STABILITY OF
A
TWO-MACHINE SYSTEM

A Dissertation
submitted in partial fulfilment
of the requirements for the Degree
of
MASTER OF ENGINEERING
in
POWER SYSTEM ENGINEERING
(ELECTRICAL ENGINEERING)

By
RABINDRA NATH CHOUDHURY

Acc. No. 64075
Date 3 - JAN 1967

DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE
(INDIA)
August, 1966

CERTIFICATE

Certified that the dissertation entitled "STABILITY OF A TWO-MACHINES SYSTEM" which is being submitted by Shri Rabindra Nath Choudhury in partial fulfillment for the award of Degree of Engineering in POWER SYSTEM ENGINEERING of the University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for a period of 7 months from Dec. '65 to June '66 for preparing dissertation for Master of Engineering Degree at the University.

Dated. 9.8. 1966.

Pmukhopadhyay

(P. Mukhopadhyay)
Associate Professor
Electrical Engineering Deptt.,
University of Roorkee,
R_o_o_r_k_e_e.

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Rabindranath Choudhury
(Rabindranath Choudhury)

Dated
Roorkee the 8|8| 1966.

C O N T E N T S.

S Y N O P S I S.

This thesis deals with the dynamic stability of a system, consisting of a synchronous generator connected to a synchronous motor through a tie-line - the tie-line having resistance and reactance. The analysis is an extension of the two-reaction theory and equations relating direct and quadrature axes quantities are derived.

The small displacements theory is introduced to linearise the non-linear equations of the system. The characteristic equations of the system after linearisation are found out and stability is checked by Routh's-Hurwitz criterion. The system equations are arranged in the proper form, to be represented by a closed-loop system. Nyquist criterion is then applied to ascertain the stability of the systems for different operating conditions.

LIST OF SYMBOLS:

δ = Angle between the direct axis of the synchronous generator and the direct axis of the synchronous motor in degrees.

f = frequency, cycles/sec. (50 c/s through out)

ω = radians per sec.

M = Angular momentum in p.u.

H = Inertia constant; $M = \frac{H}{\pi f}$

T_i = Input torque

T_m = Electromagnetic torque.

e_{fd} = Excitation voltage.

E = open circuit terminal voltage at normal speed.

p = $\frac{d}{dt}$ = time-derivative operator.

θ = rotor angle in Electrical radians.

$p\theta$ = Rotor angular velocity.

Δ = small change in a quantity.

e = system voltage.

e_a, e_b, e_c = phase voltages.

i_a, i_b, i_c = phase currents.

ψ = flux-linkages.

Subscripts d and q indicate direct and quadrature axes components respectively.

e_d and e_q = d and q axis voltages

i_d and i_q = d and q axis currents

x_d and x_q = d and q axis synchronous reactances.

r = total armature circuit resistances p.u.

r_e = total tie-line resistance in p.u.

x_e = total tie-line reactance in p.u.

$x_d(p)$ = impedance operator relating to the direct-axis armature flux-linkages with the d-axis armature current.

$X_{q(p)}$ = impedance operator relating the q-axis armature flux linkages with the q-axis armature current.

$G_{(p)}$ = operator relating the d-axis armature linkages with the d-axis field-excitation voltage of the generator.

$G(p)$ = operator relating the d-axis armature linkages with the d-axis field excitation voltage of the Exciter.

T'_d = d-axis transient short-circuit time constant of the machine.

T''_d = d-axis sub-transient short-circuit time constant of the machine.

$T_{d'_o}$ = d-axis transient open-circuit time constant.

$T_{d''_o}$ = d-axis is subtransient open-circuit time constant.

T_r = d-axis transient short-circuit time constant of the Exciter.

D = System damping, including effect of damper circuits.

The suffix "1" stands for the Generator and suffix "2" stands for the synchronous motor through out.

CHAPTER 1.

INTRODUCTION.

INTRODUCTION:

The importance of power system stability does not require any introduction. The problem is well known to the Electrical Engineers. With the growth of Grid Systems at voltages of 400kV and above, it is to be expected that leading power factor operations of synchronous generators under highly loaded conditions becomes increasingly difficult to avoid. With generators unit size increasing, loss of stability in any subsequent dynamic region of operation of any one machine becomes more serious problem. Greater reliance is therefore, is placed on well designed control system which extend the operating margin of stability.

It is well known that by the inclusion of voltage regulator equipment, the unit of stable operation of the synchronous machines can be extended to the "dynamic zone" of operation, and a number of authoritative papers(1,2,3,4,6,7) have been published describing theoretical and computer- assisted investigations into the maintenance of stability in that zone.

Messerle,(9) investigates the effect of individual alternator parameters and controller constants on the dynamic stability limit (i.e. the steady state stability limit of the controlled alternator) and the optimum operating condition for various regulators and governors are established.

The steady state stability(9) limit of synchronous alternators can be modified by controlling prime-mover torque and the field voltage. The steady state stability limit of a controlled alternator is defined there as dynamic stability limit. This limit is defined largely by the feedback parameters by the governors and continuous acting regulators. Very important also are the alternators characteristics, the load and the system to which the alternator is connected.

In the recent years particular attention has been focussed on the dynamic limit of leading power factor operation following the trends of modern power systems. The effects of several types of load system on the dynamic limit of an alternator have been investigated in some recent publications[7,8] considering the leading power factor region. In particular, the improvement that can be achieved by using fast continuous output-voltage regulator has been the specific interest. The effects of some of the feedback parameters introduced by regulators has been discussed for a few typical machines operating at unity power factor[3,5] and the method investigating the combined effects of governors and control of the alternator field excitation have been investigated in separate papers.[5]

Messerle[5,9] concludes that the operations of an alternator under dynamic condition is better for machines with low per-unit inertia constant and large field line constant; changes in alternator impedances also affect the dynamic stability limit and have to be allowed for any general investigation. Higher speeds improve stability and with increasing output ratings if they are compared on a per-unit basis, stability is improved.

The most recent approach to study the stability limit is forwarded by Gove[10] and Laughton,[11] the first one being the geometrical construction of the stability limits of synchronous machines and the second one being the matrix analysis. A simple geometrical construction on a "Capacity chart" of machine is used to display the operating limits.

Laughton[11] presents an obvious extension of one method where the frequency response method can be avoided, and by means of matrix algebra, a set of general co-efficients related to the well known Heffron and Phillip[11] constants is derived, which allows complex impedance terms to be included in the analysis. A simple alternative in keeping with modern systems analysis is to formulate the linearised problems, nor in terms of non linear differential equations, but in terms of linearised differential equations. It is the investigation of the latter approach using matrix algebra, which is followed there, with the basic equations established, under various circumstances.

In this thesis, the author deals with the problem of stability of a synchronous generator connected to a synchronous motor through a tie line- the tie line having inductance and resistance and the influence of fast acting automatic voltage regulator on the system stability is taken into consideration in particular.

The system equations are deduced assuming the tie-line reactance and resistance to be included as leakage values of the synchronous motor. In the initial analysis of the problem, the author takes the works of concordia, C[1] and Mukhopadhyay,[12] as guiding works but has avoided all assumptions. The system equations are deduced relating direct and quadrature axis[1] quantities considering the effect of voltage regulator.

The small displacement theory[14] is applied and the stability of the system is initially determined by applying Routh's criterion[13] The analysis is based on the frequency response method of the system equations forwarded by Aldred and Shack Shaft,[4] and Concordia.[1]

The Routh's criterion does not provide with the margin of stability or unstability of the system, and to ascertain the limit of stability, the general system torque equation is represented by a closed-loop system[4] and the stability is ascertained by Nyquist-Criterion. The effect of damper windings is neglected but the system equations take care of all other factors, such as armature resistances and tie-line resistance, etc.

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CHAPTER 2

ANALYSIS OF A SYNCHRONOUS GENERATOR CONNECTED TO A SYNCHRONOUS MOTOR THROUGH A TIE-LINE

ANALYSIS OF A SYNCHRONOUS GENERATOR CONNECTED TO A SYNCHRONOUS MOTOR THROUGH A TIE-LINE.

2.1. INTRODUCTION.

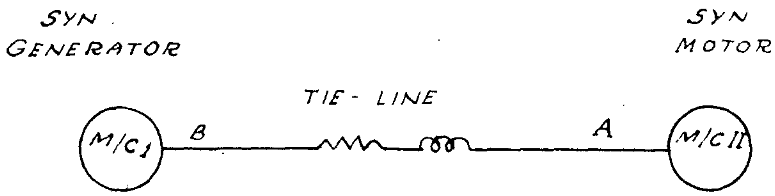
Extensive work has been done on the analysis of the synchronous machine, so far. Established work is also there, for the analysis of the inter-connections of a number of machines. Hence, in this work, the author has analysed a synchronous generator connected to a synchronous motor through a tie-line, the tie-line having the resistance and inductance. The entire analysis is based upon the two-reaction theory of synchronous machine.[13] The two machines, connected directly by a tie-line are, however, assumed to be ideal, as defined by Park.[13]

2.2. MATHEMATICAL ANALYSIS.[20]

The relations between the voltages and currents of the two machines are analysed relating to the direct and quadrature axes of the two machines. The synchronous generator and the synchronous motor are denoted as machine I and machine II for all further analysis. The phasor diagram with respect to d and q-axis are shown in fig.2.2. An initial separation of angle δ is assumed between the two direct-axes of the machines, and "a" phase is taken as a common axis of the two machines. Here, the current and voltages of machine II will be expressed in terms of the machine I.

These equations equally will apply for currents, voltages and fluxes.

SYNCHRONOUS.
FIG. 2.1 GENERATOR CONNECTED TO A SYNCHRONOUS MOTOR THROUGH A TIE-LINE.



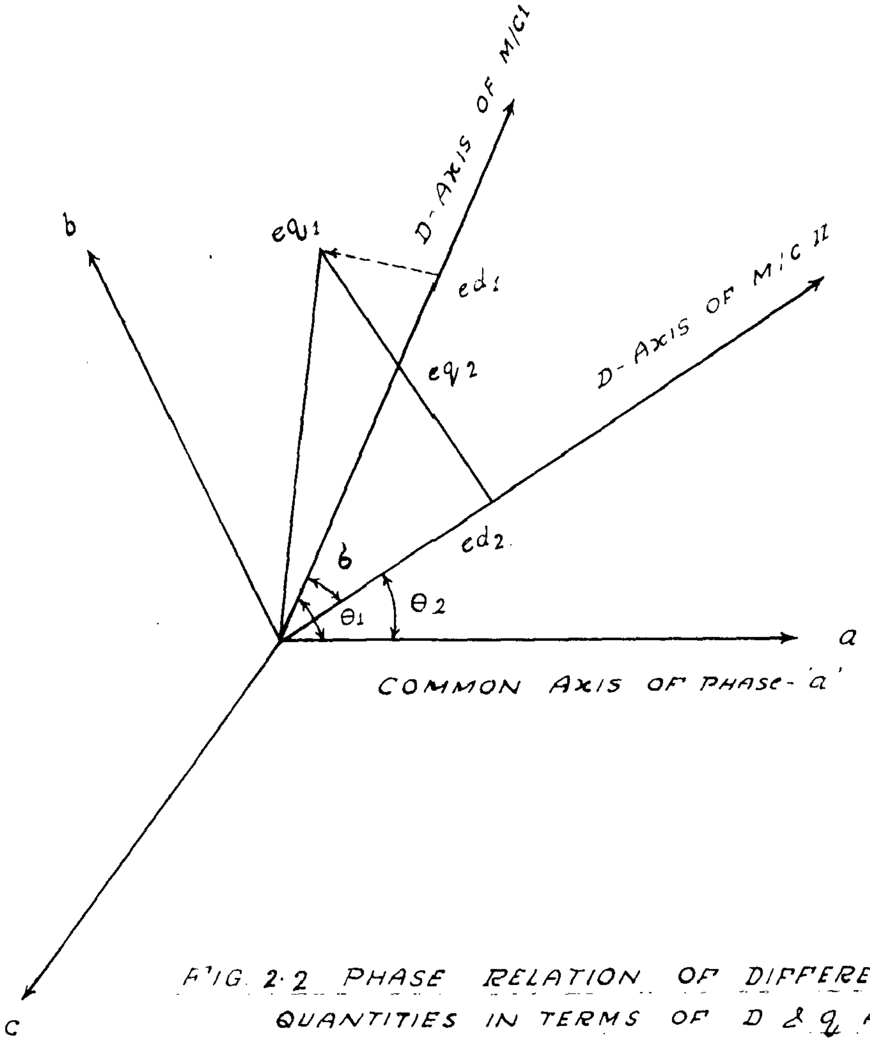


FIG. 2.2 PHASE RELATION OF DIFFERENT QUANTITIES IN TERMS OF D & q AXIS.

The terminal voltage relations are,

$$e_{a1} = e_{a2}$$
$$e_{b1} = e_{b2} \quad \ldots \quad \ldots \quad (2.1)$$
$$e_{c1} = e_{c2}$$

The armature current relations are-

$$i_{a1} = - i_{a2} \quad \ldots$$
$$i_{b1} = - i_{b2} \quad \ldots \quad \ldots \quad (2.2)$$
$$i_{c1} = - i_{c2}$$

The angle relation is as shown in the fig.No.2.2.

$$\theta_1 = \delta + \theta_2 \quad \ldots \quad \ldots \quad (2.3)$$

The following relations are well known-

$$i_a = i_d \cos\theta - i_q \sin\theta + i_0$$
$$i_b = i_d \cos(\theta-120°) - i_q \sin(\theta-120°) + i_0 \quad (2.4)$$
$$i_c = i_d \cos(\theta+120°) - i_q \sin(\theta+120°) + i_0$$

and similarly for the voltages-

$$e_a = e_d \cos\theta - e_q \sin\theta + e_0$$
$$e_b = e_d \cos(\theta-120°) - e_q \sin(\theta-120°) + e_0 \quad (2.5)$$
$$e_c = e_d \cos(\theta+120°) - e_q \sin(\theta+120°) + e_0$$

From the above equations (2.1), (2.2), (2.3), (2.4) and (.25), all the currents and voltages of machine II can be expressed in terms of the machine I and the vice-versa. And the following sets of equations are obtained:

$$e_{d1} = e_{d2} \cos\delta_0 + e_{q2} \sin\delta_0$$
$$e_{q1} = -e_{d2} \sin\delta_0 + e_{q2} \cos\delta_0 \quad \ldots \quad \ldots \quad (2.6)$$

$$e_{d2} = e_{d1} \cos\delta_0 - e_{q1} \sin\delta_0$$
$$e_{q2} = e_{d1} \sin\delta_0 + e_{q1} \cos\delta_0 \quad \ldots \quad \ldots \quad (2.6a)$$

$$i_{d1} = -i_{d2} \cos\delta_0 - i_{q2} \sin\delta_0$$
$$i_{q2} = i_{d2} \sin\delta_0 - i_{q2} \cos\delta_0 \quad \ldots \quad \ldots \quad (2.6b)$$

and

$$i_{d2} = -i_{d1} \cos\delta_0 + i_{q1} \sin\delta_0$$
$$i_{q2} = -i_{d1} \sin\delta_0 - i_{q1} \cos\delta_0 \quad \ldots \quad \ldots \quad (2.6c)$$

<u>The steady-state current in the system</u>

Here, an expression will be found out for the d and q components of the current flow in the system.

We know that-

$$i_d = \frac{2}{3}\left[i_a \cos\theta + i_b \cos(\theta-120^\circ) + i_c \cos(\theta+120^\circ)\right]$$
$$i_q = -\frac{2}{3}\left[i_a \sin\theta + i_b \sin(\theta-120^\circ) + i_c \sin(\theta+120^\circ)\right] \quad (2.7)$$

The i_d and i_q of the above relations show that, they may be regarded as projections of the magnitudes of e_a, e_b, e_c, i_a, i_b, i_c on the mutually perpendicular direct and quadrature axes for the two machines, if e_a, e_b, e_c etc. are plotted along three equally spaced a, b, c axes common to the two machines.

From the Park's original equations[13] for the either machines.

$$e_d = e_m \sin \delta_0$$

$$= p\,\psi_d - \nu\,\psi_q - r i_d \qquad \ldots \qquad \ldots \qquad (2.8)$$

$$e_q = e_m \cos \delta_0$$

$$= p\,\psi_q + \nu\,\psi_d - r i_q \qquad \ldots \qquad \ldots \qquad (2.9)$$

where,

$$\nu = p\,(\omega_0 t - \delta) = \omega_0 - p\delta \ldots \qquad \ldots \qquad (2.10)$$

In the steady-state,

$$p = 0, \quad \omega_0 = 1.$$

then for either machines,

$$e_d = -\,\psi_q\ r i_d$$

$$= x_q\, i_q - r i_d \qquad \ldots \qquad \ldots \qquad (2.11)$$

$$e_q = \psi_d - r i_d$$

$$= E - x_d\, i_d - r i_d \qquad \ldots \qquad \ldots \qquad (2.12)$$

where,

r is the armature resistance.

Then from equations (6), it is found,

$$x_{q1}\, i_{q1} - r_1 i_{d1} = x_{q2}\, i_{q2} \cos\delta_0 + (E_2 - x_{d2} i_{d2}) \sin\delta_0$$

$$- r_2 i_{d2} \cos\delta_0 - r_2 i_{q2} \sin\delta_0 \qquad (2.13)$$

and

$$E_1 - x_{d1}\, i_{d1} - r_1 i_{q1} = - x_{q2} i_{q2} \sin\delta_0 + (E_2 - x_{d2} i_{d2}) \cos\delta_0$$

$$+ r_2 i_{d2} \sin\delta_0 - r_2 i_{q2} \cos\delta_0 \qquad (2.14)$$

now, substituting the equations (6c) in the above two equations, it is obtained,

$$x_{q1}\, i_{q1} - r_1 i_{d1} = (-i_{d1}\ \mathrm{Sin}\,\delta_0 - i_{q1}\mathrm{Cos}\,\delta_0)x_{q2}\ \mathrm{Cos}\,\delta_0 + E_2 \mathrm{Sin}\,\delta_0 - x_{d2} x$$
$$(i_{d1}\ \mathrm{Cos}\,\delta_0 + i_{q1}\mathrm{Sin}\,\delta_0)\mathrm{Sin}\,\delta_0 - r_2(-i_{d1}\ \mathrm{Cos}\,\delta_0 +$$
$$i_{q1}\mathrm{Sin}\,\delta_0)\mathrm{Cos}\,\delta_0 - r_2(-i_{d1}\mathrm{Sin}\,\delta_0 - i_{q1}\mathrm{Cos}\,\delta_0)\mathrm{Sin}\,\delta_0 . .$$

or, $$i_{d1}(-r_1 + x_{q2}\ \mathrm{Sin}\,\delta_0 \mathrm{Cos}\,\delta_0 - x_{d2}\ \mathrm{Cos}\,\delta_0\ \mathrm{Sin}\,\delta_0 - r_2\ \mathrm{Cos}^2\delta_0 - r_2\mathrm{Sin}^2\delta_0) +$$
$$i_{q1}(x_{q1} + x_{q2}\mathrm{Cos}^2\delta_0 + x_{d2}\ \mathrm{Sin}^2\delta_0)$$
$$= E_2\mathrm{Sin}\,\delta_0$$

(2.15)

Similarly,

$$E_1 - x_{d1}i_{d1} - r_1 i_{q1} = x_{q2}\ (i_{d1}\ \mathrm{Sin}\,\delta_0 + i_{q1}\ \mathrm{Cos}\,\delta_0)\mathrm{Sin}\,\delta_0 + E_2\mathrm{Cos}\,\delta_0 +$$
$$x_{d2}(i_{d1}\mathrm{Cos}\,\delta_0 - i_{q1}\mathrm{Sin}\,\delta_0)\ \mathrm{Cos}\,\delta_0 + r_2(i_{d1}\ \mathrm{Cos}\,\delta_0 -$$
$$i_{q1}\ \mathrm{Sin}\,\delta_0)\mathrm{Sin}\,\delta_0 + r_2(i_{d1}\mathrm{Sin}\,\delta_0 + i_{q1}\mathrm{Cos}\,\delta_0)\mathrm{Cos}\,\delta_0$$

or, $$i_{d1}(x_{d1} + x_{q2}\mathrm{Sin}^2\delta_0 + x_{d2}\ \mathrm{Cos}^2\delta_0) + i_{q1}\ (x_{q2}\ \mathrm{Sin}\,\delta_0\ \mathrm{Cos}\,\delta_0 + r_1 -$$
$$x_{d2}\ \mathrm{Sin}\,\delta_0\ \mathrm{Cos}\,\delta_0 + r_2\ \mathrm{Cos}\ 2\delta_0) = E_1 - E_2\ \mathrm{Cos}\,\delta_0$$

... (2.16)

or, $$i_{d1}\left(x_{d1} + \frac{x_{d2} + x_{q2}}{2} + \frac{x_{d2} - x_{q2}}{2}\ \mathrm{Cos}\ 2\delta_0\right) + i_{q1}\left(-\frac{x_{d2} - x_{q2}}{2}\ x\right.$$
$$\left.\mathrm{Sin}\ 2\delta_0 + r_1 + r_2\mathrm{Cos}\ 2\delta_0\ \right) = E_1 - E_2\ \mathrm{Cos}\,\delta_0$$

or, $$i_{q1} = \frac{E_2\mathrm{Sin}\,\delta_0 + i_{d1}\ \left(r_1 + r_2 + \frac{x_{d2} - x_{q2}}{2}\mathrm{Sin}\ 2\delta_0\right)}{\left(x_{q1} + \frac{x_{d2} + x_{q2}}{2} - \frac{x_{d2} - x_{q2}}{2}\ \mathrm{Cos}\ 2\delta_0\right)} \qquad (2.17)$$

$$\therefore i_{d1} = \frac{E_1 - E_2 \cos\delta_o - \frac{B}{A} E_2 \sin\delta_o}{x_{d1} + \frac{x_{d2}+x_{q2}}{2} + \frac{x_{d2}-x_{q2}}{2} \cos 2\delta_o + (r_1 + r_2 + \frac{x_{d2}-x_{q2}}{2} \sin\delta_o)\frac{B}{A}} \quad ...(2.18)$$

where,

$$A = x_{q1} + \frac{x_{d2} + x_{q2}}{2} - \frac{x_{d2} - x_{q2}}{2} \cos 2\delta_o$$

$$B = r_1 + r_2 \cos 2\delta_o - \frac{x_{d2} - x_{q2}}{2} \sin 2\delta_o$$

The above two equations give the expressions for currents and these are similar to the equations derived by C.Concordia[20]

THE EFFECT OF VOLTAGE REGULATORS ON STABILITY LIMIT-

3.1. INTRODUCTION-

The steady-state stability limit of an alternator defines the maximum steady load the alternator can carry without falling out of synchronism. This limit can be found experimentally by loading the alternator, increasing the load in small steps, untill it becomes unstable. Theoretically, the load steps should be infinitesimally small to avoid transient disturbances which obscure the results.

The quick acting voltage regulator can increase the stability limit of a system is a well discussed subject. Kimbark[22] and Crary[26] stressed the need of automatic quick acting voltage regulators to improve the steady-state stability limit. These discussions & suggestions revealed the need of the voltage regulator to improve the stability limit, but, proper attempt was not noticed to formulate the equipment in conjunction with the other components of the system.

Later, in a Trans. paper, Concordia[1] co-related the voltage regulator action with the fundamental system equations. The author came to important conclusions that a properly designed voltage regulator can increase the steady state stability limit of a synchronous machine by a considerable amount. In the 2nd part of his paper, it was discussed the stability of a system-(a generator being connected to a synchronous motor) under the influence of automatic voltage regulators. He has shown that the power limit can be increase by 150% with the quick acting

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The divisions of transient reactances between two otherwise similar machines (acting as motor and generator) makes little difference, but if the field time constants of two otherwise similar machines are different, the maximum power limit is obtained if the field of the machine with smaller time constant is regulated.

Four years later, the same author, Concordia[2] tried to justify the use of an angle regulator in conjunction with an automatic voltage regulator to improve the system stability. An angle regulator was defined by the author as applied to synchronous machines as a regulator that varies the machine excitation voltage in response to changes in the angle between the rotor interpole axis and the effective system voltage so as to tend to restore the initially set angle.

The principal conclusions of that paper is that the dynamic action of the angle and voltage regulators near the system stability limit are essentially equivalent, so that their effects on stability limit can by proper regulator design by made practically indistinguishable. The need of angle regulator can not be therefore demonstrated on the basis of stability requirements.

In the normal operation of a synchronous generator over its load range, it is customary and indeed, it seems highly desirable and almost necessary to maintain voltage rather load angle constant.

Another difference in performance over the load angle is that as the operating angle increases the voltage drop for a given angle increases, so that the voltage regulator tends to act more and more in response to power input or system changes as the stability limit is approached.

Messerle and Bruck[5] in a paper states that when using continuously acting regulators and governors, negative feed-back can be introduced, and the alternator can be loaded beyond its ordinary steady-state stability limit and its load angle may increase past the normal maximum.

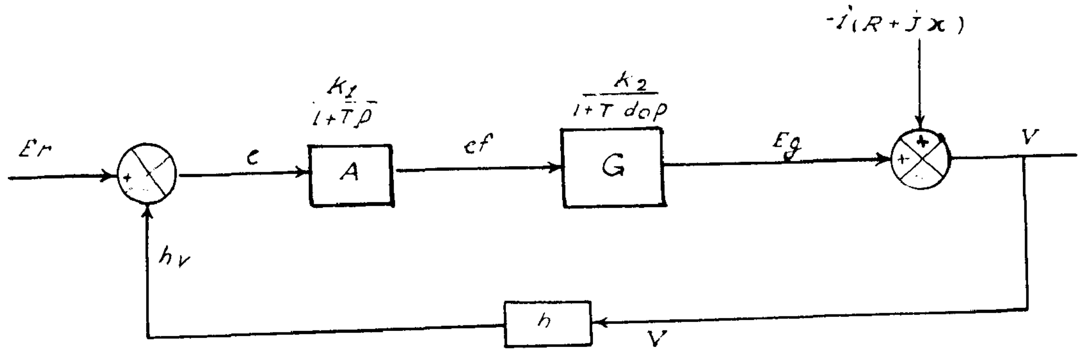
The over all steady-state or the dynamic stability range is to some extent inversely proportional to the synchronous reactance of the alternator, this applies particularly to the condition when the external reactance x_e is small. Thus the stability limit without continuous regulation can be achieved only by reducing the synchronous reactances which involves an increase in the frame size of the machine, with a considerable increase in the cost.

When using a continuously acting regulator, expensive alternators with large frame can be avoided, which reduces the over-all cost. In general, the stability range of a machine can be increased by 50-100% with continuous voltage regulators, even for a power factor as low as 0.3.

Aldred & Shackshaft[6] also showed that the steady-state stability limit is improved by the voltage regulator action; while the transient stability limit is not affected much.

SCHEME OF A VOLTAGE REGULATOR

(BLOCK DIAGRAM).



A = AMPLIFIER.

G = GENERATOR.

h = FEED - BACK.

FIG. 3.1 .

It treats the flow of load current as the disturbances as shown on the block-diagram.

The reference voltage is taken as constant,

$$\therefore \Delta E_g = \Delta e_f \times \frac{K_2}{1+T'_{d_0}p}$$

$$= \Delta e \times \frac{K_1}{1+T_r p} \times \frac{K_2}{1+T'_{d_0}p}$$

$$= -h\,\Delta V \times \frac{K_1}{1+T_r p} \times \frac{K_2}{1+T'_{d_0}p} \quad \ldots \quad (3.1)$$

for the steady-state condition-

$$\Delta E_g = -\Delta V h.\, K_1 K_2 \quad \ldots \quad \ldots \quad (3.2)$$

$\therefore$ The amplification factor is defined as-

$$K_1 = 'a' = -\frac{\Delta E_g}{h.\,\Delta V.\,K_2} \ldots \quad \ldots \quad (3.3)$$

In the process of the analytical analysis in this thesis, the author found it difficult to ascertain the value of the feed-back gain h without proper experimentation. Ofcourse, the other factors can be formulated & realistic data can be selected.

Concordia[1] in his investigation of two machine stability has drawn a series of graph between the power angle and the amplification factor. The most suitable value for "a" is between -1 to -35 and the maximum power limit is obtained at a = - 2.5.

Here, in this analytical analysis the value for the amplification factor is selected as a = -2.5.

3.2. THE REGULATOR ACTION:

The regulator is responsive to the changes in terminal voltage or in voltage at any specified point on the system and acts on the field voltage of the synchronous machine. The regulator introduces a change in the field voltage as a function of the change in magnitude of the terminal voltage e_a.

$$\therefore \Delta E = G_{(p)} \, \Delta e_a \qquad \ldots \qquad \ldots (3.4)$$

where,

$$e_a = \sqrt{e_d^2 + e_q^2} \qquad \ldots \qquad \ldots (3.5)$$

and $G(p) = \frac{a}{T_r p + 1}$ which is most simple operational expression for the action of the regulator and does not have time lag.

Differentiating (3.5) with respect to e_d & e_q,

$$\Delta e_a = e_d' \Delta e_d + e_q' \Delta e_q \qquad \ldots \qquad \ldots (3.6)$$

where, $e_d' = \frac{e_{d_0}}{e_{a_0}}$; $e_q' = \frac{e_{q_0}}{e_{a_0}}$.

The change in terminal voltage is-

$$\therefore \Delta E = G(p) \left[e_d' \Delta e_d + e_q' \Delta e_q \right] \qquad \ldots (3.7)$$

where,

$$e_{q_0} = \psi_{d_0} - r \, i_{q_0} \qquad \ldots \qquad \ldots (3.8)$$

and,

$$e_{d_0} = -\psi_{q_0} - r \, i_{d_0} \qquad \ldots \qquad \ldots (3.9)$$

CHAPTER 4.

THE SMALL DISPLACEMENT THEORY

THE SMALL DISPLACEMENT THEORY

4.1. INTRODUCTION:

The stability of synchronous machines has been studied by workers like Kimbark[22] and Crary[26] with the help of equal area criterion. In that method, the power angle characteristic is drawn either with constant field flux linkage or constant voltage behind the transient reactance. The limit of stability is found by a purely geometrical method, i.e. equating the accelerating and retarding energies. The method is good for first few swings; later on, due to the field decrement, the power angle characteristic is changed and it is not possible to ascertain stability. Moreover, some paradoxical results are also obtained, i.e. transient stability limit is greater than steady-state stability limit[22] When very/short clearing times are used, the transient stability limit, calculated on the basis of constant field flux (constant voltage behind the transient reactance) may even exceed the steady state limit with the faulted circ it switched out, calculated on the basis of constant field current (constant voltage behind saturated synchronous reactance) of value determined the prefault condition. This is possible because the transient reactance of the synchronous machine is considerably less than the saturated synchronous reactances.

Now-a-days, the method of small oscillations is commonly used in stability studies. The small displacement of motion of the machines is obtained by assuming that each variable changes by a very small amount during any transient disturbance. It is well known that a synchronous machine is analogous to a mass damper spring system. The small oscillation theory[14] effectively eliminates the non-linear terms in the equations which otherwise, is a non-linear differential equation of higher order.

The initial steady state conditions are denoted by a suffix '0'. Actually, the small displacement theory, initially described by Park[14] in relation to synchronous machine and to organise them into correct form for representating by a closed-loop system.

Initial values are found out first and then small displacements are applied to the equations of the voltages & torques.

The addition of damper windings may be may not have an effect on stability when a voltage regulator is used[4,6] Since it appears that damping created by voltage regulators action and subsidiary feed-backs may well outweigh any damping introduced by ammortisseur windings. The voltage regulator actions and the system load makes the system to behave almost as a critically damped. Thus in this analysis, the damper winding is not considered which reduces the further complication in the otherwise, tedious & complicated characteristic equation of the system studied.

The small displacement equations are derived and stability may now be tested by any of the criterion. Since it is well known, due to Lyapounoff[12] that a non-linear system be studied after linearising and that-

1. If the resulting linear system is stable, the original system is also stable, and

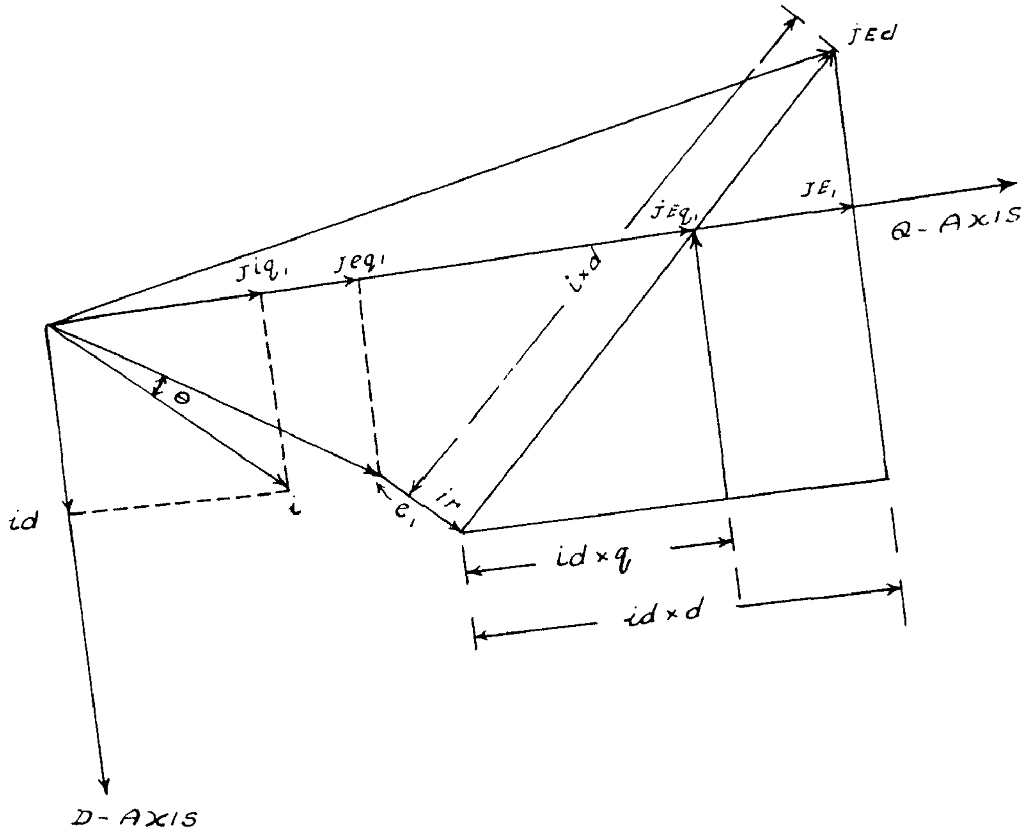
2. if the resulting system is unstable, the original system is unstable.

4.2. INCREMENTAL EQUATIONS:-

For a salient pole synchronous machine, the voltage equations in the absence of zero-sequence terms for balanced operation are as follows:

THE STEADY-STATE VECTOR DIAGRAM FOR THE SYNCHRONOUS MACHINES.[20]

POWER FACTOR ·8 (LAGGING)



FOR THE GENERATOR.

FIG. 4·1

NOT TO SCALE

NOT TO SCALE

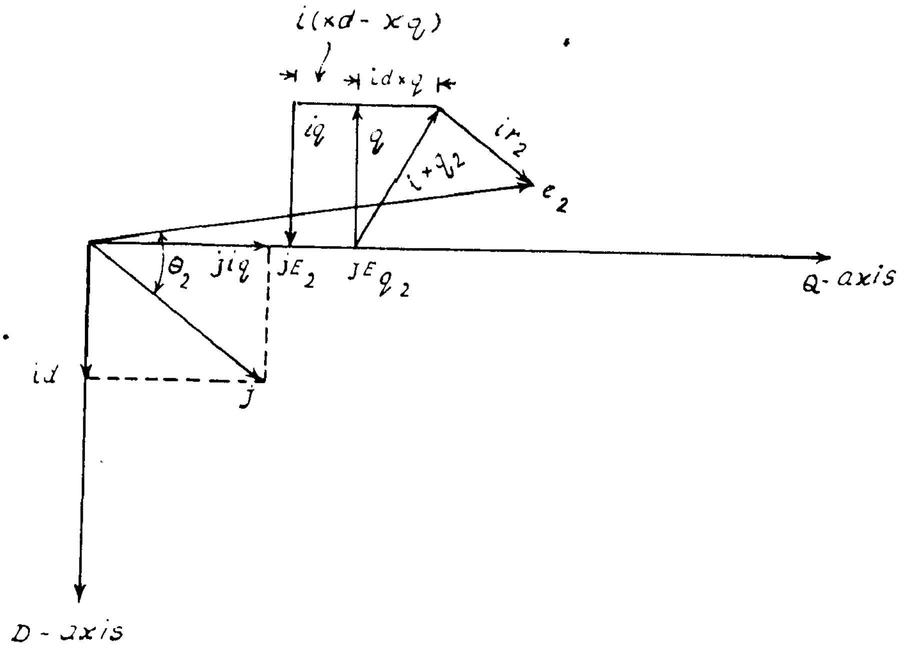


FIG. 4.2 PHASOR DIAGRAM OF THE SYN. MOTOR (STEADYSTATE) AT A LAGGING P.F.8

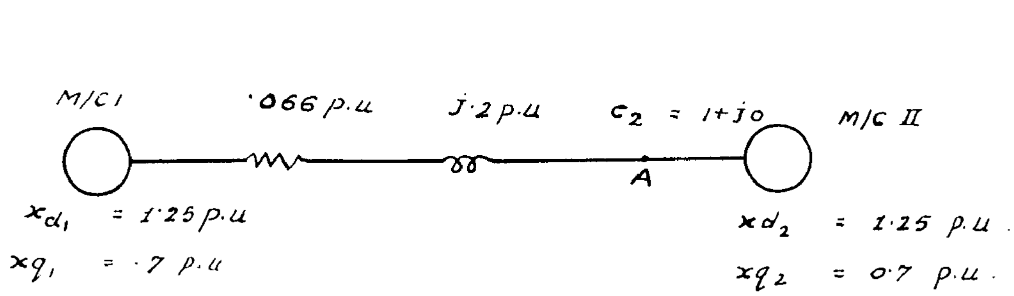


FIG. 4.3.

$$e_d = e_m \sin\delta$$

$$= p\psi_d - \nu\psi_q - r\, i_d \qquad \ldots \qquad \ldots (4.1)$$

$$e_q = e_m \cos\delta$$

$$= p\psi_q + \nu\psi_d - r\, i_q \qquad \ldots \qquad \ldots (4.2)$$

The quantities referred to the synchronous motor of the system under considered are denoted by suffix '2', hence, the voltages of the synchronous motor are as follows:

$$e_{d2} = p\psi_{d2} - \nu\psi_{q2} - r_2\, i_{d2} \qquad \ldots \qquad \ldots (4.3)$$

$$e_{q2} = p\psi_{q2} + \nu\psi_{d2} - r_2 i_{q2} \qquad \ldots \qquad \ldots (4.4)$$

where, $\nu = p(\,wt - \delta\,) = w_0 - p\delta \qquad \ldots \qquad \ldots (4.5)$

As all the flux linkage relations and all the rotor, circuit voltage relations are linear, the equations can be written in the operational form as follows:-

$$\psi_d = G_{(p)} e_f - i_d\, x_{d(p)} \qquad \ldots \qquad \ldots (4.6)$$

$$\psi_q = -\, x_{q(p)}\, i_q \qquad \ldots \qquad \ldots (4.7)$$

where, $G_{(p)} = \dfrac{1}{pT'_{d0} + 1} \qquad \ldots \qquad \ldots (4.8)$

$$x_{d(p)} = \frac{x_d(1+pT'_d)}{1+pT'_{d0}} \quad \ldots (4.9) \quad \& \quad x_{q(p)} = x_q \qquad \ldots (4.10)$$

Putting the values of the equations (4.6) and (4.7) in the equations (4.1) and (4.2), the voltage equations of the synchronous motor are-

$$e_{d2} = pG_{2(p)}\, e_{f2} - pi_{d2}\, x_{d2(p)} + x_{q2(p)} i_{q2} - r_2 i_{q2} \qquad \ldots (4.11)$$

$$e_{q2} = -\, p\, x_{q2(p)} i_{q2} + (G_{2(p)} e_{f2} - i_{d2} x_{d2(p)}) - r_2 i_{q2} \;. \; (4.12)$$

Differentiating with respect to δ and taking the steady-state value as '0' & incremental 'Δ' subscriptions-

$$\Delta e_{d2} = pG_{2(p)} \Delta E_2 - px_{d2(p)} \Delta i_{d2} + \nu_0 x_{q2(p)} \Delta i_{q2} + x_{q2(p)} i_{q2} \Delta \nu - r_2 \Delta i_{d2} \quad \ldots \quad \ldots (4.13)$$

and,

$$\Delta e_{q2} = - e_{m2} \sin \delta_0 \Delta \delta$$
$$= - p x_{q2(p)} \Delta i_{q2} + \nu_0 G_{2(p)} \Delta E_2 + G_{2(p)} E_2 \Delta \nu - x_{d2(p)} \Delta i_{d2} - x_{d2(p)} i_{d2} \Delta \nu - r_2 \Delta i_{d2} \quad \ldots (4.14)$$

C. Concordia(1) in his analysis of stability of two machines connected through a tie-line, has analysed the effect of voltage regulator action in both the machines. In his discussions and analysis of stability limit, he justifies that one voltage regulator is equally good from the point of view of stability limit. From this important inference, here in this analysis, the author will use only one regulator in the system. The entire analysis is based on the use of an automatic voltage regulator at the generator terminal only.

This states that the terms -

$$pG_{2(p)} \Delta E_2 = 0$$

&

$$G_{2(p)} \Delta E_2 = 0$$

in the equations (4.13) and (4.14). Thus the above two equations reduce to -

$$\Delta e_{d2} = - p x_{d2(p)} \Delta i_{d2} + v_0 x_{q2(p)} \Delta i_{q2} - x_{q2(p)} i_{q2} p \Delta \delta - r_2 \Delta i_{d2} \quad \ldots \quad \ldots (4.15)$$

$$e_{q2} = - p x_{q2(p)} \Delta i_{q2} - p G_{2(p)} E_2 \Delta \delta - v_0 x_{d2(p)} \Delta i_{d2} + x_{d2(p)} i_{d2} p \Delta \delta - r_2 \Delta i_{q2} \quad \ldots \quad \ldots (4.15a)$$

As the synchronous speed is maintained constant through out-

from
$$\nu = p(wt - \delta) = w - p\delta$$

$$\therefore \nu_0 = w_0 \text{ and } \Delta\nu = -p\Delta\delta \qquad \ldots \qquad \ldots (4.16)$$

Now differentiating the equations (2.6a) and (2.6c) of chapter 2, the following equations are obtained-

$$\Delta e_{d2} = \Delta e_{d1} \cos\delta_0 - \Delta e_{q1} \sin\delta_0 - (e_{d1} \sin\delta_0 + e_{q1} \cos\delta_0)\Delta\delta \qquad \ldots \qquad \ldots (4.17)$$

$$\Delta e_{q2} = \Delta e_{d1} \sin\delta_0 + \Delta e_{q1} \cos\delta_0 + (e_{d1} \cos\delta_0 - e_{q1} \sin\delta_0)\Delta\delta \qquad \ldots \qquad \ldots (4.18)$$

$$\Delta i_{d2} = -\Delta i_{d1} \cos\delta_0 + \Delta i_{q1} \sin\delta_0 + (i_{d1} \sin\delta_0 + i_{q1} \cos\delta_0)\Delta\delta \qquad (4.19)$$

$$\Delta i_{q2} = -\Delta i_{d1} \sin\delta_0 - \Delta i_{q1} \cos\delta_0 - (i_{d1} \cos\delta_0 - i_{q1} \sin\delta_0)\Delta\delta \qquad \ldots \qquad \ldots (4.20)$$

Now, the equations (4.17), (4.18), (4.19) & (4.20) are substituted in the equations (4.10) and (4.15), the following equations are obtained -

$$\Delta e_{d1} \cos\delta_0 - \Delta e_{q1} \sin\delta_0 = (\Delta i_{d1} \cos\delta_0 - \Delta i_{q1} \sin\delta_0) p x_{d2(p)} - (i_{d1} \sin\delta_0 + i_{q1} \cos\delta_0) x\, p\, x_{d2(p)} \Delta\delta - (\Delta i_{d1} \sin\delta_0 + \Delta i_{q1} \cos\delta_0)\, x\, w_0\, x_{q2(p)} - (i_{d1} \cos\delta_0 - i_{q1} \sin\delta_0)\, w_0\, x_{q2(p)} \Delta\delta - (-i_{d1} \sin\delta_0 - i_{q1} \cos\delta_0)\, x_{q2(p)}\, p\Delta\delta - r_2 \left[-\Delta i_{d1} \cos\delta_0 + \Delta i_{q1} \sin\delta_0) + (i_{d1} \sin\delta_0 + i_{q1} \cos\delta_0)\Delta\delta \right] + (e_{d1} \sin\delta_0 + e_{q0} \cos\delta_0)\Delta\delta \qquad \ldots \qquad \ldots (4.21)$$

Rearranging the terms, the equation is written as -

$$\Delta e_{d1} \cos\delta_0 - \Delta e_{q1} \sin\delta_0$$

$$= p\, x_{d2(p)} (\Delta i_{d1} \cos\delta_0 - \Delta i_{q1} \cos\delta_0) - v_0 x_{q2(p)}\, x$$

$$(\Delta i_{d1} \sin\delta_0 + \Delta i_{q1} \cos\delta_0) - r_2(-\Delta i_{d1} \cos\delta_0 + \Delta i_{q1} \sin\delta_0) +$$

$$\Big[-p\, x_{d2(p)} (i_{d1} \sin\delta_0 + i_{q1} \cos\delta_0) - v_0\, x_{q2(p)} (i_{d1} \cos\delta_0 -$$

$$i_{q1} \sin\delta_0) + x_{q2(p)} p\, (i_{d1} \sin\delta_0 + i_{q1} \cos\delta_0) - r_2 (i_{d1} \sin\delta_0 +$$

$$i_{q1} \cos\delta_0) + (e_{d1} \sin\delta_0 + e_{q1} \cos\delta_0)\Big] \Delta\delta \qquad \ldots (4.22)$$

Similarly for the equation (4.15)

$$\Delta e_{d1} \sin\delta_0 + \Delta e_{q1} \cos\delta_0$$

$$= \Delta e_{q2} - (e_{d1} \cos\delta_0 - e_{q1} \sin\delta_0)\, \Delta\delta$$

$$= p\, x_{q2(p)} \big[\Delta i_{d1} \sin\delta_0 + \Delta i_{q1} \cos\delta_0\big] + p x_{q2(p)} (i_{d1} \cos\delta_0 -$$

$$i_{q1} \sin\delta_0)\Delta\delta - p G_{2(p)} E_2 \Delta\delta + v_0\, x_{d2(p)} \big(\Delta i_{d1} \cos\delta_0 -$$

$$\Delta i_{q1} \sin\delta_0) - v_0\, x_{d2(p)} (i_{d1} \sin\delta_0 + i_{q1} \cos\delta_0)\Delta\delta +$$

$$x_{d2(p)}\, p\, (-i_{d1} \cos\delta_0 + i_{q1} \sin\delta_0)\Delta\delta - r_2 \big[-\Delta i_{d1} \sin\delta_0 -$$

$$\Delta i_{q1} \cos\delta_0 - (i_{d1} \cos\delta_0 - i_{q1} \sin\delta_0)\Delta\delta\big] - (e_{d1} \cos\delta_0 -$$

$$e_{q1} \sin\delta_0)\, \Delta\delta \qquad \ldots \qquad \ldots (4.23)$$

Rearranging the terms, the equation can be written as -

$$\Delta e_{d1} \sin\delta_0 + \Delta e_{q1} \cos\delta_0$$

$$= p\, x_{q2(p)} (\Delta i_{d1} \sin\delta_0 + \Delta i_{q1} \cos\delta_0) + v_0 x_{d2(p)} (\Delta i_{d1} \cos\delta_0 -$$

$$\Delta i_{q1} \sin\delta_0) + r_2 (\Delta i_{d1} \sin\delta_0 + \Delta i_{q1} \cos\delta_0) + \Big[p\, x_{q2(p)}\, x$$

$$(i_{d1} \cos\delta_0 - i_{q1} \sin\delta_0) - p\, G_{2(p)} E_2 - v_0\, x_{d2(p)} (i_{d1} \sin\delta_0 +$$

$$i_{q1} \cos\delta_0) + x_{d2(p)} p \big(-i_{d1} \cos\delta_0 + i_{q1} \sin\delta_0\big) + r_2 (i_{d1} \cos\delta_0 -$$

$$i_{q1} \sin\delta_0) - (e_{d1} \cos\delta_0 - e_{q1} \sin\delta_0)\Big] \Delta\delta \qquad \ldots (4.24)$$

Again, in per unit, the torque of the machine 1 is given by

$$T_1 = \psi_{d1}\, i_{q1} - \psi_{q1}\, i_{d1} + M_1 p^2 \theta_1 \qquad \ldots \qquad \ldots (4.25)$$

where $M_1 = \dfrac{H_1}{\pi f}$

Differentiating the above equation-

$$\Delta T_1 = \psi_{d10} \Delta i_{q1} - \psi_{q10} \Delta i_{d1} + i_{q10} \Delta \psi_{d1} - i_{d10} \Delta \psi_{q1} +$$
$$M_1 p^2 \Delta \theta_1 \qquad \ldots \qquad \ldots (4.26)$$

Now, replacing the values of ψ_{d01} and ψ_{q01} from the equations-

$$\psi_{d01} = E_1 - x_{d1}\, i_{d10} \qquad \ldots \qquad \ldots (4.27)$$

$$\psi_{q01} = - x_{q1}\, i_{q01} \qquad \ldots \qquad \ldots (4.28)$$

the incremental torque equation is obtained as -

$$\Delta T_1 = \psi_{d01} \Delta i_{q1} + i_{q01}\left(G_{(p)} \Delta E_1 - x_{d1(p)} \Delta i_{d1}\right) - \psi_{q01} \Delta i_{d1} -$$
$$i_{d0}\left(-x_{q1(p)} \Delta i_{q1}\right) + M_1 p^2 \Delta \theta_1 \qquad \ldots \qquad \ldots (4.29)$$

Now, replacing ΔE_1 from the equation (3.7) of chapter 3 and rearranging the terms, the equation reduces to -

$$\Delta T_1 = i_{q01} G_1(p) G_1(p) \left(e_d^0\, \Delta e_{d1} + e_q^0\, \Delta e_{q1}\right) - \left[\psi_{q01} + x_{d1(p)} i_{q01}\right] \times$$
$$\Delta i_{d1} + \left[\psi_{d01} + i_{d0}\, x_{q1(p)}\right] \times \Delta i_{q1} + M_1 p^2 \Delta \theta_1 \qquad \ldots (4.30)$$

As the electrical torque of the two machines are equal and opposite (as the machines of same ratings are assumed) and if there are no mechanical damping as explained in chapter 3, the accelerating torques may be equated-

$$M_1 p^2 \Delta \theta_1 = - M_2 p^2 \Delta \theta_2$$

$$= \frac{M_1 M_2}{M_1 + M_2}\, p^2 \left(\Delta \theta_1 - \Delta \theta_2\right)$$

$$= M' p^2 \Delta \delta \qquad \ldots \qquad \ldots (4.31)$$

Therefore, the ultimate incremental torque equation is-

$$\Delta T_1 = i_{q0} G_1(p) G_1(p) (e'_d \Delta e_{d1} + e'_q \Delta e_{q1}) - [\psi_{q0} + x_{d1}(p) i_{q1}] \times$$

$$\Delta i_{d1} + [\psi_{d0} + i_{d01}\, x_{q1}(p)] \Delta i_{q1} + M' p^2 \Delta \delta \qquad \ldots (4.32)$$

Now the incremental voltage equations of machine 1 can be written similar to equations (4.15) and (4.16)-

$$\Delta e_{d1} = -p\, x_{d1}(p) \Delta i_{d1} + \omega_0 x_{q1}(p) \Delta i_{q1} - x_{q1}(p)\, i_{q1} p \Delta \delta -$$

$$r_1 \Delta i_{d1} + p\, G_{1(p)}\, \Delta E_1 \qquad \ldots \qquad \ldots (4.33)$$

and

$$\Delta e_{q1} = -p\, x_{q2}(p) \Delta i_{q1} + \omega_0 G_1(p) \Delta E_1 - p\, G_{1(p)} E_1 \Delta \delta - \omega_0 x_{d1}(p) \times$$

$$\Delta i_{d1} + x_{d1}(p) i_{d1}\, p \Delta \delta - r_1 \Delta i_{q1} \qquad \ldots \qquad \ldots (4.34)$$

Now replacing ΔE_1 from the equation (3.7) of chapter 3, the voltage equations are obtained as.

$$\Delta e_{d1} = -p x_{d1}(p) \Delta i_{d1} + \omega_0 x_{q1}(p) \Delta i_{q1} - x_{q1}(p) i_{q1} p \Delta \delta - r_1 \Delta i_{d1} +$$

$$p G_1(p) G_1(p) (e'_d \Delta e_{d1} + e'_q \Delta e_{q1}) \qquad \ldots \qquad \ldots (4.35)$$

$$\Delta e_{q1} = -p\, x_{q1}(p) \Delta i_{q1} + \omega_0 G_1(p) G_1(p) \times (e'_d \Delta e_{d1} + e'_q \Delta e_{q1}) -$$

$$p G_{(p)} E_1 \Delta \delta - \omega_0 x_{d1}(p) \Delta i_{d1} + x_{d(p)}\, i_{d1} p \Delta \delta - r_1 \Delta i_{q1}$$

$$\ldots \qquad \ldots (4.35a)$$

Now, writing the equations (4.30), (4.22), (4.24), (4.34), (4.35a) in the matrix form, it is obtained that:-

Δe_{d1}	Δe_{q1}	Δi_{d1}	Δi_{q1}	$\Delta \delta$	
$i_{q1}G_{1(p)}g_{1(p)}e'_{d1}$	$i_{q1}G_{1(p)}g_{1(p)}e'_{q1}$	$-[\psi_{q1}+x_{d1(p)}i_{q1}]$	$\psi_{d1}+x_{q1(p)}i_{d1}$	$M'p^2$	$=\Delta T_1$
$-\cos\delta_0$	$\sin\delta_0$	$px_{d2(p)}\cos\delta_0-$ $w_0x_{q2(p)}\sin\delta_0+$ $r_2\cos\delta_0$	$-p\,x_{d2(p)}\sin\delta_0-$ $w_0x_{q2(p)}\cos\delta_0-$ $r_2\sin\delta_0$	$-px_{d2(p)}(i_{d1}\sin\delta_0+i_{q1}\cos\delta_0)-$ $w_0x_{q2(p)}(i_{d1}\cos\delta_0-i_{q1}\sin\delta_0)+$ $px_{q2(p)}(i_{d1}\sin\delta_0+i_{q1}\cos\delta_0)-$ $r_2(i_{d1}\sin\delta_0+i_{q1}\cos\delta_0)+$ $(e_{d1}\sin\delta_0+e_{q1}\cos\delta_0).$	$=0$
$-\sin\delta_0$	$-\cos\delta_0$	$p\,x_{q2(p)}\sin\delta_0+$ $w_0x_{d2(p)}\cos\delta_0+$ $r_2\sin\delta_0$	$p\,x_{q2}\cos\delta_0-$ $w_0x_{d2(p)}\sin\delta_0+$ $r_2\cos\delta_0$	$p\,x_{q2(p)}(i_{d1}\cos\delta_0-i_{q1}\sin\delta_0)+$ $px_{d2(p)}(-i_{d1}\cos\delta_0+i_{q1}\sin\delta_0)+$ $r_2(i_{d1}\cos\delta_0-i_{q1}\sin\delta_0)-pG_{2(p)}E_2-$ $x_{d2(p)}w_0(i_{d1}\sin\delta_0+i_{q1}\cos\delta_0)-$ $(e_{d1}\cos\delta_0-e_{q1}\sin\delta_0).$	$=0$
$pG_{1(p)}g_{1(p)}e'_{d1}-1$	$pG_{1(p)}g_{1(p)}e'_{q1}$	$-px_{d1(p)}-r_1$	$w_0x_{q1(p)}$	$-p\,x_{q1(p)}i_{q1}$	$=0$
$w_0G_{1(p)}g_{1(p)}e'_{d1}$	$w_0G_{1(p)}g_{1(p)}e'_{q1}-1$	$-w_0x_{d1(p)}$	$-px_{q1(p)}-r_1$	$-pG_{1(p)}E_1+p\,x_{d1(p)}i_{d1}$	$=0$

... (4.36)

The determinant of the characteristic matrix is the characteristic equation of the system. The determinant is a five by five one and each factor contains a number of terms. The expansion of the characteristic equation becomes very complicated and lengthy. So, the characteristic equation is left at this stage for the general case. The characteristic equations, ofcourse, will be found out in the succeeding calculations for different operating conditions of the system.

4.3. SAMPLE CALCULATIONS FOR DIFFERENT OPERATING CONDITIONS TO CALCULATE THE CHARACTERISTIC E UATIONS.

The following characteristic machine and system constants[25] have been selected for the analysis of the analytical system.

FOR THE MACHINE 1		FOR THE MACHINE 2	
x_{d1}	= 1.25 p.u.	x_{d2}	= 1.25 p.u.
x_{q1}	= 0.7 p.u.	x_{q2}	= 0.7 p.u.
r_1	= 0.01 p.u.	r_2	= 0.01 p.u.
T'_{d01}	= 4 secs.	T'_{d02}	= 4 secs.
T'_{d1}	= 1.5 secs.	T'_{d1}	= 1.5 secs.
H_1	= 5.3 p.u.	H_2	= 5.3 p.u.
f_1	= 50 c/s.	f_2	= 50 c/s.

$x_e = 0.2$ p.u., $r_e = 0.066$ p.u.

$T_r = 1$ secs. , $w_0 = 1$

The external resistances r_e and the external reactance x_e which are due to the tie-line are included in the machine 2 for all calculations. These values are assumed to be the leakage values of the machine number 2.

Excepting the condition of operation = 0, the expansion of the characteristic déterminants are not shown in this chapter as it becomes very lengthy and complicated. However, all the cases of expansion have been shown in theAppendix for references.

The expansion of the characteristic determinant of equation (4.36) is shown here by denoting the terms of the determinant by series of terms as shown below:-

D	E	A	B	C
d	e	a	b	c
d_1	e_1	a_1	b_1	c_1
d_2	e_2	a_2	b_2	c_2
d_3	e_3	a_3	b_3	c_3

=0 ... (4.37)

The expansion of equation (4.37) gives the characteristic equation.

The expansion is shown on the next page.

The characteristic equation is -

$$
\begin{array}{lll}
D & e & \left[a_1(b_2c_3-b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - b_2a_3)\right] - \\
D & a & \left[e_1(b_2c_3-c_2b_3) - b_1(e_2c_3 - c_2e_3) + c_1(e_2b_3 - b_2e_3)\right] + \\
D & b & \left[e_1(a_2c_3-c_2a_3) - a_1(e_2c_3 - c_2e_3) + c_1(e_2a_3 - e_3e_2)\right] - \\
D & c & \left[(a_2b_3 - a_2b_3) - a_1(e_2b_3 - b_2e_3) + b_1(e_2a_3 - a_2e_3)\right] - \\
E & d & \left[a_1(b_2c_3-b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - b_2a_3)\right] + \\
E & a & \left[d_1(b_2c_3-b_3c_2) - b_1(d_2c_3 - c_2d_3) + c_1(d_2b_3 - b_2d_3)\right] - \\
E & b & \left[d_1(a_2c_3-a_3c_2) - a_1(d_2c_3 - c_2d_3) + c_1(d_2a_3 - d_3a_2)\right] + \\
E & c & \left[d_1(a_2b_3-a_3b_2) - a_1(d_2b_3 - d_3b_2) + b_1(d_2a_3 - d_3a_2)\right] + \\
A & d & \left[e_1(b_2c_3-c_2b_3) - b_1(e_2c_3 - e_3c_2) + c_1(e_2b_3 - e_3b_2)\right] - \\
A & e & \left[d_1(b_2c_3-c_2b_3) - b_1(d_2c_3 - c_2d_3) + c_1(d_2b_3 - d_3b_2)\right] + \\
A & b & \left[d_1(e_2c_3-e_3c_2) - e_1(d_2c_3 - d_3c_2) + c_1(d_2e_3 - d_3e_2)\right] - \\
A & c & \left[d_1(e_2b_3-b_2e_3) - e_1(d_2b_3 - b_2d_3) + b_1(d_2e_3 - e_2d_3)\right] - \\
B & d & \left[e_1(a_2c_3-a_3c_2) - a_1(e_2c_3 - e_3c_2) + c_1(e_2a_3 - e_3a_2)\right] + \\
B & e & \left[d_1(a_2c_3-a_3c_2) - a_1(d_2c_3 - d_3c_2) + c_1(d_2a_3 - a_2d_3)\right] - \\
Ba & & \left[d_1(e_2c_3-c_2e_3) - e_1(d_2c_3 - d_3c_2) + c_1(d_2e_3 - d_3e_2)\right] + \\
Bc & & \left[d_1(e_2a_3-a_2e_3) - e_1(d_2a_3 - d_3a_2) + a_1(d_2e_3 - d_3e_2)\right] + \\
Cd & & \left[e_1(a_2b_3-b_2a_3) - a_1(e_2b_3 - b_2e_3) + b_1(e_2a_3 - a_2e_3)\right] - \\
Ce & & \left[d_1(a_2b_3-b_2a_3) - a_1(d_2b_3 - b_2d_3) + b_1(d_2a_3 - d_3a_2)\right] + \\
Ca & & \left[d_1(e_2b_3-b_2e_3) - e_1(d_2b_3 - d_3b_2) + b_1(d_2e_3 - d_3e_2)\right] - \\
Cb & & \left[d_1(e_2a_3-a_2e_3) - e_1(d_2a_3 - d_3a_2) + a_1(d_2e_3 - d_3e_2)\right]
\end{array}
$$

$$= 0 \qquad \ldots \qquad \ldots(4.38)$$

CASE I

THE OPERATING CONDITIONS

(NO-LOAD)

$\delta_0 = 0°$

OPERATING CONDITIONS

(No-load on the System).

MACHINE I

$\psi_{m1} = \sqrt{2}$ p.u.

$E_1 = 1.5\sqrt{2}$ p.u.

MACHINE II

$\psi_{m2} = \sqrt{2}$ p.u.

$E_2 = 1.5\sqrt{2}$ p.u.

$$\delta_0 = 0$$

The above values are assumed to be at no-load condition of the system.

Calculations of currents:

$$i_{d1} = \frac{1}{4.319}\left[\left\{.7 + .5\,(.9 + 1.45) - .5(1.45 - .9)\right\} \times 1.5\sqrt{2} - 1.5\sqrt{2}\,(.7 + .9)\right]$$

$$= \frac{1.5\sqrt{2}}{4.319}\,(.7 + 1.175 - .275 - 1.6\ )$$

$$= \frac{1.5\sqrt{2}}{4.319}\,(\ 1.875 - 1.875\ )$$

$$= 0$$

$i_{q1} = 0$

$\psi_{d01} = 1.5\sqrt{2} = 2.12$ $\qquad$ $\psi_{q01} = 0$

$e_{q01} = 2.12$ $\qquad$ $e_{d01} = 0$

$e_{t01} = 2.12$ p.u.

$e'_{d1} = \frac{e_{d0}}{e_{t01}} = 0,$ $\qquad$ $e'_{q1} = \frac{e_{q01}}{e_{t0}} = 1$ p.u.

$H' = 2.65$

$M' = \frac{H'}{\pi f} = 0.0168$ p.u.

1st Column

$c_0 = 0$, $\qquad$ $d = -1$, $\qquad$ $d_1 = 0$

$d_2 = -1$, $\qquad$ $d_3 = 0$

2nd Column-

$$d_0 = 0$$

$$e_2 = \frac{-2.5p}{(4p+1)(p+1)}, \qquad e_3 = -\frac{4p^2 + 5p + 3.5}{(4p+1)(p+1)}$$

3rd Column-

$$A = 0, \qquad a = p\,\frac{1.45 + 2.18\,p}{p+1} + .076$$

$$= \frac{1}{4p+1}\left(1.45p + 2.18p^2 + .296p + .076\right)$$

$$a = \frac{1}{4p+1}\left(2.18p^2 + 1.746p + .076\right)$$

$$a_1 = \frac{1.45 + 2.18p}{4p+1}, \quad a_2 = \frac{-1}{4p+1}\;1.87p^2 + 1.3p + .01$$

$$a_3 = -\frac{1.25 + 1.87\,p}{4p+1}$$

4th Column-

$$B = 2.12 \qquad b_1 = 0.9\,p + .076$$

$$b = -0.9 \qquad b_2 = 0.7, \qquad b_3 = -(0.7p + .01)$$

5th Column-

$$C = 0.0168\,p^2$$

$$c = \sqrt{2} = 1.413$$

$$c_1 = -p\,\frac{2.12}{4p+1} = -\frac{2.12p}{4p+1}$$

$$c_2 = 0$$

$$c_3 = -\frac{2.12p}{4p+1}$$

THE CHARACTERISTIC MATRIX IS

Δe_{d1}	Δe_{q1}	Δi_{d1}	Δi_{q1}	$\Delta \delta$	
0	0	0	2.12	$.0168p^2$	$= \Delta T_1$
-1	0	$\frac{2.18p^2+1.746p+.076}{4p+1}$	-0.9	1.413	$= 0$
0	-1	$\frac{2.17p+1.45}{4p+1}$	$.9p + .076$	$-\frac{2.12\,p}{4p+1}$	$= 0$
-1	$-\frac{2.5p}{(4p+1)(p+1)}$	$-\frac{(1.87p^2+1.3p+.01)}{4p+1}$	.7	0	$= 0$
0	$\frac{-(4p^2+5p+3.5)}{(4p+1)(p+1)}$	$-\frac{1.87p+1.25}{4p+1}$	$-(.7p+.01)$	$-\frac{2.12\,p}{4p+1}$	$= 0$

THE CHARACTERISTIC DETERMINANT IS.

Δe_{d1}	Δe_{q1}	Δi_{d1}	Δi_{q1}	$\Delta \delta$	
0	0	0	2.12	$0.0168p^2+.0672p^3$	= 0
-1	0	$2.18p^2+1.746p+.076$	-0.9	$1.413 + 5.652p$	= 0
0	$-(4p^2 +5p +1)$	$2.17p + 1.45$	$.9p + .076$	$-2.12p$	= 0
-1	$-2.5p$	$-(1.87p^2+1.3p+.01)$	0.7	0	= 0
0	$-(4p^2+5p+3.5)$	$-(1.87p + 1.25)$	$-(.7p +.01)$	$-2.12p$	= 0

The characteristic equation is -

$$2.12 \Big[-(4p^2 + 5p+1) \Big\{ (1.87p^2 + 1.3p + .01) \times 2.12p \Big\{ -(2.17p+1.45)$$

$$2.5p \times 2.12p \quad -2.12p \Big\{ 2.5p\ (1.87p + 1.25) - (4p^2 + 5p + 3.5)\times$$

$$(1.87p^2 + 1.3p + .01) \Big\}\Big] - 2.12\ (2.18p^2 + 1.746p + .076) \Big[(4p^2+5p+1)$$

$$2.12p \ - 2.12p\ (4p^2 + 5p + 3.5) \Big] + 2.12(5.62p + 1.413) \Big[(4p^2 + 5p+1)\times$$

$$(1.87p + 1.25) + (2.17p + 1.45)\ (4p^2 + 5p + 3.5) \Big] + (0.672p^3 + .0168p^2)\times$$

$$(2.18p^2 + 1.746p + .076) \Big[(4p^2 + 5p+1)(.7p + .01) + (.9p+.076) \times$$

$$(4p^2 + 5p + 3.5) \Big] + (.0672p^3 + .0168p^2) \times .9 \Big[(4p^2 + 5p + 1) \times$$

$$(1.87p + 1.25) + (2.17p + 1.45)\ (4p^2 + 5p + 3.5) \Big] \quad = \quad 0$$

or

$$2.12\ (-12.08\ p^3 - 8.47p^2 + .053p - .0212) + 5.30p(4.62p^2 + 3.73p +$$

$$.161) + (1.2p + 2.99)\ (16.16p^3 + 31p^2 + 22.97p + 6.325) + (0.146p^5 +$$

$$.144p^4 + .08p^3 + .0013p^2)(6.4p^3 + 8.344p^2 + 4.28p + .366) +$$

$$(.0605p^3 + .015\ p^2)\ (16.16\ p^3 + 31p^2 + 22.97p + 6.325) \quad = \quad 0$$

or

$$0.935p^8 + 2.134p^7 + 2.311p^6 + 3.462p^5 + 21.862p^4 + 84.963p^3 +$$

$$122.095p^2 + 77.165p + 18.855 \quad = \quad 0$$

CASE II

OPERATING CONDITION; 100% FULL-LOAD.

$\delta_0 = 66.7°$

The voltage regulator put at the generator end does the full load compensation, when it delivers, power at the motor and with the working power factor 0.8 (lagging) and the voltage and current at the motor terminal is 1 p.u.

$i = 1$ p.u. $= 1\underline{/-36.9°}$ $\quad e_2 = 1 + j0 = 1 \underline{/0°}$

$= 0.8 - j.6$

referring to fig.4.3-

$E_{q1} = e_2 + i\,(r_e + j\,x_{q1})$

$= 1 + j0 + (.8 - j.6)\,(0.076 + j\,0.9)$

$= 1 + .0608 + .54 + j\,(.72 - .0456\ )$

$= 1.6008 + j\,.6744$

$= 1.74 \underline{/+22.7°}$

$i_{d1} = i$ Sin $22.7° = 1 \times .3859 = .3859$

referring to fig.4.1-

$E_1 = E_{Q1} + (x_{d1} - x_{q1})\,i_{d1}$

$= 1.74 + (1.25 - .7) \times .3859$

$= 1.74 + .55 \times .3859$

$= 1.74 + .212$

$= \underline{1.952 \text{ p.u.}}$

For the motor -

$i = 1 \underline{/-36.9°} = .8 - j.6$

referring to fig.4.2-

$E_{Q2} = e_2 - i\,(r_2 + j\,x_{q2})$

$= 1 + j0 - (.8 - j.6)\,(.01 + j.7)$

$= 1 - .008 - .42 - j(.56 + .006)$

$= 1 - .428 - j.554$

$= .572 - j.554$

$= 0.82 \angle -44.0°$

when figures 4.1 & 4.2 are super imposed -

$\therefore \quad = 22.7 + 44.0$

$= 66.7°$

$i_{d2} = i \, \text{Sin}\, 44.7°$

$= 0.7034$

$E_2 = E_{q2} - i_{d2} (x_{d2} \cdot x_{q2})$

$= 0.82 - 0.7034\ (.55)$

$= 0.82 - .386$

$= \underline{0.484 \text{ p.u.}}$

<u>THE OPERATING CONDITIONS</u> - (Full-load)

<u>MACHINES I</u>	<u>MACHINES II</u>
$e_{m1} = 1.59\sqrt{2}$ p.u.	$e_{m2} = \sqrt{2}$ p.u.
$E_2 = 1.95\sqrt{2}$ p.u.	$E_2 = 0.432\ \sqrt{2}$ p.u.

$\delta_o = 66.7°$

<u>Calculations of currents</u>-

$A = 0.7 + 1.175 + .275 \times .6871 = 1.875 + .189 = 2.064$

$B = .01 - .076 \times .6871 - .275 \times .7266 = -.0423 - .2 = -0.2423$

$$i_{d1} = \frac{2.76 - .226 + .298}{1.25 + 1.175 - 1.89 - (.076 + .252) \times 0.1175}$$

$$= \frac{2.792}{2.425 - .228} = \frac{2.792}{2.197} = 1.32 \text{ p.u.}$$

$$i_{q1} = \frac{.397 + 1.32 \times (.076 + .2)}{.7 + 1.175 + .189}$$

$$= \frac{.397 + .364}{2.064} = \frac{.761}{2.064} = 0.368 \text{ p.u.}$$

$$\psi_{d01} = E_1 - x_{d1} i_{d01} \qquad \psi_{q01} = -x_{q1} i_{q01}$$

$$= \sqrt{2} \times 1.952 - 1.25 \times 1.32 \qquad = -0.7 \times .368$$

$$= 2.78 - 1.65 \qquad = -.2576$$

$$= 1.13$$

$$e_{q01} = 1.13 - .00368 \qquad e_{d01} = .2576 - .0132$$

$$= 1.126 \qquad = 0.2444$$

$$e_{a01} = \sqrt{e_{d01}^2 + e_{q01}^2} = \sqrt{1.25 + .06} = \sqrt{1.31} = 1.145$$

$$e'_{d1} = \frac{e_{d01}}{e_{a01}} = 0.214 \qquad e'_{q1} = \frac{e'_{q01}}{e_{a01}} = 0.981$$

$$H' = \frac{H_1 H_2}{H_1 + H_2} = \frac{5.3 \times 5.3}{10.6} = 2.65$$

$$M' = \frac{H'}{\wedge f} = 0.0168 \text{ p.u.}$$

<u>1st Column-</u>

$$D = .368 \frac{1}{4p+1} \times \frac{-2.5}{1+p} \times .214 = \frac{-0.197}{(4p+1)(1+p)}$$

$$d = -\cos_0 = -0.3795 \quad d_1 = -\sin_0 = -.9184$$

$$d_2 = \frac{-.535p - 4p^2 - 5p - 1}{(4p+1)(p+1)} = \frac{-(4p^2 + 5.535p + 1)}{(4p+1)(p+1)}$$

$$d_3 = -\frac{0.535}{(4p+1)(p+1)}$$

<u>2nd Column-</u>

$$E = \frac{-0.904}{(4p+1)(p+1)}, \qquad e = .0184$$

$$e_1 = -0.3795$$

$$e_2 = \frac{-2.448\ p}{(4p+1)(p+1)}\ ,\quad e_3 = \frac{-(4p^2 + 5p + 3.448)}{(4p+1)(1+p)}$$

3rd Column-

$$A = -\left(-0.2578 + \frac{1.25 + 1.87p}{4p+1} \times .368\right)$$

$$= -(-1.03p - .2576 + .46 + .688p)\frac{1}{4p+1}$$

$$= \frac{-1}{4p+1}(-.343p + .202)$$

$$a = p\,\frac{.55 + .825p}{4p+1} - .826 + .022$$

$$= \frac{1}{4p+1}(.55p + .825p^2 - 3.39p - .848)$$

$$= \frac{1}{4p+1}(.825p^2 - 2.84p - 0.848)$$

$$a_1 = .826p + \frac{.55 + .825p}{4p+1} + .0699$$

$$= \frac{1}{4p+1}(3.31p^2 + .826p + .55 + .825p + .28p + .07)$$

$$= \frac{1}{4p+1}(3.31p^2 + 1.931p + .62)$$

$$a_2 = -p\,\frac{1.25 + 1.87p}{4p+1} - .01 = \frac{-1}{4p+1}(1.25p + 1.87p^2 + .04p + .01)$$

$$= \frac{-1}{4p+1}(1.87p^2 + 1.29p + .01)$$

$$a_3 = -\frac{1.25 + 1.87\,p}{4p+1}.$$

4th Column-

$$B = (1.13 + .916) = 2.046$$

$$b = -p\,\frac{1.33 + 2.0\,p}{4p+1} - .341 - .07$$

$$= -(1.33p + 2.00p^2 + 1.645p + .411)\frac{1}{4p+1}$$

$$= -\frac{1}{4p+1}(2p^2 + 2.98p + .411)$$

$$b_1 = .341p - \frac{1.33 + 2.00p}{4p+1} + .022$$

$$= \frac{1}{4p+1}\,(1.365p^2 + .341p - 1.330 - 2.00p + .022)$$

$$= \frac{1}{4p+1}\,(1.37p^2 - 1.66p - 1.31)$$

$$b_2 = 0.7$$

$$b_3 = -\,p0.7 - .01 = -(0.7p + .01)$$

<u>5th Column</u>-

$$C = .0175p^2$$

$$c = -p\,\frac{1.45 + 2.17P}{4p+1}\,(1.225 + .139) - 0.9\,(.514 - .338) +$$

$$.9p\,(1.225 + .138) - .076 \times 1.364 + 1.143\ .24$$

$$= -p\,\frac{1.97 + 2.97P}{4p+1} - .158 + p\ 1.1227 - .1035 + 1.383$$

$$= \frac{1}{4p+1}(-1.97p - 2.96p^2 + 4.5p^2 + 1.123p + 4.84p + 1.212)$$

$$= \frac{1}{4p+1}\,(1.54p^2 + 3.953p + 1.2)$$

$$c_1 = p \times .9\,(.500 - .338) + p\,\frac{1.45 + 2.17P}{4p+1} \times (-0.162) +$$

$$.076(0.162) - p\,\frac{.432}{4p+1} - \frac{1.45 + 2.17P}{4p+1} \times 1.364$$

$$= p \times .146 + \frac{-.235p - .352p^2}{4p+1} + .0123 - \frac{.432P}{4p+1}\ \frac{1.97+2.97P}{4p+1}$$

$$= \frac{1}{4p+1}\,(.584p^2 + .146p - .235p - .352p^2 + .049p + .0123 -$$

$$1.97 - 2.97p - .432p)$$

$$= \frac{1}{4p+1}\,(0.232p^2 - 3.207p - 1.96)$$

$$c_2 = -\ .258p$$

$$c_3 = -p\,\frac{1.952}{4p+1} + \frac{1.65p + 2.48p^2}{4p+1}$$

$$= \frac{1}{4p+1}(2.48p^2 - .302p)$$

THE CHARACTERISTIC MATRIX IS-

Δe_{d1}	Δe_{q1}	Δi_{d1}	Δi_{q1}	$\Delta \delta$	
$\frac{-0.197}{(4p+1)(p+1)}$	$\frac{-0.904}{(4p+1)(1+p)}$	$\frac{0.343p^2-.202}{4p+1}$	2.046	$0.0168p^2$	$= \Delta T_1$
-0.3795	0.9184	$\frac{.825p^2-2.84p-.848}{4p+1}$	$-\frac{(2p^2+2.98p+.411)}{4p+1.}$	$\frac{1.54p^2-3.95p+1.2}{4p+1}$	$= 0$
$-.9184$	-0.3795	$\frac{3.31p^2+1.93p+.62}{4p+1}$	$\frac{(1.37p^2-1.66p-1.31)}{4p+1}$	$\frac{(.232p^2-3.207p-1.96)}{4p+1}$	$= 0$
$\frac{-(4p^2+5.535p+1)}{(4p+1)(p+1)}$	$\frac{-2.448p}{(4p+1)(1+p)}$	$-\frac{(1.87p^2+1.29p+.01)}{4p+1}$	0.7	$-.258p$	$= 0$
$-\frac{0.535}{(4p+1)(p+1)}$	$\frac{-(4p^2+5p+3.448)}{(4p+1)(p+1)}$	$-\frac{1.87p+1.25}{4p+1}$	$-(.7p+.01)$	$\frac{2.48p^2-.302p}{4p+1}$	$= 0$

THE CHARACTERISTIC DETERMINANT IS

Δo_{d1}	Δo_{q1}	Δi_{d1}	Δi_{q1}	$\Delta \delta$
-0.197	-0.904	$(.343p - 0.202)$	$(8.184p + 2.046)$	$(0.0372p^3 + .0168p^2)$
$-(1.52p^2 + 1.9p + .38)$	$3.67p^2 + 4.59p + .92$	$(0.825p^2 - 2.84p - .85)$	$-(2p^2 + 2.98p + .411)$	$1.54p^2 + 3.95p + 1.2$
$-(3.67p^2 + 4.59p + .92)$	$-(1.52p^2 + 1.9p + .38)$	$3.31p^2 + 1.93p + .62$	$1.3p^2 - 1.66p - 1.31$	$0.232p^2 - 3.207p - 1.96$
$-(4p^2 + 5.54p + 1)$	$-2.448p$	$-(1.87p^2 + 1.3p + .01)$	$2.8p + .7$	$-(1.03p^2 + .258p)$
-0.538	$-(4p^2 + 5p + 3.45)$	$-(1.9p + 1.25)$	$-(2.8p^2 + .74p + .01)$	$2.48p^2 - 0.302p$

$= 0$

CASE III

OPERATING CONDITIONS

(150% Full load.)

$\delta_o = 86.1°$

OPERATING CONDITIONS:

(150% of Full-load current)

At full load, the angle between the motor and generator terminal voltage-

referring to the fig.4.3.-

$$e_1 = e_2 + i\,(r_0 - jx_0)$$

$$= 1 + j0 + (.8 - j.6)\,(.066 + j.2)$$

$$= 1 + .0528 + .12 + j.16 - j\,.0396$$

$$= 1.1728 + j.081$$

$$= 1.39\,\angle{+4^\circ}$$

$i = 1.5$ p.u. or $i = 1.5\,\angle{-40.9^\circ}$

$$= 1.5\,(.7559 - j.6547)$$

$$= 1.134 - j0.98.$$

The terminal voltage ofthe generator, i.e. the machine no.1 is maintained at the rated full-load terminal voltage by the voltage regulator. The other voltages are calculated with the generator terminal voltages as the reference voltage.

$$\therefore e_1 = 1.39 \text{ p.u.} = 1.39\,\angle{0^\circ}$$

$$e_2 = e_1 - i\,(r_0 + jx_0)$$

$$= 1.39 + j0 - (1.134 - j.98)\,(.066 + j0.2)$$

$$= 1.39 - .0815 - .196 - j.2268 + j.0638$$

$$= 1.39 - .2775 - j.163$$

$$= 1.1125 - j\,0.163$$

From the steady-state vector diagram fig.4.2

$$E_{q1} = e_2 + i(r_2 + j\,x_2)$$

$$= 1.113 - 0.163 + (1.134 - j.98)\,(.076 + j0.9)$$

$$= 1.113 - j0.163 + .0862 + .882 + j(1.022 - .0745)$$

$$= 2.051 + j.948 - j.163$$

$$= 2.051 + j.785 = 2.2\ \underline{/+20.90}$$

$$i_{d1} = i\ \text{Sin}\ 20.9° = 1.5 \times .3567 = .534\ \text{p.u.}$$

$$\therefore\ E_1 = E_{q_1} + (x_{d1} - x_{q1})\ i_{d1}$$

$$= 2.2 + 0.55 \times .534$$

$$= 2.2 + 0.294 = 2.494\ \text{p.u.}$$

referring to the fig.4.2-

$$E_{q2} = e_2 - i\ (r_2 + jx_{q2})$$

$$= 1.1185 - j0.163 - (1.2 - j.9)(.01 - j.7)$$

$$= 1.1125 - j0.163 - .012 - .63 - j.84 + j.009$$

$$= 1.1125 - .642 - j1.03 + j.009$$

$$= 0.471 - j1.021$$

$$= 1.14 - 65.2°$$

$$i_{d2} = i\ \text{Sin}\ 65.2 = 1.5 \times .9078 = 1.36\ \text{p.u.}$$

$$\therefore E_2 = E_{Q2} - i_{d2}\ (x_{d2} - x_{q2})$$

$$= 1.14 - 1.36 \times .55$$

$$= 1.14 - .749$$

$$= 0.391\ \text{p.u.}$$

when the two vector diagrams are super-imposed-

$$\delta_o = 65.2 + 20.9$$

$$= 86.1°$$

THE OPERATING CONDITIONS (150% Full-load)

MACHINE I

$e_{m1} = 1.39 \times \sqrt{2}$ p.u.

$E_1 = 2.494 \times \sqrt{2}$ p.u.

MACHINE II

$e_{m2} = 1.14 \times \sqrt{2}$ p.u.

$E_2 = 0.39 \times \sqrt{2}$ p.u.

$$\delta_0 = 86.1°$$

Calculations of Currents etc.

$A = 2.147$, $B = -0.1028$

$i_{d1} = 1.635$ p.u., $i_{q1} = .37$ p.u.

$d01 = 1.50$ $q01 = -.259$

$e_{q01} = 1.496$, $e_{d01} = 0.243$

$e_{m01} = 1.51$ p.u. $e'_{d1} = .161$ p.u.

$e'_{q1} = 0.99$ p.u. $M' = .0168$ p.u.

Columns of the characteristic determinant-

1st Column

$D = \dfrac{-0.149}{(4p+1)(1+p)}$ $d = -0.068$

$d_1 = -.9977$ $d_2 = \dfrac{-(4p^2+5.402p+1)}{(4p+1)(p+1)}$

$d_3 = \dfrac{-0.402}{(4p+1)(p+1)}$

2nd Column

$E = \dfrac{-.915}{(4p+1)(p+1)}$ $e = 0.9977$

$e_1 = -0.068$, $e_2 = \dfrac{-2.47\,p}{(4p+1)(p+1)}$

$e_3 = \dfrac{-(4p^2+5p+3.47)}{(4p+1)(1+p)}$

3rd Column

$$A = -\frac{1}{4p+1}(-0.345p + 0.20\,)$$

$$a = \frac{1}{4p+1}(0.148p^2 - 3.4687p - 0.892)$$

$$a_1 = \frac{1}{4p+1}(3.588p^2 + 1.3486\,p + .1744)$$

$$a_2 = -\frac{1}{4p+1}(1.87p^2 + 1.29p + .01)$$

$$a_3 = -\frac{1.87p + 1.25}{4p+1}$$

4th Column-

$$B = 2.645$$

$$b = -\frac{1}{4p+1}(2.17\,p^2 + 1.993\,p + 0.137)$$

$$b_1 = \frac{1}{4p+1}(0.244p^2 - 2.088p - 1.44)$$

$$b_2 = 0.7, \qquad b_3 = -(0.7p + .01)$$

5th Column-

$$C = 0.0168p^2$$

$$c = \frac{1}{4p+1}(2.37p^2 + 6.066p + 1.744)$$

$$c_1 = \frac{1}{4p+1}(1.488p^2 - 3.452p - 2.42)$$

$$c_2 = -0.259p$$

$$c_3 = \frac{1}{4p+1}(3.06p^2 - .454p)$$

THE CHARACTERISTIX MATRIX IS-

Δe_{d1}	Δe_{q1}	Δi_{d1}	Δi_{q1}	$\Delta \delta$	
$\frac{-0.149}{(4p+1)(p+1)}$	$-\frac{0.915}{(4p+1)(p+1)}$	$\frac{0.345p - 0.203}{4p+1}$	2.645	$0.0168p^2$	$= \Delta T_1$
-0.068	0.9977	$\frac{0.148p^2-3.469p-0.892}{4p+1}$	$\frac{-(2.17p^2+1.99p+0.137)}{4p+1}$	$\frac{2.37p^2+6.066p+1.744}{4p+1}$	= 0
-0.9977	-0.068	$\frac{3.588p^2+1.349p+1.744}{4p+1}$	$\frac{0.244p^2-2.088p-1.44}{4p+1}$	$\frac{1.488p^2-3.452p-2.42}{4p+1}$	= 0
$\frac{-(4p^2+5.402p+1)}{(4p+1)(p+1)}$	$-\frac{2.47p}{(4p+1)(p+1)}$	$\frac{-(1.87p^2+1.29p+.01)}{4p+1}$	0.7	-0.259p	= 0
$-\frac{0.402}{(4p+1)(p+1)}$	$-\frac{(4p^2+5p+3.47)}{(4p+1)(p+1)}$	$\frac{-(1.87p+1.25)}{4p+1}$	-(0.7p + .01)	$\frac{3.06p^2-.454p}{4p+1.}$	= 0

THE CHARACTERISTIC DETERMINANT IS -

Δe_{d1}	Δe_{q1}	Δi_{d1}	Δi_{q1}	$\Delta \delta$
-0.149	-0.915	$(0.345p - 0.203)$	$10.58p + 2.645$	$.0672p^3 + 0.0138p^2$
$-(0.272p^2 + .34p + 0.068)$	$3.99p^2 + 4.988p + .998$	$0.148p^2 - 3.47p - .892$	$-(2.17p^2 + 1.99p + .137)$	$2.37p^2 + 6.066p + 1.744$
$-(3.491p^2 + 4.988p + .998)$	$-(.272p^2 + .34p + .068)$	$3.588p^2 + 1.35p + 1.74$	$.244p^2 - 2.088p - 1.44$	$1.488p^2 - 3.452p - 2.42$
$-(4p^2 + 5.402p + 1)$	$-2.47p$	$-(1.87p^2 + 1.3p + .01)$	$2.8p + 0.7$	$-(1.036p^2 + .26p)$
-0.402	$-(4p^2 + 5p + 3.47)$	$-(1.87p + 1.25)$	$-(2.8p^2 + .74p + .01)$	$3.06p^2 - 0.454p$

$= 0$

CASE IV

OPERATING CONDITIONS

(175% OF FULL LOAD)

$\delta_0 = 99.3°$

THE OPERATING CONDITIONS

(175% of full-load, p.f. .8 lagging)

$i = 1.75 \angle -40.9°$

$= 1.75\,(.7559 - j0.6547)$

$= 1.323 - j1.145$

The terminal voltage of the generator, i.e. the machine no.1 is maintained at the rated full-load load terminal voltage by the voltage regulator. The other voltages are calculated with the generator terminal voltages as the reference voltage.

$e_1 = 1.39$ p.u. $= 1.39 \angle 0°$

referring to fig.4.3,

$e_2 = e_1 - i(r_0 + j\,x_0)$

$= 1.39 + j0 - (1.323 - j1.145)(0.066 + j0.2)$

$= 1.39 - 0.087 - j.264 + j0.075 - 0.229$

$= 1.39 - 0.316 - j0.189$

$= 1.074 - j0.189$

From the steady-state phasor diagram fig.4.1,

$E_{q1} = e_2 + i\,(r_0 + jx_2)$

$= 1.074 - j0.189 + (1.323 - j1.145)(.076 + j0.9)$

$= 1.074 - j0.189 + 0.1 + j1.19 - j0.087 + 1.03$

$= 2.204 + j1.19 - j.276$

$= 2.204 + j0.914$

$= 2.38 \angle 22.5°$

$$i_{d1} = i \text{ Sin } 22.5^\circ = 1.75 \times .3827$$

$$= 0.67$$

$$E_1 = E_{q1} + (x_{d1} - x_{q1})\, i_{d1}$$

$$= 2.38 + .55 \times 0.67$$

$$= 2.38 + .368$$

$$= 2.748 \text{ p.u.}$$

referring to fig.4.2-

$$E_{q2} = e_2 - i\,(r_2 + jx_{q2})$$

$$= 1.074 - j.189 - (1.326 - j1.145)(.01 + j0.7)$$

$$= 1.074 - j.189 - .013 + j.0114 - j.928 - .801$$

$$= 1.074 - j1.117 - .814 + j.0114$$

$$= 0.260 - j1.106$$

$$= 1.141 \quad -76.8^\circ$$

$$i_{d2} = i \text{ Sin } 76.8$$

$$= 1.75 \times .9736 \quad = 1.7$$

$$\therefore E_2 = E_{q2} - i_{d2}\,(x_{d2} - x_{q2})$$

$$= 1.141 - 1.7 \times .55$$

$$= 1.141 - .935$$

$$= 0.206 \text{ p.u.}$$

$$\therefore \delta_0 = 76.8 + 22.5 \quad = 99.3^\circ$$

NEW OPERATING CONDITIONS (175% Full-load)-

MACHINE I	MACHINE II
$e_{m1} = 1.39\sqrt{2}$ p.u.	$e_{m2} = 1.05 \times \sqrt{2}$ p.u.
$E_1 = 2.748 \times \sqrt{2}$ p.u.	$E_2 = 0.206 \times \sqrt{2}$ p.u.

$$\delta_0 = 99.3^\circ$$

Calculations of currents etc.

$i_{d1} = 2.37$ p.u. $\qquad i_{q1} = -0.191$ p.u.

$\psi_{d01} = 0.82$ $\qquad \psi_{q01} = .133$ p.u.

$e_{q01} = 0.822$ p.u., $\qquad e_{d01} = -.157$ p.u.

$e_{a01} = .885$ p.u. $\qquad e'_{d1} = -0.176$ p.u.

$e'_{q1} = .937$ p.u. $\qquad M' = 0.0168$ p.u.

Terms of the different columns of the characteristic determinant:-

<u>1st Column-</u>

$$D = \frac{-0.084}{(4p+1)(p+1)} \qquad d = 0.1616$$

$$d_1 = -.9869 \qquad d_2 = -\frac{4p^2 + 4.56p+1}{(4p+1)(1+p)}$$

$$d_3 = \frac{.44}{(4p+1)(1+p)}$$

<u>2nd Column-</u>

$$E = \frac{0.445}{(4p+1)(p+1)} \qquad e = 0.9869$$

$$e_1 = 0.1616 \qquad e_2 = \frac{-2.34p}{(4p+1)(p+1)}$$

$$e_3 = -\frac{4p^2 + 5p + 3.47}{(4p+1)(p+1)}$$

<u>3rd Column-</u>

$$A = \frac{1}{4p+1}(-.176p + 0.116)$$

$$a = -\frac{1}{4p+1}(.35p^2 + 3.70p + .889)$$

$$a_1 = \frac{1}{4p+1}(3.55p^2 + .837p - .159)$$

$$a_2 = -\frac{1}{4p+1}(1.87p^2 + 1.3p + .01)$$

$$a_3 = -\frac{1.87p + 1.25}{4p+1}$$

4th Column

$$B = 2.48,$$

$$b = \frac{1}{4p+1}(-3.14\ p^2 - 1.15p + .07)$$

$$b_1 = -\frac{1}{4p+1} (.58p^2 + 2.334p + 1.442)$$

$$b_2 = 0.7$$

$$b_3 = -(0.7p + .01)$$

5th Column

$$C = 0.0168p^2$$

$$c = \frac{1}{4p+1} (4.072p^2 + 5.362p + 1.62)$$

$$c_1 = -\frac{1}{4p+1}(5.408p^2 + .1459 + 3.45)$$

$$c_2 = 0.134p$$

$$c_3 = \frac{1}{4p+1}(4.4p^2 + 0.192p)$$

THE CHARACT.RISTIC MATRIX IS.

Δe_{d1}	Δe_{q1}	Δi_{d1}	Δi_{q1}	$\Delta\delta$	
$\frac{-0.084}{(4p+1)(p+1)}$	$\frac{0.445}{(4p+1)(p+1)}$	$\frac{1}{4p+1}(-.176p+0.116)$	2.48	$0.0168p^2$	$=\Delta T_1$
0.1616	0.9869	$-\frac{1}{4p+1}(.35p^2+3.79p+.889)$	$\frac{1}{4p+1}(-2.14p^2-1.15p+.07)$	$\frac{1}{4p+1}(4.072p^2+5.362p+1.62)$	$=0$
-0.9869	0.1616	$\frac{1}{4p+1}(3.55p^2+.837p-.159)$	$-\frac{1}{4p+1}(.58p^2+2.334p+1.442)$	$-\frac{1}{4p+1}(5.408p^2+.145p+3.45)$	$=0$
$-\frac{4p^2+4.56p+1}{(4p+1)(p+1}$	$-\frac{2.34p}{(4p+1)(p+1)}$	$-\frac{1}{4p+1}(1.87p^2+1.3p+.01)$	0.7	$0.134\ p$	$=0$
$\frac{.44}{(4p+1)(p+1)}$	$-\frac{4p^2+5p+3.46}{(4p+1)(p+1)}$	$-\frac{1}{4p+1}(1.87p+1.25)$	$-(.7p+.01)$	$\frac{1}{4p+1}(4.4p^2+0.192p)$	$=0$

CHAPTER 5.

THE ROUTH-HURWITZ CRITERION ANALYSIS

THE ROUTH-HURWITZ CRITERION[23]

5.1. INTRODUCTION.

Basically the design of feed-back control systems can be regarded as a problem of arranging the location of the characteristic equation, roots in such a way that the corresponding system will perform according to the prescribed specifications. A system is defined as stable if the output response to any bounded input disturbance is finite. This implies that all the roots of the characteristic equation must be located in the left half of the P-plane. Roots on the right half of the plane give rise to transients which tends to diverge from the steady state, and the system said to be unstable. Thus the stability of a linear feed-back control system is also uniquely determined by the location of the roots of its characteristic equation.

5.2. ROUTH-HURWITZ CRITERION.

It is established that the problem of determining the stability of a linear system is one of finding the roots of the characteristic equation. However, for polynomials of the 3rd order or higher, the task of finding the roots is very tedious and time consuming. Hence, it is desired that an alternate method be used, so that the system stability can be determined without actually solving for the roots of the characteristic equation.

Suppose that the characteristic equation of linear system is written in the general form-

$$F_{(p)} = 1 + G_{(p)} H_{(p)}$$

$$= a_0 p^n + a_1 p^{n-1} \quad \ldots \quad + a_{n-1} p + a_n = 0 \qquad (5.1)$$

In order that no roots of the last equation with positive real parts, it is necessary but not sufficient that-

(1) All the coefficients of the polynomial have the same signs.

(2) none of the co-efficients vanish.

The necessary and the sufficient condition that all the roots of an nth. order polynomial lie in the left half of the P-plane is that the polynomial's Hurwitz determinant must be all positive.

5.3. SAMPLE CALCULATIONS & DETERMINATION OF STABILITY:

The characteristic equations have been calculated for the following cases-

1. No load, $\delta_0 = 0$
2. 100% Full load $\delta_0 = 66.7°$
3. 150% Full load $\delta_0 = 86.1°$
4. 175% Full load $\delta_0 = 99.3°$

The detailed calculations has been shown on the next pages.

Case 1. $\delta_o = 0$

The characteristic equation is-

$$0.935p^8 + 2.134p^7 + 2.311\,p^6 + 3.46p^5 + 21.862p^4 + 84.963p^3 + 122.095p^2 + 77.165p + 18.855 = 0. \quad \ldots \quad \ldots \quad (5.1)$$

The Routh's Array can be arranged as follows:

p^8	0.935	2.311	21.862	122.095	18.855
p^7	2.134	3.462	84.963	77.165	0
p^6	0.855	-15.062	89.695	18.855	0
p^5	40.862	-189.04	30.365	0	0
p^4	-11.102	89.06	18.855	0	0
p^3	138.96	99.765	0	0	0
p^2	98.05	18.855	0	0	0
p^1	73.065	0	0	0	0

There are changes of signs in the 1st column of the Routh's array. This states that the system is unstable at this operating condition.

Case 2.

$\delta_o = 66.70$

The characteristic equation is-

$$13.04\,p^{11} + 75.2\,p^{10} + 2434.842\,p^9 + 12834.833p^8 + 30388.842p^7 + 37605.746p^6 + 29879.815p^5 + 15032.370p^4 + 6331.482p^3 + 1797.021\,p^2 + 294.663p + 21.097 = 0.$$

(5.2)

The Routh's Arrays can be written as-

p^{11}	13.04	2434.842	30380.842	29879.815	6331.48	294.66
p^{10}	75.04	12834.833	37605.795	15032.370	1797.021	21.1
p^{9}	214.00	23389.842	27269.21	6019.2	291.8	0
p^{8}	4734.00	27055.45	12914.5	1695.50	21.10	0
p^{7}	22161.00	26684.35	5252.78	290.00	0	0
p^{6}	21355.00	11793.8	1633.04	21.10	0	0
p^{5}	14480.00	4562.07	268.7	0	0	0
p^{4}	5092.00	1249.00	21.10	0	0	0
p^{3}	972.00	207.00	0	0	0	0
p^{2}	169.00	21.10	0	0	0	0
p	84.00	0	0	0	0	0

There is no change of signs and all the terms on the first column are positive, hence, the system is stable.

Case 3.

$\delta_0 = 86.1$

The characteristic Equation is-

$12.02\,p^{11} + 60.2p^{10} + 5162.364\,p^{9} + 24433.6p^{8} + 48494p^{7} + 51984p^{6} + 31197.6p^{5} + 9876.6p^{4} + 1422p^{3} + 58.6p^{2} + 10.5p + 0.01 = 0.$ ... ... (.5.3)

The Routh's array can be arranged as -

p^{11}	20.544	6777.698	34276.9	- 928.44	-404.4	-646.6
p^{10}	77.792	20108.566	3596.65	-11785.5	-3489.9	- 37.07
p^{9}	1477.7	33331.9	2171.56	+515.6	-636.95	0
p^{8}	18353.5	3482.1	-11812.7	-3456.3	-37.07	0
p^{7}	33051.9	3121.156	792.6	-633.88	0	0
p^{6}	1732.0	-12252.7	-3104.5	-37.07	0	0
p^{5}	26521.5	6710.6	-563.37	0	0	0
p^{4}	-12691.2	-3067.5	-37.07	0	0	0
p^{3}	310.6	-640.87	0	0	0	0
p^{2}	-29252.5	-37.07	0	0	0	0
p^{1}	-641.265	0	0	0	0	0

The system is unstable and has three roots with real parts positive.

The Routh's Array can be arranged as-

p^{11}	12.02	5162.364	48494.0	311976.0	1422.0	10.5
p^{10}	60.2	24433.6	51984.0	9876.6	58.6	0.01
p^{9}	302.4	38144.0	29232.6	1410.3	10.5	0
p^{8}	16878.6	46164.0	9595.6	57.5	0.01	0
p^{7}	37316.0	27512.0	1419.3	10.48	0	0
p^{6}	33764.0	8955.6	44.56	0.01	0	0
p^{5}	17262.0	1368.3	10.45	0	0	0
p^{4}	6293.0	25.06	0.01	0	0	0
p^{3}	1353.0	4.53	0	0	0	0
p^{2}	4.16	0.01	0	0	0	0
p	1.28	0	0	0	0	0

There is no change of sign in the 1st column, hence, the system is stable.

<u>Case 4.</u>

$\delta_o = 99.3°$

The characteristic equation is,

$$20.544p^{11} + 77.792p^{10} + 6777.698p^{9} + 20108.566p^{8} + 34276.965p^{7} + 3596.635\,p^{6} - 928.445p^{5} - 11705.458p^{4} - 404.388p^{3} - 3489.903p^{2} - 646.576p - 37.073 = 0. \quad \ldots (5.4)$$

The Routh's array can be arranged as -

p^{11}	20.544	6777.698	34276.9	- 928.44	-404.4	-646.6
p^{10}	77.792	20108.566	3596.65	-11785.5	-3489.9	- 37.07
p^{9}	1477.7	33331.9	2171.56	+515.6	-636.95	0
p^{8}	18353.5	3482.1	-11812.7	-3456.3	-37.07	0
p^{7}	33051.9	3121.156	792.6	-633.88	0	0
p^{6}	1732.0	-12252.7	-3104.8	-37.07	0	0
p^{5}	26521.5	6710.6	-563.37	0	0	0
p^{4}	-12691.2	-3067.5	-37.07	0	0	0
p^{3}	310.6	-640.87	0	0	0	0
p^{2}	-29252.5	-37.07	0	0	0	0
p^{1}	-641.265	0	0	0	0	0

The system is unstable and has three roots with real parts positive.

CHAPTER 6

FREQUENCY RESPONSE ANALYSIS.

FREQUENCY RESPONSE ANALYSIS

6.1. INTRODUCTION(4,16)

The frequency response method of determining stability is a step ahead of previous analytical work on electrical system in this field. It is based upon a rigorous analysis of system dynamics, both electrical and mechanical.in this field, the system is realised as a closed loop configuration and then the Nyquist criterion of stability is applied. Realisation as a closed-loop system is necessary as some form of feed-back is essential for any system to be unstable. A simple closed loop system consists of one main loop and one feedback loop. These are determined after studying the equation of motion obtained from small displacement theory.

The main advantage of this method lies, not in being able to tackle the problem easily but in the fact that it provides for rapid appreciation of the effects of modification of the system equation. In the stability studies, many times, the actual solution is not as much important as the "why and how" of the solution. As the degree of stability can be ascertained and also the effect of modification - it is sure that the method will be of great help in the design of voltage regulators.

Aldred and Shackshaft(4) present a new concept for the pre-determination of the synchronous machine stability. The concept is based on the realisation of a basic closed loop pattern for a synchronous generator, which when established can be subjected to the frequency response procedure of Nyquist stability criterion. The basic closed-loop pattern emerges from the application of small displacement theory of Park's equation for a synchronous generator.[13] This method is applicable to a machine or a system with or without a voltage regulator, but is more useful in the later case, and this forms the majority of the application.

A more exact method which has been employed by Concordia[1] is to determine the stability by the application of Routh's criterion to the co-efficients of the characteristic equation of the motion of the system. Logical extension of Park's small oscillation theory[14] led naturally to the application of Nyquist criterion with the attended advantages of providing informations concerning the degree of stability and a rapid appreciation of the effects of modification of the machine equations owing to the addition of damper windings or voltage regulator.

The technique enables stability problems to be tackled with no more apparatus THAN PENCIL AND PEN. However, it is difficult to insert the appropriate critical conditions into the small oscillationequation. The difficulty can be avoided by measurement of initial conditions in the machines. Failing this, the author has found the references[1], [25] to be most useful for determining the initial condition and the work described has been framed around those characteristics.

6.2. THE NYQUIST CRITERION OF STABILITY[23]

Consider a closed loop system as shown in fig.6.1. If $G_{(p)}$ is the forward loop transfer function and $H_{(p)}$ is the feed back

CLOSED - LOOP SYSTEM.

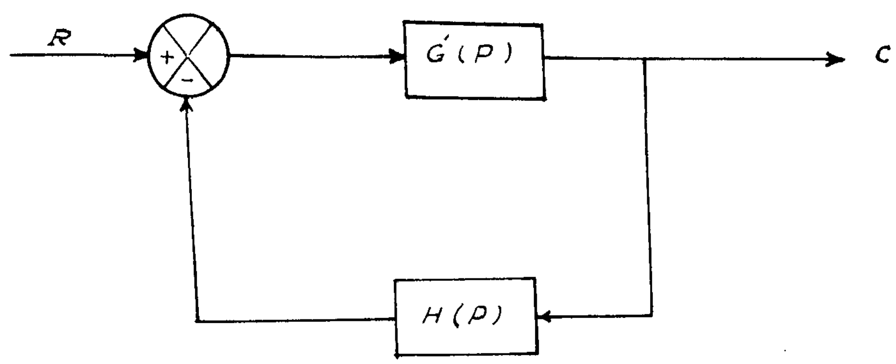


FIG. 6·1.

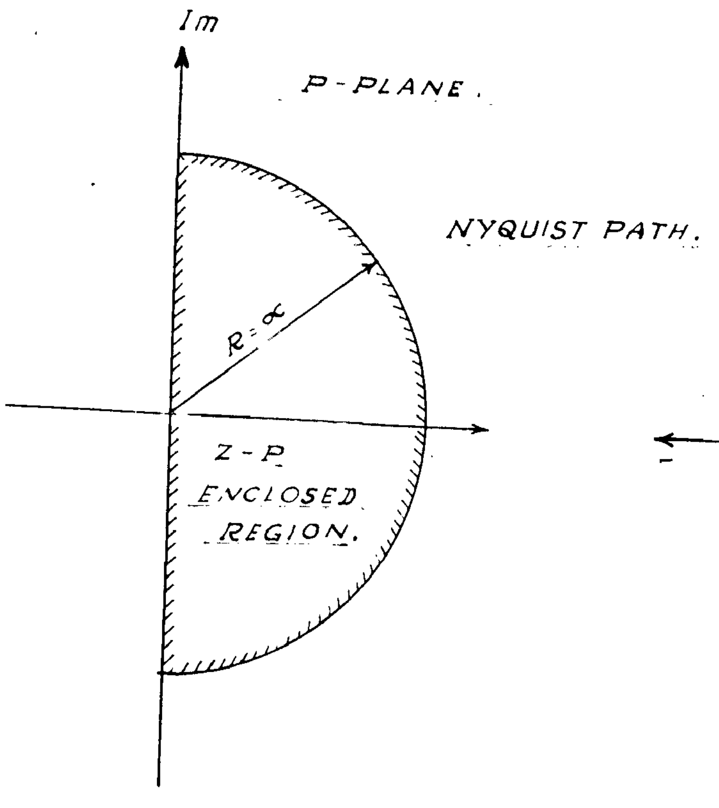


FIG. 6·2.

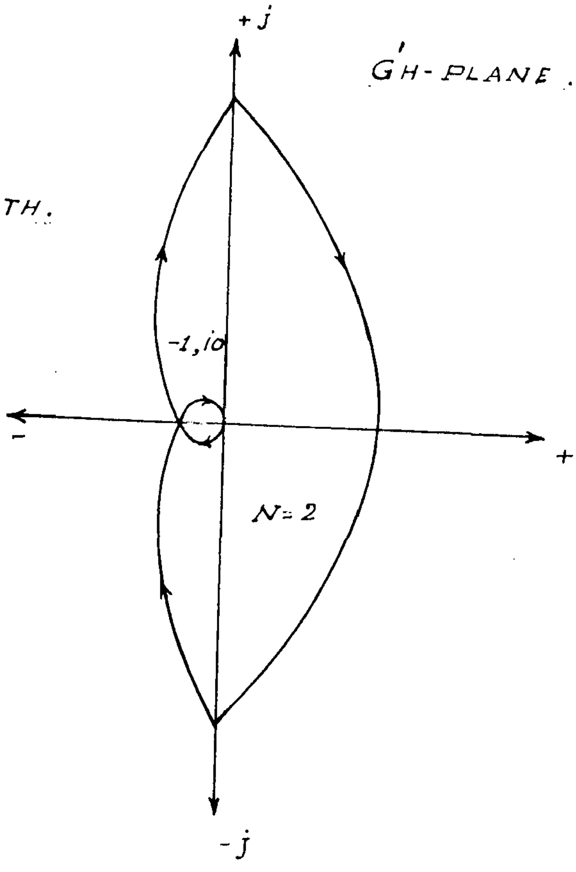


FIG. 6·3.

transfer function, the closed loop transfer function of the system is given by-

$$\frac{C_{(p)}}{R_{(p)}} = \frac{G'_{(p)}}{1+G'_{(p)}H_{(p)}}$$

Hence, it is clear that if the characteristic equation $1+G'_{(p)}H_{(p)} = 0$, has any positive real roots, then the system will have an infinitely increasing response to a finite input, i.e. the system will be unstable.

Now consider the semi circle of radius R = on right hand of a p-plane having real and imaginary axes. The completely encloses the right hand portion of p-plane. So, if any of the roots of the characteristic equation is enclosed by this Nyquist path, then the system is unstable.

Let,
Z = number of zeros of the characteristic equation, enclosed by Nyquist path.

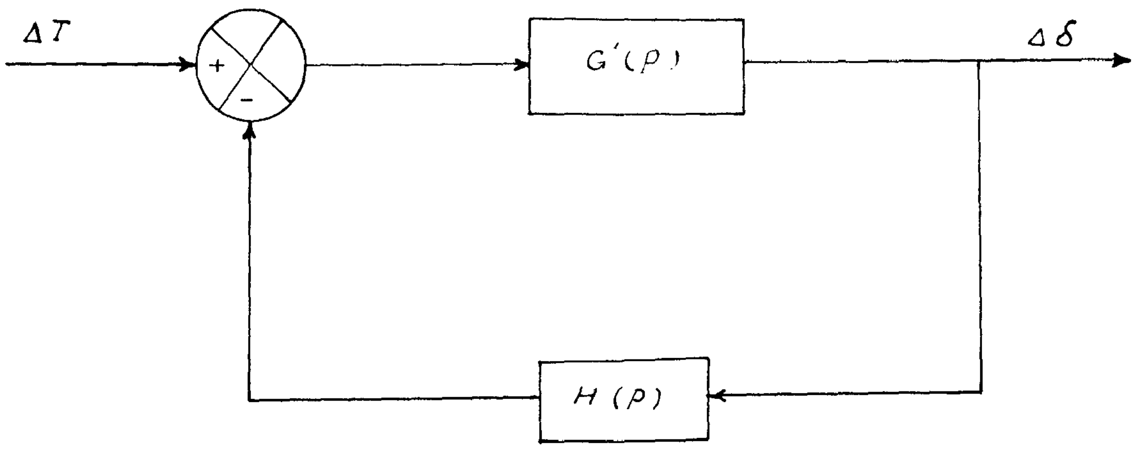
P = number of poles of the characteristic, equation enclosed by Nyquist path.

It is not easy always to find out Z and for that only, Nyquist criterion is to be used. Referring to this path, we can draw the corresponding plot of the function,

$$D_{(p)} = 1 + G'H$$

For that, travel along the path in the p-plane in the clock-wise direction and plot the corresponding values of $D_{(p)}$ along the real and imaginary axes. From the property of conformal mapping, we know that the number of encirclements of the origin by the plot of $D_{(p)}$ in clock-wise direction, N = Z - P, P is also

CLOSED-LOOP SYSTEM FOR THE SYSTEM



$$|A| = 1 + G'(p)\,H(p).$$

FIG. 6·4.

the number of poles of GH- which is the open loop transfer function - enclosed by the path. P can be found easily. So by counting N, Z can be determined; system is unstable if Z =N+P is more than zero. Instead of taking encirclements of origin by 1 + GH, generally encirclements of (-1, j0) by GH are counted.

6.3. SYSTEM AS A CLOSED LOOP:

If the characteristic matrix equation (4.36) is considered and the determinant of the matrix is denoted by $|\Delta|$, the said matrix equation (4.36) of the 2nd chapter can be written in the form-

$$\begin{bmatrix} \Delta e_{d1} \\ \Delta e_{q1} \\ \Delta i_{d1} \\ \Delta i_{q1} \\ \Delta \delta \end{bmatrix} = \frac{\Delta Adj.A}{|\Delta|} \begin{bmatrix} \Delta T_{m1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \ldots \quad (6.1)$$

from the above equation, it is evident that-

$$\frac{\Delta \delta}{\Delta T_{m1}} = \frac{D_{(p)}}{|\Delta|} \quad \ldots \quad \ldots \quad (6.2)$$

where, $D_{(p)}$ denotes term in the new matrix of one column only.

If we further denote-

$$D_{(p)} = G'_{(p)}$$

$$|\Delta| = 1 + G'_{(p)} H_{(p)}.$$

$$\therefore \frac{\Delta \delta}{\Delta T_{m1}} = \frac{G'_{(p)}}{1+G'_{(p)}H_{(p)}} \quad \ldots \quad (6.3)$$

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This denotes a closed loop transfer function of the system where $\Delta \omega_{m}$ is the input and $\Delta\delta$ is the output.

6.4. OPEN LOOP TRANSFER FUNCTION FOR THE CASES STUDIED AND THE NYQUIST PLOT:

The open loop transfer functions for the cases studied in the chapter 5 by applying Routh's criterion will now be obtained together with their Nyquist Plots.

Case 1. $\delta_0 = 0.$

From the equations (5.1) and (6.3), the open loop transfer function is-

$$G'_{(p)}H_{(p)} = -\frac{(0.935p^8-1021.866p^7-3325.7p^6-4220.54p^5-2698.14p^4-975p^3-78.9p^2+55.165p+17.8)}{1024p^7+3328p^6+4224p^5+2720p^4+980p^3+201p^2+22p+1} \quad \ldots (6.4)$$

Case 2. $\delta_0 = 66.7°$

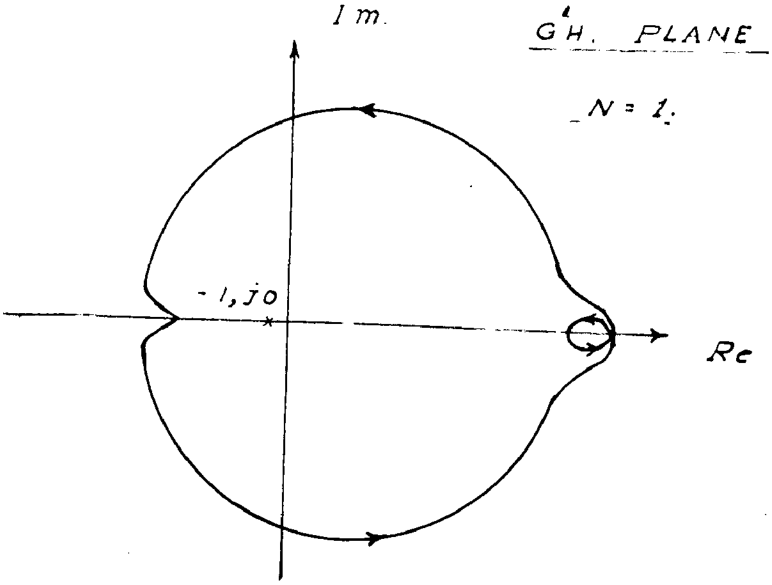
From the equations (5.2) and (6.3), the open-loop transfer function for the system is -

$$G'_{(p)}H_{(p)} = \frac{(13.04p^{11}+75.2p^{10}+2434p^9+12834.8p^8+29364.8p^7+34277.746p^6+25655.8p^5+12312.37p^4+5361.5p^3+1596p^2+272.6p+20.097)}{1024p^7+3328p^6+4224p^5+2720p^4+980p^3+201p^2+22p+1} \quad \ldots (6.5)$$

Case 3.

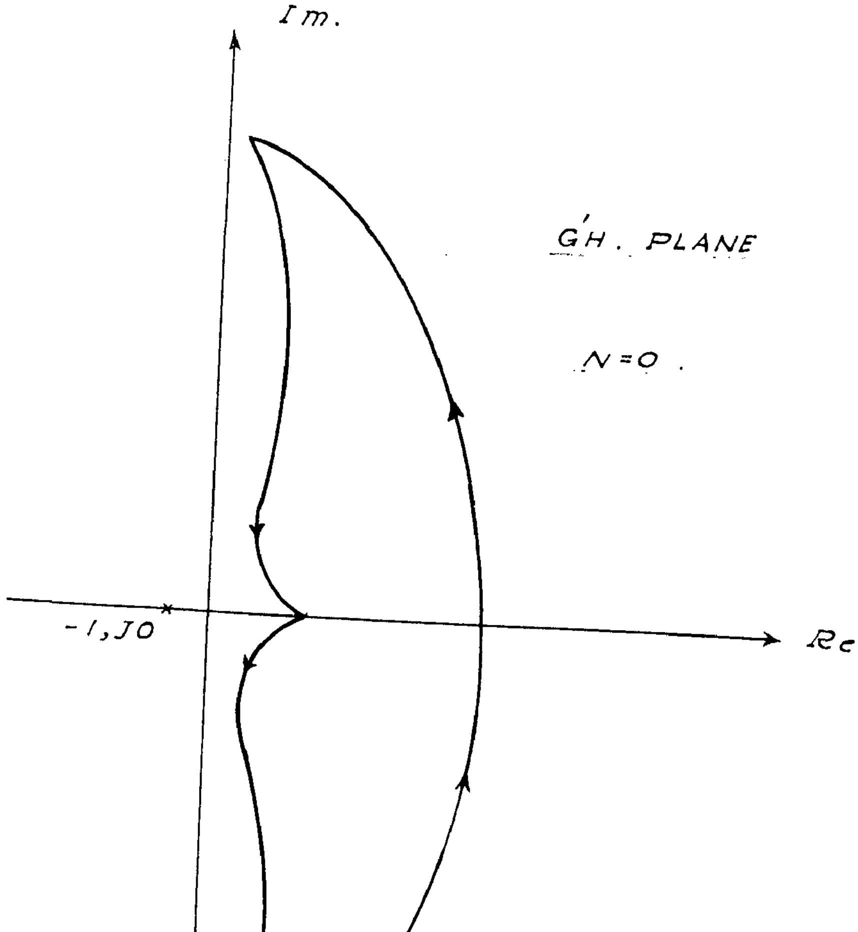
$\delta_0 = 86.1°$

From the equations (5.3) and (6.3), the open-loop transfer functions of the system is-



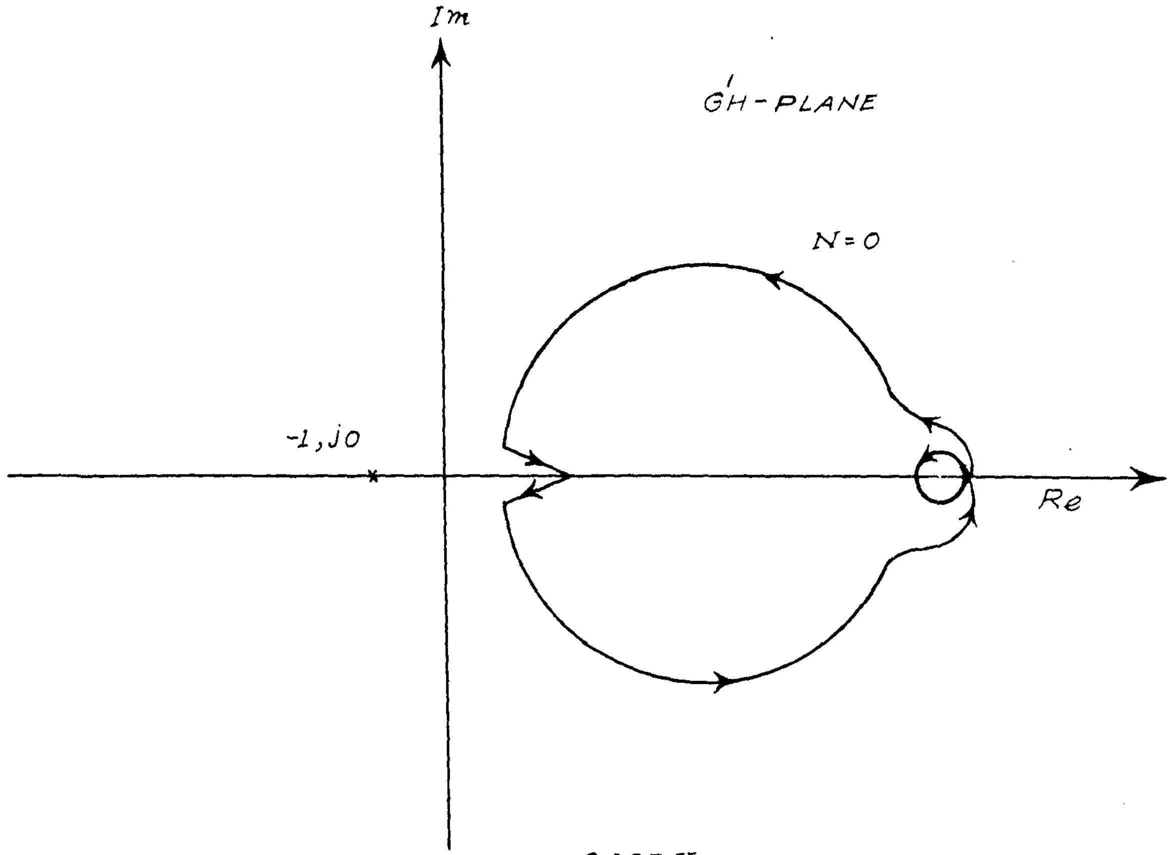
CASE IV SHAPE OF NYQUIST ENCLOSURE

FIG. 6·8.



CASE I. SHAPE OF NYQUIST ENCLOSURE

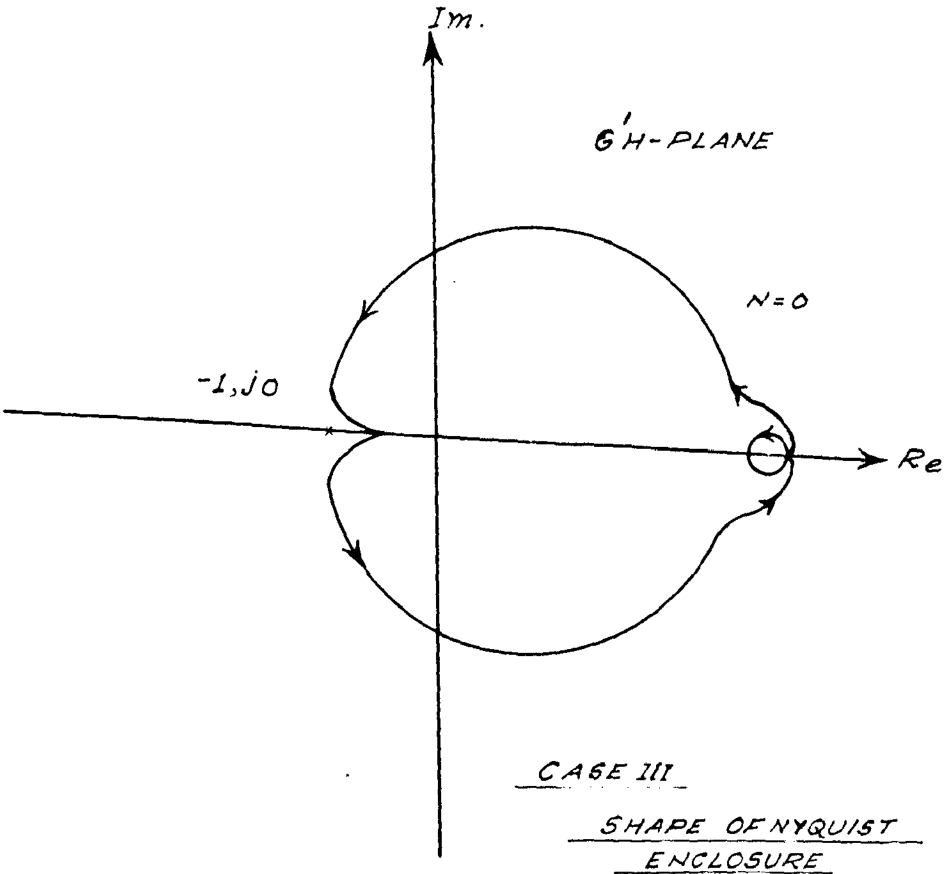
FIG. 6·5.



CASE II

SHAPE OF NYQUIST ENCLOSURE

FIG. 6.6



CASE III

SHAPE OF NYQUIST ENCLOSURE

$$G'_{(p)}H_{(p)} = \frac{(12.02p^{11}+60.2p^{10}+5162.364p^{9}+24433.6p^{8}+47070p^{7}+48656p^{6}+26973.6p^{5}+7156.6p^{4}+482\,p^{3}-142.4p^{2}-11.5p-0.99)}{1024p^{7}+3328p^{6}+4224p^{5}+2720p^{4}+980p^{3}+201p^{2}+22p+1} \quad \ldots (6.6)$$

<u>Case 4.</u>

$\delta_o = 99.3°$

From the equations (5.4) and (6.3), the open loop transfer function for the system is -

$$G'_{(p)}H_{(p)} = \frac{(20.54p^{11}+77.8p^{10}+677.7p^{9}+20108.5p^{8}+33252.9p^{7}+25866p^{6}+5152.445p^{5}-14465.5p^{4}-1384.4p^{3}-3690.9p^{2}-668.6p-38.703)}{1024p^{7}+3328p^{6}+4224p^{5}+2720p^{4}+980p^{3}+201p^{2}+22p+1.} \quad \ldots (6.7)$$

The different shapes of the Nyquist plotshave been shown in the figures. 6.3, 6.4, 6.5 and 6.6.

<u>Case 1-</u>
<u>No-load $\delta_o = 0$</u>

From the Routh's criterion, as shown in chapter 5, the characteristic equation has two real positive roots. Hence,

Z = 2 and P = 0

N = Z - P = 2

But the Nyquist plot does not encircle the critical point at all. Thus the system is <u>unstable</u> (fig.6.3)

<u>Case 2</u>
<u>Full-load. $\delta_o = 66.7°$</u>

The Routh's criterion showsthat the characteristic equation does not have any real positive roots. As there is no pole on the

R.H.S. too, which is evident from the equation (4.36).

$Z = 0, \quad P = 0 \quad \therefore N = 0$

The Nyquist plot as shown in fig. (6.4) does not encircle the critical point at all in either directions, so, the system is <u>stable</u>.

<u>Case 3</u>

<u>150% Full load, $\delta = 86.1°$</u>

From the Routh's criterion, it is seen that the characteristic equation, does not have any roots on R.H.S.

$\therefore Z = 0, \quad P = 0, \quad \therefore N = 0$

The Nyquist plot shown in fig.6.5 shows that, it does not encircle, the critical point which showsthat the system is <u>stable.</u>

<u>Case 4.</u>

<u>175% Full-load $\delta = 99.3°$</u>

The Routh's criterion analysis shows that the characteristic equation has three positive real roots.

As,

$P = 0; \quad Z = 3$

$\therefore N = Z - P = 3.$

The Nyquist plot as shown in fig.6.6 shows that the plot encircles the critical point only once in the anti-clockwise direction. Therefore, the system is <u>unstable</u>.

<u>Plotting of actual Frequency Response-</u>

The sample calculation for case 1 ($\delta = 0$) is shown here-

$$GH'_{(jw)} = \frac{(0.935w^8 + 332.5w^6 - 2698.14w^4 + 78.9w^2) + 17.8 +}{(-3328\,w^6 + 2720\,w^4 - 201\,w^2) + 1 +}$$

$$\frac{jw(1021.86w^6 - 4220.54w^4 + 975w^2 + 55.165)}{jw(-1024w^6 + 4224w^4 - 980w^2 + 22)}$$

At $w = 0$

$$GH'_{(jw)} = 17.8\angle 0^\circ = 17.8 + j0$$

At $w = 0.1$

$$GH'_{(jw)} = \frac{-.2698 + .789 + j10^{-1}(-.42 + 9.75 + 55.165) + 17.8}{(-.003 + .272 - 2.01) + 1 + j\,10^{-1}(.001 + .422 - 9.8 + 22)}$$

$$= \frac{18.319 + j6.4}{-1.841 + j\,1.262}$$

$$= \frac{(18.319 + j6.4)(-1.841 - j1.262)}{(-1.891)^2 - (1.262)^2}$$

$$= \frac{-33.7 - j23.1 - j11.8 \pm 8.51}{3.4 - 1.544}$$

$$= \frac{-25.19 - j34.9}{1.956} = -12.85 - j\,17.84$$

At $w = 0.05$

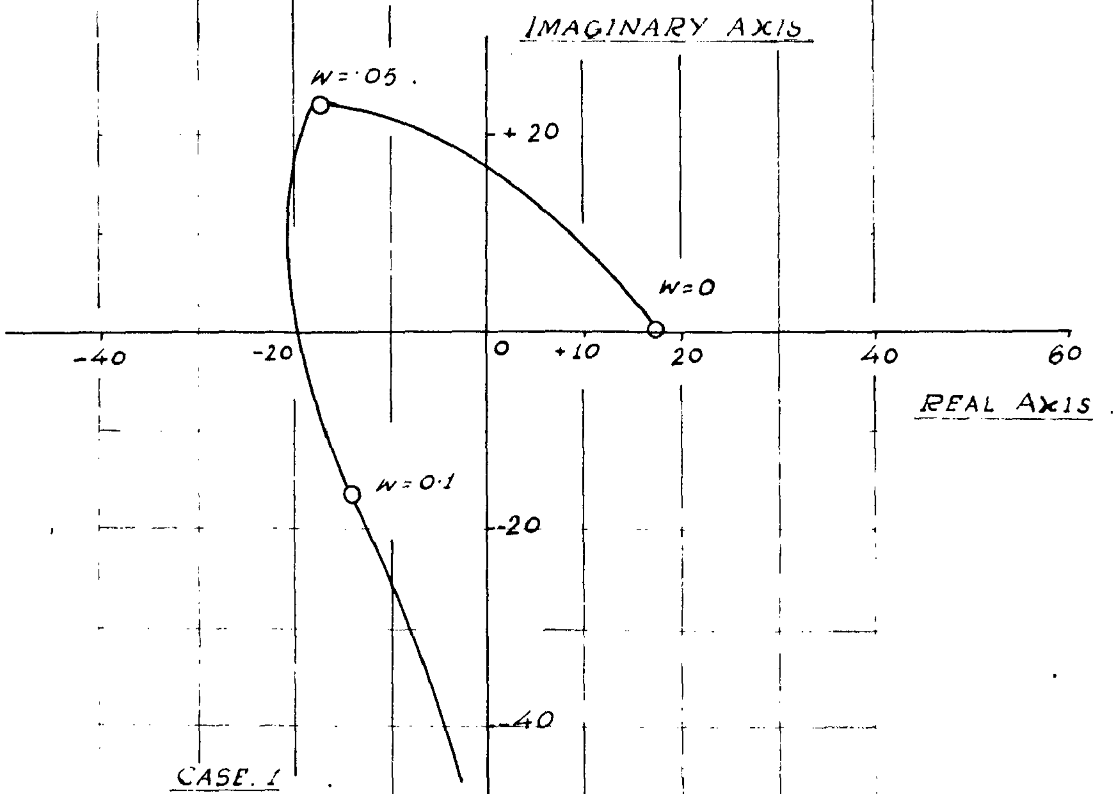
$$GH'_{(jw)} = \frac{-.0168 + .1972 + j \times .05(-.026 + 2.64 + 55.165) + 17.8}{(.017 - .502 + 1) + (0.026 - 2.45 + 22) \times j.05}$$

$$= \frac{17.98 + j2.99}{0.515 + j0.9788}$$

$$= \frac{9.25 + 2.92 + j(-17.58 + 1.54)}{.261 - 0.955}$$

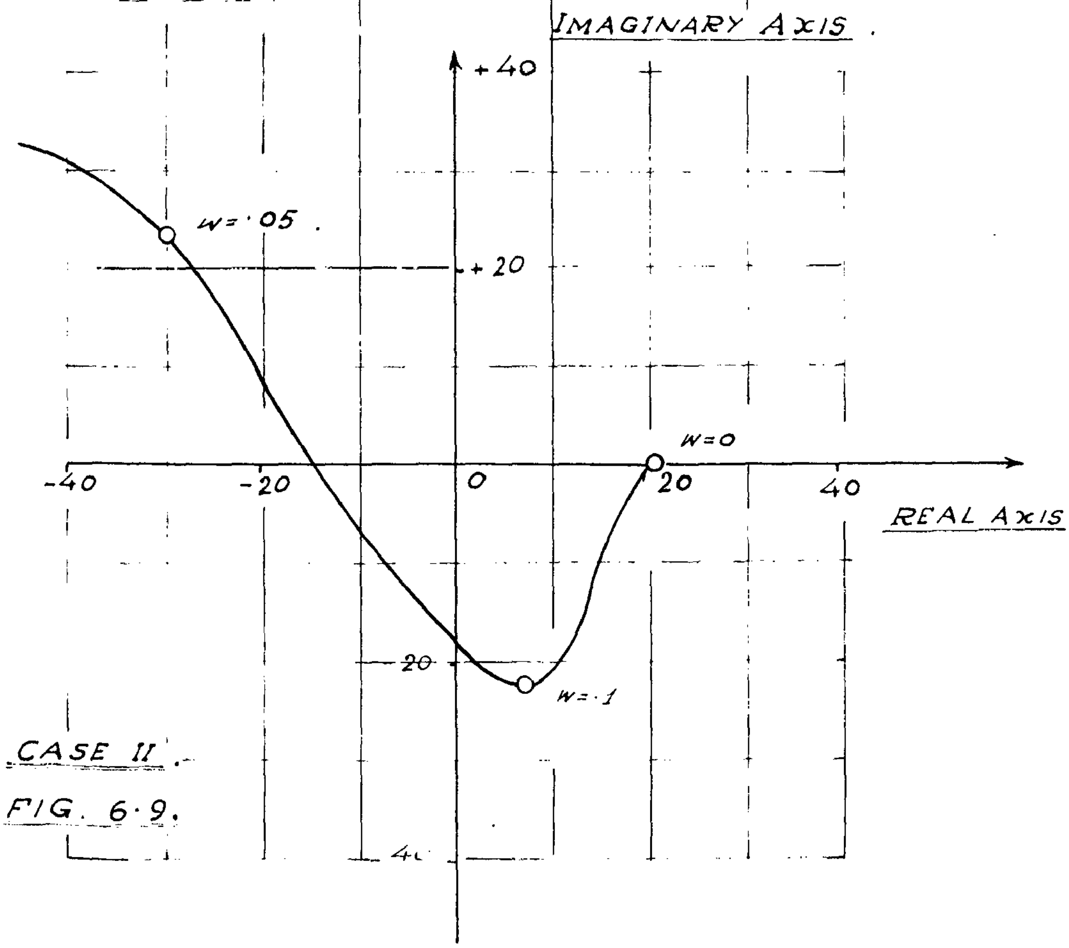
$$= \frac{12.17 - j16.04}{-.694} = -17.55 + j23.17.$$

NYQUIST - PLOT



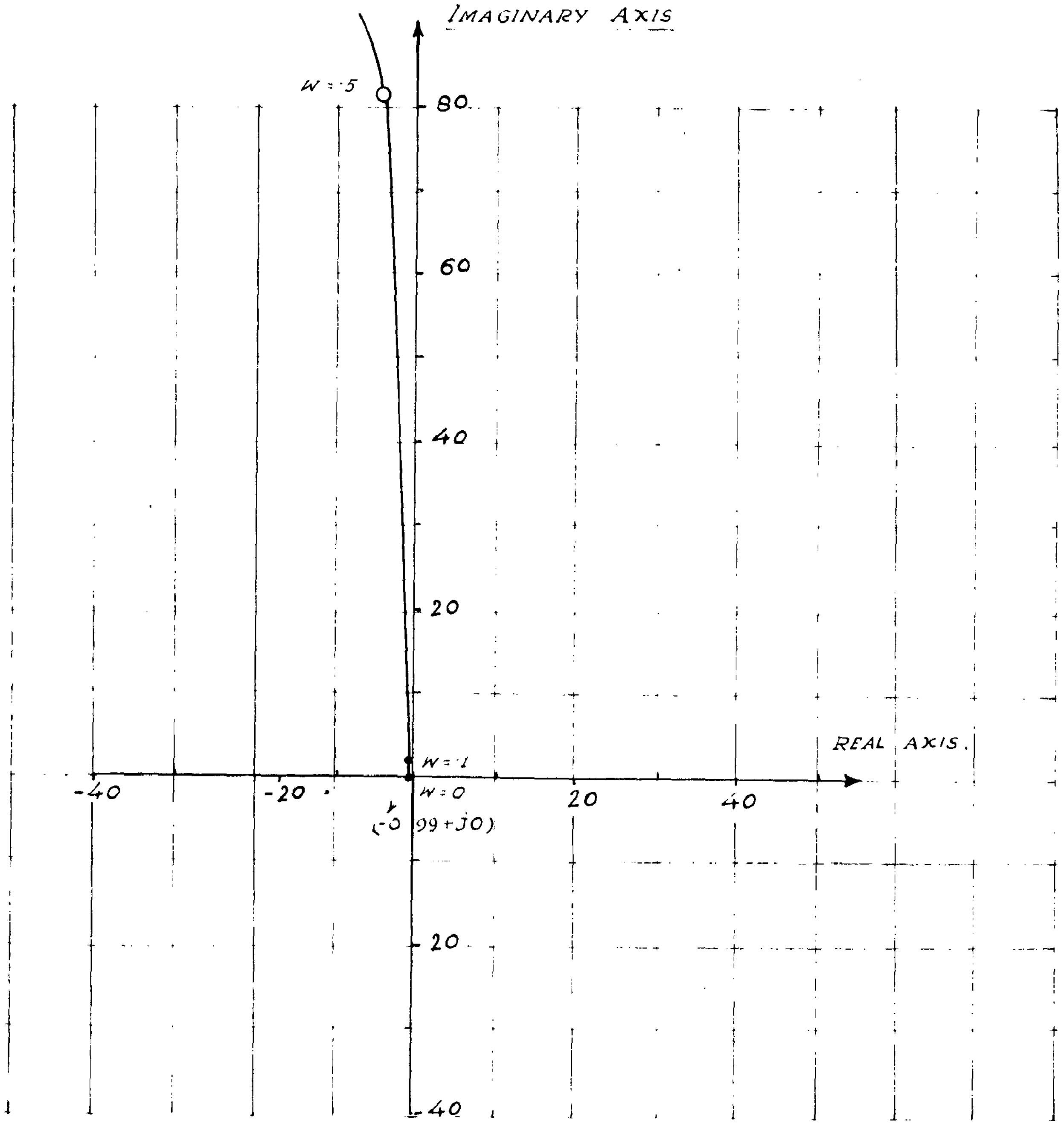
CASE. I

FIG. 6·8



CASE II

FIG. 6·9.



CASE III

FIG. 6·10.

The Nyquist plot is shown in figure no.6.8, which encloses the critical point (-1, j0) and the system is <u>unstable.</u>

Similarly, the Nyquist plots i.e. the actual frequency responses to the scale are plotted in the figures 6.9, 6.10 and 6.11 for the cases 2, 3 and 4 respectively. From these figs., the systems stability or unstability can be seen and at the same time, the phase margins and the gain margins can be ascertained in the cases of the systems which are stable.

CHAPTER 7

C O N C L U S I O N S.

CONCLUSIONS

The analysis of a system consisting of a synchronous generator connected to a synchronous motor, through a tie-line is the representation of an actual system and it is an important aspect of power system stability. The author has analysed the system with the help of two-reaction theory and the effect of quick acting automatic voltage regulator has been taken into consideration, in particular.

An expression for the system torque equation was derived avoiding any assumption, the equation included the effect of voltage regulator too. The electro-mechanical stability was studied by the application of small displacement theory to the equations of the synchronous machines system. The advantages of this method are that the effects of dampers and voltage regulators can be included and secondly all the techniques of control systems can now be applied to the electrical system.

The Routh's criterion is applied to the system equations in the following four cases and the following conclusions are drawn-

(1) At no-load ($\delta_0 = 0$)

The system is unstable as the sustained oscillations are not damped out in the no-load condition.

(2) Full load ($\delta_0 = 66.7°$)

The system is stable.

(3) 150% Full-load ($\delta_0 = 86.1°$):

The system is stable. The steady-state stability limit of a system consisting of salient pole synchronous machines without the effect of voltage regulator is less than 90°.

The effect of voltage regulator is significant here, to increase the stability limit.

(4) 175% Full-load ($\delta_0 = 99.3°$):

The system is found unstable.

The frequency response criterion can illustrate the influence of different parameters of the system on its stability. The author has derived the expressions for open-loop transfer functions of the system under different conditions and calculated these cases. It was not possible to draw any general conclusions as the system equations were very complex. Ofcourse, the individual inferences drawn from Routh's criterion were supported by this analysis too and the case II which was infered to be stable from Routh's criterion seems to be a conditionally stable system.

The author neglected the saturation effect and the effect of damper windings. The damping produced by the load and the feed-back loop of the automatic voltage regulators compensates the effect of damper winding. This helps to some extent to reduce the further complex nature of the system equations which are already complex enough.

This method of analysis has got the limitations - that it requires the most tedious algebric calculations. Secondly, it should be noted that the changes assumed in the variables are infinitely small. So, it is not strictly correct to apply these results to changes of big amplitudes. But there will surely be some correspondence between the two cases.

Finally, the selection of the "amplification factor" of the voltage regulator has the maximum influence on the steady-state stability which in turns depends, to a great extent on the feed-back of the regulating system.

Irrespective of all these short comings, and limitations, it is hoped that this work will prove to be of value in the field of stability.

APPENDIX

EXPANSION OF THE CHARACTERISTIC EQUATIONS

CASE II

THE CHARACTERISTIC EQUATION IS:

$$-0.197\ (3.67p^2 + 4.5p + .92)\ [(3.31\ p^2 + 1.93p + .62)\ \{(2.8p+.7)\times$$
$$(2.48p^2 - .302p) - (2.8p^2 + .74p + .01)(1.03p^2 + .258p)\} -(1.3p^2-$$
$$1.66p -1.31)\ \{-(2.48p^2 -.302p)\ (1.87p^2 +1.3p + .01) -(1.9p+1.25p)\times$$
$$(1.03p^2 + .258p)\} +(0.232p^2 - 3.207p - 1.96)\ \{(1.87p^2 +1.3p +.01)\times$$
$$(2.8p^2 + .74p + .01)+(2.8p +.7)(1.9p+1.25)\}]+0.197(.825p^2-2.84p-$$
$$.85)\ [-(1.52p^2 +1.9p +.38)\ \{(2.8p+.7)(2.48p^2 -302p)-(2.8p^2 +.74p+$$
$$.01)(1.03p^2 +.258p)\} -(1.3p^2 -1.66p -1.31)\ \{-2.45p\ (2.48p^2-302p)-$$
$$(4p^2 + 5p + 3.45)(1.03p^2 +.258p)\} +(0.232p^2 -3.207p -1.96)\ \{2.45p\times$$
$$(2.8p^2 + .74p +.01)+(2.8p + .7)\ (4p^2+5p +3.45)\}]+0.197(2p^2+2.98p +$$
$$.411)\ [-(1.52p^2 + 1.9p +.38)\ \{-(1.87p^2 +1.3p +.01)(2.48p^2-.302p)-$$
$$(1.03p^2 +.258p)(1.9p+1.25)\} -(3.81p^2 +1.93p + .62)\ \{-2.45p(2.48p^2-$$
$$.302p) - (4p^2 +5p +3.45)(1.03\ p^2 + .258p)\} +(0.232p^2 -3.207p -$$
$$1.96)\ \{2.45p(1.9p +1.25)- (4p^2+5p+3.45)\ (1.87p^2 +1.3p +.01)\}]+$$
$$.197\ (1.54p^2 +3.95p + 1.2)\ [-(1.52p^2 + 1.9p +.38)\{(1.87p^2 +$$
$$1.3p +.01)\ (2.8p^2 +.74p +.01) +(1.9p + 1.25)(2.8p +.7)\} -(3.31p^2+$$
$$1.9p +.62)\ \{2.45p\ (2.8p^2 + .74p +.01)+(4p^2 +5p+3.45)(2.8p +.7)\}+$$
$$(1.3p^2 -1.66p -1.31)\ \{2.45p(1.9p +1.25) -(4p^2+5p+3.45)(1.87p^2 +$$
$$1.3p +.01)\}]- 0.904\ (1.52p^2 + 1.9p+ .38)\ [(3.31p^2 +1.93p+.62)\ \times$$
$$\{(2.8p +.7)\ (2.48p^2 -.302p) - (2.89p^2 +.74p +.01)\ (1.03p^2 +$$
$$.258p)\} - (1.3p^2 - 1.66p -1.31)\ \{- (1.87p^2 + 1.3p + .01)\ \times$$
$$(2.48p^2 - .302p)-(1.9p+1.25)(1.03p^2 + .258p)\}+(0.232p^2-3.207p -$$
$$1.96)\ \{(1.87p^2+1.3p+.01)(2.8p^2+.74p+.01)+(1.9p+1.25)(2.8p+.7)\}]$$

$$-.904(0.825p^2 -2.84p -.06) \Big[-(3.67p^2 +4.59p +.92)\{(2.8p+.7) \times$$

$$(2.48p^2 - .302p)-(2.8p^2 +.74p +.01) (1.03p^2 +.258p)\} -(1.3p^2 -$$

$$1.66p - 1.31)\{-(4p^2 +5.54p +1)(2.48p^2 -.302p)-0.535(1.03p^2 +$$

$$.258p)\} +(0.232p^2 -3.207p -1.96) \{(4p^2+5.54p +1)(2.8p^2+.74p +$$

$$.01) +.535 (2.8p +.7)\}\Big]-0.904 (2p^2+2.98p +.411)\Big[-(3.67p^2 +4.59p+$$

$$.92)\{-(1.87p^2 +1.3p +.01) (2.48p^2 -0.302p)-(1.9p+1.25)(1.03p^2+$$

$$.258p)\} -(3.31p^2 +1.93p +.62) \{-(4p^2 +5.54p+1)(2.48p^2-0.302p)-$$

$$.535(1.03p^2+.258p)\}+(0.232p^2-3.207p -1.96) \{ (4p^2+5.54p+1) \times$$

$$(1.9p +1.25) -(1.87p^2 +1.3p+.01) \times .535\}\Big] -0.904 (1.54p^2 +3.95p+$$

$$1.2) \Big[-(3.67p^2 + 4.59p +.92) \{(1.87p^2 +1.3p+.01)(2.8p^2+.74p+$$

$$.01) +(1.9p +1.25) (2.8p +.7)\}-(3.31p^2+1.93p +.62)\{(4p^2+5.54p+1)\times$$

$$(2.8p^2 +.74p +.01)+ .535(2.8p+.7)\} +(1.3p^2-1.66p-1.31) \{(4p^2+$$

$$5.54p +1)(1.9p +1.25)-0.535(1.87p^2 +1.3p+.01)\}\Big]-(.343p-.202) \times$$

$$(1.52p^2+1.9p +.38) \Big[-(1.52p^2+1.9p+.38)\{(2.8p+.7)(2.48p^2-0.302p)-$$

$$(2.8p^2 +.74p +0.01)(1.03p^2 +.258p)\} -(1.3p^2-1.66p-1.31)\{-2.448p$$

$$(2.48p^2-0.302p) -(1.03p^2 +.258p)(4p^2+5p+3.48)\}+(0.232p^2-3.207p-$$

$$1.96) \{2.448p(2.8p^2 +.74p+.01)+ (4p^2 +5p+3.48)(2.8p+.7)\}\Big]-$$

$$(0.343p -.202) (3.67p^2 +4.59p +.92)\Big[-(3.5p^2+4.59p +.92) \times$$

$$\{(2.8p+.7)(2.48p^2-0.303p)-(2.8p^2+.74p+.01)(1.03p^2+.258p)\}-(1.3p^2-$$

$$1.66p-1.31) \{-(4p^2+5.54p+1)(2.48p^2-0.302p)-.535(1.03p^2+.258p)\}+$$

$$(0.232p^2 -3.207p-1.96) \{(4p^2+5.54p+1)(2.8p^2+.74p+.01)+0.535(2.8p+$$

$$.7)\}\Big] -(.343p -.202) (2p^2+2.98p+.411)\Big[-(3.5p^2+4.59p+.92)\{-2.448p \times$$

$$(2.48p^2 -0.302p)-(4p^2 +5p+3.48)(1.03p^2 +.258p)\} +(1.52p^2+1.9p+38)$$

$$\{-(4p^2+5.54p+1)(2.48p^2-0.302p)-0.535(1.03p^2+.258p)\}+(0.232p^2-$$

$$3.207p-1.96)\{(4p^2+5.54p+1)\times(4p^2+5p+3.48)-.535\times 2.448p\}]-$$

$$(.343p-.202)(1.54p^2+3.95p+1.2)[-(3.57p^2+4.59p+.92)\{2.448p\times$$

$$(2.8p^2+.74p+.01)+(4p^2+5p+3.45)(2.8p+.7)\}+(1.52p^2+1.9p+.38)\times$$

$$\{(4p^2+5.54p+1)(2.8p^2+.74p+.01)+.535(2.8p+.7)\}+(1.3p^2-1.66p-$$

$$1.31)\{(4p^2+5.54p+1)(4p^2+5p+3.45)-0.535(2.448p)\}]+(8.184p+$$

$$2.046)(1.52p^2+1.9p+.38)[-(1.52p^2+1.9p+.38)\{-(1.87p^2+1.3p+$$

$$.01)(2.48p^2-.302p)-(1.9p+1.25)(1.03p^2+.258p)\}-(3.31p^2+$$

$$1.93p+.62)\{-(2.48p^2-.302p)\times 2.448p-(4p^2+5p+3.45)(1.03p^2+$$

$$.258p)\}+(.232p^2-3.207p-1.96)\{2.448p(1.9p+1.25)-(4p^2+5p+$$

$$3.45)(1.87p^2+1.3p+.01)\}]\ (8.184p+2.046)(3.67p^2+4.59p+.92)[-$$

$$(3.67p^2+4.59p+.92)\{-(1.87p^2+1.3p+.01)(2.48p^2-0.302p)-$$

$$(1.9p+1.25)(1.03p^2+.258p)\}-(3.31p^2+1.93p+.62)\{-(4p^2+$$

$$5.54p+1)(2.48p^2-0.302p)-.535(1.03p^2+.258p)\}+(.232p^2-3.21p-$$

$$1.96)\{(4p^2+5.54p+1)(1.9p+1.25)-.535(1.87p^2+1.3p+.01)\}]-(8.184p+$$

$$2.046)(.825p^2-2.84p-.85)[-(3.67p^2+4.59p+.92)\{-2.448p(2.48p^2-$$

$$0.302p)-(1.03p^2+.258p)(4p^2+5p+3.45)\}+(1.52p^2+1.9p+.38)\{-(2.48p^2-$$

$$302p)(4p^2+5.54p+1)-.535(1.03p^2+.258p)\}+(0.232p^2-3.207p-1.96)\times$$

$$\{(4p^2+5.54p+1)(4p^2+5p+3.45)-.535\times 2.448p)\}]+(8.184p+2.046)\times$$

$$(1.54p^2+3.95+1.2)[-(3.67p^2+4.59p+.92)\{2.448p(1.9p+1.25)-$$

$$(4p^2+5p+3.45)(1.87p^2+1.3p+.01)\}+(1.52p^2+1.9p+.38)\{(4p^2+$$

$$5.54p+1)(1.9p+1.25)-.535(1.87p^2+1.3p+.01)\}+(3.31p^2+1.93p+$$

$$.62)\{(4p^2+5.54p+1))4p^2+5p+3.45)-.535(2.448p)\}]\ -$$

$$-(.0672p^3 + .0168p^2)\ (1.52p^2 + 1.9p + .38)\ \Big[-(1.52p^2 + 1.9p + .38)\ \times$$

$$\Big\{(1.87p^2 + 1.3p + .01)(2.8p^2 + .74p + .01) + (1.9p + 1.25)\ (2.8p + .7)\Big\} -$$

$$(.825p^2 - 2.84p - .85)\ \Big\{2.448p\ (2.8p^2 + .74p + .01) + (4p^2 + 5p + 3.45)\ \times$$

$$(2.8p + 0.7)\Big\} + (1.3p^2 - 1.66p - 1.31)\ \Big\{2.448p(1.9p + 1.25)$$

[illegible] $$- (4p^2 + 5p + 3.48)\ (1.87p^2 + 1.3p + .01)\Big\}\Big] - (.0672p^3$$

$$.0168p^2)\ (3.67p^2 + 4.59p + .92)\ \Big[-(3.67p^2 + 4.59p + .92)\ \Big\{(1.87p^2 +$$

$$1.3p + .01)\ (2.8p^2 + .74p + .01) + (1.9p + 1.25)\ (2.8p + .7)\Big\} - (3.31p^2 +$$

$$1.93p + .62)\ \Big\{(4p^2 + 5.54p + 1)(2.8p^2 + .74p + .01) + .535(2.8p + .7)\Big\} +$$

$$(1.3p^2 - 1.66p - 1.31)\Big\{(4p^2 + 5.54p + 1)\ (1.9p + 1.25) - .535(1.87p^2 +$$

$$1.3p + .01)\Big\}\Big] + (.0672p^3 + .0168)\ (0.825p^2 - 2.84p - .85)\ \Big[-(3.67p^2 + 4.59p +$$

$$.92)\ \Big\{2.448p(2.8p^2 + .74p + .01) + (2.8p + .7)\ (4p^2 + 5p + 3.45)\Big\} + (1.52p^2 +$$

$$1.9p + .38)\ \Big\{(2.8p^2 + .74p + .01)\ (4p^2 + 5.54p + 1) + .535(2.8p + .7)\Big\} +$$

$$(1.3p^2 - 1.66p - 1.31)\Big\{(4p^2 + 5.54p + 1)(4p^2 + 5p + 3.45) - 535 \times 2.448p\Big\}\Big] +$$

$$(.0672p^3 + .0168p^2)\ (2p^2 + 2.98p + .411)\ \times\ \Big[-(3.67p^2 + 4.59p + .92)\ \times$$

$$\Big\{2.448p(1.9p + 1.25) - (4p^2 + 5p + 3.45)\ (1.87p^2 + 1.3p + .01)\Big\} + (1.52p^2 +$$

$$1.9p + .38)\ \Big\{(4p^2 + 5.54p + 1)\ (1.9p + 1.25) - 0.535\ (1.87p^2 + 1.3p +$$

$$.01)\Big\} + (3.31p^2 + 1.93p + 0.62)\ \Big\{(4p^2 + 5.54p + 1)\ (4p^2 + 5p + 3.45) -$$

$$.535 \times 2.448p\Big\}\Big] = 0$$

CASE III

THE CHARACTERISTIC EQUATION IS

$$= -0.149(3.99p^2 + 4.988p + .909)\left[(3.588p^2 + 1.35p + 1.74)\left\{(2.8p + 0.7)\times\right.\right.$$

$$(3.06p^2 - .454p) - (2.8p^2 + .74p + .01)(1.036p^2 + .26p)\Big\} - (.244p^2 -$$

$$2.088p - 1.44)\Big\{-(1.87p^2 + 1.3p + .01)(3.06p^2 - .454p) - (1.87p + 1.25)\times$$

$$(1.036p^2 + .26p)\Big\} + (1.488p^2 - 3.452p - 2.42)\Big\{(1.87p^2 + 1.3p + .01)\times$$

$$(2.8p^2 + .74p + .01) + (2.8p + .7)(1.87p + 1.25)\Big\}\Big] + 0.149(0.148p^2 -$$

$$3.47p - .892)\Big[-(.272p^2 + .34p + .068)\Big\{(2.8p + .7)(3.06p^2 - .454p) -$$

$$(2.8p^2 + .74p + .01)(1.036p^2 + .26p)\Big\} - (.244p^2 - 2.088p - 1.44)\Big\{-$$

$$2.47p(3.06p^2 - .454p) - (4p^2 + 5p + 3.47)(1.036p^2 + .26p)\Big\} + (1.488p^2 -$$

$$3.452p - 2.42)\Big\{2.47p(2.8p^2 + .74p + .01) + (4p^2 + 5p + 3.47)(2.8p + .7)\Big\}\Big] +$$

$$0.149(2.17p^2 + 1.99p + .137)\Big[-(.272p^2 + .34p + .068)\Big\{-(1.87p^2 +$$

$$1.3p + .01)(3.06p^2 - 0.454p) - (1.87p + 1.25)(1.036p^2 + .26p)\Big\} -$$

$$(3.588p^2 + 1.35p + 1.74)\Big\{-2.48p(3.06p^2 - 0.454p) - (4p^2 + 5p + 3.47)\times$$

$$(1.036p^2 + .26p)\Big\} + (1.488p^2 - 3.452p - 2.42)\Big\{2.47p(1.87p + 1.25) -$$

$$(4p^2 + 5p + 3.47)(1.87p^2 + 1.3p + .01)\Big\}\Big] + 0.149(2.37p^2 + 6.066p +$$

$$1.744)\Big[-(.272p^2 + .34p + .068)\Big\{(1.87p^2 + 1.3p + .01)(2.8p^2 + .74p +$$

$$.01) + (2.8p + 0.7)(1.87p + 1.25)\Big\} - (3.558p^2 + 1.35p + 1.74)\times$$

$$\Big\{2.47(2.8p^2 + .74p + .01) + (4p^2 + 5p + 3.47)(2.8p + 0.7)\Big\} + (.244p^2 -$$

$$2.088p - .44)\Big\{2.47p(1.87p + 1.25) - (4p^2 + 5p + 3.47)(1.87p^2 +$$

$$1.3p + .01)\Big\}\Big] - 0.915(0.272p^2 + .34p + 0.068)\Big[(3.588p^2 + 1.35p +$$

$$1.74)\Big\{(2.8p + 0.7)(3.06p^2 - 0.454p) - (2.8p^2 + .74p + .01)(1.036p^2 +$$

$$.26p)\Big\} - (.244p^2 - 2.088p - 1.44)\Big\{-(1.87p^2 + 1.3p + .01)(3.06p^2 -$$

$$.454p) - (1.87p + 1.25)(1.036p^2 + .26p)\Big\} + (1.488p^2 - 3.425p -$$

→

$2.42) \{(1.87p^2 + 1.3p + .01)(2.8p^2 + .74p + .01) + (1.87p + 1.25) \times$

$(2.8p + .7)\}] - 0.915 (.148p^2 - 3.47p - .892) [-(3.99p^2 + 4.99p + .998) \times$

$\{(2.8p + .7)(3.06p^2 - .454p) - (2.8p^2 + .74p + .01)(1.036p^2 + .26p)\} -$

$(.244p^2 - 2.088p - 1.44) \{-(4p^2 + 5.402p + 1)(3.06p^2 - 0.459p) -$

$.402 (1.036p^2 + .26p)\} + (1.49p^2 - 3.45p - 2.42) \{(4p^2 + 5.402p + 1) \times$

$(2.8p^2 + .74p + .01) + 0.402(2.8p + .7)\}] - 0.915 (2.17p^2 + 1.99p + .137) [-$

$(3.99p^2 + 4.99p + .998)\{-(1.87p^2 + 1.3p + .01)(3.06p^2 - 0.454p) -$

$(1.87p + 1.25)(1.036p^2 + .26p)\} - (3.588p^2 + 1.35p + 1.74) \{-(4p^2 +$

$5.402p + 1)(3.06p^2 - 0.454p) - 0.402(1.036p^2 + .26p)\} + (1.488p^2 -$

$3.452p - 2.42) \{(4p^2 + 5.402p + 1)(1.87p + 1-25) - .402(1.87p^2 + 1.3p +$

$.01)\}] - 0.915 (2.37 p^2 + 6.066p + 1.744) [-(3.99p^2 + 4.99p + .998) \times$

$\{(1.87p^2 + 1.3p + .01)(2.8p^2 + .74p + .01) + (2.8p + .7)(1.87p + 1.25)\} -$

$(3.588p^2 + 1.35p + 1.74) \{(4p^2 + 5.402p + 1)(2.8p^2 + .74p + .01) + 0.402 \times$

$(2.8p + 0.7)\} + (.244p^2 - 2.088p - 1.44) \{(4p^2 + 5.402p + 1)(1.87p + 1.25) -$

$0.402 (1.87p^2 + 1.3p + .01)\}] + (10.58p + 2.645)(.272p^2 + .34p + 0.068)[-$

$(.272p^2 + .34p + 0.068) \{-(1.87p^2 + 1.3p + .01)(3.06p^2 - 0.454p) -$

$(1.87p + 1.25)(1.036p^2 + .26p)\} - (3.588p^2 + 1.35p + 1.74)\{-2.47p(3.06p^2 -$

$0.454p) - (4p^2 + 5p + 3.47)(1.036p^2 + .26p)\} + (1.488p^2 - 3.452p - 2.42) \times$

$\{2.47(1.87p + 1.25) - (4p^2 + 5p + 3.47)(1.87p^2 + 1.3p + .01)\}] + (10.58p +$

$2.645)(3.99p^2 + 4.99p + .998) [-(3.99p^2 + 4.99p + .998)\{-(1.87p^2 +$

$1.3p + .01)(3.06p^2 - .454p) - (1.87p + 1.25)(1.036p^2 + .26p)\} - (3.588p^2 +$

$1.35p + 1.74)\{-(4p^2 + 5.402p + 1)(3.06p^2 - .454p) - .402 \times (1.036p^2 + .26p)\} +$

$(1.488p^2 - 3.452p - 2.42)\{(4p^2 + 5.402p + 1)(1.87p + 1.25) - .402(1.87p^2 +$

$$1.9p+.01)\}\Big] - (10.58p + 2.645)\ (.148p^2 - 3.47\ p-.892)\ \Big[-\ (3.99p^2+$$

$$4.99p + .998)\ \{-2.47p(3.06p^2-.454p)-(4p^2+5p+3.47)(1.036p^2+.26p)\}\ +$$

$$(.272p^2 + .34p + .068\ )\ \{-(4p^2 + 5.402p + 1)(3.06p^2 - .454p) - .402\ x$$

$$(1.036p^2 + .26p)\} + (1.488p^2 - 3.452p - 2.42)\ \{(4p^2+5.402p + 1)(4p^2+$$

$$5p+3.47) - .402\ x\ 2.47p)\}\Big] + (10.58p + 2.645)\ (2.37p^2+6.066p+1.744)\Big[-$$

$$(3.99p^2 + 4.99p + .998)\ \{2.47p\ (1.87p + 1.25)-(4p^2+5p + 3.47)(1.87p^2+$$

$$1.3p + .01)\}\ + (.272p^2 + .34p + .068)\ \{(4p^2+5.402p+1)(1.87p+1.25)-$$

$$0.402(1.87p^2 + 1.3p + .01)\} + (3.588p^2 + 1.35p + 1.74)\ \{(4p^2+5.402p +$$

$$1)(4p^2 + 5p + 3.47)\}\Big] - 0.402\ x\ 2.47p\quad -(0.345p-0.203)(.272p^2+.34p+$$

$$.068)\Big[-(.272p^2 + .34p + .068)\ \{(2.8p + .7)(3.06p^2 - .454p)-(2.8p^2+.74p+$$

$$01)(1.036p^2 + .26p)\}\ -(.244p^2-2.088p - 1.44)\ \{-2.47p\ (3.06p^2-$$

$$.454p)-(4p^2 + 5p + 3.47)(1.036p^2+.26p)\} + (1.488p^2-3.452p-2.42)\{2.47px$$

$$(2.8p^2+.74p+.01)+(4p^2+5p+3.47)\ (2.8p+0.7)\}\Big] - (0.345p-0.203)(3.99p^2+$$

$$4.99p^2 + .998)\ \Big[-(3.99p^2+4.99p^2+.998)\ \{(2.8p + .7)(3.06p^2 - 0.454p)-$$

$$(2.8p^2+.74p+.01)(1.036p^2+.26p)\}\ -(.244p^2-2.088p-1.44)\ \{-(4p^2+5.402p+$$

$$1)(3.06p^2 - 0.454p) - .402(1.306p^2 + .26p)\}\ + (1.488p^2-3.452p - 2.42)x$$

$$\{(4p^2 + 5.402p+1)\ (2.8p^2 + .74p + .01)\ + .402(2.8p + .8)\}\Big] - (0.345p-.203)x$$

$$(2.17p^2 + 1.99p + .137)\ \Big[-(3.99p^2+4.99p+.998)\ \{-2.47p(3.06p^2-0.454p)-$$

$$(4p^2+5p+3.47)(1.036p^2++.26p)\}\ + (.272p^2+.34p+.068)\{-(4p^2+5.402p+1)x$$

$$(3.06p^2-.454p) - 0.402(1.036p^2 + .26p)\}\ + (1.488p^2 - 3.452p-2.42)\ x$$

$$\{(4p^2 + 5.402p + 1)(4p^2 + 5p+3.47) - 0.402x2.47\ p\}\Big]\ -.345p - .203)\ x$$

$$(2.37p^2+6.066p + 1.744)x\Big[-\ (3.99p^2 + 4.99p+.998)\ \{2.47p(2.8p^2+.74p+$$

$$.01)+(4p^2 + 5p+3.47)\ (2.8p+.7)\}\ + (.272p^2+.34p + .068)\{(4p^2+5.402p+1)x$$

→

$$(2.8p^2+.74p+.01)+0.402(2.8p+.7)\} -(2.17p^2+1.99p+1.37) \times$$
$$\{(4p^2+5.402p+1)(4p^2+5p+3.47)-.0402 \times 2.47p\}] -(0.0672p^3+$$
$$.0168p^2)(.272p^2+.34p+0.068)[-(.272p^2+.34p+0.068)\{(1.87p^2+$$
$$1.3p+.01)(2.8p^2+.74p+.01)+(.187p+1.25)(2.8p+0.7)\}-(3.588p^2+$$
$$1.35p+1.74)\{(2.8p^2+.74p+.01)\times 2.47p+(4p^2+5p+3.47)(2.80p+$$
$$.7)\}+(.244p^2-2.088p-1.44)\{2.47p(1.87p+1.25)-(4p^2+5p+3.47)\times$$
$$(1.87p^2+1.3p+.01)\}]-(0.0672p^3+.0168p^2)(3.99p^2+4.99p+$$
$$.998)[-(3.99p^2+4.99p+.998)\{(1.87p^2+1.3p+.01)(2.8p^2+$$
$$.74p+.01)+(1.87p+1.25)(2.8p+.7)\}-(3.588p^2+1.35p+1.74)\times$$
$$\{(4p^2+5.402p+1)(2.8p^2+.74p+.01)+0.402(2.8p+.7)\}+$$
$$(244p^2-2.088p-1.44)\{(1.87p+1.25)(4p^2+5.402p+1)-.402(1.87p^2+$$
$$1.3p+.01)\}]+(0.0672p^3+0.0168p^2)(.148p^2-3.47p-.892)[-(3.99p^2+$$
$$4.99p+.998)\{(2.47p(2.8p^2+.74p+.01)+(4p^2+5p+3.47)(2.8p+$$
$$.7)\}+(.272p^2+.34p+.068)\{(4p^2+5.402p+1)(2.8p^2+.74p+.01)+$$
$$.402(2.8p+.7)\}+(.244p^2-2.088p-1.44)\{(4p^2+5p+3.47)(4p^2+$$
$$5.402p+1)-.402\times 2.47p\}]+(0.0672p^3+.0168p^2)(2.17p^2+1.99p+$$
$$.137)[-(3.99p^2+4.99p+.998)\{2.47p(1.87p+1.25)-(1.87p^2+$$
$$1.3p+.01)(4p^2+5p+3.47)\}+(.272p^2+.34p+.068)\{(4p^2+5.402p+1)\times$$
$$(1.87p+1.25)-.402(1.87p^2+1.3p+.01)\}+(3.588p^2+1.35p+$$
$$1.74)\{(4p^2+5.402p+1)(4p^2+5p+3.47)-0.402\times 2.477p\}]=0$$

CASE IV

THE CHARACTERISTIC EQUATION IS:

$$= -\,0.084\ (3.94p^2+4.94P+.987)\ \Big[(3.53p^2+.837p-.159)\Big\{(2.8p+.7)\times$$
$$(4.4p^2+.192p)+(2.8p^2+.74p+.01)(.536p^2+.134p)\Big\}+(5.8p^2+2.334p+$$
$$1.44)\ \Big\{-(1.87p^2+1.3p+.01)(4.4p^2+.192p)+(1.87p+1.25)(.536p^2+$$
$$.134p)\Big\}-(5.408p^2+.145p+3.45)\ \Big\{(1.87p^2+1.3p+.01)(2.8p^2+.74p+.01)+$$
$$(1.87p+1.25)(2.8p+0.7)\Big\}\Big]-0.084(.35p^2+3.79p+.889)\Big\{(.64p^2+.808p+$$
$$.162)\ \Big\{(2.8p+0.7)(4.4p^2+.192p)+(.536p^2+.134p)(2.8p^2+.74p+.01)\Big\}+$$
$$(.58p^2+2.33p+1.44)\ \Big\{-2.34p(4.4p^2+.192p)+(4p^2+5p+3.46)(.536p^2+$$
$$.134p)\Big\}-(5.408p^2+.145p+3.45)\ \Big\{2.34p(2.8p^2+.74p+.01)+(4p^2+5p+$$
$$3.46)(2.8p+0.7)\Big\}\Big]-0.084(-2.14p^2-1.15p+.07)\ \Big[(.64p^2+.808p+.162)$$
$$\Big\{-(1.87p^2+1.3p+.01)(4.4p^2+.192p)+(1.87p+1.25)(.536p^2+$$
$$.134p)\Big\}-(3.53p^2+0.837p-.159)\ \Big\{-2.34p\ (4.4p^2+.192p)+(4p^2+5p+$$
$$3.96)\ (.536p^2+.134p)\Big\}-(5.408p^2+.145p+3.45)\ \Big\{2.34p(1.87p+1.25)-$$
$$(4p^2+5p+3.46)\ (1.87p^2+1.3p+.01)\Big\}\Big]+0.084(4.072p^2+5.362p+$$
$$1.62)\ \Big[(0.64p^2+.808p+.162)\ \Big\{(1.87p^2+1.3p+.01)\ (2.8p^2+.74p+$$
$$.01)+(1.87p+1.25)\ (2.8p+0.7)\Big\}-(3.53p^2+.837p-.159)\ \Big\{2.34p\times$$
$$(2.8p^2+.74p+.01)+(4p^2+5p+3.46)\ (2.8p+0.7)\Big\}-(.58p^2+$$
$$2.334p+1.44)\ \Big\{2.34p\ (1.87p+1.25)-(4p^2+5p+3.46)(1.87p^2+$$
$$1.3p+.01)\Big\}\Big]-0.445\ (.64p^2+.808p+.162)\ \Big[(3.53p^2+.837p-$$
$$.159)\ \Big\{(2.8p+.7)\ (4.4p^2+.192p)+(2.8p^2+.74p+.01)\times$$
$$(.536p^2+.134p)\Big\}+(.58p^2+2.334p+1.44)\ \Big\{-(4.4p^2+.192p)\times$$
$$(1.87p^2+1.3p+.01)+(1.87p^2+1.25)\ (.536p^2+.134p)\Big\}-$$
$$(5.408p^2+.145p+3.45)\ \Big\{(1.87p^2+1.3p+.01)\ (2.8p^2+.74p+$$
$$.01)+(1.87p+1.25)\ (2.8p+.7)\Big\}\Big]-0.445\ (.35p^2+3.79p+$$
$$.889)\ \Big[-(3.94p^2+4.94p+.987)\ \Big\{(2.8p+0.7)\ (4.4p^2+.192p)+$$
$$(.536p^2+.134p)\ (2.8p^2+.74p+.01)\Big\}+(.58p^2+2.334p+$$

$$1.44)\ \{(4.4p^2 + .192p)(4p^2 + 4.56p + 1) - .44)(.536p^2 + .134p)\} -$$

$$(5.408p^2 + .145p + 3.45)\ \{(4p^2 + 4.56p + 1)(2.8p^2 + .74p + .01) - 0.44 \times$$

$$(2.8p + 0.7)\}] - 0.445\ (-2.14p^2 - 1.15p + .07)\ [-(3.94p^2 + 4.94p +$$

$$.987)\ \{-(1.87p^2 + 1.3p + .01)(4.4p^2 + .192p) + (1.87p + 1.25) \times$$

$$(.536p^2 + .134p)\} - (3.53p^2 + .837p - .159)\ \{(-4.4p^2 - .192p)(4p^2 + 4.56p +$$

$$1) - 0.44(.536p^2 + .134p)\} - (5.408p^2 + .145p + 3.45)\{(4p^2 + 4.56p + 1) \times$$

$$(1.87p + 1.25) + 0.44\ (1.87p^2 + 1.3p + .01)\}] + 0.445(4.072p^2 + 5.362p +$$

$$1.62)\ [-(3.94p^2 + 4.94p + .987)\ \{(1.87p^2 + 1.3p + .01)(2.8p^2 + 74p +$$

$$.01) + (2.8p + .7)(1.87p + 1.25)\} - (3.53p^2 + .837p - .159)\ \{(4p^2 +$$

$$4.56p + 1)(2.8p^2 + .74p + .01) - 4\ .44(2.8p + 0.7)\} - (.58p^2 + 2.33p +$$

$$1.44)\ \{(4p^2 + 4.56p + 1)(1.87p + 1.25) + 0.44(1.87p^2 + 1.3p + .01)\}] +$$

$$(-.176p + .116)(.64p^2 + .808p + .162)\ [(.64p^2 + .808p + .162) \times$$

$$\{(2.8p + .7)(4.4p^2 + .192p) + (2.8p^2 + .74p + .01)(.536p^2 + .134p)\} +$$

$$(.58p^2 + 2.334p + 1.44)\ \{-2.34p(4.4p^2 + .192p) + (4p^2 + 5p + 3.46)(.536p^2 +$$

$$.134p)\} - (5.408p^2 + .145p + 3.45)\ \{2.34p(2.8p^2 + .74p + .01) + (4p^2 +$$

$$5p + 3.46)(2.8p + .7)\}] + (-.176p + .116)(3.94p^2 + 4.94p + .987)\ [-(3.94p^2 +$$

$$4.94p + .987)\ \{(2.8p + 0.7)(4.4p^2 + .192p) + (2.8p^2 + .74p + .01) \times$$

$$(.536p^2 + .134p)\} + (.58p^2 + 2.334p + 1.44)\ \{-(4p^2 + 4.56p + 1)(4.4p^2 +$$

$$1.92p) - .44(.536p^2 + .134p)\} - (5.408p^2 + .145p + 3.45)\{(4p^2 + 4.56p + 1) \times (2$$

$$(2.8p^2 + .74p + .01) - .44\ (2.8p + .7)\}] - (-.176p + .116)(2.14p^2 + 1.15p - .07)\ [-$$

$$(3.94p^2 + 4.94p + .162)\ \{-2.34p(4.4p^2 + .192p) + (4p^2 + 5p + 3.34) \times$$

$$(.536p^2 + .134p)\} - (.64p^2 + .808p + .162)\ \{-(4p^2 + 4.56p + 1)(4.4p^2 +$$

$$.192p) - .44(.536p^2 + .134p)\} - (5.408p^2 + .145p + 3.45)\ \{(4p^2 + 4.56p + 1) \times$$

$$(4p^2 +5p +3.46)+.44 \times 2.34p\}] - (-.176p + .116)\ (4.072p^2 +$$

$$5.3682p +1.62)\ [-(3.94p^2+4.94p +.162)\ \{2.34p(2.8p^2 +.74p +.01)+$$

$$(2.8p +.7)\ (4p^2 +5p +3.46)\} -(.64p^2 + .808p +.162)\ \{(4p^2+4.56p+$$

$$1)\ (2.8p^2 +.74p+.01)-.44(2.8p+.7)\} -(.58p^2 +2.334p +1.44)\{(4p^2+$$

$$5p +3.46)+.44 \times 2.34p\}]-(9.92p +2.48)\ (.64p^2 + .808p +.162)\times$$

$$[(.64p^2 + .808p +.162)\ \{-(1.87p^2 + 1.3p +.01)(4.4p^2 +.192p) +$$

$$(1.87p +1.25)\ (.536p^2 + .134p)\} -(3.53p^2 + .837p -.159)\{-2.34p \times$$

$$(4.4p^2 +.192p) +(4p^2 + 5p + 3.46)\ (.536 p^2 + .134p)\} -(5.408p^2 +$$

$$.145p +3.95)\ \{2.34p(1.87p + 1.25) -(4p^2+5p +3.46)\ (1.88p^2+1.3p+$$

$$.01)\}] +(9.92p +2.48)\ (3.94p^2 +4.94p +.987)[-(3.94p^2 +4.94p +$$

$$.987)\ \{-(4.4p^2 +.192p)\ (1.87p^2 +1.3p +.01) +(1.87p +1.25) \times$$

$$(.536p^2 + .134p)\}- (3.53p^2 + .837p - .159)\{-(4p^2+4.56p +1)\times$$

$$(4.4p^2 + .192p) -0.44(.536p^2 + .134p)\} -(5.408p^2 +.145p+3.45) \times$$

$$\{(4p^2 +4.56p +1)\ (1.87p +1.25) +0.44(1.87p^2+1.3p +.01)\}]+(9.92p+$$

$$2.48)\ (.35p^2 + 3.79p +.089)\ [-(3.94p^2 + 4.94p + .937)\ \{-2.34p \times$$

$$(4.4p^2 + .192p) + (4p^2 + 5p+3.34)\ (.536p^2 +.134p)\} -(.64p^2 +$$

$$.808p +.162)\ \{-(4p^2 +4.56p +1)\ (4.4p^2 +.192p)-.44(.536p^2 +$$

$$.134p)\} -(5.408p^2 +.145p +3.45)\ \{(4p^2 +4.56p +1)(4p^2+5p+3.46)+$$

$$.44 \times 2.34p\}]+(9.92p + 2.48)\ (4.072p^2 + 5.36p +1.62)\ [-(3.94p^2 +$$

$$4.94p + .987)\ \{2.34p\ (1.87p +1.25) - (4p^2 +5p+3.46)(1.87p^2 +$$

$$1.3p +.01)\} -(.64p^2 + .808p +.162)\ \{(4p^2 + 4.56p + 1)\ (1.87p +$$

$$1.25) +0.44(1.87p^2 +1.3p +.01)\} +(3.53p^2 +.837p -0.159)\ \{(4p^2+$$

$$4.56p +1)\ (4p^2+5p+3.34) + .44 \times 2.34 p \}] + (0.067p^3+0.0168p^2)\times$$

$$
\begin{aligned}
&(.64p^2 + .808p + .162)\Big[(.64p^2 + .808p + .162)\big\{(1.87p^2 + 1.3p + \\
&.01)(2.8p^2 + .74p + .01) + (1.87p + 1.25)(2.8p + .7)\big\} - (3.53p^2 + \\
&.837p - .159)\big\{2.34p(2.8p^2 + .74p + .01) + (4p^2 + 5p + 3.34)(2.8p + \\
&.7)\big\} - (.58p^2 + 2.33p + 1.44)\big\{2.34p(1.87p + 1.25) - (4p^2 + 5p + \\
&3.46)(1.87p^2 + 1.3p + .01)\big\}\Big] - (0.0672\,p^3 + 0.0108p^2)(3.94p^2 + \\
&4.94p + .987)\Big[-(3.94p^2 + 4.94p + .987)\big\{(1.87p^2 + 1.3p + .01)\times \\
&(2.8p^2 + .74p + .01) + (1.87p + 1.25)(2.8p + .7)\big\} - (3.53p^2 + .837p - \\
&.159)\big\{(4p^2 + 4.56p + 1)(2.8p^2 + .74p + .01) - .44(2.8p + .7)\big\} - (.58p^2 + \\
&2.33p + 1.44)\big\{(4p^2 + 4.56p + 1)(1.87p + 1.25 + .44)(1.87p^2 + 1.3p + \\
&.01)\big\}\Big] - (0.0672\,p^3 + .0168p^2)(.35p^2 + 3.79p + .889)\Big[-(3.94p^2 + \\
&4.94p^2 + .987)\big\{2.34p(2.5p^2 + .74p + .01) + (4p^2 + 5p + 3.34)\times \\
&(2.8p + .7)\big\} - (.64p^2 + .808p + .162)\big\{(4p^2 + 4.56p + 1)(2.8p^2 + .74p + \\
&.01) - .44\times(2.8p + .7)\big\} - (.58p^2 + 2.33p + 1.44)\big\{(4p^2 + 4.56p + 1)\times \\
&(4p^2 + 5p + 3.46) + 2.34p \times .44\big\}\Big] + (0.0672\,p^3 + 0.0168p^2)(2.14p^2 + \\
&1.15p + .07)\Big[-(3.44p^2 + 4.94p + .987)\big\{2.34p(1.87p + 1.25) - \\
&(4p^2 + 5p + 3.34)(1.87p^2 + 1.3p + .01)\big\} - (.64p^2 + .808p + .162)\times \\
&\big\{(4p^2 + 4.56p + 1)(1.87p + 1.25) + .44(1.87p^2 + 1.3p + .01)\big\} + \\
&(3.53p^2 + .837p - 1.59)\big\{(4p^2 + 4.56p + 1)(4p^2 + 5p + 3.34 + \\
&.44 \times 2.34p\big\}\Big] = 0
\end{aligned}
$$

R_E_F_E_R_E_N_C_E_S

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