USE OF REDUCED ORDER MODELS IN POWER SYSTEMS

A DISSERTATION

Submitted in partial fulfilment of the requirements for the award of the degree
of
MASTER OF ENGINEERING
in
ELECTRICAL ENGINEERING
(Power Systems Engineering)

by

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CANDIDATE'S DECLARATION

I hereby certify that the work which is being presented in the dissertation entitled "USE OF REDUCED ORDER MODELS IN POWER SYSTEMS" in partial fulfilment of the requirements for the degree of MASTER OF ENGINEERING IN ELECTRICAL ENGINEERING (Power Systems Engineering) submitted in the Department of Electrical Engineering, University of Roorkee, Roorkee is an authentic record of my own work. carried out during a period of six months from August 1984 to January 1985 under the supervision of Dr. Jayanta Pal, Reader and Sri R.Prasad, Lecturer, Electrical Engineering Department, University of Roorkee, Roorkee.

The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

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This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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ABSTRACT

In the present work existing model order reduction techniques are applied to power system simplification and to controller design. The thesis deals with frequency domain model order reduction techniques and controller design based on transfer function description of the power system. Brief conclusions are given at the end of each chapter.

In chapter one we have listed various techniques for model order reduction. Only methods based on Pade approximation techniques are applied in the thesis.

In the same chapter we discribe the Pade approximation technique and its various modified versions as given by number of researchers. The methods described in this chapter are successfully extended to multivariable cases also. For simplicity the methods are applied to single input single output systems only.

The second chapter is devoted to develop the transfer function from a given state space description. The classical Faddeeva algorithm is applied to the state space equations. This method gives erronous results when the dimension of matrix A is large. To eradicate this difficulty, a modified algorithm of Faddeeva is given in the same chapter.

The third chapter describes the development of a power system model. The problem is taken from [17] and the development of the model is also from reference [17] .

In the fourth chapter describes the reduction techniques based on Pade, modified Pade, Routh - Hurwitz array are applied to an actual power system model. The transfer functions obtained from different methods are given in this chapter and all the reduced models are summarised in the last, of the same chapter. The final responses show the validity of each method and their relative drawbacks.

In the fifth chapter a method for sub optimal controller design is given. This design method applied to the reduced order models chosen from chapter four. The time response comparison of reduced order case with that of the original system shows the importance of reduced models in controller design.

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INTRODUCTION

Physical systems such as aircraft, chemical plants, electrical power systems etc. can be described mathematically by state space models or transfer function models. Electric power systems may be modelled by large number of differential equations that lead to high order state-space or transfer function models. From analysis point of view, these high order models present formidable problems. Thus a need exists for a systematic procedure to derive a reduced order dynamic equivalent model in the state-space on in transfer function form from the corresponding high order description.

Several schools of approach to the model reduction problem have been developed either in the time domain or in the frequency domain. As far as possible, the lower order approximates certain dominant characteristics of original system.

Broadly speaking, the present work consists of three main parts. In the first chapter the reduction techniques based on Pade approximation and its variants are described. In chapter two methods to obtain transfer functions from state-space equations have been discussed. All the reduction techniques discussed are for the continuous time case.

Chapter three describes the development of system model for a single area power system; i.e. a synchronous machine connected to an infinite bus through a transmission line. In the fourth chapter we obtain the reduced order models for the power system using the methods of chapter one. Time response comparisons are made to show the closeness of the reduced model to the original system.

The use of transfer functions and their reduced order models in design of controllers is dealt within chapter five. The entire chapter is devoted to the controller design based on sub optimal criterion. The time response comparison of optimal controller to the sub-optimal controller shows the validity of sub-optimal controller design criterion. The time response graph attached to this chapter shows the closeness. The sub-optimal controller design criterion in controller design overcomes the difficulties in solving the equations and it has the viability in the analysis of complex problems such as Electrical Power Systems where all the control parameters are not available for measurement. So, the strong point goes in favour os sub-optimal controller design and it resolves the measurement problem of all control parameters.

The last chapter; i.e. chapter six deals with the merits and demerits of the methods which are described in the different chapters.

NOTATIONS

U or U(t)	Input vector or control vector of (rx1) dimensions
x or x(t)	n^{th} order state vector
A	System matrix of order (nxn)
B	Control matrix of (nxr) dimension
C	Measurement matrix of order (nxr) dimention
Y or Y(t)	Output vector of (mx1) dimension
n	System order
r	Number of inputs
m	Number of outputs
G(s)	System transfer function of appropriate order
~~A_R or A^*~~	~~Reduced model system~~ matrix
B_R or B^*	Reduced model control matrix
C_R or C^*	Reduced model measurement matrix
l	Reduced model system order
$G_R(s)$	Reduced model system transfer function
i	i^{th} eigenvalues of system
z	Transformed state vector of system
v	Infinite bus voltage
v_r	Terminal voltage of the m/c
V_d, V_q	Direct & quadrature axis voltage at the terminals of the m/c
I_d, I_q	Direct & quadrature axis currents of the m/c

E_q	Excitation voltage or open circuit voltage of the m/c
E_{ex}	Exciter input signal
ω	Angular velocity deviation of m/c rotor in rad/sec
δ	Rotor angle of the m/c in rad
P	Electrical power output of the m/c
M	Moment inertia of the m/c
D	Damping coefficient of the m/c
p	Change in input power of m/c due to governor action
T_1, T_2	Time constants of the prime mover governor
a	Product of governor time constants
b	Summation of governor time constants
c	Governor gain constant
y_{11}	Self admittance of the network at the internal bus of the m/c
y_{12}	Mutual admittance of the network between the internal bus of the m/c and the infinite bus
x_d	Direct axis synchronous reactance
x_d'	Direct axis transient reactance of m/c
T_{do}	Direct axis field time constants
	Incremental operator

CHAPTER - 1

DESCRIPTION OF PADE REDUCTION TECHNIQUES

The Pade approximation technique is recognised to be a powerful tool for obtaining reduced order models. But occassionally it leads to unstable models for originally systems. Mixed methods are available that give a stable model using partial Pade approximations. In this chapter we give a brief description of Pade type methods for model order reduction.

METHOD 1

1.1 PADE APPROXIMATION TECHNIQUE FOR SYSTEM REDUCTION

The Pade approximation technique for system reduction as proposed by Shamash [3, 7, 8, 9] is described below. This technique is also useful in case of systems described by state space equations.

DEFINITION OF PADE APPROXIMANT

Shamash [7] has defined the Pade approximant as:

'A Pade approximant is a rational function $P_m(x)/Q_n(x)$, when $P_m(x)$ and $Q_n(x)$ are polynomials in x of degree m, n where [m, n] is said to be the Pade approximant of the function f(x), if and only if, the power series expansion of [m, n] is identical with that of f(x) upto and including terms of order x^{m+n}'.

Let the function to be approximated be defined by the power series

$$f(x) = c_0 + c_1 x + c_2 x^2 + \ldots \tag{1}$$

and the Pade approximant be defined by

$$\frac{P_m(x)}{Q_n(x)} \triangleq \frac{a_0 + a_1 x + a_2 x^2 + \ldots + a_m x^m}{b_0 + b_1 x + b_2 x^2 + \ldots + b_n x^n} \tag{2}$$

$m \leq n$

Since the power series expansion of (2) is to agree with (1) as far as and including the terms in x^{m+n}, we have the following set of linear simultaneous equations.

$$\begin{aligned}
a_0 &= b_0 c_0 \\
a_1 &= b_0 c_1 + b_1 c_0 \\
a_2 &= b_0 c_2 + b_1 c_1 + b_2 c_0 \\
&\vdots \\
a_m &= b_0 c_m + b_1 c_{m-1} + \ldots + b_m c_0 \\
&\vdots \\
0 &= b_0 c_{m+n} + b_1 c_{m+n-1} + b_2 c_{m+n-2} + \ldots + b_n c_m
\end{aligned} \tag{3}$$

which serves uniquely to derive the coefficients of (2). It should be noted that either b_0 or b_n is to be taken as unity. In the present work b_n has been taken to be unity

In the above analysis, the Pade approximation was made about the point $x = 0$. The generalization of Pade approximation technique to two (or more) points was first introduced by Baker. Sometimes information about the function to be approximated is available at two or more points. It is suggested that

This additional information about the function may be taken into account by requiring the Pade approximant to satisfy, exactly the conditions at the origin and other prescribed points.

METHOD 2

1.2 MODEL REDUCTION USING ROUTH-HURWITZ ARRAY

Krishnamurthi et. al [13] have presented an interesting method for the reduction of dimension of a high order transfer function. Their method makes use of the classical Routh-Hurwitz stability array and is applicable to SISO systems. The natural extension of this method to multivariable systems is given below.

The common denominator polynomal of a general multivariable (1 x m) transfer function [G(s)] may be reduced to a lower dimension. The numerator polynomials of each of the scalar functions $g_{ij}(s)$ may then be reduced by the method Krishnamurthi et. al [13] . This natural extension is included here as it can form the basis of a computer aided model reduction technique as given is the steps below.

Step 1 Find the transfer matrix [G(s)] from the state space description (A, B, C, D) of a high order system.

Step 2 Find the reduced model [R(s)] of dimension r as given above

Step 3 Find the numerator by Routh-Hurwitz array as for the denominator.

METHOD 3

1.3 PADE APPROXIMATION AND DOMINANT MODE RETENTION

Consider the following high order system transfer function $G(s)$ as shown below:

$$G(s) = \frac{a_0 + a_1 s + a_2 s^2 + \dots + a_{n-1} s^{n-1}}{(s+\lambda_1)(s+\lambda_2)\dots(s+\lambda_n)} \tag{4}$$

$$= \frac{N(s)}{b_0 + b_1 s + b_2 s^2 + \dots + b_n s^n} \tag{5}$$

where $N(s)$ denotes numerator of (4).

$G(s)$ can be expanded into a power series about $s=0$ of the form

$$G(s) = c_0 + c_1 s + c_2 s^2 + \dots \tag{6}$$

where $c_0 = \frac{a_0}{b_0}$

$$c_k = \frac{1}{b_0} \left[a_k - \sum_{j=1}^{k} b_j c_{k-j} \right], \quad \forall K \neq 0 \tag{7}$$

with $a_k = 0, \quad \forall K > n-1$

The b_i are directly proportional to the time moments of the system.

Assume that a reduced model $G_R(s)$ of order l, is required which retains the pole at $s = -\lambda_1$ say,

$$\text{Let } G_R(s) = \frac{a_0^* + a_1^* s + a_2^* s^2 + \dots + a_{l-1}^* s^{l-1}}{b_0^* + b_1^* s + b_2^* s^2 + \dots + b_l^* s^l} \tag{8}$$

The order of the numerator of $G_R(s)$ and $G(s)$ have been assumed to be one less than the denominators to simplify the notation. Then for $G_R(s)$ to be Pade approximant of $G(s)$ we have from equation (3), following set of equation.

$$
\begin{aligned}
a_0^* &= b_0^* c_0 \\
a_1^* &= b_0^* c_1 + b_1^* c_0 \\
a_2^* &= b_0^* c_2 + b_1^* c_1 + b_2^* c_0 \\
&\ldots \\
0 &= b_0^* c_l + b_1^* c_{l-1} + \ldots + b_{l-1}^* c_1 + b_l^* c_0 \qquad (9) \\
&\ldots \\
0 &= b_0^* c_{2l-1} + b_1^* c_{2l-2} + \ldots + b_l^* c_{l-1}
\end{aligned}
$$

with $b_l^* = 1$

But since $G_R(s)$ is to have a pole at $s = -\lambda_1$, then using the concept of Pade approximation about more than one point, the last equation of (9) is replaced by the following equation.

$$0 = b_0^* - b_1^* \lambda_1 + b_2^* \lambda_1^2 - b_3^* \lambda_1^3 + \ldots + (-1)^l \lambda_1^l \qquad (10)$$

Hence these equations are solved for the coefficients of b_i^*, a_i^*; $(i = 0, 1, \ldots, (l-1))$ of equation (8).

Now suppose that reduced order model $G_R(s)$, retains the l dominant poles (l poles nearest the origin) of the high order system. Further suppose that the l dominant poles are known, $G_R(s)$ can then be written as

$$G_R(s) = \frac{a_0^* + a_1^* s + a_2^* s^2 + \ldots + a_{l-1}^* s^{l-1}}{(s+\lambda_1)(s+\lambda_2)\ldots(s+\lambda_l)} \qquad (11)$$

From denominators of (11) and (8) all b_i for i = 0, 1, 2, ...(l-1) can be calculated. Then if $G_R(s)$ is to approximate G(s) in the Pade sense about s = 0, then the a_i(i = 0, 1, 2,..., (l-1) may be determined using the first l equations of (9).

So far it has been assumed that the dominant poles of the system are known which in most cases is not necessarily true. In such cases, the roots of the denominator polynomial can be obtained by LIN's method.

If the system is described in state-vector form

$$\begin{aligned} \dot{x} &= A\underset{\sim}{x} + B\underset{\sim}{u} \\ \underset{\sim}{y} &= C\underset{\sim}{x} + D\underset{\sim}{u} \end{aligned} \qquad (12)$$

The system transfer function is given by

$$\begin{aligned} G(s) &= C(s_I - A)^{-1} B + D \\ &= (C A^{-1} B + D) + C A^{-2} B s + C A^{-3} B s^2 + \ldots \quad (13) \\ &= c_0 + c_1 s + c_2 s^2 + c_3 s^3 + \ldots \end{aligned}$$

where

$$\begin{aligned} c_0 &= C A^{-1} B + D \\ c_i &= C A^{-(i+1)} B \qquad i > 0 \end{aligned} \qquad (14)$$

Hence the reduction algorithm is applied to the expantion (13) where the coefficients are obtained using equation (14).

If the system being modelled is unstable, then it is important that reduced model should be unstable as well. Hence, unstable mode of G(s) must be retained in the reduced model. Koenig's theorem and its generalization [7] may be used to compute the unstable modes as follws:

Given G(s), the following transformation is made

$$s = (z-1) / (z+1) \tag{15}$$

to get G(z).

The unstable poles of G(s) are mapped outside the unit circle in z plane, expand G(z) in the form

$$G(z) = d_0 + d_1 z + d_2 z^2 + \ldots \tag{16}$$

Then applying LIN's method we get all the large magnitude poles of G(z) which in this case will be the poles outside the unit circle. Having computed the unstable poles, the coeffieient of $G_R(s)$ are computed as before.

1.4 REDUCTION OF MULTIVARIABLE SYSTEM

The Pade approximation technique has been extended to the reduction of multivariable systems [7, 8]. However, the method may involve large amounts of computation which may make the reduction of the system less desirable.

For a multivariable system

$$Y(s) = G(s)\, U(s) \tag{17}$$

Where Y(s) is the output m-vector and U is the input r-vector and G(s) is the transfer function matrix of the system. Eq. (17) may be rewritten in the form:

$$\left[G(s)\right] = \frac{A_0 + A_1 s + A_2 s^2 + \ldots + A_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \ldots + b_n s^n} \qquad (18)$$

$$d(s) = b_0 + b_1 s + b_2 s^2 + \ldots + b_n s^n \qquad (18a)$$

Where A_i $(i = 0, 1, 2, \ldots, (n-1))$ are (m x r) constant matrices and b_i $(i = 0, 1, 2, \ldots, n)$ are scalar constants. G(s) can be expanded in the power series of the dorm:

$$\left[G(s)\right] = c_0 + c_1 s + c_2 s^2 + \ldots \qquad (19)$$

where c_i $(i = 0, 1, 2, \ldots)$ are (m x r) constant matrices, which satisfy the relation

$$c_i = \frac{1}{b_0} \left[A_i - \sum_{j=0}^{i-1} b_{i-j} c_j \right] ; \quad i = 0,1,2,\ldots \qquad (20)$$

with $c_{-1} = 0$ and $a_i = 0, \forall\, i \geq n$

Thus using Eq. (20), the matrix transfer function may be expanded into a power series. Let the reduced order model have a matrix transfer function of the form:

$$\left[G_R(s)\right] = \frac{A_0^* + A_1^* s + A_2^* s^2 + \ldots + A_{l-1}^* s^{l-1}}{b_0^* + b_1^* s + b_2^* s^2 + \ldots + b_l^* s^l} \qquad (21)$$

Where $A_i (i = 0,1,\ldots,(l-1))$ are (m x r) constant matrices and b_i^* $(i = 0,1,\ldots,l)$ are constant scalars. By applying the LIN's method, the roots of the denominator polynomial are computed. The dominant poles (or any other desirable poles) of $[G(s)]$ are retained in $[G_R(s)]$. The numerator coefficients of $[G_R(s)]$ are chosen such othat $[G_R(s)]$ approximates $[G(s)]$ in Pade sense.

The procedure is as follows:

(i) The common denominator d(s) of $[G(s)]$ is found.

(ii) By applying LIN's method or Koenig's theorem and its generalization to power series expansion of (1/d(s)); the l dominant poles of $[G(s)]$ are found. This then determines the coefficients b_i^* (i = 0,1,2,...,(l-1)) in (21).

(iii) The numerator matrices $[A_i^*]$ of $[G_R(s)]$ are then computed as follows:

$$\begin{aligned}
A_0^* &= b_0^* \, c_0 \\
A_1^* &= b_0^* \, c_1 + b_1^* \, c_0 \\
A_2^* &= b_0^* \, c_2 + b_1^* \, c_1 + b_2^* \, c_0 \qquad (22) \\
&\;\;\vdots \\
A_{l-1}^* &= b_0^* \, c_{l-1} + b_1^* \, c_{l-2} + \ldots + b_{l-1}^* \, c_0
\end{aligned}$$

Thus $[G_R(s)]$ can be chosen such that its first l time moments are equal to the first l time moments of $[G(s)]$.

The Pade approximation approach has three basic difficulties; viz.

(i) The reduced model may be unstable (stable) although the parent system is stable (unstable).

(ii) The Pade approximant often shows poor metching in the transient zone, although the steady state values are the same as for the high order system.

(iii) The Pade approximant often shows non-minimum phase characteristics (i.e. inverse response due to zeros in the right half s-plane).

The techniques reported hereafter attempt to remove such difficulties.

METHOD 4

1.5 PADE APPROXIMANT USING A MIXED METHOD

The mixed mathod for deriving stable low-order equivalents of high order systems, given below, is computationally easy to program and conceptually simple. It combines the Pade's approximation technique and the Routh - Hurwitz array method[13].

1.5.1 Case 1

SINGLE INPUT SINGLE OUTPUT SYSTEMS

Let the n^{th} order system transfer function G(s) and its r^{th} order reduced equivalent R(s) be described by

$$G(s) = \sum_{j=1}^{n} a_{2,j}\, s^{j-1} / \Delta(s) \tag{23}$$

$$= c_0 + c_1 s + c_2 s^2 + \ldots \tag{24}$$

and

$$R(s) = \sum_{j=1}^{r} b_{2,j}\, s^{j-1} / \Delta_r(s) \tag{25}$$

where

$$\Delta(s) = \sum_{j=1}^{(n+1)} a_{1,j}\, s^{j-1} ;$$

$\Delta_r(s)$ = denominator polynomial of degree r.

Eqn. (24) is the power series expansion of (23) about $s = 0$. The method consists of the following steps:

(i) For convenience, the even and odd terms of $\Delta(s)$ may be separated and rewritten as:

$$\Delta(s) = \sum_{j} a_{1,j+1}\, s^{n-2j} + \sum_{k} a_{2,k+1}\, s^{n-(2k+1)} \tag{26}$$

where

$j = 0,1,2,\ldots,n/2$ and $k = 0,1,2,\ldots,(n-2)/2$ for n even

$j = 0,1,2,\ldots,(n-1)/2$ and $k = 0,1,2,\ldots,(n-1)/2$ for n odd

From the R-H stability array for the denominator polynomial in (26) as follows:

$$\begin{matrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots \\ a_{31} & a_{32} & a_{33} & a_{34} & \cdots \\ \vdots & & & & \end{matrix}$$

$$a_{n-1,1} \qquad a_{n-1,2}$$
$$a_{n,1}$$
$$a_{n+1,1} \tag{27}$$

The above array is formed by the well known algorithm:

$$a_{i,j} = a_{i-2,j+1} - (a_{i-2,1} \cdot a_{i-1,j+1})/(a_{i-1,1}) \tag{28}$$

Where $i \geq 3$ and $1 \leq j \leq [(n-i+3)/2]$, $[.]$ stands for the integral part of the quantity. A polynomial of lower order r may be easily constructed [13] with $(n+1-r)$th and $(n+2-r)$th rows of the above array, to give

$$\Delta_r(s) = a_{(n+1-r),1} s^r + a_{(n+2-r),1} s^{r-1} + a_{(n+1-r),2} s^{r-2} + \ldots \tag{29}$$

Eqn. (29) may be put in the convenient form:

$$\Delta_r(s) = (s-\lambda_1)(s-\lambda_2) \ldots (s-\lambda_r)$$
$$= \sum_{j=1}^{r+1} b_{1,j} s^{j-1} \tag{30}$$

with $b_{1,r+1} = 1$.

where the $b_{1,j}$ coefficients are now known from eqn. (29).

(ii) For $R(s)$ of Eqn. (25) to be the Pade approximant of $G(s)$, we have [14]

$$b_{21} = b_1 c_0$$
$$b_{22} = b_1 c_1 + b_2 c_0$$
$$\vdots$$

$$b_{2r} = b_1 c_{r-1} + b_2 c_{r-2} + \dots + b_{r-1} c_1 + b_r c_0 \qquad (31)$$

After substituting the values of $c_i (i=0,1,\dots,(r-1))$ and $b_j (j=1,2,\dots,(r+1))$ from Eqns. (24) and (30) respectively, the $b_{2j} (j=1,2,\dots,r)$ can be found by solving the above r eqns. Now R(s) is completely determined.

1.5.2 Case 2

MATCHING A COMBINATION OF TIME MOMENTS AND MARKOV PARAMETERS

The Pade approximation technique basically matches the initial few time monents of the original and reduced systems and hence a good matching is achieved in the steady - state zone where as the transient response will only be approximate. To have overall good approximations in the low and high frequency regions, the above mentioned method is modified below to match a combination of Markov parameters and time - moments of the original and reduced systems.

Let the n^{th} order system transfer function G(s) and its r^{th} order reduced equivalent R(s) be described by

$$G(s) = \sum_{j=1}^{n} a_{2,j} s^{j-1} / \Delta(s) \qquad (32)$$

$$= \sum_{i=0}^{\infty} M_i \, s^{-i-1} \qquad (33)$$

$$= \sum_{i=0}^{\infty} T_i \, s^{i} \qquad (34)$$

and
$$R(s) = \sum_{j=1}^{r} b_{2,j}\, s^{j-1}/\Delta_r(s) \tag{35}$$

where
$$\Delta(s) = \sum_{j=1}^{(n+1)} a_{1,j} s^{j-1}; \quad \Delta_r(s) = \text{denominator polynomial of degree } r.$$

Eqns. (33) and (34) are the power series expansions of Eqn. (32) about $s = \infty$, and $s = 0$, respectively. M_i and T_i are proportional to the i^{th} Markov parameter and the i^{th} time-moment of G(s) respectively and may easily be obtained by expanding G(s) either in negative or positive power of s. The reduction method consists of the following steps.

(i) Same as step 1 of previous method.

(ii) Assume that the first time-moments and first Markov parameters are identical for the original system and the reduced models. Then one may solve the following relations to determine the coefficients $b_{2,j}$ of Eqn.(35).

$$b_{2,1} = b_1 T_0$$

$$b_{2,2} = b_1 T_1 + b_2 T_0$$

$$\vdots$$

$$b_{2,\alpha} = b_1 T_{\alpha-1} + b_2 T_{\alpha-2} + \ldots + b_{\alpha-1} T_1 + b_\alpha T_0$$

$$b_{2,(r-\beta+1)} = b_{r+1} M_{\beta-1} + b_r M_{\beta-2} + \ldots + b_{(r-\beta+3)} M_1 + b_{(r-\beta+2)} M_0$$

$$\vdots$$

$$b_{2,r-2} = b_{r+1}M_2 + b_r M_1 + b_{r-1}M_0$$

$$b_{2,r-1} = b_{r+1}M_1 + b_r M_0 \qquad (36)$$

$$b_{2,r} = b_{r+1}M_0$$

where

$$\alpha + \beta = r$$

and

α = number of time moments matched.

β = number of Markov Parameters matched.

On substituting the values of M_i, T_i and b_j from Eqns. (33), (34) and (30) respectively, the r linear relations in Eqn. (36) may be solved to find r numerator coefficients $b_{2,j}$. R(s) is then completely determined.

1.5.3 Case 3

1.5.3.1 MULTIVARIABLE SYSTEMS

Let the n^{th} order transfer function $[G(s)]$ and its r^{th} order reduced equivalent $[R(s)]$ be represented as

$$[G(s)] = \frac{[A(s)]}{\Delta(s)} = \frac{\sum_{j=1}^{n} A_{2,j}s^{j-1}}{\Delta(s)} \qquad (37)$$

$$[R(s)] = \frac{[B(s)]}{\Delta_r(s)} = \frac{\sum_{j=1}^{r} B_{2,j}s^{j-1}}{\Delta_r(s)} \qquad (38)$$

Where [A(s)], [B(s)] are (m xq) polynomial matrices and $\Delta(s), \Delta_r(s)$ are scalar denominator polynomials. r = deg $\Delta_r(s)$, and n = deg. $\Delta(s) > r$. The n^{th} order denominator polynomial of [G(s)] may be broken into the even and odd terms in the powers of s as shown in (26).

(i) Same as the first step of previous method, we finally form the denominator polynomial of [R(s)] as given in Eqn. (30).

(ii) Expand [G(s)] into a power series expansion (about s = 0) to hget

$$[G(s)] = \sum_{i=0}^{r-1} c_i \, s^i \tag{39}$$

where,

c_i (i=0,1,2,...,(r-1)) are (m x q) constant matrices.

(iii) For [R(s)] to be a Pade's approximant to [G(s)], we have

$$\begin{aligned} B_{2,1} &= b_1 c_0 \\ B_{2,2} &= b_1 c_1 + b_2 c_0 \\ &\vdots \\ B_{2,r} &= b_1 c_{r-1} + b_2 c_{r-2} + \dots + b_{r-1} c_1 + b_r c_0 \end{aligned} \tag{40}$$

From the above equation, $B_{2,j}$ (j = 1,2,...,r) may be easily determined by substituting the values of c_i(i=0,1,..., (r-1)) and b_j(j=1,2,3,...,r) from Eqns.(39) & (30) respectively.

1.5.3.2 MULTIVARIABLE SYSTEM REDUCTION BY MATCHING TIME MOMENTS AND MARKOV PARAMETERS

In this case the transfer functions $[G(s)]$ and the reduced approximant $[R(s)]$ are given by Eqns (37) and (38), where

$$[G(s)] = \sum_{i=0}^{\infty} M_i\, s^{-i-1} \qquad (41)$$

$$= \sum_{i=0}^{\infty} T_i\, s^{i} \qquad (42)$$

and unlike to quantities defined in (33) and (34), here the M_i and T_i are matrices.

1.6 METHOD 5

STABLE BIASED REDUCED ORDER MODELS USING A MODIFIED ROUTH - HURWITZ ARRAY

The method of Krishnamurthi et.al. [13] makes use of the well established Routh - Hurwitz stability array and is applicable to single input single output systems. Through an example Krishamurthi reported [13] that the poles of R(s) approximate the poles of G(s) that are closest to the origin. Thus it follows that this method of approximation is about s = 0. Thus, this method gives stable reduced models but has the following disadvantages:

The reduced models match the steady state response well but the transient response matching may be poor. To overcome this disadvantage a modified algorithm is suggested [21]. which is basically based on a modified Routh - Hurwitz array.

Let the n^{th} order system transfer function G(s) and its r^{th} order reduced equivalent R(s) be given by

$$G(s) = \frac{\sum_{j=1}^{n} a_{2,j}\, s^{j-1}}{\Delta(s)} \tag{43}$$

$$= \sum_{i=0}^{\infty} M_i\, s^{-i-1} \tag{44}$$

$$= \sum_{i=0}^{\infty} T_i\, s^{i} \tag{45}$$

and

$$R(s) = \frac{\sum_{j=1}^{r} b_{2,j}\, s^{j-1}}{\Delta(s)} \tag{46}$$

Let $\Delta(s)$ be given by

$$\Delta(s) = \sum_{j=1}^{(n+1)} a_{i,j}\, s^{j-1}$$

$$= a_{11} + a_{12}\, s^{1} + a_{13}\, s^{2} + \ldots + a_{1,n-1}\, s^{n-2} + a_{1,n}\, s^{n-1} + a_{1,n+1}\, s^{n} \tag{47}$$

The Routh Hurwitz stability array of $\Delta(s)$ in (47) may be formed as in the first step of single input single output systems (page 10). The poles of reduced model are found by this method. The poles nearest to the origin are retained while the poles which are away from the origin are neglected, because, the poles which are near to the origin decide the transient behaviour of the system and are the dominant poles. However, in order to obtain better approximation of the initial transient response (i.e. near t = 0) it is important that the roots of $\Delta_r(s)$ should be chosen to approximate the large magnitude poles of G(s), as well as the small magnitude poles. This is achieved by the modified Routh - Hurwitz array which is described below:

The basic idea behind this modification is to retain the large magnitude poles which are redundant elements in general. For this, the reciprocal polynomial $\tilde{\Delta}(s)$ defined by

$$\tilde{\Delta}(s) = s^n \Delta(1/s)$$

$$= a_{11}s^n + a_{12}s^{n-1} + a_{13}s^{n-2} + \ldots + a_{1,n-1}s^2 + a_{1,n}s + a_{1,n+1}$$

It simply reverses the order of the polynomial coefficients of $\Delta(s)$. The basic property of this reciprocal transformation is that it inverts the roots of the original polynomial. Therefore, if $\Delta(s)$ has all its roots in the left half plane, then so will $\tilde{\Delta}(s)$. The small magnitude poles of $\Delta(s)$ will become the large magnitude poles in $\tilde{\Delta}(s)$ and vice-versa. In fact, the small magnitude poles correspond the large magnitude poles of $\Delta(s)$. Thus of $\tilde{\Delta}(s)$ is used to form the standard Routh - Hurwitz stability array, the method of single input single output systems step - 1 can be used to arrive at the dominant poles of $\tilde{\Delta}(s)$ i.e. it contains the large magnitude poles of $\Delta(s)$ because it is inverted.

Once this is done, the appropriate number of small as well as large magnitude poles may be retained to form $\Delta_r(s)$. The numerator terms $b_{2,j}$ $(j=1,2,\ldots,r)$ of R(s) may then be obtained by matching a number of Markov parameters and time moments as shown in Eqn. (36).

1.7 CONCLUSION

In this chapter we have described the classical Pade approximation technique and its various modifications for obtaining reduced order models. In the mixed methods we find the stable denominator polynomial and the numerator terms are then found by matching the appropriate number of time moments and/or Markov parameters of the original system and its reduced equivalent. This process eliminates the problem of Pade approximants often given unstable models for stable systems.

CHAPTER - 2

ALGORITHM FOR TRANSFER FUNCTION

In this chapter, the algorithm due to Leverrier is described with modifications highlighted in [20]. The Leverrier algorithm gives numerical errors when the dimension of matrix A increases. The modified algorithm increases the accuracy.

2.1 FADDEEVA LEVERRIER ALGORITHM

The algorithm widely used to calculate the coefficients of the characteristic polynomial is the algorithm of Leverrier, alternatively called the algorithm of Souriau, Frame or Faddeeva. The algorithm calculates the coefficients a_i of the characteristic polynomial p(s) of matrix A :

$$p(s) = \text{Det}\ (sI-A) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_n \qquad (48)$$

and the matrices B_i of the adjoint of (sI-A):

$$\text{adj}\ (sI-A) = B_0 s^{n-1} + B_1 s^{n-2} + \ldots + B_{n-1} \qquad (49)$$

then:

$$B_0 = I \ ; \quad a_0 = 1$$

$$a_i = \frac{1}{i}\ \text{Trace}\ (A\ B_{i-1}) \quad \text{for} \quad i = 1,n \qquad (50$$

$$B_i = A\ B_{i-1} + a_i\ I$$

A nice additional test on the accuracy of this algorithm is given by the equality $B_n = 0$.

Though the method is easy to program but it is a well established fact that nearly all arithmatic operations on a digital computer introduce an error due to the limited accuracy with which the nos. are represented. From equation (50) it can be calculated that these errors will accumulate from a_1 to a_n and from B_1 to B_n, so that a_{i+1} and B_{i+1} will be less accurate than a_i and B_i respectively.

2.2 MODIFIED FADDEEVA LEVERRIER ALGORITHM

Due to the above mentioned deficiency of the ordinary algorithm, the latter coefficients should be obtain in a different manner. Such an approach is possible by using the coefficients b_i of the characteristic polynomial q(s) of the inverse of A and the matrices D_i of the adjoint of $(sI-A^{-1})$,

$$q(s) = \det\ (sI - A^{-1}) = b_0 s^n + b_1 s^{n-1} + \ldots + b_n \qquad (51)$$

$$\text{adj}\ (sI-A^{-1}) = D_0 s^{n-1} + D_1 s^{n-2} + \ldots + D_{n-1} \qquad (52)$$

Then, the following relations between a_i and b_i and between B_i and D_i can be used:

$$a_n = (-1)^n \det A$$

$$a_{n-i} = a_n b_i\ ;\quad B_{n-i} = -a_n A^{-1} D_i;\quad \text{for}\quad i = 1,n \qquad (53)$$

For

$$q(s) = \det\ (sI-A^{-1}) = \det\ [(-s\ A^{-1})\ (s^{-1}I-A)]$$

$$= (\det A^{-1})\ (-1)^n\ (a_0+a_1 s+a_2 s^2+\ldots+a_n s^n) \qquad (54)$$

Moreover,

$$\text{adj}\,(sI-A^{-1}) = \det\,(sI-A^{-1})\,(sI-A^{-1})$$

$$= (\det A^{-1})\, A\, (-1)^{n-1}\,(I+B_1 s+\ldots+B_{n-1}s^{n-1}) \tag{55}$$

So, from above analysis, it is evident that by using one additional matrix inversion and one determinant evaluation, the same Faddeeva Leverrier algorithm can be used. First to calculate a_i and B_i from A and then b_i and D_i from A^{-1}. Only the first $(m-1)$ elements a_i and B_i of A and the first $(n-m)$ elements b_i and D_i of the A^{-1} need to be calculated. The value of m has to be selected between $(n/2)$ and n. The critical value on average comes as $2n/3$ offers good results [20].

The modified algorithm is now:

$$B_0 = I\;;\quad a_0 = 1.0\,,\quad m = 2n/3$$

$$a_i = \frac{1}{i}\,\text{Trace}\,(A\,B_{i-1})\ \text{for}\ i = 1,\ m-1$$

$$B_i = A\,B_{i-1} + a_i\,I$$

$$D_0 = I\;;\quad a_n = (-1)^n\,\det A$$

$$b_i = -\frac{1}{i}\,\text{Trace}\,(A^{-1}\,D_{i-1})$$

$$D_i = A^{-1}\,D_{i-1} + b_i\,I\quad \text{for}\ i = 1,\ n-m \tag{56}$$

$$a_{n-i} = a_n\,b_i$$

$$B_{n-i} = -\,a_n\,A^{-1}\,D_i$$

2.3 TRANSFER FUNCTION

$$\dot{x} = Ax + Bu$$

$$Y = cx$$

$$G(s) = \frac{C[\sum_{i=0}^{n-1} s^{n-i-1} B_i] B}{\sum_{i=0}^{n} a_i s^{n-i}} \tag{57}$$

$$= \frac{C[\sum_{i=0}^{n-1} s^{n-i-1} B_i] B}{\Delta(s)}$$

where,

$$\Delta(s) = \sum_{i=0}^{n} a_i s^{n-i}$$

The B_i and a_i are calculated from (50). If the determinant value of A is non zero then the modified algorithm can be applied to calculate a_i and B_i from (56).

2.4 CONCLUSIONS

The entire chapter is devoted fully to the development of transfer function $[G(s)]$ from state space equations. The idea of accuracy is also taken into account which arose due to the cumulative error in the calculation of a_i and B_i.

CHAPTER - 3

MODEL FOR SINGLE MACHINE POWER SYSTEM CONNECTED TO AN INFINITE BUS

The development of this model is based on [18] and taken from [17]. The single machine power system is connected to an infinite bus and shown in Fig. (1). In this power system, generator is provided with a double time constants speed governor.

3.1 MODEL DEVELOPMENT

The electro mechanical oscillation of synchronous generator about a steady state operating point δ_o can be given by

$$M\Delta\ddot{\delta} + D\Delta\dot{\delta} + \Delta p = p \qquad (58)$$

where

$$p = c_1\Delta\delta + b_1\Delta E_q \qquad (59)$$

$$c_1 = \frac{\partial P}{\partial \delta} = -E_q\, V\, Y_{12}\ \text{sing}\ (\delta_0 - \theta_{12}) \qquad (60)$$

$$b_1 = \frac{\partial P}{\partial E_q} = -E_q\, Y_{11}\cos\theta_{11} + VY_{12}\cos(\delta_0 - \theta_{12}) \qquad (61)$$

and

$$p = E_q^2\, Y_{11}\cos\theta_{11} + E_q\, V\, Y_{12}\cos(\delta_0 - \theta_{12}) \qquad (62)$$

The electro magnetic oscillation of the power system can be expressed by

$$\Delta E_q + p\, T_{d0}\ \Delta E_q' = \Delta E_{ex} \qquad (63)$$

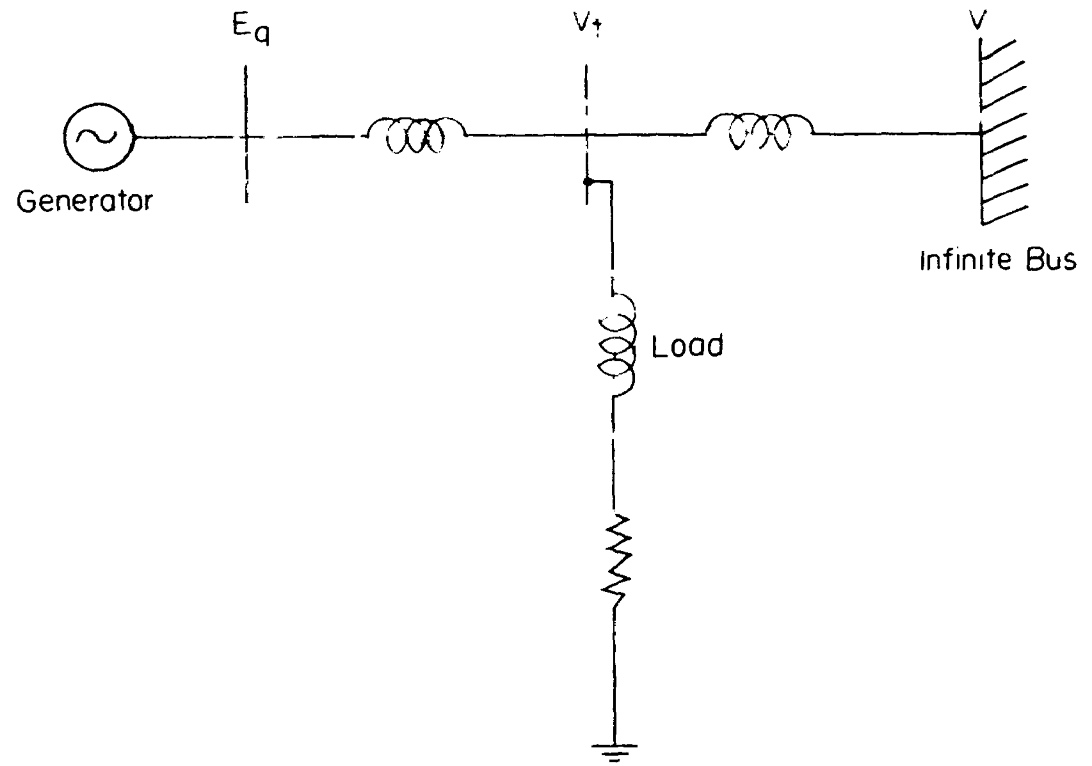


FIG.1 SINGLE MACHINE POWER SYSTEM CONNECTED TO AN INFINITE BUS.

where

$$\Delta E_q' = E_q - (x_d - x_d') I_d \qquad (64)$$

and

$$I_q = E_q Y_{11} \cos \theta_{11} + V Y_{12} \cos (\delta_o - \theta_{12}) \qquad (65)$$

$$I_d = - E_q Y_{11} \sin \theta_{11} - V Y_{12} \cos (\theta_{12} - \delta_0) \qquad (66)$$

From equations (64) and (66) $\Delta E_q'$ can be given by

$$\Delta E_q' = - (x_d - x_d') V Y_{12} \cos (\theta_{12} - \delta_0) \Delta \delta + 1 + (x_d - x_d') Y_{11} \sin \theta_{11} \Delta E_q$$

$$E_q' = c_2 \Delta \delta + b_2 \Delta E_q \qquad (67)$$

where

$$c_2 = - (x_d - x_d') V Y_{12} \cos (\theta_{12} - \delta_0) \qquad (68)$$

$$b_2 = 1 + (x_d - x_d') Y_{11} \sin \theta_{11} \qquad (69)$$

The terminal voltage V_t is given by

$$V_t^2 = V_d^2 + V_q^2$$

$$V_t = (V_d + V_q)^{1/2} \qquad (70)$$

where

$$V_q = E_q - x_d I_d \qquad (71)$$

$$V_d = x_q I_q \qquad (72)$$

Substituting for I_d and I_q in the above equation we get

$$V_t = \frac{1}{V_t} [V_q \frac{\partial V_q}{\partial \delta} + V_d \frac{\partial V_d}{\partial \delta}] \Delta \delta + \frac{1}{V_t} [V_q \frac{\partial V_q}{\partial E_q} + V_d \frac{\partial V_d}{\partial E_q}] \Delta E_q$$

$$V_t = c_3 \Delta \delta + b_3 \Delta E_q \qquad (73)$$

where

$$c_3 = \frac{1}{V_t}\left[V_q \frac{\partial V_q}{\partial \delta} + V_d \frac{\partial V_d}{\partial \delta}\right] \quad (74)$$

$$b_3 = \frac{1}{V_t}\left[V_q \frac{\partial V_q}{\partial E_q} + V_d \frac{\partial V_d}{\partial E_q}\right] \quad (75)$$

The governor output 'p' in equation (58) can be given by

$$a \frac{d^2 p}{dt^2} + b \frac{dp}{dt} + p = - c\Delta\delta \quad (76)$$

Defining

$$w = \Delta \dot{\delta} \qquad p_1 = \dot{p} \qquad U = \frac{\Delta E_{ex}}{T_{d0}} \quad (77)$$

We get the following state equations from (58), (59), (63), (67) and (76).

$$\begin{bmatrix} \Delta \dot{E}_q \\ \Delta \dot{\delta} \\ \dot{w} \\ \dot{p} \\ \dot{p}_1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_{d0} b_2} & 0 & -\frac{c_2}{b_2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -b_1/M & -c_1/M & -D/M & 1/M & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -c/a & -1/a & -b/a \end{bmatrix} \begin{bmatrix} \Delta E_q \\ \Delta \delta \\ w \\ p \\ p_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u \quad (78)$$

3.2 CONCLUSION

The model of single machine power system connected to an infinite bus is developed in well known

$\dot{x} = Ax + Bu$, state space equation form. The matrix equation contains the matrix A of order (5 x 5). In this model the five variables are taken into account while the power system contains more variables. The central idea is to analyse the system with the help of model reduction techniques and check its effectiveness.

CHAPTER - 4

APPLICATION OF REDUCTION METHODS IN A POWER SYSTEM

The chapter is devoted entirely for the development of transfer function from given power system model (78) by [20] and then the reduced order models are found out by applications of reduction techniques especially Pade approximation technique, Krishnamurthi's [12] Routh - Hurwitz array method, the modified method of Pal [14], and other mixed methods and by the retention of dominant poles. The methods applied and the developed reduced order models are presented here.

4.1 SYSTEM MATRIX A, B, C

The values of described parameters of A are taken from [17] as:

M	=	1.000	D	=	0.50
E_q	=	1.482	V_o	=	1.00
P_o	=	2.105	δ_o	=	60^o
T_{do}	=	5.0 sec	y_{11}	=	0.266 - j 1.530
x_d'	=	0.084	y_{12}	=	0.180 + j 1.080
x_d	=	0.320	a	=	$T_1 T_2$ = 0.05
T_1	=	0.100 sec	b	=	T_1+T_2 = 0.6 sec
T_2	=	0.500 sec	c	=	0.05

with the help of these values the matrix A is found as:

$$A = \begin{bmatrix} -0.183 & 0.0 & 0.227 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ -1.815 & -0.57 & -0.50 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & -1.0 & -20.0 & -12.0 \end{bmatrix} \qquad (79)$$

$$B^T = [\,1.0 \quad 0.0 \quad 0.0 \quad 0.0 \quad 0.0\,]$$

and

$$C^T = [\,1.0 \quad 0.0 \quad 0.0 \quad 0.0 \quad 0.0\,]$$

The state vector

$$X = [\Delta E_q \quad \Delta\delta \quad w \quad p \quad p_1\,]$$

Using the set of equations from (50), we get the transfer function of the power system with given matrices A, B and C as:

$$G(s) = \frac{11.4+17.8s+26.57s^2+12.5s^3+s^4}{1+12.6881s+29.332s^2+27.7791s^3+22.9928s^4+2.1432s^5} \qquad (80)$$

This open loop transfer function can be written in the form given below taking the coefficient of s^5 as unity.

$$G(s) = \frac{5.31915+8.3240s+12.39735s^2+5.83240s^3+0.46659s^4}{0.46659+5.92012s+13.68608s^2+12.96146s^3+10.72822s^4+s^5} \qquad (81)$$

The power series about $s = 0$ (time moments)

$$G(s) = 11.4-126.804s+1301.08s^2-13093.0s^3+131222.0s^4- \ldots \quad (82)$$

$$= c_0+c_1s+c_2s^2+c_3s^3+ \ldots\ldots$$

The power series about $s = \infty$ (Markov Parameter)

$$G(s) = 0.46659s^{-1}+0.82672s^{-2}-2.51957s^{-3}+18.2532s^{-4}-171.9250s^{-5}+ \quad (83)$$

$$= m_0s^{-1} + m_1s^{-2} + m_2s^{-3} + \ldots$$

4.2 APPLICATION OF METHODS

4.2.1 METHOD 1:

Using the equations (9) reduced model of third order by Pade approximation technique

Matching time moments only

$$G(s) = \frac{0.46659s^4+5.8324s^3+12.39735s^2+8.3240s+5.31915}{s^5+10.72822s^4+12.96146s^3+13.68608s^2+5.92012s+0.46659} \quad (84)$$

$$= c_o+c_1s+c_2s^2+c_3s^3+c_4s^4+ \ldots$$

$$= 11.4-126.8s+1301.08s^2-13093s^3+131222s^4-1314160s^5+\ldots \quad (85)$$

or

$$= m_os^{-1}+m_1s^{-2}+m_2s^{-3}+ \ldots$$

$$= 0.46659s^{-1}+0.82672s^{-2}-2.51957s^{-3}+18.25320s^{-4}-171.925s^{-5}+\ldots \quad (86)$$

$$G_R(s) = \frac{A_o+A_1s+A_2s^2}{B_o+B_1s+B_2s^2+B_3s^3} \quad (87)$$

Using Pade set of linear simultaneous equations

$$A_o = c_o B_o$$

$$A_1 = c_1 B_o + c_o B_1$$

$$A_2 = c_2 B_o + c_1 B_1 + c_o B_2$$

$$0 = c_3 B_o + c_2 B_1 + c_1 B_2 + c_o B_3$$

$$0 = c_4 B_o + c_3 B_1 + c_2 B_2 + c_1 B_3$$

$$0 = c_5 B_o + c_4 B_1 + c_3 B_2 + c_2 B_3$$

Taking B_3 as unity and solving last three equations, we get

$$B_3 = 1.0$$

$$B_2 = 1.144932$$

$$B_1 = 0.415374$$

$$B_o = 0.0310591$$

Using these values of B_o, B_1, B_2 and B_3 in first three equations of above set, we get:

$$A_o = 0.3540737$$

$$A_1 = 0.7969697$$

$$A_2 = 0.7931754$$

so

$$G_R(s) = \frac{0.3540737 + 0.7969697s + 0.7931754s^2}{0.0310591 + 0.415374s + 1.144932s^2 + s^3} \qquad (88)$$

Results are in table 1 and response is shown in Fig.(2).

TABLE - 1

TIME RESPONSE (METHOD 1)

Time (sec)	Original	Reduced
0	0.000000 E+00	0.000000 E+00
1	0.704090 E+00	0.757423 E+00
2	0.155366 E+01	0.149002 E+01
3	0.231412 E+01	0.222202 E+01
4	0.297397 E+01	0.294820 E+01
6	0.426036 E+01	0.433077 E+01
8	0.555613 E+01	0.555615 E+01
9	0.612406 E+01	0.609962 E+01
10	0.662115 E+01	0.659699 E+01
12	0.746061 E+01	0.746212 E+01
14	0.817164 E+01	0.817431 E+01
16	0.876170 E+01	0.875837 E+01
18	0.923733 E+01	0.923676 E+01
20	0.962751 E+01	0.962850 E+01
30	0.107474 E+02	0.107474 E+02
40	0.111596 E+02	0.111596 E+02
50	0.113114 E+02	0.113114 E+02

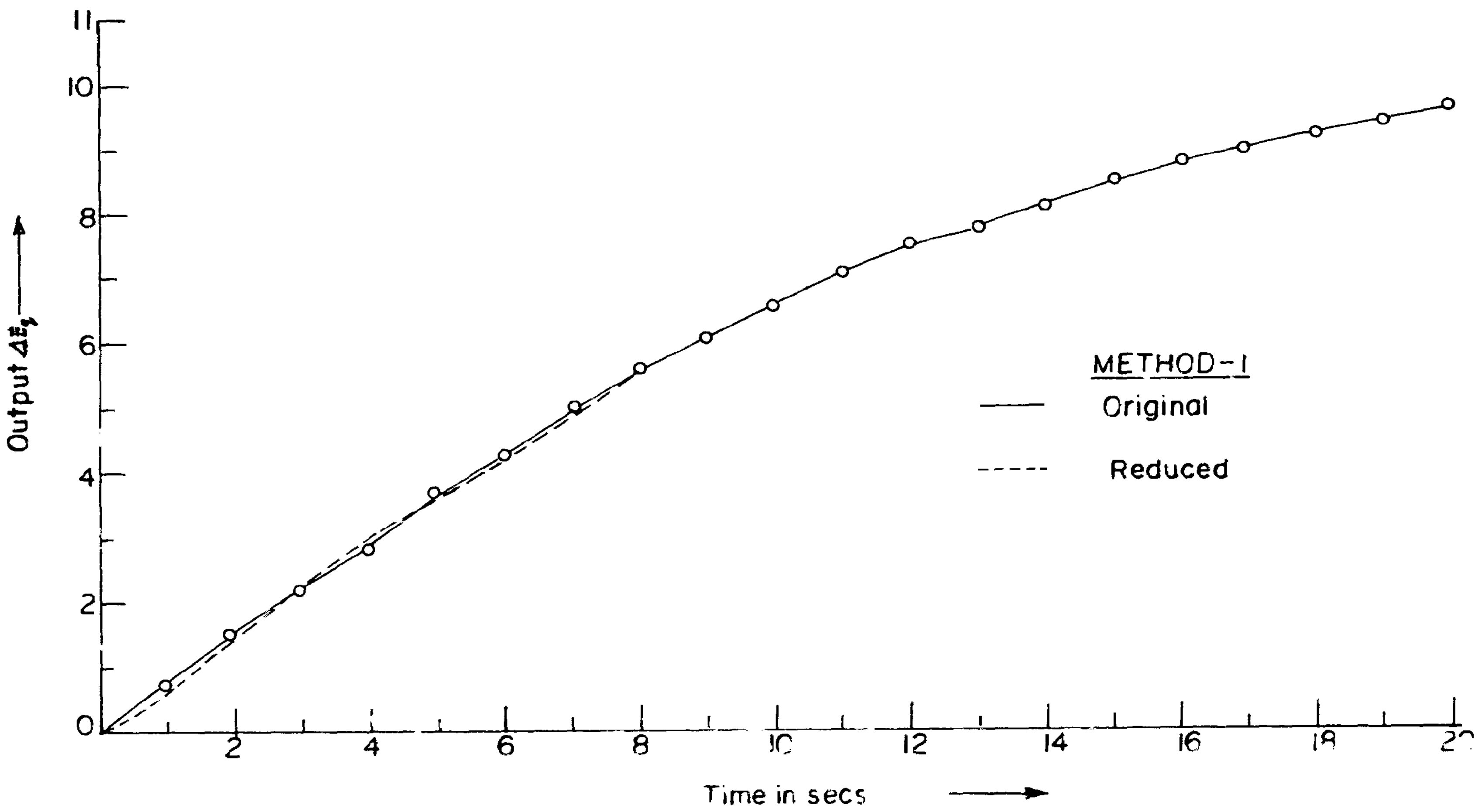


FIG.2 COMPARISON OF UNIT STEP RESPONSES.

4.2.2 METHOD 2

Using the method of Routh Hurwitz array (stability) and technique of reduction highlighted by Krishnamurthi [12], the reduced order model of third order is found as:

Reduced Model by Routh Hurwitz array-

The numerator stability array of (84)

s^4	0.46659	12.39735	5.31915	
s^3	5.83240	8.3240		
s^2	11.73143	5.31915		(89)
s^1	5.67953			
s^0	5.31915			

The denominator stability array (84)

s^5	1	12.96146	5.92012	
s^4	10.72822	13.68608	0.46659	
s^3	11.68575	5.87663		(90)
s^2	8.29098	0.46659		
s^1	5.21899			
s^0	0.46659			

So numerator for third order reduced model is

$$N_R(s) = 5.31915 + 5.67953s + 11.73143s^2 \quad (91)$$

and denominator from (90) is

$$D_R(s) = 0.46659 + 5.87663s + 8.29098s^2 + 11.68575s^3 \quad (92)$$

$$G_R(s) = \frac{0.45518 + 0.48602s + 1.00391s^2}{0.03993 + 0.50289s + 0.70950s^2 + s^3} \quad (93)$$

The results are in table 2 and time response is shown in Fig. (3).

TABLE - 2

TIME RESPONSE (METHOD 2)

Time (sec)	Original	Reduced
0	0.000000 E+00	0.000000 E+00
1	0.704090 E+00	0.908732 E+01
2	0.155366 E+01	0.169081 E+01
3	0.231412 E+01	0.241615 E+01
4	0.297397 E+01	0.311818 E+01
6	0.426036 E+01	0.445695 E+01
8	0.555613 E+01	0.563169 E+01
9	0.612406 E+01	0.613899 E+01
10	0.662115 E+01	0.659473 E+01
12	0.746061 E+01	0.737660 E+01
14	0.817164 E+01	0.802676 E+01
16	0.876170 E+01	0.857597 E+01
18	0.923733 E+01	0.903870 E+01
20	0.962751 E+01	0.942560 E+01
30	0.107474 E+02	0.105904 E+02
40	0.111596 E+02	0.110680 E+02
50	0.113114 E+02	0.112636 E+02

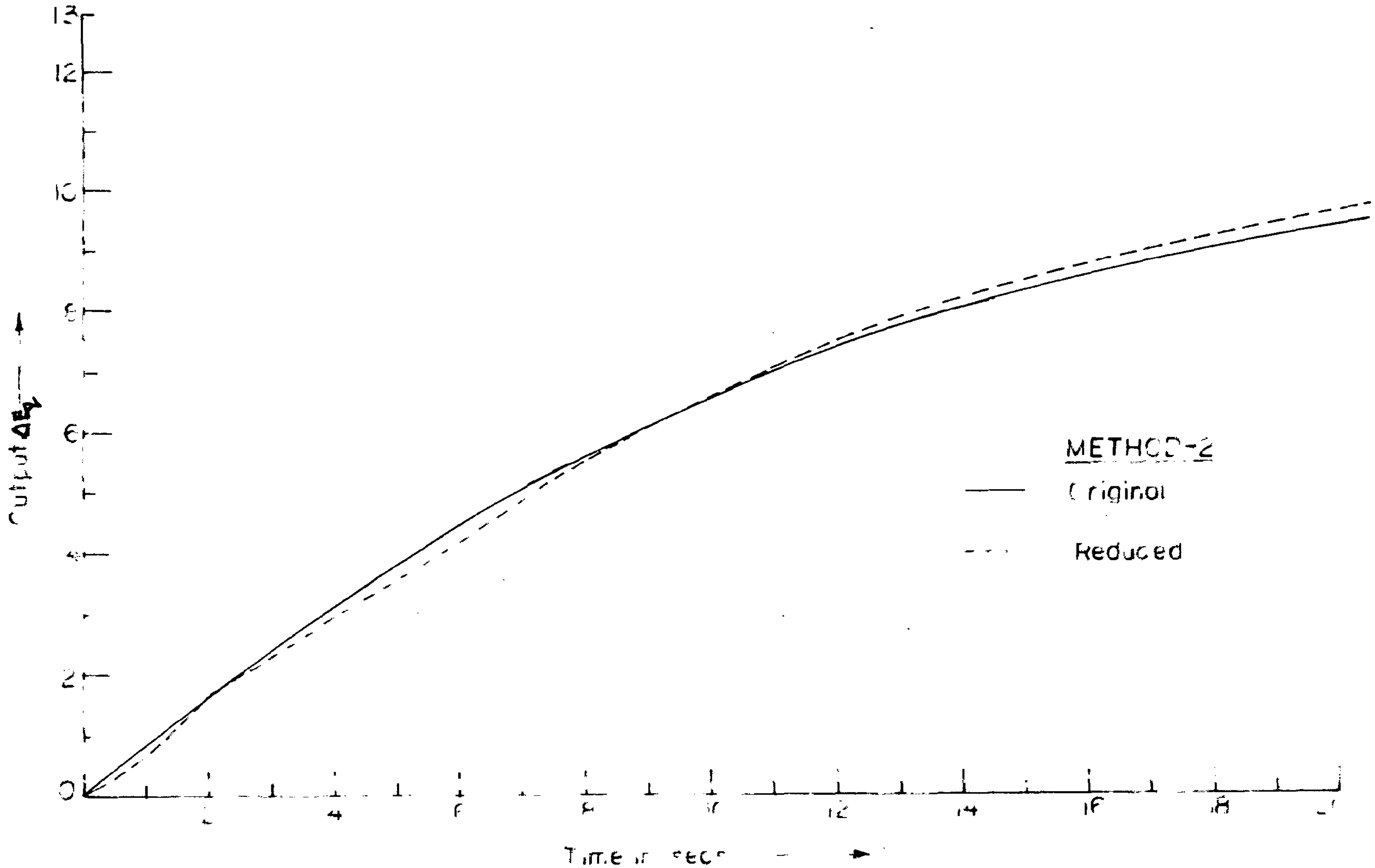


FIG. 3 COMPARISON OF UNIT STEP RESPONSES

4.2.3 METHOD 3

It is better to consider the poles which are nearer to the imaginary axis or origin. It is best established fact that the poles which are near to the origin have the dominant role on the overall behaviour of the system. With the help of equations from (87) to (89) we get:

Pade Approximant + Dominant Roots.

Dominant roots (poles) of transfer function (84) are

$$s_1 = 0.099870356$$
$$s_2 = 0.30111677 \tag{94}$$
$$s_3 = 0.30111677$$

From (94) the denominator of reduced order model is

$$D_R(s) = (s + 0.099870356)\,(s + 0.30111677)^2$$
$$= 0.0090554 + 0.692905s + 0.702104s^2 + s^3$$

and numerator

$$N_R(s) = A_o + A_1 s + A_2 s^2$$

and

$$A_o = C_o B_o = 0.1032315$$
$$A_1 = C_1 B_o + C_o B_1 = 6.7508561$$
$$A_2 = C_2 B_o + C_1 B_1 + C_o B_2 = -68.07734$$

$$G_R(s) = \frac{0.1032315+6.7508561s-68.07734s^2}{0.0090554+0.6929050s+0.7021040s^2} \tag{95}$$

The results are in table 3 and time response is in Fig. (4).

TABLE - 3

TIME RESPONSE (METHOD 3)

Time (sec)	Original	Reduced
0	0.000000 E+00	0.000000 E+00
1	0.704090 E+00	- 0.429067 E+02
2	0.155366 E+01	- 0.371236 E+02
3	0.231412 E+01	- 0.126633 E+02
4	0.297397 E+01	0.107601 E+02
6	0.426036 E+01	0.233578 E+02
8	0.555613 E+01	0.121409 E+02
9	0.612406 E+01	0.876475 E+01
10	0.662115 E+01	0.821330 E+01
12	0.746061 E+01	0.107821 E+02
14	0.817164 E+01	0.119512 E+02
16	0.876170 E+01	0.113877 E+02
18	0.923733 E+01	0.110601 E+02
20	0.962751 E+01	0.111851 E+02
30	0.107474 E+02	0.112584 E+02
40	0.111596 E+02	0.112736 E+02
50	0.113114 E+02	0.112885 E+02

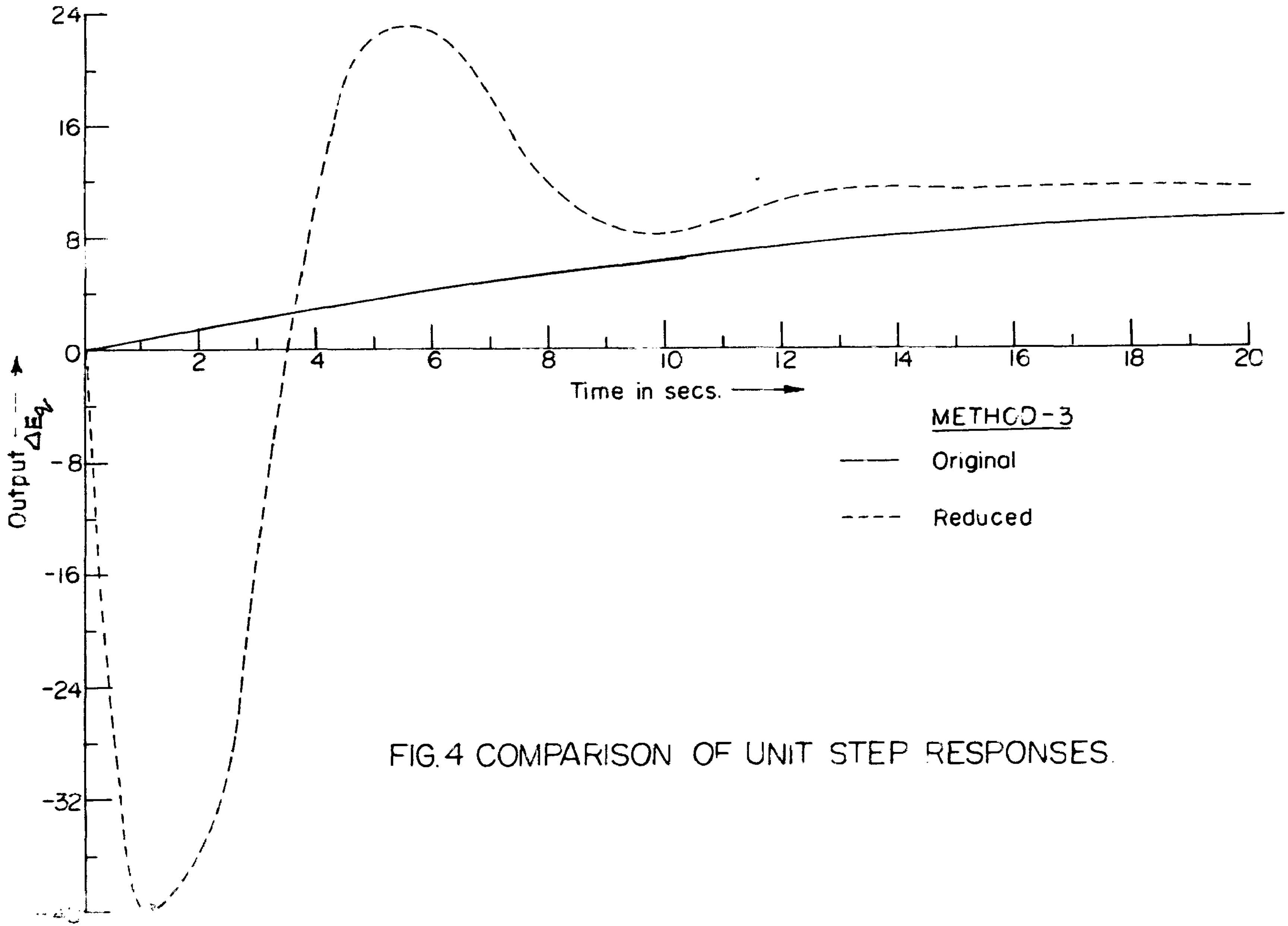


FIG.4 COMPARISON OF UNIT STEP RESPONSES.

4.2.4 METHOD 4

A mix method which utilizes the combination of Pade approximants and Routh Hurwitz array. The denominator of reduced order model is taken from Routh Hurwitz array while numerator is by partial Pade approximation. The Case 1 considers only the time moments while Case 2 and Case 3 take the combination of time moments and Markov parameters to find out the numerator of reduced model by partial Pade approximation.

Case 1 Matching all time moments

From equation (84) and equation (90) the denominator comes out to be such as (92) and numerator becomes such as

$$N(s) = A_o + A_1 s + A_2 s^2$$

$$A_o = C_o B_o$$

$$A_1 = C_1 B_o + C_o B_1$$

$$A_2 = C_2 B_o + C_1 B_1 + C_o B_2$$

The B_o, B_1 and B_2 are known from equation (92) and C_o, C_1 and C_2 from (85)

so

$$A_o = 5.319126$$

$$A_1 = 7.82997$$

$$A_2 = -43.568595$$

so

$$G_R(s) = \frac{5.319126 + 7.82997s - 43.568595s^2}{0.46659 + 5.87663s + 8.29098s^2 + 11.68575s^3} \qquad (96)$$

or

$$= \frac{0.45518 + 0.6700442s - 3.728353s^2}{0.0399281 + 0.502889s + 0.70950s^2 + s^3}$$

The results are in table 4 and time response is shown in Fig. (5)

Case 2: Matching one time moment and two Markov parameters

The denominator of third order reduced model from Routh Hurwitz array is:

$$D_R(s) = 11.68575s^3 + 8.29098s^2 + 5.87663s + 0.46659 \qquad (97)$$

$$= b_4s^3 + b_3s^2 + b_2s + b_1 \quad \text{(say)}$$

using set of equations of (36) from chapter one

$$\alpha = 2, \quad \beta = 1, \quad r = 3$$

$$b_{21} = b_1T_o = 0.46659 \times 11.4 = 5.319126$$

$$b_{22} = b_4M_1 + b_3M_o$$

$$= 11.68575 \times 0.82672 + 8.29098 \times 0.46659$$

$$= 13.529332$$

$$b_{23} = b_4M_o = 11.68575 \times 0.46659 = 5.4524541$$

$$G_R(s) = \frac{b_{21} + b_{22}s + b_{23}s^2}{D_R(s)}$$

$$= \frac{5.319126 + 13.529332s + 5.4524541s^2}{0.46659 + 5.87663s + 8.29098s^2 + 11.68575s^3}$$

$$= \frac{0.45518 + 1.15776s + 0.46659s^2}{0.03993 + 0.502889s + 0.709495s^2 + s^3} \qquad (98)$$

The results are in table 5 and time response is in Fig. (5).

Case 3 Matching two time moments and one Markov parameter

The denominator is same as (92);

$$b_{21} = b_1 T_o = 5.319126$$

$$b_{22} = b_1 T_1 + b_2 T_o = 0.46659 \times (-126.8) + 8.29098 \times 11.4$$

$$= 7.82997$$

$$b_{23} = b_4 M_o = 5.4524541$$

$$G_R(s) = \frac{5.319126 + 7.82997s + 5.4524541s^2}{0.46659 + 5.87663s + 8.29098s^2 + 11.68575s^3}$$

$$G_R(s) = \frac{0.45518 + 0.6700442s + 0.46659s^2}{0.039928 + 0.50289s + 0.709495s^2 + s^3} \qquad (99)$$

The results are in table 6 and time response is shown in Fig. (5).

TABLE - 4

TIME RESPONSE (METHOD 4, CASE 1)

Time (sec)	Original	Reduced
0	0.000000 E+00	0.000000 E+00
1	0.704090 E+00	- 0.212908 E+01
2	0.155366 E+01	- 0.160812 E+01
3	0.231412 E+01	0.399478 E+01
4	0.297397 E+01	0.277078 E+01
6	0.426036 E+01	0.609554 E+01
8	0.555613 E+01	0.708464 E+01
9	0.612406 E+01	0.718951 E+01
10	0.662115 E+01	0.728549 E+01
12	0.746061 E+01	0.771859 E+01
14	0.817164 E+01	0.836928 E+01
16	0.876170 E+01	0.895354 E+01
18	0.923733 E+01	0.937504 E+01
20	0.962751 E+01	0.968893 E+01
30	0.107474 E+02	0.106953 E+02
40	0.111596 E+02	0.111107 E+02
50	0.113114 E+02	0.112811 E+02

TABLE - 5

TIME RESPONSE (METHOD 4 CASE 2)

Time (sec)	Original	Reduced
0	0.000000 E+00	0.000000 E+00
1	0.704090 E+00	0.813818 E+00
2	0.155366 E+01	0.204604 E+01
3	0.231412 E+01	0.332986 E+01
4	0.297397 E+01	0.443240 E+01
6	0.426036 E+01	0.586477 E+01
8	0.555613 E+01	0.663993 E+01
9	0.612406 E+01	0.695895 E+01
10	0.662115 E+01	0.728052 E+01
12	0.746061 E+01	0.793557 E+01
14	0.817164 E+01	0.853405 E+01
16	0.876170 E+01	0.902064 E+01
18	0.923733 E+01	0.940743 E+01
20	0.962751 E+01	0.972689 E+01
30	0.107474 E+02	0.107143 E+02
40	0.111596 E+02	0.111187 E+02
50	0.113114 E+02	0.112844 E+02

TABLE - 6

TIME RESPONSE (METHOD 4 CASE 3)

Time (sec)	Original	Reduced
0	0.000000 E+00	0.000000 E+00
1	0.704090 E+00	0.626468 E+00
2	0.155366 E+01	0.149915 E+01
3	0.231412 E+01	0.247357 E+01
4	0.297397 E+01	0.341802 E+01
6	0.426036 E+01	0.494922 E+01
8	0.555613 E+01	0.600802 E+01
9	0.612406 E+01	0.643389 E+01
10	0.662115 E+01	0.682539 E+01
12	0.746061 E+01	0.754579 E+01
14	0.817164 E+01	0.818305 E+01
16	0.876170 E+01	0.871955 E+01
18	0.923733 E+01	0.915974 E+01
20	0.962751 E+01	0.952352 E+01
30	0.107474 E+02	0.106304 E+02
40	0.111596 E+02	0.110843 E+02
50	0.113114 E+02	0.112703 E+02

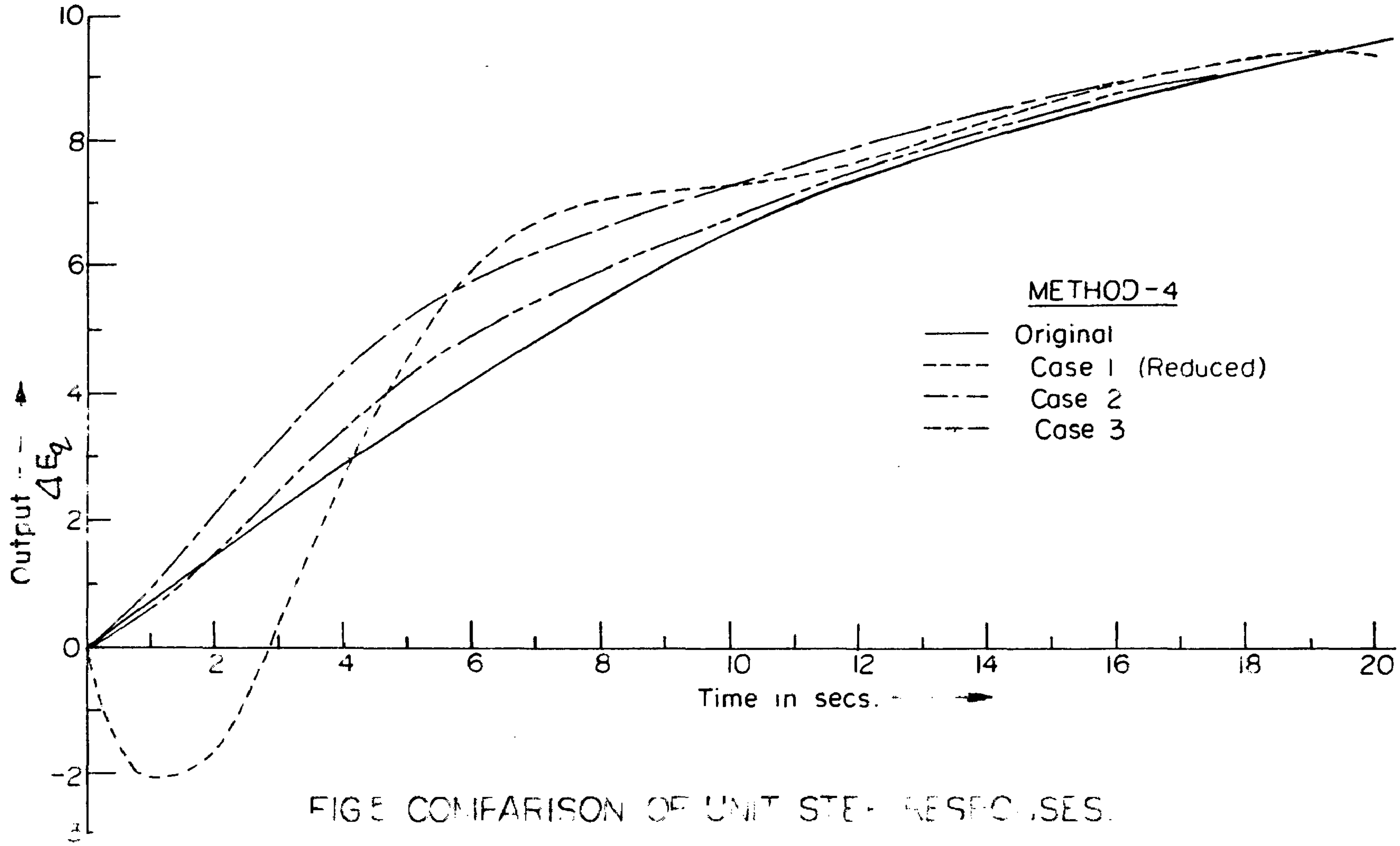


FIG 5 COMPARISON OF UNIT STEP RESPONSES.

4.2.5 METHOD 5

The reduced order models are obtained by using the stable biased technique. The method is briefly described in chapter one and from equation (48) to (51) and (36) and (3). We get the reduced models as given below. Here Case 1 discribes the matching of time moments only while cases (2) ~~and (~~ and (3) are the combination of time moments and Markov parameters.

Stability Based Reduced Order Models

$$D(s) = s^5 + 10.72822s^4 + 12.96146s^3 + 13.68608s^2 + 5.92012s + 0.46659 \qquad (100)$$

Now inverted denominator is:

$$D(s) = 0.46659s^5 + 5.92012s^4 + 13.68608s^3 + 12.96146s^2 + 10.72822s + 1 \qquad (101)$$

R H array of D(s) gives us

$$D_3(s) = 11.68575s^3 + 8.29098s^2 + 5.87663s + 0.46659$$
$$= 11.68575\ (s^3+0.7094949s^2+0.5028885s+0.0399281)$$

$$D_2(s) = 8.29098s^2 + 5.21899s + 0.46659 \qquad (102)$$
$$= 8.29098\ (s^2+0.629478s+0.0562768)$$

$$D_1(s) = 5.21899s + 0.46659$$
$$= 5.21899\ (s+0.0894023)$$

And RH array of (101) gives

$$\tilde{D}_3(s) = 12.664532s^3 + 7.9833242s^2 + 10.649406s + 1.0$$
$$= 12.664532\ (s^3+0.6303686s^2+0.8408842s+0.0789606)$$

$$\tilde{D}_2(s) = 7.9833242s^2 + 9.0630328s + 1.0$$
$$= 9.9833242\ (s^2+1.1352455s+0.1252611) \qquad (103)$$

$$\tilde{D}_1(s) = 9.0630328s + 1.0$$
$$= 9.0630328\ (s+0.1103383)$$

Using the reciprocal transformation again, we have the equivalent reduced polynomials for D(s) as

$$D_1'(s) = s + 9.0630328$$
$$D_2'(s) = s^2 + 9.0630328s + 7.9833242 \qquad (104)$$
$$D_3'(s) = s^3 + 10.649406s^2 + 7.9833242s + 12.664532$$

Retaining two dominant poles from $D_2(s)$ and one from $D_1'(s)$ far off pole, we have a third order denomiantor polynomial as

$$D_R(s) = (s^2+0.629478s+0.0562768)\ (s+9.0630328) \qquad (105)$$
$$= s^3 + 9.6925108s^2 + 5.7612566s + 0.5100384$$
$$= b_4s^3 + b_3s^2 + b_2s + b_1$$

Case 1 Matching Time Moments Only

$$A_o = c_ob_1 = 5.8144378$$
$$A_1 = c_1b_1 + c_ob_2 = 1.003416$$
$$A_2 = c_2b_1 + c_1b_2 + c_ob_3 = 43.545003$$

$$C_1R(s) = \frac{5.8144378 + 1.003416s + 43.545003s^2}{0.5100384 + 5.7612566s + 9.6925108s^2 + s^3} \tag{106}$$

The results are shown in table 7 and Fig. (6)

Case 2 Matching one time moment and two Markov parameters

$$G_R(s) = \frac{5.8144378 + 5.3491486s + 0.46659s^2}{0.5100384 + 5.7612566s + 9.6925108s^2 + s^3} \tag{107}$$

The results are shown in table 8 and Fig. (6)

Case 3 Matching two time moments and one Markov parameter

$$G_R(s) = \frac{5.8144378 + 1.003416s + 0.46659s^2}{0.5100384 + 5.7612566s + 9.6925108s^2 + s^3} \tag{108}$$

The results are shown in table 9 and time response in Fig. (6).

The reduced models are

(1) Now reduced order models from Method 1 is (using Pade approximant and time moments only)

$$R(s) = \frac{0.354074 + 0.7969697s + 0.793175s^2}{0.031059 + 0.415374s + 1.44932s^2}$$

(2) From method 2, using Routh Hurwitz criterion

$$R(s) = \frac{0.45578 + 0.48602s + 1.00391s^2}{0.03993 + 0.50289s + 0.70950s^2 + s^3}$$

(3) From method 3, reduced model by retaining the dominant roots and Partial Pade approximants is:

$$R(s) = \frac{0.10323 + 6.75086s - 68.07734s^2}{0.00906 + 0.69291s + 0.70210s^2 + s^3}$$

(4) From method 4, using the mixed methods:

A combination of Routh Hurwitz array and Partial Pade approximants gives the reduced models as:

Case 1 Matching time moments only

$$R(s) = \frac{0.45518 + 0.67004s - 3.72835s^2}{0.03993 + 0.50289s + 0.70950s^2 + s^3}$$

Case 2 Mateching one time moment and two Markov parameters

$$R(s) = \frac{0.45518 + 1.15776s + 0.46659s^2}{0.03993 + 0.50289s + 0.70950s^2 + s^3}$$

(5) From method 5, using stability biased criterion

Case 1 Matching time moments only

$$R(s) = \frac{5.81444 + 1.00342s + 43.5450s^2}{0.51004 + 5.76126s + 9.69251s^2 + s^3}$$

Case 2 Matching one time moment and two Markov parameters

$$R(s) = \frac{5.81444 + 5.34915s + 0.46659s^2}{0.51004 + 5.76126s + 9.69251s^2 + s^3}$$

Case 3 Matching two time moments and one Markov parameters

$$R(s) = \frac{5.81444 + 1.00342s + 0.46659s^2}{0.51004 + 5.76126s + 9.69251s^2 + s^3}$$

4.3 TIME RESPONSE

From the above we find that the steady state values of all the methods match with original are upto second place of decimal. Method 1 gives least error in the described problem.. The various methods presented herein - are algebraic in nature and require simple calculations that can be easily automated. These methods do not require finding the eigen-values and eigen vectors of high order system. The solution of high order non-linear equations is not required. Time response of each method and model requires almost same computation time (CPU) about 0.87 seconds.

4.4 CONCLUSION

The method 1, viz, Pade approximant reduced order model with matching time moments only gives poor response in transient zone while matching Markov parameters gives poor response in steady state zone. Although it is a good tool for model reduction yet in some cases it gives stable (unstable) models for unstable (stable) original systems. The method is computationally simple and it occupies less memory location.

Method 2, makes, use of classical Routh - Hurwitz stability array and is applicable to single input single output systems. This method is also computationally simple and gives satisfactory results. The unit step responses of original to reduced model shown poor matching in the transient zone. This is because the method, in effect, retains the most dominant poles in the reduced models and thus matching in the transient zone may be sometimes quite in error. However, the transient response of synchronous machines in electrical power systems may be of much importance.

Method 3, retains all dominant poles. The time response of original with reduced model is in error in the transient zone and the cause is the same as in method 2.

Method 4, makes use of method 2 and method 1. This method also takes into consideration the combination of time moments and Markov parameters and hence reduces the errors in the transient zone as well as in the steady - state zone.

Method 5, is the duplication of method 2. In method 2, the poles of the reduced model R(s) approximate the poles of the original system G(s) that are closest to the origin. Thus it follows this method of approximation is essentially about $s = 0$, which in the time domain corresponds to approximating the system response about $t = \infty$ (i.e. all the emphasis is placed on approximating the system steady state response). Though the reduced models are stable, it may suffer from the disadvantage that the reduced models approximate the steady state portion of the response but the transient response may be quite in error.

TABLE - 7

TIME RESPONSE (METHOD 5 CASE 1)

Time (sec)	Original	Reduced
0	0.000000 E+00	0.000000 E+00
1	0.704090 E+00	0.296520 E+01
2	0.155366 E+01	0.214942 E+01
3	0.232412 E+01	0.209906 E+01
4	0.297397 E+01	0.245880 E+01
6	0.426036 E+01	0.367053 E+01
8	0.555613 E+01	0.498655 E+01
9	0.612406 E+01	0.559868 E+01
10	0.662115 E+01	0.616660 E+01
12	0.746061 E+01	0.716287 E+01
14	0.817164 E+01	0.797360 E+01
16	0.876170 E+01	0.862943 E+01
18	0.923733 E+01	0.917207 E+01
20	0.962751 E+01	0.960755 E+01
30	0.107474 E+02	0.107196 E+02
40	0.111596 E+02	0.111740 E+02
50	0.113114 E+02	0.113249 E+02

TABLE - 8

TIME RESPONSE (METHOD 5 CASE 2)

Time (sec)	Original	Reduced
0	0.000000 E+00	0.000000 E+00
1	0.704090 E+00	0.640314 E+00
2	0.155366 E+01	0.143212 E+01
3	0.231412 E+01	0.226839 E+01
4	0.297397 E+01	0.309360 E+01
6	0.426036 E+01	0.460957 E+01
8	0.555613 E+01	0.589366 E+01
9	0.612406 E+01	0.644883 E+01
10	0.662115 E+01	0.695046 E+01
12	0.746061 E+01	0.780989 E+01
14	0.817164 E+01	0.850524 E+01
16	0.876170 E+01	0.906658 E+01
18	0.923733 E+01	0.951930 E+01
20	0.961751 E+01	0.988426 E+01
30	0.107474 E+02	0.108846 E+02
40	0.111596 E+02	0.112247 E+02
50	0.113114 E+02	0.113404 E+02

TABLE - 9

TIME RESPONSE (METHOD 5 CASE 3)

Time (sec)	Original	Reduced
0	0.000000 E+00	0.000000 E+00
1	0.704090 E+00	0.317261 E+00
2	0.155366 E+01	0.920076 E+00
3	0.231412 E+01	0.167693 E+01
4	0.297397 E+01	0.248440 E+01
6	0.426036 E+01	0.404937 E+01
8	0.555613 E+01	0.541778 E+01
9	0.612406 E+01	0.601586 E+01
10	0.662115 E+01	0.655837 E+01
12	0.746061 E+01	0.749091 E+01
14	0.817164 E+01	0.824706 E+01
16	0.876170 E+01	0.885807 E+01
18	0.923733 E+01	0.935119 E+01
20	0.962757 E+01	0.974878 E+01
30	0.107474 E+02	0.108380 E+02
40	0.111596 E+02	0.112093 E+02
50	0.113114 E+02	0.113350 E+02

TABLE - 10

COMPARATIVE STUDY

Method No.		Output yr AT Time = 50 sec	Cumulative error J
1		0.113114 E+02	0.264703 E-01
2		0.112636 E+02	0.993987 E+00
3		0.112885 E+02	0.455290 E+04
4	Case 1	0.112811 E+02	0.343709 E+02
	Case 2	0.112844 E+02	0.140725 E+02
	Case 3	0.112703 E+02	0.215544 E+01
5	Case 1	0.113249 E+02	0.803555 E+01
	Case 2	0.113404 E+02	0.212877 E+01
	Case 3	0.113351 E+02	0.166341 E+01

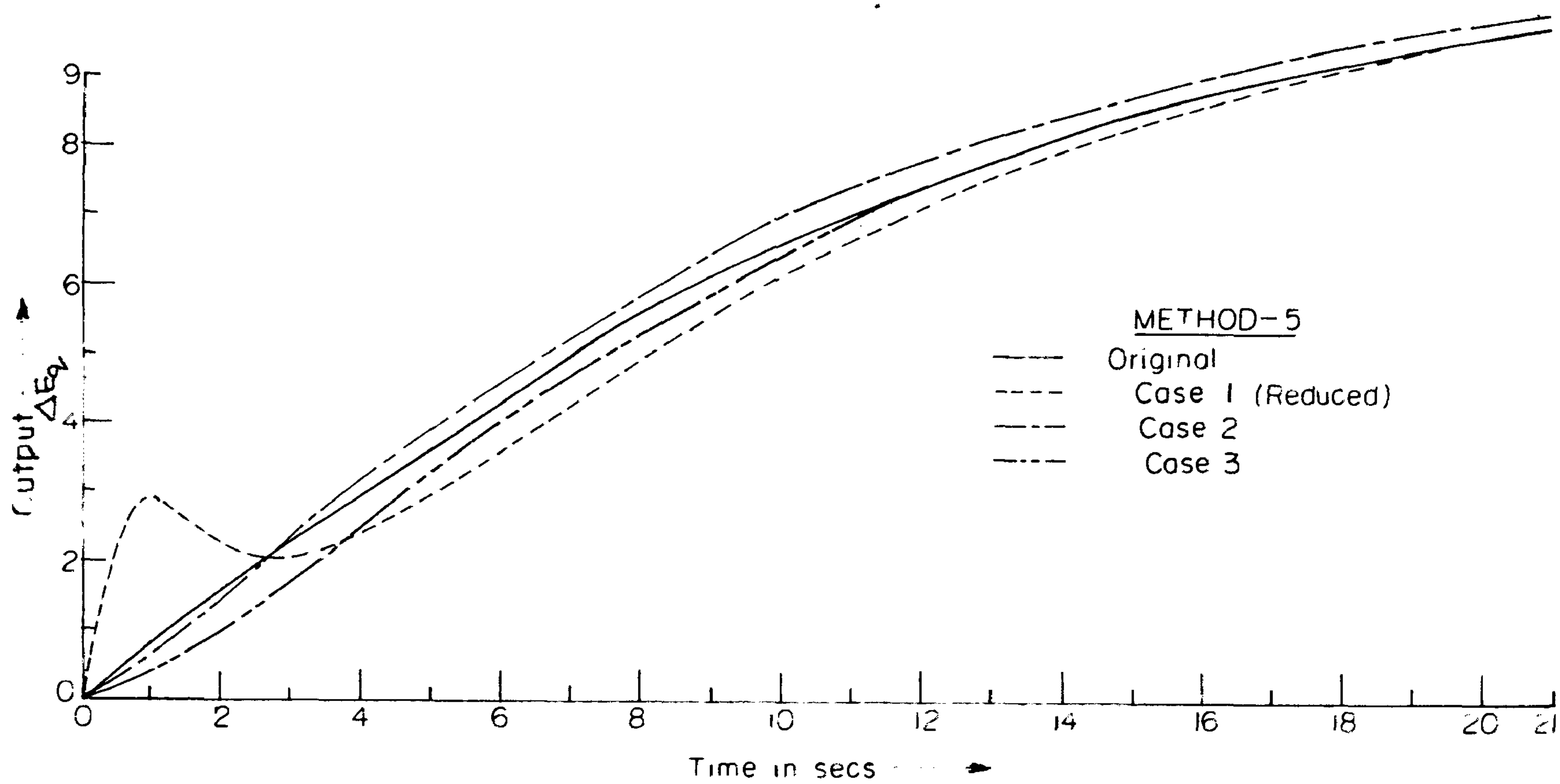


FIG.6 COMPARISON OF UNIT STEP RESPONSES

CHAPTER - 5

SUBOPTIMAL CONTROLLER DESIGN USING MODEL REDUCTION TECHNIQUE

The optimal control theory has a major drawback that it requires feedback from all the state variables that are defined to describe the dynamics of the plant. Unfortunately, the entire state - vector is never available for measurement. So, the question often arises whether there is any alternative scheme to control the system in the absense of one or some states. The entire chapter is devoted to optimal control for the specified power system problem and then the parameters (unknown) are found out by suboptimal control.

However, in practice it is often impossible, very expensive and too difficult to measure all the state variables that are defined to describe the dynamics of the system. The optimal controllers are very complex and their use in higher dimensional systems could be prohibitive as the cost of controller increases with its dimension.

5.1 SUBOPTIMAL CONTROL USING PADE APPROXIMATION TECHNIQUE

The closed loop transfer function of the plant with the optimal controller with all states feedback is first designated as the model transfer function. And, then the control structure with available states is specified i.e. some states in feedback are missing. The closed loop transfer function of the plant with this specified controller is determined and it contains the feedback parameter as unknown quanties. This closed loop transfer function is matched with the model transfer function using frequency domain model reduction techniques and thus the unknown control parameters are determined.

5.1.1 DESIGN METHOD:

The design based on [19 , pp. 1007]

Consider the n^{th} order SISO linear dynamic system described by

$$\dot{x}(t) = Ax(t) + bU(t)$$

$$y(t) = C^T x(t) \qquad (109)$$

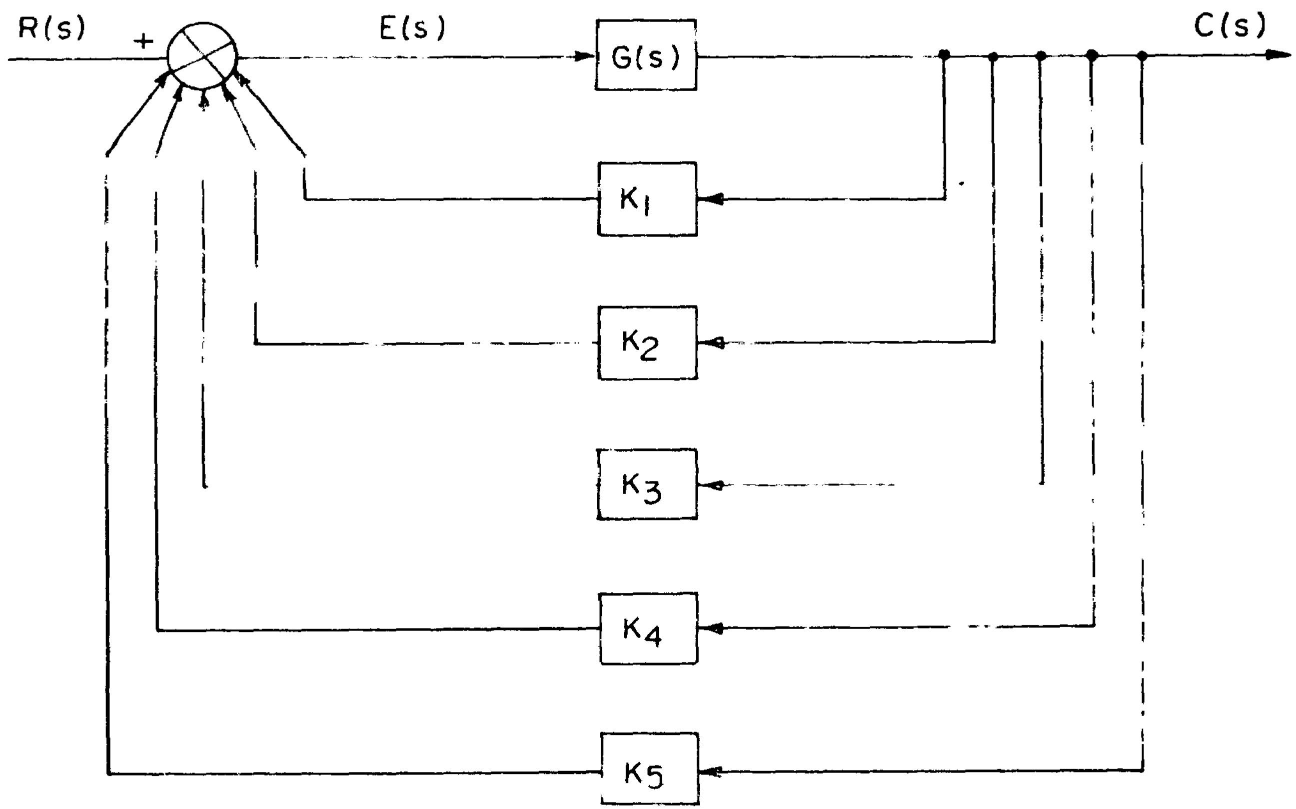


FIG.7 SYSTEM WITH FEED-BACK CONTROL.

The quadratic cost function from optimal control theory

$$J = \int_0^{\infty} [x^T(t)\ Qx(t) + rU^2(t)]\ dt \qquad (110)$$

where

$Q = (n \times n)$ positive semidefinite matrix

r = positive weight

It is well known that the optimal feedback control law is a linear combination of the state variables

$$u(t) = -r^{-1}\ b^T\ P\ x\ (t) = -K^T\ x\ (t) \qquad (111)$$

where

P = Symmetric positive definite matrix, the elements of it may be found by Riecati equation:

$$A^TP + PA - Pbr^{-1}b^TP + Q = 0 \qquad (112)$$

The closed loop transfer function with the optimal controller of (111) is

$$T^*(s) = c^T\ [sI - A + bK^T]^{-1}\ b$$

$$= \frac{b_o + b_1s + b_2s^2 + \ldots + b_{m-1}s^{m-1} + b_ms^m}{a_o + a_1s + a_2s^2 + \ldots + a_{n-1}s^{n-1} + a_ns^n} \qquad (113)$$

$$= d_o + d_1s + d_2s^2 + \ldots \qquad (114)$$

E_q (114) is the expansion of (113) about $s = 0$

Now utilizing only the states available for feedback, the suboptimal controller may be specified as:

$$\tilde{u} = -\tilde{K}^T\ x\ (t) \qquad (115)$$

Assuming that such a suboptimal controller exists. The transfer function for suboptimal controller is

$$\tilde{T}(s) = c^T [sI - A + b\tilde{K}^T]^{-1} b$$

$$= \frac{b_o + b_1 s + b_2 s^2 + \ldots + b_{m-1} s^{m-1} + b_m s^m}{f_o + f_1 s + f_2 s^2 + \ldots + f_{n-1} s^{n-1} + f_n s^n} \qquad (116)$$

The incomplete state feedback problem is concerned with finding the elements of $\tilde{K}^T$ on some basis. For suboptimal system response to be favourably comparable with that of the optimal $\tilde{T}(s)$ should approximate $T^*(s)$ in (113) in some sense. The design technique utilizes the Pade approximation method to find unknown $\tilde{K}^T$.

For $\tilde{T}(s)$ to approximate $T^*(s)$ in the Pade sense, we have

$$\begin{aligned}
b_o &= f_o d_o \\
b_1 &= f_o d_1 + f_1 d_o \qquad (117) \\
&\vdots \\
b_m &= f_o d_m + f_1 d_{m-1} + \ldots + f_m d_o \\
0 &= f_o d_{m+1} + \ldots + f_{m+1} d_o \\
&\ldots\ldots \\
0 &= f_o d_{m+n} + f_1 d_{m+n-1} + \ldots + f_n d_m
\end{aligned}$$

Assuming that l state variables ($l < n$) are available for feedback, the l elements of $\tilde{K}^T$ can be explicitly determined in solving the first l linear relations in (117).

5.1.2 APPLICATION

Our power system, synchronous machine excitation control problem analysed, where

$$A = \begin{bmatrix} -0.188 & 0.0 & 0.227 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ -1.815 & -0.57 & -0.5 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & -1.0 & -20.0 & -12.0 \end{bmatrix} ; \quad b = \begin{bmatrix} 1.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix} \qquad (118)$$

An optimal controller with the following gains is found suitable [17]

$$K^T = [-1.9068 \quad 0.4101 \quad 1.133 \quad 0.5694 \quad 0.0450]$$

From (113) we have

$$T^*(s) = \frac{N(s)}{D(s)} = 0.29406211 - 0.3441475s + 0.8394748s^2 - 1.4713089s^3 + \ldots \qquad (119)$$

where;

$$N(s) = s^4 + 12.5s^3 + 26.57s^2 + 17.84s + 11.4$$

$$D(s) = s^5 + 14.5948s^4 + 55.223396s^3 + 103.78227s^2 + 106.03773s + 38.767319$$

Assuming only first three states x_1, x_2, x_3 are available for feedback, i.e. $K_4 = K_5 = 0$ we get

$$\tilde{K}^T = [K_1 \quad K_2 \quad K_3 \quad 0 \quad 0]$$

The overall transfer function of equation (8) becomes as

$$\tilde{T}(s) = \frac{N(s)}{f_o + f_1 s + f_2 s^2 + f_3 s^3 + f_4 s^4 + f_5 s^5}$$

where

$$f_o = 11.4\tilde{K}_1 - 36.3\tilde{K}_2 + 2.76565E - 05\ \tilde{K}_3 + 2.143199$$

$$f_1 = 17.84\tilde{K}_1 - 21.78\tilde{K}_2 - 36.3\tilde{K}_3 + 22.994003$$

$$f_2 = 26.57\tilde{K}_1 - 1.815\tilde{K}_2 - 21.78\tilde{K}_3 + 27.779205$$

$$f_3 = 12.5\tilde{K}_1 - 1.815\tilde{K}_2 + 29.332001$$

$$f_4 = \tilde{K}_1 + 12.688$$

$$f_5 = 1.0$$

Finally, Eqns. (117) becomes:

$$10.769766 = 3.352307\tilde{K}_1 - 10.674454\tilde{K}_2 + 8.13272E - 06\tilde{K}_3$$

$$11.815911 = 1.3227863\tilde{K}_1 + 6.0876831\tilde{K}_2 - 10.6744\ \tilde{K}_3$$

$$24.515355 = 11.243651\tilde{K}_1 - 23.511125\tilde{K}_2 + 6.0879047\tilde{K}_3$$

Solving the above three equations;

$$\tilde{K}_1 = 1.9345531$$

$$\tilde{K}_2 = -0.4013842$$

$$\tilde{K}_3 = -1.0961199$$

with the above control parameters

$$\tilde{T}(s) = 0.29406211 - 0.3441475s + 0.8394743s^2 - 1.4734319s^3 + 2.1904581s^4 - \ldots \quad (120)$$

$$f(s) = s^5 + 14.622553s^4 - 55.503372s^3 + 103.78228s^2 + 106.03773s + 38.45732$$

The time responses for unit step of optimal and sub-optimal are given in Fig. (8).

5.2 CONCLUSION

The major drawback of optimal control theory is that it requires feedback from all the state variables that are defined to describe the dynamics of the plant. Unfortunately, the whole state vector is seldom available for measurement. The suboptimal controller design is free from this constaint. The Pade approximation technique for model order reduction is used for arriving at the controller parameters.

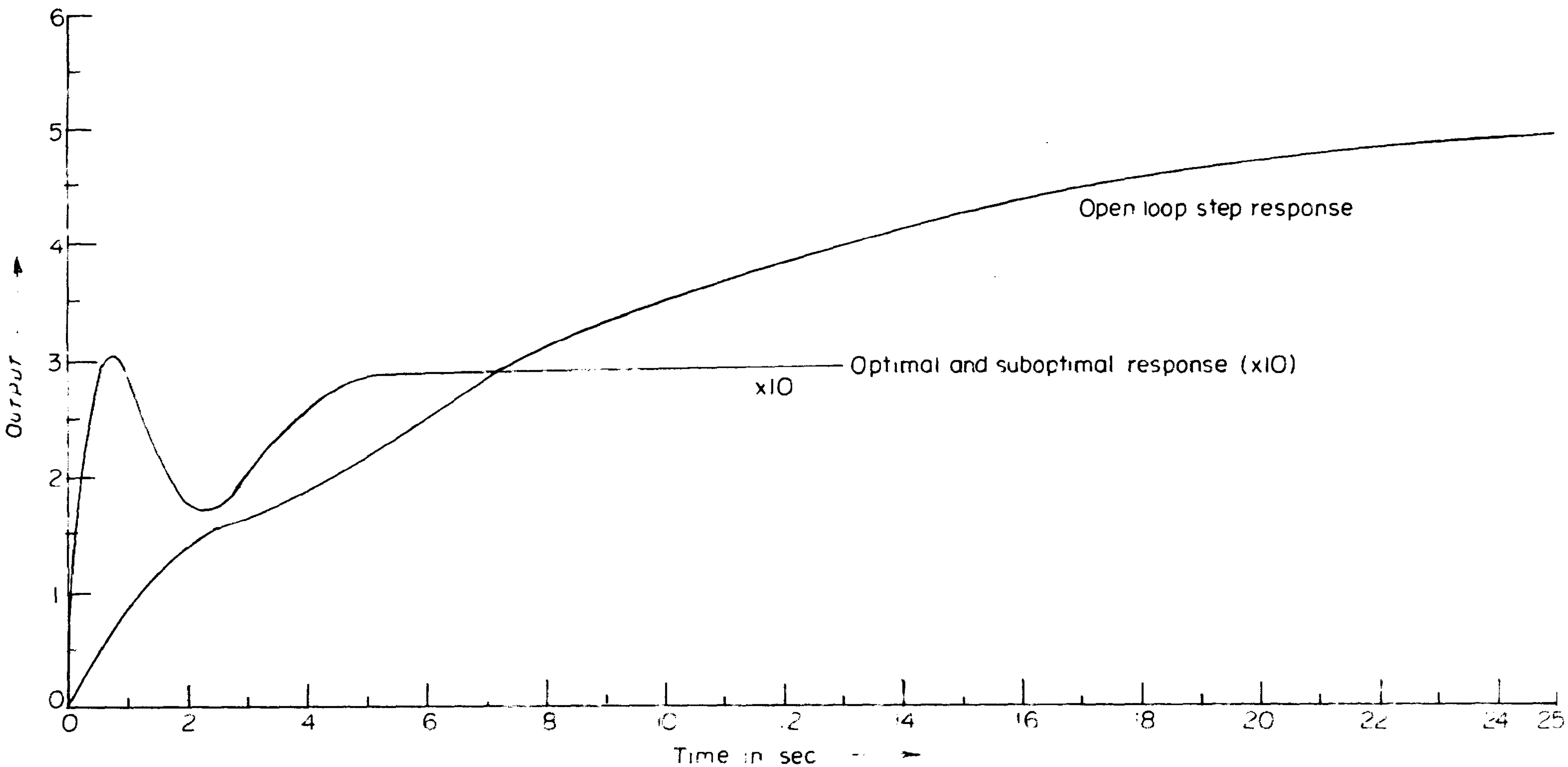
The suboptimal controller design in this chapter is based on partial state feedback. The design is suboptimal in the sense that the closed loop transfer function $\tilde{T}(s)$ approximates the optimal about s = 0, i.e. for lower frequencies. Thus, a useful feature of this method is that the steady state values of the output of the suboptimal and optimal systems are the same for polynomial inputs of the form $\alpha_i t^i$ (i = 0, 1, 2,..., v). This is because the method, in effect, matches the first v time moments of the corresponding systems. On comparison of power series expansion of $T^*(s)$ and $\tilde{T}(s)$, viz, equations (19) and (20), we find that the first v(v = 3) terms are the same. It shows identical first three time moments. This procedure shows exact matching in the steady state region (Fig. 8). The time response comparison in transient zone is quite close to optimal one. For overall good approximations in the transient and steady state behaviour, the method could be extended by matching a combination of Markov parameters and time moments of the optimal and suboptimal systems. The optimal ratio of the Markov parameters and time moments to be matched depends on particular problem and is open to investigate in future.

The novel feature of this method is that when only few states are available for feedback, dynamic compensators may be included to increase the number of design parameters.

TABLE - 11

TIME RESPONSE (CONTROLLER)
TIME RESPONSE

Time (sec)	Optimal	Sub Optimal
0.1	8.985214 E-02	8.974176 E-02
0.2	1.607567 E-01	1.604091 E-01
0.3	2.147953 E-01	2.141892 E-01
0.4	2.540782 E-01	2.532584 E-01
0.5	2.806877 E-01	2.797349 E-01
0.6	2.966331 E-01	2.956417 E-01
0.8	3.039888 E-01	3.031856 E-01
1.0	2.895769 E-01	2.892120 E-01
2.0	1.767807 E-01	1.778623 E-01
3.0	1.996104 E-01	1.993799 E-01
4.0	2.584664 E-01	2.580350 E-01
5.0	2.829635 E-01	2.830567 E-01
6.0	2.877358 E-01	2.878899 E-01
7.0	2.898196 E-01	2.898148 E-01
8.0	2.918996 E-01	2.918487 E-01
9.0	2.931270 E-01	2.931058 E-01
10.0	2.936192 E-01	2.936160 E-01
20.0	2.940614 E-01	2.940614 E-01
30.0	2.940620 E-01	2.940620 E-01
40.0	2.940620 E-01	2.940620 E-01



NOTE--OPTIMAL & SUBOPTIMAL RESPONSES ARE INDISTINGUISHABLE

FIG 8

CHAPTER - 6

CONCLUSION

The development of reduced order models for the analysis and synthesis of high order systems has been an area of active research during the past decade. The present work deals with the application of methods for model order reduction to a power system model and the design of a suboptimal controller using a model reduction technique. The work included herein deals with frequency domain model reduction techniques and suboptimal controller design based on the transfer function description of the system. Detailed discussions and conclusions are given at the end of each chapter and hence this concluding chapter will be primarily devoted to summarizing the main contributions of this work.

The first introductory chapter describes in brief some reduction techniques. Model order reduction techniques have been developed both in the time and frequency domains. In this chapter the Pade approximation techniques and its various variants are first described. The Pade approximation technique has an advantage of computational simplicity. The matching of time moments only gives a reduced model, the power series expansions of which agree with that of the original system about s = 0 and hence steady state responses are accurately reproduced; whereas matching of Markov parameters ensures good matching about $s = \infty$ i.e. the transient response matching is good. Hence by appropriate matching of some time moments and Markov parameters results in overall good approximation. However, this technique sometimes leads to an unstable model for a stable system. So mixed methods have been proposed by various methods to overcome the stability problem. In such methods the denominator polynomial is predetermined by using various stability criteria or by retaining appropriate poles of the original system. The numerator terms are then determined by the classical Pade approximation technique to match a combination of time moments and Markov parameters. The choice of the number of time moments and Markov parameters to be matched can not be decided a-priori and is normally determined by trial and error procedure and depends on the type of the original system being reduced. The related steps in obtaining the reduced order model by such techniques are described in this chapter.

In chapter two we describe different methods for obtaining the transfer function description from given state variable equations. The classical Faddeeva approach is first described. This method is known to give erronous results if the system matrix A is of a high order. A modified algorithm is introduced that removes this problem of inaccuracy. Computer programmes have been developed for these methods.

The third chapter describes the development of state space model for a power system which consists of synchronous machine connected to an infinite bus. The system model is developed using well known Parks equations.

In chapter four the various model reduction techniques described in ch. one are applied to the power system model developed in the previous chapter. A comparative study has been made and the merits and demerits of the various models have been brought out in table 10.

Chapter five deals with the design of suboptimal controller using the Pade approximation technique. It is found that a suboptimal controller using restricted state feedback using the above technique gives a time response that cannot be distinquished from the optimal one. Hence it is felt that this method for suboptimal design may be used in practice that will lead to simple controllers as well as requiring feedback from measurable variables.

Overall frequency domain reduction techniques have been found to yield reduced order models for power systems and practical controllers may also be designed for such systems. This dissertation has been restricted to the application of above techniques in model order reduction and controller synthesis for single input single output systems only. These techniques are also applicable to the multivariable case and is left as an exercise for future workers. It is felt that this application of frequency domain model order reduction techniques to a power system problem is reported for the first time.

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