SINGLE-PHASE SYNCHRONOUS MACHINES

A Dissertation
submitted in partial fulfilment
of the requirements for the Degree
of

MASTER OF ENGINEERING
in
ADVANCED ELECTRICAL MACHINES

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CERTIFICATE

Certified that the dissertation entitled, "Single-phase Synchronous Machines" which is being submitted by Sri Himayat Ullah Beg, in partial fulfilment for the award of the Degree of Master of Engineering in Advanced Electrical Machines of University of Roorkee, Roorkee, is a record of candidate's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for 7 months from January 1966 to July, 1966 for preparing dissertation for Master of Engineering Degree at the University.

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C_O_N_T_E_N_T_S

S Y N O P S I S

On the basis of theory and equations developed for three-phase synchronous machine, the single-phase synchronous machine may be analysed. By taking the effect of positive and negative sequence components of fluxes currents and voltages etc., the steady-state analysis is carried out. Due to presence of negative sequence current in the armature even harmonic currents are induced in the field and in the additional rotor circuits, if any. With the help of the equivalent circuit the expression for field current is derived.

The equation of the field current in transient state is also derived and test is made to confirm the wave form obtained theoretically with that recorded experimentally.

In the steady-state and transient state analysis of the single-phase synchronous machine the equivalent three-phase constants appear always together with the negative sequence reactance. These constants are taken as single-phase constants Their theoretical and physical descriptions are given and they are named as single-phase synchronous machine constants. Methods to determine these constants are also discussed. From the experiments performed for the purpose, it is evident that these constants can be determined accurately.

LIST OF SYMBOLS

a	=	Operator, to rotate the vector by 120° $(-\frac{1}{2} + j\sqrt{3}/2)$.
a, b, c	=	Phases of three-phase machine.
e, e_d	=	Terminal voltage per phase.
e_a, e_b, e_c	=	Respective phase voltages.
e_d	=	Excitation voltage in the direct-axis
e_q	=	Excitation voltage in the quadrature-axis.
i_a, i_b, i_c	=	Phase currents in phases a, b, & c respectively
i_{a1}, i_{a2}	=	Positive and negative sequence currents of phase 'a' respectively.
$i_{as}(t)$	=	Steady-state short circuit armature current in phase a of single-phase synchronous machine at any instant t.
$i_a(t)$	=	Armature current at any instant - t.
i_d	=	Direct axis armature current.
i_{d1}	=	Positive Sequence direct-axis armature current
i_{d2}	=	Negative sequence quadrature-axis armature current
i_f	=	Field current
I_{fo}	=	No load field current
i_{fs}	=	Steady-state field current on armature short-circuit.
$i_{fs}(t)$	=	Steady-state field current on armature short-circuit at any instant t.
$i'_{fs}(t)$	=	Transient field current on sudden armature short-circuit at any instant t.
$i_f(t)$	=	Steady-state field current at any instant t.
$i'_f(t)$	=	Transient field current at any instant t.
i_q	=	Quadrature axis armature current
i_{q1}	=	Positive sequence quadrature axis armature current
i_{q2}	=	Negative sequence quadrature axis armature current
j	=	Operator, to rotate the vector by 90°.

L_{ffd}, L_{fd}	=	Field circuit inductance
L_o	=	Average self inductance.
M_{fd}	=	Mutual inductance between field and phase a.
M_o	=	Amplitude of variation of self inductance ($L_a = L_o + M_o \cos 2\theta$), M_o is not a mutual inductance.
M_s	=	Average value of mutual inductance between phases.
p	=	Laplace operator
p.u.	=	per unit
r_a	=	Armature resistance per phase.
r_f	=	Field resistance.
R_L	=	Load resistance per phase.
s	=	Slip.
t	=	Time in radians and seconds
T'_a	=	Short-circuit transient armature time constant of three-phase synchronous machine.
T'_A	=	Short-circuit transient armature time constant of single-phase synchronous machine.
T'_d	=	Short-circuit transient field time constant of three-phase synchronous machine.
T'_D	=	Short-circuit transient field time constant of single-phase synchronous machine.
T'_{do}	=	Open circuit transient field time constant of three-phase synchronous machine.
T'_{Do}	=	Open circuit transient field time constant of single-phase synchronous machine.
X_2	=	Negative sequence reactance of three-phase synchronous machine.
X_{ad}	=	Direct-axis armature reaction reactance.
X_d	=	Direct-axis synchronous reactance of three-phase synchronous machine.
X_D	=	Direct axis synchronous reactance of single-phase synchronous machine.

X_d' = Direct axis transient reactance of three-phase synchronous machine.

X_D' = Direct axis transient reactance of single-phase synchronous machine.

X_d'' = Direct axis subtransient reactance of three-phase synchronous machine.

X_D'' = Direct axis subtransient reactance of single-phase synchronous machine.

X_{fl} = Field leakage reactance.

X_{ffd} = Field self reactance.

X_l = Armature leakage reactance.

X_L = Load reactance

X_o =, Zero sequence reactance.

X_q = Quadrature axis synchronous reactance, of three-phase synchronous machine.

X_Q = Quadrature axis synchronous reactance, of single-phase synchronous machine.

Z_L = Load impedance.

δ = Load angle

ϕ = Power factor angle

ψ = Flux linkage (suffixes a, b & c denote the phases of the machine).

θ = Angle between the axis of pole and the axis of phase a.

θ_o = Angle θ when machine is at stand-still i.e. at t = 0.

INTRODUCTION

The synchronous machine generally used in power station, industrial plants and for commercial purposes, are three-phase machines. There are, however, a number of applications both as motors and generators in which single-phase power is used quite advantageously. The most important and the largest use of single-phase power is in railway electrification. Other uses of commercial frequency single-phase power from 25-60 cycles are confined to small isolated plants and for testing facilities. There are a few low frequency applications and a number of high frequency applications of single-phase power. The low frequency power use is almost confined to low frequency induction furnaces, The high frequency single-phase power is generally used for testing and in high frequency induction furnaces. Another use of single-phase power is for carrier current supply for radio and similar purposes.

The low frequency and usual commercial frequency machines are salient pole type and high frequency machines are usually cylindrical rotor type. The very high frequency machines used for carrier current applications are inductor type. In a single-phase synchronous machine the armature magnetic field pulsates at line frequency. It can be considered as comprised of two rotating components, one rotating at synchronous speed in the forward direction i.e. the positive sequence component, and the other rotating at synchronous speed in the backward direction i.e. the negative sequence component. The backward rotating component or the negative sequence component induces double frequency current in the rotor causing rotor heating which is well known limiting factor in the design of single-phase synchronous machine. To provide low resistance path to the induced currents all single-phase synchronous machines are equipped with dampers usually in the form of pole face winding for salient pole machine. In the cylindrical rotor machine, the conducting wedges are electrically connected at machine ends and function as pole face damper windings.

As the field- in a single-phase synchronous machine is pulsating, the torque is also pulsating. This phenomena impresses variable forces upon the supporting structure. In large units, these forces may be so great that spring mounting between frame and foundation may be necessary to eliminate the fatigue stresses. Since rotor inertia is large there is no variation in speed.

The starting of a single-phase synchronous motor as an induction motor is not possible. The methods normally employed to start a single-phase two-winding induction motor are also used for this machine. The capacitor start method is most common. Or otherwise the machine may be brought to synchronous speed by another motor and than power is switched to synchronous motor.

Single-phase synchronous generators are usually wound for three-phase Y-connected winding with suitable rotor construction. One of the stator phases is left idle. The idle phase may be considered as an insurance or protection against any injury to one of the active phases. But with the reliability of the present day apparatus has reduced the utility of the additional phase. So machines are manufactured with empty slots corresponding to the third phase winding, consequently there is a saving in the cost. If the armature is rotating, leaving one third slots empty, may result in mechanical unbalance. In the otherway the machine may specially be wound for single-phase operation with the windings having space angle of 90°. The performance of the machine in both the cases is quite similar.

In this dissertation, the steady-state performance of single-phase salient pole synchronous machine has been analysed An attempt has been made to define, calculate and experimentally determine the constants of a single-phase salient pole synchronous machine. The physical explainations have also been rendered to the various phenomena under steady and transient conditions of operation of the machine.

The analysis is on symmetrical component theory. The machine has been treated as three-phase synchronous machine with one winding left idle and having direct and quadrature axis symmetry. The assumptions of no saturation, no hysteresis and eddy current loss in stator, necessary for linear super imposition have been made.

CHAPTER I

STEADY STATE ANALYSIS

The machines considered are of the salient pole type having axes of symmetry as the pole centres and also midway between the poles. Current, voltage, flux and reactances can be resolved into direct and quadrature components. Further the armature winding is assumed to be sinusoidally distributed thereby fulfiling the consideration of the "ideal" synchronous machines.

It is to be understood that the magnitudes of all quantities are expressed in per unit term.

The Flux linkage equations previously developed for the three-phase machine completely define the operation of that type, and since the same equations are immediately adaptable to the single-phase synchronous machines with the phase winding having 120° space angle, the treatment here will be based on them.

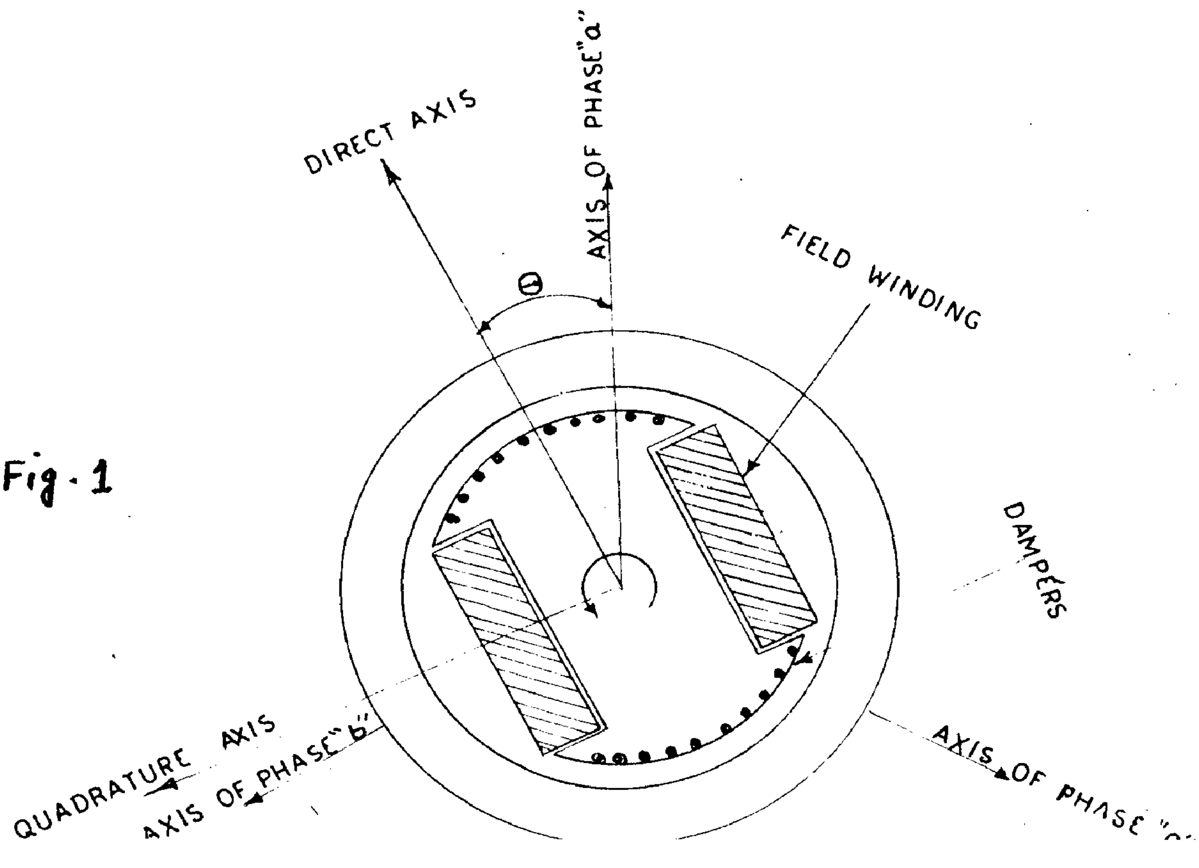


Fig. 1

A two pole synchronous machine is shown in fig.1. The per unit flux linkages of phase of an ideal three-phase synchronous machine has been given by R.H. Park[1] as

$$\Psi_a = e_d \cos \theta - e_q \sin \theta - \frac{X_0}{3} (i_a + i_b + i_c) -$$

$$\frac{X_d + X_q}{3} \left(i_a - \frac{i_b + i_c}{2} \right) - \frac{X_d - X_q}{3} \{ i_a \cos 2\theta +$$

$$i_b \cos (2\theta - 120^\circ) + i_c \cos (2\theta + 120^\circ) \} \quad \ldots (1.1)$$

where,

θ = Angle between the axis of phase winding and the pole-axis.
(θ_0 being the initial angle when the machine is at stand-still).

e_d = Direct-axis excitation voltage.

e_q = Quadrature-axis excitation voltage.

i_a, i_b, i_c = Instantaneous values of phase currents.

X_d, X_q = Synchronous reactances in direct and quadrature axis respectively of three-phase machine.

X_0 = Zero sequence reactance of three-phase machine.

The corresponding linkages of phase b and c are stated in similar manner. Only positive sequence currents are considered.

The single-phase machine due to its method of construction may be treated as a three-phase machine with one-phase left idle in operation or it is initially unwound. Hence the single-phase armature currents are-

$$i_a = i \text{ Cos } t \qquad w = 1 \text{ p.u.}$$

$$i_b = -i_a = -i \text{ Cos } t \qquad \ldots (1.2)$$

$$i_c = 0 \quad \text{(phase C considered idle)}$$

These currents can be resolved into their positive and negative sequence components, and hence become-

$$i_a = i \text{ Cos } t$$

$$= \frac{i}{\sqrt{3}} \left[\text{Cos } (t - 30) + \text{Cos } (t + 30) \right]$$

$$i_b = - i \text{ Cos } t$$

$$= \frac{i}{\sqrt{3}} \left[\text{Cos } (t - 150) + \text{Cos } (t + 150) \right] \qquad \ldots (1.3)$$

$$i_c = 0$$

$$= \frac{i}{\sqrt{3}} \left[\text{Cos } (t + 90) + \text{Cos } (t - 90) \right]$$

The first trignometric function of each of these three current expressions of equation 1.3 form the positive sequence system, the last set of these terms forming the negative sequence group. The positive sequence components of current are corporated with synchronous reactances under steady-state operation. The negative sequence components meet with the sub-transient reactances which are to be denoted by double primes. The appropriate factor is therefore to be used with each sequence current. Substituting the current relations, result in-

$$i_a + i_b + i_c = i \text{ Cos } t - i \text{ Cos } t + 0 = 0 \qquad \ldots (1.4)$$

$$i_a - \frac{i_b + i_c}{2} = \frac{3}{2} \cdot \frac{i}{\sqrt{3}} \left[\text{Cos}(t-30) + \text{Cos } (t+30) \right]. \quad (1.5)$$

The first term of equation 1.5 is positive sequence component hence it will be multiplied by synchronous reactance and second term is the negative sequence component hence, it will be multiplied by subtransient reactance. Hence-

$$\frac{X_d + X_q}{3}\left(i_a - \frac{i_b + i_c}{2}\right) = \frac{i}{\sqrt{3}} \left[\frac{X_d + X_q}{2} \text{Cos } (t-30) + \frac{X''_d + X''_q}{2} \times \text{Cos } (t+30) \right] \quad \ldots (1.6)$$

Now taking the last term of the equation 1.1 and solving similarly,

$$i_a \text{ Cos } 2\theta + i_b \text{ Cos } (2\theta - 120°) + i_c \text{ Cos } (2\theta + 120°) = \frac{3}{2} \cdot \frac{i}{\sqrt{3}} \left[\text{Cos } (t + 2\theta + 30) + \text{Cos } (t - 2\theta + 30) \right] \ldots (1.7)$$

Multiplying with appropriate factors and adding 1.6 and 1.7, it is obtained-

$$\Psi_a = e_d \text{ Cos } \theta - e_q \text{ Sin } \theta - \frac{i}{3} \left[\frac{X_d + X_q}{2} \text{ Cos } (t - 30) + \frac{X_d - X_q}{2} \text{ Cos}(t+2\theta+30) + \frac{X''_d + X''_q}{2} \text{ Cos } (t + 30) + \frac{X''_d - X''_q}{2} \text{ Cos } (t - 2\theta + 30) \right] \ldots (1.8)$$

Since at $t = 0$, $\theta = \theta_o$ (initial angle between the pole axis and phase axis when machine is at stand-still) and the speed of the backward component with reference to pole is twice the synchronous ~~machine~~ speed backwards, hence Ψ_a may be written as,

$$\Psi_a = e_d \text{ Cos } \theta - e_q \text{ Sin } \theta - \frac{i}{\sqrt{3}} \left[\frac{X_d + X_q}{2} \text{ Cos } (t-30) + \frac{X_d - X_q}{2} \text{ Cos } (t + 2\theta_o + 30) + \frac{X''_d + X''_q}{2} \text{ Cos}(t+30) + \frac{X''_d - X''_q}{2} \times \text{Cos } (3t + 2\theta_0 + 30) \right] \quad \ldots (1.9)$$

The flux linkage equations of phase b and phase c are obtained similarly.

The resultant voltage of each armature phase is given by-

$$e_a = p\psi_a - i_a r$$

$$= -e_d \sin \theta - e_q \cos \theta + \frac{1}{\sqrt{3}} \Big[\frac{X_d+X_q}{2} \sin (t-30) +$$

$$\frac{X_d - X_q}{2} \sin (t + 2\theta_o + 30) + \frac{X_d'' + X_q''}{2} \sin(t+30) +$$

$$3.\frac{X_d'' - X_q''}{2} \sin (3t + 2\theta_o + 30)\Big] - i\, r \cos t$$

$$\ldots (1.10)$$

$\left(\frac{X_d'' + X_q''}{2}\right)$ may be replaced by X_2, the negative sequence reactance of the machine, connected to a system through a large external impedance. The resistance drop for phase b becomes a positive quantity because $i_b = - i_a = i \cos t$. The resistance drop for phase c is ofcourse zero.

The line-to-line voltage under consideration is the difference between the voltage of phase a and b-

The flux linkage equation for phase b will be -

$$\psi_b = e_d \cos (\theta - 120°) - e_q \sin (\theta - 120) - \frac{X_o}{3}(i_a + i_b + i_c) -$$

$$\frac{X_d + X_q}{3} \left(i_b - \frac{i_a + i_c}{2}\right) - \frac{X_d - X_q}{3} \Big[i_b \cos 2\theta +$$

$$i_c \cos (2\theta - 120) + i_a \cos (2\theta + 120)\Big] \quad \ldots (1.11)$$

Substituting for currents and proceeding on the same lines as before, final expression for ψ_b comes as follows:

$$\Psi_b = e_d \operatorname{Cos}(\Theta - 120) - e_q \operatorname{Sin}(\Theta - 120) - \frac{1}{\sqrt{3}} \left[\frac{X_d + X_q}{2} \operatorname{Cos} x \right.$$
$$(t - 150) - \frac{X_d - X_q}{2} \operatorname{Cos}(t + 2\Theta_b - 30) + \frac{X_d'' + X_q''}{2} \operatorname{Cos} x$$
$$\left.(t + 150) - \frac{X_d'' - X_q''}{2} \operatorname{Cos}(t - 2\Theta_b + 30) \right]$$

Since $\Theta_b = \Theta_o - 120$ at $t = 0$ and considering for backward component.

$$\Psi_b = e_d \operatorname{Cos}(\Theta - 120) - e_q \operatorname{Sin}(\Theta - 120) - \frac{1}{\sqrt{3}} \left[\frac{X_d + X_q}{2} \right. x$$
$$\operatorname{Cos}(t-150) - \frac{X_d - X_q}{2} \operatorname{Cos}(t + 2t_o + 90°) + \frac{X_d'' + X_q''}{2} x$$
$$\left. \operatorname{Cos}(t+150) + \frac{X_d'' - X_q''}{2} \operatorname{Cos}(rt + 2\Theta_o + 90) \right] \quad \ldots (1.12)$$

$$\therefore e_b = p\Psi_b - i_b r$$
$$= - e_d \operatorname{Sin}(\Theta - 120) - e_q \operatorname{Cos}(\Theta - 120) + \frac{1}{\sqrt{3}} \left[\frac{X_d + X_q}{2} \right. x$$
$$\operatorname{Sin}(t-150) - \frac{X_d - X_q}{2} \operatorname{Sin}(t + 2\Theta_o + 90) + \frac{X_d'' + X_q''}{2} x$$
$$\left. \operatorname{Sin}(t + 150) - \frac{X_d'' - X_q''}{2} . 3 \operatorname{Sin}(3t + 2\Theta_o + 90) \right] - ir \operatorname{Cos} t \quad \ldots (1.13)$$

Hence the line-to-line voltage will be given by

$$e_a - e_b = - e_d \left[\operatorname{Sin}\Theta - \operatorname{Sin}(\Theta - 120) \right] - e_q \left[\operatorname{Cos}\Theta - \right.$$
$$\left. \operatorname{Cos}(\Theta - 120) \right] + \frac{1}{\sqrt{3}} \left\{ \frac{X_d + X_q}{2} \left[\operatorname{Sin}(t-30) - \right. \right.$$
$$\left. \operatorname{Sin}(t-150) \right] + \frac{X_d - X_q}{2} \left[\operatorname{Sin}(t + 2\Theta_o + 30) + \right.$$
$$\left. \operatorname{Sin}(t + 2\Theta_o + 90) \right] + \frac{X_d'' + X_q''}{2} \left[\operatorname{Sin}(t+30) - \right.$$
$$\left. \operatorname{Sin}(t+150) \right] + 3\frac{X_d'' - X_q''}{2} \left[\operatorname{Sin}(rt + 2\Theta_o + 30) + \right.$$
$$\left. \left. \operatorname{Sin}(3t + 2\Theta_o + 90) \right] \right\} - 2 ir \operatorname{Cos} t \quad \ldots (1.14)$$

Solving quantities in brackets in equation 1.14 final expression comes as-

$$e_a - e_b = -\sqrt{3}\, e_d \operatorname{Sin}(\Theta - 30) - \sqrt{3}\, e_q \operatorname{Cos}(\Theta + 30) + i \left| \frac{X_d + X_q}{2} \operatorname{Sin} t + \frac{X_d - X_q}{2} \operatorname{Sin}(t + 2\Theta_o + 60) + X_2 \operatorname{Sin} t + 3\left(\frac{X_d'' - X_q''}{2}\right) \operatorname{Sin}(3t + 2\Theta_o + 60) \right| - 2ir \operatorname{Cos} t \quad \ldots (1.15)$$

When t is measured in per unit and is in radians, it can be put as-

$$t = (\Theta - \Theta_o), \quad w = 1 \text{ p.u.}$$

Therefore,

$$e_a - e_b = -\sqrt{3}\, e_d \operatorname{Sin}(\Theta + 30) - \sqrt{3}\, e_q \operatorname{Cos}(\Theta + 30) + i \left| \frac{X_d + X_q}{2} \operatorname{Sin}(\Theta - \Theta_o) + \frac{X_d - X_q}{2} \operatorname{Sin}(\Theta + \Theta_o + 60) + X_2 \operatorname{Sin}(\Theta - \Theta_o) + 3\frac{X_d'' - X_q''}{2} \operatorname{Sin}(3t + 2\Theta_o + 60) - 2r \operatorname{Cos}(\Theta - \Theta_o) \right|$$

Expanding the above equation-

$$e_a - e_b = -\sqrt{3}\, e_d \operatorname{Sin}(\Theta + 30) - \sqrt{3}\, e_q \operatorname{Cos}(\Theta + 30) + i \left| \frac{X_d + X_q}{2} \left| \operatorname{Sin}(\Theta + 30) \operatorname{Cos}(\Theta_o + 30) - \operatorname{Cos}(\Theta + 30) \times \operatorname{Sin}(\Theta_o + 30) \right| + \frac{X_d - X_q}{2} \left| \operatorname{Sin}(\Theta + 30) \operatorname{Cos}(\Theta_o + 30) + \operatorname{Cos}(\Theta + 30) \operatorname{Sin}(\Theta_o + 30) \right| + X_2 \left| \operatorname{Sin}(\Theta + 30) \times \operatorname{Cos}(\Theta_o + 30) - \operatorname{Cos}(\Theta + 30) \operatorname{Sin}(\Theta_o + 30) \right| + 3\left(\frac{X_d'' - X_q''}{2}\right) \times \operatorname{Sin}(3t + 2\Theta_o + 60) - 2r \left| \operatorname{Cos}(\Theta + 30) \operatorname{Cos}(\Theta_o + 30) - \operatorname{Sin}(\Theta + 30) \operatorname{Sin}(\Theta_o + 30) \right| \right|$$

or

$$e_a - e_b = -\sqrt{3}\, e_d \operatorname{Sin}(\Theta + 30) - \sqrt{3}\, e_q \operatorname{Cos}(\Theta + 30) + i \left| X_d \operatorname{Sin}(\Theta + 30) \times \operatorname{Cos}(\Theta_o + 30) - X_q \operatorname{Cos}(\Theta + 30) \operatorname{Sin}(\Theta_o + 30) + X_2 \right| \operatorname{Sin}(\Theta + 30) \times$$

$$\cos(\theta_o + 30) - \cos(\theta + 30)\sin(\theta_o + 30) \Big] + 3\left(\frac{X''_d - X''_q}{2}\right)\sin(3t + 2\theta_o + 60) - 2r\Big[\cos(\theta + 30)\cos(\theta_o + 30) - \sin(\theta + 30)\sin(\theta_o + 30)\Big]\Big] \quad \ldots (1.16)$$

The armature currents have been expressed in terms of i, so far, but now it is desirable to express in terms of their direct and quadrature positive and negative sequence components. The positive sequence direct current, for a three-phase machine is given in terms of the phase currents as-

$$i_d = \frac{2}{3}\Big[i_a \cos\theta + i_b \cos(\theta - 120) + i_c \cos(\theta + 120)\Big] \quad \ldots (1.17)$$

Considering only the direct positive sequence component of single phase current-

$$i_{d1} = \frac{2}{3}\,\frac{1}{\sqrt{3}}\Big[\cos(t-30)\cos\theta + \cos(t-150)\cos(\theta - 120) + \cos(t+90)\cos(\theta + 120)\Big]$$

On solving it reduces to-

$$i_{d1} = \frac{1}{\sqrt{3}}\cos(\theta_o + 30) \quad \ldots (1.18)$$

Also, for the quadrature positive sequence component of current,

$$i_{q1} = -\frac{1}{\sqrt{3}}\sin(\theta_o + 30) \quad \ldots (1.19)$$

The negative sequence components are-

$$i_{d2} = \frac{1}{\sqrt{3}}\cos(\theta_o - 30) \quad \ldots (1.20)$$

$$i_{q2} = -\frac{1}{\sqrt{3}}\sin(\theta_o - 30) \quad \ldots (1.21)$$

Substituting equations 1.18, 1.19, & 1.20, 1.21 into equation 1.16 for components of current.

$$e_a - e_b = \sqrt{3}\ \Big| -e_d \text{ Sin } (\Theta + 30) - e_q \text{ Cos } (\Theta + 30) + i_{d1}(X_d + X_2) \times \text{Sin}(\Theta + 30) + i_{q1}(X_q + X_2) \text{ Cos } (\Theta + 30) + \sqrt{3}\, i\left(\frac{X''_d - X''_q}{2}\right) \times \text{Sin}(3t + 2\Theta_o + 60) - 2\, i_{d1} r \text{ Cos } (\Theta + 30) + 2 i_{q1} r \text{ Sin}(\Theta + 30)\Big| \quad \ldots (1.22)$$

or,

$$e_a - e_b = \sqrt{3} \cdot e \text{ Cos } (t + \phi) \quad \ldots (1.23)$$

Where ϕ is the power factor angle of the machine. Similar equations for $(e_b - e_c)$ can be written by inspection.

<u>Vector Diagram</u>:-

From equation 1.22 the vector diagram for single-phase synchronous machine can be drawn as follows, assuming δ as the angle between the excitation voltage in direct axis and the terminal voltage.

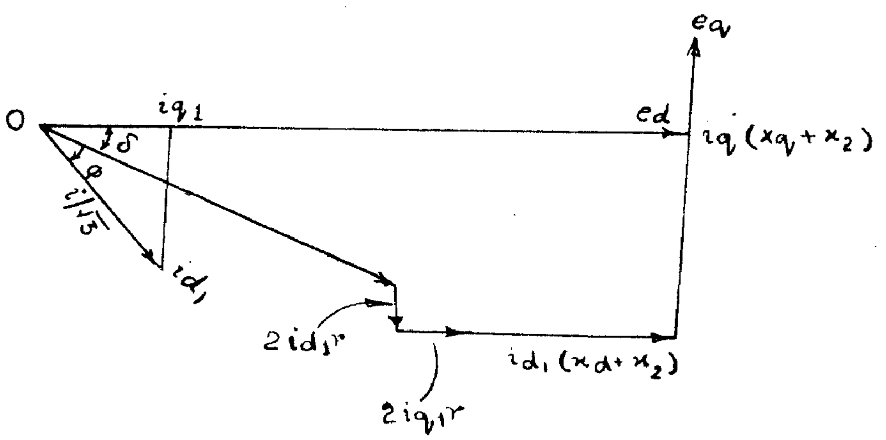


Fig.2. Vector diagram of single-phase synchronous machine.

The diagram includes armature resistance, although often it can be neglected because it is small compared to the synchronouse reactances. The third harmonic voltage of equation 1.28

is neglected in the diagram.

The vector diagram represents the generator supplying a lagging current. All quantities are in effective values and have been divided by $\sqrt{3}$, the terminal voltage e being measured from line to neutral. It should be noted that voltage drops are induced quantities, and arrows reversed in the vector diagram, will conform to the equation 1.22. When treated as the components of the terminal voltage, the arrows on the resistance and reactance drops will be as shown in Fig.2.

Excitation Diagram:-

For the purpose of calculating the excitation, the vector diagram corresponding to the cylindrical rotor machine may be used. With usual values of voltage, current, power factor and machine constants, this form of diagram gives an excitation very close to that considering the effect of salient poles.

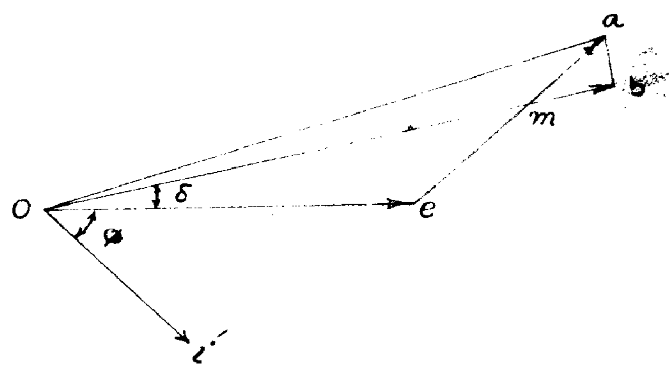


Fig.3. Excitation diagram of single-phase synchronous machine.

Fig.3 shows the excitation diagram for the single-phase machine in terms of that for an equivalent cylindrical rotor machine, the diagram being similar in steps to that for a three-phase machine. Point m divides the total synchronous reactance drop ea, in the ratio $\frac{X_q + X_2}{X_d + X_2}$. The projection of the excitation vector oa, on the line om extended gives the excitation ob for the salient pole machine.

Torque-Angle Characteristics:-

From the vector diagram of fig.2.

$$e_d = e \cos\delta + 2\, i_{q1}\, r + i_{d1}\, (X_d + X_2)$$

$$e_q = -e \sin\delta - 2\, i_{d1}\, r + i_{q1}\, (X_q + X_2) \qquad \ldots (1.24)$$

Neglecting armature resistance, these quantities may be solved simultaneously for currents which are-

$$i_{d1} = \frac{e_d - e \cos\delta}{X_d + X_2} \qquad \ldots (1.25)$$

$$i_{q1} = \frac{e_q + e \sin\delta}{X_q + X_2} \qquad \ldots (1.26)$$

and,

$$e_d \cos\delta = i_{q1}\, (X_q + X_2) - e_q \qquad \ldots (1.27)$$

$$e_d \sin\delta = e_d - i_{d1}\, (X_d + X_2) \qquad \ldots (1.28)$$

The positive sequence power is given by the expression-

$$P = e_d \cos\delta\, i_{d1} + e_q \sin\delta\, i_{q1} \qquad \ldots (1.29)$$

Substituting in equation 1.29 valuesof i_{d1}, i_{q1} in terms of terminal voltage and e_d and e_q from 1.25 and 1.26 and $e_d \cos\delta$ and $e_q \sin\delta$ from 1.27 and 1.28 and adding-

$$P = \frac{e . e_d \sin\delta}{X_d + X_2} + \frac{e\, e_q \cos\delta}{X_q + X_2} + \frac{e^2}{2} \left[\frac{X_d - X_q}{(X_d + X_2)(X_q + X_2)} \right] \sin 2\delta \qquad \ldots (1.30)$$

Putting,

$$X_d + X_2 = X_D$$

and $X_q + X_2 = X_Q$

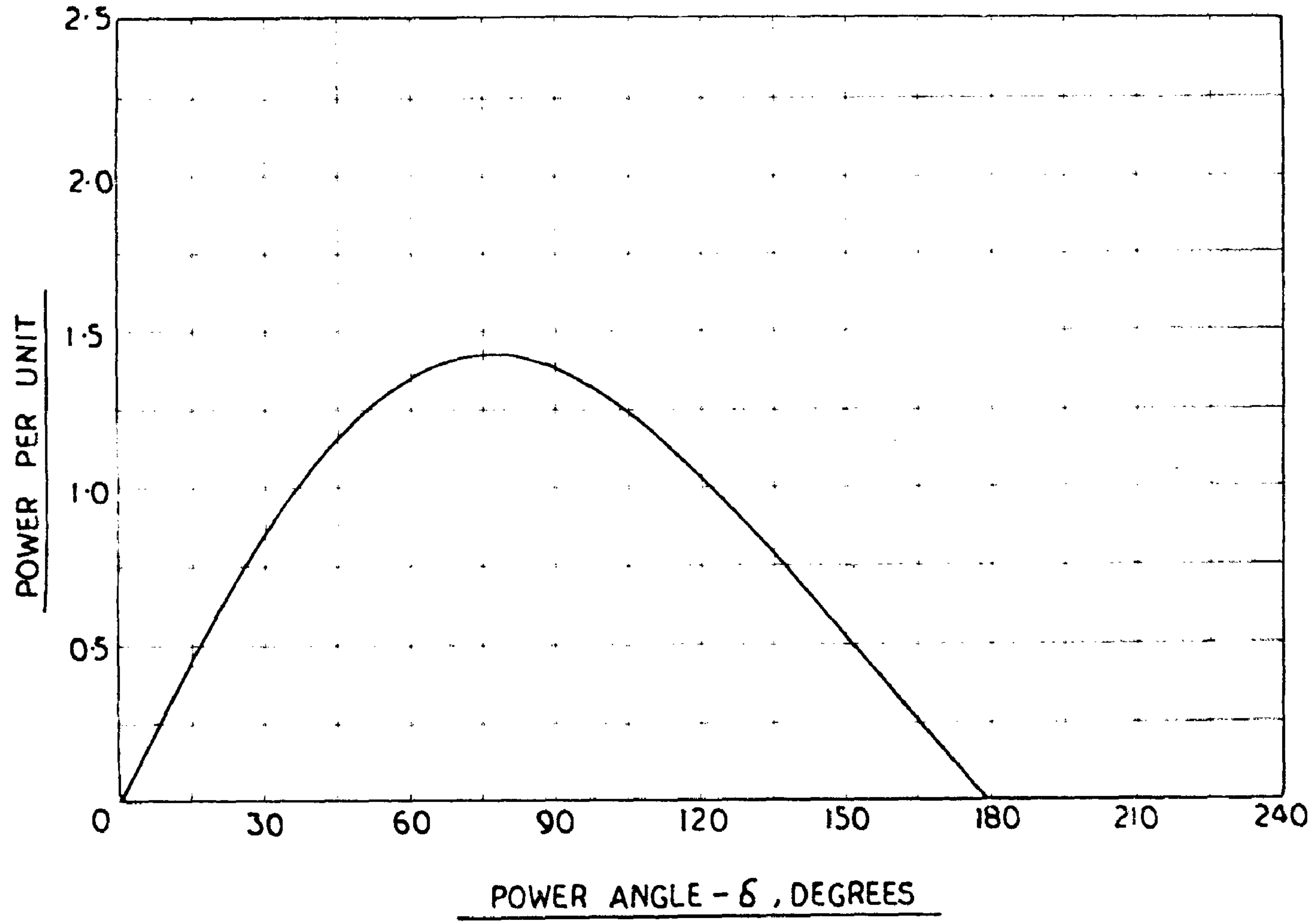

2·5
2·0
1·5
1·0
0·5
POWER PER UNIT
0
30
60
90
120
150
180
210
240
POWER ANGLE - δ , DEGREES

FIG. 4

The power equation becomes-

$$P = \frac{e.e_d \ \mathrm{Sin}\delta}{X_D} + \frac{e.e_q \ \mathrm{Cos}\delta}{X_Q} + \frac{e^2}{2} \left[\frac{X_D - X_Q}{X_D X_Q} \right] \mathrm{Sin}\ 2\delta \quad \ldots (1.31)$$

In case of the salient pole synchronous machines there is no quadrature axis excitation, hence e_q will become zero. Therefore,

$$P = \frac{e.e_d \ \mathrm{Sin}\delta}{X_D} + \frac{e^2}{2} \left[\frac{X_D - X_Q}{X_D X_Q} \right] \mathrm{Sin}\ 2\delta \quad \ldots (1.32)$$

Equation 1.32 is identical to the power equation of a three-phase synchronous machine.

Fig.4 shows power-angle curve for a typical single-phase salient pole machine. Power is expressed in per unit of three-phase ratings. The maximum power is obtained from the foregoing power equation by setting $\frac{dP}{d\delta} = 0$, solving for and then substituting it in the power expression. Fig.4 has been obtained from the equation:

$$P = 1.385 \ \mathrm{Sin} \quad + 0.1995 \ \mathrm{Sin}\ 2 \quad \ldots (1.33)$$

with $e_d = 1.44$ p.u. $e = 1.0$ p.u.

$X_D = 1.04$ p.u. and $X_Q = 0.735$ p.u.

It will be observed that the second term in the power equation is significant in magnitude and affects considerably the pull-out angle of the machine. It increases the pull-out angle by about 5% compared to three-phase machine because the percentage reluctance power is comparatively greater in case of single-phase synchronous machine. Hence single-phase synchronous machines are more stable than the three-phase machines. For this machine the pull-out angle is about 75° (Elect.). The machine on which the test was made is rated 4 KVA, 3.2 KW, 0.8 power factor, 1000 r.p.m., and $X_D = 1.04$, $X_Q = 0.735$ as will be seen latter in Chapter 3.

From the help of the vector diagram of fig.2, the load-angle δ in terms of the machine constants and load can be expressed as-

$$\delta = \tan^{-1} \frac{iX_Q . \cos \phi}{\sqrt{3}\, e + i\, X_Q \sin\phi} \qquad \ldots (1.34)$$

No considerable error is expected in the load-angle analysis if the armature resistance is neglected for large synchronous machines. However, for small size machines, resistance usually can not be disregarded.

STEADY-STATE PERFORMANCE OF SINGLE-PHASE SYNCHRONOUS MACHINE:

The vector diagram, which portrays the steady-state performance of the machine, and which in itself is a graphical solution corresponding to the equation developed previously for line-to-line voltage. In salient pole machines there is no excitation in quadrature axis hence e_q becomes zero and also neglecting the third harmonic component of voltage the line-to-line voltage equation reduces to-

$$e_a - e_b = \sqrt{3} \left[e_d \sin(\theta + 30) - i_d X_D \sin(\theta + 30) - i_q X_Q \times \cos(\theta + 30) + 2\, i_d r \cos(\theta + 30) - 2 i_q r \sin(\theta + 30) \right]$$

$$= \sqrt{3}\, e_t \cos(t + \phi) \qquad \ldots (1.35)$$

In complex notation and for the single-phase synchronous motor with lagging power factor the equation 1.35 becomes-

$$e_t = e_d + i_d X_D + j\, i_q X_Q + i\, i_q\, r - j 2 i_d r \qquad \ldots (1.36)$$

and,

$$i = i_q - j\, i_d \qquad \dots (1.37)$$

Method of Calculation:

The calculation procedure will depend upon the vector diagram shown in the Fig.3. To start with the terminal voltage e_t in p.u. is laid off as the reference vector. The line current divided by the root of 3 and expressed as per unit of rated current is drawn from the orign at proper power-factor angle. Next is laid off the resistance drop

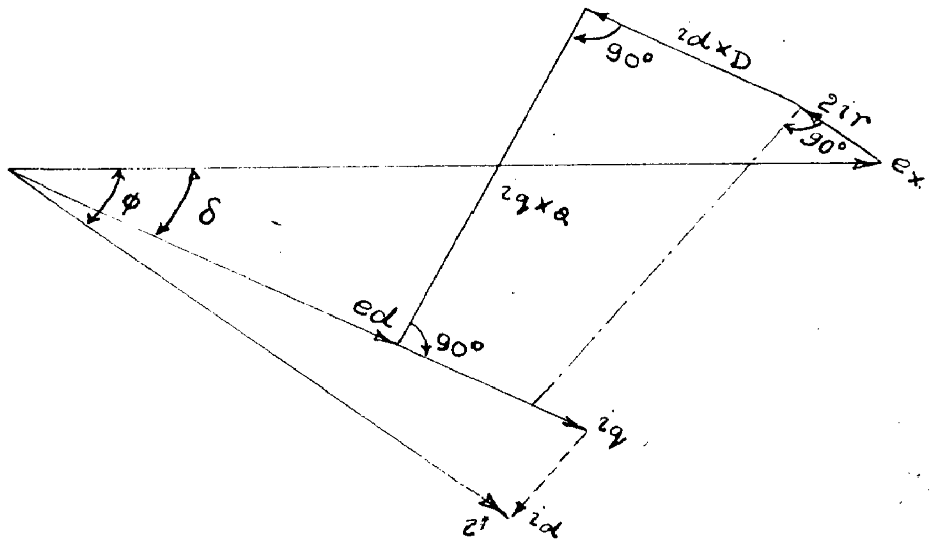


Fig.5- Vector diagram for single-phase synchronous motor.

ir parallel to vector i and starting from the tip of vector e_t. Perpendicular to ir vector is laid off the vector corresponding to $i_q\, X_Q$ which locates the direct axis. The torque angle can immediately be read as shown. The reactance drops determine the length of the excitation voltage e_d, which can be obtained from the nommal projection of tip of the vector

$i_d\ X_D$ upon the direct-axis. Knowing the field current corresponding to e_t when carried to air-gap line, the field current corresponding to the excitation voltage e_d can easily be calculated.

Although the armature resistance is usually neglected in synchronous machine studies, there are, nevertheless, many instances in which it has pronounced effect and should be taken into account. This is particularly true for small machines. Because for a single-phase synchronous machine the armature resistance appears as twice its value in the calculations. Therefore, it may be appreciable in magnitude.

The procedure described above may also be used for calculation of performance of generators. The vector diagram drawn on the same lines as in case of synchronous motor is shown in Fig.6.

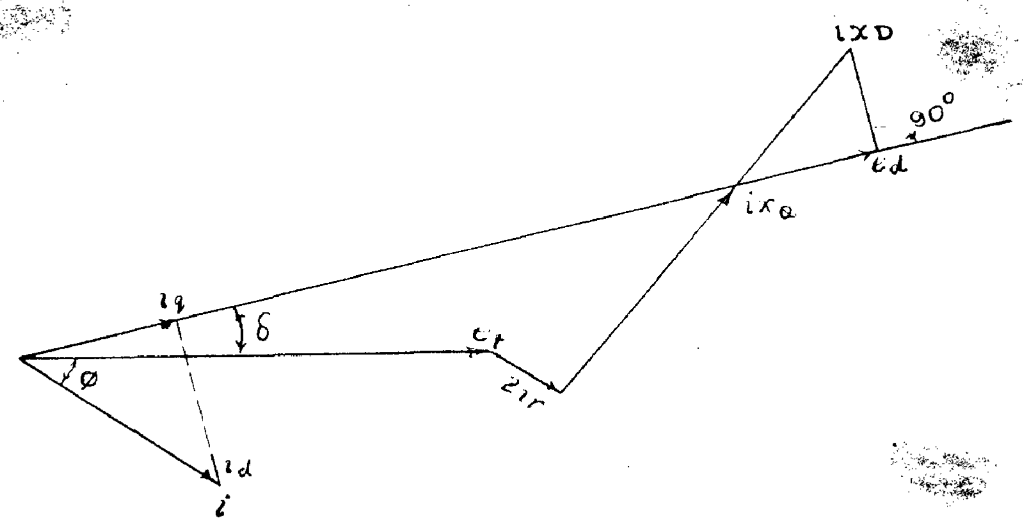


Fig.6. Vector diagram for single-phase synchronous generator.

To check the method experimental was performed on 220V, 3.2 KW generator, whose constants are-

$X_D = 1.04$ p.u., $X_Q = 0.735$ p.u., r = neglected

The observations were-

Voltage = 220 V line-to-line, 127V per phase.

Current = 9A

Power = 1.73 KW

$$\text{Power factor} = \frac{1.73 \times 1000}{9 \times 220 \times \frac{2}{3}} = \underline{0.758}$$

To compare the power angle with that obtained from vector diagram, it may either be measured directly or calculated from equation 1.34.

The vector diagram was drawn as follows:

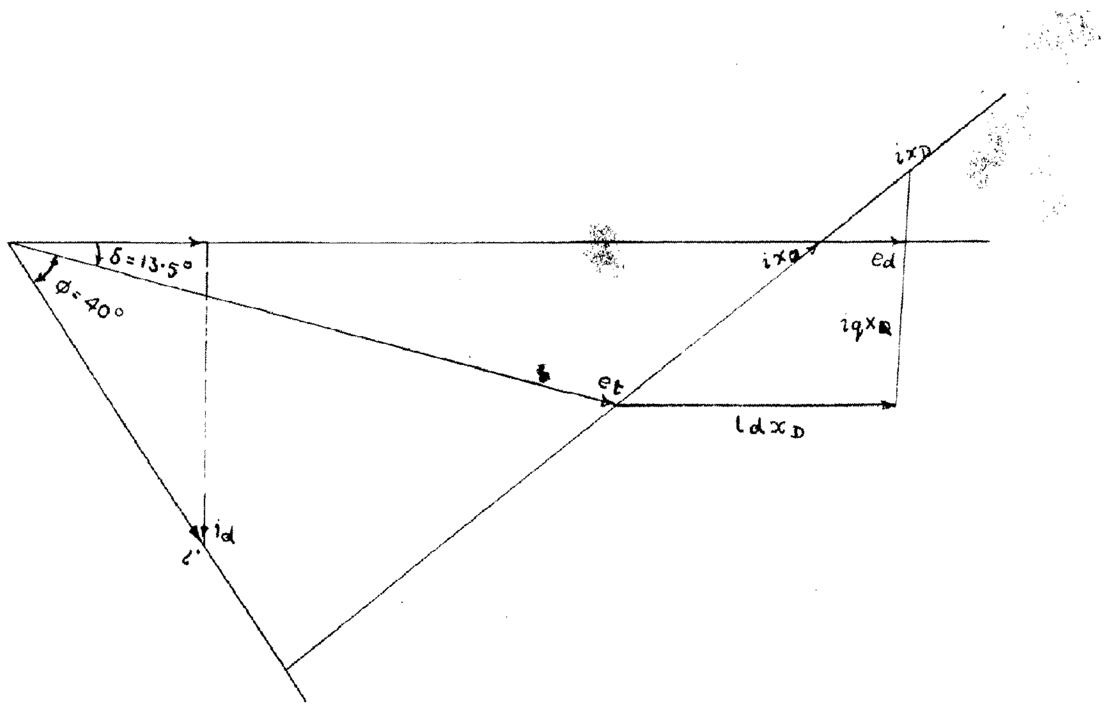


Fig.7. Vector diagram for single-phase synchronous generator

Power factor angle ϕ = 40°

$e_t = \frac{127}{127} = 1.0$ p.u.

$i = 0.546$ p.u.

$iX_D = 1.04 \times 0.546 = 0.573$ p.u.

$iX_Q = 0.73 \times 0.546 = 0.40$ p.u.

From the vector diagram of fig.7 the following results are calculated-

$\delta = 13.5°$

$e_d = 1.44$ p.u. $= 183$ V per phase.

$i_d X_D = 0.450$ p.u.

$i_q X_Q = 0.236$ p.u.

also $\tan \delta = \frac{i_q X_Q \cos \phi}{e_t + i_q X_Q \sin \phi} = \frac{0.4 \times 0.758}{1 + 0.4 \times 0.65}$

$= 0.24$

Therefore,

$\delta = 13.5°$

excitation corresponding to 127V per phase given from the air-gap line = 1.575 A

Therefore,

Excitation corresponding to $e_d = 1.44 \times 1.575$

$= 2.275$ A

The field current recorded = 2.3 A

Thus there is close agreement in the observed and calculated values.

Under the assumed conditions of no saturation, the theory and actual performance check to a very close extent. The effect of armature resistance may not always be negligible as explained earlier. It seems that the third harmonic which enters in the line-to-line voltage for the single-phase machine has no significant effect upon the steady-state performance and may be neglected with safety.

STEADY-STATE EXCITER CURRENT:

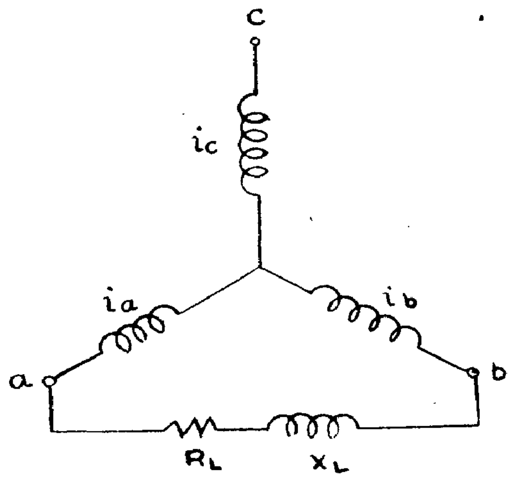


Fig.8.

From the analysis of the single-phase synchronous machine we have seen that-

positive sequence direct component of armature current,

$$i_{d1} = \frac{i}{\sqrt{3}} \text{Cos} (\theta_o + 30) \qquad \ldots (1.38)$$

negative sequence direct axis component of armature current

$$i_{d2} = \frac{i}{\sqrt{3}} \text{Cos} (\theta_o - 30) \qquad \ldots (1.39)$$

Similarly,

$$i_{q1} = \frac{-i}{\sqrt{3}} \text{Sin} (\theta_o + 30) \qquad \ldots (1.40)$$

$$i_{q2} = \frac{-i}{\sqrt{3}} \text{Sin} (\theta_o - 30) \qquad \ldots (1.41)$$

Since there is no negative sequence voltage source present in the machine but a negative sequence current as given by equation 1.39 in the direct axis, this can be shown that this current is caused by the potential difference of induced voltage e_d and drop $(Z_L + X_d)i_{c1}$ in a direction opposite to that of positive sequence current. The equivalent circuit is as shown in Fig.9.

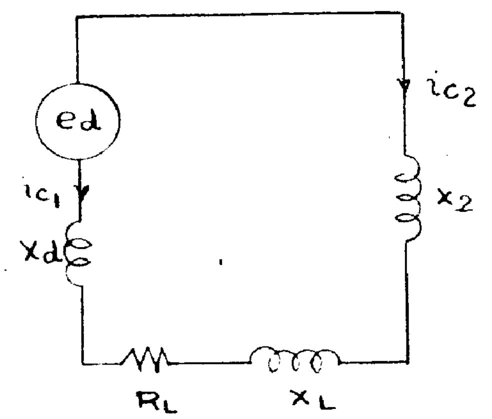


Fig.9

$$i_a = - i_b, \quad i_o = 0$$

$$\overline{i_{c1}} = j\frac{i}{\sqrt{3}}$$

$$\overline{i_{c2}} = -j\frac{i}{\sqrt{3}}$$

Therefore,

$$\overline{i_{c1}} = - \overline{i_{c2}}$$

and, $$\overline{i_{c1}} = \frac{e_d}{X_d + X_2 + Z_L}, \qquad \overline{i_{c2}} = - \frac{e_d}{X_d + X_2 + Z_L}$$

$$| i | = \frac{\sqrt{3}\, e_d}{X_d + X_2 + Z_L}$$

Therefore,

$$| i_{d2} | = \frac{e_d}{X_d + X_2 + Z_L} \qquad \dots (1.42)$$

The negative sequence current flowing in the armature has a slip equal to 2 with respect to positive sequence current since rotating in opposite direction with synchronous machine. speed. The current i_{d2} will induce additional rotor currents as with respect to rotor also it has slip 2. In such conditions the equivalent circuit becomes as follows.

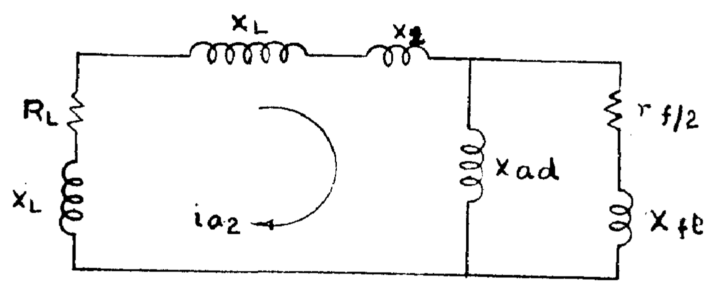


Fig.10

$$\overline{i_{d2}} = \frac{e_d}{X_d + X_2 + Z_L} = \frac{I_{fo} \cdot jX_{ad}}{j(X_d+X_2)+(R_L+jX_L)} \quad \ldots (1.43)$$

where,

I_{fo} = Steady-state field current when machine runs on no load.

The double frequency exciter current due to negative sequence field is given by-[4]

$$\overline{i_{f2}} = \frac{i_{d2} \cdot j\,X_{ad}}{jX_{ad} + \frac{r_f}{2} + jX_{fl}}$$

$$= \frac{I_{fo} \cdot jX_{ad}}{j(X_d+X_2)+(R_L+jX_L)} \times \frac{jX_{ad}}{r_f/2 + jX_{ffd}}$$

$$= \frac{I_{fo}}{(R_L+jX_2)+j(X_d+X_2)} \times \frac{j(X_d - X_d')}{\frac{r_f}{j2X_{ffd}} + 1}$$

or,

$$\overline{i_{f2}} = \frac{I_{fo} \cdot j(X_d - X_d')}{(R_L + jX_L) + j(X_d + X_2)} \qquad \ldots (1.44)$$

Therefore,

$$i_{f2} = \frac{I_{fo} \cdot j(X_d - X_d')}{(R_L + jX_L) + j(X_d + X_2)} \operatorname{Cos} 2(t + \theta) \qquad \ldots .(1.45)$$

Since $r_{f/2}$ is very small compared to X_{ffd}, hence $\frac{r_f}{2X_{ffd}}$ is neglected. θ is the angle at which load is switched on to the armature.

i_{f2} rotates at twice synchronous speed with respect to rotor, therefore it will induce a harmonic current in the armature of the order three. Let it be i_{d3}.

$$i_{d3} = \frac{e_d \cdot j\,(X_d - X_d')}{\left|(R_L + jX_L) + j(X_d + X_2)\right|^2} \cdot \operatorname{Cos} 3(\theta_o - 30) \qquad \ldots (1.46)$$

i_{d3} will in turn induce in the rotor windings a fourth harmonic current. Let it be i_{f4}-

$$\overline{i_{f4}} = I_{fo} \left|\frac{j(X_d - X_d')}{(R_L + jX_L) + j(X_d + X_2)}\right|^2 \qquad \ldots (1.47)$$

$$i_{f4} = I_{fo} \left|\frac{j\,(X_d - X_d')}{(R_L + jX_L) + j(X_d + X_2)}\right|^2 \operatorname{Cos} 4(t + \theta) \ldots (1.48)$$

By observation of equations 1.45 and 1.48 we can generalize the expression for total exciter current in steady-state as-

$$i_f(t) = I_{fo} + i_{f2} + i_{f4} + \ldots.$$

or,

$$i_f(t) = I_{fo} \left| 1 + \frac{j(X_d - X_d')}{(R_L + jX_L) + j(X_d + X_2)} \operatorname{Cos} 2(t + \theta) + \left|\frac{j(X_d - X_d')}{(R_L + jX_L) + j(X_d + X_2)}\right|^2 \operatorname{Cos} 4(t + \theta) + \left|\frac{j(X_d - X_d')}{(R_L + jX_L) + j(X_d + X_2)}\right|^3 \operatorname{Cos} 6(t + \theta) \ldots \right| \qquad \ldots (1.49)$$

Rotor current i_{f4} will induce in the armature a current of order 5.

$$\overline{i_{d5}} = \frac{e_d \cdot | (X_d - X_d') |^2}{| (R_L + jX_L) + j(X_d + X_2) |^3} \quad \ldots (1.50)$$

$$i_{d5} = \frac{e_d \cdot | j(X_d - X_d') |^2}{| (R_L + jX_L) + j(X_d + X_2) |^3} \cos 5(\theta_o - 30) \ldots (1.51)$$

From the equations 1.46 and 1.51 we can generalize the equation for direct axis negative sequence armature current as-

$$i_{d2}(t) = e_d \left| \frac{1}{|(R_L + jX_L) + j(X_d + X_2)|} \cos(\theta_o - 30) + \frac{j(X_d - X_d')}{|(R_L + jX_L) + j(X_d + X_2)|^2} \cos 3(\theta_o - 30) + \frac{|j(X_d - X_d')|^2}{|(R_L + jX_L) + j(X_d + X_2)|^3} \cos 5(\theta_o - 30) + \ldots\ldots\ldots\ldots \right| \quad \ldots (1.52)$$

Since,

$$i_{d1}(t) = \frac{e_d}{(R_L + jX_L) + j(X_d + X_2)} \cos(\theta_o + 30) \ldots (1.53)$$

By adding equations 1.52 and 1.53 we get the direct axis armature current as,

$$i_d(t) = e_d \left| \frac{\sqrt{3}}{|(R_L + jX_L) + j(X_d + X_2)|} \cdot \cos\theta_o + \frac{j(X_d - X_d')}{|(R_L + jX_L) + j(X_d + X_2)|^2} \cdot \cos 3(\theta_o - 30) + \frac{|j(X_d - X_d')|^2}{|(R_L + jX_L) + j(X_d + X_2)|^3} \cdot \cos 5(\theta_o - 30) + \ldots\ldots\ldots\ldots\ldots \right| \quad \ldots (1.54)$$

Putting-

$$X_D = X_d + X_2$$

&

$X_D' = X_d' + X_2$ in equation 1.54, we get,

$$i_d(t) = e_d \left| \frac{\sqrt{3}}{R_L + j(X_L + X_D)} \cos \theta_o + \frac{j(X_D - X_D')}{\left| R_L + j(X_L + X_D) \right|^2} \cos 3(\theta_o - 30) + \frac{\left| j(X_D - X_D') \right|^2}{\left| R_L + j(X_L + X_D) \right|^2} \cdot \cos 5(\theta_o - 30) + \ldots\ldots\ldots\ldots\ldots\ldots \right| \quad \ldots (1.55)$$

Similarly, we can rewrite the equation 1.49 also

$$i_f(t) = I_{fo} \left| 1 + \frac{j(X_D - X_D')}{\left| R_L + j(X_L + X_D) \right|} \cos 2(t + \theta) + \left| \frac{j(X_D - X_D')}{R_L + j(X_L + X_D)} \right|^2 \cos 4(t + \theta) + \left| \frac{j(X_D - X_D')}{R_L + j(X_L + X_D)} \right|^3 \cos 6(t + \theta) + \ldots\ldots\ldots\ldots\ldots\ldots \right| \quad \ldots (1.56)$$

Since there is no field winding in the quadrature axis, hence no reflections of negative sequence current. At any instant t, i_q is given by,

$$i_q = - i \sin \theta_o$$

$$= - \frac{\sqrt{3}\, e_d \sin \theta_o}{R_L + j(X_L + X_D)} \quad \ldots (1.57)$$

Since the machine is inherently a three-phase machine in which the one-phase winding is either left ideal or it is

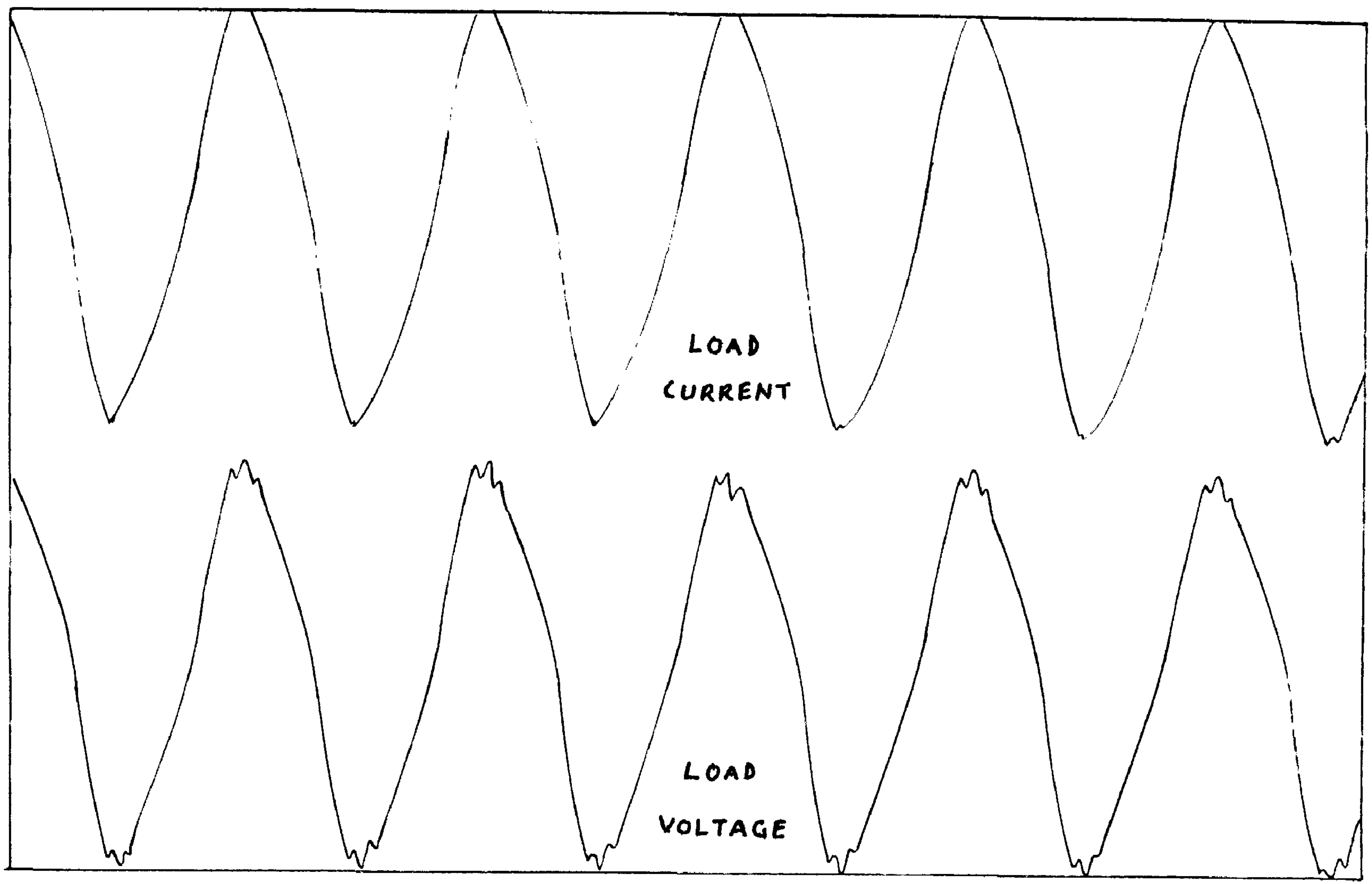


FIG. 11

unwound, therefore,

$$i_a = i_d \cos\theta - i_q \sin\theta + i_o \quad \ldots (1.58)$$

$i_o = 0$, and substituting i_d and i_q from equations 1.55 and 1.57 in equation 1.58 we get,

$$i_a(t) = e_d \left\{ \frac{\sqrt{3}}{R_L + j(X_L + X_D)} \cdot \cos(\theta - \theta_o) + \frac{j(X_D - X_D')}{|R_L + j(X_L + X_D)|^2} \cdot \cos\theta \cos 3(\theta_o - 30) + \frac{|j(X_D - X_D')|^2}{|R_L + j(X_L + X_D)|^3} \cdot \cos\theta \cos 5(\theta_o - 30) + \ldots\ldots \right\} \quad \ldots (1.59)$$

From equation 1.49 it is obvious that even harmonics are present in the field current due to the presence of negative sequence current in the armature. Magnitude of the harmonics reduces successively. Hence for purpose of calculation first few harmonic terms may be accounted. Similar is the case with the armature current also. There odd harmonics are present which reduce successively for increasing order. When precise calculations are required few initial harmonics may be taken into consideration.

Windings with 90° Displacement:

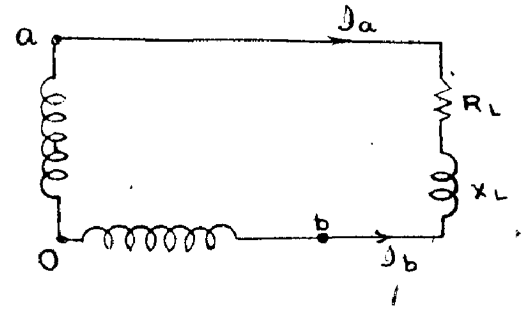


Fig.12

$i_a = - i_b = i \cos t.$

To determine the sequence components of the current for such an arrangement the operator will be j

$$I_{a1} = \frac{1}{2} (I_a + j\, I_b)$$
$$= \frac{1}{2} (I - jI) \quad = \frac{I}{\sqrt{2}} \angle{-45^\circ}$$

Hence instantaneous positive sequence current in phase a,

$$i_{a1} = \frac{1}{\sqrt{2}} \cos (t - 45^\circ) \qquad \ldots (1.60)$$

and

$$I_{a2} = \frac{1}{2} (I_a - j\, I_b) = \frac{I}{\sqrt{2}} \angle 45^\circ$$

$$i_{a2} = \frac{1}{\sqrt{2}} \cos (t + 45^\circ) \qquad \ldots (1.61)$$

Similarly,

$$i_{b1} = \frac{1}{\sqrt{2}} \cos (t + 135^\circ) \qquad \ldots (1.62)$$

$$i_{b2} = \frac{1}{\sqrt{2}} \cos (t - 135^\circ) \qquad \ldots (1.63)$$

Therefore,

$$i_a = \frac{1}{\sqrt{2}} \left\{ \cos (t - 45) + \cos (t + 45^\circ) \right\} \qquad \ldots (1.64)$$

$$i_b = \frac{1}{\sqrt{2}} \left\{ \cos (t + 135) + \cos (t - 135^\circ) \right\} \qquad \ldots (1.65)$$

Direct and Quadrature Axis Components of Armature Current:

Proceeding on the same lines as in case of three-phase synchronous machine with 120° space displacement,

$$i_a = i_d \cos \theta - i_q \sin \theta \qquad \ldots (1.66)$$

Then

$$i_b = i_d \cos (\theta - 90^\circ) - i_q \sin (\theta - 90^\circ)$$

or $$i_b = i_d \sin \theta + i_q \cos \theta \qquad \ldots (1.67)$$

By solving 1.66 and 1.67 simultaneously for i_d and i_q, we get,

$$i_d = i_a \cos\theta + i_b \sin\theta \quad \ldots (1.68)$$

$$i_q = -i_a \sin\theta + i_b \cos\theta \quad \ldots (1.69)$$

Substituting in equation 1.68 and 1.69 the positive and negative sequence component of current from 1.64 and 1.65 we get, at t = 0-

$$i_{d1} = \frac{i}{\sqrt{2}} \cos(\theta_0 + 45°), \quad i_{d2} = \frac{i}{\sqrt{2}} \cos(\theta_0 - 45°) \quad \ldots (1.70)$$

$$i_{q1} = -\frac{i}{\sqrt{2}} \sin(\theta_0 + 45°), \quad i_{q2} = -\frac{i}{\sqrt{2}} \sin(\theta_0 - 45°) \quad \ldots (1.71)$$

These currents, have been calculated by the same procedure as in case of single-phase machine with windings displaced 120°.

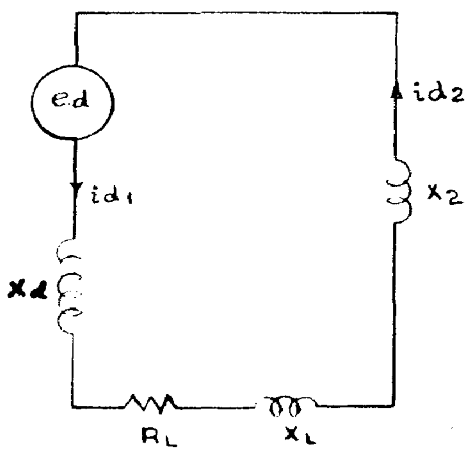


Fig.13

The magnitude of direct axis positive sequence current-

$$\overline{i_{d1}} = \frac{e_d}{|R_L + j(X_L + X_D)|} = \overline{i_{d2}}$$

Therefore,

$$i_{d2} = \frac{e_d}{|R_L + j(X_L + X_D)|} \cos(\theta_0 - 45°) \quad \ldots (1.72)$$

The double frequency current in the field will be

$$i_{f2} = I_{fo} \left| \frac{j(X_D - X_D')}{R_L + j(X_L + X_D)} \right| \cos 2(t+\theta) \qquad \ldots (1.73)$$

Proceeding on the same lines—

$$i_f(t) = I_{fo} \left| 1 + \frac{j(X_D - X_D')}{R_L + j(X_L + X_D)} \cdot \cos 2(t+\theta) + \left| \frac{j(X_D - X_D')}{R_L + j(X_L + X_D)} \right|^2 \cos 2(t+\theta) + \left| \frac{j(X_D - X_D')}{R_L + j(X_L + X_D)} \right|^3 \cos 6(t+\theta) + \ldots\ldots\ldots\ldots\ldots \right| \qquad \ldots (1.74)$$

and,

$$i_{d2}(t) = e_d \left| \frac{1}{R_L + j(X_L + X_D)} \cos(\theta_o - 45°) + \frac{j(X_D - X_D')}{|R_L + j(X_L + X_D)|^2} \cos 3(\theta_o - 45°) + \frac{|j(X_D - X_D')|^2}{|R_L + j(X_L + X_D)|^3} \cos 5(\theta_o - 45°) + \ldots\ldots\ldots\ldots\ldots \right| \qquad \ldots (1.75)$$

Finally,

$$i_d(t) = e_d \left| \frac{\sqrt{2}}{R_L + j(X_L + X_D)} \cos \theta_o + \frac{j(X_D - X_D')}{|R_D + j(X_L + X_D)|^2} \cos 3(\theta_o - 45°) + \frac{|j(X_D - X_D')|^2}{|R_L + j(X_L + X_D)|^3} \cos 5(\theta_o - 45°) + \ldots\ldots\ldots\ldots\ldots \right| \qquad \ldots (1.76)$$

The total armature current will be—

$$i_a(t) = i_d \cos \theta - i_q \sin \theta$$

or,

$$i_a(t) = e_d \Bigg| \frac{\sqrt{2}}{R_L + j(X_L + X_D)} \cos(\theta - \theta_o) + \frac{j(X_D \mp X_D')}{|R_L + j(X_L + X_D)|^2} \cos\theta \cos 3(\theta_o - 45^\circ) + \frac{|j(X_D - X_D')|^2}{|R_L + j(X_L + X_D)|^3} \cos\theta \cos 5(\theta_o - 45^\circ) + \ldots\ldots\ldots \Bigg| \qquad \ldots (1.77)$$

CHAPTER 2

SHORT CIRCUIT TRANSIENT FIELD CURRENT

There are two m.m.f. waves in the armature, one rotating in the forward direction at synchronous speed produced by the positive sequence current in the armature and the other rotating in the backward direction at synchronous speed resulted by the negative sequence current in the armature. The positive sequence m.m.f. will produce the similar effects as the m.m.f. wave of three-phase synchronous machine ie. during short circuit of single phase synchronous machine the flux linkages due to positive sequence component are identical to that of three-phase synchronous machine, when its all the three terminals are shorted. Therefore, the transient current in the field of single-phase synchronous machine due to positive sequence armature m.m.f. only, can be expressed exactly in the same manner as that of a three-phase synchronous machine. The only difference will be that the reactances applicable in this case will be of a single-phase synchronous machine i.e. including the negative sequence reactance. The expression for transient field current at sudden short circuit of armature terminals of a three-phase machine has been given by Doherty & Nickle[2] It can be very easily adopted for a single-phase machine using single-phase reactances. This equation is stated as follow:

$$i'_{f1}(t) = I_{fo} \left| \frac{X_D - X'_D}{X'_D} \epsilon^{-t/T'_D} - \frac{X_D - X'_D}{X'_D} \cos t \; \epsilon^{-t/T'_A} \right| \quad \ldots (2.1)$$

First term of the equation 2.1 is the direct component of positive sequence transient field current which decays according to single-phase short circuit transient time constant T_D'. The other term is the alternating component of the transient field current. This is produced by the direct flux in the armature resulted by the direct component of transient current in the armature. This component decays according to the armature time constant T_A'.

It can very easily be verified that[8]

$$T_D' = \frac{X_D'}{X_D} T_{do}'$$

Where T_{do}' is the open circuit armature time constant for a three-phase synchronous machine.

The negative sequence m.m.f. will result in even harmonic transient field currents. This can very easily be obtained from the expression of the additional short-circuit field current in steady-state by replacing X_D with X_D'. Hence, from equation 1.56, the total steady-state field current at armature dead short-circuit is obtained by putting $Z_L = 0$.

$$\text{i.e.}\quad i_{fs_2}(t) = I_{fo}\left| 1 + \frac{X_D - X_D'}{X_D}\text{Cos } 2(t+\theta) + \left(\frac{X_D - X_D'}{X_D}\right)^2 \text{Cos}4(t+\theta) + \left(\frac{X_D - X_D'}{X_D}\right)^3 \text{Cos } 6(t+\theta) + \ldots\ldots \right| \quad \ldots (2.2)$$

Therefore, field current due to negative sequence armature current will be-

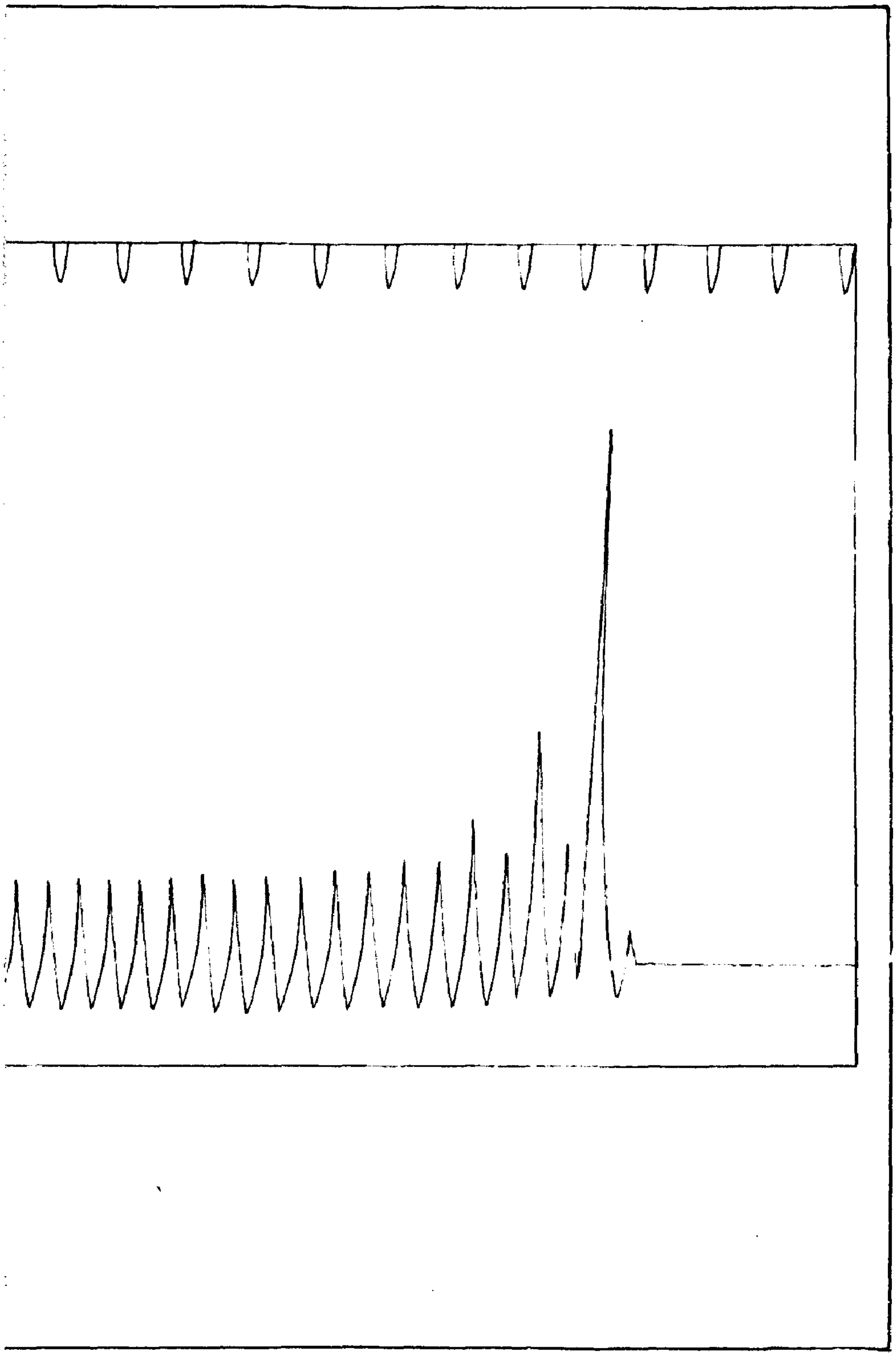
$$i_{f2}(t) = I_{fo}\left(\frac{X_D - X_D'}{X_D}\right)\left| \text{Cos } 2(t+\theta) + \frac{X_D - X_D'}{X_D}\text{Cos } 4(1+\theta) + \left(\frac{X_D - X_D'}{X_D}\right)^2 \text{Cos } 6\ (t+\theta) + \ldots \right| \quad \ldots (2.3)$$

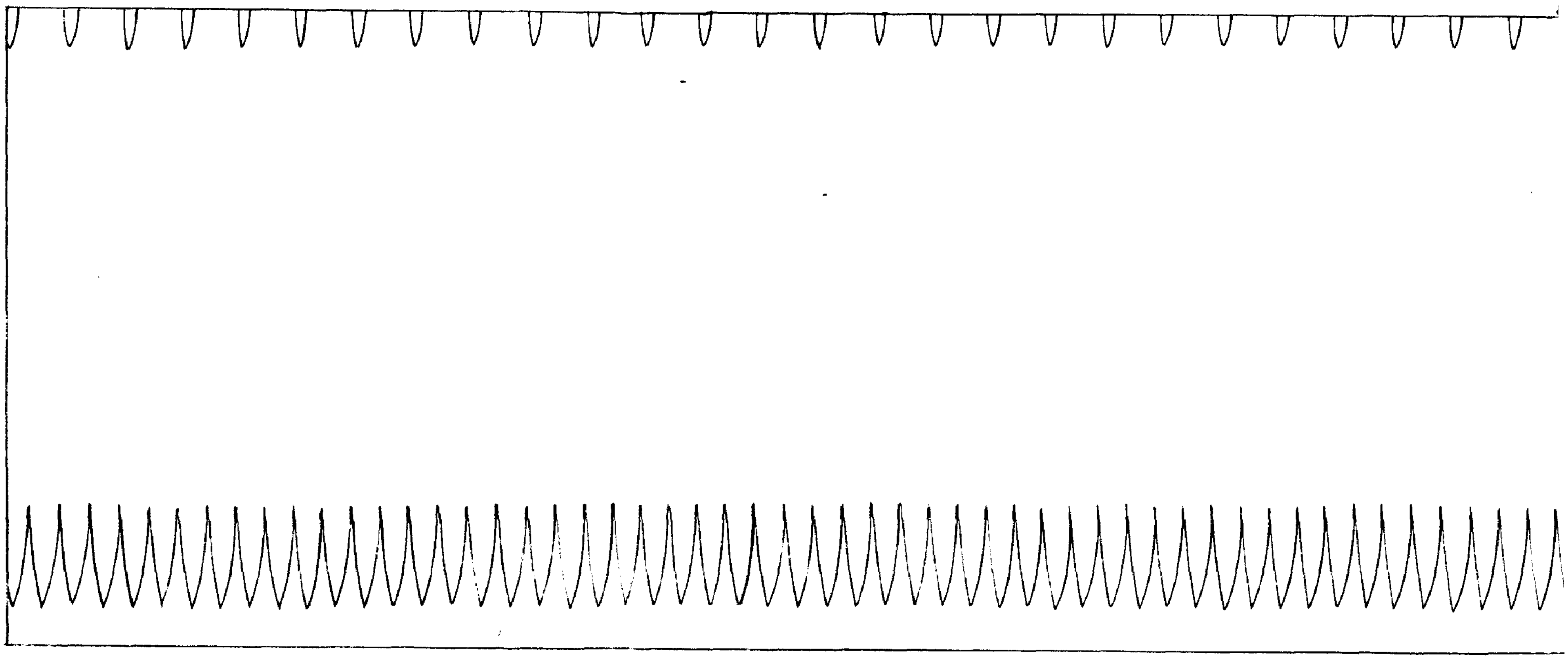
Replacing X_D by X'_D, the transient reactance in the direct-axis, the transient field current due to the negative sequence current in the armature is obtained as-

$$i'_{f2}(t) = I_{fo} \frac{X_D - X'_D}{X'_D} \cdot \epsilon^{-t/T'_D} \left| \cos 2(t+\theta) + \frac{X_D - X'_D}{X'_D} \cos 4(t+\theta) + \left(\frac{X_D - X'_D}{X'_D}\right)^2 \cos 6(t+\theta) + \ldots\ldots \right| \quad \ldots (2.4)$$

$i'_{f2}(t)$ will decay according to the short circuit field time constant T'_D.

It should be noted that in the position of maximum flux linkage between armature and field i.e. when $\theta = 0$ viz, pole-axis coincides with the axis of the armature phase winding the direct component of transient armature current will be present in the armature[2] which will be stationary in space and decaying according to the short-circuit armature time constant T'_A. This component being stationary will induce a transient alternating current of normal frequency in the field which will decay according to T'_A. When the flux linkage between the armature and field is minimum i.e. $\theta = \pi/2$ viz. when the axis of the phase winding coincides with the quadrature-axis, there will be no direct component present in the transient armature current and hence no normal frequency transient alternating current will be present in the field. That means the term $\frac{X_D - X'_D}{X'_D} \cos t \cdot \epsilon^{-t/T'_A}$ in equation 2.1 will be absent, leaving only the direct component in the transient field current and even harmonic components due to the negative sequence current in the armature.

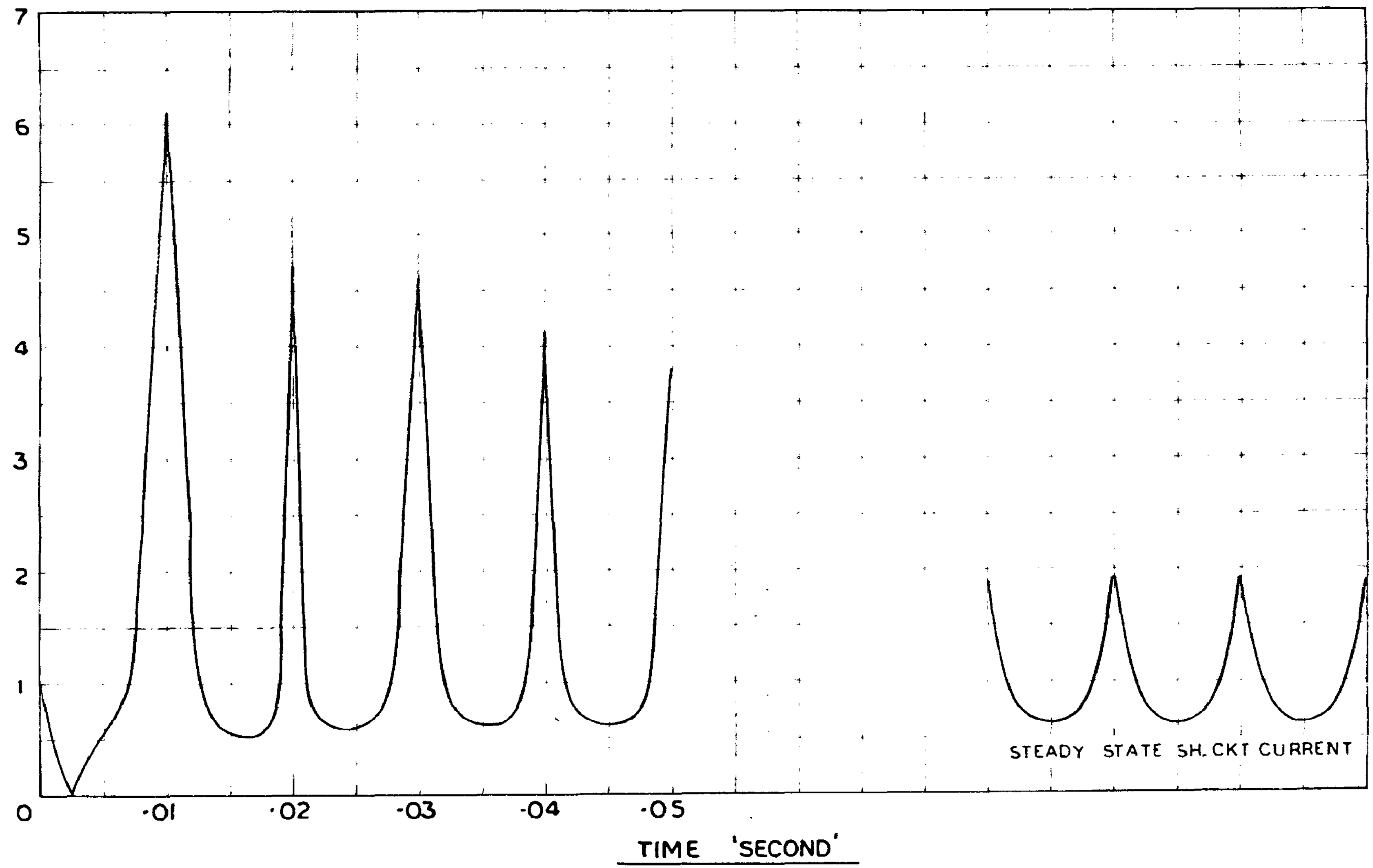




FIELD CURRENT ON SUDDEN SINGLE — PHASE

LINE -TO- LINE SHORT CIRCUIT

FIG. 14 (TEST)



TRANSIENT FIELD CURRENT

SH. CKT. EFFECTED ON MAX.
FLUX – LINKAGE POSITION

FIG. 15A

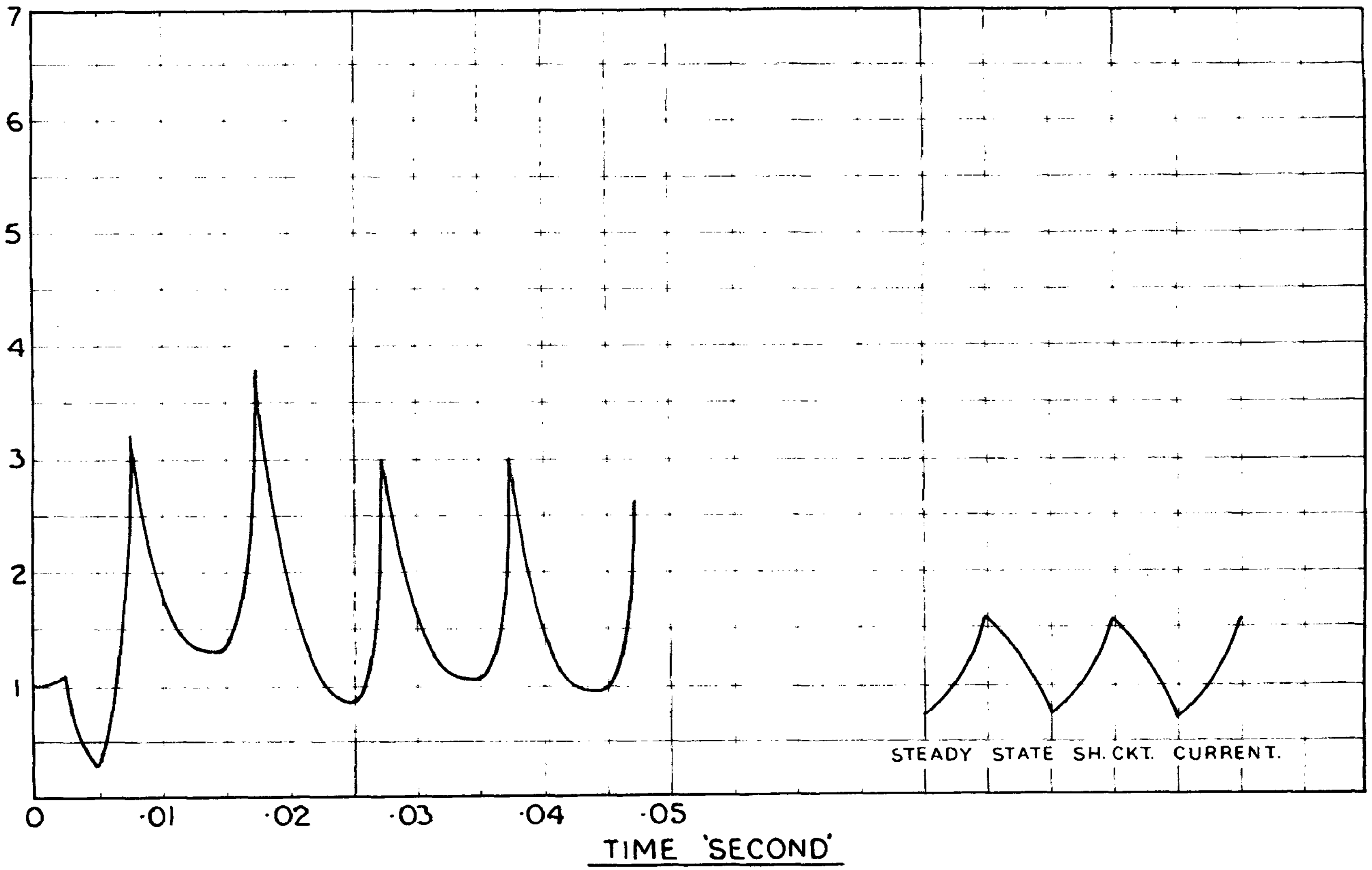
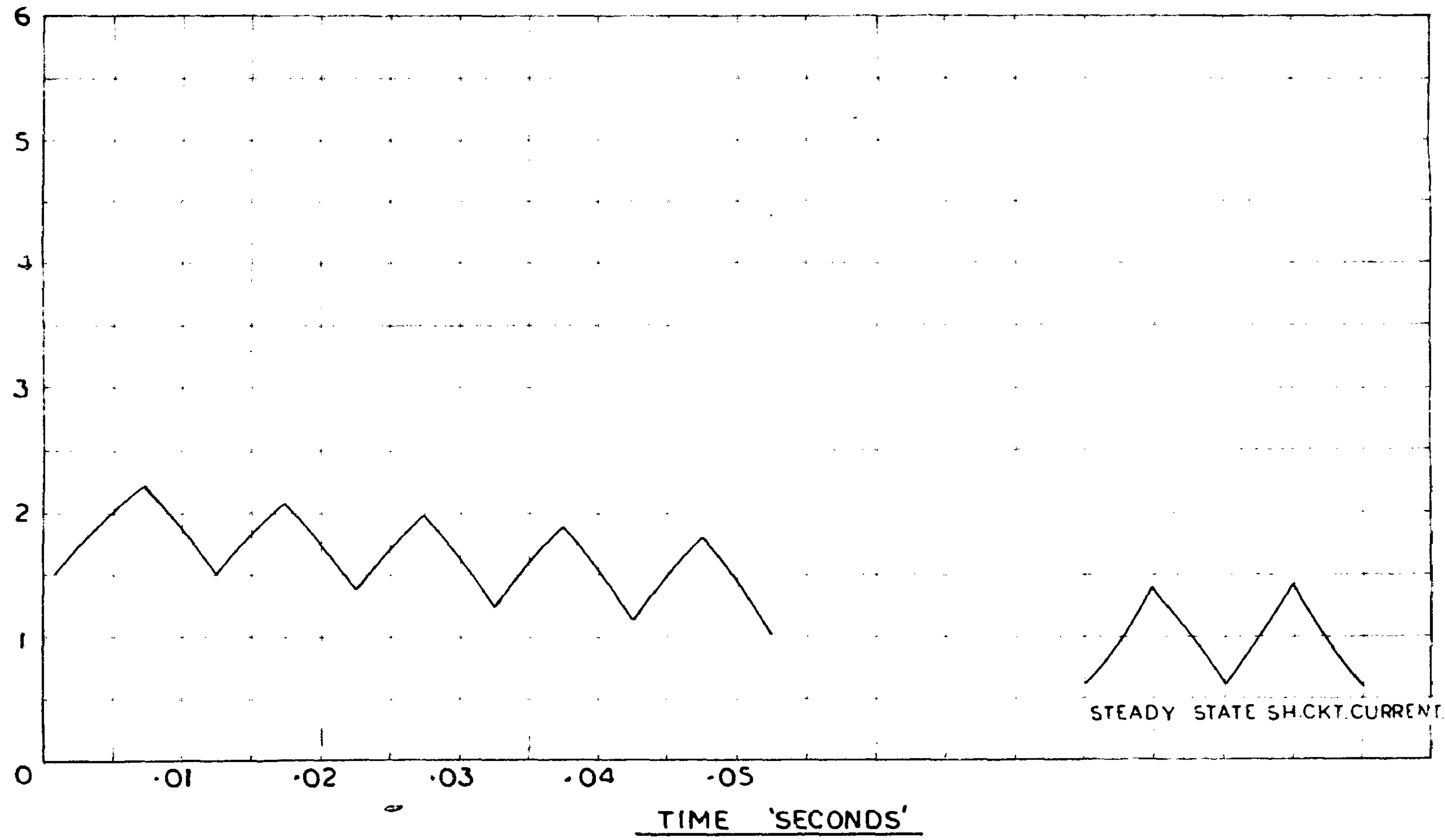


FIG. 15. b

TRANSIENT FIELD CURRENT

(SH. CKT. EFFECTED ON θ=60)



TRANSIENT FIELD CURRENT

SH.CKT. EFFECTED ON MIN. FLUX.-LINKAGE POSITION

FIG. 16

Hence total transient field current in position of maximum flux linkage will be-

$$i_f'(t)\Big|_{\theta = 0} = I_{fo}\left| 1 + \frac{X_D - X_D'}{X_D'}\,\mathrm{Cos}\,2t + \left(\frac{X_D - X_D'}{X_D'}\right)^2 \mathrm{Cos}\,4\,t + \ldots \right| +$$

$$I_{fo}\left| \frac{X_D - X_D'}{X_D'} \cdot \varepsilon^{-t/T_D'} - \frac{X_D - X_D'}{X_D'}\,\mathrm{Cos}\,t\ \varepsilon^{-t/T_A'} \right| +$$

$$I_{fo}\left(\frac{X_D - X_D'}{X_D'}\right) \cdot \varepsilon^{-t/T_D'} \left| \mathrm{Cos}\,2t + \frac{X_D - X_D'}{X_D'}\,\mathrm{Cos}\,4t + \left(\frac{X_D - X_D'}{X_D'}\right)^2 \mathrm{Cos}\,6\,t + \ldots\ldots\ldots \right|$$

or

$$i_f(t)\Big|_{\theta = 0} = I_{fo}\left| 1 + \frac{X_D - X_D'}{X_D}\,\mathrm{Cos}\,2t + \left(\frac{X_D - X_D'}{X_D'}\right)^2 \mathrm{Cos}\,4\,t + \ldots \right| +$$

$$I_{fo} \cdot \frac{X_D - X_D'}{X_D'} \cdot \varepsilon^{-t/T_D'} \left| 1 + \mathrm{Cos}\,2t + \frac{X_D - X_D'}{X_D'}\,\mathrm{Cos}\,4t + \left(\frac{X_D - X_D'}{X_D'}\right)^2 \mathrm{Cos}\,6\,t + \ldots\ldots \right| +$$

$$\frac{X_D - X_D'}{X_D'}\,\mathrm{cos}\,t \cdot \varepsilon^{-t/T_A'} \cdot I_{fo} \qquad \ldots(2.5)$$

in the position of minimum flux linkage-

$$i_f'(t)\Big|_{\theta = /2} = I_{fo}\left| 1 - \frac{X_D - X_D'}{X_D}\,\mathrm{Sin}\,2\,t - \left(\frac{X_D - X_D'}{X_D}\right)^2 \mathrm{Sin}\,4t + \ldots \right| +$$

$$I_{fo} \cdot \frac{X_D - X_D'}{X_D'} \cdot \varepsilon^{-t/T_D'} \left| 1 - \mathrm{Sin}\,2\,t - \frac{X_D - X_D'}{X_D'}\,\mathrm{Sin}\,4t - \left(\frac{X_D - X_D'}{X_D'}\right)^2 \mathrm{Sin}\,6\,t + \ldots\ldots \right| \qquad \ldots(2.6)$$

To confirm the validity of foregoing equations of transient field current, oscillogram of the same was recorded which is shown in fig.14. It was not possible with the available apparatus to control the instant of switching. With the help of equations three curves at different instants of switching were calculated and plotted as shown in figs. 15A, 15B & 16. The wave form of transient field current at the instant when the angle between the axis of pole and the axis of phase a is 60°, is very similar to that obtained by actual test on the machine. From the inspection of three calculated curves it can be concluded that the wave forms of transient field current obtained theoretically and that obtained practically are quite similar.

CHAPTER 3

SINGLE-PHASE CONSTANTS

From the theory of Chapter 1 and Chapter 2 it is obvious that for a single-phase synchronous machine there always appears negative sequence reactance together with the three-phase equivalent direct and quadrature axis constants. And so the single-phase constants are taken as addition of the equivalent negative sequence reactance with the three-phase equivalent direct or quadrature axis constants as the case may be. In the following analysis, theoretical and physical concept of these single-phase constants has been brought out.

The single-phase constants X_D, X_Q, X_D' ... etc. employed in the theory can easily be explained theoretically and physically. The experimental methods, to determine these constants are discussed along with the analysis of the respective constants.

Direct-axis single-phase Synchronous reactance (X_D):-

The flux linkage equation, in terms of self and mutual inductances, given by B.R. Prentice[5] for a three-phase synchronous machine is as follows:

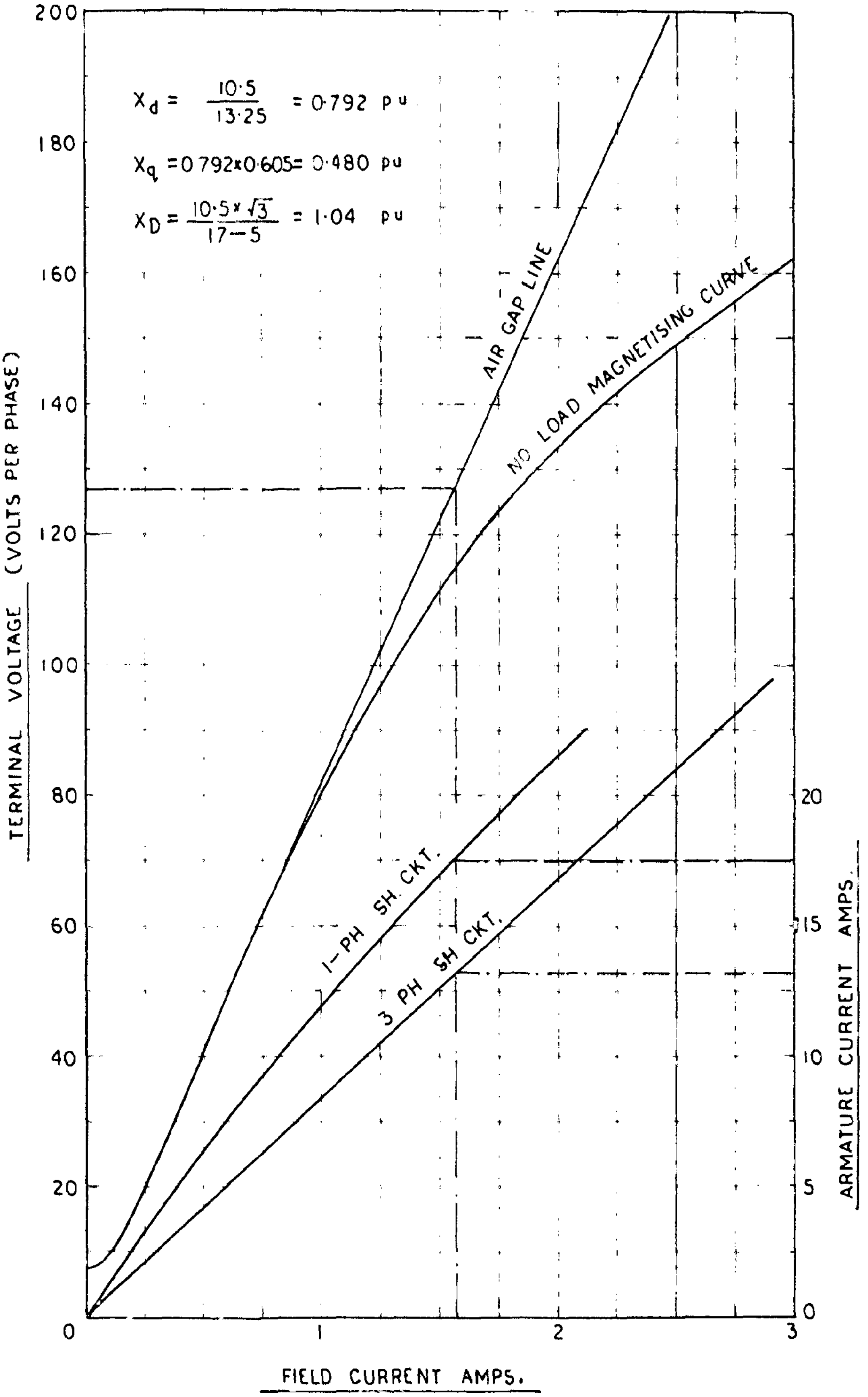
$$\psi_a = L_0 i_a - M_s (i_b + i_c) + M_o \{ i_a \cos 2\theta + i_b \times \cos (2\theta - 120°) + i_c \cos (2\theta + 120°)\} + i_{fd} M_{fd} \cos\theta \quad ... (3.1)$$

where,

ψ_a = Flux linkages in phase 'a'

L_o = Average self inductance

M_o = Amplitude of variation of self inductance ($L_a = L_o + M_o \cos 2\theta$)



$X_d = \frac{10 \cdot 5}{13 \cdot 25} = 0 \cdot 792$ pu
$X_q = 0 \cdot 792 \times 0 \cdot 605 = 0 \cdot 480$ pu
$X_D = \frac{10 \cdot 5 \times \sqrt{3}}{17 - 5} = 1 \cdot 04$ pu
AIR GAP LINE
NO LOAD MAGNETISING CURVE
1-PH SH. CKT.
3 PH SH. CKT.
TERMINAL VOLTAGE (VOLTS PER PHASE)
ARMATURE CURRENT AMPS.
FIELD CURRENT AMPS.
200
180
160
140
120
100
80
60
40
20
0
1
2
3
20
15
10
5
0

FIG. 17

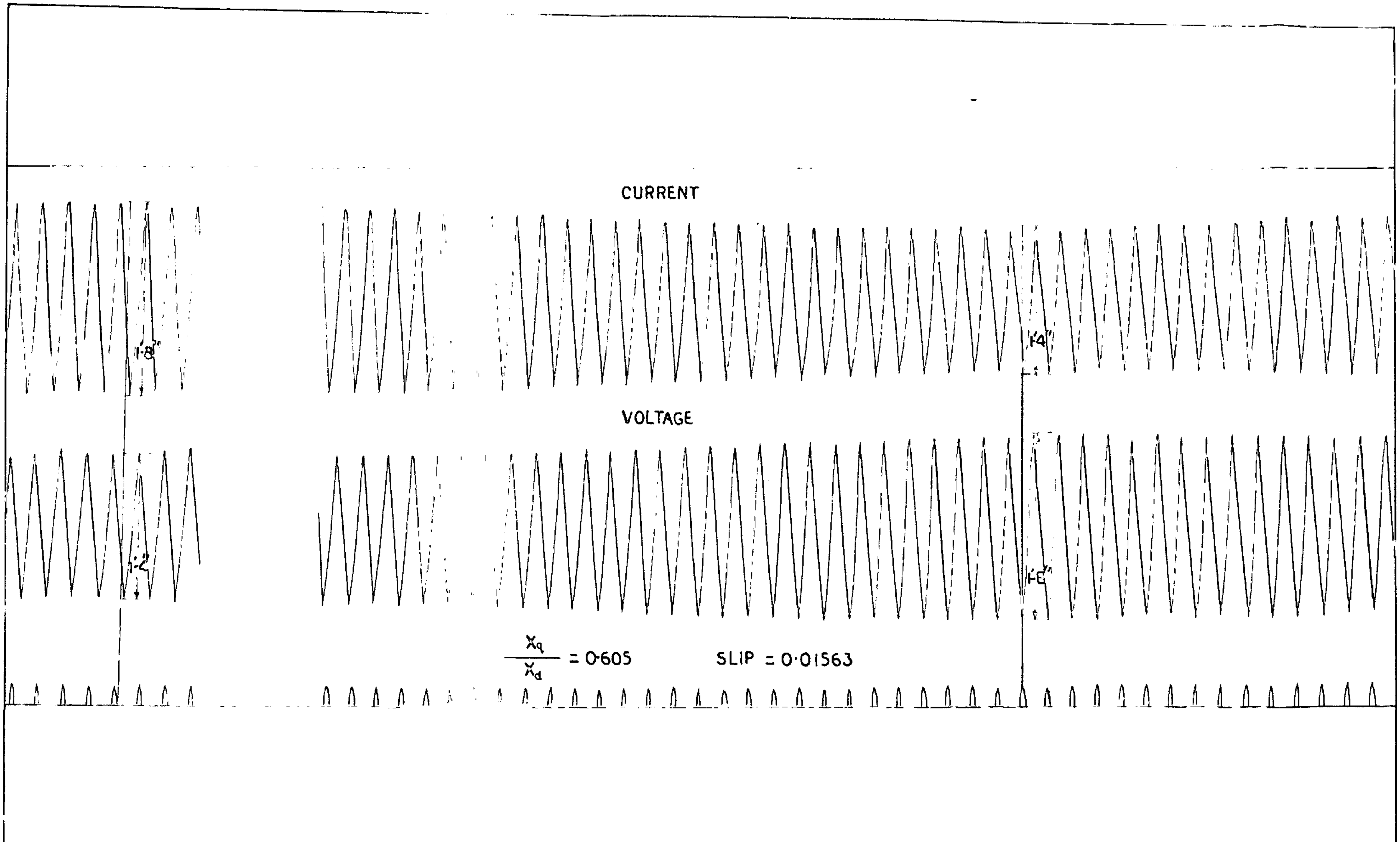
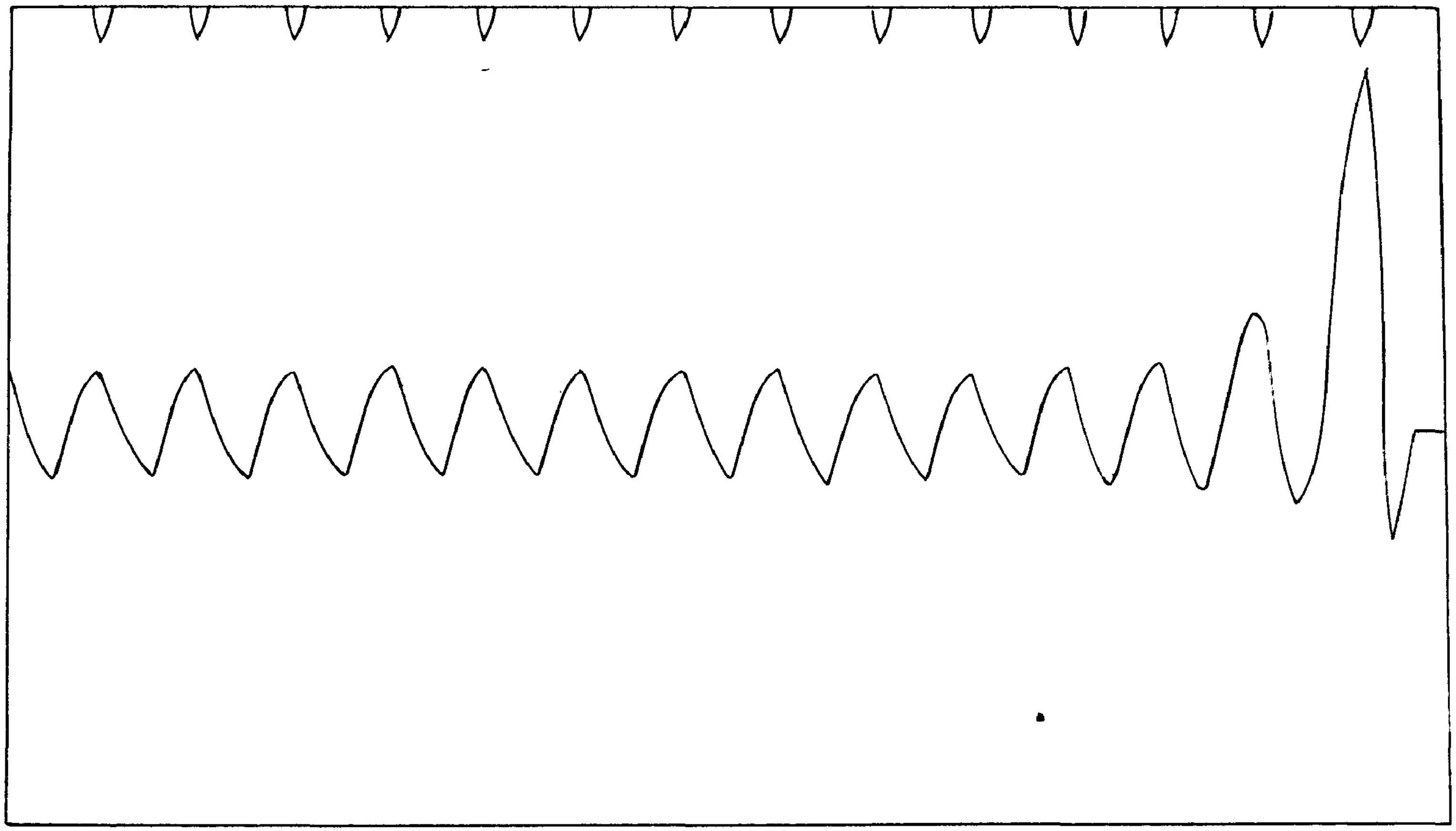


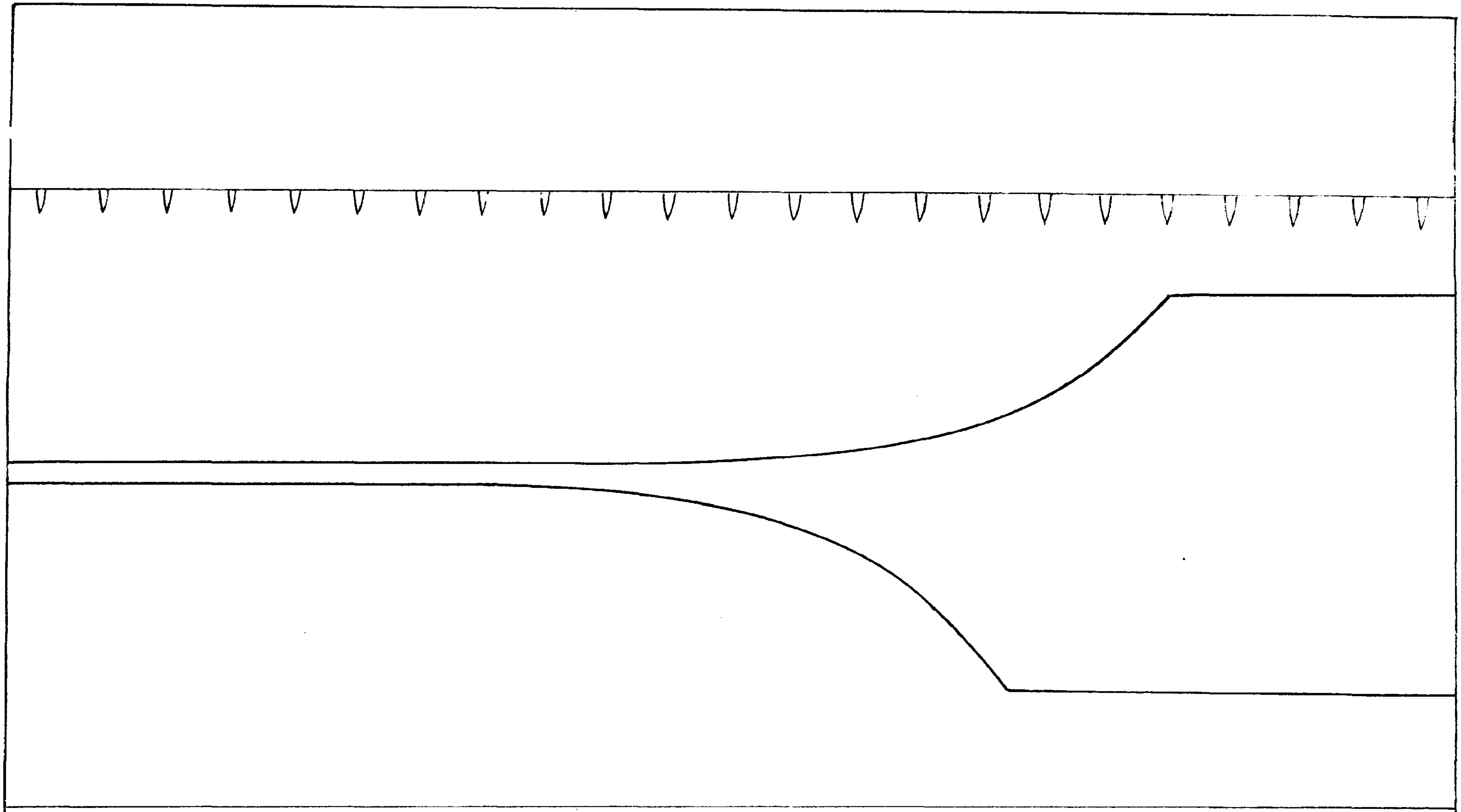
FIG. 18



ARMATURE CURRENT ON SUDDEN THREE-PHASE

SHORT CIRCUIT

FIG. 19



FIELD CURRENT ON SUDDEN

FIELD SHORT CIRCUIT

FIG. 20

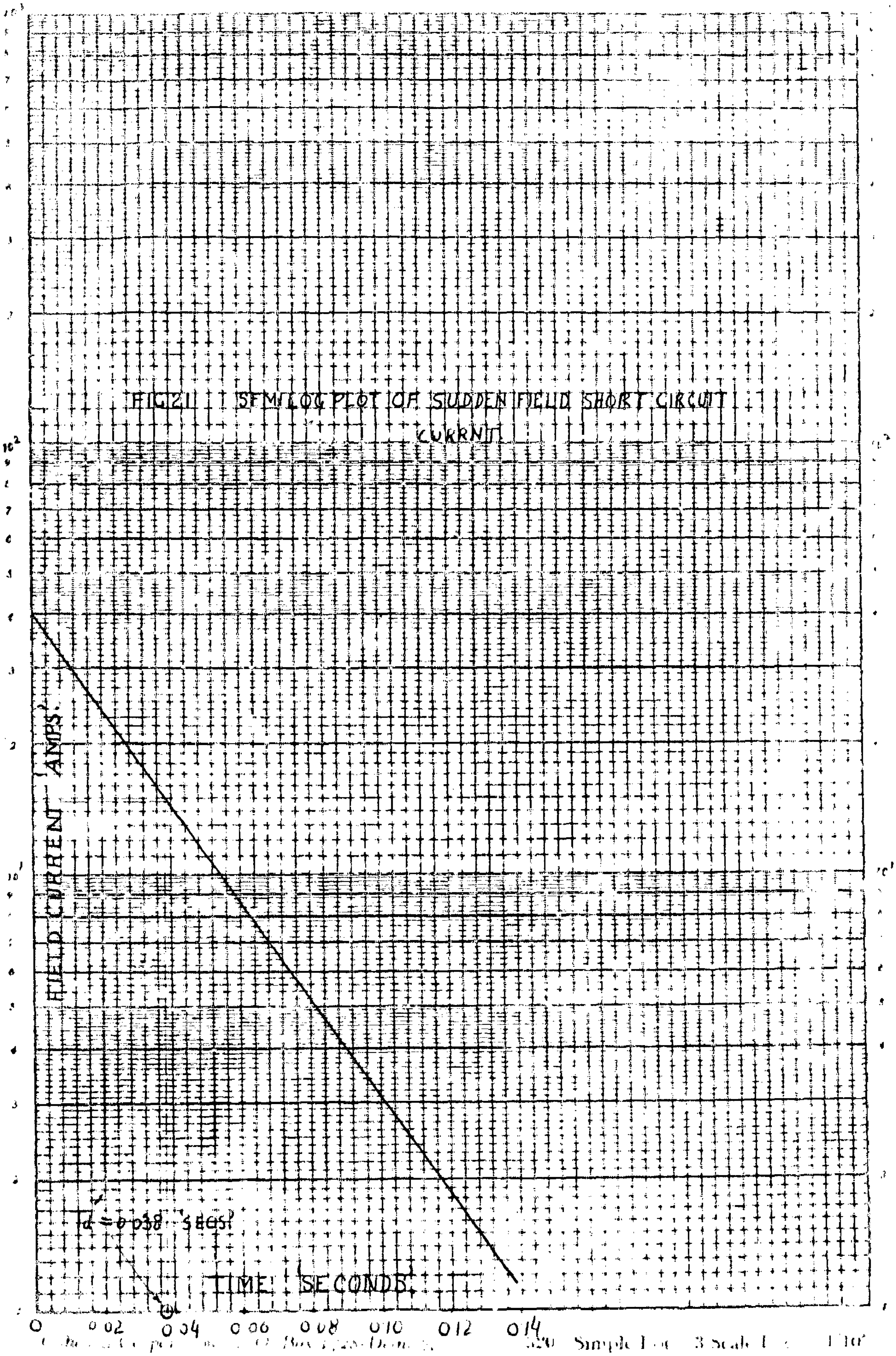


FIG21 SEMILOG PLOT OF SUDDEN FIELD SHORT CIRCUIT CURRENT
FIELD CURRENT AMPS.
Td = 0.038 SECS.
TIME SECONDS.
0
0.02
0.04
0.06
0.08
0.10
0.12
0.14

M_s = Average Value of mutual inductance between phases (M_o is not a mutual inductance).

θ = Angle between axis of pole and axis of phase 'a'

i_{fd} = Field current in the direct axis.

M_{fd} = Mutual inductance between field and phase a (Maximum value).

The direct axis synchronous reactance will be given when axis of the pole coincides with the axis of the winding of phase 'a'. In such position the positive sequence armature currents will be as given by the equation 1.3.

$$i_{a1} = \frac{1}{\sqrt{3}} \text{ Cos } (t - 30) \qquad \text{... (3.2)}$$

$$i_{b1} = \frac{1}{\sqrt{3}} \text{ Cos } (t - 150°), \qquad i_b + i_c = - i_a$$

$$i_{c1} = \frac{1}{\sqrt{3}} \text{ Cos } (t + 90°)$$

and if the field is open

$$i_{fd} = 0$$

Field is rotated at synchronous speed so that the direct axis is in the line with the axis of armature magnetomotive force wave,

$$\theta = t, \qquad w = 1 \text{ p.u.}$$

Substituting these conditions in the equation 3.1

$$a = \frac{1}{\sqrt{3}} \Big\{ L_o \text{ Cos } (t-30) + M_s \text{ Cos } (t-30) + M_o \big\{ \text{Cos}(t-30) \times \text{Cos } 2t + \text{Cos } (t-150) \text{ Cos}(2t - 120°) + \text{Cos } (t+90°) \text{ Cos } (2t + 120°) \big\} \Big\}$$

Which upon trignometrical reductions becomes

$$\Psi_a = I\,(L_o + M_s + 3/2\,M_o)\cdot\frac{1}{\sqrt{3}}\ \mathrm{Cos}\,(t-30)\ I$$

or

$$\Psi_a = (L_o + M_s + 3/2\,M_o)\cdot i_{a1}$$

and as defined, X_D in per unit is the armature flux linkage per ampere, so

$$X_D = \frac{I\Psi_a I}{I\,i_{a1} I} = \frac{I\Psi_a I}{1/\sqrt{3}}$$

$$= (L_o + M_s + 3/2\,M_o)$$

Physical View-point:-

In the armature of a single-phase synchronous machine currents exist which are pulsating in time at line frequency.

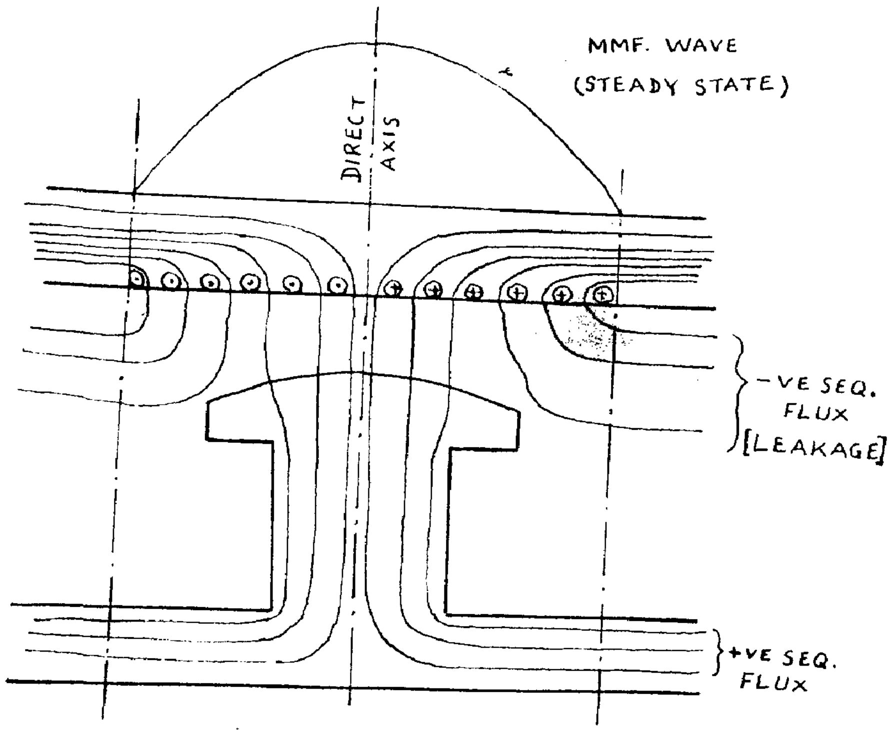


Fig.22 Direct axis synchronous reactance flux paths.

The magnetomotive force affected by these currents can be resolved into two components, one positive sequence and the other negative sequence component. These m.m.fs. produce fluxes of positive and negative sequence nature. For the positively rotating flux wave the machine behaves exactly as a three-phase synchronous machine.

X_D is the reactance offered to the flow of current when the axis of pole coincides the axis of m.m.f. wave and rotor rotates synchronously. In this case the flux linkage will be maximum and all the positive flux will link with the main field winding. The negative sequence flux exists in the air and does not link with windings of the machine to create any mutual effects. Hence it can be considered to add with the leakage flux of the machine. The flux paths are shown in the fig. 22. Thus the single-phase direct axis synchronous reactance will be obtained by dividing positive sequence flux linkage by the positive sequence armature current whose magnitude is $1/\sqrt{3}$ times the magnitude of the phase current ie. $\psi_a / i/\sqrt{3}$.

For determination of X_D experimentally open circuit and short circuit tests were performed. The machine used for single-phase operation was the same three-phase machine whose three-phase constants are given in the Appendix. One phase of the machine was left idle and remaining two terminals were used as armature terminals.

The open circuit magnetizing curve for single-phase operation will be the same as for three-phase operation since on open circuit there is no current in the armature in both the cases. The magnetizing curve for the particular machine is shown in fig.17.

It can be observed that the short-circuit curve of a single-phase synchronous machine is not a straight line like that of three-phase synchronous machine. The phenomena is easily explained as the short circuit armature current in a single-phase synchronous machine will comprise of positive and negative sequence components. These two components added together according to the laws of symmetrical components to give armature currents for different excitations, will not result in a straight line curve.[6]

To determine X_D the same procedure is adopted as in case of three-phase synchronous machines. The single-phase short-circuit current corresponding to per phase rated voltage is determined by dropping a perpendicular from the point on air gap line corresponding to rated per phase voltage on to single-phase short circuit curve as shown in fig.17. The rated current devided by $1/\sqrt{3}$ times the current thus obtained gives the X_D in per unit.

From the result of the experiment on single-phase machine-

$X_D = 1.04$ per unit

From three-phase results-

$X_D = X_d + X_2 = 1.042$ p.u.

Quadrature axis Single-phase Synchronous reactance (X_Q):-

If the quadrature axis is in line with the axis of armature magnetomotive force wave, and the field is rotated at synchronous speed-

$\theta = t - 90°$ also $i_{fd} = 0$.

Substituting all these quantities in equation 3.1 and solving trignometrically, the following equation is obtained:

$$\Psi_a = (L_o + M_s - 3/2\ M_o) \cdot \frac{1}{\sqrt{3}} \operatorname{Cos}(t - 30)$$

$$= (L_o + M_s - 3/2\ M_o) \cdot i_{a1}$$

and,
$$X_Q = \frac{|\Psi_a|}{|i_{a1}|} = \frac{|\Psi_a|}{1/\sqrt{3}}$$

$$= (L_o + M_s - 3/2\ M_o)$$

Physical View-point:

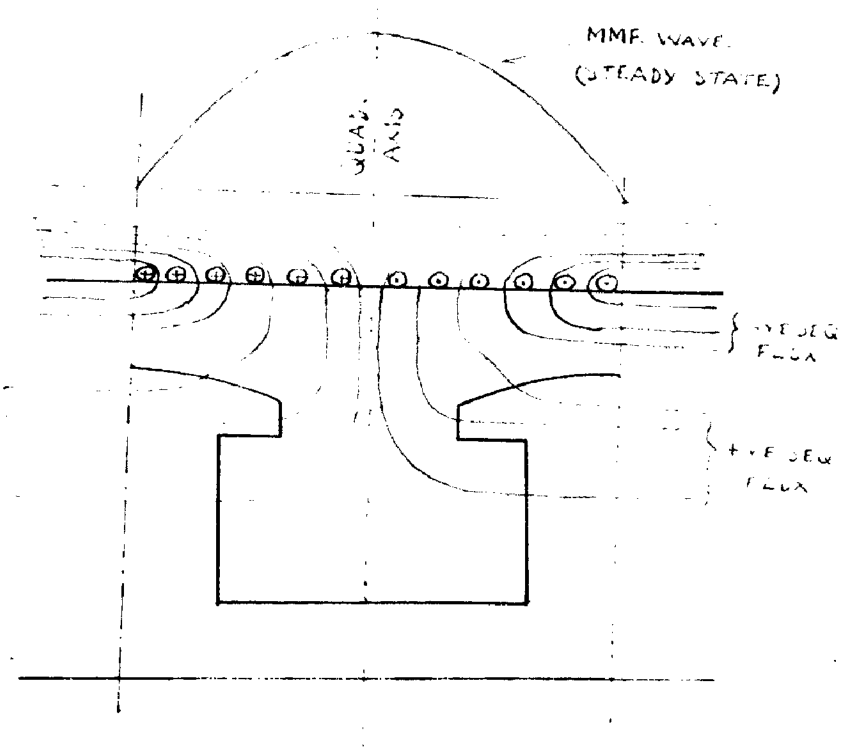


Fig.23. Quadrature axis synchronous reactance flux paths.

The quadrature axis i.e. the axis midway between the poles coincides with the axis of m.m.f. wave and rotor rotates at synchronous speed. The positive sequence flux passes through the quadrature axis and therefore the flux linking to the main field winding is minimum in this case. The negative sequence flux here also is regarded as the leakage flux. The flux paths are shown in the fig.23. Thus the quadrature axis single-phase synchronous reactance will be obtained by dividing positive sequence flux linkage in quadrature axis by positive sequence armature current i.e.

$$X_Q = \frac{I\psi_a I}{i/\sqrt{3}}$$

When determining X_d and X_q assoc8ated with three-phase machine, the well known "Slip Test" is employed and the ratio X_q/X_d is determined and X_q is calculated. Determination of X_Q associated with single-phase machine by performing the slip test does not seem possible because the current present in the armature is of pulsating nature and so there may not any slip between waves of voltage and current applied to armature terminal from external source, of a little different frequency than that of the armature, and those of the armature itself. If we observe the effect by the way of positive and negative sequence components i.e. logically appears that due to presence of negative sequence component the wave may be too complicated to give any accurate results regarding slip value and the required ratio $\frac{X_Q}{X_D}$. Keeping in view the above difficulties, the"negative Excitation Method" was employed for determining X_Q.

The machine to be tested was run as a synchronous motor. It should be recalled that the machine being a single-phase one, has got to be brought up to near synchronous speed by some other motor. The constant rated terminal voltage was applied to armature terminals. The polarity of the applied excitation was reduced to zero, reversed and increased in the negative direction causing an increase in the armature current. By increasing the negative excitation in small increments, the maximum stable per unit line current corresponding to the maximum stable negative excitation is found. X_Q is determined from the formula.

$$X_Q = \frac{\text{Applied voltage per phase}}{\text{Maximum Stable armature current per phase}}$$

The experiment gave the results as follows:

Maximum stable armature current		= 26A
Volts applied to the armature terminal	...	= 200 V.
Volts/phase	...	$= \frac{200}{\sqrt{3}}$ V
Current/phase	...	$= \frac{26}{2} = 13$A

Therefore,

$$X_Q = \frac{200}{\sqrt{3} \times 13} = 8.9 \text{ ohms}$$

$$= \underline{0.735} \text{ per unit}$$

From three-phase results-

$$X_Q = X_q + X_2$$

$$= \underline{0.732} \text{ per unit}$$

Hence result confirms very closely.

Direct Axis Single-phase Transient Reactance (X_D'):

The positive phase sequence currents are suddenly applied to armature, expressions for which are given in equation 3.2.

Field circuit is closed and by constant flux linkage Theorem,

$$\Psi_d = 0$$

Field is rotated at constant speed so that the direct axis is in line with the peak of newly established armature m.m.f. wave,

$$\theta = t \qquad (w = 1 \text{ p.u.})$$

Then,

$$i_{d1} = \frac{2}{3} \left| i_{a1} \cos \theta + i_{b1} \cos (\theta - 120) + i_{c1} \cos(\theta + 120) \right|$$

$$= \frac{2}{3} \cdot \frac{1}{\sqrt{3}} \left| \cos (t - 30°) \cos t + \cos(t - 150°) \times \cos (t - 120°) + \cos(t + 90°) \cos(t + 120°) \right|$$

$$= \frac{1}{2} \qquad \ldots (3.3)$$

Similarly, $i_{q1} = -\frac{2}{3} \left| i_{a1} \sin \theta + i_{b1} \sin(\theta - 120°) + i_{c1} \sin(\theta + 120°) \right|$

$$= 0$$

Flux linkage equation in the direct axis is given by

$$\Psi_d = M_{fd}\, i_{fd} + (L_o + M_s + 3/2\, M_o)\, i_d \qquad \ldots (3.4)$$

and in the quadrature axis-

$$\Psi_q = (L_o + M_s - 3/2\, M_o)\, i_q \qquad \ldots (3.5)$$

Substituting the positive sequence currents in equation (3.4) and (3.5)-

$$_d = M_{fd}\, i_{fd} + (L_o + M_s + 3/2\, M_o)\, i/2 \qquad \ldots (3.6)$$

Also,

$$\Psi_d = M_{fd} \left| i_a \cos \theta + i_b \cos (\theta - 120) + i_c \cos(\theta + 120) \right| + L_{fd} i_{fd}$$

$$= 3/2\, M_{fd}\, i_{d1} + L_{fd}\, i_{fd}$$

or,

$$\psi_d = 3/2\, M_{fd} \cdot i/2 + L_{fd}\, i_{fd} \qquad \ldots (3.7)$$

But

$$\psi_d = 0$$

Therefore,

$$3/2\, M_{fd}\, \frac{i}{2} + L_{fd} \cdot i_{fd} = 0$$

or,
$$i_{fd} = -\frac{3}{2} \cdot \frac{M_{fd}}{L_{fd}} \cdot \frac{i}{2}$$

Therefore,

$$\psi_d = -\frac{3}{2} \cdot \frac{M_{fd}^2}{L_{fd}} \cdot \frac{i}{2} + (L_o + M_o + \frac{3}{2} M_o)\, \frac{i}{2}$$

$$= \frac{i}{2} \left(X_D - \frac{3}{2} \frac{M_{fd}^2}{L_{fd}}\right)$$

Hence,

$$X_D' = \frac{d}{i/2} = X_D - 3/2 \cdot \frac{M_{fd}^2}{L_{fd}}$$

Physical View-point:

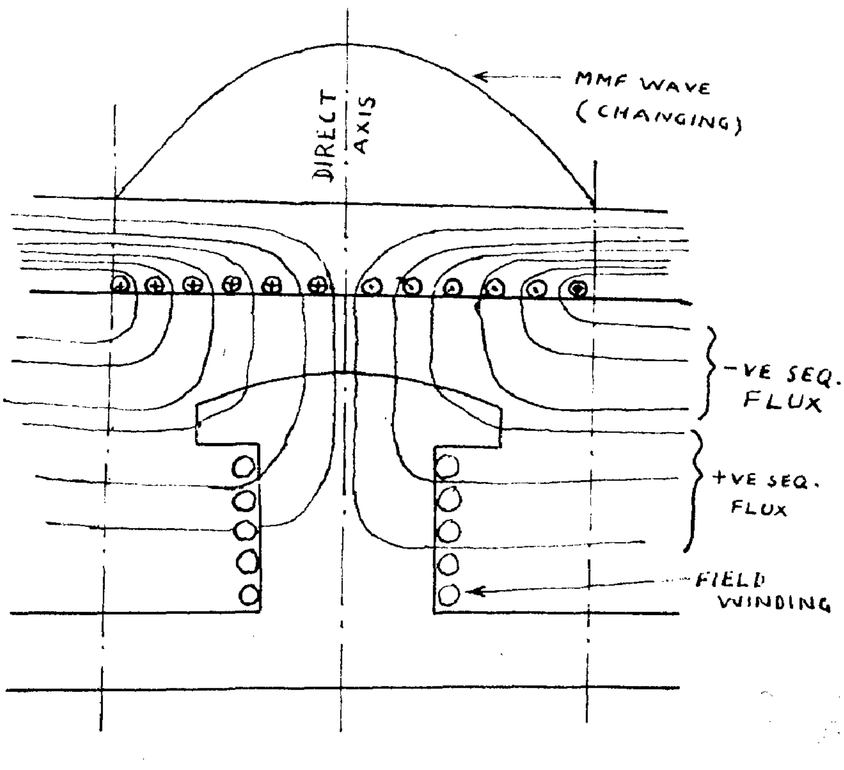
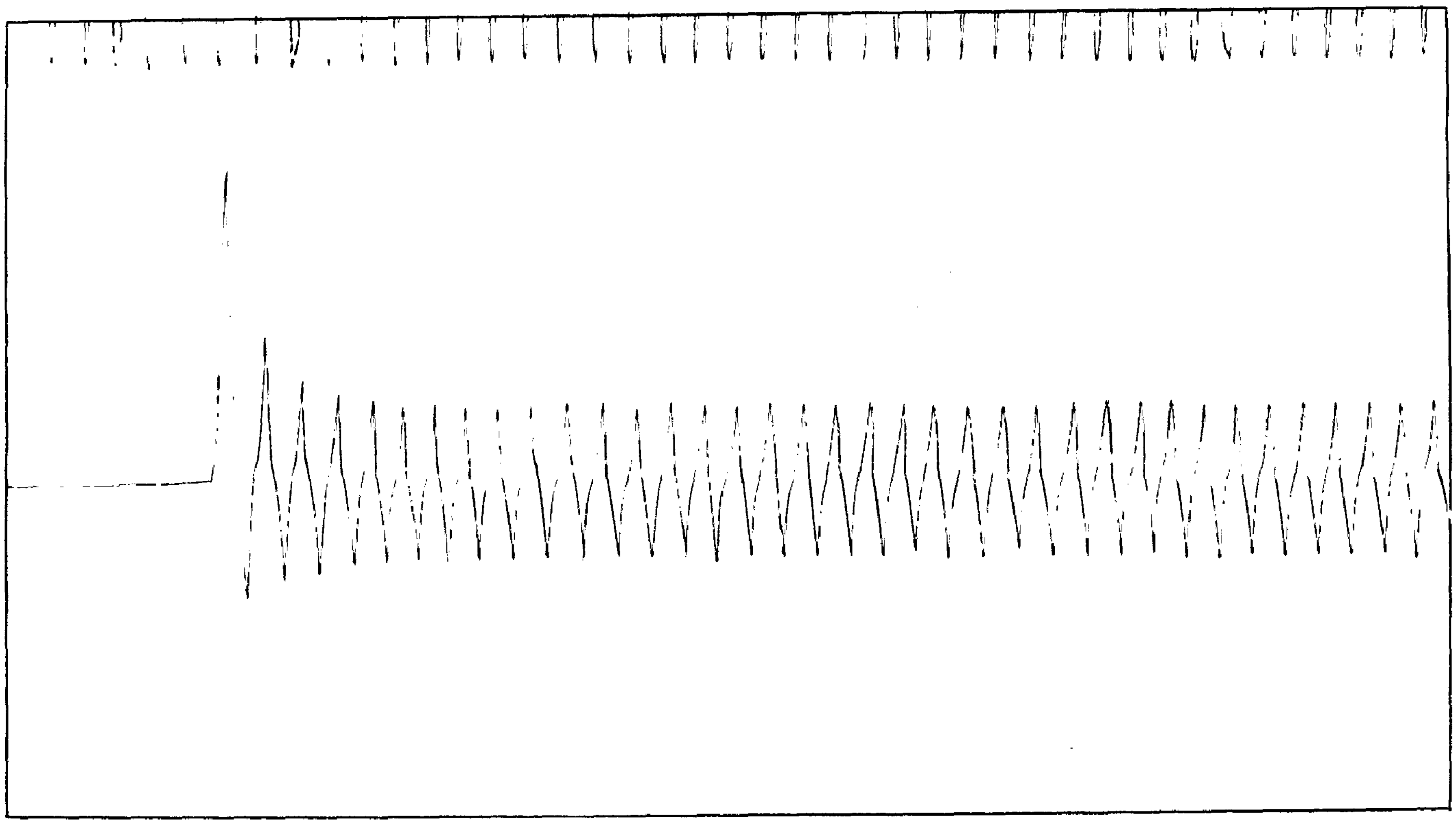


Fig.24. Direct axis Single-phase transient reactance flux paths.



ARMATURE CURRENT ON SUDDEN SINGLE-PHACE

LINE-TO-LINE SHORT CIRCUIT

FIG. 25

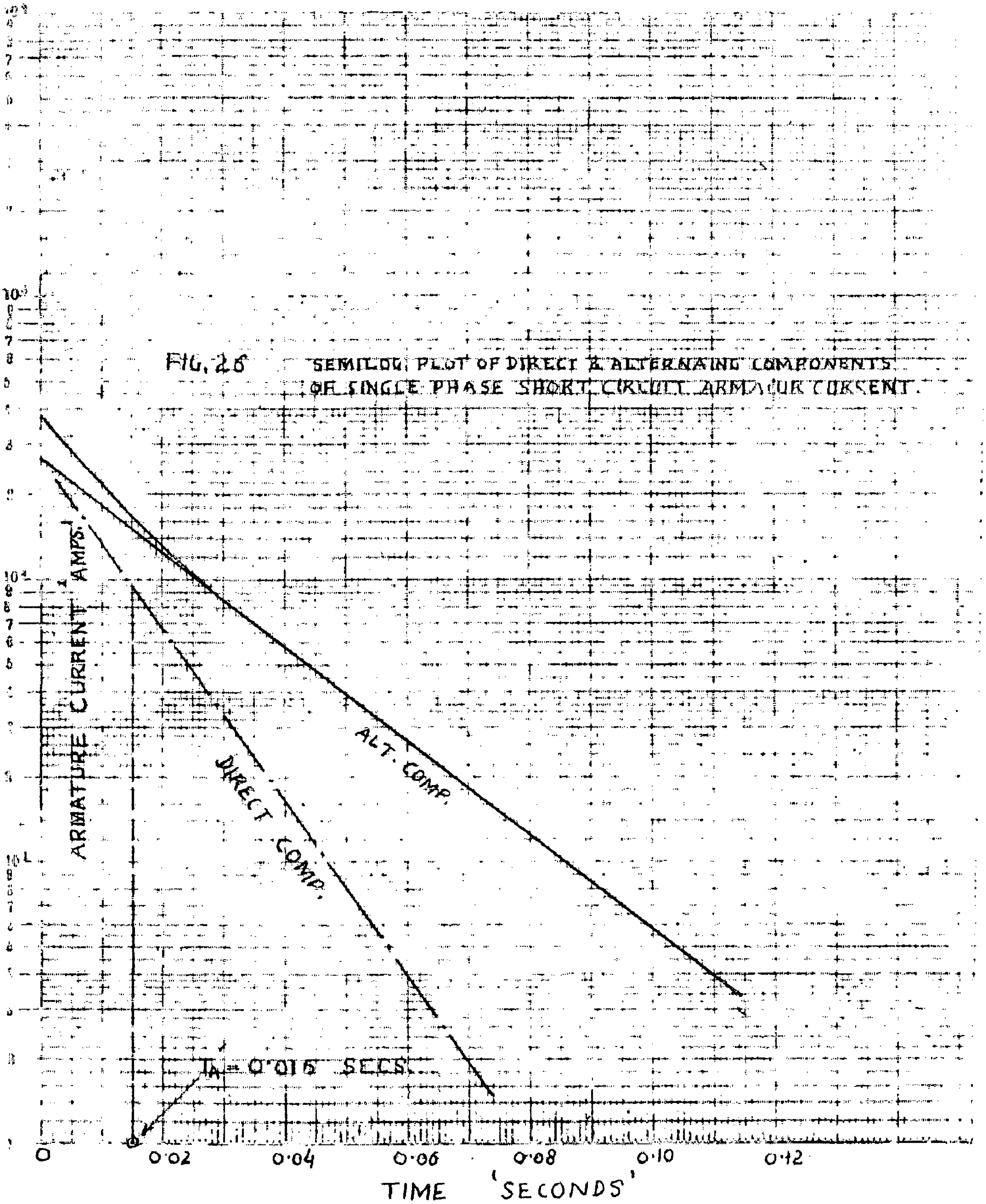


FIG. 26 SEMILOG PLOT OF DIRECT & ALTERNATING COMPONENTS OF SINGLE PHASE SHORT CIRCUIT ARMATURE CURRENT.

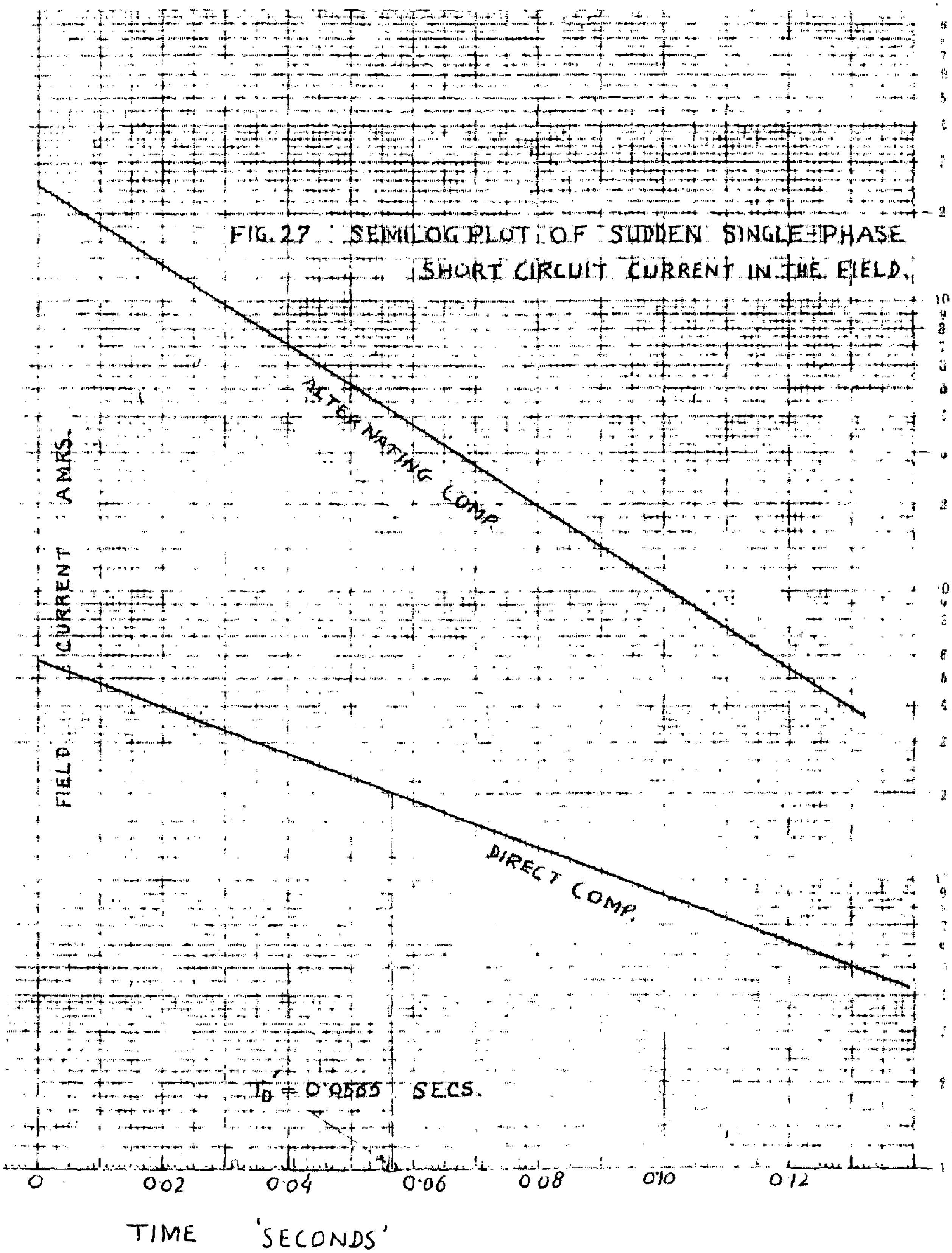


FIG. 27 SEMILOG PLOT OF SUDDEN SINGLE-PHASE SHORT CIRCUIT CURRENT IN THE FIELD.
ALTERNATING COMP.
DIRECT COMP.
FIELD CURRENT AMPS.
T_D = 0·0565 SECS.
0
0·02
0·04
0·06
0·08
0·10
0·12
TIME 'SECONDS'

Direct-axis Single-phase Sub-transient Reactance (X_D''):

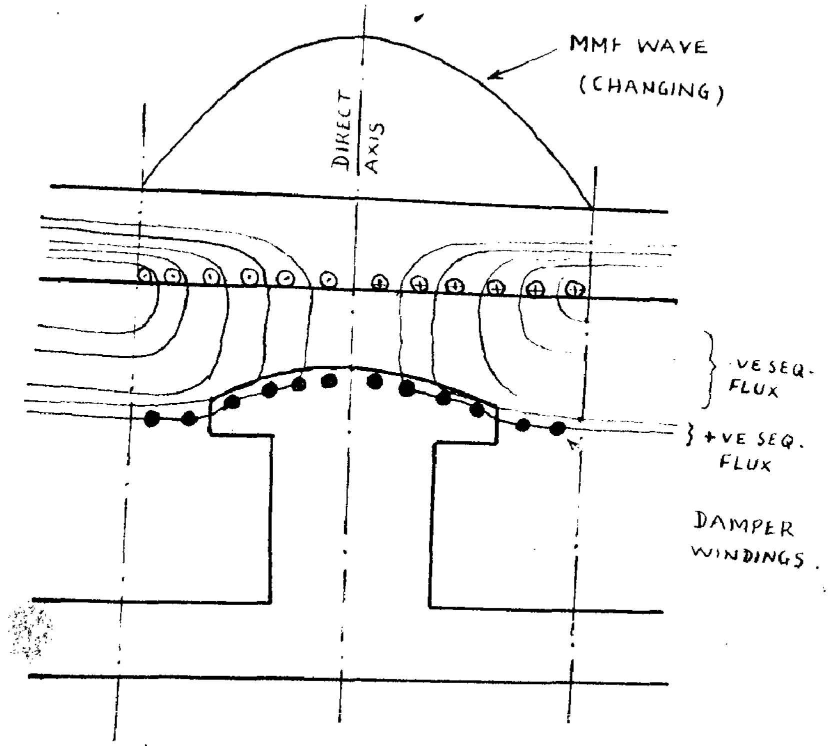


Fig. 28 Direct axis single-phase sub transient reactance flux paths.

The additional field winding is the damper winding is considered short circuited. The positve sequence flux links with these additional rotor windings. Since the resistance of the dampers is large the current in it dies out very rapidly. The negative sequence flux acts as a leakage flux.

The single-phase armature short-circuit oscillogram of fig.25 was used to determine the X_D''. Subtransient current was determined from the semi log plot of alternating component of armature current by usual method. X_D'' was calculated as follows[7]

Subtransient current I = 41.3A

Voltage before short circuit= 116V.

Therefore,

$$X_D'' = \frac{\sqrt{3} \times 116}{41.3} = \underline{0.402} \text{ per unit}$$

From three-phase results-

$$X_D'' = X_d'' + X_2$$

$$= \underline{0.397} \text{ per unit}$$

Time Constants:

Open Circuit Transient time constant for single-phase Machine (T'_{DO})

The method used for determining the open circuit transient time constant for three-phase machine (T'_{do}) was that the armature terminals were kept open and decay of field current was recorded after sudden short circuit was applied to field terminals. The armature terminals were open and thus no effect of armature circuit on the decay of field current. The same phenomena occurs in case of single-phase synchronous machine. For the machine undertest both three-phase and single-phase open circuit time constant will be the same.

$$T'_{Do} = T'_{do} = \underline{0.117} \text{ secs.}$$

Single-phase short circuit transient time constant (T'_D):

For the purpose of determining single-phase short circuit field time constant the oscillogram of transient field current was recorded when sudden single-phase short circuit was applied to the armature terminals. The Oscillogram is shown in fig.14. The semi log plot of direct component of field current was made and from there the short circuit time constant was determined by taking the time of decay of 36.8% of maximum value of the direct component of field current in transient condition. It comes out to be-

$$T'_D = \underline{0.0565} \text{ secs.}$$

It can be shown that[8]

$$T'_D = \frac{X'_D}{X_D} \cdot T'_{do}$$

Therefore,

$$T'_{do} = \frac{X_D}{X'_D} \cdot T'_D$$

$$= \frac{1.04}{0.51} \times 0.0565$$

$$= \underline{0.117} \text{ secs.}$$

The results is in confirmation with the experimental value.

Single-phase armature time constant:

T'_A can easily be determined from the armature short circuit oscillogram of fig.25. The semi-log curve of direct component is plotted. From that it comes-

$$T'_A = \underline{0.015} \text{ secs.}$$

C_O_N_C_L_U_S_I_O_N_S

The steady-state analysis of the single-phase synchronous machine, on the basis of the classical assumptions of no saturation, no stator eddy and hysteresis losses shows that the line-to-line voltage equation is identical to the voltage equation of a three-phase synchronous machine with the addition of the third harmonic term. In actual practice the effect of this third harmonic voltage is not at all pronounced and does not effect in any way the performance of the machine and may safely be neglected. However, when precise calculations are needed the effect of third harmonic voltage is to be taken into account.

The single-phase synchronous machine voltage equation leads to a vector diagram which is similar to that of a three-phase synchronous machine. The excitation diagram applicable to the single-phase machine is also drawn exactly in the same manner as for three-phase machines. In the construction of the vector diagram the third harmonic voltage, appearing in the voltage equation, is not considered and as stated earlier it does not introduce any appreciable error.

The power equation for the single-phase synchronous machine also proves to be exactly identical to that of three-phase machine. Since the negative sequence reactance always occurs together with the equivalent three-phase direct and quadrature axis reactances in the denominator, it reduces the over all power of the machine. The percentage reluctance power is increased. It helps to hold the machine under the over load conditions more efficiently than the three-phase machine. In other words it increases the pull-out angle of the single-phase synchronous machine. Experiment shows that the pull-out angle is increased by about 5%. This leads to a more stable machine than a three-phase synchronous machine.

Performance of the single-phase synchronous machine is calculated with the help of the vector diagram of fig.3. The check is made by measuring δ experimentally and noting the field current at particular load. The values of the field current calculated from the vector diagram and that measured experimentally agree to a very close extent. In all the performance calculations the resistance was neglected.

Due to the presence of the negative sequence current in the armature even harmonic currents are induced in the field and odd harmonic currents in the armature. From the expression derived for them, it isobvious that the harmonic currents go on reducing with the increase of order. For the purpose of calculations, only first few harmonics terms are considered. The similarity between the expression of field and armature currents of the winding with 90° displacement with those for windings with 120° space displacement is quite obvious. Hence it may be possible to construct an economical single-phase synchronous machine with windings of the armature having 90° angle between them.

The equation for the transient field current is obtained with the help of the expression for field-current in steady-state and considering the effect of transient positive sequence armature current on the field current. The final expression includes a direct component, an alternative component of normal frequency and even harmonic components resulted by the negative sequence armature current, in the maximum flux linkage position ($\theta = 0$). For the rotor position when the flux linkage is minimum ($\theta = \pi/2$) the normal frequency alternating component will be absent since there is no direct component in the transient armature current in this particular position.

To check the validity of the transient field current expressions, oscillogram of field current is recorded. A comparision of the calculated wave-form of transient field-current for $\theta = 60°$ and the experimental oscillogram of fig.14 shows that the waveforms obtained practically and theoretically are the same. The calculated waveforms at maximum and minimum flux linkage positions are also given (Fig.15A & 16). If oscillograms are recorded in these particular positions, they must conform to the calculated wave forms.

Through-out the theory of the single-phase synchronous machine it is observed that the equivalent three-phase reactances always appear together with the negative sequence reactance. The single-phase reactances are defined as the addition of equivalent three-phase reactance and negative sequence reactance. From the theoretical considerations it is obvious that they can be measured as a whole when running a three-phase machine as a single-phase machine. The results thus obtained are in close agreement to those obtained from equivalent three-phase results.

The negative sequence flux paths are in the air and so they do not link with any of the windings to create any mutual effects. Hence the negative sequence flux can be considered to add with the leakage flux of the machine. In dealing with the physical signfficance of the single-phase reactances the negative sequence flux is taken as the leakage flux. The positive sequence flux plays the role of the main flux of the machine. Methods have been suggested to determine the constants. The close agreement between the results obtained from the suggested methods and otherwise confirm the validity of the experimental determinations.

APPENDIX

THREE-PHASE CONSTANTS

The machine under test is rated as-

4 kva	3.2 kw.
220/110 Volts	10.5/21 Amps.

0.8 power factor, 3 phase, 1000 r.p.m.

The magnetizing curve is hown in the fig.17. The short circuit test was performed and armature short circuit current was plotted against field-current. The straight line is shown in fig.17. The machine is used as a three-phase synchronous machine and all the constants are determined employing the usual methods of testing the synchronous machine for different constants. The three-phase constants thus determining are as follows: (7,9,10)

X_d = 0.79 p.u.

X_q = 0.48 p.u.

X_q/X_d = 0.605

X'_d = 0.256 p.u.

$X'_q = X''_q$ = 0.36 p.u.

X''_d = 0.145 p.u.

X_2 = 0.252 p.u.

T'_d = 0.038 secs.

T'_{do} = 0.117 secs.

The short-circuit time constant for three-phase machine was determined by short circuiting the field suddenly when the armature was short-circuited at its terminals and machine running at synchronous speed. The oscillogram is shown in fig.20 and semilog plot of the decaying field current is shown in the fig.21.

R_E_F_E_R_E_N_C_E_S.

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....

was submitted by

..................HIMAYAT ULLAH BEG..................

and accepted for the award of Degree of Master of Engineering in..................

..................ADVANCED ELECTRICAL MACHINES..................

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vide Notification No. Ex/150/P-65(Degree)..................

dated October 31, 1966.

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Assistant Registrar (Exam.)

5.XI.66

Dated November 3, 1966.

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The negative sequence flux here also is considered a purely leakage flux. In the transient state the field is considered short circuited and the transient positive sequence armature flux links with this field winding and behaves exactly in the same manner as that in case of three-phase, synchronous machine. The alternating component of current in armature in transient state will be of positive and negative sequence nature. The values of transient sequence currents can be determined from oscillogram of sudden single-phase armature short circuit.

For determination of single-phase transient reactance X_D', single-phase sudden armature short circuit oscillogram was recorded as shown in fig.25, and transient alternating current was calculated therefrom in usual manner. The semi-log plot of the transient alternating component was drawn as shown in fig.26. Thus the total transient alternating current was known. The positive sequence transient current wastaken as $1/\sqrt{3}$ times the total transient current.

Total transient current = 32.7A

Positive sequence transient alternating current $= \frac{32.7}{\sqrt{3}}$

Voltage before short circuit = 116V.

Therefore,

$$X_D' = \frac{\sqrt{3} \times 116}{32.7} = 6.14 \text{ ohms}$$

$$= \underline{0.507} \text{ per unit}$$

From three-phase results-

$$X_D' = X_d' + X_2$$

$$= \underline{0.508} \text{ per unit}$$