APPLICATION OF MODELLING AND ORDER REDUCTION IN AIRCRAFT INSTRUMENTATION SYSTEM

A DISSERTATION

submitted in partial fulfilment of the requirements for the award of the degree
of
MASTER OF ENGINEERING
in
ELECTRICAL ENGINEERING
(With Specialization in Measurement and Instrumentation)

BY

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DEDICATED
TO
MY PARENTS

CANDIDATE'S DECLARATION

I hereby certify that the work is being presented in the dissertation entitled **"APPLICATION OF MODELLING AND ORDER REDUCTION IN AIRCRAFT INSTRUMENTATION SYSTEM"** in partial fulfilment of the requirement for the award of degree of **MASTER OF ENGINEERING** in **ELECTRICAL ENGINEERING** with specialization in Measurement and Instrumentation submitted in the department of Electrical Engineering of the University is an authentic record of my own work carried out during a period from December, 1989 to June, 1990 under the supervision of **Dr.R.N.Mishra**, Prof. of Electrical Engineering.

The matter embodies in this dissertation has not been submitted by me for the award of any other degree.

Date : 9th July, 1990 **(MANOJ KUMAR JAISWAL)**

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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Roorkee

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ABSTRACT

Measured engine speed regulation and synchronization of various engines used in multi-engined aircraft is discussed. Designing the PID controller for engine speed regulation to attain the satisfactory transient response, which results in matching the dynamic behaviour of speed regulation of non-identical engines which has slight changes in parameter used in multi-engine aircraft. Thus for synchronization to achieve the same steady state value.

Reduced order model of high order (i.e. Aircraft Blind Landing) linear system is obtained by marshall method and predicting the transient response sensitivity using reduced order model of Aircraft Blind landing system. A new low order model i.e. marshall method with equivalent lag is introduced and when compared with the exact response, it is found to give an encouraging accuracy of prediction.

CONTENTS

chapter : 1

introduction

CHAPTER-1

INTRODUCTION

1.1 GENERAL

Safe, economical, and reliable operation of modern aircraft is dependent upon the use of instruments. The first aircraft instrument were fuel and oil pressure instruments to warn of engine trouble so that the aircraft could be landed before the engine failed. As aircraft that could fly over considerable distances were developed, weather became a problem. Instruments are developed that helped to fly through bad weather conditions. Under adverse weather conditions, it had already found that pilots soon lost their sense of equilibrium and had difficulty in controlling an aeroplane when external references were obscured. Instruments were therefore required to assist the pilot in circumstances which became known as 'Blind Flying Conditions', to fly 'Blind' by means of a small group of instruments.

Instrumentation is basically the science of measurement. Speed, distance, altitude, attitude, direction, temperature, pressure and rpm are measured and these measurements are displayed on dials in the cockpit.

1.2 ADVANTAGE OF MULTI-ENGINE AIRCRAFT

An aircraft can be single engined or multi-engined. If single engined, the engine is usually mounted at the nose of the fuselage. If the aircraft has two or four engines, they are usually housed in the leading edge of the wing and are equally disposed of on either side of the fuselage.

These are several reasons of using more than one engine in an aircraft. First reason is that the power of the machine and its weight carrying capacity are multiplied. Another reason is reliability. It is quite difficult to produce an engine which is so reliable that it can never break down. If this happens in a single engine aircraft, it is compulsory that the aircraft must land. It is likely that a satisfactory landing ground may not be available with in the reach. Under these conditions, it may be difficult for the pilot to avoid crash. The chances of accidents are reduced if the aircraft has two or more engines, since it will continue to fly satisfactorily even if one engine has failed.

1.3 SYSTEM CONTROL

Broadly a system can be thought of as a collection of interacting components, although some times interest might lie just in one single component. Two broad classes of control system are available, open loop control and closed loop control and these are depicted schematically in Fig. 1.1

(a) Open loop control : On the basis of knowledge about the system and of past experience, a prediction is made of what input should be to give the desired output; the input is adjusted accordingly.

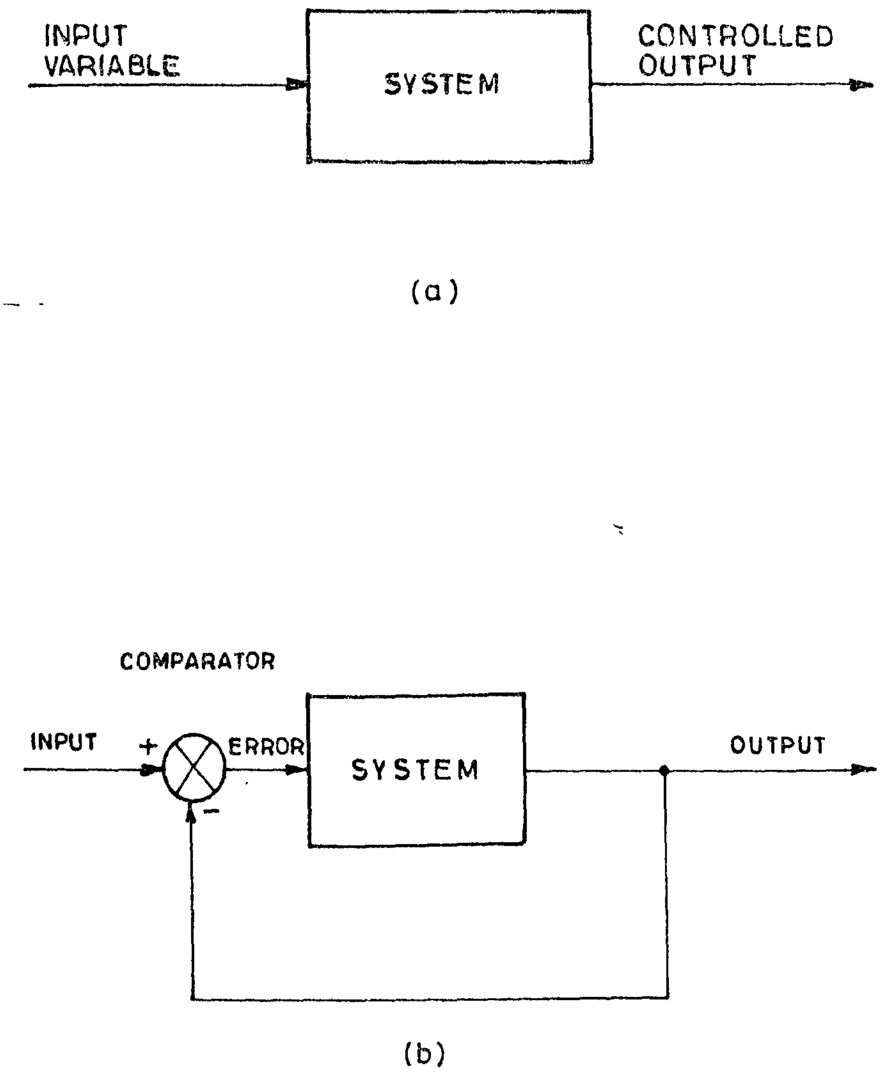

FIG. 1.1 OPEN LOOP AND CLOSED LOOP CONTROL
(a) OPEN LOOP (b) CLOSED LOOP

(b) Closed loop control : The system output is measured and compared with the desired value; the system continually attempts to reduce the error between the two.

1.4 METHOD OF ANALYSIS

The analysis of all physical systems start by building up of a model. The reason is that once a physical phenomenon has been adequately modelled so as to be a faithful representation of reality, all further analysis can be done on the model so that experimentation on the process is no longer required. The advent of the digital computer has meant that relatively complex models can be manipulated.

The output response as a function of time can be obtained for any specific forcing (Step, ramp or impulsive) function by computer solution of the differential equations. High order system can be approximated to reduce low order model by marshall method and the output response of reduced model as a function of time thus obtained which considerably reduces the computation time of computer.

1.5 ORGANISATION OF A DISSERTATION

Chapter-2, describes the speed control system and its various components i.e. Engine and tachometers used in aircraft. In chapter-3, it is shown how the synchronization is maintained in multi engine aircraft at 'on speed' conditions with the help of synchroscope and thus minimising the effects of structural vibration and noise.

Chapter-4, is concerned broadly with designing or modifying a speed control system to ensure that its dynamic behaviour is acceptable. It also describes the root locus method of analysis, a technique which asists in understanding of system behaviour by showing what effect variation of system gain or some other variable has on the transient response. It shows that increase in system gain is accompanied by a tendency towards more oscillatory behaviour and might give to instability. Integral and Derivative action has been also incorporated with controller to improve the transient and steady state behaviour for attaining the synchronization of various engines in aircraft.

Prediction of transient response and sensitivity using the low order model of high order (Aircraft Blind Landing) system using marshall's approximation method is describe in chapter-5. The conclusion is drawn in the last chapter-6.

chapter :2

control system and components

CHAPTER-2

CONTROL SYSTEM AN COMPONENTS

2.1 SPEED CONTROL SYSTEM

2.1.1 Proportional Control System

The Fig.(2.1) shows a typical speed control system [10] for gas turbines, steam turbines or diesel engines. The position of the throttle lever sets the desired speed of the engine. The speed control is drawn in some reference operating position so that the values of all the lower case parameters are zero. The positive direction of motion of these parameters is indicated by the arrowhead on each.

The centre of mass of the flyweights is at distance $R = R_i + r$ from the center of rotation. The fly weights are geared directly to the output shaft, so that the speed of the flyweights is proportional to the output speed. A lever which is pivoted as indicated in Fig.(2.1) transmits the centrifugal force from the flyweights to the bottom of the lower spring seat. The pivot and lever rotate with the flyweights as a unit. If the speed of the engine should drop below its reference value, then the centrifugal force of the flyweights decreases, thus decreasing the force exerted on the bottom of the spring. This causes x to move downward, which in turn moves e downward. Fluid then flows to the bottom of the big piston to increase y and thus open wider the flow control valve. When more fuel is supplied, the speed of the engine will increase until equilibrium is again reached. For steam turbines, the flow control valve controls the flow of steam rather than fuel as in the case with gas turbines and diesels.

Suppose that the throttle lever is moved to a higher speed setting, which in turn causes z to move downward. This in turn causes x to move downward. As just discussed, moving x downward opens the fuel flow valve, which increases the speed.

2.1.2 Integral Control System

By eliminating the linkage between x and y of Fig.(2.1) and using the hydraulic integrator shown in Fig.(2.2), the proportional control system is converted to an integral control system [12].

The operation of an integral control system may be visualized as follows : From Fig.(2.2), it can be seen that if x momentarily changes and then returns to its line-on-line position, the position of y has been changed permanently and so has the amount of flow going to the engine. Therefore, changing the amount of flow to account for a new operating torque does not change the steady-state position of x, which must be line-on-line. Because neither x nor the spring compression changes, the output speed must always be equal to the desired value in order that the flyweight force balances the spring force. While for the proportional control system,

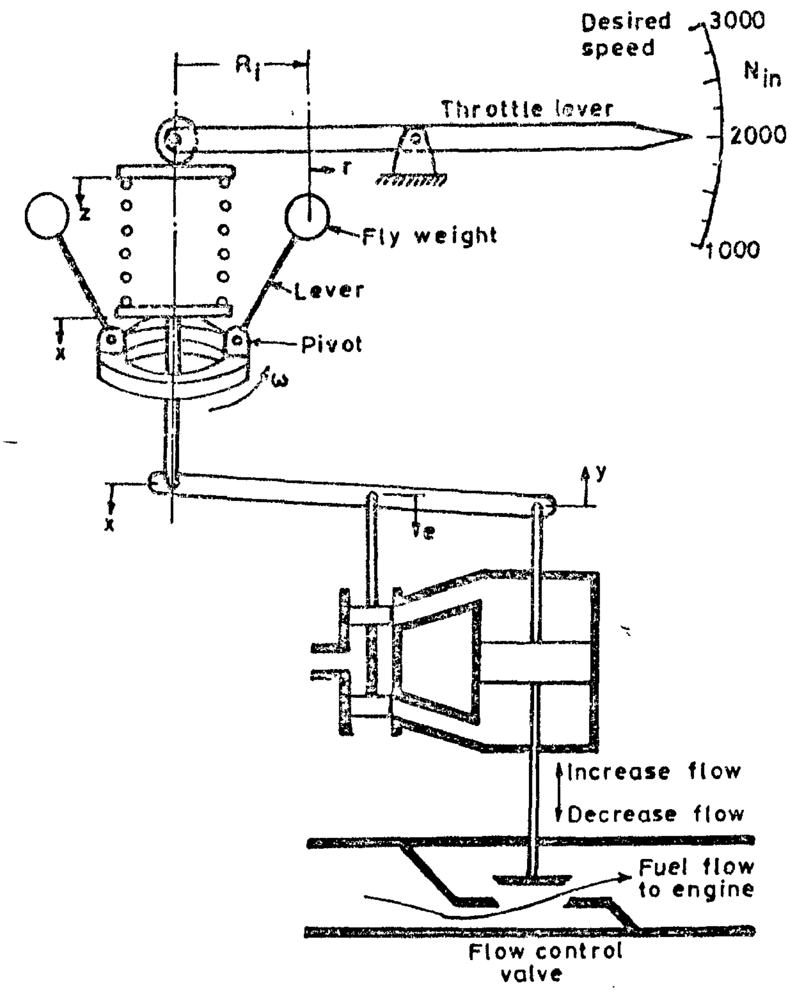

FIG. 2.1 SPEED CONTROL SYSTEM

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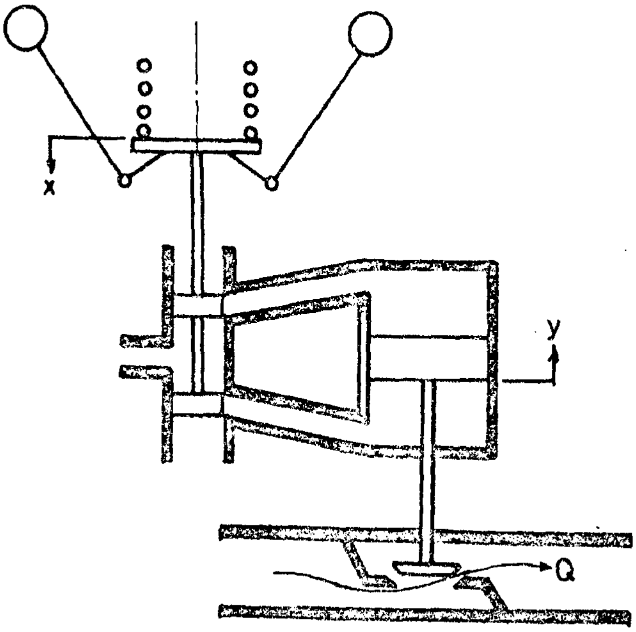


FIG.2.2 INTEGRAL CONTROLLER

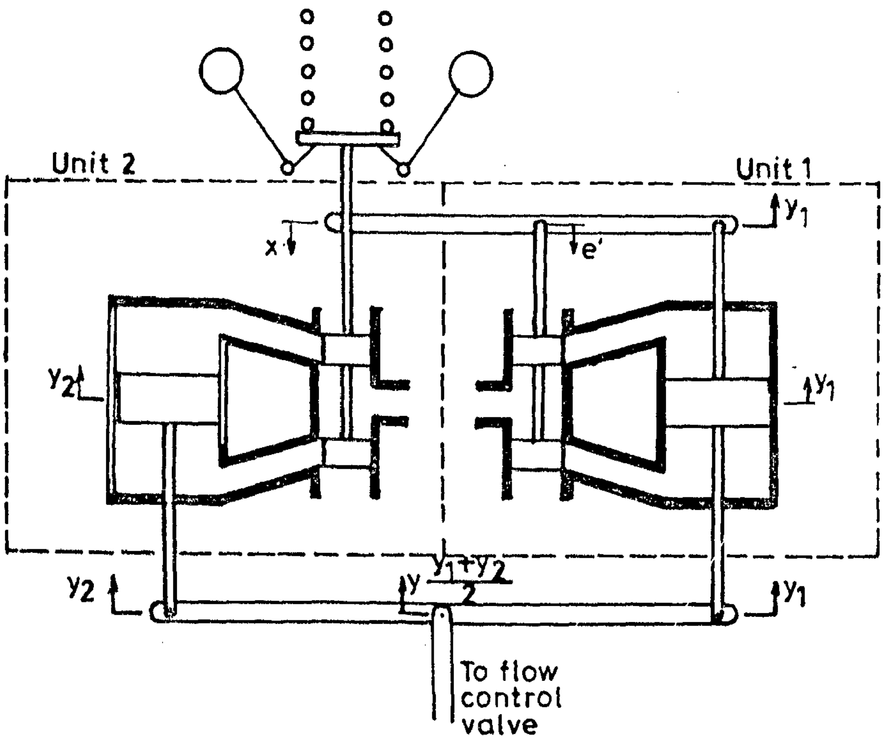


FIG. 2.3 PROPORTIONAL PLUS INTEGRAL CONTROLLER

changing the fuel flow requires a permanent change in the position x.

An integral control is also called a floating controller because of the floating action of the position y of the flow setting valve.

2.1.3 Proportional Plus Integral Control System

From a consideration of steady state operation only, integral control systems seem preferable to proportional systems. However, it is generally easier to achieve good transient behaviour with a proportional system than with an integral system. It is possible to combine the basic features of a proportional controller and an integral controller to form a proportional plus integral controller.

The action of a proportional plus integral controller in response to a change in the input or external disturbance is initially similar to that of a proportional controller, but as the new equilibrium point is reached, the control action becomes the same as that of an integral controller.

A proportional plus integral controller combines the desirable transient characteristics of a proportional controller and the feature of no steady-state error of the integral controller.

A proportional plus integral controller [12] is shown in Fig.(2.3). The proportional action is provided by unit 1, which is the same as that for the proportional controller shown in Fig.(2.1). The integral action is provided by unit 2, which is the same as that for the integral controller shown in Fig.(2.2). The proportional and integral actions are added by a walking-beam linkage such that $y = (y_1+y_2)/2$.

The action of this controller is as, suppose the throttle lever is moved to increase the speed. This causes the position x to move down, as does e'. Hence y_1 changes rapidly to increase the flow setting. The resulting motion of y_1, returns e' to its line-on-line position.

For integrating unit, y_2 continues to move at slower rate to provide corrective action. As the speed increases, the position x moves up. The integrating unit continues to provide corrective action until x is returned to its line-on-line position (that is $x = 0$). In summary, for proportional plus integral control, the initial effect is provided primarily by the proportional action, and the final effect is provided by the integrater.

2.1.4 Proportional Plus Integral Plus Derivative Control

In addition to proportional, integral and proportional plus integral control, another mode of control is derivative or rate action. The output of a derivative controller is proportional to the rate of change of error. For any constant value of the actuating signal e, the output of the control elements is zero. Thus, steady state may exist in a derivative control system with any constant value of error signal. Because a derivative controller operates on the rate of change of error and not the error itself, the derivative mode of control is never used alone, but rather in combination with a proportional, or integral or proportional plus integral controller. The advantage of using derivative action is that the derivative is a measure of how fast the signal is changing and thus tends to give the effect of anticipation.

2.2 ENGINE

The main purpose of an aircraft engine [5] is to provide a force for propelling the aircraft through the air. Aircrafts can be classified according to their propulsion as follows.

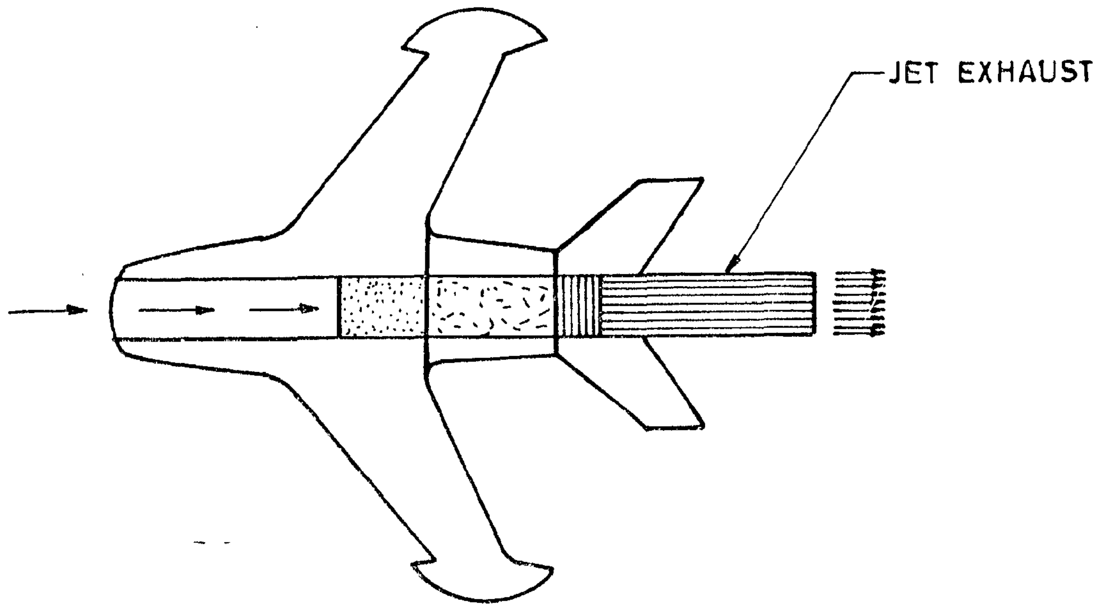
(1) Piston Engine

(2) Turbo jet

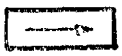
(3) Turbofan or Turboprop.

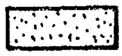
2.2.1 Piston Engine

It is powered by gasoline fed reciprocating engine and is driven by propeller or airscrew. In this system the engine rotates a shaft with a considerable amount of torque. A propeller is mounted on the shaft to absorb the torque. When



LEGEND


OUT-SIDE AIR TAKEN INSIDE


INSIDE AIR IS COMPRESSED


BURNING FUEL CAUSES COMPRESSED AIR TO EXPAND


TURBINE ROTATES THE COMPRESSOR

FIG. 2.4 PRINCIPLE OF TURBO JET PROPULSION the rotating propeller attains its rated speed, huge masses of air hurled rearward, there by pulling the aircraft forward and creating lift on the wings. These conventional aircraft engines are suitable to operate at low altitudes and moderate speeds.

2.2.2 Turbojet

The Schematic sketch of turbo jet engine aircraft is given in Fig.(2.4). To start the machine, the compressor is rotated with a motor. As the compressor gains its rated speed, it sucks in air through the air intake and compresses it in the compression chamber. The air is ignited by a fuel. The expanding gases pass through the fan like blades of the turbine. The turbine extracts that much power from the gases which is sufficient to keep the compressor rotating. The compressor rotates at the same speed as the turbine because the two are fastened solidly to one shaft. The hot gases, with the remaining energy escape through the tail pipe which become smaller is diameter at the exit end. The hot gases, having high velocity, give a forward thrust to the engine.

2.2.3 Turboprop

It is similar to the turbo jet engine except that a propeller is provided in it. Main difference is in the design of turbines. The turbine in turboprop extracts enough power to drive both the compressor and the propeller. Only a small amount of power is left as a jet thrust. Principal advantage of the turbo prop is its high degree of performance over a general range of altitude. While the turbo jet has a considerable lower performance ratio at moderate altitudes than at high altitudes, the turbo prop performs well at both.

2.3 TACHOMETER

The tachometer indicator is an instrument for indicating the speed of the crankshaft of a reciprocating engine and the speed of the main rotor assembly of a gas turbine engine. The dials of tachometer indicators used with reciprocating engines are calibrated in r.p.m.; those used with turbine engines are calibrated in percentage of r.p.m. being used, based on the take off r.p.m. [1].

There are two types of tachometer systems in wide use :

1. The mechanical indicating systems.
2. The electrical indicating systems.

2.3.1 Mechanical Indicating Systems

Mechanical indicating system consist of a magnet which is continually rotated by a flexible shaft coupled to a drive outlet at the engine. An alloy-cup shaped element (known as a drag cup) fits around the magnet such that a small gap is left between the two. The drag cup is supported on a shaft to which is attached a pointer and a controlling spring. As the magnet

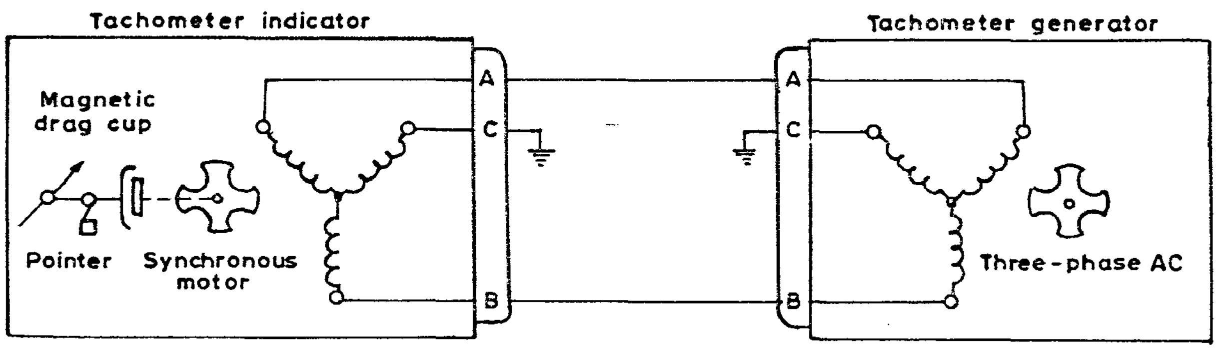


FIG. 2. 5 SCHEMATIC OF A TACHOMETER SYSTEM

rotates it induces eddy currents in the drag cup which tend to rotate the cup at the same speed as the magnet. This, however, is restrained by the controlling spring in such a manner that for anyone speed, the eddy current drag and spring tension are in equilibrium and the pointer then indicates the corresponding speed on the tachometer dail.

2.3.2 Electrical Indicating Systems

A number of different types and sizes of tachometer generators and indicators are used in aircraft electrical tachometer-systems. Generally, the various types of tachometer indicator and generators operate on the same basic principle.

The typical tachometer system Fig. (2.5) is at three-phase a.c. generator coupled to the aircraft engine, and connected electrically to an indicator mounted on the instrument panel. These two units are connected by a current-carrying cable. The generator transmits three-phase power to the synchronous motor in the indicator. The frequency of the transmitted power is proportional to the engine speed. Through use of magnetic drag principle, the indicator furnishes an accurate indication of engine speed [1].

chapter :3

open loop control

CHAPTER-3

OPEN LOOP CONTROL

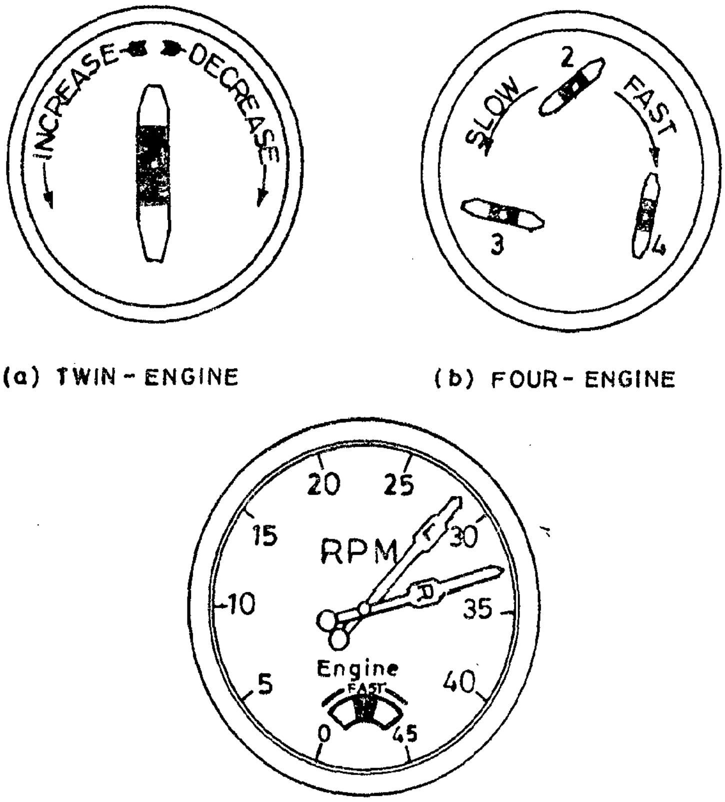
In aircraft powered by multi-arrangement of either piston engines or turbo propeller engines, the problem arises of maintaining the engine speeds in synchronism at 'ON-SPEED' conditions and so minimizing the effects of structural vibration and noise.

3.1 MANUAL

The simplest method of maintaining synchronism between engines would be to manually adjust the throttle and speed control systems of the engines until the relevant tachometer indicators read the same. This, however, is not very practical for the simple reason that individual instruments can have different permissible indication errors; therefore, when made to read the same operating speeds, the engines would in fact be running at speeds differing by the indication errors. In addition, the synchronizing of engines by a direct comparision of tachometer indicator readings is made some what difficult by the sensitivity of the instruments causing a pilot or engineer to overshoot or undershoot an on-speed condition by having to 'Chase the pointer' [11].

3.2 SYNCHROSCOPE

In order to faciliate manual adjustment of speed an



(a) TWIN - ENGINE

(b) FOUR - ENGINE

(c) COMBINED DUAL TACHOMETER AND SYNCHROSCOPE

FIG. 3.1 DIAL PRESENTATIONS OF SYNCHROSCOPES additional instrument known a synchroscope is introduced. The synchroscope is an instrument that indicates whether two (or more) engines are synchronized; that is, whether they are operating at the same r.p.m. It provides a qualitative indication of the differences in speeds between two or more engines, and by using the technique of setting up the required on-speed conditions on a selected master engine, the instrument also provides a clear and unmistakable indication of whether a slave engine is running faster or slower than the master.

The instrument is designed at the outset for operation from the alternating current generated by the tachometer system, and it therefore forms an electrical part of this system. The dial presentations of synchroscopes designed for use in twin and four-engined aircraft are shown in Fig.(3.1) (a) and (b) respectively, while a combination dual r.p.m and synchroscope presentation is shown at (c).

3.2 CONSTRUCTION AND PRINCIPLE

The operation is based on principle of the induction motor. Which, for this application, consists of a three-phase star-wound laminated stator and a three-phase star-wound laminated rotor pivoted in jewelled bearing with in the stator. The stator phases are connected to the tachometer generator of the slave engine while the rotor phases are connected to the master engine generator via slip rings and wire brushes. A disc at the front end of the rotor shaft provides for balancing of the rotor. The pointer, which is double-ended to symbolize a propeller, is attached to the front end of the rotor shaft and can be rotated over a dial marked INCREASE at its left-hand side and DECREASE at its right-hand side [11].

On some synchroscopes the left-hand and right-hand sides may be marked SLOW and FAST respectively. Synchroscopes designed for use in four-engined aircraft employ three separate induction motors, the rotor of each being connected to the master engine tachometer generator while each stator is connected to one of the three other generators.

3.4 OPERATION

For understanding the operation of a synchroscope let us consider the installation of a typical twin-engined aircraft tachometer system, the circuit of which is shown in Fig.(3.2). Let us assume that the master engine, and this is usually the No.1, has been adjusted to the required 'on-speed' condition and that the slave engine has been brought into synchronism with it.

Now, both generators are producing a three-phase alternating current for the operation of their respective indicators, and this is also being fed to the synchroscope, generator No.1 feeding the rotor and No.2 the stator. Thus, a

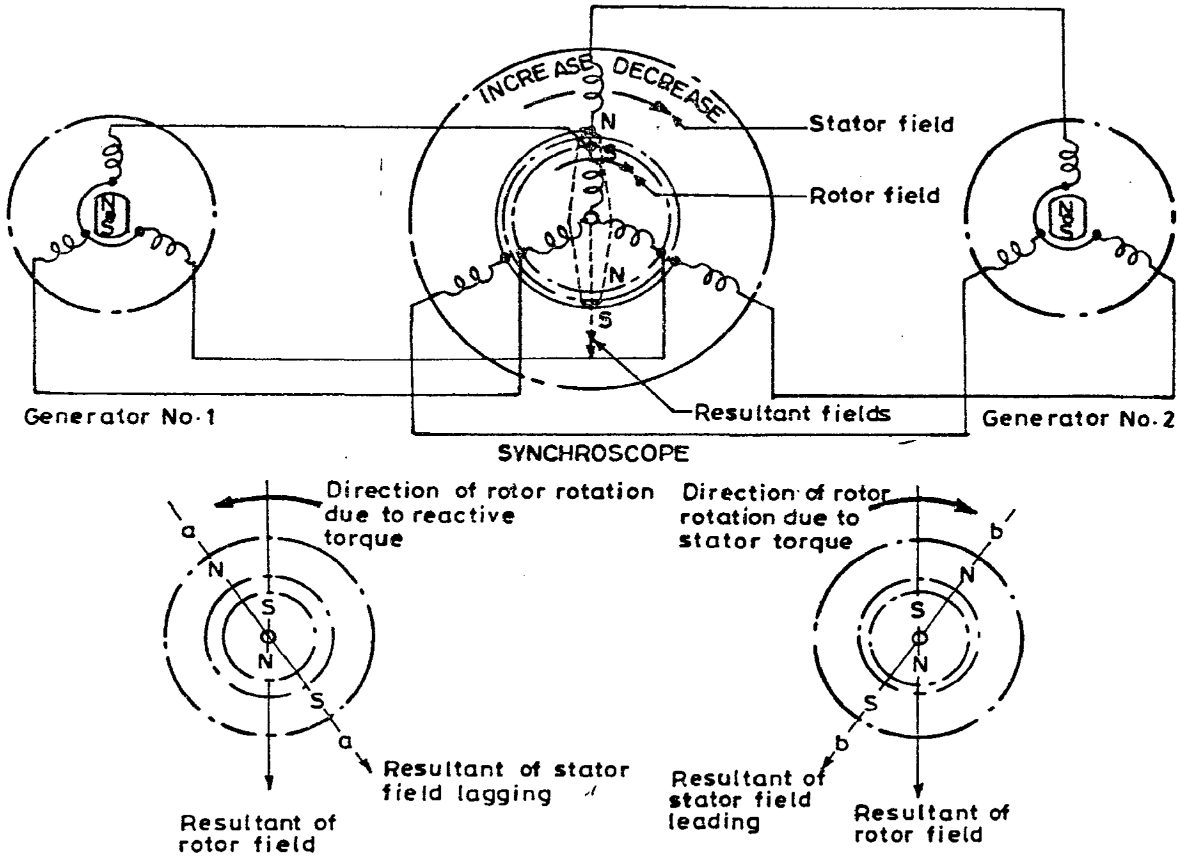


FIG. 3-2 OPERATION OF A SYNCHROSCOPE

magnetic field is set up in the rotor and stator, each field rotating at a frequency proportional to its corresponding generator frequency, and for the phase rotation of the system, rotating in the same direction. For the conditions assumed, and because generator frequencies are proportional to speed, it is clear that the frequency of the synchroscope stator field is the same as that of the rotor field. This means that both fields reach their maximum strength at the same instant; the torques due to these fields are in balance, and the attraction between opposite poles keeps the rotor 'locked' in some stationary position, thus indicating synchronism between engine speeds.

Consider now the effect of the slave engine generator running slower than the master engine generator, and consequently the stator field will be lagging behind the rotor field; in other words, reaching its maximum strength at a latter strength at, say point a in Fig. (3.2). The rotor, in being magnetized faster than the stator, tries to rotate the stator and bring the stator field into alignment, but the stator is a fixed unit, therefore, a reactive torque is set up by the interaction of the greater rotor torque with the stator. This torque causes the rotor to turn in a direction opposite to that of its field so that it is forced to continually realign itself with lagging stator field. The continuous rotation of the rotor drives the propeller-shaped pointer round to indicate that the slave engine is running-SLOW and that on INCREASE of speed is required to bring it into synchronism with the master engine.

If the slave engine should run faster than the master then the synchroscope stator field would lead the rotor field, reaching maximum strength at say point b. The stator field would then produce the greater torque, which would drive the rotor to realign itself with the leading stator field, the pointer indicating that the slave engine is running FAST and that a DECREASE of speed is required to synchronize it.

As the speed of the slave engine is brought into synchronism once again, the generator frequency is changed so that a balance between fields and torques is once more restored and the synchroscope rotor and pointer take up a stationary position.

From the foregoing description we see that a synchroscope is, in reality, a frequency meter, its action being due only to the relative frequencies of two or more generators. The generator voltages play no part in synchroscope action except to determine the operating range above and below synchronism.

chapter : 4

design of closed loop control system

CHAPTER-4

DESIGN OF CLOSED LOOP CONTROL SYSTEM

4.1 THE GENERAL APPROACH TO DESIGN

The requirements for a system will be described by some appropriate performance specification, expressed as the time domain requirements (defining the transient and steady state response for a step change or other forcing function).

There are two basic approaches to design. The older, but still very widely used one is an orderly trial and observation intuitive approach aimed at finding an acceptable, but not necessarily the best possible, design solution. The alternative approach is one of true synthesis, where an attempt is made to determine a unique solution in accord with a rigidly defined specification in some optimal way [13].

The simplest and most widely used arrangement is series compensation where the controller or compensation device is positioned in the forward loop as shown in Fig.(4.1). For studying the response of a closed loop system where the loop gain can be adjusted but where otherwise the dynamic characteristic are fixed such an arrangement with a series controller which is simple a gain element gives what is termed proportional control action, where no suitable value of gain can be found to achieve the specification, additional loop elements are needed i.e. the use of integral action and derivative action.

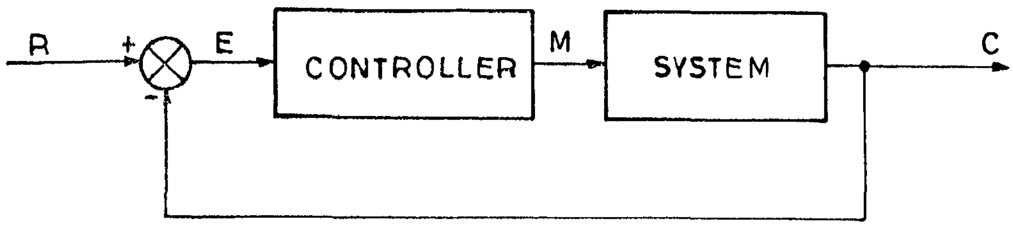


FIG. 4.1 CONTROL LOOP CONFIGURATIONS SERIES OR CASCADE COMPENSATION

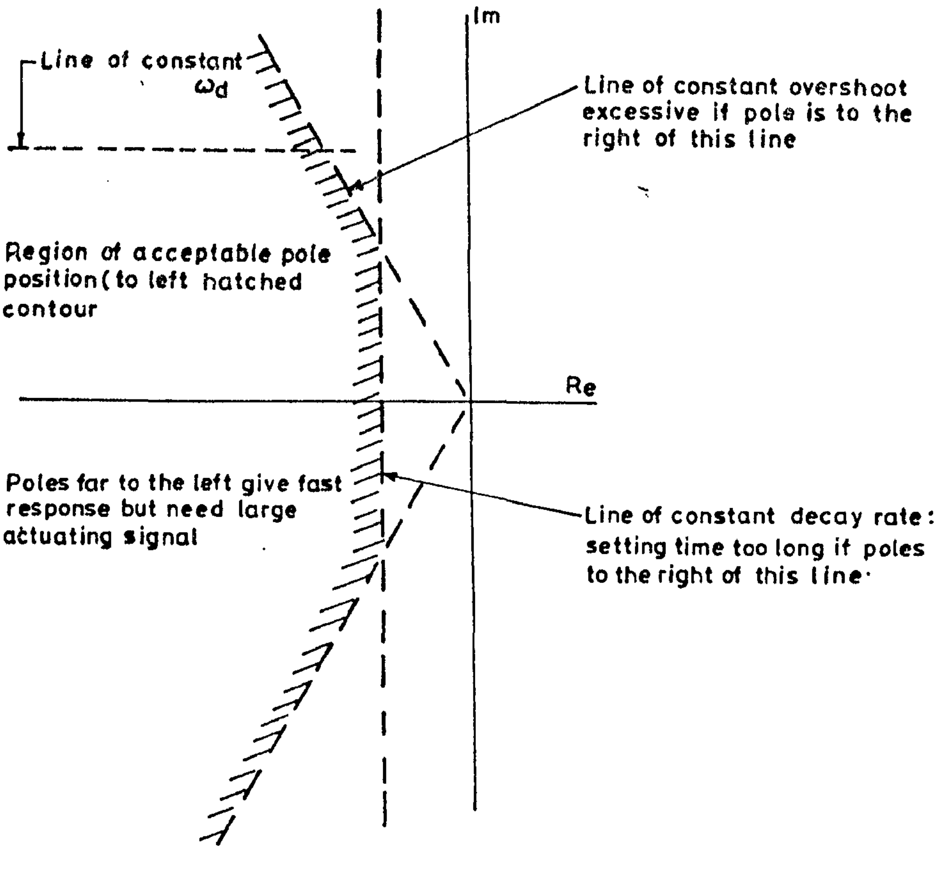


FIG. 4.2 ACCEPTABLE REGION FOR DOMINANT ROOTS

The steady state accuracy requirement dictates the form of open loop transfer function and the value of gain needed. Design in time domain to achieve specific transient response characteristic is facilitated by studies of root locus plots for the system. The dominant poles are the most critical, and considerations of settling time and maximum overshoot in response to a step change dictate the area which should give acceptable behaviour as indicated in Fig. (4.2). The response is, however, influenced both by the secondary poles and by any system zeros which are present, and hence root locus studies must be supplemented by simulation studies to confirm that the specifications are satisfied.

For automatically tuning the PID controller, the most well known method is that of Ziegler and Nichols [17]. Their method determines the parameters for the given plant. Proportional gain, integral time constant and derivative time constant by observing the gain at which the plant becomes oscillatory and the frequency of oscillation. Thompson [14] presented a procedure for designing multivariable controllers for unidentified plant. For such plant no mathematical model is required in order to generate multivariable PID controllers, but it has assumed that the open loop plant is stable and its response to step inputs are basically nonoscillatory. Gawthrop and Nomikos [3] very recently has developed the continuous time self-tuning algorithms which are capable of generating tuning parameters for commercial PID controllers.

4.2 CONSTRUCTION OF ROOT LOCI

The root locus program is used to obtain and plot the zeroes of the equation

$$1 + G(s)H(s) = 0 \qquad ..4.1$$

as a function of K. Here G(s)H(s) is assumed to be rational function of the form

$$G(s)H(s) = K \frac{N(s)}{D(s)} \qquad ..4.2$$

So that we may also consider the problem of obtaining the root locus of the polynomial

$$D_K(s) = D(s) + KN(s) \qquad ..4.3$$

as a function of K

Theory : The polynomials N(s) and D(s) are written as

$$N(s) = n_1 + n_2 s + \ldots + n_m s^{m-1} + s^m \qquad ..4.4$$

$$D(s) = d_1 + d_2 s + \ldots + d_n s^{n-1} + s^n \qquad ..4.5$$

Where $m<n<20$, then $D_K(s)$ is given by

$$D_K(s) = (Kn_1+d_1) + (Kn_2+d_2)s + \ldots + (K+d_{m+1})s^m + \ldots + s^n \qquad ..4.6$$

Now as K vary $D_K(s)$ is computed.

An algebraic plus linear progression is used to vary K in the following manner.

$$K_{new} = 1.15(K_{old} + 0.05) \qquad ..4.7$$

This procedure has been found to lead to a reasonably uniform spacing of the roots. The use of this procedure assumes that K takes only positive value. If K is to range through negative values, the algebraically smaller (less negative) value must be used as the minimum value. Hence the routine starts at the maximum (less negative) value and becomes increasly negative until the lower limit is reached. If both positive and negtive values are desired, then two separate runs must be made with only positive and only negative values [9].

When the entire desired range of K has been exhausted, the subroutine SPLIT is used to plot the resulting root locus. This program also has an option which allows one to plot only the portion of the root locus which lies in a specified rectangular region in the s-plane. If this option has been selected a rectangular region of s-plane is specified by giving σ_{max}, σ_{min}, ω_{max} and ω_{min}. Only the portion of the root locus which lies in this region will be plotted, when this option is selected, the progression of gain values is calculated by the rule

$$K_{new} = 1.04(K_{old} + 0.02) \qquad ..4.8$$

This expression leads to a larger number of gain values and hence a more refined plot. The basic use of option is to refine a portion of the root locus which is of particular interest.

4.3 DIGITAL CONTINUOUS SYSTEM SIMULATION

As a consequence of the very rapid development of digital computer hardware and software giving ever greater capability and flexibility at decreasing cost, system simulation is inevitably being carried out more and more on the digital computer. There is effectively no problem of overloading, so very wide ranges of parameter variation can be readily accommodated, any desirable accuracy can be attained and with the aid of appropriate high level languages program writing is straight forward.

The solution of a differential equation involves the process of integration, and for the digital computer analytical integration must be replaced by some numerical method which yields an approximation to the true solution. A continuous signal $x(t)$ is represented by a series of number x_0, x_1, x_2, $x_3 \ldots x_n$, say, which define the signal amplitude at times $t_0, t_1, t_2, t_3 \ldots t_n$. These sample values are normally at equally spaced time intervals, and if the sampling interval is chosen to be small enough then no information about the signal is lost [13].

A complex and efficient algorithm which uses only the current value to estimate the next value, but estimates three or more usually four derivative values to do so, is the Runge-Kutta Method, which is discussed in appendix-B.

From Runge-kutta method, the solution is ascertained in discrete form i.e. signal is sampled at equally spaced interval, which can be plotted by a CalComp Electromechanical Plotter.

The CalComp Host Computer Basic software (HCBS) package consists of a set of subroutines written in FORTRAN and/or assembly language which control elementary operations of the plotter and provide certain aids in plotting graphs. The subroutines are called by CalComp (and user) written applications programs and host computer functional software. All output to the plotting system should go through the basic software package.

4.4 PERFORMANCE CHARACTERISTICS (TIME DOMAIN)

What is a 'good' type of transient response ? How should the transient response of a practical system, or the type of response desired for a system be described ? The algebraic equation is not very helpful since the form of the response is not readily apparent. A plot of the response is not satisfactory since a numerical description is required for analysis.

A first order system can be completely described by specifying the value of the time constant. A second order system can be clearly described by specify the two time constant values if overdamped, or the values of ζ and ω_n if underdamped. For a higher order system, values of ζ , and ω_n cannot be specified since they do not exist though values of these can be given for the dominant roots.

In general the following parameters, shown in Fig.(4.3) give an adequate description and are used to describe the step response of a system :

(i) maximum overshoot - this is usually expressed as a percentage of the step size.

(ii) number of oscillations,

(iii) rise time - this is usually defined as the time taken from 5% to 95% of the step size, or over some similar range; defining rise time thus avoids the practical difficulty of having to determine the exact start of the transient, and the finish, if over damped.

(iv) peak time - this is defined as the time required to reach the peak of time response or the peak overshoot.

(v) settling time - the time taken until the output falls within and remains with in $\pm$5%, say of the steady state value.

(vi) steady state error.

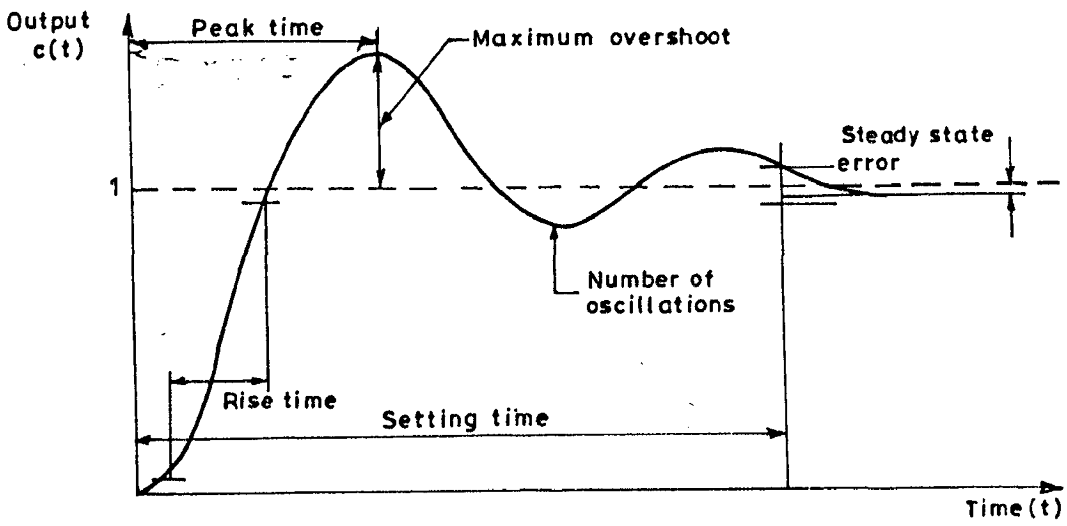


FIG. 4.3 PARAMETERS DESCRIBING UNIT STEP RESPONSE

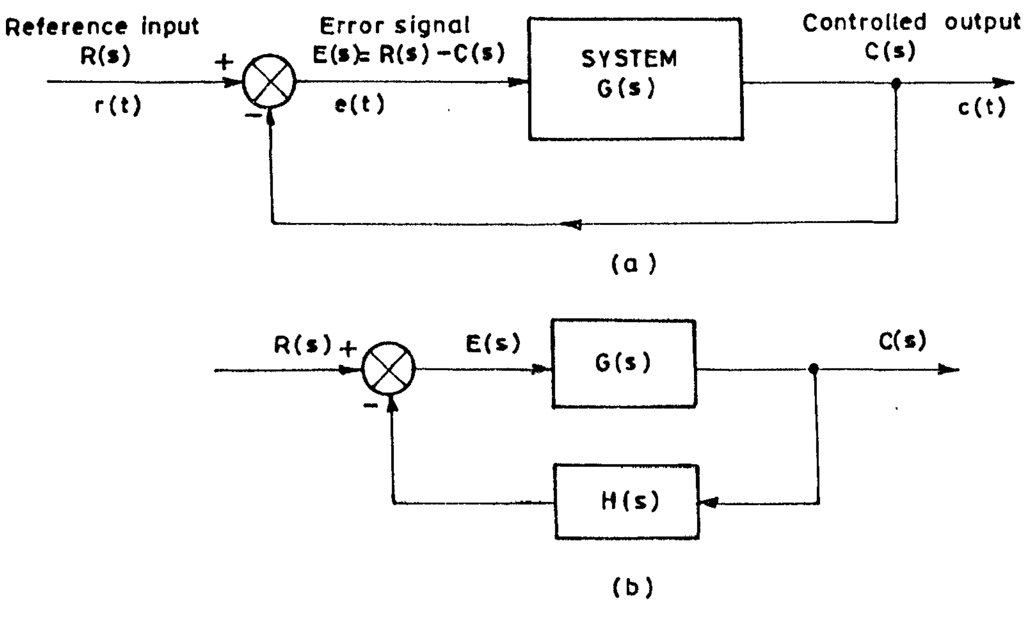


FIG. 4.4 BLOCK DIAGRAM OF SIMPLE FEEDBACK SYSTEM WITH UNITY FEEDBACK

These parameters are interrelated and requirements tend tc conflict. The maximum over shoot can generally only be decreased at the expense of an increase in rise time, steady state error can generally only be reduced at the expense of making the transient mcre oscillatory.

4.5 STEADY STATE ERROR

Consider the unit feed back system of Fig.(4.4a). The error signal e(t) is the difference between the reference input and the controlled output :

$$e(t) = r(t) - c(t) \qquad ..4.9$$

The reference input signal can thus thought of as desired output. The steady state error e_{ss} is the limiting value of the error e(t) as time t becomes very large.

$$\text{Steady error} = e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) \qquad ..4.10$$

the final value theorem of Laplace transform analysis

$$E(s) = R(s) - C(s)$$

$$= R(s) - G(s).E(s)$$

$$\therefore E(s) = \frac{R(s)}{1+G(s)} \qquad ..4.11$$

Hence the steady state error is

$$e_{ss} = \lim_{s \to 0} \left[\frac{sR(s)}{1 + G(s)}\right] \qquad ..4.12$$

This equation shows that the steady state error is a function both of the type of system as described by G(s) and of the type of input R(s).

For unit step input function i.e. a constant input for values of time $t > 0$. The Laplace transform of input is then

$R(s) = 1/s$

Inserting this in Eq.4.12 gives

$$e_{ss} = \lim_{s \to 0} \left[\frac{1}{1 + G(s)}\right] = \frac{1}{1 + \lim_{s \to 0} G(s)} = \frac{1}{1 + K_p} \qquad ..4.13$$

Where $K_p = \lim_{s \to 0} G(s)$ is called the positional error coefficient or positional error constant. Rearranging the expression for e_{ss} gives

$$K_p = \frac{1 - e_{ss}}{e_{ss}}$$

Hence $K_p = \lim_{s \to 0} G(s)$

$$= \frac{\text{desired output - allowable steady state error}}{\text{allowable steady state error}} \qquad ..4.14$$

Systems are classified as being Type 0,1, or 2, where the type number is the value of α , which corresponds to the number of open loop poles at the origin. Table 4.1 summarised the steady state error for these type of system

Table 4.1

System	**Steady state error to step input**
Type 0	Finite
Type 1	0
Type 2	0

4.6 PROPORTIONAL CONTROL

Consider the following model, which controls the speed of engine by varying the governor setting in closed loop system [13]. This arrangement assumed is that shown in block diagram Fig. (4.5a). The governor is simple one with the proportional gain term k_1, only the actual engine speed is sensed by an electrical tachometer with a first order transfer function and a reciprocating engine can for such a study be adequately described by a second order transfer function.

The open loop transfer function can be written as

$$G(s)H(s) = \frac{k_1 K_2 K_3}{(1+\tau s)(s^2+2\xi\omega_n s+\omega_n^2)}$$

$$= \frac{k_1 K_2 K_3/\tau}{(s+1/\tau)(s^2+2\xi\omega_n s+\omega_n^2)}$$

$$= \frac{K}{(s+1/\tau)(s^2+2\xi\omega_n s+\omega_n^2)} \quad ..4.15$$

$$= \frac{K}{\omega_n^2/\tau+(2\xi/\tau+\omega_n)\omega_n s+(1/\tau+2\xi\omega_n)s^2+s^3} \quad ..4.16$$

Where $K = k_1 K_2 K_3/\tau$ and $H(s) = 1$ as it is a unit feed back system, the closed loop transfer function is then

$$T(s) = \frac{C(s)}{R(s)} = \frac{K}{(s+1/\tau)(s^2+2\xi\omega_n s+\omega_n^2)+K} \quad ..4.17$$

$$= \frac{K}{(K+\omega_n^2/\tau)+(2\xi/\tau+\omega_n).\omega_n s+(1/\tau+2\xi\omega_n)s^2+s^3} \quad ..4.18$$

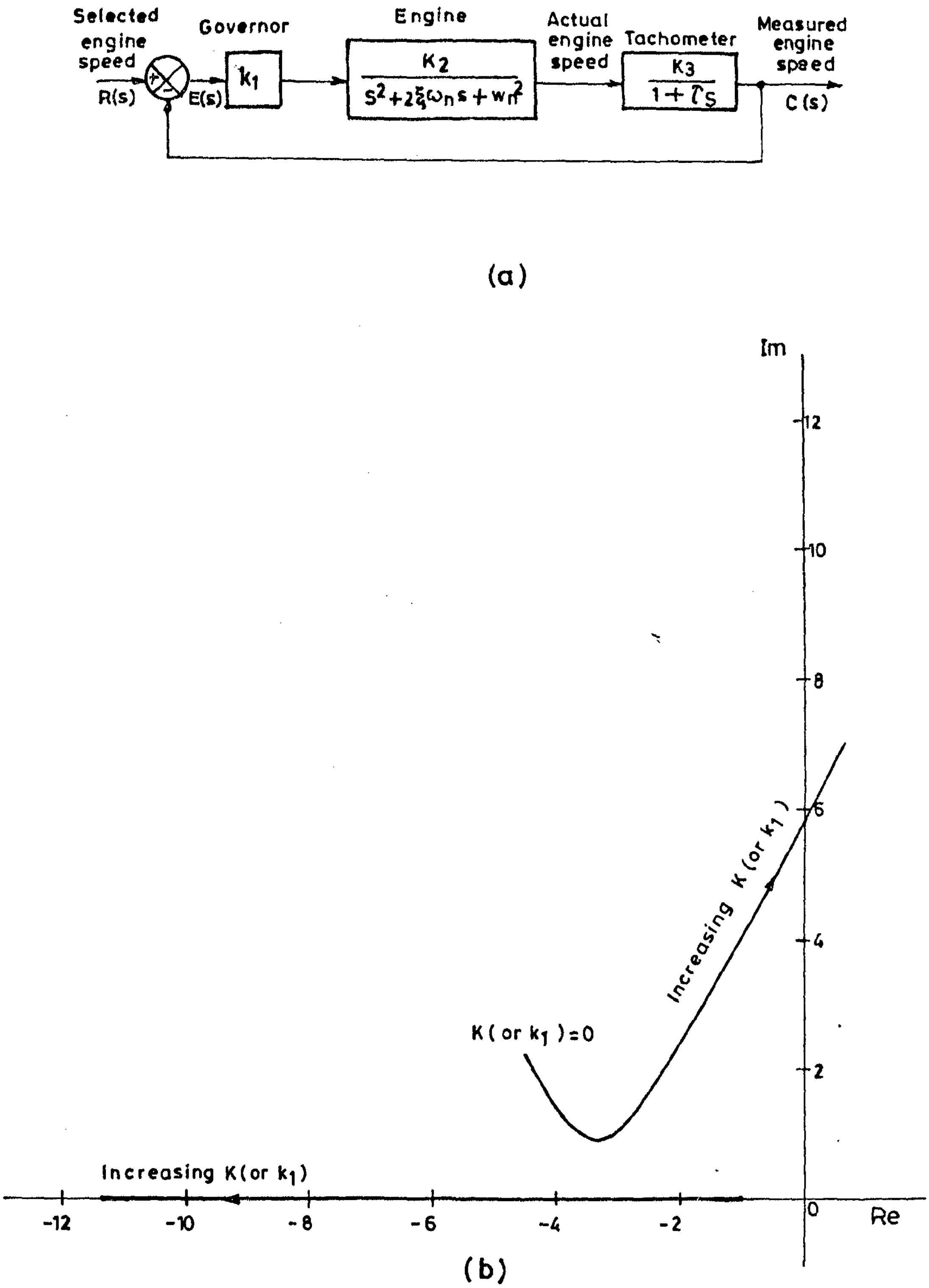


FIG. 4.5 ENGINE SPEED REGULATING SYSTEM
(a) BLOCK DIAGRAM (b) ROOT LOCUS DIAGRAM
FOR VARIATION OF PROPORTIONAL ACTION (k_1)

The multi-engine aircraft different engines which are used are made identical but as a matter of fact, however hard, we go for identical engines there will be slight changes in the parameters. This is due to design problem or may be due to wear and tear of machines. Thus for the same setting of governors the speed of engines will be not same or in other words the steady state value is closed loop system will be different.

In general G(s)H(s) can be expressed as from Eq.4.2

$$G(s)H(s) = K \frac{n_1+n_2s+ \text{---} + n_m s^{m-1}+s^m}{d_1+d_2s+ \text{---} + d_n s^{n-1}+s^n} \qquad ..4.19$$

Substituting value of G(s)H(s) in Eq.4.13

$$e_{ss} = \frac{1}{1 + K\frac{n_1}{d_1}} = \frac{d_1}{Kn_1+d_1} \qquad ..4.20$$

or

$$\text{Gain}(K) = \frac{d_1}{n_1}\left[\frac{1-e_{ss}}{e_{ss}}\right]$$

$$= \frac{d_1}{n_1} * \frac{\text{desired output - allowable steady state error}}{\text{allowable steady state error}} \qquad ..4.21$$

Eq.4.21 shows if d_1 and n_1 are different values, the value of gain(K) have to be different for same steady state error. The Eq.4.21 can also be expressed as

$$\text{Gain}(K) = \frac{d_1}{n_1} * \left[\frac{T(0)}{1-T(0)}\right] \qquad ..4.22$$

Where T(0) is steady state value = $1-e_{ss}$ (in closed loop).

Thus it is obvious from Eq.4.22 for the same steady state value of different engines, the gain(K) has to be different. Thus for all engines have different governor setting (k_1) for the same steady state value i.e. for same speeds. Thus, in other words, governor setting should be such as for each engine the steady state value should be identical so that all engine should in synchronism.

Let us consider the four engine aircraft, which has parameters and the corresponding tachometers time constant as shown in Table 4.2. Thus the G(s)H(s) of four engines are evaluated by using Eq.4.16, which are shown in Table 4.3.

Since K_2, K_3 τ are constant for particular engine, so gain(K) can be vary by simply varying the k_1. Thus the numerical value of this constant k_1 determines the amount of corrective effort which is applied for a given magnitude of error. By varying the value of k_1, the dynamic behaviour of the overall system can be altered. For very low values of k_1, the corrective effort is small and hence the response would likely to be sluggish; as k_1 is increases the response of the system for the same magnitude of error becomes more rapid and, if k_1 is very large, instability would likely to result, or the oscillatory response would be so lightly damped that it would be unsatisfactory.

Table 4.2

Engine Identification	Natural freuency(ω_n)rad./sec.	Damping Factor(ξ)	Tachometer time constant (τ)	K_2	K_3
A	5.0	0.90	1.00	1.0	1.0
B	5.2	0.92	1.20	1.0	1.0
C	5.1	0.91	1.30	1.0	1.0
D	4.8	0.93	0.98	1.0	1.0

Table 4.3

Engine Identification	Coefficients of G(s)H(s) in ascending order of s		Gain (K) = $k_1K_2K_3/\tau$
	Denominator	Numerator	
A	25-000, 34.000, 10.000, 1.000	1.000	1.000xk_1
B	22.524, 35.010, 10.401, 1.000	1.000	0.833*k_1
C	20.002, 33.148, 10.051, 1.000	1.000	0.769*k_1
D	23.501, 32.147, 09.948, 1.000	1.000	1.020*k_1

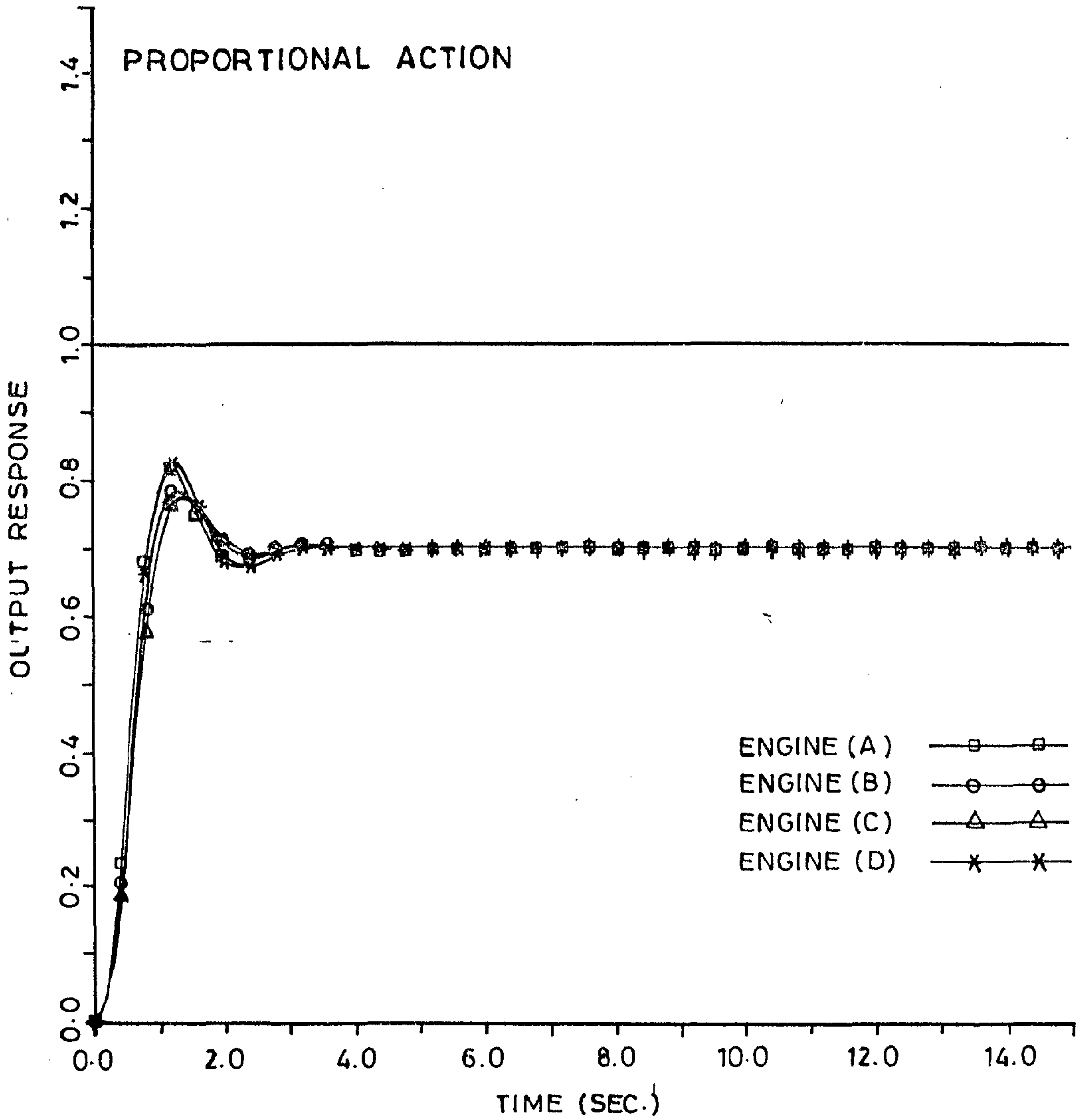


FIG. 4.6 MEASURED ENGINE SPEED RESPONSE TO UNIT STEP INPUT AT DIFFERENT (k_1)

To make the steady state same of all the engines i.e. the same speed for synchronization of engines. The gain(K) is calculated from eq.no.4.22 for the steady state value of 0.7. From the root locus diagram Fig. 4.5b, it is clearly that dominant complex roots damping factor is in between 0.4 to 0.6 for all engines.

The C(s)/R(s) of four engines are calculated from Eq.4.18 for the particular value of gains which are calculated from Eq.4.22 to make the steady state value equal to 0.7 is shown in Table 4.4. The simulation study is done for unit step input from the table 4.4 for each engine which is shown in Fig.4.6 whose performance is listed in table 4.5.

The response shown for this typical speed control system would in engineering practice normally be considered 'satisfactory' having maximum overshoot of 20% and a peak time of 1.5 second and a setting time of 4.0 seconds.

The major disadvantage for this control is the steady state error i.e. 30% for this example, increase in the gain(K), however, causes the dominant complex roots to move closer to the imaginary axis and to the instability associated with the root positions in the right half of the complex s-plane. Provided a value of gain(K) can be chosen which gives both an acceptable transient response and a small enough steady state

Table 4.4

Engine Identification	Gain (K)	Coefficients of C(s)/R(s) in ascending order of s: Denominator	Numerator	k_1	Daming factor of Dominant complex root
A	58.32	83.320, 34.000, 10.000, 1.000	1.000	58.32	0.4568
B	52.56	75.080, 35.010, 10.401, 1.000	1.000	63.10	0.4442
C	46.67	66.673, 33.148, 10.051, 1.000	1.000	60.69	0.5496
D	54.84	78.337, 32.147, 09.948, 1.000	1.000	53.76	0.4473

Table 4.5

Engine Identification	Peak over Shoot (M_p) in%	Peak time (t_p) in sec.	Settling time (t_s) in sec.	Steady State Value	Steady state error
A	17.0	1.2	3.6	0.7	0.3
B	12.0	1.2	3.6	0.7	0.3
C	11.0	1.4	3.6	0.7	0.3
D	18.0	1.2	3.6	0.7	0.3

then only the design problem will be solved. Since these requirements can not be satisfied simultaneously then the loop must be modified by the inclusion of some other form of control action i.e. integral control action and derivative control action.

4.7 INTEGRAL AND DERIVATIVE CONTROL ACTION

4.7.1 Integral Action

To over come the steady state error or zero steady state positional error would require the system to be of Type-1, and this can be achieved by introducing integral action with in the controller. To the proportional term is added a signal proportional to the time integral of error i.e. The controller output m(t) is $[k_1 e(t)+k_2 \; e(t)dt]$ and it is this signal which actuates the system. The block diagram is now in the form shown in Fig.4.7. Since the error signal is integrated with in the controller, even the smallest error eventually produces a corrective signal of sufficient magnitude to actuate the system to eliminate the error.

Consider the analytically the effect of integral action on the system of Fig. 4.7. From table 4.2, the transfer function of engine (A) and tachometer (A) can be written as

$$G1(s)G2(s) = \frac{1}{(s+1)(s^2+9s+25)} \qquad ..4.23$$

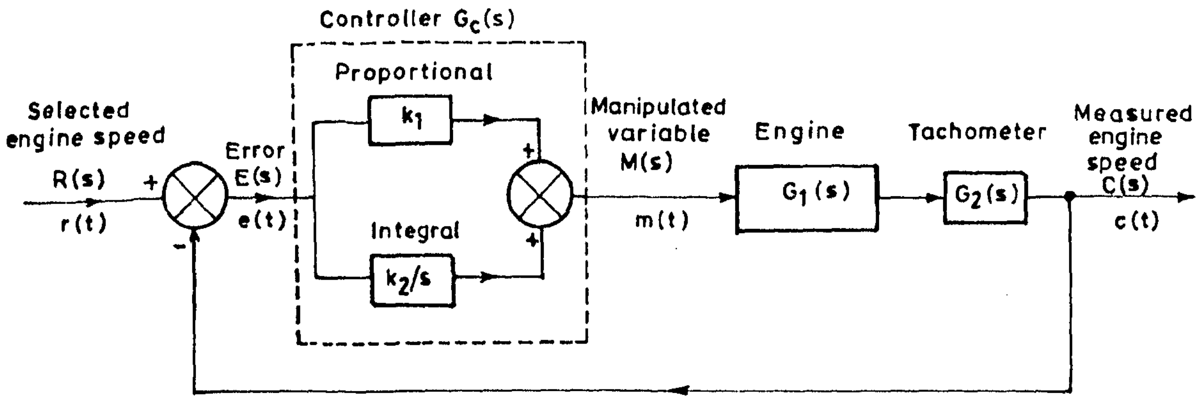


FIG. 4.7 FEEDBACK SYSTEM WITH P+I CONTROL ACTION

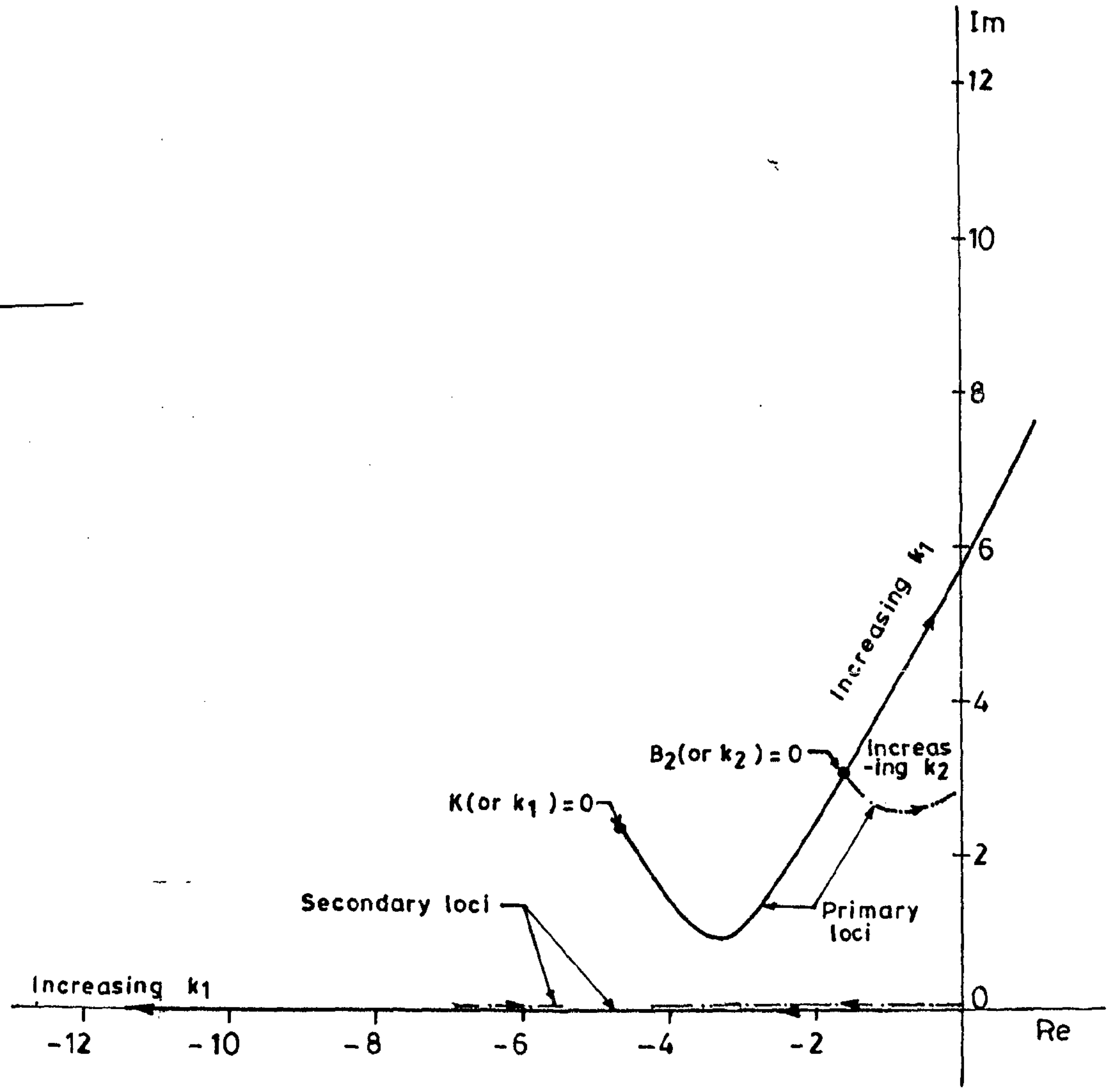


FIG. 4.8 ROOT LOCUS DIAGRAM FOR VARIATION OF PROPORTIONAL ACTION (k_1) AND INTEGRAL ACTION (k_2)

With proportional action (k_2 = 0) the closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K}{K+25+34s+10s^2+s^3} \qquad ..4.24$$

Where $K = k_1K_2K_3/\tau$

Hence the steady state error for a unit step input is

$$e_{ss} = 1 - \lim_{s\to0} s[\frac{1}{s}.\frac{K}{K+25+34s+10s^2+s^3}] = \frac{25}{25+K} \qquad ..4.25$$

With proportional plus integral (P+I) control

$$\frac{C(s)}{R(s)} = \frac{Ks + B_2}{B_2+(25+K)s+34s^2+10s^3+s^4} \qquad ..4.26$$

Where $B_2 = k_2K_2K_3/\tau$

and the steady state error for a unit input is

$$e_{ss} = 1 - \lim_{s\to0} s[\frac{1}{s}.\frac{Ks + B_2}{B_2+(25+K)s+34s^2+10s^3+s^4}] = 0$$

4.7.2 Root Contour

The numerical values for the parameters k_1 and k_2 can be selected by use of a root locus or root contour plot. The root contour technique is extended to allow additional loci to be added to a root locus diagram. Such loci representing variation of roots with a second in dependent variable, and each corresponding to a given and each corresponding to a given value of first in dependent variable K(or k_1) are known as root contours. This approach also enables a root locus diagram to be plotted for an independent variable which is not normally a simple multiplying factor. The method employed to construct root contours necessitates the rearrangement of the system equations to an equivalent transfer function in which the independent variables does in fact appear as a simple multiplying factor; after this the plot can be drawn by root locus program.

To illustrate the method of approach, consider the block diagram shown in Fig. 4.7 where B_2 (or k_2) is taken as independent variable of interest and the K(or k_1) has been used as previously specified in proportional control. The closed loop transfer from Fig.4.26 must be arranged as follows

$$\frac{C(s)}{R(s)} = \frac{Ks + B_2}{B_2+(25+K)s+34s^2+10s^3+s^4}$$

Where $K = k_1\ K_2\ K_3/\tau$ and $B_2 = k_2\ K_2\ K_3/\tau$

From table from 4.4 for engine A,K = 58.32 or k_1= 58.32, substituting this in above eqn. 4.26

$$\frac{C(s)}{R(s)} = \frac{B_2 + 58.32s}{B_2+83.32s+34s^2+10s^3+s^4} \qquad ..4.27$$

Divide the eq.4.27 through by terms not containing the independent variable B_2 to yield

$$\frac{C(s)}{R(s)} = \frac{(B_2+58.32s)/(83.32s+34s^2+10s^3+s^4)}{1+B_2/(83.32s+34s^2+10s^3+s^4)} \qquad ..4.28$$

The transfer function written in this form corresponds to an equivalent system with different values of G(s)H(s) but with the same characterisitc equation. By inspection it can be seen that

$$[G(s)H(s)]_{equiv} = \frac{B_2}{83.32s+34s^2+10s^3+s^4} \qquad ..4.29$$

Where the independent variable B_2(or k_2) is now a simple multiplying factor and hence a root locus diagram can be plotted in normal way. A closer inspection of Eq.4.29 reveals the poles of this equation are the roots of the characteristic equation of Eq.4.26 when $k_2 = 0$. Hence the starting point points of the root contours are points on the root loci for G(s)H(s) for specific value K(or k_2).

To match the transient response characteristic such as rise time, initial over shoot etc. then the roots on the dominant loci must lie in between the damping factor (ξ) 0.4 and 0.6. Thus the equivalent G(s)H(s) of each engine is shown in table 4.6 for proportional action K(or k_1) as selected in table 4.4.

Now to select the B_2(or k_2) root locus program is run for equivalent G(s)H(s) from the table 4.6 for each engine and from the plot on the primary loci, we choose the B_2(or k_2) on the basis of the damping factor of dominant complex root which should lie in between 0.4 to 0.6. From the simulation study, we can able to choose still better value of B_2(or k_2) which will give the desired transient response.

Table 4.6

Engine Identification	K	Coefficients of Equivalent G(s)H(s) in ascending order of s: Denominator	Numerator	Gain(B_2) $=k_2K_2K_3/\tau$
A	58.32	0.000, 83.320, 34.000, 10.000, 1.000	1.000	1.000*k_2
B	52.56	0.000, 75.080, 35.010, 10.401, 1.000	1.000	0.833*k_2
C	46.67	0.000, 66.673, 33.148, 10.051, 1.000	1.000	0.769*k_2
D	54.84	0.000, 78.337, 32.147, 09.948, 1.000	1.000	1.020*k_2

Table 4.7

Engine Identification	B_2	Coefficients of C(s)/R(s) in ascending order of s: Denominator	Numerator	k_2	Damping factor of dominant complex root
A	50.67	50.670,83.320, 34.000,10.000,1.000	50.67, 58.32	50.67	0.4246
B	44.01	44.010,75.080, 35.010,10,401,1.000	44.01, 52.56	52.83	0.5111
C	38.22	38.220, 66.673, 33.148,10.051,1.000	38.22, 46.67	49.70	0.5385
D	45.00	45.000 , 78.337 32.147,09.948,1.000	45.00, 54.84	44.12	0.4100

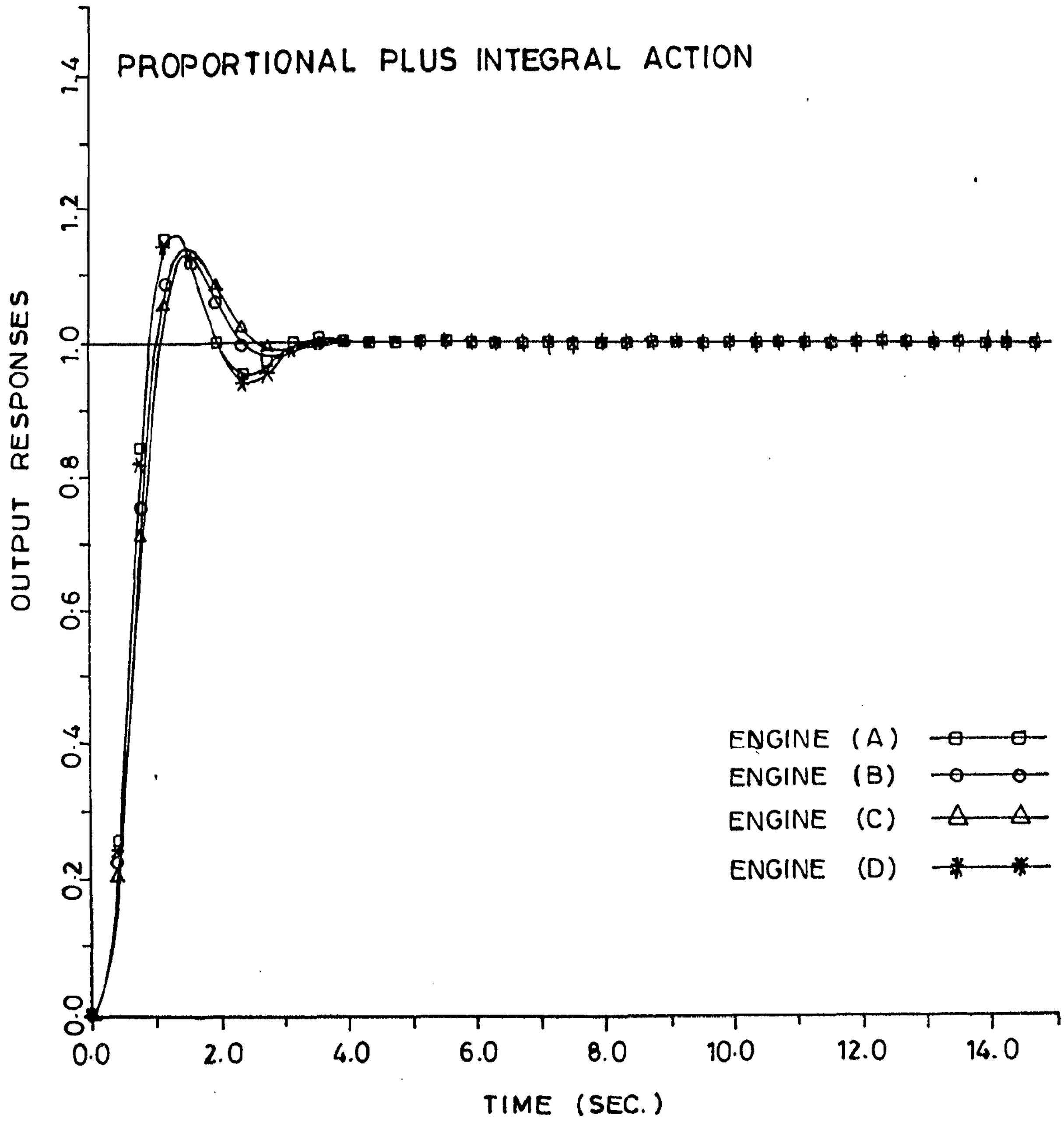


FIG.4.9 MEASURED ENGINES SPEED RESPONSE TO UNIT STEP INPUT AT DIFFERENT, (k_1 & k_2)

On the basis of Eq.4.27 the C(s)/R(s) of each engine is shown in table 4.7 after selecting the value of B_2(or k_2). From the simulation study, unit step response is plotted from the table 4.7 of C(s)/R(s) for each engine which is shown in Fig. (4.9) and the performance is listed in table 4.8.

From the table 4.8 which shows the initial overshoot is less than 17% and the settling time is 5.5 sec. Thus the transient of all engines almost matches, which is from practical consideration is also satisfactory having maximum overshoot not more than 20% and peak time is 1.5 seconds and settling time is 5.5 second.

To still reduce the maximum overshoot and settling time we will go for the derivative action.

4.7.3 Derivative Action

This form of control action which can increases the effective damping in derivative action, this is not used by itself but in conjuction with proportional or proportional plus integral action. To the normal error signal is added signal proportional to its derivative giving a 2-term or 3-term controller shown in Fig.4.10. The 3-term controller has a transfer function

$$G_c(s) = k_1 + \frac{k_2}{s} + k_3 s \qquad .. \; 4.30$$

Table 4.8

Engine Identification	Peak over Shoot (M_p) in %	Peak time (t_p) in sec.	Settling time(t_s) in sec.	Steady state Value	Steady state error
A	16.0	1.3	5.5	1.0	0.0
B	15.0	1.5	5.5	1.0	0.0
C	14.0	1.6	5.5	1.0	0.0
D	16.0	1.4	5.5	1.0	0.0

Table 4.9

Engine Identification	K	B_2	Coefficients of Equivlent G(s)H(s) in ascending order of s: Denominator	Numerator	Gain(B_3) $=k_3K_2K_3/\tau$
A	50.67	58.32	50.670,83.220,34.000 10.000,1.000	0.000,0.000 1.000	1.000*k_3
B	52.56	44.01	44.010,75.080,35.010 10.401,1.00	0.000,0.000 1.000	0.833*k_3
C	46.67	38.22	38.220,66.673,33.148 10.051,1.00	0.000,0.000 1.000	0.769*k_3
D	54.84	45.00	45.000,78.337,32.147 09.948,1.000	0.000,0.000 1.000	1.020*k_3

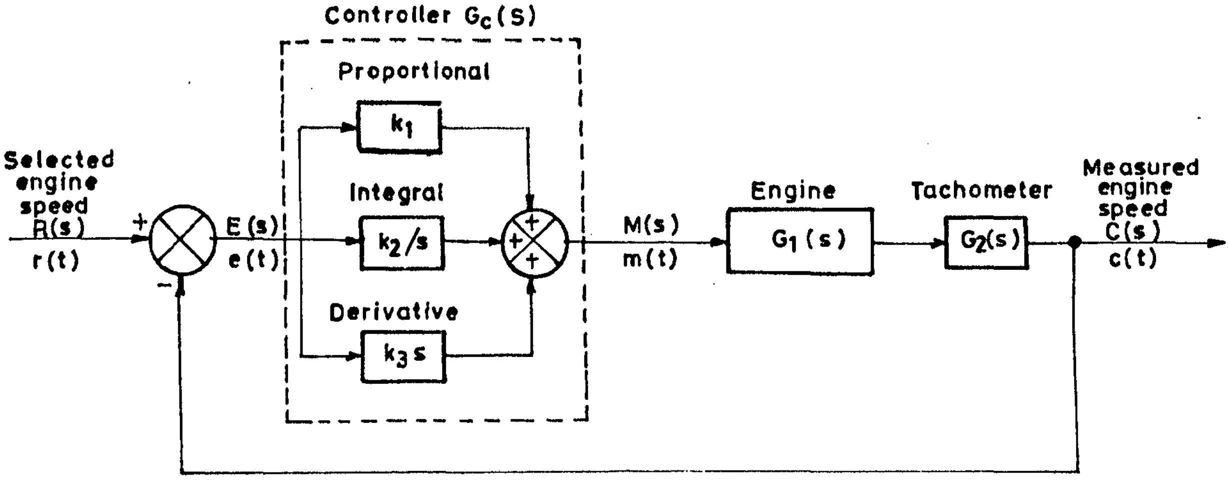


FIG. 4.10 FEEDBACK SYSTEM WITH P+I+D CONTROL ACTION

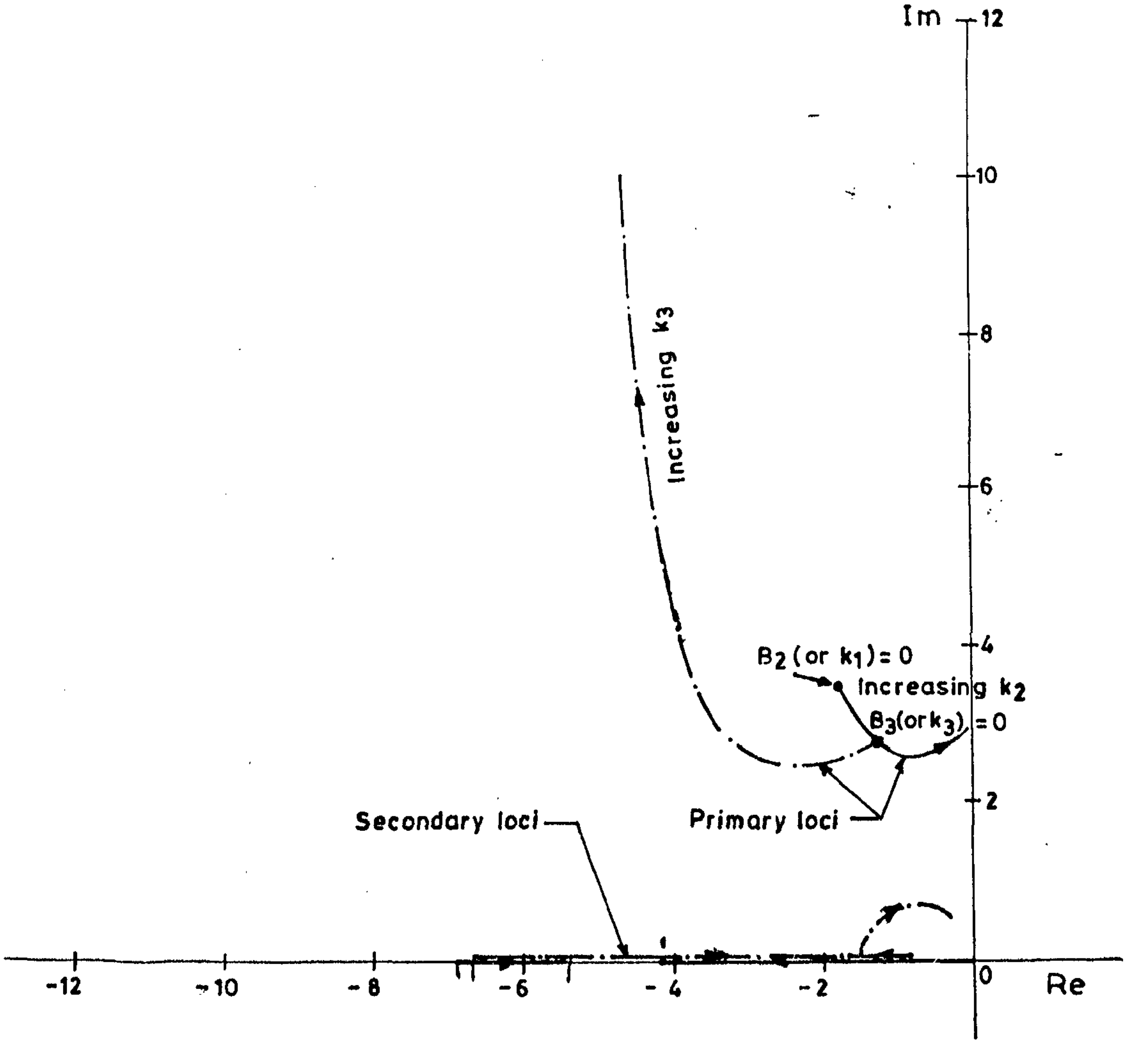


FIG.4.11 ROOT LOCUS DIAGRAM FOR VARIATION OF INTEGRAL ACTION (k_2) AND DERIVATIVE ACTION (k_3)

The effect of derivative action on the position of the roots of the characteristic equation can be seen in fig.4.11 which shows that the addition of derivative action has as expected improved the relative stability of the system. This is always a highly desirable feature in the design of a control system, since any changes in the values of parameters over a period of time is less likely to cause the system to drift into instability. The root locus is plotted again by root contour method as earlier discussed. The equivalent G(s)H(s) is shown in table 4.9

Inspection of fig. 4.11 suggests that a useful value of k_3 to reduce the overshoot with minimum affect on the other dynamic characteristic. This is study by simulation and will be selected best values to match the transient behaviour. For large value of B_3(or k_3) whose dominent root damping factor between 0.4 to 0.6 will give the large rise time or sluggish response which is not desired. So small B_3(or k_3) is selected. After selecting the B_3(or k_3) the C(s)/R(s) transfer function is shown in table 4.10 for different engines. From table 4.10, the simulation study is done for unit step input the plot is shown in Fig. 4.12. The performance index shown in table 4.11. which shows the overshoot is now reduced to less than 10% and settling time to 4.0 and peak time is 1.5 seconds. This response is quite satisfactory as it matches reasonably good dynamic behaviour and steady state error to zero.

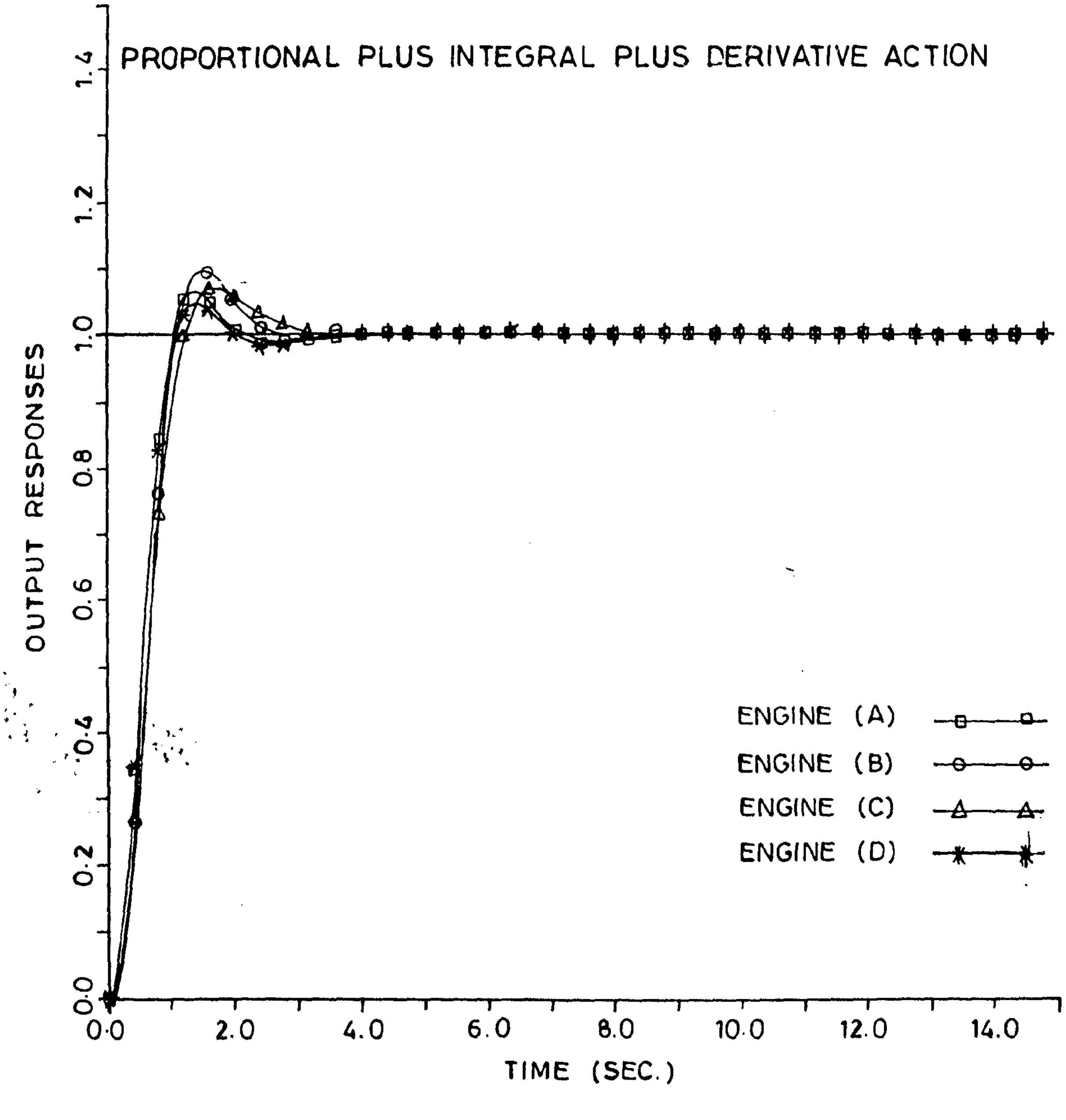


FIG. 4.12 MEASURED ENGINES SPEED RESPONSE TO UNIT STEP INPUT AT DIFFERENT, k_1, k_2 & k_3

Table 4.10

Engine Identification	K	B_2	B_3	Coefficients of C(s)/R(s) in ascending order of s: Denominator	Numerator	k_3
A	58.32	50.67	5.072	50.670,83.320,39.072 10.000,1.000	50.67,58.32 5.072	5.072
B	52.56	44.01	2.329	44.010,75.080,37.339 10.401,1.000	44.01,52.56 2.329	2.796
C	46.67	38.22	3.742	38.218,66.673,36.890 10.051,1.000	38.22,46.67 3.742	4.866
D	54.84	45.00	5.891	45.000,78.337,38.030 09.948,1.000	45.00,54.84 5.891	5.775

Table 4.11

Engine Identification	Peak over Shoot (M_p) in %	Peak time (t_p) in sec.	Settling time(t_s) in sec.	Steady state Value	Steady state error
A	6.3	1.4	4.0	1.0	0.0
B	9.5	1.5	4.0	1.0	0.0
C	7.0	1.7	4.0	1.0	0.0
D	4.5	1.4	4.0	1.0	0.0

chapter :5

analysis by order reduction

CHAPTER-5

ANALYSIS BY ORDER REDUCTION

5.1 MARSHALL'S APPROXIMATION METHOD

The reduced-order model of higher order systems are obtained by Marshall's method of model reduction technique [6,7] when the model is represented by a large set of simultaneous non-linear partial differential equations, then it is approximated by a set of linear time invariant differential equations. The system can be represented by the vector differential equation

$$\dot{\mathbf{x}}(t) = \mathbf{Ax}(t) + \mathbf{Bu}(t) \qquad ..5.1$$

$$y(t) = \mathbf{Cx}(t) \qquad ..5.2$$

Where **x** is the n-state vector of the system. **A**, **B** & **C** are respectively nxn, nxr and qxn constant coefficient matrices and **u** is the r-input vector and **Y** is the q-output vector of higher order system.

In this technique for reducing the large set of equations by neglecting the dynamic effects associated with the small time constants of the system. The eigen values of **A** should have negative real parts and distinct. This is a valid approximation where the spread of eigen values is large, because it is easy to distinguish which mode have a more dominant effect in the transient or steady state, since the eigen values of **A** to be distinct and have negative real parts and to be arrange in the order of increasing moduli, $\lambda_1, \lambda_2, \ldots \lambda_n$. Suppose now that the dynamics associated with the last n-m eigen values are to be neglected, i.e. it is required to reduce the order of model from n to m.

For convenience, partition equ. 5.1 so that the m variables to be retained in the reduced model are the first m variables of the state vector **X**, the eqn.5.1 becomes-

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \qquad ..5.3$$

Consider now the transformation

$$\mathbf{x} = \mathbf{Uz} \qquad ..5.4$$

Where **U** is the modal matrix of **A** and apply it to eq.5.1, giving

$$\dot{\mathbf{z}}(t) = \mathbf{U}^{-1}\mathbf{AUz}(t) + \mathbf{U}^{-1}\mathbf{Bu}(t)$$

i.e. $\dot{\mathbf{z}}(t) = \mathbf{Jz}(t)+\mathbf{U}^{-1}\mathbf{Bu}(t)$ ..5.5

Where **J** is the (nxn) diagonal matrix whose elements are eigen values of **A**. Eq.5.5 in the partitioned from becomes

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} V_1 & V_2 \\ V_3 & V_4 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \qquad ..5.6$$

Where $\mathbf{V} = \mathbf{U}^{-1}$

The first m eigen values are contained in the submatrix J_1 and the remaining (n-m) eigen values in J_2. Since J_2 contains only non-dominant eigen values resulting in small time constants, the state variables associated with them reach their steady state after a very short time. Hence, for the case of a step input, they may be approximated by an instantaneous step change.

Mathematically the approximation is equivalent to putting

$$\dot{z}_2 = 0 \qquad ..5.7$$

then eq.5.6 becomes

$$\dot{z}_1 = J_1 z_1 + (V_1 B_1 + V_2 B_2)u \qquad ..5.8$$

and $$0 = J_2 z_2 + (V_3 B_1 + V_4 B_2)u \qquad ..5.9$$

Now from eq.5.4

$$z = U^{-1}x = Vx$$

or

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} V_1 & V_2 \\ V_3 & V_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad ..5.10$$

giving-

$$z_2 = V_3 x_1 + V_4 x_2 = -J_2^{-1} (V_3 B_1 + V_4 B_2)u$$

$$x_2 = -V_4^{-1} V_3 x_1 - V_4^{-1} J_2^{-1} (V_3 B_1 + V_4 B_2)u \qquad ..5.11$$

Substituting equation 5.11 into eq.5.3 and using the relationships between U_i and V_i, one obtains-

$$\dot{x}_1 = U_1 J_1 U_1^{-1} x_1 + [B_1 - A_2 V_4^{-1} J_2^{-1} (V_3 B_1 + V_4 B_2)]u \qquad ..5.12$$

This set of m equation approximates to the original set of n equations and is called the reduced system. One important aspect is that the steady state values of the reduced system are identical to the original system.

5.2 INTER-RELATIONSHIP BETWEEN STATE SPACE AND FREQUENCY DOMAIN

The equation 5.12 can also be written as

$$\dot{x}_1 = A_r x_1 + B_r u \qquad ..5.13$$

$$y_1 = C_r x_1 \qquad ..5.14$$

Where x_1 is the m-state vector of the system. A_r, B_r and C_r are respectively mxm, mxr and qxm constant coefficient matrices.

The transfer function matrix of the system is

$$G(s) = C_r[sI_m - A_r]^{-1} B_r \qquad ..5.15$$

Where s is the complex variable. Since the original higher order systems are generally identified in frequency domain, thus it is subsequently converted to state space in Bush form [10], because the software simulation for output response is developed using state space approach.

5.3 RESPONSE OF ENGINE SPEED REGULATION SYSTEM OF LOW ORDER MODEL

In previous chapter, the engine speed regulation system with P+I and P+I+D action has Fourth order system. The

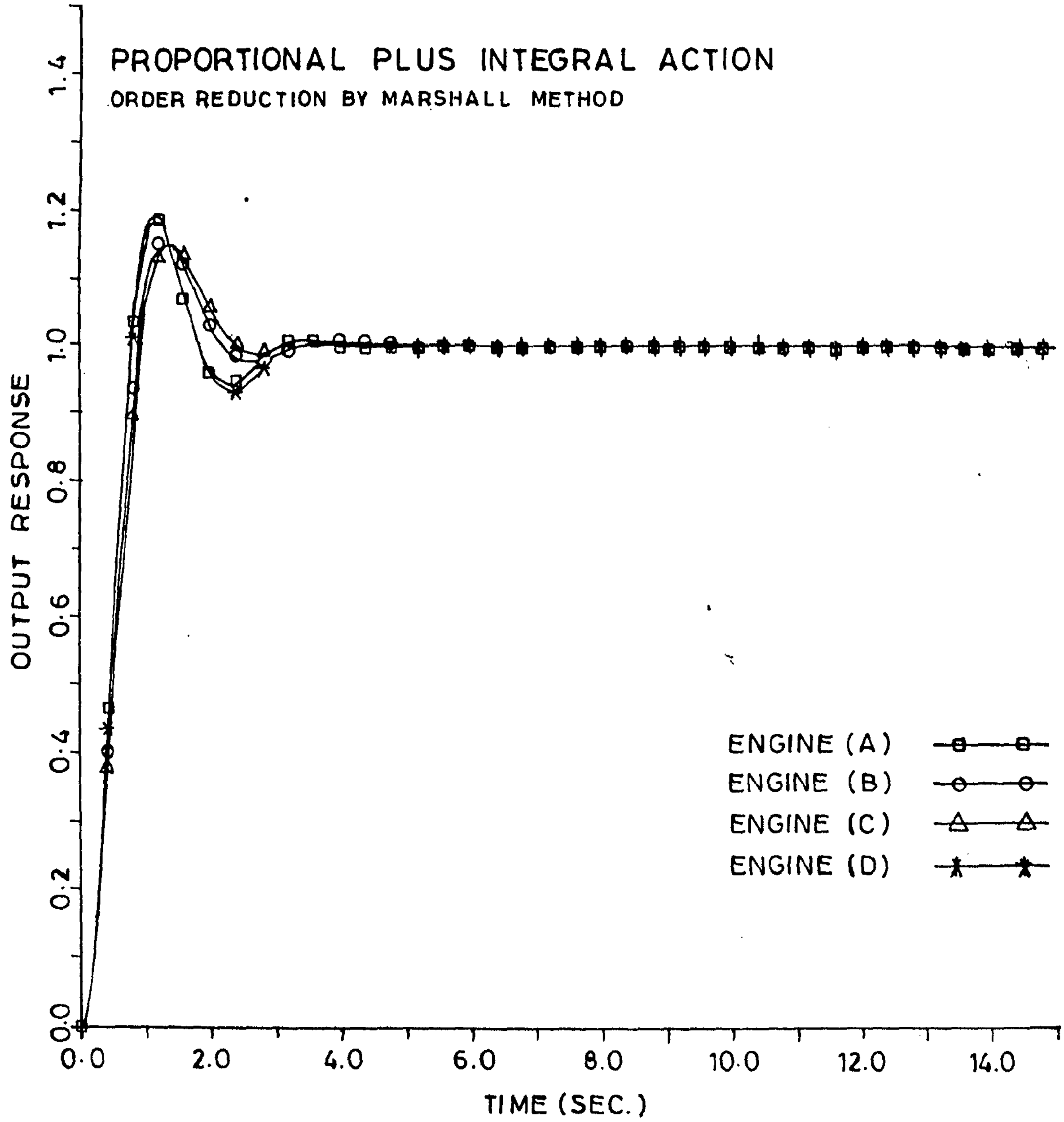


FIG. 5.1 REDUCED ORDER MODEL RESPONSE OF P+I ACTION

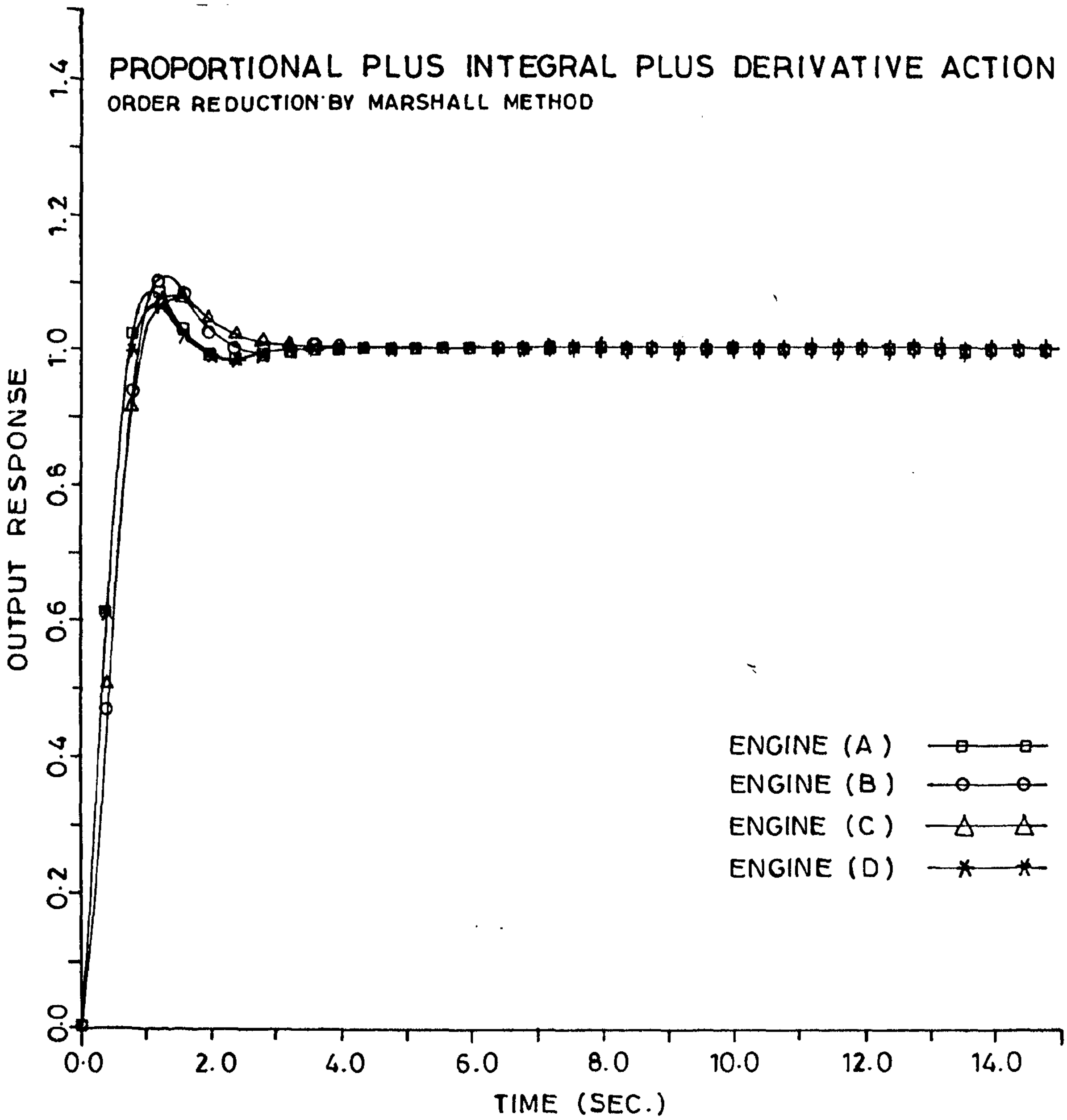


FIG. 5.2 REDUCED ORDER MODEL RESPONSE OF P+I+D ACTION

computation time of computer can be reduced, when the higher order system is approximated to low order model for analysis purpose in control.

The unit step response of third order model of engine speed regulation system is shown in Fig.5.1 and Fig.5.2 respectively for P+I and P+I+D action. Table 5.1 and Table 5.2 shows the performance index respectively for P+I and P+I+D action. It can be seen from table 4.8 and table 4.11, that performance index is matching respectively from table 5.1 and table 5.2.

5.4 AIRCRAFT BLIND LANDING SYSTEM

The block diagram of Aircraft Azimuth channel Blind landing system [16] is shown in fig.(5.3) where K is forward gain (in degree/foot), T_r is beam rate time constant (in sec), T_n is noise filter time constant (in sec), T_a is aircraft-autopilot time constant (in sec), Y_i is aircraft blind landing system azimuth displacement demand (in feet) and Y_o is aircraft blind landing system azimuth displacement (in feet). This is the sixth order zero-velocity lag system. Using the usual algebraic manipulation, it is readily shown that

$$G(s) = \frac{Y_o}{Y} = \frac{K(1+T_r s)}{s^2(1+T_n.s)^3(1+T_a s)} \qquad ..5.16$$

Table 5.1

Engine Identification	Peak over Shoot(M_p) in%	Peak time (t_p) in sec.	Settling time(t_s) in sec.	Steady state Value	Steady state error
A	18.0	1.2	5.5	1.0	0.0
B	15.0	1.3	5.5	1.0	0.0
C	14.5	1.4	5.5	1.0	0.0
D	18.5	1.1	5.5	1.0	0.0

Table 5.2

Engine Identification	Peak over Shoot(M_p) in%	Peak time (t_p)in sec	Settling time(t_s) in sec.	Steady state value	Steady state error
A	8.0	1.1	4.0	1.0	0.0
B	11.0	1.3	4.0	1.0	0.0
C	7.5	1.4	4.0	1.0	0.0
D	5.0	1.2	4.0	1.0	0.0

For the design value T_n = 0.73 sec., T_a=0.50 sec, T_r=10sec. The eqn. (5.16) becomes

$$G(s) = \frac{K(1+10s)}{s^2(1+0.73s)^3(1+0.50s)} = \frac{51.41K(s+0.1)}{s^2(s+1.37)^3(s+2)^2}$$

$$= \frac{K'(s+0.1)}{s^2(s+1.37)^3(s+2)^2} \quad ..5.17$$

Fig.5.4 shows the system root locus for variable gain, and for the design value K'_{nom} = 0.9652 degree/foot. The system transfer function is

$$T(s) = \frac{Y_o}{Y_i} = \frac{0.9652\ (s+0.1)}{0.097+0.965s+5.143s^2+13.833s^3+13.851s^4+6.110s^5+s^6} \quad ..5.18$$

The eqn. 5.18 can also be written in factorised form

$$T(s) = \frac{0.965\ (0.1+s)}{(s+0.283)(s+2.379)[(s+0.112)^2+(0.1784)^2][(s+1.612)^2+(0.7961)^2]}$$

$$= \frac{1+10s}{(1+3.51s)(1+0.42s)(1+2\cdot0.53/0.21s+s^2/0.21^2)(1+2\cdot0.9/1.8s+s^2/1.8^2)} \quad ..5.19$$

5.5 MARSHALL'S APPROXIMATION METHOD WITH EQUIVALENT LAG

It is clear from fig.5.5 that for systems such as aircraft blind landing system the prediction of transient response by low order model from marshall method is not good. In the marshall approximation method of order reduction, J_2

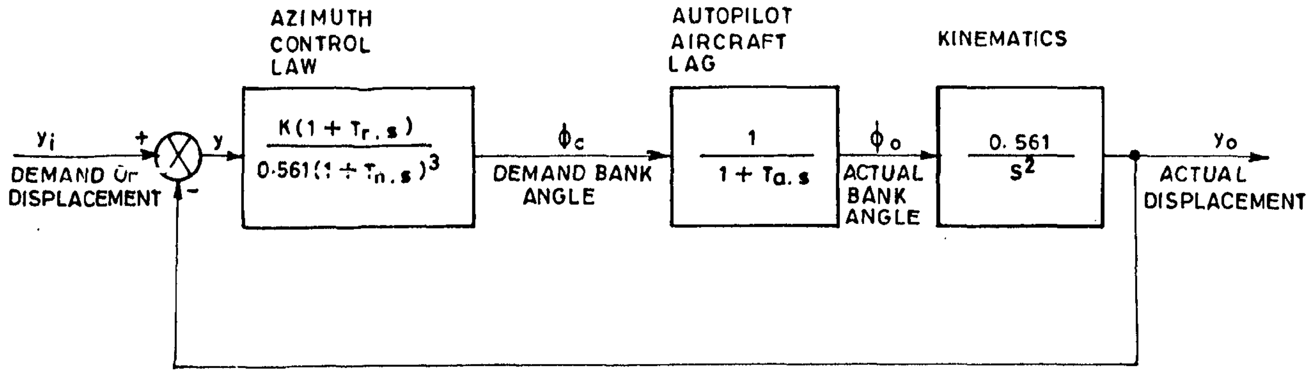
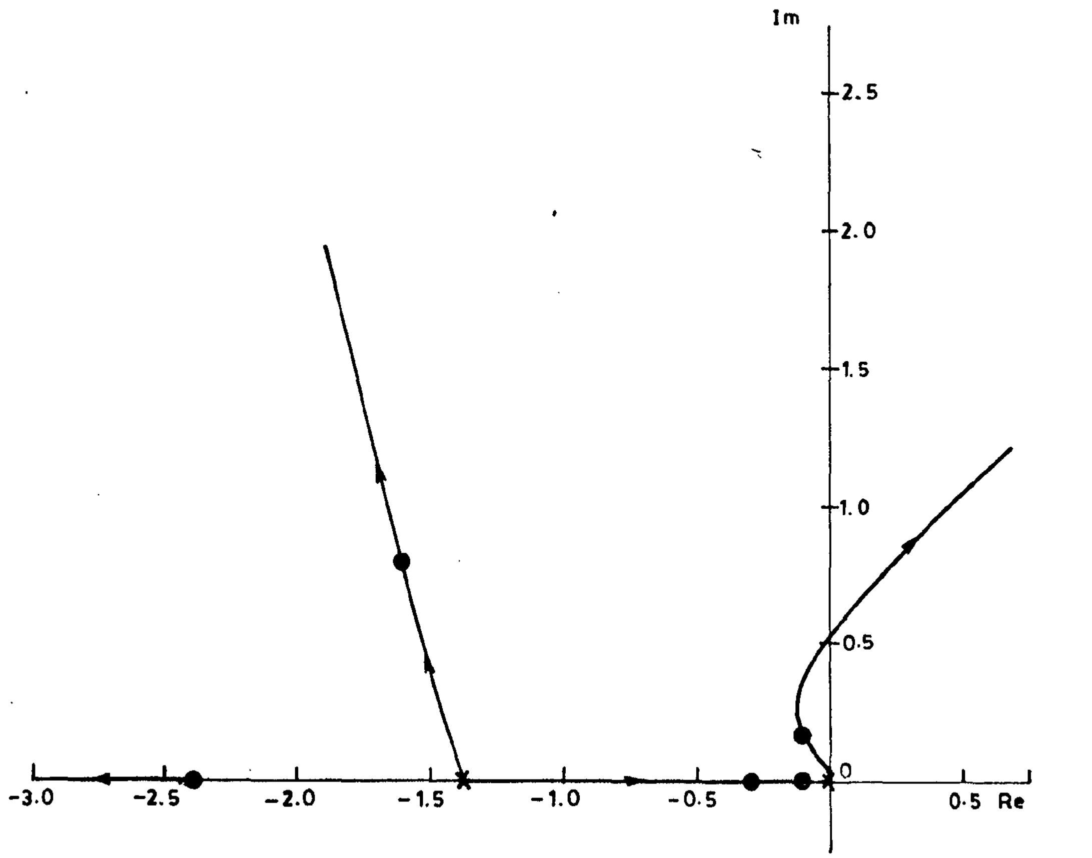


FIG. 5.3 BLIND DIAGRAM OF AIRCRAFT AZIMUTH CHANNEL
BLIND LANDING SYSTEM



System Poles shown are for K_{nom} (or K'_{nom}) = 0.0188 (or 0.9652)

FIG. 5.4 ROOT LOCUS FOR AIRCRAFT BLIND LANDING SYSTEM
WITH FEED-FORWARD GAIN AS VARIABLE

contains only non dominant eigen values resulting in small time constants, thus neglecting the effect of far off poles and zeroes from the dominant poles and zeroes. But now we will take account of the effect of the far off poles and zeroes by equating them to equivalent time delay term [8] $e^{-\tau s}$ with C_r output matrix, where τ is evaluated as

$$\tau = \sum_{i=m}^{i=n} T_i - \sum_{j=r}^{j=q} T_j \qquad ..5.20$$

Where T_i and T_j are time constant associated with discorded poles and zeroes respectively. The equivalent lag, provided the complex poles and zeroes are sufficiently far removed from the dominant poles is simply $2\xi/\omega_n$.

Thus from eq.5.15, If the low order model from marshall method is

$$G(s) = C_r[sI_m - A_r]^{-1}B_r$$

Then the low order model for the marshall method with equivalent lag will be

$$G(s) = e^{-\tau s}\, C_r[sI_m - A_r]^{-1}B_r \qquad ..5.21$$

5.6 TRANSIENT RESPONSE OF AIRCRAFT BLIND LANDING SYSTEM

Inspection of fig.5.4 suggests that the transient response is dominated by three poles one zero near the origin with the remaining three poles, approximately equal to time delay $e^{-\tau s}$.

Table 5.3

Type	System transfer function
Exact	$$\frac{0.097 + 0.965s}{0.097+0.965s+5.143s^2+13.833s^3+13.851s^4+6.110s^5+s^6}$$
Marshall Method	$$\frac{0.126 + 0.1256s}{0.0126 + 0.1076s + 0.5071s^2 + s^3}$$
Marshall Method with Equivalent lag	$$\frac{0.0126 + 0.1081s - 0.1789s^2}{0.0126 + 0.1076s + 0.5071s^2 + s^3}$$

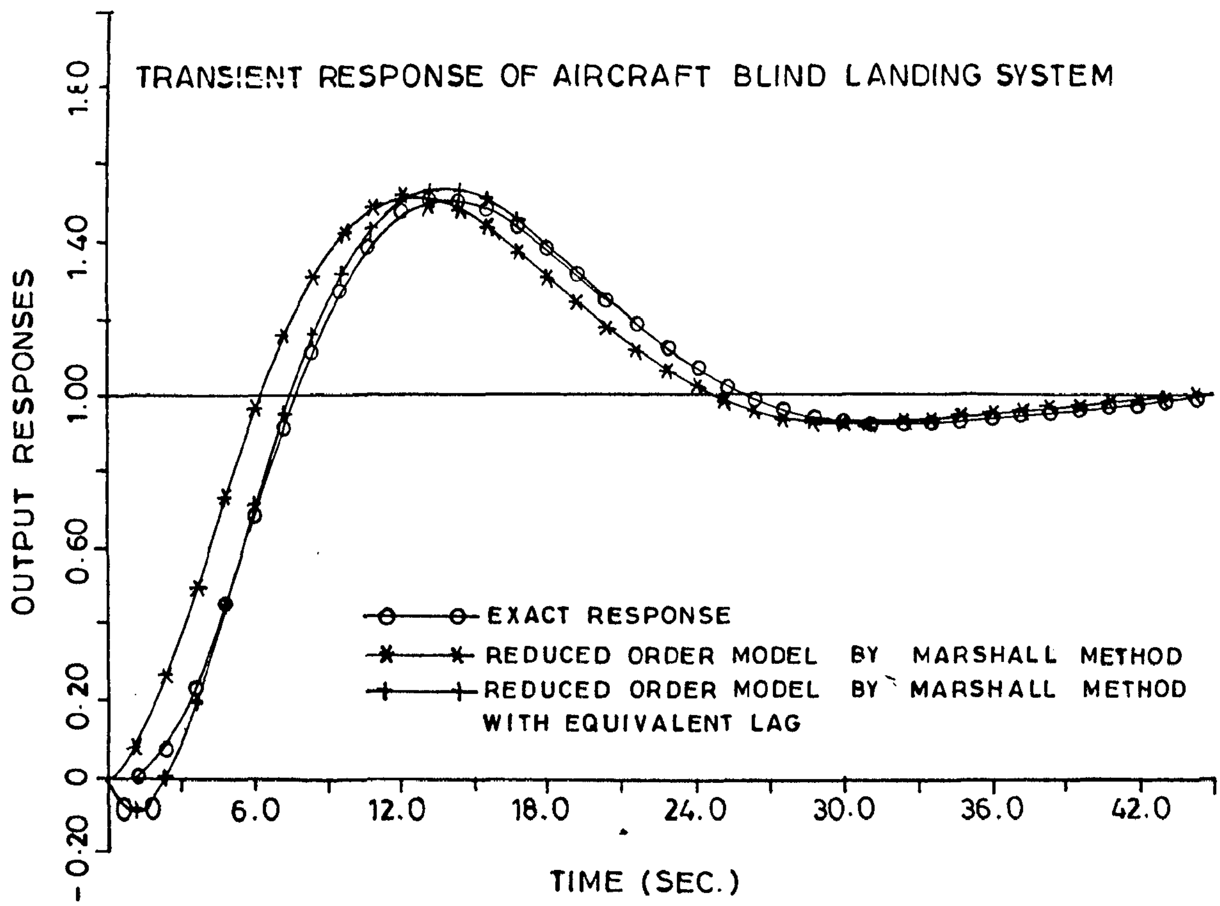


FIG. 5.5 TRANSIENT RESPONSE OF AIRCRAFT BLIND SYSTEM LOW ORDER MODELS

From eq. 5.19, & eq. 5.20, can be evaluated as

$\tau = 0.42 + 2 \times 0.9/1.8 = 1.42$

$$e^{-\tau s} = e^{-1.42s} \cong (1-1.42s) \qquad ..5.22$$

Table 5.3 lists the system transfer function of exact and low order model. Fig.5.5 shows that transient response is best predicted by marshall method with equivalent lag while marshall method is sufficient accurate for realistic prediction of important features such as, overshoot, rise time and settling time.

5.7 CLASSICIAL TRANSIENT RESPONSE SENSITIVITIES

It is shown by Horowitz [4] that the basic small perturbation sensitivity of system transient response to incremental change in scalar plant characteristics is given, in the case of unity feed back system by

$$\frac{\partial u(t)}{\partial p/p} = h(t)*[1-u(t)] \qquad ..5.23$$

Where h(t) is system impulse response, u(t) is system step response, p is generic term for controlled element.

The practical determination of $\frac{\partial u(t)}{\partial p/p}$ can be achieved by simulating

$$\frac{\partial U(s)}{\partial p/p} = [\frac{\theta_o}{\theta_i}(s)][1-\frac{\theta_o}{\theta_i}(s)] \qquad ..5.24$$

$$= [\frac{\theta_o}{\theta_i}(s)]\ [\frac{\theta}{\theta_i}(s)] \qquad ..5.25$$

Where θ_i and θ_o are system command variable and system controlled variable and $\theta = (\theta_i - \theta_o)$. The time solution can be obtained by simulating on digital computer for unit step input.

5.8 USE OF LOW ORDER MODELS FOR SENSITIVITY PREDICTION

Tomovic[15] has suggested the use of classical sensitivity functions, which predict the incremental change in system response for incremental change in system parameters. In general, the use of classical sensitivity functions eq. 5.25 requires the solution of a differential equation of at least twice the order of the system. Thus use of low order models for sensitivity prediction is potentially of even greater practical significance than the use of models for response prediction alone due to the immense saving in computer facilities.

It is proposed to replace the actual system impulse response h(t) and step response u(t) by the responses $u_m(t)$ and $h_m(t)$ of a suitable low order model, such that estimated sensitivity is now

$$\frac{\partial u(t)}{\partial p/p} = h_m(t)*[1-u_m(t)] \qquad ..5.26$$

It is evident from comparison of equations 5.23 and 5.26 that the requirement for adequacy of the model is simply that transient response be satisfactorily matched to the transient response of high order system i.e. $u_m(t) \cong u(t)$.

Table 5.4

Model	Basic Sensitivity Transfer Function
Exact	$\dfrac{(0.097+0.965s)\ (5.143s^2+13.883s^3+13.851s^4+6.110s^5+s^6)}{(0.097+0.965s+5.143s^2+13.833s^3+13.851s^4+6.110s^5+s^6)^2}$
Marshall Method	$\dfrac{(0.0126+0.1256s)\ (-0.018s+0.5071s^2+s^3)}{(0.0126+0.1076s+0.5071s^2+s^3)^2}$
Marshall Method with Equivalent lag	$\dfrac{(0.0126+0.1081s-0.1789s^2)\ (-0.0005s+0.686s^2+s^3)}{(0.0126+0.1076s+0.5071s^2+s^3)^2}$

Table 5.5

Model	Coefficients of basic sensitivity transfer function in ascending order of s	
	Numerator	Denominator
Exact	0.00,0.00,0.496,6.299 14.689,13.959,5.994, 0.965	0.009,0.187,1.929,12.554, 55.835,170.204,345.810, 447.979,371.175,196.925, 65.034,12.22
Marshall Method	0.00,-0.227×10^{-3}, 0.413×10^{-2},0.0763, 0.1256	0.159×15^{-3},0.27×10^{-2},0.0244, 0.1343,0.4724,1.0142
Marshall Method with Equivalent lag	0.00,-6.3×10^{-6}, 0.00859,0.0865, -0.0146,-0.1789	0.159×10^{-3},0.27×10^{-2},0.0244, 0.1343,0.4724,1.0142

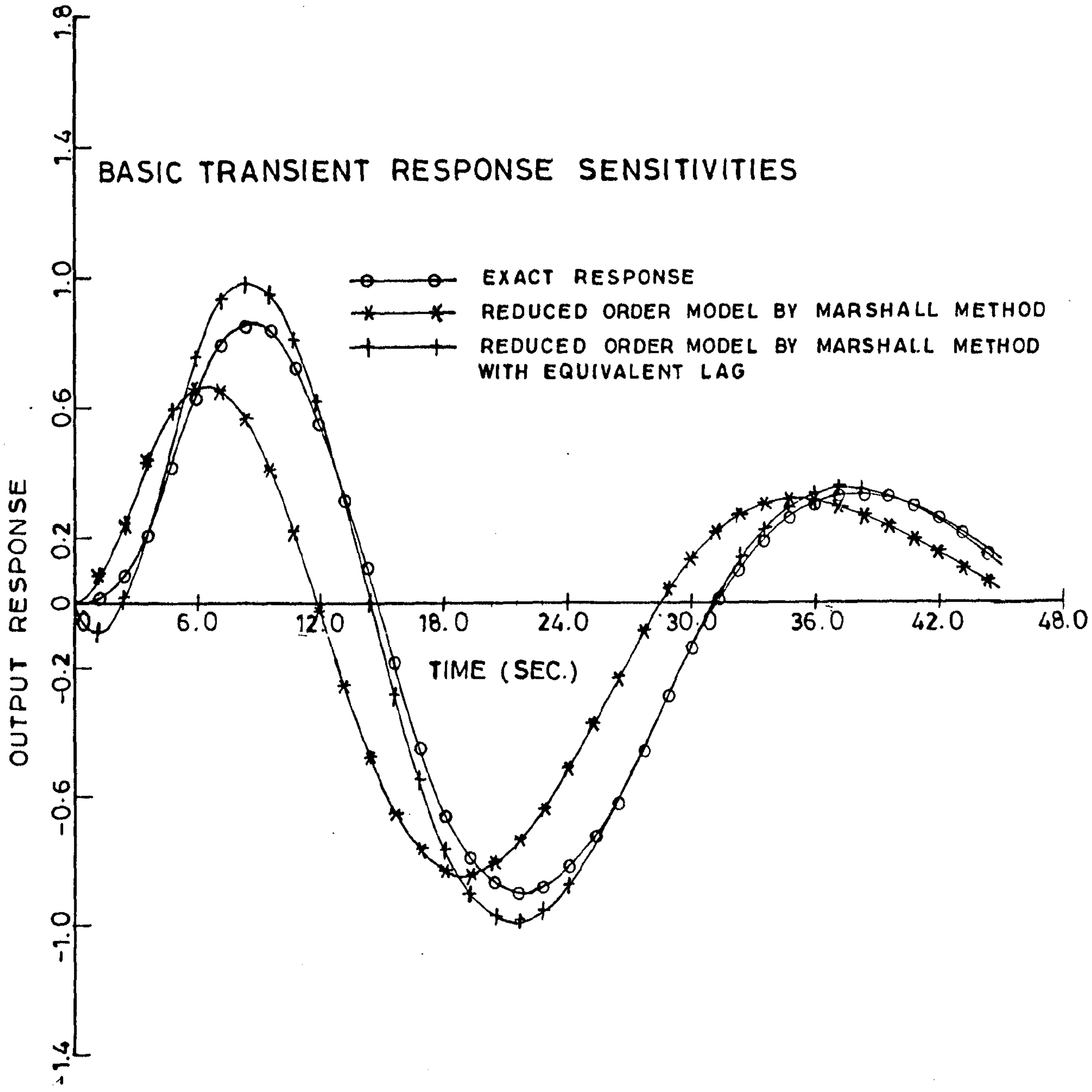


FIG. 5.6 COMPARISON OF BASIC TRANSIENT RESPONSE SENSITIVITIES OBTAINED FROM LOW ORDER MODELS

5.9 SENSITIVITY MODELS FOR AN AIRCRAFT BLIND LANDING SYSTEM

From eq. 5.24, we can easily determine the sensitivity functions associated with various low order model (from table 5.3) which is listed is table 5.4. Numerator and denominator coefficients of Basic Sensitivity transfer function from various model islisted in table 5.5.

From table 5.5, the time solution is obtained for sensitivity transfer function, which is shown in fig.5.6. It is clear from the fig.5.6 that better result is obtained by using the marshall method with equivalent lag than marshall method this is due to the better matching of transient response to exact response from fig.5.5.

chapter :6

conclusion

CHAPTER-6

CONCLUSION

It has been found that synchronization of engines in multi-engine aircraft is achieved in closed loop system with proportional action, proportional plus integral action and proportional plus integral plus derivative action. In proportional action system, the major drawback is the steady state error. This has been eliminated in proportional plus integral action but the settling time and the maximum overshoot has increased. To further reduce the settling time and maximum overshoot while satisfactorly matching the transient as well steady state has been done with proportional plus integral plus derivative action. These scheme can be used in the autopilot because of closed loop speed synchronization. While open loop scheme of synchronization is achieved by the instrument known as synchroscope.

It has been established that low order models adequately predict the response and transient response sensitivity of higher order linear systems, leading to considerable reduction in computation time and complexity. The use of low order models for sensitivity prediction is potentially of even greater practical significance than the use of models for response prediction alone due to the immense savings in computer facilities because a third order model of a tenth order system reduces the basic sensitivity function from twentieth order to sixth order.

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appendix - A

APPENDIX-A

CONSTRUCTION OF ROOT LOCI

Construction of Root Loci

The method is discussed in section 4.2 that how the roots are found for the root locous. The flow chart is shown in Fig. (A.1).

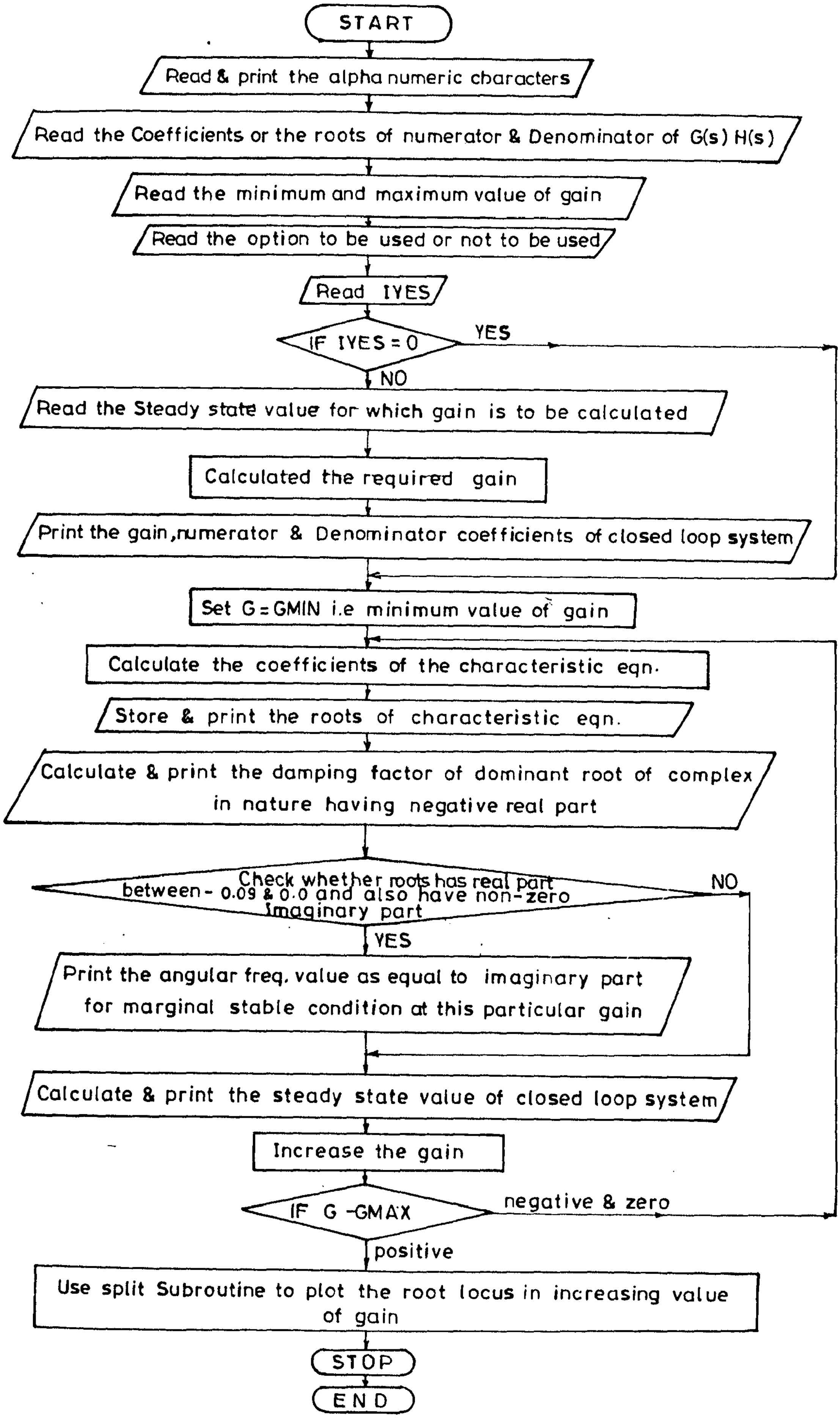

FIG. A.1 FLOW CHART OF ROOT LOCUS PROGRAM

appendix — B

APPENDIX-B

NUMERICAL SOLUTION

B.1 Numerical Integration

State equation may be solved numerically by Runga-Kutta method. The method is illustrated first for a single differential equations and then for simultaneous equations.

A numerical solution of a first order differential equation could be described as a process of finding a series of points on this line. If we substitute known set of values x_n and t_n into the equation

$$\frac{dx}{dt} = f(t,x) \qquad \text{B.1}$$

We in effect get the slope of the curve at t_n.

The gist of this numerical integration method is finding an approximate slope of the function at a known point, and using this approximate slope and a small enough increment of time to proceed to the next point. Then, assuming that the new point is now a known point on the line, one proceeds to repeat the operation of getting an approximate slope and incrementing to the next point.

The Runge-kutta method finds the approximate slope to a higher degree of accuracy instead of using the slope found at the beginning of the interval and assuming it constant for the entire interval, we find approximate values of the function at the beginning, the middle, and the end of the interval in an

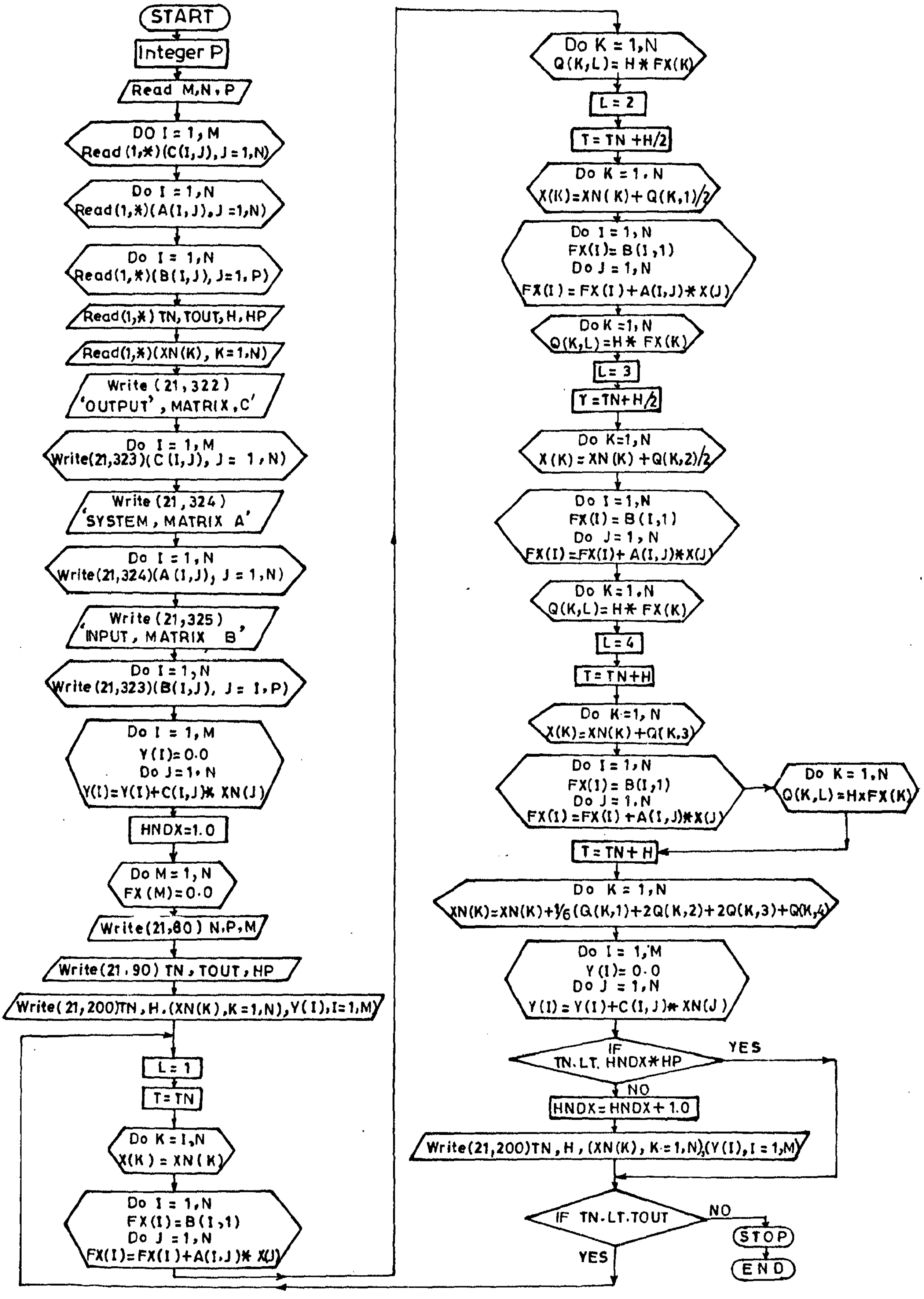

FIG. B.1 FLOW CHART OF RUNGA KUTTA METHOD iterative procedure. The final approximate value of $f(x_{n+1}, t_{n+1})$ is an weighted average of these.

We define terms.

FX is the slope at point.

H is the increment of t.

Q1 is the first trial value for x_{n+1} found by using the initial conditions.

Q2 is the second trial value for x_{n+1} found at the half interval using for a slope a value of FX found by substituting Q1.

Q3 is a third trial value found at the half interval, using for a slope a value of FX found with the aid of Q2.

Q4 is a fourth trial value found at the end of the time interval, using for a slope the value of FX found with the substitution of Q3.

Each Q produced is used to produce the next one.

TN is the initial value of t.

XN is the initial value of x.

H is the increment of t.

FX is the number obtained by substituting into

$$\frac{dx}{dt} = f(t,x) \qquad \text{..B.2}$$

the 'present value' of x and t.

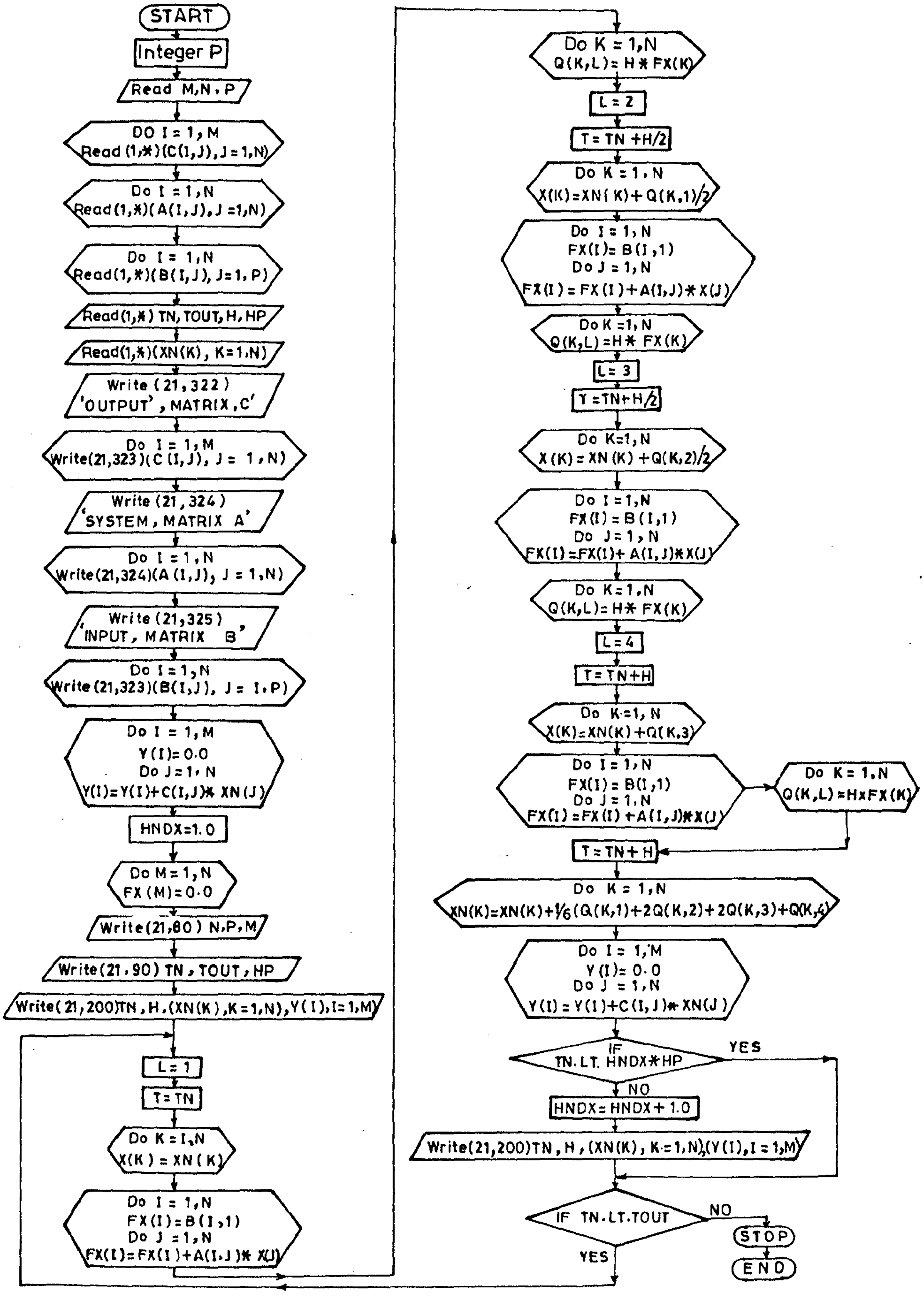

FIG. B.1 FLOW CHART OF RUNGA KUTTA METHOD

The new value x_{n+1} at the end of time interval H is x_n plus a weighted average of the Q's

Calling $\frac{dx}{dt} = FX(t,x)$ ..B.3

We find the Runge-kutta equations are

$$Q1 = H.FX(t_n, x_n) \quad ..B.4$$

$$Q2 = H.FX(t_n+\frac{H}{2}, x_n+\frac{Q1}{2}) \quad ..B.5$$

$$Q3 = H.FX(t_n+\frac{H}{2}, x_n+\frac{Q2}{2}) \quad ..B.6$$

$$Q4 = H.FX(t_n+H, x_n+Q3) \quad ..B.7$$

$$x_{n+1} = x_n+\frac{1}{6}(Q1+2Q2+2Q3+Q4) \quad ..B.8$$

B.2 Simulatenous Equations

For solving simultaneous equations is much the same except that there are sets of Q's like the above for each equation in the set. These Q's must be found in the proper order, first, all the Q1's must be found, then all the Q2's and soon, because all the Q1's are needed to find the first Q2 and all the Q2's are needed to find the first Q3 and soon.

A flow chart for solving two or more state equations is shown in Fig. (B.1).