage levels. Its characteristic is non-linear and its use is restricted to magnetic amplifiers.

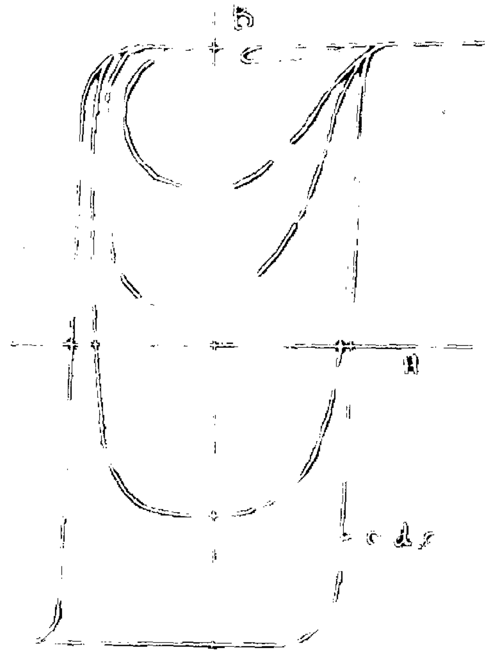
Another family of minor loops is obtained with combined d.c. and a.c. excitation (Fig. 3.22) The applied voltage is adequate to drive the core through 2 B_s , when no d.c. magnetizing force is present. As the d.c. current is increased, the flux-swing becomes less than 2 B_s though one of the saturation level is still reached. The minor loops for Deltamax with sine-flux are shown in Fig- 3.23. The operation of many magnetic amplifiers, if the minor loops are known, the volt-amp. conditions and the performance of the circuit can be predicted.

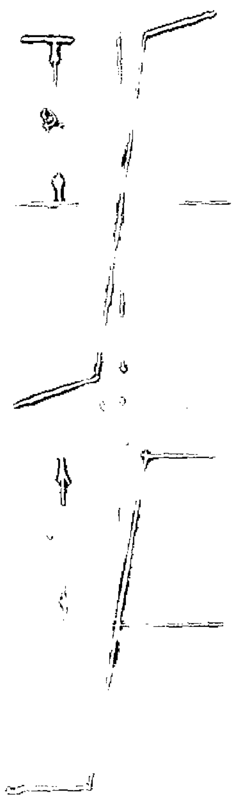
3.3.7 Operation of S.C.R.

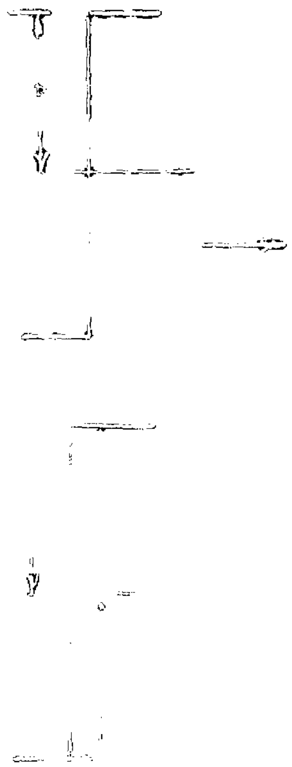
To explain the operation of SCR and other magnetic amplifiers, numbers of authors have asumed the different forms of ideal magnetization characteristics for the cores (Fig. 3.24).

The form shown in Fig. 3.24 (a) is based on the assumption that the slope of the characteristics is finite before and after saturation, and has been used by Lamm[13] in explaining the operation of 3-phase magnetic amplifiers. Krabbe[14] has also adopted a similiar apreach for explaining the operation of self excited magnetic amplifiers.









The form shown in Fig. 3.24 (b) is based on the assumption that the slope of the magnetization curve is infinite before saturation and zero after saturation, and has been used by Gale and Atkinson[15] and others. With these assumptions the operation of magnetic amplifiers are explained fairly well both in the transient and steady state. However, the resutls of the analysis are not valid when complete excitation is used, because infinite permeability means zero magnetizing currents, which leads to infinite gain.

The form of Fig. 3.24 (c) is based on the assumption of the infinite slope to the magnetization curve with additional assumption about the control circuit impedence, and has been used by Milnes[16] in obtaining relations for the self excited magnetic amplifiers, taking into account the magnetizing current also.

The form of Fig. 3.24 (d) is based on the assumption that although the mmf is reduced to zero, the residue flux in the core is nearly equal to the saturation value. This form is applicable to many practical core materials, particularly those having a rectangular hysteresis loop.

3.3.8 Single Core Reactor

The two coil reactor of Fig. 3.25 can function as a SCR The example does not have much practical value but serves to illustrate the relations among the electric and magnetic variables. The load circuit has N_L turns and is in series with line voltage and the load resister R_L. A d.c. current I_c flows through the control coil N_c from the d.c. source E_c. $N_L = N_c$, and R_c is sufficiently high to establish constant current in the control winding. The load current is controlled by I_c . When $I_c = 0$, the reactor traces the minor loop of Fig. 3.27 (with no saturation), the full line voltage across the coils, and only the magnetizing current .

flows in the load. Since R_s is assumed large, the currents induced in the control circuit through the reactor core are negligible.

As I_c increases, its mmf adds to the magnetic circuit. If I_c is + ve into the dot, it produces a flux change toward + Bs. Since the positive half cycle of line voltage drives the flux in the same direction, the core is saturated by the end of the positive line swing.

Figs. 3.26 & 3.27 show the significant waveshapes and the minor loop traced. The circuit action is explained below:

(i) Just before the end of the positive half cycle, the core is saturated and the load current equals e_1/R_L

(ii) At the reversal t_o, the flux is still saturated, coil voltage and line current are zero, and H = N_c $N_c I_c$ / 1

(iii) As the line goes - ve the core remains saturated between t_o and t_1 (the right flank of the loop), the load absorbs all the line voltages and coil voltage is still zero. At t_2, the load current only slightly smaller than I_c, with their difference determined by the width of the loop; $H_m = N_c (I_c - I_L) / 1$

(iv) Immediately after t_1, the net H on the core is smaller than necessary for saturation, the load current increases to $I_c + i_m$ and thereafter remains constant; the flux begins to move toward B_s, and coil voltage appears. Between t_1 and t_2, the coil voltage is the difference between the line and the constant voltage drop R_L ($I_c + i_m$) set up by the load current.

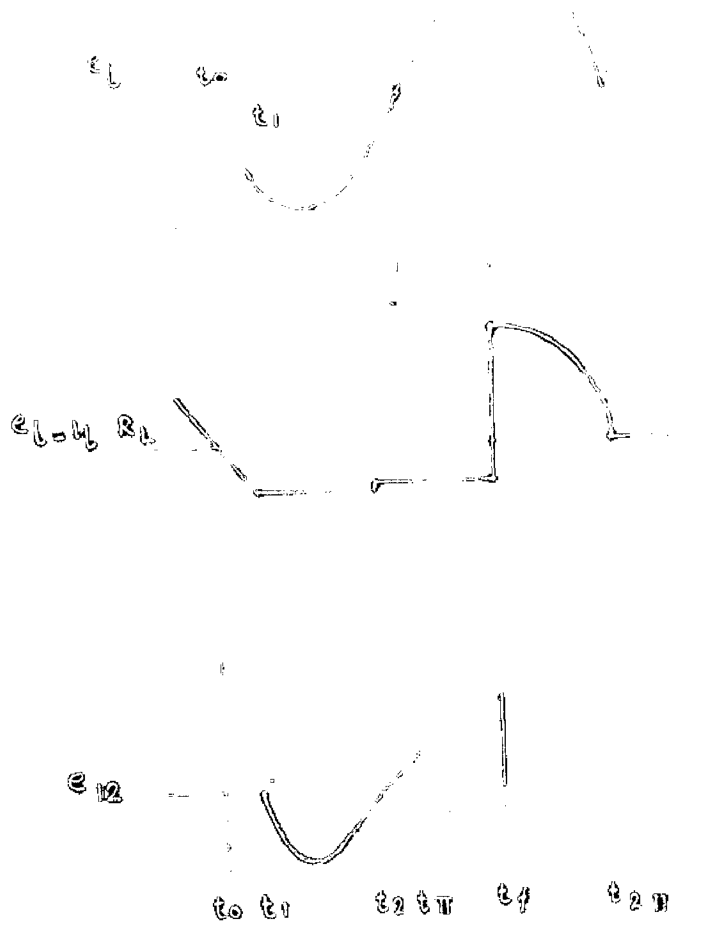


FIG 3·26

WAVE SHAPES, SIMPLE SCR

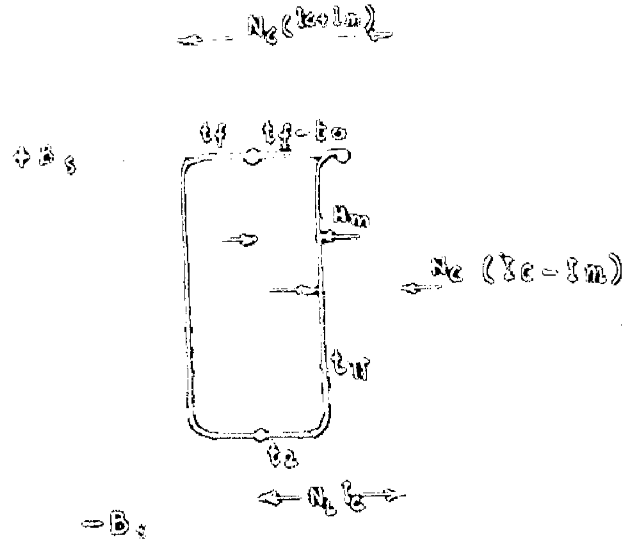


FIG. 3·27

B-H LOOP SIMPLE SCR

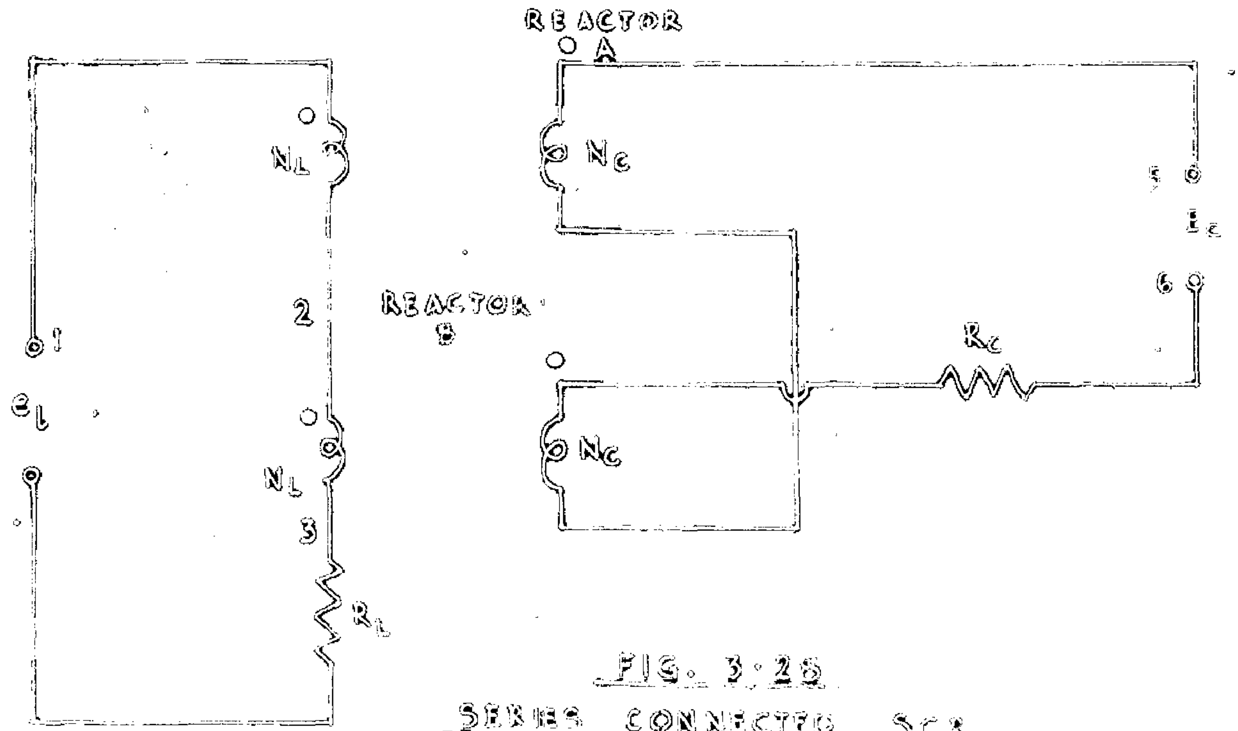


FIG. 3·28

SERIES CONNECTED SCR.

(v) At t_2, $e_1 = R_L$, $e_c = 0$, $B = B_e$, and the bottom of the loop is reached. This occurs before the reversal at t_π. Also $t_1 - t_e = t_\pi - t_2$.

(vi) Immediately after t_2, e_L e_1, coil voltage is positive the flux moves toward $+ B_s$, and $i_L = I_c - i_m$

(vii) The flux can swing upward only by the exact amount that it moved downward. It saturates at $+ B_s$ and t_f, and following holds

$$A_c \Big|_{t_1}^{t_2} = A_c \Big|_{t_2}^{t_f}$$

where A_c == area of the coil voltage- second

(viii) Between t_f and t_1 core flux remains at $+ B_s$ and $i_L = \frac{e_1}{R_L}$

As I_e increases, the portion of line voltage appearing across the Coil becomes smaller and the flux swing decreases when $I_e = E_m / R_L$, the core remains saturated, load and line voltages are identical, and no further action takes place.

Some power gain can be obtained if N_c N_L and the control coil is wound with many turns of fine wire. However, this circuit is rarely used due to following disadvantages.

(1) A high voltage is induced in the control circuit.

(2) If R_c is made large, to limit the induced current, most of the control power is dissipated in the control circuit and the gain of the circuit is negligible.

(3) If a linear inductor replaces R_c to limit the induced currents, it dissipates no control power and hence raises the power gain, but its size may be prohibitive, because of the d.c. current it must carry without saturating.

3.3.9 The Series- Connected SCR

If the control circuit resistance is relatively low, when the d.c. control signal is zero, the current in the load will equal

$$i_L = I_m + e_1 / R_c \, (N_C / N_L)^2$$

The second term is the reflected component caused by the control circuit resistance. Unless $R_c \, (N_C / N_L)^2$ is very high compared to RL, this current component will be large, and the amplifier action will be very poor.

The SCR of Fig. 3.28 does not require a high control circuit resistance and can operate with a wide range of control circuit resistance and can operate with a wide range of control circuit impedences. It consists of two identical reactors with load coils in series aiding and control coils in series opposition. The significant features are :

(i) The output is a non reversible a.c. or, with a load rectifier, single polarity d.c.

(ii) When the control voltage $E_c = 0$, both cores trace minor loops without saturating (Fig, 3.29.) and the output current - is minimum.

(iii) When $E_c \neq 0$, the core saturates on alternate half cycles of line voltage (Fig. 3.30), and power is delivered to the load.

(iv) The coil of a saturated reactor short circuits the control circuit.

(v) The circuit action in each half cycle is identical, with the two reactors changing roles.

Fig. 3.31 shows the electric and magnetic variables for the circuit at a value $E_c \neq 0$. The detailed circuit action

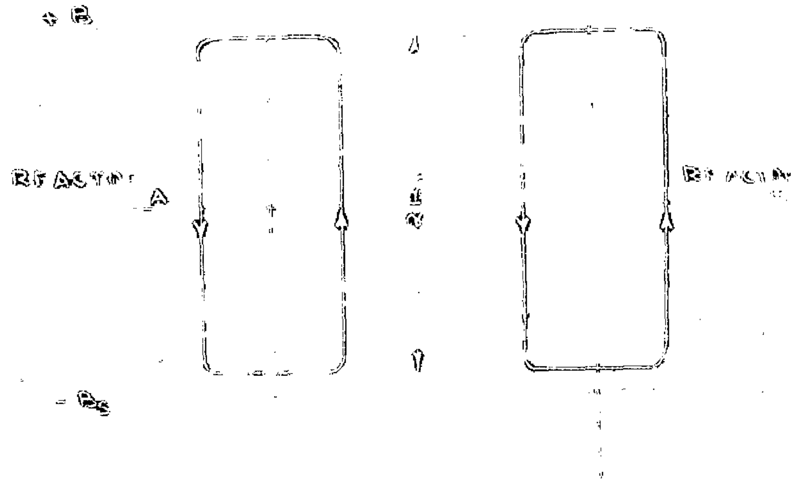


FIG 3.9

MINOR LOOPS $(E_c = 0)$

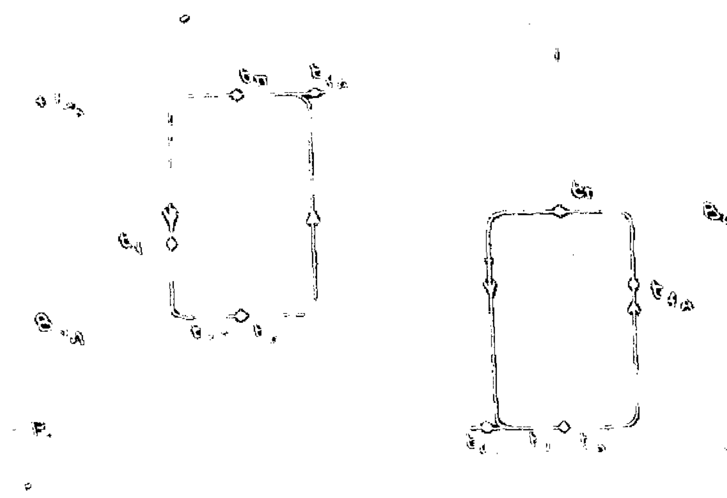


FIG 3.10

MINOR LOOPS $(E_c \neq 0)$

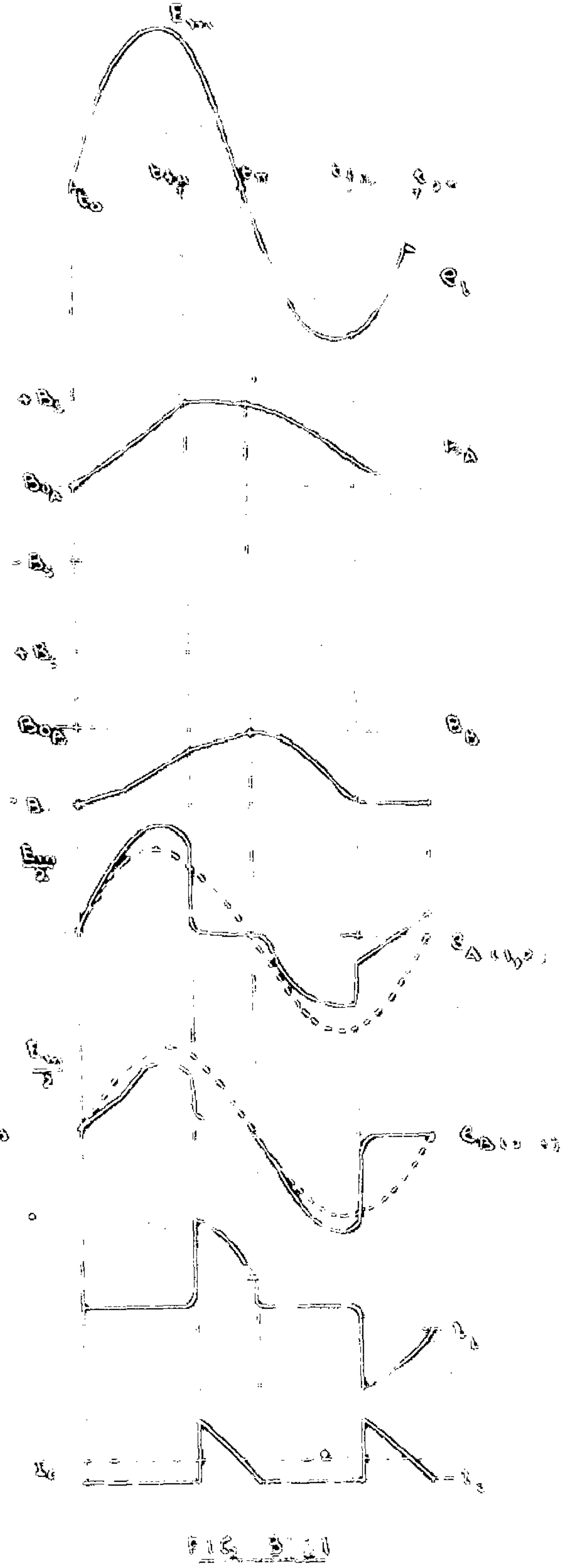


FIG 3.11

STEADY STATE WAVEFORMS

is as follows:

(a) A +ve control voltage into terminal 5 saturates A in the + ve half cycle and B in the - ve one.

(b) At t_o, A is at B_s, and B at $-B_s$.

(c) Between t_o and t_{fA}, both fluxes move toward + B_s. E_c adds to the core voltage of A and substracts from that of B, since the control circuit current is substantially zero and the drop across R_c negligible.

(d) If identical reacters are assumed, because of E_c the line voltage does not divide equally between the two load coils, This holds only for this period.

$$e_{A(1,2)} > e_1/2 > e_{B(2,3)}, \text{ and}$$
$$e_{A(1,2)} - e_{B(2,3)} = E_c N_L / N_c$$

(e) At t_{fA}, reacter A saturates, and no further flux change occurs upto t_π. Its coil acts as a short circuit in the control circuit.

(f) Between t_π and t_{fA}, $\emptyset_B$ increases for two reasons :
(1) the full value of E_c is available to change its flux &
(2) the load current sets up a voltage drop across its terminals equal to $i_L R_c (N_L / N_c)^2$.

(g) Immediately after t_π both flux densities move towards $- B_s$, and the action repeats itself with the two reacters changing roles.

(h) If magnetizing currents are neglected, the instantaneous control current must always be related to the load current during the intervals t_{fA} to t_π and t_{fB} to t_2. Because of the connections in the control circuit, I_c is a double frequency train of pulses with the wave shape of a full wave rectified portion of i_L.

Since the control circuit is linear, in so far as d.c. values are concerned, the average values of these pulses must equal $I_c = E_c / R_c$.

The following relations can be derived.

$I_c N_c = I_L N_L$ (Law of equal ampere-turns)

where I_c = average control current, and

I_L = average load current.

The average load voltage is

$$E_L = I_L R_L = E_C (N_c / N_L)(R_L / R_c)$$

The power gain, if the load responds only to the average value of load current, is

$$P = (E_L^2 / R_L) / (E_c^2 / R_c) = R_L / R_c (N_c / N_L)^2$$

If the useful load power is determined by the rms value of load current, the power vain, is

$$P' = (R_L / R_c)(N_c / N_L)^2 \cdot K_f^2$$

where K_f = form factor of the load current.

Fig. 3.32 shows the parallel connected SCR. The load windings are in parallel with a series conection for the control circuit, the basic relationship, similar to series connected, are applicable, and voltage, and current waveshapes are almost identical.

In practice, the performance of series of parallel connected reactor is obtained with the common control coil linking both cores (Fig. 3.33). With this arrangment R_c is lower. The load windings are on the outside limbs of three limbed core structure and the control winding on the centre limb. The a.c. windings have equal ampereturns effect, and are so connected (either in series or parallel) that the a.c. flux due to these windings cancel out in the centre limb, and does not induce alternating voltages of the fundamental frequency in the central

FIG. 3.38

PARALLEL CONNECTION

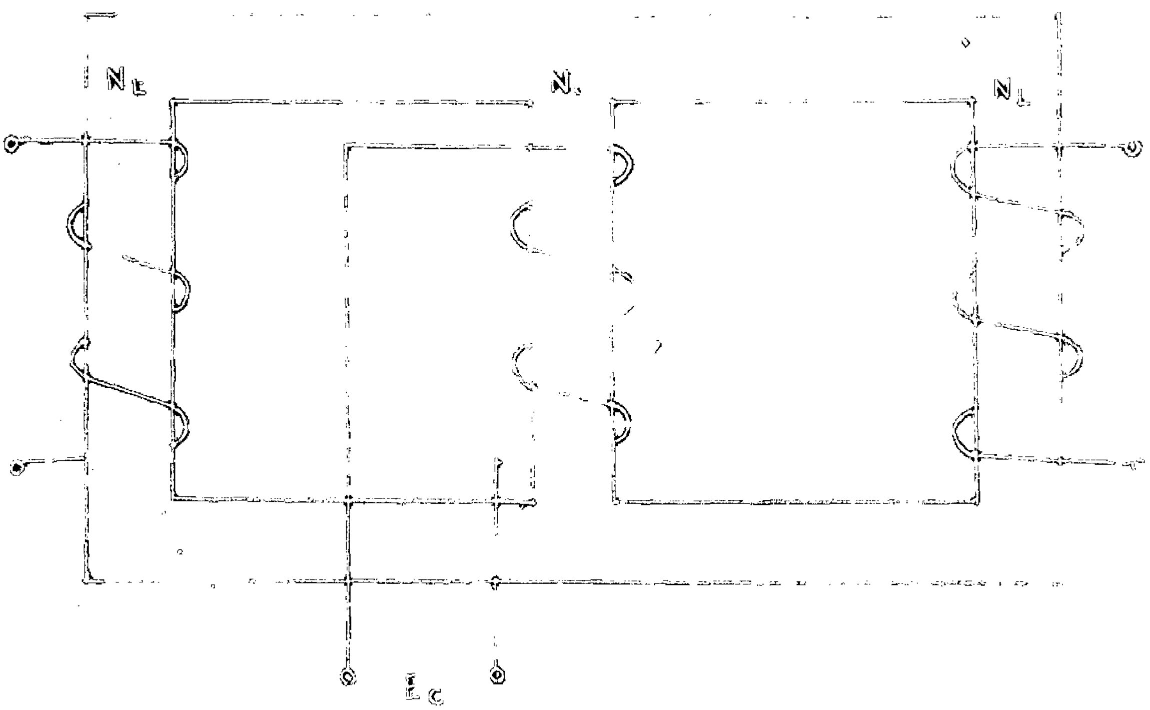


FIG. 3.39

SCR WITH COMMON CONTROL WINDING

winding. When a control voltage E_c is applied, it sets up a flux change in the two cores, or halves, in opposite directions.

3.3.10 Characteristics of SCR

Figs 3.34 and 3.35 show a family of curves relating E_a average SCR voltage, average load current I_L , and average control current I_c . Fig. 3.34 shows an idealized characteristic based on ideal cores, and Fig. 3.35 is a practical characteristic.

The curves are for no load conditions (a.c. circuit resistance being very small). The slope of the line OZ corresponds to the resistance in the a.c. circuit windings. The line passing through ABCD defines the circuit when operating with a constant average SCR voltage E_a and different values of I_c. The load line YX corresponds to a supply voltage OY, the slope of line YX = $-R_L$. The point G is the limiting point and maximum average load current is

$$I_{L max.} = E_I / R_a$$

where $R_a = R_L + r_x$, r_x = winding resistance.

The characteristic for a given load may be plotted as shown in Fig. 3.36 in which the average load current I_L is plotted against the average control current I_c, for the two load conditions of Fig. 3.35. The slope of this curve is the current, gain which is equal to N_c / N_L and $I_{L max.} = E_1 / R_a$.

The control ratio m, is defined approximately as the ratio of maximum average load current, to the minimum average load current with zero control current, and is used as a criterion of the range of control.

$$m = I_{L max} / I_{L o} = (E_1 / R_a) / (E_1 / 2x_a)$$

$$= R_a / 2 x_a$$

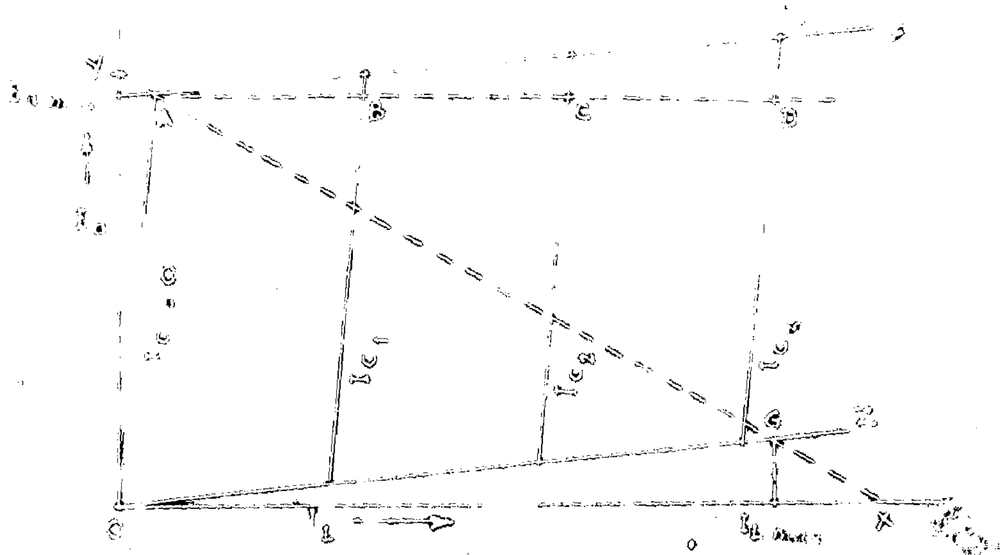


FIG. 3.34
CHARACTERISTIC FOR IDEAL CASE

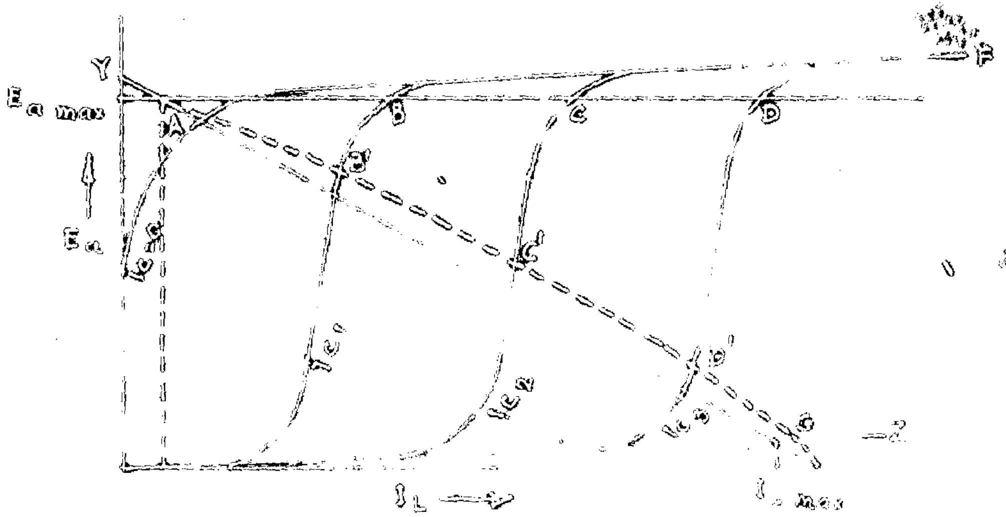


FIG. 3.35
CHARACTERISTIC FOR PRACTICAL CASE

where $2X_a$ = reactance of the a.c. windings in the unsaturated state.

3.3.11 Self Excited SCR

The SCR circuit described previously relies entirely on the ampere-turns on the centre limb to saturate the core. This requires large control coils, which adds to the physical size of the structure, and therefore to the cost. The large control coils also have an unfavourable time constants L/R, when considered in relation to the low time constants required for regulating equipments. The application of this type of device to regulating system has been limited, because of its low power gain, its slow rates of response, and its cost. The self excited SCR (Fig. 3.37) has, to a great extent, overcome the above inherent disadvantages of the SCR. The load coils provide 100% feed back, and assist in saturating the core. The control coil ampereturns and watts can be relatively low.

The equivalent circuit for self exciting SCR is shown in Fig. 3.38. The energy drawn from the control source during a conducting period is supplied partly or wholly from the main power source by using self excitation. This is a form of positive feed back. The load current is rectified and made to flow in feed back windings F_B similarly disposed to the control windings. A typical control current characteristics is shown in Fig. 3.39.

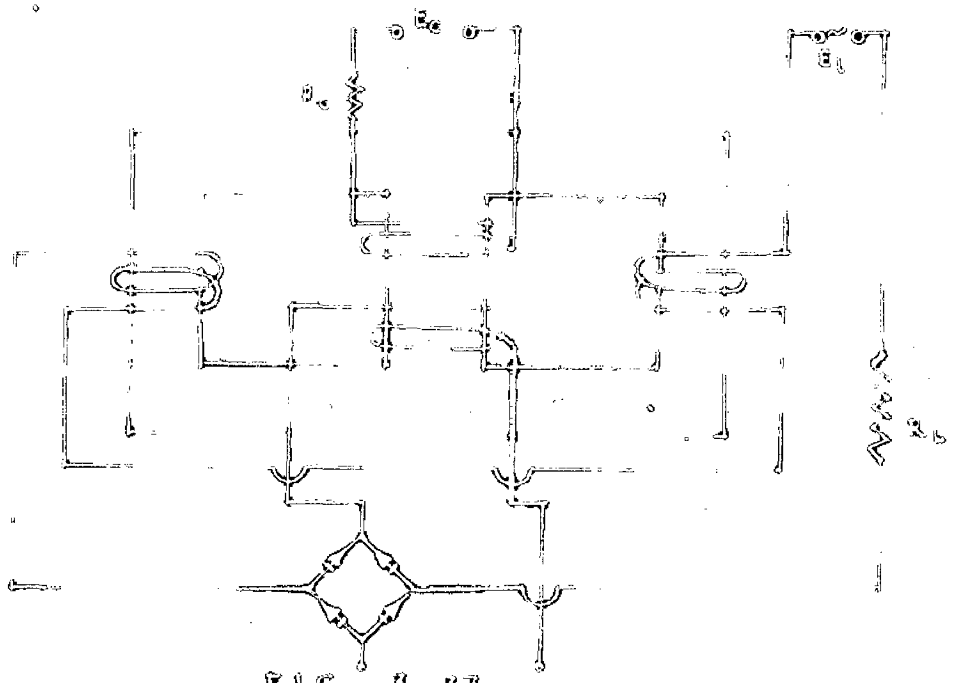


FIG. 3·37

SELF EXCITED SATURABLE CORE REACTOR

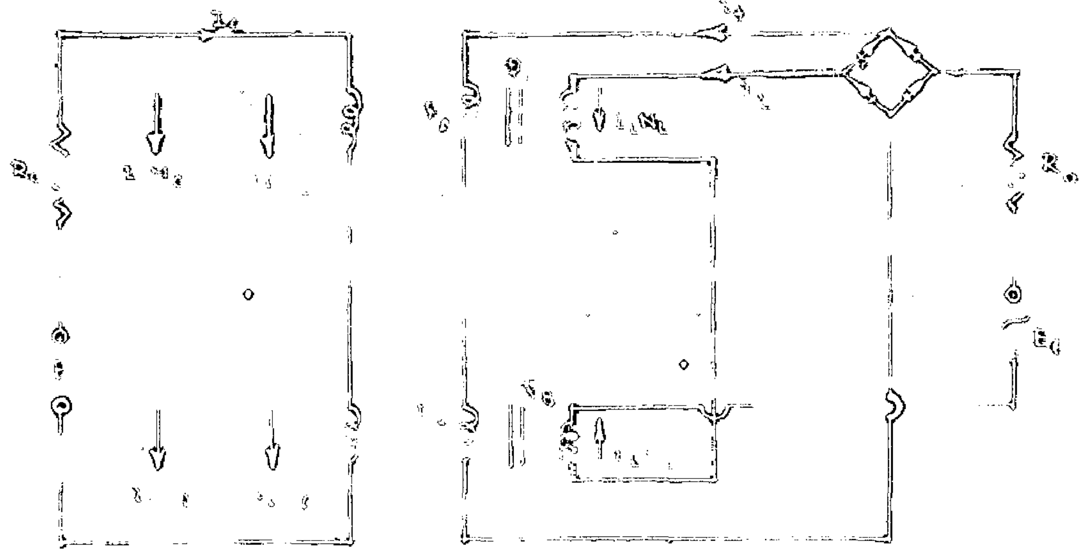


FIG. 3·38

EQUIVALENT CIRCUIT OF FIG. 3·37

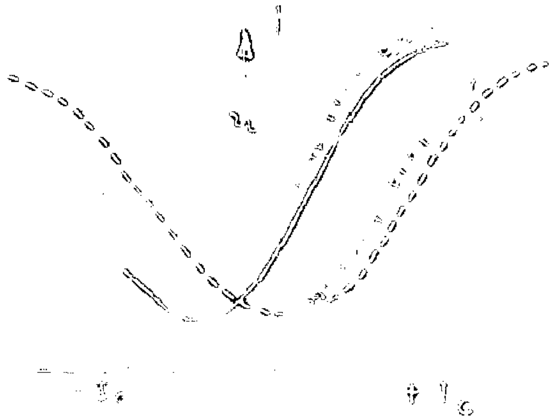


FIG. 3·[illegible]

SECTION - 4

4. APPLICATION OF SCR FOR CONTROL OF SPEED OF INDUCTION MOTOR

4.1 Slip - ring Induction Motor

Fig. 4.1 shows a simple arrangement of speed control of slip-ring induction motor. The a.c. winding of the SCR is connected in one of the supply lines, and the d.c. for the control winding is either obtained with the help of rectifiers or directly from a d.c. source, if available. In the secondary of the induction motor a variable impedance, resistance, inductance, and capacitance either singly or combined, is connected. As explained in 3.3. by varying the control current of SCR, the relative magnitude of negative sequence voltage can be varied, and with the variation of secondary resistance along with the ration of $|V_1|/|V_2|$ different speed - torque characteristics can be obtained. A wider range of speed control can be obtained by using capacitance in addition to resistance in the secondary circuit, or by using a combination of resistance capacitance and inductance. This can be expalined by the fact that the current in the secondary of a slip-ring induction motor is determined by the induced voltage and impedance of the secondary circuit. As the motor approached synchronous speed, its slip approached zero. The induced voltage decreases in proportion to the slip. When the secondary impedance is capacitive, it varies inversely as the slip, increasing to infinity as the slip approached to zero. The combination of decreased voltage and increased impedance causes a rapid decrease in current as the motor accelarates from standsill. In addition, the power factor is low and leading because of high capacitive reactance. The combination of low secondary current and

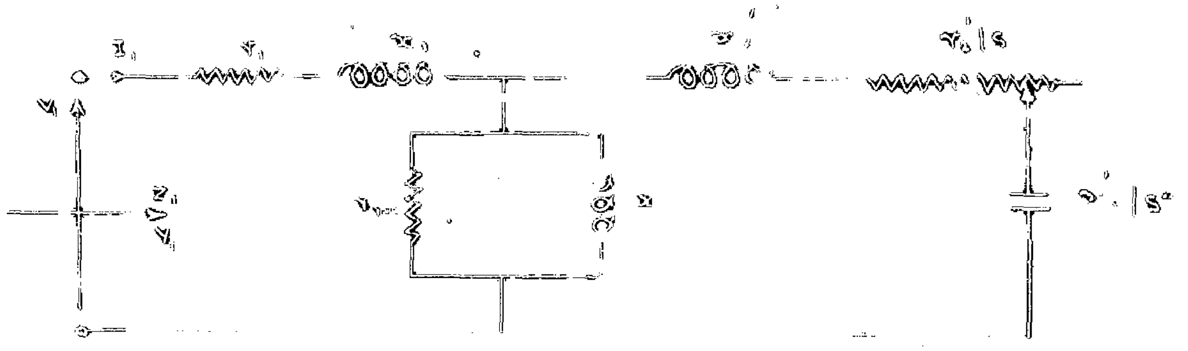
FIG. 11

SIMPLE ARRANGEMENT OF SPEED CONTROL OF SLIP-RING INDUCTION MOTOR USING SCR and low power factor results in less than 10% torque from slip less than 50%. The disadvantage of these characteristics is that there is still an appreciable torque at standstill. However, with this method full load lowering speeds, in hoists, as low as 18% can be obtained.

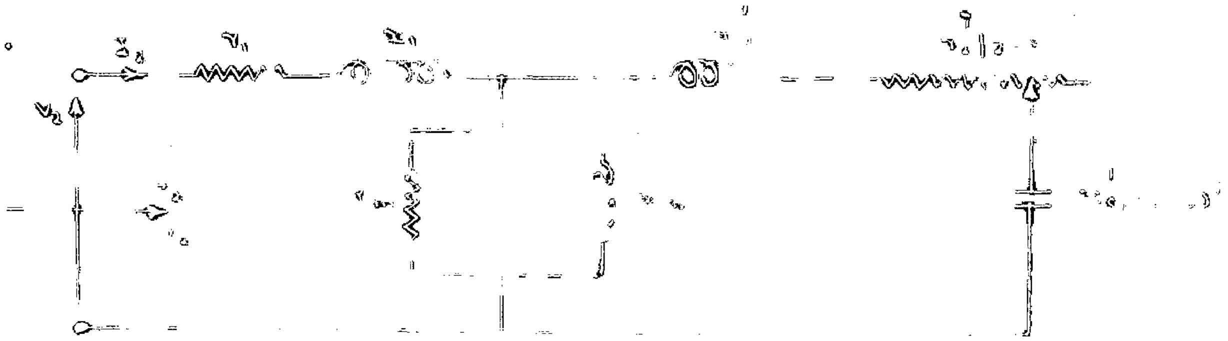
The equivalent circuit for positive, and negative-sequence systems, with capacitive reactance in the secondary are shown in Fig. 4.2. The current loci with secondary capacitance may be obtained in a manner similar to that used in constructing the conventional circle diagram of an induction motor. The circle diagram of an induction motor represents the current locus obtained when a variable resistance is connected in series with a fixed reactance.

With the capactitance in secondary the reactance of the motor changes as a function of slip, and thus requires a different diameter constant reactance circle for each value of slip. The diameter is equal to $E_2/(x-x_c/S^2)$ (for positive-sequence system). If a series of constanct resistance and constant reactance circles are drawn for a number of values of slip, there, intersection can be used to determine the current with capacitance in the secondary circuit.

At large values of slip the diameters of the constant secondary resistance and constanct secondary impedqnce circles are large. As slip decreases, the constant resistance circle becomes smaller. The constant impqdence increase in diameter until resonance is reached, at which value the diameter is infinity, and the locus becomes a vertical line. Further reduction in slip result in successively smaller circles of constant resistance. The constant impedqnce circles become capacitive current loci and their diameter also



(a) POSITIVE-SEQUENCE IMPEDANCE



(b) NEGATIVE-SEQUENCE IMPEDANCE

FIG. 4.3

EQUIVALENT CIRCUIT WITH SECONDARY
CAPACITIVE REACTANCE

decrease at zero slip, or synchronous speed the diameters are zero, as would be expected since secondary current is also zero for this condition. The method of obtaining the current loci with secondary capacitive reactance is illustrated in Fig. 4.3.

Another ingenious method, suggested by Wickerham uses a feature, namely variable unbalanced primary voltage automatically responsive to speed with this method it is possible to: -

i) employ unbalanced voltage without high input current which might be expected.

ii) obtain less than synchronous speed at no load.

iii) obtain characteristics providing any degree of starting torque from 0 to 75% full load torque, without the adjustment of rotor resistance.

iv) obtain characteristics providing retarding torques which are zero at zero speed, and which cover a speed range from 0 to 35% at light, or no load and 10 to 15% at full load.

The essential elements of control are :-

(1) a variable impedence reactor

(2) a phase shifter

(3) speed detector with amplifier, and

(4) a standard reversing controller equipment.

For all sub-synchoronous speed control this system uses a single relativity high external resistance in the rotor circuit which

(a) within the operating range, gives a stable motor operation speed-torque characteristics; and

(b) limits the input current, this being the fundamental and one of the prime importance of the system.

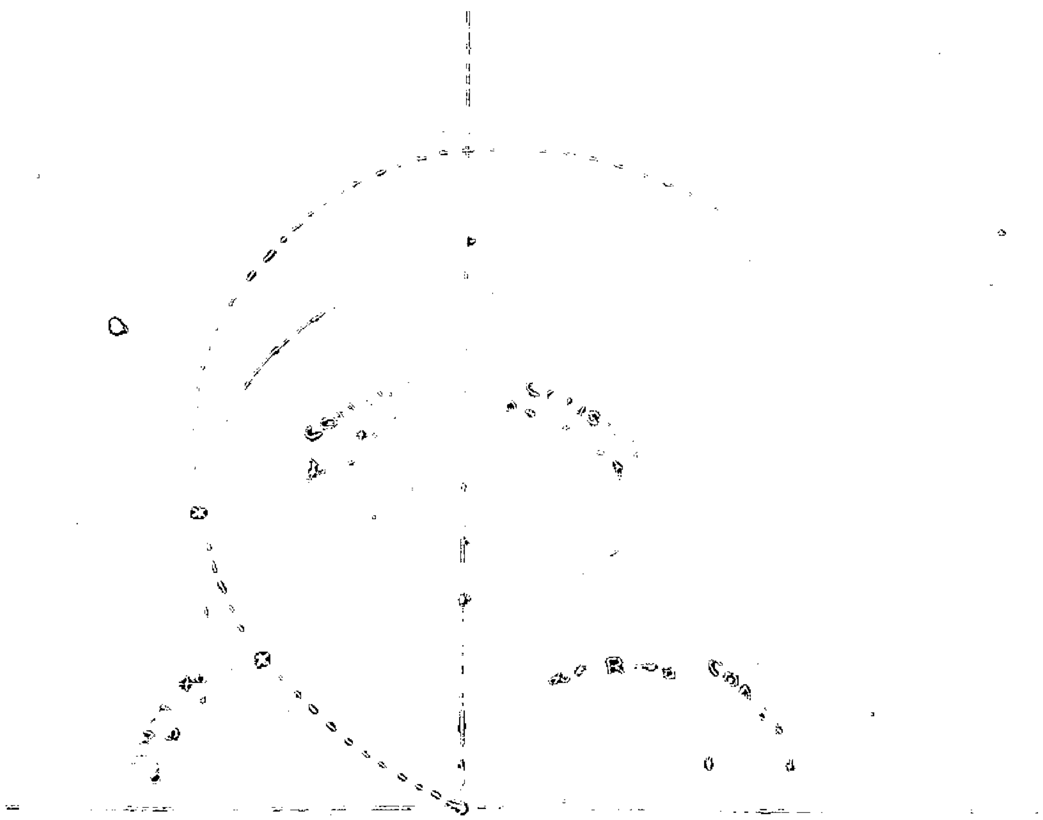


FIG. 4[illegible]

[illegible]

Typical speed - torque characteristics TQ-1 and speed-input current characteristics of a motor with a high resistance in the rotor circuit are shown in Fig. 4.4.(a). From the Fig. 4.4(a), it is apparent that withing the operating range in the IV quadrant the motor operation is stable i.e., retarding torque always increases with the increase in speed. The characteristic TQ-1 may be regarded as the positive sequence torque component where the negative-sequence torque is zero.

For a minimum unbalance in voltage, used in the system, caused by the displacement voltage vector EX_1 in Fig.4.5(b) where EX_1= 0.23E we have

(a) the positive-sequence speed-torque characteristic TP-2 Fig.4.4(b), which is similar to TQ-1, except that the magnitude is reduced along the torque-axis.

(b) the negative sequence component, TN-2 Fig.4.4(b), which is similar to the positive component, except that the magnitude is lower, since the unbalanced is not great and is inverted on the speed axis.

The motor shaft torque is the algebric sum of the two components or the characteristics TQ-2, Fig. 4.(b).

If the motor primary is further unbalanced by increasing the displacement vector EX_2 to 0.866 E, Fig.4.5(b), the applied single phase. The positive (TP-3), and negative (YN-3) sequence components are equal in maginitude for equal degree of relative slip, Fig. 4.4(c). The shaft torque or net torque characteristic passes through zero torque at zero speed and the shaft torque opposes the roration at all speeds.

If EX is increased in magnitude still further by EX_3 (Fig.4.5 "b"), the phase sequence at the motor t e r m i n a l s is reversed and the negative-sequence torque component becomes

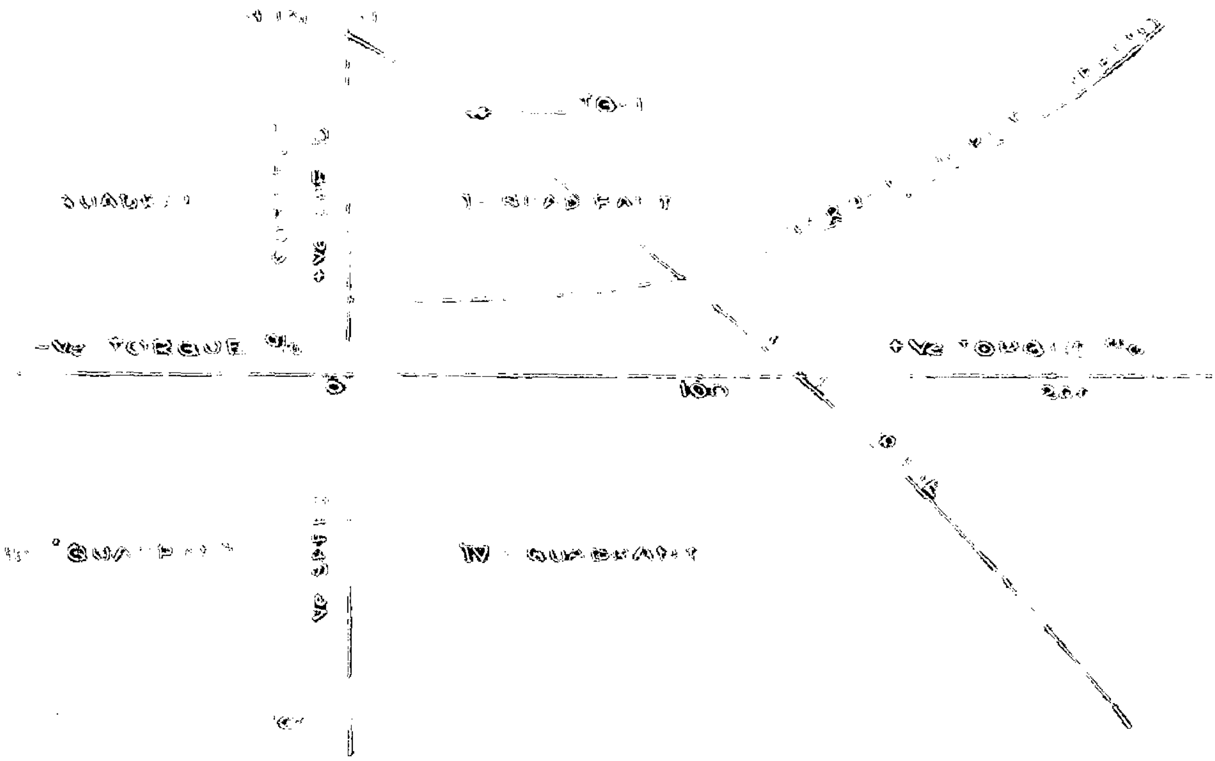


FIG. 4.2 (a)

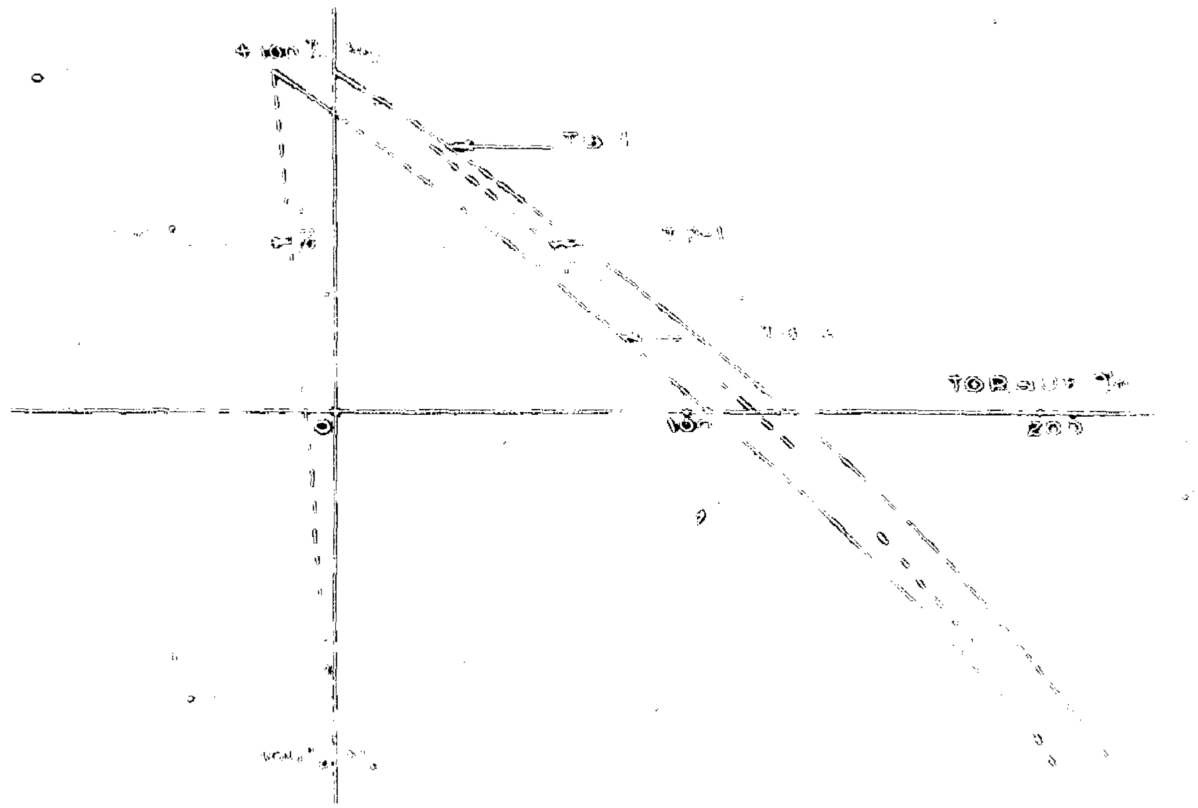


FIG. 4.2 (b)

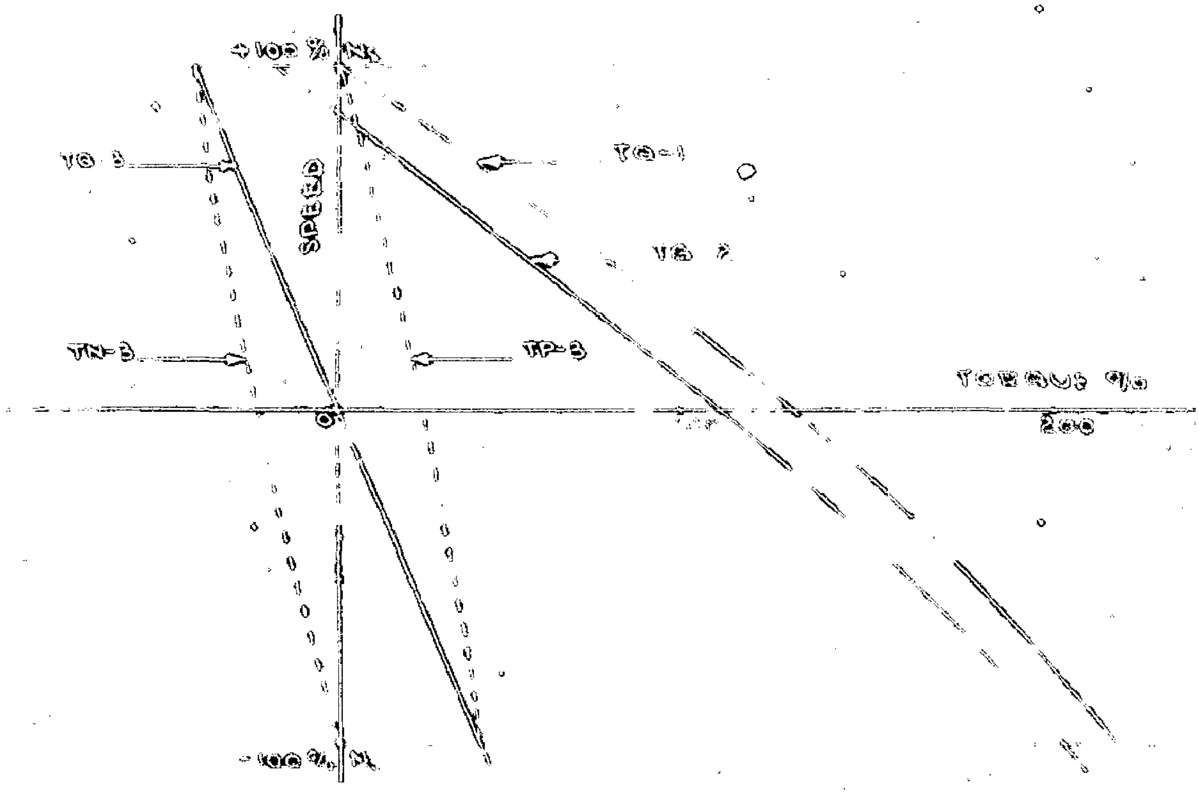


FIG. 4.4 (c)

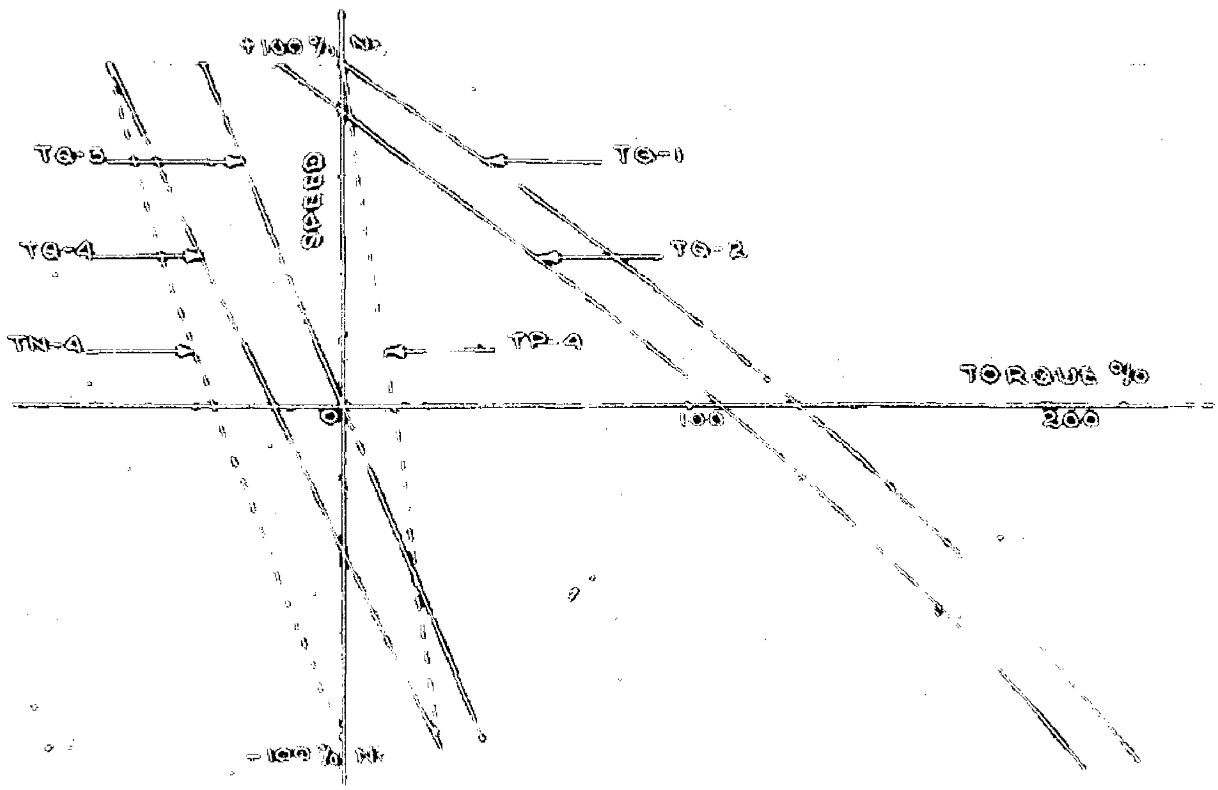
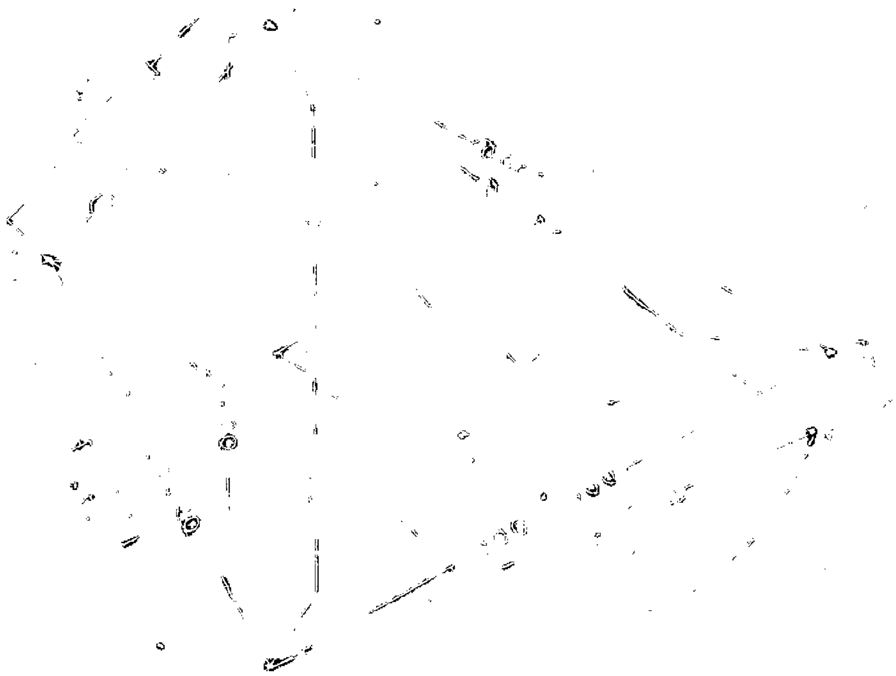


FIG. 4.4 (d)



predominant as shown in Fig. 4.4(d). The corresponding motor shaft torque passes through zero at minus 35% speed and shows that negative torqueis developed at zero speed.

4.1.1 Speed Responsive Variable Unbalance

The four speed-torque characteristics curves TQ-1,TQ-2, TQ-3, and TQ-4 each resulting from the application of different values of fixed unbalanced voltage and with input current limited do not cover effectively the area of the IV quadrant and are therefore unsuitable in themselves for speed control at all loads. This system provides the necessary coverage by effecting an automatic transfer in response to speed from either of the characteristics TQ-3 or TQ-4 at zero speed or to the characteristics TQ-2 at any selected negative speed. The curves are of general shape shown in Fig.6. They are quite different, particularly in the low torque regions, then any speed- torque characteristic attained so far. There may be as many intermediate curves as desired and they may originate from either TQ-3 or TQ-4 at zero speed.

4.1.2 Variable Impedance Reactor

The vector coltages EX_1 and EX_2 and any intermediate value shown between L_1 and T_1 shown in Fig. 4.5 are produced by a single saturable core reactor connected between these two points. The vector voltages EX_3 and EX_{14} and all intermediate values are produced by the same reactor in combination with a phase shifter.

4.1.3 Phase- Shifter

The phase shifter is a series reactor and resistor combination indicated by XPD and RPD in Fig. 4.5. As connected the three elements of the star connected load are :

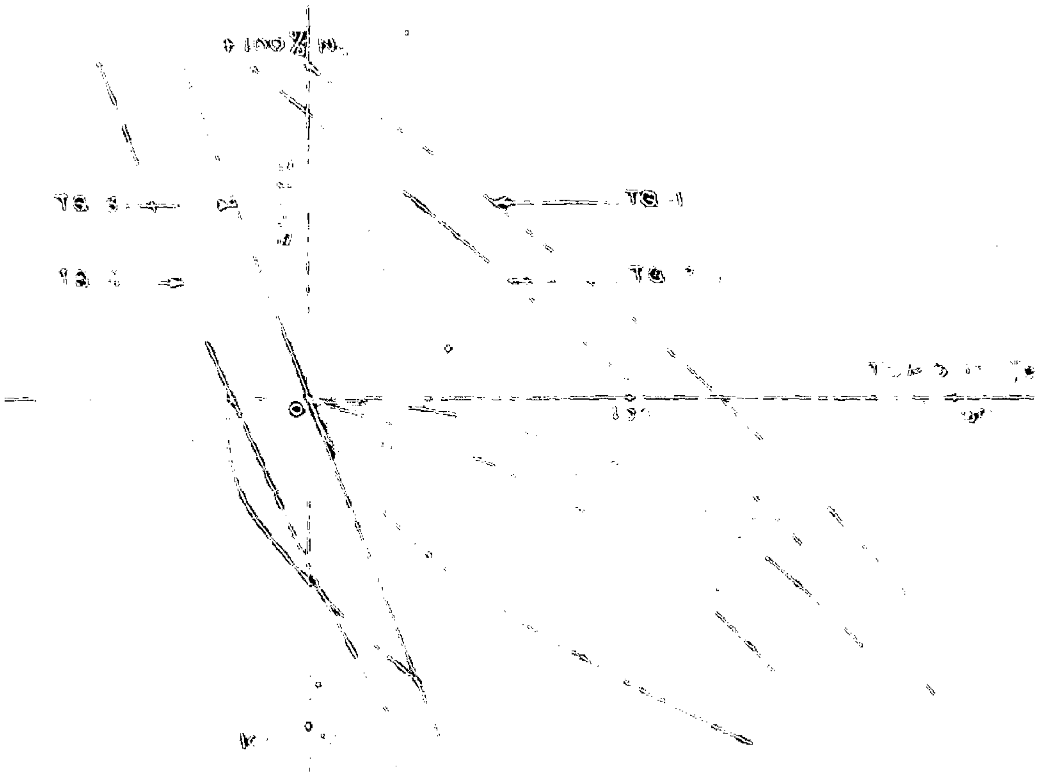


FIG. [illegible]

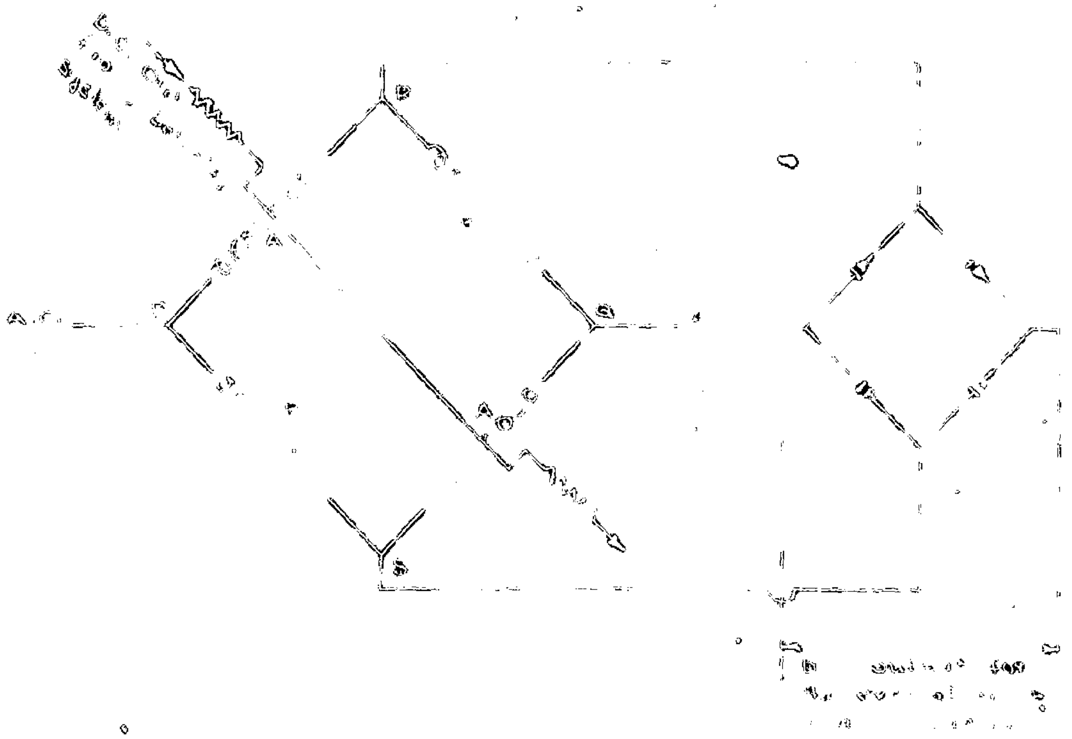


FIG. [illegible]

AMI[illegible]

(a) the resistor RPD and winding W_1

(b) the reactor XPD and winding W2 , and

(c) the reactor (voltage drop EX_3)

The neutral of the star connected load is therefore the motor terminal T_1 and the displacement of the neutral with respect to L_1, L_2, L_3 determined the unbalance of voltage applied to the motor Windings.

4.1.4. Speed Detector and Secondary Voltage Phenomena

To make the primary unbalanced voltages a function of motor speed the power for the control winding (d.c. coil) of SCR, used for unbalancing, is taken from the motor secondary voltage.

When operating the motor with unbalanced primary voltages, the voltages and frequencies developed in the rotor are:-

(a) For positive - sequence voltages

$$\left.\begin{aligned} V_{1r} &= s\ V_1 \\ \text{and}\quad f_{1r} &= s\ f \end{aligned}\right\} \quad \ldots\ldots (4.1)$$

(b) For negative - sequence voltages

$$\left.\begin{aligned} V_{2r} &= s'\ V_2 = (2-s)\ V_2 \\ f_{2r} &= s'\ f = (2-s)\ f \end{aligned}\right\} \ldots\ldots (4.2)$$

Where V_1 , V_2, s, and s' are primary positive and negative sequence voltages and slips respectively, and f is primary supply frequency.

In the speed range 0 to - 100 % the negative sequence component (4-2), with its descending magnitude and frequency produces modulation in magnitude of the positive sequence component (4.1) but does not affect its fundamental frequency. Besides these modulations there is still another variable in the secondary voltage resulting from the variable unbalance. This variation is indirectly associated with speed in such a way that a rise in speed is accompanied by a rise in secondary voltage, however only in proportion to the torque being developed. Thus as the speed rises from 0 to - 100 % the magnitude of secondary voltage is indeterminate but its frequency has a perfect definite value for any definite speed. The speed detector used, therefore, should have minimum response to magnitude of applied voltages and maximum response to the frequency there of.

The detector consists of

(1) a three-phase transformer operating at high flux density.

(2) a Capacitor in series with each primary coil, and

(3) a rectifier to convert the output to direct current

The reactance of the capacitor ($-j/2\pi f c$) varies with the change of frequency and therefore the voltage drop across the capacitor will vary with the change of the primary current of frequency.

The transformer primary voltage (counter e.m.f.) is given by

$$E_1 = 4.44 N_1 \phi_{max} f = K f \quad \text{..........} \quad (4.2)$$

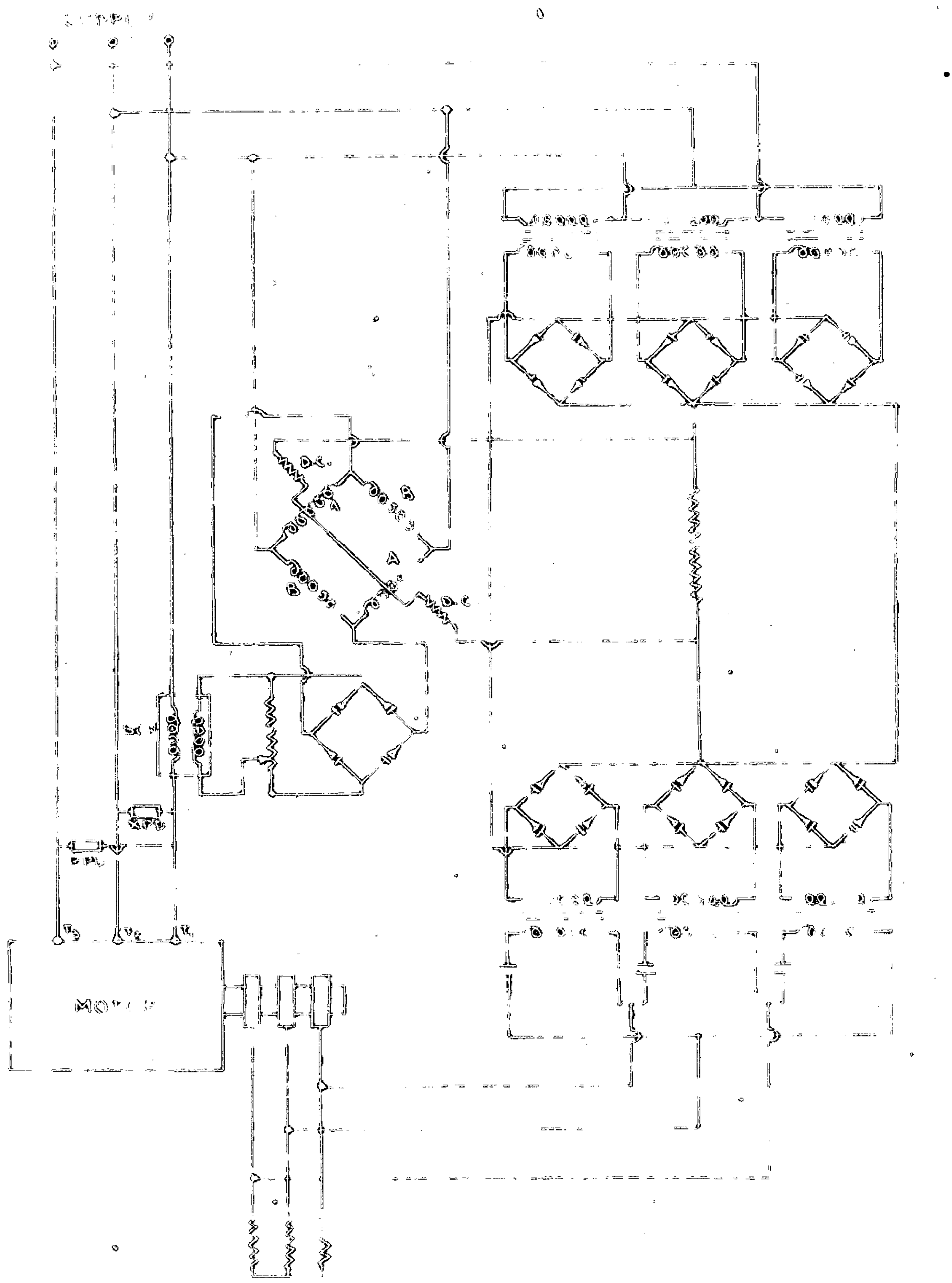
where N_1 = number of primary turns

ϕ_{max} = maximum flux in the core

f = frequency of the voltage applied

K = constant = $4.44 N_1 \phi_{max}$

Due to core saturation, ϕ_{max}, after a particular value of applied voltage, remains constant, and hence E_1 the transformer primary voltage (Eq. 4.2) varies with the variation of frequency only.



[illegible] 4.8

ARR[illegible]ENT OF W[illegible] SYSTEM

Also, the wave-form of the speed detector transformer secondary voltage is peaked due to core saturation and the amplitude of the peaks increases with the increase of primary voltage. The increase in amplitude is accompanied with lessened duration, so that the energy per peak remains approximately constant. As the frequency rises, there are more peaks per unit time hence the output of the system varies directly as the frequency except as modified by the relatively minor meter secondary voltage variations from other causes. These minor variations are taken care of by the capacitors, with the result that the output of the detector system responds fairly well to frequncy and consequently to speed.

At zero speed, , the excitation of the main reactor (SCR) must be zero, hence the detector output voltage existing at zero speed must be nullified. This is obtained by paralleling the output of the detector with another rectified constant voltage and completing the circuit for both through a common resistor.

4.1.6 Amplifier

As the input energy to the detector system is small and most of it output energy is consumed by the loading resister, the excetation energy, obtained only from the detector system, is not sufficient for the efficent working of the system. Therefore the use of an amplifier to obtain the excitation energy for the main reactor makes it possible

(i) to keep the speed detector components small, and

(ii) to standardize the detector system which may bo applied to all ratings of moter with the variable being taken care of by the amplifier.

The amplifier used, is a single a.c. bridge, composed of two saturable core reactors as shown in Fig. 4.7.

A-A and B-B and a.c. coils of the two saturable core reactors. The control winding of only one saturable core reactor is excited from the output of the detector system. The saturable core reactor with d.c. excitation has variable impedence while the other without d.c. excitation has fixed impedence. Out put energy from the detector varies the impedence of coils A-A, to upset the balance of the bridge and produce a potential difference at R and S consequent to the exciting coil of the main reactor.

4.1.6 Input Current and Heating

Application of unbalanced voltages to the prmiary winding of a motor of-course produces unbalanced current input.

Fig. 4.9 curves show the line currents and percentage heating in the following three different cases, the motor developing the same given torque (100 %)

i) when the motor is operating with balanced input voltages.

ii) when the motor is running at slow speed (low balance)

iii) when the motor is running at high speed, FAST (greater unbalancing)

Fig. 4.10 gives a comparision of the system with other systems of speed control as regards the percentage average heating.

4.1.7 Reversal of Motor Connections

A simple reversal of motor terminal connections produces performance in the first and second quadrants equal to that developed in the fourth quadrant.

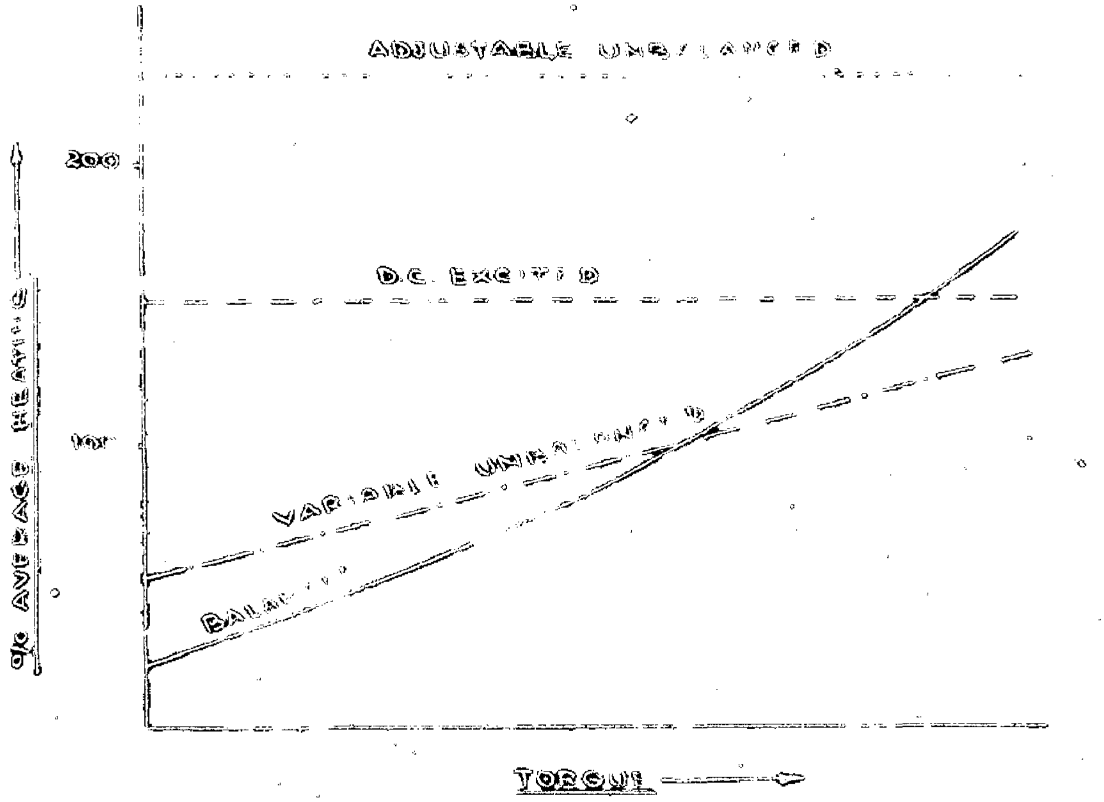


FIG. 4·10

COMPARISON OF DIFFERENT SYSTEMS OF SPEED CONTROL AS REGARDS % AVERAGE HEATING.

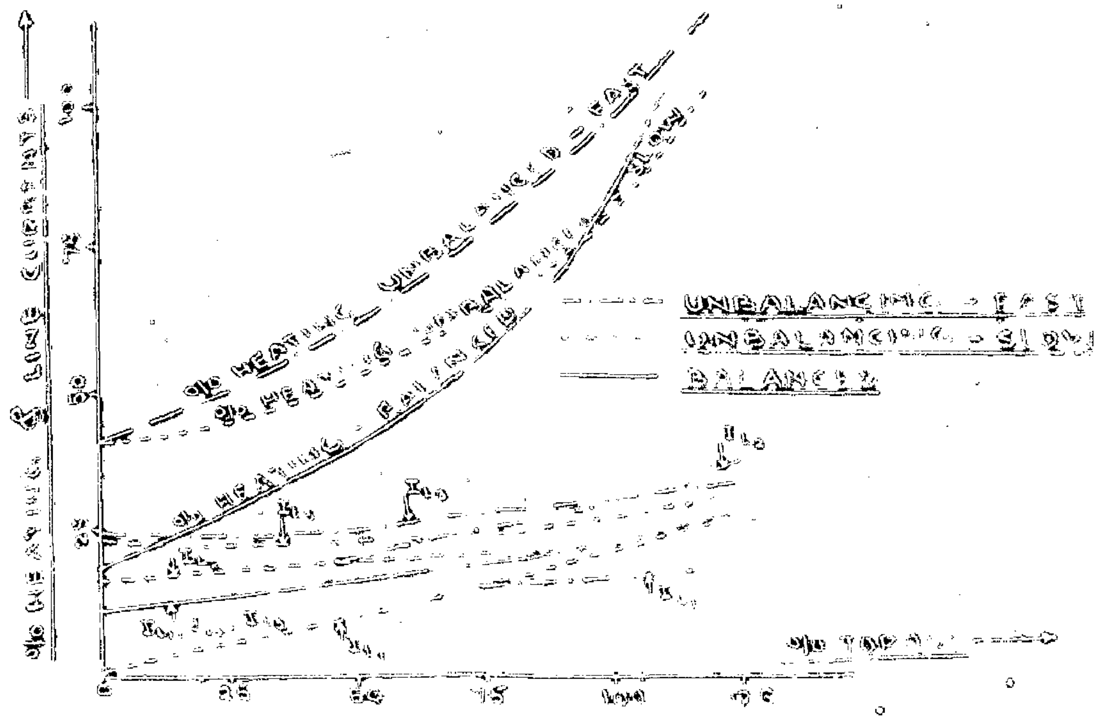


FIG. 4·9

LINE CURRENTS AND % HEATING UNDER DIFFERENT CONDITIONS

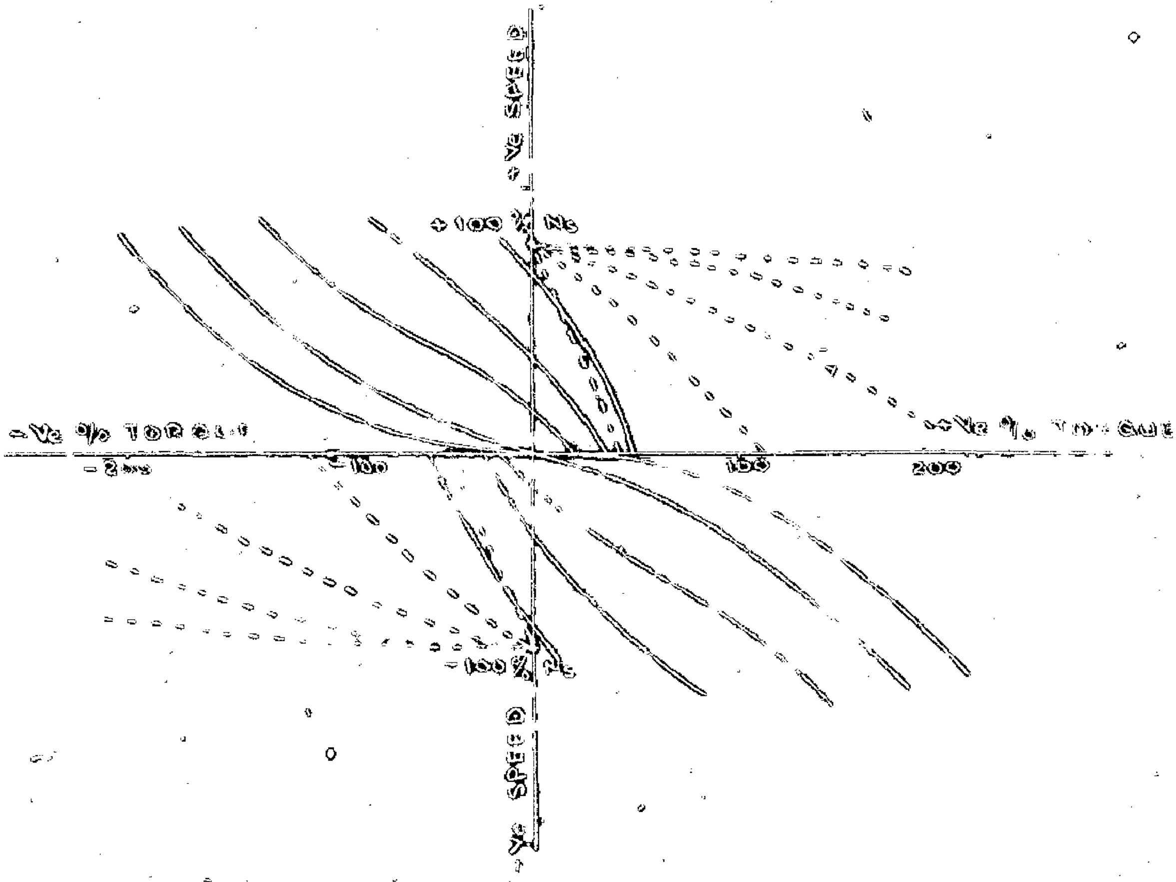


FIG 4·11

······ SPEED-TORQUE CHARACTERISTICS OBTAINED BY VARYING ROTOR RESISTANCE ONLY

—— SPEED-TORQUE CHARACTERISTICS OBTAINED BY VARIABLE UNBALANCING

4.2 Squirrel Cage Induction Motor

The system discussed in 4.1, for the speed control of slip-ring induction motors can also be applied to the squirrel cage induction motors with the following modifications:

i) to limit the input current high resistance rotor may be used.

ii) to eliminate zero-sequence system currents (3rd harmonic) causing excessive heating, the stator winding may be

a) connected in star,

b) wound either with a coil-span of two-thirds the pole-pitch, or each phase winding is distributed in two -thirs of the pole pitch.

iii) to make the primary unbalanced voltages a function of motor speed, the power for the control winding (d.c. coil) of main reactor (SCR), used for unbalancing, is taken from a d.c. tachometer generator fixed to the shaft of the motor.

Fig. 4.12 shows a system applicable to the control of speed of squirrel cage induction motor. The current in the control winding of the saturable core reactor is automatically varied by the d.c. tachometer generator (fixed on the shaft of the induction motor) and a reference direct voltage Vd.

The typical speed torque characteristics are shown in Fig. 4.13. The steepness of the speed-torque characteristic will depend on the normal characteristic of the motor and the design of the reactor and control circuit.

By using a capacitor in parallel with the saturable core reactor,

i) The direction of rotation of the motor can be reversed without interrupting the mains circuit,

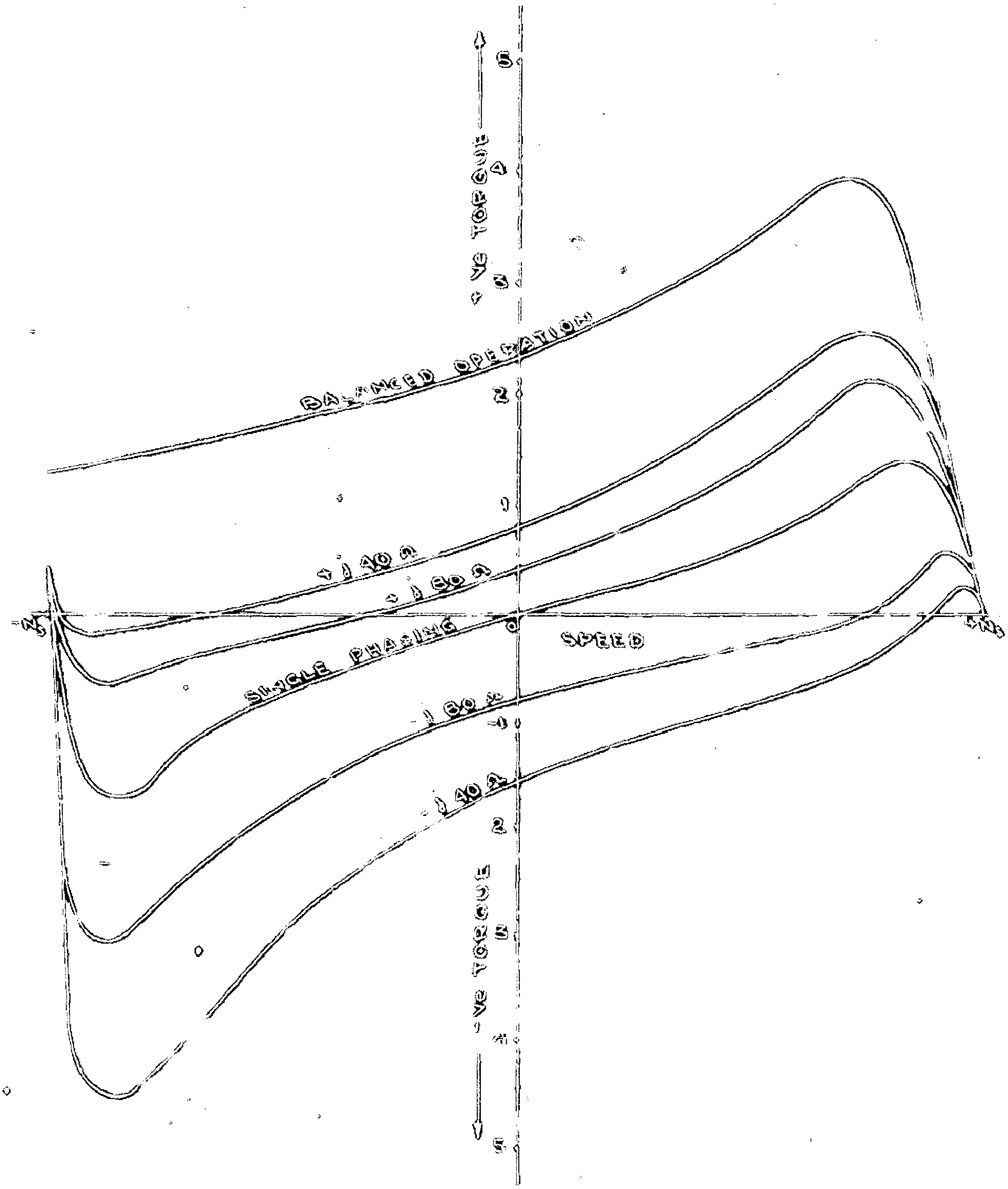


FIG. 4·13

SPEED - TORQUE CHARACTERISTICS

ii) the external reactance may be varied upto a greater extent along-with the facility of reversing its sign too, and

iii) dynamic braking is possible without an extra equipment.

However, the speed control of squirrel cage induction motor is somewhat more difficult than that of slip-ring induction motor.

4.3 General Discussion of Speed control by Unbalancing

4.3.1 Magnetic Saturation

The positive and negative-sequence magnetic fields, proportional to the positive -sequence voltage V_1 and negative-sequence voltage V_2, coexist in the machine. They rotate at different speeds so that sometimes they oppose and sometimes they did. At the latter instants, even though the peak flux densities of the individual fields are reasonable, their combination results in gross saturation. Due to saturation, there is a considerable increase in the magnetising currents and a deterioration in their wave form. It is to be expected that saturation will begin to be apparent when the sum of the peak flux-densities of the two component fields exceeds the normal peak flux-density in the machine. Neglecting second order effects, this will be when the algebric sum of the voltages V1 and V2 exceeds the normal phase voltage.

The magnetic conditions may be alleviated by the use of the following methods.

1) reduction in supply voltage- the delta connected motor may be operated in star.

2) introduction of additional secondary impedence- the addition of impedence in series with the secondary phases reductes the negativive sequence flux without appreciably

affecting the positive-sequence flux. The secondary impedence should be resistance since reactance, although giving the desired drop in voltages contributes nothing to the output.

3) introduction of additional primary impedences.

4) increase in the frame size of the motor above that necessary for normal operation.

4.3.2. Vibration.

The interaction of the positive-sequence mmf with the negative-sequence field, and the negative-sequence mmf with positive-sequence field produces torques which alternate at the relative slip frequency of the two components and havezero mean. value. These torques are responsible for the cibration in machines carrying unsymmetrical currents. Each of the two alternating torques is proportional to the product of the amplitude of the field and mmf waves producing it, but since they alternate at the same frequency their resultant is the vector sum of the two components.

4.3.3. Line Currents, Losses, and Heating of the Motor.

The unbalanced primary voltages have the following effects as regards the line currents and losses.

1) The negative-sequence currents produced by unbalancing causes unbalanced line currents and extra losses.
2) Negative-sequence current losses are in addition to the normal lossos at the same slip with balanced voltages, that is the losses may be superimposed.
3) The input current to the motor isincreased.
4) The additional losses due to operation on unbalanced voltages are large for motor with multiple cage rotors.
5) In addition to increasing motor losses, unbalanced

line voltages cause non-uniform distribution of Copper losses.

6) Small unbalance in voltage cause much larger unbalance in line currents.

Any consideration of heating involves not only the current in the motor but also the duty cycle upon which the motor operates. the currents encountered with the unbalanced primary type of hoists and draw bench controls are no greater than those in d.c. hoist motors. These currents reach upto about 125% of normal in the variable unbalanced voltages, and upto 230 % in adjustable fixed unbalanced voltages. The satisfactory service records of such motors have shown that heating is not excessive.[23]

The reason that the d.c. and a.c. hoist motors do not overheat is that the duty cycle is intermittent in all cases. The most severe duty cycle included in specifications is 15 secs. on out of 45 secs. Even with a more severe duty cycle, undue heating is not likely because:

1) Coil adjacent to the hot winding are cooler, facilitating heat transfer.

2) The temperature equalizes rapidly when hoisting with balanced current and also when the motor is at rest.

Two similar Cargo winch drives were tested in Cutler Hammer, I nc, Milwakee Wisconsin. One employed a 50 H.P., 440 V, 60 C/s, 575 r.p.m. to tally enclosed slipring induction motor. This motor was used in the unbalanced primary control scheme. Other employed a 50 H.P., 230V,600 r.p.m. totally enclosed d.c. motor. This motor was used in a constant potential d.c. Cargo winch control scheme. These motors were mounted in turn on a standard Cargo winch arranged to hoist and lower full load. The duty cycle used was:

a standard cycle of 17 secs. hoist, 17 secs. lower, and 20 secs. off. The highest temperature rise of the a.c. motor was for all particular purposes the same as that of the d.c. motor. Temperature of the armature, interpole, and shut field windings were comparable to those of the less heavily loaded a.c. motor windings.

SECTION 5

5. EXPERIMENTAL WORK

5.1 Details of Apparatus Used and Procedure.

To verify the theory discussed in previous chapters, experiments were performed on two induction motors whose name plate specifications are given below:-

a) Slip-ring Induction Motor

5 B.H.P., 3-phase, 400 volts., 50c/s

7.0 amps., 1440 r.p.m.

b) Squirrel-Cage Induction Motor

5 B.H.P., 3-phase, 400 volts, 50 c/s

7.3. amps., 1440 r.p.m.

The specifications of other main apparatuses were as follows:-

c) Saturable-Core Reactor

A.C. windings - 110 volts, 0.21 - 6 amps.,

D.C. windings - 110 volts, 0 - 0.5 amps.

d) Single phase variac

230 V , 60 C/s 9 A

First, standard tests were performed to obtain the parameters of the motor windings. Then connections were made as shown in Fig. 5.1. The torque developed by the induction motor, under test, at different speeds was measured by the Ward-Leonard method. The torque-speed, and torque-input current characteristics were measured under different conditions discussed in the following sections. Also, to avoid the effects of saturation, the tests on both the motors were carried out at reduced voltages.

5.1.1 Tests on slip-ring Induction Motor.

Standard tests gave the following results:-

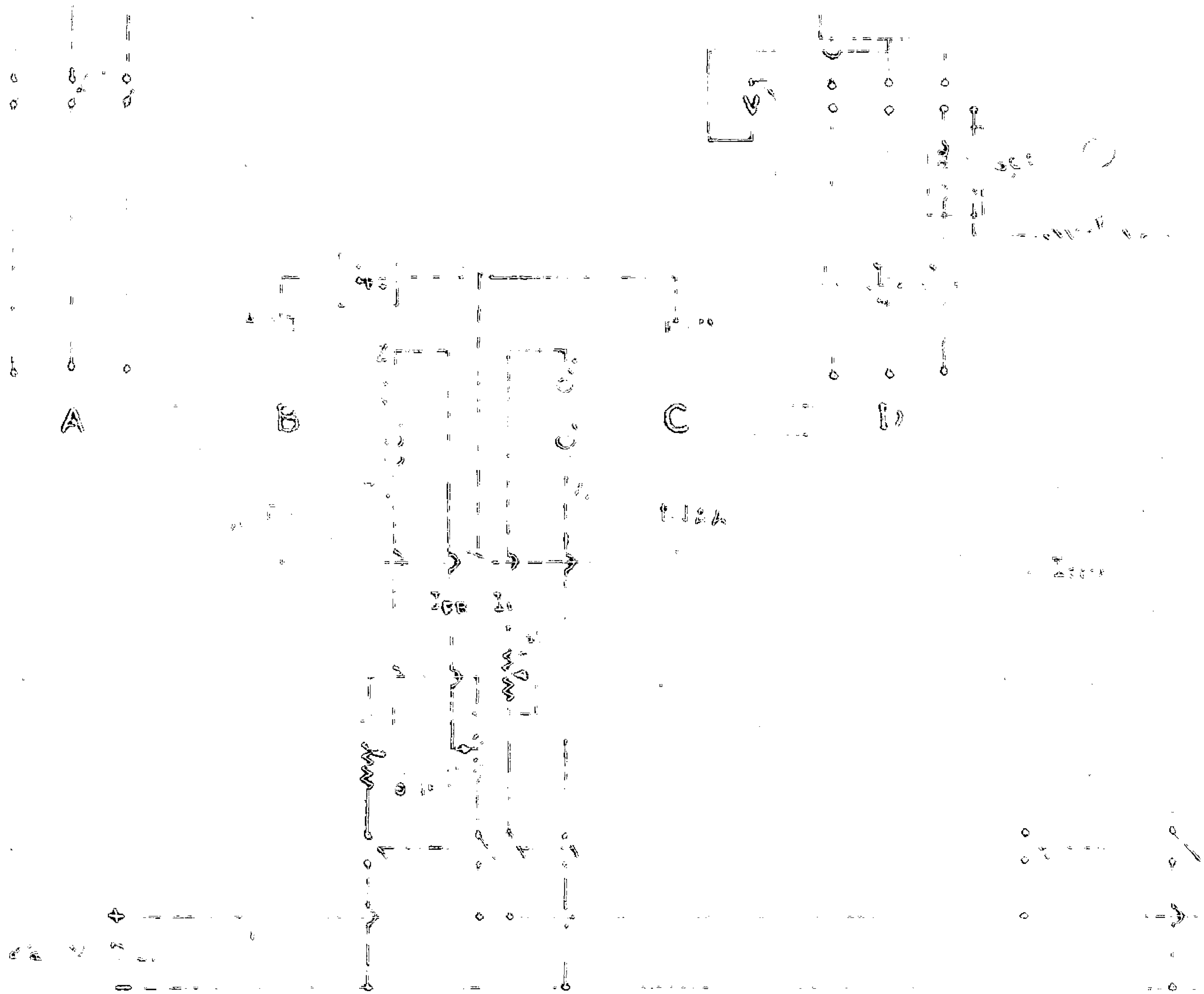


FIG. 92

a) <u>Turns Ratio Test</u>

It is found that the turns ratio of an induction motor is not same for the different positions of its rotor with respect to the stator. This is due to the effect of differential leakage flux. The turns ratio, therefore, was calculated as suggested by Pustcin Lloyd and others.

S.No.	Voltage applied to the stator V Volts	Rotor voltage in different positions of the rotor E_2 Volts
1.	400	330
2.	400	332
3.	400	328
4.	400	333
Mean	400	330.75

S.No.	Vetage applied to the the rotor E^1_2 Volts	Stator voltage in different positions of the rotor V' Volts
1.	300	360
2.	300	358
3.	300	362
4.	300	364
Mean	300	361

Turn Ratio $a = \frac{V}{E_2}\sqrt{\frac{V}{V'} \frac{E^1_2}{E_2}}$

$$= \frac{400}{330.75}\sqrt{\frac{400}{361} \times \frac{300}{330.75}} = 1.215$$

(b) No Load Test

S.No.	Voltage applied per phase Volts.	Input current amps.	Power Wattmeter Readings W_1	W_2	Power watts
1.	390	1.82	455	-220	235

(c) Blocked Rotor Test

S.No.	Voltage applied per phase Volts	Input current amps.	Power Wattmeter Readings W	W	Power Watts
1.	$155/\sqrt{3}$	7.0	690	-100	590

(d) Direct Measurements

Stator resistance = 2.1 ohms/phase

Rotor resistance = 1.13 ohms/phase

From the above tests the following parameters were obtained

Equivalent resistance referred to primary

R'_e = 4.01 ohms/phase

Equivalent reactance referred to primary

X'_e = 12.1 ohms/phase

Secondary (rotor) resistance R_2 = 1.39 ohms/phase

(e) The torque-speed, and torque-input current characteristics were measured with balanced applied voltages of 150 volts/phase, and without any additional resistance in the rotor circuit. These characteristics are shown in Fig. 5.2

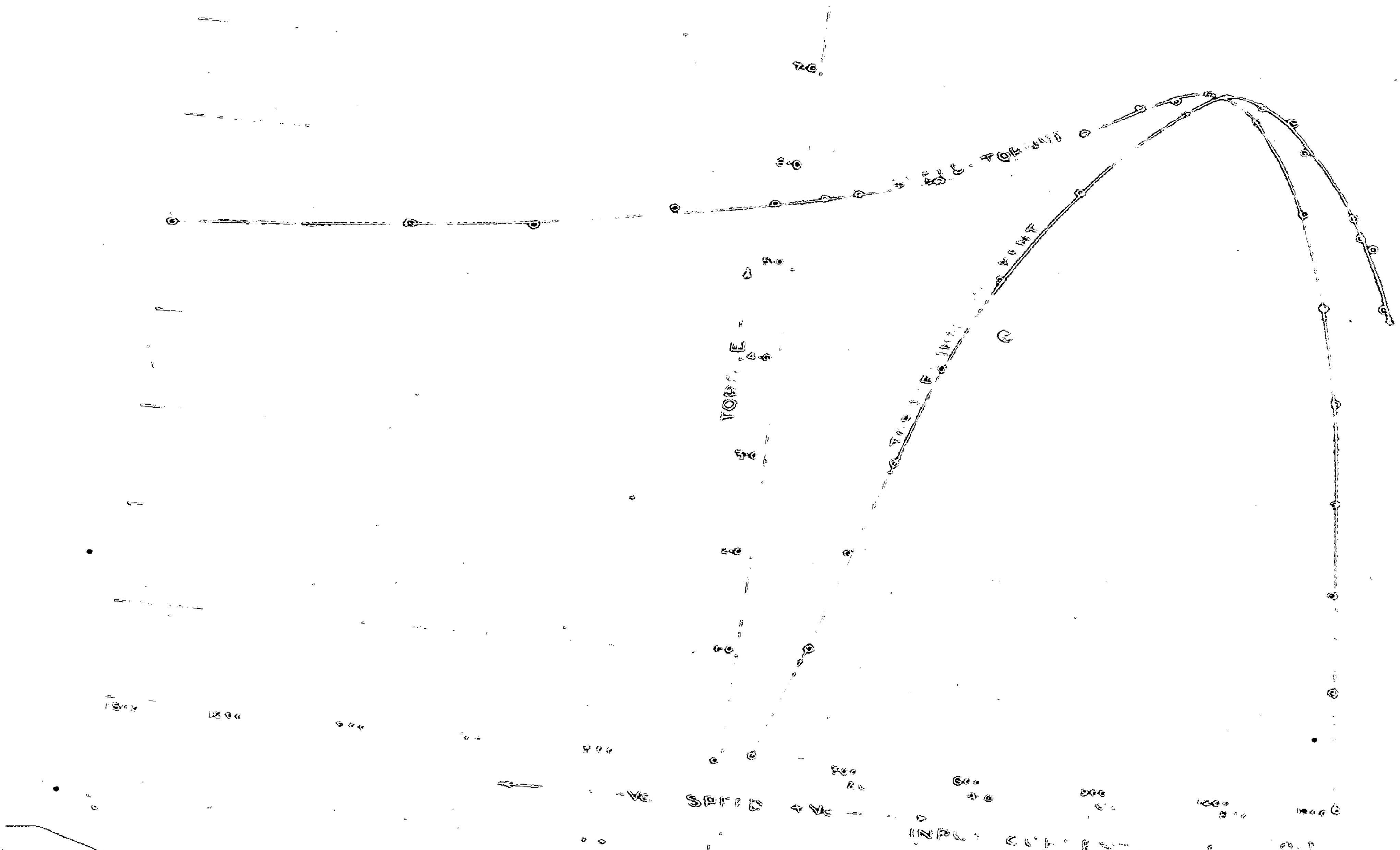
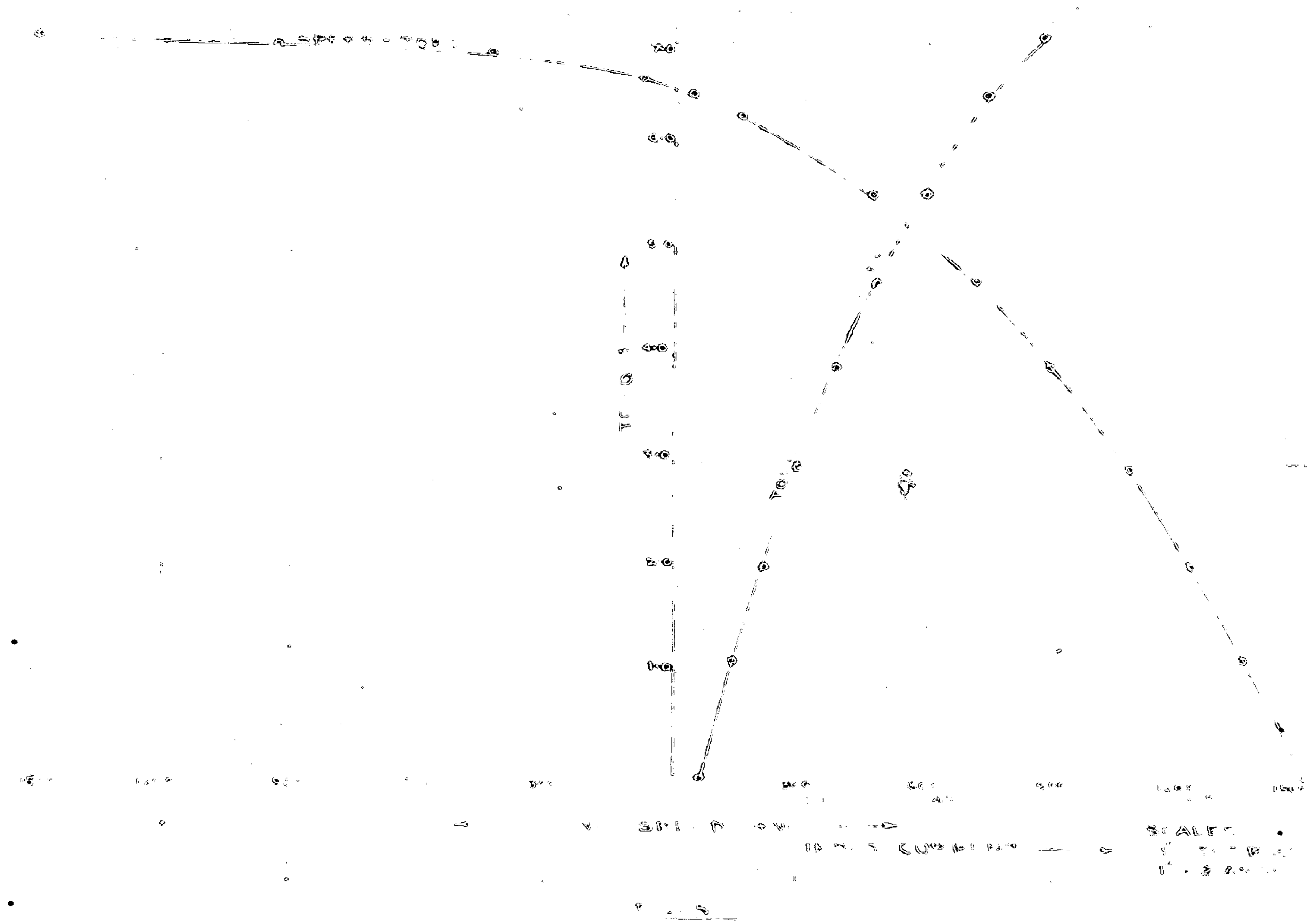


FIG.


SCALE

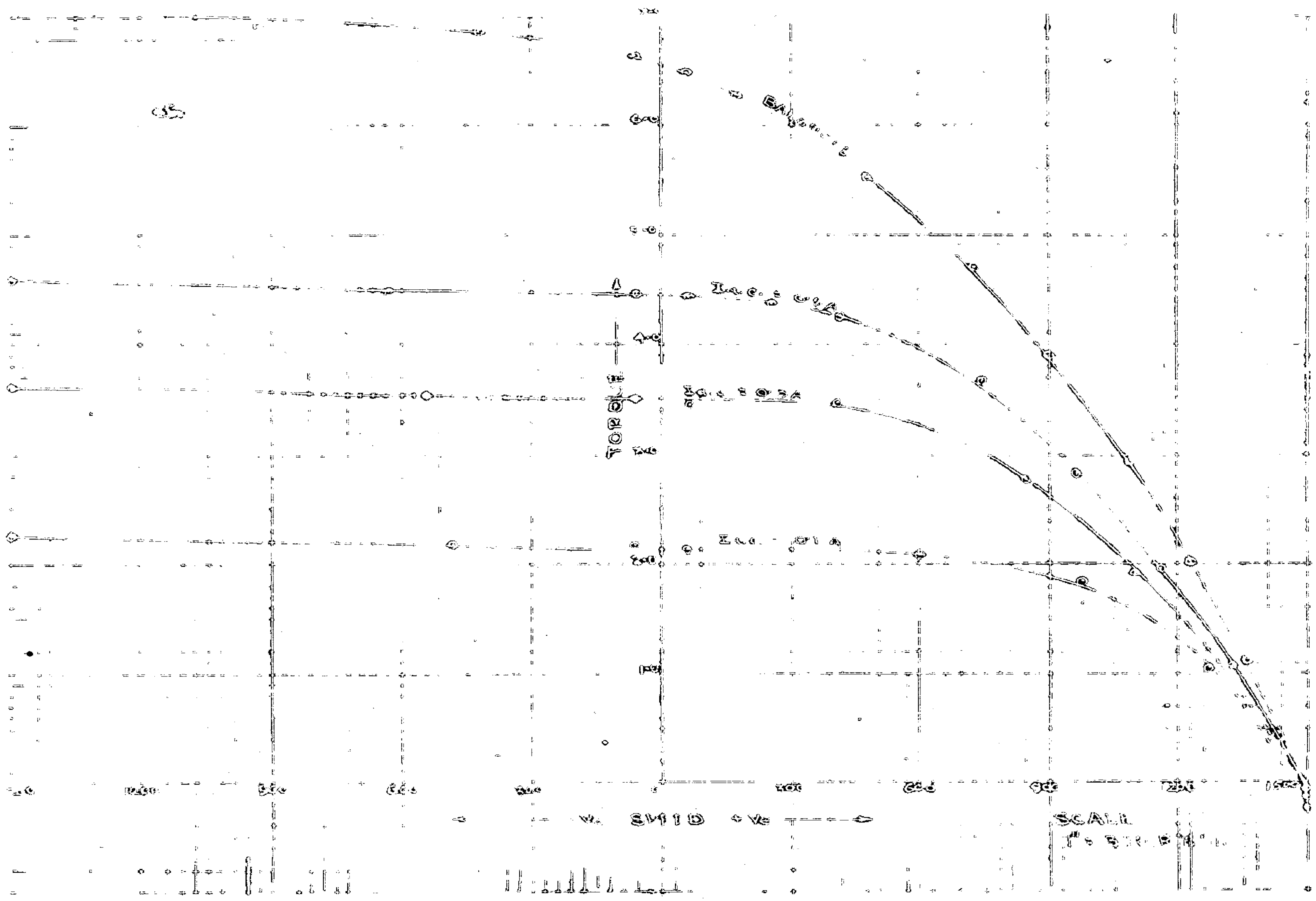
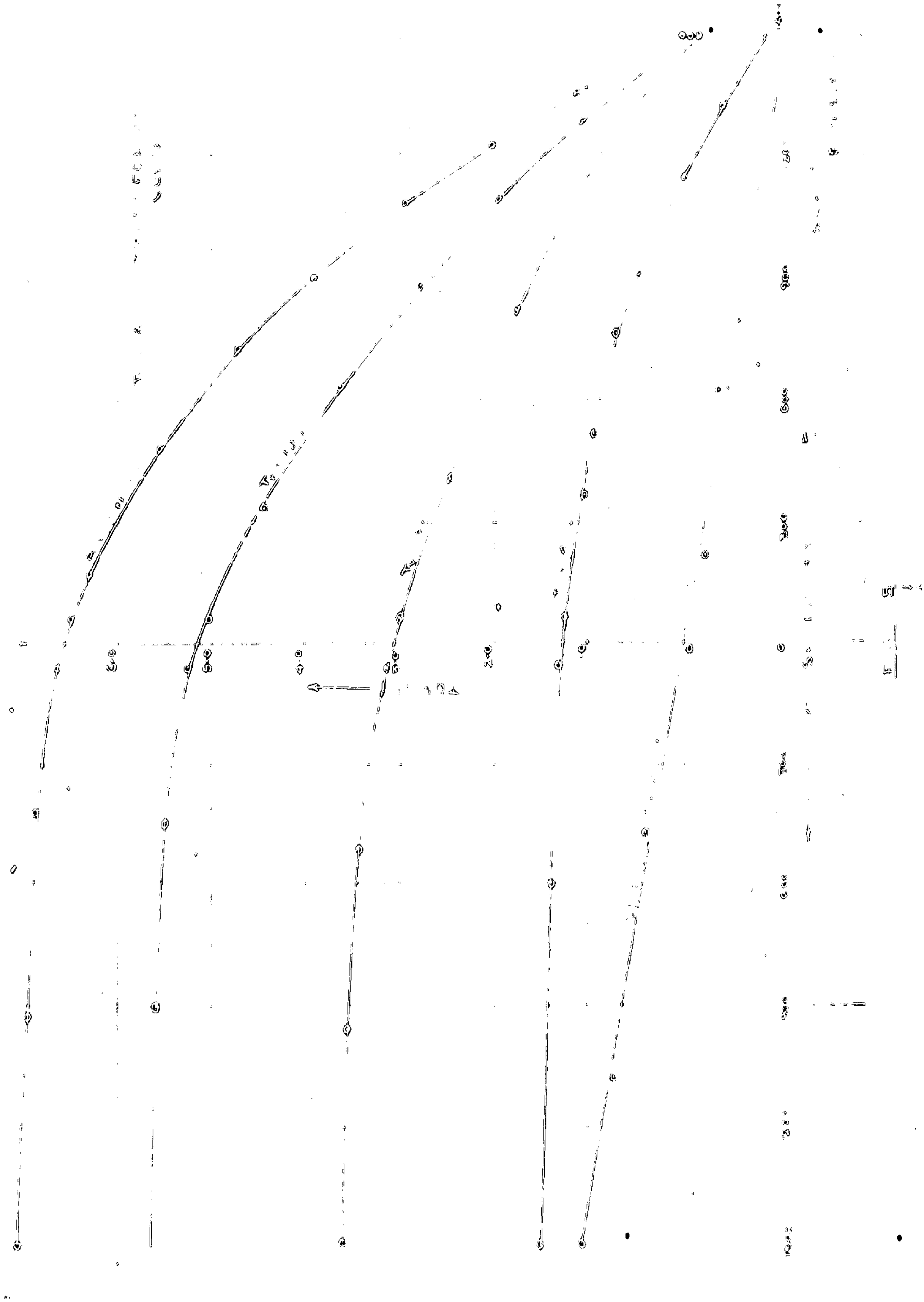


FIG. 5.4



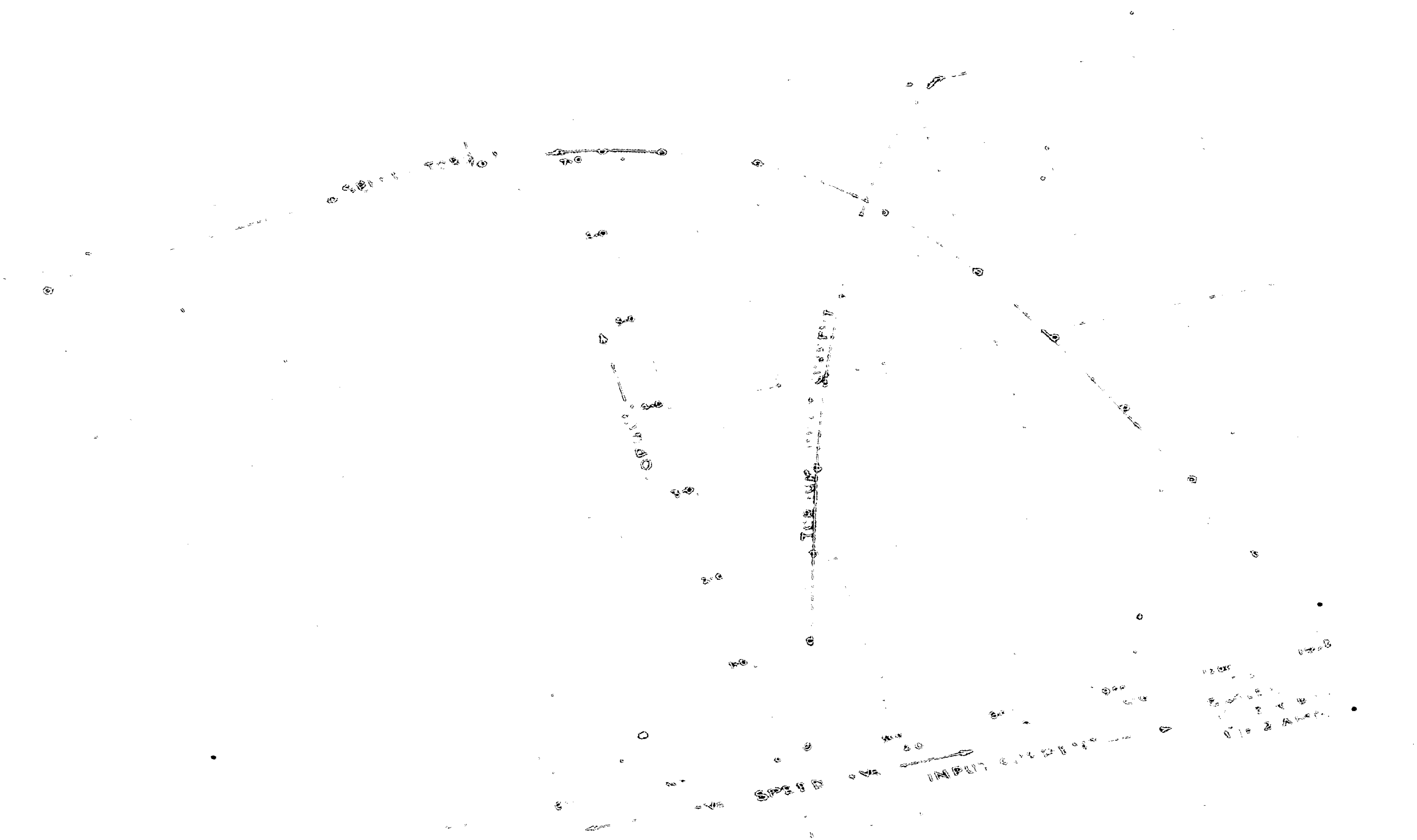


FIG 5.6

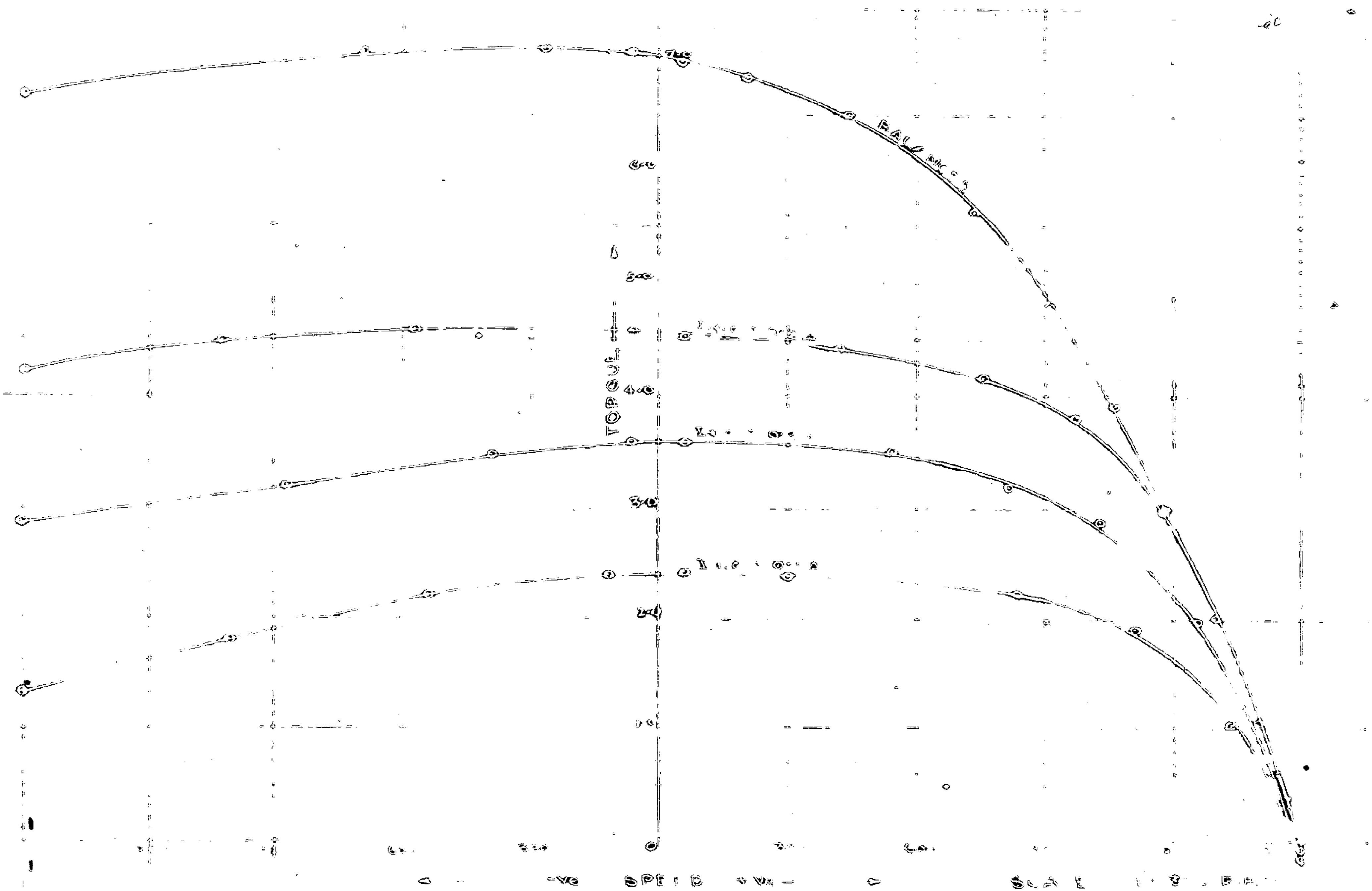
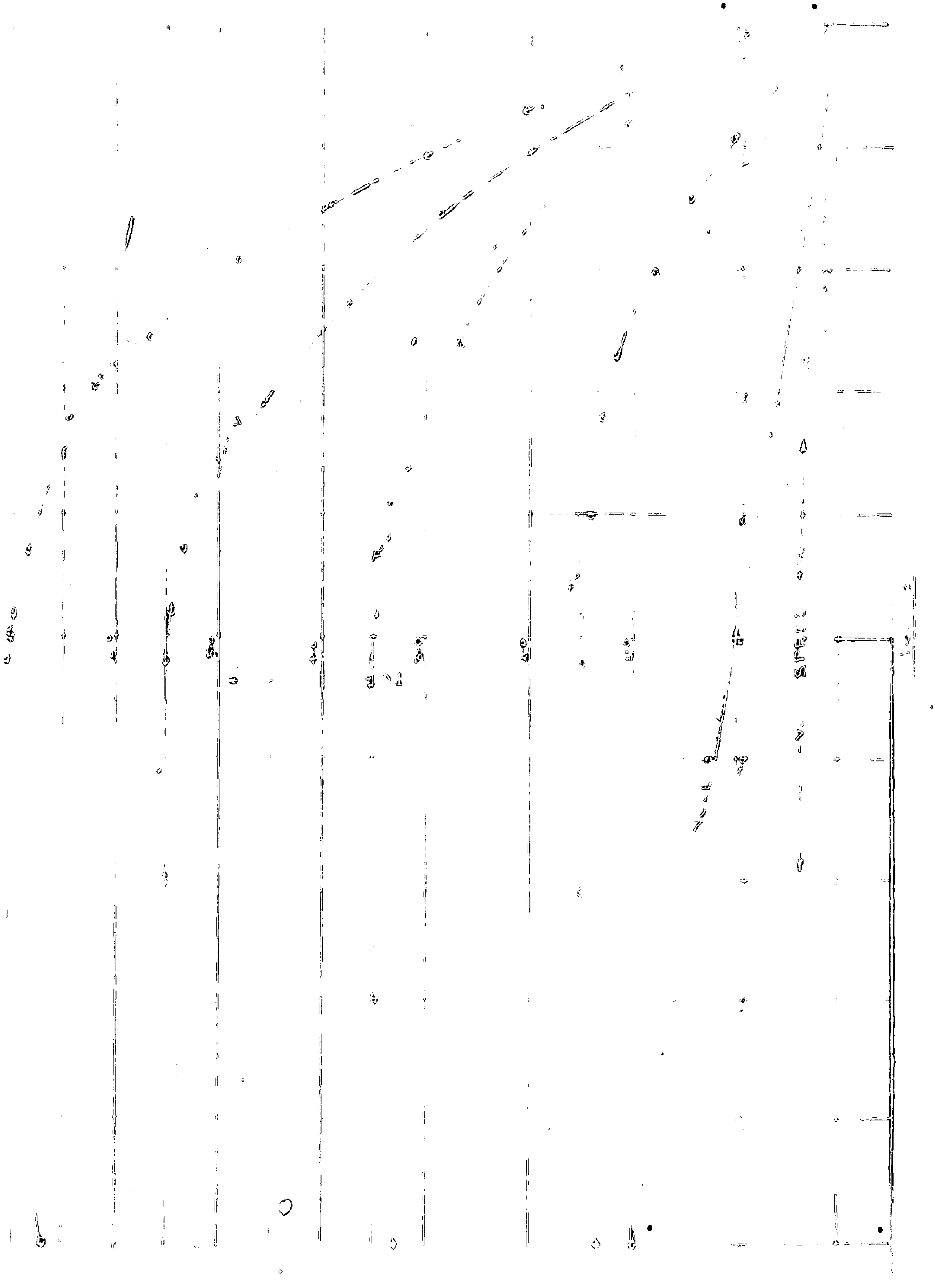


FIG. 5



(f) An external resistance, of the value to give the stable operation of the motor in the complete range of the speed from + 100 % N to - 100 % N, was added in the rotor circuit, and torque-speed, and torque-input current characteristics were measured with

(i) balanced voltage of 150 volts/phase

(ii) various unbalanced voltages, obtained with the SCR

and (iii) various unbalanced voltages, obtained with the phase variac.

The various characteristics are respectively shown in Fig. 5.3, 5.4, and 5.5.

(g) The external resistance in the rotor circuit was reduced and the torque-speed and torque-input current characteristics were measured with

i) balanced voltage of 150 volts/phase

ii) various unbalanced voltages obtained with the SCR

iii) various unbalanced voltages obtained with the single-phase variac.

The various characteristics are respectively shown in Fig. 5.6, 5.7, and 5.8.

5.1.2 Tests on squirrel-Cage Induction Motor

(a) No Load Test

S.No.	Voltage applied per phase Volts.	Input current amps.	Power Wattmeter Readings		
			W_1	W_2	Power Watts
1.	380	2.75	360	-140	220

(b) <u>Blocked Rotor Test</u>

S.No.	Voltage applied per phase volts	Input current amps.	Power Wattmeter Readings W_1	W_2	Power watts
1.	$220/\sqrt{3}$	7.30	1160	-60	1100

(c) Stator resistance= 2.8 ohms/ phase

From the above tests the following parameters were obtained. Equivalent resistance referred to primary

R'_e = 6.88 ohms/phase

Equivalent reactance referred to primary

X'_e = 16.0 ohms/phase

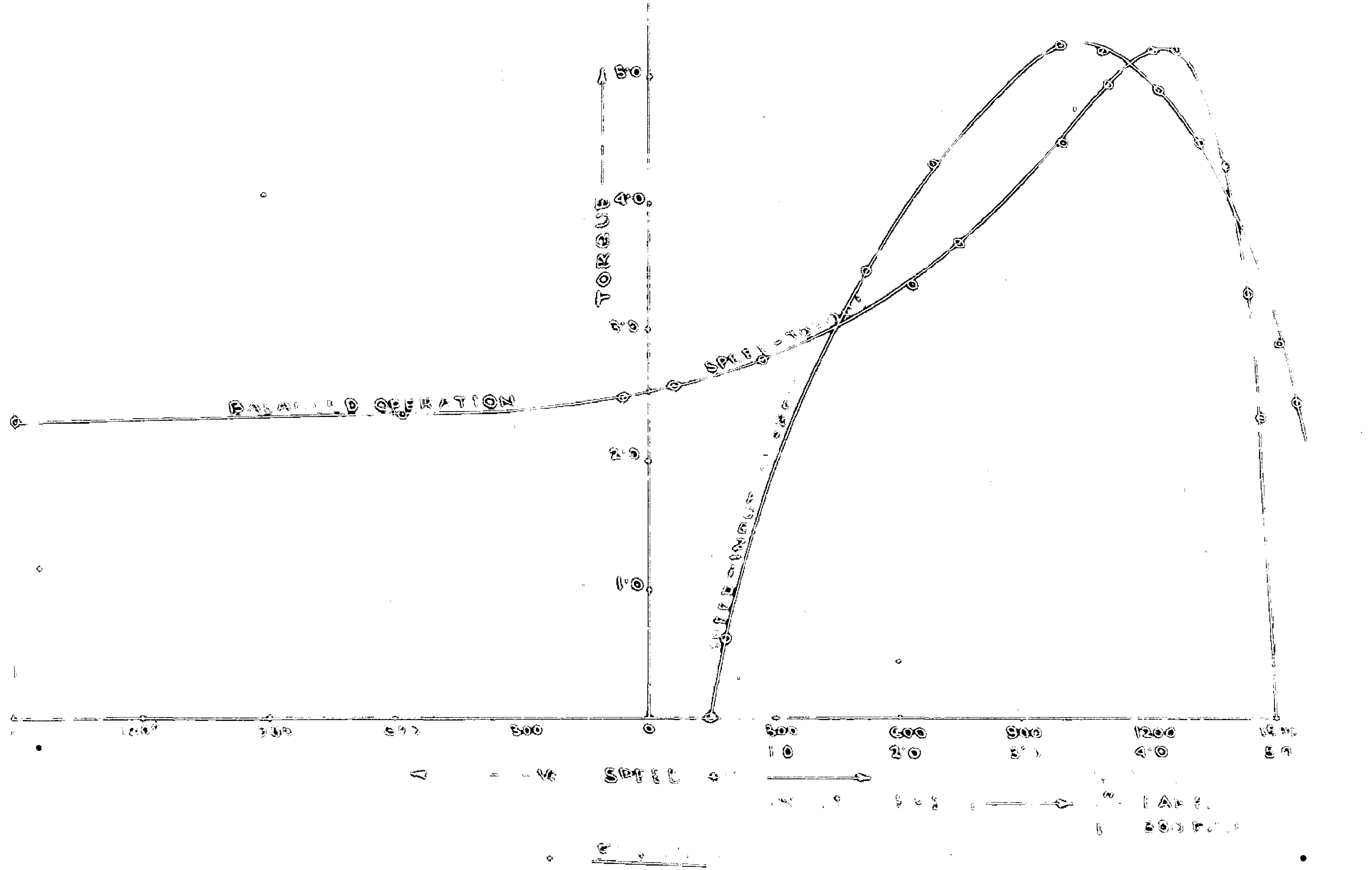
(d) The stator of the motor was connected in star to eliminate 3rd harmonics torque. The torque-speed, and torque-input current characteristics were measured with

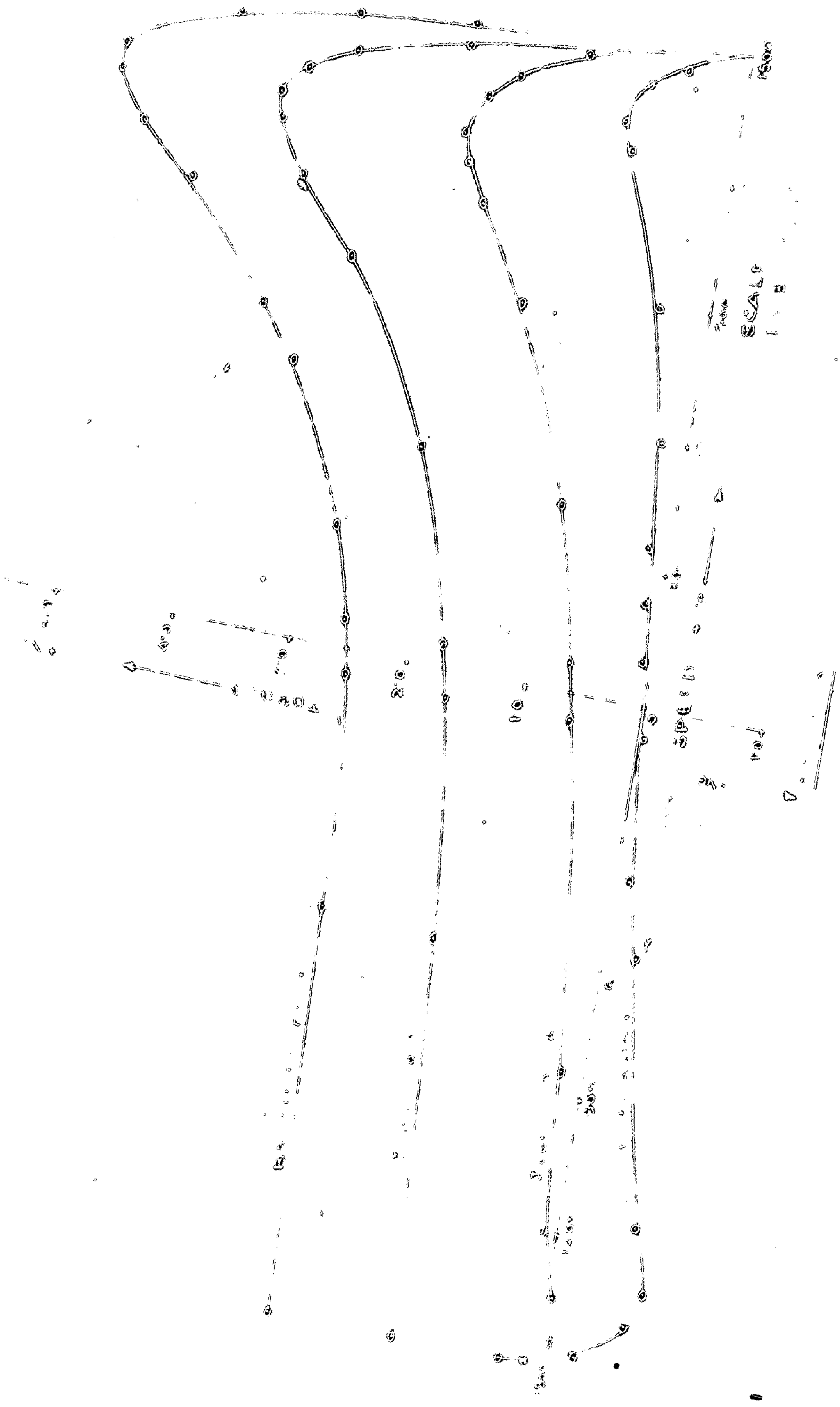
i) balanced applied voltage of $220/\sqrt{3}$

ii) various unbalanced voltages obtained with the SCR

iii) various unbalanced voltages obtained with the single-phase variac.

The various characteristics are respectively shown in Fig. 5.9, 5.10, and 5.11.


TORQUE
5·0
4·0
3·0
2·0
1·0
0
300
600
900
1200
1·0
2·0
4·0
SPEED
OPERATION



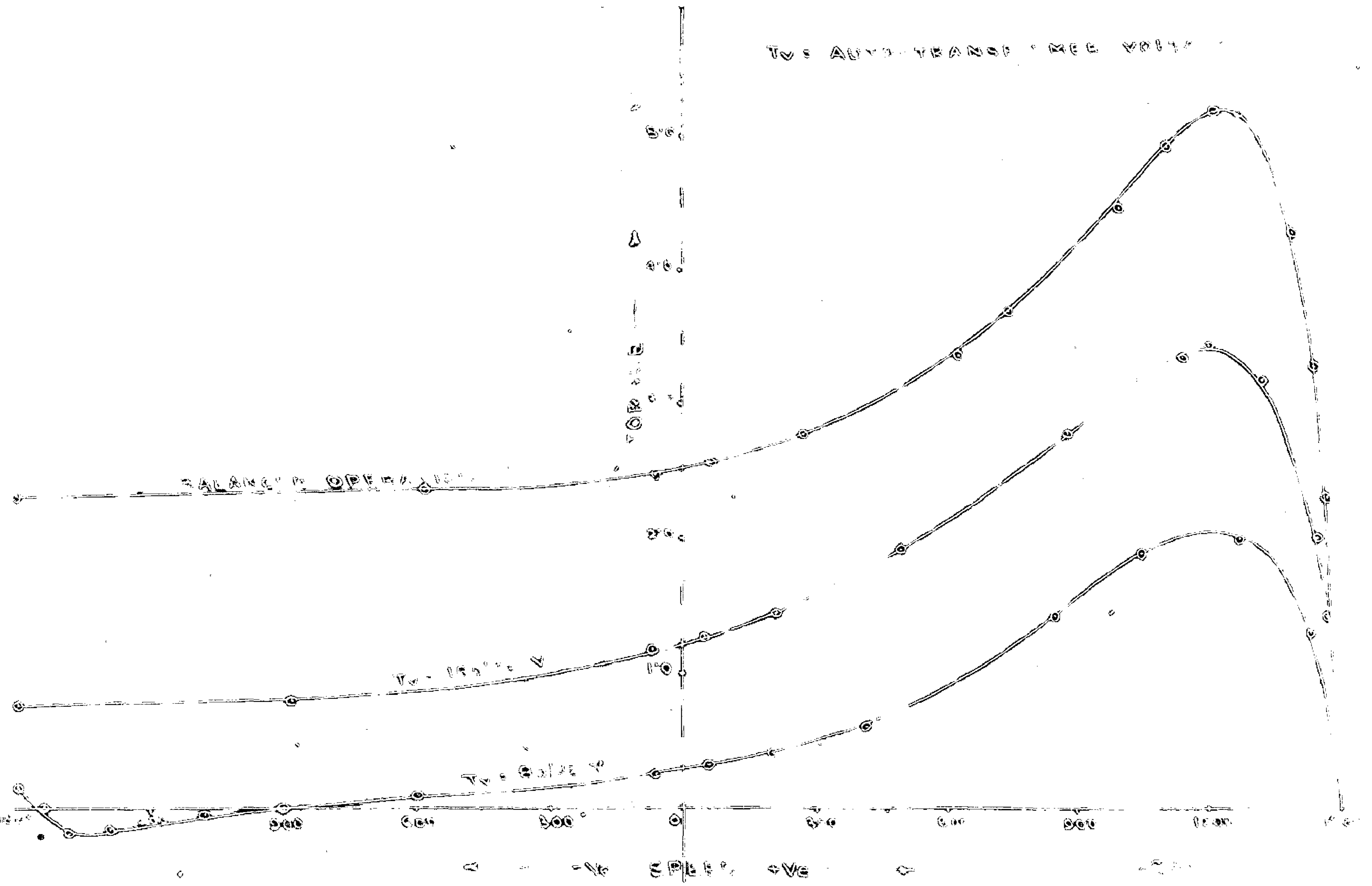


FIG 8.11

5.2 Discussion of Results

From Figs. 5.2, and 5.3 it is apparent that by adding an external resistance in the rotor circuit of the slip-ring induction motor, the motor becomes stable in the complete range from -Ns to + Ns, and also the maxm. input current reduces from 9.8 amps. to 5.8 amps. Torque-speed characteristics, obtained by unbalancing, with the help of SCR, gives a poor speed control, in the vicinity of the positive synchronous speed, and torque at positive synchronous speed is practically zero, under different control currents in the control winding of the SCR. However, unbalancing with single phase variac gives a quite appreciable speed control in the vicinity of the plus synchronous speed, and also at + Ns the negative torque developed, under different conditions of single phase variac voltage, is different and satisfactory.

In actual practice with the variable unbalancing (with a phase shifter, and amplifier, 4.1) the speed control is better than that of adjusted unbalancing (with single phase variac). One of the main reasons for getting poor speed control with the variable unbalancing, obtained with the SCR, is that the SCR used, was assembled in the laboratory from ordinary choke stampings. The SCR with such stampings, does not work efficiently and properly when the alternating current in the load winding is less because the B-H loop of the ordinary choke stamping is not rectangular, which is necessary for a SCR core (3.3.1). However as soon as the a.c. current in the load winding becomes sufficiently more, the SCR performance is satisfactory. But, all the speed torque characteristics, Figs. 5.4 to 5.8, suggest that the speed of the slip-ring induction motor can be controlled smoothly by unbalancing i.e. by applying the positive, and negative sequence systems simultaneously, and varying the relative magnitudes of the two systems.

During the experiment, it was also observed that the motor under single phase opration, becomes hot, and it cannot be operated normally for a longer period under this condition. Magnetic vibrations, under single phase opration in this case, were not appreciable. Due to low voltage applied, the magnetic saturation, at normal voltage, can not be predicted, as the magnetic conditions at this low voltage, and at normal voltage of operation, will be quite different.

Figs. 5.9 to 5.11 give the torque-speed characteristics for the squirrel cage induction motor, under test, under different conditions of unbalancing, and these characteristics are similar to that of Fig 4.13, discussed in chapter 4. The speed control, in the vicinity of the stable region is very poor, and these characteristics for the squirrel cage induction motor, under test, cannot be used for any practical purpose, leaving the method, as such, only of an academic interest.

Alongwith the poor speed control, the squirrel cage induction motor becomes very hot, under single phasing, and an additional fan had to be used to cool it for completing the experiment, under this condition. Vibrations were also quite appreciable, possibly due to production of higher harmonics.

6. CONCLUSIONS

As explained in chapter 4, the variable unbalancing gives a satisfactory method of stepless speed control of induction motors, and in combination with normal wound rotor motor speed control by varying the secondary resistance (Fig. 4.11), the torque-speed characteristics cover completely all the four quadrants, and meet the requirements for many drives e.g. hoist crane drive, winch drive, slow speed or inching operation of conveyors etc., and machines formerly requiring armature shunted d.c. operations. But the addition of quite a number of apparatus, capacitors, transformers, and rectifiers makes the system complicated, and is not very desirable. The speed control of squirrel cage induction motor by the above method is not very useful for practical purposes, and the problems of vibrations, noise, heating, and high input line current are more considerable, and difficult. However, in future, if the above mentioned difficulties are overcome, and a simpler feed-back control circuit, avoiding the use of number of apparatus and to give the similar characteristics as obtained by the method discussed, is developed, the method will be commercially economical and could be used for all practical purposes making the d.c. motors altogether obsolete.

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8 APPENDICES

8.1. Calculations of V_1 and V_2 Produced by the Insertion of an Impedance in One of the Lines

Inspection equations are given by (3.1), (3.2) and (3.3) and symmetrical component theory yields

$$V_a = V_o + V_1 + V_2 \quad \text{(a)}$$
$$V_b = V_o + a^2 V_1 + a V_2 \quad \text{(b)} \quad \ldots (8.1)$$
$$V_c = V_o + a V_1 + a^2 V_2 \quad \text{(c)}$$

$$I_a = I_o + I_1 + I_2 \quad \text{(a)}$$
$$I_b = I_o + a^2 I_1 + a^2 I_2 \quad \text{(b)} \quad \ldots (8.2)$$
$$I_c = I_o + a I_1 + a^2 I_2 \quad \text{(c)}$$

$$V_o = I_o Y_o \quad \text{(a)}$$
$$V_1 = I_1 Y_1 \quad \text{(b)} \quad \ldots (8.3)$$
$$V_2 = I_2 Y_2 \quad \text{(c)}$$

Equations (3.3), (8.2), and (8.3.a) give

$$I_o = 0, \text{ and } V_o = 0 \quad \ldots (8.4)$$

Equations (3.1), (8.2.a), (8.3.b) (8.3.c), and (8.4) give

$$V_b - V_a = (V_1 Y_1 / Y + V_2 Y_2 / Y) - V \quad \ldots (8.5)$$

Equations (8.1.a), (8.1.b), and (8.5) give

$$V_1 (1 - a^2 + Y_1 / Y) + V_2 (1 - a + Y_2 / Y) = V \quad \ldots (8.6)$$

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Equations (3.2), (8.1.b), and (8.1.c) yield

$$V_1 \quad (a - 1) \quad + \quad V_2 \,(1 - a) \qquad = \quad a\,V \quad .. \; (8.7)$$

Equations (8.6), and (8.7), give the solution

$$\left.\begin{aligned} V_1 &= -\,a\;V\left[\frac{-\,(3 + Y_2 / Y)}{(1-a)\;(3 + Y_1 / Y + Y_2 / Y)}\right] \\ V_2 &= \;V\left[\frac{(3 + Y_1 / Y)}{(1 - a)\;(3 + Y_1 / Y + Y_2 / Y)}\right] \end{aligned}\right\} ..(8.8)$$

The ratio $\left|\frac{V_2}{V_1}\right|$ is given by $\left|\frac{V_2}{V_1}\right| = \left|\frac{3 + Y_1 / Y}{3 + Y2/ Y}\right|$ ... (8.9)

8.2 <u>Calculations of V_1 , and V_2 Produced by a Single Phase Auto-Transformer</u>

The inspection equations (3.6), (3.7), and (3.8) are similar to that or (3.1), (3.2), and (3.3), except that Z has been replaced by Z , and V by K V . The characteristics equations, and equations (8.1), (8.2), (8.3), and (8.4) will yield

$$V_1 \;(1 - a^2 + Y_1 / Y_s\,) + V\;(1 - a + Y_2 / Y_s) = K\,V \;......(8.10)$$

$$V_1 \quad (a - 1) + V_2 \;(\,1 - a\,) \qquad = a\,V \quad .... \;(8.11)$$

Equations (8.10), and (8.11) give the solution

$$\left.\begin{aligned} V_1 &= V\left[\frac{(K - 1) - a\;(\,K + 2) - a\,Y_2 / Y_s}{(1 - a)\;(3 + Y_1 / \;Y_s + Y_2 / Y_s\,)}\right] \\ \text{and } V_2 &= V\left[\frac{(2 + K\,) + a\;(1 - K) + Y_1 / \;Y_s}{(1 - a)\;(3 + Y_1 / Y_s + Y_2 / \;Y_s\,)}\right] \end{aligned}\right\} .....(8.12)$$

The ratio $\left|\frac{V_2}{V_1}\right|$ is given by

$$\left|\frac{V_2}{V_1}\right| = \left|\frac{(2 + K) \;+ a\;(1 - K) \;+\; Y_1 \;/\; Y_s}{(K - 1\,) - a\;(K + 2) \;- a\,Y_2 \;/\; Y_s}\right| .....(8.13)$$