"PARALLEL OPERATION OF MULTI-WINDING TRANSFORMERS"

Dissertation submitted in partial fulfillment of

the requirements for the Award of the

Degree of Master of Engineering

in Electrical Machine Design

By

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University of Roorkee,

Roorkee.

C_E_R_T_I_F_I_C_A_T_E.

Certified that the dissertation entitled "PARALLEL OPERATION OF ~~THREE~~ MULTI-WINDING TRANSFORMERS" which is being submitted by Shri Om Parkash Malik in partial fulfilment for the award of the Degree of Master of Engineering in Electrical Machine Design of University of Roorkee is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other Degree or Diploma.

This is further to certify that he has worked for a period of three months from 1st June, 1962 to 31st August, 1962 for preparing dissertation for Master of Engineering Degree at the University.

Dated, Roorkee.

10th September, 1962.

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Designation of Supervisor........ R.E.E

Seal.................

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P R E F A C E.

The growth in size and complexity of power systems has resulted in a trend towards multi-winding power-transformers. With this has arisen the problem of operating these transformers in parallel with two-circuit or other multi-circuit transformers. Multi-circuit transformers show peculiar characteristics due to the interlinked magnetic circuit of various windings and this presents special problems in their operation.

So far no attempt seems to have been made to study the behaviour of three-circuit transformers when operating in parallel and to lay down conditions for their satisfactory operation. On the suggestion of Shri D.R. Kohli, M.E., Deptt. of Electrical Engineering, Roorkee University, this topic was therefore, selected for study. In this dissertation, a general study has been made of the performance of three-circuit transformers when operating in parallel and certain conclusions arrived at. The conditions which must be satisfied for their satisfactory operation and the methods that can be adopted, to operate three-circuit transformers in parallel successfully and economically have been indicated.

I wish to express my deep sense of gratitude to Shri D.R. Kohli for his initiating this topic for study, and the active support, ever available guidance and invaluable suggestions offered by him during the course of these studies and in reviewing the manuscript. I am also indebted to the staff of Electrical Engineering Department, Roorkee University for their help from

time to time.

Roorkee,

September, 1962.

O.P. Malik.
L.D.C

C_O_N_T_E_N_T_S.

LIST OF SYMBOLS

Z_a, Z_b, Z_c; Z_ℓ, Z_m, Z_n; Z_1, Z_2, Z_3.	Equivalent star network branch impedances of transformers with terminals a b c, ℓ m n, and 1 2 3 respectively.
Z_{ab}, Z_{bc}, Z_{ca}	Equivalent short-circuit impedances of respective pair of windings.
R_{ab}, R_{bc}, R_{ca}; X_{ab}, X_{bc}, X_{ca}.	Equivalent short-circuit resistances and leakage reactances between respective pair of windings.
R_a, R_b, R_c; X_a, X_b, X_c.	Equivalent star network branch resistances and leakage reactances.
$x_{12}^{(1)}$, $x_{23}^{(1)}$, $x_{31}^{(1)}$	Equivalent short-circuit leakage reactances of respective pair of windings as seen from winding 1.
$x_{23}^{(2)}$	Equivalent short-circuit leakage reactance of windings 2 & 3 as seen from winding 2.
$x_{31}^{(3)}$	Equivalent short-circuit leakage reactance of windings 3 & 1 as seen from winding 3.
N_1, N_2, N_1', N_2', N_3.	Various turns ratios.
I_a, I_b, I_c; I_ℓ, I_m, I_n; I_1, I_2, I_3.	Equivalent circuit branch currents of transformers with terminals a b c, ℓ m n and 1 2 3 respectively.

CHAPTER I.

I_N_T_R_O_D_U_C_T_I_O_N.

Transformers having more than two windings are now being more frequently employed whenever transformation of power is desired between three or more circuits. The practice of using multiwinding transformers with windings of different voltage ratings is growing on account of the resulting economy, not only in first cost, but also in operation due to smaller losses. By using one transformer bank rather than two in the case of a three-winding transformer, double transformation of power is avoided and the total amount of transformer KVA for a given service is correspondingly reduced.

Multi-winding transformers when operated alone or in parallel with other transformers, present some peculiar but very interesting problems. These relate to leakage impedance phenomena, such as (a) voltage regulation, (b) efficiency, (c) load division between circuits, (d) parallel operation with other transformers, and (e) short-circuit characteristics. On a multiwinding transformer, the open-circuit no-load voltage of a winding will change with variation of loading on another separate winding or windings, although it may be unloaded itself. Load currents in one circuit affect voltages in another and a lagging current in one winding can, under certain circumstances, cause a voltage rise in one or more of the other windings depending upon the arrangement of windings and their impedances.

Due to the peculiar behaviour of multiwinding transformers, the various problems connected with their operation can not be solved directly by two-winding transformer formulae. So far, in the entire technical literature in English only casual references have been made to the operation in parallel of two or more of such transformers and at no time any attention has been paid to a thorou study of this aspect. As a practical proposition, it is only the three-winding transformer that is of any importance and the study of transformers with more than three windings-though used at times-is more of academic interest. It is therefore proposed to include in the scope of these discussions only three-winding transformers and consider in detail the division of load between the various windings of these transformers connected in parallel on both primary and secondary sides. A comparative study of the merits and demerits of various methods that are possible to be adopted for satisfactory load sharing will also be made.

The three windings of a three-winding transformer are each interlinked with the magnetic leakage fields of the other windings, and as such load currents in different circuits affect each other's voltages in complicated and un-expected manner. It is thus very desirable to have a clear conception of the leakage impedance relationships of a multi-circuit transformer. The equivalent circuit for a three-circuit transformer has been developed in Appendix A and its knowledge is essential before actually tackling the problems involved in parallel operation of three-winding transformers.

CHAPTER II

PARALLEL OPERATION - GENERAL

2.1 It is well-known from the study of parallel operation of two-winding transformers, that their satisfactory performance in parallel requires the following conditions to be fulfilled:

(a) the same voltage ratio;

(b) the same percentage impedance both in magnitude and quality;

(c) the same polarity;

(d) the same phase sequence; and

(e) the same phase angle difference between the primary and secondary terminals.

If all the above conditions are satisfied for the two-winding transformers to be operated in parallel, the load will be divided among the units in proportion to their KVA ratings and the secondary currents in each transformer will be in phase with the load current.

2.2. Parallel operation of three-winding transformers.

2.2.1. General relations:

The conditions required to be satisfied for satisfactory parallel operation of two-winding transformers are also essential to be satisfied for a proper division of load between paralleled three-winding transformers. However, due to the peculiar impedance characteristics of these transformers, as described in Appendix A, their case requires a special study.

Three-circuit transformers can be represented by an equivalent circuit as developed in Appendix A. Equivalent circuit for two three-circuit transformers in parallel will be as shown in fig.2.1 and for 3 three-circuit transformers in parallel as shown in fig. 2.2. By solving such an equivalent circuit, current flowing in the individual windings of paralleled three-winding transformer banks can be determined. The terminal loads, as well as winding ratios and impedances affect the division of currents among the windings of a three-winding transformers, so all these factors must be known before a solution is attempted.

(a) Two three-circuit transformers in parallel

Expressing all the impedances as per unit or percentage values based on an assumed common KVA load, and assuming one to one turns ratio, fig. 2.1 will be simplified as shown in fig. 2.3 and distribution of load in such a network will be as given below:

Let i_5 and i_6 be the given two loads in KVA, then the total input will be their vector sum:

$$i_4 = i_5 + i_6 \qquad \ldots\ 2.1$$

The distribution of i_4, i_5 and i_6 in the network obtained by solving the network of fig. 2.3 (see appendix B for solution) will be

$$i_1 = \frac{i_4.Z_a\,(1 + k_1) + i_5.Z_b + k_1.i_6.Z_c}{(Z_1 + Za)\,(1 + k_1) + (Z_2 + Z_b)} \qquad \ldots\ 2.2$$

$$\text{where } k_1 = \frac{Z_2 + Z_b}{Z_3 + Z_c} \qquad \ldots\ 2.2(b)$$

$$i_a = i_4 - i_1 \qquad \ldots\ 2.3$$

$$i_2 = \frac{i_4.Z_a + i_5.Z_b\,(1 + k_2) - k_2.i_6.Z_c}{(Z_1 + Z_a)(1 + k_1) + (Z_2 + Z_b)} \qquad \ldots\ 2.4$$

$$\text{where } k_2 = \frac{Z_1 + Z_a}{Z_3 + Z_c} \qquad \ldots\ 2.4(b)$$

$$i_3 = i^1 - i_2 \qquad \ldots\ 2.5$$

$i_c = i_6 - i_3$ ... 2.6

$i_b = i_5 - i_2$ ... 2.7

While substituting KVA values for the i_4, i_5 & i_6 above, KW values are to be substituted first to determine the distribution of active power, and then the calculations are to be repeated for the reactive KVA values. The total load in any branch is of-course

Total load in a branch $= \sqrt{(\text{KW of branch})^2 + (\text{Reactive KVA of branch})^2}$ ..2.8

(b) Three three-circuit transformers in parallel.

Solution of equivalent circuit for more than two three-circuit transformers in parallel becomes too involved to be solved by ordinary methods. As derived in Appendix C, currents in various branches of three three-circuit transformers in parallel (see fig.2.2.) can be calculated by the solution of the four equations 2.9 to 2.12 given below:

$$i_a Z_a + i_b.Z_b - i_l.Z_l - i_m.Z_m = 0 \quad ..2.9$$

$$i_a Z_1 + i_b.Z_2 + i_l(Z_1 + Z_l) + i_m(Z_m + Z_2) = i_4 Z_1 + i_5.Z_2 \quad ..2.10$$

$$i_a(Z_a + Z_c) - i_b.Z_c - i_l(Z_l + Z_n) + i_m.Z_n = 0 \quad ..2.11$$

$$i_a(Z_1 + Z_3) - i_b.Z_3 + i_l(Z_1 + Z_l + Z_3 + Z_n) - i_m(Z_3 + Z_n) = i_4 Z_1 + i_6.Z_3 \quad ..2.12$$

Equations 2.9 to 2.12 can be solved by the use of determinants. Expression for branch current i_a will be

$$i_a = \frac{\begin{vmatrix} 0 & Z_b & -Z_\ell & -Z_m \\ (i_4Z_1 + i_5Z_2) & Z_2 & (Z_1 + Z_\ell) & (Z_2 + Z_m) \\ 0 & -Z_c & -(Z_\ell + Z_n) & Z_n \\ (i_4Z_1 + i_6Z_3) & -Z_3 & (Z_1 + Z_\ell + Z_3 + Z_n) & -(Z_3 + Z_n) \end{vmatrix}}{\begin{vmatrix} Z_a & +Z_b & -Z_\ell & -Z_m \\ Z_1 & Z_2 & (Z_1 + Z_\ell) & (Z_2 + Z_m) \\ (Z_a + Z_c) & -Z_c & -(Z_\ell + Z_n) & Z_n \\ (Z_1 + Z_3) & -Z_3 & (Z_1 + Z_\ell + Z_3 + Z_n) & -(Z_3 + Z_n) \end{vmatrix}} \quad .. \; 2.13$$

As would be observed, it is a very cumbersome equation and its solution will be very laborious. In the case of more than three such transformers in parallel, solution by ordinary mathematics will be even more difficult. The best method to solve all problems involving more than two three-circuit transformers in parallel would be to use an A.C. calculating Board, a Network Analyzer or a digital computer.

2.2.2. Equal ratios of transformation - Effect of impedances on load sharing.

It is evident from 2.2.1 that the distribution of load between various windings and transformers is controlled by the impedance of different branches of the equivalent circuit:

$$i_1 = \frac{i_4.Z_a\,(1 + k_1) + i_5.Z_b + k_1.i_6.Z_c}{(Z_1 + Z_a)(1 + k_1) + (Z_2 + Z_b)} \quad .. \; 2.2$$

$$= \frac{i_4Z_a\{(Z_2+Z_b)+(Z_3+Z_c)\} + i_5Z_b(Z_3 + Z_c) + i_6.Z_c(Z_2+Z_b)}{(Z_1+Z_a)(Z_2+Z_b) + (Z_2 + Z_b)(Z_3 + Z_c) + (Z_3+Z_c)(Z_1+Z_a)} \; ..$$

also $i_a = i_4 - i_1$ .. 2.3

$$= \frac{i_4Z_1(1+k_1)+i_5Z_2+k_1.i_6.Z_3}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)} \qquad .. \ 2.14$$

(see appendix B)

$$= \frac{i_4Z_1\{(Z_2+Z_b)+(Z_3+Z_c)\}+i_5Z_2(Z_3+Z_c)+i_6Z_3(Z_2+Z_b)}{(Z_1+Z_a)(Z_2+Z_b)+(Z_2+Z_b)(Z_3+Z_c)+(Z_3+Z_c)(Z_1+Z_a)}$$

Dividing equation 2.2 by equation 2.14

$$\frac{i_1}{i_a} = \frac{i_4Z_a\{(Z_2+Z_b)+(Z_3+Z_c)\}+i_5Z_b(Z_3+Z_c)+i_6Z_c(Z_2+Z_b)}{i_4Z_1\{(Z_2+Z_b)+(Z_3+Z_c)\}+i_5Z_2(Z_3+Z_c)+i_6Z_3(Z_2+Z_b)}$$

Now supposing $Z_1 = k.Z_a$

$Z_2 = k.Z_b$

$Z_3 = k.Z_c$

$$\text{then } \frac{i_1}{i_a} = \frac{i_4Z_a(1+k)(Z_b+Z_c)+i_5Z_b(1+k).Z_c+i_6Z_c(1+k).Z_b}{k.i_4Z_a(1+k)(Z_b+Z_c)+k.i_5Z_b(1+k).Z_c+k.i_6Z_c(1+k).Z_b}$$

$$= \frac{(1+k)\{i_4.Z_a(Z_b+Z_c)+i_5.Z_b.Z_c+i_6.Z_c.Z_b\}}{k(1+k)\{i_4Z_a(Z_b+Z_c)+i_5.Z_b.Z_c+i_6.Z_c.Z_b\}}$$

$$= \frac{1}{k} \qquad .. \ 2.15$$

This shows that three-winding transformers of the same voltage ratio will parallel with each other and divide their loads properly under all conditions of loading only if the impedances under those conditions are the same for all transformers to be paralleled. If the impedance of even one branch is different, the sharing of load will not be proper. The effect, of varrying different impedances, on the distribution of loads between paralleled transformers will be discussed in detail in Chapter III.

2.2.3. <u>Effect of equivalent reactance to equivalent resistance ratio on load sharing</u>:

For best results, transformers in parallel should all have the same ratio of equivalent reactance to equivalent resistance. Fig. 2.4 shows the effect of this ratio being not equal for the two transformers. This figure has been drawn for the particular case of the two transformers in parallel having the same per unit impedance. Therefore, the currents in the two transformers will be equal in magnitude, but will not be in phase unless the ratios of equivalent reactance to resistance are the same for both the transformers.

Since the currents are not in phase, the current in each transformer is greater than half of the total current and therefore the kilovolt-ampere output of the pair is less than the sum of the kilovolt-ampere outputs of the individual transformers. However, the effect of a comparatively considerable difference in the ratios of equivalent reactance-to-resistance of two paralleled transformers is so small in practice that the currents in the transformers are very nearly in phase and their vector sum substantially equals their numerical sum. This requirement is thus of only a minor importance and particularly so in all modern power transformers, whose ratio of equivalent reactance-to-resistance is very nearly the same. In these transformers the resistance is so small compared to the reactance that reactance of the transformer can be taken as equal to the impedance without any appreciable error in the results. In the formulae given in para 2.2.1 it will therefore, be quite in order to use percentage reactance instead of percentage impedance to calculate the division of load.

2.2.4 Unequal ratios of transformation:

As in the case of two-winding transformers, a circulating current will be superposed on the no-load current, equal to

$$\frac{\text{difference of induced voltage}}{\text{sum of leakage reactances}}$$

The division of this circulating current among different branches depends upon the impedances of the various branches in the equivalent circuit and the branch in which the voltage unbalance exists. To a very limited extent this fact can be utilised to alter the flow of KVA in different branches of paralleled three-circuit transformers, although no universal use of the same can be made, as shown in Chapter IV subsequently.

2.3. Parallel operation of three-circuit transformer with a two-circuit transformer:

2.3.1. Load sharing:

The equivalent circuit of a three-winding transformer paralleled with a two-winding transformer is given in fig.2.5. Division of currents may be calculated for this circuit, if the load currents 2 & 3 are known. In this figure the parallel connection is between primaries A & 1 and between secondaries C & 2 of the three-circuit and two-circuit transformers respectively.

Considering the load 3 on the bus C2, it is evident that this will be furnished over two paths in parallel (shown by solid lines), i.e. path with the impedance $(Z_a + Z_c)$ in parallel with path having impedance Z_{12}. The division of load supplied to 2 depends upon the impedance of the path Z_a in parallel with the path $(Z_{12} + Z_c)$. In each case, the currents divide in the parallel paths inversely as the impedances of the paths.

Therefore, $(KVA)_2$ is supplied in such a way that the propotion of the current flowing through Z_{12} and Z_a is

$$(12)_s' \text{ share of } (KVA)_2 = \frac{Z_a}{Z_a + Z_c + Z_{12}} (KVA)_2 \qquad .. \quad 2.16$$

$$(Z_a)_s' \text{ share of } (KVA)_2 = \frac{Z_c + Z_{12}}{Z_a + Z_c + Z_{12}} . (KVA)_2 \qquad .. \quad 2.17$$

Similarly load 3 divides so that the portion supplied through Z_c & Z_a in series is

$$(Z_a)_s' \text{ share of } (KVA)_3 = \frac{Z_{12}}{Z_a + Z_c + Z_{12}} . (KVA)_3 \qquad .. \quad 2.18$$

and that supplied through 12 is

$$(12)_s' \text{ share of } (KVA)_3 = \frac{Z_a + Z_c}{Z_a + Z_c + Z_{12}} . (KVA)_3 \qquad .. \quad 2.19$$

In branches a & 12, the two component loads add vectorially as shown by the relative directions of the firm and dotted arrows. In branch C, link Z_c, they subtract vectorially. As a consequence of this, we have the seemingly absurd condition that assuming the loads 2 & 3 to have the same kind of power factor (i.e. both lagging er both leading), the removal of one of the loads increases the load in C. The reason for this is, of-course, the fact that C acts like a secondary for load 3, but as a primary for load. 2.

2.3.2 <u>Special operating conditions:</u>

In the light of the above, it can be seen that it may be impossible to operate two and three winding transformers in parallel with a satisfactory division of current under all conditions. The tertiary load may have any value independent of the secondary load (fed by both transformers), and, as it must flow through the impedance of the primary, it will produce a voltage drop there, and any change in tertiary load will alter the distribution of load between the other two windings. If the impedances are proportioned to divide the load properly for one load condition, the load division between transformers at some other loading is likely to be unsatisfactory. An exception is the case wherein the circuit A represents a delta tertiary winding in a 3-phase bank, with no load connected to the tertiary; in this instance the transformers can be made to divide currents similarly at all loads.

It is possible to design a three-winding transformer so that the load taken from the tertiary winding does not seriously affect the load division between the paralleled windings of the two transformers. If the impedance Z_a is made equal to zero, then current division at the C2 bus will be determined by Z_c and Z_{12} only and this impedance ratio will remain independent of tertiary loading. It is difficult to have zero as the value for Z_a particularly if this winding is of high voltage, however, values near zero can be obtained with specific designs at increased cost. Such a design may result in a value of Z_b which is undesirable from other reasons.

Further, in such a case, if the direction of power flow reverses making the normal secondary act as a primary, the load distribution is greatly affected. Even with an individual analysis of each operating case, it is sometimes impossible to design for parallel operation with ideal load division when conflicting characteristics are required.

An interesting feature to observe in this case is that for a load in the circuits of A & C acting like two primaries in parallel;

The division of load between them is completely determined by the impedances of the network as indicated above, independent of prime-mover and generator control apparatus. If desired, it would be possible of-course, to control this by suitable regulators between C & 2 or between A & bus A1 or between 1 and bus A1. Also sometimes balance coils are used to force the proper division of load between groups, just like the case of two-winding transformers as shown in fig.2.6.

2.4. Expressions for the reactances in terms of the geometric arrangements of the windings for simple concentric windings:

In section 2.3.2. above it has been mentioned that when the individual impedance of the primary is zero, the tertiary load will have no effect on the load division. The impedance of various coils is a function of the relative location of the various windings on the core, for example, in a 3-phase concentric-coil core-type transformers, there is no alternative but to submit to a high reactance between one pair of windings in as much as all the windings for one phase is on one leg. Thus to design a three-winding transformer to meet certain specified values of reactances between each pair of windings would impose conditiohs which are extremely difficult and sometimes practically impossible to fulfill.

However, to illustrate how it is sometimes possible to obtain zero impedance for a particular winding, computation of reactance on the basis of the physical location of the windings on the core is given below:

Let the windings be arranged on the core in the order 1, 2, 3 (fig.2.7)

The reactances in ohms seen from sides 1, 2 & 3 are respectively

$$x_{12}^{(1)} = a.b_{12}.n_1^2 \qquad x_{23}^{(2)} = a.b_{23}.n_2^2 \qquad x_{31}^{(3)} = a.b_{31}.n_3^2 \quad ..2.20$$

$$\left(a = 8f \times 10^{-8}, \qquad b = \frac{l_m}{h}\left(g + \frac{e_1 + e_2}{3}\right)\right)$$

and $$\frac{x_{31}^{(3)}}{n_3^2} = \frac{x_{31}^{(1)}}{n_1^2} \quad ..2.21$$

so that $$x_{31}^{(1)} = \frac{n_1^2}{n_3^2} \cdot x_{31}^{(3)} = a.b_{31}.n_1^2 \quad ..2.22$$

The reactance between 2 & 3 seen from the winding is defined by

$$\frac{x_{23}^{(2)}}{n_2^2} = \frac{x_{23}^{(1)}}{n_1^2}$$

so that $$x_{23}^{(1)} = \frac{n_1^2}{n_2^2} \cdot x_{23}^{(2)} = a.b_{13}.n_1^2 \quad ..2.23$$

Hence $$x_{12}^{(1)} = a.b_{12}.n_1^2; \quad x_{23}^{(1)} = a.b_{23}.n_1^2; \quad x_{31}^{(1)} = a.b_{31}.n_1^2 \quad ..2.24$$

The reactance in ohms of the three-winding transformer seen from side 1 consequently depend only on n_1^2.

From the equations A.4 to A.6

$$x_2^{(1)} = \tfrac{1}{2}\left(x_{12}^{(1)} + x_{23}^{(1)} - x_{31}^{(1)}\right)$$

and since b_{31} is already almost equal to $b_{12} + b_{23}$, we can write

$$x_{31}^{(1)} = x_{12}^{(1)} + x_{23}^{(1)} \quad ..2.25$$

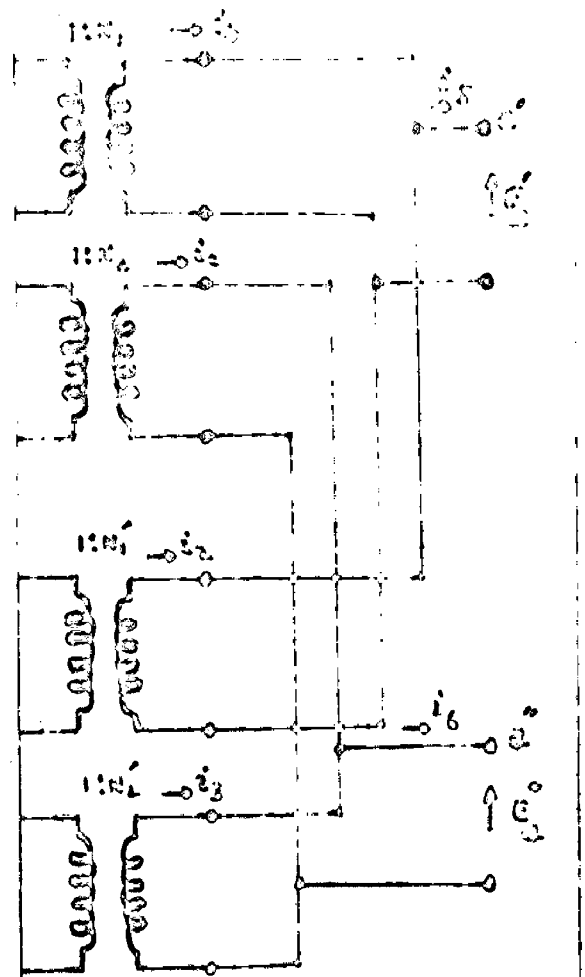
that is $x_2^{(1)} = 0$ (Zero reactance is in the centre branch)

then $x_{12}^{(1)} = x_1^{(1)}$ ..2.26

$x_{32}^{(1)} = x_3^{(1)}$

and we obtain the figure 2.7(b).

Thus by a proper selection of the arrangement of windings, it is possible to control within certain limits the impedance of various branches of the equivalent circuit. It may however, be mentioned that the relations of eqn. 2.26 hold in transformers having the three windings placed successively on the core without interleaving. With interleaved windings, however, whether on the shell-type or core-type design, the straight line reactance circuit no longer holds true and it is generally possible to obtain nearly any desired relationship between the values of x_1, x_2 and x_3.



TRANSFORMERS IN PARALLEL.

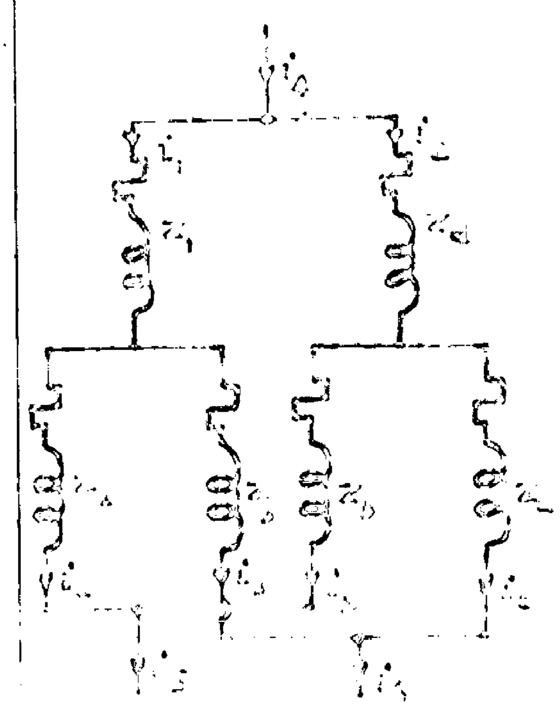
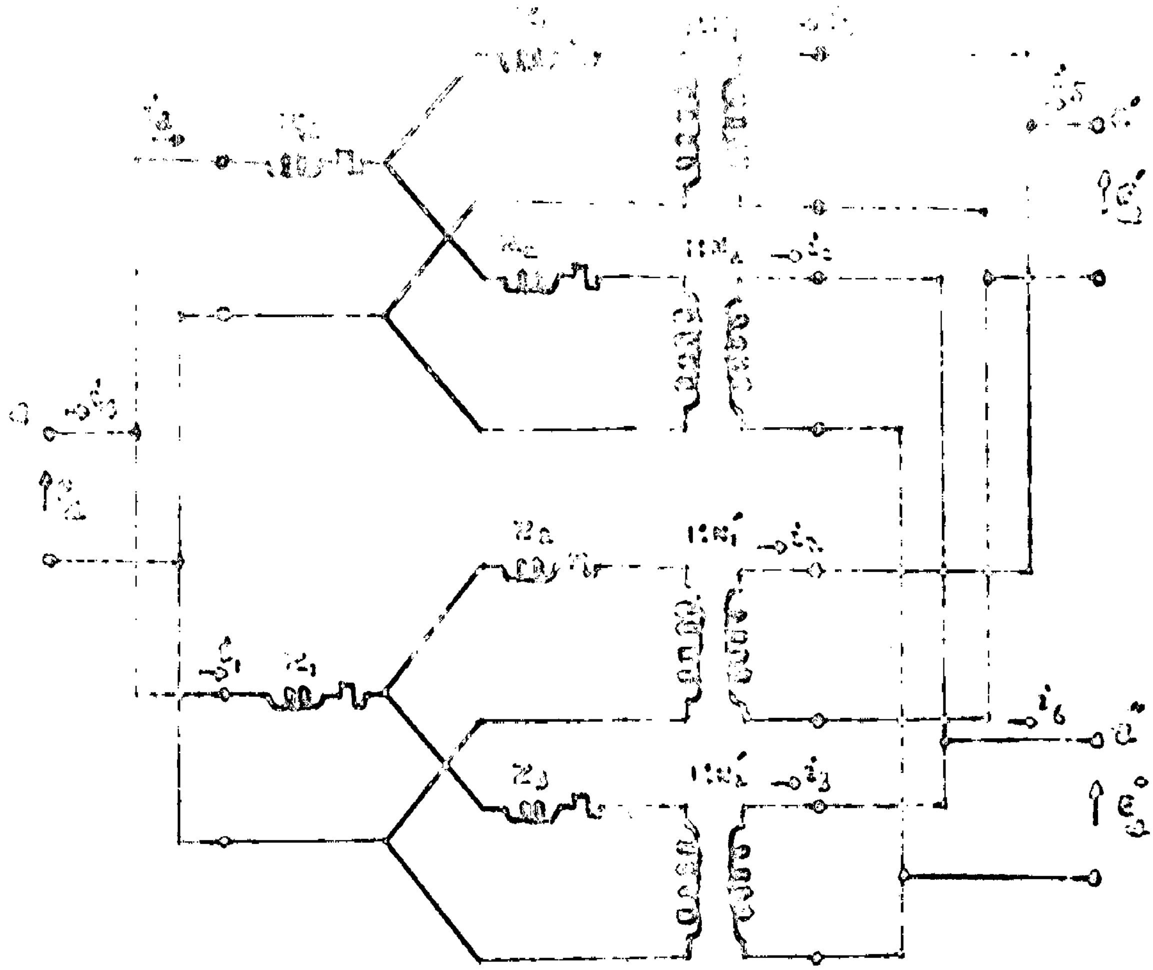
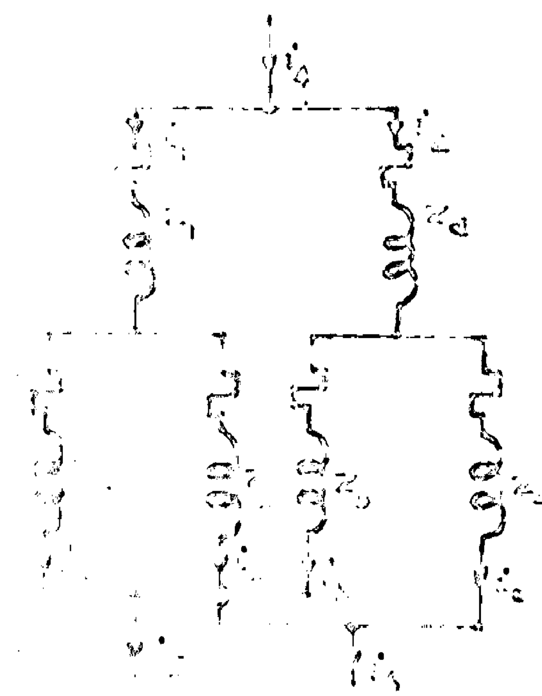


Fig. EQUIVALENT CIRCUIT FOR TWO THREE-CIRCUIT TRANSFORMERS IN PARALLEL (SIMPLIFIED).

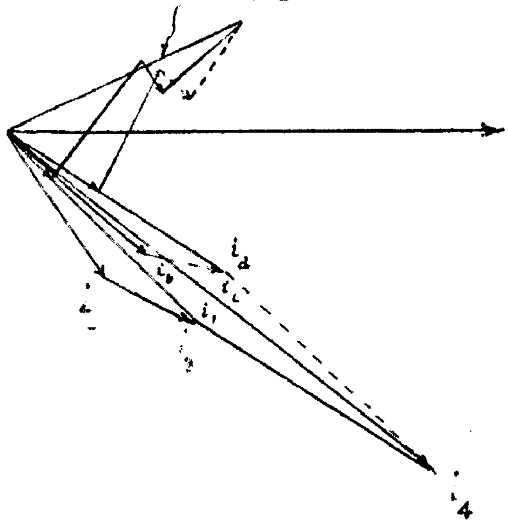


EQUIVALENT CIRCUIT OF TWO THREE-WINDING TRANSFORMERS IN PARALLEL.

EQUIVALENT CIRCUIT FOR THREE THREE-CIRCUIT TRANSFORMERS IN PARALLEL.



EQUIVALENT CIRCUIT FOR TWO THREE-CIRCUIT TRANSFORMERS IN PARALLEL (SIMPLIFIED).



PHASOR DIAGRAM FOR TWO THREE-WINDING TRANSFORMERS IN PARALLEL, SHOWING THE EFFECT OF UNEQUAL EQUIVALENT REACTANCE TO RESISTANCE RATIOS. (THE VOLTAGE DROPS ARE GREATLY EXAGGERATED.)

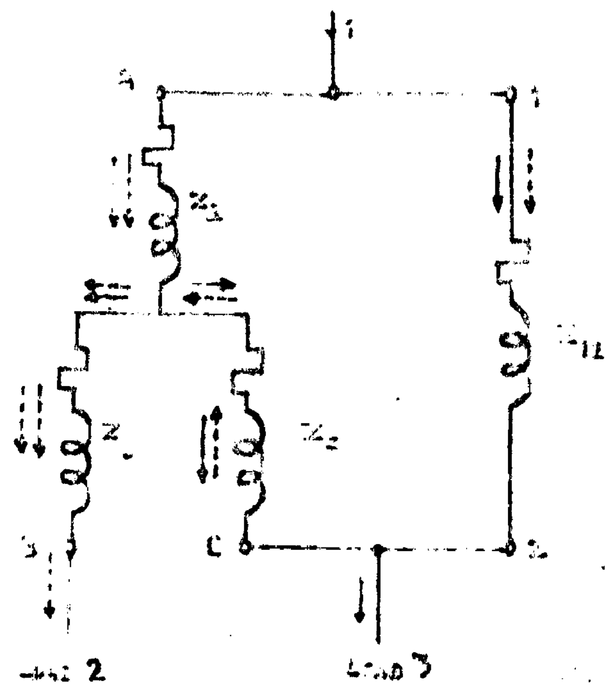


FIG. 2.5 NETWORK OF A THREE-CIRCUIT TRANSFORMER IN PARALLEL WITH A TWO-CIRCUIT TRANSFORMER.

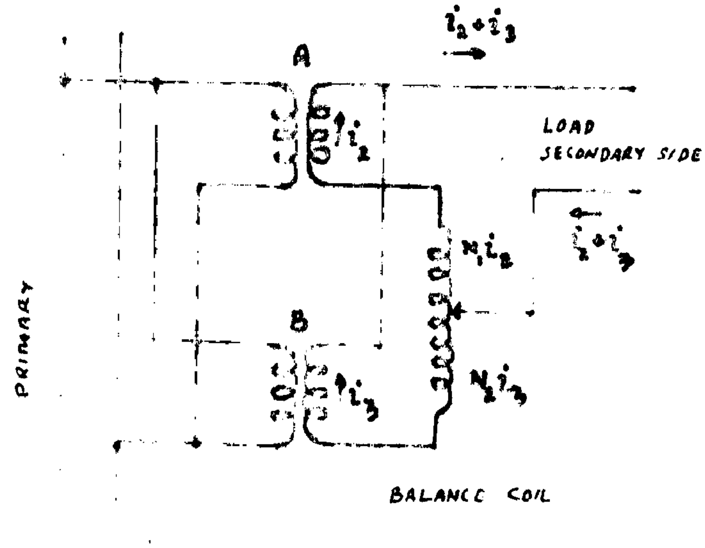


FIG. 2.6 CIRCUIT SHOWING THE USE OF BALANCE COILS TO OBTAIN PROPER LOAD DIVISION BETWEEN TWO TRANSFORMERS OPERATING IN PARALLEL.

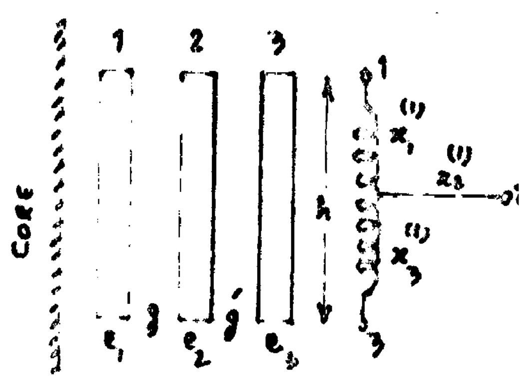


FIG. 2.7 ARRANGEMENT OF WINDINGS ON THE CORE.

CHAPTER III

PARALLEL OPERATION-EFFECT OF EXTERNAL IMPEDANCES.

3,1 When transformers operating in parallel are to share the load in proportion to their KVA ratings, their equivalent-impedance voltage drops at full-load must be equal. To arrange for satisfactory sharing of loads between paralleled two-winding transformers, having unequal equivalent-impedance voltage drops, one of the most commonly adopted methods is the addition of impedance in series with the transformer having the smaller value of per-unit impedance.

A similar arrangement can be adopted in the case of three-circuit transformers. However, it is possible that all the three branch impedances of one transformer may be different from the corresponding three branch impedances of the other three-winding transformer to be connected in parallel, each branch varying by a different amount. In that case, the effect of connecting impedances in series with one or more branches will have to be considered to find out the most economical method for the satisfactory sharing of load.

Equations 2.1 to 2.7 show the division of load between various branches of two paralleled three-circuit transformers having the same voltage ratio. A perusal of these equations shows that they are dependent on 8 variables and are thus too involved to provide a general solution as to the magnitude of impedances required for satisfactory parallel operation. Each concrete case can be easily solved with the help of equivalent circuit given in fig.2.3. However, to have a general idea as to the various requirements, two concrete examples were taken and their detailed solution attempted for various loads and power-factors. Of the two cases selected for study, one was to study the parallel operation of two three-circuit transformers of the same capacity and slightly different percentage impedances, and the other case was of two three-circuit transformers of different capacities and widely varying percentage impedances. The transformers selected for study in these two cases had the following specifications:

Case 1

	Transformer I.	Transformer II
Capacity KVA	10,000/6,000/4,000	10,000/6,000/4,000
Voltage Volts	63,500/13,200/6,600	63,500/13,200/6,600
%IX on 10,000 KVA base		
63,500/13,200 Volts windings	13	11
13,200/6,600 Volts windings	10	9
63,500/6,600 Volts windings	11	10

Load i_5 = load on 13,200 Volts winding = 12,000 KVA

i_6 = load on 6,600 Volts winding = 8,000 KVA

Case II:

Capacity KVA	10,000/7,500/5,000	5,000/3,750/2,500
Voltage Volts	63,500/13,200/6,600	63,500/13,200/6,600
%IX on 10,000 KVA base		
63,500/13,200 Volt windings	13	16
13,200/6,600 Volt windings	10	12
63,500/6,600 Volt windings	11	10

Load i_5 = load on 13,200 Volt winding = 8,000 KVA

i_6 = load on 6,600 Volt winding = 6,000 KVA loads and power-factors. Of the two cases selected for study, one was to study the parallel operation of two three-circuit transformers of the same capacity and slightly different percentage impedances, and the other case was of two three-circuit transformers of different capacities and widely varying percentage impedances. The transformers selected for study in these two cases had the following specifications:

Case 1

	Transformer I.	Transformer II
Capacity KVA	10,000/6,000/4,000	10,000/6,000/4,000
Voltage Volts	63,500/13,200/6,600	63,500/13,200/6,600
%IX on 10,000 KVA base		
63,500/13,200 Volts windings	13	11
13,200/6,600 Volts windings	10	9
63,500/6,600 Volts windings	11	10

Load i_5 = load on 13,200 Volts winding = 12,000 KVA

i_6 = load on 6,600 Volts winding = 8,000 KVA

Case II:

Capacity KVA	10,000/7,500/5,000	5,000/3,750/2,500
Voltage Volts	63,500/13,200/6,600	63,500/13,200/6,600
%IX on 10,000 KVA base		
63,500/13,200 Volt windings	13	16
13,200/6,600 Volt windings	10	12
63,500/6,600 Volt windings	11	10

Load i_5 = load on 13,200 Volt winding = 8,000 KVA

i_6 = load on 6,600 Volt winding = 6,000 KVA

3.2 Studies carried out:

To find out the effect on load sharing of putting external impedances in series, extensive computations were carried out on the two sets of three-circuit transformers mentioned in para 3.1. These computations were made for a number of different loads and power-factors which may be grouped as under:

i) Load on one set of secondary windings was taken as zero, one quarter, half, three-quarter and full-load capacity while that on the other set of secondary windings was kept at its full-load capacity. Load division between different branches for each of these loading conditions was calculated in detail.

ii) Power-factor of the load on one set of secondary windings was changed from zero lag to zero lead and load division under full-load conditions calculated for power-factors zero lag, 0.5 lag, 0.8 lag, unity, 0.5 lead and zero lead. Load on the other secondary windings was taken at full-load capacity and a constant power-factor.

Calculations were carried out for all these conditions of loading with different magnitudes of impedance put in series with each of the three branches of the heavily loaded transformer, in turn-one branch at a time-, and also for two impedances simultaneously i.e. one in series with the primary and the other in series with one of the two secondaries. Load division among various branches for all these sets of computations was calculated.

3.3 Inferences:

The various sets of calculations showed a very similar general trend for all the cases. Graphs 3.1 to 3.10 have been drawn as samples for different conditions of loading to show the general behaviour. These graphs fairly represent the trend of all the computations and the conclusions drawn from the same may be summarised as below:-

i) The impedance shall be put in series with the transformer having the lower percentage impedance.

ii) The same magnitude of series impedance is most effective when put in the primary circuit as compared to its effect when put in either of the two secondary circuits. This is because the kilovolt-amperes flowing to both the secondaries are passing through the primary and thus any impedance in the primary circuit affects both the secondary circuits.

iii) Impedance in series with either of the two secondary circuits has a very small effect on the division of load between the primaries.

iv) Series impedance has a pronounced effect in reducing the load in the secondary circuit in which it is connected, but its effect on the second secondary circuit of the same transforme[r] is just the opposite.

Supposing the three windings of a three-circuit transform[er] are denoted as primary, secondary and tertiary. Then if the impedance is put in series with secondary to decrease the loa[d] in this branch, the load in tertiary will increase and vice versa. This is the reason why load in primary changes insignificantly as mentioned in (iii) above.

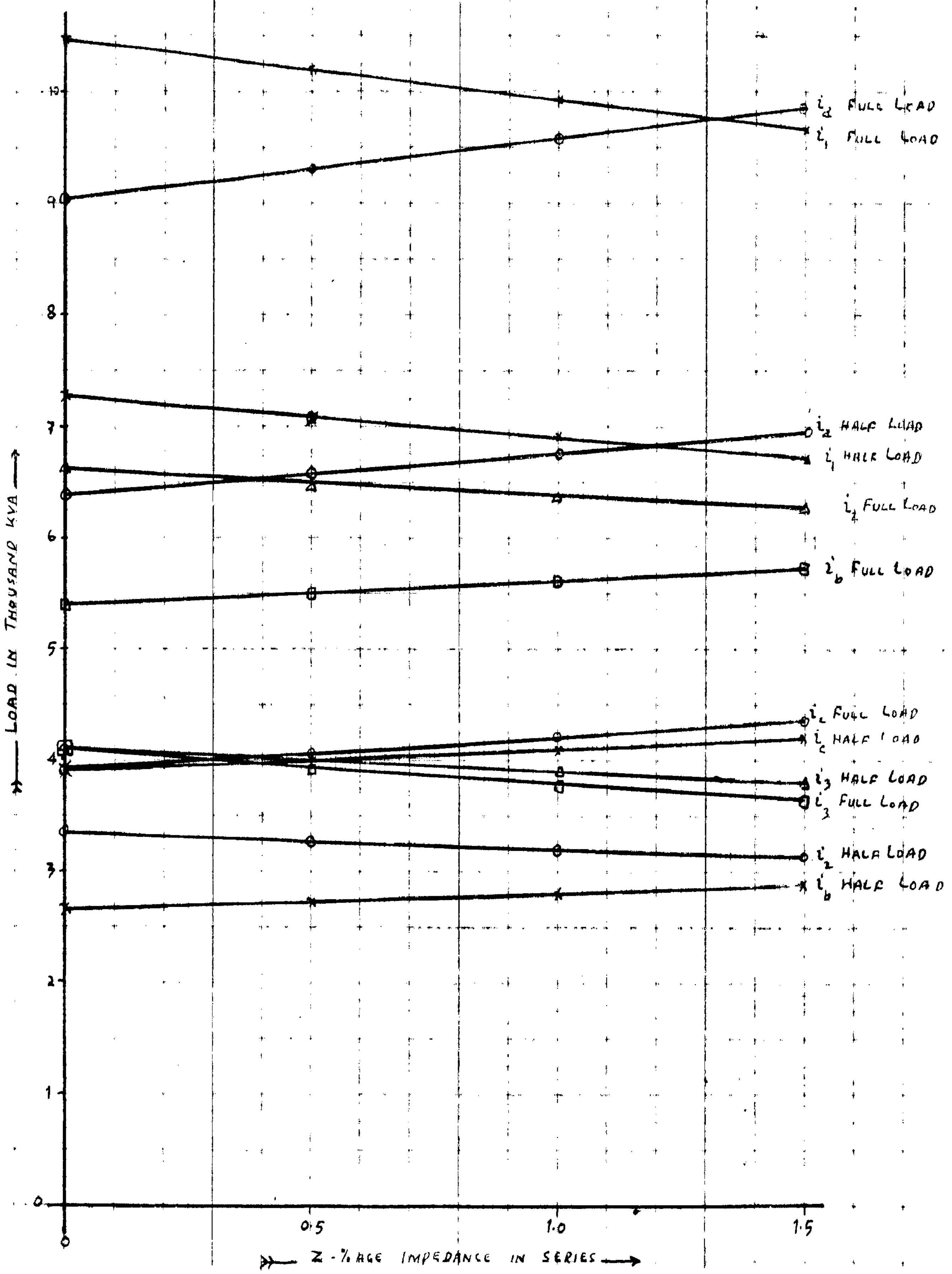
(iv) For a rough balance of loads, a suitable impedance only in the primary circuit is enough. For a large number of cases and over a wide range of load, only one impedance will be required for a reasonably good load sharing.

However, to have a perfect balance in all the branches, suitable impedances in series with the primary and one of the two secondaries must be provided. In general, two impedances will be required only when the transformers are operating near their full-load capacity.

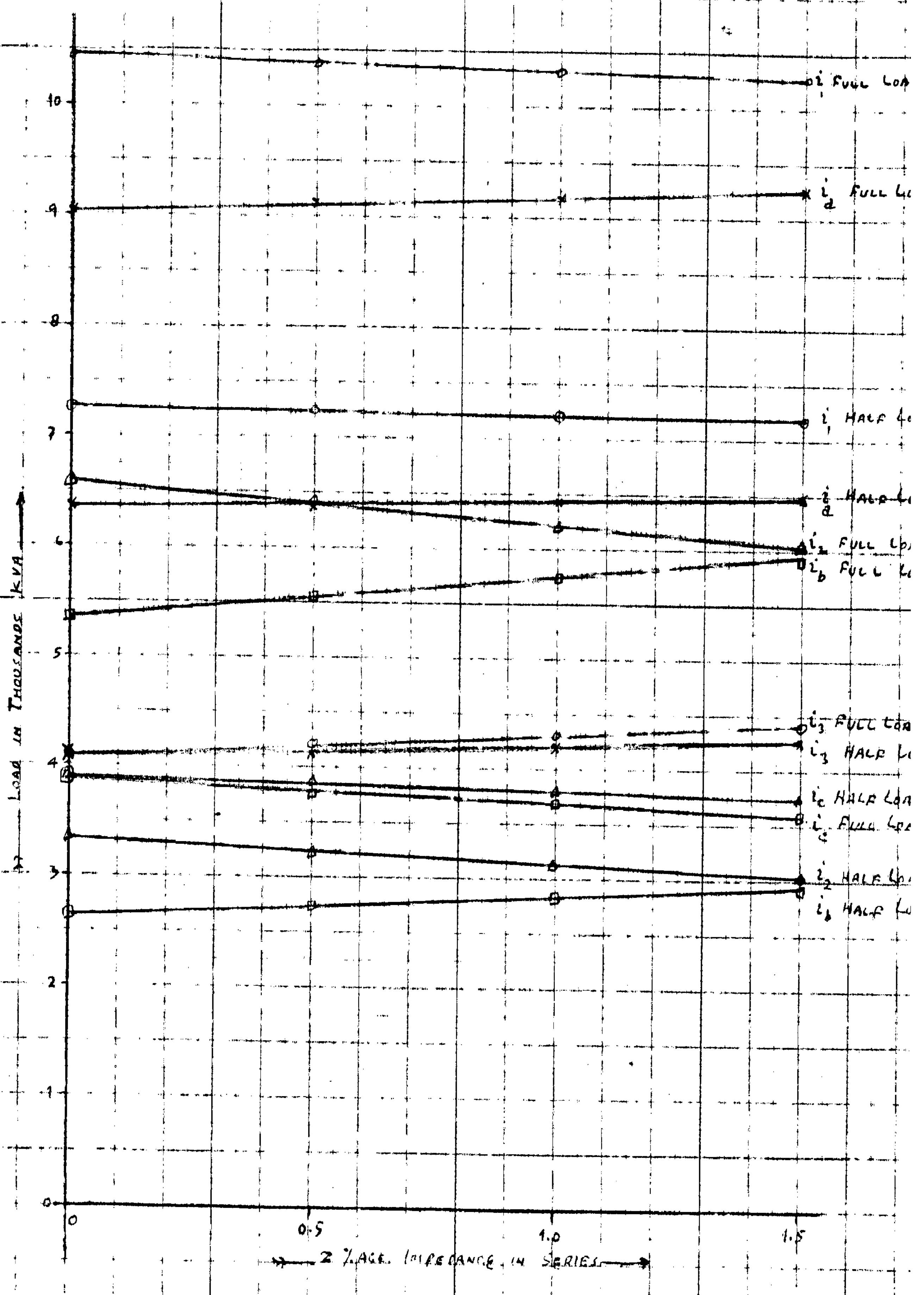
(vi) The magnitude of impedance required is proportional to the variation of the actual impedances from those required under ideal conditions.

(vii) An analysis of the short-circuit impedances and the equivalen circuit branch impedances shows that it is not the values of short-circuit impedances that are of greater importance in deciding the load sharing between paralleled three-circuit transformers, but the values of equivalent circuit branch impedances that are of greater importance.

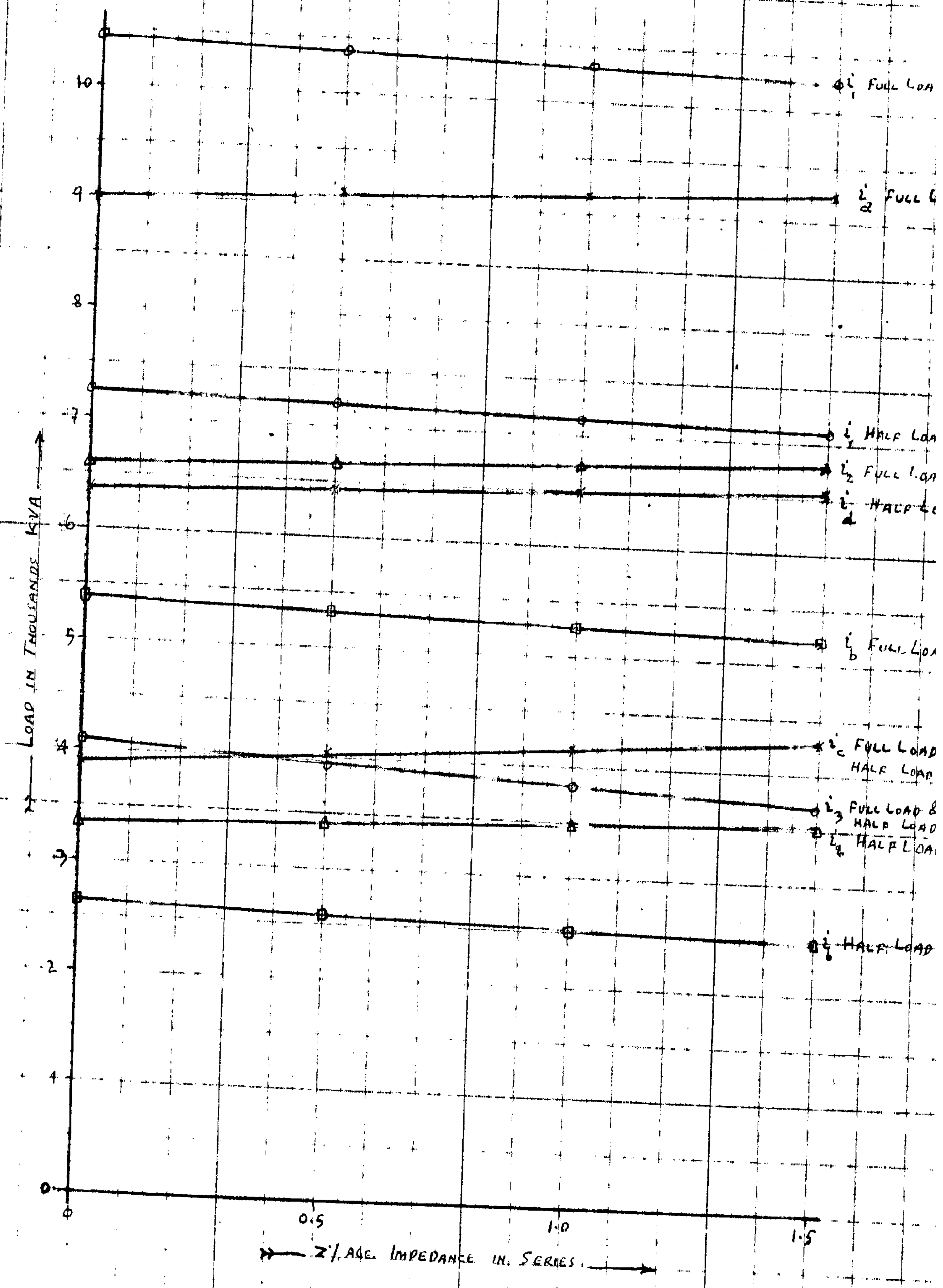
GRAPH 3·1. SHOWING THE EFFECT OF AN EXTERNAL IMPEDANCE Z CONNECTED IN SERIES WITH PRIMARY ON LOAD SHARING AT DIFFERENT LOADS FOR CASE I.
LOADS: i_5 = 12,000 KVA AT UNITY P.F.
i_6 = 8,000 KVA AT 0.9 P.F. LAG.



GRAPH 3.2 SAME AS GRAPH 3.1, BUT WITH IMPEDANCE Z IN SERIES WITH SECONDARY LEG 2 INSTEAD OF WITH PRIMARY.



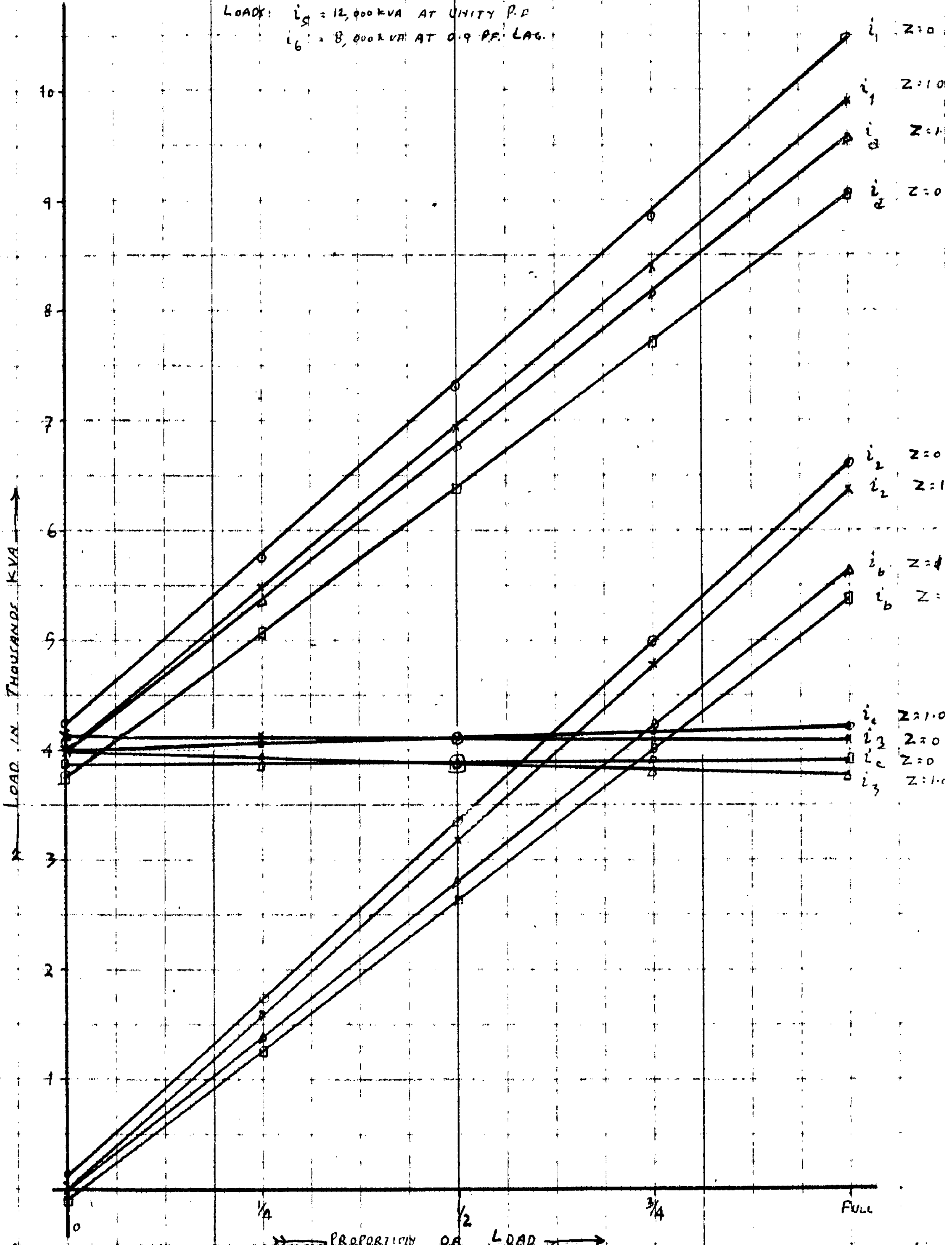
GRAPH 3.2 SAME AS IN GRAPH 3.1 BUT WITH IMPEDANCE Z IN SERIES WITH SECONDARY LEG 3 INSTEAD OF WITH PRIMARY.



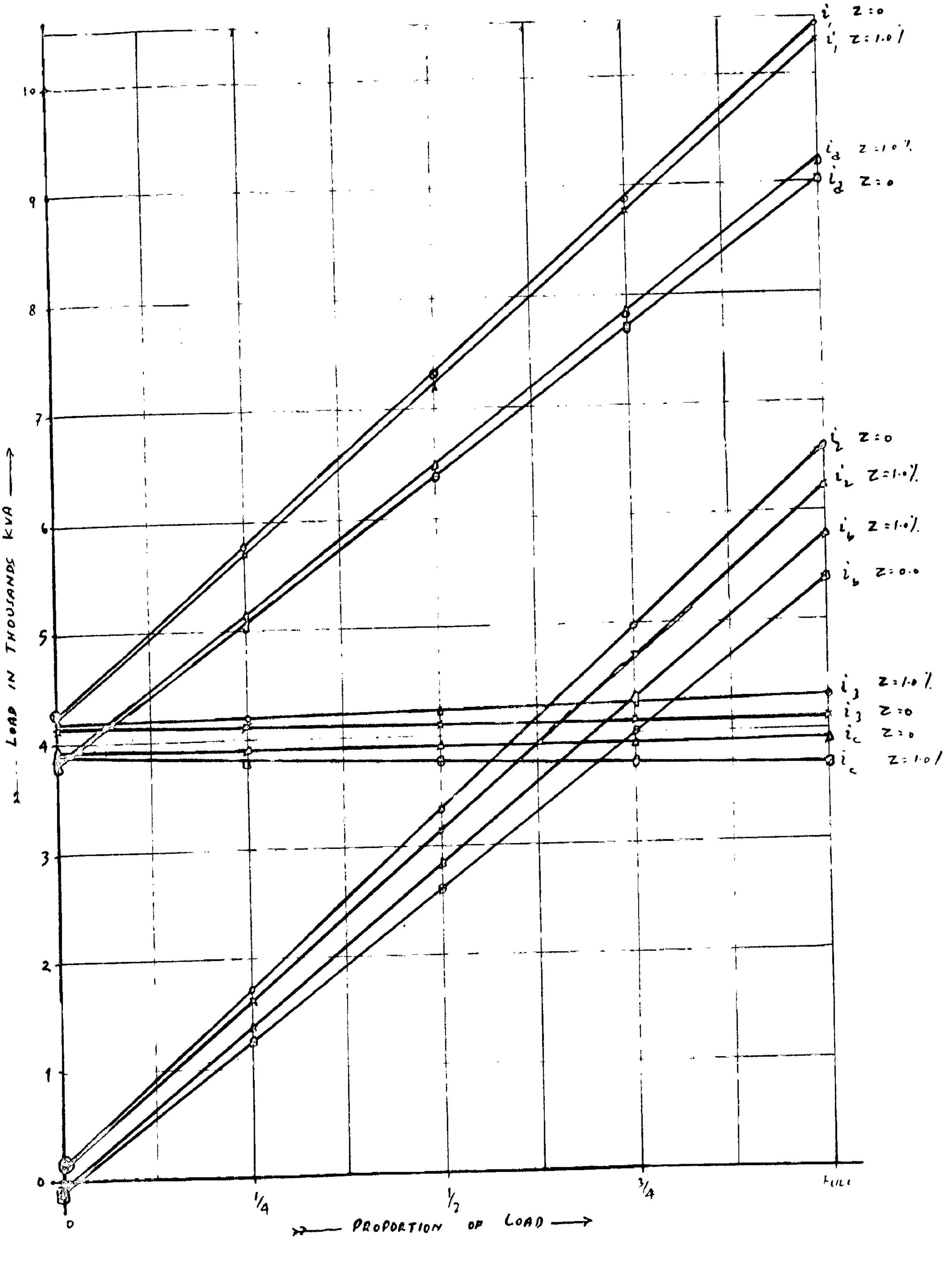
GRAPH 3.4 SHOWING THE LOAD SHARING AT DIFFERENT LOADS DUE TO VARIOUS EXTERNAL IMPEDANCES, Z CONNECTED IN SERIES WITH PRIMARY FOR CASE I.

LOADS: i_a = 12,000 KVA AT UNITY P.F

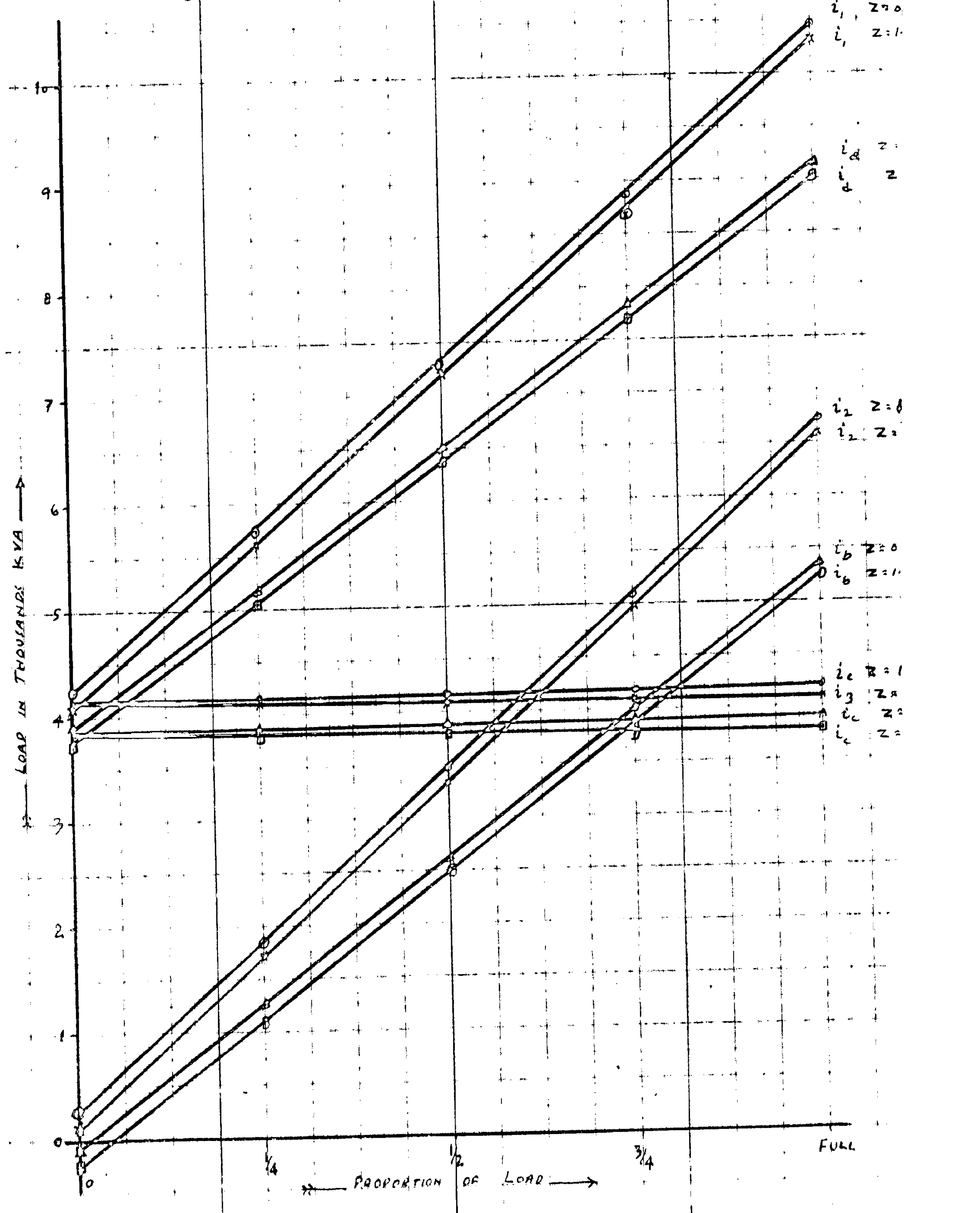
i_b = 8,000 KVA AT 0.9 P.F. LAG.



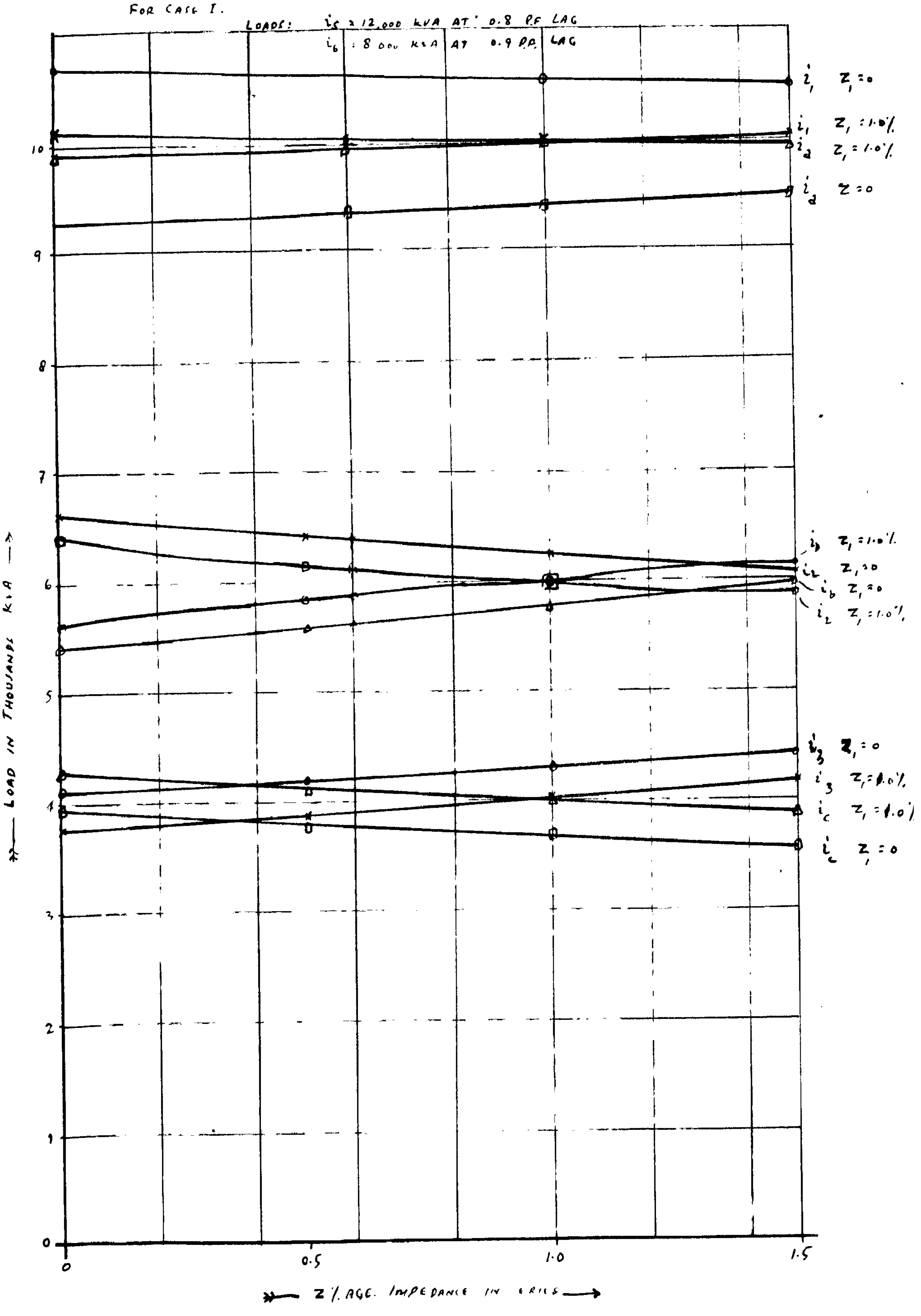
GRAPH 3.5 SAME AS GRAPH 3.4, BUT WITH IMPEDANCE Z IN SERIES WITH SECONDARY LEG 2 INSTEAD OF WITH PRIMARY.



GRAPH 3.6 SAME AS GRAPH 3.4, BUT WITH IMPEDANCE Z IN SERIES WITH SECONDARY LEG 3 INSTEAD OF WITH PRIMARY.

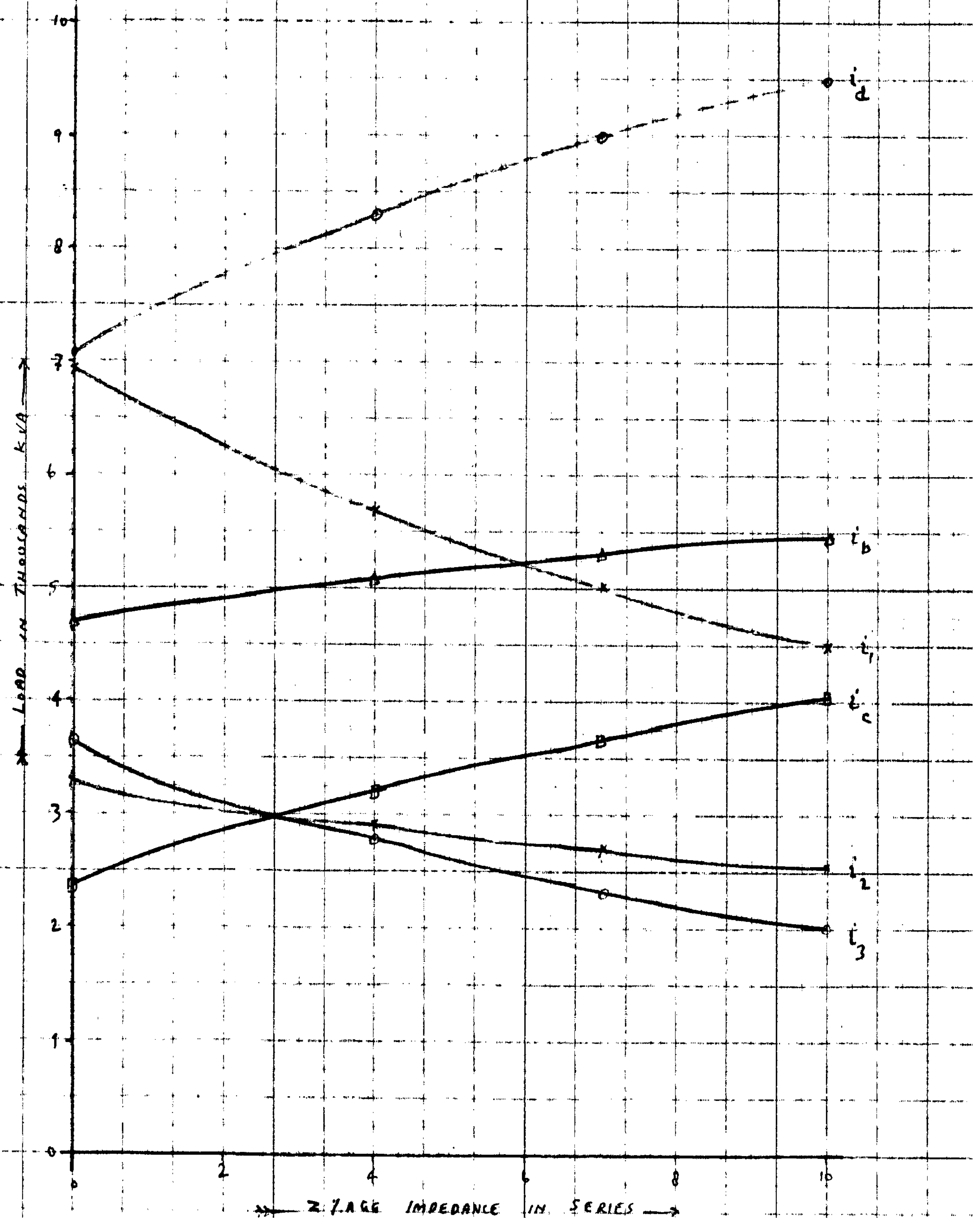


Graph 3.7 Showing the effect of having a suitable external impedance Z_1 in series with the primary and simultaneously a second impedance in series with the secondary leg 2 for Case I.

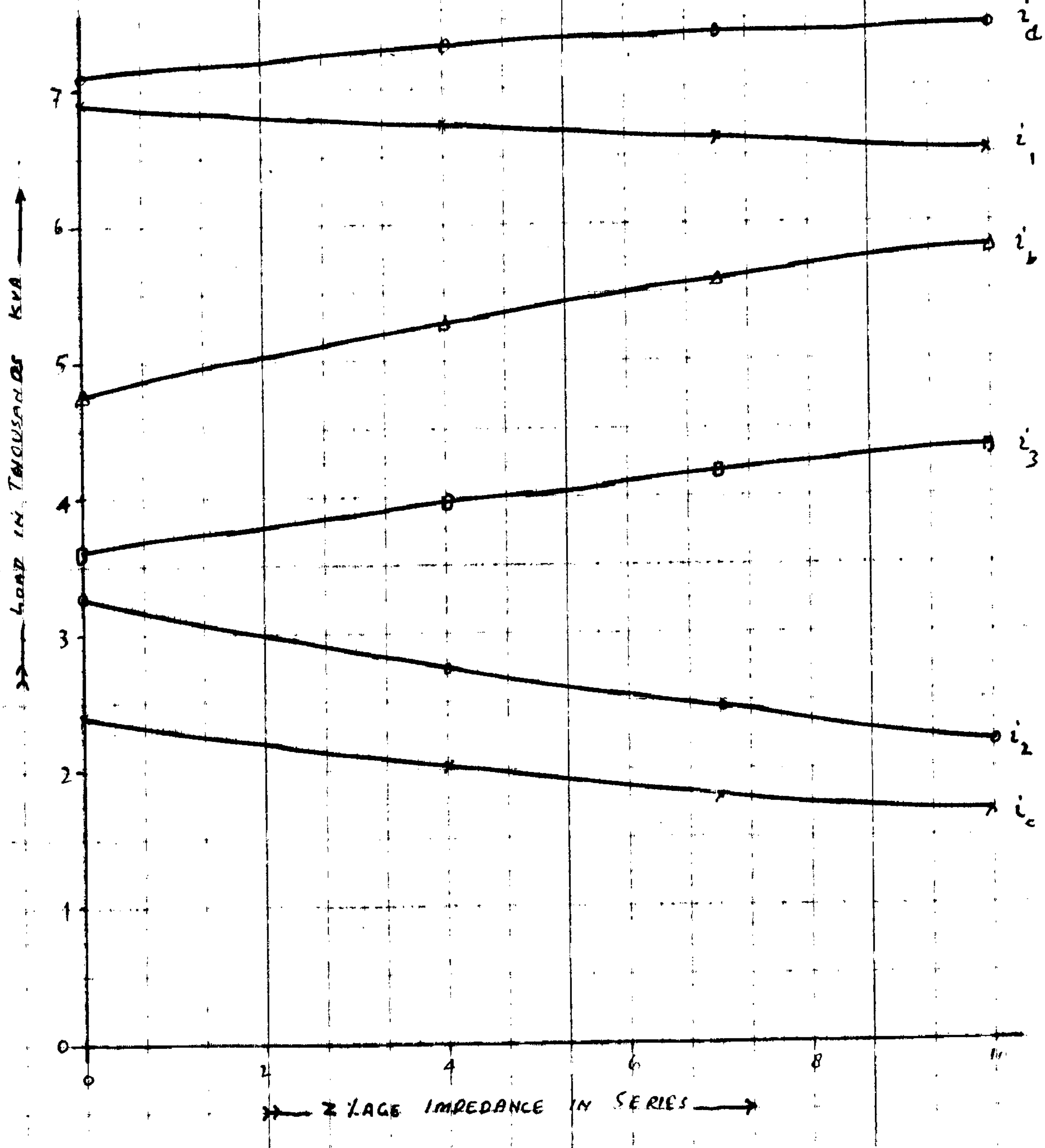


GRAPH 3.8 SHOWING THE EFFECT ON LOAD SHARING OF EXTERNAL IMPEDANCE Z CONNECTED IN SERIES WITH PRIMARY FOR CASE II.

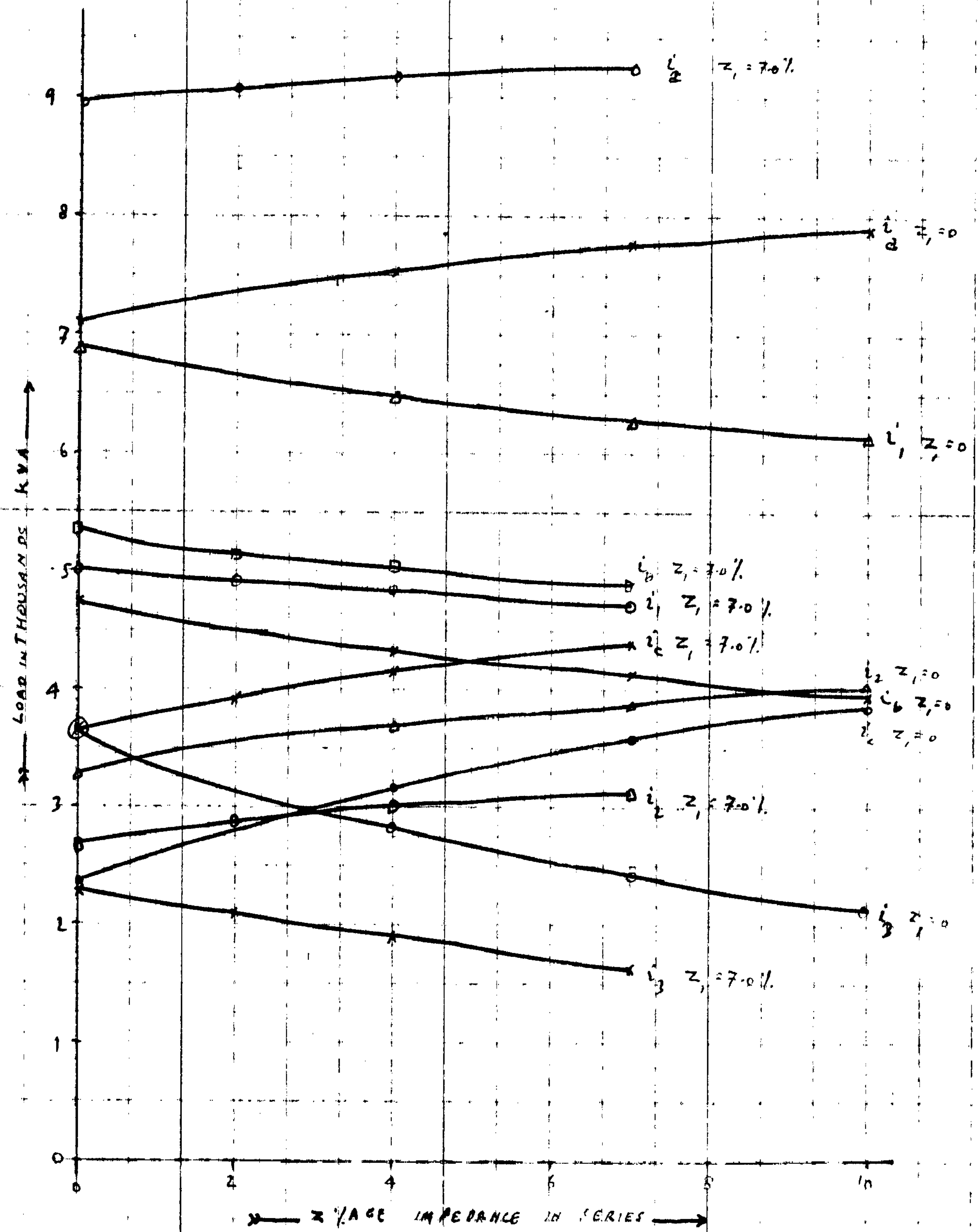
LOADS: i'_5 = 8,000 KVA AT 0.8 PF LAG
i'_6 : 6,000 KVA AT 0.85 PF LAG



GRAPH 3.9 SAME AS IN GRAPH 3.8 BUT WITH IMPEDANCE Z IN SERIES WITH THE SECONDARY LEG 2 INSTEAD OF WITH PRIMARY.



GRAPH 3·10 SAME AS IN GRAPH 3·8 BUT WITH IMPEDANCE Z IN SERIES WITH SECONDARY LEG 3 INSTEAD OF WITH PRIMARY.
IT ALSO SHOWS THE EFFECT OF HAVING TWO IMPEDANCES i.e. A SUITABLE IMPEDANCE Z IN SERIES WITH PRIMARY AND SIMULTANEOUSLY A SECOND IMPEDANCE Z IN SERIES WITH THE SECONDARY LEG 3.



CHAPTER IV.

PARALLEL OPERATION - EFFECT OF TAP-CHANGING.

4.1 When two units (or banks) having different voltage ratios are connected in multiple, a circulating current equal to

$$\frac{\text{difference of induced voltages}}{\text{sum of the leakage reactances}}$$

superposed on the no-load current, flows between the two transformers in both primary and secondary windings. This circulating current is entirely independent of load and load division and flows even before the transformers are connected-up to any external load. In the case of two-winding transformers, this circulating current vectorially adds up to the load current in one transformer and subtracts from the load current in the other transformer. If the bank is delivering full-load, one of the transformers, especially if the load power-factor is low, may thus considerably be overheated. This lowers the efficiency as well as decreases the maximum safe load which the bank can carry. In general, efforts are therefore, made to restrict this circulating current and it is not considered good practice to operate transformers in parallel when the circulating current flowing in any transformer exceeds ten percent of the full-load rated value.

4.2 Nature of circulating current produced by tap-changing:

Sometimes in meeting emergency conditions which necessiate the paralleling of transformer banks whose percentage impedances are not equal, ratio of transformation of the bank with the lower impedance is deliberately changed in order to prevent its being overloaded when the total load approaches the combined capacity of the two banks.

An on-load tap-changer can modify the reactive load division between two transformers in parallel, but it can not modify the active load division. This can be seen from fig. 4.1

Let "e" be the difference between the open-circuit secondary voltages of the two transformers resulting from different positions of the tap-changers. It is in phase with V_2 and causes a circulating current I_c in quadrature.

Supposing the two transformers have the same reactances, then the respective currents I_2 & I_2' are in phase: I_2 = OB and I_2' = BD are in line, so long as e = 0. When e is not zero, a circulating current I_c adds vectorially to I_2 & I_2'.

The resultant currents are

$I_2 + I_c$ = OC in transformer 1

$I_2' - I_c$ = BE in transformer 2

If e increases due to action of the tap-changer, the point C is displaced towards the right on line F_1 and point E towards the left on line F_2. The reactive components are changed, but not the in-phase components.

4.3 Effect of tap-changing-three-winding transformers.

4.3.1. Circulating current

The equivalent circuit of a three-circuit transformer has three branches as shown in fig. A.3 instead of one branch as for a two-circuit transformer. A tap in the primary will affect both the ratios of transformation, whereas a tap in either of the secondary circuits will affect the ratio of transformation for that circuit only. However, due to interconnected nature of the various impedances, a tap-changer in any circuit will affect all the three-circuits differently so far as the circulating current is concerned. In appendix D expressions have been derived for the circulating current that will flow in the various branches due to the change of taps i.e. difference of voltage existing in the different branches respectively. The circulating currents for the various alternatives are:

(a) <u>Tap change on the primary circuit a</u>

Circulating current in branch "a" $= \dfrac{(1+k_1).e}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)}$ ..4.1

Circulating current in branch "b" $= \dfrac{e}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)}$ ..4.2

Circulating current in branch "c" $= \dfrac{k_1.e}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)}$ ..4.3

(b) <u>Tap change on the secondary circuit b</u>

Circulating current in branch "a" $= \dfrac{e}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)}$ ...4.4

Circulating current in branch "b" $= \dfrac{(1+k_2).e}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)}$ ...4.5

Circulating current in branch "c" $= -\dfrac{k_2.e}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)}$ ...4.6

(c) <u>Tap change on the secondary circuit c</u>

Circulating current in branch "a" $= \dfrac{k_1.e}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)}$ ...4.7

Circulating current in branch "b" $= -\dfrac{k_2.e}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)}$ ...4.8

Circulating current in branch "c" $= \dfrac{(k_1+k_2).e}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)}$ ..4.9

where e is the difference in voltage between the respective windings of the two transformers,

impedances Z_1, Z_2 and Z_3 are in ohms

$$k_1 = \frac{Z_2 + Z_b}{Z_3 + Z_c}$$

and $$k_2 = \frac{Z_1 + Z_a}{Z_3 + Z_c}$$

Equations 4.1 to 4.9 show that

i) the magnitude of circulating current is directly proportional to the difference in voltage introduced by the tap-change;

ii) due to a tap change in the primary winding, circulating current in both the secondary circuits flows in the same direction;

and iii) tap change in any one of the secondary windings has a reverse effect on the second secondary winding of the same transformer i.e. the circulating current in the second secondary winding flows in a direction opposite to that in the secondary winding in which tap change has been introduced.

4.3.2 Reactance:

The presence of tappings has a direct effect on the reactance of a transformer. The percentage reactance varies directly as the current and turns, and inversely as volts per turn and axial length of windings In addition the cutting out of appreciable portions of the winding by means of tappings tends to distort the leakage field, thereby affecting the reactance.

Reference 2 has given a table (table 4.1) to show the variations

TABLE 4.1

Showing the variation in reactance due to changes in current, no. of turns & volts per turn, caused by the presence of tappings.

Tappings on Primary or Secondary	Variations of primary or secondary	Constant output or current	Multiplying factor for: Current		Turns		Ampere turns	Volts per turn	Reactance	Output
			Prim.	Sec.	Prim.	Sec.				
Primary	Primary	Output	$\frac{1}{r}$	1	r	1	1	1	1	1
"	"	Current	1	r	r	1	r	1	r	r
"	Secondary	Output	1	r	r	1	r	$\frac{1}{r}$	r^2	1
"	"	Current	1	r	r	1	r	$\frac{1}{r}$	r^2	1
Secondary	"	Output	1	$\frac{1}{r}$	1	r	1	1	1	1
"	"	Current	r	1	1	r	r	1	r	r
"	Primary	Output	r	1	1	r	r	$\frac{1}{r}$	r^2	1
"	"	Current	r	1	1	r	r	$\frac{1}{r}$	r^2	1

NOTE:- Multiplying factors applicable when primary or secondary turns are reduced in the ratio 1: "r" by means of a Tapping.

in reactance due to changes in current, number of turns and volts per turn, caused by the presence of tappings. For the purposes of this table, it has been assumed that the normal condition is when all turns are in circuit, and under this condition the two windings are equal in length. The table shows the factors by which the various items are multiplied when the turns in one winding are reduced to "r" times the original value by means of tapping.

The effect upon reactance, of leakage field distortion due to the cutting out of portions of the windings by means of tappings is illustrated by curves in fig. 4.2 (reference 2). These curves are only representative, as the percentage variations in reactance, due to a given percentage reduction in the number of turns in circuit, varies with different designs. In this reference, it has however been suggested that these curves may be taken as a fair approximation for all transformers of normal output and voltage.

The changes in reactance indicated by table 4,1 and fig.4.2 are cumulative.

4.3.3 <u>Losses:</u>

Use of tappings tends to slightly increase the losses in a transformer due to changes in current, the total lengths of conductors in circuit and the magnetic flux density. Reference 2 has given a table to calculate the exact variation in losses when primary or secondary currents are reduced by means of a tapping. However, as the range of tapping that can possibly be employed in compatibility with allowable circulating currents for satisfactory division of load between two paralleled transformers is small, the variation in losses will be very small and may be neglected for ordinary calculations.

4.4 Studies carried out

The transformers taken in case I & II for the studies in Chapter III were also adopted for various computations under this group. To investigate the effect of tap-changing on the sharing of load by two three-circuit transformers connected in parallel, a number of computations were carried out with different taps.

Transformers were assumed to be operating on taps so that a circulating current would flow in such a direction that it would add to the load current in the lightly loaded transformer and subtract from the load current in the heavily loaded transformer. The various sets of calculations were performed with 2½% and 5% taps in each branch in turn and its effect on the distribution of load in different branches investigated. Some of the sets of calculations were made both for full-load and half-load conditions.

Since the circulating current flows independently of the load current, the results were computed by calculating the load current and circulating current in different equivalent circuit branches separately and than superposing the two vectorially.

4.5 Inferences:

The various sets of computations showed a fairly uniform trend, and the conclusions drawn therefrom can be summarised as below:

i) Variation in tapping has a predominant effect on the equivalent circuit branch impedance in which the tapping is used and very small effect on the other two equivalent circuit branch impedances. As the transformer with higher percentage impedance will have a smaller share of load, it will have to be operated on a lower primary/secondary turns ratio so that its higher secondary voltage helps in increasing the load on this transformer. As per the effect mentioned in section 4.3.2 above, this increases the impedance of this transformer, thereby further increasing the active power on the already heavily loaded transformer. The flow of circulating current, which is reactive, changes the KVA flow through the transformer.

ii) The effects of circulating currents depend upon the power factor of the load, and are greater for lagging power-factors than for power-factors near unity. Depending upon the load power-factor, the circulating current may in certain cases even increase the KVA flow through certain branches.

Unlike the effect of series impedances, effect of tappings is not uniform in all the branches and is dependent upon load, power-factor etc.

iii) Effect of tappings in the primary circuit is much more pronounced than that of the tappings in either of the two secondary circuits. Also the circulating current due to tappings in any one secondary circuit acts in a manner just reverse to the desired affect in the second secondary circuit of the same transformer, thereby further loading the already overloaded winding. This is self evident from the equations 4.4 to 4.9.

In the case of series impedances, this trend can be easily countered by the addition of series impedance in the desired branch, but it can not be economically done with tappings, as that will require tappings both plus and minus on all the three windings. This will make the transformer very bulky, costly as well as more susceptible to faults.

(iv) Even if a satisfactory division of load can be arranged in a particular case by suitably fixing the tappings, that solution will be correct for only that one load, and the currents will shift in relative magnitude and in relative phase position as the magnitude and power-factor of the load are changed.

As compared to this, in the case of series impedances, only one value of impedance is required for a set of transformers and it will always give satisfactory division of load irrespective of the magnitude of load and power-factor.

(v) The effect of circulating currents is to increase the total current flowing through the windings, thereby increasing the total loss as well as heating of the transformer. Also the higher current decreases the maximum safe load which the bank can carry.

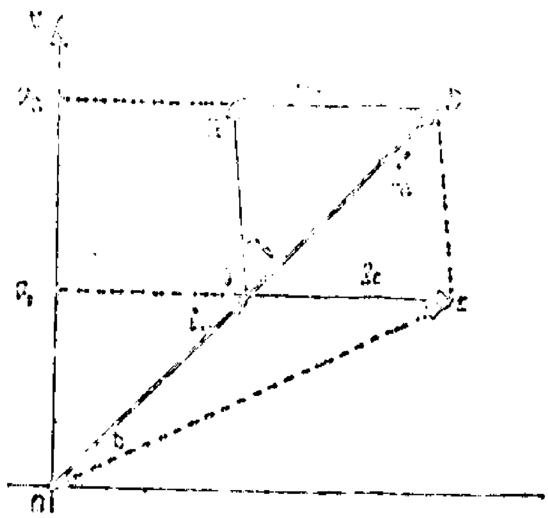


FIG.(a) [illegible] DIAGRAM [illegible] THE EFFECT OF TAPCHANGING

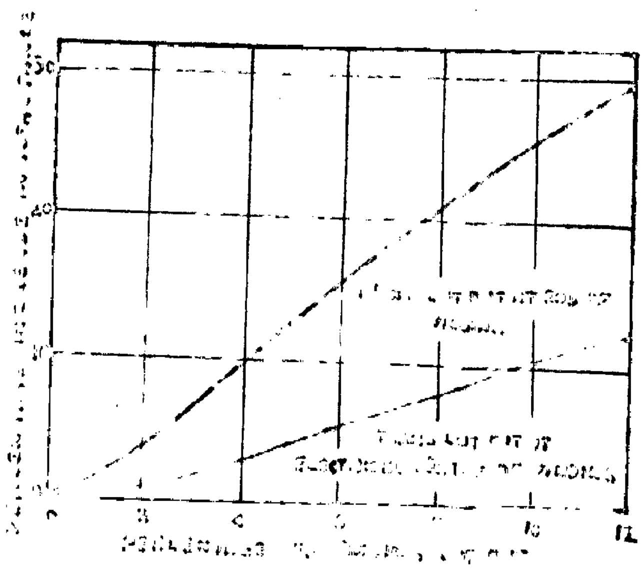


Fig.(b) INCREASE IN [illegible] DUE TO LEAKAGE FIELD DISTORTION CAUSED BY CUTTING OUT A PORTION OF ONE WINDING BY MEANS OF A TAPPING.

CHAPTER V.

C O N C L U S I O N S.

5.1 Three-circuit transformers in parallel

1. Currents flowing in the individual windings of parallel three-winding banks can be determined by solving an equivalent circuit. The terminal loads, as well as winding ratios and impedances affect the division of currents among the windings of three-winding transformers, so all these factors must be known before a solution is attempted.

2. Three-circuit transformers of the same voltage ratio will parallel with each other and divide their loads properly under all conditions of loading only if the impedances under these conditions are the same for all transformers to be paralleled.

3. Two three-circuit transformers having different impedances can be operated in parallel and made to share the loads in all the windings properly by connecting two impedances in series with the transformer having lower percentage impedance. One of the impedances must be donnected in series with the primary circuit and the other in series with either of the two secondary circuits depending upon the equivalent circuit branch impedances and division of loads.

For a large number of cases and over a wide range of load, only one impedance will be required for a reasonably good load sharing and in general, two impedances will be required only when the transformers are operating near their full load capacity.

4. Only one value of impedances is required for a particular set of transformers to properly divide the load between the two and this will give satisfactory results ir-respective of the total load and its power-factor.

5. The greater the variation of actual impedances from those required under ideal conditions, the greater the magnitude of external impedances required.

6. In deciding the load sharing between paralleled three-circuit transformers, it is the values of equivalent circuit branch impedances and not the short-circuit impedances which are of greater importance.

7. The loads in various branches can to a certain extent be adjusted by the variation of tape in the primary circuit. Variation of taps in either of the secondary circuits has an adverse effect on the other secondary circuit, with a much smaller effect on the primary circuit.

8. Tap-changer can only modify the reactive load division between two transformers in parallel, but it can not appreciably alter the active load division. In fact, the active load on the already heavily loaded transformer increases still further.

9. One value of tap can correct the load division for one particular load only. Any change in magnitude of load or power-factor will require a new tap position for a satisfactory sharing of load.

10. As compared to the method of using series impedances, the alternative method employing tap-changing for satisfactory division of load between three-winding transformers in parallel is in-efficient and uneconomical. Such an expedient is a make-shift, justifiable only in meeting emergency conditions when maintenance of service is the paramount consideration and efficiency is for the time being of secondary importance.

5.2. <u>Three-winding transformer in parallel with two-winding transformer</u>

1. Parallel operation of two such transformers is not in general satisfactory. If the impedances are proportioned to divide the load properly for one load condition, the load division between transformers at some other loading is likely to be unsatisfactory.

2. Only when the individual impedance of the primary is zero will the tertiary load have no effect on the load division. In certain cases this is possible to be arranged, but the requirement of certain conflicting characteristics may make it impossible to design a transformer to meet with this condition.

3. Balance coils can be used to force the proper division of load between groups.

APPENDIX A.

EQUIVALENT CIRCUIT OF A THREE-CIRCUIT TRANSFORMER.

A.1 In a two-winding transformer, the internal impedance is usually expressed in terms of one value. The leakage reactance between the primary and secondary, and the resistances of the two windings are lumped into one and this value of impedance is sufficient to express the voltage consumed internally by the transformer.

On the other hand in three-winding transformers, the current flowing in one winding is not necessarily equivalent to, nor in phase with a current in one of the other windings. The three windings are so related to each other that impedance and regulation cannot be so simply expressed as in ordinary transformers, although by means of a simple equivalent circuit, equivalent impedance values can be determined for each winding and these values used in the ordinary formulae to obtain regulation or current division for any given condition.

It is convenient and in fact possible to represent completely and rigorously the leakage impedance characteristics of a three-circuit transformer by assuming that each winding possesses an individual leakage reactance belonging to itself just as distinctly as its resistance and that they are connected to-gether in the equivalent circuit represented either as a star network or as a mesh network. The star network, however, is decidedly more convenient since the impedances representing the transformer can then be combined in series with the impedances of the external circuits, and is therefore, discussed below.

A.2. Star equivalent network of three-circuit transformer.

A single phase three-circuit transformer is shown diagramatically in fig.A.1, as a two line diagram in fig. A.1(a), as a single line diagram in fig. A.1(b), with connected apparatus which may be generators motors, lighting load, or any other kind of electrical apparatus. All that the transformer does between the circuits of A, B & C is to link them with a transformation in voltage and current. This transformation is accomplished at the expense of a magnetising current taken by the transformer, core and copper losses in the transformer, and an impedance or impedances introduced between the various circuits. In the derivation of the equivalent circuit to be used to determine division of load and parallel operation, the following assumptions are made:

i) Magnetising current has been ignored.
Because of this assumption, all the reactances considered will refer exclusively to those reactances which the transformer offers to the load currents (not those which apply to the magnetizing current.)

ii) Transformer and load constants i.e. impedances are all expressed as percentages of rated values of corresponding circuits (all based on an assumed standard KVA load).

iii) All impedances used in equations are those effective at the external circuit terminals.

iv) Because of assumption (ii), turn-ratio drops out of consideration completely, and as such one to one turns ratio is assumed.

With the above assumptions, the magnetically interlined circuits of a three-circuit transformer may be completely represented by the electrically interlinked circuits of fig. A.2. Both figures A.2(a) and A.2(b) are considerably simplified by the use of a single line diagram as in fig. A.3(a). Fig. A.3(a), therefore, applies to all single phase and symmetrical polyphase transformers interconnecting three-circuits per phase. Fig. A.3(b) is essentially the same, but may sometimes be preferred so as to segregate the primary and secondary circuits from each other. It will be noted that the equivalent network amounts to the connection of the three-circuits or systems A, B & C to the same bus-bars through impedances Z_a, Z_b and Z_c, equivalent to the impedance effect of the interconnecting transformer.

A.3. Impedances of the equivalent network:

The impedances Z_a, Z_b, & Z_c of the equivalent network (fig.A.3) are not as a rule equal to each other, and although they originate in the commonly recognised leakage impedances between pairs of windings of the transformer, they are not numerically equal to them, but are determined as follows:

The impedance to the flow of kilovolt-amperes between A & B is $(Z_a + Z_b)$ as seen from the equivalent network. Hence if the equivalent network is to represent the performance of the transformer correctly, it must satisfy the condition that

$$Z_a + Z_b = Z_{ab} \qquad .. \text{A.1}$$

Similarly it must satisfy the conditions that

$$Z_a + Z_c = Z_{ac} \qquad .. \text{A.2}$$

$$Z_b + Z_c = Z_{bc} \qquad .. \text{A.3}$$

Equations A.1 to A.3 can be solved for the branch impedances are the conventional two-winding impedances), yielding the following expressions:

$$Z_a = \tfrac{1}{2}(Z_{ab} + Z_{ac} - Z_{bc}) \qquad \ldots \text{A.4}$$

$$Z_b = \tfrac{1}{2}(Z_{ab} + Z_{bc} - Z_{ca}) \qquad \ldots \text{A.5}$$

$$Z_c = \tfrac{1}{2}(Z_{bc} + Z_{ca} - Z_{ab}) \qquad \ldots \text{A.6}$$

These equations are naturally vectorial. The resistance and reactance components of the equivalent impedances are, therefore,

$$X_a = \tfrac{1}{2}(X_{ab} + X_{ac} - X_{bc}) \qquad \ldots \text{A.7}$$

$$X_b = \tfrac{1}{2}(X_{ab} + X_{bc} - X_{ca}) \qquad \ldots \text{A.8}$$

$$X_c = \tfrac{1}{2}(X_{bc} + X_{ca} - X_{ab}) \qquad \ldots \text{A.9}$$

$$R_a = \tfrac{1}{2}(R_{ab} + R_{ac} - R_{bc}) \qquad \ldots \text{A.10}$$

$$R_b = \tfrac{1}{2}(R_{ab} + R_{bc} - R_{ca}) \qquad \ldots \text{A.11}$$

$$R_c = \tfrac{1}{2}(R_{bc} + R_{ca} - R_{ab}) \qquad \ldots \text{A.12.}$$

A.4 Determination of the equivalent circuit parameters.

A simple method of determining the impedances of the star equivalent circuit is to evaluate them in terms of the short-circuit or equivalent impedances of each pair of windings acting as a two-circuit transformer. The equivalent impedance of each pair of windings can be computed from well known two circuit design formulas, or can be determined experimentally by means of three simple short-circuit tests as described below:

Short circuit tests are performed by applying a low voltage A.C. to winding 1 with winding 2 short-circuited and winding 3 open-circuited. fig.A.4(a). Under these conditions, the equivalent circuit of fig. A.3(b) reduces to that shown in fig. A.4(b). Let V_1, I_1, and P_1 be the measured values of the voltage, current and power supplied to winding 1. Then the magnitude of the equivalent short-circuit impedance Z_{12} of windings 1 & 2 is

$$Z_{12} = \frac{V_1}{I_1} \quad \ldots \text{A.13}$$

and its resistance and reactance components are

$$R_{12} = \frac{P_1}{I_1^2} \quad \ldots \text{A.14}$$

$$X_{12} = \sqrt{Z_{12}^2 - R_{12}^2} \quad \ldots \text{A.15}$$

Inspection of fig. A.4(b) shows that with winding 1 excited and winding 2 short-circuited, the short-circuit impedance Z_{12} is the series combination of the two branch impedances Z_1 and Z_2 of the equivalent circuit. Therefore,

$$Z_{12} = Z_1 + Z_2 \quad \ldots \text{A.16}$$

Similarly the relations among the short-circuit impedances Z_{23} & Z_{31}, and the equivalent [illegible]t-circuit impedances Z_1, Z_2 and Z_3 are

$$Z_{23} = Z_2 + Z_3 \quad \ldots \text{A.17}$$

$$Z_{31} = Z_3 + Z_1 \quad \ldots \text{A.18}$$

Equations A.16 to A.18 can be solved for the branch impedances Z_1, Z_2 & Z_3 of the equivalent circuit in terms of short-circuit impedances Z_{12}, Z_{23} & Z_{31} yielding equations A.4 to A.6.

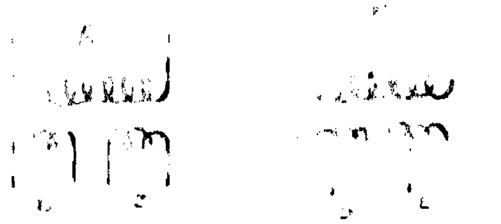
A.5. Physical significance of equivalent circuit Resistances and Reactances:

From the above, it is clear that the characteristics of the three-circuit transformer viz. load sharing, regulation, efficiency etc. depend upon the parameters Z_{ab}, Z_{bc} and Z_{ca} which are a function of the arrangement of the windings. It is however, not necessary to have a knowledge of the physical arrangement of three windings in order to be able to pre-determine the performance. The only thing needed to be known is the reactance and resistances of the various winding arrangements to be used in the equivalent circuit. These impedances can be very readily calculated by simple experiments given in section. A.4.

It is possible that for some arrangement of windings, either of the values of impedance Z_a, Z_b or Z_c in the star equivalent circuit as obtained from equations A.4 to A.6 may be zero or negative, even though the equivalent impedance, Z_{ab}, Z_{bc} or Z_{ca} must all have positive resistances and positive (i.e. inductive) reactances. Equations A.4 to A.6 do not contradict this possibility and actual practical designs confirm it. However, only one of the branches can have a negative reactanne, because the net impedance between every pair of windings of the transformer must be positive or inductive and cannot be negative or capacitive. Moreover, the negative reactance of any branch must be less than the positive reactance of either of the other two branches.

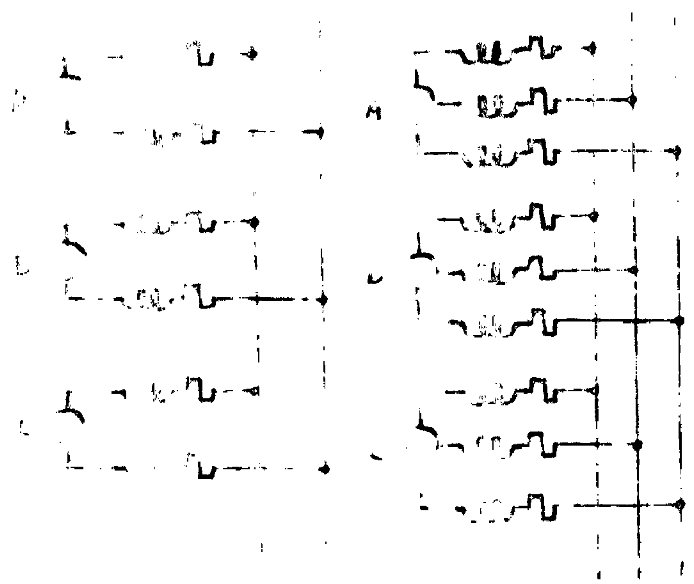
One of the branches of the equivalent circuit may, in fact, have a negative resistance, but it should be borne in mind that the equivalent circuit shows only the external behaviour of the transformer and a negative resistance in one branch of the equivalent circuit does not signify a negative load loss in one of the windings. The equivalent circuit gives the correct total load loss for any load conditions, but it does not show the way in which the load losses are distributed among the windings. The negative resistance ordinarily appears only in auto-transformers with separate tertiaries.

This equivalent negative reactance is responsible for most of the peculiar phenomena in multi-circuit transformers. Apparently no practical use can be made of the negative resistances or reactances of three-circuit transformers as negative impedances by themselves. These negative impedances are virtual values. They reprdduce faithfully the terminal characteristics of the circuits of the transformer, but they cannot necessarily be applied or assigned directly to any internal coils.



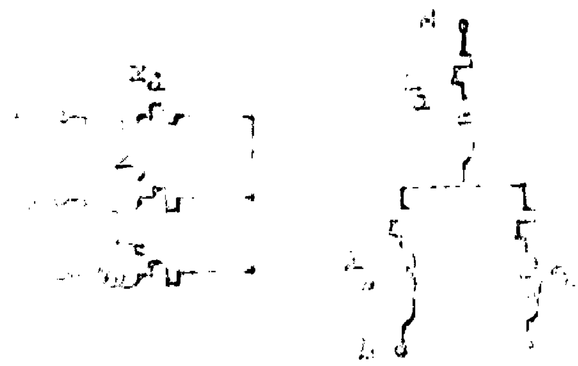
TWO WIRE DIAGRAM.   ONE-WIRE DIAGRAM

THREE CIRCUIT TRANSFORMER

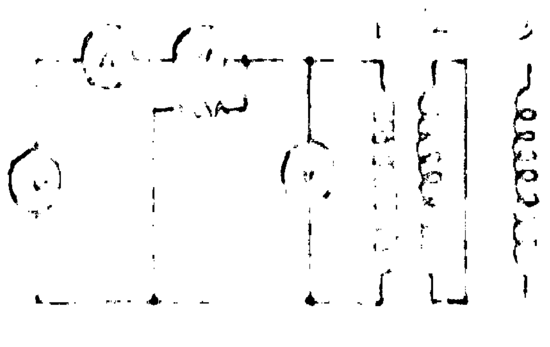


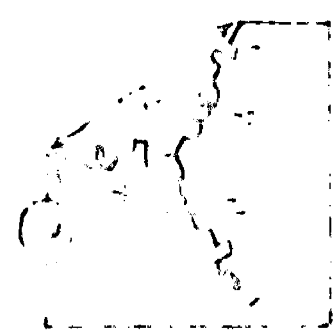
SINGLE PHASE UNIT (OR BANK) OF THREE-CIRCUIT TRANSFORMER.

THREE PHASE UNIT (OR BAN[K]) OF THREE CIRCUIT TRANSFORMER.



EQUIVALENT NETWORK OF THREE CIRCUIT TRANSFORMER.





SHORT-CIRCUIT TEST FOR MEASURING THE EQUIVALENT CIRCUIT IMPEDANCES.
CIRCUIT DIAGRAM, AND
b, THE CORRESPONDING EQUIVALENT CIRCUIT.

APPENDIX B.

DIVISION OF LOAD BETWEEN TWO THREE-CIRCUIT TRANSFORMERS IN PARALLEL.

Given the two loads, i_5 and i_6 (refer fig. 2.3) then the total input i_4 will be their vector sum:

$$i_4 = i_5 + i_6 \qquad ..B.1$$

From the circuit applying Kirchoff's laws,

$$i_a.Z_a + i_b.Z_b = i_1.Z_1 + i_2.Z_2 \qquad ..B.2$$

$$i_a.Z_a + i_c.Z_c = i_1.Z_1 + i_3.Z_3 \qquad ..B.3$$

$$i_a = i_4 - i_1 \qquad ..B.4$$

$$i_b = i_5 - i_2 \qquad ..B.5$$

$$i_c = i_6 - i_3 \qquad ..B.6$$

$$i_1 = i_2 + i_3 \qquad ..B.7$$

Substituting from equations B.4 and B.5 in eqn. B.2, we get

$$(i_4 - i_1)Z_a + (i_5 - i_2)Z_b = i_1 Z_1 + i_2.Z_2$$

or

$$i_4.Z_a + i_5.Z_b - i_2(Z_2 + Z_b) = i_1(Z_1 + Z_a) \qquad ..B.8$$

Substituting from equations B.4 and B.6 in eqn. B.3, we get

$$(i_4 - i_1)Z_a + (i_6 - i_3)Z_c = i_1.Z_1 + i_3.Z_3$$

or

$$i_4 Z_a + i_6 Z_c - i_3(Z_3 + Z_c) = i_1(Z_1 + Z_a) \qquad ..B.9$$

Substituting the value of i_3 from eqn. B.7 in eqn. B.9,

$$i_4 Z_a + i_6 Z_c - (i_1 - i_2)(Z_3 + Z_c) = i_1(Z_1 + Z_a)$$

or

$$i_4 Z_a + i_6 Z_c + i_2(Z_3 + Z_c) = i_1 \{(Z_1 + Z_a) + (Z_3 + Z_c)\}$$

or

$$i_2(Z_3 + Z_c) = i_1(Z_1 + Z_a + Z_3 + Z_c) - i_4 Z_a - i_6 Z_c$$

or $i_2 = \dfrac{i_1 (Z_1 + Z_a + Z_3 + Z_c) - i_4 Z_a - i_6 Z_c}{(Z_3 + Z_c)}$ ..B.10

Substituting the value of i_2 from eqn B.10 in eqn.B.8, we get

$$i_4 Z_a + i_5 Z_b - \frac{Z_2 + Z_b}{Z_3 + Z_c} \{(Z_1 + Z_a + Z_3 + Z_c) i_1 - i_4 Z_a - i_6 Z_c\} = i_1 (Z_1 + Z_a)$$

Putting $\dfrac{Z_2 + Z_b}{Z_3 + Z_c} = k_1$ in the above equation, gives

$$i_4 Z_a + k_1 . i_4 Z_a + i_5 Z_b + k_1 . i_6 . Z_c = i_1 (Z_1 + Z_a) + k_1 . i_1 (Z_1 + Z_a + Z_3 + Z_c)$$

$$= i_1 (Z_1 + Z_a) + k_1 i_1 (Z_1 + Z_a) + k_1 . i_1 (Z_3 + Z_c)$$

$$= i_1 (Z_1 + Z_a)(1 + k_1) + i_1 (Z_2 + Z_b)$$

or

$$i_4 Z_a (1 + k_1) + i_5 Z_b + k_1 i_6 Z_c = i_1 \{(Z_1 + Z_a)(1 + k_1) + (Z_2 + Z_b)\}$$

or

$$i_1 = \frac{i_4 Z_a (1 + k_1) + i_5 Z_b + k_1 i_6 Z_c}{(Z_1 + Z_a)(1 + k_1) + (Z_2 + Z_b)} \quad ..B.11$$

Substituting the value of i_1 from eqn. B.11 in eqn. B.8 we get

$$i_4 Z_a + i_5 Z_b - i_2 (Z_2 + Z_b) = i_1 (Z_1 + Z_a)$$

$$= (Z_1 + Z_a) . \frac{i_4 Z_a + i_6 . Z_c + i_2 (Z_3 + Z_c)}{(Z_1 + Z_a) + (Z_3 + Z_c)}$$

$$= \frac{i_4 Z_a + i_6 Z_c}{1 + \frac{1}{k_2}} + i_2 . \frac{(Z_1 + Z_a)(Z_3 + Z_c)}{(Z_1 + Z_a) + (Z_3 + Z_c)}$$

Putting $\dfrac{Z_1 + Z_a}{Z_3 + Z_c} = k_2$

or

$$(1 + \frac{1}{k_2}) \{ i_4 Z_a + i_5 Z_b - i_2 (Z_2 + Z_b)\} = i_4 Z_a + i_6 Z_c + i_2 (Z_3 + Z_c)$$

or

$$(1+k_2)\{i_4 . Z_a + i_5 Z_b - i_2(Z_2+Z_b)\} = k_2 . i_4 Z_a + k_2 i_6 Z_c + k_2 i_2 (Z_3+Z_c)$$

or

$$i_4 Z_a + (1+k_2) i_5 Z_b - k_2 i_6 Z_c = i_2(1+k_2)(Z_2+Z_b) + k_2 i_2 (Z_3+Z_c)$$

$$= i_2\left\{(Z_2+Z_b)+(Z_1+Z_a)+(Z_2+Z_b)\frac{(Z_1+Z_a)}{(Z_3+Z_c)}\right\}$$

$$= i_2\{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)\}$$

or

$$i_2 = \frac{i_4 Z_a + (1+k_2) i_5 Z_b - k_2 . i_6 . Z_c}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)} \qquad ..B.12$$

$$i_a = i_4 - i_1 = \frac{i_4 . Z_1 (1+k_1) + i_5 Z_2 + i_6 . Z_3 . k_1}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)} \qquad ...B.13$$

$$i_b = i_5 - i_2 = \frac{i_4 Z_1 + (1+k_2) . i_5 . Z_2 - k_2 . i_6 . Z_3}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)} \qquad ..B.14$$

$$i_3 = i_1 - i_2 = \frac{i_4 Z_a . k_1 - i_5 . Z_b k_2 + i_6 . Z_c (k_1+k_2)}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)} \qquad ..B.15$$

$$i_c = i_a - i_b = \frac{i_4 Z_1 . k_1 - i_5 . Z_2 . k_2 + i_6 Z_3 (k_1+k_2)}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)} \qquad ..B.16$$

Equations B.13 to B.16 can be very easily derived by simple substitution of the values of various quantities in equations B.4 to B.7

APPENDIX C.

DIVISION OF LOAD BETWEEN THREE THREE-CIRCUIT TRANSFORMERS IN PARALLEL.

Given the two loads i_5 & i_6 (refer fig. 2.2) then the total input i_4 will be their vector sum:

$$i_4 = i_5 + i_6 = i_1 + i_a + i_\ell \qquad ..C.1$$

Also

$$i_5 = i_2 + i_b + i_m \qquad ..C.2$$

$$i_6 = i_3 + i_c + i_n \qquad ..C.3$$

$$i_1 = i_2 + i_3 \qquad ..C.4$$

$$i_a = i_b + i_c \qquad ..C.5$$

$$i_\ell = i_m + i_n \qquad ..C.6$$

From the circuit, applying Kirchoff's laws,

$$i_1 Z_1 + i_2 Z_2 = i_a Z_a + i_b Z_b = i_\ell Z_\ell + i_m . Z_m \qquad ..C.7$$

$$i_1 Z_1 + i_3 Z_3 = i_a Z_a + i_c Z_c = i_\ell Z_\ell + i_n . Z_n \qquad ..C.8$$

Multiplying eqn. C.1 by Z_1 and eqn. C.2 by Z_2,

$$i_4 Z_1 = i_1 Z_1 + i_a Z_1 + i_\ell Z_1 \qquad ..C.9$$

$$i_5 Z_2 = i_2 Z_2 + i_b Z_2 + i_m Z_2 \qquad ..C.10$$

Adding equations C.9 and C.10

$$i_1 Z_1 + i_2 Z_2 = i_4 Z_1 - i_a Z_1 - i_\ell Z_1 + i_5 Z_2 - i_b Z_2 - i_m Z_2$$

$$= i_a Z_a + i_b Z_b \qquad ..C.11$$

$$= i_\ell Z_\ell + i_m Z_m \quad \text{(from eqn.C.7)} \qquad ..C.12$$

Again multiplying eqn. C.1 by Z_1 and C.3 by Z_3,

$$i_4 Z_1 = i_1 Z_1 + i_a Z_1 + i_\ell Z_1 \qquad ..C.13$$

$$i_6 Z_3 = i_3 Z_3 + i_c Z_3 + i_n Z_3 \qquad ..C.14$$

Adding equations C.13 and C.14,

$$i_1Z_1 + i_3Z_3 = i_4Z_1 - i_aZ_1 - i_{\ell}Z_1 + i_6Z_3 - i_cZ_3 - i_nZ_3$$

$$= i_aZ_a + i_cZ_c \qquad \text{.. C.15}$$

$$= i_{\ell}Z_{\ell} + i_nZ_n \quad \text{(from eqn.C.8)} \qquad \text{.. C.16}$$

By substitution, eqn.C.15 and C. 16 become:

$$i_4Z_1 - i_aZ_1 - i_{\ell}Z_1 + i_6Z_3 - (i_a - i_b)Z_3 - (i_{\ell} - i_m)Z_3$$

$$= i_aZ_a + (i_a - i_b)Z_c \qquad \text{.. C.17}$$

$$= i_{\ell}Z_{\ell} + (i - i_m)Z_n \qquad \text{.. C.18}$$

By rearranging equations C.11, C.12, C.17 and C.18,

$$i_a(Z_1 + Z_a) + i_b(Z_2 + Z_b) + i_{\ell}Z_1 + i_mZ_2 = i_4Z_1 + i_5Z_2 \qquad \text{...C.19}$$

$$i_aZ_1 + i_bZ_2 + i_{\ell}(Z_1 + Z_{\ell}) + i_m(Z_2 + Z_m) = i_4Z_1 + i_5Z_2 \qquad \text{...C.20}$$

$$i_a(Z_1 + Z_a + Z_3 + Z_c) - i_b(Z_3 + Z_c) + i_{\ell}(Z_1 + Z_3) - i_mZ_3 = i_4Z_1 + i_6Z_3 \qquad \text{...C.21}$$

$$i_a(Z_1 + Z_3) - i_bZ_3 + i_{\ell}(Z_1 + Z_{\ell} + Z_3 + Z_n) - i_m(Z_3 + Z_n) = i_4Z_1 + i_6Z_3 \qquad \text{...C.22}$$

Subtracting Equation C.20 from C.19 and equation C.22 from eqn.C.21,

$$i_aZ_a + i_bZ_b - i_{\ell}Z_{\ell} - i_mZ_m = 0 \qquad \text{..C.23}$$

$$i_aZ_1 + i_bZ_2 + i_{\ell}(Z_1 + Z_{\ell}) + i_m(Z_2 + Z_m) = i_4Z_1 + i_5Z_2 \qquad \text{..C.24}$$

$$i_a(Z_a + Z_c) - i_bZ_c - i_{\ell}(Z_{\ell} + Z_n) + i_mZ_n = 0 \qquad \text{..C.25}$$

$$i_a(Z_1 + Z_3) - i_bZ_3 + i_{\ell}(Z_1 + Z_{\ell} + Z_3 + Z_n) - i_m(Z_3 + Z_n) = i_4Z_1 + i_6Z_3 \qquad \text{..C.26}$$

By the use of determinants, the values of i_a, i_b, i_{ℓ} & i_m can be evaluated from equations C.23 to C.26 and thus all the branch currents can be calculated.

APPENDIX D.

EQUATIONS FOR CIRCULATING CURRENT DUE TO UNEQUAL TURNS-RATIO.

(a) Tap-change on the primary circuit "a"

Equivalent circuit for this condition is shown in fig. D.1

Applying Kirchoff's laws to two closed circuits:

$$(Z_3 + Z_c)i_c + (Z_1 + Z_a)(i_b + i_c) = e \qquad \text{..D.1}$$

$$(Z_2 + Z_b)i_b + (Z_1 + Z_a)(i_b + i_c) = e \qquad \text{..D.2}$$

From the equations D.1 and D.2,

$$(Z_3 + Z_c)i_c = (Z_2 + Z_b)i_b \qquad \text{..D.3}$$

Substituting from eqn. D.3 the value of i_b in eqn. D.1,

$$(Z_3 + Z_c)i_c + (Z_1 + Z_a)\left(\frac{Z_3 + Z_c}{Z_2 + Z_b} i_c + i_c\right) = e$$

or

$$i_c \left\{(Z_3 + Z_c) + \frac{(Z_1 + Z_a)(Z_2 + Z_b + Z_3 + Z_c)}{(Z_2 + Z_b)}\right\} = e$$

or

$$i_c\left\{(Z_3+Z_c)(Z_2+Z_b)+(Z_1+Z_a)(Z_3+Z_c) + (Z_1 + Z_a)(Z_2+Z_b)\right\} = (Z_2 + Z_b).e$$

or

$$i_c = \frac{(Z_2 + Z_b).e}{(Z_1+Z_a)(Z_2 + Z_b)+(Z_2+Z_b)(Z_3+Z_c)+(Z_3 + Z_c)(Z_1 + Z_a)}$$

$$= \frac{k_1.e}{(Z_1 + Z_a)(1+k_1) + (Z_2 + Z_b)} \qquad \text{..D.4}$$

$$i_b = \frac{(Z_3 + Z_c).e}{(Z_1 + Z_a)(Z_2 + Z_b) + (Z_2+Z_b)(Z_3+Z_c) + (Z_3+Z_c)(Z_1+Z_a)}$$

$$= \frac{e}{(Z_1 + Z_a)(1+k_1) + (Z_2 + Z_b)} \qquad \text{..D.5}$$

and $$i_a = i_b + i_c = \frac{(1+k_1).e}{(Z_1 + Z_a)(1 + k_1) + (Z_2 + Z_b)} \qquad \text{..D.6}$$

(b) Tap-change on the secondary circuit "b".

Equivalent circuit for this condition is shown in fig.D.2

Applying Kirchoff's laws to the two closed circuits,

$$(Z_3 + Z_c)i_c + (Z_1 + Z_a)(i_b + i_c) = 0 \qquad ..D.7$$

$$(Z_2 + Z_b)i_b + (Z_1 + Z_a)(i_b + i_c) = e \qquad ..D.8$$

From equations D.7 and D.8,

$$(Z_2 + Z_b)i_b - (Z_3 + Z_c)i_c = e \qquad ..D.9$$

or

$$i_b = \frac{e + (Z_3 + Z_c)i_c}{Z_2 + Z_b} \qquad ..D.10$$

Substituting the value of i_b from eqn. D.10 in Eqn. D.7,

$$(Z_3 + Z_c)i_c + (Z_1 + Z_a)\left\{\frac{e + (Z_3 + Z_c)i_c}{Z_2 + Z_b} + i_c\right\} = 0$$

or

$$i_c\{(Z_2 + Z_b)(Z_3 + Z_c) + (Z_1 + Z_a)(Z_3 + Z_c) + (Z_1 + Z_a)(Z_2 + Z_b)\} = -e.(Z_1 + Z_a)$$

or

$$i_c = -\frac{e.(Z_1 + Z_a)/(Z_3 + Z_c)}{(Z_1 + Z_a)(1 + k_1) + (Z_2 + Z_b)} \qquad ..D.$$

$$= -\frac{k_2.e}{(Z_1 + Z_a)(1 + k_1) + (Z_2 + Z_b)} \qquad ..D.11$$

Substituting the value of i_c from equation D.9 in eqn.D.8,

$$(Z_2 + Z_b)i_b + (Z_1 + Z_a)\left\{i_b + \frac{(Z_2 + Z_b)i_b - e}{(Z_3 + Z_c)}\right\} = e$$

or

$$i_b\{(Z_1 + Z_a)(Z_2 + Z_b) + (Z_2 + Z_b)(Z_3 + Z_c) + (Z_3 + Z_c)(Z_1 + Z_a)\} = e\{(Z_1 + Z_a) + (Z_3 + Z_c)$$

or

$$i_b = \frac{e(1 + k_2)}{(Z_1 + Z_a)(1 + k_1) + (Z_2 + Z_b)} \qquad ..D.12$$

$$i_a = i_b + i_c = \frac{e}{(Z_1 + Z_a)(1 + k_1) + (Z_2 + Z_b)} \qquad ..D.13$$

(c) <u>Tap-change on the secondary circuit "c"</u>

Equivalent circuit for this condition is shown in fig.D.3

Applying Kirchoff's laws to the two closed circuits:

$$(Z_3+Z_c)i_c + (Z_1+Z_a)(i_b+i_c) = e \qquad ..D.14$$

$$(Z_2+Z_b)i_b + (Z_1+Z_a)(i_b+i_c) = 0 \qquad ..D.15$$

From equations C.14 & C.15,

$$(Z_3+Z_c)i_c - (Z_2+Z_b)i_b = e$$

or

$$i_c = \frac{e + (Z_2+Z_b)i_b}{(Z_3+Z_c)} \qquad ..D.16$$

and

$$i_b = \frac{(Z_3+Z_c)i_c - e}{(Z_2+Z_b)} \qquad ..D.17$$

Substituting the value of i_c from eqn. D.16 in eqn. D.15,

$$(Z_2+Z_b)i_b + (Z_1+Z_a)\left\{ i_b + \frac{e+(Z_2+Z_b)i_b}{Z_3+Z_c} \right\} = 0$$

or

$$i_b\left\{(Z_2+Z_b)(Z_3+Z_c)+(Z_1+Z_a)(Z_3+Z_c)+(Z_2+Z_b)(Z_1+Z_a)\right\} = -e.(Z_1+Z_a)$$

or

$$i_b = -\frac{e.k_2}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)} \qquad ..D.18$$

Substituting the value of i_b from Eqn. D.17 in Eqn.D.14,

$$i_c(Z_3+Z_c) + (Z_1+Z_a)\left\{ \frac{(Z_3+Z_c)i_c - e}{Z_2+Z_b} + i_c \right\} = e$$

or

$$i_c\left\{(Z_3+Z_c)(Z_2+Z_b)+(Z_1+Z_a)(Z_3+Z_c)+(Z_1+Z_a)(Z_2+Z_b)\right\} = e\left\{(Z_1+Z_a)+(Z_2+Z_b)\right)$$

or

$$i_c = \frac{e.(k_1+k_2)}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)} \qquad ..D.19$$

$$i_a = i_b + i_c = \frac{e.k_1}{(Z_1+Z_a)(1+k_1)+(Z_2+Z_b)} \qquad ..D.20$$

In the above equations

"e" is the difference in voltage between the respective windings of the two transformers,

Z_1, Z_2 & Z_3 are in ohms,

i_a, i_b & i_c are circulating currents in various branches caused by the voltage difference "e".

$$k_1 = \frac{Z_2 + Z_b}{Z_3 + Z_c}$$

and $$k_2 = \frac{Z_1 + Z_a}{Z_3 + Z_c}$$

FIG. EQUIVALENT CIRCUIT FOR TAP-CHANGE IN PRIMARY CIRCUIT.

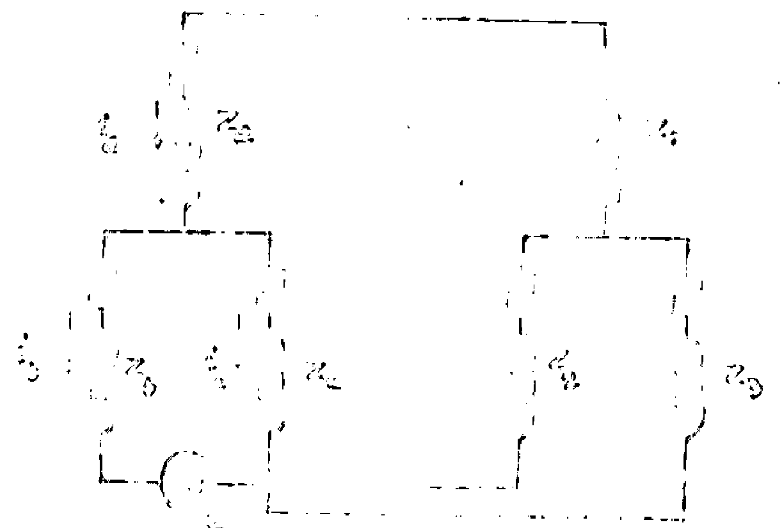


FIG. EQUIVALENT CIRCUIT FOR TAP-CHANGE IN SECONDARY CIRCUIT 'b'.

FIG. EQUIVALENT CIRCUIT FOR TAP-CHANGE IN SECONDARY CIRCUIT 'c'.

R_E_F_E_R_E_N_C_E_S.

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