TRANSIENT & STEADY STATE PERFORMANCES OF AMPLIDYNE

By
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C E R T I F I C A T E

CERTIFIED that the dissertation entitled " TRANSIENT AND STEADY STATE PERFORMANCE OF AMPLIDYNE" which is being submitted by Sri Munish Bhargava in partial fulfilment for the award of Degree of Master of Engineering in "Advanced Electrical Machines" of University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of eight months, from January 1965 to August , 1965 for preparing thesis for Master of Engineering Degree at the University.

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A C K N O W L E D G E M E N T S

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Munish Bhargava.

(Munish Bhargava)

SYNOPSIS

The dissertation mainly deals with the Describing Function analysis of the effects of saturation and hysteresis in amplidyne voltage regulation system. The input output characteristic incorporating the non-linearity has been represented by various mathematical expressions for the derivation of the describing function. The Nyquist diagrams have been plotted to study the stability of the system.

It also describes the steady state and transient characteristics of the machine.

To the best of the knowledge of the author the following analysis approaches are original :

1) Use of modified bessel function in the derivation of describing function for saturation by an exponential rise and derivation of Describing Function therefrom.

2) Representing of Hysteresis loop of the machine considering the non-linear brush contact resistance and derivation of Describing Function.

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NOTATIONS

A	Amplidyne.
C	Capacitance.
E	Voltage.
G	Generator
G(s)	Transfer Function.
$G_d(s)$	Describing Function.
I'	Moment of Inertia,
I	Current
K	Slope of the Hysteresis Loop.
L	Inductance
M	Mutual Inductance
N	Speed in Revolution per Minute.
R	Resistance
T(s)	Transfer function of Feedback Path.
W	Power
Z	Impedance Matrix
a	Constant
b	Constant
i	Current
j	$= \sqrt{-1}$
p	$\frac{d}{dt}$
f	Frequency
α, β, γ & θ	Angles

C O N T E N T S

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CHAPTER 1

INTRODUCTION .

INTRODUCTION

Industrial revolution brought a great improvement in the availability of power for the use of mankind. To utilize this power effectively and economically, it became necessary to learn how to control and regulate this vast power. This led the way to a consolidated effort at devicing servomechanisms to suit the requirements and resulted in very substantial contributions in this field. Of comparable importance are the development of improved power sources, error indicators and other components for supplying the practical needs of the servomechanism designer. Typical of these are such devices as the Amplidyne for obtaining a high gain rapid-response d-c generator.

1.1. HISTORICAL DEVELOPMENT:

The word Amplidyne is a trade name used for a special type of direct current generator known as Cross-field generator or Armature reaction generator. The first of this kind was the Resenberg-generator[(1)] originally developed in 1905 for train lighting service. The Metadyne, the direct antecedent of the Amplidyne was developed in France around 1930 by J.M.Pestarini.

Pestarini[(2)] defined Metadyne as a generalized d.c, machine consisting of a d.c. armature and commutator, any number of field poles each of which may be excited in any manner, and any number of brushes arranged to bear on the commutator in position for which satisfactory communication can be obtained. According to this definition all rotating d.c. machines including the ordinary d.c. generator are metadynes. But generally the term

"Metadyne" refers only to those forms of d.c machines which convert constant potential energy in-to constant current energy. Then it was around 1938 that Edwards and Alexanderson of the General Electric Co. of America introduced Amplidyne as a machine having full compensation and thus providing a constant voltage characteristic together with the greatest possible amplification.

1.2. WORKING PRINCIPLE:

Amplidyne is basically a d.c. generator and may be regarded as a two stage generator but both stages using same armature winding. The working principle of which can be explained as follows :

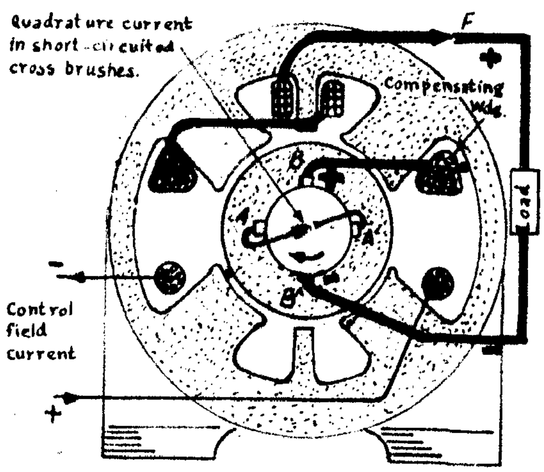


FIG. 1_ AMPLIDYNE

The generator rotor which is wound similar to a d.c. motor is driven at constant speed by means of a suitable motor. Four brushes are placed on the commutator at AA' and BB' (Fig.1)[3]. The control field is excited by the control voltage. (corresponding to which a current i_c . and a flux ϕ_d is carried by d-axis). Since the armature winding is rotating at full speed, an e.m.f. eq. is generated between the brushes AA'. By short circuiting these brushes and using an armature winding resistance of low value, very large values of armature current can be made to flow. Generally a series winding is provided in this circuit, the purpose of which being to establish quadrature axis flux with smaller quadrature-axis armature current. As a result of the relatively high current which flows in the short circuited path, the armature reaction of this generator is very high and a very strong field ϕ_q is produced in quadrature to the original field ϕ_d. By placing a second set of brushes at BB' normal to the ϕ_q field, a high level voltage source is obtained between the terminals of the brushes BB'. It is important to note that a cumulative winding is provided on the direct-axis in series with the direct-axis load current. This winding is called a compensating winding and is very carefully designed to produce a flux as nearly as possible equal and opposite to the flux produced by the direct-axis armature current. The negative feed back effect of the load current is thereby cancelled, and the field winding has complete control over the direct-axis flux. It should be noted that the armature reaction due to the current flowing in the circuit ADA' should under no circumstances be compensated by any device (compensating windings, offset brushes etc.)

whereas in order to procure maximum efficiency at the second stage, (i.e in the circuit BFB') the secondary armature reaction should be compensated. It is the percentage of compensation which differentiates the metadyne from the amplidyne. The metadyne is fully under-compensated so that it is a constant current generator while the amplidyne is hundred percent compensated. Amplidyne is generally provided with more than one control field winding. These allow more independent signals to control the output. These independent signals produce independent fields which combine in the field core to produce one resultant field and it is on this resultant field that the amplidyne acts. Sometimes amplidyne is also provided with an antihunt coil which reduces the hunting of the machine to a large extent. Hunting means, oscillation of output voltage about some mean value. This is not at all desirable and is eliminated by feeding back the output voltage through the capacitor which offers low impedance for alternating (oscillating) voltage (referring to diagram 2&3). The current flowing in the antihunt coil is proportional to the output voltage and produces a field along d-axis in a direction opposing the output voltage. Thus the output voltage oscillations are reduced. Often antihunting transformers are provided in the system.

The amplidyne which is used in the present work incorporates antihunt coil and six control fields. The specifications of various coils and machine are as mentioned below.

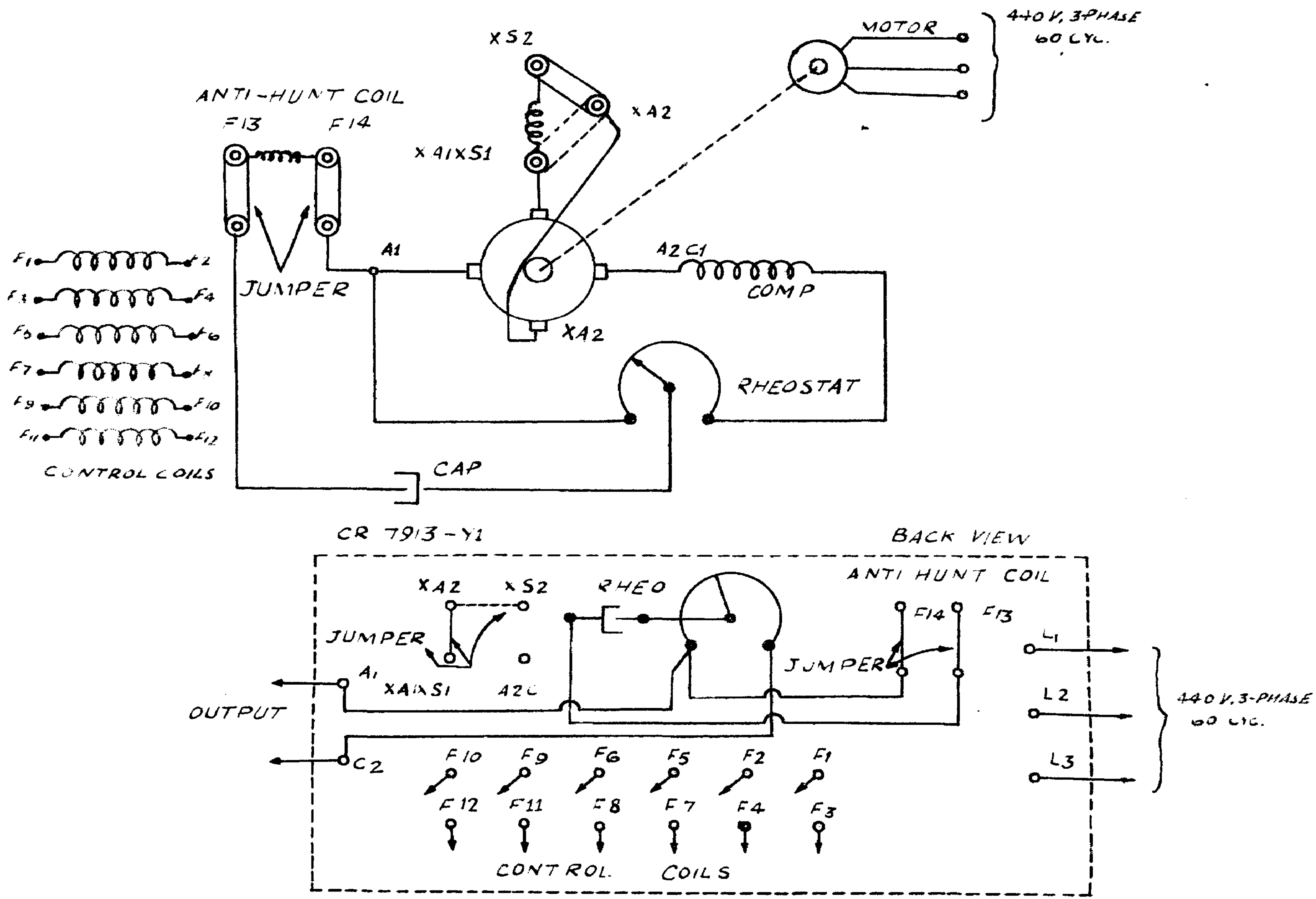


FIG. 3 WIRING DIAGRAM OF TIPICAL ANTIHUNT CIRCUIT AND TERMINAL BOARD

Watts 1500 , Volts - 125, R.P.M. 1800

Field	Resistance at 25°C	Max. Amps.
F_1F_2	980 Ohms	0.120
F_3F_4	980 Ohms	0.120
F_5F_6	43 Ohms	0.598
F_7F_8	43 Ohms	0.598
F_9F_{10}	2.6 Ohms	2.2
$F_{11}F_{12}$	2.6 Ohms	2.2
$F_{13}F_{14}$	55.5 Ohms	0.477

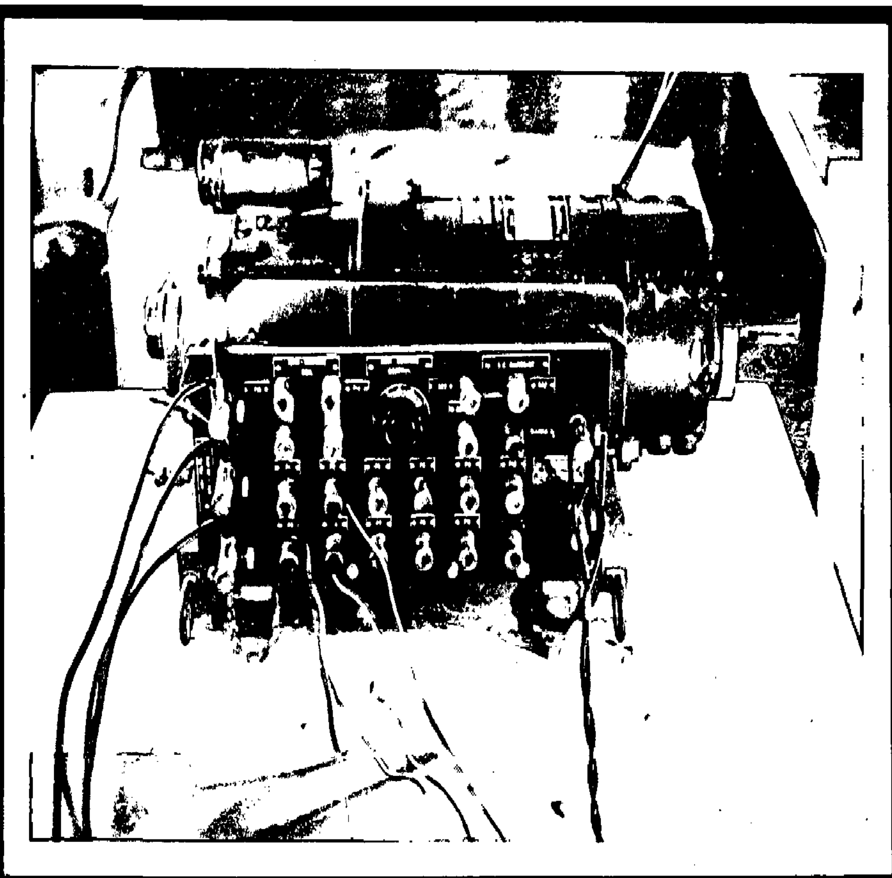
The amplidyne is directly coupled to and on the same base with the induction motor whose specifications are as mentioned below :

Type K , Frame 77 , Volts 220
3 phase , 60 cycles , 1800 r.p.m.

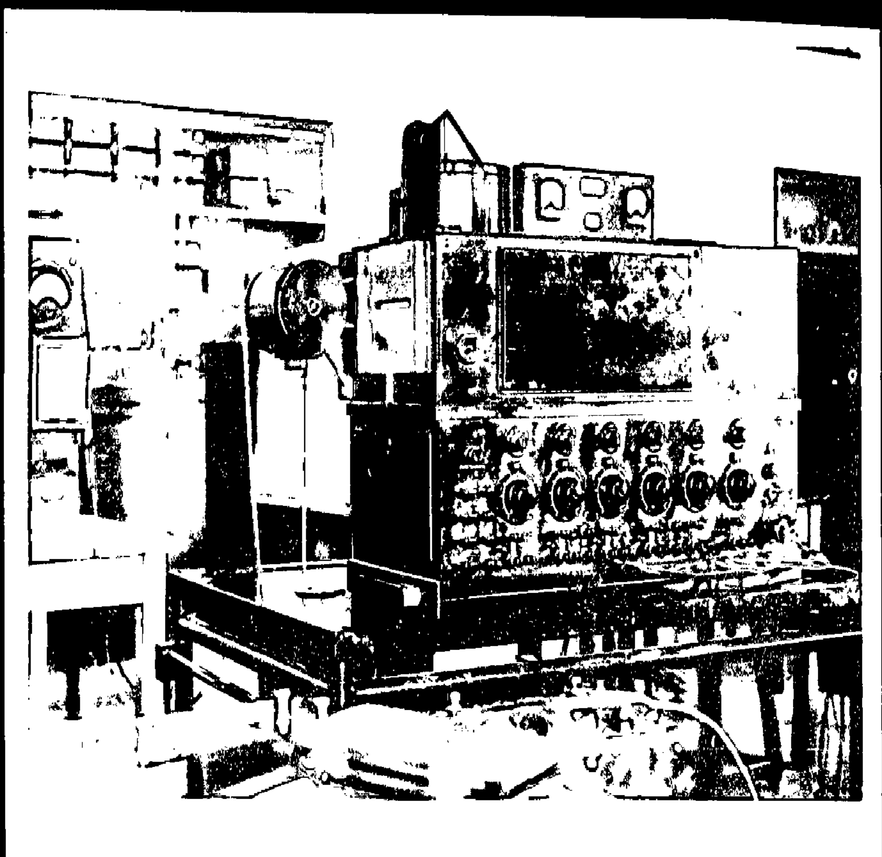
The attached photographs show the pictorial view of the amplidyne and the experimental set up.

1.3. APPLICATIONS:

The amplidyne generator has been widely applied and is recognized as an important device in the electrical industry. From the application point of view the amplidyne may be looked upon as a D.C. generator which, by virtue of special features of design and construction will produce its rated output with an extremely low net excitation. Amplidyne applications(4) mainly may be devided in to two groups, one in which the amplidyne is used as a regulator for automatically controlling certain quantities and those in which it is used simply as an amplifier.



AMPLIDYNE.



When used with a regulating circuit, the amplidyne will supply a constant excitation for generators whose output voltage must be kept constant. When used for current control, the amplidyne supplies a constant current to a motor or generator even when the speed of the motor or the load on the generator changes. It is also used for speed control of electrical machinery . Thus important industrial uses of amplidyne as regulator are for Power factor control, position control, voltage control and current control.

One of the important use of amplidyne is as an amplifier. The American Standard Association definition of an amplfier is "Amplifier is a device for increasing the power associated with a phenomenon with-out appreciably altering its quality, through control by the amplifier input of a larger amount of power supplied by a local source to the amplifier output." There are several rotating machines that can be called as amplifiers according to the above definition. They are : separately excited d-c generator, the amplidyne, the Rototrol and the Regulex. In each of these machines a large amount of power can be controlled by a small control field current . Power amplification of the order of 20,000 : 1 can easily be obtained with the help of amplidyne in comparison to that obtained by a d.c. generator which is of the order of about 100 : 1. So where the power required to drive a load is large but only a small amount of power is available as a control signal, rotating or dynamo-electric amplifiers can be used. Rotating amplifiers are nothing but prime-mover driven d-c generators whose out put can be controlled by small field power inputs. Generally electronic amplifiers are used to bring the power level upto 50 to 400 watts and above this rotary amplifiers are often used.

The difficulties peculiar to the rotating amplifier are the hysteresis effect, saturation and commutation effects. But these difficulties can be over come to large extent by using high grade low loss magnetic material and periodically checking brushes and commutator.

Thus we see that the above mentioned and many other applications make the amplidyne generator a versatile and important machine in the industry.

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CHAPTER 2.

STEADY STATE CHARACTERISTICS.

STEADY STATE CHARACTERISTICS

The steady state characteristics are the characteristics of the particular machine obtained under conditions of gradual or slow changes. The steady state characteristics[5] of the amplidyne generator are obtained by plotting the following curves:

1. Total Saturation.
2. Total Regulation.

2.1. TOTAL SATURATION:

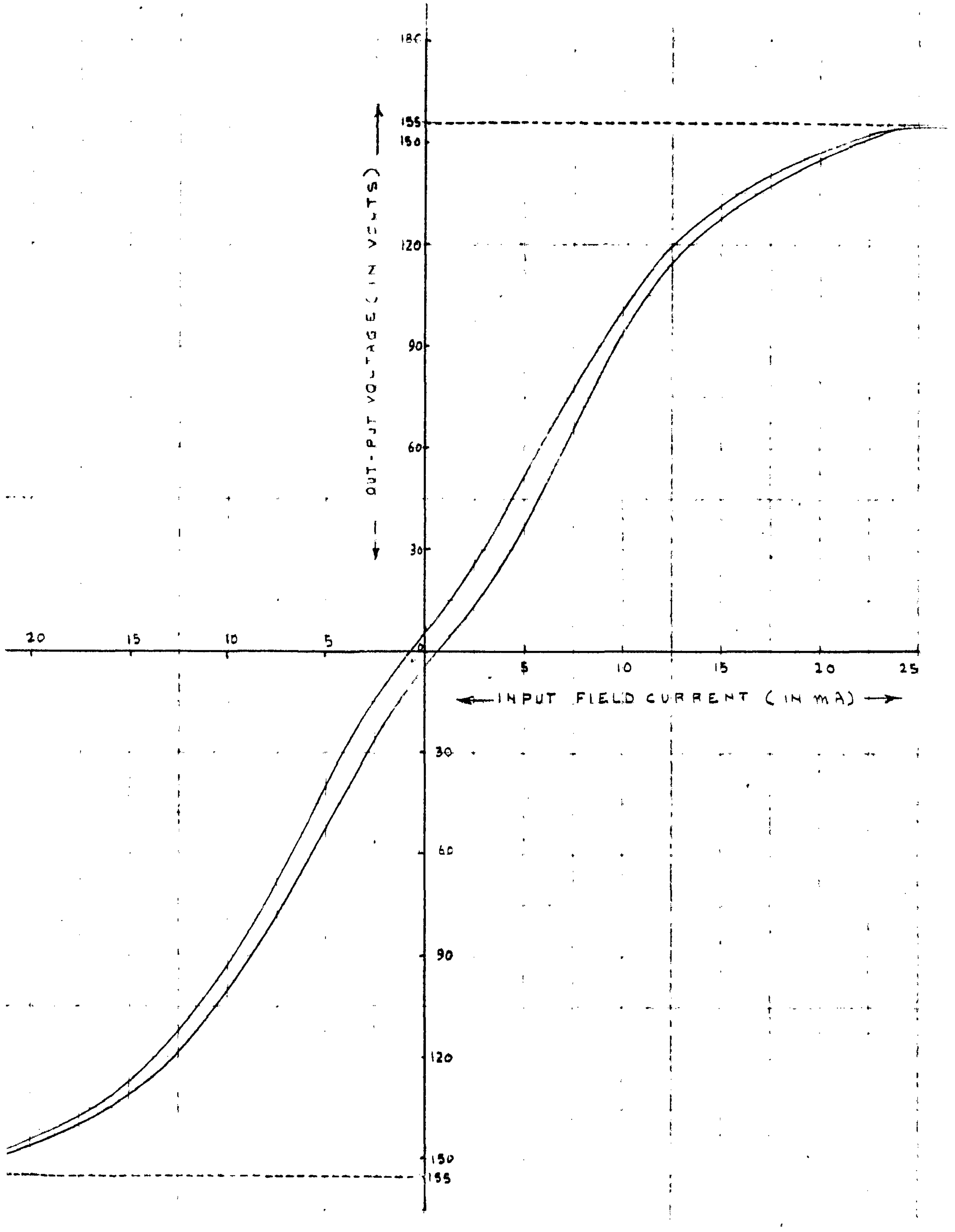
A commonly accuring phenomenon in the field of electrical engineering is the phenomenon of saturation. The saturation can be either magnetic saturation or di electric saturation. In this analysis magnetic saturation is considered and the total saturation curve is a plot between the direct axis no-load voltage and the control field current with quadrature circuit closed.

The complete hysteresis loops were found out by exciting the control field F_1F_2 for the following configuration.

(i) Without antihunt coil and with quadrature series field in the circuit.

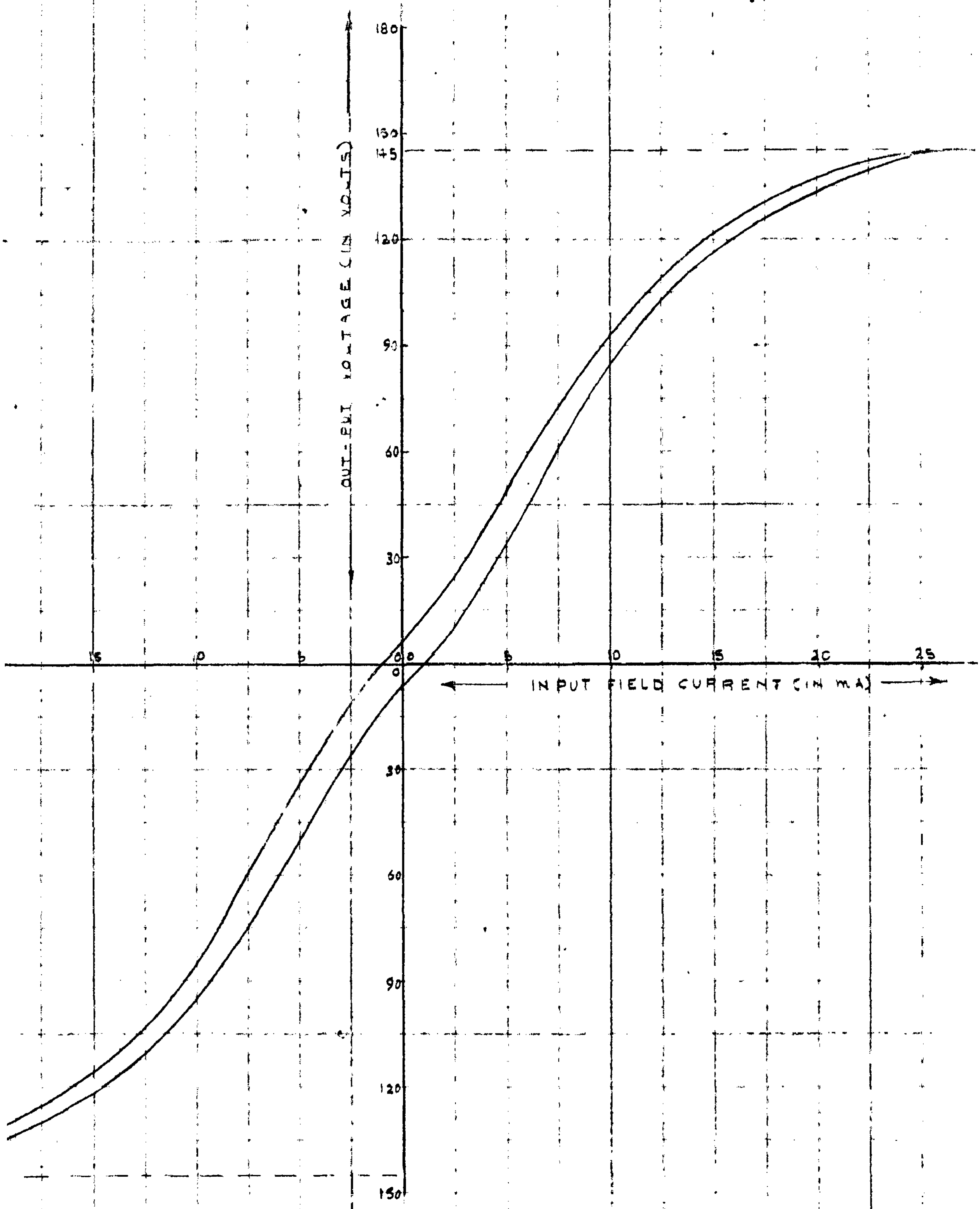
(ii) Without antihunt coil and without quadrature series field in the circuit.

(iii) With antihunt coil and with quadrature series field in the circuit.



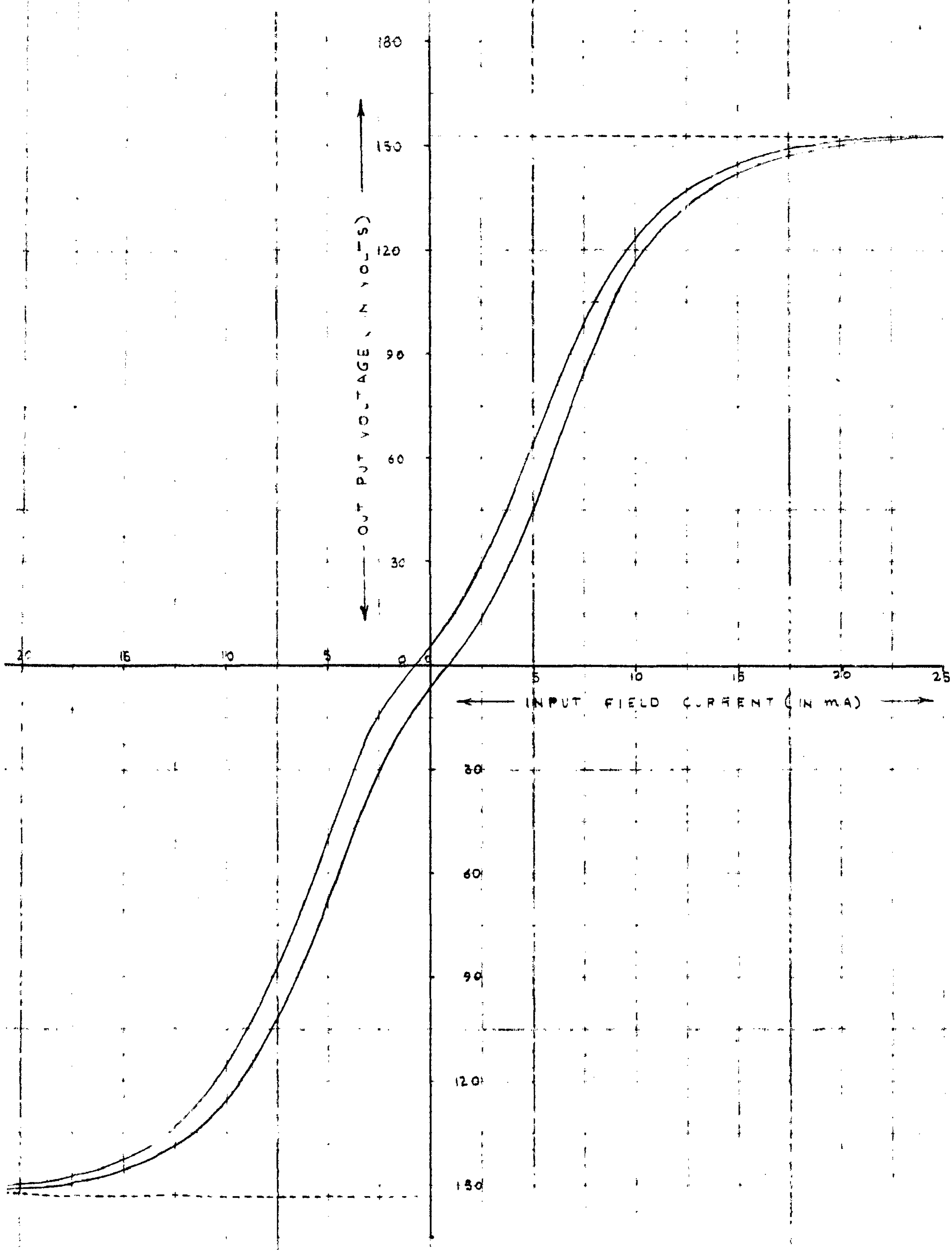
UT-OUTPUT CHARACTERISTIC WITH Q AXIS SERIES FIELD AND WITH OUT ANTIHUNT COIL.

Fig. 4



PUT-OUTPUT CHARACTERISTIC WITH OUT Q-AXIS SERIES FIELD AND WITH ANTI HUNT COIL.

Fig. 5



OUTPUT - INPUT CHARACTERISTIC WITH Q - AXIS SERIES WINDING AND ANTIHUNT COIL

Fig. 6

The complete hysteresis loops for the above connections are as shown in the Figures (4), (5) and (6).

The function of the quadrature field is clear from the no-load curves of Figs. (4) and (5). Because with the quadrature series field in the circuit we obtain a higher output voltage than what is obtained without the quadrature series field. Thus quadrature series field increases the over all gain of the amplidyne.

From the mean no -load curve (mean curve of the hysteresis.loop), we see that as soon as the point is reached where the saturation in the iron becomes noticeable; it is found that the effect does not appear gradually but occurs a bit adruptly so that it may be described in terms of ceiling voltage which the machine has reached. This phenomenon can be explained by the fact that the internal currents produce saturation under one control field pole and not the other. The loss in flux due to this saturation is such that it requires more control field magneto motive force. Now because both these currents are quite large compared to the original control field current, the demagnetizing effect is large and accounts for the adrupt saturation

Referring[7] to the figure 7, the validity of the following equations is easily understood.

$$V_q = K_c I_c = \frac{K_c}{R_c} V \qquad \ldots (1)$$

$$V_e = K_q I_q = \frac{K_q}{R_q} V_q \qquad \ldots (2)$$

where K_c and K_q are the constants depending upon the respective slope of saturation curves and respective number of turns.

AMPLIDYNE

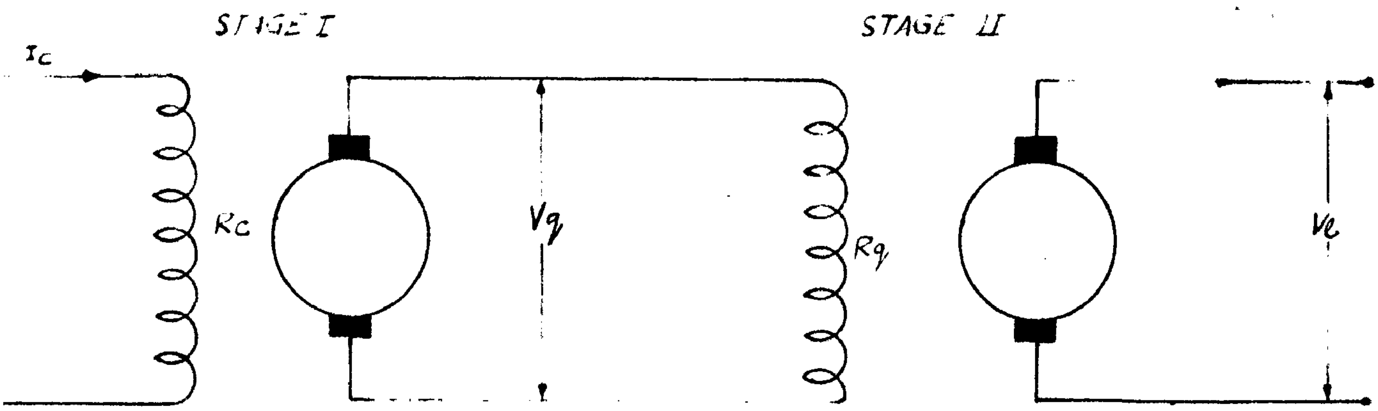


FIG. 7.

Let I_e be the load current which when flowing sets up an m.m.f. on the same axis as that of control field current but in opposite direction. And therefore,

$$V_q = \frac{K_c}{R_c} V - K_e I_e \qquad --- \qquad (3)$$

where K_e is a constant depending on the slope of the saturation curve for the horizontal axis and the number of armature turns. In an uncompensated machine the m.m.f. due to the load current is very large in comparison to the m.m.f. which is required to generate V_q. And therefore for a given output voltage the control field current has to be greatly increased in order to neutralize the effect of the load current. And hence by neutralizing partially or fully the effect of armature reaction due to load current by compensating winding, the control field current can be reduced to a very large extent. If C is the ratio of the mean ampere turns of the compensating winding to the mean ampe: turns of the armature, then the equation (3) can be written as

$$V_q = \frac{K_c}{R_c} V - K_e I_e (1-C) \qquad ---(4)$$

and equation (2) can be written as

$$V_e = \frac{K_q}{R_q} V_q - I_e R_e \qquad ---(5)$$

where $I_e R_e$ is the resistance drop in the Armature circuit including compensating winding.

Combining equations (4) and (5) we get

$$V_e = \frac{K_q K_c}{R_q R_c} V - \frac{K_q K_e}{R_q} (1-C) + R_e \quad I_e$$

$$= KV - K_1 (1-C) + R_e \quad I_e \qquad ---(6)$$

Where $K = \dfrac{K_q K_c}{R_q R_c}$ and $K_1 = \dfrac{K_q K_e}{R_q}$

The equation (6) describes the load characteristics of the amplidyne. When the compensation is 100% then the equation (6) reduces to

$$V_e = KV - R_e I_e \qquad ---(7)$$

Equation (7) shows the characteristic similar to the separately excited D.C. generator. The voltage remains nearly constant over the load range and hence the constant voltage characteristic . In a fully compensated amplidyne, the power amplification ratio which is defined as the ratio of output

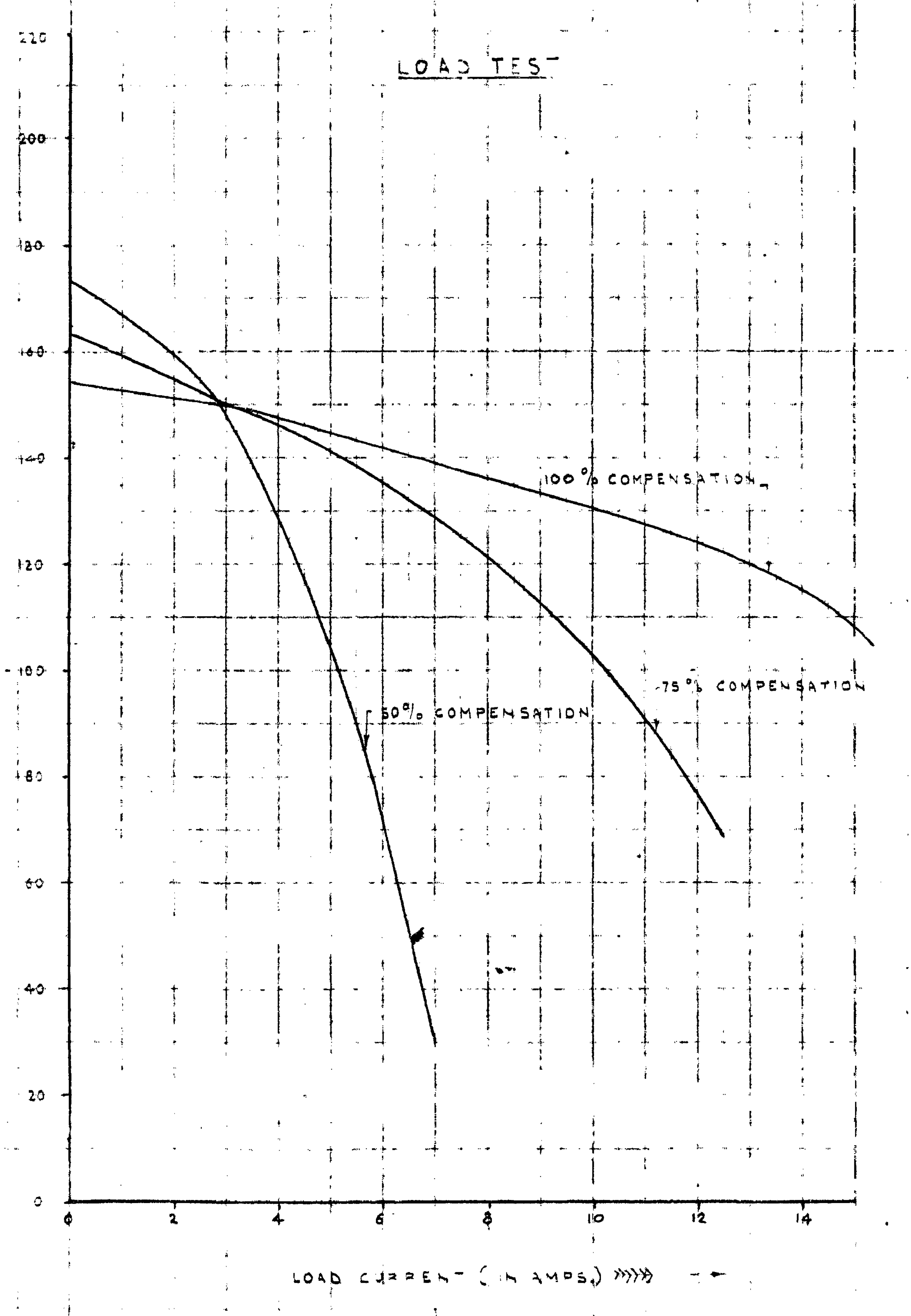

LOAD TEST
100 % COMPENSATION
75 % COMPENSATION
50% COMPENSATION
220
200
180
160
140
120
100
80
60
40
20
0
0
2
4
6
8
10
12
14
LOAD CURRENT (IN AMPS.)

Fig 8.

power $V_e I_e$ to the input power VI_c can be of the order of 2×10^4 or more.

The load test was performed on the experimental amplidyne and the curves for over, under and 100 % compensations are plotted as shown in the Fig. 8.

If the compensation is very low then the constant current characteristic is obtained.

2.2. TOTAL REGULATION:

The total regulation curve shows the variation of the direct axis terminal voltage with the direct -axis current.

One of the major use of amplidyne is in closed cycle controllers or regulating system. A closed cycle control system[8] is one in which the controlling agency is actuated by some function of the final output in such a manner as to minimize any deviation of the output from the ideal values This system mainly can be devided in to three parts :

(1) A standard of ideal performance against which the output to be regulated can be compared.

(2) An amplifier to amplify any deviation of output from the standard.

(3) A means of feeding the amplifier output into the system in such a manner so as to minimize the deviation.

One of the control field winding termed as the reference field is supplied with a current which represents the standard of performance (Fig. 9). The second winding is supplied with a current which is a function of the quantity to be controlled.

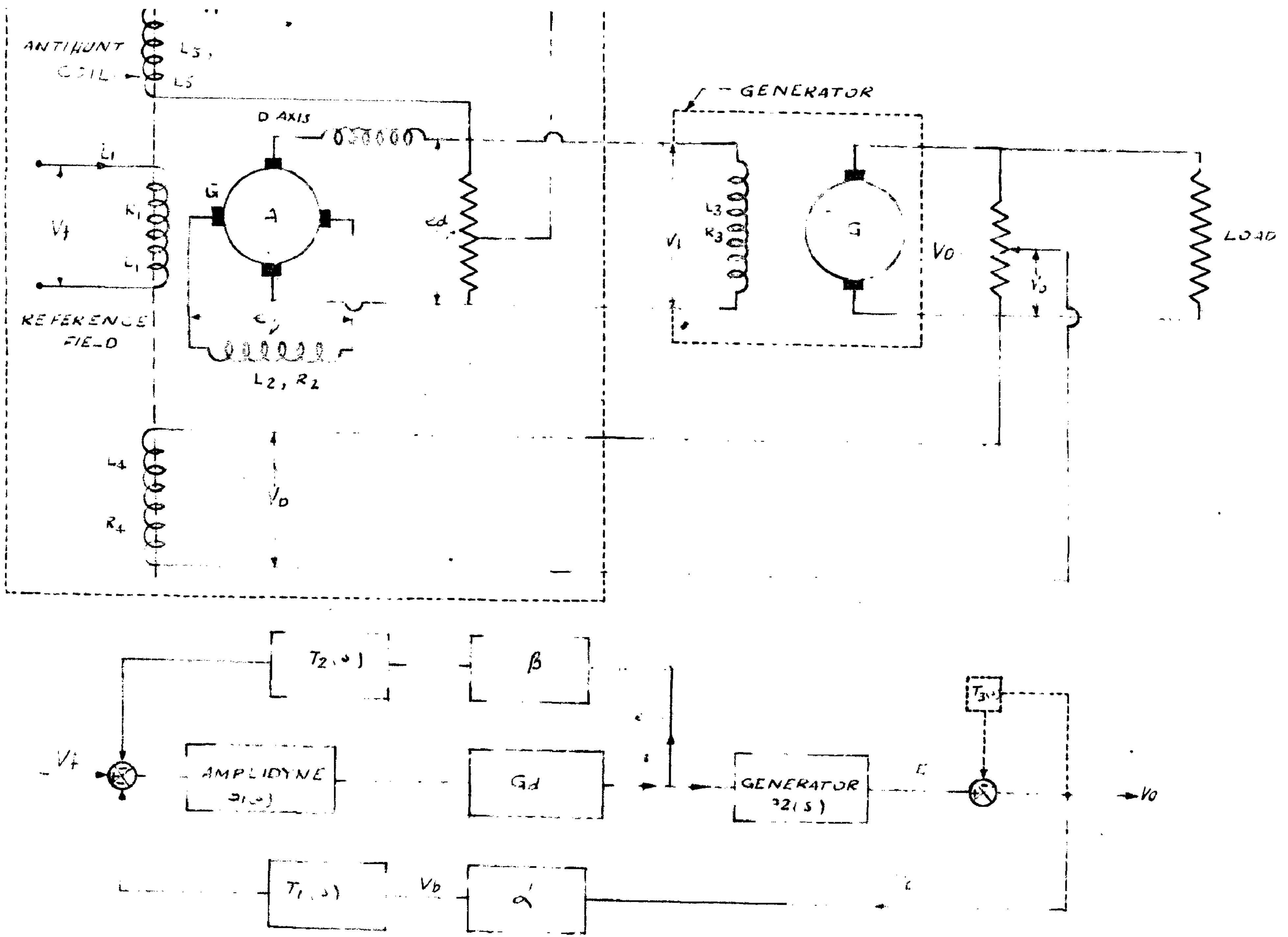


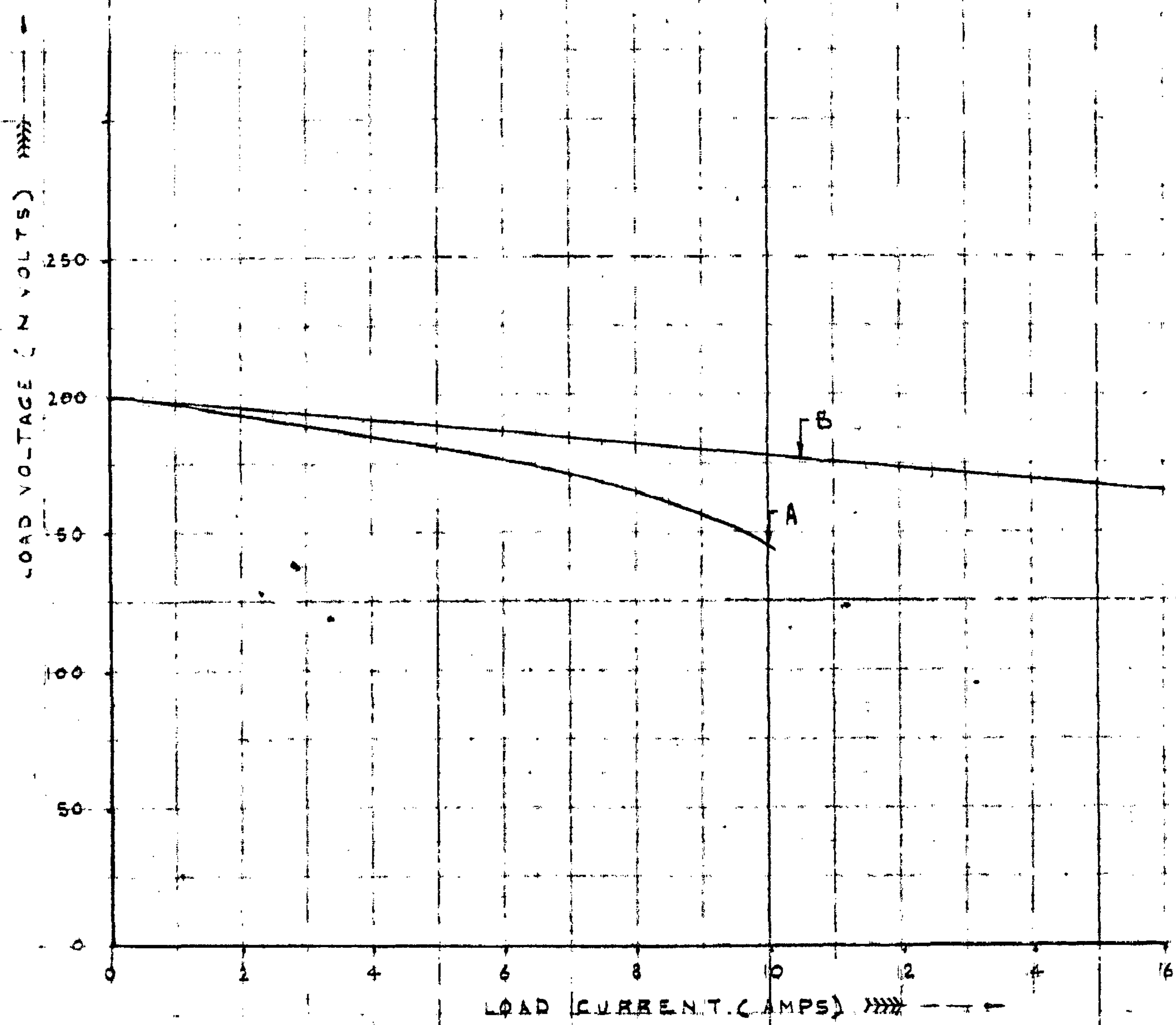
FIG. - AMPLIDYNE AS VOLTAGE REGULAT... WITH ANTIHUNT ...

This winding is so connected so that the m.m.f. developed by this winding is in opposition to that developed by reference field. And it is only the difference between the ampereturns of these two windings by which the performance of the equipment differs from the ideal as dictated by the reference field.

The Fig. 10. shows the complete set up of the voltage regulation of the generator by differential flux method using amplidyne. Now the loading was done on the generator and the observations were noted as shown in the table. The Fig. 11 shows the curves showing the voltage regulator action of the amplidyne. From the curve we see that the deviation of the voltage with the load is not much. According to British Standard 205[9], a voltage regulator is defined as " a device for varying, at will the voltage of a circuit or for automatically maintaining it at or near a prescribed value". And hence in this system amplidyne acts as automatic voltage regulator.

The form of total regulation curve is affected by speed, degree and nature of compensation and brush settings. And the form of total saturation curve is affected by speed, quadrature brush settings but it is not affected by the degree or nature of compensation and direct brush settings.

It is clear from the no-load curve of amplidyne that nonlinearity is quite pronounced. And therefore consideration of the nonlinearity is very essential for the analysis and design of the machine. Although mostly all practical systems are nonlinear to some extent and then it depends upon the degree of nonlinearity and the performance region of the machine that whether the existing nonlinearity can be neglected or not. One of the various methods known for linearizing the system is the describing function method



CURVE (A).... OBTAINED WHEN AMPLIDYNE IS USED AS VOLTAGE REGULATOR.

CURVE (B)..... SHOWS VARIATION OF THE OUT PUT VOLTAGE WITH LOADING.

Fig. 11

This method is usually applied to those devices in which there is a large departure from linearity.

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CHAPTER 3.

DESCRIBING FUNCTION ANALYSIS.

DESCRIBING FUNCTION APPROACH

3.1. DEFINITION:

The term non-linearity is interpreted as a phenomenon described by a nonlinear differential equation. A nonlinear system is one which includes one or more non-linear components. Non-linearities can be of two types[10] namely, incidental and intentional. Incidental nonlinearities occur in the system due to the limitations of physical equipment and include such phenomenon as saturation, backlash etc. Intentional non-linearities are introduced in the system in order to modify system characteristics. Non-linearities can also be classified according to the rate of change of the characteristics of the nonlinear element. Slow nonlinearities are those in which the system remains linear over a time interval which is long compared to the response time of the system. And fast non-linearities are those in which the mode of operation of the system changes rapidly compared to the response time. Saturation is the non-linearity of this kind.

Mathematical methods for the analysis of nonlinear systems are very very tedious and therefore the recent developed method of describing function has proved to be a very useful tool in the design and analysis of non-linear systems.

"Describing function by definition[11] is an equation expressing the ratio of amplitudes and the phase angle between the fundamental component of the output and the input sinusoid, for all frequencies from zero to plus infinity". Therefore it is a type of linearization, because the steady-state periodic output of the non linear device is considered to be a sine wave of the same frequency as the input sine wave. The following are the four basic assumptions on which the describing function analysis is based:

(1) The input to the non-linear device is a pure sine wave.

(2) The higher harmonics in the output of the non-linear device are neglected.

(3) The output of the non-linear element depends only on the present value and the past history of the input.

(4) There is only one non-linear element in the system. If there are more than one then all non-linear elements can be grouped as a single non-linear component.

These assumptions are generally satisfied in most of the feed back control systems. Generally linear elements used have one or more time constants in their denominators and hence are equivalent to low pass filter, attenuating higher harmonics except the fundamental, and secondly the amplitude of harmonics is much smaller in comparison to the amplitude of the fundamental.

Representing the non-linear system by the block diagram as shown in the Fig. (12) where $G_1(S)$, $G_2(S)$ and $G_b(S)$ are the transfer functions of the linear elements in the forward and backward path respectively. G_d being the describing function. Then the overall response(12) of the system can be expressed as

$$\frac{c}{y} = \frac{G_1(S)G_2(S)G_d}{1+ G_1(S)\ G_2(S)\ G_dG_b(S)}$$

The Nyquist plot is based on the characteristic equation which is

$$1 + G_1(S) G_2(S) \; G_d G_b(S) = 0.$$

Now denoting G(S) as the product of all the linear transfer functions around the closed loop.

$$G(S) = G_1(S) \; G_2(S) \; G_b(S)$$

The characteristic equation can be written

$$1 + G(S) \; G_d = 0$$

$$\text{or} \quad G(S) = -\frac{1}{G_d} \qquad \text{----(A)}$$

Now the Nyquist plots of the locus of the negative reciprocal of the describing function $-\frac{1}{G_d}$ and the locus of the product of the linear transfer functions around the loops G(S) can be plotted, and the degree and nature of stability of the Amplidyne (machine) can be studied.

Thus the describing function, obtained for the known non-linearity can be used to determine the stability of the system. The describing function for the component is found by evaluating fourier series for the output wave shape and then obtaining the complex ratio of the fundamental-frequency term in the series to the expression for the input signal.

Expressing mathematically, if sinusoidal input is $i(t) = I \operatorname{Sin}(wt)$ then fourier series of the output is

$$O(t) = \frac{A_o}{2} + A_1 \operatorname{Cos} wt + B_1 \operatorname{Sin} wt + A_2 \operatorname{Cos}(wt)$$

$$+ B_2 \operatorname{Sin} 2(wt) + \text{----} \qquad \text{(B)}$$

Where A_n and B_n are given by

$$A_n = \frac{2}{\pi} \int_0^{\pi} O(\theta) \text{ Cos } nwt \ dt.$$

$$B_n = \frac{2}{\pi} \int_0^{\pi} O(\theta) \text{ Sin } nwt \ dt$$

We only require A_1 and B_1, the first coefficients of the series, because in evaluating describing function expression we neglect all harmonies except the fundamental one. If we write the equation(13) in the form as shown below :

$$O(\theta) = \frac{C_o}{2} + C_1 \text{ Sin } (wt + \phi_1) + C_2 \text{Sin}(2wt + \phi_2) + \ldots\ldots.$$

Where $C_i = \sqrt{a_i^2 + b_i^2}$

and $\phi_i = \tan^{-1} \dfrac{a_i}{b_i}$

then the describing function can be expressed as

$$Gd = \frac{C_1}{I} \angle \phi .$$

3.2. DERIVATION OF THE DESCRIBING FUNCTION FOR VARIOUS APPROACHES:

It is difficult to know the practical saturation characteristics in the exact analytical form and therefore the most accurate method of determining the amplitudes of the various components of the output is to perform fourier analysis. This method is very tedious and an alternative method is to approximate the actual saturation characteristic by a curve which may be described analytically. Thereby making the fourier analysis more accurate and easier.

The following are the various approaches(14) considered for deriving the describing function:

I. Piecewise Linearization :

The Amplidyne system is linearized by approximating the output-input curve of the actual non-linearity with two straight line segments as shown in the figure (13).

II. Froelich's Equation Curve:

Replacing the noload saturation curve by the Froelich's equation $\phi = \frac{i}{a + bi}$ and then finding the describing function.

III. Exponential Rise:

Denoting the open-circuit characteristic of amplidyne by exponential equation. $\phi = K(1 - e^{-ai})$

That is, assuming the rise and decay of the total saturation curve exponentially and then finding the describing function.

IV. Rectangular Hysteresis Loop :

The describing function is found for the hysteresis loop as shown in the figure (14).

V. Hysteresis loop Considering the Effect of Brush Contact Resistance drop:

The describing function is found for the hysteresis loop considering the effect of brush contact resistance drop. The considered shape of the loop as shown in the figure (15).

This shape of the hysteresis loop has been taken for the analysis due to the fact that the various obtained amplidyne loops are characterized by the shown peculiarity.

Now the describing function expression for these

I PIECEWISE LINEARIZATION

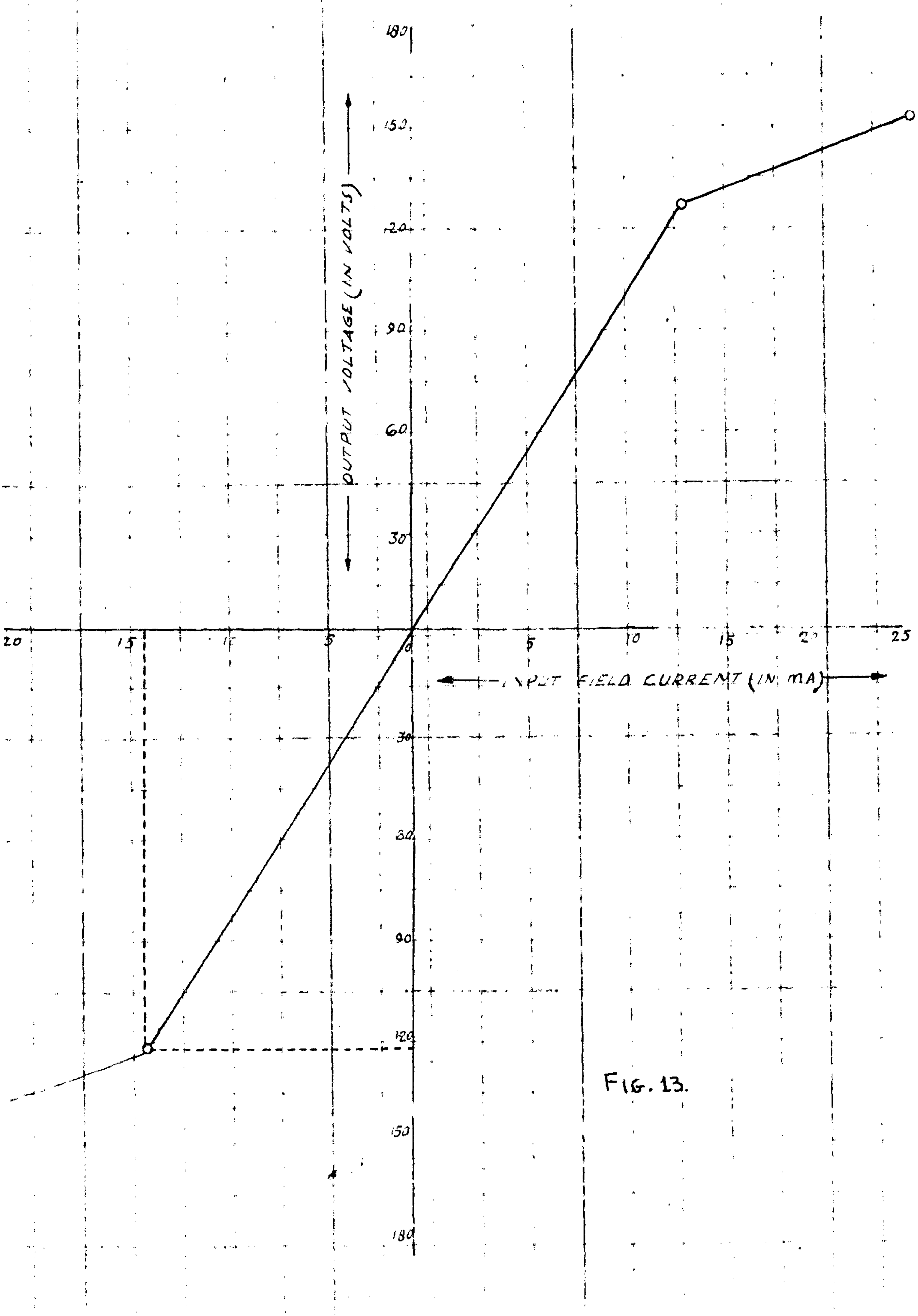


FIG. 13.

RECTANGULAR HYSTERESIS LOOP

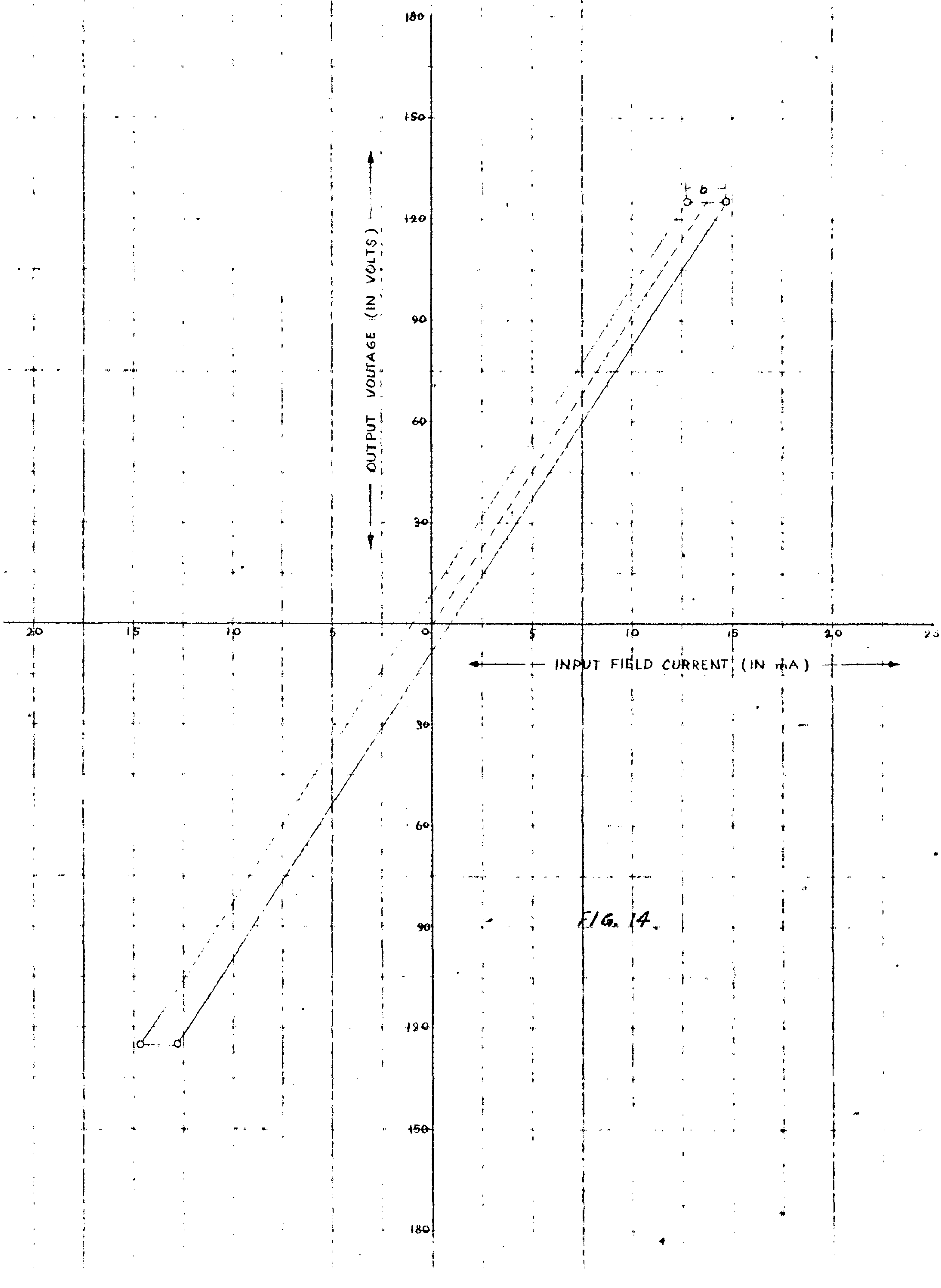
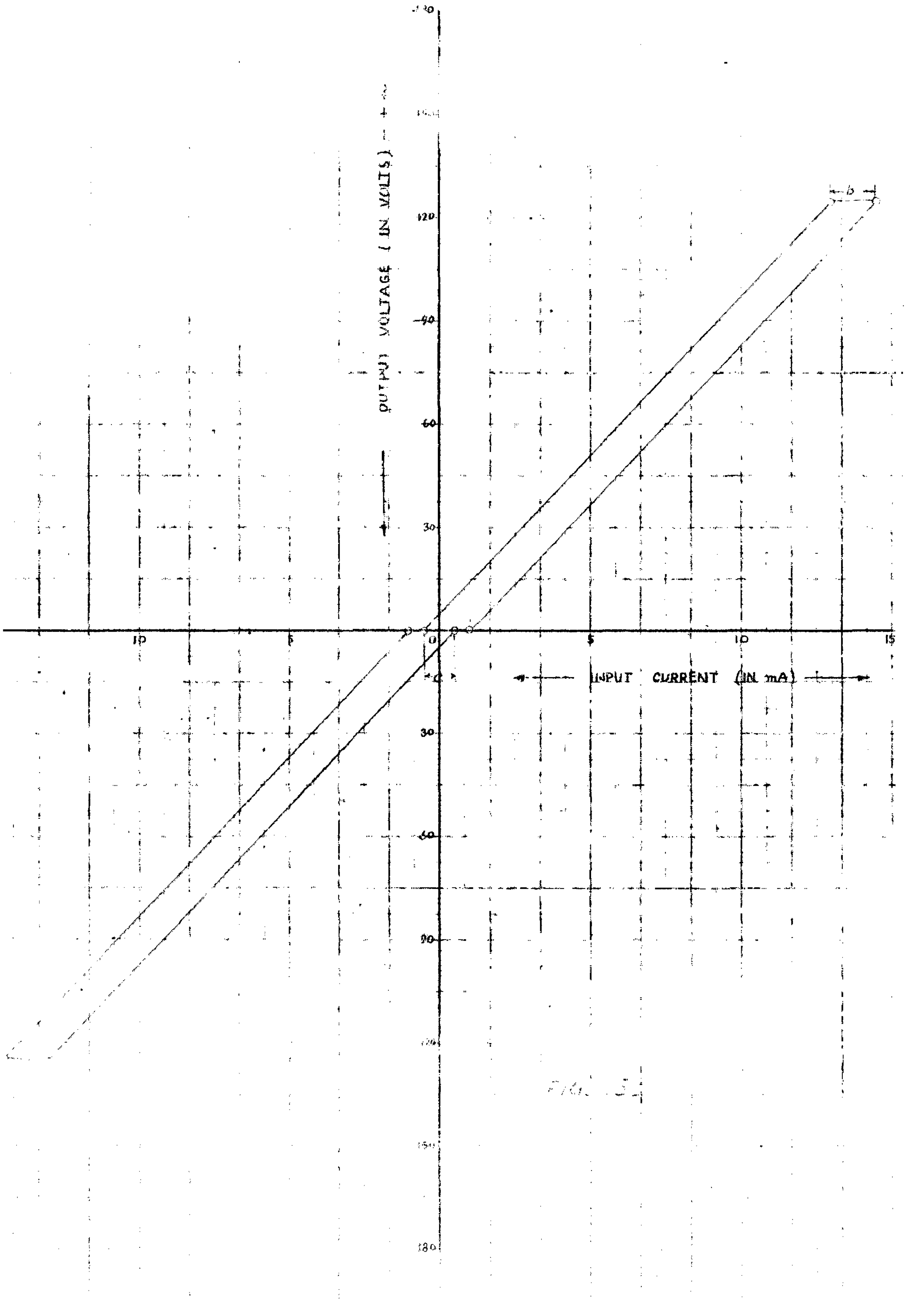


FIG. 14.



OUTPUT VOLTAGE (IN VOLTS)
INPUT CURRENT (IN mA)
120
90
60
30
0
30
60
90
180
10
5
5
10
15
a
b

FIG. 5

approaches are found one by one.

1. <u>Piecewise-Linearization</u>:

As mentioned earlier, the no-load saturation curve is replaced by two line segments. The initial slope is assumed as unity.

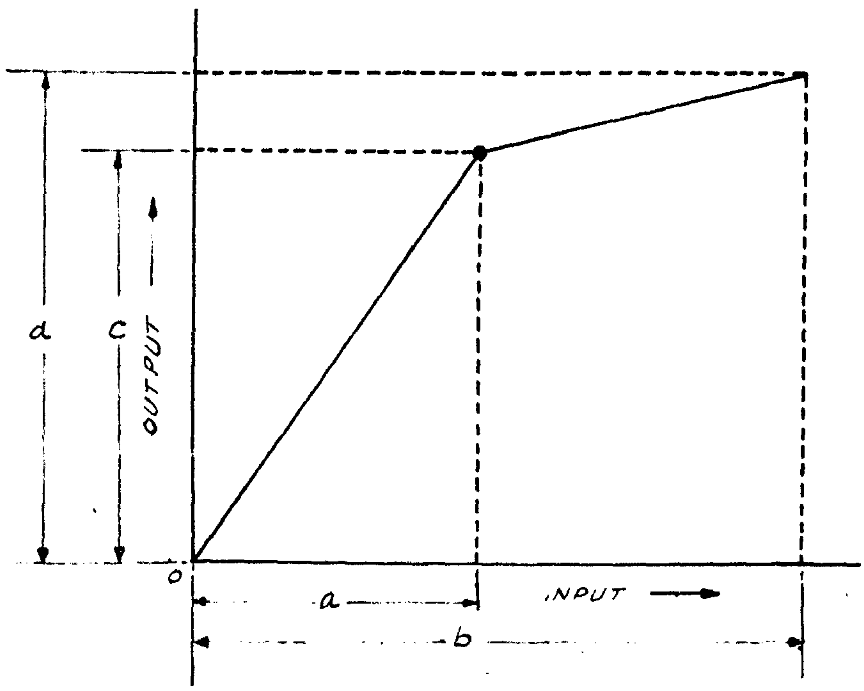


FIG. 16

$O(\theta)$ is the output of the non-linear element in response to the sinusoidal input $i(\theta)$.

$\therefore \; i(\theta) = I \sin\theta \quad , \quad \theta = wt$

Now the out-input characteristic of this type can mathematically be expressed as follows . (Referring to the Fig. 16).

$$O(\theta) = \frac{C}{a}\, i \qquad 0 < i < a$$

$$= \frac{C}{a}\, I \sin\theta \qquad 0 < \theta < \sin^{-1}\frac{a}{I}$$

$$(\because I \sin\theta_1 = a \therefore \theta_1 = \sin^{-1}\frac{a}{I})$$

$$\therefore \quad \phi(\theta) = \frac{c}{a} I \sin\theta \qquad 0 < \theta < \sin^{-1}\frac{a}{I}$$

$$\phi(\theta) = \frac{bc-ad}{b-a} + \frac{d-c}{b-a} I \sin\theta \qquad \sin^{-1}\frac{a}{I} < \theta < \frac{\pi}{2}$$

Now $a_1 = 0$, as curve is symmetrical about y - axis

and $b_1 = \frac{4}{\pi} \int_0^{\pi/2} \phi(\theta) \sin\theta \, d\theta$

$$= \frac{4}{\pi}\left[\int_0^{\theta_1} \frac{c}{a} I \sin^2\theta \, d\theta + \int_{\theta_1}^{\pi/2} \left(\frac{bc-ad}{b-a}\right)\sin\theta + \frac{d-c}{b-a} I\sin^2\theta d\theta\right]$$

$$= \frac{4}{\pi}\left\{\frac{cI}{2a}\left[\int_0^{\theta_1}(1-\cos 2\theta)d\theta\right] + \frac{bc-ad}{b-a}\cos\theta_1 + \left(\frac{d-c}{b-a}\right)\right.$$

$$\left.\frac{I}{2}\left[\int_{\theta_1}^{\pi/2}(1-\cos 2\theta)d\theta\right]\right\}$$

$$= \frac{4}{\pi}\left\{\frac{cI}{2a}(\theta_1 - \tfrac{1}{2}\sin 2\theta_1) + \frac{bc-ad}{b-a}\cos\theta_1 + \left(\frac{d-c}{b-a}\right)\frac{I}{2}(\pi/2 - \theta_1)\right.$$

$$\left. + \left(\frac{d-c}{b-a}\right)\left(\frac{I}{2}\right)\left(\frac{\sin 2\theta_1}{2}\right)\right\}$$

$$= \frac{2}{\pi}\left\{\frac{cI}{a}(\theta_1 - \tfrac{1}{2}\sin 2\theta_1) + \left(\frac{bc-ad}{b-a}\right)2\cos\theta_1\right.$$

$$\left. + I\frac{(d-c)}{(b-a)}\left(\frac{\pi}{2} - \theta_1\right) + \frac{(d-c)}{(b-a)}\frac{I}{2}\sin 2\theta_1\right\}$$

Now substituting the value of $\theta_1 = \sin^{-1}\frac{a}{I}$, we get

$$b_1 = \frac{2}{\pi}\left\{\frac{cI}{a}\left(\sin^{-1}\frac{a}{I} - \tfrac{1}{2}\sin 2\sin^{-1}\frac{a}{I}\right) + \left(\frac{bc-ad}{b-a}\right)2\cos\sin^{-1}\frac{a}{I}\right.$$

$$\left. + I\left(\frac{d-c}{b-a}\right)\left(\pi/2 - \sin^{-1}\frac{a}{I}\right) + \left(\frac{d-c}{b-a}\right)\frac{I}{2}\sin 2\sin^{-1}\frac{a}{I}\right\}$$

but $Gd = \frac{\sqrt{a_1^2 + b_1^2}}{I} \angle \tan^{-1} \frac{a_1}{b_1}$

$= \frac{b_1}{I} \angle 0$

$$\therefore Gd = \frac{2}{\pi}\left\{ \frac{c}{a} \left(\sin^{-1}\frac{a}{I} - \tfrac{1}{2}\sin 2 \sin^{-1}\frac{a}{I}\right) + \left(\frac{bc-ad}{b-a}\right)\frac{2}{I}\cos \sin^{-1}\frac{a}{I} \right.$$

$$\left. + \left(\frac{d-c}{b-a}\right)\left(\frac{\pi}{2} - \sin^{-1}\frac{a}{I}\right) + \tfrac{1}{2}\left(\frac{d-c}{b-a}\right)\sin 2 \sin^{-1}\frac{a}{I} \right\}$$

II. Froelich's Equation:

The mathematical equation for the curve in this approach is $E = \frac{i}{a + ib}$

Substituting $i = I \sin \theta$. , we get

$$E = \frac{I \sin \theta}{a + b I \sin \theta}$$

$a_1 = 0$, and $b_1 = \frac{2}{\pi} \int_0^{\pi} O(\theta) \sin \theta \, d\theta$

or $b_1 = \frac{2}{\pi} \int_0^{\pi} \frac{I \sin \theta}{a + bI\sin\theta} \sin\theta \, d\theta$

$$= \frac{2I}{\pi} \int_0^{\pi} \frac{\sin^2 \theta}{a + bI\sin\theta} \, d\theta.$$

$$= \frac{2I^2b^2}{\pi b^2 I} \int_0^{\pi} \frac{\sin^2 \theta}{a + bI\sin\theta} \, d\theta .$$

$$= \frac{2}{\pi I b^2} \left[\int_0^{\pi} \left\{ \frac{(bI\sin\theta + a)(bI\sin\theta - a)}{a + bI\sin\theta} \, d\theta + \frac{a^2 \, d\theta}{a + bI\sin\theta} \right\} \right]$$

$$= \frac{2}{\pi b^2 I} \left[\int_0^{\pi} \frac{a^2 \, d\theta}{a + bI\sin\theta} + \int_0^{\pi} (bI \sin\theta - a) \, d\theta \right]$$

$$= \frac{2}{I\pi b^2}\left[\left\{-bI\cos\theta - a\theta\right\}_0^{\pi} + \int_0^{\pi}\frac{a^2\, d\theta}{a+bI\sin\theta}\right]$$

$$= \frac{2}{I\pi b^2}\left[\left\{bI - a \quad +bI\right\} + \int_0^{\pi}\frac{a^2\, d\theta}{a+bI\sin\theta}\right]$$

Let $X = \int_0^{\pi}\frac{1}{a+bI\sin\theta}\, d\theta$

put $\theta = \frac{\pi}{2} + y$, $\therefore$ $d\theta = dy$

$$\therefore X = \int_0^{\pi}\frac{d\theta}{a+bI\sin\theta} = \int_{-\pi/2}^{+\pi/2}\frac{dy}{a+bI\cos y}$$

$$X = \int_{-\pi/2}^{+\pi/2}\frac{dy}{a(\cos^2 y/2 + \sin^2 y/2) + bI(\cos^2 y/2 - \sin^2 y/2)}$$

$$= \int_{-\pi/2}^{\pi/2}\frac{dy}{(a+bI)\cos^2 y/2 + (a-bI)\sin^2 y/2}$$

$$= \int_{-\pi/2}^{\pi/2}\frac{1/\cos^2 y/2 \;.\; dy}{(a+bI) + (a-bI)\dfrac{\sin^2 y/2}{\cos^2 y/2}}$$

$$= \int_{-\pi/2}^{\pi/2}\frac{\sec^2 y/2}{(a+bI) + (a-bI)\tan^2 y/2}$$

$$= \frac{2}{(a-bI)}\int_{-\pi/2}^{\pi/2}\frac{\frac{1}{2}\sec^2 y/2 \;\; dy}{\dfrac{a+bI}{a-bI} + \tan^2 y/2}$$

putting $\tan y/2 = t$ $\therefore$ $\tan\frac{\pi}{4} = 1$

$$X = \frac{2}{a-bI}\int_{-1}^{+1}\frac{dt}{\dfrac{a+bI}{a-bI} + t^2}$$

Case (i) if $a > bI$

$$X = \frac{2}{a-bI}\left[\frac{1}{\sqrt{\frac{a+bI}{a-bI}}}\tan^{-1}\frac{t}{\sqrt{\frac{a+bI}{a-bI}}}\right]_{-1}^{+1}$$

$$= \frac{2}{a-bI}\sqrt{\frac{a-bI}{a+bI}}\left[\tan^{-1}\sqrt{\frac{a-bI}{a+bI}} - \tan^{-1}(-1)\sqrt{\frac{a-bI}{a+bI}}\right]$$

$$= \frac{2}{\sqrt{(a-bI)(a+bI)}}\left[2\tan^{-1}\sqrt{\frac{a-bI}{a+bI}}\right]$$

$$= \frac{4}{(a^2-(bI)^2)^{\frac{1}{2}}}\tan^{-1}\sqrt{\frac{a-bI}{a+bI}}$$

$$\therefore b_1 = \frac{2}{I\,b^2\pi}\left[2bI - a\pi + \frac{4a^2}{\sqrt{a^2-b^2I^2}}\tan^{-1}\sqrt{\frac{a-bI}{a+bI}}\right]$$

We know that $Gd = \dfrac{\sqrt{a_1^2 + b_1^2}}{I}, \ \angle \tan^{-1}\dfrac{a_1}{b_1}$

$$\therefore Gd = \frac{2}{I^2\pi b^2}\left[2bI + \frac{4a^2}{\sqrt{a^2-b^2I^2}}\tan^{-1}\sqrt{\frac{a-bI}{a+bI}} - a\pi\right] \angle 0$$

Case (ii) When $a < bI$

$$X = \frac{2}{a-bI}\left[\int_{-1}^{+1}\frac{dt}{\frac{a+bI}{a-bI}+t^2}\right]$$

$$= \frac{2}{bI-a}\left[\int_{-1}^{+1}\frac{dt}{\frac{a+bI}{bI-a}-t^2}\right]$$

$$= \frac{2}{bI - a}\left[\frac{1}{2\sqrt{\frac{a+bI}{bI-a}}} \cdot \log\left|\frac{\frac{a+bI}{bI-a} + t}{\frac{a+bI}{bI-a} - t}\right|\right]_{-1}^{+1}$$

$$= \frac{1}{\sqrt{(bI)^2 - a^2}}\left[\log\left|\frac{\frac{a+bI}{bI-a} + 1}{\frac{a+bI}{bI-a} - 1}\right| - \log\left|\frac{\frac{a+bI}{bI-a} - 1}{\frac{a+bI}{bI-a} + 1}\right|\right]$$

$$= \frac{1}{\sqrt{(bI)^2 - a^2}}\left[\log\frac{\left|\frac{a+bI+bI-a}{a+bI-bI+a}\right|}{\left|\frac{a+bI-bI+a}{a+bI+bI-a}\right|}\right]$$

$$= \frac{1}{\sqrt{(bI)^2 - a^2}}\left[\log\frac{\frac{2bI}{2a}}{\frac{2a}{2bI}}\right]$$

$$= \frac{1}{\sqrt{(bI)^2 - a^2}}\left[\log\frac{bI}{a} \times \frac{bI}{a}\right]$$

$$= \frac{1}{\sqrt{(bI)^2 - a^2}}\left[\log\frac{(bI)^2}{a^2}\right]$$

$$b_1 = \frac{2}{I\ b^2 \pi}\left[\frac{2a^2}{\sqrt{(bI)^2 - a^2}}\log\left(\frac{bI}{a}\right) + 2bI - a\pi\right]$$

$$\text{and } Gd = \frac{2}{I^2 \pi\ b^2}\left[\frac{2a^2}{\sqrt{(bI)^2 - a^2}}\log\left(\frac{bI}{a}\right) + 2bI - a\pi\right]$$

III Exponential rise :

In this approach, mean no load saturation curve is approximated by the exponential curve. The analytical equation governing the curve is as mentioned below.

$$E = K\ (1 - e^{-ai})$$

where K is the ceiling value of the voltage, "a" is a constant and i is sinusoidal input current

$$\therefore\ i = I \sin\theta$$

hence $E = K\ (1 - e^{-aI \sin\theta})$

here $a_1 = \theta_1$ and $b_1 = \frac{2}{\pi}\int_0^{\pi} O(\theta)\ \sin\theta\ d\theta.$

$$\therefore\ b_1 = \frac{2K}{\pi}\int_0^{\pi}(1 - e^{-aI\sin\theta})\ \sin\theta\ d\theta.$$

now $e^{-aI \sin\theta} = e^{j.jaI\sin\theta}$

putting $jaI = x$, we get

$$e^{-aI\sin\theta} = e^{jx\sin\theta}$$

and we know that $e^{jx\sin\theta} = J_o(x) + 2J_2(x)\cos 2\theta$
$+ 2J_4(x)\cos 4\theta + 2J_4(x)\cos 6\theta + \ldots$
$+ 2j\left[J_1(x)\sin\theta + J_3(x)\sin 3\theta + \ldots\right]$

$$\therefore\ b_1 = \frac{2K}{\pi}\int_0^{\pi}\left[1 - \left[J_o(x) + 2J_2(x)\cos 2\theta + 2J_4(x)\cos 4\theta \ldots\right] + 2j\left[J_1(x)\sin\theta + 2jJ_1\sin 3\theta + ..\right]\right]\sin\theta d\theta.$$

$$\text{or } b_1 = \frac{2K}{\pi}\Big[\, 2 - 2J_0(x) + \frac{4}{3}J_2(x) + \frac{4}{15}J_4(x) + \frac{4}{35}J_6(x) + \frac{4}{63}J_8(x) + \frac{4}{99}J_{10}(x) + \ldots - jJ_1(x)\pi \Big]$$

$$Gd = \frac{\sqrt{a_1^2 + b_1^2}}{I} \angle \tan^{-1}\frac{a_1}{b_1}$$

$$\therefore \; Gd = \frac{2K}{\pi I}\Big[\, 2 - 2J_o(x) + \frac{4}{3}J_2(x) + \frac{4}{15}J_4(x) + \frac{4}{35}J_6(x) + \frac{4}{63}J_8(x) + \frac{4}{99}J_{10}(x) + \ldots + I_1(x)\pi \Big] \angle 0$$

IV Rectangular Hysteresis Loop :

The shape of the hysteresis loop is approximated as shown in the fig. 17. The slope of the loop is assumed unity for the derivation.

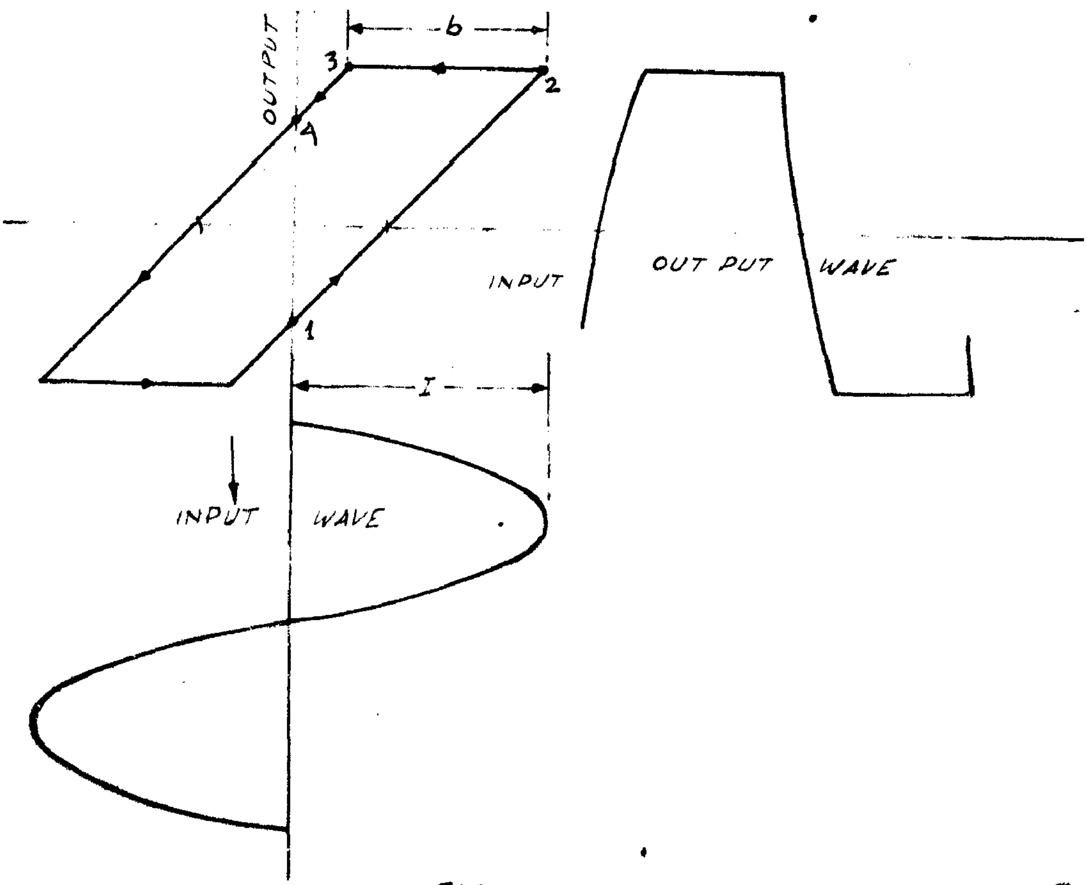


FIG. 17.

Expressing the output in-put characteristic mathematically we get, From (1) to (2) $O(t) = (I \sin\theta - b/2)$ $\quad 0 < \theta < \pi/2$

From (2) to (3) $O(t) = (I - b/2)$ $\quad \pi/2 < \theta < \pi - \beta$

from (3) to (4) $O(t) = (I\sin\theta + b/2)$ $\quad \pi - \beta < \theta < \pi$

$$a_1 = \frac{2}{\pi}\int_0^{\pi/2}(I\sin\theta - b/2)\cos\theta\, d\theta.$$

$$+\frac{2}{\pi}\int_{\pi/2}^{\pi-\beta}(I - b/2)\cos\theta\, d\theta.$$

$$+\frac{2}{\pi}\int_{\pi-\beta}^{\pi}(I\sin\theta + b/2)\cos\theta\, d\theta.$$

$$= \frac{2}{\pi}\left[\frac{I\sin^2\theta}{2} - \frac{b}{2}\sin\theta\right]_0^{\pi/2} + \left[(I - \frac{b}{2})\sin\theta\right]_{\pi/2}^{\pi-\beta}$$

$$+\left[\frac{I\sin^2\theta}{2} + \frac{b}{2}\sin\theta\right]_{\pi-\beta}^{\pi}$$

$$= \frac{2}{\pi}\left[(\frac{I}{2} - \frac{b}{2}) + (I - \frac{b}{2})\sin(\pi-\beta) - (I - \frac{b}{2}) + 0 - \frac{I\sin^2(\pi-\beta)}{2} - \frac{b}{2}\sin(\pi-\beta)\right]$$

$$= \frac{2}{\pi}\left(\frac{I}{2} - \frac{b}{2} + (I - \frac{b}{2})\sin\beta - (I - \frac{b}{2}) - \frac{I}{2}\sin^2\beta - \frac{b}{2}\sin\beta\right)$$

$$= \frac{2}{\pi}\left[-\frac{I}{2} + I\sin\beta - b\sin\beta - \frac{I}{2}\sin^2\beta\right]$$

$$= \frac{2}{\pi}\left[-\frac{I}{2}\left\{1 + \sin^2\beta\right\} + \sin\beta.(I - b)\right]$$

$$= \frac{2}{\pi}\left[-\frac{I}{2}\left(1 + \left(\frac{I-b}{I}\right)^2 + \frac{I-b}{I}(I-b)\right)\right]$$

$\because$ $I - b = I \operatorname{Sin}\beta$ $\qquad \therefore\ \beta = \operatorname{Sin}^{-1}\dfrac{I-b}{I}$

$$a_1 = -\frac{2}{\pi}\left[\frac{I^2 + (I-b)^2}{2I} - \frac{(I-b)^2}{I}\right]$$

$$= -\frac{1}{\pi I}\left[-b^2 + 2\,Ib\right]$$

$$= \frac{1}{\pi}\left[\frac{b^2}{I} - 2b\right]$$

$$= \frac{4\,I}{\pi}\left[\frac{b^2}{4I^2} - \frac{b}{2I}\right]$$

$$= \frac{4\,I}{\pi}\left[\frac{(b/2)^2}{I^2} - \frac{(b/2)}{I}\right]$$

Now $$b_1 = \frac{2}{\pi}\int_0^{\pi/2}\left(I\operatorname{Sin}\theta - \frac{b}{2}\right)\operatorname{Sin}\theta\, d\theta$$

$$+ \frac{2}{\pi}\int_{\pi/2}^{\pi-\beta}(I - b/2)\operatorname{Sin}\theta\, d\theta.$$

$$+ \frac{2}{\pi}\int_{\pi-\beta}^{\pi}\left(I\operatorname{Sin}\theta + \frac{b}{2}\right)\operatorname{Sin}\theta\, d\theta.$$

$$= \frac{2}{\pi}\left\{\left[I\left(\frac{\theta}{2} - \frac{\operatorname{Sin}2\theta}{4}\right) + \frac{b}{2}\operatorname{Cos}\theta\right]_0^{\pi/2}\right.$$

$$+ \left[-I\operatorname{Cos}\theta + \frac{b}{2}\operatorname{Cos}\theta\right]_{\pi/2}^{\pi-\beta}$$

$$\left. + \left[I\left(\frac{\theta}{2} - \frac{\operatorname{Sin}2\theta}{4}\right) - \frac{b}{2}\operatorname{Cos}\theta\right]_{\pi-\beta}^{\pi}\right\}$$

$$= \frac{2}{\pi}\left[\frac{I}{4} + \frac{I}{2} - \frac{I\pi}{2} + \frac{I\beta}{2} + I\cos\beta - \frac{b}{2}\cos\beta - \frac{b}{2}\cos\beta - \frac{I\sin(\pi-\beta)\cos(\pi-\beta)}{2}\right]$$

$$= \frac{I}{\pi}\left[\frac{\pi}{2} + \beta + 2\cos\beta - b|I\cos\beta - \frac{b}{I}\cos\beta - \sin(\pi-\beta) \cdot \cos(\pi-\beta)\right]$$

$$= \frac{I}{\pi}\left[\frac{\pi}{2} + \beta + 2\cos\beta - \frac{2b}{I}\cos\beta + \sin\beta\cos\beta\right]$$

$$= \frac{I}{\pi}\left[\frac{\pi}{2} + \sqrt{\frac{2b}{I} - \frac{b^2}{I^2}}\left(\frac{2I - 2b + I - b}{I}\right)\right]$$

$$= \frac{I}{\pi}\left[\frac{\pi}{2} + \frac{b}{I}\sqrt{\frac{2I}{b} - 1}\left(\frac{I-b}{I}\right)\right.$$

$$= \frac{I}{\pi}\left[\frac{\pi}{2} + \frac{b(I-b)}{I^2}\sqrt{\frac{2I}{b} - 1}\right.$$

we know that $G_d = \dfrac{\sqrt{A_1^2 + b_1^2}}{I} \angle\phi$

Now putting $R = \dfrac{b/2}{I}$, we get.

$$Gd = \frac{1}{\pi}\left\{\left[\frac{\pi}{2} + \sin^{-1}(1-2R) + \sqrt{\frac{1-R}{R}} \times 2R(1-2R)\right]^2 + \left[4(R^2 - R)\right]^2\right\}^{1/2}$$

$$= \frac{1}{\pi}\left\{\left[\frac{\pi}{2} + \mathrm{Sin}^{-1}(1-2R)\right]^2 + \frac{4R^2(1-2R)^2\,(1-R)}{R}\right.$$

$$+\left[\pi + 2\,\mathrm{Sin}^{-1}(1-2R)\right]\left[2R\,(1-2R)\sqrt{\frac{1-R}{R}}\right]$$

$$\left. + 16\,R^2\left[R\,(R-2) + .1\right]\right\}^{1/2}$$

$$= \left\{\left[\frac{\pi}{2} + \mathrm{Sin}^{-1}\,\overline{(1-2R)}\right]^2 + \left[\pi + 2\,\mathrm{Sin}^{-1}(1-2R)\right]\left[2R(1-2R)\right.\right.$$

$$\left.\left.\sqrt{\frac{1-R}{R}}\right] + \left[-16R^4+32R^3-20R^2+4R+16R^4-32R^3+16R^2\right]\right\}^{1/2}$$

$$\therefore\quad Gd = \left[\left(\frac{\pi}{2} + \mathrm{Sin}^{-1}\,\overline{1-2R}\right)^2 + 4\,R(1-R) + (\pi + 2\mathrm{Sin}^{-1}\,\overline{1-2R})\,2R\right.$$

$$\left.(1-2R)\sqrt{\frac{1-R}{R}}\right]^{1/2}$$

$$= \left\{\left[\frac{\pi}{2} + \mathrm{Sin}^{-1}\,(1-2R)\right]^2 + 4\,R\,(1-R) + \left[\pi + 2\,\mathrm{Sin}^{-1}(1-2R)\right]\right.$$

$$\left.2R(1-2R)\sqrt{\frac{1-R}{R}}\right\}^{\frac{1}{2}}$$

$$\phi = \tan^{-1}\frac{A_1}{b_1} = \tan^{-1}\frac{4\,R(R-1)}{\frac{\pi}{2} + \mathrm{Sin}^{-1}(1-2R) + \sqrt{\frac{1-R}{R}} \times 2R(1-2R)}$$

V. Hysteresis Loop Considering the Effect of Brush Contact Drop :

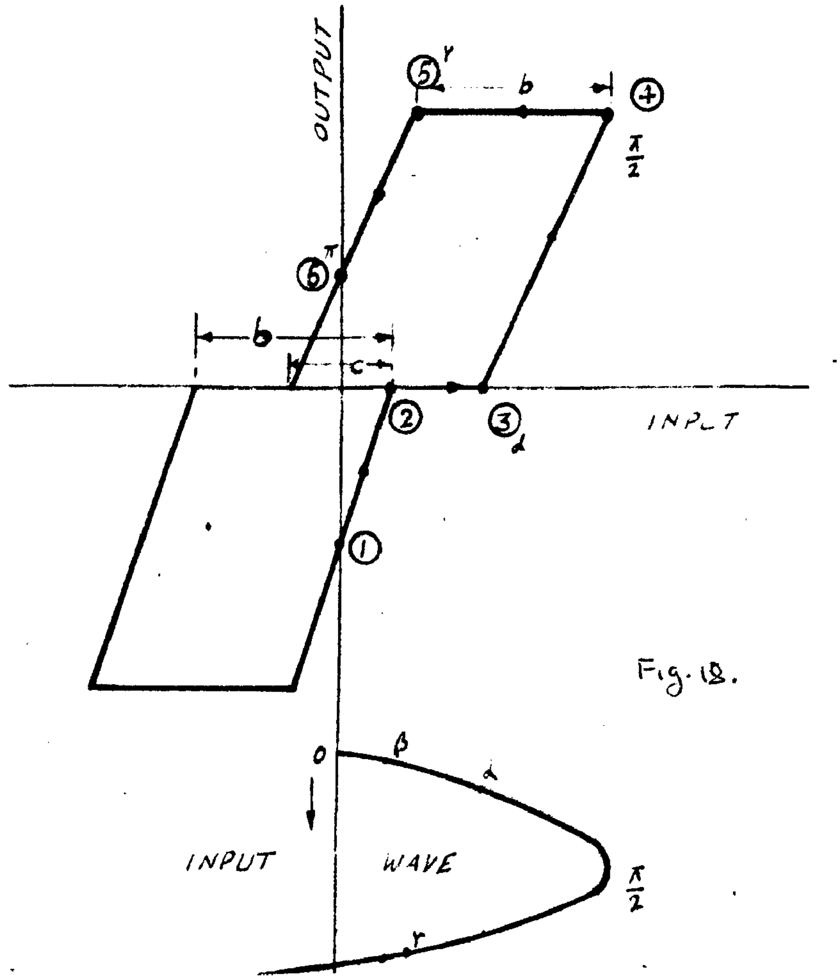


Fig. 18.

Input-output characteristic as shown in the above diagram is expressed analytically by the equations mentioned below :

$$\begin{cases} O(\theta) = (I\sin\theta - \frac{c}{2}) & \ldots \quad 0 < \theta < \beta \\ O(\theta) = 0 & \beta < \theta < \alpha \\ O(\theta) = (I\sin\theta - b + \frac{c}{2}) & \alpha < \theta < \pi/2 \\ O(\theta) = (I - b + c/2) & \frac{\pi}{2} < \theta < \gamma \\ O(\theta) = (I\sin\theta + \frac{c}{2}) & \gamma < \theta < \pi \end{cases}$$

Where $\beta = \sin^{-1}\frac{c}{2I}$

$$\alpha = \sin^{-1}\frac{b - c/2}{I}$$

$$\gamma = \sin^{-1}\frac{I - b}{I}$$

Now $$a_1 = \frac{2}{\pi}\int_0^{\pi} O(\theta)\ \mathrm{Cos}\ \theta\ d\theta$$

$$\therefore a_1 = \frac{2}{\pi}\int_0^{\beta}\left(I\mathrm{Sin}\ \theta - \frac{c}{2}\right)\mathrm{Cos}\ \theta\ d\theta + + \frac{2}{\pi}\int_{\alpha}^{\pi/2}\left(I\mathrm{Sin}\theta - b + \frac{c}{2}\right)\mathrm{Cos}\theta\ .d\theta$$

$$+ \frac{2}{\pi}\int_{\pi/2}^{\gamma}\left(1-b+\frac{c}{2}\right)\mathrm{Cos}\ \theta\ d\theta + \frac{2}{\pi}\int_{\gamma}^{\pi}\left(I\mathrm{Sin}\theta + \frac{c}{2}\right)\mathrm{Cos}\ \theta\ d\theta$$

$$= \frac{2}{\pi}\left[\frac{I}{2}\mathrm{Sin}^2\beta - \frac{c}{2}\mathrm{Sin}\ \beta\right] + \frac{2}{\pi}\left[(I/4 - b + \frac{c}{2}) + (b - \frac{c}{2})\mathrm{Sin}\ \alpha + \frac{I}{4}\mathrm{Cos}\ 2\alpha\right]$$

$$+ \frac{2}{\pi}\left[(I-b+\frac{c}{2})\ \mathrm{Sin}\gamma - I + b - \frac{c}{2}\right] + \frac{2}{\pi}\left[-\ \frac{I}{2}\mathrm{Sin}^2\gamma - \frac{c}{2}\mathrm{Sin}\gamma\right]$$

Substituting the values α , β and γ we get ,

$$a_1 = \frac{2}{\pi}\left[-\frac{c^2}{8I} + \frac{4I^2+4b^2+c^2-8bI+4cI-4bc}{8\ I} + \frac{2\ b^2\ -bc\ -2bI}{2I} + \frac{bc-b^2-I^2+2bI-Ic}{2I}\right].$$

$$= \frac{2}{\pi}\left[\frac{8\ b^2-8bI-4bc}{8I}\right]$$

$$a_1 = \frac{2\ b^2\ -\ 2bI\ -\ bc}{\pi\ I}$$

Now $$b_1 = \frac{2}{\pi}\int_0^{\pi} O\ (\theta)\ \mathrm{Sin}\ \theta\ d\ \theta$$

$$\therefore b_1 = \frac{2}{\pi}\left[\int_0^{\beta}\left(I\sin\theta - \frac{c}{2}\right)\sin\theta\, d\theta + \int_{\alpha}^{\pi/2}\left(I\sin\theta - b + \frac{c}{2}\right)\sin\theta\, d\theta.\right.$$

$$\left. + \int_{\pi/2}^{\gamma}\left(I - b + \frac{c}{2}\right)\sin\theta\, d\theta + \int_{\gamma}^{\pi}\left(I\sin\theta + \frac{c}{2}\right)\sin\theta\, d\theta.\right]$$

$$= \frac{2}{\pi}\left[\frac{I}{2}(\beta - \sin\beta\cos\beta) + \frac{c}{2}(\cos\beta - 1)\right] + \frac{2}{\pi}\left[\frac{I}{2}\left(\frac{\pi}{2} - \alpha + \sin\alpha\cos\alpha\right)\right.$$

$$\left. + \cos\alpha\left(\frac{c}{2} - b\right)\right] + \frac{2}{\pi}\left[-I\cos\gamma + b\cos\gamma - \frac{c}{2}\cos\gamma\right] + \frac{2}{\pi}\left[\frac{I}{2}(\pi - \gamma + \sin\gamma\cos\gamma) + \frac{c}{2}(1 + \cos\gamma)\right]$$

After substituting the values of α, β and γ, we get.

$$b_1 = \frac{2}{\pi}\left[\frac{4I^2\sin^{-1}\left(\frac{c}{2I}\right) + c\sqrt{4I^2 - c^2} - 4Ic}{8I} + \frac{2\pi I - 4I\sin^{-1}\left(\frac{b - \frac{c}{2}}{I}\right)}{8}\right.$$

$$+ \frac{\sqrt{2Ib - b^2}\,(8b - 4c - 8I)}{8I}$$

$$\left. + \frac{4\pi I^2 - 4I^2\sin^{-1}\left(\frac{I-b}{I}\right) + (4I - 4b)\sqrt{2Ib - b^2} + 4IC + 4c\sqrt{2Ib - b^2}}{8I}\right]$$

$$b_1 = \frac{1}{4\pi I}\left[4I^2\left\{\sin^{-1}\left(\frac{c}{2I}\right) - \sin^{-1}\left(\frac{b - \frac{c}{2}}{I}\right) - \sin^{-1}\left(\frac{I-b}{I}\right)\right\} + 6\pi I^2\right.$$

$$\left. + c\sqrt{4I^2 - c^2} + 4.\sqrt{2Ib - b^2}\,(b - I)\right]$$

$$\text{and } Gd = \frac{\sqrt{a_1^2 + b_1^2}}{I} \angle \tan^{-1}\frac{a_1}{b_1}$$

The value of Gd is found after knowing the numerical values of a_1 and b_1 which are found after substituting the values of constants in their respective expressions.

The complete expressions for various describing function approaches are as given below :

I Piece wise Linearization:

$$Gd. = \frac{2}{\pi}\left\{ \frac{c}{a}\left(\mathrm{Sin}^{-1}\frac{a}{I} - \tfrac{1}{2}\,\mathrm{Sin}\,2\,\mathrm{Sin}^{-1}\frac{a}{I}\right) + \left(\frac{bc-ad}{b-a}\right)\frac{2}{I}\right.$$

$$\left. .\ \mathrm{Cos}\,\mathrm{Sin}^{-1}\frac{a}{I} + \left(\frac{d-c}{b-a}\right)\left(\frac{\pi}{2} - \mathrm{Sin}^{-1}\frac{a}{I}\right) + \tfrac{1}{2}\left(\frac{d-c}{b-a}\right)\mathrm{Sin}\,2\,\mathrm{Sin}^{-1}\frac{a}{I}\right\}$$

II Froelich's Equation:

Case (i) if $a > bI$

$$Gd = \frac{2}{\pi I^2 b^2}\left[2bI + \frac{4a^2}{\sqrt{a^2-(bI)^2}}\tan^{-1}\sqrt{\frac{a-bI}{a+bI}} - a\pi\right] \angle 0$$

Case (ii) if $a < bI$

$$Gd = \frac{2}{I^2\,\pi\,b^2}\left[\frac{2a^2}{\sqrt{(bI)^2 - a^2}}\log\left(\frac{bI}{a}\right) + 2bI - a\pi\right] \angle 0$$

III Exponential Rise:

$$Gd = \frac{2K}{\pi I}\left[2 - 2J_0(jaI) + \frac{4}{3}J_2(jaI) + \frac{4}{15}(jaI)\right.$$

$$+ \frac{4}{35}J_6(jaI) + \frac{4}{63}J_8(jaI) \ldots\ldots$$

$$\left. - jJ_1(jaI)\pi\right] \qquad \angle 0$$

IV Rectangular Hysteresis loop:

$$Gd = \frac{K}{\pi}\left\{\left[\frac{\pi}{2} + \mathrm{Sin}^{-1}(1-2R)\right]^2 + 4R(1-R) + \left[\pi + 2\mathrm{Sin}^{-1}\right.\right.$$

$$\left.\left.(1-2R)\right]\ 2R(1-2R)\sqrt{\frac{1-R}{R}}\right\}^{1/2}$$

$$\phi = \tan^{-1} \frac{4R(R-1)}{\frac{\pi}{2} + \sin^{-1}(1-2R) + 2R(1-2R)\sqrt{\frac{1-R}{R}}}$$

V. Hysteresis Loop Considering the Effect of Brush Contact Resistance Drop:

$$a_1 = \frac{2b^2 - 2bI - bc}{\pi I}$$

$$b_1 = \frac{1}{4\pi I} \left[\left\{ 4I^2 \left(\sin^{-1}\left(\frac{c}{2I}\right) - \sin^{-1}\left(\frac{b - \frac{c}{2}}{I}\right) - \sin^{-1}\left(\frac{I-b}{I}\right) \right) \right\} + 6\pi I^2 + c\sqrt{4I^2 - c^2} + 4\sqrt{2Ib - b^2}\,(b - I) \right]$$

$$\therefore \quad Gd = K \frac{\sqrt{a_1^2 + b_1^2}}{I} \quad , \quad \angle \tan^{-1} \frac{a_1}{b_1}$$

3.3. DERIVATION OF THE TOTAL TRANSFER FUNCTION OF THE SYSTEM :

The describing functions for the nonlinearities such as saturation and hysteresis existing in the amplidyne have been found. Now the total loop transfer function of the system is found out, assuming all system elements to be linear. Because in describing function analysis non-linear block has been separated from linear components.

The system under consideration is as shown in the fig.19 Amplidyne is serving the purpose of automatic voltage regulator in this system. First the transfer functions of the various system blocks are found out separately and then by suitably reducing the block diagram of the system, the overall G(s) of the system is known.

G(s) of Voltage Regulation System:

Referring to the fig. 19, the forward loop comprises of three blocks namely Amplidyne, Generator and describing function block.

Transfer function of the Amplidyne :

In deriving this transfer function the following assumptions [16] are made:-

(i) Zero - coupling between the flux in the Direct axis and the flux in the quadrature axis.

(ii) Non-linearity is neglected as it has been taken in to account in the describing function analysis.

Referring to the figure 20, the voltage equations for both the windings are as follows :

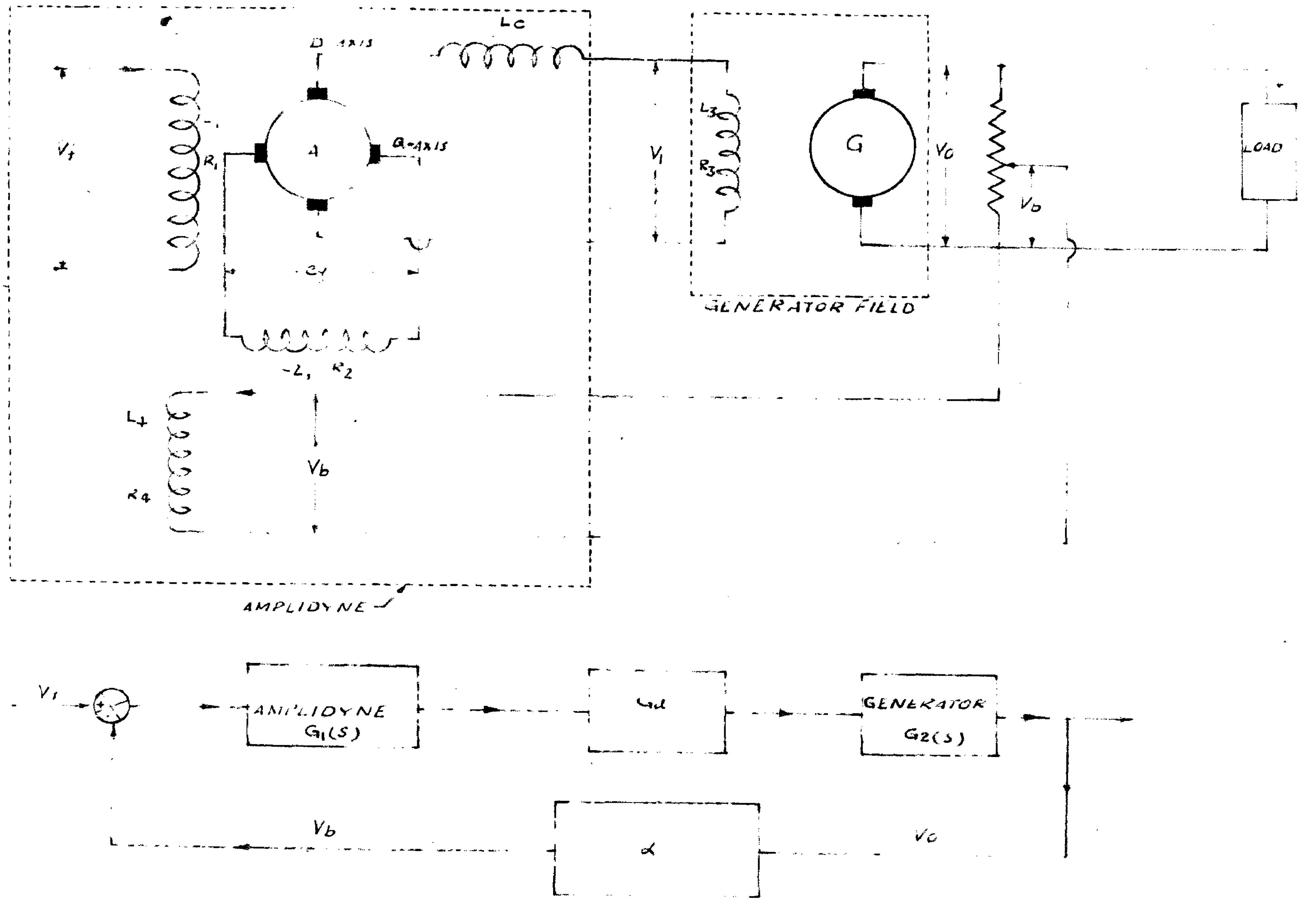


FIG. 19. VOLTAGE REGULATION SYSTEM UNDER CONSIDERATION

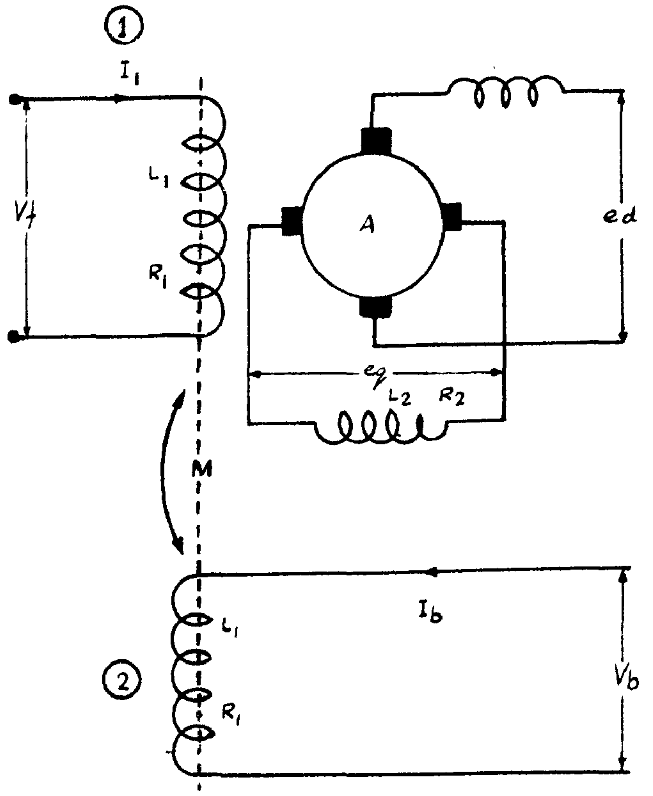


FIG. 20- AMPLIDYNE

$$V_f = (R_1 + SL_1)I_1 + S\,MI_b \quad --- \quad (1)$$

$$V_b = (R_1 + SL_1)I_b + SMI_1 \quad --- \quad (2)$$

Windings (1) and (2) are identical in nature.

$$\therefore I_1 = \frac{V_f(R_1+SL_1) - V_bSM}{(R_1+SL_1)^2 - S^2 M^2} \quad --- \quad (3)$$

$$I_b = \frac{V_b(R_1+SL_1) - V_fSM}{(R_1+SL_1) - (SM)^2} \quad ---- \quad (4)$$

$$I_1 - I_b = \frac{(V_f - V_b)\ (R_1 + SL_1 + SM\ )}{R_1^2 + 2\ SL_1 R_1 + S^2\ (\ L_1^2 - M^2)} \qquad \text{-- (5)}$$

$$\frac{(I_1 - I_b)}{(V_f - V_b)}\ (S) = \frac{(R_1 + SL_1 + SM\ )}{(R_1^2 + 2\ SL_1\ R_1 + S^2\ (L_1^2 - M^2\ )} \qquad \text{--(6)}$$

$$e_q = K_q\ \phi_d \qquad \text{--- (7)}$$

$$= K_1 (i_1 - i_b) \qquad \text{--- (8)}$$

Where K_1 = no. of volts induced in the q axis per unit control fld. current in amps.

also $e_q = i_2 R_2 + L_2 \dfrac{di_2}{dt}$ --- (9)

or $K_1\ (\ i_1 - i_b\ ) = i_2\ R_2 + L_2 \dfrac{di_2}{dt}$ --- (10)

or $\dfrac{I_2}{(I_1 - I_b)}\ (S) = \dfrac{K_1}{R_2(1 + S\tau_2)}$ --- (11)

Now $e_d = K_2\ i_2$ --- (12)

where K_2 = number of volts induced in the d- axis circuit for per ampere in the a axis.

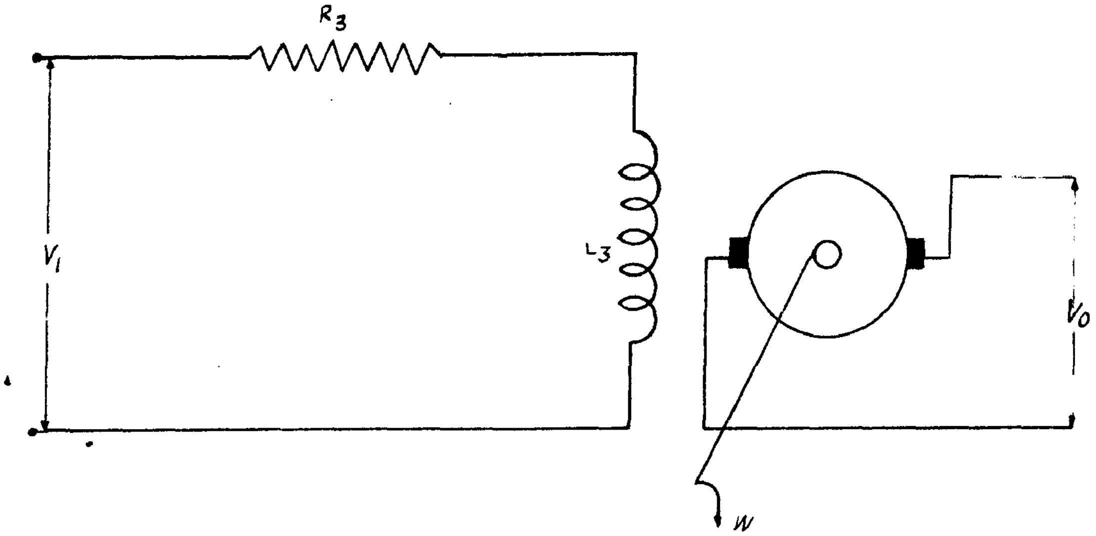
or $\dfrac{E_d}{I_2}\ (S) = K_2$ --- (13)

Multiplying equations (6), (11) and (13)we get.

$$\frac{Ed}{I_2}(S) \times \frac{I_2}{(I_1 - I_b)}(S) \times \frac{(I_1 - I_b)}{(V_f - V_b)}(S) = \frac{Ed}{(V_f - V_b)}(S)$$

$$\therefore T.F = G_1(s) = \frac{Ed}{V_f - V_b}(S) = \frac{K_2 K_1 (R_1 + SL_1 + SM)}{R_2(1+ST_2)(R_1^2+2SL_1R_1+S^2(L_1^2-M)} \quad \text{---(14)}$$

Transfer Function of the Generator The schematic diagram of the generator is as shown below



GENERATOR

.... 21

Referring to the figure 21 the equation for the field can be written as

$$V_1 = i_3R_3 + L_3 \frac{di_3}{dt} \quad \text{--- (1)}$$

and $$V_0 = K_g\, i_3 \quad \text{---(2)}$$

Where Kg is the generator constant number of volts generated for per unit field current in ampere.

Transforming we get,

$$K_g = \frac{V_o}{I_3}(S) \qquad \text{----(3)}$$

and $V_1(S) = R_3 I_3(S) + SL_3 I_3(S)$ ----(4)

$$\therefore \frac{V_o}{V_1}(S) = \frac{Kg\ I_3(S)}{R_3 I_3(S) + SL_3 I_3(S)} \qquad \text{----(5)}$$

$$= \frac{K_g\ I_3\ (S)}{(R_3 + SL_3)I_3(S)} = \frac{Kg}{R_3(1+ST_3\ )}$$

Putting $Kg/R_3 = K$, then the transfer function of the generator is

$$\frac{V_o}{V_1}(S) = \frac{K}{1+ST_3} = G_2(S)$$

Now the total transfer function G(S) of the system is found by block diagram reduction. And knowing both Gd(describing function) and G(S) - (Total transfer function) of the system, Nyquist diagrams can be plotted to study the stability of the machine.

Referring to the diagram, we get,

$$\text{Over all T.F.} = \frac{G_1(S)\ G_2(S)\ G_d(S)}{1+ G_1(S)\ G_2(S)\ G_d(S)}$$

$$\therefore G(S) = G_1(S)\ G_2(S)$$

$$G(S) = \frac{K_1\ K_2\ Kg\ \alpha\ (R_1 + SL_1 + SM)}{R_2\ R_3\ (1 + S\ _2)\ (\ 1+S\ _3)\left[(R_1^2 + 2SL_1R_1 + S^2(L_1^2 - M^2)\right]} \qquad \text{--(A)}$$

Hence the complete expressions for the describing function and for the overall transfer function are known.

3.4. EVALUATION OF VARIOUS CONSTANTS:

The values of constants needed in various describing functions and in over all transfer functions are found either practically or from total saturation curve and hysteresis loop.

Amplidyne:

The resistance self-inductance and mutual inductance of various windings are found and noted as shown below :

D.C. Resistance of control field winding $f_1 f_2$ and $f_3\ f_4$

$$= 980 \text{ Ohms.}$$

$\therefore\ R_1 = 980$ Ohms.

Resistance of quadrature circuit winding = .966 Ohms.

Resistance of the series coil inserted in the quadrature circuit = 0.311 Ohms.

$\therefore$ Total resistance of the quadrature circuit =0.966+0.311

= 1.277 Ohms

$\therefore\ R_2 = 1.277$ Ohms.

Self inductance of control field winding $f_1 f_2$ and f = 3.1 henery.

$\therefore$ Time constant of control field winding $= \frac{3.1}{980} = .0031$

Total self inductance of quadrature field circuit = .1048 henery

$\therefore$ Time constant of quadrature circuit $= \frac{.1048}{1.277} \approx .082$

Resistance of antihunt coil $f_{13}\ f_{14}$ = 55.5 Ohms.

Self inductance of antihunt coil = 1.56 H.

Resistance of field winding $f_3 f_4$ = 980 Ohms.

Self inductance of field winding $f_3 f_4$ = 3.1 henery.

Mutual inductance between $f_1 f_2$ and antihunt coil $f_{13} f_{14}$ =5.23 H.

Mutual inductance between field winding $f_3 f_4$ and $f_1 f_2$ = 2.7 H.

Value of $\alpha = \frac{V_b}{V_o} = 0.353$.

Value of the rheostatic resistance in the antihunt circuit = 3.5 Ohms.

Value of the capacitance in the antihunt circuit = C.= 120 μ F

K_1 = number of volts induced in the q - axis for per unit control field current in amps. = 540 volts.

Self inductance of direct axis circuits = 0.0446 H

Resistance of direct axis circuit = 0.886 Ohms.

K_2 = number of volts induced in the d- axis circuit per ampere in the Q axis = 35 Volts.

Resistance of compensating winding = .621 Ohms.

Self inductance of compensating winding =.0338 H.

Total time const. of D- axis circuit = .052.

Moment of Inertia of the Machine:

The value of moment of inertia is required in the motional impedance matrix (V Chapter) for finding the condition of electromechanical oscillations.

We know that moment of inertia I at any moment. $= \frac{W}{0.0149 N \frac{dN}{dt}}$ --(1)

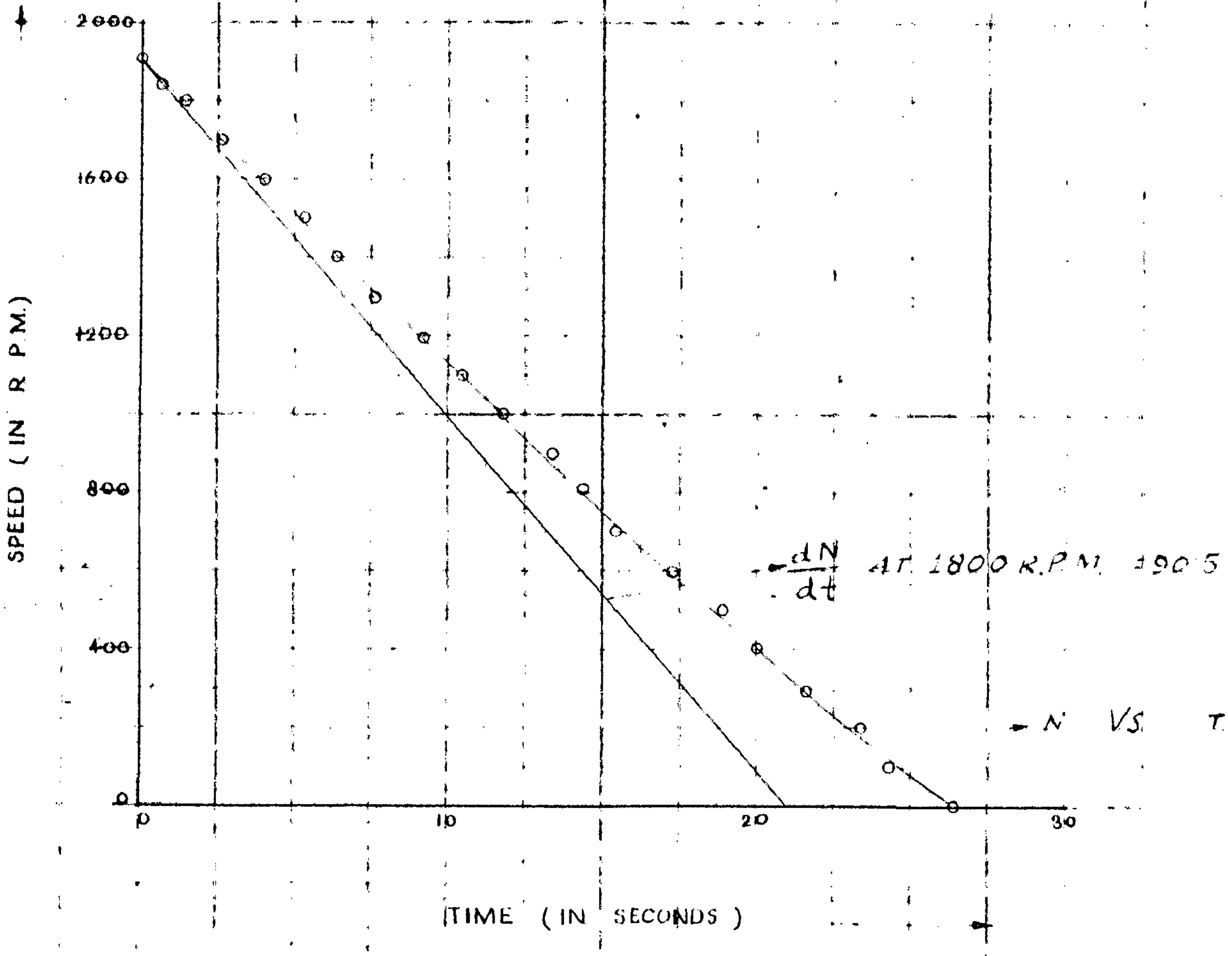
where

W = Wattage at any moment.

I' = Moment of inertia in lb. ft^2.

N = r.p.m.

Retardation curve is plotted as shown in the Fig. 26



2000
1600
1200
800
400
0
SPEED (IN R P M.)
0
10
20
30
TIME (IN SECONDS)
dN/dt AT 1800 R.P.M. 190.5
N VS. T

FIG. 20.

to find out $\frac{dN}{dt}$ at N r.p.m., w is also the no load power at N (rated no load speed) . Substituting the numerical values in the above equation 1, we get

$$I' = \frac{1000}{.0149 \times 1800 \times 90.2} = .421 \text{ lb. ft}^2$$

Generator:

The following are values of winding parameters needed in the transfer function of the generator.

Generator field resistance = 102.5 Ohms.

Generator field inductance = 7.6 henery.

Generator field time constant= $\frac{7.6}{102.5}$ = .074

Armature winding self inductance = .915 henery.

Armature winding resistance = 1.01 Ohm.

Armature winding time constant = $\frac{.915}{1.01}$ =.905

Kg = volts per field ampere of generator = 100 volts.

Values of Constants required in Various Describing Function approaches:-

(i) In piecewise linearization:

a = 13.7 mA.

b = 25 mA

c = 125 V = $\frac{135 \times 10^3}{980}$ mA = 127.55

d = 151 V = $\frac{151 \times 10^3}{980}$ mA = 154.

(ii) Froelich's Equation:

The mean saturation curve is mathematically expressed as :

$$E = \frac{i}{a + bi} \quad \text{or} \quad \frac{i}{E} = a + bi$$

let $\frac{i}{E} = y$ and $i = x$

$\therefore$ Modified equation is $y = a + bx$

Now if y and x are plotted then intercept on y-axis will give the value of "a" and intercept on x-axis will be equal to $-\frac{a}{b}$.

From the figure 22 , a = .09 , b = .0025.

It is clear from the figure 23 that curve (B) which is drawn with calculated value of E closely fits the mean saturation curve. The following intuitive criteria should be kept in mind while fitting the curve.

(a) The vertical distance between the two curves must be kept to a minimum at all points.

(b) The net area between the two curves should be as small as possible.

(c) The slopes of the curves near the origin should be as close as possible.

(iii) Exponential Rise:

The mathematical equation representing the curve is $E = K\ (1 - e^{-ai})$

or $\frac{E}{K} = (1 - e^{-ai})$ or $(1 - \frac{E}{K}) = e^{-ai}$

CURVES DRAWN FOR THE EVALUTION OF CONSTANTS.

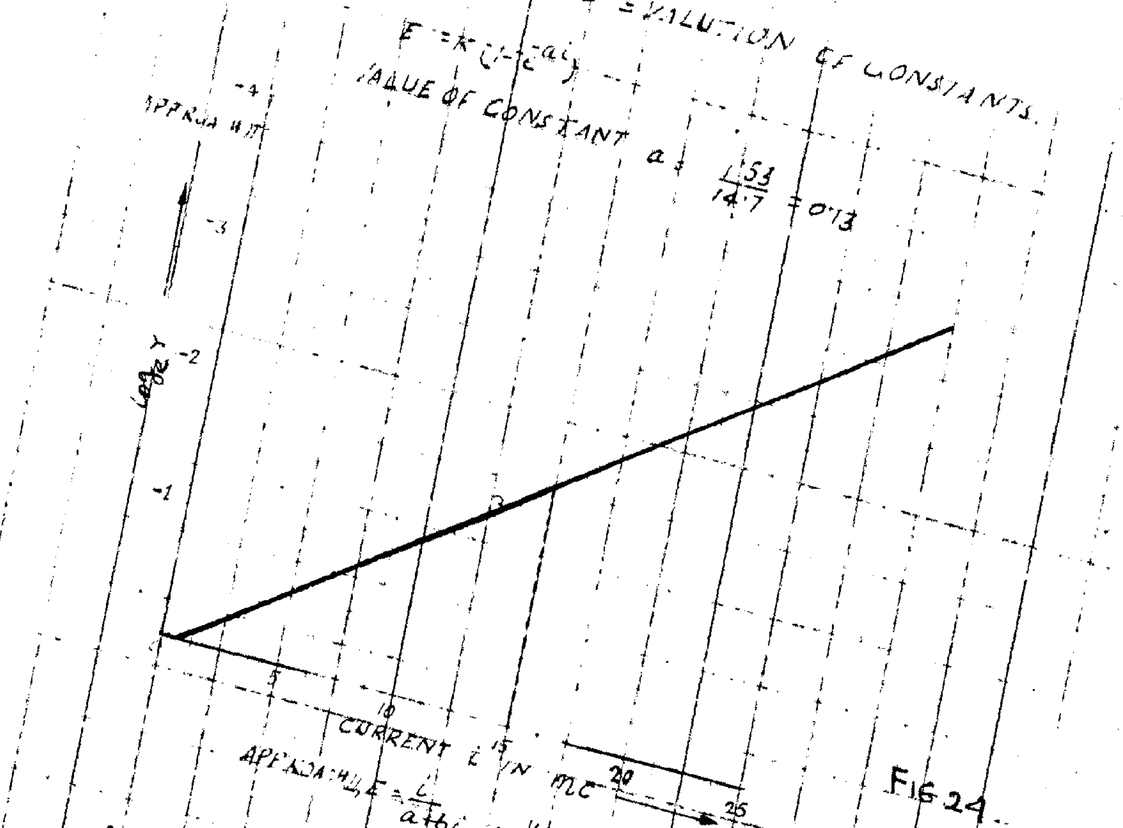


FIG 24

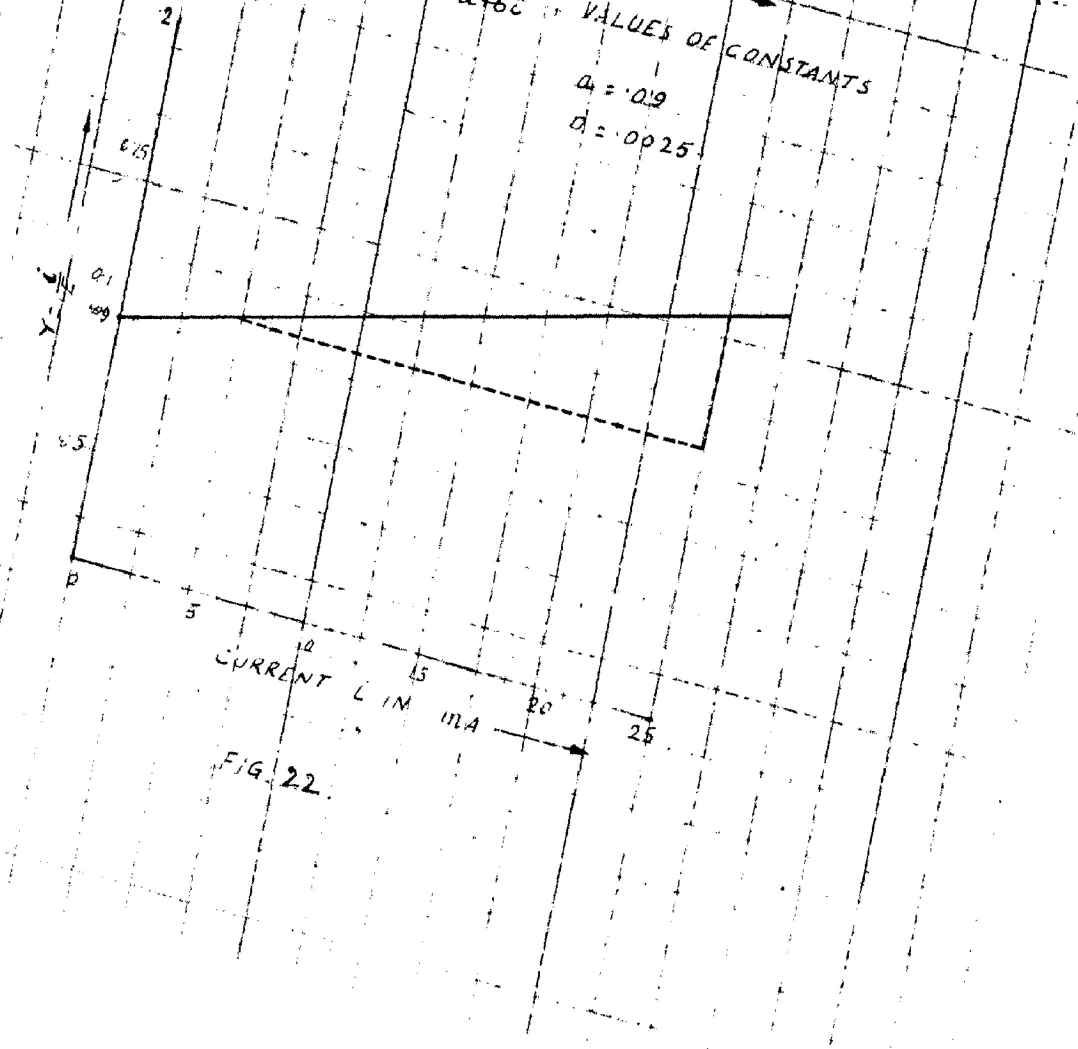
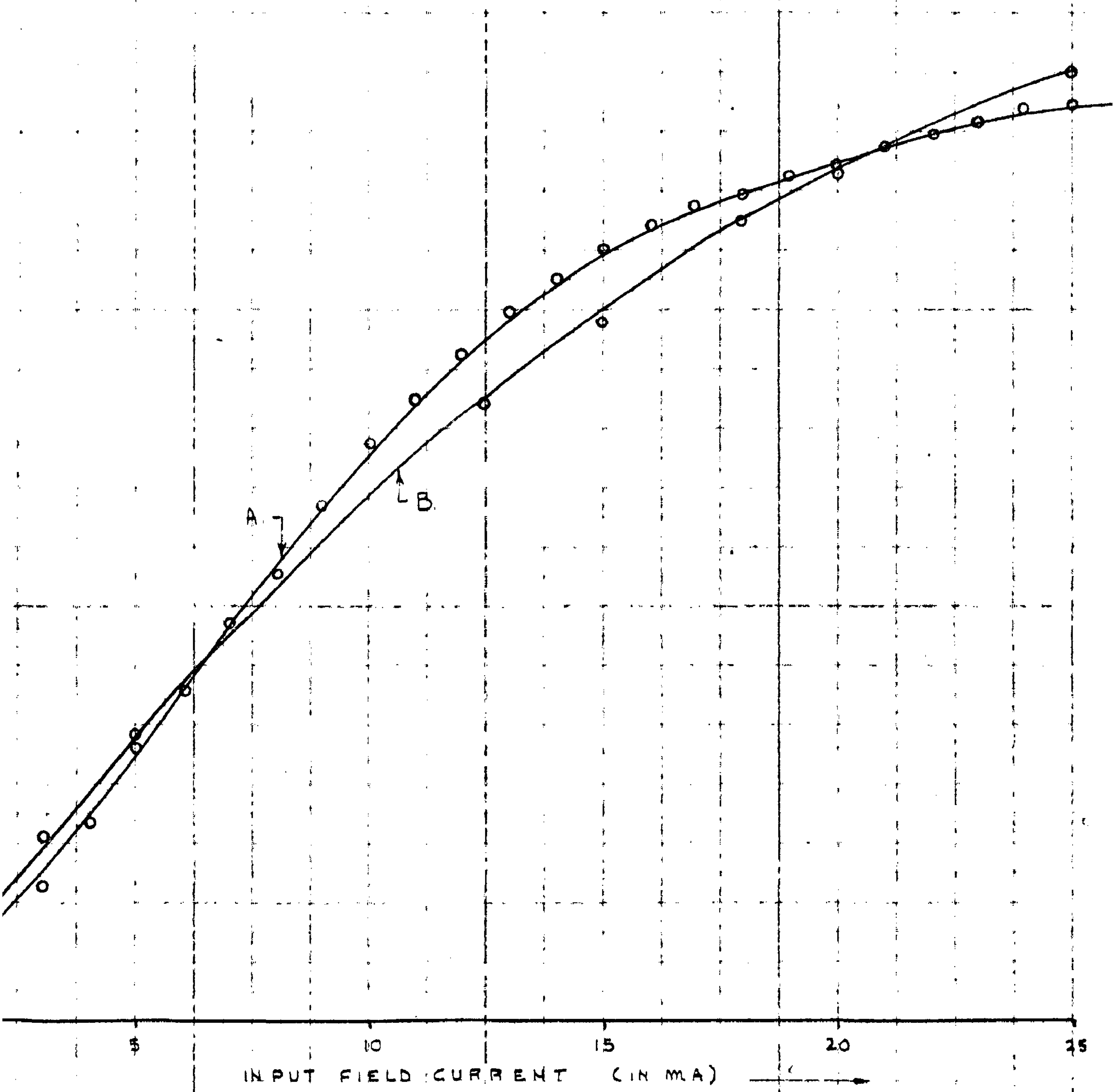


FIG 22.

CURVE FITTING FOR APPROACH NO. II $E = \frac{i}{a + bi}$



A.....ACTUAL CURVE.
B.....OBTAINED BY CURVE FITTING

Fig. 23

Putting $(1-\frac{E}{K}) = y$ and $i = x$

Modified equation becomes

$$y = e^{-ax}$$

$$\text{or } \log_e y = -ax$$

where K = ceiling value of voltage = 154 volts.

Plotting the curve between $\log_e y$ and x, the value of 'a' is found out. Reference Figure 24 , the slope of curve $= -a = -.13$ therefore $a = 0.13$. Figure 25 shows the curve fitting of this approach. The curve (B) quite closely fits the curve (A) thereby confirming the value of a = 0.13.

(iv) Rectangular Hysteresis Loop:

The following are the values of constants needed in this approach.

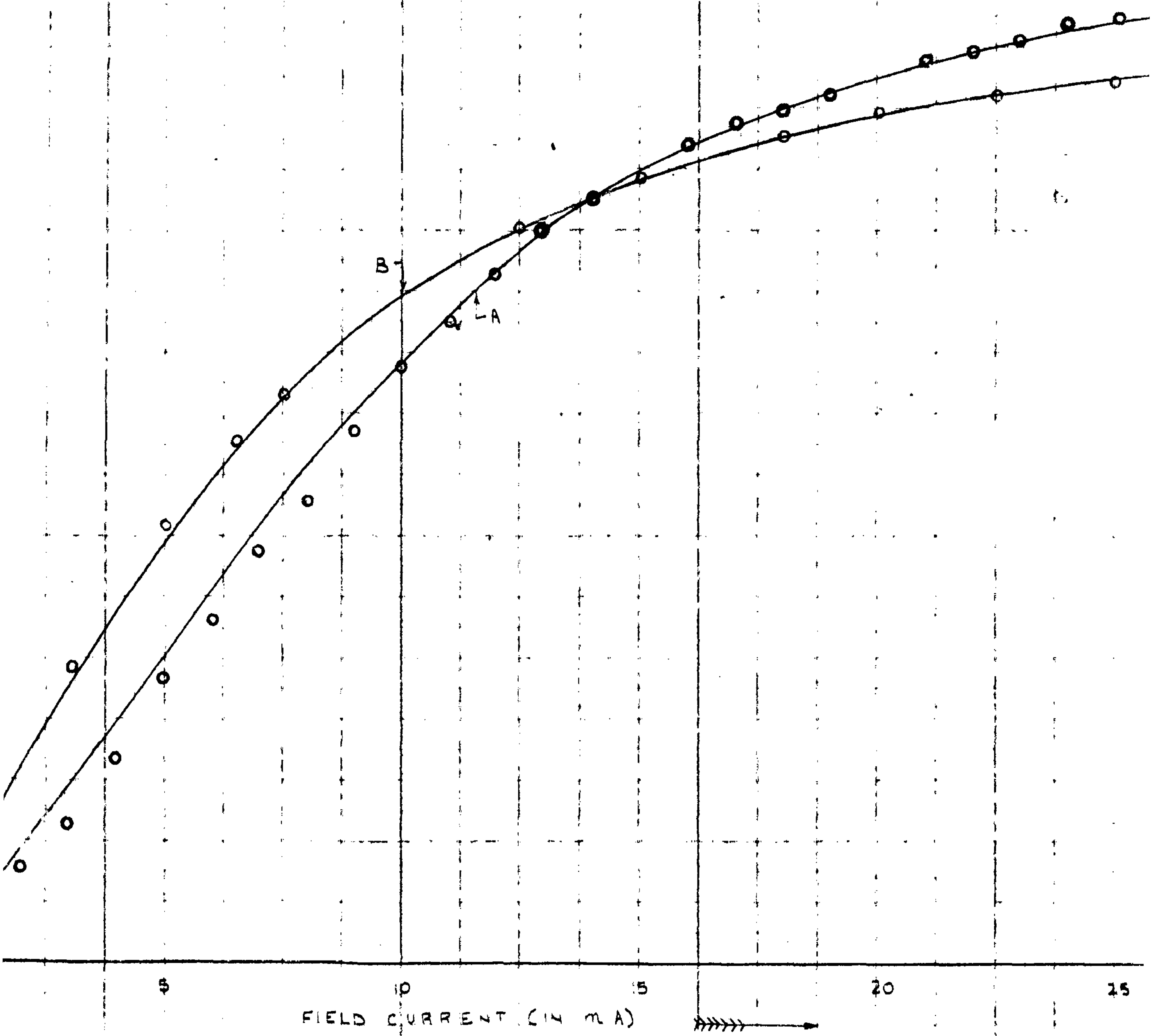
b = 2 mA.

$$\text{Slope} = \frac{50 \times 10^3}{4.5 \times 980} = 11.35$$

(v) Hysteresis Loop considering the Brush contact resistance drop:

In this approach, the hysteresis loop of the machine has been represented as already shown in the Fig. 15 and the required values of the constants are as given below :

b = 1.5 mA c = 1 mA slope = 11.35.



CURVE (A)......ACTUAL MEAN INPUT-OUTPUT CURVE

CURVE (B)......OBTAINED BY CURVE FITTING

Fig. 2.

CHAPTER 4.

NYQUIST PLOT ANALYSIS.

NYQUIST PLOTS ANALYSIS

4.1. PLOTTING:

On substituting the numerical values of the constants in the G(jw)& describing function expressions we get

$$G(jw) = \frac{1785000}{(S+12.2)(S+2107)(S+13)}$$

$$= \frac{1.785 \times 10^6}{(S+12.2(S+2107)(S+13)}$$

I Approach:

$$Gd = \left(4.41 \operatorname{Sin}^{-1}\frac{13.75}{I} - 2.207 \operatorname{Sin} 2 \operatorname{Sin}^{-1}\frac{13.75}{I} + \frac{121.5}{I} \times \operatorname{Cos}\operatorname{Sin}^{-1}\frac{13.75}{I} + 2.35\right)$$

II. Approach:

Case (i) $\quad .09 > .0025I$

$$Gd = \frac{2}{(.0025)^2 I^2}\left(.005I + \frac{4(.09)^2}{\sqrt{.0081-(.0025)^2 I}}\tan^{-1}\sqrt{\frac{.09-.0025I}{.09+.0025I}} - .09\pi\right)$$

Case (ii) $\quad .09 < .0025I$

$$Gd = \frac{2}{(.0025)^2 I^2}\left(.005I + \frac{2(.09)^2}{\sqrt{(.0025)^2 I^2-(.09)^2}}\log\frac{.0025I}{.09} - .09\pi\right)$$

III. Approach:

$$Gd = \frac{100}{I}\Big[(2-2J_o(.13jI) + \frac{4}{3} J_2(.13jI) + \frac{4}{15} J_4(.13jI)$$

$$+ \frac{4}{35} J_6(.13jI) + \frac{4}{63} J_8(.13jI) + \frac{4}{99} J_{10}(.13jI) + \ldots.$$

$$- jJ_1(.13jI)\Big]$$

IV. Approach:

$$Gd. = \frac{11.35}{\pi}\left\{\left[(\frac{\pi}{2} + Sin^{-1}(1-2R)\right]^2 + 4R(1-R) + \left[(\pi + 2Sin^{-1}(1-2R)\right](2R-4R^2) \times \sqrt{\frac{1-R}{R}}\right\}^{\frac{1}{2}}$$

Where $R = 1/I$

$$\phi = \frac{\tan^{-1} 4R(R-1)}{\frac{\pi}{2} + Sin^{-1}(1-2R) + \sqrt{\frac{1-R}{R}} (2R-4R^2)}$$

V. Approach:

$$a_1 = \frac{0.955(1-I)}{I}$$

$$b_1 = \frac{.0795}{I}\left\{\left[4I^2(Sin^{-1}\frac{.5}{I} - Sin^{-1}\frac{1}{I} - Sin^{-1}(\frac{1-1.5}{I}) + 1.5\pi)\right] + \left[\sqrt{4I^2-1} + 4(\sqrt{3I-2.25})(1.5-I)\right]\right\}$$

$$\therefore Gd = \frac{11.35\sqrt{a_1^2 + b_1^2}}{I} \quad \text{and} \quad \phi = \tan^{-1}\frac{a_1}{b_1}$$

The table on the next page shows the values of G(jw) for various values of w.

TABLE .

W	G(S)	Angle
0.0	5.35	0.0
0.5	5.35	- 4.548
1.0	5.35	- 9.1
3.0	4.95	- 26.5
5.0	4.52	- 43.036
10.0	3.23	- 76.572
12.0	2.945	- 88.53
13.0	2.59	- 92.153
20.0	1.51	-116.193
50.0	0.313	-153.36
75.0	0.1465	-163.76
100.0	0.0834	-168.37
200.0	0.0211	-178.2
250.0	0.0134	-180.98
300.0	0.0093	-183.3
400.0	0.0052	-187.15
500.0	0.0034	-190.46
1000.0	0.00076	-203.95
1500.0	0.0003	-214.535
2107.0	0.000135	-224.32
3000.0	0.000054	-234.65
5000.0	0.0000132	-246.98
10000.0	0.0000017	-258.0
50000.0	0.00000029	-267.8
∞	0	-270.0

Now G(jw) and - 1/Gd terms are plotted to the same scale for all the approaches. These curves may fit into one of three classifications[11]. (i) if there are no intersections between G(jw) curve and the - 1/Gd curve and if the G(jw) curve encloses all the -1/Gd curve, then the system is absolutely unstable (ii) if there are no intersections and the G(jw) curve does not enclose - 1/ Gd curve, then the system is absolutely stable (iii) if there are intersections then the system may be unstable or may (exhibit limit cycle) be stable depending on the region of operation.

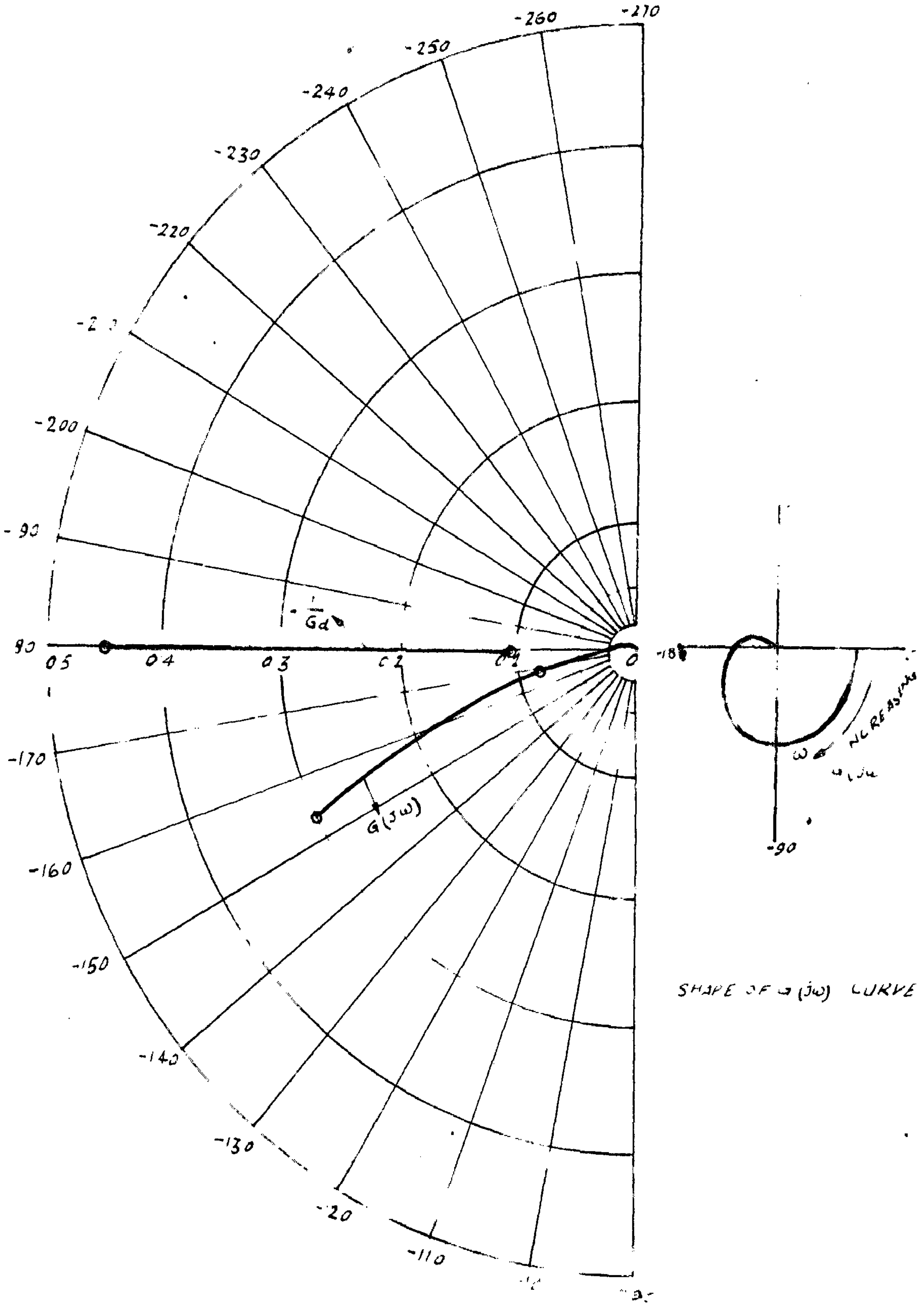
Referring to the plotting of first approach in the Figure 27 , it is concluding that the system is absolutely stable.

The Fig. 28, shows the plotting for the second approach. This analysis clearly shows the absolute stability of the system.

The third approach (exponential curve approximation) also proves the system to be absolutely stable. (Fig. 29).

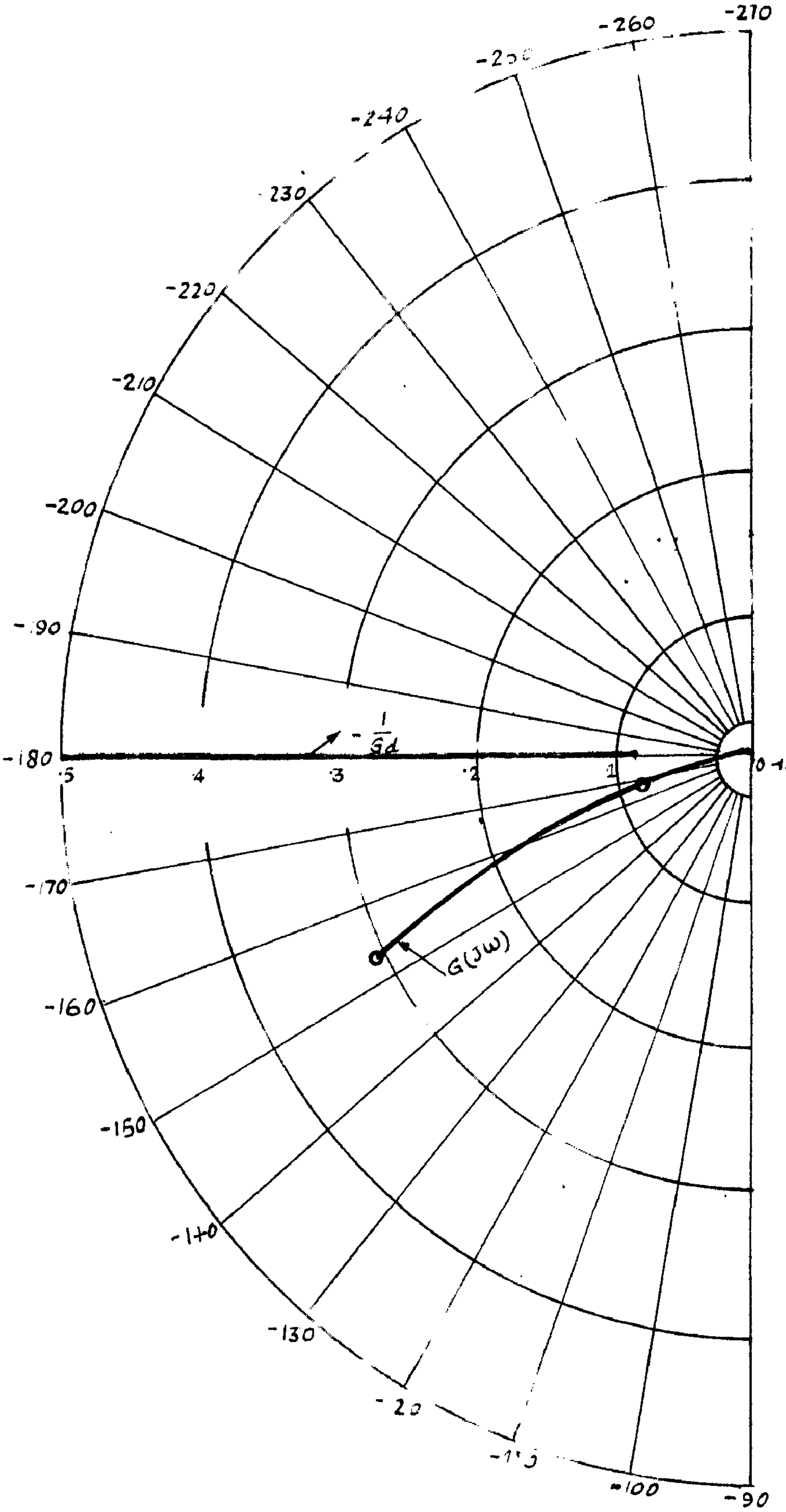
Hence the first three approaches come to the same conclusion of the absolute stability of the system.

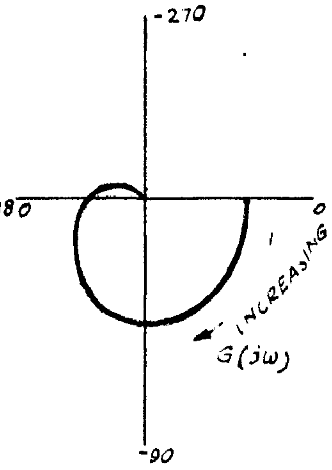
The plotting of fourth approach (Fig.30) shows that there are two intersection points A and B between the plots of G(jw) and - 1/Gd , thereby indicating the possibility of unstability of the machine. PA is the stable range of the machine. As soon as the point A is crossed then the machine B. becomes unstable and a periodic oscillation of increasing amplitude is set up. If the oscillation amplitude is increased beyond that defined at B, then amplitude of oscillation will go on decreasing and ultimately the machine will exhibit limit cycle. The amplitude of periodic oscillation will be defined by the value of - 1/Gd at point B and of fundamental frequency by the value of w associated with G(jw) at the point B.



POLAR PLOT FOR DESCRIBING FUNCTION ANALYSIS IN FIRST APPROACH

FIG. 21.

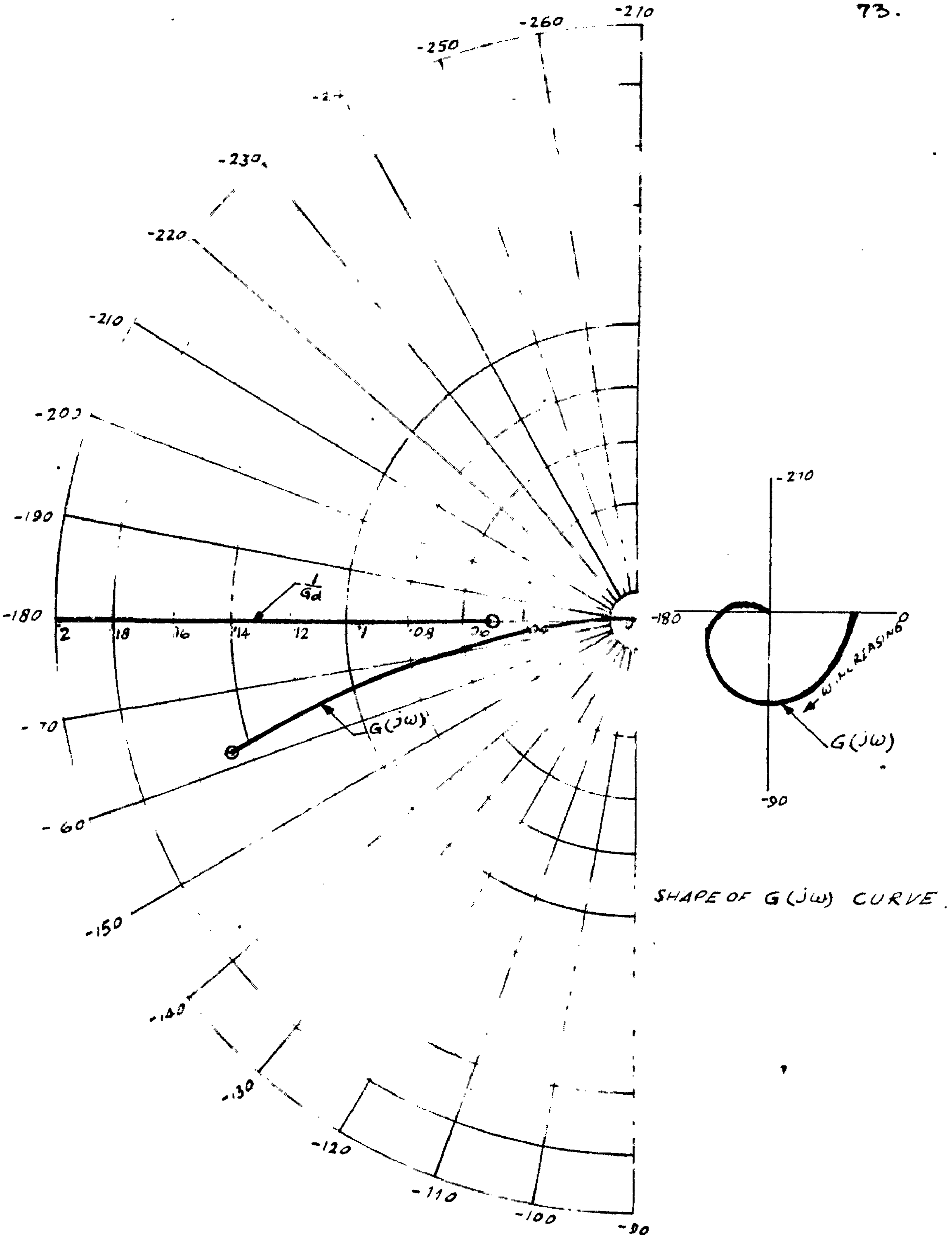




SHAPE OF G(jω) CURVE

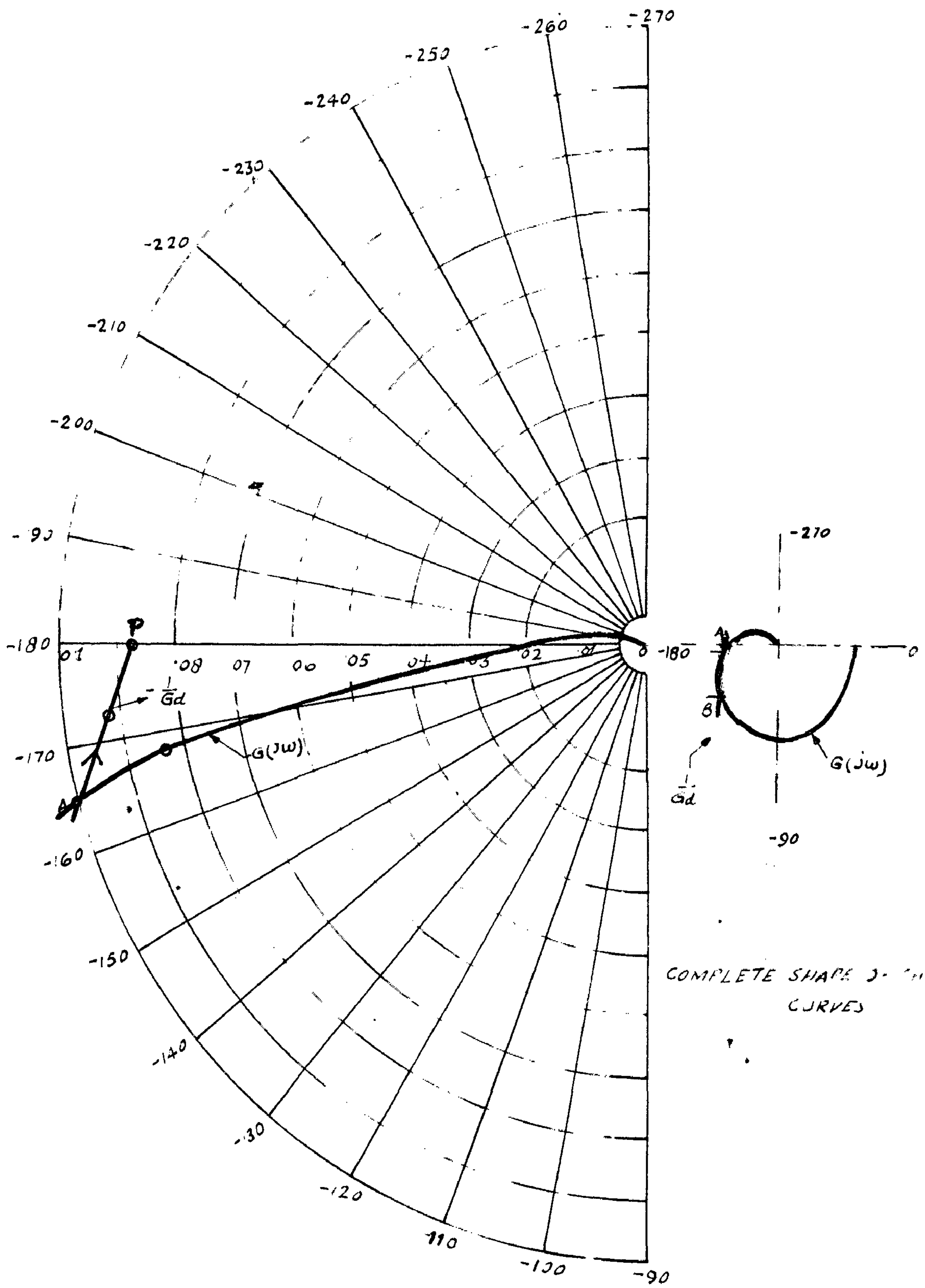
POLAR PLOT FOR DESCRIBING FUNCTION ANALYSIS
IN SECOND APPROCH.

FIG. 28



POLAR PLOT FOR DESCRIBING FUNCTION ANALYSIS
IN THIRD APPROCH.

FIG. 29

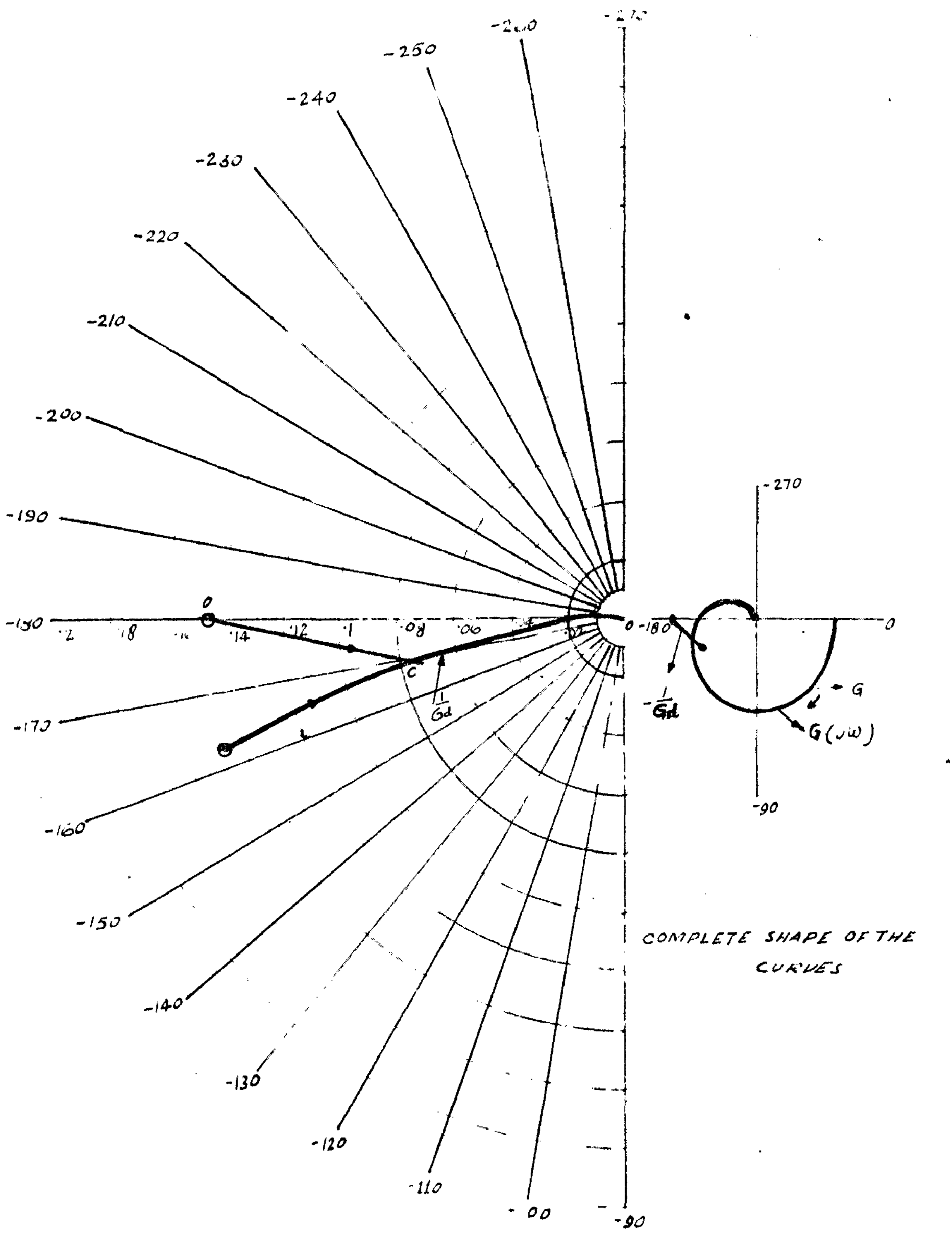


POLAR PLOT FOR DESCRIBING FUNCTION ANALYSIS IN FOURTH APPROCH.

MAGUITUDE AT A = 0.1

ω AT A 110

FIG. 30.



POLAR PLOT FOR DESCRIBING FUNCTION ANALYSIS IN FIFTH APPROCH.

MAGNITUDE AT C = ·076

ω AT C 120

FIG. 11.

The fifth analysis also shows the possibility of unstability as obtained in the fourth approach. It is seen from Fig. 31 that the two loci intersect at the point C. The system is stable in the region OC and beyond the point C of intersection periodic oscillation of amplitude defined by the value of - 1/Gd at point C and of fundamental frequency defined by the value of w associated with G(jw) at point C will be obtained.

Thus it is observed that the last two approaches predict the possibility of the unstability of the machine and thereby depict the correct behaviour of the machine under consideration. Hence it can be concluded that it is the hysteresis effect and the excessive brush contact resistance drop which are responsible for the probability of unstability.

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4.2. TRANSIENT CHARACTERISTICS OF LINEAR SYSTEM:

These are the characteristics of the system under sudden changing conditions. The rapidity of the response and the accuracy under transient conditions are more important in practice than the steady state. Anyway, the system should not become unstable either under steady state or under transient state. And therefore it is essential to study the behaviour of the control system under transient conditions as well when the voltage and currents vary with time in any manner.

The oscillograms show the transient response of amplidyne The oscillogram No.1 is for the sudden application of the control voltage. The rises in the control voltage wave shape are for 9.5 V and 90 volts respectively and their corresponding output voltage being 150 V and 182 volts. The comparative less rise in voltage at record time is due to the saturation of the machine.

The output voltage build up obeys the law[17]

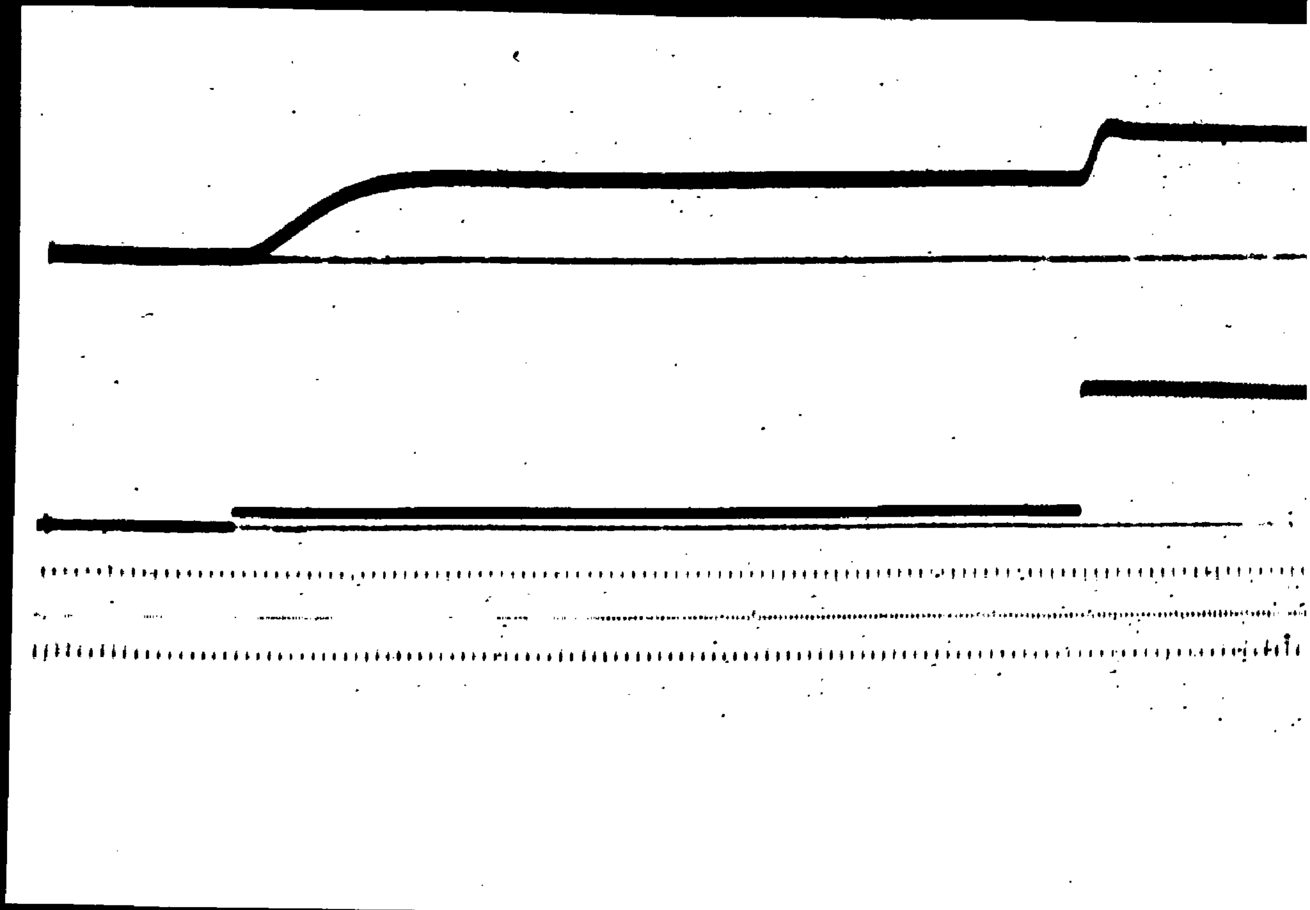
$$e_2 = E_2 \left(1 - e^{\frac{-R_2'}{L_2'} t_2'} \right)$$

The oscillogram No. 2 gives the picture when the load is suddenly applied. Referring to the diagram we find that there is sudden rise of load current along with the dip in the output voltage. The input voltage remains constant. The fourth wave is 50 c/s time wave. The values of voltage and currents are as shown.

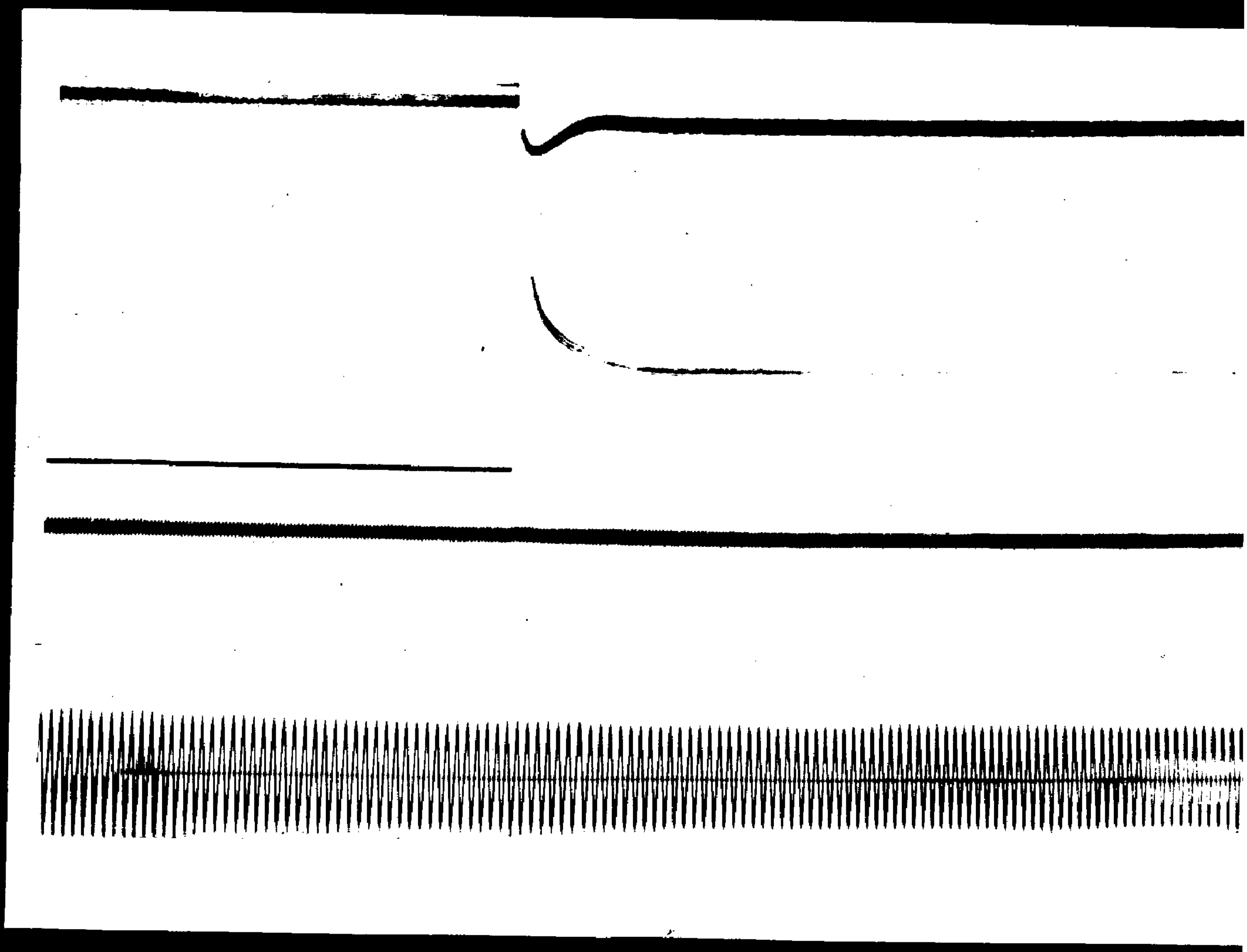
Control field voltage = 145 volts.

Control field current = 20 mA

$(f_1 f_2)$



OSCILLOGRAM No-1.



unloaded output voltage = 160 volts.

output voltage under
loaded condition = 140 volts.

Load current = 3.7 amps.

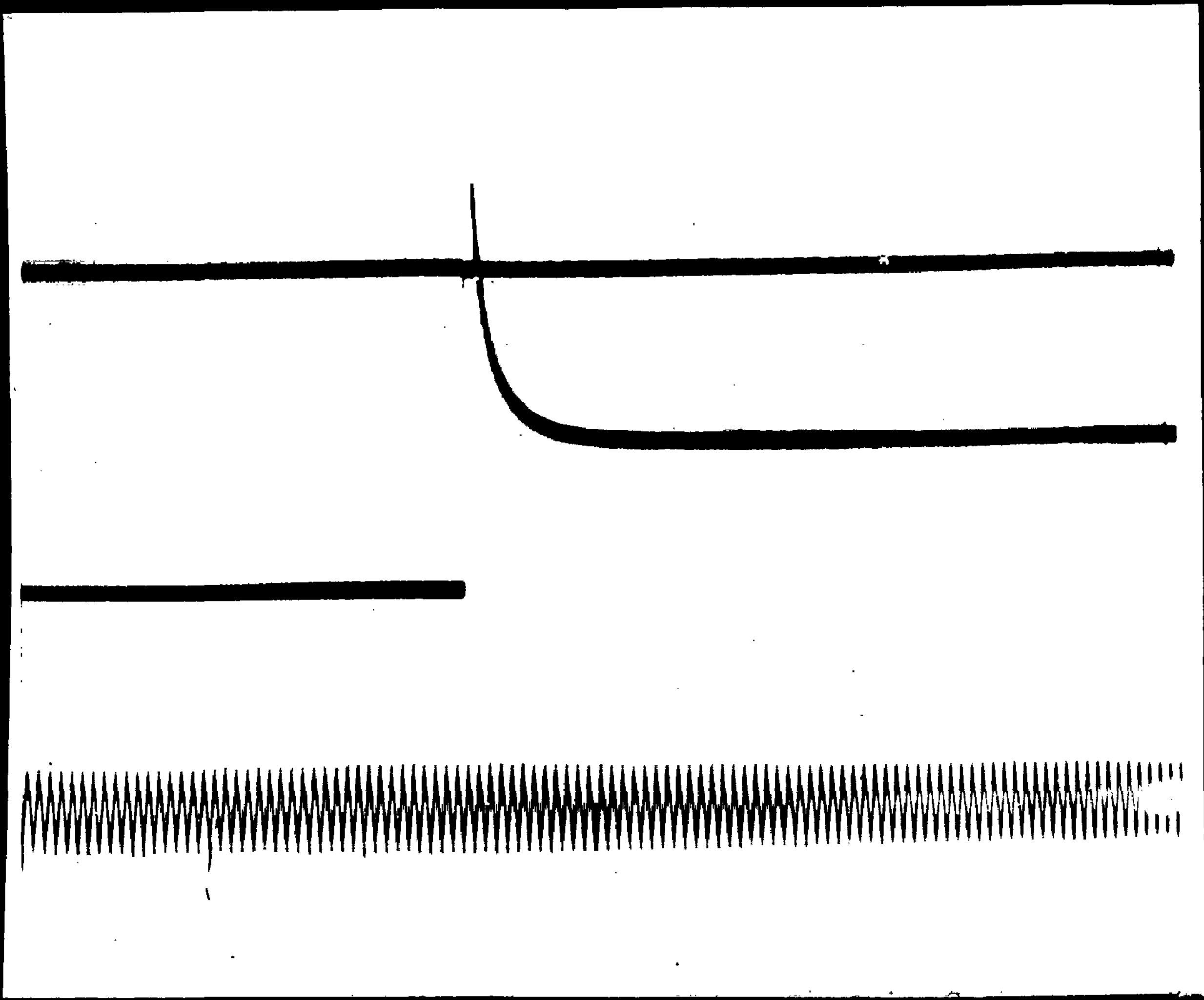
The oscillogram No. 3 shows the action of amplidyne as voltage regulator . Referring to the oscillograms Nos. 2 and 3., we find that on sudden application of load there is much less drop in the output voltage wave of No.3 in comparision to that of No. 2. The voltage drop in this case is of the order of 4.9 volts in comparison to 8 volts obtained in case 2. The values of voltages and currents are as shown. Load Current = 3.5 amps.

The oscillogram No. 4 shows the hunting of the amplidyne when some disturbance is introduced. in the system. This is achieved by varying the resistance in the antihunt circuit. It is clear from the oscillogram that with the decrease in feedback voltage, the amplitude and frequency of the oscillations increase upto a certain point .

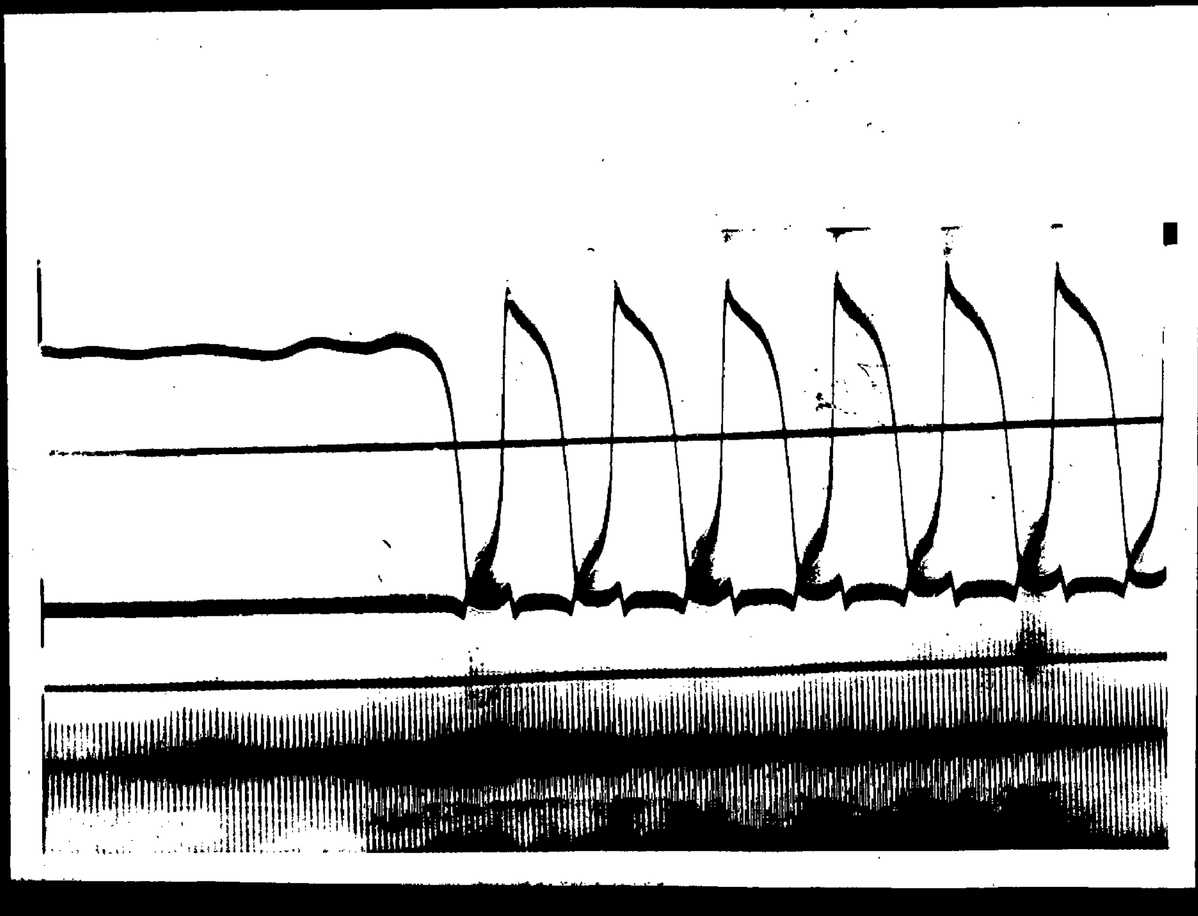
The condition for electromechanical oscillations can be obtained by framing the motional impedance array and then applying Routh's criterion to the characteristic equation. The figure 32 shows the system (Amplidyne with antihunt circuit) required for this analysis.

The impedance matrix for the configuration of figure 32 can be derived as follows. The following assumptions are made in deriving the impedance matrix.

1. System is fully compensated.
2. All mutual impedances are equal.
   i.e. $M_1 = M_2 = M_{12} = M$



Oscillogram № - 3.



Considering the primitive machine, the impedance matrix can be written as [18]

$$
Z_p = \begin{array}{c|cccc} & 1 & 2 & d & q \\ \hline 1 & R_1 + L_1 p & Mp & - & - \\ 2 & Mp & R_2 + L_2 p & - & - \\ d & Mp & Mp & R_d + L_d p & L_q p\theta \\ q & -M_p\theta & -M_p\theta & - & R_q + L_q p \end{array}
$$

The relation of currents in various paths for the connection as shown in Fig. 32 can be expressed as :

$$
\begin{bmatrix} i_1 \\ i_2 \\ i_d \\ i_q \end{bmatrix} = \begin{bmatrix} 1 & & \\ & -r & \\ & 1 & \\ & & 1 \end{bmatrix} \times \begin{bmatrix} i_1 \\ i_d \\ i_q \end{bmatrix}
$$

$$
\text{where} \quad r = \frac{R\alpha}{R_2 + L_2 p + \dfrac{1}{c_2 p}}
$$

Therefore connection Matrix c

$$
= \begin{bmatrix} 1 & & \\ & -r & \\ & 1 & \\ & & 1 \end{bmatrix}
$$

We know that the impedance matrix is given by $Z = C_t Z_p C$ and hence

$$Z = \begin{bmatrix} R_1 + L_1 p & -r\,Mp & \\ -rMp + Mp & r^2(R_2 + L_2 p) - rMp + (R_d + L_d p) & L_q p\theta \\ -Mp\theta & rMp\theta & R_q + L_q p \end{bmatrix}$$

Let G be the torque matrix ...

$$\text{Torque} = (Gi) = \begin{bmatrix} & & \\ & & L_q \\ -M & rM & \end{bmatrix} \times \begin{bmatrix} i_1 \\ i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \\ L_q i_d \\ -Mi_1 + i_d Mr \end{bmatrix}$$

and $-i_m(G+G_t)$

$$\begin{bmatrix} -i_1 & -i_d & -i_q \end{bmatrix} \times \begin{bmatrix} & & -M \\ & & L_q + rM \\ -M & L_q + rM & \end{bmatrix}$$

$$= \begin{bmatrix} i_q M & -i_q(L_q + rM) & i_i M - i_d(L_q + rM) \end{bmatrix}$$

and we know that motional impedance matrix is as given below :

$$\begin{bmatrix} e \\ T \end{bmatrix} = \begin{bmatrix} Z & G \\ iG & I'p \end{bmatrix} \times \begin{bmatrix} i \\ p\theta \end{bmatrix}$$

Hence the motional impedance array :-

R_1+L_1p	$-rMp$	-	-
$-rMp+Mp$	$r^2(R_2+L_2p)-rMp$ $+(R_d+L_dp)$	$L_q(p\theta)_o$	L_qi_d
$-M(p\theta)_o$	$rM(p\theta)$	R_q+L_qp	$-Mi_1+i_drM$
i_qM	$-i_q(rM+L_q)$	$i_1M-i_d(rM+L_q)$	$I'p$

Now after substituting the various numerical values in the above matrix, and taking the value of α in the expression of r equal to 0.2, we get.

$580+3.1p$	$\frac{-1.89p^2}{1.56p^2+56.2p+834}$	0	0
$\frac{-1.891p^2}{1.56p^2+56.2p+834}+2.7p$	$\left(\frac{.7p}{1.56p^2+56.2p+834}\right)^2(55.5+1.56p) - \frac{.7p \times 2.7p}{1.56p^2+56.2p+834} + (1.5+078p)$	$.1048 \times 120$	3.1×5.08
-2.7×120	$\frac{227p}{1.56p^2+56.2p+834}$	$1.277+.1048p$	$-2.97\times10^{-2} + \frac{9.6p}{1.56p^2+56.2p+834}$

3.55 x2.7	$\dfrac{-6.7p}{1.56p^2+56.2p+834} - 0.372$	$29.7\times10^{-3}-5.08\,(.1048) + \dfrac{2.7\times0.7}{1.56p^2+56.2p+834})$	.01735p

The characteristic equation obtained after opening the above determinant is as follows :

$$-0.312p^{12}-30.94p^{11}+267.3p^{10}+10.57\times10^{4}p^{9}+45.8\times10^{5}p^{8}$$
$$+4.96\times10^{7}p^{7}+11.86\times10^{8}p^{6}+17.93\times10^{9}\times p^{5}+21.4\times10^{10}p^{4}$$
$$+26.7\times10^{11}p^{3}+19.46\times10^{12}p^{2}+50.56\times10^{12}p+54.6\times10^{12}=0$$

The application of the Routh's criterion to the above equation clearly shows the unstability of the system as there is change of sign from second term to third term.

When machine is operated near the value of $\alpha = 0.2$, the electromechanical oscillations were observed, thereby confirming the above analysis.

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CHAPTER 5.

C O N C L U S I O N S.

CONCLUSIONS

The actual mean curve is approximated by two analyticaly defined curves namely (1) Exponential curve (ii) Curve represented by Froelich's equation and also linearization. It can be concluded from these curves that the piecewise linearization technique is best suited for systems which have sudden limiting of their output, while Froelich's equation very well fits for gradual saturation curves (magnetization curves). The exponential curve lies in between the above mentioned curves and hence can be used at many places. Thus it is seen that depending on the nonlinearity of the system, the best suited analytically defined curves can be used.

Secondly a peculiarity has been observed in all the hysteresis loops obtained for the various configuration of the machine. And that peculiarity is the slight flattening of the loop near the origin. (Reference Figures 4,5,6). And this is the reason for the probability of the unstability. The flattening of the loop near the origin is due to the excessive brush contact drop occuring in the machine. Hence the hysteresis loop can be modified as shown earlier.

Thirdly, the residual voltage in the case of amplidyne is usually larger than that of a normal d.c. generator because the hysteresis effects in the first stage are amplified in the second stage.

Fourthly, the stability analysis plays an important role in the study and design of the machine. For example the variation in the gain of unstable machine may result in an stable machine or vice versa.

Finally, the electromechanical oscillation, may, it is guessed, be induced by electrical oscillation. However, it is seen that electrical oscillation has a comparatively higher frequency (110 rad/sec.) whereas the electromechanical one has a much lower frequency (about 10 rad/sec.). It may be pointed out, without the mathematical analysis, that electromagnetic torque fluctuating at about 110 rad/sec., may give rise to subharmonic response of 10 rad/sec. since the system is essentially non-linear. The exact analysis of the nonlinear system, derivation of transient performance seems to be quite interesting and further work may be done in this line.

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