MULTI-ITEM MULTI-PERIOD INVENTORY CONTROL

A DISSERTATION

submitted in partial fulfilment of the
requirements for the award of the degree
of
MASTER OF ENGINEERING
in
ELECTRICAL ENGINEERING
(System Engineering and Operation Research)

By

CAPT. DEEPAK SRIVASTAVA

DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF ROORKEE
ROORKEE-247 667 (INDIA)

JULY, 1986

CERTIFICATE

Certified that the dissertation entitled MULTI-ITEM MULTI-PERIOD INVENTORY CONTROL which is being submitted by Capt. DEEPAK SRIVASTAVA in the partial fulfilment of the requirements for the degree of Master of Engineering in Electrical Engineering (Systems Engineering and Operations Research) of the University of Roorkee, is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is to further certify that he has worked for a period of Eleven Months from August 25 th, 1985 to July 25 th, 1986 for preparing this dissertation for the Master of Engineering at this University.

(Dr. J.D. SHARMA)
PROFESSOR
DEPARTMENT OF ELECTRICAL ENGINEERING,
UNIVERSITY OF ROORKEE,
ROORKEE (U.P.)

Dated : 29th July, 1986

CONTENTS

ACKNOWLEDGEMENTS

I wish to express my sincere gratitude for the keen interest, which my guide Dr. J.D. Sharma, Professor, Department of Electrical Engineering, has taken in getting my dissertation completed. Inspite of his highly tight schedules of working, he has been a constant source of encouragement and invaluable help. His overwhelming interest and sharp criticism at every stage have been a great assistance in bringing this work in the final shape.

Thanks are also due to Mr. Vinay Pant, Lecturer, Department of Electrical Engineering and all other Faculty members and friends, who have directly or indirectly helped in completing this dissertation.

Dated : 29 July, 1986

Roorkee

DEEPAK SRIVASTAVA

ABSTRACT :

An inventory problem is a problem of making optimal decisions with respect to an inventory system or in other words it is a problem concerned with decisions which minimise the total cost of an inventory system. If cost of one parameter is increased/decreased, the connected cost of another parameter may decrease/increase. Therefore, we have to endeavour to minimise the sum cost. We can also say that an inventory problem is a problem of making optimal decisions with respect to various inventory costs.

A mathematical model for multi-item inventory control has been developed in Chapter II. A practical example has been discussed for the efficient and speedy repair of Truck 1 Tonne Nissan vehicle used in the Army. The optimal inventory that must be maintained for this (of course with the constraint that a certain level of user satisfaction is maintained) has been found with the help of a computer.

Reliability based inventory control problems, formulated by considering the failure rates of the components, have been developed in Chapter III. The number of spares required to be maintained are found out by maximising the reliability of the system in which these components are used. The identical components are bunched together into component groups. In this chapter, the mathamatical model developed has been transformed into a zero-one programming

problem which has a special structure. By exploiting the special structure, the problem is solved by a partial enumeration technique. This method requires less computational time than earlier methods. The drawback of the method, however, lies in the fact that the number of variables in this type of formulation is larger than in other methods. An algorithm for this method has been discussed in this chapter alongwith a practical example.

Chapter IV deals with the formulation of a multi-item multi- period inventory control problem and developing the optimality conditions for a 'No fixed cost of ordering' and a 'Positive fixed cost of ordering'.

1.1 ROLE OF INVENTORY

The term Inventory refers to the stocking of items used in the operation of an organisation. This organisation may be a factory, workshop, departmental store, hospital etc. In reality, inventory is made use of in all walks of life knowinglyor unknowingly. The inputs in the complete scope of inventory would include human, financial, energy, equipment and raw materials. Outputs would be parts, components, finished goods, partially finished goods or work in progress. The choice of items to be included with inventory would rest solely on the organisation. For example, a manufacturing organisation may have an inventory of personnel, machines, working capital, raw materials and finished goods. A workshop can have an inventory of spare parts, machines, men and repair tools. Similarly an airline can have an inventory of seats, or farm an inventory of uncut produce, and an engineering firm an inventory of talent. To classify an item as inventory, it must satisfy two basic requirements,viz

(a) The item must be specifically identified as different from all other items.

(b) The item should be storable or stockable.

1.2 TYPES OF INVENTORY

Broadly, there are two types of inventory, viz product inventory and service inventory. The differentiation between the two is generally made along the lines stating that a product offers a service to the consumer, while a service is being consumed at the same rate at which it is being produced. A major difference between the two is that service inventory is not storable. Thus manufacturing inventory can be defined in terms of product output and service inventory in terms of service capacity.

In manufacturing, inventory generally refers to inanimate physical entities that contribute to or become part of the firms output. These may include raw materials, finished products, component parts, supplies, work in process, etc.

In services, inventory refers to the administrative backing available to render the service. These may include physical space, numbers of channels or work places, service personnel, productive equipment, parts, supplies, etc.

Thus a repair facility would have an inventory of spare parts and supplies alongwith the service personnel and available space to perform the repair service.

1.3 OBJECTIVE OF INVENTORY ANALYSIS

In manufacturing, the objective of inventory analysis is to specify the following : -

(a) When should an order for an item be placed.

(b) How large should this order be.

In services, on the other hand, the objective of inventory analysis is to specify

(a) The unit of productive capacity which should be available to perform the service.

(b) The numbers of units which should be available in each time period in order to provide some specified level of service. Decisions in inventory thus become complicated by the varied purposes of inventory and the varied costs involved.

1.4 PURPOSES OF INVENTORY

A stock of inventory is kept to satisfy the following requirements :-

(a) To maintain independence of operations

If a supply of required materials is kept at a work centre and if the output of that centre is not immediately required anywhere else, then we have some flexibility in operating that centre. Since costs are involved in establishing new production setups, this flexibility in operating the centre allows the management to consider economic production lot sizes. An assembly line that is fed raw materials to correspond with the line speed, with no work in process inventory except on which each worker is working on, is an example of completely dependent operations. The unit in process passes from one person' to the next.

(b) To meet variations in product demand

If the precise demand of any product is known then it is feasible (although not necessarily economical) to produce the product to satisfy the demand exactly. In practice, whowever, demands are not completely or exactly known and a safety or buffer stock must, therefore, be maintained to cater for variations. Increases in demand as a result of promotional campaigns or seasonal demand can be catered for. Such seasonal inventory permits a more stable employment level with lower capital investment since it allows a more gradual build up of stock in anticipation of this higher demand.

(c) To allow flexibility in production scheduling

To relieve the pressure on the production system to send the finished products outside, there is a requirement of maintaining higher levels of finished goods inventory. This not only enables lower cost operation through more economic lot size production, but also permits longer lead times for production planning. High set up costs, for example, favour the production of a large number of units once the set up has been made.

(d) To provide a safeguard for variation in raw material delivery time

When ordering a material from a supplier, delays can occur for a number of reasons, viz, the normal transhipment/transportation time, which occasionally will be great, a shortage of material at the supplier's plant, causing him to backlog orders, an unexpected strike at the supplier's plant or at one of the transhipment/transport companies, a lost order, or incorrect or defective material. A safety stock level is thus determined depending on the severity of the consequences of material shortage. Normally a high level or stock of materials or supplies, crucial to an operation of the production system, will be maintained.

(e) To take advantage of economic purchase order size

The larger the size of an order, the fewer the number of orders that need be placed, since there are procedural costs for placing an order for goods. Also non-linearity of transhipment/transportation costs favours placement of a large order, that is, the larger the shipment quantity, the lower the per unit cost.

1.5 INVENTORY COSTS

The various costs involved in making any decision affecting inventory size are as follows :-

(a) Production change (or Setup) costs

For large scale inventory systems, the replenishment quantity is usually sizeable and certainly greater than unity. There are various reasons for this. The main reason is that very small orders would result in frequent reordering and thereby incurring a considerable expense associated with processing and receiving the order. This expense is often referred to as setup or reorder cost. The other and less obvious reason is that a large order protects the company against running out too soon. Other reasons that are some-times significant include quantity discounts or minimal order sizes stipulated by the supplier, or the firm's forecast of rising supplier prices.

Similarly to affect a change in production would involve obtaining raw materials, arranging specific equipment setups, appropriately charging time and materials, disposing off the previous stock of material, etc. Other costs on account of hiring, training, or layoff of workers, overtime, etc., may be involved. Even if there were no costs or loss of time in changing from the production of one product to another, change over costs will normally exist.

(b) Holding (or Carrying) costs

Keeping items in stock is costly because inventories tie up capital that might otherwise be profitably employed. Also they incur the expenses on account of storage, maintenance, insurance, etc. Other limiting reasons that arise in real situations include pilferage, breakage, obsolescence, depreciation, budgetary, taxes, space restrictions, etc. For these reasons it is obvious that high holding cost tend to favour low inventory levels and frequent replenishments.

(c) Ordering costs

These costs refer to the managerial and clerical costs involved in preparing the purchase or production order. These are subdivided into two categories, viz,

(i) Header Cost

This is the cost of identifying and issuing an order to a single supplier.

(ii) Line cost

This is the cost for computing each separate item order from the same supplier.

Thus ordering three items from a supplier entails one header cost and three line costs.

(d) Shortage or Penalty costs or Profit Loss

When the stock of an item is depleted, an order for that item must either wait until the stock is replenished, or be cancelled. There is a trade off between carrying stock to satisfy demand and the costs resulting from stockout. This balance is mostly difficult to assess since it may not be possible to place a value on lost profits, lost customers, or lateness penalties. Frequently, the amount of shortage cost is little more than a guess.

Whenever the wholesaler is out of stock of an item a customer requests, there is a penalty cost or profit loss. Obviously a lost sale means less revenue. But there is a penalty even if the customer is willing to have his order backlogged, for then the wholesaler must incur some extra expense from keeping backlog order records and filling the order in a later shipment. As a consequence, the firm will inventory an item if the 'out of stock' cost is high.

The determination of quantities purchased from other suppliers of the size of lots submitted to the firm's productive facilities involves a search for the minimum total cost resulting from the combined effects of three individual costs, viz holding costs production or ordering costs and shortage costs. This minimisation, obtained by using mathematical models, is traditionally conceded to be the essence of inventory theory.

1.6 DEMAND PARAMETERS

(a) Demand size

The quantity required to satisfy the demand for inventory will be called the demand size. Inventory systems where demand size is known are referred to as deterministic systems whereas those in which the size is not known are referred to as probabilistic systems

PROBABILISTIC SYSTEMS

In probabilistic systems,

P(X) = Probability of distribution of demand

X min = least possible demand

X max = Maximum demand

Therefore,

$$\sum_{X = X\,min}^{X\,max} P(X) = 1$$

Now let

F (S) = the cumulative distribution of demand

$$\therefore F(S) = \sum_{X = X\,min}^{S} P(X)$$

The average demand size or mean size is designated by $\bar{X}$

$$\therefore \bar{X} = \sum_{X = X\,min}^{X\,max} X\,P(X)$$

(b) Demand rate (R)

Demand size per unit time is known as demand rate. If a demand of size X quantity units occurs over a period of time T, demand rate R is given by

$R = X/T$

In probabilistic systems we use the average rate of demand. Therefore, average rate of demand is given by

$R = \bar{X}\,(T)/T$

(c) Demand Paterns

Demand patterns can be of numerous types depending upon whether the quantities in the inventory are taken out

(i) Uniformly throughout the period

(ii) At the beginning of the period

(iii) At the end of the period

These different ways by which demand occurs during a period will be referred to as demand patterns. Fig I.1 represents a few patterns in which there are 'S' quantity units in inventory at the beginning of the period, the length of period is 't' time units, and demand size is 'x' quantity units. The nature of the patterns is determined by 'n', the pattern index.

When $n = 1$, the demand is uniform

$n = (\infty)$ the demand is instantaneous

Other power patterns are shown when $n = 1/2$ & 2. When $n<1$ a large portion of demand occurs at the end of the period, when $n = 0$, the entire demand occurs at the end of the period and the quantity 's' is carried in inventory throughout the period. Fig 2 shows the replenishment patterns.

1.7 IMPLEMENTATION OF INVENTORY MODELS

With the help of models we can obtain solution(s) to the problems they represent. The solutions thus obtained are only a step, a part of the 'real' solution to the actual problem. The actual solution is obtained only when the recommendations are put to work and the models producing the solutions are continuously updated during implementation. On the other hand, if the results are not put into use

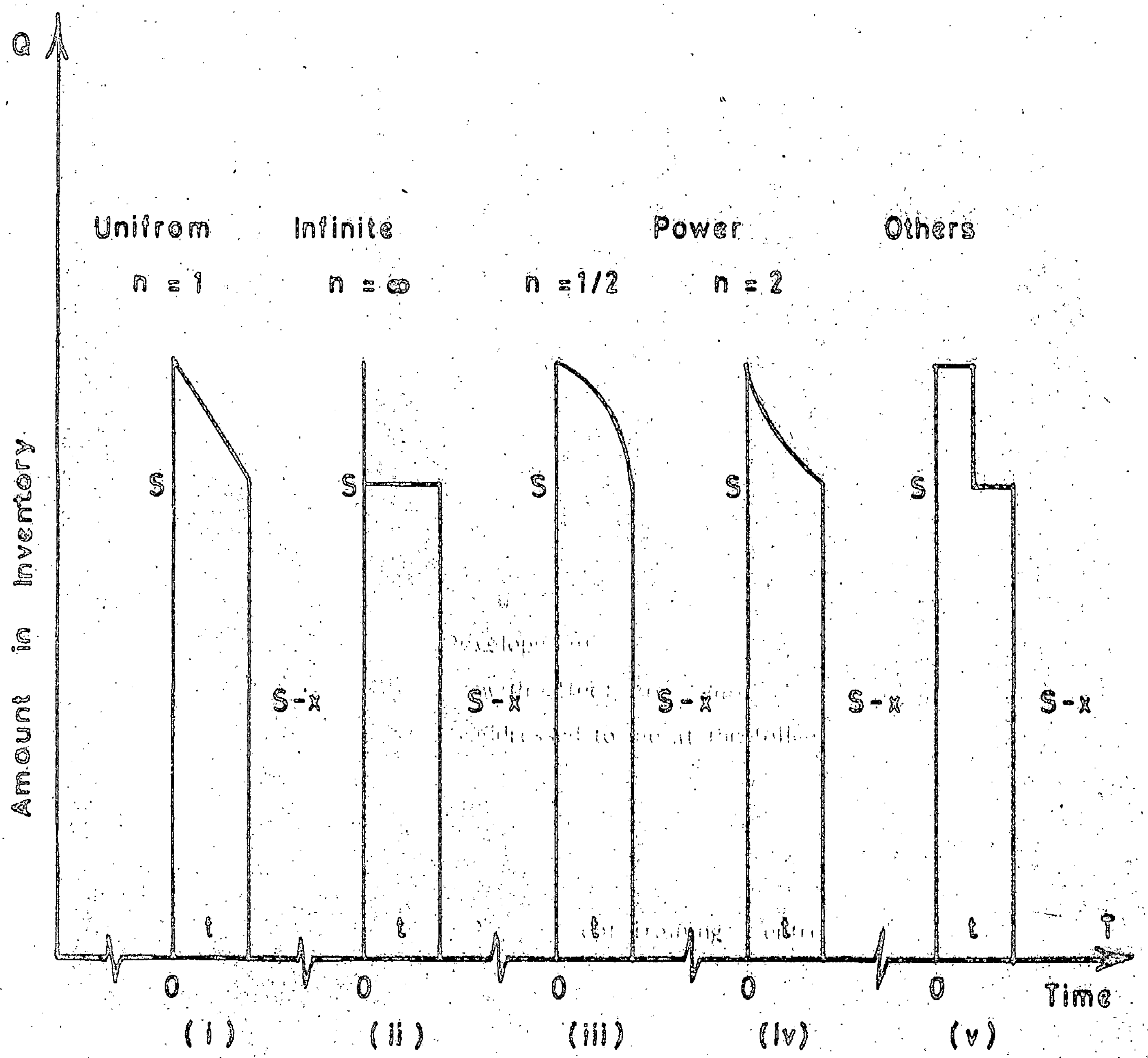

Fig.1·1· Demand Patterns

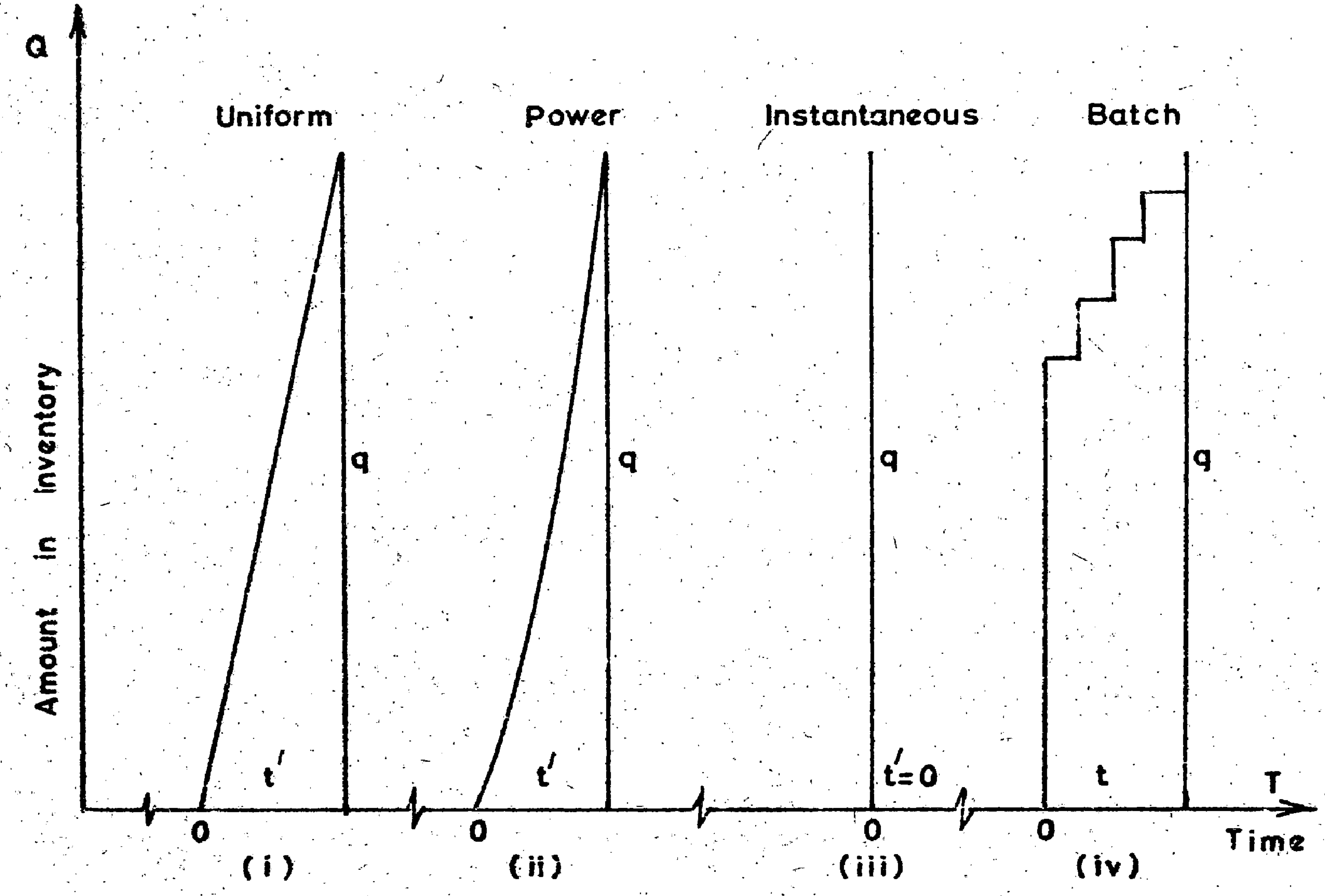

Fig. 1·2 - Replenishment Patterns

efficiently and effectively, it is difficult to claim that a solution has been provided to the real world problem.

Some factors, from the implementation point of view, have to be taken into account in order to improve the chancesof successful application of the study results. The principal factors are :-

(a) Technical difficulty in obtaining a solution from the model.

(b) Estimation effort required to find the values of the parameters which appear in the model.

(c) Maintaining effort needed to keep the model continuously updated for implementation.

The selection of an appropriate model greatly affects the chances of successful implementation of the results. A model which requires highly advansed solution techniques and a great amount of estimation effort is less likely to be implemented. Similarly, if adapting the model to changing conditions is very difficult and the information support system to implement the model is costly and complex, one cannot have high expectations for its applicability. On the other hand, if a model can easily lead to a solution with little estimation and updating efforts, then that model is of great interest and is likely to be effectively implemented.

2.1 INTRODUCTION

The inventory control problem considered here is of a multi-item periodic review type with a budgetory constraint. Inventory levels are reviewed at the end of every T units of time and necessary orders are placed to the suppliers. A transport mechanism is established with the suppliers over a period of time and we shall assume that it operates in a manner independent of individual replenishment decisions. Therefore, we can say that, individual orders do not influence either the number of vehicles used in transporting the items for the frequency of transportation during or given period of time since there is a common transportation mechanism for all items. Moreover, since many items are ordered simultaneously therefore individual orders do not affect replenishment decisions. Because of these facts, ordering cost is found to be negligible and irrelevant, and therefore an 'order upto S' policy is employed in making individual ordering decisions. According to this policy, a quantity equal to the difference between the target level S for an items and its inventory insight is ordered at the end of T units of time.

2.2 PROBLEM DESCRIPTION

This problem was based on the study of spares consumption for the efficient and speedy repair of truck

1 tonne 4x4 NSN vehicle used in the Army. About 500 different items are regularly carried in stock and the corresponding amount of money invested in inventories averages several lakhs of rupees.

Now rather than designing an inventory control system for all of 500 items right at the beginning, . the position of first concentrating on a group of items, implementing the results, and then gradually increasing the number of items in the domain of implementation (of course with the proviso that the implementation results were encouraging to do so). For this purpose a group of 23 fast moving spares were selected as the subject of this study taking into consideration their usefulness and importance in terms of giving a certain satisfaction level of service to the users. The reason for choosing these 23 items was solely based on the policy that these items were fast moving and were invariably required in large quantities for efficient repair of the vehicle.

Since many items are ordered simultaneously ordering cost for individual items was found to be negligible. There was also a common transporation system for all items, and therefore individual demands influenced neither the lorries used in transporting the items nor their frequency of shipments during a fixed period of time. 'Order-up-to-S' replenishment policy is the optimal policy if either the ordering cost is negligible or a common transportation facility is used. Hence 'Order-up-to-S' replenishment policy was chosen to be employed in making inventory control decisions. According to this policy a quantity equal to the difference between the MSP (Monthly stock potential which is calculated for a period of 3months requirement) or target level S and inventory-in-sight is ordered at the end of every quarter. The same replenishment policy is used for all items. A statistical analysis reveals that the quarterly demand for the selected items were independent random variables having either a gamma or normal distribution.

The objective of the study was to find the optimal target level, S, for each item such that expected monthly operational cost is minimised. It may, however, be noted that demand for any item vital to the repair of the equipment may be made at any time. This in between demand may be due to a variety of reasons like mobilisation schemes, seasonal requirements, inherent defects of the equipment, etc. But for mathematical analysis we shall assume that no demands are placed before the end of the quarter.

2.3 MATHEMATICAL MODEL

For ease of mathematical formulation of this problem, we shall define some variables as under, viz,

ξ_i = Monthly demand for item i, i= 1,2,.......,n

$F_i(x) = P\{\xi_i \leq x\}$ is the distribution function of the monthly demand for item i, i =1,2,........,n

$f_i(x)$ = density function of the monthly demand for item i.

S_i =Target level for item i (decision variable)

S = $(S_1, S_2, \ldots S_n)$ decision vector

h_i = Monthly inventory holding cost of item i (in Rs)/unit.

π_i =Unit shortage cost of item i (in Rs)

C_i =Unit cost of item i to the organisation.

β = Maximum amount of money that can be tied up in inventories at any one time.

n =Number of items : in this case n= 23

The objective function here is to minimise the total holding costs subject to the budgetary constraint.

Now,

C_h = Total holding cost

$$= h_i \int_0^{S_i} (S_i - x)\, f_i(x)\, dx$$

and,

C_s = Total shortage cost

$$= \pi_i \int_{S_i}^{\infty} (x - S_i)\, f_i(x)\, dx$$

Therefore, the inventory control problem can now be expressed as follows :-

$$\underset{S}{\text{Minimise}} \quad Z(s) = \sum_{i=1}^{n} \left\{ h_i \int_0^{S_i} (S_i - x)\, f_i(x)\, dx \right.$$

$$+ \pi_i \int_{S_i}^{\infty} (x - S_i) f_i (x) dx$$

$$S_i \geq 0, \quad \text{for } i = 1,2,\ldots\ldots\ldots,n.$$

2.4 OPTIMAL POLICY

Consider a linear holding and penalty cost case in which p(q) is a probability density function, the variable y is continuous and

$$L(y) = \begin{cases} \int_0^y h(y-q) p(q) dq + \int_y^{\infty} \pi (q-y) p(q) dq & \text{for } y \geq 0 \\ \int_0^{\infty} \pi (q-y) p(q) dq & \text{for } y < 0 \end{cases}$$

Then if,

$$\frac{dL(y)}{dy} \equiv L'(y) \text{ exists,}$$

then y = S that minimises [C(y) + L(y)] must satisfy

$$C + L'(y) = 0 \qquad \ldots.(2.1)$$

It can be shown by advanced calculus that for y> 0

$$L'(y) = h (y-y) p(y) + \int_0^y h p(q) dq$$

$$- \pi (y-y) p(y) - \int_y^{\infty} \pi p(q) dq$$

$$= (h + \pi) \int_0^y p(q) dq - \pi \qquad \ldots.(2.2)$$

Therefore, from (2.1) and (2.2) above, S satisfies

$$P(S) \equiv \int_0^S p(q)\,dq = \frac{\pi - C}{\pi + h} \qquad \ldots.(2.3)$$

The value of S is found by solving

$L(S) < K + C(S - s) + L(s)$ expressed as an equality. This is also the determination of reorder point s.

The holding cost formulae in the above discussion have been placed on the value of inventory at the end of the period. If the holding cost is linear and assessed on the value of y, then

$$R = \frac{\pi - (c + h)}{\pi}$$

If the holding cost is linear and assessed on the expected average value of inventory, viz,

$[\ .5y + .5\ (\ y-q\ )\ ]$, then

$$R = \frac{\pi - (\ c + .5h\ )}{\pi + .5h}$$

2.5 EXAMPLE

The example considered here was based on the actual data collected from an Army unit (Station Workshop,EME Roorkee), for the efficient repair of Truck 1 Tonne, Nissan vehicle used in the Army, for the year 1985. Since it was not possible to consider all the spare parts utilised for the repair of the vehicle, only 23 fast moving spares (which were used the maximum) were considered. The items considered (alongwith their abbreviated names, as used in the computer programming) are listed in Table 1.

Since it was not feasible to assess the holding and penalty costs (being an army unit), these have been assumed based on the existing availability of space and criticality of the spare parts respectively. The consumption of these spare parts for one year alongwith their holding costs and penalty costs have been listed in Table 3, whereas Table2 gives only holding and penalty costs.

Based on the mathematical model discussed in Section 2.3, the mean demand and standard deviation was obtained for these 23 items. These are given in Table 4.

2.6 RESULTS

The minimum inventory which has to be maintained for these 23 items, as calculated with the help of the Computer programme, is given in Table 5.

TABLE 1

Ser No.	Nomenclature of items	Abbreviated nomenclature as used in the computer programme
(a)	(b)	(c)
1.	Speedometer Cable assembly	SPDCABASSY
2.	Sparking plug	SPKPLG
3.	Fanbelt	FANBELT

(a)	(b)	(c)
4.	Rotor Distributor	ROTORDIST
5.	Head Lamp Bulb	HDLMPBULB
6.	Clutch Repair Kit	CLUREPKIT
7.	Needle Valbe Assembly	NDLVLVASY
8.	Contact Breaker Point Assembly	CBPTASSY
9.	Solenoid Switch	SLNDSWTCH
10	Radiator Hose Upper	RADTRHOSUP
11	Radiator Hose Lower	RADTRHOSLR
12	Spring Ball Crank	SPGBALCRNK
13	Exhaust Neck Gasket	EXHNEKGKT
14	Starter Motor Bush	SMBUSH
15	DE Bearing Dynamo	DEBRGDYN
16	Wiper Motor Assembly	WPRMRASSY
17	Battery Terminal	BTYTRML
18	Wiper Arm Assembly	WPRARMASSY
19	Ignition Coil	IGNCOIL
20	Armature Dynamo	ARMTRDYN
21	Bowel Glass	BOWELGLAS
22	Fuel Filter	FUELFLTR
23	CE Bearing	CEBRG

TABLE 2.

Srl No	Item	Holding Cost	Penalty Cost
1.	Speedometer Cable Assy	75.0	78.0
2.	Sparking Plug	35.0	72.0
3.	Fan Belt	80.0	85.0
4.	Rotor Distributor	37.0	86.0
5.	Head Lamp Bulb	32.0	65.0
6.	Clutch Repair Kit	55.0	67.0
7.	Needle Valve Assembly	32.0	65.0
8.	Contact Breaker Point Assy	33.0	83.0
9.	Solenoid Switch	56.0	76.0
10	Radiator Hose Upper	74.0	79.0
11	Radiator Hose Lower	74.0	79.0
12	Spring Ball Crank	42.0	48.0
13	Exhaust Neck Gasket	48.0	50.0
14	Starter Motor Bush	42.0	53.0
15	DE Bearing Dynamo	44.0	65.0
16	Wiper Motor Assembly	54.0	55.0
17	Battery Terminal	46.0	63.0
18	Wiper Arm Assembly	70.0	72.0

Srl No	Item	Holding Cost	Penalty Cost
19	Ignition Coil	65.0	87.0
20	Armature Dynamo	71.0	77.0
21	Bowel Glass	40.0	45.0
22	Fuel Filter	65.0	67.0
23	CE Bearing	44.0	65.0

ITEM	JAN	FEB	MAR	NOV	DEC	HOLD COST	PENL COST
SPDCABASSY	5.	8.	7.	0.	1.	75.0000	78.0000
SPKPLG	11.	14.	21.	21.	24.	35.0000	72.0000
FANBELT	1.	1.	[illegible]	7.	9.	80.0000	85.0000
ROTORDIST	5.	0.	3.	3.	4.	37.0000	86.0000
HDLMPBULB	6.	16.	18.	37.	24.	32.0000	65.0000
CLUREPKIT	2.	2.	[illegible]	0.	0.	55.0000	67.0000
NDLVLVASY	4.	7.	4.	3.	1.	32.0000	65.0000
CBPTASSY	2.	2.	[illegible]	5.	0.	[illegible]	83.0000
SLNDSWTCH	2.	[illegible]	[illegible]	1.	0.	[illegible]	76.0000
RADTRHOS P	1.	8.	6.	5.	0.	74.0000	79.0000
RADTRHOSLR	[illegible]	5.	0.	1.	0.	[illegible]	79.0000
SPGBALCRNK	2.	2.	2.	2.	1.	42.0000	48.0000
FXHGEKGKT	2.	5.	9.	1.	2.	43.0000	50.0000
SMBUSH	6.	3.	[illegible]	0.	0.	42.0000	53.0000
DEBRGDYN	1.	3.	4.	3.	1.	44.0000	65.0000
WPRMRASSY	3.	3.	[illegible]	0.	0.	51.0000	55.0000
BTYTRML	2	7.	1.	12.	7.	46.0000	63.0000
WPRARMASSY	4.	5.	8.	0.	2.	76.0000	72.0000
IGNCOIL	4.	2.	2.	2.	0.	55.0000	87.0000
ARMTRDYN	1.	2.	3.	0.	4.	71.0000	77.0000
BOWELGLA.	2.	2.	[illegible]	6.	5.	40.0000	45.0000
FUELFLTR	4.	3.	0.	5.	0.	65.0000	67.0000
CEBRG	1.	0.	5.	7.	0.	64.0000	65.0000

TABLE 4

ITEM	MEAN	STD.DEV
SPDCARASSY	5.417	0.6396
SPKPLG	17.583	1.9345
FANBELT	4.750	0.86[illegible]4
ROTORDIST	3.167	0.6437
HOLHPBULB	13.083	2.647[illegible]
CHURRPKIT	0.833	0.2591
SOLVLVASY	4.250	0.7372
COPTASSY	2.917	0.6062
SLDDSWTCH	1.667	0.3191
RADTRHOSUP	4.333	1.0161
RADTRHOSLR	2.583	0.5946
SPGBALCRNK	1.750	0.2394
EXHPCKGKT	3.083	0.6504
SHBUSH	1.583	0.5565
DEARGOYL	4.250	0.9869
WPRMRASSY	0.833	0.3081
STYTRAL	9.583	1.[illegible]495
WPRARMASSY	1.833	0.7152
IGNCOIL	2.833	0.61[illegible]5
APMTRDYN	1.250	0.3560
BOWLGLAS	3.333	0.5813
FUELFLTR	2.333	0.5813
CAPRG	5.667	0.9[illegible]22

TABLE 5

RESULTS

SL. NO.	ITEM	MONTHLY INVENTORY
1	SPDCARASCY	6
2	SPKPLG	19
3	FANBELT	5
4	ROTORPIST	4
5	HDLMPBULB	15
6	CLUREPKIT	1
7	HDLVLVASY	5
8	CBPTASSY	4
9	SLNDSWTCH	2
10	PADTRHOSUP	5
11	RADTRHOSLR	3
12	SPGRALCRNK	2
13	EXHAEAGKT	4
14	SHBUSH	2
15	DEFRGDYH	5
16	PRKRASSY	1
17	BTYTRML	1[illegible]
18	PRARMASSY	2
19	IGNCOIL	3
20	ARMTRDYN	2
21	BOWLGLAS	4
22	FUELFLTR	3
23	CEBRG	4

CHAPTER III

RELIABILITY BASED INVENTORY CONTROL

3.1 INTRODUCTION

With equipment spares being consolidated at the production source and/or at specific field locations, the need exists for efficient spares provisioning. Also for any successful maintenance, major replacement effort will be necessary. Because of space and cost limitations it is necessary to minimise the number of spare parts to be kept in inventory without affecting the specified user satisfaction level. Over or under spares provisioning can lead to unacceptable costs and/or unacceptable system operation. The selection of spares provisioning should be based on criteria such as

(a) criticality of the replaceable unit to the system.

(b) failure and repair rates of replaceable units.

(c) necessary spare adequacy

(d) whether the units (army) served are within easy reach of Ordnance Depots

(e) whether the repair facility is in the workshop or in situ.

(f) the total number of army units to be served.

(g) the additional cost of serving far flung army units from Ordnance Depots.

(h) retrieval capability of the repairing agency.

The turn - around time depends on the replaceable unit's failure rate , the repair location, repair costs or spare replaceable unit costs, etc. Therefore, there is a requirement to design systems so that they can use interchangeable components.

This work aims at examining, comparing and assessing the practicality of the several techniques available for apportioning the number of spares for a particular equipment under single or multiple constraint such as space, cost, etc., in order to maximise or at least obtain a good value for the system reliability.

3.2 PROBLEM FORMULATION

The system reliability can be broadly classified into fixed portions consisting of

(a) Non replaceable components

(b) Replaceable components

Assuming component independence, the system is the product of reliability of the fixed portion and the reliability of the replaceable portion. We will concentrate on the replaceable portion here. We shall assume that the standardisation has been introduced so as to reduce the number of spare parts that must be stocked.

Say, the replaceable structure requires ji items of component type i and that ki items of component type i have already been stocked. Considering a planning period of length t, let us define a few variables, viz,

$R(t; k_1, k_2, \text{------------}, K_n)$ = reliability of the replaceable portion

Ci = Cost of the i th component type

V_i = Volume of the ith component type

The problem now is to find the numbers of each spare to be stocked, $k_1, k_2, \text{---------}, k_n$, in order to maximise the reliability, ie,

max : $R(t ; k_1, k_2, \text{------------}, k_n)$ subject to the constraints,

$$C_1K_2 + C_2K_2 + \text{--------------------} + C_nK_n < C \quad \text{....(3.1)}$$

$$V_1K_2 + V_2K_2 + \text{--------------------} + V_nK_n < V \quad \text{....(3.2)}$$

Where C and V (the resources) are the maximum cost and volume allowed for spares respectively. In general form these constraints can be written as

$$\sum_{j=1}^{n} Cij\ ki \leq bj$$

For the most part, we will assume that the replaceable structure consists of independent component groups (of indentical components) as shown in fig 3.1 in series, and, therefore, the system reliability can be expressed as the product of the component group reliabilities :

$$R\ (t;\ k_1,\ k_2,\ldots\ldots,\ k_n) = \prod_{i=1}^{n} R_i\ (t;\ k_i)$$

The problem now is to find the integers

$K_1 \geq 0$, $k_2 \geq 0$,........., $k_n \geq 0$ in order to

maximise, for a given planning period, the nonlinear transcendental expression for the system reliability given by

$$R\ (t\ ;\ k_1,\ k_2,\ldots\ldots k_n) = \prod_{i=1}^{n} R\ (t\ ;\ k_i)$$

$$= \prod_{i=1}^{n} e^{-\lambda_i t} \sum_{l=0}^{ki} \frac{(\lambda_i t)^l}{l\,!} \qquad \ldots.(3.3)$$

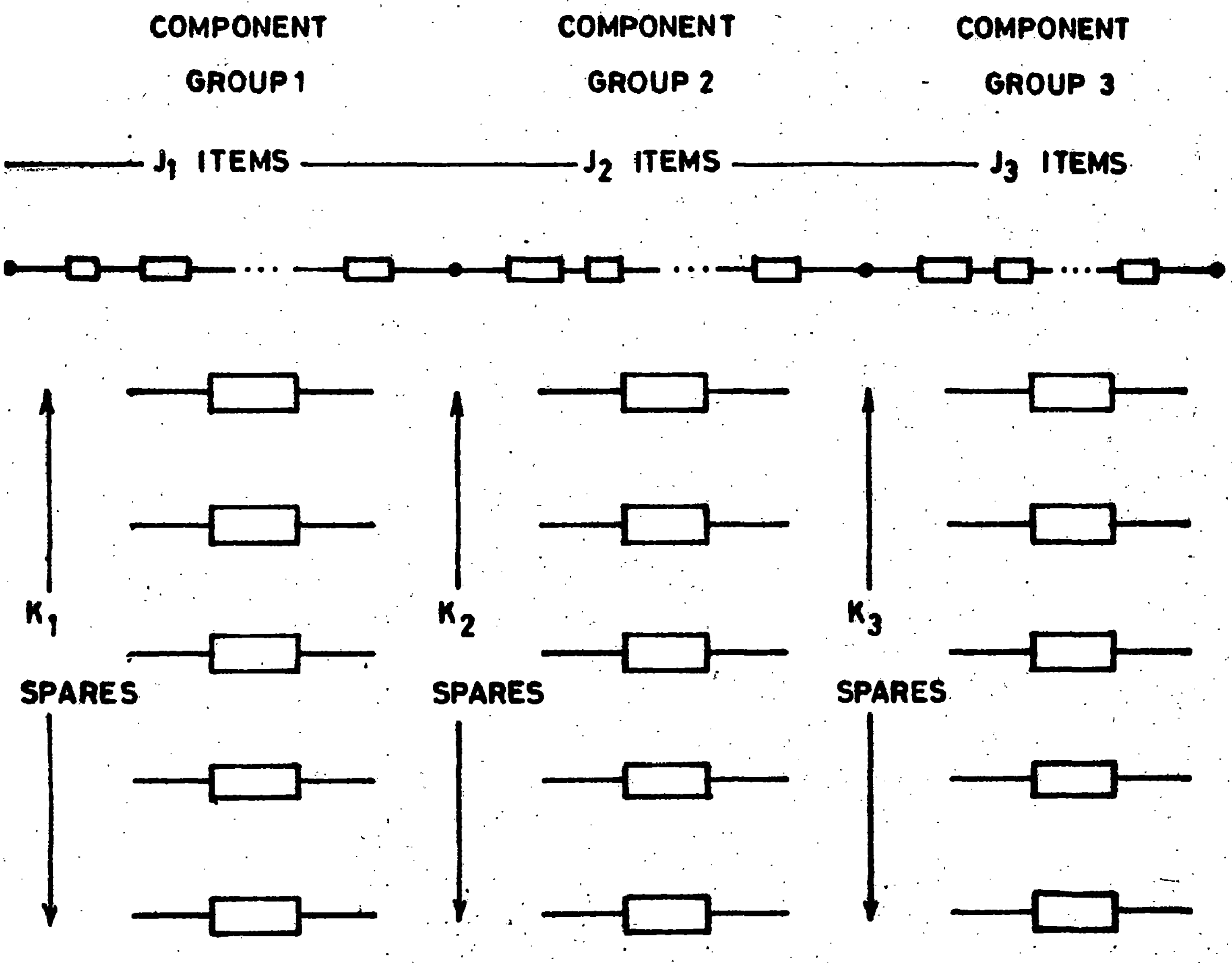

FIG. 3·1 - Replaceable Structure Consisting of three Component Types, Forming three Component Groups.

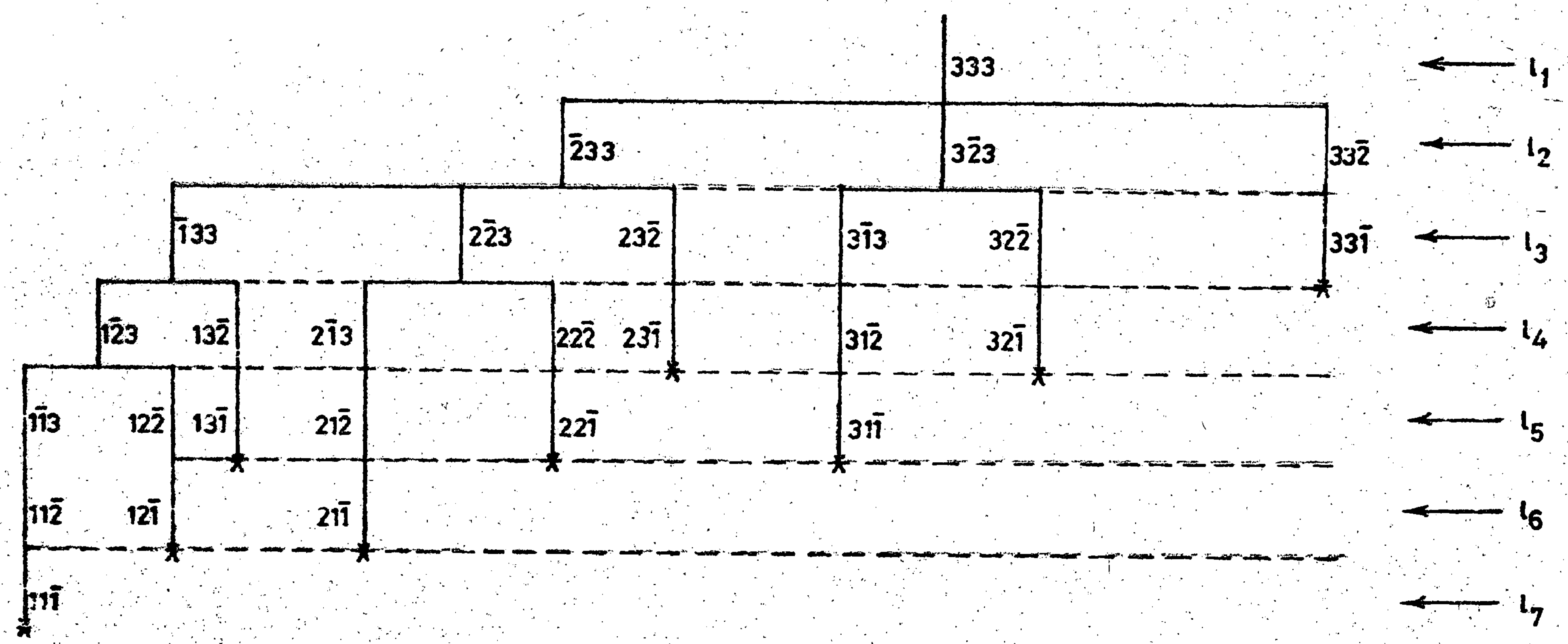

Fig. 3·2 - Tree Diagram for A Three Element Network with Three Discrete Points Each

Where λ_i = group hazard associated with the failure in the i th component

subject to constraints as given by(3.1) and (3.2).

Taking the natural logrithm yields a sum rather than a product and is computationally more convenient.

$$\log R(t; k_1, k_2, \ldots, k_n) = \sum_{i=1}^{n} \log R_i(t; k_i)$$

$$= -t \sum_{i=1}^{n} + \sum_{i=1}^{n} \log \sum_{l=0}^{k_i} \frac{(\lambda_i t)^l}{l!} \qquad \ldots.(3.4)$$

Since the natural logarithm is monotonic, maximisation of the logarithm is equivalent to maximisation of the argument.

Now this problem can be summarised as

$$\text{Log } R(t, k) = \sum_{i=1}^{n} \text{Log} \sum_{l=0}^{k_i} \frac{(\lambda_i t)^l}{l!} \qquad \ldots.(3.5)$$

subject to

$$\sum_{j=1}^{n} C_{ji} k_i \leq b_j \qquad \ldots.(3.6)$$

$$j = 1, 2, \ldots\ldots, M$$

where M is the total number of constraints. The first term is eqnation (3.4) is dropped as it is constraint with respect to problem var-iables.

3.3 TRANSFORMATION INTO ZERO - ONE PROGRAMMING PROBLEM

To convert the problem as stated in equation (3.4), we redefine the var-iables of equation (3.4) in terms of binary variables x_{il} as

$$K_i = \sum_{\ell=1}^{k_{mi}} \ell \, x_{i\ell} \qquad \text{....(3.7)}$$

and

$$\sum_{\ell=1}^{k_{mi}} x_{i\ell} = 1 \qquad \text{....(3.8)}$$

where,

k_{mi} = maximum number of spares in component group i.

Therefore, now the problem can be expressed in terms of these binary variables as

Maximise

$$\text{Log } R(t, k) = \sum_{i=1}^{n} \sum_{\ell=1}^{k_{mi}} C_{i\ell} \, x_{i\ell} \qquad \text{....(3.9)}$$

subject to the constraints,

$$\sum_{i=1}^{n} \sum_{\ell=1}^{k_{mi}} d_{ji\ell} \, x_{i\ell} \leq b_j \qquad \text{....(3.10)}$$

$$j = 1, 2, \ldots\ldots\ldots\ldots, C$$

and $$\sum_{\ell=1}^{k_{mi}} x_i = 1 \qquad \text{....(3.11)}$$

and

$$d_{ji\ell} = \ell \, c_{ji} \qquad \text{....(3.12)}$$

where

$$C_{i\ell} = \log \left(\sum_{j=0}^{\ell} \frac{(\ t)^j}{j!} \right) \qquad \text{....(3.13)}$$

and

$j = 1, 2, \ldots\ldots, M$

$i = 1, 2, \ldots\ldots, n$

$\ell = 1, 2, \ldots\ldots k_{mi}$

3.4 SOLUTION TECHNIQUE

The crudest way to solve the above problem is by the total enumeration technique. In this technique the total enumeration of binary vectors $x_{i\ell}$ is carried out. This will result in the generation of

$$2^{\sum_{i=1}^{n} K_{mi}} \quad \text{binary vectors}$$

and the testing of their feasibility. Here a technique, exploiting the special structure of the problem, viz, any one variable out of variables $x_{i1}, x_{i2}, \ldots\ldots, x_{iK_{mi}}$ should have value equal to unity and rest of them should be zero, is used. In addition, certain feasibility tests are also used, thereby resulting in reduction of the number of test vectors which are to be generated. With the use of this strategy, the efficiency of this solution technique is enhanced enormously. The test vectors are generated by a systematic approach such that the constraints given by equation (3.11) are always satisfied.

3.4.1 Developement of the tree by exploiting the special structure of the problem

Test vectors are generated in a special way known as the coded test vector (CT Vector). The number of elements in a CT Vector are equal to the number of component groups in the system. A typical CT vector for a system having three component groups can be written as [333]. The meaning of this code is that the variables $x_{13} = x_{23} = x_{33} = 1$ and other variables are zero. Let us denote this vector by $\overset{v}{k}$.

For generation of the tree, the top node is assigned k(1), k(2), ...,k(n). For the ease of explanation let us assume that there are three component groups and each component group has got three discrete spare parts. In this case the top node is formed by the subvector [333]. From this subvector other subvectors are generated and these are called the descendents of subvector [333].

3.4.2 Steps for this particular problem

The steps for this particular problem are given below :-

Step 1

Assign top node by subvector [333]

Step 2

Obtain first descendent by setting $x_{13} = 0$ and $x_{12} = 1$ and denote this by subvector [$\bar{2}33$]. The first component group assumes its second discrete value, the discrete values of the other two remaining unaltered.

Step 3

Put a bar on the element 2 in order to avoid duplication or redundancy of subvectors. This bar is also used to generate the descendent vector.

Step 4

Reduce the element of the descendent test vector till it reaches its minimum value (ie equal to 1).

Step 5

Once the first element of the first component group reaches its maximum, the bar is shifted to the next element of the test vector on its right hand side.

Step 6

Stop once all elements of the test vectors reach their minimum.

A typical network having three component groups, each having three sets of spare parts, is shown by means of a tree diagram (Fig 3.2).

3.4.3 Computerised generation of the tree

For generating the tree with the help of a computer, the subvectors are represented by three dimensional vectors $y_{i,j,k}$ where

i - index denotes the level of tree

j - index denotes the test vector and

k - index denotes the element number of the test vector.

In order to generate the test vector systematically, instead ofputting a bar on the element of the subvector, a negative sign is attached with it. The number NOD, of the descendents of a test vector can be calculated with the help of the following relation:-

$$\text{NOD} = \begin{cases} n - k_1 + 1 & \text{if } y_{i,j,k_1} \text{ is not unity} \\ n - k_1 & \text{otherwise} \\ 0 & \text{if } k_1 \text{ is the last element of the subvector and } | y_{i,j,k} | \text{ is unity.} \end{cases} \quad \ldots(3.14)$$

where the k_1 th element of the j th subvector at the i th level have negative value.

Therefore, the descendent subvectors will be given by the following relation :-

$$y_{i+1,j,k} = \left\{ \begin{array}{ll} [\, | y_{i,j,k+j-1} | - 1 \,], & \text{if } | y_{i,j,k} | \text{ is not unity} \\ y\,(i,j,k) & \text{otherwise} \\ j=1,\ \text{NOD} & \end{array} \right\} \begin{array}{l}\text{and} \\ k=k_1\end{array} \quad \ldots(3.15)$$

3.4.4 Skipping tests

In order to reduce the number of vectors to be enumerated, feasibility test is applied to the test vectors. The value of the objective function corresponding to the I th level will always be less than the value of the objective function corresponding to its descendent vectors at the (I + 1) th level. Therefore, once a feasible test vector is obtained at any level I , then all the descendent test vectors of this test vector can be skipped.

3.5 ALGORITHM

The various steps involved for the computerisation to solve the problem are given below :-

STEP 1

Read the problem data.

STEP 2

Set i = 1

Initialise test vector

$$Y_{1,1,K} = \begin{cases} -J_K , & \text{for } K = 1,2,\ldots\ldots,n \\ J_K & \text{otherwise} \end{cases}$$

STEP 3

If test vector feasible, stop ; otherwise go to next step.

STEP 4

Set i = i + 1 . Generate the descendents by equations (3.14) and (3.15).

STEP 5

Check if any solution vector at the i th level is feasible. If yes, go to next step, otherwise stop.

STEP 6

Calculate the objective function value for the feasible solution. Find the feasible test vector corresponding to which the value of the objective function is minimum. Set this value equal to Z_{min} and store this test vector. If Z_{min} is less than the value of the objective function corresponding to the other test vector at the i th level, then stop. Otherwise go to next step.

STEP 7

Apply skipping rule and go to step 4.

3.6 Example

Say we have a subsystem comprising of three component groups. The failure rates of these component groups is as shown below:

Subsystem	Failure Rate (1/yr),
Component Group 1	0.5
Component Group 2	0.05
Component Group 3	0.2

The detailed description of the problem is given in Table below :-

Table

Sub-system	Number of spare parts	Associated variables	Objective function cost coeff	Reli const coeff	Vol const coeff
Component Group 1	1	x_1	2.00	0.13369	2.00
	2	x_2	4.00	0.03677	4.00
	3	x_3	6.00	0.00990	6.00
	4	x_4	8.00	0.00259	8.00
Component Group 2	1	x_5	0.50	0.30685	1.00
	2	x_6	1.00	0.08371	2.00
	3	x_7	1.50	0.01917	3.00
	4	x_8	2.00	0.00367	4.00
Component Group 3	1	x_9	1.50	0.03278	0.5
	2	x_{10}	2.25	0.00643	1.0
	2	x_{11}	3.00	0.00163	1.0

From design consideration, the number of spare parts for each group is known. The decisions are to remain valid for a period of five years . The entire space should not exceed 12 units, the available resources and the system reliability should be 0.98480. The cost of obtaining the required schedule is to be minimised.

The optimal solution obtained is

$$x_3 = x_8 = x_{11} = 1$$

$$x_1 = x_2 = x_4 = x_5 = x_6 = x_7 = x_9 = x_{10} = 0$$

$$\therefore Z^* = 11$$

The optimal level is obtained at level 7 of the tree.

The results show that the spare components for component group 1 and 2 are 3and 4 respectively. Component group 3 is to be supported with 2 spare components.

CHAPTER IV

MULTI-ITEM MULTI-PERIOD INVENTORY CONTROL

4.1 INTRODUCTION

In a strict sense, steady state conditions are a fiction in the real world. The essential characteristic of all economic systems is that they are continually changing with time. For inventory systems, the processes generating demands and lead times change with time, as do the various costs of interest, and even the items carried by the system. When both demand and lead times are variable, there is an increase in the problem complexity. A joint probability distribution of demand during the replenishment period can be developed. The range of joint probability distribution is from the level indicated by the product of the smallest demand and the shortest lead time to the level indicated by the product of the largest demand and the largest lead time. In many cases, however, the changes occur slowly enough so that for a considerable length of time the system can be treated as if it were in a steady state mode of operation. In other instances,however, the changes occur with such rapidity that they must be explicitly accounted for.

As might be expected, the difficulty of formulating and obtaining numerical solutions to realistic dynamic inventory models is cosiderably greater than for the case where it was permissible to assume that the system was in steady state. In fact, when demand is treated as a stochastic variable whose mean is time dependent, only the most trivial problems can be solved manually. Usually a large digital computer is required to obtain numerical results. It is assumed that there is no set up cost at any period. The inclusion of set up costs in the multi period case generally leads to difficult computations. Normally multi period models are formulated by dynamic programming.

Unlike the single period models, a multi-period model should take into account the discounted value of money. Thus if α (< 1) is the discount factor per period, an amount of money S after n periods ($n \geq 1$) is equivalent to $\alpha^n S$ now.

It is the purpose of this chapter to study multi period models in which the mean rate of demand changes with time.

4.2 PROBLEM FORMULATION

In a classical inventory problem a purchasing is made at the beginning of a regularly spaced period of time, say a week. This decision will be based on several factors such as

(a) level of inventory at that time

(b) ordering costs

(c) holding costs

(d) backorder penalty costs during the period

(e) the effect the decision will have on future periods.

Let us now define a set of variables as under

Z = purchase quantity, $Z \geqslant 0$

$C(Z)$ = cost of purchasing Z units,

λ = number of time periods lag between an order and its delivery, the possible values for λ being 0, 1, 2,......

$\phi(t)$ = probability density function of demand during a period, where demand is a continuous random variable and is independent from period to period.

U = inventory on hand at the end of a period, where $-\infty < \lambda < \infty$ (a negative value indicates that demand occurred during the period that could not be filled)

$h(U)$ = holding cost charged on inventory on hand at the end of the period.

$p(U)$ = shortage cost charged for failure to meet demand during a period.

x_n = inventory on hand at the beginning of the n th period before an order is received, ie, the inventory on hand at the end of the previous period.

y_n = inventory on hand at the beginning of the n th period, immediately after an order is received, and

$L(y_n)$ = expected holding and shortage cost during the n th period (hereafter referred to as the period cost)

$$L(y_n) = \begin{cases} \int_0^{y_n} h(y_n - t)\, \phi(t)\, dt + \int_{y_n}^{\infty} p(t - y_n)\, \phi(t)\, dt, & y_n \geq 0 \\ \int p(t - y_n)\, \phi(t)\, dt, & y_n < 0 \end{cases}$$

If a delivery is to be received during a given period as a result of an order placed λ periods before, then it is assumed that

(a) this order arrives at the beginning of the period and before the purchase decision is made for that period.

(b) the supplier carries an infinite supply of the item, ie , the supplier never backorders the installation.

(c) in the dynamic formulation of the problem $\lambda = 0$, ie , the delivery is instantaneous and that excess demand is backordered.

Let

$C_n(x_n, y_n)$ = total expected discounted inventory cost for a problem lasting n periods, when inventory on hand at the beginning of period n, prior to ordering, is x_n and immediately after ordering is y_n , n= 1,2,............ and,

α = discount factor.

The periods are numbered backwards in time ; thus period number one is the last period of the problem and it is assumed that units on hand at the end of the last period have no salvage value.

4.3 OPTIMALITY CONDITIONS

Considering we are at the beginning of the n th period of the problem and x_n is the inventory on hand before an ordering decision is made. The optimal policy for the n th period is the policy which minimises $C_n (x_n , y_n)$. The well known dynamic programming recursive relation for this problem is

$$\tilde{C}_n (x_n) = \left\{ \min_{y_n \geq x_n} \left\{ c (y_n - x_n) + L (Y_n) + \alpha \int_0^{\infty} \tilde{C}_{n-1} (y_n - t) \phi(t) \, dt \right\} \right.$$

$n = 1, 2, \ldots\ldots$

where $\tilde{C}_n(x_n)$ equals minimum total expected discounted cost for a problem lasting n periods, $n = 1,2,\ldots\ldots\ldots$ In this equation, $\tilde{C}_n(x_n)$ has been broken down into three components, viz,

(a) the purchasing cost for the n th period

(b) the period cost for the n th period

(c) the total expected discounted cost for (n - 1) periods of operation

of course with the assumption that an optimal inventory policy is followed during the last (n - 1) periods of operation. This recursive relation is often used to find the optimal value of y_n, which we call S_n. Clearly, the desired inventory level at the beginning of the n th period, S_n, has an effect on all future levels S_i, $i = 1, 2, \ldots\ldots, n - 1$.

From this model we can establish two well known results, viz ,

(a) <u>No fixed cost of ordering</u>

Assume the purchasing cost is linear with no fixed cost of ordering, then

$$C(Z) = C \cdot Z, \quad Z > 0$$

Assume $L(y_n)$ is convex, then it can be shown that the optimal policy for an n - period problem can be characterised by a sequence of critical numbers $S_1, S_2, \ldots\ldots S_n$. The policy for the kth period is

if $x_k \leq S_k$, order $S_k - x_k$

if $x_k > S_k$, order nothing.

(b) <u>Positive fixed cost of ordering</u>

Assume the purchasing cost is linear with a fixed cost of ordering equal to K, then

$$C(Z) = C \cdot Z + K, \quad Z > 0$$
$$= 0, \quad Z = 0$$

Assume $L(y_k)$ is convex. Then it can be shown that the optimal oplicy for the k th period is defined by a pair of critical numbers, (S_k , s_k). The policy for the k th period is

if $x_k \leq s_k$, order $S_k - x_k$

if $x_k > s_k$, order nothing.

CONCLUSIONS

Very few, if any, areas of management decision-making offer more potential for rich theory than problems involving the design and/or operation of a multi-item multi-period inventory systems.

In this study various inventory problems are discussed and their mathematical models are developed. A study on effecient and effective repair of Truck 1 Tonne Nissan used in the Army, was carried out and an optimal inventory to be maintained for this purpose has been worked out. Emphasis has also been laid on inventory problems formulated by considering the failure rates of the components. The number of spares required to be maintained is calculated by maximising the reliability system in which these components are used. The identical components were bunched into a group. The limits on the space required are also considered in the problem by exploiting the special structure of the zero-one programming problem.

The optimality conditions for a multi-item multi-period inventory control problem have been obtained for a 'No fixed cost of ordering' and 'Positive fixed cost of ordering.'

The various mathematical models/formulations discussed in this study offer an instrument to reduce estimation efforts considerably, and hence increase the effectiveness and success of implementation. Moreover, it strongly suggests that inventory control problems which lead to models where holding and shortage costs appear should not be formulated. Therefore, in real world cases, inventory control problems should be considered from different points of view in order to arrive at a model which will not get in the way of implementation.

REFERENCES

1. Muhittin Oral., Multi-Item Inventory Management With Monetary Objective Function, AIIE Transactions, Vol 13, No. 1, March 1981, pp 41-43.
2. Muhittin Oral., Equivalent Formulations of Inventory Control Problems, AIIE Transactions, Vol 13, No. 4, December 1981, pp 325-329.
3. Harvey M. Wagner., Principles of Operations Research With Applications to Managerial Decisions, Prentice Hall of India Private Limited, New Delhi, 1973.
4. Eliezer Naddor., Inventory Systems, John Wiley and Sons, Inc, 1966.
5. Henry A. Malec and Daniel Steinhorn., A New Technique for Depot and Sub-depot Spares, IEEE Transactions, Vol R-29, No. 5, December 1980, pp 381-382.
6. Martin Messinger and Martin L Shooman, Techniques For Optimum Spares Allocation : A Tutorial Review., IEEE Transactions on Reliability, Vol R-19, No. 4, November 1970, pp 157-158.
7. G. Hadley and T. M. Whitkin., Analysis of Inventory Systems , Prentice Hall, Inc, Englewood Cliffs, N. J.1963
8. Richard J. Tersine., Materials Management and Inventory Systems, Elsevier North- Holland, Inc, 1976.
9. L.B. Schwarz (Editor), Multi-Level Production/ Inventory Control Systems : Theory and Practise, Elsevier North-Holland, Inc, 1981.
10. Handy A. Taha., Operations Research : an Introduction Macmillan Publishing Co,Inc., New York.