OPTIMAL SCHEDULING OF SYSTEM MAINTENANCE PERSONNEL

A DISSERTATION

Submitted in partial fulfilment
of the requirements for the award of the degree
of
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in
ELECTRICAL ENGINEERING

By

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CERTIFICATE

Certified that the dissertation entitled OPTIMAL SCHEDULING OF SYSTEM MAINTENANCE PERSONNEL which is being submitted by Mr Naresh Kumar Goel in partial fulfilment of the requirements for the award of the degree of Master of Engineering in Electrical Engineering (System Engineering and Operation Research) of the University of Roorkee, Roorkee (U.P.) is the record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

It is further certified that he has worked for a period of eight months from September 1984 to April 1985 for preparing this dissertation at this University.

Dated: 29.4.85

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Naresh Kumar Goel

SYNOPSIS

Present time is time of rationalization. In every field of life we have to take the decisions to perform the work efficiently, intelligently. Efficiency and the art of performing the job plays a good role in the industry.

Scheduling of the jobs and personnel is the way out to achieve the efficiency of man power and proper utilization of the man-hours. Introduction covered in first chapter throws the light on various aspect of man-power scheduling or planning in various fields of application such as Man power planning in a plant (chemical, Textile , semi-conductor, switchgear etc.), man-power scheduling of airline cleaning crews, Nurse scheduling, Man power planning for Hospital maintenance.

Scheduling of maintenance personnel in a system (plant) has been covered in the second chapter. The objective here is to minimize the idle time of man power or to make much busy of man-power, while the constraints are to see every possibility of scheduling each job over the time horizon and to see the every possibility of scheduling each skill over the time horizon. To solve this problem 0-1 integer linear programming implicity Enumeration method algorithm is used.

Man power utilization model for Health maintenance organisation has been covered in third chapter. It describes the development and validation of a set of mathematical programming models. An application of model in planning context of an emerging HMO is presented. The model treats the interaction between effective man power utilization, facility requirements and available capital. It can be used effectively in planning aids.

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CHAPTER I

INTRODUCTION

With the increasing pace of industrialization, Man confronts the various managerial problem on the way. Efficiency is the prime aim. Cost factor is an important part in accomplition of a job.

In practice, scheduling of jobs and personnel is an art. The scheduling decisions are commonly left in the hands of the scheduler who by experience and knowledge of the system arrives at a schedule. His objectives are not clearly spelled out and the quality of his schedule is difficult to assess.

Specifically, here the scheduling of personnel has been covered. It depends upon the nature of the problem. Despite the vast literature on scheduling, very little has been published on the scheduling of Plant Maintenance Personnel. Rothestein[2] schedules man power to account for the employess reguler days off. Roberts, S.M. and Escudero, L.F.[1] has contributed a lot of attention in formulating the problem and to get it solved by optimization techniques. We consider the following situation to go through the idea used in formulating the problem. Suppose a list of jobs is available, all or some of which may be schedules within the next horizon. Each job has been broken down into finite time intervals, and the skills required for each time interval are known. Over the specified time Horizon, the number of personnel in each skill available is known. Having picked a set of jobs to be scheduled, the objective of the scheduling is to schedule the jobs and personnel in such a way that all the jobs are scheduled with the number of personnel idle hours minimized. In such situation we have three types of matching between skill requirements and skill availabilities. These are: (1) total skill requirements of the jobs exceed the total skill availabilities; (2) total skill requirements of the jobs equal the total skill availabilities; and (3) the total skill requirement of the jobs are less than the total skill availabilities. In the first case there is no solution. One must either decrease the number of jobs to be scheduled or increase the skill supply. A quick check on the total skill requirement against the total skill availabilities skill by skill can decide this issue. In the second case, although the skill requirements and skill availabilities match, this does not assure that feasible schedule exsists.

This case cam represent, however, the ideal case where a schedule exists and there is no idle time. The third case is the usual situation where a schedule exists and there is some idle time. In fact, we can be reasonable sure that, if one schedule exists, it is not unique. Under some circumstances, it may not be possible to find a schedule for the job mix skill requirements and skill availabilities. When this occurs, jobs may be dropped and/or added.

Helman I Stern and Marvin Harsh[4] has given the idea of scheduling for the Air line cleaning crews. It also helps in making the background of scheduling of the man-power. Its basic structural elements are related to the variable schedule minimum fleet size problem treated by Gerts Bakh and Stern[20]. In the minimum fleet size problem, one resource unit is required to process each activity uniformily over the activities duration. In this problem each activity sequires a different fixed quantity of resources, and these must be distributed in a specific pattern over the available time span of the activity. Moreover this schedule horizon is partitioned into segments (shifts), each with a different crew cost. Activities overlapping shifts must be processed within a single shift. A further difference is that each crew member must process a special activity (the meal activity) which is only available during specified portions of the schedule Horizon. All of these complexities were handled within the frame work of our integer programming formulation of the crew scheduling problem. This integer program contained specially devised constraints and variables able to guarantee the avoidance of certain undesirable phenomena, such as job interruptions and shift interruptions. Concerning worker interruptions, it was felt that most of these could be avoided through the use of non-decreasing job loading profiles (fine span of the activity). Such profiles require that the number of workers assigned to a particular job does not decrease over time until the job is completed. This tends to reduce the possibility of the worker, once assigned to a job, leaving it before its completion (Worker interruption).

In the case considered the introduction of the loading profile constraint led to a crew schedule for which worker assignments have no worker interruptions were easily found by inspection for this the following job requirements is considered. (1) The cleaning of a Boeing 707 aircraft required 9 standard man-hours (2) The Boeing 747 required 17 standard man-hours. The number, type and availability of jobs was determined by the aircraft flight schedule. The job availabilities corresponds to the time the aircraft were on the ground between flights less the time allocated for passenger loading and unloading.

Uninterrupted Working Schedule for Shift I of the Crews

jobs considered M, 4,7,9,11 for shift 1(evening)
7 and 11 are the overlapping
jobs assignment

TABLE 1.1

Crew Scheduling Problems

	e	n	d
(4)	-	-	-
(7)	-		-
(9)	-		
(11)	-	-	

e = evening

d = day

n = night,

(-)= shows the job window (plane availability for cleaning)

TABLE 1.2

WORKER	Period t								J(T)	M(T)	I(T)
	1	2	3	4	5	6	7	8			
1	9	9	M	4	I	I	11	11	5	1	2
2	9	9	M	4	I	I	11	11	5	1	2
3	9	9	M	4	I	I	I	11	4	1	3
4	9	9	M	4	I	I	I	I	3	1	4
5	9	9	M	4	I	I	I	I	3	1	4
6	9	9	M	4	I	I	I	I	3	1	4
7	9	9	M	4	I	I	I	I	3	1	4
8	9	9	M	4	I	I	I	I	3	1	4
9	I	9	M	4	I	7	7	7	5	1	2
10	I	M	4	4	I	7	7	7	5	1	2
11	I	M	4	4	I	7	7	7	5	1	2
12	I	M	4	4	I	I	11	11	4	1	3
13	I	M	4	4	I	I	11	11	4	1	3
$D_t(J)$	8	9	4	13	0	3	7	8	52	-	-
$D_t(M)$	0	4	9	0	0	0	0	0	-	13	-
$D_t(I)$	5	0	0	0	13	10	6	5	-	-	31

J(T) = no. of Periods worker assigned to jobs,
M(T) = no. of Periods worker assigned to meals
I(T) = no. of worker idle, 4,7,9,11 = job number assignment
M = Meal Assignment and I = idle time assignment

Going through the paper of Panwalker,S.S. and Iskander, W.[7] on scheduling survey, it gives the scope of the job scheduling approach for different kind of problems such as (1) Processing time (2) Due dates (3) no. of operations (4) costs (5) Set up times (6) Arrival time (and random) (7) Slack (based on processing time and due dates) (8) Machines (Machines Oriented rules) Approach.

The chapter third covered the topic on Optimum man power utilization model for Health maintenance Organisations . This topic is added in this dissertation in support of the idea of man-power planning considering the costs involved to be minimized.

The basic objective of Health Maintenance Organisation is to provide to specified subscriber population a comprehensive set of services in an organised manner for a fixed annual fee.

The HMO concept has been proposed as a potential cure for a number of problems present in the American Health Care System. Among the most frequently cited are rising costs, the episodic rather than preventive nature of health care delivery and mal distribution of services resulting in inadequate access to care in inner city and rural areas. HMO have the incentive and resources to contain costs through greater use of auxiliary personnel, increased levels of technology, more effecient organisation, and effective use of services purchased externally. The planning decisions required to determine the proper mix of services staffing patterns, and level of technology for a specified subscriber population are complex and interrelated to the point of initial service delivery.

In support of the third chapter the work published by Warner Michael D.[15] is very useful. Three important decisions has to be taken staffing in nurse scheduling (1) Staffing Decision: made annually specifying the number of full time equivalent nursing personnel of each skill class. (2) Scheduling Decision: made each four or six weeks, specifying when each nurse will be on and off duty in a scheduling horizon in such a way that some specified minimum number of each skill class is provided each shift of each day (3) Allocation decision: made each shift a fine tuning of the scheduling decision allocating a pool of available float nursing personnel among nursing units to account for largely unforseeable variability of demand for nursing care and for absenteeism (The float pool i.e. nuses who can be allocated to any unit after they arrive for duty, can be considered a seperate unit for the scheduling decision).

The quality of a sechedule refers to an expression of the industrial nurses judgement as to how well the schedule, She is assigned conforms with her desires to be off and on duty this scheduling period and with her feelings about other properties of the schedule, such as work stretch, rotation pattern, etc.

CHAPTER II

SCHEDULLING OF PLANT MAINTENANCE PERSONNEL

2.1 Nature of the Problem:

We are here concerned with the scheduling of personnel in a plant to carry out the maintenance function. In these plants, a maintenance department is charged with the responsibility of accepting maintenance requests and processing them. This includes analysing the requests and deciding what is to be done, how it is to be done, by whom it is done, and what materials and parts are required, for each job. For each job, the estimated time for each subtask and the skills required are determined.

The maintenance jobs may be broadly characterized as preventive maintenance of jobs (which are nominally scheduled to be carried out periodically) and unplanned maintenance jobs (which are accepted as the need arises). The unplanned jobs of course are analyzed here.

The schedule and management decide which jobs are to be processed by setting up a priority schedule and sequencing the jobs within priority level. The jobs in the queue therefore, consists both of preventive maintenance jobs and the unplanned jobs. Each day, the scheduler checks the personnel reaster to determine how many persons are available in each skill type. With this in mind, he selects from the queue of jobs, those jobs to be worked on by the personnel available and estabishes a schedule. Since the number of jobs always exceeds the capacity of the personnel to process them in a given day, the scheduler has flexibility in matching jobs and personnel. Both the list of jobs selected to be worked on and the list of personnel are volatile. The schedule of jobs and personnel can be dis rupted by emergencies. Under these circumstances, one or more jobs currently being worked on may be abandoned or halted and personel switched among jobs to accomodate the needs of the emergency

In this dissertation the problem faced in coal Industries has been considered. Where an Elect. and Mechanical department exists. Under this department the maintenance of the equipments used in mining is performed. Many kind of processes exists in between mining the coal to despatching of the coal. Each process goes under the following sections 1. Locomotive/Section 2. underground mining section 3. Water Pump Section 4. Mining car section 5. Coal Handling Plant Section, 6. Workshop 7. Substation Maintenance 8. Battery charging section.

To illustrate the use of optimisation technique to solve the problem of scheduling the man-power, the LOCOMOTIVE SECTION is considered. The list of the jobs is following

1. Locomotive number 10 and 11 are under maintenance
2. Locomotive under running 13 is under maintenance.
3. Coal cutting machines is under maintenance
4. Drilling machine is under maintenance
5. Switch-gears is under maintenance.

The number of skill hours required for each job is given under the following tables

TABLE 1

JOB					
1	1M 2H	1M 2H	1E 1H	1E 1H	1M 4H
2	1M 1H	1E OH	M 2H		
3	M 4H	M 4H	M 4H	M 2H	M 2H
4	M 1H	E 1H	E OH		
5	E 2H	E 2H	E 2H	E 2H	E 2H

M designates for mechanic

E designates for Electrician

H designates for Helpers

2.2 Formulation of Mathematical Model

Scheduling Problems are combinational problems and as such, are nonlinear. The non linearity manfests itself in the fact that job j can begin at say, hour 1, or hour 2, or .... or hour k, but at only one hour; and similarly for other jobs. This O ring situation, where each job can occupy a number of potential time slots, but can fill one and only one of them, is the cause of the nonlinearity. The solution of the scheduling problem selects when each job shall occur.

To by pass the O ring nature of the problem, we will introduce two types of binary variables, Y_{jkl} and Z_{jl}. The Z_{jl} variable determine at what hour job j shall start. For each Z_{jl} variable, there is a corresponding set of Y_{jkl} variables that determine which skills shall be employed at what times. The scheduling problem consists of three kinds of linear expressions: (i) the job-hour balance inequalities; (ii) the skill-hour balance inequalities; (iii) The selection equations that by pass the O ring condition.

For convenience we will assume the following:

(1) The jobs are broken into time intervals of one hour. Smaller interval can be used at the expense of more variables and equations.

(2) Within any interval, one or more persons of the same skill may be used, for example during the third hour of a job. This assumptions can be relaxed to include one or more persons of the same skill, with one or more skills.

(3) The time Horizon is fixed for the scheduling e.g. the time Horizon could be 8 hours, 1 day, or 1 week.

(4) The scheduler has more jobs to schedule than the personnel available to process the jobs over the specified time Horizon. This is realistic, since there is always a back log of jobs.scheduling too far in advance is not possible due to the plant operations and conditions.

(5) The no. of persons available in each skill is known for each time interval over the time Horizon.

Let us define the following terms:

i = subscript, for the ith skill;

j = subscript, for the jth job;

k = subscript, for the kth hour clock time;

l = subscript, for the lth starting time of a job;

t = subscript, for the tth hour from the start of a job, incremental time;

h_{ijt} = hours required for the ith skill on the jth job, at the tth hour from the start of a job;

H = horizon of scheduling, hours;

n = number of jobs to be scheduled;

s = total no. of skill types available;

S_{ik} = skill hours available for the ith skill, kth hour clock time;

Y_{ijkl} = binary variable for the ith skill on, jth job, kth hour clock time , lth starting time for the job; if $Yjkl= 1$, the job is scheduled if $Y_{jkl} = 0$, the job is not scheduled.

Z_{jl} = binary variable for the jth job, lth starting time for the job; $Z_{jl}= 1$, job is scheduled

Z_{jl} = 0, job is not scheduled;

D_j = job duration, number of clock hours for job j;

$\{I_j\}$ = index set for skill type required for job j;

$\{I_k\}$ = index set of skill types that are used in hour k;

$\{J_{ik}\}$ = index set for jobs that use skill i in hour k, for all t and l subscript;

$\{K_i\}$ = index set of hours where skill i is used.

The index set notation is used, since not all terms may be available. For example, if we scan the skill types for a job, we may assume that skill 1 is not used in job 1. Assuming the skill types 1,2 and 3 are available for all jobs, we find in this case:

$$\{I_1\} = \{2,3\}$$

The list of the jobs with the required skill hours is given in Table 1. Here Each square represents one hour. We code the skills as Mechanic (M) skill type 1, Electrician (E) skill type 2, and Helper (H) - skill type 3.

We assume the scheduling time Horizon is 8 hours, As a result, job 1 may begin in hour 1 or 2 or 3 or 4. It can not begin in hour 5, because it is a five-hour job and starting in hour 5 would cause the job to terminate in hour 9 which is beyond on horizon. Similarly, job 2 can begin in hour 1 or 2 or 3 or 4 or 5 or 6.

We consider first the job-hour inequalities, which represent the balance on the hours required for each job, skill by skill.

<u>Job-hour Inequalities for job 1</u>

<u>TABLE-12</u>

$$h_{111}Y_{1111} + h_{311}Y_{3111} + h_{112}Y_{1121} + h_{312}Y_{3122} + h_{213}Y_{2131} + h_{313}Y_{3131} + h_{314}Y_{3141} + h_{115}Y_{1151} + h_{315}Y_{3151} \leq 15, \text{ job 1. hour 1;}$$

$$h_{111}Y_{1122} + h_{311}Y_{3122} + h_{112}Y_{1132} + h_{312}Y_{3132} + h_{213}Y_{2142} + h_{313}Y_{3142} + h_{214}Y_{2152} + h_{314}Y_{3158} + h_{115}Y_{1162} + h_{315}Y_{3162} \leq 16, \text{ job 1, hour 2;}$$

$$h_{111}Y_{1123}+h_{311}Y_{3133}+h_{112}Y_{1143}+h_{312}Y_{3143}+h_{213}Y_{2153}+h_{313}Y_{3153}$$
$$+h_{214}Y_{2163}+h_{314}Y_{3163}+h_{115}Y_{1173}+h_{315}Y_{3173}\leq 15,$$

job 1, hour 3.

$$h_{111}Y_{1144}+h_{311}Y_{3144}+h_{112}Y_{1154}+h_{312}Y_{3154}+h_{213}Y_{2164}+h_{313}Y_{3164}$$
$$+h_{214}Y_{2174}+h_{314}Y_{3174}+h_{115}Y_{1184}+h_{315}Y_{3184}\leq 15$$

job 1, hour 4.

Likewise the job-hour balance inequalities can be written for other jobs.

Here the first line says that, if job 1 began in hour 1, the skill hours for skill 1 in hour 1 times its associated binary variable, plus the skill hours for skill 3 in hour 1 times its associated binary variable, plus the skill hours for skill 1 in hour 2 times its associated binary variable, plus the skill hours for skill 3 in hour 2 times its associated binary variable, plus the skill hours for skill 2 in hour 3 times its associated binary variable, plus the skill hours for skill 3 in hour 3 times its associated binary variable, plus the skill hours for skill 2 in hour 4 times its associated binary variable, plus the skill hours 3 in hour 4 times its associated binary variable, plus the skill hours for skill 1 in hour 5 times its associated binary variable, plus the skill hours for skill 3 in hour 5 times its associated variable is less than or equal to the sum of all the skill hours (h_{111} + h_{311} + h_{112} + h_{312} + h_{213} + h_{313} + h_{214} + h_{314} + h_{115} + h_{315}), in this inequality, 15 hours. The second equation gives the same information as the first equations, except job 1 now begins in hour 2. Since job 1 is a 5-hour job ($\theta_1 = 5$), and since the planning Horizon is 8 hours (H = 8), the number of starting times (number of inequalities) for job 1 equals $H - \theta_1 + 1$ or $8 - 5 + 1 = 4$. Note by the definition of h_{ijt}, that these terms are constant down the columns. The subscript 't' represents incremental time from the start of a job; so, no matter when the job begins, the incremental time index assumes values 1,2,3,4,5 to correspond to the five-hour job duration of job 1.

The Y_{ijkl} binary variables are used inconjunction with the Z_{jt} variables to determine which of the four equations for job 1 will be in the schedule. The Y_{ijkl} variables will be coupled so that, if one inequality in the job-hour expressions is selected, all the Y_{ijkl} variables will be 1 in that row and the remaining Y_{ijkl} variables in the other inequalities for job j will be zero. This assumes that job j (or 1 in this case) will be scheduled once and only once within the time Horizon.

We make a distinction between clock time to complete the job, job duration (θ_j for job j), and the skill hours utilized within that clock time. For example, if job 1 requires two type of skill rather than one and the no. of skill 3 is 2 for the first hour, then $h_{111} = 1$, $h_{311} = 2$. Likewise the skill hours in other conservcative hours, then $h_{112} = 1$, $h_{312} = 2$, $h_{213} = 1$, $h_{313} = 1$, $h_{214} = 1$, $h_{314} = 1$, $h_{115} = 1$, $h_{315} = 4$ and the total skill hours for job 1.

$$h_{111} + h_{311} + h_{112} + h_{312} + h_{213} + h_{313} + h_{214} + h_{314} + h_{115} + h_{315} = 15$$

While the clock time to do the job remains equal to 5.

The skill hour balance gives, for each skill, the hour by hour skill requirements, which must be less than or equal to the skill availability at each hour. The skill-hour balance inequalities are extracted from the job-hour inequalities for eabh skill by summing the skill requirements for each hour across all the job-hour inequalities.

<u>Skill 1 - hour balance inequalities</u>

<u>TABLE 3</u>

$h_{111}Y_{1111}+h_{121}Y_{1211}+h_{131}Y_{1311}+h_{141}Y_{1411}\leq S_{11}$, Skill 1, hour 1.

$h_{111}Y_{1122}+h_{112}Y_{1121}+h_{121}Y_{1221}+h_{132}Y_{1321}+h_{132}Y_{1322}+h_{141}Y_{1421}$
$\leq S_{12}$, Skill 1, hour 2.

$h_{111}Y_{1133}+h_{112}Y_{1132}+h_{121}Y_{1233}+h_{123}Y_{1231}+h_{131}Y_{1335}+h_{132}Y_{1332}$
$+h_{133}Y_{1331}+h_{141}Y_{1433}\leq S_{13}$, Skill 1, hour 3.

$h_{111}Y_{1144}+h_{112}Y_{1143}+h_{123}Y_{1242}+h_{122}Y_{1243}+h_{134}Y_{1341}+h_{133}Y_{1342}$
$+h_{132}Y_{1343}+h_{131}Y_{1344}+h_{141}Y_{1444}\leq S_{14}$, Skill 1, hour 4.

$h_{112}Y_{1154}+h_{115}Y_{1151}+h_{121}Y_{1255}+h_{123}Y_{1253}+h_{132}Y_{1354}+h_{133}Y_{1353}$
$+h_{134}Y_{1352}+h_{135}Y_{1351}+h_{141}Y_{1455}\leq S_{15}$, Skill 1, hour 5.

$h_{115}Y_{1162}+h_{121}Y_{1266}+h_{123}Y_{1264}+h_{133}Y_{1364}+h_{134}Y_{1363}+h_{135}Y_{1362}$
$+h_{141}Y_{1466}\leq S_{16}$, Skill 1, hour 6

$h_{115}Y_{1173}+h_{123}Y_{1275}+h_{134}Y_{1374}+h_{135}Y_{1373}\leq S_{17}$, Skill 1, hour 7

$h_{115}Y_{1184}+h_{123}Y_{1286}+h_{135}Y_{1384}\leq S_{18}$, Skill 1, hour 8.

The first equation in table 3 represents the skill-hours of skill 1 times its associated Y_{ijkl} variable summed over all jobs that used skill 1 in hour 1. The second equalities represent the skill hours of skill 1 times its associated Y_{ijkl} variable summed over all jobs that used skill 1 in hour 2, etc. The skills are found from the job-hour inequalities from the subscript i on h_{ijt}, and the clock time hours are found from the subscript k on Y_{ijkl}. The right hand side of these inequalities are the S_{ik} terms which are input data, independent of job selected, and represents the scheduler's resources of the ith skill at the kth hour.

Since each job can be run once and only once, we need a way to select from the job-hour inequalities when the jobs (and the skills) are to be scheduled. To do this, we introduce another class of binary variables Z_{jl}, which specify the jthm job, begining at the l th starting time. To illustrate the use of the Z_{jl} variables, we recall that job 1 may begin at hour 1,2,3 or 4. We therefore express this

$$Z_{11} + Z_{12} + Z_{13} + Z_{14} = 1$$

which enforces the reqdirement that only one $Z_{11} = 1$. We note that Z_{11} is associated with the first equation of the job-hour equation for job 1; Z_{12} is associated with the second equation of the job-hour equation for job 1 etc. Further more to assure that all the Y_{ijkl} associated with Z_{11} will be 1 When $Z_{11} = 1$ and will be 0 when $Z_{11} = 0$, we write the following equations recognising that job 1 is a five hour job, that is k= 1,2,3,4,5

Corresponding to the first row of job-hour inequalities of Table 2, we have

$Y_{1111} = Z_{11}$, $Y_{3111} = Z_{11}$, $Y_{1121} = Z_{11}$, $Y_{3121} = Z_{11}$, $Y_{2131} = Z_{11}$,
$Y_{3131} = Z_{11}$, $Y_{3141} = Z_{11}$, $Y_{3141} = Z_{11}$, $Y_{1151} = Z_{11}$, $Y_{3151} = Z_{11}$.

Corresponding to the second row of Table 2, we have

$Y_{1122} = Z_{12}$, $Y_{3122} = Z_{12}$, $Y_{1132} = Z_{12}$, $Y_{3132} = Z_{12}$, $Y_{2142} = Z_{12}$,
$Y_{3142} = Z_{12}$, $Y_{2152} = Z_{12}$, $Y_{3152} = Z_{12}$, $Y_{1162} = Z_{12}$, $Y_{3162} = Z_{12}$.

In a similar manner we may write for the remaining rows

$Y_{1133} = Z_{13}$, $Y_{3133} = Z_{13}$, $Y_{1143} = Z_{13}$, $Y_{3143} = Z_{13}$, $Y_{2153} = Z_{13}$,
$Y_{3153} = Z_{13}$, $Y_{2163} = Z_{13}$, $Y_{3163} = Z_{13}$, $Y_{1173} = Z_{13}$, $Y_{3173} = Z_{13}$

— for 3rd row.

$Y_{1144} = Z_{14}$, $Y_{3144} = Z_{14}$, $Y_{1154} = Z_{14}$, $Y_{3154} = Z_{14}$, $Y_{2164} = Z_{14}$
$Y_{3164} = Z_{14}$, $Y_{2174} = Z_{14}$, $Y_{3174} = Z_{14}$, $Y_{1184} = Z_{14}$, $Y_{3184} = Z_{14}$.

The equation

$$\sum_{l=1}^{H-\theta_j+1} Z_{jl} = 1, \quad j = 1,2,\ldots,n,$$

and

$$Y_{ijkl} = Z_{jl} \qquad \begin{array}{l} j = 1,\ 2,\ldots,n. \\ l = 1,\ H - \theta_j + 1 \\ k = 1,\ \theta_j \\ i \in \{I_j\} \end{array}$$

Will be referred to collectively as the selection equations, for their utilization makes it possible to select the appropriate inequalities and variables in the job-hour inequalities and the skill hour inequalities to create a schedule.

Total Overall Problem Formulation

Objective

Our objective is to minimise the idle time. This objective function may be expressed as

$$\min F = \min\left(\sum_{i}\sum_{k} S_{ik} - \sum_{i}\sum_{j}\sum_{k}\sum_{l}\sum_{t} h_{ijt}\, Y_{ijkl}\right),$$

Where the first term on the right side represents all the skill resources over all the hours available and the 2nd terms represents the skill hours required. Because $\Sigma_i \Sigma_k\, S_{ik}$ is a constant, instead of minimizing F, we are equivalently maximizing F,

$$\text{Where } F = \sum_{i}\sum_{j}\sum_{k}\sum_{l}\sum_{t} h_{ijt} Y_{ijkl}$$

$$\max F = \max \sum_{i}\sum_{j}\sum_{k}\sum_{l}\sum_{t} h_{ijt} Y_{ijkl}$$

To generalize we may write this as follows

$$\max F = \max \sum_{j}\sum_{k}\sum_{l} C_{jkl} Y_{jkl} \qquad \ldots \qquad (2\text{-}1)$$

Where C_{jkl} are the values associated with Y_{ijkl}

Subjected to Constraints:

(1) Job-Hour Balance Constraints

$$\sum_{i\in\{I_j\}} \sum_{k=P}^{P+\theta_j-1} \sum_{t=1}^{\theta_j} h_{ijt} Y_{ijkl} \leq \sum_{i\in\{I_j\}} \sum_{t=1}^{\theta_j} h_{ijt}, \quad j = 1,2,\ldots,n; \quad l = 1, H-\theta_j+1, \ldots N \qquad (2\text{-}2)$$

(ii) Skill-Hour Balance Constraints

$$\sum_{j\epsilon(J_{ik})} h_{ijt}Y_{ijkl} \leq S_{ik}, \qquad (2\text{-}3)$$

$$i\epsilon(I_k),\ K\epsilon(K_i),$$

(iii) Selection Equation Constraints

$$\sum_{l=1}^{H-\theta_j+1} Z_{jl} = 1, \quad j = 1,2,\ldots,n. \qquad (2\text{-}4)$$

and $Y_{ijkl} - Z_{jl} = 0,\ \ j= 1,2,\ldots,n, l = 1,H-\theta_j+1,\ k= 1,\theta_j$

(2.3) Reduced Size Model I

We reduce the number of variables by making the following observations:

When job j starting in the lth hour, is chosen for scheduling, all the Y_{ijkl} variables in the lth row of the job hour inequalities are equal to 1 and the remaining Y_{ijkl} variables in the other rows of the job-hour inequalities for job j are equal to zero.

Our observation means that for each row of the job-hour inequalities, we can employ one Y_{ijkl} to play the role of each of the other Y_{ijkl}. We have chosen Y_{2j11} to replace all the Y_{ijkl} terms in row 1 of the job-hour balance. Similarly, in the skill-hour balances and the selection equations Y_{ij11} may be used.

The modified job-hour balance inequalities are

$$(h_{111}+h_{112}+h_{115})Y_{1111}+(h_{311}+h_{312}+h_{313}+h_{314}+ h_{315})Y_{3111}$$
$$+(h_{213} + h_{214})Y_{2111} \leq 15, \text{job 1, hour 1.}$$

Here again we see that three different binary variable Y_{1111}, Y_{2111} and Y_{3111} in first row but if Y_{ij11} in this row is equal to one, the remaining Y_{ij11} shall be zero. It means these variables can be modified again by Y_{j11}, irrespective of skill type used. Now the job-hour inequalities are:

TABLE-4

$(h_{111}+h_{112}+h_{113}+h_{311}+h_{312}+h_{313}+h_{314}+h_{315}+h_{213}+h_{214})Y_{111} \leq 15$, job 1, hour 1

$(h_{111}+h_{311}+h_{112}+h_{312}+h_{213}+h_{313}+h_{214}+h_{314}+h_{113}+h_{315})Y_{122} \leq 15$, job 1, hour 2.

$(h_{111}+H_{112}+h_{113}+h_{311}+H_{312}+H_{313}+H_{314}+H_{315}+h_{213}+h_{214})Y_{133} \leq 15$, job 1, hour 3.

$(h_{111}+H_{112}+h_{113}+h_{311}+H_{312}+h_{313}+h_{314}+h_{315}+h_{213}+h_{214})Y_{144} \leq 15$, job 1, hour 4.

The skill-hour balance inequalities for skill type 1 are

TABLE-5

$h_{111}Y_{111} + h_{121}Y_{211} + h_{131}Y_{311} + h_{141}Y_{411} \leq S_{11}$ Skill 1, hour 1.

$h_{111}Y_{122} + h_{112}Y_{111} + h_{121}Y_{212} + h_{131}Y_{312} + h_{132}Y_{322}$
$+ h_{141}Y_{422} \leq S_{12}$, Skill 1, hour 2.

$h_{111}Y_{113} + h_{112}Y_{122} + h_{121}Y_{213} + h_{123}Y_{211} + h_{131}Y_{313} +$
$+ h_{132}Y_{322} + h_{133}Y_{331} + h_{141}Y_{433} \leq S_{13}$, Skill 1, hour 3.

$h_{111}Y_{144} + h_{112}Y_{133} + h_{123}Y_{222} + h_{121}Y_{244} + h_{134}Y_{311} +$
$+ h_{133}Y_{322} + h_{132}Y_{333} + h_{131}Y_{344} + h_{141}Y_{444} \leq S_{14}$, Skill 1, hour

$h_{112}Y_{144}+ h_{115}Y_{111}+ h_{121}Y_{255}+h_{123}Y_{233}+ h_{132}Y_{344}+ h_{133}Y_{333}+ h_{134}Y_{322}$

$+ h_{135}Y_{311}+ h_{141}Y_{455} \leq S_{15}$, Skill 1, hour 5.

$h_{115}Y_{122}+ h_{121}Y_{266}+ h_{123}Y_{244}+ h_{133}Y_{344} + h_{134}Y_{333} + h_{135}Y_{322}$

$+ h_{141}Y_{466} \leq S_{16}$, Skill 1, hour 6

$h_{115}Y_{133}+ h_{122}Y_{255}+ h_{134}Y_{344} + h_{135}Y_{333} \leq S_{17}$, Skill 1, hour 7.

$h_{135}Y_{144}+ h_{123}Y_{266}+ h_{135}Y_{344} \leq S_{18}$, Skill 1, hour 8.

<u>Selection Equation 1</u> (For job 1)

$$Z_{11} + Z_{12} + Z_{11} + Z_{14} = 1$$

$$Z_{11} = Y_{111}$$

$$Z_{12} = Y_{122}$$

$$Z_{13} = Y_{133}$$

$$Z_{14} = Y_{144}$$

(2.4) <u>Reduced Size Formulation II</u>

In reduced size formulation I, we find the job-hour balance inequalities as

$$Y_{j11} \leq 1, \quad j = 1, 2, \ldots, n \text{ and } 1 = 1, H- \theta_j+ 1$$

where h for the first job-hour inequalities

$h_{111}=1$, $h_{112}= 1$, $h_{115} = 1$, $h_{311}= 2$, $h_{312} = 2$, $h_{313}= 1$, $h_{314}= 1$.

$h_{315}= 4$, $h_{213} = 1$, $h_{214}= 1$.

$$(h_{111} + h_{112} + h_{115} + h_{311}+ h_{312} + h_{313} + h_{314} +h_{315}+ h_{213}+h_{214})=15$$

Like wise this can be seen with other jobs-hour inequalities

Since $Y_{jl1} \leq 1$ really do not tell(1) any more than we know originally, i.e. Y_{jl} being a binary variable, we can drop the job-hour inequalities from the Integer Linear Programming problem optimization.

We can not, however drop the job-hour expressions from the overall problem formulation because we need those expressions to create the skill hour balance inequalities and to create the index sets $\{I_j\}$, $\{I_k\}$, $\{J_{ik}\}$, $\{K_i\}$. Further more we need there job-hour balance inequalities to convert the solution of the linear program in terms of Y_{jl1} back to Y_{jkl}. In other words we use the job-hour expressions Off-line or outside of the optimization to prepare the skill-hour inequalities and to interpret the solution

The selection equations are

$$\sum_{l=1}^{H-\theta_j+1} Z_{jl} = 1, \quad j = 1, 2, \ldots, n.$$

$$Y_{jl1} = Z_{jl} \qquad j = 1, 2, \ldots, n.$$

$$l = 1, \; H-\theta_j+1$$

While $Y_{jl1} \leq 1$

It means that $Z_{jl} \leq 1$ which is originally true that Z_{jl} is a binary variable so we can use Z_{jl} in place of Y_{jl1} in the formulation.

Now the overall reduced size formulation II becomes as it i.e.

<u>Skill-Hour Balance Inequalities Constraints</u>

$$\sum_{j\varepsilon\{J_{ik}\}} h_{ijt} Z_{jl} \leq S_{ik} \qquad i\varepsilon\{I_k\}, \; K\varepsilon\{K_i\} \qquad \ldots \qquad (2.5)$$

Selection Equation Constraints

$$\sum_{j=1}^{H-\theta_j+1} Z_{jl} = 1 , \qquad j = 1,2,\ldots,n. \tag{2-6}$$

maximize

$$F = \max \sum_j \sum_k \sum_l C_{jkl} Z_{jl} \tag{2-7}$$

(2.5) Comparisions

Variables, inequalities, and equations count

TABLE 6

Variables	General Formulation	Reduced Size Formulation I	Reduced Size Formulation II
Y_{ijkl}	$\sum_{j=1}^{n} (\theta_j)(H-\theta_j+1)$	$\sum_{j=1}^{n} (H-\theta_j+1)$	0
Z_{jl}	$\sum_{j=1}^{n} (H-\theta_j+1)$	$\sum_{j=1}^{n} (H-\theta_j+1)$	$\sum_{j=1}^{n} (H-\theta_j+1)$
Total	$\sum_{j=1}^{n} (\theta_j+1)(H-\theta_j+1)$	$\sum_{j=1}^{n} (H-\theta_j+1)$	$\sum_{j=1}^{n} (H-\theta_j+1)$

Expressions	General Formulation	Reduced Size formulation I	Reduced size formulation II
Job-Hour	$\sum_{j=1}^{n} (H-\theta_j+1)$	$\sum_{j=1}^{n} (H-\theta_j+1)$	0
Skill-Hour	sH	sH	sH
$\Sigma Z_{jl}= 1$	n	n	n
$Y_{ijkl}-Z_{jl}=0$	$\sum_{j=1}^{n} \theta_j(H-\theta_j+1)$	$\sum_{j=1}^{n} (H-\theta_j+1)$	0
Total	$\sum_{j=1}^{n} (\theta_j+1)(H-\theta_j+1) + n + sH$	$2\sum_{j=1}^{n} (H-\theta_j+1) + n + sH$	n+SH

(2.6) Numerical Problem

Hours*

1	1M 2H	1M 2H	1E 1H	1E 1H	1M 4H
2	1M 1H	1E 0H	1M 2H		
3	1M 4H	1M 4H	1M 4H	1M 2H	1M 2H
4	1M 1H	1E 1H	1E 0H		
5	1E 2H	1E 2H	1E 2H	1E 2H	1E 2H

Now writing the overall reduced size formulation II

$$\begin{aligned}\text{Max } F = \ & \max(Z_{11} + Z_{12} + Z_{13} + Z_{14}).15 \\ & + (Z_{21} + Z_{22} + Z_{23} + Z_{24} + Z_{25} + Z_{26}).6 \\ & + (Z_{31} + Z_{32} + Z_{33} + Z_{34}).21 \\ & + (Z_{41} + Z_{42} + Z_{43} + Z_{44} + Z_{45} + Z_{46}).5 \\ & + (Z_{51} + Z_{52} + Z_{53} + Z_{54}).15 \qquad (2\text{-}8)\end{aligned}$$

Subject to the following constraints

Skill 1. - hour balance inequalities

$$Z_{11} + Z_{21} + Z_{31} + Z_{41} \leq 2 \qquad (1)$$

$$Z_{12} + Z_{11} + Z_{22} + Z_{32} + Z_{31} + Z_{42} \leq 2 \qquad (2)$$

$$Z_{13} + Z_{12} + Z_{21} + Z_{23} + Z_{33} + Z_{32} + + Z_{31} + Z_{43} \leq 2 \qquad (3)$$

$$Z_{14} + Z_{13} + Z_{22} + Z_{24} + Z_{34} + Z_{33} + Z_{32} + Z_{31} + Z_{44} \leq 2 \qquad (4)$$

$$Z_{14} + Z_{11} + Z_{25} + Z_{23} + Z_{34} + Z_{33} + Z_{32} + Z_{31} + Z_{45} \leq 2 \qquad (5)$$

$$Z_{12} + Z_{26} + Z_{24} + Z_{34} + Z_{33} + Z_{32} + Z_{46} \leq 2 \qquad (6)$$

$$Z_{13} + Z_{25} + Z_{34} + Z_{33} \leq 2 \qquad (7)$$

$$Z_{14} + Z_{26} + Z_{34} \leq 2 \qquad (8)$$

(2-9)

Skill 2.- hour balance inequalities

$$Z_{51} \leq 2 \qquad (9)$$

$$Z_{21} + Z_{41} + Z_{52} + Z_{51} \leq 2 \qquad (10)$$

$$Z_{11} + Z_{22} + Z_{42} + Z_{41} + Z_{53} + Z_{52} + Z_{51} \leq 2 \qquad (11)$$

$$Z_{12} + Z_{11} + Z_{23} + Z_{43} + Z_{42} + Z_{54} + Z_{53} + Z_{52} + Z_{51} \leq 2 \qquad (12)$$

$$Z_{13} + Z_{12} + Z_{24} + Z_{44} + Z_{43} + Z_{54} + Z_{53} + Z_{52} + Z_{51} \leq 2 \qquad (13)$$

$$Z_{14} + Z_{13} + Z_{25} + Z_{45} + Z_{44} + Z_{54} + Z_{53} + Z_{52} \leq 2 \qquad (14)$$

$$Z_{14} + Z_{26} + Z_{46} + Z_{45} + Z_{54} + Z_{53} \leq \qquad (15)$$

$$Z_{46} + Z_{54} \leq \qquad (16)$$

(2-10)

Skill 3- Hour balance inequalities

$$2Z_{11} + Z_{21} + 4Z_{31} + Z_{41} + 2Z_{51} \leq 6 \qquad (17)$$

$$2Z_{12} + 2Z_{11} + Z_{22} + 4Z_{32} + 4Z_{31} + Z_{42} + Z_{41} + 2Z_{51} \leq 6 \qquad (18)$$

$$2Z_{13}+2Z_{12}+1.Z_{11}+1.Z_{23}+0.Z_{22}+2.Z_{21}+4Z_{33}+4Z_{32}+4Z_{31}$$

$$+1.4Z_{43}+1.Z_{42}+0.Z_{41}+2Z_{53}+2Z_{53}+2Z_{52}+2Z_{51}\leq 6 \quad (19)$$

$$2Z_{14}+2Z_{13}+1.Z_{12}+1.Z_{11}+1.Z_{24}+0.Z_{23}+2Z_{22}+4Z_{34}+4Z_{33}+4Z_{32}$$

$$+2Z_{31}+1.Z_{44}+1.Z_{43}+0.Z_{42}+2Z_{54}+2Z_{53}+2Z_{52}+2Z_{51}\leq 6 \quad (20)$$

$$2Z_{14}+1.Z_{13}+1.Z_{12}+4.Z_{11}+1.Z_{25}+0.Z_{24}+2.Z_{23}+4.Z_{34}+4.Z_{33}$$

$$+2Z_{32}+2Z_{31}+1.Z_{45}+1.Z_{44}+0.Z_{43}+2Z_{54}+2Z_{53}+2Z_{52}+2Z_{51}\leq 6 \quad (21) \quad (2\text{-}11)$$

$$1Z_{14}+1Z_{13}+4Z_{12}+1Z_{26}+0Z_{25}+2Z_{24}+4Z_{34}+2Z_{33}+2Z_{32}+1Z_{44}+1Z_{45}$$

$$+0Z_{44}+2Z_{54}+2Z_{53}+2Z_{52}\leq 6 \quad (22)$$

$$1Z_{14}+4Z_{13}+0Z_{26}+2Z_{25}+2Z_{34}+2Z_{33}+1Z_{46}+0Z_{45}+2Z_{54}+2Z_{53}\leq 6 \quad (23)$$

$$2Z_{14}+2Z_{26}+2Z_{34}+0.Z_{46}+2Z_{54}\leq 6 \quad (24)$$

(SELECTION EQUATIONS)

$$Z_{11}+Z_{12}+Z_{13}+Z_{14}=1 \quad (25)$$

$$Z_{21}+Z_{22}+Z_{23}+Z_{24}+Z_{25}+Z_{26}=1 \quad (26)$$

$$Z_{31}+Z_{32}+Z_{33}+Z_{34}=1 \quad (27) \quad (2\text{-}12)$$

$$Z_{41}+Z_{42}+Z_{43}+Z_{44}+Z_{45}+Z_{46}=1 \quad (28)$$

$$Z_{51}+Z_{52}+Z_{53}+Z_{54}=1 \quad (29)$$

(2.7) Algorithm

The 0-1 integer program has the form

$$\text{minimize} \quad Z = \sum_{j=1}^{N} C(J) \quad X(J)$$

subject to $Q(I) = - B(I) + \sum_{J=1}^{N} A(I, J) \; X(J) \geq 0$

$I = 1, 2, \ldots, M$

$X(J) = 0$ or 1

$J = 1, 2, \ldots, N$

where $C(J) \geq 0$; $J = 1, 2, \ldots, N$

The notation used is:

FREE = the set of subscripts of the variables that have not been specified to be 0 or 1.

NFREE = the set of subscripts of the variables that have been specified to be 0 or 1. If an element of NFREE is negative, the corresponding variable has been specified to be 0, otherwise, it has been specified to be 1. The first element (left most) in NFREE corresponds to the first variable specified to be 0 or 1. Likewise the second element in NFREE corresponds to the second variable specified to be 0 or 1 etc.

ZMIN = the value of the objective function corresponding to the best feasible solution to date

VC = the set of violated constraints.

T = the variables in FREE that have

(a) An objective function coefficient less than BOUND,

where, $\text{BOUND} = \text{ZMIN} - \sum_{I \epsilon \text{NFREE}} C(I)\,X(I)$

(b) A positive coefficient in some constraint in VC

$\sum_{I \epsilon \text{FREE}}$ = sum over all subscripts in NFREE

STEP 1

Set FREE = (1, 2, ..., N)

NFREE = ϕ, the empty set

ZMIN = 10^{10}

STEP 2

Calculate $Z = \sum_{I \in NFREE} C(I)\, X(I)$

Note that some of the X(I)'s in the above sum may be specified to have the value 0.

STEP 3

Evaluate each constraint Q(I), (I = 1, 2, ...,M) using the NFREE variables each set equal to 0. If each of the constraint are feasible (satisfied), then the values of variables used to evaluate the constraints constitute a feasible solution.

Let VC denote the set of violated constraints.

STEP 4

If VC is empty, go to step 12; otherwise, go to step 5.

STEP 5

Set BOUND = ZMIN - Z

STEP 6

Select the FREE variables that have a chance to make all of the constraints feasible. That is let T be the set of variables in FREE that have

1. A positive coefficient in some constraint in VC
2. An objective function coefficient < BOUND

A violated constraint can only be made 'more' infeasible by setting to 1 a variable with negative coefficient in the constraint, so only variables with a positive coefficient in a given constraint have a chance to make the constraint feasible (≥ 0). Likewise, a variable X(K) in FREE such that

$$\sum_{I \epsilon NFREE} C(I)\ X(I) + C(K) \geq ZMIN$$

should not be considered for inclusion in NFREE since the feasible solution corresponding to ZMIN is already atleast as good.

STEP 7

If T is empty, go to step 1; otherwise go to step 8.

STEP 8

For each constraint in VC

Set to 1 the FREE variables in T that have positive coefficients in the given constraint.

Set the NFREE variables equal to their specified values.

STEP 9

If any of the constraints are still violated, go to step 11; otherwise go to step 10.

STEP 10

Remove from FREE and add to NFREE the variable in T that would minimize the total distance from feasibility over all constraints. This process is covered in detail in steps 10A - 10C.

STEP 10-A

For each variable, say X(K), in T, evaluate each constraint

Q(I), (I = 1, 2, ...,M), using the NFREE variables with their specified values, X(k) = 1, and the remaining FREE variables each set equal to 0.

STEP 10-B

Sum the negative results from step 10-A and let ASUM be the absolute value of sum. The absolute value of each negative result is the amount the corresponding constraint must be increased to be feasible. Hence, ASUM represents in some sense the total distance from feasibility using X(k) = 1.

STEP 10-C

Remove from FREE and add to NFREE the variable in T that has the smallest total distance from feasibility (the smallest ASUM). Go to step 2.

STEP 11

If NFREE is empty, go to step 21; otherwise no feasible completion of the partial solution represented by NFREE has a smaller value than the current ZMIN, so go to step 16.

STEP 12

The variables in NFREE with their specified values, along with the variables in FREE set equal to 0, from a complete solution. Go to step 13.

STEP 13

If Z <ZMIN, go to step 14; otherwise, go to step 15.

STEP 14

Set ZMIN = Z. Save the complete solution and go to step 15.

STEP 15

Back track. If NFREE is empty, the feasible solution $X(I) = 0 (I = 1, 2, ..., N)$ is optimal, so go to step 20; otherwise go to step 16.

STEP 16

If the last element in NFREE is negative, go to step 18; otherwise go to step 17. The rightmost element in NFREE is considered to be the last element in NFREE.

STEP 17

Make the last (rightmost) element in NFREE negative and go to step 2. The variable corresponding to the last element has been specified to be 1 (corresponding subscript in NFREE has been positive). We now specify the variable to be zero (change the sign of the last element in NFREE to minus).

STEP 18

If all ements in NFREE are negative, an optimal solution has been reached, so go to step 20; otherwise go to step 19.

STEP 19

Make the rightmost positive element in NFREE negative and remove the remaining elements to the right from NFREE. Add the dropped elements to FREE. Go to step 2.

STEP 20

The complete solution corresponding to ZMIN is optimal. If $ZMIN = 10^{10}$, no feasible solution. Print results. Stop.

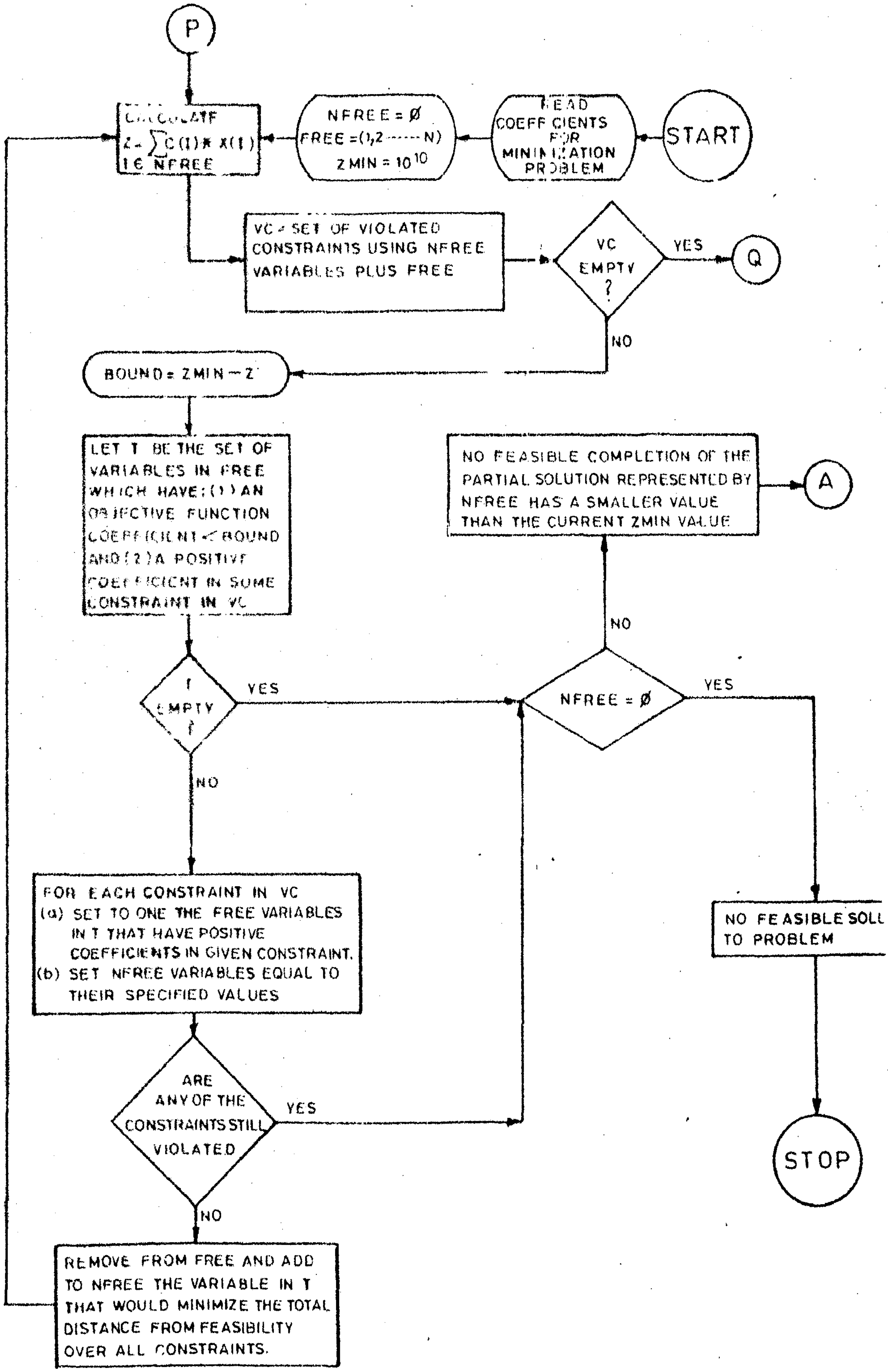

FIGURE 2.7.1 FLOWCHART FOR ALGORITHM

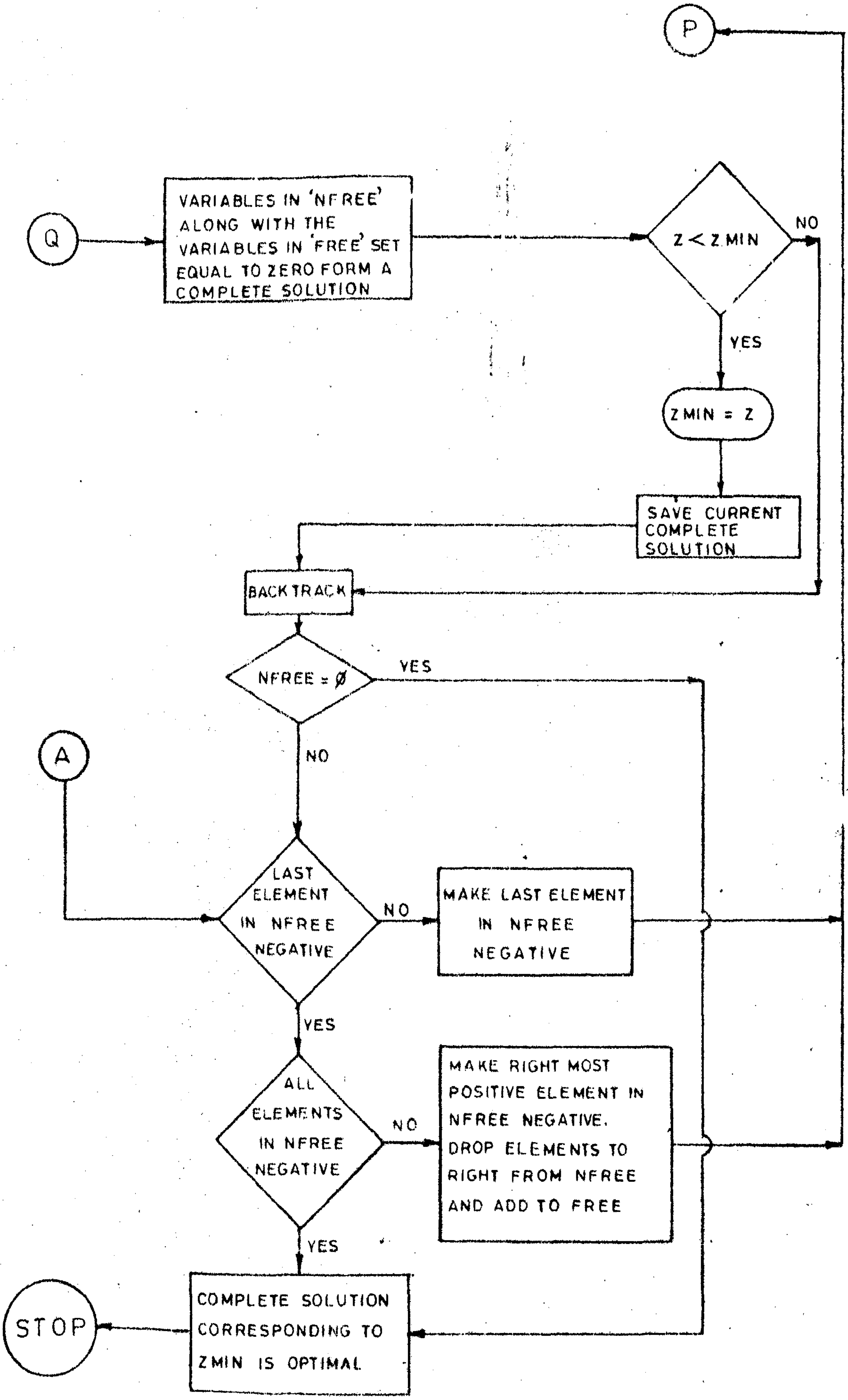

FIGURE 2.7.1 FLOWCHART FOR ALGORITHM

STEP 21

No feasible solution to the problem. Stop.

The flow chart for the Algorithm is given in Fig.2.7.1.

Computer Program for the algorithm:

The program used fwom reference(18) is given as Appendix. The following notations are used in the program.

M - Total number of constraints

K - No. of variables

NLET - Number of < OR = constraints

NGET - Number of > OR = constraints

NET - Number of = constraints

NTYPE - 0 for minimization problem
1 for maximization problem

CODE(I)- 0 if < OR = constraint
1 if > OR = constraint
2 if = constraint

B(I) = Constant in the constraint

A(I,J) = Coefficients in the constraints

C(J) = Cost coefficients of objective function

The program will solve both maximization and minimization problems with positive or negative cost coefficients. The problem is presented to the program in its original form and the program takes care of the necessary bookwork to get it in the standard form for the implicit enumeration algorithm. The only requirement is that the objective function and/or the constraints must be

THE ORIGINAL COEFFICIENTS

C ODE 0 > R=CO
CODE 1 > >OR=(
CO DE 2 >

1 CODE	CONSTANT	A(1,1)	A(1,2)	...	(1,8)
+ 1	0	2	1 0 0		0 0 0
+ 2	0	2	1 0 0		0 1 0
+ 3	0	2	0 0 1		0 0 0
+ 4	0	2	0 0 0		1 0 0
+ 5	0	2	1 1 0		0 0 0
+ 6	0	2	0 0 0		1 0 0
+ 7	0	2	0 1		0 0

OUT-PUT LISTING 2.8.1

THE ORIGINAL COEFFICIENTS OF THE CONSTRAINTS

C ODE 0 > R=CONSTRAINT
CODE 1 > >OR=CONSTRAINT
CO DE 2 > =CONSTRAINT

I	CODE	CONSTANT	A(I,1)	A(I,2)	A(I,3)	A(I,4)	A(I,5)	A(I,6)	A(I,7)	A(I,8)
1	0	2								
+			1	0	0	0	1	0	0	0
			0	0	1	0	0	0	1	0
			0	0	0	0	0	0	0	0
2	0	2								
+			1	1	0	0	0	1	0	0
			0	0	1	1	0	0	0	1
			0	0	0	0	0	0	0	0
3	0	2								
+			0	1	1	0	1	0	1	0
			0	0	1	1	1	0	0	0
			1	0	0	0	0	0	0	0
4	0	2								
+			0	0	1	1	0	1	0	1
			0	0	1	1	1	1	0	0
			0	1	0	0	0	0	0	0
5	0	2								
+			1	0	0	1	0	0	1	0
			1	0	1	1	1	1	0	0
			0	0	1	0	0	0	0	0
6	0	2								
+			0	1	0	0	0	0	0	1
			0	1	0	1	1	1	0	0
			0	0	0	1	0	0	0	0
7	0	2								
+			0	0	1	0	0	0	0	0
			1	0	0	0	1	1	0	0

			0	0	0	0	0	0	0	0
8	0	2								
+			0	0	0	1	0	0	0	0
			0	1	0	0	0	1	0	0
			0	0	0	0	0	0	0	0
9	0	2								
+			0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0
			0	0	0	0	1	0	0	0
10	0	2								
+			0	0	0	0	1	0	0	0
			0	0	0	0	0	0	1	0
			0	0	0	0	1	1	0	0
11	0	2								
+			1	0	0	0	0	1	0	0
			0	0	0	0	0	0	1	1
			0	0	0	0	1	1	1	0
12	0	2								
+			1	1	0	0	0	0	1	0
			0	0	0	0	0	0	0	1
			1	0	0	0	1	1	1	1
13	0	2								
+			0	1	1	0	0	0	0	1
			0	0	0	0	0	0	0	0
			1	1	0	0	1	1	1	1
14	0	2								
+			0	0	1	1	0	0	0	0
			1	0	0	0	0	0	0	0
			0	1	1	0	0	1	1	1
15	0	2								
+			0	0	0	1	0	0	0	0
			0	1	0	0	0	0	0	0
			0	0	1	1	0	0	1	1
16	0	2								
+			0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0

			0	0	0	1	0	0	0	1
17	[illegible]	6								
+			2	0	0	0	1	0	0	0
			0	0	4	0	0	0	1	0
			0	0	0	0	2	0	0	0
18	0	6								
+			2	2	0	0	0	1	0	0
			0	0	4	4	0	0	1	1
			0	0	0	0	2	0	0	0
19	0	6								
+			1	2	2	0	2	0	1	0
			0	0	4	4	4	0	0	1
			1	0	0	0	2	2	2	0
20	0	6								
+			1	1	2	2	0	2	0	1
			0	0	2	4	4	4	0	0
			1	1	0	0	2	2	2	2
21	0	6								
+			4	1	1	2	0	0	2	0
			1	0	2	2	4	4	0	0
			0	1	1	0	2	2	2	2
22	0	6								
+			0	4	1	1	0	0	0	2
			0	1	0	2	2	4	0	0
			0	0	1	1	0	2	2	2
23	0	6								
+			0	0	4	1	0	0	0	0
			2	0	0	0	2	2	0	0
			0	0	0	1	0	0	2	2
24	0	6								
+			0	0	0	4	0	0	0	0
			0	2	0	0	0	2	0	0
			0	0	0	0	0	0	0	2
25	2	1								
+			1	1	1	1	0	0	0	0
			0	0	0	0	0	0	0	0

			0	0	0	0	0	0	0	0
26	2	1								
+			0	0	0	0	1	1	1	1
			1	1	0	0	0	0	0	0
			0	0	0	0	0	0	0	0
27	2	1								
+			0	0	0	0	0	0	0	0
			0	0	1	1	1	1	0	0
			0	0	0	0	0	0	0	0
28	2	1								
+			0	0	0	0	0	0	0	0
			0	0	0	0	0	0	1	1
			1	1	1	1	0	0	0	0
29	2	1								
+			0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0
			0	0	0	0	1	1	1	1

THE COEFFICIENTS IN THE ORIGINAL OBJECTIVE FUNCTION TO BE MINIMIZED ARE:

15	15	15	15	6	6	6	6
6	6	21	21	21	21	5	5
5	5	5	5	15	15	15	15

OPTIMAL SOLUTION

VARIABLE 1 HAS VALUE OF 0
VARIABLE 2 HAS VALUE OF 0
VARIABLE 3 HAS VALUE OF 1
VARIABLE 4 HAS VALUE OF 0
VARIABLE 5 HAS VALUE OF 0
VARIABLE 6 HAS VALUE OF 0
VARIABLE 7 HAS VALUE OF 0

```
VARIABLE  8 HAS VALUE OF  0
VARIABLE  9 HAS VALUE OF  0
VARIABLE 10 HAS VALUE OF  1
VARIABLE 11 HAS VALUE OF  1
VARIABLE 12 HAS VALUE OF  0
VARIABLE 13 HAS VALUE OF  0
VARIABLE 14 HAS VALUE OF  0
VARIABLE 15 HAS VALUE OF  1
VARIABLE 16 HAS VALUE OF  0
VARIABLE 17 HAS VALUE OF  0
VARIABLE 18 HAS VALUE OF  0
VARIABLE 19 HAS VALUE OF  0
VARIABLE 20 HAS VALUE OF  0
VARIABLE 21 HAS VALUE OF  0
VARIABLE 22 HAS VALUE OF  0
VARIABLE 23 HAS VALUE OF  0
VARIABLE 24 HAS VALUE OF  1

THE OPTIMAL VALUE OF THE OBJECTIVE FUNCTION IS        62
```

multiplied by proper constants to make all the input data in the integer form.

2.8 Results and Discussion:

The results obtained is shown in output listing 2.8.1. The optimal value of the objective function is 62.

We have used 0-1 integer linear programming approach with implicit Enumeration method. The total number of variables are 24. Each has to be assigned the value 0 or 1. The optimal solution found gives the value 1 to some of the binary variables while keeping the remaining values to 0. Interpreting the binary variable we get $Z_{13} = 1$, $Z_{26} = 1$, $Z_{31} = 1$ $Z_{41} = 1$ and $Z_{54} = 1$. While the other variables have the value zero. The Table 7 exhibits the understanding of the solution obtained.

It provides a schedule for the jobs and schedule for the skills. Second, it may be used for man-power planning by constructively using the excess skill supply table. If excess skill hours are available, management can elect to utilize the man-power by scheduling more or different jobs, or it can reassign the excess skill supply to other area of the plant.

The C P U time for the problem is 34.2 sec and the no. of iterations are 1064.

Schedule for skill requirements and supply

TABLE - 7

HOURS →

	S.NO.	1	2	3	4	5	6	7	8
Job ↓	1			1 M 2 H	1 M 2 H	1 E 1 H	1 E 1 H	1 M 4 H	
	2						1 M 1 H	1 E 0 H	1 M 2 H
	3	1 M 4 H	1 M 4 H	1 M 4 H	1 M 2 H	1 M 2 H			
	4	1 M 1 H	1 E 1 H	1 E 0 H					
	5				1E 2H	1 E 2H	1 E 2H	1 E 2 H	1 E 2 H
Skill required	M	2	1	2	2	1	1	1	1
	E	0	1	1	1	2	2	2	1
	H	5	5	6	6	5	4	6	4
Skill Supply	M	2	2	2	2	2	2	2	2
	E	2	2	2	2	2	2	2	2
	H	6	6	6	6	6	6	6	6
Escess Skill Supply	M	0	1	0	0	1	1	1	1
	E	2	1	1	1	0	0	0	1
	H	1	1	0	0	1	2	0	2

CHAPTER III

AN OPTIMUM MAN POWER UTILIZATION MODEL FOR HEALTH MAINTENANCE ORGANISATION

3.1 Nature of the Problem

The basic objective of a Health Maintenance Organisations (HMO's), While specifically formulated is to provide to specified subscriber population a comprehensive set of services in an organised manner for a fixed annual fee.

An HMO is an entity:

1) that serves an enrolledpopulation that contracts with the delivery system for provision of a pre-negotiated package of comprehensive health services,

2) that provides these services to all subscribers for a fixed payment on a periodic basis without regard to frequency, extent, or kind of service actually provided during the period,

3) that is managed in a manner to ensure legal, fiscal and professional accountability.

Principal trade offs to be considered in developing the planning model concern the delegation to auxiliary personnel of services that do not require the extended training of the physician, This process, however, is complicated.

The delegation of services to auxiliary personnel in the models is accomplished by introducing physician Extenders(PE's) in each primary care department. The physcian extenders considered are the two types. The first type is Physician Assistant(PA). The other type is nurse and is trained in a Program designed to accepteNurses(RN's).

Utilization of auxiliary personnel in primary-care delivery is usually done in a context of a health care Team'. Auxiliary personnel is to be supervised by an M.D.

The three team is comprised of

(a) a physician, an RN, and possible LPN (Licenced Practical N).

(b) a physcian, a physcian extend-or, and a RN.

(c) a physician extender, an RN and possibly an LPN.

Ultimate responsibility for the patient, however, still world reside with the physcian.

3.2 Development of Model

Initially we describe a very simple model to motivate the structural relations. We then under take a step-by-step embellishment that culiminates in a more complex but more flexible and realistic model. The inpatient care and dental care portions of an HMO are not specifically addressed. This implies not that there is no relation of ambulatory care to in patient and dental care but that these relation can be handeled externally.

Also within the HMO structure, there are services and management activities (such as laboratory and radiology services clerical, administrative, and financial function) that support the medical care function. These are included in the models to permit an examination of the full costs of providing a specified set of services to a subscriber group.

Within the medical care system there are two main populations to consider the subscribers and the providers.

The subscriber services (requirements) must be expressed in some taxonomy of health care delivery, and manpower (resources) must be expressed in terms of provides types and capabilities.

Two objectives present themselves for this elementary model:

(1) Minimize the medical power for a given no. of subscribers.

or (2) Maximise the subscribers for a given no. of medical personnel.

The first objective takes the requirements as fixed and the resources as variable, while the second objective is the converse of the first.

We use the following set of names:

I = set of medical personnel of type i,

I = set of M.D.'s,

I = set of Non M.D.'s

J = set of medical services of type j,

x_{ij} = number of manpower type i performing service j, (man-years)

d_j = demand for service j (medical services per year).

b_{ij} = rate at which manpower type i produces service j at a given level of technology (annual medical services per man-year)

N_i = number of manpower type i employed (man-years)

S_i = salary for type i personnel (dollars/year)

We can meet the medical care requirements with the constraint:

$$\Sigma_i \, b_{ij} \, x_{ij} \geq d_{ij} \qquad \forall_j \epsilon J \tag{3-1}$$

We can also define the number of man power type i employed as

$$\Sigma_j \, x_{ij} - N_i \leq 0 \qquad \forall i \epsilon I \tag{3-2}$$

and objective (1) becomes

$$\min \ \Sigma_i \ N_i \tag{3-3}$$

This model does not allow a full exploitation of the structure of the medical care process. It also ignors the cost structure in which the medical system operates

3.2. Development of the Medical Care Aspects of the Model:

Medical care can be provided either by individual or by teams. Within a health care team the leader is typically an M.D.; with the following additional definations the previous model supposed it is to be called M1 can be revised to provide the needed generality.

Let I^s = set of individual health care personnel

I^t = set of health care teams

I = set of man power including teams,

q_{kij} = man-years of man-power type i per man-year of team k providing service j.

$$\Sigma_k = \Sigma_{k \epsilon I^t} \quad \text{and } I = I^* \cup \bar{I}^* \cup I^t$$

The particular introduction and definition of q_{kij} is the key to the generalization of the model. Since expressions (1) and (3) are unaffected, the only change occurs in (2), which becomes

$$\Sigma_j \; x_{ij} + \Sigma_k \Sigma_j \; q_{kij} \; x_{kj} - N_i \leq 0, \quad \forall_i \; \epsilon I \qquad (3\text{-}4)$$

In addition, a method of incorporating indirect supervision must be introduced, not only for PE-nurse teams but also for PE's or nurses in an individual capacity. Defining

f_{ij} = level of independence for type i personnel to perform service j;

f_{ij} = 0 implies no indirect supervision required while f_{ij}= 1 implies full indirect supervision,

J_{i^*} = Set of services class $i^* \epsilon I$ personnel supervise,

$x \; i^* j^*$ = Supervisory level manpower i^* assigned to the supervisory service j^* (man-years) for each i^*. there is only one j^*, the double subscript is retained for notational symmetry.

The supervisory constraint involves legal restrictions and possibly the quality of health care in that it constraints the number of personnel a proffessional staff member may supervise and the level of independence exercised by assistants. The general supervisory constraint is given by

$$\Sigma_{j \in J_i^*} \Sigma_{i \in I^*} f_{ij}\, b_{ij}\, x_{ij} \leq b_{i^* j^*} \beta\, x_{i^* j^*} \quad \forall_{i^*} \in I^* \qquad (3\text{-}5)$$

It is generally accepted that M.D.'s do not want to be burdened by an excessive amount of administrative work or supervision of ancillary personnel. The amount of supervision can be limited without increasing the difficulty of the solution by making $x_{i^* j^*}$ an upper bounded variable. Thus, defining SU_i as the maximum fraction of time type i personnel will engage in indirect supervision of ancillary personnel, we give the upper bound by

$$x_{i^* j^*} \leq SU_i N_i \qquad \forall_{i^*} \in I^* \qquad (3\text{-}6)$$

The resources considered thus far in the model are the available health manpower. Since an HMO has as a principal objective the delivery of effective health care in a cost conscious manners, the cost of providing the manpower resources must also be evaluated. Also, in many cases the employees will be full-time employees only; this can be represented by integer variables in the model. Thus, Let N_i be restricted to be an integer variable and define

x_i' = fractional part of personnel group i that is slack or idle,

C_{ij} = labour costs for type i personnel or team to perform one type j services (dollars/medical service)

The result is

$$C_{ij} = S_i/b_{ij} \quad \forall \ i \epsilon I^s,$$

$$C_{ij} = \left(\frac{1}{b_{ij}}\right)\left(\Sigma\ n \epsilon I^s \ S_n q_{inj}\right. \quad \forall \ i \epsilon I^t \qquad (3\text{-}7)$$

To represent teams the definition for b_{ij} mustbe modified to

$$b_{ij} = \frac{\text{Units of time worked by team leader/year}}{\text{Units of team leader time/medical service}}$$

$$\forall \ i \epsilon I^t$$

Incorporating the integer manpower constraint for N_i and noting that summation over j includes the supervising index, we see that constraint(4)

$$\Sigma_j x_{ij} + \Sigma_k \Sigma_j \ q_{kij} x_{kj} + x_i' - N_i = 0$$

$$\forall \ i \epsilon I \qquad (3\text{-}8)$$

and the objective becomes

$$\min \Sigma_j \Sigma_i (b_{ij} C_{ij}) x_{ij} + \Sigma_i \ S_i x_i' \qquad (3\text{-}9)$$

It is also possible to generalize and refine the requirements constraint, which is given by constraint (1). Physically, It is impossible for a HMO to deliver more services than are demanded by the patients. It is, however, possible to deliver fewer services, but this would not be consistent with the concept of prepaid practice. The remaining option is to provide some health services by extra ordinary means.

Thus, a new variable n_j can be defined as
n_j = number of type j services demanded per year and not provided by ordinary means.

This variable can be incorporated in constraint (1) to give

$$\Sigma_i \; b_{ij}x_{ij} + n_j = d_j \;\forall_{j\epsilon J} \qquad j \neq j^* \qquad (3\text{-}10)$$

The n_j variable has several possible interpretation with an HMO structure. In a multiclinic HMO It may be desirable to schedule some patients into a more heavily staffed clinic during peak periods. Another interpretation is that the HNO may contract out to other health care providers for their overload patients. A third possibility is that n_j represents those services performed on an overtime basis.

Several restrictions must be placed on the n_j variable for the solutions to be reasonable. To develop these restrictions, define the coefficients:

U_j = per unit cost for provision of type j services by extraordinary income, and

MX_j = max. fraction of services provided by extra ordinary means

Constraint(10) shows the requirements cna be provided by n_j, which has no cost in the objective function. To rectify this problem, (9) can be modified to

$$\min \Sigma_i\Sigma_j(b_{ij}C_{ij})x_{ij} + \Sigma_i \; S_i x_i^* + \Sigma_j \; U_j n_j \qquad (3\text{-}11)$$

In addition, from a standpoint of convenience to the patient and paper work problem for the HMO, it is desirable to limit the services to be provided by extraordinary means. This limitation can easily be included in the model by using an Upper bound constraint

$$n_j \leq Mx_j d_j \quad \forall_{j\epsilon J} \qquad (3\text{-}12)$$

3.3 Further Development of Financial Aspects of the Model

Thus for the financial aspects of the model have not been fully developed. Health man-power costs have been considered, but there are significant cost elements to be included. To determine the total cost of medical service, It is necessary to include manpower overhead cases, facility overhead costs, and ancillary costs.

In most cases, an HMO will be organized in departments along medical speciality lines such as adult medicine, pediatrics, and OB/GYN. This departmentalization and patients ready access to any department lead to little overlap in the functions carried out in the various departments. Defining:

V_m = Zero-one decision variable; if

V_m = 1 department m is offered and if

V_m = 0 department in is not offered;

M = Set of all medical departments under consideration

I^m = Set of personnel in department m;

J = Set of services considered for provision;

Let $J = (1, 2, \ldots, j_1^*, \ldots, j_{m-1}^*, j_m^*)$

where j_m^* is the supervisory service provided by set I^* personnel to supervise set I^* personnel independment m;

j^* = Subset of j for which $V_m = 1$; and

j^m = Set of services considered for provision or to be provided in department m.

Thus, $J = (U_m J^m) \cup (U_m \{\dot{J}^*_m\})$. The V_m variable can thus be used as a decision variable to decide whether it is economically desirable to include a department in one or more clinies of an HMO or whether these services can best be obtained elsewhere . If an HMO decided not to maintain a given department, it could contract out to another health came provides or for an optional service remove it from the list of benefits. If the departments are fixed, than the V_m would be prescribed, which makes the model easier to solve.

In the development of the cost model there will be three main overhead cost categories manpower overhead costs, facility overhead costs, and laboratory and x-rays costs. The costs for these functions will be reflected in the model, but no optimization w.r. to them will be done. Three major variables are used to allocate overhead costs to the revenue cost centers: departmental service performed departmental budget, and the number of departmental employees. For the purposes of defining the model , the manpower overhead costs are broken down and allocated in three basic parts:

(a) medical records, membership, appointments, etc. (allocated on the basins of patients visits to department).

(b) management, planning, legal service etc. [allocated on the basic of full time equivalent proffessionals is a dependent];

(c) personnel service, administrators of employee benefits, etc. (allocated on the basis of FTE employees in a department)

With the above guidelines, define:

AD_m = manpower overhead or adminstrative costs for department m (dollers per year)

MAN = management and planning cost per FTE(full time Employee) professional (dollers/man-year)

PER = personnel cost per FTE employee (dollers/man-year)

PV = man power overhead cost per patient visit (dollers per patient visit),

$b_{j'}$ = number of type j services physician can perform per year by a given specialty and for a given level of technology.

The number of FTE proffessionals needed to provide medical services for a department is given by

$$FTE_m = \Sigma_{j \in J^m} (d_j V_m - n_j)/b_{j'} \qquad (3\text{-}13)$$

which can then be used to define the manpower overhead costs for the nth department

$$AD_m = PV\Sigma_{j \in J^m} \Sigma_{i \in I^m} b_{ij} x_{ij} + MAN \Sigma_{j \in J^m} (d_j V_m - n_j)/b_j + PER \Sigma_{i \in I^m} N_i \qquad (3\text{-}14)$$

To continue with the model development, define the following terms:

g = initial cost per unit area for construction (dollers per unit area)

g' = amortized cost per unit area for construction (dollars per unit area per year)

O_c = maintenance and utility costs (dollars per unit area per area per year)

P_c = amortization rate for initial capitalizations

t_m = cost of equipment in department m per FTE professional (dollars/man-years)

t'_m = amortized cost of equipment in department m per FTE professional (dollars /man-years/year)

w_i = Space required per type i person (units of area per man-years),

Y^* = initial capitalization (dollars).

If an HMO is in a planning stage, there may be an upper limit on capital available for construction, which requires the constraint

$$g \Sigma_i w_i N_i + \Sigma_m t_m FTE_m \leq Y^* \qquad (3\text{-}15)$$

The capitalization costs must also be amortized and appear in the objective function, which will be shown shortly. The plant operation and maintenance cost must also be developed. This cost is

$$O_c \Sigma_i w_i N_i \qquad (3\text{-}16)$$

The overhead costs, except the ancillary costs for the laboratory and x-ray departments, have none all been included. With suppicient data these departments could be treated like the medical departments and the following development excluded. However, since optimal stiffing patterns would not be affected, by additional detail in those departments, they are treated separately.

Define the following coefficients:

CLT = average cost per laboratory test (dollars/ test)

CXR = average cost per X-ray service (dollars/service)

NLT_m = no of Laboratory tests ordered in department m per service provided.(tests per medical services)

NXR_m = number of x-ray services ordered in department m per service provided (x-ray services/medical service).

From the above definition the laboratory and x-ray costs are thus

$$\Sigma_m (CLT.NT_m + CXR.NXR_m)\ \Sigma_{j \in J^m}\ \Sigma_{i \in I^m}\ b_{ij}\ x_{ij} \qquad (3\text{-}17)$$

3.4 Additional Model Refinements

Previously indirect supervision of Physician extenders and nurses, constraints (5) and (6) arose mainly from time requirements ofor Physician to supervise auxiliary personnel. Moreover, from the patient care was point a PE is not qualified to diagnose and triat every patient under the indirect supervision mode. For many health services, a PE can be used only under direct supervision. That limitations results in a upper bounded variables, MAX_{ij}, the maximum percent of service j that can be carried out by personnel i in the indirect,supervision role. This coefficient will be used for those indices $i \in I^P$ that refer to PE's. The upper bound is thus

$$x_{ij} \leq MAX_{ij} d_j \quad \forall_{j \in J}\ i \in I^P \qquad (3\text{-}18)$$

Two further refinement involve the cost structure. Even though

HMO's will operate on a capitation basis, a small fee for service (e.g., $ 100 to $ 200 per visit) may be corrected. Let r_j denote the fee for service j; the income from the fees is then

$$\Sigma_j \, r_j \, (\Sigma_i \, b_{ij} x_{ij}) \qquad (3\text{-}19)$$

Because HMO's provide medical care on a capitation basis and are further constrained to operate within a budget defined by the Capitation income plus any external source of revenue, the yearly operating expenditures must be less than the budgeted amount to cover cost of operation. The budget constraint is developed for a non profit institution. For a profit making institution appropriate changes can easily be made. Define the coefficients.

B^* = Upper limit on yearly budget for operating and salary expenses for medical departments included in the optimization (dollars per year).

OM = Other medical costs, such as costs of operating departments that are not included in the optimization, inpatient care, out of area professional services, additional cost of premium plans, etc.(dollars).

P = rate at which capital fund accumulates as a function of gross income from subscriber capitation fees.

R = Capitation rate (dollars/subscriber/year).

EXT= external source of revenue such as planning grants or endowments (dollars/year).

Now since the initial capitalization, Y^*, is being amortized at the rate P_c, the result is $B^* = (1 - P)S.R - P_c Y^* - OM + EXT$ (3-20)

3.5 Over All Planning Model

The entire model can now be summarized. The objective function must be revised to reflect the income from fees, and FTE_m and AD_m will be replaced by their definitions given by (13) and (14) respectively. The result is the over all planning model (OPM):

$$\min[g^* \Sigma_i v_i N_i + \Sigma_j \Sigma_i (b_{ij} C_{ij}) x_{ij} + \Sigma_i S_i x_i^* + \Sigma_m (1-V) \Sigma_{j \in J^m} d_j U_j$$

$$+ \Sigma_j U_j n_j + \Sigma_m t_m^* (\Sigma_{j \in J^m} (d_j V_m - n_j)/b_{j*}) + O_c \Sigma_i v_i N_i - \Sigma_j r_j (\Sigma_i b_{ij} x_{ij}) +$$

$$+ \Sigma_m (CLT.NLT_m + CXR.NXR_m) . \Sigma_{j \in J^m} \Sigma_{i \in I^m} b_{ij} x_{ij} +$$

$$+ \Sigma_m (PV \, \Sigma_{j \in J^m} \Sigma_{i \in I^m} b_{ij} x_{ij} + MAN \, \Sigma_{j \in J^m} (d_j V_m - n_j)/b_{j*} +$$

$$+ PER \, \Sigma_{i \in I^m} N_i)] \qquad (3\text{-}21)$$

Subject to

$$\Sigma_j x_{ij} + \Sigma_k \Sigma_j q_{kij} x_{kj} + x_i^* - N_i = 0 \quad \forall_{i \in I} \qquad (3\text{-}8)$$

$$\Sigma_i b_{ij} x_{ij} + n_j - d_j V_m = 0 , \forall j \in J, \; j \neq J^*. \qquad (3\text{-}10)$$

$$\Sigma_{j \in J_{i^*}} \Sigma_{i \in I^*} f_{ij} b_{ij} x_{ij} - b_{i^* j^*} x_{i^* j^*} \leq 0, \forall i^* \in I^* \qquad (3\text{-}5)$$

$$\Sigma_j \Sigma_i (b_{ij} c_{ij}) x_{ij} + \Sigma_i S_i x_i^* + O_c \Sigma_i v_i N_i -$$

$$\Sigma_j r_j (\Sigma_i b_{ij} x_{ij}) + \Sigma_m (CLT.NTM_m + CXR.NXR_m).$$

$$\Sigma_{j \in J^m} \Sigma_{i \in I^m} b_{ij} x_{ij} + \Sigma_m (PV \Sigma_{j \in J^m} \Sigma_{i \in I^m} b_{ij} x_{ij} + MAN \, \Sigma_{j \in J^m} (d_j V_m - n_j)/b_{j*}$$

$$+ PER \, \Sigma_{i \in I^m} N_i) \leq B^*$$ (3-24), (3-16), (3-17), (3-20).

$$g \cdot \Sigma_i \, w_i N_i + \Sigma_m \, t_m \, \Sigma_{j \in J} m(d_j V_m - n)/b_j \leq Y \, , \qquad (3\text{-}13,\ 3\text{-}15)$$

$$x_{i\ j} \leq Su_i N_i \quad \forall \, i \in I \, . \qquad (3\text{-}6)$$

$$n_j \leq Mx_j d_j \quad \forall \, j \in J \qquad (3\text{-}17)$$

$$x_{ij} \leq MAX_{ij} \, d_i \quad \forall \, j \in J, \, i \in I^P \qquad (3\text{-}18)$$

N_i, V_m Integer

Thus, the problem is to find the x_{ij}, N_i, n_j V_m to minimize(21) subject to the indicated constraints. Note that OPM has about 25 percent more variables and about 5 percent more constraints than the M1. Thus the model was enriched principally through the refinement of constraints that existed in the M1 or the addition of upper bounded variables. Upper bounds, added to a model, cause only small increases in solution time.

<u>CONCLUSION</u>

The various aspects of scheduling man power in various field of applications have been discussed in first chapter. The mathematical model for the Plant Maintenance personnel was developed in second chapter. It was found that it could in corporate different values for time Horizon such as day, week etc. and accordingly the jobs could be broken in interval of more than 1 hour. It was found that the size of the original model could be reduced by using the physical interpretation of the variables and mathematically replacing the binary variable Y_{ijhl} by Y_{jll}. It was also discussed that the job-hour inequalities reduced to $Y_{jll} \leq 1$. This is true by the defination, So the job-hour inequalities vanishes from the formulation.

Again It was discussed that if Y_{jll} replaced by Z_{jl}, the formulation again reduced. The selection equations $\sum_{j=1}^{1} Z_{jl} = 1$, $Y_{jll} = Z_{jl}$ reduced to $\sum_{j=1}^{1} Z_{jl} = 1$. The comparision of each model formulation has shown the difference generated by reducing the total [unclear] no. of variables and total expressions.

The precedence constraints could also be incorporated in the reduced size model if it was needed in certain type of jobs scheduling.

The preparation of the data file was a tedious job particularly when the no. of variables and no. of expression increased.

In the discussion of the result, it was found that we could get a picture of each type of skill used in every hour of the time horizon. The excess skill supply could be accomodated in other works of the plant in the specified hour.

The mathematical model for the man-power planning for the maintenance of health organisation was developed in third chapter. To make it more relevant to the actual practice the various points have been added to develop the over all planning model.

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