UNIVERSITY OF ROORKEE,
ROORKEE (U.P.)

Certified that the attached dissertation on "STUDIES OF [illegible] CIRCUITS"

was submitted by SURESH [illegible]

and accepted for the award of Degree of Master of Engineering in POWER SYSTEM [illegible]

vide Notification No. Ex/168/P-65(Degree)

dated December 21,1966.

(S. S. SRIVASTAVA)

Dated December 30,1966.

Assistant Registrar (Exam.)

S.S. 30.XII.1966

PSUP (R) 291 Ch. 1964—1,000.

STUDIES OF FERRO-RESONANCE CIRCUITS

A Dissertation
submitted in partial fulfilment
of the requirements for the Degree
of
MASTER OF ENGINEERING
in
ELECTRICAL ENGINEERING (Power Systems)

by
SURESH CHANDRA

DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE
November 1966

C E R T I F I C A T E

CERTIFIED that the dissertation entitled "STUDIES OF FERRO-RESONANCE CIRCUITS", which is being submitted by Shri Suresh Chandra in partial fulfilment for the award of the Degree of Master of Engineering in Electrical Power Systems of the University of Roorkee, is a record of students own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of six months from Feb., 1966 to July, 1966 for preparing this dissertation for Master of Engineering Degree at the University.

L. M. Ray

Dr. L.M. Ray
Associate Professor in
Electrical Engineering
University of Roorkee,
ROORKEE.

DATE 11/11/66
ROORKEE.

A C K N O W L E D G E M E N T

The author is very much indebted to his respected Professor Dr. L.M. Ray for initiating this topic for the dissertation and wishes to express his deep sense of gratitude for his able guidance and affectionate encouragement throughout.

Sincere thanks are due to Professor C.S. Ghosh, Head of the Electrical Engineering Department, University of Roorkee, Roorkee for the various facilities offered in the Department in connection with this work.

The author also acknowledges the help of his colleague Shri C.L. Wadhwa.

DATED
ROORKEE.

SURESH CHANDRA

C O N T E N T S

CHAPTER I

INTRODUCTION

INTRODUCTION

In case of resonance circuits, we assume that the inductance of the circuit is independent of the current and is constant. This is actually true if the magnetic field of the inductance flows preponderantly in air, for example around over head lines, underground cables or in the leakage paths of rotating machines or transformers. However if the magnetic field flows entirely or to a large part through iron, the circuit has no longer a constant inductance; rather this will decrease with larger currents and magnetic field strengths. Hence the phenomena in literature occuring at a critical value of applied voltage where the non linear inductance variable with the magnitude of the forcing function reaches a value so as to reasonate with the fixed capacitor is known as ferroreasonance. This is probably the reason for using the term ferroresonance to describe the hysteresis behavior.

The present problem is to study the behavior of oscillatory circuits under the influence of magnetic saturation in the iron core of the transformers connected to the long transmission line to deliver power at the load centre. Here it is tried to consider the effect of the variation of the distance at which the fault occurs in the transmission line. It is known that for a perticular system the line parameters are fixed and they are taken in per unit length. The loss component of capacitance of the line is neglected. Now as the distance of the fault is increased from the sending end, then the magnitude of the inductance resistance and capacitance increases. The abnormal voltages which may occur at those instants will be dependent on the magnitude of the capacitance which comes across the transformer. The transformer may be assumed to be a non linear inductor. A graphical method is suggested in Chapter II for calculating abnormal voltages and the conclusion is drawn that the effect of the distance of fault is directly related to the abnormal voltage and currents produced. As the distance is increased the magnitudes of the over voltages also increases. This analysis is based under the assumption of sinusoidal variation of current and a steady state voltage characteristic is obtained from the circuit equation, though it is true that the current in the non linear circuit is not a sinusoidal function but it is composed of harmonics.

In Chapter III attempt is made to give mathematical treatment for the non linear circuit arising under above mentioned circumstances. In the study of these non linear circuits with a.c. applied voltage, regorous mathematical treatment has been found practically impossible, but under certain simplifying assumptions, however, the solution is being given which serve to describe the nature of the phenomena, in general.

In Chapter III & IV certain simplifying assumptions are suggested which lead to Duffing's equation for which standard solution is available. Initially a sinusoidal flux linkages is assumed which give rise to the 3rd harmonic current after expansion i of the non linear equation for inductor characteristic. Further the approximation is corrected and the solution is assumed for the flux, linkages to contain fundamental and 3rd harmonic. Then it gives rise to 5th, 7th and 9th harmonics too. The coefficients of fundamental and 3rd harmonics are found under certain simplifying assumptions.

The further correction can be made in the assumed solution for the flux linkages, which may contain upto 5th harmonic, so the solution of the Duffings equation for current is a odd multiple series of fundamental having their amplitudes different, actual magnitudes of which can be determined only with further simplifying assumptions. The extension is made upto fifth harmonic in the flux linkages for more accurate solution. It is not advisible to go for higher correction because the magnitude of fifth harmonic itself is very small with respect to fundamental, so the higher harmonic will be more smaller.

In Chapter V & IV it is shown that the solution for Duffings' equation can also exist in subharmonic form, under certain initial conditions. Subharmonics are defined as the components having their frequency an integral submultiple of the driving frequency. Most commonly observed subharmonics are defined as the components having their frequency an integral submultiple of the driving frequency. Most commonly observed subharmonic in the system described by Duffing's equation is at one third the driving frequency.

Mr. Irven Travis and C.N. Weygandt[6] showed that the initial conditions are most important if the subharmonic oscillation is to exist in the physical system. Under different initial conditions different solutions may exist. These initial conditions should exist with in the system itself. These conditions are found for a perticular examples.

CHAPTER II

GRAPHICAL METHOD

REPRESENTATION OF THE SYSTEM

The effect of the line length is considered on the nonlinearity in the following. This nonlinearity is due to transformers in the transmission circuits.

Let the transmission system be represented by a single line diagram as shown in figure (2.1). The line is represented by a T equivalent circuit with lumped parameters. Moreover the transformer is represented by a nonlinear inductor with a voltage source behind it. Now with these types of representation the transmission circuit takes the form as shown in figure (2.2). In the circuit R L and C are the resistance, Inductance and capacitance per unit length of the transmission line l is the length of the transmission line.

During the switching out of a fault on the transmission line at a certain distance say x from the sending end the system may be represented by a series circuit shown in figure (2.3)

Under these circumstances the circuit becomes a nonlinear oscillatory one which may cause high magnitude of current and voltages.

FERRO-RESONANCE CIRCUITS: 1,2,3, 3

Ferro-resonance may occur in a circuit consisting of resistance, capacitance and an iron core inductance. In the study of ferro-resonant circuits regorous mathematical treatment has been found practically impossible. With certain simplifying assumptions, it is possible to compute performance data which describe the nature of ferro-resonance

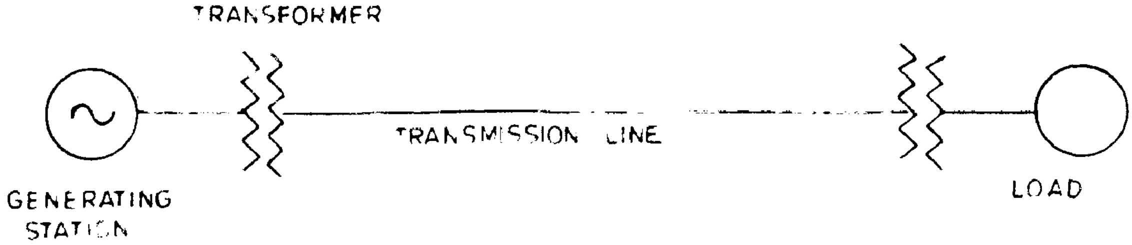


FIG. 2.1

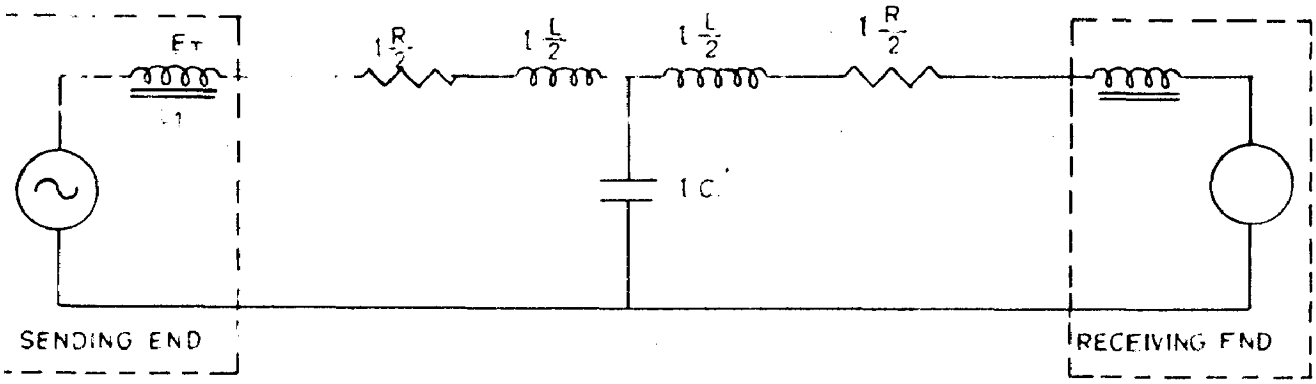


FIG. 2.2

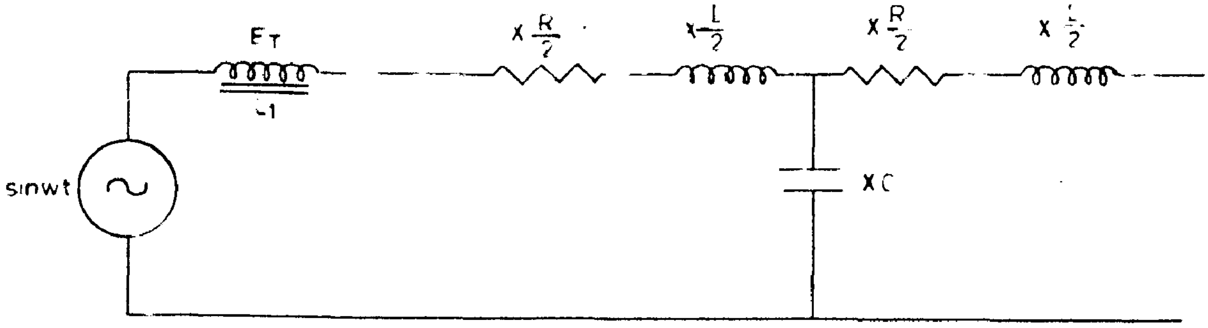


FIG. 2.3

phenomena. The methods followed in treating the series circuits in figure (2.3) needs determination of a functional relationship between current and inductance of the iron core reactor. Under the assumption of sinusoidal variation of current a continuous voltage characteristic is obtained from the well known circuit equation,

$$E = I\sqrt{\left(\frac{xR}{2}\right)^2 + \left(wL1 + \frac{xwL}{2} - \frac{1}{xwC}\right)^2} \quad \text{--------(2.1)}$$

where the functional relation is defined by

$$L_1 = \phi\ (I) \quad \text{-----(2.2)}$$

GRAPHICAL SOLUTION[2]

The graphical solution for the series - ferro - resonant circuit figure (2.3) follows from two independent relations for transformer voltage, namely the volt ampere-characteristic of the transformer,

$$E_T = f\ (I) \quad \text{------(2.3)}$$

and the circuit equation (assuming sinusoidal quantities)

$$E_T = \pm\sqrt{E^2 - \left(\frac{xRI}{2}\right)^2} + \frac{I}{xwC} - \frac{xwL\ I}{2}$$

$$\text{or} \quad E_T + \frac{xwL}{2} I = \pm\sqrt{E^2 - \left(\frac{xRI}{2}\right)^2} + \frac{I}{xCw} \quad \text{----(2.4)}$$

$$\text{or} \quad E_T + E_L = V_1 + E_c \quad \text{--------(2.5)}$$

where

$$V_1 = \pm \sqrt{E^2 - (\frac{XB}{2} I)^2} \qquad \text{———— (2.6)}$$

$$E_c = \frac{I}{Xw\epsilon} \qquad \text{———— (2.7)}$$

$$E_L = Xw \frac{L}{2} I \qquad \text{———— (2.8)}$$

Here first of all the study is made for the different components of equation (2.4) individually. The equation (2.6) represents an ellipse between voltage and current whose principal axes have values E and $\frac{E}{\frac{XB}{2}}$. For this ellipse the principal axis $\frac{E}{X\frac{B}{2}}$ is dependent upon the distance of the line where the fault had occured. The principal axis is inversely proportional to the distance. There are two limiting values for the principal axis $\frac{E}{X\frac{B}{2}}$, one corresponding to the fault condition at the sending end, in that case the ellipse reduces to two lines parallel to the current axis at distances ~~In that case the ellipse~~ of ± E. Other extreme fault condition may occur at receiving end at a greater distance corresponding to an infinite line, the ellipse collapses to a straight line corresponding to the voltage axis figure 2.4.

Considering the voltage drop equation 2.7, the relation between voltage and current can be represented by a straight line of different slopes depending on the line length involved. The slope is given by the relation

$$\tan \gamma = \frac{1}{X w\epsilon} \qquad \text{———— (2.9)}$$

The two limiting values, one corresponding to sending end at which the distance is zero and the other at receiving end at which the distance is maximum and is infinite in case of infinite

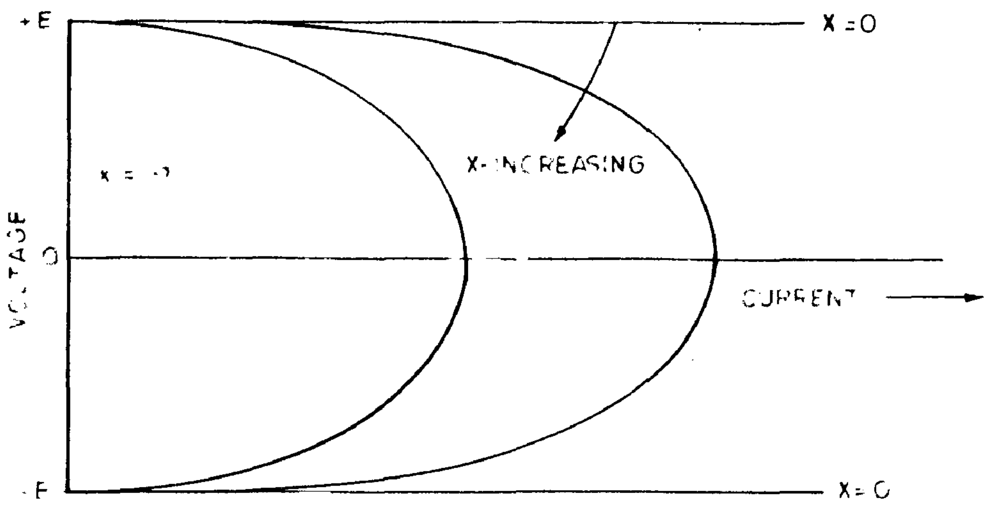


FIG. 2.4

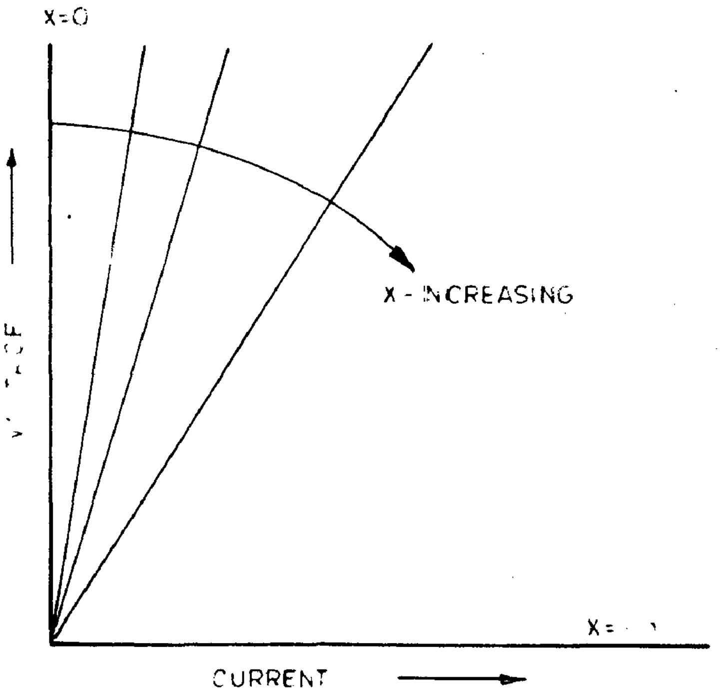


FIG. 2.5

line, are 90° and zero degree. The rotation of the line is clockwise as the fault moves from the sending end towards receiving end having its one end fixed at the origin (figure 2.5).

The sum of the equations 2.6 and 2.7 between the limits $0 \angle X \angle \infty$ is an ellipse (a fact that may be readily ascertained). The position of this ellipse changes as the distance of the fault changes fig. 2.6.

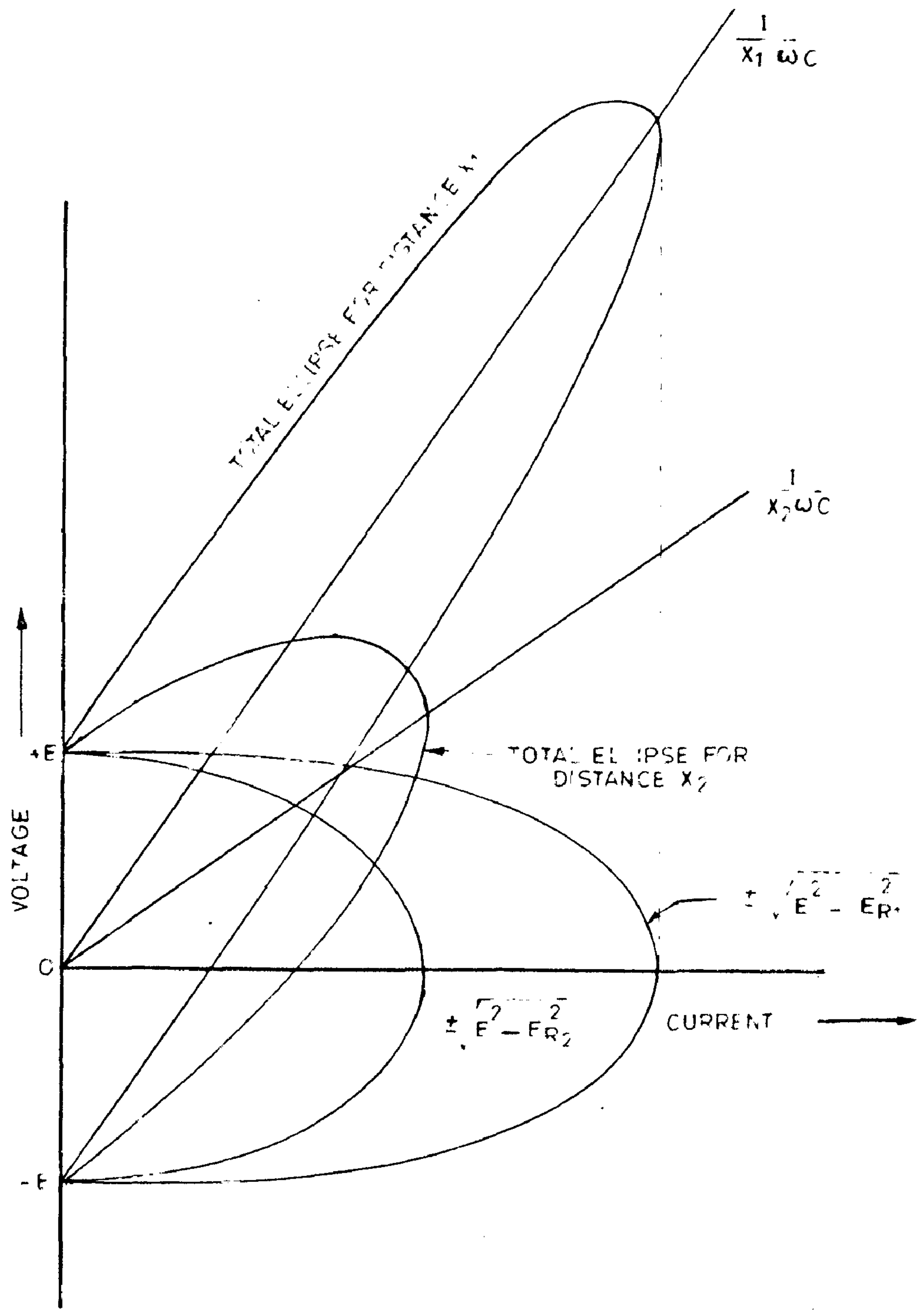
On the left hand side of equation 2.4 the first term is independent of the fault distance x, it represents the transfo mer volt ampere characteristic (O. c. c.) fig. 2.7

The equation 2.8 gives a striaght line relation between voltage and current. The slope of the line is given by the following

$$\tan \beta = X w \frac{L}{2} \qquad \text{———————} (2.10)$$

The above relation shows that the slope of the line is directly proportional to the fault distance X. In this case the rotation of the line will be anticlockwise (opposite to that of capacitive drop characteristic represented by equation 2.7) as the fault moves from the sending end towards receiving end having its one end fixed at the origin. Figure 2.8.

The addition of the volt ampere characteristic of transformer to the inductive drop characteristic of the transmission line (equation 2.8) gives the total inductive drop characteristic. The total characteristic is lifted upwards as the fault proceeds from sending end towards receiving end and has got the same tendency of rotation as the straight line represented by equation 2.8 i.e. in an anticlockwise direction fig. 2.9.



$\frac{1}{X_1 \omega C}$
$\frac{1}{X_2 \omega C}$
VOLTAGE
CURRENT
+E
O
-E
TOTAL ELLIPSE FOR DISTANCE X_2
$\pm\sqrt{E^2 - E_{R_1}^2}$
$\pm\sqrt{E^2 - E_{R_2}^2}$

FIG. 2.6

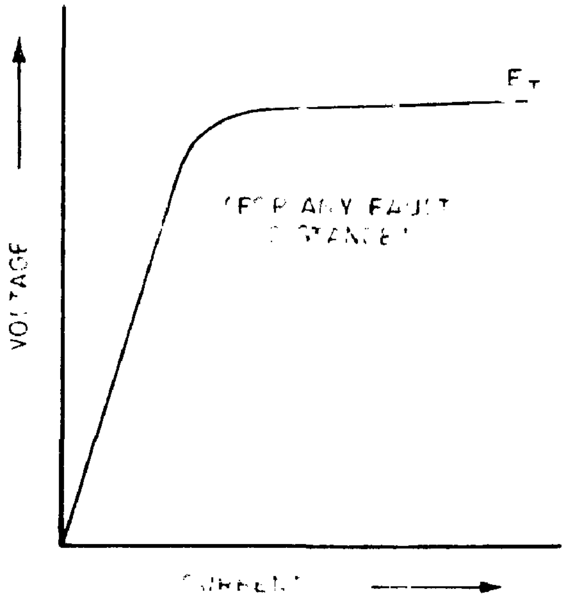


FIG. 2.7

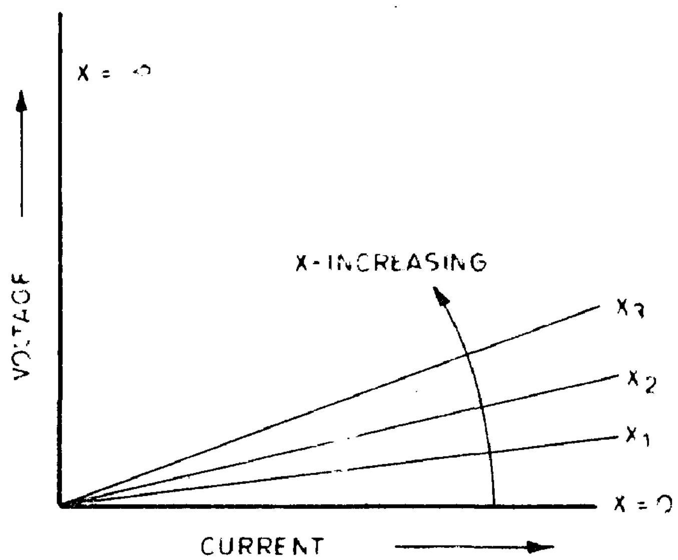


FIG. 2.8

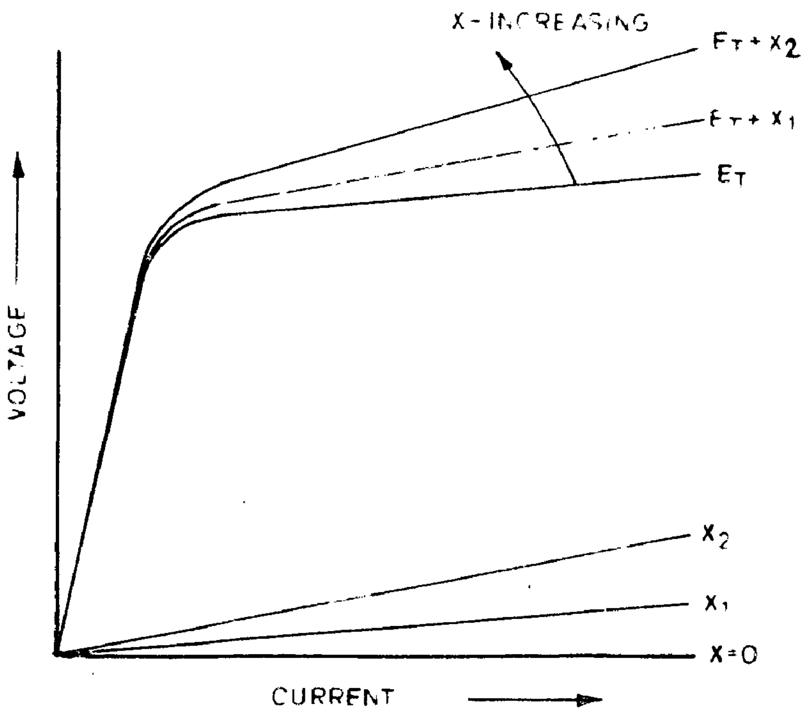


FIG. 2.9

The intersection of this total magnetic characteristic with the total ellipse are the graphical solutions for the current ((points 1, 2 and 3 in fig. 2.12). The representation in this form, because of its component nature, permits a simple graphical study of the circuit characteristic with a variation of fault distance X.

Multiple current[5] values are due to the discontinuties or jumps. As the voltage is gradually increased accross the combination of the ideal reactor and the capacitor, the reactor does not conduct until a critical voltage is reached at which the operation point suddenly jumps to A_1. With further gradual increase in voltage the current increases smoothly along the curve 'a' (fig.2.10) upto same point A_2. With the decrease in voltage the current reduces in a similar manner upto point A_1, where the voltage is again equal to its critical value. Now if the voltage is further reduced the current reduces smoothly upto point A_3 which corresponds to a second critical point at which the further reduction in voltages reduces the current abruptely to zero. Curves b and c of figure 2.10 corresponds to less ideal ferro-resonance associated with the B - H curve b and c of figure 2.11.

At the critical points A_1 and A_3 there is an abrupt change in current which takes place and is shown by the dotted lines in figure 2.10. The path between these extremities is not known, it may have any shape. Hence for the voltage between these extremities there may be atleast as many values as three for current. Out of these one is unstable and does not exist in the system, so we call that value unstable one. Actually current values are dependent

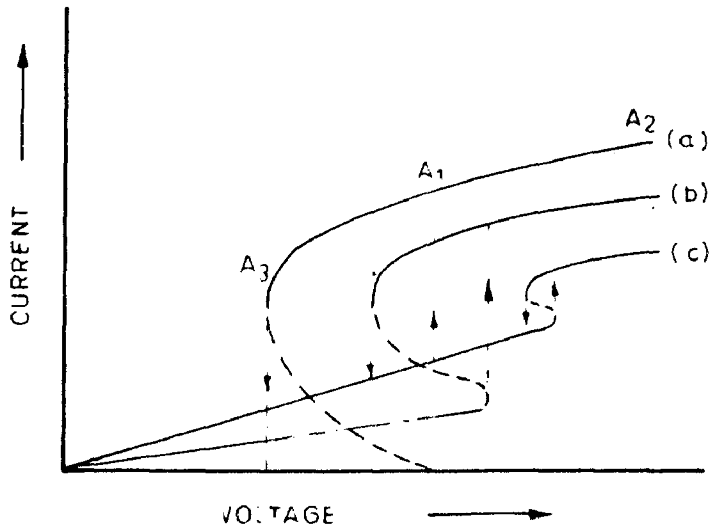


FIG. 2.10

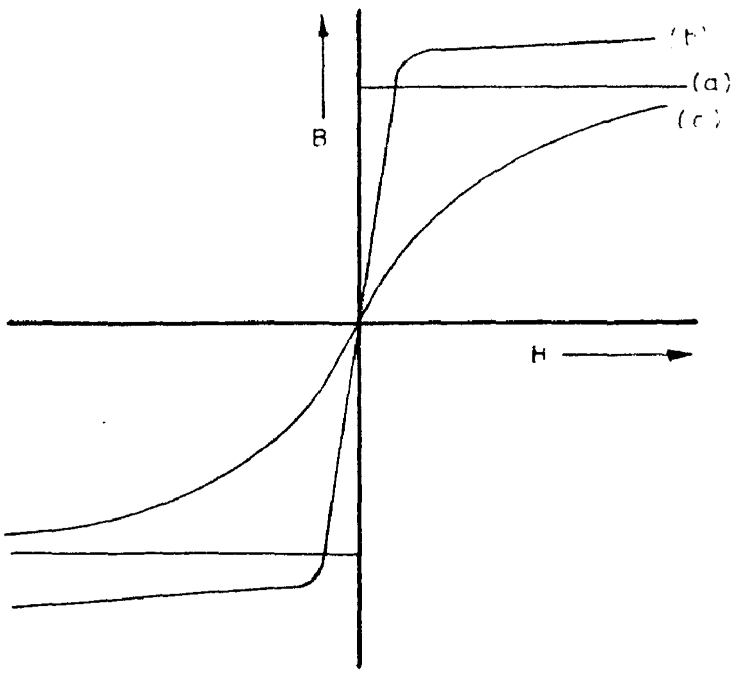


FIG. 2.11

upon non linearity curve.

To avoid ambiquity of multiple current values however it is necessary to distinguish the real or stable solutions from the unstable solutions.

STABILITY TEST[2]- Margand had given a method to test the stability of the multiple current values. Point 1 and 2 corresponds to stable solutions whereas point 3 gives an unstable solutions. The reason for this distinction is as follows:-

If we leave a stable point the current suddenly changes by a small amount I and its effect on the circuit can be considered from the following relation.

$$\left(\frac{XR}{2} I \right)^2 = E^2 - \left[f(I) - \frac{I}{XwC} + Xw \frac{L}{2} I \right]^2 \quad \text{———}(2.11)$$

Due to a slight increase in current, if the quantity $\left[f(I) + Xw \frac{L}{2} I - \frac{I}{XwC} \right]$ increases in magnitude then $\left(\frac{XR}{2} I \right)^2$ will tend to decrease according to equation no. 2.11. The sense of variation of $\left(\frac{XR}{2} I \right)^2$, however is opposed to the assumed change of current, therefore the point is indicative of a circuit condition in favour of stability. Now with the increase of current, if the quantity $\left[f(I) + X \frac{wL}{2} I - I/XwC \right]$ decreases in magnitude then $\left(\frac{XR}{2} I \right)^2$ will tend to increase according to equation 2.11, indicating a sense of variation similar to that of current. Hence the point is indicative of a circuit condition which is not in favour of stability. In a similar manner arguments can be given if the current is assumed to decrease slightly.

The stability of multiple current values can be observed -

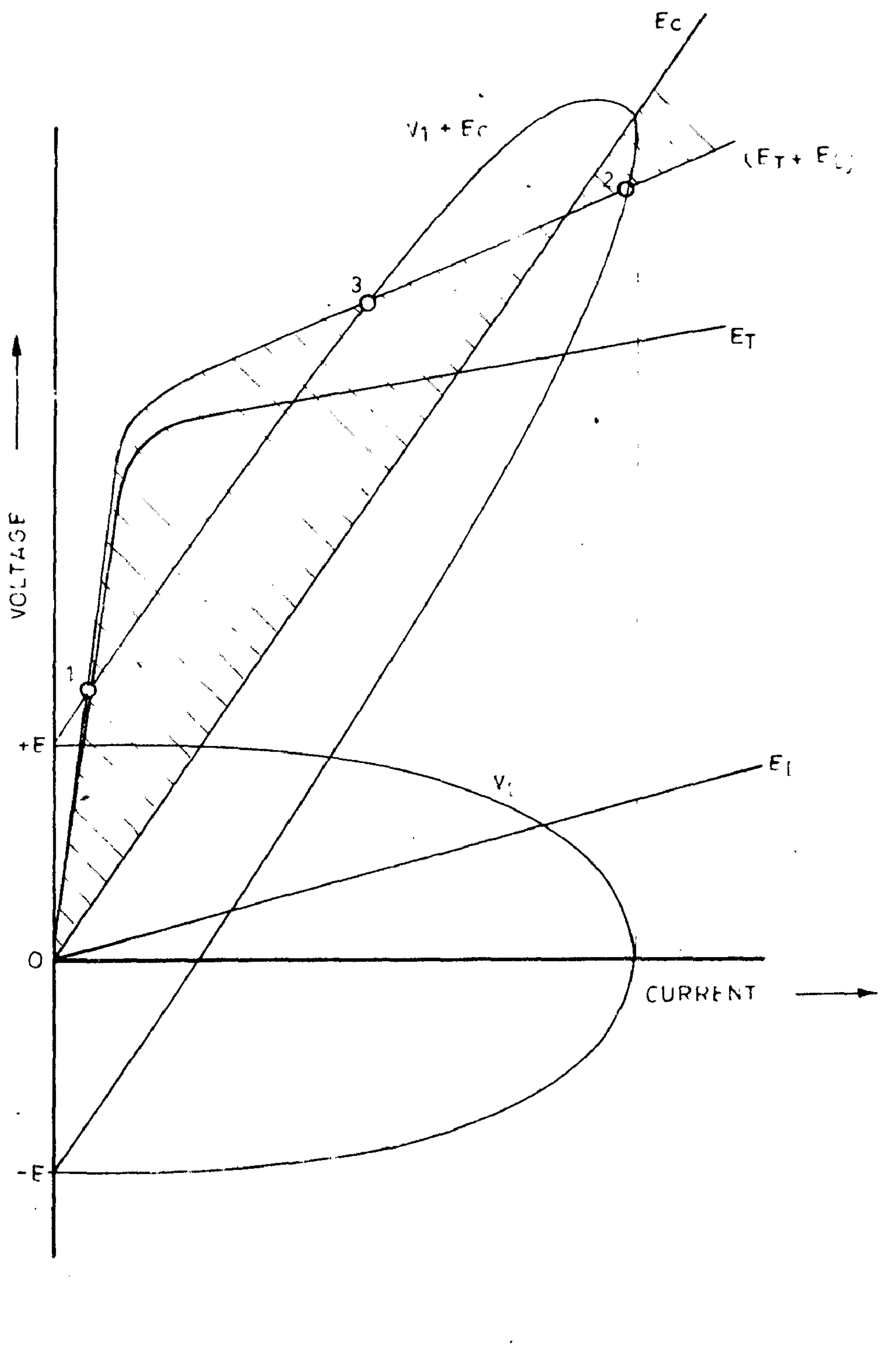


FIG. 2.12

graphically by seeing the variation in magnitude of the quantity $\left[f(I) + \frac{wX_L}{2} I - \frac{I}{X\,wC}\right]$ as is shown by the hatched ordinates in figure 2.12.

EFFECT OF DISTANCE:- As the length of the line is increased, all parameters of transmission line increases in magnitude. The effect of resistance will be to shorten the principal axis and hence it is of slight influence on the position of intersection point 1 figure 2.12, permitting small values of currents and voltages in the circuit. At point 2 however with large currents permitted to flow, the resistance prevents an excessive increase of currents and voltages. It is possible that the increased resistance may shorten the ellipse so far that the intersection 2 entirely disappears. Now only a single state of oscillation can develop, with small current and voltages, and a turning over to high values is no longer possible. Hence the insertion of ohmic resistance is the safest means for stabilising of ferro-resonant circuit which tend to over turn.

The effect of increased inductance due to the transmission line will be to lift up the saturation characteristic of the transformer It has also vary slight effect on the position of intersection point 1, while it has got an appreciable effect on the intersection point 2. The value of current and voltage increases considerably by the increased inductance for point 2.

The increased capacitance has a slight effect on the position of point 1 and the values for current and voltages are slightly reduced, while there is an appreciable increase in the value of current and voltage corresponding to point 2.

It is of theoritcal importance to note that in case of a ideal line i.e. having no losses in the transmission system, as the distance of the fault is increased the saturation characteristic and the condenser line are rotating in opposite directions between zero degree to 90°. However the intersection of the condenser line and the magnetic characteristic now may move to infinity. This will occur if the slopes according to equations 2.9 and 2.10 are equal.

$$\therefore \tan\gamma = \tan\beta = \frac{X\omega L}{2} = \frac{1}{X\omega C}$$

Since the slope of total magnetic characteristic for large values of current is governed by the slope of inductance line.

Hence a true resonance can develop if,

$$X = \frac{1}{\omega\sqrt{LC}} \qquad \text{-------}(2\text{-}12)$$

From the above considerations it is clear that in oscillatory circuits containing elements having nonlinear behaviour due to saturation there never can develop sustained resonance between capacitance and inductance. However there may occur a change in the state of circuit due to the jump from one point of the characteristic to another far distant one. Hence in case of ferroresonant circuit there is no frequency which is extremely dangerous for the circuit. This is due to the fact that oscillatory circuit with magnetic saturation do not have a natural frequency.

For an unsymmetrical fault the same procedure will be adopted except that the source voltage E will now be modified to E'.

EXAMPLE:

The graphical method is explained with the help of following example. The power of 100 MVA is required to be transmitted through a distance of 200 miles. At the sending end the step up three bank transformer of 100 MVA, 13.2/132 KV is installed. The line has got the following constants.

Resistance r	=	0.275 /mile/phase.
Inductance L	=	3.48 m H/mile/phase
Capacitance C =	=	0.0218 μF/mile/phase.

The standard magnetization curve in per unit is taken from AIEE August 1954 pp 1027-32

	p.u.Magnatizing current	p.u. voltage
1	0.5	0.800
2	1.0	1.000
3	2.0	1.060
4	3.0	1.100
5	4.0	1.120
6	5.0	1.140
7	6.0	1.160
8	7.0	1.180
9	8.0	1.200
10.	9.0	1.210
11	10.0	1.220
12	11.0	1.230
13	12.0	1.240

14	13.0	1.245
15	14.0	1.250
16	15.0	1.255
17	16.0	1.260
18	17.0	1.265
19	18.0	1.270
20	19.0	1.275
21	20.0	1.280

Full load current per phase = 437 amps.

Generally the magnetization current is less than 5%. So the magnetization current for such a transformer may be taken as equal to 20 amps.

Now actual magnetization curve becomes as under taking secondary side voltage to be base voltage

Sl. No.	Magnetizing current amps	Voltage in KV.
1	10	105.6
2	20	132.10
3	40	139.9
4	60	145.2
5	80	147.8
6	100	150.5
7	120	153.1
8	140	155.8

9	160	158.4
10	180	159.8
11	200	161.0
12	220	162.3
13	240	163.7
14	260	164.2
15	280	165.0
16	300	165.8
17	320	166.2
18	340	167.0
19	360	167.7
20	380	168.4
21	400	168.9

For different distances the curves are drawn.

CASE (I)

when X = 100 miles

$$\therefore E_R = X.\frac{R}{2}.I = 100.\frac{0.275}{2}.I \text{ volts.}$$

$$= 13.75.\ 10^{-3}\ I\ KV$$

$$E_L = Xw\frac{L}{2}I = 100.100\pi.\frac{3.48}{2}.10^{-3}\ I.\ \text{volts}$$

$$= 54.7.10^{-3}\ I\ KV.$$

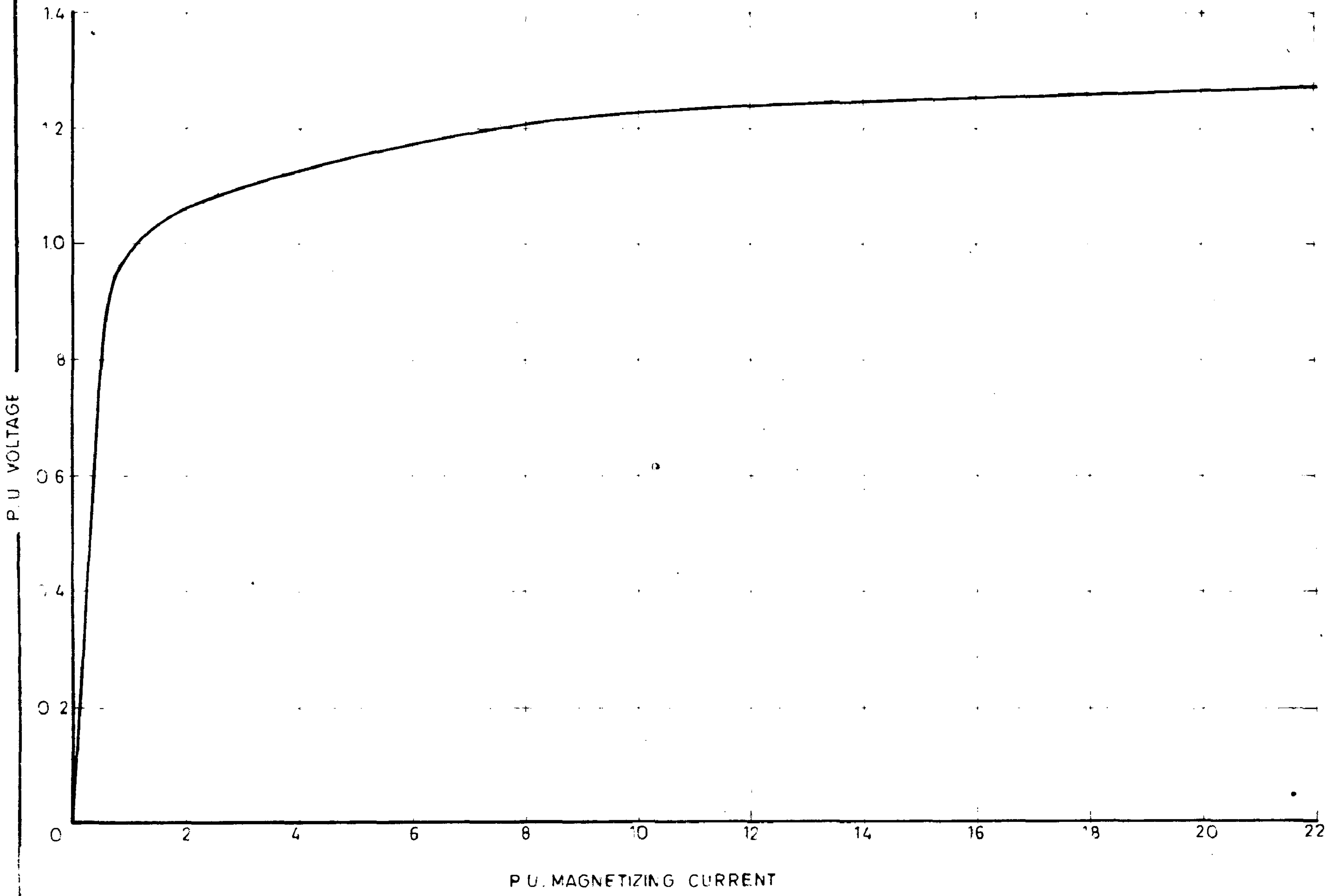

P.U VOLTAGE
1.4
1.2
1.0
8
0 6
4
0 2
0
2
4
6
8
10
12
14
16
18
20
22
P U. MAGNETIZING CURRENT

FIG. 2.13

$$E_c = \frac{I}{Xwc} = \frac{I}{100 \,.\, 100\pi . 0.0218 \,.\, 10^{-6}} \text{ volts.}$$

$$= 1.46 \text{ I KV.}$$

CASE II

When X = 125 mile

$$E_R = X . \frac{R}{2} I = 125 \,.\, \frac{0.275}{2} \text{ I volts.}$$

$$= 17.2 \,.\, 10^{-3} \text{ I KV.}$$

$$E_L = Xw \frac{L}{2} I = 125.100\pi \,.\, \frac{3.48}{2} .10^{-3} \text{ I Volts.}$$

$$= 68.4 \,.\, 10^{-3} \text{ I . KV.}$$

$$E_c = \frac{I}{X\,wC} = \frac{I}{125.100\pi .0.218 \,.\, 10^{-6}} \text{ Volts.}$$

$$= 1.168 \text{ I . KV.}$$

CASE III

When X = 150 miles.

$$E_R = X \frac{R}{2} I = 150 \,.\, \frac{0.275}{2} \text{ I volts.}$$

$$= 20.62 \,.\, 10^{-3} \text{ I KV.}$$

$$E_L = X\,w \frac{L}{2} I = 150 \,.\, 100\pi . \frac{3.48}{2} \quad 10^{-3} \text{ I volts.}$$

$$= 82.2 \,.\, 10^{-3} \text{ I KV.}$$

$$E_c = \frac{I}{X\,Cw} = \frac{I}{150.100\pi .0.0218.10^{-6}} \text{ Volts.}$$

$$= 0.974 \text{ I KV.}$$

<u>CASE IV</u>

When $X = 175$ miles.

$E_R = X \frac{R}{2} I = 175 \cdot \frac{0.275}{2}$ I Volts.

$= 24.05 \cdot 10^{-3}$ I XXX KV.

$E_L = X w \frac{L}{2} I = 175.100 \pi \cdot \frac{3.48}{2} \cdot 10^{-3}$ I volts.

$= 95.6 \cdot 10^{-3}$ I KV.

$E_c = \frac{I}{X wC} = \frac{I}{175.100 \pi \cdot 0.0218 \cdot 10^{-6}}$ Volts

$= 0.835 \cdot I$. KV.

<u>CASE V</u>

When $X = 200$ miles

$E_R = X \frac{R}{2} I = 200 \cdot \frac{0.275}{2}$ I Volts.

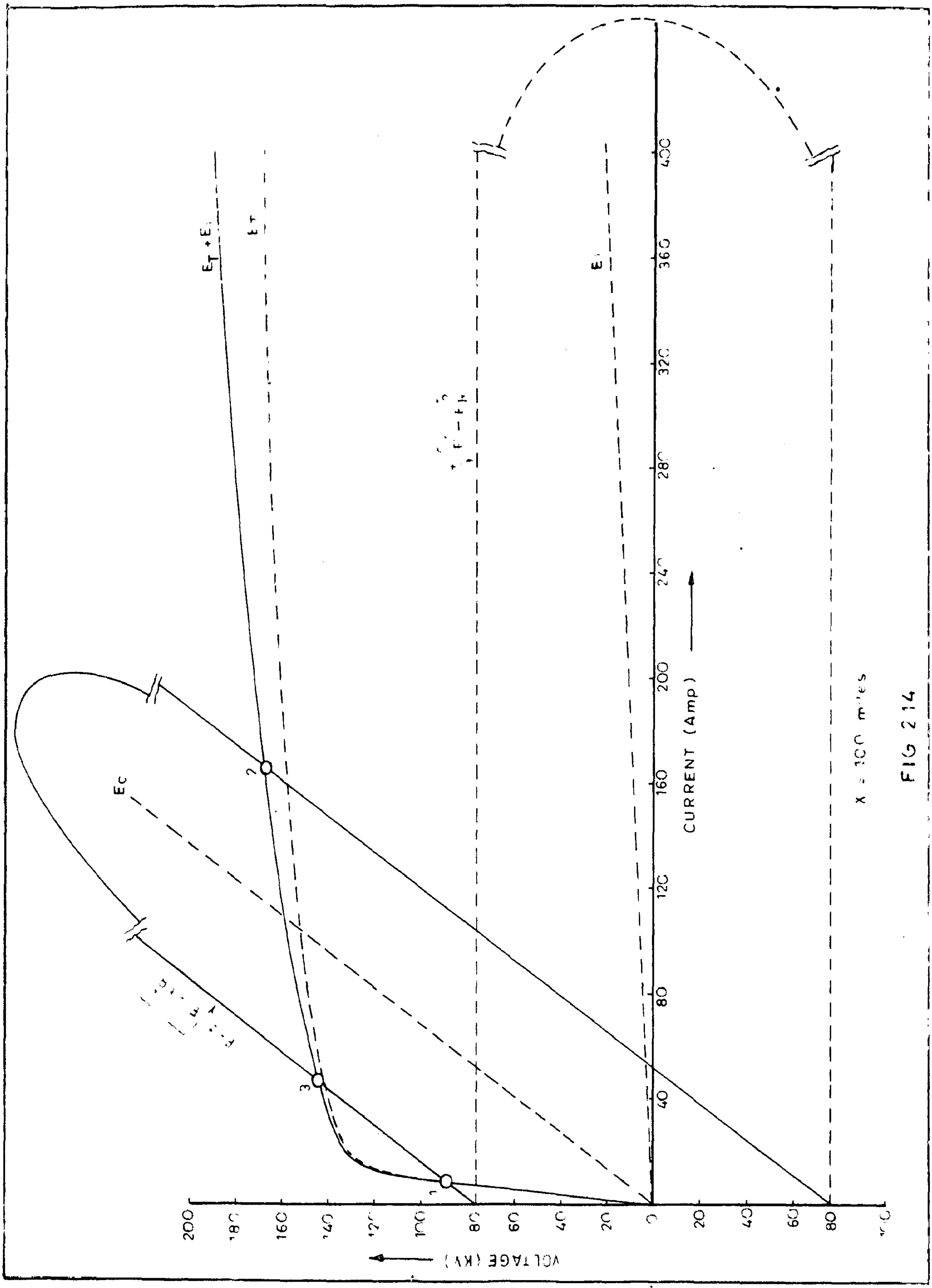
$= 27.5 \cdot 10^{-3}$ I KV.

$E_L = X w \frac{L}{2} I = 200 \cdot 100 \pi \cdot \frac{3.48}{2} \cdot 10^{-3}$ I Volts

$= 109.4 \cdot 10^{-3}$ I KV.

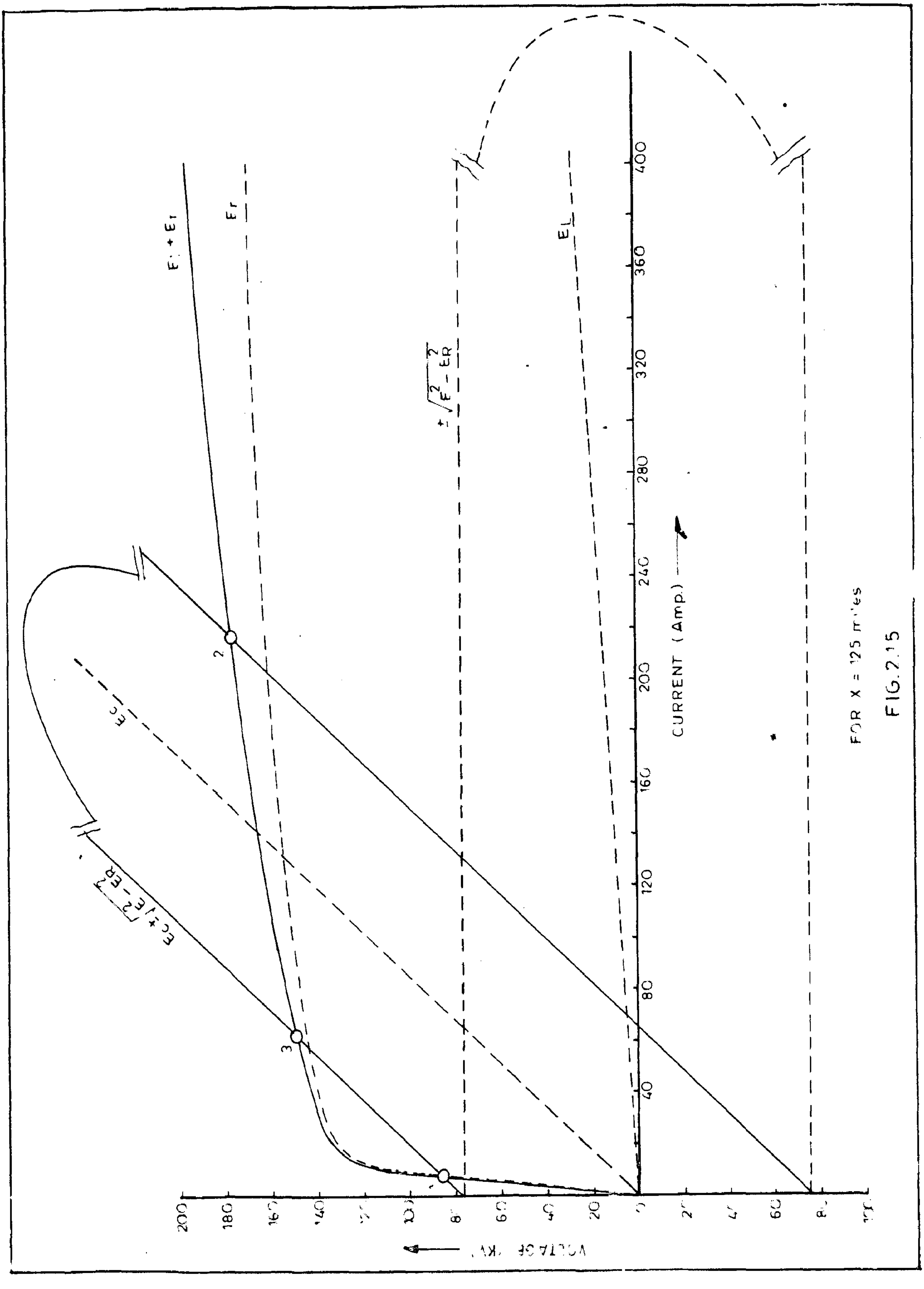
$E_c = \frac{I}{Xw^C} = \frac{I}{200 \cdot 100 \pi \cdot 0.0218 \cdot 10^{-6}}$ Volts.

$= 0.73$ I KV.



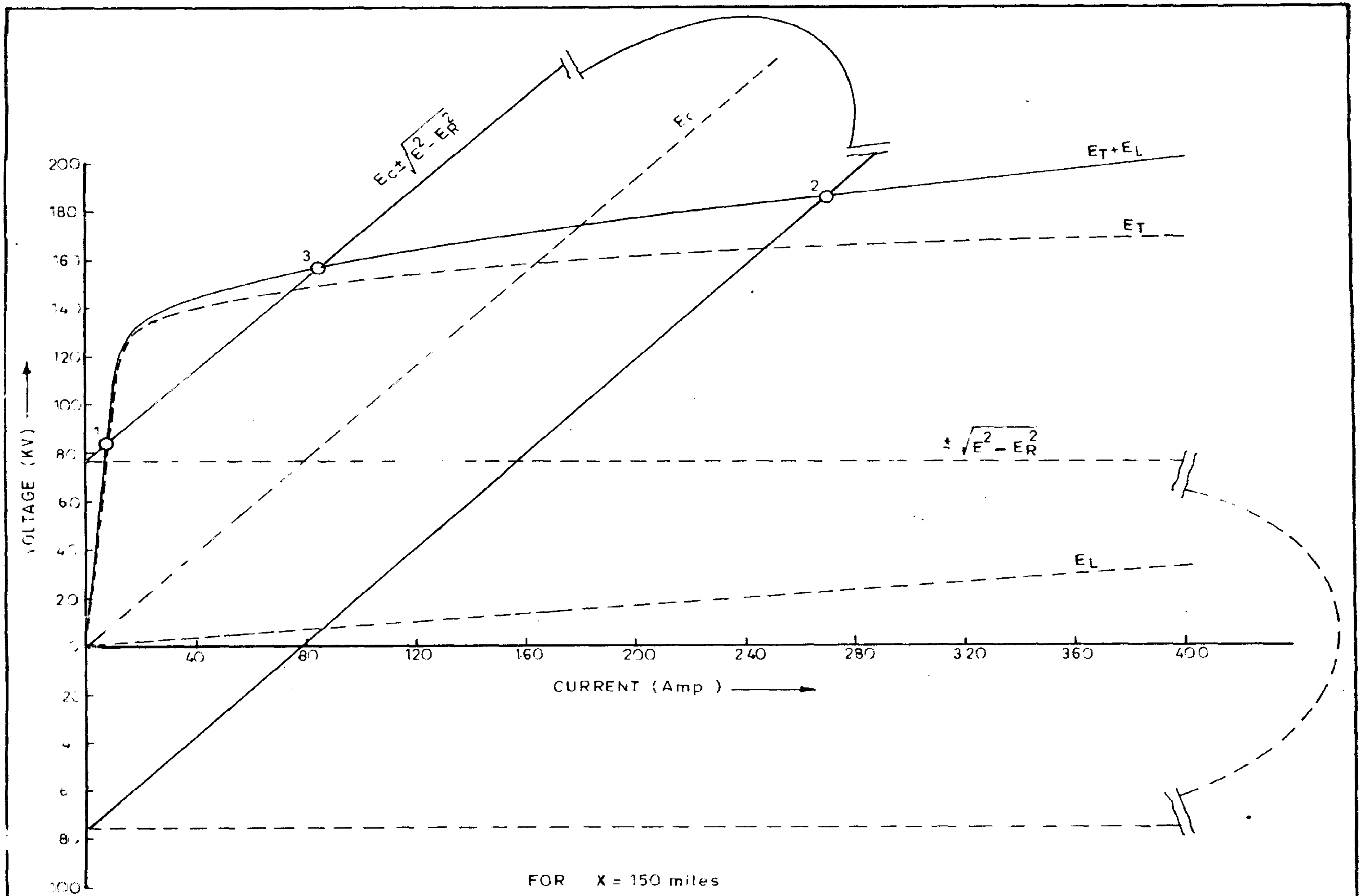
X = 100 miles

FIG 2.14



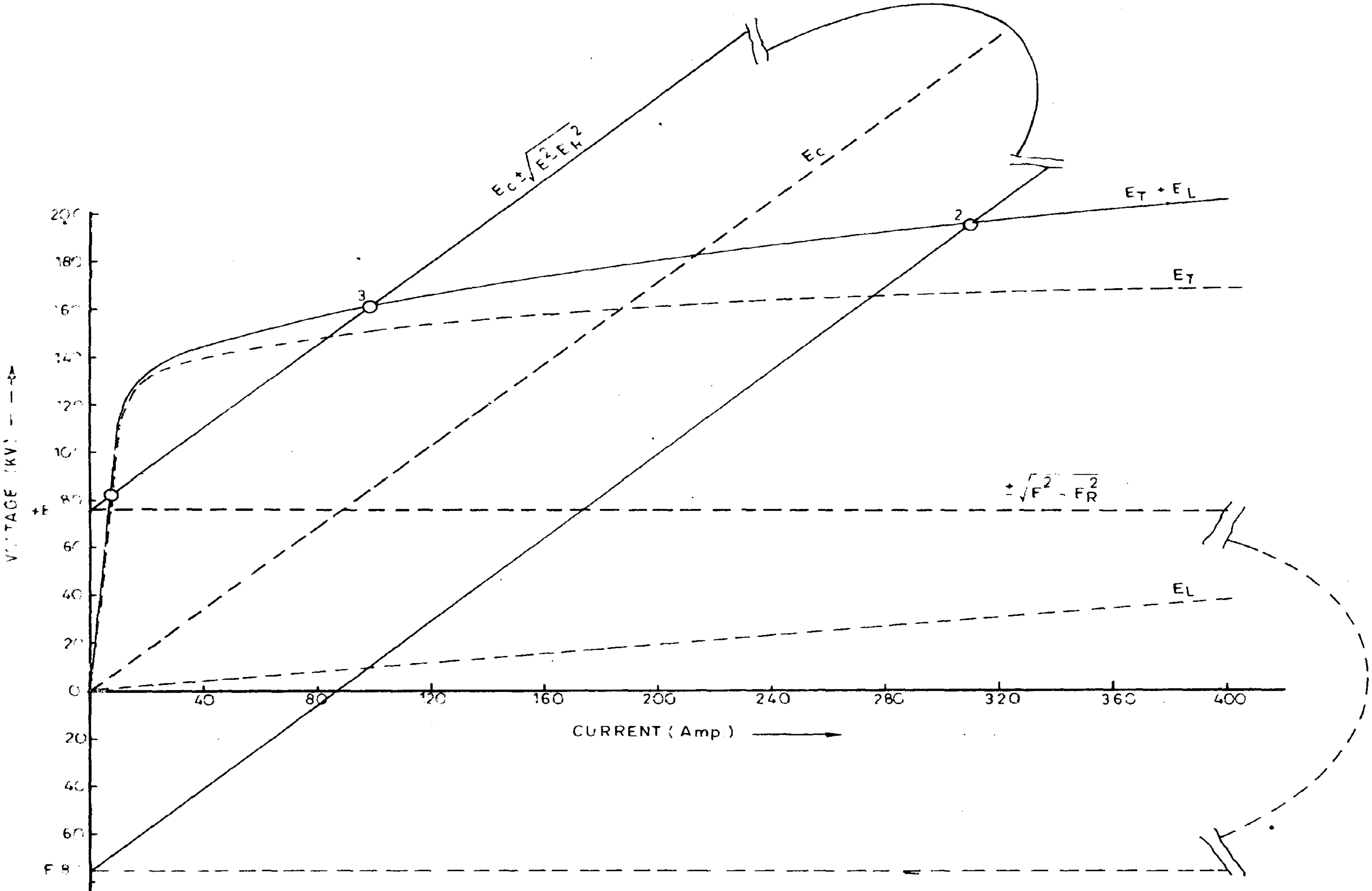
FOR X = 125 miles

FIG.2.15



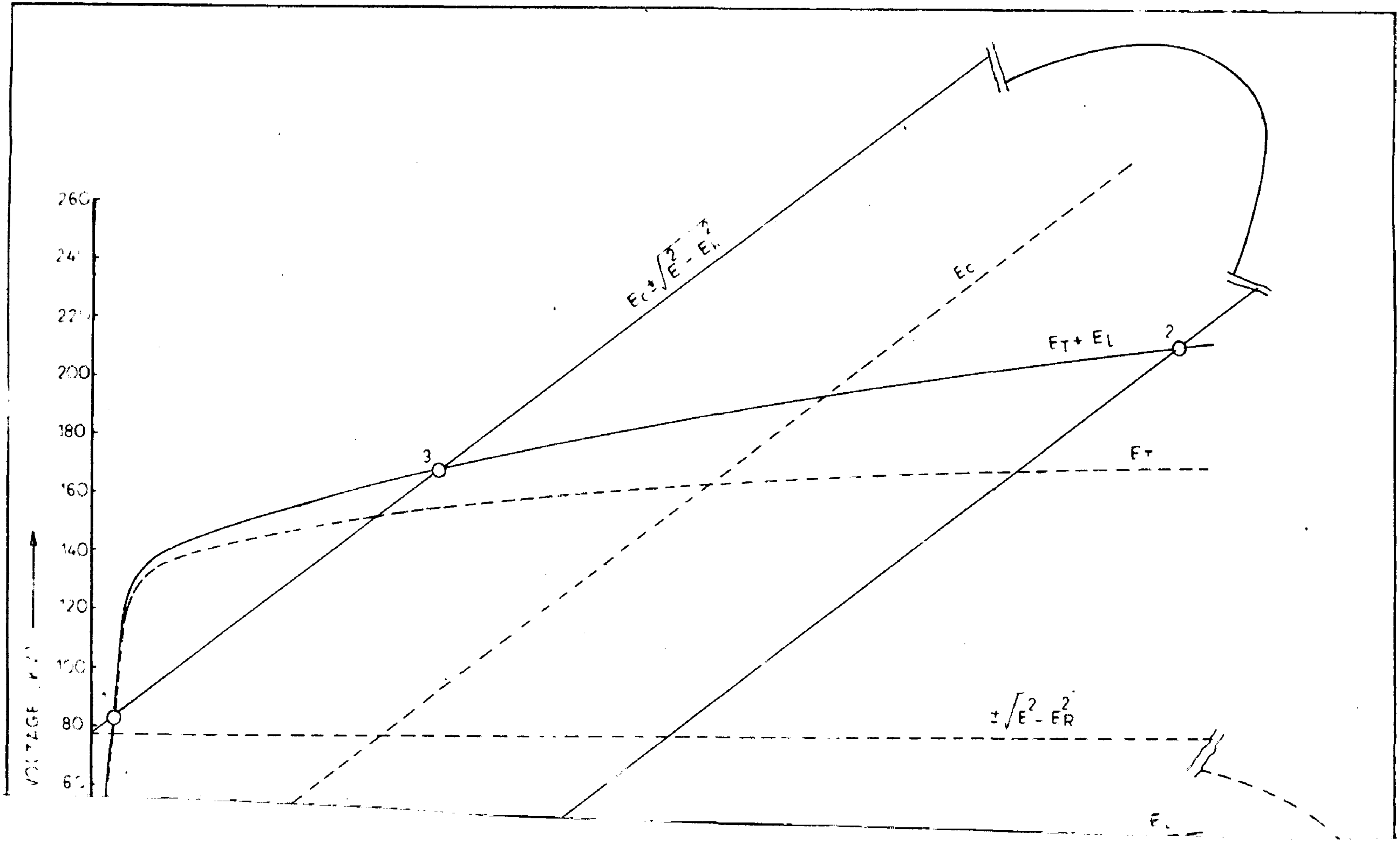
FOR X = 150 miles

FIG. 2.16



FOR X = 175 miles

FIG 2.17



260
200
180
160
140
120
100
80
VOLTAGE
$E_c \pm \sqrt{E^2 - E_R^2}$
E_c
$E_T + E_L$
E_T
$\pm \sqrt{E^2 - E_R^2}$
3
2

CONCLUSIONS.

From the comparative study of the graphs the following conclusions are drawn:-

i) The effect of line resistance only on points 1 & 2 is negligible as the distance of the fault is increased.

ii) The effect of line inductance only on point 2 is to increase the magnitudes of current and voltage to an appreciable extent with the increase of the fault distance. Whereas point 1 has got a tendency to decrease the magnitude of current and voltage, of course, to a negligible extent.

iii) The effect of line capacitance only on point 2 is to increase the magnitude of current and voltage to an appreciable extent with the increase of the fault distance whereas point 1 has got a tendency to decrease the magnitude of current and voltage, of course, to a negligible extent.

iv) The net effect of line parameters on point 2 is to increase the mangitude of current and voltage to an appreciable extent with the increase of the fault distance. Whereas point 1 has got a tendency to decrease the magnitude of current and voltage, of course, to a negligible extent.

The equivalent circuit representing the transmission line as formulated in the Chapter 2, is utilized here for calculating the circuit abnormal current and voltage.

The following assumptions are made here.

1. The line inductance and capacitances are considered to be lumped one.
2. The resistance of the line is assumed to be negligible.
3. The loss component of the capacitance of the line is also assumed to be zero.

The circuit parameters of the transmission line are such that the ferroresonance phenomena may occur under certain conditions. Let the abnormal condition occurs at a distance X from the sending end and L and C are the inductance and capacitance per unit length of the line. At this distance the lumped values for the parameters are expressed in a T form. The expression for the voltage can be written from the circuit (figure 3.1).[8,12]

$$V_m \sin (wt + \theta) = N. \frac{d\phi}{dt} + X\frac{L}{2} . \frac{di}{dt} + \frac{1}{XC}\int i dt \quad \text{———(3.1)}$$

$$\left[\frac{d\phi}{dt} = \frac{d\phi}{di} \cdot \frac{di}{dt}\right]$$

Let

$$\phi = Ai + Bi^3 \quad \text{———(3.2)}$$

$$\therefore \quad \frac{d\phi}{di} = A + 3Bi^2 \quad \text{———(3.3)}$$

Putting the value of $d\phi/dt$ in (3.1), the result is

$$V_m \sin (wt + \theta) = N.(A+3Bi^2) . \frac{di}{dt} + X\frac{L}{2}\frac{di}{dt} + \frac{1}{Xc}\int i dt$$

$$= (NA+3NBi^2+X\frac{L}{2})\frac{di}{dt} + \frac{1}{Xc}\int i dt \quad \text{———(3.4)}$$

CONCLUSIONS.

From the comparative study of the graphs the following conclusions are drawn:-

i) The effect of line resistance only on points 1 & 2 is negligible as the distance of the fault is increased.

ii) The effect of line inductance only on point 2 is to increase the magnitudes of current and voltage to an appreciable extent with the increase of the fault distance. Whereas point 1 has got a tendency to decrease the magnitude of current and voltage, of course, to a negligible extent.

iii) The effect of line capacitance only on point 2 is to increase the magnitude of current and voltage to an appreciable extent with the increase of the fault distance whereas point 1 has got a tendency to decrease the magnitude of current and voltage, of course, to a negligible extent.

iv) The net effect of line parameters on point 2 is to increase the mangitude of current and voltage to an appreciable extent with the increase of the fault distance. Whereas point 1 has got a tendency to decrease the magnitude of current and voltage, of course, to a negligible extent.

CHAPTER III

ANALYTICAL METHOD

The equivalent circuit representing the transmission line as formulated in the Chapter 2, is utilized here for calculating the circuit abnormal current and voltage.

The following assumptions are made here.

1. The line inductance and capacitances are considered to be lumped one.

2. The resistance of the line is assumed to be negligible.

3. The loss component of the capacitance of the line is also assumed to be zero.

The circuit parameters of the transmission line are such that the ferroresonance phenomena may occur under certain conditions. Let the abnormal condition occurs at a distance X from the sending end and L and C are the inductance and capacitance per unit length of the line. At this distance the lumped values for the parameters are expressed in a T form. The expression for the voltage can be written from the circuit (figure 3.1).[8,12]

$$V_m \sin (wt + \theta) = N. \frac{d\phi}{dt} + X \frac{L}{2} \cdot \frac{di}{dt} + \frac{1}{XC} \int i dt \quad \text{———(3.1)}$$

$$\left[\frac{d\phi}{dt} = \frac{d\phi}{di} \cdot \frac{di}{dt}\right]$$

Let

$$\phi = Ai + Bi^3 \quad \text{———(3.2)}$$

$$\therefore \quad \frac{d\phi}{di} = A + 3Bi^2 \quad \text{———(3.3)}$$

Putting the value of $d\phi/dt$ in (3.1), the result is

$$V_m \sin (wt + \theta) = N.(A+3Bi^2) \cdot \frac{di}{dt} + X \frac{L}{2} \frac{di}{dt} + \frac{1}{Xc} \int i dt$$

$$= (NA+3NBi^2+X \frac{L}{2}) \frac{di}{dt} + \frac{1}{Xc} \int i dt \quad \text{———(3.4)}$$

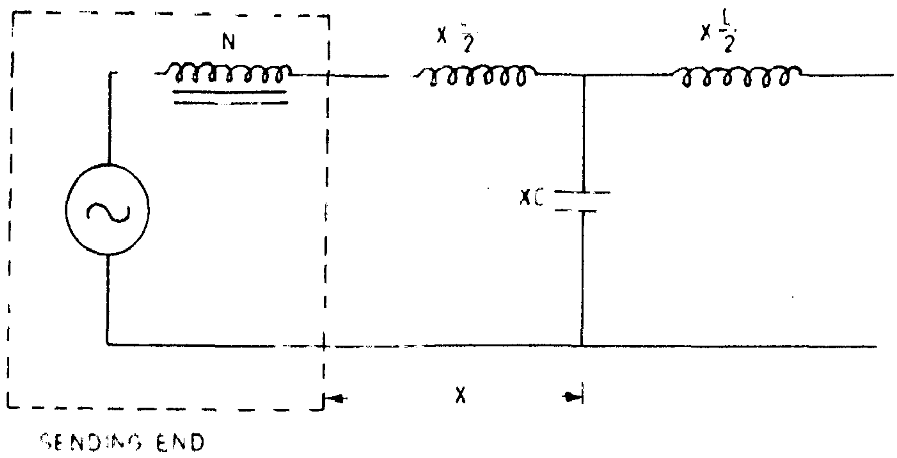


FIG. 3.1

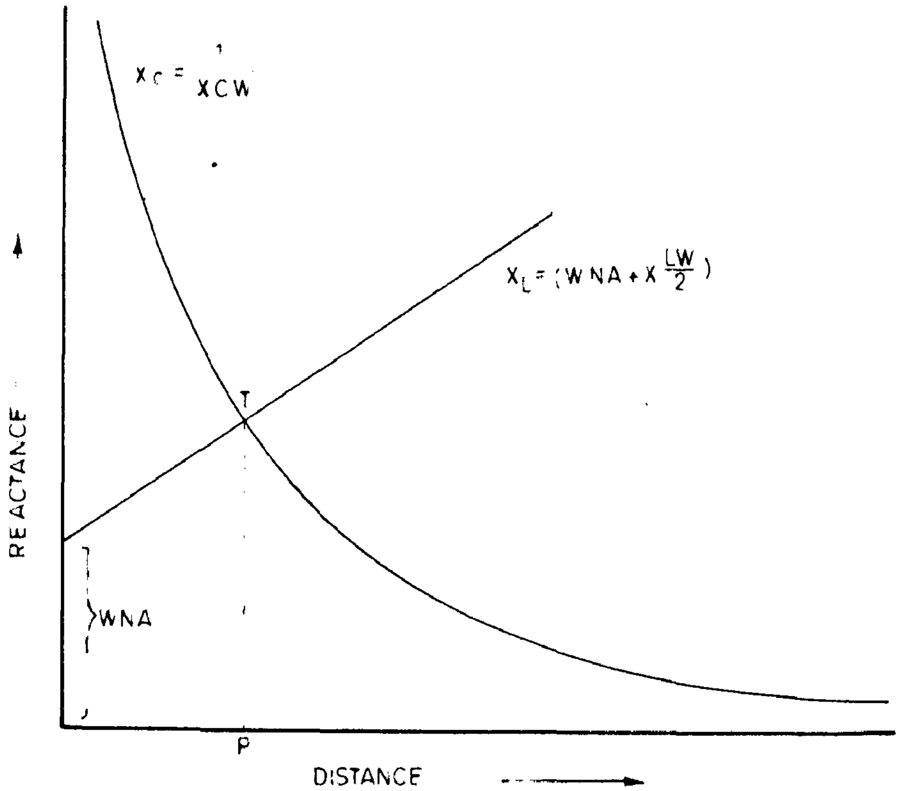


FIG. 3.2

In order to obtain an approximate steady state solution by Duffing's method a periodic solution of the following form is assumed

$$i = I_m \sin wt \quad \text{———(.3.5)}$$

$$\frac{di}{dt} = I_m . w . \cos wt.$$

and $$\int i dt = - \frac{I_m}{w} \cos wt + Q$$

Putting these values in equation (3.4), the result is

$$V_m \sin (wt + \theta) = \left[NA + \frac{XL}{2} + 3\, NB\, I_m^2 \sin^2 wt \right] I_m . w . \cos wt$$

$$+ \frac{1}{XC} \left[Q - \frac{I_m}{w} \cos wt \right]$$

or $$V_m \sin wt \cos \theta + V_m \cos wt \sin \theta = \left[(NA + \frac{XL}{2}) I_m\, w - \frac{I_m}{XCw} \right] \cos wt$$

$$+ \frac{3}{2}\, NB\, I_m^3 \sin wt \sin 2\, wt + \frac{Q}{XC}$$

or

$$V_m \sin wt \cos \theta + V_m \cos wt . \sin \theta = \left[(NA + \frac{XL}{2})\, W\, I_m - \frac{I_m}{XCW} + \frac{3}{4}\, NB\, w\, I_m^3 \right]$$

$$\cos wt. - \frac{3}{4}\, w\, NB\, I_m^3 \cos 3\, wt + \frac{Q}{XC} \quad \text{———(.3.6)}$$

The undetermined coefficients I_m and θ may be adjusted to make an approximate solution by equating the constant term and the coefficients of sin wt and cos wt on both sides of the above equation.

$$\therefore \frac{Q}{XC} = 0 \quad \text{———(3.7)}$$

$$V_m \cos \theta = 0 \quad \text{———(3.8)}$$

$$\left[(NA + XL) w I_m - \frac{I_m}{XCw} + \frac{3}{4} NBw I_m^3 \right] = V_m \sin \theta \quad \text{———}(3.9)$$

θ can be calculated from equation No. (3.8) and it is approximately equal to 90° and value of I_m can be found from equation No.(3.9). It is a cubic equation and it can be written as $3/4\ NBw\ I_m^3 - I_m \left[1/XCw - w\ (NA + XL/2) \right] - V_m \sin \theta = 0$

$$\text{or } I_m^3 - I_m \left[\frac{4}{3\ NBw} \left\{ \frac{1}{XCw} - \left(w\ NB + \frac{wXL}{2} \right) \right\} \right]$$

$$- \frac{4\ V_m \sin \theta}{3\ NBw} = 0 \quad \text{———}(3.10)$$

This equation is of the form

$$I^3 - q\,I - r = 0 \quad \text{———}(3.11)$$

$$\text{where } q = \frac{4}{3NBw} \left\{ \frac{1}{XCw} - \left(wNA + \frac{wXL}{3} \right) \right\} \quad \text{———}(3.12)$$

$$\text{and } r = \frac{4\ V_m \sin \theta}{3NBw} \quad \text{———}(3.13)$$

There are two principal cases to be considered.

<u>CASE (A)</u>[8] where $4\ q^3 > 27\ r^2$

In this case the cubic equation has three real roots. The smallest positive angle β is found such that

$$\cos \beta = \left(\frac{3}{q} \right)^{3/2} . \frac{r}{2} \quad \text{————} \quad (3.14)$$

Then the real roots are given by

$$I_1 = \frac{2}{\sqrt{3}} \cdot q^{1/2} \cdot \cos \frac{\beta}{3}$$

$$I_2 = -\frac{2}{\sqrt{3}} \cdot q^{1/2} \cdot \cos \frac{\pi - \beta}{3} \qquad \text{------(3-15)}$$

$$I_3 = -\frac{2}{\sqrt{3}} \cdot q^{\frac{1}{2}} \cdot \cos \frac{\pi + \beta}{3}$$

CASE (B)

Where $4q^3 < 27 r^2$

In this case the cubic equation has one real root and two complex roots. Then if q and r are both positive. Angle β is found such that

$$\cosh \beta = \left(\frac{3}{q}\right)^{3/2} \cdot \frac{r}{2} \qquad \text{----(3.16)}$$

Then the real root is given by the equation

$$I_o = \frac{2}{\sqrt{3}} \; q^{1/2} \cosh \frac{\beta}{3} \qquad \text{------(3.17)}$$

Dividing equation No. (3.11) by $(I - I_o)$ the equation is reduced to a quadratic one and the pair of complex roots is obtained by solving the resulting quadratic equation.

If q is negative and r is positive, the angle β is found such that

$$\sinh \beta = \left(\frac{3}{-q}\right)^{3/2} \cdot \frac{r}{2} \qquad \text{------(3.18)}$$

Then the real root is given by the equation

$$I = \frac{2}{\sqrt{3}} \cdot (-q)^{\frac{1}{2}} \cdot \sinh \frac{\beta}{3} \qquad \text{------(3.19)}$$

It may be noted that r may always be supposed to be positive since if the sign of r is changed the sign of the roots is merely changed.

Now for equation No.(3.10) to have the three real roots, the following condition must be satisfied

$$4\left[\frac{4}{3NBw}\left\{\frac{1}{XCw}-\left(wNA+\frac{XLw}{2}\right)\right\}\right]^3 > 27\left(\frac{4V_m \sin\theta}{3NBw}\right)^2$$

$$\text{or} \quad \frac{16}{3NBw}\left\{\frac{1}{XCw}-\left(wNA+\frac{XLw}{2}\right)\right\}^3 > 27\,V_m^2 \sin^2\theta \qquad \text{——(3.20)}$$

The quantity $\left\{\frac{1}{XCw}-\left(wNA+\frac{XLw}{2}\right)\right\}^3$ is changing with the distance. The first term in the bracket corresponds to the capacitive reactance and the term within the small bracket represents the inductive reactance. The curves are drawn for these two types of reactances against distance, as shown by figure (3.2.)

Generally the constant B has a negative value, if the current is to have three real values, then the inductive reactance should be more than the capactive reactance, then and then only the condition given by equation No.(3.20) may be satisfied. The inductive reactance is a straight line characteristic having an intercept with the reactance axis as (wNA). The capacitive reactance v/s distance graph is of the hyperbolic nature. The inductive reactance is increasing with the distance, while the capactive reactance is decreasing with the distance, so the two curves will definitely cut at a point T as shown in the graph at which the values of the two reactances will be equal. Before the distance P, the current will have one real value and a pair of conjugate complex values, But as the distance is gradually increased beyond distance P, at some point the expression (3.20) will be satisfied resulting in all real values for the current. For all the three real values of current, the distance can be found out by the expression (3.20) and this is given by

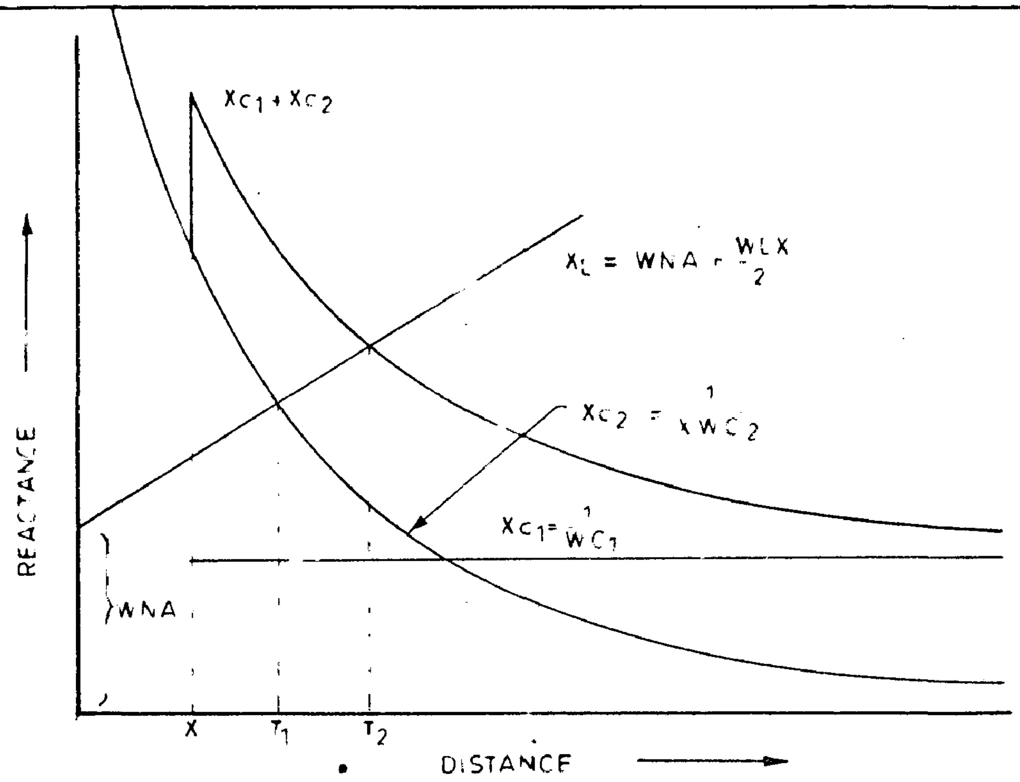

Xc1 + Xc2
XL = WNA + WLX/2
Xc2 = 1/xWC2
Xc1 = 1/WC1
WNA
REACTANCE
X
T1
T2
DISTANCE

FIG. 3.3

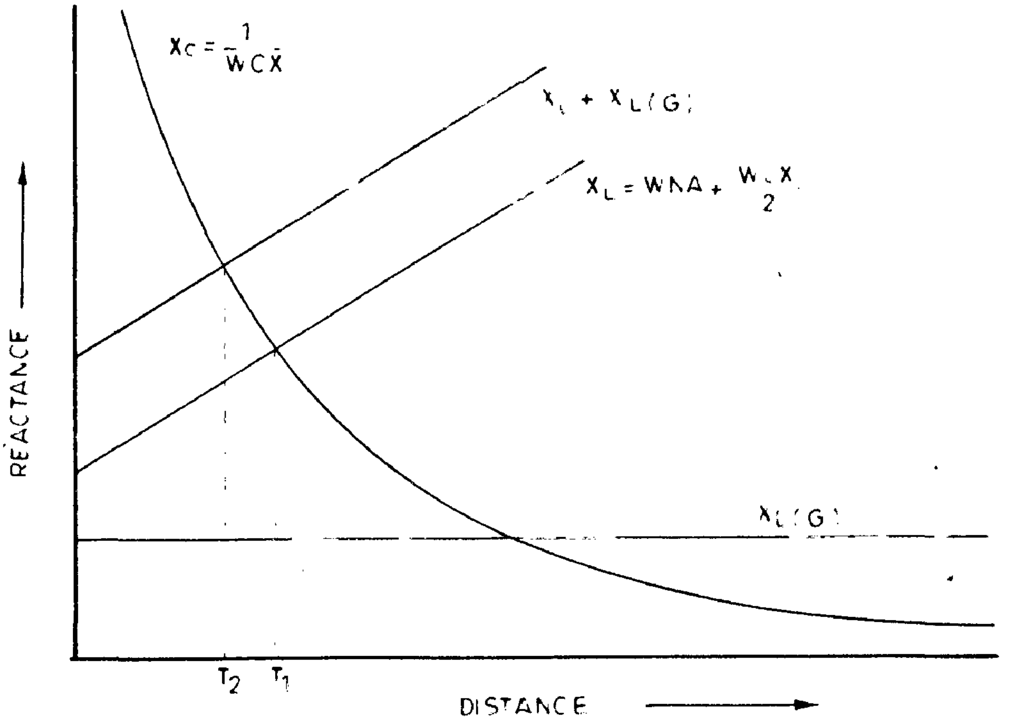

Xc = 1/WCX
XL + XL(G)
XL = WNA + WLX/2
XL(G)
REACTANCE
T2
T1
DISTANCE

FIG. 3.4

$$\frac{-\left[w^2 CNA + Cw\left\{\frac{81}{16} NBw V_m^2 \sin^2\theta\right\}^{1/3}\right] + \sqrt{\left[w^2 cNA - Cw\left\{\frac{81}{16} NBwV_m^2 \sin^2\theta\right\}\right] + 2w^2 LC}}{2\, w^2 LC}$$

EFFECT OF CAPACITANCE

Generally when the power is transmitted through long distances, it is required to install static capacitors or synchronous condensers at the receiving end at some times intermediate stations are also required, where these equipments are also installed, otherwise the stability has got much effect over the distance through which power can be transmitted. In such cases, if the conductor gets opened beyond these stations then static capacitors also come in picture. The effect can be studied by adding this value to the capacitive reactance curve as shown in the figure (3.3). The initial intersection point when the effect of static capacitor is not considered, is at a distance T_1, and after considering the effect of capacitor which is installed at a distance X from the sending end, the total capacitive reactance curve takes the form as is represented by the thick line. The point of intersection shifts towards right at a distance T_2. It shows that the real values for current now would be obtained at a larger distance.

EFFECT OF INDUCTANCE

So far we have considered the system to be solidly grounded, if the system is inductance grounded (assuming here that the inductance is linear one) then the grounded inductive reactance will be constant through out the length of line. This can be added to the inductive reactance characteristic to get the net effect and the shape of the curve will be of the form as shown in the figure (3.4) by the thick line.

Here the intersection point is shifted towards the left hand side and hence all the three real values of current will be obtained at a lesser distance.

CHAPTER IV

HARMONIC SOLUTION

DUFFING'S EQUATION

A power system having generators connected to transformers to transmit power at high voltage through transmission lines to the load centre having low power factor, which is improved with the help of static capacitors or synchronous condensers, may be converted into a simple non linear oscillatory circuit, in case a fault at the load centre had occured and is being cleared.

The following assumptions for converting this circuit to the ordinary non linear oscillatory circuit are made.

1. The line parameters are assumed to be lumped one.
2. The line resistance is assumed to be negligible.
3. The line inductive drop is added to the volt - ampere characteristic of the transformer.
4. The transformer may be assumed to be a non linear inductor having a voltage source behind it.

Harmonic solution may be given for such a circuit.[12,16]

Let the magnetization curve is given by

$$i = \frac{N\,\phi}{L_0} + A\,.\,\phi^3 \quad \text{———} (4.1)$$

where N is the no. of turns on the coil carrying current i and L_0 and A are positive constants. Constant L_0 represents the self inductance of transformer together with the line, which would exist, in case the saturation effect is absent, so that $A = 0$

The circuit equation can be written from fig. (4.2) as

$$\frac{q}{C_*} + e_L = E \sin w_1 t \quad \text{———} (4.2)$$

Here q represents the instantaneous charge on the capacitor. C_* is the total capacitance of the line and the p.f. correction

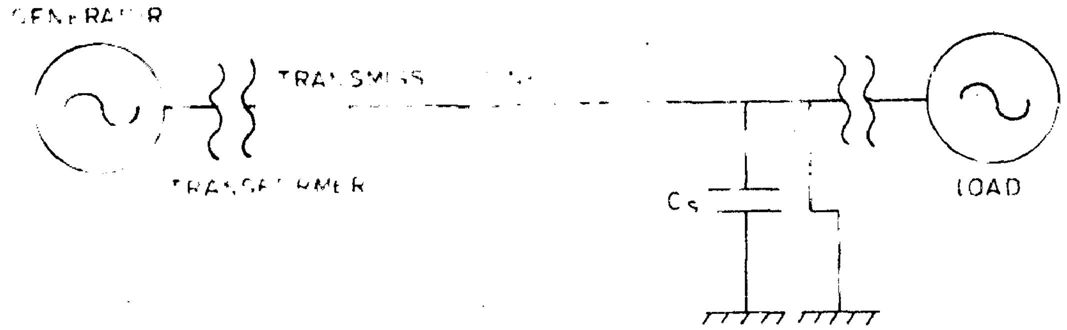


FIG. 4.1

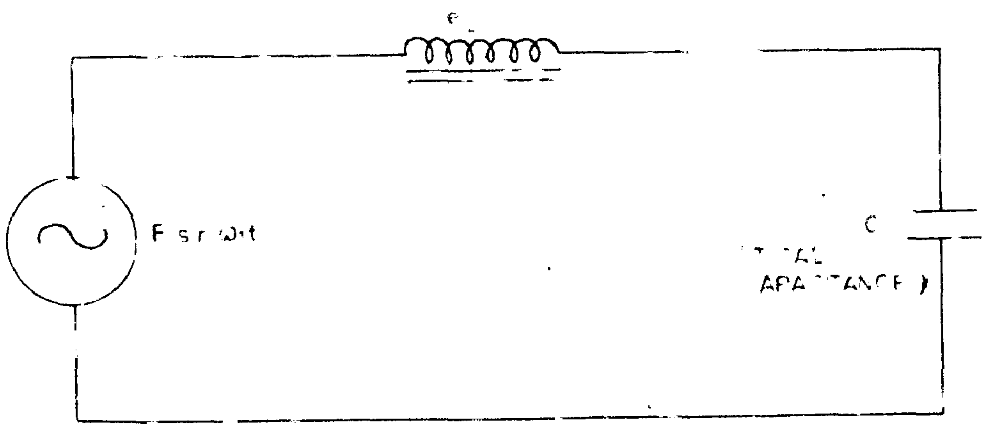


FIG. 4.2

equipment.

e_L is the instantaneous voltage across the transformer taking into account the inductive drop of the line also.

E sin $w_1 t$ is the driving voltage.

It is well known that the rate of change of charge is the current i in the circuit and this is related to the instantaneous flux ϕ in the core by equation (4.1)

$$dq/dt = i = \frac{N\phi}{L_0} + A\,\phi^3 \quad \text{———}(4.3)$$

Further the instantaneous voltage across the transformer is proportional to the rate of change of flux, $e_L = N.d\phi/dt$. The circuit equation (4.2) is differentiated with respect to time and these relations substituted.

$$\frac{1}{C}\cdot\frac{dq}{dt} + \frac{d}{dt}\cdot(e_L) = Ew_1 \operatorname{Cos} w_1 t \quad \text{———}(4.4)$$

or $$\frac{1}{C}\cdot\left[\frac{N\phi}{L_0} + A\phi^3\right] + N\cdot\frac{d^2\phi}{dt^2} = Ew_1 \cos w_1 t$$

put $\psi = N\phi$

$$\therefore \frac{1}{C}\left[\frac{\psi}{L_0} + \frac{A\psi^3}{N3}\right] + \ddot{\psi} = Ew_1 \cos w_1 t$$

Again put

$$w_0^{\,2} = \frac{1}{L_0 C} \quad \text{———}(4\text{-}5)$$

$$\text{and } h = \frac{A}{N^3 C} \quad \text{———}(4.6)$$

Hence

$$\ddot{\psi} + w_0^2\psi + h\psi^3 = Ew_1 \cos w_1 t \quad \text{———}(4.7)$$

Now equation (4.7) is of the same form as Duffing's equation i.e.

$$X + w_0^{\,2}\,X + h\,X^3 = Q_\cdot \cos w_1 t \quad \text{———}(4.8)$$

One of the kinds of phenomena appearing in systems governed by Duffing's equation is that of discontinuous jumps in amplitude, as the magnitude of the driving function is varied smoothly and continuously. At the same time, harmonics of the driving frequency are generated.

SOLUTION (I)[12].

The solution for equation (4.7) in the form of

$$\psi = X \cos w_1 t \quad \text{———}(4.9)$$

is assumed.

Now in equation no.(4.7) there is no term representing dessipation which is included, and hence it is expected that ψ will be exactly in or out of phase with the driving force. So The phase ~~with the driving~~ angle is not included. The equation No.(4.9) is differentiated twice and is substituted into equation no.(4.7) giving $- X w_1^2 \cos w_1 t + w_0^2 X \cos w_1 t + hX^3 \cos^3 w_1 t = Ew_1 \cos w_1 t$ or $\left(-Xw_1^2 + xw_0^2 + \frac{3hX^3}{4} \right) \cos w_1 t + \frac{hX^3}{4} \cos 3 w_1 t = Ew_1 \cos t$ ——(4.10)

Now it is required that only terms of fundamental frequency should satisfy the equation (4.10) so all the terms in $\cos w_1 t$ are collected. The term $\cos^3 w_1 t$ is ignored until later. Equating the coefficients of $\cos w_1 t$ on both sides following results in

$$-w_1^2 X + w_0^2 X + \frac{3h}{4} X^3 = Ew_1 \quad \text{———}(4.11)$$

Equation no.(4.11) is a cubic in the unknown amplitude X. It is evident that it will always yield one real value of X.

and there may be as many real values as three. Hence it is possible to obtain the multiple valued solution needed to allow jumps in amplitude. The roots of cubic equation are discussed in chapter 3.

Therefore

$$\psi = X \cos w_1 t$$

But $\psi = N\,\phi$

$$\therefore \phi = X/N \,.\, \cos w_1 t \quad \text{———} (4.12)$$

Now putting the value of ϕ in equation no.(4.1) we have

$$i = \frac{X}{L_o} \cos w_1 t + A\left[\frac{X}{N} \cos w_1 t\right]^3$$

$$i = \left[\frac{X}{L_o} + \frac{3AX^3}{4N^3}\right] \cos w_1 t + \frac{AX^3}{4N^3} \cdot \cos 3 w_1 t \quad \text{———} (4.13)$$

This is the required solution of Duffing's equation, though it is not accurate because a third harmonic term was ignored.

<u>SOLUTION (II)</u>

The solution No.(I) is based upon just the fundamental component of what actually is a nonsinusoidal solution for the original nonlinear differential equation. In reality a third harmonic term has been completely ignored, which arised when the assumed solution was substituted into the equation. So the solution is approximately correct.

Now to improve the accuracy of the solution, an obvious way is to assume the solution of the form as

$$\psi = X_1 \cos w_1 t + X_3 \cos 3 w_1 t \quad \text{———} (4.14)$$

This solution is composed of both the fundamental and third harmonic term, instead of equation no.(4.9) having only the fundamental. The coefficients X_1 and X_3 must be determined. Now substituting the assumed solution given by equation no. (4.14) into equation no.(4.7) applicable to the system without dissipation. The coefficients of $\cos w_1 t$ and $\cos 3 w_1 t$ are collected and are equated on both sides. The result is a pair of simultaneous algebric equations for the coefficients.

$$-X_1 w_1^2 \cos w_1 t - 9X_3 w_1^2 \cos 3 w_1 t + w_o^2 (X_1 \cos w_1 t + X_3 \cos 3 w_1 t) + h \left[X_1 \cos w_1 t + X_3 \cos 3 w_1 t\right]^3 = E w_1 \cos w_1 t \quad \text{——(4.15)}$$

or

$$(-X_1 w_1^2 + X_1 w_o^2) \cos w_1 t + (-9 X_3 w_1^2 + w_o^2 X_3) \cos 3w_1 t + h \left[X_1^3 \cos^3 w_1 t + X_3^3 \cos^3 3w_1 t + 3X_1^2 X_3 \cos^2 w_1 t . \cos 3w_1 t + 3X_1 X_3^2 \cos w_1 t \cos^2 w_1 t\right]$$

$$= E w_1 \cos w_1 t.$$

or $$(-X_1 w_1^2 + X_1 w_o^2) \cos w_1 t + X_3 (w_o^2 - 9w_1^2) \cos 3w_1 t + hX_1^3 \left[\frac{3\cos w_1 t + \cos 3w_1 t}{4}\right] + hX_3^3 \left[\frac{3\cos 3w_1 t + \cos 9w_1 t}{4}\right] + 3hX_1^2 X_3 \left[\tfrac{1}{4} \cos w_1 t + \tfrac{1}{2} \cos 3w_1 t + \tfrac{1}{4} \cos 5w_1 t\right] + 3hX_1 X_3^2 \left[\tfrac{1}{2} \cos w_1 t + \tfrac{1}{4} \cos 5w_1 t + \tfrac{1}{4} \cos 7w_1 t\right] = Ew_1 \cos w_1 t$$

or $$\left[(w_o^2 - w_1^2)X_1 + \frac{3h}{4} X_1^3 + \frac{3hX_1 X_3}{4} (X_1 + 2X_3)\right] \cos w_1 t + \left[(w_o^2 - 9w_1^2) X_3 + \frac{3hX_3^3}{4} + \frac{hX_1^2}{4} (X_1 + 6X_3)\right] \cos 3w_1 t + \tfrac{3}{4} hX_1 X_3 (X_1 + X_3) \cos 5w_1 t + \tfrac{3}{4} hX_1 X_3^2 \cos 7w_1 t + \frac{h}{4} X_3^3 \cos 9 w_1 t = Ew_1 \cos w_1 t$$

$$\therefore \text{Cos } w_1 t: (w_o^2 - w_1^2)X_1 + \frac{3h}{4} X_1^3 + \frac{3h}{4} X_1 X_3(X_1 + 2X_3) = Ew_1 \quad \text{---(4.16)}$$

$$\text{Cos } 3w_1 t: (w_o^2 - 9w_1^2)X_3 + \frac{3h}{4} X_3^3 + \frac{3h}{4} X_1^2 (X_1 + 6X_3) = 0 \quad \text{---(4.17)}$$

In theory, these equation can be solved for X_1 and X_3, but in reality their solution is not so simple. Numerical solution is possible in a particular case, but a general representation of the solution can not be obtained.

It is supposed that the conditions are such that the third harmonic is relatively small in comparision to fundamental i.e. $X_3/X_1 \ll 1$, then the equation no.(4.17) can be rewritten as

$$\frac{X_3}{X_1} = \frac{-hX_1^2}{4} \Big/ (w_o^2 - 9w_1^2) \quad \text{---(4.18)}$$

and equation no.(4.16) becomes the same as equation no.(4.11). Hence X_1 can be found from equation (4.11) as before and X_3 is then calculated from equation (4.18)

Now if $w_1 > w_o/3$, the algebric signs for X_1 and X_3 are the same so that the third harmonic component is of such phase as to accentuate the peaks of the wave.

Hence $\psi = X_1 \cos w_1 t + X_3 \cos 3w_1 t$

But $\psi = N\phi = X_1 \cos w_1 t + X_3 \cos 3w_1 t$

$$\therefore \phi = \frac{X_1}{N} \cos w_1 t + \frac{X_3}{N} \cos 3 w_1 t \quad \text{---(4.19)}$$

Now putting the value of ϕ in equation (4.1)

$$i = \left[\frac{X_1}{L_o} \cos w_1 t + \frac{X_3}{L_o} \cos 3w_1 t\right] + \frac{A}{N^3}\left[X_1 \cos w_1 t + X_3 \cos 3w_1 t\right]^3$$

After expansion and simplification the result is as under:-

$$i = \left[\frac{X_1}{L_o} + \frac{A}{N^3}\left(\tfrac{3}{4}X_1^3 + \tfrac{3}{4}X_1^2 X_2 + \tfrac{3}{2} X_1 X_3^2\right)\right] \cos w_1 t + \left[\frac{X_3}{L_o} + \frac{A}{N^3}\left(\frac{X_1^3}{4} + \tfrac{3}{2} X_1^2 X_3 + \tfrac{3}{4}X_3^3\right)\right] \cos 3w_1 t + \tfrac{3}{4} \cdot \frac{A}{N^3} X_1 X_3 (X_1 + X_3) \cos 5w_1 t + \tfrac{3}{4} \frac{A}{N^3} \cdot X_1 X_3^2 \cos 7 w_1 t + \tfrac{1}{4} \cdot \frac{A}{N^3} X_3^3 \cos 9 w_1 t \quad \text{———(4.20)}$$

Now this is the required solution of Duffing's equation and it is more accurate than solution No. I. In this derivation the fifth, seventh and ninth harmonic terms, arising when the assumed solution was substituted into the equation, have been completely ignored.

Further more accurate solution may be obtained by considering the fifth harmonic also in the assumed solution.

SOLUTION (III)

The analysis is now extended to include the fifth harmonic of the flux or 3rd approximation. With the third approximation the accuracy of the system is improved, though the analysis becomes more complicated. Here the solution is assumed in the form

$$\psi = X_1 \cos w_1 t + X_3 \cos 3w_1 t + X_5 \cos 5w_1 t \quad \text{———(4.21)}$$

The coefficients X_1, X_3 and X_5 must be determined. The way to cetermine these coefficients is the same as is used in previous solutions. Upon substitution of the assumed solution given by equation (4.21) into equation (4.7) applicable to the system without dissipation, one obtains

$$\ddot{\psi} = -X_1 w_1^2 \cos w_1 t - 9X_3 w_1^2 \cos 3w_1 t - 25X_5 w_1^2 \cos w_1 t \quad \text{———(4.22)}$$

$$w_o^2 \psi = w_o^2 (X_1 \cos w_1 t + X_3 \cos 3w_1 t + X_5 \cos 5w_1 t) \quad \text{———(4.23)}$$

$$h\,\psi^3 = h.(X_1 \cos w_1 t + X_3 \cos 3w_1 t + X_5 \cos 5w_1 t)^3 \quad \text{———(4.24)}$$

Here first of all ψ^3 is expanded in the multiples of $w_1 t$. It is known by expansion that

$$(a+b+c)^3 = a^3+b^3+c^3+6abc+3a^2b+3b^2c+3c^2a+3ab^2+3bc^2+3ca^2 \quad \text{———(4.25)}$$

Now put

$$a = X_1 \cos w_1 t$$

$$b = X_3 \cos 3w_1 t$$

$$c = X_5 \cos 5\, w_1 t$$

$$\therefore \quad a^3 = \tfrac{3}{4} X_1^3 \cos w_1 t + \tfrac{1}{4} X_1^3 \cos 3\, w_1 t$$

$$b^3 = \tfrac{3}{4} X_3^3 \cos 3w_1 t + \tfrac{1}{4} X_3^3 \cos 9\, w_1 t$$

$$c^3 = \tfrac{3}{4} X_5^3 \cos 5w_1 t + \tfrac{1}{4} X_5^3 \cos 15\, w_1 t$$

$$6abc = 3/2.X_1 X_3 X_5 \left[\cos w_1 t + \cos 3w_1 t + \cos 7w_1 t + \cos 9w_1 t\right]$$

$$3a^2b = \tfrac{3}{4} X_1^2 X_3 \left[\cos w_1 t + 2\cos 3w_1 t + \cos 5w_1 t\right]$$

$$3b^2c = \tfrac{3}{4} X_3^2 X_5 \left[\cos w_1 t + 2\cos 5w_1 t + \cos 11w_1 t\right]$$

$$3c^2a = \tfrac{3}{4} X_5^2 X_1 \left[2\cos w_1 t + \cos 9w_1 t + \cos 11\, w_1 t\right]$$

$$3ab^2 = \tfrac{3}{4} X_1 X_3^2 \left[2\cos w_1 t + \cos 5\, w_1 t + \cos 7\, w_1 t\right]$$

$$3\,bc^2 = \tfrac{3}{4} X_3 X_5^2 \left[2\cos 3w_1 t + \cos 7w_1 t + \cos 13\, w_1 t\right]$$

$$3\,ca^2 = \tfrac{3}{4} X_5 X_1^2 \left[\cos 3w_1 t + 2\cos 5w_1 t + \cos 7\, w_1 t\right]$$

Now adding these and collecting the coefficient of $\cos w_1 t$ $\cos 3w_1 t$ ......seperately one obtains

$$(a+b+c)^3 = \cos w_1 t\left[\tfrac{3}{4}X_1^3 + 3/2 X_1X_3X_5 + \tfrac{3}{4}X_1^2X_3 + \tfrac{3}{4}X_3^2X_5 + 3/2 X_5^2X_1 + 3/2 X_1X_3^2\right]$$

$$+\cos 3w_1 t\left[\tfrac{1}{4}X_1^3 + \tfrac{3}{4}X_3^3 + 3/2 X_1X_3X_5 + 3/2 X_1^2X_3 + 3/2 X_3X_5^2 + \tfrac{3}{4}X_5X_1^2\right.$$

$$+\cos 5w_1 t\left[\tfrac{3}{4}X_5^3 + \tfrac{3}{4}X_1^2X_3 + 3/2 X_3^2X_5 + \tfrac{3}{4}X_1X_3^2 + 3/2 X_5X_1^2\right]$$

$$+\cos 7w_1 t\left[3/2 X_1X_3X_5 + \tfrac{3}{4}X_1X_3^2 + \tfrac{3}{4}X_3X_5^2 + \tfrac{3}{4}X_5X_1^2\right]$$

$$+\cos 9w_1 t\left[\tfrac{1}{4}X_3^3 + 3/2 X_1X_3X_5 + \tfrac{3}{4}X_5^2X_1\right]$$

$$+\cos 11w_1 t\left[\tfrac{3}{4}X_3^2X_5 + \tfrac{3}{4}X_5^2X_1\right] + \cos 13w_1 t\left[\tfrac{3}{4}X_3X_5^2\right]$$

$$+\tfrac{1}{4}X_5^3\cos 15w_1 t \quad \text{------} (4.26)$$

Upon substitution of the values of $\ddot{\psi}$, ψ and ψ^3 in equation (4.7) and after making simplifications the following results in

$$\left[-X_1w_1^2 + w_0^2X_1 + h\left(\tfrac{3}{4}X_1^3 + 3/2X_1X_3X_5 + \tfrac{3}{4}X_1^2X_3 + \tfrac{3}{4}X_3^2X_5 + 3/2X_5^2X_1 + 3/2X_1X_3^2\right)\right]\cos$$

$$+\left[-9X_3\omega_1^2 + \omega_0^2X_3 + h\left(\tfrac{1}{4}X_1^3 + \tfrac{3}{4}X_3^3 + \tfrac{3}{2}X_1X_3X_5 + \tfrac{3}{2}X_1^2X_3 + \tfrac{3}{2}X_3X_5^2 + \tfrac{3}{4}X_5X_1^2\right)\right]\cos 3\omega_1 t$$

$$+\left[-25X_5\omega_1^2 + \omega_0^2X_5 + h\left(\tfrac{3}{4}X_5^3 + \tfrac{3}{4}X_1^2X_3 + \tfrac{3}{2}X_3^2X_5 + \tfrac{3}{4}X_1X_3^2 + \tfrac{3}{2}X_5X_1^2\right)\right]\cos 5\omega_1 t$$

$$+h\left[\tfrac{3}{2}X_1X_3X_5 + \tfrac{3}{4}X_1X_3^2 + \tfrac{3}{4}X_3X_5^2 + \tfrac{3}{4}X_5X_1^2\right]\cos 7\omega_1 t + h\left[\tfrac{1}{4}X_3^3 + \tfrac{3}{2}X_1X_3X_5 + \tfrac{3}{4}X_5^2\right.$$

$$\cos 9\omega_1 t + h\left[\tfrac{3}{4}X_3^2X_5 + \tfrac{3}{4}X_5^2X_1\right]\cos 11\omega_1 t + \tfrac{3}{4}h.X_3X_5^2\cos 13\omega_1 t$$

$$+\frac{h}{4}X_5^3\cos 15\omega_1 t$$

$$= Ew_1\cos w_1 t \quad \text{------} (4.27)$$

Equating the coefficients of $\cos w_1 t$, $\cos 3w_1 t$ and $\cos 5w_1 t$ on both side and ignoring the other higher harmonics, the following relations are obtained.

$$-X_1 w_1^2 + w_o^2 X_1 + h\left(\tfrac{3}{4}X_1^3 + 3/2\,X_1X_3X_5 + \tfrac{3}{4}X_1^2 X_3 + \tfrac{3}{4}X_3^2 X_5 + 3/2\,X_5^2 X_1 + 3/2\,X_1 X_3^2\right)$$

$$= Ew_1 \quad \text{———}(4.28)$$

$$-9X_3 w_1^2 + w_o^2 X_3 + h\left(\tfrac{1}{4}X_1^3 + \tfrac{3}{4}X_3^3 + 3/2\,X_1X_3X_5 + 3/2\,X_1^2 X_3 + 3/2\,X_3 X_5^2 + \tfrac{3}{4}X_5 X_1^2\right) = 0 \quad (4.29)$$

$$-25X_5 w_1^2 + w_o^2 X_5 + h\left(\tfrac{3}{4}X_5^3 + \tfrac{3}{4}X_1^2 X_3 + 3/2\,X_3^2 X_5 + \tfrac{3}{4}X_1 X_3^2 + 3/2\,X_5 X_1^2\right) = 0 \quad \text{——}(4.30)$$

In theory these equations can be solved for X_1, X_3 and X_5 but in reality their solution is not so simple. Numerical solution is possible under certain simplifying assumptions, in a particular case, but the general representation of the solution can not be obtained.

It is supposed that the conditions are such that the third harmonic is relatively small in comparision to fundamental and fifth harmonic is relatively small in comparision to third harmonic i.e.

$$\frac{X_3}{X_1} \ll 1$$

and

$$\frac{X_5}{X_3} \ll 1$$

Then the equation nos. 4.28, 4.29 and 4.30 can be rewritten as follows respectively.

$$\tfrac{3}{4}hX_1^3 + (w_o^2 - w_1^2)X_1 - Ew_1 = 0 \quad \text{———}(4.31)$$

$$\frac{X_3}{X_1} = -\frac{hX_1^2}{4(w_o^2 - 9w_1^2)} \quad \text{————}(4.32)$$

$$\frac{X_5}{X_3} = -\frac{3hX_1\,(X_1 + X_3)}{\left[4(w_o^2 - 25w_1^2) + 6hX_1^2\right]} \quad \text{————}(4.33)$$

Hence X_1 can be calculated from equation 4.31 as before and X_3 and X_5 are then found from equations (4.32) and (4.33) respectively.

To get the values of X_1, X_3 and X_5 analytically for compairing their amplitudes the values for h and w_0^2 are taken from chapter V for the system discussed there.

$$h = 10.37$$

$$w_0^2 = 1.061 \quad 10^4$$

Upon substitution of these values in equation 4.31 the cubic equation gives the root for X_1 as

$$X_1 = 119.2$$

From equation 4.32 the value of X_3 is found

$$X_3 = 5.08$$

and from equation 4.33 the value of X_5 is

$$X_5 = 0.262$$

The magnitude of the 3rd and 5th harmonics are 4.2% and 0.22% respectively.

Now the expression for current will be derived

$$\because \psi = X_1 \cos w_1 t + X_3 \cos 3 w_1 t + X_5 \cos 5 w_1 t$$

But $\psi = N\phi$

$$\therefore \quad \phi = \frac{X_1}{N} \cos w_1 t + \frac{X_3}{N} \cos 3w_1 t + \frac{X_5}{N} \cos 5w_1 t \quad \text{---}(4.31)$$

Substituting the value of ϕ in equation 4.1 to get the value of current the following relation is obtained.

$$i = \frac{X_1}{L_0} \cos w_1 t + \frac{X_3}{L_0} \cos 3w_1 t + \frac{X_5}{L_0} \cos 5w_1 t + \frac{A}{N^3}\Big[X_1 \cos w_1 t + X \cos w_1 t + X_5 \cos 5 w_1 t\Big]^3$$

After expansion and simplification of the above relation the result is as under:-

$$\begin{aligned} i = & \left[\frac{X_1}{L_0} + \frac{A}{N^3}\left(\frac{3}{4}X_1^3 + \frac{3}{2}X_1X_3X_5 + \frac{3}{4}X_1^2X_3 + \frac{3}{4}X_3^2X_5 + \frac{3}{2}X_5^2X_1 + \frac{3}{2}X_1X_3^2\right)\right]\cos\omega_1 t \\ & + \left[\frac{X_3}{L_0} + \frac{A}{N^3}\left(\frac{1}{4}X_1^3 + \frac{3}{4}X_3^3 + \frac{3}{2}X_1X_3X_5 + \frac{3}{2}X_1^2X_3 + \frac{3}{2}X_3X_5^2 + \frac{3}{4}X_5X_1^2\right)\right]\cos 3\omega_1 t. \\ & + \left[\frac{X_5}{L_0} + \frac{A}{N^3}\left(\frac{3}{4}X_5^3 + \frac{3}{4}X_1^2X_3 + \frac{3}{2}X_3^2X_5 + \frac{3}{4}X_1X_3^2 + \frac{3}{2}X_5X_1^2\right)\right]\cos 5\omega_1 t \\ & + \frac{A}{N^3}\left[\frac{3}{2}X_1X_3X_5 + \frac{3}{4}X_1X_3^2 + \frac{3}{4}X_3X_5^2 + \frac{3}{4}X_5X_1^2\right]\cos 7\omega_1 t + \frac{A}{N^3}\left[\frac{1}{4}X_3^3 + \frac{3}{2}X_1X_3X_5 \right. \\ & \left. + \frac{3}{4}X_5^2X_1\right]\cos 9\omega_1 t + \frac{A}{N^3}\cdot\left[\frac{3}{4}X_3^2X_5 + \frac{3}{4}X_5^2X_1\right]\cos 11\omega_1 t \\ & + \frac{3}{4}\frac{A}{N^3}\cdot X_3X_5^2 \cos 13\omega_1 t + \frac{A}{4N^3}X_5^3 \cos 15\omega_1 t \ldots\ldots\ldots\ldots \quad (4.35) \end{aligned}$$

CONCLUSION

So far the solution of Duffing's equation was found upto second approximation i.e. only the fundamental and third harmonic frequencies were included. Here in this work the solution has been obtained upto the third approximation i.e. fifth harmonic has been included with the condition that $X_3/X_1 < 1$ and $X_5/X_3 < 1$ which generally is the case.

It has been shown by taking one example that the magnitude of fifth harmonic is much smaller than the third and it itself is a very small quantity, therefore even though Duffing's equation could be solved for an infinite series containing odd harmonics, it is not wise to go beyond fifth harmonic practically because the complete solution is very much complicated.

CHAPTER V

SUBHARMONIC SOLUTION FOR TRANSMISSION LINE USING SHUNT CAPACITORS

The solution for Duffing's equation formed in chapter IV can also exist in subharmonics form,[6,7,12] under some suitable initial condition. Subharmonics are defined as the components having their frequency an integral submultiple of the driving frequency. Most commonly observed subharmonic in the systems described by Duffing's equation is at one third the driving frequency.

The initial conditions are quite important if a subharmonic oscillation is to exist in a physical system. The frequency and amplitude of the driving force must fall with in certain definite limits. Exact initial condition must exist with in the system itself.

The subharmonics in the solution for Duffing's equation can be predicted with the help of perturbation method.[12]

$$\ddot{\psi} + w_0^2 \psi + \mu h.\psi^3 = Ew_1 \cos w_1 t \quad \text{———}(5.1)$$

In this case both the free and forced oscillations are desired in the generating solution. The small parameter is associated with the non linear term only. The driving frequency will be at an integral multiple of the natural frequency. A solution for equation (5.1) is assumed in the form as

$$\psi = \psi_0(t) + \mu\psi_1(t) \quad \text{———} \quad (5.2)$$

$$w^2 = w_0^2 + \mu\, b_1(E) \quad \text{———}(5.3)$$

Here only first order corrections are used. w is the actual fundamental frequency of the solution and this frequency should be an integral submultiple of the driving frequency w_1.

Say $w = w_1/n$, where n is an integer.

The generating solution is found from the relation.

μ_0 : $$\ddot{\psi}_0 + w^2 \psi_0 = Ew_1 \cos w_1 t \quad \text{———}(5.4)$$

and is $$\psi_0 = B \cos wt + D \cos w_1 t \quad \text{———}(5.5)$$

Here the amplitude of the subharmonic component is B

and $$D = \frac{Ew_1}{(w^2 - w_1^2)} \quad \text{———}(5.6)$$

is the amplitude of the component of driving frequency and moreover $w^2 \neq w_1^2$

This solution satisfies the initial condition $\psi_0 = 0$ at $t = 0$ but the amplitude of the subharmonic component B is not being determined till now. One thing more is to be noted over here is that the sum of the two cosine functions with their frequencies integrally related is a function of even symmetry as must be the case for a solution for equation (5.1) since this equation is unchanged if t is replaced by − t

The first order correction terms are found

The terms in first order correction are not dependent only on one frequency so it is now necessary to choose the relation between these frequencies. The most commonly observed subharmonic in this type of the system is of one third order, in which case n = 3 .

Now the condition required to avoid a secular term is

$$b_1 B - \frac{3h}{4} B^3 - \frac{3h}{4} B^2 D - \frac{3h}{2} B.D^2 = 0 \quad \text{——}(5.7)$$

Equation (5.7) can be satisfied if either B = 0 or

$$b_1 = \frac{3h}{4}\left[B^2 + BD + 2D^2\right] \quad \text{———}(5.8)$$

The first possibility to satisfy the equation (5.10) is trivial, since it removes the subharmonic, and second given by equation (5.8) is interesting one.

Now substituting the value of b_1 from equation (5.8) to equation (5.3) determines the necessary condition.

$$w_0^2 = w^2 - 3h/4\ (B^2+BD+2D^2) \quad \text{------}(5.9)$$

Here μ has been set equal to unity. It is already known from equation (5.6) that $D = Ew_1/w^2-w_1^2$ or

here $D = -\frac{9E}{8w_1}$ or $-\frac{3E}{8w}$

Since

$$\psi_0 = B \cos wt + D \cos w_1t$$

$$\psi 0 = B \cos w_1t/3 + D \cos w_1t$$

But $\psi_0 = N\phi$

$$\therefore \phi = B/N \cos \frac{w_1t}{3} + \frac{D}{N} \cos w_1t$$

Putting this value in equation 4.1 to get the value of current the following results in

$$i = \left(\frac{B}{L_0} \cos\frac{w_1t}{3} + \frac{D}{L_0} \cos w_1t\right) + A\left(\frac{B}{N} \cos\frac{w_1t}{3} + \frac{D}{N} \cos w_1t\right)^3$$

$$= \left(\frac{B}{L_0} + \frac{3}{4}\frac{B^3}{N^3} + \frac{3}{2}\frac{B}{N^3}D^2 + \frac{3}{4}\frac{B^2D}{N^3}\right) \cos\frac{w_1t}{3} + \left(\frac{D}{L_0} + \frac{1}{4}\frac{B^3}{N^3} + \frac{3}{2}\frac{B^2D}{N^3} + \frac{3}{4}\frac{D^3}{N^3}\right)$$

$$\cos w_1t + \left(\frac{3}{4}\frac{B^2D}{N^3} + \frac{3}{4}\frac{BD^2}{N^3}\right) \cos\frac{5w_1t}{3} + \frac{3}{4}\frac{BD^2}{N^3} \cos\frac{7w_1t}{3} + \frac{1}{4}\frac{D^3}{N^3} \cos 3w_1$$

Till now only the amplitudes are determined but the charge on the capacitor and the voltage across it is not determined. Moreover the phase difference is also not considered. Here it is assumed that a phase difference of θ exists and the relation for current will remain same except that w_1t is replaced by $w_1t + \theta$.

So

$$i = \left(\frac{E}{L_0} + \frac{3}{4}\frac{E^3}{N^3} + \frac{3}{2}\frac{ED^2}{N^3} + \frac{3}{4}\frac{E^2D}{N^3}\right)\cos\frac{(w_1t+\theta)}{3} + \left(\frac{D}{L_0} + \frac{1}{4}\frac{E^3}{N^3} + \frac{3}{2}\frac{E^2D}{N^3} + \frac{3}{4}\frac{D^3}{N^3}\right)\cos(w_1t+\theta) + \left(\frac{3}{4}\frac{E^2D}{N^3} + \frac{3}{4}\frac{ED^2}{N^3}\right)\cos\frac{5(w_1t+\theta)}{3} + \frac{3}{4}\frac{ED^2}{N^3}\cos\frac{7(w_1t+\theta)}{3} + \frac{1}{4}\frac{D^3}{N^3}\cos 3(w_1t+\theta)$$

It is assumed that the origin of time is the instant at which the current is passing through zero, then at that instant i.e. at $t = 0$ $i = 0$, and the total charge on the line and static capacitor should be zero, putting $i = 0$ at $t = 0$ in the above equation

$$0 = \left(\frac{E}{L_0} + \frac{3}{4}\frac{E^3}{N^3} + \frac{3}{2}\frac{ED^2}{N^3} + \frac{3}{4}\frac{E^2D}{N^3}\right)\cos\frac{\theta}{3} + \left(\frac{D}{L_0} + \frac{1}{4}\frac{E^3}{N^3} + \frac{3}{2}\frac{E^2D}{N^3} + \frac{3}{4}\frac{D^3}{N^3}\right)\cos\theta + \left(\frac{3}{4}\frac{E^2D}{N^3} + \frac{3}{4}\frac{ED^2}{N^3}\right)\cos\frac{5\theta}{3} + \frac{3}{4}\frac{ED^2}{N^3}\cos\frac{7\theta}{3} + \frac{1}{4}\frac{D^3}{N^3}\cos 3\theta$$

This gives, $\theta = \frac{3\pi}{2}$

The expression for total charge can be obtained by integrating the expression for the current with respect to t. These conditions are more specifically explained in the following example.

<u>EXAMPLE</u>:

Here a system is considered where it is required to trainsit the power of 100 MVA at 0.85 p.f. to the load centre through a high voltage transmission line of 100 miles. At the sending end we are having a bank of transformers to supply this power. The generation voltage is 11 KV and the transmission voltage is 132 KV. Since the load power factor is low so it is required to install static capacitors at the receiving end of the load centre to improve it by 0.9. line parameters are given below:-

Line capacitance = 1.27 μ.F./phase = 0.0127 μF/mile/phase
line Inductance = 0.2 H./phase = 2.0 mH./mile/phase
Line Resistance = 25 ohms/phase = 0.25 ohms/mile/phase.

The capacity of the static capacitors to improve the power factor from 0.75 to 0.9 is calculated and is found to be 4.58 μF per phase (see Appendix).

Now it is required to get the saturation curve for the transformer bank of 100 MVA, 11/132 KV. Through it is quite difficult to get the exact saturation curve for such a transformer, but the approach is that the shape of the saturation curve is dependent upon the material of the core. The standard B, H curves are available for the materials used for transformer core. Only it is required to change the scales of B and H into ϕ and i. To convert the B and H scales to ϕ and i the following relations are used.

$$i = \frac{\text{Amp. turns/m} \times L}{N} \text{ amps.} \quad \text{———} (5.10)$$

and ϕ = B.A. Wb.———(5.11)

where N is the no. of turns

L is the mean length of magnetic path

A is the area of X-section of the core

So here the number of turns in primary and secondary, the area of X-section and mean length of the magnetic path are required. It is rather very difficult to design such a transformer, as it does not only requires the emperical relation but more practical experience in design. Here the selection for these parameters is some what based on the procedure given in the book 'PERFORMANCE & DESIGN OF ELECT. MACHINES' written by 'M.G. Say. Though not exactly the same. The main aim is to show the method how the initial conditions which are necessary for the production of subharmonics are deduced.

All calculations are done on per phase basis and more over all the circuit parameters are transferred to the primary side. The standard B.H curve for stalloy is given below-

TABLE 5·1.

Sl.No.	A.T./METRE	B.Wb/m^2
1	100	0.60
2	150	0.90
3	200	1.05
4	250	1.15
5	300	1.21
6.	350	1.26
7.	400	1.30
8.	450	1.33
9	500	1.36
10	550	1.38
11	600	1.40
12	650	1.42
13	700	1.44
14	750	1.45

Now the following values for the transformer are used

Voltage per turn E_t = 60 V

Primary turns N_1 = 106

Secondary turns N_2 = 1270

Flux density B_m = 1.4 Wb/m^2

Area of X-section of core A_i = 0.193 sq.meter

Area of window A_w = 2.78 sq.meter.

Mean length of magnetic path

l = 9 meters

Now making use of the relations (5.10) and (5.11) the scales of B and H are changed to ϕ and i. These values are tabulated below:

TABLE 5.2

Sl.No.	Current i in amps.	Flux ϕ in Wb
1.	8.49	0.0966
2	12.72	0.1450
3	16.98	0.1681
4	20.85	0.1850
5	25.44	0.1942
6	29.70	0.2025
7	33.95	0.2090
8	38.20	0.2133
9	42.45	0.2183
10	46.70	0.2217
11	50.90	0.2250

TABLE No 5·2.

12	55.20	0.2283
13	59.40	0.2317
14	63.60	0.2330

The line reactance characteristic can also be obtained with the help of following relation

$$L.i = N.\phi \quad \text{———}(5.12)$$

Here N is one since the transmission line has only one turn

$$\therefore i = \frac{\phi}{L} = 720\,\phi \quad \text{———}(5.13)$$

Now the corresponding value of current is found to the value of fluxes in Table (5.2). These values are tabulated in table No. (5.3).

TABLE NO. 5.3

S.No.	Current i in amps.	Flux ϕ in Wbs.
1	69.5	0.0966
2.	104.3	0.1450
3	121.8	0.1691
4	133.2	0.1850
5	139.9	0.1942
6	145.8	0.2025
7	150.5	0.2090
8	153.7	0.2133
9	157.2	0.2183
10	159.4	0.2217

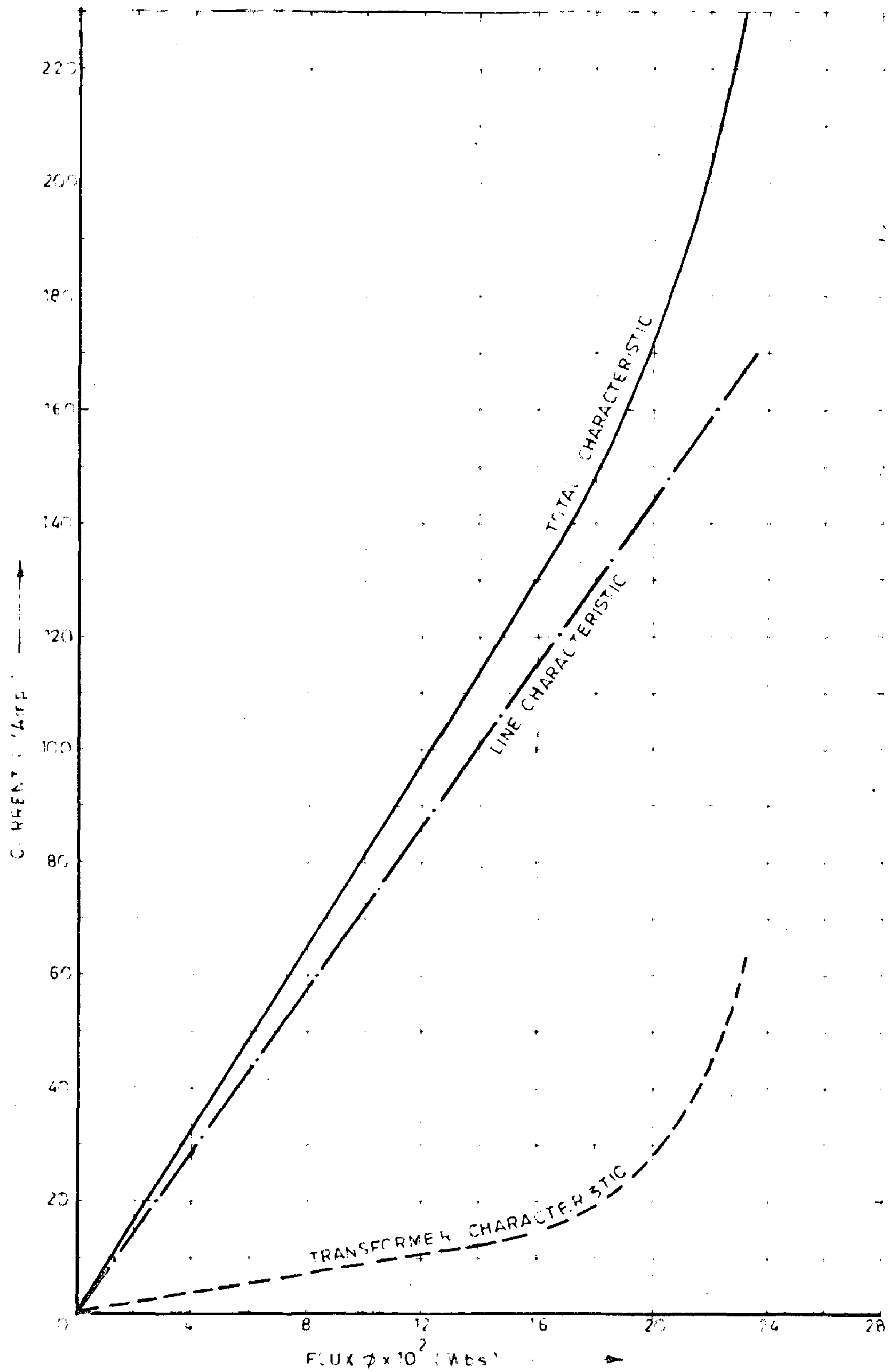
TABLE No 5·3.

11.	162.0	0.2250
12	164.7	0.2283
13	166.8	0.2317
14	167.9	0.2330

Now adding the values of current to the corresponding value of flux, the total characteristic is obtained. This is tabulated in table 5.4.

TABLE 5.4

S.No.	Current i in amps.	Flux 0 in Wbs.
1	77.99	0.0966
2	117.02	0.1450
3	138.78	0.1691
4	154.05	0.1850
5	165.35	0.1942
6	175.50	0.2025
7	184.45	0.2090
8	191.90	0.2133
9	199.65	0.2183
10.	206.10	0.2217
11	212.90	0.2250
12	219.90	0.2283
13	226.20	0.2317
14	231.50	0.2330



TOTAL CHARACTERISTIC
LINE CHARACTERISTIC
TRANSFORMER CHARACTERISTIC
FLUX Ø x 10² (Wbs)
0
4
8
12
16
20
24
28
20
40
60
80
100
120
140
160
180
200
220

After getting the total characteristic, now it is required to get the equation for this curve in the form of odd powers series, so that the equation to the curve fits in well. The best possible way to get the best equation to the curve is the method of least squares. Now here the odd power series is restricted to the 3rd power only. If the equation to the curve is of the form

$$i = a\,\phi + b\,\phi^3 \quad \text{———} (5.14)$$

then the requirement is to determine the coefficients a and b such that they give the best possible fitting to the original curve. To find out the values of a and b, two simultaneous equation are formed as under

$$\Sigma i.\,\phi = a\,\Sigma\phi^2 + b\,\Sigma\phi^4 \quad \text{———} (5.15)$$

$$\Sigma i\;\phi^3 = a\Sigma\phi^4 + b\;\Sigma\phi^6 \quad \text{———} (5.16)$$

These equations are formed from the least square method for curve fitting.

Now the values of $\Sigma i\phi$, $\Sigma\phi^2$, $\Sigma\phi^4$, $\Sigma\phi^6$ and $\Sigma i\phi^3$ are found in table 5.5. The equations (5.15) and (5.16) becomes

$$518.78 = 57.86.10^{-2}\,a + 255.04.10^{-4}.\,b$$

$$23479.15.10^{-2} = 255.04.10^{-4}\,a + 1185.78.10^{-6}.b.$$

After simplification these equations turn out to be as under

$$a + 4.41.10^{-2}b = 896$$

$$a + 4.65.10^{-2}b = 921$$

TABLE 5.5

S. No.	Current i in amps.	Flux ϕ in Wbs.	$\phi^2.10^{-2}$	$\phi^3.10^{-3}$	$\phi^4.10^{-4}$	$\phi.i$	$\phi^3.i.10^{-3}$	$\phi^6.10^{-6}$
1	77.99	0.0966	0.935	0.900	0.875	7.530	75.150	0.810
2	117.02	0.1450	2.105	3.050	4.425	16.980	357.000	9.320
3	138.78	0.1691	2.865	4.840	8.200	23.450	671.000	23.450
4	154.05	0.1850	3.425	6.340	11.750	28.490	976.000	40.200
5	165.35	0.1942	3.780	7.350	14.300	32.100	1217.500	54.100
6	175.50	0.2025	4.110	8.320	17.900	35.500	1489.000	69.200
7	184.45	0.2090	4.375	9.140	19.190	38.580	1684.000	83.500
8	191.90	0.2133	4.550	9.690	20.700	40.800	1857.500	92.000
9	199.65	0.2183	4.780	10.450	22.850	43.600	2085.000	109.500
10	206.10	0.2217	4.855	10.730	23.600	45.600	2210.000	115.600
11.	212.40	0.2250	5.080	11.440	25.900	47.850	2435.000	131.000
12	219.90	0.2283	5.210	11.900	27.200	50.200	2619.000	141.900
13	226.20	0.2317	5.360	12.410	28.650	52.350	2809.000	154.700
14	239.50	0.2330	5.430	12.680	29.500	55.750	3029.000	160.500
			57.860		255.040	518.780	23479.150	1187.780

On further simplification the values of a and b are found

$a = 437$

$b = 10400$

Hence the required equation to the curve becomes

$i = 437\ \phi + 10400\ \phi^3$ ——(5.17)

This equation no. (5.17) corresponds to equation (4.1). Now the values of constants will be determined by equating the coefficients of both the equations. One thing is to be noted over here is that the equation (5.17) shows some deviation to the actual curve in the linear region, while it is approxmately well fits in the original curve in the saturation region. Hence the constant A can be equated to the corresponding coefficient in equation (5.17).

i.e. $A = 10400$

But to get N/L_o it is better to get the slope from the original curve, because this constant L_o is the self inductance of transformer and line when the saturation effect is neglected. Hence the slope of the curve come out to be

$$\frac{N}{L_o} = 947$$

$\therefore L_o = 0.112$ H (Here N = 106)

Total capacitance referred to primary side $C = 144.5.85.10^{-6}$ F

Now the values of w_o^2 and h are calculated from equations (4.5) and (4.6) respectively.

$$w_o^2 = \frac{1}{L_o C} = \frac{1}{0.112.144.5.85.10^{-6}} = 1.061.10^4$$

$$h = \frac{A}{N^3 C} = \frac{10400}{(106)^3.5.85.144.10^{-6}} = 10.37$$

Value of D the amplitude of the component of driving frequency is obtained from equation (5.6).

$$D = \frac{Ew_1}{w^2 - w_1^2} = -\frac{9E}{8w_1} = -\frac{9.11\sqrt{2}.10^3}{8\sqrt{3}.100\pi} = -32.18$$

Now the amplitude B of the subharmonic is still left to be determined. This value is obtained from the necessary condition given by equation (5.9)

$$B^2 + BD + 2D^2 = \frac{4}{3h}\left(\frac{w_1^2}{9} - w_o^2\right)$$

$$B^2 - 32.18B - 2900 = 0$$

$\therefore$ $B = 75, - 38$

Now substituting the values of B and D in equation (5.5)

$$\psi_o = 75 \cos wt - 32.18 \cos w_1 t$$

or $\psi_o = N\phi = 75 \cos \frac{w_1 t}{3} - 32.18 \cos w_1 t$

$$\therefore \quad \phi = 0.708 \cos \frac{w_1 t}{3} - 0.3035 \cos w_1 t \quad \text{———} (5.18)$$

The value of instaneous current i is obtained from equation (4.1)

$$i = \frac{N\phi}{L_o} + A\phi^3$$

$$= \frac{1}{0.112}\left[75\cos\frac{w_1t}{3} - 32.18\cos w_1t\right] + 10400\left[0.706\cos\frac{w_1t}{3} - 0.3035\cos w_1t\right]$$

$$\text{or } i = \left(671.0\cos\frac{w_1t}{3} - 287\cos w_1t\right) + 10.4\left[7.06\cos\frac{w_1t}{3} - 3.035\cos w_1t\right]^3 \qquad \text{——(5.19)}$$

It is know that

$$X\cos\frac{w_1t}{3} + Y\cos w_1t = \left[0.75X^3 + 1.5XY^2 + 0.75X^2Y\right]\cos\frac{w_1t}{3}$$

$$+\left[0.25X^3 + 1.5X^2Y + 0.75Y^3\right]\cos w_1t$$

$$+\left(0.75X^2Y + 0.75XY^2\right)\cos\frac{5w_1t}{3} + 0.75XY^2\cos\frac{7w_1t}{3}$$

$$+ 0.25Y^3\cos 3w_1t.$$

Here put

$X = 7.06$

$Y = -3.035$

Therefore

$X^3 = 356.0$

$Y^3 = -27.9$

$X^2Y = -152.3$

$XY^2 = 65.0$

Hence

$10.4\left[0.75X^3 + 1.5XY^2 + 0.75X^2Y\right] = 2600$

$10.4\left[0.25X^3 + 1.5X^2Y + 0.75Y^3\right] = -1670$

$10.4\left[0.75(X^2Y + XY^2)\right] = -682.0$

$10.4\left[0.75XY^2\right] = 508$

$10.4\left[0.25\,Y^3\right] = -72.6$

$$\therefore 10.4\left[7.09\cos\frac{w_1t}{3}-3.035\cos w_1t\right]^3 = 2600\cos\frac{w_1t}{3}-1670\cos w_1t$$

$$-682.0\cos\frac{5w_1t}{3}+508\cos\frac{7w_1t}{3}$$

$$-72.6\cos\frac{9w_1t}{3}$$

Substituting this value in equation (4.29) we have

$$i = 3271.5\cos\frac{w_1t}{3}-1957\cos w_1t-682.0\cos\frac{5w_1t}{3}$$

$$+508\cos\frac{7w_1t}{3}-72.6\cos\frac{9w_1t}{3} \quad \text{———(5.20)}$$

Till now the amplitudes are determined. The initial conditions are still left to be determined for the subharmonics. The initial phase difference is not considered in the previous analysis. So here introducing the phase difference of θ, the value of the instanaous current i can be obtained simply by replacing w_1t by $(w_1t+\theta)$ in equation (5.20)

$$\therefore i = 3271.5\cos\frac{(w_1t+\theta)}{3}-1957\cos(w_1t+\theta)$$

$$-682.0\cos\frac{5(w_1t+\theta)}{3}+508\cos\frac{7(w_1t+\theta)}{3}-72.6\cos\frac{9(w_1t+\theta)}{3} \quad \text{———(5.2}$$

To determine the initial condition for the subharmonic it is assumed that the origin of time as the instant at which the current is passing through zero, then at that instant i.e. at $t = 0$, $i = 0$ and the total charge on the line and static capacitors should be zero. Putting $i = 0$ at $t = 0$ in equation (5.21) the following results in

$$0 = 3271.5 \cos\frac{\theta}{3} - 1957 \cos\theta - 682.0 \cos\frac{5\theta}{3} + 508 \cos\frac{7\theta}{3}$$

$$- 72.6 \cos 3\,\theta.$$

This gives $\theta = \frac{3\pi}{2}$

The expression for the total charge can be found by integating equation (5.21) with respect to t.

$$q = \int i\,dt$$

$$= \frac{3271.5}{\frac{w_1}{3}} \sin \frac{(w_1 t+\theta)}{3} - \frac{1957}{w_1} \sin (w_1 t + \theta)$$

$$\frac{682}{\frac{5w_1}{3}} \sin \frac{5(w_1 t+\theta)}{3} + \frac{508}{\frac{7w_1}{3}} \sin \frac{7(w_1 t+\theta)}{3} - \frac{72.6}{3w_1} \sin 3(w_1 t+\theta)$$

$$+ q_o \text{ ——— } (5.32)$$

where q_o is the initial charge on the system. Now to obtain this value of charge the limits are substituted i.e. at $t = 0$ total charge is zero and $\theta = \frac{3\pi}{2}$

$$\therefore 0 = \frac{3271.5}{100\pi} \cdot 3 - \frac{1957}{100\pi} - \frac{682.3}{500\pi} + \frac{508}{700\pi} \cdot 3 - \frac{72.6}{300\pi} + q_o$$

$$\therefore q_o = - 21.5$$

The voltage generated in the system will be

$$V = \frac{Q}{C}$$

$$= \frac{21.5}{144.5.85.10^{-6}} \text{ . Volts.}$$

$$= 25.5 \text{ KV.}$$

$\therefore$ The r.m.s. value $= \frac{25.5}{\sqrt{2}} = 18$ KV.

$= 31.1\ KV_{LL}$ ([illegible])

CONCLUSION

This figure of the overvoltage for the system is not impractical. Because overvoltages of two to four times normal value are produced on the open phase or phases following the di-energization of one or two phases.

Now from all this we conclude that the subharmonic will be produced in the above system if the initial conditions exists with in the system. The system should be able to produce the above mentioned voltage.

The solution of the Duffing's equation shows that subharmonics due to non-linearity of the transformer will exists on the transmission line when a fault on the receiving end is removed, only and only if certain initial conditions exist after the fault is removed.

It has been shown by taking one example also. It is shown there that at $t = 0$ i.e. at the instant of switching off the current wave should pass through zero and the phase difference between voltage current should be $\frac{3\pi}{2}$ and the voltage on the system should be 31.1 KV rms. This overvoltage[9,10,14] as is known can ~~be sub-harmonics on~~ exist on a system of 11 KV r.m.s. and hence there can be subharmonics on such systems under the above mentioned initial conditions . ~~The over voltages from two to four times normal value are produced on the open phase or phases following the di-energization of one or two phases.~~

CHAPTER VI

SUBHARMONIC SOLUTION FOR TRANSMISSION LINE USING SERIES CAPACITORS

The series capacitors are used to improve the voltage regulation in case of radial feeders.[13,18] Under 3 Ø fault on the feeder, the inductance of the feeder in series with the capacitor forms an oscillatory circuit and hence with particular initial conditions on the feeder, sub-harmonics may be generated. Here the aim is to determine those initial conditions.

Let the system be as shown in Fig. 6.1

Let base KVA be = 10^5 KVA

Base KV = 132 KV.

$\therefore$ Base current = 437 amps.

Inductive reactance = 18.05 %

and ratio $\frac{R}{X} = \frac{9.4}{31.4} = 0.3$

From the curve (Fig. 6.2) to limit the voltage regulation to 5% at full load the series capacitor must be rated at 17% of the circuit rating i.e. this is 17% of 10^5 KVA.

KVA rating of capacitor = $3I^2X_c$

$$\therefore X_c = \frac{17000 \times 10^3}{3 \times 437 \times 437} = 29.7$$

$$\therefore C = 107.2 \, \mu F$$

The equivalent diagram after neglecting the resistance of the feeder will be as shown in Fig. 6.3. This circuit is of the same form as shown in Chapter IV. The analysis for producing sub-harmonics under certain initial conditions has already been

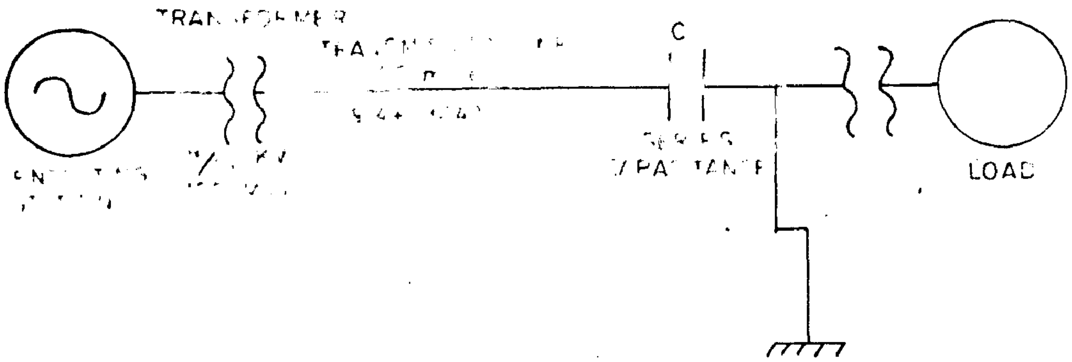


FIG. 6.1

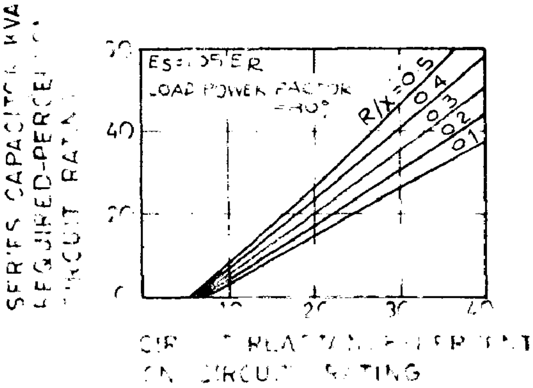


FIG. 6.2

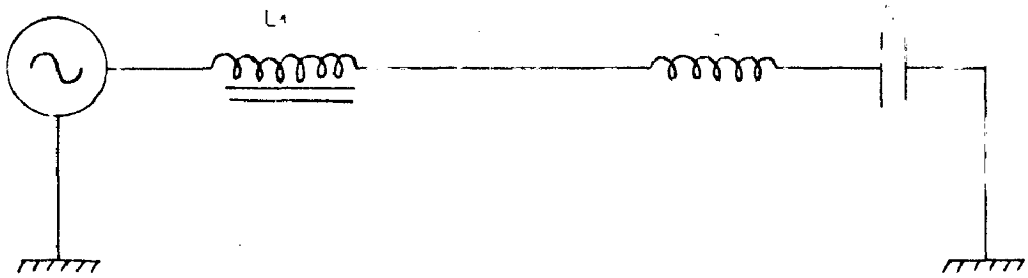


FIG. 6.3

dealt with in Chapter V. Here the same procedure will be followed.

The characteristic for the line reactance between current and flux according to equation (5.12) will be (ref. to primary side)

$i = 1440\ \phi$

The corresponding value of current is found to the value of the flux in table 5.2 and these are tabulated in table 6.1.

~~Adding the~~

TABLE 6.1

Sl. No.	Current i in Amps.	Flux ϕ in Wbs.
1.	139.0	0.0966
2.	208.6	0.1450
3.	243.6	0.1691
4.	266.4	0.1850
5.	279.8	0.1942
6.	291.6	0.2025
7.	301.0	0.2090
8.	307.4	0.2133
9.	314.4	0.2183
10.	318.8	0.2217
11.	324.0	0.2250
12.	329.4	0.2283
13.	333.6	0.2317
14.	335.8	0.2330

Adding the values of current of table 5.2 and 6.1 to the corresponding value of flux the total characteristic is obtained in table 6.2.

TABLE 6.2.

Sl. No.	Current i in Amps.	Flux ϕ in Wbs.
1.	147.49	0.0966
2.	221.32	0.1450
3.	260.58	0.1691
4.	287.25	0.1850
5.	305.24	0.1942
6.	321.30	0.2025
7.	334.95	0.2090
8.	345.60	0.2133
9.	356.85	0.2183
10.	365.50	0.2217
11.	374.90	0.2250
12.	384.60	0.2283
13.	393.00	0.2317
14.	399.40	0.2330

The equation (5.14) relating current and flux is re-written

$$i = a\phi + b\,\phi^3$$

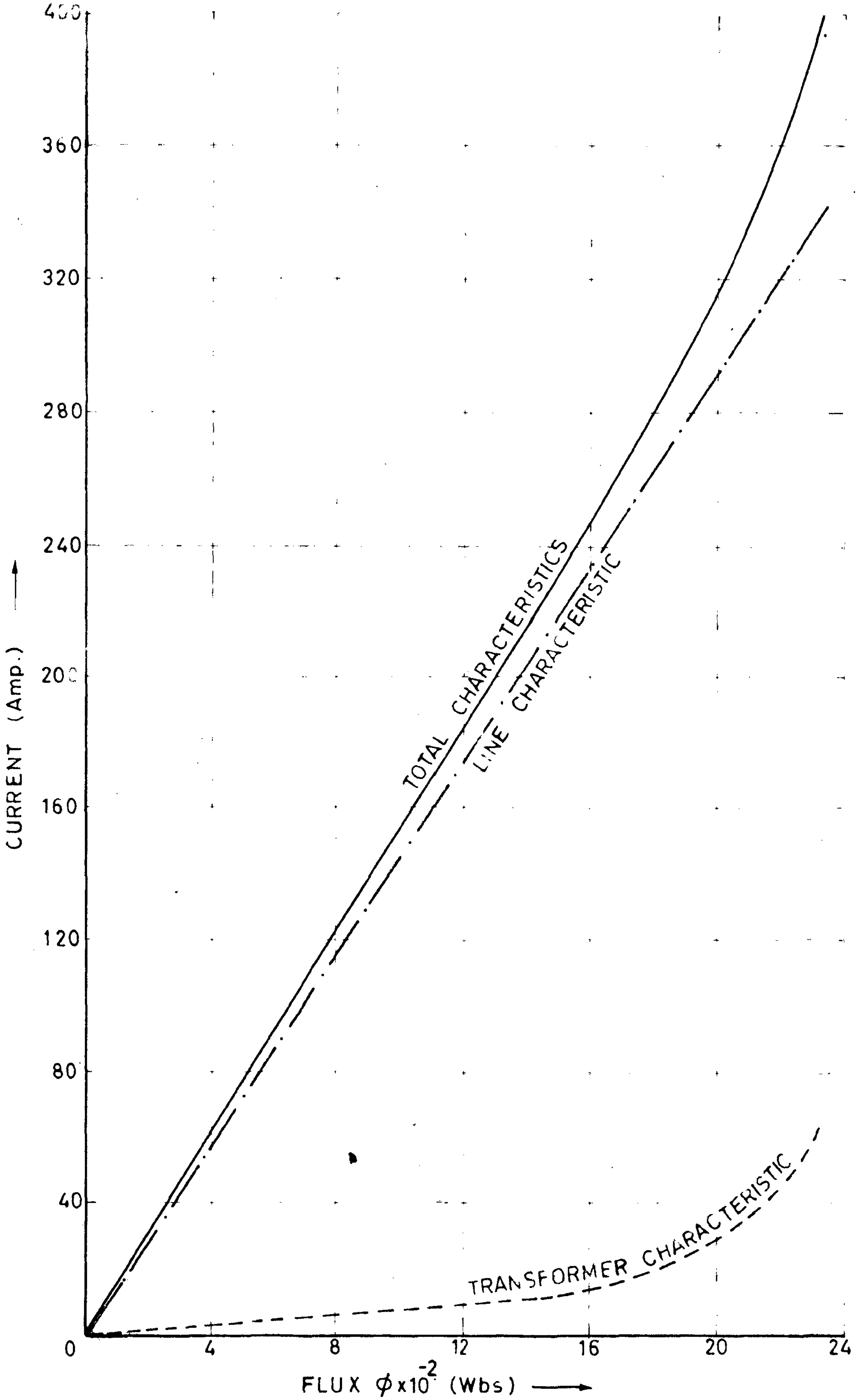
The co-efficient a and b are found with the help of least square method so that this characteristics fits in well the actual curve so calculated. To calculate a and b the following equation are used.

$$\sum i.\phi = a\sum \phi^2 + b \sum \phi^4$$

$$\sum i\,\phi^3 = a \sum \phi^4 + b\sum \phi^6$$

These equations are tabulated in Table 6.3.

Sl. No.	Current i in Amps.	Flux ϕ in Wbs.	$\phi^2 .10^{-2}$	$\phi^3 .10^{-3}$	$\phi^4 .10^{-4}$	$\phi . i$	$\phi^3 . i . 10^{-3}$	$\phi^6 .10^{-6}$
1	147.49	0.0966	0.935	0.900	0.875	14.22	132.80	0.810
2	221.32	0.1450	2.105	3.050	4.425	32.04	674.00	9.320
3	260.58	0.1691	2.865	4.840	8.200	44.10	1262.00	23.450
4	287.25	0.1850	3.425	6.340	11.750	53.10	1820.00	40.200
5	305.24	0.1942	3.780	7.350	14.300	59.20	2240.00	54.100
6.	321.30	0.2025	4.110	8.320	17.900	65.00	2665.00	69.200
7	334.95	0.2090	4.375	9.140	19.190	70.00	3060.00	83.500
8	345.60	0.2133	4.550	9.690	20.700	73.40	3340.00	92.000
9	356.85	0.2183	4.780	10.450	22.850	77.80	3725.00	109.500
10.	365.50	0.2217	4.855	10.730	23.600	80.80	3920.00	115.600
11.	374.90	0.2250	5.080	11.440	25.900	84.40	4285.00	131.000
12.	384.60	0.2283	5.210	11.900	27.200	87.80	4575.00	141.900
13	393.00	0.2317	5.360	12.410	28.650	91.00	4880.00	154.700
14	399.40	0.2330	5.430	12.660	29.500	93.00	5050.00	160.590
			57.860		255.040	925.86	41628.80	1187.780


400
360
320
280
240
200
160
120
80
40
0
4
8
12
16
20
24
CURRENT (Amp.)
FLUX $\phi \times 10^{-2}$ (Wbs)
TOTAL CHARACTERISTICS
LINE CHARACTERISTIC
TRANSFORMER CHARACTERISTIC

$$925.86 = 57.86 \times 10^{-2}\ a + 255.04 \times 10^{-4}\ b$$

$$41628.8 \times 10^{-3} = 255.04 \times 10^{-4}\ a + 1187.78 \times 10^{-6}\ b$$

or

$$1600 = a + 4.41 \times 10^{-2}\ b$$

$$1632 = a + 4.65 \times 10^{-2} b$$

Solving these equations

$$\therefore b = \frac{32}{0.24} \times 100 = 13.334$$

$$\text{and } a = 1600 - 4.41 \times 10^{-2} \times 13334$$

$$= 1600 - 588 = 1012$$

$$\therefore i = 1012\ \phi + 13334\ \phi^{3}$$

Now equating the co-efficients of the above equation with the equation 4.1 the constants of equation 4.1 are determined.

$$A = 13334$$

$$\text{and } \frac{N}{L_o} = 1012 \quad (\text{ Here } N = 106)$$

$$\therefore L_o = \frac{106}{1012} = 0.1049\ H$$

$$w_o^2 = \frac{1}{L_s C} = \frac{1}{0.1049 \times 144 \times 107.2 \times 10^{-2}} = 618$$

$$h = \frac{A}{N^3 C} \qquad \frac{4 \times 10^4}{3(106)^3 \times 107.2 \times 144 \times 10^{-6}} = 0.727$$

$$D = \frac{E w_1}{W - w_1^2} = -32.18$$

The amplitude of B of the subharmonic is still left to be determined. This value is obtained from the necessary condition given by equation 5.9 i.e.

$$\therefore \quad B^2 + BD + 2D^2 = \frac{4}{3h} \left(\frac{w_1^2}{9} - w_o^2 \right)$$

$$B^2 - 32.18B + 2070 = \frac{4}{3 \times 0.727} \quad 1.098 \times 10^4 = 618$$

$$B^2 - 32.183 - 16920 = 0$$

$$\therefore \quad B = 146.09 \; , \; -113.91$$

Now substituting the values of B and D in equation (5.5)

$$\psi_o = 146.09 \cos \frac{w_1 t}{3} - 32.18 \cos w_1 t$$

$$\psi_o = N \phi$$

$$\therefore \quad \phi = 1.38 \cos \frac{w_1 t}{3} \quad -0.3035 \cos w_1 t$$

The value of instantaneous current i is obtained from equation 4.1

$$i = \frac{N\phi}{L_o} + A\phi^3$$

and is

$$i = 24392 \cos \frac{w_1 t}{3} - 3426 \cos w_1 t - 4520 \cos \frac{5w_1 t}{3} + 1270 \cos \frac{7w_1 t}{3}$$

$$- 93.1 \cos 9 \frac{w_1 t}{3}$$

To get the initial charge on the system the current expression is integrated and equated to zero, since the total charge at $t = 0$ should be zero. After integration the value of w_1 is substituted and the following expression is obtained

$$\frac{1}{100\pi}\left[73176 - 3426 - 2710 + 545 - 31.1\right] + q_o = 0$$

$$\therefore q_o = - 215 \text{ coulombs.}$$

$$\therefore \text{voltage} = V = \frac{215}{144 \times 107.2} \times 10^6 = 13.9 \text{ KV}$$

$$= 16.75 \text{ KV L-L}$$

The solution of the Duffings' equation shows that sub-harmonics on the system will exist when a 3ϕ fault on the receiving end occurs only and only if certain initial conditions are prevailing in the system. It is shown in the example that at $t = 0$ i.e. at the instant of switching off the current wave should pass through zero and the phase difference between voltage and current should be $3\pi/2$ and the voltage on the system should be 16.75 KV r.m.s. This overvoltage as is known can exist on a system of 11 K and hence there can be subharmonics on such systems under the above mentio initial conditions.

CHAPTER VII

CONCLUSION

CONCLUSIONS

Here in this work a graphical solution for non linear circuits arising out of the fault at different distances over the long transmission line has been studied under certain simplyfing assumptions. Lumped parameters for the line are used. The improvement can be done by finding the equivalent impedance and admitance in T form using distributed parameters for the length of line involved. The over voltages and currents so calculated assuming lumped parameters are larger than the actual distributed parameters, so we are on safer side as far as any protective device applied to the system is considered. Moreover the analysis is based under the assumption of sinusoidal variation of current, which is not true, hence the method gives the approximate solution.

Analytical method is also tried to consider the effect of the fault distance for long transmission line neglecting the effect due to resistance of the line. An expression is derived for the fault distance X and it is shown that if the value of X is greater than the derived expression then all the three roots of cubic equation in I_m are real otherwise they contain one real and two complex roots.

Till now, the Duffing equation was solved for second approximation. Here a third approximation is attempted i.e. fifth harmonic has been included in the assumed solution for flux linkages under the assumption that $X_3/X_1 < 1$ and $X_5/X_3 < 1$ which generally is the case, resulting in more accurate solution for flux linkages which will result for the current to contain upto 21st harmonic. This procedure becomes more laborious for

for higher harmonic correction to obtain more and more accurate solution. Hence practically it is not proper to have an infinite series solution of Duffing's equation which of course theoretically can be derived. As approximation neglecting higher harmonics is acceptable. It is shown by one example that even the fifth harmonic magnitude of flux linkages itself is small in comparision to the fundamental.

Transmission lines using shunt capacitors or series capacitors are represented by Duffing's equation under simplifying assumptions when a symmetrical fault occurs at the receiving end. The solution for Duffing's equation is obtained in subharmonic form under certain initial conditions. The solution is valid only when the initial conditions exists with in the system. With different initial conditions different solutions will exist.

Further work can be done on this topic by considering the effect of resistance on the non linear circuit formed under fault condition. Although rigirous mathematical treatment is impossible but an approximate solution under certain assumptions can be achieved. More over different type of faults may also be studied.

APPENDIX

APPENDIX I

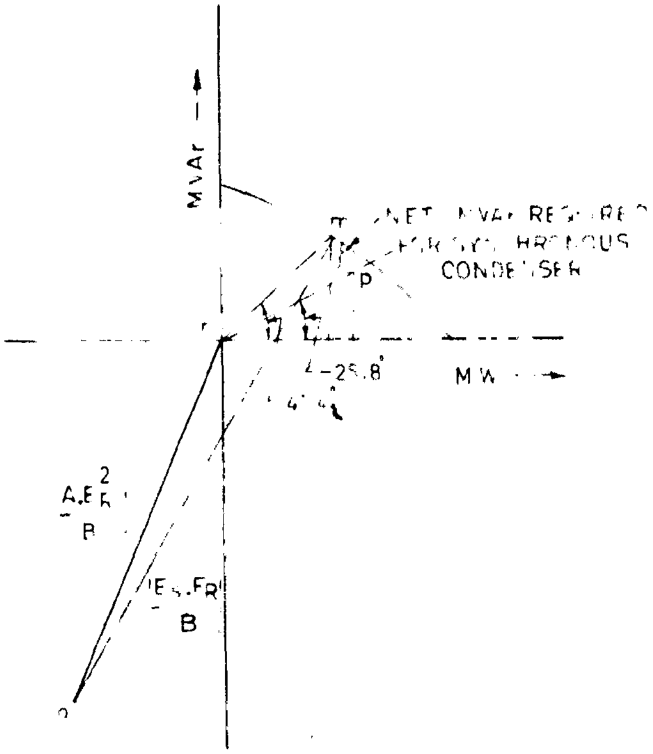
The capacity of the synchronous or static conductor used to improve the power factor is calculated by power circle diagram as follows.

On the coordinate axis with origin 'n' the centre of power circle '0' is located at a distance of $\left|\frac{A\,E_R^2}{B}\right|$ with an angle $(\beta - \alpha)$ to a scale of 1 cm = 50 MVA as shown. With 'n' as origin a line nm equal to 100 MVA at an angle 41.4° i.e. at a p.f. of 0.75 is drawn. Now another line 'np' of the same MVA but at an angle of 25.8° i.e. at a p.f. of 0.9 is drawn. The difference in the ordinates of these two lines gives the required MVA_r for the synchronous or static condenser.

$$\text{Hence} \quad \frac{V^2}{X_C} = 0.5 \text{ cm} = 25 \text{ MVA}_r$$

$$\text{or} \quad X_C = \frac{132.132}{25} = 695$$

$$\therefore \quad C = \frac{1}{wX_C} = \frac{1}{695.314} = 4.58\,\mu F.$$



POWER CIRCLE DIAGRAM
SCALE 1cm :50 MVA

LIST OF SYMBOLS USED

LIST OF SYMBOLS USED

R	=	Resistance of the line per unit length
L	=	Inductance of the line per unit length
C	=	Capacitance of the line per unit length
l	=	Length of the line
x	=	Distance of the fault from sending end.
L_1	=	Non linear inductance of Transformer.
I	=	Current in the circuit
E_T	=	Transformer voltage being the function of current
E_R	=	Drop in voltage due to resistance
E_C	=	Drop in voltage due to capacitance
~~EL~~ E_L	=	Drop in voltage due to Inductance.
ϕ	=	Flux in wbs.
E, V_m	=	Source voltage (maximum)
N	=	Number of turns of the inductor.
N_1	=	Number of turns of in primary of Transformer.
N_2	=	Number of turns in secondary of Transformer.
/	=	Flux linkages
B	=	Flux density
A_1	=	Area of X section of the core
A_w	=	Area of X section of window.
E_t	=	Voltage per turn.

REFERENCES

R E F E R E N C E S

1. C.G.SUITS: "Studies in non linear circuits." Transactions AIEE Vol. 50, 1931. p. 724

2. E. Weber and P.H.Odessey: - "Critical conditions in ferroresonance Transactions AIEE Vol.57,1938, p.444.

3. J.D.Ryder:- "Ferro inductance as a variable electric circuit eleme Transactions AIEE Vol.64, 1945, p.671.

4. C.M.Summers:- "An unstable non linear circuits". Transactions AIEE. Vol.59, 1940, p.273.

5. SABIHI: "Theory of Ferroresonance" Transactions AIEE Vol.78, 1959, p.755-62(Communication and Electronics).

6. Irven Travis & C.N.Weygandt. "Subharmonics in circuits containing Iron cored Reactors. Transactions AIEE Vol. 57, 1938. p-423-31.

7. C.Hayashi:- "Subharmonic Oscillations in Non-Linear Systems" Journal of Applied Physics Vol.24, 1953, p-521-29.

8- Louis A.Pipes, "Applied Mathematics for Engineers and Physicists" (Book) McGraw-Hill.

9. G.G.Auer & A.J.Schultz:-"An Analysis of 14.4/24.9 KV. Grounded Y Distribution system Overvoltages". AIEE August 1954 pp. 1027-32-

10. L.B.Crann & R.B.Flickinger: "Over voltages on 14.4/24.9 KV Rural Distribution systems". AIEE October 1954. pp-1208-12.

11. M.G.Say: "Performance and Design of A.C.Machines" (Book) McGraw Hill.

12. W.J.Cunningham: "Non Linear Analysis p-173,191(Book)

13. Westing House:- "Transmission and Distribution Reference Book" Ch. 5.p-96, Ch.8, p-256.

14. R.H.Hopkinson: "Ferroresonance During Single Phase Switching of 3 Phase Distribution Transformer Banks". Transactions AIEE April 1965 pp.289.

15. Clarke, Peterson, Hight: "Abnormal Voltage Conditions" AIEE Vol.60, 1941 pp.329-39.

16. Sheldon Plotkin: "Discontinuous Transition Time Between Stable States in Ferroresonant Circuits" AIEE. Vol.76, Pt.I 1957, p-410(Comm. & Electronics)

17. J.W.Butler and C.Concordia;- "Analysis of series capacitors application Problem". Transactions AIEE Vol.56, 1937, p-975.

18. "Abnormal loads on power Systems" IEE Conference report series No. 8. p.27, 30.