ON STABILITY OF ONE MACHINE SYSTEM

A Dissertation
submitted in partial fulfilment
of the requirements for the Degree
of

MASTER OF ENGINEERING
in
POWER SYSTEM ENGINEERING

By
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CERTIFICATE

Certified that the dissertation entitled ' ON STABILITY OF ONE MACHINE SYSTEM' which is being submitted by Shri S.C.Bhargava, in partial fulfilment for the award of the degree of Master of Engineering in Electrical Power Systems of University of Roorkee, is a record of candidate's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is to certify that he has worked for a period of 7 months from Dec '65 to June '66 for preparing dissertation for Master of Engineering Degree at the University.

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Matter has been adopted freely in direct form or otherwise from the various references. The author expresses his sincere thanks to authors of all such literature.

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S.C. Bhargava

C_O_N_T_E_N_T_S

- o -

S_Y_N_O_P_S_I_S.

This thesis deals with the STEADY-STATE and TRANSIENT-STABILITY problems of a Synchronous machine. The first part deals with the 'steady-state' stability limit of a synchronous machine connected to infinite bus bar, (i) directly, (ii) through a tie line, and (iii) through a transmission line based on dynamic relations. Special reference has been made to the effect of voltage regulator on the steady-state stability limit which is improved by a considerable amount through the use of automatic voltage regulators.

The second part deals with the transient performance of a synchronous machine connected to infinite bus bar. A new method of approach to transient stability problems has been introduced.

In all the above work the basic equations make use of Park's two reaction theory and equations and attempts have been made to do away with various assumptions which although allow a simple analysis of the problem, do not give very accurate results, even giving paradoxical results in some cases.

LIST OF SYMBOLS

δ = Angle between rotor axis and axis of stator voltage in electrical radians.

f = frequency, c/sec.

ω = electric speed, rad/sec. or unity.

ν = electric speed, p.u.

M = Angular momentum, p.u.

T_e = Electrical torque, p.u.

T_m = Mechanical or shaft torque, p.u.

T' = Additional torque suddenlly applied, p.u.

e = Infinite bus voltage, p.u.

e_f = Excitation voltage, p.u.

E_o = Open circuit terminal voltage at normal speed and no-load, p.u.

e_t = Machine terminal voltage, p.u.

I = load current, p.u.

r_m = total armature circuit resistance, p.u.

r = tie-line resistance, p.u.

r_f = field winding resistance, p.u.

x = tie-line reactance, p.u.

ψ = flux-linkages

Subscript d and q indicate direct and quadrature axis components respectively. Subscript f refer to field winding. A further subscript 0 indicates initial steady-state value of a quantity.

e_d and e_q = d and q axis voltages.

i_d and i_q = d and q axis currents.

x_d and x_q = d and q axis synchronous reactances, p.u.

$x_d(p)$ = Impedance operator relating the d-axis armature flux-linkages with the d-axis armature current.

$x_q(p)$	=	Impedance operator relating the q-axis armature flux-linkages with the q-axis armature current.
x'_d	=	Machine direct-axis transient reactance, p.u.
$G(p)$	=	Operator relating the d-axis armature linkages with the d-axis field-excitation voltage.
$g(p)$	=	Operator for the voltage regulator.
T'_d	=	d-axis transient short-circuit time constant
T'_{do}	=	d-axis transient open-circuit time constant.
$p = \frac{d}{dt}$	=	time-derivative operator.
1	=	Heaviside unit function.
Δ	=	small change in a quantity.

Chapter 1.

I_N_T_R_O_D_U_C_T_I_O_N

INTRODUCTION

The problem of power system stability is not new to the engineers. However, the subject became of more importance as the transmission systems grew more and more complex with several generating stations interconnected together through long transmission lines. This has led to continuous investigations in the subject and several authors have contributed towards the study of power system stability and methods to solve stability problems. One of the aspects of stability problems ~~is~~ to calculate the steady state stability limit of a synchronous machine and improving this limit by suitable means when the synchronous machine is connected to an infinite bus either directly or through a tie line or a transmission line.

One of the means employed to improve the steady-state stability limit of a synchronous machine is the use of quick acting automatic voltage regulators. The fact that properly designed voltage regulators may on occasion increase the stability limit is more or less well known. Crary[3] and Kimbark[4] in their discussions of the factors affecting system stability have emphasized on the importance of automatic voltage regulators in improving the stability limit considerably. They, however, did not enter into detailed analytical methods to calculate the steady state stability limit as affected by voltage regulators.

It was Concordia[7] who possibly first time discussed the subject at length and starting from the basic machine equations derived the expressions for, firstly ascertaining the stability of the system (a synchronous machine connected to an infinite bus-bar through a tie line) under the effect of voltage regulator and, secondly, to obtain the steady-state stability limit. Although simplified expressions for the final results have been used, the author has dealt with the subject quite thoroughly and comes to several important conclusions which constitute the 'back-bone' of the design aspects of a voltage regulator for a particular system. The results obtained by Concordia show that with a properly designed voltage regulator, the steady-state stability limit of a system can be increased to as much as 1.6 times its value without regulators and that the system remains stable for a value of δ as high as about 115 degrees.

Aldred and Shackshaft[8] in their paper have tried to show the effect of voltage regulators on the steady-state and transient stability of a synchronous generator. Doing away with complicated analytical expressions they have used an electronic analogue computer to solve the system equations. The effect of the main regulator loop parameters, such as gain, exciter and main field time constants, etc., on the stability of the system are examined and curves obtained to that effect. They, too, conclude that while the steady-state stability limit is increased considerably by the use of voltage regulators, the transient stability limit remains practically unaffected.

The subject has, similarly, been discussed by Nickle and Carothers[12] under the head of "Automatic Voltage Regulators". In this paper, the authors have considered automatic voltage regulators of the rheostatic and vibration contact types and have tried to show their effects on steady-state stability limit of a generator.

Further, electro-mechanical stability is studied mainly with the help of power-angle characteristic drawing of which is based, in general, on several assumptions. Thus, a rigrous solution of problem is never achieved or the stability without the usual assumptions can not be ascertained. By the application of small displacement theory it has been possible to linearize the non-linear equations of a synchronous machine with or without a voltage regulator. These linearized equations help in arriving at a characteristic equation of the system to which is applied Routh's well known criterion to test stability. If the linearized system is stable, then the original system is also stable, otherwise, unstable.

Mukhopadhyay[(9)] in his paper has discussed the applicability of Routh's criterion in synchronous machine problems and has come to various useful conclusions.

The application of frequency-response method to machine theory is only a recent development. In this connection much work has already been done by various authors.[(10),(11)] The papers deal with the possibility of applying Nyquist criterion to synchronous machine stability problems and obtaining Nyquist plots in various cases.

In this thesis, attempts have been made to deal with the synchronous machine stability problem in general and with the effects of voltage regulator in particular. The analysis is based on two reaction theory. Equations are derived relating the direct and quadrature axis quantities considering the effect of voltage regulators when the machine is connected to an infinite bus, directly or through a tie line. Making use of small displacement theory, Routh's criterion is applied to check stability. The general machine torque equation is then represented by a closed-loop system and the stability is ascertained by Nyquist criterion. Expressions have been derived for

Chapter 2.

SMALL DISPLACEMENT THEORY AND MACHINE TORQUE EQUATION

SMALL DISPLACEMENT THEORY AND MACHINE TORQUE EQUATION.

2.1. Introduction

So far, the most common method to study stability problems has been the use of power-angle characteristics. The use of these curves while simplifies the analysis, seldom provides with accurate and most desirable results. This is because when deriving the system torque-angle equation several assumptions are made and the factors like armature and interconnector resistance, effect of saturation, damper windings, effect of voltage regulators etc. are usually neglected. These factors though not so important for a practical and approximate analysis, nevertheless, affect the system stability limit considerably and must be taken into consideration when an exact solution of the problem is required.

The introduction of "small displacement theory" initially due to Lyapounov has opened a way to consider the effect of such elements as voltage regulators, damper windings etc. Making use of this theory, the original differential equation of the system which is non-linear and of second order is linearized by taking small changes in the dependent variable, thus assuming that each variable changes by a very small amount during any change in shaft power or under other circumstances. In this case, the initial steady-state conditions, i.e., prior to changes, are denoted by a suffix '0' while the small change in any variable quantity by the symbol Δ. Thus the main feature of small displacement theory is to allow linearization of original non-linear differential equations and to organise them into correct form for representation by a closed loop system.

Initial values of quantities are found out first and then small displacements are applied to the equations of voltages, currents and flux-linkages to obtain final torque equations.

The characteristic equation of the system, i.e., the denominator of the expression of the quiotient of power angle and torque is tested by Routh's or any other standard criterion and it may be concluded that:

1. if the resulting linearized system is stable, the original system is stable and
2. if the linearized system is unstable, original system is unstable.

2.2. Small Displacement Equations:[(7), (9)]

The electro-mechanical equation of a synchronous machine neglecting mechanical damping, i.e. friction and windage, is given by,

$$Mp^2\delta + T_e = T_m \qquad \dots \quad \dots (1)$$

where, $M = H/\pi f$, H being the inertia constant.

As discussed, considering small displacements on the above equation,

$$Mp^2\Delta\delta + \Delta T_e = \Delta T_m \qquad \dots \quad \dots (2)$$

If now ΔT_e is obtained as a function of p multiplied by $\Delta\delta$, the characteristic equation of the system is available and Routh's criterion can be applied to test stability.

Next few steps show how incremental torque (ΔT_e) can be related to the incremental torque angle, $\Delta\delta$.

Using Park's two reaction theory and convention, the voltage equations for a salient pole synchronous machine connected to an infinite bus bar, in the absence of zero sequence terms for balanced operation, are given by:

$$\left.\begin{aligned} e_d &= e \sin\delta = p\,\psi_d - \nu\psi_q - r_m i_d \\ e_q &= e \cos\delta = p\psi_q + \nu\psi_d - r_m i_q \end{aligned}\right\} \qquad \ldots \qquad \ldots (3)$$

In per unit, the developed electrical torque is given by,

$$T_e = \psi_d i_q - \psi_q\, i_d \qquad \ldots \qquad \ldots (4)$$

and the per unit speed at any instant is given by,

$$\nu = p\,(\omega t - \delta) \qquad \ldots \qquad \ldots (5)$$

Again, the flux linkages in the direct and quadrature axes are related to the axes currents, by-

$$\left.\begin{aligned} \psi_d &= G(p)E - x_d(p)\; i_d \\ \psi_q &= -\, x_q(p)\; i_q \end{aligned}\right\} \qquad \ldots \qquad \ldots (6)$$

It is now assumed that all the quantities in equations (3) to (6) may be expressed by a sum of a steady-state value with subscript '0' and an incremental value by Δ. In other words considering small displacements on the above quantities, we derive at following equations in which the action of voltage regulator is neglected initially and constant excitation is assumed-

$$\left.\begin{aligned} \Delta e_d &= e \cos\delta_o . \Delta\delta = p\Delta\psi_d - \nu_o \Delta\psi_q - \psi_{q_o}\Delta\nu - r_m \Delta i_d \\ \Delta e_q &= -e \sin\delta_o . \Delta\delta = p\,\Delta\psi_q + \nu_o \Delta\psi_d + \psi_{d_o}\Delta\nu - r_m \Delta i_q \end{aligned}\right\} \qquad \ldots \qquad \ldots (7)$$

$$\Delta\nu = -p\Delta\delta \qquad \ldots \qquad \ldots (8)$$

$$\left.\begin{aligned} \Delta\psi_d &= -\, x_d(p)\Delta i_d \\ \Delta\psi_q &= -\, x_q(p)\Delta i_q \end{aligned}\right\} \qquad \ldots \qquad \ldots (9)$$

Substitution of $\Delta\nu$, $\Delta\psi_d$ and $\Delta\psi_q$ from equations (8) and (9) in equations (7) for Δe_d and Δe_q results in the equations.

$$\left.\begin{aligned} \Delta e_d &= e \cos\delta\,\Delta\delta = -Z_d(p)\Delta i_d + x_q(p)\Delta i_q - \psi_{qo} p\Delta\delta \\ \Delta e_q &= -e \sin\delta . \Delta\delta = -x_d(p)\Delta i_d - Z_q(p)\Delta i_q + \psi_{do} p\Delta\delta \end{aligned}\right\} \qquad \ldots \qquad \ldots (10)$$

Assuming the usual per unit value of ν_o as unity, the solution of equations (10) for Δi_d and Δi_q yields

$$\left.\begin{aligned} \Delta i_d &= \frac{\left[-Z_q(p)\left\{e \cos\delta + \psi_{qo}p\right\} + x_q(p)\left\{e \sin\delta + \psi_{do}p\right\}\right]}{Z_d(p)\, Z_q(p) + x_d(p)\, x_q(p)}\,\Delta\delta \\ \Delta i_q &= \frac{\left[x_d(p)\left\{e \cos\delta + \psi_{qo}p\right\} + Z_d(p)\left\{e \sin\delta + \psi_{do}\, p\right\}\right]}{Z_d(p)\, Z_q(p) + x_d(p)\, x_q(p)}\,\Delta\delta \end{aligned}\right\} \qquad \ldots \quad \ldots (11)$$

where,

$$Z_d(p) = r_m + x_d(p).p.$$
$$Z_q(p) = r_m + x_q(p).p.$$

Small displacement in T_e in equation (4) gives,

$$\Delta T_e = \psi_{do}\Delta i_q + i_{qo}\Delta\psi_d - \psi_{qo}\Delta i_d - i_{do}\Delta\psi_q \qquad \ldots \quad \ldots (12)$$

2.3. Determination of initial or steady-state values:

Under intial steady-state operating conditions the system voltage and flux equations are as follows:

$$\left.\begin{aligned} e_d &= e \sin\delta_o = -\, r_m\, i_{do} - \psi_{qo} \\ e_q &= e \cos\delta_o = -\, r_m\, i_{qo} + \psi_{do} \end{aligned}\right\} \qquad \ldots \quad \ldots (13)$$

$$\left.\begin{aligned} \psi_{do} &= E_o - x_d\, i_{do} \\ \psi_{qo} &= -\, x_q\, i_{qo} \end{aligned}\right\} \qquad \ldots \quad \ldots (14)$$

Substituting ψ_{do} and ψ_{qo} from equation (14) in equations (13) we get:

$$\left.\begin{aligned} e \sin\delta_o &= -\, r_m\, i_{do} + x_q\, i_{qo} \\ e \cos\delta_o &= -\, r_m\, i_{qo} + E_o - x_d\, i_{do} \end{aligned}\right\} \qquad \ldots \quad \ldots (15)$$

Equations (15) are then solved to give required initial currents i_{do} and i_{qo} as follows:

$$i_{do} = \frac{-r_m\ e \sin\delta_o - x_q\ (e \cos\delta_o - E_o)}{r_m^2 + x_d\ x_q}$$

$$i_{qo} = \frac{+x_d\ e \sin\delta_o - r_m\ (e \cos\delta_o - E_o)}{r_m^2 + x_d\ x_q} \qquad \ldots \quad \ldots (16)$$

2.4. <u>The Characteristic Equation:</u>

The values of initial or steady-state quantities i_{do}, i_{qo} and ψ_{do}, ψ_{qo} from equations (16) and (14) respectively together with the incremental values Δi_d, Δi_q and $\Delta\psi_d$, $\Delta\psi_q$, from equations (11) and (9) are substituted in equation (12) for incremental electrical torque to give, finally,

$$\Delta T_e = \frac{\psi_{do}\left[x_d\ (p)\left\{e \cos\delta + p\psi_{qo}\right\} + Z_d(p)\left\{e \sin\delta + p\ \psi_{do}\right\}\right]}{Z_d(p)\ Z_q(p) + x_d(p)\ x_q(p)}\Delta\delta$$

$$+\frac{i_{qo}\ x_d(p)\left[Z_q(p)\left\{e \cos\delta + \psi_{qo}p\right\} - x_q(p)\left\{e \sin\delta + p\psi_{do}\right\}\right]}{Z_d(p)\ Z_q(p) + x_d(p)\ x_q(p)}\Delta\delta$$

$$+\frac{\psi_{qo}\left[Z_q(p)\left\{e \cos\delta + \psi_{qo}p\right\} - x_q(p)\left\{e \sin\delta + p\ \psi_{do}\right\}\right]}{Z_d(p)\ Z_q(p) + x_d(p)\ x_q(p)}\Delta\delta$$

$$+\frac{i_{do}x_q(p)\left[x_d(p)\left\{e \cos\delta + p\ \psi_{qo}\right\} + Z_d(p)\left\{e \sin\delta + p\psi_{do}\right\}\right]}{Z_d(p)\ Z_q(p) + x_d(p)\ x_q(p)}\Delta\delta$$

or $$\Delta T_e = \frac{\Delta\delta}{D}\Big[\left\{i_{do}\ x_q(p) + \psi_{do}\right\}\left\{x_d(p)\ (e \cos\delta + \psi_{qo}p) + Z_d(p)(e \sin\delta + \psi_{do}p)\right\}$$

$$+\left\{i_{qo}x_d(p) + \psi_{qo}\right\}\left\{Z_q(p)\ (e \cos\delta + \psi_{qo}p) - x_q(p)(e \sin\delta + \psi_{do}p)\right\}\Big]$$

$$= f(p)\,.\,\Delta\delta \qquad \ldots \quad \ldots (17)$$

where,

$$D = Z_d(p)\ Z_q(p) + x_d(p)\ x_q(p)$$

The electro-mechanical equation of the system, then, reduces to the form:

$$\Delta T_m = \left[Mp^2 + f(p)\right].\,\Delta\delta \qquad \ldots \quad \ldots (18)$$

This can be written:

$$\frac{\Delta\delta}{\Delta T_m} = \frac{1}{Mp^2 + f(p)} \qquad \ldots \quad \ldots (19)$$

in which the characteristic equation is

$$Mp^2 + f\,(p) = 0 \qquad \ldots \quad \ldots (20)$$

This equation is linear and is obtained in powers of polynominal 'p' with constant coefficients. To test the stability of the system, roots of the characteristic equation $Mp^2 + f(p) = 0$ are investigated by applying Routh's criterion or, as shown later, Nyquist criterion may be applied using frequency response method.

2.5. Machine Connected to Infinite Bus Through Tie Line:

In last few sections, small displacement theory has been applied to derive the characteristic equation when synchronous machine is directly connected to an infinite bus-bar. In the present section the case of machine connected to infinite bus through a tie line of resistance r and reactance x will be considered. For this, it is best to assume the tie-line with infinite bus as a second machine(7) and write the voltage equations.

Thus, neglecting the effect of voltage regulators, we have for the synchronous machine,

$$e_d = -\,p\,x_d(p)\,i_d - r_m\,i_d + x_q(p)\,i_q.\nu$$

$$e_q = -\,x_d(p)\,i_d.\nu - px_q\,(p)\,i_q - r_m\,i_q$$

or

$$\left.\begin{aligned} e_d &= -\,Z_d(p)\,i_d + x_q(p)\,i_q.\nu \\ e_q &= -\,x_d(p)\,i_d.\nu - Z_q(p)\,i_q \end{aligned}\right\} \qquad \ldots \quad \ldots (21)$$

For the tie line,

$$\left.\begin{aligned} e_d &= e\sin\delta + Z(p)\,i_d - xi_q.\nu \\ e_q &= e\cos\delta + xi_d.\nu + Z(p)\,i_q \end{aligned}\right\} \qquad \ldots \quad \ldots (22)$$

Machine torque equation is,

$$Mp^2\delta + T_e = T_m \qquad \ldots \qquad \ldots (23)$$

where,

$$T_e = \psi_d\, i_q - \psi_q\, i_d \qquad \ldots \qquad \ldots (24)$$

Small displacements in equations (21), (22), (23), (24) give,

$$\Delta e_d = -Z_d(p)\Delta i_d + x_q(p)\Delta i_q - \psi_{qo} p\Delta\delta \qquad \ldots \qquad \ldots (25)$$

$$\Delta e_q = -x_d(p)\Delta i_d - Z_q(p)\Delta i_q + \psi_{do}\, p\Delta\delta \qquad \ldots \qquad \ldots (26)$$

$$\Delta e_d = Z(p)\Delta i_d - x\Delta i_q + (e\cos\delta - x i_{qo}\, p)\Delta\delta \qquad \ldots \qquad \ldots (27)$$

$$\Delta e_q = x\Delta i_d + Z(p)\Delta i_q - (e\sin\delta - x\, i_{do}\, p)\Delta\delta \ldots \qquad \ldots (28)$$

$$\Delta T_m = -\left[\psi_{qo} + i_{qo}\, x_d\,(p)\right]\Delta i_d + \left[\psi_{do} + i_{do}x_q(p)\right]\Delta i_q + p^2 M\Delta\delta. (29)$$

The above equations can be arranged in matrix form as given below:

$$\begin{array}{ccccc|c}
\Delta e_d & \Delta e_q & \Delta i_d & \Delta i_q & \Delta\delta & \\
\hline
0 & 0 & -\left[\psi_{qo}+i_{qo}x_d(p)\right] & \psi_{do}+i_{do}x_q(p) & p^2M & =\Delta T_m \\
-1 & -0 & -Z_d(p) & x_q(p) & -\psi_{qo}p & = 0 \\
0 & -1 & -x_d(p) & -Z_q(p) & \psi_{do}p & = 0 \\
-1 & 0 & Z(p) & -x & e\cos\delta - x i_{qo}p & = 0 \\
0 & -1 & x & Z(p) & -(e\sin\delta - x i_{do}p) & = 0
\end{array}$$

$$\ldots \qquad \ldots (30)$$

To obtain the characteristic equation, the determinant of the coefficients of above equations is expanded in powers of p. Stability may then be tested by applying Routh's criterion.

2.5.2. Calculation of initial steady-state values:

Following the method of section 2.3, the values of i_{do} and i_{qo} are given by:

$$\left.\begin{aligned} i_{do} &= \frac{-(r_m+r)\ e\ \sin\delta_0 - (x+x_q)(e\ \cos\delta_0 - E_0)}{(r_m+r)^2 + (x_1+x_d)(x+x_q)} \\ i_{qo} &= \frac{(x+x_d)\ e\ \sin\delta_0 - (r_m+r)\ (e\ \cos\delta_0 - E_0)}{(r_m+r)^2 + (x+x_d)(x+x_q)} \end{aligned}\right\} \quad \ldots \quad \ldots (31)$$

$$\left.\begin{aligned} \psi_{do} &= E_o - x_d\ i_{do} \\ \psi_{qo} &= -\ x_q\ i_{qe} \end{aligned}\right\} \quad \ldots \quad \ldots (32)$$

$$\left.\begin{aligned} e_{qo} &= \psi_{do} - r_m\ i_{qo} \\ e_{do} &= -\psi_{qo} - r_m\ i_{do} \end{aligned}\right\} \quad \ldots \quad \ldots (33)$$

2.6. <u>Machine Connected to Infinite Bus Through Transmission Line:</u>

The transmission line may be represented by a π circuit with half of its total capacitance assumed lumped at the two ends, or by a T circuit. But, if use is made of the A, B, C, D constants of the transmission line, expressed in per unit values, a more general analysis of the system can be obtained.

If e_s is the sending end voltage at the machine terminals, its direct-axis and quadrature-axis components e_{ds} and e_{qs} respectively are related to it as follows:

$$e_s = e_{ds} + j\ e_{qs} \quad \ldots \quad \ldots (34)$$

Also,

$$\begin{aligned} e_s &= AE_r + BI_r \\ &= A\ (e_{dr} + je_{qr}) + B(i_{dr} + ji_{qr}) \\ &= (A\ e_{dr} + B\ i_{dr}) + j\ (A\ e_{qr} + B\ i_{qr}) \end{aligned}$$

Therefore, for the transmission line,

$$\left.\begin{aligned} e_d &= A\ e\ \sin\delta + Bi_d \\ e_q &= A\ e\ \cos\delta + Bi_q \end{aligned}\right\} \qquad \ldots \quad \ldots (35)$$

since

$$e_d = e\ \sin\delta \qquad \text{and} \qquad e_q = e\ \cos\delta$$

For the synchronous machine, neglecting any voltage regulator action,

$$\left.\begin{aligned} e_d &= -Z_d(p)i_d + x_q(p)i_q\,.\nu \\ e_q &= -x_d(p)i_d\nu - Z_q(p)i_q \end{aligned}\right\} \qquad \ldots \quad \ldots (36)$$

Small displacements in equations (35) and (36) result in the following equations:

$$\left.\begin{aligned} \Delta e_d &= A\ e\ \cos\delta\,.\,\Delta\delta + B\Delta i_d \\ \Delta e_q &= -A\ e\ \sin\delta\,.\,\Delta\delta + B\Delta i_q \\ \Delta e_d &= -Z_d(p)\Delta i_d + x_q(p)\Delta i_q - \psi_{qo}p\,\Delta\delta \\ \Delta e_q &= -x_d(p)\Delta i_d - Z_q(p)\Delta i_q + \psi_{do}p\,.\,\Delta\delta \end{aligned}\right\} \qquad \ldots \quad \ldots (37)$$

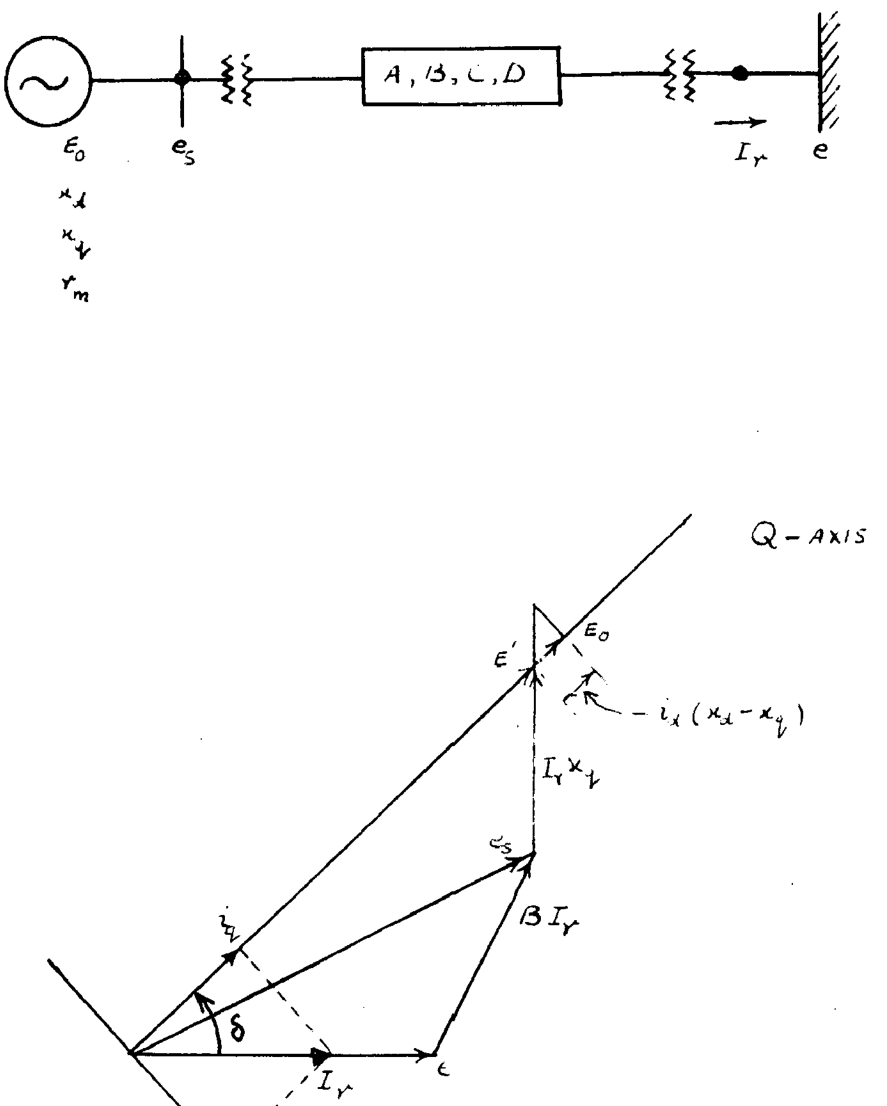
Small increments on machine torque equation give,

$$\Delta T_m = -\left[\psi_{qo} + i_{qo}x_d\ (p)\right]\Delta i_d + \left[\psi_{do} + i_{do}x_q(p)\right]\Delta i_q + p^2M\,\Delta\delta \quad \ldots (38)$$

From equations (37) and (38),

Δe_d	Δe_q	Δi_d	Δi_q	$\Delta\delta$	
0	0	$-\left[\psi_{qo} + i_{qo}x_d(p)\right]$	$\psi_{do} + i_{do}x_q(p)$	p^2M	$= \Delta T_m$
-1	0	$-Z_d(p)$	$x_q(p)$	$-\psi_{qo}p$	$= 0$
0	-1	$-x_d(p)$	$-Z_q(p)$	$\psi_{do}p$	$= 0$
-1	0	B	0	$Ae\ \cos\delta$	$= 0$
0	-1	0	B	$-Ae\ \sin\delta$	$= 0$

$\ldots \quad \ldots (39)$



A, B, C, D
E_0
e_s
I_r
e
x_d
x_q
r_m
Q - AXIS
E_0
E'
$i_d(x_d - x_q)$
$I_r x_q$
e_s
$B I_r$
i_q
δ
I_r
e
i_d
D - AXIS

FIGURE I.

Again, the stability of the system can be ascertained by applying Routh's criterion to the characteristic equation obtained by expanding the determinant formed by the coefficient of equations (39).

2.6.2. Calculation of initial steady-state values:

As a first step, E_o is calculated as follows:

Referring to phasor diagram of Fig.1, there is,

$e_s = A_e + BI_r$, for any receiving end load current I_r.

Then neglecting resistance, E' is given by

$$E' = e_s + j\, I_r\, x_q$$

and

$$E_o = E' + i_d\,(x_d - x_q) \qquad \ldots \quad \ldots (40)$$

The values of machine currents and flux-linkages are given by,

$$\left.\begin{aligned} i_{do} &= \frac{-r_m\, e \sin\delta_o - x_q\,(e \cos\delta_o - E_o)}{r_m^2 + x_d\, x_q} \\ i_{qo} &= \frac{x_d\, e \sin\delta_o - r_m\,(e \cos\delta_o - E_o)}{r_m^2 + x_d\, x_q} \end{aligned}\right\} \qquad \ldots \quad \ldots (41)$$

$$\left.\begin{aligned} \psi_{do} &= E_o - x_d\, i_{do} \\ \psi_{qo} &= -\, x_q\, i_{qo} \end{aligned}\right\} \qquad \ldots \quad \ldots (42)$$

The sample calculations for each of the above cases have been illustrated in Chapter 4 where the stability is also tested by frequency response method.

Chapter 3.

EFFECT OF VOLTAGE REGULATOR

EFFECT OF VOLTAGE REGULATOR

3.1. Introduction and Preliminary Remarks:

As has been stated earlier, the steady-state stability limit of a synchronous machine is increased considerably when the effect of voltage regulators is taken into account. Much useful work has already been done in this connection by authors like Concordia, Aldred and Shackshaft, Nickle and Carothers and others.

This Chapter will consider in details the effect of a voltage regulator on the steady-state stability limit of a synchronous machine connected to an infinite bus bar, (i) directly. (ii) through an impedance tie. The regulator is responsive to changes in terminal voltage of the machine so as to maintain it at some constant value (unity in this case) and acts on the field voltage of the synchronous machine. The notation and assumptions of Park are followed:

3.1.2. Preliminary remarks:

Before entering into detailed mathematical analysis, it is desirable to have a general review of the fundamental concepts involved.

Consider a single round-rotor synchronous machine connected to an infinite bus through a tie-line. Neglecting machine and tie-line resistance, the steady-state power transfer is given by

$$P = \frac{E_o e}{x_d + x} \sin \delta \qquad \ldots \quad \ldots (1)$$

It may be pointed out here that in the steady-state (or generally, if the effects of rotor circuits are neglected) there is analytically no distinction between a tie line with its infinite bus and a second machine. The distinction lies rather in the values to be assigned to the various parameters and in the

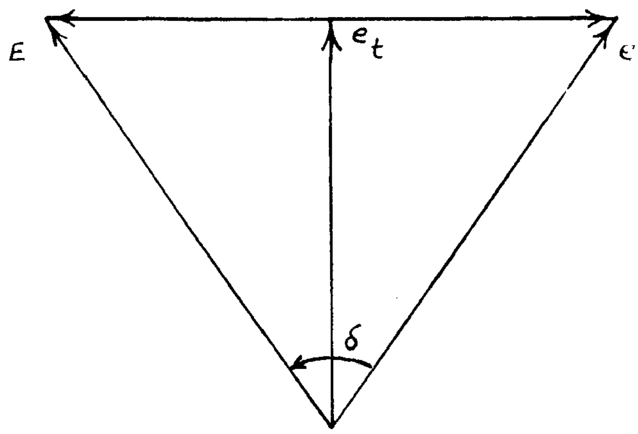


FIGURE 2.

interpretation of the quantitative results. This assumption, however, simplifies the matter when writing voltage equations for the tie line, based on two reaction theory, on the lines of machine equations.

Now, if in equation (1),

$$x_d = x = 1.0 \text{ p.u.}$$

with excitation of machine set at some fixed value, for example, to give rated current at unity power factor and unit terminal voltage, referring to phasor diagram of Fig.2,

$$E_o = e = \frac{1}{\cos 45^o} = 1.414$$

and the power transfer is

$$P = \frac{(1.414) \times (1.414)}{1 + 1} \sin\delta$$

or,

$$P = \sin\delta$$

The power limit occurs at $\delta = 90^o$ and is $P_{max} = 1.0$ p.u.

(Note that the calculations above give infinite bus voltage e as 1.414 instead of conventional value of 1.0 p.u. This is because of the assumption that the infinite bus together with the tie line can be looked upon as a second round rotor machine. Also, the terminal voltage at the terminals of actual machine is assumed to be unity, with current at unity power factor. Though difficult to follow, it makes little difference as far as the calculation of stability limit by analytical methods is concerned).

If on the other hand, the excitations are not constant but are controlled (through the use of voltage regulators) so as to maintain unit terminal voltage and power factor, then

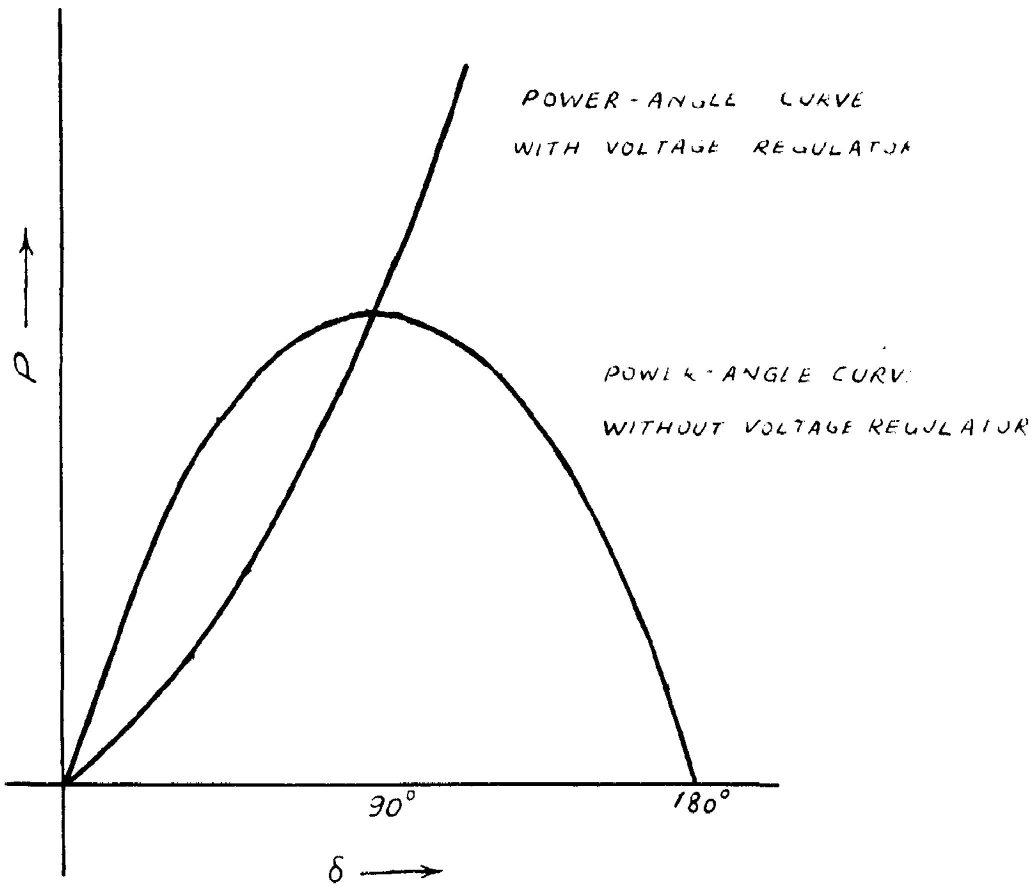

POWER - ANGLE CURVE
WITH VOLTAGE REGULATOR
POWER - ANGLE CURVE
WITHOUT VOLTAGE REGULATOR
P
90°
180°
δ

FIGURE 3.

$$E_o = e = \frac{1}{\cos\delta/2}$$

and the power transfer is

$$P = \frac{E_o e}{x_d + x}\sin\delta = \frac{\sin\delta}{2\cos^2\delta/2} = \tan\delta/2$$

Now the power transfer increases without limit as the angle δ is increased from 0 to 180 degrees. The new torque-angle curve crosses the old torque-angle curve at $\delta = 90$ degrees (Fig.3). The question of the meaning of this result, particularly of thepossibility of stable operation in the region $\delta > 90$ degrees, now naturally arises; the answer is found in the kind of control used. If the excitations are controlled slowly, for example, by hand, so as to return the terminal voltage to unity only after deviation is noticed, the power limit again will be found at $\delta = 90$ degrees. One may imagine the load increased in small steps. At each step the terminal voltage is returned to unity by an adjustment of the excitations, and then the system is tested for steady-state stability in the conventional way by noting whether or not a small increase in angle with fixed excitations results in an increase or decrease in power transfer. However, if the excitations are controlled automatically to maintain constant terminal voltage continuously and instantaneously, regardless of load or angle, and even during the stability test, the relation $P = \tan\delta/2$ will hold instantaneously, and there no longer is a stability limit. In order to maintain constant terminal voltage required for such stable operation it is necessary to have a flat regulator (that is one with infinite amplification factor, $a = -\infty$) and no time lags in the regulator, exciter, or even in the main machine fields, so that the excitations are corrected instantaneously. Such action is, of course, practically unrealizable because of the time lags inherent in any system. Moreover, both excitations also increase without limit as the angle approaches 180 degrees. Thus, in practice a stability limit is reached for some value of δ greater than 90 degrees but less than 180 degrees, beyond which the system becomes unstable.

3.2. Mathematical Analysis:

In the following sections, a methamatical analysis of the system including effect of voltage regulator will be presented for the two cases viz. (i) machine directly connected to infinite bus, (ii) machine connected through tie line to infinite bus. The second case will be considered first as the expressions for first case can be derived from it.

3.2.1. Machine connected to infinite bus through tie-line.[7]

The equations of axes voltages for synchronous machine are,

$$\left.\begin{aligned} e_d &= p\,\psi_d - r_m i_d - \psi_q \cdot \nu \\ e_q &= p\,\psi_q - r_m i_q + \psi_d \cdot \nu \end{aligned}\right\} \qquad \ldots \quad \ldots (2)$$

Similarly, the equations for flux linkages are:

$$\left.\begin{aligned} \psi_d &= G(p)E - x_d(p)\, i_d \\ \psi_q &= -x_q(p)\, i_q \end{aligned}\right\} \qquad \ldots \quad \ldots (3)$$

Substitution of ψ_d and ψ_q from equations (3) in equations (2) gives

$$e_d = p\,G(p)E - px_d(p)i_d - r_m i_d + x_q(p)i_q \cdot \nu$$

or

$$e_d = p\,G(p)E - Z_d(p)i_d + x_q(p)\, i_q \cdot \nu \qquad \ldots \quad \ldots (4)$$

$$e_q = -p\,x_q(p)i_q - r_m i_q + G(p)E \cdot \nu - x_d(p)\, i_d \cdot \nu$$

or

$$e_q = G(p)E \cdot \nu - x_d(p)\, i_d \nu - Z_q(p)\, i_q \qquad \ldots \quad \ldots (5)$$

where $Z_d(p) = r_m + px_d(p)$ and $Z_q(p) = r_m + px_q(p)$

On similar lines, for the tie line, (Impressed currents)

$$e_d = e \sin\delta + Z(p)\, i_d - x i_q \cdot \nu \qquad \ldots \qquad \ldots (6)$$

$$e_q = e \cos\delta + x i_d \cdot \nu + Z(p)\, i_q \qquad \ldots \qquad \ldots (7)$$

where $Z(p) = r + px$ for the tie line.

Machine torque equation is,

$$Mp^2\delta + T_e = T_m \qquad \ldots \qquad \ldots (8)$$

where T_e is the electrical torque given by

$$T_e = \psi_d\, i_q - \psi_q\, i_d \qquad \ldots \qquad \ldots (9)$$

From equations (4), (5), (6), (7), and (8), the equations for small changes from a steady-state operating position are:

$$\Delta e_d = pG(p)\Delta E - Z_d(p)\Delta i_d + x_q(p)\Delta i_q - \psi_{qo} p \cdot \Delta\delta \qquad \ldots \qquad \ldots (10)$$

$$\Delta e_q = G(p)\Delta E - x_d(p)\Delta i_d - Z_q(p)\Delta i_q + \psi_{do}\, p\Delta\delta \ldots \qquad \ldots (11)$$

$$\Delta e_d = Z(p)\Delta i_d - x\Delta i_q + (e\cos\delta - x i_{qo} p)\cdot\Delta\delta \qquad \ldots \qquad \ldots (12)$$

$$\Delta e_q = x\Delta i_d + Z(p)\Delta i_q - (e\sin\delta - x i_{do} p)\cdot\Delta\delta \qquad \ldots \qquad \ldots (13)$$

$$\Delta T_m = i_{qo} G(p)\Delta E - \left[\psi_{qo} + i_{qo} x_d(p)\right]\Delta i_d + \left[\psi_{do} + i_{do} x_q(p)\right]\Delta i_q + p^2 M\Delta\delta \, (14)$$

The regulator introduces a change in field voltage E as a function of change in <u>magnitude</u> of the terminal voltage e_a. This is given by,

$$\Delta E = g(p)\cdot\Delta e_a \qquad \ldots \qquad \ldots (15)$$

where

$$e_a^2 = e_d^2 + e_q^2 \qquad \ldots \qquad \ldots (16)$$

and g(p) is the operation expression for the action of the voltage regulator.

From (16),

$$\Delta e_a = e_d' \cdot \Delta e_d + e_q' \cdot \Delta e_q \qquad \ldots \qquad \ldots (17)$$

where $e'_d = e_{do}/e_{ao}$ and $e'_q = e_{qo}/e_{ao}$

Then, $\Delta E = g(p)\left[e'_d \Delta e_d + e'_q \Delta e_q\right]$ ... ... (18)

when equation (18) is substituted in equations, (10), (11) and (14), these become-

$$\Delta e_d = pG(p)g(p)e'_d \Delta e_d + pG(p)g(p)e'_q \Delta e_q - Z_d(p)\Delta i_d + x_q(p)\Delta i_q - \psi_{qo} p \Delta\delta$$ ... ... (19)

$$\Delta e_q = G(p)g(p)e'_d \Delta e_d + G(p)g(p)e'_q \Delta e_q - x_d(p)\Delta i_d - Z_q(p)\Delta i_q + \psi_{do} p \Delta\delta$$ ... ... (20)

$$\Delta T_m = i_{qo}G(p)g(p)e'_d \Delta e_d + i_{qo}G(p)g(p)e'_q \Delta e_q - \left[\psi_{qo} + i_{qo}x_d(p)\right]\Delta i_d + \left[\psi_{do} + i_{do}\, x_q(p)\right]\Delta i_q + p^2 M \Delta\delta$$ ... ... (21)

The equations (21), (19), (20), (12), and (13) when arranged in a matrix form, after necessary transposition, become,

Δe_d	Δe_q	Δi_d	Δi_q	$\Delta\delta$	
$i_{qo}G(p)g(p)e'_d$	$i_{qo}G(p)g(p)e'_q$	$-\psi_{qo} - i_{qo}x_d(p)$	$\psi_{do} + i_{do}x_q(p)$	p^2M	$=\Delta T_m$
$pG(p)g(p)e'_d - 1$	$pG(p)g(p)e'_q$	$-Z_d(p)$	$x_q(p)$	$-\psi_{qo}p$	$=0$
$G(p)g(p)e'_d$	$G(p)g(p)e'_q - 1$	$-x_d(p)$	$-Z_q(p)$	$\psi_{do}p$	$=0$
-1	0	$Z(p)$	$-x$	$e\cos\delta - x i_q p$	$=0$
0	-1	x	$Z(p)$	$-e\sin\delta + x i_d p$	$=0$

... ... (22)

in which, Δe_d, Δe_q, Δi_d, Δi_q, and ΔT_m are the variable quantities and the terms within the matrix are obtained as the simple or fractional expressions in polynominal p.

In order to test the stability of the system, the determinant of the efficients of equations (22) is expressed as a polynominal in p, and the signs of the real parts of the p roots are investigated by Routh's criterion.

It is to be seen that expression of the determinant of the coefficients of equations (22) results in an expression of the form,

$$\Delta T_m = F(p) . \Delta\delta$$

in which F(p), the determinant expressed as a polynominal in p, is obviously the characteristic equation of the system whose roots are tested by applying Routh's criterion.

Derivation of Characteristic Equation:

Let the equations (22) be represented as:

Δe_d	Δe_q	Δi_d	Δi_q	$\Delta\delta$	
a_1	b_1	c_1	d_1	e_1	$= \Delta T_m$
a_2	b_2	c_2	d_2	e_2	$= 0$
a_3	b_3	c_3	d_3	e_3	$= 0$
a_4	b_4	c_4	d_4	e_4	$= 0$
a_5	b_5	c_5	d_5	e_5	$= 0$

. . . . . . (23)

The values of a_1, a_2.... , b_1, b_2 .... etc. are given by corresponding terms in the original matrix of equations (22).

Expansion of determinant results in

$$D_1 = a_1 \begin{vmatrix} b_2 & c_2 & d_2 & e_2 \\ b_3 & c_3 & d_3 & e_3 \\ b_4 & c_4 & d_4 & e_4 \\ b_5 & c_5 & d_5 & e_5 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 & d_2 & e_2 \\ a_3 & c_3 & d_3 & e_3 \\ a_4 & c_4 & d_4 & e_4 \\ a_5 & c_5 & d_5 & e_5 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 & d_2 & e_2 \\ a_3 & b_3 & d_3 & e_3 \\ a_4 & b_4 & d_4 & e_4 \\ a_5 & b_5 & d_5 & e_5 \end{vmatrix}$$

$$- d_1 \begin{vmatrix} a_2 & b_2 & c_2 & e_2 \\ a_3 & b_3 & c_3 & e_3 \\ a_4 & b_4 & c_4 & e_4 \\ a_5 & b_5 & c_5 & e_5 \end{vmatrix} + e_1 \begin{vmatrix} a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \\ a_5 & b_5 & c_5 & d_5 \end{vmatrix}$$

The first term of D_1, which is a_1 multiplied by a fourth order determinant, is similarly expanded further until we have,

$$\text{I term} = a_1\Big[(b_2c_3 - c_2b_3)(d_4e_5 - e_4d_5) - (b_2d_3 - d_2b_3)(c_4e_5 - e_4c_5) + (b_2e_3 - e_2b_3)(c_4d_5 - d_4c_5) + (c_2d_3 - d_2c_3)(b_4e_5 - e_4b_5) - (c_2e_3 - e_2c_3)(b_4d_5 - d_4b_5) + (d_2e_3 - e_2d_3)(b_4c_5 - c_4b_5)\Big]$$

Proceeding likewise,

$$\text{II term} = -b_1\Big[(a_2c_3 - c_2a_3)(d_4e_5 - e_4d_5) - (a_2d_3 - d_2a_3)(c_4e_5 - e_4c_5) + (a_2e_3 - e_2a_3)(c_4d_5 - d_4c_5) + (c_2d_3 - d_2c_3)(a_4e_5 - e_4a_5) - (c_2e_3 - e_2c_3)(a_4d_5 - d_4a_5) + (d_2e_3 - e_2d_3)(a_4c_5 - c_4a_5)\Big]$$

$$\text{III term} = c_1\Big[(a_2b_3 - b_2a_3)(d_4e_5 - e_4d_5) - (a_2d_3 - d_2a_3)(b_4e_5 - e_4b_5) + (a_2e_3 - e_2a_3)(b_4d_5 - d_4b_5) + (b_2d_3 - d_2b_3)(a_4e_5 - e_4a_5) - (b_2e_3 - e_2b_3)(a_4d_5 - d_4a_5) + (d_2e_3 - e_2d_3)(a_4b_5 - b_4a_5)\Big]$$

$$\text{IV term} = -d_1\Big[(a_2b_3 - b_2a_3)(c_4e_5 - e_4c_5) - (a_2c_3 - c_2a_3)(b_4e_5 - e_4b_5) + (a_2e_3 - e_2a_3)(b_4c_5 - c_4b_5) + (b_2c_3 - c_2b_3)(a_4e_5 - e_4a_5) - (b_2e_3 - e_2b_3)(a_4c_5 - c_4a_5) + (c_2e_3 - e_2c_3)(a_4b_5 - b_4a_5)\Big]$$

$$\dot{V} \text{ term} = e_1 \Big[(a_2b_3 - b_2a_3)(c_4d_5 - d_4c_5) - (a_2c_3 - c_2a_3)(b_4d_5 - d_4b_5) + (a_2d_3 - d_2a_3)(b_4c_5 - c_4b_5) + (b_2c_3 - c_2b_3)(a_4d_5 - d_4a_5) - (b_2d_3 - d_2b_3)(a_4c_5 - c_4a_5) + (c_2d_3 - d_2c_3)(a_4b_5 - b_4a_5) \Big]$$

The final expression for the characteristic determinant is simplified further through elimination of some common terms and there results,

the characteristic determinant

$$D_1 = A + B + C + D + E + F + G + H + I + J \qquad \ldots \quad \ldots (24)$$

where,

$$A = (d_4e_5 - e_4d_5)\left\{a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)\right\}$$

$$B = -(c_4e_5 - e_4c_5)\left\{a_1(b_2d_3 - d_2b_3) - b_1(a_2d_3 - d_2a_3) + d_1(a_2b_3 - b_2a_3)\right\}$$

$$C = (c_4d_5 - d_4c_5)\left\{a_1(b_2e_3 - e_2b_3) - b_1(a_2e_3 - e_2a_3) + e_1(a_2b_3 - b_2a_3)\right\}$$

$$D = (b_4e_5 - e_4b_5)\left\{a_1(c_2d_3 - d_2c_3) - c_1(a_2d_3 - d_2a_3) + d_1(a_2c_3 - c_2a_3)\right\}$$

$$E = -(b_4d_5 - d_4b_5)\left\{a_1(c_2e_3 - e_2c_3) - c_1(a_2e_3 - e_2a_3) + e_1(a_2c_3 - c_2a_3)\right\}$$

$$F = (b_4c_5 - c_4b_5)\left\{a_1(d_2e_3 - d_3e_2) - d_1(a_2e_3 - e_2a_3) + e_1(a_2d_3 - d_2a_3)\right\}$$

$$G = -(a_4e_5 - e_4a_5)\left\{b_1(c_2d_3 - d_2c_3) - c_1(b_2d_3 - d_2b_3) + d_1(b_2c_3 - c_2b_3)\right\}$$

$$H = (a_4d_5 - d_4a_5)\left\{b_1(c_2e_3 - e_2c_3) - c_1(b_2e_3 - e_2b_3) + e_1(b_2c_3 - c_2b_3)\right\}$$

$$I = -(a_4c_5 - c_4a_5)\left\{b_1(d_2e_3 - e_2d_3) - d_1(b_2e_3 - e_2b_3) + e_1(b_2d_3 - d_2b_3)\right\}$$

$$J = (a_4b_5 - b_4a_5)\left\{c_1(d_2e_3 - e_2d_3) - d_1(c_2e_3 - e_2c_3) + e_1(c_2d_3 - d_2c_3)\right\}$$

The expression, though appearing quite complicated at first glance, is not so when one proceeds with actual calculations. In fact, if some assumptions are made while trying to obtain the characteristic determinant, the work can be much simplified and a simple form of characteristic determinant obtained. For example, if the components of voltages caused by the rates of change of flux linkages and of angle are neglected together with armature and tie line resistance, then the characteristic determinant obtained will have only fourth and lower powers of p in its final expression.(7)

Calculation of Steady State Quantities:

The starting point for calculation of initial steady-state values of various quantities will be the torque angle δ. Assuming a value for δ, the values of current I, machine internal voltage E_o and infinite bus voltage e will be calculated with the help of steady state vector diagram based on two reaction theory and taking into account saturation. In all the cases, a constant terminal voltage (achieved by voltage regulator) of 1.0 p.u. and unity power factor at the terminals will be assumed unless otherwise stated. Knowing the values of E_o, e and I at a particular angle, the other quantities such as i_{do}, i_{qo}, ψ_{do}, ψ_{qo}, etc. can be calculated.

Calculation of E_o, e and I:

Referring to phasor diagram of Fig.4(a), there are,

$$\tan(\delta-\theta) = \frac{x_q I}{1+r_m I} \text{ and } \tan\theta = \frac{I.x}{1-rI}$$

or

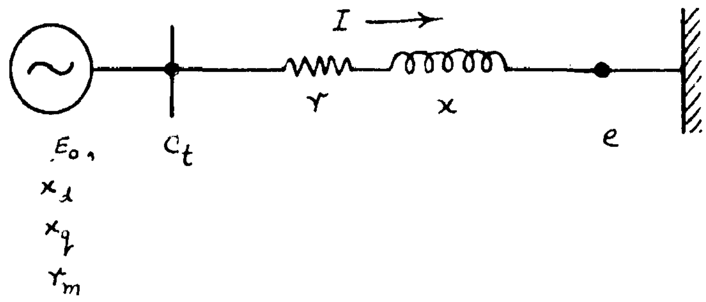
$$\frac{\tan\delta-\tan\theta}{1+\tan\delta\tan\theta} = \frac{x_q I}{1+r_m I}$$

or

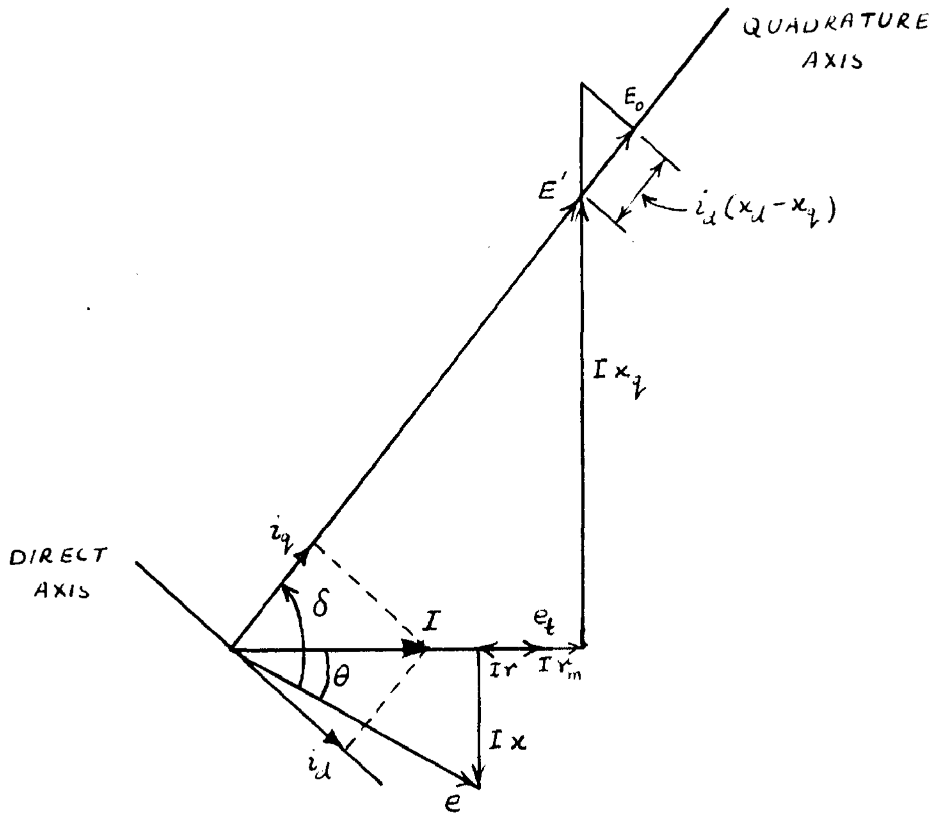
$$\frac{\tan\delta-\frac{Ix}{1-Ir}}{1+\tan\delta\frac{Ix}{1-Ir}} = \frac{Ix_q}{1+Ir_m}$$

or

$$\frac{\tan\delta - r.\tan\delta I - xI}{1 - Ir + x\tan\delta I} = \frac{Ix_q}{1+r_m I}$$



I
r
x
e
E_o,
x_d
x_q
r_m
e_t



QUADRATURE AXIS
E_o
E'
$i_d(x_d - x_q)$
$I x_q$
DIRECT AXIS
i_q
δ
I
e_t
θ
Ir
Ir_m
Ix
i_d
e

FIGURE 4 (a).

or

$$\tan\delta + r_m \tan\delta \cdot I - r \cdot \tan\delta \cdot I - r \cdot r_m \tan\delta I^2 - x \cdot I - x \cdot r_m \cdot I^2$$

$$= I \cdot x_q + x_q (x \cdot \tan\delta - r) \cdot I^2$$

or

$$\left\{x_q (x \tan\delta - r) + x \cdot r_m + r\, r_m \tan\delta\right\} I^2 + (x_q + x + r \cdot \tan\delta - r_m \tan\delta) I - \tan\delta = 0 \quad \ldots \quad \ldots (25)$$

For known values of x_q, x, r_m, r and δ equation (24) gives the value of current I.

Again from the vector diagram, [Fig.4(a)]

$$E' = \frac{I\, x_q}{\sin(\delta - \theta)} \quad (\theta \text{ is now known since I is known})$$

and

$$E_o = E' + i_d (x_d - x_q), \quad i_d = I \sin(\delta - \theta) \quad \ldots (26)$$

Also,

$$e = \frac{Ix}{\sin\theta} \quad \ldots \quad \ldots (27)$$

Calculation of i_{do}, i_{qo}, ψ_{do}, ψ_{qo}:

It is assumed that values of i_{do} and i_{qo} are affected by line resistance and reactance in addition to machine resistance and reactance. But the values of e_{do}, e_{qo} and ψ_{do} and ψ_{qo} are assumed to remain unaffected by line parameters. Thus proceeding as in Chapter 2, the values of i_{do} and i_{qo} are now modified by r and x and given by,

$$\left.\begin{aligned} i_{do} &= \frac{-(r_m + r) e \sin\delta - (x + x_q)(e \cos\delta - E_o)}{(r_m + r)^2 + (x + x_d)(x + x_q)} \\ i_{qo} &= \frac{(x + x_d) e \sin\delta - (r_m + r)(e \cos\delta - E_o)}{(r_m + r)^2 + (x + x_d)(x + x_q)} \end{aligned}\right\} \quad \ldots \quad \ldots (28)$$

$$\left.\begin{aligned} \psi_{do} &= E_o - x_d\, i_{do} \\ \psi_{qo} &= -x_q\, i_{qo} \end{aligned}\right\} \quad \ldots \quad \ldots (29)$$

$$\left.\begin{aligned} e_{qo} &= \psi_{do} - r_m\, i_{qo} = e'_q \\ e_{do} &= -\psi_{qo} - r_m i_{do} = e'_d \end{aligned}\right\} \qquad \ldots \quad \ldots (30)$$

since e_a is assumed to be unity.

Expressions for $x_d(p)$, $x_q(p)$, $G(p)$ and $g(p)$:

When damper windings are neglected, the operational impedances are given by:

$$x_d(p) = \frac{x_d\,(1 + T'_d p)}{1 + T'_{do} p}\,, \quad x_q(p) = x_q$$

$$G(p) = \frac{1}{1 + T'_{do} p}$$

If it is assumed that the regulator and exciter performance can be expressed as a static change (a) in field voltage E_o per unit change in terminal voltage e_a, together with a time lag expressible as an equivalent single time constant T_r, then

$$g(p) = \frac{a}{1 + T_r p}$$

where the value of a (taken negative) has to be coordinated properly with a particular system.

3.2.2. Machine directly connected to infinite bus bar:

The equations of small displacements in this case, too, remain unchanged for the machine i.e. the equations (19), (20) and (21) of preceding section are applicable again. The only difference is that equations (12) and (13) are modified to give:

$$\text{and} \quad \left.\begin{aligned} \Delta e_d &= e \cos\delta . \Delta\delta \\ \Delta e_q &= -e \sin\delta . \Delta\delta \end{aligned}\right\} \qquad \ldots \quad \ldots (31)$$

which are obtained easily by putting $r = x = Z(p) = 0$ in equations (12) and (13).

The matrix form of the equations is:

$$
\begin{array}{ccccc|c}
\Delta e_d & \Delta e_q & \Delta i_d & \Delta i_q & \Delta\delta & \\
\hline
i_{qo}G(p)g(p)e_d' & i_{qo}G(p)g(p)e_q' & -\psi_{qo}-i_{qo}x_d(p) & \psi_{do}+i_{do}x_q(p) & p^2M & =\Delta T_m \\
pG(p)g(p)e_d'-1 & pG(p)g(p)e_q' & -Z_d(p) & x_q(p) & -\psi_{qo}p & =0 \\
G(p)g(p)\,e_d' & G(p)g(p)e_q'-1 & -x_d(p) & -Z_q(p) & \psi_{do}p & =0 \\
-1 & 0 & 0 & 0 & e\cos\delta & =0 \\
0 & -1 & 0 & 0 & -e\sin\delta & =0
\end{array}
$$

... ... (32)

Again, the stability can be tested by applying Routh's criterion to the characteristic determinant of the above equations.

Characteristic equation:

Referring to equations (23) of the preceding section, there is, in this case,

$$b_4 = c_4 = d_4 = 0 = a_5 = c_5 = d_5$$

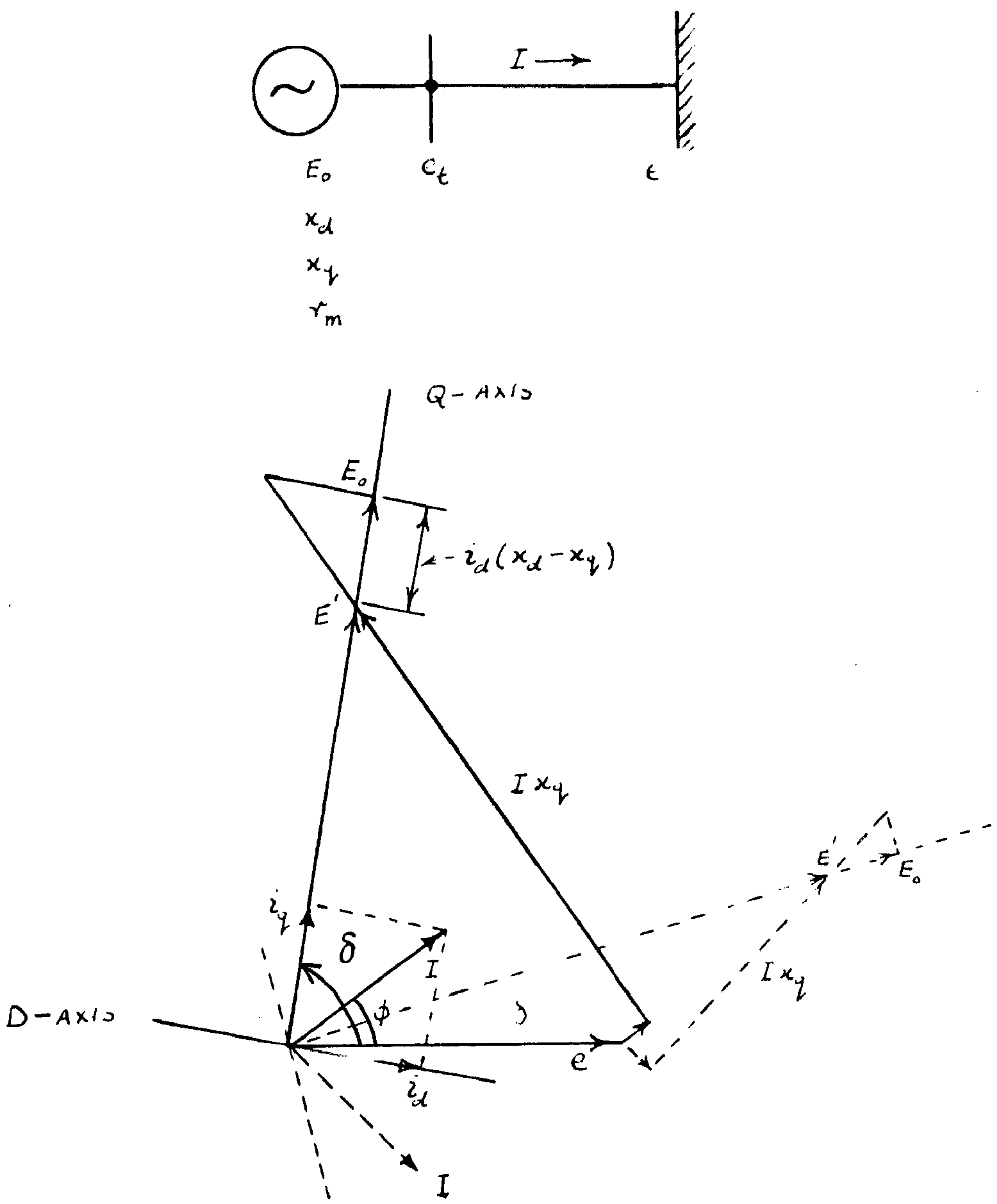
and

$$a_4 = b_5 = -1$$

If these values are substituted in the expression for characteristic equation obtained in the preceding section, there results,

$$F(p) = A_1 + B_1 + C_1 = 0 \qquad \dots \quad \dots (33)$$

which is the characteristic equation in this case. The terms A_1, B_1, C_1 are given by,



—— FOR LEADING POWER FACTOR

- - - - FOR LAGGING POWER FACTOR

FIGURE 4 (b).

$$A_1 = e_4 \left\{ a_1(c_2d_3 - d_2c_3) - c_1(a_2d_3 - d_2a_3) + d_1(a_2c_3 - c_2a_3) \right\}$$

$$B_1 = e_5 \left\{ b_1(c_2d_3 - d_2c_3) - c_1(b_2d_3 - d_2b_3) + d_1(b_2c_3 - c_2b_3) \right\}$$

$$C_1 = a_4b_5 \left\{ c_1(d_2e_3 - e_2d_3) - d_1(c_2e_3 - e_2c_3) + e_1(c_2d_3 - d_2c_3) \right\}$$

Calculation of initial steady-state values:

The value of E_o and I can be calculated as shown in the preceding section for a given value of δ. The vector diagram, however, is now modified as shown in Fig.4(b), since the value of e is now fixed at unity. It is to be noted that the power factor may not be held at unity in all the cases, but may have to be changed from lagging to unity and then to leading in order to keep E_o at a suitable value - particularly at higher values of δ. The examples considered in latter sections will illustrate this requirement.

The steady-state values of currents, voltages and flux linkages in the machine are given by-

$$\left.\begin{aligned} i_{do} &= \frac{-r_m\, e \sin\delta - x_q\,(e\cos\delta - E_o)}{r_m^2 + x_d\, x_q} \\ i_{qo} &= \frac{x_d\, e \sin\delta - r_m\,(e\cos\delta - E_o)}{r_m^2 + x_d\, x_q} \end{aligned}\right\} \qquad \ldots \quad \ldots (34)$$

$$\left.\begin{aligned} \psi_{do} &= E_o - x_d\, i_{do} \\ \psi_{qo} &= -x_q\, i_{qo} \end{aligned}\right\} \qquad \ldots \quad \ldots (35)$$

$$\left.\begin{aligned} e_{qo} &= \psi_{do} - r_m\, i_{qo} = e_q' \\ e_{do} &= -\psi_{qo} - r_m\, i_{do} = e_d' \end{aligned}\right\} \qquad \ldots \quad \ldots (36)$$

since $e_a = 1.0$ p.u.

3.3. Sample Calculations:

In this section a few calculations are shown for a particular machine under the effect of voltage regulator. For various values of torque angle δ , characteristic equations will be obtained to which Routh's criterion will be applied to test system stability.

The following values for machine constants and tie line resistance and reactance are assumed.

$x_d = 1.2$

$x_q = 0.8$

$x_{ad} = 1.0$

$x_l = 0.2$

$x'_d = 0.3$

$r_m = 0.02$

$T'_{do} = 5$ sec.

$T'_d = 1.25$ sec.

$T_r = 2$ sec.

$a = +2$

$x = 0.3$

$r = 0.06$

3.3.1. Machine connected to infinite bus through tie line:

(a) $\delta_o = 45^o$, p.f. = 1.0

Following the method discussed in section 3.2.1., the values of E_o, I and e are obtained as-

$I = 0.78$, $E_o = 1.37$, $e = 0.98$

o,

$i_{do} = 0.423$ $\qquad$ $i_{qo} = 0.665$

$\psi_{do} = 0.86$ $\qquad$ $\psi_{qo} = -0.53$

$e_{do} = e'_d = 0.53$ $\qquad$ $e_{qo} = e'_q = 0.85$

Proceeding with the calculations as shown in section .1., the exact characteristic equation is

$$p^9+14.92p^8+422p^7+734p^6+957p^5+872p^4+494p^3+151.7p^2+26.6p+1.846=0$$

th's array is set up as,

6.6	422	958	494	26.6
14.92	734	872	151.7	1.846
74	572	427	25.85	0
618	786	146.5	1.846	0
478	409.4	25.63	0	0
259	113.5	1.846	0	0
200	22.23	0	0	0
84.7	1.846	0	0	0
17.88	0	0	0	0
1.846	0	0	0	0
0	0	0	0	0

e the system is stable

(b) The operating conditions are:

. $\delta_0 = 90^\circ$, $\quad$ p.f. $= 1.0$

values of I, E_o and e are obtained as

$I = 1.906$, $\qquad E_o = 2.44$, $\qquad e = 1.05$

$i_{do} = 1.57$ $\qquad$ $i_{qo} = 1.07$

$\psi_{do} = 0.56$ $\qquad$ $\psi_{qo} = -0.86$

$e_{do} = e'_d = 0.83$ $\qquad$ $e_{qo} = e'_q = 0.54$

The system characteristic equation is:

$$6.6p^9+14.3p^8+416p^7+507p^6+607.8p^5+474p^4+228p^3+47p^2+13.3p+0.848=0$$

Routh's array is:

6.6	416	607	228	13.3
14.3	507	474	47	0.848
192	390	202	13.0	0
478	460	46	0.848	0
206	192	13	0	0
19	16	0.848	0	0
28	4	0	0	0
13.3	0.848	0	0	0
2.2	0	0	0	0
0.848	0	0	0	0
0	0	0	0	0

The system is, therefore, stable.

(c) $\delta_o = 100^\circ$, $\qquad$ p.f. = 1.0

Values of other steady-state quantities are:

$I = 2.34$ $\qquad$ $E_o = 2.957$ $\qquad$ $e = 1.11$

$i_{do} = 2.04$ $\qquad$ $i_{qo} = 1.14$

$\psi_{do} = 0.50$ $\qquad$ $\psi_{qo} = -0.91$

$e_{do} = e'_d = 0.98$ $\qquad$ $e_{qo} = e'_q = 0.477$

and the characteristic equation is-

$$6.6p^9+14.55p^8+332p^7+330p^6+326p^5+200p^4+63p^3+29.8p^2-2.6p-0.707=0$$

The Routh's array is:

6.6	332	326	63	-2.6
14.55	330	200	29.8	-0.707
181	236	49.5	-2.17	0
311	196	29.97	-0.707	0
122	32.1	-1.76	0	0
111.5	34.47	-0.707	0	0
-5.4	-1.0	0	0	0
-13.87	-0.707	0	0	0
-1.275	0	0	0	0
-0.707	0	0	0	0

Hence the system is unstable and there is one root of the characteristic equation with positive real part.

3.3.2. Machine directly connected to infinite bus:

The same machine is now assumed to be connected to infinite bus bar directly. The stability of the system under the effect of voltage regulator at different operating torque angles is investigated in this section:

(a) $\delta_o = 45^\circ$, $\qquad$ p.f. = 1.0

The initial values are:

$I = 1.28$, $\qquad$ $E_0 = 1.73$ $\qquad$ $e = 1.0$ p.u.

$i_{do} = 0.845$ $\qquad$ $i_{qo} = 0.855$

$\psi_{do} = 0.72$ $\qquad$ $\psi_{qo} = -0.682$

$e_{do} = e'_d = 0.665$ $\qquad$ $e_{qo} = e'_q = 0.703$

The system characteristic equation is:

$$0.24p^7+0.368p^6+27.11p^5+31.28p^4+39.3p^3+31.4p^2+10p+1.051=0$$

Routh's array is:

0.24	27.11	39.3	10
0.368	31.28	31.4	1.051
6.7	18.8	9.32	0
30.25	30.9	1.051	0
12.0	9.08	0	0
8.1	1.051	0	0
7.52	0	0	0

The operating point is, therefore, stable.

(b) $\delta_o = 90^\circ$, p.f. = 0.9 leading.

The steady-state quantities are:

$I = 2.87$ $\qquad$ $E_o = 3.1$ $\qquad$ $e = 1.0$

$i_{do} = 2.55$ $\qquad$ $i_{qo} = 1.31$

$\psi_{do} = 0.04$ $\qquad$ $\psi_{qo} = -1.05$

The characteristic equation in this case is:

$$0.24p^7+0.378p^6+29.37p^5+16.7p^4+27.5p^3+15p^2+0.15p-0.353 = 0$$

Routh's array is set up as:

0.24	29.37	27.5	0.15
0.378	16.7	15	-0.353
18.7	18	0.375	0
16.4	15	-0.353	0
0.9	0.775	0	0
0.9	0.353	0	0
1.128	0	0	0
-0.353	0	0	0
0	0	0	0

Hence the system is unstable, there being one root of the characteristic equation with positive real part.

(c) $\delta_o = 100^\circ$ p.f. = 0.8 leading

The initial values are:

$I = 2.73$ $E_o = 2.79$ $e = 1.0$

$i_{do} = 2.44$ $i_{qo} = 1.29$

$\psi_{do} = -0.14$ $\psi_{qo} = -1.03$

The characteristic equation is-

$$0.24p^7 + 0.368p^6 + 23.0p^5 + 8.94p^4 + 19.2p^3 + 5.8p^2 - 3.3p - 0.493 = 0$$

The system is seen to be unstable, there being one root with real positive part.

Chapter 4.

FREQUENCY RESPONSE ANALYSIS OF LINEARIZED SYSTEMS

FREQUENCY RESPONSE ANALYSIS OF LINEARIZED SYSTEMS

4.1. Introduction

A more complete picture of a synchronous machine stability problem is obtained by frequency response method which can be considered as a step ahead of the analytical methods discussed earlier. The method is based upon a rigrous analysis of synchronous machine dynamics; both electrical as well as mechanical. The characteristic feature of this method is the realization of a synchronous machine as a closed loop configuration as shown later and then the application of Nyquist criterion to test stability of the system. Since some kind of feed-back is always necessary to make a system stable, the necessity of representing a synchronous machine as a closed loop system is obvious. The simplest closed loop system may consist of one main loop or forward loop and one feed-back loop as shown in Fig.5. In the case of a synchronous machine these are obtained after studying the equations of motion available as a result of applying small displacement theory to actual machine equations.

It may be pointed out here that while the Routh's criterion or other criteria determine only whether a particular system is stable or not, the frequency response method not only does so, but enables one to know to what degree the system is stable. Hence the importance of the method. Thus, the main advantage of the method lies, not in being able to tackle the problem easily but in the fact that it provides for rapid appreciation of the effects of modification of the machine equation.

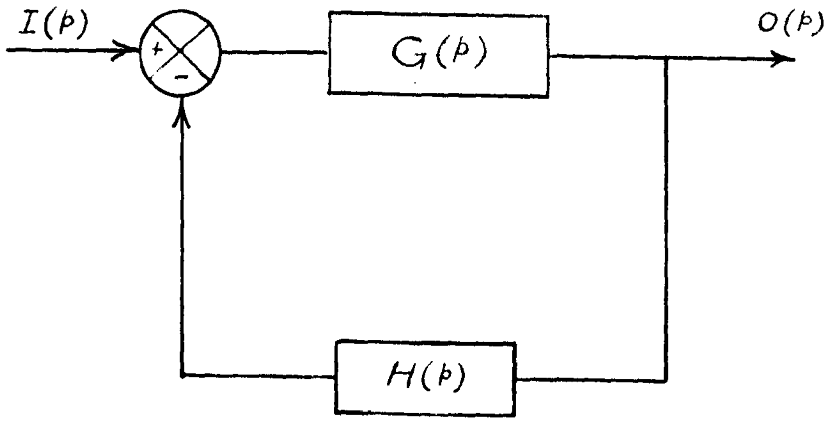


FIGURE 5.

4.2. The Nyquist Criterion of Stability:[5]

Consider a closed-loop feed-back control system as shown in Fig.5. If $G(p)$ is the forward loop transfer function and $H(p)$ is the feed-back transfer function, the closed-loop transfer function of the system is given by:

$$\frac{O(p)}{I(p)} = \frac{G(p)}{1+G(p).H(p)} \qquad \ldots \qquad \ldots (1)$$

Let the denominator of equation (1) be represented by $F(p)$; that is,

$$F(p) = 1 + G(p)\ H(p)$$

The zeros of $F(p)$ are the roots of the characteristic equation of the system. If this equation has any roots with positive real parts, the system will have an infinitely increasing response to a finite input or in other words, the system will be unstable.

Now consider the locus defined by the semi-circle of radius $R = \infty$ an right hand side of a p-plane having real and imaginary axes. This locus is described in a counter-clockwise direction so that it ENCLOSES the entire finite right half of the p-plane; this path is called the NYQUIST PATH. If any of the roots of the characteristic equation is enclosed by this Nyquist path, then system is unstable. It will be shown how this can be realized exactly:

Let Z = number of zeros of the characteristic equation enclosed by Nyquist path.

P = number of poles of the characteristic equation enclosed by Nyquist path (this is also the number of poles of the open loop transfer function $G(p)\ H(p)$.)

Now, the determination of Z is our main concern while P is usually known and for stable system is equal to zero. Referring to the Nyquist path, a corresponding plot of the function $1 + G(p)\,H(p)$ can be obtained for various values of p. For this, travel along the Nyquist path in p-plane in the counter-clockwise direction and plot the corresponding values of F(p) along real and imaginary axes in the F-plane. Then, the number by which F(p) plot ENCLOSES the origin of F-plane determines Z by the relation:

$$N = Z - P$$

where N is the number of enclosures.

Now, since for a stable system P is always zero and Z has to be zero, N must be zero too. This means that 'F(p) plot must <u>not</u> enclose the origin at all for a stable closed loop system'. This apparently is, what isknown as, Nyquist Criterion of Stability.

Usually, the plot of $G(p)\,H(p)$, i.e. open loop transfer function, is obtained instead of $1 + G(p)\,H(p)$ or F(p) and the critical point then becomes $(-1, j0)$ instead of the origin.

4.3. <u>Closed-loop System for a Synchronous Machine:</u>

In Chapter 2, the dynamical equation of motion of a synchronous machine is derived to be

$$\Delta T_m = M\,p^2 \Delta\delta + f(p)\Delta\delta$$

This can be arranged in two ways to give two different closed-loop systems.

(1) $$\frac{\Delta T_m - f(p)\Delta\delta}{Mp^2} = \Delta\delta$$

i.e. the difference of the two inputs $-\Delta T_m$ and $f(p)\Delta\delta$ - operated upon by a direct transfer function of $\frac{1}{Mp^2}$ gives

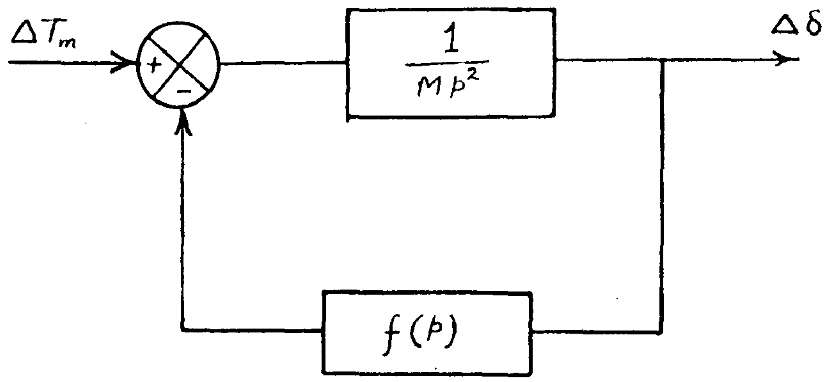


FIGURE 6 (a).

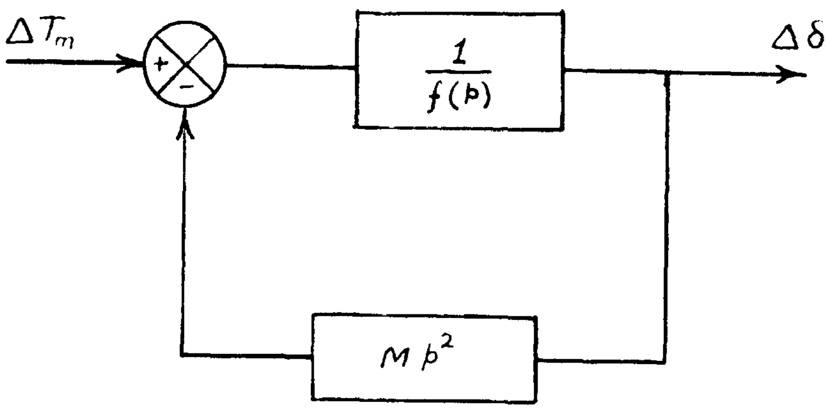


FIGURE 6 (b).

the output of small change in rotor angle $\Delta\delta$.

The system will be as shown in Fig.6 (a)

(ii) $$\frac{\Delta T_m - Mp^2\Delta\delta}{f(p)} = \Delta\delta$$

Here the inputs are ΔT_m and $Mp^2\Delta\delta$ and the direct transfer function is $\frac{1}{f(p)}$. The system is shown in Fig.6(b). The analysis to follow makes use of first representation.

4.4. <u>The Open Loop Transfer Function</u>:

The small displacement equation of electro-mechanical torque, referring to equation (21) of Chapter 3, section 2.1, is:

$$\Delta T_m = i_{qo}\, G(p)\; g(p)\; e_d'\,\Delta e_d + i_{qo} G(p)\; g(p)\; e_q' \Delta e_q -$$
$$\{\psi_{qo} + i_{qo}\, x_d(p)\}\Delta i_d + \{\psi_{do} + i_{do}\, x_q(p)\}\Delta i_q + p^2 M\Delta\delta$$

If $p^2M\Delta\delta$ term is exclused, we have the electric torque ΔT_e given by:

$$\Delta T_e = i_{qo}\, G(p)\; g(p)\; e_d'\,\Delta e_d + i_{qo}\, G(p)\; g(p)\; e_q'\Delta e_q -$$
$$\{\psi_{qo} + i_{qo}\, x_d(p)\}\Delta i_d + \{\psi_{do} + i_{do}\, x_q(p)\}\Delta i_q \qquad \ldots \quad \ldots (2)$$

which can be obtained in a form,

$$\Delta T_e = f(p)\cdot\Delta\delta \qquad \ldots \quad \ldots (3)$$

making use of voltage and current small displacement equations.

Thus, if in equations (22) or (32) of Chapter 3, (depending whether the machine is connected to infinite bus through tie line or directly), e_1 is put equal to zero instead of p^2M, then the determinant of the coefficients of these equations gives the expressions for $f(p)$. Dividing $f(p)$ by Mp^2, the open loop transfer function is obtained.

4.5. Open Loop Transfer Functions for Cases Studied and the Nyquist Plots:

The open loop transfer functions for the cases studied in Chapter 3 by applying Routh's criterion will now be obtained together with their Nyquist Plots.

4.5.1. Machine connected to infinite bus through tie-line: (considering effect of voltage regulator)-

(a) $\delta = 45^{\circ}$,

Following the procedure outlined in section 4.4, the open loop transfer function is obtained as,

$$\frac{f(p)}{Mp^2} = \frac{402p^7 + 717p^6 + 942p^5 + 867p^4 + 473p^3 + 151.7p^2 + 26.55p + 1.846}{.02p^2\,(1+5p)^3\,(1+2p)^2}$$

$$= \frac{402p^7 + 717p^6 + 942p^5 + 867p^4 + 473p^3 + 151.7p^2 + 26.55p + 1.846}{10p^7 + 16p^6 + 9.7p^5 + 2.8p^4 + 0.38p^3 + 0.02p^2}$$

By applying Routh's criterion, it can be shown that the open loop is stable i.e. $Z = 0$, and since $P = 0$ too, for a closed loop stable system N must be zero. This means that the Nyquist plot must not enclose $(-1, j0)$ point.

Writing $p = j\omega$,

$$\frac{f(j\omega)}{-\omega^2} = G(j\omega)\,H(j\omega)$$

$$= \frac{(717\omega^6 - 867\omega^4 + 151.7\omega^2 - 1.846) + j(402\omega^7 - 942\omega^5 + 473\omega^3 - 26.55\omega)}{(16\omega^6 - 2.8\omega^4 + 0.02\omega^2) + j(10\omega^7 - 9.7\omega^5 + 0.38\omega^3)}$$

At $\omega = 0$,

$$\frac{f(j\ )}{-\omega^2} = \underset{\omega\to 0}{\frac{-1.846}{.02\omega^2}} = -\infty$$

At $\omega = \infty$,

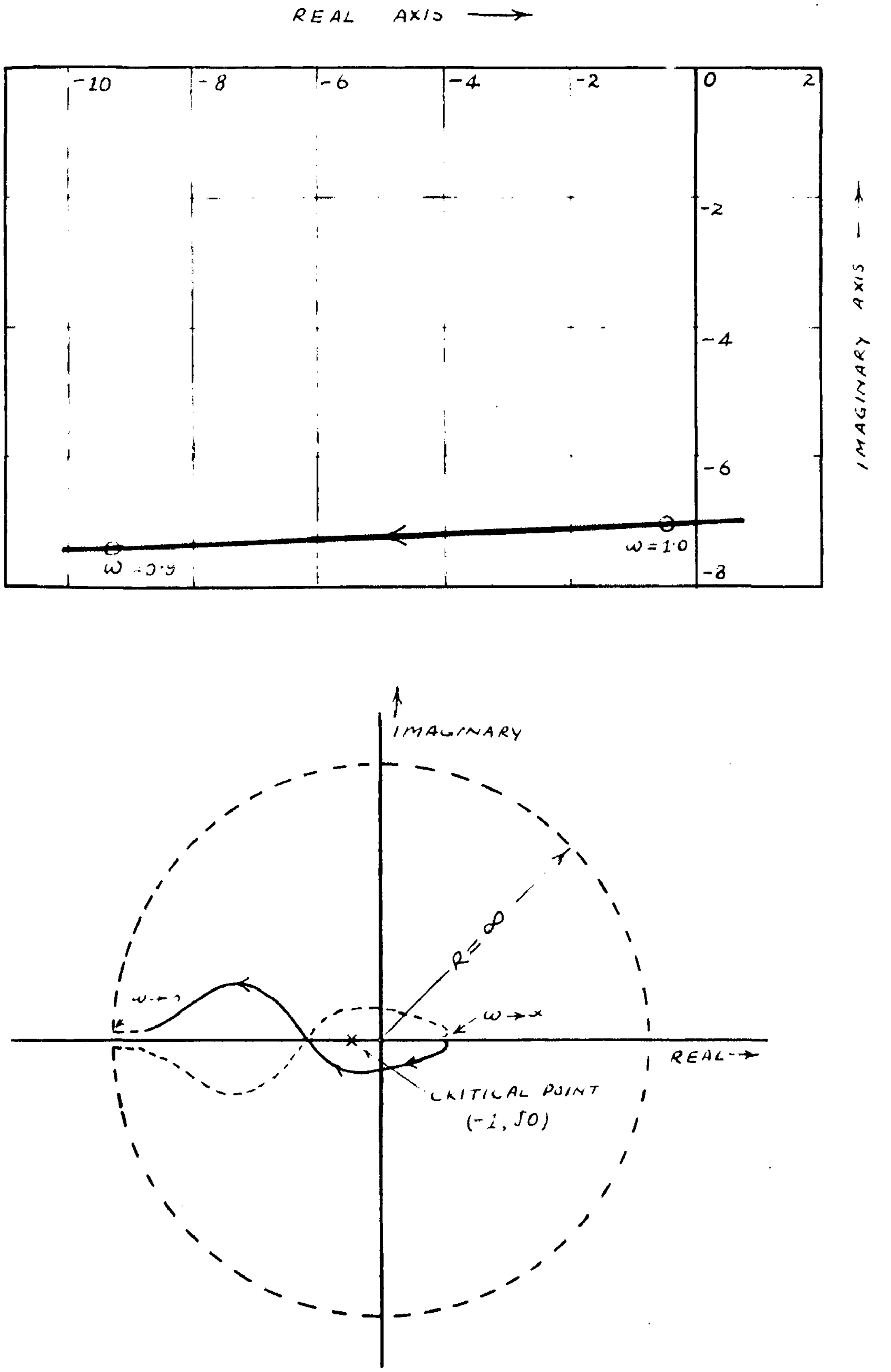


FIGURE 7. MACHINE CONNECTED THROUGH TIE LINE, $\delta = 45^{\circ}$

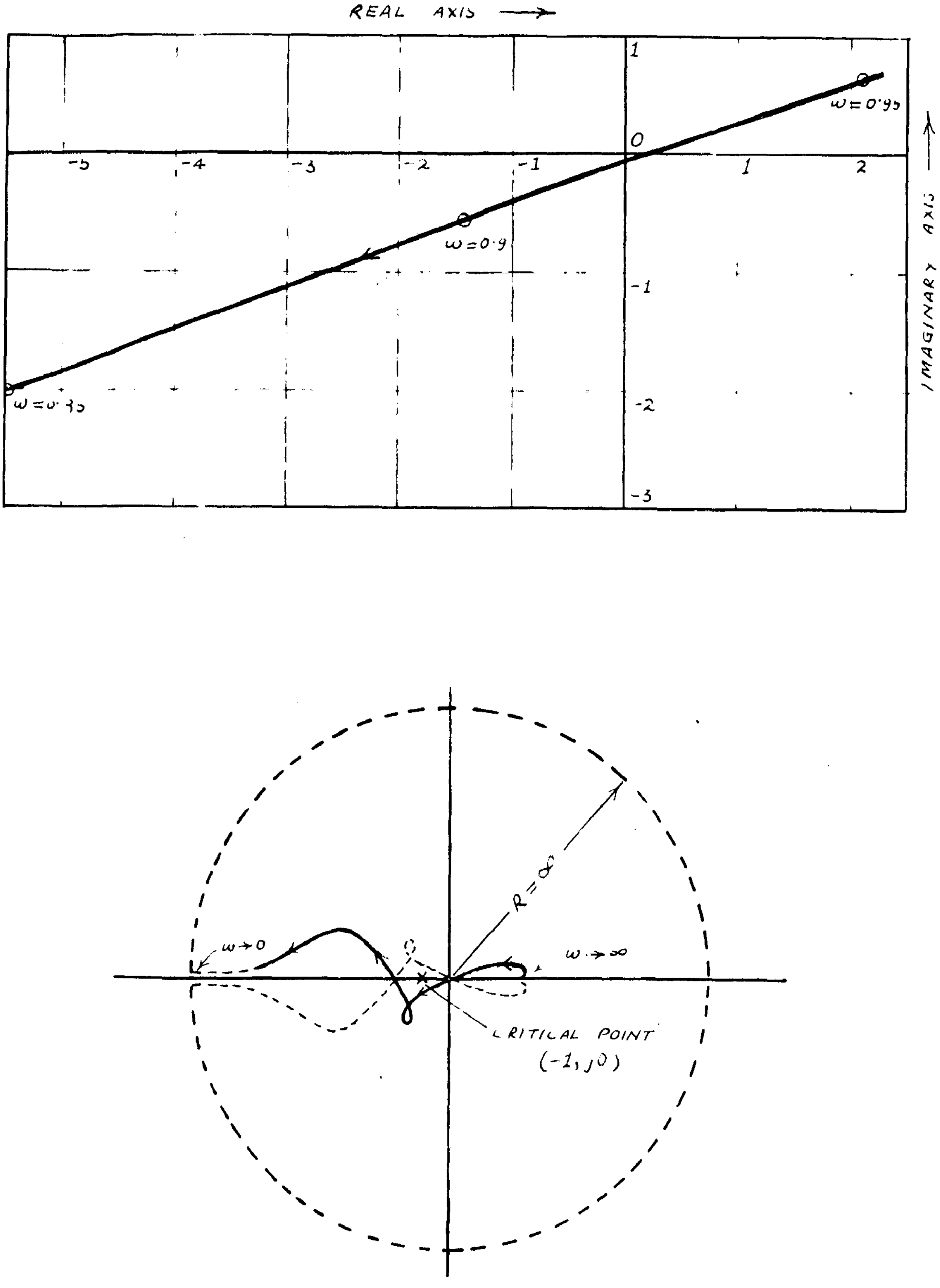


FIGURE 8. MACHINE CONNECTED THROUGH TIE LINE, $\delta = 90^{\circ}$

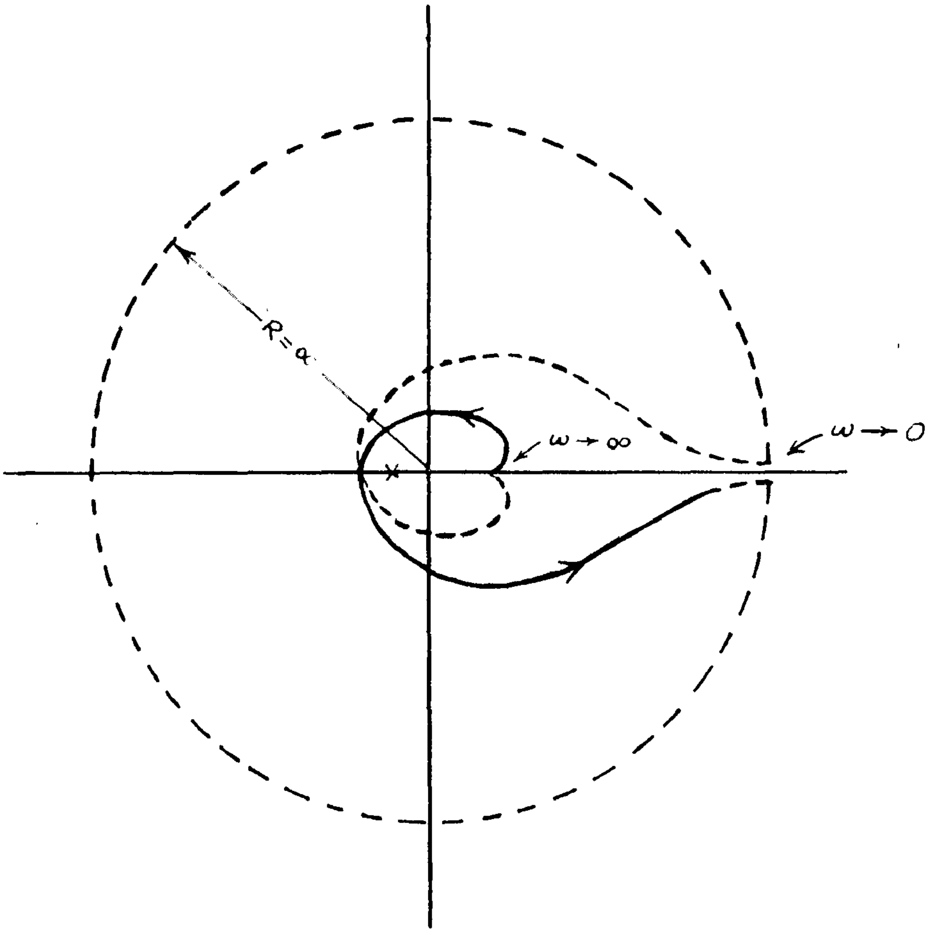


FIGURE 9. MACHINE CONNECTED THROUGH TIE LINE

$\delta = 100^\circ$

$$\frac{f(j\omega)}{-\omega^2} = \frac{402}{10} = 40.2$$

The values of $G(j\omega)\ H(j\omega)$ for other values of ω are listed in table 1 obtained with the help of a digital computer. The Nyquist plot is given in Fig. 7(a) and (b). Fig. (a) shows enlarged portion of the plot near the critical point (-1, jo) whereas Fig. (b) shows general shape of the plot. It can be seen that the plot does not enclose the critical point and hence the system is stable.

(b) $\delta = 90^{o}$

Here $\frac{f(p)}{Mp^2} = G(p)H(p) =$

$$= \frac{401p^7 + 494p^6 + 592p^5 + 472p^4 + 229p^3 + 47p^2 + 13.3p + .848}{.02p^2\ (1+5p)^3\ (1+2p)^2}$$

and $$G(j\omega)H(j\omega) = \frac{(494\omega^6 - 472\omega^4 + 47\omega^2 - 0.848) + j(401\omega^7 - 592\omega^5 + 229\omega^3 - 13.3\omega)}{(16\omega^6 - 2.8\omega^4 + 0.02\omega^2) + j(10\omega^7 - 9.7\omega^5 + 0.38\omega^3)}$$

$$\lim_{\omega \to 0} G(j\omega)H(j\omega) = -\infty$$

and $$\lim_{\omega \to \infty} G(j\omega)H(j\omega) = 40.1$$

The Nyquist plot is shown in Fig.8 from which it is seen that the critical point is not enclosed and the system is stable.

(c) $\delta = 100^{o}$

$$\frac{f(p)}{Mp^2} = G(p)H(p)$$

$$= \frac{316p^7 + 321p^6 + 317p^5 + 187p^4 + 56p^3 + 37.5p^2 - 2.91p - 0.29}{.02p^2\ (1+5p)^3\ (1+2p)^2}$$

$$G(j\omega)H(j\omega) = \frac{(321\omega^6 - 187\omega^4 + 37.5\omega^2 + 0.29) + j(316\omega^7 - 317\omega^5 + 56\omega^3 + 2.91\omega)}{(16\omega^6 - 2.8\omega^4 + 0.02\omega^2) + j(10\omega^7 - 9.7\omega^5 + 0.38\omega^3)}$$

$$\lim_{\omega \to 0} G(j\omega)H(j\omega) = +\infty$$

$$\lim_{\omega \to \infty} G(j\omega)\, H(j\omega) = 31.6$$

In this case, applying Routh's criterion, it is seen that the system has one root with real positive part, i.e., Z = 1. Therefore P being zero, N must be one or the Nyquist plot must enclose the critical point once in order that the system be stable. However, the actual plot shows that N=2 and hence the system is unstable. The plot is shown in Fig.9

4.5.2. Machine directly connected to infinite bus:

Equations (32) of Chapter 3 are used with $e_1 = 0$ to obtain open loop transfer functions in each case.

(a) $\delta = 45^o$,

The open loop transfer function is given by,

$$\frac{f(p)}{Mp^2} = \frac{26.7p^5 + 30.9p^4 + 39.1p^3 + 31.4p^2 + 10\,p + 1.051}{0.02\,p^2\,(1+5p)^2\,(1+2p)}$$

and $$\frac{f(j\omega)}{-\omega^2} = G(j\omega)H(j\omega) = \frac{(30.9\omega^4 - 31.4\omega^2 + 1.051) + j(26.7\omega^5 - 39.1\omega^3 + 10\omega)}{(0.9\omega^4 - 0.02\omega^2) + j(\omega^5 - 0.36\omega^3)}$$

$$\lim_{\omega \to 0} G(j\omega)\, H(j\omega) = \frac{1.051}{-0.02\omega^2} = -\infty$$

$$\lim_{\omega \to \infty} G(j\omega)\, H(j\omega) = 26.7$$

The Nyquist plot is shown in Fig.10. It does not enclose the critical point and hence the system is stable.

(b) $\delta = 90^o$

System open loop transfer function is,

$$\frac{f(p)}{Mp^2} = \frac{28.95p^5 + 17p^4 + 26.7p^3 + 15\,p^2 + 0.2p - 0.353}{.02p^2\,(1+5p)^2\,(1+2p)}$$

$$= \frac{28.95p^5 + 17p^4 + 26.7p^3 + 15p^2 + 0.2p - 0.353}{p^5 + 0.9p^4 + 0.36p^3 + 0.02p^2}$$

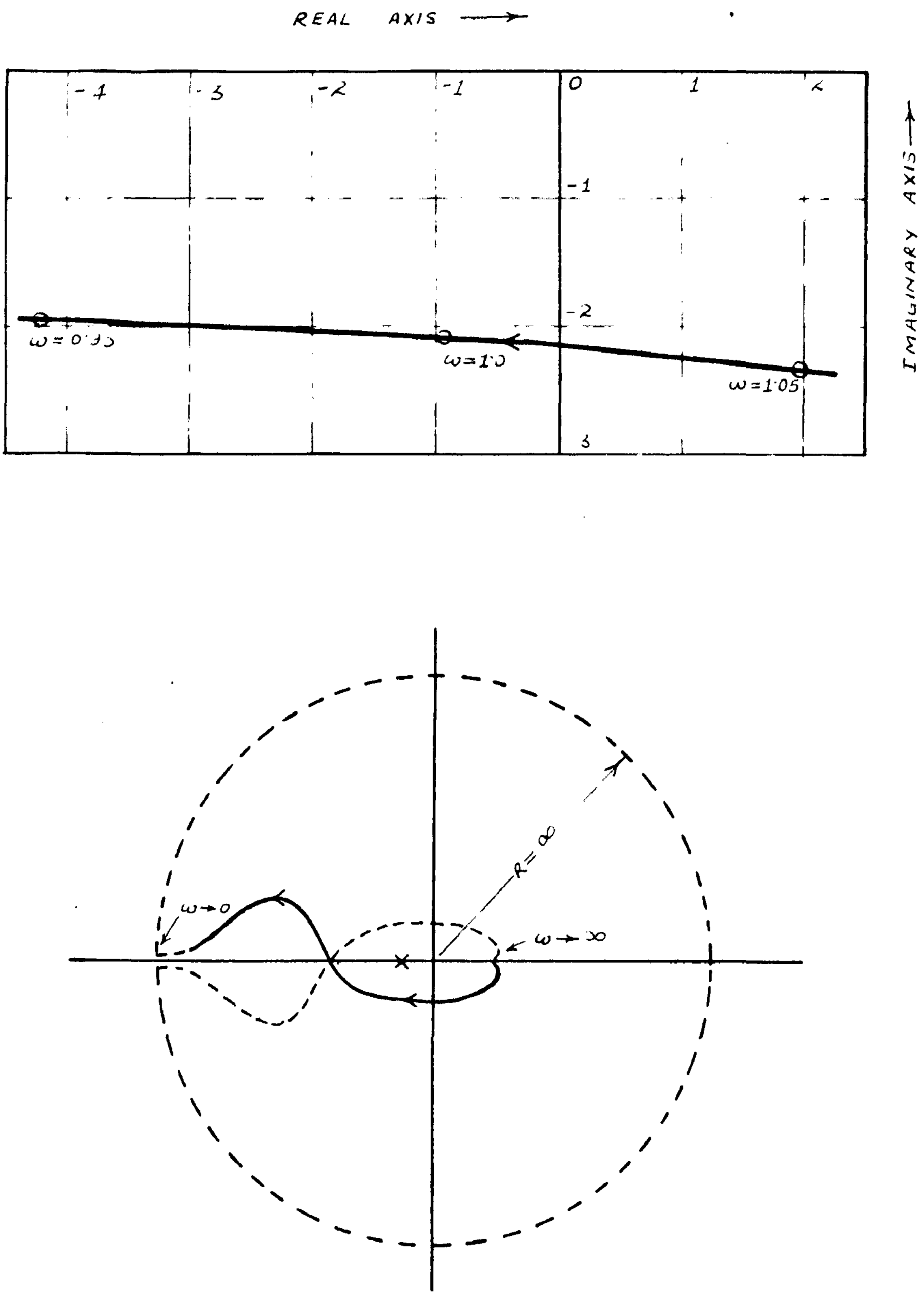


FIGURE 10. MACHINE DIRECTLY CONNECTED, $\delta = 45^\circ$

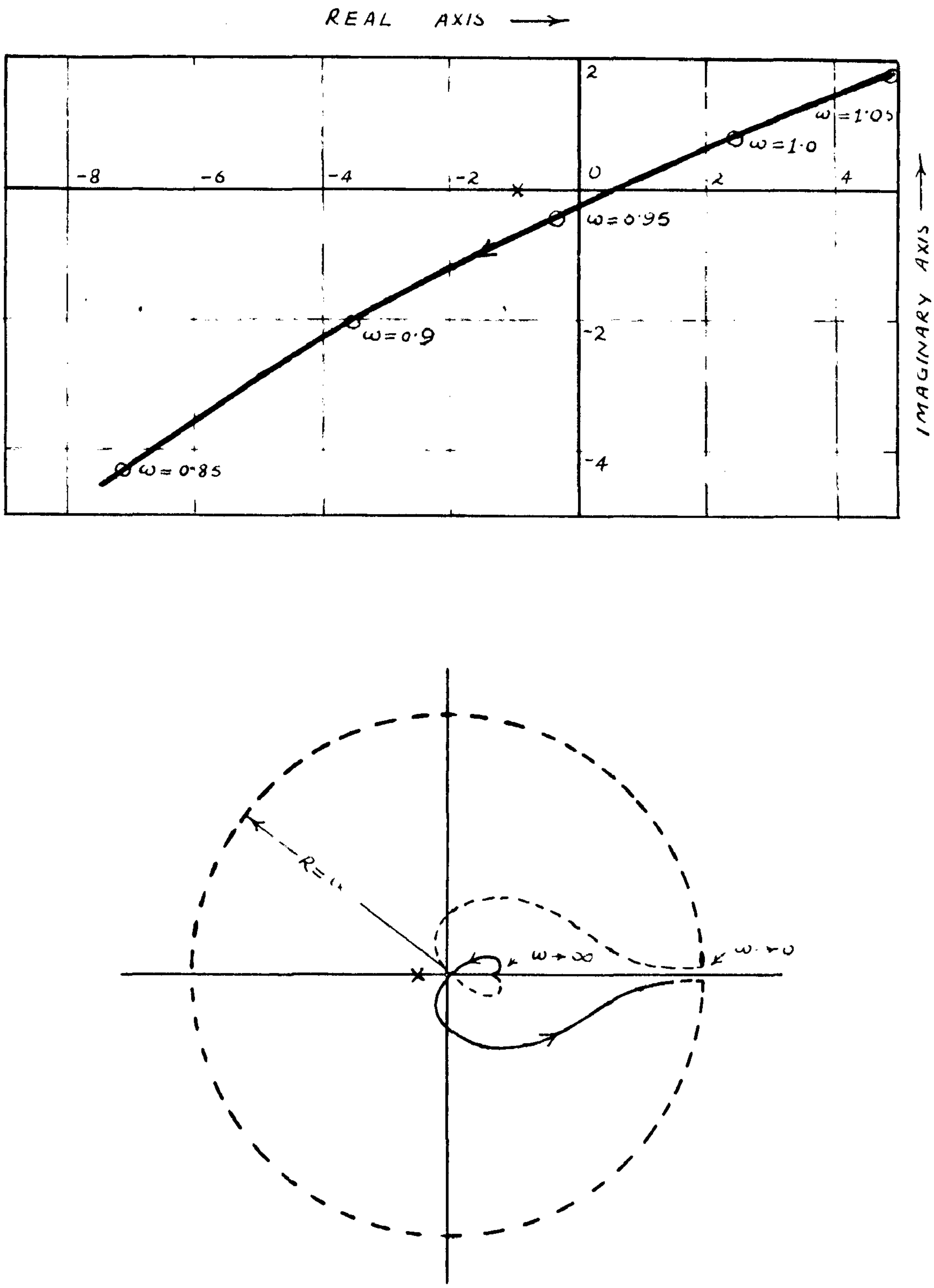


FIGURE 11. MACHINE DIRECTLY CONNECTED, δ = 90°.

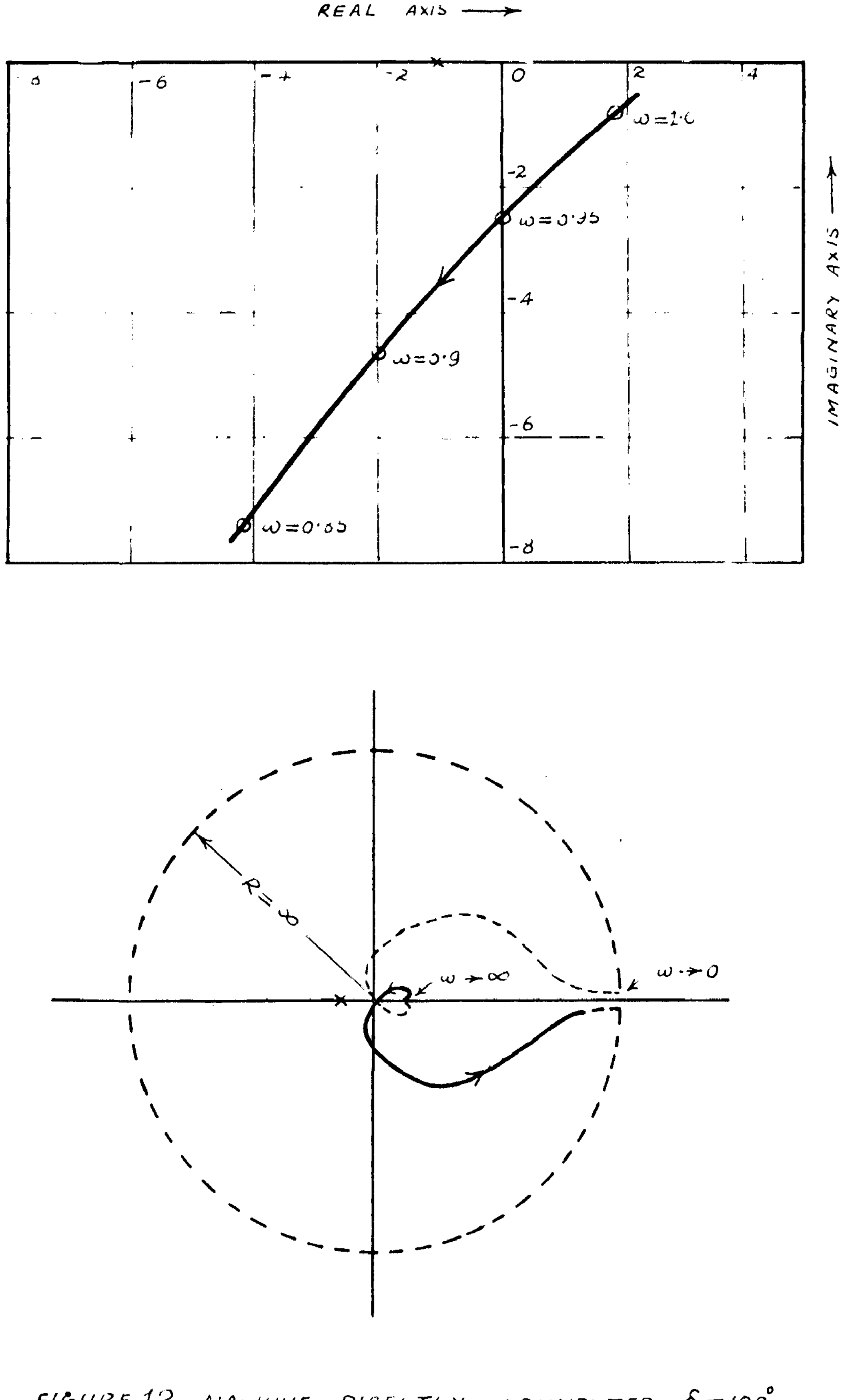


FIGURE 12. MACHINE DIRECTLY CONNECTED, $\delta = 100^\circ$

and $$\frac{f(j\omega)}{-\omega^2} = G(j\omega)H(j\omega) = \frac{(17\omega^4-15\omega^2-0.353)+j(28.95\omega^5-26.7\omega^3+0.02\omega)}{(0.9\omega^4-0.02\omega^2)+j(\omega^5-0.36\omega^3)}$$

$$\lim_{\omega \to 0} G(j\omega)H(j\omega) = \frac{-0.353}{-0.02\omega^2} = +\infty$$

$$\lim_{\omega \to \infty} G(j\omega)H(j\omega) = 28.95$$

Other values of $G(j\omega)\ H(j\omega)$ for various values of ω are given in Table 5.

The Routh's criterion in this case reveals one root with positive real part. However, the Nyquist plot shown in Fig.11 does not enclose (-1, jo) point and hence the system is unstable, as ascertained already in Chapter 3 by applying Routh's criterion to closed loop characteristic equation.

(c) $\delta = 100^o$

The open loop transfer function is,

$$\frac{f(p)}{Mp^2} = \frac{22.6p^4+ 8.51p^4+19.0p^3 +5.8p^2 - 3.14p - 0.493}{p^5 + 0.9p^4 + 0.36p^3 + 0.02p^2}$$

and $$\frac{f(j\omega)}{-\omega^2} = G(j\omega)H(j\omega) = \frac{(8.51\omega^4-5.8\omega^2-0.493)+j(22.6\omega^5-19\omega^3-3.14\omega)}{(0.9\omega^4-0.02\omega^2)+j(\omega^5-0.36\omega^3)}$$

$$\lim_{\omega \to 0} G(j\omega)H(j\omega) = \frac{-0.493}{-0.02\omega^2} = +\infty$$

$$\lim_{\omega \to \infty} G(j\omega)H(j\omega) = 22.6$$

Here again the Nyquist plot does not enclose the critical point which it must enclose once in order that the system be stable for in this case application of Routh's criterion reveals one root of the characteristic equation with positive real part. Since it does not, the system is unstable.

In all the above cases it may be noted that instead of deriving the actual characteristic equation, as in Chapter 3, only the open loop transfer function be obtained which is rather simpler to do (the order of powers in p is reduced by 2) and

Nyquist criterion applied to it to test stability. This conclusion naturally favours the use of frequency response method.

4.6. Systems Without Voltage Regulator:

In this section, open loop transfer functions will be obtained for the system neglecting effect of voltage regulator. The following cases would be considered:

(i) Machine directly connected to infinite bus,

(ii) Machine connected to infinite bus through an impedance tie,

(iii) Machine connected to infinite bus through a transmission line with generalized A, B, C, D constants.

In each case, Routh's criterion will first be applied to open loop transfer function and then frequency response method to test system stability.

4.6.1. Machine directly connected to infinite bus:

Let $\delta_0 = 80^\circ$, p.f. = 0.9 leading, $e = 1.0$

It can be shown that the system remains in stable equilibrium under these conditions of operation when effect of voltage regulator is considered. Operation without regulator will be studied.

Proceeding as in case (b) section 3.3.2., the values of E_o and I are obtained to be:

$$I = 2.14 \text{ p.u. and } E_o = 2.156$$

$$\therefore i_{do} = \frac{-0.02 \times 1.0 \times 0.9848 - 0.8(0.173 - 2.156)}{(.02)^2 + 1.2 \times 0.8} = 1.63$$

$$i_{qo} = \frac{1.2 \times 1.0 \times 0.9898 - 0.02\ (0.173 - 2.156)}{(0.02)^2 + 1.2 \times 0.8} = 1.28$$

$$\psi_{do} = 2.156 - 1.2 \times 1.63 = 0.206$$

$$\psi_{qo} = - 0.8 \times 1.28 = - 1.02$$

The above values together with other quantities are then substituted in f(p) part of equation (17), Chapter 2 i.e. in

$$f(p) = \frac{1}{D}\Big[\Big\{i_{do}x_q(p)+\psi_{do}\Big\}\Big\{x_d(P)\ (e \cos\delta + \psi_{qo}p)+Z_d(P)\ x$$

$$(e \sin\delta + \psi_{do}p)\Big\} + \Big\{i_{qo}x_d(p) + \psi_{qo}\Big\}\ Z_q(p)(e\cos\delta + \psi_{qo}p) -$$

$$x_q(p)\ (e \sin\delta + \psi_{do}\ p)\Big\}\Big]$$

the open loop 'characteristic' equation is obtained as

$$3.067p^3 + 0.08p^2 + 2.988p - 0.06 = 0$$

For this, Routh's array is set up as:

3.067	2.988
0.08	-0.06
0.688	0
-0.06	0
0	0

There is, obviously, one root with real positive part and hence the system is unstable.

The open loop transfer function is given by,

$$\frac{f(p)}{Mp^2} = \frac{3.067p^3 + 0.08p^2 + 2.988p - 0.06}{Mp^2 \times (1 \times 5p)}$$

or $$\frac{f(p)}{Mp^2} = \frac{3.067p^3 + 0.08p^2 + 2.988p - 0.06}{0.02p^2\ (1 + 5p)}$$

$$= \frac{153p^3 + 4p^2 + 149p - 3.0}{5p^3 + p^2}$$

Putting $p = j\omega$,

$$\frac{f(j\omega)}{-M\omega^2} = F(j\omega)H(j\omega) = \frac{-j153\omega^3 - 4\omega^2 + j149\omega - 3.0}{(-j5\omega^3 - \omega^2)}$$

or $$F(j\omega)H(j\omega) = \frac{(+4\omega^2 + 3) + j(+153\omega^3 - 149\omega)}{(\omega^2 + j5\omega^3)}$$

$$\underset{\omega \to 0}{F(j\omega)H(j\omega)} = \frac{+3}{+\omega^2} \to +\infty$$

$$\underset{\omega \to \infty}{F(j\omega)H(j\omega)} = \frac{153}{5} = 30.6$$

The Nyquist plot is shown in Fig.13 from which it will be noted that it does not enclose the critical point (which it must enclose once as $Z = 1$) and hence unstability is again ascertained.

4.6.2. Machine connected to infinite bus through tie line:

The operating conditions are assumed to be,

$$\delta_o = 90^\circ, \text{ p.f.} = 1.0, e_t = 1.0$$

As given in case (b), section 3.3.1, the values of I, E_o and e are obtained to be,

$$I = 1.906 \qquad E_o = 2.44, \qquad e = 1.05$$

Also,

$$i_{do} = 1.57 \qquad i_{qo} = 1.07$$

$$\psi_{do} = 0.56 \qquad \psi_{qo} = -0.86$$

Now making use of equations (30), Chapter 2 and following the technique given in section 3.2.1, of Chapter 3 and 4.4, the open loop transfer function is obtained as,

$$\frac{f(p)}{Mp^2} = \frac{4.1p^3 - 0.21p^2 + 3.08p - 0.333}{0.02p^2(1 + 5p)}$$

$$= \frac{205p^3 - 10.5p^2 + 154p - 16.7}{5p^3 + p^2}$$

and $$\frac{f(j\omega)}{-M\omega^2} = \frac{(-10.5\omega^2 + 16.7) + j(205\omega^3 - 154\omega)}{\omega^2 + j5\omega^3}$$

Applying Routh's criterion it can be seen that the open

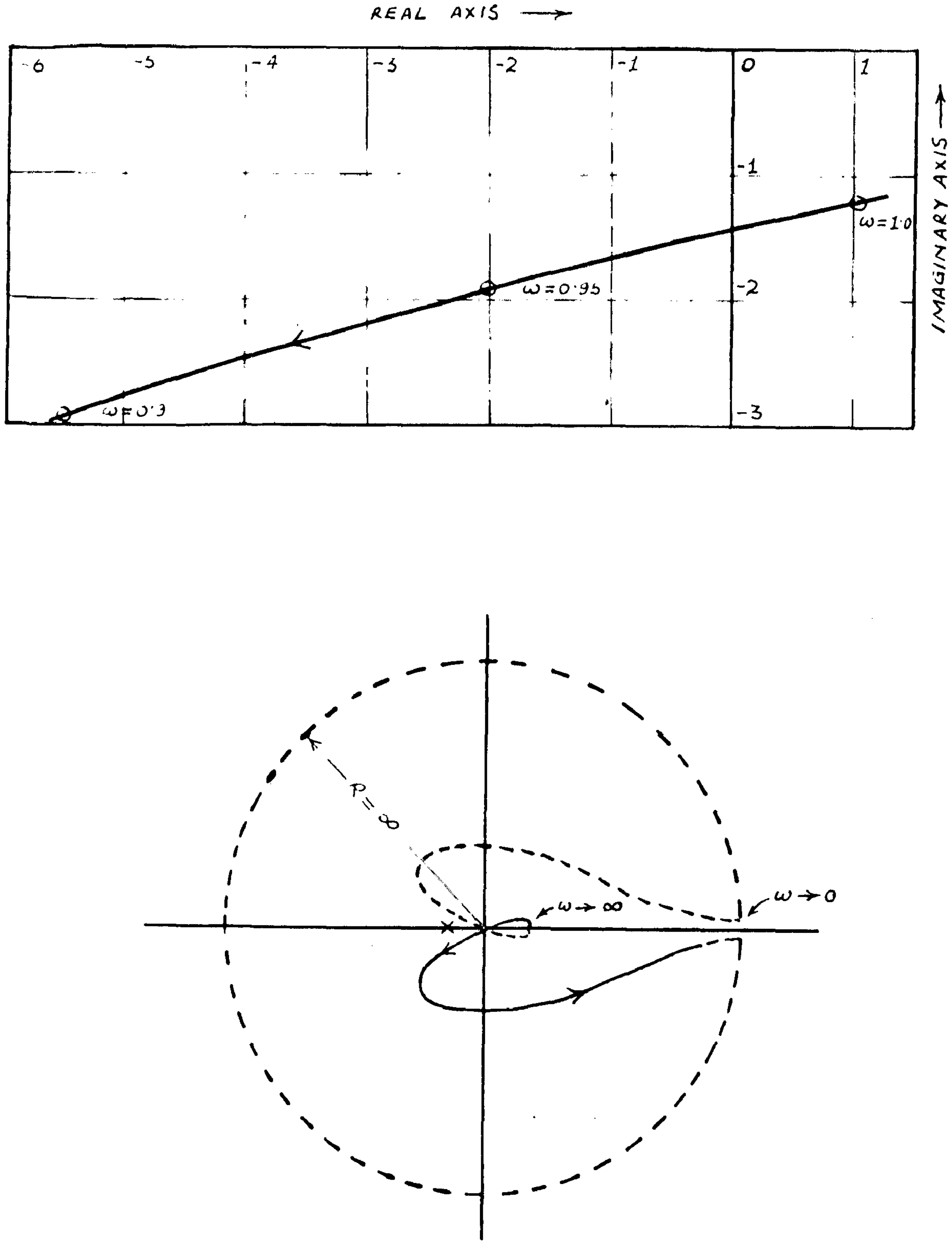


FIGURE 13. MACHINE DIRECTLY CONNECTED, $\delta = 80^{\circ}$

(WITHOUT REGULATOR)

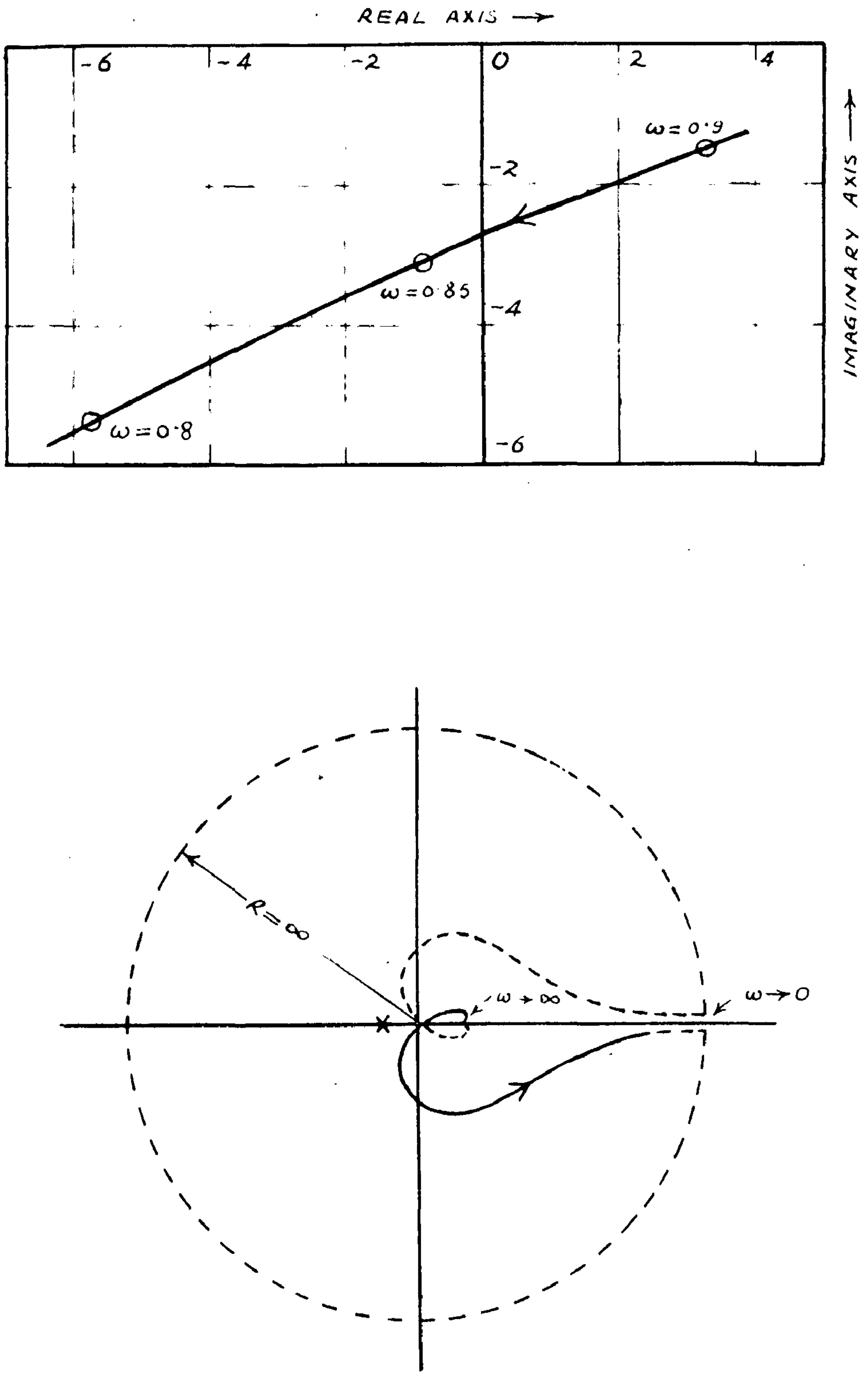


FIGURE 14. MACHINE CONNECTED THROUGH TIE LINE, δ = 90°

(WITHOUT REGULATOR)

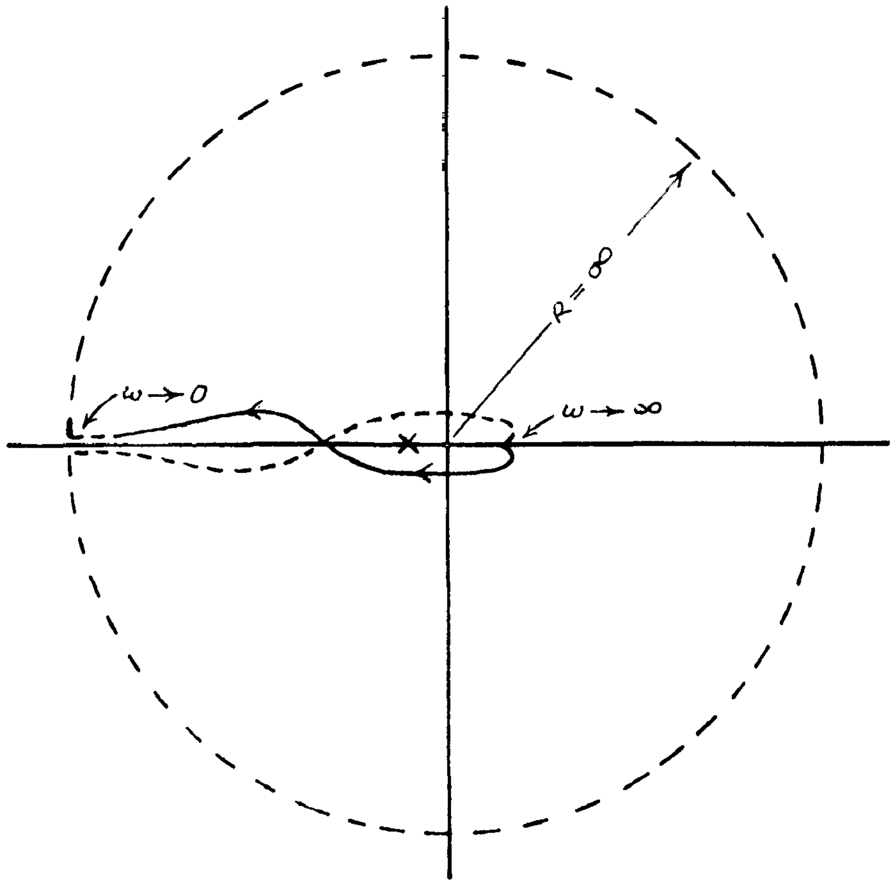


FIGURE 15. MACHINE CONNECTED THROUGH TRANSMISSION LINE, $\delta = 68{\cdot}4^{\circ}$

(WITHOUT REGULATOR)

loop system is unstable having one root with real positive part. The Nyquist plot shown in Fig.14, also confirms the result.

It may be noted that the system under the effect of voltage regulator was found to be stable under similar operating conditions.

4.6.3. System connected to infinite bus through transmission line:

The constants for the transmission line in per unit value are assumed to be:

$$A = 1.0 \ \underline{/0^\circ}, \qquad B = 0.735 \ \underline{/74.4^\circ}, \ C = 0, \ D = 1.0\underline{/0^\circ}.$$

Let the receiving end load current be $I_r = 2.5$ p.u. and p.f.=1.0

Then, $\quad e_s = 1.0 + 2.5 \times 0.735 \ \underline{/74.4^\circ} = 2.3 \ /50^\circ$

and referring to phasor diagram of Fig.1, Chapter 2,

$$E' = e_s + jIx_q = 4.05 \ \underline{/68.4^\circ}$$

$$\therefore \delta = 68.4^\circ$$

Also, $E_o = 4.05 + 2.5 \sin 68.4^\circ (1.2 - 0.8) = 4.98$

The steady state currents and flux linkages, using equations (41) and (42) of Chapter 2, are given by,

$$i_{do} = 3.87, \qquad i_{qo} = 1.07$$
$$\psi_{do} = 0.34, \qquad \psi_{qo} = -0.855$$

Now proceding as in section 2.6.1, using equations (39) in which Mp^2 is replaced by 0, the open loop transfer function is obtained as:

$$\frac{f(p)}{Mp^2} = G(p)\ H(p) = \frac{3.59p^3 + 4.37p^2 + 10.8p + 3.44}{0.02\ p^2\ (1 + 5p)}$$

The Routh's criterion reveals stable operation.

Again, $\dfrac{f(j\omega)}{-M\omega^2} = G(j\omega)\ H(j\omega) = \dfrac{-j179\omega^3 - 219\,\omega^2 + j540\omega + 172}{-j5\omega^3 - \omega^2}$

$$= \frac{(219\omega^2 - 172) + j(179\omega^3 - 540\omega)}{\omega^2 + j5\omega^3}$$

$$\underset{\omega \to 0}{G(j\omega)\ H(j\omega)} = \frac{-172}{\omega^2} = -\infty$$

$$\underset{\omega \to \infty}{G(j\omega)\ H(j\omega)} = \frac{179}{5} = 36.$$

The Nyquist plot shown in Fig.15 does not enclose (-1, j0) point and thus confirms stability.

Chapter 5.

LARGE DISPLACEMENT THEORY

LARGE DISPLACEMENT THEORY

5.1. Introduction

While much work is available(3),(4),(14),(15) concerning the transient performance of a synchronous machine and the criteria to test machine stability under transient operating conditions, almost every author has made use of the assumption of either constant field flux linkages or constant voltage behind transient reactance when deriving equation of power transfer. With the use of most common method for ascertaining machine stability, i.e. equal-area criterion, this assumption does not lead to accurate results in most cases. The method is good for first few swings and can be justified when the fault is cleared within first few cycles by the use of very high speed breakers. But in cases where prolonged oscillations have to be considered and high speed breaking is not available or desirable (as in a one-line-to-ground fault sometimes), change in field flux linkages has to be taken into account and a different approach to test machine or system stability adopted.

The present chapter has been devoted to derive a torque-angle characteristic equation assuming variable field-flux linkages. Park's equations are used(4) and a machine directly connected to infinite bus is considered. In the basic equations and wherever required the armature and field resistances are neglected so also the voltages induced through transformer action.

As will be seen, the torque angle equation now obtained is a second order non-linear equation, exact or approximate solution of which is not possible and some kind of graphical solution has to be adopted. For this purpose, the well known method of 'phase-plane construction'(15),(16),(17),(19),(20) has been used.

Aylett[15] has dealt with the method quite thoroughly and has not only demonstrated how stability of the system under faulty conditions can be tested, but also shows how the critical switching time can be calculated by performing only one integral. Ku[16] in his paper has solved stability problem under sudden loading condition for a round-rotor and a salient-pole synchronous machine and has tried to develop a new graphical construction in the phase-plane which makes the proposition simple.

McLachlan[20] in his book entitled "Ordinary Non-Linear Differential Equations" has used the method of "isoclines" or lines of equal slope to draw the phase-plane trajectories and solve the stability problems. His approach is quite simple and easy to understand and is being followed here.

Before proceeding to derive the torque-angle characteristic equation a brief description of phase-plane and nature of phase-plane trajectories in relation to stability-problem solutions will be given.

5.2. Phase-Plane and Phase-Plane Trajectories:

A non-linear differential equation of second order may be represented as,

$$\frac{d^2\theta}{dt^2} + F_1(\theta)\,\frac{d\theta}{dt} + f_2(\theta) = C \qquad \dots \quad \dots (1)$$

where $f_1(\theta)$ and $f_2(\theta)$ are non-linear functions of θ or may be constant quantities and C is a constant.

Let $$\frac{d\theta}{dt} = v$$

Then, $$\frac{d^2\theta}{dt^2} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d\theta}{dt} \cdot \frac{d}{d\theta}\left(\frac{d\theta}{dt}\right) = v \cdot \frac{dv}{d\theta}$$

Substituting the transformations in equation (1), there results,

$$v \frac{dv}{d\theta} + f_1(\theta)\, v + f_2(\theta) = C \qquad \dots \dots (2)$$

This equation differs from equation (1) in following two ways,

(i) the equation is non-linear but of first degree only.

(ii) the time variable does not appear in the equation but instead the two variables occuring are θ and v which may be identified as displacement and velocity respectively.

In the nomenclature adopted by writers in the field of non-linear mechanics[(17),(18),(19),(20)] the plane of the variables v, θ, is the 'phase-plane', and a curve corresponding to equation (2) (which may be obtained following any method possible) is a 'phase-plane trajectory'.

It will be noticed that the phase-plane trajectories, as the name implies, are not solutions of the original differential equation (1), since time does not appear as a variable. On the contrary, a trajectory is the path of a representative point, There is an infinity of solutions corresponding to different time-origins for a given trajectory. The motion of a representative point along a trajectory corresponds to one of the solutions.

Two types of trajectories can be obtained, normal trajectories and degenerate trajectories, or singular points. When the value of $dv/d\theta$ is determinate, we get first kind of trajectories, but when this value is indeterminate, second kind of trajectories are obtained. The latter are of special interest to all stability problems.

To find singular points of equation (2), there is,

$$\frac{dv}{d\theta} = \frac{C - f_2(\theta) - f_1(\theta)v}{v} = 0$$

or

$$C - f_2(\theta) = 0 \qquad \dots \quad \dots (3)$$

There could be three cases related to three types of singular points when referred to stability problems.

(i) When a singular point is such that a phase-plane trajectory converges to it finally, the singular point is known as a VORTEX point and the trajectory corresponds to a stable operation.

(ii) When a singular point is such that a phase-plane trajectory just passes through it, the singular point is known as a SADDLE point and trajectory corresponds to a critical operation or critically stable equilibrium.

(iii) When a trajectory in the phase-plane is of such a nature that it only dips near the saddle point but never converges round it or passes through it, the system is said to be unstable.

The trajectory described in (ii) is known as a SEPARATRIX as well since it divides the region of stable operation and unstable operation in a phase-plane.

5.3. Method of Isoclines:

When the equation (2) is of the type shown in which a term like $f_2(\theta)\cdot v$ occurs, direct integration is not possible and an indirect method of drawing phase-plane trajectories is desirable such that the trajectory obtained not only satisfies equation (2) for various values of θ and v but also has the same slope as required in equation (2) at a particular point. Method of isoclines. is such an approach which satisfies these requirements.

The method of isoclines originally due to Hobson consists of the following steps:

(a) Equation (2) is reduced to the form:

$$v = \frac{C - f_2(\theta)}{f_1(\theta)+\frac{dv}{d\theta}} \quad \ldots \quad \ldots (3)$$

(b) For various values of θ and one particular value of $\frac{dv}{d\theta}$, corresponding values of v are calculated from (3). These values of v are plotted against corresponding value θ in the phase-plane. The value of assumed slope $\frac{dv}{d\theta}$ is marked at the end of curve.

This step is repeated for different proper values of $\frac{dv}{d\theta}$: The usual value of $\frac{dv}{d\theta}$ range from $\pm$ 0.1 to $\pm$ 10. The curves so obtained are known as ISOCLINES.

(c) Small lines at equal intervals which correspond to the slope value for a curve are drawn. Every isocline is thus marked with these small lines.

(d) Knowing initial conditions of operation, the value of θ_o is known which becomes the starting point for the trajectory.

(e) A smooth curve guided at crossing of every isocline by the small slope lines will result in the required trajectory, the nature of which will comment upon the stability of the system.

Although appearing laborious and time consuming, the method does not take much time in practice and a trajectory can be easily constructed. It might be that an extract solution of equation (1), though providing accurate results, would have been quite complicated and repulsive to adopt. It may be commented here that transient stability problems involve considerations not of the exact solution of the differential equations for a system, but of the general nature of these solutions.

5.4. <u>Derivation of Electrical Torque Equation</u>:

The equations of flux linkages in armature and field circuit are:[4]

$$\left.\begin{aligned}\psi_d &= L_{ad}\, i_f - L_d\, i_d \\ \psi_q &= - L_q i_q\end{aligned}\right\} \qquad \ldots \quad \ldots (4)$$

$$\left.\begin{aligned}\psi_f &= L_f\, i_f - L_{ad}\, i_d \\ e_f &= r_f\, i_f + p.\psi_f\end{aligned}\right\} \qquad \ldots \quad \ldots (5)$$

From second of equations (5),

$$\psi_f = \frac{e_f - r_f\, i_f}{p}$$

$$\therefore\ L_f i_f - L_{ad} i_d = \frac{e_f - r_f\, i_f}{p}$$

$$\text{or } e_f - r_f i_f = L_f\, p i_f - L_{ad}\, p i_d$$

$$\text{or } e_f + L_{ad}\, p i_d = i_f\,(r_f + pL_f)$$

$$\text{or } i_f = \frac{e_f + L_{ad}\, p i_d}{r_f + pL_f} \qquad \ldots \quad \ldots (6)$$

From first of equations (4),

$$\psi_d = - L_d\, i_d + \frac{L_{ad}\, e_f + L_{ad}\, p i_d}{r_f + pL_f}$$

$$\text{or } (r_f + pL_f).\psi_d = -r_f L_d i_d - (L_f L_d)\, p i_d + L_{ad}\ \ e_f + L_{ad}\, p i_d$$

Neglecting $L_d p i_d$ and $L_{ad}^2 p i_d$ terms which denote voltage induced due to transformer action,

$$(r_f + pL_f).\psi_d = - r_f\, L_d i_d + L_{ad} e_f \qquad \ldots \quad \ldots (7)$$

Now, neglecting armature resistance and terms denoting voltages induced through transformer action, the machine voltage equations in direct-and quadrature axes are:

$$\left.\begin{aligned}e_d &= e \sin\delta = - \omega.\psi_q \\ e_q &= e \cos\delta = \ \ \omega.\psi_d\end{aligned}\right. \qquad \ldots \quad \ldots (8)$$

Equations (8) give,

$$\psi_q = \frac{-e \sin\delta}{\omega}$$

and $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \ldots \quad \ldots \; (9)$

$$\psi_d = \frac{e \cos\delta}{\omega}$$

Substituting value of ψ_d from equations (9) into equation (7),

$$(r_f + pL_f).\frac{e \cos\delta}{\omega} = - r_f L_d i_d + L_{ad} e_f$$

or

$$L_{ad} e_f - \frac{r_f e \cos\delta}{\omega} + \frac{L_f e \sin\delta . \dot{\delta}}{\omega} = r_f L_d i_d$$

or

$$i_d = \frac{\omega L_{ad} e_f + L_f e \sin\delta . \dot{\delta}}{\omega L_d r_f}$$

where the term $\frac{r_f e \cos\delta}{\omega}$ is neglected since r_f is very small.

or

$$i_d = \frac{x_{ad} e_f + L_f e \sin\delta . \dot{\delta}}{x_d r_f} \qquad \ldots \quad \ldots \; (10)$$

From second of equations (4),

$$i_q = - \frac{\psi_q}{L_q}$$

or

$$i_q = \frac{e \sin\delta}{\omega L_q} = \frac{e \sin\delta}{x_q} \qquad \ldots . \quad \ldots \; (11)$$

Now, the electrical torque T_e is given by

$$T_e = \psi_d i_q - \psi_q i_d$$

Substituting values of ψ_d, ψ_q, i_d, i_q from equations (9),(10),(11)

$$T_e = \frac{e \sin\delta}{x_q} . \frac{e \cos\delta}{\omega} + \frac{e \sin\delta}{\omega} \left\{ \frac{x_{ad} e_f + L_f e \sin\delta . \dot{\delta}}{x_d r_f} \right\}$$

$$= \frac{1}{2} \frac{e^2}{\omega x_q}(2 \sin\delta . \cos\delta) + \frac{e . e_f x_{ad} \sin\delta}{\omega x_d r_f} +$$

$$\frac{1}{2} \frac{e^2}{\omega x_d} \frac{L_f}{r_f}(2 \sin^2\delta).\dot{\delta}$$

But $2 \sin\delta . \cos\delta = \sin 2\delta$, $2 \sin^2\delta = 1 - \cos 2\delta$ and $\frac{L_f}{r_f} = T'_{do}$

$$\therefore T_e = \frac{e^2 \sin 2\delta}{2\omega x_q} + \frac{e . e_f x_{ad}}{\omega x_d r_f} \sin\delta + \frac{e^2 T'_{do}}{2 \omega x_d} (1 - \cos 2\delta) . \dot{\delta}$$

or $$T_e = \frac{e.e_f\, x_{ad}}{\omega x_d\, r_f} \sin\delta + \frac{e^2}{2\omega x_q} \sin 2\delta + \frac{e^2 T'_{do}}{2\omega x_d}(1-\cos 2\delta)\cdot\dot{\delta} \qquad \ldots \quad \ldots (12)$$

Equation (12) represents the required torque-angle equation in which all quantities are in per unit values except that when T'_{do} is substituted in seconds, ω has to be in radians, per second otherwise per unit.

The first two terms in equation (12) represent the electric torque generated by the machine. The value of e_f in first term has to be calculated for every initial state of operation. This corresponds to the voltage E_o is steady-state. The last term of the equation denote the damping produced within the machine by induction motor action and is of considerable importance in damping out small oscillations in the machine.

5.5. The Electro-mechanical Equation:

The electro-mechanical equation of the system can now be written;

$$Mp^2\delta + T_e = T_m$$

or $$M\frac{d^2\delta}{dt^2} + \frac{e\ e_f\ x_{ad}}{\omega x_d\ r_f} \sin\delta + \frac{e^2}{2\omega x_q} \sin 2\delta + \frac{e^2 T'_{do}}{2\omega x_d}(1-\cos 2\delta)\dot{\delta} = T_m$$

or $$M\frac{d^2\delta}{dt^2} + \frac{e^2 T'_{do}}{2\omega x_d}(1-\cos 2\delta)\frac{d\delta}{dt} + \frac{e.e_f\ x_{ad}}{\omega x_d r_f} \sin\delta + \frac{e^2}{2\omega x_q} \sin 2\delta = T_m \qquad \ldots \quad \ldots (13)$$

which can be compared to the general equation (1) to show that actual electro-mechanical torque equation of a salient pole synchronous machine is a second order non-linear equation.

Equation (13) can be written,

$$\frac{d^2\delta}{dt^2} + a(1-\cos 2\delta)\frac{d\delta}{dt} + b \sin\delta + c \sin 2\delta = T_o \ldots (14)$$

where,

$$a = \frac{e^2\, T'_{do}}{M\, 2\omega x_d}, \quad b = \frac{e\, e_f x_{ad}}{M \omega x_d r_f}, \quad c = \frac{e^2}{M\, 2\omega x_q}, \quad T_o = \frac{T_m}{M}$$

5.6. Sample Calculations:

In this section the practical case of a synchronous machine will be considered. Assuming average values for the machine constants, the electro-mechanical torque equation will be obtained and a few problems of transient operation of the machine will be solved by phase-plane method.

Machine Constants:

The following values are assumed:

x_d = 1.2 p.u.

x_q = 0.8 p.u.

x_{ad} = 1.0 p.u.

r_f = 0.001 p.u.

T'_{do} = 5 sec.

ω = $2\pi f$ radians per second or 1.0 p.u.

f = 50 cycles per second.

x'_d = 0.3 p.u.

Using equation (14) we have,

$$a = \frac{e^2\, T'_{do}}{M\, 2\omega x_d} = \frac{(1.0)^2 \times 5}{0.02 \times 2 \times 314 \times 1.2} = 0.34$$

$$b = \frac{e\, e_f\, x_{ad}}{M \omega x_d\, r_f} = \frac{1.0 \times 1.0 \times 1.0}{0.02 \times 314 \times 1.2 \times 0.001} = 133$$

(it is being assumed that machine is operating at no load i.e. $\delta_o = 0$ and hence $e_f = 1.0$ p.u. corresponding to terminal or infinite bus voltage $e = 1.0$ p.u.).

$$c = \frac{e^2}{M\, 2\omega x_q} = \frac{(1.0)^2}{0.02 \times 2 \times 1.0 \times 0.8} = 31.2$$

(The value of ω is to be taken unity here to have proper balance of dimentions).

$$T_o = \frac{T_m}{M} = \frac{T_m}{0.02} = 50\, T_m$$

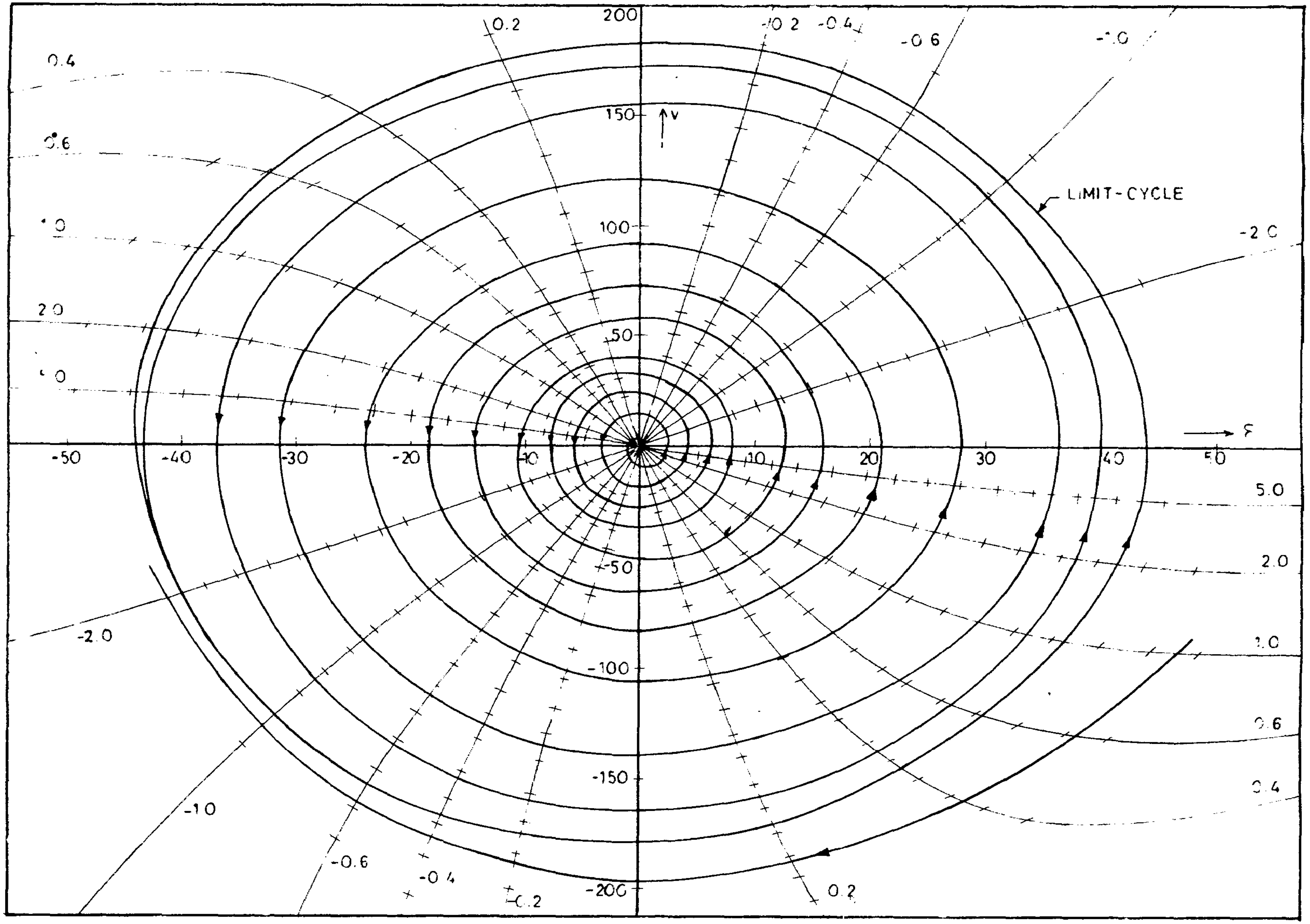


FIGURE.16_ FREE OSCILLATIONS

The torque equation is, therefore, given by:

$$\frac{d^2\delta}{dt^2} + 0.34(1-\cos 2\delta)\frac{d\delta}{dt} + 133 \sin\delta + 31.2 \sin 2\delta = 50T_m + 50(1)T' \qquad \ldots \quad \ldots (15)$$

where T' is any additional torque suddenly applied to the machine and maintained at that value thereafter.

The following cases are analyzed:

Case I - Free Oscillations:

In this case, $\delta_o = 0$ and $T_m = 0$.

The problem is to determine whether the machine is stable or not and to what maximum value of δ it oscillates.

The torque equation in this case is,

$$\frac{d^2\delta}{dt^2} + 0.34\,(1-\cos 2\delta)\frac{d\delta}{dt} + 133 \sin\delta + 31.2 \sin 2\delta = 0.. \qquad \ldots (16)$$

writing $\frac{d\delta}{dt} = v$, equation (16) becomes,

$$v\frac{dv}{d\delta} + 0.34\,(1-\cos 2\delta).v + 133 \sin\delta + 31.2 \sin 2\delta = 0.$$

On transposition, the equation for drawing isoclines is obtained as:

$$v = \frac{-(133 \sin\delta + 31.2 \sin 2\delta)}{0.34\,(1-\cos 2\delta) + \frac{dv}{d\delta}} \qquad \ldots \quad \ldots (17)$$

Table 10 gives the values of v corresponding to different values of δ and the slope $\frac{dv}{d\delta}$. The isoclines are drawn on the phase-plane and following the method described earlier, the required trajectory is constructed which is shown in Fig.16. It is seen that the curve spirals out from the origin and merges into a limit cycle showing that the system is stable and also the maximum degree of oscillations for the machine is about $\pm 44^\circ$.

Case II - Load suddenly applied:

The machine is assumed to be operating in no-load condition initially, i.e. $\delta_o = 0$ and $T_m = 9$. Then a sudden load equal in magnitude to the value obtained as transient limit by using equal area criterion as shown in Fig.17, is applied. The torque angle characteristic used for the purpose of calculating the transient limit makes use of the equation,

$$T = \frac{E.e}{x_d'} \sin\delta$$

where E is the internal voltage of the machine under initial steady-state operating conditions.

The transient limit by equal area criterion referring to Fig.17 is obtained as follows:

We have- $$T = \frac{E.e \sin\delta}{x_d'} = \frac{1.0 \times 1.0}{0.3} \sin\delta = 3.334 \sin\delta$$

Equating the two shaded areas,

$$T'\delta_1 - \int_0^{\delta_1} 3.334 \sin\delta \, d\delta = \int_{\delta_1}^{\pi-\delta_1} 3.334 \sin\delta \, d\delta - T'(\pi - 2\delta_1)$$

or

$$T'\delta_1 + 3.334 \left[\cos\delta\right]_0^{\delta_1} = -3.334 \left[\cos\delta\right]_{\delta_1}^{\pi-\delta_1} - \pi T' + 2T'\delta_1$$

or

$$3.334 \left[\cos\delta_1 - 1\right] = -3.334\left[-2 \operatorname{Cos}\delta_1\right] - \pi T' + T'\delta_1$$

or

$$-3.334 = 3.334 \cos\delta_1 - \pi T' + T'\delta_1 \qquad \dots \quad \dots (18)$$

Also,

$$T' = 3.334 \sin\delta_1$$

$$\therefore \quad -3.334 = 3.334 \cos\delta_1 - \pi \times 3.334 \sin\delta_1 + 3.334\delta_1 \sin\delta_1$$

whence, by trial, δ_1 is obtained

$$\delta_1 \simeq 46^\circ$$

Substitution of δ_1 in equation (18) gives,

$$T' = 2.42 \text{ p.u.}$$

The electro-mechanical equation, then, takes the form:

$$\frac{d^2\delta}{dt^2} + 0.34(1 - \cos 2\delta)\frac{d\delta}{dt} + 133 \sin\delta + 31.2 \sin 2\delta = 0 + 50T' (1)$$

or

$$\frac{d^2\delta}{dt^2} + 0.34(1 - \cos 2\delta)\frac{d\delta}{dt} + 133 \sin\delta + 31.2 \sin 2\delta = 121 \quad \dots (19)$$

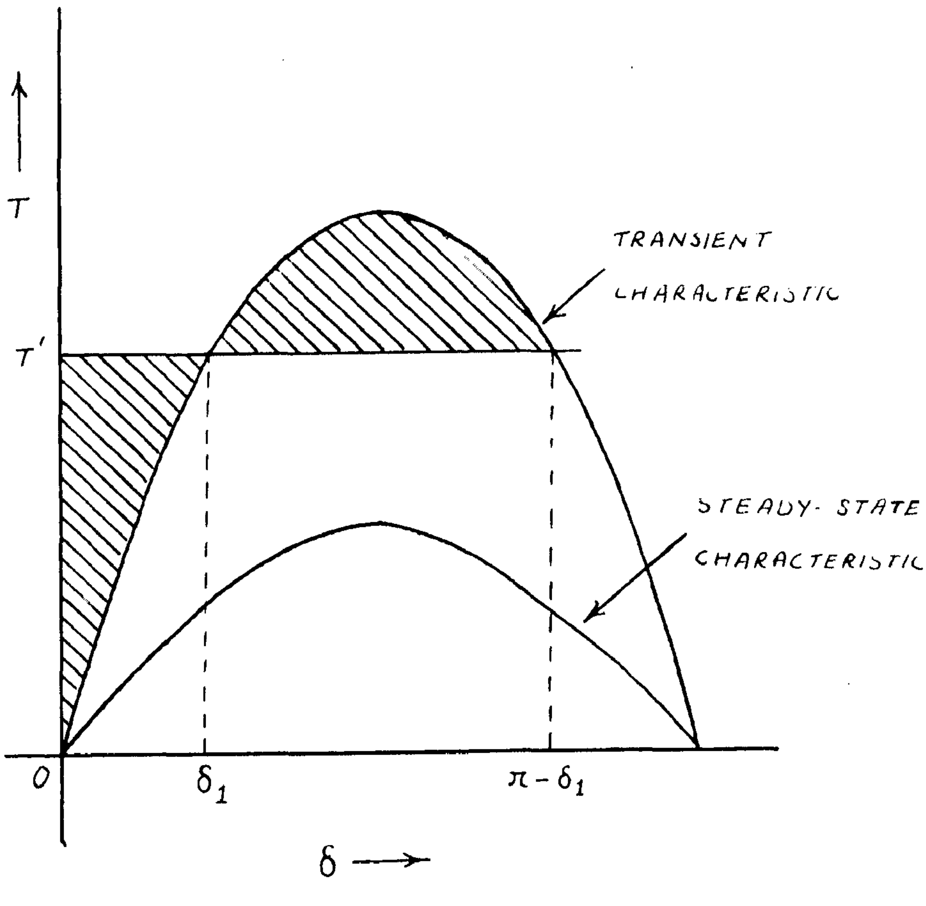

T
T'
TRANSIENT
CHARACTERISTIC
STEADY-STATE
CHARACTERISTIC
0
δ_1
$\pi - \delta_1$
δ

FIGURE 17.

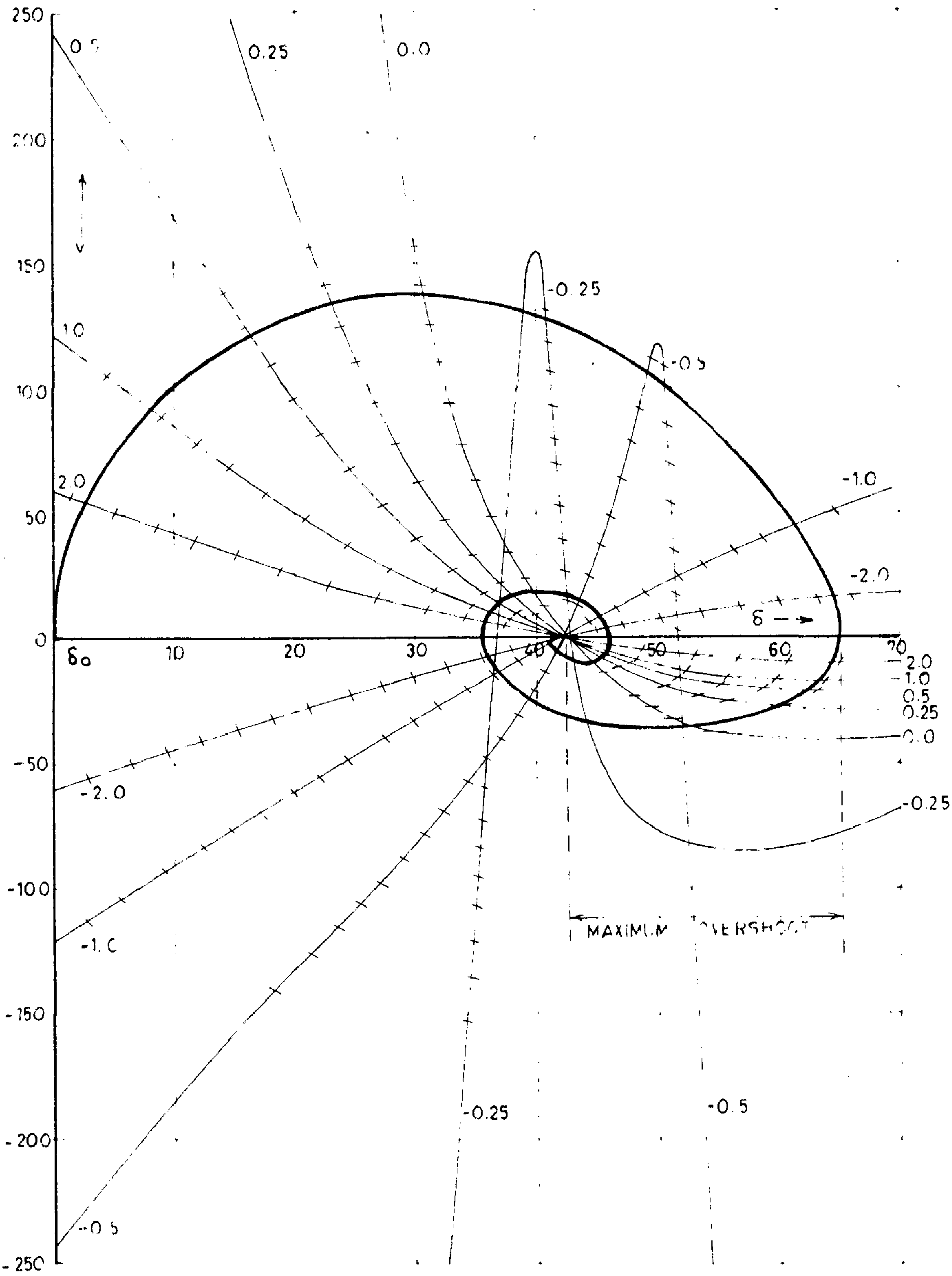


FIGURE.18_LOAD SUDDENLY APPLIED

With $\frac{d\delta}{dt} = v$, the equation for phase-plane is given by

$$v\frac{dv}{d\delta} + 0.34\ (1-\cos 2\delta)v + 133 \sin\delta + 31.2 \sin 2\delta = 121$$

and the equation for isoclines is

$$v = \frac{121 - (133 \sin\delta + 31.2 \sin 2\delta)}{0.34\ (1-\cos 2\delta) + \frac{dv}{d\delta}}$$

Calculations for drawing isoclines are shown in Table 11. The phase-plane trajectory is drawn which is seen to be converging to a vortex point $= 43^o$. The machine remains in stable equilibrium and the maximum overshoot is about $63^o - 43^o = 20^o$. The trajectory is shown in Fig.18.

Case III. Machine operating under normal full load condition-sudden load applied:

In this case the initial operating conditions are:

$I = 1.0$ p.u. p.f. = 0.8 lag, $e = 1.0$ p.u.

Calculation of e_f or E_o:

Referring to phasor diagram of Fig.19 (a),

$$E' = e + j\,I\,x_q$$
$$= 1.0 + j(0.8 - j0.6) \times 0.8 = 1.61 \angle 23.2^o$$

$$\therefore E_o \text{ or } e_f = E' + i_d\ (x_d - x_q)$$
$$= 1.61 + 1.0 \times \sin\ (\delta_o + \cos^{-1}\phi) \times (1.2 - 0.8)$$
$$= 1.955$$

Transient Limit by Equal-Area Criterion:

Referring to Fig.19 (b) and proceeding as in case II, the angle δ_1 is found to be approximately equal to 60 degrees and the additional torque T' to reach the transient limit is

$$T' = 3.09 \text{ p.u.}$$

In the electro-mechanical equation (15), the coefficient of

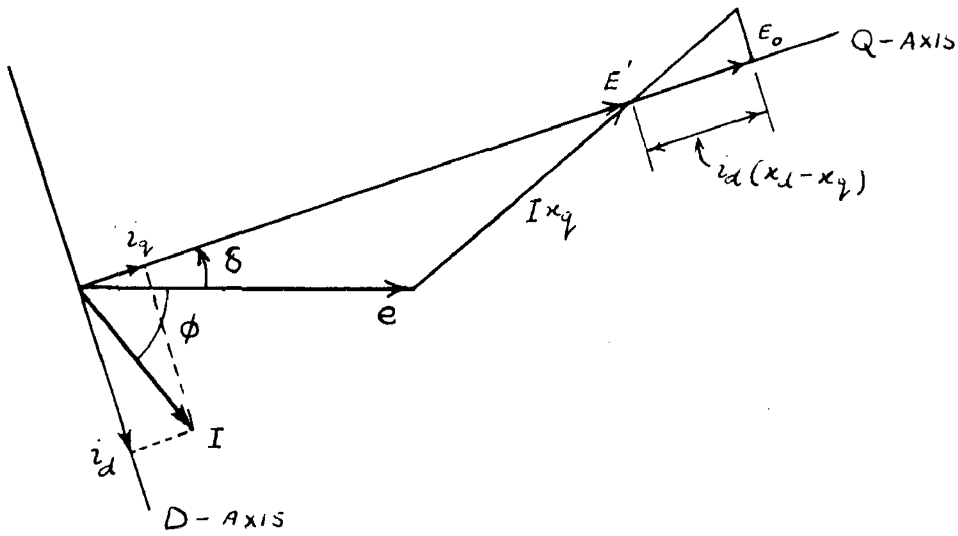


FIGURE 19(a).

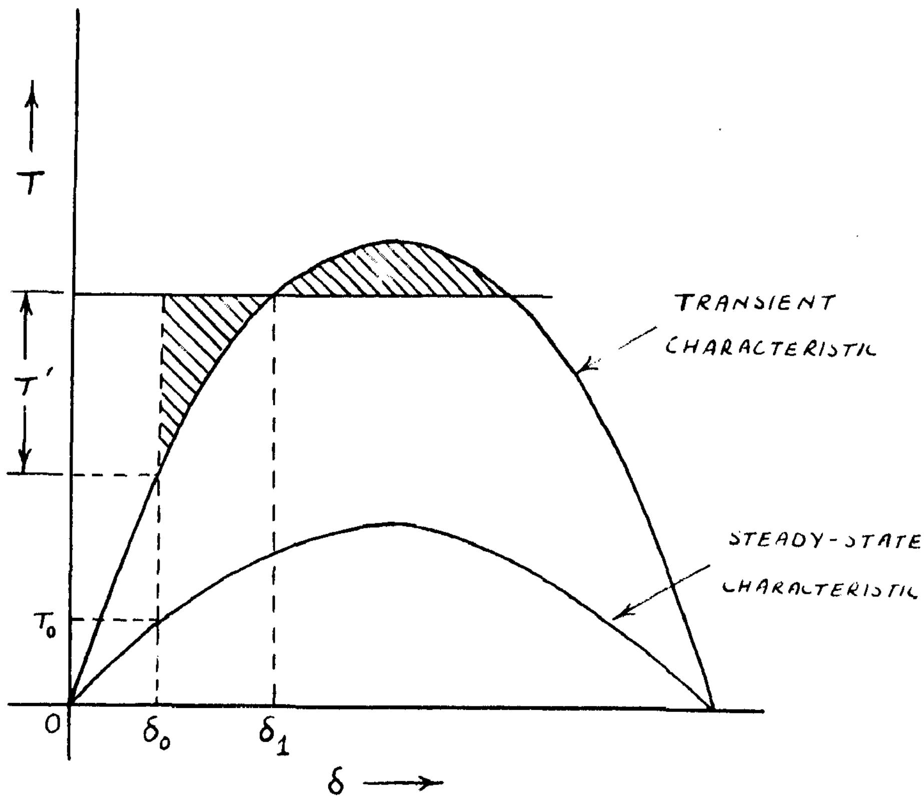


FIGURE 19(b).

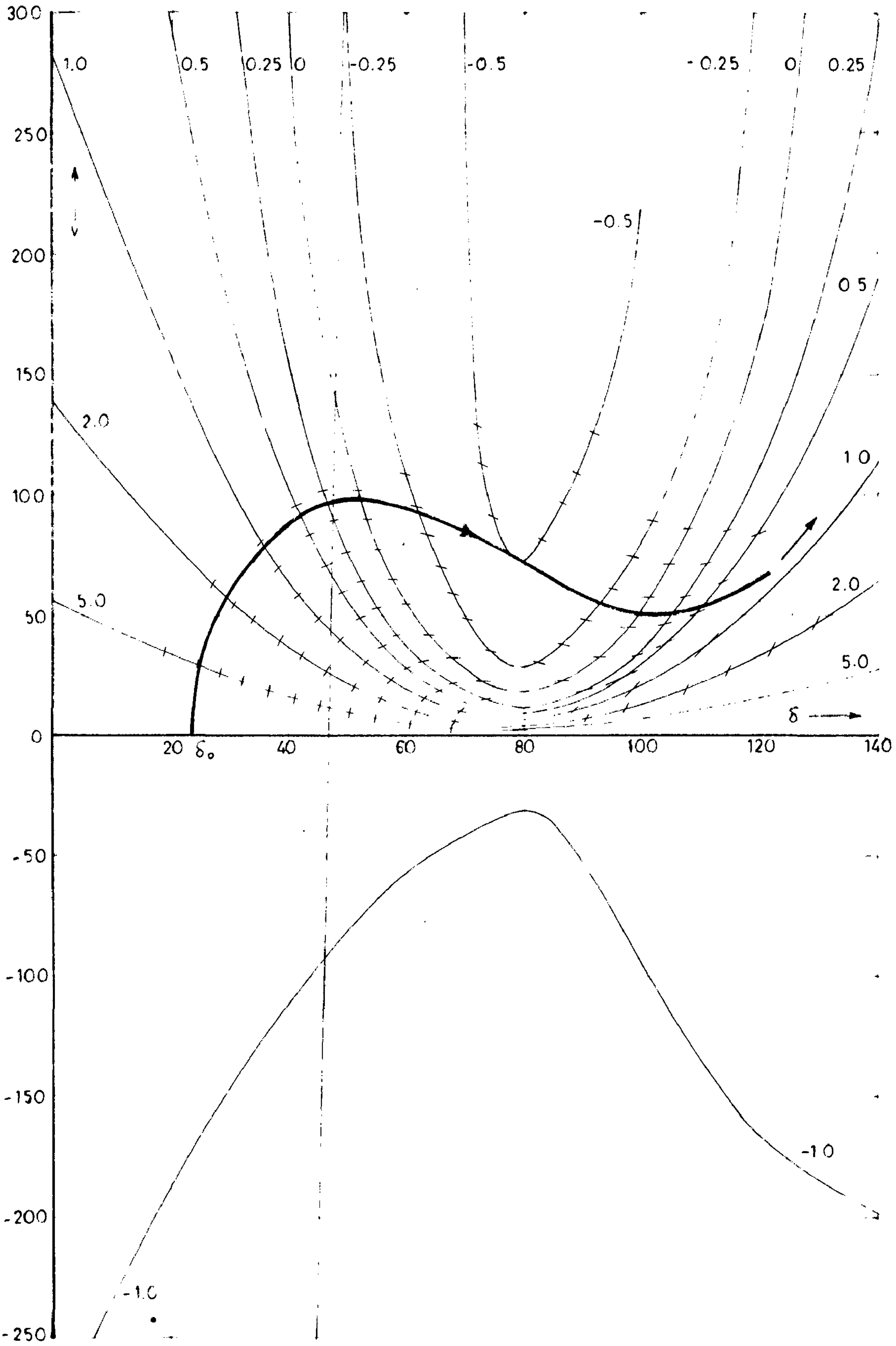

300
250
200
150
100
50
0
-50
-100
-150
-200
-250
20
40
60
80
100
120
140
δ_o
δ
1.0
0.5
0.25
0
-0.25
-0.5
-0.25
0
0.25
-0.5
0.5
1.0
2.0
5.0
2.0
5.0
-1.0
-1.0

FIGURE. 20

$\sin\delta$ term changes, since e_f is changed. The new coefficient is given by-

$$b = \frac{e.e_f\, x_{ad}}{M\omega x_d r_f} = \frac{1.0 \times 1.955 \times 1.0}{0.02 \times 314 \times 1.2 \times 0.001} = 262$$

The torque equation then becomes,

$$\frac{d^2\delta}{dt^2} + 0.34(1-\cos 2\delta)\frac{d\delta}{dt} + 262 \sin\delta + 31.2 \sin 2\delta = 5=.T_m + 50T'(1)$$

where $T_m = 6.5 \sin 23.2^\circ = 2.56$ p.u.(corresponding to transient characteristic).

$$\therefore \frac{d^2\delta}{dt^2} + 0.34(1-\cos 2\delta)\frac{d\delta}{dt} + 262 \sin\delta + 31.2 \sin 2\delta = 5 = x2.56 + 50x3.09(1)$$

In the phase-plane, the equation reduces to,

$$v\frac{dv}{d\delta} + 0.34\,(1-\cos 2\delta)v + 262 \sin\delta + 31.2 \sin 2\delta = 281.$$

and the equation for drawing isoclines is

$$v = \frac{281 - (262 \sin\delta + 31.2 \sin 2\delta)}{0.34\,(1-\cos 2\delta) + \frac{dv}{d\delta}}$$

The isoclines and the phase-plane trajectory starting from the initial operating point $\delta_0 = 23.2$, degrees are drawn in Fig.20. From the nature of the trajectory, which travels away from the saddle point (about 95 degrees) after a slight dip there, it can be concluded that the system loses stability in this case.

Table 12 gives the values for drawing isoclines.

Case IV. Machine operating at normal full load-additional load equal to full load value suddenly applied:

Here the operating conditions remains the same initially as in case III i.e. $I = 1.0$ p.u. p.f. = 0.8 lag, $E_0 = 1.955$

The full load torque, obtained by transient characteristic is-

$$T_0 = 2.56 \text{ p.u.}$$

$$\therefore \quad T' = 2.56 \text{ p.u.}$$

so that,

$$T = T_o + T' = 5.12 \text{ p.u.}$$

The machine torque equation is given by,

$$\frac{d^2\delta}{dt^2} + 0.34(1- \cos 2\delta)\frac{d\delta}{dt} + 262 \sin\delta + 31.2 \sin 2\delta = 5.12 \times 50 = 256$$

at $t = 0$, $\delta_o = 23.2^o$ and $T_o = 2.56$ p.u.

The vortex point is given by,

$$262 \sin \delta_v + 31.2 \sin 2 \delta_v = 256$$

whence,

$$\delta_v \simeq 60^o$$

The equation in phase-plane is

$$v\frac{dv}{d\delta} + 0.34 (1-\cos 2\delta).v + 262 \sin\delta + 31.2 \sin 2 \delta = 256$$

and the equation for isoclines is

$$v = \frac{256 - (262 \sin\delta + 31.2 \sin 2\delta)}{0.34 (1-\cos 2\delta) + \frac{dv}{d\delta}}$$

Fig.21 shows the isoclines and the trajectory for this case. It is seen that the trajectory converges round the vortex point (61 degrees approximately) and the system is stable. The maximum overshoot is very large, however, being 94.5-61 = 34 degrees approximately.

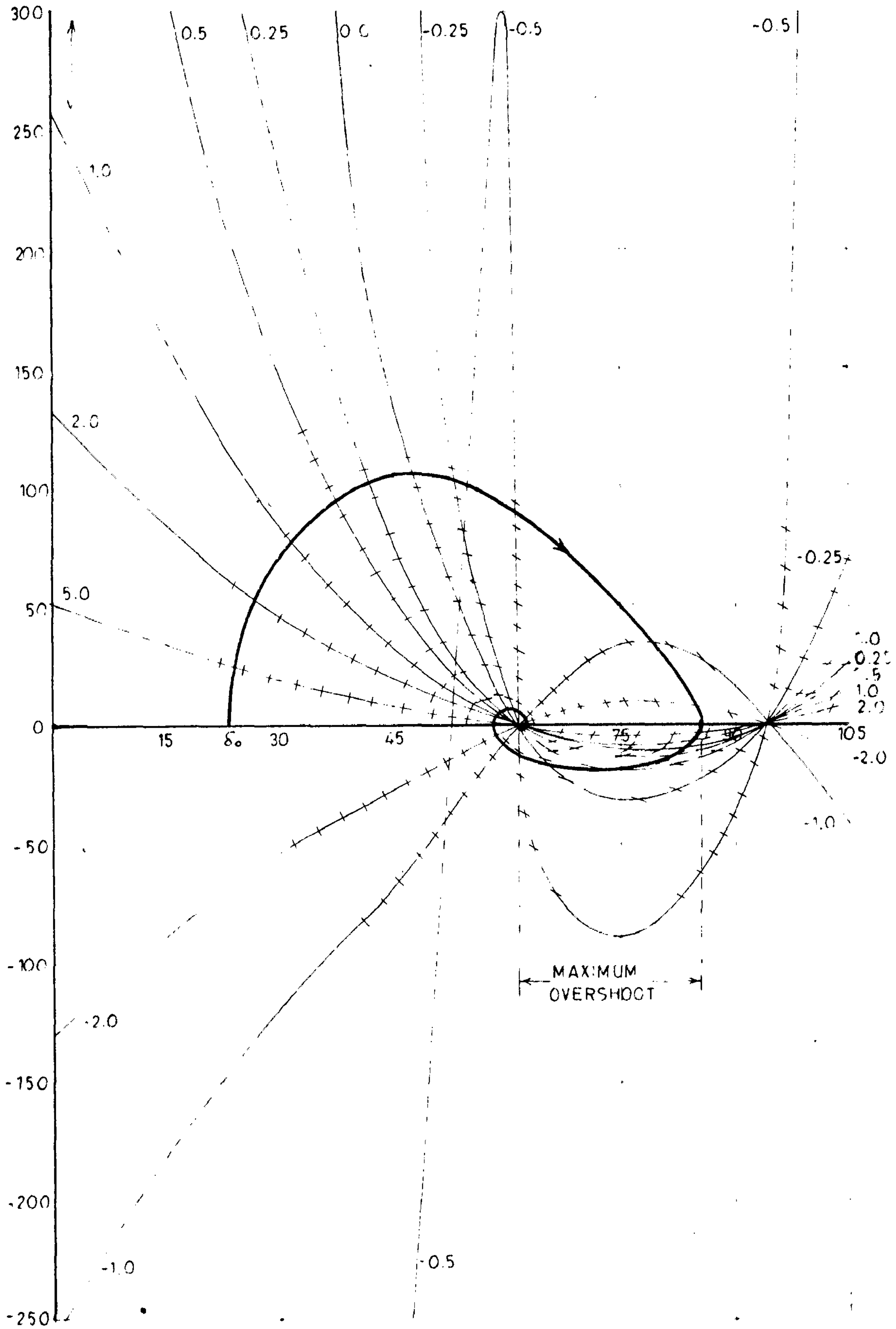
Values used to draw isoclines are given in Table 13.

Case V. The Operating conditions are:

$$\delta_o = 30^o, \text{ p.f.} = 1.0, \quad e = 1.0 \text{ p.u.}$$

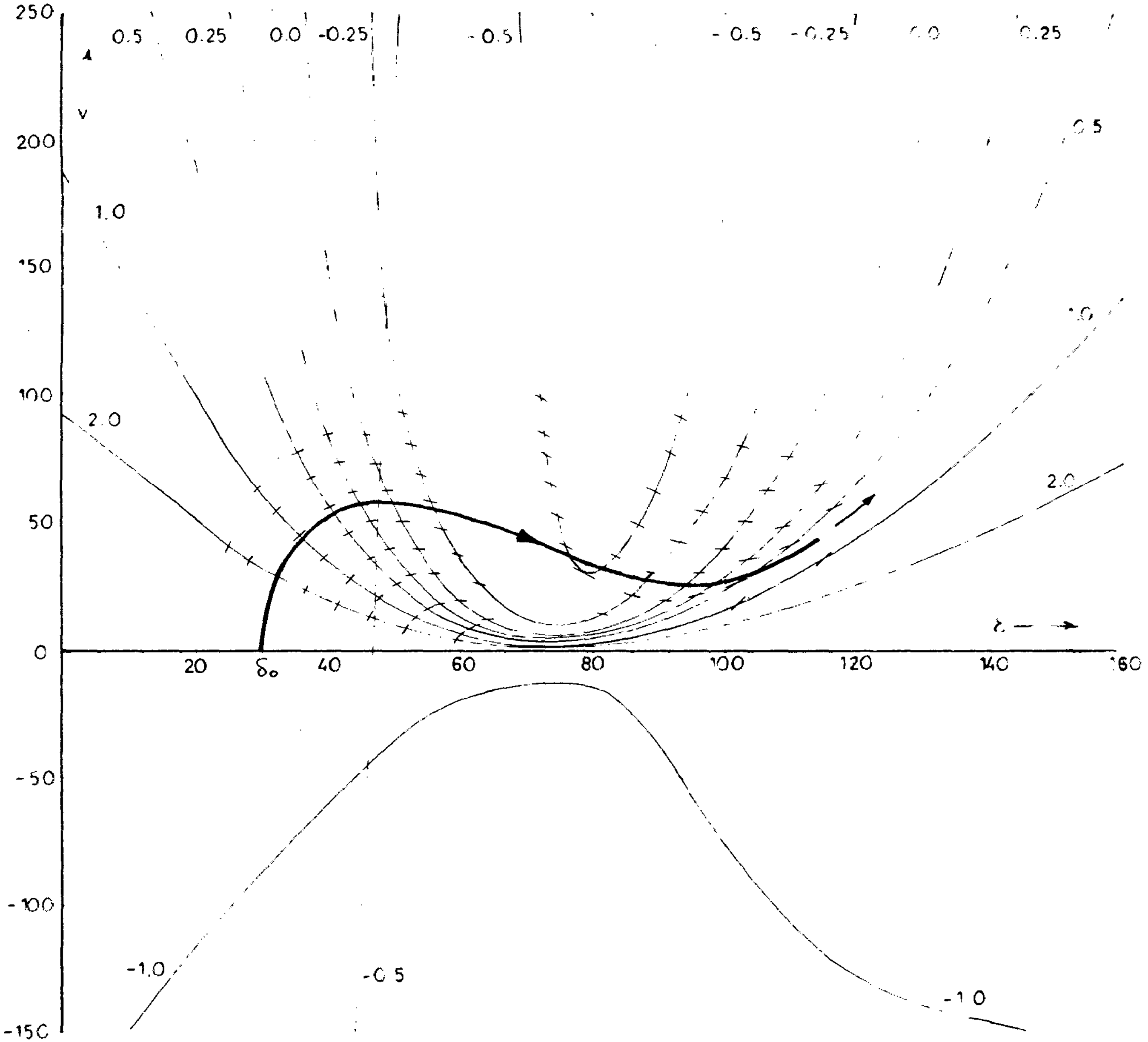
Then on additional load to reach the transient limit (calculated using transient characteristic) is suddenly applied and maintained thereafter. To test the stability.

Using the phasor diagram of Fig.19(a) given in case III the value of E_o ~~or~~ e_f is determined which is-



300
250
200
150
100
50
0
-50
-100
-150
-200
-250
15
δ_o
30
45
75
90
105
0.5
0.25
0.0
-0.25
-0.5
-0.5
1.0
2.0
5.0
-0.25
0.25
.5
1.0
2.0
-2.0
-1.0
-2.0
-1.0
-0.5
MAXIMUM
OVERSHOOT

FIGURE. 21



250
200
150
100
50
0
-50
-100
-150
v
20
40
60
80
100
120
140
160
δ_0
δ
0.5
0.25
0.0
-0.25
-0.5
1.0
2.0
-1.0

FIGURE. 22

$E_0 = 1.3$ p.u.

The transient torque-angle characteristic (assuming a round rotor machine) is given by

$$T = \frac{1.3 \times 1.0}{0.3} \sin\delta = 4.34 \sin\delta$$

$$T_0 = 4.34 \sin 30^\circ = 2.17 \text{ p.u.}$$

The additional torque T' to reach the transient limit can be calculated with the help of transient torque -angle characteristic as in case III. Thus,

$$\delta_1 \simeq 60^\circ$$

and

$$T' = 1.58 \text{ p.u.}$$

Again, the coefficient of sin term in equation (15) is modified to,

$$b = \frac{1.3 \times 1.0 \times 1.0}{0.02 \times 314 \times 1.2 \times 0.001} = 175$$

The torque-equation for the system is given by

$$\frac{d^2\delta}{dt^2} + 0.34\,(1- \cos 2\delta)\frac{d\delta}{dt} + 175 \sin\delta + 31.2 \sin 2\delta = 2.17 \times 50 + 1.58 \times 50\ (1).$$

Putting $\frac{d\delta}{dt} = v$, the phase-plane equation is

$$v\frac{dv}{d\delta} + 0.34\,(1- \cos 2\delta)v + 175 \sin\delta + 31.2 \sin 2\delta = 187.5$$

while the equation for isoclines is

$$v = \frac{187.5 - (175 \sin\delta + 31.2 \sin 2\delta)}{0.34(1- \cos 2\delta) + \frac{dv}{d\delta}}$$

Applying the expression (3), it can be seen that no vortex point is available and so the system appears to be unstable. However, the isoclines and the trajectory are drawn which are shown in Fig.22. The nature of trajectory confirms that the system is unstable, the saddle point being at about 101 degrees.

Chaper 6.

C_O_N_C_L_U_S_I_O_N_S.

CONCLUSIONS

Consideration of voltage regulator action while calculating steady-state stability of a system specially when the system consists of one machine connected to infinite bus, is becoming more and more important. Formerly when only hand-operated voltage regulators were in use, the question of improving stability limit was not so useful and promising and calculations for stability limit neglected the voltage regulator because the analysis became simple and time saving. But with the invent of automatic voltage regulator equipped with almost every modern alternator, it has become desirable to take into account the regulator action when calculating steady-state stability limit of a synchronous machine.

The following general conclusions can be derived based on the calculated cases in the preceding chapters:

1. A salient pole synchronous machine which is otherwise unstable at angles of operation much less than 90 degrees becomes stable upto about 90 degree (torque angle) when provided with a suitably designed voltage regulator.

2. The stability limit of a synchronous machine when connected to the infinite bus-bar through a tie line of proper resistance and reactance values is slightly higher than when the machine is directly connected to the bus. This shows that a synchronous machine may be connected to infinite bus through a tie line or a reactor of similar characteristics in order to improve stable operation of system in dynamic stability region.

3. Neglecting armature and tie-line resistance and voltages induced through transformer action from the final equations

would have resulted in much simpler analysis, but giving rather optimistic results.

4. While ascertaining stability under various cases, it was noticed that the amplification factor 'a' has great influence on stability. The higher value may result in unstable operation even at much small values of torque angle. Similarly, a low value of regulator time constant T_r is desirable, which means a fast acting voltage regulator.

In the cases studied, the saturation was taken into account by using modified phasor diagrams as shown at suitable places. This again makes the analysis more rigrous.

The effect of damper windings was neglected. It was assumed that while making the analysis very complicated, the damper windings have little effect on the steady-state stability limit of a synchronous machine when operating in dynamic stability region i.e. under the influence of voltage regulator.

Both, the Routh's as well as Nyquist's criteria, have been applied, the latter in the course of frequency response analysis, while ascertaining stability in each case. It was noted that frequency response method using Nyquist criterion is better suited and less laborious to apply when computing aids are available, This ~~is~~ because only open loop transfer function is required. In any case it can be concluded that it is the small displacement theory which has made it possible to linearize the otherwise non-linear electro-mechanical equation and use of either of the above criteria applicable. Without the use of this theory it would have been impossible to consider the effect of voltage regulator specially in mathematical analysis.

Again, in the case of large displacement theory (Chapter 5) it was found that an electro-mechanical torque equation can be obtained starting from basic voltage and flux equations in which change of field flux linkages could be considered. The equation, which is non-linear in nature, when solved in the phase-plane trajectory forms gives quite convincing results. The following conclusions can be derived referring to the cases studied:

1. The machine in free oscillation state ($\delta = 0$, $T_m = 0$) always comes to stable equilibrium, the oscillations being finally damped out. The extent to which the machine may oscillate depends upon the degree of damping available.

2. The application of a sudden large load from initial no load condition may be thought upon to result in an unstable operation, in general. However, the machine may remain in stable equilibrium as can be seen from case II and IV, Chapter 5.

3. Application of equal-area criterion with the assumption of a round rotor and constant voltage behind transient reactance does not always give correct results and when stability is predicted by this method in some of the cases. The system may actually run to unstable operation. This can be noticed by studying case III and V, Chapter 5.

Thus, in the present work, attempts have been made to derive analytical methods to solve some of the stability problems in a one machine system with particular emphasis on the use of voltage regulator in the steady-state operation and to derive and use a more general torque equation to study the system under large oscillation operations.

It is hoped that the investigations should prove to be of value in the field of stability.

APPENDIX - Tables

TABLES FOR NYQUIST PLOTS:

Table 1.

Omega, ω	0.1	0.5	0.9	1.0	2.0	5.0	10.0
Real Part	-7029.83	-106.13	-9.25	-0.37	29.55	38.46	39.76
Imaginary Part	3045.39	8.93	-7.36	-7.05	-3.78	-1,48	-0.74

	Table 2.		Table 3.	
Omega, ω	Real Part	Imaginary Part	Real Part	Imaginary Part
0.01	-424880.34	14336.28	146391.91	-13352.31
0.10	-3388.38	2259.02	663.97	-2426.46
0.20	-125.23	378.12	-560.24	-364.63
0.30	-41.31	-41.51	-195.05	52.73
0.50	-53.90	-36.99	1.55	16.16
0.80	-10.14	-4.01	10.17	0.79
0.85	-5.48	-2.07	10.79	1.49
0.90	-1.42	-0.56	11.53	2.25
0.95	2.11	0.63	12.33	3.00
1.00	5.23	1.59	13.18	3.70
5.00	38.36	2.79	30.39	3.52
10.00	39.66	1.45	31.29	1.82
100.00	40.09	0.14	31.59	0.18

Table 4.

Omega ω	0.01	0.10	0.20	0.50	0.90	0.95	1.0	1.05	10.0
Real Part	-518433.98	-3104.14	-637.73	-93.22	-8.19	-4.23	-0.88	1.95	26.46
Imaginary Part	43507.52	1138.95	98.66	-6.53	-1.82	-1.95	-2.08	-2.20	-0.67

Table 5.

Omega ω	0.19	0.20	0.50	0.85	0.90	0.95	1.00	1.05	10.0	100.0
Real Part	473.15	-11.43	-43.51	-7.16	-3.53	-0.34	2.45	4.90	28.70	28.94
Imaginary Part	-1283.57	-358.78	-58.94	-4.25	-2.07	-0.43	0.79	1.72	0.90	0.09

Table6.

Omega ω	0.10	0.20	0.50	0.85	0.90	0.95	1.0	1.05	10.0	100.0
Real Part	1314.91	218.72	-9.86	-4.16	-1.98	0.04	1.89	3.57	22.38	22.59
Imaginary Part	-1158.59	-332.56	-65.84	-7.42	-4.66	-2.52	-0.85	0.44	1.17	0.11

Table 7.

Omega ω	0.01	0.10	0.50	0.90	0.95	1.0	10.0	100.00
Real Part	29186.11	-346.68	-74.17	-5.53	-2.00	1.03	30.29	30.59
Imaginary Part	-16357.77	-1301.36	-36.06	-2.94	-1.96	-1.19	0.52	0.05

Table 8

Omega ω	0.01	0.10	0.50	0.80	0.85	0.90	1.0	10.0	100.0
Real Part	165805.10	719.80	-63.09	-5.78	-0.88	3.31	10.04	40.67	40.99
Imaginary Part	-23688.20	-1879.40	-47.75	-5.34	-3.17	-1.51	0.76	1.02	0.10

Table 9

Omega ω	0.01	0.1	0.5	1.0	1.5	10.0	100.0
Real Part	-1718185.40	-45737.64	-406.24	-67.61	-9.49	34.79	35.78
Imaginary Part	31911.06	2486.72	25.10	-22.92	-20.27	-3.64	-0.36

Table 10

δ (degrees)	0	±5	±10	±15	±20	±30	±40	±50	±60
-(133 sinδ + 31.2 sin 2δ)	0	∓17.0	∓33.7	∓50.0	∓65.6	∓93.8	∓116.2	∓132.7	∓142
.34(1-cos 2δ)	0	.0052	.021	.046	.08	.17	.281	.4	.51
dv/dδ =0 v =		∓3270	∓1610	∓1090	∓820	∓550	∓413	∓330	∓278
dv/dδ =.2 v =	0	∓82.8	∓152	∓203	∓234	∓253	∓241	∓220	∓200
dv/dδ =.4 v =	0	∓42	∓80	∓112	∓136	∓164	∓170	∓165	∓156
dv/dδ = .6 v =	0	∓28	∓ 54	∓77.5	∓96.5	∓122	∓131.5	∓132.7	∓128
dv/dδ = 1.0 v =	0	∓17	∓33	∓ 48	∓60.7	∓80	∓90.5	∓94	∓ 94
dv/dδ=2.8 v =	0	∓ 8	∓16.6	∓24	∓31.5	∓43.2	∓50.6	∓55	∓56.5
dv/dδ=5.0 v =	0	∓3.4	∓6.7	∓ 9.9	∓12.9	∓18.2	∓22	∓24.5	∓25.7
dv/dδ=-.2 v =	0	±87	±168	±325	±548				
dv/dδ=-.4 v =	0	±43	±89	±142	±205	±407			
dv/dδ=-.6 v =	0	±28.6	±58	±90	±126	±218	±364		
dv/dδ=-1.0 v =	0	±17.1	±34.4	±52.5	±71.5	±113	±162	±220	±290
dv/dδ=-2.0 v =	0	± 8	±17.1	±25.7	±34.2	±51.2	±67.5	±82.5	±95

Table 11

δ (degrees)	0	10	20	30	40	50	60	70
$133 \sin\delta + 31.2 \sin 2\delta$	0	33.7	65.6	93.8	116.2	132.7	142	145
$121 - (133 \sin\delta + 31.2 \sin 2\delta)$	121	87.3	55.4	27.2	4.8	-11.7	-21	-24
$.34(1-\cos 2\delta)$	0	.021	.08	.17	.281	.4	.51	.6
$dv/d\delta =$ v= 0	∞	4150	692	160	17.1	-29.2	-41	-40
v = .25	483	321	168	65	9.1	-18	-27.5	-28.2
v = .5	242	168	195.7	40.6	6.15	-13	-21	-21.8
v = 1.0	121	85.5	51.2	23.2	3.75	-8.35	-14	-15
v= 2.0	60.5	43.2	26.6	12.6	2.1	-4.86	-8.4	-9.25
v= -0.25	-483	-380	-326	-340	+155	-78	-84	-68.5
v= -0.5	-242	-182.5	-132	-83	-21.8	+170		
v= -1.0	-121	-89.5	-60.2	-32.8	-6.6	+19.5	+42.8	+60
v= -2.0	-60.5	-49.2	-28	-14.9	-2.8	+ 7.3	14.1	+17.2

Table 12

δ (degrees)		0	20	40	60	80	100	120	140
$262 \sin\delta + 31.2 \sin 2\delta$		0	109.5	198.7	253	269.7	247.3	199	137.3
$281-(262 \sin\delta + 31.2 \sin 2\delta)$		281	171.5	82.3	28	11.3	33.7	82	143.7
$.34(1-\cos 2\delta)$		0	.08	.28	.51	.66	.66	.51	.28
$dv/d\delta=$	$v=0$	∞	2150	294	55	17.1	51	161	510
	$v=.25$	1120	518	162	37	11.8	35	108	280
	$v=.5$	560	294	106	28	7.75	29	82	183
	$v=1.0$	281	158	64	18.8	6.68	20.2	54	112
	$v=2.0$	140	82	36	11.2	4.25	12.6	32.7	62.5
	$v=5.0$	56	33.7	15.6	5.1	2.0	6.0	14.7	27.2
	$v=-.25$	-1120	-1000	+2460	+ 108	+27.5	+ 82	+315	+4750
	$v=-.5$	-560	-408	-375	+2800	+70.5	+210	+8200	+510
	$v=-1.0$	-281	-188	-114	-57	-31.4	-93.5	-168	-198

Table 13

δ (degrees)	0	15	30	45	60	75	90	120
262 sinδ+ 31.2 sin 2δ	0	83.4	158	209.2	253	268.6	262	199
256-(262 sinδ +31.2 sin2δ	256	172.6	98	46.8	3.0	-12.6	-6	57
.34(1-cos2δ)	0	.046	.17	.34	.51	.635	.68	.51
dv/dδ= v= 0	∞	3750	576	134	5.9	-20.2	-8.8	112
v= .25	1020	580	240	78.5	3.94	-14.4	-6.5	75
v= .5	510	316	146	55	3.0	-11.1	-5.1	57
v= 1.0	256	165	84	34.5	2.0	-7.7	-3.6	37.7
v= 2.0	128	84	45	19.8	1.2	-4.7	-2.25	22.7
v= 5.0	51	34	19	8.7				
v= -.25	-1020	-820	(-1220	+ 513	+11.6	-32.8	-13.4	+220
v= -.5	-510	-375	-297	-287	+300	-89	(-33.4	+5700
v= -1.0	-256	-180	-118	-70	-6.1	+34.5	+18.8	-116
v= -2.0	-128	- 88	-53.5	-28	-2.0	+9.25	+4.55	-38.2

Table 14

δ (degrees)	0	20	40	60	80	100	120	140
$175 \sin\delta + 31.2 \sin 2\delta$	0	78.1	143.7	178	182.7	161.3	124	82.3
$187.5-(175\sin\delta + 31.2 \sin 2\delta)$	187.5	109.4	43.8	9.5	4.8	26.2	63.5	105.2
$.34(1-\cos 2\delta)$	0	.08	.28	.51	.66	.66	.51	.28
$dv/d\delta =$ v= 0	∞	1360	156	18.6	7.3	40	124	375
v= .25	750	330	86	12.5	5.3	27.8	83.5	198
v= .5	375	189	56	9.5	4.15	22.6	63.5	134
v= 1.0	187.5	101	34.2	6.3	2.9	15.8	42.0	82
v= 2.0	93.7	52.5	19.3	3.8	1.8	10.0	25.2	46
v= -.25	-750	-643	1460	36.6	11.7	64	244	3500
v= -.5	-375	-260	-200	950	30	164	6350	-480
v= -1.0	-187.5	-119	-61	-19.4	-14.1	-77	-130	-146
v= -2.0	-93.7	-57	-25.3	-6.4	-3.6	-19.6	-42.5	-61

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