# EFFECT OF DAMPER WINDINGS ON THE TRANSIENT STABILITY OF SYNCHRONOUS MACHINES

A

DISSERTATION

**Submitted in partial fulfilment of the requirements for the degree of**

MASTER OF ENGINEERING

in

ELECTRICAL MACHINE DESIGN



BY

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## ...C E R T I F I C A T E...

Certified that the dissertation entitled "Effect of Damper Windings on the Transient Stability of Synchronous Machines" which is being submitted by Sri T.B. Partha Sarathy in partial fulfilment for the award of the Degree of Master of Engineering in Electrical Machine Design of University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for a period of six months from 5th January, 1963 to 5th July, 1963 for preparing dissertation for Master of Engineering Degree at the University.

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## ...C O N T E N T S...

Pages

## ...S Y N O P S I S...

The scope of this dissertation is to study the dynamic behaviour of a synchronous machine under transient conditions. In order to make a detailed study of the transient stability, equations showing the variation of torque and load angle of a synchronous machine connected to a fixed supply voltage under various types of aperiodic disturbances have been developed with the assumption of constant field flux linkages. The behaviour of the synchronous machine is investigated by the step-by-step method using the equations developed. The effect of damper windings from stability considerations alone is analysed in detail. Details of experimental tests conducted to verify the theoritical analysis are given at the end.

## ...NOMENCLATURE...

$A$ = Armature decrement factor

$a, b$ = Damping coefficients.

$F$ = A factor describing the flux linkage decay.

$G(p)$ = Operational function.

$H$ = Inertia constant.

$i$ = Instantaneous current.

$i_{dt}, i_{qt}$ = Components of axis currents.

$i_{dc}$ = Instantaneous D.C. component of armature

$I_r$ = Effective value of rotor current.

$I_{r2}$ = Negative sequence rotor current.

$I_s$ = Effective value of stator current

$I_{s2}$ = Negative sequence stator current.

$K$ = Relative damping coefficient.

$L_a, L_f, L_{kd}, L_{kq}$ = Leakage inductances.

$L_e$ = External inductance.

$L_{md}, L_{mq}$ = Magnetizing inductances.

$p$ = Heaviside operator $\frac{d}{dt}$

$r$ = Winding resistance.

$r_1$ = Positive sequence resistance

$r_2$ = Negative sequence resistance

$r_s$ = Stator resistance

$r_r$ = Rotor resistance

$S$ = Slip

$T_{ac}$ = Accelerating torque

$T_b$ = Braking torque.

$T_D$ = Positive sequence damping torque.

$T_d$ = Damping torque coefficient.

$T_e$ = Electrical torque of alternator.

$T_i$ = Instantaneous torque developed at any angle $\delta$

$T_j$ = A coefficient which when multiplied by acceleration gives torque

$T_L$ = Abtupt shaft torque

$T_m$ = Input torque from prime mover.

$T_o$ = Initial shaft torque.

$T_u$ = Unidirectional torques.

$T_a$ = Armature time constant.

$T_d'$ = Short circuit transient time constant

$T_{do}'$ = Open circuit transient time constant.

$T_d'', T_q''$ = Short circuit subtransient time constants.

$T_{do}'', T_{qo}''$ = Open circuit subtransient time constants.

$T_{kd}$ = Damper winding time constant.

$\dot{u}$ = Instantaneous speed in radians/second.

v = Instantaneous voltage.

$V, V_m$ = R.M.S. and maximum voltages of fixed supply.

$V_o$ = Voltage behind synchronous reactance

$V_q'$ = Voltage behind transient reactance.

$X_{d(p)}, X_{q(p)}$ = Operational impedances.

$X_d, X_q$ = Synchronous reactances.

$X_d', X_q'$ = Transient reactances.

$X_d'', X_q''$ = Subtransient reactances.

$X_e$ = External reactance

$X_2$ = Negative sequence reactance

$X_o$ = Zero sequence reactance

$\delta$ = Instantaneous load angle

$\Psi$ = Armature flux linkages.

$\omega$ = Synchronous speed.

Suffixes:

a, b, c = Armature phases.

d = direct axis

$e$ = Parameter of modified alternator

$f$ = field winding

$K_d$, $K_q$ = Damper windings on direct & quadrature axes.

$q$ = Quadruature axis,

$r$ = Rotor

$S$ = Stator.

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# ..INTRODUCTION..

Stability of a power system is the ability of a system to remain in synchronous equilibrium under steady operating conditions and to regain the state of equilibrium after a disturbance has taken place.

The criterion of stability of a power system can be studied under two broad headings - steady state stability and transient state stability. The former deals with the stability of the system under steady load conditions and with strictly constant armature and field currents in all synchronous machines. The latter criterion investigates the stability of the system under transient or aperiodic disturbances.

Under transient disturbances, the currents and potentials of the system do not change abruptly and assume new values to suit the new operating conditions. There is a transition period during which transient values of these quantities appear, each consisting of several components. Usually, the transient components diminish more or less rapidly at different rates and then disappear. However, during this process, the resultant current or potential may reach an abnormal or even disastrous value. It is for this reason that the study of the transient behaviour and how they can be controlled is an important engineering problem. Moreover, the present trend towards larger individual generating units and towards greater interest in their automatic control increases the need for detailed knowledge of this aspect of power-system performance.

A power-system transient stability study is mainly concerned with the determination of the maximum load that the system can carry without loss of synchronism, after a sudden change in the operating conditions. The most important quantity determining the transient behaviour is the transient torque. In the study of the transient behaviour of synchronous machines, it is essential to know the torque - angle characteristic of the given machine under conditions resulting from disturbances of various kinds.

Whether the machine will stay in synchronism under the disturbed conditions can be studied by the swing curves which show the variation of rotor angle with time. These indicate whether the machine is liable to lose synchronism under the disturbed conditions.

Transient analysis of system dynamics by manual methods is impracticable for all but the simplest cases, because of time and labour involved. Some form of model or computer is normally essential in a serious study.

In this field, there is a division of interest between, on the one hand, studies of complete power systems with many generators operating in parallel and, on the other hand, detailed studies of only a single machine or of a few interconnected machine groups.

The A.C. network analyser is extensively used in the study of various disturbances that arise in power systems. Because the analyser is essentially a steady - state representation of a power system, a step-by-step process must be used for transient investigations i.e. the transient period is divided into a evaluated by double numerical integration of the accelerating torque at each interval to form the swing curves of the system. This procedure is, however, tedious and therefore other methods have been developed. The most significant of these is the use of the digital computer to solve transient stability problems, with the network analyser calculating detailed load flows.

Modelling of the power system with specially designed micro-machines and micro-networks which simulate larger systems is a technique that is attracting attention. While it is difficult to carry out detailed investigations on large-scale equipment to obtain a full theoritical explanation of many phenomena, these small models can be tested more fully and their basic parameters can be measured more precisely than is possible with large-scale equipment. Transient studies may be carried out directly using this equipment, without recourse to step-by-step methods. The model is energized from a 3 phase supply and unbalanced and fault conditions can be investigated with relative ease. This method is particularly suitable for some studies like pole slipping, asynchronous running and resynchronization which cannot be undertaken by computer techniques because of the continuous and protracted nature of the changing conditions. The micro machines lend themselves more readily for investigations of this kind as they are continuously running.

In the present dissertaion, a small synchronous machine of 4 KVA capacity available in the laboratory is chosen to study its behaviour when connected to a fixed supply voltage under transient conditions subsequent to a system disturbance and also to study the effect of the damper windings on transient stability.

The main types of disturbances that have been discussed are-

1. Occurrence of system faule.
2. A sudden change in external reactance due to switching in of a reactance.
3. Sudden application of shaft loads.

and 4. Asynchronous operation owing to a system disturbance such as a severe short circuit.

Equations for torque-angle characteristic of the machine and swing curves are formed, the latter by double numerical integration of the accelerating torque using the step-by-step method.

In chapter I, the transient conditions due to system faults have been studied. Detailed analysis is given for the most severe fault i.e. a 3-phase short-circuit. The effect of the damper windings both during the fault and after the fault is cleared is clearly shown.

Chapter II deals with the transient behaviour due to sudden change in reactance and Chapter III deals with disturbances arising from sudden application of loads to the shaft of a synchronous motor. The effect of the damper windings is again discussed fully.

A method of calculating the currents and torque in a synchronous motor running at a constant speed away from synchronism is given by Linville.[23*] In chapter IV this method is extended to study the criteria for resynchronising in the case of an alternator running continuously out-of-step. How the damper windings play an important role in resynchronization without disconnecting the machine is shown by means of curves.

A detailed and thorough experimental varification of the theoritical results could not be undertaken because of lack of equipment and facilities. However within the limitations as above, an attempt has been made to correlate the test data with the conclusions arrived at from theoritical considerations.

...CHAPTER I...

...TRANSIENT BEHAVIOUR OF AN ALTERNATOR DURING SYSTEM FAULTS...

1.1 Three phase short-circuit of an unloaded alternator..

Although the majority of faults occuring in practice on a power system are unsymmetrical between the phases, the symmetrical fault is important, because, although rare, it is more severe, It is, moreover a simpler condition to analyze and therefore forms a suitable starting point for a study of fault conditions. The short circuit test, in which the three terminals of an unloaded alternator are all short-circuited simultaneously, is a well-established method of checking its transient characteristics. The present section gives a full analysis of a sudden symmetrical short circuit of an unloaded alternator, and the solution is then extended in section 1.7 to a loaded machine.

When a generator is suddenly short circuited, a transient electrical torque consisting of unidirectional and alternating components is suddenly impressed on the generator rotor. Each component of this torque decays exponentially to a value determined by the steady state terminal conditions of the machine. In the case of a salient-pole generator, there is an infinite series of alternating components of harmonic frequency; whereas, for the turbo-alternator, the only alternating torques of appreciable magnitude are those at fundamental and second harmonic frequency. The turbo generator and the salient-pole machine with connected dampers are close approximations during transient and sub-transient periods. The saleint-pole machine with no dampers will have torques containing all odd and even harmonics.

The calculation of shaft and frame stresses in turbo generators during short circuits also is dependent upon a knowledge of the electrical torque developed at the rotor air gap. The torque discussed here is the electrical torque developed at the air gap of the machine and not the torque at the coupling or that transmitted to the foundation. The torque in couplings, shafts and foundations must be calculated from a consideration of masses and mechanical spring constants involved.

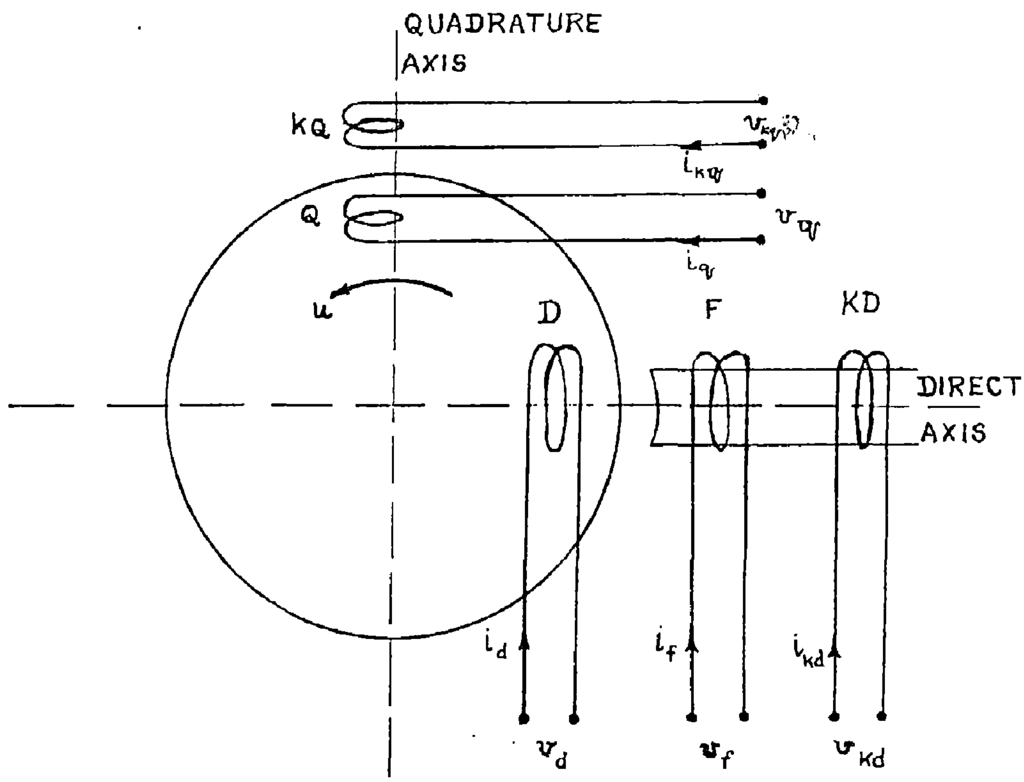
As stated above, the electrical torque developed at the rotor contains unidirectional, fundamental frequency and second harmonic components, all of which are damped. It is the steady components of torque which are of interest in the calculation of system stability, while the mechanical design engineer will be mainly interested in the alternating torques.

In the practical range of machine design, the fundamental frequency component has the dominant effect on the resulting transient shaft torques for the more severe types of disturbances. It was found that for short circuits at no load, with no external resistance, a line-to-line fault results in the most severe shaft torques. For faults with low external resistance, a double-line-to-ground fault produces the highest shaft torques, which, for mechanical systems of low natural frequency, may exceed the torques produced by a line-to-line fault with zero resistance. However, for values of external resistance as high as machine sub-transient reactance, the shaft torques are lower than with zero resistance. The most severe fault torques are produced by line-to-line short circuits when delivering full load with no external impedance.

The electrical torques resulting from unsymmetrical faults will always be smaller than the 3-phase short circuit, and hence the latter type of faults are important from stability point of view.

When the armature is short circuited, a portion of normal armature flux is trapped in the armature circuit, this portion being dependent upon the time of application and the type of short circuit. The decay of the trapped flux depends upon the time constant of the armature circuit containing the flux. This flux manifests itself as a D.C. component in the short circuit current. If the short circuit is unbalanced, the armature current will contain, in addition to the D.C. component, positive and negative sequence components of fundamental frequency. These components of current will set up positive and negative sequence armature MMF's and (neglecting saturation) positively and negatively rotating fields. The sudden appearance of the positve sequence armature MMF over the direct-axis of rotor causes rotor currents to rise suddenly, counterbalancing this additional direct-axis excitation. This follows from the fact that the flux linking the rotor circuits can not change instantly[1]. The additional rotor currents, induced by the sudden appearance of the positive sequence field, are unsupported by exciter voltage and they decay with decrements which depend upon the short circuit rotor time constants and the type of fault. The net excitation on the direct-axis decays and as a consequence, the air gap flux also decays.

R.H. Park's formulae[2,3,4] for the armature and field flux linkages of an ideal synchronous machine are used as a starting point. The machine is considered to have two field circuits in the direct-axis, referred to as the main and additional field (amortisseur winding). In the quadrature axis, there is no main field; so only the additional field is present. Direct current excitation is applied only in the main field.




[illegible]

## ...1.2 Basic Assumptions:...

To simplify the mathematical work and to use the operational solutions, certain reasonable and practical assumptions have been made.

A synchronous machine idealized to the following extent will be considered. (In the ideal machine, linear relationships exist between magnetic flux and currents in any part of the machine, so that the principle of superposition applies).

1. The air gap is uniform.
2. Saturation and Hysteresis are neglected in calculating the alternating components of torque.
3. The stator has a sinusoidally distributed 3 phase winding.
4. The air gap flux is sinusoidally distributed in space.
5. Armature and field resistances are negligible except in determining the decrements.
6. The effect of higher harmonics are neglected.
7. The short circuit is assumed to occur at the instant of maximum interlinkage.
8. Synchronous speed is maintained during short circuit.
9. The short circuit is assumed to occur directly at the terminals of the machine at a time when it is carrying no load.

Let $t = 0$ at the instant of short circuit and let $\lambda$ be the angle between the axis of phase A and the direct-axis at that instant. The angle $\lambda$ defines the point in the A.C. cycle at which the short circuit occurs. Then from assumption 8,

$$\theta = \omega t + \lambda$$

By the principle of superposition, the voltages and currents after the short circuit are each equal to the sum of the original value and the change resulting from the short circuit. The original values are denoted thus - $i_{a_o}$ and the change due to short circuit thus - $i_a'$.

The solution is best obtained by using the principle of superposition. The equations giving the changes in the currents are obtained by putting $v_f' = 0$ $v_d'=0$ and $v_q' = -v_q 1$ in the equations for the superimposed quantities corresponding to equations A.1. Speed remains constant at $\omega$.

$$\begin{aligned} \therefore 0 &= p\psi_d' - \omega\psi_q' - r_a i_d \\ -v_q 1 &= +\omega\psi_d' + p\psi_q' - r_a i_q \end{aligned} \quad \ldots 1.1$$

$i_d$ & $i_q$ have the same values as $i_d'$ and $i_q'$ because $i_{do}=i_{qo}=0$. The values of $\psi_d'$ and $\psi_q'$ are given by equations (A5) with $v_f = 0$. Hence, equations 1.1 become:

$$\begin{aligned} 0 &= \left[r_a + \frac{p}{\omega} x_d(p)\right] i_d - x_q(p)\, i_q \\ v_q 1 &= x_d(p)\, i_d + \left[r_a + \frac{p}{\omega} x_q(p)\right] i_q \end{aligned} \quad \ldots 1.2$$

<u>1.3 Solution for the short circuit current:</u>

Eliminating $i_q$ from equations 1.2:

$$v_q 1 = \left[p^2 + \omega^2 + p\omega r_a \left\{ \frac{1}{x_d(p)} + \frac{1}{x_q(p)} \right\} + \frac{\omega^2 r_a^2}{x_d(p) x_q(p)}\right] \frac{x_d(p)}{\omega^2} i_d \quad 1.3$$

The expression in the square brackets can be simplified by using the assumption 5. The term in $r_a^2$ can be neglected entirely and, in the term in $r_a$, $x_d(p)$ and $x_q(p)$ can be simplified by neglecting the resistances $r_f$, $r_{kd}$, $r_{kq}$. This is equivalent to replacing in this term all the factors of the form $(1+Tp)$

by $T\,p$. With this approximation $X_d(p)$ and $X_q(p)$ reduce to the sub transient reactances $X_d''$ and $X_q''$.

Then, $i_d = \dfrac{(1+T_{do}'p)(1+T_{do}''p)}{(1+T_d'p)(1+T_d''p)} \; \dfrac{\omega^2}{(\;^2+2\;\;+\;^2)} \; \dfrac{V_{q1}}{X_d}$ (approx)

where $\alpha = \dfrac{\omega\, r_a}{2}\left(\dfrac{1}{X_d''} + \dfrac{1}{X_q''}\right)$ ... ... 1.4

The operational expression can be evaluated by the partial fraction method if $(p^2+2\alpha p+\omega^2)$ is factorised as $(p+\alpha_1)(p+\alpha_2)$

$$i_d = \frac{V_m}{X_d}\left[1 - \frac{\left(1-\frac{T_{do}'}{T_d'}\right)\left(1-\frac{T_{do}''}{T_d'}\right)}{\left(1-\frac{T_d''}{T_d'}\right)\left(1-\frac{2\alpha}{\omega^2 T_d'}+\frac{1}{\omega^2 T_d'^2}\right)}e^{-t/T_d'}\right.$$

$$-\frac{\left(1-\frac{T_{do}'}{T_d''}\right)\left(1-\frac{T_d''}{T_{do}''}\right)}{\left(1-\frac{T_d'}{T_d''}\right)\left(1-\frac{2\alpha}{\omega^2 T_d''}+\frac{1}{\omega^2 T_d''^2}\right)}e^{-t/T_d''}$$

$$+\frac{(1-\alpha_1 T_{do}')(1-\alpha_1 T_{do}'')}{(1-\alpha_1 T_d')(1-\alpha_1 T_d'')}\;\frac{\omega^2}{\alpha_1(\alpha_1-\alpha_2)}\,e^{-\alpha_1 t}$$

$$\left.-\frac{(1-\alpha_2 T_{do}')(1-\alpha_2 T_{do}'')}{(1-\alpha_2 T_d')(1-\alpha_2 T_d'')}\;\frac{\omega^2}{\alpha_2(\alpha_1-\alpha_2)}\,e^{-\alpha_2 t}\right] \quad \ldots\ 1.5$$

Now $T_d''$, $T_{do}''$ are small compared with $T_d'$, $T_{do}'$. In addition $T_{do}'$, $T_{do}''$, $T_d'$, $T_d''$ are all large compared with $\dfrac{1}{\omega}$ and $\alpha$ is small compared with $\omega$.

Hence

$\alpha_1 = \alpha + j\omega$ (approx.)

$\alpha_2 = \alpha - j\omega$ (approx.)

and $\dfrac{\omega^2}{(\alpha_1-\alpha_2)}\left(\dfrac{e^{-\alpha_1 t}}{\alpha_1} - \dfrac{e^{-\alpha_2 t}}{\alpha_2}\right) = -\,e^{-\alpha t}\cos\omega t$ (approx)

Hence, approximately, putting $\alpha = 1/T_a$

$$i_d = \frac{V_m}{X_d}\left[1 + \frac{(T_{do}' - T_d')}{T_d'} e^{-t/T_d'} + \frac{T_{do}'}{T_d'} \frac{(T_{do}'' - T_d'')}{T_d''} e^{-t/T_d''} - \frac{T_{do}' \, T_{do}''}{T_d' \, T_d''} e^{-t/T_a} \cos\omega t\right]$$

$$= \frac{V_m}{X_d} + \left(\frac{V_m}{X_d'} - \frac{V_m}{X_d}\right) e^{-t/T_d'} + \left(\frac{V_m}{X_d''} - \frac{V_m}{X_d'}\right) e^{-t/T_d''} - \frac{V_m}{X_d''} e^{-t/T_a} \cos\omega t. \quad \ldots\ldots 1.6$$

The quadrature current $i_q$ may be found in a similar way.

$$i_q = + \frac{(1 + T_{qo}''p)}{(1 + T_q''p)} \frac{p\omega}{(p^2 + 2\alpha p + \omega^2)} \frac{V_{q1}}{X_q}$$

$$= + \frac{V_m}{X_q''} e^{-t/T_a} \sin \omega t \text{ (approx.)} \quad \ldots\ldots 1.7$$

Substitution of the values of $i_d$ and $i_q$ in the transformation equations A.17 with $\theta = (\omega t + \lambda)$ gives the expressions for $i_a$, $i_b$ and $i_c$.

$$i_a = \left[\frac{V_m}{X_d} + \left(\frac{V_m}{X_d'} - \frac{V_m}{X_d}\right) e^{-t/T_d'} + \left(\frac{V_m}{X_d''} - \frac{V_m}{X_d'}\right) e^{-t/T_d''}\right] \times \cos(\omega t + \lambda) - \frac{V_m}{X_m} e^{-t/T_a} \cos\lambda - \frac{V_m}{X_n} e^{-t/T_a} \cos(2\omega t + \lambda)$$

$$\ldots 1.8$$

where $X_m = \dfrac{2 X_d'' X_q''}{X_d'' + X_q''}$

$X_n = \dfrac{2 X_d'' X_q''}{X_q'' - X_d''}$

The values of $i_b$ and $i_c$ are obtained by replacing $\lambda$ by $(\lambda - \frac{2\pi}{3})$ and $(\lambda - \frac{4\pi}{3})$ respectively in the expression for $i_a$.

...1.4 The short circuit torque...

to determine the quantities $\Psi_d$ and $\Psi_q$. Before the short circuit, the values are, from equations A.5

$$\Psi_{do} = + \frac{V_m}{\omega}$$

$$\Psi_{qo} = 0.$$

After the short circuit, the values are:-

$$\Psi_d = \Psi_{do} + \Psi_d'$$

$$\Psi_q = \Psi_{qo} + \Psi_q'$$

Where $\Psi_d'$, $\Psi_q'$ are the superimposed quantities. Using equations A 5

$$\Psi_{d'} = \frac{X_d(p)}{\omega} i_d = \frac{\omega}{p^2 + 2\alpha p + \omega^2} V_{q1}$$

$$= - \frac{V_m}{\omega}(1 - e^{-t/T_a}\cos\omega t)$$

$$\Psi_q' = - \frac{V_m}{\omega} e^{-t/T_a} \sin\omega t$$

Hence, $\Psi_d = + \frac{V_m}{\omega} e^{-t/T_a} \cos\omega t$

Similarly, $\Psi_q = - \frac{V_m}{\omega} e^{-t/T_a} \sin\omega t$ ... 1.1.9

The two flux waves represented by $\Psi_d$ and $\Psi_q$ combine to form a foward rotating flux wave, which rotates at speed $\omega$ and is therefore stationary relative to the armature. Its magnitude dies away with the armature time constant $T_a$, which depends on the armature resistance. Thus, the effect of the short circuit can be explained by imagining that the flux, which rotates relatively to the armature during normal operation, is frozen in position relative to the armature at the instant of short circuit and then dies away with time constant $T_a$.

The torque is obtained by substituting the expression for $\psi_d, \psi_q, i_d, i_q$ in Eqn. A4.

$$\text{A 4.} \quad \therefore T_e = \frac{V_m^2}{2} \Big[ e^{-t/T_a} \cos \omega t \left\{ \frac{1}{X_q''} e^{-t/T_a} \sin \; t \right\} + e^{-t/T_a} \sin \omega t \left\{ \frac{1}{X_d} + \left( \frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/T_d'} + \left( \frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-t/T_d''} - \frac{1}{X_d''} e^{-t/T_a} \cos \omega t \right\} \Big]$$

$$= V^2 e^{-t/T_a} \sin \omega t \left\{ \frac{1}{X_d} + \left( \frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/T_d'} + \left( \frac{1}{X_d''} - \frac{1}{X_d'} \right) \times e^{-t/T_d''} \right\} + \frac{V^2}{2} e^{-2t/T_a} \sin 2\omega t \left( \frac{1}{X_q''} - \frac{1}{X_d''} \right) \quad \ldots \; 1.10$$

where $V$ = RMS voltage equal to $\dfrac{V_m}{\sqrt{2}}$

The double-frequency torque represented by the second term of Eqn. 1.10 is relatively small. This term depends upon the difference in the subtransient reactances of the direct and quadrature axes and may be considered as a "reluctance torque" due to the variation in the permeance which the rotor offers to the trapped armature flux. This may be practically eliminated by the presence of a good amortisseur winding. Hence the principal component of torque oscillates at normal frequency and has an initial amplitude $\dfrac{V^2}{X_d''}$.

The effect of a solid rotor in a turbo alternator is to make the subtransient reactance in the quadrature axis more nearly equal to the corresponding reactance in the direct axis than would be the case for a perfectly laminated rotor. This will ~~be~~

of course reduce considerably the second harmonic torque components. The turbo generator and salient pole machine with connected dampers are close approximations during transient and sub-transient periods. The salient - pole machine with no dampers is furthest removed in its character from the ideal machine and will have torques containing all odd and even harmonic components.

From eqn. 1.10, it is evident that the peak value of electrical torque is limited chiefly by the sub-transient reactance $X_d''$, although for machines without damper windings, the difference between the direct and quadrature axis reactances $(X_q'' - X_d'')$ increases the peak torques.

Eqn. 1.10 can also be written as

$$T = \frac{V^2}{X_d''} FA \sin\omega t \quad \ldots \quad \ldots \quad 1.11$$

$$\text{where } F = \left[ \frac{X_d''}{X_d} + \frac{X_d''(X_d - X_d')}{X_d X_d'} e^{-t/T_{do}' \frac{X_d}{X_d'}} + \frac{X_d' - X_d''}{X_d'} e^{-t/T_{do}'' \frac{X_d'}{X_d''}} \right]$$

= Rotor flux linkage as a fraction of its initial value.

$$A = e^{-2\pi f . \frac{ra}{X_d''} t}$$

= Armature decrement factor.

## ...1.5 The field current after a 3 phase short circuit...

The field current before short circuit is given by substituting $i_d = i_q = 0$ in the expressionsA 1 and A 2 for $V_d$ $V_q$ & $V_f$. $i_{fe} = +\frac{V_{qo}}{X_{md}} = +\frac{V_m}{X_{md}}$

The field current after the short circuit is obtained by the equations already used to calculate the armature current. A relation between $i_f'$ and $i_d$ can be obtained by eliminating $i_{kd}$ from the second and third eqns. of A2 with $V_f = 0$

$$\left[\left\{r_f + (L_{md} + l_f)p\right\}\left\{r_{kd}+(L_{md}+l_{kd})p\right\} - L^2_{md}\; p^2\right] i_f'$$

$$- L_{md}\, p\, (r_{kd} + L_{kd}\, p) i_d = 0$$

Hence, substituting the value of $i_d$ from Eqn. 1.4,

$$i_f' = + \frac{L_{md}\, p\, (1 + T_{kd} p)}{r_f\, (1+T_{do}' p)(1+T_{do}'' p)}\, i_d$$

$$= + \frac{(1 + T_{kd}\, p)}{(1+T_d' p)(1+T_d'' p)} \; \frac{\omega p}{(p^2+2\alpha p + \omega^2)} \; \frac{X_{md}}{X_d} \times \frac{V_q}{r_f}\, 1 \quad \ldots \quad \ldots \quad 1.12$$

The following operational solution is obtained by using the partial fraction method and making the same approximations as in the solution of the short circuit current:

$$i_f' = + \frac{V_m X_{md}}{\omega\, T_{do}' r_f X_d}\left[e^{-t/T_d'} - (1 - \frac{T_{kd}}{T_d''}) - \frac{T_{kd}}{T_d''} e^{-t/T_a} \cos \omega t\right]$$

$$\ldots \quad \ldots \quad 1.13$$

Now, $+\dfrac{V_m X_{md}}{\omega T_d' r_f X_d} = \dfrac{i_{fo}}{T_d'} \cdot \dfrac{X^2_{md}}{\omega r_f X_d} = i_{fo}\dfrac{T_{do}'-T_d'}{T_d'} = i_{fo}\dfrac{X_d - X_d'}{X_d'}$

Hence the total field current after the short circuit is given by:

$$i_f = i_{fo} + i_f'$$

$$= i_{fo} + i_{fo}\left(\frac{X_d - X_d'}{X_d'}\right)\left[e^{-t/T_d'} - (1 - \frac{T_{kd}}{T_d''})\, e^{-t/T_d''} - \frac{T_{kd}}{T_d''}\, e^{-t/T_a} \cos \omega t\right] \quad \ldots \quad 1.14$$

### ...1.6 Explanation of short circuit currents...

Each armature current given by eqn. 1.8 consists of a symmetrical or alternating component and an asymmetrical or direct component. The A.C. component consists of a sustained component; a transient component which decays with a long time constant $T_d'$ and a subtransient component which decays with a very short time constant $T_d''$.

The D.C. components of the three armature currents all decrease to zero exponentially with the same time constant $T_a$.

The field current, given by eqn. 1.14, like the armature current, consists of d.c. & a.c. components. The d.c. component consists of sustained, transient and subtransient components. The transient and subtransient d.c. components of field current decrease with the same time constants $T_d'$ & $T_d''$ respectively as the corresponding a.c. components of armature current. The a.c. component of field current decays with the time constant $T_a$, as do the d.c. components of armature currents. The initial crest value of the a.c. component is equal to the initial value of the transient d.c. component. Hence the d.c. components of armature current correspond to A.C. components of armature current and vice versa.

### ...1.7 UNSYMMETRICAL FAULTS...

#### 1.7.1 Single- phase short circuits...

Single phase short circuits may be either line to line or line to neutral. The only essential difference between the two types is that the line-to-neutral short circuit involves also the zero-phase sequence impedances of the machine and of any impedance connected between neutral and ground if the fault is line to ground. As in the previous case, constant rotor speed is assumed in the analysis.

...1.7.2. Line-to-line short circuit on phases B-C...

The analysis of a single phase short circuit on a synchronous generator was first given by Doherty and Nickle[5] who derived expressions for the transient currents in the armature and field circuits and verified them experimentally. A very full theoretical treatment of the three alternative types of short circuit of a 3 phase generator (line-to-line, line-to-neutral and double-line-to-neutral) is given by Concordia[6]. For each case, expressions are given for the transient torque as well as for the currents. The method used by these authors is to derive the initial values of the components of the currents and to estimate a time constant appropriate to each component.

For a L-to-L short circuit on phases B-C, the expressions for $i_d$ and $i_q$ will be

$$i_d = \frac{2\,V(F\ \sin\theta - A\ \sin\lambda)\ \sin\theta}{X_d'' + X_q'' - (X_d''-X_q'')\cos 2\theta}$$

$$i_q = \frac{2V(F\ \sin\theta - A\ \sin\lambda)\cos\theta}{X_d''+X_q'' - (X_d''-X_q'')\cos 2\theta} \qquad \ldots \quad \ldots \quad 1.15$$

where F = A factor describing the flux-linkage decay.

$$= \left(\frac{X_d' - X_d''}{X_d' + X_2}\right)e^{-t/T_d''} + \left(\frac{X_d''+X_2}{X_d'+X_2} - \frac{X_d''+X_2}{X_d+X_2}\right)e^{-t/T_d'} + \left(\frac{X_d''+X_2}{X_d+X_2}\right)$$

and $A = e^{-t/T_a}$

= Armature decrement factor.

The electrical torque is given by

$$T = \frac{2V^2F\ (F \sin\theta - A \sin\lambda) \cos\theta}{X_d'' + X_q'' - (X_d'' - X_q'')\cos 2\theta}$$

$$+ \frac{2(X_q' - X_d'')\left[V(F \sin\theta - A \sin\lambda)\right]^2 \sin 2\theta}{\left[X_d'' + X_q'' - (X_d'' - X_q'')\cos 2\theta\right]^2} \quad \ldots\ 1.16$$

To compute the resulting shaft torques, equation 1.16 has to be expressed in terms of its harmonic components. This can be done by resolving equation 1.16 into a Fourier Series. Hence, we obtain,

$$T = \frac{2\ V^2}{X_d'' + X_2}\Big[FA \sin\lambda\,(-\cos\theta + 3b \cos 3\theta - 5b^2 \cos 5\theta\ )$$

$$+ \left\{F^2 \frac{X_2}{X_d'' + X_2} - A^2 . \frac{X_d'' - X_2}{X_2} \sin^2\lambda\right\} x\ (\sin 2\theta - 2b \sin 4\theta)\Big] \quad \ldots 1.17$$

where $b = \dfrac{X_2 - X_d''}{X_2 + X_d''}$

Neglecting terms of higher harmonics.

$$T = \frac{2\ V^2}{X_d'' + X_2}\left[-FA \sin\lambda \cos\theta + \left(F^2 \frac{X_2}{X_d'' + X_2} - A^2 \frac{X_d'' - X_2}{X_2} \sin^2\lambda\right) \times \sin 2\theta\right] \quad \ldots\ 1.18$$

For turbine generators and for machines with damper windings, $X_d'' = X_q''$. Hence $X_2 = X_d''$.

$$\therefore T = \frac{2V^2}{X_2 + X_d''}\left[-FA \sin\lambda \cos\theta + F^2 . \frac{X_2}{X_d'' + X_2} \sin 2\theta\right]$$

$$= \frac{V^2}{X_2 + X_d''}\left[-2FA \sin\lambda \cos\theta + F^2 \sin 2\theta\right] \quad \ldots\ 1.19$$

If $\lambda$ = Angle between the quadrature axis of rotor and phase A,

$$T = \frac{V^2}{X_d'' + X_2}\left[+2FA\,\cos\lambda\,\sin(\omega t + \lambda) - F^2 . \sin 2\,(\omega t + \lambda)\right] \quad \ldots 1.20$$

For the particular case of maximum initial armature flux linkages, $\lambda = 0$

$$T_{\ell-\ell} = \frac{V^2}{X_d'' + X_2}\left[2FA\ \sin(\omega t + \lambda) - F^2\ \sin 2(\omega t + \lambda)\right] \quad \ldots 1.21$$

Thus, the maximum fundamental-frequency torque, immediately following a short circuit with maximum armature flux linkages is $\frac{2\,V^2}{X_d'' + X_2}$.

### ...1.7.3 Line-to-neutral short circuit on phase A...

The Line-to-neutral case can easily be obtained directly from the Line-to-Line case by simply replacing $X_d''$ by $X_d'' + \frac{X_o}{2}$ and $X_q''$ by $X_q'' + \frac{X_o}{2}$ and replacing $\theta$ by $(\theta + 90^o)$ Explain

$$T_{\ell-n} = \frac{V^2}{X_d'' + X_2 + X_o}\left[2FA . \sin(\omega t + \lambda) - F^2\ \sin 2\,\overline{\omega t + \lambda}\right] \quad \ldots 1.22$$

$$\text{where } F = \left[\frac{X_d'' + X_2 + X_o}{X_d + X_2 + X_o} + \frac{(X_d'' + X_2 + X_o)(X_d - X_d')}{(X_d' + X_2 + X_o)(X_d + X_2 + X_o)} \times e^{-\frac{t}{T_{do}'}}\,\frac{X_d + X_2 + X_o}{X_d' + X_2 + X_o} + \frac{X_d' - X_d''}{X_d' + X_2 + X_o}\,e^{-\frac{t}{T_{do}''}}\,\frac{X_d' + X_2 + X_o}{X_d'' + X_2 + X_o}\right]$$

$$A = e^{-2\pi f\,\frac{3ra}{X_d'' + X_2 + X_o}\,t}$$

Where $\lambda$ = angle between the d-axis of rotor and phase A.

### ...1.7.4 Double-line-to-ground short circuit on phases B & C...

In the case of double-L-to-ground short circuits, two armature time constants have to be considered because there may be many times as much resistance in the ground connection as in the armature winding. Hence, there can not be single characteristic armature time constant for this type of fault.

The symmetrical component method is indispensable fo[r]
attainment of a clear picture of the double-line-to-groun[d]
particularly in relation to rotor decrement factors.

Again, eqns. can be developed for $i_d, i_q$ and the elec[trical]
torque.

The short circuit torques for different types of fau[lts are]
shown in Table 1.1.

It can be seen that for the 3 phase fault, the curre[nts]
$i_d$ and $i_q$ are functions of time t (measured from the inst[ant]
of short circuit) directly rather than of the rotor angle
and thus do not depend on the initial rotor angle $\lambda$ (the [angle]
of the direct axis ahead of the axis of phase a' at the in[stant]
of short circuit). This is due to the complete symmetry [of]
the stator winding. The initial rotor angle determines o[nly]
the location and not the magnitude of the flux trapped in [the]
stator. For all the unbalanced faults, the currents and [conse-]
quently the torques depend on the initial rotor angle and [have]
therefore been expressed as functions of $\theta$ where $\theta = \omega t$

## ...1.8 Short circuits from full Load...

It is more probable for a short circuit to occur fro[m full load]
than from no load. When the short circuit is 3-phase, th[e]
entire external load is dropped and the only torque will [be]
the unidirectional torques (explained in sec. 1.10) given [by]
equations in Table. 1.1 This is also true in the case of [a]
double-line-to-ground fault on a machine with low zero se[quence]
reactance.

The short circuit occuring on a loaded alternator is similar to a no load short circuit, the only difference being in the initial conditions. In deriving the expressions for short circuit under load, it is only necessary to substitute the new value of the entrapped flux to make the expressions hold for a short circuit under load.

The new expressions for $i_d$ and $i_q$ will be

$$i_d = \left[\left(\frac{1}{X_d''} - \frac{1}{X_d'}\right) e^{-t/T_d''} + \left(\frac{1}{X_d'} - \frac{1}{X_d}\right) e^{-t/T_d'} + \frac{1}{X_d}\right] V_m \cos\delta$$

$$- \frac{V_m}{X_d''} e^{-t/T_a} \cos\ (\omega t + \delta) \quad \ldots$$

$$i_q = -\left[\left(\frac{1}{X_q''} - \frac{1}{X_q}\right) e^{-t/T_q''} + \frac{1}{X_q}\right] V_m \sin\delta + \frac{V_m}{X_q''} e^{-t/T_a} \sin(\omega t + \delta) \quad \ldots \; 1.23$$

This is the current for short circuit from any load condition, but with small armature resistance. If the short circuit occurs at no load, $\delta = 0$ and the equations reduce to those given by eqns. 1.6 and 1.7.

The transient rotor air gap torques for short circuits under full load are of the same form as those from no load with the addition of a transient term to represent the rapid dropping of part of the load. With the assumption that for a 3 phase short-circuit, the total load is dropped, the most severe shaft torques are produced by line-to-line faults from full load with no external resistance.

...1.9 Effect of external impedance...

The presence of external-system reactance decreases the magnitude of the fault currents and therefore, the rotor air-gap torque. However, additional external resistance without increasing the reactance can increase some components of the air gap torque. It increases the crest magnitude of the D.C. component, increases rapidly the damping of the fundamental frequency component and increases more slowly the damping of the d.c. component.[7]

Since the addition of external resistance increases the magnitude of the d.c. component of electrical torque, such a condition will produce more severe shaft torques at the lower system frequency, where the d.c. component has its greatest effect. This is true for low values of external resistance.

Further increase of d.c. component with greater amount of external resistance is more than offset by increased damping.

Thus the effect of an external resistance during a fault is two fold; the unidirectional torque is increased and the decrement of the fundamental frequency torque is also increased. This may or may not result in increased shaft stress, depending upon the masses and spring constants of the generator.

## ...1.10 Loss torques or unidirectional components of torques...

During some types of short circuits, the machine seems to "load up" during the first few cycles rather than dump load as might be expected. The tendency of a machine to "load up" during short circuit is due to loss torques, which have been neglected in the present analysis. These loss torques have been investigated by Kilgore[8], and later by Whitney and Criner[9] and Kirschbaun[7]. The unidirectional torques are important for the calculation of stability.

### 1.10.1 Determination of unidirectional torques:

It is possible to determine the unidirectional torques from the general relation applied to the average power[7].

Power output is equal to power input to shaft plus rate of decrease of K.E. in the machine plus rate of decrease of stored magnetic energy in the machine minus total ohmic losses in the machine. The sum of the first two terms on the R.H. side of the equation are proportional to the unidirectional torque acting on the rotor. During a short circuit, power output is equal to zero. Unidirectional torque is equal to sum of average ohmic losses minus rate of decrease of average stored magnetic energy in the machine. In other words, some of the ohmic losses are supplied by decaying magnetic fields. Of the losses in the machine, two components are to be supplied: the loss caused by the flow of D.C. in the short circuited armature and the loss caused by the flow of the additional D.C. in the rotor circuits. The remainder of the D.C. ohmic loss in the rotor field is supplied by the exciter. The only machine losses which appear as unidirectional torques are those which are caused by the flow of alternating current in the various machine circuits.

## ...1.11 Effect of damper windings upon stability:..

The principal effects of damper windings upon power system stability may be listed as follows:-

1. Positive-sequence damping
2. Negative - sequence braking during an unsymmetrical fault.
3. Effect of Negative - sequence impedance upon positive sequence electric power output of the machine during an unsymmetrical fault.
4. D.C. Braking

### ...1.11.1 Positive - sequence damping...

This results from the torque caused by interaction of the damper currents with the positive - sequence (forward - rotating) magnetic field in the air gap. Except during starting or pulling out of step, the slip of the rotor with respect to the positive-sequence field is low and oscillatory. Positive- sequence damping causes the oscillations of the machine rotors, after an aperiodic shock that does not cause loss of synchronism, to decrease in amplitude and ultimately to die out entirely. Low-resistance amortisseurs greatly increase the amount of positive-sequence damping. Positive - sequence damping is present both during and after a fault. However, it is much more effective after clearing of a fault than during the fault because the positive-sequence voltage and flux are greater after clearing.

By absorbing energy from the oscillation, positive-sequence damping may prevent a machine which has survived the first swing from going out of step on the second or subsequent swings because great enough to increase significantly the power that can be carried through the first swing. Its effect is almost always neglected in calculating power limits.

...1.11.2 Negative - sequence braking...

This results from the torque caused by interaction of the damper currents with the negative - sequence (backward-rotating) magnetic field in the air gap. Such a field is present in a polyphase machine only when an unbalance - usually an unsymmetrical short circuit - is present on the poly-phase circuit to which the armature winding is connected. The slip of the rotor with respect to the negative - sequence field is $2-S$, where $S$ = positive-sequence slip. Since $S$ is small (except during starting and out-of-step operation), the Negative sequence slip is very nearly equal to 2. It never reverses, nor does the resulting torque reverse. The torque always retards the rotor and hence called a "braking" torque. Its effect is therefore equivalent to a reduction of mechanical input torque. In a generator, which tends to speed up during a fault, the Negative - sequence torque decreases the accelerating torque. The braking effect is present only as long as an unsymmetrical fault is on the system. The greatest braking torque is afforded by high-resistance dampers and very little by low-resistance dampers. This method of improving transient stability is less important where high-speed fault clearing is generally used. The higher the speed of clearing, the less important is the shock caused by the fault itself in comparison with the shock of opening the faulted line to clear the fault. This is true even if high-speed reclosing is employed.

...1.11.3 Negative - sequence impedance...

A damper winding decreases the negative sequence reactance of the machine on which it is installed and may either increase or decrease the negative - sequence resistance. In general, the negative-sequence impedance is lowered by a damper winding, especially by a lower - resistance damper. Lowering the negative-sequence impedance of any machine on a network lowers the negative sequence impedance of the network viewed from the point of fault and thus also lowers theimpedance of the fault shunt representing any type of short circuit except a 3-phase short circuit. The effect is greater for a line-to-line short circuit than for any other type and is greatest for a fault located near the machine equipped with the dampers. Usually a fault location near the principal equivalent generators is taken as the most severe one and the one used as a criterion of system stability.

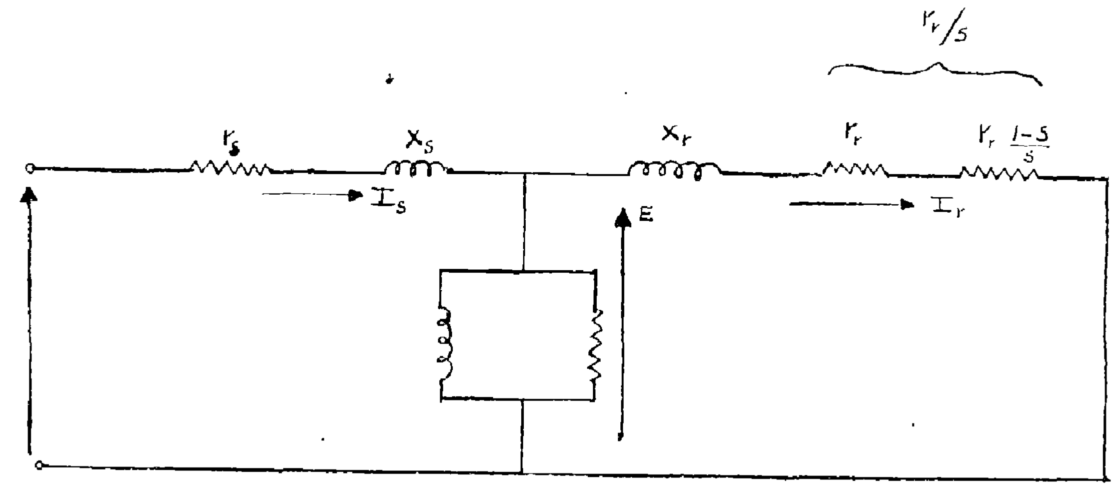
Thus, while an unsymmetrical fault is on the system, the damper winding on a generator gives rise to two opposing effects. (1) braking torque, which is beneficial to stability and (2)decrease of negative sequence impedance, which is detrimental to stability. Which of thesetwo effects predominates depends largely on the resistance of the damper winding. With a low-resistance damper, the decrease of negative-sequence impedance usually predomintes, while with a high-resistance damper the braking torque usually predominates. This is true because increasing the resistance of the damper both increases the negative-sequence impedance and increases the braking torque.

The more rapidly faults are cleared, the less important are both of these effects. The use of high-resistance dampers to.

increase the stability limits of generators is therefore less important now than it was when slower clearing speeds were in general use.

### ...1.11.4 Damper action explained by induction motor theory...

In as much as a damper winding is similar to the squirrel cage rotor winding of an induction motor, and the armature winding of a synchronous machine is like the stator winding of an induction motor, the various effects of the damper windings on stability (discussed above) can be analysed (approximately) by induction motor theory.




If a synchronous machine with dampers hunts or swings, the slip is alternately positive and negative. The torque is likewise alternately positive and negative and always tends to bring the speed back to synchronous speed.

Negative-sequence applied voltage causes a wave of air-gap flux to rotate backward at synchronous speed. If the slip of the rotor with respect to the positive-sequence field is S, the slip with respect to the negative-sequence field is 2-S and the torque is positive though the speed is negative. In other words, the torque opposes the rotation; it is a braking torque. For S=2, the mechanical power output $= I_r^2 r_r \times \frac{1-2}{2} = -\frac{1}{2} I_r^2 r_r$ watts/phase ... 1.24. This is negative (i.e. it is actually input) and is equal to half the rotor copper loss $I_r^2 r_r$. The other half of the rotor copper loss is supplied electrically via the stator terminals. Thus, there is at the same time both electric and mechanical power input, all of which goes into losses, principally copper losses.

The negative-sequence rotor currents have a frequency (2-S)f or very nearly 2f, where f is the stator frequency. In a synchronous machine, such currents flow both in the field winding and in the damper windings if damper windings are provided, otherwise, in the field winding and in the field pole faces.

In order to give high damping torque at small values of slip, the dampers should have as low a resistance as possible. In order to give high negative-sequence braking torque, the dampers should have a much higher resistance, such that maximum torque occurs at or near S=2. A damper which is designed for one of these purposes is poorly suited for the other purposes.

Negative-sequence impedance of a machine may be found by putting S=2 in the equivalent circuit. Fig. 1.2. Then negative sequence resistance $r_2 = r_s + \frac{1}{2} r_r$ ... ... 1.25

where $r_s$ = stator resistance

$r_r$ = Rotor resistance.

...1.10.5 Calculation of damping...

Induction-motor theory assumes constant slip and a symmetrical rotor with only one set of windings. A synchronous machine even though equipped with damper windings, has an unsymmetrical rotor, with a field winding on only one axis and the slip of the machine varies during the disturbances considered in stability studies. Nevertheless, induction-motor theory is considered applicable to the calculation of negative-sequence braking. For the calculation of positive-sequence damping, however, synchronous machine theory is invoked.

...1.12 Calculation of unidirectional torques, taking damper action into account...

...1.12.1 Negative-sequence braking torque...

The slip of the rotor with respect to negative sequence field is very nearly 2. Its value may be taken as 2 with little error, even though the machine is actually swinging or starting to pull out of step, because the torque is nearly independent of slip in this vicinity.

If the rotor circuits in the two axes are unlike (i.e. if $x_d'' \neq x_q''$ or if $T_{do}'' \neq T_{qo}''$), then the negative-sequence torque pulsates at twice the rated frequency. This fluctuation is so rapid compared with the frequency of electro mechanical swings that the average torque, as calculated by induction-motor theory, can be used with negligible error in stability calculations. The mechanical power corresponding to this torque is

$$T_b = \frac{1}{2} I_{r2}^2 r_r \quad \ldots \qquad \ldots \quad 1.26$$

where $I_{r2}$ = Negative- sequence rotor current.

$r_r$ = Rotor resistance.

As the negative sequence rotor current is directly proportional to the negative sequence stator (armature) current $I_{S2}$, $T_b = K_b . I_{S2}^2$. where $K_b$ = a constant. Also, $I_{r2} \approx I_{S2}$ and $r_r \approx 2 (r_2 - r_1)$ where $r_1$ = Positive - sequence resistance equal to $r_s$. Then, $T_b \approx I_{S2}^2 \ (r_2 - r_1)$ ... .. 1.27. This form is convenient for computing the negative-sequence braking power of synchronous machines because the positive and negative sequence resistances of the machines are usually given rather than the rotor resistance. The negative-sequence armature current is present only during unsymmetrical faults.

Thus, negative sequence braking torque $= I_{S2}^2 2(r_2 - r_1)$ for a 1-phase short circuit because for a 1_phase short circuit all the loss appears as torque because of transformer action between the sequences.

$$= \left(\frac{VF}{x_d'' + x_2}\right)^2 2\ (r_2 - r_1)$$ for 1 phase short circuit or L-to-L short circuit.

$$= \left(\frac{VF}{x_d'' + x_2 + x_0}\right)^2 2(r_2 - r_1)$$ for L-to-G fault

$$= 0$$ for 3 phase short circuit.

The negative-sequence torque is always a retarding or braking torque. It has therefore the same effect on the angular motion of the rotor as a decrease of mechanical input torque. In the calculation of swing curves, negative-sequence damping may be handled as a reduction of the input.

When damper action was neglected, the accelerating power (or torque) (In P.U. system of representation, torque and power will be same, As P.U. system of representation is adopted in this dissertation, torque and power will be same and hence they are not distinguished seperately from each other) is taken as the difference between the mechanical power input and the electric power output (both internal values)

$$T_{ac} = T_m - T_e.$$

when negative-sequence braking is taken into account, another term is included.

$$T_{ac} = T_m - T_e - T_b \quad \ldots \quad \ldots \quad 1.28$$

where $T_b$ = braking power, The term $T_e$, represents only the positive-sequence power output.

Negative-sequence braking can be neglected except in machines having a high-resistance damper winding. Even in such machines, it exists only during the existence of an unbalanced fault.

### 1.12.2 D.C. braking Torque

For 3 phase faults at or very near the armature terminals of a machine, the d.c. components of short circuit armature current persist long enough to have an appreciable effect. The d.c. components of armature current induce in the rotor circuits currents of fundamental frequency, which give rise to a braking torque similar to that caused by negative-sequence armature currents. The D.C. braking torque (or power), like the negative sequence braking torque, decreases the accelerating torque and may be considered as all or part of $T_b$ in eqn. 1.28.

Although the rotor currents induced by direct armature currents are of fundamental frequency, whereas those induced by negative-sequence armature currents are of twice fundamental frequency, nevertheless for a given magnitude of armature current, the magnitudes of the various rotor currents are substantially the same in the two cases, being determined mainly by the reactances. There is, however, this important difference: that whereas in the negative-sequence case, half the rotor copper loss was supplied mechanically and therefore produced braking torques, in the d.c. case, all the rotor copper loss is supplied mechanically. This loss is $I_r^2 \, r_r$ or twice the value given by eqn. 1.26. Using the same value of $r_r$ as before, we get for the d.c. braking power twice the value given by eqn. 1.27, namely: $T_b \approx 2I_S^2 \, (r_2 - r_1)$ ... 1.29. Here $I_S$ = effective value of armature currents, which can be imagined to be polyphase currents of very low frequency, whose instantaneous values equal the direct armature currents. The crest value of these polyphase currents corresponds to the total direct current, that is, to the value of d.c. component that would exist in one phase of the armature if the switching angle were such as to give this component its maximum value. For an unloaded machine the total direct current is initially equal to the initial crest value of the a.c. component of armature short circuit current and decays exponentially with time constant $T_a$. If the total instantaneous d.c. component of armature current is denoted by $i_{dc}$, the exprn. for d.c. braking power becomes $T_b \approx i_{dc}^2 \, (r_2 - r_1)$ ... 1.30 As $i_{dc}$ decreases exponentially, $T_b$ also decreases.

For types of fault other than 3 phase and for faults not very near the armature terminals, $i_{dc}$ decays so rapidly that its braking effect is negligible.

Also, assuming a solid rotor, structure, the rotor resistance & reactance vary very closely as the square root of frequency of rotor current. If $r_2$ = Negative-sequence resistance of the machine (as measured with double frequency rotor currents flowing), rotor resistance = $2(r_2-r_1)$ at double frequency and $=\sqrt{2}(r_2-r_1)$ at fundamental frequency. In turbine generators, the rotor losses vary as 1.8 power of rotor current. For salient-pole machines, this will be 2. ∴ The d.c. braking torque due to fundamental frequency rotor currents induced by d.c. armature currents is given by

$$T_b = \left(\frac{\sqrt{2}\,VA}{2\,X_d''}\right)^{1.8 \text{ or } 2}\sqrt{2}(r_2-r_1) \text{ for a 3 phase short-circuit.}$$

$$= \left(\frac{\sqrt{2}\,VA}{X_d''+X_2}\right)^{1.8 \text{ or } 2}\sqrt{2}(r_2-r_1) \text{ for a L-to-L short-circuit.}$$

$$= \left(\frac{\sqrt{2}\,VA}{X_d''+X_2+X_0}\right)^{1.8 \text{ or } 2}\sqrt{2}(r_2-r_1) \text{ for a L-to-N short-circuit.}$$

These terms correspond to d.c. components of <u>armature</u> current and flux linkages or to fundamental frequency components of <u>rotor</u> currents and fluxes. In effect, it is just as though the armature were supplied with d.c. excitation, so that the resulting rotor losses must be supplied by mechanical torque, while the stator $i^2r$ losses are supplied by the decay of the energy of the stator magnetic field.

...<u>1.12.3 Unidirectional torque due to armature winding copper loss</u>...

The 3rd component of unidirectional torque is due to the flow of the additional d.c. in the rotor circuit. This is now the stator (armature) $i^2r_a$, where i is now the A.C. component of armature current. This is equal to $\frac{1}{2}\,\frac{V^2{}_m F^2 r_a}{X_d''^2}$

$$= \left(\frac{VF}{X_d''}\right)^2 r_1 \text{ for a 3 phase short circuit and } \left(\frac{VF}{X_d''+X_2}\right)^2 2r_1 .$$

## TABLE .1

## Short-circuit torques.

**Type of fault: 3 phase**

$$T = \frac{V^2}{X_d''} FA \sin\omega t + \sqrt{2}\left[\frac{\sqrt{2}\, VA}{2X_d''}\right]^{1.8 \text{ or } 2}(r_2 - r_1) + \left(\frac{VF}{X_1''}\right)^2 r_1 \text{ P.U.}$$

$$F = \left[\frac{X_d''}{X_d} + \frac{X_d''(X_d - X_d')}{X_d X_d'}\, e^{-t/T_{do}'}\,\frac{X_d}{X_d'} + \frac{X_d' - X_d''}{X_d'}\, e^{-t/T_{do}''}\,\frac{X_d'}{X_d''}\right];$$

$$A = e^{-2\pi f \frac{r_a}{X_2''} t} \; ; \quad X_2 = \frac{X_d'' + X_q''}{2}$$

**Line-to-Ground phase A**

$$T = \frac{V^2}{X_d'' + X_2 + X_o}\left[2FA \sin(\omega t + \lambda) - F^2 \sin 2(\omega t + \lambda)\right] + 2\left[\frac{VF}{X_d'' + X_2 + X_o}\right]^{1.8 \text{ or } 2}(r_2 - r_1)$$

$$+ \sqrt{2}\left[\frac{\sqrt{2}\, VA}{X_d'' + X_2 + X_o}\right]^{1.8 \text{ or } 2}(r_2 - r_1) + 2\left[\frac{VF}{X_d'' + X_2 + X_o}\right]^{1.8 \text{ or } 2} \text{P.U.}$$

$$F = \left[\frac{X_d'' + X_2 + X_o}{X_d + X_2 + X_o} + \frac{(X_d'' + X_2 + X_o)(X_d - X_d')}{(X_d' + X_2 + X_o)(X_d + X_2 + X_o)}\, e^{-t/T_{do}'}\,\frac{X_d + X_2 + X_o}{X_d' + X_2 + X_o} + \frac{X_d' - X_d''}{X_d' + X_2 + X_o} \times e^{-t/T_{do}''}\,\frac{X_d' + X_2 + X_o}{X_d'' + X_2 + X_o}\right]$$

$$A = e^{-2\pi f \cdot \frac{3r_a}{X_d'' + X_2 + X_o} t} ; \quad X_2 = \sqrt{\left(X_d'' + \frac{X_o}{2}\right)\left(X_q'' + \frac{X_o}{2}\right)} - \frac{X_o}{2} ; \quad \lambda = \text{angle between d-axis of rotor and phase A}$$

**Line-to-Line on B & C**

$$T = \frac{V^2}{X_d'' + X_2}\left[2FA \sin(\omega t + \lambda) - F^2 \sin 2(\omega t + \lambda)\right] + 2\left[\frac{VF}{X_d'' + X_2}\right]^{1.8 \text{ or } 2}(r_2 - r_1)$$

$$+ \sqrt{2}\left[\frac{VA}{X_d'' + X_2}\right]^{1.8 \text{ or } 2}(r_2 - r_1) + 2\left[\frac{VF}{X_1'' + X_2}\right]^2 r_1 \text{ P.U.}$$

Contd...

TABLE [illegible]

Short-circuit torques.

| ...of fault | |
|---|---|
| phase | $$T = \frac{V^2}{X_d''} FA \sin\omega t + \sqrt{2}\left[\frac{\sqrt{2}\,VA}{2X_d''}\right]^{1.8 \text{ or } 2}(r_2-r_1) + \left(\frac{VF}{X_d''}\right) r_1 \text{ P.U.}$$ $$F = \left[\frac{X_d''}{X_d} + \frac{X_d''(X_d - X_d')}{X_d X_d'} e^{-t/T_{do}'} \frac{X_d}{X_d'} + \frac{X_d'-X_d''}{X_d'} e^{-t/T_{do}''} \frac{X_d'}{X_d''}\right];$$ $$A = e^{-2\pi f \frac{r_a}{X_d''} t}; \quad X_2 = \frac{X_d''+X_q''}{2}$$ |
| ...-to-Ground phase A | $$T = \frac{V^2}{X_d''+X_2+X_0}\left[2FA\sin(\omega t+\lambda) - F^2 \sin 2(\omega t+\lambda)\right] + 2\left[\frac{VF}{X_d''+X_2+X_0}\right]^{1.8 \text{ or } 2}(r_2-r_1)$$ $$+ \sqrt{2}\left[\frac{\sqrt{2}\,VA}{X_d''+X_2+X_0}\right]^{1.8 \text{ or } 2}(r_2-r_1) + 2\left[\frac{[illegible]}{X_d''+X_2+X_0}\right]^{1.8 \text{ or } 2} \text{ P.U.}$$ $$F = \left[\frac{X_d''+X_2+X_0}{X_d+X_2+X_0} + \frac{(X_d''+X_2+X_0)(X_d-X_d')}{(X_d'+X_2+X_0)(X_d+X_2+X_0)} e^{-t/T_{do}'} \frac{X_d+X_2+X_0}{X_d'+X_2+X_0} + \frac{X_d'-X_d''}{X_d'+X_2+X_0} \times e^{-t/T_{do}''} \frac{X_d'+X_2+X_0}{X_d''+X_2+X_0}\right]$$ $$A = e^{-2\pi f \cdot \frac{3r_a}{X_d''+X_2+X_0} t}; \quad X_2 = \sqrt{\left(X_d''+\frac{X_0}{2}\right)\left(X_q''+\frac{X_0}{2}\right)} - \frac{X_0}{2}; \quad \lambda = \text{angle between d-axis of rotor and phase A}$$ |
| ...-Line on B & C | $$T = \frac{V^2}{X_d''+X_2}\left[2FA\sin(\omega t+\lambda) - F^2 \sin 2(\omega t+\lambda)\right] + 2\left[\frac{VF}{X_d''+X_2}\right]^{1.8 \text{ or } 2}(r_2-r_1)$$ $$+ \sqrt{2}\left[\frac{VA}{X_d''+X_2}\right]^{1.8 \text{ or } 2}(r_2-r_1) + 2\left[\frac{VF}{X_d''+X_2}\right]^2 r_1 \text{ P.U.}$$ |

Contd...

Contd...Table 1.1

$$F = \left[\frac{X_d'' + X_2}{X_d + X_2} + \frac{(X_d''+X_2)(X_d-X_d')}{(X_d'+X_2)(X_d+X_2)}\, e^{-t/T_{do}'}\, \frac{X_d + X_2}{X_d'+X_2} + \frac{X_d' - X_d''}{X_d' + X_2} \cdot e^{-t/\ldots}\right.$$

$$A = e^{-2\pi f \frac{2\, ra}{X_d''+X_2} t}; \quad X_2 = \frac{X_d'' + X_q''}{2}; \quad \lambda$$ – angle between q-axis of roto[r] and phase A.

Double-line-to-ground on phases B, C

$$T = \frac{V^2}{X_d'' + \frac{X_2 X_0}{X_2+X_0}} \left[ 2A_1 F \sin(\omega t + \lambda) - 2A_2 F \cos(\omega t + \lambda) - F^2 \frac{X_0}{X_2+X_0} \sin \ldots \right.$$

$$+ 2 \left[\frac{VF}{X_d'' + X_2 + \frac{X_d'' X_2}{X_0}}\right]^{1.8 \text{ or } 2} (r_2 - r_1) + \sqrt{3}\left\{\left[\frac{\sqrt{2}\, VA_1}{X_d'' + \frac{X_2 X_0}{X_2+X_0}}\right]^{1.8 \text{ or } 2}\right.$$

$$\left. + \left[\frac{\sqrt{2}\, VA_2}{X_d'' + \frac{X_2 X_0}{X_2+X_0}}\right]^{1.8 \text{ or } 2}\right\}(r_2 - r_1) + 2 \left[\frac{VF \sqrt{X_2^2 + X_2 X_0 + X_0^2}}{X_d'' X_2 + X_2 X_0 + X_0 X_d''}\right]^2$$

$$F = \left[\frac{X_d'' X_2 + X_2 X_0 + X_0 X_d''}{X_d X_2 + X_2 X_0 + X_0 X_d} + \frac{(X_d'' X_2 + X_2 X_0 + X_0 X_d'')(X_2 + X_0)(X_d - X_d')}{(X_d' X_2 + X_2 X_0 + X_0 X_d')(X_d X_2 + X_2 X_0 + X_0 X_d)}\right.$$

$$e^{-t/T_{do}'}\, \frac{X_d X_2 + X_2 X_0 + X_0 X_d}{X_d' X_2 + X_2 X_0 + X_0 X_d'} + \frac{(X_2 + X_0)(X_d' - X_d'')}{(X_d' X_2 + X_2 X_0 + X_0 X_d')} \times$$

$$\left. e^{-t/T_{do}''}\, \frac{X_d' X_2 + X_2 X_0 + X_0 X_d'}{X_2 X_d'' + X_2 X_0 + X_0 X_d''}\right]$$

$$A_1 = \left[\frac{(X_2 + 2X_0)\cos\lambda}{2(X_2 + X_0)}\right] e^{-2\pi f \cdot \frac{2\, ra}{X_d'' + X_2} t}. \qquad A_2 = \left[\frac{X_2 \sin\lambda}{2(X_2 + X_0)}\right] e^{-2\pi f \cdot \frac{ra}{X_d''}\ldots}$$

for a L-to-Line short circuit and $3\left(\frac{VF}{X_{d''}+X_2+X_0}\right)^2 r_1$ for a L-to-N short circuit.

### 1.1.12.4 Additional loss due to subtransient saliency...

There may be an additional loss torque in the armature circuit due to subtransient saliency. (where $X_d'' \neq X_q''$). Generally, this can be neglected.

In table 1.1 equations are given for the fundamental and double frequency components of Electrical torque developed at the rotor and also the unidirectional components of torque for various types of faults.

During a 3-Ø-short circuit, the electrical torque output is zero i.e. $T_e = 0$, and the only unidirectional torques will be those given by equations in Table 1.1

### 1.13 Transient Stability Analysis...

After a fault occurs on a system, the asymmetrical and subtransient components of the current die away in a small fraction of a second, but the transient effect of the disturbance continues for several seconds. If the system is stable under this transient condition, the generators ultimately settle down to a new condition of steady operation at synchronous speed. On the other hand, a severe disturbance may lead to instability, so that the generators pull out of step and have to be disconnected. The phenomenon depends greatly on the mechanical characteristics of the generators, since the rotors accelerate and decelerate, causing the instantaneous load angles to oscillate. Stability is usually determined by the 1st swing of the machine and it is assumed that if the generator does not pull out of step during the 1st swing, it will ultimately settle down to a steady condition.

The transient power angle characteristic is usually calculated by one of the following assumptions, depending upon the accuracy & simplicity of calculation needed:-

1. The voltage behind the d- axis transient reactance may be assumed constant (constant $V_i'$) Gr.

2. The flux linkage of the field winding may be assumed constant (constant $V_q'$) i.e. the voltage behind the transient reactance may be assumed constant.

With the two different assumptions, the power-angle curves can be plotted by the respective equations -

$$T_e = \frac{V_q' V}{X_d'} \sin\delta - V^2 \frac{X_q - X_d'}{2X_d' X_q} \sin 2\delta \quad \ldots \quad \ldots \quad 1.31$$

$$\text{or } T_e = \frac{V_i' V}{X_d'} \sin\delta_i \quad \ldots \quad \ldots \quad 1.32$$

The transient stability limit in a single machine problem can be estimated by the maximum angle of fault clearing by simply balancing positive & negative areas on the power-angle diagram. This is possible because the movement of only one machine rotor is to be followed. In the case of two or more machines, the equal area method can not be applied; stability or instability is determined by the relative angular position of two machines.

Transient stability studies are also made for determining the ability of a system to maintain synchronism following a disturbance such as short circuit. Whether the system stays .

in synchronism or not is determined from the swing curve. The swing curve is drawn from the following eqn. by a step-by-step method using a small interval of time.

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = T_m - T_e - T_b \qquad \ldots \qquad \ldots \qquad 1.33$$

where $T_m$ = Mechanical torque input of Prime Mover and is assumed constant.

$T_e$ = Positive sequence synchronous torque output.

$T_b$ = Braking torque due to unidirectional torques.

From the swing curves, the time $t_c$ corresponding to the maximum angle of fault clearing can be obtained.

If the actual time of clearing the fault is less than $t_c$, the system will be stable; if it is greater, the system will be unstable. The time that it takes the rotor to swing to this critical angular displacement is the maximum time that the fault can be left on with stability.

## 1.14 Calculations for a synchronous machine during a 3 Ø short-circuit...

The constants of the machine measured by standard test methods are (in P.U.):-

$X_d = 0.766$ $\qquad$ H = .381 sec.

$X_d' = 0.26$

$X_d'' = X_q'' = X_2 = 0.245$

$X_o = 0.035$

$X_q = X_q' = 0.452$

$r_1 = .0195$

$r_2 = .076$

$T_{do}' = 1.3$

$T_{do}'' = .0235$

$T_{d}' = 0.49$

$T_{d}'' = T_{q}'' = .02$

$T_{qo}'' = .0367$

$X_d$ & $X_q$ are unsaturated values.

...1.14.1 3 phase short circuit on a Loaded alternator...

Before the short circuit, the alternator is driven by the prime mover and supplies power to the fixed supply. Under this steady conditions, the P.U. values of the quantities are:-

V= 1.00; I = 1.00; P.F. = unity

$\delta_o = 25^o$ $V_{q_o}' = 1.02$

From equation 1.31, the power output is given by-

$$T_e = 3.93 \sin\delta - .817 \sin 2\delta \quad \ldots \quad \ldots \quad 1.34$$

The torque angle diagram is calculated from the calculations. tabulated in Table 1.2

The torque angle diagram is shown plotted in Fig.1.3 In the figure, the constant Prime Mover input, which is taken equal to 1 P.U., is also drawn. When the short circuit is applied at the stator terminals, the power output is zero i.e. the torque angle diagram now reduces to the x-axis. The critical angle of fault clearing can be found by the equal-area method. In this case, the critical angle has been found to be 119°.

TABLE [illegible]

Computation [illegible] Characteristic.

| δ (deg) | 2δ (deg) | Sin δ | Sin 2δ | 3.99 x Sin δ | -.817 x Sin 2δ | $T_e$ (P.U.) |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 30 | .258 | .500 | 1.02 | -.408 | .612 |
| 30 | 60 | .500 | .866 | 1.96 | -.708 | 1.252 |
| 45 | 90 | .707 | 1.000 | 2.78 | -.817 | 1.963 |
| 60 | 120 | .866 | .866 | 3.40 | -.708 | 2.692 |
| 75 | 150 | .966 | .500 | 3.80 | -.408 | 3.392 |
| 90 | 180 | 1.00 | 0 | 3.99 | 0 | 3.990 |
| 105 | 210 | .966 | -.500 | 3.80 | .408 | 4.210 |
| 120 | 240 | .866 | -.866 | 3.40 | .708 | 4.110 |
| 135 | 270 | .707 | -1.000 | 2.78 | .817 | 3.590 |
| 150 | 300 | .500 | -.866 | 1.96 | .708 | 2.668 |
| 165 | 330 | .258 | -.500 | 1.02 | .408 | 1.428 |
| 180 | 360 | 0 | 0 | 0 | 0 | 0 |

TABLE 1.2

Computation of torque-angle characteristic.

| δ (deg) | 2δ (deg) | Sin δ | Sin 2δ | 3.93 x Sin δ | -.817 x Sin 2δ | Te |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 30 | .259 | .500 | 1.02 | -.408 | . |
| 30 | 60 | .500 | .866 | 1.96 | -.708 | 1. |
| 45 | 90 | .707 | 1.000 | 2.78 | -.817 | 1. |
| 60 | 120 | .866 | .866 | 3.40 | -.708 | 2. |
| 75 | 150 | .966 | .500 | 3.80 | -.408 | 3. |
| 90 | 180 | 1.00 | 0 | 3.93 | 0 | 3. |
| 105 | 210 | .966 | -.500 | 3.80 | .408 | 4. |
| 120 | 240 | .866 | -.866 | 3.40 | .708 | 4. |
| 135 | 270 | .707 | -1.000 | 2.78 | .817 | 3. |
| 150 | 300 | .500 | -.866 | 1.96 | .708 | 2. |
| 165 | 330 | .259 | -.500 | 1.02 | .408 | 1. |
| 180 | 360 | 0 | 0 | 0 | 0 | 0 |

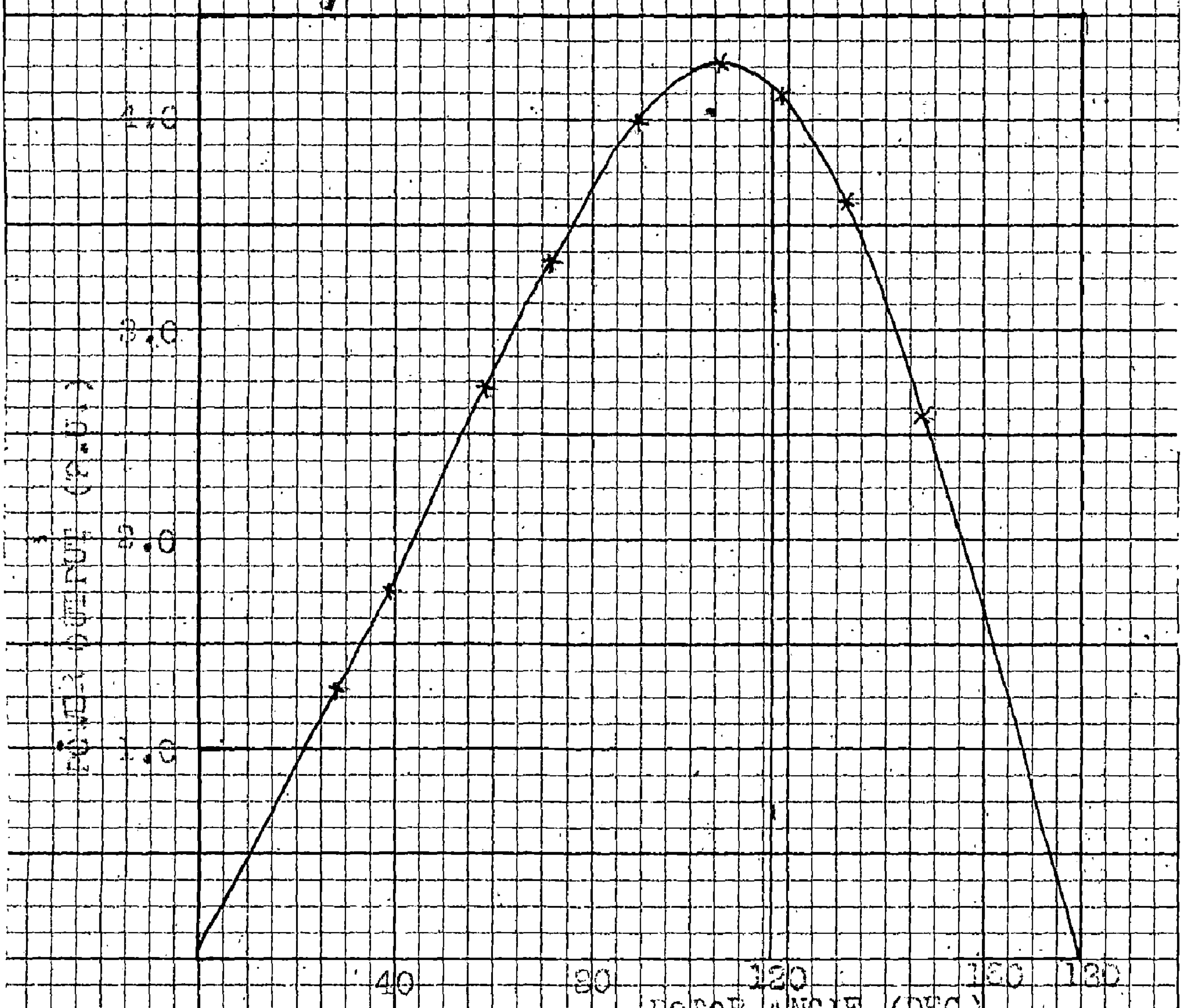



**Fig.1.3**

DETERMINATION OF CRITICAL ANGLE OF FAULT CLEARING DURING A 3 PHASE SHORT CIRCUIT FROM THE POWER-ANGLE DIAGRAM.

...1.14.1.1 Calculation of the swing curve...

The braking torques are calculated as shown below-

$$T_b = T_{u1} + T_{u2}$$

where $T_{u1} = r_a \left[ (I_{do} + I_{dt})^2 + (I_{qo} + I_{qt})^2 \right]$

$$I_{do} = I \sin (\phi + \delta) = .422$$

$$I_{qo} = I \cos (\phi + \delta) = .905$$

$$I_{qt} = V_{do} \left( \frac{1}{X_q} + \frac{X_q - X_q''}{X_q \, X_q''} e^{-t/T_q''} \right)$$

$$= 1 \times \sin 25 \left[ \frac{1}{.452} + \frac{.452 - .245}{.452 \times .245} e^{-t/.02} \right]$$

$$= (.93 + .79 e^{-50t})$$

$$I_{dt} = \frac{V_{qo} \, F}{X_d''}$$

$$= \frac{1 \times \cos 25}{.245} \left[ .32 + .622 e^{-2.27t} + .0577 e^{-45t} \right]$$

$$= (1.18 + 2.34 e^{-2.27t} + .214 e^{-45t})$$

$$T_{u1} = .115 + .103 e^{-4.54t} + .000895 e^{-90t} + .148 e^{-2.27t} + .0192 e^{-47.27t} + .0134 e^{-45t} + .0121 e^{-100t} + .0565 e^{-50t}$$

= Torque due to armature copper loss.

$$T_{u2} = \sqrt{2} \left[ \frac{\sqrt{2} \times 1 \times e^{-25t}}{2 \times .245} \right]^2 (.076 - .0195)$$

$$= 0.66 e^{-50t}$$

= Torque due to rotor losses.

$$T_b = T_{u1} + T_{u2}$$

$$=(.115+.103e^{-4.54t}+ .148e^{-2.27t}+.0192e^{-47.27t}+.012\ e^{-100t} + .716\ e^{-50t}) \quad \ldots \quad 1.35$$

The variation of unidirectional torque after short circuiting a loaded alternator calculated from equation 1.35 is plotted in fig. 1.4.

$T_m = 1$

$T_e = 0$ during 3 phase short circuit.

From equation 1.33,

$$\frac{.381}{\pi f} \frac{d^2\delta}{dt^2} = 1- (.115 +.103e^{-4.54t} + .148e^{-2.27t}+ .0192e^{-47.27t} + .1175e^{-50t}+.0121e^{-100t} +.716\ e^{-50t})$$

At $t = 0$, $\frac{d\delta}{dt} = 0$ and $\delta = 25^0$

$$\therefore \delta=(-42.16t + 182t^2+39 -2.05\ e^{-4.54t}-11.85e^{-2.27t} -.1175\ e^{-50t}) \quad \ldots \quad \ldots \quad 1.36$$

The swing curve is calculated as shown in table 1.3

The swing curve is plotted in fig. 1.5 From the curve, it is seen that the maximum time for which the fault can be allowed to persist is 0.8 seconds. The dip in the curve during the first few cycles can be explained as follows. After the short circuit, the sudden transient torque produced by the alternator temporarily exceeds the torque of the prime mover, resulting in a drop in speed. Subsequently, when the alternator torque decays, the machine accelerates because of the Prime mover torque.

1.1.14.1.2. 3 Phase short circuit on a loaded alternator (without damper windings)...

much greater than the positive sequence resistance and its value depends largely on the value of the resistance of the damper windings.

With this assumption, $T_b = T_{ul}$ and $X_q = X_q' = X_q''$

Now $I_{do} = .422$

$I_{qo} = .905$

$I_{qt} = .93$

$I_{dt} = (1.18 + 2.3\ e^{-2.04t})$

$$\therefore \quad T_b = .0195 \left[ (1.60 + 2.3\ e^{-2.04t})^2 + (1.83)^2 \right]$$

$$= (.115 + .1035\ e^{-4.08t} + .1435\ e^{-2.04t}) \quad \ldots \quad 1.37$$

The variation of unidirectional torque calculated from equation 1.37 is plotted in fig. 1.4.

$$\frac{d^2\delta}{dt^2} = 412 \left[ 1 - (.115 + .1035\ e^{-4.08t} + .1435 e^{-2.04t}) \right]$$

$$= (364 - 42.6\ e^{-4.08\ t} - 59.1\ e^{-2.04t})$$

At $t = 0$, $\frac{d\delta}{dt} = 0$ and $\delta = 25^{o}$

$$\therefore \delta = (182\ t^2 - 39.4t + 41.75 - 2.55e^{-4.08t} - 14.2e^{-2.04t}$$

$\ldots \quad \ldots 1.38$

The swing curve is calculated in table 1.4 as shown. The swing curve is plotted in Fig. 1.5.

It is seen that the damper windings provide an additional braking torque on a generator and thus reduce the accelerating torque during the fault.

The modern trend is to clear the faults as rapidly as possible. clearing times greater than 0.2 sec. are seldom used in important circuits where system stability is a consideration. Moreover, the

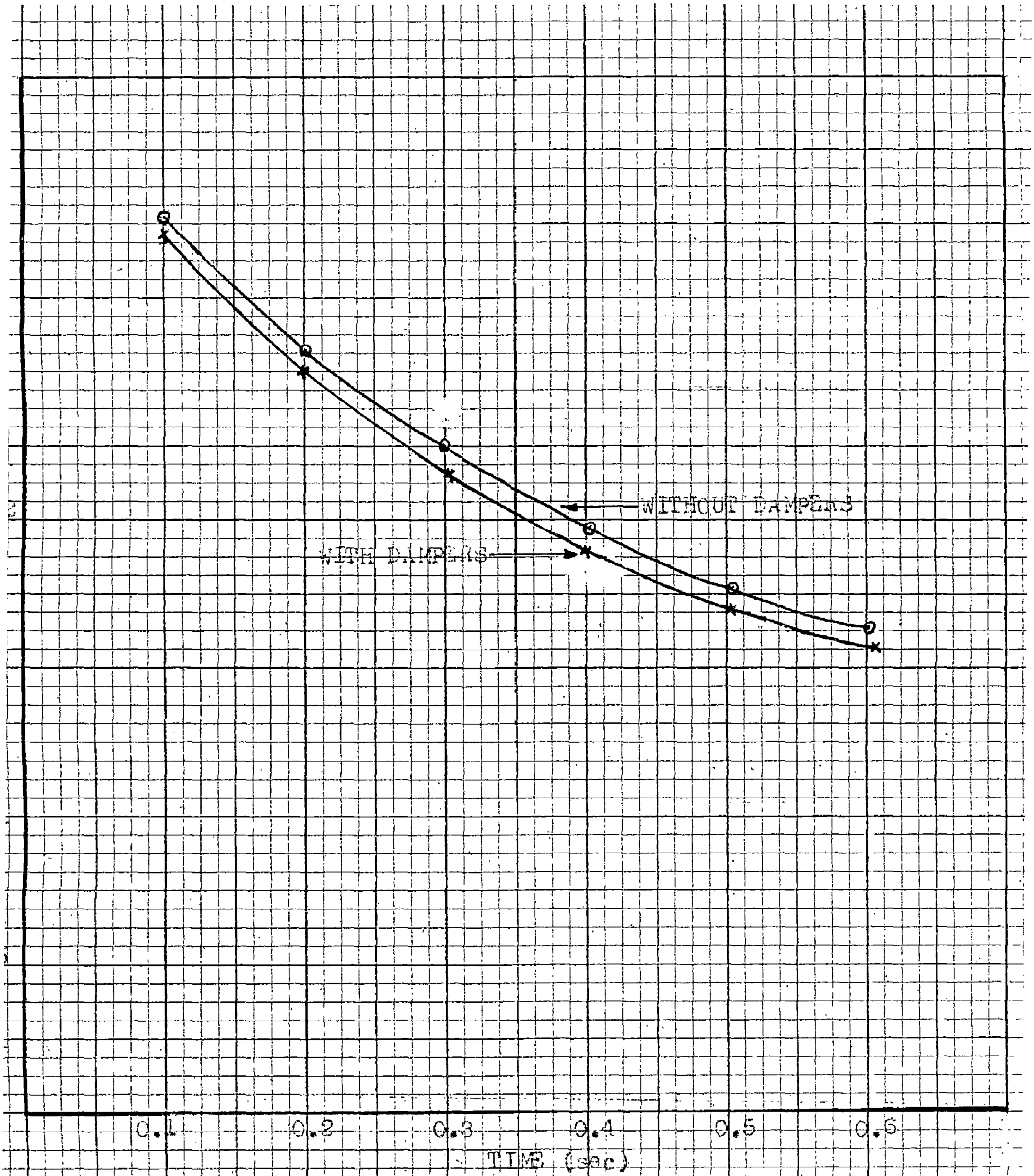



FIG. 1.4

VARIATION OF UNIDIRECTIONAL TORQUE AFTER SHORT CIRCUITING
A LOADED ALTERNATOR

TABLE 1.3

Computation of swing curve during 3 Ø S/C of a loaded alternator (with damper windings).

| t (sec) | -42.16t | 152 $t^2$ | $-2.05e^{-4.54t}$ | $-11.85e^{-2.27t}$ | $-.1175e^{-50t}$ | δ (deg) |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | -2.05 | -11.85 | -.1175 | 25 |
| .02 | - .84 | .072 | -1.87 | -11.3 | -.11 | 24.89 |
| .10 | -4.216 | 1.32 | -1.30 | -9.4 | - | 25.91 |
| .20 | -8.432 | 7.28 | -.826 | -7.5 | - | 29.56 |
| .30 | -12.64 | 16.4 | -.528 | -6.0 | - | 36.24 |
| .40 | -16.85 | 29.2 | -.334 | -4.8 | - | 46.22 |
| .50 | -21.08 | 45.5 | -.213 | -3.78 | - | 59.43 |
| .60 | -25.2 | 61.5 | -.133 | -3.03 | - | 72.14 |
| .70 | -29.5 | 89.0 | -.09 | -2.42 | - | 96.0 |
| .80 | -33.7 | 116.5 | -.053 | -2.02 | - | 119.7 |
| .90 | -37.9 | 147.0 | -.032 | -1.54 | - | 146.56 |

TABLE 1.4

Computation of swing curve during three-phase short circuit of a loaded alternator (without damper windings)

| t (sec) | $182\ t^2$ | $-39.4t$ | $-2.55e^{-4.08t}$ | $-14.2e^{-2.04t}$ | $\delta$ (deg) |
|---|---|---|---|---|---|
| 0 | 0 | 0 | -2.55 | -14.2 | 25 |
| 0.1 | 1.82 | -3.94 | -1.7 | -11.6 | 26.33 |
| 0.2 | 7.28 | -7.88 | -1.13 | -9.45 | 30.57 |
| 0.3 | 16.4 | -11.92 | -0.75 | -7.70 | 37.78 |
| 0.4 | 29.1 | -15.76 | -0.49 | -6.26 | 48.34 |
| 0.5 | 45.5 | -19.70 | -0.33 | -5.11 | 62.11 |
| 0.6 | 65.5 | -23.60 | -0.22 | -4.17 | 79.26 |
| 0.7 | 89.0 | -27.60 | -0.14 | -3.51 | 99.50 |
| 0.8 | 116.0 | -31.50 | -0.09 | -2.78 | 123.38 |
| 0.9 | 147.0 | -35.40 | -0.07 | -2.27 | 151.02 |

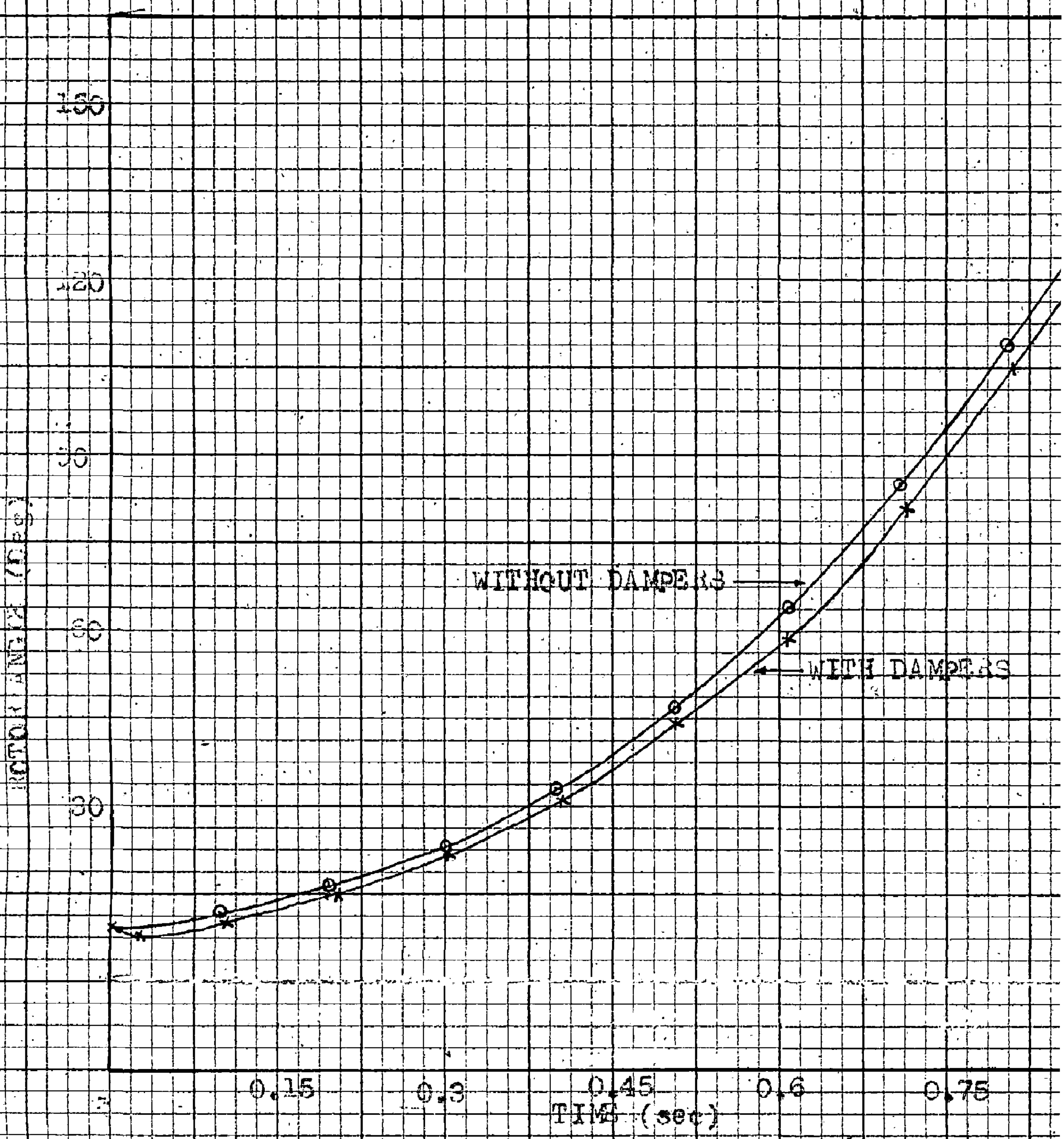



**Fig.1.5: VARIATION OF ROTOR ANGLE DURING 3 PHASE SHORT CIRCUIT OF A LOADED ALTERNATOR**

majority of faults occuring in practice are unsymmetrical between phases, which are less severe than the 3 phase short-circuit. A less severe type of fault would permit either slower clearing or transmission of more power without loss of synchronism.

### ...1.14.2 3 Phase short circuit on an unloaded alternator...

The alternator is unloaded before short circuit and is run as a synchronous motor. Under this condition, the speed falls continuously after the short circuit.

Under these conditions, $T_{u1}$ and $T_{u2}$ are given by the expressions in Table 1.1.

$$T_b = T_{u1} + T_{u2}$$

$$= .0195 \left[ \frac{.32 + .622\, e^{-2.27t} + .0577 e^{-45t}}{.245} \right]^2$$

$$+ \sqrt{2} \left[ \frac{\sqrt{2} \times 1 \times e^{-25t}}{2 \times .245} \right]^2 (.076 - .0195)$$

$$= (.66 e^{-50t} + .033 + .125 e^{-4.54t} + .001075 e^{-90t} + .129 e^{-2.27t}$$

$$+ .01195\, e^{-45t} + .0234\, e^{-97.27t}). \quad \ldots \quad 1.39$$

The variation of unidirectional torque after short circuiting on unloaded alternator calculated from eqn. 1.39 is plotted in fig. 1.6.

$$T_m = 0$$

$$\therefore \frac{d^2\delta}{dt^2} = -412\, (.66 e^{-50t} + .033 + .125 e^{-4.54t} + .001075 e^{-90t}$$

$$+ .129 e^{-2.27t} + .01195 e^{-45t} + .0234 e^{-47.27t})$$

$$= -(13.6 + 272 e^{-50t} + 51.5\, e^{-45.4t} + .442\, e^{-90t}$$

$$+ 53.1\, e^{-2.27t} + 4.92\, e^{-45t} + 9.65 e^{-47.27t})$$

when $t = 0$, $\frac{d\delta}{dt} = 0$ and $\delta = 0$

with these conditions

$$\delta = (-6.8t^2 - 40.46t + 13 - .108e^{-50t} - 2.5e^{-4.54t} - 10.3e^{-2.27t})$$

... .. 1.40

The swing curve is calculated as shown in Table 1.5

The swing curve is plotted in Fig. 1.7.

...<u>1.14.2.1 3 Phase short circuit on an unloaded alternator (without damper windings)</u>...

With the same assumptions as for a loaded alternator without damper windings,

$$T_b = T_{ul} = \left(\frac{VF}{X_d'}\right)^2 r_1$$

$$F = \left[\frac{X_d'}{X_d} + \frac{X_d - X_d'}{X_d} e^{-t/T_d'}\right]$$

$$\therefore T_b = \left(\frac{.34 + .66e^{-2.04t}}{.26}\right)^2 \times .0195$$

$$= (.0316 + .126\, e^{-4.08t} + .13\, e^{-2.04t}) \quad \text{.... ..1.41}$$

The variation of unidirectional torque calculated from eqn.1.41 is plotted in fig. 1.6.

$$\frac{d^2\delta}{dt^2} = -412 \quad (.0316 + .126\, e^{-4.08t} + .13e^{-2.04t})$$

$$\text{At } t = 0, \frac{d\delta}{dt} = 0, \quad \delta = 0$$

Under these conditions,

$$\delta = (-6.5\, t^2 - 38.9t + 15.92 - 3.12\, e^{-4.08t} - 12.8e^{-2.04t})$$

.... ... 1.42

The swing curve is calculated as shown in table 1.6. The swing curve is plotted in Fig. 1.7.

It can be seen that in the case of an unloaded alternator with damper windings, when the 3 phase short circuit is applied, the rotor falls more quickly than in the case of an unloaded alternator without damper windings.

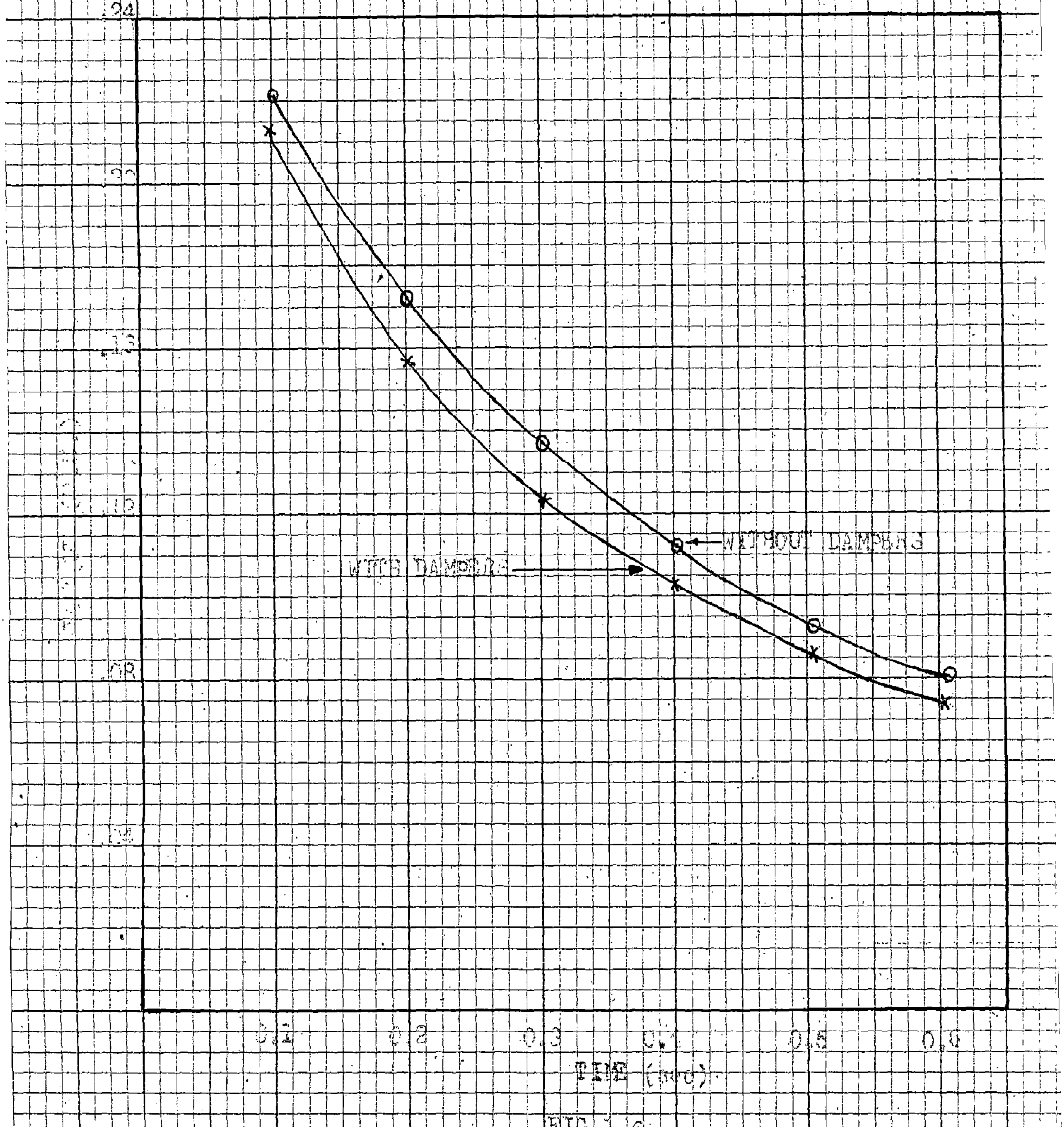



FIG 16

VARIATION OF UNIDIRECTIONAL TORQUE AFTER SHORT CIRCUITING AN UNLOADED ALTERNATOR

majority of faults occuring in practice are unsymmetrical between phases, which are less severe than the 3 phase short-circuit. A less severe type of fault would permit either slower clearing or transmission of more power without loss of synchronism.

### 1.14.2 3 Phase short circuit on an unloaded alternator.

The alternator is unloaded before short circuit and is run as a synchronous motor. Under this condition, the speed falls continuously after the short circuit.

Under these conditions, $T_{u1}$ and $T_{u2}$ are given by the expressions in Table 1.1.

$$T_b = T_{u1} + T_{u2}$$

$$= .0195 \left[ \frac{.32 + .622\, e^{-2.27t} + .0577 e^{-45t}}{.245} \right]^2 + \sqrt{2} \left[ \frac{\sqrt{2} \times 1 \times e^{-25t}}{2 \times .245} \right]^2 (.076 - .0195)$$

$$= (.66 e^{-50t} + .033 + .125 e^{-4.54t} + .001075 e^{-90t} + .129 e^{-2.27t} + .01196\, e^{-45t} + .0234\, e^{-97.27t}). \quad \ldots \quad 1.39$$

The variation of unidirectional torque after short circuiting on unloaded alternator calculated from eqn. 1.39 is plotted in fig. 1.6.

$$T_m = 0$$

$$\therefore \frac{d^2\delta}{dt^2} = -412\, (.66 e^{-50t} + .033 + .125 e^{-4.54t} + .001075 e^{-90t} + .129 e^{-2.27t} + .01195 e^{-45t} + .0234 e^{-47.27t})$$

$$= -(13.6 + 272 e^{-50t} + 51.5\, e^{-45.4t} + .442\, e^{-90t} + 53.1\, e^{-2.27t} + 4.92\, e^{-45t} + 9.65 e^{-47.27t})$$

when $t = 0$, $\frac{d\delta}{dt} = 0$ and $\delta = 0$

with these conditions

$$\delta = (-6.8t^2 - 40.46t + 13 - .108e^{-50t} - 2.5e^{-4.54t} - 10.3e^{-2.27t}) \quad \ldots \quad \ldots 1.40$$

The swing curve is calculated as shown in Table 1.5

The swing curve is plotted in Fig. 1.7.

### 1.14.2.1 3 Phase short circuit on an unloaded alternator (without damper windings)

With the same assumptions as for a loaded alternator without damper windings,

$$T_b = T_{ul} = \left(\frac{VF}{X_d'}\right)^2 r_1$$

$$F = \left[\frac{X_d'}{X_d} + \frac{X_d - X_d'}{X_d} e^{-t/T_d'}\right]$$

$$\therefore T_b = \left(\frac{.34 + .66e^{-2.04t}}{.26}\right)^2 \times .0195$$

$$= (.0316 + .126\, e^{-4.08t} + .13\, e^{-2.04t}) \quad \ldots \quad \ldots 1.41$$

The variation of unidirectional torque calculated from eqn. 1.41 is plotted in fig. 1.6.

$$\frac{d^2\delta}{dt^2} = -412 \quad (.0316 + .126\, e^{-4.08t} + .13e^{-2.04t})$$

$$\text{At } t = 0, \frac{d\delta}{dt} = 0, \quad \delta = 0$$

Under these conditions,

$$\delta = (-6.5\, t^2 - 38.9t + 15.92 - 3.12\, e^{-4.08t} - 12.8e^{-2.04t}) \quad \ldots \quad \ldots 1.42$$

The swing curve is calculated as shown in table 1.6. The swing curve is plotted in Fig. 1.7.

It can be seen that in the case of an unloaded alternator with damper windings, when the 3 phase short circuit is applied, the rotor falls more quickly than in the case of an unloaded alternator without damper windings.

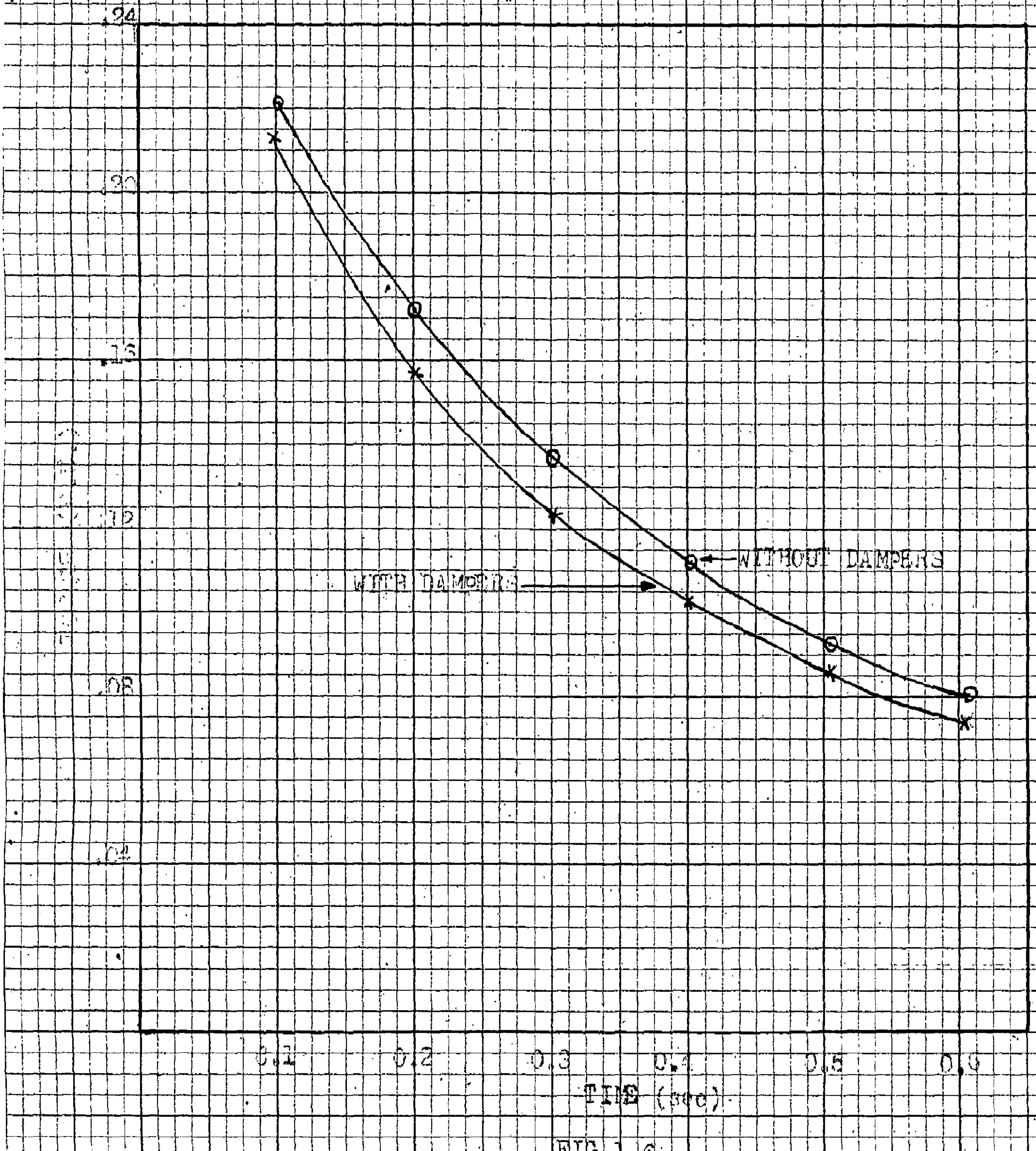



FIG 1.6

VARIATION OF UNIDIRECTIONAL TORQUE AFTER SHORT CIRCUITING AN UNLOADED ALTERNATOR

TABLE 1.5

Computation of swing curve during 2Ø short circuit of an unloaded alternator (with damper windings).

| t (sec) | $-6.8t^2$ | $-40.46t$ | $-.109e^{-50t}$ | $-2.5e^{-4.54t}$ | $-10.3e^{-2.27t}$ | δ (deg) |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | -.109 | -2.5 | -10.3 | 0 |
| 0.1 | -.068 | -4.04 | -.0017 | -1.59 | -8.2 | -0.89 |
| 0.2 | -.272 | -8.09 | -.00004 | -1.01 | -6.55 | -2.92 |
| 0.3 | -.61 | -12.1 | - | -.64 | -5.16 | -5.51 |
| 0.4 | -1.09 | -16.3 | - | -.404 | -4.16 | -8.95 |

TABLE 1.6

Computation of swing curve during 3 phase short circuit of an unloaded alternator (without damper windings).

| t (sec) | $-6.5\ t^2$ | $-38.9t$ | $-3.12e^{-4.08t}$ | $-12.8e^{-2.04t}$ | δ (deg) |
|---|---|---|---|---|---|
| 0 | 0 | 0 | -3.12 | -12.8 | 0 |
| 0.1 | -.065 | -3.89 | -2.07 | -10.4 | -.5 |
| 0.2 | -.26 | -7.78 | -1.38 | -8.5 | -2.0 |
| 0.3 | -.585 | -11.67 | -.916 | -6.92 | -4.09 |
| 0.4 | -1.04 | -15.5 | -.611 | -5.55 | -6.78 |

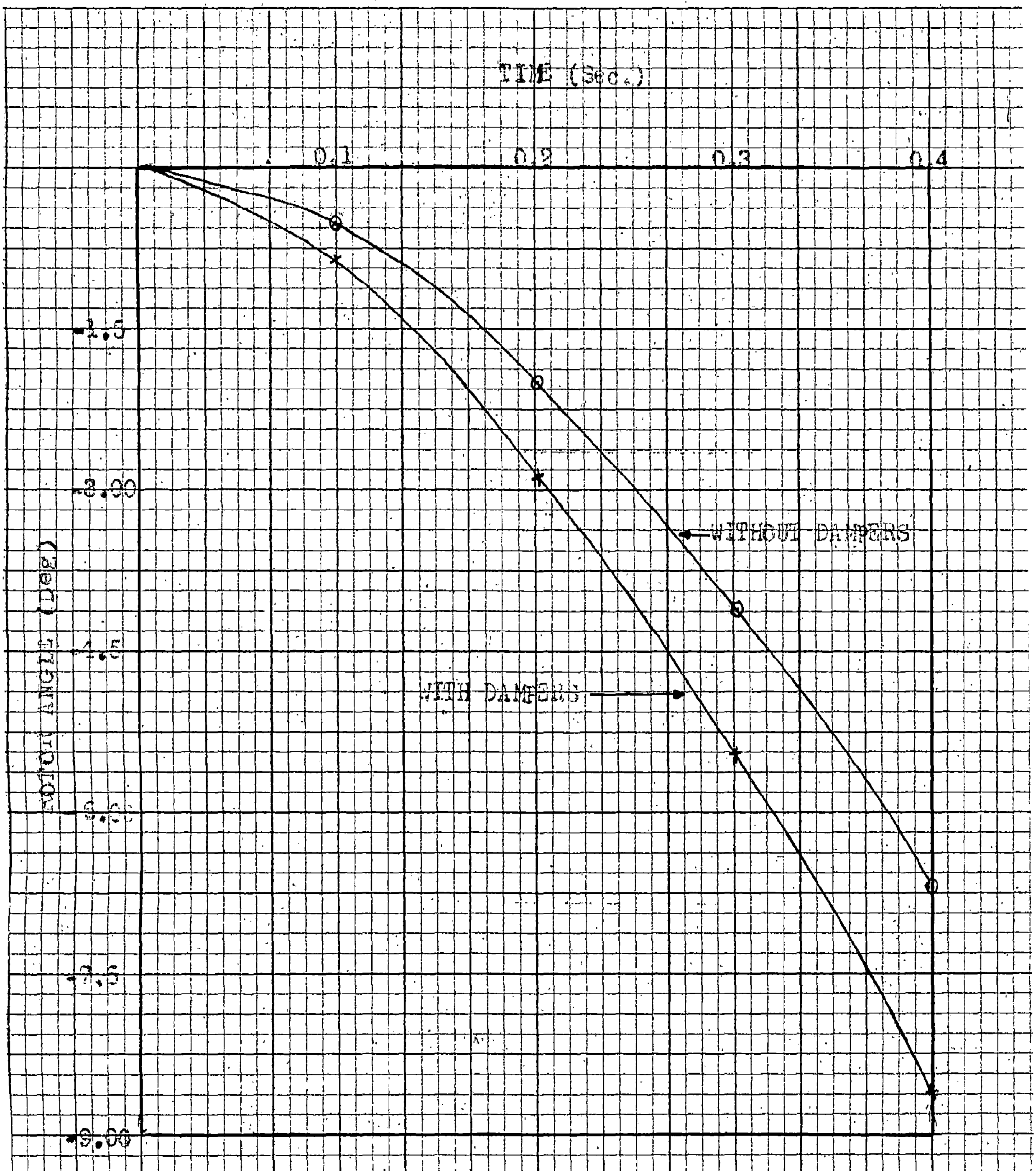



FIG 1.7

VARIATION OF ROTOR ANGLE DURING 3 PHASE SHORT CIRCUIT

OF AN UNLOADED ALTERNATOR

majority of faults occuring in practice are unsymmetrical between phases, which are less severe than the 3 phase short-circuit. A less severe type of fault would permit either slower clearing or transmission of more power without loss of synchronism.

...1.14.2 3 Phase short circuit on an unloaded alternator...

The alternator is unloaded before short circuit and is run as a synchronous motor. Under this condition, the speed falls continuously after the short circuit.

Under these conditions, $T_{u1}$ and $T_{u2}$ are given by the expressions in Table 1.1.

$$T_b = T_{u1} + T_{u2}$$

$$= .0195 \left[ \frac{.32 + .622\, e^{-2.27t} + .0577 e^{-45t}}{.245} \right]^2 + \sqrt{2} \left[ \frac{\sqrt{2} \times 1 \times e^{-25t}}{2 \times .245} \right]^2 (.076 - .0195)$$

$$= (.66e^{-50t} + .033 + .125e^{-4.54t} + .001075e^{-90t} + .129e^{-2.27t} + .01195\, e^{-45t} + .0234\, e^{-97.27t}). \quad \ldots \quad 1.39$$

The variation of unidirectional torque after short circuiting on unloaded alternator calculated from eqn. 1.39 is plotted in fig. 1.6.

$$T_m = 0$$

$$\therefore \frac{d^2\delta}{dt^2} = -412\, (.66e^{-50t} + .033 + .125e^{-4.54t} + .001075e^{-90t} + .129e^{-2.27t} + .01195e^{-45t} + .0234e^{-47.27t})$$

$$= -(13.6 + 272e^{-50t} + 51.5\, e^{-45.4t} + .442\, e^{-90t} + 53.1\, e^{-2.27t} + 4.92\, e^{-45t} + 9.65e^{-47.27t})$$

when $t = 0$, $\frac{d\delta}{dt} = 0$ and $\delta = 0$

with these conditions

1.1.15 Positive Sequence damping...

In the above analysis, positive sequence damping torque, which results from the torque caused by interaction of the damper currents with the positive sequence (forward rotating) magnetic field in the air gap, has been neglected. When damping occurs, the positive-sequence power output of the machine is the algebraic sum of the damping power and the synchronous power (as though the machine had no damper windings). If the former is much smaller than the latter, it may be sufficiently accurate to solve the positive sequence network as though only the synchronous components of power existed. Then cognizance of the damping power would be taken only in the calculation of accelerating torque from the eqn. $T_{ac} = T_m - T_e - T_b - T_D$ ... 1.43.

where $T_m$ = Mechanical torque input.

$T_e$ = Positive sequence synchronous power output.

$T_b$ = braking torques due to unidirectional torques.

$T_D$ = Positive sequence damping torques.

If however, the damping power should be as great as 5 to 10% of synchronous power, it might affect conditions in the network sufficiently to warrant taking it into account in the network solution[10].

There is no damping effect during the period the fault is on as already explained.

Then eqn. 1.43 becomes $T_{ac} = T_m - T_e - T_D$ ... 1.44

The damping torque is given by

$$T_D = V^2 s\omega\left[\frac{X_d' - X_d''}{X_d'^2} T_{do}'' \sin^2\delta + \frac{X_q' - X_q''}{X_q'^2} T_{qo}'' \cos^2\delta\right]$$

the proof of which is given in Chapter II.

are made:-

1. No resistance in armature circuit.
2. No resistance in field circuit.
3. Small slip.
4. Damping action caused by only one set of rotor windings (damper windings).

...1.15.1 Calculations...

$$T_D = V^2 s\omega \left[ \frac{X_d' - X_d''}{X_d'^2} T_{do}'' \sin^2\delta + \frac{X_q' - X_q''}{X_q^2} T_{qo}'' \cos^2\delta \right]$$

$$= 1 \times \frac{\Delta\delta}{\Delta t} \left[ \frac{(.26 - .245)}{(.26)^2} \times .0235 \sin^2\delta + \frac{(.45 - .245)}{(.45)^2} \times .036 \cos^2\delta \right]$$

$$= \frac{\Delta\delta}{\Delta t} \left[ .00522 \sin^2\delta + .0372 \cos^2\delta \right]$$

$$= \frac{\Delta\delta}{\Delta t} \left[ .02121 + .01599 \cos 2\delta \right]$$

$=\Delta\delta(0.2121 + 0.1599 \cos 2\delta)$ for a time interval of 0.1sec.

$\Delta\delta$ is taken as the average value for the time intervals immediately preceding and following the instant for which $T_D$ and $T_a$ are to be computed.

After the fault is cleared at 0.8 sec. after occurance of the fault, the swing curve is calculated using eqn. 1.44

$$T_{ac} = T_m - T_e - T_D$$

where $T_m = 1$

$T_e = 3.93 \sin\delta - 0.817 \sin 2\delta$

$T_D = \Delta\delta(0.2121 + 0.1599 \cos 2\delta)$

$\approx 0.2121 \ \Delta\delta$

Hence, the eqn. of motion is given by

$$\frac{\Delta^2\delta}{(\Delta t)^2} = 1 - (3.93 \ \mathrm{Sin}\,\delta - 0.817 \ \mathrm{Sin}\ 2\delta) - 0.2121\,\Delta\delta$$

The swing curve after the fault is cleared in 0.8 sec, is calculated as shown in Table 1.7.

The swing curve is plotted in Fig. 1.8

The calculations are repeated neglecting damping. The eqn. of motion will now be-

$$\frac{\Delta^2\delta}{(\Delta t)^2} = 1 - (3.73 \ \mathrm{Sin}\,\delta - .817 \ \mathrm{Sin}\ 2\delta)$$

The swing curve is calculated as shown in table. 1.8.

The swing curve is plotted in Fig. 1.8.

It is seen from the figure that positive sequence damping causes the oscillations of the machine rotors to decrease in amplitude. If the fault had been cleared sooner, damping would have been more effective in reducing the amplitude of swing.

TABLE 1.7

Calculation of swing curve after the 3 phase short circuit is cleared. Field Flux linkage assumed constant. (Damping included)

| t (sec) | Sin δ | 3.93x Sin δ | Sin 2δ | -.817x Sin 2δ | $T_o$ | Δδ (est) | $T_D$ | $T_{ac}$ | $\Delta^2 \delta$ | Δδ | δ (deg) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.8- | - | - | - | - | 0 | - | 0 | 1.00 | - | 23.7 | - |
| 0.8+ | 0.87[illegible] | 3.44 | -.846 | +.691 | 4.132 | 16 | 3.39 | -6.52 | - | - | - |
| 0.9 av | - | - | - | | - | - | - | -2.76 | -11.5 | - | 110° |
| | | | | | | | | | | 12.2 | - |
| 0.9 | 0.75 | 2.[illegible] | -0.99 | +.[illegible] | 3.76 | 4 | .24 | -3.6 | -14.8 | | 131.2 |
| | | | | | | | | | | -2.6 | - |
| 1.0 | 0.78 | 3.[illegible]7 | -.97 | +.79 | 3.86 | -6 | -1.26 | -1.6 | -6.6 | | 128.6 |
| | | | | | | | | | | -9.2 | |
| 1.1 | .875 | [illegible] | -.85 | +.692 | 4.12 | -10 | -2.12 | -1.0 | -4.12 | | 113.4 |
| | | | | | | | | | | -13.3 | |
| 1.2 | .96 | 3.78 | -.53 | +.432 | 4.21 | -13 | -2.76 | -.45 | -1.86 | | 106.1 |
| | | | | | | | | | | -15.1 | |
| 1.3 | 1.0 | 3.93 | -.034 | +.027 | 3.95 | -14 | -2.81 | -.14 | -.58 | | 91.07 |
| | | | | | | | | | | -15.6 | |
| 1.4 | .97 | 3.8 | -.5 | +.408 | 4.2 | -15 | -3.15 | -.05 | -.216 | | 75.4 |
| | | | | | | | | | | -15.3 | |
| | | | | | | | | | | | 59.6 |

TABLE 1.2

Calculation of swing curve after the 3 phase short circuit is cleared.

Field flux linkage assumed constant. (Damping neglected).

| t | $\sin\delta$ | $2.93 \times \sin\delta$ | $\sin 2\delta$ | $-.317 \times \sin 2\delta$ | $T_e$ | $T_{ac}$ | $\Delta^2\delta$ | $\Delta\delta$ | $\delta$ (deg) |
|---|---|---|---|---|---|---|---|---|---|
| .8− | | | | | 0 | 1.0 | | | |
| .8+ | .875 | 2.44 | −.846 | +.691 | 4.132 | −3.132 | | 23.7 | |
| .8av | | | | | 2.07 | −1.07 | −4.43 | | 119 |
| | | | | | | | | 19.3 | |
| 0.9 | .665 | 2.61 | −.9945 | .91 | 3.42 | −2.42 | −10 | | 138.3 |
| | | | | | | | | +9.3 | |
| 1.0 | [illegible] | 2.10 | −.902 | .736 | 2.86 | −1.86 | −7.66 | | 147.6 |
| | | | | | | | | −1.7 | |
| 1.2 | [illegible] | [illegible] | −.994 | .512 | 3.54 | −2.54 | −10.5 | | 145.9 |
| | | | | | | | | −20.1 | |
| 1.3 | [illegible] | 2.55 | −.780 | .24 | 4.17 | −3.17 | −12.1 | | 116.2 |
| | | | | | | | | −33.2 | |
| 1.4 | .336 | 3.38 | −.24 | .19 | 4.07 | −3.07 | −12.5 | | 83.0 |
| | | | | | | | | −45.3 | |
| 1.5 | | | | | | | | | 37.2 |

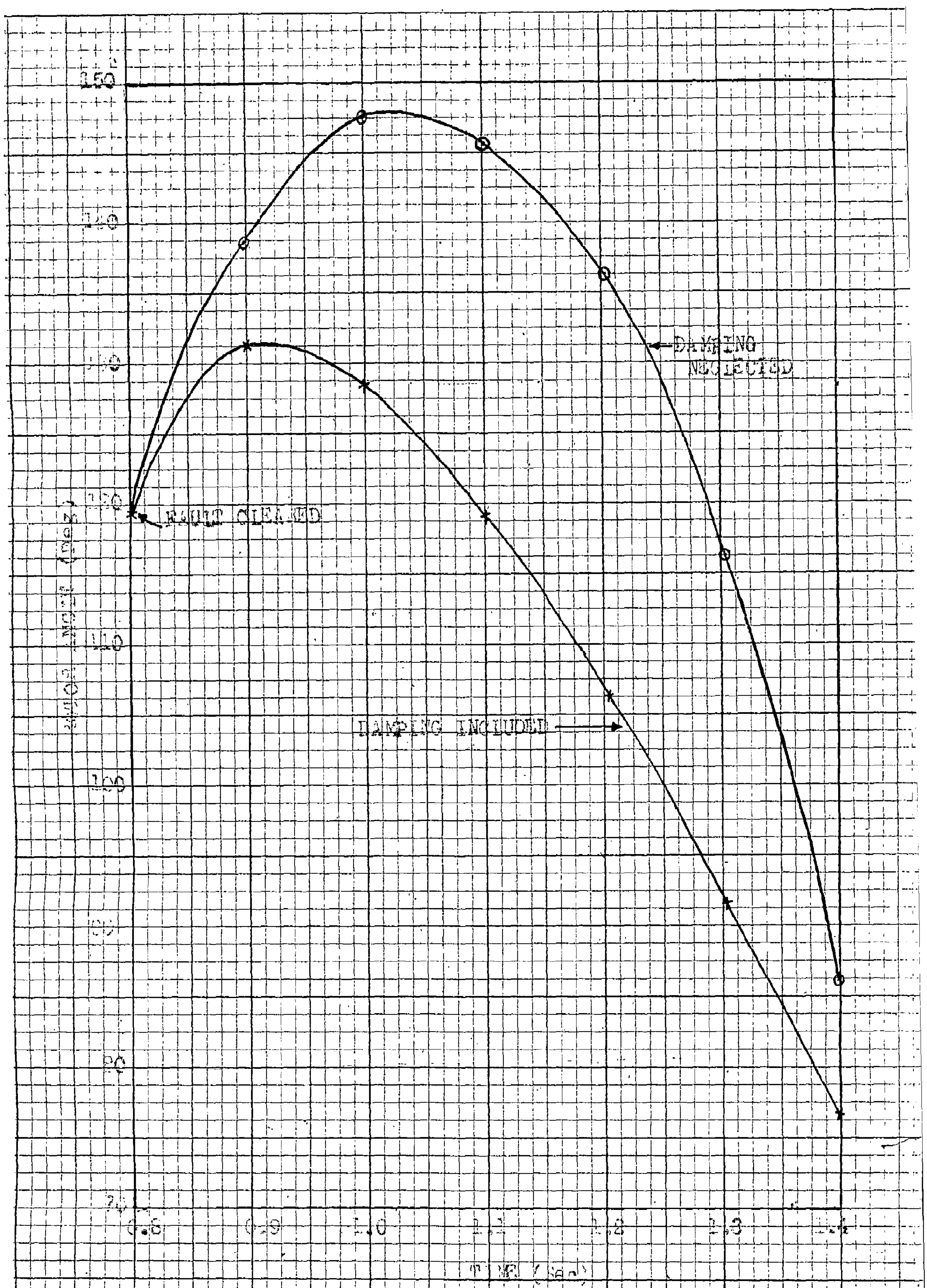



FIG 1.8

VARIATION OF ROTOR ANGLE OF A LOADED ALTERNATOR

# .....CHAPTER II.....

## .....TRANSIENT BEHAVIOUR OF AN ALTERNATOR DUE TO SWITCHING OPERATIONS RESULTING IN SUDDEN INCREASE IN REACTANCE.....

### ...2.1 Sudden insertion of reactance between an alternator and infinite bus...

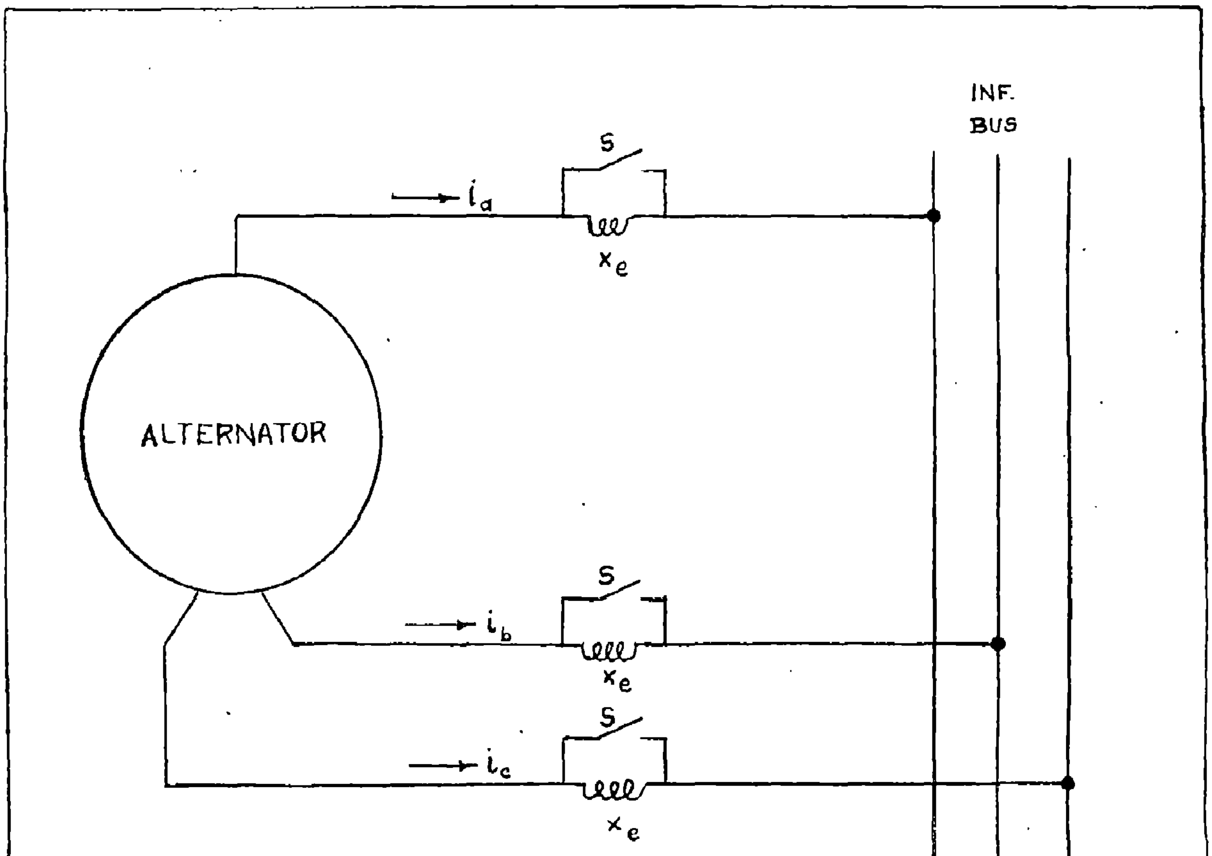



FIG.2.1 ALTERNATOR SUPPLYING POWER TO AN INF.BUS

A 3 phase alternator is supplying power to an infinite bus as shown in fig. 2.1. A reactance $X_e$ is suddenly inserted in each phase by opening the switches S. After a time, the machine may either settle down to a new condition of synchronous operation or it may lose synchronism and operate asynchronously above synchronous speed.

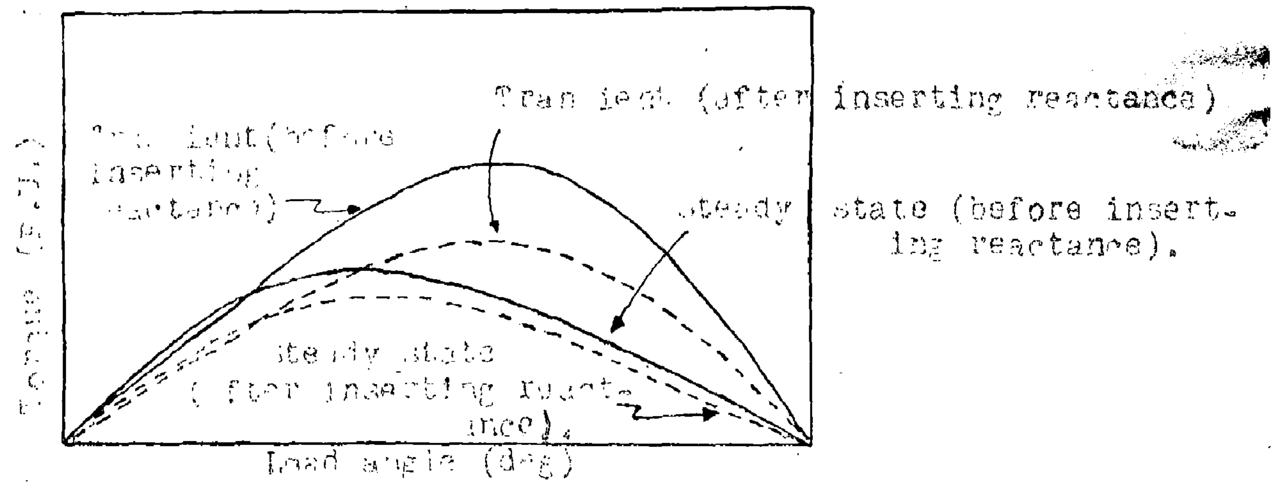



FIG. 2.2

TORQUE ANGLE CHARACTERISTICS OF SALIENT POLE ALTERNATOR CONNECTED TO AN INFINITE BUS.

Fig. 2.2 shows the transient and steady state torque-angle characteristics before and after inserting the reactance. Fig.2.3

is an expanded plot of a portion of Fig.2.2 to illustrate the resulting torque-angular oscillation when the reactance is inserted.

When the reactance is inserted, the electrical torque drops from the initial value at point O to a value approaching that given by the transient curve after inserting reactance. However, because of subtransient or additional rotor circuits,

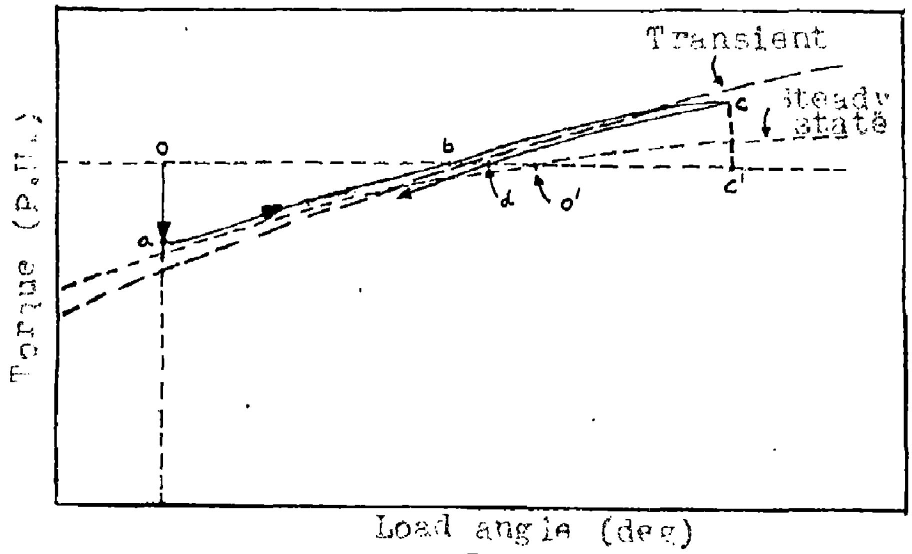



FIG.2.3

PORTION OF FIG.2.2 SHOWING NATURE OF OSCILLATION FROM TRANSIENT TO STEADY STATE CHARACTERISTICS

the torque drops to point 'a' only part of the way. As the angular displacement increases, because of the accelerating torque represented by the ordinate oa, currents are induced in the additional rotor circuits so as to produce a component of electrical torque which adds to the synchronous component. At point b, the rotor velocity is a maximum. The rotor swings to point C so that the area bcc' equals oab according to equal area criterion. As the rotor then decelerates, the rotor velocity relative to the infinite bus voltage is negative so that the electrical torque due to rotor motion changes sign and the electrical torque follows the curve as indicated by the arrow. Because of this component of torque developed as a result of the motion of the rotor, called damping torque, the area cc'd is less than bcc' so that the magnitude of the rotor oscillation decreases. In this way, the oscillation is damped out and the path of motion is along a spiral until it finally comes to rest at O', corresponding to the steady state torque.

### ...2.2 Angle-time or swing curves...

Several methods of calculating the swing curves are given by Crary[11], ranging from the simple method based on constant flux linkage and no damping, to an accurate but laborious method described in a paper by Crary and Waring.[12] Making certain simplifying assumptions, a simplified method has been presented[13], by which a clear picture can be obtained of the part played by damper windings.

In order to study the conditions fully, it is necessary to calculate the electrical torque and the load angle of the machine after the disturbance.

### ...2.3 Expression for Electrical torque...

For slow transients, the approximate eqns. given by eqns. A 12 can be used. If $\delta$ is known as a function of time, the currents and flux linkages and hence the torque can be calculated. However, as $\delta$ is not a known function of time, it is necessary to formulate a general expression for the electrical torque $T_e$ as a function of $\delta$ and use the expression to solve the dynamical equations of motion of the machine rotor by a step-by-step method. By this means, both $T_e$ and $\delta$ can be determined as functions of time.

Substituting the value of $\Psi$ [illegible] from equations A 12 into equations A 2,

$$+\frac{V_m}{\omega}\cos\delta_o = L_{md}\, i_{fo} + L_{md} i_{kdo} - (L_{md} + L_a)\, i_{do}$$

$$V_f = \left[r_f + (L_{md} + L_f)p\right]\; i_{fo} + L_{md}\, p\, i_{kdo} - L_{md}\, p\, i_{do}$$

$$0 = L_{md} p\, i_{fo} + \left[r_{kd} + (L_{md} + L_{kd})p\right]\, i_{kdo} - L_{md}\, p\, i_{do}$$

.. .. 2.1

Here $i_{do}$ and $i_{fo}$ are constant and $i_{kdo} = 0$. After the switch is opened, the inductance increases by an amount $L_e$. The combination of alternator and inductance can be treated as a modified alternator having a leakage reactance $X_a + X_e = \omega(L_a + L_e)$. The field voltage is unchanged. The equations of the modified alternator are:

$$+\frac{V_m}{\omega}\cos\delta = L_{md}\, i_f + L_{md}\, i_{kd} - (L_{md} + L_a + L_e)i_d$$

$$V_f = \left[r_f + (L_{md} + L_f)p\right]\; i_f + L_{md} p\, i_{kd} - L_{md} p\, i_d$$

$$0 = L_{md}\, p\, i_f + \left[r_{kd} + (L_{md} + L_{kd})p\right]\, i_{kd} - L_{md}\, p\, i_d$$

... .. 2.2

The first of equations 2.1 can be rearranged as:

$$+\frac{V_m}{\omega}\cos\delta_o - L_e\, i_{do} = L_{md}\, i_{fo} + L_{md}\, i_{kdo}$$

$$-(L_{md} + L_a + L_e)i_{do} \quad \ldots \quad .. \; 2.3$$

In equations 2.2 and 2.3, the term on the L.H.S. can be regarded as an impressed voltage which changes suddenly at $t = 0$. Hence the superimposed changes of current, denoted by symbols with dashes, are determined by the equations obtained by subtracting equations 2.1 and 2.3 from 2.2.

$$+\left[\frac{V_m}{\omega}(\cos\delta - \cos\delta_o) + L_e\, i_{do}\right] 1 = L_{md}\, i_f' + L_{md} i_{kd}' - (L_{md}+L_a+L_e) i_d'$$

$$0 = \left[r_f + (L_{md} + L_f)p\right] i_f' + L_{md}\, p\, i_{kd}' - L_{md} p i_d'$$

$$0 = L_{md} p i_f' + \left[r_{kd} + (L_{md} + L_{kd})p\right] i_{kd}' - L_{md} p i_d' \quad \ldots \; 2.4$$

The currents $i_f'$ and $i_{kd}'$ are now eliminated from equations 2.4 by the method used for obtaining the 1st line of equations. A.5. Here the field voltage is zero.

$$\therefore +\left[\frac{V_m}{\omega}(\cos\delta - \cos\delta_o) + L_e\, i_{do}\right] 1 = -\frac{X_{de}(p)}{\omega} i_d'$$

$$\ldots \quad \ldots \; 2.5$$

where $X_{de}(p)$ = operational impedance of modified alternator.

The total current $i_d$ is obtained by adding $i_{do}$ to $i_d'$ given by equation 2.5.

$$\therefore i_d = i_{do} - \frac{1}{X_{de}(p)}\left[V_m(\cos\delta - \cos\delta_o) + X_e i_{do}\right] 1$$

Similarly

$$i_q = i_{qo} + \frac{1}{X_{qe}(p)}\left[V_m(\sin\delta - \sin\delta_o) - X_e i_{qo}\right] 1 \quad \ldots 2.6$$

Eqns.2.6 are used to calculate the electrical torque developed. For the modified alternator,

$$X_{de} = (X_d + X_e); \quad T_{de}' = T_{do}' \frac{X_{de}'}{X_{de}} \text{ etc} \quad \ldots 2.7$$

For the modified alternator, the flux linkages are given by equations A 12 and the currents by equations 2.6. The electrical torque is obtained by substituting these values in equation A (4).

$$T_e = + \frac{V_m}{2}(i_d \sin\delta - i_q \cos\delta) \quad \ldots \quad \ldots 2.8$$

By using the partial-fraction forms of equations A(9) and A(8), the currents given by equation 2.6 can each be split up into three parts as follows:

$$i_{d1} = i_{do} - \frac{1}{X_{de}'}\left[V_m(\cos\delta - \cos\delta_o) + X_e i_{do}\right] 1 \quad \ldots 2.9$$

$$i_{q1} = i_{qo} + \frac{1}{X_{qe}}\left[V_m(\sin\delta - \sin\delta_o) - X_e i_{qo}\right] 1$$

$$i_{d2} = \left(\frac{1}{X_{de}'} - \frac{1}{X_{de}}\right)\frac{1}{1+T_{de}'p}\left[V_m(\cos\delta - \cos\delta_o) + X_e i_{do}\right] 1 \quad \ldots 2.10$$

$$i_{q2} = 0$$

$$i_{d3} = -\left(\frac{1}{X_{de}''} - \frac{1}{X_{de}'}\right)\frac{T_{de}''p}{1+T_{de}''p}\left[V_m(\cos\delta - \cos\delta_o) + X_e i_{do}\right] 1$$

$$i_{q3} = \left(\frac{1}{X_{qe}''} - \frac{1}{X_{qe}}\right)\frac{T_{qe}''p}{1+T_{qe}''p}\left[V_m(\sin\delta - \sin\delta_o) - X_e i_{qo}\right] 1 \quad \ldots 2.11$$

Corresponding to each pair of currents $(i_{d1} i_{q1})$ $(i_{d2}, i_{q2})$ and $(i_{d3}, i_{q3})$ torque components $T_{e1}, T_{e2}\ T_{e3}$ can be calculated.

...2.3.1 First Torque component...

In equation 2.9, symbol 1 can be omitted because the operator does not appear in these equations. By using the relations of equations A(13) - A(16),

$$i_{d1} = \frac{\sqrt{2}}{X_{de}'} (V_{q_o}' - V \cos\delta)$$
$$i_{q1} = \frac{\sqrt{2}}{X_{qe}} V \sin\delta \qquad \text{... ... 2.12}$$

$$T_{e1} = + \frac{V V_{qo}'}{X_{de}'} \sin\delta - \frac{V^2}{2} \left(\frac{1}{X_{de}'} - \frac{1}{X_{qe}}\right) \sin 2\delta$$
... ... 2.13

$T_{e1}$ is the torque produced if the field flux linkages are constant.

...2.3.2 Second Torque component...

After the switch is closed, the field flux linkages change and $V_q'$ (the voltage behind transient reactance) varies with $\delta$, the relation being given by equation A(16) without the suffix o:

$$V_q' = V \cos\delta + X_d' I_d \qquad \text{... ... 2.14}$$

Let $V_q'$ change from the original value $V_{qo}'$ $V_{q2}'$

$$V_q' = V_{qo}' + V_{q2}' \qquad \text{... 2.15}$$

The torque associated with the change of $V_q'$ is

$$T_{e2} = + \frac{V V_{q2}'}{X_{de}'} \sin\delta \qquad \text{... ... 2.16}$$

...2.3.3. Third torque component...

This is due to the presence of damper winding. The current components are calculated from equation 2.2.11 by using the approximate standard integral form of Duhamel's Theorem (equation A21)

$$i_{d3} = \left( \frac{1}{X_{de}''} - \frac{1}{X_{de}'} \right) \left[ -X_e i_{do}\, e^{-t/T_{de}''} + V_m T_{de}'' \sin\delta \frac{d\delta}{dt} (1 - e^{-t/T_{de}''}) \right]$$

$$i_{q3} = \left( \frac{1}{X_{qe}''} - \frac{1}{X_{qe}} \right) \left[ -X_e\, i_{qo}\, e^{-t/T_{qe}''} + V_m T_{qe}'' \cos\delta \frac{d\delta}{dt} (1 - e^{-t/T_{qe}''}) \right] \quad \ldots 2.17$$

Substituting equation 2.17 in equation 2.18 and using the equations A(13) and A(14)

$$T_{e3} = +(a \sin^2\delta + b \cos^2\delta) S - V \sin\delta \left( \frac{1}{X_{de}''} - \frac{1}{X_{de}'} \right) x$$
$$(X_e I_{do} + \omega S\, T_{de}'' V \sin\delta) e^{-t/T_{de}''} - V \cos\delta \left( \frac{1}{X_{qe}''} - \frac{1}{X_{qe}} \right) x$$
$$(X_e I_{qo} + \omega S\, T_{qe}'' V \cos\delta) e^{-t/T_{qe}''} \quad \ldots \quad \ldots 2.18$$

where

$$a = \omega V^2 \; T_{de}'' \left( \frac{1}{X_{de}''} - \frac{1}{X_{de}'} \right)$$

$$b = \omega V^2 \; T_{qe}'' \left( \frac{1}{X_{qe}''} - \frac{1}{X_{qe}} \right) \quad \ldots \quad \ldots 2.19$$

$$\omega S = \frac{d\delta}{dt}$$

The 2nd and 3rd terms die away rapidly and if these are neglected, the damping torque is

$$T_{e3} = +(a \sin^2\delta + b \cos^2\delta) S \ldots \quad 2.20$$

If a and b are nearly equal,

$$T_{e3} = + \frac{a + b}{2} S \quad \ldots \quad \ldots \quad 2.21$$

The damping torque coefficient $T_d = \frac{d}{dS} T_{e3} = a \sin^2\delta + b\cos^2\delta$

$$= \frac{a + b}{2} \ldots \quad 2.22.$$

following the disturbance is given by

$$T_e = T_{e1} + T_{e2} + T_{e3}$$

$$= + \frac{V\,V_{qo}'}{X_{de}'}\sin\delta - \frac{V^2}{2}\left(\frac{1}{X_{de}'} - \frac{1}{X_{qe}}\right)\sin 2\delta$$

$$+ \frac{V\,V_{q2}}{X_{de}'}\sin\delta + (a\sin^2\delta + b\cos^2\delta)S \quad ..2.23$$

The 1st torque component is obtained on the assumption of constant flux linkage. For network analyzer studies, a further assumption is made that $Xd' = X_q$. This method gives a pessimistic result, but is adequate for determining whether the machine is likely to go out of synchronism on the 1st swing. This method is inaccurate for determining the behaviour after the first swing.

The 2nd torque component allows for variation of field flux linkage but ignores the effect of damper winding. This becomes zero with the usual assumption of constant flux linkage in stability studies.

The 3rd torque component is a damping torque which is proportional to the slip and is produced by the action of the damper winding. The value of the damping torque coefficient depends on the instantaneous load angle but if a & b in equation 2.23 are equal, the damping coefficient is approximately constant as in equation 2.22.

...2.4 Calculations...

Under the initial steady conditions (before the reactance is inserted):

$$V = 1\text{ P.U.}$$

$\cos \phi = 0.8$ (lag)

$V_{qo}'$ = Voltage behind the transient reactance

$= 1.166$

$\delta_0 = 16.5^\circ$

Power $= 0.8$ P.U.

The value of reactance inserted is $X_e = 0.924$ P.U. After the reactance is inserted, the constants of the modified alternator as per equation 2.7 are:

$X_{de} = 1.69$

$X_{de}' = 1.184$

$X_{de}'' = 1.169$

$X_{qe} = 1.376$

$X_{qe}'' = 1.169$

$T_{de}' = 0.91$

$T_{de}'' = .0232$

$T_{qe}'' = .0313$

From equation 2.13, $$T_{e_1} = \frac{V\,V_{qo}'}{X_{de}'} \sin\delta - \frac{V^2}{2}\left(\frac{1}{X_{de}'} - \frac{1}{X_{qe}}\right)\sin 2\delta$$

$$= (0.984 \sin\delta - .585 \sin 2\delta)$$

$T_{e2} = 0$ with the assumption of constant flux linkages.

From equation 2.19, $$a = \omega V^2 T_{de}''\left(\frac{1}{X_{de}''} - \frac{1}{X_{de}'}\right) = (.000255\,\omega)$$

$$b = \omega V^2 T_{qe}''\left(\frac{1}{X_{qe}''} - \frac{1}{X_{qe}}\right) = (.00398\,\omega)$$

From equation 2.20, $$T_{e3} = (a \sin^2\delta + b \cos^2\delta)\, S$$

$$= \omega S(.000255 \sin^2\delta + .00398 \cos^2\delta)$$

$$= \frac{\Delta\delta}{\Delta t}\; .000255 \sin^2\delta + .00398 \cos^2\delta$$

$\therefore T_e = (.984 \sin\delta - .585 \sin 2\delta) + (.02117\Delta\delta)$

The equation of motion is given by

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = T_m - T_e$$

$$= 412\ (T_m - T_e)$$

The swing curve is calculated by the step-by-step method as shown in table 2.1

From table 2.1, the following curves are plotted -

swing curve ... ... Fig. 2.4

Variation of transient torque ... Fig. 2.5

Torque-angle characteristic..Fig. 2.6

The swing curve calculations are repeated in table 2.2, when the damping effect due to damper windings is absent. From table 2.2, the swing curve, the transient torque and torque-angle characteristic are again plotted in Figs. 2.4, 2.5 & 2.6 respectively.

Fig. 2.4, 2.5 and 2.6 correspond to the oscillation of the rotor during the first swing.

The following points can be observed from the graphs:

Figs. 2.4 and 2.5: when damping is included, the load angle and transient torque reach maximum values of 121.5 and 1.44(P.U.) respectively during the first swing and then oscillate with decreasing magnitude in each subsequent swing about the final values. Thus the system finally settles down to new values of load angle and torque and hence stable. When damping is absent,

TABLE 2.1

Computation of swing curve after inserting reactance

(Damping included) $x_e$ = 0.924 P.U.

| t (sec) | .984x sin δ | -.585x sin 2δ | $T_{e_1}$ | Δδ (est) | $T_{e_2}$ | $T_e$ | $T_{ac} = T_m - T_e$ | $\Delta^2\delta$ | Δδ | δ (deg) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 0- | | | 0.8 | | 0 | | | | 0 | |
| 0+ | .279 | -.318 | -.039 | 1 | .0211 | -.018 | .818 | | | |
| 0ave | | | | | | | .409 | 1.68 | | 16.5 |
| | | | | | | | | | 1.68 | |
| 0.1 | .304 | -.345 | -.041 | 3 | .0635 | .022 | .778 | 3.2 | | 18.18 |
| | | | | | | | | | 4.88 | |
| 0.2 | .384 | -.421 | -.037 | 6 | .1265 | .089 | .712 | 2.94 | | 23.06 |
| | | | | | | | | | 7.78 | |
| 0.3 | .502 | -.512 | -.01 | 9 | .1905 | .18 | .62 | 2.56 | | 30.84 |
| | | | | | | | | | 10.34 | |
| 0.4 | .646 | -.576 | .07 | 12 | .253 | .323 | .477 | 1.97 | | 41.18 |
| | | | | | | | | | 12.31 | |
| 0.5 | .79 | -.56 | .23 | 13 | .274 | .504 | .296 | 1.22 | | 53.49 |
| | | | | | | | | | 13.53 | |
| 0.6 | .905 | -.42 | .385 | 14 | .296 | .681 | .129 | .531 | | 67.02 |
| | | | | | | | | | 14.06 | |
| 0.7 | .97 | -.181 | .789 | 13 | .275 | 1.064 | -.264 | -1.09 | | 81.08 |
| | | | | | | | | | 12.97 | |
| 0.8 | .93 | .0811 | 1.06 | 12 | .253 | 1.31 | -.51 | -2.1 | | 94.05 |
| | | | | | | | | | 10.87 | |
| 0.9 | .965 | .292 | 1.247 | 10 | .2117 | 1.458 | -.658 | -2.7 | | 104.9 |
| | | | | | | | | | 8.17 | |
| 1.0 | .904 | .42 | 1.324 | 7 | .148 | 1.472 | -.672 | -2.78 | | 113.09 |
| | | | | | | | | | 5.39 | |
| 1.1 | .875 | .489 | 1.364 | 4 | .084 | 1.448 | -.648 | -2.67 | | 118.48 |
| | | | | | | | | | 2.72 | |
| 1.2 | .84 | .516 | 1.356 | 1 | .0211 | 1.377 | -.577 | -2.38 | | 121.2 |
| | | | | | | | | | .34 | |
| 1.3 | .836 | .52 | 1.356 | -1 | -.0211 | 1.355 | -.535 | -2.2 | | 121.54 |
| | | | | | | | | | -1.86 | |
| 1.4 | .856 | .501 | 1.357 | -3 | -.063 | 1.294 | -.494 | -2.04 | | 119.68 |
| | | | | | | | | | -3.9 | |
| 1.5 | .886 | .456 | 1.342 | -5 | -.1055 | 1.237 | -.437 | -1.8 | | 115.78 |
| | | | | | | | | | -5.7 | |
| 1.6 | .920 | .376 | 1.296 | -7 | -.148 | 1.148 | -.348 | -1.43 | | 110.08 |
| | | | | | | | | | -7.13 | |
| 1.7 | .964 | .255 | 1.219 | -8 | -.168 | 1.051 | -.205 | -.84 | | 102.95 |
| | | | | | | | | | -7.97 | |
| 1.8 | .98 | .116 | 1.096 | -8 | -.168 | .928 | -.128 | -.526 | | 94.98 |
| | | | | | | | | | -8.49 | |
| 1.9 | .984 | -.073 | .911 | -8 | -.168 | .743 | +.06 | .247 | | 86.49 |
| | | | | | | | | | -8.25 | |
| 2.0 | .960 | -.255 | .705 | -7 | -.148 | .557 | +.25 | 1.03 | | 78.24 |

Table 2.1 Contd..

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.1 | .93 | -.36 | .57 | -6 | -.125 | .45 | .35 | 1.44 | -7.22 | 71.02 |
| 2.2 | .89 | -.446 | .45 | -5 | -.105 | .35 | .45 | 1.85 | -5.78 | 65.24 |
| 2.3 | .86 | -.496 | .37 | -3 | .063 | .31 | .49 | 2.02 | -3.93 | 61.31 |
| 2.4 | .852 | -.499 | .353 | -1 | -.021 | .332 | .468 | 1.93 | -1.91 | 59.4 |
| 2.5 | .852 | -.499 | .353 | 1 | .02 | .373 | .427 | 1.76 | .02 | 59.42 |
| 2.6 | .86 | -.495 | .365 | 3 | .063 | .428 | .372 | 1.53 | 1.78 | 61.2 |
| 2.7 | .885 | -.454 | .431 | 4 | .084 | .535 | .265 | 1.1 | 3.31 | 64.5 |
| 2.8 | | | | | | | | | 3.41 | 67.9 |

## TABLE 2.2

### Computation of swing curve after inserting the reactance

( Damping absent) $X_e$ = 0.924 P.U.

| t (sec) | Sin δ | .984 x Sin δ | Sin 2δ | -.585 x Sin 2δ | $T_e$ | $T_{ac}$ | $\Delta^2\delta$ | Δδ | δ (deg) |
|---|---|---|---|---|---|---|---|---|---|
| 0- | | | | | 0.8 | 0 | | 0 | |
| 0+ | .284 | .279 | .544 | -.318 | -.039 | 0.839 | | | |
| 0av | | | | | | 0.419 | 1.73 | | 16.50 |
| | | | | | | | | 1.73 | |
| 0.1 | .312 | .306 | .594 | -.346 | -.040 | 0.840 | 3.46 | | 18.23 |
| | | | | | | | | 5.19 | |
| 0.2 | .398 | .391 | .730 | -.426 | -.035 | 0.835 | 3.44 | | 23.42 |
| | | | | | | | | 8.63 | |
| 0.3 | .53 | .521 | 0.9 | -.526 | -.005 | 0.805 | 3.32 | | 32.05 |
| | | | | | | | | 11.95 | |
| 0.4 | .695 | .683 | 1.0 | -.585 | +.098 | 0.702 | 2.90 | | 44.00 |
| | | | | | | | | 14.85 | |
| 0.5 | .855 | .840 | .886 | -.519 | +.321 | 0.479 | 1.98 | | 58.85 |
| | | | | | | | | 16.83 | |
| 0.6 | .968 | .952 | .49 | -.286 | +.666 | 0.134 | 0.55 | | 75.68 |
| | | | | | | | | 17.38 | |
| 0.7 | 1.0 | .984 | -.069 | +.0403 | 1.024 | -.224 | -.975 | | 92.06 |
| | | | | | | | | 16.41 | |
| 0.8 | .95 | .932 | -.599 | +.350 | 1.282 | -.482 | -1.99 | | 108.47 |
| | | | | | | | | 14.42 | |
| 0.9 | .84 | .826 | -.91 | +.530 | 1.356 | -.556 | -2.30 | | 122.89 |
| | | | | | | | | 12.12 | |
| 1.0 | .706 | .693 | -1.0 | +.585 | 1.278 | -.478 | -1.97 | | 135.01 |
| | | | | | | | | 10.15 | |
| 1.1 | .57 | .560 | -.94 | +.549 | 1.109 | -.309 | -1.27 | | 145.16 |
| | | | | | | | | 8.88 | |
| 1.2 | .437 | .43 | -.789 | +.460 | .89 | -.09 | - .37 | | 154.04 |
| | | | | | | | | 8.51 | |
| 1.3 | .30 | .2852 | -.574 | +.336 | .621 | .179 | .739 | | 162.55 |
| | | | | | | | | 9.24 | |
| 1.4 | .144 | .142 | -.286 | +.167 | .309 | .491 | 2.04 | | 171.79 |
| | | | | | | | | 11.28 | |
| 1.5 | -.052 | -.0511 | +.104 | -.0609 | -.111 | .911 | 3.76 | | 183.07 |
| | | | | | | | | 15.04 | |
| 1.6 | -.31 | -.304 | .616 | -.360 | -.664 | 1.464 | | | 198.11 |

62,640

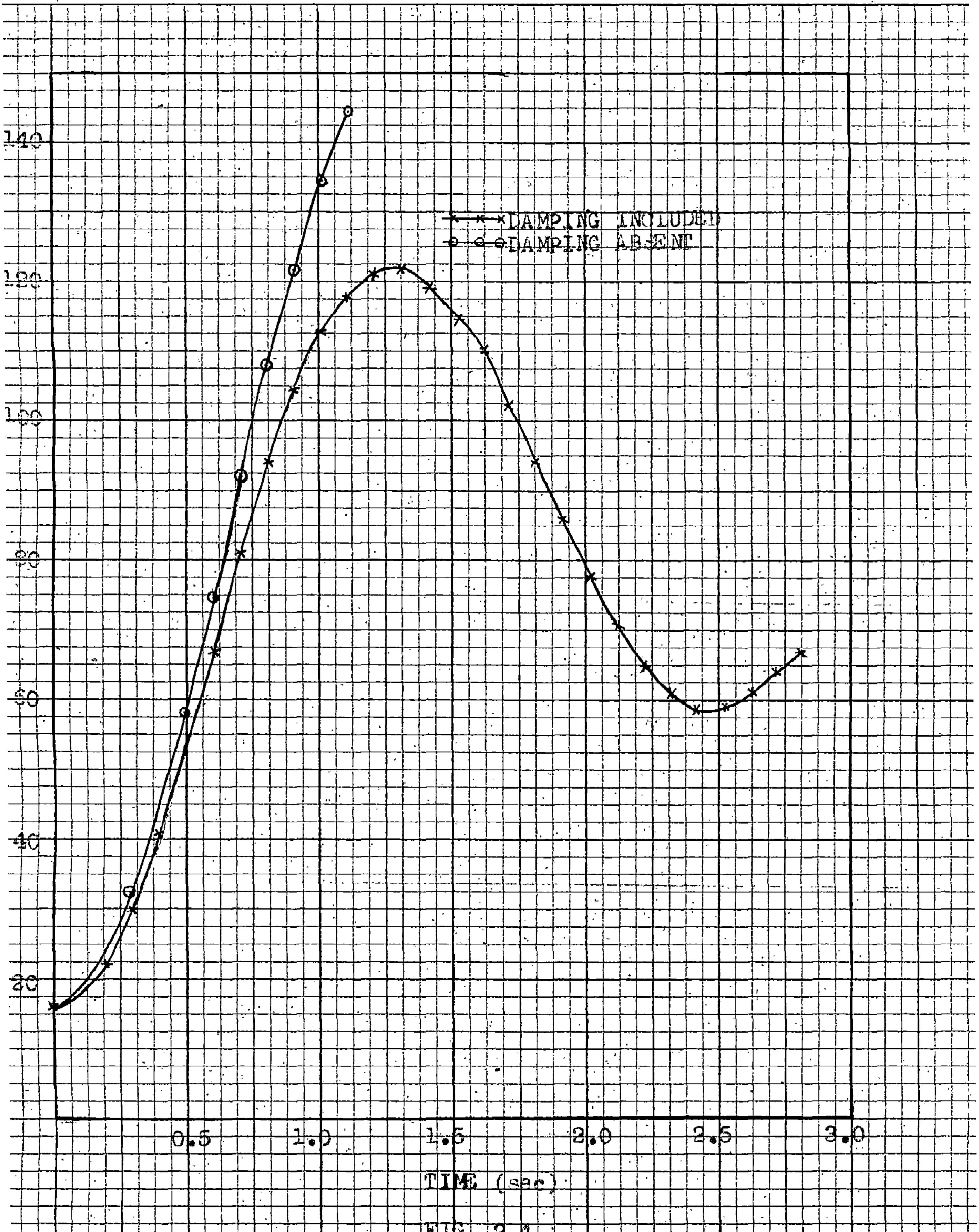



FIG. 2.4

VARIATION OF LOAD ANGLE AFTER INSERTING A REACTANCE

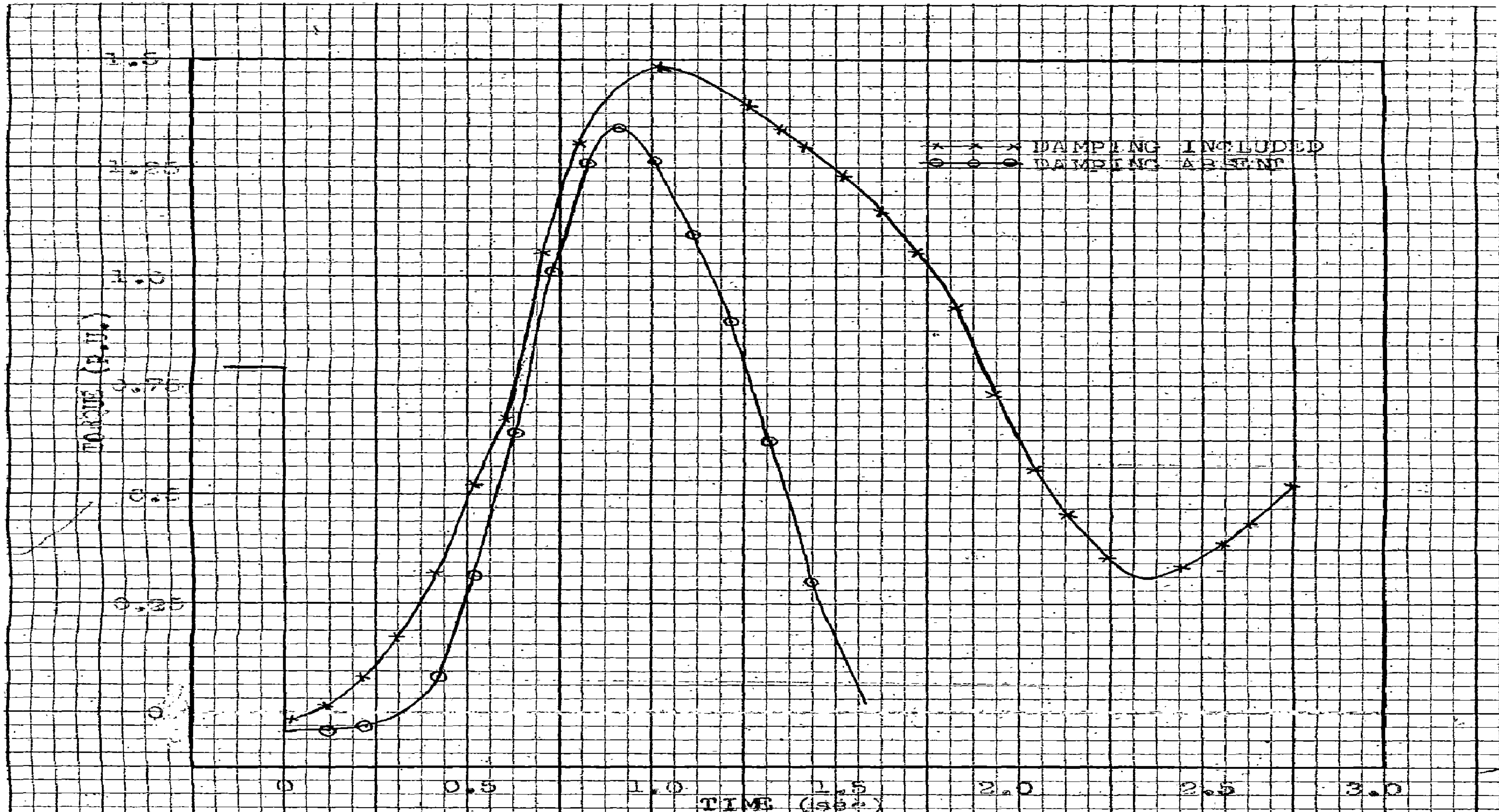



FIG.2.5

VARIATION OF TRANSIENT TORQUE AFTER INSERTING A REACTANCE

($X_e$ = 0.924 P.U.)

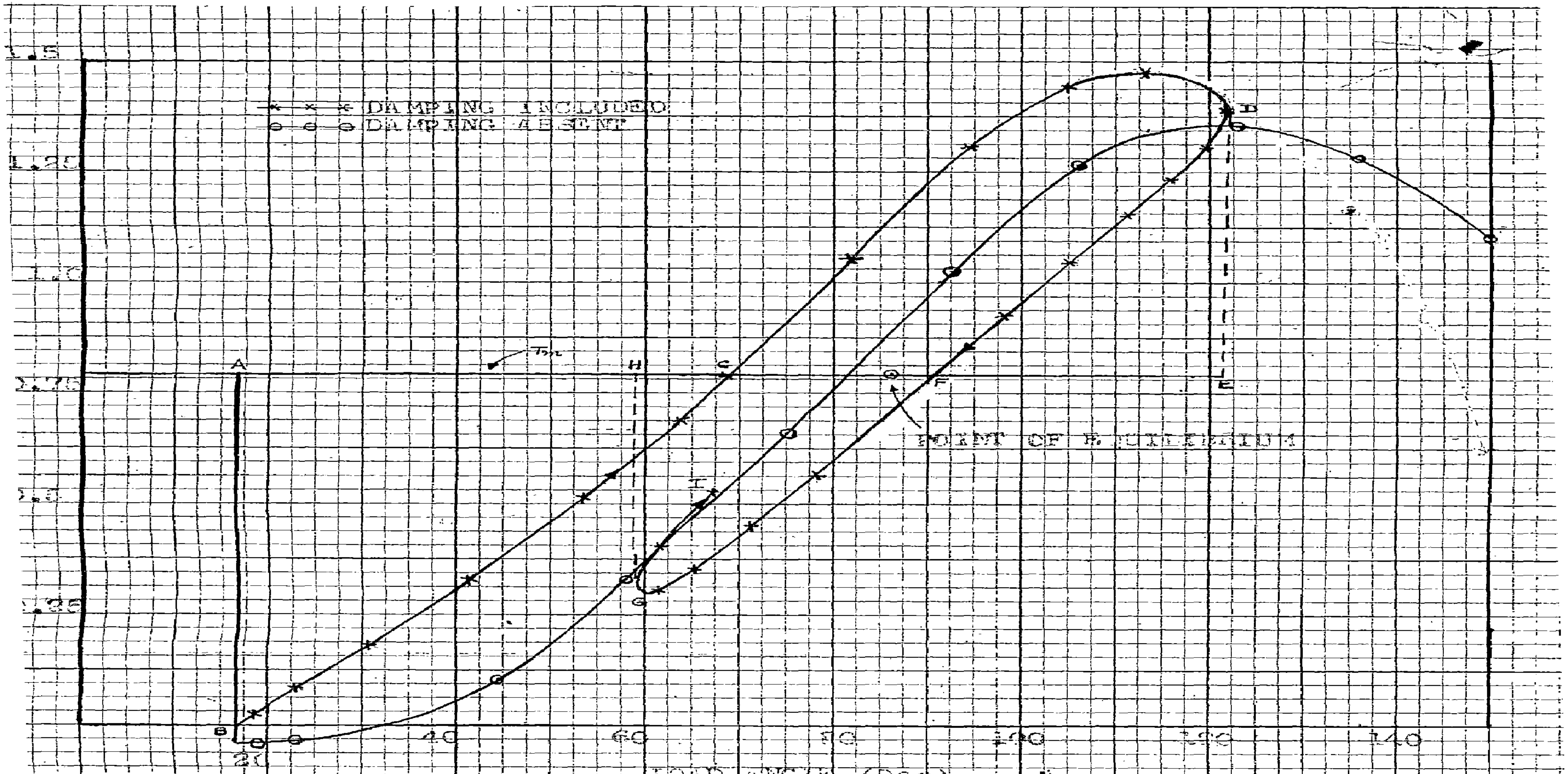



FIG. 2.8

TORQUE-ANGLE CHARACTERISTIC AFTER INSERTING A REACTANCE

($X_e$ = 0.984 P.U.)

decreases continuously after the maximum value is reached. The system does not show any tendency to oscillate about a final value and settle down to a steady operating condition, and hence falls out of step.

<u>Fig. 2.6:</u> When damping is included, the torque angle characteristic shows that during the first swing, the operating point moves along BC, reaches a maximum value at point D such that area ABC = area CDE; then moves along DF and reaches a minimum value at G such that area DEF = area FGH. The operating point then moves along GI during the second swing. Each successive oscillation will be smaller and the curve converges on the point of equilibrium (about 85$^{o}$), which is the intersection of the horizontal line $T_m$ and the torque-angle characteristic. The operation is thus stable. When damping is absent, the torque-angle characteristic does not show any tendency to converge to a point of equilibrium and hence the system is not stable.

When the value of external reactance inserted is reduced to 0.25 P.U., the constants are:-

$X_e$ = 0.25 P.U.

$X_{de}$ = 1.01

$X_{de}'$ = 0.51

$X_{de}''$ = 0.49

$X_{qe}$ = 0.70

$X_{qe}''$ = 0.49

$T_{de}'$ = 0.524

$T_{de}''$ = .0283

$T_{qe}''$ = .0259

$T_{e1}$ = (2.28 sin $\delta$ - .267 sin 2$\delta$ )

$$T_e = (2.28 \sin\delta - .267 \sin 2\delta + .0851 \Delta\delta)$$

The equation of motion is given by

$$\frac{d^2}{dt^2} = 412\ (T_m - T_e)$$

The swing curves are calculated by the step-by-step method in tables 2.3 and 2.4 respectively, when damping is included and when damping is absent. From tables 2.3 and 2.4, the swing curve, transient torque curve, and the torque-angle characteristic are plotted as shown in Figs. 2.7, 2.8 and 2.9. As in the previous case, Figs 2.7, 2.8 and 2.9 correspond to the oscillation of the rotor during the 1st swing. From these figures, it is observed that the system will be stable in both cases i.e., when damping is included and also when damping is absent. However, when the damping is included, it can be seen, that the maximum load angle and torque during the first swing are very much less than when the damping is neglected.

From the above, it can be concluded that an alternator running on an infinite bus is likely to fall out of step when a sufficiently large value of reactance is suddenly inserted in the line; but if the alternator is provided with damper windings, stability may be restored. When the suddenly inserted reactance is small, the system will be stable even though the alternator is not provided with amortisseur windings; but if the amortisseur windings are provided, they will help to damp oscillations and hence to reduce the maximum angle of swing.

[illegible]

## Computation of swing curve after inserting the reactance (Damping included).

$X_a$ = 0.25P.U.

| t (sec) | Sinδ | 2.28x Sinδ | Sin 2δ | -.267x Sin2δ | $T_{el}$ | Δδ (est) | $T_{a2}$ =.0851 : Δδ | $T_e$ =$T_{el}$+ $T_{e2}$ | $T_{ac}$ | $\Delta^2\delta$ | Δδ | δ (deg) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 0- 0+ 0aver | .284 | .646 | .545 | -.145 | .800 .501 | .5 | .042 | .543 | 0 .257 .128 | .53 | 0 .53 | 16.5 |
| 0.1 | .292 | .665 | .56 | -.149 | .516 | 1.0 | .085 | .601 | .199 | .822 | 1.352 | 17.03 |
| 0.2 | .314 | .716 | .596 | -.159 | .557 | 1.5 | .127 | .684 | .116 | .48 | 1.83 | 18.38 |
| 0.3 | .346 | .79 | .66 | -.176 | .623 | 2.00 | .17 | .799 | .001 | .00412 | 2.24 | 20.21 |
| 0.4 | .388 | .885 | .715 | -.191 | .694 | 2.0 | .17 | .864 | -.064 | -.264 | 1.98 | 22.86 |
| 0.5 | .42 | .955 | .76 | -.203 | .752 | 1.5 | .127 | .879 | -.079 | -.325 | 1.66 | 24.84 |
| 0.6 | .445 | 1.01 | .8 | -.213 | .80 | 1.0 | .085 | .885 | -.085 | -.35 | 1.31 | 26.50 |
| 0.7 | .466 | 1.065 | .825 | -.22 | .84 | 1.0 | .085 | .925 | -.125 | -.51 | 0.80 | 27.81 |
| 0.8 | .48 | 1.09 | .84 | -.224 | .87 | 0.5 | .042 | .912 | -.112 | -.46 | 0.34 | 28.61 |
| 0.9 | .482 | 1.1 | .845 | -.224 | .876 | 0.2 | .017 | .892 | -.092 | -.88 | -.04 | 28.95 |
| 1.0 | .484 | 1.1 | .845 | -.225 | .875 | -0.1 | -.0085 | .867 | -.067 | -.276 | -.316 | 28.91 |
| 1.1 | .48 | 1.09 | .84 | -.224 | .87 | -0.3 | -.02553 | .85 | -.05 | -.2 | -.516 | 28.60 |
| 1.2 | .47 | 1.07 | .826 | -.22 | .85 | -0.5 | -.042 | .81 | -.01 | -.04 | -.55 | 28.09 |
| 1.3 | .465 | 1.05 | .82 | -.213 | 0.84 | -.4 | -.032 | .81 | -.01 | -.04 | -.59 | 27.54 |
| 1.4 | .452 | 1.02 | .806 | -.215 | 0.82 | -.4 | -.032 | .73 | +.01 | +.04 | | [illegible] |

Table 2.3 Contd...

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | -.55 | |
| 1.5 | .431 | .982 | .78 | -.207 | .775 | -.4 | -.032 | .743 | .057 | .235 | -.315 | 25.60 |
| 1.6 | .426 | .97 | .766 | -.204 | .77 | -.2 | -.016 | .76 | +.04 | .16 | -.155 | 25.29 |
| 1.7 | .425 | .97 | .765 | -.204 | .77 | -.1 | -.0085 | .762 | .038 | .157 | +.002 | 25.14 |
| 1.8 | .425 | .97 | .765 | -.204 | .77 | +.05 | +.004 | .774 | .026 | .107 | .109 | 25.142 |
| 1.9 | .425 | .97 | .765 | -.204 | .77 | +.1 | +.008 | .778 | .002 | .008 | .117 | 25.24 |
| 2.0 | .426 | .97 | .766 | -.204 | .77 | +.15 | .012 | .782 | .018 | .0074 | .124 | 25.35 |
| 2.D | .427 | .97 | .767 | -.204 | .77 | +.15 | .012 | .782 | .018 | .0074 | .131 | 25.47 |
| 2.1 | .431 | .982 | .78 | -.207 | .775 | +.1 | .008 | .783 | .017 | .007 | .138 | 25.60 |
| 2.2 | .433 | .989 | .78 | -.207 | .782 | +.15 | .012 | .792 | .018 | .007 | .145 | 25.73 |
| 2.3 | .435 | .995 | .784 | -.209 | .786 | .15 | .012 | .798 | .002 | .008 | .153 | 25.87 |
| 2.4 | .439 | 1.0 | .786 | -.21 | .79 | .15 | .012 | .802 | -.002 | -.008 | .145 | 26.02 |
| 2.5 | .439 | 1.0 | .786 | -.21 | .79 | .15 | .012 | .802 | -.002 | -.008 | .137 | 26.16 |
| 2.6 | .443 | 1.01 | .791 | -.212 | .80 | .15 | .012 | .812 | -.012 | -.049 | .133 | 26.29 |
| 2.7 | .445 | 1.01 | .8 | -.213 | .8 | .1 | .008 | .808 | -.008 | -.032 | .130 | 26.42 |
| 2.8 | .445 | 1.01 | .8 | -.213 | .8 | .1 | .008 | .808 | -.008 | -.032 | .1 | 26.55 |
| 2.9 | .445 | 1.01 | .8 | -.213 | .8 | .08 | .0068 | .8068 | -.0068 | -.028 | .08 | 26.65 |
| 3.0 | .45 | 1.02 | .804 | -.214 | .81 | .06 | .0048 | .814 | -.014 | -.057 | .03 | 26.73 |
| 3.1 | .45 | 1.02 | .804 | -.214 | .81 | .01 | .0008 | .81 | -.01 | -.04 | -.01 | 26.76 |
| 3.2 | .45 | 1.02 | .804 | -.214 | .81 | -.02 | -.0016 | .81 | -.01 | -.04 | -.05 | 26.75 |
| 3.3 | | | | | | | | | | | | 26.70 |

TABLE 2.4

Computation of swing curve after inserting a reactance

(Damping absent) X = 0.25 P.U.

| t (sec) | Sin δ | 2.78 x Sin δ | Sin 2δ | -.237x Sin 2δ | $T_{e1}$ | $T_{ac}$ | $\Delta^2 \delta$ | $\Delta \delta$ | δ (deg) |
|---|---|---|---|---|---|---|---|---|---|
| 0- | | | | | [illegible] | [illegible] | | 0 | |
| 0+ | [illegible] | .646 | [illegible] | -.145 | [illegible] | [illegible] | [illegible] | | |
| 0 av | | | | | | [illegible] | [illegible] | | 16.5 |
| | | | | | | | | .61 | |
| 0.1 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | 17.11 |
| | | | | | | | | 1.72 | |
| 0.2 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| 0.3 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| 0.4 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | .946 | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |
| | | | | | | | | [illegible] | |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | | [illegible] |

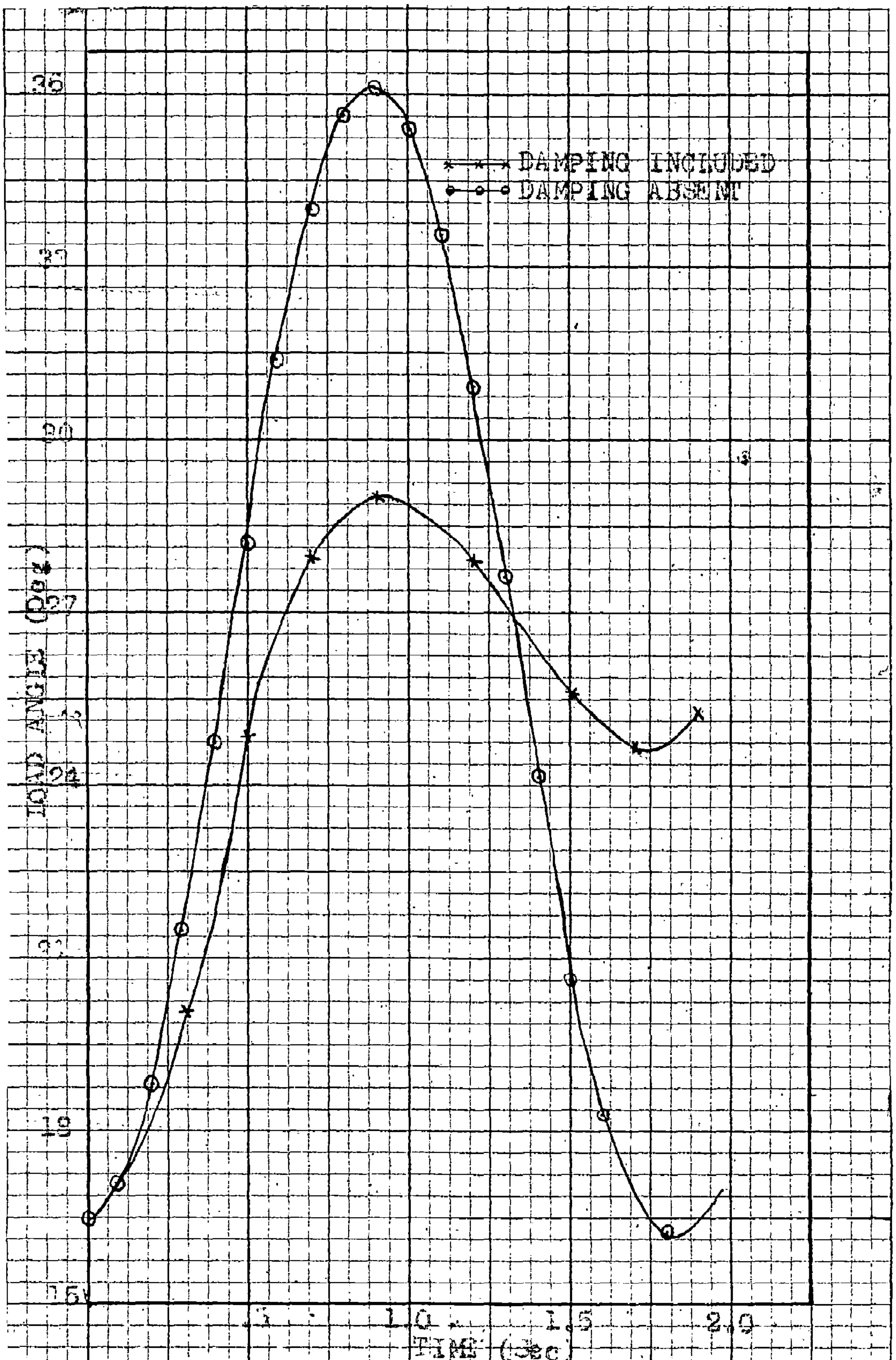



FIG. 3.7 VARIATION OF LOAD ANGLE AFTER INSERTING A REACTANCE ( Xe = 0.25 P.U.)

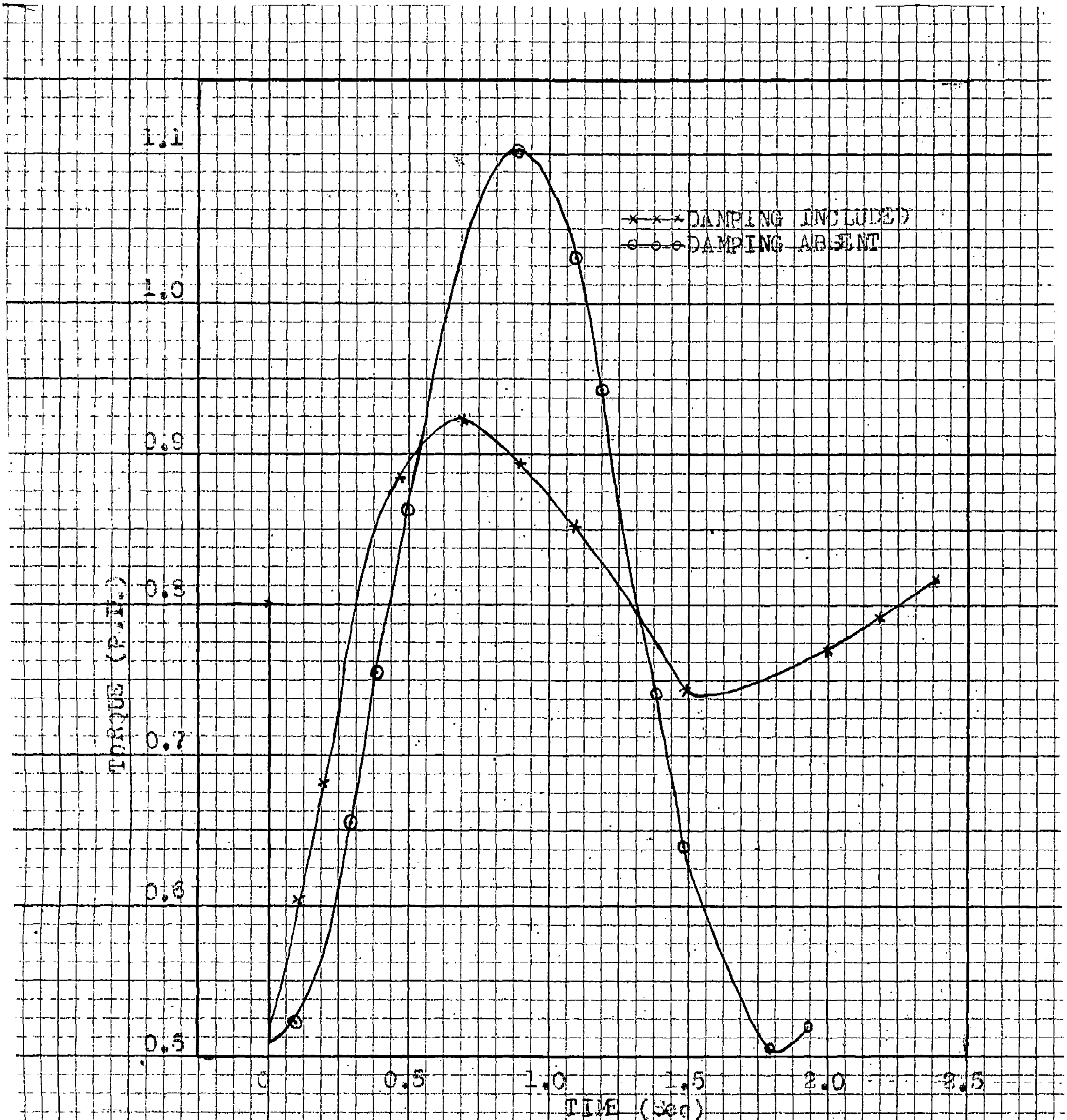



FIG.2.5

VARIATION OF TRANSIENT TORQUE AFTER INSERTING A REACTANCE ($X_e$ = 0.25 P.U.)

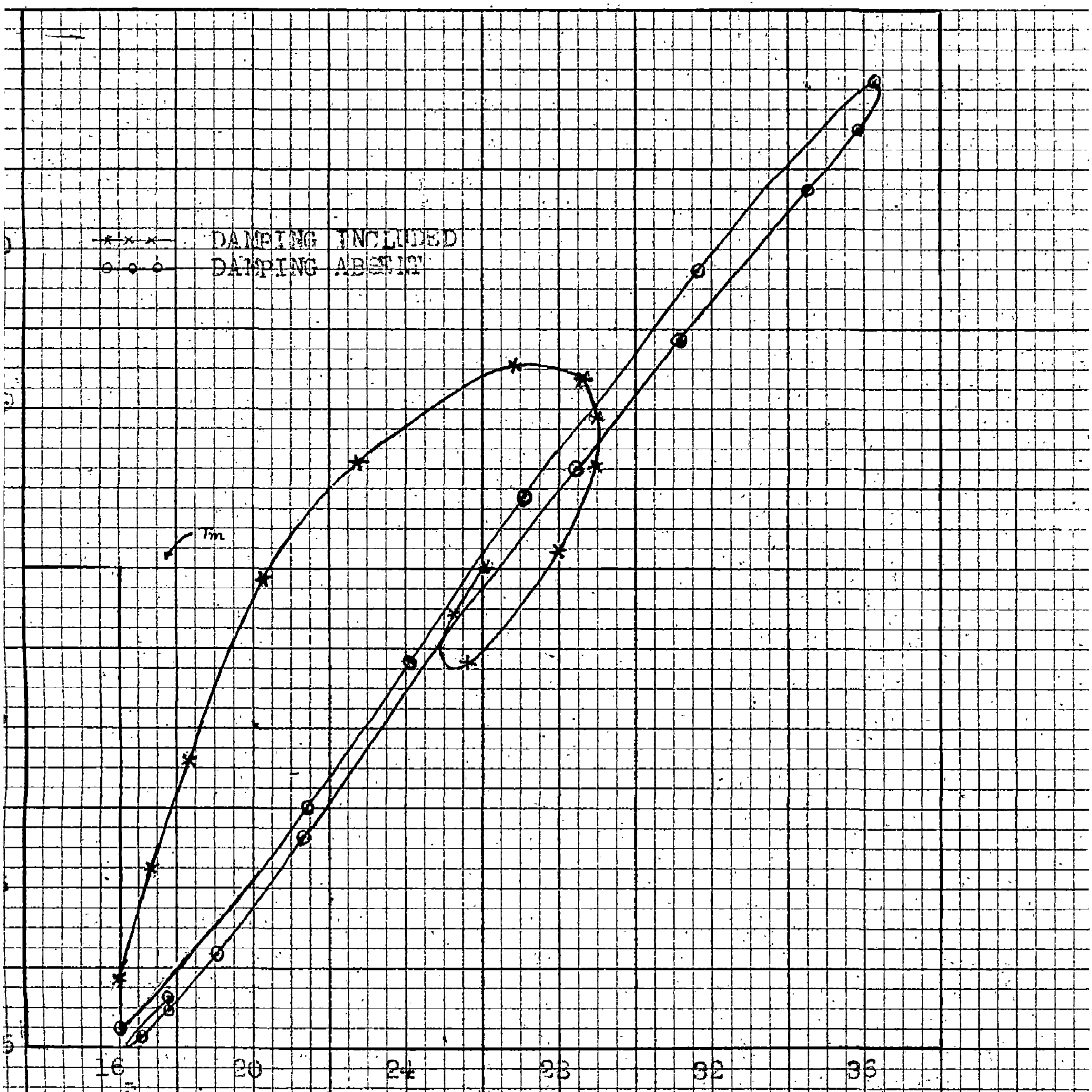



FIG.2.9

TORQUE ANGLE CHARACTRAISTIC AFTER INSERTING A REACTANCE ($X_e$ = 0.25 P.U.)

# .....CHAPTER III.....

## ...TRANSIENT BEHAVIOUR OF A SYNCHRONOUS MOTOR DUE TO SUDDEN APPLICATION OF SHAFT LOADS

### ...3.1 Formulation of the problem and the significance of the various Parameters...

Sudden load increases can result in transient disturbances that are important from stability stand point if (1) the total load exceeds the steady-state stability limit for specific voltages and circuit reactances conditions, or (2) if the load increase sets up an oscillation that causes the system to swing beyond the critical point from which recovery would be impossible.

The problem of sudden application of an additional shaft load on a salient-pole alternator running as synchronous motor and the determination of the stability limit leads to a non-linear differential equation. This problem was first formulated by Lyon and Edgerton[14] and a solution obtained for the non-salient pole machine by means of the Integraph developed at the Massachusetts Institute of Technology. The problem of Lyon and Edgerton was later attacked by Mclachlan[15] utilising the method of isoclines, Stoker[16] by the method of finite differences and by ku[17] by a new graphical method devised by him involving velocity-acceleration plot. Later, the problem has also been solved by Ganapathy by applying Runge-kutta method. Though tedious, the problem will be solved here by the step-by-step method and certain conclusions are drawn based on theoritical results obtained.

Due to the sudden application of a load torque $T_L$, the machine momentarily slows down and the characteristic equation which determines the performance of a synchronous machine is

$$T_j \frac{d^2\delta}{dt^2} + T_i = T_o + T_L\,1 \qquad \ldots \quad \ldots \; 3.1$$

(assuming that the motor is operating in a steady-state condition at the time when the abrupt load is applied i.e., at $t = 0$, $\delta = \delta_o$ and $\frac{d\delta}{dt} = 0$).

where $T_j$ = A coefficient which when multiplied by acceleration gives torque.

$$= \frac{H}{\pi f}$$

$T_i$ = Instantaneous torque developed at any value of $\delta$.

$T_i$ consists of synchronous motor torque as well as induction motor torque, since the speed is no longer synchronous.

$T_o$ = Initial shaft torque.

$T_L$ = Abrupt shaft torque.

1 = Heaviside's unit function which indicates that the quantity to which it is applied is zero before time equals zero and is multiplied by one thereafter.

The initial shaft torque

$$T_o = T_m \sin\delta_o + T_r \sin 2\delta_o \qquad \ldots \quad \ldots \; 3.2$$

where $T_m = \dfrac{V_o V}{X_d}$

$$T_r = \frac{V^2}{2} \; \frac{X_d - X_q}{X_d X_q}$$

$\delta_o$ = Initial load angle

The synchronous motor torque = $T_m \sin\delta + T_r \sin 2\delta$

Induction motor torque = $T_D$

$$= \omega S V^2 \left[ \frac{X_d' - X_d''}{X_d'^2} T_{do}'' \sin^2 \delta + \frac{X_q' - X_q''}{X_q'^2} T_{qo}'' \cos^2 \delta \right]$$

$$= \frac{T_d}{\omega} \frac{d\delta}{dt}$$

$$= T_\rho \frac{d\delta}{dt}$$

Hence equation 3.1 becomes

$$T_j \frac{d^2\delta}{dt^2} + T_m \sin\delta + T_r \sin 2\delta + T_\rho \frac{d\delta}{dt} = T_o + T_L 1 \quad \ldots 3.3$$

Equation 3.3 can be written as

$$\frac{T_j}{T_m} \frac{d^2\delta}{dt^2} + \frac{T_\rho}{T_m} \frac{d\delta}{dt} + \sin\delta + \frac{T_r}{T_m} \sin 2\delta = \frac{T_o}{T_m} + \frac{T_L}{T_m} 1 \quad \ldots 3.4$$

Introducing a new variable for time,

$$\tau = t \sqrt{\frac{T_m}{T_j}},$$

equation 3.4 can be written as -

$$\frac{d^2\delta}{d\tau^2} + \frac{T_\rho}{\sqrt{T_m T_j}} \frac{d\delta}{d\tau} + \sin\delta + \frac{T_r}{T_m} \sin 2\delta = \frac{T_o}{T_m} + \frac{T_L}{T_m} 1 \quad \ldots 3.5$$

$\frac{T_\rho}{\sqrt{T_m T_j}}$ is called the relative damping coefficient and is denoted by K.

$$\therefore K = \frac{T_\rho}{\sqrt{T_m T_j}} \quad \ldots \quad \ldots \quad \ldots \quad 3.6$$

$\frac{T_o}{T_m}$ is called the initial load ratio.

$\frac{T_L}{T_m}$ is called the abrupt load ratio

Equation 3.5 can be written as -

$$\frac{d^2\delta}{d\tau^2} + K.\frac{d\delta}{d\tau} + \sin\delta + \frac{T_r}{T_m}\sin 2\delta = \frac{T_o}{T_m} + \frac{T_L}{T_m} - 1 \quad \ldots 3.7$$

This equation characterises the angular transients that follow a suddenly applied shaft load. It is non-linear and a solution can not be obtained directly. The equation is discussed below in detail.

From equation 3.7, it can be seen that the performance of a synchronous motor when a sudden load is applied to the shaft depends upon four factors -

1- K, the relative damping coefficient.
2- $\frac{T_o}{T_m}$, the initial load ratio
3- $\frac{T_L}{T_m}$, the abrupt load ratio.

and 4- $\frac{T_r}{T_m}$

If K=0, then the equation 3.7 represents undamped motion and the angular variation is an oscillation between the initial value and some maximum value. This equation is solvable for the critical load and the maximum angle of swing for any load can be determined by equal-area method, as indicated below.

Consider a system operating under the conditions shown in figure 3.1 with the shaft load $T_1$ at the angle $\delta_1$ and the shaft load abruptly increased to $T_2$. Because of the inertia of the rotating machine, the internal voltage of the motor does not immediately swing to $\delta_2$, which would permit transfer of power $T_2$. Instead, the initial difference of power input and shaft output are used in decelerating the motor rotating elements. This causes the rotor to depart from synchronous speed and to increase the angular difference. Thus, when the system reaches $\delta_2$, the motor is travelling below synchronous speed.

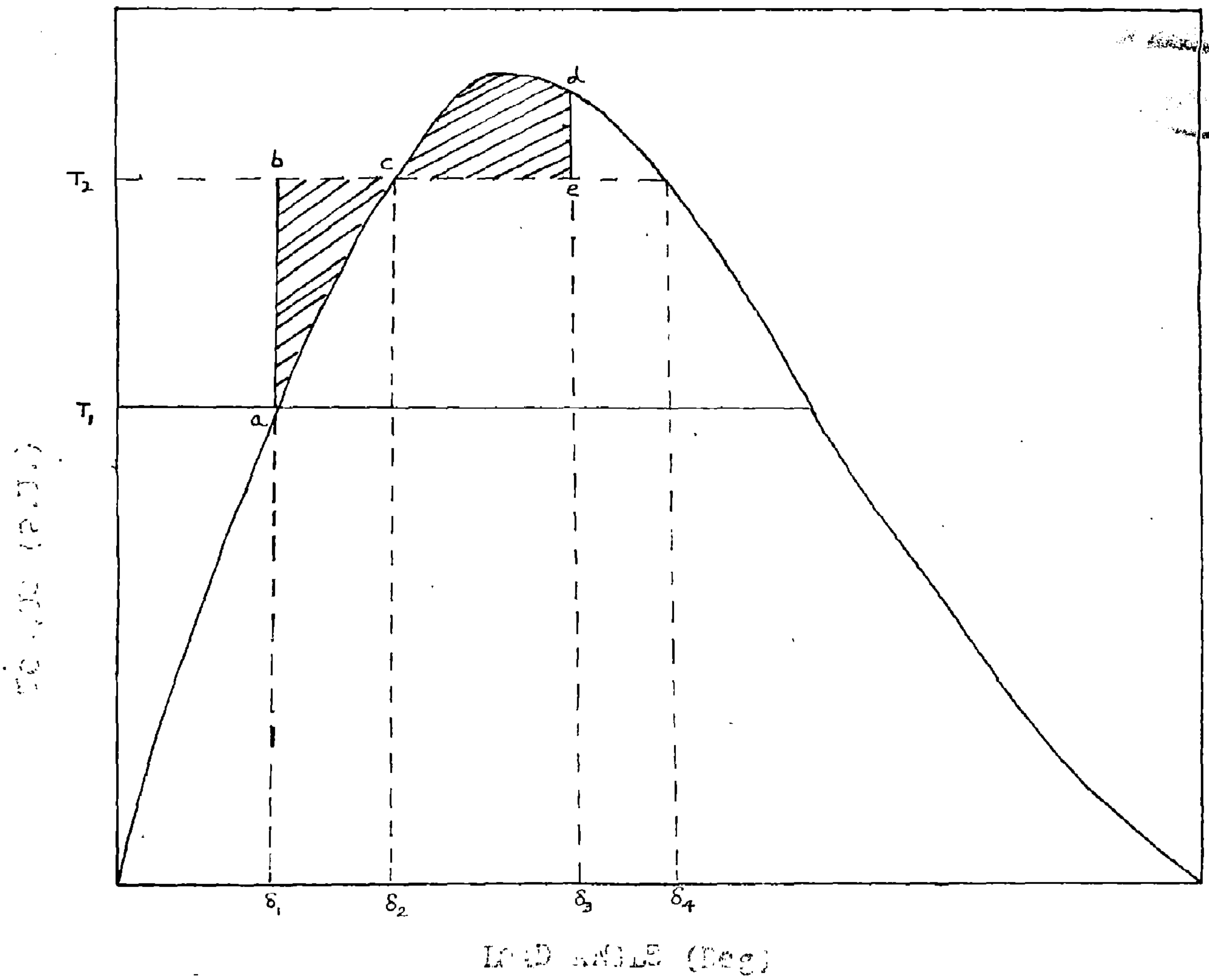



FIG.3.1

TORQUE-ANGLE CHARACTERISTICS OF A SYNCHRONOUS MOTOR DUE TO ABRUPT SHAFT LOADS WHEN DAMPING IS ZERO

The difference in the stored energy cannot instantly be absorbed, and as a result, the system overshoots $\delta_2$ and reaches some larger angle such as $\delta_3$ such that the shaded area cde is equal to the area abc. Neglecting losses, these two areas are equal. The oscillation will not exceed the angle $\delta_3$ and because of losses in an actual system, equilibrium will ultimately be reached at $\delta_2$. For the case illustrated in Fig.3.1, the system oscillates to

$\delta_3$ is less than $\delta_4$, the critical angle for the load $T_2$. With a greater increment of load, the maximum point reached in the oscillation would be greater than $\delta_3$, shown in the figure. With increasingly severe conditions, a point is reached where the critical angle is equalled and this represents the critical load i.e. the maximum load that can be abruptly applied to the shaft. If the maximum angle of swing exceeds the angle corresponding to this critical load in the unstable part of the torque-angle characteristic, stability will be lost because the synchronizing torque is less than the load torque. The amount of load increase that a system can withstand depends upon the steady. state limit of the system and the initial load.

K will not be zero if damper windings are present. The relative damping coefficient K becomes larger, as the induction motor effect becomes greater. It will be seen that the motion represented by equation 3.7 is beneficially influenced by increasing the value of K, as the damping term reduces the maximum angle of swing. In other words, as the value of K becomes larger, the amount of sudden load ratio that may be applied becomes greater with a constant initial load ratio, However, a limit will be reached when K has such a value that the sudden load ratio plus the initial load ratio equals one. Further increase of the relative damping coefficient K cannot allow the motor to stand more sudden load because the final steady-state conditions are impossible. The value of K that just allows the sudden load ratio plus the initial load ratio to equal one is called the "critical damping coefficient". This coefficient has different numerical values for different initial load ratios. Relative damping factors larger than those for critical damping produce no beneficial effects but only slow up the rate of swinging. They do not allow greater loads to be applied without loss of synchronism.

Curves of angular position against time can be plotted by the step-by-step method.

Small sudden load ratios cause the swing (angle-time) curves to resemble damped sinusoids, all having approximately the same frequency. For larger sudden load ratios, the frequency of oscillation is considerably reduced and the time required for one oscillation is increased.

From the swing curves a particular load ratio can be found for which there will be no oscillation. For this condition, the rotor angle continues to increase and the motor falls out of step. For a slightly lesser value of the sudden load ratio, the angular deviation reaches a maximum value and then oscillates with decreasing amplitude about the final value. Between these two values of sudden load ratio, there is one value for which the angular deviation would neither continue to increase nor decrease. This value gives the "unstable equilibrium condition" for this particular equation. Physically, such a solution means that the slip and acceleration for the rotor of a synchronous machine both become zero when the rotor reaches the angle giving an unstable steady-state solution. This angle will be on the unstable portion of the torque-angle curve, thus being greater than $90^{\circ}$.

## ...3.2. Calculations...

For the machine under investigation, the initial load on the machine is found to be 0.19 P.U., this being the core loss of the

the two machines (the other machine being mechanically coupled to the first)

Hence $T_0 = 0.19$ P.U.

$\delta_0$ = Initial Load angle

$= 3^o37'$

$V_0$ = voltage behind the synchronous reactance

$= 1.111$ P.U.

$$T_j = \frac{2H}{\omega}$$

$= .00242$ P.U.

$$T_m = \frac{V\,V_0}{X_d} = 1.45 \text{ P.U.}$$

$$T_r = \frac{V^2}{2} \quad \frac{X_d - X_q}{X_d X_q}$$

$= 0.452$ P.U.

$\therefore$ Synchronous motor torque $= (1.45 \sin\delta + .452 \sin 2\delta)$ ...3.8

Induction motor torque $= .0212 \dfrac{d\delta}{dt}$

Relative damping coefficient $= \dfrac{.0212}{\sqrt{.00242 \times 1.45}} = 0.357$

...<u>3.2.1. Determination of critical load and the maximum angle of swing for any load for the undamped case</u>...

$K = 0$ for the undamped case.

The torque - angle characteristic is given by equation 3.8

Torque $= (1.45 \sin\delta + .452 \sin 2\delta)$ and is plotted in Fig.3.2 For a suddenly applied shaft load of $(0.8-0.19)=0.61$P.U., the maximum angle of swing is found by equal-area method. The maximum angle of swing for the case considered is $42^o$.

The torque angle characteristic given by equation 3.8 is again plotted in Fig.3.3. The critical load is found by equal-

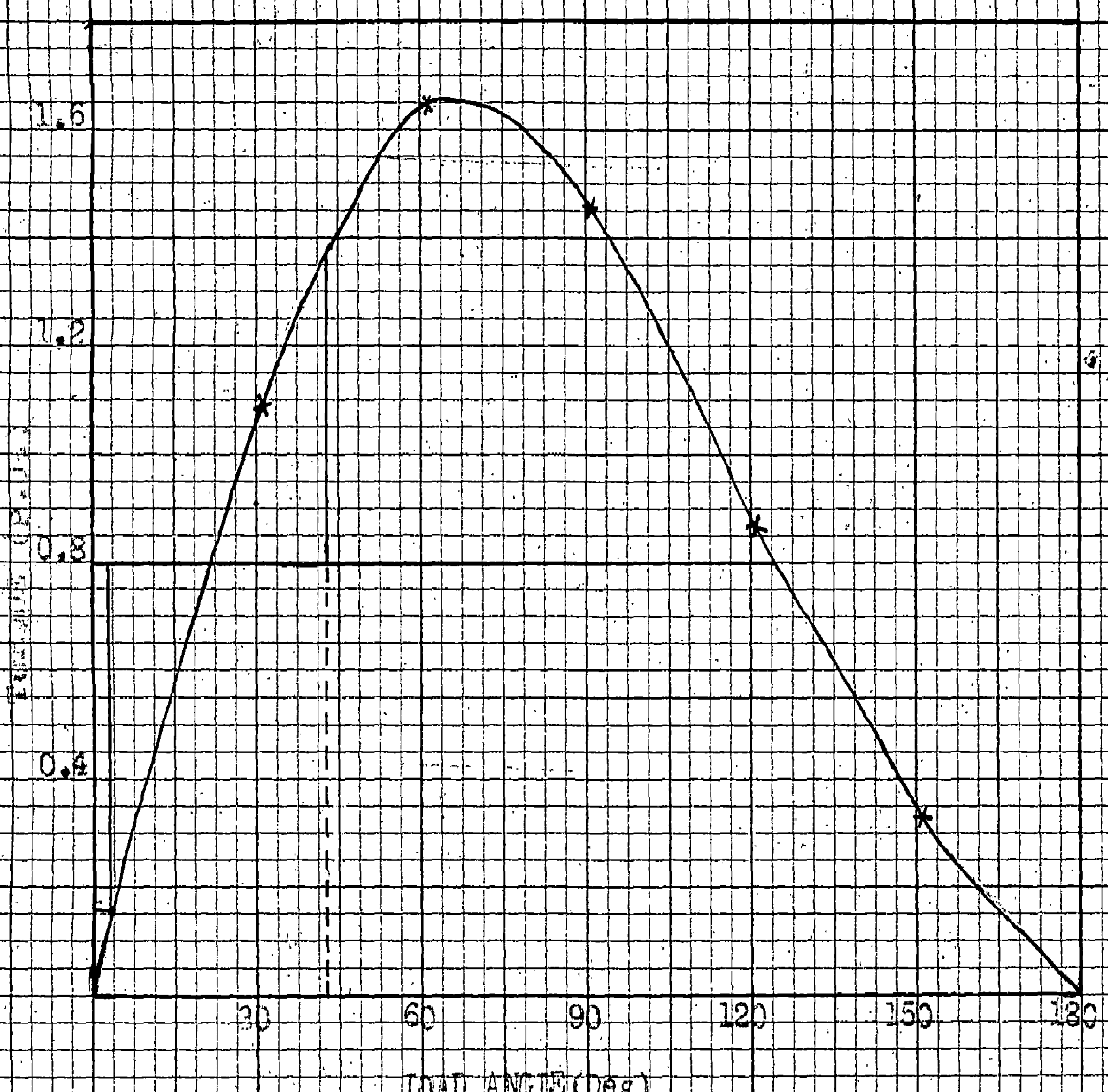



FIG.3.2

DETERMINATION OF MAXIMUM ANGLE OF SWING WHEN A SUDDEN LOAD OF 0.61 P.U. IS APPLIED TO SHAFT

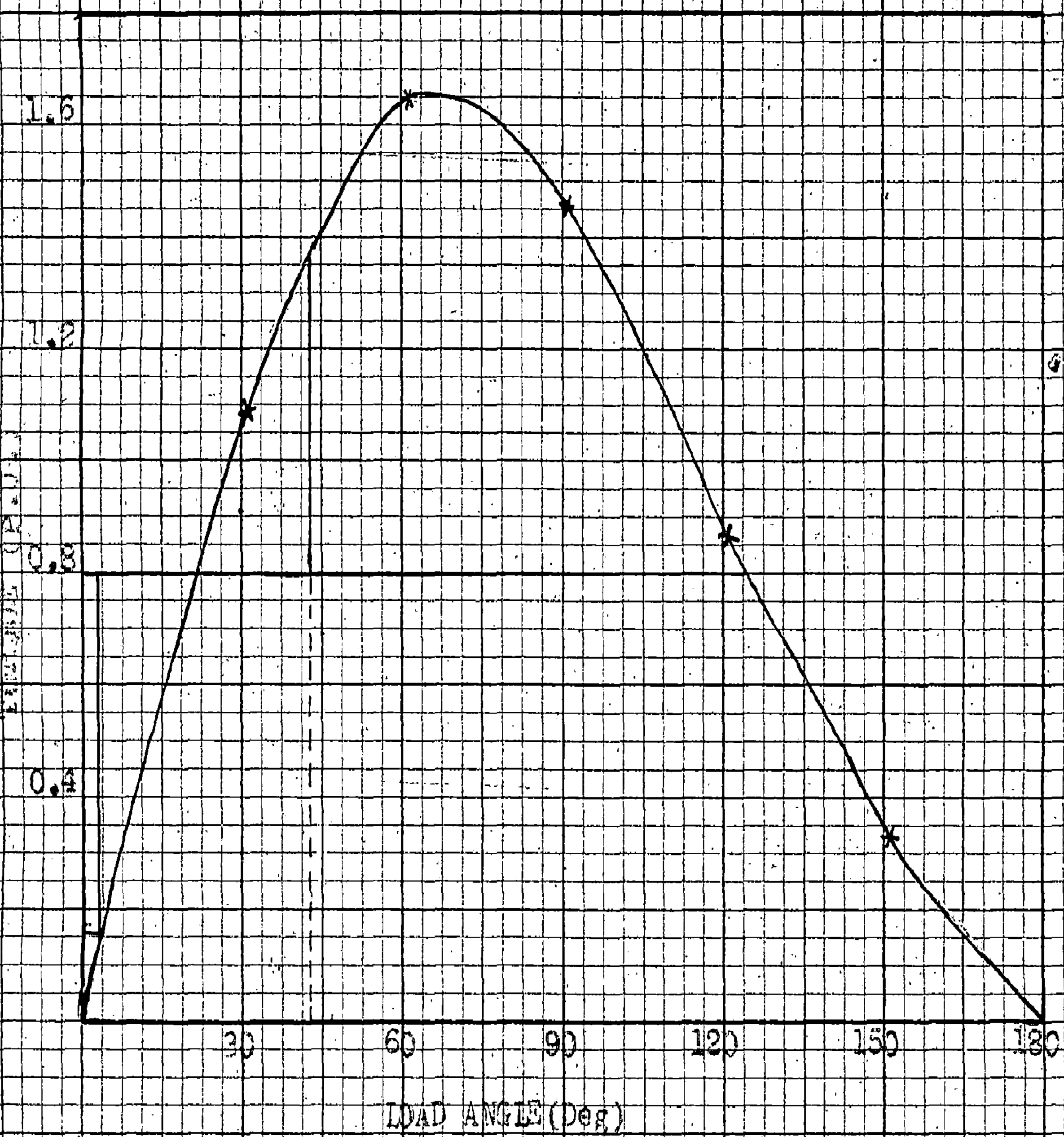



FIG.3.2

DETERMINATION OF MAXIMUM ANGLE OF SWING WHEN A SUDDEN LOAD OF 0.61 P.U. IS APPLIED TO SHAFT

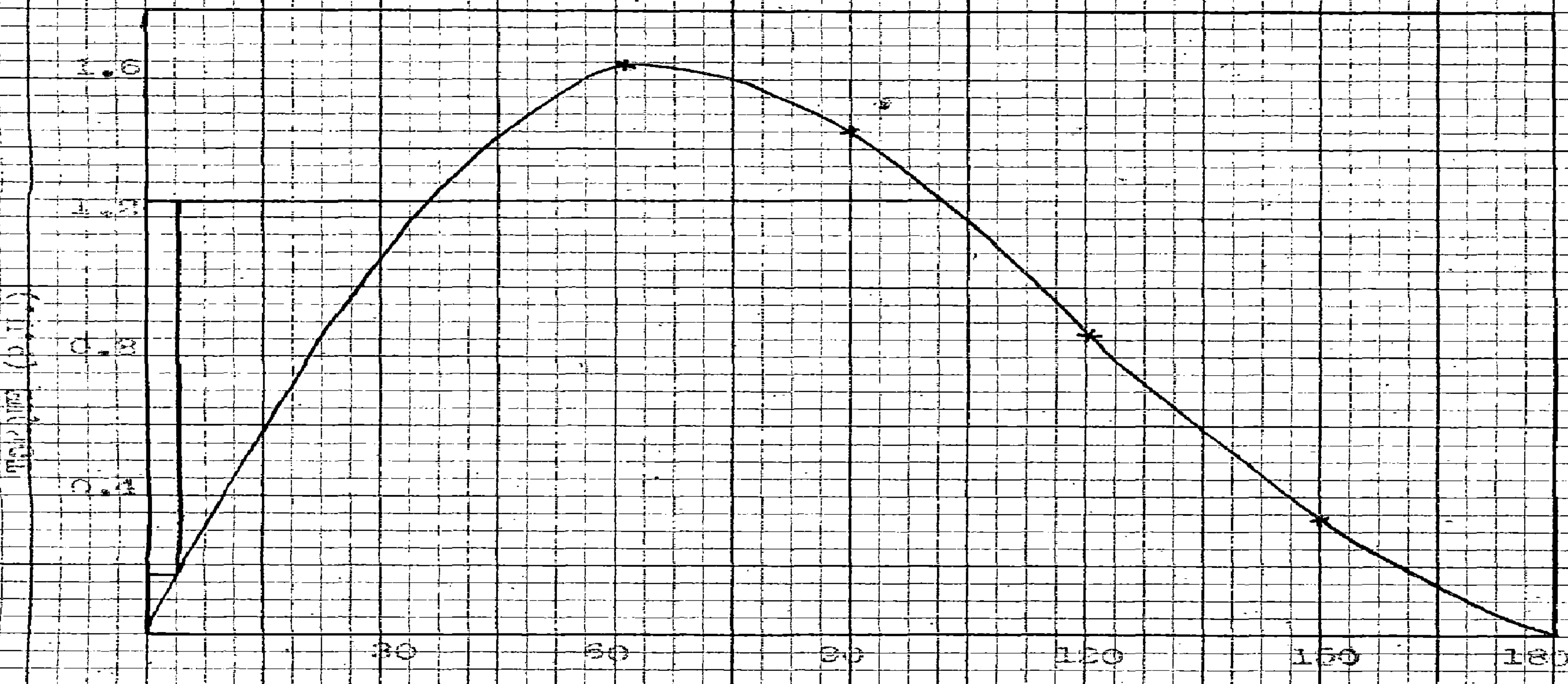



FIG. 2.3

DETERMINATION OF CRITICAL LOAD

area method. The maximum load that can be abruptly applied to shaft is 1.26 - .19 = 1.07 P.U.

...3.2.2. Effect of the value of relative damping coefficient at various suddenly applied loads for a constant initial load ratio...

To show the effect of relative damping coefficient, three values of K are chosen - K = 0, K = .0357 and K = .357. Corresponding to these three values of K, the induction motor torque will be zero; $.00212 \frac{d\delta}{dt}$ and $.0212 \frac{d\delta}{dt}$.

The corresponding equations of motion will be -

$$.00242 \frac{d^2\delta}{dt^2} + 1.45 \sin\delta + .452 \sin 2\delta = .19 + T_L 1.$$

$$.00242 \frac{d^2\delta}{dt^2} + .00212 \frac{d\delta}{dt} + 1.45 \sin\delta + .452 \sin 2\delta = .19 + T_L 1$$

$$.00242 \frac{d^2\delta}{dt^2} + .0212 \frac{d\delta}{dt} + 1.45 \sin\delta + .452 \sin 2\delta = .19 + T_L 1 \qquad ...3.9$$

With the constant initial load ratio $\frac{T_o}{T_m} = \frac{0.19}{1.45} = .131$, two values of suddenly applied loads are considered -

$T_L = 0.61$ and $T_L = 1.11$

<u>$T_L = 0.61$</u> Equations 3.9 become -

$$.00242 \frac{d^2\delta}{dt^2} + 1.45 \sin\delta + .452 \sin 2\delta = 0.8$$

$$.00242 \frac{d^2\delta}{dt^2} + .00212 \frac{d\delta}{dt} + 1.45 \sin\delta + .452 \sin 2\delta = 0.8 \qquad ..3.10$$

$$.00242 \frac{d^2\delta}{dt^2} + .0212 \frac{d\delta}{dt} + 1.45 \sin\delta + .452 \sin 2\delta = 0.8$$

Equations 3.10 are solved in Tables 3.1, 3.2 & 3.3 by the step-by-step method and plotted in Figs. 3.4 and 3.5. Fig.3.4 shows the variation of load angle with time and Fig.3.5 shows

TABLE 3.1

Swing curve calculations (Suddenly applied load 0.61 P.U. Relative Damping coefficient zero)

| t (sec) | $T_e$ | $T_o+T_L$ | $T_{ac}$ | $\Delta^2 \delta$ | $\Delta\delta$ | $\delta$ (deg) |
|---|---|---|---|---|---|---|
| 0- | | .19 | | | | |
| 0+ | .139 | .80 | .661 | | | |
| 0aver | | | .33 | 1.36 | | 3.616 |
| | | | | | 1.36 | |
| 0.1 | .2 | " | .6 | 2.47 | | 4.97 |
| | | | | | 3.83 | |
| 0.2 | .358 | " | .442 | 1.82 | | 8.8 |
| | | | | | 5.65 | |
| 0.3 | .54 | " | .26 | 1.07 | | 13.45 |
| | | | | | 6.72 | |
| 0.4 | .738 | " | .02 | .082 | | 20.17 |
| | | | | | 6.8 | |
| 0.5 | 1.019 | " | -.219 | .905 | | 26.97 |
| | | | | | 7.7 | |
| 0.6 | 1.243 | " | -.443 | -1.82 | | 34.67 |
| | | | | | 5.88 | |
| 0.7 | 1.385 | " | -.585 | -2.42 | | 40.55 |
| | | | | | 3.46 | |
| 0.8 | 1.46 | " | -.66 | -2.72 | | 44.01 |
| | | | | | 1.16 | |
| 0.9 | 1.48 | " | -.68 | -2.8 | | 45.17 |
| | | | | | -1.64 | |
| 1.0 | 1.45 | " | -.65 | -2.68 | | 43.53 |
| | | | | | -4.32 | |
| 1.1 | 1.367 | " | -.567 | -2.34 | | 39.21 |
| | | | | | -6.66 | |
| 1.2 | 1.19 | " | -.39 | -1.6 | | 32.55 |
| | | | | | -8.26 | |
| 1.3 | .932 | " | -.132 | -.54 | | 24.29 |
| | | | | | -7.72 | |
| 1.4 | | | | | | 17.57 |

TABLE 3.1

Swing curve calculations (Suddenly applied load 0.61 P.U.

Relative Damping coefficient zero)

| t (sec) | $T_e$ | $T_o+T_L$ | $T_{ac}$ | $\Delta^2 \delta$ | $\Delta\delta$ | $\delta$ (deg) |
|---|---|---|---|---|---|---|
| 0- | | .19 | | | | |
| 0+ | .139 | .80 | .661 | | | |
| 0aver | | | .33 | 1.36 | | 3.616 |
| | | | | | 1.36 | |
| 0.1 | .2 | " | .6 | 2.47 | | 4.97 |
| | | | | | 3.83 | |
| 0.2 | .358 | " | .442 | 1.82 | | 8.8 |
| | | | | | 5.65 | |
| 0.3 | .54 | " | .26 | 1.07 | | 13.45 |
| | | | | | 6.72 | |
| 0.4 | .788 | " | .02 | .082 | | 20.17 |
| | | | | | 6.8 | |
| 0.5 | 1.019 | " | -.219 | .905 | | 26.97 |
| | | | | | 7.7 | |
| 0.6 | 1.243 | " | -.443 | -1.82 | | 34.67 |
| | | | | | 5.88 | |
| 0.7 | 1.385 | " | -.585 | -2.42 | | 40.55 |
| | | | | | 3.46 | |
| 0.8 | 1.46 | " | -.66 | -2.72 | | 44.01 |
| | | | | | 1.16 | |
| 0.9 | 1.48 | " | -.68 | -2.8 | | 45.17 |
| | | | | | -1.64 | |
| 1.0 | 1.45 | " | -.65 | -2.68 | | 43.53 |
| | | | | | -4.32 | |
| 1.1 | 1.367 | " | -.567 | -2.34 | | 39.21 |
| | | | | | -6.66 | |
| 1.2 | 1.19 | " | -.39 | -1.6 | | 32.55 |
| | | | | | -8.26 | |
| 1.3 | .932 | " | -.132 | -.54 | | 24.29 |
| | | | | | -7.72 | |
| 1.4 | | | | | | 17.57 |

## TABLE 3.2

Swing curve calculations (Suddenly applied load 0.61 P.U.

Relative damping coefficient 0.02571.

| t (sec) | $T_e$ | $T_D$ = -.0212 $\Delta\delta$ | $T_e + T_D$ | $T_o + T_L$ | $T_{ac}$ | $\Delta^2\delta$ | $\Delta\delta$ | $\delta$ (deg) |
|---|---|---|---|---|---|---|---|---|
| 0- | | | | 0.19 | | | 0 | |
| 0+ | .139 | .0126 | .351 | 0.8 | .449 | | | |
| 0aver | | | | | .224 | .924 | | 3.616 |
| | | | | | | | .924 | |
| 0.1 | .189 | .042 | .231 | " | .569 | 2.35 | | 4.54 |
| | | | | | | | 3.27 | |
| 0.2 | .312 | .084 | .396 | " | .404 | 1.66 | | 7.81 |
| | | | | | | | 4.93 | |
| 0.3 | .512 | .117 | .629 | " | .171 | .705 | | 12.74 |
| | | | | | | | 5.63 | |
| 0.4 | .723 | .117 | .84 | " | -.04 | -.164 | | 18.37 |
| | | | | | | | 5.44 | |
| 0.5 | .919 | .105 | 1.02 | " | -.204 | -.845 | | 23.84 |
| | | | | | | | 4.63 | |
| 0.6 | 1.03 | .084 | 1.11 | " | -.318 | -1.31 | | 27.47 |
| | | | | | | | 3.32 | |
| 0.7 | 1.13 | .053 | 1.19 | " | -.391 | -1.61 | | 30.79 |
| | | | | | | | 1.71 | |
| 0.8 | 1.19 | .021 | 1.21 | " | -.41 | -1.69 | | 32.50 |
| | | | | | | | - .02 | |
| 0.9 | 1.18 | -.021 | 1.16 | " | -.36 | -1.50 | | 32.48 |
| | | | | | | | -1.52 | |
| 1.0 | 1.14 | -.042 | 1.10 | " | -.3 | -1.23 | | 30.96 |
| | | | | | | | -2.75 | |
| 1.1 | 1.06 | -.053 | .907 | " | -.187 | -.44 | | 28.21 |
| | | | | | | | -2.31 | |
| 1.2 | | | | | | | | 25.90 |

TABLE 3.3

Swing curve calculations (Suddenly applied load 0.61P.U.

Relative damping coefficient .357)

| t (sec) | $T_e$ | $T_D = .2121\,\Delta\delta$ | $T_e+T_D$ | $T_o+T_L$ | $T_{ac}$ | $\Delta^2\delta$ | $\Delta\delta$ | $\delta$ (deg) |
|---|---|---|---|---|---|---|---|---|
| 0- | | | .19 | .19 | | | | |
| 0+ | .139 | .106 | .245 | 0.8 | .56 | | | |
| 0aver | | | | | .28 | 1.15 | | 3.616 |
| | | | | | | | 1.15 | |
| 0.1 | .181 | .318 | .489 | " | .311 | 1.28 | | 4.76 |
| | | | | | | | 2.43 | |
| 0.2 | .29 | .53 | .82 | " | -.02 | .082 | | 7.19 |
| | | | | | | | 2.35 | |
| 0.3 | .387 | .424 | .811 | " | -.011 | .045 | | 9.54 |
| | | | | | | | 2.31 | |
| 0.4 | .478 | .424 | .902 | " | -.102 | .42 | | 11.85 |
| | | | | | | | 1.89 | |
| 0.5 | .552 | .318 | .87 | " | -.07 | -.287 | | 13.74 |
| | | | | | | | 1.61 | |
| 0.6 | .612 | .318 | .93 | " | -.13 | -.535 | | 15.35 |
| | | | | | | | 1.08 | |
| 0.7 | .655 | .212 | .867 | " | -.067 | -.276 | | 16.43 |
| | | | | | | | .81 | |
| 0.8 | .683 | .126 | .809 | " | -.009 | -.04 | | 17.24 |
| | | | | | | | .77 | |
| 0.9 | .713 | .147 | .86 | " | -.06 | -.27 | | 18.01 |
| | | | | | | | .5 | |
| 1.0 | .713 | .106 | .819 | " | -.019 | -.08 | | 18.51 |
| | | | | | | | .42 | |
| 1.1 | .748 | .084 | .832 | " | -.032 | -.132 | | 18.93 |
| | | | | | | | .29 | |
| 1.2 | .758 | .063 | .821 | " | -.021 | -.08 | | 19.22 |
| | | | | | | | .21 | |
| 1.3 | .764 | .042 | .806 | " | -.006 | -.02 | | 19.43 |
| | | | | | | | .19 | |
| 1.4 | .772 | .038 | .81 | " | -.01 | -.04 | | 19.62 |
| | | | | | | | .15 | |
| 1.5 | .772 | .031 | .803 | " | -.003 | -.01 | | 19.77 |
| | | | | | | | .14 | |
| 1.6 | .782 | .027 | .809 | " | -.009 | -.03 | | 19.91 |
| | | | | | | | .11 | |
| 1.7 | .786 | .021 | .807 | " | -.007 | -.028 | | 20.02 |
| | | | | | | | .08 | |
| 1.8 | .786 | .021 | .807 | " | -.007 | -.03 | | 20.11 |
| | | | | | | | .06 | |
| 1.9 | .788 | .014 | .802 | " | -.002 | -.008 | | 20.17 |

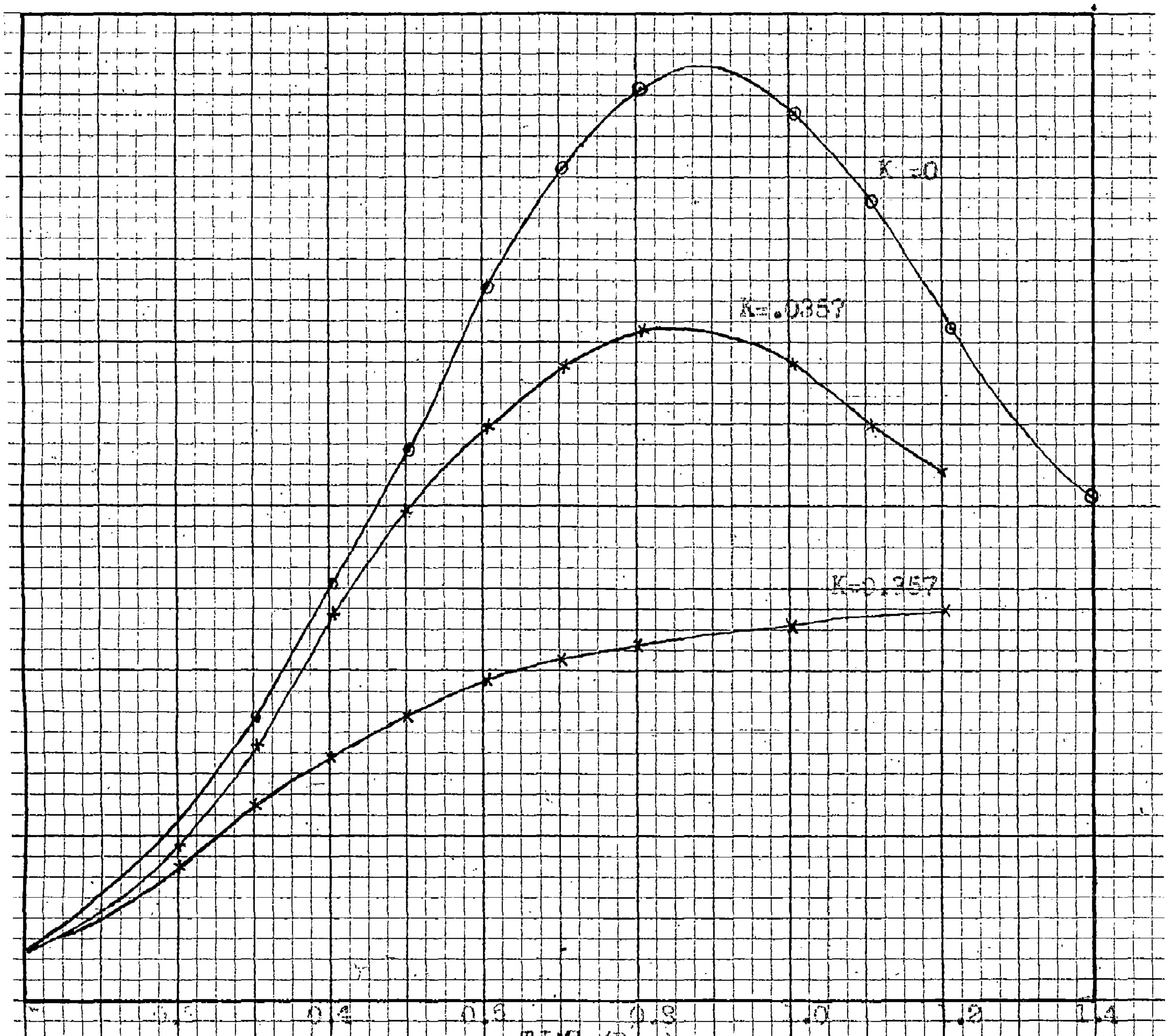



FIG. 3.4

VARIATION OF LOAD ANGLE WHEN A SUDDEN LOAD OF 0.61 P.U. IS APPLIED TO SHAFT

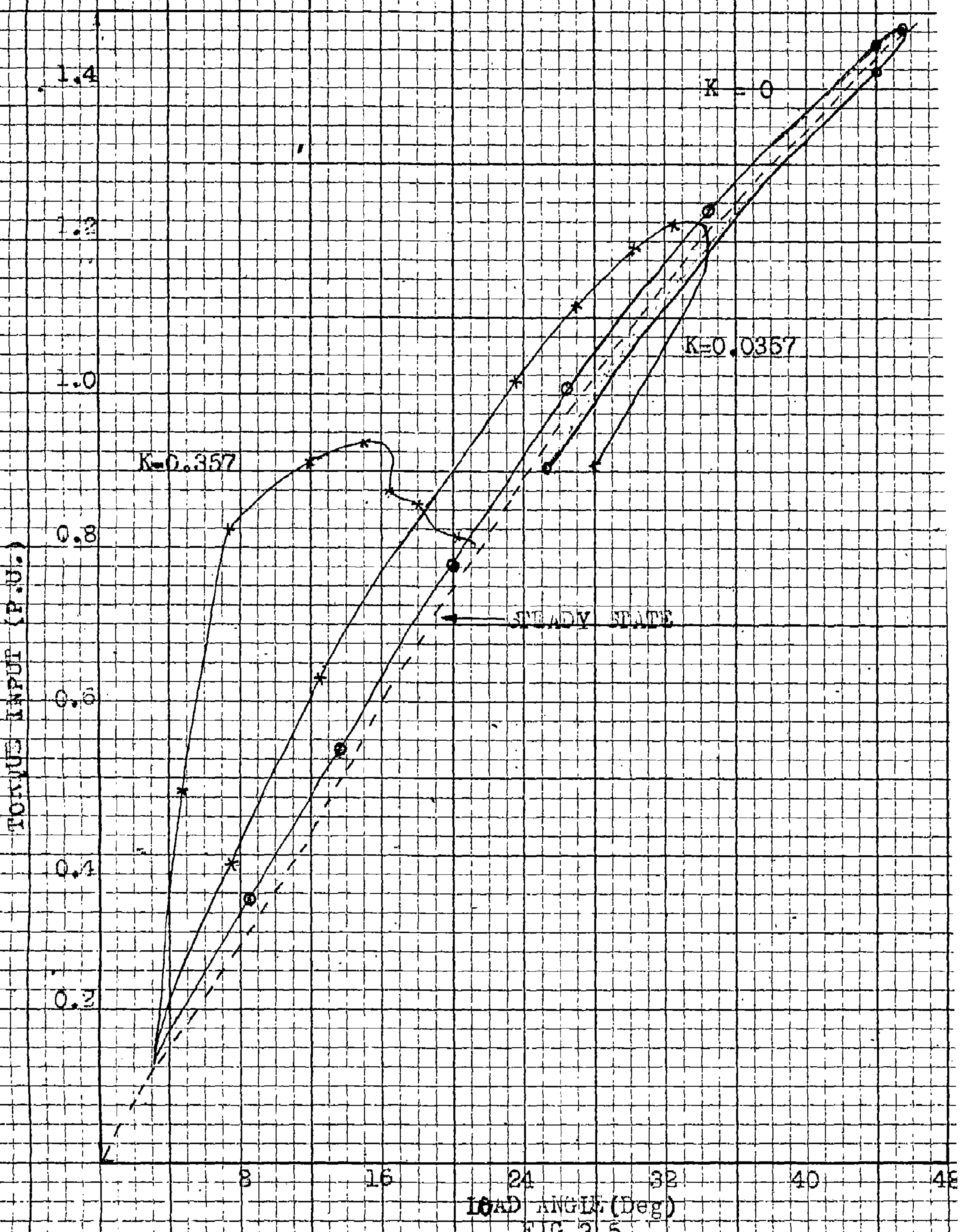



FIG.3.5

TORQUE -ANGLE CHARACTERISTIC WHEN A SUDDEN LOAD OF 0.61P IS APPLIED TO SHAFT that the system is stable, when $K = 0$ as $(T_0 + T_L) < 1.26$, the critical load for the undamped case. Increasing the value of K to .0357 reduces the maximum angle of swing. Further increase in the value of K to .357 reduces the maximum angle of swing further; slows up the rate of swinging and at the same time the motor will not stand greater loads and synchronism will be lost.

$T_L = 1.11$ Equations 3.9 become -

$$.00242 \frac{d^2\delta}{dt^2} + 1.45 \sin\delta + .452 \sin\delta = 1.3$$

$$.00242 \frac{d^2\delta}{dt^2} + .00212 \frac{d\delta}{dt} + 1.45 \sin\delta + .452 \sin 2\delta = 1.3$$

$$.00242 \frac{d^2\delta}{dt^2} + .212 \frac{d\delta}{dt} + 1.45 \sin\delta + .452 \sin 2\delta = 1.3$$

...3.11

Equations 3.11 are solved in tables 3.4, 3.5 and 3.6 by the step-by-step method and plotted in Figs. 3.6 and 3.7 Fig.3.6 shows the variation of load angle with time and Fig.3.7 shows the torque-angle characteristics. It is seen from the figures that the system is unstable when $K = 0$, as $(T_0 + T_L) > 1.26$, the critical load for the undamped case. Increasing the value of K to .0357 makes the system stable. Further increase in the value of K to 0.357, reduces the maximum angle of swing; slows up the rate of swinging and at the same time, the motor will not stand greater loads and synchronism will be lost.

It can also be seen from figs. 3.5 & 3.7 that the transient torque - angle curve for the unstable condition is always above the corresponding steady state curve; whereas for the stable condition, the curve will be above the steady-state curve during the outward swing and falls below it on the return swing.

TABLE 3.4

Swing curve calculations (Suddenly applied load 1.11 P.U.

Relative damping coefficient zero)

| t (sec) | $T_e$ | $T_o + T_L$ | $T_{ac}$ | $\Delta^2\delta$ | $\Delta\delta$ | $\delta$ (deg) |
|---|---|---|---|---|---|---|
| 0- | | .19 | | | 0 | |
| 0+ | .139 | 1.3 | 1.161 | 4.76 | | |
| 0aver | | | | 2.38 | | 3.616 |
| | | | | | 2.38 | |
| 0.1 | .244 | " | 1.056 | 4.35 | | 5.99 |
| | | | | | 6.73 | |
| 0.2 | .512 | " | .788 | 3.25 | | 12.72 |
| | | | | | 9.98 | |
| 0.3 | .882 | " | .418 | 1.72 | | 22.70 |
| | | | | | 11.70 | |
| 0.4 | 1.24 | " | .06 | .247 | | 34.40 |
| | | | | | 11.94 | |
| 0.5 | 1.50 | " | -.2 | -.82 | | 46.34 |
| | | | | | 11.12 | |
| 0.6 | 1.63 | " | -.33 | -1.36 | | 57.46 |
| | | | | | 9.76 | |
| 0.7 | 1.656 | " | -.356 | -1.47 | | 67.22 |
| | | | | | 8.29 | |
| 0.8 | 1.619 | " | -.319 | -1.31 | | 75.51 |
| | | | | | 6.88 | |
| 0.9 | 1.55 | " | +.05 | .206 | | 82.39 |
| | | | | | 7.08 | |
| 1.0 | 1.46 | " | +.16 | .66 | | 89.47 |
| | | | | | 7.74 | |
| 1.1 | | | | | | 97.21 |

TABLE 3.5

Swing curve calculations (Suddenly applied load 1.11 P.U. Relative damping coefficient .0357)

| t (sec) | $T_e$ | $T_D = .0212\,\Delta\delta$ | $T_e+T_D$ | $T_o+T_L$ | $T_{ac}$ | $\Delta^2\delta$ | $\Delta\delta$ | $\delta$ (deg) |
|---|---|---|---|---|---|---|---|---|
| 0− | | | .19 | .19 | | | | |
| 0+ | .139 | .042 | .181 | 1.3 | 1.119 | | | |
| 0aver | | | | | .559 | 2.3 | | 3.616 |
| | | | | | | | 2.3 | |
| 0.1 | .241 | .084 | .325 | " | .975 | 4.02 | | 5.91 |
| | | | | | | | 6.32 | |
| 0.2 | .492 | .168 | .66 | " | .64 | 2.64 | | 12.23 |
| | | | | | | | 8.96 | |
| 0.3 | .823 | .189 | 1.012 | " | .288 | 1.19 | | 21.19 |
| | | | | | | | 10.75 | |
| 0.4 | 1.257 | .212 | 1.469 | " | −.169 | −.696 | | 31.34 |
| | | | | | | | 9.46 | |
| 0.5 | 1.39 | .189 | 1.579 | " | −.279 | −1.15 | | 40.80 |
| | | | | | | | 8.31 | |
| 0.6 | 1.544 | .149 | 1.693 | " | −.393 | −1.62 | | 49.11 |
| | | | | | | | 6.69 | |
| 0.7 | 1.61 | .105 | 1.715 | " | −.415 | −1.71 | | 55.8 |
| | | | | | | | 4.98 | |
| 0.8 | 1.645 | .084 | 1.729 | " | −.428 | −1.77 | | 60.78 |
| | | | | | | | 3.21 | |
| 0.9 | 1.656 | .063 | 1.719 | " | −.419 | −1.77 | | 63.99 |
| | | | | | | | 1.44 | |
| 1.0 | 1.652 | .024 | 1.673 | " | −.373 | −1.6 | | 65.43 |
| | | | | | | | 1.14 | |
| 1.1 | 1.65 | .008 | 1.65 | " | −.35 | −1.44 | | 66.57 |
| | | | | | | | −.3 | |
| 1.2 | 1.656 | −.016 | 1.64 | " | −.34 | −1.40 | | 66.27 |
| | | | | | | | −1.7 | |
| 1.3 | 1.66 | −.04 | 1.62 | " | −.32 | −1.32 | | 64.57 |
| | | | | | | | −3.02 | |
| 1.4 | | | | | | | | 61.55 |

TABLE 3.6

Swing curve calculations (suddenly applied load 1.11 P.U.

Relative damping coefficient .357)

| t (sec) | $T_e$ | $T_D = .2121 \Delta\delta$ | $T_e + T_D$ | $T_o + T_L$ | $T_{ac}$ | $\Delta^2\delta$ | $\Delta\delta$ | $\delta$ (deg) |
|---|---|---|---|---|---|---|---|---|
| 0- | | | .19 | .19 | 0 | | 0 | |
| 0+ | .139 | .212 | .351 | 1.3 | .95 | | | |
| 0aver | | | | | .47 | 1.9 | | 3.616 |
| | | | | | | | 1.9 | |
| 0.1 | .17 | .53 | .70 | " | .6 | 2.472 | | 5.51 |
| | | | | | | | 4.37 | |
| 0.2 | .39 | .955 | 1.34 | " | -.04 | -.164 | | 9.88 |
| | | | | | | | 4.21 | |
| 0.3 | .56 | .84 | 1.40 | " | -.1 | -.415 | | 14.09 |
| | | | | | | | 3.80 | |
| 0.4 | .70 | .764 | 1.46 | " | -.16 | -.66 | | 17.89 |
| | | | | | | | 3.14 | |
| 0.5 | .82 | .63 | 1.45 | " | -.15 | -.62 | | 21.03 |
| | | | | | | | 2.52 | |
| 0.6 | .90 | .486 | 1.38 | " | -.08 | -.32 | | 23.55 |
| | | | | | | | 2.20 | |
| 0.7 | .98 | .42 | 1.40 | " | -.1 | -.41 | | 25.76 |
| | | | | | | | 1.79 | |
| 0.8 | 1.04 | .34 | 1.38 | " | -.08 | -.32 | | 27.54 |
| | | | | | | | 1.49 | |
| 0.9 | 1.09 | .298 | 1.388 | " | -.08 | -.32 | | 29.01 |
| | | | | | | | 1.14 | |
| 1.0 | 1.12 | .231 | 1.35 | " | -.05 | -.206 | | 30.15 |
| | | | | | | | 0.94 | |
| 1.1 | 1.15 | .189 | 1.33 | " | -.03 | -.12 | | 31.09 |

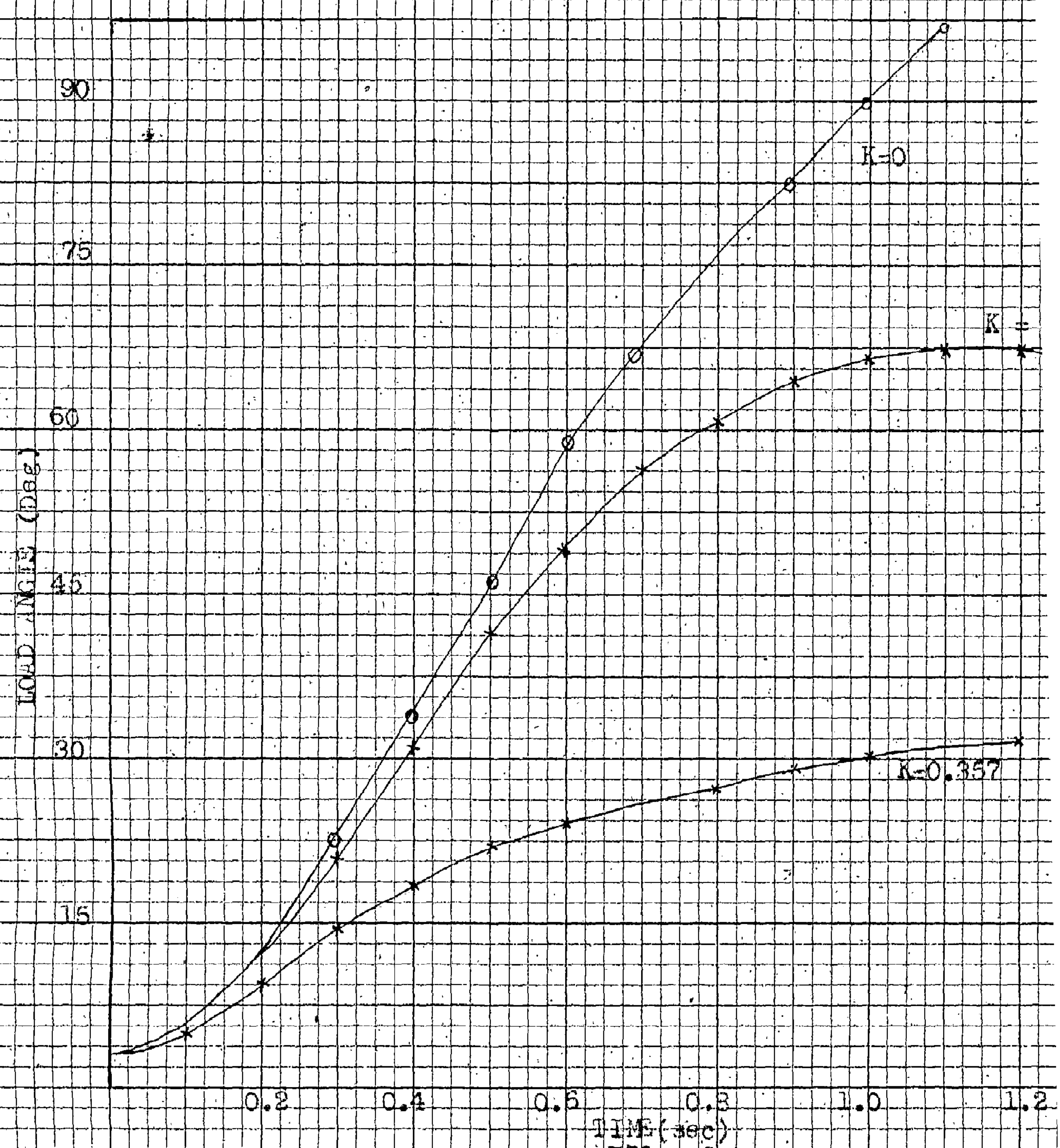



FIG.3.6

VARIATION OF LOAD ANGLE WHEN A SUDDEN LOAD OF 1.11 P.U. IS APPLIED TO SHAFT

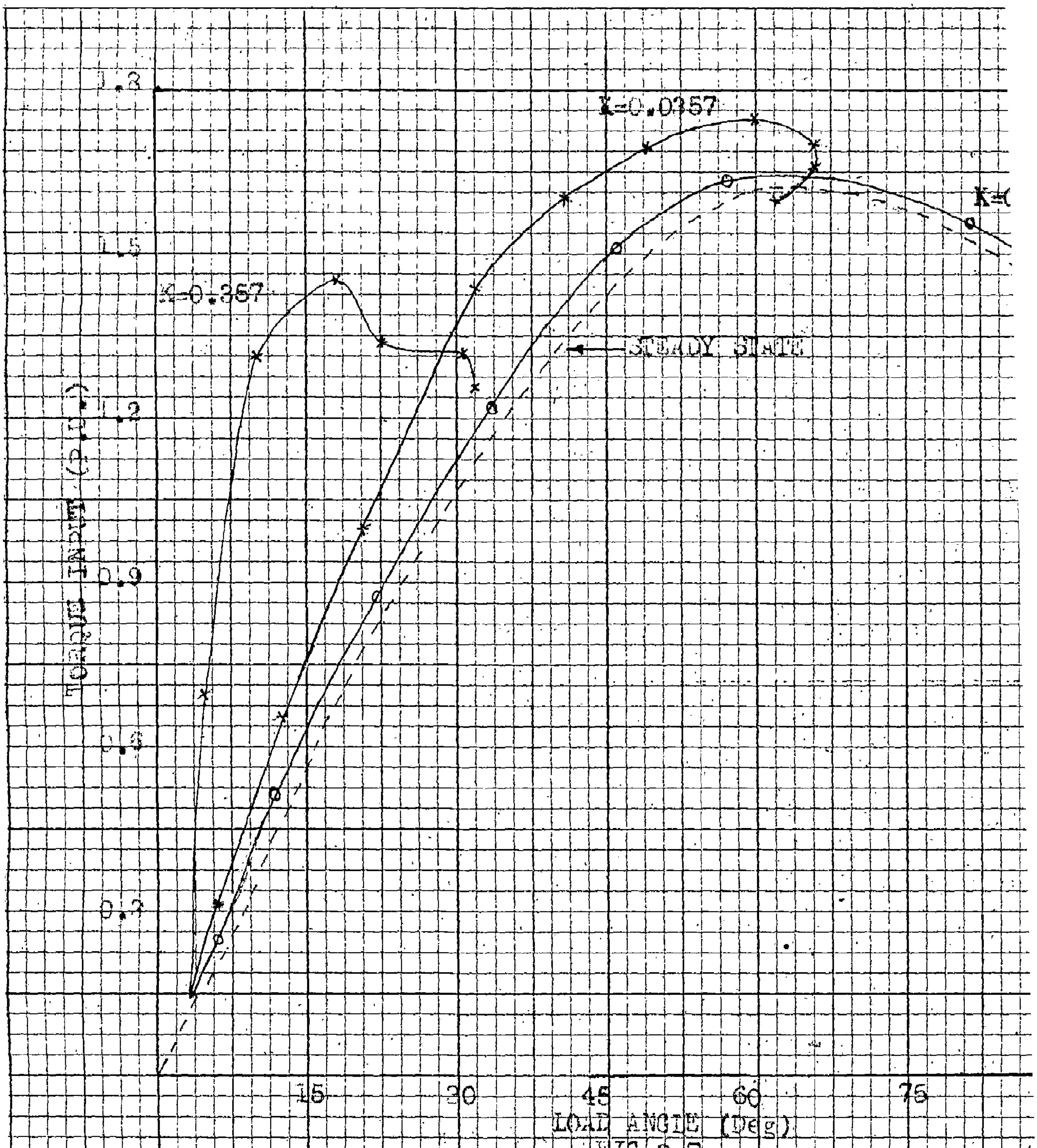



FIG.3.7

TORQUE-ANGLE CHARACTERISTIC WHEN A SUDDEN LOAD OF 1. IS APPLIED TO SHAFT

...3.2.3. Variation of angular displacement with various suddenly applied shaft loads for a constant initial load ratio and for a fixed value of relative damping coefficient...

For a relative damping coefficient of .0357, the variation of Angular displacement with time for various suddenly applied shaft loads can be studied from the following equations:-

$.00242 \frac{d^2\delta}{dt^2}$ + ~~$.00212 \frac{d}{dt^2}$~~ $+.00212 \frac{d\delta}{dt} + 1.45 \text{ Sin}\,\delta + .452 \text{ Sin } 2\delta$

$= 0.8$ for a suddenly applied load of 0.61 P.U.

$$.00242 \frac{d^2\delta}{dt^2} + .00212 \frac{d\delta}{dt} + .145 \text{ Sin}\,\delta + .452 \text{ Sin } 2\delta = 1.3$$

for a suddenly applied load of 1.11 P.U.

$$.00242 \frac{d^2\delta}{dt^2} + .00212 \frac{d\delta}{dt} + 1.45 \text{ Sin}\,\delta + .452 \text{ Sin } 2\delta = 1.4$$

for a suddenly applied load of 1.21 P.U.

$$.00242 \frac{d^2\delta}{dt^2} + .00212 \frac{d\delta}{dt} + 1.45 \text{ Sin}\,\delta + .452 \text{ Sin } 2\delta = 1.6$$

for a suddenly applied load of 1.41 P.U.

... ... 3.12

Equations 3.12 are solved in Tables 3.2, 3.5, 3.7 & 3.8 by the step-by-step method and plotted in fig.3.8. From the figure it can be seen that the sudden load which can be applied without the machine falling out of step is about 1.45 - .19 = 1.26 P.U.

When K = 0, the maximum sudden load which could be applied to the shaft was 1.26 -.19 = 1.07 P.U.

Hence it can be concluded that reasonable amounts of damping torque increase the amount of load that can be suddenly applied to the shaft without loss of synchronism.

## TABLE 2.7

Swing curve calculations (suddenly applied load 1.2

Relative damping coefficient .0357)

| t (sec) | $T_e$ | $T_D = .0212\,\Delta\delta$ | $T_e+T_D$ | $T_o+T_L$ | $T_{ac}$ | $\Delta^2\delta$ | $\Delta\delta$ | ( |
|---|---|---|---|---|---|---|---|---|
| 0- | | | | .19 | | | 0 | |
| 0+ | .139 | .021 | .16 | 1.4 | 1.24 | 5.10 | | |
| 0aver | | | | | | 2.50 | | 3. |
| | | | | | | | 2.5 | |
| 0.1 | .283 | .1 | .383 | " | 1.017 | 4.16 | | 6. |
| | | | | | | | 6.66 | |
| 0.2 | .512 | .16 | .672 | " | .728 | 3.00 | | 12. |
| | | | | | | | 9.66 | |
| 0.3 | 1.166 | .212 | 1.378 | " | .022 | .09 | | 22. |
| | | | | | | | 9.75 | |
| 0.4 | 1.75 | .18 | 1.355 | " | .05 | .206 | | 32. |
| | | | | | | | 9.95 | |
| 0.5 | 1.413 | .18 | 1.59 | " | -.19 | -.78 | | 41. |
| | | | | | | | 9.17 | |
| 0.6 | 1.552 | .16 | 1.712 | " | -.312 | -1.29 | | 51. |
| | | | | | | | 7.88 | |
| 0.7 | 1.64 | .14 | 1.78 | " | -.38 | -1.56 | | 59. |
| | | | | | | | 6.32 | |
| 0.8 | 1.656 | .12 | 1.976 | " | -.37 | -1.52 | | 65. |
| | | | | | | | 5.80 | |
| 0.9 | 1.648 | .10 | 1.748 | " | -.348 | -1.44 | | 71. |
| | | | | | | | 4.36 | |
| 1.0 | 1.626 | .08 | 1.70 | " | -.3 | -1.226 | | 75. |
| | | | | | | | 3.14 | |
| 1.1 | 1.596 | .06 | 1.65 | " | -.5 | -.206 | | 78. |
| | | | | | | | 2.94 | |
| 1.2 | 1.55 | .04 | 1.59 | " | +.01 | +.041 | | 81. |

TABLE 3.8

Swing curve calculations (suddenly applied load 1.41 P.U.

Relative damping coefficient .0357)

| t (sec) | $T_e$ | $T_D$ =.0212 | $T_e+T_D$ | $T_o+T_L$ | $T_{ac}$ | $\Delta^2 \delta$ | $\Delta \delta$ | $\delta$ (deg) |
|---|---|---|---|---|---|---|---|---|
| 0- | | | | .19 | | | 0 | |
| 0+ | .139 | .03 | .169 | 1.6 | 1.43 | | | |
| 0aver | | | | | .71 | 2.92 | | 3.616 |
| | | | | | | | 2.92 | |
| 0.1 | .264 | .01 | .364 | " | 1.236 | 5.06 | | 6.53 |
| | | | | | | | 7.98 | |
| 0.2 | .581 | .212 | .793 | " | .807 | 3.32 | | 14.51 |
| | | | | | | | 11.30 | |
| 0.3 | .984 | .242 | 1.226 | " | .38 | 1.57 | | 25.81 |
| | | | | | | | 12.87 | |
| 0.4 | 1.345 | .263 | 1.608 | " | -.008 | -.03 | | 38.68 |
| | | | | | | | 12.84 | |
| 0.5 | 1.57 | .242 | 1.81 | " | -.21 | -.866 | | 51.52 |
| | | | | | | | 11.78 | |
| 0.6 | 1.655 | .231 | 1.866 | " | -.266 | -1.1 | | 63.30 |
| | | | | | | | 10.68 | |
| 0.7 | 1.63 | .212 | 1.84 | " | -.24 | -.99 | | 73.98 |
| | | | | | | | 9.69 | |
| 0.8 | 1.54 | .189 | 1.72 | " | -.12 | -.49 | | 83.67 |
| | | | | | | | 9.20 | |
| 0.9 | 1.4 | .168 | 1.568 | " | .032 | .132 | | 92.87 |
| | | | | | | | 9.33 | |
| 1.0 | 1.226 | .189 | 1.415 | " | .185 | .076 | | 102.20 |
| | | | | | | | 9.40 | |
| 1.1 | | | | | | | | 111.60 |

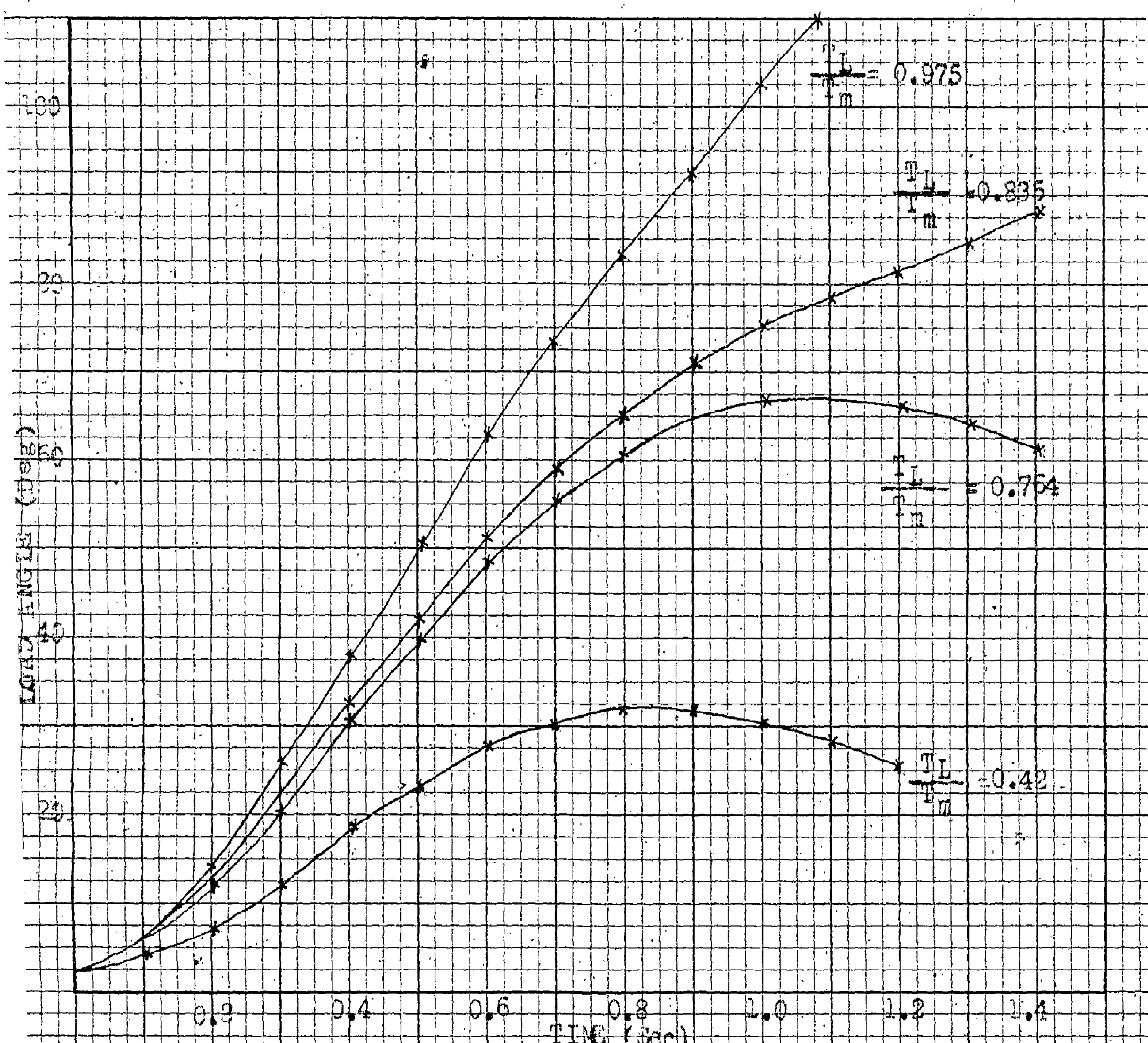



FIG. 7.8

VARIATION OF ANGULAR DISPLACEMENT FOR VARIOUS SUDDENLY APPLIED SHAFT LOADS. RELATIVE DAMPING COEFFICIENT 0.0357 INITIAL LOAD 0.[illegible] P.U.

...3.2.4. Effect of the relative damping coefficient on the time required for the maximum angle of swing...

For large values of sudden load ratio, the time required for the angle to change to its maximum value increases for a given value of K; but this time is reduced for larger values of K. This is shown in fig. 3.9 which shows a plot of the time required for the angle to reach its maximum value against sudden load ratio. Curves are plotted for two different values of K- K = 0 and K = .0357. It is seen that for a given value of K, the time required for the angle to reach its maximum value increases with the sudden load ratio but is reduced for larger values of K. For very large values of sudden load ratio, the time increases very sharply.

...3.2.5. Effect of the relative damping coefficient on the maximum angle of swing...

Fig. 3.10 shows a plot of the maximum angle of swing against sudden load ratio for three different values of K - K = 0, K = .0357 and K = .357. It is seen that the maximum angle of swing increases with sudden load ratio but is reduced with larger values of K.

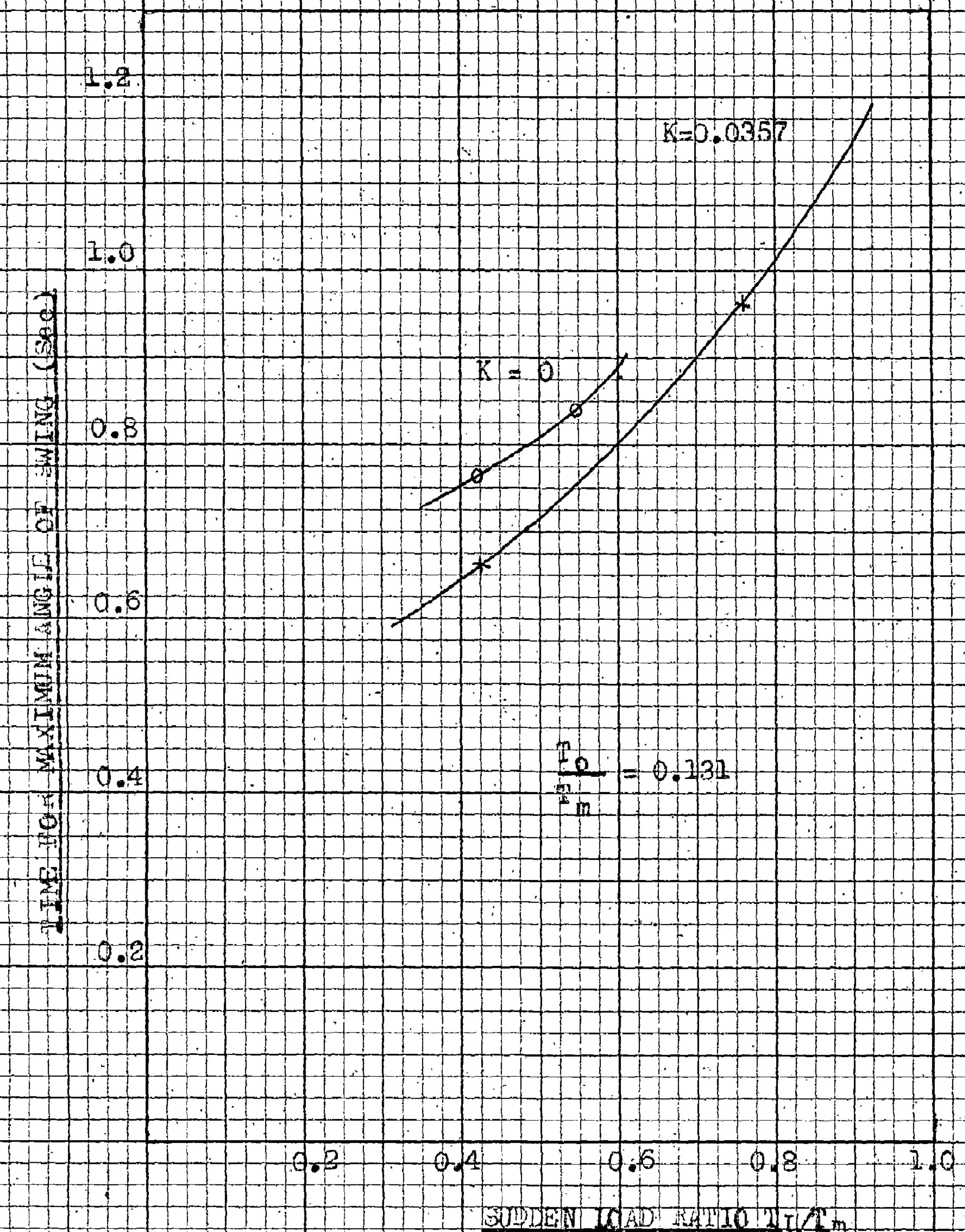



FIG. 3.9

PLOT OF THE TIME REQUIRED FOR THE ANGLE TO REACH ITS MAXIMUM VALUE FOR VARIOUS SUDDENLY APPLIED SHAFT LOADS.

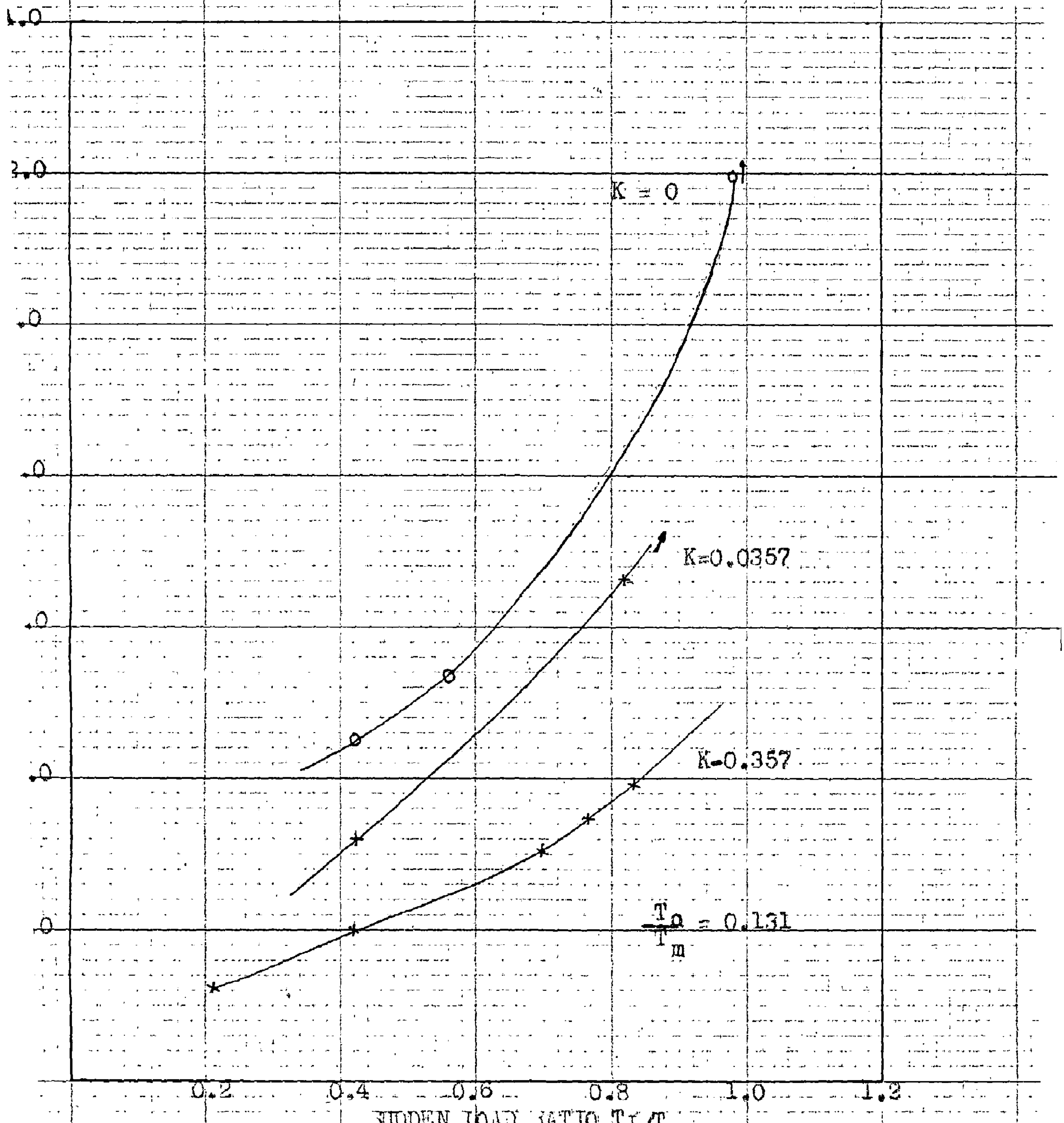



FIG.3.10

PLOT OF THE MAXIMUM ANGLE OF SWING FOR VARIOUS SUDDENLY APPLIED SHAFT LOADS

# ...CHAPTER IV...

## ...ASYNCHRONOUS OPERATION AND RESYNCHRONIZATION...

### ...4.1. Asynchronous operation...

Following a system disturbance, such as a severe short circuit which is finally cleared, a synchronous machine may find itself operating out of step. An alternator which loses synchronism but remains connected to the supply system may settle down after a time to a steady condition of operation as an asynchronous generator. For a given power output, the currents are increased compared with the values during normal synchronous operation and the currents in the supply leads pulsate. The speed also pulsates about a mean value.

In the past it was considered that an alternator which loses synchronism must be disconnected forthwith. However, recent experience has indicated that under certain conditions and for a limited period it is permissible to leave the machine connected, until some action can be taken to cause it to resynchronize. What happens when an alternator runs asynchronously, what are the criteria for resynchronization and how the damper windings assist in resynchronization are dealt in this chapter.

When the machine loses synchronism, if the field excitation is removed, the current and torque pulsations are much less severe. The alternator is then an induction generator with unsymmetrical secondary circuit. The theory in the following chapter starts by analysing this condition during which the field circuit may be closed or open or closed through a resistance. In the next step, the effect of field voltage is studied. Then the conditons for resynchronization are studied.

The analysis given below is an approximate one. During asynchronous operation, the speed of the machine pulsates and an exact analysis would require the solution of non-linear differential equations. The 1st part of the analysis is therefore based on the assumption that the speed is constant, for which condition the equations are linear. In the 2nd part of the analysis the torque pulsations, calculated from formulae derived in the 1st part, are used to determine the pulsations of speed.

## ...4.2. Theoritical analysis based on constant slip...

With a constant slip S, the load angle increases uniformly with time i.e. $\delta = - S\omega t$. Equations A 11 then become (neglecting $r_a$)

$$\left.\begin{aligned} - V_m \sin S\omega t &= p\psi_d - (1-S)\,\omega\,\psi_q \\ V_m \cos S\omega t &= + (1-S)\omega\psi_d + p\psi_q \end{aligned}\right\} \quad ...4.1$$

Equations 4.1 are used in conjunction with equations A5. As the equations are linear, the solution may be obtained by superimposing two separate parts for which additional suffixes are used; Suffix 1, solution with applied terminal voltage but no field voltage; suffix 2, solution with applied field voltage but no terminal voltage.

The field resistance $r_f$ must include any external resistance in the circuit.

## ...4.3. Calculation of torque with no field voltage...

During steady operation with $v_f = 0$, the axis currents and the flux linkages obtained by the solution of equations 4.1 and A5 are sinusoidal quantities at slip frequency. The equations can be converted into vector equations by substituting $p = jS\omega$ and replacing the variables by the corresponding vectors, indicated by symbols in bold type.

$$\left.\begin{aligned} +j\mathbf{V} &= jS\omega\psi_{d1} - (1-S)\omega\psi_{q1} \\ \mathbf{V} &= +(1-S)\omega\psi_{d1} + jS\omega\psi_{q1} \end{aligned}\right\} \quad \ldots 4.2$$

$$\left.\begin{aligned} \omega\psi_{d1} &= -X_d(jS\omega)\, I_{d1} \\ \omega\psi_{q1} &= -X_q(jS\omega)\, I_{q1} \end{aligned}\right\} \quad \ldots 4.3$$

$$\left.\begin{aligned} \omega\psi_{d1} &= +\mathbf{V} \\ \omega\psi_{q1} &= +j\mathbf{V} \end{aligned}\right\} \quad \ldots \quad \ldots 4.4$$

$$\left.\begin{aligned} I_{d1} &= \frac{-\mathbf{V}}{X_d(jS\omega)} = -\mathbf{V}(Y_a + Y_b) \\ I_{q1} &= \frac{-j\mathbf{V}}{X_q(jS\omega)} = \mathbf{V}(Y_d - jY_c) \end{aligned}\right\} \ldots 4.5$$

The instantaneous values of the flux linkages and currents are therefore given by

$$\left.\begin{aligned} \omega\psi_{d1} &= +V_m \cos S\omega t \\ \omega\psi_{q1} &= -V_m \sin S\omega t \end{aligned}\right\} \quad \ldots 4.6$$

$$\left.\begin{aligned} i_{d1} &= -V_m(Y_a \cos S\omega t - Y_b \sin S\omega t) \\ i_{q1} &= +V_m(Y_d \cos S\omega t + Y_c \sin S\omega t) \end{aligned}\right\} \ldots 4.7$$

Substituting equations 4.6 and 4.7 in equation A-4, the corresponding torque component is:

$$T_{e1} = +\frac{V^2}{2}\Big[(Y_b + Y_d) + (Y_c - Y_a)\sin 2S\omega t + (Y_d - Y_b)\cos 2S\omega t\Big] \quad \ldots 4.8$$

The mean torque is

$$T_e \text{ (mean)} = + \frac{V^2}{2} (Y_b + Y_d) \quad \dots \quad 4.9$$

### 4.4. Calculation of torque with a field voltage.

The torque component $T_{e2}$ due to the application of a field voltage is calculated by putting $V_m = 0$ in equation 4.1 i.e. a short circuited armature, running at constant speed. The axis currents and flux linkages are constant quantities and the solution is obtained by putting $p = 0$.

$$\left.\begin{aligned} \psi_{d2} &= 0 \\ \psi_{q2} &= 0 \end{aligned}\right\} \quad \dots \quad 4.10$$

$$\left.\begin{aligned} i_{d2} &= + \frac{X_{md}}{r_f X_d} v_f \\ i_{q2} &= 0 \end{aligned}\right\} \quad \dots \quad 4.11$$

The components of equations 4.10 and 4.11 by themselves produce no torque but there is a torque due to the interaction of $i_{d2}$ and $\psi_{q1}$.

The total torque of a machine running asynchronously with supply voltage V and field voltage $v_f$ is

$$T_e = + \frac{V^2}{2} \Big[ (Y_b + Y_d) + (Y_c - Y_a) \sin 2 S\omega t + (Y_d - Y_b) \cos 2 S\omega t \Big] + \frac{V V_o}{X_d} \sin S\omega t \quad \dots \quad 4.12$$

$$\text{where } V_o = + \frac{1}{\sqrt{2}} \frac{X_{md}}{r_f} v_f \quad \dots \quad 4.13$$

= Open circuit voltage induced by the excitation at synchronous speed.

Equation 4.8 gives the torque developed by a synchronous machine when operating at constant, asynchronous speed, with

This torque is the slip torque or induction- motor torque and is seen to have an average and a twice - slip-frequency component.

From equation 4.12 it is seen that the rotor excitation does not affect the torque component of twice slip frequency. The torque component due to rotor excitation is the braking torque tending to slow down the machine. Thus it opposes the induction motor torque which is tending to accelerate the machine.

In equation 4.12, the coefficients $Y_a, Y_b, Y_c$ and $Y_d$ are functions of S. They can, however, be evaluated approximately by putting $p = jS\omega$ in the partial fractions of equations (A.7) and (A.8) and rationalizing the seperate terms.

$$Y_a + j\,Y_b = \frac{1}{X_d(jS\omega)} = \frac{1}{X_d} + \left(\frac{1}{X_d'} - \frac{1}{X_d}\right)\frac{jS\omega T_d' + S^2\omega^2 T_d'^2}{1 + S^2\omega^2 T_d'^2}$$

$$+\left(\frac{1}{X_d''} - \frac{1}{X_d'}\right)\frac{jS\omega T_d'' + S^2\omega^2 T_d''^2}{1 + S^2\omega^2 T_d''^2}$$

$$Y_c + jY_d = \frac{1}{X_q} + \left(\frac{1}{X_q''} - \frac{1}{X_q}\right)\frac{jS\omega T_q'' + S^2\omega^2 T_q''^2}{1 + S^2\omega^2 T_q''^2}$$

For the range of slips under consideration $S^2\omega^2 T_d'^2$ is much larger than unity and $S^2\omega^2 T_d''^2$ and $S^2\omega^2 T_q''^2$ are much less than unity. Hence, approximately,

$$Y_a = \frac{1}{X_d'}$$

$$Y_b = \left(\frac{1}{X_d''} - \frac{1}{X_d'}\right) S\omega T_d''$$

$$Y_c = \frac{1}{X_q}$$

$$Y_d = \left(\frac{1}{X_q''} - \frac{1}{X_q}\right) S\omega T_q''$$

.

$$T_e = + \frac{V V_0}{X_d} \sin\delta - \frac{V^2}{2}\left(\frac{1}{X_d'} - \frac{1}{X_q}\right) \sin 2\delta + \frac{(a+b)}{2} S$$

$$- \frac{(a-b)}{2} S \cos 2\delta \quad \ldots \quad \ldots 4.14$$

Where a & b are the damping constants. Equations of this type have been studied by the methods of non-linear mechanics.[13]

## 4.5. Calculation of slip pulsations.

The behaviour of the machine when running asynchronously under practical operating conditions differs from that considered above, because the torque pulsations cause the slip to oscillate about a mean value. The method given below for calculating the slip variations is based on the assumptions that the prime-mover torque has the constant value given by $T_{mean} = + \frac{V^2}{2}(Y_b + Y_d)$ and that the electrical torque is still given by equation 4.12 with $S\omega t$ replaced by $\delta$.

Then, the equation of motion is

$$\frac{2H}{\omega}\frac{d^2\delta}{dt^2} = - \frac{VV_0}{X_d} \sin\delta - \frac{V^2}{2}(Y_c - Y_a) \sin 2\delta - \frac{V^2}{2}$$

$$(Y_d - Y_b) \cos 2\delta \ldots \quad \ldots 4.15$$

Multiplying by $\frac{d\delta}{dt}$ and integrating, using $S\omega = \frac{d\delta}{dt}$

$$\omega H S^2 = \frac{VV_0}{X_d} \cos\delta + \frac{V^2}{4}(Y_c - Y_a) \cos 2\delta$$

$$- \frac{V^2}{4}(Y_d - Y_b) \sin 2\delta + X \ldots 4.16$$

X is a constant of integration which is equal to the mean value, taken w.r.t. $\delta$, of the function on the R.H. side and is given by $X = \omega H S_0^2$. Hence,

$$S^2 = S_0^2 + \frac{1}{\omega H}\left[\frac{VV_0}{X_d}\cos\delta + \frac{V^2}{4}(Y_c - Y_a)\cos 2\delta\right.$$

$$\left. - \frac{V^2}{4}(Y_d - Y_b) \sin 2\delta\right] \ldots 4.17$$

## ...4.6. Resynchronization...

The severity of conditions when operating out of step depends on whether or not the slip is such that operation is beyond the peak of the asynchronous power/slip curve. Beyond this peak resynchronization is difficult and is possible only if the power output is reduced.

To study the conditions under which the synchronous machine resynchronizes, it is assumed that no sudden change in the conditions takes place i.e. taht the load torque, the field voltage and the external circuit remain fixed or change slowly. For example, a sudden increase of field voltage at certain instants, would make the machine more likely to synchronize. The criterion suggested above covers the worst condition. To apply the criterion, the mean slip for any specified load is first calculated from equation 4.9 and the minimum slip is calculated from equation 4.17. If the minimum value of $S^2$ is zero or negative, the machine will synchronize.

An error occurs because $S_o$, which should strictly be the mean value with respect to $\delta$, is not necessarily the same as the value calculated. The method is therefore empirical since the theoritical basis is not rigorous.

## ...4.7. Calculations...

For the machine under investigation,

$$Y_a = \frac{1}{X_d'} = 3.84$$

$$Y_b = \left(\frac{1}{X_d''} - \frac{1}{X_d'}\right) S\omega T_d'' = 1.59\ S.$$

$$Y_c = \frac{1}{X_q} = 2.16$$

$$Y_d = \left(\frac{1}{\ldots} - \frac{1}{\ldots}\right) S\omega T_{\ldots}'' = 11.7\ S.$$

For the rated F.L. torque $S_0$ = .151 from equation 4.9 From equation 4.17, the equation for slip pulsations is given by $S^2 = .0229 + .0133 \cos\delta - .0035 \cos 2\delta - .0032 \sin 2\delta$ ..4.18

This is plotted as curve (1) in fig.4.1. The values of mean slip corresponding to 10%, 20% and 30% increase in damping torque coefficient will be 0.137, 0.1255 and 0.116 respectively.

The corresponding equations for slip pulsations will be:-

$$S^2 = .0188 + .0133 \cos\delta - .0035 \cos 2\delta - .0032 \sin 2\delta$$
$$S^2 = .0158 + .0133 \cos\delta - .0035 \cos 2\delta - .0032 \sin 2\delta$$
$$S^2 = .0134 + .0133 \cos\delta - .0035 \cos 2\delta - .0032 \sin 2\delta$$

These are plotted as curves (2), (3) and (4) respectively in fig. 4.1.

It can be seen from the figure, that with normal damping torque coefficient, the machine will not resynchronize when operating out of step. But with 20% increase in damping torque coefficient, the machine will just resynchronize after slipping through nearly half a slip cycle, which gives a critical value.

From this it is seen that provision of suitable damper windings help to pull the generator back into step after synchronism has been lost because of a fault.

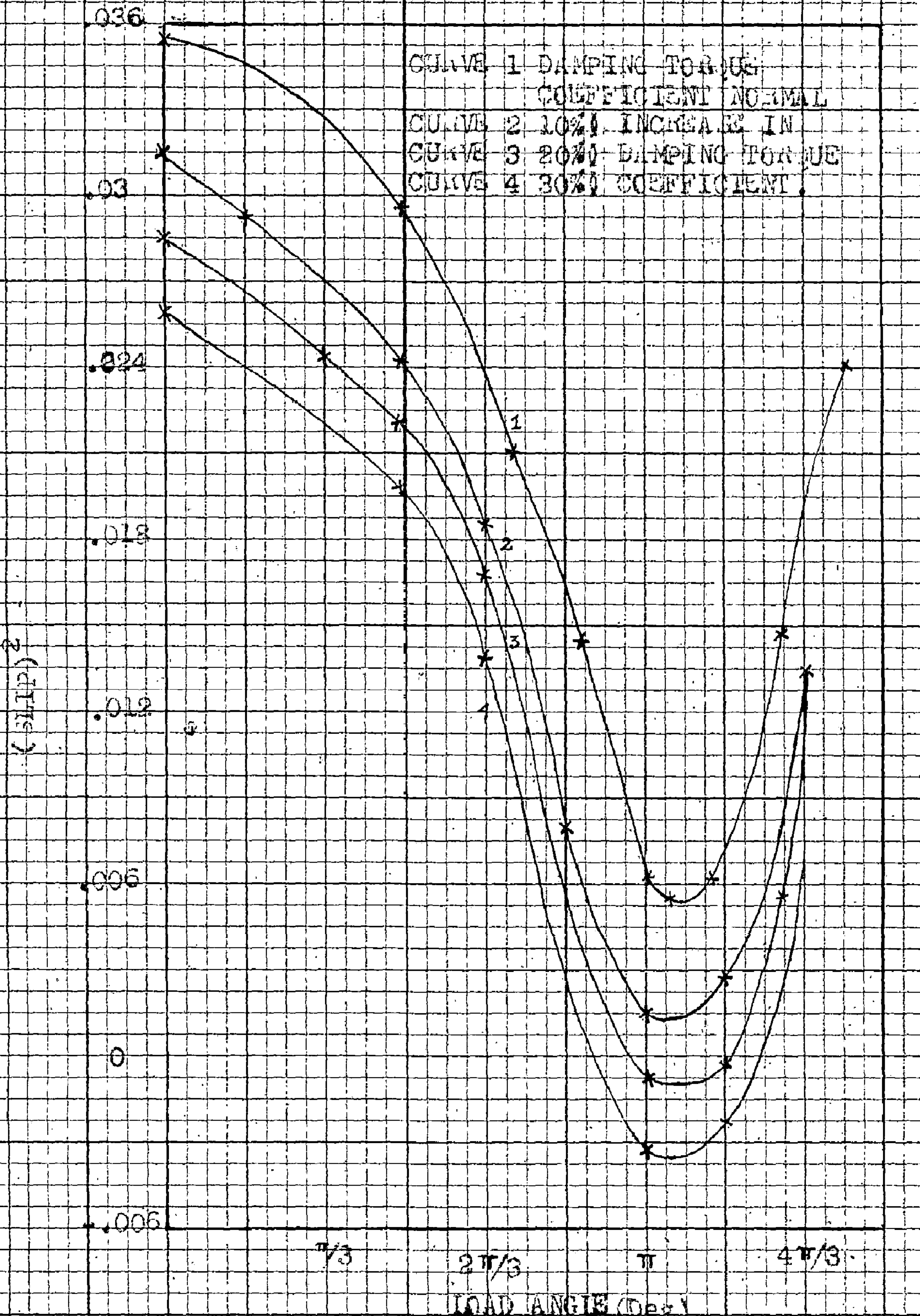



FIG.4.1

PLOT OF VARIATION OF (SLIP)$^2$ WITH LOAD ANGLE TO APPLY THE CRITERION FOR RESYNCHRONIZATION

# ...CHAPTER V...

## .....EXPERIMENTAL TEST RESULTS.....

This chapter deals with the experimental verification of some of the theoritical results discussed so far.

The transient electrical torque and the load angle are the two most important quantities that have to be determined for transient stability studies. The transient electrical torque could not be measured in the laboratory as a torque meter was not available. Fabrication of a torque meter would have taken considerable time and hence was not attempted. Hence the experimental verification of the theoritical results is confined to measurement of load angle only.

Another factor which influenced the test results is the non-availability of equipment for recording purposes. The cathode-ray-oscilloscope (C.R.O.) used was not equipped with a recording camera and hence considerable difficulty was experienced in recording. Photographs taken with a high-precision camera also were not very successful. Hence the experimental work has been limited to visual observation only.

The third limitation imposed is the non-availability of two similar salient-pole machines one with damper windings and the other without damper windings. To compare and verify all the theoritical results, such a set of two similar machines is essential. In the absence of a machine without damper windings, the experimental investigation is limited to tests on a machine with damper windings.

To obtain the load angle of an alternator, an electrical position of the rotor and has to be compared in phase with the generator terminal voltage. There are a variety of ways of obtaining the electrical signal such as using -

a) an A.C. generator fixed to the end of the generator rotor shaft.

b) a brush and an auxiliary commutator system mounted somewhere on the rotor shaft.

c) photo electric systems employing alternate light and dark bands on the rotor.[30]

d) electro-magnetic systems employing a pick up head placed close to the rotor to which is attached a permanent magnet.[31]

and e) a stroboscope[32] and a ciné camera for recording purposes.

For method (a) extra mounting framework is required. For method (b) the rate of wear will be very high. Method (c) is not satisfactory for long-term use and method (e) suffers from the disadvantage that viewing of results is delayed until the film is developed. Hence these methods are unattractive.

The method that has been adopted for experimental investigation is the one suggested in method (d). This method is described in detail below.

...5.1. Load angle meter...

The schematic diagram is shown in Fig. 5.1. A pulse is obtained from a magnetic pick-up M which consists of a permanent magnet with a small air gap and wound with about 4000 turns of 29 SWG. The pick-up is mounted near to a rotating disc D has 6 poles) such that these steel projections pass through the air gap of the permanent magnet. The output from the pick-up is applied to an amplifier A (the connections of which are shown in Fig. 5.2) in order to obtain a large pulse voltage for applying to the terminal Z of the C.R.O. By employing the amplifier, the pulse voltage has been amplified about 16 times.

The terminal voltage of the machine, which provides the reference signal, is applied to the Y plate of the C.R.O. through a phase shifter P. If the oscillograph beam is continuously energized, a sine wave would appear on the screen. ~~as shown in Fig. 5.3~~. But as a result of the modulation of the beam intensity by the pulses obtained from the pick-up, as in fig 5.3, only the spots appear at the instants when the steel projections pass through the air gap of the permanent magnet. This is shown in Fig. 5.4.

Thus, the displacement of the spot is a measure of the load angle. For a given location of the pick-up, in relation to the machine windings and the steel projections, there is one setting of the phase shifter (the neutral position) for which zero displacement corresponds to zero load angle. The displacement is then proportional to sin $\delta$ .

If the phase shifter is moved through an angle $\delta_o$ from the neutral position, the displacement of the oscillograph beam is proportional to sin $(\delta - \delta_o)$. For the measurement of small oscillations relative to a steady operating condition of the synchronous machine, the phase shifter is set so that the oscillograph deflection is zero for the angle $\delta_o$ corresponding to the steady load. During a small oscillation, the oscillograph deflection is very closely proportional to $\Delta\delta = \delta - \delta_o$.

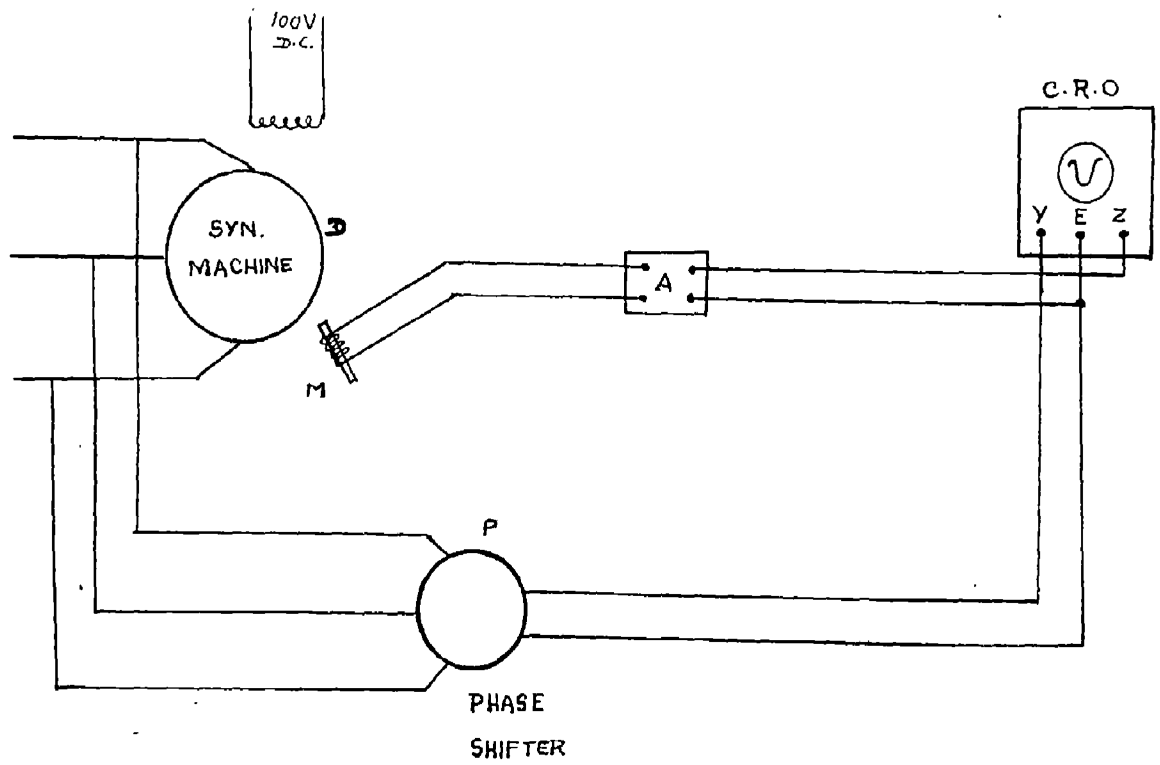



FIG.5.1

DIAGRAM SHOWING THE CONNECTIONS OF A LOAD ANGLE METER.

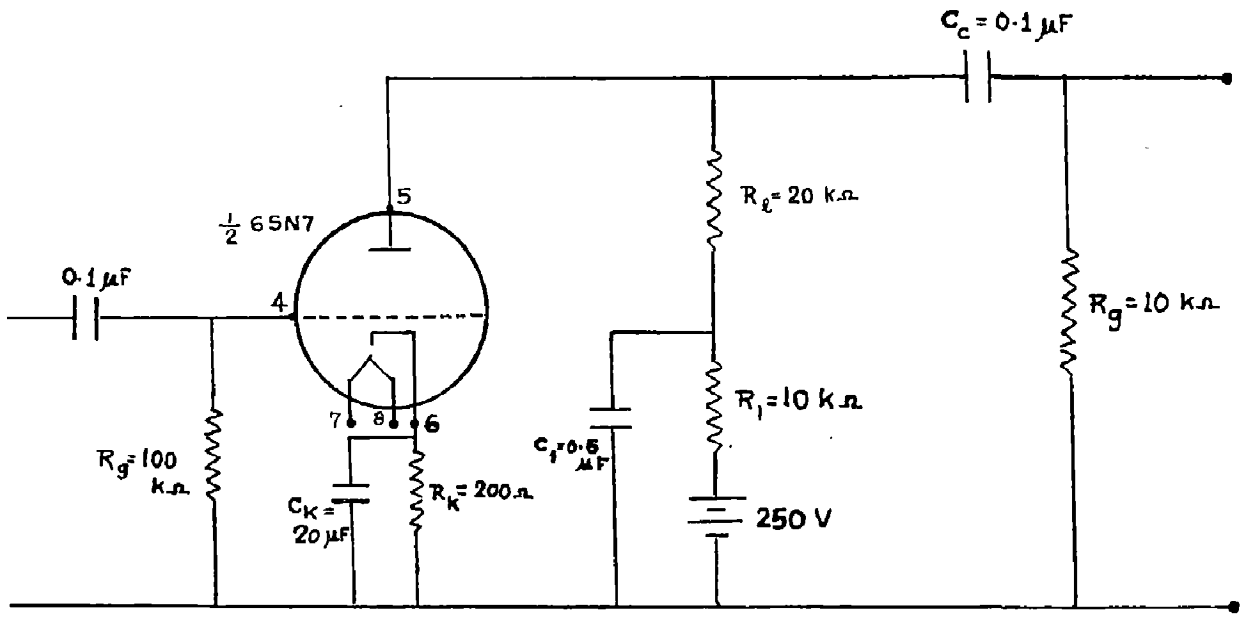



FIG.5.2

DIAGRAM SHOWING THE CONNECTIONS OF AN AMPLIFIER

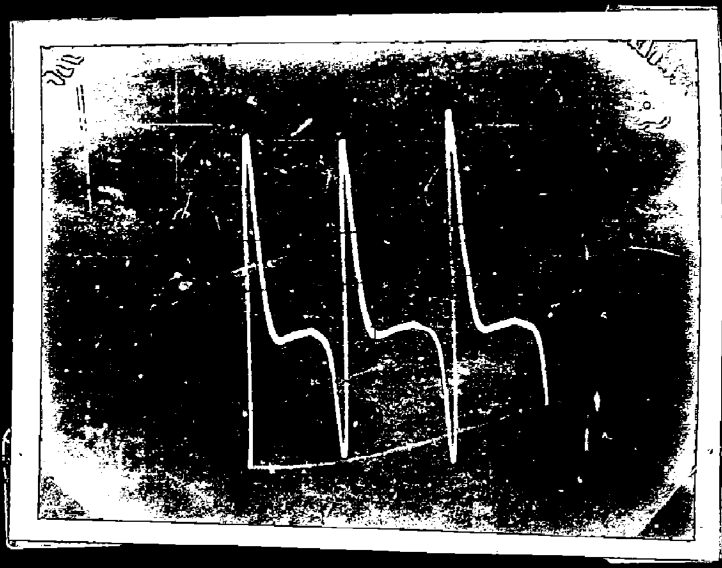


**Fig.5.3** WAVE FORM WHEN THE OSCILLOGRAPH BEAM IS CONTINUO
ENERGIZED. by the pulses.

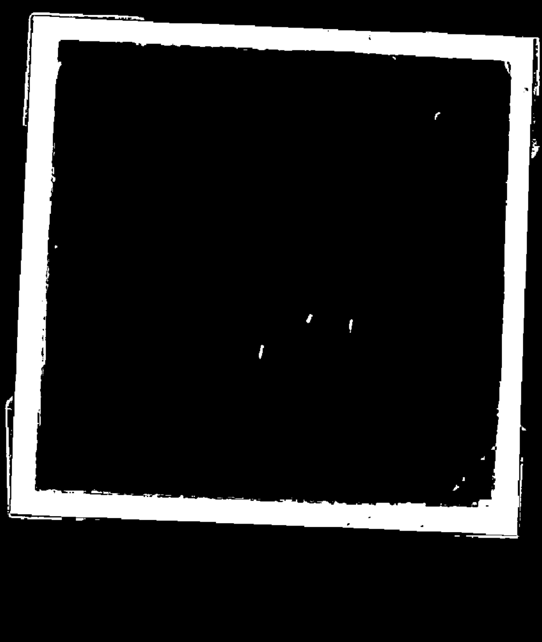


**FIG.5.4** RESULT OF THE MODULATION OF THE BEAM INTENSITY BY THE PULSES OBTAINED FROM THE PICK-UP.

The tests were carried out on a 3 phase 4 KVA 110/220V, 21/10.5 Amps 1000 RPM salient pole synchronous generator equipped with damper windings. A similar machine coupled to this machine mechanically acted as the prime mover for the generator.

The experiment consists of two parts -

i) determination of the constants of the machine and

ii) verification of the theoritical results using the constants determined in part 1.

...5.2. Determination of the constants of the machine...

The constants of the machine in P.U. determined by standard test methods (and not described here in detail) are as follows-

| | |
|---|---|
| $X_d = 0.766$<br>$X_q = X_q' = 0.452$ | determined from slip test. |
| $X_d' = 0.26$<br>$X_d'' = X_q'' = 0.245$<br>$T_d' = 0.49$<br>$T_d'' = T_q'' = 0.02$ | determined from 3 phase short circuit test. |
| $X_2 = 0.245$<br>$r_2 = 0.076$ | determined by applying the negative sequence voltage and measuring the voltage, current and power. |
| $r_1 = 0.0195$ ... | determined by applying a small d.c. voltage and measuring the voltage and current. |
| $H = 0.381$ sec. | determined from retardation test. |

$$T_{qo}'' = \frac{X_q}{X_q''} T_q'' = 0.0367$$

$$T_{do}' = \frac{X_d}{X_d'} T_d' = 1.3$$

$$T_{do}'' = \frac{X_d'}{X_d''} T_d'' = 0.0235$$

Calculated from the above constants using the formuale as shown.

...5.3. Verification of theoritical results using the constants determined in sec. 5.2...

The general arrangement for all the tests conducted is shown in Fig.5.5.

..5.3.1. Transient Behaviour of an alternator during system faults..

.5.3.1.1. 3 Phase short circuit on an unloaded alternator.

This test is conducted mainly to verify the presence of unidirectional torques during a 3 phase short circuit.

For this test the generator was run as a synchronous motor on no load and the prime mover was disconnected from the supply. The schematic diagram of connections is shown in Fig.5.6. Z is a 3 phase protective impedance. S is a switch which is normally open. The 3 phase short circuit was applied at the terminals of the machine by closing the switch S by means of a relay by energizing the relay coil. Under this condition, the speed fall continuously after the short-circuit. The pulses recorded during fixed specified intervals of time from the instant of applying the short circuit is shown in Fig.5.8.

Next, a deceleration test was performed on the machine under this condition, the set slowed down owing to friction only. The pulses recorded during the same intervals of time as before is shown in Fig.5.7.

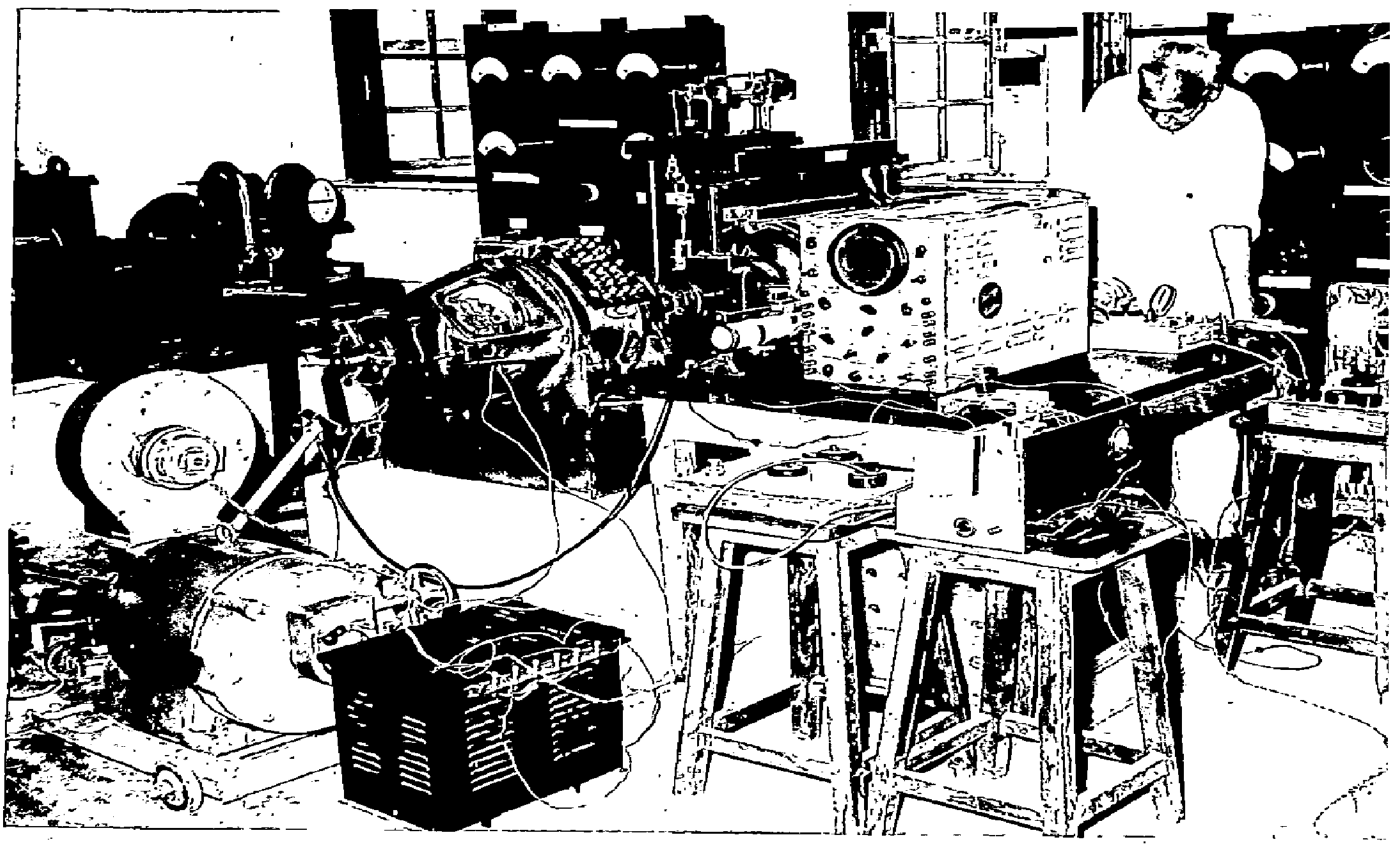


FIG.5.5 SET UP OF THE EXPERIMENT.

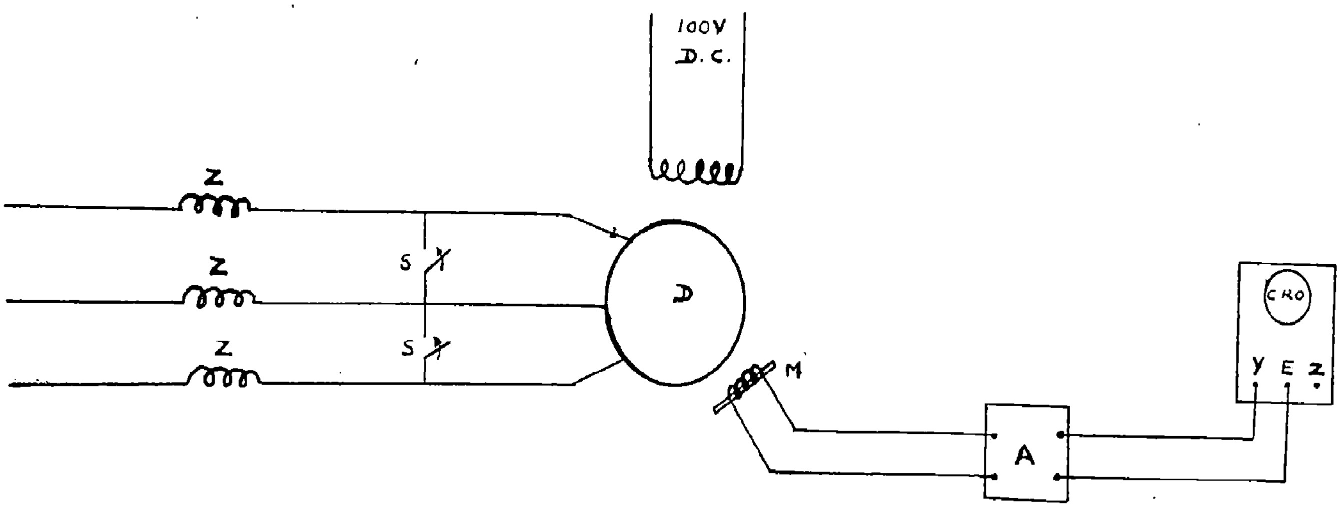



FIG.5.6

DIAGRAM OF CONNECTIONS SHOWING THE ARRANGEMENT FOR CONDUCTING A 3 PHASE SHORT CIRCUIT TEST ON AN UNLOADED ALTERNATOR

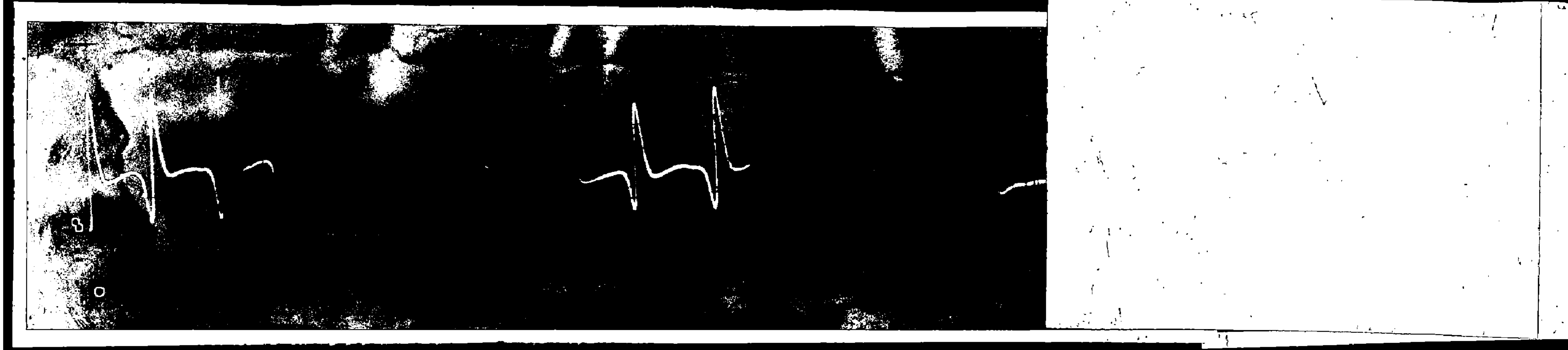


Fig.5.7 PULSES RECORDED DURING THE DECELERATION TEST.

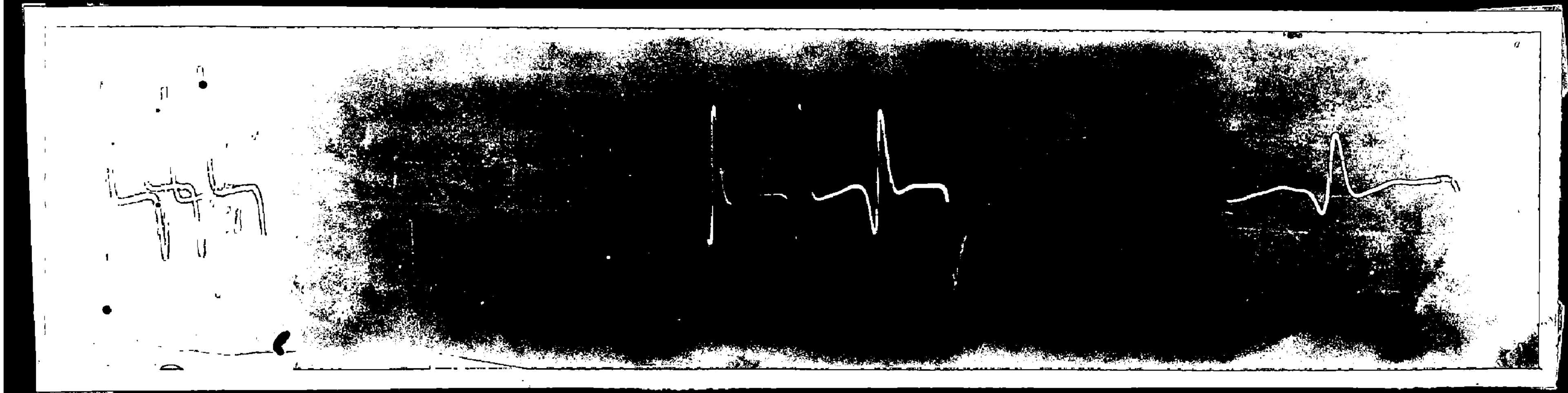


Fig.5.8 PULSES RECORDED FROM THE INSTANT OF SHORT CIRCUITING THE TERMINALS OF AN UNLOADED ALTERNATOR RUNNING AS A SYNCHRONOUS MOTOR.

It can be seen from Figs. 5.7 and 5.8 that when the machine is short circuited, the rotor angle falls more quickly. This is because the machine comes to rest more quickly when the 3 phase short circuit is applied than when it slows down due to frictional resistance only. The greater fall in rotor angle is due to unidirectional components of torque. This confirms the results plotted in Fig.1.7.

### 5.3.1.2 3 Phase short circuit on a loaded alternator

Tests on a loaded alternator could not be conducted owing to lack of suitable protective devices.

### 5.3.2. Transient behaviour of an alternator due to switching operations resulting in sudden increase in reactance

The schematic diagram of connections is shown in Fig.5.9

S is a 3 phase switch normally closed but is opened by means of a relay when the relay coil is energized. $Z_1$, $Z_2$ and $Z_3$ are three single phase reactances each having a reactance of 0.924 P.U.

The alternator was excited to generate balanced 3 phase voltages of 220V and then parallelled with the infinite bus with the help of tthe synchroscope. The phase shifter was set so that the oscillograph deflection is zero for the angle corresponding to the steady load. Under these conditions, the original steady condition P.U. values were -

$$V = 1 \text{ P.U.}$$

$$I = 1 \text{ P.U.}$$

$$\cos \phi = 0.8 \text{ (lag).}$$

$$\delta_0 = 16.5^0$$

The three reactances were suddenly inserted in the

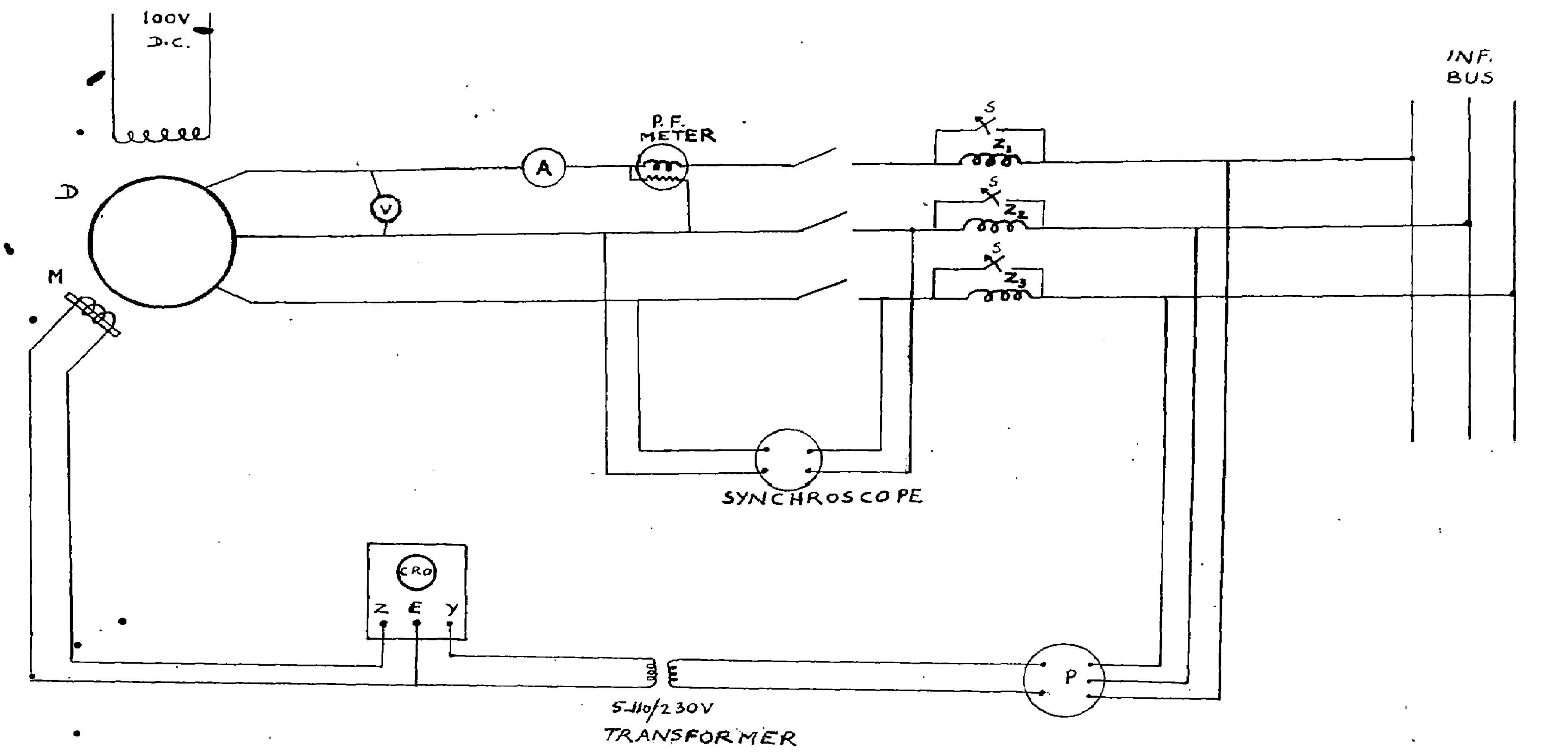



FIG.5.9

DIAGRAM OF CONNECTIONS SHOWING THE ARRANGEMENT FOR SUDDENLY INSERTING A REACTANCE WHEN AN ALTERNATOR IS SUPPLYING POWER TO AN INFINITE BUS.

line by energizing the relay coil and thus opening the 3 phase switch S. Under these conditions, the rotor angle varied and settled down to a new value. The system was stable, which confirms the results plotted in Fig.2.4.

### ...5.3.3. Transient behaviour of a synchronous motor due to sudden application of shaft loads...

The machine under investigation is now the synchronous motor which run as the prime mover so far. The alternator is now used as a load to the machine under test. The connections are as shown in Fig. 5.10.

The machine under investigation was started as an induction motor and pulled into synchronism when its speed was nearly synchronous. The machine then ran as a synchronous motor. The field winding of the alternator, coupled to this machine, was excited and the armature connected to three variable rheostats, which were connected in star to form a balanced load on this alternator. The load on the alternator was steadily increased upto a certain value $T_L$. The load was then taken off by opening the switch S keeping the position of the rheostats unchanged.

When steady conditions were obtained, the phase shifter was set so that the oscillograph deflection is zero for the angle $\delta_0$ corresponding to no load. Under these conditions, the steady condition P.U. values were - $V = 1$ P.U.; $\delta_0 = 3^\circ 38'$; $T_0$ (which is due to windage and friction losses of the two machines and the core loss of one machine) was measured and found to be about 0.19 P.U.

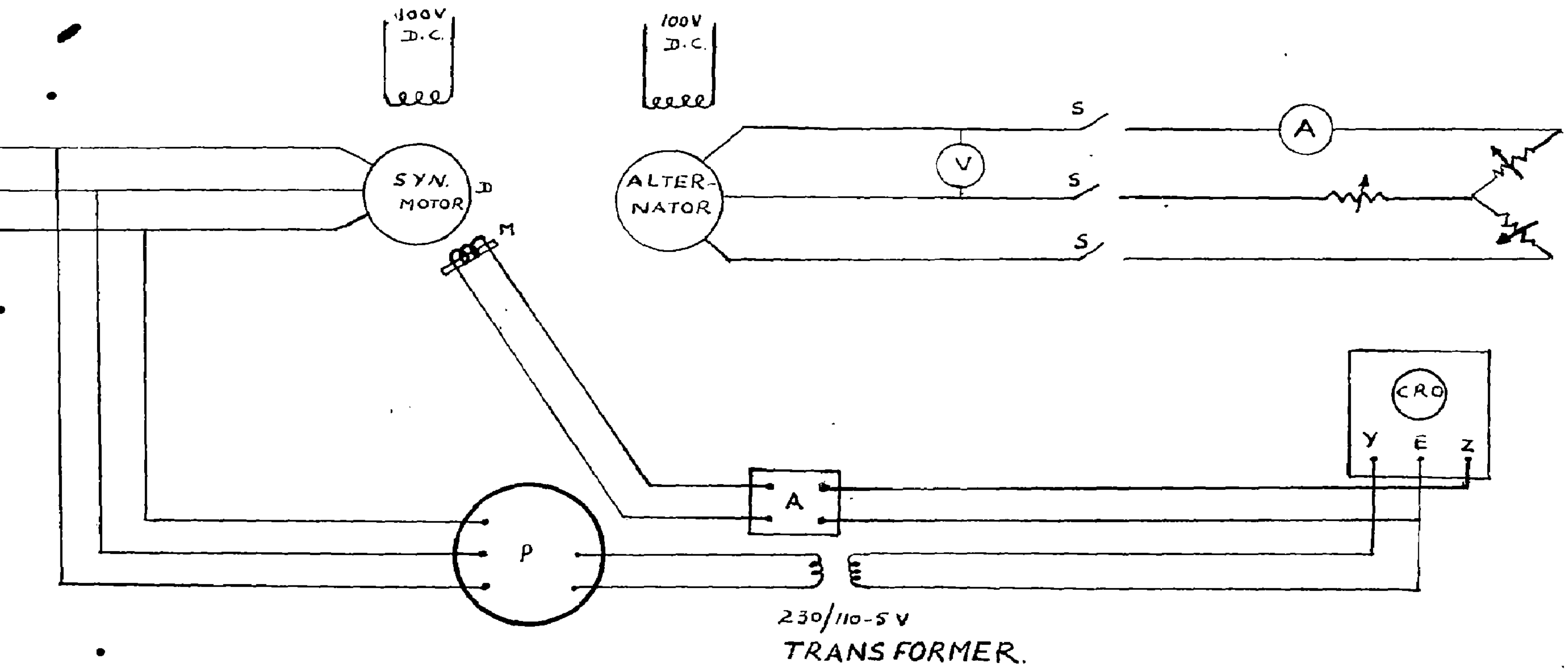



FIG. 5.10

DIAGRAM OF CONNECTIONS SHOWING THE ARRANGEMENT FOR SUDDEN APPLICATION OF SHAFT LOADS.

The switch can be suddenly closed so that the load $T_L$ can be suddenly applied to the shaft. This method of loading is justified as long as the time constants due to the inductance of the machine are very small compared to the mechanical time constant of the rotating system. When a load of $T_L = 1.11$P.U. (about 9.5 amps) was suddenly applied to the shaft, the machine pulled out of step, confirming the results plotted in Figs. 3.6 (for K = 0.357) and 3.7.

...5.3.4. Asynchronous operation...

No tests were conducted in the laboratory to verify the theoritical results.

...CHAPTER VI...

.....C O N C L U S I O N S.....

6.1. The principal effects of damper windings upon system stability under transient conditions due to system faults are-

...a) Positive sequence Damping...

This is more effective after a fault is cleared than during the fault. Positive sequence damping causes the oscillations of the machine rotor to decrease in amplitude after an aperiodic shock (such as 3 phase short circuit) that does not cause loss of synchronism. This is shown in Fig. 1.8.

...b) D.C. braking...

The currents induced in the rotor circuits due to D.C. components of armature current during system faults produce a braking torque, which decreases the accelerating torque and thus slows down a generator which tends to speed up during a fault. The D.C. braking torque is less important for faults other than 3 phase short circuits.

...c) Negative sequence braking torque...

This is due to interaction of the damper currents with the negative sequence magnetic field in the air gap and is present only when an unsymmetrical short circuit occurs. This torque also retards the rotor and decreases the accelerating torque during the fault.

The magnitudes of D.C. braking torque and negative sequence braking torque for various types of faults are given in Table 1.1 and their effect in providing the additional braking torque is shown in Figs. 1.4, 1.5, 1.6 and 1.7.

However, both of these means of improving transient stability are less important where high-speed fault clearing are used.

Damper windings damp out oscillations caused by aperiodic shocks such as switching as shown in Figs. 2.7, 2.8 and 2.9. They also restore stability when the system loses synchronism due to switching operations as shown in Figs. 2.4, 2.5 and 2.6.

When sudden loads are applied to synchronous motor shafts, the damper windings restore stability when the system is likely to fall out of step; increase the maximum load that can be abruptly applied to the shaft and reduce the maximum angle of swing as well as the time required to reach the maximum angle. However much advantage can not be taken of this fact, because values of relative damping coefficient greater than the critical damping coefficient produce no beneficial effects but only slow up the rate of swinging. These conclusions are arrived at from Figs. 3.4 to 3.7, Figs. 3.9 and 3.10.

Following a system disturbance (such as a severe short circuit) which is finally cleared, a synchronous machine may be operating out of step. Suitably designed damper windings help to pull the generator back into step as shown in Fig.4.1.

A full-scale experimental investigation of the conclusions reached above could not be under taken because of lack of facilities and equipment.

However, from the theoritical analysis and from the tests that were conducted, it can be concluded, in general, that the damper windings have a beneficial effect on the transient stability

# APPENDIX III

## The equations of a synchronous machine

The fundamental differential equations relating the instantaneous values of the voltages, currents, flux linkages, torque and speed of a synchronous machine are as follows:

ref 9.

$$\left.\begin{aligned} V_d &= p\psi_d - u\psi_q - r_a i_d \\ V_q &= +u\,\psi_d + p\psi_q - r_a i_q \end{aligned}\right\} \quad \ldots \ldots \text{A.1}$$

$$\left.\begin{aligned} \psi_d &= L_{md}\, i_f + L_{md}\, i_{kd} - (L_{md} + L_a) i_d \\ V_f &= \left[ r_f + (L_{md} + L_f) p \right] i_f + L_{md}\, p\, i_{kd} - L_{md}\, p\, i_d \\ 0 &= L_{md}\, p\, i_f + \left[ r_{kd} + (L_{md} + L_{kd}) p \right] i_{kd} - L_{md}\, p\, i_d \end{aligned}\right\} \ldots \text{A.2}$$

$$\left.\begin{aligned} \psi_q &= L_{mq} i_{kq} - (L_{mq} + L_a)\, i_q \\ 0 &= \left[ r_{kq} + (L_{mq} + L_{kq}) p \right] i_{kq} - L_{mq}\, p\, i_q \end{aligned}\right\} \ldots \text{A.3}$$

$$T_e = \frac{\omega}{2} (\psi_d\, i_q - \psi_q\, i_d) \quad \ldots \quad \ldots \quad \ldots \text{A.4}$$

By eliminating $i_f$ and $i_{kd}$ from equations A.2 and $i_{kq}$ from equation A.3

$$\left.\begin{aligned} \omega\psi_d &= -X_d(p) i_d + G(p) V_f \\ \omega\psi_q &= -X_q(p) i_q \end{aligned}\right\} \quad \ldots \quad \ldots \text{A.5}$$

where $X_d(p)$ and $X_q(p)$ are operational impedances and $G(p)$ is the transfer function.

$$\left.\begin{aligned} X_d(p) &= \frac{(1 + T_d' p)(1 + T_d'' p)}{(1 + T_{do}' p)(1 + T_{do}'' p)} X_d \\ X_q(p) &= \frac{(1 + T_q'' p)}{(1 + T_{qo}'' p)} X_q \end{aligned}\right\} \quad \ldots \text{A.6}$$

## Partial fraction forms

The reciprocals of the operational impedances may be smaller than $T_d'$ and $T_{do}'$, the approximate expressions are:

$$\frac{1}{X_d(p)} = \frac{1}{X_d} + \left(\frac{1}{X_d'} - \frac{1}{X_d}\right)\frac{T_d' p}{1+T_d' p} + \left(\frac{1}{X_d''} - \frac{1}{X_d'}\right)\frac{T_d'' p}{1+T_d'' p} \quad \text{..A.7}$$

$$\frac{1}{X_q(p)} = \frac{1}{X_q} + \left(\frac{1}{X_q''} - \frac{1}{X_q}\right)\frac{T_q'' p}{1+T_q'' p} \quad \text{... .. ..A.8}$$

Alternative form of $X_d(p)$ is:

$$\frac{1}{X_d(p)} = \frac{1}{X_d'} - \left(\frac{1}{X_d'} - \frac{1}{X_d}\right)\frac{1}{1+T_d' p} + \left(\frac{1}{X_d''} - \frac{1}{X_d'}\right)\frac{T_d'' p}{1+T_d'' p} \quad \text{.. ..A.9}$$

**...<u>Simplified equations for a machine connected to a fixed supply</u>...**

For a machine connected to a fixed supply, of which the voltage of phase A is $e_a = V_m \sin\omega t$, equation A.1 may be rearranged so as to include the load angle $\delta$ as a variable instead of the speed u, Since $u = \frac{d}{dt}(\omega t + \delta) = \omega + p\delta$ ...A.10

The rearranged equations are:

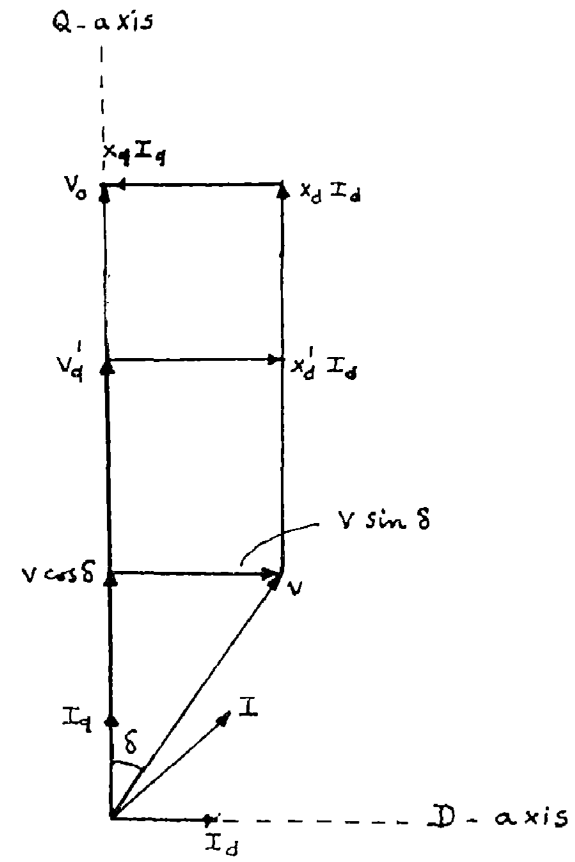
$$\left.\begin{aligned} +V_m \sin\delta &= p\psi_d - \omega\psi_q - r_a i_d - \psi_q\, p\delta \\ V_m \cos\delta &= +\omega\psi_d + p\psi_d - r_a i_q + \psi_d\, p\delta \end{aligned}\right\} \quad \text{...A.11}$$

For the slow transient changes, the terms which depend on the rate of change of $\psi_d$, $\psi_q$, or $\delta$ can be neglected, as well as the resistance drops.

Equations A.11 simplify to:

$$\left.\begin{aligned} -V_m \sin\delta &= \omega\psi_q \\ +V_m \cos\delta &= \omega\psi_d \end{aligned}\right\} \quad \text{... ...A.12}$$

The two voltages in equation A.12 correspond to the components of the terminal voltage in the steady state vector diagram shown below:




[illegible]

Thus a vector diagram can be used to represent the conditions at any instant during a slow transient, but the diagram changes from one instant to another.

...Steady Synchronous operation...

During steady operation, the axis voltages and currents are constant quantities and are related to the components of the vectors in the above vector diagram. Symbol O is used to represent the original steady values existing before the disturbance. Resistance $r_a$ is neglected.

current vector $I_o$ as follows:

$$i_{do} = \sqrt{2}\ I_{do} \ ; \ i_{qo} = + \sqrt{2}\ I_{qo} \quad \ldots \quad A.13$$

The RMS value of the terminal voltage is

$$V = V_m / \sqrt{2} \quad \ldots \quad A.14$$

In the vector diagram

$$\left.\begin{aligned} V \cos \delta_o &= V_o - X_d\, I_{do} \\ V \sin \delta_o &= + X_q\, I_{qo} \end{aligned}\right\} \quad \ldots \quad A.15$$

$V_o$ = voltage behind the synchronous reactance. The diagram also shows $V_{qo}' = V\cos \delta_o + X_d'\, I_{do}$ ... A.16

where $V_{qo}'$ = Voltage behind the transient reactance.

<u>Transformation equations:</u>

$$\left.\begin{aligned} i_a &= i_d \cos \theta - i_q \sin \theta + i_o \\ i_b &= i_d \cos (\theta - 120^o) - i_q \sin (\theta - 120^o) + i_o \\ i_c &= i_d \cos (\theta + 120) - i_q \sin (\theta + 120^o) + i_o \end{aligned}\right\} \quad \ldots A.17$$

Similarly,

$$\left.\begin{aligned} v_a &= v_d \cos \theta - v_q \sin \theta + v_o \\ v_b &= v_d \cos (\theta - 120) - v_q \sin (\theta - 120) + v_o \\ v_c &= v_d \cos (\theta + 120) - v_q \sin (\theta + 120) + v_o \end{aligned}\right\} \quad \ldots A.18$$

<u>Duhamel's Theorem</u>

The following equations, in which f(t) is any function of time are based on Duhamel's theorem:

$$\frac{p}{p+\beta}\, f(t)1 = e^{-\beta t} f(o) + \sum_{n=1}^{n=n} \left\{ f(n\Delta t) - f\left[ (n-1)\Delta t \right] \right\} e^{\beta(t-n\Delta t)} \quad \ldots A.19$$

In equation A.19, t is a small internal of time and n is the no. of intervals. f $(n\Delta t)$ is the value of the function after n intervals.

The integral form of equation A19 is

$$\frac{p}{p+\beta}\, f(t)1 = e^{-\beta t} f(o) + \int_{\tau=0}^{\tau=t} e^{-\beta(t-\tau)} f'(\tau)\,d\tau$$

... A.20

Now, if $1/\beta$ is a short time constant, and f(t) is a slowly--changing function, the integral in equation A.20 may be evaluated approximately by giving $f'(\tau)$ its value at $\tau = t$ and bringing it outside the integral.

$$\frac{p}{p+\beta}\, f(t)1 = e^{-\beta t} f(0) + \frac{1}{\beta}(1-e^{-\beta t}) f'(t)$$

...A.21

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