"EXCITER RESPONSE - ITS CALCULATIONS AND FIELD OF USE

Dissertation submitted in partial
fulfilment of the requirements for
the degree of Master of Engineering
(ELECTRICAL MACHINE DESIGN)

By

S. KUMAR

September 1961

University of Roorkee
ROORKEE.

ACKNOWLEDGEMENTS

The author wishes to acknowledge his indebtedness in connection with this work to Dr. G. C. Jain, formerly Reader in the Department of Electrical Engineering, University of Roorkee, and now on the staff of the Department of Electrical Engineering, Purdue University, U.S.A., for his guidance, supervision and personal interest without which this work would not have been possible. He is also particularly indebted to Prof. C. S. Ghosh, Head of the Electrical Engineering Department, University of Roorkee, for permitting use of laboratory facilities and for his advice and inspiration.

September 1961
Roorkee.

S. KUMAR

INDEX

INTRODUCTION

While finding out the transient response of a direct current machine the field winding is represented by a resistance and an inductive reactance of constant magnitude. The differential equation of voltage around the field circuit is easily solved corresponding to any type of input, either step, rate or sinusoidal. The response in case of a step input or discontinuity is found to be exponential. This however, gives the response only approximately. The field winding changes its inductance as the field current changes depending upon the saturation. Change in inductance can be found from the magnetisation characteristics of the winding. In the differential equation which is of the form $L_f \frac{di_f}{dt} + R_f i_f = V$, L_f changes with i_f and hence we obtain a differential equation with variable coefficients. 3 methods of solution have been outlined viz. (a) Point by point solution (b) Method of graphical integration and (c) Actual solution of the differential equation representing magnetisation curve by an approximate equation. The method involving use of differential analyser has not been indicated as its use is limited by the availability of such a machine. In chapter IV the effect of load and the procedure for calculation of response in case of loaded exciters have been discussed. In the next chapter the various factors which effect the response as also the methods of improving the rate of response have been discussed.

In the chapter which follows, the method involving point by point solution for the alternator field current when the exciter response is applied has been indicated. Also the extent to which exciter response increases the transient and dynamic stability of the system has been discussed.

Last ly it has been attempted to calculate the response of an exciter machine using all the three methods outlined in the earlier chapters. The response as calculated by these methods has been compared to the response obtained by actual oscillographic record. Using the same machine as a separately excited exciter the effect of varying the ceiling voltage on response has been studied.

CHAPTER I.

DEFINITIONS AND DETERMINATION OF EXCITER RESPONSE OF CONVENTIONAL MAIN EXCITERS USING POINT BY POINT SOLUTION

1.1. Definitions:

A. I. E. E., defines 'exciter response' as the rate of increase or decrease of the main exciter voltage when resistance is suddenly removed from or inserted in the main exciter field circuit.

The response may be expressed either in volts/sec., or in per unit voltage per second. When expressed in per unit voltage per second the base voltage is not usually the rated voltage of the exciter but the nominal collector ring voltage of the main generator.

Nominal collector ring voltage is the 'voltage required across the collector rings to generate rated kilovolt-amperes in the main machine at rated voltage, speed, frequency and power factor with the field winding at a temperature of 75 degrees centigrade.'

For a particular exciter the rate of volt-age build up or the response depends upon (i) Type of exciter (ii) the speed of rotation (iii) Whether the exciter is self excited or separately excited (iv) If separately excited, its initial field circuit voltage and resistance (v) The initial armature vol-tage and current (vi) The amount of resistance cut in or out of the field circuit (vii) The load on the exciter during build up or build down. This load is determined not only by the characteristics and the field circuit of the main generator but also by the transient field current induced by sudden changes and the armature current of the main generator, (viii) Whether the rate of response is for build up or for build down.

Even with the foregoing conditions fixed the response varies from instant to instant during the process of build up or build down. The exciter response for a particular exciter, therefore, depends upon a number of factors. For approximate comparison of the effectiveness of diff. excitation systems it is, therefore, necessary to define a '<u>nominal excit-er response</u>' which is a definite quantity for a par-ticular excitation system.

__Nominal exciter response__:- This is defined as the 'numerical value obtained when the nominal collector ring voltage is divided into the slope, expressed in volts per second, of that straight line voltage-time curve which begins at nominal collector ring voltage and continues for one half second, under which the area is the same as the area under the no-load voltage increase time curve of the exciter star-ting at the same initial voltage and continuing for the same length of time.'

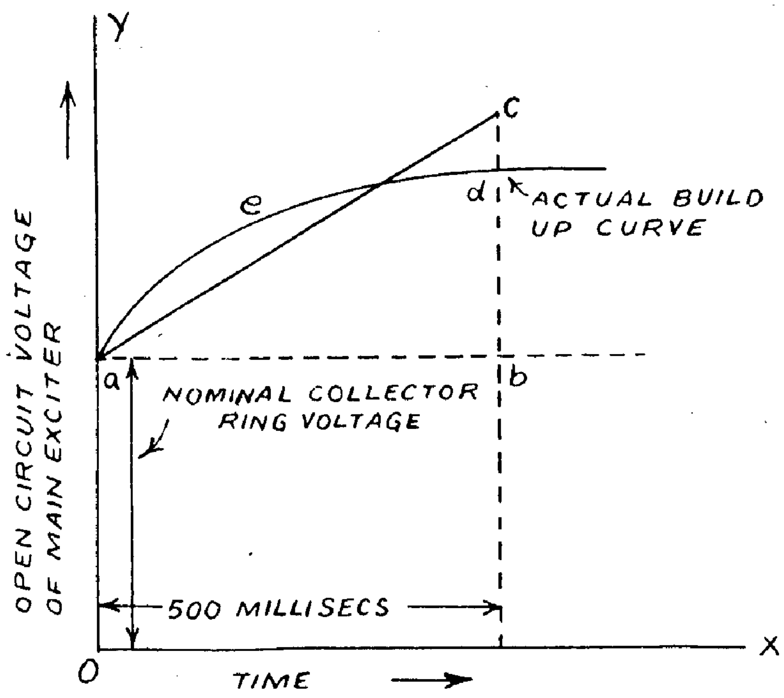


FIG. 1.1

The actual voltage build up curve a e d, starts at nominal collector ring voltage o a, where the reference time represented by the ordinate o r is reckoned as the instant at which which the regulating resistance is short circui-ted. Straight line a c, is drawn so that the area under it is equal to that under the actual build up curve during the first half seco--nd i.e. area a b c = area a b d e. The nominal exciter response is then the slope of line a c, in volts per second divided by the nominal collector ring voltage in volts.

While defining the nominal exciter response, the area under the voltage build up curve is taken ; since when the exciter voltage is applied to the field of the main generator of which the resistance is small the change in flux produced is given by

$$\Delta\phi = \int_{o}^{t} e\,dt$$

In practice the quick response excitation is applied for a time of the order of 500 msecs , and hence t is taken as 500 msecs.

The nominal response is defined for build up rather than build down because during and immediate-ly after the presence of a fault it is necessary to incr--ease field linkages of the main generator to improve system stability which necessitates an increase in exciter voltage.

Also, since change of voltage caused by load is not great whereas exciter response can be more easily calculated or determined from test in case of unloaded exciters, the above definition of nominal exciter response is on the basis of unloaded exciter.

1.2. DETERMINATION OF EXCITER RESPONSE FROM DESIGN DATA USING POINT BY POINT SOLUTION

From considerations of power system stability the excitervoltage build up is more impor-tant than voltage build down because in case of faults or other system disturbances it is necessary for the field flux linkages of the main generator to be increased. Though the exciter response for load-ed exciter is what is actually wanted, the calculations are much simpler for unloaded than for the loaded exciter. In case of separately excited exciters the voltage-time curves for the loaded and un-loaded conditions differ by a few per cent only.

For these reasons the build up of voltage of unloaded exciters would be analysed first and that of loaded exciters will be considered later.

RESPONSE OF UNLOADED EXCITER:- P.T.O.

When a shunt or compound wound d.c. generator is used as exciter, the exciters may operate either self excited or separately excited. The excitation of a separately excited exciter may be supplied by a rectifier or storage battery but ordinarily a pilot exciter is used. The pilot exciter is either a flat compounded d.c. generator or a rotating amplifier. The main exciter is directly coupled to the main generator and the system is known as <u>unit exciter scheme</u>. The circuit diagrams for separate excitation and self excitation of the main exciter are given below :

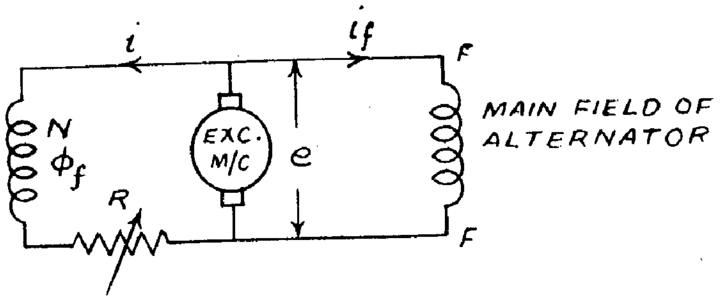


Fig. 1.2A: <u>Self excited exciter.</u>

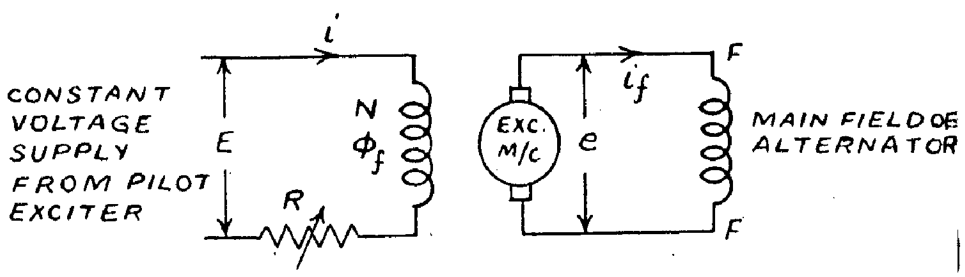


Fig. 1.2B: <u>Separately excited exciter.</u>

The equations of the voltages around the field circuit and the main exciter will be

$$N \frac{d\phi f}{dt} + Ri = e \quad \text{for self excited case.} \quad \text{------ 1.1.}$$

and

$$N \frac{d\phi f}{dt} + Ri = E \quad \text{for separately excited case.} \quad \text{------ 1.2.}$$

where N is the number of field turns in series

ϕf is field flux in webers

R= is the field circuit resistance in ohms

i= field current in amperes

E= pilot exciter e.m.f.

e = main exciter armature e.m.f.

Presently it is assumed that the pilot exciter is ideally flat compounded so that E is constant. ϕf is the total field flux per core. Part of it does not cross the air gap and link the armature conductors and constitutes the leakage flux, ϕ_L. The rest of the flux ($\phi f - \phi_L$) crosses the air gap and links the armature conductors thus constitut-ing the useful flux ϕa. This induces an e.m.f. $e = \frac{\phi_a ZN}{60} \cdot p/a$ volts in the armature of the main exciter or $e = K.\phi a$. where $K = \frac{ZNP}{60a}$

Where Z = total number of armature condtrs.

N = speed of rotation of exciter(RPM)

p = number of poles

a = number of parallel paths through the armature.

Assuming leakage fluxes to be confined who-lly in air paths, effect of saturation to a certain approximation can be neglected. Leakage flux can, therefore, be considered as being proportional to the field current its effect being taken into account by assuming an equivalent constant leakage inductance L_L henries. Since $\phi f = \phi a + \phi_L$

$$\phi f = \phi a + \frac{L_L . i}{N}$$

$$\frac{d\phi f}{dt} = \frac{d\phi a}{dt} + \frac{L_L}{N} \cdot \frac{di}{dt}$$

or $$\frac{d\phi f}{dt} = \frac{1}{k} \cdot \frac{de}{dt} + \frac{L_L}{N} \cdot \frac{di}{dt}$$

Combinwing this in equations (1.1) and (1.2) gives.

$$\frac{N}{K} \cdot \frac{de}{dt} + L_L \frac{di}{dt} + Ri = e \quad \ldots 1.3$$ for self excited case.

and

$$\frac{N}{K} \cdot \frac{de}{dt} + L_L \frac{di}{dt} + Ri = E \quad \ldots 1.4$$ for separately excited case.

e is a nonlinear function of i given by the magnetisation characteristics A solution for <u>e</u> as a function of time would give the required response.

SOLUTION BY POINT BY POINT CALCULATION

1.2A. SELF EXCITED EXCITER:-

The differential equation is given by

$$\frac{N}{K} \cdot \frac{de}{dt} + L_L \frac{di}{dt} + Ri = e$$

On replacing the time derivative $\frac{de}{dt}$ and $\frac{di}{dt}$, b ratios of finite increments,

$\frac{\Delta e}{\Delta t}$ and $\frac{\Delta i}{\Delta t}$ respectively

and solving for Δt gives

$$\frac{N}{K} \cdot \frac{\Delta e}{\Delta t} + L_L \cdot \frac{\Delta i}{\Delta t} + Ri = e$$

$$\text{or } \frac{1}{\Delta t}\left[\frac{N}{K} \cdot \Delta e + L_L \Delta i\right] = e - Ri$$

$$\text{or } \Delta t = \frac{\frac{N}{K} \cdot \Delta e + L_L \Delta i}{e - Ri} \qquad \text{-------.. 1.5}$$

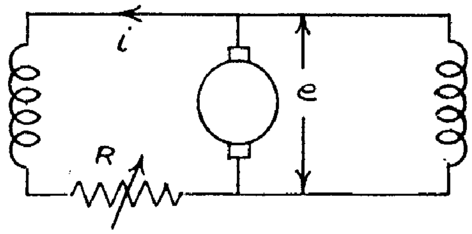


Fig. 1.3.A.

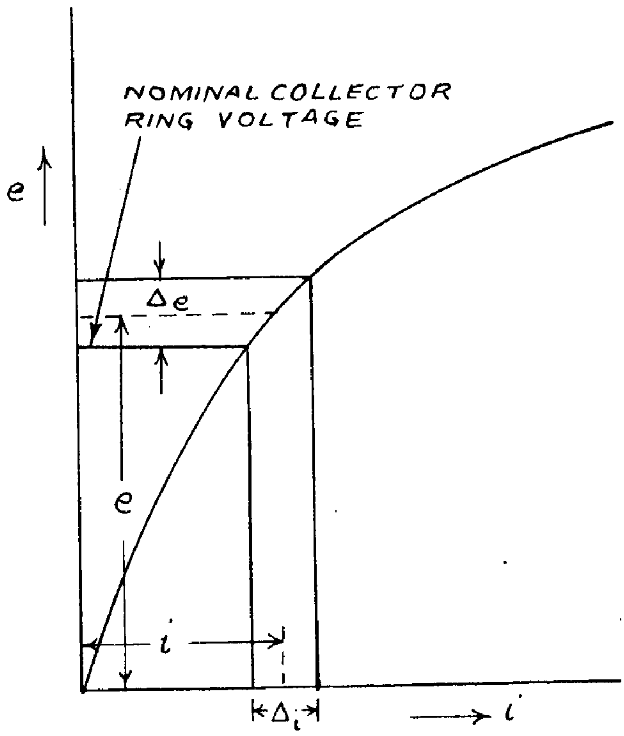


Fig. 1.3.B.

The ordinates of the magnetization curve are divided into increments Δe. above the nominal collector ring voltage. The corresponding increments in i i.e. Δi are found out. The average values for the interval for e and i are used in the de-nominator of equation 1.5, to find out the time t , required for the voltage to rise by Δe. The curve of voltage versus time can be plotted from the incre-ments of voltage and time starting at zero time with the known initial value of voltage. During the last intervals smaller intervals of Δe, can be taken for obtaining an accurate response. The use of average values of e and i instead of values at the begi-nning of an interval greatly reduces the cumulative error.

1.2B. SEPARATELY EXCITED CASE:-

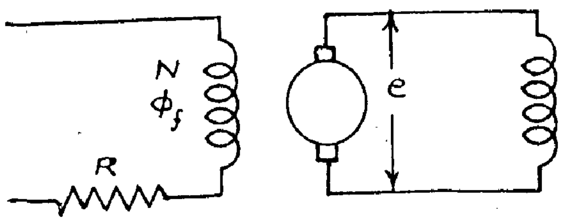


Fig. 1.4

The differential equation is given by

$$\frac{N}{K} \cdot \frac{de}{dt} + L_L \frac{di}{dt} + Ri = E \ldots\ldots.$$

Replacing $\frac{de}{dt}$ by $\frac{\Delta e}{\Delta t}$ and $\frac{di}{dt}$ by $\frac{\Delta i}{\Delta t}$ and for Δt gives

$$\Delta t = \frac{(N/K).\,\Delta e + L_L \,.\, \Delta i}{E - Ri} \quad \text{-----1.6}$$

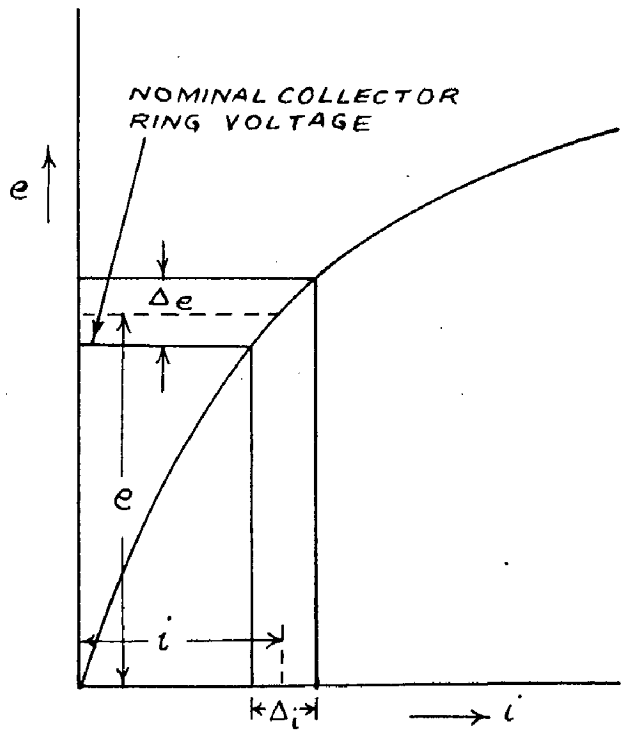


Fig. 1.5

Constant value for E is taken. The time required Δt for the armature voltage to change by an amount Δe is found and the response can be plotted.

R is the field circuit resistance in ohms and remains in circuit after a certain amount of regu--lating resistance r has been cut out by the act--ion of the voltage regulator.

1.3. CALCULATION OF FIELD LEAKAGE INDUCTANCE L_L.

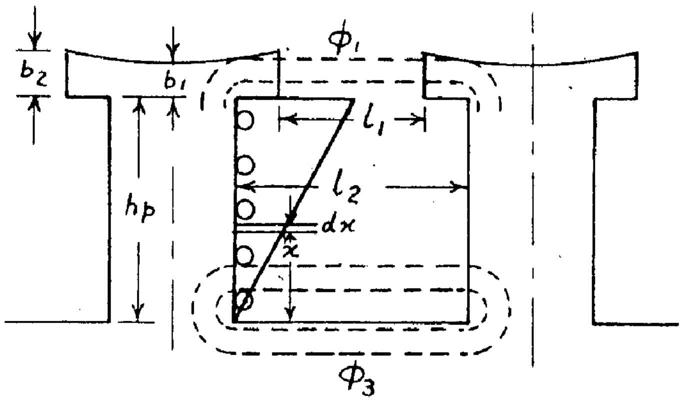


Fig. 1.6.

Let 0.4π AT = $0.4\pi\, I_f.T_f$. be the total m.m.f. due to one pole = M_L say. This would incr--ease linearly from zero to M_L at the pole shoe as shown.

1.31. Inductance due to leakage flux between Pole shoe to pole shoe

$$\phi_{1/pole} = 2X \frac{M_L}{\text{Reluctance}} = 2X \frac{M_L}{L_1/bL_p} = \frac{0.8\pi I_f T_f . bL_p.}{L_1}$$

This links with all the field turns T_f. and hence linkages due to this $= \frac{0.8\pi \; I_f \; T_f^2 \; b.L_p}{L_1}$.

Inductance due to this = $\frac{\text{Linkages per}}{\text{ampere of currents CAUSING THE flux}}$

$$= \frac{0.8\pi T_f^2 \; b.L_p}{L_1} \text{henries}$$

--------..1.7.

1.3.2. Inductance due to leakage flux between shaft to shaft

Considering a strip of height dx at a distance X from the bottom, m.m.f. = $0.4\pi \; I_f T_f . \frac{X}{h_p}$.

Reluctance of the strip = $\frac{L_p. \;\; dx}{L_2}$.

Leakage flux in the strip = $0.4\pi I_f T_f . \frac{L_p}{h_p L_2} . X dX$.

This links with T_f. X/h_p number of field turns.

Linkages = $0.4\pi \; I_f T_f^2 \; \frac{L_p.}{h_p^2 \; L_2} . X^2 . dx.$

$$\text{Total linkages} = 0.4\pi I_f T_f^2 \frac{L_p}{h_p 2L_2} \int_0^{h_p} X^2 dx$$

$$= 0.4\pi I_f T_f^2 . \frac{L_p . h_p^3 .}{3h_p^2 . L_2}$$

$$\text{Total linkages per pole} = \frac{0.4\pi I_f T_f^2 L_p . h_p}{3 . L_2}$$

$$= \frac{0.8\pi I_f T_f^2 L_p h_p}{3L_2}$$

Inductance due to this

$$= \frac{0.8\pi T_f^2 L_p . h_p}{3L_2} \text{ henries}$$

---------..1.8.

1.3.3. Inductance due to Leakage flux between Pole shoe to pole shoe at end.

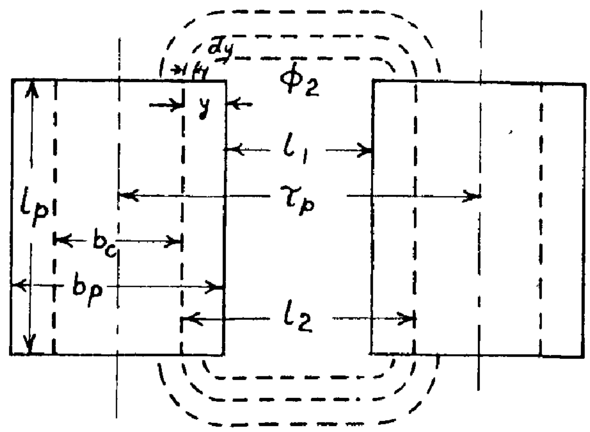


Fig. 1.7.

For purposes of calculation it is assumed that flux lines follow paths which are circular of radius y and then straight lines of lengths L_1. Such leakages take place at both ends and hence at any distance y.

Leakage flux per pole in the strip of width by

$$= 2 \quad ML. \quad \frac{b\,dy}{\pi y + L_1}$$

$$= \frac{0.8\pi \, I_f T_f^2 \, b \, dy}{\pi y + L_1}.$$

For a strip dy considered at a distance y from the other end there will be linkages =

$$\frac{0.8\pi \, I_f . T_f^2 \, b \, dy}{\pi y + L_1}$$

Total linkages per pole

$$= 1.6\pi \, I_f T_f^2 \, b \int_0^{b_p/2} \frac{dy}{\pi y + L_1}$$

$$= 1.6\pi \, I_f T_f^2 \, b \, \frac{1}{\pi} \Big[\log_e(\pi y + L_1) \Big]_0^{b_p/}$$

$$= 1.6 \; I_f \, T_f^2 \, \underline{b} \times 2.3.\log_{10}(1+\pi/2.b_p/L_1)$$

$$= 3.6 \; I_f \, T_f^2 \, b \, \log_{10}(1+ \pi/2 \; b_p/L_1)$$

Inductance due to this

$$= 3.6 \; T_f^2 \, b \, \log_{10} \; (1+ \pi/2.b_p/L_1) \qquad \text{--------.. 1.9.}$$

When a constant air gap pole is not used b is not the same at the centre and the ends and is taken $= \frac{b_1 + b_2}{2}$.

1.3.4. <u>Inductance due to leakage flux at ends from pole shaft to pole shaft:</u>

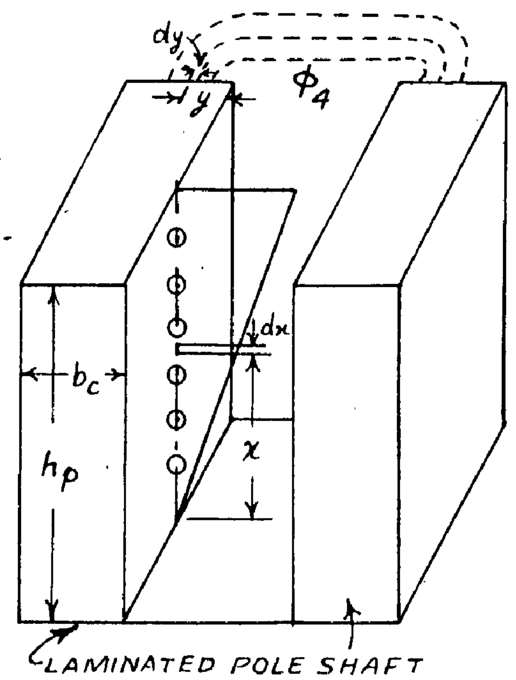


Fig. 1.8.

Leakage fluxes such as $\phi 4$ would be present along the length h_p of the pole shaft.

At ht. x the mmf. causing leakage flux in strip dx

$$= \frac{0{\cdot}4\pi \, I_f T_f \, x}{h_p}$$

Reluctance offered to this leakage flux

$$= \frac{\pi y + L_2}{dx.\ dy.}$$

Flux in width dy $= 0.4\pi\ I_f T_f . \frac{X}{h_p} . \frac{dz\ .\ dy}{\pi y + L_2}$

This links with $\frac{T_f . X}{h_p}$ no. of field turn

and hence linkages

$$= 0.4\pi\ I_f . T_f^2 . \frac{X^2}{h_p^2} .\ dx\ .\ \frac{dy}{\pi y + L_2}$$

Linkages in width bc/2 at a ht. x

$$= 0.4\pi\ I_f . T_f^2 . \frac{X^2 dx}{h_p^2} . \int_0^{bc/2} \frac{dy}{\pi y + L_2}$$

$$= 0.4\pi\ I_f T_f^2 . \frac{X^2 dX}{h_p^2} . \frac{2.3}{\pi} \log_{10}(1 + \pi/2\ bc/L_2)$$

$$= \frac{0.9\ I_f . T_f^2 \log_{10}(1+\pi/2\ bc/L_2) . X^2 dX}{h_p^2}$$

For ht hp the linkages

$$= \frac{0.9\ I_f T_f^2 \log_{10}(1+\pi/2\ bc/L_2)}{h_p^2} \frac{h_p^3}{3}$$

$$= 0.9 . I_f . T_f^2 . \frac{h_p}{3} \log_{10}(1+\pi/2 . bc/L_2)$$

Per pole core there will be 4 times of this linkages

$$= 3.6\, I_f\, T_f^2 . \frac{h_p}{3.} \log_{10} (1 + \pi/2\, bc/L_2)$$

--------.. 1.10.

$\therefore$ Inductance due to this =

$$3.6. T_f^2 \frac{h_p}{3}. \log (1 + \pi/2 \frac{bc}{L_2})$$

Total leakage inductance per pole

$$L_L = 0.8\pi\, T_f^2 (\frac{b. L_p}{L_1} + \frac{L_p\ h_p}{3L_2}) + 3.6. T_f^2 .$$

$$\left[b \log_{10}(1+\pi/2 \frac{bp}{L_1}) + \frac{hp}{3.} \log_{10}(1+\pi/2 \frac{bc}{L_2}) \right]$$

henries

---------1.11

1.4. CALCULATION OF RESPONSE BY POINT BY POINT CALCULATION ASSUMING CONSTANT COEFFICIENT OF DISPERSION

In the foregoing analysis leakage flux was assumed proportional to the field current and its effect was taken by assuming a constant equivalent leakage inductance. Effect of saturation on leakage flux was neglected by assuming leakage flux limes to be confined mostly in air paths. This assumption is not wholly justified because leakage flux is to some extent affected by saturation though not to the s extent as armature flux. The exact extent to which leakage flux is affected by saturation is not easily estimatable and hence implifying assumptions are made. To take into account the effect of saturation the leakage flux is taken proportional to the armat=-ure flux. This definitely over estimates the effect of saturation on leakage flux because the leakage flux is less affected by saturation than is the arma-ture flux because the highest saturation is found in the teeth and the former passes mostly over air paths. The exciter response however calculated by this assum-ption does not lead to much error.

The field flux $\phi_f = \phi_a + \phi_l$ is taken $= \sigma \phi_a$ ----- 1.12.

Where σ is the coefficient of dispersion, its value ranging from 1.1 to 1.2.

The equations for voltage around the field circuit of the exciter were

$$N \frac{d\phi_f}{dt} + Ri = E \text{ for separately excited excit}$$

and $$N \frac{d\phi_f}{dt} + Ri = e \text{ for self excited exciter.}$$

On substituting equation 1.12 we get

$$N\sigma \frac{d\phi_a}{dt} + Ri = E \text{ or } e.$$

Also $$e = \frac{\phi_a . Zn}{60} . p/a = K \phi_a.$$

$$\therefore \frac{N\sigma}{K} . \frac{de}{dt} + Ri = E \text{ or } e$$

On replacing $\frac{de}{dt}$ by $\frac{\Delta e}{\Delta t}$

$$\frac{N\sigma}{K} \cdot \frac{\Delta e}{\Delta t} + Ri = E \text{ or } e.$$

$$\therefore \Delta t = \frac{\frac{\sigma N}{K} \Delta e}{e - Ri} \quad \text{for self excited case} \; \ldots 1.$$

$$\text{and } \Delta t = \frac{\frac{\sigma N}{K} \cdot \Delta e}{E - Ri} \quad \text{for separately excited case} \ldots 1.14$$

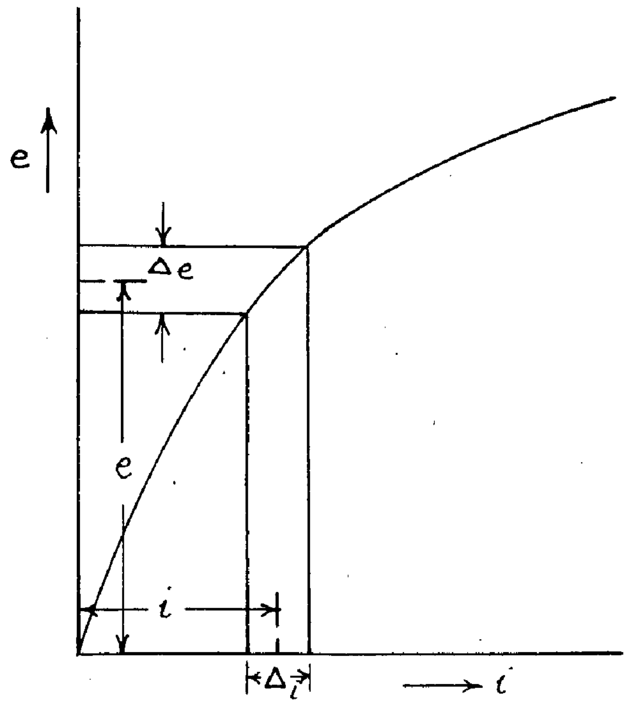


Fig. 1.9.

Finite increments of say 10 or 5 volts are given to Δe and corresponding average values of e and i are found.

Δt for each increment is determined 7 which is the time required for armature voltage to change by an amount Δe. The curve of voltage versus time can be plotted from the increments of voltage and time starting at zero time with the known initial value of voltage.

In case of separately excited exciter e need not be determined since the constant value E of the pilot exciter is to be used in the denomenators of the equation

$$t = \frac{(6N/K)\ \Delta e}{E - Ri}$$

In a similar way response for this can also be plotted.

1.5. CALCULATION OF NOMINAL EXCITER RESPONSE

By using either of the methods described earlier it is possible to obtain the voltage build up curve starting at nominal collector ring voltage of the main machine. The area under this curve during the first 500 milliseconds can be measured by a Planimeter or found by counting squares on the graph paper. If the curve has been calculated by the point by point method, the area under it is readily calculated by summing the products of each Δt by the average value of voltage e for the interval.

Referring to the definition of nominal exciter response it is necessary to find the slope of that straight line under which the area during 0.5 secs. is equal to the area under the voltage time curve during the same time. The slope of such a straight line is given by $\frac{bc}{ab} = \frac{bc}{0.5} = 2bc. = m$ say (Ref. Fig. 1.1.)

If the areas under the line and under the curve are equal they will still be equal if the area of the rectangle below nominal collector ring voltage is substracted from each. Let the remaining area abdea under the curve and above this voltage be denoted by A.

$$A = \frac{1}{2}(ab)(bc) = \frac{1}{2}(0.5)\ bc = \frac{bc}{4}.$$

The slope of the straight line ac. expressed in turns of the area A is m = 8A.

It is not necessary actually to draw line ac, because its slope can be found by measuring area abdea under the exciter build up curve and multiplying it by 8.

Finally the nominal exciter response is the slope thus found divided by nominal collector ring voltage.

CHAPTER II.

CALCULATION OF RESPONSE BY METHOD OF GRAPHICAL INTEGRATION

2.1 SEPARATELY EXCITED EXCITERS:-

2.1A Assumption of constant coefficient of dispersion:

The differential equation for such a case was given by

$$\frac{\sigma N}{K} \cdot \frac{de}{dt} + Ri = E$$

where N = number of field turns in series

$K = \frac{Znp}{60.a.}$

and σ = constant coefficient of dispersion.

Rearranging the above gives

$$\frac{\sigma N}{K} \cdot \frac{de}{dt} = E - Ri.$$

Let the armature voltage and field current build up from a point on the magnetisation curve having coordinates i_1 and e_1 to a point having coordinates i_2 and e_2

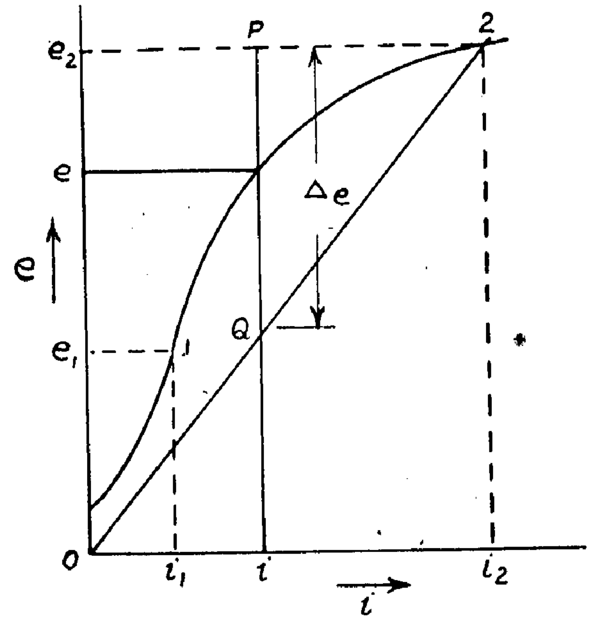


Fig. 2.1.

Multiplying the above equation on both sides by e_2/E gives

$$\frac{\sigma N}{K} \cdot \frac{e_2}{E} \cdot \frac{de}{dt} = e_2 - \frac{Re_2}{E} \cdot i.$$

R is the resistance of the field circuit ar and remains in field circuit after the regulating resistance has been short circuited.

$$E/R = i_2$$

$$\therefore \quad \frac{\sigma N}{K} \cdot \frac{e_2}{E} \cdot \frac{de}{dt} = e_2 - \frac{e_2}{i_2} \cdot i \qquad \text{--------..2.1}$$

If a straight line is drawn from the origin to the final operating point (i_2, e_2) then the slope of such a line would be e_2/i_2. Corresponding to any pt(e,i) on the magnetisation curve QR would represent $\frac{e_2}{i_2} \cdot i$ and PQ would represent $e_2 - \frac{e_2}{i_2} \cdot i$. This can be determined for different values of i and this may be designated by Δe . The quantity $\frac{\sigma N e_2}{KE}$ has the dimensions of time and may, therefore, be considered as the ' time constant'. Due to the magnetisation curve being nonlinear the build up of the armature voltage e or the field current i is not exponential and hence T can not be given the same interpretation as the time constant of a linear circuit.

The differential equation may therefore, be put in the form $T \frac{de}{dt} = \Delta e$

where Δe is a function of e.

$$\text{Therefore} \quad dt = T \frac{de}{\Delta e}$$

$$\text{and} \quad t = T \int_{e_1}^{e} \frac{de}{\Delta e} \quad \text{------..2.2.}$$

This is the time required for the voltage to build up from its initial value e_1 to any value e. As explained earlier Δe for different values of e can be obtained and hence $1/\Delta e$ can be plotted as a function of e.

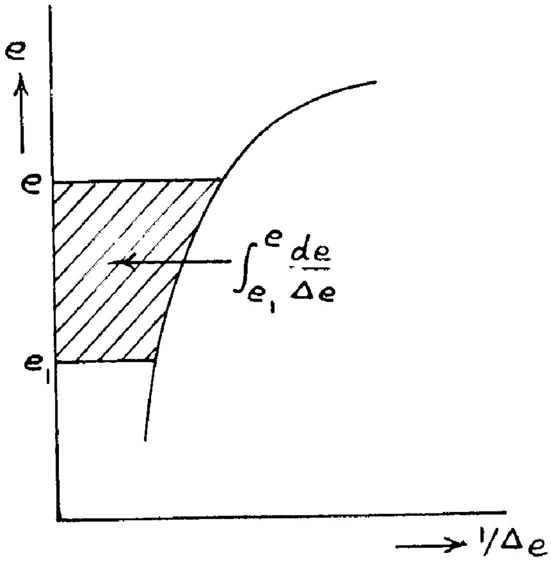


Fig. 2.2.

The integral $\int_{e_1}^{e} \frac{de}{\Delta e}$ is given by the area between the curve and the e axis between the lower limit e_1 and a running upper limit e. Thus the above integral can be graphically integrated and the time required t for the voltage to build up to a value e can be determined and the build up response can be plotted.

2.1B EFFECT OF SATURATION ON FIELD LEAKAGE NEGLECTED:

Assumption of condtant leakage inductance:

The differential equation on this assumption for separately excited exciters was derived earlier and was found to be

$$\frac{N}{K} \cdot \frac{de}{dt} + L_L \frac{di}{dt} + Ri = E$$

On rearranging this may be written as

$$\frac{N}{K} \cdot \frac{d}{dt} \left(e + \frac{L_L K}{N} . i \right) = E - Ri$$

$\frac{L_L K}{N}$ is a constant which when multiplied by i will give a certain voltage which may be designated by e'.

Hence $\frac{N}{K} \cdot \frac{d}{dt} (e + e') = E - Ri$

Multiplying both sides by e_2/E gives

$$\frac{Ne_2}{KE} \cdot \frac{d}{dt} (e + e') = e_2 - \frac{Re_2}{E} . i.$$

or $$T \frac{d}{dt}(e + e') = \Delta e. \qquad \text{-------.. } 2.3$$

Where $T = \frac{Te_2}{KE}$ and $\Delta e = e_2 - \frac{Re_2}{E} . i.$

The time constant in this case is defined differently than it was for the analysis based on constant coefficient of dispersion. Δe is however the same and may be given the same interpretation.

A straight line such as ON having a slope of $\frac{L_L K}{N}$ is drawn. Corresponding to any value of i, PQ gives Δe and MS gives $e + \frac{L_L K}{N} i$ $= (e + e')$.

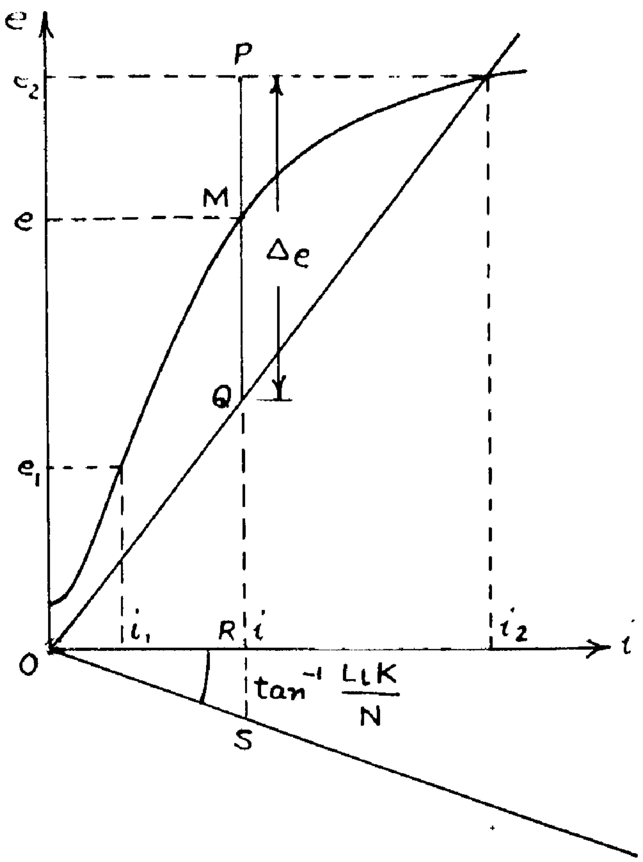


Fig. 2.4.

$\frac{1}{\Delta e}$ can be plotted as a function of $e + e'$

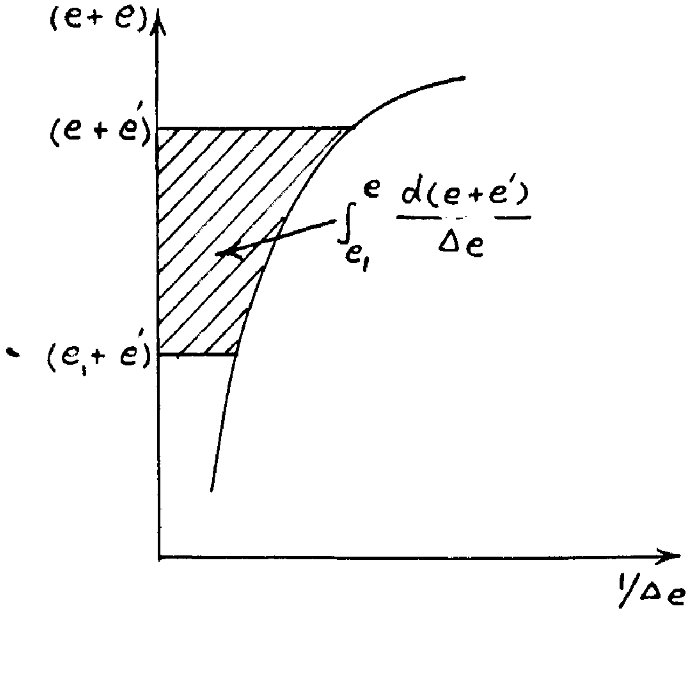


Fig. 2.5.

The integral $t = T \int_{e_1}^{e} \frac{d(e \ +e')}{\Delta e}$

can be evaluated by measuring the hatched area , for any value of e and subsequently the response can be plotted.

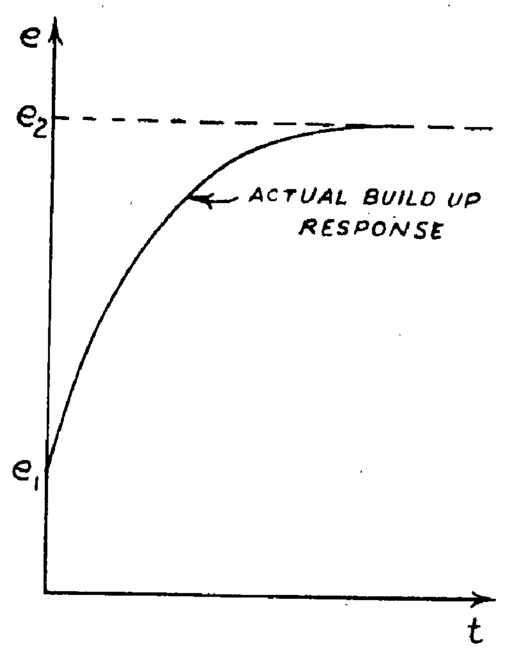


Fig. 2.6.

2.2. RESPONSE OF SELF EXCITED EXCITERS BY THE METHOD OF GRAPHICAL INTEGRATION

2.2A Assumption of constant coefficient of dispersion:

The differential equation for such a case was found to be

$$\frac{N}{K} \cdot \frac{de}{dt} + Ri = e.$$

$$\text{or} \quad \frac{N}{K} \cdot \frac{de}{dt} = e - Ri \quad \text{or} \quad T\frac{de}{dt} = \Delta e \qquad \text{--------..2.4}$$

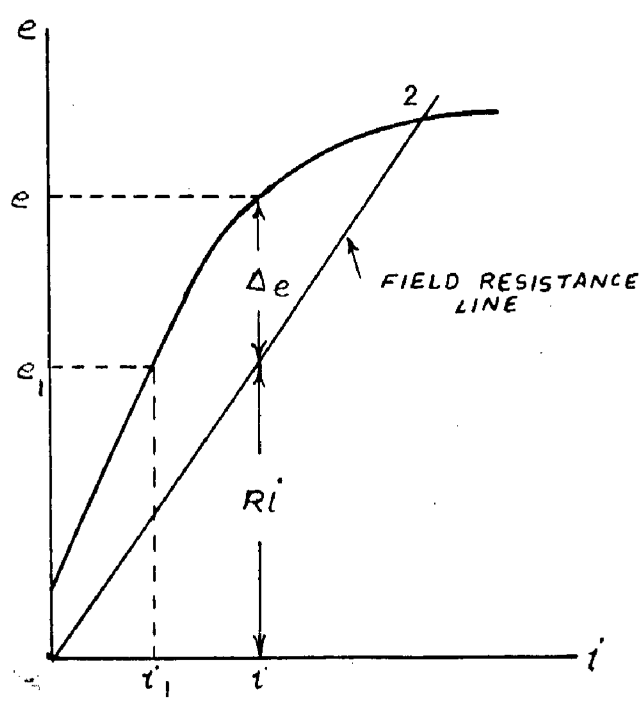


Fig. 2.7.

If the regulating resistance is suddenly short circuited at a point on the magnetisation curve given by e_1 , i_1 then depending upon the actual field circuit resistance R, voltage would build-up, up to the point given by the intersection of the magnetisation curve and the field resistance line. This point of intersection gives the final point (2) upto which build up in case of self excited exciters would take place.

Corresponding to any field current e, Ri, and hence Δe can be found. And $1/\Delta e$ can be plotted as a function of e

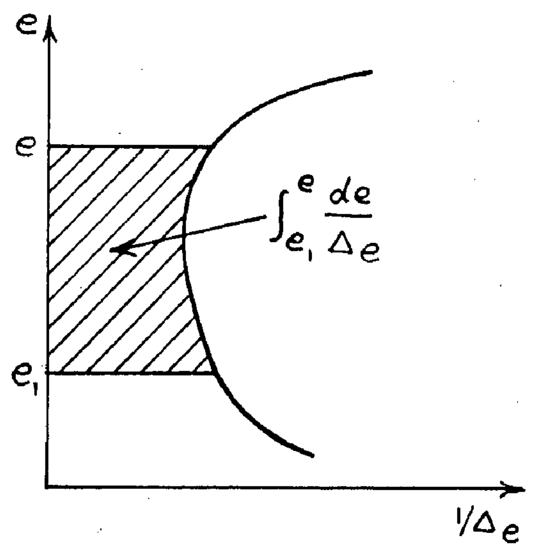


Fig. 2.8.

The time $t = T \int_{e_1}^{e} \frac{de}{\Delta e}$ required for the voltage to build up from the initial value e_1 to a certain value e can be determined by finding out the hatched area shown above and multiplyigg it by T.

The time constant for the self excited exciter is different from that of the separately excited exciter in that it is in-dependant of the final value of the armature voltage. The response can subsequently be plotted.

2.2B ASSUMPTION OF CONSTANT LEAKAGE INDUCTANCE

The differential equation for such a case was derived as

$$\frac{N}{K} \cdot \frac{de}{dt} + L_L \frac{di}{dt} + Ri = e$$

$$\text{or} \quad \frac{N}{K} \frac{dt}{dt} \left(e + \frac{L_L . K.}{N} i \right) = e - Ri$$

$$\text{or} \quad \frac{N}{K} \cdot \frac{d}{dt} (e + e') = (e - Ri)$$

$$\text{or} \quad T \frac{d(e + e')}{dt} = \Delta e. \quad \text{---------..2.5}$$

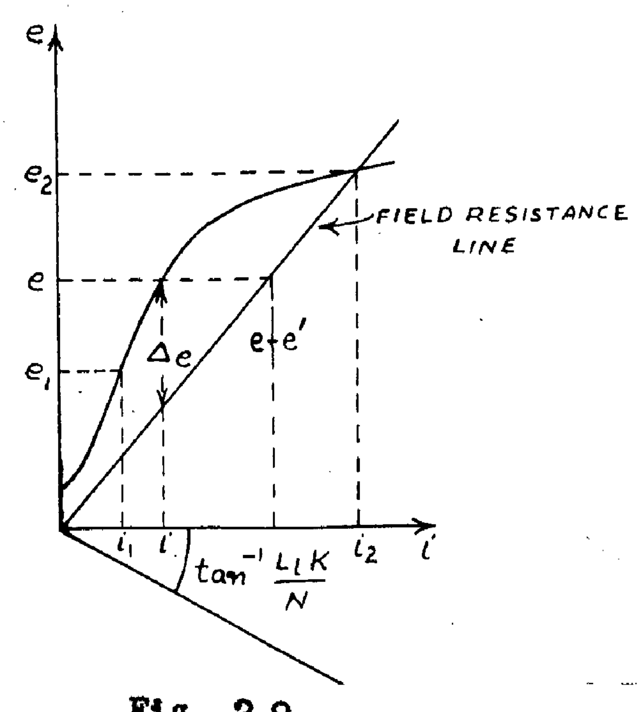


Fig. 2.9.

Corresponding to any value of i, Δe and $(e + e')$ can be found.

$\frac{1}{\Delta e}$ can be plotted against $(e + e')$

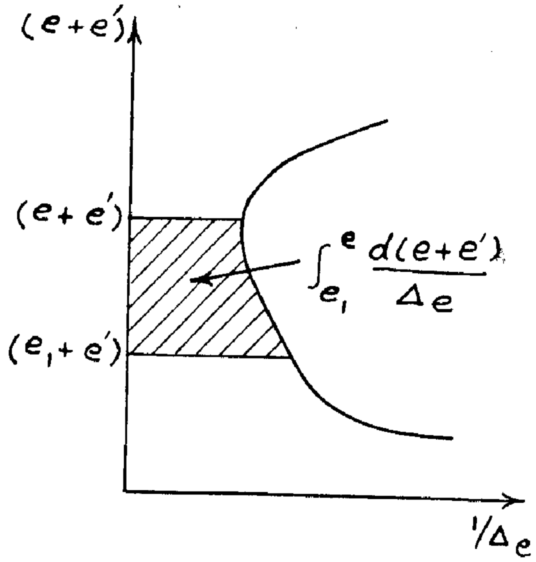


Fig. 2.10.

$$t = T \int_{e_1}^{e} \frac{d(e+e')}{\Delta e}$$

The time t required for the exciter armature voltage to rise from e_1 to e is determined and subsequently the build up exciter response can be plotted.

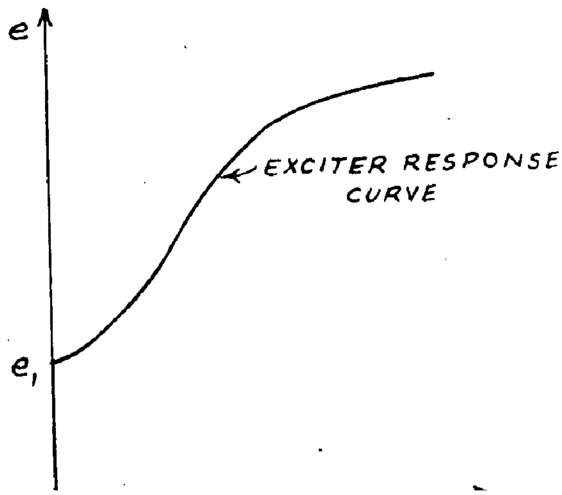


Fig. 2.11.

2.8. ALTERNATIVE METHOD OF CALCULATION OF EXCITER RESPONSE USING STEP BY STEP INTEGRATION:

The differential equation for the voltage around the field circuit of the exciter can be expressed as

$$Ri + \frac{d\psi}{dt} = e \text{ or } E$$

E is the voltage of the pilot exciter and is to be used in case of separately excited exciters

$$\frac{d\psi}{dt} = e \text{ or } E - Ri \quad \text{...............} 2.6$$

ψ is the total field flux linkages and consists of the useful air gap flux linkages which is proportional to the exciter armature e.m.f. e and the field leakage flux which upon assumption of no saturation may be considered as proportional to the field current.

$\therefore \psi = K_1 e + K_2 i$ where K_1 and K_2 are constants.

Equation 2·6 shows that the time rate of rise of ψ is proportional at any instant to a forcing voltage which is given by the vertical distance between the terminal voltage curve and the field resistance line corresponding to any given field current.

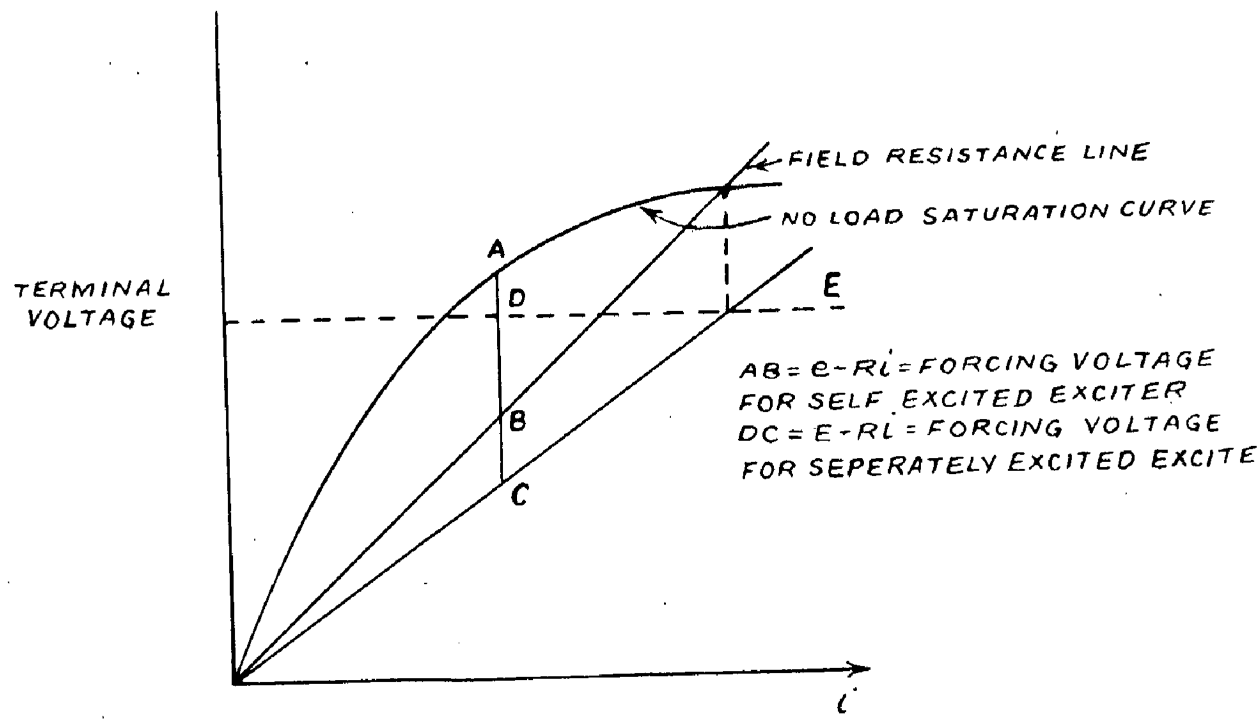


Fig. 2.12.

Since $\frac{dt}{d\psi} = \frac{1}{e \text{ or } E-Ri}$ from eqn. 2.6 ---------..2.7

and $\psi = K_1 e + K_2 i$ ---------..2.8

for different values of i, ψ and $\frac{dt}{d\psi}$ can be obtained. $\frac{dt}{d\psi}$ can be plotted as function of ψ

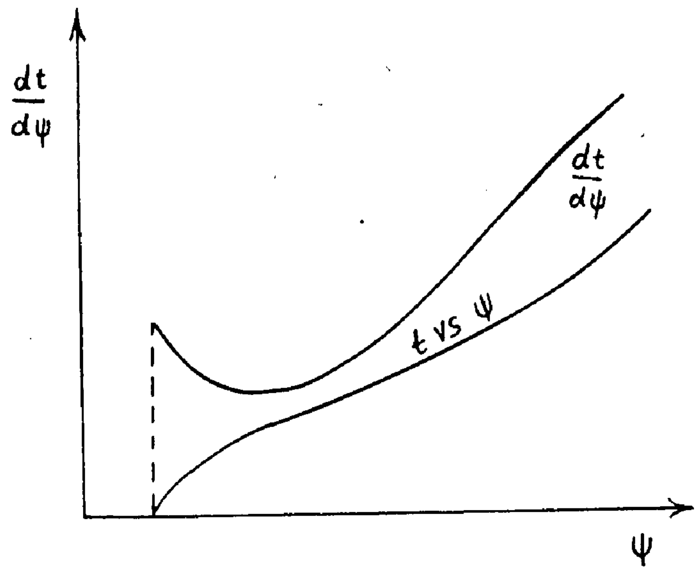


Fig. 2.13.

In order to integrate this curve ψ is divided into increments of unit width. Increments of time Δt are determined. From this, time required for the flux linkage to increase to a certain value ψ can be found. A curve of ψ against t can be plotted.

After ψ is obtained e can be plotted as a function of time which will give the required exciter repponse.

CHAPTER III.

SOLUTION BY FORMAL INTEGRATION

The differential equations of exciter armature voltage and field current were derived earlier as

$$\sigma \frac{N}{K} \cdot \frac{de}{dt} + Ri = E \text{ or } e \quad \text{(assumed constant coefficient of dispersion)} \text{----3.1.}$$

and
$$\frac{N}{K} \cdot \frac{de}{dt} + L_L \cdot \frac{di}{dt} + Ri = E \text{ or } e \quad \text{(assumed const. leakage inductance)} \text{-----3.2.}$$

E is the voltage of the pilot exciter and i to be used in case of separetely excited exciters.

In these equations e is a nonlinear function of i as given by the magnetisation curve and hence represent differential equations with variable coefficients. A solution by formal integration requires that the variable coefficients can be expressed in terms of one of the variables and that the resulting expression can be integrated by formal methods. This means that the magnetisation curve must be represented by an emperical equation such as the

Fröhlich equation $e = \frac{ai}{b+i}$ ----------3.3.

or the modified Fröhlich equation

$$e = \frac{ai}{b+i} + ci \text{ ---------3.4.}$$

Formal Analytical Solutions in Case of Separately excited exciters would be attempted:

1.1. Constant leakage Inductance:

Frohlich equation gives $e = \dfrac{ai}{b+i}$

On differenting w.r.t. time we get

$$\frac{de}{dt} = \frac{ab}{(b+i)^2} \cdot \frac{di}{dt}.$$

Substituting this in equation 3.2 gives

$$\left[\frac{Nab}{K(b+i)^2} + L_L\right]\frac{di}{dt} + Ri = E \text{ ------.3.5}$$

This when integrated would yield a solution in turns of current. On rearranging equation 3.5 we get

$$\left[\frac{Nab}{K(b+i)^2} + L_L\right]di = (E - Ri)\, dt$$

or $$\frac{Nab}{K(E - iR)(b+i)^2}\, di + \frac{L_L}{E-Ri} \cdot di = dt.$$

The first term $\dfrac{1}{(E - iR)(b+i)^2}$ can be split into partial fractions

$$\text{as} = \frac{A}{E - iR} + \frac{B}{(b+i)^2} + \frac{C}{(b+i)}$$

The coefficients A, B, C can be evaluated from the following equations $A - CR = 0$

$$A\,2b - BR + C(E - bR) = 0$$

$$A\,b^2 + BE + CEb = 1$$

The coefficients are solved for as

$$A = \frac{R^2}{(E + Rb)^2}$$

$$B = \frac{1}{(E + Rb)}$$

and $$C = \frac{R}{(E + Rb)^2}$$

Substituting in the main equation we get

$$\frac{Nab\ R^2}{K(E + Rb)^2(E - iR)^2}\,di + \frac{Nab}{K(E + Rb)(b + i)^2}\,di$$

$$+ \frac{Nab\ R}{K(E + Rb)^2(b + i)}\,di + \frac{L_L}{(E - iR)}\,di = dt.$$

When integration is carried out we get

$$- \frac{Nab\ R}{K(E + Rb)^2}\log(E - iR) - \frac{Nab}{K(E+bR)(b+i)}$$

$$+ \frac{Nab\ R}{K(E + Rb)^2}\log(b + i)$$

$$- \frac{L_L}{R}\log(E - iR) = t + C$$

Where C is a constant of integration.

The equation can be rewritten as

$$\frac{Nab}{-K(E+R_b)(b+i)} + \frac{Nab\ R}{K(E+R_b)^2}\log(b+1)$$

$$-\left[\frac{Nab\ R}{K(E+Rb)^2} + \frac{L_L}{R}\right]\log(E-iR) = t + C. \text{---} 3.$$

To simplify let

$$K_1 = \frac{Nab}{K(E+Rb)}$$

$$K_2 = \frac{Nab\ R}{K(E+Rb)^2}$$

$$K_3 = \frac{Nab\ R}{K(E+R_b)^2} + \frac{L_L}{R.}$$

Inserting these constant the equation 3.6 reduces to $-\frac{K_1}{(b+i)} + K_2 \log(b+i) - K_3 \log(E-iR) = t+C$ --------.. 3.7.

The integration constant C can be determined assuming the field current to be i_1 at $t = 0$ when the discontinuity is impressed

i.e. at $t = 0$ $i = i_1$

On substituting

$$C = -\frac{K_1}{b+i_1} + K_2\log(b+i_1) - K_3\log(E-i_1R)$$

Substituting this in the equation gives the final solution as

$$t = \left(\frac{K_1}{b+i_1} - \frac{K_1}{b+i} + K_2 \log \frac{b+i}{b+i_1} + K_3 \log \frac{E-Ri_1}{E-Ri} \right) -$$

This gives the time cunsent relationship or the response of exciter field current. By referring to the magnetisation curve the voltage build up curve can also be readily found.

Instead of sloving the equation for a current it could be possible to solve for e and by procedure similar to above a solution for e can alsc be found. 3.2 CONSTANT COEFFICIENT OF DISPERSION

Frohlich equation gives $e = \dfrac{ai}{b+i}$

This substituted in equation 3.1. gives

$$\frac{\sigma Nab..}{K(b+i)^2} \frac{di}{dt} + Ri = E \quad \text{--------} \quad 3.9$$

This is identical in form with equation 3.5 with $L_L = 0$. Hence the constants in the genera] solution may be directly written as

$$K_1 = \frac{\sigma Nab}{cK(E+Rb)} \quad \text{and } K_2 = K_3 = \frac{\sigma ab\ RN}{K(E+Rb)^2} = \frac{RK_1}{E+Rb.}$$

The solution for time current relationship becc

$$t = \left[\frac{K_1}{b+i_1} - \frac{K_1}{b+i} + K_2 \log \frac{(b+i)(E-Ri_1)}{(b+i_1)(E-Ri)} \right]$$

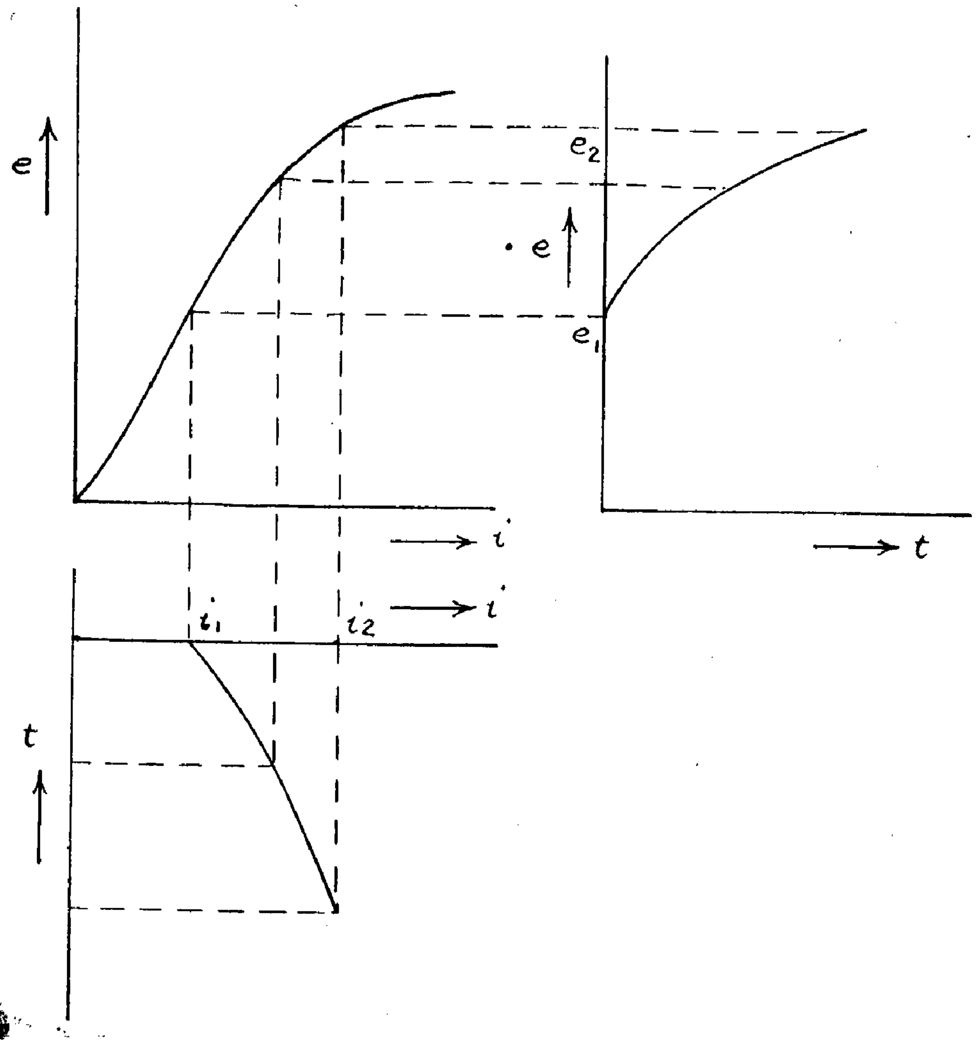


Fig. 3.1.

Determination of Voltage response curve from Magnetisation current and field current response curve.

Similar formal solutions for (i) or (e) can be obtained in case of self excited exciters also by using e instead of E in the differential equations 3.1 and 3.2 and using Frohlich equation.

However, this method involving formal integration is not so much used as the point by point calculation method or the method involving graphical integration. This method demands the mangetisation curve to be represented by the Frohlich equation and hence accurate estimation of the constant a and b. The two methods described earlier do not involve ccumbersome calculations and accurate results are obtained with more ease and lesser calculations.

CHAPTER IV.

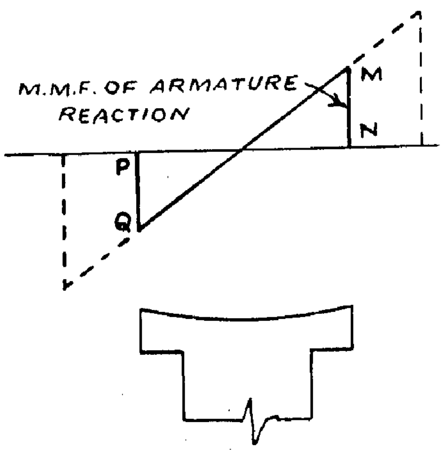
4.1. CALCULATION OF RESPONSE UNDER LOADED CONDITION

Most of the cases for which the exciter response is desired are concerned with sudden changes such as sudden short circuits in the armature circuit of the synchronous machine. In case of such sudden changes the field linkages of the synchronous machine would tend to decrease till the circuit breaker operates and the fault is cleared off. Under these conditions high rate of exciter response keeps the field flux linkages of the alternator nearly constant. The field current of the atternator and hence the armature current of the exciter increase a considerable amount. This high value of exciter armature current may be assumed to remain substantially constant during most of the period of exciter build up not only because of the high inductance and therefore the time constant of the alternator field but also because of the increasing effect of the exciter build up. By assumption of constant armature current during build up the analysis is facilitated.

The terminal voltage of a loaded exciter differs from the voltage of the unloaded exciter with the same field current because of the effects of armature resistance, brush drop, armature inductance and armature reaction. If the armature current of the exciter is constant which is assumed during build up then there is no effect of armature inductance. The armature resistance drop and brush drop are constant.

Modern exciters have interpoles and the brushes are set on neutral zones consequently the magnitising or demagnetising effects of armature reaction are nulified. If compensating windings are also provided then the cross magnetising effect of armature reaction is also counter-acted. However when compensating windings are not provided there is a demagnetising effect of cross magnetising ampere turns of armature reaction. This effect is to be taken into account for determining the response under loaded conditions and is discussed below:

The m.m.f. of armature reaction produces an m.m.f. that varies linearly from the polar axis, one side being positive, the other being negative.



MN represents the maximum magnetising m.m.f. at one pole edge and PQ the maximum demagnetising m.m.f. at the other edge.

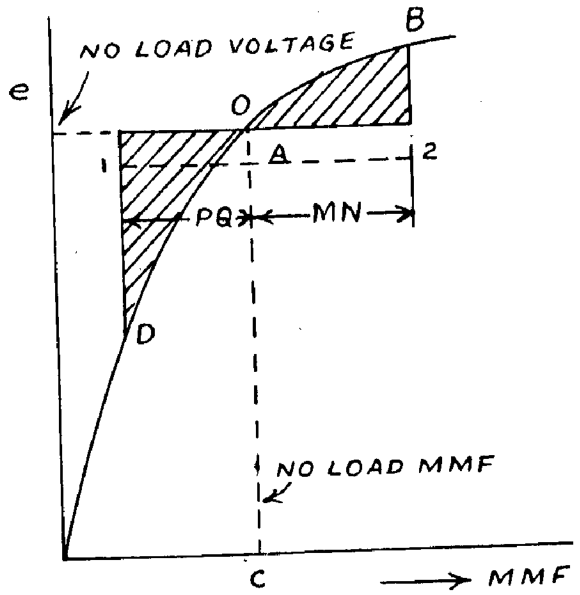


FIG. 4.2.

Due to saturation it is observed that the higher m.m.f. dōes not increase the flux on the right hand side as muchas the lower m.m.f. decreases the flux on the left hand side. As a result the total flux per pole and the generated voltage are decreased from the value indicated by OC to that indicated by AC which is obtained by integrating the area unde the curve DOB and drawing 12 so that the two triang ular areas are equal. The extent to which the averag flux or voltage is decreased can be indicated by a 'distortion curve', such as indicated in the curve as below, Fig. 4.3. This effect is most pronounced in the region of the knee of the saturation curve, as at both higher and lower field currents, there is tendency to add on the one side of the pole just as much flux as is subtracted on the other.

From the distortion curve if the constant armature resistance and brush drops are substracted the load saturation curve for constant exciter armature current is obtained Fig. 4.3.

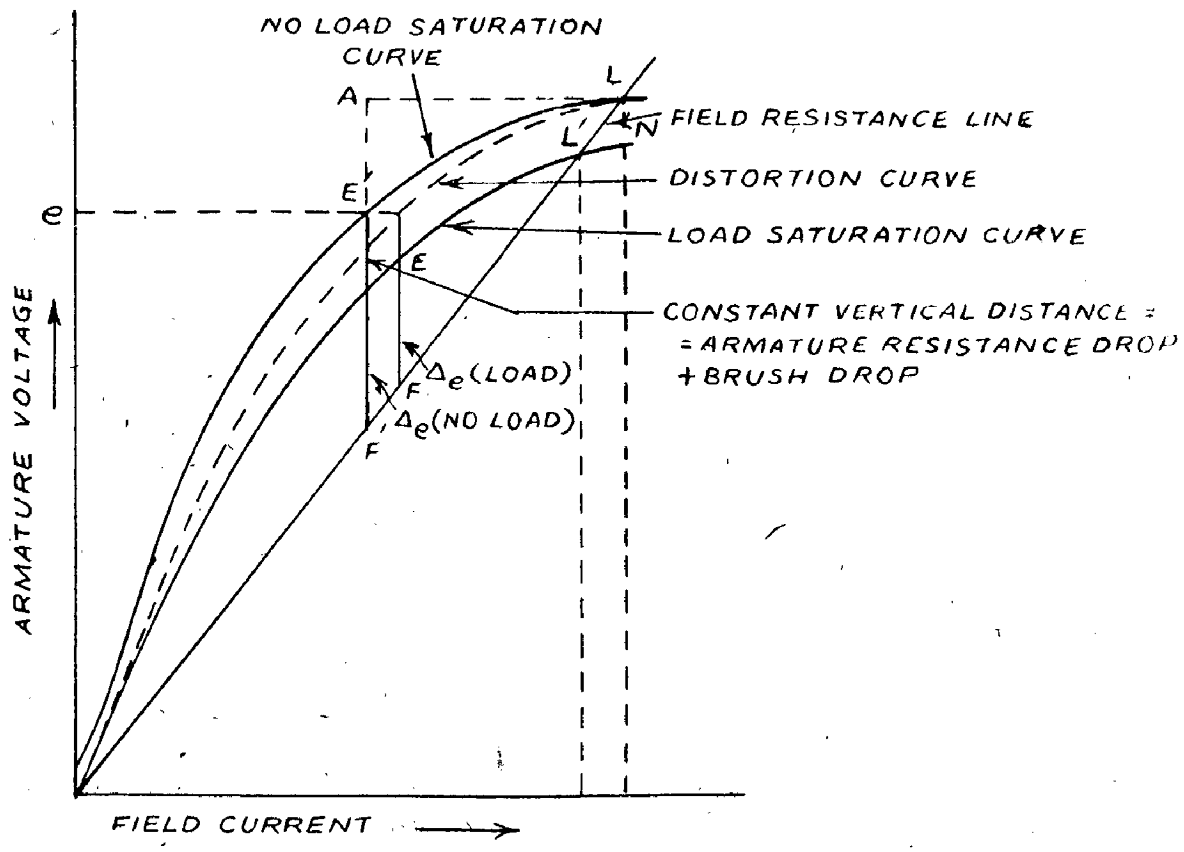


Fig. 4.3.

While calculating the load response, the load saturation curve and the distortion curve are used instead of the no-load saturation curve. Deta-ils of calculation of exciter response under loaded condition by method of graphical integration as are follows:

a) The variable of integration e is taken from the distortion curve instead of the no-load saturation curve.

b) For an internally induced armature voltage e, the noload field forcing voltage is given by E'F' (self excited) A'F'(separately excited) Under load the field forcing voltage is given by EF (self excited) and AF(separately excited). This is on assumption that the noload ceiling voltage is same for both kinds of excitation.

c) By method of graphical integration or by point by point calculation respense of the internal voltage e of the exciter can be determined. This is usually sufficient. If however a curve of exciter terminal voltage versus t is wanted it can be obtained by substracting the constant armature resistance and brush drops from e - t curve.

4.2. SUMMARY OF EFFECTS OF LOAD UPON BUILD-UP EXCITER RESPONSE

The effect of both armature resistance and armature reaction is to decrease the exciter response during build up, especially that of a self excited exciter. This can be seen from figure 4.3. which shows that for the same value of internal voltag e the field forcing voltage Δe is reduced from E'F' on no load to DF on load for self excited case. This is due to armature resistance drop ~~and the demagnetising effect of the cross magnetising armature reactance drop~~ and the demagnetising effect of the cross magnetising armature reaction as discussed earlier.

When the method of graphical integration is used and either constant coefficient of dispersion is assumed or constant leakage inductance is assumed, the the time required for the internally induced voltage to build up to any value e was given by

$$t = T\int_{e_1}^{e} \frac{de}{\Delta e} \quad \text{or} \quad t = T\int_{e_1}^{e} \frac{d(e+e')}{\Delta e}$$

$$\text{where} \quad T = \frac{\sigma N}{K} \qquad \text{where} \quad T = \frac{N}{K}$$

In both cases the field forcing voltage Δe is in the denomenator of the integrand and the integra representing the time required for the voltage to buil up to e is increased. The effect of load is therefo to decrease the response of self excited exciters considerably.

In case of separately excited exciter assuming some ceiling voltage as in self excited exciter the field forcing voltage is A'F' on no load and AF on load. Δe is only slightly decreased from no-load to load and therefore the response is also slightly effected.

Thus with either kind of excitation the exciter response is decreased by load. The effect is much greater in the self excited exciter than in the separately excited one however.

Besides reducing the response, load also reduces the ceiling voltage. In case of self excited exciter it is reduced from MN to M'N'. In case of separately excited exciter it is reduced from LM to NM

CHAPTER V

5.1. FACTORS AFFECTING RATE OF RESPONSE OF EXCITER

The differential equation used for calculation by graphical integration as derived earlier, was

$$\frac{de}{dt} = \frac{\Delta e}{T} \quad \text{or} \quad t = T \int_{e_1}^{e} \frac{de}{\Delta e}$$

This was on assumption of constant coefficient of dispersion and was applicable to both kinds of excitation i.e. self excited exciter as well as separately excited exciter. The time constant T for self excitation was $\frac{\sigma N}{K}$

and for separate excitation $\frac{\sigma N}{K} \cdot \frac{e_2}{E}$

Where σ, the dispersion coefficeint, N, the number of field turns in series and $K = \frac{Znp}{60.a.}$ are design constants of the machine.

With the separately excited machine the ceiling voltage e_2 can be varied between wide limits by suitable change of pilot exciter voltage, external shunt field resistance etc. With self excitation the optimum ceiling voltage is limited by field resistance itself.

With self excitation the time constant is fixed by the design while in separate excitation the time constant can be controlled over wide limits by suitable selection of pilot exciter voltage, external field resistance and the field circuit resistance itself. The time constant can be decreased at will and the response rates can be increased in separately excited exciter.

The response rate also depends on the quantity Δe. This quantity for the two types of excitation is given by

$$\Delta e = e_2 - R_e i = e_2 - \frac{e_2}{i_2} . i. \quad \text{--- separate excitation.}$$

$$\text{and } \Delta e = e - Ri = e - \frac{e_2}{i_2} . i. \quad \text{---- self excitation.}$$

Assuming the same ceiling voltage it is evident that Δe is larger for separate excitation than for self excitation during the entire process of build up or build down since e is always less than e_2 except at the final operating point where the two field forcing voltages Δe become zero. Separately excited exciter machine has a higher response rate ove practically the entire range. The difference is parti cularly pronounced during the first part of the transi ent. Providing separate excitation therefore,increase exciter response.

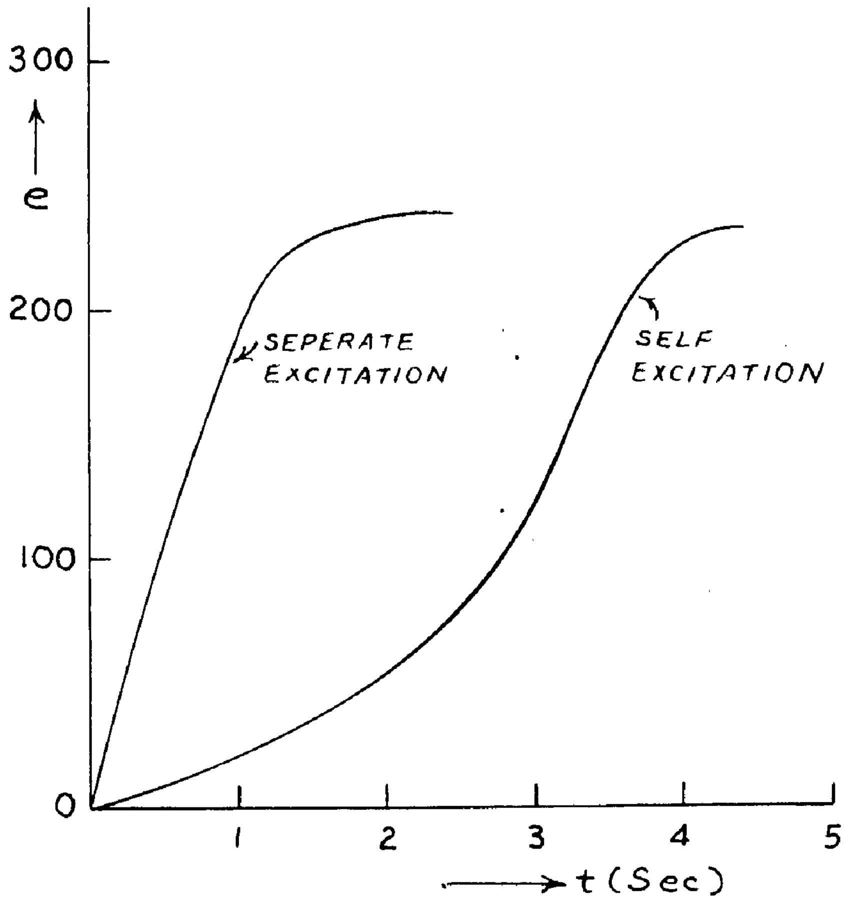


Fig. 5.1

In a separately excited exciter the response can be increased by increasing the ceiling voltage e_2. The amount of this increase is however limited by saturation. The field current required to raise the ceiling becomes large and may have to be limited by considerations of heating.

Response rate can also be increased by decreasing the time constant of the exciter field circuit which is

$$= \frac{\sigma N}{K} \cdot \frac{e_2}{E}.$$

This can be achieved by increasing the no.

of parallel paths or by providing a new field winding with larger conductors and fewer turns. This would reduce the number of field turns in series and therefore, the time constant T.

By increasing the excitation voltage E, the time constant can be reduced. But an increase in E means an increase in $i_2 = E/R$ and hence of e_2 By increasing R also, e_2 can remain unchanged, so that an increase both in the excitation voltage E and of the field resistance R in the same proportion would decrease the time constant and hence increase the response.

Although decreasing the time constant also decreases the time required for the exciter voltage to build up from one value to another, the nominal response is not affected in the same proportion.

5.2. SCREENING EFFECT OF EDDY CURRENTS ON CALCULATION

In the foregoing analysis of calculation of exciter response the possible screening effect of eddy currents set up in the iron due to the flux alternations was neglected. This effect is uncertain and depends upon the nature of the magnetic circuit of the exciter machine and also upon the rapidity of flux variation. The effect is noticeable particularly with thick solid cores of low resistivity but is negligibly small with laminated cores of lowloss irons, especially when the frequency of flux reve-rsals is also small. Since the eddy currents con-tract the change in flux during the transient, their effect is always to lengthen the transient process and virtually to add to the time constant of the circuit.

If the voltage time curve was found by calculation in which the effect of eddy currents was neglected, the calculated nominal response will be too high. The exact calculation of eddy currents is a complicated proposition and regorous inclusion in the analysis of their effect therefore is not easy Rudenberg suggested a practical way of approximating its effect by increasing the time constants slightly upwards. W.A. Lewis in his paper 'Quick Response Excitation' published 1934 in 'Elec. Journal' found values of nominal exciter response by test on exciter with rolled steel frames and laminated pole pieces and compared them with calculated values and thereby found correction factors for different values of nomi nal response. A relation is expressed in a graph of the following form.

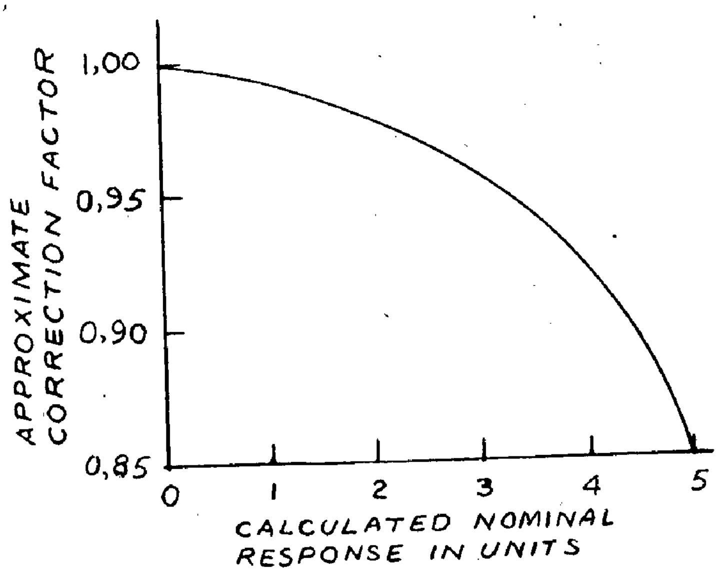


Fig. 5.2.

The effect of eddy currents on the response is small except at high values of response. If the no-load saturation curve is obtained by test, the accuracy of the calculated nominal response after application of the correction factors can be expected to be within $\pm 5\%$ of the value found by test.

These correction factors apply to the overall effect of eddy currents on nominal response and not to individual ordinates of the voltage time curve. The actual shape of the curve as found by test may differ greatly from the corresponding calculated curve.

5.3. EFFECT OF DIFFERENTIAL FIELDS ON RESPONSE:

Differential windings consist of a small number of turns wound on each pole so connected that the m.m.f. produced thereby is opposite to that of the main windings. Figure below shows their schematic arrangement. The extent to which differential circuit reduces the response of the exciter may be calculated as follows:-

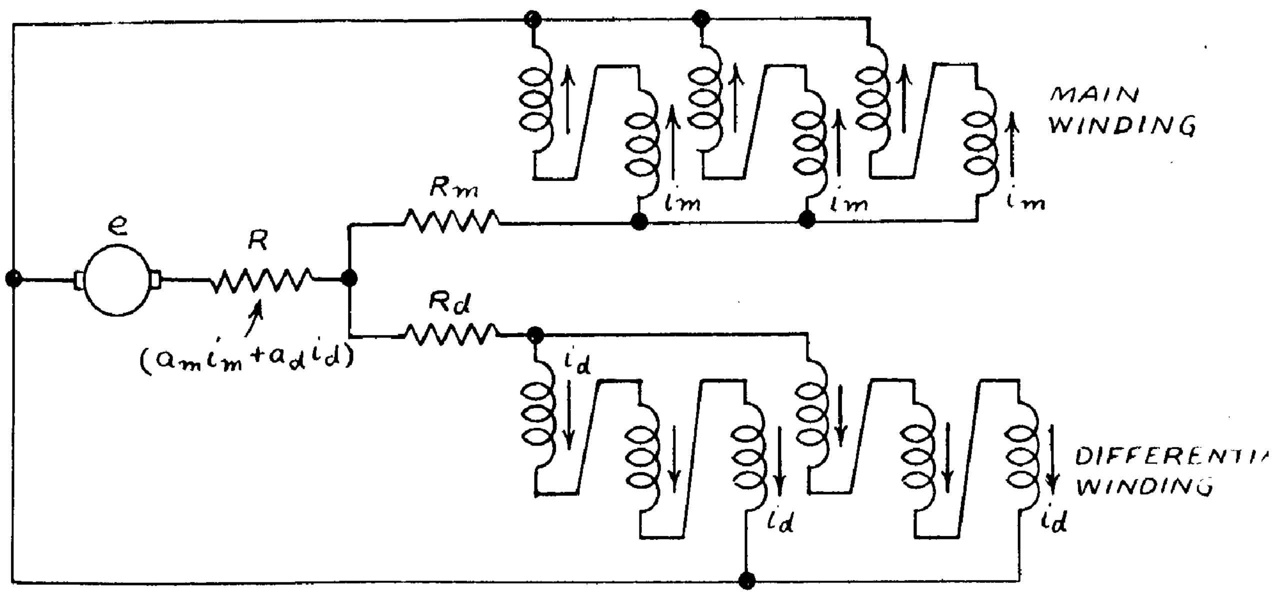


Fig. 53.

Let i_m and i_d be the currents per circuit in the main and differential windings.

a_m and a_d be the number of parallel paths in the two windings ; T_m and T_d be the number of turns per pole on the two windings + p be the number of poles of the exciter.

The resistors R_m and R_d in series with the combined main and differential windings respectively may be included in the calculation by increasing the actual resistances in each of the main and differential circuits by $a_m R_m$ and $a_d R_d$. Then the resistances will be designated by the symbols r_m and r_d respectively.

Referring to figure above the following equations can be written

$$e = R(a_m i_m + a_d i_d) + R_m i_m + \frac{d\psi_m}{dt} \text{ ----5.1}$$

$$e = R(a_m i_m + a_d i_d) + r_d i_d + \frac{d\psi_d}{dt} \text{ ----5.2}$$

ψ_m and ψ_d are the flux linkages of the two respective circuits.

If all the field flux cuts all turns then

$$\psi_m = \frac{P.T_m}{a_m} \text{ x (Flux per pole webers)}$$

$$\psi_d = \frac{P\ T_d}{a_d} \text{ x (Flux per pole in webers)}$$

$$\text{or} \quad \frac{\psi_d}{\psi_m} = \frac{T_d\ a_m}{T_m\ a_d}$$

$$\text{or} \quad \psi_d = \frac{T_d\ a_m}{T_m\ a_d} . \psi_m \text{ --------5.3.}$$

If it be assumed that the two windings be replaced by another winding having the same number of turns and circuit connections as the main windings, then the instantaneous m.m.f. of this winding is the same as that of the combination if

$$T_m i = T_m i_m - T_d i_d$$

or $$i = i_m - \frac{T_d}{T_m} \cdot i_d$$

or $$i_m = i + \frac{T_d}{T_m} \cdot i_d \quad \text{-----} \quad \text{---} \; 5.4.$$

On substituting 5.3 and 5.4 in equations 5.1 and 5.2, we have

$$e = (Ra_m + r_m)\left(i + \frac{T_d}{T_m} i_d\right) + Ra_d\, i_d + \frac{d\psi_m}{dt} \quad \text{---} \; 5.5.$$

and $$e = Ra_m\left(i + \frac{T_d}{T_m} i_d\right) + (Ra_d + r_d) i_d + \frac{T_d\, a_m}{T_m\, a_d} \frac{d\psi_m}{dt} \quad \text{----} \; 5.6$$

Multiplying 5.6 by $\frac{T_m\, a_d}{T_d\, a_m}$ gives

$$\frac{T_m\, a_d}{T_d\, a_m} e = \frac{T_m\, a_d}{T_d \cdot a_m} R.a_m\left(i + \frac{T_d}{T_m} i_d\right)$$

$$+ \frac{T_m\, a_d}{T_d\, a_m} i_d (Ra_d + r_d) + \frac{d\psi_m}{dt} \quad \text{------} \quad \text{---} \; 5.7$$

Substracting 5.7 fpom 5.5 gives

$$e\left(1 - \frac{T_m\, a_d}{T_d\, a_m}\right) = i\left[Ra_m + r_m - \frac{T_m\, a_d}{T_d}\, R.\right]$$

$$+\; i_d\left[\frac{Ra_m\, T_d}{T_m} + \frac{T_d}{T_m} .\; r_m + R\, a_d - a_d . R.\right.$$

$$\left. - \frac{T_m\, a_d}{T_d\, a_m}\left(R . a_d + r_d\right)\right]$$

On solving for i_d in turns of i and substituting the expression for i_d in equation 5.5 we obtain

$$\frac{1 - \dfrac{T_d}{T_m} . \dfrac{r_m}{r_d}}{A} . e = \frac{1 + R\left(\dfrac{a_m}{r_m} + \dfrac{a_d}{r_d}\right)}{A} . r_m i + \frac{d\phi}{dt} \qquad \text{---------. 5.8.}$$

where $A = 1 - \dfrac{a_m . T_d^{\,2}}{a_d . T_m^{\,2}} . \dfrac{1}{r_d}\left[r_m + \left(\dfrac{a_d^{\,2} T_m^{\,2}}{a_m T_d^{\,2}} - a_m\right) R\right]$

Equation 5.8 shows that the ordinary flux linkage curve for the exciter and method of calculatic indicated in section 2.3 can be used if the coefficier of i be used as the equivalent resistance of each circuit, i be the current read from the saturation curve and the voltage across each circuit be multipli‹ by the coefficient of e.

In other words, the calculations for exciter response can be carried out as though the differential winding were not present except that instead of using the expeession

$$\frac{d\psi}{dt} = e - Ri$$ to determine the forcing voltage, e should be multiplied by

$$\frac{\left(1 - \frac{T_d}{T_m} \cdot \frac{r_m}{r_d}\right)}{A}$$

and R by $$\frac{1 + R\left(\frac{a_m}{r_m} + \frac{a_d}{r_d}\right)}{A}$$

Where $$A = 1 - \frac{a_m \, T_d^2}{a_d \, T_m^2} \, \frac{1}{r_d} \left[r_m + \left(\frac{a_d^2 \cdot T_m^2}{a_m \, T_d^2} - am \right) R \right]$$

5.4. RESPONSE OF UNLOADED EXCITER HAVING ROTATING AMPLIFIER EMF IN SERIES WITH ITS SHUNT FIELD CIRCUIT

Instead of exciting the main exciter separately from the output of a rotating amplifier, it is desirable to use some combination of self and separate excitation of the main exciter so that the rotating amplifier can be removed from service for maintenance without shunting down the generator unit W.A. Hunter and M. TEMOSHOK in a paper published in A.I.E.E. Transactions 1952 Oct. suggested such a method by connecting the rotating amplifier in series with the self excited shunt field. The field rheostat is so adjusted that when the amplifier voltage is zero the exciter operates self excited to supply the proper field current for average load on the a.c. generator. The voltage of the rotating amplifier either bucks or boosts that of the exciter armature, as required for proper control of the alternating voltage. The amplifier may be disconnected by a transfer switch, the main exciter then being manually controlled. The circuit of such an excitation system is given below and we shall proceed to find out the response of such an excitation system.

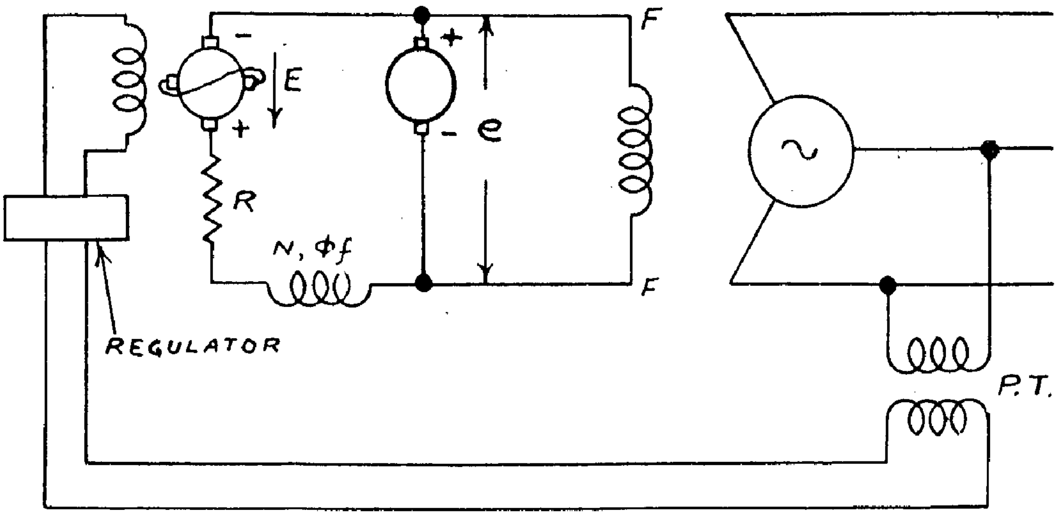


Fig. 5.4.

The equation of voltages around the closed path is $N \frac{d\phi f}{dt} + Ri = e + E.$

The armature e.m.f. of the rotating amplifier E is positive when boosting the armature e.m.f. e of the main exciter. If constant coefficient of dispersion is used the equation becomes

$$\frac{\sigma N}{K} \cdot \frac{de}{dt} = e - (Ri - E)$$

$$\text{or} \quad \frac{\sigma N}{K.} \frac{\Delta e}{\Delta t} = e - (Ri - E)$$

$$\text{or} \quad \Delta t = \frac{(\frac{\sigma N}{K}) \Delta e}{(e + E - Ri)} \quad \text{-----5.9.}$$

Δe is the change in e in time interval Δt. Exciter response curve by point by point calcul-tion similar to that already described can be determined.

CHAPTER VI

FIELD OF USE OF EXCITER RESPONSE

6.1. ANALYSIS OF AN EXCITER SUPPLYING THE FIELD CURRENT OF AN UNLOADED SYNCHRONOUS GENERATOR:

SOLUTION FOR ALTERNATOR FIELD CURRENT:-

The simplest exciter-alternator problem results when the alternator is open circuited. This eliminates completely the effect of the alternator armature circuit and the system which. The alternator can be represented by a resistance and variabl inductance and in fact the problem reduces to findin the response in case of a loaded exciter, the load being a fixed resistance and a variable inductance. Self excited case is only considered here while the method can be extended conveniently to separately excited case too.

When the machine is loaded the consta ts of the armature circuit i.e. the armature resista ce and inductance as well as armature reaction enter into the problem in addition to those of the load. The effect of the armature resistance is usually the most important and the leakage inductance of the armature is ordinarily quite small and the reactance voltage during the transients is rather insignifican

The demagnetising effect of the cross magnetising amperetturns of armature reaction has been discussed earlier and is accounted for by using the load distortion curve instead of the magnetisation curve. The circuit to be analysed is shown below:

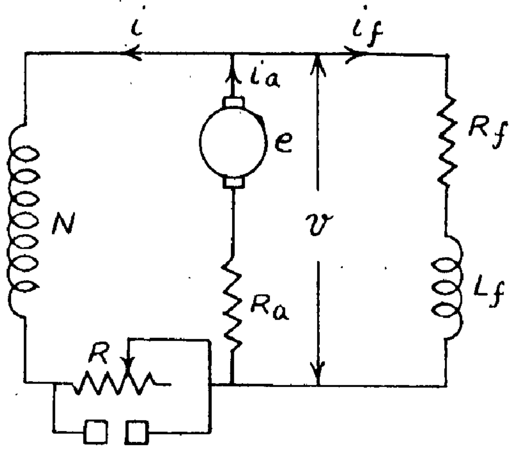


Fig. 6.1.

The differential equation on assumption of constant coefficient of dispersion will be

$$T \frac{de}{dt} + Ri = e - R_a i_a$$

Also $i_a = i + i_f$

Therefore $$T \frac{de}{dt} = e - Ri - R_a i_a$$

or $$T \frac{de}{dt} = e - Ri - R_a i - R_a i_f.$$

or $$T \frac{de}{dt} = e - (R + R_a) i - R_a i_f$$

$$= \Delta e - R_a i_f \quad \text{--------..} \quad 6.1.$$

Where T is the exciter time constant discussed earlier.

R_a is the armature resistance of the exciter.

R_f is the field resistance of the alternator.

L_f is the inductance of the field winding ~~winding~~ of the alternator.

The equation for voltage around the alternator field ~~atw~~ circuit would give

$$L_f \frac{di_f}{dt} + R_f\, i_f = e - R_a\, i_a$$

or

$$L_f \frac{di_f}{dt} = e - iR_a - i_f\, R_a - R_f i_f$$

$$= e - R_a . i - (R_a + R_f)\, i_f .$$

$$= \Delta e' - (R_a + R_f)\, i_f \text{-------} 6.$$

We obtain therefore the relations

$$\frac{de}{dt} = \frac{\Delta e - R_a\, i_f}{T} \quad \text{---------.. } 6.3$$

and

$$\frac{di_f}{dt} = \underline{\Delta e' - (R_a + R_f)\, i_f} \quad \text{--------.. } 6.4$$

Where $\Delta e = e - (R + R_a) i$

and $\Delta e' = e - R_a . i$

where $T = \frac{\sigma N}{K}$

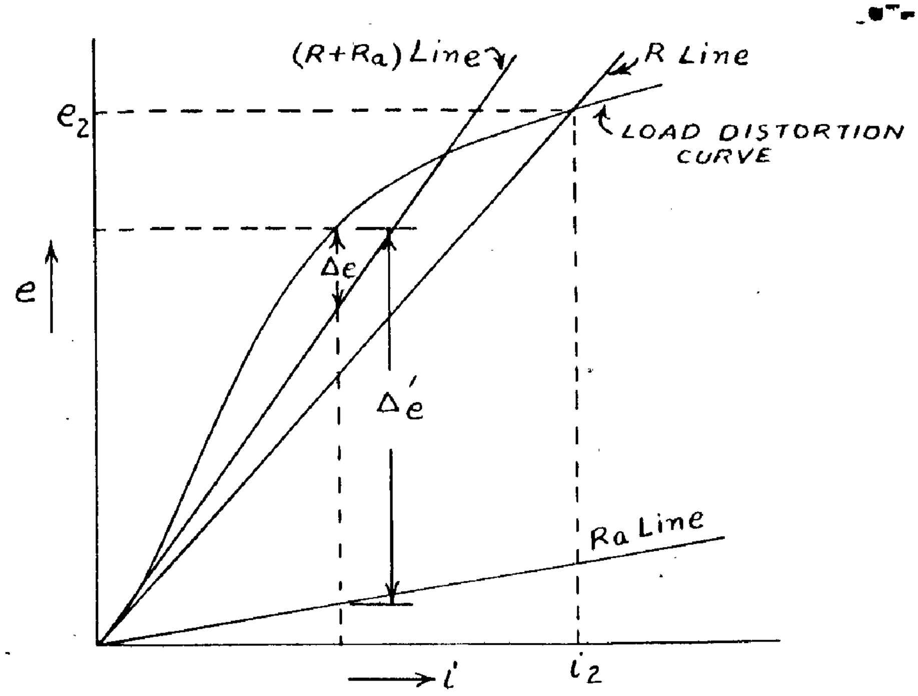


Fig. 6.2.

The graphical interpretation of Δe and $\Delta e'$ is shown above. Based on the above two equations a point by point solution can be easily carried throug[h] The response of alternator field current as well as of the exciter e.m.f. can be obtained.

The alternator field inductance L_f is the total self inductance of the field winding and in carrying out the solutions congnizance must b e taken of the fact that the load inductance is variable and is a function of the field current. The relationship between the field inductance and the field current must be known so that the proper induc- tance can be used for each interval in the point by point calculations. In order to obtain the inductance as a function of current a magnetisation curve or its equivalent must in general be available. The inductance at a particular value of current is then proportional to the slope of the curve at the point being given by

$$L_f = N \frac{d\,\phi_f}{d\,i_f}.$$

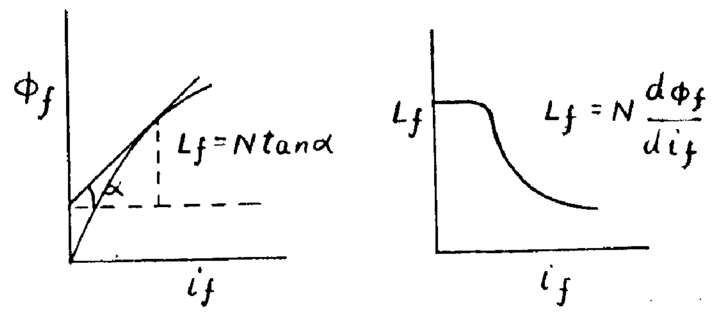


Fig. 6.3.

From the forgoing analysis it is possible to determine the response of the alternator field current and from the open circuit magnetisation curve the corresponding armature voltage may be read directly.

6.2. ANALYSIS OF EXCITER SUPPLYING FIELD CURRENT OF A LOADED ALTERNATOR

The problem involving an exciter connected to a loaded alternator e.g. an alternator, supplying power to a system is considerably more complicated than the one previously discussed with the alternator on open circuit. In this case the effect of varying armature reaction upon field current has to be taken into account in the solution. Furthermore the nonlinear characteristics of alternator as well as of the exciter should be included. However, unless simplifications are made, the problem becomes quite unwieldy and a rigorous solution is seldom attempted. The most complete methods are those described by V. Bush and R.D. Booth in 'Power System Transients" in Trans. A.I.E.E., 1925. Their analysis particularly relates to the problem of sudden load changes (balanced loads) but can be extended to other types of discontinuities. Their methods allow the effect of varying armature current and nonlinearity and involve point by point computations utilizing primarily rather elaborate graphical methods. In the approximations used in this problem the field transient time constant in the direct axis on open circuit T_{do}' is considered constant. Theoretically it could be possible to take into account the effect of saturation on Tdo' in the point by point analysis. Usually an average value of Tdo' over the range of saturation encountered may be used.

The general method of handling exciter action in connection with alternators connected to power systems is discussed in the pages which follow. It is shown how exciter response may be included in point by point solutions of problems involving sudden load changes and faults and consequently oscillations of the synchronous machines.

6.3. RELATION OF EXCITER RESPONSE WITH TRANSIENT STABILITY

Upon the occurence of a 3 phase sudden short circuit the field flux linkages immediately after and before the occurence of the fault are the same according to Doherty's 'Constant flux linkage theorem'. During the fault, however, the flux linkages decay at a rate described by short circuit subtransient and transient time constants Td" and Td' if damper windings are present. If the fault is sustained for a long time, a machine may survive the first suring of its rotor but because of the continued decrease of its field flux linkage it may pull out of step on the second suring or on subsequent surings.

If the excitation system is provided with an automatic voltage regulator then the exciter emf builds up, the time rate of build up being as discussed in the foregoing analysis. In the present analysis the effect of exciter response on field flux linkages and therefore on the stability of the system would be investigated.

CHANGE OF FIELD FLUX LINKAGE UNDER TRANSIENT CONDITIONS AS AFFECTED BY EXCITER RESPONSE :

Kirchoff's voltage law applied to the field circuit gives the equation

$$E_{ex}' = R_f\, i_f + \frac{d\psi_f}{dt}$$

where E_{ex}' is the exciter armature emf in volts

R_f is the alternator field circuit resistance in ohms.

i_f is the field current in amperes.

ψ_f is the field flux linkage in weber turns.

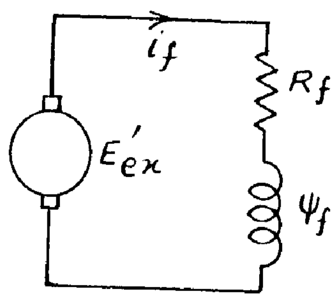


Fig. 6.4

Multiplying both sides by $\frac{WM_f}{R_f}$ gives

$$\frac{\omega M_f E_{ex}'}{R_f} = WM_f\, i_f + \frac{WM_f}{R_f}\frac{d\psi_f}{dt} \qquad \text{------------.6.5.}$$

Where M_f is the mutual inductance between the field and the armature winding.

$WM_f \, i_f = E_q$ is the steady state interval voltage and is directly proportional to the field current.

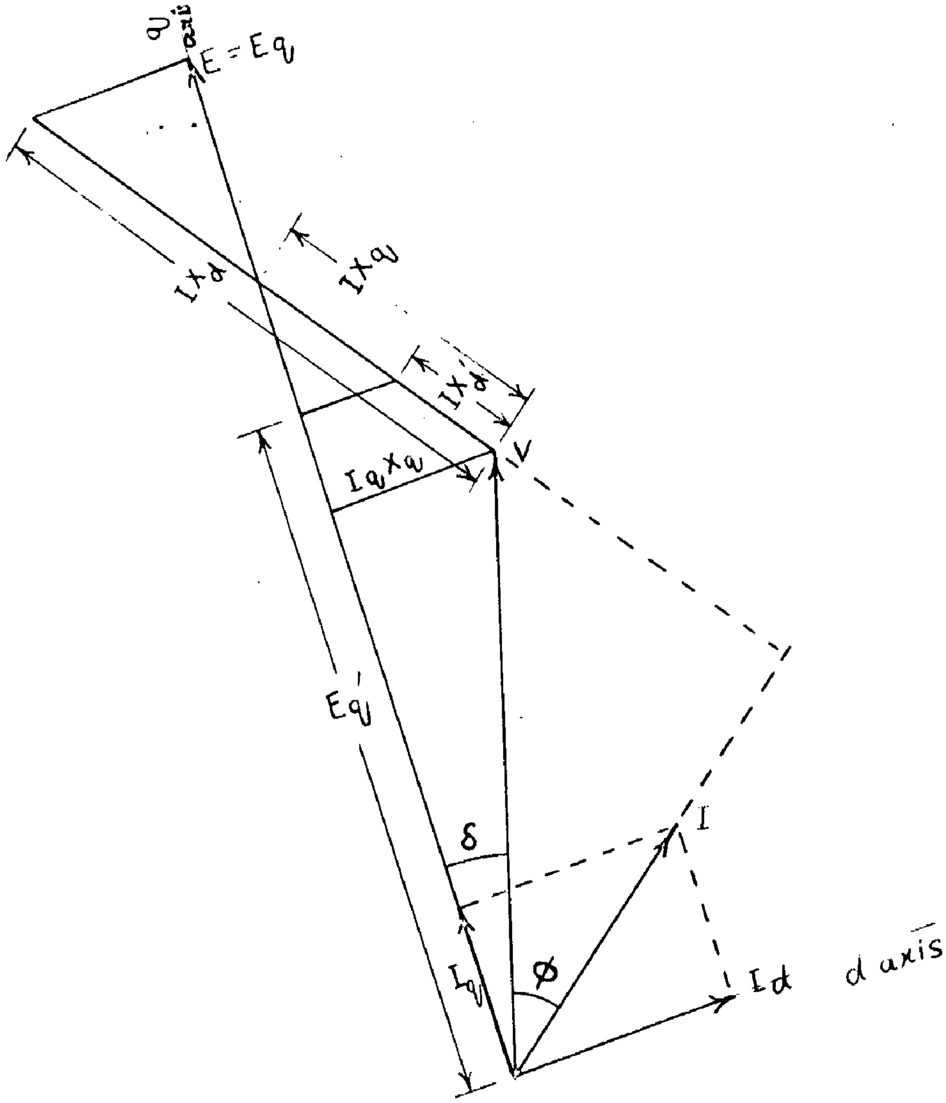


FIG. 6.5. VECTOR DIAGRAM OF A SALIENT POLE SYNCHRONOUS MACHINE UNDER TRANSIENT STATE

Fig. 6.5

Vector diagram of a salient pole synchronous machin

Under

Referring to the vector diagram of the synchronous machine under transient state a new fictitious interval armature voltage E_q' appears in the quadrature axis.

This is the voltage behind the transient reactance in the direct axis and is

$$E_q' = \frac{WM_f}{L_{ff}} \psi_f$$ It is directly proportional to the field flux linkage. During any transient changes since the flux linkage ψ_f of the field winding remains constant immediately after the change ; the voltage E_q' would also remain constant.

$$\frac{WM_f \psi_f}{R_f} = \frac{L_{ff}}{R_f} \cdot \frac{WM_f \psi_f}{L_{ff}} = Tdo' \; E_q' \text{ -----..6.6}$$

Since $\frac{L_{ff}}{R_f}$ is = Tdo' the open circuit transient time constant in the direct axis.

On differentiating equation 6.6. w.r.t. time we have

$$\frac{WMf}{Rf} \cdot \frac{d\psi f}{dt} = Tdo' \cdot \frac{dE_q'}{dt} \text{ ------- -----6.7}$$

Under steady state the exciter voltage Eex' would cause a field current $i_f = \frac{Eex'}{Rf}$ and an open circuit armature voltage $= \frac{WMf \; Eex'}{Rf} = Eex$ say--6.8.

Substituting eqns. 6.7 and 6.8 in eqn. 6.5 gives

$$Eex = E_q + Tdo' \frac{d\, E_q'}{dt} \text{ - - - - - - - - .. 6.9}$$

Solving for the derivative, we get

$$\frac{dE_q}{dt} = \frac{E_{ex} - E_q}{T_{do}'} \quad \text{---------- ----- } 6.10$$

Thus the rate of change of E_q' and therefore of the field flux linkage can be determined in tarms of T_{do}' and in turms of E_{ex} which is a known function of time t.

In case of three phase sudden short circuit at the armature terminals of a synchronous generator the transient a.c. component of armature current and the transient d.c. component of field current both decay exponentially with the transient short circuit time constant T_d'.

$$T_d' = \frac{X_d'}{X_d} \cdot T_{do}'.$$

During the short circuit the terminal voltage of the machine is zero and the direct axis short circuit current

$$I_d = \frac{E_q'}{X_d'}$$

$$\text{Also} \quad I_d X_d = E_q$$

$$\text{Hence} \quad E_q' = E_q \cdot \frac{X_d'}{X_d}$$

$$\text{or} \quad E_q = E_q' \cdot \frac{X_d}{X_d'} \quad \text{------- ---} 6.11$$

Substituting this in equation 6.10 gives

$$\frac{d\,E_q'}{dt} = \frac{E_{ex} - E_q'}{T_{do}'} \cdot \frac{X_d}{X_d'} \quad \text{---------} 6.12.$$

Multiplying the numerator and denomenator of R.H.S. by $\frac{Eq'}{Eq} = \frac{Xd'}{Xd}$ we get

$$\frac{dEq'}{dt} = \frac{\left(\frac{Eq'}{Eq}\right) \quad Eex - Eq'}{\left(\frac{Eq'}{Eq}\right) \quad Tdo'} \qquad \text{------} 6.1$$

For a fixed ratio $\frac{Eq'}{Eq}$ this equation shows that Eq' varies with a time constant (Eq'/Eq)Tdo'.

For the condition of short circuit $\frac{Eq'}{Eq} = \frac{Xd'}{Xd}$ and hence the time constant is Td'. On open circuit $\frac{Eq'}{Eq} = 1$ and the time constant is Tdo.

Using point by point calculation the quadrature axis transient voltage Eq' can be obtained as functions of time if the exciter rate of build up i.e Eex(t) is known

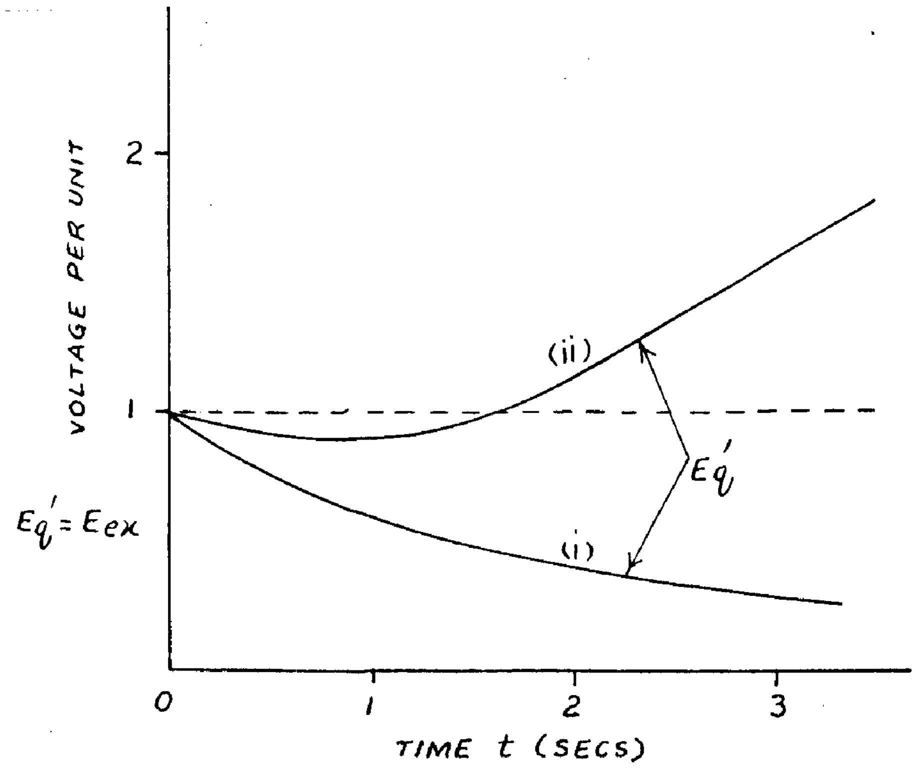


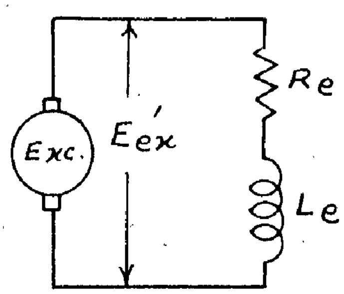
Figure 6.6 shows the plot of Eq' as f(t) for two cases. (i) when the exciter voltage Eex is assumed constant i.e. voltage regulator action is absent and (ii) when voltage regulator action is present and the exciter voltage is assumed to build up linearly.

In the second case the voltage regulator causes the flux linkage first to decrease more slowly and then to increase. As a result a machine which does not go out of step on the first few surings will not go out of step on subsequent surings of the same distrubance.

The standard value of exciter response is about 1.0. unit i.e. 100 volts per second for a 125 volt exciter. More rapid rates up to 2·0 units i.e. 200 volts/sec. or still faster rates up to about 5·0 units or 500 volts per sec. can be provided! Super excitation' with response as high as 6000 to 7000 volt sec. on a 250 volt field (30 to 35 units) has been attained. This ultra high rate of response has been employed on synchronous condensers. R.H.Park and E.H. Banker in A.I.E.E. Trans.1929 have shown that a moderate exciter response of about 1 unit i.e. 150 to 250 v/sec. for 250 volt fields is usually sufficient to prevent loss of synchronous on other than the first suring. They further state that the gain in stability limit due to a higher response than this is small in comparison with the gain detained at these moderate values of response. The power which could be carried without loss of synchronesin on the first suring after the occurrence of a fine to ground fault on the generator bus with fast enough exciter response(about 600 volt/sec) to maintain constant field linkages was found to be only from 4 % to 6 % greater than the power that could be carried with exciter response (of 150 - 200 volts/sec.) sufficient to prevent pull out on the second suring. Nevertheless, it is still true that the higher the speed of exciter response the greater is the power limit.

Floyd and Sills have also shown that the gain in stability by using a response rate higher than 1.0 unit is very small.

6.4 EFFECT OF EXCITER RESPONSE ON DYNAMIC STABILIT



For an open circuited alternator if an exciter emf Eex' is suddenly applied to the field terminals of the alternator then the field current would be given by

$$L_e \frac{di_e}{dt} + R_e \, i_e = Ee_x'$$

or $$\frac{L_e}{R_e} \cdot \frac{di_e}{dt} + i_e = \frac{Eex'}{R_e}$$

or $$T_{do}' \frac{di_e}{dt} + i_e = \frac{Eex'}{R_e} \quad \text{--------} 6.14.$$

Where L_e and R_e are the e-quivalent inductance and resistance of the field winding.

Also when per unit system of quantities is used then i_e can be replaced by E where unit field current is such as to produce unit voltage on the air gap line.

Under steady state the exciter voltage Ee_x' would cause a field current $= \frac{E_{ex}'}{R_e}$ which may be replaced by the U_e in per unit system. U_e is the open circuit armature voltage in p.u. produced by exciter voltage Ee_x' in steady state.

Equation 6.14 may therefore be written as

$$T_{do}' \frac{dE}{dt} + E = U_e \quad \text{------- -------} 6.15.$$

If the alternator is loaded then the transient load time constant T_B' has to be used in place of T_{do}' and the equation 6.15 becomes

$$T_B' \frac{dE}{dt} + E = U_e(t) \qquad \text{-----------..6.1}$$

$U_e(t)$ is a function of E_{ex}' and is the exciter voltage referred to the air gap line of the alternator armature circuit, this does not take into account the effect of saturation. In order to simplify an otherwise complicated problem this effect is neglected and $U_e(t)$ is considered to be an exponential function of time. This is only approximately true since as was seen earlier E_{ex}', the exciter voltage was also not an exponential function due to saturation in the exciter magnitic circuit. Point by point solution of a complicated nature has to be resorted to if saturation effects of both exciter and alternator have to be considered. As a simplified theoretical treatment $U_e(t)$ is taken to be an exponential function of time and can be expressed as

$$U_e(t) = (U_{e2} - U_{e1}) \, \epsilon^{-t/\tau} \qquad \text{--------6.17}$$

Where U_{e2} = ceiling voltage

U_{e1} = initial exciter voltage.

= exciter voltage build up time constant depending upon exciter response.

At time $t = 0$ if there is a disturbance in the system causing the voltage regulator to act then depending upon the regulator time log after time t_1, the exciter voltage starts building up. The voltage given by equation 6.17 is applied after time t_1 then using. Heairside delayed unit function $H(t-t_1)$ the exciter voltage becomes

$$U_e(t) = U_{e1} + (U_{e2} - U_{e1}) \left[1 - \epsilon^{-\frac{(t-t_1)}{\tau}} \right] H(t-t_1) \qquad \text{.........6.18}$$

So that for $t < t_1$

$$U_e(t) = U_{e1}$$

and for $t > t_1$ $\quad U_e(t) = U_{e1} + (U_{e2} - U_{e1}) \left[1 - \epsilon^{-\frac{(t-t}{\tau}} \right.$

Substituting equation 6.18 in eqn. 6.16 gives

$$\frac{dE}{dt} + \frac{E}{T_B'} = \frac{1}{T_B'} \left[\frac{U_{e1}}{p} + (U_{e2} - U_{e1}) \left(1 - \epsilon^{-\frac{(t-t_1)}{\tau}}\right) H(t-t_1) \right]$$

Taking Laplace transforme and putting $(E)p = 0 = E_1$

we have

$$\left(p + \frac{1}{T_B'}\right) \bar{E} = \frac{1}{T_B'} \left[\frac{U_{e1}}{p} + (U_{e2} - U_{e1}) \left\{ \frac{\epsilon^{-pt_1}}{p} - \frac{\epsilon^{-pt_1}}{p + \frac{1}{\tau}} \right\} \right] + E_1$$

$$\bar{E} = \frac{1}{T_{B'}}\left[\frac{U_{e1}}{p(p+\frac{1}{T_{B'}})} + (U_{e2}-U_{e1})\left\{\frac{\epsilon^{-pt_1}}{p(p+\frac{1}{T_{B'}})} - \frac{\epsilon^{-pt_1}}{(p+\frac{1}{T_{B'}})(p+\frac{1}{\tau})}\right\}\right] + \frac{E_1}{p+\frac{1}{T_{B'}}}$$

or

$$\bar{E} = \frac{1}{T_{B'}}\left[U_{e1}\cdot T_{B'}\left\{\frac{1}{p} - \frac{1}{p+\frac{1}{T_{B'}}}\right\} + (U_{e2}-U_{e1})\epsilon^{-pt_1}\left\{\frac{T_{B'}}{p} - \frac{T_{B'}}{p+\frac{1}{T_{B'}}} - \left(\frac{\tau T_{B'}}{\tau - T_{B'}}\cdot\frac{1}{p+\frac{1}{\tau}} - \frac{1}{p+\frac{1}{T_{B'}}}\right)\right\}\right] + \frac{E_1}{p+\frac{1}{T_{B'}}}$$

On taking Laplace inverse we get

$$E(t) = U_{e1}\left\{1-\epsilon^{-t/T_{B'}}\right\} + (U_{e2}-U_{e1})\left\{1-\epsilon^{-\frac{t+t1}{T_{B'}}} - \frac{\tau}{\tau-T_{B'}}\left(\epsilon^{-\frac{t+t1}{\tau}} - \epsilon^{-\frac{t+t1}{T_{B'}}}\right)\right\} + E_1\epsilon^{-t/T_{B'}} \text{-----} 6.$$

This gives the time variation of the internally induced emf if U_{e2}, U_{e1}, $T_{B'}$, τ and t_1 are known.

U_{e2}, U_{e1} and τ can be known from exciter response characteristics.

T_B' is a function of the machine constants and of the loading parameters.

t_1 is the time delay of the regulator and will depend upon the type of regulator used. Equation 6.19 which gives the internally induced emf as a time varying function if substituted in the generalised power equation of a salient pole synchronous machine would give

$$p(t) = \frac{E(t).V}{X_d + X_e} . \operatorname{Sin}\delta + \frac{V^2}{2} \left\langle \left[\frac{X_d - X_q}{(X_d+X_e)(X_q+X_e)} \right] \right\rangle \operatorname{Sin}2\delta \quad \text{-------.3.20}$$

Where X_d and X_q are the direct and quadrature axis synchronous reactances

X_e is the external reactance

and δ is the load angle.

At $t = 0$ Let $E(t) = Eo$

and $\delta = \delta_1$

then at $t = 0$

$$p = \frac{E_o.V}{X_d + X_e} \cdot \operatorname{Sin}\delta_1 + \frac{V^2}{2} \cdot \left\langle \left[\frac{X_d - X_q}{(X_d+X_e)(X_q+X_e)} \right] \right\rangle \operatorname{Sin}2\delta \quad \text{-------3.21}$$

The torque tending to suring the rotor would be

$$p(t) - p = \frac{V}{X_d - X_e} \left\langle \left[E(t)\operatorname{Sin}\delta - E_o \operatorname{Sin}\delta_1 \right] \right\rangle + \frac{V^2}{2} \left\{ \frac{X_d - X_q}{(X_d+X_e)(X_q+X_e)} \right\} (\operatorname{Sin}2\delta - \operatorname{Sin}2\delta_1) + \text{----3.22}$$

Where E(t) is given by 6.19 and is a function of the exciter response characteristics.

The equation of a motion of the rotor in turns of its mechanical constants can be expressed as

$$I \frac{d^2\delta}{dt^2} + T_f \frac{d\delta}{dt} + K\delta \quad \text{----------} 3.23.$$

If equations 3.22 and 3.23 are equated and a solution for δ as a function of time is obtained then the dynamics of the stability can be analysed and it could be reckoned if the system would remain in dynamic equilibrium.

CHAPTER VII

DETERMINATION OF RESPONSE OF AN EXCITER MACHINE BY DIFFERENT METHODS.

Specifications of the exciter machine.

General Electric Comp. Wound D.C. Generator.

125 volts 5 KW. 40 amps. Service Factor 1.15 at rated volts.

M/c. No. 2464 230 NG.

Enclosure Open type

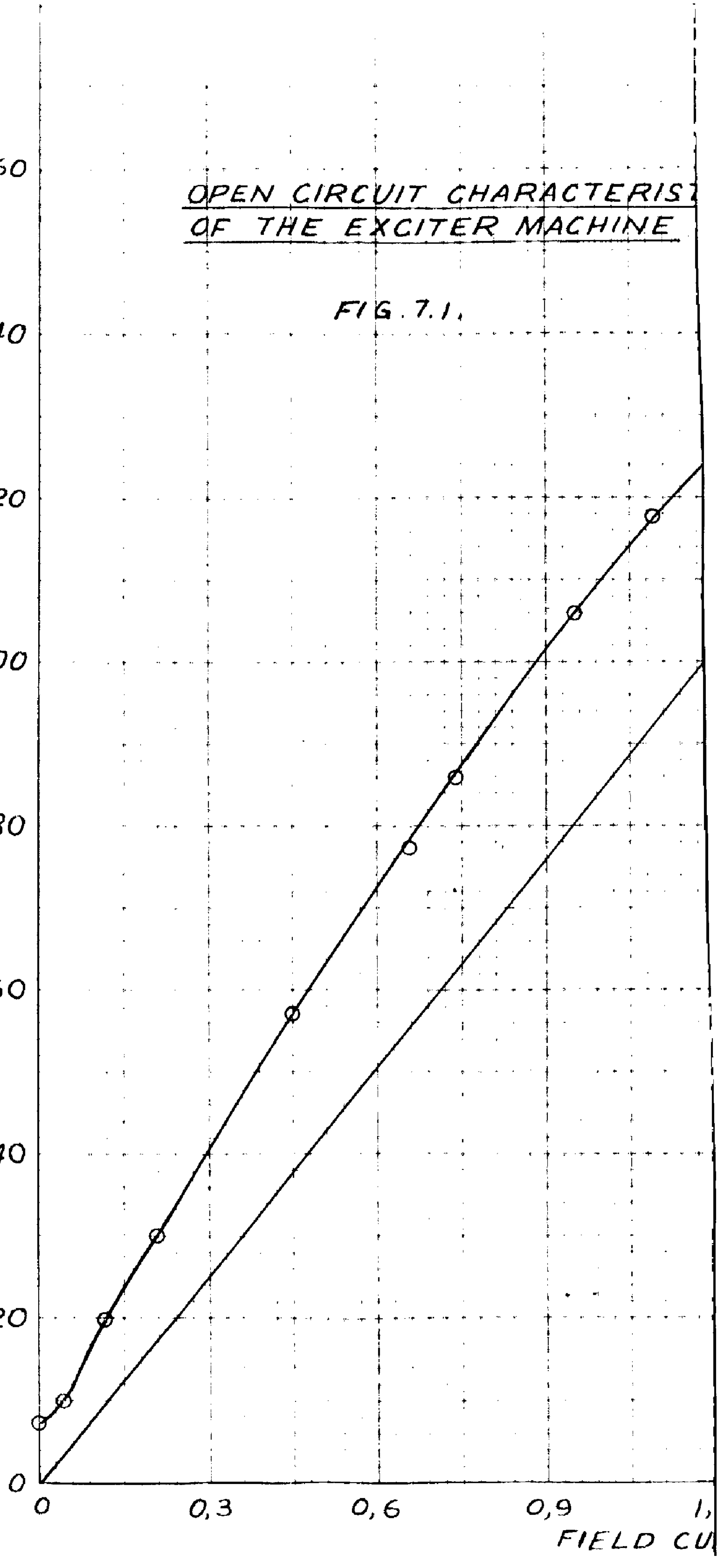
TEST DATA FOR OPEN CIRCUIT CHARACTERISTICS

TABLE I.

FIELD CURRENT AMPS	ARMATURE OPEN CIRCUIT VOLTS
0	7·1
0·05	10·0
0·12	20·0
0·21	30·0
0·45	56·6
0·65	77·4
0·74	85·5
0·95	105·0
1·10	117·0
1·21	125·4
1·35	135·0
1·50	144·0
1·63	150·6
1·83	160·0
1·94	163·0

D.C. Resistance of Field winding (hot)

$$= \frac{163}{1.94} = 84 \text{ ohms.}$$


OPEN CIRCUIT CHARACTERIST
OF THE EXCITER MACHINE
FIG. 7.1.
0
20
40
60
80
100
120
140
160
0
0,3
0,6
0,9
1,
FIELD CU

7.1. POINT BY POINT SOLUTION

In chapter I, this method of determining the response of a self or a separately excited exciter was indicated.

For the self excited case assuming constant coefficient of dispersion equation 1.13 gives the incremental time Δt required for the voltage to rise by Δe and is

$$\Delta t = \frac{\frac{\sigma N}{K} \cdot \Delta e}{e - Ri} = \frac{T . \Delta e}{e - Ri}$$

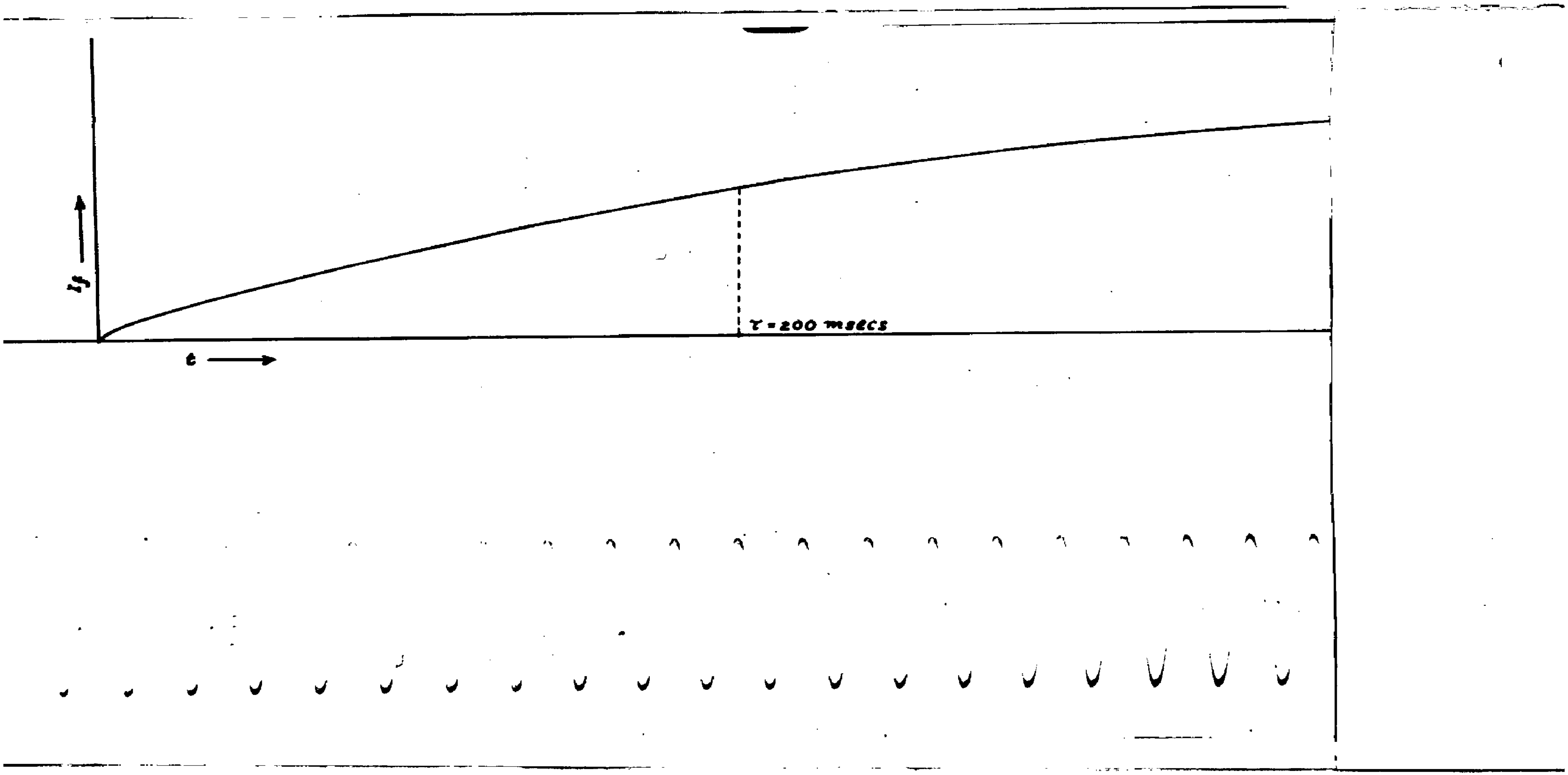
where T is time constant of the exciter machine and has been defined earlier.

Using the O.C.C. obtained by test, the time required Δt for the armature voltage to rise by Δe is indicated in the following table. The discontinuity in the field circuit is introduced when the armature voltage is 100 volts.

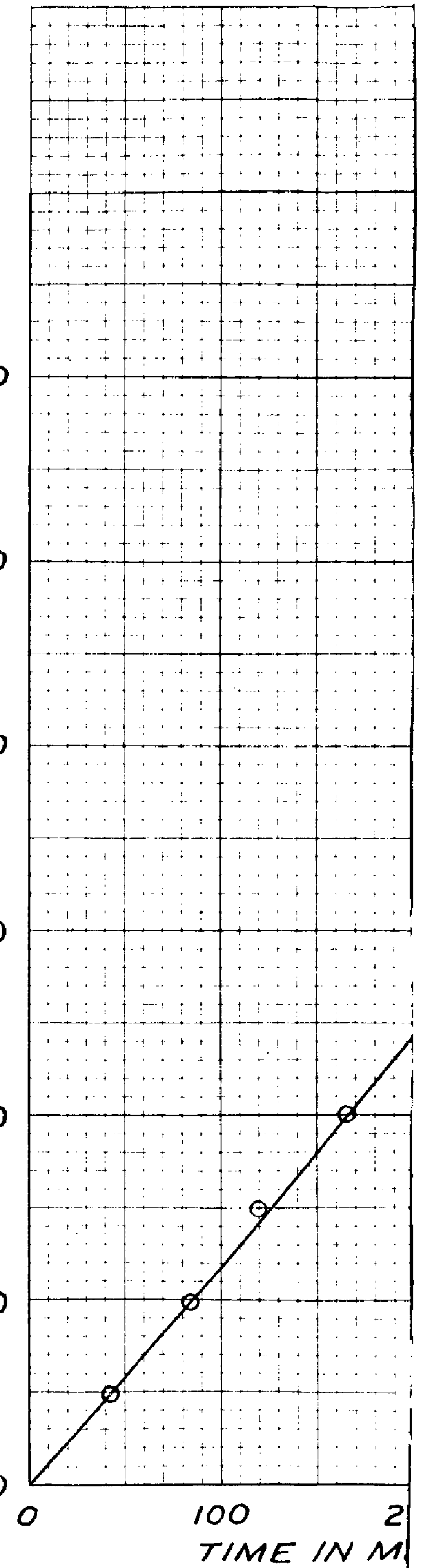
P.T.O.

TABLE II

Δ_e	e	i	$R \times i = 84i$	$e - Ri$	$\frac{\Delta e}{e - Ri}$	$\Sigma \frac{\Delta e}{e - Ri}$	$T \Sigma \frac{\Delta e}{e - Ri}$ T = 0·2 Sec.
5	102·5	0·933	78·3	24·2	0·207	0·207	41·4
5	107·5	0·995	83·5	24·0	0·208	0·415	83·0
5	112·5	1·055	88·5	24·0	0·208	0·623	124·6
5	117·5	1·11	93·2	24·3	0·206	0·829	165·8
5	122·5	1·18	99·0	23·5	0·213	1·042	208·4
5	127·5	1·245	104·5	23·0	0·217	1·259	251·8
5	132·5	1·32	111·0	21·0	0·238	1·497	299·4
5	137·5	1·39	116·8	20·7	0·242	1·739	347·8
5	142·5	1·475	124·0	18·5	0·270	2·009	401·8
5	147·5	1·575	132·5	15·0	0·333	2·342	468·4
2	151·0	1·64	137·8	13·2	0·152	2·494	498·8
2	153·0	1·665	140·0	13·0	0·154	2·648	529·6
2	155·0	1·70	142·7	12·3	0·163	2·811	562·2
2	157·0	1·735	145·7	11·3	0·177	2·988	597·6
2	159·0	1·80	151·2	7·8	0·256	3·244	648·8
1	160·5	1·855	156·0	4·5	0·222	3·466	693·2
1	161·5	1·92	161·2	0·3	3·330	6·796	1359·2



OSCILLOGRAPH NO. 1



0
100
2
TIME IN M

7.2 SOLUTION USING GRAPHICAL INTEGRATION

The method was detailed out in chapter II and the time required for the emf to rise from e_1 to e is given by $t = T \int_{e_1}^{e} \frac{de}{\Delta e}$ in case of self excited exciters where $T = \frac{\sigma N}{K}$ is the time constant on assumption of constant coefficient of dispersion e is obtained from the O.C.C. and the field resistance line.

$\int_{e_1}^{e} \frac{de}{\Delta e}$ is evaluated by measuring the area under the curve of $1/\Delta e$ vs e from e to e_1. This evaluation is done by counting the no. of squares in that area. The detailed calculations are carried out in the following table.

TABLE III

e	Δe	$\frac{1}{\Delta e} \times 10^{-3}$	$\int_e^{e_1} de/\Delta e$ NO. OF SQUARES	$\int_e^{e_1} de/\Delta e$ MILLISECS MULTIPLYING FACTOR 1	$T\int_e^{e_1} \frac{de}{\Delta e}$ T = 0.2 Sec
100	26	38.4	0	0	0
110	26	38.4	402	402	80.4 msecs
120	26	38.4	784	784	156.8
130	23	43.5	1190	1190	238.0
140	20	50.0	1655	1655	331.0
145	18	55.5	1920	1920	384.0
150	15	66.6	2525	2525	505.0
155	11	90.8	2895	2895	579.0
157	8	125.0	3087	3087	617.4
159	7	143.0	3320	3320	664.0
161	2	500.0	3670	3670	734.0
163	0	∞			

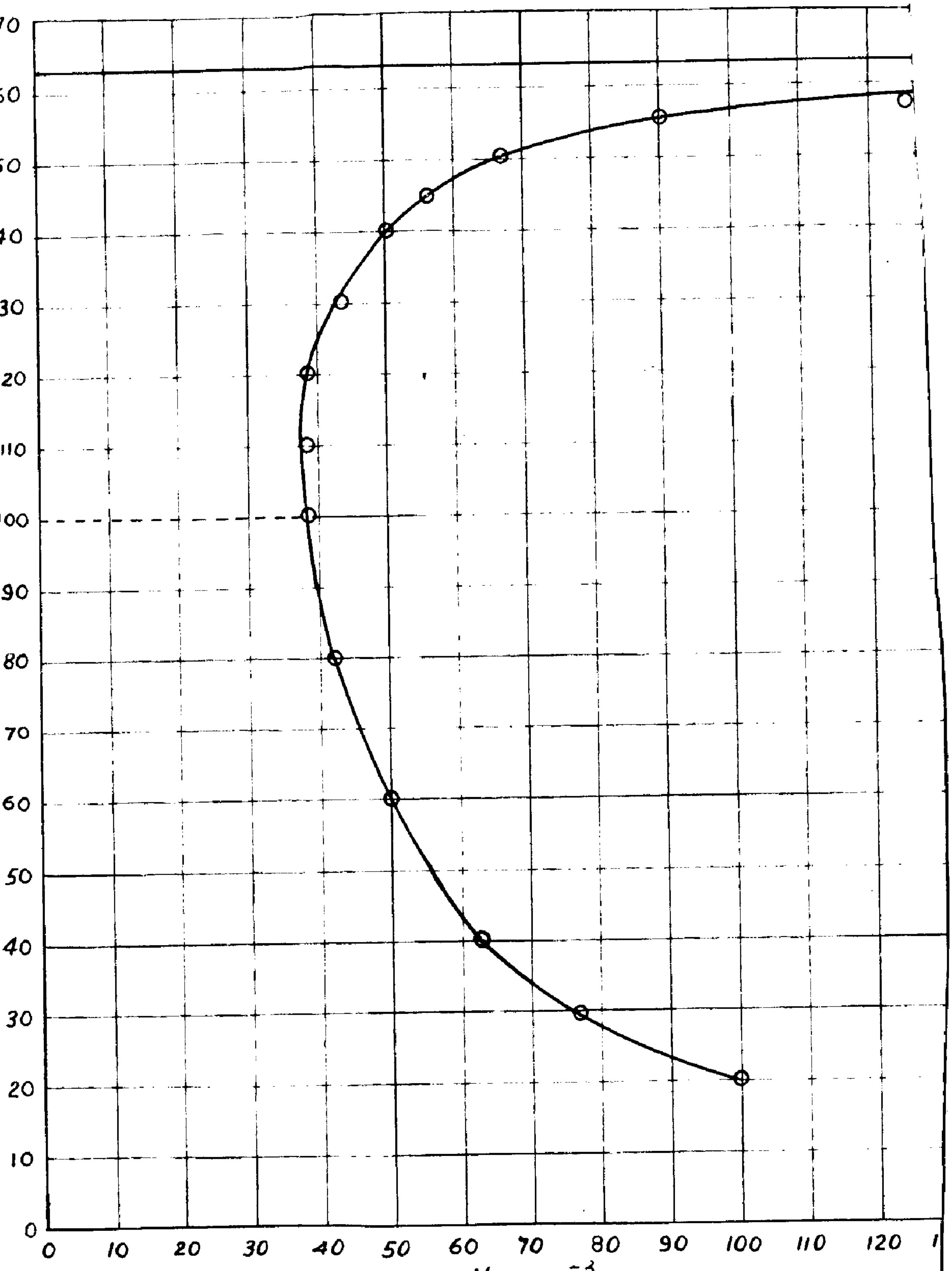
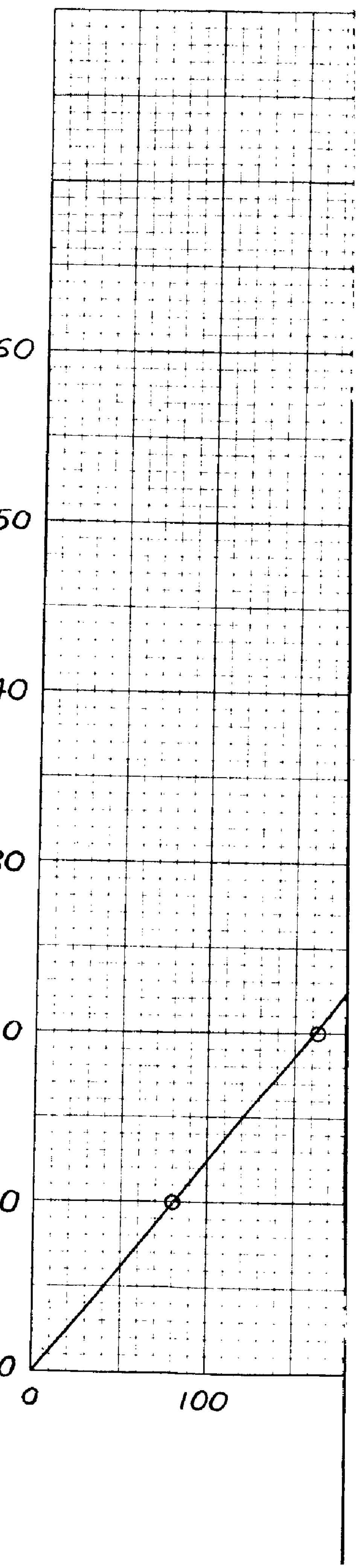
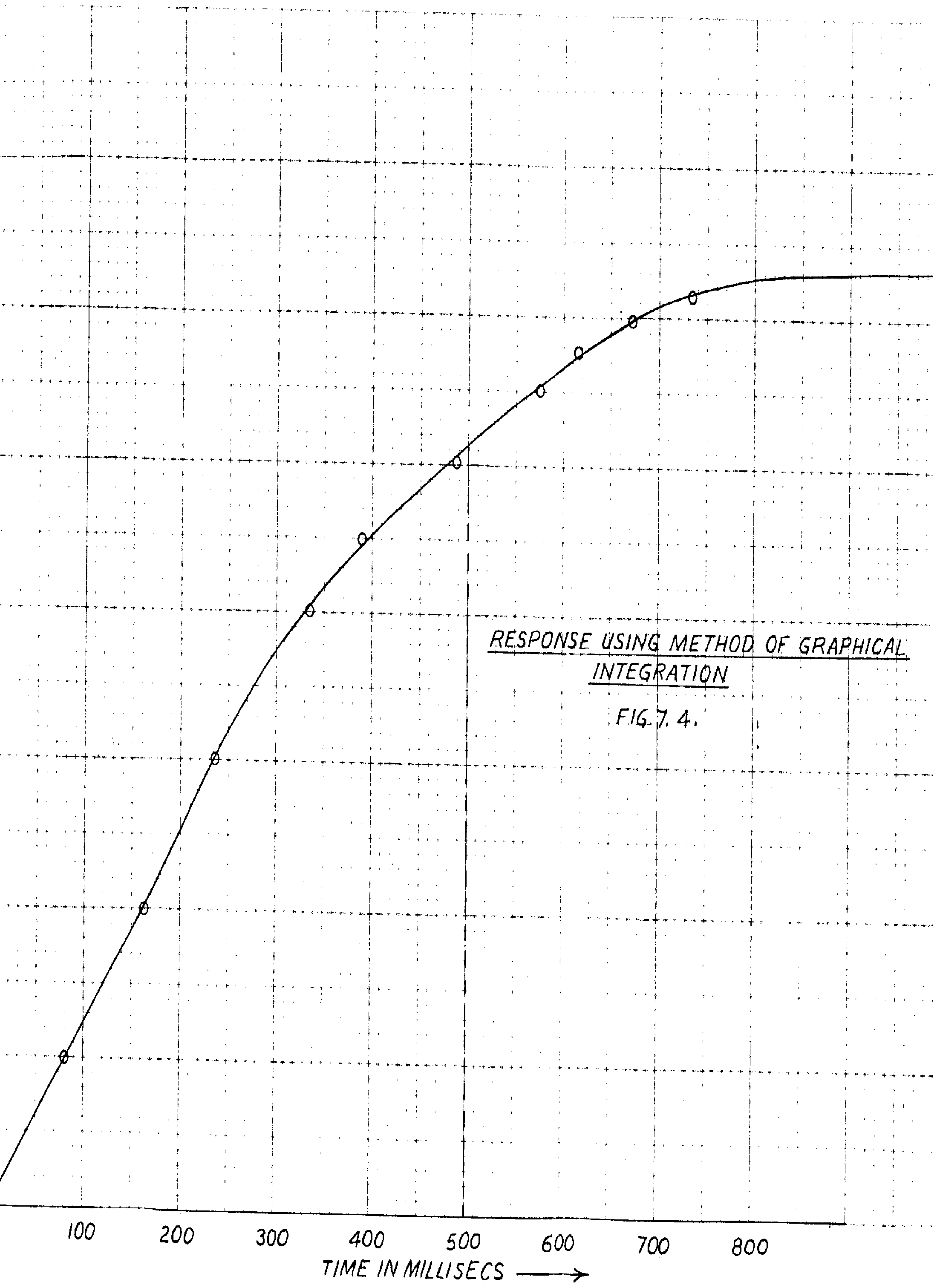

0 10 20 30 40 50 60 70 80 90 100 110 120
$1/\Delta_e \times 10^{-3}$ →
0 10 20 30 40 50 60 70 80 90 100 110 120

FIG. 7.3.


0
100



RESPONSE USING METHOD OF GRAPHICAL INTEGRATION

FIG. 7. 4.

The time constant T used in the above two solutions was taken from the oscillograph no. I. of the field current of the exciter machine when a d.c. voltage was suddenly impressed on it. The time constant $T = \frac{\sigma N}{K} = \frac{\sigma N.\phi}{E} = \frac{\sigma N\phi.}{Ri}$

ϕ is the total flux produced by the field and $\frac{\sigma\phi}{I}N$ is the flux linkages per ampere produced by the field, and may be called the Nominal field inductance of the machine. This time constant L/R is taken approximately to be the same as the time reqd. for the field current to reach 63.2 percent of its final value. This later time is the time constant of the exciter field on assumption of linear magnetisation curve so that the inductance does not change. In our calculation of response cognizance is taken of the nonlinearily of the magnetisation curve and hence the time constant can not be given the same interpretation as the time constant of a linear circuit. The actual time constant should be slightly different and the error can be estimated by comparing the response calculated by the foregoing methods with the response obtained by actual oscillograph.

7.3. SOLUTION USING FORMAL INTEGRATION

7.3.1. Representing magnetisation characteristics by Fröhlich equation:

The constants a and b of the Fröhlich equation $e = \frac{ai}{b+i}$ are evaluated by taking two points on the O.G.C. corresponding to i = 0.5 amps and i = 1.5 amps.

For i = 0.5 and for i = 1.5

e = 60 and e = 144

$$60 = \frac{a \times 0.5}{b + 0.5}$$

or $60b + 30 = 0.5a$ ------(i)

and $$144 = \frac{1.5a}{b + 1.5}$$

or $144b + 216 = 1.5a$ -----(ii)

Solving (i) and (ii) for a and b gives

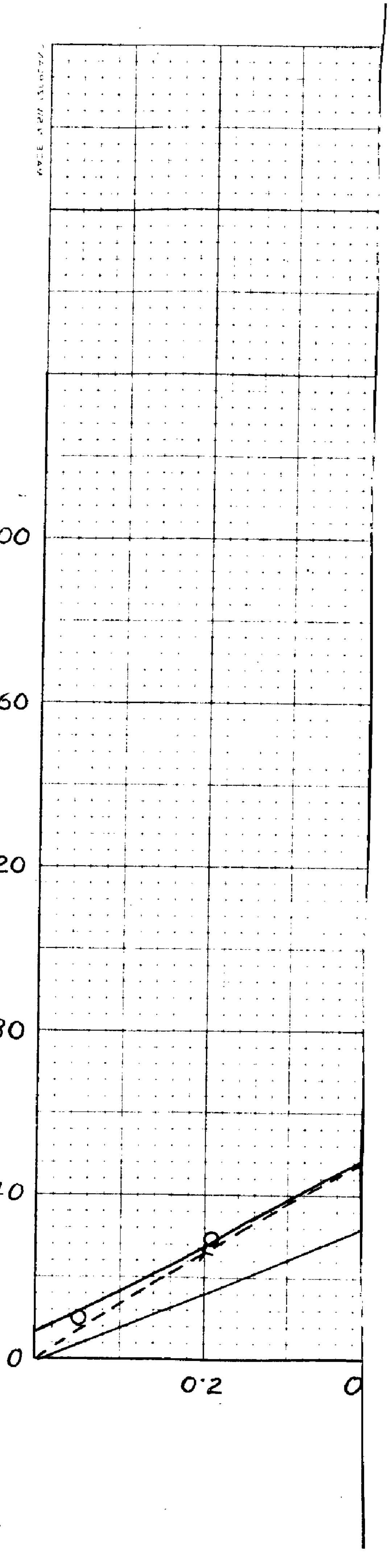
a = 480

and b = 3.5.

Fröhlich equation therefore becomes $e = \frac{480i}{3.5 + i}$.

TABLE IV

i	$480i$	$3.5+i$	$e=\frac{480i}{3.5+i}$
0.2	96	3.7	26
0.5	240	4.0	60
0.7	336	4.2	80
1.0	480	4.5	107
1.3	625	4.8	130
1.5	720	5.0	144
1.8	865	5.3	163
1.9	912	5.4	169


0
0·2

The curve of e vs. i is plotted on the sar graph as the O.C.C. in fig. 7.5.

7.3.2. Representing magnetisation curve by modified Fröhlich equation:

The constants a, b and c of the modified Frohlich equation $e = \frac{ai}{b+i} + ci$ are evaluated by taking 3 points on the O.C.C.

For i = 0.4 amps e = 50 volts.

i = 1.0 amp e = 108 volts.

i = 1.6 amp e = 150 volts.

Three simultaneous equations in the constant are obtained whose solution gives a,b and c.

$$50 = \frac{0.4a}{b + 0.4} + 0.4c.$$

or $50b + 20 = 0.4a + 0.4bc + 0.1bc$ --- (i)

$$108 = \frac{a}{b+1} + c$$

or $108b + 108 = a + bc + c$ ---------- (ii)

Multiplying (i) by 2.5 and substracting from (ii) gives

$-17b + 58 = 0.6c$

or $17b - 58 = -0.6c$ -------------- (a)

Also $150 = \frac{1.6a}{b+1.6} + 1.6c.$

or $150b + 240 = 1.6a + 1.6\,bc + 2.56c$ -- (iii

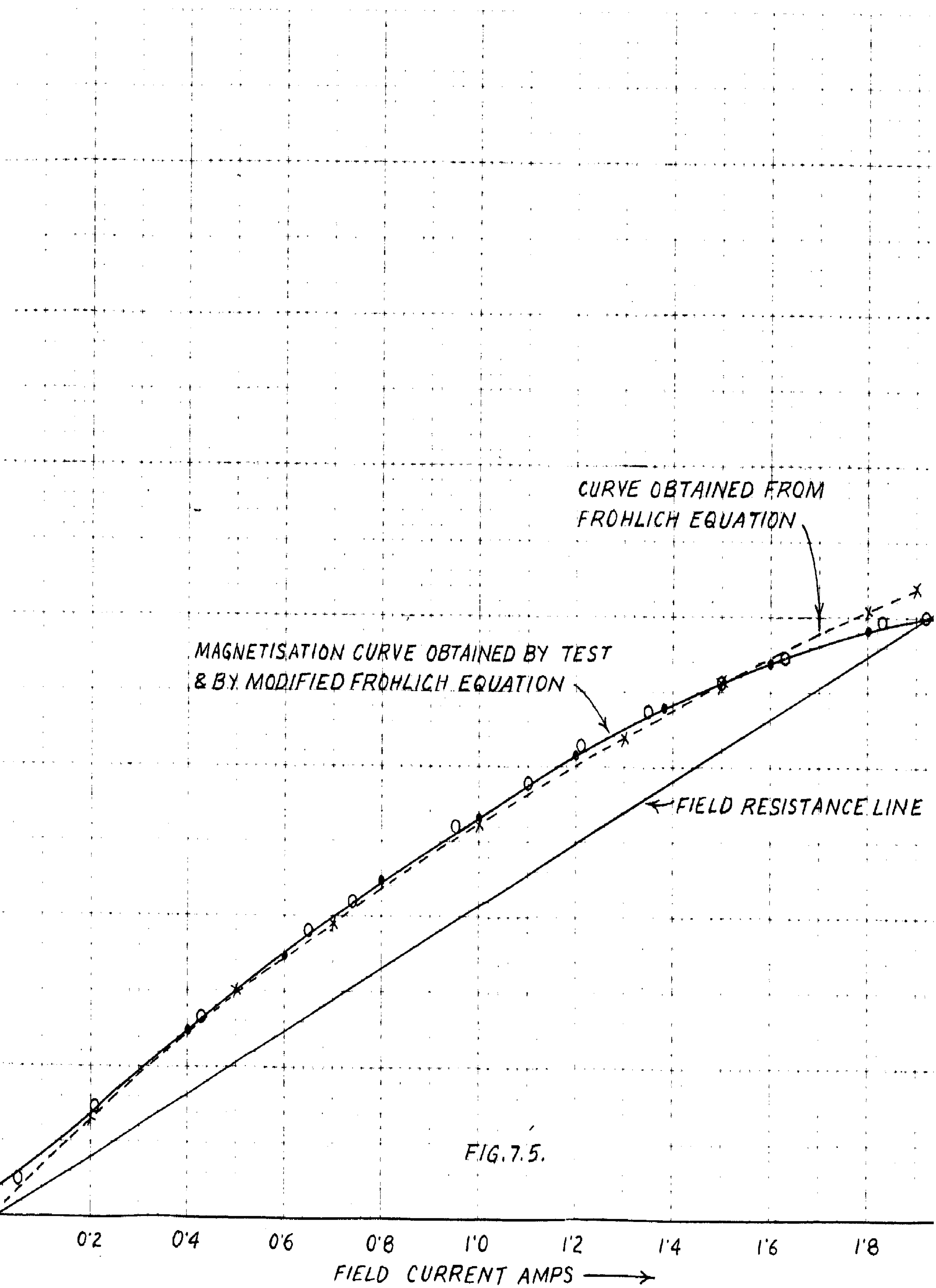

CURVE OBTAINED FROM
FROHLICH EQUATION
MAGNETISATION CURVE OBTAINED BY TEST
& BY MODIFIED FROHLICH EQUATION
FIELD RESISTANCE LINE
0'2
0'4
0'6
0'8
1'0
1'2
1'4
1'6
1'8
FIELD CURRENT AMPS

FIG. 7.5.

The curve of e vs. i is plotted on the sar graph as the O.G.C. in fig. 7.5.

7.3.2. Representing magnetisation curve by modified Fröhlich equation:

The constants a, b and c of the modified Frohlich equation $e = \frac{ai}{b+i} + ci$ are evaluated by taking 3 points on the O.C.C.

For i = 0.4 amps e = 50 volts.

i = 1.0 amp e = 108 volts.

i = 1.6 amp e = 150 volts.

Three simultaneous equations in the constant are obtained whose solution gives a,b and c.

$$50 = \frac{0.4a}{b + 0.4} + 0.4c.$$

or $50b + 20 = 0.4a + 0.4bc + 0.1bc$ ---(i)

$$108 = \frac{a}{b+1} + c$$

or $108b + 108 = a + bc + c$ ----------(ii)

Multiplying (i) by 2.5 and substracting from (ii) gives

$-17b + 58 = 0.6c$

or $17b - 58 = - 0.6c$ -------------- (a)

Also $150 = \frac{1.6a}{b + 1.6} + 1.6c.$

or $150b + 240 = 1.6a + 1.6\,bc + 2.56c$--(iii)

Multiplying (i) by 4 and substracting from

(iii) gives $-50b + 160 = 1.92c$

or $50b - 160 = -192c$ ------------(b)

Simultaneous solution of (a) and (b) gives

$b = 5.9$ and $c = -80.6$

From (i) $a = 1300.$

The modified Fröhlich equation representing the open circuit characteristics would be

$$e = \frac{1300\,i}{5.9 + i} - 80.6\,i.$$

TABLE V

i	$1300\,i$	$5{\cdot}9+i$	$\frac{1300\,i}{5{\cdot}9+i}$	$80{\cdot}6\,i$	$e=\frac{1300\,i}{3{\cdot}5+i}-80{\cdot}6\,i$
0·2	260	6·1	42·6	16·1	24·5
0·4	520	6·3	82·5	32·25	50·25
0·6	780	6·5	120·0	48·3	71·7
0·8	1040	6·7	155·0	64·5	90·5
1·0	1300	6·9	188·6	80·6	108·0
1·2	1560	7·1	220·0	96·7	123·3
1·4	1820	7·3	250·0	112·8	137·2
1·6	2080	7·5	279·0	129·0	150·0
1·8	2340	7·7	304·0	145·2	158·8

On the same graph as the O.C.C., the curve of e vs. i as obtained in Table V is plotted. The modified Fröhlich equation therefore represents the magnetisation characteristics with greater accuracy and the equation $e = \frac{1300\ i}{5.9 + i} - 80.6 i$ almost accurately represents the O.C.C. Since however not much error is introduced by using the simple Fröhlich equation, this is substituted in the differential equation and a solution of current response is obtained. Therefore the actual voltage response is obta-ined. In chapter III solution by formal integration using the Fröhlich equation was attempted in case of separately excited exciters.

7.3.3. SOLUTION BY FORMAL INTEGRATION USING FRÖHLICH EQUATION:-

Derivation of time current relationship in case of self excited exciters:

The differential equations ~~was~~ was

$$T \frac{de}{dt} + Ri = e.$$

$$\text{where } T = \frac{\sigma N}{K}$$

Also

$$e = \frac{ai}{b + i}$$

Hence.

$$\frac{de}{dt} = \frac{(b+i)a - ai}{(b+i)^2} \cdot \frac{di}{dt}.$$

$$= \frac{ab.}{(b+i)^2} \cdot \frac{di}{dt}$$

On substitution

$$\frac{T\ ab}{(b+i)^2} \frac{di}{dt} + Ri = \frac{ai}{b + i}$$

$$\text{or } \frac{Tab}{(b+i)^2} \cdot \frac{di}{dt} = \left[\frac{ai}{b+i} - Ri \right]$$

Or

$$\frac{Tab}{(b+i)} \cdot \frac{di}{dt} = \frac{(ai - Rbi - Ri^2)}{(b + i)}$$

Or

$$\frac{Tab}{(b+i)\ i\ (a-Rb - Ri)} di = dt$$

To integrate this expression it is resolved into partial fractions as $\frac{A}{b + i} + \frac{B}{i} + \frac{C}{a-Rb-Ri}$

The coefficicnts A, B and C can be determined from the following three simultaneous equations.

$$- AR - BR + C = 0$$

$$A(a - Rb) + B\ (a - 2Rb) + Cb = 0.$$

$$B(a - Rb)\ b = 1$$

Solution of these gives

$$A = - \frac{1}{ab}$$

$$B = \frac{1}{(a - Rb)b}$$

$$C = \frac{R^2}{(a - Rb)\ a}$$

Substituting these in the main expression gives

$$T \left[- \frac{di}{b + i} + \frac{a}{a-Rb} \cdot \frac{di}{i} + \frac{bR^2}{a-Rb} \cdot \frac{di}{a-Rb-Ri} \right. = dt$$

Integration gives

$$T\left[-\log(b+i_1)+\frac{a}{a-Rb}\log i-\frac{bR}{a-Rb}\log(a-Rb-Ri)\right]=t+k$$

The integration constant K can be evaluated by using the initial condition that at $t=0\ \ i=i_1$

$$\therefore T\left[-\log(b+i_1)+\frac{a}{a-Rb}\log i_1-\frac{bR}{a-Rb}\log(a-Rb-Ri_1)\right]=k$$

$\therefore$ The final solution for the time current relationship is therefore

$$t=T\left[\frac{a}{a-Rb}\cdot\log\frac{i}{i_1}-\log\frac{b+i}{b+i_1}-\frac{Rb}{a-Rb}\log\frac{a-Rb-Ri}{a-Rb-Ri}\right]$$

The above time current relationship can be used to determine the current response.

In our case, constants to be used in the above expression have already been evaluated and are

$T = 0.2$ secs.

$a = 480$

$b = 3.5$

$R = 84$ ohms.

$i_1 = 0.9$ amps.

$$\frac{a}{a-Rb}=\frac{480}{480-84\times3.5}=\frac{480}{480-294}=\frac{480}{186}=2.58$$

$$\frac{Rb}{a-Rb} = \frac{84x3.5}{480-84x3.4} = \frac{294}{480-294} = \frac{294}{186} = 1.58.$$

The time current relationship becomes

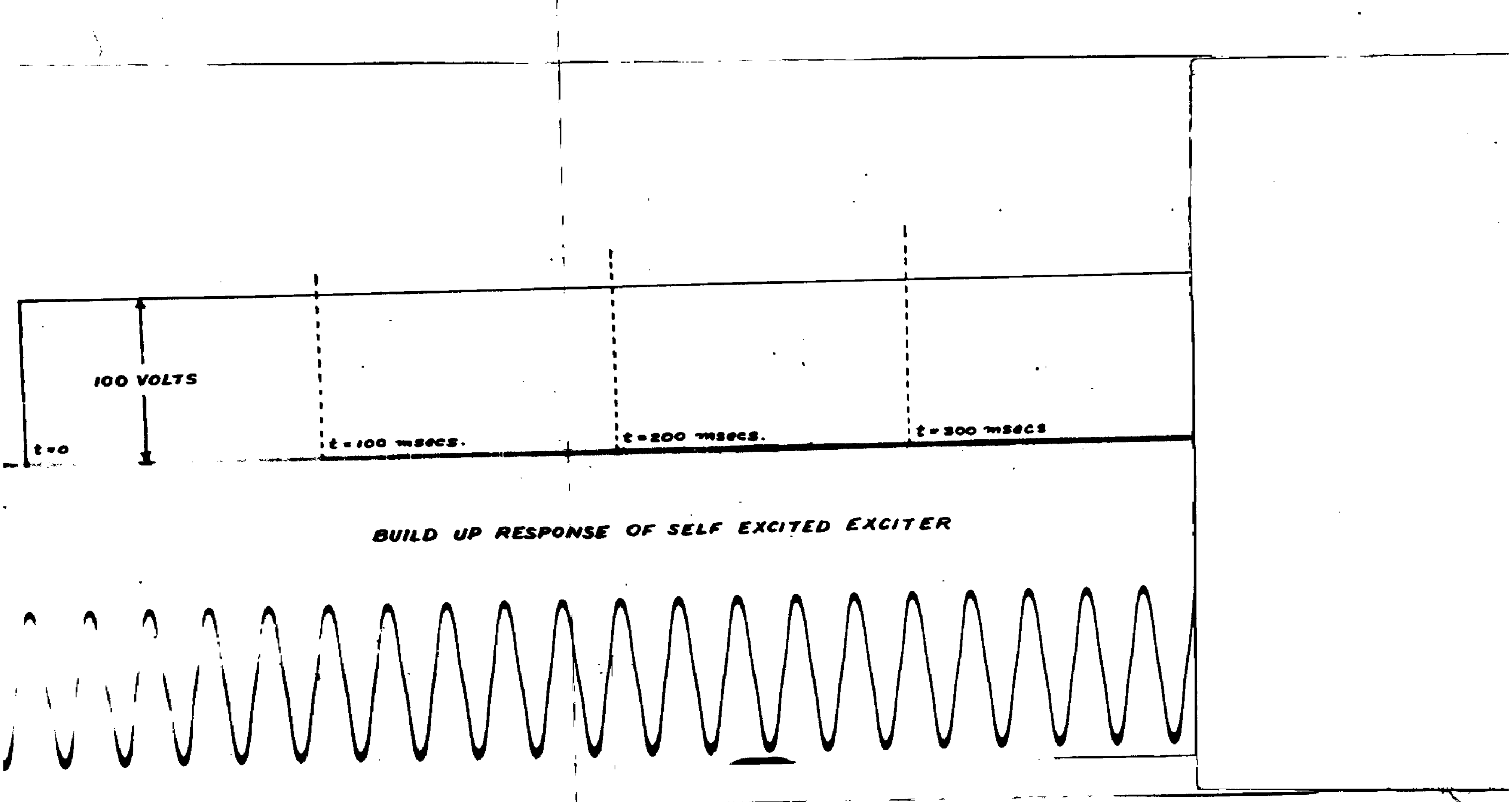
$$t = 0.2\left[2.58\ \log_e 1.11\, i - \log_e \frac{3.5+i}{4.4} - 1.58 \log_e \frac{186-84\, i}{112.5}\right]$$

Assuming different values for i, the time t required for the current to rise from $i_1 = 0.9$ amps to i can be found. Details of calculation are shown in the following table.

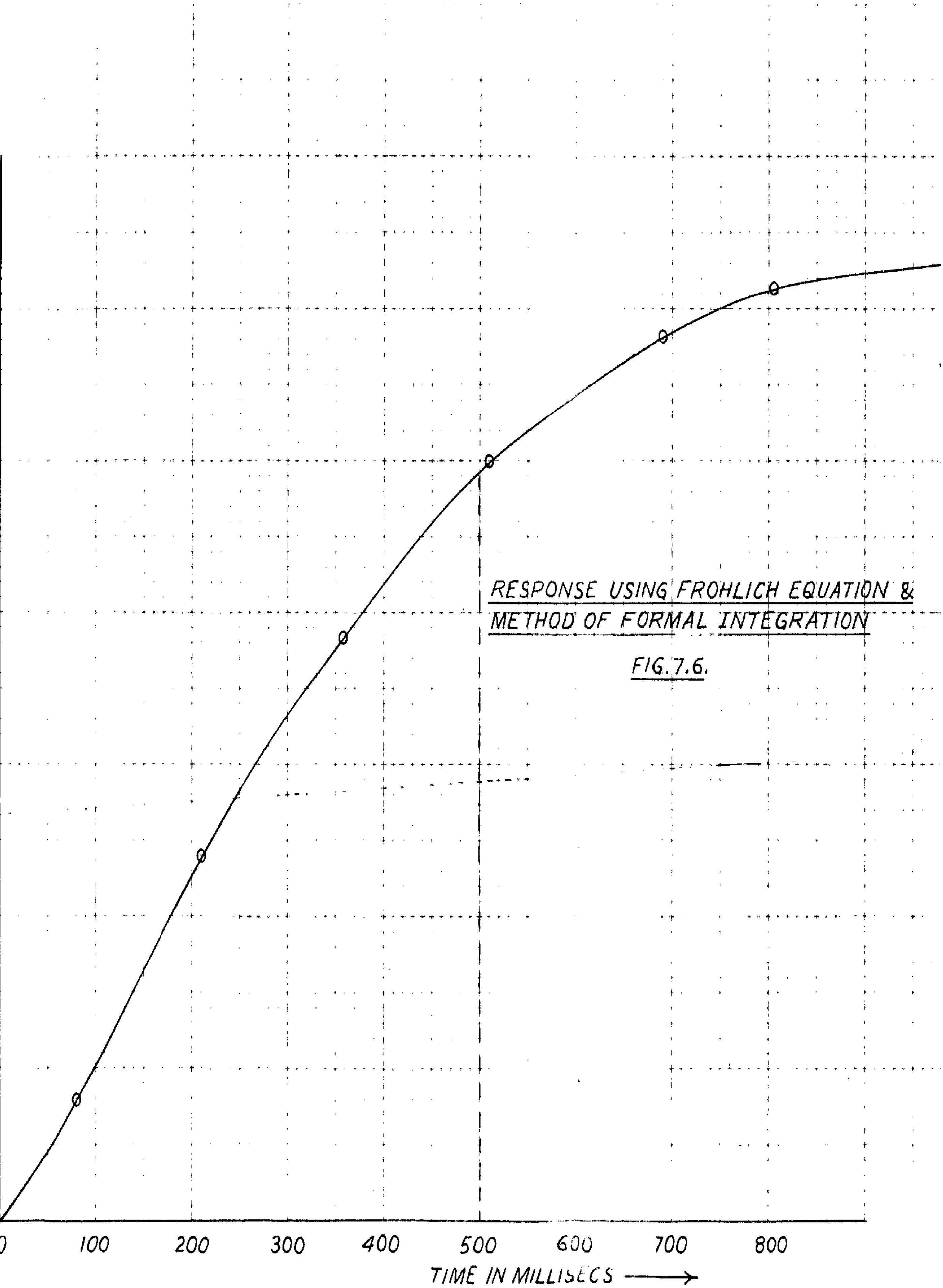
TABLE VI

i	$\log_e 1.11\, i$	(i) $2.58 \log_e 1.11$	$\frac{3.5+i}{4.4}$	(ii) $\log_e \frac{3.5+i}{4.4}$	$\frac{186-84i}{112.5}$	(iii) $1.58 \log_e \times \frac{186-84i}{112.5}$	$[i-ii-iii] \cdot 2\times10^3 = t$ in m secs	e (volts)
1.0	0.104	~~0.268~~	~~1.02~~	~~0.02~~	0.906	-0.153	80.2	108
1.2	0.285	0.665	1.07	0.068	0.755	-0.440	207.4	124
1.4	0.438	1.130	1.115	0.140	0.610	-0.777	353.4	138
1.6	0.577	1.490	1.17	0.157	0.462	-1.220	510.6	150
1.8	0.639	1.790	1.20	0.182	0.312	-1.850	691.6	158
1.9	0.747	1.930	1.23	0.207	0.231	-2.310	806.6	161

In the last column e has been written corresponding to the current i from magnetisation curve. The actual response curve can be plotted with reference to the last two columns of e and t.



OSCILLOGRAPH NO. 2



RESPONSE USING FROHLICH EQUATION & METHOD OF FORMAL INTEGRATION

FIG. 7.6.

7.4. COMPARISON OF RESPONSE AS CALCULATED BY DIFFERENT METHODS

The following table gives the rise in armature voltage of the exciter after a certain time as calculated by 3 different methods and as obtained by actual oscillograph.

TABLE VII

t millisecs	e 1st Method	e 2nd Method	e 3rd Method	e From Oscillograph
100	112	112	110	112
200	124	125	123	123
300	135	137	133	135
400	145	145	142	144
500	152	151	149	152
600	158	156	154	158
700	161	160	158	-
800	162	162	161	-

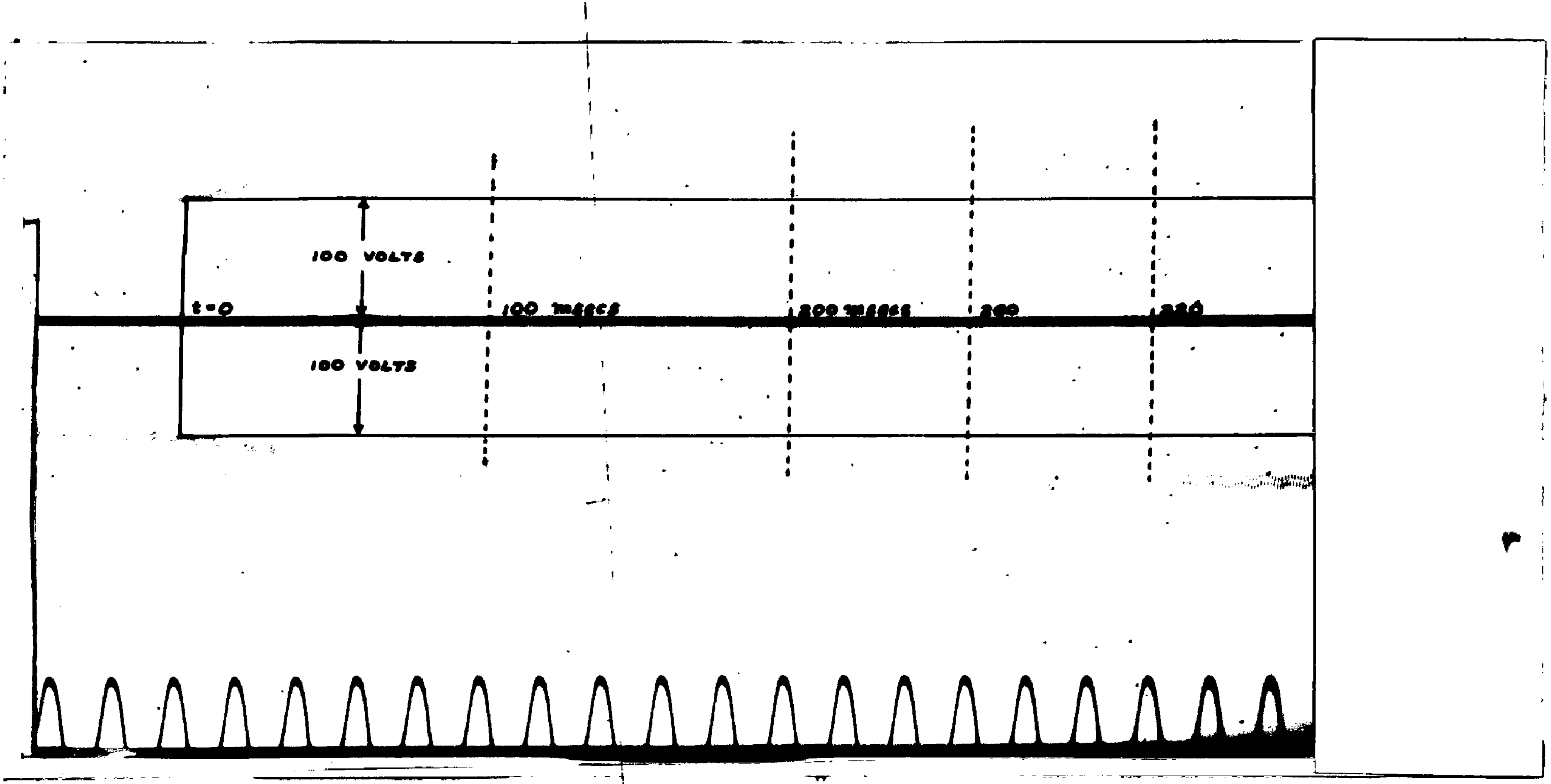
Following the method given in chapter I section 1.5 the nominal response ratio has been found by all the three methods and has been calculated out as

1.14 units by 1st method

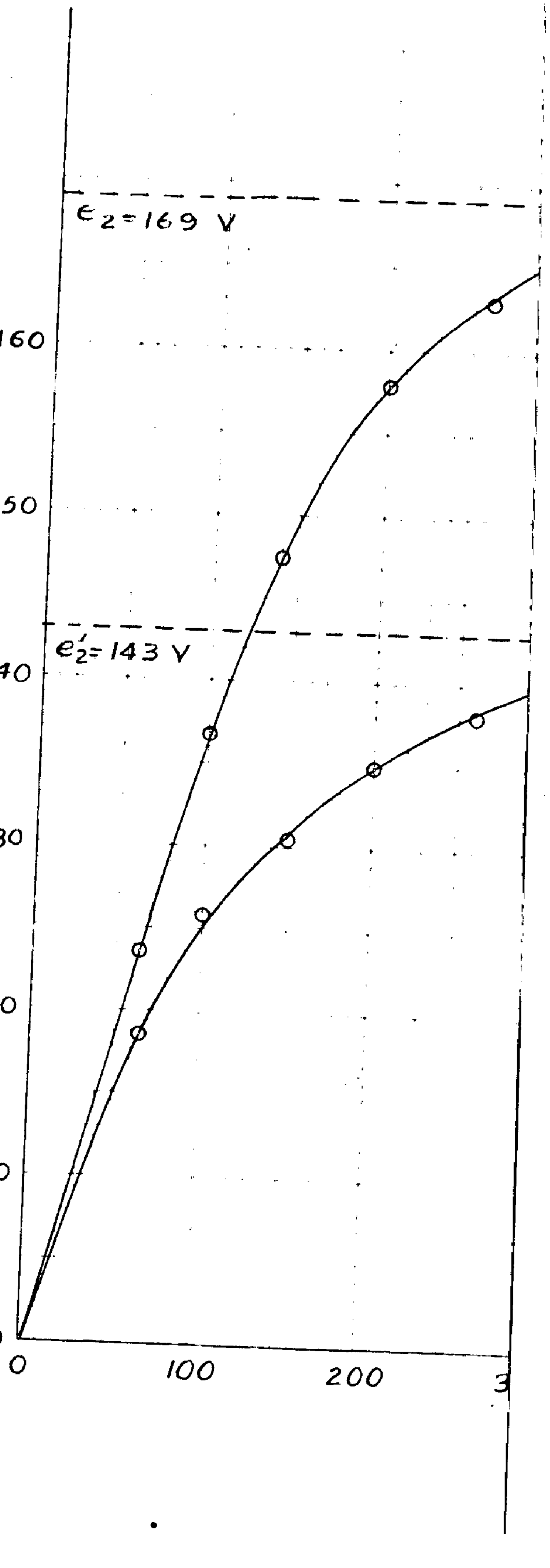
1.16 units by 2nd method

1.08 units by 3rd method.

1.1 units by oscillograph.



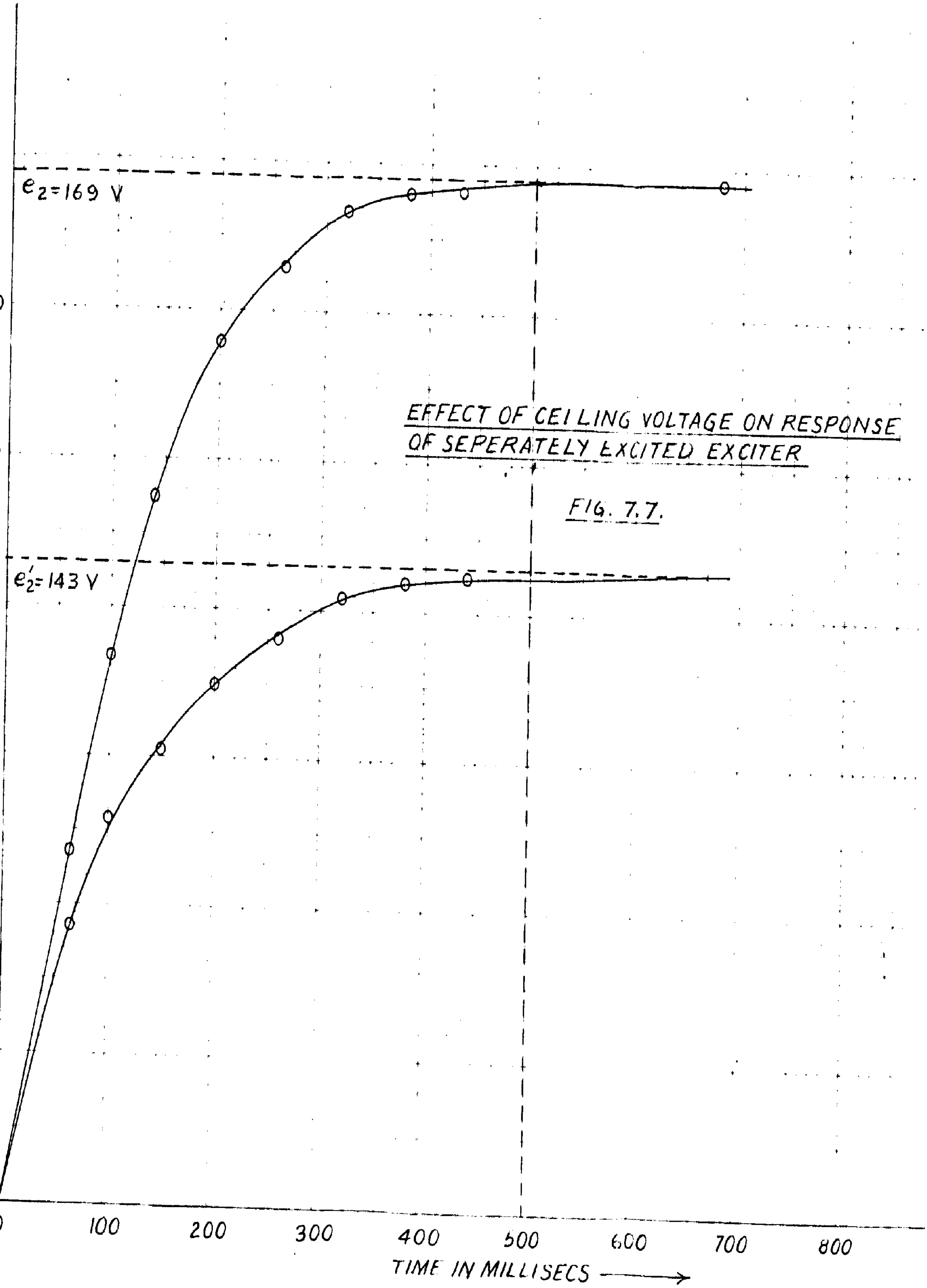
OSCILLOGRAPH NO.3



$e_2 = 169$ V
$e_2' = 143$ V
160
50
40
30
0
100
200

7. EFFECT OF CEILING VOLTAGE ON RESPONSE OF SEPARATELY EXCITED EXCITER

Using the exciter machine as a separately excited exciter oscillographs of the responses were taken corresponding to ceiling voltages of 169 volts and 143 volts. In chapter IV it was seen that the response rate can be varied over wide limits by varying the ceiling voltage in case of separately excited exciter while in case of self excited exciters it is limited by the field resistance of the machine. The nominal response ratio was calculated in both the cases and was found to be 1.23 units for cieling voltage of 143 volts and 2.13 units for ceiling voltage of 169 volts. A study of fig. 7.7 which is a plot of the two responses shows that the rate of response is also higher than that in case of self excited exciter. The shape of response curves is in comformity with the theoretical consideration outlined in chapter IV while comparing the response of self and separately excited exciters.



EFFECT OF CEILING VOLTAGE ON RESPONSE OF SEPERATELY EXCITED EXCITER

FIG. 7.7.

7. EFFECT OF CEILING VOLTAGE ON RESPONSE OF SEPARATELY EXCITED EXCITER

Using the exciter machine as a separately excited exciter oscillographs of the responses were taken corresponding to ceiling voltages of 169 volts and 143 volts. In chapter IV it was seen that the response rate can be varied over wide limits by varying the ceiling voltage in case of separately excited exciter while in case of self excited exciters it is limited by the field resistance of the machine. The nominal response ratio was calculated in both the cases and was found to be 1.23 units for cieling voltage of 143 volts and 2.13 units for ceiling voltage of 169 volts. A study of fig. 7.7 which is a plot of the two responses shows that the rate of response is also higher than that in case of self excited exciter. The shape of response curves is in comformity with the theoretical consideration outlined in chapter IV while comparing the response of self and separately excited exciters.

CONCLUSIONS

The scope of a dissertation on 'Calculation of Exciter Response and its Field of Use' was more in the nature of a systematic presentation of all the methods of calculation and the factors which effect the calculation rather than a concentrated study of a certain aspect of this quite vast field. Before an actual calculation could be attempted it was necessary to outline methods of solution of differential equations of nonlinear circuits which are characterised by variable coefficients whereby the calculation of response could be carried out. The method which uses a constant leakage inductance of the field winding neglecting the effect of saturation on leakage field flux necessitates an accurate estimation of L_ℓ. This can be accurately calculated only if the details of field poles and their stampings are available. For a machine which has been assembled, the assumption of constant coefficient of dispersion needs only the open circuit characteristics of the machine and the nominal field inductance to be known for the exciter response to be calculated. The nominal field inductance could be easily calculated by knowing the field time constant of the machine from an oscillograph of the build-up of field current in case of a step input of the voltage to the field winding. Basically the three methods e.g. point by point solution method of graphical integration, and the method of Formal integration are different mathematical approaches to the same solution. The analytical method incorporating actual solution of the differential equation by representing the magnetisation characteristics by an approximate equation becomes quite lengthy. This requires an exact representation of magnetisation curve by an equation. Fröhlich equation represents the curve only approximately and at higher saturations it becomes quite inaccurate. Analytical method using Fröhlich equation gives only an approximate solution though the calculations involved are quite cumbersome. If however it is attempted to represent the characteristics by modified Fröhlich equation or some other accurate representative equation then the integration of the differential equation and its subsequent calculation would become too cumbersome to be carried out. The graphical solution involved in adapting the form of the differential equation to a convenient form and subsequently plotting the integrand and evaluating the integral by measuring areas. This could be carried out with sufficient ease and accuracy by counting little squares on the graph paper. The method involving point by point solution is more readily and easily adaptable to any transient problem of a linear or a nonlinear circuit. In case of exciter analysis this gives the easiest solution even when finding out solutions for the alternator field current or for E_q' etc.

Results obtained by the three methods and also that obtained by actual oscillograph record were quite in conformity with each other.

It was only possible to consider theoretically the effect of exciter response on transient and dynamic system stability. Both these forms of stability are affected considerably by the build up and build down response rates of the exciter. Since it was not possible to take an oscillogram of E_q' in case of a sudden symmetrical three phase short circuit both with and without exciter response, further experimental work in studying the effect of exciter response on transient stability could not be carried out. It has been indicated in section 6.4 that if Ue_2, Ue, and T can be known from exciter response characteristics then the internally induced e.m.f. can be known as a time varying function. This when substituted in the power equation and equated to the equation of motion of the rotor would enable us to study the dynamic stability of the system when exciter response has been applied.

REFERENCES

1. Kimbark E. W., 'Power System Stability' Volume III. John Wiley and Sons, Inc., New York.

2. Dahl O. G. C., Electric Power Circuits: Theory and applications, Vol.II. Power System Stability. McGrawHill Book Co. New York.

3. Doherty R. E., Trans. A.I.E.E. "Excitation System." Vol.47, pp 944-56, July 1928.

4.9 Park R. H and Bancher E. H., 'System Stability as design problem'. A.I.E.E. Trans. Vol. 48, pp 170-94, 1929.

5. Jain G. C., Design operation and Testing of Synchronous Machines.

6. Lewis W.A., 'Quick Response excitation'. Elec.Journal Vol. 31, pp 308-12, August 1934.

7. Crary S., Power System Stability Vol.II.

8. Barkle J.E., Jr. Westinghouse Transmission & Distribution Reference Book "Excitation systems". Pp. 195.