STATE ESTIMATION OF POWER SYSTEMS

A DISSERTATION

submitted in partial fulfilment of the
requirements for the award of the degree
of
MASTER OF ENGINEERING
in
ELECTRICAL ENGINEERING
(With Specialization in Power System Engineering)

By

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CANDIDATE's DECLARATION

I hereby certify that the work presented in this dissertation entitled STATE ESTIMATION OF POWER SYSTEMS in partial fulfilment of the requirements for the award of the degree of Master of Engineering (Electrical) with specialization in Power System Engineering, University of Roorkee, is an authentic record of my own work carried out during the period August 1989 to February 1990 under supervision of Dr. J.D. Sharma, Professor and Dr. H.O. Gupta, Reader, Electrical Engineering Department, University of Roorkee.

The matter embodied in this dissertation has not been submitted by me for the award of any other degree or diploma.

DATED 28-2-1990

(SUBHASH KUMAR JOSHI)

This is to certify that the above statement made by the candidate is correct to the best of our knowledge.

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SYNOPSIS

Utilities in India are now largely equipped with Computerized Load Despatch Centres. These Load Despatch Centres are to function as Supervisory Control and Data Acquisition Centres (SCADA), which collect status data of breakers and switches, and analog data of active and reactive power flow, injection and bus voltages. These data when combined with supervisory control system allows operator to control circuit breakers and transformer taps and disconnect switches remotely. Presently in our country, the telemetered data is used by the operators to send the switching commands,and filtering of data errors is not yet carried out. This motivated the author, who is an utility engineer to work on State Estimation so as to have a feeling of the problem and build confidence to incorporate it as an integeral part of the SCADA for real time application.

The computational speed and efficiency is the main criteria in implementing State Estimation for real time security, monitoring and assessment. Therefore, the latest reported Hachtel's Augmented Matrix method has been compared and implemented with Normal Equation method, the basic method. Experience of implementing both the methods in their Fast Decoupled version using single precision was not favourable. Hachtel's Augmented Matrix method has been developed using full Jacobian in single precision.

The growing size of system has diverted the efforts to the Hierarchical State Estimation for gaining computational speed and reliability. The State Estimation by Network Decomposition has been presented.

The thesis concludes with findings during this work and areas identified as future scope of work.

A C K N O W L E D G E M E N T

Economy of words in acknowledgement is nevertheless poverty of feelings. It is only the insufficiency of the vocubalory to find true manifestation.

It has been a privilege to work under the guidance of Dr. J.D. Sharma, Professor and Dr. H.O. Gupta, Reader, Electrical Engineering Department, University of Roorkee, whose discussions were critical, thought provoking but always encouraging.

This work is an outcome of the encouragement from PARENTS to achieve all that, denied to them by destiny; inspiration from COLLEAGUES during service career; the difficulties of this detuned engineer solved by dedicated TEACHERS; motivation from author's FAMILY which righteously needed his association; FRIENDS and COLLEAGUES who always interrupted author's depression before point of no return.

This venture would not have been possible without the permission of MPEB to undertake higher studies.

Thanks are due to all those who worked for this presentation.

S.K. JOSHI

C O N T E N T S

S Y M B O L S

The symbols used in this thesis are listed below. Any minor departure from these symbols and specially used symbols are explained in the text itself.

PRINCIPAL SYMBOLS

A	Incidence matrix
B^I	Set of buses in I^{th} area
B_i^I	Set of internal buses in I^{th} area
$B_{b_{L,I}}^L$	Set of external boundary of I^{th} area connected to L^{th} area
C	Derivative of c(x) w.r.t. vector x.
E[ ]	Trace of matrix
H	Full Jacobian of measured functions w.r.t. State vector
H_0 and H_1	Hypothesis
I	Area identification
J	Cost function
K	Iteration count
L	Lower traingular matrix (factorized)
L^I	Set of lines in I^{th} area
L_i^I	Set of internal tie lines of I^{th} area

$L_{t_{L,I}}^{L}$	Set of tie lines of I^{th} area connecting L^{th} area
M^{I}	Set of measurements in I^{th} area
M_{i}^{I}	Set of internal measurements in I^{th} area
$M_{t_{L,I}}^{L}$	Set of tie line measurements of I^{th} area connected to L^{th} area
P_i	Active power injected in i^{th} bus
P_{ij} or P_l	Active power line flow in i-j^{th} line
P_e	Probability of rejecting H_L
P_d	Probability of accepting H_L
Q	Orthogonal matrix
Q_i	Reactive power injected in i^{th} bus
Q_{ij} or Q_l	Reactive power flow in ij^{th} line
R	Right hand traingular matrix of QR transformation
$\mathcal{R}$	Residual senstivity matrix
T	Time
U	Upper traingular matrix (factorized)
W	Weightage diagonal matrix
W_{ii}	Weight corresponding to the measurement
Y_{ii}	Self admittance of i^{th} bus
Y_{ij}	Transfer admittance i-j^{th} line
C(x)	Constraint vector as a function of state
e	Error vector

$f(x)$	Vector of measured quantities calculated as a function of states
m	Number of measurements
n	Number of states
r_N	Normalized residual
r_ω	Weighted residual
x	State vector
x_K	K^{th} estimate of state vector
x_0	Initial estimate of state vector
x_c	Corrected state vector after removing bad data
w.r.t.	With respect to
y_1 and y_2	Variables
z	Measurement vector
Δc	Mismatch of constraints
Δx	State correction vector
Δz	Measurement mismatch
δ_i	Bus voltage angle of i^{th} bus
θ_{ij}	Impedence angle of i^{th} bus
θ_{ij}	Impedence angle of i-j^{th} line
Σ	Gain matrix
Σ_j	Summation
λ	Lagrangian multiplier

α	Controlling parameter
β	Detection threshold for sum of the squares of the weighted residuals
γ_i	Detection threshold of the residual of i^{th} measurement
σ_{ii}	Variance of i^{th} measurement
ϕ	Angle vector for observability analysis
ε	Convergence tolerance

CHAPTER I

INTRODUCTION

Attempts of precise control of engineering systems proved the validity of the paradox "What it appears is not as it is." The crude information obtained by various measurements is insufficient to explain the state of operation of the system due to its inherent errors. This has led to the evolution of Statistical Estimation Theory as a concept to approximate the state variables of a system from its erroreous measurements.

Estimation Theory has been extensively used for navigation of air-craft and space-craft, as well as post experimental analysis. But it was first applied to power systems by Scheweppe et al [1,2,3] in 1970 followed by a series of papers [4-11] in the same year.

Load despatcher in power system control centres is required to know at all times the values of voltages, currents and power throughout the network. Some of the values such as bus voltage magnitude and power line flows can be measured within a certain degree of variance. Difficulties are further encountered when some of the data is missing either due to meter being out of order or missing transmission. Moreover, the size of the present day power system (PS) is prohibitive to manual calculations or even on a small computer to generate online missing information.

State Estimation (SE) utilizes the available redundancy, for systematic cross checking of the measurements, to approximate the states as well as generate information in respect of missing observations or gross measurement errors called Bad Data (BD). The prerequisite for state estimation is that the system must be observable with the available measurements.

The states of a power system can also be computed with the Load Flow calculations, based on equal number of measurements, assuming them to be accurate. However, the implicit error will lead to imperfect data base and prejudice the security monitoring, whereas, the State Estimator is a data processing algorithm for use on a digital computer to transform meter readings (measurement vector) into an estimate of the system's states (State Vector), which is not accurate but best reliable estimate. A comparison between Load Flow Calculation and State Estimation has been shown in Fig. 1.1.

The State Estimator, apart from security monitoring, bad data and topological error detection and identification has wider applications in central control of power systems as shown in Fig. 1.2. The State Estimator is an essential tool of load despatchers. The State Estimators are classified in three categories.

(i) Static State Estimator: It converts observation vector into state vector without regard to past information [8]. Here system changes are considered to be slow enough to be assumed static. This is discussed in detail in Chapter II.

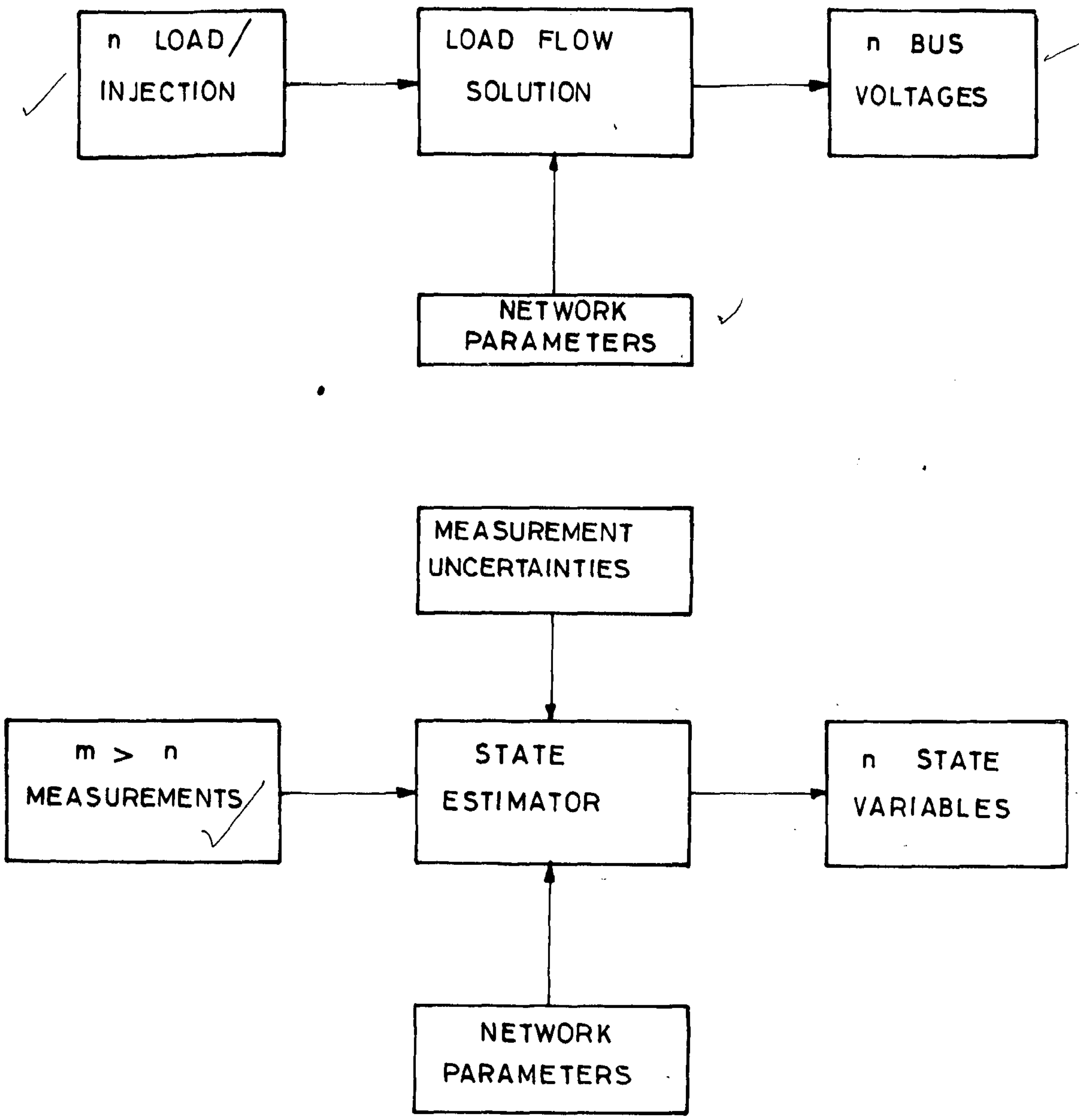


FIG. 1.1 COMPARISON BETWEEN LOAD FLOW CALCULATIONS AND STATE ESTIMATOR

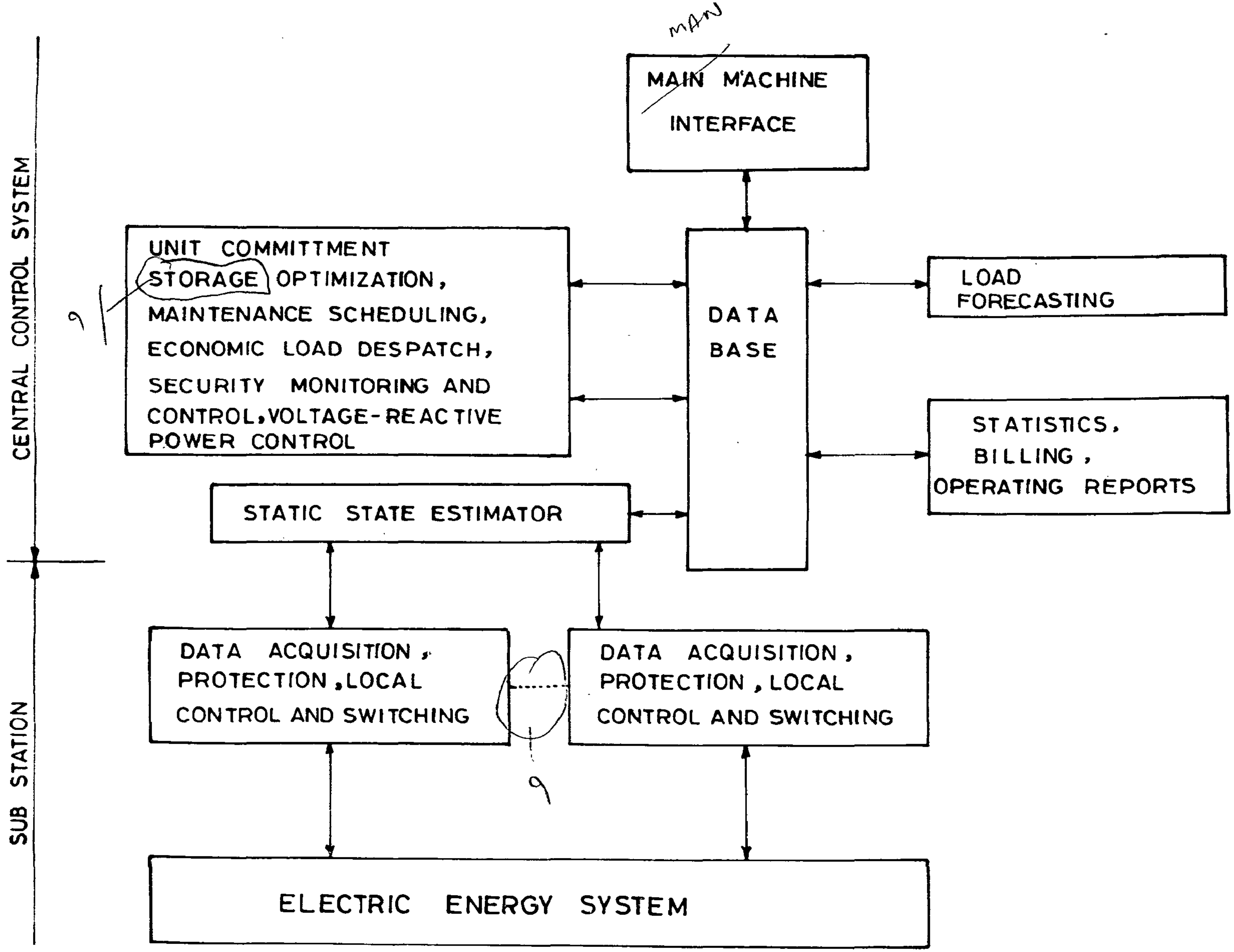


FIG. 1.2 SCHEMATIC OF LOAD CONTROL CENTRE

(ii) Tracking State Estimator: It is a discrete feed back loop which uses real time measurements to track the static state as it varies during the daily load cycle [12, 13]. The comparison of Static and Tracking State Estimator is shown in Fig. 1.3. In real sense tracking state estimator extends techniques developed for static state estimation to the time varying case without explicit definition of the dynamic models.

(iii) Dynamic State Estimator: It is based on time behaviour of the State Vector and requires knowledge of past states alongwith the present measurement vector [13, 14]. Power system under normal operating conditions since behave in quasi-static manner, the state trajectory is discretised in small time intervals. It has been considered that state vector obeys linear dynamic model [25]. The dynamic state estimation approach is based on Kalman filtering technique, using simplified model of the dynamic behaviour of the power system [26]. This dynamic state estimator in real sense is a tracking estimator with memory, because model is not sufficiently accurate under rapidly changing conditions [13]. A true dynamic state estimation in power system must be based on dynamic models, using magnetic flux linkages in all the synchronous generators in the network as state vector. The complexity of this model has, perhaps, been a bottleneck in its on line application.

The use of static state estimator in real time operation, security and monitoring has received such a wide acceptance that, unless dynamic or tracking state estimation is specified, State Estimation is synonym to Static State Estimation. The state estimator with its functional constituents is illustrated in Fig. 1.4.

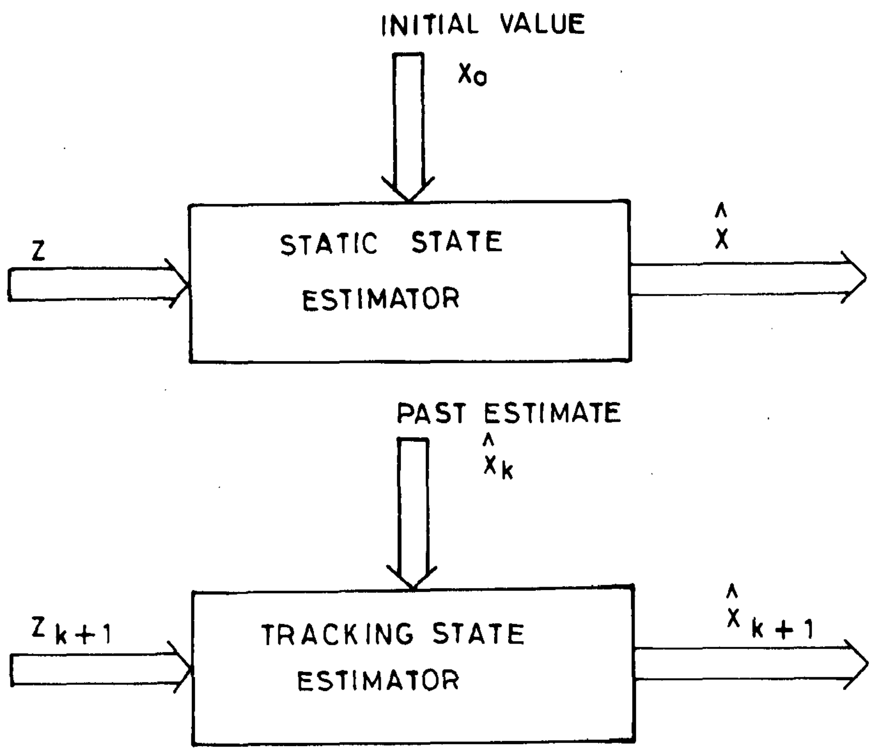


FIG. 1. 3 COMPARISON BETWEEN STATIC AND TRACKING STATE ESTIMATOR

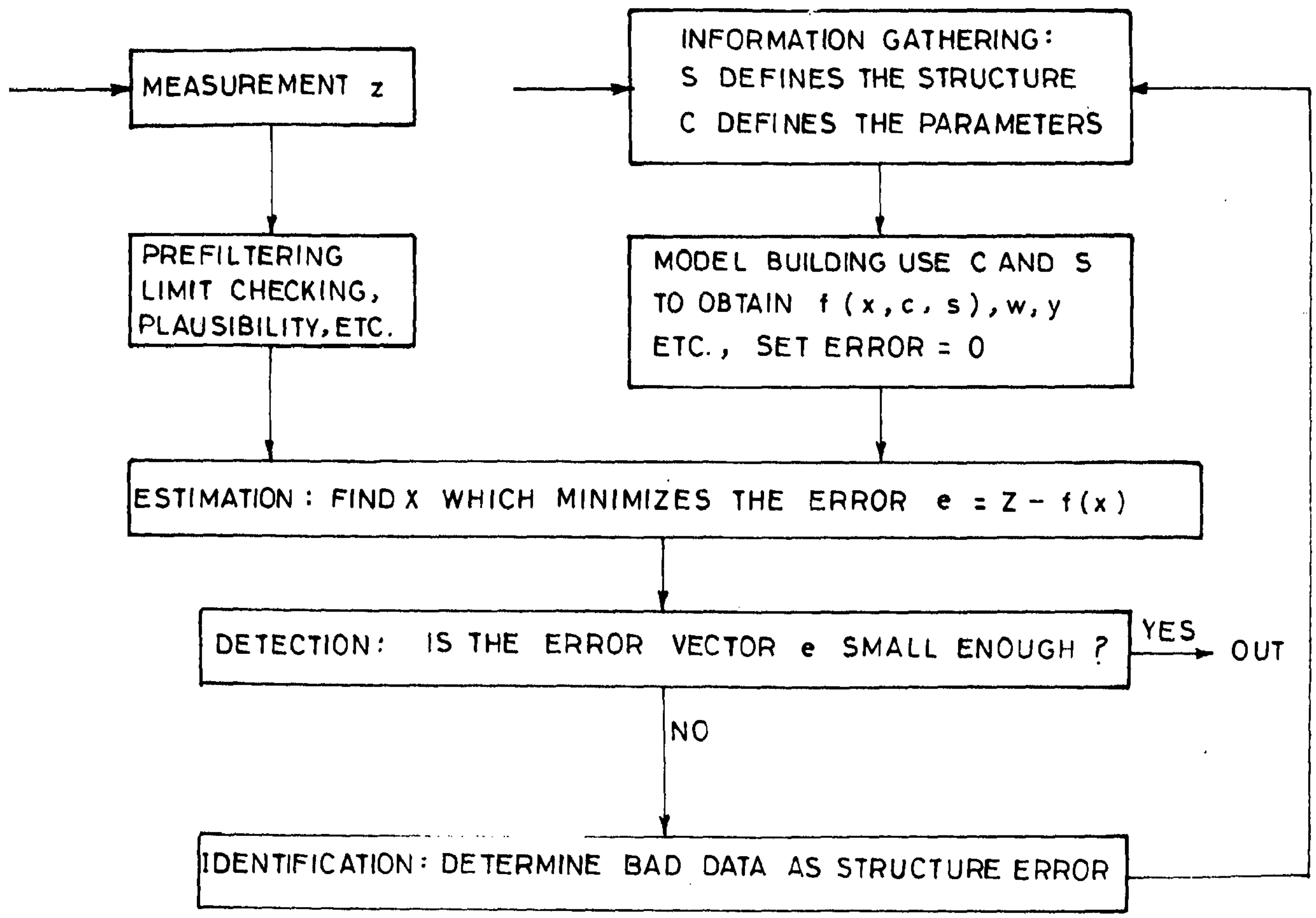


FIG. 1. 4 BASIC STATE ESTIMATOR

The state estimator has since to cater the needs of on-line application, computation speed plays a vital role specially when systems are large. Newer methods of state estimation are being reported to optimize on (i) numerical stability, (ii) computation efficiency, and (iii) implementation complexity [15]. Further, methods of decomposition of a large system to achieve overall computation speed have been developed. The contents of this thesis in remaining chapters are briefly as under -

Chapter - II - The state of art of State Estimation has been brought out. It describes various methods of State Estimation, alongwith bad data detection and observability analysis.

Chapter - III - Briefly summarizes difficulties encountered in implementation of fast decoupled version of Normal Equation method and Hachtel's Augmented Matrix method in single precision. The implementation of both these methods using full Jacobian in single precision and comparison thereof has been reported.

Chapter - IV - The growing dimension of system has its own computational intricasies. This chapter discusses some of the reported methods of hierarchical State Estimation, and new algorithm for State Estimation by decomposition of the network has been presented.

Chapter - V - Software developed for the state estimation has been detailed in this chapter.

Chapter - VI - This chapter concludes the thesis with future scope of work.

Author's Contribution

The fast decoupled version of Hachtel's Augmented Matrix method and Normal Equation methods reported in double precision, since did not yield successful results in single precision, hence these methods were implemented using full Jacobian in single precision. Superiority of Hachtel's method has been confirmed. Adequacy of the recently reported Decomposition Approach for load flow [48], for State Estimation has been presented.

CHAPTER - II

STATIC STATE ESTIMATION : STATE OF ART

The load flow calculations, indeed are an inevitable tool for off-line studies and planning exercises. But incomplete and erroreous measurement is a real time proposition. Solution for such a situation is provided by Static State Estimator, which ignores the slow changes in the system and utilizes redundant set of measurements for cross checking and approximating to most reliable estimates of the state.

2.1 The fundamental equation for the measurement vector is

$$z = f(x) + e \qquad \ldots (2.1)$$

where z - is the measurement vector

x - is the state vector

$f(x)$ - vector corresponding to z i.e. measured quantities calculated as a function of x.

e - error vector

2.1.1 Errors

The errors are broadly classified as (i) Measurement error and (ii) Modelling error, which are detailed below.

2.1.1.1 Measurement Error

The in-situ measurement are telemetered to load despatch centres. This complete process is susceptible to the following discrepancies:

(i) Error in transducer calibration

(ii) Noise in communication channel

(iii) Non-simultaneity of the data

and defaults -

(i) Failure of the communication channel

(ii) Meter being defective or out of order

2.1.1.2 Modelling Error

The model of the network constitutes its topology and its parameters. The topological errors can be caused by missing information in respect of disconnector or the reconnected line, while parameter errors are due to its wrong initial estimation. Moreover, the system representation considered in such studies is single phase, while unbalance conditions cause significant error [16].

2.1.2 Measurements

The non-availability of measurement may create conditions of unobservability and therefore it is important to maintain sufficient redundancy. This leads to the following classification of measurements [15].

Telemetered Measurements - are on line telemetered data of line flows, bus injections and voltages. They are assigned weightage in inverse proportion of their variance and is expressed as

$$R_{ii} = \frac{1}{T} \int_0^T e_i^2 (t)\, dt \qquad \lim T \rightarrow \infty$$

$$W_{ii} = R_{ii}^{-1} \qquad \dots (2.2)$$

Pseudo Measurements - are the guess in respect of generation or substation loads based on historical data and are assigned least weightage. It is used in the event of missing data or bad data.

Virtual Measurements - In network there could be switching stations with zero injection and therefore do not require measurement. However, they are used to create redundancy. These measurements are assigned highest weightage.

2.1.2.1 The measurements are further classified in two categories based on their purpose [4].

Basic Measurement - are the measurements, equal to the number of the unknown states and sufficient to determine these states. If number of measurements m is equal to n then it is the load flow solution and suffers from the aforesaid inaccuracies.

Redundant Measurements - when m is greater than n then m - n measurements are redundant measurements and are used to cross check and compute the correction vector to approximate the reliable estimates.

The measurements are never simultaneous, they are sequential, however at a very close interval and therefore the static state estimator assumes it to be snap-shot measurement [3], i.e. all measurements assumed to be taken simultaneously.

2.2 NON-LINEAR ESTIMATION THEORY

The electrical power in the network is since a non-linear function of states and therefore, Linear Estimation Theory [4, 17] is only of classical interest. The Taylor's series expansion of f(x) in equation (2.1) is

$$f(x) = f(x_0) + f'(x_0)\,\Delta x + f''(x_0)\,\frac{\Delta x^2}{2} + \cdots$$

The second and higher order terms when neglected, the equation (2.1) can be rewritten as

$$z = f(x_0) + f'(x_0)\,\Delta x + e$$

This equation can be written as

$$[\Delta z - H\,\Delta x] = e \qquad \ldots (2.3)$$

where $\Delta z = z - f(x_0)$ is the measurement errors, $f(x_0)$ is the measurement estimate vector, x_0 is the estimated state vector and H is the rectangular Jacobian of $f(x_0)$. The vector e is the residual error vector.

Since the constituents of the measurement vector are attached different weightage as discussed in para 1.3, the cost function shall be

$$J(\Delta x) = e^T W e = [\Delta z - H\,\Delta x]^T\, W\,(\Delta z - H\,\Delta x] \qquad \ldots (2.4)$$

Minimizing cost function

$$\frac{\partial J(\Delta x)}{\partial \Delta x} = 0 = 2H^T\, W\,[\Delta z - H\,\Delta x] \qquad \ldots (2.5)$$

Solving above equation for Δx we get

$$\Sigma\,\Delta x = H^T\, W\,\Delta z \qquad \ldots (2.6)$$

Here $\Sigma = H^T\, WH$

$$x_{K+1} = x_K + \Delta x \qquad \ldots (2.7)$$

The representation of equation (2.3) for zero residual error vector in power system would follow -

$$\begin{bmatrix} \Delta P_i \\ \Delta P_l \\ \Delta Q_i \\ \Delta Q_l \end{bmatrix} = \begin{bmatrix} \\ H \\ \\ \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \frac{\Delta |E_i|}{|E_i|} \end{bmatrix} = \begin{bmatrix} H_A & H_B \\ H_C & H_D \\ H_E & H_F \\ H_G & H_H \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \frac{\Delta |E_i|}{|E_i|} \end{bmatrix} \quad .. (2.8)$$

Here P_i and Q_i are the bus injections, P_l and Q_l are the line flows, $|E_i|$ and δ_i are the bus voltage magnitude and angle and $H_A = \frac{\partial P_i}{\partial \delta_i}$, $H_B = \frac{\partial P_i}{\partial |E_i|/|E_i|}$, $H_C = \frac{\partial P_l}{\partial \delta_i}$, $H_D = \frac{\partial P_l}{\partial |E_i|/|E_i|}$,

$H_E = \frac{\partial Q_i}{\partial \delta_i}$, $H_F = \frac{\partial Q_i}{\partial |E_i|/|E_i|}$, $H_G = \frac{\partial Q_l}{\partial \delta_i}$, $H_H = \frac{\partial Q_l}{\partial |E_i|/|E_i|}$.

2.3 COMPUTATIONAL PROCEDURES

(i) Complete Set of Measurements

It requires computation of gain matrix which is by post and pre multiplication of W matrix, with rectangular matrix H and its transpose, followed by inversion as brought out in equation (2.6). The complexity of the computation and numerical stability are the general problems in this approach [15]. It is also known as Weighted Least Square (WLS) method.

(ii) Partitioning of the Measurements

It suggests partitioning of the measurement as shown in equation (2.8). The basic measurements are solved using load flow program. The correction term is produced by m-n measurements vector which is of lesser dimension than in method (i) above [4, 7], and can be expressed by the following

$$\begin{bmatrix} \Delta P_{kl} \\ \\ \Delta Q_{kl} \end{bmatrix} = \begin{bmatrix} H_C & H_D \\ \\ H_G & H_H \end{bmatrix} \begin{bmatrix} \Delta\delta_i \\ \\ \Delta E_i/|E_i| \end{bmatrix} \quad \ldots (2.9)$$

This method is also known as solution by Independent Equation [7]. This concept has also been used to suggest methods for State Estimation from line flow measurements [18, 19].

(iii) Sequential Processing

This concept suggests use of each redundant measurement sequentially. It replaces inversion of (m-n) x (m-n) matrix by (m-n) scalar divisions [4].

2.4 SOLUTION ALGORITHMS

The Weighted Least Square method is based on (2.4). The growing importance of on-line state estimation has provided impetus to development of solution algorithm which is fast, numerically stable and provide solution of even ill conditioned systems. The reported algorithms [15, 20] are - (i) Normal Equation (NE), (ii) Orthogonal Transformation (ORTHO), (iii) Hybrid Method (HYBRID), (iv) Normal Equation with Constraints (NE/C), (v) Hachtel's Augumented Matrix Method (HACHTEL).

2.4.1 Normal Equation Method (NE)

The coefficient of Δx in equation (2.6) is termed as gain matrix Σ which is a square and symmetric matrix. Therefore, instead of its inversion, its traingular factorisation is done.

$U^T U = \Sigma$, and thus the (2.6) can be rewritten as

$$U^T U \, \Delta x = H^T \, W \, \Delta z \qquad \ldots (2.10)$$

The solution steps by back substitution follow -

(i) $U^T \Delta\tilde{x} = \Delta\tilde{z}$, where $\Delta\tilde{x} = U \, \Delta x$ and $\Delta\tilde{z} = H^T W \, \Delta z$

(ii) $U \, \Delta x = \Delta\tilde{\tilde{z}}$

(iii) $\Delta x = \Delta\tilde{\tilde{\tilde{z}}}$

2.4.2 Orthogonal Transformation Method (ORTHO)

The cost function of WLS in (2.4) can be rewritten as

$$J(\Delta x) = [\Delta\tilde{z} - \tilde{H} \, \Delta x]^T \; [\Delta\tilde{z} - \tilde{H} \, \Delta x] \qquad \ldots (2.11)$$

$$= ||\Delta\tilde{z} - \tilde{H} \, \Delta x||^2 \qquad \ldots (2.12)$$

where $\tilde{H} = W^{1/2} H$ and $\Delta\tilde{z} = W^{1/2} \, \Delta z$

An Orthogonal Matrix Q, i.e., $Q^T Q = I$ be such that

$$Q \tilde{H} = \begin{bmatrix} R \\ 0 \end{bmatrix} \qquad \ldots (2.13)$$

where R is the upper traingular matrix and the (2.11) can be rewritten as

$$J(\Delta x) = [\Delta\tilde{z} - \tilde{H} \, \Delta x]^T \, Q^T Q [\Delta\tilde{z} - \tilde{H} \, \Delta x]$$

$$= [Q \, \Delta\tilde{z} - Q \, \tilde{H} \, \Delta x]^T \, [Q \, \Delta\tilde{z} - Q \, \tilde{H} \, \Delta x]$$

$$= ||Q \, \Delta\tilde{z} - Q \, \tilde{H} \, \Delta x||^2$$

$$= ||\Delta y_1 - R \, \Delta x||^2 + || \, \Delta y_2 \, ||^2 \qquad \ldots (2.14)$$

where $Q\,\Delta\tilde{z} = \begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix}$ ... (2.15)

The minimum cost function occurs at

$$R\,\Delta x = \Delta y_1 \qquad \text{... (2.16)}$$

The Given's method for orthogonal transformation has been used in (2.13) and solution of (2.16) obtained by back substitution [15, 20].

2.4.3 Hybrid Method (HYBRID)

It is an improvement on ORTHO and uses (2.6) as basic equation and can be rewritten as

$$\tilde{H}^T \tilde{H}\,\Delta x = H^T W\,\Delta z \qquad \text{... (2.17)}$$

where $\tilde{H} = W^{1/2}$ H and orthogonalization of $\tilde{H}$ will lead to

$$\begin{bmatrix} 0 \\ R \end{bmatrix} Q^T Q \begin{bmatrix} R \\ 0 \end{bmatrix} \Delta x = H^T W\,\Delta z \qquad \text{... (2.18)}$$

and it can be written as $R^T R\,\Delta x = H^T W\,\Delta z$ ... (2.19)

Here R is the upper traingular matrix and thus exploits sparsity alongwith advantages of ORTHO. The (2.19) is solved by back substitution.

2.4.4 Normal Equation With Constraints Method (NE/C)

In power network there are some nodes with zero injection i.e. switching substations as constant load. Such buses are called constrained buses and can be included in the cost function by the method of Lagrangian Multiplier.

$$J(\Delta x, \lambda) = [\Delta z - H\,\Delta x]^T W[\Delta z - H\,\Delta x] + [\Delta c(x) - C\,\Delta x]^T \lambda \quad \ldots (2.20)$$

where λ represents the Lagrangian Multiplier and c(x) the constraints, such that

$$c(x) = c(\hat{x}) + C\,\Delta x \quad \ldots (2.21)$$

Using optimality condition -

$$\frac{\partial J}{\partial \Delta x} = 0 \Rightarrow H^T WH\,\Delta x + C^T \lambda = H^T W\,\Delta z \quad \ldots (2.22)$$

Here $C = \dfrac{\partial c(x)}{\partial x}$ ... (2.23)

The (2.22) and (2.23) can be expressed as

$$\begin{bmatrix} H^T WH & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \lambda \end{bmatrix} = \begin{bmatrix} H^T W\,\Delta z \\ \Delta c \end{bmatrix} \quad \ldots (2.24)$$

The coefficient matrix is symmetric and can be solved by $U^T U$ factorisation and back substitution. The constraint buses are used to increase the redundancy by considering them as virtual measurements. These virtual measurements could be considered in earlier method (2.4.1 to 2.4.3) using high weightages, but it may cause instability problems.

?

✓2.4.5 Hachtel's Aug-mented Matrix Method (HACHTEL)

This method has been used in solution of sparse ~~equations~~, but was first applied to power system by Gjelsvik [21] and its treatise by Wu [15, 20]. This method alongwith (2.22) and (2.23) uses error vector discussed in (2.3) re-expressed as -

$$\Delta r = \Delta z - H\,\Delta x \quad \ldots (2.25)$$

✓

It can also be written as

$$\alpha W^{-1} (\alpha W \Delta r) + H \Delta x = \Delta z \qquad \ldots (2.26)$$

Here α is the parameter used to control the numerical stability and W the weightage diagonal matrix. The (2.22), (2.23) and (2.26) can be expressed as

$$\begin{bmatrix} 0 & 0 & C \\ 0 & \alpha W^{-1} & H \\ C^T & H^T & 0 \end{bmatrix} \begin{bmatrix} \alpha^{-1}\lambda \\ \alpha^{-1} W \Delta r \\ \Delta x \end{bmatrix} = \begin{bmatrix} \Delta c \\ \Delta z \\ 0 \end{bmatrix} \qquad \ldots (2.27)$$

$$\begin{bmatrix} 0 & 0 & C \\ 0 & \alpha W^{-1} & H \\ C^T & H^T & 0 \end{bmatrix} \begin{bmatrix} \lambda' \\ \Delta r' \\ \Delta x \end{bmatrix} = \begin{bmatrix} \Delta c \\ \Delta z \\ 0 \end{bmatrix} \qquad \ldots (2.28)$$

where $\Delta r' = \alpha^{-1} W \Delta r$ and $\lambda' = \alpha^{-1} \lambda$

This method is good compromise between numerical stability, computational efficiency and implementation complexity. In this method the dimensions of the coefficient matrix is large. However, it is quite sparse and symmetric in structure and also it does not require computation of $H^T WH$, unlike methods in 2.4.1 to 2.4.4. Thus this method offers high speed and requires less memory. It is solved by back substitution.

2.5 COMPARISON OF STATE ESTIMATION ALGORITHMS

Comparison of State Estimation algorithms has been made in Table 2.1. The part I of it compares the major computational

Normal Equation Method (NE)	Orthogonal Transformation Method (ORTHO)	Hybrid Method (HYBRID)	Normal Equation with constraints (NE/C)	Hachtel's Augmented matrix method (HACHTEL)
1	2	3	4	5
I. COMPUTATION STEPS				
(i) Form H $G = H^T W H$	Form $\tilde{H} = W^{1/2} H$	Form $\tilde{H} = W^{1/2} H$	Form $F = \begin{bmatrix} H^T W H & C^T \\ C & 0 \end{bmatrix}$	Form $K = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \alpha W^{-1} & H \\ C^T & H^T & 0 \end{bmatrix}$
(ii) Factorize G (Traingular Factorization) $G = U^T U$	Factorize $\tilde{H}$ (QR Factorization) $\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \tilde{H} = \begin{bmatrix} R \\ 0 \end{bmatrix}$	Factorize $\tilde{H}$ (QR Factorization) $Q\tilde{H} = \begin{bmatrix} R \\ 0 \end{bmatrix}$	Factorize F (Traingular Factorization) $F = U_f^T U_f$	Factorize K (Traingular Factorization) $K = U_K^T U_K$
(iii) Solve for Δx (Back substitution) $(U^T U)\Delta x = H^T W \Delta z$	Solve for Δx (back substitution) $R \Delta x = Q W^{1/2} \Delta z$	Solve for Δx (Back substitution) $(R^T R)\Delta x = H^T W \Delta z$	Solve for Δx (Back substitution) $(U_f^T U_f) \begin{bmatrix} \Delta x \\ \\ \end{bmatrix} = \begin{bmatrix} H^T W \Delta z \\ \Delta c \end{bmatrix}$	Solve for Δx (Back substitution) $(U_K^T U_K) \begin{bmatrix} \lambda' \\ \Delta r' \\ \Delta x \end{bmatrix} = \begin{bmatrix} \Delta c \\ \Delta z \\ 0 \end{bmatrix}$

contd....

1	2	3	4	5
II. NUMERICAL STABILITY AND EFFICIENCY				
- Condition number of gain matrix G is square of H and is therefore less numerically stable. - Fast decoupled version can be implemented effectively and thus it requires factorization of coefficient matrix only once, thereby increases computation speed.	- Condition number is same as that of H which is therefore numerically more stable than NE This method has rather highest numerical stability. - It does not lend advantage of real and reactive power decoupling. This orthogonalization is required at each iteration and thus enormous requirement of computation time looses its benefits of numerical stability	- The large weights on virtual measurements, are likely to cause loss of information in respect of telemetered measurements with large errors and assigned less weights, due to rounding error. Thus it suffers disadvantage with respect to ORTHO. - Fast decoupling implementation is effective.	- The cause of ill-conditioning due to weights on virtual measurements is remedied by using them as constraints. - The G matrix in NE is positive definite, has numerical stability for pivoting in any order. But this method has since indefinite coefficient matrix and thus requires use of sophisticated factorization method. - It allows effective, active and reactive power decoupling. - Computation requirements are not extensive.	- Like NE/C method, this also requires sophisticated factorization method. - It is a good compromise between numerical stability, computation speed and implementation complexity.

steps. The part II high-lights relative merits and demerits in respect of numerical stability, computation speed and implementation complexity [15]. The Hachtel's method is judged to be the most suitable method for state estimation since it makes a good compromise between numerical stability, computation speed and implementation complexity.

2.6 BAD DATA PROCESSING

A data which is more inaccurate than is assumed by mathematical model is called Bad Data (BD). The presence of BD can be due to number of reasons, viz. failure of communication link, intermittent fault in meters, change of system states far off from that assumed for pseudo-measurements. The presence of BD causes very poor estimates. The later effort on development of state estimation for practical application has deserving share on bad data processing. The bad data processing is a three tier exercise (i) Detection, (ii) Identification, and (iii) Estimate correction.

2.6.1 Detection

The (2.1) when expressed in terms of the estimated state vector x, then

$$z = f(\hat{x}) + r, \quad \text{or} \quad r = z - f(\hat{x}) \qquad \ldots (2.29)$$

where r is the estimation residual. The deviation in r can be expressed as

$$\Delta r = \frac{\partial r}{\partial z}\Delta z - \frac{\partial r}{\partial x}\Delta x$$

$$= I\,\Delta z - H\,\Delta x \qquad \ldots (2.30)$$

since $\frac{\partial r}{\partial z} = I$ and $\left(\frac{r}{x}\right) = f\,(x) = H.$

The sensitivity of the residual to the measurements is called the residual sensitivity matrix and is expressed as $\frac{\partial r}{\partial z}$, expression for which can be developed from (2.30) using (2.6) as under [27].

$$\frac{\partial r}{\partial z} = I - H.\frac{\partial x}{\partial z}$$

$$= I - H\ (H^T\ WH)^{-1}\ H^T W = \mathcal{R} \qquad \ldots (2.31)$$

This residual sensitivity matrix is of vital importance in bad data processing. The properties of this matrix are shown in Appendix A.[28].

In absence of BD the measurement residual vector is distributed N $(0, \mathcal{R} W^{-1} \mathcal{R}^T)$, or $N(0, W\mathcal{R})$. The presence of BD is currently detected through one of the variables below [28 - 29] -

(i) Weighted residual vector $r_\omega = \sqrt{W}\ r$

(ii) Normalized residual vector $r_N = \sqrt{D^{-1}}\ r$
where $D = \text{diag}\ (\mathcal{R} W^{-1})$

(iii) Quadratic cost function

$$J(x) = r^T\ W\ r = r_\omega^T\ r_\omega$$

The detection of BD is based on a hypothesis testing with two hypotheses H_0 and H_1.

where H_0 no bad data are present

H_1 H_0 is not true i.e. there are bad data.

Denoting by P_e the probability of rejecting H_0 when H_0 is actually true and P_d the probability of accepting H_1 when H_1 is true. The hypothesis consists of testing $J(x)$, $|r_{\omega i}|$ or $|r_{N i}|$

with a detection threshold γ which depends upon P_e. For example, considering the normalized residuals, one is led to:

- accept H_0 if $|r_{Ni}| < \gamma$, i = 1, 2 ...... m
- reject H_0 (and hence accept H_1) otherwise.

The r_N has some interesting properties for acceptance as detection test i.e. the R_N - test [29].

(i) For a same detection threshold, the r_N-test is more sensitive since $|r_{N_i}| > |r_{\omega_i}|$.

(ii) r_N provides a more powerful test than r_ω because $|E[r_{Ni}]| > |E[r_{\omega i}]|$.

(iii) Within linearized approximation and provided $e = 0$, the largest normalized residual, $|r_{Ni}|_{max}$ corresponds to the erroreous measurements in the presence of a single bad data. This is generally not true for $|r_{\omega i}|_{max}$

(iv) For $\eta = m/n \to \infty$, $\mathcal{R} \to I$ and therefore $r_\omega \to r_N$

(v) In presence of multiple BD the property (iii) above does not hold true. In this case

$$E[r_{Ni}] = \frac{\Sigma_j \mathcal{R}_{ij} e_j}{\sigma_i \sqrt{\mathcal{R}_{ii}}} \qquad \text{... (2.32)}$$

2.6.2 <u>Identification</u>

A set of BD, being known, it is interesting to determine whether the measurement configuration is rich enough to allow their proper identification. A set of BD is said to be topologically identifi-able if their suppression does not cause:

- System's unobservability
- Creation of basic or critical measurements, i.e. those measurements whose errors are undetectable.

It is desired that the if f is BD then f < m-n. It is a necessary condition but not sufficient, as it must satisfy the observability criteria discussed in pare 2.7. The techniques of BD identification are broadly classified in three categories [30] -

(i) Identification by Estimation (IBE)

(ii) Non-Quadratic Criteria (NQC)

(iii) Hypothesis Testing Identification (HTI).

2.6.2.1 Identification By Estimation (IBE)

Conceptually it is the continuation of BD detection step implying residual vector r_N or (r_ω). In the event of positive detection test, a first list of candidate BD is drawn up on the basis of an R_N test, then successive cycles of elimination - reestimation - redetection are performed until the test becomes positive. Two sub-classes can be distinguished corresponding to the elimination of single or of group BD. The former consists in eliminating at each cycle the measurements having the largest magnitude of the normalized or weighted residual as introduced by Schweppe et al [1, 2]. While for grouped elimination a grouped search has been proposed by Handschin et al [31]. It consists in eliminating a group of suspected measurements which supposedly include all BD, and reinserting them afterwards one-by-one. Another variaton to this procedure is correction of suspected measurements. in (2.29) [30]. The work by Xiang et al [28] has proved that correcting measurements amount to their elimination.

2.6.2.2 Identification by NQC

This methodology has bearing on minimizing the cost function

$$J(x) = \sum_{i=1}^{m} f_i(r_i/\gamma_i) \qquad \ldots (2.33)$$

where f_i is equal to r_i^2/σ_i^2 when $|r_{xi}| < \gamma$, here r_{xi} denotes either r_{Ni} or $r_{\omega i}$ and γ is a properly chosen threshold. When $|r_{xi}| \geq \gamma$, f_i takes one of the non-quadratic criteria detailed by Handschin et al [31].

Applying the Guass-Newton algorithm to (2.33) gives the following iterative algorithm (K = 0, 1, 2 ....)

$$H^T PH[x(K+1)-x(K)] = H^T Q[z-x(K)] \qquad \ldots (2.34)$$

Here P and Q are diagonal weighting matrices. Comparison of (2.34) with (2.6) show that the method consists in modifying the weights of the measurements according to their residuals.

2.6.2.3 Identification by HTI

This method comprises ~~of~~ three main steps -

(i) At the end of detection test, which presumably has shown presence of BD, the measurements are arranged in decreasing value of $|r_{Ni}|$, i.e. in decreasing suspicion. A list 's' the suspected measurements is drawn up and an estimate $\hat{e}_s$ of the measurement error vector is computed as under

$$\hat{e}_s = \mathcal{R}_{ss}^{-1} r_s \qquad \ldots (2.35)$$

$$J(\hat{x}_c) = J(\hat{x}) - r_s^T W_s \hat{e}_s \qquad \ldots (2.36)$$

By means of (2.36), the $J(\hat{x}_c)$ test allows verifying whether all the BD have been selected.

(ii) On the basis of variance of $\hat{e}_{si}$ of the i^{th} measurement assumed to be valid and for fixed risk α, a threshold is computed.

(iii) Comparing $|e_{si}|$ with λ_i allows deciding whether i^{th} measurement is valid ($|e_{si}| \leqslant \lambda_i$) or false. It is important to note that unlike detection test, this identification test is particularized to each processed measurement.

The HTI method can be exploited through either of the two strategies [29].

Strategy α : The decision is taken with a fixed type α error probability of deciding false a measurement which is valid.

Strategy β : The decision is taken with a fixed type β error probability of declaring valid a measurement which is false.

2.6.3 Comparison of Identification Methods

A comparison of the above three methods of identification with their relative advantages and disadvantages is given in Table 2.2.

Slutsker [32] has utilized the best features of the above methods and has suggested that the presence of erroroeous measure-

Identification by Elimination	Non-Quadratic Crietria	Hypothesis Testing Identification
Advantages	**Advantages**	**Advantages**
(i) It is simple, since only Computation it needs besides estimation is that of residuals.	(i) It's simplicity is main advantage. It can be implemented through a simple transformation of the basic WLS algorithm.	(i) This method is generally able to identify all BD within a single step.
(ii) It is capable to warn the operator that the BD are topologically identifiable.	(ii) Estimation and identification are carried out in a single procedure which avoids successive re-estimations.	(ii) This method is able to identify strongly interacting BD.
		(iii) This method treats properly topologically unidentifiable BD.
Disadvantages	**Disadvantages**	**Disadvantages**
(i) It is heavy since it requires series of restimation - detection after each elimination, and may lead to incompatibility to on-line implementation.	(i) It has strong tendency to slow convergence or even to divergence.	(i) There is a risk of poor identification corresponding to the case where one or several BD are not selected.
(ii) It leads to a degeneration of the measurements configuration and a subsequent drop of the power of detection test.	(ii) High risk of wrong identification.	(ii) The method requires computation of $\mathcal{R}_{ss}$ matrix whereas the other procedures merely need the diagonal of the $\mathcal{R}$ matrix.
(iii) In the event of multiple bad data it can provoke an undue elimination of valid measurements	(iii) No recognition of topological unidentifiable BD situations.	
	(iv) Partial rejection of BD except for QC criteria.	

ment is assumed when at least one of the two conditions is violated.

$$J(x) = \sum_{i=1}^{m} (r_{\omega_i})^2 < \beta \qquad \ldots (2.37)$$

$$|r_{N_i}| < \gamma_i \qquad i = 1 \ldots\ldots m \qquad \ldots (2.38)$$

It has been reported that individual requirements in (2.37) and (2.38) do not always generate an error free measurement set which is assured by meeting both the conditions. This method performs identification in two phases. In phase 1, measurements with the largest absolute normalized residual are successively eliminated and added to the suspected measurement set, referred to as compensated set. The remaining measurements are analysed for the presence of bad data by computing new values of J(x) and r_{N_i}, and comparing them with thresholds. Each cycle of this process is referred to as identification pass. When the identification tests (2.37) to (2.38) is negative the suspected measurements are assumed to include all bad data.

As a product of measurement elimination the estimated errors of the suspected measurements become available. In phase 2 of the method the final classification of the suspected measurements is performed by comparing their normalized estimated error against statistically derived threshold. The measurements deemed to be valid data are returned to the measurement set.

2.7 OBSERVABILITY

A system is said to be observable if with the available set of measurements it is possible to determine the states of the system. It requires the measurements to be well distributed geographically. Sufficient redundancy in measurements will allow processing of BD as discussed in section 2.6. Thus at the stage of design of a state estimator following questions must be positively replied.

(i) Are there sufficient measurements to make state estimation possible

(ii) If not, where additional meters should be placed so that state estimation is possible

Temporary unobservability may still occur due to unanticipated network topology changes or failure of communication link. However, a system is designed to be observable for most operating conditions. Therefore the observability test algorithm must satisfy following requirements -

(i) Test whether there are enought real time measurements to make state estimation possible.

(ii) If requirement (i) is not met, it should provide information in respect to the part of the network whose states can still be estimated with available measurements i.e. observable islands.

(iii) It should assist in estimation of the states of observable islands.

(iv) Selection of pseudo-measurements to be included in the measurement set to make the state estimation possible.

(v) It should guarantee that inclusion of additional pseudo-measurements will not contaminate the results of the state estimation.

These considerations lead to redefinition as under [33].

A network is said to be observable if for all ϕ such that $H\phi = 0$, $A^T\phi = 0$. Any state ϕ^* for which $H\phi^* = 0$, $A^T\phi^* \neq 0$ is called unobservable state. For an unobservable ϕ^*, let $\delta^* = A^T \phi^*$ if $\delta_i^* \neq 0$ then the corresponding branch is an unobservable branch.

Here H is the B' matrix of the fast decoupled load flow. A is the incidence matrix and ϕ is the angle vector.

Mathematically network observability is related to the rank of the Jacobian matrix. The rank of matrix is very sensitive to the numerical values of its elements, where-as the observability should not. Therefore most of the methods proposed on network observability are combinatoric in nature and use no floating point calculation. Clements and Wollenberg [34] proposed a heuristic procedure to process measurements for observability. Allemong et al [35] proposed a modified version of the Clement's method as it was conservative in the sense that it may label an observable systems as unobservable. Handschin et al [36] proposed a method which tests connectivity of the Jacobian matrix. Krumpholz et al [37, 38, 39] utilized concept of graph theory to develop a theoretic topological basis of a algorithm for network observability. These combinatoric methods were since very complex and computationally expensive Monticelli et al [33, 40] developed an observability algorithm on Traingular Factorisation. This algorithm has theoretical basis on five theorems , brought out in Appendix B.

2.8 CONCLUSION

The NS method being the basic one has been initially taken up for implementation. The HACHTEL'S method for its superiority discussed in para 2.4.5 and 2.5 has also been selected for implementation. Bad data processing and observability are essential constituents of a state estimator. However, these features have not been included in the present software because, these areas in themself are of special study and it has been considered as next stage of the development of state estimator, and included in future scope of work.

CHAPTER - III

IMPLEMENTATION OF POWER SYSTEM STATE ESTIMATION METHODS

3.0 INTRODUCTION

The Normal Equation method and Hachtel's Augmented Matrix method have been selected for implementation. The former being basic method and the later one is recently reported method for Power System State Estimation. Fast decoupled version of these methods have been reported by Wu et al [15], using double precision computations for solution of equations. Fast decoupled implementation of these methods were developed using single precision. It was seen that in single precision computation these methods were non-convergent. This necessiated the development of implementation using full Jacobian of Newton-Raphson method in single precision. The natural decoupling of voltage angle and magnitude with active and reactive power, facilitates decoupling of equations [24]. Therefore all the equations have been developed in polar co-ordinates for facilitating subsequent extension into decoupled version.

3.1 FUNDAMENTAL EQUATIONS

The state variables of power system are V and δ, while measurements are active and reactive power injections at buses and line flows. The equations for bus power injections and line flow are -

$$Y_{BUS} : \bar{Y}_{ii} = |Y_{ii}| \angle -\theta_{ii}, \quad \bar{Y}_{ij} = |Y_{ij}| \angle -\theta_{ij} \quad \ldots (3.1)$$

Power injection : $S_i = P_i + j\,Q_i$ ... (3.2)

$$P_i = |V_i|^2 |Y_{ii}| \cos\theta_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} |Y_{ij} V_i V_j| \times \cos(\delta_i - \delta_j + \theta_{ij}) \quad \ldots (3.3)$$

$$Q_i = |V_i^2 Y_{ii}| \sin\theta_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} |Y_{ij} V_i V_j| \times \sin(\delta_i - \delta_j + \theta_{ij}) \quad \ldots (3.4)$$

Line Flows : $P_{ij} = |V_i V_j Y_{ij}| \cos(\delta_i - \delta_j + \theta_{ij}) - |V_i^2 Y_{ij}| \cos\theta_{ij}$ ... (3.5)

$$Q_{ij} = |V_i V_j Y_{ij}| \sin(\delta_i - \delta_j + \theta_{ij}) - |V_i^2 Y_{ij}| \sin\theta_{ij} \quad \ldots (3.6)$$

The Jacobian H in (2.8) is derived from (3.1) to (3.6). The elements of H are initially derived considering measurement of active and reactive power injections on all the buses and line flows at both the ends. In case some measurements are missing, then rows and columns corresponding to these measurements could be deleted while structuring H matrix.

The Jacobian partitions H_A, H_B, H_E and H_F in (2.8) are square matrices and their diagonal elements are -

$$\frac{\partial P_i}{\partial \delta_i} = - \sum_{\substack{j=1 \\ j \neq i}}^{n} |Y_{ij} V_i V_j| \, \text{Sin}(\delta_i - \delta_j + \theta_{ij}) \qquad \ldots (3.7)$$

$$\frac{\partial P_i}{\partial V_i / |V_i|} = |2V_i^2 Y_{ii}| \, \text{Cos}\, \theta_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} |Y_{ij} V_i V_j| \times \text{Cos}(\delta_i - \delta_j + \theta_{ij}) \qquad \ldots (3.8)$$

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} |Y_{ij} V_i V_j| \, \text{Cos}(\delta_i - \delta_j + \theta_{ij}) \qquad \ldots (3.9)$$

$$\frac{\partial Q_i}{\partial V_i / |V_i|} = 2|V_i^2 Y_{ii}| \, \text{Sin}\, \theta_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} |Y_{ij} V_i V_j| \times \text{Sin}\,(\delta_i - \delta_j + \theta_{ij}) \qquad \ldots (3.10)$$

The off diagonal terms of H_A, H_B, H_E and H_F are not symmetrical as shown below.

$$\begin{bmatrix} \cdots (i,i) \cdots & \cdots (i,j) \cdots \\ \vdots & \vdots \\ \cdots (j,i) \cdots & \cdots (j,j) \cdots \end{bmatrix}$$

The $(i,j)^{th}$ element corresponding to H_A will be

$$\frac{\partial P_i}{\partial \delta_j} = |Y_{ij} V_i V_j| \, \text{Sin}(\delta_i - \delta_j + \theta_{ij})$$

$$= |Y_{ij} V_i V_j| \, \text{Sin}(\theta_{ij} + \Delta\delta_{ij}) \qquad \ldots (3.11)$$

where $\Delta\delta_{ij} = \delta_i - \delta_j$

and $(j,i)^{th}$ of H_A will be

$$\frac{\partial P_j}{\partial \delta_i} = |Y_{ij} V_i V_j| \operatorname{Sin}(\theta_{ij} - \Delta\delta_{ij}) \qquad \ldots (3.12)$$

Likewise in all the off diagonal elements of H_B, H_E and H_F, the $\Delta\delta_{ij}$ will appear with opposite signs and distort their symmetry. However, matrix H_A, H_B, H_E and H_F are structurally symmetrical. The off diagonal elements of H_B, H_E and H_F are -

$$\frac{\partial P_i}{\partial V_j/|V_j|} = |Y_{ij} V_i V_j| \operatorname{Cos}(\theta_{ij} + \Delta\delta_{ij}) \qquad \ldots (3.13)$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|Y_{ij} V_i V_j| \operatorname{Cos}(\theta_{ij} + \Delta\delta_{ij}) \qquad \ldots (3.14)$$

$$\frac{\partial Q_i}{\partial V_j/|V_j|} = |Y_{ij} V_i V_j| \operatorname{Sin}(\theta_{ij} + \Delta\delta_{ij}) \qquad \ldots (3.15)$$

The line flow measurements contribute to H_C, H_D, H_G and H_H partitions in (2.9) and their respective elements are -

$$\left.\begin{aligned} \frac{\partial P_{ij}}{\partial \delta_i} &= -|Y_{ij} V_i V_j| \operatorname{Sin}(\delta_i - \delta_j + \theta_{ij}) \\ \frac{\partial P_{ij}}{\partial \delta_j} &= |Y_{ij} V_i V_j| \operatorname{Sin}(\delta_i - \delta_j + \theta_{ij}) \end{aligned}\right\} \qquad \ldots (3.16)$$

$$\left.\begin{aligned} \frac{\partial P_{ij}}{\partial V_i/|V_i|} &= |V_i V_j Y_{ij}| \operatorname{Cos}(\delta_i - \delta_j + \theta_{ij}) - \\ &\quad |2V_i^2 Y_{ij}| \operatorname{Cos} \theta_{ij} \\ \frac{\partial P_{ij}}{\partial V_j/|V_j|} &= |V_i V_j V_{ij}| \operatorname{Cos}(\delta_i - \delta_j + \theta_{ij} \end{aligned}\right\} \qquad \ldots (3.17)$$

$$\left.\begin{aligned}\frac{\partial Q_{ij}}{\partial \delta_i} &= |Y_{ij}\ V_i\ V_j| \cos(\delta_i - \delta_j + \theta_{ij}) \\ \frac{\partial Q_{ij}}{\partial \delta_j} &= -|Y_{ij}\ V_i\ V_j| \cos(\delta_i - \delta_j + \theta_{ij})\end{aligned}\right\} \quad \ldots (3.18)$$

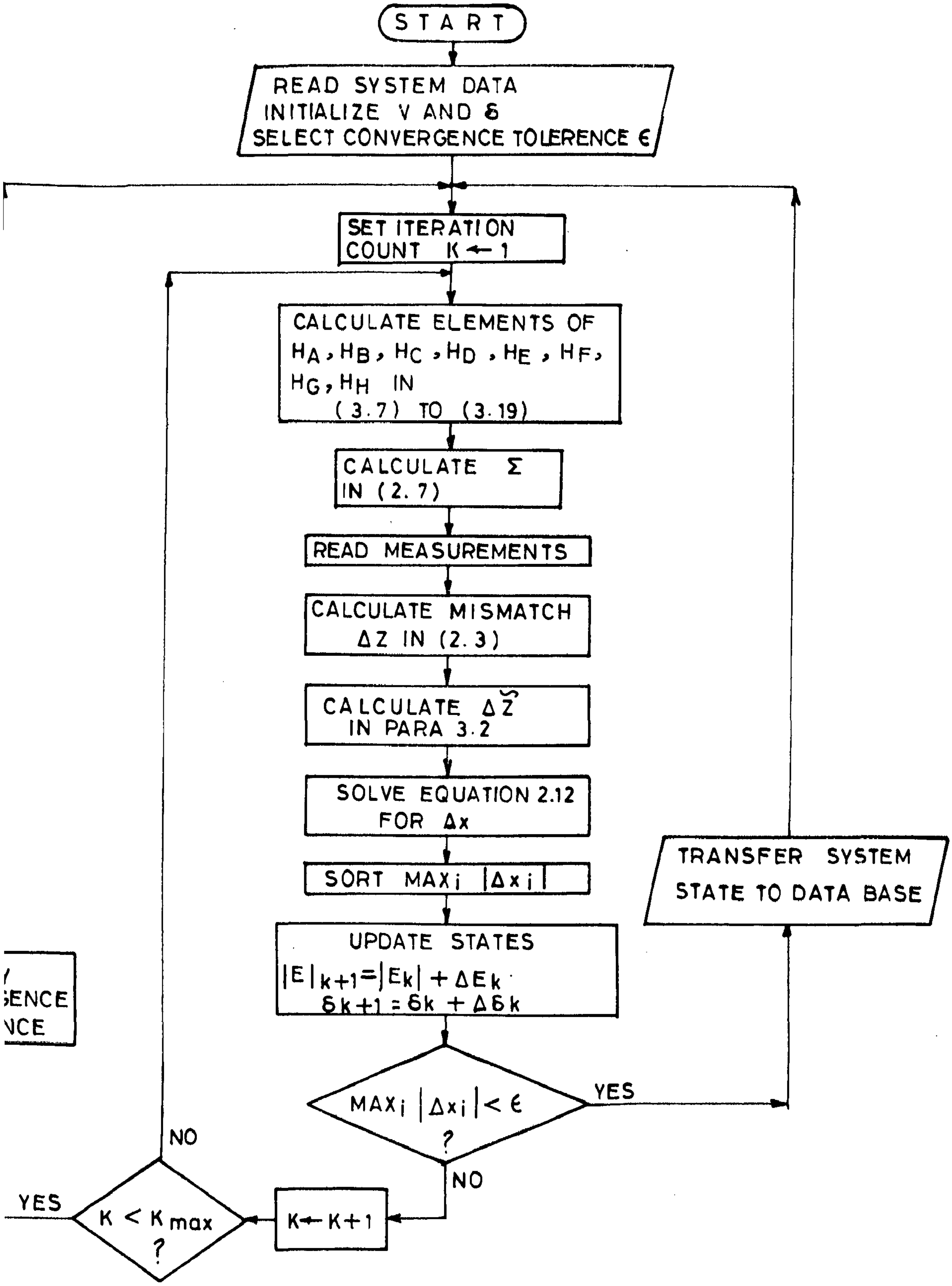
$$\left.\begin{aligned}\frac{\partial Q_{ij}}{\partial V_i/|V_i|} &= |V_i\ V_j\ Y_{ij}| \sin(\delta_i - \delta_j + \theta_{ij}) - |2V_i^2\ Y_{ij}| \sin\theta_{ij} \\ \frac{\partial Q_{ij}}{\partial V_j/|V_j|} &= |V_i\ V_j\ Y_{ij}| \sin(\delta_i - \delta_j + \theta_{ij})\end{aligned}\right\} \quad \ldots (3.19)$$

3.2 NORMAL EQUATION (NE) METHOD

The mathematical formulation of NE method has been discussed in 2.4.1. A pre-requisite for determining state correction vector by solving equations (2.12), is computation of $\Sigma(= H^T WH)$ and $\Delta\tilde{z}(= H^T W\Delta z)$. Algorithm for state estimation by NE method is given in Fig. 3.1.

The solution steps of this method are as follows -

1. Read system data and compute Y_{BUS} and initialize state V and δ. Select convergence tolerance ϵ.
2. Set $K \leftarrow 1$.
3. Compute Jacobian H in (2.8) using (3.7) to (3.19).
4. Compute gain matrix Σ ($= H^T WH$) in (2.6) and store non-zero elements.
5. Read measurements and calculate mismatches Δz in (2.3).



1 ALGORITHM FOR STATE ESTIMATION BY NORMAL EQUATION METHOD

6. Compute $\Delta\tilde{z}$ ($= H^T W\Delta z$) in para 3.2.

7. Solve (2.6) by LU factorisation for state correction vector Δx.

8. Correct the estimates $x_{K+1} \leftarrow x_K + \Delta x$. If $\max_i |\Delta x_i| < \varepsilon$ then output the state to data base and go to Step 2.

9. Increment iteration count $K \leftarrow K+1$. If $K < K_{max}$, then go to step 4. Otherwise modify convergence tolerance and initialize iteration count $K \leftarrow 1$, and go to Step 3.

The results of state estimation comparing with base case values and measured values for Two bus and Six bus (SPC) systems are shown in Table 3.1(a) and 3.1(b), respectively.

3.3 HACHTEL's AUGMENTED MATRIX (HACHTEL) METHOD

The mathematical aspects of this method are discussed in para 2.4.5. This method has advantage of saving computation of Σ and $\Delta\tilde{z}$. It, however, has higher order matrix due to augmentation. Coefficient matrix in HACHTEL method is quite sparse, while in NE method the matrix is quite dense.

The set of equations in HACHTEL method are defined in (2.28) The diagonal elements corresponding to the constrained buses are zero. Thus controversy or any ill-conditioning due to excessive weights on these virtual measurements is averted. It is, further, noteworthy that all other row and column elements outside C and C^T respectively are zero. Moreover rows of C matrix are rows of H_A, H_B, HE and HF sub-matrices in (2.8). Therefore, while implementing the algorithm, instead of clubbing them separately in C and C^T

TABLE 3.1(a)

NE method - 2 Bus system

----STATE ESTIMATION RESULTS----

PARTICULARS		*BASE CASE*	*MEASURED*	*ESTIMATED*
BUS POWER INJECTIONS -				
NO.	TYPE			
ACTIVE POWER:				
1	0	1.4999	1.4569	1.5106
2	3	1.4996	1.5370	1.5103
REACTIVE POWER:				
1	0	.0127	-.0061	-.0059
2	3	.4482	.4742	.4436
LINE POWER FLOWS -				
FROM	TO			
ACTIVE POWER:				
1	2	-1.5000	-1.5446	-1.5107
2	1	1.4997	1.4973	1.5104
REACTIVE POWER:				
1	2	-.0127	.0191	.0059
2	1	.1482	.1326	.1511
BUS VOLTAGE MAGNITUDE -				
NO.				
1		.9862		.9750
2		1.0000		.9875
BUS VOLTAGE ANGLE -				
NO.				
1		-.2587		-.2671
2		.0000		.0000

N.B. 1. ANGLES IN RADIANS
2. POWER AND VOLTAGE MAGNITUDES IN P.U.

Summary of errors:

Sum of bus active power measurements errors	=	5.55 E-03
Sum of bus reactive power measurements errors	=	-1.99 E-02
Sum of line active power measurements errors	=	4.70 E-02
Sum of line reactive power measurements errors	=	-3.50 E-03
Sum of all active power measurements errors	=	5.26 E-02
Sum of all reactive power measurements errors	=	-2.34 E-02
Sum of all measurement errors	=	2.92 E-02

TABLE 3.1(b)

NE method - 6 Bus (SPC) system

----STATE ESTIMATION RESULTS----

PARTICULARS		*BASE CASE*	*MEASURED*	*ESTIMATED*
1		2	3	4
BUS POWER INJECTIONS -				
NO.	TYPE			
ACTIVE POWER:				
1	0	.5631	.5201	.5600
2	0	.9852	.9478	.9637
3	2	-.4165	-.4217	-.4213
4	2	1.3285	1.3223	1.3340
5	2	1.4999	1.4553	1.4913
6	3	-.6380	-.6404	-.6524
REACTIVE POWER:				
1	0	.0391	.0203	.0324
2	0	.1022	.0762	.1011
3	2	-.0131	-.0485	-.0228
4	2	.0277	.0684	.0649
5	2	.0746	.1064	.0692
6	3	1.3406	1.3250	1.3214
LINE POWER FLOWS -				
FROM	TO			
ACTIVE POWER:				
1	2	.3586	.3086	.3496
2	1	-.3378	-.3812	-.3291
1	5	-1.4427	-1.4594	-1.4329
5	1	1.4999	1.5320	1.4913
1	6	.5209	.4798	.5233
6	1	-.5128	-.5108	-.5148
2	4	-.4544	-.4688	-.4542
4	2	.5121	.4704	.5124
2	6	-.1929	-.1565	-.1804
6	2	.2029	.2097	.1899
3	4	-.7484	-.7741	-.7526
4	3	.8165	.8592	.8216
3	6	.3319	.3746	.3313
6	3	-.3281	-.3055	-.3275
REACTIVE POWER:				
1	2	-.0320	-.0370	-.0377
2	1	.0470	.0364	.0527
1	5	.0615	.0845	.0820
5	1	.0746	.0495	.0692
1	6	-.3454	-.3422	-.3454
6	1	.1270	.1502	.1379

contd..

1	2	3	4
2 4	.0488	.0564	.0416
4 2	-.0287	-.0538	-.0164
2 6	-.1981	-.2243	-.1954
6 2	.1905	.1903	.1873
3 4	.0772	.0497	.0630
4 3	-.2436	-.2249	-.2187
3 6	-.0903	-.0962	-.0859
6 3	.0731	.0256	.0696
BUS VOLTAGE MAGNITUDE -			
NO.			
1	.9605		.9463
2	.9204		.9094
3	1.0000		.9881
4	1.0000		.9998
5	1.0000		.9839
6	1.0000		.9876
BUS VOLTAGE ANGLE -			
NO.			
1	.1197		.1237
2	-.0329		-.0302
3	.0464		.0474
4	.4645		.4709
5	.3800		.3910
6	.0000		.0000

N.B. 1. ANGLES IN RADIANS
2. POWER AND VOLTAGE MAGNITUDES IN P.U.

Summary of errors:

Sum of bus active power measurements errors	=	1.39 E-03
Sum of bus reactive power measurements errors	=	-2.07 E-02
Sum of line active power measurements errors	=	4.77 E-02
Sum of line reactive power measurements errors	=	4.09 E-03
Sum of all active power measurements errors	=	1.86 E-02
Sum of all reactive power measurements errors	=	2.02 E-02
Sum of all measurement errors	=	3.88 E-02

submatrices of the augmented coefficient matrix, they have been retained in respective positions of H_A, H_B, H_E and H_F sub-matrices, and their corresponding diagonal elements are made zero. This is equivalent to re-ordering of row and column and simplifies structuring of coefficient matrix.

The stepwise procedure of the method is as under -

1. Read system data and compute Y_{BUS} and initialize states V and δ. Select convergence tolerance ϵ.
2. Set $K \leftarrow 1$.
3. Compute Jacobian H in (2.8) using (3.7) to (3.19). Store non-zero elements.
4. Construct Hachtel's Augmented matrix in (2.28).
5. Read measurements and calculate mismatches Δz in (2.3). Construct right hand side vector in (2.28).
6. Solve (2.28) using LU factorisation and sort state correction vector Δx.
7. Correct state estimates. If $\max_i |\Delta x_i| < \epsilon$ then output the state to data base and go to Step 2.
8. Increment iteration count $K \leftarrow K+1$. If $K < K_{max}$, then go to Step 4. Otherwise modify convergence tolerance and initialize iteration count $K \leftarrow 1$, and go to Step 3.

The algorithm for state estimation by this method is given in Fig. 3.2. The results of state estimation comparing base case values and measured values for Two bus and Six bus (SPC) systems

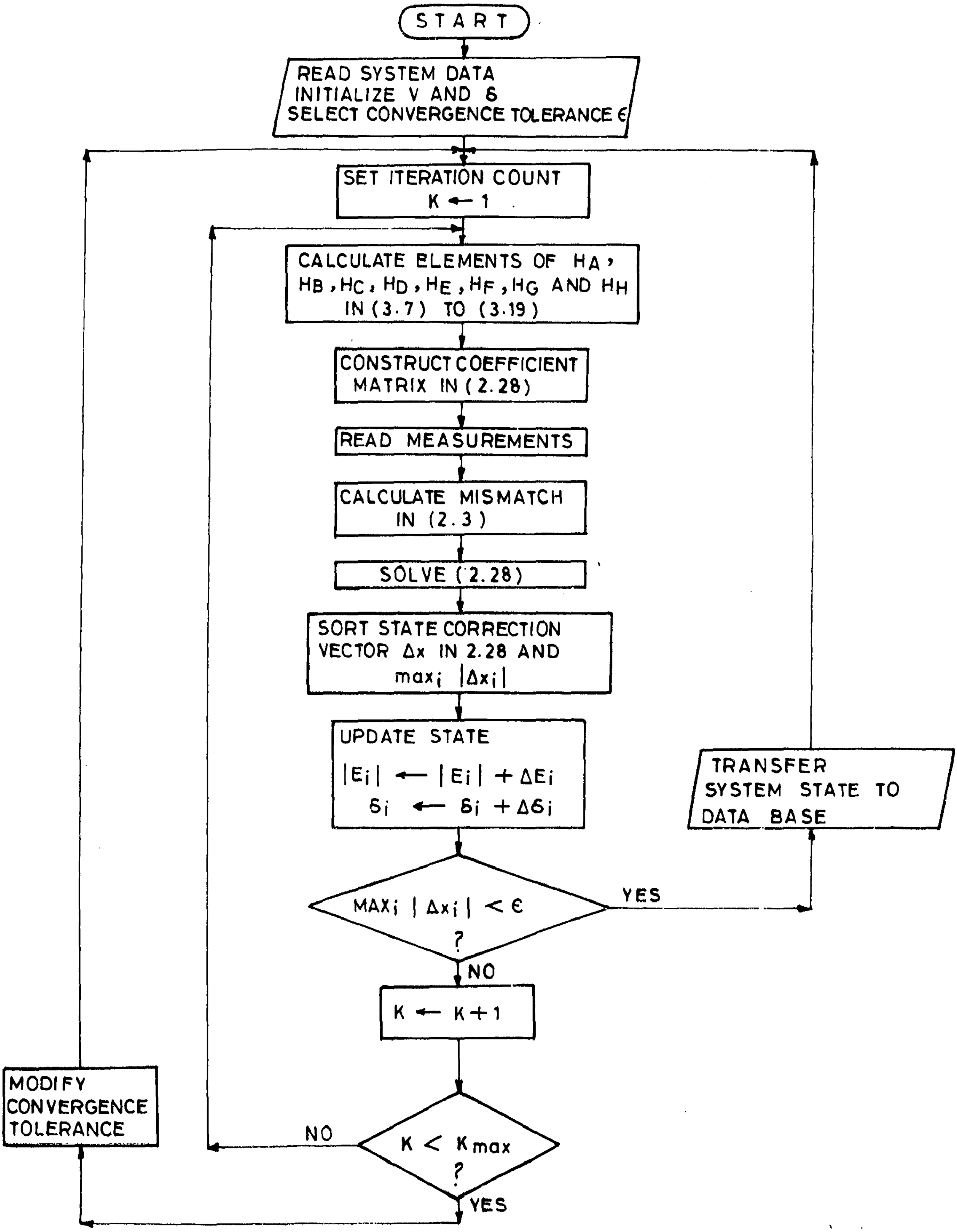


FIG. 3.2 ALGORITHM FOR STATE ESTIMATION BY HACHTEL'S METHOD

are shown in Tables 3.2(a) and 3.2(b), respectively. The Table 3.2(c) shows similar results for 14 bus IEEE system.

3.4 DISCUSSION

The results of State Estimation by these two methods have been obtained on the identical base case values and measured values for Two bus and Six bus systems. It is seen from the results that the state estimates are practically identical in both the methods. Since there is biased error in measurements, as depicted in summary of errors shown with tabulated results, there is slight drift from base case values in approximated states by the State Estimators.

The computation time of the two methods for Two bus and Six bus systems are shown in Table 3.3. It is seen that for two bus system the computation time by NE method is marginally less than HACHTEL method. But for Six bus system there is significant gain in computation time by HACHTEL method. Computation time for Six bus system by NE method was since inordinately high, State Estimation for Fourteen bus system was made only by HACHTEL method. It is seen that inspite of increase in system size by about 2.5 times the computation time has increased by about 2.2 times. This confirms that notwithstanding to the increase in order of matrix in HACHTEL method, there is considerable saving in computation time. Large computation time in NE method is attributed to computation of $\Sigma (= H^T WH)$ and $\Delta\tilde{z}$ $(= H^T W\Delta z)$. The matrix H is since stored in sparse form, the matrix operation are further time consuming.

TABLE 3.2(a)

HACHTEL's method - 2 Bus system

----STATE ESTIMATION RESULTS----

PARTICULARS		*BASE CASE*	*MEASURED*	*ESTIMATED*
BUS POWER INJECTIONS -				
NO.	TYPE			
ACTIVE POWER:				
1	0	1.4999	1.4569	1.5106
2	3	1.4996	1.5370	1.5103
REACTIVE POWER:				
1	0	.0127	-.0061	-.0059
2	3	.4482	.4742	.4436
LINE POWER FLOWS -				
FROM	TO			
ACTIVE POWER:				
1	2	-1.5000	-1.5446	-1.5107
2	1	1.4997	1.4973	1.5104
REACTIVE POWER:				
1	2	-.0127	.0191	.0059
2	1	.1482	.1326	.1511
BUS VOLTAGE MAGNITUDE -				
NO.				
1		.9862		.9750
2		1.0000		.9875
BUS VOLTAGE ANGLE -				
NO.				
1		-.2587		-.2671
2		.0000		.0000

N.B. 1. ANGLES IN RADIANS
2. POWER AND VOLTAGE MAGNITUDES IN P.U.

Summary of errors:

Sum of bus active power measurements errors = 5.55 E-03
Sum of bus reactive power measurements errors = -1.99 E-02
Sum of line active power measurements errors = 4.70 E-02
Sum of line reactive power measurements errors = -3.50 E-03
Sum of all active power measurements errors = 5.26 E-02
Sum of all reactive power measurements errors = -2.34 E-02
Sum of all measurement errors = 2.92 E-02

TABLE 3.2(b)

HACHTEL's method - 6 Bus (SPC) system

----STATE ESTIMATION RESULTS----

PARTICULARS		*BASE CASE*	*MEASURED*	*ESTIMATED*
BUS POWER INJECTIONS -				
NO.	TYPE			
ACTIVE POWER:				
1	0	.5631	.5201	.5600
2	0	.9852	.9478	.9637
3	2	-.4165	-.4217	-.4213
4	2	1.3285	1.3223	1.3340
5	2	1.4999	1.4553	1.4913
6	3	-.6380	-.6404	-.6524
REACTIVE POWER:				
1	0	.0391	.0203	.0324
2	0	.1022	.0762	.1011
3	2	-.0131	-.0485	-.0228
4	2	.0277	.0684	.0649
5	2	.0746	.1064	.0692
6	3	1.3406	1.3250	1.3214
LINE POWER FLOWS -				
FROM	TO			
ACTIVE POWER:				
1	2	.3586	.3086	.3496
2	1	-.3378	-.3812	-.3291
1	5	-1.4427	-1.4594	-1.4329
5	1	1.4999	1.5320	1.4913
1	6	.5209	.4798	.5233
6	1	-.5128	-.5108	-.5148
2	4	-.4544	-.4688	-.4542
4	2	.5121	.4704	.5124
2	6	-.1929	-.1565	-.1804
6	2	.2029	.2097	.1899
3	4	.7484	-.7741	-.7526
4	3	.8165	.8592	.8216
3	6	.3319	.3746	.3313
6	3	-.3281	-.3055	-.3275
REACTIVE POWER:				
1	2	-.0320	-.0370	-.0377
2	1	.0470	.0364	.0527
1	5	.0615	.0845	.0820
5	1	.0746	.0495	.0692
1	6	-.3454	-.3422	-.3454
6	1	.1270	.1502	.1379

contd...

1		2	3	4
2	4	.0488	.0564	.0416
4	2	-.0287	-.0538	-.0164
2	6	-.1981	-.2243	-.1954
6	2	.1905	.1903	.1873
3	4	.0772	.0497	.0630
4	3	-.2436	-.2249	-.2187
3	6	-.0903	-.0962	-.0859
6	3	.0731	.0256	.0696

BUS VOLTAGE MAGNITUDE

NO.			
1	.9605		.9463
2	.9204		.9094
3	1.0000		.9881
4	1.0000		.9998
5	1.0000		.9839
6	1.0000		.9876

BUS VOLTAGE ANGLE -

NO.			
1	.1197		.1237
2	-.0329		-.0302
3	.0464		.0474
4	.4645		.4709
5	.3800		.3910
6	.0000		.0000

N.B. 1. ANGLES IN RADIANS
2. POWER AND VOLTAGE MAGNITUDES IN P.U.

Summary of errors:

Sum of bus active power measurements errors = 1.39 E-03
Sum of bus reactive power measurements errors = -2.07 E-02
Sum of line active power measurements errors = 4.77 E-02
Sum of line reactive power measurements errors = 4.09 E-03
Sum of all active power measurements errors = 1.86 E-02
Sum of all reactive power measurements errors = 2.02 E-02
Sum of all measurement errors = 3.88 E-02

TABLE 3.2(c)

HACHTEL's method 14 Bus (IEEE) system

----STATE ESTIMATION RESULTS----

PARTICULARS		*BASE CASE*	*MEASURED*	*ESTIMATED*
1		2	3	4
BUS POWER INJECTIONS -				
NO.	TYPE			
ACTIVE POWER:				
1	3	2.3450	2.3500	2.3343
2	2	.1830	.2310	.2147
3	2	-.9420	-.9920	-1.0038
4	0	.4780	.4440	.4384
5	0	.0760	.0520	.0467
6	2	-.1119	-.1019	-.1087
7	0	.0000	.0110	-.0168
8	2	.0000	-.0330	-.0309
9	0	.2950	.3202	.3034
10	0	.0900	.1630	.1138
11	0	.0350	.0201	.0432
12	0	.0607	.0656	.0522
13	0	.1354	.0604	.1070
14	0	.1490	.1639	.1618
REALTIVE POWER:				
1	3	-.0342	-.0008	.0006
2	2	.2692	.2432	.2568
3	2	.1487	.1167	.1144
4	0	.0390	.0900	.0841
5	0	.0160	-.0240	-.0359
6	2	.0291	-.0449	-.0093
7	0	.0000	-.0148	-.0102
8	2	.0841	.1601	.0898
9	0	.1659	.1539	.1473
10	0	.0580	.0532	.0359
11	0	.0179	.0329	.0386
12	0	.0152	.0204	.0114
13	0	.0586	.0666	.0549
14	0	.0501	.0271	.0511
LINE POWER FLOWS -				
FROM	TO			
ACTIVE POWER:				
1	2	1.5863	1.5903	1.5800
2	1	-1.5427	-1.5395	-1.5376
1	5	.7587	.7558	.7543
5	1	-.7304	-.6854	-.7267
2	3	.7416	.7464	.7710

contd.

1		2	3	4
7	4	.0450	.0470	.0630
4	9	.0169	-.0210	.0117
9	4	-.0029	-.0008	.0038
5	6	.1630	.1960	.1786
6	5	-.1067	-.0777	-.1309
6	11	.0421	.0468	.0363
11	6	-.0408	-.0568	-.0345
6	12	.0255	.0307	.0204
12	6	-.0239	-.0289	-.0192
6	13	.0759	.0738	.0649
13	6	-.0715	-.0755	-.0612
7	8	-.0829	-.0811	-.0884
8	7	.0841	.0311	.0898
7	9	.0369	.0351	.0356
9	7	-.0285	-.0270	-.0269
9	10	.0358	.0376	.0409
10	9	-.0355	-.0369	-.0404
9	14	.0322	.0862	.0413
14	9	-.0298	-.0346	-.0388
10	11	-.0225	-.0035	.0045
11	10	.0228	-.0422	-.0040
12	13	.0088	-.0142	.0078
13	12	-.0087	-.0071	-.0077
13	14	.0216	-.0064	.0141
14	13	-.0203	-.0543	-.0123

BUS VOLTAGE MAGNITUDE -

NO.		
1	1.0600	1.0686
2	1.0400	1.0469
3	1.0100	1.0113
4	1.0055	1.0119
5	1.0091	1.0174
6	1.0500	1.0553
7	1.0359	1.0455
8	1.0500	1.0604
9	1.0324	1.0422
10	1.0279	1.0369
11	1.0353	1.0401
12	1.0348	1.0423
13	1.0295	1.0371
14	1.0127	1.0206

BUS VOLTAGE ANGLE -

NO.		
1	.0000	.0000
2	-.0869	-.0847
3	-.2256	-.2267
4	-.1799	-.1766

contd..

1		2	3	4
3	2	-.7177	-.7467	-.7454
2	4	.5650	.5650	.5666
4	2	-.5478	-.5178	-.5495
2	5	.4191	.4210	.4147
5	2	-.4097	-.4087	-.4056
3	4	-.2243	-.2463	-.2585
4	3	.2286	.2906	.2636
4	5	-.6095	-.6655	-.6368
5	4	.6146	.6176	.6422
4	7	.2832	.3033	.3064
7	4	-.2832	-.2854	-.3064
4	9	.1631	.1391	.1735
9	4	-.1631	-.1646	-.1735
5	6	.4465	.4447	.4405
6	5	-.4466	-.4626	-.4405
6	11	.0751	.1281	.0941
11	6	-.0744	-.0724	-.0933
6	12	.0783	.0764	.0702
12	6	-.0775	-.0754	-.0696
6	13	.1786	.1803	.1646
13	6	-.1763	-.1796	-.1628
7	8	.0000	.0870	.0309
8	7	.0000	.0017	-.0309
7	9	.2832	.2922	.2923
9	7	-.2832	-.3022	-.2923
9	10	.0509	.0496	.0642
10	9	-.0508	-.0466	-.0640
9	14	.0931	.0911	.0907
14	9	-.0919	-.0864	-.0896
10	11	-.0392	-.0407	-.0498
11	10	.0394	-.0156	.0500
12	13	.0168	.0183	.0174
13	12	-.0167	-.1047	-.0174
13	14	.0576	.0586	.0731
14	13	-.0570	-.0586	-.0722

REACTIVE POWER:

1	2	-.1207	-.1233	-.0885
2	1	.1954	.1404	.1590
1	5	.0865	.0896	.0891
5	1	-.0224	-.0252	-.0288
2	3	.0089	.0969	.0351
3	2	.0460	.0441	.0265
2	4	.0225	.0206	.0263
4	2	-.0093	.0036	-.0139
2	5	.0424	.0457	.0365
5	2	-.0493	-.0343	-.0450
3	4	.1028	.1518	.0879
4	3	-.1271	-.0631	-.1103
4	5	.1105	.0245	.0727
5	4	-.1074	-.1091	-.0689
4	7	-.0300	-.0290	-.0443

contd...

1	2	3	4
5	-.1530		-.1495
6	-.2522		-.2460
7	-.2355		-.2358
8	-.2355		-.2407
9	-.2646		-.2653
10	-.2676		-.2691
11	-.2623		-.2599
12	-.2678		-.2601
13	-.2691		-.2617
14	-.2848		-.2835

N.B. 1. ANGLES IN RADIANS
2. POWER AND VOLTAGE MAGNITUDES IN P.U.

SUMMARY OF ERRORS:

SUM OF BUS ACTIVE POWER MEASUREMENT ERRORS	=	-2.00E-04
SUM OF BUS REACTIVE POWER MEASUREMENT ERRORS	=	4.00E-04
SUM OF LINE ACTIVE POWER MEASUREMENT ERRORS	=	1.41E-02
SUM OF LINE REACTIVE POWER MEASUREMENT ERRORS	=	5.10E-03
SUM OF ALL ACTIVE POWER MEASUREMENT ERRORS	=	1.39E-02
SUM OF ALL REACTIVE POWER MEASUREMENT ERRORS	=	5.50E-03
SUM OF ALL POWER MEASUREMENT ERRORS	=	1.94E-02

TABLE 3.3

COMPARISON OF COMPUTATION TIME

Sl. No.	System	Normal Equation Method		HACHTEL's Method	
		Iterat-ions	Time/ Iteration	Iterat-ion	Time/ Iteration
1	2 Bus System	2	3.93 sec	2	4.07 sec
2	6 Bus (SPC) System	2	8 min 25.18 sec	2	21.81 sec
3	14 Bus (IEEE) System	-	?	2	48.1 sec

3.5 CONCLUSION

The superiority of HACHTEL'sAugmented Matrix method is confirmed. It is efficient for state estimation studies. It is also concluded that the gain in computation by HACHTEL method increased with the increase in size of the system.

CHAPTER - IV

HIERARCHICAL STATE ESTIMATION

4.0 INTRODUCTION

The growth in size of power systems poses computational time restriction on state estimator. In view of this, in recent years considerable amount of effort has been devoted to both the theoretical and practical aspects of State Estimation. These include SE technique, numerical methods, programming techniques and hardware development. Hierarchical State Estimation (HSE) is one of the analytical aspects of the problem, where local state estimation is carried out for each area by decomposition of the network and coordinating area corrects the states of the boundary buses.

Kurzyn [43] in his survey on methods for Hierarchical State Estimation has summarized the desired characteristics as under -

1. Low Applicability Constraints - These concern the way a large system can be decomposed for control purposes, and possibility of using SE, observability analysis, ill-conditioning treatment and bad data handling.

2. High Reliability - Minimum data transfer leading to state estimation even under condition of failure of one of the areas to supply the data.

3. High Robustness - This means good convergence characteristics under a wide range of system conditions.

4. Sufficient Accuracy - The HSE is not as accurate as Integrated State Estimation (ISE). But degradation in accuracy of HSE must be within acceptable limits.

5. Efficient Bad Data Handling - It is applicable at both the levels.

6. Low Complexity - It concerns application convenience.

7. Gain in CPU Time - This is an essential feature and is compared with ISE.

8. Easy Observability Analysis - It is applicable at both the levels.

9. Reduction in Core Memory - In view of the reducing costs this is less important criterion.

Van Custem et al [44] and Tripathy et al [45] have suggested two level HSE in which a network if divided into K sub-networks, K+1 solutions are obtained. One solution for each area and K+1th solution for interconnecting area formed by boundary nodes and tie lines. The First Level State Estimation (FSE) provides estimates of local area utilizing its own measurements. The Second Level State Estimator (SSE) uses the states of boundary buses as pseudo measurements and the measurements of the tie line flows for State Estimation. In case, information of tie line of one of the area is not available to the SSE, it continues operating with all the remaining available information of the tie line of the remaining area.

Seidu et al [46] have stretched the logic further, to develop coupling equations in respect of the inter-connection so that overall effect of the system is reflected on boundary parameters. They have used sparsity oriented optimally ordered traingular factorisation to solve the system of equations. This algorithm, however, requires large data transfers to arrive at the correction vector and thus defeats requirements of reliability discussed above.

Iwamato et al [47] have developed HSE mainly based on Second Order Load Flow method. The first level State Estimator in each iteration computes sub-optimal correction vector of the internal boundary buses using pre-estimated states of the external boundary buses. The sub-optimal correction vector associated with coupling information is transmitted to SSE, where optimal correction vector of the external boundary buses is computed and returned to FSE. This scheme requires repetitive data transfer, however, less than that suggested by Seidu et al [46]. Recently a Decomposition Approach for Load Flow Solution of large systems has been reported.

4.1 DECOMPOSITION METHOD

In this method with the pre-estimated states of external boundary buses State Estimation is carried out for an area. This provides sub-optimal states of internal boundary buses of the area under iteration. These sub-optimal estimates serve as pre-estimated states of external boundary buses of the connected area. One complete cycle of inter-area changes forms one coordinating cycle or global solution.

The set of buses used in local state estimation of I^{th} area comprise of the internal buses of the I^{th} area and external boundary buses of the connected area and is expressed as

$$B^I = B_i^I \cup B_{b_{1,I}}^1 \;\ldots\ldots\; \cup B_{b_{I-1,I}}^{I-1} \;\cup B_{b_{I+1,I}}^{I+1} \;\ldots\ldots\; \cup B_{b_{n,I}}^n \qquad \ldots (4.1)$$

Similarly set of lines used in local State Estimation of I^{th} area are the internal lines of the I^{th} area and tie lines connected to the th area -

$$L^I = L_i^I \cup L_{t_{1,I}}^1 \;\ldots\ldots\; \cup L_{t_{I-1,I}}^{I-1} \cup L_{t_{I+1,I}}^{I+1} \;\ldots\ldots\; \cup L_{t_{n,I}}^n \qquad \ldots (4.2)$$

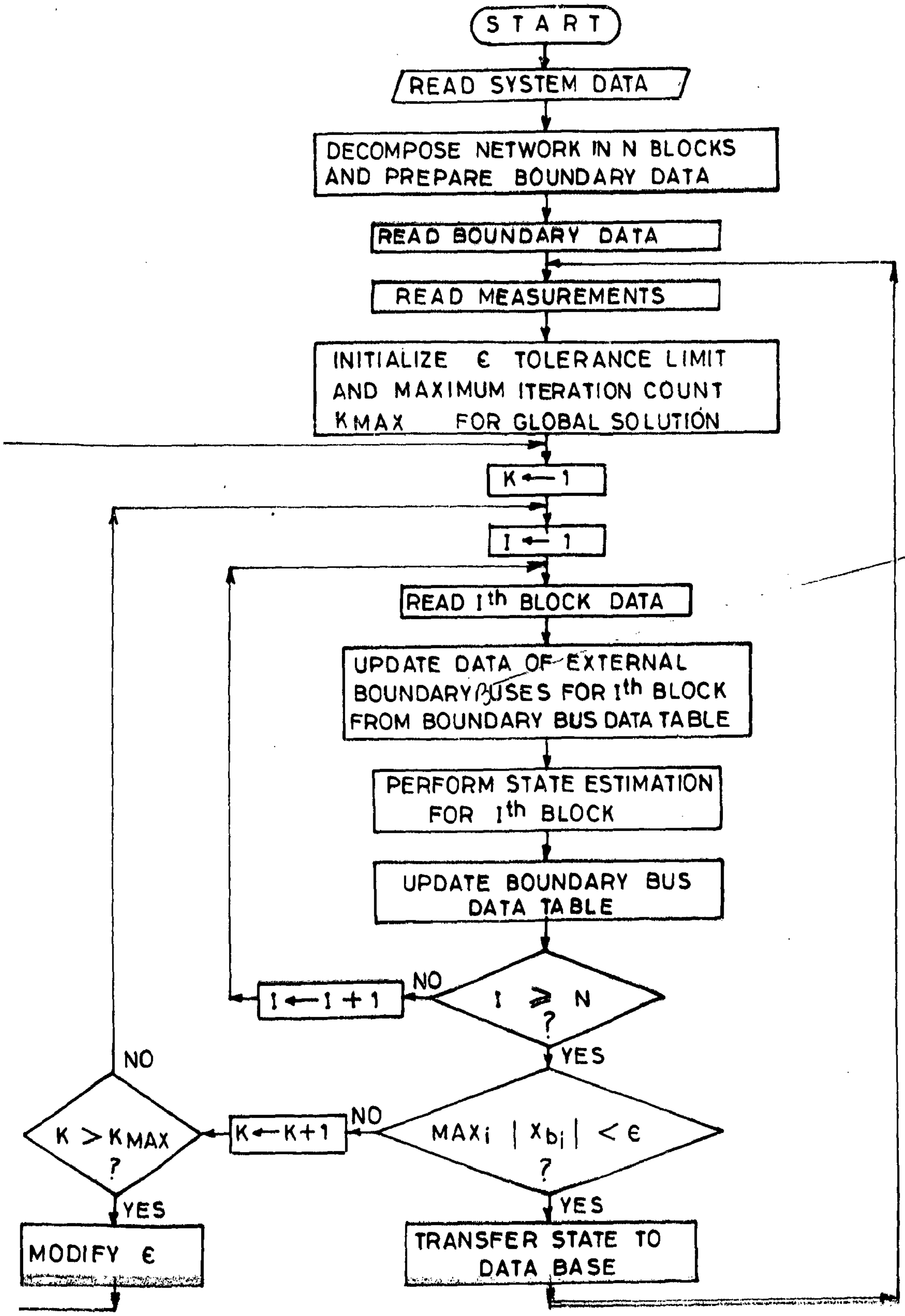
Likewise measurements are the internal measurements of the area, measurements on the lines connected to the K^{th} area and pre-estimated states of the external boundary buses as pseudo-measurements.

$$M^I = M_i^I \cup (M_{t_{1,I}}^1 \;\ldots\ldots\; \cup M_{t_{I-1,I}}^{I-1} \;\cup M_{t_{I+1,I}}^{I+1} \;\ldots\ldots\; \cup M_{t_{n,I}}^n)$$

$$\cup(\hat{x}_{b_{1,I}}^1 \cup \;\ldots\ldots\; \cup \hat{x}_{b_{I-1,}}^{I-1} \;\cup \hat{x}_{b_{I+1,I}}^{I+1} \;\ldots\ldots\; \cup \hat{x}_{b_{n,I}}^n) \qquad \ldots (4.3)$$

The algorithm has been shown in Fig. 4.1. Stepwise solution procedure of this method is -

1. Read system data and decompose the network into N blocks, prepare block data and boundary data.



IG. 4. 1 ALGORITHM OF STATE ESTIMATION BY DECOMPOSITION

2. Read boundary data, internal and external boundary buses of each area and tie lines.

3. Set maximum iteration count for global solution K_{MAX}.

4. Set $K \leftarrow 1$.

5. Initialize convergence tolerance ϵ for global solution.

6. Read measurement data and sort them blockwise.

7. Read data of I^{th} block.

8. Update V and δ of external boundary buses of I^{th} block from boundary data table.

9. Perform State Estimation of I^{th} block using solution steps given in para 3.3.

10. Update boundary bus data V and δ corresponding to the internal boundary buses of I^{th} block.

11. If $I \neq N$ then $I \leftarrow I + 1$ go to Step 7.

12. Increment global iteration count $K \leftarrow K + 1$.

13. If $\max_i |\Delta x_b^i| < \epsilon$ then transfer states to data bsse. Initialize global iteration count $K \leftarrow 1$ go to Step 6.

14. If $K > K_{MAX}$ modify convergence tolerance. Go to Step 7.

In this algorithm states of each area are estimated sequentially and updated states of the boundary buses are used in sequence, while updating the states of the remaining area. This algorithm can also be used on parallel processing.

4.2 RESULTS AND DISCUSSION

The implementation of the method developed in para 4.1 was made on Fourteen bus IEEE system. The network is shown in Appendix C . This network was decoupled in two areas as shown in Fig. 4.2. The results of state estimation obtained by the decomposition method are shown in table 4.1, which provides comparison with the base case values, measured values and estimates of Integral State Estimation (ISE). Each area sub-network required two iterations. However, after one complete cycle i.e. Global Solution no further state correction was needed.

Kurzyn [43] has reported that the estimates of HSE are not as accurate as that of ISE. However, it is seen that the results of State Estimation by Decomposition (DSE) are reasonably close to the estimates of ISE as reported in Table 4.1.

4.3 CONCLUSION

The effect of gain in computation time could not be demonstrated on Fourteen bus system since it was not large enough to justify the decomposition. The area B has Eleven buses out of Fourteen buses in the system. However, the aim was to demonstrate the method.

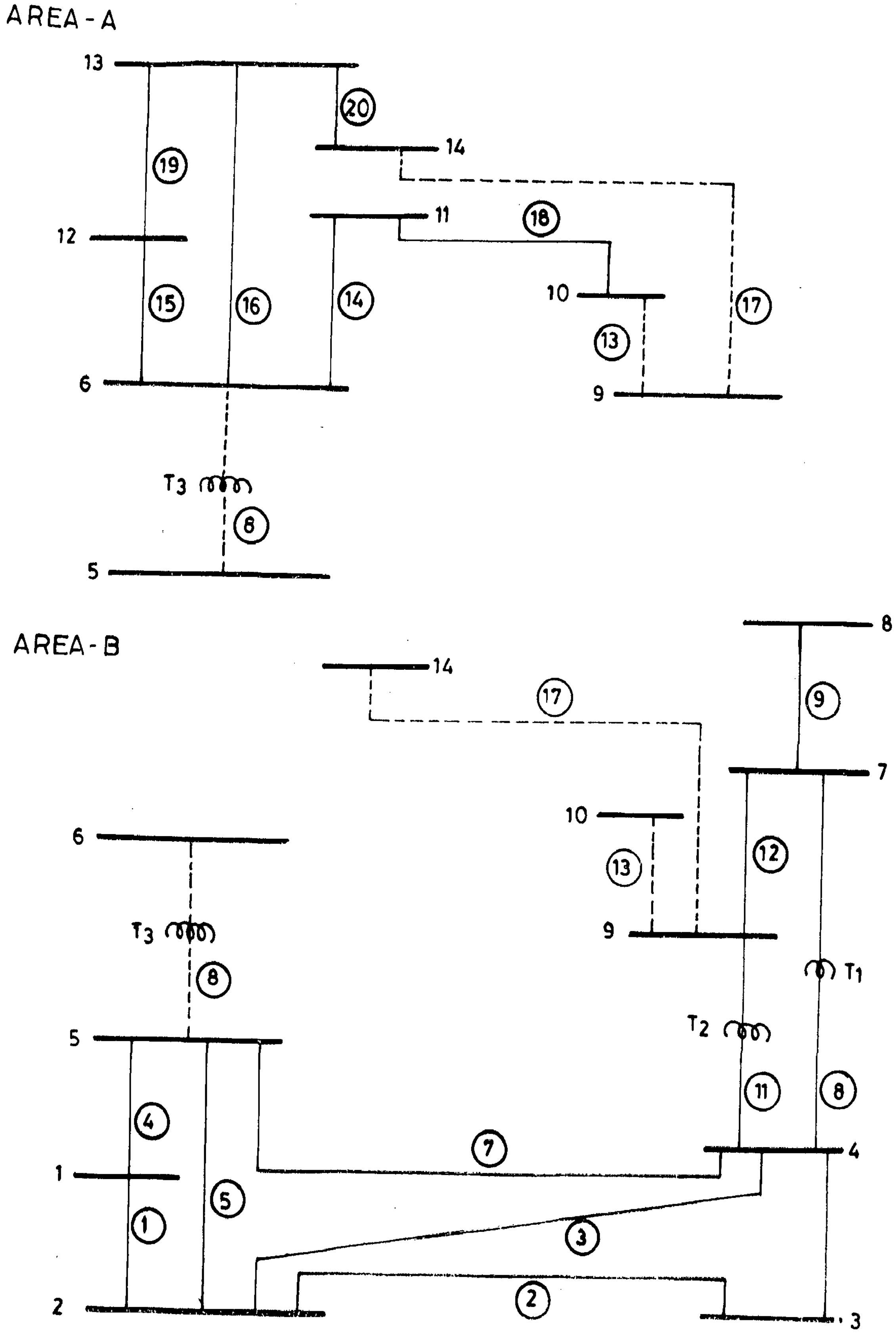


FIG. 4.2 DECOMPARED NETWORKS OF 14- BUS (IEEE) SYSTEM

TABLE 4.1

COMPARISON OF INTEGERARED STATE ESTIMATION

AND DECOMPOSITION METHOD

(14 BUS IEEE SYSTEM)

----STATE ESTIMATION RESULTS----

PARTICULARS		*BASE CASE*	*MEASURED*	*ESTIMATED*	
				ISE *	HSE **
BUS POWER INJECTIONS					
NO.	TYPE				
ACTIVE POWER:					
1	3	2.3450	2.3500	2.3343	2.3390
2	2	.1830	.2310	.2147	0.2240
3	2	-.9420	-.9920	-1.0038	-1.0031
4	0	.4780	.4440	.4384	0.3389
5	0	.0760	.0520	.0467	0.0658
6	2	-.1119	-.1019	-.1087	-0.1099
7	0	.0000	.0110	-.0168	-0.0198
8	2	.0000	-.0330	-.0309	-0.0176
9	0	.2950	.3202	.3034	0.3236
10	0	.0900	.1630	.1138	0.0780
11	0	.0350	.0201	.0432	0.0377
12	0	.0607	.0656	.0522	0.0640
13	0	.1354	.0604	.1070	0.1353
14	0	.1490	.1639	.1618	0.1645
REACTIVE POWER:					
1	3	-.0342	-.0008	.0006	0.0340
2	2	.2692	.2432	.2568	0.2953
3	2	.1487	.1167	.1144	0.1107
4	0	.0390	.0900	.0841	0.0791
5	0	.0160	-.0240	-.0359	-0.0140
6	2	.0291	-.0449	-.0093	0.0266
7	0	.0000	-.0148	-.0102	-0.0152
8	2	.0841	.1601	.0898	0.0898
9	0	.1659	.1539	.1473	0.1557
10	0	.0580	.0532	.0359	0.0360
11	0	.0179	.0329	.0386	0.0289
12	0	.0152	.0204	.0114	0.0282
13	0	.0586	.0666	.0549	0.0698
14	0	.0501	.0271	.0511	0.0594

contd...

1		2	3	4	5
LINE POWER FLOWS:					
FROM	TO				
ACTIVE POWER:					
1	2	1.5863	1.5903	1.5800	1.5813
2	1	-1.5427	-1.5395	-1.5376	-1.5406
1	5	.7587	.7558	.7543	0.7577
5	1	-.7304	-.6854	-.7267	-0.7306
2	3	.7416	.7464	.7710	0.7706
3	2	-.7177	-.7467	-.7454	-0.7461
2	4	.5650	.5650	.5666	0.5680
4	2	-.5478	-.5178	-.5495	-0.5515
2	5	.4191	.4210	.4147	0.4260
5	2	-.4097	-.4087	-.4056	-0.4165
3	4	-.2243	-.2463	-.2585	-0.2570
4	3	.2286	.2906	.2636	0.2619
4	5	-.6095	-.6655	-.6368	-0.5836
5	4	.6146	.6176	.6422	0.5806
4	7	.2832	.3033	.3064	0.3060
7	4	-.2832	-.2854	-.3064	-0.3060
4	9	.1631	.1391	.1735	0.1837
9	4	-.1631	-.1646	-.1735	-0.1837
5	6	.4465	.4447	.4405	0.4496
6	5	-.4466	-.4626	-.4405	-0.4496
6	11	.0751	.1281	.0941	0.0924
11	6	-.0744	-.0724	-.0933	-0.1010
6	12	.0783	.0764	.0702	0.0822
12	6	-.0775	-.0754	-.0696	-0.0817
6	13	.1786	.1803	.1646	0.1918
13	6	-.1763	-.1796	-.1628	-0.1889
7	8	.0000	.0870	.0309	0.0276
8	7	.0000	.0017	-.0309	-0.0276
7	9	.2832	.2922	.2923	0.3082
9	7	-.2832	-.3022	-.2923	-0.3082
9	10	.0509	.0496	.0642	0.0451
10	9	-.0508	.0466	.0640	0.0449
9	14	.0931	.0911	.0907	0.0955
14	9	-.0919	-.0864	-.0896	-0.0941
10	11	-.0392	-.0407	-.0498	-0.0331
11	10	.0394	-.0156	.0500	0.0333
12	13	.0168	.0183	.0174	0.0178
13	12	-.0167	.1047	-.0174	-0.0177
13	14	.0576	.0586	.0731	0.0712
14	13	-.0570	-.0588	-.0722	-0.0704
REACTIVE POWER:					
1	2	-.1207	-.1233	-.0885	-0.0915
2	1	.1954	.1404	.1590	0.1542
1	5	.0865	.0896	.0891	0.0955
5	1	-.0224	-.0252	-.0288	-0.0290
2	3	.0089	.0969	.0351	0.0321

1		2	3	4	5
3	2	.0460	.0441	.0265	0.0226
2	4	.0225	.0206	.0263	0.0263
4	2	-.0093	.0036	-.0139	-0.0177
2	5	.0424	.0457	.0365	0.0327
5	2	-.0493	-.0343	-.0450	-0.0411
3	4	.1028	.1518	.0879	0.0881
4	3	-.1271	-.0631	-.1103	-0.1128
4	5	.1105	.0245	.0727	0.1125
5	4	-.1074	-.1091	-.0689	-0.1027
4	7	-.0300	-.0290	-.0443	-0.0398
7	4	.0450	.0470	.0630	0.0563
4	9	.0169	-.0210	.0117	0.0287
9	4	-.0029	-.0008	.0038	-0.0115
5	6	.1630	.1960	.1786	0.1883
6	5	-.1067	-.0777	-.1309	-0.1300
6	11	.0421	.0468	.0363	0.0653
11	6	-.0408	-.0568	-.0345	-0.0624
6	12	.0255	.0307	.0204	0.0469
12	6	-.0239	-.0289	-.0192	-0.0449
6	13	.0759	.0738	.0649	0.0840
13	6	-.0715	-.0755	-.0612	-0.0851
7	8	-.0829	-.0811	-.0884	-0.0865
8	7	.0841	.0311	.0898	0.0858
7	9	.0369	.0351	.0356	0.0461
9	7	-.0285	-.0270	-.0269	-0.0334
9	10	.0358	.0376	.0409	0.0532
10	9	-.0355	-.0369	-.0404	-0.0528
9	14	.0322	.0862	.0413	0.0590
14	9	-.0298	-.0346	-.0388	-0.0559
10	11	-.0225	-.0035	.0045	0.0368
11	10	.0228	-.0422	-.0040	-0.0364
12	13	.0088	-.0142	.0078	0.0066
13	12	-.0087	-.0071	-.0077	-0.0066
13	14	.0216	-.0064	.0141	0.0151
14	13	-.0203	-.0543	-.0123	-0.0035

BUS VOLTAGE MAGNITUDE :

NO.				
1	1.0600		1.0686	1.0714
2	1.0400		1.0469	1.0493
3	1.0100		1.0113	1.0152
4	1.0055		1.0119	1.0035
5	1.0091		1.0174	1.0103
6	1.0500		1.0553	1.0603
7	1.0359		1.0455	1.0383
8	1.0500		1.0604	1.0729
9	1.0324		1.0422	1.0380
10	1.0279		1.0369	1.0324
11	1.0353		1.0401	1.0483
12	1.0348		1.0423	1.0497
13	1.0295		1.0371	1.0347
14	1.0127		1.0206	1.0136

contd...

1	2	3	4	5
BUS VOLTAGE ANGLE :				
NO.				
1	.0000		.0000	-0.0035
2	-.0869		-.0847	-0.0874
3	-.2256		-.2267	-0.2304
4	-.1799		-.1766	-0.1827
5	-.1530		-.1495	-0.1565
6	-.2522		-.2460	-0.2522
7	-.2355		-.2358	-0.2393
8	-.2355		-.2407	-0.2435
9	-.2646		-.2653	-0.2732
10	-.2676		-.2691	-0.2751
11	-.2623		-.2599	-0.2666
12	-.2678		-.2601	-0.2659
13	-.2691		-.2617	-0.2678
14	-.2848		-.2835	-0.2900

N.B. 1. ANGLES IN RADIANS
2. POWER AND VOLTAGE MAGNITUDES IN P.U.

* ISE - INTGERATED STATE ESTIMATION
** HSE - HIERACHICAL STATE ESTIMATION

CHAPTER - V

SOFTWARE DEVELOPMENT

Software for State Estimation has been written in Fortran IV language and tested on IBM compatible PC. The computations of various parameters have been preferred in polar coordinates for the reasons discussed in para 3.0. This necessiated development of programs right from scratch. However, direct use of LU factorisation subroutines for solution of equations, have been made. Software has been designed in Modular structure such that output of one module is compatible to the input of other module for direct use.

5.1 SOFTWARE DESIGN

The software entitled "STATST" has been designed for static State Estimation. The modular structure of the software is shown in Fig. 5.1. Function of each module is as under -

LFLOW - It performs LOAD FLOW solution for obtaining base case values of power injection in each bus and line flow.

MESVCT - Simulates random measurement error and by adding them to base case power injections and line flows generates measurement vector.

JHCHTL - Performs State Estimation by HACHTEL's method using full Jacobian.

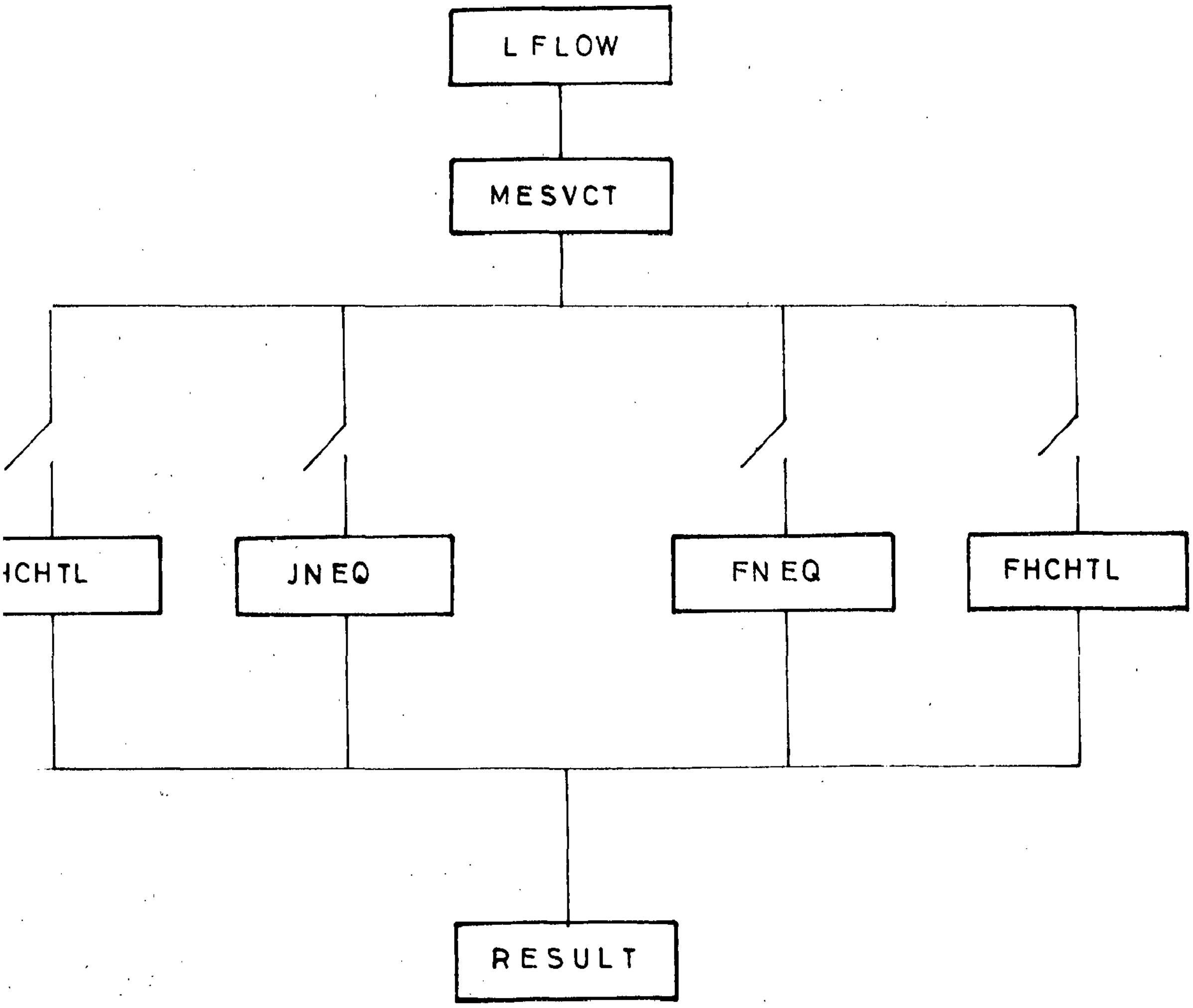


FIG. 5.1 MODULAR STRUCTURE OF "STATST"

FNEQ - Performs State Estimation by Normal Equation method in fast decoupled version. This module, however, did not produce convergence for the reasons discussed in para 3.0.

FHCHTL - Performs State Estimation by HACHTEL's method in fast decoupled version. This module too did not produce convergence for the reasons discussed in para 3.0.

JNEQ - Performs State Estimation by Normal Equation method using full Jacobian.

RESULT - It produces results in a format which facilitates comparison of base case values, measured values and estimated values.

The switches in Fig. 5.1 indicate choice of the method of State Estimation. The choice of some controlling variables has been provided in an interactive mode in some modules. The LFLOW module provides following choices -

(i) Convergence tolerance
(ii) Maximum iteration count
(iii) Retrial with different valuesof choices(i) and (ii) above.

Similarly, JHCHTL, JNEQ, FNEQ and FHCHTL modules also provide following choices -

(i) Convergence tolerance
(ii) Maximum iteration count
(iii) Acceleration/Deceleration factor
(iv) Retrial with different values of choices at (i), (ii) and (iii) above.

The JHCHTL and FHCHTL module further provide choice of controlling parameter 'α'.

5.2 SUBROUTINE FUNCTIONS

The subroutines of LFLOW, JHCHTL, JNEQ, FNEQ and FHCHTL modules are shown in Fig. 5.2 to 5.6. The subroutines having similarity of purpose bear common titles but are designed to cater needs of individual module. The description of each subroutine is as under -

READ - It reads the system data. The data structure available in the group, with whom the author has worked, has been retained with identical variable names to facilitate utilization of available data bank.

LSORT - Sorts lines in ascending order of bus numbers.

JYBUS - Computes elements of Y_{BUS}, B' and B" matrices.

MESVCT - Reads measurement data.

ELEMENT - Calculates elements of Jacobian.

HMAT - Constructs fast decoupled H matrices using B' and B" matrices.

STORAG - Stores non-zero elements of Jacobian.

COEFF - Stores elements of coefficient matrices.

SLACK - Modifies B' and B" matrices for slack bus for load flow solution.

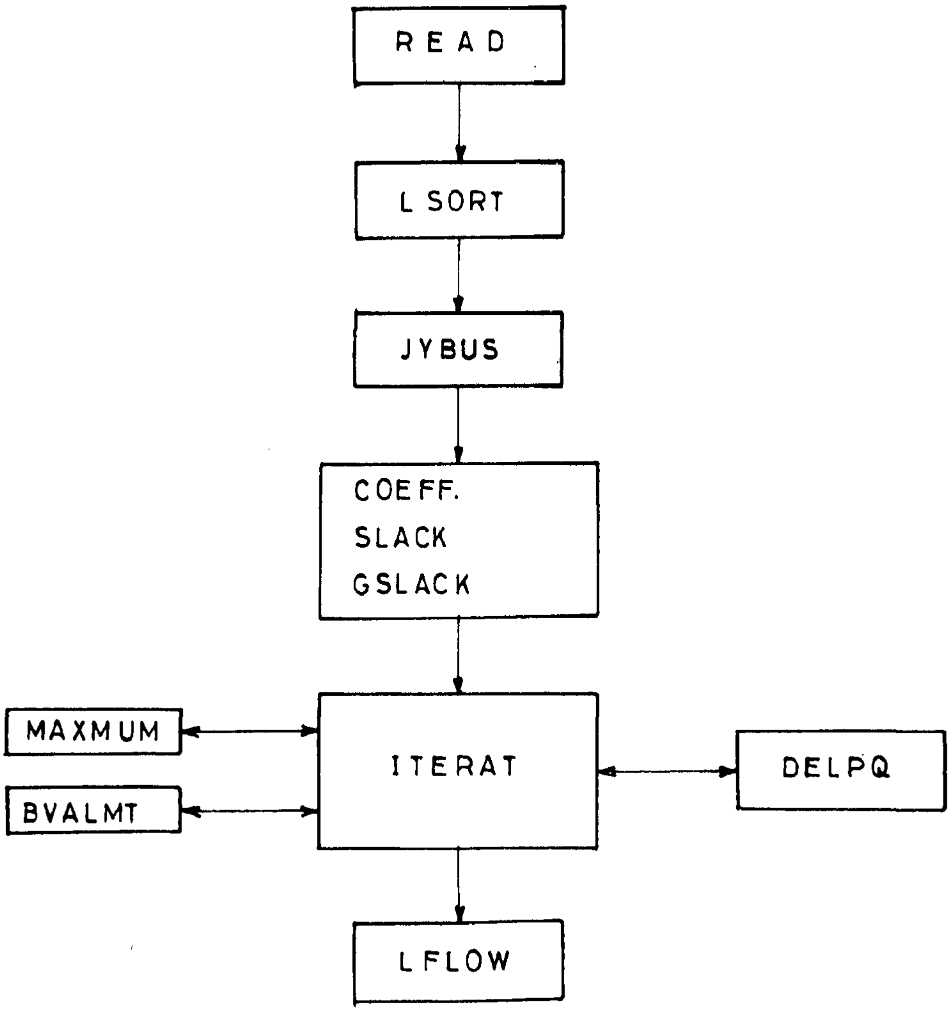


FIG. 5.2 BLOCK STRUCTURE OF MODULE "LFLOW"

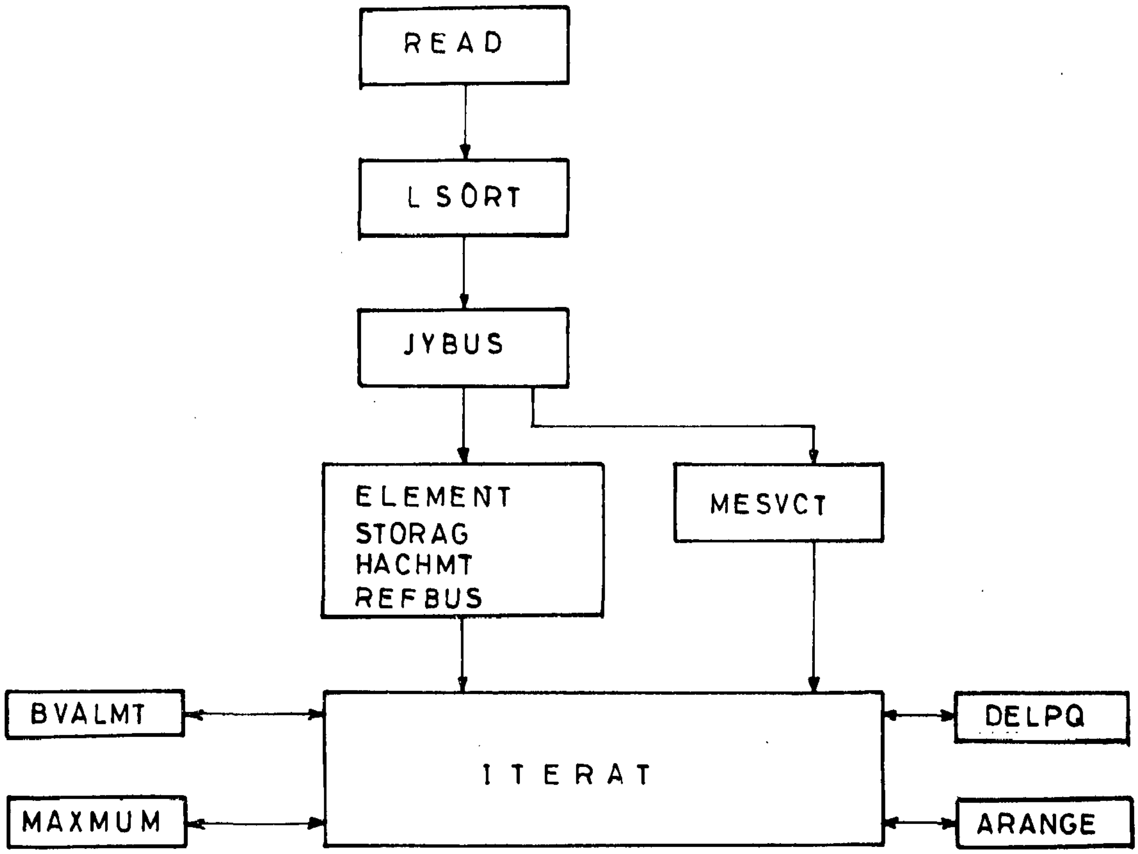


FIG. 5.3 STRUCTURE OF MODULE "JHCHTL"

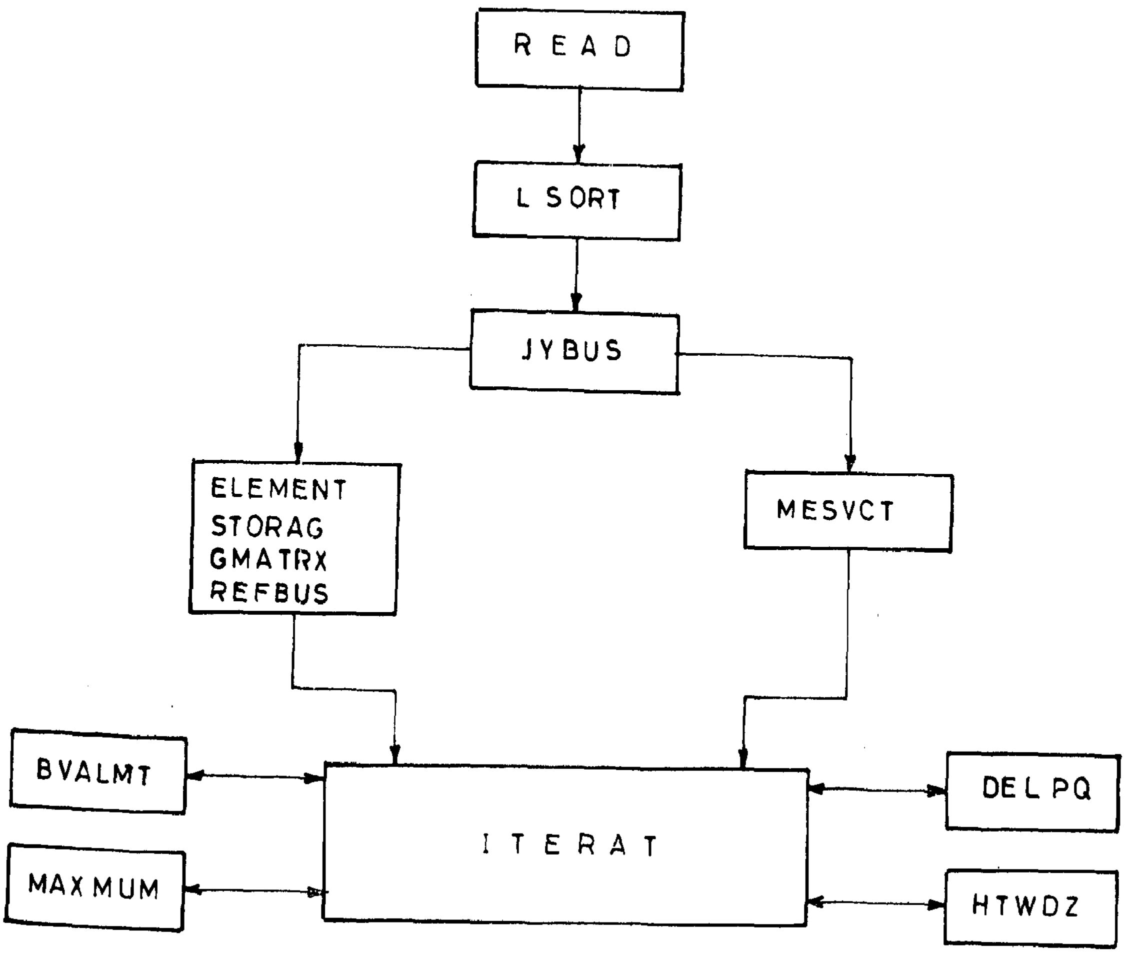


FIG. 5.4 STRUCTURE OF MODULE "J N E Q"

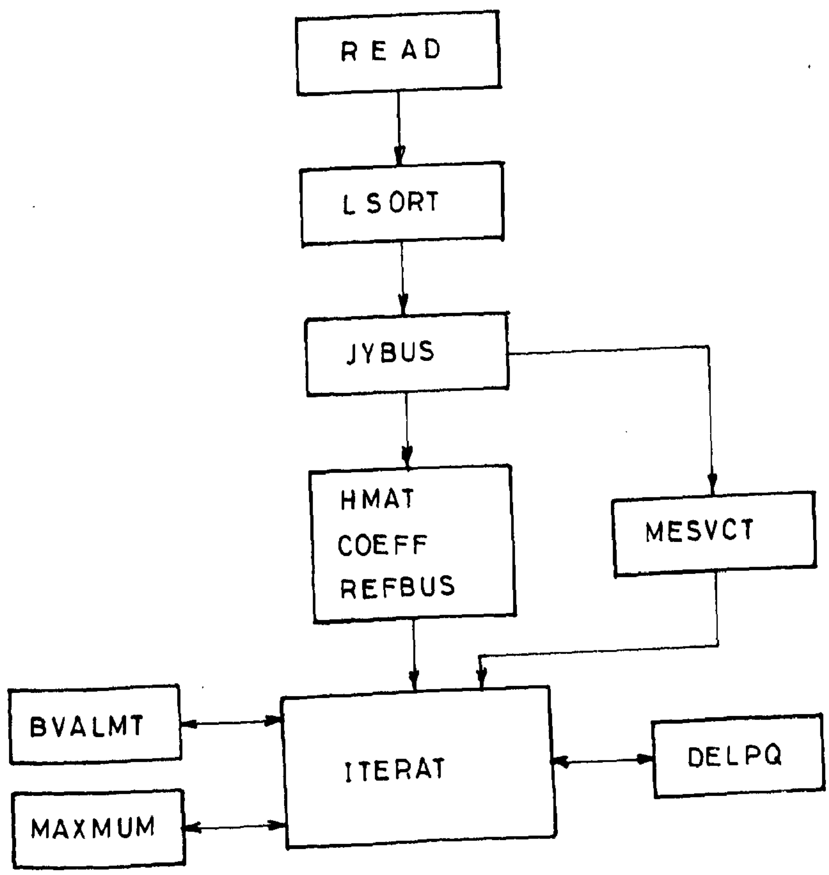


FIG. 5.5 STRUCTURE OF MODULE "FNEQ"

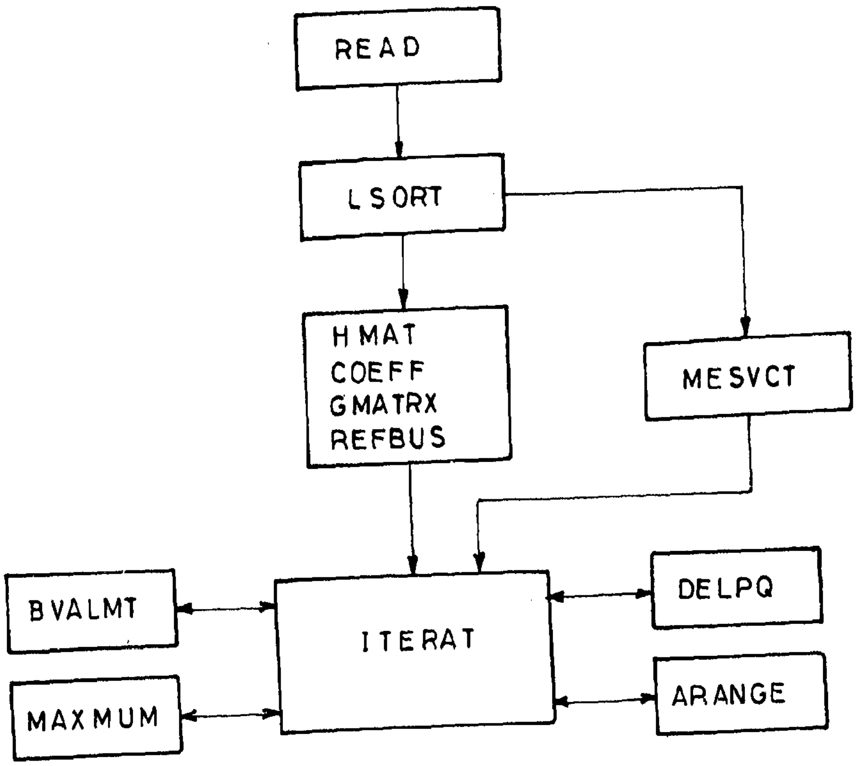


FIG. 5.6 STRUCTURE OF MODULE "FHCHTL"

REFBUS	-	Modifies coefficient matrices for the reference bus.
GSLACK	-	Modifies B'' matrix for PV buses.
GMATRX	-	Computes H^TWH for Normal Equation method.
HTWDZ	-	Computes $H^TW\Delta z$ for Normal Equation method.
ITERAT	-	Controls iteration till convergence.
DELPQ	-	Calculates active and reactive power mismatches.
LFLOW	-	Calculates line flows.
BVALMT	-	Imposes bounds on voltage angles.
ARANGE	-	Sorts state correction vector from the solution of system of equations for HACHTEL's method.
MAXMUM	-	Sorts maximum correction vector for comparison with convergence tolerance.

CHAPTER - VI

CONCLUSION AND FUTURE SCOPE OF WORK

The State Estimation by Hachtel's Augmented Matrix method has been found to be very fast as compared to Normal Equation method. Implementation of this method in fast decoupled version using single precision did not yield convergence. In view of this Hachtel's Augmented Matrix method is modified and Jacobian Matrix is computed in each iteration under single precision. A new method can be developed which uses advantages of fast decoupling as well as computation in single precision.

The growing size of the systems severly calls upon the computation time to maintain feasibility of state estimator for real time operation and monitoring. The State Estimation by Decomposition of network has worked successfully and it can be used on larger systems.

The programmes developed in this thesis can be extended for inclusion of bad data handling and observability analysis.

APPENDIX - A

PROPERTIES OF RESIDUAL SENSITIVITY MATRIX- $\mathcal{R}$

1. $\mathcal{R}$ is an idempotent matrix i.e.

$$\mathcal{R}^2 = \mathcal{R} \qquad \text{... (A.1)}$$

2. The eigen values of $\mathcal{R}$ matrix must be either 1 or 0, i.e. it is semi-positive definite.

3. $\mathcal{R}$ is a matrix with eigen values of K set of ones an n set of zeros. Where K is the degree of freedom (m-n) and n is the number of state variables.

4. $\mathcal{R}$ is a singular matrix of rank K.

5. The weighted residual sensitivity matrix $\mathcal{R}_\omega$ is symmetrical.

$$\mathcal{R}_\omega^T = \mathcal{R}_\omega$$

6. If there is no redundancy i.e. number of measurements m = n, then

$$\mathcal{R} = 0$$

7. If it is assumed that measuring points are evenly distributed in a network and $m \to \infty$ then

$$\lim_{m \to \infty} \mathcal{R} = I$$

8. Utilizing above properties

$$r = \mathcal{R}r$$

and when $m \to \infty$, then $r = e$

9. The value of diagonal elements $\mathcal{R}_{ii}$ may have the range of

$$0 < \mathcal{R}_{ii} < 1$$

It has been reported that performance of identification of bad data are better at measurement points where $\mathcal{R}_{ii} \geqslant 0.5$.

APPENDIX - B

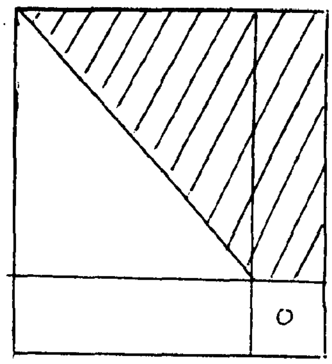
NETWORK OBSERVABILITY THEOREMS

Theorem 1 - Assume that there is no voltage measurement, then the following statements are equivalent.

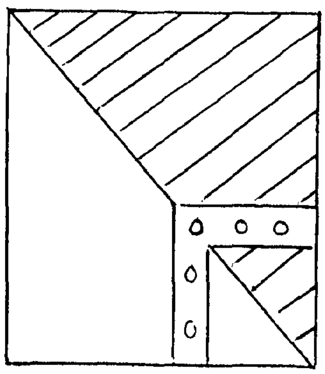
(i) The network is observable.

(ii) Let $\bar{H}$ be obtained from H by deleting any column, then $\bar{H}$ is of full rank.

(iii) The taingular factorisation reduces the gain matrix $G = H^T H$ in the following form.



Theorem 2 - In the traingular factorisation of the gain matrix G, if a zero pivot is encountered, then the remaining elements of row and column are all zeros, i.e., G is reduced to the form.



APPENDIX - C

TEST SYSTEM CONFIGURATIONS

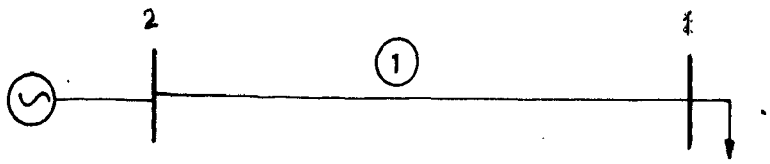


FIG. C.1 2- BUS SYSTEM

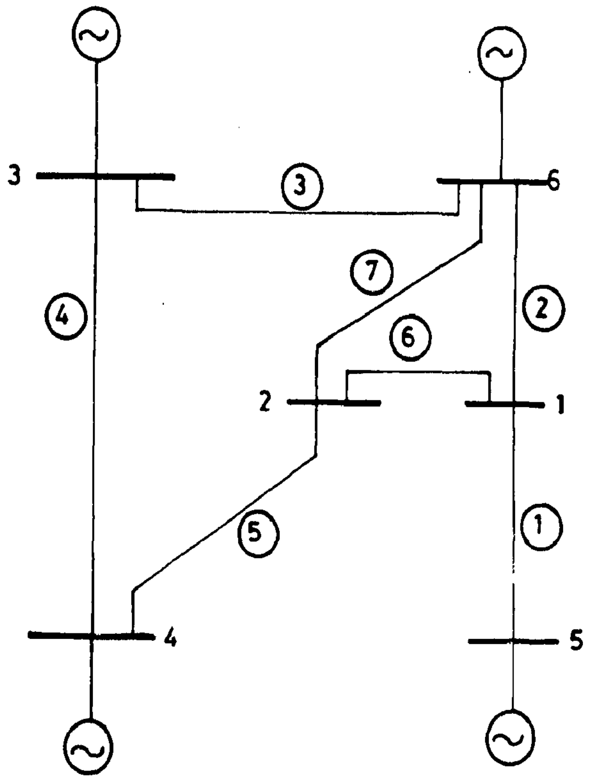
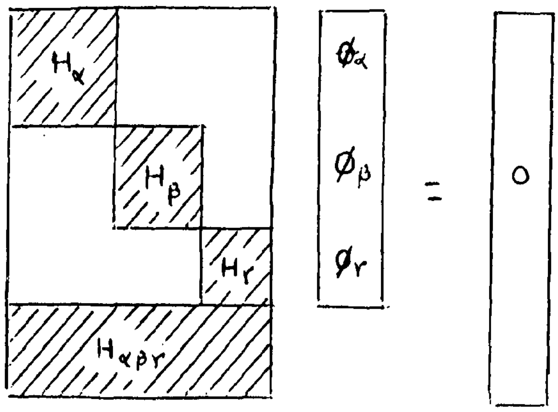


FIG. C.2 6 - BUS (SPC) SYSTEM

Theorem 3 - $\phi\alpha$ is not an unobservable state for the subnetwork α with measurement $H\alpha$, similarly $\phi\beta$ and ϕr



Theorem 4 - Consider State Estimation model

$$\zeta = H\phi + r$$

Suppose that the measurement set consists of the ϕs pseudo measurements and all other measurements equal to zero, then the residual $r = 0$.

Theorem 5 - If minimal set of additional non-redundant (pseudo) measurements is so selected that they make the network barely observable, then the estimated states of the already observable islands will not be affected by these pseudo measurements.

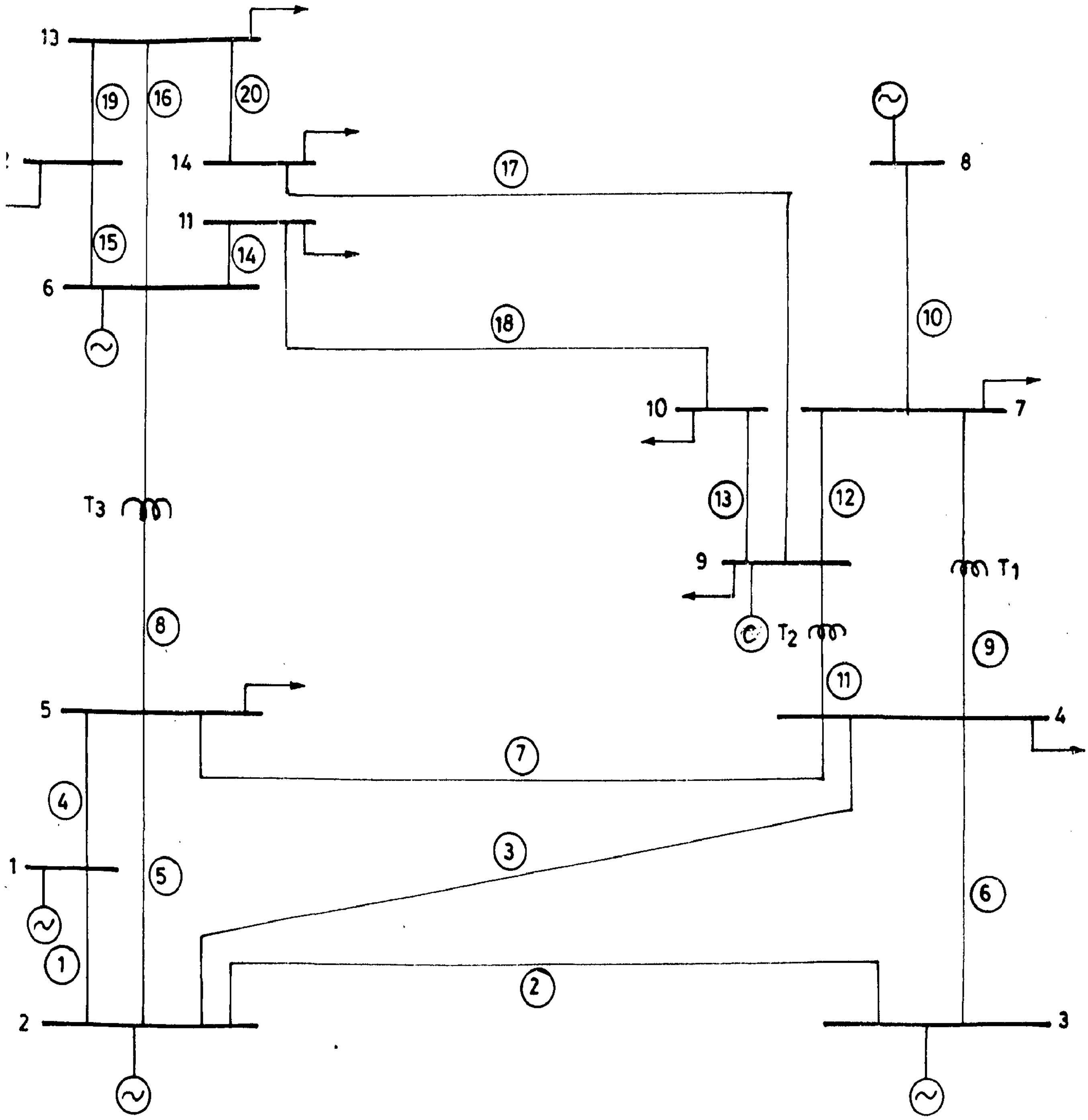


FIG. C.3 14 - BUS (IEEE) SYSTEM

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