INVESTIGATION OF THE EFFECT OF ROTOR BAR SKEW ON THE PERFORMANCE OF AN INDUCTION MOTOR

By

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CERTIFICATE

CERTIFIED that the dissertation entitled "Investigation of the Effect of Rotor Bar Skew on the Performance of an Induction Motor" which is being submitted by Shri O.P.Garg in partial fulfilment for the award of the Degree of Master of Engineering in 'Advanced Electrical Machines' of the University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for a period of 5-months from November 1965 to March 1966 for preparing this dissertation for Master of Engineering Degree at University of Roorkee, Roorkee.

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S Y N O P S I S

Harmonic effects on skewed Induction machine are analysed by means of digital computer using punched card technique. Analytical methods are developed to ascertain the effect of skew over current, power factor, torque and other undesirable features like dead points, crawling, cogging etc., resulting from wide variety of harmonics of machine . In addition to the conventional space harmonics, permeance harmonics which are likely to produce appreciable effects are also considered.

Attention has also been paid to the influence of space harmonics on noise too.

It is shown here that, the skewing improves the performance and reduces the noise to appreciably very low degree.

INTRODUCTION

Harmonics have been known to be the source of trouble in electrical machines for many years.

For almost as long as Engineers have been designing induction machines, they have attempted to suppress harmonics to make it trouble free. The interest has given rise to large number of articles on the subject. Most of the significant articles appearing before 1947 are limited to phase belt harmonics only.

A review of the literature of the subject shows, the wide acceptance of the view that slot ripples in the electrical machines are caused by the interaction of rotor permeance harmonics with those set up by non-sinusiodal distribution of stator winding, non-uniformity of air gap and slotted surfaces of stator and rotor. It is shown for that very high slot frequency currents are induced in rotor circuit, but these high frequency currents are not the problem as the magnitude of such currents is not generally appreciable so these can be overlooked easily. The old reasoning of non-integral number of slots being the cause of higher harmonics was based on incomplete understanding and has been ruled out by Carter, Kron(25), Hearty(16), Walker(19), supported by mathematical theory.

Attempts have been made by Alger(17,18), to study the effects of principal harmonics of the air gap field neglecting saturation and saturation harmonics by means of equivalent circuit. The harmonics deteriorate the performance in respect of current, torque, power factor. Not so much so but sometimes, it is impossible to start the motor due to stand still locking (dead points). Even if the motor starts, it fails to come to speed due to harmonic parasite torques and soon gets heated up due to poor power factor and high currents. Therefore, the problem of reduction or elimination of harmonics have gathered large attention and, one of these suggestions in addition to others frequently advocated for squirrel cage rotor is skewing of rotor bars. The interest in the subject is being revived at present by the need for precision electric machines free from magnetic (32) noise and starting torque trouble etc.

The phenomena of harmonics in 3 phase induction machines can be better understood by Block diagram showing, the relations of harmonic fluxes and voltages. The block diagram (see adjacent page) shows considerable similarity with induction machine free from harmonics when operating, in the balanced steady state. The diagram gives a diagrammatic representation of quantitative equations, where the relationship between the quantities such as flux, emf, current etc., are treated as cause and effect sequence. Quantities which react back to modify earlier quantities are treated as cases of feed back.

The generality of the concepts used here has the important advantage that by a process of physical reasoning, it provides a means of arriving quickly at a qualitative understanding of the machine harmonic phenomena, which arises to predetermine the exact behaviour of the machine. The machine is further assumed to work in unsaturated region.

The recent developments in computing techniques over computer have made possible large scale computations pertaining to machines which otherwise would be out of the question. These techniques make a frontal assault on long standing problems feasible; this work is a report of such an investigation. The basic purpose of this work is to present qualitative as well as quantitative approach to the behaviourial complexity of harmonics.

Performance curves for useful torques, parasitic torques due to a number of harmonics, currents, power factor and speed have been computed for full range of the skewed rotor machine. Similar calculations are also performed, neglecting the skew of the rotor. The results so obtained draws attention to the improvments which the skewing can introduce in starting as well as in running conditions.

Noise[30] in electrical machines is also equally retrogative effects of space harmonics. All the harmonics produce air gap forces and there by contribute to noise and vibration. The author has attempted to predict the noise level of the machine inspite of the voluminious calculation involved. Noise spectrum and sound intensity curves have been computed.

And lastly, the influence of further skewing of rotor bar over parasitic torques starting torque, etc., is analytically studied and thus criteria for most proper skew angle in general and for the 'test - model' is suggested.

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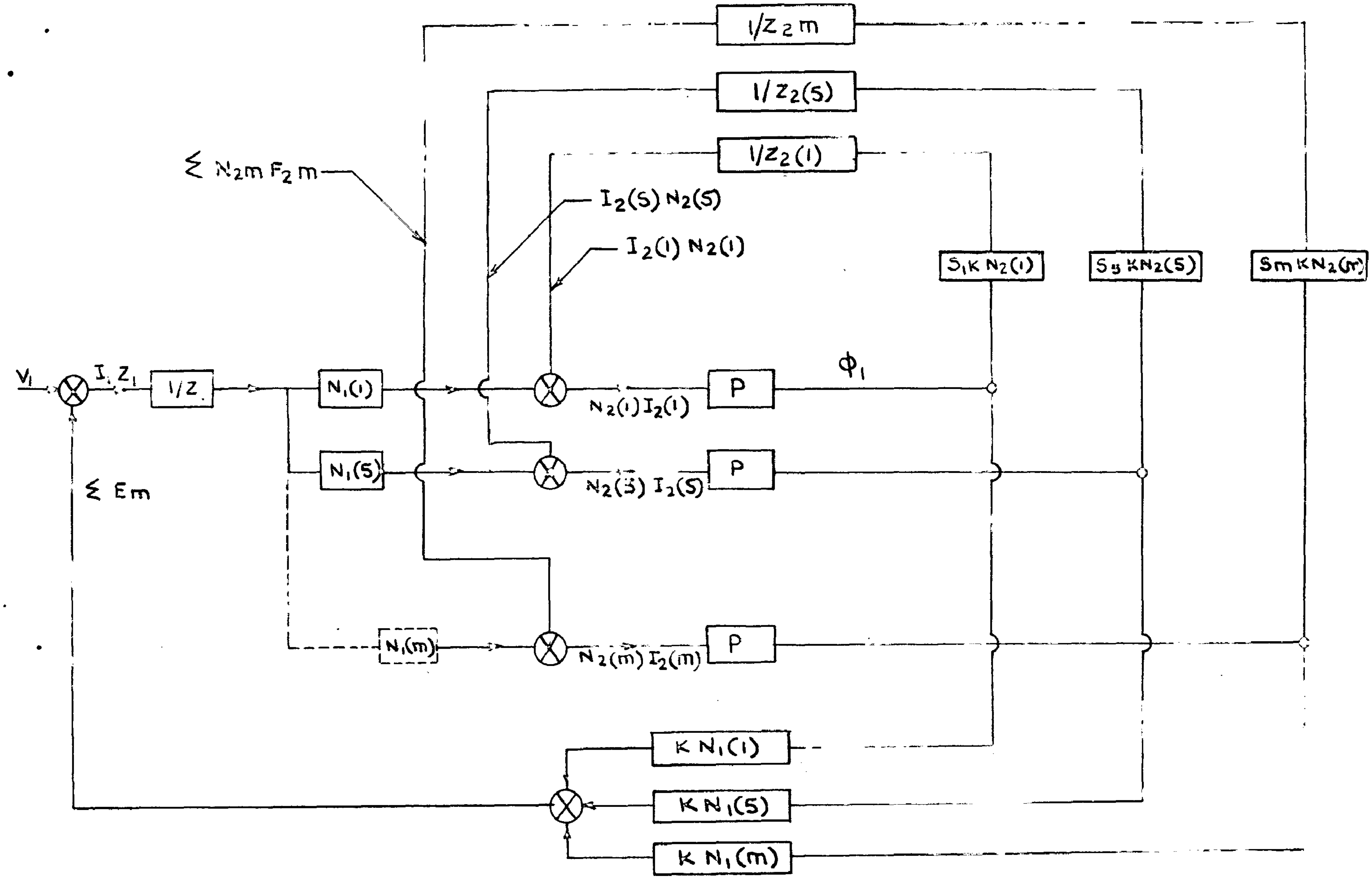


BLOCK DIAGRAM OF HARMONIC FLUXES

C O N T E N T S

LIST OF SYMBOLS

F, f	Magnato motive force
I, i	Current.
$(A)_i$	mmf. constant.
θ	Space angle
mb, mf	Forward, backward harmonic (mth)
m	Order of space harmonic
mp	order of permeance harmonic
m_s	Order of slot harmonic
w	Radians / sec.
t	Time.
γ	Slot angle
k	number of phase
g_o, l_g	air gap length
B, b	flux density
R	number of rotor slots
S	Number of stator slots
p	Pair of poles
P	Number of Poles
P_s	Stator permeance factor
P_r	Rotor permeance factor
v_1, v_2	Order of permeance wave.
e_s, e_r	Ratio of R.M.S. to average values of stator and Rotor resp.
K	Constant
ϕ	Flux
ψ	Flux linkage
c	Integral number
K_1, K_2	Average Mean Permeance.

s	Slip
T_p	Number of stator turn per phase.
k_{w1}	Primary winding factor
k_{w2}	Secondary winding factor
D	Diameter of stator
D_m	Damping factor for mth harmonic (primary currents)
$G_{(m)}$	Damping factor for mth harmonic (Rotor current)
K_1 , K_2	Average mean permeance.
k'	Integral constant
n, N	Number of turns
X_M	magnetising reactance
X_{l2}, X_2	Secondary leakage reactance referred to primary
X_{l1}, X_1	Primary leakage reactance.
L	Length of Stator Core
y	Distance along y - axis
α, σ, θ_{sk}	Skew angle
m_1, m_2	Number of stator and rotor phases
λ_g	Air gap flux linkage
β	Coil span
K_s	Skew factor
μ	Permeability factor
L_{11}, L_{22}	Self inductances
M_{12}, M_{22}	Mutual inductance
e, E	Induced emf
v, V	Applied voltage
N_{sm}	Speed of mth harmonic
X_z	Zig Zag leakage reactance

$K_{(m)}$	Skin factor for mth harmonic resistance.
$J_{(m)}$	Reactance, skin factor
CSC	Cosec
n	Order of rotor harmonics
∇_c	Coil span angle
$P(\theta)$	Permeance
r_1, R_1	Stator resistance per phase
r_2, R_2	Rotor resistance per phase (refer to primary)
X_e	Coil end leakage
$X\alpha$	skew leakage flux
K_R	$\frac{\beta}{2-\beta}$
α_s	Slot angle
h_1, h_2, h_3, h_4	Height of sections of slots.
w_s	Width of slot
w_o	Opening of slot
λ	Slot pitch cms.
C	Constant defined by equation (4.26)
P_d	Power developed (syn watts)
$T_{(m)}$	Torque developed of mth harmonic (Syn Watts).
N_s	Syncronous speed (Syn. r.p.m.)
N_m	Speed of mth harmonic
$s_{mP.o}$	Pull out slip (for mth harmonics)
V_{1na}	Speed of nath harmonic
$\mathcal{T}_m$	X_2/X_M
τ	POLE pitch
q	Number of phase belts

Part II

p	r.m.s. pressure in dynes $/cm^2$
d'	Density of air
v	Velocity of sound in air cms/sec.
w	Watts/cm^2
h	Radial depth of the stator core behind the slot in inches.
D_s	Mean diameter of the stator core
W	Load in lbs.
k_{ro}	Correction factor.
F_r, f_r	Radial force lbs.
d	Deflection in micro inch
I_d	Sound intensity (dbs)
f	Frequency
p'	Force wave poles
E	Modulas of Elasticity
r	Phase number.

CHAPTER I

HARMONIC ANALYSIS OF INDUCTION MACHINE.

Since induction motors were the first built, it has been noticed that some motors behave erratically, some do not go up to full speed but 'crawl' at a lower speed or are noisy in certain regions and are useless for all commercial application. This behaviour was attributed to the presence of harmonic in the air gap but the conclusions drawn from these analysis were at variance with the observed facts. The slot openings have also been suggested as a possible cause of these irregularities. However, in the analysis of slot openings either one of the surfaces was assumed to be smooth as emphasis is paid only on the change in the fundamental flux due to the presence of slots.

Analysists arrived at some emperical relations for the selection of rotor and stator slots some of which were mathematically supported(25) at a latter stage. By using such emperical or semi-emperical rules they were reasonably safe from unpleasant surprises.

The present chapter presents the review of what has been achieved till now in the field of treatment of harmonics of non-salient electrical machines. For our purpose of determining the different types of parasitic torques e.g., syncronous, crawling torques, dead points, vibrating forces and other desirable effects produced by harmonics, we will consider all the possible harmonics of desirable orders and magnitudes likely to effect the normal performance of the machine to an appreciable degree.

Figures written in paranthesis denotes the serial number of references given at the end.

The possible harmonics are:

1. Space Harmonics produced by the non-sinusiodal distribution of the winding known as phase belt harmonics.

2. Space Harmonics due to slotting of stator and rotor surfaces.

3. Permeance Harmonics introduced by the non-uniform air gap of the machine. As these harmonics are produced in collusion of air gap variance harmonics and higher order harmonics of stater and rotor, these harmonics are also function of number of slots of both sides. Generally permeance harmonics are of very low amplitude.

4. And lastly the time harmonics present in the supply wave and which consequently give rise to other higher order of space and time harmonics. But for our purpose we will assume the supply a sinusiodal free from any time harmonics.

1.1. PHASE BELT HARMONICS

1.1.1. Order of Harmonics

It is shown that each distribution of winding may be resolved into space harmonics, beginning with a fundamental and the higher harmonics. Fig. (1.1) represents the third and fifth harmonics in addition to fundamental of a winding in which number of slots per pole per phase $q = 4$. slot angle $\gamma = 15°$ with no chording.

Taking 0 as reference point, the space distribution of any harmonic mmf due to current i_1 in phase I may be represented by

$$f_1 = (A)_1 \cos (m\theta)$$

where $i_1 = I_m \cos (wt)$

$$\therefore \; f_1 = (A)_1 \cos(wt) \cos (m\theta)$$

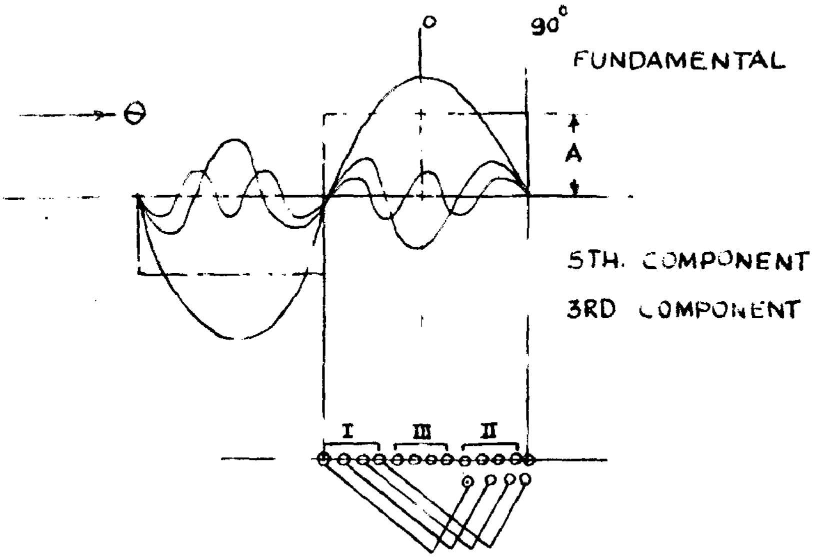


FIGURE 1.1

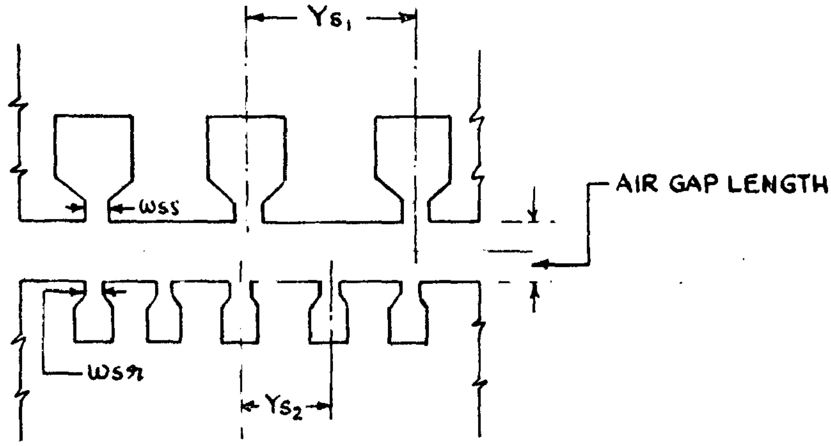


FIGURE 1.2

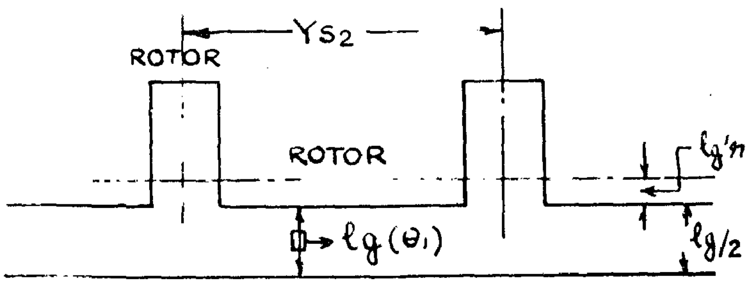


FIGURE 1.3

Similarly
$$f_2 = A_1 \cos\left(m\theta - \frac{2\pi}{k}\right) \cos\left(wt + \frac{2\pi}{k}\right)$$

$$f_3 = A_1 \cos\left(m\theta + \frac{2\pi}{k}\right) \cos\left(wt - \frac{2\pi}{k}\right)$$

The resultant distribution for mth harmonic is

$$F_{(m)} = A_m \sum_{r=1}^{r=k} \left\{ \cos\left[m\theta + wt - (r-1)(m+1)\frac{2\pi}{k} \right] + \cos\left[m\theta + wt - (r-1)(m-1)\frac{2\pi}{k} \right] \right\}$$

Summing it up for 'k' phases

$$F = \frac{A_m}{2} \left\{ \frac{\sin(m+1)\pi}{\sin(m+1)\frac{\pi}{k}} \cos\left[m\theta + wt - \frac{(k-1)(m+1)\pi}{k} \right] \right\}$$

$$+ \frac{A_m}{2} \left\{ \frac{\sin(m-1)\pi}{\sin(m-1)\frac{\pi}{k}} \cos\left[m\theta - wt - \frac{(k-1)(m-1)\pi}{k} \right] \right\} \quad \ldots.(1-1)$$

Where

m = order of space harmonic due to distributed winding.

k = number of phases

θ = space angle

A_m = Maximum magnitude of mth harmonic

Finding the limiting conditions of the equation (1-1)

$$\frac{\sin(m-1)\pi}{\sin(m-1)\frac{\pi}{k}} = \frac{0}{0} = k \quad \ldots\ldots(1-2)$$

If $(m-1)$ is multiple of k

Similarly $$\frac{\sin(m+1)\pi}{\sin(m+1)\frac{\pi}{k}} \quad \frac{0}{0} = k \quad \ldots.(1-3)$$

If $(m+1)$ is multiple of k.

1.1.2. Velocity and Direction of rotation of Harmonics.

Differentiating the angular function of cosine terms of equation (1-1)

$$m\frac{d\theta}{dt} = -w \qquad \therefore \frac{d\theta}{dt} = -\frac{w}{m} \tag{1-4}$$

$$m\frac{d\theta}{dt} = +w \qquad \frac{d\theta}{dt} = \frac{w}{m} \tag{1-5}$$

Therefore the first cosine term of equation (1-1) gives only Backward rotations and second cosine term gives 'forward rotating' harmonies . Tabulating the possible harmonics thus produced in space with $k = 3$ (number of phase) with respective speeds and direction of rotation.

Table (1-1) Speeds and Directions of Rotation of Components of Armature mmf of 3 Phase windings.

Order of space Harmonics	Speed & Direction.	Order of space Harmonic	Speed & Direction
1	+1	29	$-\frac{1}{29}$
5	$-\frac{1}{5}$	31	$+\frac{1}{31}$
7	$+\frac{1}{7}$	35	$-\frac{1}{35}$
11	$-\frac{1}{11}$	37	$+\frac{1}{37}$
13	$+\frac{1}{13}$	41	$-\frac{1}{41}$
17	$-\frac{1}{17}$	43	$+\frac{1}{43}$
19	$+\frac{1}{19}$	47	$-\frac{1}{47}$
23	$-\frac{1}{23}$	49	$+\frac{1}{49}$
25	$+\frac{1}{25}$	53	$-\frac{1}{53}$

1.2. PERMEANCE HARMONICS

1.2.1. Order of Stator Permeance Harmonics

First it is assumed that both the stator and rotor teeth are removed. The permeance of the resulting gap is Po (constant) and is represented on figure by unit height.

Let the stator teeth be introduced. At certain places the length of the gap is decreased and permeance increased. This change of permeance can be considered as if an additional permeance Ps consisting of rect angles had been introduced with the stator teeth. Similarly let now the stator teeth be removed and revolving rotar teeth be introduced This is equivalent to introducing an additional revolving permeance Ps.

If both teeth are introduced, at those points along the circumference where no stator teeth exist, permeance due to the rotor teeth is the same as Ps but at those points where both the teeth exist the permeance would be much different. The net permeance is given by the product of two permeance functions. Writing for the mmf F as $f(\theta_1 t)$ as it is function of θ and time. Assuming unsaturated conditions, the air gap flux density at the stator surface is

$$b(\theta_1,\theta_2,t) = f(\theta_1, t)\, P(\theta_1, \theta_2) = \frac{f(\theta_1, t)}{l_g(\theta_1, \theta_2)} \qquad (1\text{-}6)$$

Where

$b(\theta_1, \theta_2, t)$ = flux density which is function of rotor position θ_2, stator position θ_1 and time t.

$l_g(\theta_1, \theta_2)$ = length of the airgap which is a function of motor position and stator position.

The permeance function (a term chosen by the author to avoid using the longer term 'reciprocal air gap length function') is defined as $1/l_g(\theta_1 , \theta_2)$

The exact solution to the equation (1-6) is very complex involving three variables θ_1 , θ_2 and t. $P(\theta_1 , \theta_2)$ permeance function itself consist of infinite series terms , but the function can be simplified by introducing an assumption. The assumption is that there exists in the air gap of any machine a magnetic equipotential surface of cylinderical fasion. This assumption is well founded in machines with semi-closed slots, it may not hold good for salient machines.

If this postulate is permitted,(16) all of the effects of stator slots may be measured with respect to the equipotential surface without reference to rotor and all of the rotor effects without reference to the stator. Thus it may be written

$$l_g (\theta_1 , \theta_2) = l_{gs}(\theta_1) + l_{gr}(\theta_2)$$

when

$l_{gs}(\theta_1)$ = length of the air gap from equipotential surface to the stator and a function of θ_1 alone.

$l_{gr}(\theta_2)$ = length of the air gap from equipotential surface to the rotor and a function of θ_2 alone as represented in Fig. (1-3).

Each of these terms l_{gs} and l_{gr} may be expanded as infinite series.

$$l_g(\theta_1,\theta_2) = l_{go} + \sum_{p=1}^{\infty} l_{gpr} \operatorname{Cos} pR\theta_2 + l_{gpr'} \operatorname{Sin} pR\theta_2$$

$$+ l_{gps} \operatorname{Cos} ps\theta_1 + l_{gps'} \operatorname{Sin} ps\theta_1 \qquad (1\text{-}7)$$

Where

l_{go} = mean effective air gap length.

l_{gpr}, l_{gps}'', l_{gps}, l_{gps}' = harmonic coefficients of air gap length.

R & S = number of rotor and stator slots respectively.

By means of binomial theorem it can be shown that

$$P(\theta_1, \theta_2) = P_o + P_r + P_s \tag{1-8}$$

where

P_o = mean effective permeance.

P_r = additional permeance due to rotor slotting.

P_s = additional permeance due to stator slotting.

If P_1 = permeance of the air gap assuming the rotor smooth and stator slotted with S slots.

P_2 = Permeance of the air gap assuming the stator smooth and rotor slotted with R slots.

The two permeance equations derived with the help of conformal - Transformation (see Appendix I) Theory are:-

$$P_1 = k_1 \left[1 + \sum_{v_1=1}^{\infty} \frac{es}{v_1} \operatorname{Cos}(v_1 S\theta) \right] \tag{1-9}$$

$$P_2 = k_2 \left[1 + \sum_{v_2=1}^{\infty} \frac{er}{v_2} \operatorname{Cos}(v_2 R\,\theta') \right] \tag{1-10}$$

Where,

v_1 = order of permeance harmonic with rotor smooth or in brief stator permeance harmonic order.

v_2 = Rotor permeance harmonic order.

es, er = Ratio of R.m.s to average values of stator and rotor respectively.

$$f(\theta_1,t) = \frac{3}{2} A_m \sum \text{Cos}\,(m\theta_1 \mp wt)$$

defining a new space variable angle in terms of number of pair of poles (p) as

$$\theta_1 = p\,\theta$$

then
$$f(\theta,t) = \frac{3}{2} A_m \sum \text{Cos}\,(mp\theta \mp wt) \tag{1-11}$$

$$\phi_s = f(\theta, t) \times P_1$$

$$\phi_s = \frac{3}{2} A_m k_1 \sum \text{Cos}\left[pm\theta \mp wt\right]\left[1 + \sum_{v_1}^{v_1=\infty} \frac{2a}{v_1} \text{Cos}(v_1 S\theta)\right]$$

$$\phi_s = \frac{3}{2} A_m k_1 \sum_m \text{Cos}\,(pm\theta \mp wt)$$

$$+ \frac{3}{2} A_m k_1 \frac{2a}{v_1} \sum_m \sum_{v_1} \text{Cos}\,(pm\theta \mp wt)\,\text{Cos}\,(v_1 S\theta)$$

$$\phi_s = \phi_0 + \phi\,P_m$$

$$\therefore \phi\,P_s = \sum_m \sum_{v_1} \tfrac{1}{2} \left\{ \text{Cos}\,(pm\theta \mp wt - v_1 S\theta) + \text{Cos}\,(pm\theta \mp wt + v_1 S\theta)\right\}$$

$$\phi_{Pm} = \sum_m \sum_{v_1} \tfrac{1}{2} \left\{ \text{Cos}\left[\left(\frac{v_1 S}{p} - m\right)p\theta \pm wt\right] + \text{Cos}\left[\left(\frac{v_1 S}{p} + m\right)\theta \mp wt\right]\right\}$$

.....(1-12)

$\therefore$ mth phase-belt harmonic when combined with v_1th permeance harmonic give rise to two new space harmonics which ~~are~~ we will cal as 'Permeance travelling wave' of the order of

$$\left(\frac{v_1 S}{p} \mp m\right) \quad \text{travelling at the speed of}$$

$$\frac{\pm w}{\frac{v_1 S}{p} - m} \quad \text{and} \quad \frac{\mp w}{\frac{v_1 S}{p} + m} \quad \text{i.e., one in forward direction}$$

and other in backward direction.

The permeance travelling waves will influence the magnitude of flux density of mth belt harmonic (b_m) with respect to which they are stand still. These are the stator -slot harmonics . To each stator harmonic b_m there corresponds a series of permeance travelling waves.

The order of the slot harmonics of stator is given by the relation

$$m_s = \pm C'\left(\frac{S}{p}\right) + 1 \quad \text{where } C' = 1,2,3,\ \ldots\ldots\ldots \tag{1-13}$$

and permeance travelling wave order is

$$m_p = v_1\left(\frac{S}{p}\right) \pm 1 \quad \text{or} \quad \pm\, v_1\left(\frac{S}{p}\right) + 1 \tag{1-14}$$

Comparing equations (1-13), (1-14) it is seen that the permeances waves of first order ($v_1 = 1$) are at stand still with respect to slot harmonics of first order ($C' = 1$) and similarly second order permeance wave are stand still with 2nd order slot harmonics ($C' = 2$).

The table (1-2) gives the order of stator permeance harmonics produced by air gap permeance harmonics ($v_1 = 1, 2, \ldots\ldots\ldots$). For our purpose we will limit ourselves to $v_1 = 1, 2$ i.e. first and second order air gap permeance components and the resultant stator permeance harmonics are listed in Table (1-2).

TABLE (1-2)

v_1	m	m_p	Speed of m_p
1	1	23	-1/23
		25	+1/25
2	1	47	-1/47
		49	+1/49
1	5	19	+1/19
		[illegible]	[illegible] (Contd.)

Table (1-2) Contd..

v_1	m	m_p	Speed of m_p
2	5	43	+1/43
		53	+1/53
1	7	17	-1/17
		31	+1/31
2	7	41	-1/41
		55	+1/55
1	11	13	+1/13
		35	-1/35
2	11	35	+1/35
		59	-1/59
1	13	11	-1/11
		37	+1/37
1	17	7	+1/7
		41	-1/41
1	19	5	-1/5
		43	+1/43
1	23	1	+1
		47	-1/47

1.2.2. Rotor Permeance Harmonics

While finding the rotor permeance harmonics, more accurately rotor permeance travelling waves, it is assumed that only rotor surface is slotted and stator surface is smooth, then permeance equation is:

$$P_2 = K_2\left[1+ \sum_{v_2} \frac{eR}{v_2} \cos\left(\frac{v_2}{p} R\theta - w_2 t\right)\right] \qquad (1\text{-}15)$$

Where

R = number of rotor slots.

w_2 = Radians/sec.

v_2 = order of permeance component.

The mmf at the stator surface is

$$f(\theta, t) = \frac{3}{2} A_m \sum_m \cos(pm\theta \mp wt)$$

$$\therefore \; \phi(r) = P_2 \times f(\theta, t) = \frac{3}{2} K_2 A_m \sum \cos(pm\theta \mp wt)$$

$$+ \frac{3}{2} K_2 A_m \frac{eR}{v_2} \cos(pm\theta \mp wt) \cos\left(\frac{v_2 R}{p}(p\theta - w_2 t)\right)$$

$$\ldots\ldots(1\text{-}16)$$

$$\phi_r = \phi_{ro} + \phi_{rp}$$

$$\therefore \phi_{rp} = K \cos(pm\theta \mp wt) \cos\left[\frac{v_2 R}{p}(p\theta - w_2 t)\right]$$

Substituting

$$\frac{w - w_2}{w} = s \text{ (slip)}$$

$$w_2 = w - ws$$

Simplifying it

$$\phi_{rp} = \frac{K}{2} \cos\left\{p\theta \left(\frac{v_2 R}{p} + m\right) \mp wt\left[\frac{v_2 R}{p}(1-s) \pm 1\right]\right\}$$

$$+ \frac{K}{2} \cos\left\{p\theta \left(\frac{v_2 R}{p} - m\right) \pm wt\left[\frac{v_2 R}{p}(1-s) \mp 1\right]\right\} \qquad (1\text{-}17)$$

$$P_2 = K_2\left[1+ \sum_{v_2} \frac{es}{v_2} \text{Cos}\left(\frac{v_2}{p} R\theta - w_2 t\right)\right] \quad (1\text{-}15)$$

Where

R = number of rotor slots.

w_2 = Radians/sec.

v_2 = order of permeance component.

The mmf at the stator surface is

$$f(\theta, t) = \frac{3}{2} A_m \sum_m \text{Cos}\,(pm\theta \mp wt)$$

$$\therefore \quad \phi\,(r) = P_2 \times f(\theta, t) = \frac{3}{2} K_2 A_m \sum \text{Cos}\,(pm\theta \mp wt)$$

$$+ \frac{3}{2} K_2 A_m \frac{es}{v_2} \text{Cos}\,(pm\theta \mp wt)\, \text{Cos}\left(\frac{v_2 R}{p}(p\theta - w_2 t)\right)$$

$$\ldots\ldots(1\text{-}16)$$

$$\phi_r = \phi_{ro} + \phi_{rp}$$

$$\therefore \phi_{rp} = K\, \text{Cos}\,(pm\theta \mp wt)\, \text{Cos}\left[\frac{v_2 R}{p}(p\theta - w_2 t)\right]$$

Substituting

$$\frac{w - w_2}{w} = s\ (\text{slip})$$

$$w_2 = w - w s$$

Simplifying it

$$\phi_{rp} = \frac{K}{2} \quad \text{Cos}\left\{p\theta \left(\frac{v_2 R}{p} + m\right) \mp wt \left[\frac{v_2 R}{p}(1-s) \pm 1\right]\right\}$$

$$+ \frac{K}{2} \quad \text{Cos}\left\{p\theta \left(\frac{v_2 R}{p} - m\right) \pm wt \left[\frac{v_2 R}{p}(1-s) \mp 1\right]\right\} \quad (1\text{-}17)$$

Giving the following values to equation (1-17)

$$R = 68 \qquad p = 2 \qquad v_2 = 1,2$$

and m = order of stator belt harmonic

The equation (1-17) shows that rotor permeance harmonics are function of rotor slots and order of air gap permeance. It also shows that each space belt harmonic will produce a series of permeance harmonics. The permeance harmonics are sometimes also called sub-harmonics too. Tabulating the useful rotor permeance harmonics likely to produce an appreciable effect. Rotor permeance Harmonic is given by

$$v_2(R/p) \pm m$$

or $$\pm\, v_2(R/p) + m \qquad (1\text{-}18)$$

Plus sign for forward rotating and minus sign for backward rotating.

Table (1-3)

v_2	m	order of Harmonic	Speed & direction
1	1	35	1/35
		33	-1/33
2	1	69	1/69
		67	-1/67
1	5	39	+1/39
		29	-1/29
1	7	41	+1/41
		27	-1/27
1	35	69	+1/69
		1	-1

Contd...

Table (1-5) contd..

v_2	m	order of Harmonics	Speed & direction
1	31	65	+1/65
		3	-1/3
1	29	63	+1/63
		5	-1/5

1.3. AMPLITUDES OF HARMONIC mmfs.(19)

1.3.1. Amplitude of Belt Harmonics

The equation (1-1) gives the amplitude as

$$F_m = \frac{A_m}{2} \quad \frac{\sin\ (m-1)\,\pi}{\sin\ (m-1)\,\pi/k} \qquad (1-18)$$

while finding the mathematical limit of the function.

$$F_m = \frac{A_m}{2}\ \frac{k}{P} \qquad \text{where } k = \text{number of phases} = 3$$

$$\therefore\ F_m = 3/2\ A_m\ ;\ A_m = 4/\pi \cdot \frac{1}{m} \cdot K_{wm}\ T_p\ \frac{I_m}{P}$$

Where

T_p = Turns per phase.

I_m = Max. current of fundamental time harmonic.

K_{wm} = Distribution factor for mth harmonic.

$$\therefore\ F_m = \frac{6}{\pi}\ \frac{1}{m}\ \frac{K_{wm}}{P}\ T_p\ I_m$$

$$= 0.9\,\frac{k}{m}\ \frac{(\,T_p\,K_{wm}\,)}{P}\ I_1 \quad \text{(mmf per pole)} \quad ..(1-19)$$

(1-19) gives the amplitude of mth harmonic in air gap. For stator mth

$$I_{1m'} + I_{2m} = I_1$$

$$\therefore F_{(m)s} = 0.9 \frac{k}{m} \left(\frac{\tau_p K_{wm}}{P}\right) I_1'$$

$$= 0.9 (k/m) \left(\frac{\tau_p K_{wm}}{P}\right) D_m I_1 \qquad (1-20)$$

When $D_m = I_1' m / I_1$ = damping factor per mth harmonic.

Similarly if $1 - D_m = G_m$ (Damping factor for rotor currents)

then $$F_{(m)r} = 0.9 \left(\frac{k}{m}\right) \left(\frac{\tau_p K_{wm}}{P}\right) G_m I_1 \qquad (1-21)$$

1.3.2 Amplitude of Permeance Harmonics

Equations (1-12) and (1-16) gives the amplitude of permeance waves as $3/4\ K_1 \frac{es}{v_1} A_m$ and $3/4\ K_2 \frac{er}{v_2} A_m$ for stator and rotor respectively.

Amplitude of permeance harmonics is function of es and K_1, K_2 and v_1, v_2.

$$F_{mp(s)} = \frac{3}{4} K_1 \frac{es}{v_1} \frac{A_{mp}}{P} \quad \text{(Stator permeance harmonic per pole)}$$

.....(1-22)

$$F_{mp(r)} = \frac{3}{4} K_2 \frac{er}{v_2} \frac{A_{mp}}{P} \qquad .....(1-23)$$

As defined earlier

es and er = $\frac{R_{ms}}{\text{Avg.}}$ value of permeance waves.

K_1 & K_2 = Average mean permeances.

The above factors are investigated in detail in Chapter 2. .

1.3.3. Amplitude of Slot Harmonics:

For slot harmonics, the fundamental mmf of the stator when acts with non-uniform permeance produced by stator slotting gives rise to slot harmonic is the same as of fundamental. Therefore mmf of slot harmonics can be written

$$F_{ms(s)} = 0.9 \frac{k}{ms} \left(\frac{T_p K_{w_1}}{p} \right) I_1 \quad \ldots\ldots\ldots\ldots \quad (1\text{-}24)$$

$$F_{ms(s)} = \frac{F_{1(s)}}{ms} \quad \ldots\ldots\ldots\ldots \quad (1\text{-}25)$$

1.4. STATOR AND ROTAR HARMONICS

Summing up the harmonic phenomena we have

1. Phase Belt harmonic tabulated in Table 1-1
2. Stator permeance Harmonics given by $\pm k^{|} (S/P) + m$
3. Rotor permeance Harmonics given by $\pm k^{|} (R/p) + m$
4. Stator slot harmonics given by $\pm k^{|} (S/P) + 1$
5. Rotor slot harmonics given by $\pm k^{|} (R/p) + 1$.

For our purpose we will consider all these. The harmonics of importance of stator and rotor are listed below:

Table (1-4) Stator Rotor Harmonics:

STATOR	ROTOR				
	$v_2 = 0$	$v_2 = +1$	$v_2 = -1$	$k^{\|} = +2$ $v_2 = +2$	$v_2 = -2$
+1	1	+35P	-35	+69	-67
-25	5	+29P	-39	+63	-73
+37	7	+41P	-27	+75	-61

Contd..

Table (1-4) Contd..

STATOR	ROTOR				
	v_2 = 0	v_2 = +1	v_2 = -1	k' = +2 v_2 = +2	v_2 = -2
-11	11	+23P	-45	+57	-79
+13	13	+47P	-21	81	-55
-17	17	+17P	-51	51	-85
+19	19	+53P	-15	87	-49
-23	23	+11P	-57	45	-91
+25	25	+59P	- 9	93	-43
-29	29	+ 5P	-63	39	-97
+31	31	+65P	- 3	99	-37
-35	35	-1P	-69	33	-103
+37	37	+71	+ 3	105	-31
-41	41	-7 P	-75	27	-109
-43	43	+77	+ 9	111	-25
-47	47	-13P	-81	21	-115
+49	49	+83	+15	117	-19
-53	53	-19P	-87	15	-121
+55	55	+89	+21	123	-13
-59	59	-25P	-93	9	-127
+61	61	+95	+27	129	- 7
-65P	65	-31P	-99	3	-133
+67	67	+101	+33	135	- 1
-71	71	-37P	-105	- 3	-139
+73	73	+107	+39	141	+ 5
-77	77	-43P	-111	- 9	-145
+79	79	+113	+45	147	11
-83	83	-49P	-117	-15	-151
+85	85	+119	+51	153	17
-89	89	-55P	-123	-21	-157
+91	91	+125	+57	159	23
-95	95	-61P	-129	-27	-163
+99	99	+131	+63	165	29

P indicates the combination for parasitic torques.

1.5. A MODEL OF THE SPACE HARMONICS

The model shown in the Fig (1-4) is based on the following assumptions[4].

1. Harmonic fluxes having different numbers of pairs of poles do not interfere with each other, they may be assumed to exist in different induction motor structures.

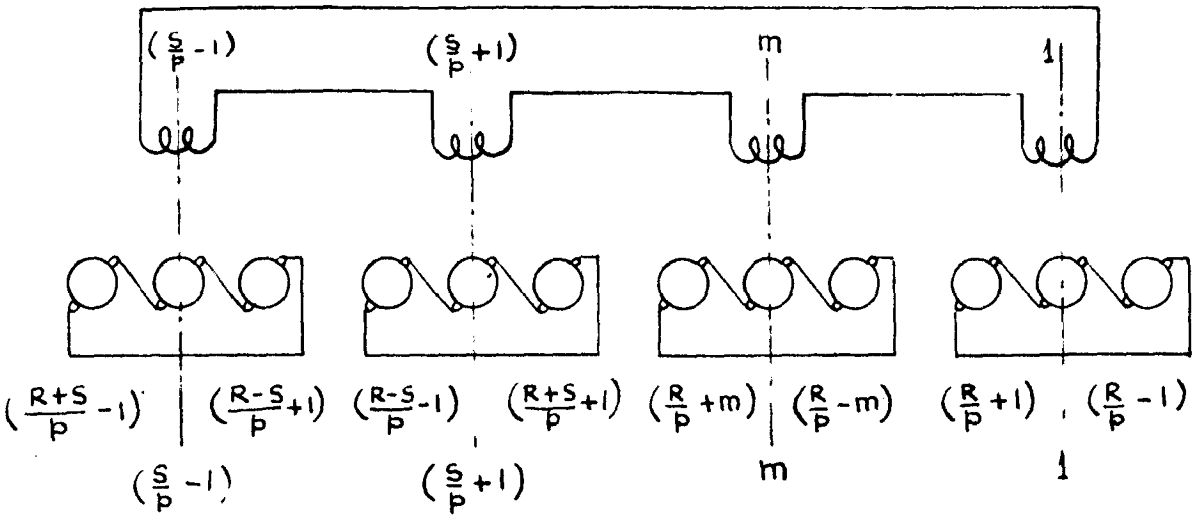
2. Each such motor has different mutual and leakage inductances, also resistances (The resistances are different for each motor because of the skin effect due to the currents of different frequencies). There is no mutual linkage between harmonics.

Since one current in stator produces a series of fluxes is represented on the model by connecting a group of stators or rotors in series. These series connected structures have the same number of pairs of poles as the fluxes produced by the current.

A model for the first stator , mth stator and first slot harmonic of stator and rotor group of harmonics is represented in Fig. (1-4). Rotor sub harmonics are not indicated.

1.6 MACHINE WINDINGS AND THEIR INTERACTION WITH THE GENERAL FLUX DENSITY COMPONENT.

1.6.1. As explained earlier, one finds, on examining a variety of harmonic, phenomena in rotating machines, that all have in common the fact that the air gap flux density cannot be expressed as a simple rotating wave function, sinusiodal in space and time but contains in general a combination of such rotating waves, harmonically related in space or in time. Attention is directed initially to the general



MODEL OF THE STATOR AND ROTOR HARMONICS

FIGURE 1.4

FIGURE 1.5

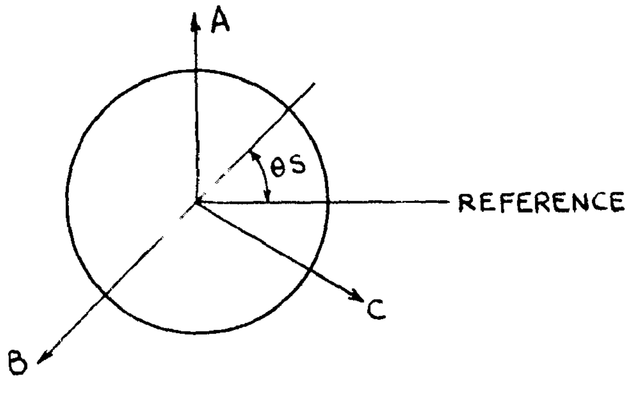
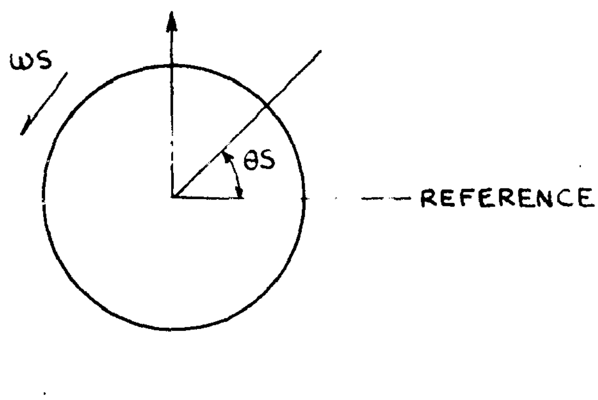


FIGURE 1.6



form of the flux density components as follows :

It is assumed that the air gap flux density is uniform in axial direction and end effects are ignored. Thus flux density b, can be written $\theta_r = \theta_s - wrt$ (taking two references winding at time zero).

$$b = B \sin (m\theta_s \pm kwt + \phi)$$

The method of approach here is that instead of considering a non-sinusiodal current in space, machine winding will be considered as sum of the harmonic component each varying sinusiodally in space.

Taking $\bar{n}$ a periodic function of space angle θ, it can be expanded in Fourier series -

$$\bar{n} = \sum_{j=1,5.} N_j \sin j \theta s$$

If $i = i(t)$

$$f = \bar{n}i = i \sum_{j=1,5.} N_j \sin j \theta s.$$

In this respect, therefore, the actual winding can be considered as equivalent to series connection of windings having conductor distribution $N_1 \sin \theta s$, $N_5 \sin 5 \theta s$ etc. when

$$N_j = N \times K_{wm}$$

1.6.2. <u>Voltage Induced in Winding</u>

The voltage $v = \frac{d \psi}{dt}$ when ψ is the flux linkage in wb time.

$$\psi = K \int_0^{2\pi} \bar{n} b \, d\theta s$$

$$\psi = \int_0^{2\pi} \left(\sum N_j \sin j\,\theta_s\right) B \sin\left(m\theta_s \pm k\omega t + \phi\right) d\theta_s$$

evaluation of this integral shows that $\psi_j = 0$ if $j \neq m$

$\psi_j = \psi_q$ if $j = m$

Therefore there is induced voltage only when mth harmonic flux links with mth harmonic winding distribution.

$$\boxed{v = v_m = -\frac{d\psi}{dt} = K_s\, k\omega\, \pi\, N_m\, B \sin(k\omega t + \phi)}$$

We reach to same result by winding distribution approach. This approach the outline of which is presented here will be further utilised in next Chapters for finding out general equivalent circuit and harmonic torque phenomena of non-salient pole electrical machine.

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CHAPTER II

GENERAL EQUIVALENT CIRCUIT FOR SKEWED ROTOR INDUCTION MACHINE AND DETERMINATION OF CIRCUIT CONSTANTS

2.1. THE GENERAL EQUIVALENT CIRCUIT

There are two general types of harmonic fields besides the fundamental magnetising field, namely the permeance harmonic fields and mmf harmonic fields. The equivalent circuit consists of the following series parallel connected impedenses[17].

1. Primary as stator winding resistance R_1.

   Primary leakage reactance X_1 resulting from all the leakage flux that does not cross the air gap.

2. X_M = Fundamental magnetising reactance corresponding to the fundamental air gap, flux wave of winding.

3. X_2 = Secondary leakage reactance.

   R_2 = Secondary resistance.

4 A Magnetising reactance and secondary impedance corresponding to the stator permeance harmonic fields at $(n_1 \pm 1)$ the harmonic frequencies, where $n_1 = \frac{S_1}{p}$

4. B Magnetising reactance and secondary impedance corresponding to the rotor permeance harmonic fields at $(n_2 \pm 1)$ the harmonic frequencies where $n_2 = S_2/p$

5 A Magnetising reactance $X_M(mb)$ resulting from stator behaviour mmf harmonic field at mb th harmonic frequency in parallel

with rotor impedance consisting of reactance.

X_2(mb), and resistance R_2(mb) /S(mb) where S(mb) represents backward harmonic slip.

5B Magnetising reactance X_M(mf) , resulting resulting down stator forward mmf harmonic field at mf th harmonic. Frequency in parallel with rotor impedance and resistance $\frac{R_2\ (mf)}{S(mf)}$ when S(mf) is harmonic slip.

6 There is still another type of harmonics under stator mmf harmonics, which is produced due to the mmf across slot openings and these are known as slot harmonics. The steps in the mmf were due to the concentration of the stator mmf across the slot openings produce (S-P) and (S+P) harmonic fields also.

7 And lastly the- those space harmonics produced by the higher order of time harmonics. The order of thespaces harmonics produced time harmonics will be determined by combinational equations, but the effect being too meager, therefore such harmonics are neglected.

2.1.1. Induction Motors in Series

As analysised in Chapter I, there are various order and type of harmonics in mmf wave of a smooth rotor machine, to establish equivalent circuit for their study, it will be assumed that each particular set of space harmonics exists in a seperate machine and so as many seperate machines of the same type have to be interconnected as there are space harmonics to be considered. But these machines, though identical in structure differ in following respect.

1. Each has a different number of pairs of poles.

2. They are connected through either stator windings or rotor windings OR both.

Hence the probelm of space harmonics consists of the interconnection of several similar machines summing at the same speed and having a different number of pairs of poles.

The physical picture presented by Kron(4) as shown in Fig. (2-1) four space harmonics with P_1 , P_2 , P_3 , P_4 pairs of poles respectively are connected in series on stator side, and each with its own similar virtual rotor rotating at the same speed v in the same direction. Kuon derived the equivalent circuit for such cases from physical consideration of flux linkage etc. As the Fig (2-1) indicates, the stator mmf is the sum of the space harmonic mmfs connected in series and for each harmonic flux linking to its corresponding virtual rotor induces harmonic rotor emf causing to circulate rotor harmonic amount as i_{r1}, i_{r2}, i_{r3} i_{r4} etc to balance the stator harmonic flux.

When the rotor windings 2.1.1(b) are connected in series (Fig. 2-2) then the absolute frequencies in their stator windings are different. This case is for sub harmonics, induced back by rotor harmonic currents in stator, as mth rotor current induced by mth stator induces back in stator kth order with different frequency.

The physical concept presented by Kron has been utilised by the author to formulate the results so obtained. The same results have been tried with conventional flux linkage theorms for harmonics of air gap mmf. The full effect of skewing of slots is also included

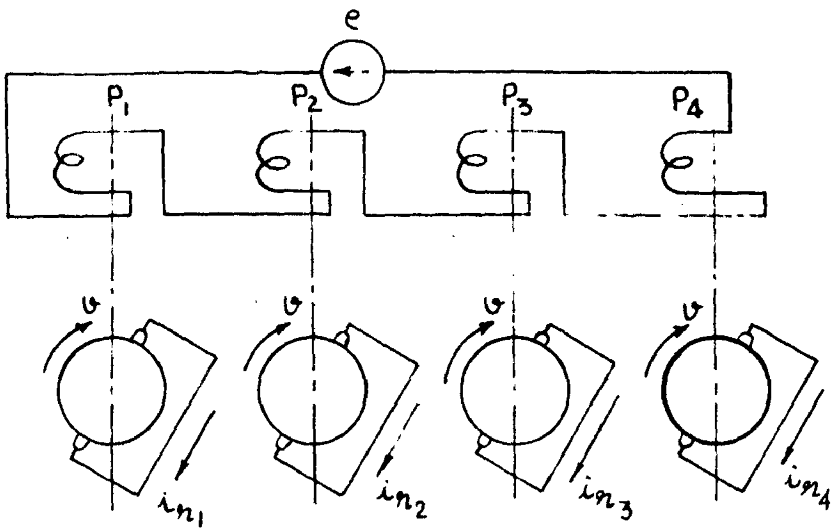


FIGURE 2.1

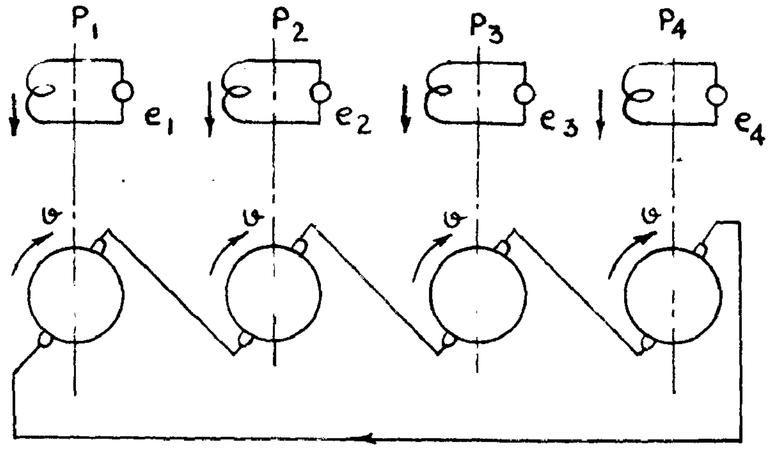


FIGURE 2.2

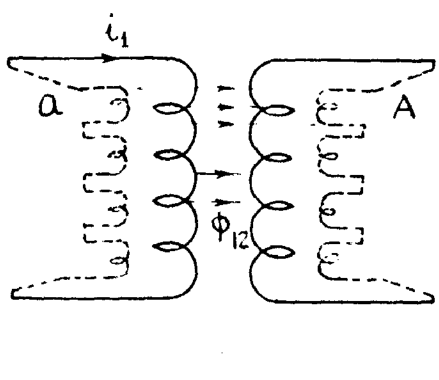


FIGURE 2.3

in analysis.

2.2. SELF INDUCTANCE OF COIL

The phase group coil 'a' and 'A' of stator and rotor respectively can be considered equivalent to harmonic coil connected in series . The total flux produced by the coil due to current i_1 is if

$$\phi_a = \phi_{ar} + \phi_{aA}$$

$$\phi_{1a} = \phi_a - \phi_{aA}$$

$$(\phi_{11} + \phi_{22} + \ldots\ldots\phi_{mm}) = (\phi_1 + \phi_2 \ldots\ldots\phi_m) - (\phi_{11'} + \phi_{22'} + \phi_{33'} + \ldots\phi_{mm'})$$

$$\ldots\ldots\ldots(2\text{-}1)$$

$$x_{1a} = \phi_{aa} - \phi_{aA} = (\phi_{11} - \phi_{11'}) + (\phi_{22} - \phi_{22'}) + \ldots\ldots(\phi_{mm} - \phi_{mm'})$$

$$= x_{11} + x_{12} + x_{13} + \ldots\ldots\ x_{1m}$$

$$= x_{1a} \qquad \ldots\ldots\ldots \quad \ldots\ldots \qquad (2\text{-}2)$$

The equation (2-1) is based on the assumption that there is a mutual coupling between same order of stator and rotor harmonics only

2.3 VOLTAGE EQUATION

The stator mmf equation as given by equation (1.11)

$$f(\theta, t) = \frac{3}{2} A_m \quad \sum \text{Cos}\,(mp\theta_1 \mp wt)$$

stator mmf is :

$$f(\theta\ ,\ t) = \frac{3}{2} A_m \sum \text{Cos}\,(mp\theta_1 \mp wt) \qquad (2\text{-}3)$$

Rotor mmf is

$$f(\theta\ ,\ t) = \frac{3}{2} A'_m \sum \text{Cos}\,(mp\theta_2 \mp w_2 t\ ) \qquad (2\text{-}4)$$

From equations (2-3) and (2-4), the net mmf acting across the air gap is the algebric sum of these two expressions[10], neglecting saturation hysteris and eddy currents, permitting superposition to be applied. The effect of parallel leakage paths prominence only in terms of voltage drops will be taken into account in this manner.

$$\text{mmf (air gap)} = \text{mmf (stator)} - \text{mmf (rotor)}$$

$$f_g(\theta,t) = f_s(\theta,t) - f_r(\theta, t) \qquad (2\text{-}5)$$

$$\theta_2 = \theta_1 - (1-s)wt \qquad \text{(If rotor not skewed)}$$

θ_{sk} = angle of skew is defined as the difference in total angular displacement in electrical radians between θ_2 and θ_1 at the two ends of the rotor stack. The origin of the coordinates is taken at the centre of the stack.

$$y'_R = y_R = y \qquad \ldots\ldots \qquad (2\text{-}6)$$

$$\theta_2 = \theta_1 = \theta_{sk}\frac{y}{L} - (1-s)\,wt \qquad (2\text{-}7)$$

Air gap mmf in terms of stator coordinates

$$f_g(\theta,t) = \frac{N_1 \quad m_2}{m\,\pi} \sum I_{2n} \; K_{w2n} \; \text{Cos}\left[\left\{ pn\theta_1 \pm wt\left\{ 1+(n-m)(1-s)\right\}\right\}\right]$$

$$\frac{N_1 I_1 m_1}{m\,\pi} \sum \; K_{w1m} \; \text{Cos}\;(pm\theta_1 \pm wt)$$

Considering only slot harmonics and belt harmonics then

$$n = m$$

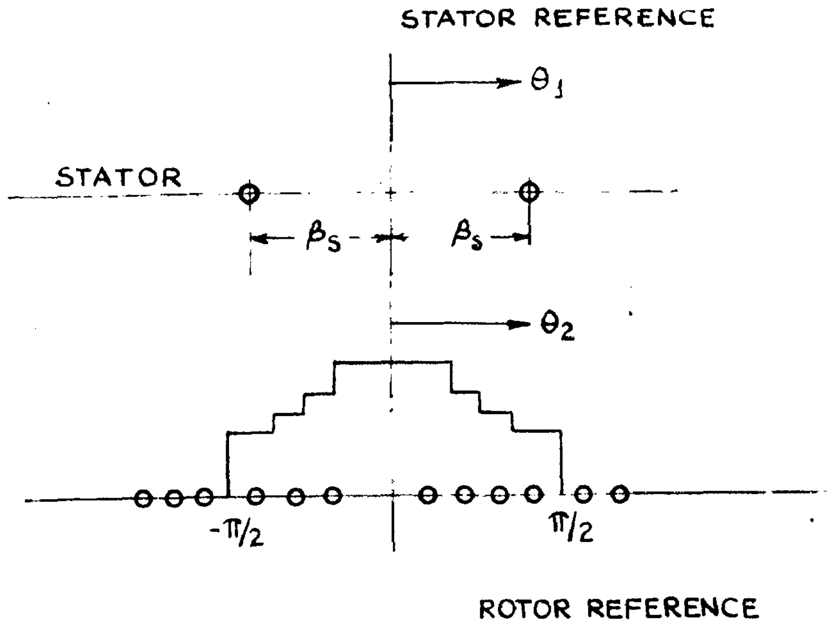


FIGURE 2.4

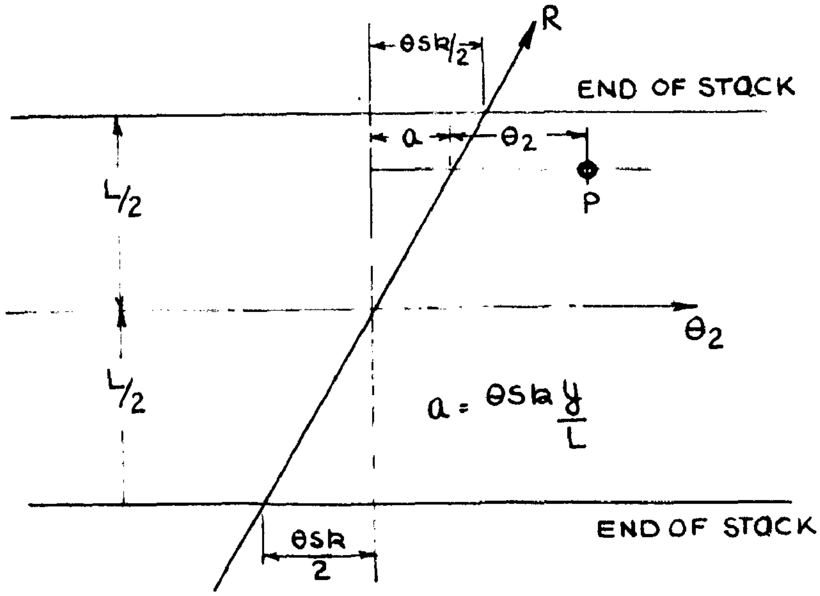


FIGURE 2.5

$$f_g(\theta, t) = \frac{N_2 \; m_2}{m \, \pi} \sum F_2 \, n \, Kw_{2m} \operatorname{Cos}\left(pm\theta_1 \pm wt \; pm\theta_{sk} \frac{y}{L} \right)$$

$$+ \frac{N_1 I_1 \; m_1}{m \, \pi} \sum k \, w_{1m} \operatorname{Cos}\left(pm\theta_1 \pm wt \right) \qquad (2\text{-}8)$$

The flux linkage of a single rotor coil turn is obtained by integrating the air gap density referred to the rotor over the area spanned by the rotor coil turn.

$$\lambda_g(s) = \int_{-\beta}^{+\beta} \int_{-L/2}^{+L/2} \frac{D}{P} \frac{\mu}{g_o} f_g(\theta, t) \, d_y . d\theta \qquad (2\text{-}9)$$

Where β = coil span

Substituting the following in simplifying equation (2-9)

$$k \, w_{1m} = k_{P(m)} \; k_{d(m)}$$

$$\theta_{sk'} = p \, \theta_{sk} \quad \text{(Elect. Skew)} \qquad (2\text{-}10)$$

$$K_{sk(m)} = \frac{\operatorname{Sin} m\theta_{sk'}/2}{m \, \theta_{sk'}/2} \quad \text{(skew factor)} \qquad (2\text{-}11)$$

$$\lambda_{g(s)} = \frac{2D\mu L}{\pi \, P g_0} N_2 m_2 \sum \frac{I_{2(m)} k_{w1(m)} k_{w2(m)} K_{s(m)} \operatorname{Cos} wt}{m^2}$$

$$- \frac{2D\mu L}{\pi \, P g_0} N_1 I_1 m_1 \sum \frac{(k_{w1(m)})^2}{m^2} \operatorname{Cos} wt \qquad (2\text{-}12)$$

Total flux linking phase I

$$\lambda_{1(g)(s)} = \lambda_{g(s)} \cdot N_1$$

$$\lambda_{g(s)} = \frac{2D\mu L}{w\, P g0} N_1 N_2 m_2 \sum \frac{I_{2(m)}\, k_{w1(m)}\, k_{w2(m)}\, K_{s(m)}}{m^2} \text{ Cos } wt$$

$$- \frac{2\, D\mu L}{w\, Pg0} N_1^2\, I_1 m_1 \sum \frac{k^2_{w1(m)}}{m^2} \text{ Cos } wt \qquad (2\text{-}13)$$

Since $e = \frac{-d\phi}{dt}$ (per turn)

$e = - \frac{d\lambda_1}{dt}$ (per phase)

Differentiating:

$$e_{1(s)} = \frac{2D\mu}{w\, P\, g0} N_2 N_1 m_2 \sum \frac{k_{w2(m)}\, k_{w1(m)}\, I_{2(m)}\, K_{s(m)}\, w \text{ Sin } (wt)}{m^2}$$

$$+ \frac{2D\mu}{w\, Pg0} N_1^2\, m_1 I_1 \sum \left(\frac{k_{w1(m)}}{m}\right)^2 w \text{ Sin } (wt) \qquad (2\text{-}14)$$

Similarly when air gap mmf is referred to rotor side using the following coordinate transformation

$$\theta_2 = \theta_1 + \theta_{sk} \frac{Z}{L} \qquad (2\text{-}15)$$

and since rotor w.r. to stator is rotating at slip speed.

$$f_2(\theta, t) = \frac{3}{2} A'_m \left(mp\theta_2 - s_m wt \right)$$

where s_m = slip harmonic

$$s_m = 1 \pm m\,(1-s) \qquad (2\text{-}16)$$

In corporating equation (2-15) and (2-16) in equation (2-5) and (2-6) the voltage induced in the rotor circuit turns out

$$e_{1(r)} = \frac{2D\mu}{w\, Pg0} N_2^2\, m_2 \sum \left(\frac{k_{w2m}}{m}\right)^2 I_{2m}(s_m) \text{Sin}\left[(s_m wt)\right]$$

$$- \frac{2\, D\mu}{\pi\, Pg0} I_1 N_1 N_2 m_1 \sum \frac{(k_{w1m})\,(k_{w2m})\, K_{s(m)}\, s_m \text{ Sin } wt}{m^2} \quad \ldots (2\text{-}17)$$

Substituting -

Total self inductance of stator phase $= L_{11}$

Total self inductance of rotor phase $= L_{22}$

and M_{12} or M_{21} is the mutual inductance

When

$$L_{11(m)} = \frac{2\,D\,\mu_L}{\pi\ g^0}\,N_1^2\,m_1\,\left(\frac{k_{w1m}}{m}\right)^2 \qquad (2\text{-}18)$$

$$L_{22(m)} = \frac{2}{\pi}\,\frac{D\,\mu_L}{g^0}\,N_2^2\,m_2\,\left(\frac{k_{w2m}}{m}\right)^2 \qquad (2\text{-}19)$$

$$M_{12(m)} = \frac{2}{\pi}\,\frac{D\mu_L}{g^0}\,N_2 N_1 m_1 k_{w1m}\,k_{w2m}\,K_{sm} \qquad (2\text{-}20)$$

$$M_{21(m)} = \frac{2}{\pi}\,\frac{D\,\mu_L}{g^0}\,N_2\,N_1\,m_2\,k_{w1m}\,k_{w2m}\,K_{sm} \qquad (2\text{-}21)$$

$$M_{21} = M_{12} \qquad (2\text{-}22)$$

Equations (2-18) through (2-22) in equation (2-19) and (2-17)

$$e_{1(s)} = w\,\mathrm{Sin}\,(wt) \sum M_{21(m)}\,I_{2m} - w\,\mathrm{Sin}(wt) I_1 \sum L_{11(m)} \qquad (2\text{-}23a)$$

$$e_{1(r)} = s_m\,\mathrm{Sin}\,(wt) \sum L_{22(m)} I_{2m} - s_m\,\mathrm{Sin}\,(wt)\,I_1 \sum L_{22(m)} \qquad (2\text{-}23b)$$

2.4 ANALOGY WITH COUPLED CIRCUITS

The well known coupled circuit equations are, for the differential equation form

$$L_{11}\,\frac{di_1}{dt} - M\,\frac{di_2}{dt} = e_1$$

$$-\,L_{22}\,\frac{di_2}{dt} + M\,\frac{di_1}{dt} = e_2$$

As far a.c. steady state form

$$jw\ L_{11}I_1 - jw\ M\ I_2 = E_1 \qquad (2\text{-}24)$$

$$jw\ M\ I_1 - jw\ L_{22}I_2 = E_2 \qquad (2\text{-}25)$$

Equations (2-22) and (2-23) does not take into account the voltage drop due to ohmic resistance of stator and rotor.

If r_1 = Stator resistance.

$r_{2\ m}$ = Harmonic rotor resistance.

The rotor resistance r_{2m} will vary as per higher harmonics, skin effects prodominates as considered further in this chapter.

If drops are taken into account, the equations for applied voltage V_1 becomes

$V_1 = e_{1(m)}$ + stator drop.

$$V_1 = I_1 r_1 + jwI_1 \quad L_{11(m)} - jwm \sum M_m\ I_{2m} \qquad (2\text{-}26)$$

$$E_R = I_{2m}\, r_{2m} - jw \sum a_m\ L_{22(m)}\ I_{2(m)} + jwI_1 \sum a_m\ M_m \qquad (2\text{-}27)$$

As the whole mmf of rotor is consumed in circulating current $I_{2(m)}$ therefore E_R is zero.

$$0 = \frac{I_{2\ m}\, r_{2m}}{a_m} - jwL_{22(m)}\, I_{2(m)} + jwI_1\ M_{(m)} \qquad (2\text{-}28)$$

Modifying (2-26) and (2-27) with (2-1) and (2-2)

$$V_1 = I_1 r_1 + I_1 \sum jwml_{1(m)} + \sum (I_1 - I_{2(m)})\ jw\ M_m \qquad (2\text{-}29)$$

$$0 = I_{2m}\frac{r_{2m}}{a_m} + jw \sum I_{2m} \times l_{2(m)} + jw \sum (I_1 - I_{2m})M_m \qquad (2\text{-}30)$$

Equations (2-29) and (2-30) gives the conventional general circuit (Fig. 2-6). This presents equivalent circuit for harmonics induced in rotor by non-sinusiodal winding distribution and due to the presence of slots on motor and stator surfaces. This does not account the harmonics produced by rotor currents which are termed as 'sub-harmonics'.

2.5 HARMONIC SLIP

2.5.1. Harmonic Slip of Rotor Currents Directly Induced by Stator mmfs.

For mth harmonic revolving forward per unit syncronous speed is

$$N_{sm} = 1/m$$

Rotor per unit speed is $= 1-s$

$$\therefore \text{ Harmonic slip is } \quad S_m = \frac{(1/m) - (1-s)}{1/m} = 1-m(1-s)$$

(for forward rotations)

$$= 1 + m(1-s) \text{ (for backward rotations)}$$

2.5.2. Hamonic Slip of Rotor permeance Harmonics

i. For example, considering kth forward rotor harmonic due to the rotor current induced by the mth backward stator harmonic field, the speed with respect to the stator fm of this mth harmonic stator field is $= -1/m$ and its speed with respect to the rotor is

$$-1/m - (1-s) = 1/m \left[ms - (m+1) \right]$$

The speed of the kth forward rotor harmonic of this field with respect to the rotor is

$$\frac{1}{mk} \left[ms - (m+1) \right]$$

and the speed of mk th harmonic with respect to the stator is

$$\frac{1}{mk} \left[ms - (m+1) \right] + (1-s) = \frac{1}{mk} \left[m(k-1)(1-s) - 1 \right] \qquad (2\text{-}31)$$

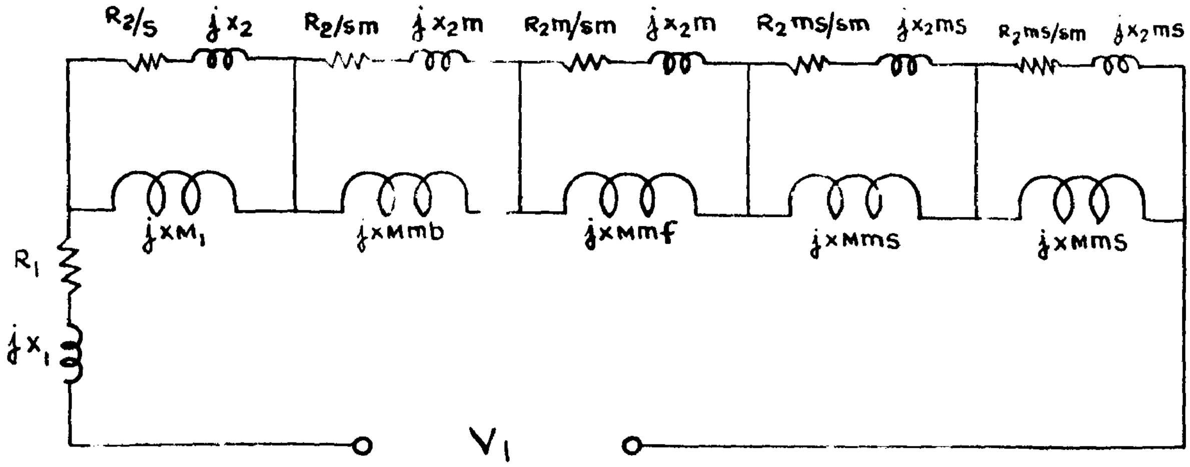


FIGURE 2.6

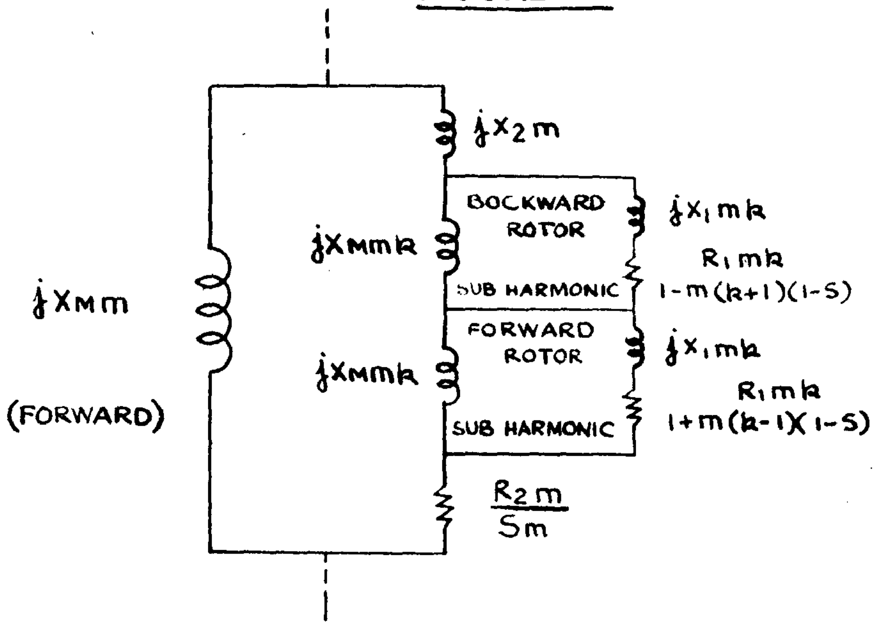


FIGURE 2.7

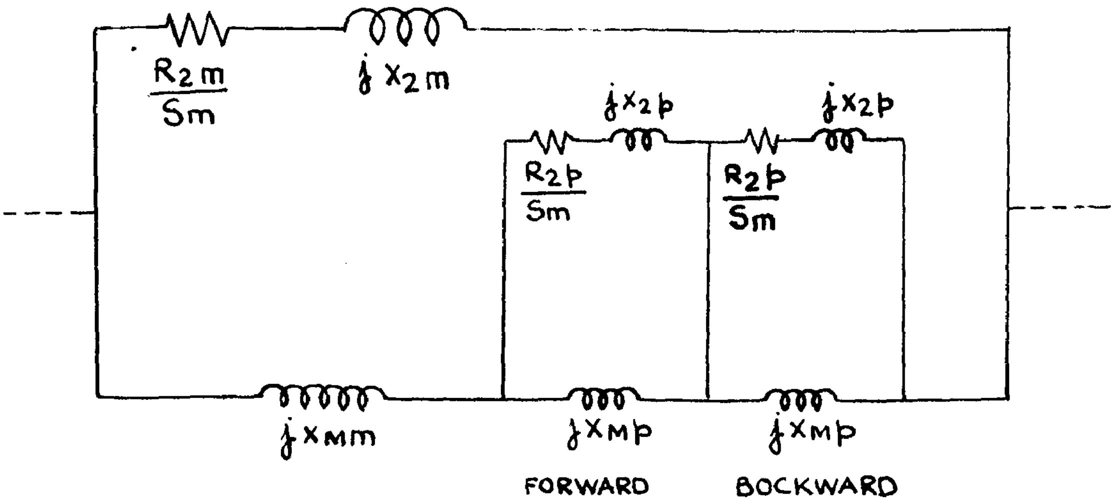


FIGURE 2.8

at $s = 1$ Speed of field is $= \frac{1}{mk}$ giving harmonic slip

$$s_{mk} = 1 - m\,(k-1)\,(1-s)$$

This per unit speed appears as the denomina or of the stator resistance term in the kth forward field circuit of the mth backward stator harmonic field in Fig. (2-7).

(ii) The case (i) has been considered for particular set of harmonics Taking m and k as general order of any direction of rotation. then

$$s_{mk} = 1 \pm m\,(k \mp 1)\,(1-s) \qquad (2\text{-}32)$$

2-6. EQUIVALENT CIRCUIT OF PERMEANCE HARMONICS

As discussed under Chapter I, effect of slot openings is to introduce permeance travelling waves of order

$\pm v_1 \frac{S}{p} + m$ in case of stator

& $\pm v_2 \frac{R}{p} + m$ in case of Rotor

As they are produced by stator and rotor harmonic including fundamental currents, such permeance fluxes have the same frequency as the mmf producing them when viewed from the stator side. Hence, the effect of the slot openings is to introduce a double set of inductors in series with stator as rotor inductor respectively as the case may be.

2.6.1 First Considering Rotor Permeance Waves

Alger puts them in equivalent form like Fig. (2-7).

2.6.2. Equivalent Circuit for Stator Permeance Waves

If exciting field is from stator of mth order then frequency of stator permeance travelling waves would be same as of mth harmonic. They are included in circuit as shown by Alger(17) (Fig. 2.8) where p stands for order of permeance wave produced by mth mmf wave p includes effect of both stator and rotor slot openings, for clarity sake only, general terms forward and backward are shown over the diagram.

2.7. DETERMINATION OF CIRCUIT REACTANCES

2.7.1. Primary and Secondary Leakage Fluxes of Belt Harmonics

Conventionally the leakage reactance takes into account of

1. Primary slot leakage.
2. Coil end leakage.
3. Zig Zag and phase belt leakage.
4. Skew leakage flux.

The zig zag reactance includes the magnetising reactances of all mmfs. Since some of the mmf harmonics are to be considered seperately in the harmonic circuit, the magnetising reactances of these harmonics would not be included in the primary (stator) leakage. To facilitate calculations, the coil end leakage reactance will be assumed to contribute equally to the primary and secondary sides.

Total primary leakage reactance is

$$X_1 = X_{s1} + \left[X_{z1} - \sum X_{M(m)} \right] + \frac{X_0}{2} + X_\alpha \qquad (2\text{-}33)$$

Where

X_{s1} = Primary or stator leakage reactance.

X_{z1} = Primary zig zag leakage reactance.

X_{Mm} = Magnetising reactance of harmonics.

X_c = Coil and leakage reactance.

X_α = Skew reactance.

The secondary leakage reactanse for the fundamental is

$$X_2 = X_{s2} + X_{z2} + \frac{X_c}{2} + X_\alpha \qquad (2\text{-}34a)$$

where

X_{s2} = Secondary slot leakage reactance of secondary.

X_{z2} = Secondary Harmonic leakage reactance.

and for mth harmonic

$$X_{2(m)} = X_{s2(m)} + X_{z2(m)} + X_{\alpha(m)} \qquad (2\text{-}34b)$$

There is no practically coil and leakage reactance. For higher harmonics only zig zag leakage reactance predominates.

<u>Slot Leakage Reactanse</u>

As well known

$$X_{s1} = \frac{2\ f\ m_1\ L\ Z^2\ P_{s1}}{S\ \times 10^7} \text{ Ohm / Phase} \qquad (2\text{-}35)$$

$$X_M = \frac{6.38\ fm_1\ N^2\ (k_{w1})^2\ DL}{l_{ge}\ p^2\ 10^8} \text{ Ohms/Phase} \qquad (2\text{-}36)$$

Where Z = number of conductors in series per phase.

S = Number of stator slots.

P_{s1} = permeance constant of slot.

l_{ge} = effective air gap length.

L = Bore length in inches.

N = Number of turns per phase.

D = Bore diameter in inches.

m_1 = Prim. No of Phases.

other of Adams , considers first time correctly the air gap leakage i.e. harmonic fluxes as source of leakage. Alger treats it as two seperate (zig zag and belt) leakages. This segregation makes Adam's[21] theory in applicable to harmonics . More over this theory does not consider any damping by squirrel cage winding in the rotor of stator fluxes. Further no due consideration is given for slots. The influense of slots is two fold as already discussed in Chapter I.

Slot Openings may considerably decrease or increase the amplitude of permeance waves.

Here the results are reproduced of the analysis presented by Liwschitz - Garik. These results are dealt in detail under Appendix II.

$$X_{s2(m)} = \left\{ \left[\frac{\frac{mp^{w}}{R}}{K_{sm}} \quad Csc\left(\frac{m^{w}p}{R}\right) \right]^{2} - 1 \right\} X_{M(m)} \qquad (2\text{-}40)$$

The skin effect factor for leakage reactance of slots is to be considered. The slot leakage position is considerably affected by high frequencies as shown by curve (Fig. II-2) of Appendix II. The skin effect factor for reactances is given by

$$J_{(m)} = \frac{1.5}{1.68 \left[\frac{(1 \pm m)\ 50}{60}\right]^{\frac{1}{2}}} \qquad (2\text{-}41)$$

for m - forward use negative sign.

m - backward use positive sign.

Therefore total leakage reactance for mth phase harmonic by substituting 2-35 to 2-41 in (2-34b)

$$X_{2(m)} = \left(\frac{k_{wm}}{k_{w1}}\right)^{2} \left[\frac{X_{s2}\,(J_{m})}{K_{sm}^{2}} + \left\{ \left(\frac{\frac{mp^{w}}{R}}{K_{sm}}\right)^{2} C_{sc}^{2}\ \frac{mp^{w}}{R} - 1 \right\} X_{M} + \frac{X_{\alpha}}{m^{2}} \right] \qquad (2\text{-}42)$$

2.7.2. Primary and Secondary Leakage Reactances of Slot Harmonics

The differential leakge reactance for slot harmonics is substituting in 2-40.

$$m = m_s = \left(\frac{S\, v_1}{p} \pm 1 \right)$$

$$X_{s2(m_s)} = \left[\frac{\left(\frac{v_1 S}{p} \pm 1 \right)^2}{K_{sm}^{\ 2}} \operatorname{Csc}^2 \frac{\left(\frac{v_1 S}{p} \pm 1 \right) p\pi}{R} - 1 \right] X_{M1}$$

....(2-43)

The other leakage components would be the same as given by (2-42)

2.7.3. Primary and Secondary Leakage Reactance for Permeance Harmonics

On the similar lines as equation (2-40), a mathematical form for the differential reactance can be got for permeance. Harmonics. Since as pointed out earlier too, permeance harmonic mmfs are of very low order. So total leakage reactance can be taken with siffucient accuracy equal to zig zag reactance, because as equation 2-40 indicates, zig zag reactance increases with increased order of harmonics.

Writing

m_p = order of permeance harmonic.

$$X_{2P} = X_{2(mp)} - X_{s2(mP)} = \left[\frac{p^2 \pi^2}{K_{sm}^{\ 2}\, R^2} \operatorname{Csc}^2 \frac{p\, \pi\, n_1}{R} - 1 \right] \frac{X_{M(m)}}{n_1^{\ 2}} \qquad (2\text{-}44)$$

$$n_1 = \left(v_2 \frac{S}{p} \pm m \right) \qquad (2\text{-}45)$$

2.7.4. Magnetising Reactance of Phase Belt Harmonics

From equation 2-29, 2-21 and current transformation

$$X_{M(m)} = \frac{X_{M1}}{m^2} \left(\frac{k_{wm1}}{k_{w1}} \right)^2 \qquad (2\text{-}46)$$

When X_{M1} = magnetising reactance of primary.

2.7.5. Magnetising Reactance of slot harmonics and permeance Harmonics

$$X_{M(ms)} = \frac{X_{M1}}{\left(\frac{v_1 s}{p} \pm 1 \right)^2} \qquad (2\text{-}47)$$

As the winding distribution for slot harmonic is the same as of fundamental so factor $\left(\frac{k_{wm}}{k_{w1}} \right)^2$ does not appear for slot harmonics.

and

Permeance Harmonic Magnetising Reactance

From equation 1-9, 1-10 it is seen that factor k_1, k_2 affects the permeance mmf and since k_1, k_2 are function of slot openings and air gap, these slot openings have a turbulent effect over permeance harmonic magnete motive force. It is directly reduced in proportional to factor K(R)

$$\text{where} \quad K_R = \frac{\text{Single amplitude of the permeance pulsation}}{\text{Average permeance}} \qquad \text{.....}(2\text{-}48)$$

(arrived on the assumption that permeance is sinusiodal over the slot)

$$K_R = \frac{\beta}{2-\beta} = \frac{\beta/2}{1-\beta/2} \qquad (2\text{-}49)$$

Where $\beta = \dfrac{\text{Double amplitude variation}}{\text{maximum permeance.}}$ (2-50)

If exciting harmonic for mpth permeance wave is mth phase harmonic where $mp = \left(\dfrac{v_2 R}{p} \pm m\right)$

then $$X_{M(p)} = \frac{K_{R2} \quad X_{M(m)}}{2v_2 \left(\dfrac{v_2 \; R}{p} \pm m\right)}$$ (2-51)

(for rotor permeance)

and $$X_{M(p)} = \frac{K_{R1} \quad X\ M(m)}{2v_1 \left(\dfrac{v_1 S}{p} \pm m\right)}$$ (for stator permance) (2-52)

3.8. SKIN FACTOR FOR HARMONIC STATOR AND ROTOR RESISTANCES

3.8.1. For Rotor bar Resistance:

The rotor resistance is composed of[15]

$$R'_2 = R_2 bar + R_{2(ring)}\ (effective)$$

$$R'_2 = R_{2b} + R_{2r(eff.)}$$

$$R'_2 = R_{2b} + \frac{R_{2r}}{2\ Sin^2(\alpha_{s/2})}$$ (2-53)

Where α_{s_2} = Angle between two bars by which bar currents are displaced.

$$\alpha_{s_2} = \frac{\pi P}{R} \ (\text{Elect. degree}).$$

Equation (2-53) applied to main wave. Considering mth the skin effect has to be taken into account in term R_{2b}. For mth the slot angle becomes :-

$$\alpha_{s(m)} = m\,\alpha_s$$

$$\therefore \quad R'_{2(m)} = \frac{k_{w1m}^{2}}{k_{w1}^{2}} \left[R_{2b\,(km)} + \frac{R_{r}}{2 \sin^{2} (\alpha_{sm}/2)} \right]$$

...(2-54)

For higher order of harmonics, Ring resistance does not form any appreciable part of resistance in comparision with bar resistance which increases very fast due to skin effect at high frequencies of harmonics. Therefore

$$R'_{2(m)} \simeq R_{2(b)} \cdot K_{(m)} \qquad (2\text{-}55)$$

Where $K_{(m)}$ = skin factor for mth harmonic dealt in detail in Appendix II.

As in case of slot harmonics, the exciting field is fundamental therefore skin factor is not important to them and slot harmonics will experiencethe same resistance as of fundamental i.,e,

$$R_{2(ms)} \simeq R_{2}$$

$$\text{Where} \quad R_{2} = \frac{R'_{2} \times 4\, m_{1} \,(N_{1}\, k_{w1m})^{2}}{R_{2}\, K_{sm}^{2}} \qquad (2\text{-}56)$$

$$\text{and} \quad \frac{4\, m_{1} \,(N_{1}\, k_{w1m})^{2}}{R_{2}\, K_{sm}^{2}} = \text{Reduction factor (RF)} \qquad (2\text{-}57)$$

2.8.2. <u>Rotor Resistance of Rotor permeance Waves:</u>

$$R_{2(mp)} = R_{2p} = R_{2(m)}$$

Where m is the exciting field for mpth permeance field. Similarly for stator permeance waves.

All the results, determined under section 2.7 and 2.8 holds good only when the fundamental and harmonic flux paths are assumed unsaturated. This assumption is valid to a greater extent as the locked rotor performance curve of the machine under test (Fig. 1-5) indicates. From this fig, we can safely assume the machine working in unsaturated region without any appreciable error creeping in.

2.9. SPECIFICATIONS OF INDUCTION MACHINE UNDER TEST AND EXPERIMENTAL DETERMINATION OF CIRCUIT CONSTANT:

2.9.1. Specifications:

The experimental skewed rotor induction machine was put to some tests and measurement to know certain relavent datas necessary for further analysis and calculations.

1. Measurements : The following figures were collected after careful measurements of the 3 ∅ , 3 H.P. , 1500 RPM , rotor skewed induction machine.

Angle of skew = 6° (Elect)

Length of Rotor = 7.1 Cms.

Length of Rotor bar = 7.3 Cms.

Diamentar at the air gap = 17.0 cms.

Length of armature = 7.5 cms.

2. Stator: Number of slots = .48

Number of Poles = 4

No. of turns per phase = 256.

Chording = full pitch coils.

Phase spread = 60° (4 slots)

speed = 1500 (syncronous)

slot width = 5.5 mm

Slot pitch = 11.1 mm.

Slot depth = 21.0 mm.

slot opening = 2.5 mm.

3. **Rotor:** No. of slots = 68

Ring size = 2.3 x 0.5 cms.

Mean dia. of Ring = 15.8 cms.

Width of rotor slot = 0.65 cms.

Rotor slot opening = 1.3 mm.

Slot pitch = 0.785 cms.

2.9.2. **Tests:**

(a) Test for Windage Friction loss:

The conventional method of variable voltage test and then determining friction windage loss by extra plotation procedure. Results are plotted in Fig. (2-9) giving windage friction loss (wf) = 185 watts.

(b) Magnetising reactance:

The magnetising reactance was determined at rated voltage, machine running at syncronous speed by means of external drive in current direction of rotation.

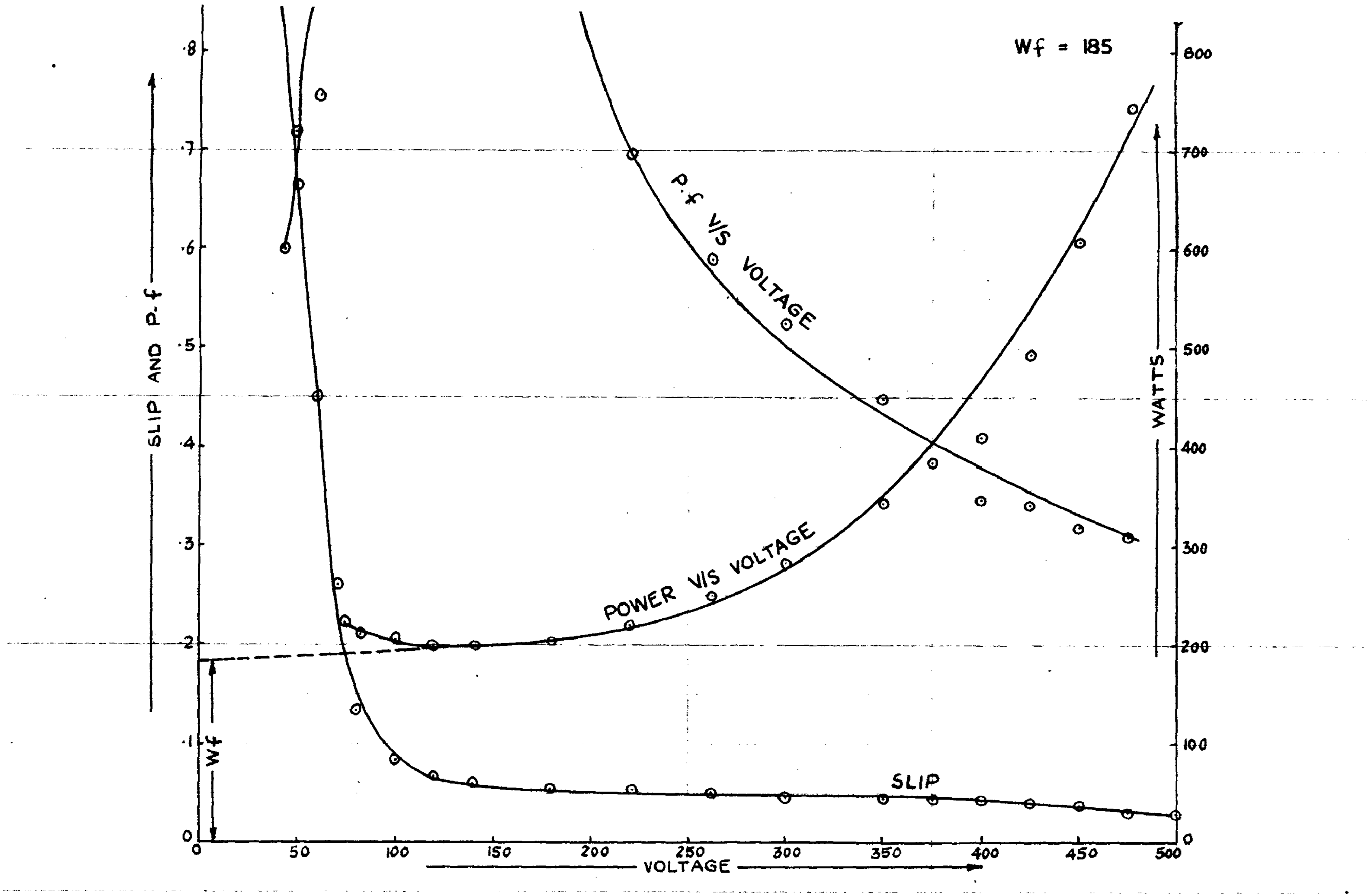
I = 2.20 Amps V = 440 Volts W = 910 Watts.

X_M = 138.0 Ohms , R_1 = 5.1 Ohms , X_1 = 3.5 Ohms

X_2 = 3.1 Ohms, (Rotor resistance referred to primary) R_2' = 1.93 Ohms.

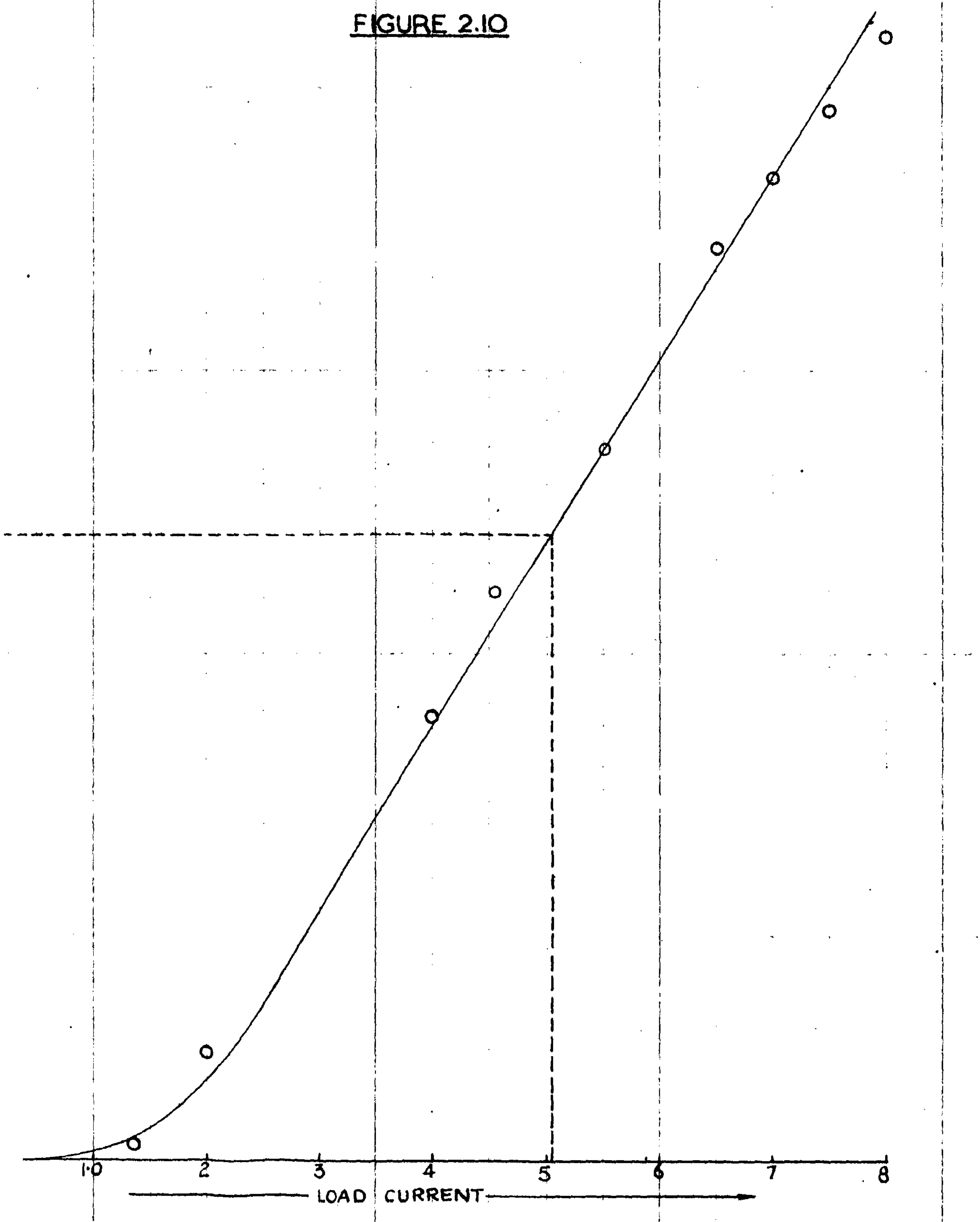
(c) Stray Load Loss Test over Induction Machine

A stray load loss curve against loss (Fig. (2-10) is drawn. Results are obtained when a reverse rotations test is performed over induction machine.


Wf = 185
P.f V/S VOLTAGE
POWER V/S VOLTAGE
SLIP
SLIP AND P.f
WATTS
VOLTAGE
Wf

STRAY LOSS CURVE

FIGURE 2.10



2.10. CALCULATION OF CIRCUIT CONSTANTS OF GENERAL EQUIVALENT CIRCUIT WITH AND WITHOUT SKEW ANGLE OF THE ROTOR

2.10.1 Calculation of Constant Factors:

Before going over to actual calculation of circuit constants, it is essential to find out certain factors required in further calculations.

(a) Distribution Factor

$$k_{w1(m)} = k\, d_{1(m)} \quad \text{(as coils are full pitched)}$$

$$k_{w1(m)} = \frac{\sin \frac{m\,\sigma_c}{2}}{g \sin \frac{m\,\sigma_c}{2g}} \tag{2-58}$$

Where g = no. of slots per pole per phase = 4.

σ_c = coil span angle = 60°

Results are tabulated in Table 2.1

Table 2.1 A.

m	1	5	7	11	13	17	19	23	25	29	31	35	37
k_{w1m}	.955	.205	.157	.126	.126	.157	.205	.955	.955	.205	.157	.126	.126

Table 2.1 B.

m	41	43	47	49	53	55	59	61	65	67	71	73	77
k_{w1m}	.157	.205	.955	.955	.205	.157	.126	.126	.157	.205	.955	.955	.205

(b) Stator slot permeance factor

From well known relation

$$P_s = \frac{h_1}{3w_s} + \frac{h_2}{w_s} + \frac{2h_3}{w_s + w_o} + \frac{h_4}{w_o}$$

y_s = 0.85 cms , h_1 = 16.0 mm, h_2 = 2.0 mm , h_3 = 2.0 mm

h_4 = 1.0 mm w_s = 5 mm, w_o = 2. 6 mm

$\therefore\ P_s = 2.2$

(c) <u>Stator slot opening factors:</u>

Fig. (2-16b) shows the dip caused in the flux density curve due to the presence of slot opening w,

$$\text{mmf} = P_o \left[1 + K_R \operatorname{Cos} (S x) \right] \quad \ldots\ldots\ldots \quad (2\text{-}59)$$

$$K_R = \frac{\beta}{2 - \beta}$$

Equation 1-9 is

$$P = K + \frac{K\ ss}{v_1} \operatorname{Cos} S\,\theta + \frac{K_{ss}}{2} \operatorname{Cos} 2\,S\theta + \ldots\ldots.$$

Considering only 1st and 2nd order of permeance wave.

If

P_g = permeance with smooth surfaces.

P'_g = Effective permeance with effective air gap length.

P_o = K = constant permeance.

P_s = half amplitude of 1st order permeance wave = $\frac{K_{ss}}{v_1}$

The maximum value which the stepped permeance can take is P_g

$$\therefore\ P_g = P + \frac{P_s}{2} + P_s$$

$$\therefore\ P_o = P_g - 1.5\,P_s \quad \ldots\ldots\ldots \quad (2\text{-}60)$$

Taking same s value

$$P_{(rms)} = P_{o(rms)} + P_s \operatorname{Cos} \theta_{(rms)} + P_s/2 \,.\, \operatorname{Cos} 2s\theta_{(rms)}$$

$$P'_g = P_o + \frac{P_s}{R} + \frac{P_s}{2(2)^{\frac{1}{2}}}$$

$$P'_g = P_g - 0.44\,P_s \quad (2\text{-}61)$$

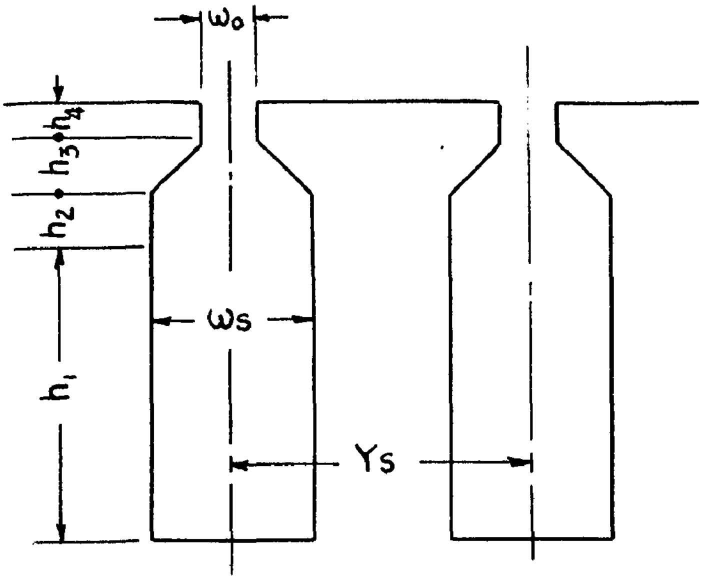


FIGURE 2.16 A

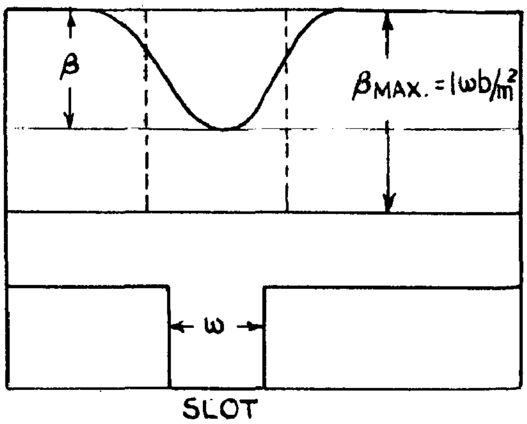


FIGURE 2.16B

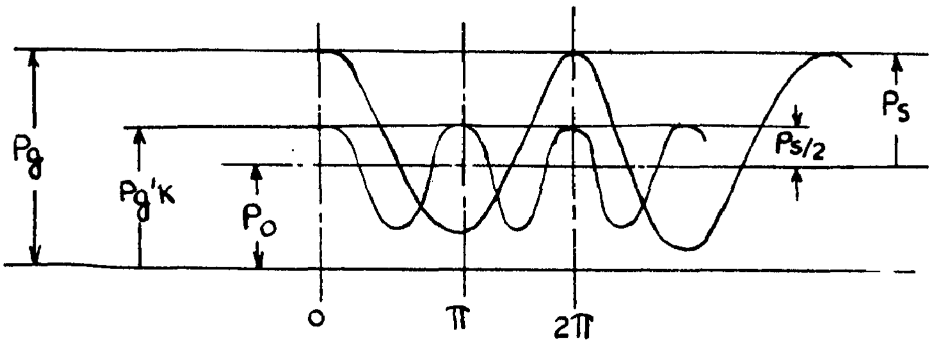


FIGURE 2.16C

Taking permeance $P \propto \dfrac{1}{\text{air gap length}}$

$A = $ Const

$$P_g = \frac{A}{l_g} = 2.5\ A \qquad l_g = 0.04 \text{ cm.}$$

and P'_g (?ms) $= A/l'_g$ when l'_g = Effective air gap with Carter Coefficient.

$= 1.99$ A. $\qquad l'_g = .048$ cms.

From (2-61) $P_s = 0.91$

$\therefore\ P_o = 1.135$

$\therefore\ P = 1.135 + 0.91 \cos S\theta + 0.455 \cos 2S\theta$

from equation (2-59)

$$K_{R1} = \frac{P_s}{P_o} = 0.889 \qquad (2.62)$$

(For stator)

Similarly finding factor K_{R2} for Rotor taking effective air gap length $l''_g = 0.424$ mms.

After using Carter's coefficient for rotor slots.

the permeance equation with rotor slotted and stator smooth becomes:

$$P_2 = 2.023 + 0.318 \cos R(\theta - w_{2t}) + 0.159 \cos 2R(\theta - w_{2t})$$

$$\therefore\ K_{R2} = \frac{0.318}{2.023} = 0.156 \qquad (2.63)$$

Comparing these results with those computed by Robinson in the form of curves

In our case , width of slot $S = 0.5$ cm. (stator)

$\lambda = 0.85$ cm.

$s/\lambda = 0.59$

$l_g = 0.04$ cms.

$s/l_g = .5/.04 = 12.5$

From Fig. (1-4) Mean flux density $= 0.49$

From Fig. (I-5) $V = K_{R1} = 0.9$

For Rotor slot width = 0.2 cms.

λ = 0.65 cms.

a/λ = 0.31 and a/l_g = 5.0 from Fig. I-5) $Y = K_{R_2} = 0.2$

The values of these factors KR_1 and KR_2 are comeiding too a fair degree.

(d) Skew leakage flux and Skew Factor:

Designers have long necognised that the space fundamental air gap flux varies with axial position because of the skewing the rotor slots with respect to stator slots, here an quantitative data are presented to demonstrate its effect, showing how much this flux may vary. It is important that variation in flux is taken into account to know the exact performance of induction machine. As Fig. (2-17) shows the skew flux path, though it crosses air gap, but it does not produce any useful torque, the skew flux simply adds to the leakage (equ. 2-33).

The skew flux is produced by unequality of stator and rotor mmf brought in by skewing of rotor bars.

Fig. (2-18) shows the stator bore surface and the rotor outside surface developed into the plane of the paper. The figure shows the stator - rotor and resultant mmf's at the ends of the stack. In the centre ($y = 0$) the resultant mmf is zero. Considering only fundamental.

If σ = Total electrical skew angle (See Fig. 2.17) $= \alpha$

$$\text{Skew factor} = \frac{\text{Emf induced in skewed bar}}{\text{Emf induced in straight bar.}}$$

$$K_s = \frac{E_{sk}}{E} \qquad (2\text{-}65)$$

The element of voltage induced in a short[3] length of the conductor at point x is $\frac{E\,dx}{\sigma} \angle x$, where E is the total voltage

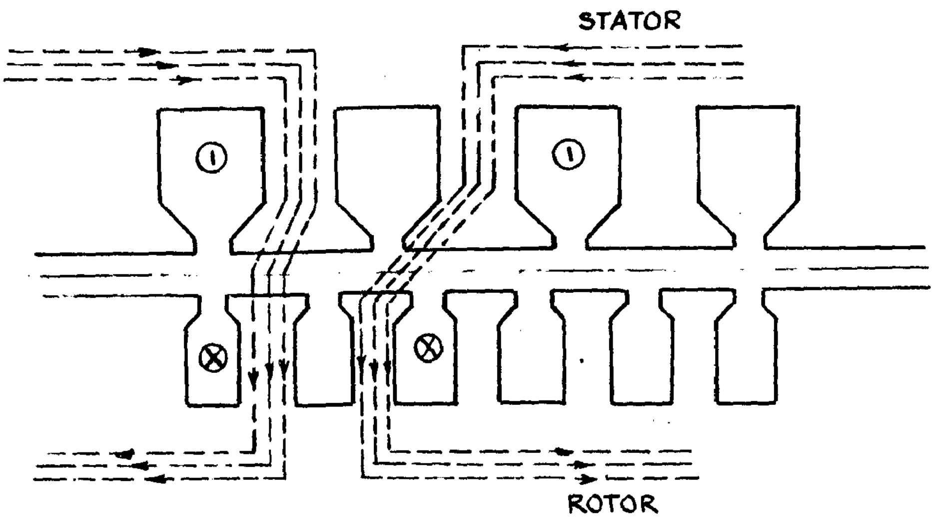


FIGURE 2.17

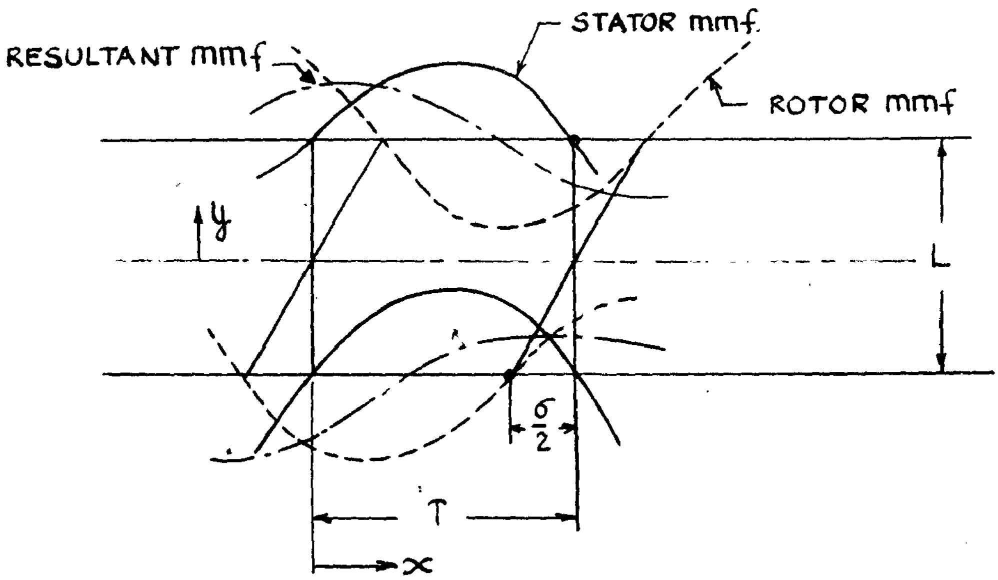


FIGURE 2.18

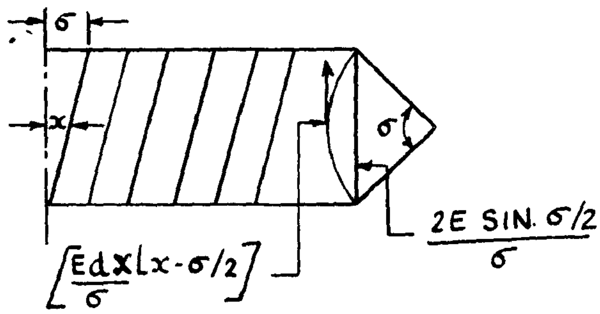


FIGURE 2.19

induced in a straight conductor. The net voltage across the bar is the sum of the projections of the elementary voltages on the mid point value or

$$E = \int_0^{\sigma} \frac{E}{\sigma} \operatorname{Cos}\left(x - \frac{\sigma}{2}\right) dx = \frac{2\,E\,\operatorname{Sin}\,\sigma/2}{\sigma}$$

$$\therefore\ K_s = \frac{2\,\operatorname{Sin}\,\sigma/2}{\sigma}$$

Similarly for mth harmonic

$$K_{s(m)} = \frac{2\,\operatorname{Sin}\,m\sigma/2}{m\,\sigma} \qquad (2\text{-}66)$$

Skew factor for harmonics are tabulated in Table 2-10 with $\sigma = 60°$ (Elect)

Table 2.10 HARMONIC SKEW FACTORS

m	1	5	7	11	13	17	19	23	25	29	31
$K_{s(m)}$	.998	.987	.98	.946	.925	.873	.845	.79	.74	.657	.616
m	35	37	41	43	47	49	53	55	59	61	65
$K_{s(m)}$	.527	.482	.391	.346	.256	.212	.129	.09	.017	.06	.076

Table (2.2) MAGNETISING REACTANCE OF PHASE BELT HARMONICS WITH AND ITHOUT SKEW

m	1	-5	7	-11	13	-17	19	-23	25	-29
$X_{M(m)}$ ohm	138.0	0.256	.077	.02	.0144	.0132	.0176	.261	.221	.0076
m	31	-35	37	-41	43	-47	-71	73	-95	97
$X_{M(m)}$ ohms	.004	.002	.002	.002	.0034	.0625	.0274	.0258	.015	.0146

Table 2.3 MAGNETISING REACTANCE OF SLOT HARMONICS WITH AND WITHOUT SKEW

m	-23	25	-47	49	-71	73	-95	97
$X_{M(ms)}$ (ohms)	.26	.221	.0625	.0575	.0274	.0256	.0153	.0147

Table 2.4 MAGNETISING REACTANCE OF PERMEANCE HARMONICS WITH AND WITHOUT SKEW

	M	$(v_1 \frac{R}{p} \pm m)$	$X_{M(p)}$	Direction of Rotation.
Stator		23	2.67	Backward
X_M = 138	1	25	2.46	Forward
Ohms.		47	1.30	Backward
		49	1.25	Forward

	m	$(v_1 \frac{R}{p} \pm m)$	$X_{M(p)}$	Direction of Rotation.
		35	.31	Forward
Rotor		33	.326	Backward
X_M = 138	1	69	.078	Forward
Ohms.		67	.08	Backward

Table 2.5 LEAKAGE REACTANCE OF PHASE BELT HARMONICS

m	K_{sm}	$J_{(m)}$	$X_{M(m)}$	with skew				without skew		
				X'_{2mm}	X'_{2m}	$X_{d,m}$	X_{2m}	X_{2mm}	X_{2m}	X_{2m}
1	.998	.895	138	2.46	.392	.21	3.1	2.44	.392	2.83
5	.987	.360	.256	1.02	.71	.003	.083	.045	.018	.054
7	.98	.36	.077	1.04	.565	.001	.044	.026	.012	.038
11	.946	.258	.02	.784	.685	.001	.026	.012	.008	.02
13	.925	.258	.014	.825	.75	.001	.028	.0123	.009	.022
17	.873	.21	.0132	.755	1.11	.002	.051	.016	.02	.035
19	.845	.21	.0176	.805	1.34	.003	.11	.031	.039	.07
23	.775	.186	.261	.845	2.36	.058	3.25	.51	1.37	1.88
25	.74	.186	.221	.93	3.72	.058	4.70	.51	1.97	2.48
29	.657	.163	.007	1.1	13.9	.002	.692	.02	.278	.298
31	.616	.163	.004	1.17	40.5	.001	1.13	.012	.42	.43
35	.527	.149	.002	1.48	5550	.03	1.97	.071	1.88	1.887
37	.482	.149	.0017	1.73	710	.032	12.5	.007	.248	.255
41	.391	.138	.002	2.46	256	.035	7.05	.01	.086	.096
43	.346	.138	.0034	3.13	246		11.5	.017	.096	.114
47	.256	.129	.0625	5.4	334		339.4	.352	1.27	1.62
49	.212	.129	.0575	7.8	470		477.8	.352	1.15	1.5

Table 2.6 SLOT HARMONIC LEAKAGE REACTANCE WITH AND WITHOUT SKEW FACTOR

(mm)	$X_{2(mm)}$ with skew	$X_{2(mm)}$ without skew
23	1.38	4.60
25	1.990	1.97
47	1.36	1.32
49	1.16	1.11
71	15.5	15.2
73	5.74	5.6
95	3.40	3.20
97	10.28	10.1

Table 2.7 PERMEANCE HARMONIC LEAKAGE REACTANCE WITH AND WITHOUT SKEW FACTOR

m	$X_{M(m)}$	$\nu_1 \frac{S}{p} \pm m$	$\sin^2 \frac{(\nu_1 \frac{S}{p} \pm m)\, p\pi}{R}$	$X_{2(mp)}$
Stator Fundamental.	138	23	0.75	1.30
		25	0.537	1.97
		47	0.875	1.27
		49	.975	1.14
m	**$X_{M(m)}$**	**$\nu_1 \frac{R}{p} \pm m$**	**$\sin^2 \frac{(\nu_1 \frac{R}{p} \pm m)\, p\pi}{S}$**	**$X_{2(mp)}$**
Rotor Fundamental	138	35	0.99	2.28
		33	0.85	2.65
		69	0.154	15.3
		67	0.415	5.28

Table 2.8 ROTOR RESISTANCE OF SLOT HARMONICS WITH AND WITHOUT SKEW

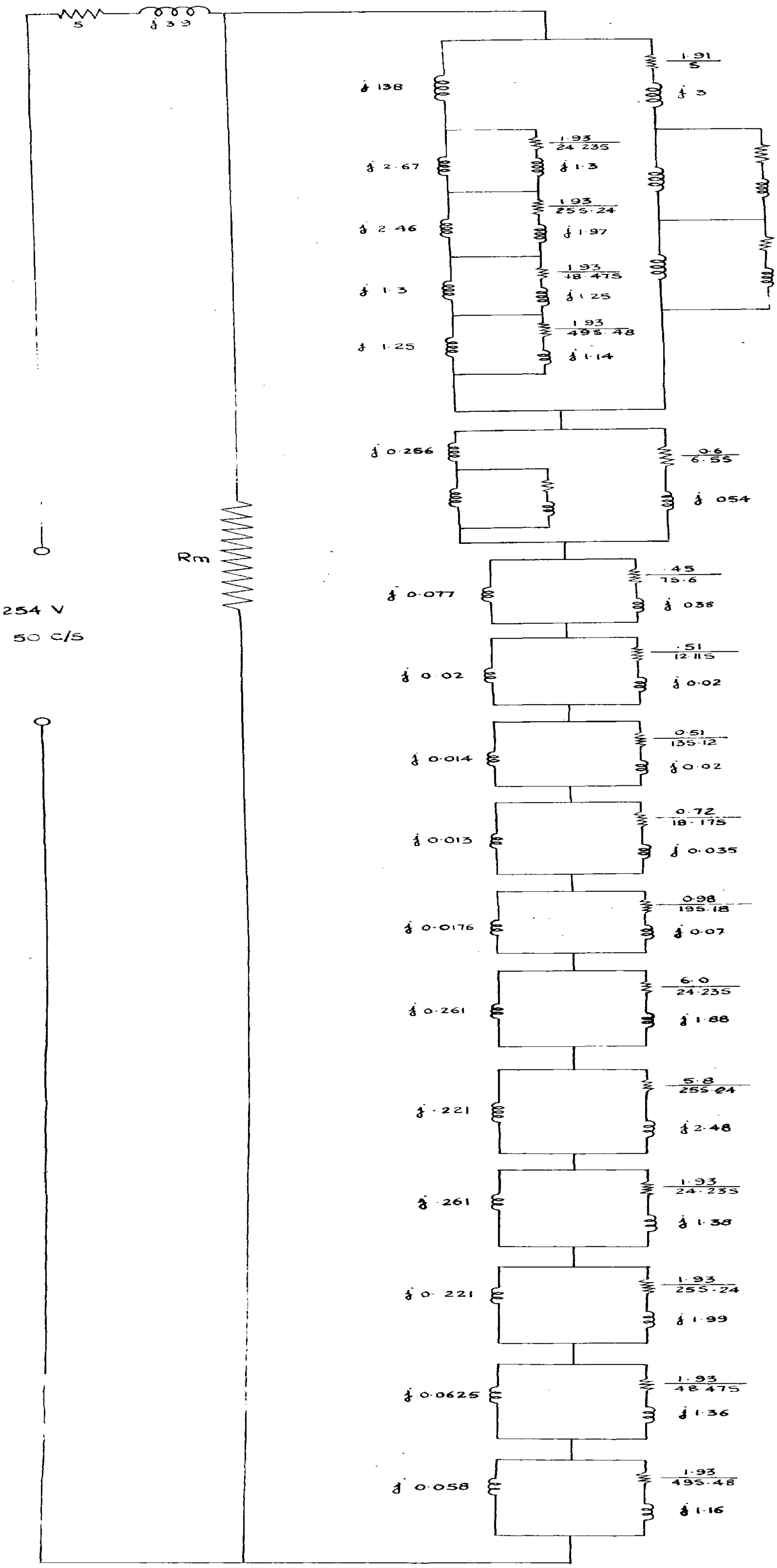
m	23	25	47	49	71	73	95	97
$R_{2(mm)}$ ohms	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93

Table 2.9 ROTOR RESISTANCE OF PERMEANCE HARMONICS

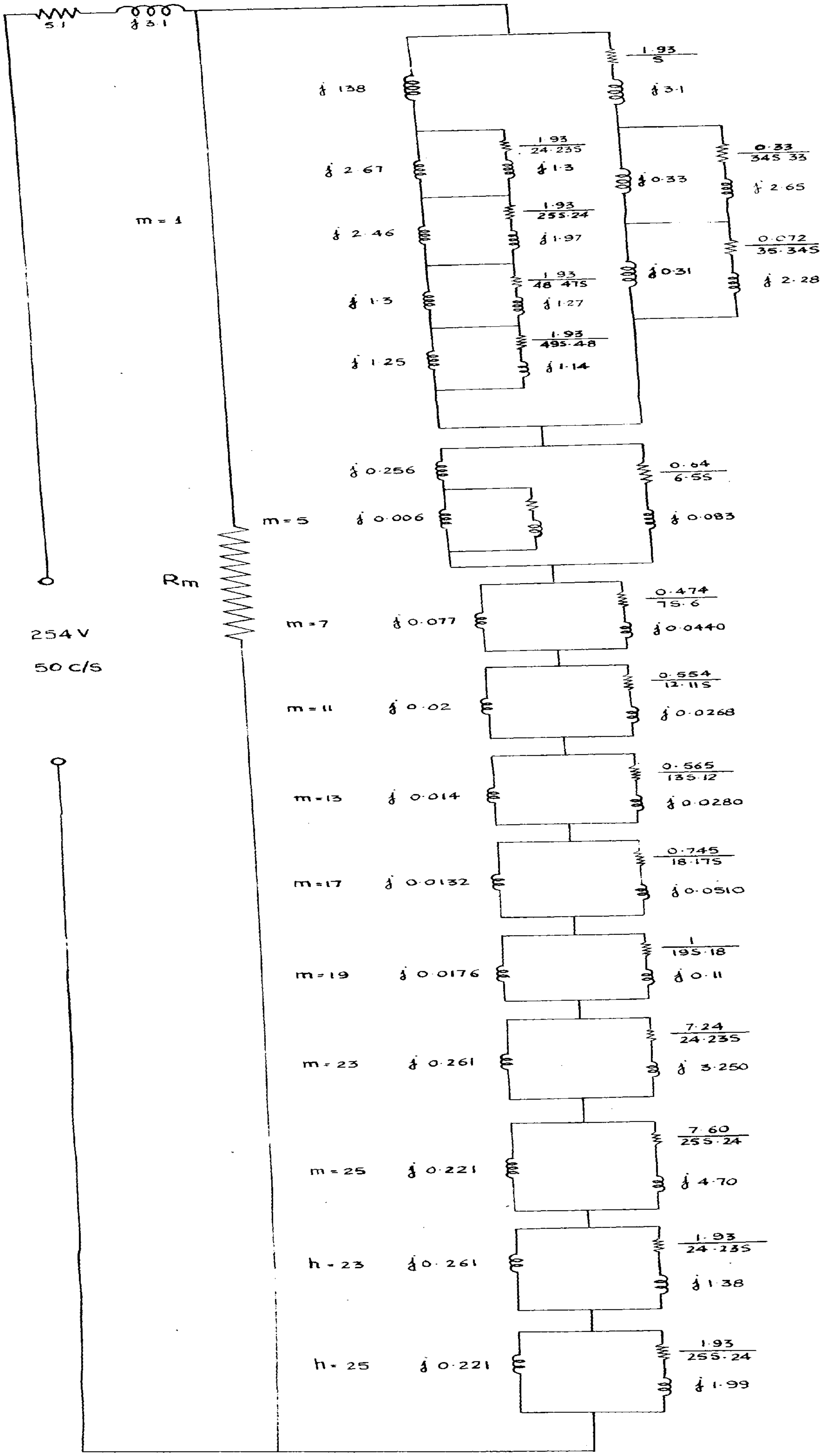
	mp	R_{2mp} with skew	R_{2mp} without skew	s_m
Stator	23	1.93	1.91	24-23s
	25	1.93	1.91	25s-24
	47	1.93	1.91	48-47s
	49	1.93	1.91	49s-48
Rotor	33	0.072	0.072	34-33s
	35	0.33	0.33	35s-34

Table 2.10 ROTOR RESISTANCE OF PHASE BELT HARMONICS WITH AND WITHOUT ROTOR SKEWED

m	α_{sm} (deg.)	$2\sin^2\alpha_{sm}/2$	R_{2bm}	K_{sm}	$R'_2(m)$ with skew ohms	$R'_2(m)$ without skew ohms	s_m	Direction of rotation.
1	10.6	.717	1.025	.998	1.93	1.91	s	+
5	53	.0307	2.523	.981	.64	.60	6-5s	−
7	74.2	.0169	2.523	.98	.474	.45	7s-6	+
11	116.6	.0085	3.56	.945	.554	.515	12-11s	−
13	137.8	.0062	3.56	.925	.565	.51	13s-12	+
17	181	.0064	4.35	.873	0.8	.72	18-17s	−
19	202	.00635	4.35	.845	1.2	0.99	19s-18	+
23	244	.0085	5.05	.77	7.24	6.0	24-23s	−
25	266	.0115	5.05	.74	7.60	5.8	25s-24	+
29	308	.0318	5.61	.657	2.04	1.0	30-29s	−
31	330	.091[illegible]	5.61	0.616	1.69	0.65	31s-30	+
35	372	.56	6.15	0.527	1.87	0.64	36-35s	−
37	394	.0717	6.15	.482	1.89	0.50	37s-36	+
41	416	.0278	6.65	.391	3.1	0.50	42-41s	−
43	437	.0109	6.65	.346	4.6	0.58	43s-42	+
47	500	.0069	7.14	.256	31.0	2.0	48-47s	−
49	521	.0063	7.14	.212	37.4	2.0	49s-48	+
53	564	.0064	7.55	.129	13.9	0.20	54-53s	−
55	585	.0072	7.55	0.09	15.25	0.13	55s-54	+
59	626	.0115	7.92	0.0169	21.[illegible]	.018	60-59s	−
61	648	.0178	7.92	.0164	21.7	.020	61s-60	+
65	690	.0915	8.24	.076	20.0	.12	66-65s	−



GENERAL EQUIVALENT CIRCUIT OF INDUCTION MACHINE
(ROTOR UNSKEWED PARAMETERS)
FIGURE 2.15



GENERAL EQUIVALENT CIRCUIT OF INDUCTION MACHINE
(ROTOR SKEWED PARAMETERS)

FIGURE 2.14

CHAPTER III

COMPUTATION OF ASYNCHRONOUS PARASITIC TORQUES OF INDUCTION MACHINE WITH ROTOR SKEWED AND UNSKEWED

The term parasitic torques includes all those torques produced by mmf wave except that of fundamental . As investigated (Chapter I) superimposed upon the currents due to the fundamental sine wave field of an induction machine, there are smaller harmonic currents produced by the myriad of harmonic fields.

In the induction motor, the rotor is not connected to the line. The main mmf wave (fundamental) of the stator produces an mmf waves in the rotor which has the same number of poles as the stator wave and which is at stand still with respect to the stator wave at any speed of the rotor and so also the other harmonics. Fundamental produces the useful torque of the machine whereas the torque produced of this kind by harmonics is called Asyncronous Parasitic torques.

In addition to asyncronous torque, there are other torques too, e.g. Syncronous torques and stray torques which will be discussed under seperate heads.

As the harmonic fields have more poles, therefore asyncronous torques occur at sub-syncronous speed. As the motor accelerates through the syncronous speed of one of these harmonic fields the harmonic torque reverses causing a dip in the resultant torque -speed curve. Unless minimised by good design, the consequent asyncronous crawling may seriously impair the motor's starting ability. As discussed further in detail, the skew reduces the asyncronous torque to some extent.

As discussed under Chapter II, the harmonic fields are equivalent to low-power motors connected to the same shaft and electrically connected in series (Fig. 2-15). The asyncronous torques increase the motor heating too much. Theoretical approach to the problem is directed here and then based upon these conventional results, extensive calculation has been undertaken to verify the results.

3.1. PRODUCTION OF ASYNCRONOUS TORQUE

The stator mmf equation from (1-11):

$$f_s(\theta,t) = F_m \sum \text{Cos } (mp\theta \mp wt) \qquad \ldots\ldots(3\text{-}1)$$

$$f_r(\theta,t) = F'_m \sum \text{Cos } \left[\pm \left\{ 1 + (n-m)(1-s) \right\} wt + np\theta \right] \qquad (3\text{-}2)$$

Power developed in air gap is $\propto B^2$ (by Kelvin's Law)

$$P_d = \bar{C} \; f_s(\theta,t) \times f_r(\theta,t)$$

$$P_d = \int_0^{2\pi} p_d \, d\theta = \int_0^{2\pi} \bar{C} \, F_m \sum \text{Cos } (mp\theta \mp wt) \, F'_m \sum \text{Cos } \left\{ np\theta \pm 1 + (n-m)(1-s) \, wt \right\} \qquad \ldots(3.3)$$

where $\bar{C}$ = Constant.

$P_d = 0$ If $m \neq n$

P_d = definite value If $m = n$

Equation 3.3. reflects that torque can be produced by harmonics of the same order existing seperately in stator and rotor respectively There is no torque possible between two harmonics of unequal order.

Since n = rotor harmonic order. $= \pm v_2 \frac{R}{p} + m \qquad \ldots.(3\text{-}4)$

Equation 3-4 (determined under Chapter I) give~~s~~ rise to two cases viz:

Case 1 : When $v_2 = 0$

then $n = m$ i,e every stator harmonic induses in rotor circuit the equal order of harmonic rotating in the same direction. The torque produced by locking of such two harmonics is named as Asyncronous torques.

Case 2 : When $v_2 = k_2$ (any integer excluding zero)

then $n = \pm v_2 \frac{R}{p} \pm m_1$ (3.5)

When $n = m_2$

Thus there is torque when n is given by (3.5) locks with stator harmonic m_2. The torque of this kind as already stated is marked as 'syncronous torque and will be dealt with fully in next Chapter.

Taking up Case 1 when $n = m$ substituting it in equation 3.1 and 3.2, Power developed is

$$P_d \propto F'_n F'_m \mathrm{Cos}^2 (n\theta \pm wt) \qquad (3.6)$$

Equation 3.6 shows, Asyncronous torque thus produced by first order stator and first order ($v_2 = 0$) rotor harmonics is independent of slip. It will be there at every speed of the motor. The nature of Harmonic Asyncronous torques is approximately the same as of fundamental torque / speed curve.

The harmonic (equation 2-31) s_m is :

$$s_m = 1 \pm m (1-s)$$

When harmonic slip becomes zero

$$1 \pm m(1-s) = 0$$

then Slip $= 1 \pm 1/m = \dfrac{N_s - N_m}{N_s} = 1 - \dfrac{N_m}{N_s}$

$$\therefore \quad N_m = \mp N_s(1/m) \tag{3.7}$$

i.e., harmonic slip is zero when rotor attains 1/m th of syncronous then torque produced by harmonic 'm' becomes zero and for speed greater than 1/m the torque reverses its direction (See Fig. 3-1).

3.2. MAGNITUDES OF HARMONIC ASYNCRONOUS TORQUES

The general equivalent circuit for the squirrel cage induction machine dealt under Chapter II, in author's view will be the correct approach to the determination of complex phenomena of torque developed by individual harmonics in certain speed range. Adnan Odok[11], simplifies the general equivalent circuit like Fig. (3-2)

$$V_1 = \bar{I}_1 (R_1 + jX_1 + + V_F + \sum_{m>1}^{\infty} V_{1m} \tag{3.8}$$

The torque of mth harmonic

$$T_{(m)} = \frac{R_e(\bar{I}\ \bar{V}_{1m}^{*})}{W_{sm}}$$

When

W_{sm} = Angular velocity of the mth harmonic.

$$\bar{V}_{1m} = \bar{I}_1 \frac{j X_{Mm}(R_{2m}/s_m + j X_{2m})}{\frac{R_{2m}}{s_m} + j(X_{Mm} + X_{2m})} \tag{3.9}$$

He assumes I_1 (current) to be constant, and then by finding the locus diagram of the voltage, torque at different speed is determined. In author's view it is over simplification of the equation (3.9) since current I_1 varies with rotor speed too much as clear from circle locus of the

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When

w_{sm} = Angular velocity of the mth harmonic.

$$\bar{V}_{1m} = \bar{I}_1 \; \frac{j\,X_{Mm}\,(R_{2m}/s_m + j\,X_{2m})}{\dfrac{R_{2m}}{s_m} + j\,(X_{Mm} + X_{2m})} \qquad (3.9)$$

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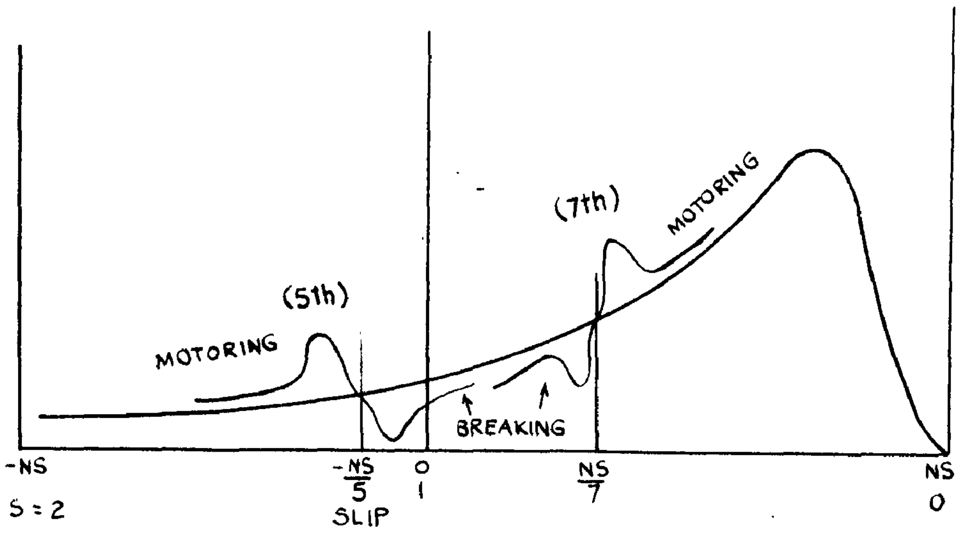


FIGURE 3.1

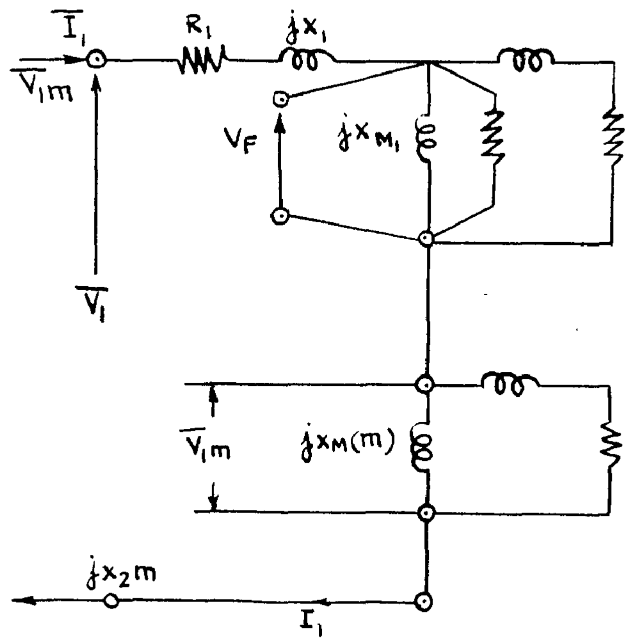


FIGURE 3.2

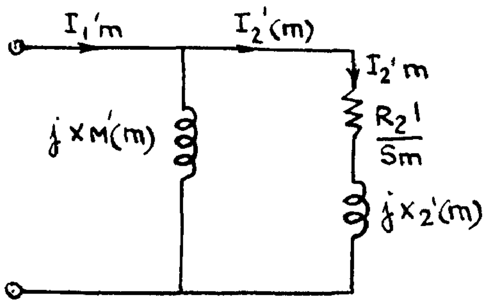


FIGURE 3.3

induction machine. Therefore the author takes the more complicated but more dependable way of finding the harmonic Asyncronous torque. The method of attack can be out lined as:

1. Solution of the General equivalent circuit for different slips.
2. Then using the currents so computed for different slips to get harmonic torques.

The torque of the main wave is :

$$T_1 = \frac{I_2^2 \; R_2}{s}$$

(Referred to primary) $T_1 = I_2'^2 \frac{R_2'}{s}$ watts (3.10)

The squirrel cage rotor does not distinguish between the main wave and the harmonics therefore (3-10) must also apply to the harmonic torques, substituting harmonic values

$$T_{(m)} = \frac{I_{2m}'^{\,2} \; R_{2m}'}{s_m} \quad \text{watts} \qquad (3.11)$$

Where all the secondary values subscripted with dash and for per bar.

If

I_{1m}' Primary current referred to secondary per bar.

I_{2m}' = Bar current (secondary)

Then from Fig. (3.3)

$$I_{2(m)}' \left[\frac{R_2'}{s_m} + j\, X_{2(m)}' \right] - \left[I_{1m}' - I_{2(m)} \quad j\, X'_{M(m)} \right] = 0$$

$$I_{2(m)}' = \frac{1}{(1 + \tau'_{2m}) - j\left(\frac{R_{2m}'}{s_m}\right) / X'_{M(m)}} \; I_{1m}' \qquad (3.12)$$

But $I_{1m}' = \left(\frac{2m_1 \; N_1 \; k_{w1m}}{R \; K_{sm}} \right) I_1$ (3.13) and reduction factor for impedance from per bar to 'total secondary referred to Primary' value is[5]

$$RF = \frac{4m_1(N_1 K_{w1m})^2}{R\ K_{sm}^2} \quad \text{with respect to any wave}(\bar{m}) \qquad (3.14)$$

Substituting (3-12) in equation (3.11), and (3.13)

$$T_{(m)} = R_{2m}\left(\frac{s_m\ X^2_{M(m)}}{R^2_{2(m)} + s_m^2\ (1 + \tau_{2m})^2\ X^2_{M(m)}}\right) m_1\ I_1^2 \quad \text{watts} \qquad (3.15)$$

3.3. <u>PULL OUT SLIP AND PULL OUT TORQUE DEVELOPED</u>

I_1 is complicated function of harmonic slip (s_m), but since the parasitic torques occur a high slips and at high slips primary current can be assumed constant[5] to a fair degree of accuracy. In order to find maximum torque (pull out torque) of mth wave, differentiate equation (3.15) taking I_1 constant. Then

$$s_{m\ p.o.} \quad \frac{R_{2(m)}}{(1 + \tau_{2m})\ X_{M(m)}} \qquad (3.16)$$

and maximum asynchronous torque:

$$T_{m(max)} = \frac{X_{M(m)}}{(1 + \tau_{2m})}\ I_1^2 \quad \text{watts per phase} \qquad (3.17)$$

Pull out slip defined in terms of fundamental slip is

$$s_{p.o.(m)} = \frac{(m - 1)(1 + \tau_{2m})\ X_{M(m)} + R_{2(m)}}{m(1 + \tau_{2m})\ X_{M(m)}} \qquad (3.18)$$

Neglecting $R_{2(m)}$

$$s_{p.o.(m)} \simeq (1 - 1/m) \qquad (3.19)$$

3.4. DETERMINATION OF CURRENT SLIP CURVE

The general equivalent circuit of Fig. (2-14) and (2-15) is solved for different values of slip in the range of slip zero to slip two (motoring and braking regions). The calculated results with partial calculations are included in Table (3.1) to Table (3.2). Calculated results are plotted in Figure (3.5) curve 2, Fig. 3.4 curve 2. Curves are found taking into account the skew, and another set considering rotor unskewed.

3.5. DETERMINATION OF HARMONIC ASYNCHRONOUS TORQUE

Calculation for relevant harmonics, which are likely to affect the performance considerably are calculated with the equations dealt 3.1. through 3.19. The calculated results so obtained are plotted on graph and partial calculations carried out are tabulated. Curves of slip, speed, power factor, current developed torques are given in Fig. 3.4 to Fig. 3.10 and Tabulation of Table (3.3) to Table (3.6) shows the results of harmonic torques. Pull out torque and pull out slip for harmonics.

Table 5.1 - DETERMINATION OF CURRENT POWER FACTOR OF GENERAL E UIVALENT CIRCUITS FOR DIFFERENT SLIPS

Slip	$Z_{equ.}$	$Z\angle\theta$	I	Cos θ	Fundamental torque	Total torque	Total harmonic torque
0.0	5.05+j143.17	143.4∠87.9	1.77	0.04	0	0	0
0.1	23.35+j9.46	25.0∠22.2	10.15	0.93	1935.0	193.0	-5.0
0.2	14.31+j9.29	17.0∠32.7	15.0	0.845	2081.0	2070	-11.0
0.3	11.21+j 8.28	14.0∠36.4	18.1	0.80	2028	2010	-18.0
0.4	9.70+j 8.21	12.65∠40.3	20.0	0.765	1887	1860	-17.0
0.5	8.58+j 8.14	11.8∠43.5	21.50	0.735	1731	1700	-31.0
0.6	8.16+j 8.11	11.6∠41	22.0	0.727	1528	1490	-38.0
0.7	7.71+j 8.22	11.3∠46.8	22.5	0.685	1377	1333	-44.0
0.8	7.42+j 8.28	11.15∠48.2	22.8	0.67	1346	1285	-61.0
0.9	7.16+j 8.34	11.0 ∠ 49.2	23.0	0.65	1193	1135	-58.0
1.0	6.95+j 8.20	10.75∠ 49.7	23.6	0.65	1105	1050	-50.0
1.1	6.73+j 8.23	10.6 ∠ 50.7	24.0	0.625	1003	966	-37.0
1.2	6.55+j 8.23	10.5 ∠ 51.5	24.2	0.675	920	905	-15.0
1.3	6.32+j 8.2	10.34∠ 52.3	24.6	0.59	844	860	+16.0
1.4	6.25+j 8.16	10.3 ∠ 52.6	24.6	0.59	763	800	+37.0
1.5	6.14+j 8.12	10.2 ∠ 53.0	24.8	0.6	719	765	+ 46.0
1.6	6.06+j 8.07	10.0∠ 53.1	25.4	0.6	695	750	+55.0
1.7	5.9 +j 8.08	10.0∠ 54.0	25.4	0.59	652	700	+48.0
1.8	5.95+j 8.0	10.0∠ 53.4	25.4	0.6	622	665	+33.0
1.9	5.98+j 8.0	9.95∠ 53.2	25.5	0.6	605	635	+30.0
2.0	5.86+j 8.0	9.91∠ 53.7	25.6	0.59	580	605	+30.0

Table 3.2 SOLUTION OF GENERAL EQUIVALENT CIRCUIT WITHOUT SKEW

Slip	X + Y	I	Cos θ
0.05	39.65 + j 17.07	5.8824	.91
0.10	23.27 + j 10.24	9.98	.91
0.15	17.31 +j8.9	13.04	.88
0.2	14.28 +j8.43	15.31	0.86
0.25	12.45 +j8.21	17.02	0.83
0.3	11.22 +j8.1	18.3	0.81
0.35	10.35 +j8.03	19.3	0.79
0.4	9.69 +j 7.9	20.2	0.77
0.45	9.18 +j 7.9	20.87	0.75
0.5	8.78 +j 7.9	21.4	0.74
0.55	8.45 +j 7.95	21.89	0.72
0.6	8.17 +j 7.9	22.2	0.71
0.65	7.9 +j 7.9	22.59	0.70
0.7	7.74 +j 7.9	22.8	0.69
0.75	7.57 + j7.9	23.29	0.68
0.8	7.43 + j7.9	23.45	0.68
0.85	7.29 + j 7.9	23.45	.67
0.9	7.18 +j 8.0	23.58	0.66
0.95	7.08 + j 8.07	23.64	0.66
1.0	7.0 +j8.0	23.7	0.65

Contd..

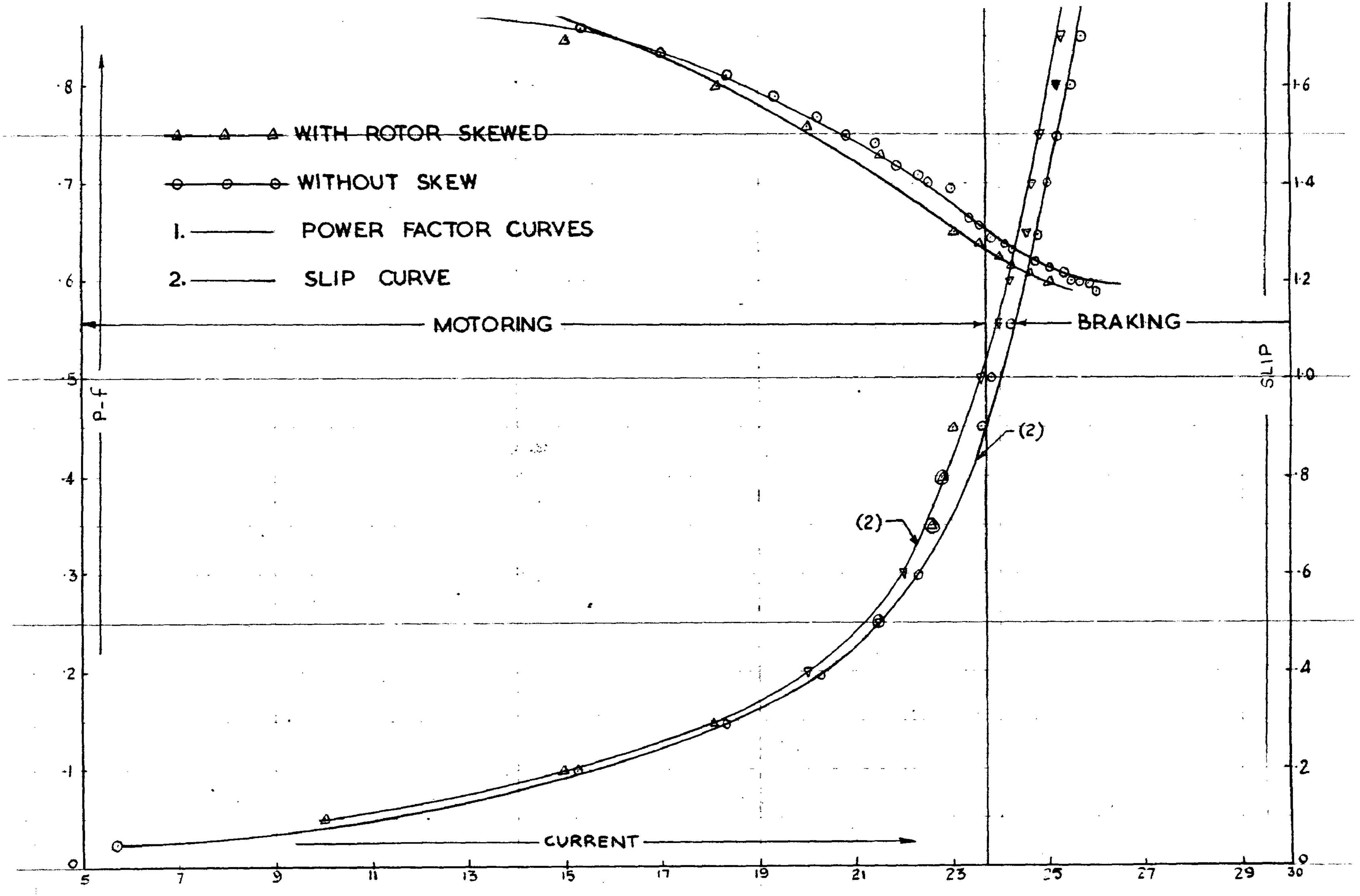
Slip	X +Y	I	Cos θ
1.05	6.866 +j 8.1	23.89	.64
1.1	6.73 +j 8.0	24.12	.64
1.15	6.64 +j 8.0	24.29	0.63
1.25	6.46 +j804	24.6	.62
1.3	6.38 +j 8.0	24.75	.62
1.35	6.31 +j 8.0	24.9	.61
1.4	6.25 + j 7.9	25.0	.61
1.45	6.19 + j 7.9	25.2	.61
1.5	6.14 + j 7.9	25.3	.61
1.55	6.1 + j 7.9	25.39	.61
1.6	6.0 + j 7.9	25.49	.6
1.65	6.03 + j 7.8	25.57	.60
1.7	6.0 +j7.8	25.65	.6
1.75	5.9 + j 7.8	25.7	.6
1.8	5.9 + j 7.85	25.78	.6
1.85	5.9 + j 7.8	25.84	.6
1.9	5.9 + j 7.8	25.89	.6
1.95	5.8 + j 7.8	25.94	.6
2.0	5.8 + j 7.8	25.99	.59

Table 3.3 HARMONIC ASYN. TORQUES (ROTOR SKEWED)

Slip	5th	7th	11th	13th	17th	23rd belt	25th belt	23rd slot	25th slot
0.0	-.17	-	-	-	-	-	-	-.2	-
0.1	-6.75	-2.42	-.2	-	-.14	-	-	-.56	-.24
0.2	-14.8	-5.5	-1.3	-	-.3	-	-	-.9	-.4
0.3	-22.2	-8.0	-1.4	-	-.44	-	-	-1.3	-.55
0.4	-30.5	-9.6	-2.0	-.7	-.55	-	-.4	-1.8	-
0.5	-38.5	-10.1	-1.7	-	-6.1	-		-2.2	-
0.6	-43.6	-9.0	-1.6	-.7	-.60	.2	-.8	-3.1	-1.0
0.7	-48.0	-6.5	1.4	-	-	-.3	-	-4.3	1.5
0.8	-54.0	-1.0	-1.0	-.3	-.5	-3.75	1.75	-7.0	-2.4
0.9	-51.5	-.5	-.75	-	-	-5.1	-2.6	-11.3	-5.6
1.0	-45.0	7.95	.3	.2	.1	-4.4	+2.5	+11.9	+6.1
1.1	-28.4	10.5	-	-	-	5.5	2.2	7.2	2.9
1.2	0	13.4	-	.73	.3	5.5	1.4	4.9	1.8
1.3	29.4	14.4	.53	-	.86	4.0	-	3.6	1.3
1.4	48.5	15.0	1.0	-	.65	-	.8	3.2	1.0
1.5	59.0	14.8	1.4	-	-	-	-	2.5	.9
1.6	67.4	15.2	1.73	1.37	.9	-	.6	2.1	.8
1.7	61.5	14.6	2.1	-	-	-	-	1.8	.6
1.8	58.0	14.0	2.2	-	.9	-	.5	1.6	.6
1.9	53.5	13.5	2.4	-	-	-	-	1.4	.5
2.0	50.0	12.7	3.0	1.64	.85	.13	.35	1.3	.5

Table 3.4. HARMONIC ASYN. TORQUES WITHOUT SKEW

ip	Fund.	5th	7th	11th	23rd belt	25th belt	23rd slot	25th slot	Total torque	Total H. torque
	1196.7	-2.23	-.84	-.17	-.13	0.0	0.0	0.0	1193.0	-3.7
	1818.6	-6.6	-2.47	-.49	-.4	-.2	-.2	0.0	1808	-10.6
	2164.2	-16.8	-6.0	-1.13	-1.0	-.5	-.6	-.2	2143	-21.0
	2075.5	-26.0	-8.6	-1.5	-1.7	-.8	-1.0	-.4	2030	-45.5
	1890.0	-34.2	-10.2	-1.78	-2.4	-1.1	-1.3	-.5	1839	-61.0
	1702	-41.6	-10.6	-1.84	-3.1	-1.5	-1.8	-.7	1641	-61.0
	1532	-48.5	-9.5	-1.7	-4.0	-2.1	-2.3	-1.0	1464	-68.0
	1385.4	-54.3	-6.9	-1.5	-5.2	-2.9	-3.1	-1.5	1310	-75.0
	1258.0	-58.4	-2.77	-1.2	-7.0	-4.1	-4.5	-2.5	1178	-80.0
	1146	-58.1	2.14	-.8	-8.7	-4.6	-7.3	-5.4	1063	-84.0
	1042.5	-49.5	6.82	-.4	-5.6	3.8	-11.5	6.18	992	-50.0
	980	-30.0	10.8	0	7.0	4.7	-13.0	3.0	972	13.0
	935	0.0	13.7	0.5	9.3	5.5	-7.9	1.9	963	28.0
	884.5	+33.4	15.5	1.0	7.7	2.7	-4.9	1.4	947	63
	839.3	56.4	16.3	1.5	6.1	2.2	-3.7	1.1	923	84
	797.1	67.5	16.5	2.0	5.1	1.8	-2.96	-.9	894	97
	757.9	70.2	16.3	2.4	4.4	1.6	-2.47	-.8	854	97
	721.7	68.6	15.8	2.7	3.7	1.4	-2.13	.7	816	95
	688	64.9	15.2	2.9	3.3	1.2	-1.89	.6	776	88
	656	60.7	14.6	3.1	3.0	1.0	-1.66	.5	738	82


WITH ROTOR SKEWED
WITHOUT SKEW
1. POWER FACTOR CURVES
2. SLIP CURVE
MOTORING
BRAKING
P-F
SLIP
CURRENT
(2)
(2)
·8
·7
·6
·5
·4
·3
·2
·1
0
1·6
1·4
1·2
1·0
·8
·6
·4
·2
0
5
7
9
11
13
15
17
19
21
23
25
27
29
30

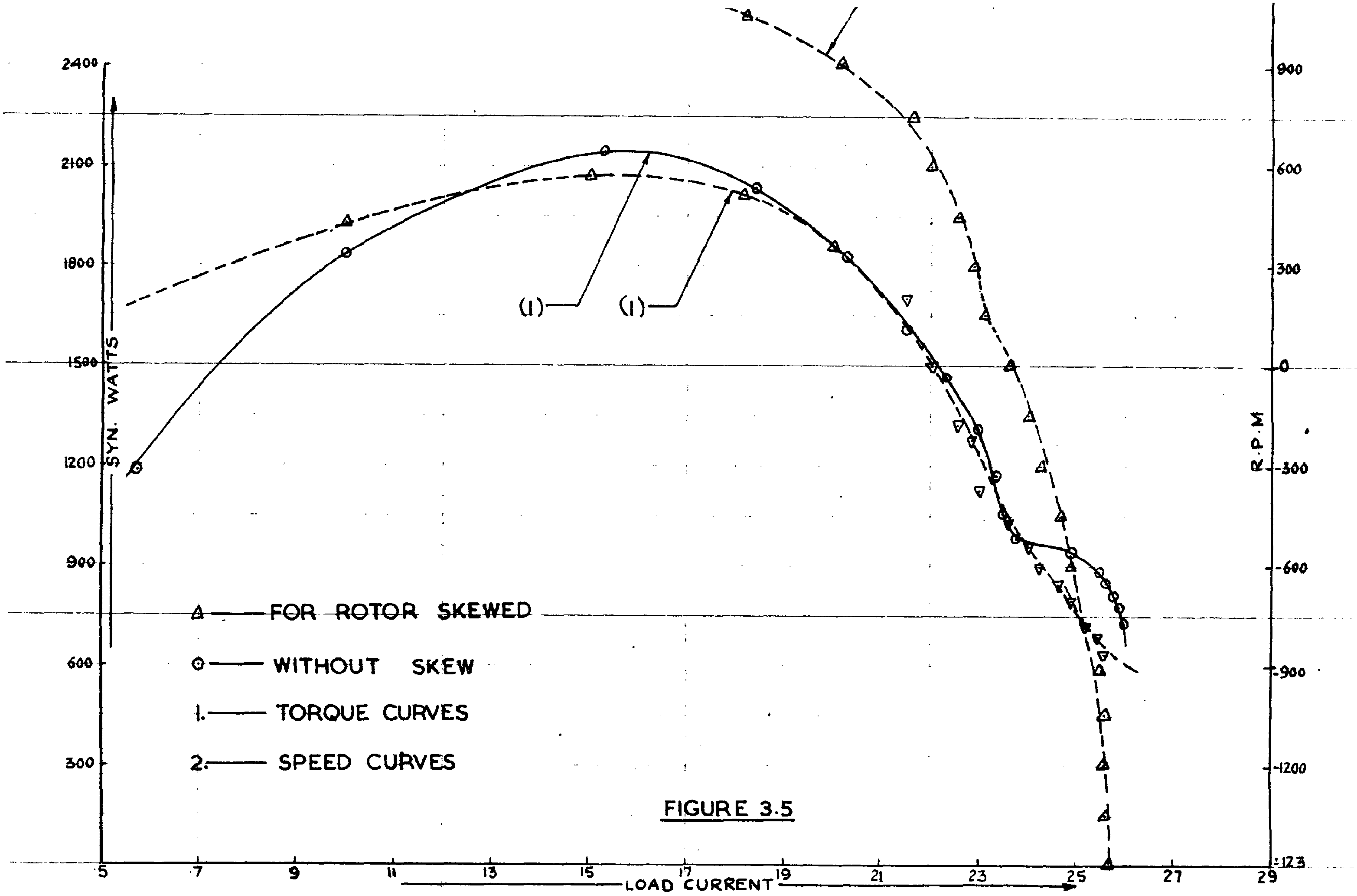
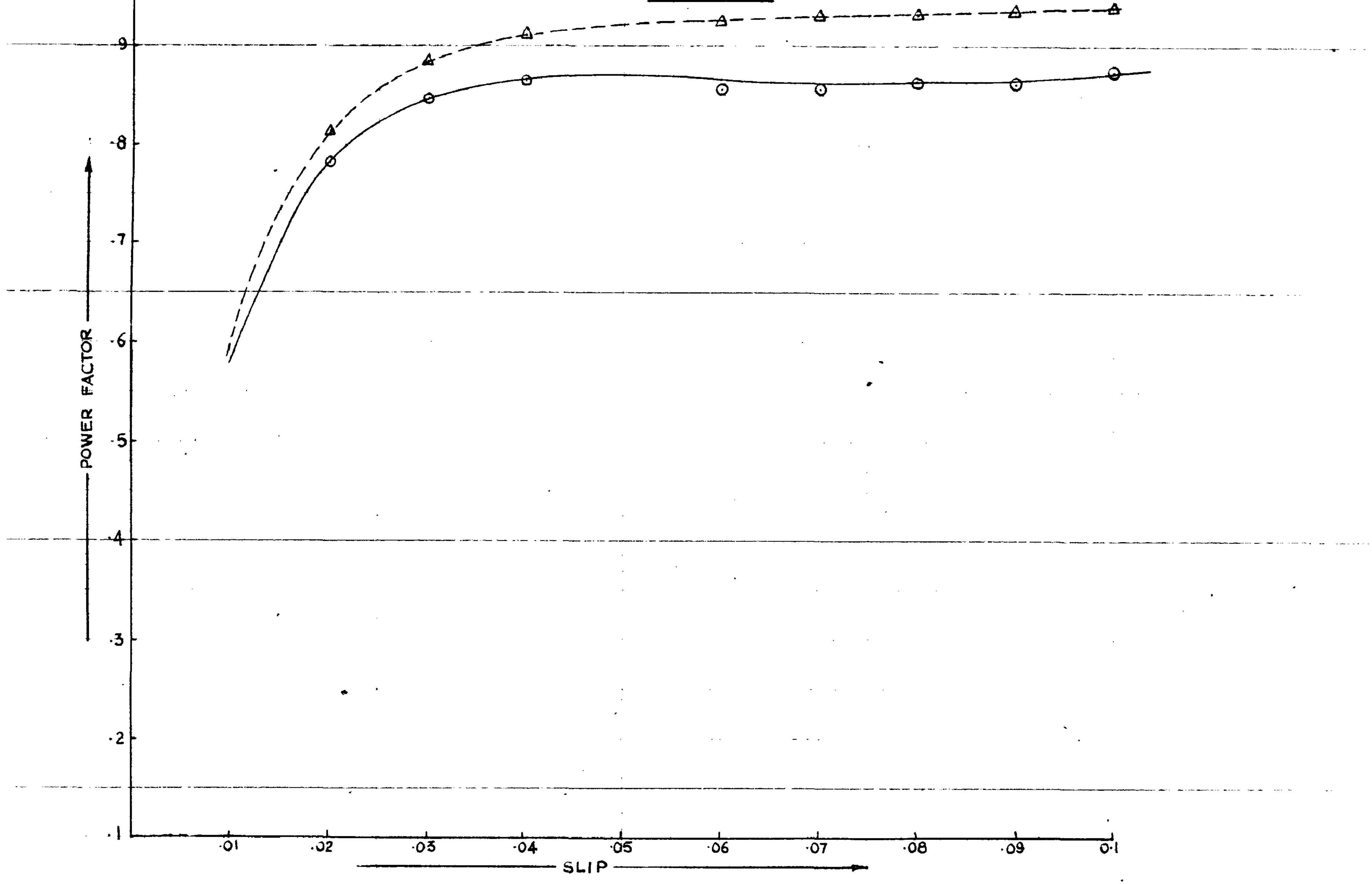

2400
2100
1800
1500
1200
900
600
300
SYN. WATTS
900
600
300
0
-300
-600
-900
-1200
-123
R.P.M
(1)
(1)
FOR ROTOR SKEWED
WITHOUT SKEW
1. TORQUE CURVES
2. SPEED CURVES
5
7
9
11
13
15
17
19
21
23
25
27
29
LOAD CURRENT

FIGURE 3.5

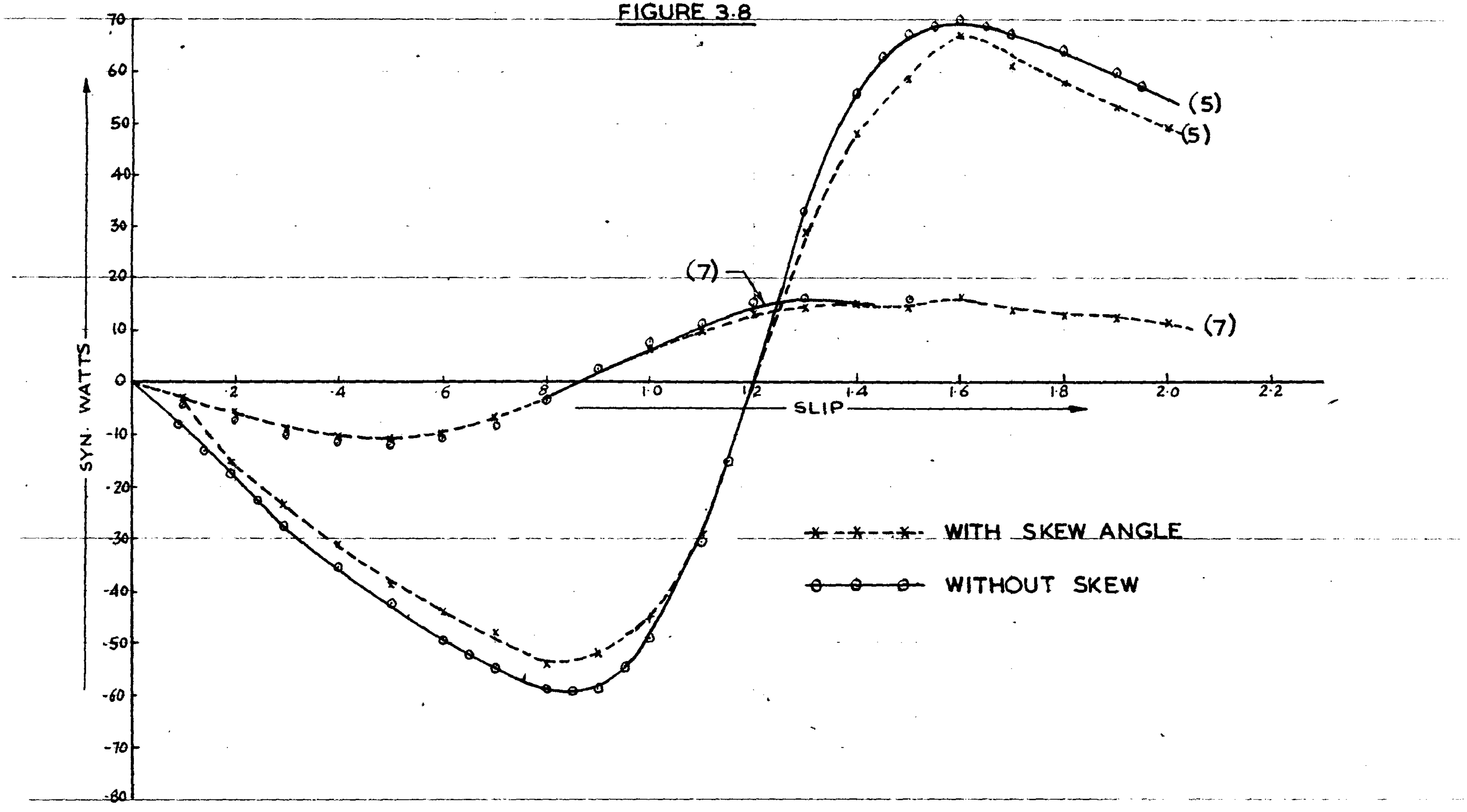
POWER FACTOR CURVES

FIGURE 3.6



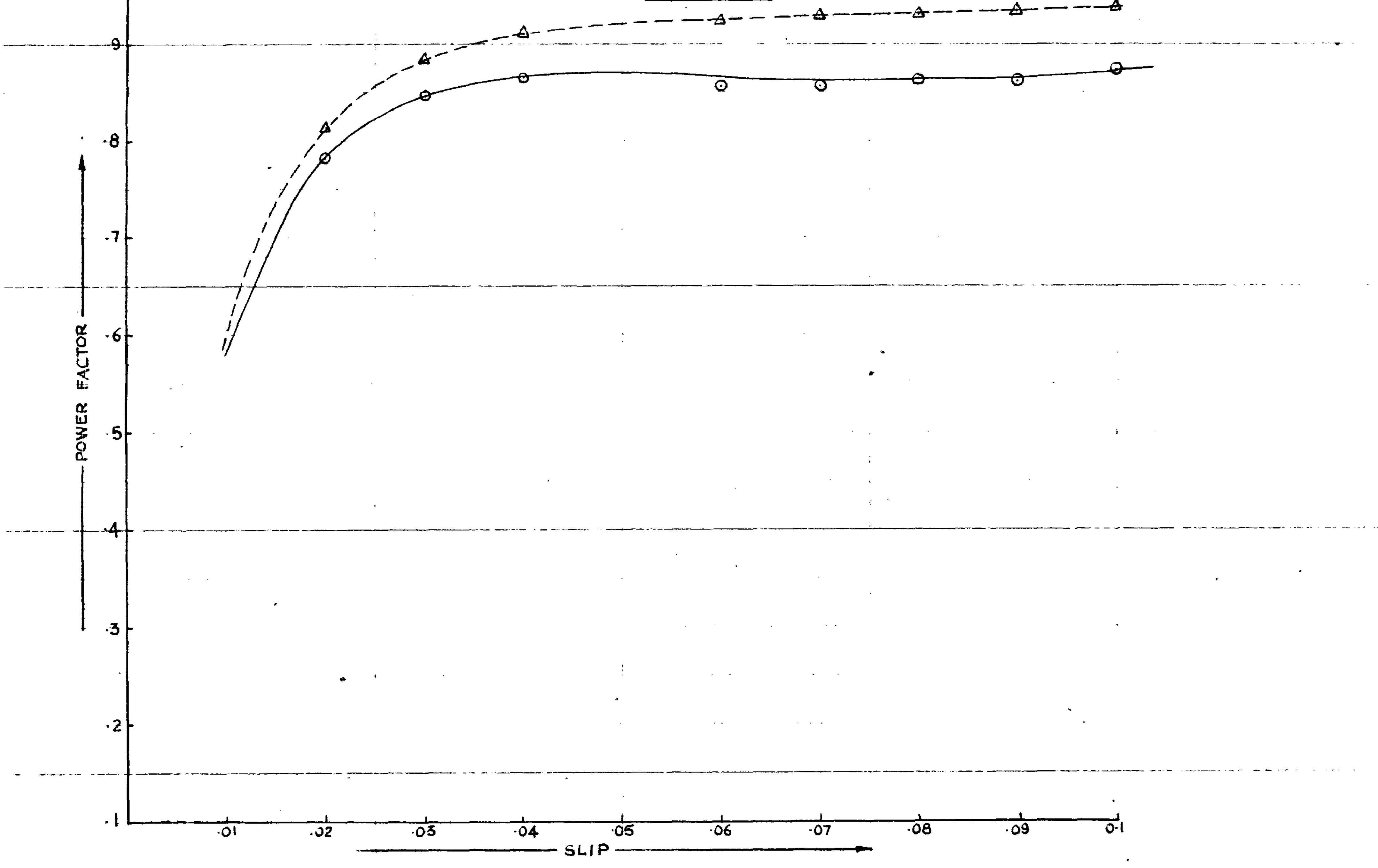
TORQUE SLIP CURVES OF 5TH AND 7TH HARMONICS

FIGURE 3.8



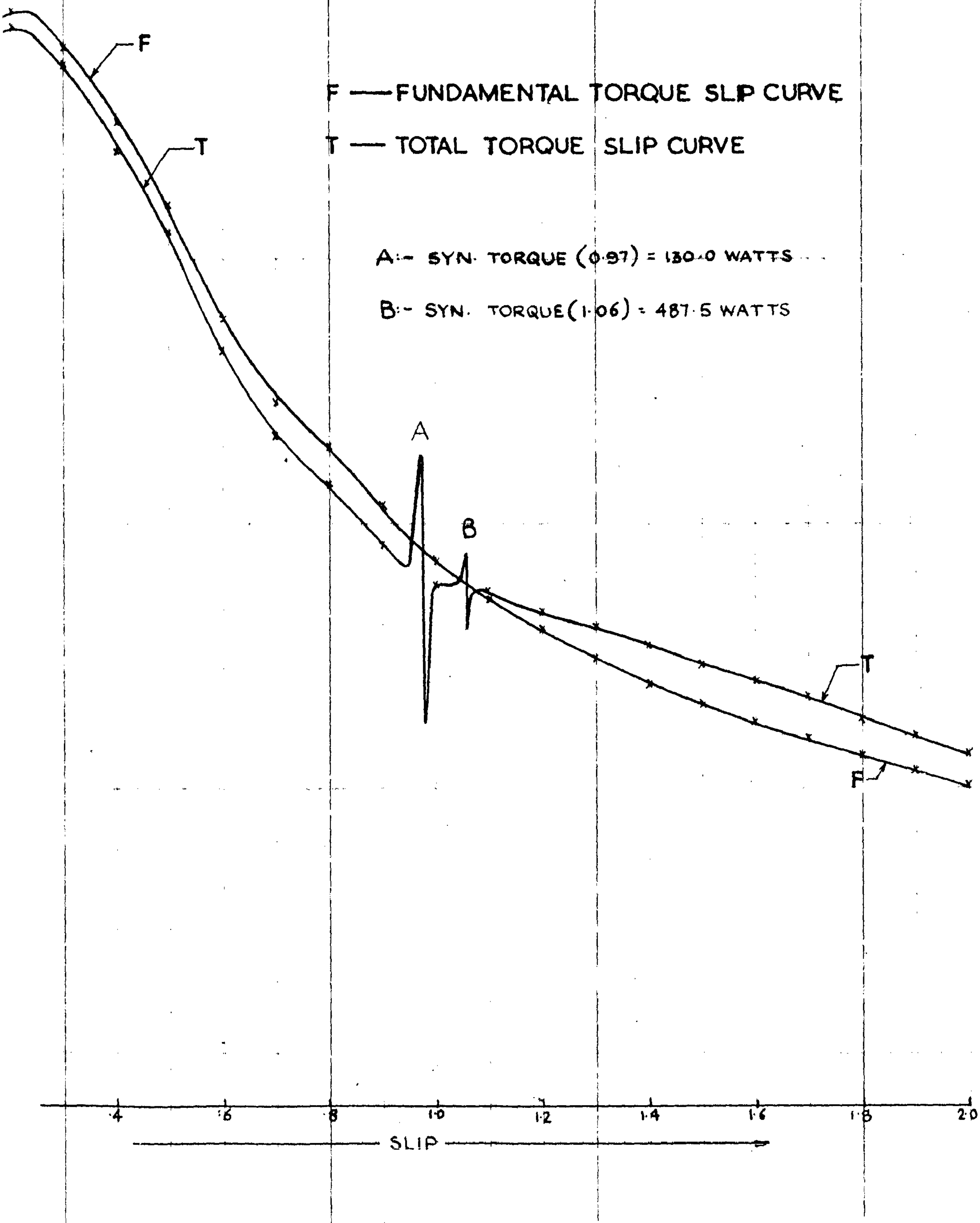
POWER FACTOR CURVES

FIGURE 3.6



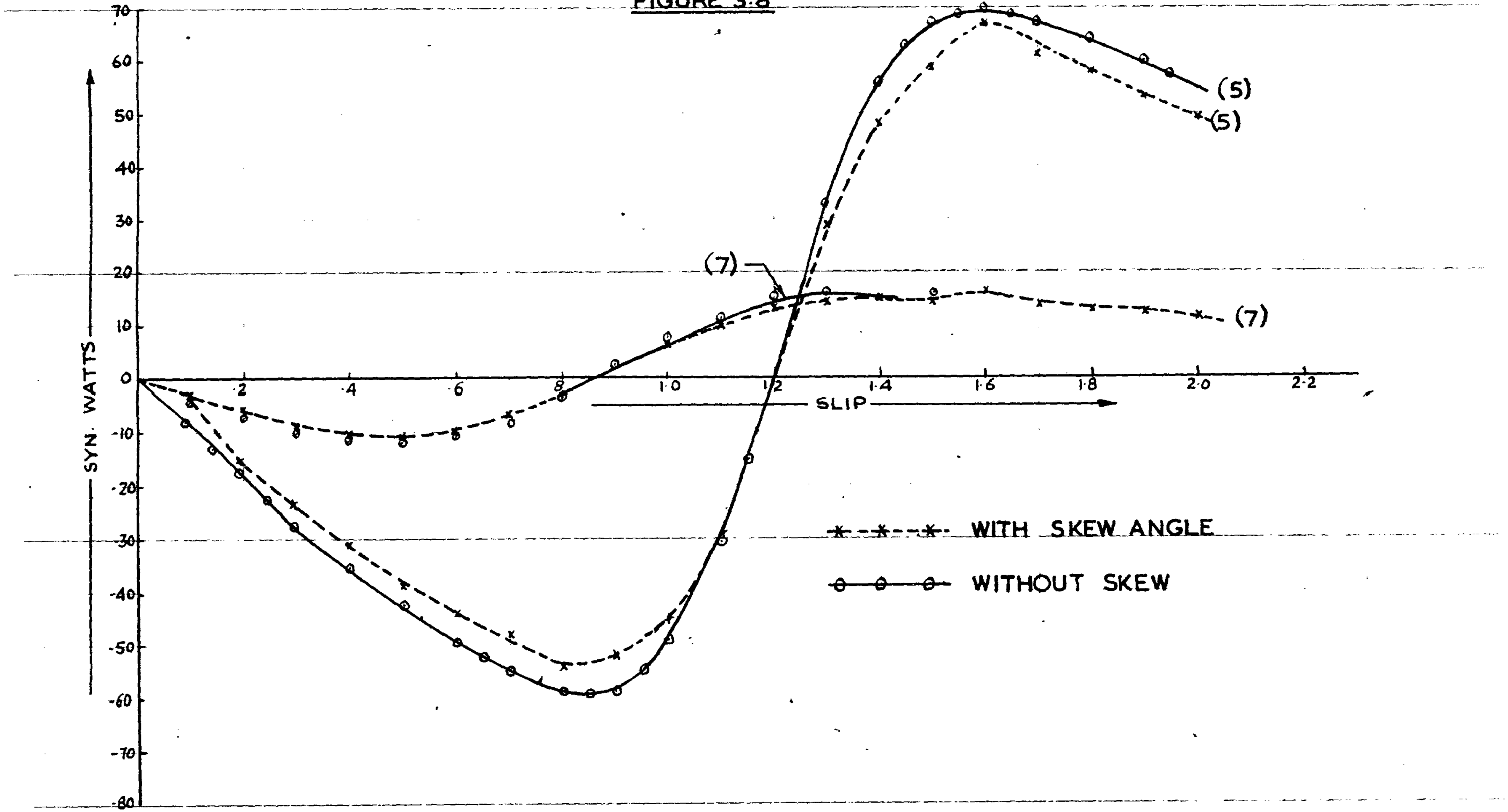
TORQUE SLIP CURVES WITH SKEW

FIGURE 3.7



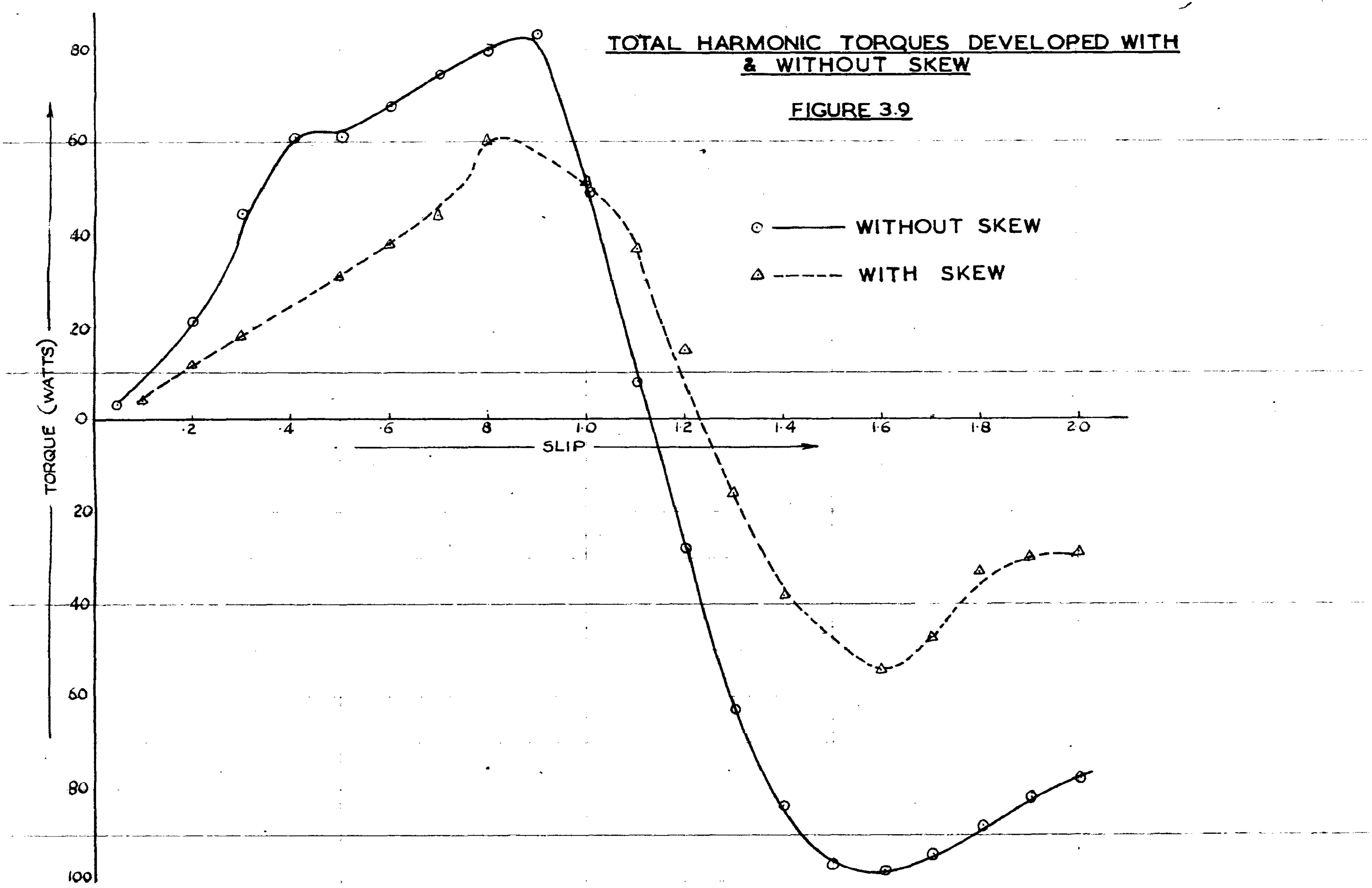
TORQUE SLIP CURVES OF 5TH AND 7TH HARMONICS

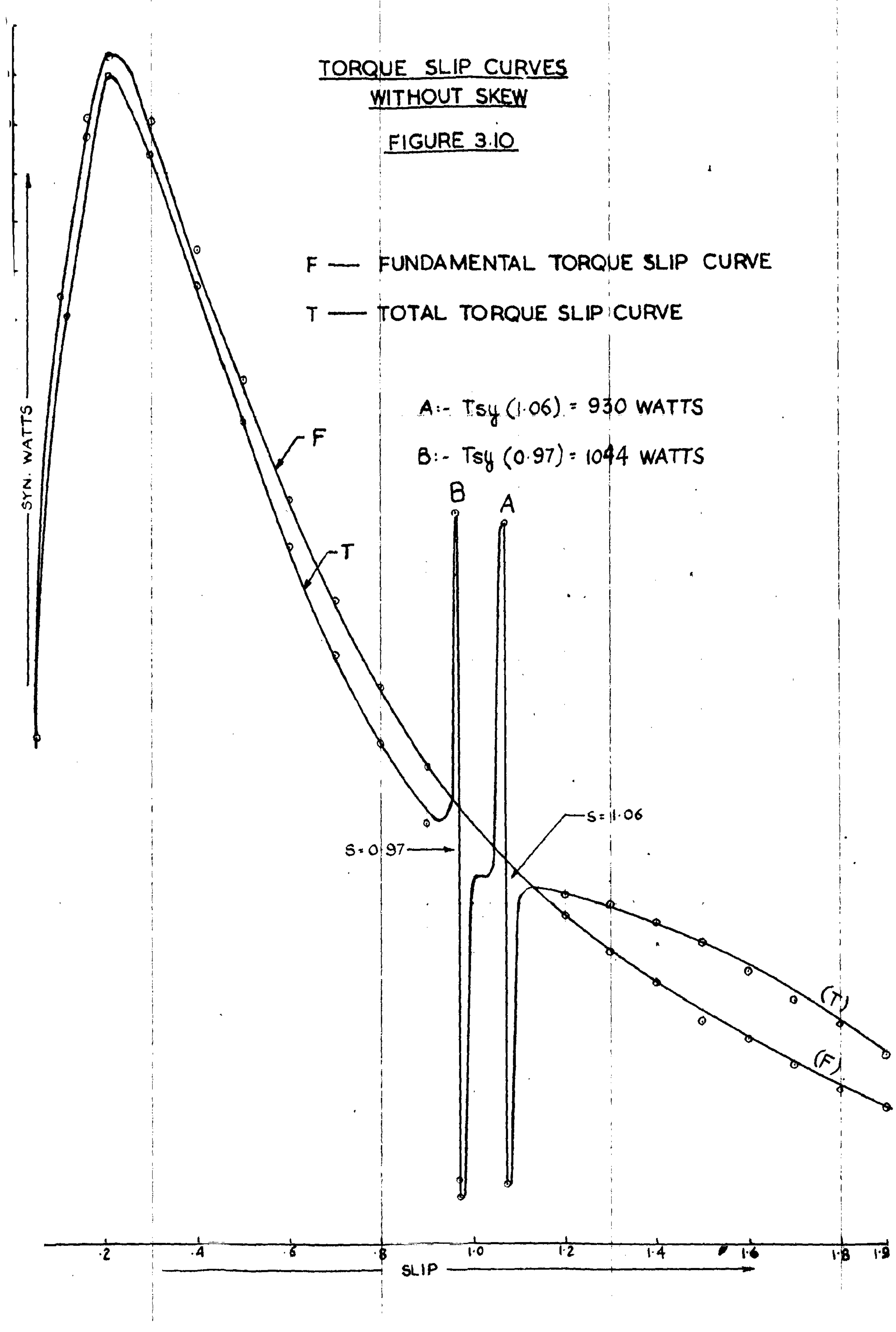
FIGURE 3.8



TOTAL HARMONIC TORQUES DEVELOPED WITH & WITHOUT SKEW

FIGURE 3.9




TORQUE SLIP CURVES
WITHOUT SKEW
FIGURE 3.10
F — FUNDAMENTAL TORQUE SLIP CURVE
T — TOTAL TORQUE SLIP CURVE
A:- Tsy (1·06) = 930 WATTS
B:- Tsy (0·97) = 1044 WATTS
F
T
B
A
S = 0·97
S = 1·06
(T)
(F)
SYN. WATTS
SLIP
·2
·4
·6
·8
1·0
1·2
1·4
1·6
1·8
1·9

CHAPTER IV

SYNCHRONOUS PARASITIC TORQUES AND DEAD POINTS OF SQUIRREL CAGE INDUCTION MACHINE WITH AND WITHOUT ROTOR SKEWED

4.1. Synchronous Crawling Torques:

4.1.1. Speed of Synchronous Crawling Torques:

Certain squirrel cage motors are known to have a wide variation in starting torque and have a tendency to run at some speed way below that normally expected. This is sometimes called 'sub-synchronous' speed ar the motor is said to be crawling. The one type of crawling called asynchronous crawling due to harmonic locking of stator and rotor has beenalready deált under Chapter III . The other cause of crawling may be 'synchronous crawling' . Sometimes the motor fails to start and in extreme cases the torque may be actually negative at certain positions making the motor entirely useless. This phenomena of the motor failing to start is called by many names such as 'dead points' cogging and 'locking' . An analysis of the causes of such dead points behaviour will be presented in next article of this Chapter.

Concentrations for the present over synchronous torques[12], as given by equation 3.3, both machine parts must have the same number of poles for a uniform useful torque to be produced by the machine. In the synchronous machine, the stator as well as the rotor are connected to sources of currents, therefore uniform torque is possible only at a single spleed of the rotor. A torque produced in this manner is called a synchronous torque.

The harmonics produce asynchronous as well as synchronous torques in the induction motor. A synchronous torque will occur when stator harmonic 'ma' produces a rotor harmonic n_a which has the same order as another stator harmonic 'm_b' linked by the equation (3.4)

$$n_a = \pm v_2 \left(\frac{R}{p}\right) + m_a$$

and $n_a = m_b$

and which at a single rotor speed is at stand still with respect to this second harmonic m_b, m_a is the source of excitation of the rotor, then only 'm_b' torque can be produced

It can be seen from harmonic chart (Table 1.4) that the rotor harmonics which correspond to $v_2 = 0$ i.e, the first column of the rotor harmonics, have the same order as the stator harmonics producing them. As pointed out earlier the torque produced by these first order harmonic is asynchronous since the speed of the mth stator harmonic with respect to stator is

$$v_{1\ 'ma'} = \frac{v_1}{m_a'}$$

The speed of the 'ma' rotor harmonic with respect to the stator is equation 3.2.

$$v_{1\ 'na'} = \frac{1}{n_a} \left[1 + (n_a - m_a)(1-s) \right] v_1 \qquad (4.1)$$

Where v_1 = fundamental speed.

In order that $v_{1\ ma} = v_{na}$ the condition is

$$(n_a - m_a)(1-s) = 0 \qquad (4.2),$$

and since $n_a = m_a$ (first order harmonic), equation 4.2, is satisfied for all values of slips, thus asynchronous torque is produced at all speeds.

In order that syncronous torque occur there must be

$$n_a = \pm m_b \quad \text{i.e.} \quad v_{ma} = \pm v_{1mb} \tag{4.3}$$

Since $v_{1mb} = v_1 / m_b$ (4.4)

Equation 4.3 shows the two possibilities

Possibility one: When $n_a = + mb$ the condition is $v_{na} = +v_{1mb}$

i.e. from equation (4.1)

$$1 = 1 + (n_a - ma)\ (1-s) \tag{4.5}$$

or $(n_a - m_a)\ (1-s) = 0$ (4.6)

Since $v_2 \neq 0$ $\therefore n_a \neq m_a$. Hence synchronous torques will occur only when $s = 1$.

This is a case of locking torque and we postpone the discussion over it for next section.

Possibility two : When $n_a = - m_b$ (4.7)

giving $v_{na} = v_{1mb}$

equation 4.1 gives $-1 = 1 + (n_a - m_a)\ (1-s)$ (4.8)

Since na $\neq m_a$ synchronous torque can occur only when

$$s = 1 + \frac{2}{(n_a - m_a)} \tag{4.9}$$

'n_a' and 'm_a' with proper sign.

as $n_a = \pm v_2 (R/p) + m_a$

$$n_a - m_a = - v_2 (R/p)$$

Speed $n = - \left(\dfrac{120 f_1}{v_2 R} \right)$ for squirrel cage rotor) (4.10)

(v_2 with proper sign)

Thus, if there exists equal number of poles fields in stator and rotor except those of first order, then syncronous torque at fixed speed given by equation (4.10) is produced. If the harmonic 'n_a' corresponds to a negative v_2, the synchronous cusp will occur at positive speed ($s < 1$), if the harmonic 'n_a' corresponds to a positive v_2, then synchronous cusp will occur at negative speed ($s > 1$).

4.1.2. Magnitude of cusp (Synchronous Parasitiv Torques) in the Torque-slip Characteristic

From equation (3.6) (Chapter III) the torque is given by

P_d (Power developed) $\propto F_{na}, F_{mb}$,

Since in this case, 'ma' is the exiter and (synchronous torque) produces flux distribution which reacts with mmf of mb of stator, therefore

$$P_d \propto F_{ma} \quad F_{mb} \tag{4.11}$$

Biot - Savart's law also gives the for one single conductor as

$$F_t = 8.85 \times 10^{-8} \text{ B I L Sin } \theta \quad \text{(lbs)} \tag{4.12}$$

$$= 1 \times 10^{-1} \text{ B I L Sin} \quad \text{Kgm} \quad \text{(m.k.s)}$$

Considering the rotor skewed, distribution of A_{mb} is directly effected by rotor skewing (See equation)

$$\therefore \quad P_d \propto B_{na} \quad F_{mb} \quad K_{smb} \tag{4.13}$$

Where A_{mb} = Ampere conductor distribution of harmonic 'mb'

Total Power developed is given by

$$T_{(mb)\,Sy.} \propto (PT) \int_0^{2\pi} b_{na} \; A_{mb} \; K_{smb} \; d\,\theta \tag{4.14}$$

where $$b_{na} = B_{na} \text{ Cos} \left[na\theta \mp \left\{ 1 + \left(\frac{v_2 R}{p} \right) (1-s) \right\} wt \right] \quad (4.15)$$

B_{na} = $F_{(m)s}$ x (Air gap permeance)

$$B_{na} = F_{(m)s} \times \frac{0.4\ \pi}{l_{g'}} \times (2.50) \text{ Lines / sq. in.} \quad (4.16)$$

$F_{(m)s}$ is given by equation (1-21) substituting it in (4.16)

$$B_{na} = 0.45 \frac{1}{(na\ )\ p} \frac{(3.19)\ m_1 N_1 \cdot k_{w1m}\ G_{(ma)} I_1}{l'_g\ K_{sma}} \quad (4.17)$$

Where $G_{(ma)}$ = Damping factor for rotor currents.

From Harmonic equivalent circuit (See fig. 3.3).

$$G_{(ma)} = \frac{s_{(ma)}\ X_{M(ma)}}{\sqrt{R_{2(ma)} + s^2_{(ma)} (1 + \tau_{2ma})^2 X^2_{M(ma)}}} \quad (4.18)$$

B b_{na} given by (4.15) is not general enough for the computation of synchronous torques. The magnitude of the synchronous torque depends upon the relative position of the stator and rotor mmf's in addition to other factors. Equation 4.15 is derived on the assumption that at $t = 0$ the references of stator (θ_1) and rotor (θ_2) coincides i.e., $\theta_1 = \theta_2$ For present analysis, it should be dropped and equation 4.15 be generalised. By introducing new system of coordinates. Let at $t=0$

$$\theta_2 = \theta_1 = \theta'_2 - v_r t \text{(at time } t) \quad (4.19)$$

Equation 4.15 in new coordinates defined by 4.19 becomes

$$b_{na} = B_{na} \text{ Cos} \left[n_a\theta - (n_a - m_a)\theta'_2 \mp \left\{ 1 + \frac{v_2 R}{p} (1-s) \right\} wt \quad ma \right] \quad ...(4.20)$$

Where $_{ma}$ in the equation of b_{na} is necessary if B_{na} is expressed in terms of the primary current I_1, since B_{na} is proportional to I_{2ma} and I_{2ma} lags V_1 by the angle

$$\psi_{ma} = \pi - \alpha_{ma} \quad \text{where}$$

$$\tan \alpha_{ma} = \frac{R_{2(ma)}}{(1+\tau_{2ma})\, S_{(ma)}\, X_{M(ma)}}$$

when current distribution super imposed over the winding groups is integrated it yields mmf curve. Therefore

$$f_\theta = \int_0^\theta a_\theta \cdot d\theta$$

Where a_θ = ampere conductor distribution and function of space angle θ.

$$\text{Conversely} \quad a_\theta = \frac{d(f_\theta)}{d\theta} \qquad (4.21)$$

$$f(\theta, t) = 0.45 \cdot m_1 \frac{K_{s(ma)} \cdot N_1}{m(p)} K_{w1(mb)}\, I_1 \cos(mb\theta \mp wt)$$

$$\therefore a(mb) = + A_{(mb)} \sin(mb\theta \mp wt) \qquad (4.22)$$

$$A_{(mb)} = \frac{2 \cdot 2^{\frac{1}{2}}}{(p\tau)} m_1 N_1 K_{w1(mb)} K_{s(ma)} I_1 \qquad (4.23)$$

$p\tau$ = Total armature circumference.

Substituting -

$$\mp 2 + v_2(\tfrac{R}{p})(1-s)\; wt - (na-ma)\,\theta_2' + \psi_{ma} = \delta_1 \qquad (4.24)$$

$$\mp v_2(\tfrac{R}{p})(1-s)wt - (na-ma)\theta'_2 + \psi\, m_a = \delta_2 \qquad (4.25)$$

$$C = \frac{0.369 \times 212.2 \times 10^{-8}}{4 \pi^2} (P \, \tau)^2 \; L_e \; B_{na} \; A_{mb} \tag{4.26}$$

Then equation (4.14) reduces to

$$T_{(mb)} \, S_y = C \int_0^{2\pi} \cos \left[na\theta \mp \left\{ \frac{v_2 R}{p} (1-s) + 1 \right\} wt - (na - ma) \, \theta'_2 + \psi_{ma} \right] \sin (mb\theta \mp wt) d\theta$$

$$T_{(mb)} \, Sy = - C \int_0^{2\pi} \sin \left[(na + mb)\theta - \delta_1 \right] d\theta - C \int_0^{2\pi} \sin \left[(na - mb)\theta - \delta_2 \right] d\theta \tag{4.27}$$

For syncronous torques

$$na = -mb \tag{4.7}$$

$$T_{(mb)} \; Sy = 2\pi \; C \sin \delta_1 = 2\pi \, C \sin \left[v_2 \left(\frac{R}{p}\right) \theta'_2 - \psi_{ma} \right] \tag{4.28}$$

Equation 4.28 indicates that syncronous torque varies with rotor position but independent of time.

$$T_{(mb)} Sy(max) = 2 \pi C \tag{4.29}$$

Therefore syncronous torque is of constant magnitude for particular order of harmonic 'mb' occuring at one slip given by

$$S = 1 + \frac{2}{na - ma}$$

Syncronous torques on torque/speed are represented by a straight line (see Fig. 4.1).

4.2. DEAD POINTS IN TORQUE SPEED CURVES

4.2.1. As already indicated under section 4.1 and 4.2, the dead points in torque speed curves are specifically the one type of synoronous torques when parasitic torque (cusp) occur at stand still or in other words in more clear terms, due to high locking torque motor does not start. As can be judged, it is more severe problem than any other. This phenomena was first named as 'Synoronous motor effect ' in induction machines by Direese(26) and latter on Kron(4), Alger(3) also warked over it. Grahm(22) first tried to explain the causes of Dead points in induction machine. The classical theory does not explain this behaviour as it purely arises due to non-sinusiodal nature of the air gap field as discussed under section 4.1 of this Chapter.

4.2.2. Variation of Torque at Stand Still Over Slot Pitch

The phase relation between harmonics is shown in Fig. (4.1) Dead point torque explained by Grahm(22) with the help of harmonic phase relation is based on the assumption that nearly 180 degrees between stator and rotor currents when locked, . The actual angle depends upon power factor of the secondary air-cuit and ratio between magnetising current and load current.

Graham , performed tests over number of motors with different stator and rotor slot combinations. The variation of standard torque over a rotor slot pitch are shown here in Fig. 4.4 and 4.5 .

4.2.3. Magnitude of Dead Point Torque:

Equation 4.5 given that dead points occur if

na = ma and for the condition the torque is given

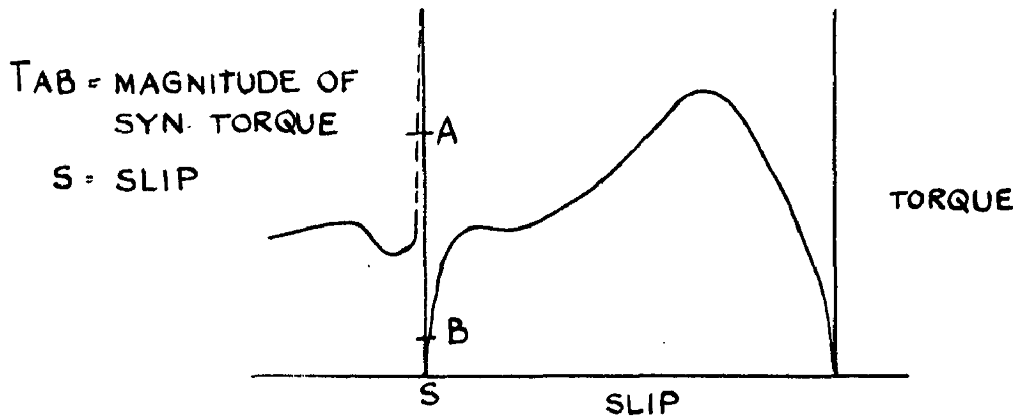
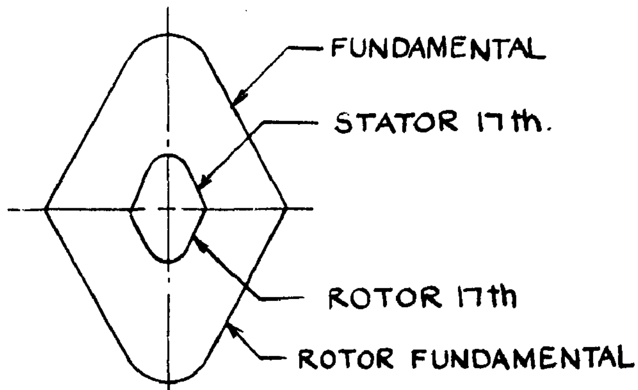
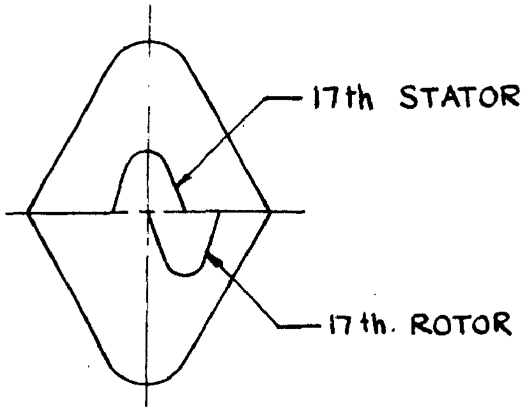


FIGURE 4.1



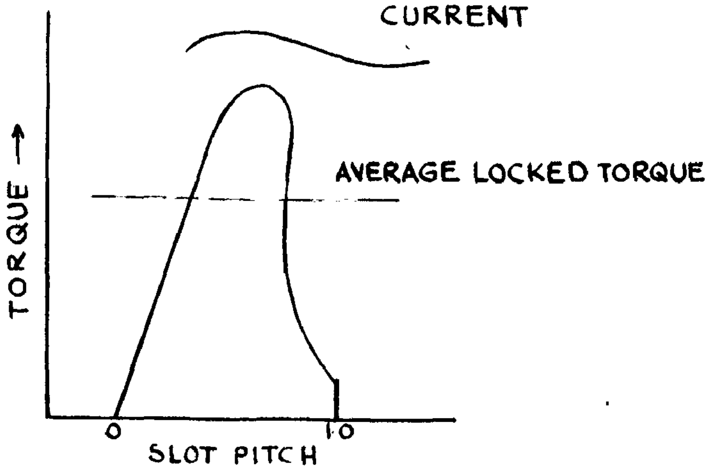
ELATION OF 17th HARMONIC WAVES
STATOR & ROTOR WITH ROTOR
ORIGINAL POSITION

FIGURE 4.2



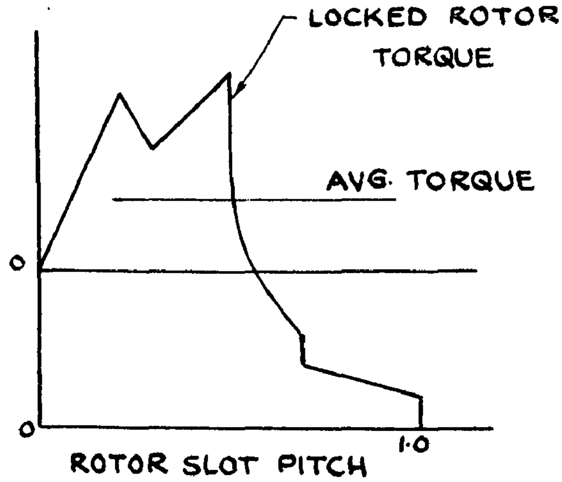
RELATION OF 17th HARMONIC
WHEN ROTOR HAS MOVED
BY $\frac{1}{4}$th OF SLOT PITCH

FIGURE 4.3



AL CURVE SHOWING VARIATION
CKED ROTOR TORQUE WITH
R POSITION.
(MACHINE ROTOR SKEWED)

FIGURE 4.4A



TYPICAL CURVE SHOWING VARIATION
OF LOCKED TORQUE WITH ROTOR
POSITION FOR MOTOR WITHOUT
SKEWED SLOTS.

FIGURE 4.4 B

by equation (4.27)

$T_{(ay)d(max)} = 2 \pi C \sin \delta_1$ and maximum torque is

$T_{ay\,d\,(max)} = 2 \pi C$

When C is given by the equation (4.26).

4.3. RELATION OF PARASITIC TORQUES WITH SLOTS:

4.3.1. Asynchronous Parasitic Torques and Slots:

As such number of slots have no direct bearing over the asynchronous torques of Phase belt harmonics, which are the product of non-sinusiodal winding distribution. But as the equation 1-22, 1-14, 1-18, shows, slot harmonics and permeance waves are function of slot numbers.

$m_s = v_1(S/p) \pm 1$, $m_p = v_1(S/p) \pm m$, $m_p = v_2(R/p) \pm m$

But by properly selecting S and R, low order of slot and permeance waves which are more harmful can be avoided. By selecting (S/p) and (R/p) on higher side, distortion in Torque - speed in the working region can be avoided.

4.3.2. Elimination of Stand Still Locking (Dead Points)

Generalising the harmonics produced in combination with slot and rotor slots we get the following equations :

STATOR	ROTOR
1. m	1. n when $n = m$
2. $\pm v_1(S/p) + m$	2. $\pm v_2(R/p) + m$
	3. $\pm v_2(R/p) \pm v_1(S/p) + m$

m is substituted with proper sign.

For stand still locking - (from equation 4.5)

$$na = mb.$$

There are number of combinations all possible.

Case(i) $\pm v_2(R/p) + ma = mb$ (4.30)

This in no way provides any relation between R and S

Case (ii) $\pm v_2(R/p) + ma = \pm v_1(S/p) + mb$

$$\pm (v_2R - v_1S) = p\,(mb - ma)$$

All are integer

$$\therefore \pm (v_2R - v_1S) = \text{EVEN or} = \text{Zero} \qquad (4.31)$$

If $v_1 = v_2 = 1$ or 2

Then (a) $\pm (R-S) =$ even.

(b) $2(R-S) =$ EVEN or Zero

Case (iii) $\pm v_2(R/p) \pm v_1(S/p) + ma = mb$

$$\therefore (v_2R + v_1S) = \text{even or Zero} \qquad (4.32)$$

Case(iv)

$$\pm \left[\pm v_2(R/p) + v_1(S/p) + ma \right] = \pm \left[\pm v_1 (S/p) + mb \right]$$

(a) $v_2(R/p) = (mb - ma)$ same as equation 4.30

(b) $\pm v_2(R/p) \mp v_1(S/p) \pm ma = \mp v_1(S/p) \pm mb$

$$\pm v_2R \pm 2 v_1 = p\,(mb - ma)$$

$$\pm (v_2R \pm 2 v_1) = \text{Even or Zero} \qquad (4.33)$$

Equations 4.30, 4.31, 4.32, 4.33 provides sufficient conditions for eliminating stand still locking.

$$ma - mb = 2\ k\ .q$$

where q = number of phase belts.

To sum up

$$(\ v_2R \pm v_1S\ ) = 2\ k\ q \quad \text{or} \qquad (4.34)$$

$$(v_2R \pm 2\ v_1S\ ) = 2\ k\ q \qquad (4.35)$$

4.3.3. Elimination of Syncronous Torque:

For syncronous torques the necessary condition given by equation 4.7 is

$na = -mb$ using this and all other relation from equattion 4.33, the following new relations are established.

$$(\ v_2R \pm v_1S) = 2\ (\ kq \pm 1) \qquad (4.36)$$

$$(v_2R + 2\ v_1S)' = 2\ (\ kq \pm 1) \qquad (4.37)$$

where k = Integer including zero.

4.4. CALCULATION OF SYNCHRONOUS TORQUES:

Employing equations 4.29 and 4.26 Table T (1-4) T(2.1)A, Table (2.2), Table (2.3), Table (2.1B), the magnitude of syncronous torques in general for any pair of harmonics is given by

$$T_{(sy)} = 1.913 \times 10^5 \left[\frac{k_{w1ma}}{K_{sma}} G_{\phi ma)}\ k_{w(mb)}\ \frac{K_{s(na)}}{na} \right] \qquad (4.38)$$

watts / phase.

The computed results are tabulated in Table (4-1) and Table (4-2) for skewed rotor and Table (4.3) for non skewed rotor induction machine. Values are indicated over the graphs in Fig. 3.8 and Fig. 3.10.

(i) At slip = 1.06

Total synchronous torque = 487.3 watts

(ii) At slip = 0.97

Total synchronous torque = 130.0 watts.

For skewed rotor machine.

(iii) at slip = 1.06

Synchronous torque = 929.9 watts.

(iv) at slip = 0.97

Synchronous torque = 1043.5 watts.

For rotor not skewed.

4.5. DETERMINATION OF DEAD POINTS:

Applying the relations of equation of 4.35 and 4.36, with R = 68

S = 48

There is no dead point. So motor is free from starting locking tendency.

Table 4.1 CALCULATION OF SYNCHRONOUS TORQUES WITH ROTOR SKEWED

ma	n=-mb	k_{wma}	K_{sma}	G_{ma}	k_{wmb}	K_{sm}	Syn.-watts/ phase.
+1	+35	0.955	.998	0.98	.126	.53	340
-5	+29	.205	.987	.445	.205	.657	83.5
+7	+41	.157	.98	.043	.157	.391	1.97
-11	+23	.126	.946	.012	.95	.775	10.4
13	+47	.126	.925	.045	.955	.256	6.1
-17	+17	.157	.873	0	.157	.873	0
+19	+53	.205	.845	.0358	.205	.129	0.84
-23	+11	.955	.775	.0142	.126	.946	36.1
+25	+59	.955	.74	.038	.126	.0169	0.34
-29	+5	. 205	.652	.0038	.205	.987	9.25
+31	+65	.157	.616	.0031	.157	.076	Negligible
-35	-1	.126	.127	2.0×10^{-5}	.955	.998	"
+37	+71	.126	.482	1.4×10^{-4}	.955	.147	"
-41	-7	.157	.391	.00036	.157	.98	"
-43	+77	.205	.346	.000298	.205	.193	"

Table 4.2. CALCULATION OF SYN. TORQUES (ROTOR SKEWED)

For $\nu = -2$

ma	n=-mb	k_{wma}	C_{ma}	k_{wmb}	K_{sm}	K_{sma}	Syn.- watts/phase.
1	-67	.955	.95	.205	.102	.998	54
-5	-73	.205	.383	.955	.165	.987	38.0
7	-61	.157	.128	.126	.0164	.98	.13
-11	-79	.126	.0455	.157	.203	.946	Negligible
13	-55	.126	.015	.157	.09	.925	"
-17	-85	.157	.027	.126	.217	.873	"
19	-49	.205	.007	.955	.212	.845	14.0
-23	-91	.955	.047	.205	.21	.775	Negligible
25	-43	.955	.0574	.205	.346	.74	23.5
-29	-97	.205	.003	.955	.184	.657	Negligible
31	-37	.151	.002	.126	.482	.616	"
-35	-103	.126	2.0×10^{-5}	.157	.144	.527	"
37	-31	.126	1.4×10^{-4}	.157	.616	.482	"
-41	-109	.157	.00036	.126	.096	.392	"
43	-25	.205	.000298	.955	.74	.346	"

Table 4.3 CALCULATED RESULTS OF SYNCHRONOUS TORQUES WITH ROTOR NOT SKEWED

For slip = 1.06					For slip = 0.97				
ma	na = -mb	k_{wma}	$k_{w(mb)}$	T_{sy}	ma	na = -mb	k_{wma}	$k_{w(mb)}$	T_{sy}
1	35	.955	.126	675.0	1	-67	.955	.205	675.0
-5	29	.205	.205	82.0	-5	-13	.205	.955	122.0
7	41	.157	.157	27.0	7	-61	.157	.126	15.7
-11	23	.126	.955	13.4	-11	-79	.126	.157	52.0
13	47	.127	.955	24.6	13	-55	.126	.157	8.5
-17	17	.197	.157	9.6	-17	-85	.157	.126	7.3
19	53	.205	.205	6.4	19	-49	.205	.955	1.3
-23	11	.955	.126	33.8	-23	-91	.955	.205	130.0
25	59	.955	.126	38.0	25	-43	.955	.205	4.2
-29	5	.205	.205	6.3	-29	.97	.205	.955	15.0
31	65	.157	.157	.63	31	-37	.157	.126	0
-35	-1	.126	.955	8.0	-35	-103	.126	.157	8.5
37	71	.126	.955	2.7	37	-31	.126	.157	.09
-41	-7	.157	.157	1.0	-41	-109	.157	.126	2.0
43	77	.205	.205	1.1	43	-25	.205	.955	0.1

CHAPTER V

ELECTROMAGNETIC NOISE AND MAGNETIC RADIAL FORCES OF SQUIRREL CAGE POLYPHASE INDUCTION MOTOR

The term 'noise' is a very vauge term defining nothing in particular of the source of noise. But to be more exact, it can be said that there is now a growing demand tht motors should have good 'sonance design', i.e. they should give out just sufficient, steady pleasing hum, just loud enough to show that they are performing their duties properly, but not loud enough to be noticed.

5.1. QUALITATIVE ANALYSIS OF NOISE

5.1.1. Definition of Noise:

Before discussing noise measurement as noise phenomena a necessary preliminary is to define what we mean by noise. The old conception was tht sounds which occupy the attention of a person could broadly classified into music and noise. This classification was based solely on the characteristics of the stimulus i.e., on physical quantities such as the relations between the component frequencies. For ou purpose we define noise as undesired or irksome sound. Sufficient work has been done by R.L. Wege, Church[36] in pulse science of Noise to which we are not much concerned at present.

In attempting, therefore to formulate a basis for noise measurement which would be acceptable for Engineering purposes, Church first considered the laws of response of the hearing system of the average individual to sounds to different characteristics, and then the choice of a practical method of noise measurement in accordance with those laws. The investigation of the laws in brief falls in three sections viz., the determination of the threshold of hearing which provides a datum from which intensity levels may be seckoned, the determination of the relation between the magnitudes of stimuli of different frequencies which produce equal loudness sensations, and finally the relation between stimulus and sensation.

5.1.2. The Assessment of Total Noise:

While the problem of analysing a complex noise into its constituent tones and stating their magnitudes and frequencies presents some difficulties, yet the work involved is essentially in the realm of physics and the results are susceptible of incontrovertible statement in absolute units. Even when the intensity values are expressed in terms of the decibles above some generally accepted values of the threshold intensities at the respective frequencies, the results are still unassailable.

The methods of determining the total loudness of a noise can be roughly divided into two classes, subjective and objective, the former involving the judgement of the human ear and the latter the interpretation of instrumental measurements in the light of the available data relating to the properties of the ear. The two classes can be further subdivided in the following way.

5.1.3. Subjective Methods:

In the 'equality' or 'balance' method a reference tone which can be specified in frequency and intensity is adjusted so that it is judged by a representative person to sound as loud as the noise in question. The reference tone may be produced in free space but is more usually generated in either one or two telephone ear pieces spaced off the head, or in one telephone placed over one ear while the other listens to the noise being measured.

5.1.4. <u>Objective Methods</u>:

Under the total noise is to be assessed by calculation from analysis, the several components of a noise are determined by analysis and their intensities weighed according to their frequencies and the sensitivity of the ear at those frequencies. The weighed components would then be summed mathematically in a way to correspond to the action of the ear. No method of summation of general validity has yet been put forward.

The methods briefly indicated above were considered in detail with a view to determining which is the most trustworthy for quantitative measurements under problems like ours of measuring noise produced by electrical machines. In the subjective and objective classes of assessment, while we have in the former a direct reference to the final judge, the human ear, we introduce the personnel factor into the observations, whereas in the latter the difficulty of stimulating instrumentally or mathematically the response of the ear is offset by the much greater consistency of instrumental readings. The relative importance of these points must be considered along with the other features, such as cost, convenience and speed of working.

Even supposing a method of summation by calculation were available the calculations would necessarily be extremely laborious owing to the complexity of the characteristic of the ear. Moreover, fairly elaborate analysing equipment is required to obtain the initial data. Calculation from analysis is therefore never likely to become a method generally applicable to engineering problems. Specifically the mathematical analysis of noise generated by rotating electrical equipment is of very complex nature, but still the author has chosen objective analysis since they are of undoubtedly of value for research purposes.

5.2. UTILITY OF NOISE MEASUREMENT:

5.2.1. Typical Loudness Values:

One point of interest is the position assigned to common noise on the decibel and loudness scales. This is illustrated in Table 5.1 below. Columns (2) and (3) have the most practical interest but it is instructive to note how they are related to columns (4) and (5). The pressure values given in column (4) are those 800-cycle field pressure which would produce the same loudness sensations as the complex noise on an observer facing the source. Column (5) gives the sound energy flow per cm^2 in an equally loud 800- cycle field. The values in column (5) are derived from those in column (4) by the relation[32]

$$W = \frac{p^2}{\bar{d}v} \times 10^{-7}$$

Where W = rate of flow of energy in watts per cm^2

p = r.m.s. pressure in dynes /cm^2

$\bar{d}$ = Intensity of air

v = velocity of sound in air cms/sec.

Decibles above threshold are given in terms of the pressure by the expression $20 \log_{10} (p/po)$ wheen po is the threshold field pressure viz. 0.000215 dynes/cm^2.

Table 5.1 THE MAGNITUDES OF COMMON NOISE ON VARIOUS SCALES.

1.	Loudness	Equivalent 800-cycle magnitudes		
		D_b s	Field pressure	Energy flow
	2	3	4	5
1. Two circular saws	160	110	73	13 x 10^{-6}
2. Loud motor horn at 100 ft.	100	100	23	13×10^{-7}
3. In suburban steam	50	84	3.6	3 x 10^{-9}
4. Conversation (3 ft)	20	69	0.65	1 x 10^{-9}
5. Quiet Elect. motor (2 ft)	5	49	0.065	1 x 10^{-11}

5.2.2. Effect of Number of Sources:

The total effect of a number of sources operating simultaneously has often to be considered in practice. It has been suggested that since two similar sources emit twice as much sound energy as one and since the number of decibles difference is given by 10 $\log_{10}$ (w/wo) where w and wo are the energy levels in the two cases, an increase of 3 db is caused whatever the initial level. This simple rule cannot apply to the equivalent 800-cycle tone outside that frequency range; The issue is further complicated in the case of a noise containing a harmonic range of frequencies e.g., as electrical rotating equipment.

5.2.3. Effect of Distance:

In considering the effect of distance from the source on the sensation of loudness experienced, it should be borne in mind that the effect is simple only in the case of a point source emitting a pure tone of a frequency between 800 to 4000 cycles per sec. in free space with zero background noise. Under point source free space conditions the accoustical pressure varies inversely as the distance from the source. With complex sounds the conditions will be again modified.

5.2.4. Effect of Enclosures:

It is not necessary to emphasize the fact that, in the case of a source placed in a building, the noise audible to a person outside may be totally different, both in composition and in amplitudes, from that heard inside. The attenuation of sound through the walls of a building depends upon the mass per unit area of the walls. A discussion of the design of enclosures is beyond the scope of this volume.

5.2.5. Effect of Noise Background:

A subject which is of vital importance in making noise measurement on machinery and in considering how much noise it is permissible for a machine to make in a given situation , is the background of noise present. The sound sensations from a given source perceived by a listener may be greatly modified if, another source is introduced, One sound can 'drown' other. Almost any motor would be unobjectionable, if not inaudible, in surroundings of loudness 85 db.

5.2.6. Problem of Windage Noise:

The problem of windage noise is of great importance particularly only in large - high speed machines. It becomes the one and only noise, but in medium sized machine, magnetic noise is the biggest problem, So sindage noise can easily be overlooked in this context. It is sufficient from the designer's point of view to know the causes, in terms of dimensions and other Physical constants of the generation of magnetic noise. This would enable him to avoid certain constructions which might magnify as radiate electromagnetic noise.

5.3. Sources of Noise in Motors:

The physical factors that make an induction motor a machine for converting electric energy into mechanical energy also make it a machine for converting this energy into accoustical energy. The problem of sonance design(32) is to reduce the accoustical efficiency of these machines so that they will produce less noise. Induction motors noise may be classified in three main groups:

1. Magnetic Noise.
2. Mechanical noise.
3. Aerodynamic noise.

Any one or more may be predominant in any motor, but here we will devote our studies to first type only to see the 'effects of Harmonics on Noise' and then subsequently demonstrate, the effect of rotor skewing.

The noise producing parts can vibrate in several ways. A torsional vibration of the stator and rotor as a whole results from periodic torque pulsations, such a vibrati n may be particularly objectionable as it is transmitted directly through the motor feet to the supporting structure. Most of the vibrations are forced vibration i.e., their frequency is not near to critical frequency. The method of determining the noise tendency of a motor is to tabulate quantitatively all the possible harmonic fields and examine for each two fields differing in number of poles.

5.3.1. Electromagnetic Noise in Electrical Rotating Machines:

Three main types of noise occur in electrical rotating machines. These are (1) Magnetic noise originating in the air gap flux density distribution on no load. This distribution remains substantially unchanged on load (2) load noise originating in the mmf wave of the stator winding. (3) Windage noise due to the rotation. Of these types of noise, the first is the one requiring most thought, since not only is it difficult to predict, but even more difficult to supress an reduce when the machine is completed. The cause and cure of noise in electrical rotating machines is of pernnial interest to both manufacturers and users.

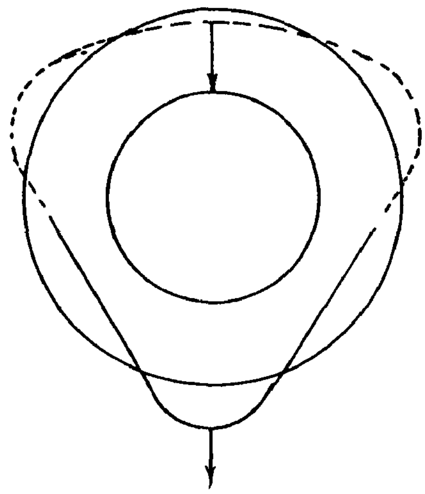
Walker and Iwschitg(28) recently gave a solution to the problem of determining the noise in syncronous as well as induction machines. The author has also attempted to find the noise level of the machine under test by exhaustive calculation carried over IBM 1620 computer. In order to predict the noise level of a electrical machines the author describes the magnetic forces that cause the magnetic noise of Polyphase induction machine, and the corresponding modes of motion of the motor cares and frames. Approximate equations are used for the decibel sound levels that motors of normal design may be expected to produce, and variations due to core and frame resonance are discussed. The principal types of magnetic noise, due to the radial forces of the fundamental harmonic air gap fields are considered.

5.3.2. Natural Behaviour of Motor:

Early research in the magnetic noise of induction motors was closely linked with investigations of other effects of higher harmonics magnetic flux density[34] waves in the air gap. Even untill recently the secommendation for low magnetic noise level induction motors was initimately related to slot combinations. Heondl , Hildebrand and Moruil have discussed in great detail the question of slot combinations and the calculations of the mode, frequency and the magnitude of the radial force waves, responsible for the vibration and noise of induction motors.

Alger[35], who was the first to provide a simplified, comprehensive treatment embracing all phases of the problem, was also the ariginator of the concept of 'sonance design'. According to him, the induction motor as regards its mechanical vibrations can be represented as a simple in extensional ring, and he analysed the motor's natural behaviour on this basis. To analyse its accoustic behaviour Alger chose to represent the motor by infinitely long vibrating cylinder. He provide the designer with formulas and curves of sound intensities of the ndise radiated from the induction motor simultaneously, Jordon in Germany provided a simplified solution. His assumptions regarding the vibrational aspect of the motor is approximated by a radiating sphere, and Gauss's theorem is used to determine the sound power level at a distance from 'stiffening effect', which is quite, justified from medium sized motors.

The magnetic field in the air gap of an induction motor creates rotating force waves with frequencies of 2 to 40 or more times the line frequency. These frequencies cause elliptical or nodal deformations of the motor laminations and frame, (See Fig. 5-1) thereby producing high frequency vibrations and noise. This analysis provides an illustration of the work that sonance designers are being called on to



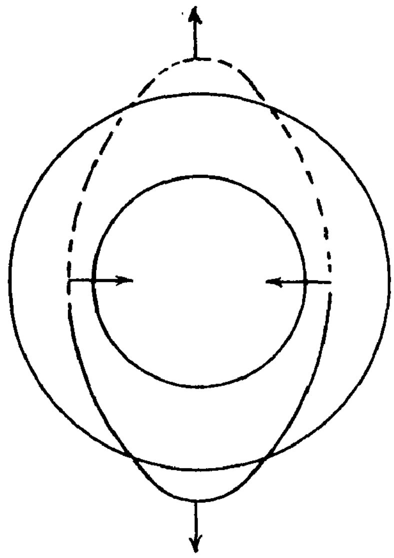
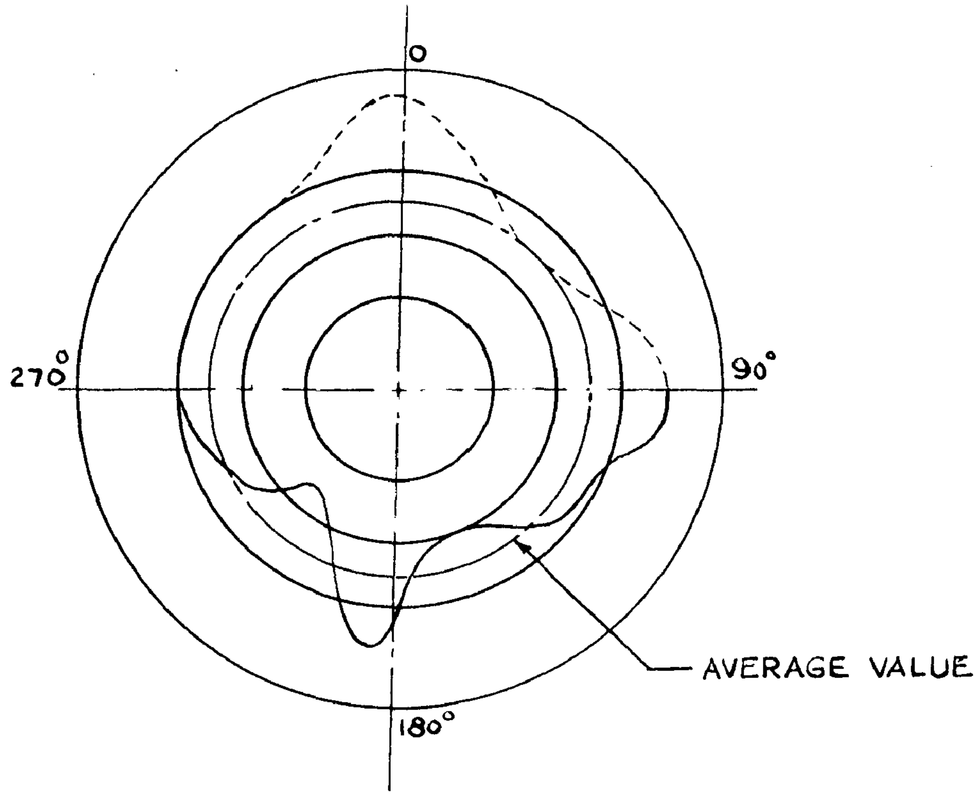


FIGURE 5.1



ELECTRO MAGNETIC DISSYMETRY OF THE ROTOR

FIGURE 5.2

do in the broad programme for the reduction of the industrial noise that is presently under way. There are three distinct types of motor magnetic noise. First, there is the double line frequency noise (transformer Hum) caused by the rotating force wave of the fundamental magnetic field. Second there is a slot frequency noise caused by the slot ripples superposed on the fundamental air gap field. Third, there is the torque noise , caused by pulsations in the motor torque that occur as investigated due to presence of harmonic field. The mathematical analysis for all these is presented here.

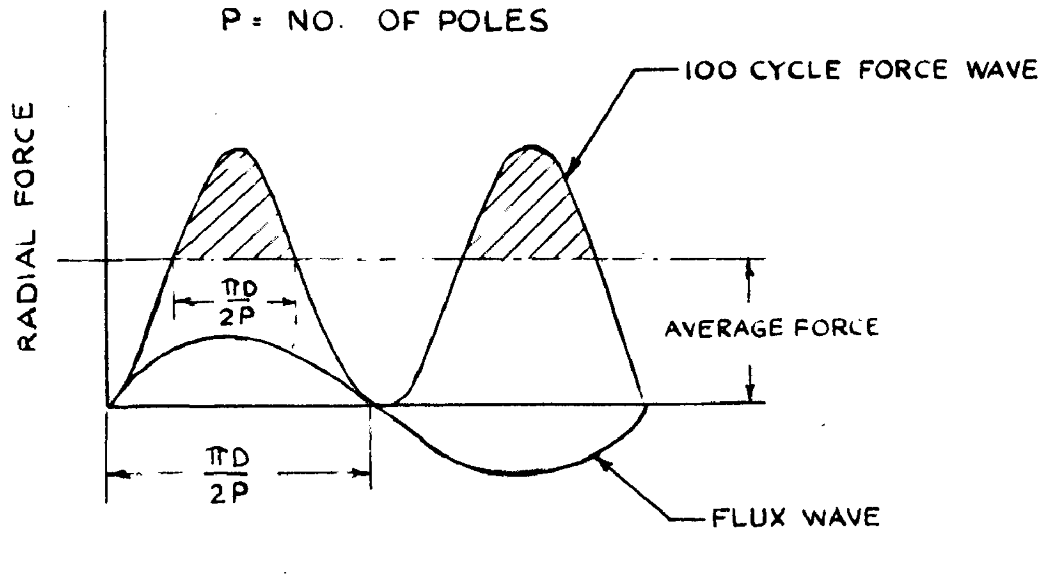
The conductors themselves can usually be neglected as a noise source as the forces on them, being situated in the relatively weak leakage field, are small likewise we can neglect whatever internal stress may exist in the magnetic parts as the rigid parts are not deformed appreciably.

$$\text{Amplitude of force wave}^{(31)} = \frac{1}{2}\,\frac{10^7}{4\pi}\;B'_r\;B'_p\;\text{Newtons/m}^2 \qquad .(5.1)$$

$$= \frac{1}{2}\,(55.6)\;B'_p\;B_r \qquad \text{lbs/in}^2$$

5.3.3. Influence of Skewing:

Skewing has two different effects on the flux distribution and consequently on the force waves. The force waves created by the interaction of flux distribution waves caused by fundamental currents are not reduced by skewing. The force wave itself appears twisted to the left and to the right along the rotor axes, as shown in Fig. 5.3 i.e., the force distribution which has been uniform along a generatrix of the rotor will vary for skewed bar rotors. The effect of skewing can be analysed by resolving the force waves into two



RADIAL MAGNETIC FLUX AND FORCE WAVE

FIGURE 5.3

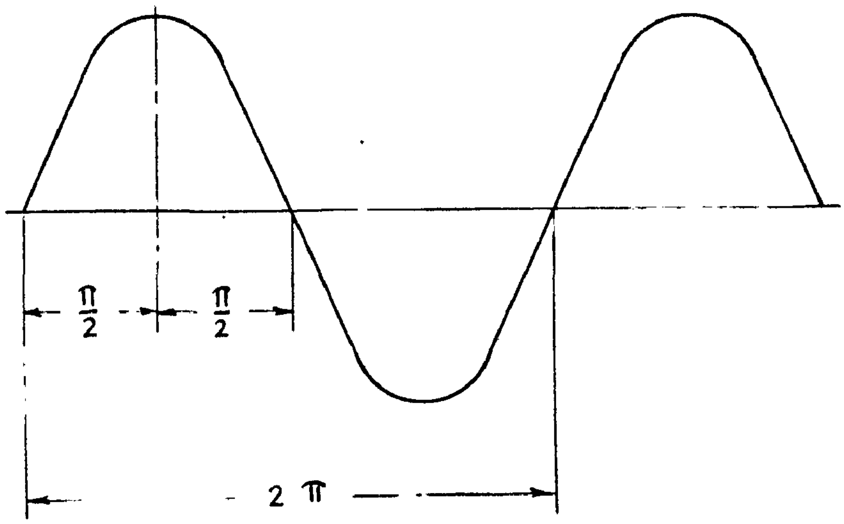


FIGURE 5.4

component waves . The force waves caused by higher harmonic rotor currents may be considerably reduced by skewing, because of the increased differential leakage reactance of the squirrel cage Induction machine.

5.5.4. Double Line Frequency Hum (100 cycle Hum)

In normal operation, with 50 cycle power supply, the 4 pole magnetic field rotates at 1500 rpm, so that the flux density at each point in the air gap varies sinusiodally. But when steel is magnetised, it expands slightly the axis of magnetisation. Hence the core are increases and decreases with flux density, causing tiny variations in peripheral and radialy axis. Since the flux distribution system revolves, so total peripheral length remains unchanged.

As shown in Fig. (5.4) the developed air gap of an induction motor, with two poles of the revolving flux wave. The magnetic flux exerts a radial pull across the air gap, proportional at each point to the square of the flux density. The force wave being square of flux wave is fully displaced sine wave of double frequency as shown in Fig. 5.4. The average force intensity is half the peak value, and is uniform around the periphery. The radial pull exerted in a 2 Pole motor by one of its 4 pole force wave (cross hatched area in 5.4) is equal to

$$W = \frac{1}{2}\ (\text{peak force intensity}) \times \frac{2}{\pi} \times (\text{area of the force Pole}).$$

The full treatment of this is given under Mathematical analysis section of this Chapter.

5.4. MATHEMATICAL ANALYSIS OF MAGNETIC NOISE IN POLYPHASE INDUCTION MOTOR(30)

5.4.1. General Approach:

The parasitic radial force is given by equation (5.1)

$$f_r = 1.39\, b^2 \times 10^{-8}\ \text{lb} - \text{in}^2$$

Where b = flux density in the air gap b lines / sq. in.

also $f_r = \dfrac{10^7}{8\pi}\, b^2$ Newton / m^2 (b in weber/m^2) writing

flux density in terms of stator and rotor harmonics.

$$b(\theta , t) = A_s(m) \sum_{m=1}^{\infty} \text{Cos}\ (m\theta' \mp w_{mt}) + A_s(n) \sum_{n=1}^{\infty} \text{Cos}\ (n\theta \mp w_{nt})$$

..(5.2)

$$\therefore\ f_r = 1.39 \times 10^{-8} \Big[A_{s(m)} \sum \text{Cos}\ (m\theta' \mp w_{mt}) + A_{s(n)} \sum \text{Cos}\ (n\theta' \mp w_{nt}) \Big]^2$$

$$f_r = C \Big[\sum^{m} \Big\{ A_{s(m)} \text{Cos}(m\theta' \mp w_{mt}) \Big\}^2 + \sum^{n} \Big[A_{r(m)} \text{Cos}(n\theta' \mp w_{nt}) \Big]^2$$

$$+ 2\, A_s(m)\, A_r(n) \sum^{h_1} \text{Cos}\ (m\theta' \mp w_{mt})\ \text{Cos}\ (n\theta' \mp w_{nt})^2$$

$$+ 2\, A^2_{s(m)} \sum^{h_2} \text{Cos}^2\ (m'\theta \mp w_{mt}) + 2A^2_{r(n)} \sum^{h_3} \text{Cos}^2\ (n\theta' \mp w_{nt}) \Big]$$

Where

$$h_1 = m \times n \qquad (5.4)$$

$$h_2 = \frac{\lfloor m}{\lfloor m-2\ \lfloor r} \qquad (5.5)$$

$$h_3 = \frac{\lfloor n}{\lfloor n-2 \; \lfloor 2} \qquad (5.6)$$

The equation 5.3 can best be studied in five groups as marked the range of each group is indicated over the summation sign

$$h_1 + h_2 + h_3 = {}^{m+n}C_2 = h \quad \text{(Total multiple terms)}$$

Each group can further be bifercated into simpler terms.

$$\mathrm{Cos}\left\{(m+n)\,\theta' - (w_n - w_m)t\right\} + \mathrm{Cos}\left\{(n-m)\,\theta' - (w_n + w_m)\,t\right\} \qquad (5.7)$$

Equation 5.7 shows that the radial force produced by any two harmonic fluxes consists of two travelling force waves, with the following velocities and pair of poles (Hence forth to be called 'Force wave poles').

$$p_1 = p\,(n-m) \text{ with velocity } (w_n - w_m) \qquad (5.8)$$

$$p_2 = p\,(n+m) \text{ with velocity } (w_n + w_m) \qquad (5.9)$$

5.4.2. Self Stator harmonic Force Waves:

A single travelling flux wave of the stator produces a force wave, this is obtained from equation 5.3 (Group I)

no. of force pole pairs = p_2

$$p_2 = 2m \quad , \text{ frequency } = 2f \qquad (5.10)$$

Equation 5.10 show that stator flux harmonic alone produce force waves of double line frequency. These force waves are seldom disturbing with respect to magnetic noise.

5.4.3. Self Rotor Harmonic Force Waves:

Equation (5.3) (Group II) also gives the similar results for rotor harmonics also, producing double line frequency force waves

$p_2 = 2\,n$, frequency $= 2f$.

5.4.4. Rotor -Stator Harmonic Force waves:

The major disturbing magnetic noise in induction motors is produced by combinations of stator and rotor harmonics . From mother equation 5.3, we get the following.

$$p_1 = n - m \qquad f_1 = (n - m)\,(1-s)f$$

$$p_2 = N+m \qquad f_2 = (\,2 + (n-m)\,(1-s)\,)\,f$$

Since

$$w_n = \left[1 + (n-m)\,(1-s)\right]\,2\,w\,f.$$

and $n = \pm\, v_2\,(R/p) + m$

$$\therefore \quad p_1 = n - m \qquad f_1 \;\; v_{2a}\,(R/p)\,(1-s)\,f \qquad (5.11)$$

$$p_2 = n + m \qquad f_2 = \left[2 + v_2a\,(R/p)\,(1-s)\right] f \qquad (5.12)$$

It is seen from the equations 5.11 and 5.12 that the frequency of the vibration produced by a stator flux harmonic in combination with rotor flux harmonic does not depend upon the order of the stator harmonic. It depends upon slip and v_2a (Rotor permeance factor)

for $v_2(a) = 0$ we get the same equation as (5.10)

5.4.5. Force Waves of Stator Harmonics only :

Group IV of equation 5.3 deals with Stator harmonics only giving

$$w_n = w_m = 0 = 2\pi f$$

and $p_1 = n - m \qquad f_1 = 0$ (5.13)

$p_2 = n + m \qquad f_2 = 2f$ (5.14)

Equation (5.13) indicates that standing force waves are also produced.

5.4.6. Force Waves of Rotor Harmonics only:

Group V of equation 5.3 deals with rotor harmonics only giving:

$n = n_a$ and $n = nb$

when $n_a = \pm v_{2a}(R/p) + m_a$ and $nb = \pm v_{2b}(R/p) + mb$

$p_1 = n_a - n_b$ Velocity $w_{na} - w_{nb}$ (5.15)

$p_2 = n_a - n_b$ Velocity $w_{na} + w_{mb}$ (5.16)

Where

$$w_{na} - w_{nb} = \left[(n_a - m_a) - (n_b - m_b)\right](1-s)f \times 2\pi$$

$$w_{na} - w_{nb} = \left[2 + \left\{(na-ma) + (nb - mb)\right\}(1-s)\right] 2\pi f$$

Simplifying

$f_1 = (v_{2a} - v_{2b})\, R/p\ (1-s)\, f$ (5.17)

$f_2 = \left[2 + (v_{2a} + v_{2b})\, R/p\ (1-s)\right] f$ (5.18)

5.47 Slot Frequency Force Waves

(a) For the squirrel cage rotor; $v_2 = \pm 1$ yields the largest amplitude for the rotor flux harmonic . For this rotor, from equation 5.17 and 5.18 with $v_2a = \pm 1$ and $v_{2b} = \pm 1$

(i) When v_{2a} 0 v_{2b} = − 1 or + 1 (both)

$$f_1 = 0 \tag{5.19}$$

$$f_2 = \left[1 \pm (R/p)\ (1-s)\right]\ 2f \tag{5.20}$$

(ii) When v_{2a} and v_{2b} are of different signs.

$$f_1 = (R/p)\ (1-s)\ 2f \tag{5.21}$$

$$f_2 = 2\,f$$

Equation 5.19 through 5.22 gives slot frequencies variable with slip (speed of the motor).

(b) <u>Slot Relations:</u>

From figure (5.1) , it is clear that force wave poles should be as large as possible to avoid its effect or p = 1 , (one pair of pole) should be avoided. Here conditions under 5.4.4. will be discussed , as this is produced in combination of stator and rotor slot harmonics. Only 2- Pole force wave conditions are discussed[38]

For $p_1 = p_2 = \pm 1$, the following equations must be satisfied.

$$p_1 = n-m \quad \text{and} \quad p_2 = n + m$$

If $n_a = \pm v_{2b}(R/p) + 1$

and $m_b = \pm v_{2a}(S/p) + 1$ (slot harmonic)

then $p_1 = \pm v_{2b}(R/p) \mp v_{2a}(S/p) = 1$

$$p_2 = 2p \pm v_{2a}(S/p) \pm v_{2b}(R/p) = 1$$

or $$\pm v_{2b}(R) \mp v_{2a}S = p\ 1 \tag{5.23}$$

$$2p \pm v_{2a}(S) \pm v_{2(b)} R = 1 \qquad (5.24)$$

for $v_{2b} = v_{2a} = \pm 1$

$$\left.\begin{array}{l} R \pm S = \pm 1 \\ P \pm R \pm S = \pm 1 \end{array}\right] \text{ should be avoided} \qquad (5.25)$$

5.5. Magnitudes of Force waves:

From equation 5.1

$$F_r = 1.39 \times 10^{-8} \times 2L \int_{-\pi/2}^{+\pi/2} b_{na} \; b_{mb} \; d\theta \qquad (5.26)$$

Fig. 5.5. shows the resultant distribution as sinusiodal, the integrand yie

$$F_r = \frac{1.39 \times 10^{-8}}{p'} DL \; B_{mb} \; B_{na} \qquad (5.27)$$

$$= 4.23 \times 10^{4} \; DL \; B_{mb} \; B_{na} \; \text{Kgm (mks)}$$

Where p' = order of force wave poles.

Flux densities can be found from equation (4.17).

5.6. DEFLECTION OF FRAME AND SOUND INTENSITIES

5.6.1. Deflection of Frame:

If we dev lop the stator lamination ring into a straight line, and consider it as a beam, we can use well known equations to calculate the deflection produced. This continuous beam will bend into a shape similar to the sinusiodal wave of applied force, with node at each zero point of the force wave. Each of the nodes may be treated as free support. The deflection of such a freely supported beam under a sinusiodally distributed load is :

$$d = \frac{3\,W D_s^3 \times 10^6}{4\,E\,m^3\,h^3\,L} \quad \text{(micro inch)} \tag{5.28}$$

where d = deflection.

W = load in lbs.

D_s = mean diameter of the stator core inches.

= D_g + 2 (slot depth) + h

h = radial depth of the stator core behind the slot in inches.

m = one half the number of nodes of core flexure.

E = modulus of Elasticity.

= the number of Poles of the force wave.

Taking equation in C.G.S. syste,, the equation 5.28 gives deflection in Cms.

Equation is a approximate equation derived from beam theory. The true equtions for the deflection of a thin ring under a sinusiodal radially applied force are

for $m = 2$ $\quad d = \dfrac{W\,D\,S^3\,10^6}{6\,E\,h^3\,L}$ (5.29)

for $m = 3$ $\quad d = \dfrac{9\,W D S^3\;10^6}{256\,E\,h^3\,L}$ (5.30)

for $m = 4$ $\quad d = \dfrac{W\,D\,S^3\;10^6}{75\,E\,h^3\,L}$ (5.31)

The natural frequency of a thin steel r ing, vibrating in 2 m nodes, is :

$$f = \frac{36,\,7.00\,m\,(m^2 - 1)\,h}{D\,S^2\sqrt{m^2 + 1}} \quad \text{C PS} \tag{5.32}$$

5.6.2. Sound Intensity of Plane Wave:

The sound intensity is defined by the equation

$$Id = 10 \log \left(\frac{I}{Io}\right) \text{ dbs} \qquad (5.33)$$

where I = actual sound Intensity in watts / cm^2 in the direction of propagation.

I_o = Reference sound intensity $= 10^{-16}$ watts / cm^2

This reference sound intensity corresponds to a sinusiodal double amplitude displacement of the air at normal pressure and temperature equal to $2.20 / f \times 10^{-6}$ cms.

This gives for the sound intensity of plane wave

$$I = 1.3 \times 10^{-16} \ (2\ df)^2 \text{ watts / cm}^2$$

thus from 5.33

$$Id = 7 + 20 \log_{10} (df) \text{ dbs} \qquad (5.34)$$

5.6.3. Correction factor:

To find the plane wave of sound that is equivalent to the wave produced by the motor frame, we can make use of the work of Morse and others as shown in Fig. 56., and Fig. 5.7 . The curves of Fig. 5.6 give the number of decibles that must be added to the sound intensity computed from equation 5.34, to obtain the intensity of sound produced by an indefinitely long cylinder vibrating with the same surface amplitude. The crossection depends upon the number of nodes (2 m) and the value of k_{ro}, which is the ratio of the cylinders periphery to the wave length of sound in air at the frequency considered.

$$k_{ro} = \frac{(2\pi\ ro)}{c/f}$$

$$= 0.00559\ fro \qquad (5.35)$$

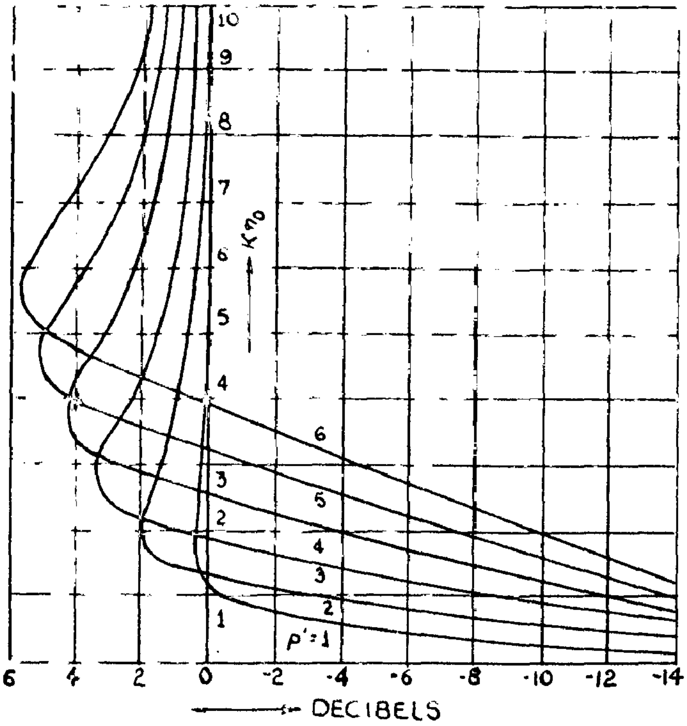


FIGURE 5·6

CORRECTION FOR AN INDEFINITELY LONG CYLINDER

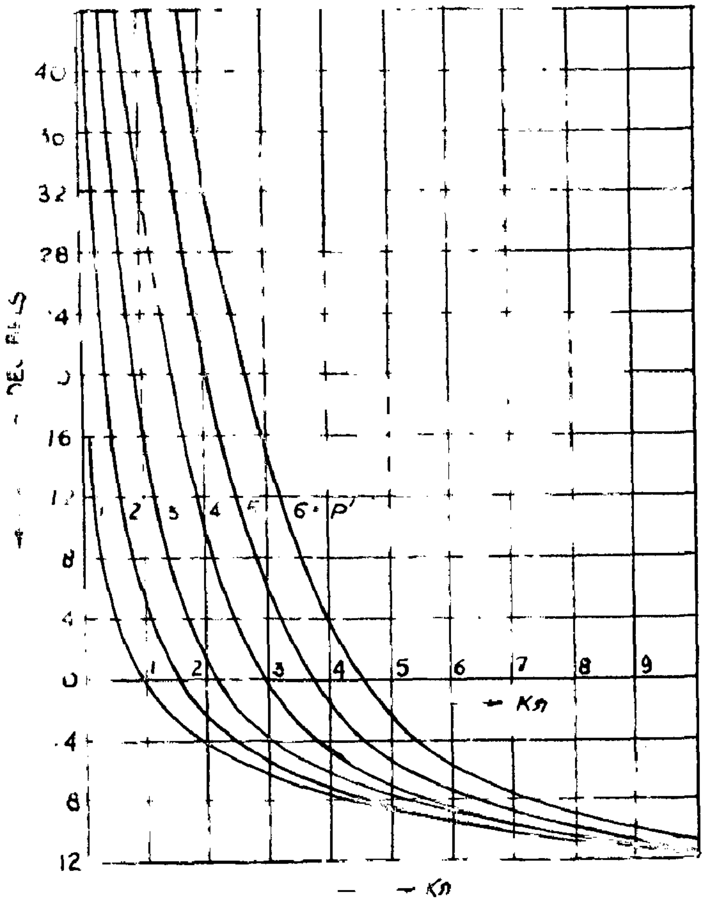


FIGURE 5.5

CORRECTION FOR DISTANCE FROM THE MOTOR SURFACE

Where C = velocity of sound in air.

ro = Cylinder radius in feet.

(b) Distance Correction:

The mate af which the sound intensity falls off with distance also depends upon on the ratio kro (equation 5.35) as shown in Fig. 5.7. When the cylinder is small, as the frequency is low, the intensity falls off very rapidly as the distance from the cylinder ar high frequency it falls off in accordance with the usual inverse square law.

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CHAPTER VI

SONANCE CALCULATIONS OF SQUIRREL CAGE INDUCTION MACHINE WITH AND WITHOUT SKEW

6.1. PROCEDURE:

Magnetic noise is produced by mechanical vibration of the motor frame or components, where the vibromotive force is supplied by the magnetic fields of the motor. The noise produced by the slot harmonics is generally the most objectionable magnetic noise, since this has a relative high frequency. It is caused by radial motions of the stator in nodal patterns of four, six, or more nodes (See Fig .). By knowing the frequencies and number of poles of these forces and the vibratory characteristics of the stator and frame, it is generally possible to avoid the coincidence of a forcing frequency and an objectionable resonance. The frequencies and pole structures of the most bothersome force waves in large motors can be calculated from the following equations:

Frequency of force waves.	Number of nodes in force waves.
$\left[\frac{R(1-s)}{P} - 2\right] f$	$(2R - 2S - 4P)$
$\left[\frac{R(1-s)}{P}\right] f$	$2(R-S)$
$\left[\frac{R(1-s)}{P} + 2\right] f$	$(2R - 2S + 4P)$

The author gives a general procedure for calculating in decibles the slot frequency and other harmonic frequency sound pressure level produced the machine under test operating at any arbitary speed and load. To do this, it is necessary, first , to calculate the frequencies, pole numbers and magnitudes of the important air gap magnetic fields; second, to calculate from these the frequencies and magnitudes of those air gap force waves with relatively few modes those are most likely to produce noise; third, to calculate the stiffness and resonant frequencies of the frame structure for vibrations of low numbers of nodes; fourth, from all these, to calculate the radial deflections of the stator core and frame that are due to the significant slot frequency force waves; and fifth, to calculate the decibel level of sound pressure at any point in space that is caused by the calculated deflection of the frame. Under this Chapter the author presents the numerical results of carried over on IBM 1620 Computer. The author posses with him the other results but only significant results are presented here.

6.2. CALCULATIONS

All the possible combinations as detailed under Section 5.4 of Chapter V are considered for calculation noise intensity, and the computed results are tabulated from Table 6.1 to 6.7.

For this purpose:

Diameter of motor at air gap (10 = 17.0 cms.

Length of armature = 7.5 cms.

$E = 3 \times 10^7$ (for steel)

$h = 3.5$ cms.

Slot depth = 2.1 cms.

$D_s = 24.7$ cms.

$N = 256$ turns The results are plotted in Fig. 6.1 to 6.6.

$$F_r = \frac{235.0}{p'} \quad \frac{k_{w1ma} \quad D_{mb} \quad G_{ma} \; k_{w1\,mb} \; I_1^2}{n_a \; k_{ama} \qquad mb} \tag{6.1}$$

Where,

$$D_{mb} = (1 - C_{mb}\ \cos\phi)^2 + (C_{mb}\ \sin\phi)^2$$

$$C_{mb} = \frac{\sqrt{\left[S_{mb}\ X_{M(mb)}(1+\tau_{2mb})\right]^2 + \left[S_{mb}\ X_{M(mb)}R_{2(mb)}\right]^2}}{R_{2mb}^2 + \left[S_{mb}\ X_{M(mb)}\ (1+\tau_{2mb})\right]^2}$$

$$\tan\phi = \frac{R_{2(mb)}}{S_{mb}\ X_{M(mb)}\ (1+\tau_{2mb})}$$

$$D_{mb} \simeq 1 - \frac{1}{1+\tau_{2mb}} \quad (\text{Neglecting } R_{2(mb)})$$

Since in for higher harmonics, only differential leakage reactance predominates.

$$\therefore\ D_{mb} \approx 1 - \frac{1}{T_{2\ h\ mb}}$$

$$D_{mb} \approx 1 - H^2\ K_{smb}^2 \qquad \ldots\ldots \qquad (6.2)$$

$$\text{Where } H = \frac{\sin\left(\frac{p\pi}{R}\right)}{\frac{p\pi}{R}}$$

$$\therefore\ G_{ma} \approx H^2\ K_{sma}^2 \qquad \text{(from Chapter II)}$$

$$\therefore\ F_r \propto \frac{(1 - H^2\ K_{smb}^2)\ H^2\ K_{sma}^2}{(K_{sma})}$$

$$F_r \propto (1 - H^2\ K_{smb}^2)\ H^2\ (K_{sma}) \qquad \ldots\ldots \qquad (6.3)$$

Equation 6.3. directly gives the effect of skewing of rotor bars. skewing reduces the radial forces and subsequently the sound emitted by magnetic fields

As deflection = d = $0.75 \frac{F_r}{L_e} \frac{D_s^3 \times 10^6}{p'^3 \; E \; h_s^3}$ Micro inch.

and Sound Intensity (I_d) = 7 + 20 log $(d \times f_s)$ dbs

6.3. NATURAL FREQUENCY

The natural frequency of the frame, considering it as simple ring

$$f_n = \frac{36{,}700 \; p' \; (p'^2 - 1) \; h_c}{Dm^2 \sqrt{p'^2 + 1}} \quad \text{CPS}$$

The minimum p' = 4 (without cage)

$\therefore$ f_n = 7560 CPS

The natural frequency being very high, there is no possibility of mechanical resonance.

Table 6.1. NOISE PRODUCED BY 17th & 19th HARMONICS ([illegible])

For Slip(S) = 0.1 to 2.0

p'	c = 0.01				c = 0.1				c = 0.2			
	f	P	d	I_d	f	P	d	I_d	f	P	d	I_d
For 17th Harmonic.												
4	100	0	0	0	100	1.1	0.05	21.0	100	2.7	.125	29
4	1600	0	0	0	1630	.2	.009	30.6	1350	.4	.018	35.6
8	100	0	0	0	100	.4	.002	-	100	1.0	.005	-
16	100	0	0	0	100	1.1	-	-	100	1.4	.007	-
16	1620	0	0	0	1630	.2	$.12 \times 10^{-3}$	-	1630	.4	-	-
For 19th Harmonic												
4	100	0	0	0	100	1.3	5.8×10^{-2}	21.3	100	2.7	.12	20.6
8	100	.2	-	-	100	2.8	.0157	11.0	100	6.2	.035	17.8
8	1530	0	0	0	1430	.1	.035	-	1200	.2	1.1×10^{-3}	11.8

Table 6.1 Contd..

p'	s = 0.5				s = 1.0				s = 2.0			
	f	F	d	I_d	f	F	d	I_d	f	F	d	I_d
For 19th harmonic												
4	100	5.6	.25	35.0	100	6.4	.285	36.0	100	10.7	.475	40.5
4	875	1.0	.04	37.8	100	1.2	.053	21.4	1700	1.3	.05	45.4
8	100	2.4	.126	9.0	100	2.6	.015	10.2	100	2.9	.016	11.0
16	100	5.0	.03	7.9	100	5.5	.03	8.0	100	6.9	.04	8.2
16	875	.8	.[illegible]	-	100	.1	-	-	1700	1.1	$.67\times10^{-3}$	8.2
For 19th harmonics.												
4	100	5.1	.23	34.2	100	6.4	.28	36.1	100	7.9	.35	38.0
8	100	12.9	.07	24.2	100	14.6	.082	25.2	100	18.1	.1	27.0
8	775	.5	2.8×10^{-3}	13.7	100	.7	-	-	1800	.8	3.9×10^{-3}	24.0

Table 6.2 NOISE INTENSITY OF 5th HARMONICS (WITH ROTOR SKEWED)

FOR SLIP .01 to 2.0

p'	s = .01				s = 0.1				s = 0.2			
	f	F	d	I_d	f	F	d	I_d	f	F	d	I_d
For 5th harmonics												
8	100	16.5	.09	25.0	100	226.5	1.26	49	100	522.0	2.9	56.2
8	1680	0	-	-	1630	1.8	$.33\times10^{-2}$	22.0	1360	8.1	1.84×10^{-2}	36.0
20	100	.6	-	-	100	4.1	-	-	100	9.0	-	-
4	100	.9	.04	5.0	100	10.3	.45	15.0	100	8.7	.39	38.0
For 23rd Harmonic												
4	100	1.0	.04	19	100	20	.89	46.0	100	44.0	1.97	52.8
8	100	.2	$.11\times10^{-2}$	-	100	2.8	1.39×10^{-2}	9.8	100	6.2	3.48×10^{-2}	17.8
16	1680	0	0	-	1630	.2	$.14\times10^{-3}$	9.9	1360	6.2	$.35\times10^{-3}$	-
20	100	0	-	-	100	.8	-	-	100	1.9	-	-
		I_d = 19.0				I_d = 55.8				I_d = 70.6		
For 25th harmonic												
4	100	1.0	.22	19	100	20.0	.021	46.0	100	43.9	.023	52.8
8	100	.1	-	-	100	1.7	-	-	100	3.8	.02	13.5
		I_d = 19.0				I_d = 46.0				I_d = 66.3		

Table 6.2 (Contd..)

For 5th Harmonic

p'	s = 0.5				s = 1.0				s = 2.0			
	f	F	d	I_d	f	F	d	I_d	f	F	d	I_d
4	100	51.4	2.3	54.0	100	67.9	3.0	56.5	100	61.3	2.74	55.0
8	875	.9	.2	18.4	100	8.6	1.8×10^{-2}	22.0	1700	8.5	1.84×10^{-2}	36.5
8	100	1083	6.0	64.6	100	1222	6.8	65.0	100	1495	8.3	66.4
20	100	19.4	-	-	100	21.3	-	-	100	201	7.2×10^{-3}	-
For 23rd Harmonic												
4	100	904	4.02	59.0	100	104	4.5	60.0	100	125.4	5.6	62.0
8	100	13.0	7.3×10^{-2}	24.2	100	14.6	8.1×10^{-2}	25.0	100	17.0	9.5×10^{-2}	26.5
16	875	1.1	$.91\times10^{-2}$	-	100	1.03	$.91\times10^{-3}$	-	1700	1.3	$.91\times10^{-3}$	13.0
20	100	42	-	-	100	5.0	-	-	100	5.3	1.9×10^{-3}	-
For 25th Harmonic												
4	100	9.06	.46	59	100	100.4	.5	60.0	100	125.5	.72	62.0
8	100	7.71	4.3×10^{-2}	-	100	8.9	4.96×10^{-2}	21.0	100	10.8	6.0×10^{-2}	22.5

TABLE 6.3 TOTAL 4 POLE SOUND INTENSITY LEVEL OF TEST MACHINE.

Slip	f	F	A	I_d
ROTOR NOT SKEWED				
.01	100	3.5	.157	31
.1	100	47.7	2.13	57
.2	100	104.7	4.67	64.0
.5	100	239.7	10.7	70.0
1.0	100	236.3	10.6	70.0
2.0	100	323.9	14.5	78.0
ROTOR SKEWED				
.01	100	2.0	.09	23
.1	100	31.4	1.41	29
.2	100	71.5	3.22	50
.5	100	138.2	6.2	63
1.0	100	178.0	8.0	65
2.0	100	205.0	9.2	68

Table 6.4. TOTAL RADIAL HARMONIC FORCE WAVE WITHOUT SCREW

p'	s = 0.01	s = 0.1	s = 0.2	s = 0.5	s = 1.0	s = 2.0
2	3.5	47.7	104.7	238.7	236.3	323.9
4	11.6	165.3	363.7	756.2	865.4	1031.6
6	17.0	140.8	342.2	792.7	1775.9	993.1
8	0	2.8	5.0	14.3	21.7	17.7
10	0	5.7	10.8	28.2	32.9	32.1
12	0.0	4.1	10.3	23.4	29.1	27.2
14	0.0	2.9	8.1	13.4	15.0	14.1
16	1.5	15.8	46.6	73.6	80.1	83.8
18	1.9	30.5	118.0	221.7	249.5	271.9
20	1.0	16.6	38.2	75.9	89.8	99.3

Table 6.5 TOTAL RADIAL HARMONIC FORCE WAVES WITH SKEW

p'	s = 0.01	s = 0.1	s = 0.2	s = 0.5	s = 1.0	s = 2.0
4	2.0	31.4	71.5	138.2	178.0	205.0
8	13.7	205.1	380.1	913.1	1015	1170.0
12	13.0	90.1	291.1	675	1053	850
16	0.0	1.7	3.1	9.3	11.6	13.7
20	0.0	3.0	7.2	21.1	29.0	27.6
24	0.0	3.2	7.3	19.1	21.1	23.2

Figures indicate lbs.

Table 6.6 HIGH FREQUENCY SOUND GENERATED BY MAGNETIC FIELDS

(ROTOR SKEWED)

Frequency	Harmonic Sound Intensity (dbs)					
	5th	17th	19th	23rd	25th	Slot Harmonics
750	-	-	13.7	-	-	7.0
850	-	10.4	57.4	-	-	-
950	-	-	-	-	-	52.0
1260	0	-	-	-	-	16.0
1350	0	36.5	36.6	-	-	-
1430	0	-	-	7.0	-	-
1460	-	-	-	-	44.0	44.0
1530	22.0	30.6	-	-	-	-
1600	0	-	-	-	-	60.0
1700	0	57.0	53.2	-	13.0	-
1800	0	-	7.0	24.0	-	-

Table 6.7 HIGH FREQUENCY SOUND GENERATED BY MAGNETIC FIELDS

(ROTOR UNSKEWED)

750	0	-	-	11.7	-	7.0
850	0	40.0	57.0	-	-	77.0
950	-	-	-	-	-	52.0
1260	0	16	21.4	-	-	16.0
1360	0	40.0	38.6	-	-	-
1430	0	-	-	7.0	-	11.0
1460	-	-	-	-	-	44.0
1530	0	33.0	30.8	-	-	-
1600	0	-	-	-	-	60.0
1650	-	-	-	-	-	52.0
1700	0	49.5	62.5	-	-	-
1800	0	-	-	27.0	14.6	30.0

6.4. REDUCTION OF PARASITIC TORQUES AND NOISE OF INDUCTION MACHINE WITH ROTOR SKEWING

Simplifying the expressions for parasitic torques and noise, we can arrive at simpler function in terms of skew factor and certain other constants of the machin . The functions thus obtained, can give the range of variations in magnitudes of the parasitic torques and noise for a corresponding change in skewing of rotor bars.

Defining Unit values:

$$\text{Unit value of (mth harmonic)} = \frac{\text{value of mth harmonic quantity with skew}}{\text{value of mth harmonic quantity without skew.}}$$

6.4.1. MAXIMUM PARASITIC TORQUES

As, our previous study reveals, for the test machine 5th, 19th and 25th harmonics are more predominant . So, effect of skewing for these harmonics will be considered primarily. From equation (3.16)

$$S_{mP.o} = \frac{R_{2(m)}}{X_{M(m)} + X_{2(m)}}$$

$$\therefore \quad S_{m\,P.o} = \frac{A_1}{1 + \tau_{2(m)}}$$

and

$$_{m(max)} = 0.369 \ \frac{m}{w} \ R \ \frac{X_{M(m)} \ I_1^{\ 2}}{1 + \tau_{2(m)}}$$

Assuming I_1 constant)

$$T_{m(max)} \propto \frac{1}{1 + \tau_{2(m)}} \tag{6.4}$$

$$\text{Unit Max. Torque} = \frac{1 + \tau_{2(m)o}}{1 + \tau_{2(m)}} \tag{6.5}$$

Results are tabulated and plotted in Fig. 6.9A.

6.4.2. Syncronous Torque:

From equation (4.29)

$$T_{sy(max)} = 2\pi C \quad B_{na} \quad A_{mb}$$

$$= \left[\frac{S_m \qquad A_2}{\left(\frac{R_{2(m)}}{X_{M(m)}}\right)^2 + sm^2 (1+ \tau_{2(m)})^2} \right] \frac{K_{sna}}{K_{sma}}$$

substituting for S_m equation (4.9)

$$T_{sy(max)} = \frac{1}{(1+ \tau_{2(m)})} \left(\frac{A_2}{H_{(m)}^2 + 1} \right) \frac{K_{sna}}{K_{sma}}$$

when $H(m) = \dfrac{R_{2(m)}}{X_{M(m)}}$

$$\therefore \; T_{sy(max)} \quad \left(\frac{K_{sna}}{K_{sma}}\right) \frac{1}{1 + \tau_{2(m)}}$$

and Unit syncronous torque $= \dfrac{K_{sna}}{K_{sma}} \left(\dfrac{1 + \tau_{2(m)o}}{1 + \tau_{2(m)}} \right)$

Where $\tau_{2(m)o}$ = Ratio of secondary leakage reactance to Magnetising reactance, with rotor unskewed.

The results are tabulated and plotted in Fig. 6.9 B.

6.4.3. Noise:

From equation (5.27) and (6.2)

$$\text{Radial force } F_r \quad \frac{\left[1 - \xi^2_{(ma)} \; K_{s(ma)}\right] K^2_{s(ma)}}{\left[1 - \xi^2_{(ma)}\right] K_{sna}} \qquad \text{.....6.7}$$

But since sound intensity is defined as

$$I_d = 7 + \log (d \times f)$$

$$\text{Where } \xi(m) = \frac{\sin\ m p \pi/12}{(m p \pi/12)^2} \qquad 6.8$$

Considering only 100 cycle hum and radial force wave of 4 poles.

$$I_d \simeq \log (d \times f)$$

As $d \propto F_r$

$$\therefore \quad I_d \propto \log (F_r \times f)$$

$$\therefore \quad \text{Unit Noise} = \frac{\log (F_r \times f)}{\log (F_{ro} \times f)} = \log \left(\frac{F_r}{F_{ro}} \right) \qquad 6.9$$

Substituting 6.7 in 6.8

$$\text{Unit Noise} = \log \left(\frac{1 - \xi^2_{(mn)} K^2_{s(mn)}}{1 - \xi^2_{(mn)}} \right) \qquad \frac{K_{smn}^2}{K_{smn}} \qquad 6.10$$

where $mn = \pm v (N/p) + mn$

equation 6.9 gives the following conclusions.

(i) For decreasing effect in noise, unit values are negative.

(ii) Increasing and decreasing effect not only depends upon skewing but the magnitude of (m) also which is again function of Rotor slots and order of curves are ppotted in Fig. 6.8. For following pairs of harmonic giving 4 pole, 100 cycle noise.

1. 5th and 7th
2. 19th and 17th
3. 25th and 23rd.

Table 6.8. HARMONIC SKEW FACTORS.

Angle Elec.	1	5	7	17	19	23	25	29	33	35	39
5	.999	.995	.993	.91	.895	.84	.815	.75	.32	.65	.214
10	.996	.97	.95	.674	.6	.45	.378	.227	.215	.029	.176
15	.995	.93	.875	.356	.246	.043	.04	.186	.088	.224	.128
20	.994	.88	.775	.06	.05	.19	.215	.186	.018	.028	.074
30	.99	.74	.53	.218	.195	.043	.076	.127	.07	.028	.017
40	.98	.56	.266	.06	.05	.123	.075	.06	.018	.028	.047
50	.97	.416	.029	.123	.11	.057	.091	.005	.039	.028	.022
60	.955	.19	.137	.056	.05	.04	.038	.033	.018	.027	.016
70	.94	.085	.24	.08	.07	.07	.027	.05	.025	.027	.027
90	.90	.18	.13	.043	.047	.039	.036	-	.034	.026	.015

Table 6.9 CALCULATION OF $\varphi_{2(m)}$ FOR DIFFERENT SKEW ANGLE

Angle	1	5	19	25
0	0	0.16	33.8	101.0
5	.002	.17	42.5	154.0
10	.01	.23	95.5	715.0
15	.015	.34	574.0	6.4×10^4
20	.015	.50	1.26×10^4	22.20
30	.02	1.10	916.0	1.74×10^4
40	.04	2.63	1.32×10^4	1.89×10^4
50	.065	5.70	2880	1.22×10^4
60	.1	30.8	1.39×10^4	7.1×10^4
70	.15	159	$.71 \times 10^4$	35.2×10^4
90	.23	35.8	1.53×10^4	7.9×10^4

Table 6.10 VARIATION OF MAXIMUM ASYN. TORQUES OF HARMONICS WITH SKEW

$T_{n(max)}$ $$\frac{1 + \tau_{2(n)o}}{1 + \tau_{2(n)}}$$

1	5	19	25
1.0	1.0	1.0	1.0
.995	.995	.0	.66
.99	.945	.35	.14
.989	.869	.06	.0016
.985	.77	2.76×10^{-3}	.046
.98	.55	3.8×10^{-2}	.006
.95	.32	2.63×10^{-3}	.0054
.94	.17	1.21×10^{-2}	.008
.91	.036	2.5×10^{-3}	.0014
.87	.007	4.9×10^{-3}	.0005
.815	.03	2.3×10^{-3}	.0015

Table 6.11 HARMONIC SYN. TORQUES WITH DIFF. SKEW ANGLE

T) $$\frac{[1 + \tau_{2(n)o}]\, K_{sn}}{[1 + \tau_{2(n)}]\, K_{o\,sn}}$$

1	5	19	25
1.0	1.0	1.0	1.0
.65	.755	.285	.174
.028	.226	.13	.067
.22	.17	very low	v.l.
.028	.16	v.l	v.l
.027	.093	v.l	v.l
.026	.096	v.l	v.l
.026	.007	v.l	v.l
.026	.06	v.l	v.l
.025	.008	v.l	v.l
.024	-	v.l	v.l

Table 6.12. VARIATION OF HARMONIC SOUND INTENSITY WITH SKEW

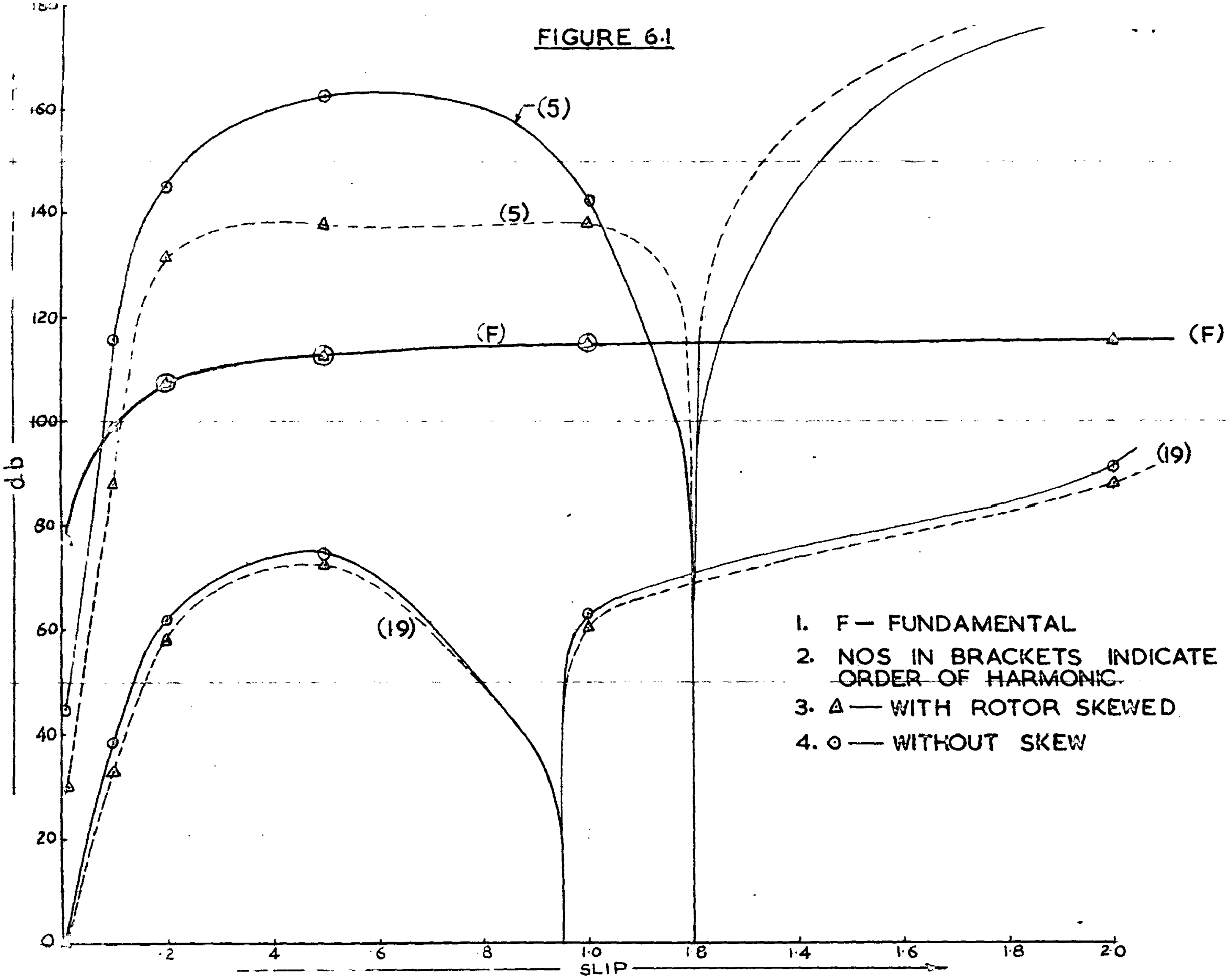
Angle	5th Harmonic (5) =0.965 na =7	19th harmonic (19)=0.55 na=17	25th Harmonic (25) =.32 na = 23
0	0.0	0.0	0.0
5	.06	-.02	-.23
10	.25	-.17	-.46
15	+ .44	-.62	+1.34
20	+.66	-1.22	-.49
30	.85	-.61	-.83
40	1.3	-1.22	-1.3
50	1.86	-.85	-.78
60	.55	-1.3	-1.42
70	.31	-1.15	-1.92
90	.53	-1.4	-1.49

$$db \qquad \log_{10} \frac{\left[1 - \xi^2_{(ma)} \quad K^2_{s(ma)}\right] K^2_{sma}}{\left[1 - \xi^2_{(ma)}\right] \quad K_{sma}}$$

Considering only 4 pole, 100 c/s hum.

	ma		na
1.	-5	with	+7
2.	19	with	17
3.	25	with	23
4.			

FIGURE 6.1



180
160
140
120
100
80
60
40
20
0
db
·2
4
6
8
1·0
1·8
1·4
1·6
1·8
2·0
SLIP
(5)
(5)
(F)
(F)
(19)
(19)
1. F — FUNDAMENTAL
2. NOS IN BRACKETS INDICATE ORDER OF HARMONIC
3. Δ — WITH ROTOR SKEWED
4. ⊙ — WITHOUT SKEW

HARMONIC SOUND INTENSITY CURVES

FIGURE 6·2

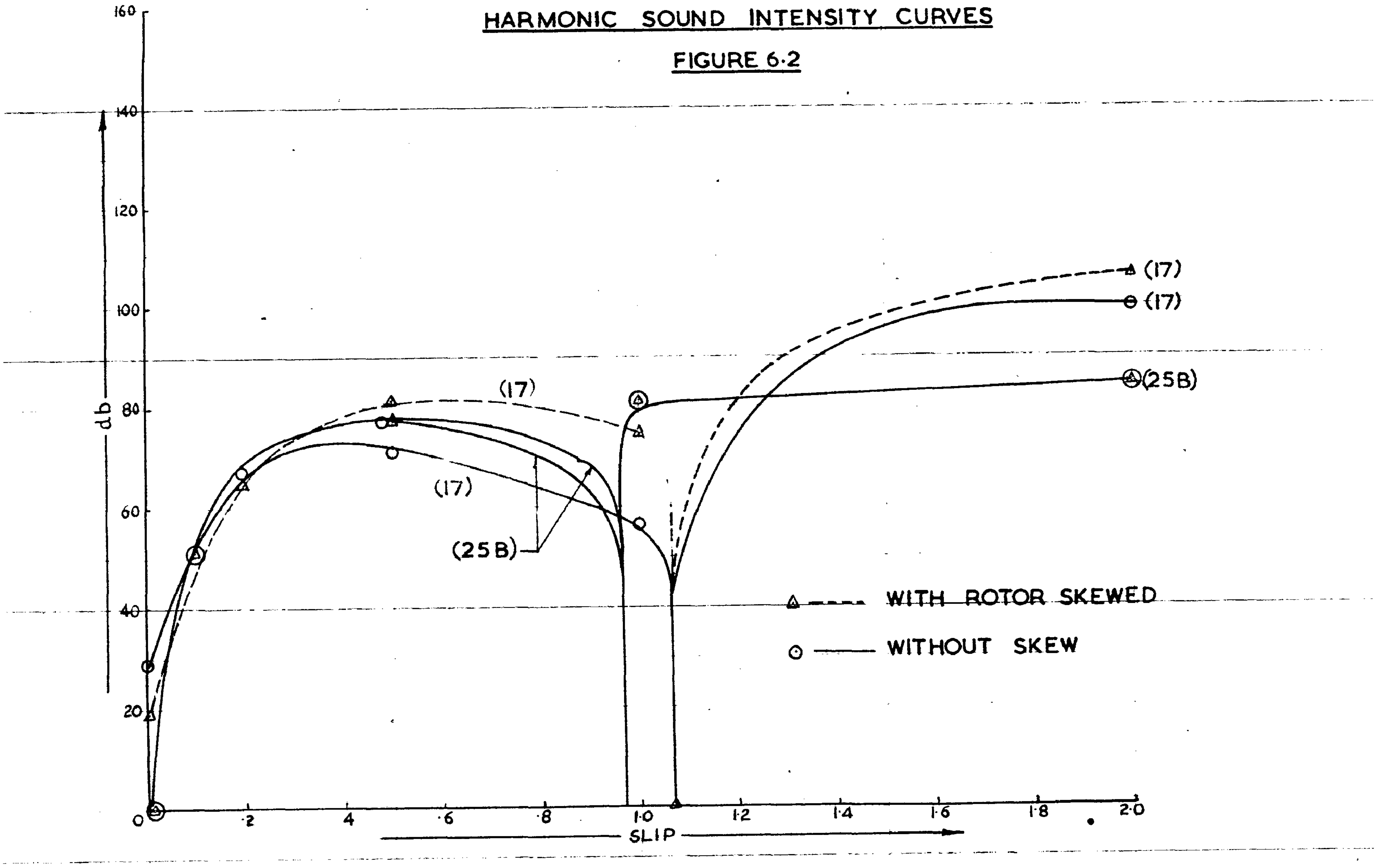
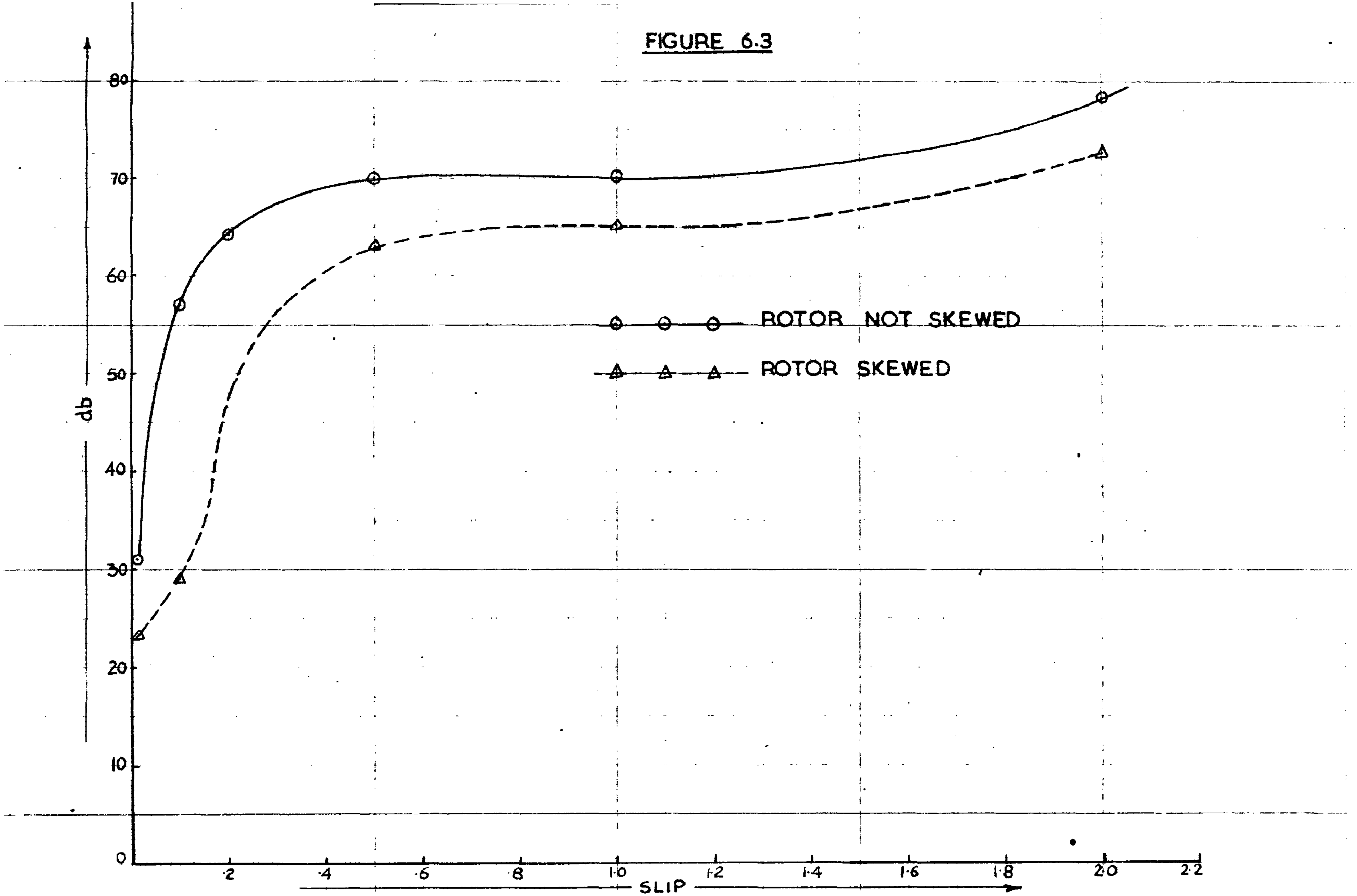


FIGURE 6.3



80
70
60
50
40
30
20
10
0
db
ROTOR NOT SKEWED
ROTOR SKEWED
·2
·4
·6
·8
1·0
1·2
1·4
1·6
1·8
2·0
2·2
SLIP

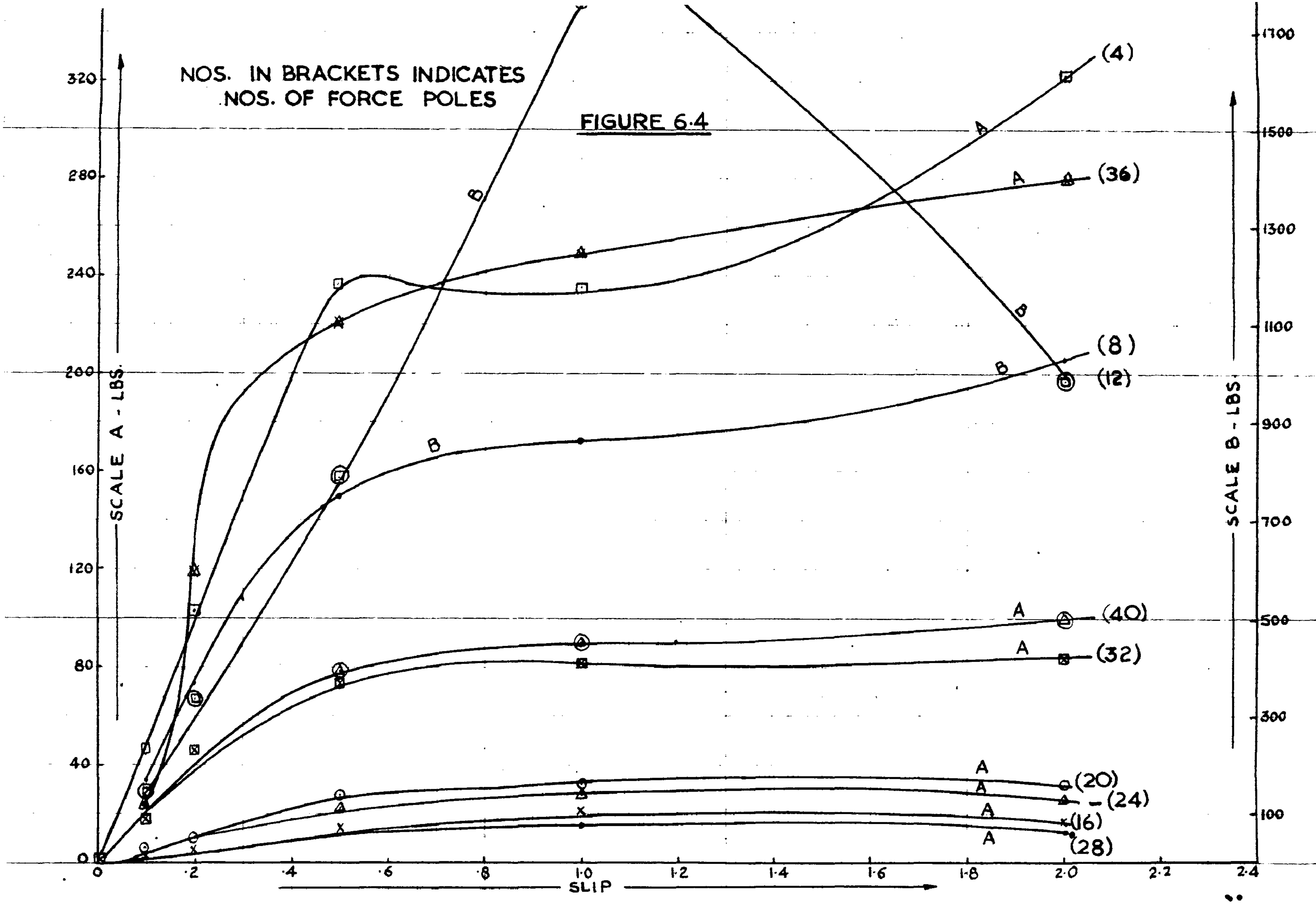
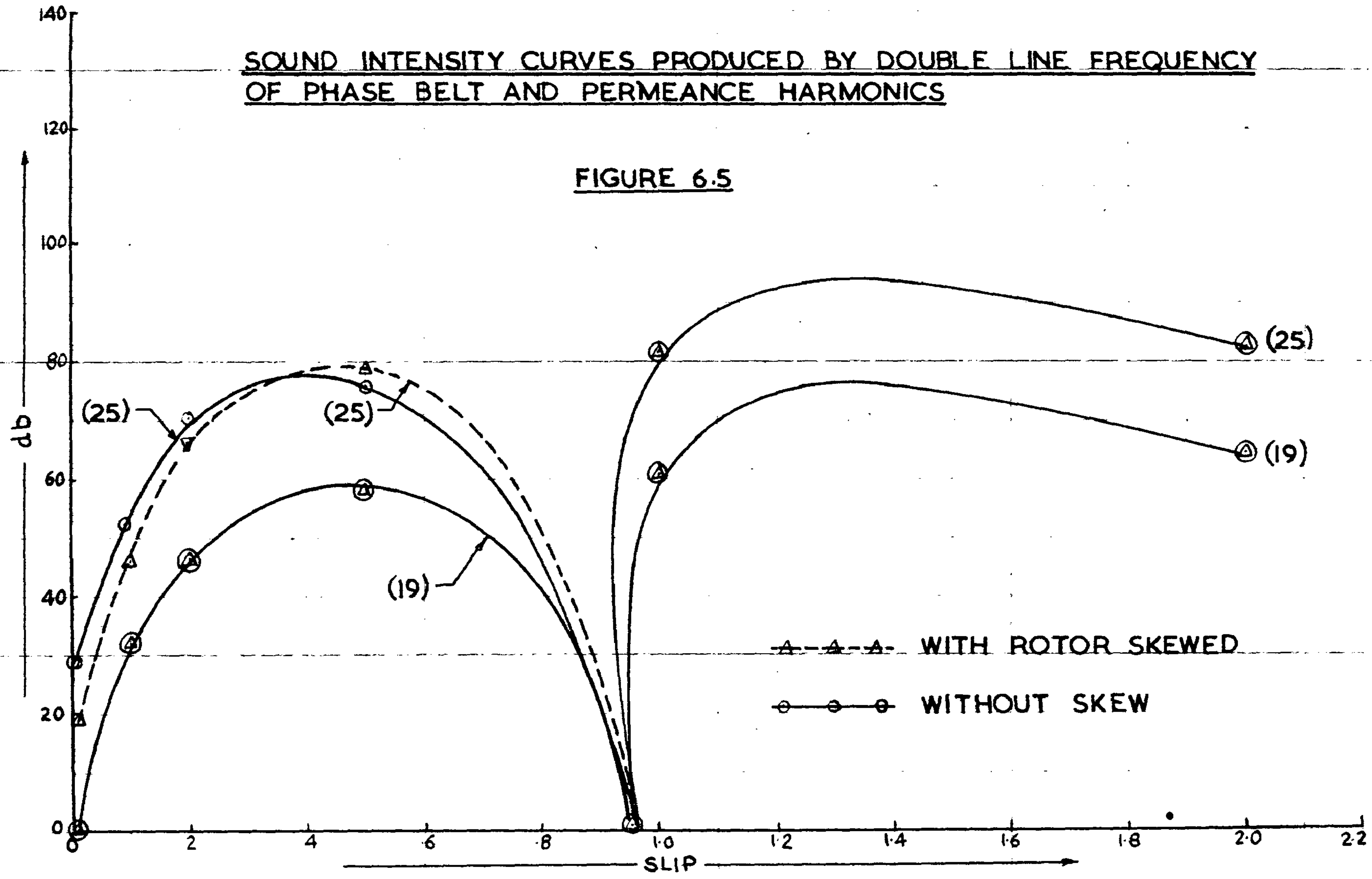
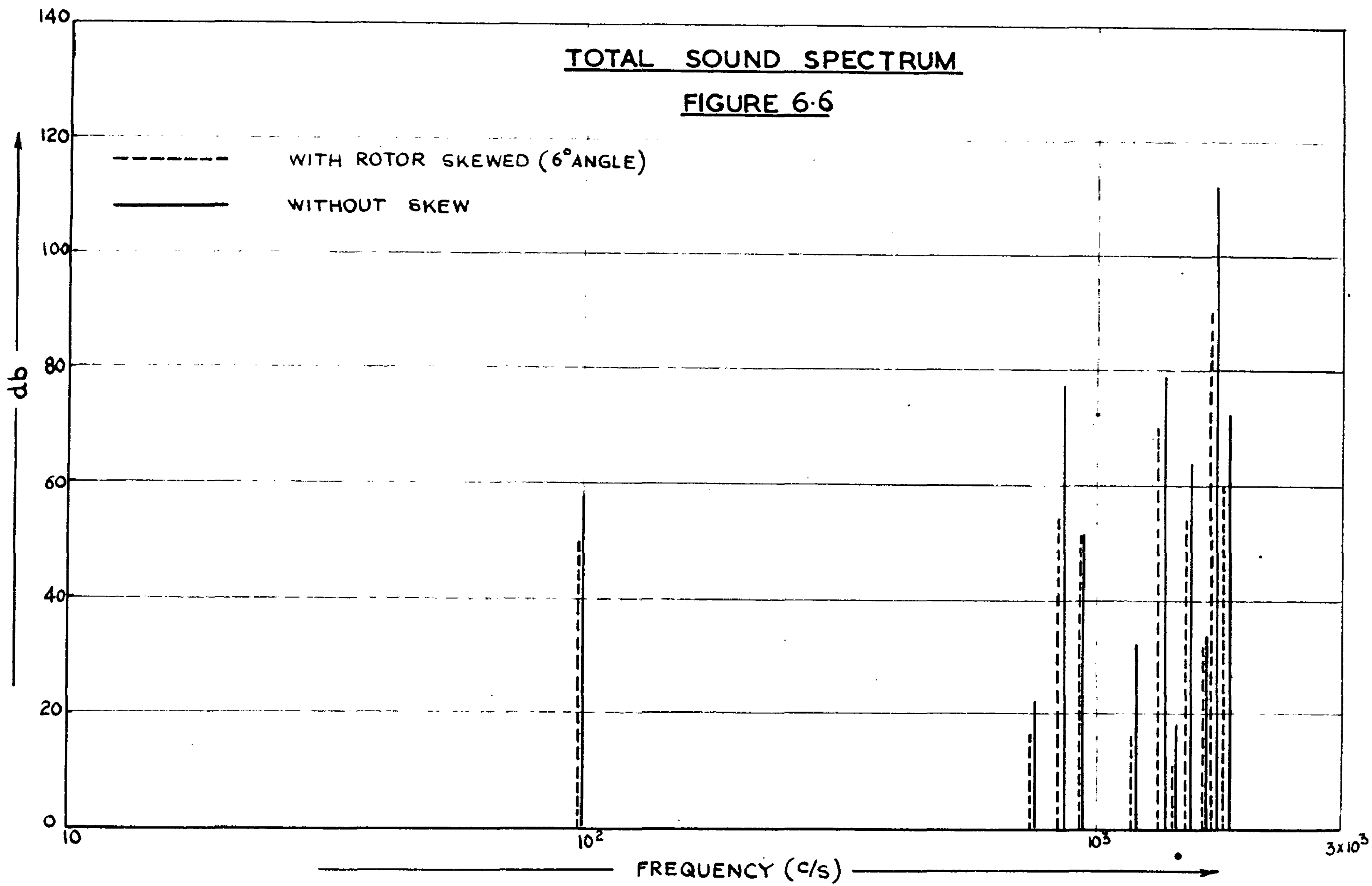

NOS. IN BRACKETS INDICATES NOS. OF FORCE POLES
SCALE A - LBS.
SCALE B - LBS.
SLIP
0
40
80
120
160
200
240
280
320
100
300
500
700
900
1100
1300
1500
1700
·2
·4
·6
·8
1·0
1·2
1·4
1·6
1·8
2·0
2·2
2·4
(4)
(36)
(8)
(12)
(40)
(32)
(20)
(24)
(16)
(28)
A
B

FIGURE 6.4



SOUND INTENSITY CURVES PRODUCED BY DOUBLE LINE FREQUENCY OF PHASE BELT AND PERMEANCE HARMONICS

FIGURE 6.5


TOTAL SOUND SPECTRUM
FIGURE 6·6
WITH ROTOR SKEWED (6° ANGLE)
WITHOUT SKEW
db
140
120
100
80
60
40
20
0
10
10^2
10^3
3×10^3
FREQUENCY (c/s)

SOUND SPECTRUM DUE TO SLOT HARMONICS (WITH AND WITHOUT SKEW)

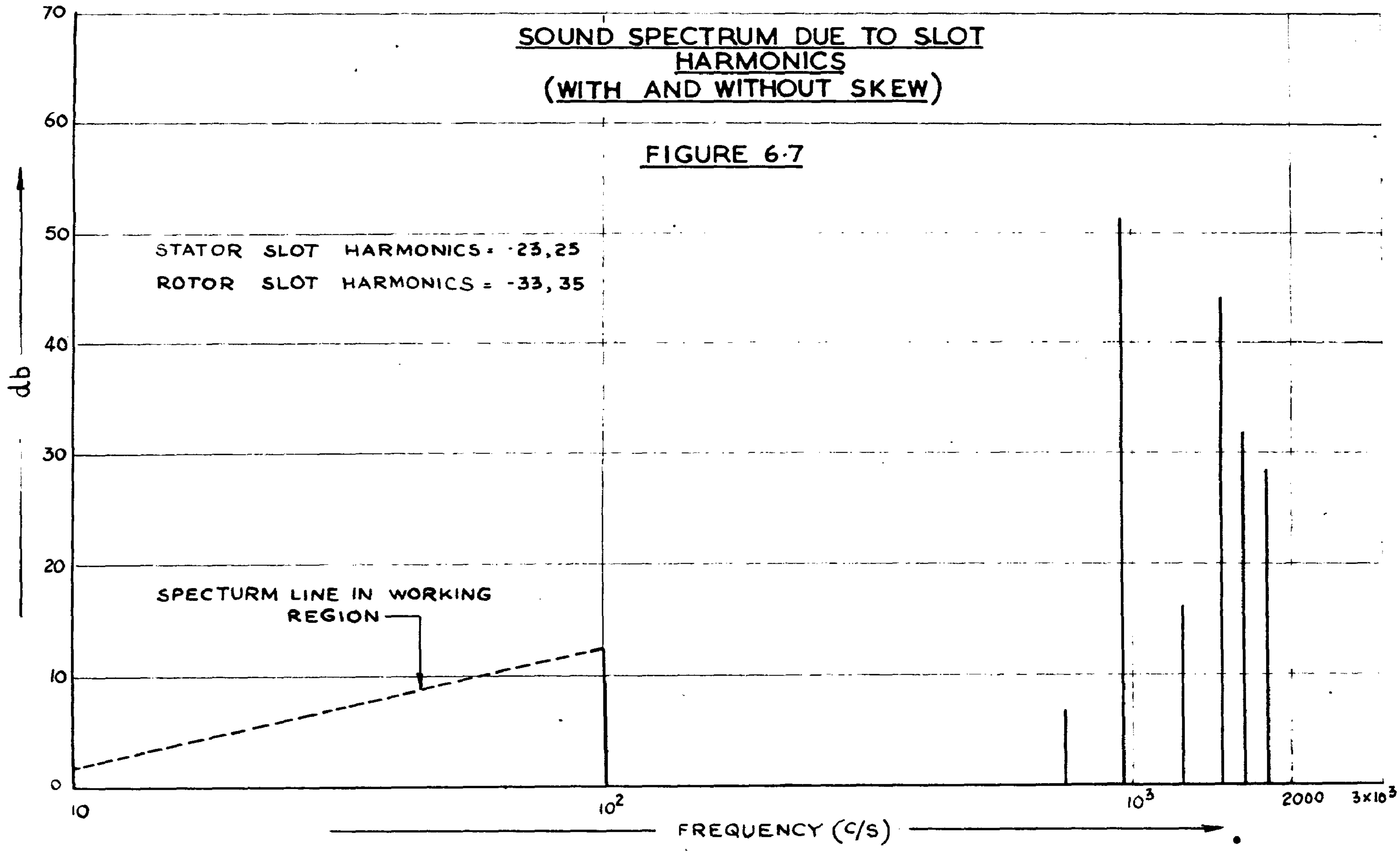
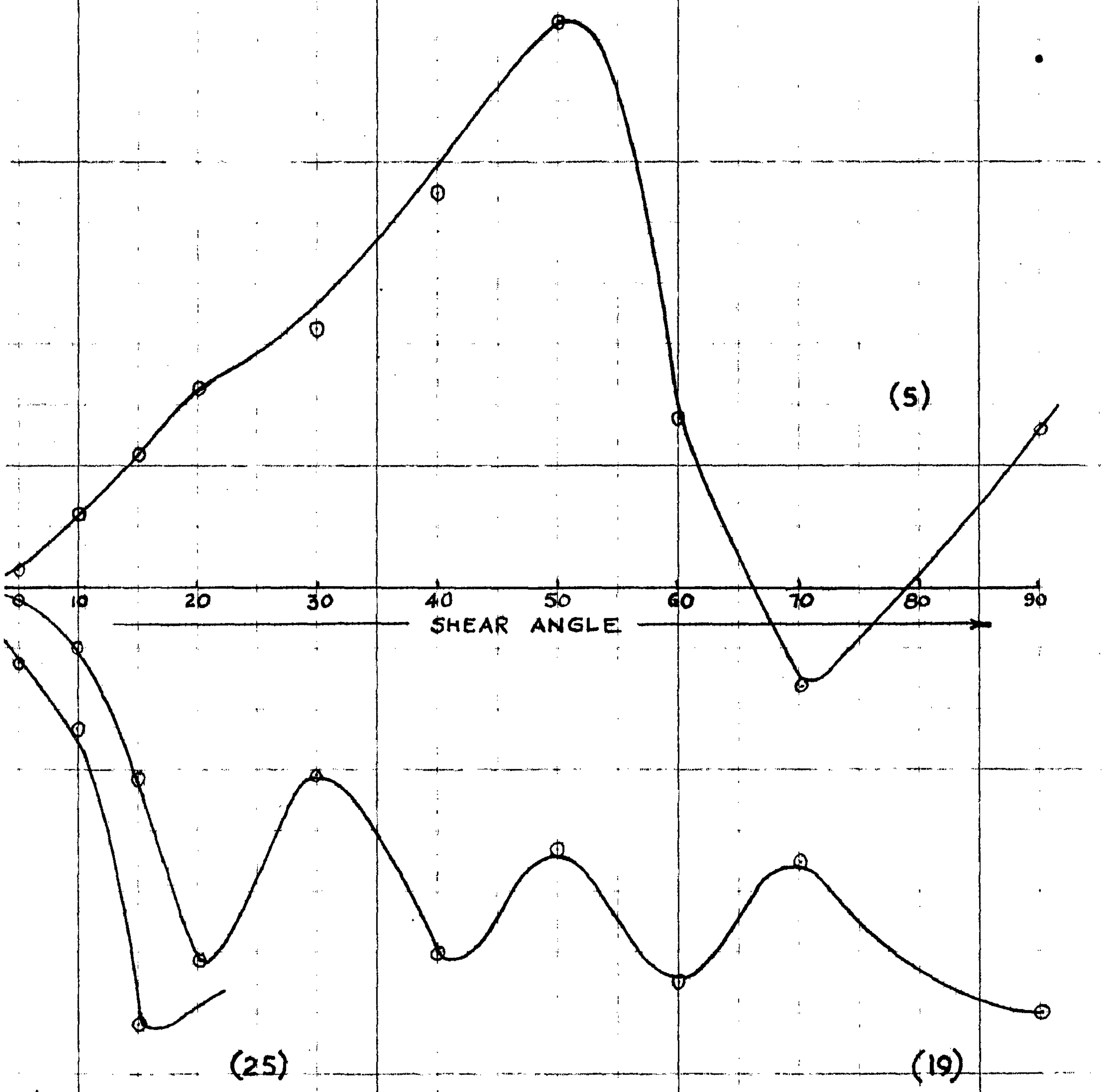


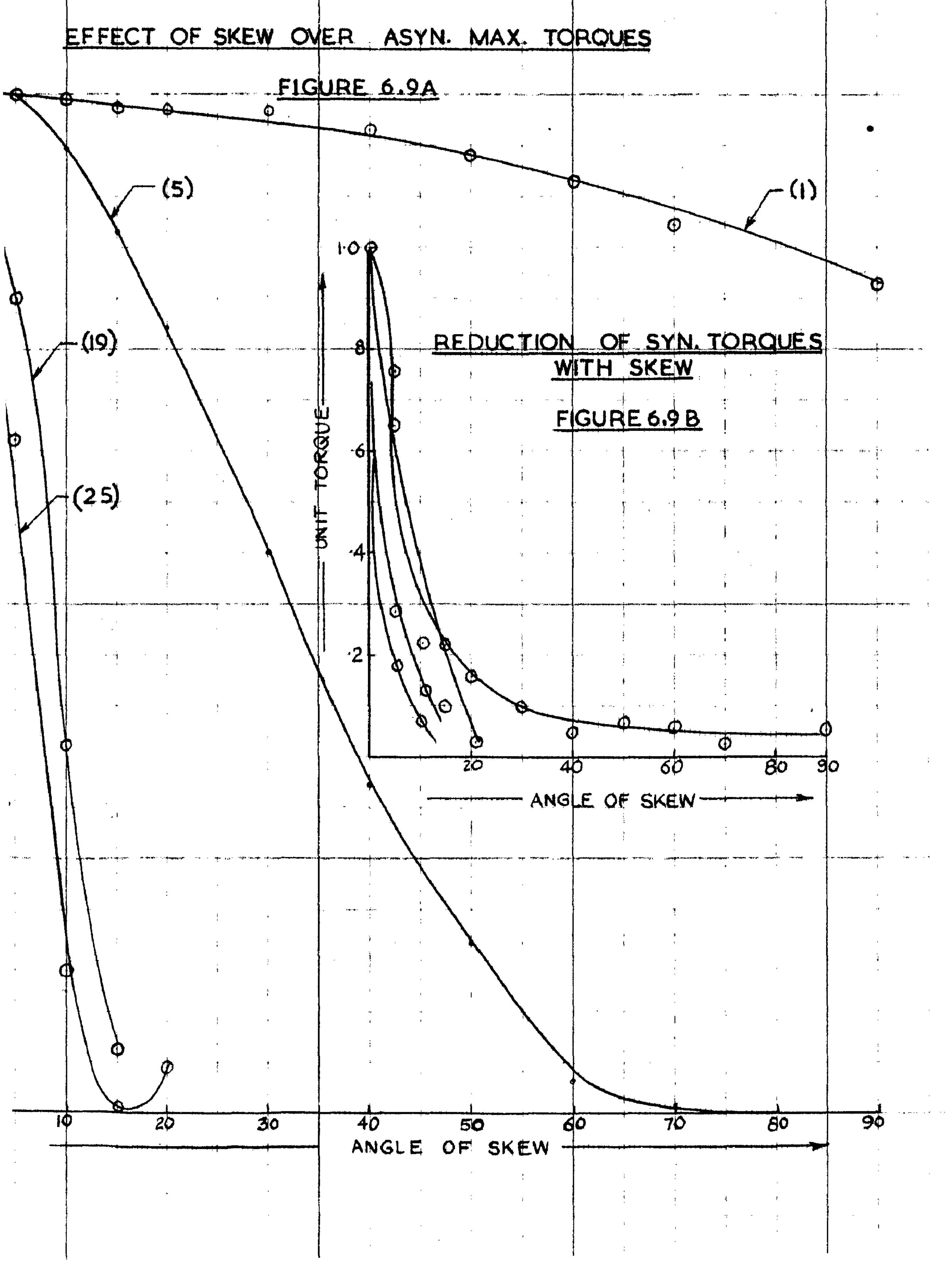
FIGURE 6·7

HARMONIC SOUND INTENSITY WITH VARIED SKEW ANGLE

FIGURE 6.8



NOS IN BRACKETS INDICATE ORDER OF HARMONIC


EFFECT OF SKEW OVER ASYN. MAX. TORQUES
FIGURE 6.9A
(1)
(5)
(19)
(25)
1·0
·8
·6
·4
·2
UNIT TORQUE
REDUCTION OF SYN. TORQUES WITH SKEW
FIGURE 6.9 B
20
40
60
80
90
ANGLE OF SKEW
10
20
30
40
50
60
70
80
90
ANGLE OF SKEW

CHAPTER VII

EXPERIMENTAL RESULTS

In addition to the tests as already specified under section 2.9, some tests over the test machine, with rotor bar skewed (6° Elect) are conducted to check the validity of the calculated performance curves equations and equivalent circuit obtained in previous Chapter.

The specification of the motor is:

440 V, 5 Amps, 1440 r.p.m., 3 H.P., 50 c/s.

3 Phase, skewed bar induction motor.

Though calculations for unskewed rotor, induction motor are also done but due to practical difficulties, unskewed motor could not be made available so observations for skewed motor only are presented.

7.1. Measurement of Resistances:

7.1.1. Measurement of Primary Resistances:

The primary resistance is measured using Kelvin's Double bridge. Since resistance is function of temperature, the measurements are carried out while the machine is hot and these hot values are used in calculation of machine characteristics.

R_1 = 5.1 Ohm.

7.1.2. Secondary Resistance:

The effective (referred to primary) rotor resistance is found by usual short-circuit test and using the value computed of R_1.

R_2 (referred to primary) = 1.93 Ohm.

7.2. TORQUE - SPEED CURVE:

By simple loading, the full range speed- torque curve can not be determined, since motor cannot develop more than the maximum torque and after that motor enters the unstable region. For this Ward Leonard speed control is adopted. In this the test machine is run by seperately excited d.c. machine, the speed of which is controlled by another seperately excited d.c. machine coupled to constant speed (syncronous motor) motor - This way speed from zero to syncronous is obtained. For accuracy sake, speed is recorded by sabatroscope.

Since machine draws very high current at low speed and inverse rotation and cannot withstand it for longer time so proper arrangement for extra cooling are made.

7.3. RECORDING OF CURRENT CHARACTERISTIC

The current slip characteristic is recorded over the C.R.O. using the circuit shown in Fig. 7.4. For speed signal, a tachogenerator generating voltage proportional to speed is used and current being alternating is fed through a rectifier bridge to the y - Plate of the oscilloscope beam. The inductance in speed signal and current signal circuits are meant for filtering any undesirable alterations caused due to vibration etc. For the change in speed corresponding change in current is recorded. (See Photograph Fig. 7.5).

DISCUSSION OF THE RESULTS:

The evidence of fairly good agreement between experimental and theoritical results of current, power factors torque of skewed rotor induction machine justifies the belief that not only is the theory of harmonics and skew within its inherent limitations basically sound but also that the methods, used for measurement of machine parameters are fairly accurate.

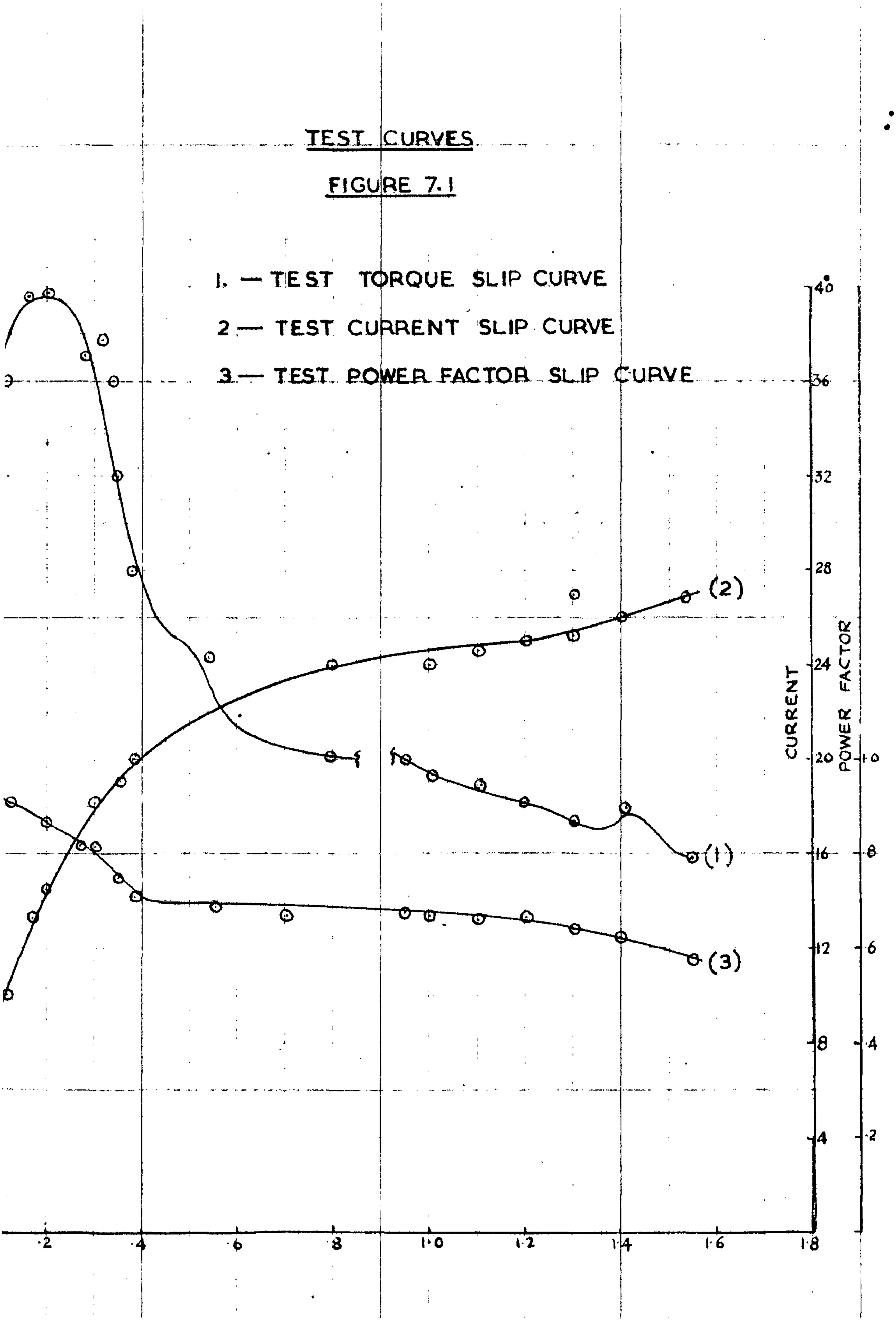
The discrepancy between the calculated and experimental curves may be due to the following reasons.

1. In calculating, it has been assumed that all the parameters are constant, but actually some parameters are dependent on the operating conditions (e.g., leakage reactance on current etc)
2. The Iron loss occuring in the motor could not be properly accounted.
3. Throughout, the frictional losses of d.c. machine has been neglected and that of induction machine assumed constant irrespective of speed.
4. Instrument errors and errors in recording might have introduced some deviation in the calculated and observed results.

—

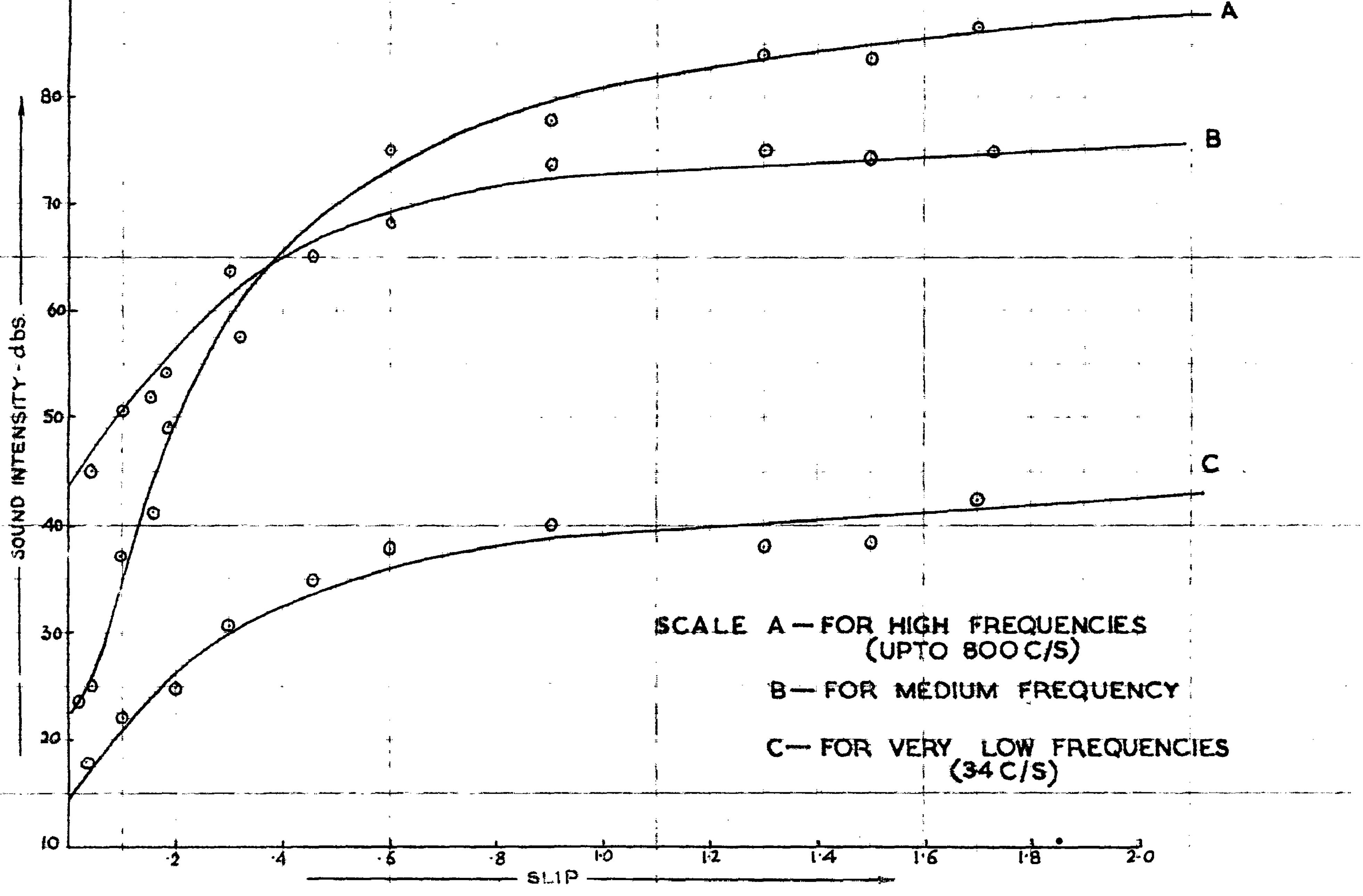
Table 7.1. PERFORMANCE TEST OBSERVATIONS

Slip	speed	Current	Watts.	Voltage	P.f.	Stator Copper loss	(per phase) Power developed	N.L.
.03	1470	5.1	1276.0	440	0.95	81.5	1000	195
.05	1420	6.4	1502	440	0.93	127.0	1180	195
.08	1380	8.9	2085	430	0.92	294.0	1590	195
.12	1320	10.4	2391	430	0.92	410.0	1800	181
.17	1240	13.5	3085	428	0.89	835.0	1970	181
.20	1200	14.6	3190	426	0.87	1060	1930	180
.27	1100	16.8	3450	425	0.82	1430	1830	180
.30	1020	18.0	3680	425	0.82	1620	1680	180
.34	990	19.0	3830	420	0.83	1850	1600	180
.38	975	19.0	3630	440	0.75	1850	1600	180
.38	930	20.0	3635	440	0.715	2040	1400	195
.53	790	22.0	3865	440	0.69	2460	1210	195
.55	675	22.0	3745	440	0.67	2460	1090	195
.8	300	24.0	4125	440	.675	2940	990	195
1.0	0	24.0	-	440	-	2940	-	195
1.1	1350	24.0	4135	440	.675	2940	1000	195
1.2	1200	24.0	4095	440	.67	2940	960	195
1.3	1050	24.4	4050	430	.60	2940	935	195
1.4	900	24.9	4255	435	.67	3160	900	195
1.5	750	85.3	4075	435	.64	3020	860	195
1.6	600	26.0	4105	455	.63	2920	990	195
1.7	375	27.0	3900	430	.58	3000	785	195


TEST CURVES
FIGURE 7.1
1. — TEST TORQUE SLIP CURVE
2 — TEST CURRENT SLIP CURVE
3 — TEST POWER FACTOR SLIP CURVE
(1)
(2)
(3)
CURRENT
POWER FACTOR
40
36
32
28
24
20
16
12
8
4
1·0
·8
·6
·4
·2
·2
·4
·6
·8
1·0
1·2
1·4
1·6
1·8

SOUND INTENSITY CURVES

FIGURE 7.2



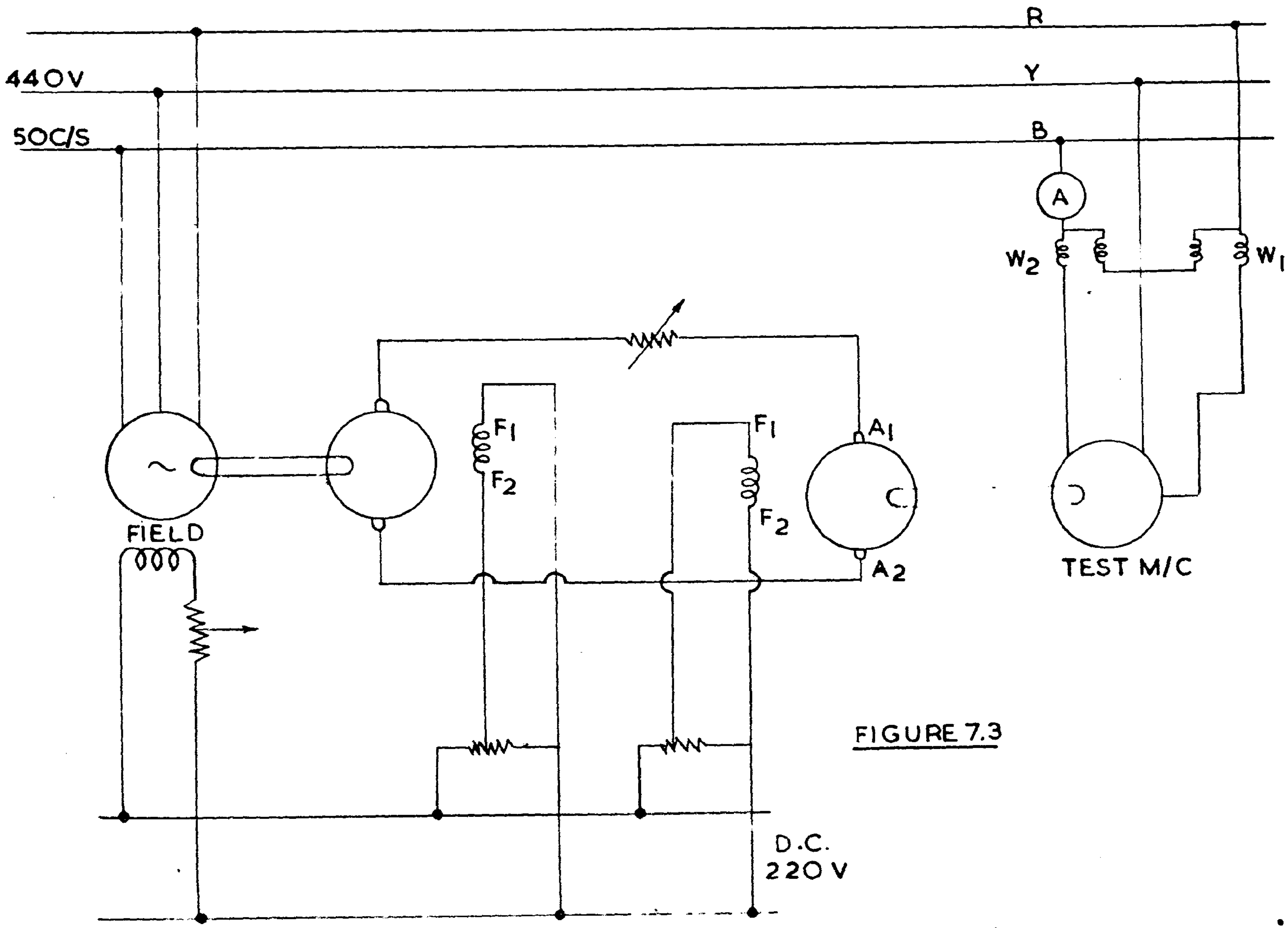


FIGURE 7.3

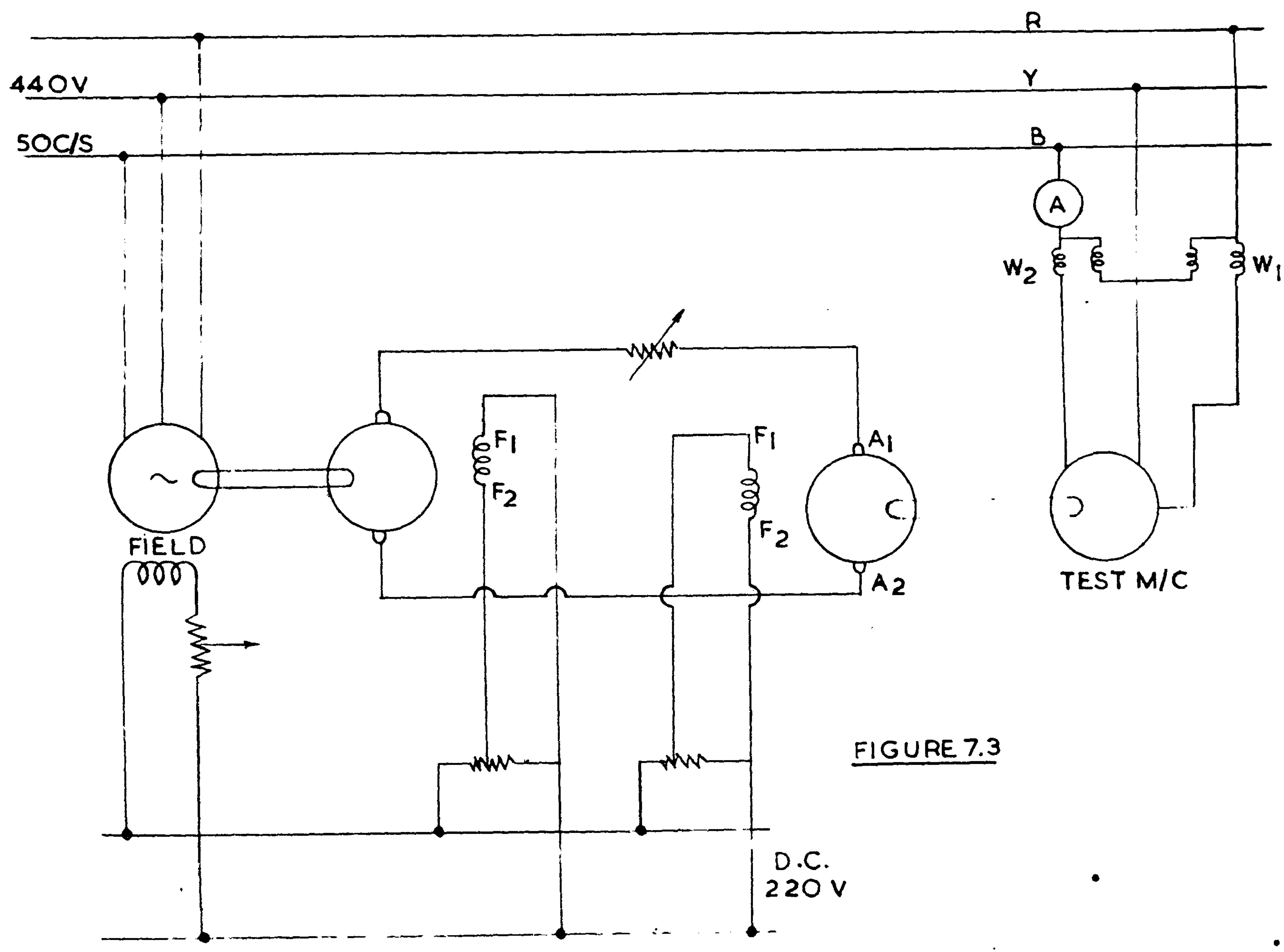
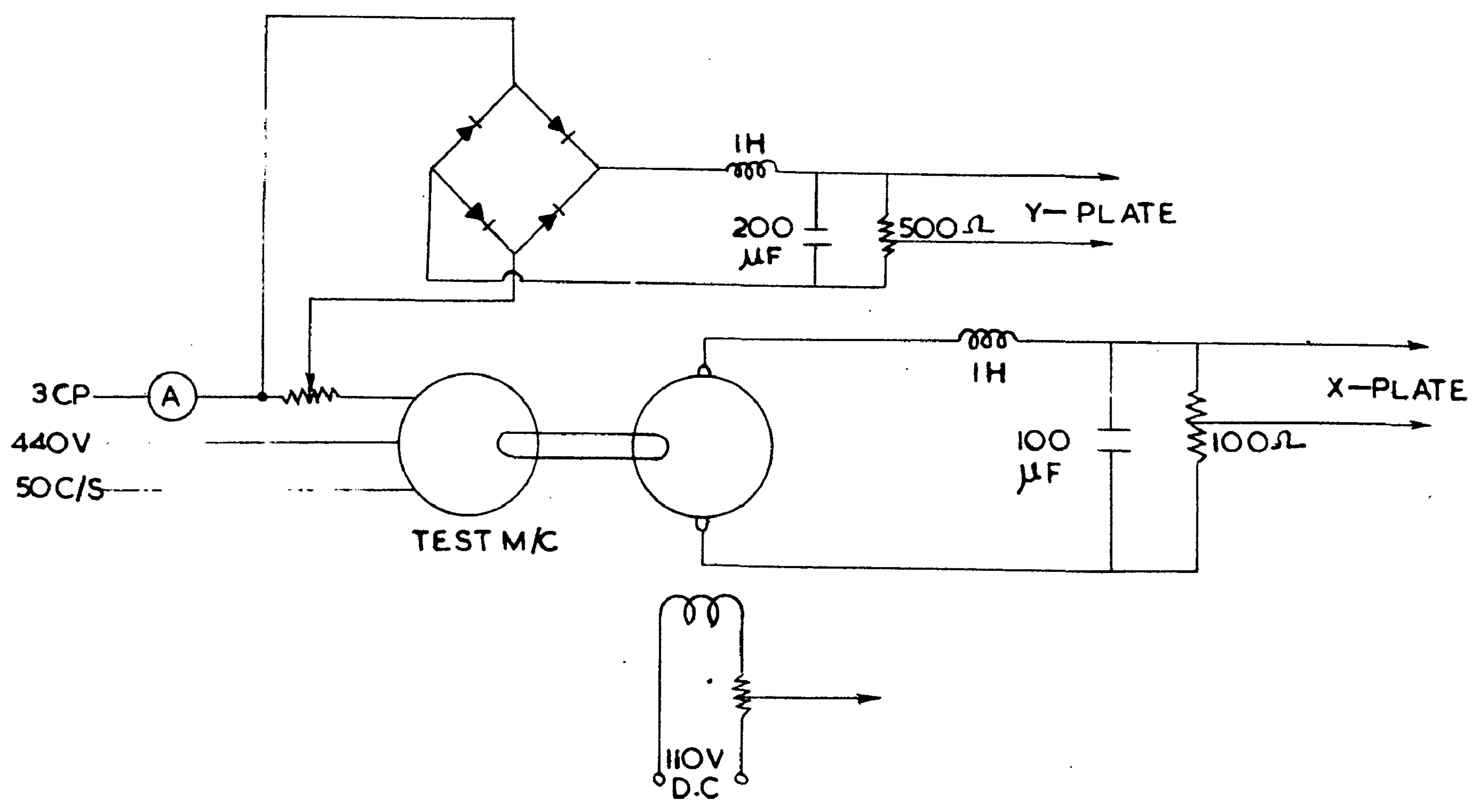
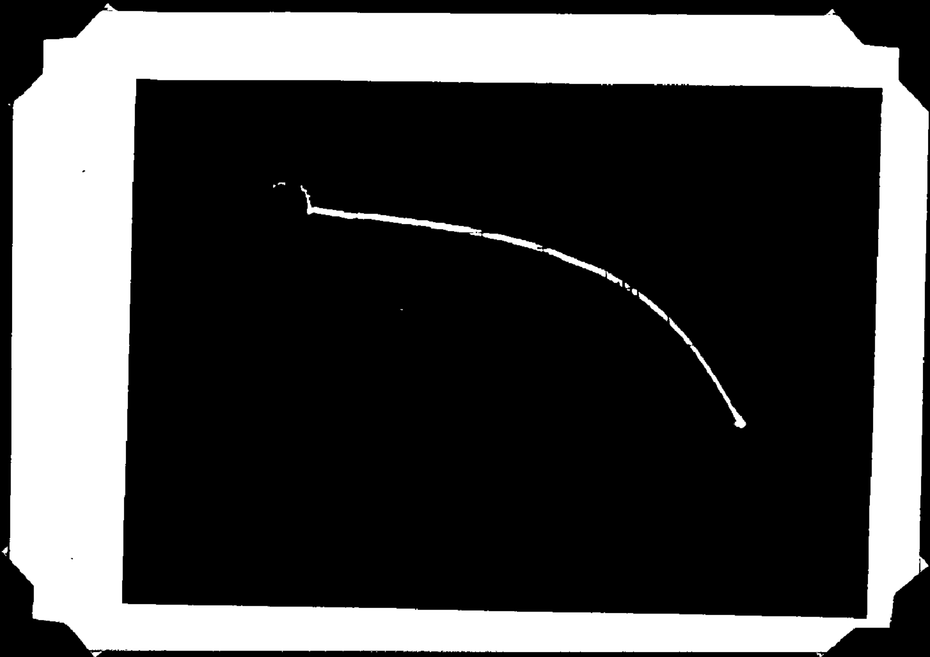


FIGURE 7.3

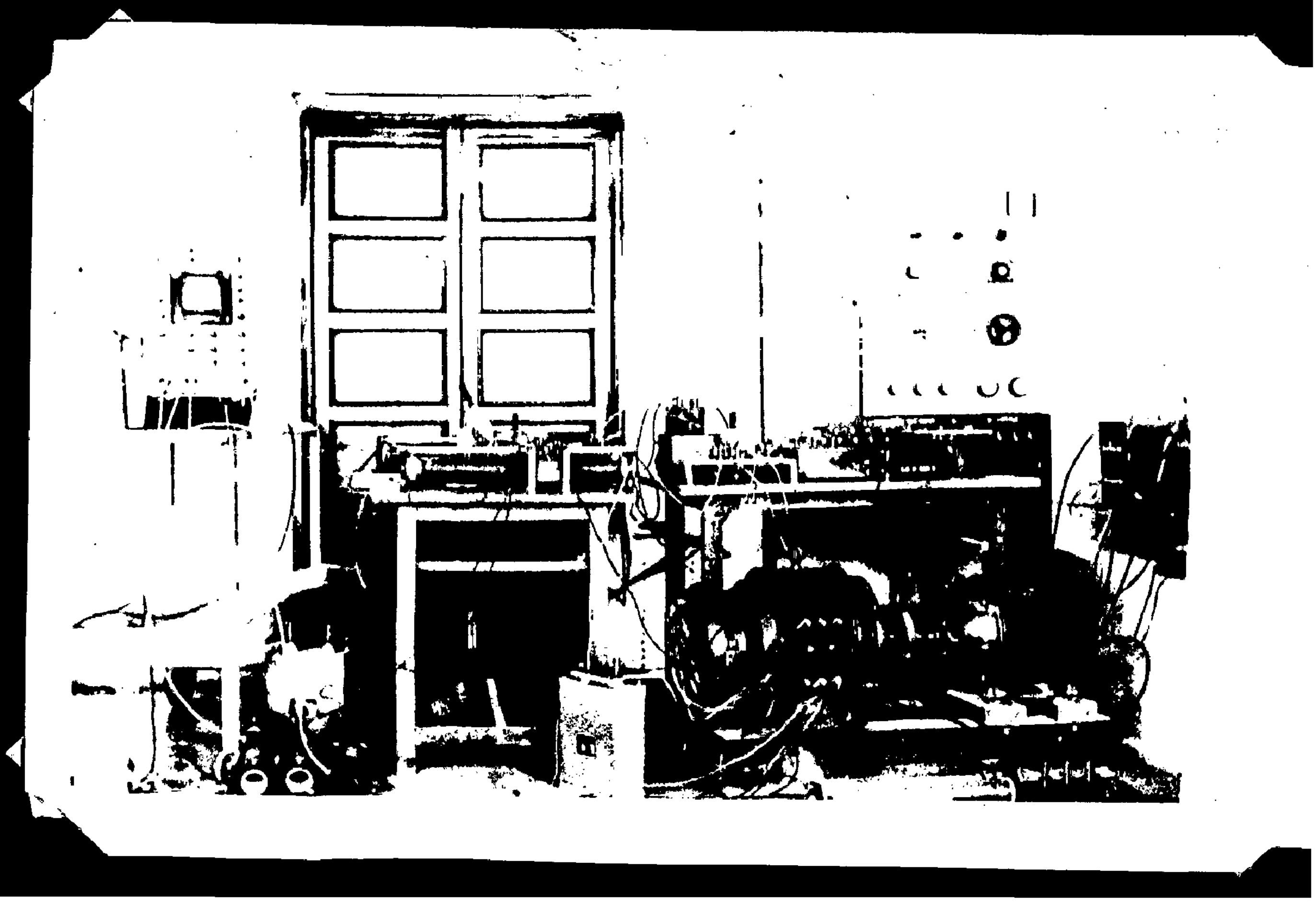


CIRCUIT FOR CURRENT RECORDING

FIGURE 7.4



Speed/current curve
(speed Zero to Syn)



C O N C L U S I O N

The performance of a induction machine is considerably affected by the skewing of rotor bars. The fact is revealed by the comparison of the figures 3.4 to 3.10.

The skewing effects on the constants of the machine are very important as detailed out in theoritical consideration of Chapter II. Except for certain portion of leakage (slot leakage) all the other (harmonic differential leakage) rotor circuit constants are inversely proportional to the square of skew factor.

The equivalent circuit of Fig. 2.14, 2.15 provides an accurate picture of the harmonic fields phenomena and of skewing effect in a squirrel cage motor. Comparison of calculated and test results confirm the usefulness of this circuit in theoritical prediction.

The two sets of calculations carried out with and without skew angle of the rotor bar shows, that spiralling or skewing, of rotor bar of an induction motor increases the reactance and affects the performance e.g. reduction in starting current; reduction in dips, peaks of torque - speed curve; reduction in cusps, improvement in power factor; reduction in maximum torque; increase in starting torque, decrease in efficiency and increase in stray load losses; besides reducing the magnetic noise and vibrations. Methods for clearly visualising and calculating its effects have been provided which should be immediately useful in machine design.

It is shown that some motors, with good relation of number of slots can as well be adopted without skew as in case of test machine or any harmful harmonic can be skewed out by providing a proper skew (Fig. 6.? A, B).

As discussed in Chapter VI, the parasitic torques and noise can be further reduced by skewing more the rotor bars. After a certain limit the performance ceases to make any substantial progress but, increases the cost of production quite comparatively.

The test machine is found to be 'Quiet' in working zone(high speed) and 'Noisy' at low speeds, producing high frequency noise. The noise being the complex function of magnetic fields and circuit constants, the exact behaviour of particular harmonic for variable skew cannot be predicted. However when noise level of 19th and 25th harmonic decreases with skew, the noise level of 5th harmonic increases considerably. So a compromisive value of skew is to be selected in such cases.

It is shown (Fig. 6.8, & 6.9) that skewing of 5° to 20° (Elect.) is most proper range.

—

APPENDIX I

I-1 AIR GAP PERMEANCE EQUATIONS FROM CONFORMAL TRANSFORMATION THEORY

Equations derived from conformal-transformation theory have been used by E.M. Freeman[9] to calculate the shape and hence the harmonic content of tooth ripple flux -density wave from large values for any value of s/g and $0 < s/\lambda < 0.8$ with the help of electronic computer.

The conformal - transformation holds good under the follo-wing assumptions:

a. Curvatures of the surfaces of air gap is neglected.

b. Depth of the slot has so little influence on the results that it can be considered infinite, this assumption is reasonable if $d > 1.65$ See Fig. (I-3).

Fig. (I-2) indicates values of Z - plane. The value 'a' is arbitarily fixed but 'b' is not independent of a.

After going through the conformal transformation from one plane to another the flux density distribution in a plane is given by the following equation (See Gibb's book)

$$B = \frac{(1 - w)\ B_{max}}{\sqrt{(a-w)\ (b-w)}}$$

where w is the independent variable $a = 1/b$ and b is determined by the equation

$$\frac{b - 1}{b^{\frac{1}{2}}} = \frac{s}{g}$$

The distance along the pole face is given in the form

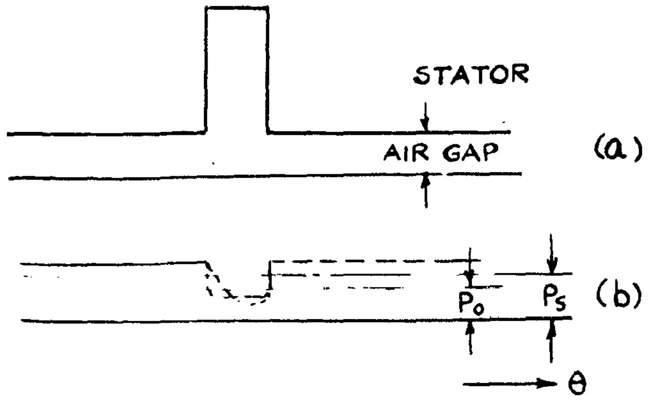


FIGURE I-1

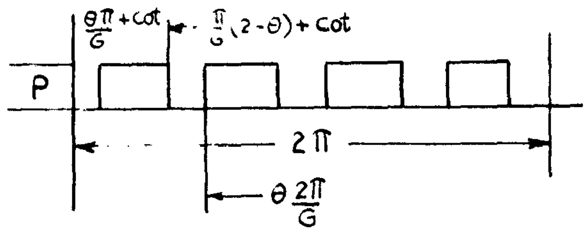


FIGURE I-2

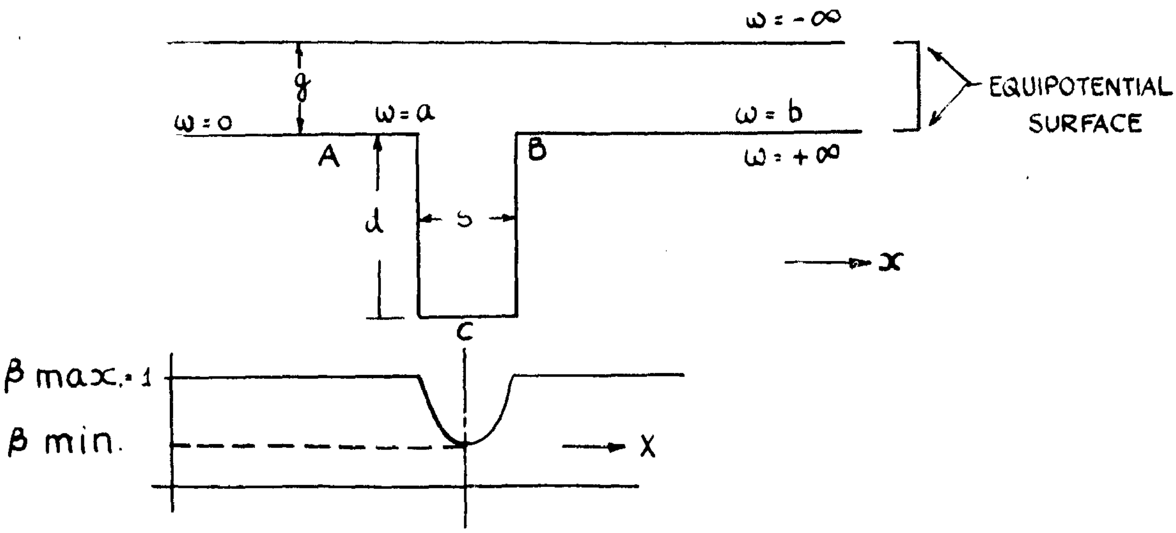


FIGURE I-3

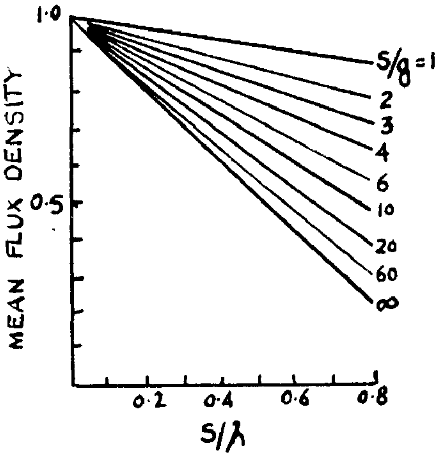


FIGURE I—4

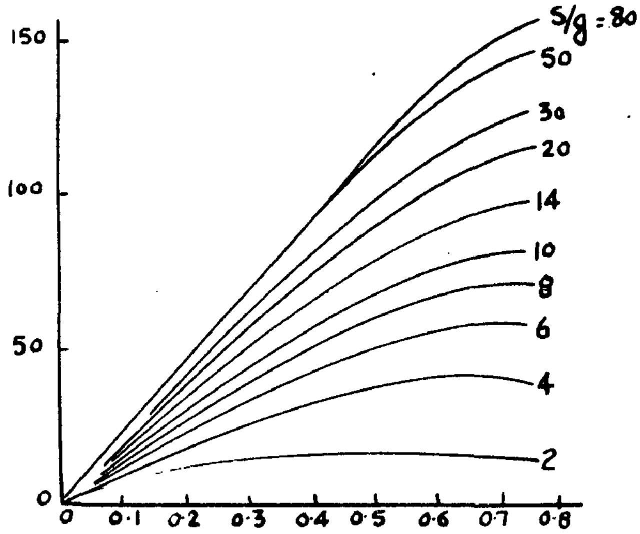


FIGURE I—5

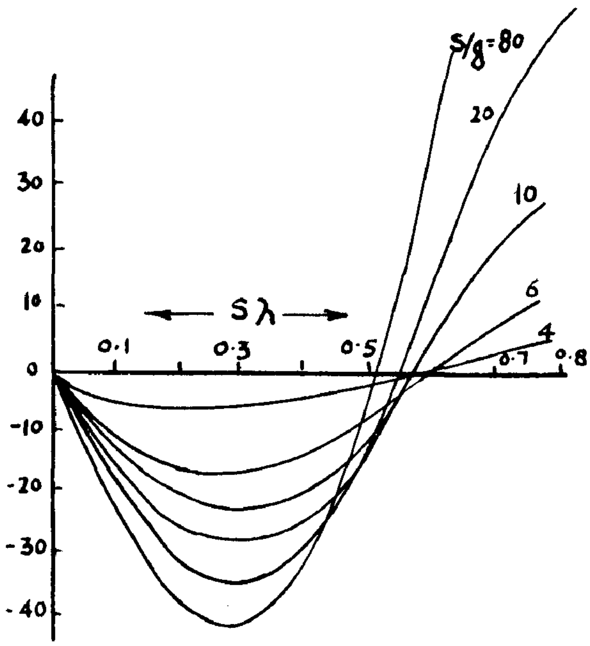


FIGURE I—6

$$x = \frac{g}{w} \left[+ \log_e \left| \frac{1+p}{1-p} \right| - \log \left| \frac{b+p}{b-p} \right| - \frac{2a}{g} \tan^{-1} \frac{p}{b^{\frac{1}{2}}} \right]$$

which is modified to

$$x = \frac{g}{w} \left[- \log_e \left| \frac{1+p}{1-p} \right| + \log \left| \frac{b+p}{b-p} \right| + \frac{2a}{g} \tan^{-1} \left(\frac{p}{b^{\frac{1}{2}}} \right) \right] - 0.55$$

where $p^2 = \dfrac{b-w}{a-w}$

The flux density curves taking B_{max} unity have been calculated for different value of w beginning from -1 to smaller values. The values of B and x thus computed are then used in a harmonic analysis sub-routine to calculate the mean value of th e flux density and the amplitudes of harmonics. The equation of flux density wave from may be written from section I-1 and I-2 as

$$B = \bar{B} \left(1 + \sum_{r=1} v_r \operatorname{Cos} vR (x - wt) \right)$$

where $\bar{B}$ = average mean flux density.

The extensive results obtained by Freeman are reproduced here.

APPENDIX II

(TO CHAPTER III)

II-1 HARMONIC LEAKAGE REACTANCE OF SQUIRREL CAGE WINDING WITH RESPECT TO THE HARMONIC

From equation (1-19) the amplitude of mth mmf wave is

$$F(m) = 0.9 \frac{k}{m} \left(\mathcal{T}P \frac{k_{wm}}{P} \right) I_1$$

If $m' = m \times p$ and $B(m') = .45\, m_1 N_1 \left(\frac{k_{w1m}}{m'} \right) \frac{3.19}{l'_g} I_1$

The corresponding flux is

$$\phi(m) = \frac{2}{\pi} \frac{\mathcal{T}}{m'} L_s B_{m'}$$

It links with $N_1\, k_{w1m}$ turns so flux linkage is

$$\psi_{m'} = \phi_{(m)} N_1 k_{w1m}$$

But harmonic leakage reactance is

$$X_{s1(m')} = 2 \pi f \frac{\psi_{m'}}{2^{\frac{1}{2}} I_1}$$

$$\therefore X_{s1(m')} = 2 \pi f_1 \frac{1}{2^{\frac{1}{2}} I_1} \sum \psi_{m'} \times 10^{-8} \text{ Ohm / Phase.}$$

After simplifying it

$$X_{s1(m')} = \frac{3.2\, m_1\, f_1 N_1^2\, \mathcal{T} L_s \times 2.54 \times 10^{-8}}{P\, l'_g} \sum \left(\frac{k_{w1m}}{m'} \right)$$

Ohms / phase.

Similarly for Rotor winding $m_2 = Q_2$; $N_2 = \frac{1}{2}$ $k_{w2m} = 1$

$$X'_{m2(m')} = \frac{0.8R\, f_1 \tau L_e}{P\, l'_g} K^2_{sm} \times 2.54 \times 10^{-8} \sum_{v_2 \neq 0} \frac{1}{\left(\frac{v_2 R}{p} + 1\right)^2} .$$

Ohm per bar.

Again the consideration of the magnetic energy in the gap yields a simple expression for the sum, as found by Liwschitz - Garik

if $\frac{2}{3} \leq (W/\tau) \leq 1$

and

$$\left(\frac{k_{w1m}}{m}\right)^2 = \frac{\pi^2}{18} \; \frac{(5q^2+1) - \left\{3q(e-q) + \frac{2}{3}\left[(e-q)/q\right] + (e-q)^2 - \frac{2}{3}\left[(e-q)^2/q\right]\right\}}{4q^2} - k^2_{w1}$$

Where e = chording angle of coil w = coil span = Pole pitch

q = Number of slots per pole per phase.

and also

$$\sum \left[\frac{1}{\left(\frac{v_2 R}{p} + m\right)}\right]^2 = \frac{1}{m'^2}\left[\frac{1}{\xi^2_m K^2 Sk} - 1\right]$$

$$\xi_m = \frac{\sin \frac{mp\pi}{R}}{\frac{m\pi p}{R}}$$

$$\therefore\; X'_{m2(m)} = 0.80Rf_1 \frac{L_e \tau}{P\, l'_g} K^2_{sm} \times 2.54 \times 10^{-8}\left[\frac{(mp\pi)^2}{(R\, K_{SM})^2} \cos^2 \frac{mp\pi}{R} - 1\right] \frac{1}{m^2}$$

Referring it to primary side

$$X_{s2(m)} = \left\{\left[\frac{m\ p^{\pi}}{R\ K_{Sm}}\ \csc\ \frac{m\ p^{\pi}}{R}\right]^2 - 1\right\} \frac{X_{M(m)}}{} \qquad (II - 1)$$

Considering the harmonic chart (T (1-3)), the mth stator harmonic for example $m = 5$ produces row of rotor harmonics $n = 5, 29, 39, 63, 73$ ----- for $v_2 = 0, \pm 1, \pm 2$ etc. It will be shown in Chapter IV that any harmonic m produces with corresponding harmonic n, when $v_2=0$ a torque characteristic of the same shape as main wave., With respect to it, all harmonic pairs all produced for diffeent value of v_2 constitute the harmonic leakage. Therefore directly the harmonic leakage reactance for permeance wave can be written as

$$X_{s_2(mp)} = \left\{\left[\frac{(mp)p^{\pi}}{S\ K_{Sm}}\ \csc\ \frac{(mp)\ p^{\pi}}{S}\right]^2 - 1\right\} \frac{X_{M(mp)}}{mp^2} \qquad (II-2)$$

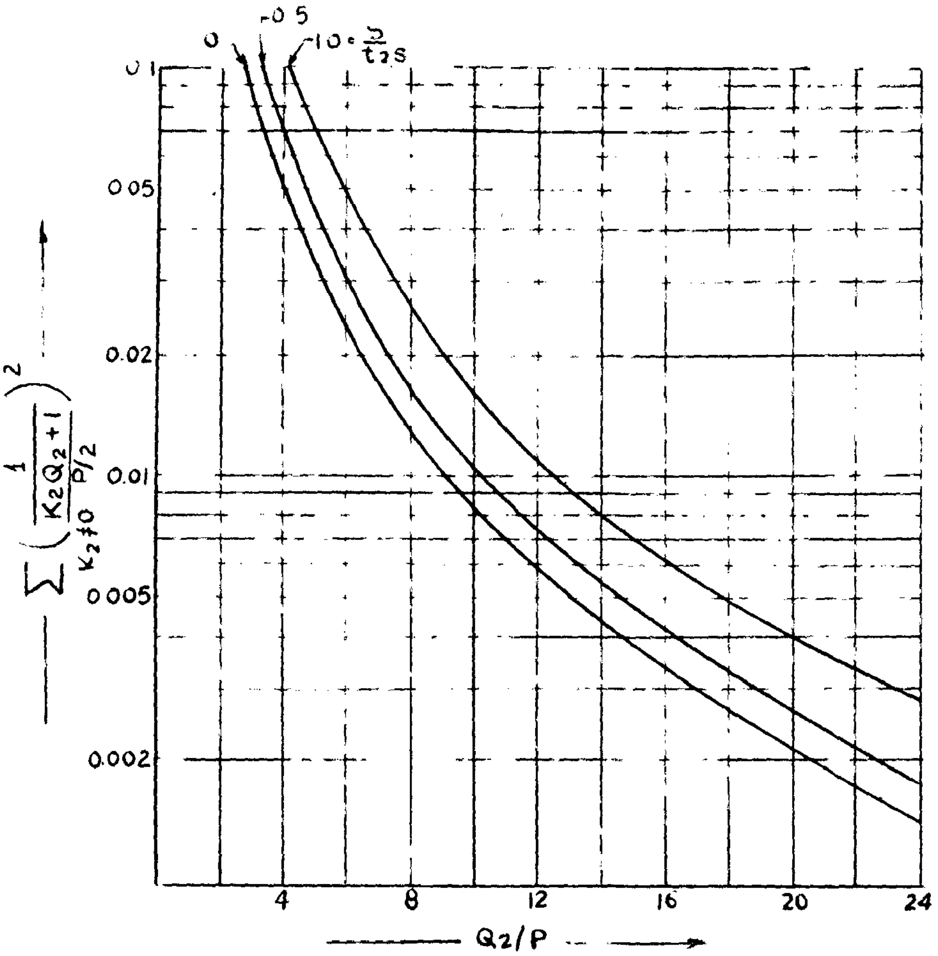
II-2 SKIN EFFECT IN BARS OF SQUIRREL CAGE ROTORS:

The aim is to give an analytical solution for the skin-effect in the most used squirrel cage bars. This interests more to us, as our problem is directly linked with calculation of Parasitic torques. For this purpose the author is reproducing some of the recent work done by Licoschitz[15] over straight simple bars of squirrel cage machine.

For resistance:

$$\frac{\text{a.c. resistance}}{\text{d.c. resistance}} = \frac{r_{ac}}{r_{dc}} = \phi\ (A)$$

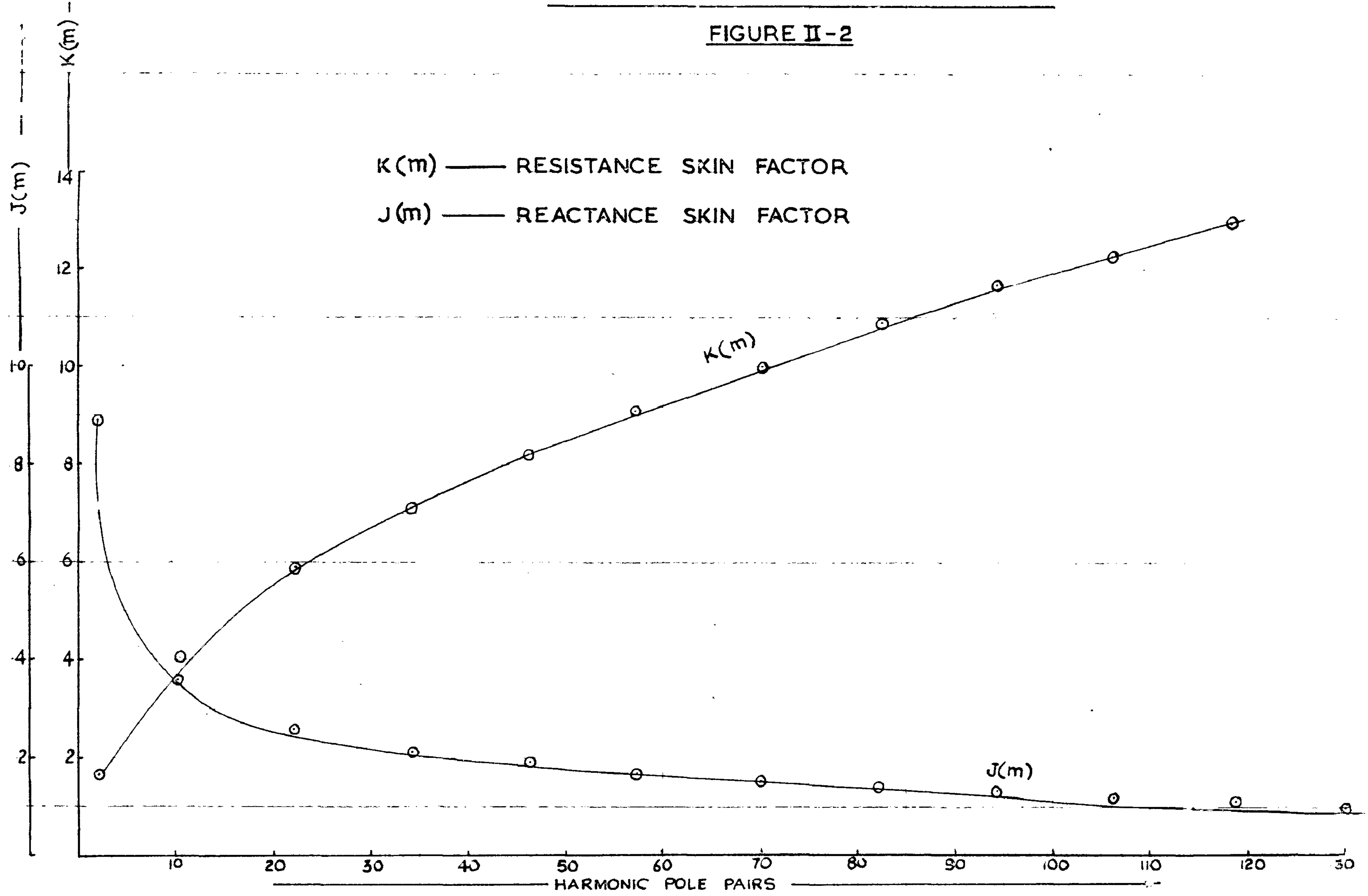
$$\text{Where } \phi\ A = A\ \frac{\operatorname{Sin h} 2A + \operatorname{Sin} 2A}{\operatorname{Cos h} 2A - \operatorname{Cos} 2A} \qquad (II-4)$$



HARMONIC DIFFERENTIAL LEAKAGE CURVE OF A SQUIRREL CAGE WINDING

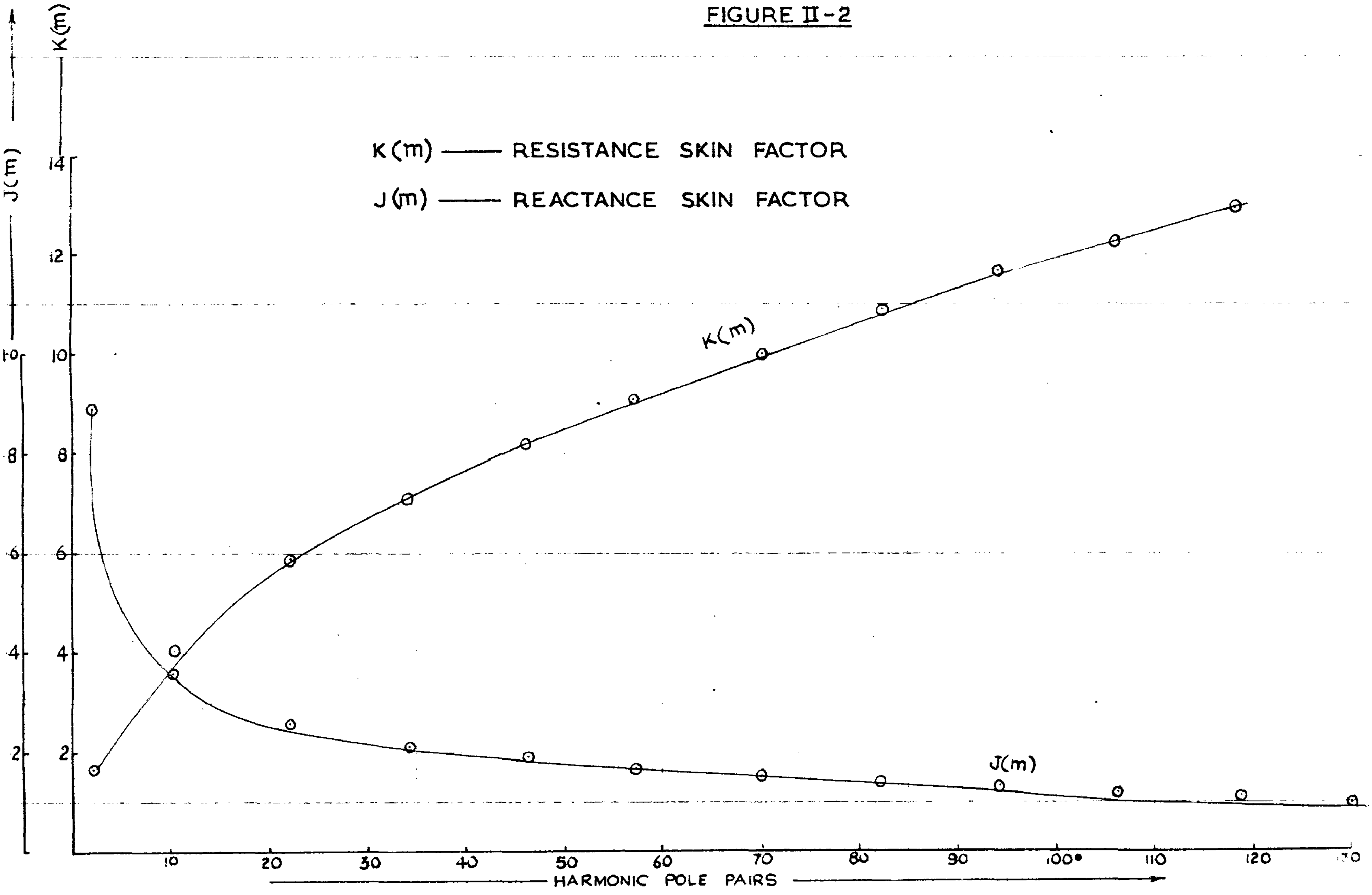
FIGURE II — I

FIGURE II-2



K(m) —— RESISTANCE SKIN FACTOR
J(m) —— REACTANCE SKIN FACTOR
K(m)
J(m)
HARMONIC POLE PAIRS
10
20
30
40
50
60
70
80
90
100
110
120
130
2
4
6
8
10
12
14

FIGURE II-2


K(m) —— RESISTANCE SKIN FACTOR
J(m) —— REACTANCE SKIN FACTOR
K(m)
J(m)
HARMONIC POLE PAIRS
10
20
30
40
50
60
70
80
90
100
110
120
2
4
6
8
10
12
14

and

$$\frac{l_{ac}}{l_{dc}} = (A) = \frac{\text{a.c. inductance}}{\text{d.c. inductance}} \text{ of bar.}$$

$$\text{When } (A) = \frac{3 \text{ Sin h } 2A - \text{Sin } 2A}{2 A \text{ Cos h } 2A - \text{Cos } 2A} \qquad \text{(II-5)}$$

$$\text{When } A = 0.316 \text{ h} \sqrt{\frac{bb}{bs} \frac{f_1}{P} \Big/ S_m} \qquad \text{(II-6)}$$

where

bb = width of bar

bs = width of slot

f_1 = line frequency

h = Height of bar

S_m = Slip of the rotor of mth wave.

P = Bar resistivity micro ohms per indh. putting the following values.

bb = 0.2 cms.

bs = 0.2 cms (approx.)

f_1 = 50 cPS

h = 2.0 cms.

S = Unity (for fundamental)

Values are tabulated in Table (II-1)

Table II-1 HARMONIC SKIN FACTORS FOR ROTOR RESISTANCE AND REACTANCE

m	1	5	7	11	13	17	19	23	25	29	31
$k_{(m)}$	1.69	4.1	4.1	5.8	5.8	7.1	7.1	8.2	8.2	9.2	9.2
(m)	.895	.350	.36	.258	.258	.21	.21	119	.18	.16	.16

m	35	37	41	43	47	49	53	55	59	61	65
$k_{(m)}$	10.0	10.0	10.9	10.9	11.7	11.7	12.3	12.3	13.0	13.0	13.5
(m)	.15	.15	.15	.138	.138	.13	.13	.122	.122	.115	.105

APPENDIX III

SAMPLE COMPUTER PROGRAMMES ARE PRESENTED HERE

III - 1 SOLUTION OF CIRCUIT

```
C  C      OPG
          DIMENSION  R(30), G(30), H(30),  T(30)
          DO  37  J  = 1,5     $   J2   = 3 * J       $ J 1  = J2 -2
37        READ  38, (R(K),G(K),H(K))  K = J1,J2 )
38        FORMAT  ( 3 (F 7.3, F7.3, F7.3))
          READ   39, V
39        FORMAT  (F 7.3)
          S  = 0.05   $
200        T(1)   =  S     $  T(2)  =  6.-5* s     $ T(3)  = 7.*s - 6.
           T(4)  = 12. - 11.*s     $  T(5)  =  13.*s - 12  $ T(6) = 18.-17.*s
           T(7)  = 19.*s = 18.  $ T(8)  = 24.-23.*s  $ T(9)  = 25.*s -24.
          T(10)   = 24.-23.*s    $ T(11) = 25.*s -24. $ T(12) = 48.-47.*s
          T(13  = 49.*s - 48.
           X1  = 5.0
           Y1  = 3.9
          DO  41  I  = 1, 13    $  A  = R(I) /T(I)  $ B  = G(I) + H(I)
           C  = A * A + B * B   $  X  = A * GI * GI/C
          Y  = (A*A*G(I)) +G(I)*H(I)*B/C
           X1 = X1+X   $  Y 1  =  Y1 + y    $
41         D  = SQRTF (X1 *X1*Y1*Y1)
           E  = V/D   $  F  =  X1/D
           PUNCH  60, S, X1 , Y1 , E , F.
```

Appendix III

```
60      FORMAT  (F4.2, 4 F 10.4)
        S = S + 0.05   $ IF (S-2.0) 200,200,201
201     STOP
        END
```

DATAS

1.930	141.250	2.830	.600	0.260	0.054	0.460	0.077	0.038
0.520	0.020	0.020	0.520	0.014	0.020	0.0750	0.013	0.035
1.0	0.018	0.070	0.0	0.261	1.880	6.0	0.221	2.480
1.930	0.261	01.380	1.930	0.221	1.980	1.930	0.062	1.360
1.930	0.058	1.160						

V = 254

III -2 SYN. TORQUES WITHOUT SKEW

```
CC      O.P.GARG  - SYN. TORQUES. W.O. SKEW
        DIMENSION R(20, G(20, H(20, Z(25) , P (25) L(20, T(20, E(20)
        DO  37  J  =  1,5   $ J2  = 3 *J   $  J1  = J2-2
37      READ 38, (R(K),G(K),H(K),K = J1, J2)
38      FORMAT  (3 (3F 7.0))
39      READ  36, (L(I)   I = 1,.15)
36       FORMAT (15I 3)
        DO 49  J  = 1,2  $ J2  = 11*J  $ J1  =  J2 - 10
49      READ  48 , (Z(I), I  = J1 , J2)
48       FORMAT  ( 7 (2F 4.0) )
         S  = 1.06
```

Appendix III

```
10 2  T(1) = S  $ T(2) = 6. -5.*S     $ T(3) = 7. *S-6.
      T(4) = 12. -11.*S    $ T( 5) = 13. * S-12.    $ T(6) =18.-1S.*S
      T(7) = 19.*S-18.   $ T(8) = 24.-23.*S    $ T(9) = 25.*S-24.
      T(10) = 30.-29.*S  $ T(11) = 31.*S-30.   $ T(12) =36.-35.*S  .
      T(13) = 37.*S-36.  $ T(14) 42.-41.*S     $ T(15) =43.*S-42.
      DO 40, I = 1,15 ,  $ K = LX (I)  $ AK = K  $ A = R(I)/T(I)
      B = G(E) + H(I)   $ C = A*A +B*B   $ D = G(I)/SQRTF(C)
40    E(I) = 1.91 x10^5 * Z(I) *Z(K)*D /AK
      E(16) = 0.0
      DO 60    J = 1,2  $ J2 = 8*J  $ J1 = J2 -7
60    PUNCH 61, S, (E(K) , K = J1, J2)
61    FORMAT (F 4.2, 8 F7.2)
      IF (S - 0.97) 100,100,101
      101 S = 0.97
      GO TO 102
      100 STOP
      END.
```

DATAS

R(K) G(K) , H(K)

1.930	141.250	2.830	0.600	0.260	0.054	0.460	0.077	0.038
0.520	0.020	0.020	0.520	0.014	0.020	0.750	0.013	0.035
1.0	0.018	0.070	8.0	0.261	1.880	6.0	0.221	2.480
1.330	0.008	0.298	01.020	0.004	0.430	0.900	0.002	1.887
0.900	0.002	0.255	1.210	0.002	0.100	1.580	0.004	0.114

Z(I)

0.955	.205	.157	.126	.126	.157	.205	.955
.955	.205	.57	.126	.126	.157	.2.5	.955
.205	.126	.157	.955	.205			

L(I)

012 010 014 008 016 0[illegible] 017 004 018 002 019 001 020 003
021

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