

OPTIMAL DISTRIBUTION SUBSTATIONS AND PRIMARY FEEDER PLANNING

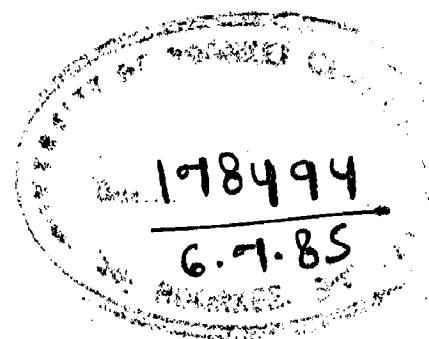
A DISSERTATION

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requirements for the award of the degree
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By

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January, 1985

DEDICATED TO MY SISTER
GUDDI
WHOSE FRAGRANT MEMORY
LINGERS-----

CERTIFICATE

Certified that the dissertation entitled 'OPTIMAL DISTRIBUTION SUBSTATIONS AND PRIMARY FEEDER PLANNING' which is being submitted by Mr. RANVIR SINGH JALTA in the partial fulfilment of the requirements for the degree of Master of Engineering in Electrical Engineering (System Engineering and Operations Research) of the University of Roorkee, is a record of student's own work carried out by him under my supervision and guidance. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of Six Months from July 16th, 1984 to January 31st, 1985 for preparing this dissertation for the Master of Engineering Degree at this University.



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ABSTRACT:

Gigantic expansion in industries and awesome growth in population have caused great demand for electrical power. So the rapidly increasing load causes changes in power flow patterns in distribution networks. The distribution feeders get overloaded and the voltages at the load buses in the expanded system get violated because of the excessive voltage drop.

A mathematical model for distribution feeder planning problem has been developed in chapter 3. In this chapter the sensitivities of line currents due to feeder addition has been calculated using A.C. adjoint method which makes use of Tellegen's theorem. The details of Tellegen's theorem and the adjoint network approach to sensitivities used in simple networks and in power system have been discussed in Chapter 2.

A heuristic method for primary distribution feeder planning has been presented in Chapter 3. It provides the feeder additions to be made for alleviating the overloads and also the capacitor installation in the expanded system to keep the voltages at the distribution substations within the prescribed limits. The effect of the decision is observed by carrying out load flow using fast decoupled load flow method.

Chapter 4 deals with the optimization of various parameters of a distribution system. A mathematical model has been described to work out the different optimal parameters, i.e. substation feed area, number of substation, feeder main length, feeder service area, and number of feeder per substation.

LIST OF SYMBOLS AND ABBREVIATIONS

- V_K - Voltage in K^{th} branch of a network.
 i_K - Current in K^{th} branch of network.
 \bar{V}_K - Voltage in K^{th} branch of adjoint network.
 \bar{i}_K - Current in K^{th} branch of adjoint network.
 n - Total number of branches in a network.
 n_L - Load branches
 n_G - Generator branches.
 Y_j - Admittance of the j^{th} feeder.
 G_j - Conductance of j^{th} feeder
 B_j - Susceptance of j^{th} feeder
 V_i - Voltage at i^{th} node
 V_{CN} - Set of nodes for which the voltage correction is high.
 FV_{ij} - The square of the voltage at i^{th} node due to the addition of j^{th} feeder.
 C_{ij} - Current index factor.
 λ_i - No of existing feeders in i^{th} right of way.
 Y_i - Number of feeders to be added across i^{th} branch.
 I_{max}^o - Maximum current in the i^{th} feeder in existing network.
 V_{max} - Maximum Voltage

- V_{min} - Minimum Current
 M_{ij} - Measure index
 E_j - Effect index
 Y_{Km} - Series admittance for a feeder connected between bus K and m
 Y'_{Km} - Charging admittance
A - Area of study system in Sq.Km.
D - Connected load density in Kw/Sq.Km.
DR - Load diversity factor at feeder mains
pf - Power Factor
UF - Utilization factor of the transformer in substation.
 N_t - Number of transformer in substation
 a' - Fixed part of transformer core loss in Kw.
 b' - Variable part of the transformer core loss in Kw per KVA capacity of the transformer.
NLS - Expected life of substations in years.
 C_{oK} - Cost of energy at Kth year
U - Annual discount rate in p.u
 C' - Fixed part of transformer full load copper loss in Kw.
 d' - Variable part of transformer full load copper loss in Kw per KVA capacity of the transformer.
LLF - Loss load factor which is a function of LF and is of the form $LLF=A(LF)^2+B(LF)$ where $A+B = 1$, $LF=$ Load Factor

- R_s - Radius of circular feed area of secondary distribution substation.
- C_f - Feeder main cost in Rs/Km.
- C'_f - Lateral feeder cost in Rs/Km.
- L_s - Length of lateral feeder in Km.
- λ - Distance between the consumers.
- f - Cost of feeder bay which is known in dependently.
- c - Substation fixed cost in Rs.
- h - Substation variable cost
- N - Number of source points feeding the primary distribution system.
- C_{fo} - Cost of infeed circuit.
- a_s - Substation feed area
- L - feeder main Length
- n_f - Number of feeders per substation.
- V - Percentage voltage regulation of radial distribution feeder.
- Z - Zig-Zag factor of feeder main.
- H - Voltage regulation constant.
- KVA - Capacity of substation.
- F' - Objective function for secondary distribution.
- F'' - Objective function for primary distribution.

CHAPTER - 1

INTRODUCTION

Electricity is modern man's most convenient and useful form of energy. It is one of the important factor which plays a plognant role in any nations economic and technical development. Particularly in developing countries (like India) the demand for power is colossal. This naturally calls for expansion of existing transmission and distribution system.

Distribution system is the network which fans out from substation (which steps down the transmission voltage) to the consumer energy meter point. In the basis of voltage, the distribution system has been divided into primary and secondary distribution system.

Power from various generating stations is carried to various primary distribution substations at 33 to 220 KV where it is stepped down to 11 or 6.6 or even 3.3 KV. The power is then delivered to different secondary distribution substations, bulk supply high voltage consumers. At secondary distribution substation the voltage is again stepped down to 440 volts. From here the distributors radiate out to feed the low voltage consumers.

1.1 Importance of planned distribution system :

The planning of distribution system plays an important role in providing high standarde of power system reliability, security, quality and ensuring maximum utilization of capital investment.

Outage or failure in the distribution system immediately affect the consumers. In fact 90% of the consumer interruptions can be attributed to the distribution system. In addition distribution systems are generally more vulnerable and have less back up capacity than bulk power supply system.

In India distribution losses vary upto about 75% of the overall system losses. Poor voltage regulation being the usual problem at the peak hour. There are frequent cycles of power shortage imposing distribution restrictions. The annual load factor at the station bus bar is quite low. It is imperative to reduce all these deficiencies for the general well being of our society.

So we see that distribution system has a lion's share in the total power. The web of feeders has to be an elaborate one because of scattered load points. The significance of the distribution system can be fathomed from the fact that in the some of the electrical utilities it shares 40% of the total investment. Now the expansion of such a large system (to meet the rapidly increasing demand) is cumbersome task and quite often it is done in an unplanned manner. The repercussions of this haphazard expansions are appalling as it results in low voltage problems, fluctuations in voltage and frequent disruption of the power supply to the consumers. Also disproportionate expansion may lead to heavy loss of equipment putting the system economy in a precarious situation. Hence the best possible solution is to have an optimal system planning and design.

There is another factor which stresses this point. It has been seen specially in context to Indian electrical utilities that loads which are low in magnitude are spilled very widely. The resulting low load densities result higher energy losses. A survey has thrown light on these losses by concluding that they constitute 85% of the total power losses of the system—definitely a loss to reckon with. The cost of energy being maximum at the distribution level the losses become more acute because of large financial losses. Thus planned optimal distribution network planning becomes a complete necessity.

1.2 Objectives of distribution system planning :

The main objective of the distribution system planning can be defined as :

- 1) Adequacy of circuit.
- 2) Quality of circuit.
- 3) Economy of the system.

Along with objective certain constraints are to be satisfied. Since the minimum expenditure is the prime concern of any electrical utility, so the objective--- 'system economy' becomes the most significant one. It takes in to account the fixed cost (capital investment) and variable cost (cost of energy losses). Also it satisfies constraints comprised by feeder voltage regulation (which has to meet consumers requirements), substation feed area (which specifies optimal network configurations) and conductor size of the feeder. On the basis of substation area and feeder voltage, voltage regulation number of feeders are also decided.

1.3 Problems in distribution network planning :

There can be various types of problems which can be considered in distribution network planning. Few of them have been discussed below :

1.3.1 Distribution system parameter optimization :

In this problem the important parameters of distribution system, i.e. substation feed area, substation size, feeder main length, feeder service area, and number of feeders per substation are optimized.

The cost of capital investment and cost of energy losses are minimized. Thus a solution is reached where the total cost of the distribution system is minimum.

The problem can be solved by number of mathematical techniques. However a mathematical model is used in chapter-4 and optimal parameters have been obtained by solving a numerical sample problem.

1.3.2 Optimal sizing and siting of substations and network routing :

Certain areas like Urban are not infested with uncertainties associated with i) future load ii) future load locations iii) feasible sites of substation and feeder routings. Thus in those areas the aforesaid factors can be assessed with satisfactory accuracy with the help of optimal problem formulation. With the help of some devised problem solutions optimal size and locations of substations and actual routing of the

feeders can be envisaged with the proper satisfaction of demand and other constraints. However these solutions are not appropriate for rural areas because of the various uncertainties which crop up in the distribution system of these areas. The various approaches to solve these problems are Integer Programming, mixed Integer Programming, Transportation and Transshipment models, Branch and Bound technique. Dynamic Programming has been used extensively. Some work has been done by using Quadratic Mixed Integer Programming.

1.3.3 Optimal choice of shunt capacitors :

Low power factor has been a bane in any distribution system because of huge energy losses. The energy loss cost due to the precarious low value of power factor is high. Thus the improvement of power factor has become an important aspect in modern distribution planning. Low power factor is because majority of the loads are inductive in nature. Also long distribution feeders with large number of distribution transformers increases the effective system reactance, thereby lowering the power factor.

Of the various measures, use of shunt capacitor has been found effective for the improvement of power factor. Selection of optimal capacitor is governed by load densities, load factor, cost of system capacities and cost of power and energy distributed. Also the power factor keeps swinging between 0.6 (during peak load) and 0.9 (during light load) in a day. This further puts constraint on the extent and type of on line current

capactors. With large number of fixed capactors the over voltage problem results, leading to energy losses. On the other hand lesser number of capactors results in under compensation. So an optimal system has to be considered having appropriate number of fixed and switching capactors satisfying various constraints.

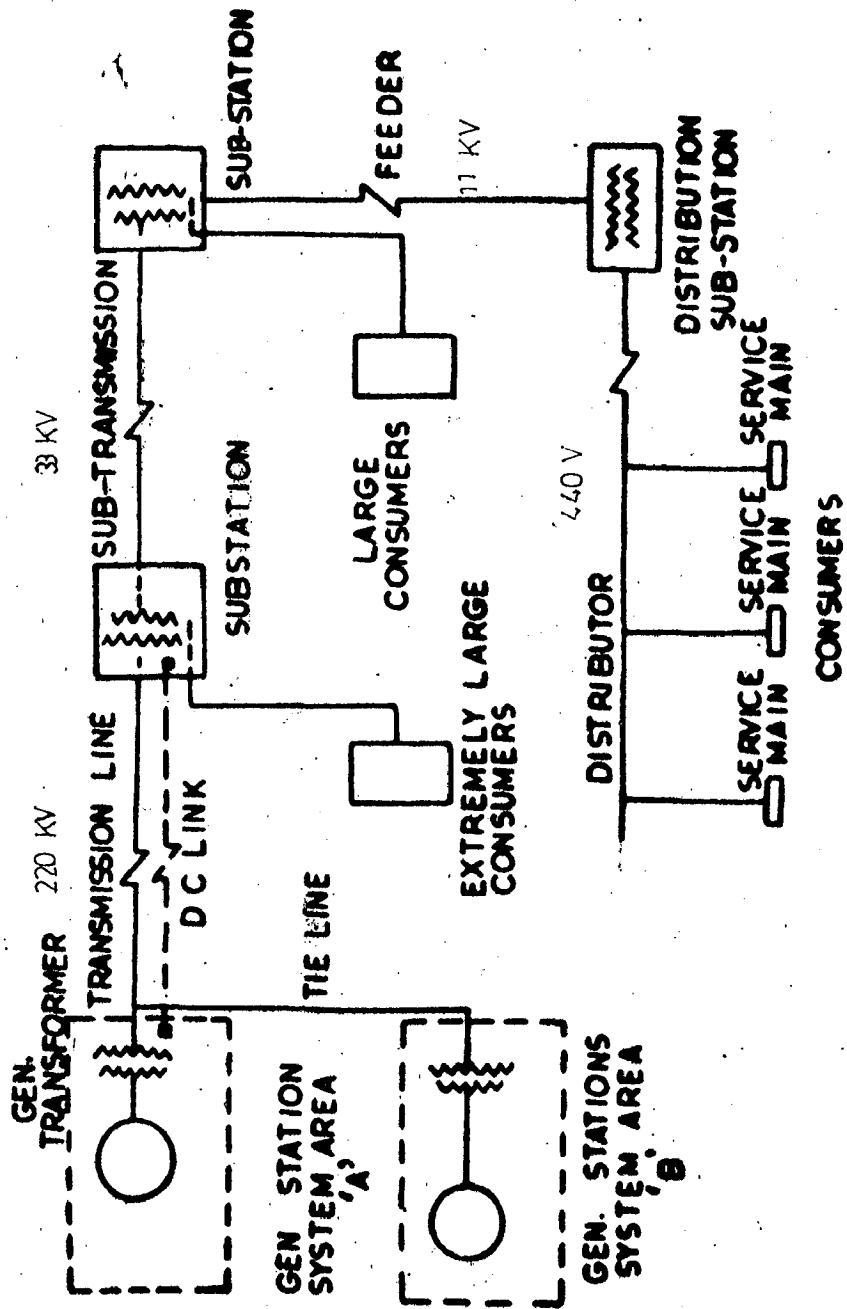
Simple but effective Dynamic Programming techniques are there to find number, location and size of shunt capactors. Method of local variation is also popular approach. The problem of shunt capacitor addition and their numbers has been solved in the third chapter using A.C. Adjoint method of sensitivities and heuristic approach.

1.3.4 Conductor gradation :

The cost of the feeder is proportional to the area of cross section of feeder conductor (both in radial as well as loop system). The sending end of the feeder (in case of radial feeder) carries maximum load where as the subsequent sections of the feeder carries lesser loads. This enables us to have different cross section along the feeder length. It definitely involves saving in system cost as compared to uniform cross section feeder.

The conductor gradation problem can be tackled by Linear Programming and Integer Programming. Some work has been done with the help of dynamic programming. Another approach which has better computational efficiency is method of local variation.

The problem of conductor gradation of loop network has been dealt in detail in third chapter using A.C. Adjoint method of sensitivity analysis and heuristic approach.



A. POWER SYSTEM

CHAPTER - 2

THE ADJOINT NETWORK APPROACH TO SENSITIVITIES

The sensitivities are used to indicate how the output varies with the element values. If the sensitivity is positive, the output increases (decreases) if the value of element increases (decreases) and when the sensitivity is negative the output decreases (increases) as the value of element increases (decreases). To find the upper limit of the output the ^{values of} elements with positive sensitivities should be increased and the ^{values of} elements with negative sensitivities should be decreased.

The technique of adjoint network is highly efficient and extremely accurate. Since we seek the sensitivity of the output to the changes of the element values, we desire some expression that relates the output to the network element. For this purpose Tellegen's theorem has been used which states:

If $v_1(t), v_2(t), \dots, v_{n_b}(t)$ are n_b branch voltages and $i_1(t), i_2(t), \dots, i_n(t)$ are n_b branch currents of a given n_b branch network comprised of arbitrary two terminal lumped element then

$$\sum_{k=1}^{n_b} v_k(t) i_k(t) = 0$$

The statement can be looked upon as statement of conservation of energy or power i.e. the total power generated by the network must be equal to the total power consumed by the network.

On generalizing, the Tellegen's theorem it can be applied to two different networks assuming same topology (structure)

even if the voltage and currents are measured at different times i.e.

If $v_1(t), v_2(t) \dots v_{n_b}(t)$ are n_b branch voltages of an n_b branch network and $I_1(\tau), I_2(\tau) \dots I_{n_b}(\tau)$ are n_b branch currents of another n_b branch network that has same topology as first then

$$\sum_{K=1}^{n_b} v_K(t) I_K(\tau) = 0 \text{ for all times } t \text{ and } \tau.$$

Also Tellegen's theorem can be applied to the voltage of network measured at time t_1 and currents of the same network measured at time t_2 .

Let v_o denote the output voltage and i_1 the input current of a given network N comprised of resistor as shown in Fig. (2. a) V_o is measured across the zero valued current source $I_o = 0$. The introduction of this source does not affect the operation of the network but merely provides a convenient branch across which to measure the output.

Assume N contains n_b branches, n of which are resistance branches numbered $1, 2, \dots, n$ and other two which are source branches numbered $n+1$ and $n+2 = n_b$. We consider second network termed the adjoint network (Fig. 2. b) denoted by \bar{N} that has exactly the same topology (structure) as N .

If v_K and i_K are used to denote the K^{th} branch current and voltage in N and \bar{v}_K and \bar{i}_K are used to denote the K^{th} branch voltage and current respectively in \bar{N} . Applying Tellegen's theorem

$$\sum_{K=1}^{n_D} v_K \bar{\phi}_K = 0 \quad \text{and}$$

$$\sum_{K=1}^{n_D} i_K \bar{v}_K = 0$$

We are interested as how variations of the element values of N affect the output v_o . The branch relationship of K^{th} ($K=1, 2, \dots, n$) branch of N is $v_K = R_K i_K$.

If each resistor is perturbed slightly, then branch current and voltage will be changed. If ΔR_K denotes the change in resistance, Δv_K the change in voltage and Δi_K the change in current of the K^{th} resistance branch then $(v_K + \Delta v_K) = (R_K + \Delta R_K)(i_K + \Delta i_K)$ for $K = 1, 2, \dots, n_D$. Expanding this expression, we have $v_K + \Delta v_K = R_K i_K + \Delta R_K i_K + R_K \Delta i_K + \Delta i_K \Delta R_K$

$$\text{but } v_K = R_K i_K$$

and $\Delta i_K \Delta R_K$ is a second order term that we assume is negligible so we have

$$v_K = \Delta R_K i_K + R_K \Delta i_K$$

For the input branch, i_1 remains fixed so that $\Delta i_1 = 0$. Similarly $\Delta i_o = 0$. Since the variations of the element values of N don't change its topology Tellegen's theorem can be still applied between varied original network and \bar{N}

$$\sum_{K=1}^{n_D} (v_K + \Delta v_K) \bar{\phi}_K = 0$$

and

$$\sum_{K=1}^{n_D} (i_K + \Delta i_K) \bar{v}_K = 0$$

But

$$\sum_{K=1}^{n_D} v_K \bar{\psi}_K = 0, \quad \sum_{K=1}^{n_D} i_K \bar{v}_K = 0$$

so that

$$\sum_{K=1}^{n_D} \Delta v_K \phi_K \quad \text{and} \quad \sum_{K=1}^{n_D} \Delta i_K v_K = 0$$

which can be combined to yield

$$\sum_{K=1}^{n_D} (\Delta v_K \phi_K - \Delta i_K v_K) = 0$$

or

$$\Delta v_1 \phi_1 - \Delta i_1 \bar{v}_1 + \Delta v_0 \phi_0 - \Delta i_0 \bar{v}_0 + \sum_{K=1}^n (\Delta v_K \phi_K - \Delta i_K \bar{v}_K) = 0$$

Recalling $\Delta i_1 = 0$ and $\Delta i_0 = 0$ and $\Delta v_I = \Delta R_K i_K + R_K \Delta i_K$

for $K = 1, 2, \dots, n$; hence

$$\begin{aligned} & \sum_{K=1}^n (\Delta v_K \phi_K - \Delta i_K \bar{v}_K) = \\ & = \sum_{K=1}^n [(\Delta R_K i_K + R_K \Delta i_K) \bar{\psi}_K - \Delta i_K \bar{v}_K] \\ & = \sum_{K=1}^n [(R_K \phi_K - v_K) \Delta i_K + (i_K \phi_K) \Delta R_K] \end{aligned}$$

Therefore we have the relationship

$$\begin{aligned} \Delta v_1 \phi_1 + \Delta v_0 \phi_0 + \sum_{K=1}^n [(R_K \phi_K - v_K) \Delta i_K \\ + i_K \phi_K \Delta R_K] = 0 \end{aligned} \quad \dots(2.1.1)$$

We desire to find an expression that relates changes in the output Δv_o , to change in the elements ΔR_k . The previous expression does contain Δv_o and ΔR_k as well as some additional terms. The additional terms can be eliminated by appropriately choosing elements in \bar{N} . Recall that to this juncture the only restriction placed as \bar{N} is that it ^{must} have the same network graph as N . Observing we find that if we choose the element of the branch in \bar{N} that corresponds to the k^{th} ($k = 1, 2, \dots, n$) resistance branch of N to be a resistor of value R_k then the branch relationship for this branch is

$$v_k = R_k \phi_k \quad k = 1, 2, \dots, n$$

Therefore,

$$R_k \phi_k - v_k = 0 \quad k = 1, 2, \dots, n$$

so that expression (2.1.1) becomes

$$\Delta v_1 \phi_1 + \Delta v_o \phi_o + \sum_{k=1}^n \Delta i_k \phi_k \Delta R_k = 0$$

$$k = 1$$

$$\dots (2.1.2)$$

v_1 can be eliminated from this expression by choosing the branch of \bar{N} that corresponds to the current source \bar{i}_1 in N to be zero valued current source, then $\phi_1 = 0$. Moreover, we can let the branch \bar{i}_o that corresponds to the current source i_o in N be unity valued current source so that $\phi_o = 1$. Finally this expression reduces to

$$\Delta v_o = \sum_{k=1}^n -i_k \phi_k \Delta R_k \quad \dots (2.1.3)$$

if only the k^{th} resistor is perturbed then

$$\Delta R_K = 0 \text{ for all } K \neq l$$

$$v_o = -i_l \phi_l - R_l$$

Therefore, the unnormalized sensitivity for R_l is

$$S_{R_l}^{v_o} = \frac{v_o}{R_l} = -i_l \phi_l$$

Normalized sensitivity for R_l is

$$S_{R_l}^{v_o} = \frac{R_l}{v_o} \frac{v_o}{R_l} = -\frac{R_l}{v_o} i_l \phi_l$$

$$l = 1, 2, 3, \dots, n$$

This result indicates that all sensitivities can be obtained after two network analysis: the analysis of the original network N yields the currents i_l , $l = 1, 2, \dots, n$; analysis of the adjoint network yields the currents ϕ_l , $l = 1, 2, 3, \dots, n$. The sensitivity results from the product of these currents.

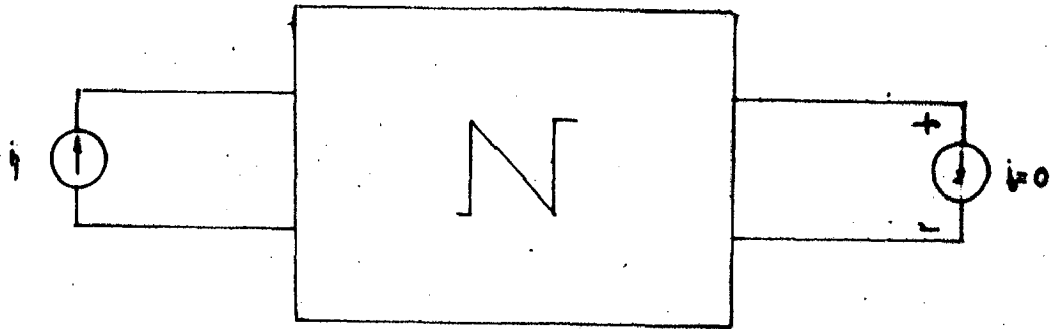


FIG 2.a

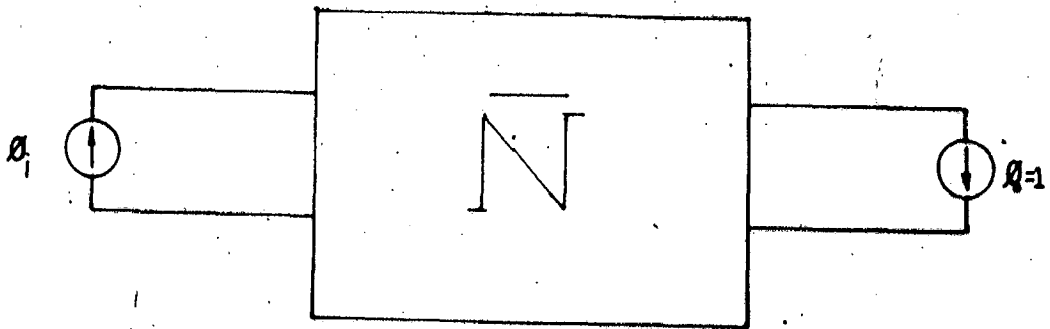


FIG 2.b

The sensitivity of V_{out} with respect to all resistors in N can be ascertained after two network analysis one on original network N and other as adjoint network \bar{N} .

2.1 ADJOINT NETWORK APPROACH TO SENSITIVITIES IN POWER SYSTEM

The adjoint network approach can be used for sensitivity analysis and gradient evaluation in power system planning and system analysis. The approach utilizes Tellegen's theorem in an augmented form which allows different power system problems to be handled based on a.c. power flow model in general and without approximations.

The approach provides the flexibility of including the line responses directly while reserving the advantages of compactness, sparsity and simplicity of the adjoint system. Fischl and Puntol [15] described the use of the adjoint network in transmission system planning problem based on linear d.c. power flow model. The d.c. power flow may be considered of sufficient accuracy for some applications. However, it is characterized by restrictive assumptions of neglecting transmission or distribution losses, excluding reactive power flows and considering flat voltages profiles which makes it inadequate for studies requiring more accurate model and more information.

Fischl and Woolley [11] have theoretically given an approach for calculating power flow sensitivities. It is based on a.c. power flow model and effectively utilizes an adjoint method to provide the gradients for several classes of functions. The system states are bus quantities so that the adjoint matrix of coefficients is the transpose of the original Jacobian matrix.

As mentioned before Tellegen's theorem is used in an augmented form to be directly applied for efficient sensitivity analysis and gradient calculations. Different types of functions can be considered. The size and the sparsity of the adjoint matrix is same as that of Jacobian matrix of the original network.

2.1.1 Use of Tellegen's Theorem in a.c. Power Model Formulation :

Let us assume that for a given network V and I represent the voltage and current variables and are complex in nature. \bar{V} and \bar{I} are the corresponding variables associated with topologically similar adjoint network. Applying the Tellegen's equation, which has been already discussed in detail, we have

$$\sum_a V_a \bar{I}_a = 0 \quad \dots \quad (2.1a)$$

$$\sum_a I_a \bar{V}_a = 0 \quad \dots \quad (2.1b)$$

where the subscript, 'a' denotes ath branch and summation is carried over all the branches in network.

As specified earlier the Tellegen's equation has certain special cases which will be used here. Taking complex conjugate terms corresponding to (2.1a) and (2.1b), namely,

$$\sum_a V_a^* \bar{I}_a^* = 0 \quad \dots \quad (2.2a)$$

$$\text{and } \sum_a I_a^* \bar{V}_a^* = 0 \quad \dots \quad (2.2b)$$

* = Complex conjugate

Considering the pair of power terms we have

$$\sum_{\alpha} S_{\alpha}^{\circ} = \sum_{\alpha} (P_{\alpha} + jQ_{\alpha}) = \sum_{\alpha} V_{\alpha}^{\circ} I_{\alpha}^{\circ} = 0 \quad \dots (2.3a)$$

$$\text{and } \sum_{\alpha} S_{\alpha} = \sum_{\alpha} (P_{\alpha} + jQ_{\alpha}) = \sum_{\alpha} V_{\alpha} I_{\alpha} = 0 \quad \dots (2.3b)$$

Observing the above equation we find that direction of power and current are same. Also, these terms are helpful in working out the generality, which we will be finding.

Writing equation (2.1) - (2.3) in terms of first order changes in voltage and current variables in the given network, and then with the help of mathematical adjustments the terms can be put in a proper manner which is

$$\begin{aligned} \sum_{\alpha} [I_{\alpha}^{\circ} \delta V_{\alpha} + I_{\alpha}^{\circ} \delta V_{\alpha}^{\circ} - V_{\alpha} \delta I_{\alpha} - V_{\alpha}^{\circ} \delta I_{\alpha}^{\circ} + \delta(V_{\alpha} I_{\alpha}) \\ + \delta(V_{\alpha}^{\circ} I_{\alpha}^{\circ})] = 0 \quad \dots (2.4a) \end{aligned}$$

or

$$\begin{aligned} \sum_{\alpha} [(I_{\alpha}^{\circ} + I_{\alpha}^{\circ}) \delta V_{\alpha} + (I_{\alpha}^{\circ} + I_{\alpha}^{\circ}) \delta V_{\alpha}^{\circ} + (V_{\alpha}^{\circ} - V_{\alpha}) \delta I_{\alpha} \\ + (V_{\alpha} - V_{\alpha}^{\circ}) \delta I_{\alpha}^{\circ}] = 0 \quad \dots (2.4b) \end{aligned}$$

From equation (2.4a) terms for different network elements can be considered. Let $l = 1, 2, 3, \dots, n_L$ identify load branches. $g = n_L + 1, \dots, n_L + n_G$ identify generator branches. $n = n_L + n_G + 1$ identify slack generator branch. Thus from equation (2.4a) a term associated with load is considered as

$$(a) \quad \bar{I}_1 \delta V_1 + \bar{I}_1^{\circ} \delta V_1^{\circ} - \bar{V}_1 \delta I_1 - \bar{V}_1^{\circ} \delta I_1^{\circ} + \delta S_1^{\circ} + \delta S_1$$

Since

$$\delta S_1 = \delta(V_1 I_1^{\circ}) = V_1 \delta I_1^{\circ} + I_1^{\circ} \delta V_1$$

We have

$$\delta I_1^{\circ} = [\delta S_1 - I_1^{\circ} \delta V_1] / V_1 \quad \dots \quad (2.5a)$$

hence,

$$\delta I_1 = [\delta S_1^{\circ} - I_1 \delta V_1^{\circ}] / V_1^{\circ} \quad \dots \quad (2.5b)$$

Substituting for δI_1° from (2.5a) and δI_1 from (2.5b) we obtain

$$\begin{aligned} & [\bar{I}_1 + \bar{V}_1^{\circ} I_1^{\circ} / V_1] \delta V_1 + [\bar{I}_1^{\circ} + \bar{V}_1 I_1 / V_1^{\circ}] \delta V_1^{\circ} \dots \\ & \dots \dots \dots (2.6) \\ & + [1 - \bar{V}_1^{\circ} / V_1] \delta S_1 + [1 - \bar{V}_1 / V_1^{\circ}] \delta S_1^{\circ} \end{aligned}$$

(b) A term of (2.4) associated with a generator is considered as

$$\bar{I}_g \delta V_g + \bar{I}_g^{\circ} \delta V_g^{\circ} - \bar{V}_g \delta I_g - \bar{V}_g^{\circ} \delta I_g^{\circ} + \delta S_g^{\circ} + \delta S_g$$

Note that

$$\delta |V_g|^2 = \delta(V_g V_g^{\circ}) = V_g \delta V_g^{\circ} + V_g^{\circ} \delta V_g \quad \dots \quad (2.7)$$

from which

$$\delta V_g^{\circ} = \delta(V_g V_g^{\circ}) / V_g - V_g^{\circ} \delta V_g / V_g \quad \dots \quad (2.8)$$

We note that the real part of S_g is expressed by

$$2\delta P_g = \delta(S_g + S_g^{\circ}) = V_g \delta I_g^{\circ} + I_g^{\circ} \delta V_g + V_g^{\circ} \delta I_g + I_g \delta V_g^{\circ} \dots (2.9)$$

from which and using (2.8), we obtain

$$\begin{aligned} \delta I_g^* &= \delta(s_g + s_g^*)/V_g - I_g \delta(V_g V_g^*)/V_g^2 \\ &\quad - (I_g^* - I_g V_g^*/V_g) \delta V_g/V_g - V_g^* \delta I_g/V_g \end{aligned}$$

Substituting for δV_g^* and δI_g^* the term associated with generator becomes

$$\begin{aligned} &[I_g - I_g^* V_g^*/V_g + [I_g^* - (I_g V_g^*/V_g)] V_g^*/V_g] \delta V_g \\ &\quad - [V_g - (V_g^* V_g^*/V_g)] \delta I_g + [I_g^*/V_g + V_g^* I_g/V_g^2] \delta(V_g V_g^2) \\ &\quad + [1 - V_g^*/V_g] \delta(s_g + s_g^*) \dots \dots \dots (2.10) \end{aligned}$$

(c) The term of (2.4) corresponding to the slack bus is, for

$$\delta V_n = 0 \dots (2.11)$$

given by

$$(V_n^* - V_n) \delta I_n + (V_n - V_n^*) \delta I_n^* \dots (2.12)$$

(d) Other elements, e.g., transmission-line elements, characterized by

$$I_t = Y_t V_t \dots (2.13)$$

lead to the first-order expression

$$\delta I_t = Y_t \delta V_t + V_t \delta Y_t$$

from which

$$\delta V_t = (\delta I_t - V_t \delta Y_t)/Y_t \dots (2.14a)$$

hence

$$\delta V_t^0 = (\delta I_t^0 - V_t^0 \delta Y_t^0) / Y_t^0 \quad \dots \quad (2.14b)$$

Substituting (2.14) into the appropriate term of (2.4) we get

$$\begin{aligned} & [V_t^0 - \bar{V}_t + (\bar{I}_t + I_t^0) / Y_t] \delta I_t \\ & + [V_t - \bar{V}_t^0 + (\bar{I}_t^0 + I_t) / Y_t^0] \delta I_t^0 - (\bar{I}_t + I_t^0) (V_t / Y_t) \delta Y_t \\ & - (\bar{I}_t^0 + I_t) (V_t^0 / Y_t^0) \delta Y_t^0 \quad \dots \quad (2.15) \end{aligned}$$

2.1.1a Adjoint network Elements and Network Sensitivity :

Let ϕ_{ti} = ith design variable e.g. parameters of phase shifting transformers or chunt control elements etc. Then we have

$$\delta Y_t = \sum_i \frac{\partial Y_t}{\partial \phi_{ti}} \phi_{ti} \quad \dots \quad (2.16a)$$

hence,

$$\delta Y_t^0 = \sum_i \frac{\partial Y_t^0}{\partial \phi_{ti}} \phi_{ti} \quad \dots \quad (2.16b)$$

Telegen summation (2.4) can be rewritten as

$$\begin{aligned} & \sum_i [\bar{I}_i + \bar{V}_i I_i^0 / V_i] \delta V_i + \sum_i [\bar{I}_i^0 + \bar{V}_i I_i / V_i^0] \delta V_i^0 \\ & + \sum_g [\bar{I}_g - \bar{V}_g^0 V_g^0 / V_g + [I_g^0 - I_g V_g^0 / V_g] \bar{V}_g^0 / V_g] \delta V_g \\ & - I [\bar{V}_g - (\bar{V}_g^0 V_g^0 / V_g)] \delta I_g + (V_n^0 - V_n) \delta I_n + (V_n - \bar{V}_n^0) \delta I_n^0 \end{aligned}$$

$$\begin{aligned}
 & + \sum_t [V_t^0 - \bar{V}_t + (\bar{I}_t + I_t^0)/V_t] \delta I_t \\
 & + \sum_t [V_t - \bar{V}_t^0 + (\bar{I}_t + I_t)/V_t^0] \delta I_t^0 + \sum_1 [1 - \bar{V}_1/V_1] \delta S_1 \\
 & + \sum_1 [1 - \bar{V}_1/V_1^0] \delta S_1^0 + \sum_g [I_g^0/V_g + \bar{V}_g I_g/V_g^2] \delta(V_g V_g^0) \\
 & + \sum_g [1 - \bar{V}_g^0/V_g^0] \delta(S_g + S_g^0) - \sum_t \sum_1 [(\bar{I}_t + I_t^0)(V_t/V_t) \frac{\partial Y_t}{\partial \phi_{t1}} \\
 & + (\bar{I}_t + I_t)(V_t^0/V_t^0) \frac{\partial Y_t^0}{\partial \phi_{t1}}] \Delta \phi_{t1} = 0 \quad \dots \quad (2.17)
 \end{aligned}$$

So, we see that if $f(v_1, V_1^0, V_g, I_g, I_t, I_t^0)$ be an explicit performance or constraint function, we can define adjoint element

$$\bar{V}_n = V_n^0 \quad \dots \quad (2.18)$$

which eliminates the expressions involving δI_n and δI_n^0 . We then rewrite the remaining components of (2.17) as

$$\begin{aligned}
 \delta f &= \sum_1 \left(\frac{\partial f}{\partial V_1} \delta V_1 + \frac{\partial f}{\partial V_1^0} \delta V_1^0 \right) + \sum_g \left(\frac{\partial f}{\partial V_g} \delta V_g + \frac{\partial f}{\partial I_g} \delta I_g \right) \\
 & + \sum_t \left(\frac{\partial f}{\partial I_t} \delta I_t + \frac{\partial f}{\partial I_t^0} \delta I_t^0 \right) \\
 & = \sum_1 \left(\frac{df}{dS_1} \delta S_1 + \frac{df}{dS_1^0} \right) + \sum_g \left(\frac{df}{d(V_g V_g^0)} \delta(V_g V_g^0) \right) \\
 & + \frac{df}{d(S_g + S_g^0)} \delta(S_g + S_g^0) + \sum_t \sum_1 \frac{df}{d\phi_{t1}} \Delta \phi_{t1} \quad \dots \quad (2.19)
 \end{aligned}$$

where we have defined the adjoint elements

$$\bar{I}_1 = \frac{\partial f}{\partial V_t} - \bar{V}_1^0 I_1^0/V_1 \quad \dots \quad (2.20)$$

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$$\bar{V}_g - \bar{V}_g^* v_g^*/v_g = - \frac{\delta f}{\delta I_g} \quad \dots \quad (2.21)$$

$$\bar{I}_g - \bar{I}_g^* v_g^*/v_g = \frac{\delta f}{\delta V_g} - (\bar{I}_g^* v_g^*/v_g)(\bar{V}_g^*/v_g/v_g) \quad \dots \quad (2.22)$$

$$v_t^* - \bar{V}_t = \frac{\delta f}{\delta I_t} - (\bar{I}_t + \bar{I}_t^*)/Y_t \quad \dots \quad (2.23a)$$

or

$$\bar{I}_t = Y_t \bar{V}_t + Y_t \frac{\delta f}{\delta I_t} - v_t^* (Y_t + Y_t^*) \quad \dots \quad (2.23b)$$

Since f is real

$$\frac{\partial f}{\partial V_1} = \left(\frac{\partial f}{\partial V_1^*} \right)^* \quad \dots \quad (2.24)$$

and

$$\frac{\partial f}{\partial I_t} = \left(- \frac{\partial f}{\partial I_t^*} \right)^* \quad \dots \quad (2.25)$$

2.1.1b Adjoint Network and their Interpretation :

Considering Fig. 2.1 and the equation associated with a load bus, namely (2.20)

For convenience, we write* (2.20) as

$$\bar{I}_t = \bar{I}_t^S + \psi_1 \bar{V}_1^* \quad \dots \quad (2.26)$$

where

$$\bar{I}_1^S \triangleq \delta f / \delta V_1 \quad \dots \quad (2.27)$$

and

$$\psi_1 \triangleq - s_1 / v_1^2 \quad \dots \quad (2.28)$$

Fig. 2.1 shows the independent source \bar{I}_1^S and the element ψ_1 .

Now considering Fig. 2.2 and the equations associated with a generator bus.

Equation (2.22) is rewritten as

$$\bar{V}_g = \phi_g \bar{I}_g + \bar{\phi}_g \bar{I}_g^* + \bar{V}_g^S \quad \dots \quad (2.29)$$

$$\phi_g \triangleq -V_g V_g^* / (j 2Q_g) \quad \dots \quad (2.30)$$

$$\bar{\phi}_g \triangleq (V_g^S)^2 / (j 2Q_g) \quad \dots \quad (2.31)$$

$$\bar{V}_g^S \triangleq - (V_g^S)^2 (\partial f / \partial V_g)^* / (j 2Q_g) \quad \dots \quad (2.32)$$

and where

$$j 2Q_g = I_g^* V_g - I_g V_g^*$$

Equation (2.21) is also rewritten (See Fig. 2.2) in the form

$$V_g \bar{V}_g - V_g^* \bar{V}_g^* = -V_g \frac{\partial f}{\partial I_g} \quad \dots \quad (2.33)$$

We observe that linear system (2.29) and (2.33) must be solved to define the adjoint element corresponding to the generator in the given network.

The slack bus constraint (2.18) is illustrated by Fig. 2.3.

Equation (2.23) for the remaining elements becomes

$$I_t = Y_t \bar{V}_t + \bar{I}_t^S \quad \dots \quad (2.34)$$

where

$$\bar{I}_t^S = Y_t \frac{\partial f}{\partial I_t} - V_t^S (Y_t + Y_t^*) \quad \dots \quad (2.35)$$

Independent sources associated with each branch are summed, as shown in Fig. 2.4, as

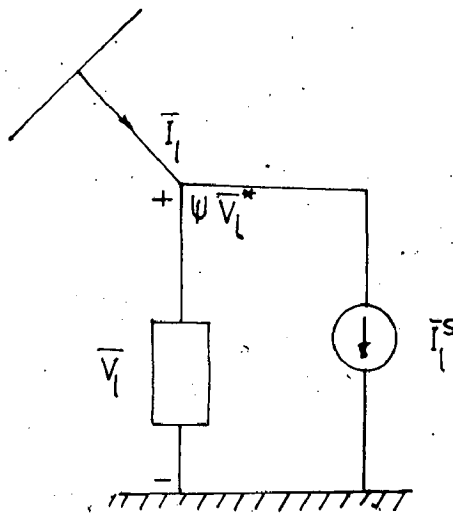


FIG 21 Adjoint element model obtained from eq.(2.20) for Load bus.

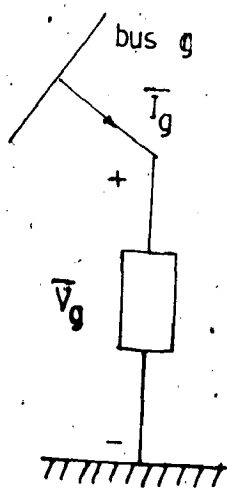


FIG 22

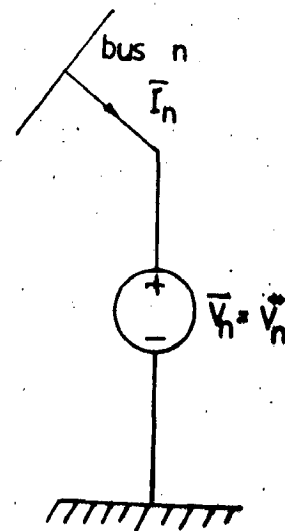


FIG 23

Adjoint element model obtained from the solution of eq.(2.29) and (2.33) for a generator bus.

Adjoint element model obtained from eq.(2.23) for a Slack bus.

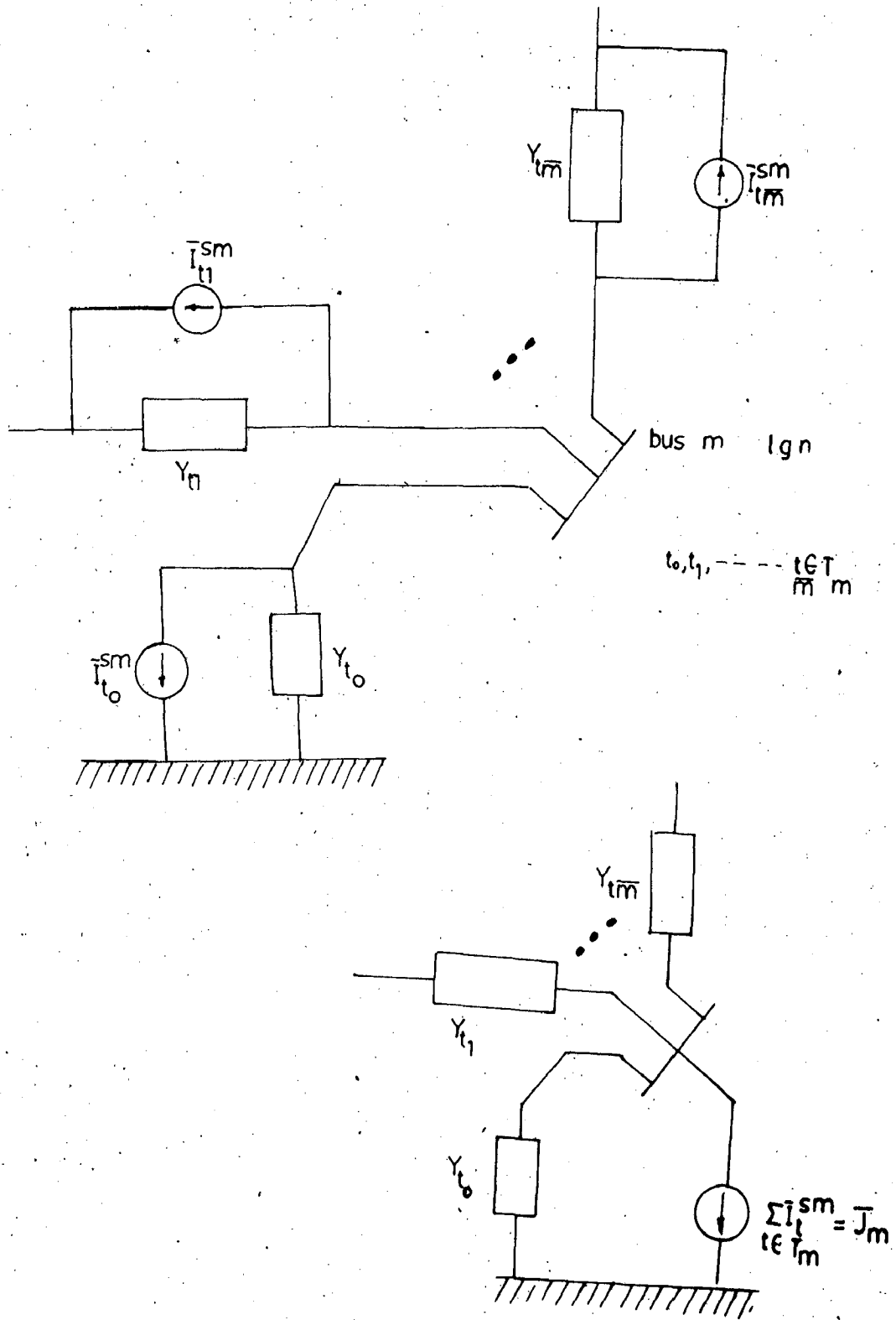


FIG 2.4

Equivalent Bus elements at Bus m

$$\bar{I}_m \quad \Sigma \quad \bar{I}_t^{S\sigma} \quad \dots \quad (2.36)$$

$\leftarrow T_m$

For any m (= 1, g or n), where T_m identifies those branches connected to bus m distribution.

2.1.1c The Adjoint Equations :

The derivation of the adjoint equations are outlined in this section. In general, they take the complex form

$$\begin{bmatrix} Y_{LL} & Y_{LG} & Y_{LN} \\ Y_{GL} & Y_{GG} & Y_{GN} \\ Y_{NL} & Y_{NG} & Y_{nn} \end{bmatrix} \begin{bmatrix} \bar{V}_L \\ \bar{V}_G \\ \bar{V}_n \end{bmatrix} = - \begin{bmatrix} \bar{I}_L + \bar{J}_L \\ \bar{I}_G + \bar{J}_G \\ \bar{I}_n + \bar{J}_n \end{bmatrix} \quad \dots \quad (2.37)$$

where the n x n bus admittance matrix has been partitioned into blocks associated with the sets of load, generator and slack buses of appropriate dimension. Note that Y_{nn} and associated variables are scalars.

For load buses we let

$$\bar{I}_L = \psi_L \bar{V}_L^* + \bar{I}_L^S \quad \dots \quad \dots \quad (2.38)$$

where \bar{I}_L , \bar{V}_L and \bar{I}_L^S are vectors of dimension n_L consisting of the \bar{I}_1 , \bar{V}_1 and \bar{I}_1^S , respectively, and ψ_L is a diagonal matrix whose diagonal elements are the corresponding ψ_1 of (2.28). For generator buses we let

$$\bar{V}_G = \bar{\phi}_G \bar{I}_G + \bar{\phi}_3 \bar{I}_G^o + \bar{V}_G^S \quad \dots \quad \dots \quad (2.39)$$

$$R_G \bar{V}_G - R_G^* V_G^* = F_G \quad \dots (2.40)$$

where \bar{V}_G , I_G and \bar{V}_G^S are vectors of dimension n_G consisting of the \bar{V}_G , I_G and \bar{V}_G^S , respectively, and $\bar{\phi}_G$, R_G and F_G are diagonal matrices whose diagonal elements are taken from (2.30), (2.31) and (2.33).

$$Y_{RS} = G_{RS} + jB_{RS} \quad \dots (2.41a)$$

$$\bar{Y}_M = \bar{Y}_{M1} + j\bar{V}_{M2} \quad \dots (2.41b)$$

$$\bar{J}_M = \bar{J}_{M1} + jJ_{M2} \quad \dots (2.41c)$$

where R, S and M can be G, L or n. Further, let

$$\bar{\Psi}_L = \bar{\Psi}_{L1} + j\bar{\Psi}_{L2} \quad \dots (2.42a)$$

$$\bar{I}_G = \bar{I}_{G1} + j\bar{I}_{G2} \quad \dots (2.42b)$$

$$\bar{\phi}_G = j\bar{\phi}_{G2} \quad \dots (2.42c)$$

$$\bar{\phi}_G = \bar{\phi}_{G1} + j\bar{\phi}_{G2} \quad \dots (2.42d)$$

$$R_G = R_{G1} + jR_{G2} \quad \dots (2.42e)$$

$$F_G = jF_{G2} \quad \dots (2.42f)$$

Using the notations as described earlier we arrive at the equation :

$G_{LL} + \psi_{L1}$	G_{LG}	$-B_{LL} + \psi_{L2}$	$-B_{LG}$	\bar{V}_{L1}
$\bar{\phi}_{G1} G_{GL} + \bar{\phi}_{G2} B_{GL}$	$\bar{\phi}_{G1} G_{GG} + \bar{\phi}_{G2} B_{GG} - 1$	$\bar{\phi}_{G1} B_{GL} + \bar{\phi}_{G2} G_{GL}$	$\bar{\phi}_{G1} B_{GG} + \bar{\phi}_{G2} G_{GG}$	\bar{V}_{G1}
$B_{LL} + \psi_{L2}$	B_{LG}	$G_{LL} - \psi_{L1}$	G_{LG}	\bar{V}_{L2}
0	$2R_{G2}$	0	$2R_{G1}$	\bar{V}_{G2}

$$\begin{aligned}
 & -\bar{I}_{L1}^S - \bar{J}_{L1} - G_{LN} \bar{V}_{n1} + B_{LN} \bar{V}_{n2} \\
 & \hline
 & -\bar{V}_{G1}^S - \bar{\phi}_{G2} \bar{J}_{G2} + \bar{\phi}_{G1} \bar{J}_{G1} + (\bar{\phi}_{G1} G_{GN} - \bar{\phi}_{G2} B_{GN}) \bar{V}_{n1} - (\bar{\phi}_{G1} B_{GN} + \bar{\phi}_{G2} G_{GN}) \bar{V}_{n2} \\
 & \hline
 & -\bar{I}_{L2}^S - \bar{J}_{L2} - G_{LN} \bar{V}_{n2} - B_{LN} \bar{V}_{n1} \\
 & \hline
 & \bar{V}_{G2}
 \end{aligned}$$

where

$$\bar{\phi}_{G2} \triangleq \phi_{G2} - \bar{\phi}_{G2} \quad \dots (2.44)$$

The rows of (2.43) corresponding to the load buses are obtained in a straightforward manner by substituting the separated forms of Y_{LL} , Y_{LG} , Y_{LN} , \bar{V}_L , \bar{I}_L^S and J_L into (2.37) and (2.38).

For the generator buses, consider the real part of (2.39) as

$$\bar{V}_{G1} = \phi_{G2} \bar{I}_{G2} + \bar{\phi}_{G1} \bar{I}_{G1} + \bar{\phi}_{G2} \bar{I}_{G2} + \bar{V}_{G1}^S \quad \dots (2.45)$$

The subset of equations (2.37) corresponding to the generator buses is

$$\bar{I}_G = -(\bar{Y}_G + \bar{J}_G) \quad \dots (2.46)$$

where

$$\bar{I}_G \triangleq Y_{GL} \bar{V}_L + Y_{GC} \bar{V}_G + Y_{GN} \bar{V}_n \quad \dots (2.47)$$

Let

$$\bar{I}_G = \bar{I}_{G1} + j\bar{I}_{G2} \quad \dots (2.48)$$

Eliminating \bar{I}_{G1} and \bar{I}_{G2} from (2.45) and (2.46) we obtain

$$(\phi_{G2} - \bar{\phi}_{G2}) \bar{I}_{G2} - \bar{\phi}_{G1} \bar{I}_{G1} - \bar{V}_{G1} = (\phi_{G2} - \bar{\phi}_{G2}) \bar{J}_{G2} + \bar{\phi}_{G1} \bar{J}_{G1} - \bar{V}_{G1}^S \quad \dots (2.49)$$

Equation (2.49) in conjunction with (2.48) separated into real and imaginary parts lead to the rows of (2.43) corresponding to the generator buses.

2.1.1d Gradient Calculations :

Comparing (2.10) with (2.17) we derive the following

Load Variables

$$\begin{aligned} \operatorname{Re} \left(\frac{df}{dS_1} \delta S_1 \right) &= \operatorname{Re} \left[(1 - \bar{V}_1^*/V_1) \delta S_1 \right] \\ &= -\operatorname{Re} (1 - \bar{V}_1^*/V_1) \delta P_1 + \operatorname{Im} (1 - \bar{V}_1^*/V_1) \delta Q_1 \end{aligned}$$

hence we can write

$$\frac{df}{dP_1} = -2\operatorname{Re} (1 - \bar{V}_1^*/V_1) \quad \dots \quad (2.50)$$

$$\frac{df}{dQ_1} = 2\operatorname{Im} (1 - \bar{V}_1^*/V_1) \quad \dots \quad (2.51)$$

Generator Variables

$$\frac{df}{d(V_g V_g^*)} = -I_g^*/V_g = -\bar{V}_g^* I_g^*/V_g^2 \quad \dots \quad (2.52)$$

$$\frac{df}{d(S_g + S_g^*)} = -1 + \bar{V}_g^*/V_g \quad \dots \quad (2.53)$$

Other Variables

$$\begin{aligned} \frac{df}{d\beta_{ti}} &= 2\operatorname{Re} \left[\frac{V_t}{Y_t} (\bar{I}_t + I_t^*) \frac{\partial Y_t}{\partial \beta_{ti}} \right] \\ &= 2\operatorname{Re} \left[V_t (\bar{V}_t + \frac{\partial f}{\partial I_t} - V_t^*) \frac{\partial Y_t}{\partial \beta_{ti}} \right] \quad \dots \quad (2.54) \end{aligned}$$

On observing the above equation we conclude that partial derivative depend on unperturbed currents and voltages in original and adjoint network. Any number of variables β_{ti} can be accommodated in these two analyses. Also if 'f' is not explicitly a function of V_1 , \bar{V}_1^* , V_g , I_g , I_t or I_t^* , the partial derivative of 'f' with respect to these variables will be zero.

It can be very well seen that all the expressions derived above are functions of voltage and current. The formulation can be done in terms of complex voltages and currents, bus or branch quantities as required for the particular problem.

2.1.1e Algorithm :

The control quantities S_1 , S_1^* , $(V_g V_g^*)$ and $(S_g + S_g^*)$ as well as the parameters ϕ_{ti} has been designated as practical designable variables. Also,

$$P_1 = (S_1 + S_1^*)/2$$

$$Q_1 = -j(S_1 - S_1^*)/2$$

$$|V_g|^2 = V_g V_g^*$$

$$P_g = (S_g + S_g^*)/2$$

Step 1

Load flow solution is obtained by fast decoupled method.

Step 2

Partial derivatives of functions f_1, f_2, \dots, f_m w.r.t $V_1, V_1^*, V_g, I_g, I_t, I_t^*$ are evaluated.

Partial derivatives of any function 'f', w.r.t. complex variables can be dealt with. Two real quantities are assigned as independent control variables at a bus. Then the required partial derivatives can be easily obtained by expressing 'f' in terms of the chosen controls and states.

Step 3

Define the adjoint parameters required for equation (2.43).

Step 4

Solve the adjoint system (2.43)

Step 5

Calculate the gradient vector using equations (2.50) - (2.54).

If the effect of line additions or removals (as it is done in the third chapter) is to be determined appropriate first-order changes are calculated using the gradient information of Step 5.

CHAPTER - 3

DISTRIBUTION FEEDER PLANNING USING A.C. ADJOINED METHOD OF SENSITIVITIES.

3.1 INTRODUCTION :

With the passage of time the load in any area is bound to increase. The existing distribution circuits, are overloaded because of increase in load. We can have two options in that case. First being the replacement of the existing feeder by another feeder of higher area of cross section. The second alternative is of reinforcing the existing feeder with another feeder so as to remove the over loads. In this chapter a problem has been formulated in which the over loads are alleviated by making suitable feeder additions or replacement.

There is every likelihood that due to the expansion of the distribution system the specified voltage magnitudes are violated. The voltage at the load bus may go down below the prescribed minimum level. Also with the increase in load there is a possibility that power factor may become low. The low power factor seriously affects the system economy because of high system losses.

Voltage correction may be carried out by shunt capacitor additions and has been used in this method. But the constraint on the number of shunt capacitors is equally important because the over compensation may result in over voltage which increases the energy losses in the system.

Enormous work on optimal distribution feeder planning has been done. Many methods for planning of primary feeder for Urban

and rural areas have been developed. Number of mathematical techniques have been evolved and used. Work has been done by using mixed Integer Programming, and Branch and bound Method. Algorithms have been developed and implemented. But these methods are only for radial and branch feeders.

The heuristic method has been presented in this section for primary loop feeder expansion planning and the associated voltage correction problem. A.C. Adjoint method is used for determining sensitivities of feeder currents and load bus voltages due to feeder additions and capacitors installations. In the ensuing problem the network expansion by the addition of feeder is the primary decision where as voltage correction part is given the priority of secondary decision.

3.2 THE MODEL :

The distribution primary feeder planning problem has been defined as follows.

For a particular area with the given load, develop a suitable distribution network so that

- (i) None of the distribution feeder is over loaded.
- (ii) Voltages at the load buses are not violated and the cost of expansion is minimized.

The model is developed with the assumption that loading is evaluated in terms of the current flowing in the distribution feeder because the current affects the heating of distribution feeders.

3.2.1 Sensitivity Analysis by A.C. Adjoint method :

Using the A.C. adjoint method of sensitivity analysis for power system the sensitivity of feeder currents and voltages of nodes with respect to various possible feeder additions and shunt capacitor installations are calculated.

3.2.1(a) Sensitivity of feeder currents due to feeder additions :

The function $f_i = (|I_i|)^2$ for $i \in \text{OVL}$ are chosen where I_i = current in the i^{th} feeder.

OVL = Set of over loaded feeders.

The function $\frac{\partial f_i}{\partial y_j}$ are calculated for every j^{th} feeder addition where

$Y_j = G_j + jB_j$ = Admittance of j^{th} feeder considered for expansion.

G_j = Conductance of j^{th} feeder

B_j = Susceptance of j^{th} feeder

From the equation (2.54) the change in the function due to the design parameter ϕ_j is given by —

$$\frac{df_i}{d\phi_j} = 2\text{Re} \left[V_j \left(\bar{V}_j + \frac{\partial f_i}{\partial I_j} - V_j^o \right) \frac{\partial Y_j}{\partial \phi_j} \right] \quad \dots(3.2)$$

Let

$$\alpha_j + j\beta_j = V_j \left[\bar{V}_j + \frac{\partial f_i}{\partial I_j} - V_j^o \right] \quad \dots(3.3)$$

If

$$i = j \quad \frac{\partial f_i}{\partial I_j} = I_i^o \quad \text{otherwise} \quad \frac{\partial f_i}{\partial I_j} = 0 \quad \dots(3.4)$$

$$\frac{df_i}{d\phi_j} = 2\text{Re} [\alpha_j + j\beta_j] = 2\alpha_j \quad \dots(3.5)$$

and

$$\frac{df_1}{dB_j} = 2R_0 [j(\alpha_j + j\beta_j)] = -2\beta_j \quad \dots(3.6)$$

The new function is given by F_{1j} = the square of the current in i^{th} feeder due to the addition of j^{th} feeder

$$F_{1j} = f_1 + \frac{df_1}{dG_j} G_j + \frac{df_1}{dB_j} B_j \quad \dots(3.7)$$

The new current due to the j^{th} feeder addition is given by

$$I_{1j} = [F_{1j}]^{\frac{1}{2}} \quad \dots(3.8)$$

3.2.1(b) Sensitivity of Node voltages due to the feeder additions :

Choose the function

$$f_{v1} = (|V_1|)^2 \quad \dots(3.9)$$

for $i \in \text{VCN}$ where VCN = Set of Nodes for which the voltage correction is necessary.

V_1 = Voltage at the i^{th} Node.

In this case

$$\frac{df_{v1}}{dG_j} \quad \text{and} \quad \frac{df_{v1}}{dB_j}$$

represent the change in the voltage function due to the j^{th} feeder addition. As f_{v1} is explicitly not a function of I_1 ,

$$\frac{\partial f_{v1}}{\partial I_1} = 0$$

The new function is given by

$$F_{V1j} = \text{The square of the voltage at } i^{\text{th}} \text{ Node due to the addition of } j^{\text{th}} \text{ feeder.}$$

$$Fv_{1j} = fv_1 + \frac{dfv_1}{dG_j} \cdot G_j + \frac{dfv_1}{dB_j} \cdot B_j \quad \dots(3.10)$$

The new voltage at i^{th} Node due to the addition of j^{th} feeder addition is given by

$$V_{1j} = (fv_{1j})^{1/2} \quad \dots(3.11)$$

3.2.1(c) Sensitivity of Node voltages due to the capacitor additions:

The constivity of node voltages due to the capacitor additions are calculated in the same way as due to the feeder additions.

For every feeder addition and capacitor location under consideration, current index factor and voltage index factor is calculated. These factors given an indication of the effect, these additions will have on distribution feeders loading and the load bus voltage.

3.2.2 Current index factor :

The current index factor for the i^{th} over loaded feeder due to the feeder addition along the j^{th} right of way is defined as follows :

$$C_{1j} = \text{Current index factor for } i^{\text{th}} \text{ over loaded feeder due to } j^{\text{th}} \text{ addition or}$$

$$C_{1j} = [I_{1j} - I_{\max i}]^2 \quad \text{If } I_{1j} > I_{\max i} \quad \dots(3.12)$$

$$= 0 \quad \text{If } I_{\max i} > I_{1j}$$

where I_{\max} which is maximum current in i^{th} feeder after power system expansion

$$I_{\max 1} = \frac{\lambda_1 + Y_1}{\lambda_1} | I_{\max}^0 | \quad \dots(3.13)$$

where λ_1 = No of existing feeders in i^{th} right of way.

Y_1 = Number of feeders to be added across i^{th} branch.

$| I_{\max}^0 |$ = Max current in the i^{th} feeder in the existing network.

For $i = j$, the new value of maximum current (after expansion) is given by

$$I_{\max 1} = | I_{\max 1}^0 | \left[\frac{1 + \lambda_1}{\lambda_1} \right] \quad \dots(3.14)$$

3.2.3 Voltage index factor :

The voltage index factor for voltage correction nodes due to j^{th} feeder addition is defined as follows

$$V_j = \sum_{i \in VCN} | (V_{\max 1} - V_i) | + \sum_{i \in VCN} | (V_i - V_{\min 1}) | \quad \dots(3.15)$$

3.2.4 Measure index :

The measure index of over loaded feeder i due to feeder j may be defined as

$$M_{ij} = C_{ij} \text{Cost} (j) \quad \dots(3.16)$$

where $\text{Cost} (j) = \text{Cost of } j^{\text{th}} \text{ feeder addition}$

The smallest of these measure indices points to the most economical feeder reinforcement to remove over loading of i^{th} feeder.

3.2.5 Effect index :

The effect index for any line addition is defined as sum of current index factor for the line and the voltage index factor for feeder.

E_j = Effect index for feeder j

$$= \sum_{i \in OVC} C_{ij} + V_j^i \quad \dots(3.17)$$

The effect index given the possible effect the feeder addition will have in removing the over loads in distribution feeder and in bringing back the voltage at the load buses within limits. Smaller the effect index greater in the effect.

3.3 ALGORITHM :

The distribution expansion planning consist of (a) Expansior logic (b) Over load logic.

3.3.1 Expansion logic :

- (1) Carry out the load flow studies using fast decoupled load flow technique. Calculate maximum current (I_{max}) and actual current flowing in the network. Actual current is given by

$$I_{Km} = (V_K - V_m) \cdot Y_{Km} + V_K Y'_{Km}/2 \quad \dots(3.18)$$

Y_{Km} = Series admittance for a feeder connected between bus K and m.

Y'_{Km} = Charging Admittance.

- (2) Using the A.C. Adjoint method of sensitivity analysis for power system, calculate the voltages of the adjoint network nodes and hence calculate the sensitivities of feeder currents in the over loaded branches and load bus voltages due to the various feeder additions and determine the current index factors and voltage index factors.

3. Calculate the measure index factors.
4. For all the feeders find the effect indices. The smallest of these effect indices indicate the greatest effect on all over loaded feeders. The most effective feeder addition not only eliminates the over load in desired feeder but also have greater effect in reducing the amount of over loads in all other loaded(over) feeders. Make the feeder addition. These steps are repeated till all over loads are eliminated.

It is observed that only reinforcement for over loaded feeders are effective in removing the over load.

3.3.2 Voltage correction logic :

When all feeder over loads have been alleviated the voltage correction logic is initiated. It follows the following steps.

- 1)
 1. Carry out the load flow studies using fast decoupled load flow method with all the feeder additions made.
 2. Using A.C. adjoint method of sensitivity analysis, calculate the voltages of adjoint network and hence calculate the sensitivity of load bus voltages due to the shunt capacitor additions at the load buses.
 3. Calculate the modified bus voltages due to these capacitor additions and find the voltage index factor.
 4. Make the capacitor addition corresponding to smallest voltage index factors.

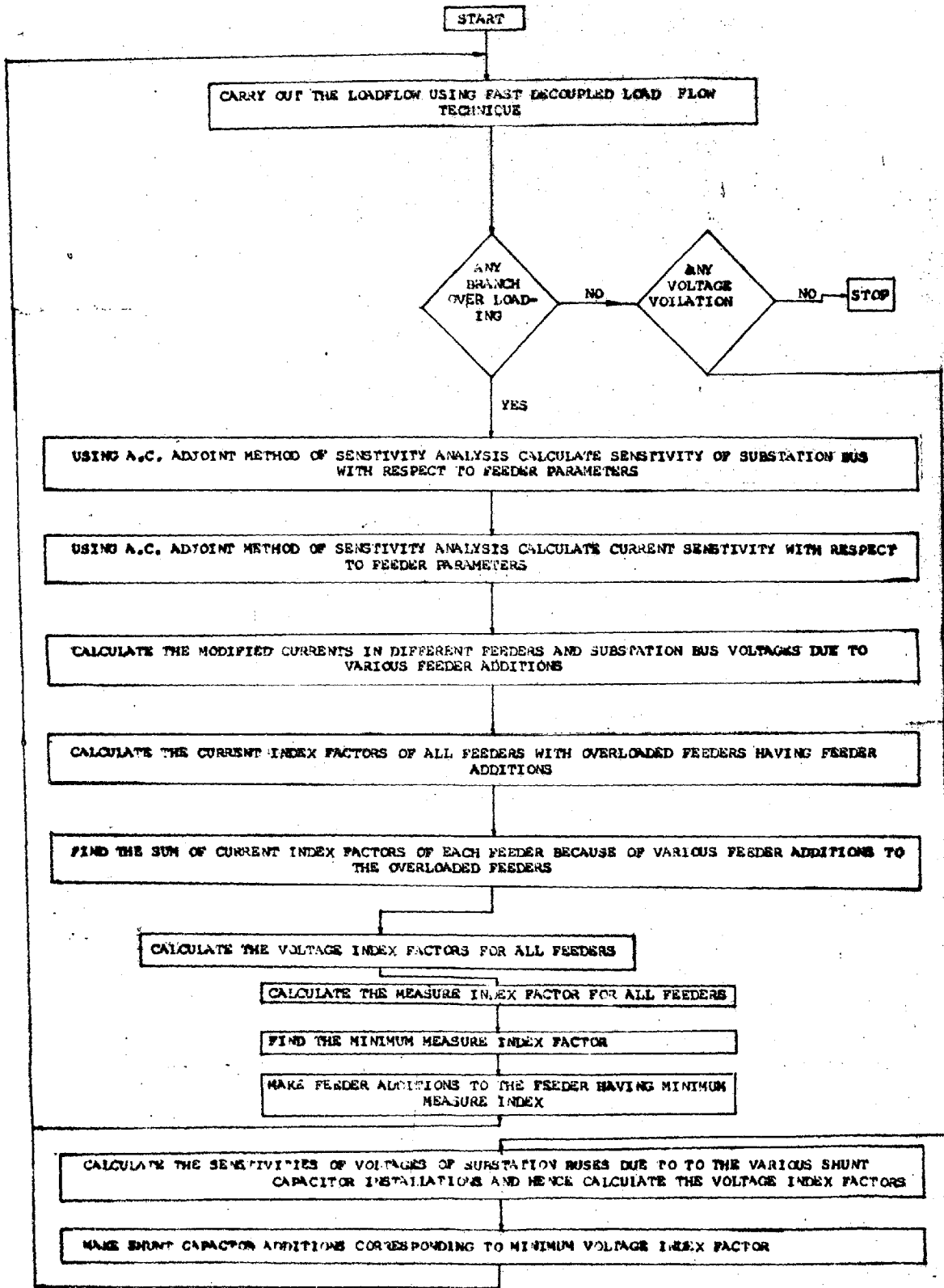


FIGURE 3.1 FLOW CHART - DISTRIBUTION PRIMARY FEEDER PLANNING AND VOLTAGE CORRECTION - HEURISTIC APPROACH.

5. Carry out the load flow study to varyfy the bus voltages.
6. Repeat the procedure till all voltage magnitudes are with in limits.

3.4 NUMERICAL EXAMPLE:

3.4.1 Five Bus System:

A five bus distribution system shown in Fig.3.2 is considered for expansion and voltage correction. The input data required for load flow is given in Table (3.1),(3.2) and (3.3). Also Number of Grid and Power Substation,NGP = 5

Number of feeders, NFED = 5

Number of Power Substation, NPSS = 4

Number of Grid Substation, NGRSS = 1

Number of Type of feeder considered,
NTFED = 3

Number of Substations at which the
Capacitor are installed, NCAF = 0

Maximum number of iterations,MAXIT = 20

Effect on all lines is considered,NEF = 1

Capacitor Additions is to be
considered NCAFAD = 1

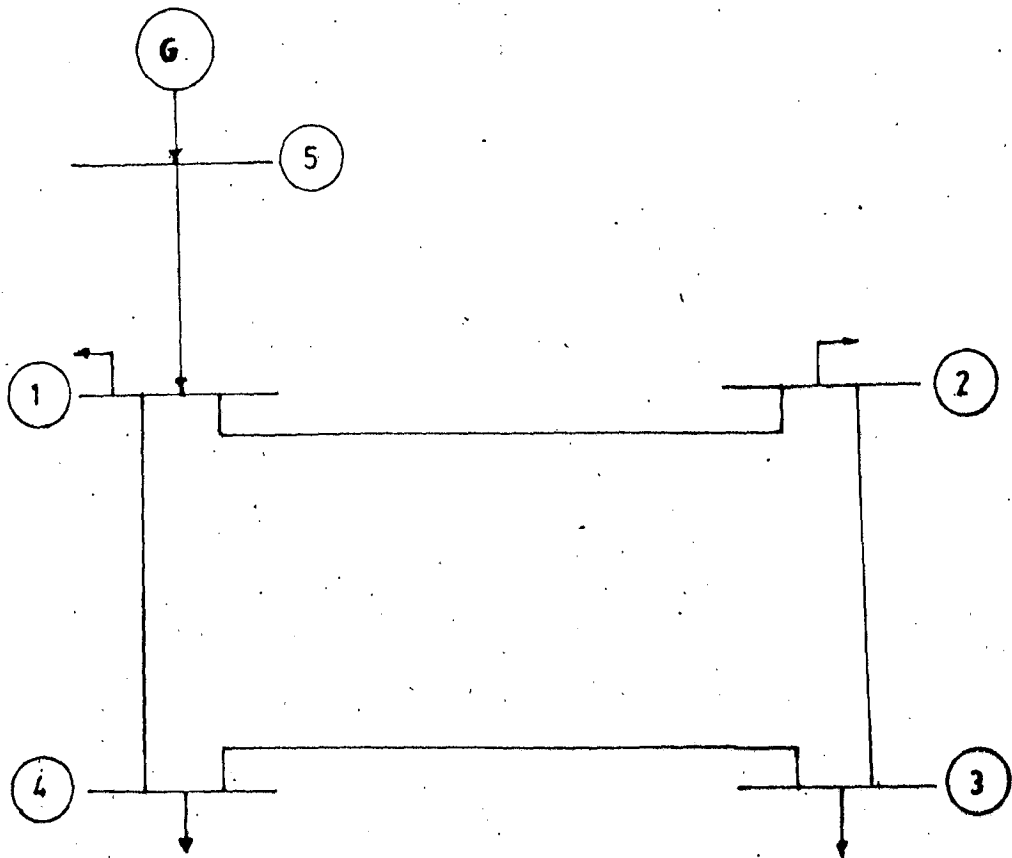


FIG-3.2 5-BUS SYSTEM

TABLE- 3.1

Power and Grid substation Data Base MVA = 20.00
 Base Voltage = 11.00 KV

Bus No.	MV capacity of substation/phase (p.u.)	MVAH capacity of substation/phase (p.u.)	V (p.u.) Specified Normal Voltage	Specified minimum Voltage of substation	Specified maximum voltage of substation	Type of substation	Grid/operation
1	0.05	0.02	1.0	0.95	1.10	0	0
2	0.02	0.01	1.0	0.95	1.10	0	0
3	0.05	0.04	1.0	0.95	1.10	0	0
4	0.16	0.09	1.0	0.95	1.10	0	0
5	0.16	0.09	1.0	0.95	1.10	3	3

TABLE - 3.2

Existing feeder Data

Feeder No.	<u>Line terminating buses</u>		Type of feeder	Length of feeder (Km.)	Number of existing feeder single/double	1, If can be expanded, otherwise zero.
	From	To				
1	1	2	1	2.0	1	1
2	2	3	2	2.0	1	1
3	3	4	3	2.0	1	1
4	4	1	1	2.0	1	1
5	5	1	3	2.0	1	2

TABLE - 3.3

Feeder Data

Sl. No.	Type of feeder	Cost/Km. (m.u.)	Resistance/phase/Km.	Reactance/phase/Km.	Susceptance/phase/Km	Current carrying capacity (Amps.)
1	1	20.0	0.254	0.053	0.002	96.00
2	2	23.0	0.203	0.052	0.002	109.00
3	3	36.0	0.100	0.049	0.002	183.00

The cost per shunt capacitor, COSCA = 20 m.u.

The susceptance of each capacitor unit of shunt capacitor BSS = 0.001 p.u.

TABLE - 3.4

Voltage of the existing network (From fast decouple load flow method)

Bus No.	Voltage magnitudes (p.u.)	Phase Angle in radians
1	0.9580	0.0034
2	0.9286	0.0152
3	0.9144	0.0216
4	0.9220	0.0197
5	1.0000	0.0000

TABLE - 3.5

Voltages of the expanded network of distribution system.

Bus No.	Voltage magnitude p.u.	Phase angle in radians
1	0.9797	0.0018
2	0.9610	0.0065
3	0.9519	0.0090
4	0.9616	0.0060
5	1.0000	0.0000

TABLE - 3.6

Loading of Distribution Feeders

Feeder Number	Existing Network		Expanded Net work	
	Maximum current (p.u.)	Actual current (p.u.)	Maximum current (p.u.)	Actual current (p.u.)
1	0.053	0.061	0.060	0.046
2	0.060	0.037	0.60	0.023
3	0.101	0.035	0.101	0.046
4	0.053	0.075	0.101	0.083
5	0.101	0.189	0.201	0.183

By observing the tables 3.4, 3.5 and 3.6 we conclude the feeders 1, 4 and 5 are over loaded and the voltages at the nodes 2, 3 and 4 are violated.

Making 2 feeder addition to feeder 4th, one each to lot and 5th feeder the over loading is alleviated. Also the load bus voltages are brought within prescribed limits. So there is now no capacitor addition.

Total Primary feeder planning Cost = 96 m.u.

3.5 CONCLUSION :

Since the A.C. Adjoint method is used to calculate the sensitivities of feeder currents and bus voltages due to various feeder additions and capacitor installation, the advantages of compactness, sparsity and simplicity of the system is capitalized. Without the use of A.C. Adjoint method in each iteration load flow studies have to be done after each and every feeder or capacitor installation which is a time consuming and cumbersome process. However in A.C. Adjoint method of sensitivity the load flow study is done only once thus making very fast and efficient. The heuristic approach used is quite practical and realistic one. Also problem of looped feeder planning has been solved contrary to the earlier attempts where mostly problem has been solved for radial and branched feeder.

REPORT OF GRID AND POWER STATIC

CENTRAL ILL. UNIVERSITY OF SCIENCE AND ENGINEERING

NUMBER OF FEEDER 1
 NUMBER OF POWER SUBSTATION 1
 NUMBER OF GRID SUBSTATION 1
 NUMBER OF TYPE OF FEEDER CONSIDERED 3
 NUMBER OF SUBSTATION AT WHICH CAPACITORS ARE INSTALLED 3
 MAXIT 2
 REF 1
 NCPAD 1

FEEDER DATA

TYPE OF FEEDER	COST	RES	REA	SUS
1	20.00	0.2510	0.53	0.002 96
2	23.00	0.23	0.52	0.002 109
3	36.00	0.1	0.49	0.002 183

RIGHT OF WAY FROM TO FEEDER TYPE LENGTH NO. OF EX FD

1	2	1	2.0	1	1
2	3	2	2.0	1	1
3	4	3	2.0	1	1
4	1	1	2.0	1	1
5	1	3	2.0	1	2

POWER AND GRID SUBSTATION DATA

0.50	0.02	1.00	0.95	1.100	
0.120	0.01	1.00	0.95	1.100	0
0.50	0.04	1.00	0.95	1.100	0
0.30	0.02	1.00	0.95	1.100	0
0.100	0.09	1.00	0.95	1.100	3
2.00000					
2.00000					
2.00000					
2.00000					
2.00000					

ITERATION NO= 1
 RANCH CURR NTS INDEX FACTORS
 0.00000E+00, 1.317823, 1.317823, 1.3
 2.339026, 2.339026, 2.339026, 0.00
 3.531644, 3.531644, 3.531644, 3.5
 5.570670, 6.388493, 6.888493, 4.8494
 VOLTAGE IND X FACTORS 0.61984 0.61984 0.61984 0.
 VALUES OF MASURE INDEX
 23.0550 345.3833 540.6000 179.97

MAKE FEEDER ADDITION WITH R.O.W.
 ITERATION NO= 2
 RANCH CURR NTS INDEX FACTORS
 0.00000E+00, 1.100278, 1.100278, 1.1
 1.745745, 1.745745, 1.745745, 0.00
 3.497250, 3.497250, 3.497250, 3.4
 5.242995, 6.343273, 6.343273, 4.5975
 VOLTAGE IND X FACTORS 0.59524 0.59524 0.59524 0.
 VALUES OF MASURE INDEX
 191.5854 319.1716 499.5729 196.57

MAKE FEEDER ADDITION WITH R.O.W.
 ITERATION NO= 3
 RANCH CURR NTS INDEX FACTORS
 1.549425, 1.549425, 1.549425, 0.00
 3.482380, 3.482380, 3.482380, 3.4
 5.031806, 5.031806, 5.031806, 3.4823
 VOLTAGE IND X FACTORS 0.57833 0.57833 0.57833 0.
 VALUES OF MASURE INDEX
 258.0660 258.0660 403.9294 154.75

MAKE FEEDER ADDITION WITH R.O.W.
 ITERATION NO= 4
 RANCH CURR NTS INDEX FACTORS
 1.518412, 1.518412, 1.518412, 0.00
 1.518412, 1.518412, 1.518412, 0.00000
 VOLTAGE IND X FACTORS 0.16372 0.16372 0.16372 0.
 VALUES OF MASURE INDEX
 77.37817 77.37817 121.1137 7.5312

MAKE FEEDER ADDITION WITH R.O.W.
 ITERATION NO= 5
 RANCH CURR NTS INDEX FACTORS
 0.00000E+00, 0.00000E+00, 0.00000E+00, 0.00000
 VOLTAGE IND X FACTORS 0.00000 0.00000 0.00000 0.
 NO FURTHER DISTRIBUTION EXPANSION NECESSARY

Line Parameters

S.No.	ACSR Conductor Code Name	Gross Aluminium area of Conductor cross-section (mm ²)	Maximum Current carrying capacity at 50°C (Amp)	Per Phase resistance at 50°C (Ohms/km)	415 V 3 phase line with conductor spacing of 12"		11 kV 3 phase line with cond. spacing of 3'		33 kV 3 phase line with cond. spacing of 4'	
					Per phase reactance (Ohms/km)	Maximum kVA capacity	Per phase reactance (Ohms/km)	Maximum kVA capacity	Per phase reactance (Ohms/km)	Max. capacity kVA
1	Squirrel	20.71	97	1.539	0.322	69	0.392	1848	-	-
2	Gopher	25.91	109	1.230	0.317	78	0.306	2076	-	-
3	Weasel	31.21	123	1.021	0.312	88	0.302	2343	-	-
4	Ferret	41.87	155	0.761	0.306	111	0.375	2953	-	-
5	Rabbit	52.21	183	0.610	0.300	131	0.369	3486	-	-
6	Mink	62.32	200	0.511	0.296	149	0.365	3963	0.383	11009
7	Beaver	74.07	235	0.430	0.291	160	0.361	4477	0.379	13432
8	Raccoon	77.83	245	0.409	0.290	176	0.360	4668	0.378	14004
9	Otter	82.85	257	0.385	0.289	184	0.358	4996	0.376	14609
10	Cat	94.21	285	0.338	0.286	204	0.355	5430	0.374	16290
11	Dog	103.60	311	0.307	0.284	223	0.353	5925	0.371	17776

Voltage regulation constants

S.No.	ACSR Conductor Code Name	M in km-kVA for 1 percent voltage drop at a pf of								
		0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
<u>A. 415 V 3 phase lines</u>										
1	Squirrel	1.458	1.383	1.317	1.259	1.209	1.165	1.129	1.102	1.119
2	Gopher	1.737	1.655	1.584	1.521	1.466	1.420	1.383	1.359	1.400
3	Weasel	1.997	1.912	1.837	1.771	1.715	1.668	1.632	1.613	1.686
4	Ferret	2.455	2.368	2.292	2.227	2.173	2.131	2.104	2.104	2.263
5	Rabbit	2.841	2.757	2.684	2.624	2.577	2.544	2.532	2.557	2.822
6	Mink	3.168	3.090	3.025	2.973	2.935	2.916	2.923	2.978	3.368
7	Beaver	3.500	3.439	3.383	3.343	3.319	3.318	3.350	3.447	4.003
8	Raccoon	3.605	3.539	3.480	3.451	3.433	3.438	3.479	3.591	4.206
9	Otter	3.728	3.667	3.621	3.590	3.579	3.594	3.647	3.700	4.477
10	Cat	3.989	3.939	3.905	3.889	3.894	3.930	4.013	4.194	5.091
11	Dog	4.183	4.193	4.119	4.115	4.136	4.191	4.300	4.523	5.601
<u>B. 11 kV 3 phase lines</u>										
1	Squirrel	978	932	892	856	825	799	778	764	786
2	Gopher	1156	1107	1065	1027	996	969	949	939	984
3	Weasel	1319	1268	1225	1188	1157	1132	1115	1111	1185
4	Ferret	1599	1552	1511	1478	1451	1433	1426	1440	1590
5	Rabbit	1830	1787	1752	1724	1705	1697	1704	1741	1903
6	Mink	2021	1985	1956	1936	1927	1930	1954	2018	2367
7	Beaver	2212	2184	2165	2155	2158	2177	2222	2321	2813
8	Raccoon	2267	2242	2225	2219	2226	2250	2303	2413	2955
9	Otter	2340	2318	2305	2304	2316	2347	2409	2536	3146
10	Cat	2485	2471	2460	2477	2502	2550	2635	2800	3577
11	Dog	2592	2585	2589	2607	2643	2705	2810	3008	3936
<u>C. 33 kV 3 phase lines</u>										
1	Mink	17760	17469	17247	17101	17047	17113	17366	17991	21299
2	Beaver	19401	19185	19045	18994	19053	19264	19715	20663	25314
3	Raccoon	19869	19678	19566	19547	19644	19902	20420	21478	26595
4	Otter	20487	20327	20251	20273	20421	20743	21351	22557	28315
5	Cat	21687	21604	21613	21734	22000	22476	23297	24857	32196
6	Dog	22620	22604	22680	22879	23242	23842	24840	26697	35422

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2	Gopher	25.91	109	1.230	0.317	78	0.306	2076	-	-
3	Weasel	31.21	123	1.021	0.312	88	0.302	2343	-	-
4	Ferrat	41.07	155	0.761	0.306	111	0.375	2953	-	-
5	Rabbit	52.21	183	0.610	0.300	131	0.369	3406	-	-
6	Mink	62.32	200	0.511	0.296	149	0.365	3963	0.383	11019
7	Beaver	74.07	235	0.430	0.291	160	0.361	4477	0.379	13432
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9	Otter	82.85	257	0.385	0.289	184	0.358	4896	0.376	14689
10	Cat	94.21	285	0.338	0.286	204	0.355	5430	0.374	16290
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5	Habbit	2.841	2.757	2.684	2.624	2.577	2.544	2.532	2.557	2.922
6	Mink	3.168	3.090	3.025	2.973	2.935	2.916	2.923	2.978	3.368
7	Beaver	3.508	3.439	3.383	3.343	3.319	3.318	3.350	3.447	4.003
8	Raccoon	3.605	3.539	3.480	3.451	3.433	3.438	3.479	3.591	4.206
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10	Cat	3.909	3.939	3.905	3.889	3.894	3.930	4.013	4.194	5.091
11	Dog	4.183	4.193	4.119	4.115	4.136	4.191	4.300	4.523	5.601
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5	Habbit	1830	1787	1752	1724	1705	1697	1704	1741	1903
6	Mink	2021	1985	1956	1936	1927	1930	1954	2018	2367
7	Beaver	2212	2184	2165	2155	2158	2177	2222	2321	2813
8	Raccoon	2267	2242	2225	2219	2226	2250	2303	2413	2955
9	Otter	2340	2318	2305	2304	2316	2347	2409	2536	3146
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11	Dog	2592	2585	2589	2607	2643	2705	2810	3008	3936
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2	Beaver	19401	19105	18945	18994	19053	19264	19715	20663	25314
3	Raccoon	19069	19678	19566	19547	19644	19902	20420	21478	26595
4	Otter	20487	20327	20251	20273	20421	20743	21351	22557	28315
5	Cat	21687	21604	21613	21734	22000	22476	23297	24857	32196
6	Dog	22620	22604	22680	22879	23242	23842	24840	26697	35422

CHAPTER - 4

OPTIMIZATION OF PARAMETERS FOR DISTRIBUTION SYSTEM

4.1 INTRODUCTION :

The distribution system is characterised by substation size, size of the conductor, load densities of the area, feeder main length, voltage regulation and number of feeders per substation. The optimization of these parameters results in the optimal distribution system.

We can have number of approaches to solve this problem. Any demand area can be served with large feed areas having lengthy feeders. This results in large amount of system losses but the capital investment is low.

The other alternative is to have the area served by more number of substations with less feed area having short feeders. This results in greater capital investment where as the energy losses are minimum. The solution which gives the minimum total system cost is taken as the optimal solution.

The earlier attempts in distribution system planning were based on defining served distribution parameters and patching up a relationship between them. Then came the algorithms in which the distribution parameters were optimized through simulation techniques. In the later works the computational efficiencies were improved remarkably. The mathematical Programming techniques like Dynamic Programming, Linear Programming, Integer Programming, Mixed Integer Programming, Transportation and Transshipment models and Branch and Bound techniques were profusely used.

In this chapter a mathematical model has been formulated to obtain the optimal distribution system parameters. The main aim in the formulation is to represent the cost function i.e. the total system cost in terms of substation feed area which is then minimized. Then the optimal substation feed area is obtained. The other parameters i.e. feeder service area, number of substation, capacity of substation, number of feeder per substation and length of the feeder, are expressed in terms of substation feed area. So the optimal substation feed area is in turn used to calculate the optimal values of other parameters. A problem in which the parameters of primary distribution system have been optimized, is considered in this chapter.

4.2 FORMULATION OF PROBLEM :

Our objective is to minimize the total distribution system cost. So, let us assume that cost function F represents to entire system cost which can be defined as follows :

$$\begin{aligned}
 F = & \text{ [Capital required for feeder mains]} && \dots(1) \\
 & + \text{ [Capital required for lateral mains]} && \dots(2) \\
 & + \text{ [Capital required for sub-stations]} && \dots(3) \quad -4.1 \\
 & + \text{ [Capital required for infeed circuits } && \\
 & \quad \text{ to the substation]} && \dots(4) \\
 & + \text{ [Cost of energy losses in transformers } && \\
 & \quad \text{ during their expected service period]} && \dots(5) \\
 & + \text{ [A constant term depicting cost of } && \\
 & \quad \text{ energy losses in feeders]} && \dots(6)
 \end{aligned}$$

The total cost of feeder depends on its length and cost in rupees per Km (which varies with the conductor size). The

substation cost can be divided into three parts which consists of fixed cost, variable cost of substation and variable cost of feeder bay in substation. The cost of in-feed circuits required to feed the distribution substation is a function of radius of the substation feed area and number of source stations. Each substation except those at source stations would require length of at least equal to twice the radius of substation feed area. As far as transformation loss cost is concern the present worth of the cost of energy losses during the expected life of the substation can be defined as [9]

$$C_{o/o} = a_1 + b_1 n_0^{-1} \quad \dots(4.2)$$

where

$$a_1 = 8760 N_t \left[a' \sum_{K=1}^{NLS} \frac{C_{oK}}{(1+u)^K} + c' (UF)^2 \right] \quad \dots(4.2.1)$$

$$\times \sum_{K=1}^{NLS} \frac{(LLF_K) C_{oK}}{(1+u)^K}]$$

$$b_1 = \frac{8760 AD}{(DF)(pf)(UF)} \left[b' \sum_{K=1}^{NLS} \frac{C_{oK}}{(1+u)^K} + d' (UF)^2 \right]$$

$$\sum_{K=1}^{NLS} \frac{(LLF_K) C_{oK}}{(1+u)^K}] \quad \dots(4.2.2)$$

with

- A - Area of study system in sq. Km.
- D - Connected load density in Kw/sq. Km.
- DF- Load diversity factor at feeder mains
- pf- Power factor
- UF- Utilization factor of the transformer in substation.

- N_t - Number of transformer in substation
- a' - Fixed part of transformer core loss in Kw.
- b' - Variable part of the transformer core loss in Kw per KVA capacity of the transformer
- NLS - Expected life of substations in years.
- C_{eK} - Cost of energy at K^{th} year
- U - Annual discount rate in p.u
- C' - Fixed part of transformer full load copper loss in Kw
- d' - Variable part of transformer full load copper loss in Kw per KVA capacity of the transformer.
- LLF - Loss load factor which is a function of LF and is of the form $LLF = A(LF)^2 + B(LF)$ where $A + B = 1$, LF = Load factor

The equation (3.6) has been obtained considering the fact that cost of energy loss is a function of the cost of energy per Kwh distributed during that year. The cost of energy keeps on increasing due to rising prices of distribution system paraphernalia, construction and maintenance. The resulting affect of increase in cost of energy losses has been considered in equation (4.2), (4.2.1) and (4.2.2).

Now consider a constant system area A sq. Km. Let us assume that load density is uniform in the area with all distribution feeder being radial, alike and having same conductor cross-section through out the feeder main. Let there be n_s substation each of KVA capacity and having a substation feed area a_s . Also let the number of feeder per substation be n_f with feeder main length equal to L. Optimal values of all these

parameters are to be calculated. The parameters, can be represented in terms of substation feed area, a_s as [9]

$$L = \left(\frac{2}{\pi^{1/2}}\right) a_s \quad \dots(4.3)$$

$$n_f = \left[\frac{1}{\pi^{3/2} (K_1)^3 V} \right] a_s^{3/2} \quad \dots(4.4)$$

where

V = Percentage voltage regulation of radial distribution feeder.

$$K_1 = \left[\frac{H(LDF)(DF)(pf)}{\pi DZ} \right]^{1/3} \quad \dots(4.5)$$

Z - Zig zag factor of the feeder main

H - Voltage regulation constant to be obtained from the specified table.

$$n_s = A a_s^{-1} \quad \dots(4.6)$$

$$a_f = a_s n_f^{-1} \quad \dots(4.7)$$

$$= \left[\pi^{3/2} (K_1)^3 V \right] a_s^{-1/2}$$

$$KVA = \frac{D a_s}{(DF)(pf)(UF)} \quad \dots(4.8)$$

Now using equation (4.1) and equations (4.3) and (4.8) the objective function F' for secondary distribution system is expressed in terms of substation feed area, a_s as [9].

$$F' = \frac{G_1 C_f}{V} a_s - G_2 C_{fe} a_s^{1/2} + G_3 + G_4 C_{fe} a_s^{-1/2} + (eA + a_1 A) a_s^{-1} \quad \dots(4.9)$$

where

$$G_1 = \frac{AZ}{\pi^2 K_1^3} \quad \dots(4.10)$$

$$G_2 = \frac{2N}{\pi^{1/2}} \quad \dots(4.11)$$

$$G_3 = \frac{ADh}{(DF)(pF)(UF)} + b_1 + \frac{AC_f^i}{\lambda} \quad \dots(4.12)$$

$$G_4 = \frac{2A}{\pi^{1/2}} \quad \dots(4.13)$$

Similarly the objective function for primary distribution is expressed as [9]

$$\begin{aligned} F'' = & \frac{G_1(C_f - C_f^i)}{\sqrt{V}} a_s + \left[\frac{G_5(f - R_s C_f^i)}{\sqrt{V}} - G_2 C_{fe} \right] a_s^{1/2} \\ & + G_6 + G_4 C_{fe} a_s^{-1/2} \\ & + (eA + a_1 A) a_s^{-1} \quad \dots(4.14) \end{aligned}$$

where

$$G_5 = \frac{A}{\pi^{3/2} (K_1)^3} \quad \dots(4.15)$$

$$G_6 = \frac{ADh}{(DF)(pF)(UF)} + b_1 + \frac{AC_f^i}{2R_s} \quad \dots(4.16)$$

where

R_s - Radius of circular feed area of secondary distribution substation.

C_f - Feeder main cost in Rs/Km.

C_f^i - Lateral feeder cost in Rs/Km.

L_s - Length of lateral feeder in Km.

λ - Distance between the consumers.

f - Cost of feeder bay which is known in dependtly.

e - Substation fixed cost in Rs.

h - Substation variable cost

N - Number of source points feeding the primary distribution system.

C_{fe} - Cost of inf,eed circuit.

To minimize (4.14), differentiate it with respect to a_s , we get

$$a_s^2 + \frac{G_7}{2G_8} a_s^{3/2} + \frac{G_4 C_{fe}}{2G_8} a_s^{1/2} = \frac{(a + a_1) A}{G_8} \quad \dots(4.17)$$

where

$$G_7 = \frac{G_3 (f - R_s C_f^i) m}{V} = G_2 C_{fe} \quad \dots(4.18)$$

$$G_8 = \frac{G_1 (C_f - C_f^i)}{V} \quad \dots(4.19)$$

The equation (3.16) is the final equation from which optimal substation feed area a_s^* can be calculated. So, using equation (4.3) to (4.8) the required optimal parameters L^* , n_f^* , N_s^* , a_f^* and substation capacity KVA^* are obtained.

4.3 NUMERICAL EXAMPLE :

Using the model described before a sample problem of primary distribution system has been solved. The data used for the problem is given below.

Area of the system (A)	=	1000 sq. Km.
Load power factor (pf)	=	0.8
Load density considered (D)	=	10 Kw/sq.Km.
The annual rate of growth of cost of energy	=	1%
Present cost of energy (C_e)	=	Rs. 0.25 per Kwh
The annual discount rate (u)	=	0.1
Life of the transformer (NLS)	=	3 years
Primary distribution voltage	=	11 KV

Subtransmission voltage	=	33 KV
Cost of infeed circuit (C_{fo})	=	Rs. 40,000 per Km.
Load factor (LF)	=	0.2
Loss load factor (LLF)	=	0.072
Average diversity factor (DF)	=	2.5
Average zig-zag factor (Z)	=	1.4
Average load distribution factor (LDF)	=	2.12
Radius of the circular feed area (R_c) of secondary distribution substation	=	0.5 Km.
Feeder main cost (C_f)	=	Rs. 18700 per Km.
Lateral feeder cost (C'_f)	=	Rs. 11,000 per Km.
Number of transformer in a substation (N_t)	=	1
Percentage voltage regulation of a radial distribution feeder (V)	=	1.5%
Voltage regulation constant (H) for conductor-Rabbit	=	1705
Utilization factor of the transformer in substation (UF)	=	0.8
Substation cost coefficients :		
Substation variable cost (h) (capacity component)	=	60.38/KVA
Substation variable cost (f) (feeder bay component)	=	Rs. 75,000
Substation fixed cost component (e)	=	Rs. 630,000
Transformer loss coefficients :		
Fixed part of transformer core loss (a')	=	0.725 Kw.
Variable part of transformer core loss (b')	=	0.001155 Kw/KVA

Fixed part of transformer full load copper loss (e')	=	3.9 Kw.
Variable part of transformer full load copper loss (d')	=	0.00605 Kw/KVA
Conductor code name for primary distribution	=	Rabbit
Conductor code name for secondary distribution	=	Gopher

Solution :

Using equation (4.5) the value of

$$K_1 = 5.477$$

From equation (4.2.1);

$$a_1 = 4206.634$$

From equation (4.10);

$$G_1 = 0.863$$

From equation (4.13);

$$G_4 = 1128.379$$

From equation (4.15);

$$G_5 = 1.093$$

From equation (4.16);

$$G_7 = 5522.331$$

From equation (4.19)

$$G_8 = 4372.532$$

Putting the values in the final equation (4.17), we obtain the equation

$$a_0^2 + 0.631 a_0^{1.5} - 5161 - 2178 a_0^{0.5} - 145043.39 = 0$$

The above equation is solved by using Newton Raphson Method. The optimal substation feed area, $a_0^o = 504.40$ sq.Km

Now from equation (4.3);

$$L = 17.73 \text{ Km}$$

From equation (4.4);

$$n_f = 8$$

From equation (4.6);

$$n_0 = 2$$

From equation (4.7);

$$a_f = 63.05 \text{ sq.Km.}$$

From equation (4.8);

$$\text{KVA} = 3152.5$$

Results :

Thus for the system area of 1000 sq.Km with load density 10 Kw/sq.Km and load factor being 0.2, the optimal parameters of the distribution system are as follows :

- | | | | |
|-------|-------------------------------|---|---------------|
| (i) | Substation feed area, a_0^o | = | 504.40 sq.Km. |
| (ii) | Feeder service area, a_f^o | = | 63.05 sq.Km |
| (iii) | Number of substation, n_0^o | = | 2 |
| (iv) | Capacity of the substation | = | 3152.5 KVA |

- (v) Number of feeder per
per substation n_f^* = 8
- (vi) Length of feeder L^* = 17.73 Km.

4.4 CONCLUSIONS :

Most of the methods, mentioned before are generally applicable to urban areas where the location of future loads is known in advance through master development plans. However the methods can't be used in rural areas because there the location of future loads is highly unpredictable. This method used in such cases with effectiveness. Various factors like load factor, average diversity factor, zig-zag factor, average load distribution factor have been considered in the model which delivers much better and realistic results. The optimal parameters give guidelines for evolving appropriate distribution system planning policies and future expansion in an optimal way.

CHAPTER-9

CONCLUSIONS

Distribution system is an integral part of power system. In the present work light has been thrown on couple of methods for optimal distribution substation and primary feeder planning.

The One of the important factor which has been taken into consideration is the rapidly increasing load growth in any area. The increased future demand may result in the overloading of feeders in distribution network and also the violation of voltage magnitudes. Model has been formulated for expansion of primary feeders and associated voltage correction. So a suitable plan is obtained for primary feeder planning.

A.C. adjoint method has been used to find the sensitivities of feeder currents and load bus voltages due to various feeder additions. A heuristic approach has been used for the expansion which is logical and is based on the rules followed in the field and is used in actual practice. The logic being- select the feeder (for reinforcement) which is most effective in alleviating overloads in distribution feeders and voltage correction at the substation.

The Load flow results are obtained with the help of fast decoupled method which is quite an efficient technique. The A.C. Adjoint method which has been used to calculate the sensitivities of feeders and node voltages, simplifies the whole problem greatly. In each iteration the Load flow studies are done once. On the other hand without its use, in each iteration load flow study has

to be done after each feeder addition. This is very time consuming and sluggish process. Another novelty in the work being that an attempt has been made on solving the looped feeder planning problem.

Some times a whole distribution system is to be planned for a new area (specially rural) or existing area of given load density. The problem has to be solved in an optimal way. One of the way is to optimize the various parameters which defined the distribution systems. A mathematical model has been used in the present work to find the optimal values of the different parameters which give guidelines for drawing strategies in distribution system policies and future expansion.

DETAILS OF THE PROGRAMME

The structure of the programme used is shown in Figure [A.1]. The subroutine INDPHE1 reads the input data to be fed. DATADR calculates the values of variables like feeder resistance, reactance for whole of the system.

The subroutine ADJ forms the adjoint L.H.S. matrix. The currents flowing in the feeders and the maximum currents in the feeders are also calculated. The subroutine ADJ1 forms the R.H.S. adjoint vector and determines the sensitivities of the feeder currents and the load bus voltage functions due to feeder additions and shunt capacitor location. Subroutine VOLT finds out if the voltage at the load buses are violated. The subroutine GSJOR inverts the L.H.S. adjoint matrix. FADELf carries out load flow using fast decoupled technique.

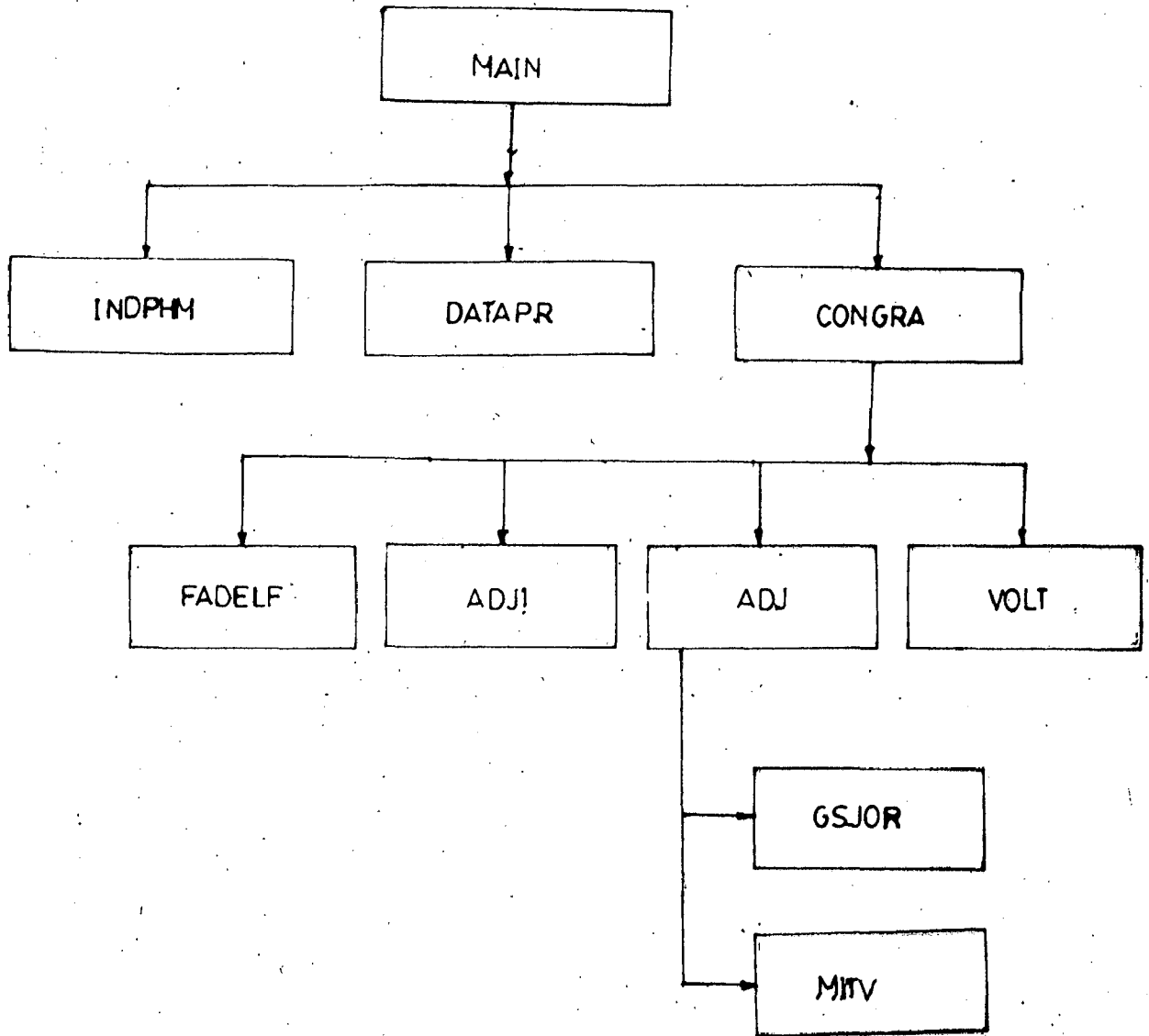


FIG A-1

STRUCTURE OF THE PROGRAMME

Input Data to be given :

- (1)
- (a) NGP - Number of grid and power substation.
 - (b) NFED- Number of feeders in the system
 - (c) NPSS- Number of power substation.
 - (d) NGRSS-Number of grid substation.
 - (e) NTFED-Number of types of feeder.
 - (f) NCAP- Number of substation at which the capacitors are installed.
 - (g) MAXIT-Maximum number of iterations
 - (h) NEF -- = 0 - Only the effect on over loaded lines is considered.
 * 0 Effect as all lines is considered.
 - (i) NCAPAD \neq 0 Capacitor additions to be considered after distribution expansion.
 - (j) NBC - *

(2) Feeder Data :

- (a) TFED - Type of feeder
 - 1 - Squirrel
 - 2 - Gopher
 - 3 - Rabbit
- (b) COSFD - Cost/Km of i^{th} type of feeder
- (c) RF - Resistance/phase/Km of i^{th} type of feeder.
- (d) XF - Reactance/phase/Km of i^{th} type of feeder

:65:

- (e) SF - Susceptance/phase/Km of i^{th} type of feeder.
- (f) CURAT - Current carrying capacity of i^{th} type of feeder
- (g) BMVA - Base MVA
- (b) BASVOL - Base Voltage in KV
- (i) LFROM, LTO - Line terminating buses.
- (j) FEDTY - Type of feeder of i^{th} R.O.W.
- (k) LENFE - Length of feeder in i^{th} R.O.W.

3. Power and grid substation Data :

- (a) QMWPS - MW capacity of substation/phase
- (b) QMVAPS - MVAR capacity of substation/phase.
- (c) V or BVM - Specified Normal Voltage
- (d) VMIN - Specified minimum Voltage of substation.
- (e) VMAX - Specified maximum voltage of substation
- (f) BTYPE - Type of substation Grid/power
If 0 - Power
OR If 3 - Grid
- (g) NUEXF - Number of existing feeder single/double.
- (h) NUEXP - 1, If can be expanded to double.
Otherwise zero.

4. Capacitor Data :

- (a) ISH - Bus number having capacitor
- (b) BS - Susceptance of the capacitors at each bus.

5. New Capacitor Data :

- (a) COSCA - Cost of installing a capacitor of size BSS
- (b) BSS - Susceptance of each capacitor unit of shunt capacitor.

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