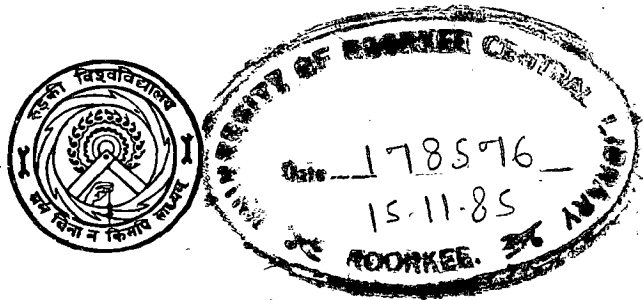


# **DESIGN, FABRICATION AND APPLICATION OF VECTORCARDIOGRAPH**

A DISSERTATION

Submitted in partial fulfilment of the requirements  
for the award of the degree  
of  
**MASTER OF ENGINEERING**  
IN  
**ELECTRICAL ENGINEERING**

By  
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**DEPARTMENT OF ELECTRICAL ENGINEERING  
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JUNE, 1985**

CANDIDATES' S DECLARATION

I hereby certify that the work which is being presented in the dissertation entitled "DESIGN, FABRICATION AND APPLICATION OF VECTORCARDIOGRAPH" in partial fulfilment of the requirements for the degree of MASTER OF ENGINEERING IN ELECTRICAL ENGINEERING (Measurement and Instrumentation) submitted in the Department of Electrical Engineering, University of Roorkee, Roorkee, is an authentic record of my own work carried out during a period of six months from January, 1985 to June, 1985 under the supervision of Dr. P. Mukhopadhyay, professor, Department of Electrical Engineering, University of Roorkee, Roorkee.

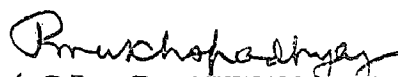
The master embodied in this disseration has not been submitted for the award of any other degree or diploma.

  
( KAVITA MISRA )

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

ROORKEE

DATE: JUNE, 26, 1985

  
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Roorkee

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( KAVITA MISRA )

A B S T R A C T

The dissertation describes the design and application of the vectorcardiogram - which is the manifestation of the electrical activity of the heart in the horizontal, sagittal and transverse planes of the body.

In the introduction the dipole concept to describe the electrical activity of the heart has been introduced. Some general definitions are also given in the same chapter. Chapter 2 contains a description of the various vectorcardiographic lead systems. A short review of the uncorrected systems used earlier is given along with a comparison between the vectorcardiographic loops obtained using various lead systems. Design of the system to pick up the electrical signal from the body and to process it is contained in chapter-3. Chapter-4 deals with the application of the vectorcardiograph. The use of the derived vectorcardiogram to solve the "inverse problem" is discussed in detail. A review of some mathematical models of the heart, as a generator, proposed previously has been done and a new model assuming the body to be a homogeneous, spherical conductor, has been proposed. Appendix 'A' deals with the mathematical analysis of 3 op-amp differential amplifier, Appendix 'B' with the derivation of surface potential due to the heart dipole and

Appendix 'C' with Langner's and Abildskov and  
Wilkinson's two reference frames\_ used to derive  
the scalar ECGs of the limb and precordial leads  
from the frontal and transverse plane vectorcardiograms.

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CHAPTER - 1I N T R O D U C T I O N

The heart is the most vital organ of the human body. This is amply borne out by the fact that it is the first major organ to develop in the embryo. It works basically as a double pump, providing the force needed to circulate the blood through the pulmonary circulation in the lungs and systemic circulation in the rest of the body. The blood in a normal individual circulates through one system before being pumped by the other section of the heart.

Elaborating slightly on the mechanical pumping action of the heart - it has four chambers, the two upper chambers, the left and right atria - are synchronised to contract simultaneously, as are the two lower chambers, the left and right ventricles. The right atrium receives venous blood from the body and pumps it to the right ventricle. This ventricle pumps the blood through the lungs, where it is oxygenated. The blood then flows into the left atrium. The contraction of the left atrium moves the blood to the left ventricle which contracts and pumps it into the general circulation, the blood passes through the capillaries into the venous system and returns it to the right atrium.

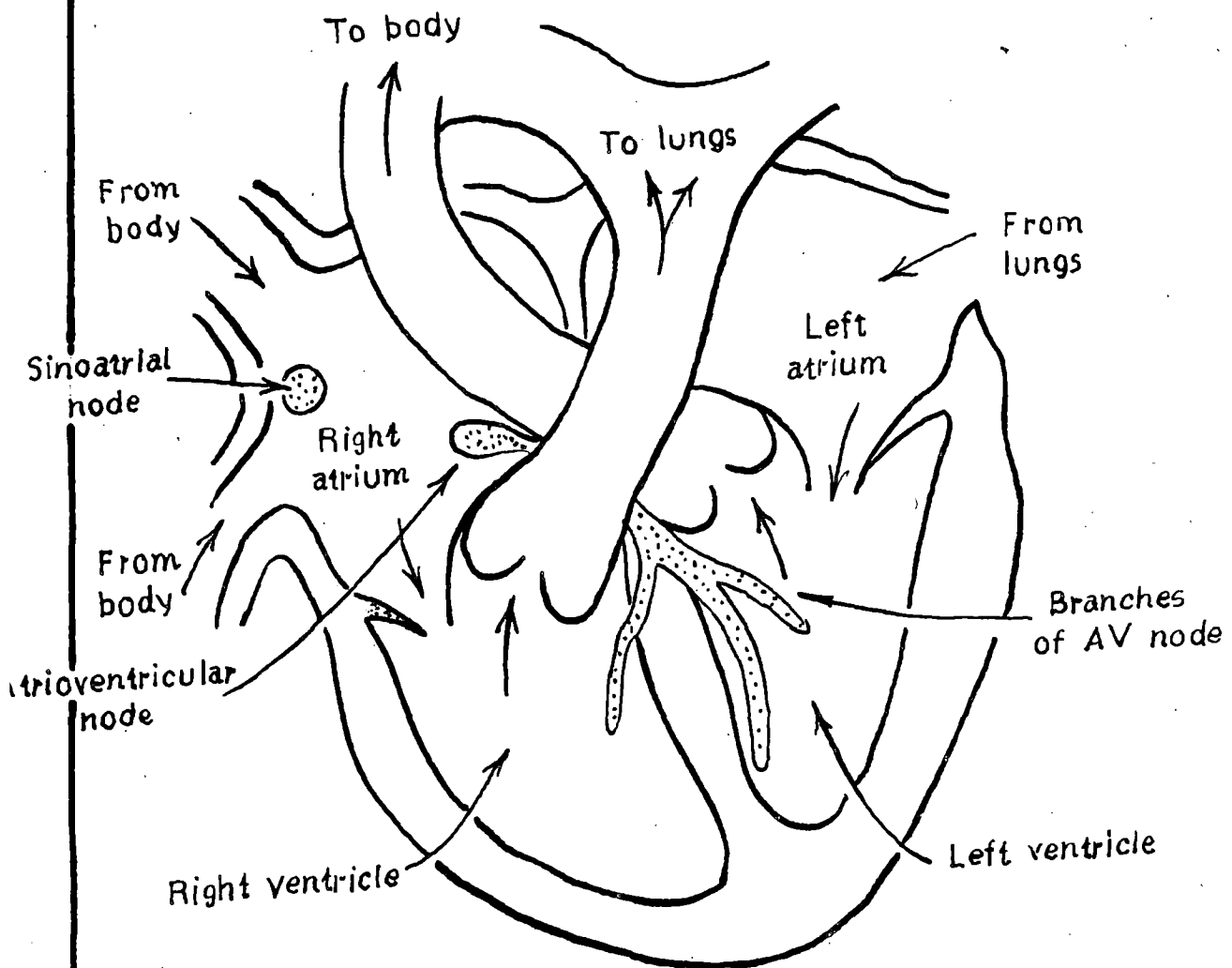


FIG. 1-1 THE HUMAN HEART

### 1.1 Electrical Activity of Heart -

This smooth rhythmic contraction of the atria and ventricles has an underlying electrical precursor in the form of a well coordinated series of electrical events that take place within the heart. That this set of electrical events is intrinsic to the heart itself, it is interesting to note, is well demonstrated when the heart is removed from the body and placed in a nutrient medium (e.g. glucose-Ringer Solution). The heart continues to beat rhythmically for many hours. The coordinated contraction of the atria and ventricles is set up by a specific pattern of electrical activation in the musculature of these structures. Moreover, the electrical activation patterns in the walls of the atria and ventricles are initiated by a coordinated series of events in the "specialized conduction system" of the heart. In relation to the heart as a whole, the specialized conduction system is very small. It constitutes only a minute portion of the total mass of the heart. The wall of the left ventricle is 2.5 to 3.0 times as thick as the wall of the right ventricle, while the intraventricular septum is nearly as thick as the left ventricular wall. The major portion of the muscle mass of the ventricle consists of the free wall of the right and left ventricles and the septum. Considering the heart as a bioelectric source, the strength of this source can be expected to be directly related to the mass of the active muscle (i.e. the number of walls and septum of the ventricles).

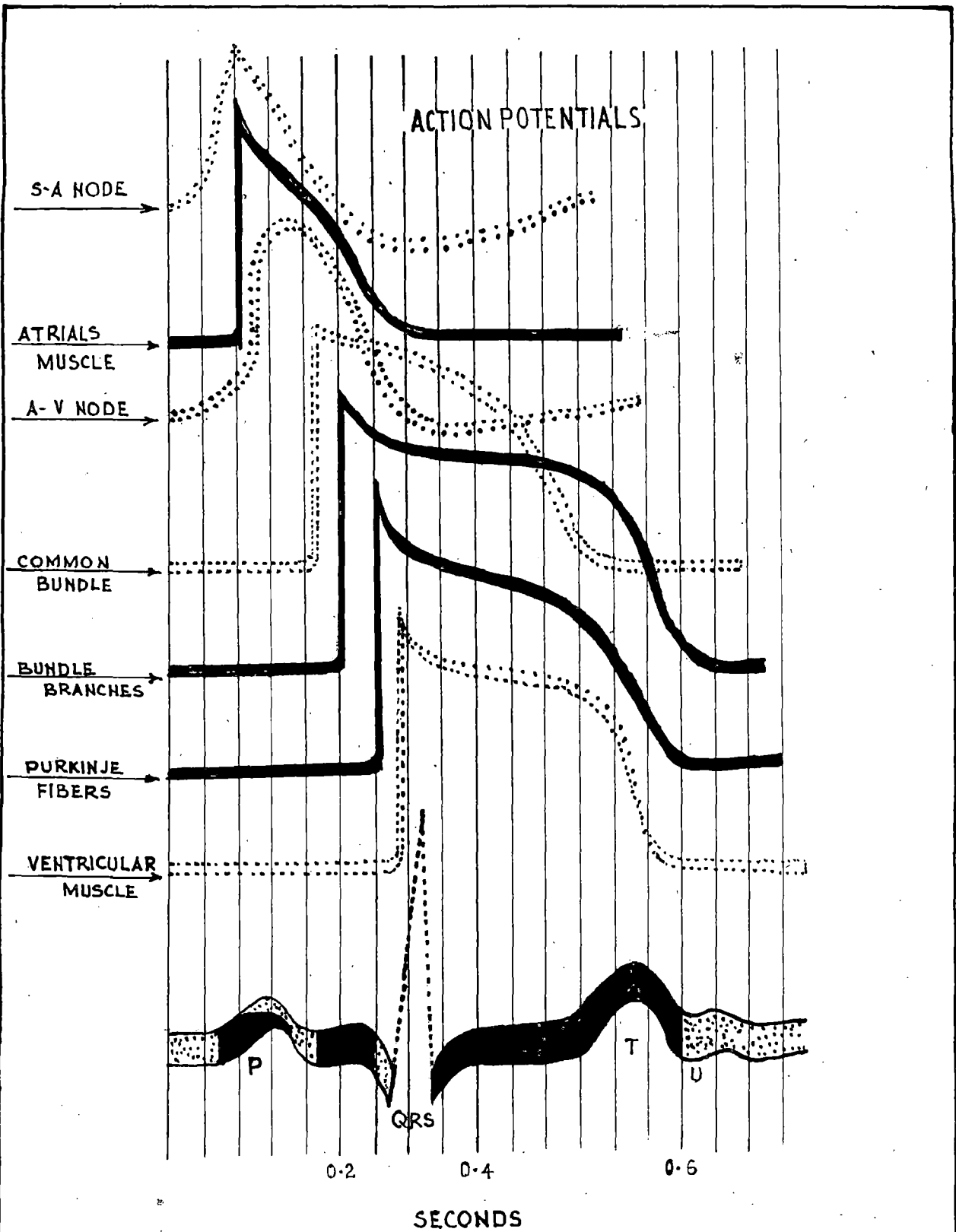


FIGURE 1-2 REPRESENTATIVE ELECTRICAL ACTIVITY FROM  
VARIOUS REGIONS OF THE HEART. BOTTOM TRACE IS SCALAR ECG.

Therefore the atria and the free walls and septum of the ventricles can be considered the major contributors to external potential fields from the heart.

The heart is comprised of several different types of tissue- Sino Atrial and Atrioventricular nodal tissue; atrial, Purkinje and ventricular tissue. All three cells are electrically excitable, with each type of cell exhibiting its own characteristic action potential.

The rhythmic cardiac impulse originates in pacemaking cells in the sinoatrial node, located at the junction of superior vena cava and right atrium. This signal initiates the depolarisation of the nerves and muscles of both atria, causing the atria to contract and pump blood into the ventricles. Repolarization of atria follows. The impulse then passes into the atrioventricular node where it is delayed before passing into the bundle of His, the right bundle branch, Purkinje network and other conducting masses, causing the ventricles to contract and force blood into the pulmonary and general circulations. The ventricle nerves and muscles then repolarize and the sequence begins again.

#### 1.2 The Dipole Concept-

In the previous paragraphs the sequence of events involved in electrical activation of the ventricles has been dealt with. This activation sequence leads to the

production of closed line action currents that flow in the thoracic volume conductor (which is considered to be a purely passive medium containing no electrical sources or sinks). The heart is viewed as an electrical equivalent generator. A common assumption is that, at each instant of time in the sequence of ventricular activation, the electrical activity of the heart can be represented by a net equivalent current dipole located at a point that is known as the "electrical centre" of the heart. This centre is assumed to lie within the anatomical boundaries of the heart. Of course, several regions of both ventricles may be simultaneously active. In this case the electrical activity of each region at any instant of time could be thought of as being represented by a current dipole and a net dipolar contribution from all active areas determined at the electrical centre. The thoracic medium can be considered as the resistive load of this equivalent cardiac generator. There is also attenuation of the field with increasing distance from the source. Following this it can be said that the electrical potential that we measure on the body's surface is merely the instantaneous projection of the electric dipole vector in a particular direction. As the vector changes with time, so does the projected potential. Figure 1.3 shows an electric dipole vector along with the three electrocardiographic body planes.

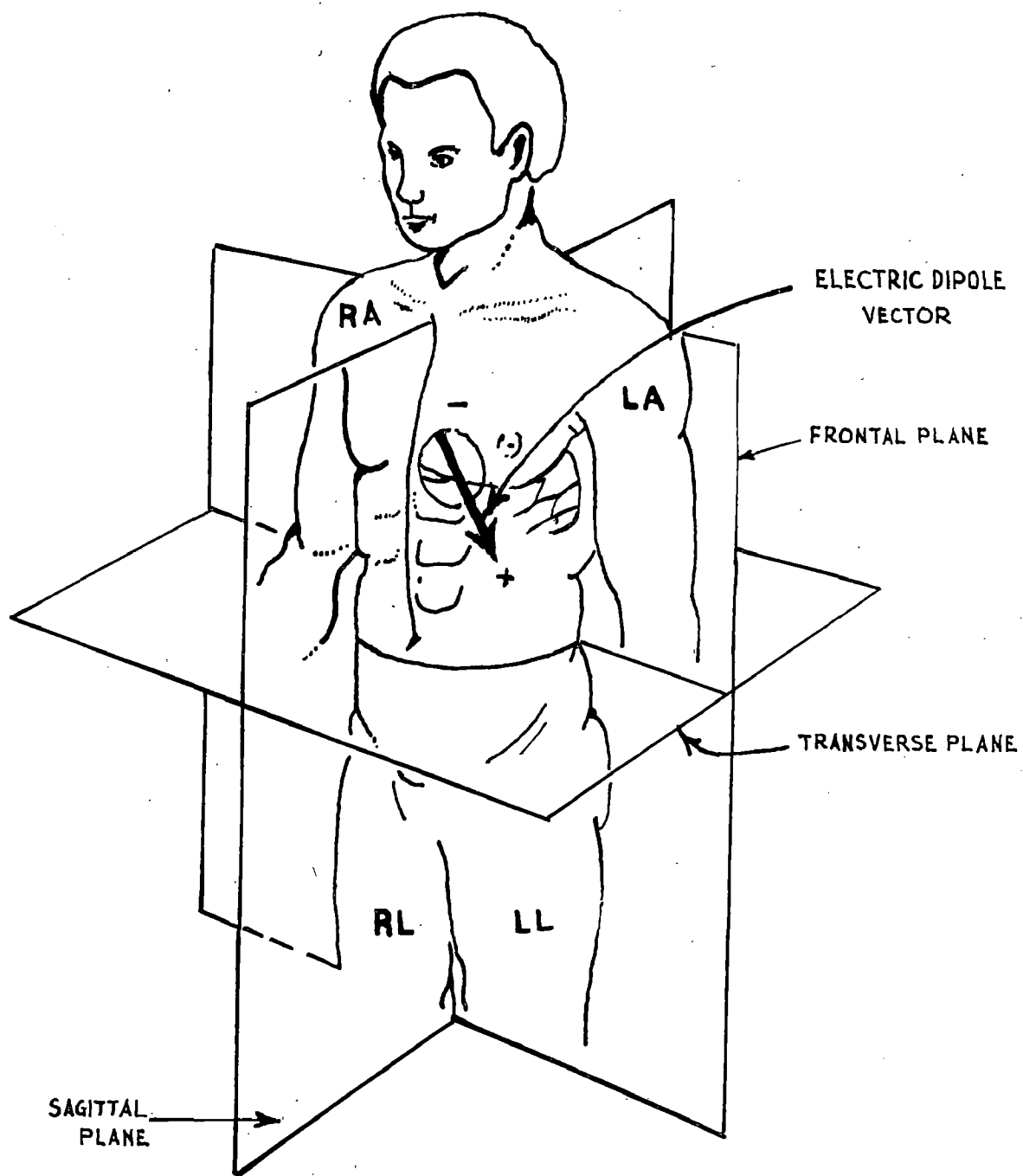


FIGURE 1-3 ELECTROCARDIOGRAPHIC PLANES AND AN ELECTRIC DIPOLE VECTOR. RA, LA, RL, AND LL INDICATE ELECTRODE LOCATIONS ON THE RIGHT AND LEFT ARMS AND LEGS.



The modern concept of the equivalent cardiac dipole and its special expression in the vectorcardiogram has been evolved over a number of years. Einthoven, Fahr, and de Waart (1) indicated diagrammatically by an arrow the manifest orientation of the electrical activity of the heart. They calculated the magnitude and orientation of the arrow by projection from reference lines representing an idealized spatial relationship between limb electrodes. These reference line came to be known as the "lead axes" and the arrow representing manifest ventricular depolarization became known as the 'QRS-axis'. Later Burger and Van Millan (1) defined the source of electrical activity in the heart as the 'heart vector' and defined the characteristics relating the electrical source to a specific surface electrode or electrode set as the "lead vector". These two vectors, if known, should permit predicting the surface potential of that lead from their scalar product.

McFee and Johnston (1) extended the lead vector concept to include variation from site to site in the heart region. Thus, in a 'leadfield', the flow lines of the field in the region of the myocardium indicate the orientation of the local lead vector at each site. All elements of the myocardium firing at a given instant were considered as local dipole sources; their vector sum should present the mean instantaneous heart vector. It became obvious that if this sum were the heart vector whose effect

is faithfully reported as a voltage in the surface electrocardiographic leads, then the leadfield must flow with uniform direction and intensity throughout the whole region of active myocardium. Put simply, all of the local lead vectors should be equal for all sites or elements of myocardium to be reported correctly.

Two major factors prevent this simple state from being attained; first, the human body is a volume conductor with an irregular boundary and secondly, the conductivity of the body is not homogeneous. Both factors prevent flow lines between any two such electrodes from following straight, evenly spaced paths through the heart. The irregularity of body contour may be compensated for in part by introducing weighting resistances behind each of the component electrodes, but the effect of inhomogeneity is so difficult to define and to counteract that it is usually ignored, at least in vectorcardiography.

### 1.3 Some Definitions -

Three instantaneous 'source' vectors, each with a slightly different meaning and operational utility may now be defined as follows -

The 'heart vector' - The vector whose moment is the resultant (vector sum) of the individual dipoles representing all the elements of the instantaneous wave of myocardial activation. An 'element' may be considered

an area of electromotive surface. Operationally, an element should be small enough to yield a surface voltage distribution which can be "completely" reproduced by a single equivalent dipole. The locus of the heart vector is the centroid of the activation surface i.e. if the surface is composed of elements of equal area, the locus is at the vectorial mean of the centre of the elements.

The 'equivalent dipole' - The single vector which best reproduces the surface voltage distribution i.e. which yields a surface voltage distribution with least departure or error from the original values.

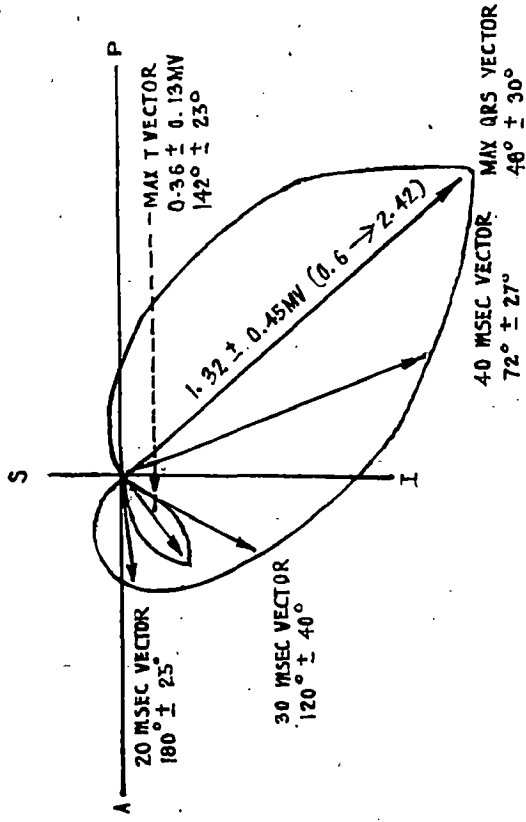
The 'VCG vector' - the vector whose moment is determined by simultaneous voltage values in three surface vector leads. This vector moment is conventionally <sup>on</sup> constructed electronically and represented as an image on an oscilloscope face or plotted on an X - Y recorder. The locus remains fixed and is usually assumed to be at the centroid of the heart.

#### 1.4 The Normal Vectorcardiogram -

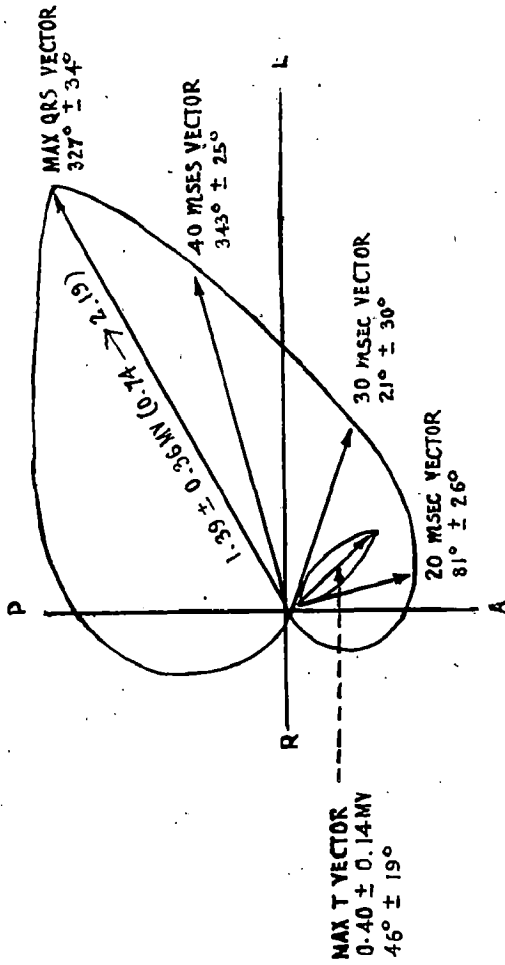
The vectorcardiogram is recorded in the transverse (horizontal), sagittal and frontal planes of the body.

The E point is the zero point on the vectorcardiogram. The X, Y and Z axes as well as the three planes of the body intersect at the E point.

LEFT SAGITTAL PLANE



TRANSVERSE PLANE



FRONTAL PLANE

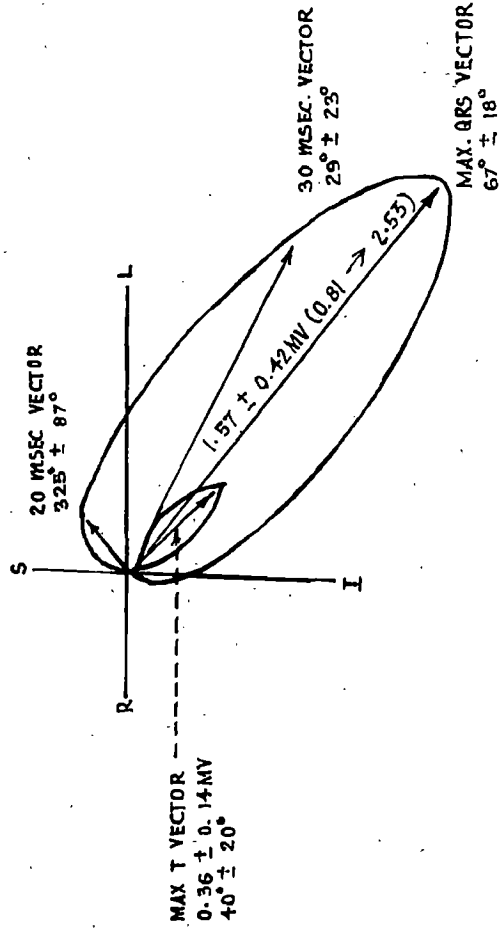


FIG. 1.4 THE NORMAL VECTORCARDIOGRAM  
THE VALUES SHOWN ARE BASED ON THE DATA  
OF H. W. DRAPER ET AL. (2)

The letter E stands for the equivalent cardiac dipole. The symbol  $\vec{E}$  is used to represent E as a vector quantity and  $\overline{SE}$  as a spatial vector quantity. The P loop is the loop of atrial depolarization, the QRS loop the loop of ventricular depolarization; and the T loop, the loop of ventricular repolarization.

$$\overline{SE} = \sqrt{E_x^2 + E_y^2 + E_z^2}$$

Where  $E_x$ ,  $E_y$  and  $E_z$  are the magnitudes of the projection of the spatial vector on the X, Y and Z leads.

#### 1.4.1- Transverse Plane -

The QRS loop in the transverse plane is usually oval in shape. Inscription of the loop is invariably counter clockwise. The initial forces are directed anteriorly and usually rightward. The 20 m-sec. vector is always directed anteriorly. The main body of the loop is located posteriorly. The average max. QRS vector is at  $327^\circ$  and its magnitude is usually less than 2.2 mV. The terminal deflection is directed posteriorly. However, less than 20 percent of the loop area is located in the right posterior quadrant.

#### 1.4.2- Left Sagittal Plane -

The QRS loop in the LSP is usually oval shaped. Inscription of the loop is invariably counterclockwise. The initial forces are directed anteriorly and usually

superiorly, but sometimes inferiorly. The 25 m-sec. vector is always<sup>S</sup> directed inferiorly. The main body of the loop is located inferiorly. The average maximum QRS vector is at  $48^{\circ}$  and its magnitude is usually less than 1.8 mV.

#### 1.4.3 Frontal Plane -

The QRS loop in the frontal plane tends to be long and narrow. Inscription of the loop is clockwise in 65 percent of cases, figures of eight in 25 percent and counterclockwise in 10 percent. As a general rule, horizontal loops rotate counterclockwise and vertical loops clockwise. The counterclockwise inscription of a vertical loop is rare but clockwise inscription of a horizontal loop sometimes occurs. The initial forces are usually directed rightward and superiorly, but the findings are variable. The average max QRS vector is at  $41^{\circ}$ , and its magnitude is less than 2 mV. The direction of the terminal forces is variable.

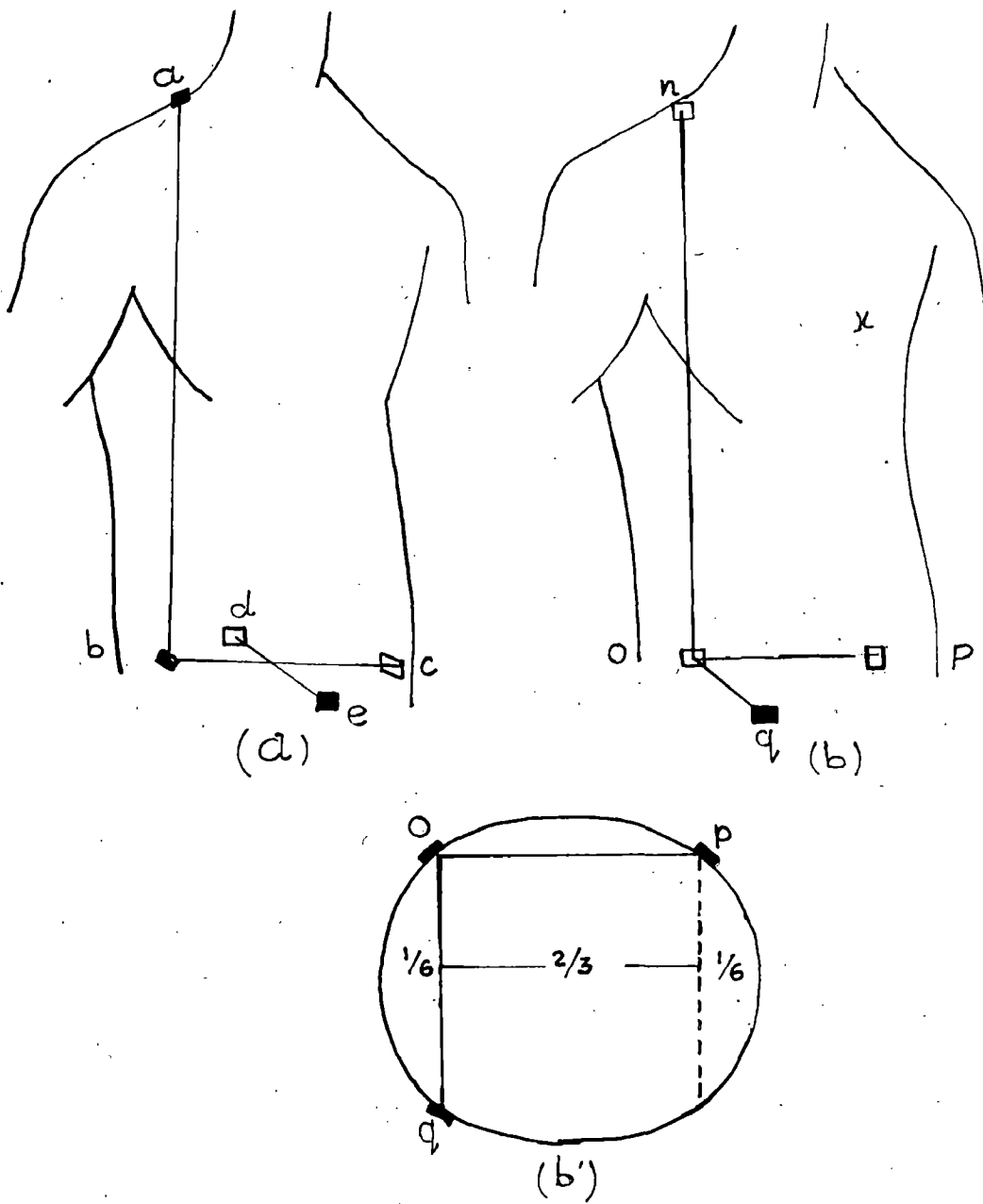


FIG. 2-1

SULZER AND DUCHOSAL'S  
REFERENCE FRAMES

vertical lead by the letter Y; and the sagittal, by the letter Z.

## 2.2 Uncorrected Lead Systems -

Before the corrected orthogonal lead systems were developed, the uncorrected lead systems were in use. Hence they are historically older and are so called because the electrode placements were determined by convenience, tradition, or the assumption that apparent right angle relationships between three lead axes on the surface would produce orthogonal reporting of the heart vector or equivalent dipole. No correction for boundary or lead field shape went into the design. On the basis of considerable clinical experience with vectorcardiography, Sulzer and Duchosal<sup>(2)</sup> advocated two electrode schemes. Although both gave similar loops, they preferred system shown in fig.(2.1)(b) which became known as the double cube system.

Wilson et al (3), after studying potential distribution of an electrically driven dipole placed in a cadaver heart, introduced the equilateral tetrahedral reference frame (fig. 2.2). With this system, limb electrodes and the central terminal (formed by joining the three limb electrodes through equal resistances) were used to obtain the frontal plane projection. The relative voltages appearing between the various electrode pairs



superiorly, but sometimes inferiorly. The 25 m-sec. vector is always<sup>s</sup> directed inferiorly. The main body of the loop is located inferiorly. The average maximum QRS vector is at  $48^{\circ}$  and its magnitude is usually less than 1.8 mV.

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## CHAPTER - II

### VECTORCARDIOGRAPHIC LEAD SYSTEMS

#### 2.1 Introduction -

The potentials of the heart need to be tapped in three the planes - frontal, transverse and sagittal to obtain a vectorcardiographic recording. For this a system of leads has to be developed in the three planes of the body. An ideal lead system for electrocardiography and vectorcardiography would consist of three leads with the following characteristics:-

- (1) The leads would be perpendicular to each other and to the horizontal, vertical and sagittal axes of the body.
- (2) The amplitude of the three lead vectors should be equal.
- (3) These leads would have the same strength and direction, not only for a single point within the heart, but for all points in the heart where electromotive forces are generated. Such leads are called corrected orthogonal leads. On theoretical grounds, such leads should contain all the information contained in the usual 12-lead electrocardiogram. By convention, the horizontal lead is designated by the letter X; the

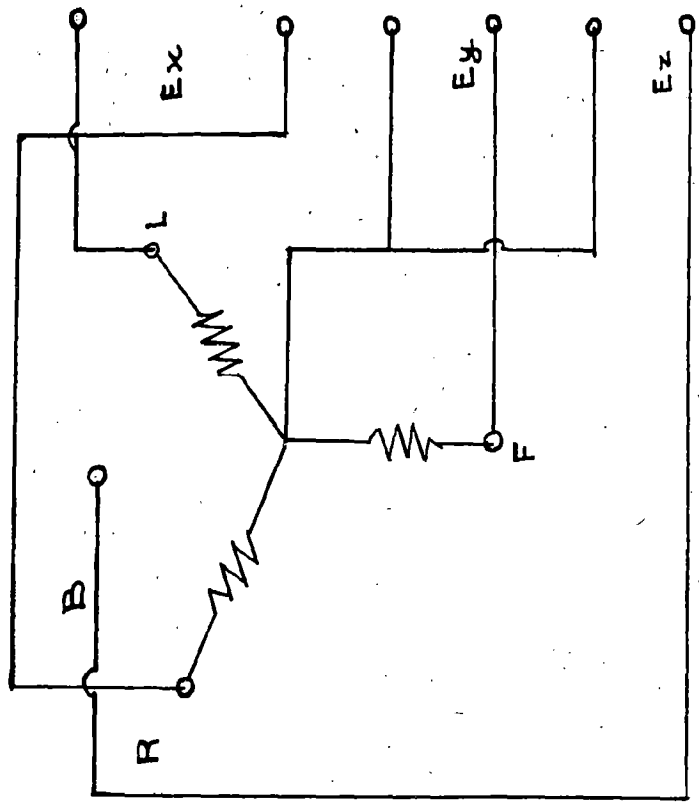
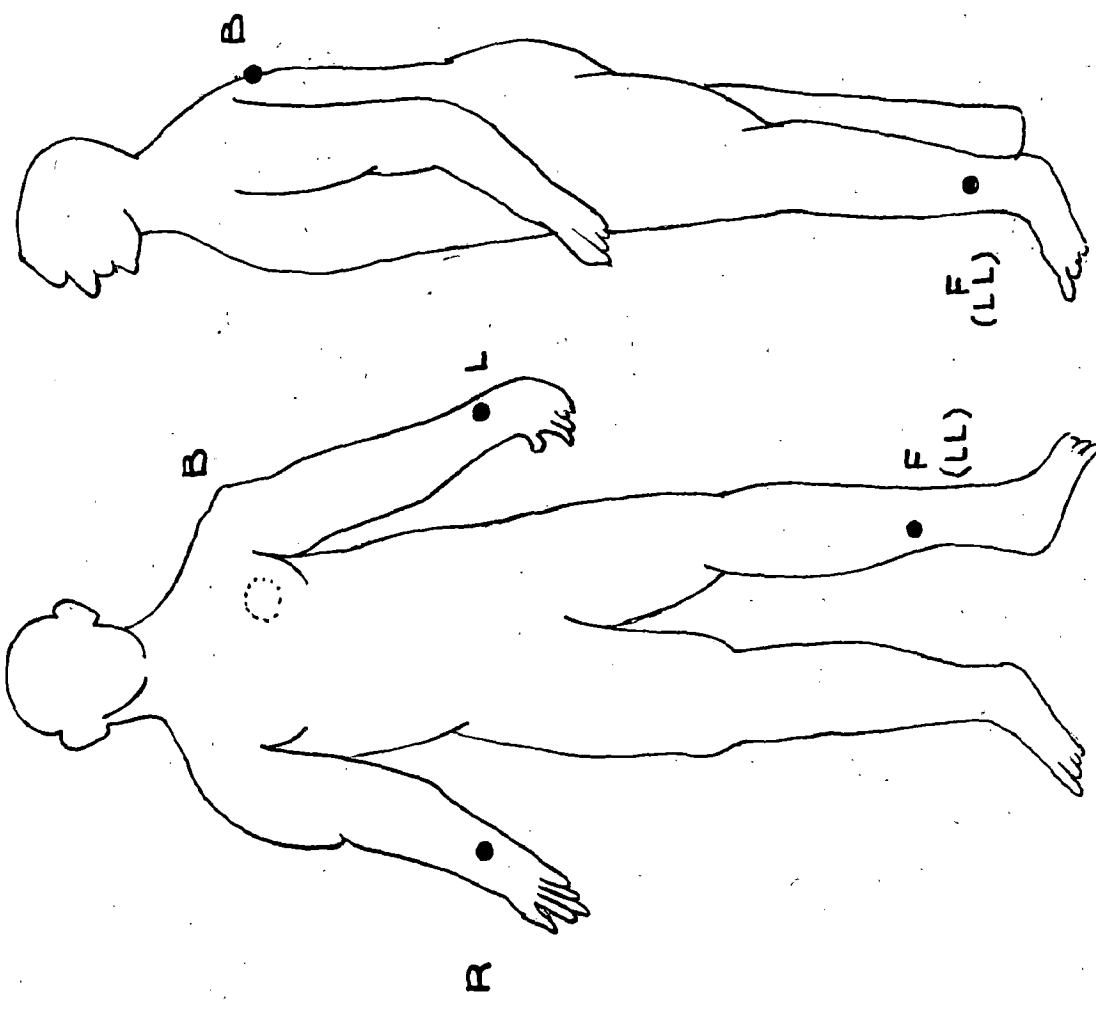


FIG. 2.2 WILSON'S TETRAHEDRAL  
REFERENCE FRAME.

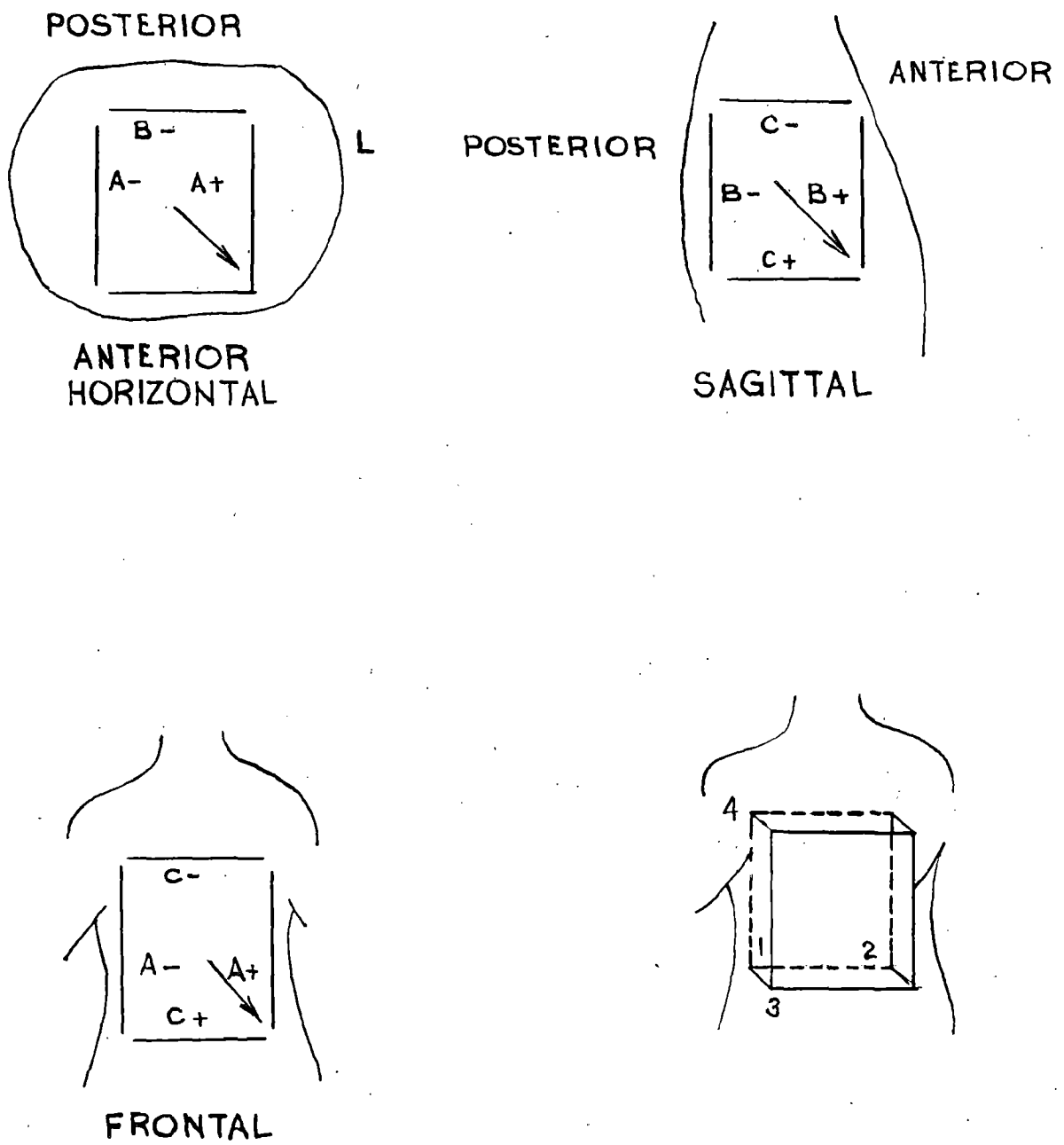


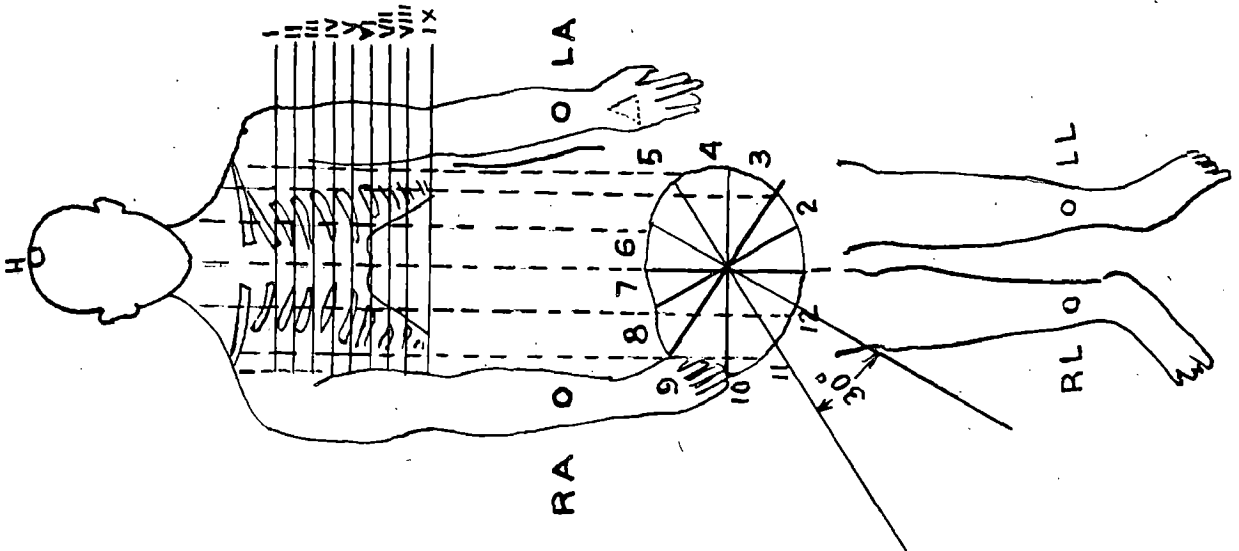
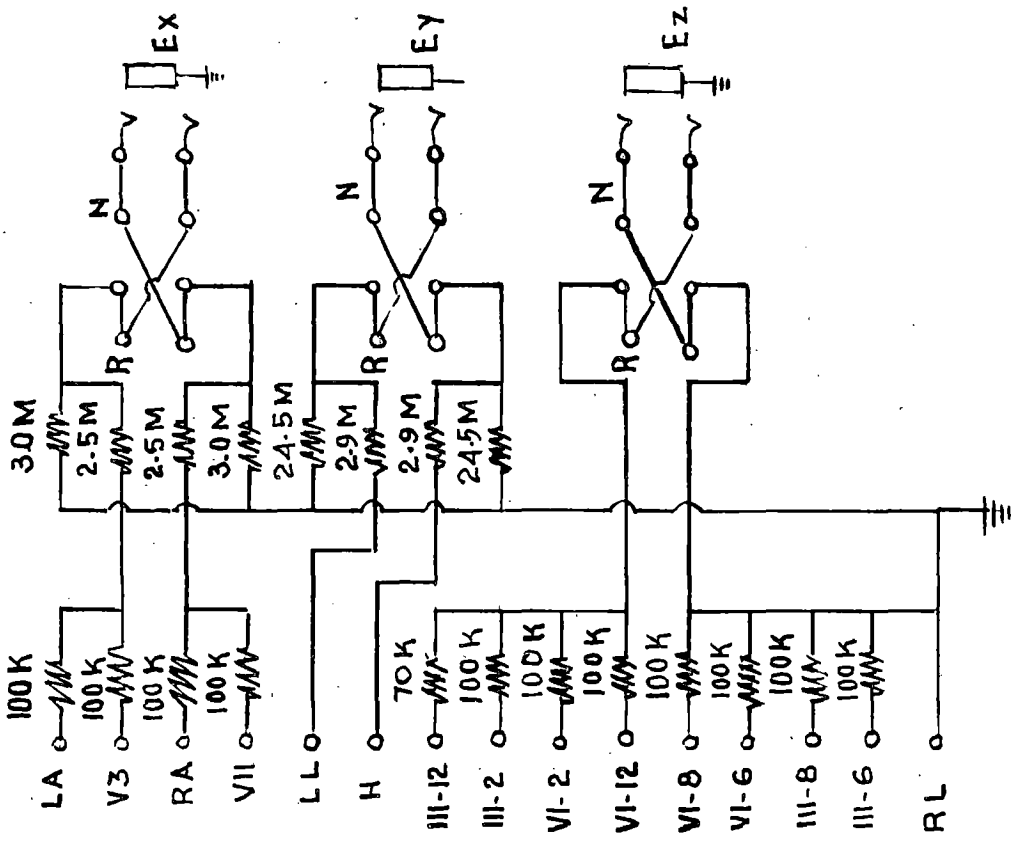
FIG.2.3 THE CUBIC REFERENCE EMPLOYED BY GRISHMAN

were in nearly all instances those predicted from the geometry of the torso. With such evidence the tetrahedral system gained considerable support. Grishman et al, (3), after obtaining clinical records with the Wilson tetrahedral and the Duchosal reference frames, were led to develop the cubic arrangement (fig. 2.3). The No. 1 electrode was placed near the right posterior axillary line at the level of the first and second lumbar vertebrae, No. 2 in the left posterior axillary line, no. 3 over the right anterior axillary line, and no. 4 over the left scapula. With this arrangement the authors claimed that the heart was as equidistant from the electrodes as the thorax allowed and that the electrodes were easily located anatomically.

### 2.3 Corrected Lead systems -

Although all these reference frames provided reasonable VCGs the QRS and T loops derived from normal subjects exhibited a remarkably wide range of magnitudes and orientations even when the same vectorcardiographic reference frame was used. In addition the data obtained with different reference frames were not easily comparable.

Hence after these attempts were made by Schmitt and Simonson (3) and Frank (3) to identify the variables by means of investigations in which electrically driven dipoles were implanted into electrolyte filled human torso models and the resulting body surface potential distributions were



**FIG. 2.4 SCHMITT'S SVEC III ORTHOGONAL LEAD SYSTEM.**

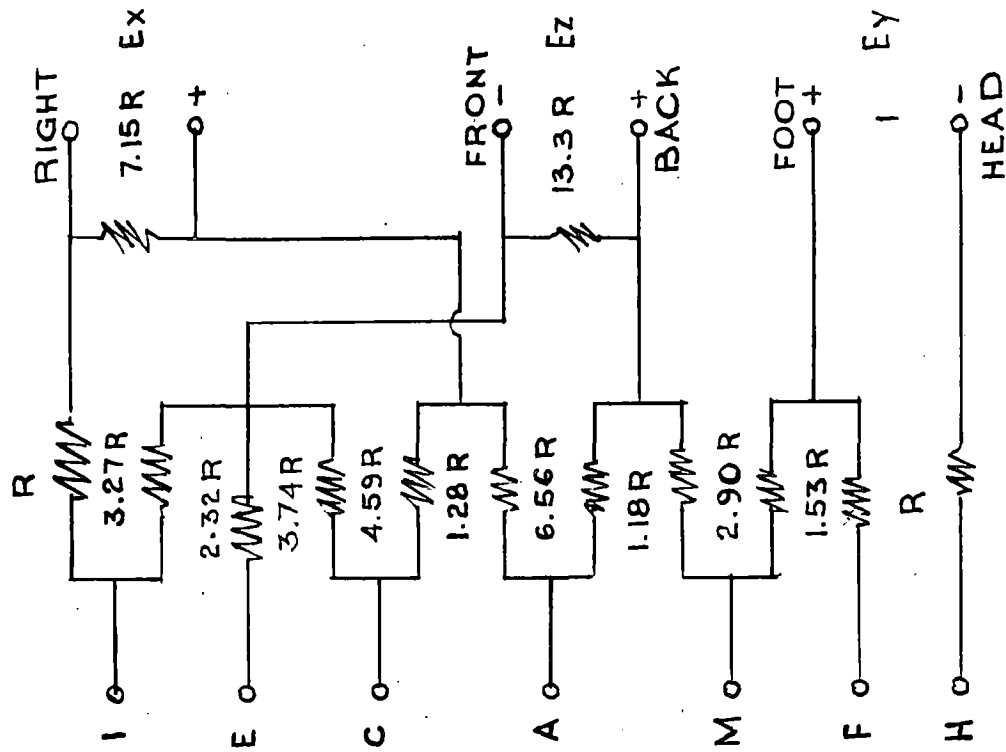
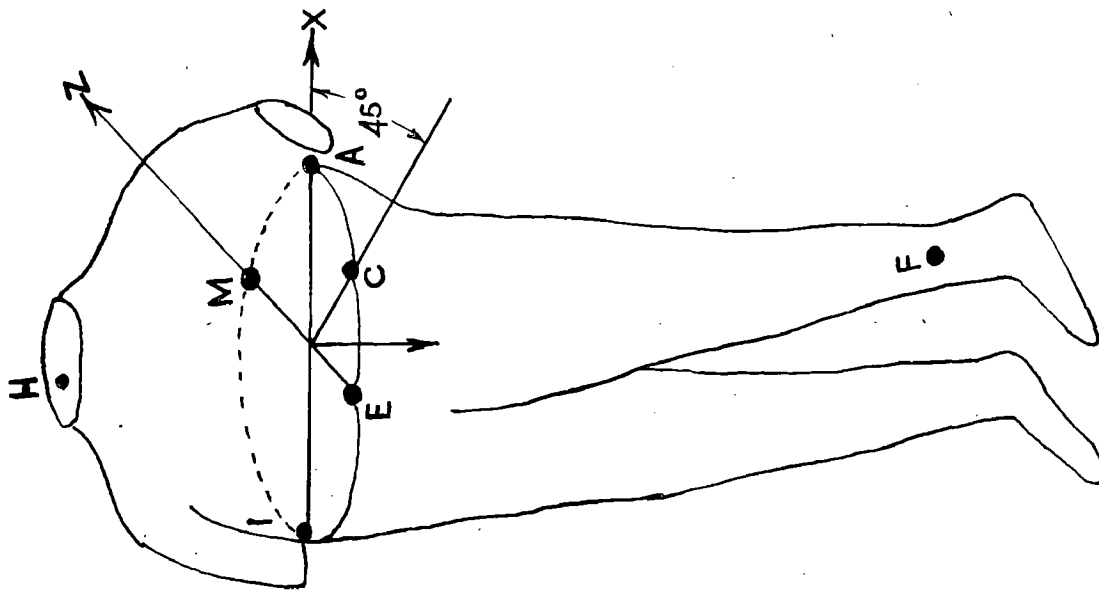


studied. From their studies, both investigators developed orthogonal lead systems for spatial vectorcardiography. Both these systems are used clinically.

In Schmitt's system (fig.2.4) fourteen active electrodes are employed. The voltage which represents the 'X' component of the cardiac vector is derived from the right and left arm electrodes, along with components derived from chest and back electrodes placed at the level of the fifth intercostal space. The 'Y' component is obtained from the head and left leg electrodes, and the 'Z' component from eight electrodes located on the chest and back at the third and sixth interspace.

Seven active electrodes are used with Frank's system (fig.2.5). The 'X' component is derived from an array of electrodes that surround the heart approximately at the level of the fifth interspace. The 'Y' component is derived from neck and left leg electrodes, and the 'Z' component from the voltage appearing between an array of three electrodes on the anterior of the chest and one electrode in the back and one on the midaxillary line. To provide a more accurate location of the level for the chest and back electrodes, Frank developed a three electrode exploring tool and gave instructions for its use.

There is also the axial system of Mcfee and Parangao (1) consisting of nine electrodes and appropriate resistances. They developed this system by adopting the



**FIG. 2.5 FRANK'S VECTORCARDIOGRAPHIC LEAD SYSTEM**

approach that if the electrodes at a given lead 'bracketed' both ends of the heart region and were appropriately spaced, the field lines between the bracketing sets would approach equal spacing and parallel orientation in the heart region. Closer the relevant surfaces of the particular lead to the heart, more is the need for weighting and separation of electrodes, thus, the head and foot electrodes were left alone on the assumption that the greater distance increased the likelihood of parallelism and uniform density in the heart region.

Several systems with greater refinement of the corrective principle should be mentioned. Rijlant (1) applied 72 electrodes and a complex resistive network presumably to obtain the effect of a surface integration. Nelson et al. (1) employed three encircling belts of eight electrodes each to obtain for each coordinate axis a surface summation. Helm introduced large saline soaked sponges applied to the chest which obtain either surface summation or uniform lead fields - depending on one's point of view. Fischmann, Barber and Weiss (1) similarly presented the chest with arrays of many electrodes precise in grid formations for each lead. The equal spacing of the elements of the grid permitted each electrode to apply to an element of surface whose projection in the direction of lead equaled the others. This corrected for torso curvature and these again 'straightened' out the lead fields. While some of these systems appear simpler to apply than the rest none

have received general clinical acceptance despite the apparent increase in precision each offers over the 'basic' corrected systems.

2.4            Comparison between loops of dipole moment  
                 and loops of various vectorcardiographic  
                 systems -

There have been several comparisons of the data obtained with the various vectorcardiographic reference frames. In considering the merits of one reference frame over another, it is useful to remember that most of the carefully examined reference frames were derived from human torso models in which electrically driven dipoles were implanted. Although this is a good starting point, the situation in the actual human subject is quite different. Not only do human torsos come in a wide range of shapes and sizes, but also the tissues between the heart and body surface electrodes have quite different electrical properties. Even within a single tissue the resistivity is not the same in all directions. Therefore there still remains the need to conduct more cadaver experiments to evaluate the magnitude of the distortions produced by the intrathoracic contents.

With such a variety of reference frames and the lack of clear cut clinical evidence to indicate the superiority of one reference system over another to identify specific myocardial diseases, it is difficult to set forth criteria which would lead to the adoption of a single method.

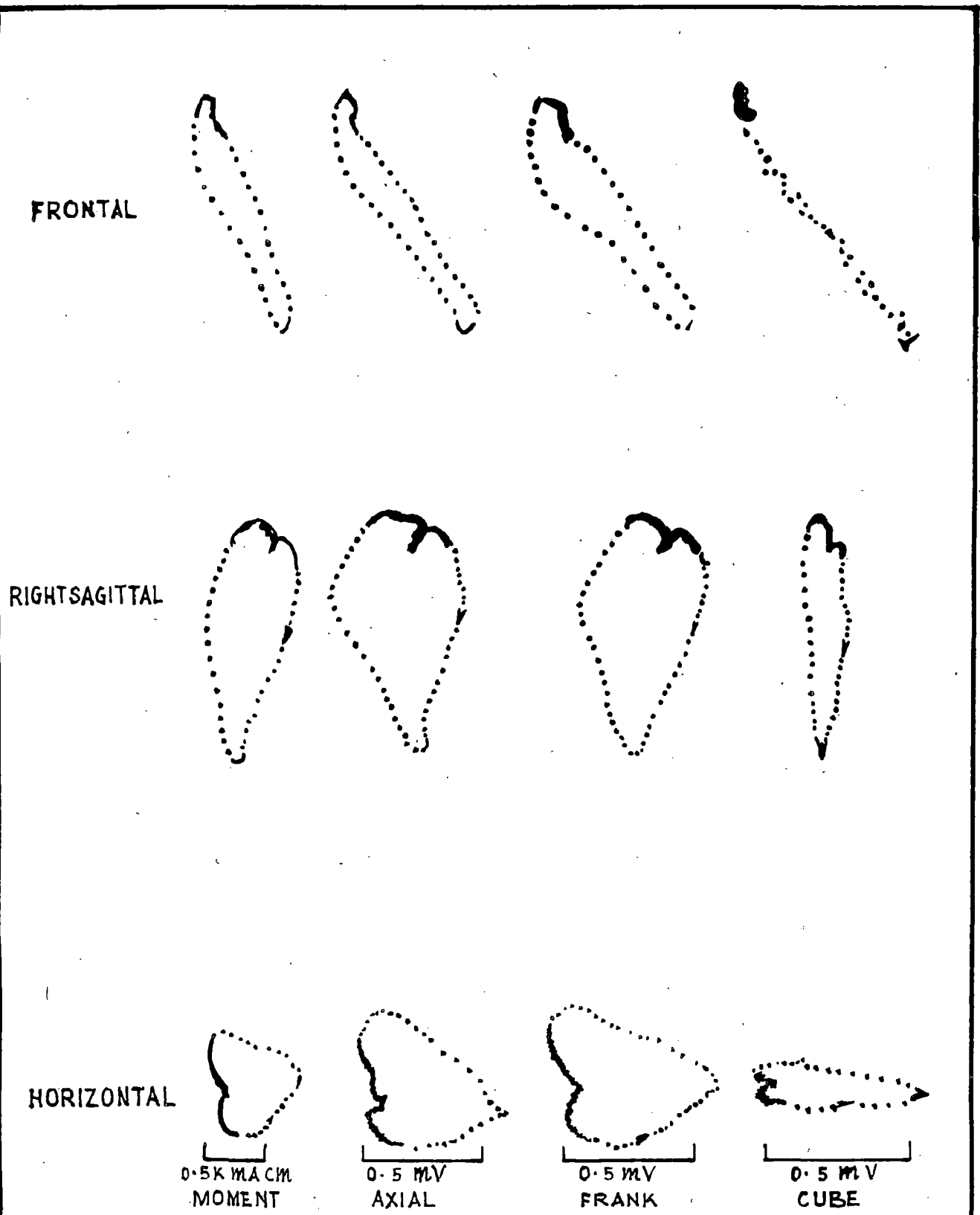
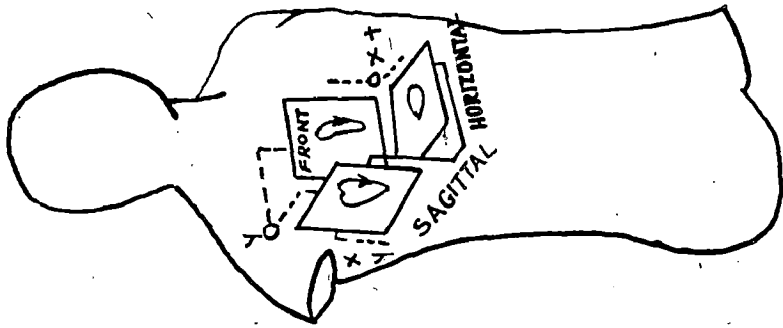


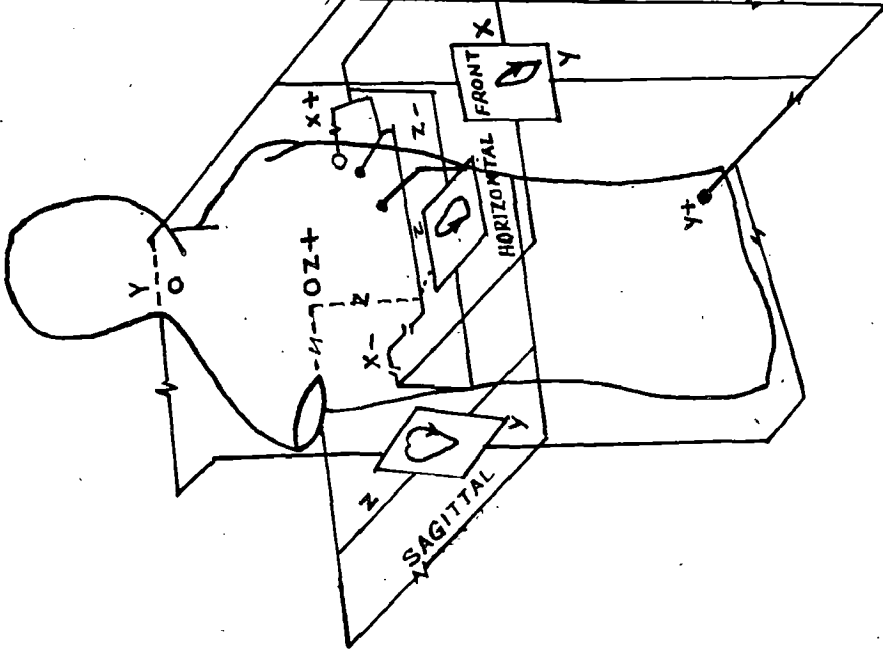
FIG. 2-6 COMPARISON OF THE LOOP OF THE CARDIAC DIPOLE  
MOMENT WITH THE RECORDED VECTORCARDIOGRAPHIC LOOPS  
DURING THE QRS COMPLEX OF A NORMAL HUMAN SUBJECT. THE  
FIRST COLUMN SHOWS THE DIPOLE MOMENT PROJECTIONS FOR THE  
FRONTAL, RIGHT SAGITTAL, AND HORIZONTAL PLANES; THE  
SUCCEEDING COLUMNS, THE RESPECTIVE PROJECTIONS FOR THE  
AXIAL, FRANK, AND CUBE VECTORCARDIOGRAPHIC SYSTEMS.

CUBE VCG SYSTEM



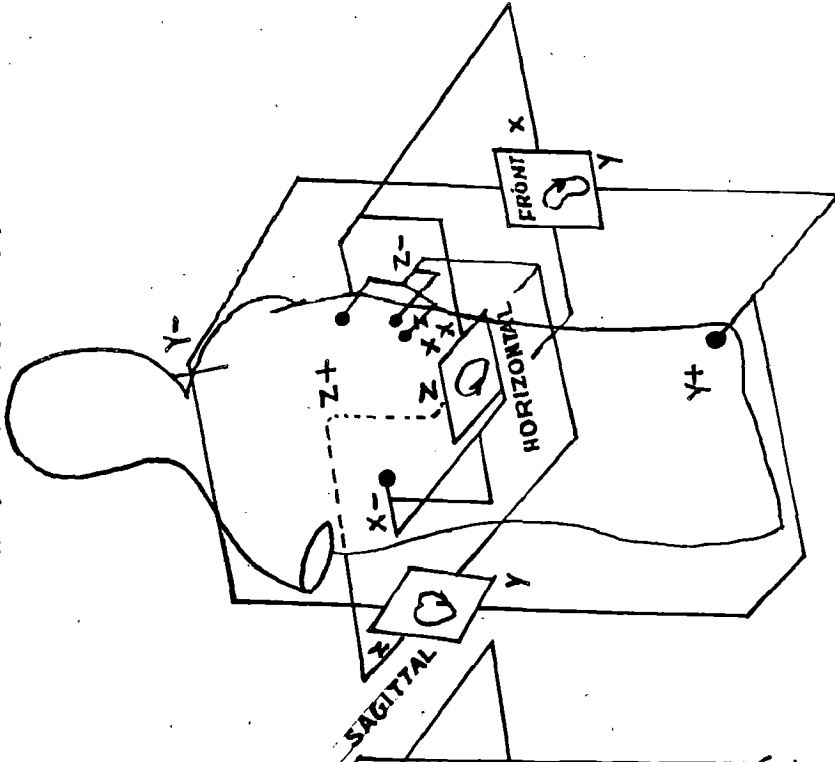
(a)

FRANK VCG SYSTEM



(b)

AXIAL VCG SYSTEM

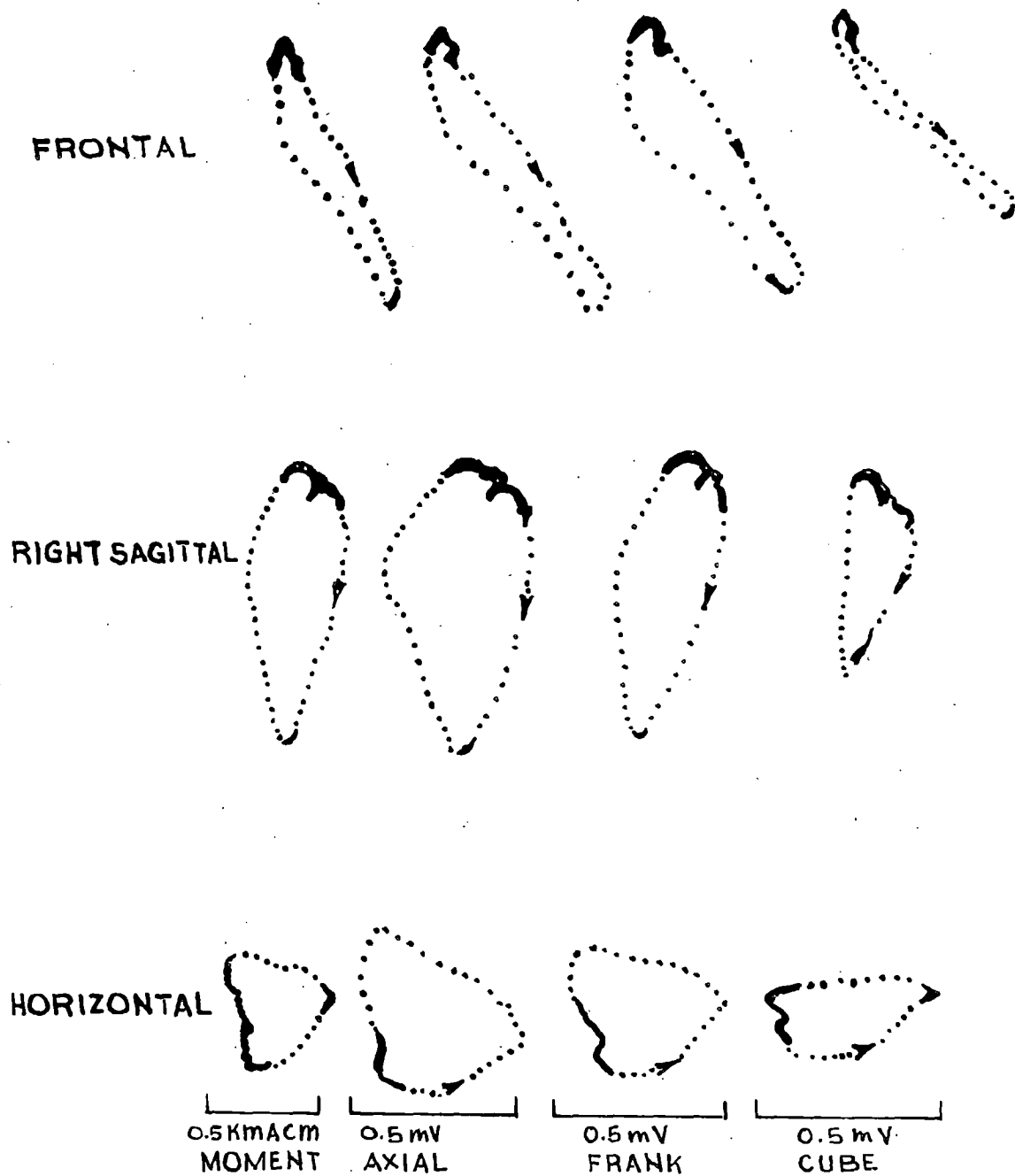


(c)

**FIG. 2.7** DIAGRAMS OF THE TORSO SHOWING COMPARATIVE ELECTRODE PLACEMENT IN THE (a) CUBE (b) FRANK AND (c) AXIAL VECTORCARDIOGRAPHIC SYSTEM.

To make a comparison between the vectorcardiographic loops obtained from different reference frames a comparison is made of each VCG loop with the "equivalent dipole moment". It is possible to obtain a relatively good approximation of the equivalent dipole (assuming a homogeneous volume conductor) by surface summation of the data used in body surface potential mapping, provided the precision of recording technique is sufficiently good and the spatial geometry of the recording sites and thoracic surface are also known. Since vectorcardiographic systems are also forced to make the assumption of homogeneity, a comparison between loops of equivalent dipole moment and of vectorcardiographic loops is revealing. Figure (2.6) indicates the comparison between equivalent dipole moment and three selected systems (Cube, Frank and Axial (fig.2.7) ) in a healthy normal subject.

Two aspects are examined when this comparison is made; firstly, how faithfully does each vectorcardiographic system report the equivalent dipole and secondly, how much contamination with non dipolar component does each system permit. The relatively good dipolar reporting by Frank and axial systems are reasonably predicated, but the discrepancies in the cube record could arise from suppression of dipolar signal, from sensitivity to non-dipolar signal or from both factors. Sorting out the roles of fidelity and contamination in vectorcardiographic reporting may be



**FIG. 2.8**

**COMPARISON OF THE LOOP OF THE DIPOLE MOMENT OF A NORMAL SUBJECT WITH THAT PRESENTED BY EACH VECTORCARDIOGRAPHIC SYSTEM WHEN NON-DIPOLAR CONTENT HAS BEEN EXCLUDED FROM THE SURFACE ELECTROCARDIOGRAPHIC SIGNAL.**



FRONTAL



RIGHT SAGITTAL



HORIZONTAL



0.5 mV  
AXIAL

0.5 mV  
FRANK

0.5 mV  
CUBE

FIG. 2.9 COMPARISON OF THE NON-DIPOLAR  
COMPONENTS OF THE VECTORCARDIO-  
GRAPHIC DISPLAY OF A NORMAL  
SUBJECT IN THE THREE LEAD SYSTEMS.

approached by (a) finding the surface distribution of the equivalent dipole, (b) finding the surface distribution of the non dipolar residual by subtraction of the potential attributable to dipole from that actually recorded at each surface site, and (c) examining the vectorcardiographic report of all three potential sets; the actual, dipolar and the non dipolar residual. Figures 2.8 and 2.9 illustrate in different ways the effect of non dipolar contamination in the usually recorded VCG's. In Fig. 2.9 the loop of dipole moment is compared with the VCG's resulting when only the dipolar component of the electrocardiographic signal was permitted to be "picked up" by the system electrodes. We noticed that the cube VCG is much more like the loop of dipole moment when non-dipolar contamination is excluded. The magnitude of the non-dipolar signal acquired by the three systems is shown in figure 2.9. The asymmetry of the human torso permits far more discrepancy in the actually recorded cube. At present, it is very difficult to predict the superiority of one reference over another. Quantitative appraisal of all systems, under the prevailing conditions, is very difficult. But there may be some sort of "pattern recognition" at work and one or more systems may be particularly helpful in dramatizing or emphasizing clinically significant patterns. While objectivity has prevailed in the comparison of the physical characteristics of many lead systems, the

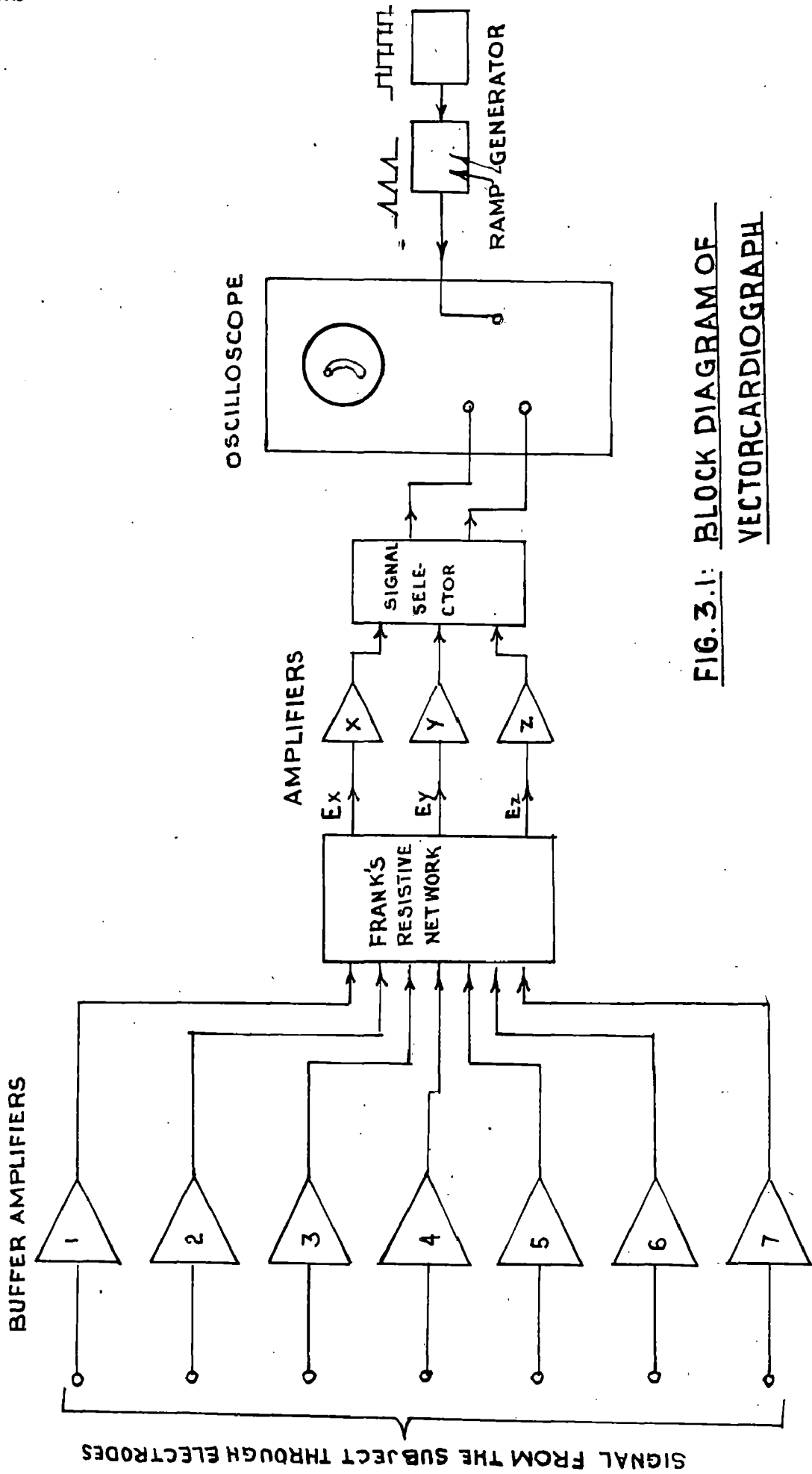
inclination of the vectorcardiographer towards a particular system and familiarity with it, has prevented any large scale "objective" comparison of clinical usefulness of the various systems.

CHAPTER - IIIDESIGN OF VECTORCARDIOGRAPH

## 3.1 Introduction :

A vectorcardiograph shows a three dimensional or at least a two dimensional picture of the orientation and magnitude of the cardiac vector throughout the cardiac cycle. It is difficult for practical machines to display the vectorcardiograph in three dimensions, but it is relatively simple to display it in two dimensions or in other words, in a particular plane of the body.

As already described in the previous chapter, special lead systems have been developed that can provide the 'X' 'Y' and 'Z' components of the ECG. Any two of these can be fed into the 'X' and 'Y' deflection circuits of a cathode ray oscilloscope or an X - Y recorder. In each heart beat a vector loop representing the locus of the tip of the cardiac vector with its tail ~~is~~ at the origin is then traced on the oscilloscope screen. But this type of display does not give any information on how the vectorcardiograph signals are changing with respect to time. The scalar ECG is best able to provide this information, but it is of interest to see the time course of the vector loop as well. For this reason, the Z (intensity) axis of the oscilloscope is modulated by a ramp generator driven by a precision clock. The ramp signals produced for every



**FIG. 3.1: BLOCK DIAGRAM OF VECTORCARDIOGRAPH**

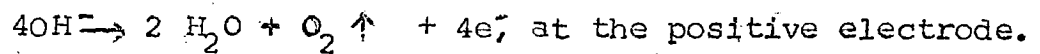
clock pulse give a segment of the vector loop that corresponds to the duration of the ramp. The viewer can tell the direction of the vector loop, as well as its time course, since the ramp causes one end of the segment to be at low intensity while the other is at high intensity- Figures 3.1 shows the block diagram of the VCG. Often triggering circuits are included in the vectorcardiograph, so that only one cardiac cycle is displayed, thereby avoiding overlap. With the type of vectorcardiograph described the VCG is displayed on the cathode ray tube screen. If we wish to get a permanent record of the VCG, we must take a photograph of the pattern.

### 3.2 Electrodes -

From the foregoing discussion it is evident that the primary requirement of the vectorcardiograph is to pick up the bio-signal from the human body and this is done by the help of electrodes. The electrodes serve as an interface between the body and the amplifying circuitry. When a measurement of the potential is made, current flows in the circuit. Ideally this current should be very small. However in practical situations, it is never zero. The electrodes must therefore have the capability of conducting a current across the interface between the body and the electronic circuitry. In addition, the electrodes also carry out a transducing function, because current is carried

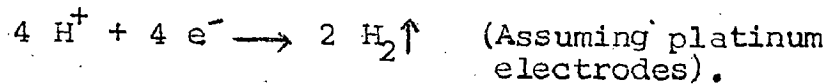
in the body by ions, while it is carried in the electrodes and its lead wire by electrons. Thus, the electrode must serve as a transducer to change an ionic current into an electronic current. This change is brought about by a chemical reaction.

If ordinary metals are used for electrodes, polarisation results from this chemical reaction as follows:-



These electrons flow from the negative to the positive electrode.

At the negative electrode ,



Hence at one or both electrodes, gas bubbles form due to electrolysis, and the resulting electrode to solution interface is electrically unstable. This instability produces electrical noise and drift which may be much larger than the  $\overset{E}{ECG}$  signal.

### 3.2.1 Polarizable and non-polarizable electrodes :-

Theoretically two types of electrodes are possible those that are perfectly polarizable and those that are perfectly non polarizable. This classification refers to what happens to an electrode when a current passes between it and the electrolyte. Perfectly polarizable electrodes

are these in which no actual charge crosses the electrode-electrolyte interface when a current is applied. Of course, there has to be current across the interface, but this current is a displacement current and the electrode behaves as if it were a capacitor. Perfectly non-polarizable electrodes are those in which current passes freely across the electrode electrolyte interface, requiring no energy to make the transition. Thus, for perfectly non polarizable electrodes there are no overvoltages.

Neither of these two electrodes can be fabricated; however, some practical electrodes come close to acquiring their characteristics. Electrodes made of noble metal come closest to behaving as perfectly polarizable electrodes. Since the materials of these electrodes are relatively inert it is difficult for them to oxidize and dissolve. Thus, current passing between the electrode and the electrolyte primarily changes the concentration of ions at the interface. The electrical characteristics of such an electrode produce a strong capacitive effect. The silver-silver chloride ( $\text{Ag} - \text{AgCl}$ ) electrode is a practical electrode that approaches the characteristics of a perfectly nonpolarizable electrode and can easily be fabricated in the laboratory.

### 3.2.2 The silver-silver chloride electrode -

A reversible electrochemical reaction occurs at the surface of the silver - silver chloride electrode.



If the electrode is operated as one half of a cell,  $\text{Cl}^-$  is deposited on the electrode when it is the anode and AgCl on the electrode surface is reduced to Ag, and  $\text{Cl}^-$  ions are freed into the electrolyte solution when it is the cathode.

The Ag - AgCl electrode is generally represented as,  $\text{Ag}/\text{AgCl}/\text{Cl}^-$ .

It consists of a metallic silver substrate coated with AgCl, and is in contact with an electrolyte solution which contains a soluble chloride such as NaCl or KCl. Silver - Silver chloride electrodes are reversible or nonpolarizing. This means that the electrode can pass electric current without changing the chemical environment in the vicinity of the electrode. Since AgCl is slightly soluble in water, the electric current at the electrode-electrolyte interface must be kept relatively small to maintain an unchanged chemical environment.

### 3.2.3 Fabrication of Ag AgCl Electrodes :-

The electrodes to be used with the vectorcardiograph have to be placed on the chest in the level of fifth intercostal space approximately, one in the neck and one at the left foot. They should hence be small enough to fit snugly into the intercostal spaces. Hence round plate electrodes of one centimetre radius are fabricated from high purity silver. A screw is provided on one side of the round plate so that the lead wire can be attached to

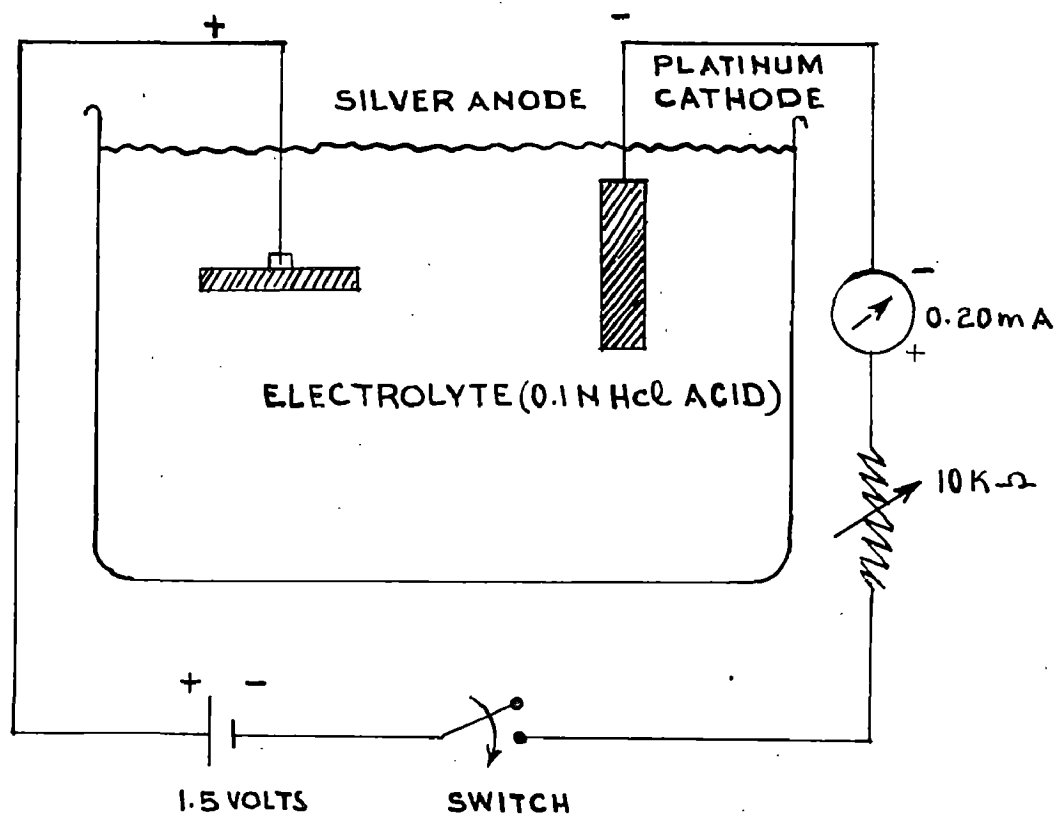


FIG.3.2 CHLORIDIZING A SILVER ELECTRODE.

the electrode. This silver electrode is, then chloridized in an electrolytic cell as follows (fig.3.2). The electrode is made the anode. The electrolyte is not critical and either dilute sodium chloride or hydrochloric acid is frequently used. In this particular case 0.1 N hydrochloric acid was used as the electrolyte. The cathode material is also not critical, its only requirement is that it should not enter into the reaction, because contamination of the electrode can occur. Offner (4) even suggests the use of copper wire though it is always better to use a platinum wire or a platinum plate for the cathode. The concentration of the electrolyte is not critical either, but to ensure a stable electrode, doubly distilled  $H_2O$  and reagent grade chemicals should be used in making the electrolyte. The silver anode is first cleaned in aqua regia and washed with distilled  $H_2O$ , taking care not to touch it by hand as it may become greased. Current in the cell should be limited to  $1mA/cm^2$  of anode surface.

Radius of electrode = 1 cm.

∴ area of electrode face =  $\pi r^2 \approx 3 \text{ cm}^2$  approx.

Hence in the given case, the current through the cell is limited to 3mA by adjusting the 10 K potentiometer.

The length of time that chloridizing is carried out determines the depth of the AgCl layer. Generally from 10 to 25% of the silver core should be converted to AgCl for stable electrodes. These electrodes were chloridized for 5 minutes and this time was found to be sufficient.

Since AgCl is photoreactive, the color of the finished electrode will depend upon the amount of light present during its fabrication. The electrodes were fabricated in lamp light and turned out a grey colour.

Silver silver chloride electrodes are sensitive to certain impurities in the electrolyte during their fabrication. Bromide is probably the most serious contaminant. As little as 0.1% bromide in the chloridizing electrolyte is sufficient to reduce electrode life and cause failure. Hence great care must be taken to see that this impurity is not present in the electrolyte.

#### 3.2.4 Aging of Ag - AgCl electrodes :-

Even with very careful preparation of the electrodes, it is found that Ag-AgCl electrodes are initially unstable and that intraelectrode potentials occur. Stabilization can be accomplished by placing two electrodes in an electrolyte solution and connecting a short circuit between them. Electrodes normally stabilize in 24 to 48 hrs. In the given case, two electrodes were placed in 0.01 NHCl acid, and short circuited for 24 hrs.

#### 3.2.5 Use of Ag-AgCl Electrodes:-

There are several considerations in the use of silver-silver chloride electrodes. In making the electrical connection to it, one must be careful to see that if soldering is done, the solder material should not come into

contact with the electrolyte in which the electrode is immersed. This is also true of the connecting wire, otherwise, contamination of the electrode may occur because of chemical reaction between the solder wire and the electrolyte.

Silver chloride is photosensitive (to ultraviolet light) and is so decomposed. It also produces a photovoltaic potential, generally, Ag-AgCl electrodes should be stored in the dark and used in subdued light. Excessive electrical noise produced by a given electrode may be indicative of light damage.

Silver - silver chloride electrodes require  $\text{Cl}^-$  ions for proper operation. When used <sup>with</sup> biological electrolytes, they have a sufficient supply of  $\text{Cl}^-$  ions available. If they are used as skin surface electrodes in such applications as ECG or EEG recording, it is necessary to use a wetting solution or paste that contains  $\text{Cl}^-$  ions. In the VCG, they are used with the Cambridge electrode jelly.

These electrodes are current limited because they are reversible electrodes. Sustained passage of high direct current results in either conversion of the electrode to pure silver (if used as a cathode) or conversion of all the silver to AgCl (if used as an anode). Generally, these electrodes are used for signal recording, as opposed to

stimulation, and operate into a high input impedance recording circuit. Electrode current is usually  $10^{-9}$  Amperes.

Because Ag - AgCl electrodes are thermodynamically reversible, they exhibit (after stabilization) low noise and theoretically zero electrode polarization effect. They do produce a steady electrode potential, however, which produces a d.c. offset in direct coupled systems. This requires compensation in the electronic circuitry used for signal processing. This situation may cause problems in the sensing of low level d.c. potentials. Because of the electrochemical nature of these electrodes, each one assumes an absolute potential. When two such electrodes are used as a sensing pair; d.c. potential difference exists (frequently of the order of a few mV). If this potential difference remains constant, any measurement of a bioelectric potential is unaffected, except for a steady baseline elevation. In the usual case, ~~however,~~ however, the resting potentials of the two electrodes change unequally with time and environmental temperature. This may result in objectionable baseline drift in experimental determinations leading to errors.

### 3.3 Signal Processing -

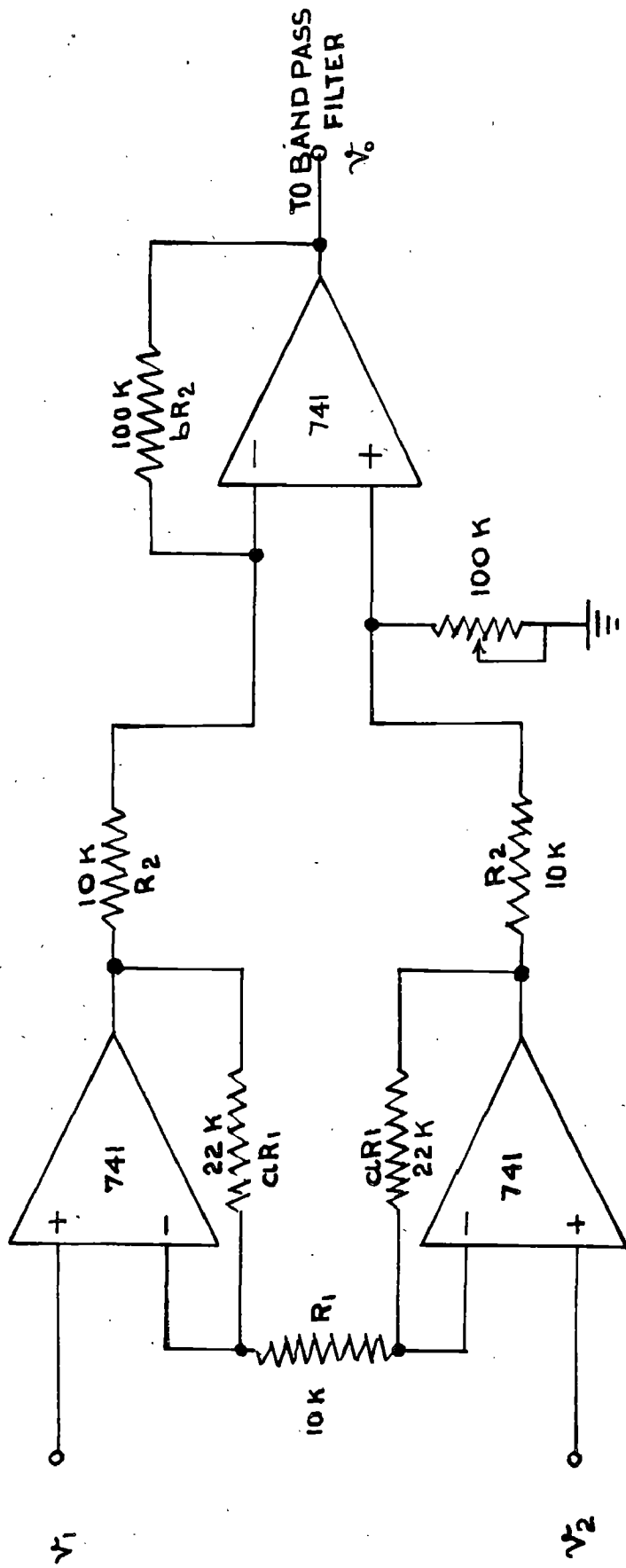
The signal picked by the seven electrodes placed on the subject's body is next fed to Frank's resistive network (fig. 2.5) to separate out the three components

of the cardiac potential. But as already mentioned the current through the electrodes should be  $10^{-9}$  Amp. Hence buffer amplifiers (unity gain, very high input impedance) are placed before the resistive network to avoid loading on electrodes and the cardiac generator. The amplitude of the signal obtained at the output of the Frank's resistive network is only of the order 1mV. However, the patient's body is capacitively coupled to the power lines and assumes a much larger 50 Hz potential. This potential is the same at all the electrodes. Hence the circuitry following must perform two functions:-

(i) amplification, (ii) common mode signal rejection.

Hence a differential amplifier <sup>(Fig. 3.3)</sup> is the next stage after the resistive network. It amplifies the differential signal picked up by the electrodes and rejects the common mode a.c. interference. This differential amplifier is fabricated from three operational amplifiers with the differential inputs given to the non-inverting terminals of two opamps, this greatly increases the input impedance of the amplifier.

The resistance of the skin varies in an unpredictable manner and may differ at different electrode locations. Thus even with a differential amplifier, unbalanced skin resistances can convert common mode voltage into a difference signal at the amplifier input. This objectionable signal can be reduced by increasing the input resistance and hence the differential amplifier should also have a very high input impedance of the order of 10 M or



**FIG. 3.3: DIFFERENTIAL AMPLIFIER**

(GAIN  $\approx 50$ ; INPUT IMPEDANCE  $> 10\text{ M}\Omega$ )



or greater. This can be illustrated mathematically (Appendix-A). A high input impedance also prevents loading of the electrodes which is very desirable.

The differential amplifier (fig.3.3) forms the first stage of the ECG amplifier and it must have low noise, since its output must be amplified through the remaining stages of the amplifier and any noise is amplified along with the signal. It must also be directly coupled to the electrodes (no series capacitors) to provide optimal low frequency response as well as to minimize charging effects on coupling capacitors from input offset current. Every attempt should be made to minimize this current, since even without coupling capacitors, it can polarize the electrodes resulting in polarizing overpotentials that produce a large d.c. offset voltage at the amplifiers input. It is for this reason that the differential amplifier must have a relatively low voltage gain. Since the offset potential is coupled directly to the input, it could saturate a high gain differential amplifier. The gain is hence kept at 50, given by the relation.

$$V_0 = (V_2 - V_1) (2a + 1)b \dots\dots\dots(3.1)$$

Let us now examine the effect of common mode voltage on the differential amplifier output. Suppose  $V_1 = V_2$ . Then both the negative inputs of the two operational amplifiers also assume this potential (considering an ideal case). This places the same voltage at both ends of

$R_1$  and hence current through it is zero. Also, no current can flow from the ideal operational amplifier inputs.

Hence current through both  $R_2$ 's is zero. So  $V_1$  appears at both op amps outputs and the common mode gain (CMG) is 1.

Now,

$$\text{CMRR} = \frac{\text{DG}}{\text{CMG}}$$

where,

CMRR = Common mode rejection ratio

DG = differential gain

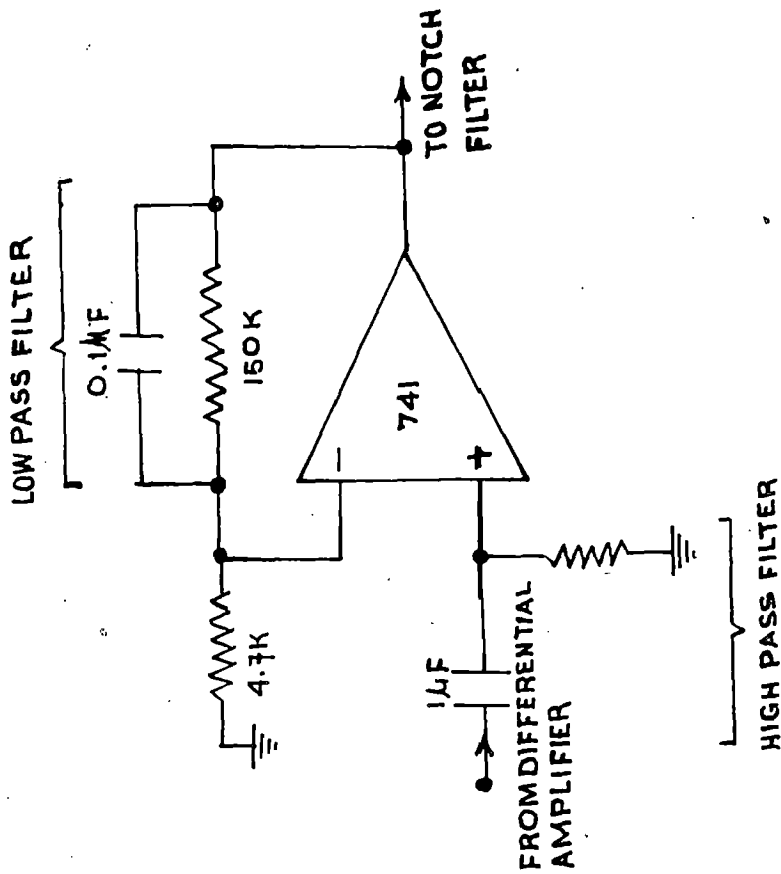
CMG = common mode gain

since CMG = 1 ,

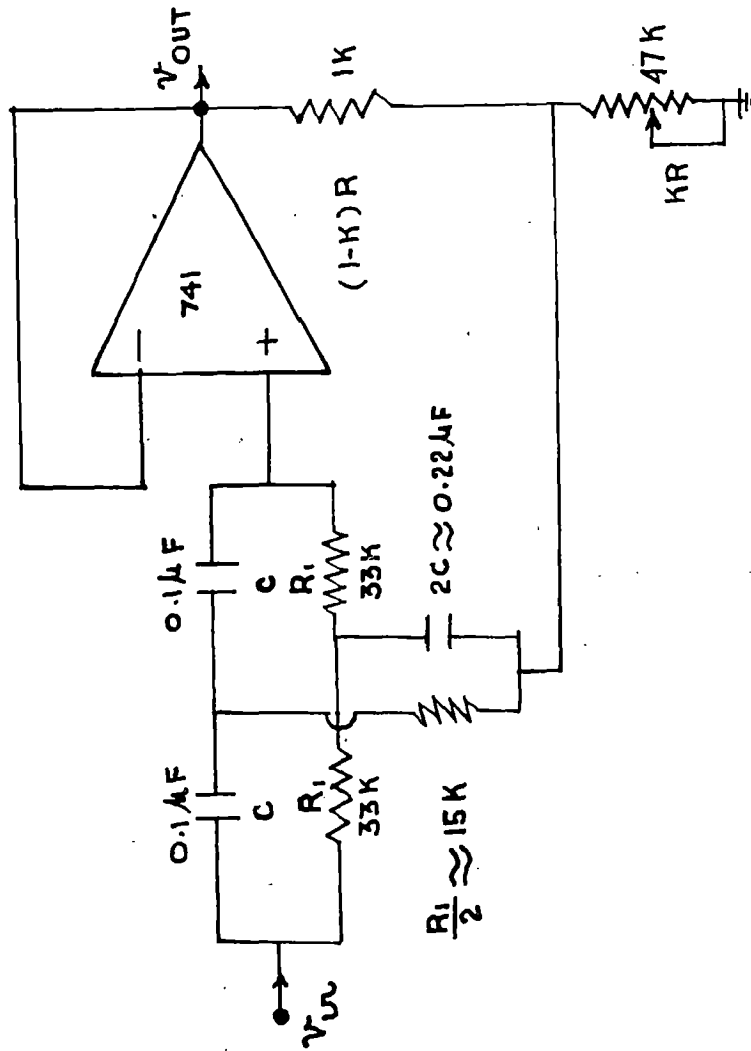
CMRR = DG

High CMRR is achieved by varying the potentiometer  $R_4$ .

The electrodes may produce an offset potential and this being coupled directly to the input, could cause saturation. To eliminate the saturation effect of the d.c. potential, the differential amplifier is capacitively coupled to the next stage which is the band pass filter (fig.3.4) ( $0.05 \text{ Hz}$  to  $100 \text{ Hz}$ ) with gain  $\approx 32$ . This band pass filter is used because the ECG signal has most of the information content in the range of  $0.05 \text{ Hz}$  to  $100 \text{ Hz}$ . It also blocks the high frequency noise present due to muscle artifacts. The d.c. blocking capacitor of  $\frac{1}{\wedge}$  micro farad together with the  $3.3\text{M}$  resistor forms the  $0.05 \text{ Hz}$  high pass filter, according to the following relation :-



**FIG. 3.4. BAND PASS FILTER**  
**0.05 TO 100Hz GAIN=32**



**FIG. 3.5. 50Hz TWIN-T NOTCH FILTER**  
**(UNITY GAIN) WITH POSITIVE FEEDBACK.**

$$\frac{1}{2\pi R_5 C_1} = f_{\text{CHP}} = \text{cut off freq. of high pass filter.}$$

or

$$\frac{1}{2\pi \times 3.3 \times 10^6 \times 1.0 \times 10^{-6}} = f_{\text{CHP}}$$

or

$$\frac{1}{6.6 \cdot \pi} = f_{\text{CHP}} = 0.05 \text{ Hz.}$$

The feedback consists of a 0.01 micro farad capacitor paralleled with 150 K resistance. This forms the 100 Hz low pass filter according to the following relation:-

$$\frac{1}{2\pi R_7 C_2} = f_{\text{CLP}} = \text{cut off freq. of low pass filter.}$$

or

$$\frac{1}{2\pi \times 150 \times 10^3 \times 0.01 \times 10^{-6}} = f_{\text{CLP}}$$

or

$$\frac{1}{30 \times 10^{-4} \times \pi} = f_{\text{CLP}} = 100 \text{ Hz.}$$

Gain is incorporated in the feed back by adding a resistance  $R_B = 4.7 \text{ k}$ , from 150 K resistance to ground. This makes the gain of the hand pass filter equal to,

$$\frac{150 \text{ K} + 4.7 \text{ K}}{4.7 \text{ K}} = 32$$

Hence the total gain is the circuit is  $50 \times 32 = 1600$ .

Even though the differential amplifier reduces the 50 Hz. common mode signal to nearly zero, after amplification in the band pass filter stage it may still be large enough to interfere with the ECG signal. Hence, a 50 Hz notch filter (fig. 3.5.) is included in the output stage of the

amplifier to reject this common mode signal. The twin-T notch filter using operational amplifier is used. Positive feedback is incorporated to improve the response of the notch filter.

R and C are arbitrarily chosen as 47 K and 0.01 micro farad.

K is defined as,

$$K = 1 - \frac{1}{4Q}, \text{ where } Q = \text{quality factor,}$$

$$F_o = 50 \text{ Hz}$$

$$BW = 5 \text{ Hz.}$$

$$\text{also, } \frac{f_o}{BW} = \frac{50 \text{ Hz.}}{5} = Q$$

$$= 10$$

$$\therefore K = 1 - \frac{1}{40} = \frac{39}{40}$$

$$\text{Hence } (1-K) R \approx 1 \text{ K}$$

$$\text{and } KR \approx 47 \text{ K.}$$

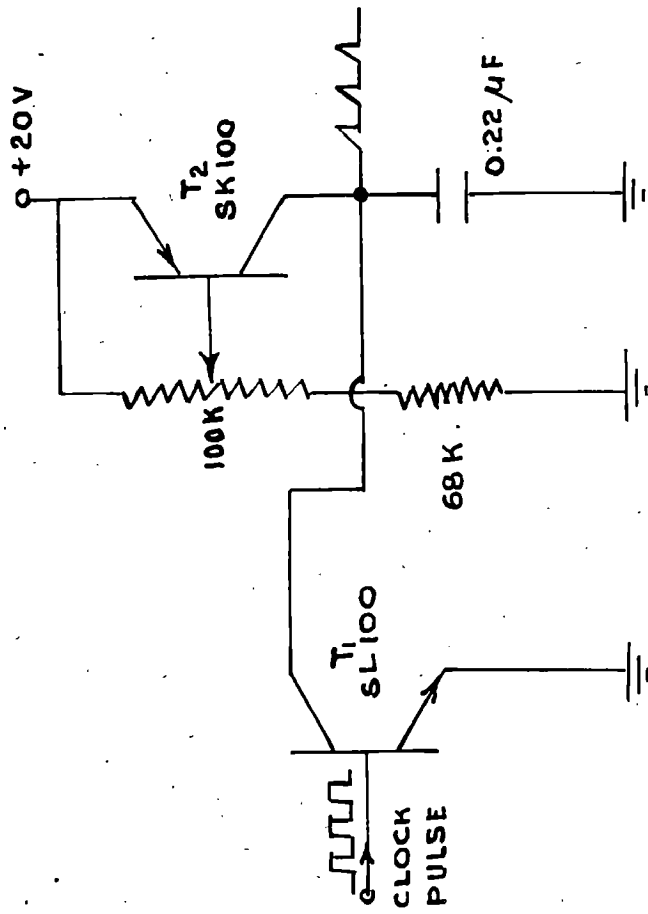
$$R_1 = \frac{1}{2\pi f_o C} = \frac{1}{2\pi \times 50 \times 0.1 \times 10^{-6}} \approx 33 \text{ K}$$

$$R_2 = R/2 \approx 15 \text{ K}$$

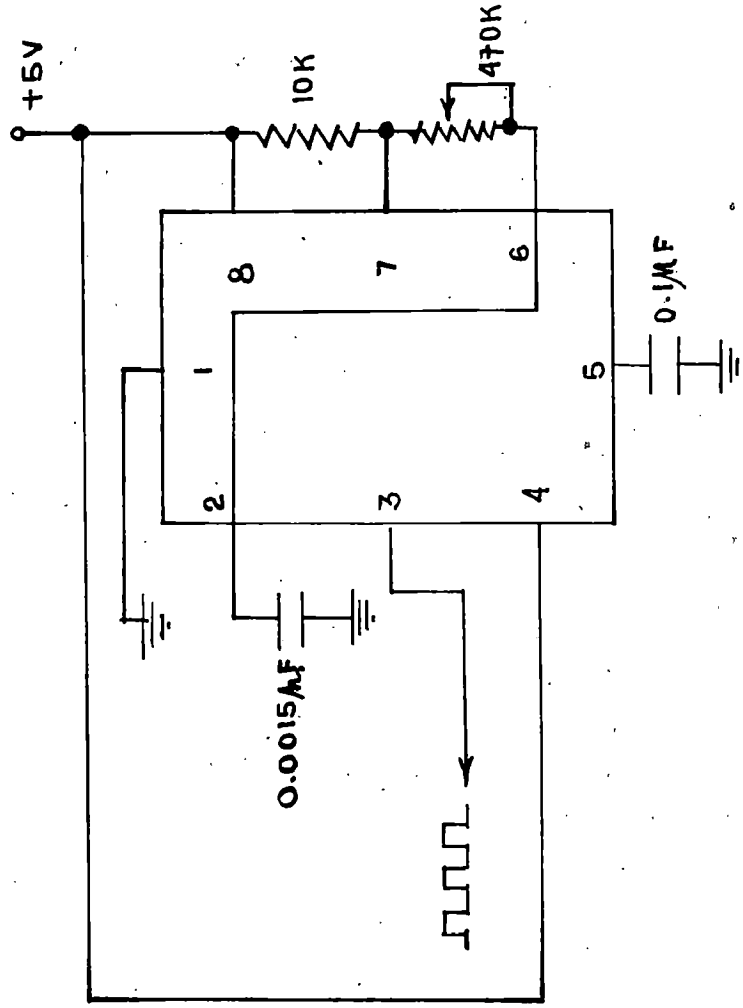
$$C_2 = 2C \approx 0.22 \text{ micro farad.}$$

As already mentioned before, to see the direction of rotation of the vectorcardiograph the Z axis of the cathode ray oscilloscope is fed with a saw-tooth waveform.

~~According to the cathode ray tube manual~~ The amplitude of the saw tooth waveform should be at least 20 volts, peak to peak. This saw tooth waveform is produced from the



**FIG. 3.6a: SAW TOOTH GENERATOR**



**FIG. 3.6b: ASTABLE MULTIVIBRATOR ( $f = 100 \text{ Hz}$ )**

two transistor circuit (fig.3.6a) in which one transistor ( $T_1$ ) works in the switching mode and the other ( $T_2$ ) in the active region. Base of  $T_1$  is fed with a clock pulse. The emitter of  $T_2$  is connected directly to the +20V supply and the base always gets a current drive. Hence this transistor is in the active region and the 0.22 micro farad capacitor charges upto 20V through the collector to emitter resistance. As soon as  $T_1$  gets a clock pulse, it goes ON and the collector is effectively grounded. The capacitor hence discharges to earth through the collector and emitter of  $T_1$ . A saw tooth waveform is therefore obtained across the 0.22 micro farad capacitor.

The square wave train (clock pulse) is obtained from a 555 timer using it in the astable mode (fig.3.5). The frequency of the saw tooth wave form should be some multiple of the frequency of the QRS complex, that is, about 100 Hertz.

$$\therefore 100 = \frac{1}{2 \pi (R_a + 2R_b) C}$$

$$R_a = 10 \text{ K}, R_b = 470 \text{ K.}$$

$$\text{or } 100 = \frac{1}{2 (10 + 470 \times 2) \times 10^3 \times C}$$

$$\text{Hence } C = 0.00168 \text{ micro farad}$$

$$\text{Hence } C \approx 0.0015 \text{ micro Farad.}$$

The slope of the saw tooth waveform is controlled from the 100 K present which changes the RC time constant and hence the charging time of the capacitor.

CHAPTER IVAPPLICATION OF VECTORCARDIOGRAPH

## 4.1 Introduction :

In the previous chapters a description of how the vectorcardiograph can be ~~derived~~<sup>derived</sup> is given. We have seen that the vectorcardiograph on all the three planes on the body can be displayed on the cathode ray oscilloscope. By looking at the vectorcardiogram of a patient and comparing it with patterns of healthy individuals certain cardiac disorders can be identified by the physician. The vectorcardiogram adds to the diagnostic accuracy of the electrocardiogram. It aids in differentiating between normal variants-right ventricular enlargement and right bundle branch block which cannot be done by looking at the electrocardiogram because both disorders reflect the same patterns in the precordial leads. The vectorcardiogram may prove to be more sensitive than the electrocardiogram in the early diagnosis of right and left ventricular enlargement and in the diagnosis of biventricular hypertrophy. It can help distinguish between ostium primum and ostium secundum defects. It is likewise helpful in the diagnosis of myocardial infarction. Patterns of myocardial infarction may be revealed more clearly in the vectrocardiogram than in the electrocardiogram. Moreover, when the electrocardiographic diagnosis of infarction is equivocal, the vectorcardiogram may establish the correct diagnosis. But all this is of interest to the doctor rather than to the Engineer.



In cardiac studies, an engineer is mainly concerned with the "forward" and "inverse" problems. The "forward problem" in vectrocardiography considers the generation of potential field within and on the torso resulting from the electrical activity of the heart. Specifically, it includes application of cardiac electrophysiology to the determination of a quantitative measure of the bioelectric generators and the determination of the current field set up in the torso due to these active sources, taking into account body inhomogeneities, anisotropy and shape. The availability of high speed computers to evaluate these potential fields has provided a powerful tool for the investigation of the forward problem.

The "inverse problem" relates to "determining something about the internal electrical generator, from measurement of potential difference made on the body surface". The vectorcardiograph can be applied to the solution of this inverse problem.

#### 4.2 The Inverse Problem :

The objective in vector cardiography is to learn something about the electrical, hence physiological, state of the heart from vectorcardiographic surface potential recordings. The forward problem aside from its intrinsic contribution to electrophysiology is important in electrocardiography since it permits the accumulation of evidence.

relating electrical events in the heart to the resultant surface potentials. Now, while knowledge of the bioelectric sources, body shape, and conductivities specifies uniquely the surface potentials, the reverse is not true. From a practical stand-point, it is not as yet completely clear how serious a limitation exists on the extent to which source information can be obtained from surface data. This is due to the fact that physiological constraints may greatly narrow the general theoretical uncertainties.

Even if it was possible to ascertain the electrical condition of each myocardial cell from surface potential measurements, the amount of data would be overwhelming, and for clinical purposes, certainly redundant. What is desirable is to define some finite number of parameters that represent the electrical condition of the heart. In general, the net dipole activity at a finite number of regions which subdivide the heart can be specified. For a subdivision of unity a great simplicity is achieved, but the description is relatively crude, and the final dipole representation has several shortcomings.

An electrocardiographic model can be formulated which assumes the primary source to be located in an inhomogeneous medium, or combined primary and secondary sources in a homogeneous medium. However, in the inverse problem, based solely on external measurements, it will be difficult to distinguish the two.

The surface potential measurements do not contain sufficient information to permit a determination of the distributed electrical sources in the heart. When the myocardial source is described as a uniform double layer, the scalar potential  $\phi_1(P)$ , over a surface  $S_1$  of volume  $V_1$  subtending a solid angle of  $\Omega_1$  at the source, of the cardiac vector is given by Plonsey (6) as,

$$\phi_1(P) = \frac{V_1 \Omega_1 \sigma_i}{4 \pi \sigma_o} \dots \dots \dots (4.1)$$

Where,  $\sigma_i$  = Conductivity in the internal part of the double layer and  $\sigma_o$  = external conductivity.

This shows that the potential depends only on the solid angle subtended by the field point and not on the shape. This example illustrates the fact that surface measurements record integrated effects produced by the sources and that there are basic difficulties in resolving the summation into its components.

Another problem that affects uniqueness in a practical way is that the intensity of the field falls off with distance from the source. Because of inevitable noise, signals that might be of importance in resolving two different distributions could be below the noise level. Thus, two sources may look alike from a practical standpoint given the ambiguity in measurement. Multipole theory gives some insight into this problem because it shows that

higher order source components involve higher inverse powers of 'r' (distance of source from field point) and, in general, have decreasing contributions to the total surface potential.

In addition to the fixed dipole, multiple dipole and multipole cardiac representations, other formulations have been proposed. One is the moving dipole- in this model the dipole location is permitted to shift so that some reflection of the activation region may occur; but in this model, no unique 'physical' representation of the significance of the shifting origin can be given theoretically.

#### 4.3 Review of mathematical models :

One analysis of the field due to electrical activity of heart has been done by Mukhopadhyay and Sharma (7), assuming the body to be homogeneous and isotropic and assigning different geometries like the sphere, circular cylinder and elliptical cylinder to it. To describe the generator, it is assumed that the current produced by each cell is small compared to the total current produced by the heart. Moreover, no net charge can build up and the net current generated by a cell is zero. It is the initial current generated in the cell that provides the first order electric field at a distance from the cell. Therefore, the current produced by each cell unit can be properly

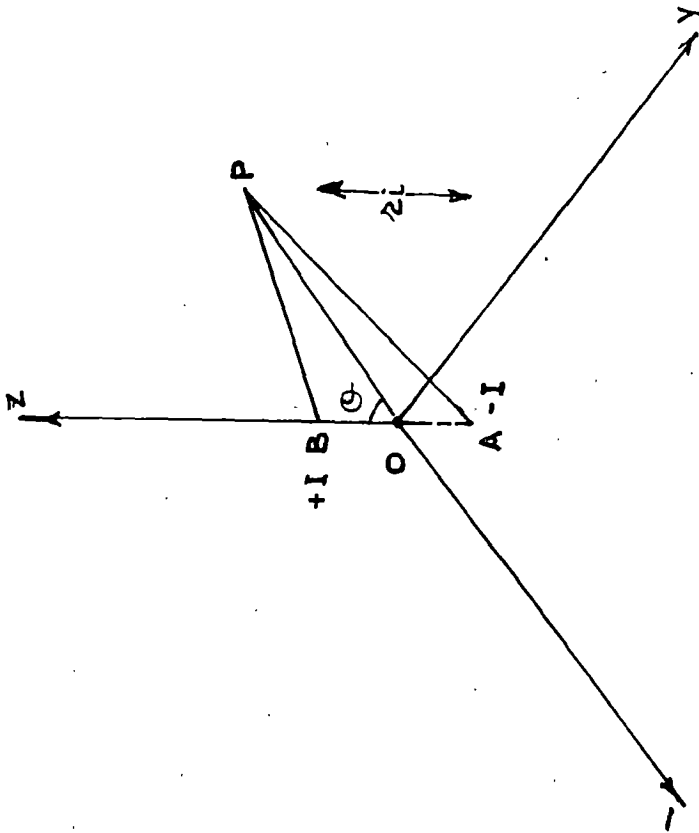


FIG. 4.1a. DIPOLE SOURCE AT THE CENTRE.

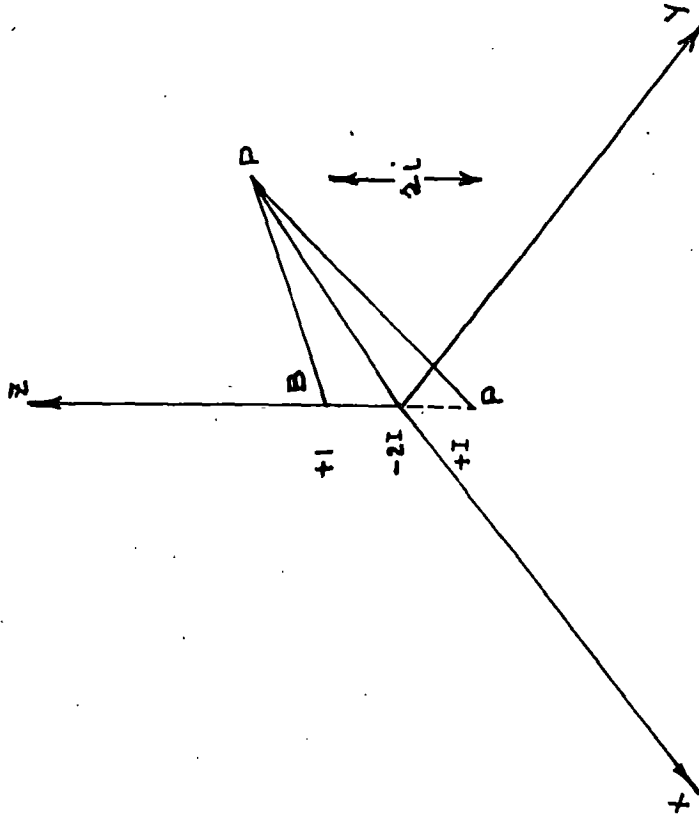


FIG. 4.1b. STRETCHED QUADRUPLE SOURCE AT THE CENTRE

designated by a current dipole moment. Since these dipoles are the only important current generators in the heart, the heart can be fully represented by a distribution of current dipole moments, each identified with the current produced by a cell. A dipolar representation carries a large part of all the information inherent in the ECG and VCG. So the electrical activity is almost adequately represented by a single dipole located anatomically at some fixed part.

#### 4.3.1 BODY TAKEN AS A SPHERE

##### 4.3.1.1 A dipole source at the centre -

The Laplace's equation is given as ,

$$\frac{\partial^2 V}{\partial r^2} + \frac{2}{r} \frac{\partial V}{\partial r} = 0 \quad \dots\dots (4.2),$$

(Where V is the scalar potential at a point in the cardiac field.)

Since, due to symmetry, V is independent of the other two coordinates  $\theta$  and  $z$ .

In case of point source, I,

$$V = \frac{I \rho}{r} \quad \dots\dots (4.3)$$

Extension to a dipole field may be carried out by assuming sources +I and -I located at distances  $\frac{r_1}{2}$  and  $\frac{-r_1}{2}$ . In fig. 4.1a,  $OA = OB = \frac{r_1}{2}$ .

Using eqn. (4.3), potential at P is obtained as,

$$V = I_p \left[ \frac{1}{\frac{r-r_i}{2}} - \frac{1}{\frac{r+r_i}{2}} \right] \dots\dots(4.4)$$

Taking  $r_i \ll r$ ,  $\frac{r_i}{r} \ll 1$ , squares and higher order terms can be neglected. And putting  $I \cdot r_i = p$ , dipole moment,

$$V = \frac{p \cos \theta}{r^2} \dots\dots(4.5)$$

#### 4.3.1.2: Stretched Quadrupole source at centre :

The sources  $+I$ ,  $-2I$  and  $+I$  are assumed at A, O and B in fig. 4.1b.

The potential at P is,

$$V = I_p \left\{ \frac{1}{AP} + \frac{1}{BP} - \frac{2}{OP} \right\} = \frac{I r_i}{4r^3} (3 \cos^2 \theta - 1) \dots (4.6)$$

#### 4.3.2 BODY TAKEN AS A CIRCULAR CYLINDER:

Source at the Axis:

In this case, the body is assumed to be a circular cylinder, an improvement in the geometry over spherical one. One single source is assumed at first at  $r=0$ ,  $z=0$ , in  $r, z$  coordinates. The potential satisfies the Laplace's equation,

In a point source located at  $r = 0$ . the potential  $V$  is independent of  $\theta$  and hence,

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0 \dots\dots(4.7)$$

Let  $\xi = (r^2 + z^2)^{1/2}$ , where  $\xi$  is the distance of a point from the origin.

$$\text{Now, } \frac{\partial V}{\partial r} = \frac{r}{(r^2 + z^2)^{1/2}} \frac{\partial V}{\partial \xi}$$

$$\text{or } \frac{1}{r} \frac{\partial V}{\partial r} = \frac{1}{\xi} \frac{\partial V}{\partial \xi}$$

$$\frac{\partial^2 V}{\partial r^2} = \frac{r^2}{\xi^2} \cdot \frac{\partial^2 V}{\partial \xi^2} + \frac{z^2}{\xi^3} \cdot \frac{\partial V}{\partial \xi}$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{z^2}{\xi^2} \frac{\partial^2 V}{\partial \xi^2} + \frac{r^2}{\xi^3} \frac{\partial V}{\partial \xi}$$

$$\text{Thus, } \frac{\partial^2 V}{\partial \xi^2} + \frac{2}{\xi} \frac{\partial V}{\partial \xi} = 0 \quad \dots\dots (4.8)$$

This eqn is identical to eqn (4.2) and hence the solution is written as,  $V = \frac{I \rho}{\xi} = \frac{I \rho}{(r^2 + z^2)^{1/2}}$

In a dipole field, sources are assumed to be placed symmetrically with respect to the centre as shown in fig.4.10.

The potential at point P where coordinates are  $(r, z)$  may be calculated as follows :-

$$\begin{aligned} V &= I \rho \cdot \left( \frac{1}{AP} - \frac{1}{BP} \right) \\ &= I \rho \left[ \frac{1}{\left( \left( r - \frac{r_1}{2} \right)^2 + z^2 \right)^{1/2}} - \frac{1}{\left( \left( r + \frac{r_1}{2} \right)^2 + z^2 \right)^{1/2}} \right] \end{aligned}$$

Expansion of the above expression binomially yields, after usual assumptions,

$$V = \frac{I \rho r_1 \cos \theta}{(r^2 + z^2)^{1/2}} \quad \dots\dots\dots (4.9)$$



#### 4.4 Present Mathematical model :

In this section a system for analytical vector-cardiography is proposed. As we have already noted in the previous section that to find a unique solution to the **inverse** vectorcardiography problem some basic aspects have to be defined. They are :-

- (1) Selection of a model of the heart as a generator of electrical current.
- (2) selection of a model of the body as a conductor of electrical current.
- (3) a method for expressing body-surface potential difference data, measured on a particular individual, in terms of free parameters of the model (the inverse problem).

The cardiac generator is viewed as a dipole in this model and consists of a source of strength  $+I$  and a sink of strength  $-I$ , length of the dipole is  $r_1$  and this is much smaller than the distance  $r$  from the origin at which the potential is measured.

Laplace's eqn in spherical coordinates is used to calculate the potential. A time independent field is assumed since the frequency of the cardiac generator is very low. The body is viewed as a sphere and assumed to be homogeneous and isotropic. The dipole source is assumed to be centered at the origin, but is not parallel to any of the axes.

Laplace's time - independent equation in spherical coordinates is given as ,

$$\frac{\partial^2 V}{\partial r^2} + \frac{2}{r} \frac{\partial V}{\partial r} + \frac{\cot \theta}{r^2} \frac{\partial V}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2 V}{\partial \theta^2} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} = 0$$

The source (+I) is placed at  $(\frac{r_1}{2}, \alpha, \beta)$  and sink (-I) is at  $(\frac{r_1}{2}, \alpha + \pi, \beta)$  the point opposite to the source.

From eqn. (4.2), we know that potential at a point distant 'r' from the origin due to a generator of strength I is,

$$V = \frac{I \rho}{r} \quad , \quad \rho = \text{resistivity of medium}$$

∴ Potential at the point P ( r,  $\theta$ ,  $\phi$  ) due to the dipole is given as,

$$V_p = I \rho \left( \frac{1}{\text{distance of P from +I}} - \frac{1}{\text{distance of P from -I}} \right)$$

Calculating the distances of the field point (r,  $\theta$ ,  $\phi$  ) from the source and the sink, the potential is shown (Appendix B ) to be,

$$V_p = \frac{I \rho r_1}{r^2} (\sin \theta \sin \alpha \cos(\phi - \beta) + \cos \theta \cos \alpha) \quad \dots (4.10)$$

This potential function satisfies the Laplace's equation (appendix B). But the expression was derived considering an infinitely extended conducting medium, whereas actually the surface is bounded. To include this boundary effect, the expression is modified to the following:-

$$V = \frac{K}{r^2} (\sin \theta \sin \alpha \cos (\phi - \beta) + \cos \theta \cos \alpha) + f(r) \cdot g(\theta, \phi) \quad \dots (4.11)$$

Where  $f(r)$  is a function of  $r$ ,  $g(\theta, \phi)$  is a function of  $\theta$  and  $\phi$  and  $K = \int \rho r_1$ .

At the interface of the body and air, there is no net radial electric field.

Hence at  $r = R$ , that is, at the surface of body.

$$\left. \frac{\partial V}{\partial r} \right|_{r=R} = 0$$

$$\text{or } \left. \frac{\partial V}{\partial r} \right|_{r=R} = \frac{-K}{R^3} \left[ \sin \theta \sin \alpha \cos (\phi - \beta) + \cos \theta \cos \alpha + f^1(r) \right]_{r=R} \cdot g(\theta, \phi) = 0$$

$$\text{where, } f^1(r) = \frac{\partial}{\partial r} f(r)$$

Since this equation is valid for all values of  $\theta$  and  $\phi$ , we have  $g(\theta, \phi) = (\sin \theta \sin \alpha \cos (\phi - \beta) + \cos \theta \cos \alpha)$

$$\therefore \left. f^1(r) \right|_{r=R} = \frac{2K}{R^3}$$

Choosing the simplest form of  $f(r)$  satisfying the above equation,

$$\left. f(r) \right|_{r=R} = \frac{2KR}{R^3}$$

∴ at  $r=R$ , that is, on the surface of the body,

$$V = \frac{3K}{R^2} (\sin \theta \sin \alpha \cos(\theta - \beta) + \cos \theta \cos \alpha)$$

Let  $V_x, V_y, V_z$  be the three components of  $V$  along the three axes.

On  $x$  axes,  $\theta = \frac{\pi}{2}, \quad \phi = 0$

On  $y$  axes,  $\theta = \frac{\pi}{2}, \quad \phi = \frac{\pi}{2}$ ,

on  $z$  axes,  $\theta = 0$

Hence,  $V_x = \frac{3K}{R^2} (\sin \frac{\pi}{2} \sin \alpha \cos(\frac{\pi}{2} - \beta) + \cos \frac{\pi}{2} \cos \alpha)$

$$= \frac{3K}{R^2} \sin \alpha \cos \beta \dots (4.12)$$

$$V_y = \frac{3K}{R^2} (\sin \frac{\pi}{2} \sin \alpha \cos(\frac{\pi}{2} - \beta) + \cos \frac{\pi}{2} \cos \alpha)$$

$$= \frac{3K}{R^2} \sin \alpha \sin \beta \dots (4.13)$$

$$V_z = \frac{3K}{R^2} \cos \alpha \dots (4.14)$$

From (4.12) and (4.13),

$$\frac{V_y}{V_x} = \tan \beta,$$

From the VCG,  $V_y$  and  $V_x$  are known hence  $\beta$  can be found from (4.13) and (4.14),

$$\frac{V_y}{V_z} = \tan \alpha \sin \beta = \frac{V_y}{\sqrt{V_y^2 + V_x^2}} (\tan \alpha)$$

$$\therefore \tan \alpha = \frac{\sqrt{V_y^2 + V_x^2}}{V_z}$$

substituting these values of  $\alpha$  and  $\beta$  in (4.13), the value of  $\frac{3K}{R^2}$  can be found also.

#### 4.5 Derivation of the scalar ECG from the VCG :

The standard and unipolar limb leads can be derived with reasonable accuracy from the frontal plane vectorcardiogram by use of the hexaxial reference diagram devised by Langer (Appendix C).

The precordial leads can similarly be derived from the transverse plane vectorcardiogram by use of the reference diagram suggested by Abildskov and Wilkinson (Appendix C).

CHAPTER - VC O N C L U S I O N

The design, fabrication and application of vectorcardiograph has been discussed in this dissertation.

The electrical activity of heart can be represented by a dipole consisting of a current source and a current sink. The thorax represents the load on this cardiac generator. For the analysis of the electrical activity of heart several geometries can be assigned to the thorax, and the cardiac generator represented by configurations other than the dipole, such as multiple dipole or moving dipole etc. One analysis of the cardiac generator has been carried out assuming a dipole source which is centred at the origin but not aligned with any of the axes. The thorax is considered to be a spherical conductor. Expressions are derived relating the orthogonal components of the scalar potential on the body's surface to the cardiac dipole moment and the angles which the dipole makes with the axes. These can be used to solve the "inverse problem" in vectorcardiography which is what an engineer is mainly concerned with.

Frank's resistive network is used to derive the three orthogonal leads. This system is used in favour of Schmitt's SVEC III resistive network mainly because of its simplicity in comparison of the two.

Silver-silver chloride electrodes are used to pick up the VCG signal from the subject's body. These electrodes have been fabricated in the laboratory by chloridizing high purity silver. The electrodes after stabilization gave a good performance with no noise.

To minimize 50 Hz power line interference with the ECG signal, a 50 Hz Twin -T notch filter has been incorporated in the ECG amplifier circuitry to reject noise. A ground loop has also been provided in the amplifier to minimize interference, the lead wires are twisted together and an electrode placed on the right leg and grounded.

A few experiments were performed with the help of the developed vectorcardiograph and it was found that the QRS loop, which is of main interest, was well defined in all the three planes and the P and T loops were slightly hazy, perhaps due to the presence of some noise. The major piece of information that is presented in the VCG is the direction of depolarization and repolarization.

The introduction of sagittal plane for complete projection of all vectors of cardiac excitation is one of the most important advantages of this corrected orthogonal lead system. Vectors that run ventrally or dorsally in the heart are particularly well projected on the sagittal plane. This is of advantage in the diagnosis of infarctions

affecting the posterior septum of heart.

Scope for further work :-

The work presented can be extended further on the following lines :-

1. Schmitt's SVEC III orthogonal lead system can be used to derive the VCG loops in the three planes and a comparison made with the loops derived from Frank's system.
2. Use of Fourier series can be made for approximate analysis of the VCG loop. Frequency analysis of the VCG loop can be done and the coefficients of the Fourier series which are significant for some diseases can be determined.
3. By recording the VCG's of a large number of subjects representing a cross-section of various age groups, sex, etc. parameters such as Beat-to-Beat variation (BBV), Day-to-Day variation (DDV), Constitutional (weight, height, chest configurations, race) variation, Year-to-year variations etc. can be studied. Computer can be used to compute the data for this purpose.



REFERENCES

1. HORAN, L.C., FLOWERS N.C., The relationship between the vectorcardiogram and actual dipole moment, in the theoretical basis of Electrocardiology. (eds: Nelson, C.V., Geselowitz, D.B.) Clarendon Press, pp:397-412, 1976.
2. FRIEDMAN, H.H., Diagnostic electrocardiography and vectorcardiography, Mcgraw Hill, 1971.
3. GEDDES, L.A., BAKER, L.E. Principles of Applied Biomedical Instrumentation, John Wiley and sons, Inc., 1968.
4. FERRIS, CLIFFORD, D., Introduction to Bioelectrodes, Plenum press, 1974.
5. CAMERON, JOHN R., SKOFRONICK, JAMES G., Medical Physics John Wiley and sons, 1978.
6. FLONSEY, ROBERT, FLEMING, DAVID G., Bioelectric Phenomena, Mcgraw Hill, 1969.
7. MUKHOPADHYAY, P., SHARMA, B., Analysis of Electro-magnetic field due to Cardiac Generator, Journal of the Institution of Engineers, Vol.54, IDGE 1, 33-38.

8. CHOWDHURY, J.K., RAKSHIT, A., MAJUMDAR, S.,  
Vectorial Detection of Abnormalities in  
Electrocardiographic recording, Journal of the  
Institution of Engineers, Vol. 65, IDGE 2, 52-55.
9. WEBSTER, JOHN G., Ed., Medical Instrumentation,  
Application and Design, Houghton Mifflin Company, 1978.

APPENDIX - ADIFFERENTIAL AMPLIFIER

- A.1 Use of a differential amplifier to eliminate common mode 50 Hz signal :

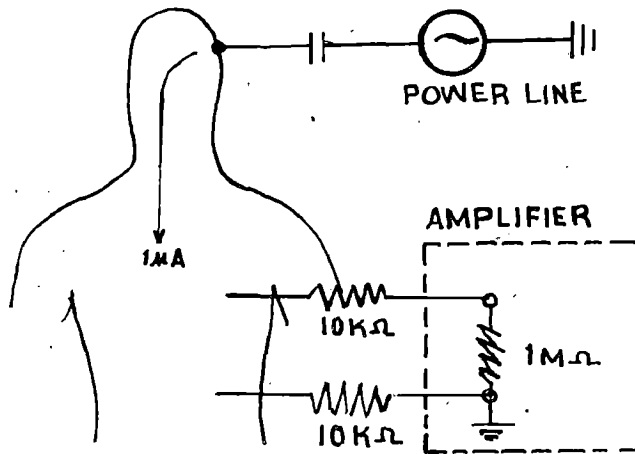
The amplitude of the typical ECG signal is only about 1mV. However, in a typical building the current capacitively coupled into the body from the power lines can produce a much larger potential (fig. A.1). The amplifier used to record the ECG or VCG must be able to eliminate interference from voltages induced in the body from such external sources.

To avoid this interference voltage, we may use the differential amplifier, which measures the difference in the two signals appearing at A and B (fig.A-2) and is insensitive to any signal that is common to A and B.

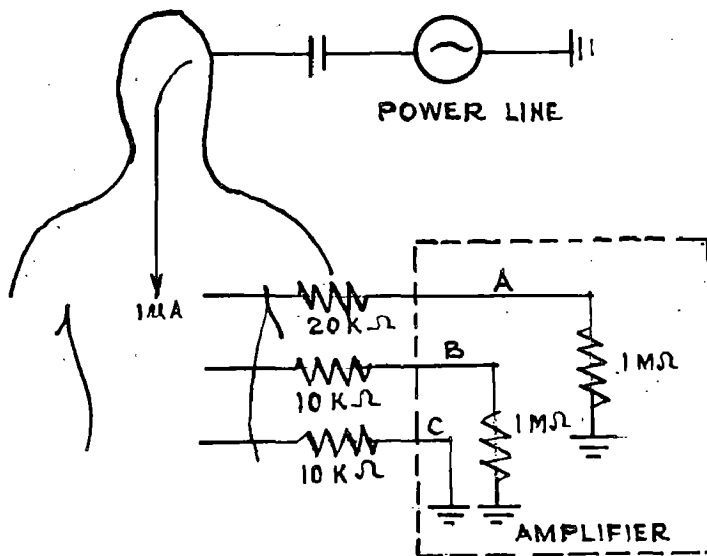
- A.2 High Input Impedance of differential amplifier to minimize interference:

The resistance of the skin is unequal at different electrode locations (fig.A.2). Thus  $V_A$  the interference voltage at point A, is determined by considering the 20K skin resistance and 1M input impedance as a simple voltage divider.

$$V_A = 10mV \left( \frac{1M}{1M + 20K} \right) = 10mV(0.99) = 9.9 mV.$$



**FIG. A.1** WHEN THE ECG IS MEASURED WITH A SINGLE ENDED OSCILLOSCOPE THERE IS MUCH INTERFERENCE FROM THE 120V POWER LINE THAT CAPACITIVELY COUPLES ABOUT 1μA INTO THE BODY. THIS CURRENT SEEKS THE PATH OF LEAST RESISTANCE TO GROUND THROUGH B TO PRODUCE AN EXTRANEOUS SIGNAL ABOUT 10 TIMES LARGER THAN THE ECG SIGNAL.



**FIG. A.2** USE OF A DIFFERENTIAL AMPLIFIER CAN MINIMIZE THE INTERFERENCE OF CAPACITIVELY COUPLED FROM THE POWER LINES. THE 1μA CURRENT FLOWS THROUGH A SEPARATE GROUND RESISTANCE (C). HOWEVER, AN UNBALANCE IN THE SKIN AND ELECTRODE RESISTANCES TO A AND B WILL CAUSE COMMON MODE VOLTAGE ON THE BODY TO BE CONVERTED TO A DIFFERENTIAL SIGNAL IN THE AMPLIFIER.

Similarly,

$$V_B = 10\text{mV} \left( \frac{1\text{M}}{1\text{M} + 10\text{K}} \right) = 10\text{mV} \cdot (0.99) = 9.9\text{mV}.$$

The differential amplifier measures the difference between these signals.

$$V_B - V_A = 9.9\text{ mV} - 9.8\text{ mV} = 0.1\text{ mV}$$

Thus, even with a differential amplifier, unbalanced skin resistances can convert common mode voltage into a difference signal at the amplifier input. This 0.1 mV interference is 10% of the ECG signal and is objectionable, if the input impedance is increased to 10M, the interference is reduced to about 1%, which is an improvement.

APPENDIX - B

Derivation of an expression for the scalar potential  $V$  at  $(r, \theta, \phi)$  from Laplace's equation :

Potential  $V$  at  $(r, \theta, \phi)$  due to a dipole with source at  $(\frac{ri}{2}, \alpha, \beta)$  and sink at  $(\frac{ri}{2}, \alpha + \pi, \beta)$  is given as,

$$V = I \int \left( \frac{1}{\text{distance of P from } +I} - \frac{1}{\text{distance of P from } -I} \right)$$

where P is a field point  $(x, y, z)$  or  $(r, \theta, \phi)$ . If source and sink are at  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  respectively,

$$V = I \int \left[ \frac{1}{\sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}} + \frac{1}{\sqrt{(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2}} \right]$$

$$= I \int \left[ \frac{1}{\sqrt{\left( r \sin \theta \cos \phi - \frac{ri}{2} \sin \alpha \cos \beta \right)^2 + \left( r \sin \theta \sin \phi - \frac{ri}{2} \sin \alpha \sin \beta \right)^2 + \left( r \cos \theta - \frac{ri}{2} \cos \alpha \right)^2}} + \frac{1}{\sqrt{\left( r \sin \theta \cos \phi - \frac{ri}{2} \sin(\alpha + \pi) \cos \beta \right)^2 + \left( r \sin \theta \sin \phi - \frac{ri}{2} \sin(\alpha + \pi) \sin \beta \right)^2 + \left( r \cos \theta - \frac{ri}{2} \cos(\alpha + \pi) \right)^2}} \right]$$

$$= I \rho \frac{1}{\sqrt{r^2 - rr_1 \sin \theta \cos \varphi \sin \alpha \cos \beta - rr_1 \sin \theta \sin \varphi \sin \alpha \sin \beta - rr_1 \cos \theta \cos \alpha}}$$

$$- \frac{1}{\sqrt{r^2 + rr_1 \sin \theta \cos \varphi \sin \alpha \cos \beta + rr_1 \sin \theta \sin \varphi \sin \alpha \sin \beta + rr_1 \cos \theta \cos \alpha}}$$

neglecting squares and higher order terms since they are small,

$$V = \frac{\rho I}{r} \left( 1 - \frac{r_1}{r} \left( \sin \theta \sin \alpha \cos (\varphi - \beta) + \cos \theta \cos \alpha \right) \right)$$

$$- \frac{\rho I}{r} \left( 1 + \frac{r_1}{r} \left( \sin \theta \sin \alpha \cos (\varphi - \beta) + \cos \theta \cos \alpha \right) \right)$$

$$= \frac{\rho I}{r} \frac{r_1}{r} \left( \sin \theta \sin \alpha \cos (\varphi - \beta) + \cos \theta \cos \alpha \right)$$

$$= \frac{K}{r^2} \left( \sin \theta \sin \alpha \cos (\varphi - \beta) + \cos \theta \cos \alpha \right) \dots (B.1)$$

$$\frac{\partial V}{\partial r} = \frac{-K r_1}{r^3} \left( \sin \theta \sin \alpha \cos (\varphi - \beta) + \cos \theta \cos \alpha \right)$$

$$\frac{2}{r} \frac{\partial V}{\partial r} = \frac{-4K}{r^4} \left( \sin \theta \sin \alpha \cos (\varphi - \beta) + \cos \theta \cos \alpha \right) \dots (B.2)$$

$$\frac{\partial^2 V}{\partial r^2} = \frac{6K}{r^4} \left( \sin \theta \sin \alpha \cos (\varphi - \beta) + \cos \theta \cos \alpha \right) \dots (B.3)$$

$$\frac{\partial V}{\partial \theta} = \frac{K}{r^2} \left( \cos \theta \sin \alpha \cos (\varphi - \beta) - \sin \theta \cos \alpha \right)$$

$$\frac{1}{r^2} \cot \theta \frac{\partial V}{\partial \theta} = \frac{K}{r^4} \left( \frac{\cos^2 \theta}{\sin \theta} \sin \alpha \cos (\varphi - \beta) - \cos \theta \cos \alpha \right) \dots (B.4)$$

$$\frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} = -\frac{K}{r^4} \sin \theta \sin \alpha \cos (\phi - \beta) - \cos \theta \cos \alpha \dots \dots (B.5)$$

$$\frac{\partial^2 v}{\partial \phi^2} = -\frac{K}{r^2} \sin \theta \sin \alpha \cos (\phi - \beta)$$

$$\frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v}{\partial \phi^2} = -\frac{K}{r^4} \frac{\sin \alpha \cos (\phi - \beta)}{\sin \theta} \dots \dots (B.6)$$

Adding equations (B.2) to B.6), we get zero.

Hence the scalar potential satisfies Laplace's equation.

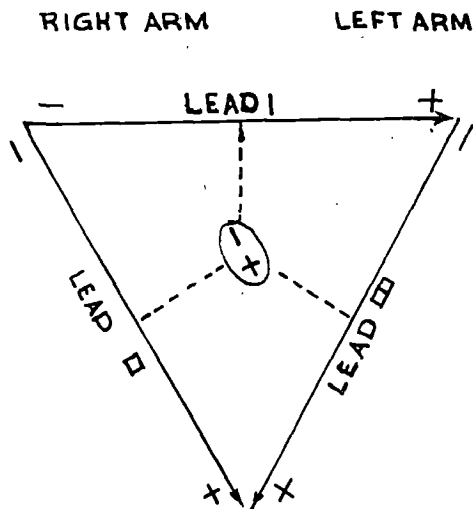


APPENDIX - CDERIVATION OF SCALAR ECG FROM THE VCG

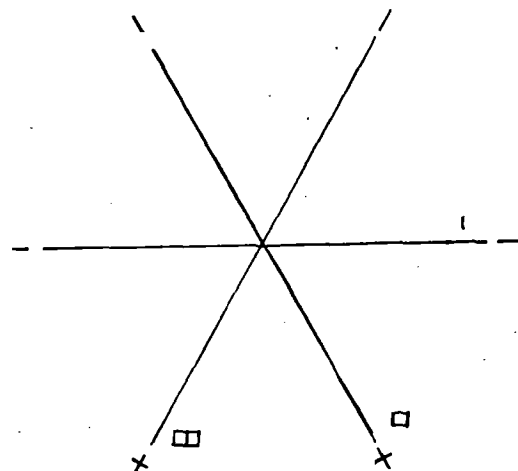
By making some simplifying assumptions Einthoven gave the hexaxial reference diagram (fig. C-1). But Burger and Van milan, by actual measurements showed that the Einthoven triangle is not equilateral as proposed by him but scalene and does not lie entirely in the frontal plane. Moreover, they demonstrated that the scalar voltage recorded by the standard leads is dependant not only on the projection of the cardiac vector on the lead axes but also on the length of the leads. Basing it on the Burger triangle, Langner devised a corrected hexaxial reference diagram.

The standard and unipolar limb leads can be derived by superimposing the frontal plane VCG loop on this hexaxial reference diagram (Fig.C-2)

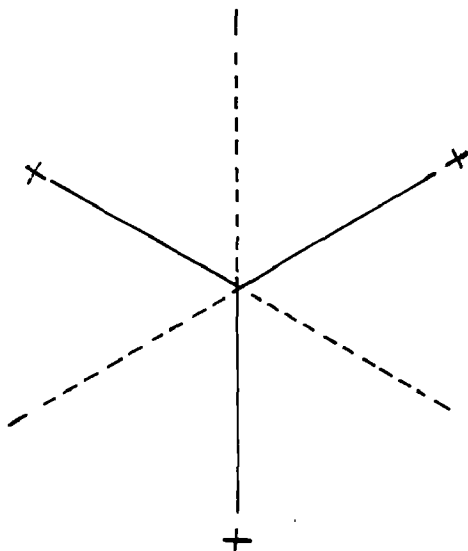
The standard precordial ECG consists of six unipolar leads,  $V_1$  through  $V_6$ . As with the standard and unipolar extremity leads, a reference diagram may be constructed for the axes of the precordial leads (fig.C.3) whereas the axes of the limb leads are located in the frontal plane, those of the precordial leads are located in the horizontal (transverse) plane. The axes of each unipolar precordial lead is on a line extending from the location of the exploring electrode to the dipole centre



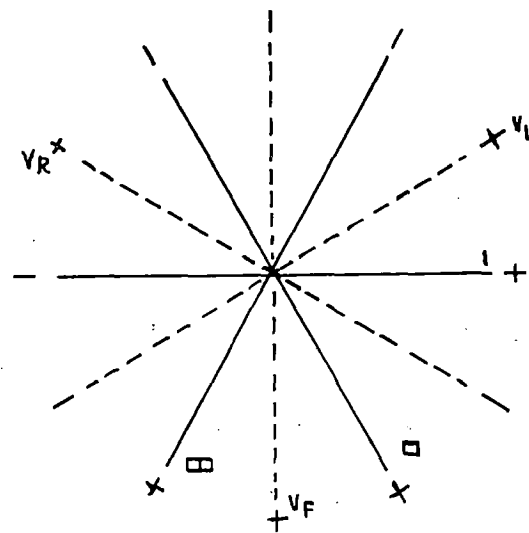
C.I.A. THE EINTHOVEN EQUILATERAL TRIANGLE SHOWING THE AXES OF THE STANDARD LIMB LEADS.



C.I.B. THE AXES OF THE LIMB LEADS MOVED TO THE CENTER OF THE TRIANGLE, FORMING A TRIAXIAL REFERENCE DIAGRAM.

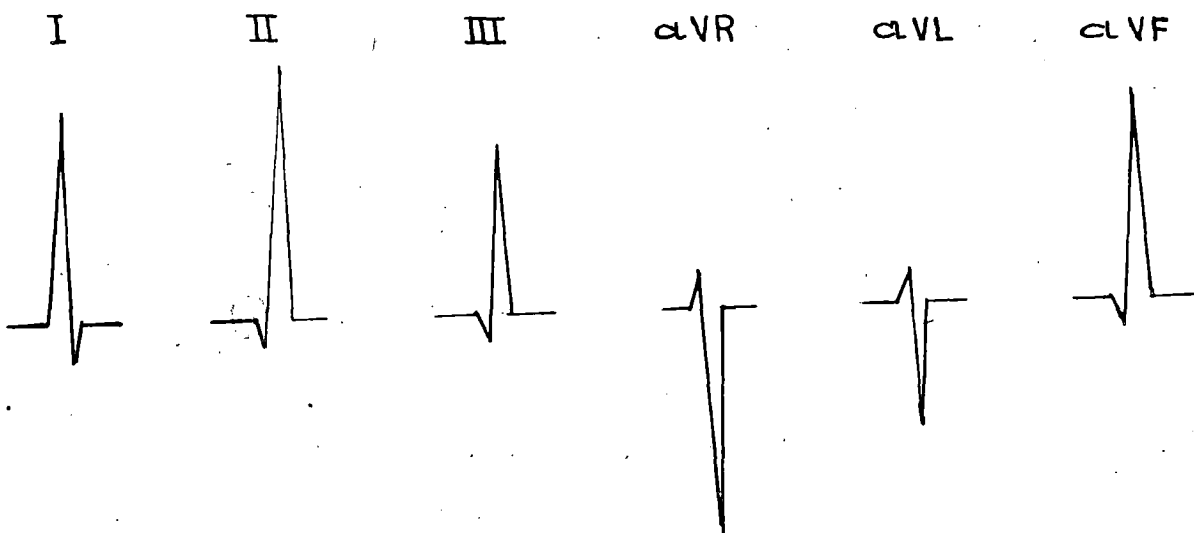
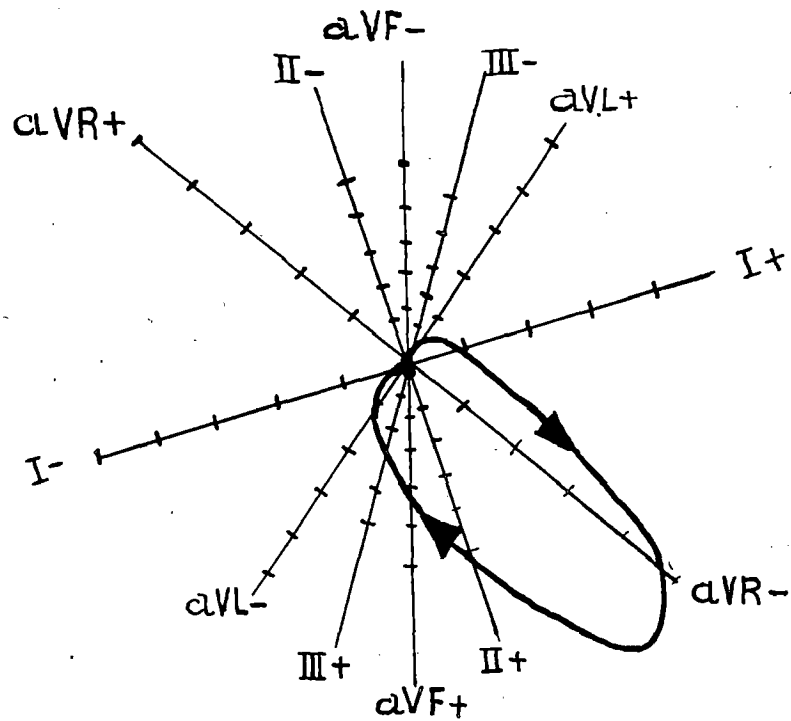


C.I.C. THE AXES OF THE UNIPOLAR LIMB LEADS.

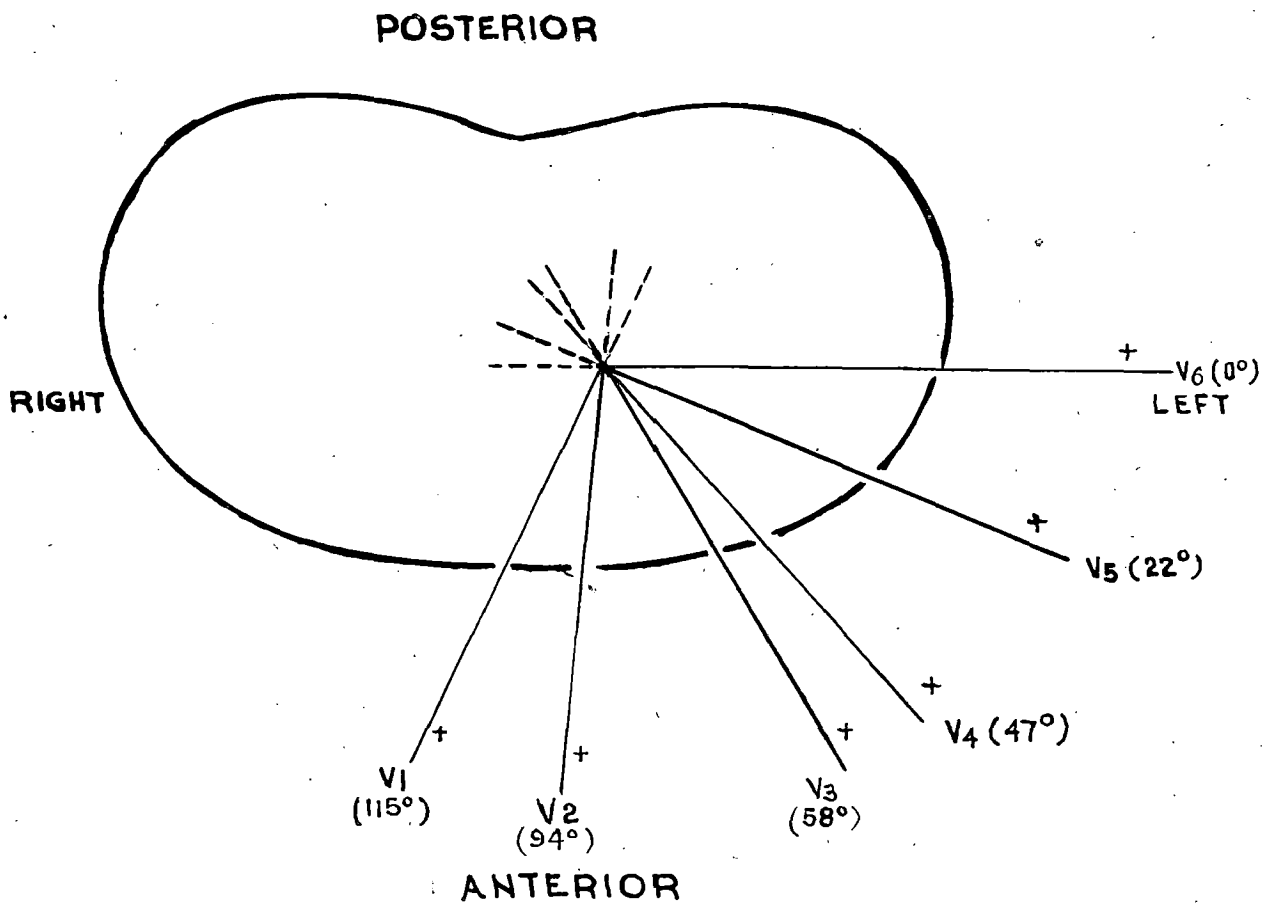


C.I.D. THE AXES OF THE STANDARD AND UNIPOLAR LIMB LEADS SUPERIMPOSED, FORMING A HEXAXIAL REFERENCE FIGURE.

**FIG.C.I. THE EINTHOVEN EQUILATERAL TRIANGLE, THE AXES OF THE BIPOLAR AND UNIPOLAR LIMB LEADS AND THE TRIAXIAL AND HEXAXIAL REFERENCE DIAGRAMS.**



**FIG. C.2** DERIVATION OF THE QRS  
COMPLEXES OF THE LIMB LEADS  
FROM THE FRONTAL PLANE QRS  
LOOP BY USING THE HEXAXIAL  
REFERENCE DIAGRAM OF LANGNER'S.



**FIG. C.3** THE AXES OF THE UNIPOLAR PRECORDIAL LEADS IN THE HORIZONTAL PLANE OF THE BODY.

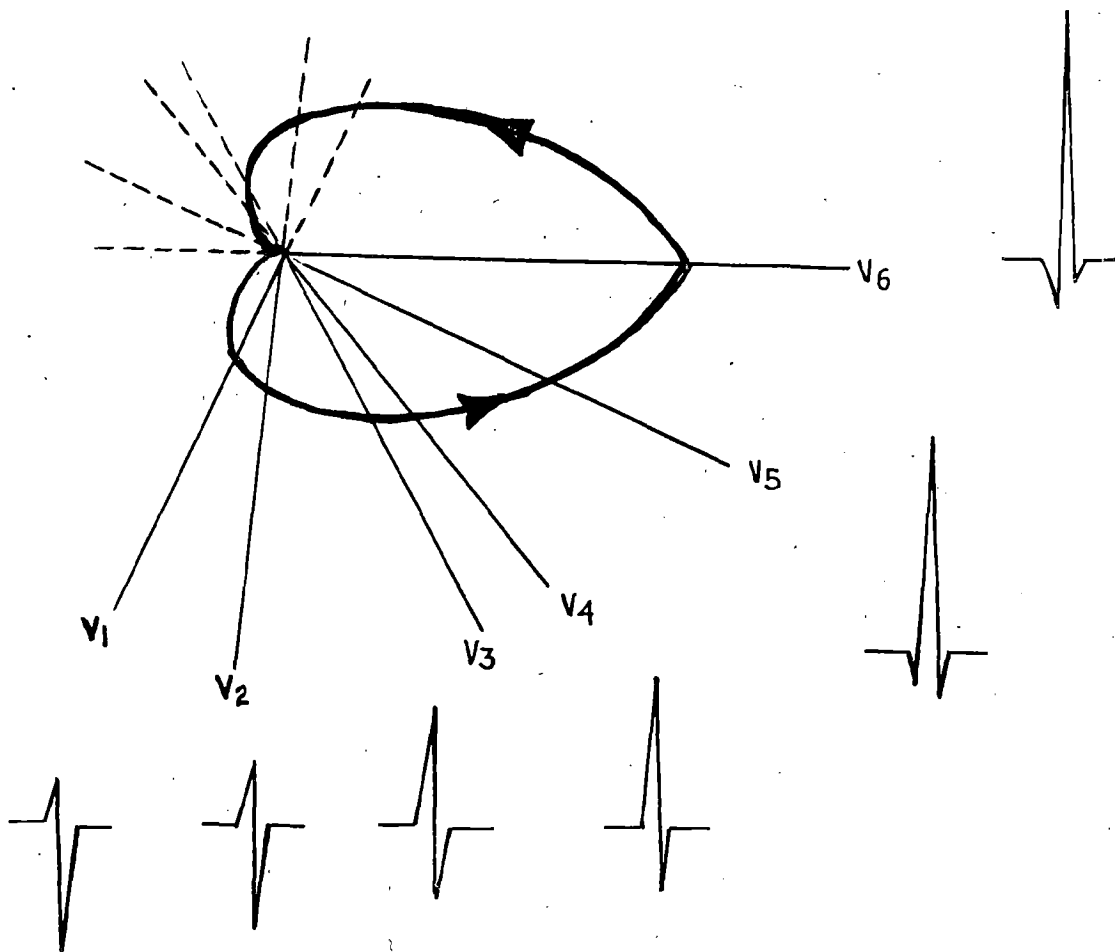


FIG. C.4 DERIVATION OF THE QRS COMPLEXES OF THE PRECORDIAL LEADS FROM THE TRANSVERSE PLANE QRS LOOP BY USING THE REFERENCE SYSTEM OF ABILDSKOV AND WILKINSON'S.

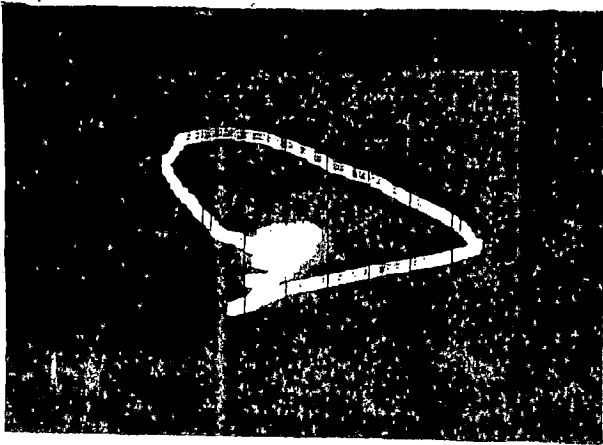
of the heart. In this horizontal reference figure the axes of the various precordial leads are approximately as follows:-

$V_1$ ,  $+115^\circ$ ,  $V_2$ ,  $+94^\circ$ ,  $V_3$ ,  $+58^\circ$ ,  $V_4$ ,  $+47^\circ$ ,  $V_5$ ,  $+22^\circ$  and  $V_6$ ,  $0^\circ$

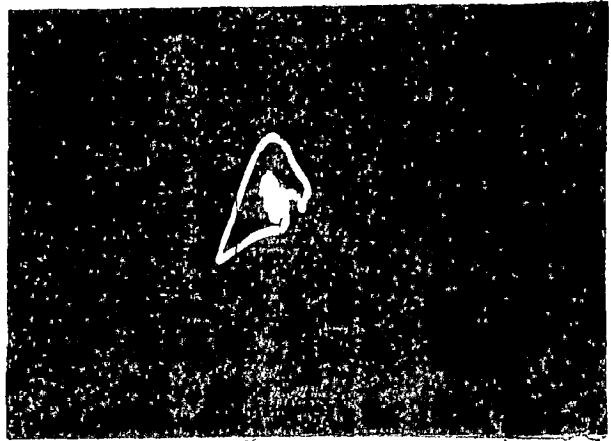
The precordial leads can hence be derived by superimposing the transverse plane VCG on Abildskov and Wilkinson's reference diagram (Fig. C.4)

APPENDIX - D

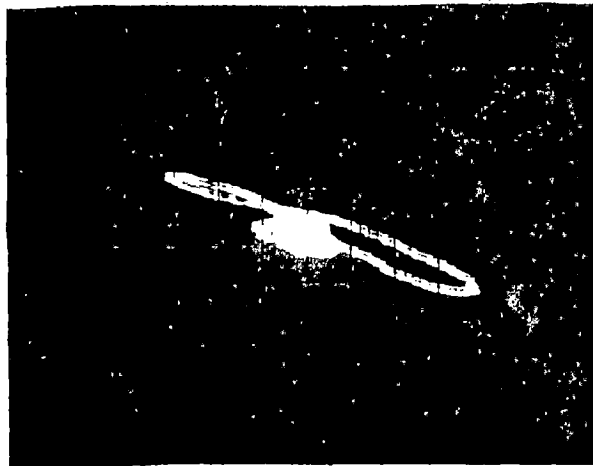
Vectorcardiogram of a normal 18 year old male subject



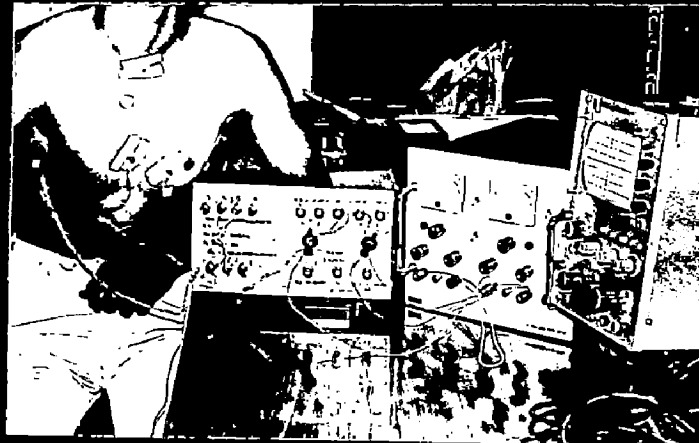
Transverse Plane



Sagittal Plane



Frontal Plane



Electrode Placement Scheme for Frank's System