MAINTENANCE SCHEDULING OF INTERCONNECTED POWER SYSTEM

A DISSERTATION
submitted in partial fulfilment of the
requirements for the award of the Degree
of
MASTER OF ENGINEERING
in
ELECTRICAL ENGINEERING
(System Engineering & Operation Research)

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February, 1984

C O N T E N T S

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CERTIFICATE

Certified that the dissertation entitled, 'Maintenance Scheduling of Interconnected Power System' submitted by Shri Bechu Rai in partial fulfilment for the award of the degree of 'Master of Engineering' in 'System Engineering And Operation Research' to the Department of Electrical Engineering, University of Roorkee, Roorkee, is a record of the student's own work carried out by him under **our** supervision. The matter embodied in this dissertation has not been submitted for the award of any other degree or diploma.

This is further to certify that he has worked for a period of seven months from August 18, 1983 to Feb. 28, 1984, for preparing this dissertation.

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ACKNOWLEDGEMENT

The author wishes to express his grateful thanks to Dr. R.B.Misra, Lecturer in Power System Engineering, Department of Electrical Engineering, University of Roorkee, Roorkee for his invaluable guidance and time devoted. in the preparation of this dissertation by the author. Without his active participation in all the aspects of the work, it would not have been possible to complete this dissertation. The author is also indebted to Sri Vinai Pant, Lecturer, Department of Electrical Engineering, University of Roorkee, Roorkee for giving his invaluable time during the preparation of this dissertation.

Lastly, I would like to express my thanks to all my friends, who rendered their help and services with great faith and dedication.

(BECHU RAI)

ABSTRACT

Preventive maintenance is required for all generating equipments in order to reduce the chances of power shortages and improve the overall availability of the system capacity. Power companies spend a lot of money every year for this purpose. The system reliability and operating costs of an electric power system are affected, when the generating units of the system are taken out for maintenance purpose. Carefully optimized maintenance schedules could potentially defer some capital expenditure for new plants in times of tightening the reserve margins and allow the critical maintenance work to be performed which might not otherwise be done. Therefore, maintenance scheduling is a significant part of the overall operations scheduling problem.

In this thesis, a method of maintenance scheduling for two interconnected power system is proposed which optimizes the expected savings due to energy interchanges. Random failures of generating units and their effect on the incremental cost of energy production is considered. Further a fast method of convolution of machines outages with the system's incremental costs for different loads is proposed, which is computationally fast. A computer programme has also been developed for the implementation of the proposed method.

The method has been applied to an interconnected system which has one fictitious system of 4100 MW connected to a modified IEEE Reliability Test System of 3400 MW capacity. The results for various maintenance schedules for different system loads are obtained.

CHAPTER I

INTRODUCTION

Preventive maintenance is required for all generating equipments in order to reduce the chances of power shortages and improve the overall availability of the system capacity. Power companies spend a lot of money every year for this purpose. In addition to these direct cost for maintenance purposes, there are certain indirect costs such as cost of replacement energy when a generating unit is out of service for maintenance, loss of sales, etc., which are also charged to maintenance. The system reliability and operating costs of an electric power system are affected, when the generating units of the system are taken out for maintenance purposes. Additionally carefully optimized maintenance schedules could potentially defer some capital expenditure for new plants in times of tightening the reserve margins and allow the critical maintenance work to be performed which might not otherwise be done. Therefore, maintenance scheduling is a significant part of the overall operations scheduling problem.

The importance of optimal maintenance scheduling problem is due to the fact that system reliability and operating cost of an electric utilities are affected by the maintenance outage of generating units and its accessories. The task of maintenance scheduling involves specifying dates at which manpower is to be allocated for overhauling a functional unit or a group of units in a power plant such that overall system security level is acceptable, cost to the system is minimized and all or most of the constraints are met. The major costs which play an important role in maintenance scheduling are energy production cost and maintenance cost. The maintenance cost becomes of importance if planned outage durations are allowed to vary within a given limit, which is mainly affected by availability of additional manpower either through increased staff or overtime.

Maintenance activities for a generating unit include followings. For a fossil-fueled generating unit, the major maintenance activities include checkup and maintenance of boiler and turbine. There are other checkups such as Regulator, generator, scrubbers, condenser etc. For nuclear units, reactors form the major component that needs maintenance checkups. Minor maintenance checkup activities for nuclear units include checkups of turbine, condenser, regulator, generator, etc. A number of uncertainties are involved in dealing with this long-term scheduling problem. These include load uncertainty, fuel supply and price uncertainty, generating unit reliability, uncertainty in resources, and crew availability.

In solving any real problem with more than one solution an objective function is defined, which is either maximized or minimized depending upon the objective criteria.

Generally, there are two objectives in the operation of an electric power system. These are : minimize the total operating costs to the utility and maximize the system reliability. Any one of these two can be considered as an objective function for maintenance scheduling and other as a constraint. Both type of the problems cost oriented as well as reliability oriented have been dealt so far [3,6,12]. Two types of costs are of importance, when the objective is to minimize the some sort of a cost function. The first is maintenance costs and second one is production costs. Production cost as an objective function has been used by Dillon [3] and the results reported in the literature show production cost as an insensitive objective. Reliability-oriented method fall under one of two categories: (i) deterministic [6,9,12] and stochastic [7]. Deterministic reliability objectives try to levelize the capacity reserves in one way or another. But the result is not accurate because of ignoring the uncertainties in demand and generating unit availabilities. Stochastic method includes load uncertainties and generating forced outages. Many researchers [6,7,10,12] have worked on maintenance scheduling of generating system considering the deterministic as well as stochastic model of the objective function.

Till now the research work on maintenance scheduling has only been limited to a single generating system.

The literature lacks in the methodologies for interconnected systems. In the present thesis a new method for maintenance scheduling of two interconnected power system is proposed. In interconnected system, energy interchange and displacement of costly and/or scarce resources is a great concern. The measure of the degree of economic benefits due to such interconnected operation has been referred to as "Tight pooling" or loose pooling". In the proposed method the objective function is the "Economic benefits" due to the energy interchanges as a function of tie line capacity. This also includes the random failures of generating units and their effect on the incremental cost of energy production.

The transfer capacity between any two systems is determined by firm sales of energy, reserve sharing, improved stability and operation under emergencies. The existence of such interconnection has made the exploitation of maximum economy possible by joint dispatch. In these days of escalating fuel prices, displacement of scarce and expensive fuel by a more abundant fuel is of great concern.

The aim of the work reported here is that of developing a method to schedule the generating units in a interconnected power system to maximize the expected savings due to energy interchanges. The planner would then have a knowledge of an economy energy interchange profile over a review period. In addition the proposed model would did in the following studies :

(i) The estimation of optimum economy energy interchange benefits between systems.

(ii) An examination of the costs associated with increasing the transfer capacity vis-a-vis the increase in expected savings.

(iii) Effects of coal conversion and other changing generation patterns on economy energy interchange benefits.

(iv) Effects of load management and time zone differences on energy interchange benefits.

The proposed method considers the effect of machine outages and their effect on incremental cost . Outages of machines with lower incremental cost will necessiate the use of more expensive generation to meet the demand. Under such circumstances, energy can be purchased from the neighbouring system if it's incremental cost is lower. The randomness between the outages of machines and the differing fuel mixes in the two interconnected systems are included in the proposed method. The essence of this method is in obtaining a bivariate probability distribution of the incremental costs in the two systems. From this distribution, the calculation of expected savings in production costs of the two system has been outlined.

Different units in both systems are considered on maintenance and then the expected savings are calculated using the method discussed. Once the saving is calculated for different sets of unit arrangement, we compare the benefit for different maintenance schedules. The maintenance schedules which gives maximum benefit is the best schedule for maintenance for the proposed time-period.

In Chapter I of this dissertation, introductory part as well as the review of the work is discussed. In Chapter II terminology as well as a method for maintenance scheduling is discussed, which gives the maintenance scheduling of the generating units by levelizing the risk [6]. Different objective functions have also been discussed. In Chapter III a method which calculate the economic benefits of interconnected power system due to interchange of energy from either of the system is discussed. A convolution process of outages of machines with hourly loads has been discussed. An idea of Bivariate Probability density function, which is the attraction of the method, to calculate the economic benefit has been given. The use of this bivariate probability density function for two interconnected system has also been discussed. The use of Bivariate Gram-Charlier expansion (BGCE), which is the core of the proposed method has been discussed. RAU et al. [14] have discussed this in detail. The use of Bivariate Gram-Charlier expansion (BGCE) is to modify the moments of $f_o(\gamma)$ (Probability density function of incremental costs) to obtain the moments of $f_n(\gamma)$, probability density function of equivalent incremental costs) to include the effect of all machine outages. This is done by using the concept of statistical moments and cumulants. Thus the use of moments and cumulants avoid the task of having to keep track of a large number of impulses as discussed in Chapter III. From the knowledge of moments of $f_n(\gamma)$, the BPDF in the γ_a γ_b plane is obtained from the BGCE discussed in Chapter III. For a given tie line capacity, the evaluation of economic benefits have also been discussed.

In Chapter IV which consist Results and Discussion, the result of different sets of data are given and relevant discussion of the result is made on the basis of the result. Different maintenance schedule of the units have been given. In addition the suggestions for the further work are given at the end of the Chapter.

CHAPTER II

GENERAL

Maintenance scheduling now a days has become a crucial factor, which directly affect the production. The maintenance scheduling for generating equipments becomes important in order to reduce the chances of power shortages and improve the overall availability of the system capacity. The system reliability and operating costs of an electric power systems are affected, when the generating units of the systems are taken out for maintenance purpose. Thus an optimal schedule which gives the minimum cost to the system utilities and meet the certain desired constraints is to be obtained to carry out scheduled maintenance. The work on maintenance scheduling of single generating system has already been done [6]. In what follows is the description requires maintenance scheduling problems.

2.1 TERMINOLOGY

Preferred Maintenance:

The nominal starting date of maintenance for a given generating unit.

Preventive Maintenance:

Regular routine or planned check ups and repair of generating equipment to increase the availability of the generating facility.

Corrective Maintenance:

Maintenance built into the system at the design stage.

Time-Horizon:

The span of time considered in a solution of the maintenance scheduling study.

Time-Increment:

Maintenance scheduling of units are scheduled in discrete time period. This time period is known as time increment.

Unit :

Device for which an outage results in a loss of generating reserve. For example, generators, boilers, turbines, auxiliary equipments and tie lines.

Reserve Loss :

A reserve loss is generally associated with each unit or device. This value, in megawatts, is the decrease in reserves during maintenance.

Gross Reserve :

The gross reserve in megawatts for a given time is the total installed generating capacity available during that time period less the peak load forecast for that time-period.

Net Reserve :

The gross reserve less the total reserve lost be cause of the maintenance scheduled.

Crew of Plant :

A crew is assigned to each or device i.e. maintenance activity. A single crew may be responsible for maintaining all the units in a given plant or the units in a given area.

Maintenance Outage:

Occurance of an outage (planned outage) are of two types, firm or flexible.

Firm Maintenance:

An occurance of an outage which has already taken place or prescheduled to begin at a specified time.

Flexible Maintenance:

An outage which is to be scheduled by the maintenance scheduling technique.

Resource :

A term applied to a pool of equipment, manpower or other quantity which is in limited supply during each time increment and is shared by more than one outage.

Frozen Time :

The time during which some given generators can not begin the maintenance.

Forced Outage :

When turbo-generators or boilers or other equipments is to be taken out of service at once or as soon as possible, called forced outage. The time during which unit is unavailable due to forced outage is known as forced outage time.

Time Exposed to Forced Outage :

The time on forced outage plus the service time.

Forced Outage Rate :

The forced outage rate for a unit is the ratio of forced outage time and the time exposed to the forced outage.

Loss-of-Load Probability (LOLP):

LOLP is the measure of generation system reliability. It can be defined as the probability that the system load exceeds the system's available generation. LOLP does not give the amount of energy lost i.e. unsupplied by utility only says that load is exceeding generation.

Expected Unsupplied Energy:

The amount of energy which cannot be supplied by the utility during some period. To obtain EUE is more time consuming than LOLP. If LDC is used to calculate the Reliability measure both involves the same computational efforts.

Variable Operating Costs :

Consists of two components i.e. variable O and M costs and fuel costs burnt in producing electric energy.

Production Cost :

The cost of burnt fuel for producing a given amount of electric energy.

Maintenance Costs :

Maintenance cost includes two components, one is fixed cost, which is constant for an unit and other component is dependent on the usage of facility. Normally this component is dependent upon the per-unit energy produced.

Objective:

The objective of a maintenance scheduling problem is to have a 'reliable' system with minimum operating costs to the utility.

Constraints:

There are restriction on maintenance activity which must be fulfilled during the maintenance. There are generally two types of constraints -

Crew Constraints:

Ensures that no two units of the same crew are scheduled for maintenance during the same time period.

Resource Constraint:

Ensures that more than available resource is not committed when planning the maintenance.

2.2 OBJECTIVE FUNCTIONS

Whenever a problem has more than one solution there is an objective function to be either maximized or minimized. Mainly there are two objective functions in the operation of an electric power system. These are :

(i) Minimize the total operating costs to the utility and

(ii) Maximize the system reliability.

Generally the system reliability is taken as a constraint rather than an objective i.e. to keep the reliability above a certain level.

2.2.1 Cost Objective Functions :

Two types of costs play an important role in cost objective function which is to be minimized. The first one is maintenance cost and the second one is production cost.

The cost to maintain a generating unit has two components i.e. undepreciated maintenance investment lost (MIL) by maintaining a unit too early and the out-of-pocket maintenance cost (PMC) in terms of time since previous maintenance. Dopazo [4] have found that the total cost is almost a convex function. Therefore for each unit there is an ideal time that the maintenance should be performed. It may be mentioned that this objective function minimizes the maintenance costs but the overall objective of operation scheduling is to minimize total operating costs when maintenance cost is one component.

Production cost as an objective function has been used by some authors [10,6] Production cost minimization requires either many approximations or heavy simulations. Yet, it has been seen that Production cost is an insensitive objective. In a discussion Garver, Happ, Dopazo and Morrie [4,7] indicated that the insensitivity of production cost is consistent with their experience in Maintenance scheduling. Overall you can say that Production cost minimization does not seems an effective objective function.

2.2.2 Reliability Objective Functions :

Reliability objective function fall under one of two categories :

(i) Deterministic and (ii) Stochastic

Deterministic reliability objectives try to levelize the capacity reserves. It has been recognized that this objective does not levelize system reliability, because of ignoring the uncertainties in demand and generating unit availabilities.

(ii) Stochastic methods include load uncertainties and generating forced outages. Garver [6] has used the idea of load carrying capability to include unit forced outages. But, due to excessive computations, the effect of taking a unit out for maintenance was not considered. But Stremel [20] overcome this problem by using the method of cumulants. However, he did not consider load uncertainties. Later on Stremel et al. [19] in a separate paper incorporated the load uncertainty.

2.3 MAINTENANCE SCHEDULES TO LEVELIZE RISK

When scheduling generating units for maintenance, one of the goals is to level the risks of capacity shortages throughout the year. Level reserves have been used as an indication of level risk. A method, i.e. The Five step method of incorporating risk considerations into existing maintenance scheduling procedures', which gives better schedules by using effective capability [7] in place of reserve is described in [6]. The procedure for leveling the risks involves these five steps.

(1) Compute the capacity outage table for all of the units in the system as would be done for a loss-of-load probability calculation.

(2) Prepare to estimate the effective capability of each unit by fitting a straight line to a semi-log plot of the capacity outage table.

(3) Compute the effective load - carrying capability of each unit using the fitted straight line.

(4) Compute the equivalent load for each maintenance period. This is the single magnitude which if encountered each day in the period would result in the same total risk as the varying loads actually forecast.

(5) Schedule the maintenance to level, throughout the year, the equivalent load plus the effective capabilities of the units on maintenance, in place of leveling the reserve.

It is clear from the literature that maintenance schedules prepared to levelize risk in place of levelizing reserves are better. Further the stochastic models consisting different uncertainties, though require large computational efforts, schedule maintenance to give more expected benefits.

CHAPTER III

A METHOD TO EVALUATE ECONOMIC BENEFITS IN INTER-CONNECTED POWER SYSTEMS

The tie line in the case of interconnected power system plays an important role in the power system economy as well as power system reliability. The description of a method to calculate the economic benefits due to energy interchange, which takes into account the random failures of generating units and their effect on the incremental cost of energy production follows.

3.1 DESCRIPTION OF METHOD

The basic steps of the method are given below.

3.1.1 Incremental Cost

It is well known that incremental cost λ increases with load and is a non-decreasing function of the load. The variations of incremental cost with load for a fictitious system is shown in Fig.3.1. Here the machines have been simulated as a multiblock units. An approximation of unit commitment has been made. In such an approximation, the units are loaded in their merit order based on their average incremental costs. In Fig.3.1, the system consists seven units in their loading order. The incremental cost λ of units have been shown to change at part load value points. Here the incremental cost λ is approximated by

$$\lambda = \beta \, e^{KL} \qquad \ldots (3.1)$$

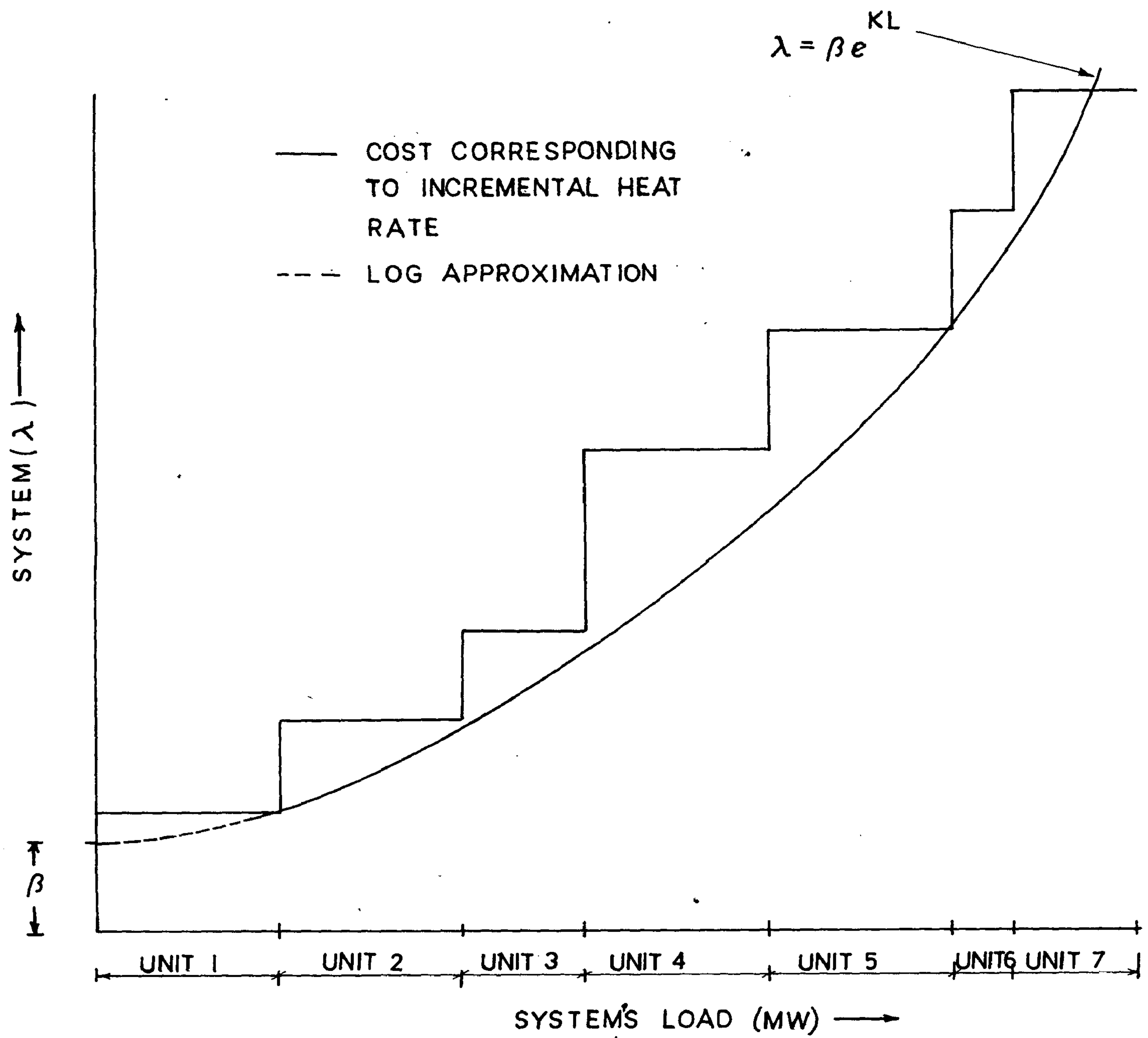


FIG. 3.1 λ VS. LOAD RELATION

The curve,shown by dotted line in Fig.3.1,represent the approximated incremental cost curve. This curve can be obtained by using a least square method or any other curve fitting technique. The eq.(3.1) may linearized by taking the natural log. Thus the equation is obtained as follows:

$$\gamma_a = \alpha_a + K_a L_a \qquad \ldots (3.2)$$

where $\gamma_a = \log_e \lambda$

$\alpha_a = \log_e \beta$

K = slope of the linearized curve

L = Load

Subscript 'a' = denotes system 1.

In the same way for system 2.

$$\gamma_b = \alpha_b + K_b L_b \qquad \ldots (3.3)$$

where $\gamma_b = \log_e \lambda$

$\alpha_b = \log_e \beta$

Subscript 'b' : denotes system 2.

These linear relations between system γ_s and loads are shown in Fig.3.2.

3.1.2 Machine Outages

The effect of machine outages is shown in Fig.3.2. The solid line of system 1 and system 2 show that all the units are operating. When unit 1 is taken out, the hourly loads that were supplied by unit 1 will now have to be supplied

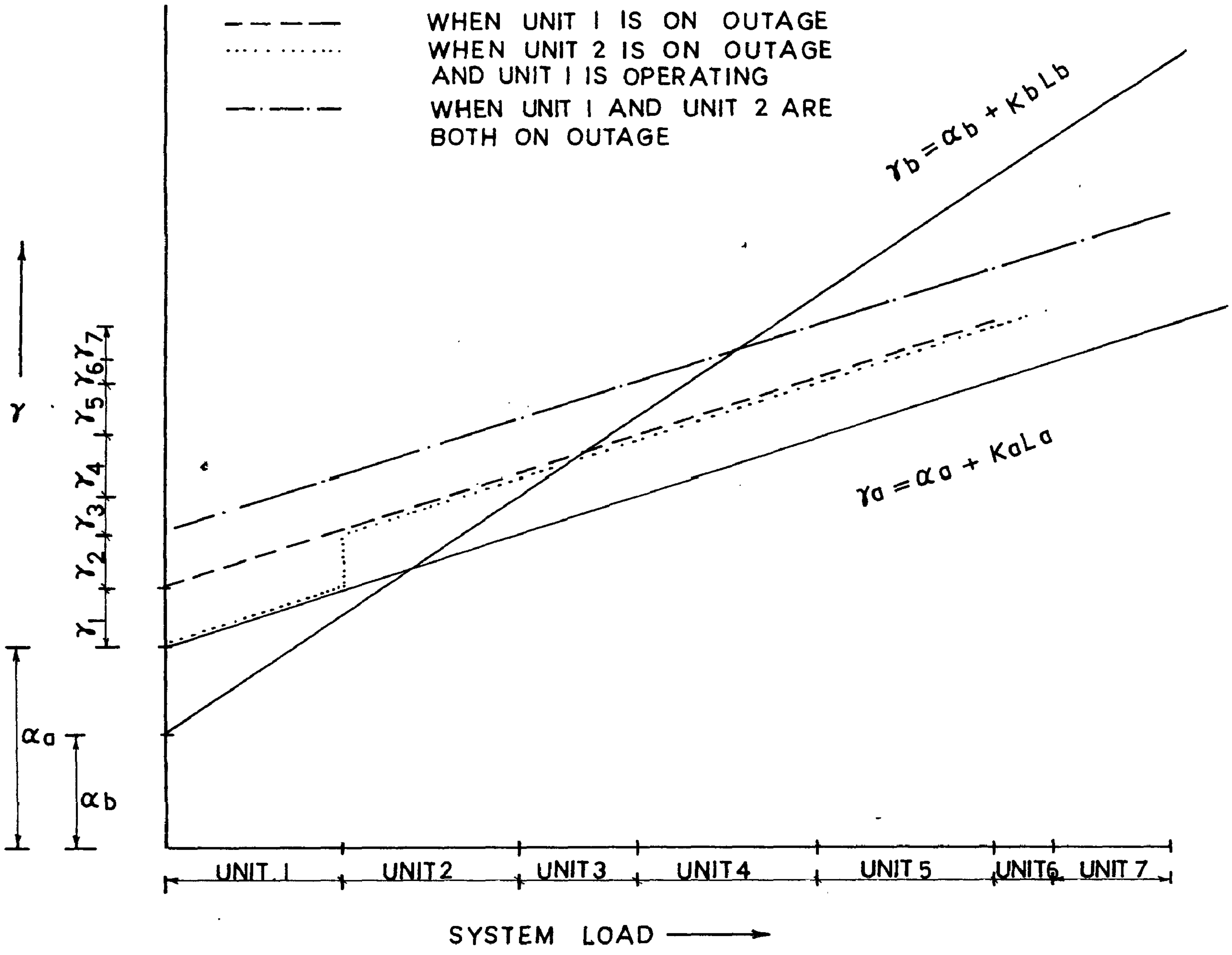


FIG. 3.2. γ VS SYSTEM LOAD

by unit 2, and those loads that were supplied by unit 2 will be supplied by unit 3, etc. such a situation is shown by dashed line, where all the hourly γ values are increased by γ_1.

If unit 2 alone is taken out when unit 1 and others are operating then the hourly loads that were supplied by unit 1 will be uneffected, But the loads supplied by unit 2 will have to be met by unit 3 and so on. Line shown by dots indicates this situation, where the γ values for hourly loads > capacity of unit 1 are increased by γ_2. Similarly the simultaneous outage of unit 1 and unit 2 will produce such a γ to load relation which is parallel to the solid line of system 1, but at a distance of $\gamma_1 + \gamma_2$ (the γ values of unit 1 and unit 2 above it. In the same way the process can be repeated for the units of system 2 also. It is difficult to draw such type of relationship for more than two units as the number of combinations of machine outage states become very large. Here aim is to obtain a bivariate density for γ in the interconnected system considering all the machine outages. From such a bivariate probability density function one can overcome the above difficulty. This process is explained as follows.

Consider the daily hourly loads for two systems. By sampling the hourly load profiles every hour and assigning each sample equal probability (1/24 in this case) a joint probability mass function is obtained as shown in Fig.3.3.

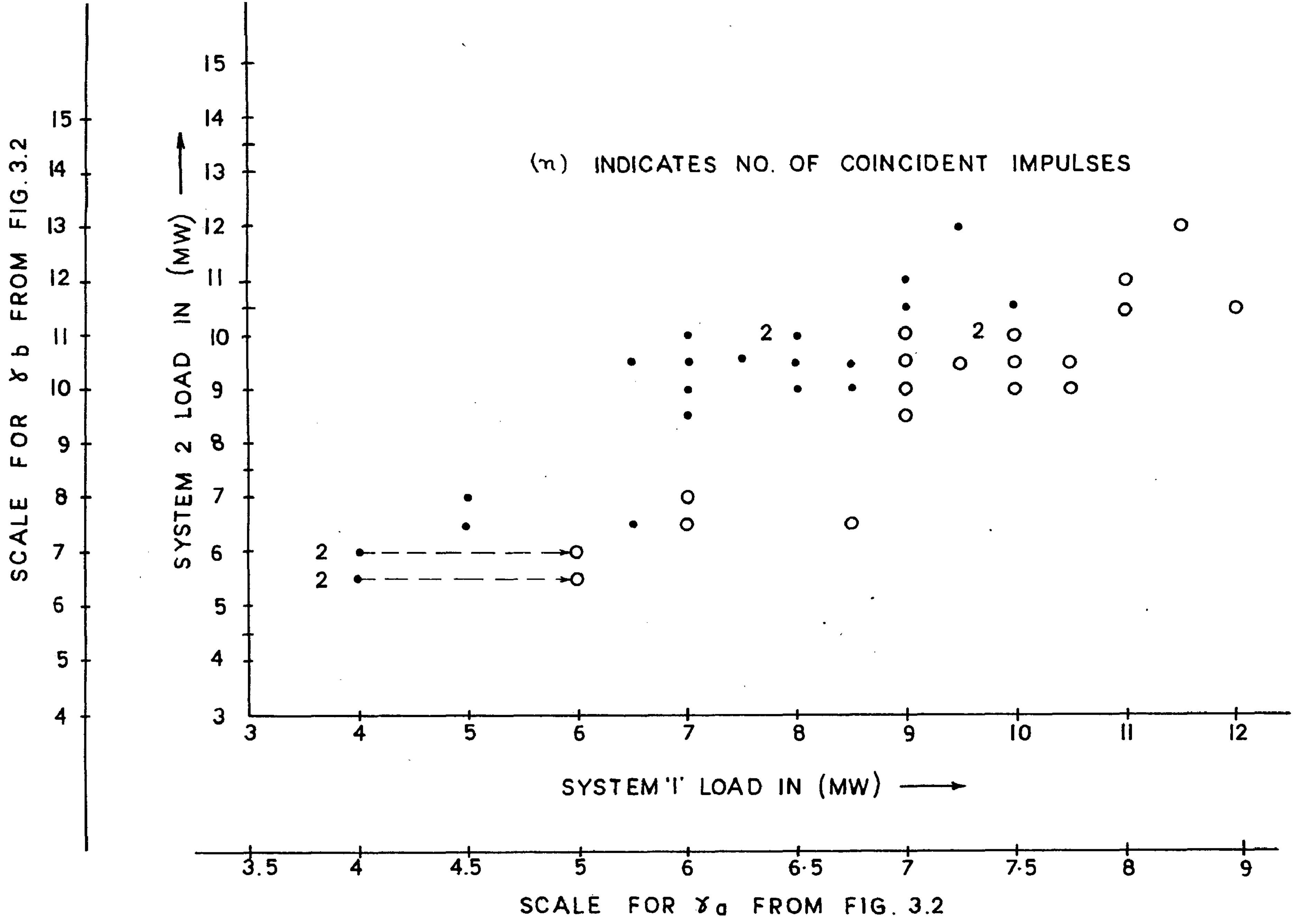


FIG. 3.3. THE BIVARIATE DENSITY

The solid dots represent the probability masses of 1/24 for a discrete bivariate probability density function of 24 hourly loads in two interconnected system. Unfilled dots represent the probability masses of outages.

The process of obtaining the joint density of equivalent load is outlined in what follows. Since the random failures of generating units are independent of system load, the convolution of random variable describing the outage capacity in terms of γ of the first machine, say in system 1 with capacity γ_1 (in terms of cost), FOR = q and availability P will modify the joint probability density function of loads to result in 48 impulses. The result will be twenty four impulses, all in the same position as before convolution but with probability masses P/24 and an additional 24 impulses, shifted by an amount γ_1 on the X-axis but retaining their positions on the y-axis, with probability masses q/24. In the same way the outage of the second unit in system 1 will transform the newly obtained 48 impulses into 96 impulses. The development for unit outages in system 2 is similar to that in system 1. However the impulses are instead displaced in Y-axis. If there are N_1 units in system 1 and N_2 units in system 2, the total number of impulses is equal to the product of load samples times $2^{N_1} \times 2^{N_2}$

i.e. Total No. of Impulses = (load sample) $\times 2^{N_1} \times 2^{N_2}$

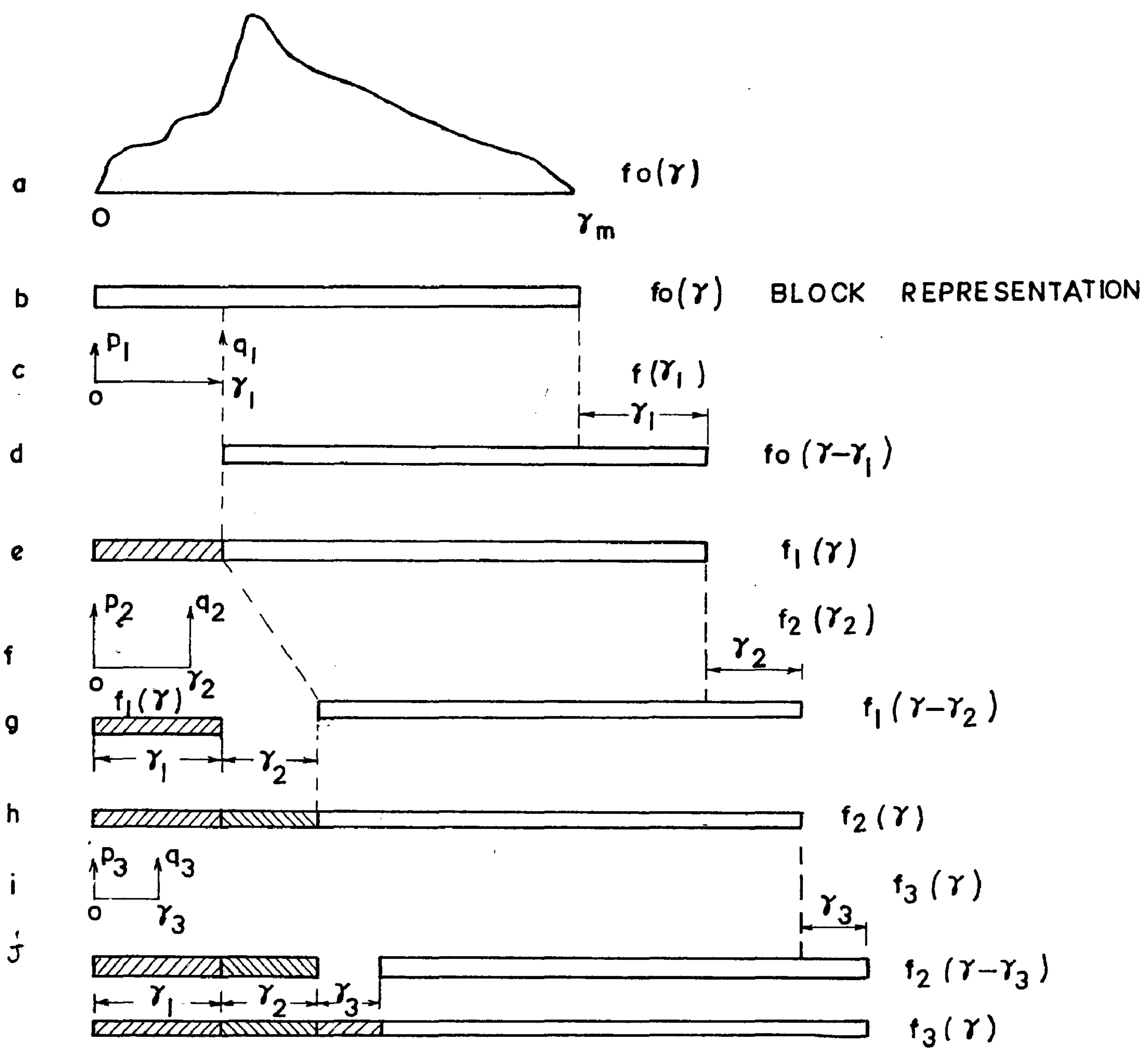


FIG. 3.4 EFFECT OF MACHINE OUTAGES ON PDF OF γ

The bivariate probability density function of 24 hourly loads in two hypothetical interconnected systems is shown in Fig.3.3. From this BPDF, the marginal probability density function of load in either system can be obtained. By using the eq.(3.2) and (3.3), the loads of system can be converted into γ_s. Thus the density of γ for system 1 is obtained and identified as $f_0(\gamma)$ as shown in Fig.3.4(a). A schematic block representation of the PDF of γ for a given pattern of load demand, when all the machines are in operation, has been given in Fig.3.4(b).

Since the failure of unit 1 will increase all γ values by γ_1, this effect is shown in Fig.3.4(d), where the whole density $f_0(\gamma)$ is shifted to the right by γ_1. Since the FOR of unit 1 is q_1, the density of γ due to loads will be changed to Fig.3.4(d). For q_1 percent of time and according to 3.4(b) for P_1 percent of time, because $(P_1 + q_1 = 1)$. The total effect of outage of unit 1 can therefore be obtained by multiplying Fig.3.4(b) by P_1 and Fig.3.4(d) by q_1 and adding the two to get $f_1(\gamma)$ in Fig.3.4(e). $F_1(\gamma)$ is nothing but is the probability density function of load of system 1, when unit 1 is convolved.

Consider the outage of unit 2, the PDF of Fig.3.4(e) is modified to $F_2(\gamma)$ as shown in Fig.3.4(h). The outage of unit 2 increases the γ values only for loads exceeding the capacity of the first machine. Outage of unit 2 does not affect the γ values for loads < the capacity of unit 1. In Fig.3.4(g) the hatched portion indicates the uneffected portion of the density. The outage of unit 2 has the effect of shifting the PDF of Fig.3.4(e) in the zone $\gamma_1 \leq \gamma \leq \gamma_m + \gamma_1$ by γ_2. This is identified as $f_1(\gamma - \gamma_2)$ and shown in Fig.3.4(g).

Consider the forced outage rate of unit 2 is q_2. Therefore, multiplying Fig.3.4(e) in the zone $\gamma_1 \leq \gamma \leq \gamma_m + \gamma_1$ by P_2 and $f_1(\gamma - \gamma_2)$ i.e. the unhatched portion of Fig.3.4(g) by q_2 and by adding the two one can obtain Fig.3.4(h) in the zone $\gamma_1 \leq \gamma \leq \gamma_m + \gamma_1 + \gamma_2$. Thus the Fig.3.4(h) represents the PDf of γ, when unit 1 and unit 2 both are convolved. It is observed that, in the region $\gamma_1 \leq \gamma \leq \gamma_m + \gamma_1 + \gamma_2$, Fig. 3.4(h) is nothing but a convolution of Fig.3.4(e) with the binary outage representation of unit 2. In the same way one can obtain $f_n(\gamma)$ for an n machines system. The $f_n(\gamma)$ spans the region $\gamma = 0$ to $\gamma = \gamma_m + \gamma_1 + \gamma_2 + \gamma_n$, consistent with the assumption of an infinite reserve capacity being available. Since the resulting $f_n(\gamma)$ does not include the constant α_a, this can be corrected by altering the first cumulant by α_a. Thus the $f_n(\gamma)$ includes the outages of all machines.

For example $f_3(\gamma)$ is given by

$$f_3(\gamma) = P_1 f_0(\gamma) \text{ for } \gamma \in (0, \gamma_1) \qquad \ldots(3.4a)$$

$$f_3(\gamma) = P_2 f_1(\gamma) \text{ for } \gamma \in (\gamma_1, \gamma_1 + \gamma_2) \qquad \ldots(3.4b)$$

$$f_3(\gamma) = P_3 f_2(\gamma) \text{ for } \gamma \in (\gamma_1 + \gamma_2, \gamma_1 + \gamma_2 + \gamma_3) \qquad \ldots(3.4c)$$

$$f_3(\gamma) = p_1p_2p_3f_o(\gamma) + q_1p_2p_3f_o(\gamma-\gamma_1)$$
$$+ p_1q_2p_3f_o(\gamma-\gamma_2) + p_1p_2q_3(\gamma-\gamma_3)$$
$$+ q_1q_2p_3f_o(\gamma-\gamma_1-\gamma_2) + q_1p_2q_3f_o(\gamma-\gamma_1-\gamma_3)$$
$$+ p_1q_2q_3f_o(\gamma-\gamma_2-\gamma_3) + q_1q_2q_3f_o(\gamma-\gamma_1-\gamma_2-\gamma_3)$$

$$\text{for } \gamma \in (\gamma_1+\gamma_2+\gamma_3,\ \gamma_m+\gamma_1+\gamma_2+\gamma_3) \qquad \ldots(3.4d)$$

3.1.3 Application of Method to Interconnected System

In the case of interconnected system the outages of machines in two systems can be considered independent while the loads and corresponding γ_s are correlated. The process of obtaining $f_1(\gamma)$ of Fig.3.4(e) is equivalent to the appropriate movement of probability masses in the bivariate case. The outage of unit 1 increases γ by γ_1, therefore, the outage of unit 1 shifts γ horizontally by γ_1 shown by unfilled dots in Fig.3.3. Thus the process of convolution is same as discussed previously. In the same way machine outages in system (2) result the movement of impulses vertically along the direction of Y axis since the resulting number of impulses, after convolution of machines of both system would be large, this method is generally not recommended. This procedure only helps to understand the basic idea of bivariate probability density function. A method of convolution using the concept of moments and cumulants has been used by Rau et al. [13]. This method is discussed in detail at the later stage.

3.1.4 Fast Method of Convolution

Here in this article the convolution of machines outage into the load moments have been described. Rau et al [13] have discussed a method of convolution of the outages of machines into loads, which is given below. In [13] authors considered the convolution of machines into the load separately for both the system. Because the equivalent machine moments are obtained independently for the two systems and then for getting the PDF of equivalent machine the convolution process is repeated twice. In this thesis a method of convolution is suggested which gives the convolution of machines outage in both the system at the same time. Thus this method of convolution is faster than the method suggested by Rau et al [13].

Let us consider that $f_o(\gamma)$ is the probability density function of the loads in system 1 and there are three machines in system 1 where incremental costs in terms of γ_s are γ_1, γ_2 and γ_3 respectively. (P_1, q_1), (P_2, q_2) and (P_3, q_3) are up and down probabilities of machines respectively as shown in Fig.3.5(a). Further, consider that the moments of $f_o(\gamma)$ of the loads of system 1 [13] divided into seven sub-blocks as shown in Fig.3.5(b).

In Fig.3.5(b), if the first machine whose γ is divided into three blocks is convolved into the load of system 1, then each block will be shifted by γ_1 as shown in Fig.3.5(c). Now

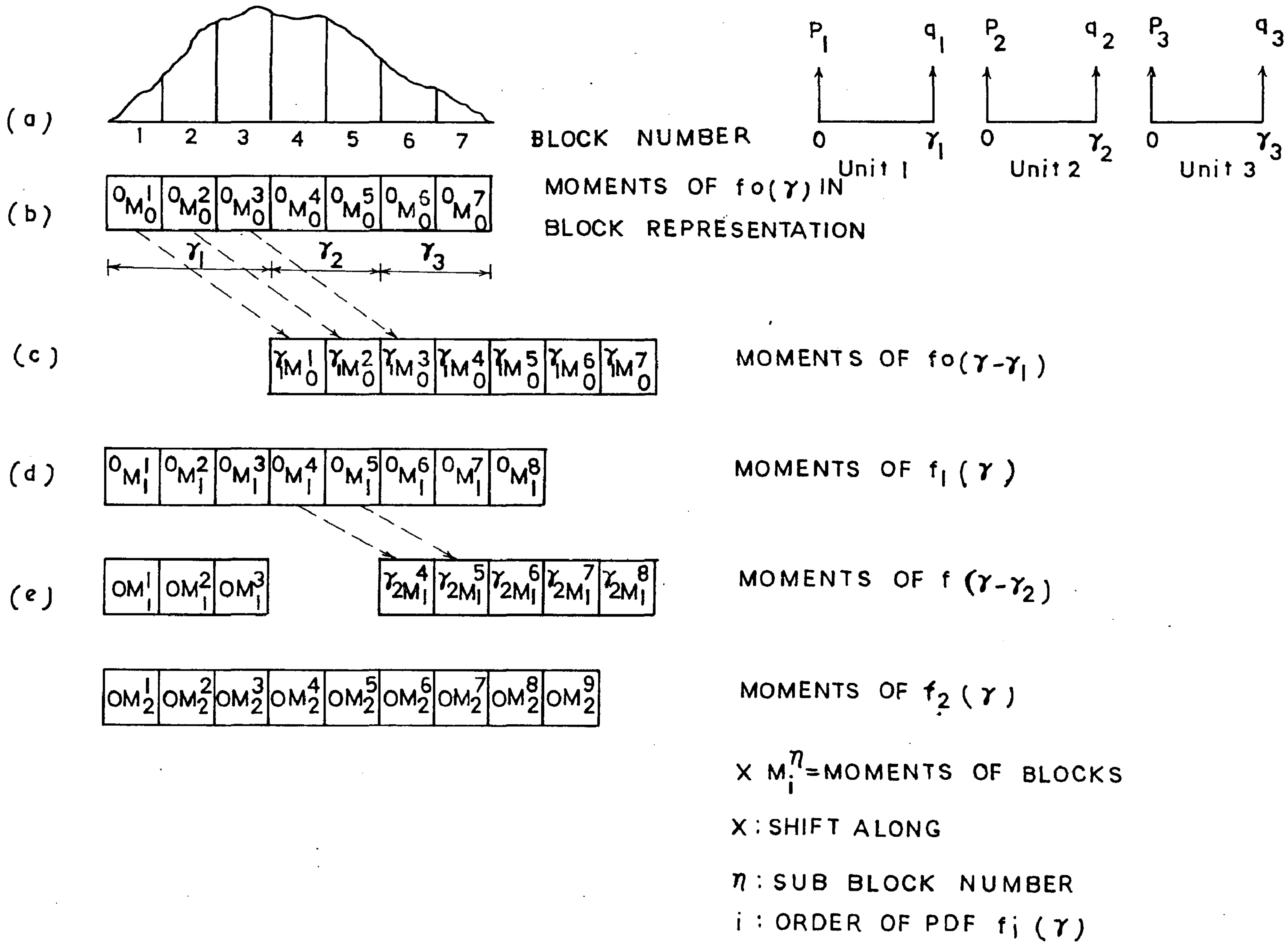


FIG. 3.5. NUMERICAL PROCEDURE TO OBTAIN MOMENTS OF $f_n(\gamma)$.

the moments of the PDf, when first machine is convolved can be found as follows :

$$^{o}M_1^1 = M_o^1 \times P_1$$

$$^{o}M_1^2 = M_o^2 \times P_1$$

$$^{o}M_1^3 = M_o^3 \times P_1$$

$$^{o}M_1^4 = M_o^4 \times P_1 + {}^{\gamma 1}M_o^1 \times q_1$$

$$^{o}M_1^5 = M_o^5 \times P_1 + {}^{\gamma 1}M_o^2 \times q_1$$

$$^{o}M_1^6 = M_o^6 \times P_1 + {}^{\gamma 1}M_o^3 \times q_1$$

$$^{o}M_1^7 = M_o^7 \times P_1 + {}^{\gamma 1}M_o^4 \times q_1$$

$$^{o}M_1^8 = ({}^{\gamma 1}M_o^5 + {}^{\gamma 1}M_o^6 + {}^{\gamma 1}M_o^7) \times q_1$$

These block wise moments are pictorially shown in Fig.3.5(d).

Since the property of moment is that it can not be added up or substracted, hence from the moment and cumulant relationship the moments are converted into cumulants and then they are added up. Once the cumulants are obtained it can be converted into moments. In the same way when the second machine is convolved, the loads of first machine will be uneffected as explained in Fig.3.2, is shown in Fig.3.5(e). Using the same process of obtaining the moments of PDF, when second machine is convolved, as discussed above the moments can be find as given below.

$${}^{o}M_2^1 = M_1^1$$

$${}^{o}M_2^2 = M_1^3$$

$${}^{o}M_2^3 = M_1^2$$

$${}^{o}M_2^4 = M_1^4 \times P_2$$

$${}^{o}M_2^5 = M_1^5 \times P_2$$

$${}^{o}M_2^6 = M_1^6 \times P_2 + {}^{\gamma 2}M_1^4 \times q_2$$

$${}^{o}M_2^7 = M_1^7 \times P_2 + {}^{\gamma 2}M_1^5 \times q_2$$

$${}^{o}M_2^8 = M_1^8 \times P_2 + {}^{\gamma 2}M_1^6 \times q_2$$

$${}^{o}M_2^9 = M({}^{\gamma 2}M_1^9 + {}^{\gamma 2}M_1^8) \times q_2$$

These moments of convolved PDf is shown in Fig.3.5(f). Similarly one by one all machines can be convolved. Thus if there are n machines the final moments of each sub-block of $f_n(\gamma)$ can be obtained. The sum of the moments of each sub-block gives the moments of $f_n(\gamma)$. Since the effect of machine outages is to change $f_o(\gamma)$ to $f_n(\gamma)$, the difference between the cumulants of $f_n(\gamma)$ and $f_o(\gamma)$ say $K_m(\gamma)$, represents the cumulants of the outage representation PDf $f_m(\gamma)$ of a fictitious and hypothetical machine. If this $f_m(\gamma)$ is convolved with $f_o(\gamma)$, $f_n(\gamma)$ is obtained. From $K_m(\gamma)$, μ_m, the moments of $f_m(\gamma)$ can be calculated.

In the case of two interconnected system the moments of bivariate probability density function are obtained.

The process of obtaining the joint moments of BPDF is discussed later on. Once the joint moments of two systems are obtained, the $f_m(\gamma_a)$ of system 1 machine outage is convolved and then the $f_m(\gamma_b)$ of system 2 machine outages is convolved which gives the $f_n(\gamma_a \gamma_b)$, the bivariate probability density function of γ_a and γ_b when the machines of both systems are convolved. In what follows is a brief description of the method of convolution.

Let X and Y be two random variables having the bivariate distribution $f_{XY}(X,Y)$ with moments μ_{rS}, then this moment can be expressed as

$$\mu_{rS} = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} (X)^r (Y)^S \ F(X,Y) \qquad \ldots (3.5)$$

in terms of expected value, this moment is given by

$$\mu_{rS} = E \left[X^r \ Y^S \right] \qquad \ldots (3.6)$$

Here X and Y represent two area loads in terms of cost, the moment μ_{rS} of the joint load probability density function can be find out by eq. 3.6. The machine outages in the two systems can be considered as random event and independent of the load. Let $f_m^{(n)}(\gamma)$ be the probability density function of the outages of n^{th} machines of system 1, then the convolution of this PDf of outage gives the PDf of equivalent load. Now the moments of equivalent loads in system 1, due to the convolution of the PDf of outage of n^{th} machines of system 1 are given by

$$\mu_{rS}^{new} = E\left[(X + f_m^{(n)})^r Y^S\right] \qquad \ldots (3.7)$$

r and S denotes the order of moments in system 1 and system 2 respectively. The development of the above equation when one machine is convolved, upto (r + S) = 3 order is given below.

$$\mu_{01}^{new} = E\left[(X + X_1)^0 Y^1\right] = E\left[Y\right]$$
$$= \mu_{01}$$

$$\mu_{02}^{new} = E\left[(X + X_1)^0 Y^2\right] = E\left[Y^2\right]$$
$$= \mu_{02}$$

$$\mu_{03}^{new} = E\left[(X + X_1)^0 Y^3\right] = E\left[Y^3\right]$$
$$= \mu_{03}$$

$$\mu_{10}^{new} = E\left[(X + X_1)^1 Y^0\right] = E(X) + E(X_1)$$
$$= \mu_{10} + \mu_1'$$

$$\mu_{20}^{new} = E\left[(X + X_1)^2 Y^0\right] = E\left[X^2 + X_1^2 + 2\, X\, X_1\right]$$
$$= E(X^2) + E(X_1^2) + 2E(X)\, E(X_1)$$
$$= \mu_{20} + \mu_2' + 2\,\mu_{10}\,\mu_1'$$

$$\mu_{30}^{new} = E\left[(X + X_1)^3 Y^0\right] = E\left[X^3 + 3X^2X_1 + 3X\, X_1^2 + X_1^3\right]$$
$$= E(X^3) + 3E(X^2).E(X_1) + 3E(X)\, E(X_1^2) + E(X_1^3)$$
$$= \mu_{30} + 3\mu_{20}.\mu_1' + 3\mu_{10}.\mu_2' + \mu_3'$$

$$\mu_{11}^{new} = E\left[(X + X_1)^1 Y^1\right] = E(XY + X_1 Y)$$
$$= E(XY) + E(X_1).E(Y)$$
$$= \mu_{10} + \mu_1' . \mu_{01}$$

$$\mu_{12}^{new} = E\left[(X + X_1)^1 Y^2\right] = E\ (XY^2 + X_1 Y^2)$$
$$= E(XY^2) + E(X_1 Y^2)$$
$$= \mu_{12} + \mu_1' \mu_{02}$$

$$\mu_{21}^{new} = E\left[(X + X_1)^2 Y^1\right] = E\left[(X^2 + 2X X_1 + X_1^2)\ Y\right]$$
$$= E(X^2 Y) + 2E(XY)\ E(X_1) + E(X_1^2).E(Y)$$
$$= \mu_{21} + 2\mu_{11}.\mu_1' + \mu_2' \mu_{01}$$

These new moments of the bivariate probability density function can also be found by expanding the power series as a binomial expansion and simplifying it, as given below.

$$\mu_{rS}^{new} = \mu_{rS}^{old} + \binom{r}{1} \mu_{(r-1)S}\, \mu_1^{(n)} + \binom{r}{2} \mu_{(r-2)S}\, \mu_2^{(n)}$$
$$\ldots\ldots + \mu_{oS}\, \mu_r^{(n)} \qquad \ldots (3.8)$$

where μ_{rS} are the moments of the load in terms of cost and $\mu_i^{(n)}$ are the moments of the machines represented by the PDF f_n. The above equation can be written as

$$\mu_{rS}^{new} = \sum_{i=0}^{r} \binom{r}{i} \mu_{(r-i)S}^{old}\, \mu_i^{(n)} \qquad \ldots (3.9)$$

This procedure is continued for all other machines in system 1. Once the moments, after convolution of the machines of system 1 is obtained, the machines of system 2 is convolved and the moments are obtained in the same way as discussed above. When the machines of system 2 is convolved, the moments obtained after considering all the outages in system 1 are further changed according to

$$\mu_{rS}^{new} = \sum_{i=0}^{r} \binom{S}{i} \mu_{r(S-i)}^{old} \mu_i^{(n)} \qquad \ldots (3.10)$$

Now these new value of moments are the moments of load and outages combined in both systems. Thus it represent the moments of the bivariate density of the two equivalent loads i.e. $f_n(\gamma)$. These moments have been used in the evaluation of the D coefficients of the bivariate Gram Charlier Expansion i.e. BGCE, which is discussed later on. Here first of all the machines of system A are convolved and then machines of system B are convolved separately. This method of convolution is used by Rau et al. [13]. In the present thesis as new method of convolution in which machines of both systems can be convolved at the same time have been discussed, which is comparatively fast as compared to [13]. In what follows is a brief description of the new convolution method.

Let us consider that X, Y are the random loads in terms of cost of system 1 and system 2, having the bivariate distribution $f_{XY}(X,Y)$ with moments μ_{rS}. X_1, Y_1 are the incremental costs of system 1 and system 2 respectively. When the outages of machines of both the systems are convolved, the moments of the convolved BPDF are evaluated as follows :

$$\mu_{rs}^{new} = E\left[(X + f_m^{(n)}(\gamma_a))^r\,(Y + f_m^{(n)}(\gamma_b))^s\right] \qquad \ldots (3.11)$$

Where $f_m^{(n)}(\gamma_a)$ and $f_m^{(n)}(\gamma_b)$ are probability density function of n machine outages in system 1 and system 2 respectively. Here the moments upto order 3 is evaluated

$$\begin{aligned}
\mu_{01}^{new} &= E\left[(X + X_1)^o (Y + Y_1)^1\right] = E\,(Y + Y_1) \\
&= E(Y) + E(Y_1) \\
&= \mu_{01} + M_{01}
\end{aligned}$$

$$\begin{aligned}
\mu_{02}^{new} &= E\left[(X + X_1)^o\,(Y + Y_1)^2\right] \\
&= E\left[Y^2 + 2Y\,Y_1 + Y_1^2\right] \\
&= E(Y^2) + 2E(Y).E(Y_1) + E(Y_1^2) \\
&= \mu_{02} + 2\mu_{01}.M_{01} + M_{02}
\end{aligned}$$

$$\begin{aligned}
\mu_{03}^{new} &= E\left[(X + X_1)^o (Y+Y_1)^3\right] \\
&= E\left[Y^3 + 3Y^2Y_1 + 3Y_1^2Y + Y_1^3\right] \\
&= E\,(Y^3) + 3E(Y^2)E(Y_1) + E(Y_1^2)E(Y) + E(Y_1^3) \\
&= \mu_{03} + 3\mu_{02}.M_{01} + 3\mu_{01}M_{02} + M_{03}
\end{aligned}$$

$$\begin{aligned}
\mu_{11}^{new} &= E\left[(X + X_1)^1\,(Y+Y_1)^1\right] \\
&= E\left[XY + XY_1 + X_1Y + X_1Y_1\right] \\
&= E(XY) + E(XY_1) + E(X_1Y) + E(X_1Y_1) \\
&= \mu_{11} + \mu_{10}.M_{01} + \mu_{01}.M_{10} + M_{11}
\end{aligned}$$

$$\mu_{12}^{new} = E\left[(X+X_1)^1 (Y+Y_1)^2\right]$$

$$= E(XY^2) + 2E(XY).E(Y_1) + E(X).E(Y_1^2)$$

$$+ E(X_1).E(Y^2) + 2E(Y).E(X_1Y_1) + E(X_1Y_1^2)$$

$$= \mu_{12} + 2\mu_{11}.M_{01} + \mu_{10}.M_{02} + \mu_{02}.M_{10} + 2\mu_{01}.M_{11} + M_{12}$$

$$\mu_{21}^{new} = E\left[(X+X_1)^2 (Y+Y_1)^1\right]$$

$$= E\left[(X^2 + 2X X_1 + X_1^2)(Y+Y_1)\right]$$

$$= \mu_{21} + 2\mu_{11}.M_{10} + \mu_{01}.M_{21} + \mu_{20}.M_{01} + 2\mu_{10}.M_{11} + M_{21}$$

$$\mu_{20}^{new} = E\left[(X+X_1)^2 (Y+Y_1)^0\right]$$

$$= E(X^2) + E(X_1^2) + 2E(X).E(X_1)$$

$$= \mu_{20} + 2\mu_{10}.M_{10} + M_{20}$$

$$\mu_{30}^{new} = E\left[(X+X_1)^3 (Y+Y_1)^0\right]$$

$$= E\left[X^3 + 3X^2X_1 + 3X X_1^2 + X_1^3\right]$$

$$= \mu_{30} + 3\mu_{20}.M_{10} + 3\mu_{10}.M_{20} + M_{30}$$

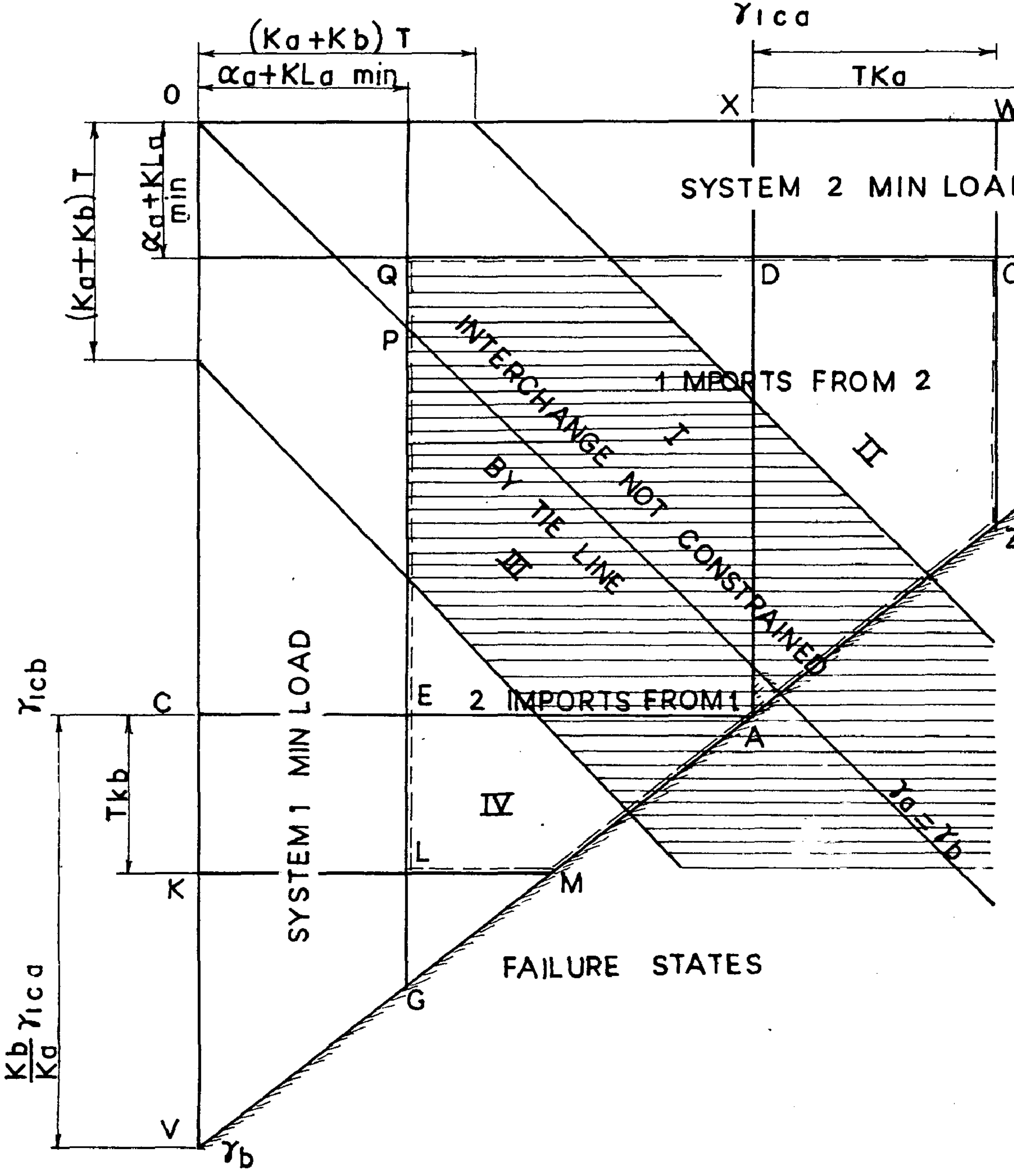
In the same fashion higher order moments can also be evaluated. M_{rs} denotes the moments of PDF of the outages of machines in other system, these moments M_{rs} can be evaluated as $M_{rs} = M_r \times M_s$. μ_{rs}^{new} are the moments of $f_n(\gamma_a \gamma_b)$, the bivariate probability density function, when the machines of both systems are convolved into the BPDF of loads. These moments now can be used in BGCE to calculate D coefficients.

However it may be noted that here it is assumed that machine outages are independent of system loads. Further, the system incremental costs as well as the system machine outages are considered correlated.

3.1.5 Use of Bivariate Probability Density Function to Calculate Economic Benefit

The convolution of machine outages in system 1 and system 2 with the bivariate probability density function of costs of system loads is discussed. The use of moment and cumulent method for convolution of machines pdf is also described. Now the use of bivariate probability density function to calculate the economic benefit is discussed. In what follows are the detailed explanation.

The plan view of BPDF $f_n(\gamma)$ for two interconnected system is shown in Fig.3.6. The probability associated with any particular values of γ is represented either by discrete impulse (or a continuous function) normal to the plane of paper. $f_n(\gamma)$ includes the effect of all machine outages in the two systems and load correlation between the two systems as discussed in the previous article. The function does not exist for γ values less than those corresponding to the minimum loads in both systems. These zones are obtained from the eq.(3.2) and (3.3) and shown c blackened in the Fig.3.6.



IN REGIONS I AND III TRANSACTIONS ARENOT LIMITED Bγ THE TIE-LINE TRANSFER CAPACITY

BENEFITS

REGIONS	TO SYSTEM '1'	TO SYSTEM '2'
I II	AVOIDED FUELCOST	EXPORT EARNINGS
III IV	EXPORT EARNINGS	AVOIDED FUELCOST
	TOTAL SAVINGS TO '1'	TOTAL SAVINGS TO '2'

FIG. 3.6. DOMAINS OF INTEGRATION OF THE PDF OF γ – Plain view of $\gamma_a \gamma_b$ plane

γ_a and γ_b are the total incremental costs of machines of system 1 and system 2. The whole region in which $f_n(\gamma)$ exists is divided in four parts as shown in Fig.3.6. In the region to the top of $\gamma_a = \gamma_b$ line, $\gamma_a > \gamma_b$ hence imports of power from system 2 to system 1 is profitable. In the same way below the $\gamma_a = \gamma_b$ line, $\gamma_b > \gamma_a$, hence in this region import of power from system 1 to system 2 is profitable. Thus in these four regions at any point the mathematical expectation of economic benefits can be calculated as given below. A summation of such benefits in the whole region of interest will give the mathematical expectation of savings due to imports and exports. This summation of benefits is similar to numerical integration, therefore the economic benefits are calculated by performing the numerical integration.

In the following paragraph, the explanation of formation of different regions in Fig. 3.6 is explained. The point 'A' in the Fig.3.6 indicates that two systems are operating at a incremental cost of γ_{ica} and γ_{icb} respectively Where γ_{ica} and γ_{icb} are the maximum value of γ corresponding to the operating capacity of system 1 and system 2. Thus A is the point at which both the systems are operating at their maximum incremental costs, and no assistance is available from either of the system. Therefore any point to the right of A with $\gamma_a > \gamma_{ica}$ and $\gamma_b > \gamma_{icb}$ indicate failure states, in which neither system can be operated.

In the same way at any point $(\gamma_a \gamma_b)$ above the line $\gamma_a = \gamma_b$, the spare or shortfall in capacity in system 1 and system 2 is calculated by using eqs.(3.2) and (3.3) as

$$\frac{\gamma_{ica} - \gamma_a}{K_a} = C_{Sa} \quad \text{and} \quad \frac{\gamma_{icb} - \gamma_b}{K_b} = C_{Sb} \quad \ldots (3.12)$$

Since the system 1 is operable only when shortfall is less or equal than the assistance available, hence assuming that the assistance is restricted by tie line this condition can be expressed as

$$\frac{\gamma_{ica} - \gamma_a}{K_a} \leq \frac{\gamma_{icb} - \gamma_b}{K_b} \quad \ldots (3.13)$$

Using equality, at γ_a axis, $\gamma_b = 0$, hence from the above equation, the distance XY in Fig.(3.6) is given by

$$XY = \gamma_{ica} - \gamma_a = \frac{K_a}{K_b} \gamma_{icb} \quad \ldots (3.14)$$

In similar way below the $\gamma_a = \gamma_b$ line, the integration is performed in the region bounded by PQGB.

If the assistance is restricted by transfer capacity of tie line then the region of integration will be different. Let us consider that maximum assistance available is T mw in either direction. This is not necessary that this will be equal in both direction. This assistance in the terms of incremental cost will be K T_a according to eq.(3.2). Thus if the operation of system 1 is considered at $\gamma = \gamma_{ica}$ i.e. the line XA, the assistance of TK_a results in the line

WCZ. Thus the region outside of line WCZ represents failure states. From the similar arguments for system 2 the integration has to be performed only in the region bounded by PQLMZC.

3.1.6 Calculation of Economic Benefits

The saving in cost due to import and export by either of the system in the described region are calculated by the following way.

Consider any point in the region PQLMZC above the $\gamma_a = \gamma_b$ line. Since in this region incremental cost of system 1 is higher than that of system 2 the power is imported from system 2. Let this amount of power be ΔL. Due to import of power from system 2 to system 1 the incremental cost of system 1 is reduced while the incremental cost of system 2 is increased due to overloading of machines of system 2. Let the new incremental cost of system 1 is λ_{anew} and of system 2 λ_{bnew}. Then the avoided fuel cost of system 1 due to import is given by

$$\text{avoided fuel cost of system 1} = \int_{L_a}^{L_{anew}} \beta_a \, e^{K_a L_a} \, dL_a$$

$$= \frac{\lambda_{anew} - \lambda_a}{K_a} \qquad \ldots (3.15)$$

The increase in cost of system 2

$$= \int_{L_b}^{L_{bnew}} \beta_b \, e^{K_b L_b} \, d\,L_b = \frac{\lambda_{bnew} - \lambda_b}{K_b} \qquad \ldots (3.16)$$

The net saving in cost is the difference between the saving in cost in system 1 and increased cost in system 2. Since the saving in system 1 will come negative, hence the global saving will be given by global saving

$$= \frac{1}{K_a} [\lambda_{anew} - \lambda_a] + \frac{1}{K_b} [\lambda_{bnew} - \lambda_b] \quad \ldots (3.17)$$

In the same way, the global saving can be calculated in the bounded region of PQLMZC below the $\gamma_a = \gamma_b$ line. Since in this region γ_b is greater than γ_a, the power is imported from system 1 which result in the saving of incremental cost of system 2 and increased in cost of system 1. The global saving can be found as given above. These global savings are shared between two system depending upon their agreement.

Since here any point (γ_a, γ_b) in the given region is associated with a probability, hence the net saving is given by the summation of expected saving at the points in the given region as given below.

Expected benefits in Region PQLMZC = Σ global savings at any point in PQLMZC x Probability value at that point x Period hours

These benefits are calculated by numerical integration.

To calculate the probability mass function, the bivariate Gram-Chartier Expansion (BGCE) has been used. The explaination of BGCE is given below.

Now having the knowledge of moments a continuous function can be obtained by the orthogonal polynomials, which is called Bivariate Gram Chartier Expansion and given by

$$f(X_1, X_2) = \frac{1}{\sigma_1 \sigma_2} \sum_{r=0}^{X} \sum_{S=0}^{X} (-1)^{r+S} D_{rS} H_{rS} \phi(Z_1, Z_2) \quad \ldots (3.19)$$

where

σ_1 and σ_2 are standard deviations.

$\phi(Z_1, Z_2)$ is the Bivariate normal function given by

$$\phi(Z_1, Z_2) = \frac{1}{2\pi \sqrt{1-\rho^2}} \operatorname{Exp} \left[- \frac{1}{2(1-\rho^2)} \times (Z_1^2 - 2\rho \, Z \, Z_2 + Z_2^2) \right]$$

with

$$Z_1 = \frac{X_1 - \mu_1}{\sigma_1} \text{ and } Z_2 = \frac{X_2 - \mu_2}{\sigma_2}$$

H_{rS} : are the Hermite polynomials in Z_1 and Z_2.

ρ : is the correlation co-efficient.

D : Polynomials in terms of the central moments.

The central moments μ_{rS} for a discrete function are given by

$$\mu_{rS} = \sum_{X_1, X_2 \in R^2} (X_1 - \mu_1)^r (X_2 - \mu_2)^S \, P(X_1, X_2) \quad \ldots (3.20)$$

Where μ_1, μ_2 are the means of Random variables X_1 and X_2 The expansion of D and H co-efficients are given in [14]. Thus from the knowledge of the moments, the probability at any point in the γ_a, γ_b plane can be obtained using the eq.(3.12).

3.1.7 Calculation of λ_{anew} and λ_{bnew}

(a) When power is imported by System 1 from System 2 and Import is not restricted by tie line :

When system 1 imports power from system 2 in that case the incremental cost γ_a of system 1 is reduced due to off-loading of machines. Similarly the incremental cost of system 2 i.e. γ_b increases due to overloading of machines. Since there is no restriction of importing the power by tie line, the import of power can be made such that the incremental cost of system 1, $\gamma_a = \gamma_b$, the incremental cost of system 2. Because for maximum gain, the machines of two systems must be loaded at equal incremental cost.

Let us consider that ΔL is the power imported by system 1, then in the case of equal loading

$$\gamma_{anew} = \gamma_{bnew}$$

or $$\alpha_a + K_a(L_a - \Delta L) = \alpha_b + K_b(L_b + \Delta L) \qquad \ldots (3.21)$$

From equation (3.21)

$$\alpha_a + K_a L_a - (\alpha_b + K_b L_b) = K_a \Delta L + K_b \Delta L$$

or $$\gamma_a - \gamma_b = \Delta L(K_a + K_b)$$

or $$\Delta L = \frac{(\gamma_a - \gamma_b)}{(K_a + K_b)} \qquad \ldots (3.22)$$

Substituting this value of ΔL in equation, the value of γ_{anew} and γ_{bnew} is calculated as follows :

$$\gamma_{anew} = \alpha_a + K_a(L_a - \Delta L)$$

$$= \alpha_a + K_a \quad L_a - K_a \frac{(\gamma_a - \gamma_b)}{(K_a + K_b)}$$

$$= \alpha_a + K_a L_a - K_a \left(\frac{\gamma_a - \gamma_b}{K_a + K_b} \right)$$

$$= \gamma_a - K_a \left(\frac{\gamma_a - \gamma_b}{K_a + K_b} \right)$$

$$= \frac{\gamma_a(K_a + K_b) - K_a(\gamma_a - \gamma_b)}{(K_a + K_b)}$$

$$= \frac{\gamma_a K_a + \gamma_a K_b - K_a \gamma_a + K_a \gamma_b}{(K_a + K_b)}$$

$$= \frac{\gamma_a K_b + K_a \gamma_b}{(K_a + K_b)}$$

$$= \frac{K_b \gamma_a + K_a \gamma_b}{K_a + K_b}$$

Thus

$$\gamma_{anew} = \gamma_{bnew} = \frac{K_b \gamma_a + K_a \gamma_b}{K_a' + K_b} \qquad \ldots (3.23)$$

From this new value of γ_{anew} and γ_{bnew} the value of λ_{anew} and λ_{bnew} can be calculated as follows :

$$\lambda_{anew} = e^{\gamma_{anew}}$$
$$\lambda_{bnew} = e^{\gamma_{bnew}} \qquad \ldots (3.24)$$

Since $\gamma = \log_e \lambda$ from equation (3.2)
When the import of power is restricted by the tie line capacity and the incremental cost of two system can not be equalized in that case the value of γ_{anew} and γ_{bnew} is given by

$$\gamma_{anew} = \gamma_a - K_a T$$
$$\gamma_{bnew} = \gamma_b + K_b T \qquad \ldots (3.25)$$

The region where this condition holds good is shown in Fig.3.6.

(b) When power is imported by System 2 from System 1 :

When the power is imported by system 2 the incremental cost of system 2 decreases due to off-loading of machines and at the same time the incremental cost of system 1 increases due to overloading of machines. Let us consider that ΔL is the power imported by system 2 from 1 then the new value of incremental cost of system 1 and system 2 is given by

$$\gamma_{anew} = \gamma_{bnew} \quad \text{(For maximum gain, equal incremental loading)}$$

i.e. $$\alpha_a + K_a(L_a + \Delta L) = \alpha_b + K_b(L_b - \Delta L) \qquad \ldots (3.26)$$

From the equation (3.26), ΔL can be calculated and given by

$$\Delta L = \frac{(-\gamma_a + \gamma_b)}{(K_a + K_b)} \qquad \ldots (3.27)$$

By substituting the value of ΔL in eq.(3.26)

$$\gamma_{anew} = \alpha_a + K_a\left(L_a + \frac{\gamma_b - \gamma_a}{K_a + K_b}\right) \qquad \ldots (3.28)$$

or
$$\gamma_{anew} = \gamma_a + K_a\left(\frac{\gamma_b - \gamma_a}{K_a + K_b}\right)$$

$$\gamma_{anew} = \frac{\gamma_a(K_a + K_b) + K_a(\gamma_b - \gamma_a)}{K_a + K_b}$$

$$\gamma_{anew} = \frac{\gamma_a K_a + \gamma_a K_b + K_a\gamma_b - K_a\gamma_a}{K_a + K_b}$$

$$\gamma_{anew} = \frac{\gamma_a K_b + K_a\gamma_b}{K_a + K_b} \qquad \ldots (3.29)$$

In the same way substituting the value of ΔL γ_{bnew} can be calculated and given by

$$\gamma_{bnew} = \frac{\gamma_b K_a + \gamma_a K_b}{K_a + K_b} \qquad \ldots (3.30)$$

From this value of γ_{anew} and γ_{bnew} the value of λ_{anew} and λ_{bnew} is calculated as follows :

$$\lambda_{anew} = e^{\gamma_{anew}}$$
$$\lambda_{bnew} = e^{\gamma_{bnew}} \qquad \ldots (3.31)$$

When the import of power is restricted by tie line capacity then ΔL becomes equal to T i.e. tie line capacity and new value of incremental costs of system 1 and 2 is given by

$$\gamma_{anew} = \gamma_a + K_a T \qquad \ldots (3.32)$$

and

$$\gamma_{bnew} = \gamma_b - K_b T \qquad \ldots (3.33)$$

Thus from these values λ_{anew} and λ_{bnew} can be calculated. Substituting these values of λ_{anew} and λ_{bnew} in equation (3.17), the global savings are calculated.

In the following Chapter this method has been applied to an interconnected system consisting of IEEE Reliability Test System and considered hypothetical system. The results for various maintenance schedules are obtained and discussed.

CHAPTER IV

RESULT AND DISCUSSION

The method of convolution of probability density function of outages of machines of system 1 and system 2 with Bivariate Probability density function of system loads have been described in Chapter III. A method of convolution has also been discussed which is faster compared to the method used by Rau et al [13]. In the present chapter economic benefits of two interconnected power system for different tie line capacity have been determined. It has also been noticed that with increasing tie line capacity the economic benefit increases. It is seen that as the megawatt outages of machines in both system increases the economic benefits decreases.

It is further noticed that the economic benefits are related with systems load. Results have been obtained for different set of machine outages in system 1 and system 2 as well as for different tie line capacities. In what follows are the detailed description of the system data as well as the result obtained.

4.1 GENERATION DATA FOR SYSTEM 1 AND SYSTEM 2

The generation data of system 1 is the data of a fictitious system. This generating system is comprised of two nuclear units, 5 coal units, three oil units, four GT units and five hydro units whose capacities along with FOR and incremental costs are given in Table 4.1. The generation data of system 2 have been taken from IEEE Reliability Test System [8]. This system consists 32 units with installed capacity of 3400 mw. However, some of the capacities of the generating units have been rounded off. The details of this modified IEEE system are given in Table 4.2. The hourly loads of [8] during the months of December to February are used for system 2 and those of May to July are used for system 1.

Table - 4.1

Generation Data of System 1

Type of Unit	Number of Unit	Unit capacity (MW)	FORs	Average incremental costs
NUC	2	500	.18	8.2
COAL	2	400	.13	17.56
COAL	1	150	08	20.85
COAL	2	350	.13	29.05
OIL	1	350	.14	33.17
OIL	2	200	.10	42.80
GT	4	50	.21	65.48
HYDRO	5	100	.01	0
	19	4300		

Table - 4.2

Generation Data of System 2

Type of unit	No.of unit	Unit capacity (MW)	FORs	Average incremental cost
NUC.	2	400	.12	5.5
COAL	4	150	.04	10.704
COAL	1	350	.08	10.883
COAL	4	80	.02	13.496
OIL	3	200	.05	20.730
OIL	3	100	.04	20.853
OIL	5	10	.02	25.875
OIL	4	20	.10	37.5
HYDRO	6	50	.01	0
	32	3400		

4.2 MODE OF TRANSACTION

Two type of agreement of sharing the economic benefit between system 1 and system 2 are considered. In first type of agreement, the economic benefit are equally divided between the two systems, where as in the second type of agreement the importing system is assumed to pay an average of the initial incremental costs i.e. $\lambda_t = \frac{1}{2}(\lambda_a + \lambda_b)$. Where λ_a and λ_b are the initial incremental cost of system 1 and system 2. Thus using the above phenomenon of sharing the economic

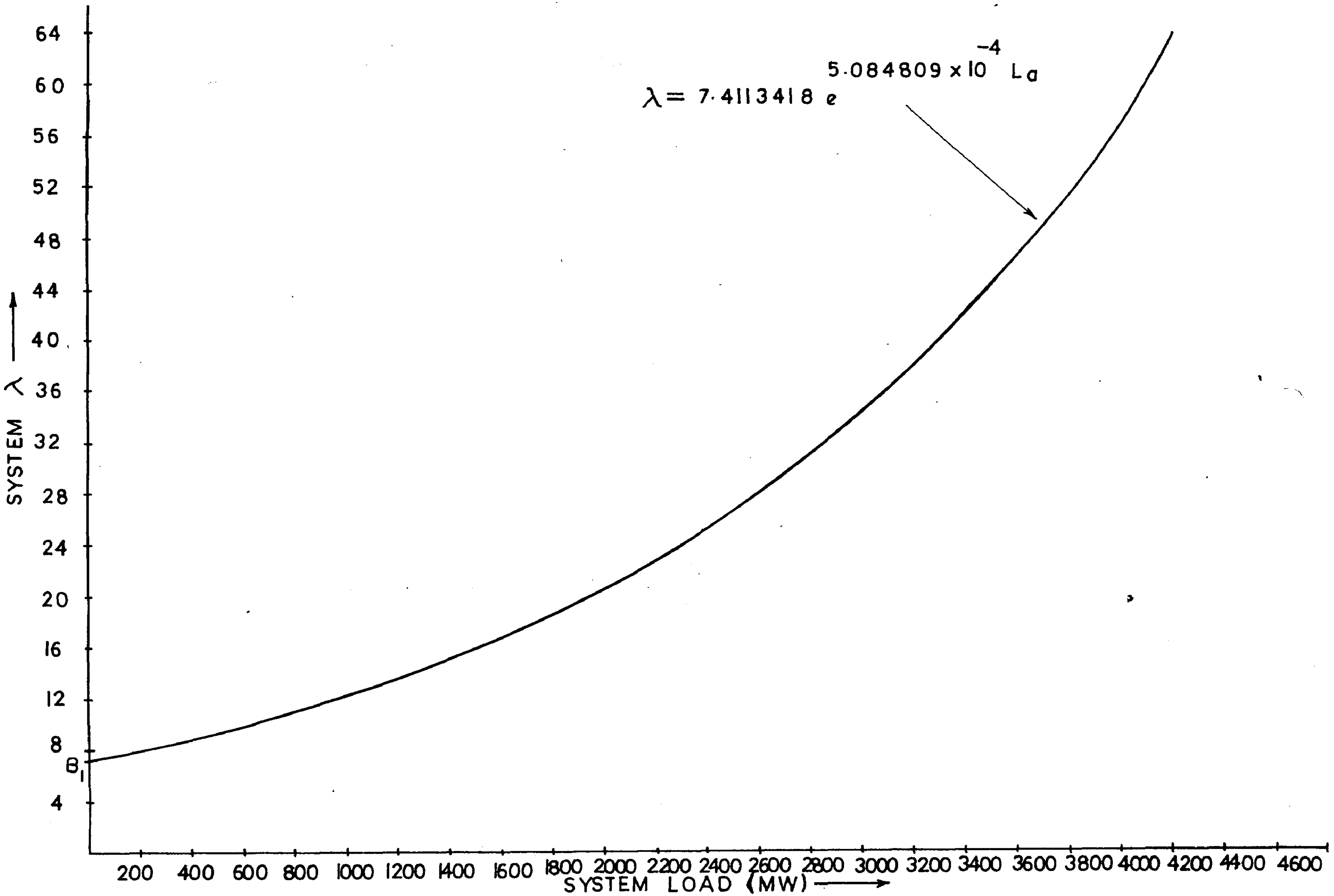


FIG. 4.1. λ VS LOAD RELATION OF SYSTEM 1

benefit, the benefit in different zones are calculated. It may be noticed that the total economic benefits are equal for either mode of transaction in Table 4.4 and 4.5.

4.3 MAINTENANCE SCHEDULE

Three different cases of maintenance schedule have been taken and the result is discussed. In what follows are the detailed description of the result for all cases.

4.3.1 Same Outage and Different Tie Capacity

The economic benefit, for a particular set of machines in system 1 and system 2 with varying tie line capacity has been calculated. In this case the peak load of system 1 and system 2 are taken as 2565 MW and 2850 MW respectively. The details of machine on scheduled outage are given in Table 4.3.

Table - 4.3

Machine Outage

Unit Capacity		No.of unit		FOR		Average incremental cost	
System 1	System 2	System 1	System 2	System 1	System 2	System 1	System 2
500	10	1	1	.18	.02	8.2	25.875
50	20	3	4	.21	.10	65.48	37.5

System 1 Peak load = 2565 MW
System 2 Peak load = 2858 MW
Total MW outage in system 1 = 650
Total MW outage in system 2 = 90

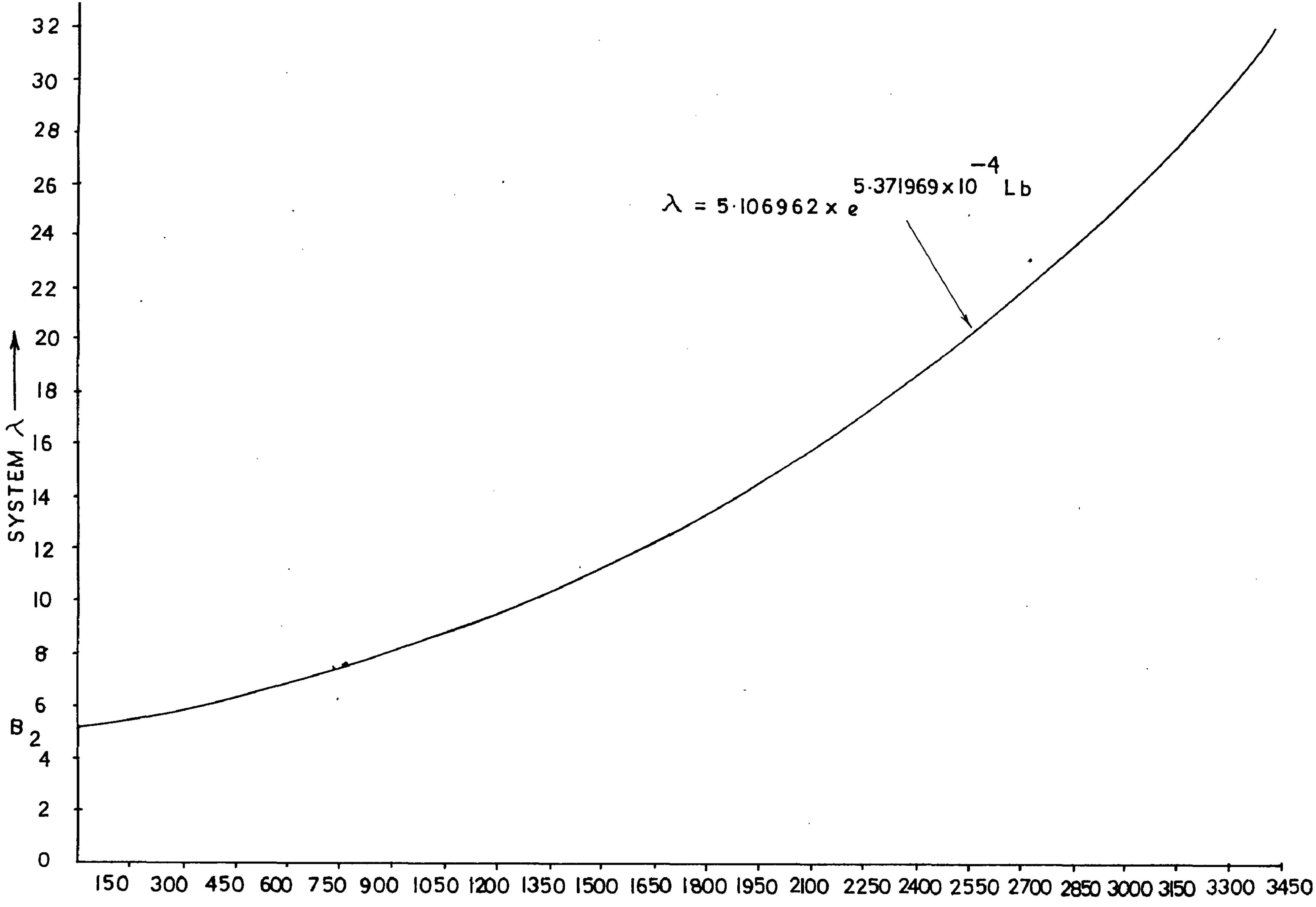


FIG. 4.2. λ VS LOAD RELATION OF SYSTEM 2

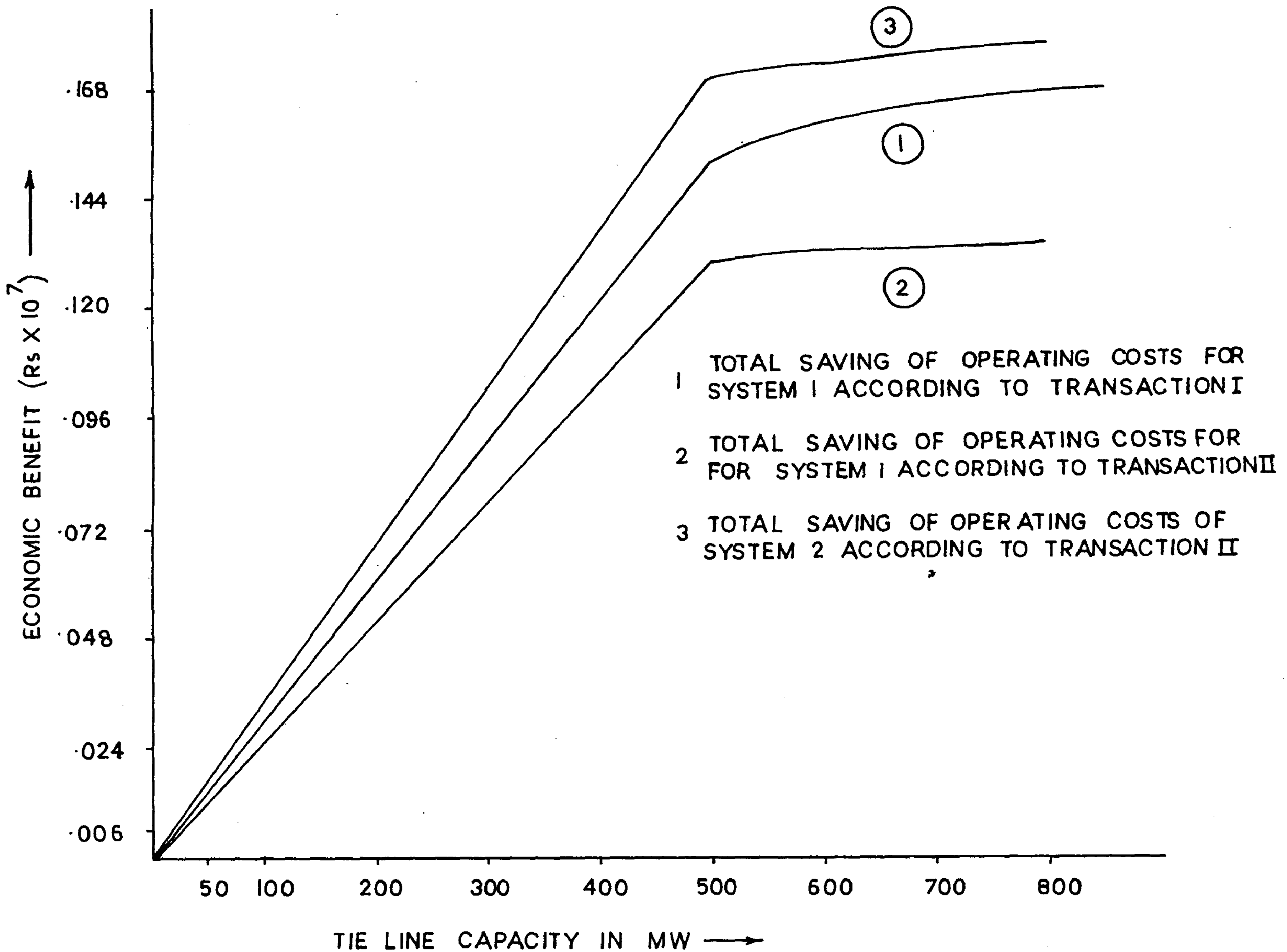


FIG. 4.3. ECONOMIC BENEFIT VS TIELINE CAPACITY

Table 4.3 depicts the detail of the data of the machines taken on scheduled outage in system 1 and system 2. For these megawatt outage the economic benefits are calculated for different tie line capacity and given in Table 4.4 and Table 4.5.

Table - 4.4

Transaction I

No. of machines		Tie Line (MW)	Economic Benefit	
System 1	System 2		System 1	System 2
15	27	50	$.681636 \times 10^5$	$.681636 \times 10^5$
15	27	100	$.21194 \times 10^6$	$.21194 \times 10^6$
15	27	200	$.520585 \times 10^6$	$.520585 \times 10^6$
15	27	300	$.855181 \times 10^6$	$.853181 \times 10^6$
15	27	400	$.145368 \times 10^7$	$.145368 \times 10^7$
15	27	500	$.152359 \times 10^7$	$.152359 \times 10^7$
15	27	600	$.159614 \times 10^7$	$.159614 \times 10^7$
15	27	700	$.164922 \times 10^7$	$.164922 \times 10^7$

Table - 4.5

Transaction II

No. of machines		Tie line	Economic Benefit	
System 1	System 2	(MW)	System 1	System 2
15	27	50	$.673794 \times 10^5$	$.689478 \times 10^5$
15	27	100	$.207207 \times 10^6$	$.216673 \times 10^6$
15	27	200	$.495506 \times 10^6$	$.543526 \times 10^6$
15	27	300	$.789491 \times 10^6$	$.928419 \times 10^6$
15	27	400	$.129699 \times 10^7$	$.160657 \times 10^7$
15	27	500	$.130536 \times 10^7$	$.170376 \times 10^7$
15	27	600	$.13225 \times 10^7$	$.172928 \times 10^7$
15	27	700	$.133693 \times 10^7$	$.176913 \times 10^7$

The economic benefits for different tie line capacity given in Table 4.4,in the case of transaction I,are plotted in Fig.4.1. Fig. 4.1 depicts that the benefit is increasing initially and after that it get saturated. The reason being that when tie line capacity increases, the benefit increases till the incremental cost of system 1 is equalized to the incremental cost of system 2. If one increases the tie line capacity beyond this, the economic benefit being not affected.

4.3.2 Different Maintenance Schedules

The economic benefits for different set of schedules keeping the same tie line capacity are calculated for both transaction I and transaction II and given in Table 4.6 and Table 4.7.

Table - 4.6

No. of machines		Tie line (MW)	Economic Benefit	
System 1	System 2		System 1	System 2
19	32	600	$.193938 \times 10^7$	$.193938 \times 10^7$
18	31	600	$.193763 \times 10^7$	$.193763 \times 10^7$
16	29	600	$.193428 \times 10^7$	$.193428 \times 10^7$
15	29	600	$.159614 \times 10^7$	$.159614 \times 10^7$
15	27	600	$.159608 \times 10^7$	$.159608 \times 10^7$

Peak load of system 1 = 2565 MW
Peak load of system 2 = 2830 MW

Table - 4.7

Transaction II

No. of machines		Tie line	Economic Benefit	
System 1	System 2	(MW)	System 1	System 2
19	32	600	$.160524 \times 10^7$	$.213948 \times 10^7$
18	31	600	$.160375 \times 10^7$	$.213726 \times 10^7$
16	29	600	$.160101 \times 10^7$	$.213281 \times 10^7$
15	29	600	$.132239 \times 10^7$	$.171917 \times 10^7$
15	27	600	$.132243 \times 10^7$	$.171928 \times 10^7$

Table 4.6 depicts that when the number of machines taken on outage increases with keeping the tie line capacity constant, the economic benefit decreases. It is also seen that the outage of large capacity unit affects more compared to the outage of smaller capacity units. The reason being that the outage of large capacity unit increases the equivalent load of corresponding system, since the tie line capacity is restricted, the import is not possible beyond the tie line capacity. Hence the extra load is to be supplied by corresponding system itself, hence due to overloading of machines the production cost is increased and result in decrease of the savings.

4.3.3 Same Megawatt and Same Machine Outages with Interchange of System's Load

The economic benefits for a particular set of machine outage with interchange of system's load are calculated. The details of machine outages are given in Table 4.3 and calculated economic benefit for two different system's load are given in Table 4.8.

Table 4.8 depicts the variation of economic benefits in system 1 and system 2 for the same outage megawatt with different peak loads. It is seen that with change of loads in two system the economic benefit changed due to the change of reserve capacity in both the systems.

Table - 4.8

Outage (MW)		No. of machines available		Peak Load(MW)		Tie Line (MW)	Economic Benefit	
System 1	System 2	System 1	System 2	System 1	System 2		System 1	System 2
650	90	15	27	2565	2850	600	$.159614 \times 10^7$	$.159614 \times 10^7$
650	90	15	27	2850	2565	600	$.219271 \times 10^7$	$.219271 \times 10^7$

4.3.4 Economic Benefit with two Different Sets of Machines Keeping the Megawatt Outage and Load Constant.

The economic benefit, for two different set of machines with same megawatt outages and keeping the load of two system unchanged is calculated. The details of data of machine outage and economic benefits are given in Table 4.9 and Table 4.10 respectively.

Table - 4.9

Unit capacity		No. of unit		FORs		Average incremental cost	
System 1	System 2	System 1	System 2	System 1	System 2	System 1	System 2
400	50	1	1	.13	.01	17.56	0
150	10	1	4	.08	.02	20.85	25.875
100	-	1	-	.01	-	0	-

Total megawatt outage in system 1 = 650
Total megawatt outage in system 2 = 90

Table 4.9 and Table 4.10 give the data of machine outage and economic benefit for two different set of machine outage while the system loads are unchanged. It is seen that for the same system loads and megawatt outage, if the machines on outage are different the economic benefit varies. The reason being due to the different forced outage rate and incremental cost of the machines.

Table - 4.10

Megawatt outage		No. of machines available		Peak Load (MW)		Tie line (MW)	Economic Benefit	
System 1	System 2	System 1	System 2	System 1	System 2		System 1	System 2
650	90	15	27	2850	2565	600	$.219271 \times 10^7$	$.219271 \times 10^7$
650	90	16	27	2850	2565	600	$.246558 \times 10^7$	$.246558 \times 10^7$

The economic benefits are calculated for four different cases discussed above, and it is seen that all the cases with different set of machines on outage result in the variation of economic benefits. Here the aim is only to justify that economic benefit changes with different set of machines taken on maintenance in both system. Now one can prepare a large number of combination of machine outage in both the system for maintenance purposes keeping the LOLP of the system at a desired value. Time period considered for maintenance may be weekly, monthly or yearly maintenance depending upon utility's requirement. The maintenance schedule which gives the maximum benefit is the optimum schedule. Therefore this method of maintenance scheduling may prove an efficient method of maintenance scheduling in interconnected system, because the optimization of global benefits is nothing but the reduction in the global fuel cost of the interconnected system.

CHAPTER V

CONCLUSION AND SUGGESTIONS FOR FURTHER WORK

Maintenance scheduling now-a-days has become a crucial factor, which directly affect the production costs. The maintenance scheduling for generating equipments becomes important in order to reduce the chances of power shortages and improve the overall availability of the system capacity. The reliability and operating costs of an electric power systems are affected, when the generating units of the systems are taken out for maintenance purpose. Thus the optimal schedule which gives the minimum cost to the generating utilities and meet the certain desired constraints is to be obtained to carry out scheduled maintenance. The maintenance scheduling of single generating system has already been discussed by Garver [6]. In this thesis a method of maintenance scheduling for interconnected system has been discussed.

The method of maintenance scheduling for interconnected power system, proposed in this thesis, evaluates the expected savings in the energy production costs due to energy interchanges as a function of tie line capacity. Random failures of generating units and their effect on the incremental cost of energy production are also taken into consideration. A detailed description of the method of convolution of machines outages in system's load is given in Chapter III. A new method of convolution of machines outages in system's load, which is computationally fast, is also given. The description of evaluation of economic benefit in two interconnected system is also given in Chapter III.

In Chapter IV the result has been taken for different maintenance schedule and discussed. The economic benefits are evaluated for different tie line capacity. It is seen that when the machines of different incremental cost and forced outage rate are taken for maintenance the economic benefit vary in all the cases, for the same system loads. Further, when the machines of larger capacity are taken out the economic benefit differ by a large amount as compared to the scheduled removal of small capacity generating unit. The variation of economic benefit with interchange of system's load is also discussed. From the different sets of results, it is clear that whenever the machines of in either or both systems are removed out for maintenance purpose, the economic benefits differ. However the change in the benefits depend upon many functions e.g. FOR of generating units, systems load, incremental cost of units etc. Therefore, one can prepare a large number of combination of machine outages for different maintenance schedule in considered time period. The maintenance schedule which gives the maximum economic benefit satisfying the desired constraints is the optimized maintenance schedule.

In the present thesis a fast method to find out the global/individual economic benefits of interconnected system has been developed. This computation time reduction is achieved through a joint convolution technique proposed in the thesis. Using a new objective function of savings (economic benefits) of the systems maintenance scheduling approach is discussed. This work may be extended by coupling this program with an optimization technique to find the optimum maintenance schedules for individual systems for given period of time. Further, in the present work bi-variate joint probability density function is obtained using BVGCE which has its own limitation. A suitable cumulant method may be developed to develop the Bivariate joint density function. This will increase the accuracy of the calculations.

APPENDIX 'A'

EVALUATION OF MOMENTS

The expressions in this section is developed in terms of two general RVs. X_1 and X_2. The product moment of order r + S of two RV_s X_1 and X_2 is defined as

$$M_{r,S} = E\left[X_1^r \quad X_2^S\right] \qquad \ldots(1)$$

Where the expectation is given for continuous RV_s by the relation

$$E\left[X_1^r \quad X_2^S\right] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X_1^r \, X_2^S \quad f_{X_1,X_2}\,(X_1,X_2)dX_1.dX_2 \qquad \ldots(2)$$

in which $f_{X_1,X_2}(.,.)$ is the joint probability density function of the two RV_s X_1 and X_2.

For two discrete RV_s, with a joint probability mass function $P_{X_1,X_2}(X_1,X_2)$, one has

$$E\left[X_1^r \; X_2^S\right] = \sum_{X_1,X,\in J} X_1^r \; X_2^S \; P_{X_1,X_2}(X_1,X_2) \qquad \ldots(3)$$

Where the set J is composed of those values of X_1 and X_2 such that $P_{X_1,X_2}(X_1,X_2) > 0$.

The central moments of order r + S of the RV_s X_1 and X_2 are defined as

$$\mu_{r,S} = E\left[\left\{X_1 - E(X_1)\right\}^r \left\{(X_2 - E(X_2)\right\}^S\right] \qquad \ldots(4)$$

The central moment $\mu_{1,0}$ and $\mu_{0,1}$ of order one vanish. The central moments $\mu_{2,0}$ and $\mu_{0,2}$ are, respectively, the variance of X_1 and X_2. The central moment $\mu_{1,1}$ is called the covariance of the RV_s X_1 and X_2. The correlation co-efficient of two jointly distributed RV_s with finite positive variance is defined in terms of the covariance as follows :

$$\rho\left[X_1, X_2\right] = \frac{\mu_{1,1}}{\sigma[X_1]\,\sigma[X_2]} \qquad \ldots(5)$$

where $\sigma[X_1]$ and $\sigma[X_2]$ are the standard deviations of X_1 and X_2 respectively. The correlation coefficient provides a measure of goodness of prediction of the value of one of the RV_s on the basis of an observed value of the other.

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