Switching Transients in Single-phase Operation of Induction Motors

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.....A C K N O W L E D G E M E N T.....

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Roorkee
Aug. 1963.

KRISHNA BIHARI VERMA

....CERTIFICATE....

Certified that the dissertation entitled, "Switching Transient in Single-Phase Operation of Induction motors" which is being submitted by Shri Krishna Bihari Verma in partial fulfilment for the award of the degree of Master of Engineering in Electrical Machine Design of University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of eight months from June, 1962 to January, 1963 for preparing dissertation for Master of Engineering degree at the University.

19/8/63

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Roorkee, U.P.

Dated, August 1963.
Roorkee.

....C O N T E N T S....

.....S_Y_N_O_P_S_I_S.....

The present thesis describes the behaviour of transient currents, voltages and torques in Induction motor at switching.

The work has been devided in to two parts:

i) Switching transients in single-phase Induction motor (Capacitor start).

ii) Switching transients in three phase Induction motor started on single-phase supply.

To predict the transient behaviour of particular motor the transients currents of motor at starting have been calculated and the result verified by actual records of transient currents.

All the values have been normalized so that the results can be utilized for the class of motors which is represented by the motors on which the present study has been made.

The difference between the calculated and experimental results is found to be with in reasonable limits of experimental error.

.....List of Symbols.....

V	Applied voltage per phase
I_m	Main winding currents.
I_a	Auxilliary winding currents.
K	Ratio of number of turns of auxilliary to main winding.
Y_1	Positive sequence admittance.
Y_2	Negative sequence admittance.
Y_s	Standstill admittance.
Y	Standstill admittance of phase converter.
y	Ratio of $\frac{Y_s}{Y}$
$r_{\alpha a}$	Resistance of main winding stator.
$r_{\beta d}$	Direct axis rotor resistance.
x_1	Direct axis leakage reactance.
x_{ma}	Magnetizing reactance of direct axis.
$i_{\beta d}$	Instantenous rotor direct axis current.
$i_{\alpha a}$	Instantenous stator direct axis current.
x_c	Reactance of phase converter.
$r_{\alpha b}$	Resistance of stator auxilliary winding.
x_{mb}	Magnetizing reactance in q-axis.
$r_{\beta q}$	Rotor resistance in q-axis.
$i_{\beta q}$	Rotor q-axis current.
$i_{\alpha b}$	Stator q-axis current.
$i_{\alpha f}$	Forward stator current.
$i_{\alpha b}$	Backward stator current.
$i_{\beta f}$	Forward rotor current.
$i_{\beta b}$	Backward rotor current.
r_c	Phase converter resistance.

x_m Magnetizing reactance for both field at standstill.

$V_f(t)$ Forward component of applied voltage.

$V_b(t)$ Backward component of applied voltage.

$i_c(t)$ Capacitor current.

$V_c(t)$ Capacitor voltage.

s Symbol for Laplace transform.

T_f Forward torque.

T_b Backward torque.

N RPM.

T Torque

T_b Torque at balance operation.

Argument for passive element such as capacitor or Inductor.

ϕ Argument for motor admittance.

$\alpha = \beta - \phi$.

C Capacitor.

E_A Voltage per phase in three-phase.

$K_{\alpha a} = \dfrac{r_{\alpha a}}{x_{\alpha a}}$

$K_{\beta d} = \dfrac{r_{\beta d}}{x_{\beta d}}$

$\sigma = 1 - \dfrac{x_m^2}{x_\alpha \; x_\beta}$

$x_{\alpha a} = x_1 + x_{ma}$

$x_{\alpha b} = x_1' + x_{mb}$

$K_\alpha = r_\alpha / x_\alpha$

$K_\beta = r_\beta / x_\beta$

$x_\alpha = x_1 + x_m$

$x_\beta = x_2 + x_m$.

.....I N T R O D U C T I O N.....

For many years engineers have used the average starting torque and pull out torque of Induction motor as the basis for mechanical design of shafts and couplings, not realizing that high alternating torques exist at the moment of starting.

In 1940 Wahl and Kilgore made an analysis of instantenous transient torque when motor is thrown on the line by sudden closing of the switch, the contacts of which close simultaneously. The fundamental frequency electrical torque which may be several times the pull out torque will be developed for the first few cycles. In 1941 Gilfillan[5] and Kaplan made the transient torque study and found that the transient torque is several times more than the steady state torque and that in design the transient torque must be considered.

In 1944 Maginniss[6] and Schultz studied the practical aspects of the problem such as the electrical and mechanical transient phenomena. They used differential analyzer for solving differential equations. In 1946 Weygandt[7] and Charp also used the differential analyzer for studying the starting characteristics of a two-phase motor. The proto type of the motors investigated was a small two-phase machine used in closed cycle control systems. The performance equations of the Induction motor were solved and summary curves bases upon the conclusion reached were presented which are useful for the designer.

Later on in 1954 P.L. Alger[10] and Y.H.'Ku presented a paper involving the study of transient that occurs when voltage is suddenly applied across the terminals of Induction motors with and without a connected capacitor. They made the study on wound rotor three-phase Induction motor and concluded that transient could be minimized by using high rotor resistance Calculated curves and oscillograms were given for currents and voltages for a 4 pole 3000 h.p. motor using external resistance in the rotor. In the same year, Habberman[10] calculated the transient for the case of single-phase operation of three-phase Induction motor. He confined his study only to theoretical derivation of equivalent circuit and calculation of currents.

In 1957 Venkata Rao[8] published a paper on single-phase operation of a three-phase Induction motor. He calculated the transient currents without taking into account the critical value of capacitor. Again in the year 1959, Venkata Rao published another paper on switching transients of single-phase Induction motor and calculated the currents and voltages with the help of electronic differential analyzer.

In the present work the author has extended the work of Venkata Rao by considering different capacitor values and supported the theoretical findings with experimental results.

The starting of Induction motor on full voltage (usual case with single-phase Induction motor) has been studied in great detail. The initial transient currents in single-phase Induction motor and three phase Induction motor operated on single phase supply have been calculated for different values of capacitor. The critical value of capacitor is determined as suggested in one of the earlier papers.[13] This value of capacitor gives satisfactory starting and also minimum unbalance.

The transient analysis in the present dissertation has been done with three different values of capacitor i.e. one higher and one lower than the critical value. The effect on the magnitude of the transient and dying out time are noted for these three values of capacitor.

The transient starting torque plays an important part in the design of mechanical shaft and coupling for the motor which has to be started and stopped frequently. The torque calculation in both cases with different values of capacitor has been made and variation of torque is plotted. The effect on transient torque with different values of capacitor is noted and results are interpreted wherever necessary.

The transient current and torque in a single-phase Induction motor varies with different points of switching on sinusoidal wave-shape of supply voltage. The transients are most severe when motor goes on line at the instant of zero crossing of the voltage wave-shape. The point on waveshape of supply is controlled experimentally with the help of electronic circuit involving relays and vacuum tubes.

A comparison of actual photographs taken under transient conditions of operation and corresponding theoretical computation of the behaviour as an ideal machine shows that generally the difference is often not more than 10-15%. However, in some cases, the difference is more due to reasons which have been pointed out wherever necessary. With such small difference, the computed results are considered satisfactory indication of actual behaviour.

When the operating conditions of an electric motor are abruptly changed, the currents in the stator and rotor windings are forced to satisfy the Kirchhoff's equations in differential form which apply to the changed conditions. In order to solve these differential equations and thus obtain analytical expressions for the currents during the transient period, it is necessary to assume that the machine has certain ideal characteristics. Although the actual characteristics of the machine of normally good design differ somewhat from the ideal particularly with regard to the effect of magnetic saturation, the agreement that can generally be obtained between the computed and measured values of transient current is usually considered satisfactory. In order to obtain this agreement, however, it is necessary to determine parameter of nearly equivalent ideal machine either from design data or from test data. It is a commonly accepted practice in transient analysis to assume that the machine has ideal characteristics.

PART ONE

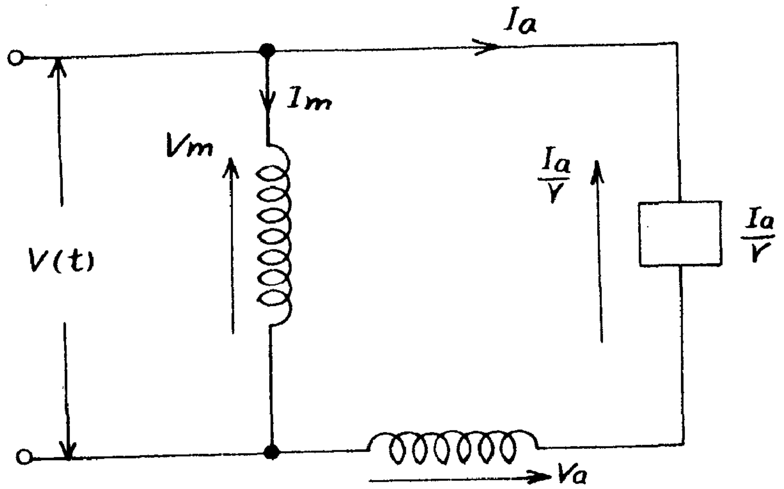
**

.....CHAPTER 1.....

1. Most single phase Induction motors with the exception of shaded pole motor employ a stator having two windings in space quadrature usually with different number of turns. One of the windings called the auxilliary winding is designed to produce either by itself or with the assistance of an external series impedance, a resistance/reactance ratio which differs considerably from that of the main winding. When the two windings are connected in parallel across the single phase supply, the currents flowing in them differ in time phase and hence produce a rotating field. In split phase and capacitor start motors the auxilliary winding is used only for starting and is disconnected once the motor has reached sufficient speed. Single phase motors can in general be considered as special cases of unbalanced operation of asymmetrical two phase motors. The device used to introduce a phase difference in the currents of the windings can be treated as a static phase converter which effectively converts the single phase supply voltage to a 2-phase voltage. The problems of operation of the motor then resolve into the choice of proper converter impedance to give satisfactory starting, run up and full load performance.

The author is interested in switching transient currents, voltages and torque of single phase Induction motor started with the above converter impedance. Therefore for evaluating the transient quantities, the value of capacitor has been determined keeping in view the satisfactory starting, run up performance and the same capacitor value (one lower and one higher also) has been used for the present analysis.

1.1. In Fig. 1.1 m and a represent respectively the main and the auxilliary stator windings of an asymmetrical 2-phase motor connected to a single phase supply with a static phase converter of admittance Y in circuit. The auxilliary winding has K times as many turns as main winding.



From the application of Kirchhoff's Law

$$I = I_m + I_a \qquad \ldots \qquad \ldots \quad 1.1$$

$$V = V_m = V_a + \frac{I_a}{Y} \ldots \qquad \ldots \quad 1.2$$

The equations can be solved by symmetrical component theory as applied to asymmetrical two phase motor.

$$V_m = V_1 + V_2 \qquad \ldots \qquad \ldots \quad 1.3$$

$$V_a = jKV_1 - jKV_2 \quad \ldots \qquad \ldots \quad 1.4$$

$$I_m = V_1Y_1 + V_2Y_2 \quad \ldots \qquad \ldots \quad 1.5$$

$$I_a = j\frac{V_1 Y_1}{K} - j\,\frac{V_2 Y_2}{K} \qquad \ldots \quad 1.6$$

Where V_1 and V_2 are respectively, the positive and negative sequence components of the voltage V_m and Y_1 and Y_2 are the respective values of the input admittance of the main winding to positive and negative sequence currents.

From equations 1.12 and 1.13

$$\frac{T}{T_b} = \frac{Ky \;\; \text{Sin}\,\alpha}{K^4 + y^2 + 2K^2 y \,\text{Cos}\,\alpha}$$

where $y = \left|\frac{Y_a}{Y}\right|$; $\alpha = \beta - \phi$ β and ϕ are the arguments of Y and Y_a respectively.

For balanced operation at starting $T = T_b$

$$K^4 + y^2 + 2K^2 y \,\text{Cos}\,\alpha = Ky \,\text{Sin}\,\alpha$$

For the motor considered here $K = 1.5$ and $\alpha = 150^0$

$$y^2 - 4.689\, y + 5.1 = 0$$

$$y = 2.972 \; ; \; 1.7075$$

$$C = \frac{1}{10.5 \times 2.9725 \times 314} = 102 \;\mu F$$

Also maximum torque per ampere of starting current capacitor value can be computed as follows:

$$x_a - x_c = \frac{1}{r_g}\left(r_g\, r_a - x_g \sqrt{r_a(x_g + r_a)}\right)$$

$$x_c = 9.6 + 20.4$$ (r_g resistance of capacitor)

$$C = 105 \;\mu F.$$

Thus we see that nearly 102 μF condenser gives maximum torque per ampere and also balanced operation. Therefore we will study the transient nature of different quantities at this particular value of capacitor say 100 μF. For the sake of comparison and the effect of capacitance on the various quantities, two

The solution of equations 1.1 and 1.2 are given by

$$V_1 = \frac{V\left[Y_2+K(K-j)Y\right]}{Y_1 + Y_2 + 2K^2Y} \quad \ldots \quad \ldots 1.7$$

$$V_2 = \frac{V\left[Y_1 +K\,(K+j)\,Y\right]}{Y_1 + Y_2 +2K^2\,Y} \quad \ldots \quad \ldots 1.8$$

from which

$$I_m = \frac{V\left[2Y_1Y_2 +K(K-j)YY_1 +K(K+j)YY_2\right]}{Y_1 + Y_2 +2K^2Y} \quad . \ 1.9$$

$$I_a = \frac{V\ \ Y_1Y\,(1+jK)+Y_2Y\,(1-jK)}{Y_1 + Y_2 +2K^2Y} \quad \ldots 1.10$$

and supply current $I = I_m + I_a$

$$I = \frac{V\left[2Y_1Y_2 + (1+K^2)Y\,(Y_1+Y_2)\right]}{Y_1 + Y_2 + 2K^2Y} \quad \ldots 1.11$$

1.2. Starting performance...

At standstill $Y_1 = Y_2 = Y_s$ where Y_s is the normal standstill admittance of the main winding. Substituting $Y_1 = Y_2 = Y_s$ in equations 1.7 and 1.8,

$$V_1 = \frac{V\left[Y_s + K(K-j)Y\right]}{2(Y_s + K^2Y)} \quad \ldots \quad \ldots 1.12$$

$$V_2 = \frac{V\left[Y_s + K(K+j)\,Y\right]}{2(Y_s + K^2Y)} \quad \ldots \quad \ldots 1.13$$

The ratio of starting torque T to the starting torque under balanced two phase operation T_b is given by the expression

$$\frac{T}{T_b} = \frac{|V_1|^2 - |V_2|^2}{|V|^2}$$

other capacitor values have been chosen i.e. one lower and one higher than critical value. These values are 80 μF and 120 μF respectively.

.....CHAPTER 2.....

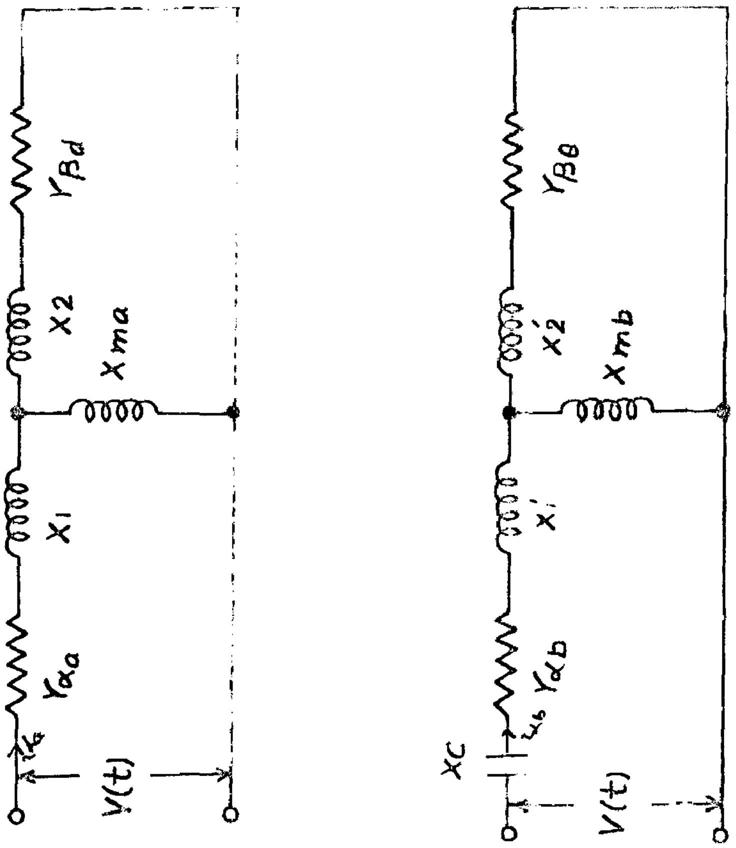
2.1. During switching operations the transient fundamental frequency torque occurs in all Induction motors. When a single phase Induction motor is started on full voltage which is usually done, this torque may be as high as two to three times the pull out torque of the motor. The transient torque plays a very important role when frequent starting and stopping of motor is necessary as in lift. Under such conditions, the mechanical failure of motor may occur if stress on the shaft exceeds the maximum allowable limit.

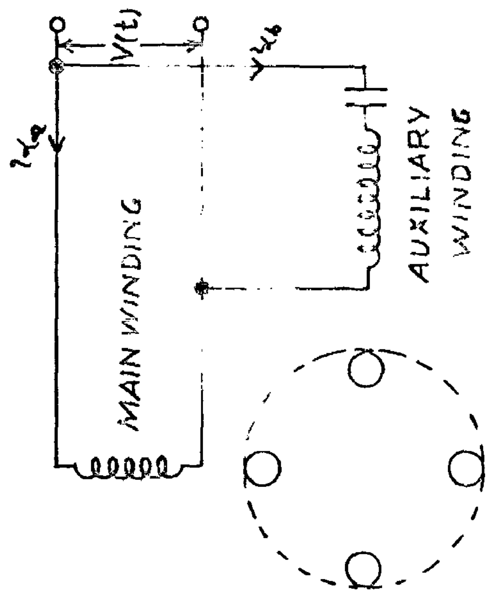
Generally designers use the average steady state starting torque of the motor as a basis for the mechanical design of coupling, shaft etc. not fully realizing that at the time of starting, the actual instantenous torque may be two to three times the average value.

In the case of capacitor-start single phase Induction motor with which the author is concerned, there is a fundamental frequency torque which persists for several cycles, if the switch is closed when the supply voltage passes through its zero value. If, on the otherhand, the switch is closed when supply voltage passes through its maximum value, the torque settles down to its steady state value in less than one cycle.

The following assumptions are made for the analysis of capacitor motor.

1. Both the rotor and stator have symmetrical windings.
2. Rotor is perfectly smooth and self inductances of windings are independent of rotor position.
3. The electrical switching is accomplished in zero time.


X1
X2
Xma
V(t)
XC
X'1
X'2
Xmb
V(t)


V(t)
MAIN WINDING
AUXILIARY
WINDING

4. The effect of saturation, hysteresis and eddy current losses are completely disregarded.

5. In the low frequency transients encountered in Induction motor, the effect of interturn capacitances of the windings may be neglected.

2.2. The general equations for the current in the main winding and auxilliary winding (capacitor motor) are developed in the operational form using the cross field theory. Fig. 2.1 shows the circuit arrangement for a capacitor motor. The general expression derived for capacitor motor can be used even for an ordinary split phase motor by putting $x_c = 0$.

2.2.1. The circuit can be analysed simply by considering the machine at standstill as equivalent to two independent circuits without any coupling between them.

The mesh equations of fig. 2.2A are

$$r_{\alpha a}\, i_{\alpha a} + x_{\alpha a} \frac{di_{\alpha a}}{dt} + x_{ma} \frac{di_{\beta d}}{dt} = V(t) \qquad .. \ 2.1$$

$$x_{ma} \frac{di_{\alpha a}}{dt} + i_{\beta d} \times r_{\beta d} + x_{\beta d} \frac{di_{\beta d}}{dt} = 0 \qquad .. \ 2.2$$

This can be expressed by taking the Laplace transform as

$$(r_{\alpha a} + s x_{\alpha a})\, \bar{I}_{\alpha a} + s x_{ma}\, \bar{I}_{\beta d} = V(s) \quad \ldots \qquad .. \ 2.3$$

$$s x_{ma}\, \bar{I}_{\alpha a} + (r_{\beta d} + s x_{\beta d})\, \bar{I}_{\beta d} = 0 \quad \ldots \qquad .. \ 2.4$$

If the voltage passes at any angle of wave α at the time of closing the switch

$$V(t) = V \sin(\omega t + \alpha)$$

$= V \sin(\omega t + \alpha)$ assuming $\omega = 1$; with this simplification, inductances can be replaced by reactances.

Thus the solution of equation 2.3 and 2.4 gives

$$\bar{I}_{\alpha a} = \frac{V(s)\,K_{\beta d}}{\sigma x_a\,p_1 p_2}\;\frac{1 + as}{(1+s^2)\,(1+T_1 s)(1+T_2 s)} \qquad ..\ 2.5$$

$$\text{where } a = \frac{1}{K_{\beta d}}\;;\quad T_1 = \frac{1}{p_1}\;;\quad T_2 = \frac{1}{p_2}$$

p_1 and p_2 are roots of the characteristic equation.

By taking inverse of Laplace transform

$$i_{\alpha a}(t) = \frac{V\cos\alpha}{\sigma x_{\alpha a}}\left[\sqrt{\frac{1 + K_{\beta d}^2}{(1+p_1^2)(1+p_2^2)}}\;\sin(\theta + t) + \frac{(K_{\beta d} - p_1)e^{-p_1 t}}{(p_2-p_1)(1+p_1^2)} + \frac{(K_{\beta d} - p_2)e^{-p_2 t}}{(p_1-p_2)(1+p_2^2)}\right] + \frac{V\sin\alpha}{\sigma x_{\alpha a}}\left[\sqrt{\frac{1+K_{\beta d}^2}{(1+p_1^2)(1+p_2^2)}}\;\cos(\theta+t) + \frac{(p_1^2 - K_{\beta d}\,p_1)\,e^{-p_1 t}}{(p_2-p_1)(1+p_1^2)} + \frac{(p_2^2 - K_{\beta d})e^{-p_2 t}}{(p_1-p_2)(1+p_2^2)}\right] \quad ..\ 2.6$$

If $\alpha = 0$, the condition corresponds to motor coming on line when voltage passes through its zero value. The current in a highly reactive circuit would be 90° lagging and will be maximum when the voltage passes through zero value. Thus transient current will be more severe for this condition. The author is interested to record and calculate the transients when $\alpha = 0$.

$$i_{\alpha a}(t) = \frac{V\,K_{\beta d}}{\sigma x_{\alpha a}}\left[\sqrt{\frac{1 + K_{\beta d}^2}{(1+p_1^2)(1+p_2^2)}}\;\sin(\theta+t) + \frac{(K_{\beta d} - p_1)\,e^{-p_1 t}}{(p_2-p_1)(1+p_1^2)} + \frac{(K_{\beta d} - p_2)\,e^{-p_2 t}}{(p_1-p_2)(1+p_2^2)}\right] \quad ..\ 2.7$$

$$\tan\theta = \frac{a(p_1p_2-1)-(p_1+p_2)}{a(p_1+p_2)+(p_1p_2-1)}$$

The above equation can be simplified by making the assumptions that for single phase Induction motor $K_{\beta d} << 1$ and $p_2 << p_1$

Thus

$$i_{\alpha a}(t) = \frac{V}{\sigma x_{\alpha a}} \left[\frac{\sin(t-\theta)}{\sqrt{1+p_1^2}} + \frac{e^{-p_1 t}}{\sqrt{1+p_1^2}} - \frac{p_2}{p_1} e^{-p_2 t} \right] \quad ..2.8$$

$$\theta = \tan^{-1}\frac{1}{p_1} + \tan^{-1}\frac{1}{p_2} - \tan^{-1}\frac{1}{K_{\beta d}}$$

After substituting the motor constants in equation 2.8, we get

$$i_{\alpha a}(t) = 18.6 \left[.632 \left\{ \sin(t-44.5) + e^{-1.214t} \right\} - 0.01313 e^{-.016t} \right] \quad 2.9$$

The maximum value of current comes out to nearly 10Amp. (peak). The steady state value recorded by an ammeter placed in series is 3.20 Amp. Thus the ratio between transient current and steady state current is $\frac{10}{3.20} = 3.13$.

The ratio of transient current peak and steady state current peak value from recorded photo (No. 1)is coming to nearly 3.8.

Thus the results obtained by two methods are within practical limits. The variation is slightly more due to the following reasons:-

1. The angle of switching was not exactly zero degree as it is not possible due to different time delay of relays connected in the circuit.

2. Load on motor was put by mechanical belt and spring

and which was not possible to keep constant during full operation due to increased friction of belt and pulley.

3. The motor constants have been determined experimentally which are also slightly different from design datas.

The mesh equations of fig.2.2B are-

$$r_{\alpha b}\, i_{\alpha b} + x_{\alpha b}\frac{di_{\alpha b}}{dt} + x_c \int_0^t i_{\alpha b}\,dt + x_m \frac{di_{\beta q}}{dt} = V(t) \qquad \ldots\ 2.10$$

$$x_{mb}\frac{di_{\alpha b}}{dt} + r_{\beta q}\, i_{\beta q} + x_{\beta q}\frac{di_{\beta q}}{dt} = 0 \qquad \ldots\ \ldots\ 2.11$$

Taking Laplace transform on both sides, the following equation is obtained,

$$\bar{i}_{\alpha b}(s) = \frac{V\, K_{\beta q}\, s\left(1 + \frac{s}{K_{\beta q}}\right)}{\sigma x_{\alpha b}\left[(1+s^2)\left\{s^3 + \frac{s^2 (K_{\alpha b} + K_{\beta q})}{\sigma} + \frac{s(K_{\alpha b}K_{\beta q} + \frac{x_c}{x})}{\sigma}\right\} + \frac{x_c\, K_{\beta q}}{x_{\alpha b}}\right]} \qquad \ldots\ \ldots\ \ldots\ \ldots\ 2.12$$

2.3.1. <u>For 100 μF:</u>

After taking inverse of Laplace transform and solving this equation; we get*

$$i_{\alpha b}(t) = 0.408\left[-7.76e^{-0.6865t}\,Cos(1.975t - 26.2) + 7.15 Cos(t-25.1) - 0.00159e^{-0.0266t}\right]$$

Simplifying further

$$i_{\alpha b}(t) = 0.408\left[7.15\,Cos(t-25.1) - 7.76\,Cos(1.975t - 26.2)e^{-0.6565t} - 0.00159e^{-0.0266t}\right] \qquad \ldots\ \ldots\ 2.13$$

The value of current at $t = 25.1^{\circ}$ (because at this instant the fundamental is maximum) is nearly 0.735A. The current after 18 cycles attains the value 2.92A due to decaying of second harmonic component of current. The centrifugal switch also opens at this instant and the current dies out completely after this time. Thus the current in auxilliary winding flows for a very small interval of time and does not effect the motor Performance much, ~~whatever its value may be,~~ ~~performance much whatever its value may be.~~

*For detailed calculation please see the appendices.

2.3.2. For 80 μF:

Auxilliary current is given by

$$i_{\alpha b}(t) = 0.408 \left[6.4 \,\text{Cos}(t-19.2) - 6.66 e^{-0.714t} \,\text{Cos}(2.18t-26) - 0.00105 e^{-0.0282t}\right] \quad \dots 2.14$$

The initial value of current at t = 19.2° is nearly 0.55A and after 12 cycles it becomes 2.62 A. At this instant centrifugal switch opens and auxilliary current completely dies out. At starting, condenser offers impedance and takes some time to change it. As condenser changes, the current increases in the auxilliary winding. This phenomena is prominent when capacitive reactance is higher. Thus with 80 μF the initial current is only 0.55A, while with 100 μF it is 0.735A.

2.3.3. For 120 μF:

Auxilliary current for 120 μF condenser would be

$$i_{\alpha b}(t) = 0.408 \left[9.16 \,\text{Cos}(t-27.2) - 9.3 e^{-0.712t} \,\text{Cos}(1.83t-29.3) - 0.00175 e^{-0.0258t}\right] \quad \dots 2.15$$

The initial value of current at t = 27.2° (when fundamental component is maximum) is nearly 1.27 A and value of current after $14\frac{1}{2}$ cycle is nearly 3.77A.

TABLE 2.1

Comparative statement showing various current components for different values of capacitors.

Capacitor	Fundamental frequency (component)	higher frequency oscillatory term	d.c. term decaying exponentially	initial current when fundamental is maximum	Frequency of higher frequency oscillatory term	Steady state current in A. (when CF switch opens)
80	6.4	6.66	0.00105	0.55	109.0	2.44
100	7.15	7.76	0.00159	0.735	97.75	2.92
120	9.16	9.3	0.00175	1.27	91.5	3.74

From the table 2.1 we can easily see the variation of auxilliary current with different capacitor values. As the capacitance increases the reactance decreases and current will go on increasing. The frequency of oscillatory term is decreasing with increased value of capacitor.

In capacitor start motor the auxilliary winding current will vanish as the centrifugal switch opens. The time taken by rotor to attain nearly 70% of the synchronous speed is comming out nearly 15 cycles i.e. 0.3 sec. This time varies with different capacitor values.

Value of capacitor		Time of operation of centrifugal switch (in cycles)
80	..	12
100	..	18
120	..	14 1/2

As the current flows in this winding only for a short duration,. ~~the transient does not play any prominent part.~~

2.4. Line Current:

The displacement between two currents is nearly 90° in time for balanced operation if the value of capacitor is suitably chosen as discussed in Chapter 1. The line current will be the vector sum of two transient currents. The main winding current is independent of capacitor value as is very clear from equivalent circuit derived in article 2.1. The steady and transient line currents will be given by vectorial sum of the two currents.

TABLE 2.2 A

Comparative statement of different currents at different capacitor values showing transient and steady state conditions

(Transient state)

Capacitor	I_a	I_m	I_L
80	0.55	10	10.009
100	0.735	10	10.01
120	1.27	10	10.05

Steady state

TABLE 2.2 B

(Aux. winding just opens)

Capacitor	I_a	I_m	I_L	Recorded value of Ammeter in line
80	2.44	3	3.86	3.7
100	2.92	3	4.19	4.0
120	3.74	3	4.8	4.2

The practically observed values of line currents by placing an ammeter in line are quite comparable with the calculated values. The error between the two is within the practical limits.

.....CHAPTER 3.....

The sudden switching of an a.c. voltage on to a highly reactive circuit of the type discussed here will produce transient asymmetrical fluxes and currents, the magnitudes of which will depend on the actual value of voltage wave at which the switch is closed. The fundamental frequency transient torque results from the reaction of the asymmetrical flux on one axis with the alternating current in other. The asymmetrical flux and the associated fundamental frequency torque decay slowly, since these depend almost entirely on the value of magnetizing reactance. The transient torque arising from the asymmetrical current decays very rapidly, since it depends on the leakage reactances.

In this chapter the author discusses the variation of transient torques with different capacitor values and also the ratios of transient torque to steady state torque value keeping the angle of switching constant.

3.1. The developed electrical torque according to cross-field theory is given by

$$T' \propto \left(i_{\alpha b} \; i_{\beta d} - i_{\alpha a} \; i_{\beta q} \right)$$ [3] reference

Similarly the expression for the instantenous torque in terms of rotor currents is

$$T \propto \left(i_{\beta d} \int_0^t i_{\beta q} \, dt - i_{\beta q} \int_0^t i_{\beta d} \, dt \right) \qquad \text{.. 3.1}$$

$$i_d(t)^* = V \left[-0.0584 e^{-1.426t} + 0.00404 e^{-0.0325t} - -0.1 \, \text{Cos} \, (t - 123.5) \right] \qquad \text{... 3.2}$$

---

*For detailed calculation, please see appendics.

$$i_{\beta q}(t) = V\left[0.0282e^{-0.713t}\,Cos(1.99t-28.3)-0.0268 \times Cos(t-20.8)\right] \qquad ..3.3$$

For 100 μF:-

$$\int_0^t i_{\beta q}\,dt = V\left[0.01335e^{-0.713t}\,Cos(1.99t-138) - -0.0268\,Sin\,(t-20.8)+0.0005\right] \qquad ..3.4$$

$$\int_0^t i_{\beta d}\,dt = V\left[0.041e^{-1.426t} - 0.1245e^{-0.0325t} - -0.1\,Sin\,(t-123.5) + 0.0012\right] \qquad ..3.5$$

The electrical torque developed

$$T \propto \left[i_{\beta d}\int_0^t i_{\beta q}\,dt - i_{\beta q}\int_0^t i_{\beta d}\,dt\right] \qquad ..3.6$$

After substituting the currents in the equation 3.6 and solving we get,

$$T = KV^2 \times 10^{-4}\left[26.2-10.8e^{-2.139t}\,Cos(1.99t-67.8) + +35.6e^{-0.7455t}\,Cos(1.99t-28.2)+19.1e^{-1.426t} \times Cos(t-25.8) - 33.4e^{-0.0325t}\,Cos(t-25.8) -0.292e^{-1.426t} + 0.0202e^{-0.0325t} - 0.523\,Cos(t-87.2)+0.338e^{-0.713t} \times Cos(1.99t-28.3) - e^{-0.713t}\left\{13.35\,Cos \times (1.99t-138.5)\,Cos(t-123.5)- -28.2\,Cos(1.99t-28.4)\,Sin \times (t-123.5)\right\}\right] \qquad ...3.7$$

Neglecting small terms the average value will be given by

$$T_{av} = K' \, 36.36 \, V^2 \quad \ldots \qquad \ldots 3.8$$

where K' is constant.

3.2 For capacitor 80 μF:

$$i_{\beta q} = V\left[0.0236e^{-0.714t}\cos(2.18t-24.2) - 0.0224\cos(t-17)\right] \qquad \ldots 3.9$$

and

$$\int_0^t i_{\beta q}\,dt = V\left[0.0104e^{-0.714t}\cos(2.18t-132.2) - 0.0224\sin(t-17) + 0.00071\right] \qquad \ldots 3.10$$

The torque will be given by

$$T = KV^2 \times 10^{-4}\Big[21.4 - 9.6e^{-2.14t}\cos(2.18t-61) + 29.2e^{-0.7465t} \times \cos(2.18t-25) + 15.9e^{-1.426t}\cos(t-72)$$
$$-27.8e^{-0.0325t}\cos(t-19) - 0.415e^{-1.426t}$$
$$+0.0286e^{-0.0325t} - 0.83\cos(t+38)$$
$$+0.2825e^{-0.714t}\cos(2.18t-24.2)$$
$$-e^{-0.714}\Big\{10.4\cos(2.18t-132)\cos(t-123.5) -$$
$$-23.6\cos(2.18t-24.2)\sin(t-123.5)\Big\}\Big] \qquad \ldots 3.11$$

Neglecting very small terms the average value will be given by

$$T_{av} = 34.49\,K'V^2 \quad \text{where } K' \text{ is constant} \qquad \ldots 3.12$$

3.3 For 120 μF:-

$$i_{\beta q}(t) = V\left[0.0324e^{-0.712t}\cos(1.83t-29.1) - 0.0316\cos(t-25.8)\right] \qquad \ldots 3.13$$

$$\int_0^t i_{\beta q}\, dt = V\left[0.0165 e^{-0.712t} \text{Cos}(1.83t-29.1) - 0.0316 \times \text{Sin}(t-25.8) - 0.001\right] \quad ..3.14$$

The torque can be calculated in the same way as per above values,

$$T = KV^2 \times 10^{-4}\Big[31.4 - 13.2e^{-2.138t} \text{Cos}\,(1.83t-72) + 40.4e^{-0.7445t} \times \text{Cos}(1.83t-29.1) - 39.4e^{-0.0375t} \text{Cos}(1.83t-29.1) - 39.4e^{-0.0375t} \text{Cos}(t-28.8) + 22.6e^{-1.426t} \text{Cos}(t-80.8) - 1.13\,\text{Cos}(t+36.2) - 0.584e^{-1.426t} - 0.404e^{-0.0325t} - e^{-0.712t}\Big\{16.5\,\text{Cos}(1.83t-140.4)\text{Cos}(t-123.5) + 32.4\,\text{Cos}(1.83t-29.1)\,\text{Sin}(t-123.5)\Big\}\Big] \quad ..3.15$$

The average value of torque will be given by

$$T_{av} = 49\ K'V^2 \quad ... \quad ..3.16$$

The relative variation of Transient torque and steady state torque has been shown in the table below:

TABLE 3.1

Comparative statement of transient torque and steady torque for different capacitor values

μF	Starting transient torque at t=0 x KV^2x 10^{-4}	T steady x KV^2 x 10^{-4}	Ratio Transient/steady	Av.transient torque x K'V^2
80	43.6	21.4	2.08	34.49
100	40.3	26.2	1.53	36.36
120	66.6	31.4	2.12	49.0

The average transient torque is responsible for turning the rotor from standstill position. Although the duration of transient torque is small but it gives rise an energy to shaft just as an impulsive force. The magnitude of torque depends on the average value of the transient torque. The average transient torque increases with increased value of capacitor as can be seen from the above table.

An important point is to be noted that ratio of transient torque at starting to steady torque is minimum in case of critical value of capacitor. Thus affect of transient is also minimum in case of balance operation and thus capacitor value chosen for balance operation also gives minimum transient behaviour.

3.4. Starting torque variation with capacitor:

The starting quality can be improved by properly selecting the value of capacitor as discussed in chapter one. The critical value which gives perfect balance operation has been used in computing the transient and steady state torques. For comparison of average transient torques two other values of capacitor have also been taken into consideration. The variation of starting torque with different values of capacitor is studied experimentally in a slightly indirect way.

A tachogenerator is connected with the shaft of Induction motor as shown in attached photograph. The output of tachogenerator depends on the speed of shaft because voltage generated is proportional to speed if other things are constant.

$$E \propto KN$$

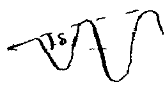
The output voltage of tachogenerator is sinusoidal if the motor is running at constant speed. During starting the shaft takes some time to come up to speed or in other words one can say that speed rises from zero to normal value at that particular load. The output voltage waveshape varies in shape and the same has been recorded by keeping voltage reference signal of motor as base. The frequency and amplitude of tachogenerator voltage varies till the motor comes up to rated speed. This variation depends on the value of capacitor chosen. The differentiation of this voltage or speed response shows the acceleration of the motor

$$\text{Acc.} \propto \frac{dN}{dt}$$

and Torque = I x Acc.

where I = moment of Inertia

$$\frac{dN}{dt} = \tan \delta$$



$$T = K \tan \delta$$

where K is constant

The variation of angle   and starting torque with different values of capacitor is tabulated below:

TABLE 3.2

The variation of angle δ with different values of capacitor

μF	δ in degrees	tan δ	$T = K_1 \tan \delta$	$T_{cal.}$(average)
80	8.0	0.140	$T = K_1 \times 0.140$	T=K' x34.49.
100	8.2	0.144	$T = K_1 \times 0.144$	T=K' x36.36
120	9.0	0.158	$T = K_1 \times 0.158$	T=K' x49.0

TABLE 3.3

The relative variation of starting torque with different values of capacitor

Capacitor	Theoretical Torque ratio	Experimental Torque ratio
$\frac{80\ \mu F}{100\ \mu F}$	0.95	0.975
$\frac{120\ \mu F}{100\ \mu F}$	1.3	1.13

There is a slight variation between two ratios that might be due to the following reasons:

1. Back-lash error in the coupling of Induction motor and tachogenerator.

2. Tachogenerator response was not very quick due to crude design of this tachogenerator.

3. As this is the indirect method of torque measurement, one can not easily determine the correct ratio. However, an approximate idea can be had of the starting torque ratio with different values of capacitor.

PART TWO

**

.....CHAPTER 1.....

1.0. Starting of Three phase Induction motor connected to one-phase supply system...

When a polyphase Induction motor is connected to a single phase supply system, no starting torque is developed. In order to produce starting torque, a phase converter is needed. The selection of phase converter depends on satisfactory starting, run up and full load performance. The author is interested only in starting transients in the motor for the above mentioned phase converter value. First, the value of phase converter for the specified motor has been determined for the best starting quality and the study of transients have been made for the same value of phase converter. It has been possible to study the relative variation in transients with different values of phase converter during the analysis.

1.1. Fig. 1.1. represents the primary windings of a star connected 3-phase Induction motor connected to a single phase

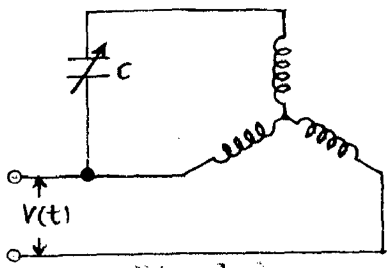


Fig.1.1
Arrangement of stator windings [illegible]

supply with an external static phase converter of admittance Y in the circuit. Inspection equations for this circuit obtained by the application of Kirchhoff's Laws are as follows:

$$I_A + I_B + I_C = 0 \qquad \ldots \qquad \ldots 1.1$$

$$V - V_A + V_B = 0 \qquad \ldots \qquad \ldots 1.2$$

$$V_A - V_C - I_C/Y = 0 \qquad \ldots \qquad \ldots 1.3$$

These equations can be solved by symmetrical component theory, by utilizing the substitution,

$$\begin{aligned} V_A &= V_o + V_1 + V_2 \\ V_B &= V_o + a^2V_1 + aV_2 \\ V_C &= V_o + aV_1 + a^2V_2 \end{aligned} \qquad \ldots \qquad \ldots 1.4$$

and

$$\begin{aligned} I_A &= V_oY_o + V_1Y_1 + V_2Y_2 \\ I_B &= V_oY_o + a^2V_1Y_1 + aV_2Y_2 \\ I_C &= V_oY_o + aV_1Y_1 + a^2V_2Y_2 \end{aligned} \qquad \ldots \qquad \ldots 1.5$$

where V_o, V_1, V_2 are zero, positive and negative sequence components of V_A respectively and Y_o, Y_1 and Y_2 are the respective values of admittances per phase of machine to zero, positive and negative sequence currents.

The solution of V_1, V_2 and V_o are given by

$$V_o = 0 \qquad \ldots \qquad \ldots \qquad \ldots 1.6$$

$$V_1 = \frac{V e^{-j30}}{\sqrt{3}} \left[\frac{\sqrt{3}\,Y\, e^{-j30} + Y_2}{3Y + Y_1 + Y_2} \right] \qquad \ldots \qquad \ldots 1.7$$

$$V_2 = \frac{V e^{j30}}{\sqrt{3}} \left[\frac{\sqrt{3}\,Y\, e^{j30} + Y_1}{3Y + Y_1 + Y_2} \right] \qquad \ldots \qquad \ldots 1.8$$

1.2. <u>Starting performance</u>...

At standstill $Y_1 = Y_2 = Y_s$

Thus $$V_1 = \frac{V e^{-j30}}{\sqrt{3}} \left[\frac{\sqrt{3}\,Y\, e^{-j30} + Y_s}{3Y + 2\,Y_s} \right] \qquad \ldots 1.9$$

$$V_2 = \frac{V e^{j30}}{\sqrt{3}} \left[\frac{\sqrt{3}\, Y e^{j30} + Y_s}{3Y + 2Y_s} \right] \quad \ldots \quad \ldots 1.10$$

The ratio of the starting torque T to the starting torque under balanced three phase operation T_b is given by the expression

$$\frac{T}{T_b} = \frac{|V_1|^2 - |V_2|^2}{\frac{1}{3} |V_1|^2} \quad \ldots \quad \ldots \quad \ldots 1.11$$

By substituting the values of V_1 and V_2 in expression 1.11,

$$\frac{T}{T_b} = \frac{2\sqrt{3}\, y \ \operatorname{Sin}\alpha}{9 + 4y^2 + 12y \operatorname{Cos}\alpha} \quad \ldots \quad \ldots \quad \ldots 1.12$$

where $\alpha = \beta - \phi$

The equation 1.12 is general expression and can be applied to any Induction motor. In practice, the angle ϕ is always negative and for normal Induction motor lies between 20 and 70°, for capacitor or inductor the angle is +90 and -90 respectively. Therefore the practical limits of the parameter α are 160° and -70°.

1.3. Unbalance factor:-

The unbalance factor is defined as the ratio negative sequence current to the positive sequence current $\left|\frac{I_2}{I_1}\right|$ which is equal to $\left|\frac{V_2 Y_2}{V_1 Y_1}\right|$.

Unbalance factor at standstill[12] will be given by

$$U = \left[\frac{3 + y^2 + 2\sqrt{3}\, y \operatorname{Cos}(\alpha + 30)}{3 + y^2 + 2\sqrt{3}\, y \operatorname{Cos}(\alpha - 30)} \right]^{1/2} \quad .. 1.13$$

For fixed value of α the unbalance factor will be minimum when $y = \sqrt{3}$

Therefore for keeping unbalance as low as possible the best form of phase converter for most of the normal Induction motor is a pure capacitor. Best performance is obtained for $y = \sqrt{3}$ and α very nearly 150°.

For the motor on which the experiment has been performed, $\alpha = 145^\circ$.

(i) With $y = \sqrt{3}$

$$\left|\frac{Y_a}{Y}\right| = y$$

$$\omega c = \left|\frac{Y_a}{y}\right|$$

$$c = \frac{1}{10.2/3 \times 314} \qquad \text{(} Y_a \text{ refer experimentation detail)}$$

$$c = 170\ \mu F$$

(ii) $\left|\frac{Y_a}{Y}\right| = y = 1.5$

$$c = \frac{10^6}{10.2 \times 1.5 \times 314} = 197\ \mu F \text{ (Say } 200\ \mu F)$$

(iii) With $y = 2$

$$c = \frac{10^6}{10.2 \times 2 \times 314} = 147\ \mu F \text{ (Say } 150\ \mu F)$$

Thus the analysis of motor for transient purpose has been made at three different capacitor values keeping in view balanced operation for $y = \sqrt{3}$.

*

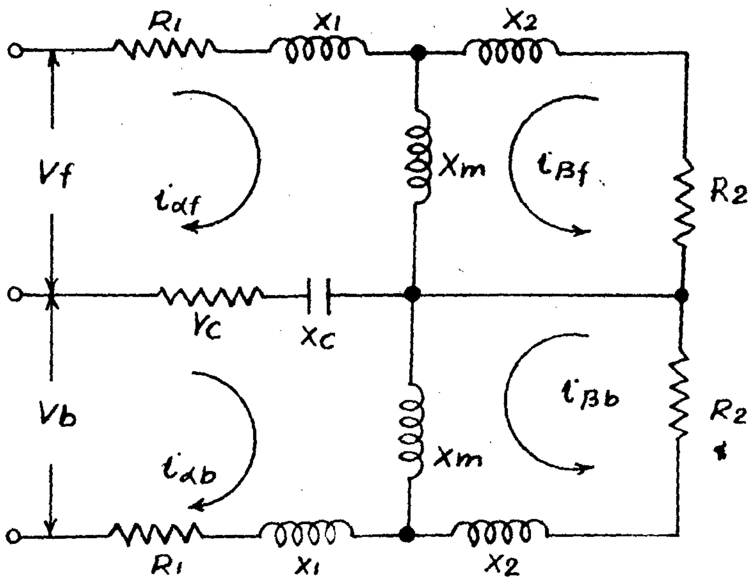
....CHAPTER 2.....

2.1. Single-phase operation of three phase Induction motors has been a subject of very great interest in the past to both the designers and operating engineers. We often come across the problem when it becomes necessary to run a three-phase motor from single-phase supply. In this case motor will develop lesser pull out torque and poor efficiency. The author's aim is to study and record the transient currents in three phase Induction motor running on single-phase supply with the help of a capacitor. The value of capacitor has been calculated in the previous chapter.

The section deals with the calculation of transient currents and determination of peak value of transient current in windings as well as steady state values. The actual records of currents of different windings have been obtained for comparision with the theoretical results.

2.2. A variable capacitor with a motor having star connected stator winding is shown in fig. No.2.1. The capacitor value can be changed according to predetermined value at starting for giving perfect balance operation. After starting, the capacitor can be removed from circuit by opening a single pole switch placed in circuit to reduce the heating of motor.

The equivalent circuit for this condition can be drawn as follows on the basis of double revolving field theory



[illegible]
Equivalent circuit [illegible]
Induction motor [illegible]
phase supply.

During transient state the differential equations can be written as

$$R_1\, i_{\alpha f} + x_\alpha \frac{di_{\alpha f}}{dt} + x_m \frac{di_{\beta f}}{dt} + r_c\,(i_{\alpha f} - i_{\alpha b}) + x_c \int_0^t (i_{\alpha f} - i_{\alpha b})\,dt = V_f(t) \quad \ldots\ 2.1$$

$$R_1 i_{\alpha b} + x_\alpha \frac{di_{\alpha b}}{dt} + x_m \frac{di_{\beta b}}{dt} + r_c(i_{\alpha b} - i_{\alpha f}) + x_c \int_0^t (i_{\alpha b} - i_{\alpha f})\,dt = V_b(t) \quad \ldots\ 2.2$$

$$x_m \frac{di_{\alpha f}}{dt} + R_2 i_{\beta f} + x_\beta \frac{di_{\beta f}}{dt} = 0 \quad \ldots \quad \ldots\ 2.3$$

$$x_m \frac{di_{\alpha b}}{dt} + R_2 \frac{i_{\alpha b}}{\sigma} + x_\beta \frac{di_{\beta b}}{dt} = 0 \quad \ldots \quad \ldots\ 2.4$$

By taking Laplace transform and rearranging suitably

$$\bar{i}_{\alpha f} \left\{ R_1 + r_c + sx + \frac{x_c}{s} \right\} + \bar{i}_{\alpha b} \left\{ - r_c - \frac{x_c}{s} \right\} + \bar{i}_{\beta f}\, s x_m + \bar{i}_{\beta b} \times 0 = V_f(s) \quad \ldots \quad \ldots\ 2.5$$

$$\bar{I}_{\alpha f}\left\{-r_c - \frac{x_c}{s}\right\} + \bar{I}_{\alpha b}\left\{R_1 + r_c + sx + \frac{x_c}{s}\right\} + \bar{I}_{\beta b} s x_m + i_{\beta b} \times 0$$

$$= V_b(s) \qquad \ldots \quad 2.6$$

$$\bar{I}_{\alpha f}\, s x_m + \bar{I}_{\alpha b} \times 0 + \bar{I}_{\beta f}\,(R_2 + sx) + \bar{I}_{\beta b} \times 0 = 0 \qquad \ldots \quad 2.7$$

$$\bar{I}_{\alpha f} \times 0 + \bar{I}_{\alpha b} \times s x_m + \bar{I}_{\beta f} \times 0 + i_{\beta b}\,(R_2 + sx) = 0 \ \ldots \quad 2.8$$

In the above equations the independent variable t(time) is in radians and not in seconds. By doing so the inductances are replaced by reactances of machine.

The equations 25 and 2.6 can be rewritten in the following way by making the simplyfing substitution.

$$A = R_1 + r_c + sx + \frac{x_c}{s}$$

$$B = -\left(r_c + \frac{x_c}{s}\right)$$

$$C = s x_m$$

$$D = R_2 + sx$$

$$\bar{I}_{\alpha f} = \frac{V_f\,(AD^2 - C^2 D) - V_b \times BD^2}{\text{Det.}\,\Delta} \qquad \ldots \quad \ldots \ 2.9$$

where

$$\text{Det.}\,\Delta = \begin{vmatrix} A & B & C & 0 \\ B & A & 0 & C \\ C & 0 & D & 0 \\ 0 & C & 0 & D \end{vmatrix}$$

$$\bar{I}_{\alpha b} = \frac{-V_f\, BD^2 + V_b\,(AD^2 - C^2 D)}{\text{Det.}\,\Delta} \qquad \ldots \quad \ldots \ 2.10$$

$$\bar{I}_{\beta f} = \frac{V_f\,(C^3 - ACD) + V_b\,BCD}{\text{Det.}\,\Delta} \qquad \ldots \qquad \ldots \qquad \ldots\ 2.11$$

$$\bar{I}_{\beta b} = \frac{V_f\,BCD + V_b\,(C^3 - ACD)}{\text{Det.}\,\Delta} \qquad \ldots \qquad \ldots \qquad \ldots\ 2.12$$

The forward and backward voltages can be written down on the assumption that the applied voltage passes through its zero value at the time of closing the switch.

$$V_f(t) = \frac{E_A}{\sqrt{3}} \sin\,(t-30)$$

$$V_b(t) = \frac{E_A}{\sqrt{3}} \sin\,(t+30)$$

$$LV_f(t) = V_f(s) = \frac{E_A}{\sqrt{3}} \left[\frac{\sqrt{3}}{2}\,\frac{1}{1+s^2} - \frac{s}{2(1+s^2)} \right] \ \ldots\ 2.13$$

$$LV_b(t) = V_b(s) = \frac{E_A}{\sqrt{3}} \left[\frac{\sqrt{3}}{2}\,\frac{1}{1+s^2} + \frac{s}{2(1+s^2)} \right] \ \ldots\ 2.14$$

On substituting the value of $V_f(s)$ and $V_b(s)$ in equation 2.9 we get (For K_α K_β refer Expt. Section)

$$\bar{I}_{\alpha f} = \frac{E_A}{2\sigma x_\alpha} \left[\frac{(s + K_\beta)}{(1+s^2)\left\{ s^2 + s\,\frac{K_\alpha + K_\beta}{\sigma} + \frac{K_\alpha K_\beta}{\sigma} \right\}} - \frac{s^2\,(s + K_\beta)}{\sqrt{3}\,(1+s^2)\left\{ s^3 + s^2\,\frac{(K_{ac} + K_\beta)}{\sigma} + \frac{s(x_c' + K_\beta K_{ac})}{\sigma} + \frac{K_\beta x_c'}{\sigma} \right\}} \right] \ldots 2.15$$

Similarly $\bar{I}_{\alpha b}$ can be written as

$$\bar{I}_{\alpha b} = \frac{E_A}{2\sigma x_\alpha} \left[\frac{(s + K_\beta)}{(1+s^2)\left\{ s^2 + \frac{s(K_\alpha + K_\beta)}{\sigma} + \frac{K_\alpha K_\beta}{\sigma} \right\}} + \frac{s^2\,(s + K_\beta)}{\sqrt{3}\,(1+s^2)\left\{ s^3 + \frac{s^2(K_{ac} + K_\beta)}{\sigma} + \frac{s(x_c' + K_\beta K_{ac})}{\sigma} \right.} \right.$$

$$\left. \right]$$

Total current i.e. phase current is the sum of the two components

$$i_A(s) = \frac{E_A}{\sigma x_\alpha}\left[\frac{(s+K_\beta)}{(1+s^2)\left\{s^2+\frac{s(K_\alpha+K_\beta)}{\sigma}+\frac{K_\alpha K_\beta}{\sigma}\right\}}\right] \quad \ldots \; 2.17$$

Taking Laplace inverse of eq^n. 2.17

$$i_A(t) = \frac{E_A}{\sigma x_\alpha}\left[\sqrt{\frac{1+K_\beta^2}{(1+p_1^2)(1+p_2^2)}}\,\sin(t-\theta) + \frac{K_\beta - p_1}{(p_2-p_1)(1+p_1^2)}\,e^{-p_1 t}\right.$$

$$\left. + \frac{K_\beta - p_2}{(p_1-p_2)(1+p_2 2)}\,\bar{e}^{\,p_2 t}\right] \quad \ldots \; 2.18$$

$$\theta = \tan^{-1} T_1 + \tan^{-1} T_2 - \tan^{-1} a$$

where $a = \frac{1}{K_\beta}$ and T_1 and T_2 are reciprocals of two roots of characteristic equation.

For usual values of the constants of three phase Induction motors $K_\beta \ll 1$ and p_2 is neglegibly small compared to p_1. The above expression can be written as

$$i_A(t) = \frac{E_A}{\sigma x_\alpha}\left[\frac{\sin(t-\theta)}{\sqrt{1+p_1^2}} + \frac{\bar{e}^{\,p_1 t}}{(1+p_1 2)} - \frac{p_2}{p_1}\,e^{-p_2 t}\right] \quad \ldots \; 2.19$$

By substituting the motor constants,

$$i_A(t) = \frac{E_A}{\sigma x_\alpha}\left[0.75\,\sin(t-49) + 0.553\,e^{-0.8865t} + 0.0111e^{-0.0135t}\right]$$

$$\ldots \; 2.20$$

At $t = \pi/2 + 49 = 139^\circ$ (because fundamental is maximum at $t = 139^\circ$).

$$t = 139 \;\; \pi/180 \text{ radian}$$

therefore $i_A(t) = \frac{E_A}{\sigma x_\alpha} \times 0.883$ (at start)

Steady state component is 75% of fundamental and the two decaying d.c. terms vanish after some time.

$$\text{Thus steady state component} = \frac{E_A}{\sigma X_\alpha} \times 0.75$$

$$\text{The ratio } \frac{\text{transient current}}{\text{steady state current}} = \frac{0.883}{0.75} = 1.11$$

Actual record of $i_A(t)$ shows the ratio to be nearly 1.13 between initial transient current and final steady state value. The difference between calculated and experimental values is negligible.

For getting numerical peak value of transient current from expression 2.20 the value of E_A ; x_α are to be substituted

$$i_A(t) = \frac{254}{0.06 \times 129.15} \times 0.883 = 29\text{A peak or } 206 \text{ rms}$$

(Assuming current wave sinusoidal)

$$\text{Steady state value} = \frac{254}{0.06 \times 129.15} \times 0.75 = 24.6\text{A or } \underline{17.4(\text{RMS})\text{A}}$$

The recorded steady state value with the standard ammeter in line was 16.8A.

2.3. Converter current:-

From equivalent circuit shown in the Fig. 2.2 the current through the capacitor during starting is the difference between the forward and backward currents.

$$i_c(t) = i_{\alpha f}(t) - i_{\alpha b}(t) \qquad \ldots \qquad 2.21$$

The converter current depends on the value of capacitor.

2.3.1. <u>For 170 μF</u>

The capacitor current will be given by

$$i_c(t) = \frac{E_A}{\sigma X_\alpha} \frac{1}{\sqrt{3}} \left[0.255 \operatorname{Cos}(t+73.4) - 0.572 e^{-0.563t} \times \operatorname{Cos}(2.11t + 82.6)\right] \quad .. \quad 2.22$$

This current consists of two terms:

i) Fundamental frequency term

ii) Nearly double frequency term dying out exponentially.

Besides these two, there is a negligibly small term decaying exponentially which has been neglected during the determination of this current.

From this expression we see that the fundamental term is free from decaying factor and the double frequency current in the capacitor though of nearly twice the value of that of the fundamental, dies out completely exponentially. The phase difference between the two components is nearly 171° and the resultant current is the direct difference of the two at time $t = 0$. The steady state value is only the fundamental component.

$$\text{The ratio } \frac{\text{starting current}}{\text{steady peak value}} = \frac{0.317}{0.255} = 1.24$$

The recorded oscillogram (No. 4 for 170 μF) gives the ratio to be nearly 1.18.

<u>For 150 μF,</u>

The capacitor current is given by

$$i_c(t) = \frac{E_A}{\sigma X_\alpha\sqrt{3}}\left[0.216\ \mathrm{Cos}(t+75.6) - 0.556\ e^{-0.578t}\ \mathrm{Cos}(2.27t+85.3)\right] \quad \ldots\ 2.23$$

For 200 μF, the capacitor current is given by

$$i_c(t) = \frac{E_A}{\sigma X_\alpha\sqrt{3}}\left[0.306\ \mathrm{Cos}\ (t+71) - 0.6365 e^{-0.5385t}\ \mathrm{Cos}(1.95t+82)\right] \quad \ldots\ 2.24$$

The variation of different components of current with the above values of the capacitor and the theoretical and practically obtained values are tabulated in Table 2.1 and 2.2 respectively.

TABLE 2.1

Comparative statement of components of current for different values of capacitor

Capacitor	Fundamental component peak	higher frequency term i.e. nearly double peak	damping factor	Remarks
150	0.216	-0.556	$e^{-0.578t}$	More damping effect i.e. Transient dies out quickly.
170	0.255	-0.572	$e^{-0.563t}$	damping effect less than first one
200	0.306	-0.6365	$e^{-0.538t}$	lowest damping i.e. transient remain for longer time

TABLE 2.2

Comparative statement of practical and theoretical results

Capacitor	Theoretical ratio of $\frac{\text{Transient}}{\text{steady}}$	Experimental ratio of $\frac{\text{Transient}}{\text{steady}}$
150	1.5	1.21
170	1.24	1.18
200	1.08	1.13

2.4. Voltage across condenser:

Voltage across the phase converter or condenser can be estimated with the help of the capacitor current and the impedance of the phase converter.

$$V_c(t) = (r_c + \frac{x_c}{s})\ (I_{\alpha f} - \bar{I}_{\alpha b}) \quad \ldots \quad .. \ 2.25$$

$$i_{\alpha f} - i_{\alpha b} = i_c(t)$$

For 170 μF

$$i_c(t) = \frac{E_A}{\sigma x_\alpha \sqrt{3}} \left[-0.572\ e^{-0.563t} \operatorname{Cos}(2.11\ t+82.6) + 0.255 \operatorname{Cos}(t+73.4)\right] \quad \ldots \quad .. \ 2.26$$

$$r_c^* = 0.94 \text{ ohms}$$

$$x_c = 18.75 \text{ ohms.}$$

$$V_c(t) = \frac{E_A}{\sigma x_\alpha \sqrt{3}} \left[5.05\ e^{-0.563t} \operatorname{Cos}(2.11t-28.4) + 4.76 \operatorname{Cos}(t-13.7)-9.12\right] \quad .. \ 2.27$$

* The value of r_c experimentally determined has been found to be nearly 5% of reactance.

for 200 μF

$$r_c = 0.795 \text{ ohms}$$

$$x_c = 15.9 \text{ ohms}$$

$$i_c(t) = \frac{E_A}{\sigma X_\alpha \sqrt{3}} \left[0.306 \, \mathrm{Cos}(t+71) - 0.6365 e^{-0.538t} \mathrm{Cos}(1.95t+82)\right] \quad .. \; 2.28$$

And
$$V_c(t) = \frac{E_A}{\sigma X_\alpha \sqrt{3}} \left[4.87 \, \mathrm{Cos}(t-16.2) + e^{-0.538t} \, 5.16 \mathrm{Cos}(1.95t-28.6) - 9.18\right] \quad .. \; 2.29$$

In a similar way for 150 μF voltage across the condenser will be given by,

$$V_c(t) = \frac{E_A}{\sigma X_\alpha \sqrt{3}} \left[4.60 \, \mathrm{Cos} \, (t-12.6) + 5.26 e^{-0.578t} \times \mathrm{Cos}(2.27 \, t - 24.6) - 9.3\right] \quad .. \; 2.30$$

The relative variation of voltage across the condenser for different values of capacitor are tabulated as shown below;

TABLE 2.3

μF	Fundamental component	higher frequency term	constant term	damping factor for higher frequency term	Frequency for damping term
150	4.6	5.26 ↑	9.3 ↑	$e^{-0.578t}$	2.27x50 =113.5c/s
170	4.76	5.05	9.12	$e^{-0.563t}$	2.11x50 =105.5c/s
200	4.87 ↓	5.16 ↓	9.18 ↓	$e^{-0.538t}$	1.95x50 =97.5c/s

From the table 2.3, we see that,

i) Fundamental component increases with increased value of capacitor.

ii) At critical value of capacitor, double frequency term is less because it corresponds to nearly balanced operation effect of backward field is minimum. As the value of capacitor is changed, the double frequency term increases due to unbalance operation of Induction motor.

iii) Constant term in the voltage expression for the voltage across the capacitor also changes, with higher values of capacitor. It is minimum for critical value of capacitor because fundamental and double frequency term increase and hence constant term decreases.

2.5. <u>Motor winding current:</u>

The current in the winding connected in parallel with the capacitor is the difference between the current and capacitor current.

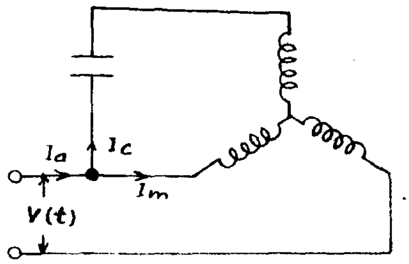


Fig.3

Arrangement of [illegible] showing different currents.

$$I_m = I_A - I_c \qquad \ldots \qquad \ldots \qquad 2.31$$

By considering instantenous values of these currents,

$$i_m(t) = i_A(t) - i_c(t) \qquad \ldots \qquad \ldots \qquad 2.32$$

$$i_c(t) = (i_{\alpha f}(t) - i_{\alpha b}(t) \qquad \ldots \qquad \ldots \qquad 2.33$$

$$i_A(t) = (i_{\alpha f} + i_{\alpha b}) \qquad \ldots \qquad \ldots \qquad 2.34$$

$$i_m(t) = 2\, i_{\alpha b} \qquad \ldots \qquad \ldots \qquad 2.35$$

By substituting the value of i_b in eq^n. 2.35 and putting the motor constants we get for a capacitance value of 150 μF,

$$i_m(t) = \frac{E_A}{\sigma X_\alpha} \left[-0.65 \, \mathrm{Cos}(t+34.8) - 0.332 e^{-0.578t} \, \mathrm{Cos}(2.27t+85.3) + 0.553 e^{-0.8865t} + 0.011 e^{-0.0135t} \right] \quad \ldots 2.36$$

for C = 170 μF, the expression for the current will be,

$$i_m(t) = \frac{E_A}{X_d}\left[-0.639\ \mathrm{Cos}(t+34.2)-0.334\ e^{-0.563t}\ \mathrm{Cos}(2.11t+82.6) + 0.553e^{-0.8865t} + 0.011e^{-0.0135t}\right] \quad ..2.37$$

for C = 200 μF,

$$i_m(t) = \frac{E_A}{X_d}\left[-0.61\ \mathrm{Cos}(t+31.5)-0.367e^{-0.538t}\ \mathrm{Cos}(1.95t+82) + 0.553e^{-0.8865t} + 0.011e^{-0.0135t}\right] \quad ..2.38$$

A relative study can be made of these currents for different capacitor values by calculating the values of the various components and arranging in a tabular form as shown in table 2.4.

TABLE 2.4

Variation of different components of current and damping factor for various values of capacitor

Capacitor	Fundamental component	higher frequency term i.e. nearly double frequency(3)	damping factor (1)	damping factor(2)	damping factor for oscillatory term 3	frequency
150	0.65	0.332	d.c.	higher	$e^{-0.578t}$	113.5c/s
170	0.639	0.334	B.6. damping	damping	$e^{-0.563t}$	105.5c/s
200	0.61	0.367	factor less & same	factor & same	$e^{-0.538t}$	97.5c/s

The following points are worth noting about these currents,

1) Fundamental component decreases with increased value of capacitor, because higher the value of capacitor, lesser the impedance of shunt branch and hence more and more current

will try to flow through the phase converter. The winding current $i_m(t)$ will decrease accordingly.

ii) The damping terms are same for any capacitor value of phase converter because damping factor depends on the constant of the winding i.e. L/R and this is constant for motor winding.

iii) As the capacitance increases, the frequency of oscillatory term decreases. For example, for 150 μF, 170 and 200 μF, these are 113 c/s, 105.5 c/s and 97.5 c/s respectively. Simultaneously the damping factor also becomes less and less.

****

.....CHAPTER 3.....

3.1. The developed electrical torque in symbolic form is given by,

$$T = KR_2 \left[2_{\beta f}^{\ 2} - i_{\beta b}^{\ 2} \right]$$

where

$$i_{\beta f}(s) = \frac{E_A x_m}{2\sigma x_\alpha\ x_\beta} \left[\frac{-s}{(1+s^2)\left\{ s^2 + \frac{s(K_\alpha + K_\beta)}{\sigma} + \frac{K_\alpha K_\beta}{\sigma} \right\}} + \frac{1}{\sqrt{3}} \frac{s^3}{(1+s^2)(\text{Ch.equation})} \right] \quad \ldots\ 3.1$$

$$i_{\beta b}(s) = \frac{E_A x_m}{2\sigma x_\alpha\ x_\beta} \left[\frac{-s}{(1+s^2)(\text{Ch.equation})} - \frac{1}{\sqrt{3}} \frac{s^3}{(1+s^2)(\text{Ch.equation})} \right] \quad \ldots\ 3.2$$

For 170 μF capacitor value, $i_{\beta f}(t)$ and $i_{\beta b}(t)$ have been calculated in detail in appendices. The final form for these currents are

$$i_{\beta f}(t) = \frac{E_A x_m}{2\sigma x_\alpha\ x_\beta} \Big[-0.565 e^{-0.8865t} + 0.0147 e^{-0.0135t} -0.626 \cos(t-35.4) - .0.394 e^{-0.563t} \cos(2.11t-5.6) \Big] \quad \ldots\ 3.3$$

$$i_{\beta b}(t) = \frac{E_A x_m}{2\sigma x_\alpha\ x_\beta} \Big[-0.565 e^{-0.8865t} + 0.0147 e^{-0.0135t} -0.875 \cos(t-47.8) + 0.394 e^{-0.563t} \cos(2.11t-5.6) \Big] \quad \ldots\ 3.4$$

T_f = Torque due to forward component of current = $KR_2 i_{\beta f}^{\ 2}$

T_b = Torque due to backward component of current = $KR_2 i_{\beta b}^{\ 2}$

$$T_f = \lambda \Big[0.32 e^{-1.773t} + 0.000216 e^{-0.027t} + 0.195(1+\cos 2\theta) + 0.0705 e^{-1.126t} \times (1+\cos 2\phi) - 0.0176 e^{-0.9t} - 0.0184 e^{-0.0135t} \times \cos(t-35.4) + 0.123 e^{-0.563t} (\cos(3.11t-41) + \cos(1.11+29.8) \ddagger 0.223 e^{-1.449t} \cos(2.11t-5.6) \Big] \quad \ldots\ 3.5$$

where $\lambda = K R_2 \left[\frac{E_A x_m}{2\sigma x_\alpha x_\beta}\right]^2$

$$T_b = \lambda \Big[0.32e^{-1.773t} + 0.000216e^{-0.027t} + 0.384(1+\cos 2\theta) + \frac{0.156}{2} \times (1+\cos 2\alpha)e^{-1.126t} - 0.0176e^{-0.9t} - 0.02570e^{-0.0135t} \times \cos(t-47.8) - 0.344e^{-0.563t} \cos(3.11t-53) + \cos(1.12t+41) - 0.223e^{-0.1449t} \cos(2.11t - 5.6) \Big] \qquad \ldots 3.6$$

where $\theta = t - 47.8$

$\alpha = 2.11\ t - 5.6 = \phi$

$$T_{net} = T_f - T_b \qquad \ldots 3.7$$

Thus by substituting T_f and T_b in equation 3.7 we get,

$$T_{net} = \lambda \Big[-0.189 + 0.195 \cos(2t-78) - 0.384 \cos(4.22t-11.2) + e^{-0.0135t} \times 0.0073 \cos(t-35.4) + 0.59e^{-0.563t} \times \cos(t-47.8) \cos(2.11t-5.6) + 0.446e^{-1.499t} \cos(2.11t-5.6) \Big] \qquad \ldots 3.8$$

The above expression gives the transient torque at starting for single phase operation of a poly-phase Induction motor.

By giving various values for t the curve for transient torque can be plotted as shown ~~below~~:

TABLE 3.1

$t = 0$	$t = \pi/2$	$t = \pi$	$t = 3\pi/2$	$t = 2\pi$	$t = 5\pi/2$
-0.268λ	0.88213λ	0.605λ	0.565λ	0.422λ	0.294λ

By plotting these points the torque curve shape will be approximately of the form shown in fig.3.1.

$$T_{steady} = \lambda\sqrt{0.239^2 + 0.195^2 + 0.384^2}$$

$$= \lambda \times 0.53$$

$$\frac{\text{Transient torque}}{\text{steady torque}} = \frac{0.8865}{0.53}$$

(Steady torque = RMS value of nondecaying term)

$$= 1.67$$

The transient torque at starting is 1.67 times the steady component and settles down to steady state with in nearly 1.5 cycles. With in this time the rotor does not attain measurable speed. Thus shaft torque is much higher at starting than the running torque.

3.2. Torque calculation with 200 μF:

As in article 3.1, the torque for different capacitor values can also be estimated. For example, the net torque for a capacitor value of 200 μF will be,

$$T_{net} = T_f - T_b$$

where

$$T_f = KR_2\, i_{\beta f}^2$$
$$T_b = KR_2\, i_{\beta b}^2$$

The forward and backward components of current change with different capacitor values and so do the torques. The forward and backward torques are given by,

$$T_f = \lambda \Big[0.32e^{-1.773t} + 0.000216e^{-0.027t} + 0.2375\,(1+\cos 2\theta) + 0.075e^{-1.076t} \times (1+\cos 2\theta) + 0.0166e^{-0.9t} - 0.02030e^{-0.0135t}\cos(t-29.1) + 0.536e^{-0.538t}\cos(t-29.1)\cos(1.95t-11.5) + 0.44e^{-1.449t}\cos(1.95t - 11.5)\Big] \quad \ldots\ 3.9$$

And

$$T_b = \lambda\Big[0.32e^{-1.773t} + 0.000216e^{-0.027t} + 0.35(1+\cos 2\delta) + 0.075e^{-1.076t}(1+\cos 2\phi) + 0.0166e^{-0.9t} - 0.02456e^{-0.0135t} \times \cos(t-53.7) - 0.648e^{-0.538t}\cos(t-53.7)\cos(1.95t-11.5) - 0.44e^{-1.449t}\cos(1.95t-11.5)\Big] \quad \ldots 3.10$$

$$T_{net} = \lambda\Big[-0.1125 + 0.2375\cos(2t-28.2) - 0.35\cos(3.9t-23) + e^{-0.0135t}\left\{0.02456\cos(t-53.7) - 0.0203\cos(t-29.1)\right\} + e^{-0.538t}\left\{0.536\cos(t-29.1)\cos(1.95t-11.5) + 0.648\cos(t-53.7)\cos(1.95t-11.5)\right\} + 0.88e^{-1.449t}\cos(1.95t-11.5)\Big] \quad \ldots 3.11$$

By giving various values for t we get,

TABLE 3.2

t	0	$\pi/2$	π	$3\pi/2$	2π
T	-1.43λ	0.9939λ	0.359λ	0.4616λ	0.07845λ

The shape of torque/time curve will be approximately as shown in fig.3.2.

$$T_{steady} = \lambda\sqrt{0.1125^2 + 0.235^2 + 0.35^2}$$
$$= 0.435\lambda$$

The ratio of transient peak torque to steady state torque is nearly 3.33 and transient torque settle down to steady state with in 1.5 cycles approx.

3.3. <u>The calculation of torque for 150 µF.</u>

The forward torque T_f will be given by

$$T_f = \lambda \Big[0.32e^{-1.773t} + 0.000216e^{-0.027t} + 0.223(1+\text{Cos } 2\theta) + 0.051e^{-1.156t}(1+\text{Cos } 1\phi) - 0.0166e^{-0.9t} - 0.0196e^{-0.0135t}\text{Cos}(t-26.3) + 0.426e^{-0.578t} \times \text{Cos}2.27t\ \text{Cos}(t-26.3) + 0.36e^{-1.4885t}\text{Cos } 2.27t \Big] \quad \ldots 3.12$$

Similarly backward torque

$$T_b = \lambda \Big[0.32e^{-1.1773t} + 0.000216e^{-0.027t} + 0.375(1+\text{Cos } 2\delta) + 0.051e^{-1.156t}(1+\text{Cos } 2\phi) - 0.0166e^{-0.9t} - 0.0254e^{-0.0135t}\text{Cos}(t-54.7) - 0.554e^{-0.578} \times \text{Cos } 2.27t\ \text{Cos}(t-26.3) - 0.36e^{-1.4665t}\text{Cos } 2.27t \Big] \quad \ldots 3.13$$

T_{net} in the direction of rotation will be the difference of T_f and T_b. Therefore net torque is

$$T_{net} = \lambda \Big[-0.152 + 0.223\ \text{Cos}(2t-52.6) - 0.375\ \text{Cos}2t.54t - e^{-0.0135t}\{0.0196\ \text{Cos}(t-26.3) - 0.0254\ \text{Cos}(t-254.3)\} + e^{-0.578t}(0.426\ \text{Cos}2.27t\ \text{Cos}(t-26.3) + 0.554\text{Cos } 2.27t \times \text{Cos}(t-54.3) + 0.72\,e^{-1.4865t}\ \text{Cos } 2.27t \Big] \quad \ldots 3.14$$

By substituting various values of t in the above expression we get,

TABLE 3.3

t	0	$\pi/2$	π	$3\pi/2$	2π
T_{net}	-1.0106λ	0.9595λ	-0.2017λ	0.03975λ	-0.3308λ

On plotting these values, the shape of the torque/time curve will be nearly as shown in fig.3.3.

$$T_{steady\ state} = \lambda\sqrt{0.152^2 + 0.223^2 + 0.375^2}$$

$$= 0.464\lambda$$

The ratio of transient peak torque to steady state torque is

$$\text{ratio} = \frac{1.0106}{0.464} = 2.18$$

The variation of transient torques and steady state torques is compared for different values of capacitor in the following table.

TABLE 3.4

μF Capacitor	Transient torque peak	steady state	ratio= $\frac{\text{Transient}}{\text{steady}}$	Transient torque at t = 0
150	+ 1.01060	0.464	2.18	- 1.4
170	+ 0.88653	0.53	1.67	- 0.289
200	+ 1.43	0.436	3.28	- 1.00

From above we note the following interesting points,

1. Transient peak torque is less for critical value of

capacitor than for any other value. Thus critical value of capacitor gives perfect balance operation at starting and minimum transient condition as well.

2. The steady state torque is also maximum for critical value of capacitor as it is very clear from the definition of balanced operation i.e. maximum torque per ampere.

3. The ratio of transient torque to steady state torque is minimum in the case of 170 μF. Thus by choosing this value of capacitor we can improve the performance under transient conditions better than with other capacitor values.

4. The time for the transient to die out is nearly the same for all values of capacitor.

3.4. Starting torque variation with capacitor:

The starting quality can be improved by properly selecting the value of capacitor as discussed in Chapter 1. The optimum value of capacitor has been calculated and steady state torque is also computed in article 3.3. The variation of starting torque has been studied experimentally in a slightly indirect way.

A tachogenerator has been connected with the shaft of Induction motor as shown in photograph attached herewith. The output of tachogenerator depends on the speed of shaft because voltage generated is proportional to speed if other things are constant.

$$E \propto KN$$

The output voltage of tachogenerator is sinusoidal if the motor is running at constant speed. During transient or at starting, the shaft takes some times to come up to speed or in otherwords one can say that speed rises from zero to rated value at that particular load. The corresponding output or voltage of tachogenerator varies and variation in shape has been recorded by keeping voltage reference signal of the motor as a base. The frequency and amplitude of tachogenerator voltage varies till the motor comes up to rated speed. This variation depends on the value of capacitor chosen. The differentiation of this voltage or speed response gives the acceleration of the motor.

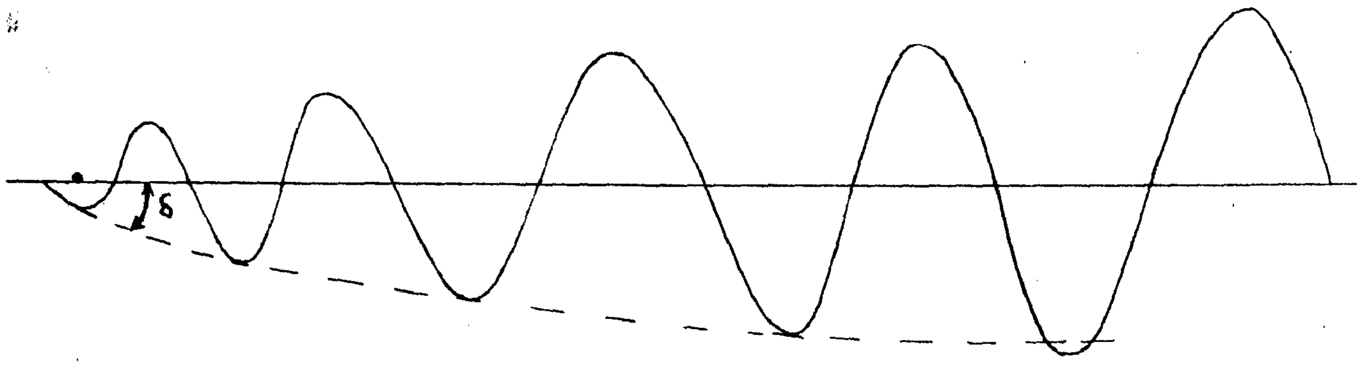
$$Acc \propto \frac{dN}{dt}$$

The torque of the motor will be T Moment of inertia acc.

$$T = K \frac{dN}{dt}$$ by assuming K (Moment of inertia in consistent system of units.

$$\frac{dN}{dt} = \tan\delta$$

$$T = K \tan\delta$$



Different photographs have been taken for speed response with different values of capacitor and the torque is calculated as shown in Table 3.5.

TABLE 3.5

μF	δ in degree	tan δ	T = K' tan δ	T cal. (average)
150	2.5	0.04361	T=0.436K'	T=0.17 λ'
170	2.6	0.04566	T=0.4566K'	T=0.214 λ'
200	2.9	0.050	T=0.050K'	T=0.245 λ'

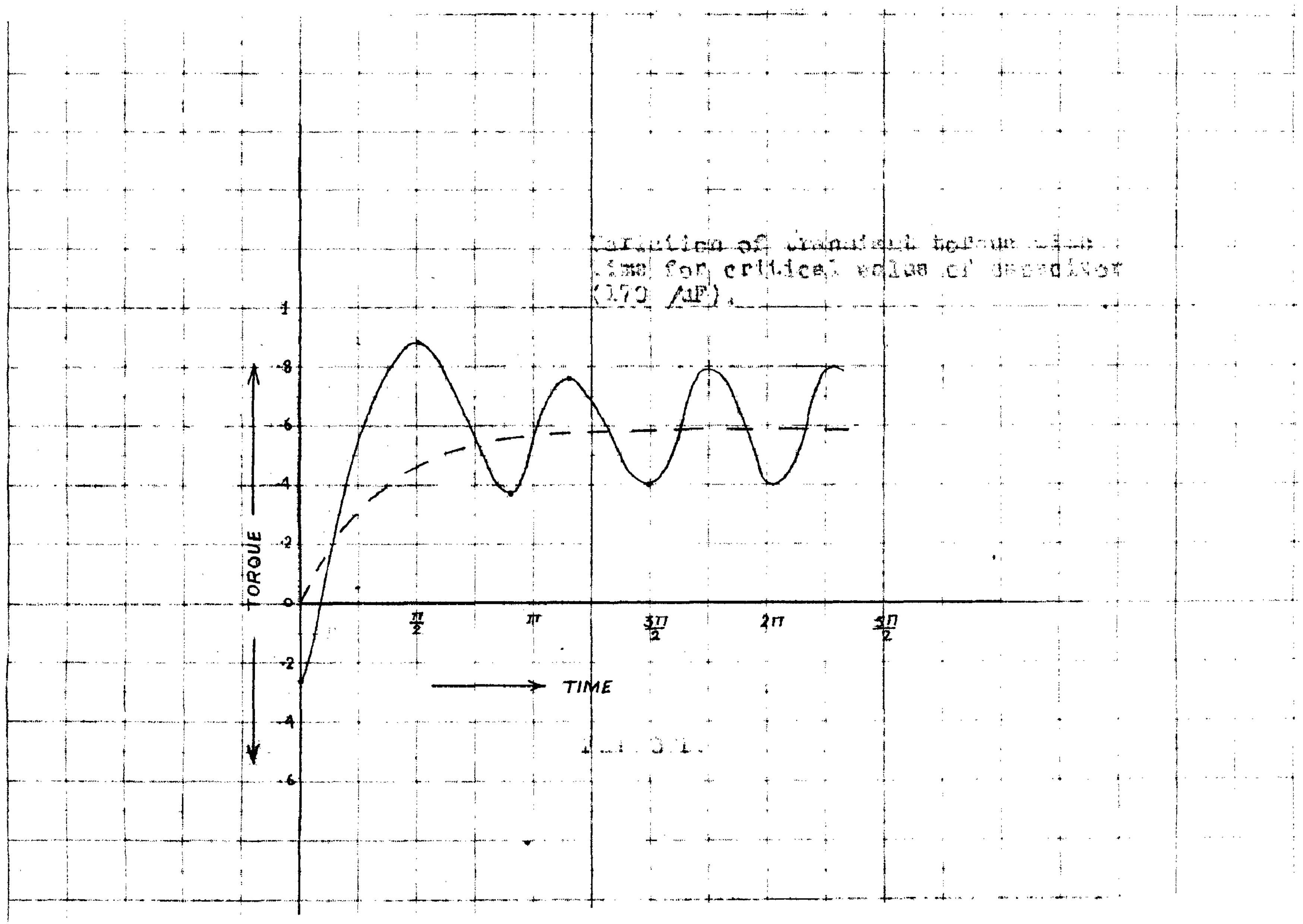
where K' and λ' are constants.

TABLE 3.6

Comparative statement of torque ratio for various capacitor ratios

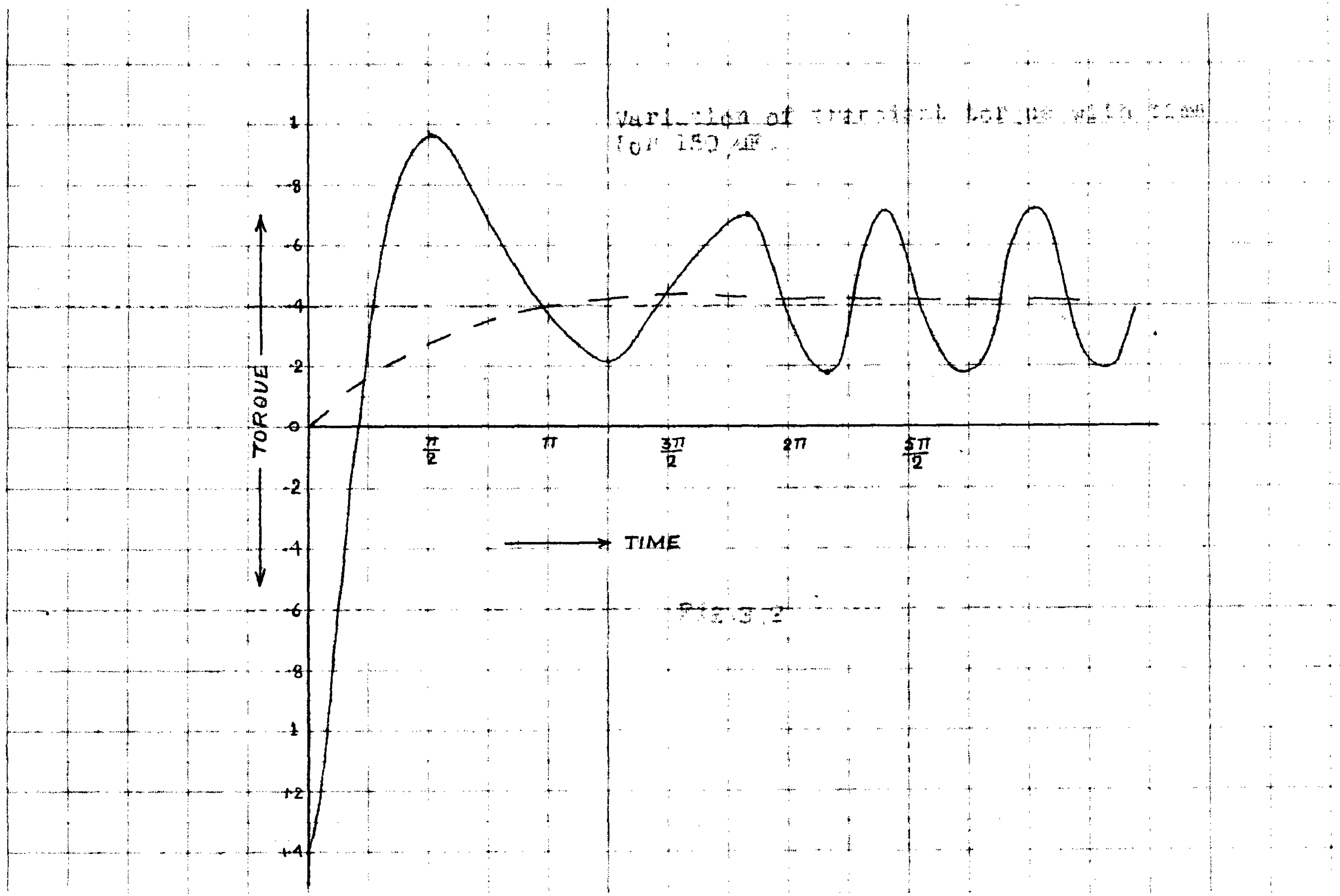
Capacitor ratio	Theoretical torque ratio	Experimental torque ratio
150 μF / 170 μF	0.952	0.835
200 μF / 170 μF	0.905	0.87

From the above, we conclude that ratio of the two starting torques with two different values of capacitor are practically same by the two methods. Thus by varying the capacitor value from 150 μF to 170 μF, the torque variation is 0.835 times according to calculation, while experimental value is 0.952. In the same way, the torque varies by 0.905 times for the variation of capacitor value from 170 μF to 200 according to calculation while experimentally ratio is 0.87. The ratios obtained by the two methods are fairly in agreement within the limits of experimental error.



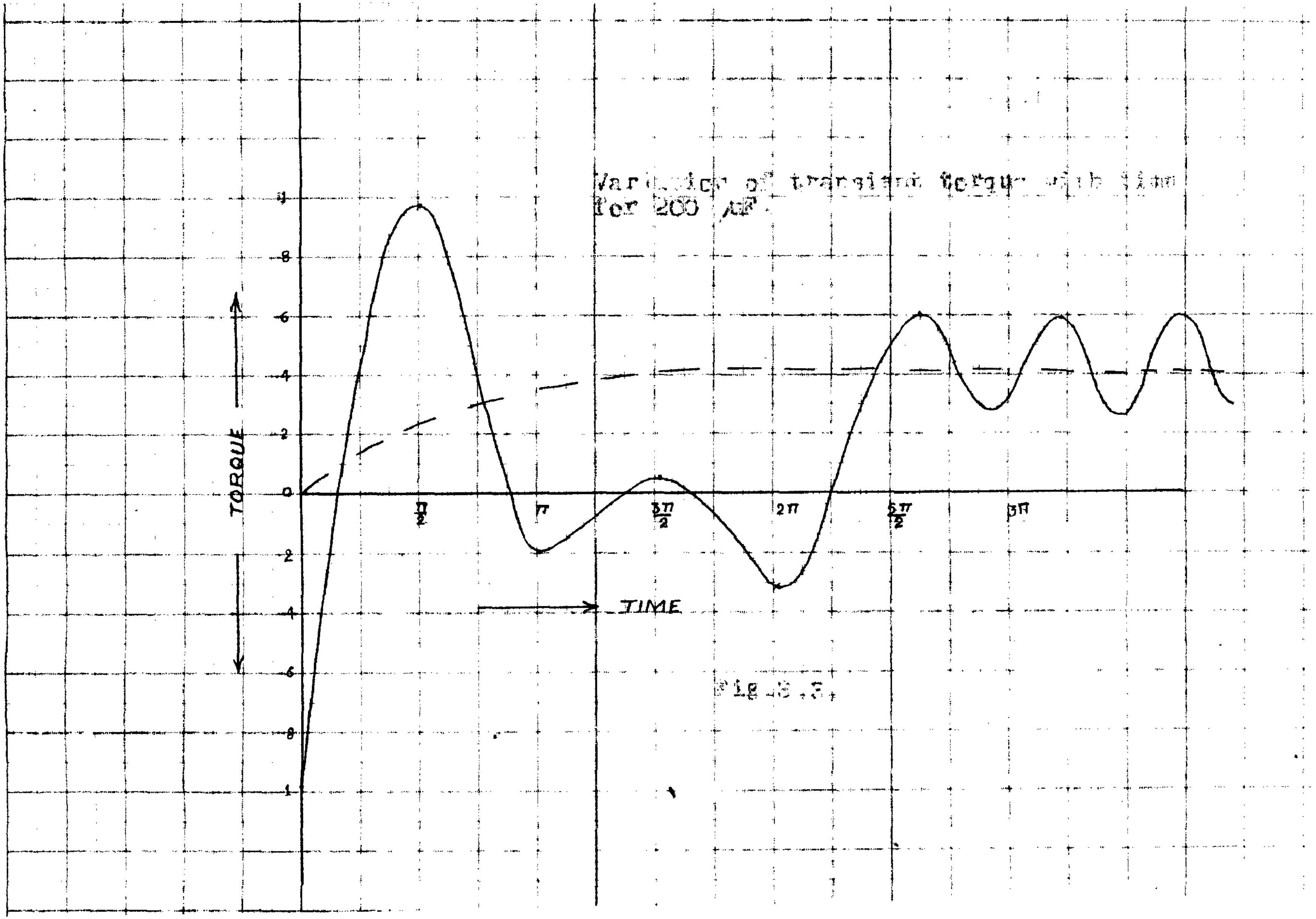
Variation of transient torque with time for critical value of capacitor (170 µF).

FIG. 3.1.



Variation of transient torque with time for 150 μF.

Fig. 3.2



Variation of transient torque with time for 200 μF.

Fig.3.5.

EXPERIMENTAL DETAILS

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.....PRACTICAL DETAILS.....

Section (i):

In following pages the results of experiments performed on two Induction motors, for verification of theoretical findings are recorded.

The specifications of motors are:

(i) Single-phase Induction motor

$\frac{1}{4}$ h.p. 110V, 50 c/s 970 rpm

5KC49 AB 589 General Electric Company

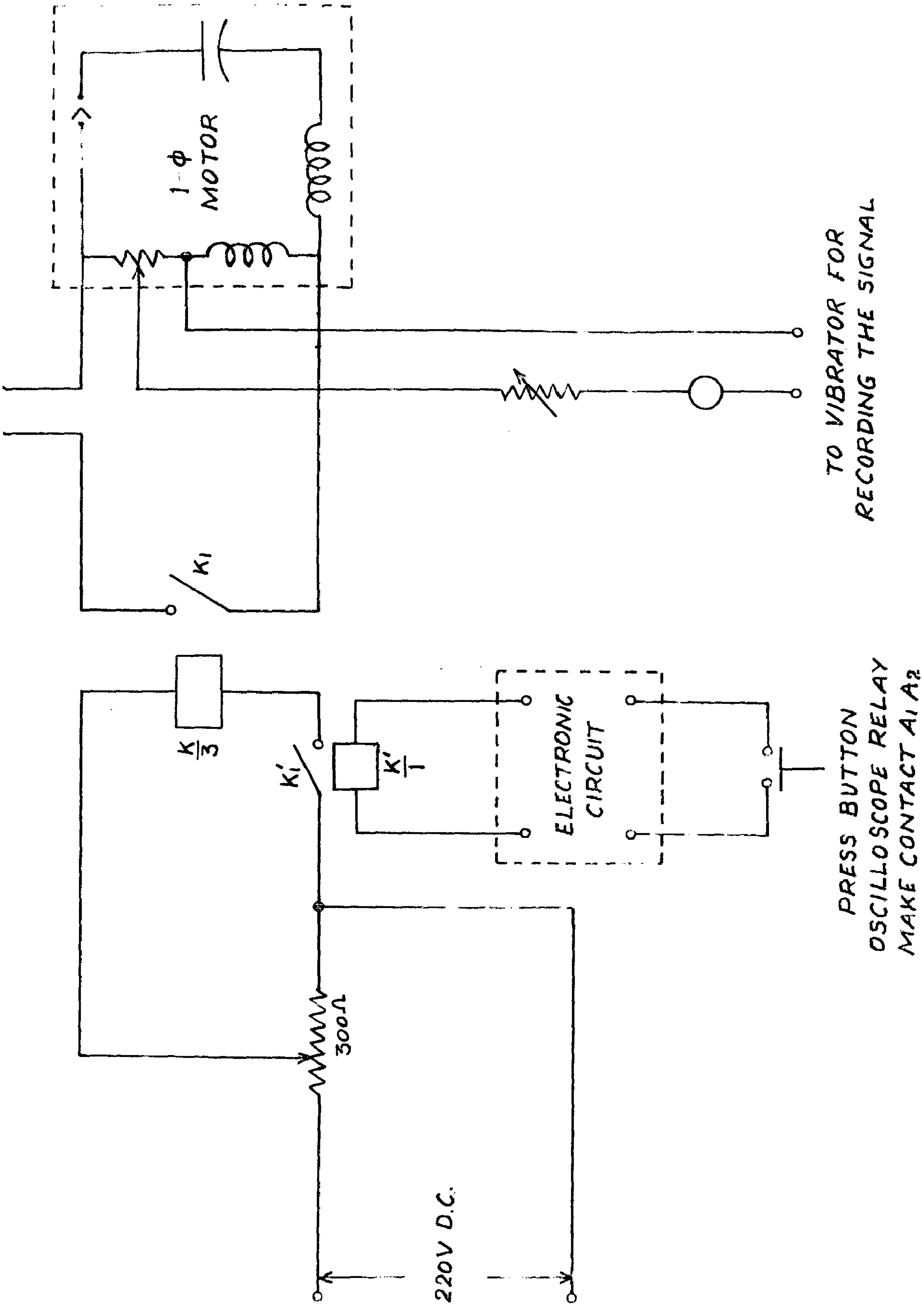
(ii) Three-phase Induction motor:

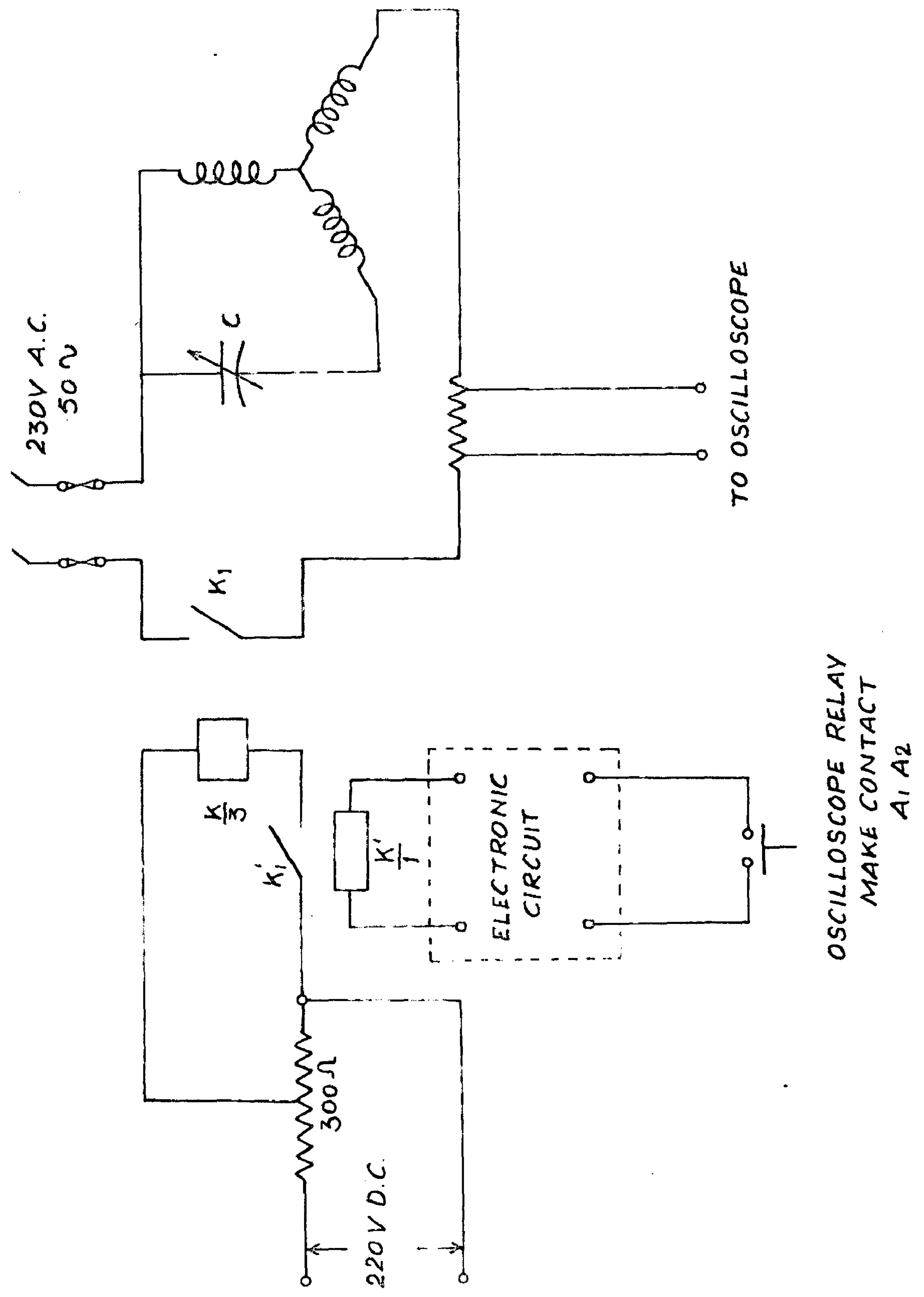
3 h.p. 440V 50 c/s 1440 rpm

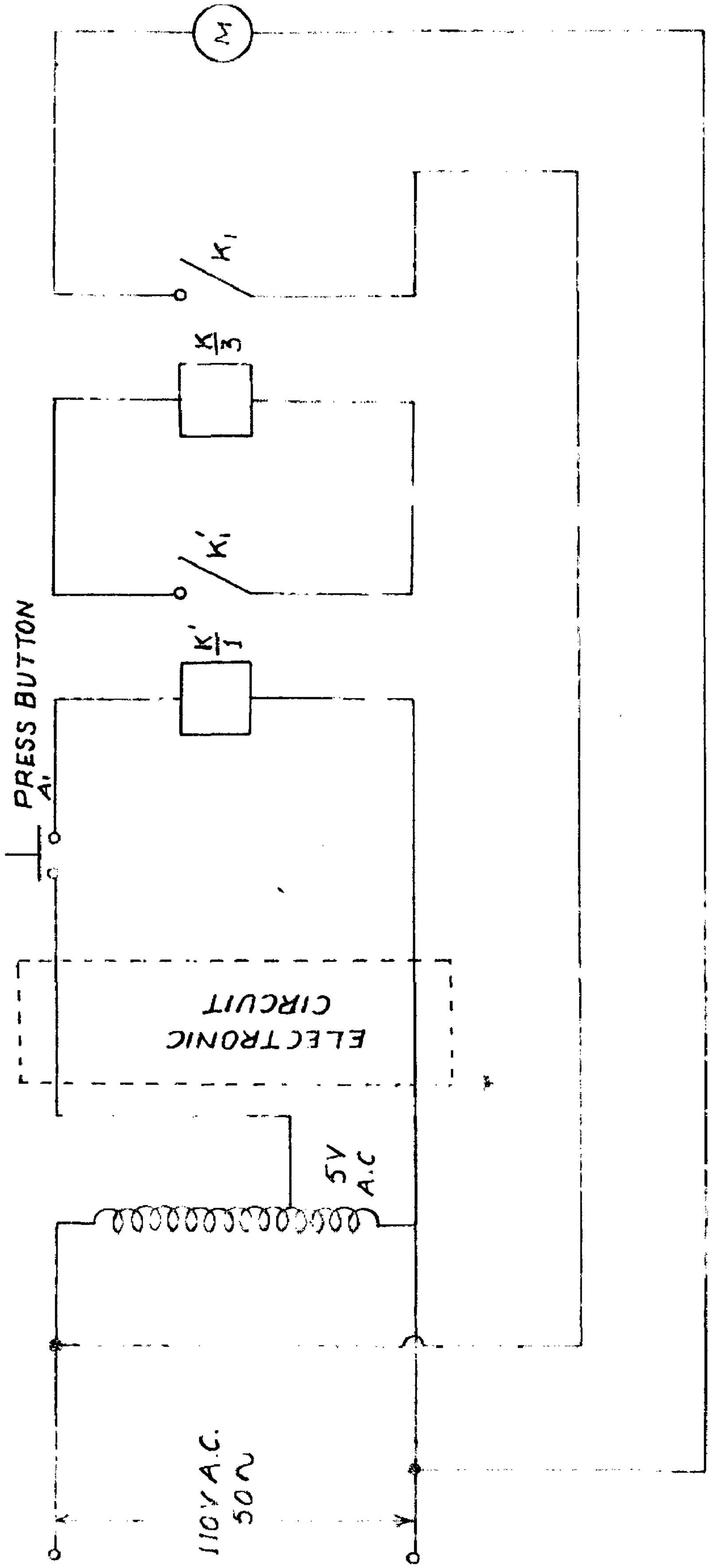
5K 254 F914 General Electric Company.

Single-phase motor		Three-phase motor	
$r_{\alpha a}$ = 2.22 ohms	$x_{\alpha a}$=54.15ohms	R_1 = 4 ohms	x_1 =4.15 ohms
$r_{\beta d}$ = 5.56 ohms	$x_{\beta d}$=54.15ohms	R_2 = 3 ohms	x_2 =4.15 ohms
$r_{\alpha b}$ = 9.5+ r_c ohms	$x_{\alpha b}$=94.3	x_α = 129.15	$x_{m_{-2}}$ = 125 ohms
$r_{\beta q}$ = 3.3 ohms	$x_{\beta q}$=94.3	$K_\alpha = \frac{r_\alpha}{x_\alpha}$ =3.1 x100	x_β = 129.15
$K_{\beta q} = \frac{r_{\beta q}}{x_{\beta q}} = 3.5\times10^{-2}$	$K_{\alpha b} = \frac{r_{\alpha b}}{x_{\alpha b}}$	$\sigma = 1 - \frac{x_m^2}{x_\alpha^2} = 0.06$	$K_\beta = \frac{r_\beta}{x_\beta} = 2.32\times10^{-2}$
$\sigma = 1 - \frac{x_{mb}}{x_{\alpha b}\, x_{\beta q}}$	$= 10.5 \times 10^{-2}$	$K_{ac} = \frac{r_1 + 2r_c}{x_\alpha}$	$x_c' = \frac{2x_c}{x_\alpha}$
$K_{\beta d} = \frac{r_{\beta d}}{x_{\beta d}} = \frac{5.56}{54.15} = 10.28\times10^{-2}$	$K_{\alpha a} = \frac{r_{\alpha a}}{x_{\alpha a}} = 4.1\times10^{-2}$; $Y_s = \frac{1}{10.5}$	$Y_s = \frac{1}{10.7}$ ℧	

In addition to the other instruments the cambridge 6-elements oscillograph has been used for recording the transients. The essential parts of oscillograph are


1-φ MOTOR
K1
TO VIBRATOR FOR RECORDING THE SIGNAL
K/3
K1'
K'/1
ELECTRONIC CIRCUIT
PRESS BUTTON OSCILLOSCOPE RELAY MAKE CONTACT A1 A2
300Ω
220V D.C.


230V A.C.
50 ~
C
TO OSCILLOSCOPE
K1
K/3
K1'
K'/T
ELECTRONIC
CIRCUIT
OSCILLOSCOPE RELAY
MAKE CONTACT
A1 A2
300Ω
220V D.C.


M
K1
K/3
K'1
PRESS BUTTON
A1
K'/1
ELECTRONIC CIRCUIT
5V A.C
110V A.C. 50~

1. Electromagnetic vibrator.
2. Optical system.
3. Commutator.
4. Drum camera
5. Motor and control Rheostat.

Brief description of oscillograph:-

The commutator mounted on main driving shaft, consists of three pairs of slip rings with three independent segments, with speed compensating scale on the left hand edge. Carbon brushes mounted beneath the commutator make contact with segments once during each revolution of the driving shaft.

Center or opening section:-

This section controls simultaneously the operation of the relay and the opening of the shutter. The position of the center segment in relation to the camera driving plate is determined by the setting of the speed scale which should be adjusted to the camera speed being used. This scale is so calibrated that at all speeds the camera may be retarded in relation to the opening segment by an amount which ensures that leading edge of the film arrives at the shutter aperture just as the shutter opens. As the inside relay A operates, the contacts A_3; A_4 open and close respectively. The contacts A_3, A_4 are brought out to the terminals mounted on the control panel, thus providing automatic switching facilities.

When the expose button is pressed, the commutator takes control and energises the open magnet at the correct instant and relay A operates which closes the contact A4. The shutter then moves up in to its open or middle position where it stays during the recording period. The commutator then energises the close magnet and shutter moves up in to its closed or upper position. The mechanism is adjusted to work from 40 rpm to 1500 rpm.

The point on waveshape of supply has been controlled with the help of electronic circuit used with the operating relay. The description of circuit and working principle is separately given in the next section no.ii.

The general circuit diagrams for both parts are more or less same as shown in fig.4 and 5 respectively. The relay K' is connected through electronic circuit and contacts of K' are in series with the motor. As soon as the expose button is pressed, the signal is impressed on electronic circuit and relay will operate only when it gets a pulse from power amplifier. This pulse position can be shifted with respect to supply voltage. In the present case pulse has been obtained when voltage wave passes through its zero position. Thus whatever may be the instant of pressing the expose button the relay K' will only operate when voltage is passing through its zero value and the motor comes on line instantenously. Actually due to the time lag in relay operation, the pulse position is not set exactly at zero but slightly less than zero on waveshape of supply, so that relay delay is taken in to account.

As discussed earlier, the opening of shutter and throwing of motor on line must be acomplished simultaneously in zero time. Practically the time can not be adjusted to zero due to the time lag in the operation of different relays. However, author used the fast operating relay with well designed electronic circuit for operation and obtained the angle of cutting very nearly at zero degree of waveshape of supply. The angle of cutting also slightly varies in different cases. These variations may be due to following reasons;

1. Supply voltage may not be purely sine function of time and fixed in magnitude.

2. Also the electronic circuit some times gives a distorted pulse and not sharp pulse due to non linear characteristic of tube.

*****

PHASE SHIFTING NETWORK FOR CONTROLLING THE OPERATING POINT OF THE RELAY.

Section (ii).

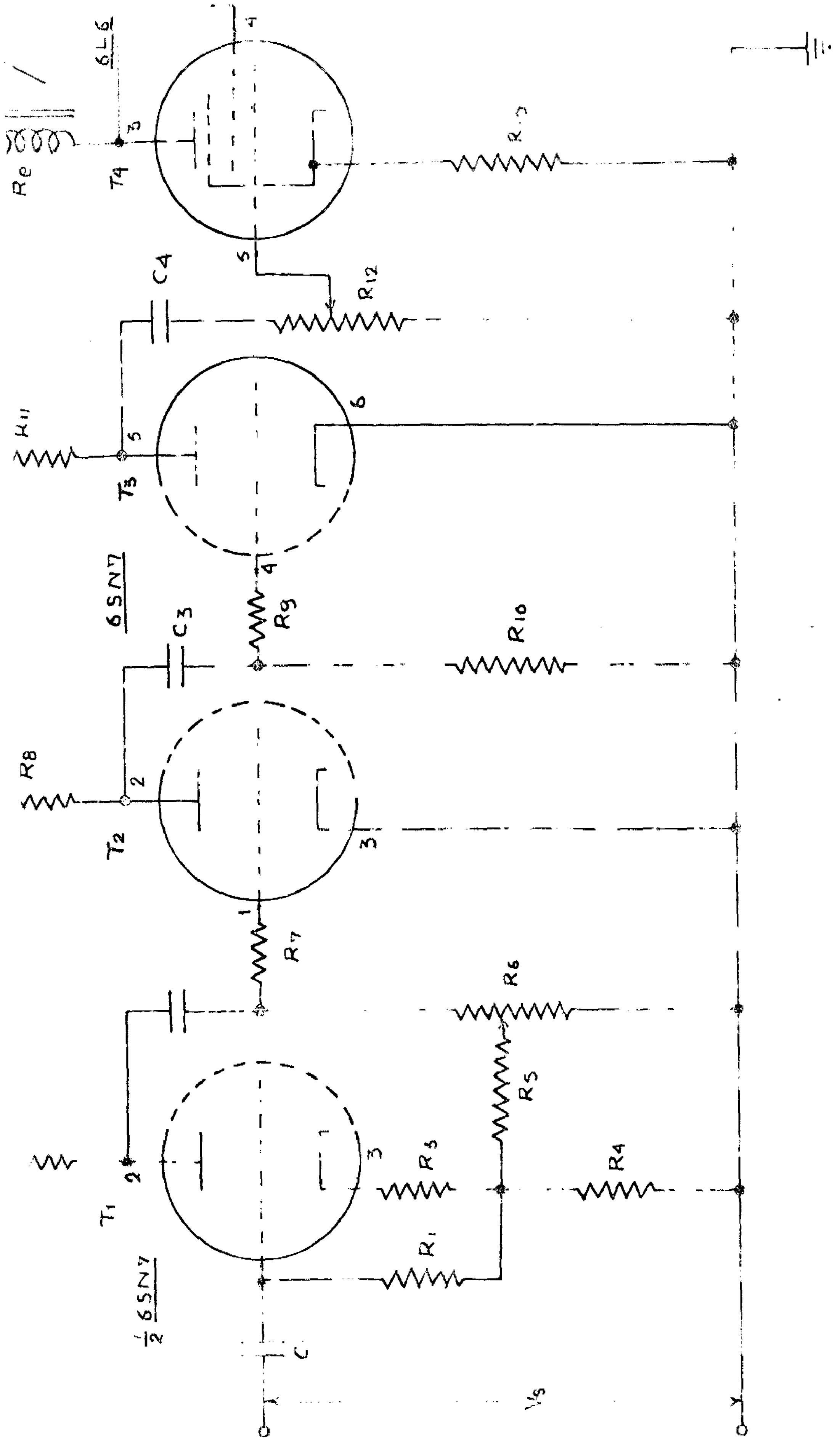
The electronic circuit as shown in the fig. has been designed for a wide-band, phase shifting purpose, to control the operating point of the relay with respect to the reference signal. The reference signal for electronic circuit has been derived from constant potential source. The same source is used for operating the Induction motor. The relay contacts are in series with motor and energized coil is driving its current from electronic circuit. The position of pulse, obtained from the circuit can be changed with respect to reference signal by adjusting the resistance of potentiometer R_4.

The circuit can be devided into three stages;

1) Phase shifting network (1/2 6SN7)
2) Clipper and differentiating network (6SN7)
3) The power amplifier (6L6).

The phase shifting network shifts the phase of the sinusoidal signal potential which is derived from the main supply for the Induction motor. The clipping circuit together with the differentiating network change the signal potential in the form of sharp peak pulses, which operate the required relay.

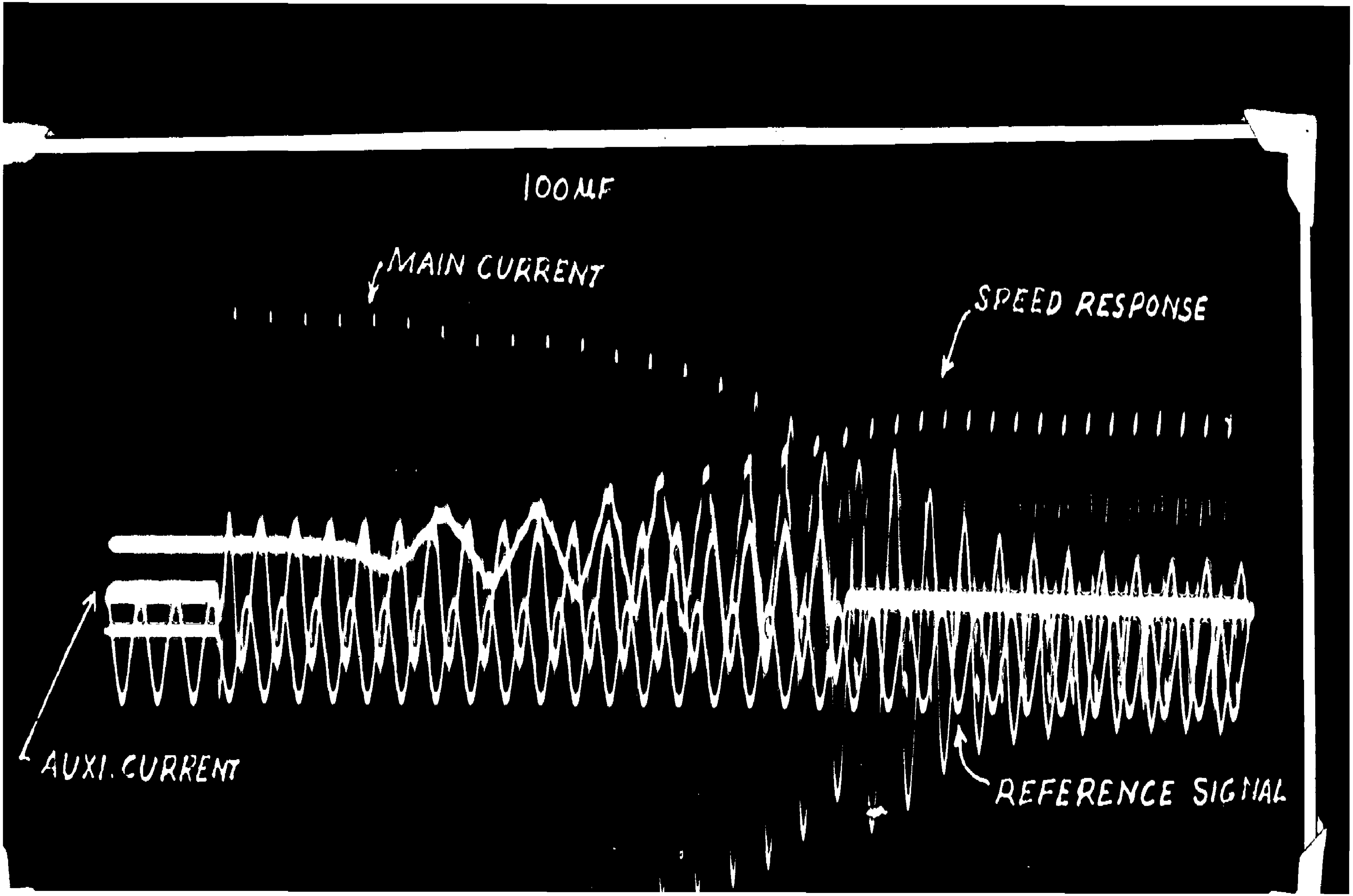
The operating and releasing currents of the 1500 ohms relay used in the experimental work were 75 ma and 15 ma respectively. In the absence of signal the current flowing

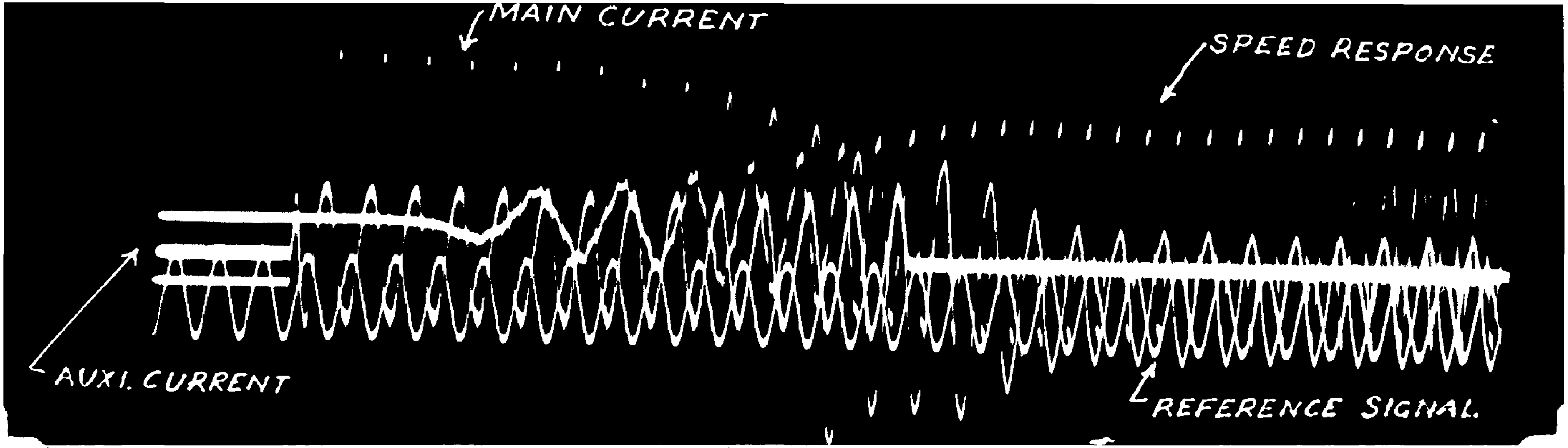


Re
T4
6L6
C4
R12
R11
T3
6SN7
C3
R9
R10
R8
T2
R7
R6
R5
T1
R3
R4
R1
½ 6SN7
C
Vs

through relay energized coil is nearly 50 ma which is not sufficient to operate the relay. As soon as the signal is impressed, the sharp positive pulse is applied to the grid of the power amplifier which increases the plate current of the power amplifier momentarily and operate the relay. Since the releasing current of relay is 15ma, the relay does not release even after pulse goes out.

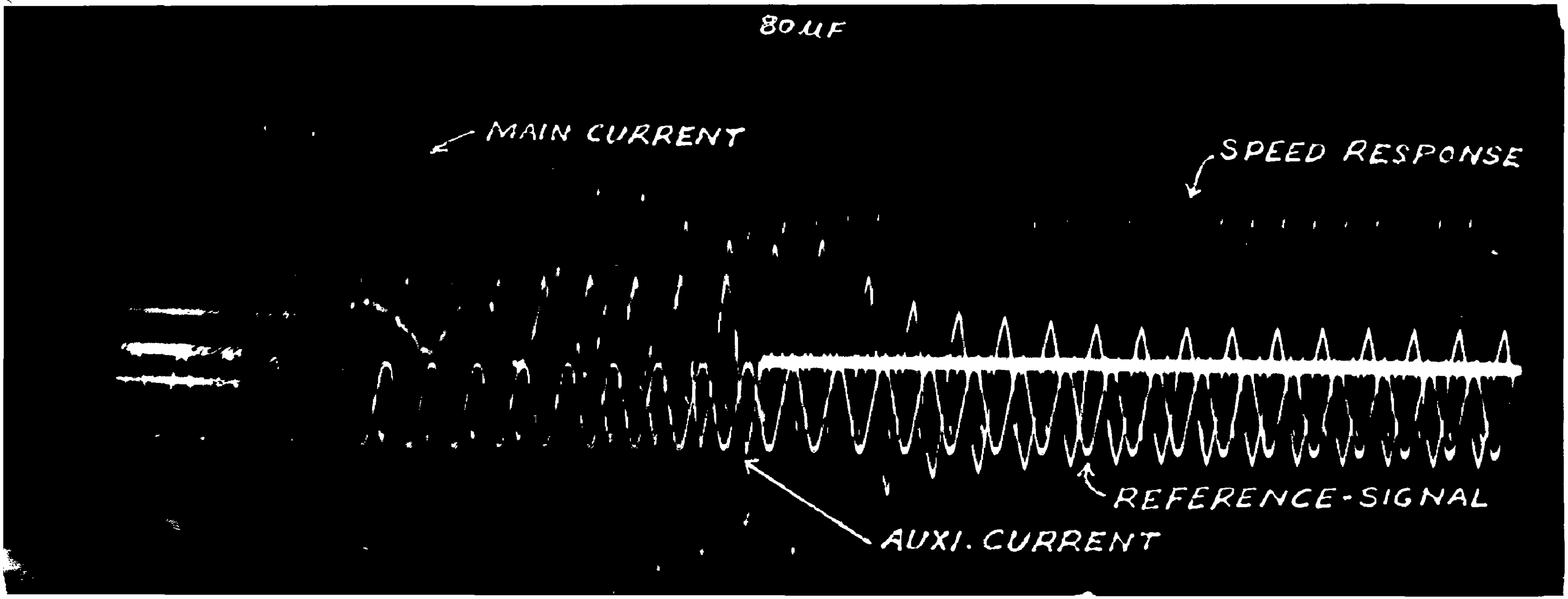
Thus the phase shifting network together with clipping circuit and differentiating network help in switching on the Induction motor at any desired angle of the applied reference voltage wave.

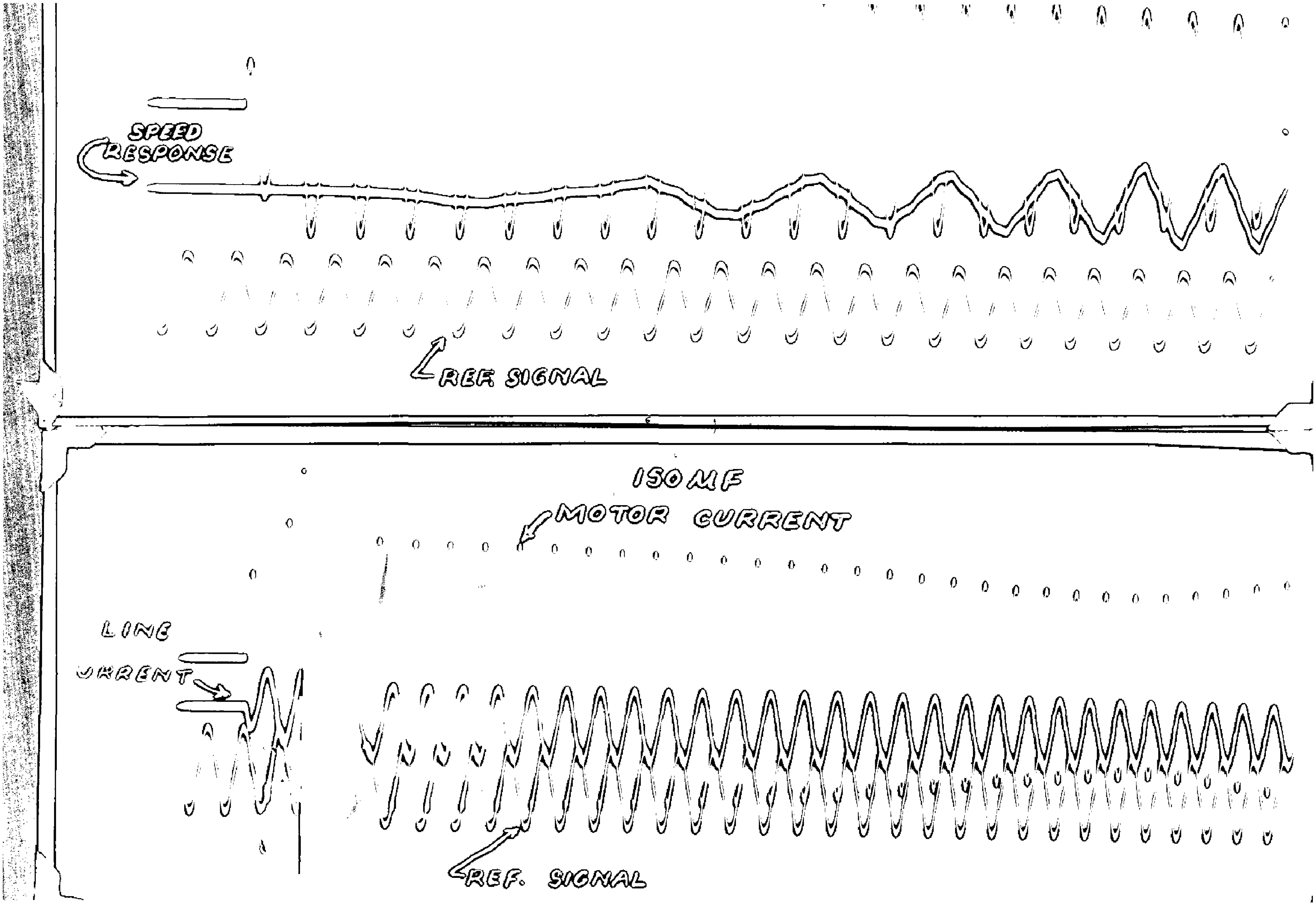

100 μF
MAIN CURRENT
SPEED RESPONSE
AUXI. CURRENT
REFERENCE SIGNAL

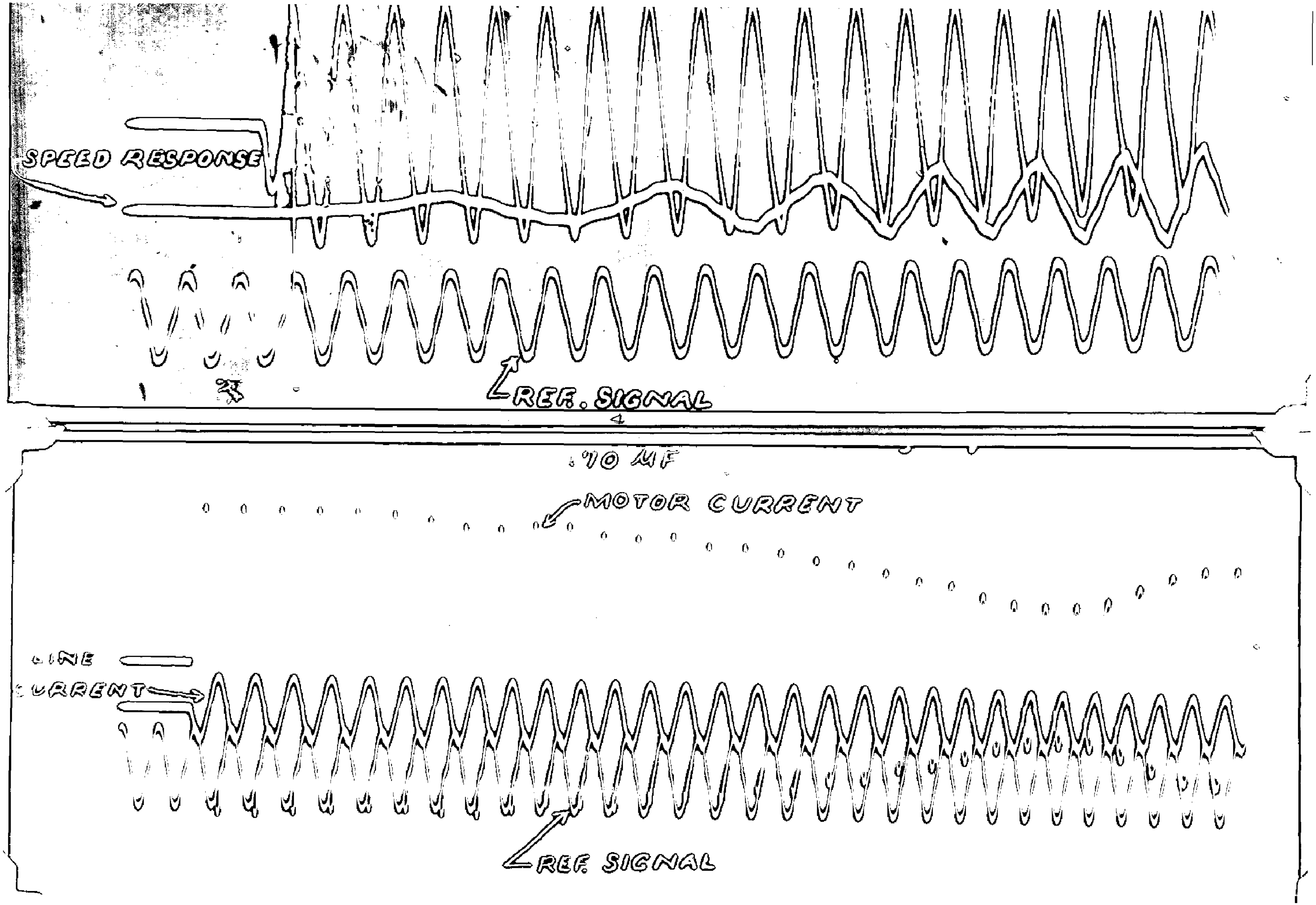

MAIN CURRENT
SPEED RESPONSE
AUXI. CURRENT
REFERENCE SIGNAL.

2

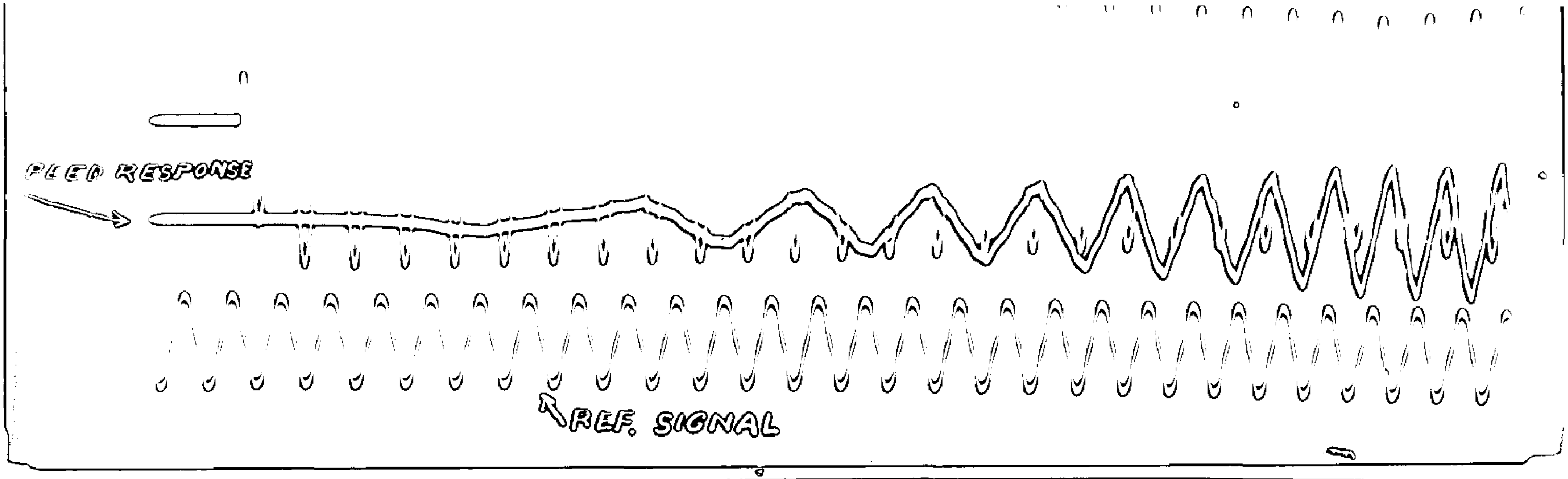
80 μF


MAIN CURRENT
SPEED RESPONSE
REFERENCE-SIGNAL
AUXI. CURRENT

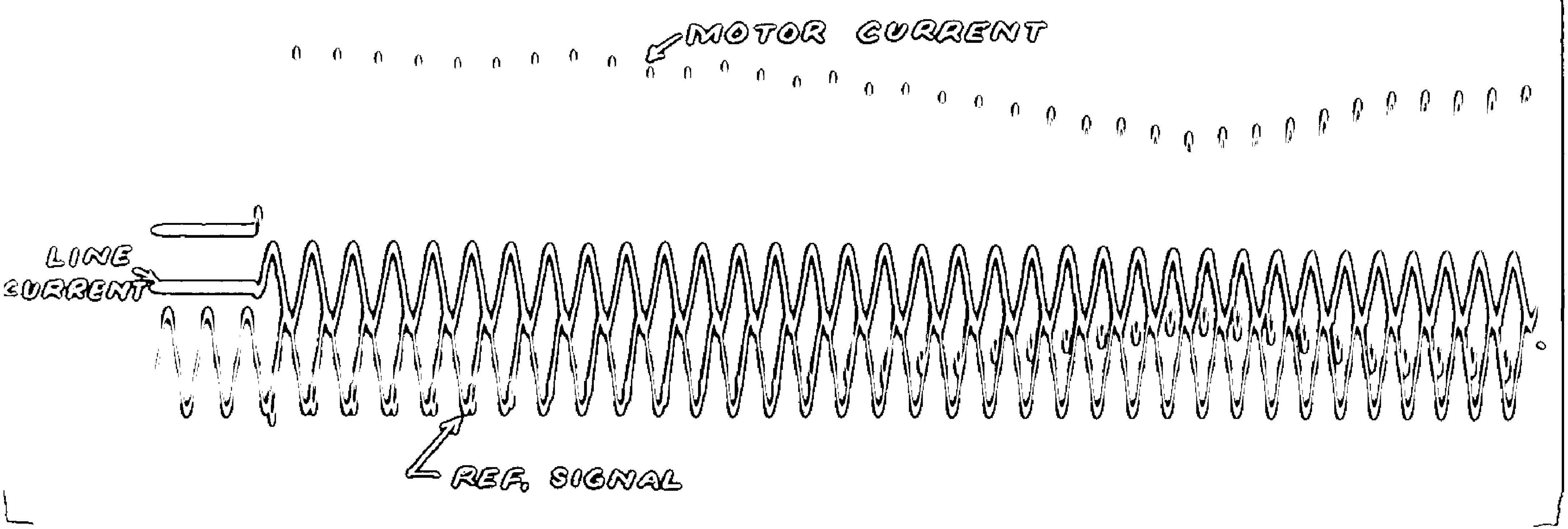

SPEED RESPONSE
REF. SIGNAL
150MF
MOTOR CURRENT
LINE
URRENT
REF. SIGNAL



SPEED RESPONSE
REF. SIGNAL
4
10 MF
MOTOR CURRENT
CURRENT
REF. SIGNAL



200 MF



10

.....CONCLUSION.....

The transient analysis of synchronous machine has been investigated thoroughly by many authors in the past but only a little amount of literature exists about the transient analysis of Induction machine.

In this dessertation the transient analysis for currents, voltages and torques on two different types of motors have been studied in detail and some interesting and useful results have been derived. The best starting value of capacitor on the single phase motor also gives minimum transient effect on the steady value of torque. The increase in average transient torque is much more in case of higher value of starting capacitor than in case of lower starting capacitor. The above statement applies for starting of both single phase Induction motor and the three-phase motor with the help of capacitor. But in the case of three-phase motor unbalance is more in case of deviation towards lower value of starting capacitor than when one uses a capacitor higher in value than the optimum starting capacitor. The actual plots of transient torque/time curve for three-phase motor show the variation of dying out time for different values of capacitor, as it increases slightly with increased values of capacitor.

In single-phase Induction motor the auxilliary winding is remaining in the circuit for very short duration and any abnormal behaviour of this current does not affect much the performance of system. However, in line current the ratio between transient and steady state is nearly four and system connected in parallel with Induction motor will be affected. The transient comes in the picture when slight variation is not tolerable even for a fraction of second such as servo system. The motor used for this purpose must be selected keeping in view its transient behaviour.

In case of three-phase Induction motor operation single-phase supply, the ratio of transient to steady state value is nearly 1.2 i.e. single-phase operation of three-phase Induction motor is itself a unbalance operation and motor does take negative sequence current and produces vibrations and heating of the motor. Although single-phase operation of three-phase Induction motor is unusual but still some work is going on unbalance operation of Induction motor in connection with speed torque control of motor and may be this transient analysis is useful in studying the unbalance operation of Induction motor.

The work presented by the author is an extension of the work of Venkata Rao in slightly indirect way. The values obtained theoretically have been compared with experimental results and found satisfactory.

.....APPENDICES.....

A. Derivation of $i_{\alpha b}(t)$

The differential equations

$$r_{\alpha b}\, i_{\alpha b} + x_{\alpha b} \frac{di_{\alpha b}}{dt} + x_c \int_0^t i_{\alpha b}\,dt + x_{mb} \frac{di_{\beta q}}{dt} = V(t) \quad ..1$$

$$x_{mb} \frac{di_{\alpha b}}{dt} + r_{\beta q}\, i_{\beta q} + x_{\beta q} \frac{di_{\beta q}}{dt} = 0 \quad .. \quad ..2$$

By taking Laplace transform and rearranging the term

$$r_{\alpha b}\, \bar{i}_{\alpha b} + x_{\alpha b}\, s x_{\alpha b} + x_c \frac{i_{\alpha b}}{s} + x_{mb}\, s i_{\beta q} = V(s) \quad ..3$$

$$x_{mb}\, s\bar{i}_{\alpha b} + r_{\beta q}\, \bar{i}_{\beta q} + x_{\beta q}\, s\bar{i}_{\beta q} = 0 \quad ..4$$

$$\frac{\bar{i}_{\alpha b}}{-V(s)\left(r_{\beta q}+s x_{\beta q}\right)} = \frac{\bar{I}_{\beta q}}{V(s)\, s x_{mb}} = \frac{1}{s^2 x^2_{mb} - (r_{\beta q}+s x_{\beta q})(r_{\alpha b}+s x_{\alpha b}+\frac{x_c}{s})}$$

$$\bar{i}_{\alpha b} = \frac{V(s)\,(r_{\beta q}+s x_{\beta q})}{(r_{\beta q}+s x_{\beta q})(r_{\alpha b}+s x_{\alpha b}+\frac{x_c}{s}) - s^2 x^2_{mb}}$$

$$\bar{i}_{\alpha b} = \frac{V(s)\, s(r_{\beta q}+s x_{\beta q})}{s^3(x_{\alpha b}x_{\beta q} - x^2_{mb}) + s^2(r_{\alpha b}x_{\beta q}+r_{\beta q}x_{\alpha b}) + s(r_{\alpha b}r_{\beta q}+x_{\beta q}x_c) + x_c r_{\beta q}}$$

$$\bar{i}_{\alpha b} = \frac{V(s)}{\sigma x_{\alpha b}\, x_{\beta q}} \cdot \frac{s(K_{\beta q}+s)}{s^3 + s^2\frac{(K_{\alpha b}+K_{\beta q})}{\sigma} + \frac{s(K_{\alpha b}K_{\beta q}+\frac{x_c}{x_{\alpha b}})}{\sigma} + \frac{x_c K_{\alpha b}}{\sigma x_{\alpha b}}}$$

$$\bar{i}_{\alpha b} = \frac{V(s)}{\sigma x_{\alpha b}}\, K_{\beta q}\, \frac{s(1+as)}{(1+T_1 s)(1+T_2 s)(1+T_3 s)}$$

$$V(s) = \frac{V\cos\alpha}{1+s^2} + \frac{V\sin\alpha\, s}{1+s^2}$$ for general value of α (phase angle)

but here angle of switching is zero

$$V_{(s)} = \frac{V}{1+s^2}$$

$$\bar{i}_{\alpha b} = \frac{V\, K_{\beta q}}{\sigma\, x_{\alpha b}} \qquad \frac{s(1 + as)}{(p_1+s)(p_2 + s)(p_3+s)(1 + s^2)}$$

By breaking in to partial fraction

$$\frac{s(1 + sa)}{(p_1+s)(p_2+s)(p_3+s)(1+s^2)} = \frac{A}{(p_1+s)} + \frac{B}{(p_2+s)} + \frac{C}{(p_3+s)} + \frac{Ds + E}{1 + s^2}$$

$$A = \frac{-p_1\,(1 - ap_1)}{(p_2-p_1)(p_3-p_1)(1 + p_1^2)}$$

$$B = \frac{-p_1\,(1-ap_2)}{(p_1-p_2)(p_3-p_2)(1 + p_2^2)}$$

$$C = \frac{-p_3\,(1 - ap_3)}{(p_1-p_3)(p_2-p_3)(1+p_3^2)}$$

$$D = \frac{\alpha + a\beta}{\alpha^2 + \beta^2}$$

$$E = \frac{\beta - a\alpha}{\alpha^2 + \beta^2}$$

where $\alpha = p_1p_2p_3 - (p_1 + p_2 + p_3)$

$\beta = (p_1p_2 + p_2p_3 + p_3p_1 - 1)$

Thus

$$i_{\alpha b}(t) = \frac{VK_{\beta q}}{\sigma\, x_{\alpha b}} \left[A\, e^{-p_1 t} + Be^{-p_2 t} + Ce^{-p_3 t} + D\cos t + E\sin t \right]$$

On substituting the values of motor constants in characteristic equation.

$$s^3 + \frac{K_{\beta q} + K_{\alpha b}}{\sigma} + \frac{s(K_{\alpha b} x_{K\beta\alpha} + \frac{x_c}{x_{\alpha b}})}{\sigma} + \frac{x_c\ K_{\beta q}}{\sigma \lambda_{\alpha b}} = 0$$

$$K_{\beta q} = 3.5 \times 10^{-2} \quad x_c = \frac{10^6}{314 \times 100} = 31.8$$

$$K_{\alpha b} = 10.5 \times 10^{-2} \quad \sigma = 1 - \frac{x_{mb}^2}{x_b\ x_q} = 1 - \frac{89.4^2}{94.2^2}$$

$$= 0.1$$

$$s^3 + s^2\ \frac{.14}{.1} + \frac{s(10.5 \times 10^{-2} x_{K\beta\alpha} + \frac{31.8}{94.3})}{.1} + \frac{31.8 \times 3.5 \times 10^{-2}}{94.3 \times .1} = 0$$

On solving this

$p_1 = .6865 - 1.975\ j$

$p_2 = .6865 + 1.975\ j$

$p_3 = .0266$

$$A = \frac{-p_1(1 - ap_1)}{(p_2 - p_1)(p_3 - p_1)(1 + p_1^2)}$$

$= -3.48 + 1.71j$

$B = -3.48 - 1.71j$

$C = -\ .00159$

$D = 6.5$

$E = 3.04$

$$i_{\alpha b}(t) = \frac{110}{28.6 \times .1 \times 94.3} \Big[(-3.48 + 1.71) e^{-(.6565 - 1.975j)t} + (-3.48 - 1.71j) e^{-(.6565t + 1.975j)t} - .00159 e^{-.0266t} + 6.5 \cos t + 3.04 \sin t \Big]$$

$$i_{\alpha b}(t) = 0.408 \Big[2e^{-.6565t} \times -\ .388 \cos(1.975t - 26.2) + 7.15 \cos(t - 25.1) - -.00159 e^{-.0266t} \Big]$$

$$= 0.408 \Big[-7.76 e^{-.6565t} \cos(1.975t - 26.2) + 7.15 \cos(t - 25.1) \cdot -.00159 e^{-.0266t} \Big]$$

B. <u>Derivation df $i_{\beta d}$</u> :-

The Laplace transform of two main differential equations.

$$(r_{\alpha a} + s x_{\alpha a})\bar{i}_{\alpha a} + s x_{ma}\, i_{\beta d} - V_{(s)} = 0 \qquad ..\ 1$$

$$s x_{ma}\, i_{\alpha a} + (r_{\beta d} + s x_{\beta d})\, \bar{i}_{\beta d} - 0 = 0 \qquad ..\ 2$$

$$\frac{\bar{i}_{\alpha a}}{-(r_{\beta d}+s x_{\beta d})\, V(s)} = \frac{\bar{I}_{\beta d}}{V(s)\, s x_{ma}} = \frac{1}{s^2 x_{ma}{}^2 - (r_{\alpha a}+s x_{\alpha a})(r_{\beta d}+s x_{\beta d})}$$

$$\bar{i}_{\beta d} = \frac{V_{(s)}\ S x_{ma}}{s^2(x_{ma}{}^2 - x_{\alpha a} x_{\beta d}) - s(r_{\alpha a} x_{\beta d} + x_{\alpha a} r_{\beta d}) - r_{\alpha a} x_{\beta d}}$$

$$= -\frac{V_{(s)}\quad S x_{ma}}{\sigma\ x_{\alpha a}\ x_{\beta d}\left\{s^2 + \dfrac{s(K_{\alpha a}+K_{\beta d})}{\sigma} + \dfrac{K_{\alpha a}K_{\beta d}}{\sigma}\right\}}$$

$$i_{\beta d} = \frac{V x_{ma}}{\sigma x_{\alpha a} x_{\beta d}} \quad \frac{s}{(p_1+s)\ (p_2+s)\ (1+s^2)}$$

After taking Laplace inverse

$$i_{\beta d}(t) = -\frac{V\ x_{ma}}{\sigma x_{\alpha} x_{\beta d}}\left[\frac{p_1 e^{-p_1 t}}{(p_1-p_2)(1+p_1{}^2)} + \frac{p_2}{(p_2-p_1)(1+p_1{}^2)} e^{-p_2 t} + \right.$$

$$\left. + \frac{\cos\ (t - \psi)}{\sqrt{(1+p_1{}^2)(1+p_2^2)}}\right]$$

$$\psi = \tan^{-1}\frac{1}{p_1} + \tan^{-1}\frac{1}{p_2}$$

$p_1\ p_2$ are the roots of characteristic

$$s^2 + \frac{s(K_{\alpha a} + K_{\beta d})}{\sigma} + \frac{K_{\alpha a}\ K_{\beta d}}{\sigma} = 0$$

$$s^2 + 1.445 + .0460 = 0$$

$$p_1 = 1.414$$

$$p_2 = .0325$$

After substituting the values of p_1 and p_2

$$i_{\beta d}(t) = V \left[-0.0584 e^{-1.426t} + 0.00404 e^{-0.0325t} - 0.1 \cos (t-123.5) \right]$$

C. Derivation of $i_{c(t)}$

$$\bar{i}_{\alpha f} = \frac{E_A}{2\sigma x_\alpha}\left\{\left[\frac{(s+K_\beta)}{(1+s^2)\;\; s^2 + \frac{s(K_\alpha + K_\beta)}{\sigma} + \frac{K_\alpha K_\beta}{\sigma}}\right] - \frac{s^2(s+K_\beta)}{\sqrt{3}(1+s^2)} x \frac{1}{(\text{Ch.equation})}\right\}$$

$$\text{Ch.eq}^n. = s^3 + \frac{s^2(K_{ac} + K_\beta)}{\sigma} + \frac{s\,(x_c' + K_\beta K_{ac})}{\sigma} + \frac{K_\beta x_c'}{\sigma}$$

$$\bar{i}_b = \frac{E_A}{2\sigma x_\alpha}\left\{\left[\frac{(s+K_\beta)}{(1+s^2)\left\{s^2 + s\frac{(K_\alpha + K_\beta)}{\sigma} + \frac{K_\alpha K_\beta}{\sigma}\right\}}\right] + \frac{s^2(s+K_\beta)}{\sqrt{3}(1+s^2)}\, x\right.$$

$$\left. x \frac{1}{\left(s^3 + \frac{s^2(K_{ac}+K_\beta)}{\sigma} + \frac{s(x_c' + x_\beta K_{ac})}{\sigma} + \frac{K_\beta x_c'}{\sigma}\right)}\right\}$$

$$s^3 + s^2\frac{K_{ac}+K_\beta}{\sigma} + \frac{s\,(x_c' + K_\beta K_{ac})}{\sigma} + \frac{K_\beta x_c'}{\sigma} = 0$$

For 170 μF

$K_\alpha = 3.1 \times 10^{-2}$

$K_\beta = 2.32 \times 10^{-2}$

$\sigma = 1 - \left(\frac{x_m}{x}\right)^2 = 1 - 0.94 = 0.06$

$K_{ac} = \frac{r_1 + 2\, r_c}{x_\alpha} = 4.56 \times 10^{-2}$

$x_c' = \frac{2\, x_c}{x} = 0.29$

$x_1 = 4.15\,\Omega$

$x_2 = 4.15\,\Omega$

$x_m = 125\,\Omega$

$x_c = 18.75\,\Omega$

$r_c = 0.94\,\Omega$

$x_\alpha = 129.15\,\Omega$

$x_\beta = 129.15\,\Omega$

$r_1 = 4$ ohms Ω

$r_2 = 3$ ohms

On substituting these constants

$$s^3 + 1.15s^2 + 4.84s + 0.112 = 0$$

$$p_1 = 0.563 - 2.11j$$

$$p_2 = 0.563 + 2.11j$$

$$p_3 = 0.0231$$

Now second half portion of $i_{\alpha f}(t)$ and $i_{\alpha b}(t)$ is same. Therefore we will take second half portion.

$$\frac{s^2(s + K_\beta)}{(1+s^2)(p_1+s)(p_2+s)(p_3+s)} = \frac{A}{(p_1+s)} + \frac{B}{(p_2+s)} + \frac{C}{p_3+s} + \frac{Ds + E}{1 + s^2}$$

$$A = \frac{p_1^2(-p_1 + K_\beta)}{(p_2 - p_1)(p_3-p_1)(1 + p_1^2)}$$

$$B = \frac{p_2^2(-p_2 + K_\beta)}{(p_1-p_2)(p_3-p_2)(1 + p_2^2)}$$

$$C = \frac{p_3^2(-p_3 + K_\beta)}{(p_1-p_3)(p_2-p_3)(1+p_3^2)}$$

$$D = \frac{-\alpha + \beta K_\beta}{\alpha^2 + \beta^2}$$

$$E = \frac{-\beta - \alpha K_\beta}{\alpha^2 + \beta^2}$$

$$\alpha = p_1p_2p_3 - (p_1 + p_2 + p_3)$$

$$\beta = p_1p_2 + p_2p_3 + p_3p_1 - 1$$

Thus the solution of second half

$$i_f(t)_2 = \frac{1}{\sqrt{3}} \frac{E_A}{\tau X_\alpha} \frac{1}{2}\left[A e^{-p_1t} + Be^{-p_2t} + Ce^{-p_3t} + D \cos t + E \sin t\right].$$

First part of $i_{\alpha f}(t)$ will be solved as follows

characteristic eq^n.

$$s^2 + \frac{K_\alpha + K_\beta}{\sigma} s + \frac{K_\alpha K_\beta}{\sigma} = 0$$

$$s^3 + 0.95 + 1.2 \times 10^{-2} = 0$$

$$p_1 = 0.8865$$

$$p_2 = 0.0135$$

$$i_f(t)_1 = \frac{E_A}{\sigma x_\alpha}\left[\sqrt{\frac{1 + K_b^2}{(1+p_1^2)(1+p_2^2)}} \text{ Sin } (t-\theta) + \right.$$

$$\left. \frac{(K_\beta - p_1)}{(p_2-p_1)(1+p_1^2)} e^{-p_1 t} + \frac{K_b - p_2}{(p_1-p_2)(1+p_2^2} e^{-p_2 t} \right]$$

The combined solution of $i_{\alpha f} t$ would be the sum of the two different parts

$$i_c(t) = i_{\alpha f}(t) - i_{\alpha b}(t)$$

Hence $i_c(t)$ would be given by only second part of the above solution.

$$i_c(t) = \frac{1}{\sqrt{3}} \frac{E_A}{\sigma x_\alpha} \left[A e^{-p_1 t} + Be^{-p_1 t} + Ce^{-p_3 t} + D \cos t + E \text{ Sin } t)\right]$$

$$A = \frac{p_1^2 (-p_1 + K_\beta)}{(p_2-p_1)(p_3-p_1)(1+p_1^2)}$$

$$= -0.037 - 0.285j$$

$$B = -0.037 + 0.285j$$

$$D = 0.0726$$

$$E = -0.244$$

$$C = 0$$

After substituting these values and rearranging in proper form

$$i_c(t) = \frac{E_A}{\sigma x_\alpha} \frac{1}{\sqrt{3}} \left[- 0.572 e^{-0.563t} \text{Cos}(2.11t+82.6) + 0.255 \text{Cos}(t+73.4)\right]$$

In a similar way for 150 and 200 μF $i_c(t)$ will be given by

for 150 μF

$$i_c(t) = \frac{E_A}{-X_\alpha} \frac{1}{\sqrt{3}} \left[0.216 \text{ Cos } (t + 75.6) - 0.556 e^{-0.578t} \times \text{Cos}(2.27t + 85.3) \right]$$

For 200 μF

$$i_c(t) = \frac{E_A}{-X_\alpha} \frac{1}{\sqrt{3}} \left[0.306 \text{ Cos } (t+71) - 0.6365 e^{-0.538t} \times \text{Cos}(1.95t + 82.1) \right]$$

****

D: Derivation of $i_{\beta f}(t)$:-

$$i_{\beta f}(s) = \frac{E_A\, x_m}{2\sigma\, x_\alpha\, x_\beta}\left[\frac{-s}{(1+s^2)\left\{s^2+\frac{s(K_\alpha+K_\beta)}{\sigma}+\frac{(K_\alpha K_\beta)}{\sigma}\right\}}+\frac{1}{\sqrt{3}}\frac{s^3}{(1+s^2)(\text{ Ch. equation })}\right] \quad \ldots 1$$

This current can be spilted up in to two parts i.e. $i_{\beta f1}$ and $i_{\beta f2}$ for simplicity.

$$i_{\beta f1}(s) = \frac{E_A\, x_m}{2\sigma\, x_\alpha\, x_\beta}\left[\frac{-s}{(1+s^2)\left\{s^2+\frac{s(K_\alpha+K_\beta)}{\sigma}+\frac{K_\alpha K_\beta}{\sigma}\right\}}\right] \quad \ldots 2$$

Thus Ch. equation for it,

$$s^2+\frac{s(K_\alpha+K_\beta)}{\sigma}+\frac{K_\alpha\ K_\beta}{\sigma}=0 \quad \ldots 3$$

After substituting the motor constants and solving we get the root of this expression as

$p_1 = 0.8865$

$p_2 = 0.0135.$

$$i_{\beta f1}(s) = \frac{E_A x_m}{2\sigma x_\alpha\, x_\beta}\left[\frac{-s}{(1+s^2)\ (p_1+s)\ (p_2+s)}\right] \ldots 4$$

By taking Laplace inverse

$$i_{\beta f1}(t) = \frac{E_A\, x_m}{2\sigma\, x_\alpha\, x_\beta}\left[-\ 0.565\ e^{-0.8865t}+0.0147e^{-0.0135t}-0.746\ \text{Sin}\ (t+312.5)\right] \ldots 5$$

For 2nd part of equation the roots of the characteristic equation are

$p_1 = 0.563 - 2.11j$

$p_2 = 0.563 + 2.11j$

$p_3 = 0.0231$

$$i_{\beta f2}(s) = \frac{E_A}{\sigma x_\alpha x_\beta} \frac{1}{\sqrt{3}} \left[\frac{s^3}{(1+s^2)(p_1+s)(p_2+s)(p_3+s)}\right] \qquad \ldots 6$$

Putting the values of p_1, p_2, p_3 and taking Laplace inverse of it,

$$i_{\beta f2}(t) = \frac{E_A}{\sigma x_\alpha x_\beta} \frac{1}{\sqrt{3}} \left[-0.147 \cos(t-254.7) - e^{-0.563t} \times 0.394 \cos(2.11t-5.6)\right] \qquad \ldots 7$$

Total $i_{\beta f}(t)$ will be the sum of two terms.

$$i_{\beta f}(t) = \frac{E_A x_m}{2\sigma x_\alpha x_\beta}\left[-0.565e^{-0.8865t} + 0.0147e^{-0.0135t} - 0.746 \times \sin(t-312.6) - 0.147 \cos(t-254.7) - 0.394e^{-0.563t} \cos(2.11t-5.6)\right] \qquad \ldots 8$$

On solving further, it can be written as

$$i_{\beta f}(t) = \frac{E_A x_m}{2\sigma x_\alpha x_\beta}\left[-0.565\, e^{-0.8865t} + 0.0147e^{-0.0135t} - 0.826 \cos(t-57.8) + 0.394e^{-0.563t} \times \cos(2.11t-5.6)\right] \qquad \ldots 9$$

$i_{\beta b}(t)$ can also be derived in the same way only it will be the difference of two terms $i_{\beta b1}$, $i_{\beta b2}$ as $i_{\beta f1}$, $i_{\beta f2}$ is sum for forward current.

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....REFERENCES....

1. Transient analysis of alternating current machinery(book) W.V. Lyon, Johnwiley & Sons.

2. Transient starting torque in Induction motors, A.M. Wahl, L.A. Kilgore, A.I.E.E. Trans. 1940 Vol.59, Page 603-607.

3. Transient conditions in Electric machinery, W.V. Lyon, A.I.E.E., 1923 pp. 157-179.

4. Transient analysis of A.C. machinery, Yu H. Ku ibid Vol.48, July 1929, pp.707.

5. Transient torques in squirrel cage Induction motor with special reference to plugging. E.S. Gilfillan, Jr. E.L. Kaplan, A.I.E.E. 1941, pp.1200-1209.

6. Transient performance of Induction motor, I.J. Maginniss, N.R. Schultz, A.I.E.E. Vol.63 Sept. 1944, pp.641-46.

7. Electromechanical transient performance of Induction motors, C.N. Weggandt, S. Charp, A.I.E.E., Vo.65 1946, pp.1000-1009.

8. Switching transient in single-phase Induction motor with constant speed. P. Venkata Rao, A.I.E.E. Trans. Vol.78, Pt.3 (power system and apparatus) No.44, Oct. 1959.

9. Induction motor switching transient in single-phase, A.I.E.E. page 1023-9, Part III.

10. Single-phase operation of three-phase Induction motor with static phase converter, A.I.E.E. Trans. 1954.

11. Journal of Indian Institute of Science, Sec.A, Vol.39 1957 page 101. (Switching transient in three-phase Induction motor run on single-phase supply) By P. Venkata Rao.

12. Proceedings I.E.E. 1959(Aug.). Jha, C.S.

13. Proceedings I.E.E. Vol. 106 Part A No.26, April 1959. Jha C.S.

14. Performance and design of A.C. commutator motors, Taylor book (Pitman & Sons. Ltd., London)pp. 306.