

DETERMINING RESERVE CAPACITY OF POWER SYSTEMS

*A Dissertation
submitted in partial fulfilment
of the requirements for the Degree
of
MASTER OF ENGINEERING*

in
POWER SYSTEM ENGINEERING (Electrical Engineering)

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CERTIFICATE

Certified that the dissertation entitled "DETERMINING
RESERVE CAPACITY OF POWER SYSTEMS" which is being submitted by
Sri M.R.Chandrasekharan in partial fulfilment for the award
of the Degree of Master of Engineering in Power System Engineering
of the University of Roorkee is a record of student's own work
carried out by him under my supervision and guidance. The
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the award of any other Degree or Diploma.

This is to further certify that he has worked
for a period of ...7....months from ¹.....^{Aug}. to¹⁹⁶⁶ for
preparing dissertation for Master of Engineering Degree at the
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SYNOPSIS

In the following pages an effort has been made to present a comprehensive picture of some of the aspects of power system reserve problems. The basic elementary ideas of the theory of probability applied to system reserve problems have been discussed and the significance of conventional methods of outage probability calculation has been illustrated. Some of the short-cut methods have been applied to a representative system and the results are compared with those obtained with exact calculations. The work deals more with analysing the factors influencing the determination of reserve capacity of power systems, than in the exact solution of a typical problem, the chief factor being the probability of outage of various capacities. The limitations and complexities involved in the treatment of hydro units have been discussed. The importance of the problem in an interconnected system has been indicated. The role of computers in probability studies has been stressed and a computer programme has been developed to determine the cumulative outage probability for various capacity outages. The need and scope for further research have been indicated.

CHAPTER - I

INTRODUCTION AND HISTORICAL DEVELOPMENT

INTRODUCTION

In the engineering of power systems one of the most important objects is the attainment of a high degree of continuity of service. In fact the life in the served area comes to a standstill when failures occur in the supply systems. Still interruptions do occur due to lack of amenities, personal negligence and even for reasons beyond prediction. Seldom, however is the proper degree of service reliability known or defined in quantitative terms and rarely does the designer employ a truly rational method to attain it. Some way of defining the goal must be found and some scientific means of reaching it sought. In this investigation an attempt is made to make an analysis of the forced outages that occur in the case of generating equipments which further leads to the determining of the measure of reliability the system can offer in meeting the demands of the customers.

Reserve requirements have in the past been based on "rule of thumb" criteria such as a fixed percentage of reserve or the outage of a number of the largest generating units. For example a current rule of thumb in the industry is that reserve capacity should not be less than 15% of the system requirements. However, the complexity of the problem often makes it difficult to find an answer by the "rule of thumb". There are no exact methods available which permit the solution of reserve problems. However a systematic approach can be made by the application of the theory of probability.

It has been observed that outages that occur are more governed by law of chance and can be treated as random events. An understanding of the kinds of regularity that occur amid random fluctuations and the use of probabilistic mathematical models to interpret the physical phenomena with appropriate measures of uncertainty leads us to a still more rigorous treatment of the problem.

than that has been used hitherto. In short the use of probability mathematics offers a means whereby some consistent account can be taken of the chances.

The probability theory predicts the average outage performance of a group of units during a long period of time. The basic information on which such predictions are made is obtained from the previous experience of the existing units. In the United States statistical data have been established on a nation wide basis covering the forced outage rate of boilers, turbines and generators. Unfortunately, such data which can be readily referred to is not available in our country.

1.2. Development in the Past:

The value of probability theory in the field of engineering has been forcefully brought out in a paper⁽¹⁾ published by S.A.Smith in 1934, entitled "Spare Capacity Fixed by Probabilities of Outage". A problem of this type which is of special interest to the telephone engineers forms the subject matter of a paper⁽¹⁰⁾ published by E.C.Moli in 1935 which explains the fundamentals of the theory by the solution of sample problems of the types frequently encountered.

Presentation of four A.E.E.E. papers^(3,4,7,11) in 1947 was a chief motivating factor in initiating the application of probability methods to solve reserve problems of many of the power systems in the United States. Moreover the inadequacy of the present system due to growth of load and increased complexities in the generation and transmission of equipments also warranted the development of a new method by which reserve problems could be tackled in a more reasonable and realistic manner. Work on this problem actually began in all its earnestness after Calabrese⁽³⁾ gave a

mathematical treatment of the pertinent factors involved in 1947. The paper was the result of investigation of forced outages on several systems of boilers and steam turbogenerator units, covering a period of six years. In the same year Lyman gave some methods which have been developed to obtain an approximate solution to complicated probability calculations. The most valuable contribution of this paper seems to be the development of a short cut method applicable to a system with any number of generating units of different sizes. The paper by Seelye⁽¹¹⁾ gave a mathematical development of relatively simple algebraic formulas. For the study of reserves, necessary to take care of forced or emergency outages of generators, results in a tangible form were developed and the method of application was indicated. In a subsequent paper⁽²⁵⁾ in 1949 he has extended the simplified approximate method to the development of a few charts whereby the percentage of reserve in terms of total system capability or total system load may be found directly. Another paper in the same series was by Loane and Watchorn⁽⁷⁾ which deals with an application of the theory of probability to a particular problem of special interest to those concerned with hydro-unit treatments.

A report by an A.I.E.E. committee⁽⁵⁾ in 1949 on "Outage Rates of Steam Turbines and Boilers and Hydro-Units" made available the forced outage data collected by the committees of the Edison Electric Institute. The data was collected by supplying questionnaires to power companies and the details presented are basic to the determination of system capacity requirements by probability computations.

In 1950⁽⁸⁾ Watchorn presented a paper⁽⁸⁾ in which he demonstrated that the complex composite effect of all the factors that affect the system capacity requirements can be evaluated by the

use of two simple basic probability principles applied arithmetically. The results showed wide differences in the capacity requirements for various conditions and he concluded that these differences can be evaluated in no other way than by the application of probability methods.

In the articles in 1947 and in subsequent papers in 1950⁽¹⁾ and in 1951⁽²⁾ Calabrese has discussed the use of load duration curve based on daily max. loads for the determination of index of reliability level. He put forward some ideas regarding the treatment of interconnected systems and suggested three methods to determine the probability of loss of load with interconnection. In the same year (1951) Standard⁽³⁰⁾ presented a new analytical method of calculating probable frequency of occurrence of forced outages and probable duration of these outages, and in 1957 Watchorn⁽¹⁹⁾ further developed a simplified method for the application of the theory of probability to reserve problems.

There seems to have been a lot of interest developed by this time in the probability methods. In 1958 as many as eight papers {14,15,20,23,24,26,27,28} were published in the A.I.E.E. Journal and the contributors were Kist, Halperin, Miller, Limmer, Mierone and Hicks to mention only a few. Kist has considered the records of emergency outages of major equipments for a period of approximately eight years, preceding the study in order to assign the actual historical outage factors to each element of the system entering into the calculations.

The nebulous and the difficult task of determining economic tie-line capacities has been attacked in a systematic manner in a paper by Jones and Wierow titled as "The Use of Probability Methods in the Economic Justification of Interconnecting Facilities Between Power Systems in South Texas"⁽²⁴⁾. The subject matter pertains to

a particular system and no attempt has been made to generalise the method. Due to the extreme complex nature of the problem, with the information available, each system is to be tackled individually, taking into account, all the pertinent factors affecting the operation of the system. Once the economic operating conditions are established, there remain only similar calculations to determine the saving in investment and annual charges.

A still more exact treatment of reserve problems was possible by this time when Halporin and Adler⁽¹⁵⁾ presented a mathematical method of reserve studies, based on the duration and frequency of outage. In an earlier paper in 1946⁽¹⁰⁾ the authors have derived some of the quite important formulas which consider the variations in frequency and duration of events. But while taking into account these variations the problem becomes so cumbersome than even with a digital computer the solutions become quite involved. To simplify the approach without sacrificing the accuracy, the authors introduced the idea of the uniform system and sub-division of a complex system into uniform sub-groups. Modified analytical expressions have been developed and methods of application suggested which invoked quite a lot of interest among those keen on the topic. A critical review of these works is given in chapter no. III, specially devoted to it.

By this time the use of digital computer has become popular and many of the power companies have begun to solve reserve capacity problems by means of probability methods aided by the computers. In the past, the time required for the manual calculations for a system of even moderate size has restricted the widespread use of probability methods. In 1958, two papers by Brennen, Galloway and Kirchmeyer discussed how a medium sized digital computer may be used to calculate the probability of capacity outage and the probability

of loss of load for a non-interconnected system. The procedure adopted in this paper affords a rapid and economic way to obtain the solutions. To develop a table of capacity outage probability for a 2-machine system having 40 possible values of capacity outage required only five minutes of computer time. The same table required about 8 hours to solve with a desk calculator. A table having 979 possible values of capacity outage required 30 minutes of computer time.

The contribution by Calabrese in applying the method to interconnected systems has already been referred to. But a classical treatment of this problem was done by Cook and Co-authors in a paper published in 1963⁽¹⁸⁾. By means of explanatory Venn-diagrams the authors have established some simple formulas applicable to systems which are non-interconnected as well as to systems with finite and infinite interconnections. The derivations are quite interesting and are discussed in Chapter No. V. In order to assess the value of interconnections and to determine the installed reserve capacity reduction by pooling, a digital computer programme is discussed. The work mainly confined to two interconnected systems; and though specific problems involving more than two areas can be solved after studying the load flow conditions, further research work is essential in obtaining a generalised solution.

A method taking the full account of the hourly, daily and seasonal load variations as well as the annual load and monthly load peaks has been suggested by Nagel and Varsel⁽¹⁶⁾. The procedure comprises

- 1) Analysis and projection of system load

- 2) Analysis of generating capacity availability.
- 3) Determination of a capacity deficiency and energy deficiency curve and
- 4) application of these deficiency curves to the specific situation.

Another paper presented by Vassol and Tibborth⁽²²⁾ discussing a short cut method to examine capacity reserve requirements seems to be the latest publication on this topic.

Many of the methods suggested have got several important advantages but have also limitations. Judgement and experience must continue to play a major role in interpreting and applying the results. The applications of the methods have been vast and a great wealth of information exists in the form of Technical Papers.

CHAPTER II

THEORY OF PROBABILITY APPLIED TO SYSTEM RESERVE PROBLEMS

CHAPTER II

2.1. Basis of Probability Study:

Probability is a measure of the chance of a certain event by the ratio between the number of events that can occur in that certain way and the number of total possible events. Consider the performance of a single generating unit during a long past period T. During this period the unit has been in operation for a period of S_1 , and out of service due to forced outages of the unit itself for a period O_1 . The average outage rate of this unit is then $\frac{O_1}{S_1 + O_1}$.

The outage and service times can be expressed in hours or days. If the 'n' units of a system are or are assumed to be similar in design and operating conditions the outage rate of the individual average unit is calculated with the formula:

$$q = \frac{O_1 + O_2 + \dots + O_k + \dots + O_n}{(S_1 + O_1) + (S_2 + O_2) + \dots + (S_k + O_k) + \dots + (S_n + O_n)} \quad 2.1$$

Where O_k is the total forced outage time and S_k the total service time of the k^{th} unit. The service rate 'p' of the average unit can be determined in the same way and is equal to

$$\frac{S_1 + S_2 + \dots + S_k + \dots + S_n}{(S_1 + O_1) + (S_2 + O_2) + \dots + (S_k + O_k) + \dots + (S_n + O_n)} \quad 2.2$$

With a given amount of the units past performance available a series of values of q are calculated by adding each year's forced outage and service times respectively to the cumulative forced outage and service times of the preceding years. If this leads to a stable value of q i.e., a value which changes little with increase in demand time this value may be used to predict the expected future performance

of the average unit. In the article published by Calabrese in 1947, he has suggested an outage factor of 0.03. However as a result of further detailed investigation a national average of 0.02, for generators, has been established in the United States.

With the records used in the study of some of the systems it has been found that there is no characteristic variation in the frequency and duration of individual generator outages with size make or age of equipment. In the analysis done in this investigation it has been assumed that the number of outages for any generator is directly proportional to its exposure. Moreover in a system consisting of a number of machines, as the growth of load occurs, new machines are installed and old and new equipment are put into service to meet the load demand. This may again lead to an intermediate value for 'a' and the correction for ageing does not seem to be essential.

In cases where equipments are connected in series relation the outage of one element causes the whole group to be out. As an example a Turbo generator outage may be caused by trouble in the boilers or in the Turbine or in the generator. In general a unit may be composed of several elements a, b, c, d in series having outage rates qa , qb , qc , etc. Each one of those outages is calculated considering only the outage caused by troubles in the particular element and not in the other elements. The corresponding service probabilities are $pa = 1 - qa$, $pb = 1 - qb$, $pc = 1 - qc$ and so forth. The service probability p , of the units as a whole by the multiplication principle is $= pa \cdot pb \cdot pc \cdot pd \dots \dots$. The outage rate q , of the units is $= 1 - pa \cdot pb \cdot pc \cdot pd \dots \dots$

The probability of occurrence of either one or the other of two mutually exclusive events is the sum of the respective probabilities. Thus the probability of having either of the two units a and b on forced outage is $(pa \cdot qb + pb \cdot qa)$. Mutually exclusive means nonoccurrence of one and

excluding the occurrence of the other event.

By applying the formal rules of the probability theory the outage probabilities of the combinations of n dissimilar units having individual outages $q_1, q_2 \dots q_n$ are obtained by developing the following products of binomial factors

$$(p_1 + q_1)(p_2 + q_2) \dots (p_k + q_k) \dots (p_n + q_n)$$

If the n units of the group are assumed to be similar, the same outage rate q is used for all of them. In that case the binomial factors degenerate into the well known Binomial formula

$$(p + q)^n = p^n + np^{n-1} \frac{q + n(n-1)}{2!} p^{n-2} q^2 + \dots$$

This expansion provides the basis for determining the probability of finding various numbers of ' n ' units available for service. In the expansion the first term represents the probability for no outage the second term for the forced outage of one unit, the third term for the forced outage of two units and so on.

2.2 Calculation of Capacity Outage Probabilities by the Conventional Methods:

A hypothetical system containing five steam units 3,25 MW units and 2,40 MW units is considered. An outage factor of 0.02 for each smaller unit and 0.03 for the larger units are attributed. Combination of the probabilities for the three smaller units is accomplished by the Binomial expansion

$$\begin{aligned} (p+q)^3 &= (0.98 + 0.02)^3 \\ &= 0.941192 + 0.057624 + 0.001176 + 0.000008 \quad \dots(1) \end{aligned}$$

Probability factors for the 40 MW are obtained from the expansion $(0.97 + 0.03)^2$

$$= 0.9409 + 0.0582 + 0.0009 \quad \dots \text{(ii)}$$

By multiplying (i) and (ii) the combined probability factors for the whole system can be obtained as tabulated below:

(1) Capacity outage in MW.	(2) Probability	(3) Units on Forced Outage.	Table 2-1.
0	0.8855675528	0	
40	0.0547773744	1	
80	0.0008470728	2	
25	0.0542184216	1	
65	0.0033537168	2	
105	0.0000518616	3	
50	0.0011064984	2	
90	0.0000684432	3	
130	0.0000019584	4	
75	0.0000075272	3	
115	0.0000004656	4	
155	0.0000000072	5	

Table 2.1 when re-arranged in the ascending order of capacity outage table 2.2 is obtained.

(1) Capacity Outage in MW.	(2) Probability	(3) Units in forced Outage
0	0.8855675528	0
25	0.0542184216	1
40	0.0547773744	1
50	0.0011064984	2
65	0.0033537168	2
75	0.0000075272	3
80	0.0008470728	2
90	0.0000684432	3
105	0.0000518616	3
115	0.0000004656	4
130	0.0000010584	4
155	0.0000000072	5

Table 2.2

2.3 Further Modifications in Outage Probability Calculations.

The validity of arithmetical methods to compute outage probabilities has been now fairly established. A convenient method which results in the reduction of the results to manageable proportions has been suggested by Kist and Thomas. The authors introduced the method of proration of the forced outage probabilities to multiples of reasonable capacity. The idea can be illustrated as given below.

Assume that it is desired to arrange the table 2.2 with a width of 25 MW capacity. The 40 MW capacity is to be eliminated. Linear proration between 25 MW and 50 MW assigns $\frac{10}{25}$ of the outage factor for 25 MW and $\frac{15}{25}$ of the outage factor to 50 MW. But the 50 MW outage condition is further modified by the proration of the 65 MW outage condition. Continuation of the process results in a new table as given below. The table is arranged in the ascending order of magnitude of various capacity outages.

<u>MW Outage</u>	<u>Probability of Outage</u>
0	0.8855675528
25	0.0761293713
50	0.0353144097
75	0.0027247927
100	0.0002521559
125	0.0000114983
150	0.0000002173
175	0.00000000144

2.4. The Concept of Cumulative Outage Probability:

The method explained in 2.2 is made use of in determining the various capacity outages and the exact outage probabilities associated with the respective capacities. But more often the designers are interested in the outage probabilities of losing a certain capacity or more and in this connection the term cumulative outage probability can be introduced. By examining the table 2.2 it is found that the exact probability of outage for '0' MW is 0.8865675528, and by simple argument it can be shown that the probability of losing '0' MW or more is the sum of all the terms in the Column No. 2, which is equal to unity. Similarly the probability of losing 25 MW or more is the sum of the terms in column No. 2, but in this case down from row no. 2 only. Table 2.2 modified to represent the cumulative outage probability is shown in Table 2.

<u>Capacity Outage in MW</u>	<u>Cumulative Probability</u>
0	1.0000000000
25	0.1144324472
40	0.0602140256
50	0.0054366512
65	0.0043301528
75	0.0009764360
80	0.0009689088
90	0.0001218360
105	0.0000533928
115	0.0001300728
130	0.0001065600
155	0.0000000072

Table 2.4

2.5. Loss of Load Probability

In the previous sections it has been shown that individual outage factors can be combined to obtain probability of various amounts of capacity on forced outage. As the load of a station undergoes hourly daily and seasonal variations, any capacity outage may or may not result in loss of load depending upon the time at which the forced outages occur. Therefore a knowledge of the probability of loss of various quantities of supply is not a measure of the effect of their loss upon the load to be served. To measure the effect of probable forced outages on the load to be served requires the use of load duration curves. If the max. system demand were constant throughout the month at 100% load factor conditions every outage in excess of installed reserve would cause some load curtailment. The maximum effect on the load, of a capacity outage can be easily determined from the load duration curve. But the expected loss from the probable forced outage will be much less because the probability factor associated with any capacity outage would evidently be a small quantity.

The above mentioned statement is supported by the very definition of the term 'Expectation'. Expectation is a mathematical method of placing an event and can be defined as "The probability of an event occurring, when multiplied by the results realized if the even occurs".

Kist and Thomas have suggested a method to evaluate the loss of load probability which leads to the determination of a term known as the "Index of Reliability". Index of reliability is defined as the ratio of load served to the load available. The authors have considered the historical index of reliability as the bench-mark with respect to which the quality of service is measured. In the present

INSTALLED CAPACITY = 224 MW.

ANNUAL LOAD DURATION CURVE

BASED ON DAILY PEAK LOADS FOR GANGA-SARDA
GRID

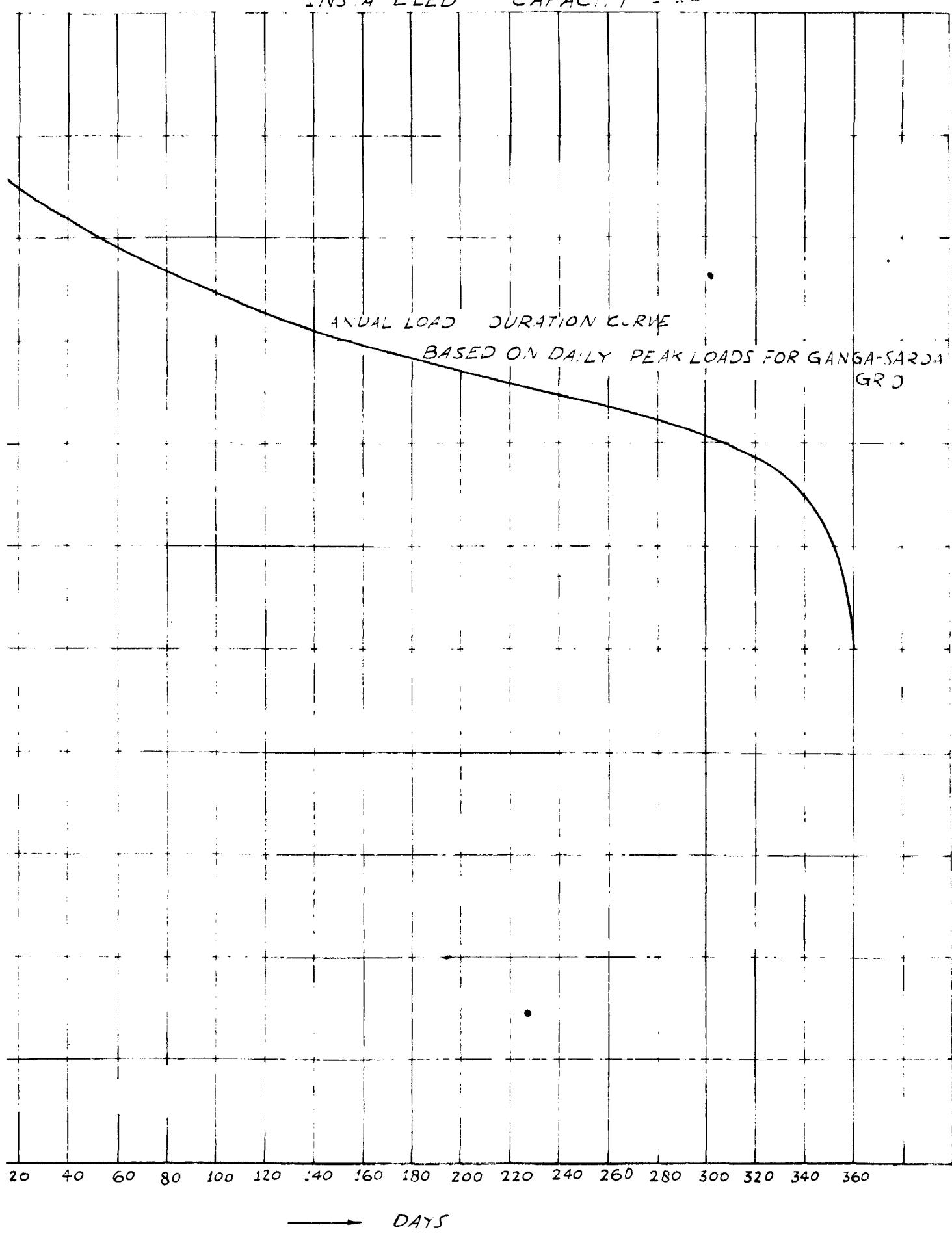
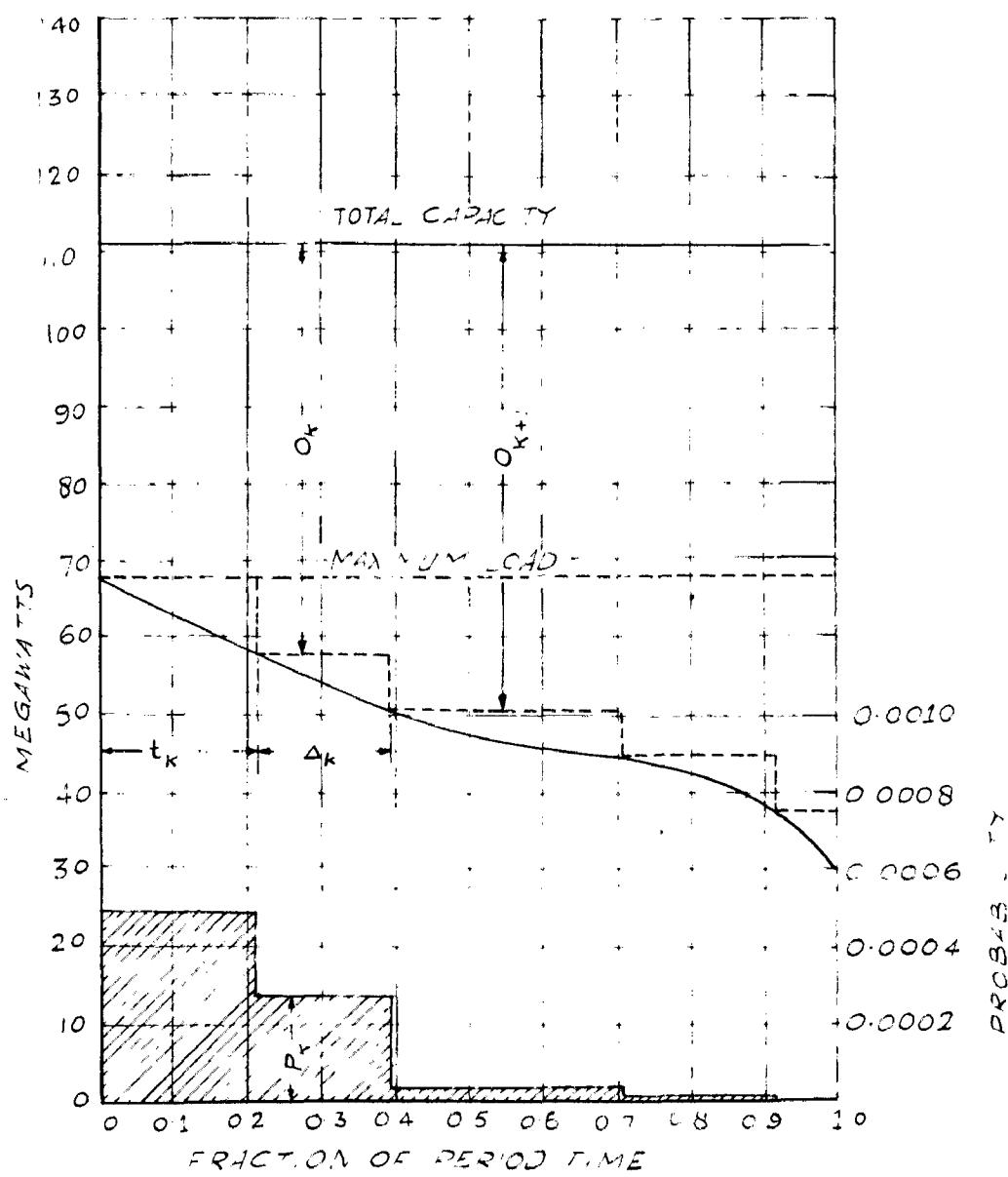


FIG. 2.1



CALCULATION OF THE PROBABILITY OF LOSS OF LOAD

F.G (22)

investigation a detailed tabular form has been presented, which illustrates the calculation of the Index of reliability. A representative annual load duration curve used for the calculation is shown in Fig. (2.1). The ordinates of the curve represent the daily peak loads and fluctuations in the load from hour to hour have not been taken into consideration. To account for the above mentioned factor load duration curves can be drawn for smaller intervals of time where the ordinates can represent the hourly peak load; or the average daily load factor can be included in the analytical expressions which are being discussed in a subsequent section.

2-6. Expected Duration of Loss of Load:

In the previous discussion we have been considering the probability of loss of energy, under the title "Loss of load probability". This approach seems to be quite rational in nature though slightly cumbersome in application. The topic has been discussed by Calabrese⁽³⁾ in his parent article itself and has evolved two different methods to determine loss of load probability. The procedure adopted can be summarised as given below:

METHOD - 1: Referring to Fig. (2.2) $0_1, 0_2, 0_3 \dots 0_k, \dots 0_{k+1}$, represent the loss in capacity values obtainable with the various unit combinations and it can be plotted down from the total capacity lines by the horizontal dashed lines. Δ_k represents the time interval between the intercepts on the load duration curve of the two successive capacity values 0_k and 0_{k+1} . For loads within the interval Δ_k loss of load occurs if the capacity outage exceeds 0_k . Let P_k be the outage probability of outages in excess of 0_k . The product $P_k \Delta_k$ is the probability of loss of load, during the whole period contributed by capacity outages exceeding 0_k . In other words

$P_k \Delta_k$ represents the total expected fraction of the whole period during which loss of load will occur, due to capacity outages in excess of O_k . In the same manner probabilities $P_1 \Delta_1, P_2 \Delta_2, \dots$ of loss of load due, respectively, to capacity outages exceeding O_1, O_2, \dots may be calculated.

$P_e = P_1 \Delta_1 + P_2 \Delta_2 + \dots + P_k \Delta_k + \dots$ gives the total probability of loss of load due to all capacity outages during the whole period. In the case of a steam plant with large number of units the probability P_k varies continuously and a continuous curve can be obtained. The expression for P_e becomes $\int_0^1 P_k dt$.

METHOD - 2.

P_e may be calculated noting that with the capacity outage O_k , loss of load occurs during the time t_k when the load exceeds the available remaining capacity. If p_k is the probability of losing exactly the capacity O_k the product $p_k t_k$ gives the probability of loss of load contributed by the capacity O_k outage. By considering all capacity values O_1, O_2, \dots, O_k a second expression can be written for the total probability of loss of load as follows:

$$P_e = p_1 t_1 + p_2 t_2 + p_k t_k + \dots$$

The numerical calculations for the method discussed above are given in Table (2.6 & 2.7). The associated graph is shown Fig. (2.2). The probability of loss of load with a reserve of 44 MW is found to be 0.000162. For various reserve values loss of load probability can be determined in a similar manner.

Table 2-5

Description of the System:-

The Fig. (2.1) represents the annual load duration curve, based on daily peak loads, for the Western U.P. Power System, for the year of 1965. The installed capacity of the system is 224 MW with a peak annual load of 200 MW. The probability of various capacity outages has been manipulated from the table the author has presented in Chapter(6). A detailed discussion of the significance of such a table and the method of framing is dealt in the Chapter on "Computer Approach". The table given below illustrates the mechanics of determination of the index of reliability of the system. Probability of outage of individual Unit = 0.02.

No.	Capacity Outage in MW	Probability of Outage	Loss of Energy in KWhr.	Expected Loss of Energy KWhr.
1	0	0.8007314	0	0
2	10	0.0326829	0	0
3	12	0.03268291	0	0
4	15	0.01634146	0	0
5	20	0.033016398	0	0
6	22	0.001333896	0	0
7	24	0.000333499	0	0
8	25	0.000666998	60000	40.01988
9	27	0.000666998	228000	152.075544
10	30	0.050358362	444000	22359.112
11	32	0.001347609	936000	1261.36

Contd....

12	34	0.000013612	1692000	23.03
13	35	0.017015261	2196000	313743.56
14	37	0.000027225	3420000	93.109
15	39	0.000006806	5076000	34.547
16	40	0.002348105	6072000	14257.693
17	42	0.0020554	8352000	17166.7
18	44	0.0000137511	10992000	151.15
19	45	0.0016947197	12456000	21109.428
20	47	0.006947782	15688000	10830.20
21	49	0.0000002778	19708000	5.28
22	50	0.0023685243	20844000	49369.52
23	52	0.0000958411	24864000	2382.993
24	54	0.0000209739	29184000	612.102
25	55	0.0007217248	31620000	22820.93
26	57	0.0000691723	36768000	2543.32
27	59	0.0000070867	42348000	300.107
28	60	0.0010959212	45372000	49724.136
29	62	0.0000966745	51804000	5008.125
30	64	0.000000978	58764000	57.471
31	65	0.00106925265	62172000	6647.75
32	67	0.000029458	70032000	2063.002
33	69	0.00000070584	78408000	55.34
34	70	0.00007583944	82788000	6278.592

36	74	0.0000098649	102828000	93.213
37	75	0.00007	107268000	7508.76
38	77	0.000043643	119184000	5201.548
39	79	0.0000003	121760000	39.52
40	80	0.00006336	138396000	8768.77
41	82	0.000003095487	152316000	471.494
42	84	0.000000456443	167040000	92.957
43	85	0.000042798	174564000	7470.99
44	87	0.00000285749	189828000	542.4316
45	89	0.000000443337	205548000	91.057
46	90	0.000009459378	213516000	2019.647

Index of Reliability = 1 - $\frac{\text{Expected Energy Loss}}{\text{Total Energy Supplied.}}$

Sum of the expected energy loss = 574861 kwhr.

Total Energy Supplied = Area under the load.

Duration Curve = 274893600 kwhr.

$$1 - \frac{574861}{274893600} = 1 - 0.002091$$

$$= 0.997909$$

(1) Cap. Out	(2) t_k	(3) Interval k	(4) Probability of greater outages P_k	(5) P_{k-k}
44	0	0.018	0.000504	0.000009
45	0.018	0.192	0.000480	0.000092
52	0.210	0.185	0.000258	0.000048
59	0.395	0.318	0.000036	0.000011
66	0.713	0.003	0.000012	0.000000
67	0.716	0.196	0.000009	0.000002
74	0.912	0.088	0.000002	0.000000
81	1.000			

Probability of loss of load = 0.000162

Table 2-6.

(1) Cap. Out	(2) No. of Units Out	(3) t_k	(4) Probability of Specified outage p_k	(5) $p_k t_k$
44	2	0	0.00239	0
45	3	0.18	0.000025	0
52	3	0.210	0.000222	0.000047
59	3	0.395	0.000222	0.000088
66	3	0.713	0.000025	0.000018
67	4	0.716	0.000002	0.00001
74	4	0.912	0.000007	0.00006
81	4	1.000	0.000002	0.000002

Probability of loss of load = 0.000162

Table 2-7.

2.7 Effect of Frequency and Duration of Outages

A number of papers have set out to determine the probability of loss of load as a ratio or probable fraction of unit time during which there will be loss of load for a given set of conditions. The feeling that this ratio is not a sufficient evaluation of the reliability of the system have made investigators to consider the probable duration and interval between outages of a given magnitude as a better criterion of system reliability. A probability of 0.01 indicates an outage of 1 % of the total demand time; but it does not state for example, whether the outage exists once in 100 days with a duration of one day or once in 500 days with a duration of 5 days. Such a difference in interpretation may be of considerable importance in the evaluation of the reliability of a system as a large loss of capacity would create a more acute problem if it should last for five days than if it should last one day.

Average duration of individual outages and average interval between individual outages can be obtained by analysing the records showing the past performance of the machines. Regarding the duration of outage, statements made by various investigators vary considerably. Instances can be cited where outage duration varied from a day to a month and even more. In the system studied by Soolye the duration of outage has been suggested to be 0.02 (one week) and interval between outages 2 (average of one outage each two years). With these values, the system under his investigation gave a probability of loss of load of once in thirty years. Santor, Baldwin and Dale of Westinghouse Electric Corporation have used the respective values as 0.06 and 3. From these it has to be concluded that the duration of outage, depends to a great extent on maintenance facilities available in the power station.

The probability calculations with the introduction of the concept of frequency and duration of outage can be simplified to a great extent by treating the system as a uniform one. A uniform system is one for which all parts are equal in size, outage rate and average duration of outage. It is usually not practicable to assume that a complex system is equivalent to a system with uniform components. Equations have been developed by Adler for a system of completely diversified units of different sizes and outage rates. However even with the help of a digital computer these equations for a completely diversified system would result in excessive calculations. For practical calculations it is found convenient and sufficiently accurate to sub-divide the system into sub groups of units which could be considered reasonably uniform. After the values for each sub-group have been calculated, the results are meshed mathematically with each other to give the values for a composite system.

2.0 Quantitative Relationships for a Uniform System:

Assume a uniform system of n units of equal size of $\frac{1}{MJD}$ with a uniform outage rate of p . Then $p = \frac{t}{T}$ 2.3.

where t is the average duration of individual outage and T the average interval of the individual outages in days. The frequency of the individual event is

$$F = \frac{1}{T} = \frac{n}{t} \quad 2.4.$$

The probability of r fold simultaneous outage is expressed by the binomial formula

$$Pr = \frac{(nt)!}{r!(nt-r)!} \left(1 - \frac{t}{T}\right)^{nt-r} \left(\frac{t}{T}\right)^r \quad 2.5$$

Expectancy of occurrence of outages of the order of 'r' or more machines is given by the formula

$$E_{Pr} = \left(\frac{n}{T} \right)^r \cdot \frac{e^{nT}}{(r-1)!} \cdot \frac{-nt}{T} \quad 2.6$$

The average duration in days of r fold simultaneous outage is

$$t_r = \frac{t(1-p)}{r + p(n-2r)} \quad 2.7.$$

The average interval in days of 'r' fold simultaneous outage is

$$T_r = \frac{tr}{Pr} \quad 2.8.$$

$$\text{Average Frequency } Pr = \frac{Pr}{tr} \quad 2.9.$$

$$\text{Magnitude of } r \text{ fold outage } M_r = rN \dots \quad 2.10.$$

2.9. Echoing of Two sub-groups of a non-uniform System:

Consider a system AB composed of two uniform Sub-groups A and B. For each of the sub groups the following values are determined for all values of r, from 0 to r according to the equations in the previous section.

PA and PB = Magnitude of r-fold outage.

PA and PB = Probability of r - fold outage.

tA and tB = Average duration of r - fold outage.

Then the values for the composite system are determined from the values of the sub groups for all possible combinations of r, according to the

matrix pattern shown below

System B

System A.

	A_0	A_1	A_2	A_3	A_4
B_0	$A_0 B_0$	$A_1 B_0$	$A_2 B_0$	$A_3 B_0$	$A_4 B_0$
B_1	$A_0 B_1$	$A_1 B_1$	$A_2 B_1$	$A_3 B_1$	$A_4 B_1$
B_2	$A_0 B_2$	$A_1 B_2$	$A_2 B_2$	$A_3 B_2$	$A_4 B_2$
B_3	$A_0 B_3$	$A_1 B_3$	$A_2 B_3$	$A_3 B_3$	$A_4 B_3$
B_4	$A_0 B_4$	$A_1 B_4$	$A_2 B_4$	$A_3 B_4$	$A_4 B_4$

The composite values are as follows:

$$M_{AB} = MA + MB$$

$$P_{AB} = P_A P_B$$

$$T_{AB} = \frac{t_{AB}}{P_{AB}}$$

$$t_{AB} = \frac{t_A t_B}{t_A + t_B}$$

$$F_{AB} = \frac{1}{T_{AB}}$$

CHAPTER - III

A CRITICAL REVIEW AND ANALYSIS OF SHORT-CUT METHODS

3-1. General

An extensive use of the probability theory has been limited by the tedious and time consuming arithmetic works involved in the computations. While considering a system consisting of various capacities, as is generally the case, the applicability of the conventional methods become complicated and short-cut methods have been attempted by investigators in this field. This chapter is exclusively being devoted to make a critical review and analysis of the several methods which have been developed to obtain an approximate solution to complicated probability calculations. As the basic data is based upon experience and judgment, in many cases even the exact calculations do not involve exact data, and the short cuts most often are expected to supplement the exact calculations.

3-2. Lyman's Method:

Lyman in his paper in 1947 has developed a valuable short-cut method. The author has made use of a most powerful statistical tool namely normal distribution curve, and the set of can be utilised to calculate the probability that any number of units selected at random from the total, will have a total capacity exceeding any given value. The basic information required in this method is the probability of 'n' units remaining in forced outage out of a total of 'm' units and this can be easily obtained by referring to the table published by C.R.A.E.C. Sub-Committee in 1952.

C. Normal Distribution Curve:-

The well known formula for the binomial frequency

$$P(x) = \frac{x!}{x!(n-x)!} p^x q^{n-x} \dots \quad (3-1)$$

With the preceding formula we find that the binomial frequency function yielding the probability of obtaining 'x' heads when an unbiased coin is tossed ten times is

$$P(x) = \frac{10!}{x!(10-x)!} (\frac{1}{2})^x (\frac{1}{2})^{(10-x)} \dots \dots \quad (3-2)$$

From an analysis of the nature of the equation it can be shown that as n increases indefinitely the Binomial histogram approaches as a limit the area under a certain bell shaped curve and this approach becomes more rapid, the more nearly equal p and q are. The approaching limit is called normal frequency function and the graph of the normal frequency function is called the normal curve.

Further discussion of the theory of statistics associated with this short-cut method is omitted here and an attempt is made to explain the relevant terms and when their application occur. We shall now apply this method to a representative system.

By calculating the mean and standard deviation of the unit sizes a normal distribution curve is fitted to the actual sizes. The standard deviation σ is the r.m.s. value of the deviation of the individual points from the average or mean value of all the points. A standard table of normal distribution curve functions⁽³⁵⁾ shows the probability that any unit selected at random from a group will exceed a given value. To illustrate the method the calculations are tabulated in Table 3-2. Probability of exactly outcome of a definite

magnitude or more has been determined for five values viz. 75, 80, 85, 90, and 95 MW, and the results are tabulated for comparison with those values obtained by the exact methods.

An Analysis of the Results Obtained by Lyman's Method:-

Basically Lyman's method involves assumptions and approximations. The fact that the actual sizes do not fit exactly the normal curve is a source of error, but unless they deviate considerably from the standard curve, the final error is relatively moderate. If it is found that the unit sizes do not fit a normal curve reasonably well they should be divided into sub-groups. On examining the results in Table (3-3) it can be observed that the method is more or less a conservative one. The values of outage probability are higher in all the cases shown in the table and this has been further supported by other typical calculations the author has made. It is only advantageous to have conservative results in a short cut method with approximations, provided the margin is within reasonable limits.

Description of the System:-

<u>Capacity of Unit Size MW</u>	<u>No. of Units.</u>
10	2
12	2
15	1
20	2
30	3
35	1
<hr/>	
Total 224 MW.	

Table 3-1.

TABLE 3-2

1	2	3	4	5	6
Unit Capacity MW	No. of Units	Col. 1 X Col. 2	Deviation from the average size	Dev. ²	Col. 2 X Col. 5
10	2	20	-10.36	107.36	214.66
12	2	24	-8.36	69.89	139.78
15	1	15	-5.36	28.73	28.73
20	2	40	-0.36	0.13	0.26
30	3	90	9.64	92.93	278.79
35	1	35	14.64	214.33	214.33
Total =		224		876.55	
average =		224 - 11		79.69	
N =		20.36		$\sigma = \sqrt{79.69} = 8.9$	

No. of Units Outage	$\frac{c/n}{(p + q)^n}$	$c/n - n$	n	$\frac{\text{Col. 9}}{\text{Col. 10}}$	$\frac{\text{Col. 9}}{\text{Col. 10}}$	$\frac{\text{Col. 11}}{\text{Col. 10}}$	$\frac{\text{Col. 11}}{\text{Col. 10}}$	$\frac{\text{Col. 11}}{\text{Col. 11}}$
2	37.5	17.14	6.2342	2.723				0.00283
3	25	4.64	6.1383	0.903				0.184

$$\begin{aligned}
 & \frac{P_n}{(p + q)^n} = \frac{\text{Col. 12} X}{\text{Col. 13} X} = \frac{\text{No. of terms in Col. 14}}{\text{Col. 14}} \\
 & 0.018343 = \frac{0.00005161068}{0.000206632} = 0.00025854268 \\
 & 0.001123 = \frac{0.00005161068}{0.000206632} = 0.00025854268
 \end{aligned}$$

13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28

Probability of outage of 75 MW or more

	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
2	40	10.64	0.2042	3.12033																		
3	26.67	6.31	5.13827	1.226																		

Probability of outage of 80 MW or more

	7	8	9	10	11	12
2	42.5	22.4	6.2942	3.55883	0.0000196	
3	28.3	7.94	5.13827	1.64526	0.0606	
4	21.25	0.89	4.45	0.2	0.421	

13 14 15 = Probability of outage of 85 MW or more.

0.018343 0.000003595228

0.001123 0.00000680538 = 0.000001015028

0.000046 0.000019366

	7	8	9	10	11	12
2	45	24.64	6.2942	3.914715	0.0000462	
3	30	9.64	5.13827	1.8761178	0.031	
4	22.5	2.14	4.45	0.450698	0.317	

13 14 15= Probability of outage of 90 MW or more.

0.018343 0.000003474466

0.001123 0.0000034813 = 0.00005024

0.000046 0.000014582

7	8	9	10	11	12
2	47.5	27.14	6.2942	4.311908	0.000022
3	31.65	11.80	5.1384	2.199183	0.0139
4	23.75	3 .39	4.45	0.761787	0.224
13	14				
0.018343	0.000001504126	15 = Probability of outage of 95 MW or more			
0.0001123	0.0000156007				
0.000046	0.000010304	= 0.000027417826			

Capacity in MW or more	TABLE 3-3		
	Cumulative Probability Method (short- cut)	Cumulative Probability Method (exact)	Cumulative (exact)
75	0.00025854268	0.00026931843	0.00026931843
80	0.00014234611	0.000155966541	0.000155966541
85	0.000091015028	0.000093553567	0.000093553567
90	0.00005024	0.000043252904	0.000043252904
95	0.00027417826	0.000031157335	0.000031157335

3.3. Strandrud's Method:-

A paper presenting a simplified method of calculating probable frequency of occurrence of forced outages involving any given generator capacity or more, and probable duration of these outages has been published by Strandrud⁽³⁰⁾. Data used are the statistics of average frequency of occurrence and average duration of forced outages of individual machines. Assumption is made that infinite interconnection facilities between all the machines are available. The procedure adopted to apply this method to the system under study, in connection with this investigation, is discussed below:

Details of the capacity distribution of the system have been given in a previous section (Table 3-1). The units of sizes 10, 12, 15 and 20 MW capacities have been grouped together and treated as a uniform system of average capacity of 14.14 MW and the remaining 4 units have an average capacity of 31.25 MW. So in the simplified system we have 7 units of 14.14 MW capacity and 4 units of 31.25 MW. Average duration of each outage has been taken to be 0.02 (one week) and average interval between outages is assumed to be one year.

$$t = 0.02, \quad T = 1, \quad T - t = 0.98,$$

$$\text{Outage factor } q = \frac{t}{T} = 0.02$$

$$\text{Service factor } p = 0.98$$

$$m_1 = 14.14$$

$$m_2 = 31.25$$

$$n_1 = 11$$

$$n_1 = 7$$

$$n_2 = 4$$

An outage of r units from the total of 11 units will include r_1 units capacity m_1 and r_2 units of capacity m_2 . r_1 and r_2 can assume all possible values such that $r_1 + r_2 = r$ holds good. An illustration of the above mentioned statement is made in the first

three columns of Table 3-4. Column 4 represents the total number of units remaining in service and columns 5 and 6 represent the no. of units of sizes m_1 and m_2 such that $s_1 + s_2 = s$

The probability that the number of machines out at a given time is r_1 of size m_1 and r_2 of size m_2 can be easily determined as follows: To illustrate, the first value in column 10 is $P_{r_1 r_2} = p^n$. Using recurrence formula

$$P(r_1+1) r_2 = \frac{s_1}{(r_1 + 1)} \quad \frac{q}{p} P_{r_1 r_2}$$

$$P_{r_1}(r_2 + 1) = \frac{s_2}{(r_2 + 1)} \quad \frac{q}{p} P_{r_1 r_2}$$

$D_{r_1 r_2}$ is the amount contributed to E_m , the average number of times per year the total capacity of machines on forced outage equals or exceeds m megawatts, by outage of r_1 machines of size m_1 and r_2 machines of size m_2 and the term is calculated from the statistical formula

$$D_{r_1 r_2} = \left(\frac{r}{t} - \frac{s}{T-t} \right) P_{r_1 r_2}$$

Column 12 represents the total megawatt capacity of machines on forced outage at a given time. In Column 13 values of m has been re-arranged in order of size and column 14 represents values of $D_{r_1 r_2}$ in order corresponding to the order of Col. 13. To obtain the values of E_m in column 15, values of $D_{r_1 r_2}$ corresponding to all values of megawatts equal to or greater than m are added.

$\frac{1}{E_m}$ represents the average interval in years between outages when megawatt capacity of units of out of service increases to 'm'

megawatts or more. In column 17 $P_{r_1 r_2}$ has been written corresponding to the order of Col. 13. Col. 18 represents the probability that the total megawatt capacity of machines out at any given time is equal to or greater than m . To find each value in column 18, values of $P_{r_1 r_2}$ corresponding to all values of megawatts equal to or greater than m , are to be added. t_{m+} in the last column represents in years the length of interval, when capacity of generators on forced outage totals m or more megawatts.

$$t_{m+} = \frac{P_{m+}}{E_{m+}}$$

The various capacity outages and the corresponding cumulative probabilities are tabulated in Table 3-5. The results obtained by the short-cut method are shown in the first two columns and column 3 shows the results calculated by the exact method. Due to the introduction of the concept of average unit, the same capacity outage may not be available in both the cases and for comparison the nearest values are chosen and shown in brackets. The difference in the two sets of readings are likely due to 1) the variation in the number of units on forced outage for the same capacity outage and ii) the variation in the number of mutually exclusive ways that might have inherently occurred in both the calculations and iii) the unavoidable selection of the nearest value of capacity outage.

TABLE 3-4

1	2	3	4	5	6	7	8	9	10	
r	r ₁	r ₂	s	s ₁	s ₂	r/t	T-t	$\frac{s}{T-t}$	$\frac{r}{t} - \frac{s}{T-t}$	P _{r1 r2}
0	0	0	11	7	4	0	11.23	-11.23	0.7982	
1	1	0	10	6	4	50	10.2	39.8	0.1140	
	0	1	10	7	3	50	10.2	39.8	0.06515	
2	2	0	9	5	4	100	9.18	90.82	0.00698	
	1	1	9	6	3	100	9.18	90.82	0.0093	
	0	2	9	7	2	100	9.18	90.82	0.001992	
3	3	0	8	4	4	150	8.17	141.83	0.0002372	
	2	1	8	5	3	150	8.17	141.83	0.00057	
	1	2	8	6	2	150	8.17	141.83	0.0002842	
	0	3	8	7	1	150	8.17	141.83	0.00002715	
4	4	0	7	3	4	200	7.15	192.85	0.000004845	
	3	1	7	4	3	200	7.15	192.85	0.00001938	
	2	2	7	5	2	200	7.15	192.85	0.00001745	
	1	3	7	6	1	200	7.15	192.85	0.00000387	
	0	4	7	7	0	200	7.15	192.85	0.0000001385	

11 D_{r_1}	12 r_2	13 m	14 D_{r_13}	15 E_m
-8.96	0	0	-8.96	0
4.54	14.14	14.14	4.54	8.96
2.592	31.25	28.28	0.634	4.419134
0.634	28.28	31.25	2.592	3.785134
0.845	45.39	42.42	0.0336	1.193134
0.181	62.50	45.39	0.845	1.159534
0.0336	42.42	56.56	0.000934	0.314534
0.0807	59.53	59.53	0.0807	0.3136
0.04025	76.64	62.50	0.181	0.23296537
0.00384	93.75	73.67	0.003738	0.5196537
0.000934	56.56	76.64	0.04025	0.04822737
0.003738	73.67	90.78	0.003362	0.0079737
0.003362	90.78	93.75	0.00384	0.0046
0.000745	107.89	107.89	0.000745	0.0007717
0.0000267	125	125	0.0000267	0.0000267

16	17	18	19
$\frac{1}{E_{m+}}$	$P_{r_1 r_2}$	P_{m+}	t_{m+}
	0.7982	1	
0.1115	0.114	0.198586	0.02218
0.227	0.00698	0.084586	0.0192
0.2645	0.06515	0.077606	0.0205
0.8395	0.0002372	0.012456	0.01045
0.862	0.0093	0.0122188	0.01055
3.18	0.000004845	0.0029188	0.00925
3.19	0.00057	0.002914	0.0093
4.29	0.001992	0.002344	0.0101
19.25	0.00001938	0.0003521885	0.00678
20.78	0.0002842	0.0003328	0.00692
126.5	0.00001745	0.0000426	0.0061
217.5	0.00002715	0.0000311585	0.00677
1295	0.00000387	0.0000040085	0.0052
37500	0.0000001385	0.0000001386	0.00518

TABLE 3-5

¹ Capacity Outage m	² Cumulative Probability ¹ m +	³
0	1	1
14.14	0.198586	0.13390281 (15 MW)
28.28	0.084586	0.082210459 (27)
31.25	0.077606	0.081543461 (30)
42.42	0.012456	0.010426481 (42)
45.39	0.0122188	0.0083572872 (45)
56.56	0.0029188	0.0027607252 (57)
59.53	0.002914	0.0026915529 (59)
62.5	0.002344	0.0015885450 (62)
73.67	0.0003521885	0.00027090492 (74)
76.64	0.0003348	0.00019990997 (77)
90.78	0.0000486	0.000043252804 (90)
93.75	0.0000311585	0.000031219523 (94)
107.89	0.0000040085	0.000030534169 (107)
125	0.0000001385	0.00000027264951 (125)

3-4. A Brief Review of Adler's Method:-

In Adler's method a system is sub-divided into sub-systems in order to increase the accuracy of the results without an excessive amount of computation. Each sub-system is composed of turbines or generators that are comparable in size and have the same outage rate. For each sub-system computations are made of the capacity lost through forced outages the duration and the interval between such occurrences, for r simultaneous forced outages where r takes on successive values $0, 1, 2, \dots, r_{\max}$. The max. number r_{\max} of simultaneous forced outages for small sub-systems is usually set equal to n . The number of machines in the sub-system. In larger sub-systems r_{\max} is assigned a value large enough for the interval between such simultaneous occurrences to be several orders of magnitude greater than the desired reliability, decided upon by management.

To obtain the probability duration etc. for the entire system, it is necessary to combine the values for the various sub-systems until the system has been re-assembled. Computations of forced outages are based upon the duration and interval between such outages for the different types of equipment as obtained from the national average and records of the system with which the author is connected. The results of these computations are the probability, capacity and the standard deviation of the capacity for the various amounts of outages.

3-5. A Short-cut Method Suggested by Hatchorn:

The simplified method given by Hatchorn involves only the determination of a few constants for use in an empirical equation. It was determined that the use of this method as applied to the particular composite system for which it was derived resulted in

almost exactly the same answers for those conditions as with the more detailed methods of handling such problems applied to them.

The investigation showed that the equation

$$L \approx B T = \frac{1}{2} q T + \frac{1}{2} n \sigma^2 / T$$

which was obtained in a manner described in the Appendix of ref. no. (19) gives the basic load for the conditions under study for which the desired objective level of service reliability is realized when the n and the B factors are appropriately determined.

n = a factor that depends on the forced outage rate, the load characteristics, and the objective level of service reliability.

n = number of generating units involved.

q = forced outage rate.

σ = unit size in MW.

B = a factor that depends on the load characteristics and the objective level of service reliability.

L = Basic load for the condition under study.

T = Capacity of the total installed generating capacity in MW.

CHAPTER - IV

PROBABILITY METHOD APPLIED TO A COMBINED HYDRO AND STEAM SYSTEM

CHAPTER - IV

4-1. Introduction:-

Management of hydro-units under probability method is still more complex than that of steam units. Steam units can be assumed to have a fixed capacity which is either in or out of service, since the entire output is lost once the unit is forced out. Moreover almost all the outages can be categorized either as scheduled maintenance outages or forced outages.

On the other hand hydro-units exhibit a variable output, the output being governed by the variation in the river flow conditions and reservoir heads. The drought and flood situations dictate the availability of hydro capacity to a great extent. Moreover the basic assumption we have made in probability analysis viz. the outages of units are independent to each other, also no longer holds good. In places where tail race is utilized for further generation purposes the working in the down stream station is dependent on the operations in the upstream stations. To cite an example, during the closure periods of Ganga Canal all the hydro units connected to Ganga grid become inactive. Thus we find that if excess hydro power is involved it is necessary to evaluate the effect of variations in river flow throughout the year or from year to year. In cases where the storage is very large compare to the annual load the problem is not much involved, but in systems where scarcity of water is felt, a detailed chronological analysis of the potential use of the available water is vital. This factor is important. To assess the reliability effect of the variation in river flow, the analysis must extend over a number of years.

The "binomial" use of storage plays an important

role in determining the influence of hydro capability on the total system reliability. The draw down of storage during short periods, in excess of that normally scheduled is meant by the term 'abnormal use'. The feasibility of such an operation is influenced by the duration and frequency of forced outages about which much has been already discussed. The other pertinent factors involved in this connection are the amount of available storage and the duration and frequency of low river flows. In short the load that the hydro storage can carry depends on its capacity and the characteristics of the emergency that might bring it into service.

During low flow periods, if steam units go on forced outage, the emergency available from the use of storage in excess of normal, limits the load carrying capability of hydro-plants. On the contrary during the higher flow period the physical capacity of the hydro plant limits the use of additional storage. Limitation is also imposed due to the reduced hydro capability because of the reduction in the head available. The reduction in the load carrying capability resulting from scheduled maintenance of hydro units can be further treated as addition to the load.

4-2. Hick's Method to Incorporate Hydro-units in Probability

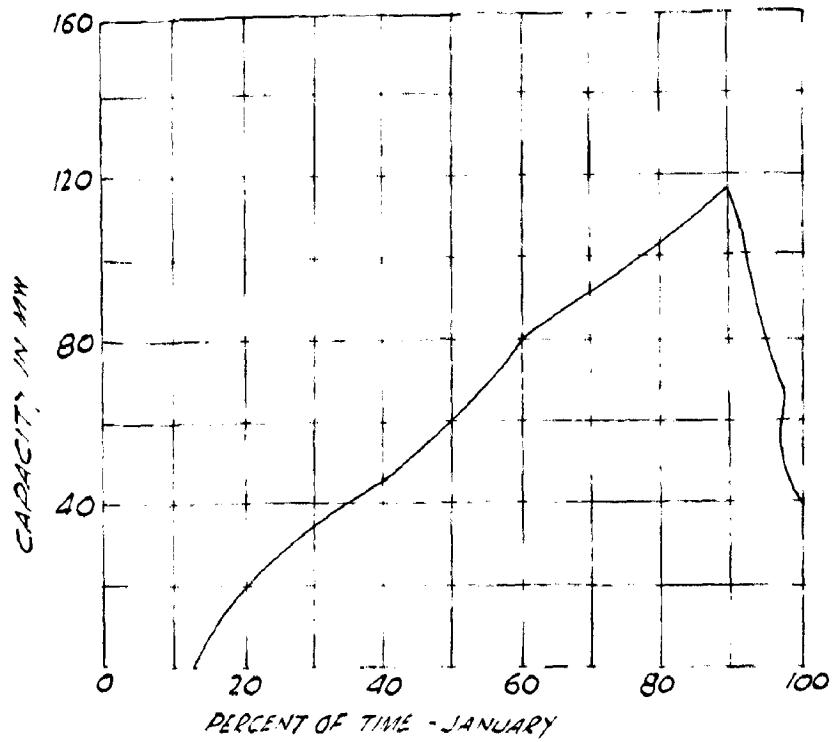
Calculations:-

Hick's (28) deals with run-of-river plants and small pondage hydro stations in systems dominated by steam stations. Hydro capability duration curves for all the 12 months, have been developed, similar to the one shown in Fig(4-1). The curves are obtained from the records of river flows for a large number of years. The maximum capability is represented by the meeting point of the two curves, drawn with two different abscissas for

each ordinate. The left hand portion shows the limitation in capability caused by the head available, while the right hand curve shows the limitation in capability by the river flow. The distance between the two curves represent the amount of time at which a given capacity is available. These curves are converted into standard capability duration curve by replotted the same data with only one abscissa for each ordinate. Fig.(4-2.) represents the standard capability duration curve drawn for the month of January.

4-3. An Example to Illustrate the Method:-

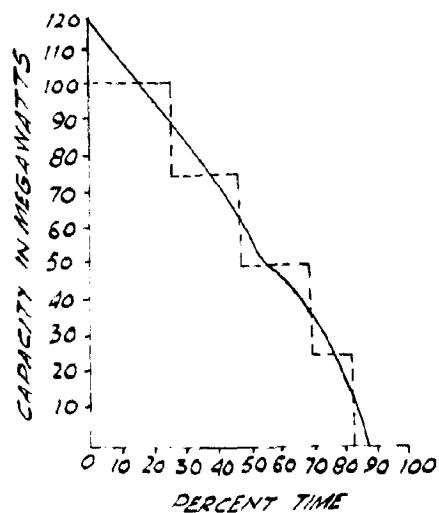
A hydro system having a capability of 100 megawatts is to be included in the probability calculations of steam units. The standard capability duration curve for the hydro units is as shown in Fig. (4-1). The variation in output is considered to occur in 25 MW increments and the same increment is used for steam plant calculation. A capability of the hydro between 0 and 12.5 is considered to be zero, from 12.5 to 37.5 is considered to be 25 MW, 37.5 to 62.5 is considered to be 50 MW and so on. Approximations are shown in dashed lines in Fig. (4-2). The probability of outage of the steam capacity determined by the conventional methods has been shown in Col. 1, of Table (4-2). Col. (2) through Col. (7) shows the calculations necessary to incorporate the hydro outage probability with the rest of the system. The system used in other calculations could not be treated with this method due to lack of availability of various data required.



HYDRO ELECTRIC STATION MONTHLY CAPABILITY

DURATION CURVE

FIG. 4-1



STANDARD CAPABILITY DURATION CURVE

FOR JANUARY

FIG. 4-2

TABLE 4-1

Probability of Capability and Outage of Hydro Plant.

	$N.C.=100$	$=75$	$=50$	$=25$	$=0$
	$\Delta RC = 90-120$	$=62.5-90$	$=37.5-62.5$	$=12.5-37.5$	$=0-12.5$
	$N.O.=0$	$=20$	$=50$	$=75$	$=100$
Jan.	0.25	0.22	0.21	0.14	0.10
Feb.	0.26	0.22	0.16	0.16	0.20
March	0.16	0.17	0.17	0.20	0.30
April	0.23	0.21	0.23	0.15	0.18
May	0.46	0.21	0.21	0.08	0.04
June	0.58	0.24	0.12	0.05	0.01
July	0.62	0.24	0.11	0.03	0.00
Aug.	0.49	0.37	0.11	0.03	0.00
Sept.	0.30	0.28	0.35	0.03	0.00
Oct.	0.29	0.29	0.36	0.06	0.00
Nov.	0.50	0.21	0.18	0.07	0.04
Dec.	0.53	0.21	0.13	0.08	0.05
Annual	0.39	0.24	0.20	0.09	0.08
Average					

N.C. = Nominal Capacity

 ΔRC = Actual Range of Capability

NO = Nominal Outage.

Limitations:- The method can be applied only when the system generation consists almost entirely of steam units. If more than one hydro station is to be considered, all the hydro stations should be considered as one large station. This takes into account that "outages" are interdependent. However, proper approach depends largely upon the particular system involved.

TABLE 4-2

Column 1 X Total Probability Columns.
 $(2+3+4+5+6)$

Limitations:- The method can be applied only when the system generation consists almost entirely of steam units. If more than one hydro station is to be considered, all the hydro stations should be considered as one large station. This takes into account that "outages" are interdependent. However, proper approach depends largely upon the particular system involved.

CHAPTER - V

PROBABILITY METHOD APPLIED TO INTERCONNECTED SYSTEMS

CHAPTER - V

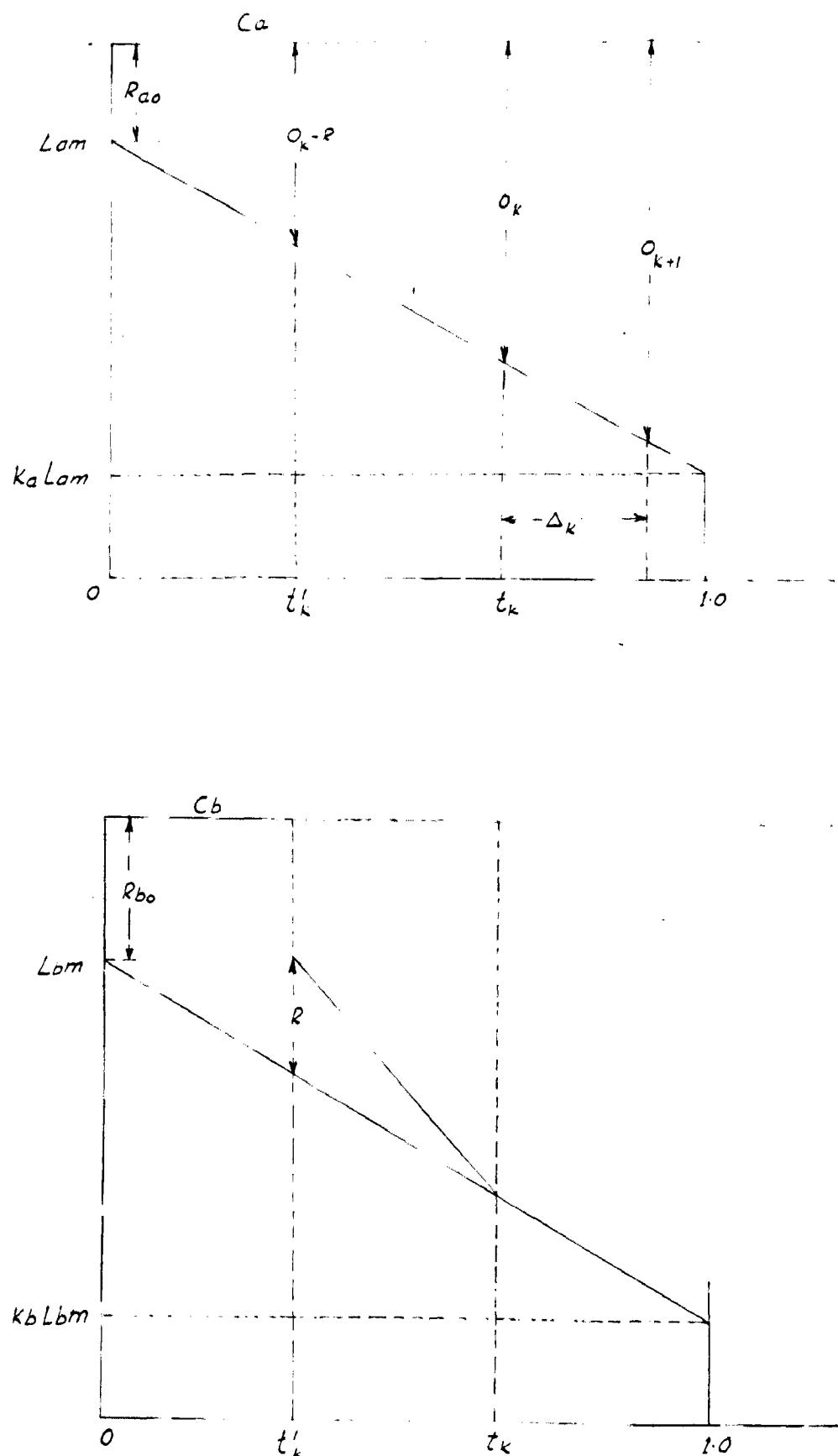
5.1. Introduction:-

More and emphasis is currently placed on achieving the benefits of interconnected operation of power systems. The extensive development of interconnections of electric power systems has been due to the following advantages.

- i) Reduction in Installed Reserve
- ii) Reduction in Spinning Reserve
- iii) Economy Loading
- iv) Staggering of Capacity Installation.

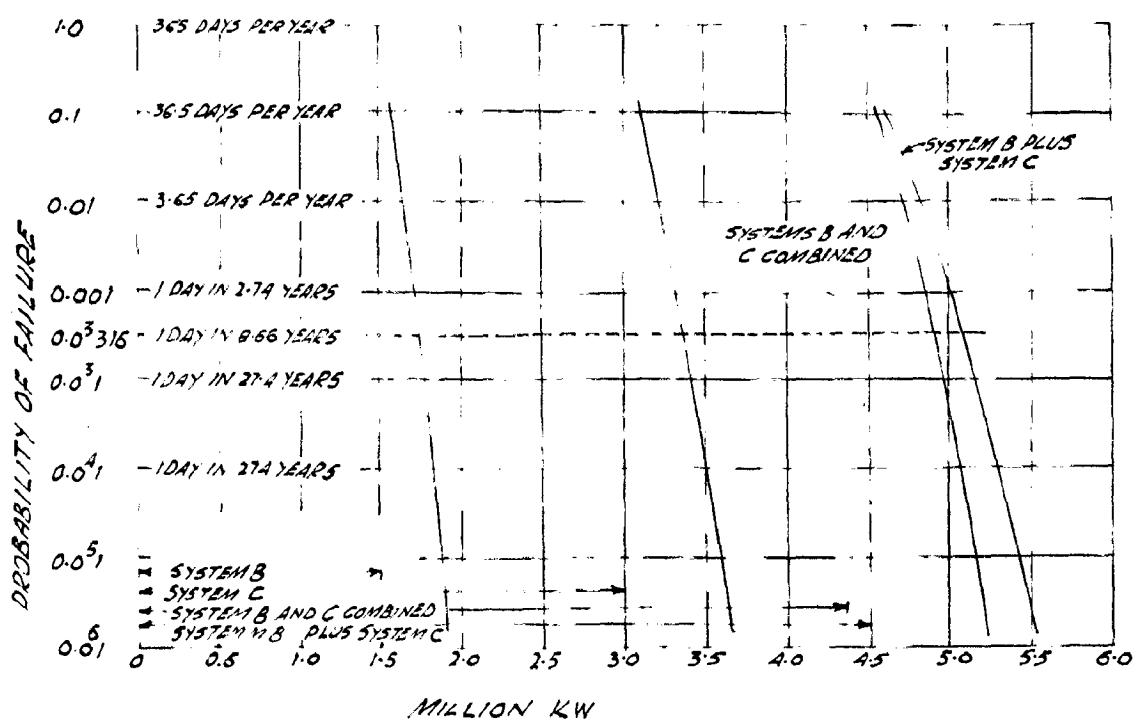
Application of probability methods in power systems planning has been primarily in the area of planning generating capacity requirements for single integrated power systems. The evaluation of loss of load probability for interconnected system is a more complex problem. The introduction of a tie line between two previously unintegrated systems to form a power pool generally has the effect of reducing the loss of load probabilities in each of the systems and reducing the total installed generating reserve capacity required to maintain a given reliability level.

Let A and B two systems, the max-loads on the two systems being represented by L_{an} and L_{bn} (Fig. 5.1). P_{oa} and P_{ob} represent the probabilities of loss of load of systems A and B respectively. Without interconnection. It is assumed that the two systems are interconnected with a tie line of capacity R. Assuming no increase in the loads L_{an} and L_{bn} the service reliabilities of both the systems will increase i.e. P_{oa} and P_{ob} will decrease. If the values of P_{oa} and P_{ob} prior to the interconnections are satisfactory the loads L_{an} and



ASSUMED STRAIGHT LINE LOAD DURATION CURVES FOR THE TWO
INTERCONNECTED SYSTEMS A AND B.

FIG. 5-1



PROBABILITY OF FAILURE TO CARRY THE LOAD VERSUS
INSTALLED CAPACITY.

FIG. 5-2

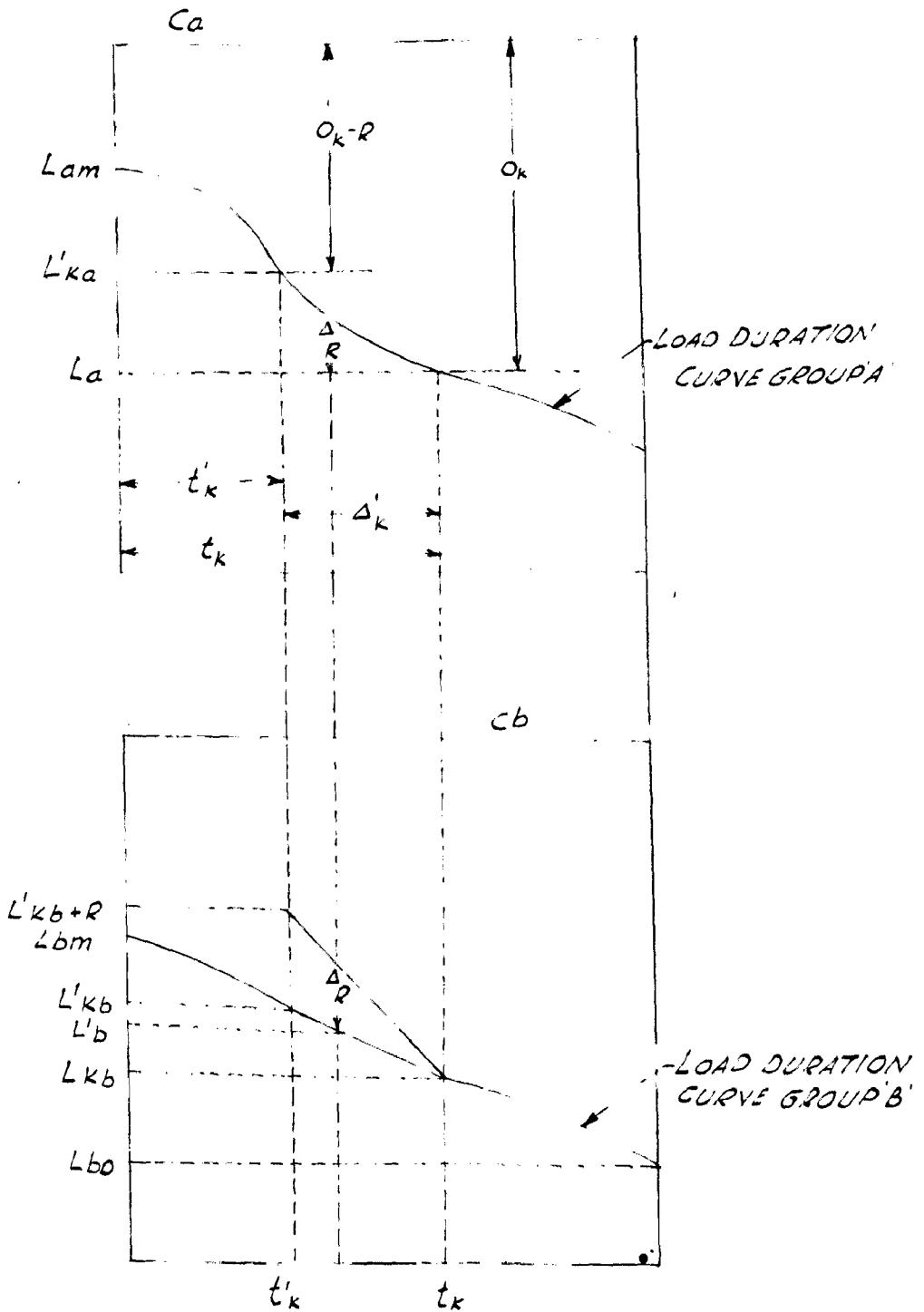
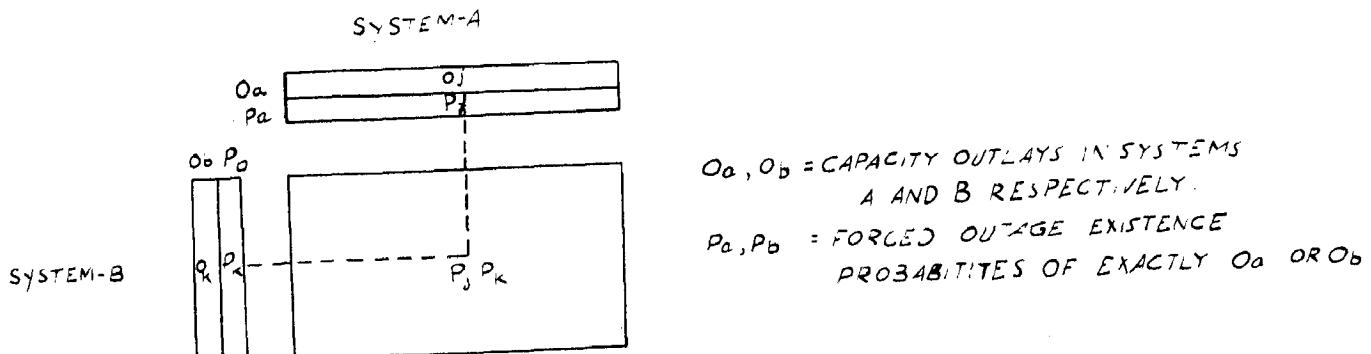
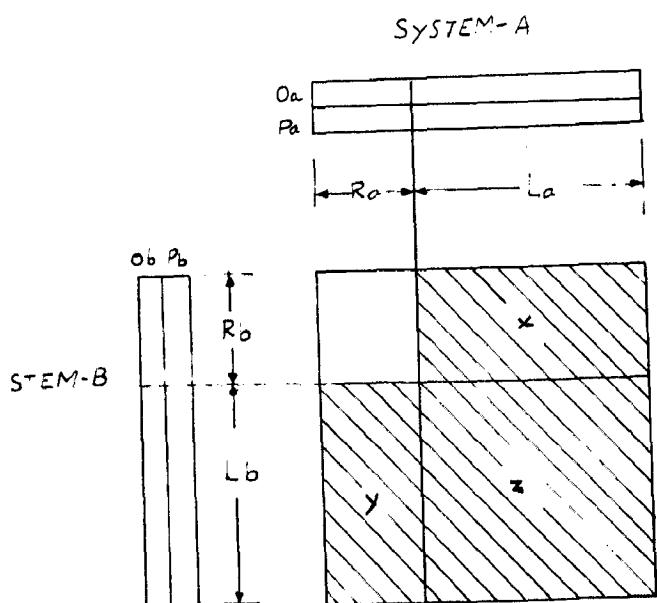


FIG. 5-3



TWO-AREA INTERCONNECTION ARRAY.

FIG. 5-4



TWO-ARRAY WITH NOTE AND SYSTEM LOADS OF L_a AND L_b MW

FIG. 5-5

L_{bm} can be increased with no addition to the installed capacity of either system. Starting with no interconnection and increasing the capacity R by small increments, these effects increase with the capacity R of the interconnection and a point of max return is reached beyond which any further increase of R will result in smaller and smaller incremental gains or even in no gains at all. From the reserve capacity stand-point there is thus a max practical limit for the capacity of interconnection.

5.2. Probability of Loss of Load with no Interconnection.

Probability of loss of load of a system A with no interconnection is given by

$$P_{ea} = \sum p_k t_k = \sum p_k \Delta_k$$

where the summations are extended to all capacity outages O_k (Fig. 5-1) in excess of reserve and

p_k = probability of a capacity outage exactly equal to O_k in system A.

P_k = Probability of a capacity outage in excess of O_k in system A

t_k = Time during which the load of system A exceeds $O_a - O_k$.

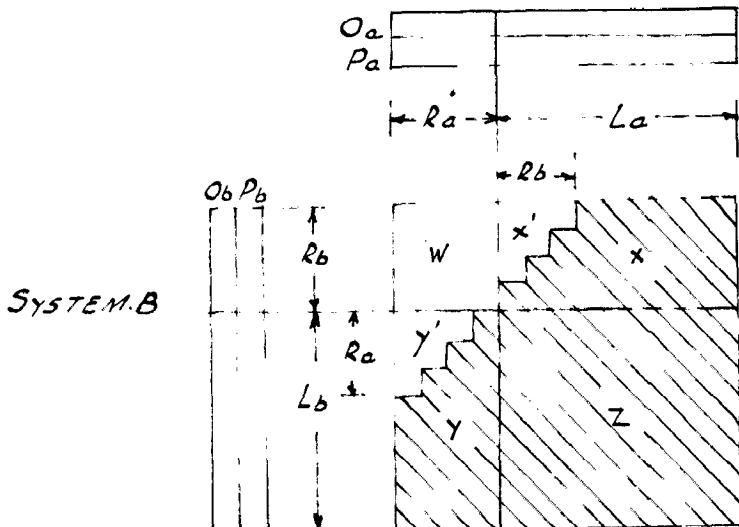
Δ_k = Time interval between two successive capacity outages
 $O_k + 1, O_k$

5.3. Effect of Interconnecting groups of units on their local reserve requirements:

As already indicated, since the two stations A and B are interconnected with a tie of capacity R the local reserve of each group can be decreased as each group can rely on the other for its reserve.

Referring to Fig. (5.3.) the probability Q_k of loss

SYSTEM-A

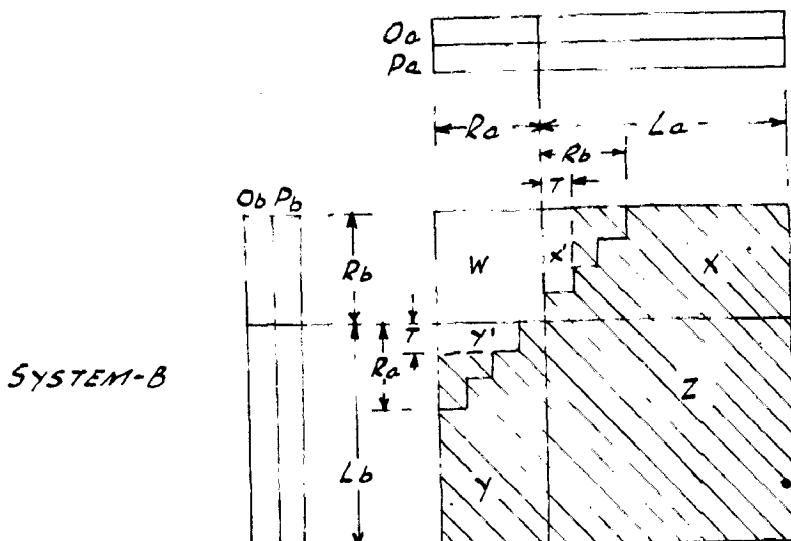


SYSTEM-B

TWO-AREA ARRAY WITH AN INFINITE TIE.

FIG. 5-7

SYSTEM-A



SYSTEM-B

TWO AREA ARRAY WITH A TIE LINE OF CAPACITY "T".

INTERCONNECTING SYSTEMS A AND B.

FIG. 5-6

of load in group A with a capacity outage O_k in the same group in (3)

$$\begin{aligned} \Sigma &= p_k \left[t'_{k'} + \int_{t'_{k'}}^{t_k} \left\{ P_b + (1-p_b) P_{E_1} \right\} dt \right] \\ &= p_k \left[t'_{k'} + \leq \left\{ P_b + (1-p_b) P_{E_1} \right\} \Delta t \right] \end{aligned}$$

The marginal capacity R_{k_0} giving the minimum value of O_k is

$$R_{k_0} = C_b - L_{k'b}$$

and

$$O_k = C_a + C_b - (L_{k'a} + L_{k'b})$$

The derivation of the above equations is not attempted here since they are lengthy and involved and can be easily obtained from the references shown.

Various phases of the application of probability methods to the problems of the type encountered here are discussed in several papers. Some of the general results obtained by an application of probability methods to the problem of determining the capacity benefits resulting from the interconnection of two systems are shown in fig.(5.2.).

5-4. Effect of Inter Connections Illustrated by Venn-diagrams.

Inherent in the outage existence probability calculation is the assumption that forced outages are independent random events. In a two area interconnected system, for any given period of time three laws of load probabilities will exist one for each of the two separate areas and for the total as a whole. If P_A is the loss of load probability in System A, P_B is the loss of load probability in system B and P_{AB} is the probability of simultaneous loss of load

on systems A and B then for a given day the loss of load probability in the total pool is

$$P_S = P_A + P_B - P_{AB} \quad (10)$$

That is the pool probability is the sum of the probabilities in each of the two areas, but since both of those probabilities contain the overlapping simultaneous probability P_{AB} , it must be taken out of the sum.

Calculation of loss of load probability is illustrated by the probability array shown in figs.

Fig(5.5) represents two area with no tie line and system loads of L_a and L_b .

Fig. (5.6) represents two area array with a tie line of capacity T.

Fig. (5.7) shows the two area array with an infinite tie.

In all the three cases it can be shown that loss of load probability of the system as a whole = loss of load probability of system A + Loss of load probability of system B - probability of simultaneous loss of load in systems A and B.

The procedures out-lined above are directly applicable to the planning of internal transmission. With some judgement the two area programme can be applied to multi-area problems. For example to evaluate the reserve benefit to system A for a pool consisting of systems A, B and C either of the following methods could be employed.

- 1) 1) Solve for reserve benefits to system A in a pool of A and B 2) Solve for reserve benefits to system A in a pool of A and C. 3) Add the

ii) Solve for reserve benefits to system A in a pool of system A and a system equal to the sum of the generation and loads of B and C.

In summary the theory and applications presented can be utilised to optimise, on an economic basis, the expansion of interconnected systems with respect to the installation of interconnecting facilities as well as generating capacity.

Another important problem in such interconnection studies is the question of the allocation of gains to the various participants in the interconnection. The value of interconnection may be greatly affected for a given company depending on how much it is required to carry on owned reserve and how much it can depend on it's neighbours. Detailed investigation of this interesting area would probably occupy as much time and space as the present work itself.

CHAPTER - VI

COMPUTER APPROACH

CHAPTER VI

6.1. General

The digital computer can be an effective tool in the calculation of the probability of simultaneous forced outages which are used in determining the generator reserve requirement of a utility system. The large amount of arithmetical work connected with an extensive study by the probability method makes the use of digital computer imperative if the results are to be obtained in reasonable length of time. Moreover the use of computer relieves the engineer of the drudgers of such a study so that he may spend more time on the technical aspect of the work.

Brennen, Gallaway and Kirchmayer⁽²⁰⁾ have discussed a method which is an example of the practical application of the digital computer for determining the system reserve requirements. The machine procedure which is followed to obtain a cumulative outage table by adding one unit to an existing system is dealt by the authors. In the present investigation the author has further developed the method formulated by Brennen so that the calculations can be done on a computer for various outage values, at a stretch. Moreover an easy to comprehend programme has been developed here, to add any number of units to an existing system, the nucleus of the work indeed being the computer flow diagram⁽²⁰⁾ published by Brennen and co-authors. The method adopted in actual computation is discussed below.

6.2. A Sample Calculation:

The existing system is assumed to have one generator of 10 MW capacity with an outage factor of 0.02. The capacity outages and corresponding cumulative probabilities can be tabulated as shown below:

<u>Capacity Outage</u>	<u>Cumulative Probability</u>
------------------------	-------------------------------

0	1.000
10	0.020

Table 6-1.

Now the object is to obtain the modified outage probability table when a new unit of 12 MW capacity with the same outage factor of 0.02 is added to the existing system. The incoming unit can set itself to two situations viz. i) The possibility for remaining in service and ii) The possibility for remaining out-of-service. In the former case the outage probability table gets changed into the one given below:

<u>Capacity Outage</u>

0 + 0

10 + 0

<u>Cumulative Probability</u>

$$1.000 \times 0.98 = 0.9800 = a^1$$

$$0.020 \times 0.98 = 0.0196 = b^1$$

Table 6-2.

And when the incoming unit is remaining on forced outage the table takes the form

<u>Capacity Outage</u>

0 + 12

10 + 12

<u>Cumulative Probability</u>

$$1.000 \times 0.02 = 0.02 = a^{11}$$

$$0.02 \times 0.02 = 0.0004 = b^{11}$$

Table 6-3.

Now the consolidated table can be made out from the above two tables and is shown below:

<u>Capacity Outage</u>	<u>Cumulative Probability</u>
0	$1.0000 = a' + a''$
10	$0.0396 = b' + b''$
12	$0.0200 = c''$
22	$0.0004 = d''$

Table 6-4.

A third unit of capacity 15 MW of outage probability 0.02 can be added in the same fashion as indicated below:

<u>Capacity Outage</u>	<u>Cumulative Probability</u>
0 + 0	$1.000 \times 0.98 = 0.9800 a''$
10 + 0	$0.0396 \times 0.98 = 0.038808 b'$
12 + 0	$0.0200 \times 0.98 = 0.0196 c'$
22 + 0	$0.0004 \times 0.98 = 0.000392 d'$

Table 6-5.

<u>Capacity Outage</u>	<u>Cumulative Probability</u>
0 + 15	$1.000 \times 0.02 = 0.02 a''$
10 + 15	$0.0396 \times 0.02 = 0.000792 b''$
12 + 15	$0.02 \times 0.02 = 0.0004 c''$
22 + 15	$0.0004 \times 0.02 = 0.000008 d''$

Table 6-6.

Consolidated Table:

<u>Capacity Outage</u>	<u>Cumulative Probability</u>	
0	1.000	= a' + a"
10	0.058808	= a' + c"
12	0.0396	= c' + a"
15	0.020392	= a" + d'
22	0.001184	= d' + b"
25	0.000792	= b"
27	0.0004	= c"
37	0.000008	= d"

Following the pattern set up above any number of units can be added and as the number of units increase the possible combinations of capacities or forced outage increase by a large proportion as is already obvious from the preceding tables. The tables presented above can now be re-arranged from the computer point of view.

Jud	A ₀	= 0	
	P ₀	= 1.00	.
	A ₁	= 10	
	P ₁	= 0.02	
	A ₂	= 0	
	P ₂	= 0	

A_n ($n = 0, 1, 2 \dots$) indicates the capacity outage being considered and P_n shows the probability that exists of an outage equal to or greater than the magnitude A_n . The addition of fictitious quantities $A_2 = 0$, and $P_2 = 0$ are essential to make the programme stop.

Re-arranged Tables

Existing System		Resultant System with Unit Added			
Index No.	Column Entry	Index No.	i Column	Index No.	j Column
0	$A_0 = 0$ $P_0 = 1.00$	i = 0	$A_0 + B_0 = 0$ $P_0 P_0 = 0.98$	j=0	$A_0 + B_1 = 12$ $P_1 P_0 =$
1	$A_1 = 10$ $P_1 = 0.02$	i = 1	$A_1 + B_0 = 10$ $P_0 P_1 = 0.0196$	j=1	$A_1 + B_1 = 22$ $P_1 P_1 = 0.0004$
2	$A_2 = 0$ $P_2 = 0$	i = 2	$A_2 + B_0 = 0$ $P_0 P_2 = 0$	j=2	$A_2 + B_1 = 12$ $P_1 P_2 = 0$

Unit Added

$$B_0 = 0$$

$$B_1 = 12$$

$$P_0 = 0.98$$

$$P_1 = 0.02$$

C. P.R. FLOW DIAGRAM

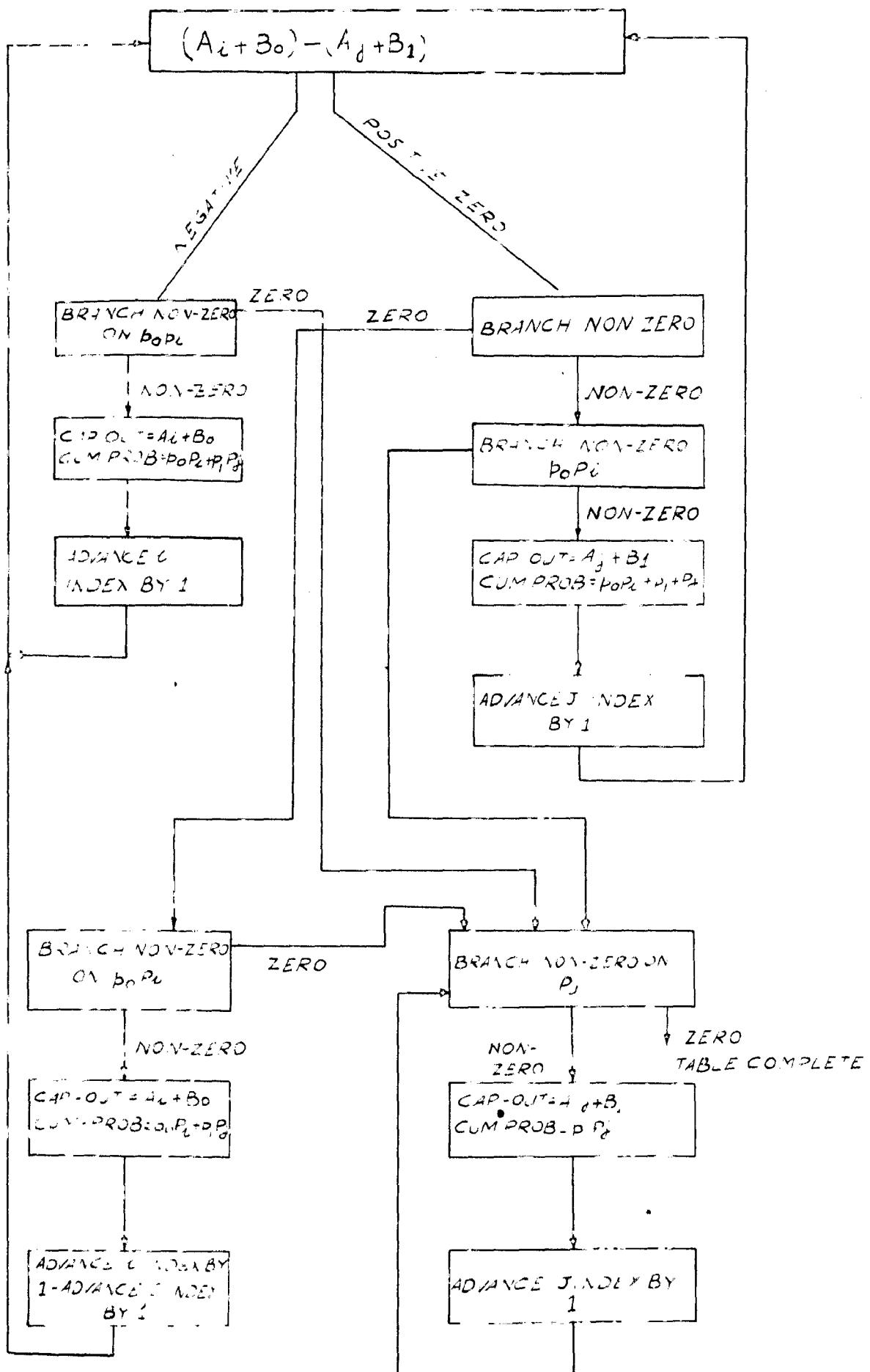


FIG (61)

The programme has been made for I.B.M. 1620. The capacities of the various units available have been tabulated in Table 3-1. The calculations are repeated for outage factors of 0.02, 0.03, 0.04, and 0.05. The programme has been made absolutely free from any variable data and can be used without any difficulty for any system, with proper dimensions. Only the data cards need be changed.

6.3. Programme for I.B.M. 1620.

CC PROBABILITY METHOD CHANDRASEKHARAN

DIMENSION A(600), AA(600), P(600), PP(600),
CO(600), CP(600)

DIMENSION B1(20), B(20), BB(20), Q(20), QQ(20)

READ 40, N, KA, LA

40 FORMAT(312)

B0 = 0.

L = 1

AKA = KA

DO 30I = 1, N

30 READ 51, B(I), BB(I), Q(I), QQ(I)

51 FORMAT (4F 10.0)

READ 52, (B1 (K), K = 1, KA)

52 FORMAT (7F 10.0)

DO 20IA = 1, LA

READ 53, FO, PT

```

53      FORMAT (2F 10.0)
54      FORMAT (2 I11 = I2)
      DO 50I = 1, N
      A(I) = B(I)
      AA(I)=BB(I)
      R(I) = Q(I)

50      IF(P(I)= Q(I))
      DO 13K = 1, KA
      AK = K
      I = 1
      J = 1
      M = 1

6       IF(A(I) + B0 - AA(J) - B1(K)) 1,2,3
      1       IF (PO* P(I)) 4,5,4
      4       CO(M) = A(I) + B0
              CP(M) = PO* P(I) + I1* PP(J)
              IF (AK = AKA) 71, 70, 70
      70      FUMCH 61, L, K, M, CO(M), CP(M)
              .
61      FORMAT (2 I11 = I2, 5 X 2 HK = I2, 5 X 2 HM = I3,
              5 X 3 HCO = E 15.8, 5 X 3 HCP = E 15.8)
              GO TO 71

71      M = K + 1
              I = I + 1
              GO TO 6

```

5 IF(PI(J)) 8,8,7

7 CO(M) = AA(J) + B1(K)

CP(M) = P0* P(I)

IF(AK - AKA) 72, 73, 73

73 PUNCH 61, L, K, M, CO(M), CP(M)

GO TO 72

72 M = M + 1

J = J + 1

GO TO 8

8 IF(P0* P(I)) 9,5,9

9 CO(M) = AA(J) + B1(K)

CP(M) = P0* P(I) + P1* PP(J)

IF (AK - AKA) 74,75,75

75 PUNCH 61, L,K,M, CO(M), CP(M)

GO TO 74

74 M = M+1

J = J + 1

GO TO 6

2 IF (P0 * P(I)) 10,5,10

10 CO(M) = A(I) + B0

CP(M) = P0* P(I) + P1* PP(J)

IF (AK - AKA) 76,77,77

77 PUNCH 61,L,K,M, CC(M), CP(M)
 GO TO 76

76 M = M + 1
 I = I + 1
 J = J + 1
 GO TO 6

8 CO(M) = 0
 CI(M) = 0
 DO 11 I = 1,M
 A(I) = CO(I)

11 I(I) = CP(I)
 DO 12 J = 1,M
 AA(J) = CO(J)

12 RP(J) = CR(J)

13 CONTINUE

20 L = L + 1
 STOP
 END

The compiled programme was run on FORTAN and the whole computation took nearly fifteen minutes. Compilation may become essential in problems of this kind since the dimensions used are likely to be high. The results are tabulated in Table 6-9.

Since cumulative outage table desired is obtained by successive modification of an original cumulative outage table the method eliminates extensive sorting and adding operations and is suited for I.B.M. 1620. Since the programme has been made extremely compact punching and card handling are reduced to a great extent. At any point in the system expansion of a table of capacity outage probability can be punched out if desired.

Table 6-9.

<u>L= 1 K= 10</u>	<u>M= 1</u>	<u>Capacity Outage</u>	<u>Cumulative Probability</u>
	M= 2	.10000000E+02	.19926862E+00
	M= 3	.12000000E+02	.16658572E+00
	M= 4	.15000000E+02	.13390281E+00
	M= 5	.20000000E+02	.11756135E+00
	M= 6	.22000000E+02	.84544952E-01
	M= 7	.24000000E+02	.63210956E-01
	M= 8	.25000000E+02	.82877457E-01
	M= 9	.27000000E+02	.82210459E-01
	M=10	.30000000E+02	.81543461E-01
	M=11	.32000000E+02	.31185099E-01
	M=12	.34000000E+02	.29837490E-01
	M=13	.35000000E+02	.29823878E-01
	M=14	.37000000E+02	.12808617E-01
	M=15	.39000000E+02	.12781392E-01
	M=16	.40000000E+02	.12774586E-01
	M=17	.42000000E+02	.10426481E-01
	M=18	.44000000E+02	.83710383E-02
	M=19	.45000000E+02	.83572872E-02
	M=20	.47000000E+02	.66625675E-02
	M=21	.49000000E+02	.59680671E-02
	M=22	.50000000E+02	.59677893E-02
	M=23	.52000000E+02	.35992650E-02
	M=24	.54000000E+02	.35034239E-02
	M=25	.55000000E+02	.34824500E-02
	M=26	.57000000E+02	.27607252E-02
	M=27	.59000000E+02	.26915529E-02
	M=28	.60000000E+02	.26844662E-02
	M=29	.62000000E+02	.15885450E-02
	M=30	.64000000E+02	.14918705E-02
	M=31	.65000000E+02	.14908925E-02
	M=32	.67000000E+02	.42163985E-03
	M=33	.69000000E+02	.39218169E-03
	M=34	.70000000E+02	.31147585E-03
	M=35	.72000000E+02	.31563641E-03

M=36	.74000000E+02	.27090492E-03
M=37	.75000000E+02	.26991843E-03
M=38	.77000000E+02	.19990997E-03
M=39	.79000000E+02	.15626700E-03
M=40	.80000000E+02	.15596641E-03
M=41	.82000000E+02	.92905797E-04
M=42	.84000000E+02	.89810310E-04
M=43	.85000000E+02	.89353867E-04
M=44	.87000000E+02	.46555632E-02
M=45	.89000000E+02	.43698141E-04
M=46	.90000000E+02	.43252804E-04
M=47	.92000000E+02	.33793426E-04
M=48	.94000000E+02	.31219523E-04
M=49	.95000000E+02	.31187935E-04
M=50	.97000000E+02	.82410842E-05
M=51	.99000000E+02	.64942177E-05
M=52	.10000000E+03	.64650595E-05
M=53	.10200000E+03	.49059770E-05
M=54	.10400000E+03	.45198799E-05
M=55	.10500000E+03	.44936155E-05
M=56	.10700000E+03	.30534169E-05
M=57	.10900000E+03	.21168003E-05
M=58	.11000000E+03	.20989851E-05
M=59	.11200000E+03	.13505677E-05
M=60	.11400000E+03	.12869318E-05
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M=62	.11700000E+03	.40955902E-06
M=63	.11900000E+03	.35077539E-06
M=64	.12000000E+03	.34121820E-06
M=65	.12200000E+03	.30384651E-06
M=66	.12400000E+03	.27329885E-06
M=67	.12500000E+03	.27264951E-06
M=68	.12700000E+03	.93485190E-07
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M=71	.13200000E+03	.36755796E-07
M=72	.13400000E+03	.35230420E-07

M=73	.13500000E+03	.34918710E-07
M=74	.13700000E+03	.20163182E-07
M=75	.13900000E+03	.12850353E-07
M=76	.14000000E+03	.12486575E-07
M=77	.14200000E+03	.88384246E-08
M=78	.14400000E+03	.80025378E-08
M=79	.14500000E+03	.79869727E-08
M=80	.14700000E+03	.18524131E-08
M=81	.14900000E+03	.12501468E-08
M=82	.15000000E+03	.11755261E-08
M=83	.15200000E+03	.87800711E-09
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M=85	.15500000E+03	.72057356E-09
M=86	.15700000E+03	.48326670E-09
M=87	.15900000E+03	.23287652E-09
M=88	.16000000E+03	.22673095E-09
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M=92	.16700000E+03	.28865336E-10
M=93	.16900000E+03	.19179343E-10
M=94	.17000000E+03	.16624340E-10
M=95	.17200000E+03	.11829532E-10
M=96	.17400000E+03	.67696850E-11
M=97	.17500000E+03	.66457703E-11
M=98	.17700000E+03	.42845084E-11
M=99	.17900000E+03	.18258846E-11
M=100	.18000000E+03	.17270478E-11
M=101	.18200000E+03	.49822777E-12
M=102	.18400000E+03	.30252133E-12
M=103	.18500000E+03	.25089022E-12
M=104	.18700000E+03	.22679571E-12
M=105	.18900000E+03	.13041764E-12
M=106	.19000000E+03	.10532964E-12
M=107	.19200000E+03	.57140630E-13
M=108	.19400000E+03	.69847040E-14
M=109	.19700000E+03	.49876992E-14
M=110	.19900000E+03	.40042496E-14

K=111	.20000000E+03	.30208000E-14
M=112	.20200000E+03	.25290752E-14
M=113	.20400000E+03	.56217600E-15
M=114	.20900000E+03	.50380800E-15
M=115	.21200000E+03	.40345600E-15
M=116	.21400000E+03	.20275200E-16
M=117	.22400000E+03	.20480000E-18
L= 2 K= 10	M= 1 .00000000E-99	.10000000E+01
	M= 2 .10000000E+02	.27732432E+00
	M= 33 .12000000E+02	.24022505E+00
	M= 4 .15000000E+02	.19552347E+00
	M= 5 .20000000E+02	.17317267E+00
	M= 6 .22000000E+02	.12801495E+00
	M= 7 .24000000E+02	.12572016E+00
	M= 8 .25000000E+02	.12502890E+00
	M= 9 .27000000E+02	.12388150E+00
	M= 10 .30000000E+02	.12249898E+00
	M= 11 .32000000E+02	.53151804E-01
	M= 12 .34000000E+02	.50358543E-01
	M= 13 .35000000E+02	.50323058E-01
	M= 14 .37000000E+02	.26575633E-01
	M= 15 .39000000E+02	.26504660E-01
	M= 16 .40000000E+02	.26483281E-01
	M= 17 .42000000E+02	.22321605E-01
	M= 18 .44000000E+02	.18032090E-01
	M= 19 .45000000E+02	.17988895E-01
	M= 20 .47000000E+02	.14696736E-01
	M= 21 .49000000E+02	.13227824E-01
	M= 22 .50000000E+02	.13226726E-01
	M= 23 .52000000E+02	.83100874E-02
	M= 24 .54000000E+02	.80526643E-02
	M= 25 .55000000E+02	.79863314E-02
	M= 26 .57000000E+02	.64609893E-02
	M= 27 .59000000E+02	.62573507E-02
	M= 28 .60000000E+02	.62346355E-02
	M= 29 .62000000E+02	.39120084E-02

:= 30	.64000000E+02	.36076364E-02
:= 31	.65000000E+02	.36099056E-02
:= 32	.67000000E+02	.33204650E-02
:= 33	.69000000E+02	.32341167E-02
:= 34	.70000000E+02	.32309356E-02
:= 35	.72000000E+02	.30145556E-02
:= 36	.74000000E+02	.27088601E-03
:= 37	.75000000E+02	.266618509E-03
:= 38	.77000000E+02	.266673715E-03
:= 39	.79000000E+02	.252590831E-03
:= 40	.80000000E+02	.252452920E-03
:= 41	.82000000E+02	.232531972E-03
:= 42	.84000000E+02	.211193353E-03
:= 43	.85000000E+02	.20971187E-03
:= 44	.87000000E+02	.17367287E-03
:= 45	.89000000E+02	.16133589E-03
:= 46	.90100000E+02	.159159361E-03
:= 47	.92000000E+02	.127175741E-03
:= 48	.94000000E+02	.11485149E-03
:= 49	.95000000E+02	.11464449E-03
:= 50	.97000000E+02	.397936054E-04
:= 51	.99000000E+02	.313833251E-04
:= 52	.10000000E+03	.311930461E-04
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:= 61	.11500000E+03	.673543091E-05
:= 62	.11700000E+03	.232603974E-05
:= 63	.11900000E+03	.214309791E-05
:= 64	.12100000E+03	.207153061E-05
:= 65	.12200000E+03	.193667451E-05
:= 66	.12300000E+03	.151241051E-05
:= 67	.12500000E+03	.150552711E-05

M= 68	.12700000E+03	.69493203E-06
M= 69	.12900000E+03	.43468103E-06
M= 70	.13000000E+03	.42872826E-06
M= 71	.13200000E+03	.28034287E-06
M= 72	.13400000E+03	.26581755E-06
M= 73	.13500000E+03	.26234968E-06
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M= 75	.13900000E+03	.10608803E-06
M= 76	.14000000E+03	.10206353E-06
M= 77	.14200000E+03	.74226213E-07
M= 78	.14400000E+03	.65047735E-07
M= 79	.14500000E+03	.64823117E-07
M= 80	.14700000E+03	.19698260E-07
M= 81	.14900000E+03	.13116645E-07
M= 82	.15000000E+03	.12645632E-07
M= 83	.15200000E+03	.96805111E-08
M= 84	.15400000E+03	.78861553E-08
M= 85	.15500000E+03	.77442200E-08
M= 86	.15700000E+03	.55733976E-08
M= 87	.15900000E+03	.27821693E-08
M= 88	.16000000E+03	.26865774E-08
M= 89	.16200000E+03	.13113227E-08
M= 90	.16400000E+03	.11234311E-08
M= 91	.16500000E+03	.10968038E-08
M= 92	.16700000E+03	.43811872E-09
M= 93	.16900000E+03	.30384104E-09
M= 94	.17000000E+03	.26067773E-09
M= 95	.17200000E+03	.19454308E-09
M= 96	.17400000E+03	.10947577E-09
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M= 98	.17700000E+03	.74101710E-10
M= 99	.17900000E+03	.33358299E-10
M=100	.18000000E+03	.31281840E-10
M=101	.18200000E+03	.10922481E-10
M=102	.18400000E+03	.68316774E-11
M=103	.18500000E+03	.55162039E-11
M=104	.18700000E+03	.51170011E-11
M=105	.18900000E+03	.31086387E-11

M=106	.19000000E+03	.24785859E-11
M=107	.19200000E+03	.14744047E-11
M=108	.19400000E+03	.21506300E-12
M=109	.19700000E+03	.15180315E-12
M=110	.19900000E+03	.12711017E-12
M=111	.20000000E+03	.96053029E-13
M=112	.20200000E+03	.83706550E-13
M=113	.20400000E+03	.21592250E-13
M=114	.20900000E+03	.21178908E-14
M=115	.21200000E+03	.17360406E-14
M=116	.21400000E+03	.97234020E-15
M=117	.22400000E+03	.11809800E-16
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	M= 2	.10000000E+02
	M= 3	.12000000E+02
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	M= 5	.20000000E+02
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	M= 7	.24000000E+02
	M= 8	.25000000E+02
	M= 9	.27000000E+02
	M= 10	.30000000E+02
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	M= 12	.34000000E+02
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	M= 14	.37000000E+02
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	M= 19	.45000000E+02
	M= 20	.47000000E+02
	M= 21	.49000000E+02
	M= 22	.50000000E+02
	M= 23	.52000000E+02
	M= 24	.54000000E+02
	M= 25	.55000000E+02

M= 64	.12000000E+03	.75816739E-05
M= 65	.12200000E+03	.67116649E-05
M= 66	.12400000E+03	.57985171E-05
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M= 75	.13900000E+03	.48104699E-06
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M= 77	.14200000E+03	.34150448E-06
M= 78	.14400000E+03	.29180807E-06
M= 79	.14500000E+03	.29029763E-06
M= 80	.14700000E+03	.10619303E-06
M= 81	.14900000E+03	.73879636E-07
M= 82	.15000000E+03	.68878321E-07
M= 83	.15200000E+03	.53084010E-07
M= 84	.15400000E+03	.43285041E-07
M= 85	.15500000E+03	.42249700E-07
M= 86	.15700000E+03	.31670874E-07
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M= 91	.16500000E+03	.66121186E-08
M= 92	.16700000E+03	.30607179E-08
M= 93	.16900000E+03	.21791491E-08
M= 94	.17000000E+03	.18595230E-08
M= 95	.17200000E+03	.14275568E-08
M= 96	.17400000E+03	.80062582E-09
M= 97	.17500000E+03	.77320516E-09
M= 98	.17700000E+03	.56157070E-09
M= 99	.17900000E+03	.26562062E-09
M=100	.18000000E+03	.24725461E-09
M=101	.18200000E+03	.99400382E-10
M=102	.18400000E+03	.63403190E-10

I= 26	.57000000E+02	.11867363E-01
I= 27	.59000000E+02	.11432445E-01
I= 28	.60000000E+02	.11381347E-01
I= 29	.62000000E+02	.74954634E-02
I= 30	.64000000E+02	.68240101E-02
I= 31	.65000000E+02	.68131892E-02
I= 32	.67000000E+02	.29907470E-02
I= 33	.69000000E+02	.27786587E-02
I= 34	.70000000E+02	.27695978E-02
I= 35	.72000000E+02	.23165652E-02
I= 36	.74000000E+02	.19927414E-02
I= 37	.75000000E+02	.19787529E-02
I= 38	.77000000E+02	.15600664E-02
I= 39	.79000000E+02	.12415297E-02
I= 40	.80000000E+02	.12371112E-02
I= 41	.82000000E+02	.79542054E-03
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I= 43	.85000000E+02	.75092149E-03
I= 44	.87000000E+02	.44741691E-03
I= 45	.89000000E+02	.41252636E-03
I= 46	.90000000E+02	.40589019E-03
I= 47	.92000000E+02	.33026452E-03
I= 48	.94000000E+02	.29345695E-03
I= 49	.93000000E+02	.29267044E-03
I= 50	.97000000E+02	.12141559E-03
I= 51	.99000000E+02	.96123548E-04
I= 52	.10000000E+03	.95384608E-04
I= 53	.10200000E+03	.74314886E-04
I= 54	.10400000E+03	.68012246E-04
I= 55	.10500000E+03	.67245422E-04
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I= 57	.10900000E+03	.35365265E-04
I= 58	.11000000E+03	.34830348E-04
I= 59	.11200000E+03	.23880575E-04
I= 60	.11400000E+03	.22123719E-04
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I= 62	.11700000E+03	.93464013E-05
I= 63	.11900000E+03	.78789916E-05

M=103	.18500000E+03	.50342128E-10
M=104	.18700000E+03	.47443025E-10
M=105	.18900000E+03	.29806817E-10
M=106	.19000000E+03	.23641191E-10
M=107	.19200000E+03	.14823089E-10
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M=111	.20000000E+03	.11429477E-11
M=112	.20200000E+03	.10221517E-11
M=113	.20400000E+03	.28730980E-12
M=114	.20900000E+03	.30618418E-13
M=115	.21200000E+03	.25585254E-13
M=116	.21400000E+03	.15518924E-13
M=117	.22400000E+03	.20971520E-15
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M= 2	.10000000E+02	.41323777E+00
M= 3	.12000000E+02	.37038081E+00
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M= 7	.24000000E+02	.21082817E+00
M= 8	.25000000E+02	.20920280E+00
M= 9	.27000000E+02	.20694717E+00
M= 10	.30000000E+02	.20369641E+00
M= 11	.32000000E+02	.19653852E+00
M= 12	.34000000E+02	.99970665E-01
M= 13	.35000000E+02	.99851947E-01
M= 14	.37000000E+02	.65685793E-01
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M= 16	.40000000E+02	.65362811E-01
M= 17	.42000000E+02	.56904203E-01
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M= 25	.55000000E+02	.22776923E-01
M= 26	.57000000E+02	.19047802E-01
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M= 34	.70000000E+02	.51732064E-02
M= 35	.72000000E+02	.43771020E-02
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M= 37	.75000000E+02	.37440429E-02
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M= 52	.10000000E+03	.22669590E-03
M= 53	.10200000E+03	.17876985E-03
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M= 55	.10500000E+03	.16108947E-03
M= 56	.10700000E+03	.12179593E-03
M= 57	.10900000E+03	.87848844E-04
M= 58	.11000000E+03	.86304335E-04
M= 59	.11200000E+03	.60735714E-04
M= 60	.11400000E+03	.55690866E-04

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M= 62	.11700000E+03	.25938734E-04
M= 63	.11900000E+03	.21802570E-04
M= 64	.12000000E+03	.20909226E-04
M= 65	.12200000E+03	.18510505E-04
M= 66	.12400000E+03	.15819071E-04
M= 67	.12500000E+03	.15686313E-04
M= 68	.12700000E+03	.86655261E-05
M= 69	.12900000E+03	.55765114E-05
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M= 71	.13200000E+03	.37327667E-05
M= 72	.13400000E+03	.34802697E-05
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M= 75	.13900000E+03	.15662471E-05
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M= 77	.14200000E+03	.11260656E-05
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M= 82	.15000000E+03	.25744122E-06
M= 83	.15200000E+03	.20093485E-06
M= 84	.15400000E+03	.16315677E-06
M= 85	.15500000E+03	.15835097E-06
M= 86	.15700000E+03	.12205990E-06
M= 87	.15900000E+03	.64830132E-07
M= 88	.16000000E+03	.61771504E-07
M= 89	.16200000E+03	.33839836E-07
M= 90	.16400000E+03	.27891797E-07
M= 91	.16500000E+03	.26897637E-07
M= 92	.16700000E+03	.13903049E-07
M= 93	.16900000E+03	.10082936E-07
M= 94	.17000000E+03	.85768893E-08
M= 95	.17200000E+03	.67147785E-08
M= 96	.17400000E+03	.37746031E-08
M= 97	.17500000E+03	.36180759E-08
M= 98	.17700000E+03	.27071151E-08

M= 99	.17900000E+03	.13392635E-08
M=100	.18000000E+03	.12387342E-08
M=101	.18200000E+03	.55551364E-09
M=102	.18400000E+03	.35950192E-09
M=103	.18500000E+03	.28212888E-09
M= 104	.18700000E+03	.26873239E-09
M=105	.18900000E+03	.17284177E-09
M=106	.19000000E+03	.13684569E-09
M=107	.19200000E+03	.88900388E-10
M=108	.19400000E+03	.16982421E-10
M=109	.19700000E+03	.11824218E-10
M=110	.19900000E+03	.10414062E-10
M=111	.20000000E+03	.78906248E-11
M=112	.20200000E+03	.71855463E-11
M=113	.20400000E+03	.21386718E-11
M=114	.20900000E+03	.24609374E-12
M=115	.21200000E+03	.20898437E-12
M=116	.21400000E+03	.13476562E-12
M=117	.22400000E+03	.19531250E-14

CHAPTER - VIICONCLUSION

CHAPTER - VIIConclusions

The apparent superfluity of the probability method has been disproved in the preceding pages. The lack of a prediction of the problem often results either in frequent failures of power supply or in substantially high costs in the initial stage. Mathematical results following systematic methods are highly desirable for all elements of economic studies pertaining to system planning, and this may be obtained by the use of probability method. For reasons already discussed a completely satisfactory solution may not be obtained, the complexity of the problem being almost unlimited.

The application of the theory has been found well adapted in the evaluation of relative reliability of alternative plans. In the typical case of system generating capacity reserves, the problem not only concerns the risk of outage, but also the economic balance between generator reserve and tie capacity in providing against local outage concentrations. The method is of considerable value in analyzing the effectiveness of interconnections,⁽²⁴⁾ studying the effect of adding large generating units on reserve requirements⁽²⁷⁾ and miscellaneous problems of power station design and system operation.

The assumptions made in the papers regarding transmission line outages were intended to mean that loss of capacity or load resulting from transmission outages is negligible. This implies that transmission systems in those areas are somewhat over-sized when judged by the reliability standard set up for the power supply. A great deal of work needs to be done in this field. The

Data now available on transmission line outages is very limited and not sufficient to provide any conclusion on transmission line outages rates. Reliability of the probability theory is not questioned but the need is recognized for improvement in the outage factors applied to the many components of the generation and transmission systems. At most factors now used are obtained from relatively short records there is an inclination to use higher factors than may be necessary. Results probably are too conservative. This points out the need for collecting more information on generation and transmission component reliabilities.

Several methods which have been developed to obtain an approximate solution to complicated probability calculations have been discussed. Even an exact determination of the probability of a certain capacity outage is, after all, only a statement of the limits which the average of future events will approach over a long period of time. The actual expectation for any one year or even for a period of few years will vary considerably from the most probable mean value. In the light of these facts the time saving approximate solutions are highly justifiable.

That a digital computer is indispensable to take the drudgery out of the preparation of tabular information as shown in Chapter VI will be immediately recognized by anyone who has ever tried it for even a single case on one simple system.

No detailed study of probability methods has so far been done in India and mostly reserve problems are solved by rules of thumb. The application of the method may not be justifiable when the grid capacity is only at the order of a few hundreds of megawatts. But in course of time when a national grid is formed and installed capacity increases considerably, necessity for solving the system

reservoir - problems are lagging in the service reliabilities in a significant manner will be felt and the application of probability method in this connection can be anticipated. It would be ^{in the} interest of Dr. Lakdawala to familiarize himself abrout of the new developments on this concept of the problem, as a lot of active research work is going on in United States where the problem has created interest. However, the study of the probability method applied to interconnected systems is more justified and long overduo in India.

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